

ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL

**AN INVESTIGATION INTO THE EFFECTS OF DIFFERENT PARAMETERS
ON HIGH-RESOLUTION GEOID MODELING ACCURACY IN THE
CONTEXT OF HEIGHT SYSTEM MODERNIZATION**



Ph.D. THESIS

Onur KARACA

Department of Geomatics Engineering

Geomatics Engineering Programme

JULY 2024

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İSTANBUL TEKNİK ÜNİVERSİTESİ ★ LİSANSÜSTÜ EĞİTİM ENSTİTÜSÜ

**YÜKSEKLİK SİSTEMİ MODERNİZASYONU BAKIMINDAN FARKLI
PARAMETRELERİN YÜKSEK ÇÖZÜNÜRLÜKLÜ GEOİT MODELLEME
DOĞRULUĞU ÜZERİNDEKİ ETKİLERİNİN İNCELENMESİ**

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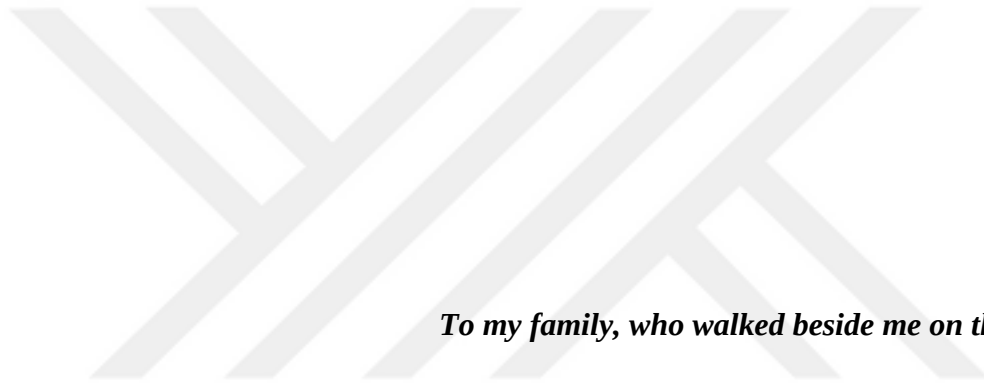
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To my family, who walked beside me on this journey,



FOREWORD

It is with great pleasure and a profound sense of accomplishment that I present this doctoral thesis, the culmination of years of dedication, hard work, and invaluable support from numerous individuals and institutions.

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July 2024

Onur KARACA



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ABBREVIATIONS

ANN	: Artificial Neural Network
BGI	: International Gravimetric Bureau
CBA	: Complete Bouguer Anomaly
CGG2013a	: Canadian Gravimetric Geoid Model of 2013 – Version A
CGVD28	: Canadian Geodetic Vertical Datum of 1928
CGVD2013	: Canadian Geodetic Vertical Datum of 2013
CORS-TR	: Continuously Operating System - Turkey
DEM	: Digital Elevation Model
DWC	: Downward Continuation Correction
EGM96	: Earth Gravitational Model 1996
EGM08	: Earth Gravitational Model 2008
EIGEN	: European Improved Gravity Model of the Earth by New Techniques
FAA	: Free-air Anomaly
FFT	: Fast Fourier Transform
GGM	: Global Geopotential Model
GGM02S	: Grace Gravity Model 02 – Satellite Only
GNSS	: Global Navigation Satellite System
GPS	: Global Positioning System
GRACE	: Gravity Recovery and Climate Experiment
GRS80	: Geodetic Reference System 1980
HTv2.0	: Height Transformation Model
IAG	: International Association of Geodesy
IDP	: Inverse Distance to a Power
ISG	: International Service for the Geoid
ITRF	: International Terrestrial Reference Frame
ITU-GRG	: Istanbul Technical University - Gravity Research Group
KMS02	: Satellite Altimetry Derived Free-air Gravity Anomalies
KTH	: Kungliga Tekniska Högskolan (Royal Institute of Technology)
LSC	: Least Squares Collocation
LSMHA	: Least Squares Modification of Hotine Integral with Additive Corrections

LSMSA	: Least Squares Modification of Stokes Formula with Additive Corrections
NAD83	: North American Datum of 1983
NATRF2022	: North American Terrestrial Reference Frame of 2022
RCR	: Remove Compute Restore
RMSE	: Root Mean Square Error
RTM	: Residual Terrain Model
SBA	: Simple Bouguer Anomaly
SRTM	: Shuttle Radar Topography Mission
TC	: Terrain Correction
TG-91	: Turkish Geoid 1991
TG-99A	: Updated Turkish Geoid 1999
TG-03	: Turkish Geoid 2003
TG-07	: Turkish Geoid 2007
TG-09	: Turkish Geoid 2009
TG-20	: Turkish Geoid 2020
TNVCN	: Turkish National Vertical Control Network (TUDKA)
UNB	: University of New Brunswick
WCS	: Web Coverage System
WGS84	: World Geodetic System of 1984

SYMBOLS

h	: Ellipsoidal height
H	: Orthometric height
N	: Geoid undulation
g_P	: Actual gravity on point P
γ_P	: Normal gravity on point P
Δg	: Gravity anomaly
Ψ_0	: Integration cap size
L	: Upper limit of the Stokes function
M	: Expansion degree of the global geopotential model
W_0	: Constant gravity potential
W_P	: Gravity potential of point P on Earth's surface
C_P	: Geopotential number
dH	: Differential height difference
P	: Point on the Earth's surface
P_0	: Point on the geoid
H_P^{dyn}	: Dynamic height
γ_0	: Normal gravity
$\bar{g}_P$	: Mean value of the gravity
H_P^H	: Helmert orthometric height
H_P^N	: Normal height
$\bar{\gamma}_P$	: Mean normal gravity
ζ_P	: Height anomaly
H_P^{No}	: Normal-orthometric height
$\widetilde{C}_P$	: Approximated geopotential number
W	: Gravity potential
V	: Gravitational potential
Φ	: Centrifugal potential
T	: Disturbing potential
U_0	: Normal potential

ρ	: Mass density
ω^2	: Angular velocity
J	: Mean curvature of level surface
γ_e	: Normal gravity at equator
g_{P_o}	: Gravity value at a point on the geoid surface
γ_{Q_o}	: Normal gravity on the ellipsoid
δg_P	: Gravity disturbance
δg_F	: Free-air reduction
Δg_{FAA}	: Free-air anomaly
δg_{TC}	: Terrain correction
Δg_{SB}	: Simple bouguer anomaly
δg_B	: Bouguer plate reduction parameter
G	: Gravitational constant
Δg_{CB}	: Complete Bouguer anomaly
δg_B^s	: Bouguer shell correction
Δg_{PH}	: Pratt-Hayford isostatic gravity anomaly
A_{top}	: Gravitational attraction of topographic masses
A_{comp}	: Gravitational attraction of compensated masses
ρ_0	: Constant density of topographic masses
ρ_w	: Density for sea
ρ_m	: Density for mantle
Δg_{AH}	: Airy-Heiskanen isostatic gravity anomaly
R	: Earth's radius
$\Delta g_{Helmert}$	: Helmert Gravity Anomaly
$\delta \Delta g$	: Indirect effect on gravity
tc	: Terrain correction
Δg_{RTM}	: RTM gravity anomaly
δA_{RTM}	: RTM reduction
Δg_{Rudzki}	: Rudzki gravity anomaly
δA_{Rudzki}	: Direct topographical effect on gravity in Rudzki reduction scheme
k	: Power parameter
Z_i	: Observed data
$\hat{Z}_j$	: Interpolated data
d_{ij}	: Distance between the interpolated and observed values

δ	: Smoothing parameter
$\gamma(d)$	: Semi-variogram value
K	: First order effect on variogram
$S(\Psi)$	: Stokes function
Ψ	: Geocentric angle
σ	: Unit sphere
$d\sigma$	: Infinitesimal surface element of the unit sphere
$\sigma_o(E)$	: Spherical cap (integration area)
$\Delta g_{red.}$	: Reduced gravity anomaly
Δg_{GGM}	: GGM-derived long-wavelength gravity anomaly contribution
Δg_T	: High-frequency component of terrain-induced gravity anomaly
$N_{ind.}$	: Indirect effect of the terrain on geoid height
$H(\Psi)$	: Hotine function
$\hat{N}$	: Geoid undulation computed through LSMSA
$\tilde{N}$	: Approximate geoid height
δN_{com}^{top}	: Combined topographic correction
δN_{DWC}	: Downward continuation effect
δN_{com}^{atm}	: Combined atmospheric correction
δN_{ell}	: Ellipsoidal correction
$P_n(\cos\Psi)$	: Legendre polynomial
s_n	: Stochastic modification parameter
dc_n	: Error degree variance
Q_n^L	: Truncation coefficient
c_n	: Signal degree variance
ρ^a	: Atmospheric density
Δg_{FAA}^{GRID}	: Gridded free-air anomaly
C	: Error variance
b_n	: Modification type
δg_B	: Bouguer reduction



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AN INVESTIGATION INTO THE EFFECTS OF DIFFERENT PARAMETERS ON HIGH-RESOLUTION GEOID MODELING ACCURACY IN THE CONTEXT OF HEIGHT SYSTEM MODERNIZATION

SUMMARY

In contemporary times, many countries have focused on obtaining orthometric heights through satellite-based positioning techniques, utilizing ellipsoidal heights and geoid heights derived from geoid models, as part of height system modernization efforts. This shift is primarily driven by several factors: the inherent limitations of traditional leveling, which is a costly, labor-intensive, and time-consuming process; the economic advantages of employing geoid models in conjunction with satellite-based positioning systems; the ability to obtain real-time physical height information; and the diminished susceptibility to the deformative effects of the Earth's crust.

Given the ability to obtain highly accurate ellipsoidal heights using GNSS, the accuracy of regional geoid models directly impacts the determination of orthometric heights. In many methods used for the computation of gravimetric geoids, similar to the least squares modification of Stokes formula with additive corrections (LSMSA) technique utilized in this thesis, gridded free-air anomaly data are employed as input. Observed gravity data from the Earth's surface is heavily influenced by the gravitational field of the Earth due to topographic masses and cannot be directly utilized in the gridding process. Consequently, observed gravity data are first transformed into anomalies and then reduced to the geoid using Bouguer reduction, where the topography is represented by a Bouguer plate (or shell). The resulting Bouguer anomalies obtained in this manner are suitable for the gridding process and create a smooth surface.

