CENTER OF GRAVITY ESTIMATION AND ROLLOVER PREVENTION USING KALMAN FILTERING TECHNIQUES

by

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ABSTRACT

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Due to its impacts on human safety and its economical cost, vehicle rollover is a very important safety issue that attracts the attention of major vehicle manufacturers and researchers. The objective of this thesis is to design a control system that only becomes active in emergency situations and prevents rollover by applying differential braking to the vehicle. More specifically, the main focus of this thesis is to estimate several unknown vehicle parameters, including the center of gravity height, that has major role in roll over, by using Kalman Filter algorithm. Subsequently, the estimated center of gravity height is used in determining the amount of differential braking force. Extensive simulations are carried out in MATLAB to demonstrate the superior performance of the proposed method.

ÖZET

KALMAN SÜZGECİ TEKNİĞİ İLE AĞIRLIK MERKEZİ TAHMİNİ VE DEVRİLME ÖNLEME

Araç devrilmesi, insan güvenliği üzerindeki etkisinden ve sebep olduğu ekonomik zararlardan dolayı araç üreticilerinin ve araştırmacıların dikkatini çeken çok önemli bir güvenlik problemidir. Bu tezin amacı, sadece acil durumlarda devreye giren ve araca diferansiyel fren uygulayarak devrilmeyi engelleyen bir denetim sistemi tasarlamaktır. Özellikle, bu tezin ana odak noktası, devrilme üzerinde büyük rolü olan ağırlık merkezi yüksekliğini ve diğer bazı bilinmeyen degişkenleri Kalman süzgeci yöntemi ile tahmin etmektir. Daha sonra tahmin edilen ağırlık merkezi yüksekliği, diferansiyel fren kuvvetinin şiddetinin belirlenmesinde kullanılmaktadır. Önerilen methodun yüksek performansını sergilemek adına MATLAB kullanılarak kapsamlı simulasyonlar gerçekleştirilmiştir.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF SYMBOLS/ABBREVIATIONS

C.G.	Center of Gravity
LTR	Load Transfer Ratio
MMST	Multi Model Switching and Tuning
TTR	Time to Rollover

1. INTRODUCTION

Rollover is a very serious safety problem which has a significant share in reported traffic accidents all over the world. In USA approximately 35000 rollover accidents are reported every year [1]. Rollover accidents threaten human safety and its economical impacts can not be underestimated.

Rollover may occur after hitting an object or when the vehicle slides and its tires get stuck in soft soil. This type of rollover is called tripped rollover [2]. On the other hand, the focus of this thesis is untripped rollover which is driver induced. Untripped rollover can occur if a vehicle enters a corner with a high speed or if the driver tries to avoid an obstacle by applying a high steering input. This type of rollover is most commonly seen on vehicles with high center of gravity such as trucks, vans and SUVs. Therefore, the center of gravity height can be considered as an important parameter in designing a controller to avoid rollover.

As a result of its economical cost and serious impact on human safety, the rollover prevention has become an important research subject. In the rollover prevention system proposed by Palkovic [3], the wheel slip difference on the two sides of the axels was used to predict the tire lift off condition. TTR (time to rollover) metric was used by some researchers to predict rollover situations [4]. In the anti roll braking method, an individual front wheel is subjected to braking action instead of full braking applied to every wheel with the same amount [5]. The Kinetic Energy measure was also used in some of the works to predict wheel lift off. Load Transfer Ratio (LTR), which gives the ratio of the difference between loads on the left and right tires, is considered to be a very good indicator before rollover situations [1], [2], [6]. Many of the previous studies make use of the lateral acceleration thresholds [5], tire lift of sensors or estimation algorithms [5], and the LTR to predict rollover situations. The roll angle, roll rate, slip angle, lateral acceleration, center of gravity height have been considered to be very important parameters which are highly corelated with vehicle rollover. Differential braking [2], active steering [7], [8], active suspension [9], active roll stabilizer bars have been used as actuators to prevent roll over.

So far some robust controllers have been designed in order to prevent rollover accidents with drive performance and efficiency costs. However better controllers can be designed which become active only in emergency situations by continuously monitoring the vehicle dynamics. Such controllers have less significant effect on the drive performance compared to that of robust controllers.

The proposed method in this thesis first estimates the CG height and other unknown parameters of the vehicle. Based on these estimates, it is then decided whether the vehicle is in an emergency situation, and if it is so, then a differential braking force is applied as the control input to the vehicle. The amount of differental braking force is proportional to the estimated CG height value.

The estimated vehicle parameters are used to calculate Load Transfer Ratio. By looking at the LTR data, it can be easily determined if the left or right tires carry most of the vehicle load. If one of these sides carries a significant portion of the total load, it means that the vehicle is very close to a rollover accident. The proposed controller becomes active in such situations.

As mentioned above, the CG height is a very important parameter that determines the rollover tendency of the vehicle. If we are in an emergency situation as described above, the controller applies differential braking to the vehicle based on the estimated height. Such a controller provides safety as much as a robust controller and it is superior to the robust controller when drive performance is considered.

The CG estimation is also a seperate research subject. Recent publications about this topic include MMST (Multi Model Switching and Tuning) methodology [2]. In this method, many different vehicle models with different parameter values (CG height and other unknown parameters) are defined. The inputs like steering angle and velocity are applied to these models and the output signals are compared with the actual measurements. The parameters of the model which is closest the real vehicle are taken as the estimated parameters which include the height of center of gravity. In another research, extended Kalman Filter, extended Luanberger observer and sliding mode observer were used to estimate vehicle side slip angle and velocity of the center of gravity [10].

In order to estimate the CG height and other unknown parameters, we propose a method based on Kalman filtering. Since the Kalman filtering aims to estimate the state of a linear system, it is a useful technique for this problem. These estimated values are then used to predict the rollover and to determine the amount of differential braking force as the controller output.

The organization of this paper is as follows: in Chapter 2, the mathematical model of the real vehicle which is used in the simulations is defined and a rollover accident is simulated. In Chapter 3, the method used for parameter estimation is discussed. In Chapter 4, the controller is presented and the proposed estimators are coupled with controller to prevent rollover. Finally some concluding remarks are given in Chapter 5.

2. SYSTEM DESCRIPTION

In this chapter the mathematical model of the vehicle is presented. This model will be used in simulations to represent the real vehicle. The model has two inputs; the steering angle induced by the driver and the differential braking force which is either applied by the driver or an automatic controller. After discussing the model parameters, the Load Transfer Ratio which is used to understand rollover situations will be defined. The lateral acceleration, an important item in this thesis, that is used in paramater estimation and determining the amount of differential braking force will be described. The availability of signals used in simulations will be discussed. Finally a simulation scenario will be carried to demonstrate the rollover situation.

2.1. Vehicle Model

As indicated in Chapter 1, the goal of this thesis is to estimate the unknown parameters of the vehicle and based on these estimations, predict and prevent rollover accidents by applying differential braking. The vehicle model which will be used as the real vehicle in simulations is described below.

On the left hand side of Figure 2.1, the bicycle model of the vehicle is shown [1],[2],[6]. It is referred to as a bicycle model because it is assumed that the left and right tires are lumped into a single one at the axle centerline. In this figure, variables of interest related with lateral dynamics of the vehicle, the longitudinal position of the center of gravity, side slip angle, yawrate, steering angle are shown (please refer to Table 2.1 for descriptions).

The side slip angle of a vehicle β is the angle its velocity vector at the center of gravity makes with the longitudinal axis of the vehicle [11]. The yaw rate $\dot{\psi}$ is vehicle's angular velocity around its vertical axis which passes through the center of gravity. The lateral and longitudinal velocities are shown as υ_x and υ_y respectively.

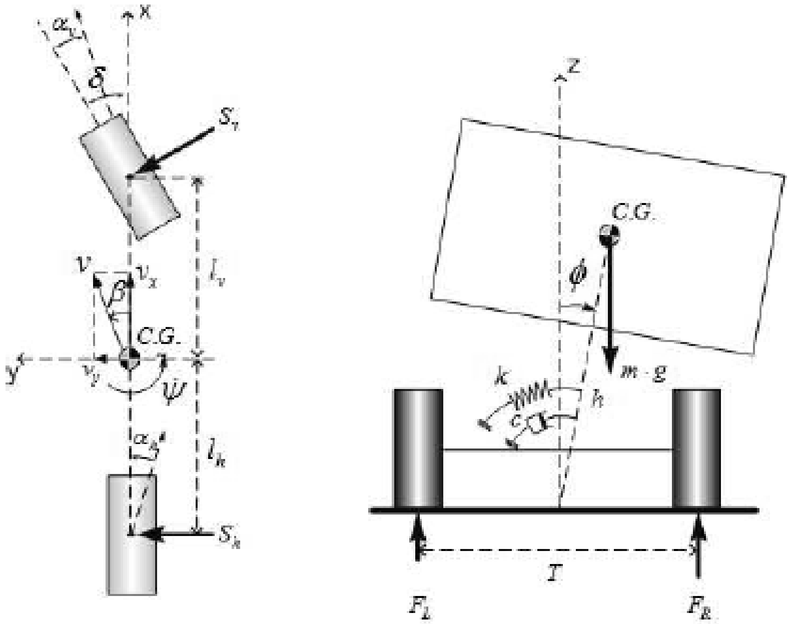

Figure 2.1. Single track model with roll degree of freedom [2]

On the right hand side of Figure 2.1, the roll dynamics of the vehicle is displayed. The roll angle ϕ is produced around the roll axis in the longitudinal plane of the vehicle [12]. In this figure, the roll motion due to lateral acceleration is shown. The center of gravity height, suspension damping coefficient and suspension spring stiffness are represented with h, c and k.

As mentioned previously, the vehicle has two inputs: differential braking force u and the steering input δ. The speed of the vehicle v decreases with braking input and can be increased with accelerating force. The speed of the vehicle affects the behavior nonlinearly.

The mathematical model of the system to be controlled is given below [1], [2], [6], [13].

$$\dot{x} = Ax + B_\delta \delta + B_u u \tag{2.1}$$

where

$$\mathbf{A} = \begin{bmatrix} -\frac{\sigma J_{x_{eq}}}{mJ_{xx}v} & \frac{\rho J_{x_{eq}}}{mJ_{xx}v^2} - 1 & -\frac{hc}{J_{xx}v} & \frac{h(mgh-k)}{J_{xx}v} \\ \frac{\rho}{J_{zz}} & -\frac{\kappa}{J_{zz}v} & 0 & 0 \\ -\frac{h\sigma}{J_{xx}} & \frac{h\rho}{J_{xx}v} & -\frac{c}{J_{xx}} & \frac{mgh-k}{J_{xx}} \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{2.2}$$

$$\mathbf{B}_\delta = \begin{bmatrix} \frac{C_v J_{x_{eq}}}{mJ_{xx}v} & \frac{C_v l_v}{J_{zz}} & -\frac{hC_v}{J_{xx}} & 0 \end{bmatrix}^T \tag{2.3}$$

$$\mathbf{B_u} = \begin{bmatrix} 0 & -\frac{T}{2J_{zz}} & 0 & 0 \end{bmatrix}^T \tag{2.4}$$

and the state x contains side-slip angle β, yaw rate $\dot{\psi}$, roll rate $\dot{\phi}$ and roll angle ϕ, i.e.,

$$x = \begin{bmatrix} \beta & \dot{\psi} & \dot{\phi} & \phi \end{bmatrix}^T \tag{2.5}$$

This model is used in order to represent the real vehicle in the simulations. The equation governing the change of side slip angle β, yaw rate $\dot{\psi}$, roll rate $\dot{\phi}$ and roll angle ϕ over time is given by (2.1). In order to simplify the equation, auxiliary parameters $\sigma = C_v + C_h$, $\rho = C_h l_h - C_v l_v$ and $\kappa = C_v l_v^2 + C_h l_h^2$ are defined. $J_{x_{eq}} = J_{xx} + mh^2$ is the equivalent roll moment of inertia. The other parameters used in the above equations are tabulated in Table 2.1.

The steering input δ directly affects both roll and lateral dynamics of the vehicle as expected. The second input u is the total differential braking force on the wheels which will be the controller's input to the system. This force is a signed quantity and

Table 2.1. Model variables [2]

Variable	Description	Value	Unit
m	Vehicle Mass	1300	kg
g	Gravitational Constant	9.81	m/s^2
υ_x	initial longitudinal speed	30	m/s
J_{xx}	Roll moment of inertia at the CG	400	kgm^2
J_{zz}	Yaw moment of inertia at at the CG	1200	kgm^2
L	Axle seperation	2.5	m
T	Track width	1.5	m
l_v	longitudinal CG w.r.t. front axle	1.2	m
l_h	longitudinal CG w.r.t. rear axle	1.3	m
h	CG height over ground	0.51	m
c	suspension damping coefficient	5000	Nms/rad
k	suspension spring stiffness	36000	Nm/rad
C_v	front tire stiffness coefficient	60000	N/rad
C_h	rear tire stiffness coefficient	90000	N/rad
δ,β,ϕ	Steering angle, Side Slip angle (at CG),and roll angle respectively	varying	rad
α_v, α_h	Side-slip angles at the front and rear tire respectively	varying	rad
$\dot{\psi}$,$\dot{\phi}$	Yaw rate and Roll rate, respectively	varying	rad/sec

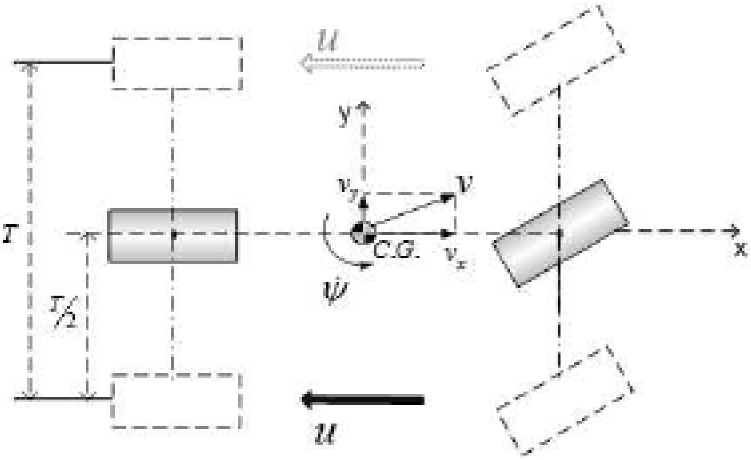

Figure 2.2. Effective differential braking [2]

is positive if the effective braking is on the right wheels and negative otherwise. The effective differential braking can be either on the left tires or on the right tires and the vehicle will tend to move around the yaw axis.

According to the Newton's law, the braking force will change the speed of the vehicle. The differential braking governs the vehicle longitudinal speed as

$$\dot{v} = \frac{F_x}{m} - \frac{|u|}{m} \tag{2.6}$$

where F_x is the accelerating force in the longitudinal direction.

2.2. Load Transfer Ratio

The Load Transfer Ratio(LTR) is defined as below [2], [1].

$$LTR = \frac{\text{Load on Right Tires} - \text{Load on Left Tires}}{\text{Total Load}} \tag{2.7}$$

The LTR is used to understand the rollover situation. It is clear that, while driving straight, the LTR is equal to 0. If the right tires carry all the load, the LTR value is 1 and this is defined to be a rollover situation in the thesis. The LTR value varies within the range [-1,1]. A threshold value like 0.6 will be used to define the severity of the rollover situation. If the LTR value is greater than the threshold value, then the controller will become active in applying the differential braking force. A dynamical approximation for the load transfer ratio is given as follows [2], [1]

$$LTR_d = -\frac{2(c\dot{\phi} + k\phi)}{mgT} \tag{2.8}$$

Since we can not use (2.7) in the simulations, the above approximation will be used as the real LTR value. We will conclude rollover accident if the LTR_d is not in the range [-1,1]. In the following chapters the above equation will be used by the controller to predict the rollover accident. The estimated c and k values will be fed to the controller. The controller will calculate the LTR_d by using measured ϕ and $\dot{\phi}$ and known m, g, T values. If the absolute value of LTR_d becomes greater than the threshold value 0.6, then the controller will become active to prevent the rollover accident by applying differential braking.

2.3. Vehicle Parameters

The vehicle parameters in (2.1) can be categorized in two groups: the lateral dynamics parameters C_v, C_h, l_v, and the roll dynamics parameters k, c, h. In this thesis, Kalman filtering and recursive least squares techniques are used to estimate these parameters. Then the LTR_d value in (2.8) can be calculated using these estimated values. This determines whether the vehicle is in emergency situation or not. If the vehicle is in an emergency situation, then a differential braking force is applied to the vehicle based on the estimated height value.

2.4. Lateral Acceleration

The lateral acceleration a_y is a measurable quantity using sensors. When its kinematic equation is considered, it is observed that it consists of two components. The first one is the derivative of v_y relative to vehicle fixed coordinate system and the second one is the acceleration component caused by the motion of the vehicle fixed coordinate system [14]. The basic kinematic equation of the acceleration vector is as follows:

$$a = \dot{v} + \omega \times v \tag{2.9}$$

where ω is the angular velocity vector describing the rotation of the vehicle fixed coordinate system relative to the inertial coordinate system.

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} \dot{v}_x \\ \dot{v}_y \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} \times \begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix} \tag{2.10}$$

$$a_x = \dot{v}_x - v_y \dot{\psi} \tag{2.11}$$

$$a_y = \dot{v}_y + v_x \dot{\psi} \tag{2.12}$$

$$a_z = 0 \tag{2.13}$$

According to Figure 2.1 the relationship between v, v_x and v_y can be expressed as:

$$v_x = v \cos \beta \tag{2.14}$$

$$v_y = v \sin \beta \tag{2.15}$$

The angle β can be assumed to be small. Therefore we have:

$$v_x = v \tag{2.16}$$

$$v_y = v\beta \tag{2.17}$$

Using v_x and v_y as above, the lateral acceleration a_y equation can be rewritten as:

$$a_y = \dot{v}\beta + v\dot{\beta} + v\dot{\psi} \tag{2.18}$$

Lateral acceleration is a very important quantity in this work. It is both used to calculate the sign and amount of differential braking force and to estimate the lateral dynamic parameters which will be studied in the following chapter.

2.5. Availability of Signals

During the simulations the signals β, a_y, v, $\dot{v}$, $\dot{\psi}$, ϕ, $\dot{\phi}$, δ will be assumed to be available with their exact values. How these signals would be obtained from a real vehicle is a very important point.

It is known that, except the side slip angle β the above signals can be obtained by some specific sensors. However the signal β can not be measured by using simple and cheap sensors.

The signal β can also be obtained from kinematic equation of lateral acceleration [11]. If we consider the equations below:

$$a_y = \dot{v_y} + v\dot{\psi} \tag{2.19}$$

$$a_y - \upsilon\dot{\psi} = \dot{\upsilon_y} \tag{2.20}$$

$$\int (a_y - \upsilon\dot{\psi})dt = \upsilon_y \tag{2.21}$$

Since we take $\upsilon_y = \upsilon\beta$;

$$\beta = \frac{\int (a_y - \upsilon\dot{\psi})dt}{\upsilon} \tag{2.22}$$

With the above equation β can be obtained. This can also be used in simulations. However we prefer to use the exact value obtained from vehicle to avoid errors during parameter estimation.

2.6. Numerical Analysis of Vehicle Dynamics without Rollover Prevention

In this section, we examine the vehicle dynamics when a time-varying driver steering input is applied. There is no control present and it is also assumed that there is no braking force applied by the driver. Therefore the vehicle is assumed to be travelling at a constant speed of 108 km/hr as shown in Figure 2.3. In the simulations the parameter values given in Table 2.1 are used. Suppose that the steering input shown in Figure 2.4 is applied by the driver. The maximum value of the input is 100 degrees that corresponds to 100/18 degrees at the wheels. In Figures 2.5 - 2.8 the state variables β, $\dot{\psi}$, $\dot{\phi}$ and ϕ are shown respectively. And finally, as seen in Figure 2.9, the LTR exceeds 1 in magnitude that corresponds to rollover by our definition.

