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ALTINBAS UNIVERSITY

Industrial Engineering

**DIRECT SEARCH BASED APPROXIMATE
DYNAMIC PROGRAMMING TECHNIQUE FOR
DYNAMIC STOCHASTIC ADVANCED
SCHEDULING**

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Master's Thesis

Supervisor

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**DIRECT SEARCH BASED APPROXIMATE DYNAMIC PROGRAMMING
TECHNIQUE FOR DYNAMIC STOCHASTIC ADVANCED SCHEDULING**

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Ahmed Hassan Abdirahman

DEDICATION

There are some people those probably won't have been composed my thesis, first of all to my mother, Amina, who keeps me on learning, grow and develop , i also have to appreciate my sisters Sadia, Nasra, Saida, Safia, and Suad, also my brother Bashir and the whole my family for their support.



ABSTRACT

DIRECT SEARCH BASED APPROXIMATE DYNAMIC PROGRAMMING TECHNIQUE FOR DYNAMIC STOCHASTIC ADVANCED SCHEDULING

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Dynamic stochastic scheduling problems are one of the most critical and challenging problems in optimization. In this thesis, we study dynamic stochastic scheduling problems with cancellations. In these problems, jobs arrive randomly at a system and have deadlines. We present a strategy for solving this problem, which involves formulating the problem through Markov Decision Process (MDP), and then solving it approximately using a direct search-based Approximate Dynamic Programming (ADP) technique. We perform the performance comparison of the direct search-based ADP policy and the Greedy policy under diverse scenarios. Our numerical results reveal that the greedy policy can be significantly improved through the implementation of the direct search-based ADP.

Keywords: Dynamic scheduling, Markov decision processes, Dynamic programming,

ÖZET

DİNAMİK STOKASTİK İLERİ PROGRAMLAMA İÇİN DOĞRUDAN ARAMA TABANLI YAKLAŞIMLI DİNAMİK PROGRAMLAMA TEKNIĞİ

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Dinamik stokastik çizelgeleme problemleri optimizasyondaki en kritik ve zorlu problemlerden biridir. Bu tez çalışmasında, dinamik stokastik çizelgeleme problemlerini iptallerle birlikte inceliyoruz. Bu problemlerde, işler rastgele bir sisteme ulaşır ve son teslim tarihleri vardır. Sorunu Markov Karar Süreci (MDP) ile formüle etmeyi ve daha sonra yaklaşık olarak doğrudan arama tabanlı bir Yaklaşık Dinamik Programlama (ADP) tekniği kullanarak çözmeyi içeren bir strateji sunuyoruz. Doğrudan arama tabanlı ADP politikasının ve miyopik politikanın performans karşılaştırmasını çeşitli senaryolar altında gerçekleştiriyoruz. Sayısal sonuçlarımız, doğrudan arama tabanlı ADP'nin uygulanmasıyla miyopik politikanın önemli ölçüde geliştirilebileceğini ortaya koymaktadır.

Anahtar Kelimeler: Dinamik çizelgeleme, Markov karar süreçleri, Dinamik programlama

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LIST OF ABBREVIATIONS

ADP: Approximate Dynamic Programming.

MDP: Markov Decision Process.

LP: Linear Programming.



1. INTRODUCTION

Job scheduling problem is an assignment problem of a number of tasks or jobs, and its execution might be limited by at least one constraint. Dynamic stochastic scheduling problems arise in numerous fields which include manufacturing, transportation, assignment control, and healthcare. In these problems, unlike static scheduling problems, jobs arrive randomly at a system relying on the sector; for example jobs could be orders, patients, and tasks. These jobs are allocated to resources such as treatment devices, operating rooms, and nurses. Dynamic stochastic scheduling problems have received extensive consideration in the operations research community over the last decade.

The process of scheduling jobs is very complicated task, due to the various resources that have to be matched, and optimized well, such as ward nurses, surgeon and other resource. Job scheduling is critical issue, for example in the surgery room longer than the planned or ordinary time of appointment or even surgery duration may result lateness not only next appointment but all the remaining appointments.