The selection of the Bouguer anomaly type (simple or complete) is performed considering the topography of the study area. Simple Bouguer anomalies can be used in studies conducted in plain areas with minimal topographic variations. However, complete Bouguer anomalies, which also incorporate the terrain correction (TC) effect, are essential for applications carried out in rough topography despite their higher computational burden. The determination of the most suitable Bouguer anomaly to be used in the gridding process in the study area of Auvergne, France, is conducted through numerical tests in the first part of this thesis. The investigations reveal differences exceeding 55.6 mGal between the two anomaly grids and 27 cm between the geoids generated from these grids. Therefore, in areas with complex topography such as Auvergne, the use of complete Bouguer anomalies incorporating terrain correction effects is crucial for obtaining precise gravimetric geoids.

In the second numerical application of the study, a series of tests are conducted to determine the optimal parameters to be used in LSMSA, selected as the computation method for gravimetric geoids. The parameters identified here are adopted as the fundamental parameters for the geoids computed throughout the entire study.

The third numerical application, constituting the main focus of this thesis, investigates the impact of interpolation methods on gravimetric geoid computation. Both the areal difference maps and absolute validation results obtained in this section demonstrate that the conventional methods such as Inverse Distance to a Power (IDP), Nearest Neighbor, and Kriging, which have been widely used for gridding gravity data in geodesy and geophysics for a long time, provide similar and reliable results. On the other hand, the Artificial Neural Network (ANN), a complex, optimization-based soft computing method with proven effectiveness in many fields, does not yield superior results compared to these three methods. As a conclusion, it is inferred that the interpolation algorithms have an impact on the gravity gridding outcomes and, consequently, the determination of the geoid model.

GPS/Leveling points are typically measured in plain terrain rather than rugged areas due to the inherent challenges of their establishment in such environments. Absolute validation processes are conducted in both plain and rugged terrain during the 4th numerical test, revealing differences of up to two-times of magnitude between the validation results obtained in these two different terrains. This disparity stems from the denser and more homogenous distribution of points in plain areas, which more accurately captures the topography.

As the final numerical application, the applicability of the ANN algorithm for gravity gridding is investigated. The conducted experiments demonstrated that the complete Bouguer anomalies gridded using the ANN approach yield distinct results each time, regardless of the employed parameters (neurons - epochs). Consequently, it is concluded that the ANN technique has not produced satisfactory results in the gravity gridding process, which is an important part of gravimetric geoid computation, in this thesis study.

In summary of the conducted studies, the findings highlight the crucial role of the interpolation algorithm employed in high-precision geoid undulation calculations. Additionally, it has been concluded that the ANN method does not perform as well in the gridding process compared to traditional interpolation methods such as Kriging, IDP, and Nearest Neighbor. Furthermore, the type of Bouguer anomaly used in gridding is crucial, especially for rugged study areas. Lastly, it is noted that the distribution of GPS/leveling points used in absolute validation affects the validation results.

YÜKSEKLİK SİSTEMİ MODERNİZASYONU BAKIMINDAN FARKLI PARAMETRELERİN YÜKSEK ÇÖZÜNÜRLÜKLÜ GEOİT MODELLEME DOĞRULUĞU ÜZERİNDEKİ ETKİLERİNİN İNCELENMESİ

ÖZET

Günümüzde birçok ülke, yükseklik sistemlerinin modernizasyonu kapsamında uydu tabanlı konumlandırma teknikleri aracılığıyla ortometrik yükseklikleri elde etmeye odaklanmaktadır. Bu kapsamda, GNSS ve benzeri sistemlerden elde edilen elipsoidal yüksekliklerden, geoit modelleri aracılığıyla türetilen geoit yükseklikleri çıkarılarak ortometrik yüksekliklere ulaşılmaktadır. Geleneksel nivelman yöntemiyle yüksek prezisyonlu ortometrik yükseklik elde edilebilmesine rağmen yöntemin doğası gereği yüksek maliyet ve yoğun işgücü gereksinimi mevcuttur. Oldukça zaman alıcı bu geleneksel yöntemle karşılık, uydu tabanlı konumlama sistemleri (GNSS) ile yükseklikler gerçek zamanlı, hızlı ve daha ekonomik bir şekilde elde edilmektedir. Ayrıca geoit modellerinin yer kabuğu değişimlerinden daha az etkilenmesi de bu modern ölçüm yönteminin avantajlarından biridir.

Şehirleşme, yol genişletme çalışmaları, insan kaynaklı tahribat, yer kabuğu hareketlerinin sebep olduğu deprem gibi afetler sebebiyle düşey kontrol ağlarının %80'inden fazlasının tahrip olduğu Türkiye, Kanada'nın 2013 yılında gerçekleştirdiği gibi geoide dayalı bir düşey referans sistemine geçme çabasıdadır. Bu çabanın sonucunda "Türkiye Yükseklik Sisteminin Modernizasyonu ve Gravite Altyapısının İyileştirilmesi Projesi" ile Türkiye Geoidi-2020 (TG-20) üretilmiştir. Bahse konu proje ile, halihazırda santimetre duyarlılığında ve gerçek zamanlı olarak erişilebilen yatay konum bilgisine ek olarak, TG-20 geoidini kullanarak TUSAGA-AKTİF sistemi üzerinden yüksek duyarlılıkta düşey konum bilgisinin elde edilebilmesi amaçlanmıştır.

Uydu tabanlı konumlama sistemleri ile yüksek doğrulukta elipsoidal yükseklik elde etme imkanı göz önüne alındığında, bölgesel geoit modellerinin doğruluğu, ortometrik yüksekliklerin belirlenmesini doğrudan etkilemektedir. Geoit modellerinden yükseklik bilgisinin hesabı Stokes fonksiyonu ile gerçekleştirilebilmektedir. Ancak klasik Stokes fonksiyonu, geoidin dışında topografik kitlelerin olmadığı ve tüm yerküre boyunca yersel gravite verisi olduğu varsayımına dayanmaktadır. Pratik olarak bu varsayımların gerçekleştirilmesi mümkün olmadığı için, klasik Stokes fonksiyonu modifiye edilmekte ve yersel gravite verileri ile uydu gravite verileri birlikte kullanılmaktadır. Hesaplamalarda gravite verileri çalışma alanıyla sınırlandırıldığından dolayı, küresel jeopotansiyel model katsayıları ile minimize edilmeye çalışılan kesme hatası meydana gelmektedir. Gravimetrik geoitlerin hesabında, Stokes fonksiyonunun dışında Hotine gibi integraller de yaygın olarak kullanılmaktadır.

Bu çalışmada kullanılan ek düzeltmeler içeren Stokes'un en küçük karelerle modifikasyonu (LSMSA) tekniğinde olduğu gibi, pek çok gravimetrik geoit belirleme yaklaşımında girdi verisi olarak gridlenmiş serbest hava anomalileri kullanılmaktadır. Topografya üzerinde gözlemlenen gravite değerleri, topoğrafik kütleler nedeniyle dünyanın yerçekim alanından önemli ölçüde etkilenmekte ve bu nedenle doğrudan

gridleme sürecinde kullanılamamaktadır. Yeryüzünde gözlemlenen gravite verileri önce gravite anomalilerine dönüştürülür ve ardından topografyanın Bouguer levhası (ya da kabuğu) ile temsil edildiği Bouguer indirgemesi ile geoide indirgenirler. Bu şekilde elde edilen Bouguer anomalileri gridleme işlemi için uygundur ve düzgün bir yüzey oluştururlar.

Basit ya da tamamlanmış Bouguer anomalilerinden hangisinin kullanılacağı, çalışma alanının topografyası göz önüne alınarak belirlenir. Topografik değişimlerin az olduğu düz alanlarda basit Bouguer anomaliler kullanılabilir. Ancak, engebeli alanlarda gerçekleştirilen uygulamalarda her ne kadar matematiksel işlem yükü artsa da arazi düzeltmesini hesaba katan tamamlanmış Bouguer anomalilerinin kullanılması gerekmektedir. Bu tezin ilk bölümünde, Fransa'nın Auvergne bölgesindeki çalışma alanında gridleme işlemi için kullanılacak en uygun Bouguer anomalisinin belirlenmesine yönelik sayısal testler gerçekleştirilmiştir. Yapılan incelemeler sonucunda, iki anomali grid arasında 55,6 mGal'i bulan farklar olduğu tespit edilmiştir. Bu gridlerden elde edilen geoit modelleri arasındaki farklar da 27 santimetreyi bulmaktadır. Sonuç olarak, Auvergne gibi karmaşık topografyaya sahip bölgelerde arazi düzeltme etkilerini de içeren tam Bouguer anomalilerinin kullanılması, yüksek prezisyonlu gravimetrik geoitlerin elde edilmesi için kritik öneme sahiptir.

Çalışmada gerçekleştirilen ikinci sayısal uygulamada, gravimetrik geoitlerin hesaplama yöntemi olarak seçilen LSMSA'da kullanılacak en uygun parametrelerin belirlenmesi adına bir dizi test gerçekleştirilmiştir. Bu testler sonucunda elde edilen parametreler, çalışma boyunca hesaplanan tüm geoitler için temel parametreler olarak benimsenmiştir.

Bu tezin ana odaklarından birini oluşturan üçüncü sayısal uygulamada ise, interpolasyon yöntemlerinin gravimetrik geoit hesaplamalarına etkisi araştırılmıştır. Gerçekleştirilen testler sonucunda elde edilen alansal fark haritaları ve mutlak validasyon sonuçları, halihazırda uzun yıllardır jeodezi ve jeofizikte gravite gridlemesinde kullanılan en yakın komşuluk, ters mesafe ağırlıklandırma (IDP) ve Kriging gibi klasik interpolasyon yöntemlerinin güvenilir ve birbirlerine benzer sonuçlar sağladığını göstermektedir. Diğer taraftan, birçok alanda etkinliği kanıtlanmış karmaşık ve optimizasyona dayalı esnek hesaplama yöntemi olan yapay sinir ağları (ANN) ile gerçekleştirilen gridlemede, bu üç yöntemde elde edilen doğruluklara ulaşamamıştır. Bu bölümde gerçekleştirilen testlere ilişkin görsel ve istatistikler incelendiğinde, interpolasyon algoritmasının grid sonuçlarına etkisi olduğu ve dolayısıyla geoit modellerinin belirlenmesinde önemli rol oynadığı anlaşılmaktadır.