2.7. Summary of the Chapter and Concluding Remarks

In this chapter, the mathematical model of the vehicle is introduced. The inputs of the system; steering angle and differential braking force and the state variables side slip angle, yaw rate, roll rate, and roll angle which represent the vehicle dynamics are defined. The load transfer ratio which helps us to understand rollover situaton is presented. The lateral acceleration is described and the availability of signals is

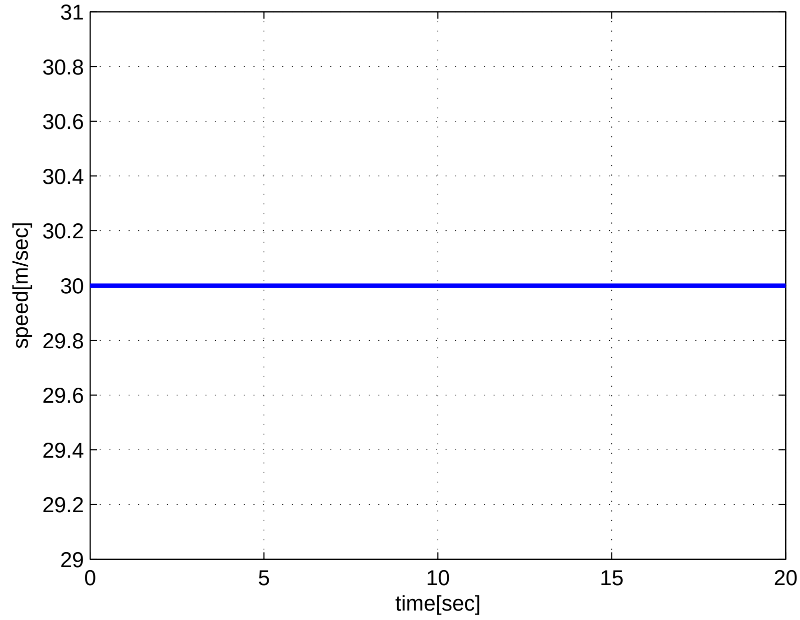

Figure 2.3. Vehicle speed without rollover prevention

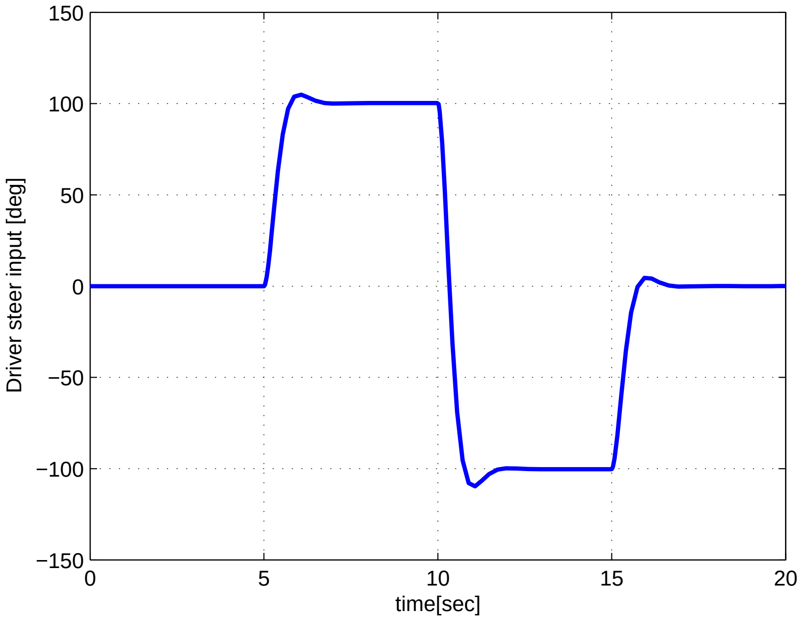

Figure 2.4. Steering input without rollover prevention

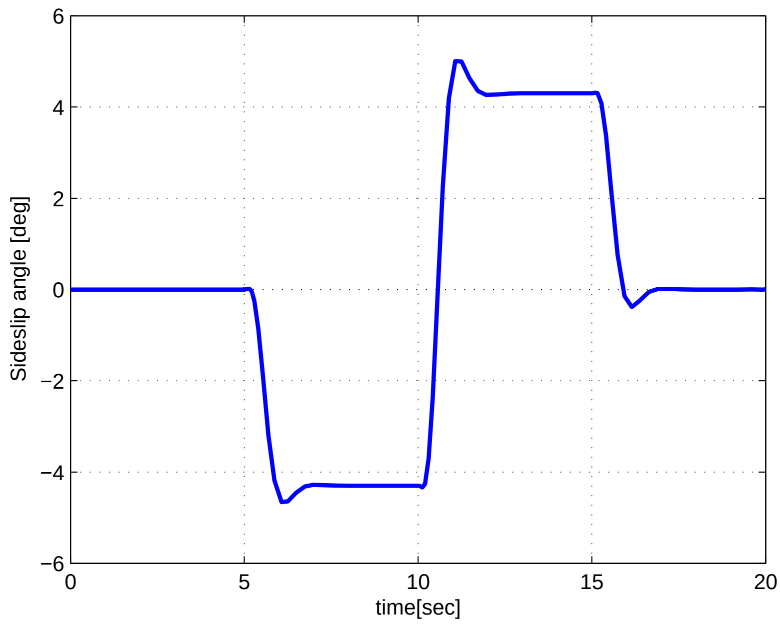

Figure 2.5. Side slip angle without rollover prevention

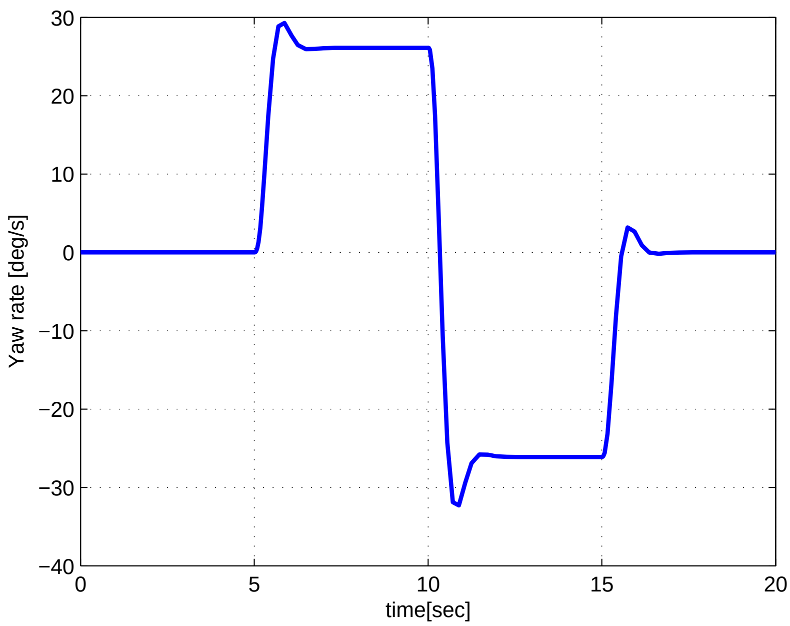

Figure 2.6. Yaw rate without rollover prevention

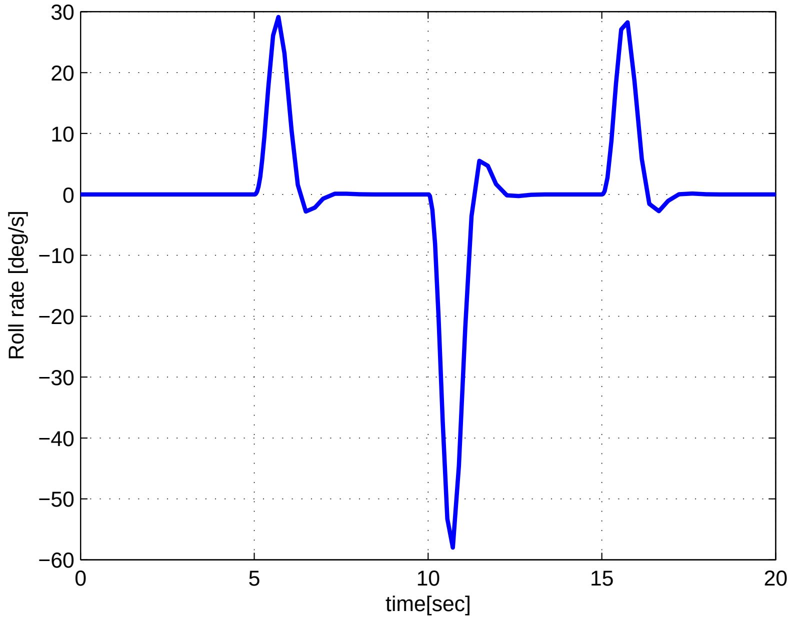

Figure 2.7. Roll rate without rollover prevention

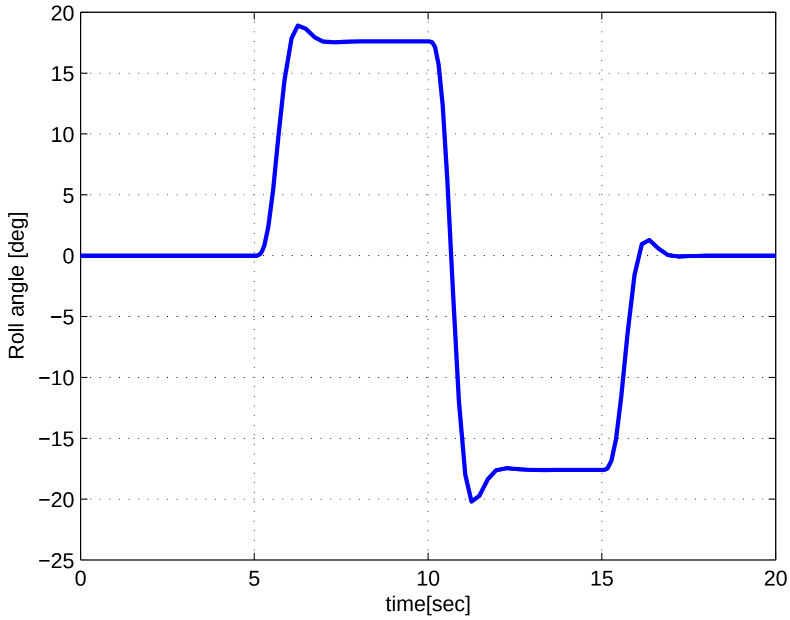

Figure 2.8. Roll angle without rollover prevention

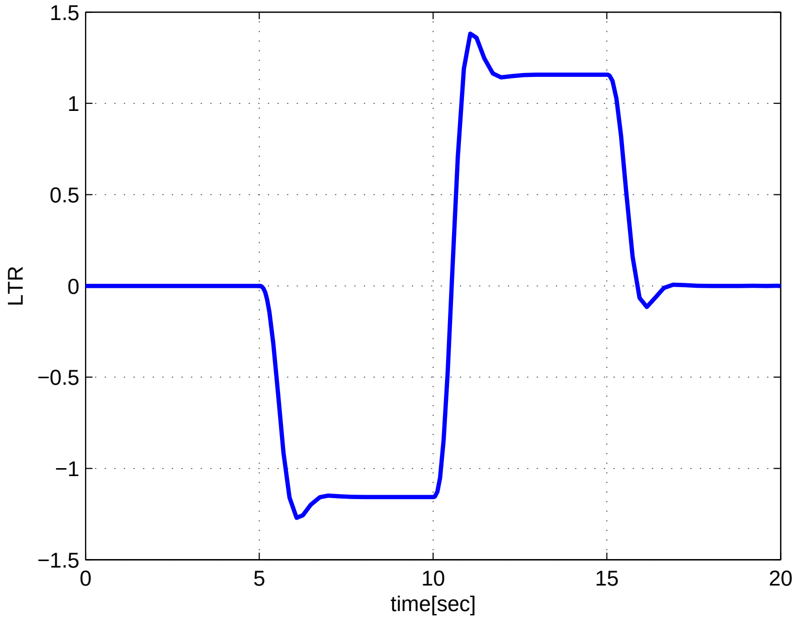

Figure 2.9. LTR without rollover prevention

discussed. At the end of the chapter, a rollover situation is simulated.

In the rest of the thesis, techniques to prevent rollover will be investigated. The proposed solution will include a controller which applies differential braking to the vehicle to prevent rollover. However, before designing the controller, the following questions should be answered. Will this controller be always active? If not, when will it become active? How will this controller predict the rollover situation or understand the emergency situation before a possible accident? What will be the amount of control force to prevent rollover situation without significant loss in the vehicle performance?

In order to answer the above questions the unknown vehicle parameters should be estimated. This is what will be covered in the next chapter.

3. VEHICLE PARAMETER IDENTIFICATION

In the previous chapter a rollover situation was simulated. Such an accident can be prevented by applying a certain amount of differential braking in the right time. In order to apply the control force in the right time, a mechanism to predict rollover situation should be designed.

The Load Transfer Ratio is a good indicator to judge the load sharing situation between left and right tires. When the absolute value of this ratio increases, one can understand that the vehicle is getting closer to a rollover accident. A good approximation of LTR was given in (2.8). The rollrate and rollangle can be estimated [15]. The mass and track width can be provided by the manufacturer and also the gravitational constant is known approximately.

If suspension damping coefficient c and suspension spring stiffness k are also known then the approximate value of LTR can be calculated in order to predict the rollover accident. However, c and k can not be measured easily.

The other roll dynamics parameter, the center of gravity height h, gives an idea about vehicle's tendency to roll. The vehicles with higher center of gravity height are more prone to such accidents. However like k and c, center of gravity height can not be measured easily.

In order to determine the roll dynamics parameters which will be used to predict roll over situation and to decide the amount of control force, the lateral dynamics parameters; longitudinal position of center of gravity l_v, the front tire stiffness C_v and rear tire stiffness C_h should be known. However these parameters are also considered as unkown parameters and should be estimated online.

In this chapter, the unknown vehicle parameters will be estimated using two methods: recursive least squares algorithm and Kalman filter. These techniques are used in different situations which will be discussed in this chapter.

3.1. Recursive Least Squares Algorithm (RLS)

Before going through the description of the RLS algorithm, let us restate the problem mathematically:

From (2.1), the state variables β, $\dot{\psi}$, $\dot{\phi}$, ϕ; the speed v and the lateral acceleration a_y are assumed to be measurable signals. The parameters m, g, J_{xx}, J_{zz}, L, T are assumed to be known with values indicated in Table 2.1. The parameters l_v, C_v, C_h, k, c and h are considered as the unknown parameters to be estimated.

Assume that there is no control force so $u = 0$. Let us consider the below equations.

$$\ddot{\psi} = \frac{C_h l_h - C_v l_v}{J_{zz}}\beta - \frac{C_v l_v^2 + C_h l_h^2}{J_{zz} v}\dot{\psi} + \frac{C_v l_v}{J_{zz}}\delta \tag{3.1}$$

$$\ddot{\phi} = -\frac{h(C_v + C_h)}{J_{xx}}\beta + \frac{h(C_h l_h - C_v l_v)}{J_{xx} v}\dot{\psi} - \frac{c}{J_{xx}}\dot{\phi} + \frac{(mgh - k)}{J_{xx}}\phi + \frac{hC_v}{J_{xx}}\delta \tag{3.2}$$

As time goes on, there will be new measurements for the state variables β, $\dot{\psi}$, $\dot{\phi}$, ϕ and driver input δ. Therefore using the measured signals and the results of their linear combinations over time, there should be a way to estimate the constant coefficients. The RLS algorithm is one method that addresses the solution of this problem.

The lateral and roll dynamic parameters can be estimated using RLS algorithm if the speed of the vehicle is constant. Constant speed guarantees that the coefficients of the measured signals are constant. Here we first estimate the lateral dynamic parameters. Subsequently the roll dynamic parameters will be estimated using the estimated lateral dynamic parameters.

The Least Squares Algorithm is a well known method to calculate the best x in the below overdetermined system [16]

$$y = Cx \tag{3.3}$$

where $y \in \mathbf{R^{nx1}}$, $x \in \mathbf{R^{mx1}}$, $C \in \mathbf{R^{nxm}}$ and $m < n$. The 'best x' refers to x which minimizes the square of the magnitude of the residual vector $r = y - Cx$. The function to be minimized is:

$$\phi(x) = \| r \|^2 = r^T r = (y - Cx)^T (y - Cx) = y^T y - 2x^T C^T y + x^T C^T C x \tag{3.4}$$

By taking the gradient of this function with respect to x and setting it equal to zero, the optimal x can be determined:

$$0 = \bigtriangledown \phi(x) = 2C^T C x - 2C^T y \tag{3.5}$$

$$x^* = (C^T C)^{-1} C^T y \tag{3.6}$$

Note that the inverse of $C^T C$ is needed in (3.6). This can be a computational problem in cases when the matrix C is very large, as new data are received. New data mean a new index for vector y and a new row for matrix C. It is not desirable to take inverse of a growing matrix at each instant since this is not computationally efficient.

To handle such cases, the recursive least squares algorithm has been developed. As new data are received or a new measurement is performed, the estimate vector x^* is updated in a recursive manner.

$$\begin{pmatrix} y(0) \\ y(1) \\ \vdots \\ y(n) \end{pmatrix} = \begin{pmatrix} \vec{c(0)}^T \\ \vec{c(1)}^T \\ \vdots \\ c(\vec{n})^T \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix}$$

The recursive Least Squares Algorithm is as follows:

$$k(t) = \frac{1}{1 + c(t)^T P(t)c(t)} P(t)c(t)$$

$$x(t+1) = x(t) + k(t)(y(t) - c(t)^T x(t))$$

$$P(t+1) = (I - k(t)c(t)^T)P(t)$$

The initial values for vector x and matrix P are also very important. The vector $x(0)$ should be chosen as a first best guess. The matrix P should be chosen as identity matrix multiplied by σ which is a very large number such as 10^9 or greater, e.g.,

$$P(0) = \sigma I, \ \sigma > 10^9$$

3.2. Lateral Dynamics Parameter Estimation Using RLS Algorithm

The RLS algorithm is used in [2] in order to estimate the lateral dynamics parameters in case when the speed is constant. Here we need to give the details of this. Therefore it will be much easier to explain why we need Kalman filter when the speed of the vehicle is not constant.

As previously discussed C_v, C_h, l_v are the lateral parameters to be estimated. The equation governing these parameters is found in (2.2). Let us check the yaw rate equation again when there is no braking or accelerating force.

$$\ddot{\psi} = \frac{C_h l_h - C_v l_v}{J_{zz}} \beta - \frac{C_v l_v^2 + C_h l_h^2}{J_{zz} v} \dot{\psi} + \frac{C_v l_v}{J_{zz}} \delta \tag{3.7}$$

In (3.7) note that we do not have the measurement for signal $\ddot{\psi}$. If we had it, then this equation would be similar to a linear equation $m = ax + by + cz$ where m is the measured output signal and x,y,z are the input signals. Consequently, the coefficients a,b and c could be estimated using techniques like least squares algorithms.

Since the output signal $\ddot{\psi}$ is not available, RLS algorithm can not be used. The equation (3.7) should be modified in a way such that all input signals and output signal are available. Taking the Laplace transform of (3.7);

$$s\dot{\Psi}(s) = \frac{C_h l_h - C_v l_v}{J_{zz}}\beta(s) - \frac{C_v l_v^2 + C_h l_h^2}{J_{zz} \upsilon}\dot{\Psi}(s) + \frac{C_v l_v}{J_{zz}}\Delta(s) \tag{3.8}$$

In order to simplify the above equation let us use c_1, c_2, c_3 for the coefficients, i.e.

$$s\dot{\Psi}(s) = c_1\beta(s) + c_2\dot{\Psi}(s) + c_3\Delta(s) \tag{3.9}$$

$$(s - c_2)\dot{\Psi}(s) = c_1\beta(s) + c_3\Delta(s) \tag{3.10}$$

we divide both sides with $(s - \lambda)$ where $\lambda < 0$;

$$\frac{s - c_2}{s - \lambda}\dot{\Psi}(s) = \frac{c_1}{s - \lambda}\beta(s) + \frac{c_3}{s - \lambda}\Delta(s) \tag{3.11}$$

$$\frac{s - \lambda + \lambda - c_2}{s - \lambda}\dot{\Psi}(s) = \frac{c_1}{s - \lambda}\beta(s) + \frac{c_3}{s - \lambda}\Delta(s) \tag{3.12}$$

$$\dot{\Psi}(s) = \frac{c_1}{s - \lambda}\beta(s) + \frac{c_2 - \lambda}{s - \lambda}\dot{\Psi}(s) + \frac{c_3}{s - \lambda}\Delta(s) \tag{3.13}$$

Now define the following filtered signals $\Omega_{l1}(s)$, $\Omega_{l2}(s)$, $\Omega_{l3}(s)$

$$\Omega_{l1}(s) = \frac{\beta(s)}{s - \lambda}, \quad \Omega_{l2}(s) = \frac{\Psi(s)}{s - \lambda}, \quad \Omega_{l3}(s) = \frac{\Delta(s)}{s - \lambda}$$

Then (3.13) can be rewritten as

$$\dot{\Psi}(s) = c_1\Omega_{l1}(s) + (c_2 - \lambda)\Omega_{l2}(s) + c_3\Omega_{l3}(s) \tag{3.14}$$

By taking the inverse Laplace, we can obtain [2]

$$\dot{\psi} = \begin{bmatrix} \omega_{l1} & \omega_{l2} & \omega_{l3} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} \tag{3.15}$$

where

$$\theta_1 = c_1,\ \ \theta_2 = (c_2 - \lambda),\ \ \theta_3 = c_3$$

Now let us consider the filtered signals ω_{l1}, ω_{l2} and ω_{l3}. How will we obtain them from β, $\dot{\psi}$ and δ ? Consider

$$\Omega_{l1}(s) = \frac{\beta(s)}{s - \lambda} \tag{3.16}$$

$$s\Omega_{l1}(s) - \lambda\Omega_{l1}(s) = \beta(s) \tag{3.17}$$

By rewriting (3.17) in time domain, we obtain

$$\dot{\omega}_{l1} = \lambda\omega_{l1} + \beta \tag{3.18}$$

and similarly;

$$\dot{\omega}_{l2} = \lambda\omega_{l2} + \dot{\psi} \tag{3.19}$$

$$\dot{\omega}_{l3} = \lambda\omega_{l3} + \delta \tag{3.20}$$

From (3.15), we have the output signal $\dot{\psi}$ and the input signals ω_{l1}, ω_{l2}, ω_{l3} which are obtained from β, $\dot{\psi}$ and δ. All these input signals and the output signal are available. It is known that when these input signals ω_{l1}, ω_{l2}, ω_{l3} are multiplied with constant coefficients θ_1, θ_2, θ_3 respectively and added, the result is equal to $\dot{\psi}$.

Note that the filtered signals and $\dot{\psi}$ are time varying. The question that remains is, given (3.15), whether we can use the measured signals to estimate the constant coefficients θ_1, θ_2, θ_3 ? If we can, then we will be able to calculate the lateral vehicle parameters C_v, C_h, l_v using the following relations [2];

$$l_v = \frac{L(\theta_1 + \theta_2) - \upsilon(-\lambda - \theta_2)}{\theta_1} \tag{3.21}$$

$$C_v = \frac{J_{zz}\theta_3}{l_v} \tag{3.22}$$

$$C_h = \frac{J_{zz}(\theta_1 + \theta_3)}{L - l_v} \tag{3.23}$$

The RLS algorithm can be used for the above problem. As new measurements are taken (3.15) is an overdetermined system similar to (3.3), i.e.,

$$\begin{bmatrix} \dot{\psi}(0) \\ \dot{\psi}(1) \\ \vdots \\ \dot{\psi}(n) \end{bmatrix} = \begin{bmatrix} \omega_{l1}(0) & \omega_{l2}(0) & \omega_{l3}(0) \\ \omega_{l1}(1) & \omega_{l2}(1) & \omega_{l3}(1) \\ \vdots & \vdots & \vdots \\ \omega_{l1}(n) & \omega_{l2}(n) & \omega_{l3}(n) \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} \tag{3.24}$$

As seen in the above equation, θ_1, θ_2 and θ_3 can be estimated using the RLS algorithm.