There are many applications of dynamic stochastic scheduling [19]. In the military applications, for example, observation and battle arranging, jobs identify with solicitations for a particular mission errands with planes, ships, and crew [20]. In correspondence systems, jobs relate to demands for information, voice, or video transmission [21]. In elite registering offices, for example, a supercomputer or a multi-processor machine with a working framework that permits dynamic resource reconfiguration (e.g., Solaris 10 on Sun), jobs relate to figuring tasks and resource to CPUs, memory, data transmission, storage space, etc. In logistics and distribution, when a client requests a transportation/moving/products conveyance administration, it might be scheduled a few weeks in the future.

Our contribution in this thesis is the following: We consider a class of dynamic stochastic advanced scheduling problems with cancellations and jobs having deadlines. Then, we formulate these problems as a Markov Decision Process (MDP). In order to get the approximate solution we utilize a direct search-based Approximate Dynamic Programming (ADP) technique. Then we

compare the performance of the direct search-based ADP approach against the greedy (myopic) policy. Our computational results indicate that the direct-search based on ADP improves the solution of the greedy policy on most of problem sets we generated.

We next provide the literature review on dynamic stochastic scheduling problems.



2. LITERATURE SURVEY

Dynamic scheduling problems have been considered extensively over the last decades. Magerlein and Martin [5] review the literature about surgical demand scheduling problems, and distinguished them in terms of two process: advanced scheduling and allocation scheduling. In advanced scheduling, jobs are given appointments to future days over a booking horizon. In allocation scheduling, jobs' appointment depends on the availability of devices and staff; this appointment can be served or rejected instantly. There are many papers [6], [7], [8] that focus on the allocation scheduling. But for us we focus on advanced dynamic scheduling problems.

Gerchak et al. [9] considered the advanced scheduling of elective surgery while the hospital's daily capacity of the operating room is engaged to the emergency surgeries. They formulated the problem as an infinite-horizon MDP Model to find the optimal number of patients who can be accepted each day to surgical department; they can be an elective and emergency surgery. Patrick [10] studied advanced scheduling problems where patients are categorized into multiple classes and scheduled them to a diagnostic imaging resource with a reasonable wait time. The author formulated the problem as an MDP model and solved it via a linear programming-based ADP technique.

Kolesar et al. [11] considered some mathematical models that deal with the problem related to the prescheduling elective admissions. They presented a new MDP model for solving the problem. They have classified inpatient admission in two categories; elective admission which is the patients who do not need an emergency hospitalization, and may be delayed to a later date, and mandatory admission which needs an emergency hospitalization. And they considered a system that deals with the elective admissions. Legrain et al. [12] studied assigning patients to the machines with respect to their priority treatment and they created it as an online fashion.

Patrick [13] designed a model which determines the outpatient of no show or open scheduling. They developed an MDP model to demonstrate a short booking window instead of no show appointment. Lamiri et al. [14] presented a stochastic model for elective surgery planning for operating rooms that deal with two types of demand for surgery: elective surgery and emergency surgery. For the elective surgery, it can be planned for the future days whereas the emergencies

arrive and served immediately. The author investigated that the elective cases can be assigned to different periods over a planning horizon. They proposed stochastic mathematical programming method. Klassen et al. [15] studied appointment scheduling policies; they considered mainly two policies ;overload rules when the arrivals is more than they use different scheduling methods like overtime, and rule delay (RD) which is when they are implanting the other rule.

Xinli et al. [16] focused on sequential appointment for scheduling the outpatients regarding the patient preferences and choices. They formulated the problem as an MDP to optimize the scheduling of sequential appointments according to the patient preferences in order to maximize the expected level of satisfaction during the booking periods. Benzaid et al. [17] Studied the planning of chemotherapy patients and patient–nurse assignment. They investigated the need of nurse and to assign the patients to the available nurses, and formulated a mathematical model to build list of waiting patients in order to compare the need of nurse and patients, then to decrease the number of nurses required. Millhiser et al. [18], Schütz et al. [19] and Conforti et al. [1] considered outpatient scheduling problem . Luh et al. [20] Examined job selection problems with random arrivals even though only one renewable resource and deterministic job period is permitted. They employed through the simplifying theory so as to every newly new jobs are of distinct types. They presented a model which considers capacity of time and resources. They formulated the problem as MDP model and worked on dynamic programming method. Gao et al. [21], and Khamisy et al. [22] consider arbitrary discrete-time arrival process with finite queue capacities.