Yüksek ve engebeli arazilerde GPS/Nivelman noktalarının tesis edilmeleri zor olduğundan, bu noktalar genellikle düz alanlarda ölçülmektedir. Gerçekleştirilen dördüncü sayısal test ile mutlak doğrulama işlemleri hem düz hem de engebeli arazide gerçekleştirilmiş olup, bu iki farklı arazi tipinde elde edilen doğrulama sonuçları arasında iki katına varan farklar olduğu ortaya konulmuştur. Bu fark, düz alanlardaki noktaların daha yoğun ve homojen dağılım göstermeleri ve dolayısıyla topografyayı daha iyi temsil etmelerinden kaynaklanmaktadır.

Tez çalışması kapsamında gerçekleştirilen son sayısal uygulamada, yapay sinir ağı algoritmasının gravite gridlemesindeki uygulanabilirliği araştırılmıştır. Gerçekleştirilen deneyler, yapay sinir ağı yaklaşımı kullanılarak gridlenen tamamlanmış Bouguer anomalilerinin, hesaplamada kullanılan parametrelerden (nöron-epok) bağımsız olarak her seferinde farklı sonuçlar ürettiğini göstermiştir. Bir

başka deyişle, aynı parametrelerle gerçekleştirilen testlerde farklı grid değerlerine ulaşılmıştır. Bu deneyim, yapay sinir ağları ile gravite gridlemesinde tek anlamlı ve standart sonuçlar elde edilememesine neden olmaktadır.

Özetle; gerçekleştirilen sayısal testler sonucunda, gridlemede kullanılan interpolasyon algoritmasının yüksek prezisyonlu geoidin belirlenmesinde kritik rol oynadığı ortaya çıkarılmıştır. Yapay sinir ağları yönteminin bu tez çalışmasında gravite verilerinin gridlenmesinde iyi sonuç vermediği görülmüştür. Ayrıca, gridlemede kullanılan Bouguer anomali tipinin seçiminde çalışma alanının topografyasının göz önüne alınması gerektiği sonucuna varılmıştır. Bunlara ek olarak, mutlak doğrulamada kullanılan GPS/Nivelman noktalarının dağılımının doğrulama sonuçlarını etkilediği tespit edilmiştir.





1. INTRODUCTION

1.1 Overview and Thesis Outline

Height information relative to a reference surface holds paramount importance across a spectrum of disciplines, including geodesy, geophysics, geodynamics, engineering, cartography, environmental science, atmospheric science, hydrology and oceanography. Several height systems exist, each grounded on distinct vertical datums, including geopotential numbers, dynamic heights, ellipsoidal heights, orthometric heights, normal heights, and normal-orthometric heights. While Global Navigation Satellite Systems (GNSS) offer the distinct advantage of readily acquiring ellipsoidal heights (h) anytime and anywhere, irrespective of weather conditions, this type of height measurement suffers from a critical limitation. Ellipsoidal height, defined as the vertical distance from a point on the Earth's surface to a reference ellipsoid measured along the ellipsoidal normal, possesses solely geometrical significance. It lacks a direct connection to the Earth's gravitational field, rendering it unsuitable for the vast majority of engineering and surveying applications (Erol & Erol, 2021; Işık et al., 2022; Sjöberg et al., 2015). In contrast, gravity-based systems, such as orthometric height (H), are physically meaningful as they account for the Earth's gravity field. Most national height systems typically employ these physical heights. Unlike ellipsoidal heights, orthometric heights, which is the distance between the geoid and the point on the Earth's surface along the plumbline, accurately represent the behavior of fluid flows (Featherstone & Kuhn, 2006; Hofmann & Moritz, 2005; Jekeli, 2000).

Orthometric heights are traditionally determined through spirit leveling surveys; however, vertical control networks established in this manner are susceptible to deformation due to factors such as crustal movements, urbanization, and road expansion/renewal works, as exemplified by Turkey, where over 80% of the height network has been deformed. Consequently, an approach that combines geoid models and modern satellite techniques, which are far less affected by these factors, is gaining prominence. In this context, orthometric heights (H) are obtained by subtracting geoid

undulations (N) derived from geoid models from GNSS based ellipsoidal heights (h). In this transformation, the utilization of high-accuracy regional geoid models derived from homogeneously and densely distributed terrestrial gravity data is crucial (Ellmann et al., 2019; Hackney & Featherstone, 2003; Hofmann & Moritz, 2005; Kiamehr, 2007). The geoid, mentioned in the definition of orthometric heights and the modernization of vertical height control networks, is described as a level surface characterized by a consistent gravity potential. It coincides with mean sea level once adjustments are made for the effects of sea surface topography across oceans (Heiskanen & Moritz, 1967).

Solving a boundary value problem forms the cornerstone for determining the geoid surface. This problem finds its solution through Stokes' formula (details are in Section 5.1), introduced by George Gabriel Stokes (Stokes, 1849). However, a critical assumption underpins this formula: the absence of any mass existing outside the geoid. To satisfy this requirement, geoid computation theory employs a mathematical approach that excludes the influence of topographical masses situated beyond the geoid's boundaries (Abbak et al., 2012; Heiskanen & Moritz, 1967; Işık et al., 2022). A key challenge in utilizing the Stokes formula lies in the requirement for global coverage of high-resolution terrestrial gravity data. Unfortunately, acquiring such dense measurements across the entire Earth is impractical. However, this data limitation is addressed through the use of a modified Stokes integral formula. This approach ingeniously integrates readily available local, dense terrestrial gravity data with global satellite gravity observations, effectively mitigating the impact of sparse data on the final results.

Two primary approaches, deterministic and stochastic, are employed to modify the Stokes formula in this context (Abbak & Ustun, 2015; Ellmann et al., 2019; Sakil et al., 2021; Sjöberg; 2003c). The deterministic approach tackles the truncation error caused by limited far-zone terrestrial gravity data. It assumes error-free Global Geopotential Model (GGM) coefficients to minimize this error. In contrast, the stochastic approach takes a more comprehensive view. It considers not only the truncation error from sparse terrestrial data but also potential errors within the GGM and terrestrial gravity data itself. Within the case study segment of this thesis, the method known as the least squares modification of the Stokes formula with additive corrections (abbreviated LSMSA or the KTH approach), employed for determining

gravimetric geoids, represents a stochastic adaptation of the Stokes integral formula pioneered by the Royal Institute of Technology in Sweden (Abbak et al., 2012a, 2012b; Abbak et al., 2022; Abdalla, 2009; Abdalla & Mogren, 2015; Agren, 2004; Agren et al., 2008; Ellmann, 2004; Ellmann et al., 2019; Işık et al., 2021; Jekeli, 2000; Märdla, 2017; Märdla et al., 2018; Sjöberg, 1981, 1984, 1991, 2003a, 2003b, 2003c, 2005, 2018b; Sjöberg et al., 2015; Sjöberg & Bagherbandi, 2017; Yildiz et al., 2012). Beyond the LSMSA approach, other commonly employed gravimetric geoid determination methods include Stokes-Helmert (Ellmann & Vanicek, 2007; Vanicek et al., 2013), least squares collocation (LSC) (Kaas et al., 2013; Tscherning, 2013, 2015), remove-compute-restore (RCR) (Abbak et al., 2012a; Barzaghi, 2016; Yildiz et al., 2012), and the least squares modification of the Hotine integral with additive corrections (LSMHA) (Abbak et al., 2022; Işık et al., 2021; Märdla et al., 2018; Sakil et al., 2021). Detailed explanations of these methods are provided in Section 5.

While the LSMSA method utilizes gridded free-air anomalies as input, their application presents a challenge. This arises due to the high correlation between free-air anomalies and the variations in topography. To address this limitation, Bouguer anomalies are employed during the gridding process because they are less dependent on topography (Bajracharya, 2003; Tziavos & Sideris, 2013). The Bouguer anomalies employed for gridding are obtained by reducing the gravity anomalies (Δg) calculated from observed gravity values (g) to the geoid using the Bouguer reduction approach. This process mathematically eliminates the gravitational attraction effects caused by the masses above the geoid (Abbak et al., 2012c; Torge, 2001). At this point, the choice between using simple and complete (refined) Bouguer anomalies in gridding emerges as a selection.

Simple Bouguer reduction involves identifying the topography as the Bouguer plate (or shell) and eliminating the masses represented by this plate above the geoid. In contrast, complete Bouguer anomalies account for deviations from this idealized model by incorporating terrain corrections (TC). These corrections mathematically capture the gravitational attraction of residual topographic masses that differ from the Bouguer plate. The most laborious aspect of computing complete Bouguer anomalies is calculating the terrain corrections. This calculation involves performing complex multiple integrals (see Eq. 3.7) using high-resolution topographic data, which consequently requires a significant amount of time and high-capacity processors. The

selection of appropriate gravity anomalies for data gridding plays a critical role in geoid determination. Using an unsuitable gridding approach for the gravity dataset can lead to a loss of the original signal within the data. This loss of information consequently reduces the quality of the resulting regional geoid model.

The suitability of simple Bouguer anomalies (SBAs) for gridding and interpolation of gravity data, compared to complete Bouguer anomalies (CBAs), has been explored in several studies (Abbak et al., 2012c; Goos & Featherstone, 2003; Kuhn et al., 2009). While CBAs are considered essential for regions with rough topography characterized by significant elevation variations, these studies suggest that SBAs can be a viable alternative in areas with smoother topography. However, this comes at the cost of omitting the influence of high-frequency gravitational effects caused by residual topographic masses.

Theoretically, complete Bouguer anomalies (CBAs) offer a significant advantage due to their smoother nature and reduced sensitivity to aliasing compared to simple and free-air anomalies. This advantage stems from removing high-frequency topographical information (terrain corrections) before gridding, effectively mitigating the aliasing problem in the input gravity data. While densifying gravity observations is another approach to minimize aliasing errors, its feasibility can be limited, particularly in areas with rough terrain or restricted observational access. In such cases, CBAs, calculated using Digital Elevation Model (DEM) data to account for the gravitational effects of topography, become a crucial alternative. DEM data also plays a vital role in the LSMSA method for geoid computation. Here, DEMs are used to calculate terrain and downward continuation corrections (DWC) that are then restored onto the low-frequency geoid component derived from a global geopotential model (GGM) (Işık et al., 2022; Kiamehr & Sjöberg, 2005; Tziavos & Sideris, 2013).