3.2.1. Numerical analysis

In this section the RLS algorithm is used to estimate the lateral vehicle parameters. No braking force is applied to the vehicle. The speed of the vehicle is assumed to be constant and the steering input shown in Figure 2.4 is applied to the vehicle.

The filtered signals are generated from β, $\dot{\psi}$ and δ as indicated. The filtered signals and $\dot{\psi}$ are fed to the RLS estimator.

For the RLS estimator the initial conditions are set as:

$$\theta(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$P(0) = 10^9 I$$

The estimation results are displayed in Figures 3.1 - 3.3. Note that these estimates converge to the values shown in Table 2.1. Moreover, the convergence is very fast.

3.3. Roll Dynamics Parameter Estimation using RLS algorithm

The roll dynamic parameters are suspension spring stiffness k, suspension damping coefficient c and CG height over ground h. To estimate these parameters we again use (2.2):

$$\ddot{\phi} = -\frac{h(C_v + C_h)}{J_{xx}}\beta + \frac{h(C_h l_h - C_v l_v)}{J_{xx} v}\dot{\psi} - \frac{c}{J_{xx}}\dot{\phi} + \frac{(mgh - k)}{J_{xx}}\phi + \frac{hC_v}{J_{xx}}\delta \qquad (3.25)$$

$$J_{xx}\ddot{\phi} + c\dot{\phi} + k\phi = h(-(C_v + C_h)\beta + (C_h l_h - C_v l_v)\frac{\dot{\psi}}{v} + mg\phi + C_v\delta) \qquad (3.26)$$

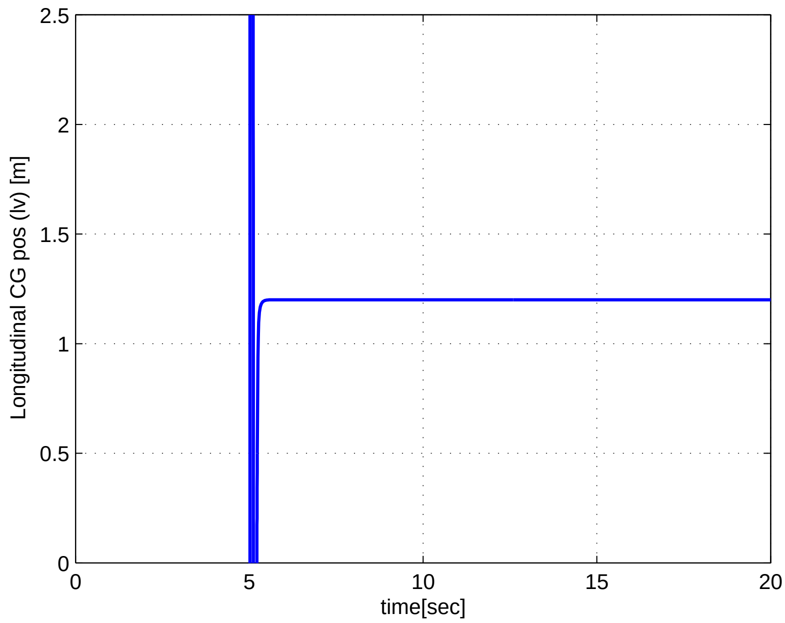

Figure 3.1. Longitudinal CG point (l_v) estimation using RLS algorithm

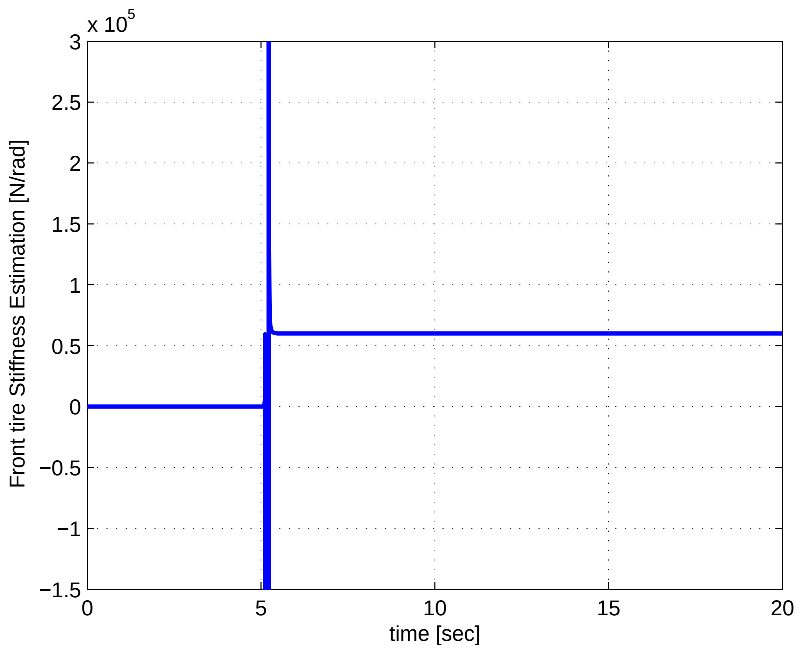

Figure 3.2. Front tire stiffness (C_v) estimation using RLS algorithm

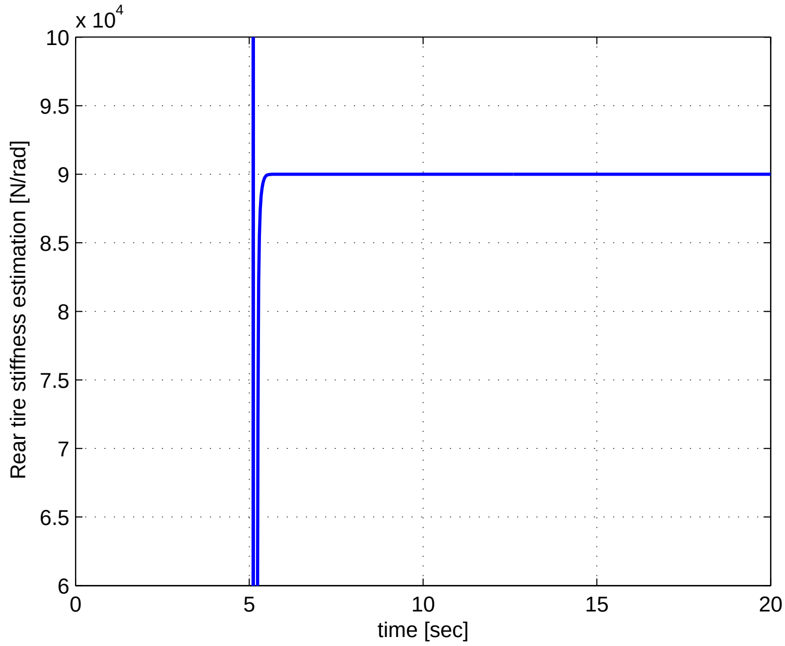

Figure 3.3. Rear tire stiffness (C_h) estimation using RLS algorithm

By letting:

$$\theta_1 = \frac{C_h l_h - C_v l_v}{J_{zz}}$$

$$J_{xx}\ddot{\phi} + c\dot{\phi} + k\phi = h(-(C_v + C_h)\beta + \theta_1 J_{zz}\frac{\dot{\psi}}{v} + mg\phi + C_v\delta) \tag{3.27}$$

and defining $\bar{\delta}$ as [2]:

$$\bar{\delta} = (-(C_v + C_h)\beta + \theta_1 J_{zz}\frac{\dot{\psi}}{v} + mg\phi + C_v\delta)/J_{xx}$$

(3.27) can be expressed as:

$$J_{xx}\ddot{\phi} + c\dot{\phi} + k\phi = J_{xx}h\bar{\delta} \tag{3.28}$$

It is observed that (3.28) includes the estimated lateral dynamic parameters which are to be fed to the roll dynamic parameter estimation mechanism.

Note that the term $\ddot{\phi}$ in (3.28) is not available. Therefore we adopt the same approach as we did for lateral dynamic parameters.

To that end we take the Laplace transform of (3.28):

$$J_{xx}s\dot{\Phi}(s) + c\dot{\Phi}(s) + k\Phi(s) = J_{xx}h\bar{\Delta}(s) \tag{3.29}$$

$$\dot{\Phi}(s)\left(s + \frac{c}{J_{xx}}\right) = -\frac{k}{J_{xx}}\Phi(s) + h\bar{\Delta}(s) \tag{3.30}$$

Dividing both sides by $s - \lambda_2$ where $\lambda_2 < 0$

$$\dot{\Phi}(s)\frac{s - \lambda_2 + \lambda_2 + c/J_{xx}}{s - \lambda_2} = -\frac{k/L_{xx}}{s - \lambda_2}\Phi(s) + \frac{h\bar{\Delta}(s)}{s - \lambda_2} \tag{3.31}$$

$$\dot{\Phi}(s) = \frac{-c/J_{xx} - \lambda_2}{s - \lambda_2}\dot{\Phi}(s) - \frac{k/L_{xx}}{s - \lambda_2}\Phi(s) + \frac{h\bar{\Delta}(s)}{s - \lambda_2} \tag{3.32}$$

Defining filtered signals as:

$$\Omega_{v1}(s) = \frac{\dot{\Phi}(s)}{s - \lambda_2}, \Omega_{v2}(s) = \frac{\Phi(s)}{s - \lambda_2}, \Omega_{v3}(s) = \frac{\bar{\Delta}(s)}{s - \lambda_2}$$

and the coefficients:

$$\xi_1 = -c/J_{xx} - \lambda_2, \xi_2 = -k/L_{xx}, \xi_3 = h$$

The equation (3.32) will be:

$$\dot{\Phi}(s) = \Omega_{v1}(s)\xi_1 + \Omega_{v2}(s)\xi_2 + \Omega_{v3}(s)\xi_3 \tag{3.33}$$

By taking the inverse Laplace and writing in the matrix form, we obtain [2]:

$$\dot{\phi} = \begin{bmatrix} \omega_{v1} & \omega_{v2} & \omega_{v3} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} \tag{3.34}$$

where;

$$\dot{\omega}_{v1} = \lambda_2 \omega_{v1} + \dot{\phi}$$

$$\dot{\omega}_{v2} = \lambda_2 \omega_{v2} + \phi$$

$$\dot{\omega}_{v3} = \lambda_2 \omega_{v3} + \bar{\delta}$$

As seen in (3.34) the roll rate can be expressed as a linear combination of the filtered signals. Using this data we can try to estimate the constant coefficients ξ_1, ξ_2, ξ_3. As new data arrives we will have the below overdetermined system:

$$\begin{bmatrix} \dot{\phi}(0) \\ \dot{\phi}(1) \\ \vdots \\ \dot{\phi}(n) \end{bmatrix} = \begin{bmatrix} \omega_{v1}(0) & \omega_{v2}(0) & \omega_{v3}(0) \\ \omega_{v1}(1) & \omega_{v2}(1) & \omega_{v3}(1) \\ \vdots & \vdots & \vdots \\ \omega_{v1}(n) & \omega_{v2}(n) & \omega_{v3}(n) \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} \tag{3.35}$$

After estimating the constant coefficients ξ_1, ξ_2, ξ_3, the roll dynamic parameters can be easily calculated:

$$c = (-\lambda_2 - \xi_1)J_{xx}, \quad k = -\xi_2 J_{xx}, \quad h = \xi_3$$

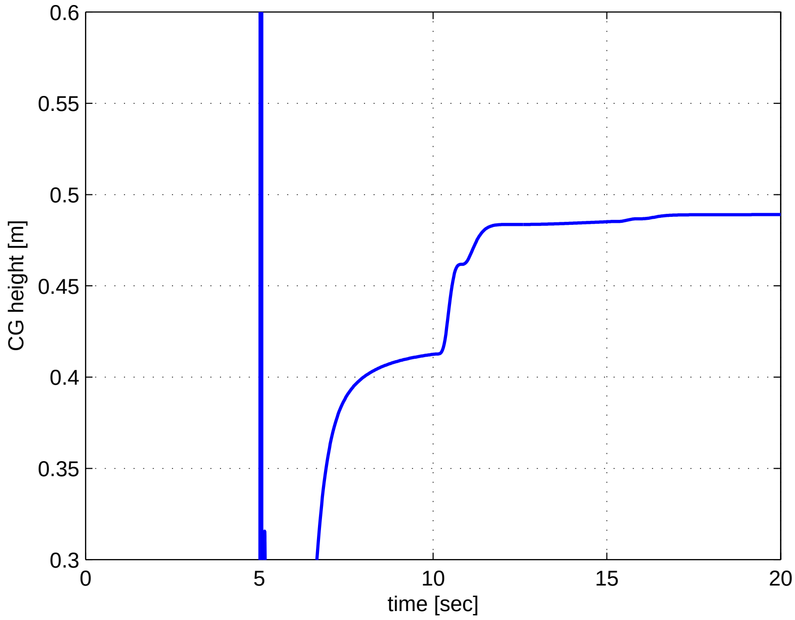

Figure 3.4. CG height estimation

3.3.1. Numerical analysis

In this section the estimated lateral vehicle parameters, the defined filtered signals and the roll rate are fed to the roll dynamics parameter estimator. The initial conditions are assumed as we did for lateral dynamics parameter estimator, i.e.,

$$\xi(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$P(0) = 10^9 I$$

In Figures 3.4 - 3.6 the roll dynamics parameter estimation results are shown. Note that the parameters converge to the values indicated in Table 2.1.

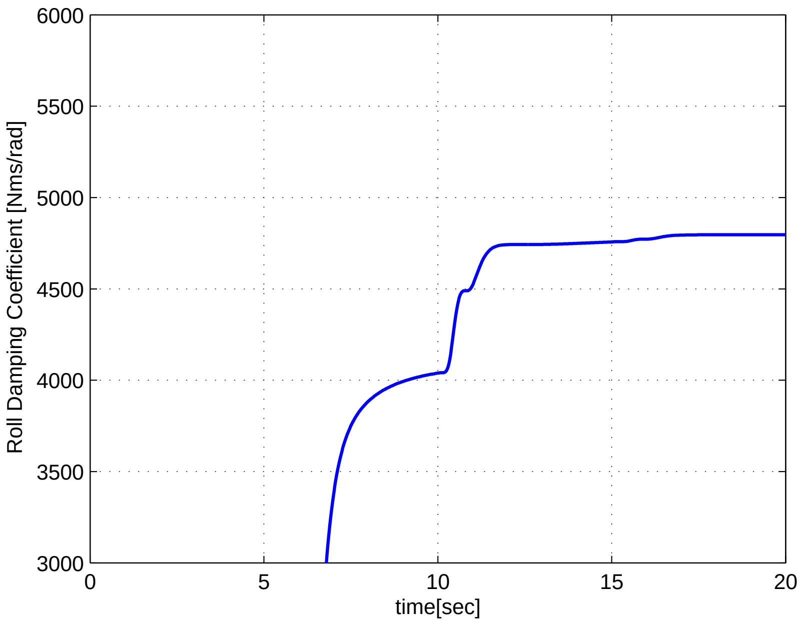

Figure 3.5. Roll damping coefficient(c)

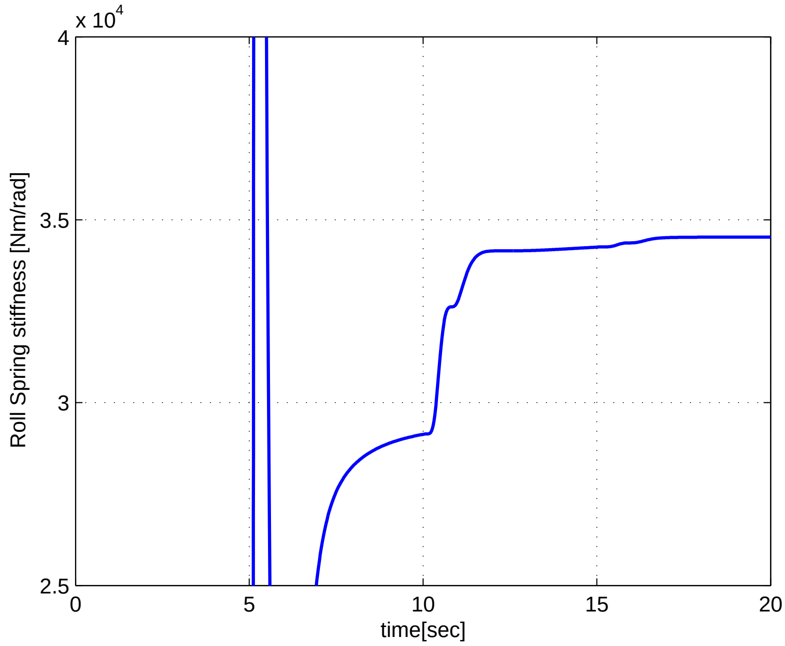

Figure 3.6. Roll spring stiffness (k)

3.4. Vehicle Parameter Identification Using Kalman Filter

In previous sections, we have used the RLS algorithm to estimate the parameters when the vehicle speed is constant. The measured signals yawrate and rollrate are linear combinations of some known filtered signals. The RLS algorithm uses these measured signals at each instant and estimates the constant coefficients of the filtered signals. It is very important that these coefficients are assumed to be constant over time. It can be easily seen that if the speed of the vehicle is constant, then the coefficients of the filtered signals (θ_1, θ_2, θ_3, ξ_1, ξ_2, ξ_3) are all constants.

What happens if the coefficients change over time? The RLS algorithm will definitely fail. As it is known the least squares algorithm is the solution for the optimization problem which minimizes the square of the the magnitiude of the residual in (3.4). In this problem there is no constraints for vector x.

If x is time-varying, this means x is different for each instant. In other words a different x is multiplied by a row of matrix C and this corresponds to the related index of vector y in (3.3). Our optimization problem has now a constraint. We try to minimize the square of magnitude of residual vector with subject to an equation governing the change of vector x over time.

3.4.1. Kalman vs Gauss

The Least Squares Algorithm was developed by Gauss to estimate the characteristics of linear systems based on the measured data [17]. For example we have a linear system in the form:

$$y = cx \tag{3.36}$$

where y is a scalar, $x \in \mathbf{R^{mx1}}$, $c \in \mathbf{R^{1xm}}$.

We try to estimate the vector x. We have c_k and the measurement value y_k at time instant k. The measurement may not be exact and we assume that there is white noise. So over time we will have the following set of equations:

$$y_k = c_k x + \omega_k \tag{3.37}$$

where k=0,1,2,...,n, $n >> m$, and ω is Gaussian white noise with zero mean.

When this set of equations are combined in a matrix form, we obtain an overdetermined system. The Linear Least Squares method then estimates the best x as described in Section 3.1. What happens if vector x changes over time? Let us consider a discrete linear time varying system [18]:

$$x_k = A_{k-1}x_{k-1} + B_{k-1}u_{k-1} + w_{k-1}$$

$$y_k = C_k x_k + v_k$$

Suppose we have the output data y_k, the input signal u_k and the coefficient matrix and vectors A_k, B_k, C_k at each instant. Moreover the measurements are not assumed to be accurate and we have the white noises w_k and v_k with zero mean, i.e.,

$$p(w) \sim N(0, Q)$$

$$p(v) \sim N(0, R)$$

where Q and R are noise covariance matrices. Based on this data Kalman filter estimates the state vector.

Table 3.1. Kalman filter algorithm [18]

Time Update ("Predict")	Measurement Update ("Correct")
(1) Project the state ahead	(1) Compute the Kalman Gain
$\hat{x}_k^- = A_k\hat{x}_{k-1} + B_k u_{k-1}$	$K_k = P_k^- C_k^T (C_k P_k^- C_k^T + R)^{-1}$
(2) Project the error covariance ahead	(2) Update estimate with measurement y_k
$P_k^- = A_k P_{k-1} A_k^T + Q$	$\hat{x}_k = \hat{x}_k^- + K_k(y_k - C_k\hat{x}_k)$
(3) Go to Measurement Update equations	(3) Update the error covariance
	$P_k = (I - K_k C_k)P_k^-$
	(4) Go to Time Update equations

3.4.2. Kalman filter algorithm

The Kalman Filter estimates state vector x_k in a recursive manner. It firstly predicts the state vector and according to the measurement data, it corrects the state vector. In the next instant, this corrected state vector is used to predict the new state and again according to the measured data, the state vector is corrected. This goes on with time and as new measurements are taken. The Kalman Filter equations are given in Table 3.1

Please note the superminus in Table 3.1. $\hat{x}_k^-$ is the priori estimate of state vector prior to instant k whereas $\hat{x}_k$ is posteriori state estimate after the measurement y_k is taken at instant k. To start the algorithm we should first determine our initial estimates $\hat{x}_0$ and P_0.