Gocgun and Puterman. [23] considered chemotherapy patient scheduling problems where arriving patients are booked an appointment at specific target dates regarding their tolerance limits. They investigated patient appointment booking problem and formulated the problem as an MDP model, and then worked on linear-programming based ADP for approximately solving the MDP model. There are a lot papers [24]–[26] that considered on optimization of healthcare and how to increase patient’s satisfaction.

Saure [10] studied dynamic scheduling for cancer patients who will be treated in a radiation therapy units. They formulated the problem as discounted infinite-horizon MDP, then utilized linear programming based on ADP technique for finding the approximate optimal policy to

utilize the available treatment capacity of the arriving patients. Tsai and Teng [27] presented that a stochastic assignment system for the outpatient clinics by utilizing of multiple resources, with assuming patients' call-in sequence. Their results show that this stochastic model is much profitable to schedule the patients compared with traditional appointment systems.

Göçgün [28] considered dynamic patient scheduling problems with cancellation with target dates and time windows. Scheduling jobs which are outside of the time windows cause a penalty. They designed the problem as a discounted infinite horizon MDP, and in order to find the approximate solution they developed a direct search method based on ADP technique. They compared the performance of the direct-search based ADP method against that of the myopic policy. Their model differs from our model in that we do not consider appointments with target dates and time windows, and hence tackle a more general problem. Further, unlike their model, our model assumes that cancelled jobs are rescheduled.

The description of dynamic stochastic advanced scheduling problems we handle along with the underling mathematical model for these problems are given next.

3. DYNAMIC STOCHASTIC ADVANCED SCHEDULING

In dynamic stochastic scheduling problems, dynamicity comes from the fact that jobs arrive dynamically at a facility; whereas stochasticity comes from random arrivals of jobs.

We consider the following scheduling problem.

- We focus on dynamic stochastic scheduling problems, with the assumption of an infinite time horizon and finite booking horizon, where the booking horizon will be through a period of N days.
- Jobs arrive randomly at a facility.
- Each job is assigned to one appointment slot.
- Arriving jobs are scheduled to future days over a booking horizon, and those jobs which are diverted can be performed through overtime.
- Jobs have deadlines. There is no cost to be incurred when jobs are scheduled to days earlier than their deadline, whereas jobs that are scheduled beyond their deadlines will cause a scheduling cost.
- Diversion cost will be incurred when arriving jobs are diverted or shifted to another facility or served through overtime.
- We assume cancellation of appointments; jobs that are cancelled are rescheduled.

Our aim is to schedule jobs to the available future days or send them to another facility (i.e., diversion). In the booking horizon the total discounted expected cost will be minimized. We consider that each type of job scheduling costs is linear. In our problem diversion is not always assumed as overtime, sometimes we utilize overtime by dealing with the problem through it on the day the decision is taken.

3.1 MARKOV DECISION PROCESS

Markov Decision Process (MDP) is a mathematical framework used to model systems that evolve over time. It has the following components: 1) state space, 2) action space, 3) probabilistic state transitions, and 4) rewards or cost.

The components of our MDP model for the abovementioned scheduling problem are given next.

3.1.1 State Space

The state space $s \in S$ is in the following form (see [28] for similar MDP components):

$$S = (x_n; y_i),$$

Where I represents the number of job types, X_{in} for $i = 1, \dots, I$, $n = 1, \dots, N$ is number of type- i jobs scheduled to day n , y_i for $i = 1; \dots; I$, is the number of type- i jobs waiting to be scheduled.

3.1.2 Action Space

At the end of each day, we need to determine jobs to be booked and jobs that are diverted.

$A_{x,y}$ denotes the set of available actions in state $s = (x; y)$. Any action in $A_{x,y}$ is represented by:

$$a = (a_{in}; z_i),$$

where a_{in} is the number of type- i jobs to book on day n , z_i is the number of diverted type- i jobs.