High-resolution digital elevation model (DEM) data acquired by the Global Shuttle Radar Topography Mission (SRTM) is employed in this thesis. SRTM3, which features a spatial resolution of 3 arc-seconds (approximately 90 meters), is freely accessible through the U.S. Geological Survey (Jarvis et al., 2006; USGS, 2018). The WGS84 datum is used for horizontal positioning of the DEM grids, while the EGM96 global geoid surface serves as the reference for height data. Local validation efforts conducted in diverse locations, including the Himalayan region of India (Mukul et al., 2017), Croatia (Varga & Bašić, 2015), and Turkey (Erol et al., 2020), have

corroborated this accuracy. Terrain corrections are computed using the TC module of the GRAVSOFIT geodetic gravity field computation software (Forsberg & Tscherning, 2014; Tscherning et al., 1992). The geoid models themselves are calculated using the ITU-LSMSA geoid computation software developed by the Istanbul Technical University Gravity Research Group (ITU-GRG).

In this thesis, the decision regarding whether to use simple or complete Bouguer anomalies is determined based on the study conducted in Section 6.2 using test data from Auvergne, France. Within this scope, an investigation is carried out to elucidate the influence of the selected Bouguer anomaly type on the gridding process and its subsequent impact on geoid model determination. This is accomplished by gridding both simple and complete Bouguer anomalies using the geostatistical Kriging interpolation algorithm, which is a prevalent technique in gravity field studies. Following the gridding process, gravimetric geoids using LSMSA technique are performed. It's important to note that the LSMSA approach in gravimetric geoid modeling requires gridded free-air anomalies on the Earth's surface. These are derived from existing gridded Bouguer anomalies by applying corrections that account for the effects of the Bouguer plate and terrain (Ellmann et al., 2019). Consequently, the statistical analysis of areal comparisons between the SBA and CBA grids at grid nodes, along with validations of the resulting geoid models within the study area, helped determine the most suitable type of Bouguer anomaly for the study.

As the second numerical test of the thesis given in Section 6.3, the determination of the most suitable parameters to be used during geoid computation in LSMSA method implemented through ITU-LSMSA software is conducted. The selection of optimal parameters in LSMSA for geoid calculation is crucial, especially concerning the integration cap size (Ψ_0), the expansion degree of the global geopotential model (M), and the upper limit of the Stokes function (L) (Abbak et al., 2012a, 2012b; Ellmann et al., 2019; Erol & Erol, 2007; Kiamehr, 2006; Sjöberg & Bagherbandi, 2017).

The impact of the interpolation method employed during the interpolation of CBAs on the resulting geoid is investigated in Section 6.4, constituting one of the central themes of the thesis. For this purpose, three commonly used techniques (Kriging, Inverse Distance to a Power, Nearest Neighbor) are implemented, along with a state-of-the-art soft computing algorithm, the Artificial Neural Network. Ordinary Kriging, a cornerstone method in geoid modeling alongside Least Squares Collocation,

represents the established approach. Software suites like KTH-Geolab and GRAVSOFT (Sjöberg, 2003a, 2003b, 2005; Tscherning, 2013, 2015; Yildiz et al., 2012) commonly integrate these methods into their data interpolation modules for precise geoid determination. Inverse Distance to a Power (IDP) and Nearest Neighbor methods offer a complementary approach, finding application not only in gravity research but also in various spatial data interpolation tasks. These simpler algorithms prioritize local data features through weighting (IDP) or boundary limitations (Nearest Neighbor), making them computationally efficient choices.

Complementing the three established interpolation methods employed in this study, a fourth approach is implemented - gridding complete Bouguer anomalies using an artificial neural network (ANN). As a learning-based soft computing algorithm, the ANN presents a distinct methodology compared to the previously tested techniques. Its inclusion serves the purpose of evaluating the performance of a recent generation interpolation algorithm within the context of gravity gridding, enabling a comparison with the conventional methods widely employed. Following this analysis, new geoid models are derived from the acquired free air anomaly grid datasets.

Here, four interpolation algorithms are tested considering statistics and difference maps derived from:

- the grid-wise comparisons of complete Bouguer anomalies on geoid surface (in mGal),
- grid-wise comparisons of free-air anomalies which have been restored from the gridded Bouguer anomalies (in mGal),
- grid-wise comparison of geoid models which have been calculated using free-air anomalies in grid form with the LSMSA geoid computation methods (in meter).

The magnitudes and statistical behavior of differences among the compared interpolation algorithms are provided by the statistics of the differences at grid nodes. Additionally, consideration is given to the difference-maps derived from these grid-wise comparisons to analyze the spatial distribution of differences in gridded values between the compared interpolation algorithms within the area. Thus, the spatial attitudes of differences between the tested gridding algorithms concerning the topographical pattern in the area could be visually interpreted.

Within these area-based (grid-wise) comparisons, Kriging is also identified as the "benchmark method" because it is one of the two common interpolation algorithms specifically employed in gravity gridding for both geodetic and geophysical purposes (the other method, least squares collocation, is quite similar to Kriging).

In addition to areal evaluations based on grid-wise comparisons, the geoid models, which serve as the final products of gridded gravity anomalies using each interpolation technique, are also assessed at GPS/leveling benchmarks. Without validation statistics using an independent dataset, the absolute accuracies of the calculated geoids cannot be estimated. In this study the GPS/leveling benchmarks are used as independent control data. GPS/leveling validation tests quantify the quality of calculated regional geoid models in an absolute manner and demonstrate their suitability and usability in large-scale mapping and engineering application projects where the models are tested.

Due to the concentration of GPS/Leveling benchmarks within low-topography regions of the study area, the obtained test statistics primarily reflect the accuracy of geoid models in plain terrains. This limitation arises from the inherent challenges associated with leveling measurements, as practical constraints make it difficult to conduct leveling procedures in areas with rough topography. In Section 6.5, where the fourth numerical tests are conducted, the investigation focuses on how the density and distribution of GPS/Leveling benchmarks affect the absolute validation process. To this end, the study area is divided into two categories: plain and rough terrain. For the validation of each category, distinct benchmarks are utilized, specifically those deemed to better represent the respective study area.

As the final numerical test of the thesis (Section 6.6), the gridding of complete Bouguer anomalies (CBAs) is conducted using both different epoch-neuron parameters and the same parameters with the Artificial Neural Network (ANN) algorithm. The consistency of the results is examined to evaluate the standardization of the method. Thus, the capability of the ANN algorithm to replace interpolation methods in gravity gridding processes is tested.

In contrast to Chapter 6, which delved into numerical tests, overview to the study area, and data through dedicated subsections, the remaining five chapters focus on other aspects of the research. In Chapter 1 (Introduction), the study's objectives and relevance are presented, along with an overview of the recent regional geoids in

Turkey and modernization efforts. Chapter 2 presents height systems and the gravity field, whereas Chapter 3 focuses on gravimetric reductions, briefly explaining the theories behind them. In Chapter 4, various gravity data interpolation algorithms are examined to assess their impact on geoid determination. Chapter 5 outlines the basics of gravimetric geoid determination. Various gravimetric geoid determination methods are briefly mentioned here, while the LSMSA technique employed in the study is thoroughly explained with its accompanying formulas. In the Conclusion part (Chapter 7), the statistics and visuals pertaining to the results of all tests conducted within the scope of the thesis are evaluated to determine whether the study objectives have been achieved and how conclusions are drawn from these results. Lastly, the regression and performance analyses of CBAs gridded with the ANN algorithm, along with the examination of the distributions of the test/validation points used during the gridding process, are conducted in the Appendices part. Additionally, visual representations of CBAs grids generated with different parameters are also provided in this section.

1.2 The Latest Regional Geoid Models in Turkey

This section delves into an examination of some of the historical and contemporary geoid models employed in Turkey. The last four regional gravimetric geoids of Turkey are TG-99A, TG03, TG07, and TG09, respectively. The geoid known as Updated Turkish Geoid 1999 (TG-99A) is a hybrid model constructed by combining TG-91 and GPS-Leveling data distributed across the country, and computed utilizing the Least Squares Collocation (LCS) technique. The accuracy of this model, which was achieved with more than 62,000 terrestrial gravity data points, is approximately $\pm 10\text{-}15$ cm (Kılıçoğlu, 2002).

The accuracy of Turkish Geoid 2003 (TG-03), calculated using a similar amount of terrestrial gravity data as TG-99A and employing the LSC technique, is also approximately ± 10 cm. In the computation of this model, the EGM96 global geopotential model and sea gravity data obtained from satellite altimetry measurements were utilized (Kılıçoğlu et al., 2005).

Afterward, Turkish Geoid 2007 (TG-07), which has an accuracy of ± 8.8 cm, was computed using the Fast Fourier Transformation (FFT) method (Yildiz et al., 2006). In the computation of this model, EGM96 and Grace Gravity Model (GGM02S),

Shuttle Radar Topography Mission-3 (SRTM3), and KMS02 gravity anomalies were utilized (Yildiz, 2012).

Following the TG-07 geoid, Turkish Geoid 2009 (TG-09) was computed using the Fast Fourier Transform (FFT) technique and incorporating over 260,000 local gravity data points and the EGM08 model. Despite employing four times more local data than its predecessors, this model's accuracy was computed as ± 8.4 cm (Kılıçoğlu et al., 2011; Simav et al., 2021). Işık et al. (2022) and Yıldiz (2012) attribute the lack of improvement in model accuracy despite the data increase to errors in local gravity measurements conducted by various institutions, the uneven distribution of gravity data, and possible inaccuracies in the Turkish National Vertical Control Network (TNVCN).

Finally, as part of the height system modernization efforts, Turkish Geoid 2020 (TG-20) was generated (HGM, 2021). Validation using historical GPS/Leveling data yielded a model accuracy of ± 9 -10 cm (Işık et al., 2022). However, Yıldiz et al. (2021) reported a model accuracy of ± 1.2 -6.3 cm when validated with newly collected GPS/Leveling data. A more detailed description of the modernization work and the TG-20 geoid is provided in Section 1.3.

1.3 Modernization of the Height Reference Systems in Turkey

A critical challenge has emerged within the Turkish National Vertical Control Network (TNVCN): its accuracy and functionality are no longer sufficient to meet modern demands. More than 80% of benchmarks are damaged due to urbanization, and crustal movements have further compromised the remaining ones. While geoid heights, representing the distance between the ellipsoid and the geoid, are used in conjunction with GNSS heights for calculating physical heights and should have an accuracy of 1-3 cm (even sub-centimeter), the accuracy of the geoid model is not at the desired level due to issues with gravity data and GNSS/levelling data. The previously used Turkish Geoid Model-2003 (TG-03) is far from achieving this accuracy (± 10 cm). Additionally, CORS-TR, which provides horizontal position information with centimeter-level accuracy in real-time, cannot be used for determining physical heights with similar sensitivity. As seen, the need for modernization of the country's height system has emerged due to its inability to meet the requirements (HGM, 2021; Simav et al., 2015; Türkezer et al., 2021).