3.4.3. Using Kalman filter when the speed is not constant

Suppose that differential braking is applied to the vehicle and the speed of the vehicle is not constant. In this case (3.7) should be reestablished since the braking or accelerating force u is not zero,i.e.,

$$\ddot{\psi} = \frac{C_h l_h - C_v l_v}{J_{zz}}\beta - \frac{C_v l_v^2 + C_h l_h^2}{J_{zz} v}\dot{\psi} + \frac{C_v l_v}{J_{zz}}\delta - \frac{T}{2J_{zz}}u \tag{3.38}$$

Again we take the Laplace transform and introduce filtered signals as we did before, therefore (3.14) becomes (here we are leaving $\frac{C_v l_v^2 + C_h l_h^2}{J_{zz} v}$ as the coefficient of $\dot{\psi}$ and do not take its Laplace transform):

$$s\dot{\Psi}(s) + \frac{T}{2J_{zz}}U(s) = \frac{C_h l_h - C_v l_v}{J_{zz}}\beta(s) - \frac{C_v l_v^2 + C_h l_h^2}{J_{zz} v}\dot{\Psi}(s) + \frac{C_v l_v}{J_{zz}}\Delta(s) \tag{3.39}$$

In order to simplify the equation use c_1, c_2, c_3 for the coefficients;

$$s\dot{\Psi}(s) + \frac{T}{2J_{zz}}U(s) = c_1\beta(s) + c_2\dot{\Psi}(s) + c_3\Delta(s) \tag{3.40}$$

$$(s - c_2)\dot{\Psi}(s) + \frac{T}{2J_{zz}}U(s) = c_1\beta(s) + c_3\Delta(s) \tag{3.41}$$

We divide both sides with $(s - \lambda)$ where $\lambda < 0$;

$$\frac{s - c_2}{s - \lambda}\dot{\Psi}(s) + \frac{T}{2J_{zz}}\frac{U(s)}{s - \lambda} = \frac{c_1}{s - \lambda}\beta(s) + \frac{c_3}{s - \lambda}\Delta(s) \tag{3.42}$$

$$\frac{s - \lambda + \lambda - c_2}{s - \lambda}\dot{\Psi}(s) + \frac{T}{2J_{zz}}\frac{U(s)}{s - \lambda} = \frac{c_1}{s - \lambda}\beta(s) + \frac{c_3}{s - \lambda}\Delta(s) \tag{3.43}$$

$$\dot{\Psi}(s) + \frac{T}{2J_{zz}}\frac{U(s)}{s-\lambda} = \frac{c_1}{s-\lambda}\beta(s) + \frac{c_2-\lambda}{s-\lambda}\dot{\Psi}(s) + \frac{c_3}{s-\lambda}\Delta(s) \tag{3.44}$$

$$\dot{\Psi}(s) + \frac{T}{2J_{zz}}\frac{U(s)}{s-\lambda} = c_1\Omega_{l1}(s) + (c_2-\lambda)\Omega_{l2}(s) + c_3\Omega_{l3}(s) \tag{3.45}$$

where

$$\Omega_{l1}(s) = \frac{\beta(s)}{s-\lambda}, \quad \Omega_{l2}(s) = \frac{\Psi(s)}{s-\lambda}, \quad \Omega_{l3}(s) = \frac{\Delta(s)}{s-\lambda}$$

and

$$c_1 = \frac{C_h l_h - C_v l_v}{J_{zz}}, \quad c_2 = \frac{C_v l_v^2 + C_h l_h^2}{J_{zz}\upsilon}, \quad c_3 = \frac{C_v l_v}{J_{zz}}$$

For simplicity we define $\bar{\dot{\Psi}}(s)$ and rewrite the equation:

$$\bar{\dot{\Psi}}(s) = c_1\Omega_{l1}(s) + (c_2-\lambda)\Omega_{l2}(s) + c_3\Omega_{l3}(s) \tag{3.46}$$

$$\bar{\dot{\Psi}}(s) = \theta_1\Omega_{l1}(s) + \theta_2\Omega_{l2}(s) + \theta_3\Omega_{l3}(s) \tag{3.47}$$

By taking inverse Laplace transform and writing it in the matrix form:

$$\bar{\dot{\psi}} = \begin{bmatrix} \omega_{l1} & \omega_{l2} & \omega_{l3} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} \tag{3.48}$$

As time goes we will have new measurements for $\bar{\dot{\psi}}$ and filtered signals. If the speed is not constant during the estimation process we can not use the least squares algorithm. If the speed changes over time, the parameter θ_2 which is the coefficient of the filtered signal ω_{l2} is no more the same for all instants, i.e.,

$$\theta_2 = -\frac{C_v l_v^2 + C_h l_h^2}{J_{zz} \upsilon} - \lambda$$

If we take the θ vector as our state vector then Kalman filter can be used. But before that the state equation should be defined first.

$$\theta_2(k-1) = -\frac{C_v l_v^2 + C_h l_h^2}{J_{zz} \upsilon(k-1)} - \lambda \tag{3.49}$$

$$\theta_2(k) = -\frac{C_v l_v^2 + C_h l_h^2}{J_{zz} \upsilon(k)} - \lambda \tag{3.50}$$

If we multiply (3.49) by $\upsilon(k-1)/\upsilon(k)$:

$$\frac{\upsilon(k-1)}{\upsilon(k)}\theta_2(k-1) = -\frac{C_v l_v^2 + C_h l_h^2}{J_{zz} \upsilon(k)} - \frac{\upsilon(k-1)}{\upsilon(k)}\lambda \tag{3.51}$$

$$\frac{\upsilon(k-1)}{\upsilon(k)}\theta_2(k-1) + \frac{\upsilon(k-1)}{\upsilon(k)}\lambda = -\frac{C_v l_v^2 + C_h l_h^2}{J_{zz} \upsilon(k)} \tag{3.52}$$

Putting this in (3.50):

$$\theta_2(k) = \frac{\upsilon(k-1)}{\upsilon(k)}\theta_2(k-1) + \frac{\upsilon(k-1)}{\upsilon(k)}\lambda - \lambda \tag{3.53}$$

rewriting;

$$\theta_2(k) = \frac{\upsilon(k-1)}{\upsilon(k)}\theta_2(k-1) + \frac{\upsilon(k-1) - \upsilon(k)}{\upsilon(k)}\lambda \tag{3.54}$$

θ_1 and θ_3 are time invariant. If we consider θ vector as the state vactor, $\bar{\dot{\psi}}$ as the output and λ as the constant input of a linear discrete time varying system, then we can write the state space equation for this system as

$$\begin{bmatrix} \theta_1(k) \\ \theta_2(k) \\ \theta_3(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{v(k-1)}{v(k)} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_1(k-1) \\ \theta_2(k-1) \\ \theta_3(k-1) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{v(k-1)-v(k)}{v(k)} \\ 0 \end{bmatrix} \lambda \tag{3.55}$$

$$\bar{\dot{\psi}}(k) = \begin{bmatrix} \omega_{l1}(k) & \omega_{l2}(k) & \omega_{l3}(k) \end{bmatrix} \begin{bmatrix} \theta_1(k) \\ \theta_2(k) \\ \theta_3(k) \end{bmatrix} \tag{3.56}$$

In order to estimate the state vector in the above linear discrete time linear varying system, Kalman filter can be used. However there is a difference with the equation which is used to describe the Kalman filter algotithm. The above equation is deterministic (noise free) wheras Kalman filter estimates the state of the linear systems with (Gaussian) noisy measurements.

Kalman Filter can also be used for deterministic system with some modifications in the algorithm as shown in Table 3.2 [19], [20], [21].

3.4.4. Numerical evaluation of Kalman filter in vehicle parameter identification

We assume that a constant braking force is applied during the estimation process as shown in Figure 3.7 which leads to speed change in Figure 3.8. The steering input is the same as in Figure 2.4. It is observed that Kalman filter successfully estimates the lateral dynamic parameters although speed is not constant. Considerably fast convergence rate can be observed in figures 3.9 - 3.11. The lateral dynamics parameter estimation results are fed to the roll dynamics parameter estimator in real time. The Ξ vector in the roll dynamic parameter equation does not change over time. So Kalman Filter is not needed in this case. RLS algorithm can be used for roll dynamic parameter estimation.

Table 3.2. Kalman filter algorithm for noise free system

Time Update ("Predict")	Measurement Update ("Correct")
(1) Project the state ahead $\hat{x}_k^- = A_k \hat{x}_{k-1} + B_k u_{k-1}$ (2) Project the error covariance ahead $P_k^- = A_k P_{k-1} A_k^T$ (3) Go to Measurement Update equations	IF $C_k P_k^- C_k^T \neq 0$ (1) Compute the Kalman Gain $K_k = P_k^- C_k^T (C_k P_k^- C_k^T)^{-1}$ (2) Update estimate with measurement y_k $\hat{x}_k = \hat{x}_k^- + K_k(y_k - C_k \hat{x}_k)$ (3) Update the error covariance $P_k = (I - K_k C_k) P_k^-$ (4) Go to Time Update equations
	IF $C_k P_k^- C_k^T = 0$ (1) $\hat{x}_k = \hat{x}_k^-$ (2) $P_k = P_k^-$ (3) Go to Time Update equations

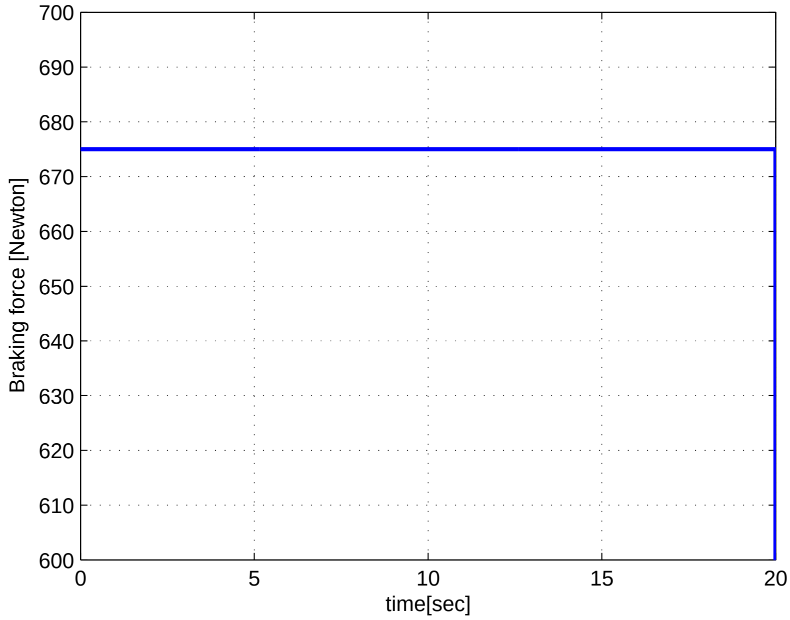

Figure 3.7. Constant differential braking force just applied to change the speed of the vehicle

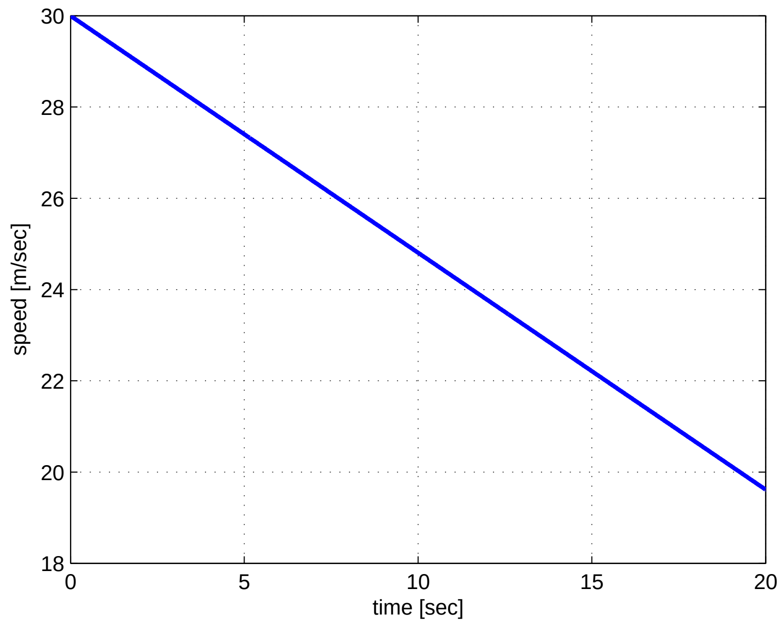

Figure 3.8. Speed of vehicle while constant diffential braking force is applied

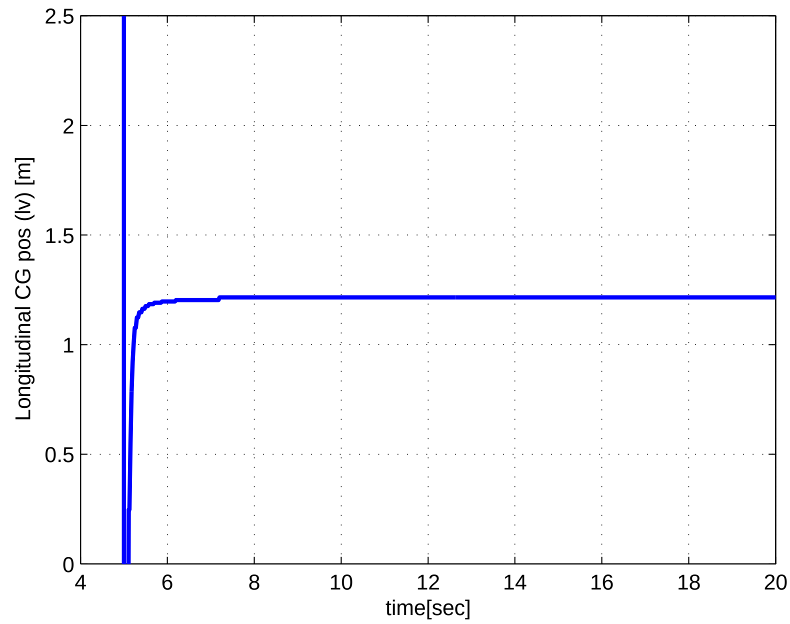

Figure 3.9. l_v estimation with Kalman filter while constant diffential braking force is being applied

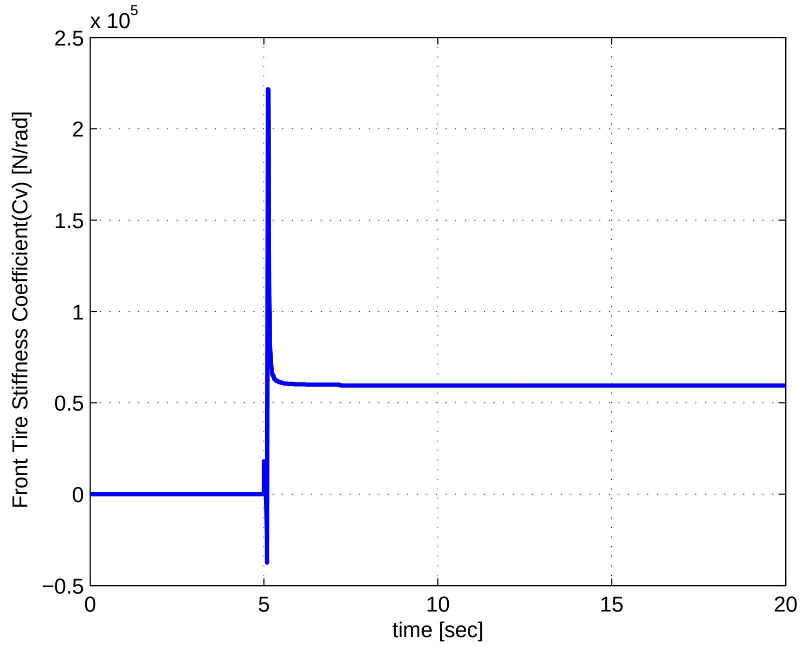

Figure 3.10. C_v estimation with Kalman filter while constant diffential braking force is being applied

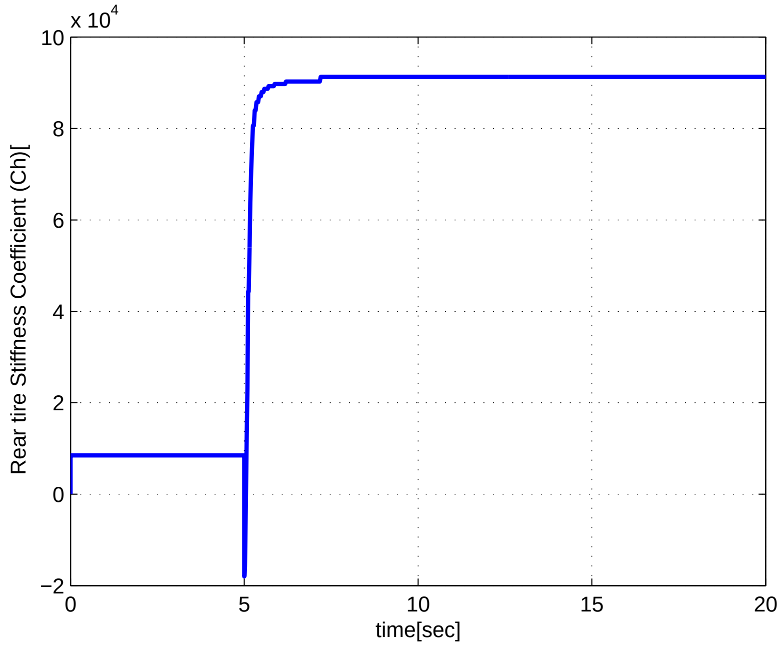

Figure 3.11. C_h estimation with Kalman filter while constant diffential braking force is being applied

However the initial results of the lateral dynamic parameter estimator may not be very accurate. When these initial invalid results are fed to the roll dynamic estimator, they cause the P vector in the RLS algorithm to become smaller. When the parameters estimated by Kalman filter converges to accurate values, these values will be fed to the roll dynamic estimator and this estimator will start to converg to accurate values. However, if the P matrix becomes very small prior to the accurate data are received from Kalman filter, the convergence speed will be very small. This means the roll dynamic parameter estimator will be too slow to estimate the parameters.

To avoid this problem, in other words; to prevent the P matrix in RLS algorithm to become very small before the accurate values are received from Kalman Filter, the roll dynamic parameter estimator is not started at the same time with the lateral dynamic parameter estimator.

The roll dynamic paremeter estimator is turned on when the lateral dynamic parameter estimator converges. This guarantees the roll dynamic parameter estimator to receive accurate values as its input. The P matrix will not be lowered using invalid

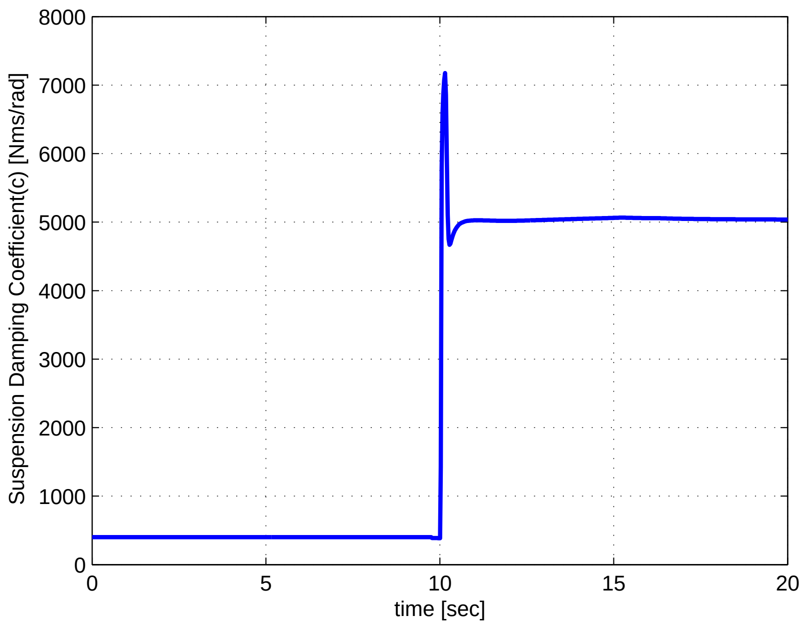

Figure 3.12. c estimation while constant diffential braking force is being applied

values. As the result, the roll dynamic parameter estimator successfully utilizes RLS algorithm and fastly converges to estimated values as shown in Figures 3.12 - 3.14.

3.5. Using the Lateral Acceleration a_y in Estimation of Lateral Vehicle Parameters

In this section we will use the kinematic equation of lateral acceleration in order to estimate the lateral vehicle parameters. We have looked for possible alternatives to estimate lateral vehicle parameters since the performance of the current approach is negatively affected by the increase in the amount of differential braking force. And also λ used in generating filtered signals affects the performance of the estimator. Therefore in order to eliminate the need for filtered signals, we have decided to use the lateral acceleration equation in order to estimate lateral vehicle parameters.