The set of feasible moves should fulfill the following constraints::

$$\sum_{i=1}^I X_{in} + \sum_{i=1}^I a_{in} \leq C_1 \quad , n = 1, \dots, N \quad (3.1)$$

$$\sum_{i=1}^I z_i \leq C_2 \quad (3.2)$$

Where C_1 is the capacity of the daily resource and C_2 is maximum number of jobs which is diverted or served through overtime in each day. Constraint 1 indicates that total number of jobs

scheduled on each day is confined through each day capability. Constraint 2 dictates that total number of jobs diverted (or served through overtime) is limited by diversion capacity.

3.1.3 Transition Probabilities

Let d_{in} the number of type- i jobs booked on day n that are cancelled, and $pr(q_i)$ be the probability that q_i new requests for type- i job arrived. The probability that d_{in} facility for type- i job scheduled to day n are cancelled is represented by the term $Q_{in}(d_{in})$ which is expressed as follows (see [28] for similar equations):

$$Q_{in}(d_{in}) = \binom{x_{in}}{d_{in}} p_{in}^{d_{in}} (1 - p_{in})^{x_{in} - d_{in}}, \quad (3.3)$$

Where p_{in} is the probability that a type- i job booked to day n is cancelled.

After decision a is selected in state s , one component of the next state is expressed as

$$(x'_{in}, y'_i) = \left(x_{i,n+1} + a_{i,n+1} - d_{i,n+1}, y_i - \left(\sum_{i=1}^I a_{in} + \sum_{i=1}^I z_i \right) + q_i \right) \quad (3.4)$$

With probability

$$Pr(q_i)Q_{in}(d_{in})$$

Hence the state changes according to

$$p(s'|s, a) = \prod_{i=1}^I Pr(q_i) \prod_{i=1}^I \prod_{n=1}^N Q_{in}(d_i)$$

If s' satisfies Eqs.(3.5)(3.6),(3.7)

$$x'_{i,n} = x_{i,n+1} + a_{i,n+1} - d_{i,n+1},$$

$$i=1, \dots, I; n=1, \dots, N-1 \quad (3.5)$$

$$X'_{i,N} = 0; i=1, \dots, N \quad (3.6)$$

$$y'_{i=y_i} - \left(\sum_{i=1}^I \sum_{n=1}^N a_n + \sum_{i=1}^I z_i \right) + q_i + \sum_{n=1}^N d_{in} \quad (3.7)$$

$i = 1, \dots, I;$

Equations (3.5) and (3.6) show the new number of type- i job which is booked on day n .

3.1.4 Cost

The cost of scheduling a type- i job to day n is denoted

$$b(i, n) = \begin{cases} 0 & \text{if } n \leq D_i \\ (n - D_i)c^i & \text{if } n > D_i \end{cases} \quad (3.8)$$

Where D_i is deadline for type- i job and C^i is scheduling cost for type- i job.

$$c(a, z) = \sum_{i,n} b(i, n)a_{i,n} + \sum_{i=1}^I pe(i) \quad (3.9)$$

Where $pe(i)$ for $i = 1, \dots, I$ is the penalty cost for diverting a type- i job.

3.1.5 Bellman's Equation

Discounting with discount factor λ is considered in our model. Bellman's equations for getting a policy which minimizes the expected infinite horizon discounted cost are defined below.

$$v(x, y) = \min_{(a,z) \in A_{x,y}} \left\{ c(a, z) + \lambda \sum_{y' \in D} p(y')v(x', y') \right\}, \quad (3.10)$$

Where D is the set of all possible demand vectors.

Our MDP model is computationally very challenging due to the fact that state and action spaces grow exponentially with the number of job types and the length of the booking horizon. Therefore we utilize approximate dynamic programming (ADP) for approximately solving our model. The ADP technique we utilized in this research is described next.

4. DIRECT-SEARCH BASED APPROXIMATE DYNAMIC PROGRAMMING

Approximate Dynamic Programming (ADP) has been considerably used for fixing intractable MDPs in various fields including manufacturing, healthcare, and revenue management [24]. ADP strategies are specially categorized as linear programming (LP)-based ADPs and simulation-based ADPs. Within the LP- based ADP technique, the fundamental MDP model is changed into the identical LP form of Bellman's equations, they worked on simulation models which include stochastic sampling and reinforcement learning technique to predict the value function.