Recognizing the limitations of the TNVCN, a collaborative effort to modernize the system was undertaken between 2015 and 2020 through the “Turkish Height System Modernization and Gravity Recovery Project”. This project involved key national agencies: General Directorate of Mapping, General Directorate of Mineral Research and Exploration, Turkish Petroleum Corporation, TÜBİTAK Marmara Research Center, and TÜBİTAK National Metrology Institute (HGM, 2021, Yildiz et al., 2021).

The “Modernization of the Turkey Height System and Improvement of Gravity Infrastructure Project” comprises ten work packages encompassing various aspects of the modernization effort (HGM, 2021):

- Procurement of measurement equipment, instruments, and ground point markers from domestic and international sources
- Establishment of national-level relative gravity measurement and evaluation standards, pre-field training, testing, and calibration activities
- Relative gravimetry studies
- Absolute gravimetry studies
- Airborne gravimetry studies
- GNSS/Levelling studies
- Quality control and verification of historical gravity data
- Establishment of the national gravity database
- Geoid modeling
- Dissemination of the geoid model via the CORS-TR system, publication of the project's final report, and official announcement of the new vertical reference system to users.

Within the scope of the project, the transformation of the TG-20 (Turkish Geoid Model - 2020), produced with 1' and 5' arcminute spatial resolutions using the Least Squares Modification of Stokes Formula with Additive Corrections (LSMSA) gravimetric geoid determination method, into the existing height system was carried out through a 4-parameter fit to 182 historical and homogeneously distributed GPS/levelling data. The absolute validation of the model utilized 278 highly accurate GPS/Levelling points, also generated as part of the project, achieving an accuracy with a standard

deviation of less than 3 cm. The standard deviation value, occasionally reaching 1.2 cm, can rise up to 6.3 cm, particularly in coastal areas. The project developed a new hybrid geoid model specifically tailored for the region between 35°N and 43°N latitude and 25°E to 45°E longitude. This model leverages a comprehensive dataset encompassing: Over 265,000 terrestrial gravity data points, GOCO06S global geopotential model, and SRTM V4.1 digital elevation model with a resolution of 7.2 arcseconds. Furthermore, for marine areas and neighboring countries, free-air gravity anomalies were calculated at a resolution of 1 arcminute by 1 arcminute using the high-degree XGM2019e model (up to degree and order 5399) (Yildiz et al., 2021).

Thanks to this currently official geoid model for Turkey (TG-20) produced by the General Directorate of Mapping, obtaining real-time physical heights with an accuracy of 1-3 cm has become possible by utilizing systems such as GNSS, Lidar, radar altimeters, thus replacing the costly and time-consuming geometric levelling. To facilitate the widespread utilization of the Turkish Geoid Model 2020 (TG-20), the General Directorate of Mapping has made it readily accessible through a Web Coverage Service (WCS). This service offers users the flexibility to download the model data at two distinct resolutions: 1 arcminute and 5 arcminutes (free of charge), catering to specific project requirements.

The accuracy of the TG-20 model mentioned above relies on the validation statistics provided by (Yildiz et al., 2021). This validation utilized recently acquired high-precision GPS/Leveling data. (Işık et al., 2022), on the other hand, stated that the model accuracy in validation using historical GPS/Leveling data is $\pm 9-10$ cm.

The project established a robust gravity infrastructure by leveraging the high-sensitivity and evenly distributed gravity data acquired during the project alongside meticulously controlled historical gravity measurements. Terrestrial and airborne gravimetry studies scheduled for the coming years aim to make necessary improvements, particularly in areas where the model's accuracy is low (Yildiz et al., 2021). The recent Turkish geoid (TG-20) is illustrated in Figure 1.1.

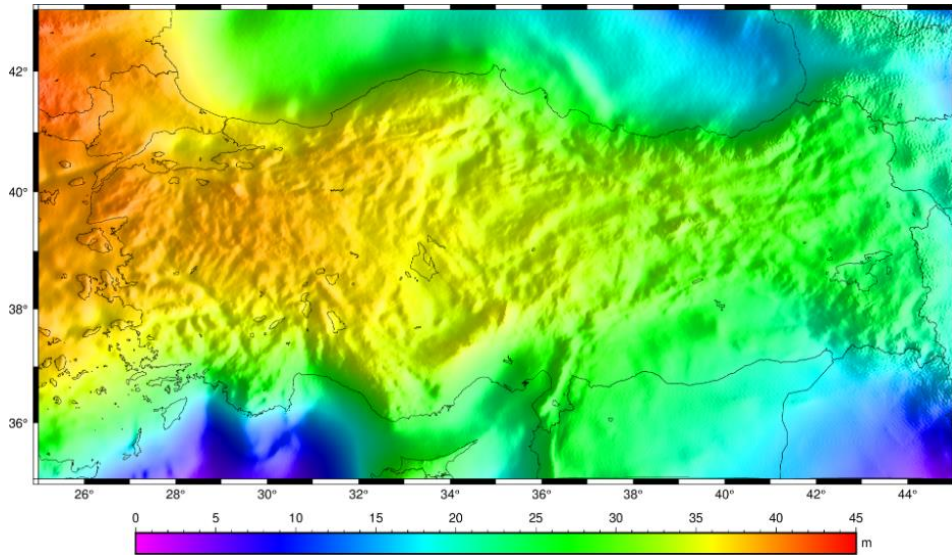


Figure 1.1 : Turkey Geoid Model-2020 (TG-20). Unit: m (Yildiz et al., 2020).

Canada stands out as one of the countries that have completed the modernization work. Until 2015, Canada utilized the Canadian Geodetic Vertical Datum of 1928 (CGVD28), which is based on leveling, along with the HTv2 geoid derived from it. Despite the high local accuracy of this datum, Canada embarked on a modernization effort to transition to a geoid-based datum due to its low national accuracy, as well as the expensive and time-consuming nature of spirit leveling.

This former reference system, employing normal-orthometric heights (CGVD28), underwent a modernization effort, resulting in the establishment of the new system, the Canadian Geodetic Vertical Datum of 2013 (CGVD2013). Notably, heights in CGVD28 continue to be published. With the implementation of the Canadian Gravimetric Geoid Model of 2013-version A (CGG2013a), an integral component of the new geoid-based datum CGVD2013, the use of orthometric heights commenced (NRCAN, 2020). For the properties of the geoid and its difference from CGG2013, see (Huang & Véronneau, 2016).

The U.S. National Geodetic Survey is currently in the process of modernizing its National Spatial Reference System (NSRS). They intend to replace NAD83 with the North American Terrestrial Reference Frame of 2022 (NATRF2022) as their national geometric reference system. This transformation, which was planned to be carried out in 2022, has been postponed to 2025 (NGS, 2024). When the U.S. transitions to this reference frame, it is expected that Canada will also adopt this reference frame due to the anticipated meter-level horizontal coordinate difference at the America-Canada border.

2. EARTH GRAVITY FIELD BASED HEIGHT SYSTEMS

2.1 An Overview of Height Systems

A height system functions as a unidimensional coordinate system, establishing the metric distance of a point relative to a designated reference surface along a predetermined vertical path (Featherstone & Kuhn, 2006). There are numerous height systems based on different vertical datums. The selection of which height system to use is determined by considering factors such as data distribution and density, ease of calculation, compatibility with GNSS, the intended applications, accuracy requirements, and the topography of the application area (Dennis & Featherstone, 2003). One of the criteria that affects the selection is whether these selected height systems are similar to those of neighboring countries or can be converted to each other's systems with high accuracy. This ensures consistency in the engineering studies they undertake collectively or in any application within the country's borders.

Height systems can be roughly divided into two according to whether they take the Earth's gravity field into account or not. Gravity based height systems are physically meaningful since they are naturally linked to the equipotential surfaces and plumb lines of the Earth's gravity field. The behavior of fluid flows is accurately displayed in these height systems. National height systems typically employ physical heights, whereas applications like local surveying, construction, and mapping often utilize geometric heights (Sjöberg & Bagherbandi, 2017).

In this context, geopotential numbers, dynamic heights, orthometric heights, normal heights, and normal-orthometric heights incorporate the Earth's gravity field, whereas ellipsoidal heights do not account for it.

The distance from a point on the Earth's surface to the ellipsoid along the ellipsoid normal is called the **ellipsoidal height (h)**. Ellipsoidal heights, a geometric height system also known as geodetic heights, can be obtained quickly and precisely using satellite based positioning systems. Heights derived from GNSS or leveling methods are referred to as geometric heights. It is essential to bear in mind that the universal

applicability of GNSS (Global Navigation Satellite Systems) is limited. Although systems such as GPS, GLONASS, Galileo, and BeiDou provide coverage over a significant portion of the Earth's surface, signal reception may be impeded or weakened in certain environments, including indoor settings, urban areas with tall buildings, dense forest regions, and deep valleys. Furthermore, atmospheric conditions and deliberate interference can lead to disruptions in signal reception. In this type of height system, the reference surface is an ellipsoid described by parameters such as location (coordinates), the semi-major (polar radius), semi-minor (equatorial radius) axes, flattening, and eccentricity.

The physically significant height systems adopt mean sea level as a reference, which delineates the approximate boundaries of the geoid, representing the Earth's shape. Mean sea level commonly aligns with an equipotential surface characterized by a constant gravity potential denoted as W_0 . It should be noted that sea levels are not consistently stable due to variations in temperature, salinity, and particle content in different locations. Additionally, differences of up to 2 meters can exist between the geoid and mean sea level in certain areas (Vaniček et al., 2012).