3.5.1. Estimation when all measurements are noise free

As mentioned before, the lateral acceleration is a measurable quantity. Its kinematic equation also includes the parameters that are to be estimated. Therefore using

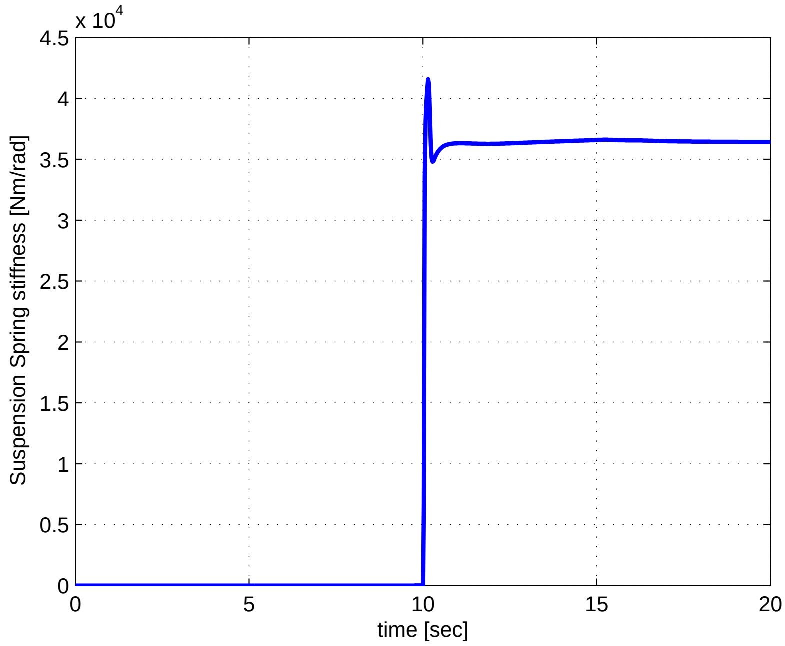

Figure 3.13. k estimation while constant diffential braking force is being applied

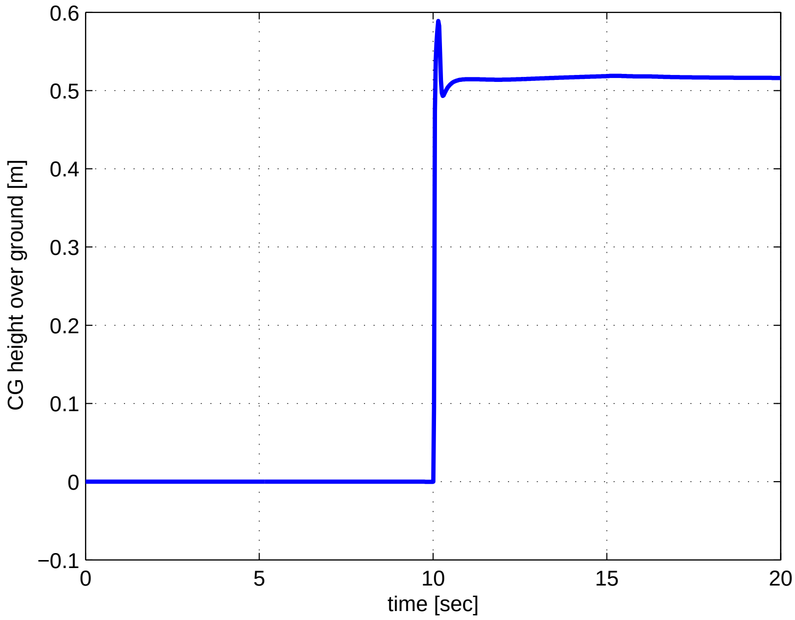

Figure 3.14. h estimation while constant diffential braking force is being applied

the measured lateral acceleration and its kinematic equation, it is possible to estimate the lateral dynamics parameters using the Kalman filter algorithm.

Let us consider the lateral acceleration equation again:

$$a_y = \dot{\upsilon}\beta + \upsilon\dot{\beta} + \upsilon\dot{\psi} \tag{3.57}$$

$$a_y = \dot{\upsilon}\beta + \upsilon(\dot{\beta} + \dot{\psi}) \tag{3.58}$$

$\dot{\beta}$ can be written by using (2.2)

$$\dot{\beta} = -\frac{\sigma J_{x_{eq}}}{mJ_{xx}\upsilon}\beta + (\frac{\rho J_{x_{eq}}}{mJ_{xx}\upsilon^2} - 1)\dot{\psi} - \frac{hc}{J_{xx}\upsilon}\dot{\phi} + \frac{h(mgh-k)}{J_{xx}\upsilon}\phi + \frac{C_\upsilon J_{x_{eq}}}{mJ_{xx}\upsilon}\delta \tag{3.59}$$

Equation (3.58) can be rewritten as:

$$a_y - \dot{\upsilon}\beta = -\frac{\sigma J_{x_{eq}}}{mJ_{xx}}\beta + \frac{\rho J_{x_{eq}}}{mJ_{xx}\upsilon}\dot{\psi} - \frac{hc}{J_{xx}}\dot{\phi} + \frac{h(mgh-k)}{J_{xx}}\phi + \frac{C_\upsilon J_{x_{eq}}}{mJ_{xx}}\delta \tag{3.60}$$

If we assume that signals a_y, β, $\dot{\upsilon}$, υ, $\dot{\psi}$, $\dot{\phi}$, ϕ, δ are available at each instant then the coefficients can be estimated using Kalman filtering. After coefficients are estimated then the lateral dynamic parameters will be available.

Before using the Kalman filtering for this purpose, firstly the state and measurement equations should be defined.

For simplicity we define θ_1, θ_2, θ_3, θ_4, θ_5 and $\bar{a}_y$

$$\bar{a}_y = \theta_1\beta + \theta_2\dot{\psi} + \theta_3\dot{\phi} + \theta_4\phi + \theta_5\delta \tag{3.61}$$

It is observed that θ_2 will not be same for all instants if the speed changes over time. Hence, the RLS algorithm can not be used. However Kalman filtering can be used if the state vector is defined as:

$$x = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix}$$

The state and measurement equations will be as follows:

$$\begin{bmatrix} \theta_1(k) \\ \theta_2(k) \\ \theta_3(k) \\ \theta_4(k) \\ \theta_5(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{v(k-1)}{v(k)} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_1(k-1) \\ \theta_2(k-1) \\ \theta_3(k-1) \\ \theta_4(k-1) \\ \theta_5(k-1) \end{bmatrix} \tag{3.62}$$

$$\bar{a}_y(k) = \begin{bmatrix} \beta(k) & \dot{\psi}(k) & \dot{\phi}(k) & \phi(k) & \delta(k) \end{bmatrix} \begin{bmatrix} \theta_1(k) \\ \theta_2(k) \\ \theta_3(k) \\ \theta_4(k) \\ \theta_5(k) \end{bmatrix} \tag{3.63}$$

Since the state and measurement equations of the noise free system has been defined, the Kalman filter algorithm described in Table 3.2 can be used to estimate the state vector. After the state vector has been estimated, using θ_1, θ_2 and θ_5 the lateral dynamics parameters are calculated as follows:

$$C_v = \frac{mJ_{xx}\theta_5}{J_{x_{eq}}} \tag{3.64}$$

$$C_h = -\frac{mJ_{xx}\theta_1}{J_{x_{eq}}} - C_v \tag{3.65}$$

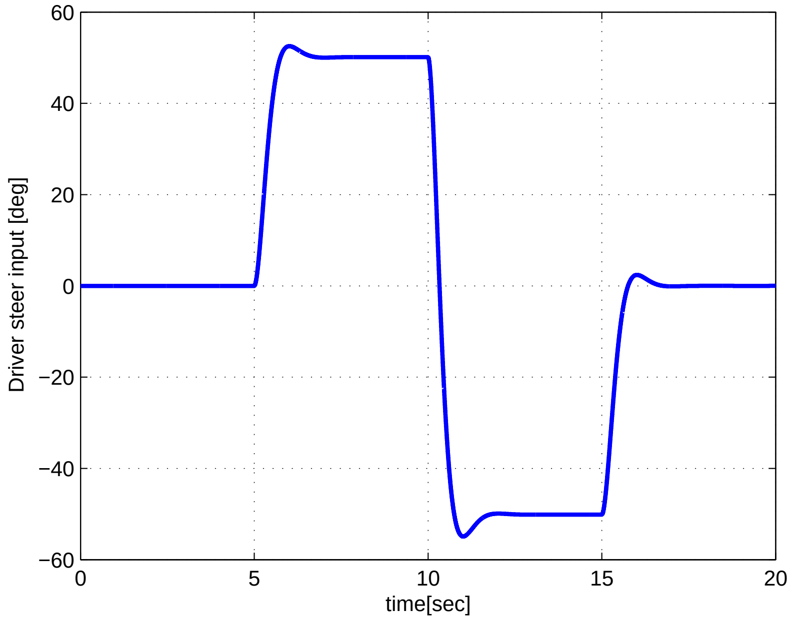

Figure 3.15. Driver steer input for parameter estimation using a_y

$$l_v = \frac{\frac{mJ_{xx}v\theta_2}{J_{xeq}} - 2.5C_h}{-C_h - C_v} \tag{3.66}$$

These estimated parameters are fed to the roll dynamics parameter estimator described in the previous sections.

3.5.1.1. Numerical analysis. This time we simulate a scenario without a rollover accident. Assume that the steering input shown in Figure 3.15 is applied by the driver. The speed of the vehicle changes as shown in Figure 3.17 as the result of applied differential braking and accelerating forces. The LTR value is considerably low as shown in Figure 3.16. As shown in Figures 3.19 - 3.21 the estimation results are very accurate and fast. Since these resuts are fed to the roll dynamics parameter estimator, roll dynamic parameter estimation performance is positively affected.

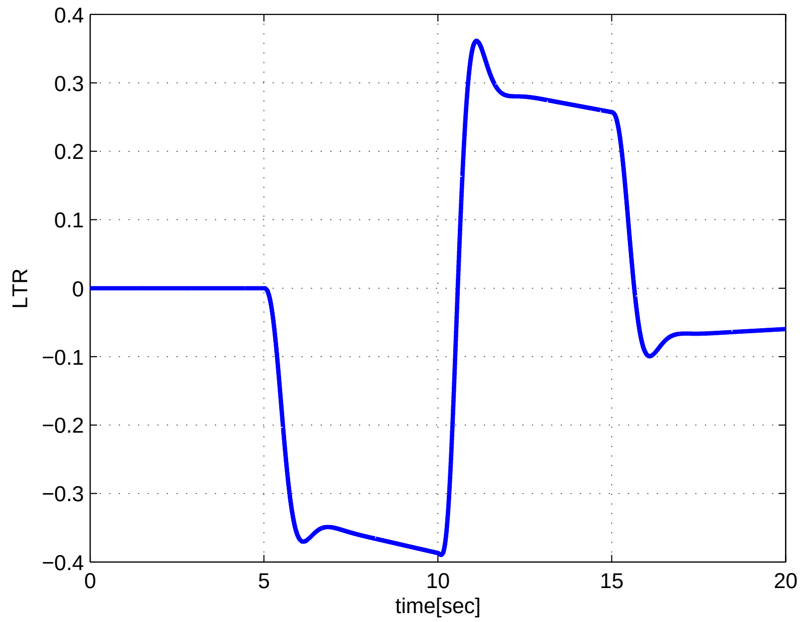

Figure 3.16. LTR value for parameter estimation using a_y

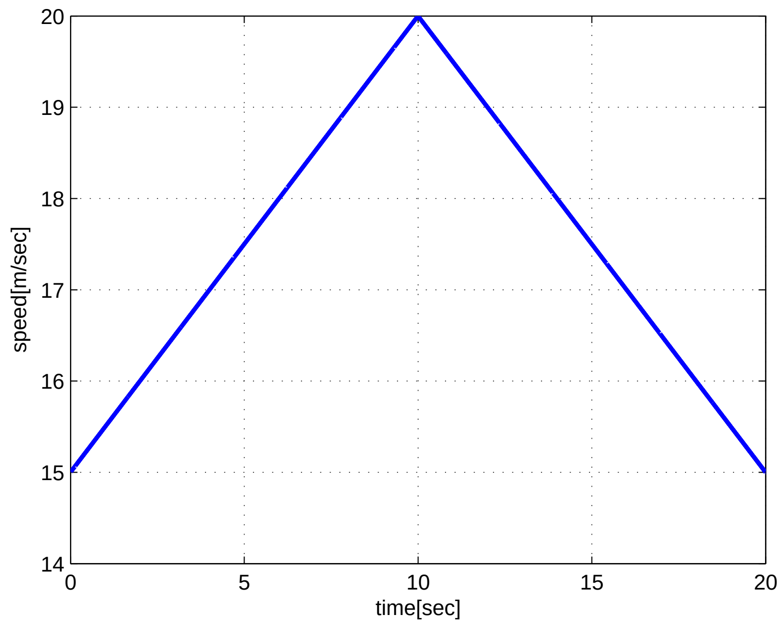

Figure 3.17. The speed of the vehicle for parameter estimation using a_y

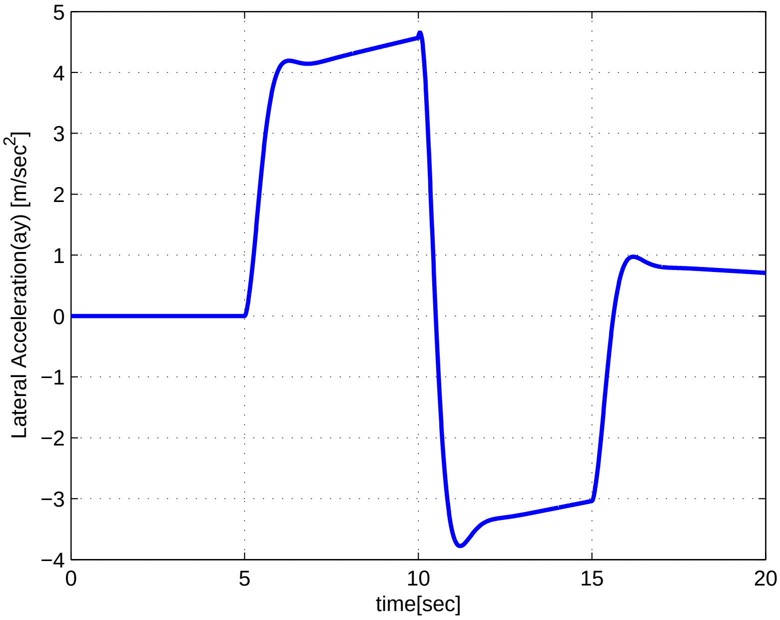

Figure 3.18. The measured lateral acceleration without noise

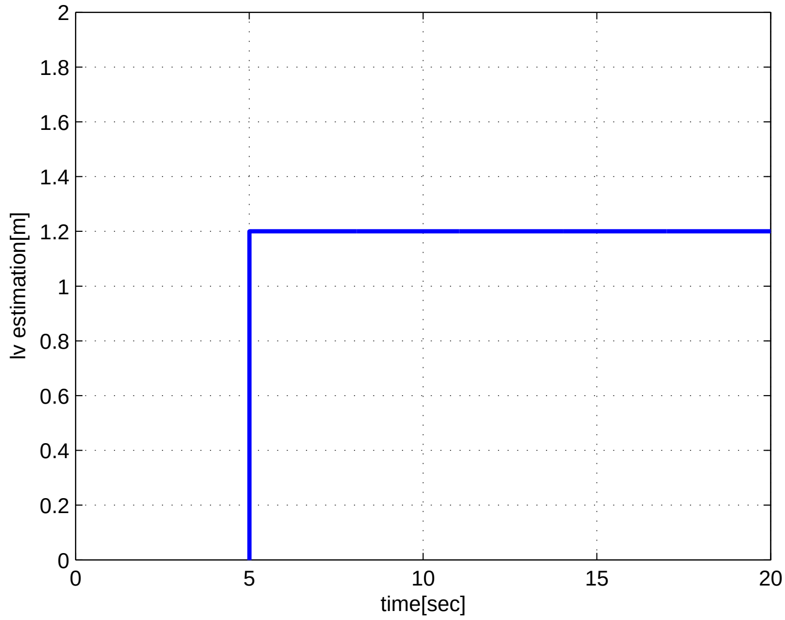

Figure 3.19. l_v estimation result for parameter estimation using a_y

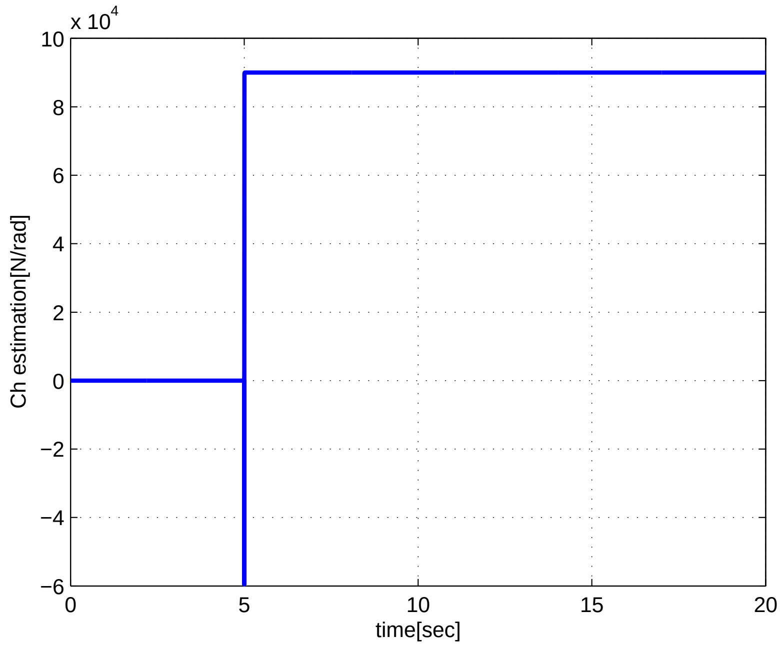

Figure 3.20. C_h estimation result for parameter estimation using a_y

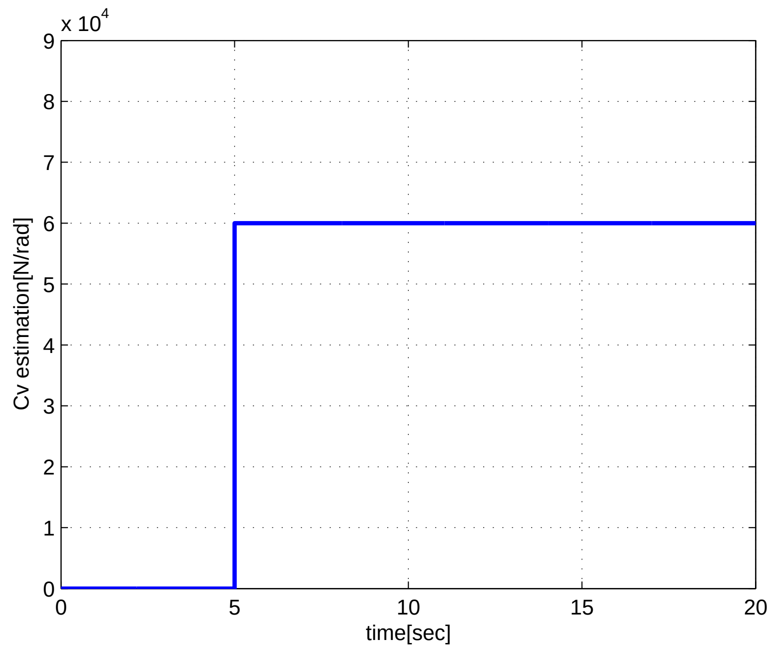

Figure 3.21. C_v estimation result for parameter estimation using a_y

3.5.2. Estimation when the lateral acceleration a_y is measured with Gaussian noise

If only the lateral acceleration a_y in (3.60) is measured with Gaussian noise with zero mean and the rest of the signals are accurate, then the estimation problem will be the state estimation of the below system:

$$x_{k+1} = A_k x_k \tag{3.67}$$

$$y_k = C_k x_k + v_k \tag{3.68}$$

where

$$x_k = \begin{bmatrix} \theta_1(k) \\ \theta_2(k) \\ \theta_3(k) \\ \theta_4(k) \\ \theta_5(k) \end{bmatrix}$$

$$A_k = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{v(k-1)}{v(k)} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C_k = \begin{bmatrix} \beta(k) & \dot{\psi}(k) & \dot{\phi}(k) & \phi(k) & \delta(k) \end{bmatrix}$$

$$y_k = a_y(k) - \dot{v}(k)\beta(k)$$

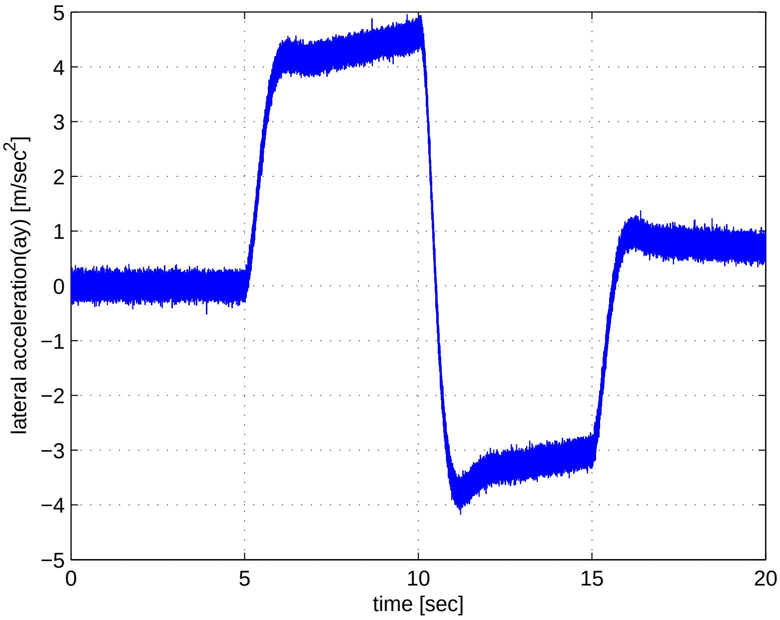

Figure 3.22. Measured lateral acceleration with Gaussian noise

v_k is the Gaussian white noise with zero mean and θ_1, θ_2, θ_3, θ_4, θ_5 are the coefficients as shown in (3.61).

3.5.2.1. Numerical analysis. As shown in (3.67), the state equation does not include noise vector. This is because it is just the state transition equation which we have defined based on the speed change. Therefore Q in Table 3.1 is zero.

We assume that the measured lateral accelaration has a Gaussian noise component with zero mean and variance 0.01. (In order to add the noise component we have used AWGN channel in MATLAB Simulink). Noisy lateral acceleration is shown in Figure 3.22.

As shown in Figures 3.23 - 3.25 the estimation results are very successful. However convergence speed is slower compared to deterministic case. In order to achieve successful estimates, the maximum step size value should be kept small. This increases the number of samples used during the estimation process.