In ADP, a set of basis functions is used to approximate the value function. Basis functions represent certain essential features of the state of the system [25]. One way of this approximation is linear approximation, as shown below:

$$V(s) \approx \sum_{k=1}^K r_k \Phi_k(s) \quad (4.1)$$

Where r_k , $k=1, \dots, K$ are tuning parameters and $\Phi_k(s)$, $k=1, \dots, K$ are basis functions. The approximation parameters are tuned iteratively to acquire an ADP policy. Specifically, ADP aims to find the optimal parameter vector that minimizes a certain objective function. In this context, we use direct search that tackles an optimization problem for tuning the ADP parameters to get the resulting best policy. Then the ADP policy is obtained through the approximate value functions.

4.1 POLICY RETRIEVAL PROCESS

The parameter tuning procedure enables us to have the approximate value of a given state. The ADP policy is then retrieved through the computation of a decision vector for any state. "That decision vector is myopic with respect to the value function approximation of our MDP." [28] The problem of retrieving decisions for a specific state s is [28]:

$$\min_{(a,z) \in A_{x,y}} \left\{ c(a,z) + \lambda \sum_{y' \in D} (y') \tilde{v}(x', y') \right\} \quad (4.2)$$

Where $\tilde{v}(x', y')$ is the approximate value of state (x', y') ,

The details about basis function of our problem is discussed next.

4.2 BASIS FUNCTION

The basis functions we use for the ADP method are given by

$$\Phi 1 = \sum \left(C_1 - \sum_{i,n} x_{i,n} - a_{i,n} \right) \quad (4.3)$$

$$\Phi 2 = - \left(\sum_{i=1}^I z_i \right) \quad (4.4)$$

The first basis function makes use of daily available capacity, whereas the second one enables us to try different values for the variable z_i in the respective optimization problem. Note that since we use two basis functions, there are two tuning parameters, r_1 and r_2 .

4.3 DIRECT SEARCH METHOD

We tune parameters by utilizing direct search to execute the direct search method. Unlike regression-based method, direct search considers achieving the critical goal of finding desirable guidelines and strategies. As stated in [28], “direct search deals with an optimization problem where the variables consist of feasible r 's and the objective function value is the expected cost of the policy induced by the corresponding parameter vector”. The underlying optimization problem is given by [28]:

$$\min_{r \in R^N} E \left[\sum_t c(s_t, \pi_r(s_t)) \right] \quad (4.5)$$

Where “ s_t is the state at step t , π_r is the policy obtained by the parameter vector r , $\pi_r(s_t)$ is the action dictated by the policy π_r in the state at stage t , and $c(s_t, \pi_r(s_t))$ is immediate cost incurred at step t as a result of choosing $\pi_r(s_t)$ ” [28].

We implemented the direct search-based ADP by ranging r_1 and r_2 from 0 to 40 with increments of 2.

The details of the numerical results for our problem are discussed next.



5. NUMERICAL RESULTS

In this section we consider the performance comparison of the direct search-based approximation dynamic programming (ADP) approach and the Greedy policy. The greedy policy is obtained by solving the problem $\min_{(a,z) \in A_{x,y}} c(a, z)$ for any state $s \in S$.

We used AMPL with CPLEX 12 for solving the integer programs.

5.1 EXPERIMENTAL DESIGN

We consider job scheduling problem where the length of booking horizon is 14. We set resource availability to (4, 8, and 12), and assumed that arrival rate is independent Poisson. Also number of stages during the simulation is 50. We have deadline DU (1, N) (DU stands for discrete uniform) where N is the length of the booking horizon. There is penalty cost DU (1,100); it is incurred when appointment is not satisfied in the deadline. There is unit delay cost DU (1,100). The probability that a job-i scheduled to day n is cancelled has three levels: 0, 0.05, and 0, 1. We also have a discounted factor Beta which has 2 levels 0.90 and 0.99. It is applied when the cost incurred in the future of step has less impact over the overall cost.

Then we will have 18 problem sets, and each problem set is determined by a specific parameter level set. We assumed number of replications is 20, and number of instances is 10, which means we will receive 200 values in each of these 18 problem sets. After that we calculate the average of each of these 18 values. We used paired t test to compare the performance of the results.