Geopotential numbers, which represent physically meaningful height systems, identify the difference in potentials between the point on the geoid (W_0) and the point P on the Earth's surface (W_P), denoted as C_P , is formulated as follows:

$$C_P = W_0 - W_P = \int_{P_0}^P g dH \quad (2.1)$$

where W_0 is the constant potential value at geoid, P is the point at the Earth's surface and W_P is its potential, P_0 is the point on the geoid, dH is differential height difference, g is observed gravity on Earth's surface and the unit of geopotential number is kGal m (Erol, 2007; Ince, 2011; Torge, 2001). Geopotential numbers are independent from leveling path (see Figure 2.1):

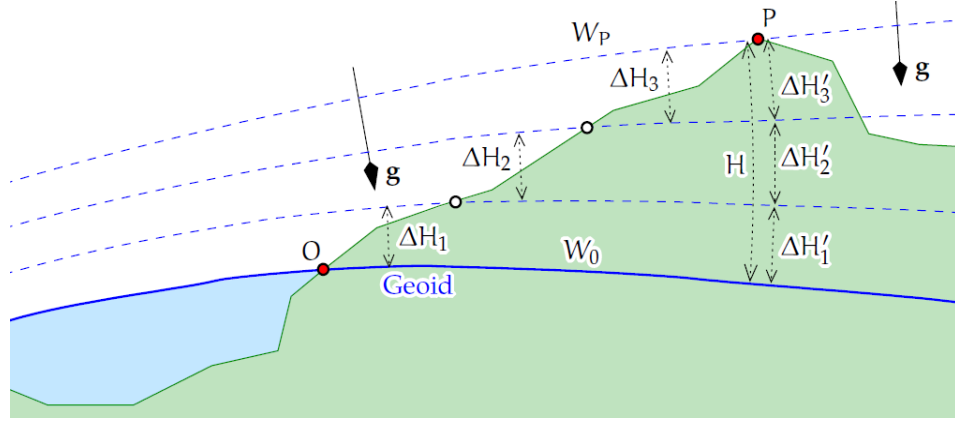


Figure 2.1 : Geopotential numbers and levelled heights (Vermeer, 2020).

In light of the fact that geopotential numbers lack dimensional units of length, **dynamic heights** are computed by dividing the geopotential number by a constant value representing gravity.

$$H_P^{dyn} = \frac{C_P}{\gamma_0} \quad (2.2)$$

γ_0 is generally the normal gravity at latitude 45° . Dynamic heights are physically meaningful as they accurately indicate the direction of water fluid, yet they lack geometrical significance. To clarify, dynamic heights on identical equipotential surfaces are equal, while geometrical distances between them differ. Moreover, dynamic height corrections to be applied to levelled heights can reach large values (Featherstone & Kuhn, 2006; Torge & Müller, 2012, Yiğit, 2003).

Orthometric heights, which are used in many engineering applications today and preferred as the national height system in many countries, are the distance between the geoid and the point on the Earth's surface along the plumblines. Height differences obtained by geometric leveling are path dependent because the level surfaces are not parallel and the differences do not form a closed loop (see Figure 2.1). Using potential differences that are independent of the path, these practical heights are described by:

$$H_P = \frac{C_P}{\bar{g}_P} \quad (2.3)$$

$\bar{g}_P$ presents the mean value of the gravity between point P on the Earth's surface and geoid along the plumblines, which is obtained by the following formula:

$$\bar{g}_P = \frac{1}{H_P} \int_{P_0}^P g dH' \quad (2.4)$$

The topographical mass density distribution is unknown along the plumbline thus the gravity value cannot be measured between the Earth's surface and the geoid. As a result, calculations are made using the mean gravity value. The Helmert, which is adopted by many countries, Niethammer and Mader models are used to compute mean gravity (Santos et al., 2006). To explore a comparison of these methods, refer to (Dennis & Featherstone, 2003). The equations for Helmert mean gravity based on Poincare-Prey reduction and Helmert orthometric heights are given below respectively (Ellmann et al., 2006; Foroughi et al., 2017; Heiskanen & Moritz, 1967):

$$\bar{g}_p^H = g_p + 0.0424H_p \quad (2.5)$$

$$H_p^H = \frac{C_p}{g_p + 0.0424H_p} \quad (2.6)$$

Normal height, defined as the distance between a projected point on the telluroid and a point on the reference ellipsoid along normal gravity plumbline, was introduced by Molodensky, who argued that orthometric height could not be theoretically employed. This measurement is obtained using the following equation:

$$H_p^N = \frac{C_p}{\bar{\gamma}_p} \quad (2.7)$$

where $\bar{\gamma}_p$ is mean normal gravity along plumbline. Normal heights can also be calculated using height anomaly (ζ_p) and ellipsoidal height (h_p) as follows:

$$H_p^N = h_p - \zeta_p \quad (2.8)$$

Height anomaly is the distance from Earth's surface to telluroid along ellipsoidal normal. The quasigeoid introduced by Molodensky, is a theoretical surface of constant gravitational potential that closely approximates the actual geoid. It is not an equipotential surface and does not require a mass density distribution (Sjöberg, 2018a). The telluroid represents a surface that closely resembles the Earth's surface but is shifted from it by the quasigeoidal height (Vaniček et al., 2012).

The distance between quasigeoid and point on Earth's surface along normal plumbline is identified as **normal-orthometric heights**. Here, orthometric heights are computed using quantities belonging to normal gravity field (Sjöberg & Bagherbandi, 2017).

$$H_p^{NO} = \frac{\widetilde{C}_p}{\bar{\gamma}_p} \quad (2.9)$$

where approximated geopotential number ($\widetilde{C}_P$) is computed as follows:

$$\widetilde{C}_P = U_{P_0} - U_P = \int_{P_0}^P \gamma dH \quad (2.10)$$

The terms such as reference surfaces, heights, height anomaly, telluroid, quasigeoid are illustrated in Figure 2.2.

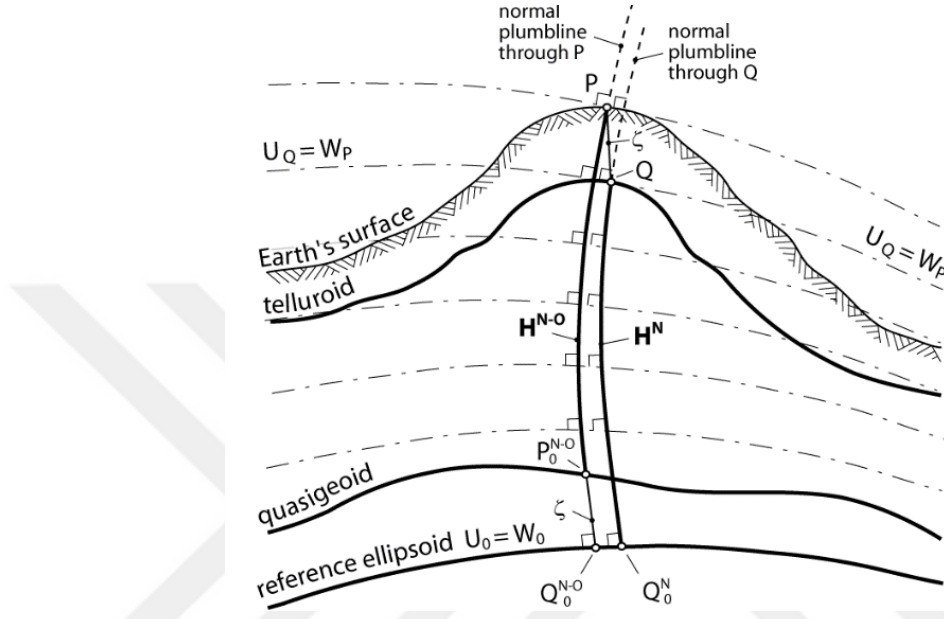


Figure 2.2 : The reference surfaces and height systems (Featherstone & Kuhn, 2006).

2.2 Gravity Field Related Quantities

Quantities such as gravity potential, disturbing potential, vertical gradient of gravity, normal gravity, gravity anomaly, gravity disturbance, and geoid height are described here, along with their basic formulas. The expression of gravity potential (W) arises from the summation of gravitational potential (V) and centrifugal potential (Φ).

$$W = V + \Phi \quad (2.11)$$

Equipotential surfaces are shown as $W(x,y,z) = W_0 = \text{constant}$. The Earth's gravity potential can be calculated as the sum of the disturbing potential (T) and the normal potential (U_0), which represents the reference potential of the gravity field assuming the Earth's surface conforms to an ellipsoidal shape.

$$W = U_0 + T \quad (2.12)$$

Vertical gradient of gravity ($\partial g/\partial z$), which describes the change in gravity with height, is computed as follows (Torge & Müller, 2012):

$$\frac{\partial g}{\partial z} = -2gJ + 4\pi G\rho - 2\omega^2 \quad (2.13)$$

where z is coordinate representing height, ρ is mass density, ω^2 is angular velocity and J is mean curvature of level surface.

The normal gravity (γ_0) computed on the reference ellipsoid is calculated using Somigliana formula in Equation (2.14) (Hackney & Featherstone, 2003; Kuhn & Featherstone, 2009):

$$\gamma_0 = \gamma_e \frac{1+k \sin^2 \varphi}{\sqrt{1-e^2 \sin^2 \varphi}} \quad (2.14)$$

In the expression above, γ_e denotes the normal gravity at the equator ($\varphi=0$), φ represents the geocentric latitude, k is the normal gravity constant, and eccentricity e is formulated as $e^2 = 1 - \frac{b^2}{a^2}$, where a and b represent the semi-major and semi-minor axes of the reference ellipsoid, respectively. The parameters of the GRS80 (Geodetic Reference System 1980) ellipsoid are employed in this investigation for calculating normal gravity; the relevant parameters can be found in Table 2.1 (Moritz, 1980).

Table 2.1 : GRS80 (Geodetic Reference System 1980) reference ellipsoid parameters.