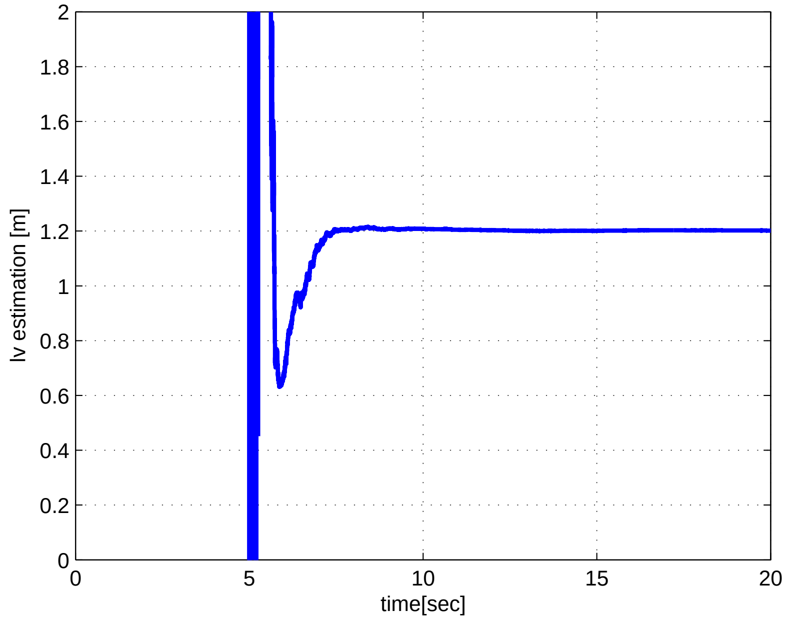

Figure 3.23. Longitudinal CG w.r.t front axle l_v estimation when a_y measurement is noisy

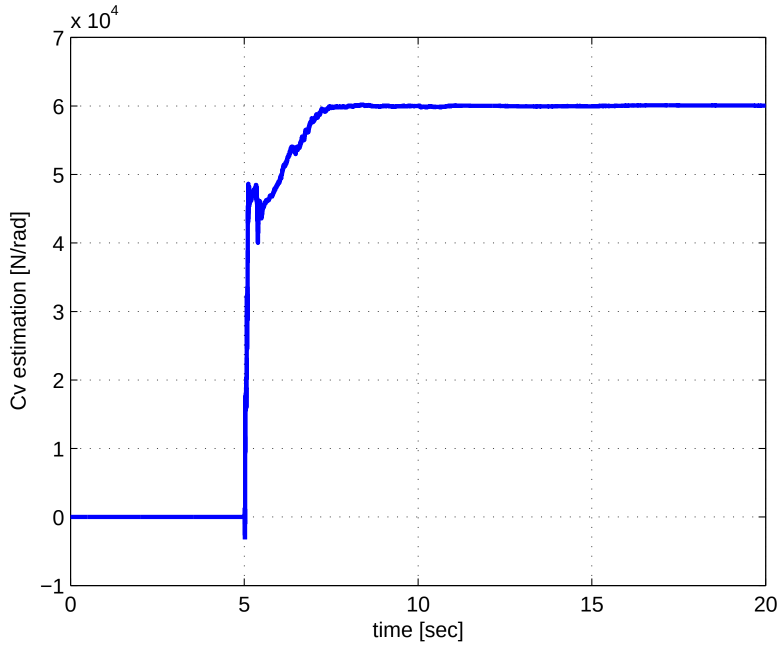

Figure 3.24. Front tire stiffness coefficient C_v estimation when a_y measurement is noisy

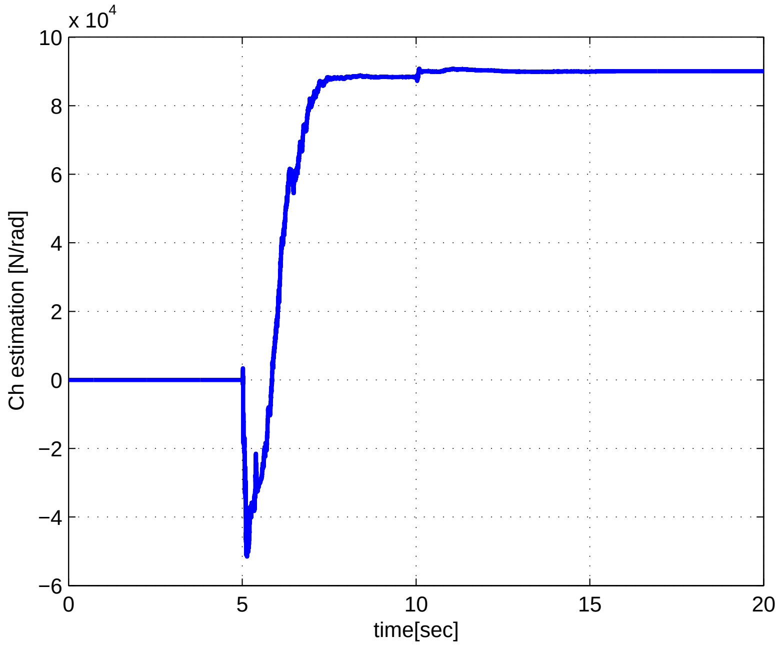

Figure 3.25. Rear tire stiffness coefficient C_h estimation when a_y measurement is noisy

3.5.3. Parameter estimation when all available signals are noisy

In the previous section we have shown that the parameter estimation can be performed successfully even if the measured value of the lateral acceleration is noisy. But what happens if the other signals β, $\dot{\psi}$, $\dot{\phi}$, ϕ, δ are all noisy?

If all signals are noisy, this means the vector C in (3.68) is not accurate. In Kalman filter algorithm this vector should be non-random function of time and should be known exactly. Otherwise the state vector x can not be estimated. This is because the vector C is used in determining the Kalman gain K, in the measurement correct equation and also in updating the error covariance P as shown in Table 3.1

This means that the proposed estimation method fails if the above signals are not accurate.

3.6. Summary of the Chapter and Concluding Remarks

In this chapter we focussed on estimating the unknown parameters of the mathematical model described in Chapter 2. In the first part it has been observed that the RLS algorithm could be used for parameter estimation when the speed of the vehicle is constant. The algorithm has been introduced. Filtered signals were defined to be able to use the RLS algorithm.

Then we tried the RLS algorithm for the case in which some amount of differential braking force was applied to the vehicle. But it failed as expected since the optimization problem which was solved by the RLS algorithm now had a constraint due to the speed change.

The need for the Kalman Filter for this case was understood well and the Kalman Filter was compared with the Least Squares algorithm. However the system whose state is to be estimated is a noise free system although the Kalman filter is used for systems with (Gaussian) noisy measurements. It is shown that the Kalman Filter algorithm can also be used for deterministic systems with small modifications in the algorithm.

To test the Kalman Filter, constant differential braking force was applied to the vehicle so the speed of the vehicle changed during estimation process. It was observed that the lateral vehicle parameters were fastly estimated. The results of these estimations were fed to the roll vehicle parameter estimator. In the end it was observed that the unknown parameters were converged to the values shown in Table 2.1 even when the speed of the vehicle changes.

In the last part we utilized the lateral acceleration to estimate the lateral vehicle parameters. This was to avoid the need for filtered signals and increase the performance. The estimation results using this method proved to be much better than the results of the estimation based on yaw rate equation. And also we examine the success of the proposed estimation method for the system with noisy measurements. We firstly assumed that the output signal y in equation (3.68) which corresponds to the lateral acceleration a_y is noisy. White Gaussian noise was added to the a_y during simulation. The simulation results showed that the parameter estimation was very successful as expected. Then we assumed that the other signals are noisy as well. But we have seen that this effects the C vector in equation (3.68). This can not be handled by Kalman filter algorithm. In conclusion, the proposed estimation method is very successful for deterministic system and also when the lateral acceleration a_y is Gaussian noisy. However the method can not perform successful estimation if the signals β, $\dot{\psi}$, $\dot{\phi}$, ϕ, δ are measured with noise.

The aim of this chapter has been to investigate the ways to estimate the unknown parameters. These estimation results are still to be used to predict the accident and to determine the amount of differential braking force in order to prevent rollover. This is the main subject of the next chapter.

4. ROLLOVER PREVENTION: A CONTROLLER COUPLED WITH ONLINE PARAMETER ESTIMATOR

In this chapter the parameter estimator and the controller are combined together to prevent the rollover situations. The estimation continues while the differential braking force is applied to the vehicle. Two different estimators will be used. The first one is the proposed estimator in the previous chapter, which uses yaw rate and filtered signals. The second estimator, which is superior to the first one, utilizes lateral acceleration to estimate lateral dynamics parameters. As mentioned in the previous chapter the second estimator does not need to generate filtered signals. However it also uses Kalman filtering as the first one.

In both cases the suspension damping coefficient c and suspension spring stiffness k will be estimated to calculate the LTR_d. As a result, the rollover situation will be predicted and based on the center of gravity height, a proportional controller output will be input to the vehicle as the differential braking force.

4.1. The Rollover Controller

As previously discussed many different actuators can be used as contoller output to prevent rollover situations. In this work, differential braking is used. The total effective differential braking can either be on the right tires or on the left tires according to the side where the vehicle tends to rollover.

To determine the amount of differential braking and its sign, two items are taken into consideration: estimated center of gravity height h and the measured lateral acceleration a_y.

The proportional controller is sufficient for our purpose. The controller output is determined with the below equation [2]:

$$u = Ka_y \tag{4.1}$$

where the controller gain K is a coefficient determined according to a performance factor in order to keep LTR_d value within the range [-1,+1] and causing minimum decrease in the performance of the vehicle as possible. For vehicles with very high center of gravity height, greater braking force might be needed to prevent rollover in dangerous situations. Therefore the coefficient K can be determined according to the center of gravity height.

In this work K is taken as below [2]. These values were calculated for different center of gravity heights and under assumption that the vehicle traveled with a speed of 108km/h and a steer input of 100 degrees was applied by the driver. Under these circumstances, the below proportional controller gains prevented rollover accidents without significant loss in the driving performance. So we have decided to use these gains during simulations after the rollover accident is predicted by checking LTR_d value.

$$K_{h>0.8} = -1550, \quad K_{0.75<h<0.8} = -1350,$$

$$K_{0.7<h<0.75} = -1170, \quad K_{0.65<h<0.7} = -1000,$$

$$K_{0.6<h<0.65} = -850, \quad K_{0.55<h<0.6} = -700,$$

$$K_{0.5<h<0.55} = -580, \quad K_{h<0.5} = -480,$$

The controller becomes active when LTR_d passes a certain threshold value which is 0.6. This means that, in non-critical situations, the vehicle will not be slowed down by the controller therefore the vehicle performance will not be reduced.

4.2. The Controller Coupled with a Parameter Estimator Based on Yawrate

In this section, the estimator proposed in Chapter 3 which utilizes yaw rate equation and uses Kalman filter algorithm is coupled with the controller described above. Simulation tests show that when the braking force is considerably high (more than 1/10 of weight of the vehicle) estimator is not very accurate. And it is known that to prevent emergency situations much greater differential braking force is needed.

To increase the performance of the estimator some adjustments have been tried. The chosen value λ in (3.42) is -1. This value is decreased to -10000 which increases the performance significantly. In other words the accuracy of the estimation is significantly improved. The drawback is that the speed of the MATLAB simulation becomes very slow.

The simulation results are shown in Figures 4.1 - 4.14 below. Suppose that the steering input shown in Figure 2.4 is applied by the driver.

Please note that the controller starts applying differential braking to the system when the LTR value becomes greater than 0.6 as shown in Figures 4.2 and 4.3. It is also important to note that the controller does not use the real LTR value. It uses the LTR_d value which is calculated by using estimated parameters. Since the parameters are estimated accurately in a timely manner, the real LTR value and the calculated LTR_d value are almost the same. Therefore the controller can act in a timely manner to prevent the rollover situation. The absolute value of LTR is always smaller than 1 which means there is no rollover accident. The vehicle slows down as shown in Figure 4.4 due to the applied braking force.

The effect of differential braking on the vehicle dynamics can be observed in Figures 4.5 - 4.8. The effect can be better understood if we compare these figures

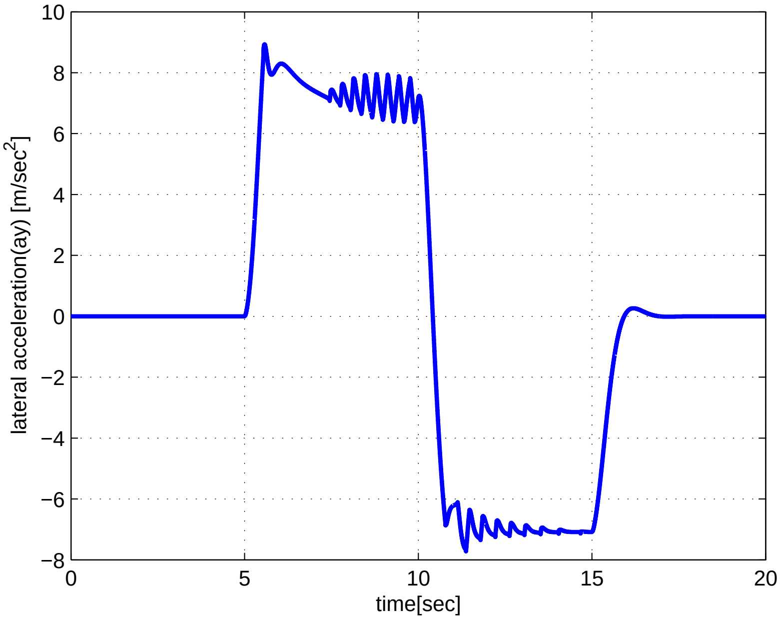

Figure 4.1. Lateral acceleration with rollover prevention using the estimator based on yawrate equation

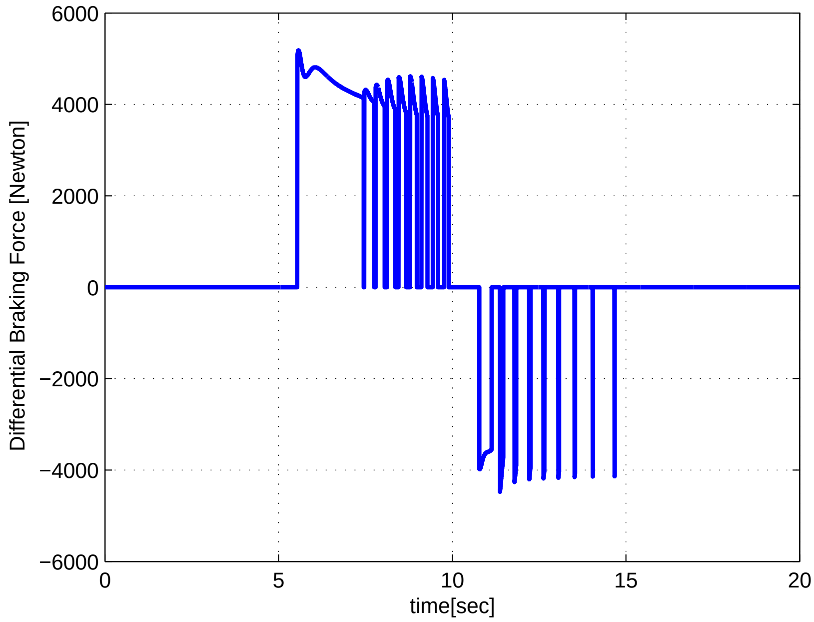

Figure 4.2. Braking force with rollover prevention using the estimator based on yawrate equation

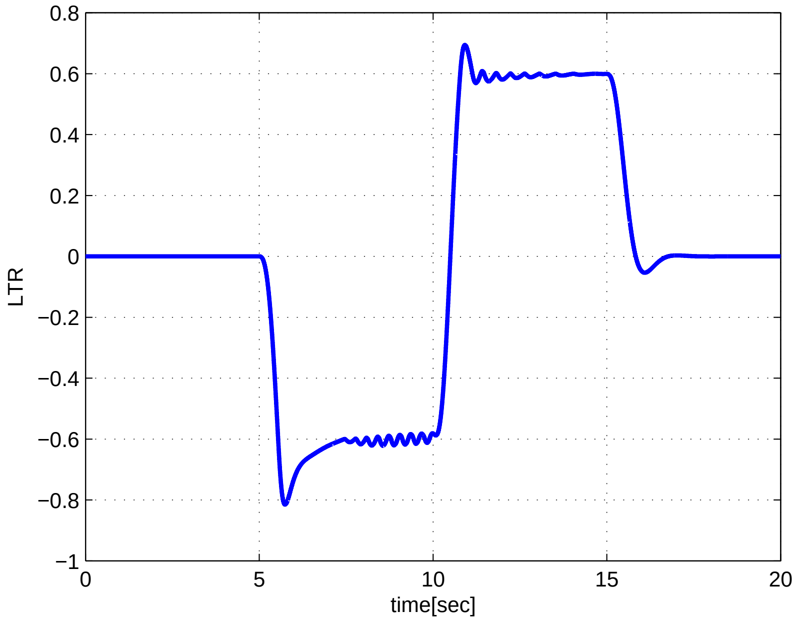

Figure 4.3. LTR with rollover prevention using the estimator based on yawrate equation

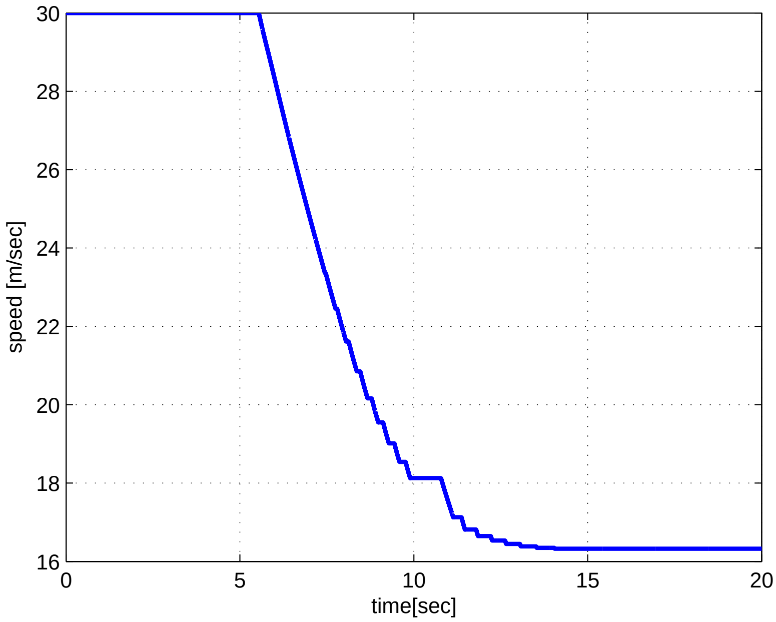

Figure 4.4. Vehicle speed with rollover prevention using the estimator based on yawrate equation

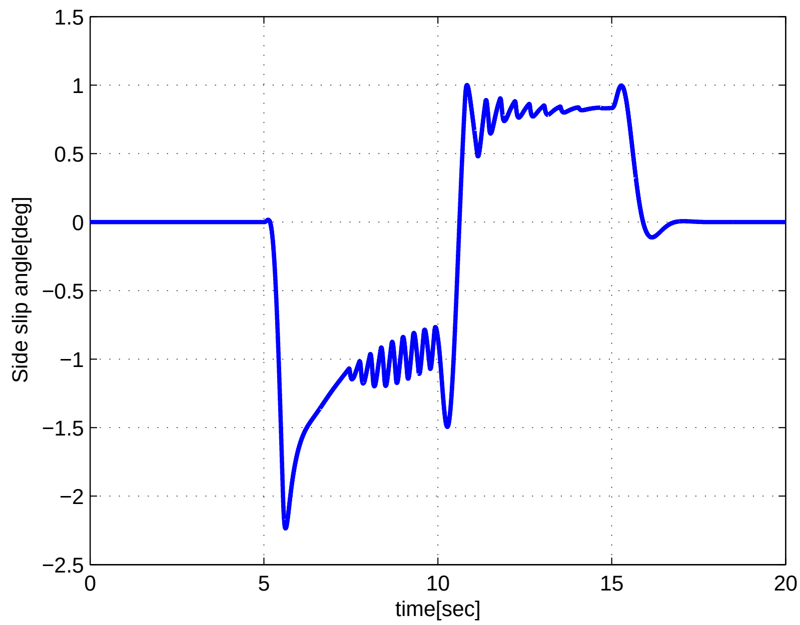

Figure 4.5. Side slip angle β with rollover prevention using the estimator based on yawrate equation

with their counterparts in Chapter 2 when there is no rollover prevention. We see a significant decrease in the magnitude of side slip angle. The absolute values of roll rate and roll angle are both decreased significantly which keep the LTR value between -1 and +1.

The speed and accuracy of the estimation can be easily observed in Figures 4.9 - 4.14. As indicated previously the λ value is decreased to -10000. This increases the accuracy and it affects the step size during simulations. The step size is decreased to small values. This also increases the convergence speed.