5.2 RESULTS

We present our results in Tables 5.1, 5.2. In each row we present a different problem set, for each of them we identify whether the direct search policy or the greedy policy performs better. We have listed the total cost which is obtained by the direct search policy and the greedy policy in the second and third column of both columns. Percentage improvement which is found by the direct search policy over the greedy policy is listed in the fourth column, and the fifth column gives us whether there is a statically improvement over the greedy policy. We have done with two types of booking horizon, N=14 days, and N= 7 days. The result of the Paired-t test reveals that P value for each problem set is almost 0.

When we increase the resource availability number and decrease level of Beta, then improvement percentage decreases.

These two tables reveal that the direct-search based on ADP improves the solution of the greedy policy on all of the problem sets that we generated.

Table 5.1: Result of each problem set when the booking horizon is N=14

$(C_1, \beta, p_{i,n})$	Greedy method	Direct search	%improvement	sign
(4, 0.99, 0)	21932	17421	20.6	YES
(4, 0.99, 0.05)	22076	17025	22.9	YES
(4, 0.99, 0.1)	21967	16798	23.5	YES
(8, 0.99, 0)	12379	8858	28.4	YES
(8, 0.99, 0.05)	11066	8264	25.3	YES
(8, 0.99, 0.1)	10010	7828	21.8	YES
(12, 0.99, 0)	5398	3556	34.1	YES
(12, 0.99, 0.05)	4727	3196	32.4	YES
(12, 0.99, 0.1)	4157	2963	28.7	YES
(4, 0.90, 0)	3878	3149	18.8	YES
(4, 0.90, 0.05)	3812	3052	19.9	YES
(4, 0.90, 0.1)	3767	2990	20.6	YES
(8, 0.90, 0)	1625	1363	16.1	YES
(8, 0.90, 0.05)	1464	1243	15.1	YES

(8, 0.90, 0.1)	1329	1143	14.0	YES
(12, 0.90, 0)	565	405	28.3	YES
(12,0.90, 0,05)	488	379	22.3	YES
(12,0.90, 0,1)	422	323	23.4	YES

Table 5.1: Result of each problem set when the booking horizon is N=7

$(C_1, \beta p_{i,n})$	Greedy method	Direct search	%improvement	sign
(4, 0.99, 0)	24685	20151	18.4	YES
(4, 0.99, 0.05)	23437	18269	22.1	YES
(4, 0.99, 0.1)	23329	18113	22.4	YES
(8, 0.99, 0)	14810	9871	33.4	YES
(8, 0.99, 0.05)	14203	9619	32.3	YES
(8, 0.99, 0.1)	13652	9341	31.6	YES
(12, 0.99, 0)	6793	4098	39.7	YES
(12, 0.99, 0.05)	6394	3978	37.7	YES
(12, 0.99, 0.1)	5978	3847	35.6	YES
(4, 0.90, 0)	4653	3705	20.4	YES
(4, 0.90, 0.05)	4626	3676	20.5	YES
(4, 0.90, 0.1)	4588	3634	20.8	YES
(8, 0.90, 0)	2391	1781	25.5	YES
(8, 0.90, 0.05)	2290	1735	24.2	YES

(8, 0.90, 0.1)	2203	1694	23.1	YES
(12, 0.90, 0)	907	639	29.5	YES
(12, 0.90, 0,05)	856	618	27.8	YES
(12, 0.90, 0,1)	799	590	26.1	YES



6. CONCLUSION

In this thesis we studied a class of dynamic stochastic advanced scheduling problems with cancellation, where the jobs are arriving randomly to the system and arrival jobs have deadlines. We formulated the problem by using MDP model; because of the intractable state and action spaces, we approximately solved the problem by using a direct search-based ADP technique. We compared the performance of direct search-based ADP technique against greedy method, and found that ADP technique can improve the solution of the greedy policy.

Scheduling problems arise in an extensive range of field of healthcare, manufacturing; most researchers of this field consider the static deterministic problems. However there is some research that focused on dynamic stochastic scheduling, and by using some mathematical programs solved these problems. In the future research dynamic stochastic scheduling problems can be modelled as MDP and can be integrated into our model and direct search based ADP can be applied for solving other dynamic stochastic scheduling problems.

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