Constant Parameters	Definitions	GRS80
a	Semi-major axis	6,378,137.0000 m
b	Semi-minor axis	6,356,752.3141 m
γ_e	Normal gravity at equator	9.7803267715 ms ⁻²
γ_p	Normal gravity at pole	9.8321863685 ms ⁻²
GM	Geocentric gravitational constant	3,986,005 × 10 ⁸ m ³ s ⁻²
e^2	First eccentricity	0.00669438002290
k	$(\frac{b\gamma_p}{a\gamma_e} - 1)$	0.001931851353
f	Flattening ($\frac{a-b}{a}$)	0.00335281068118

Gravity anomaly or gravity disturbance is utilized in geoid determination studies; further investigations are conducted in Section 5. The difference between the observed gravity value at a point on the geoid surface (g_{P_o}) and the normal gravity at its corresponding point on the ellipsoid (γ_{Q_o}) is described as the gravity anomaly on the geoid (Δg); see Equation (2.15) below (Heiskanen & Moritz, 1967; Torge, 2001; Tziavos & Sideris, 2013):

$$\Delta g = g_{P_o} - \gamma_{Q_o} \quad (2.15)$$

The difference between actual (g_P) and normal gravity (γ_P) values on point P is defined as gravity disturbance (δg_P). Figure 2.3 illustrates the gravity anomaly and gravity disturbance on reference surfaces.

$$\delta g_P = g_P - \gamma_P \quad (2.16)$$

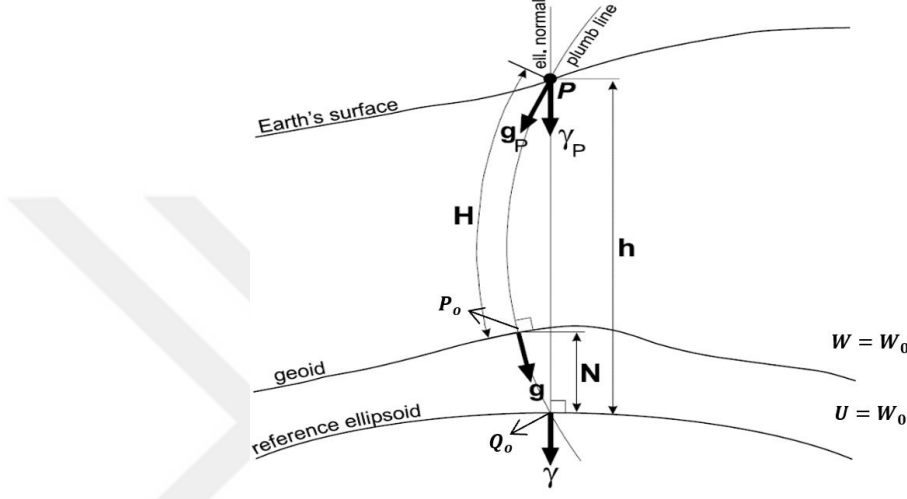


Figure 2.3 : Depiction of parameters delineating gravity anomalies and the gravity disturbance (Hackney & Featherstone, 2003).

As a final gravity field quantity, geoid height can be computed using the Bruns formula, which describes the relationship between geoid height and disturbing potential.

$$N = \frac{T_P}{\gamma} \quad (2.17)$$

For detailed representations of the formulas provided in this section, refer to Torge (2001), Sjöberg & Bagherbandi (2019) and Vermeer (2020).



3. GRAVITY REDUCTION METHODS

3.1 Bouguer and Free-air Reductions

Observed gravities on the Earth's surface are influenced by topographical masses. Hence, they must be continued downward to the geoid, which serves as a reference surface, in order to collectively assess them. The predominant techniques within the discipline of geodesy for this process, referred to as reduction, encompass free-air and Bouguer reductions which are utilized for examining geoid determination, interpolating gravity data, and geophysical explorations (Torge & Müller, 2012).

In the process of free-air reduction, observed gravity is downwardly adjusted to the geoid through the utilization of the vertical gradient of gravity. When computing free-air reduction (δg_F), the normal gradient of gravity linked with the ellipsoidal height ($\partial\gamma/\partial h$) is employed instead of $\partial g/\partial H$, offering satisfactory precision for numerous practical applications (Hofmann & Moritz, 2005, p. 134). The following equations present free-air reduction and the free-air anomaly (Δg_{FAA}), respectively (Tziavos & Sideris, 2013).

$$\delta g_F = -\frac{\partial g}{\partial H}H \cong -\frac{\partial \gamma}{\partial h}H_P = 0.3086H_P \quad (3.1)$$

$$\Delta g_{FAA} = g_{P_o} - \gamma_{Q_o} + \delta g_F \quad (3.2)$$

In most gravimetric geoid determination algorithms, gridded free-air anomalies serve as the primary input instead of observed gravity values. Although pointwise gravity data observed on Earth's surface cannot be directly interpolated due to their inclusion of high-frequency components of the Earth's gravitational field, they are transformed into simple or complete (refined) Bouguer anomalies, which possess smoother characteristics suitable for interpolation processes. Furthermore, Bouguer anomalies play a significant role in elucidating geological structures within the field of geophysics (Abbak et al., 2012b; Abbak & Ustun, 2015; Karaca et al., 2024; Kuhn et al., 2009; Märdla, 2017). In Bouguer reduction, the topography is conceptualized as a plate (or shell), and the gravitational effects of all masses within this plate, positioned

above the geoid, are computed and subtracted from the observed gravity. In principle, while free-air anomalies (FAAs) are subject to greater influence from the aliasing effect in contrast to simple Bouguer anomalies, complete Bouguer anomalies are posited to exhibit heightened smoothness and reduced susceptibility to adverse aliasing effects compared to both types of anomalies.

An illustration of a Bouguer plate is provided in Figure 3.1 below (Heiskanen & Moritz, 1967; De Gaetani et al., 2021). In this figure, the thickness of the Bouguer plate, denoted as H_P , corresponds to the height of the topographic point P (Bajracharya, 2003; Tziavos & Sideris, 2013).

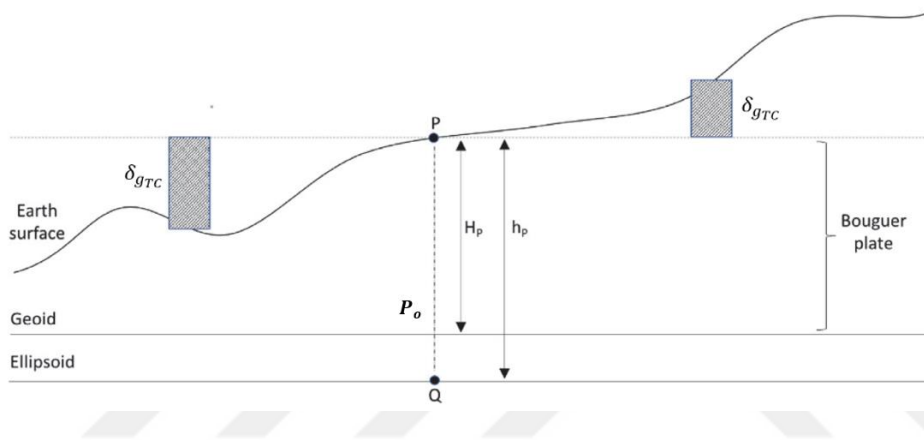


Figure 3.1 : Depiction of a Bouguer plate with a thickness denoted as H_P , featuring residual topographical masses deviating from the plate, which necessitate the computation of terrain corrections (δg_{TC}) during the Bouguer reduction process.

Simple Bouguer anomalies (Δg_{SB}) can be determined as demonstrated in Equation (3.3) (Torge, 2001; Kiamehr & Sjöberg, 2005; Sjöberg & Bagherbandi, 2017):

$$\begin{aligned}\Delta g_{SB} &= g_{P_o} - \gamma_{Q_o} + \delta g_F - \delta g_B \\ &= g_{P_o} - \gamma_{Q_o} + 0.1967H_P\end{aligned}\quad (3.3)$$

Here, δg_B represents the Bouguer plate reduction parameter, which is computed using the Equation (3.4) provided below for its planar version.

$$\delta g_B = 2\pi G\rho H_P \quad (3.4)$$

where G represents the gravitational constant ($G = 6.672585 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$). In the calculations, the topographical mass density, denoted as ρ is commonly presumed to remain constant at $\rho = 2670 \text{ kg m}^{-3}$. Consequently, upon substituting these constants into the above formula, it is reformulated as demonstrated in Equation (3.5).

$$\delta g_B = 0.1119H_P \quad (3.5)$$

Complete Bouguer Anomalies (Δg_{CB}) refine the concept of simple Bouguer anomalies by incorporating a terrain correction term (δg_{TC}) to account for the gravitational influence of residual topography deviating from the idealized Bouguer plate model (Figure 3.1). This correction, as formulated in Equation (3.6), addresses the limitations of the flat reference surface assumption in the Bouguer plate model (Torge, 2001; De Gaetani et al., 2021).

$$\Delta g_{CB} = g_{P_o} - \gamma_{Q_o} + \delta g_F - \delta g_B + \delta g_{TC} \quad (3.6)$$

The most troublesome (laborious) part of complete Bouguer anomalies' computations is calculating the "terrain corrections since its calculation requires running complex multiple-integrals (see Eq. 3.7) using high resolution topographic data, and therefore takes long time and requires high capacity processors etc. Equation (3.7) presents the mathematical formulation for terrain correction (δg_{TC}) (Forsberg, 1984; Işık et al., 2022).

$$\delta g_{TC} = G \iint_{\sigma} \int_{H_P}^z \frac{\rho(x, y, z)(z - H_P)}{\ell^3(x_P - x, y_P - y, z - H_P)} dz d\sigma \quad (3.7)$$

In this equation, H_P represents the topographic height at the computation point. The variable z corresponds to the height of a running point within the integration area (σ) and ρ denotes the density of the topographical masses. Equation (3.8) below details the calculation of ℓ , the distance separating the computation point and the running point.

$$\ell = \sqrt{(x_P - x)^2 + (y_P - y)^2 + (H_P - z)^2} \quad (3.8)$$

The planar Bouguer reduction formulas are utilized in this thesis study since the computation area possesses a reasonable size, thereby fulfilling the objectives in this research. A spherical approximation, however, is also employed to formulate the gravitation of topographic masses using Bouguer reduction. The Bouguer shell correction (δg_B^s) within the spherical approach is presented in Equation (3.9).

$$\delta g_B^s = 4\pi G \rho H_P \quad (3.9)$$

The computation of terrain corrections in spherical Bouguer reduction relies on global topography data, presenting a considerable challenge. For a comprehensive understanding of the spherical Bouguer reduction framework, refer to Kuhn et al.

(2009). Comparative analyses of planar and spherical approaches in Bouguer reduction, along with discussions, are provided in studies by Abbak et al. (2012c), Vaniček et al. (2001, 2004), Erol et al. (2023), Hackney & Featherstone (2003), and Tziavos & Sideris (2013).

The accuracy of the gridding process in terrain reduction is directly influenced by the resolution and accuracy of the Digital Elevation Model (DEM) data used to obtain height information for Bouguer plate correction and terrain correction parameter computations (Bajracharya, 2003; Goos & Featherstone, 2003; Goyal et al., 2021; Tziavos & Sideris, 2013). Furthermore, as elaborated in Section 5.2, DEM data serves as a crucial input for calculating the downward continuation effect within the LSMSA method employed for geoid modeling (Kiamehr & Sjöberg, 2005, Varga et al., 2019).