As seen in the figures, the estimation results are very accurate and the convergence is very fast. As a result, LTR value can be estimated before a rollover situation and differential braking can be applied in a timely manner. The LTR graph shows that the rollover situation is avoided. The estimation process also continues while the braking force is applied. But in this case the parameters are estimated (converged) before the braking force is applied. So one can think that the parameter estimation converges to certain values before the speed change. And then speed change will not make difference

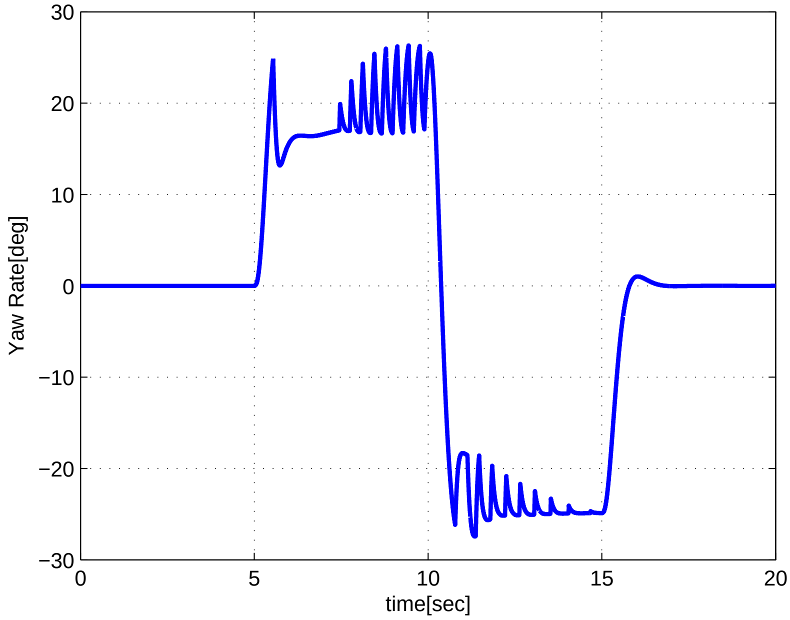

Figure 4.6. Yawrate $\dot{\psi}$ with rollover prevention using the estimator based on yawrate equation

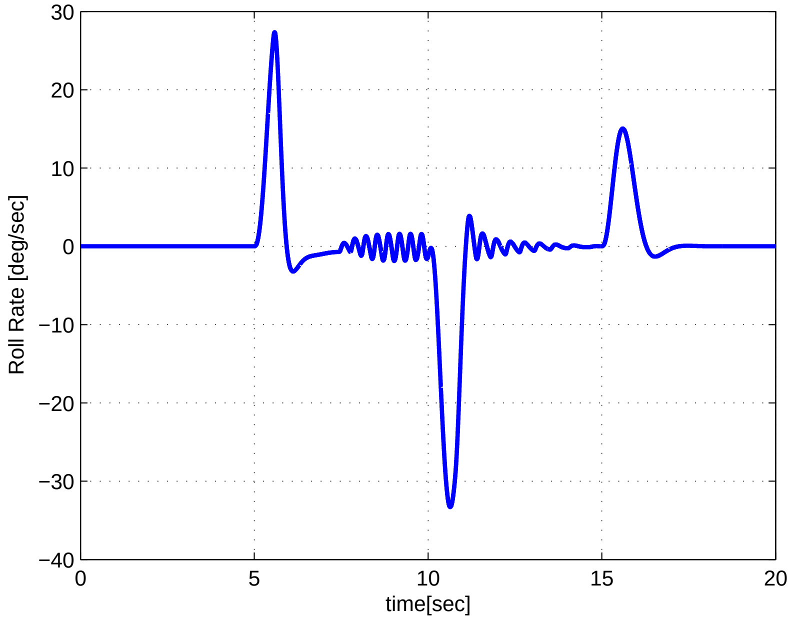

Figure 4.7. Roll rate $\dot{\phi}$ with rollover prevention using the estimator based on yawrate equation

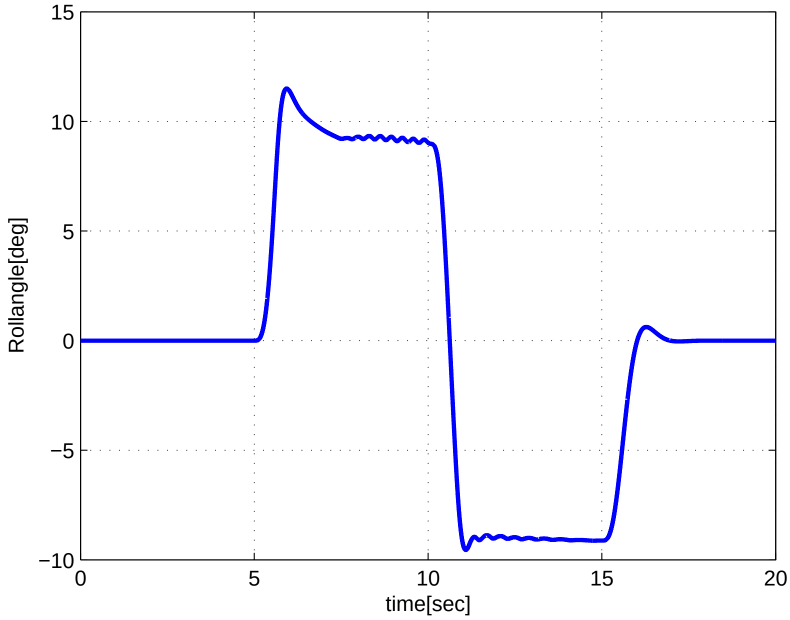

Figure 4.8. Roll angle ϕ with rollover prevention using the estimator based on yawrate equation

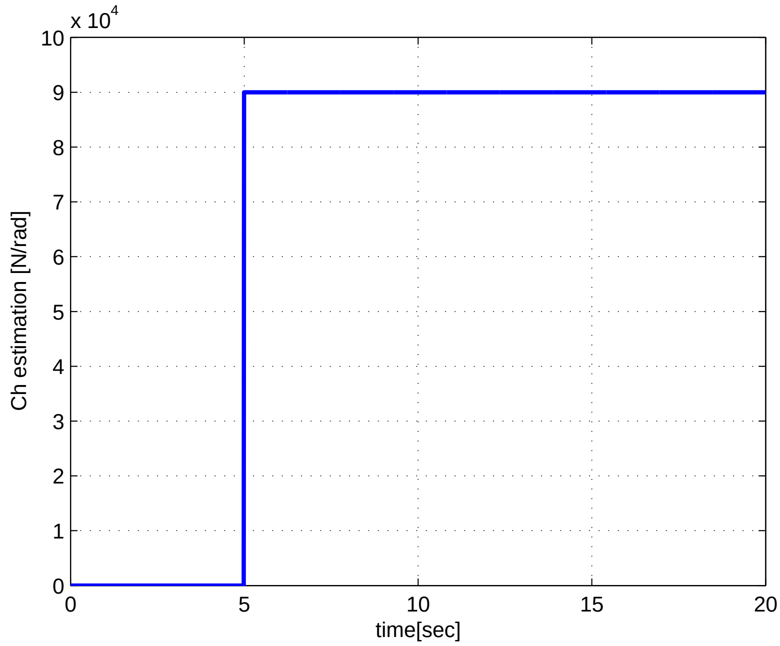

Figure 4.9. Rear Tire Stiffness (C_h) estimation with rollover prevention using the estimator based on yawrate equation

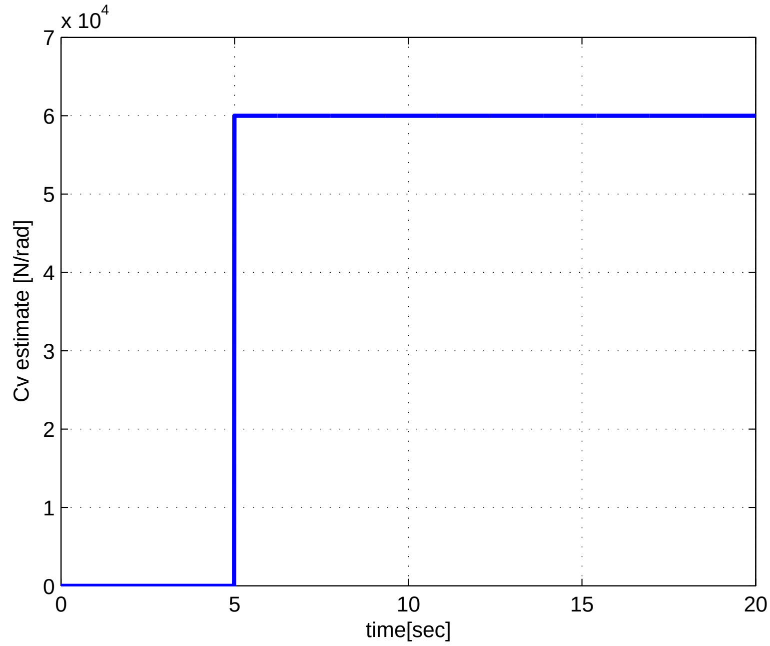

Figure 4.10. Front Tire Stiffness (C_v) estimation with rollover prevention using the estimator based on yawrate equation

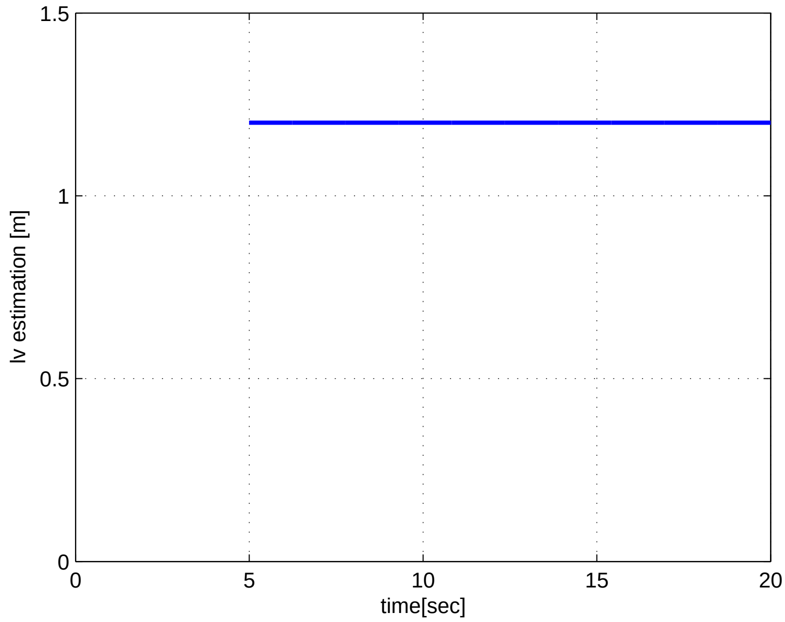

Figure 4.11. Longitudinal CG w.r.t. front axle (l_v) estimation with rollover prevention using the estimator based on yawrate equation

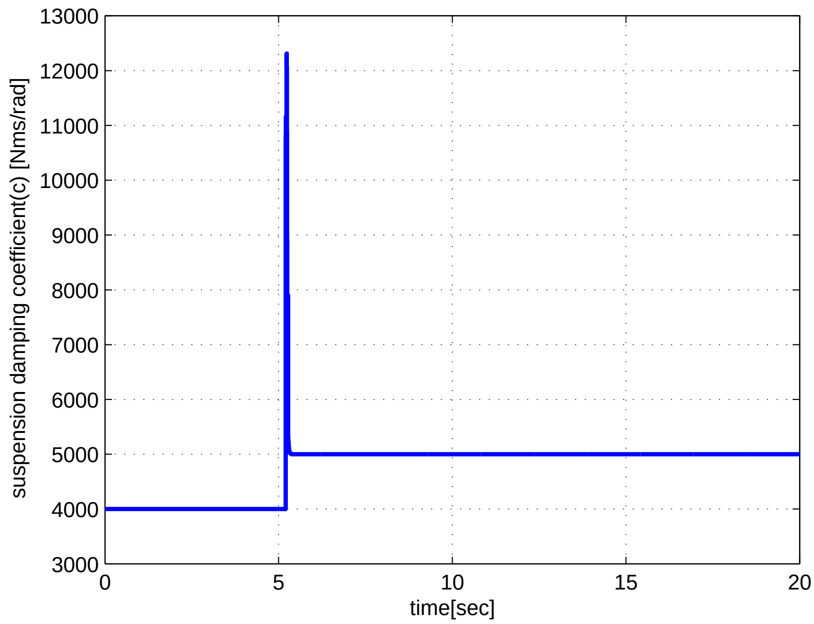

Figure 4.12. Suspension damping coefficient(c) estimation with rollover prevention using the estimator based on yawrate equation

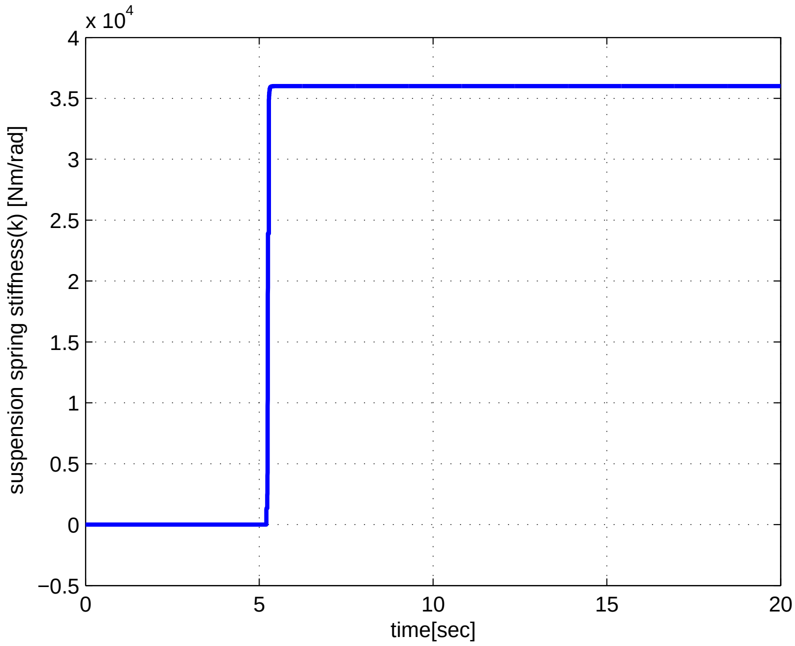

Figure 4.13. Suspension spring stiffness (k) estimation with rollover prevention using the estimator based on yawrate equation

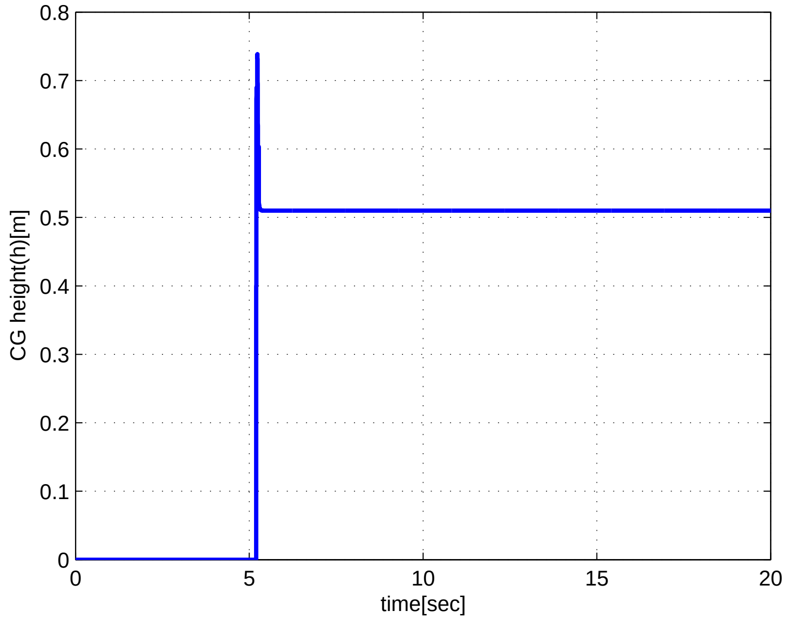

Figure 4.14. C.G. height (h) estimation with rollover prevention using the estimator based on yawrate equation

since the parameters have already converged.

Actually the estimator has been tried for different sitations to avoid this question mark. As shown in Chapter 3, the estimator is successfull even when we start applying braking force at the beginning of the simulation and keep the braking force ON till the end of simulation. The estimation is also successfull for such cases.

The drawbacks of the estimator simulated in this chapter are as follows:

- It uses filtered signals generated by using some user defined parameters (λ) and the accuracy of the estimation is significantly effected by this parameter. It is observed that if λ is taken as a great negative value such as -10000, the results are much better.
- The Matlab simulation becomes very slow when we take λ as indicated above.
- Some concerns might be raised to what is done in (3.39). The Laplace transform of the coefficient of yawrate ($\dot{\psi}$) has not been taken and it is considered as a constant coefficient of the yawrate signal. Since the speed is involved, this assumption may

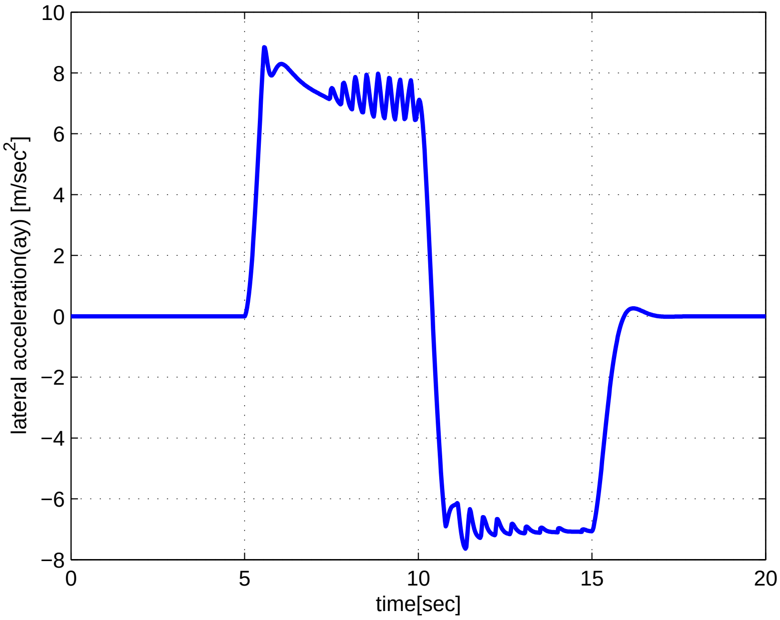

Figure 4.15. Lateral Acceleration with rollover prevention using the estimator based on a_y equation

not be true.

These drawbacks have motivated us to use the lateral acceleration equation to estimate the lateral dynamics parameters as discussed in the previous chapter.

4.3. The Controller Coupled with a Parameter Estimator Based on Measured Lateral Acceleration

The method described in Section 3.5 is used here to estimate the unknown vehicle parameters before a possible rollover accident. We expect the estimator to estimate the parameters in a timely manner and predict the rollover. Simulation results are shown below. Very fast convergence has been achieved since the system is noise free. The exact values are estimated in a very short time as expected from the noise free system. Moreover, the LTR value is always between -1 and +1.

Various tests have been performed for the estimator based on lateral acceleration. Different steering inputs and also braking forces have been applied. Test scenarios also

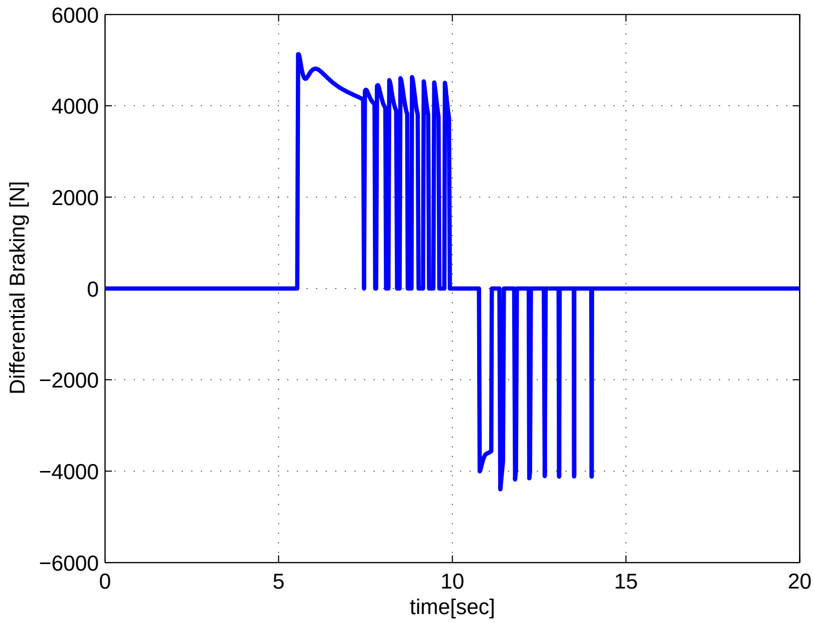

Figure 4.16. Differential braking with rollover prevention using the estimator based on a_y equation

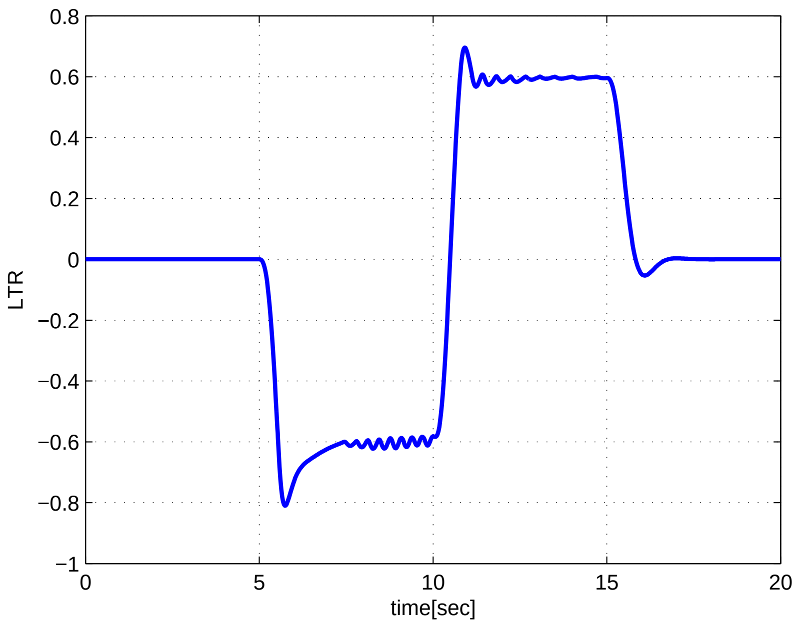

Figure 4.17. LTR with rollover prevention using the estimator based on a_y equation

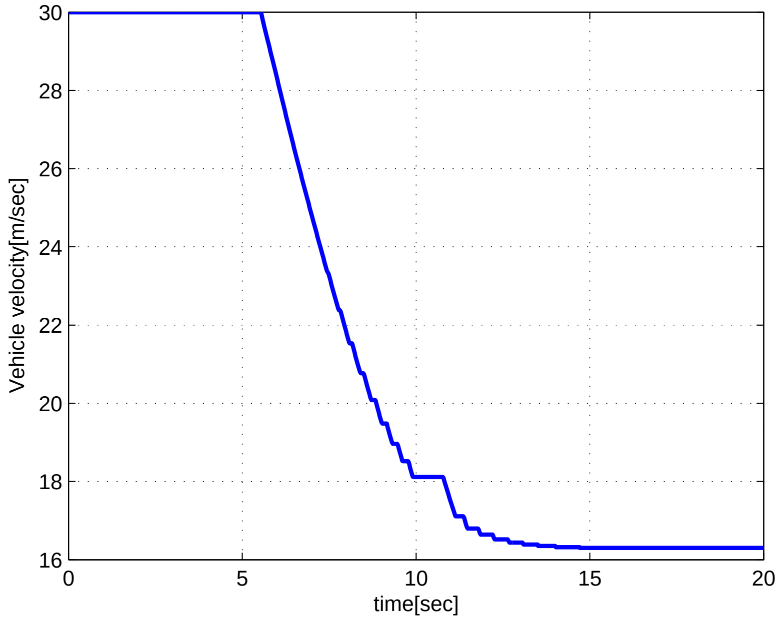

Figure 4.18. Vehicle speed with rollover prevention using the estimator based on a_y equation

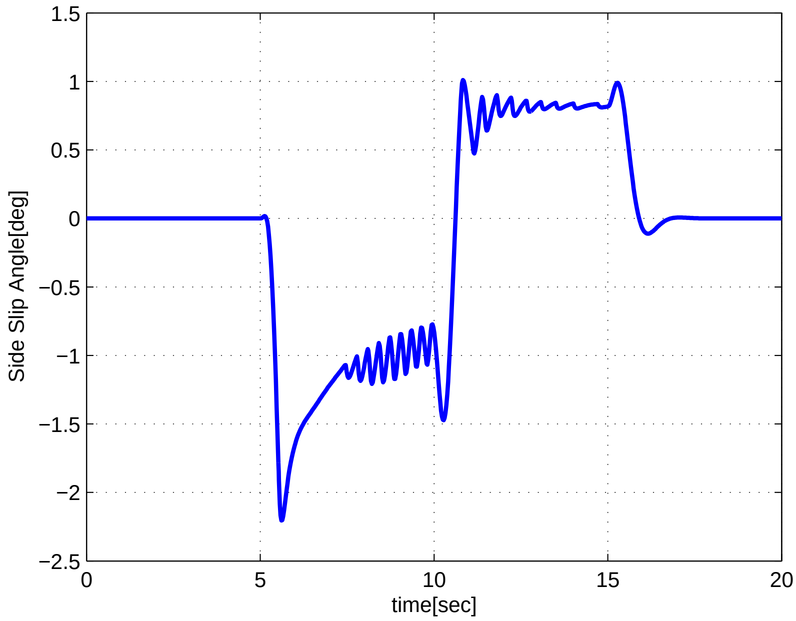

Figure 4.19. Side slip angle (β) with rollover prevention using the estimator based on a_y equation

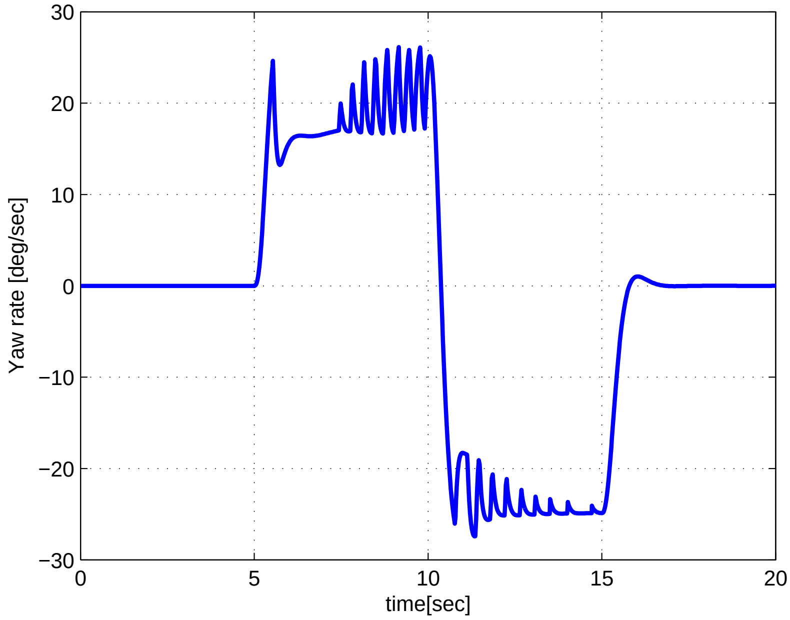

Figure 4.20. Yaw rate ($\dot{\psi}$) with rollover prevention using the estimator based on a_y equation

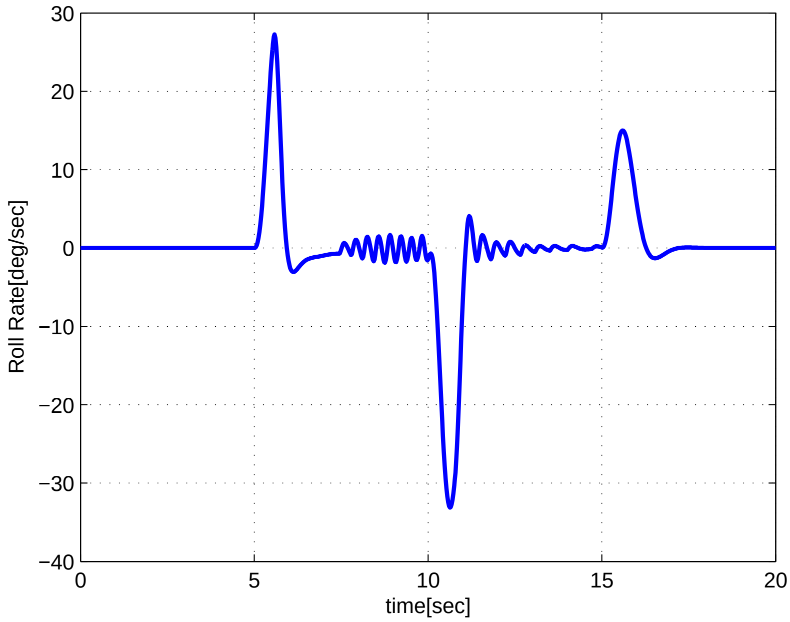

Figure 4.21. Roll rate ($\dot{\phi}$) with rollover prevention using the estimator based on a_y equation

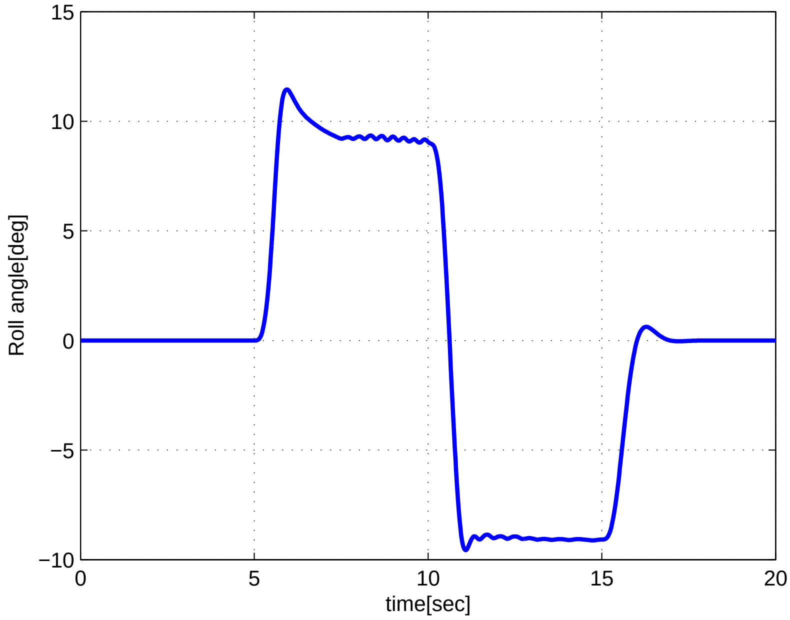

Figure 4.22. Roll angle (ϕ) with rollover prevention using the estimator based on a_y equation

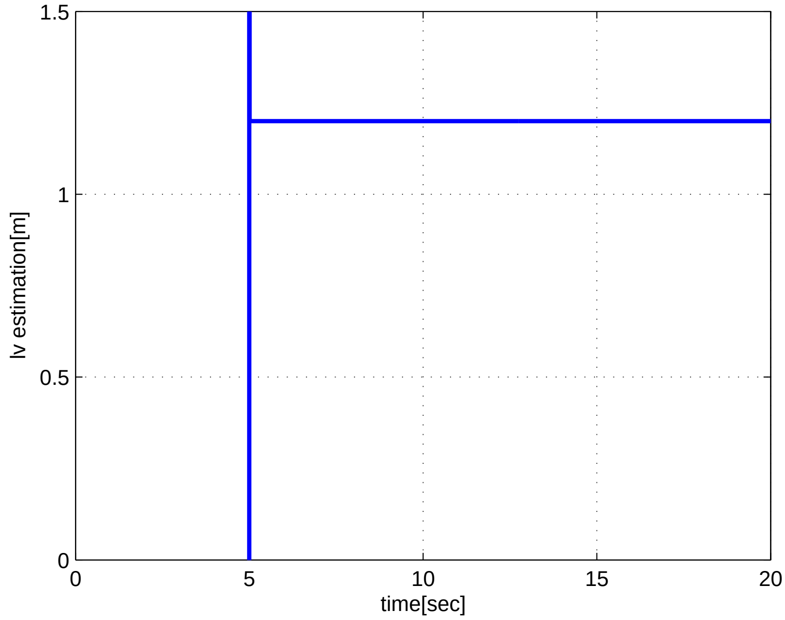

Figure 4.23. Longitudinal CG w.r.t front axle (l_v) estimation with rollover prevention using the estimator based on a_y equation

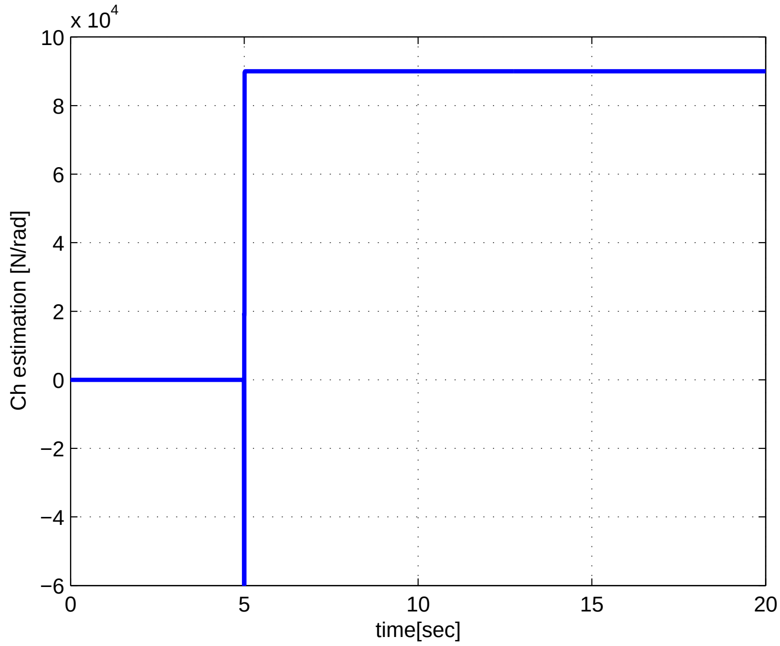

Figure 4.24. Rear tire stiffness coefficient (C_h) estimation with rollover prevention using the estimator based on a_y equation

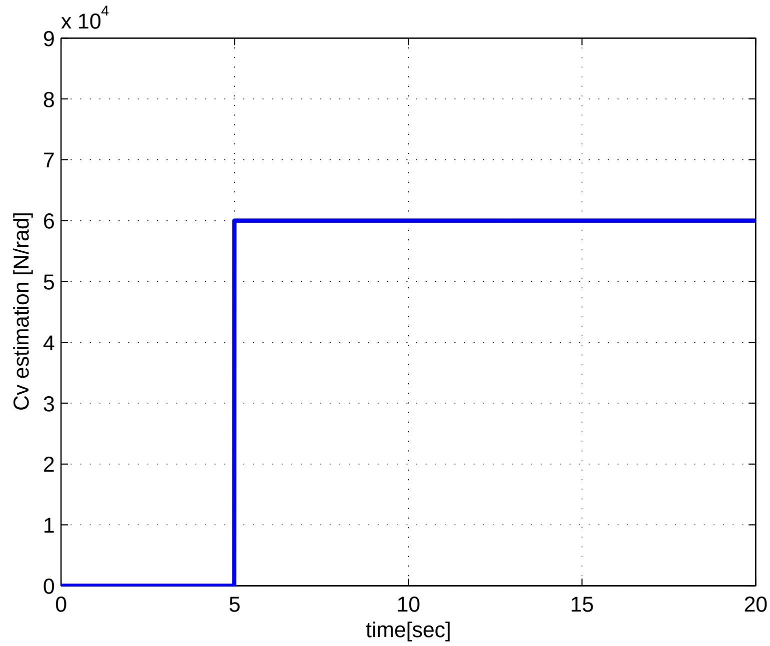

Figure 4.25. Front tire stiffness coefficient (C_v) estimation with rollover prevention using the estimator based on a_y equation

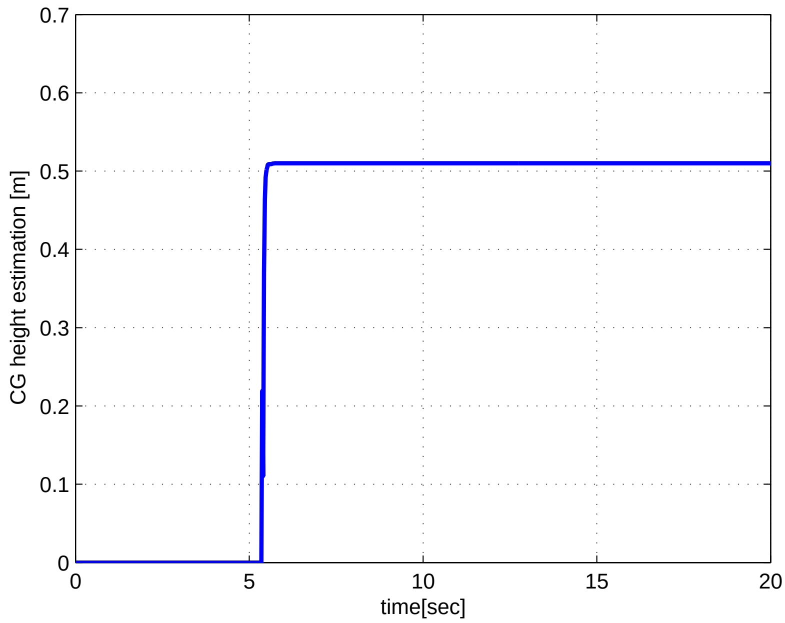

Figure 4.26. CG height (h) estimation with rollover prevention using the estimator based on a_y equation

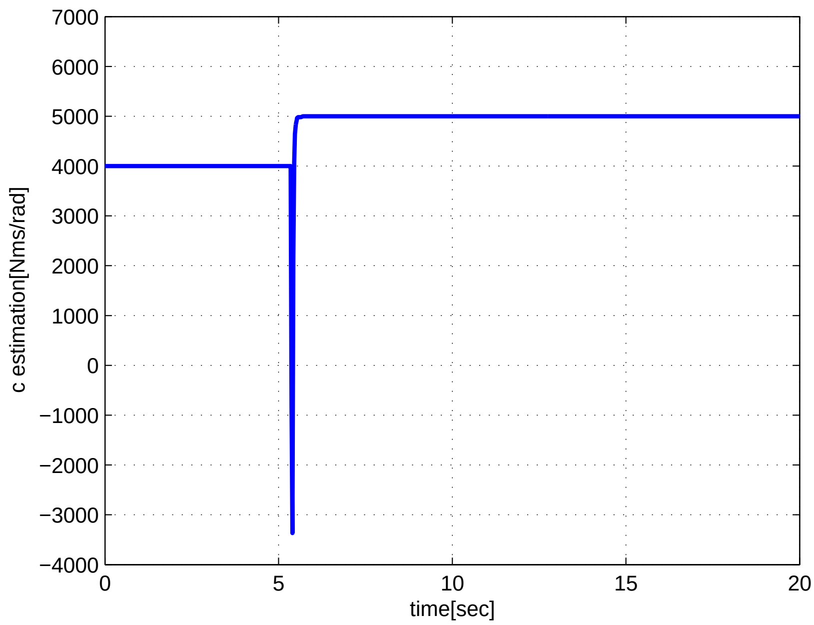

Figure 4.27. Suspension damping coefficient (c) estimation with rollover prevention using the estimator based on a_y equation

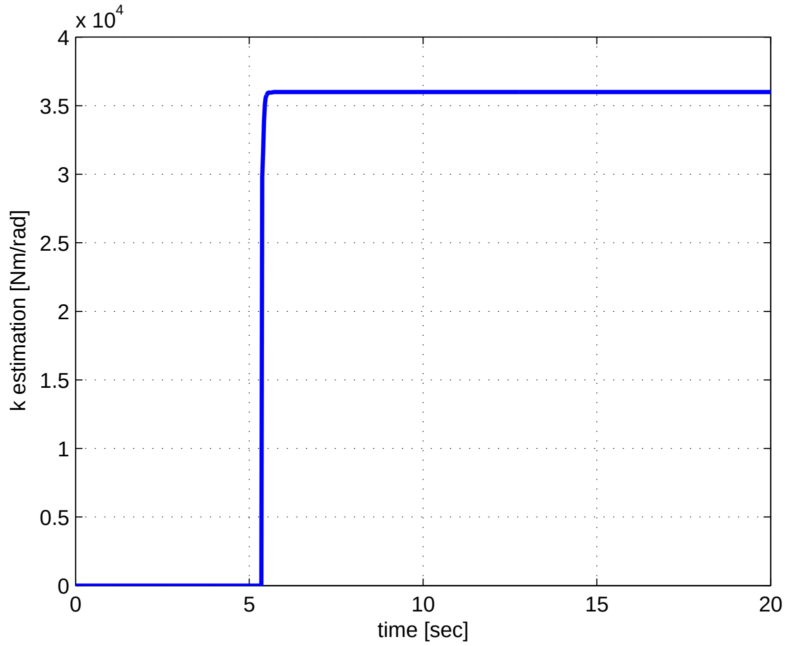

Figure 4.28. Suspension spring stiffness (k) estimation with rollover prevention using the estimator based on a_y equation

include such cases at which braking force is always ON during the simulation. In every case, very fast convergence to the exact values has been achieved as expected since the system is noise free and there is no filtered signals. Furthermore MATLAB has also been faster in simulating this second estimator.

4.4. Summary of the Chapter and Concluding Remarks

In this chapter, the main goal of this thesis, which is to predict and to prevent a rollover accident, has been achieved for the noise free system. The estimated values have been used to predict the accident and to determine the amount and sign of differential braking force in order to prevent an accident.

In the first part the controller has been discussed. Then two different estimators, introduced in Chapter 3, are coupled with the controller. The first estimator is based on the yawrate equation whose performance requires, negative large λ which is used to generate filtered signals. The second estimator is based on the lateral acceleration measurements.

Both of these estimators converge very fast. This enables the controller to calculate LTR_d in a timely manner. When the LTR_d becomes larger than the threshold value, the controller starts applying some amount of differential braking force based on the estimated center of gravity height and measured lateral acceleration value. In both cases the controller becomes active just in time, therefore the LTR value was always kept between -1 and +1.

5. CONCLUDING REMARKS

In this thesis we propose recursive algorithms in order to estimate the unknown vehicle parameters, which are subsequently used to predict and prevent rollover accidents. The recursive algorithms are based on the RLS and the Kalman filter. The RLS algorithm has already been used for this purpose before, however it is observed in this thesis that the RLS algorithm can not estimate the vehicle parameters when the speed of the vehicle changes due to braking or accelerating forces.

In order to address the above problem, the use of Kalman Filter algorithm is proposed which is the main contribution of this thesis. Using the Kalman filter algorithm, the parameters can be estimated even when the speed of the vehicle changes. This enables us to couple the estimator with the controller that prevents the vehicle from rollover accident by appplying differential braking force which reduces the speed of the vehicle.

Firstly the parameter estimation problem is converted to a state observation problem for a discrete time, time-varying, noise free system. Based on the state space equation of the state estimation problem, the possibilty of using different types of observers like Luenberger observer has been considered. However this option has been eliminated since the system is a time-varying one. The Kalman filter best fits to estimate the state however the system is a noise free system. It is known that the Kalman Filter can estimate the unknown state of the systems with Gaussian noisy measurements. But previous works show that Kalman Filter can also be used for deterministic systems with the small modifications in the algorithm.

Additionally, we have proposed a method of estimating the lateral vehicle parameters by using the measured lateral acceleration. The kinematic equation of the lateral acceleration is used and with Kalman filter algorithm the lateral vehicle parameters are estimated. Using the lateral acceleration in parameter estimation is another important contribution of this thesis. Also a discussion on the impact of measurement noise has been carried out.

Taking all these points into consideration the main contributions of this thesis are; estimating the unknown vehicle parameters by using Kalman filter while the speed changes, reminding that the Kalman Filter approach can also be used for deterministic systems with small modifications in the algorithm and utilizing the lateral acceleration in estimating unknown lateral vehicle parameters.

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