3.2 Isostatic Reduction

Considering that Bouguer anomalies don't approach zero, with substantial negative values observed in mountainous regions and considerable positive values detected in seas, it becomes apparent that topographic masses do not maintain uniform density and thickness. Surplus and missing topographic masses are balanced through the equilibrium of the Earth's crust floating on the mantle, a phenomenon known as isostasy. In geodetic applications, there are two main isostatic reduction methods commonly used, denoted as Pratt-Hayford and Airy-Heiskanen (Kılıçoğlu et al., 2010).

In the Pratt-Hayford approach, as asserted by J. H. Pratt and formulated by J. F. Hayford, there exists a compensation layer with a constant thickness value ($D=100$ km) between compensation level and geoid (mean sea level). The height of compensation layer, denoted as $H = 0$ (normal column), maintains a constant density value (ρ_0), while oceanic columns have higher densities and continental columns have lower densities (see Figure 3.2 below) (Torge & Müller, 2012; Vermeer, 2020).

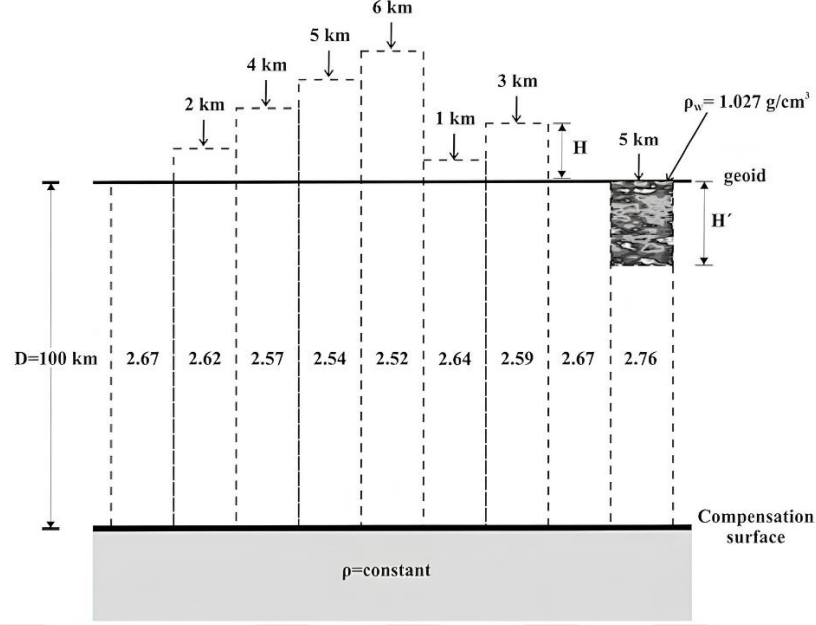


Figure 3.2 : Pratt-Hayford isostatic reduction scheme where ρ_w represents the density of water, H' is height of oceanic column and H is height of continental column (Tziavos & Sideris, 2013).

Pratt-Hayford isostatic gravity anomalies can be computed using following Equation (3.10):

$$\Delta g_{PH} = g_p - \gamma_{Q_0} + \delta g_F - A_{top(PH)} + A_{comp(PH)} \quad (3.10)$$

where g_p is observed gravity at Earth's surface, γ_{Q_0} is normal gravity at ellipsoidal surface, $A_{top(PH)}$ is gravitational attraction of topographical masses and $A_{comp(PH)}$ denotes gravitational attraction of compensated masses whose equations are given as follows, respectively (Sjöberg & Bagherbandi, 2017; Tziavos & Sideris, 2013).

$$A_{top(PH)} = G \iint_E \int_0^H \frac{\rho(x, y, z)(H_p - z)}{[(x_p - x)^2 + (y_p - y)^2 + (H_p - z)^2]^{3/2}} dx dy dz \quad (3.11)$$

$$A_{comp(PH)} = G \iint_E \int_{-D-H_p}^{-H_p} \frac{\Delta \rho(x, y, z)(H_p - z)}{[(x_p - x)^2 + (y_p - y)^2 + (H_p - z)^2]^{3/2}} dx dy dz \quad (3.12)$$

In Airy-Heiskanen method, whose hypothesis was developed by G.B. Airy and mathematical formulas were formulated by V.A. Heiskanen, normal crust thickness (T_0) is 30 km between compensation surface and geoid ($H = 0$). The density of topographic masses is constant ($\rho_0 = 2.67 \text{ g/cm}^3$) whereas it is 1.027 g/cm^3 for sea (ρ_w) and 3.27 g/cm^3 for mantle (ρ_m). Here, crusts have varying thickness and higher masses such as mountain sink deeper. In this respect, it can be said that the Airy-Heiskanen

method is a more realistic method than Pratt-Hayford (Erol, 2007; Sjöberg & Bagherbandi, 2017; Torge, 2001).

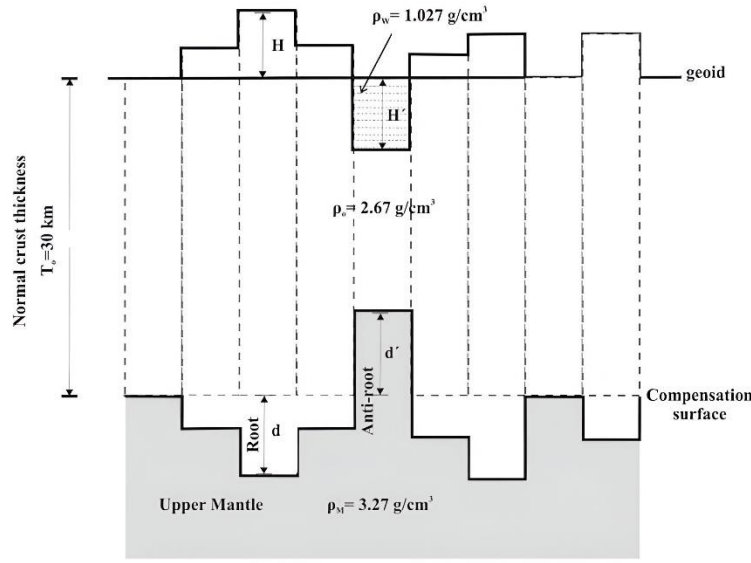


Figure 3.3 : Airy-Heiskanen isostatic reduction scheme where d represents the thickness of the root and d' is thickness of anti-root (Tziavos & Sideris, 2013).

As seen in Figure 3.3, there are roots (d) below the mountains and anti-roots (d') below the oceans. The following Equation (3.13) gives Airy-Heiskanen isostatic gravity anomalies:

$$\Delta g_{AH} = g_p - \gamma_{Q_0} + \delta g_F - A_{top(AH)} + A_{comp(AH)} \quad (3.13)$$

The gravitational attraction of topographical masses ($A_{top(AH)}$) and gravitational attraction of compensated masses ($A_{comp(AH)}$) are computed as follows, respectively (Tziavos & Sideris, 2013):

$$A_{top(AH)} = G \iint_E \int_0^H \frac{\rho(x, y, z)(H_p - z)}{[(x_p - x)^2 + (y_p - y)^2 + (H_p - z)^2]^{3/2}} dx dy dz \quad (3.14)$$

$$A_{comp(AH)} = G \iint_E \int_{-T_0-d-H_p}^{-T_0-H_p} \frac{\Delta \rho(x, y, z)(H_p - z)}{[(x_p - x)^2 + (y_p - y)^2 + (H_p - z)^2]^{3/2}} dx dy dz \quad (3.15)$$

3.3 Helmert's Second Method of Condensation

Stoke's integral, founded by G.G. Stokes in 19th century and used for geoid determination using gravity measurements, requires gravity data on geoid. When applying this method, which asserts the absence of any mass on the geoid, gravity

anomalies measured over the topography are reduced to the geoid using various techniques. Helmert's second method of condensation, asserted by Friedrich Robert Helmert, is one of the reduction approaches applied. In this method, topographic masses are shifted in an infinitesimal condensation layer on the geoid along the plumbline (Bajracharya, 2003; Ellmann & Vaniček, 2007; Ellmann et al., 2006; Janak et al., 2017; Vermeer, 2020).

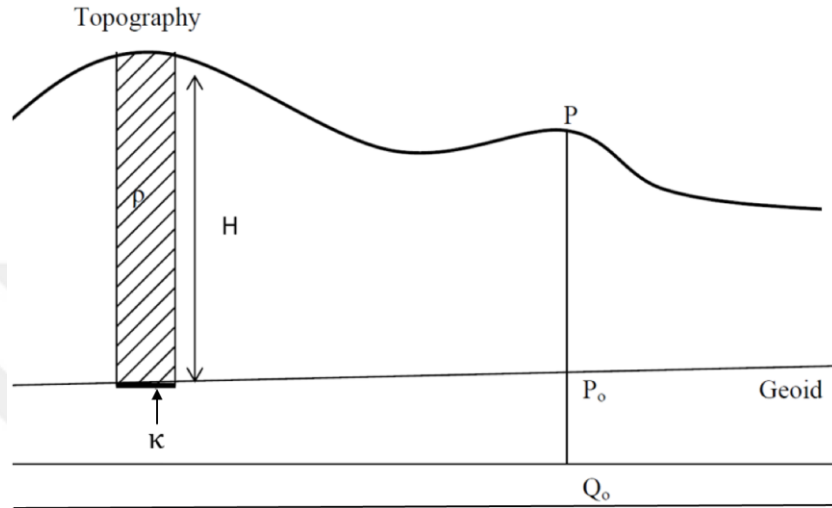


Figure 3.4 : Helmert's second condensation model (Erol, 2007)

The density relationship between the condensation layer and topographical masses can be established using height as provided in the Equation (3.16) below (Vermeer, 2020):

$$\kappa = H\rho\left(1 + \frac{H}{R}\right) \quad (3.16)$$

where κ is density of condensation layer, ρ is mean density of topographic masses, H is height of the topography and R is Earth's radius. There is no mass removal as in Bouguer reduction and Helmert gravity anomalies, known as Faye anomalies, diverge from free-air anomalies due to the inclusion of terrain correction. Furthermore, Helmert gravity anomalies exhibit a stronger correlation with height compared to free-air gravity anomalies due to their minimal indirect impact on the geoid (Tziavos & Sideris, 2013).

$$\Delta g_{Helmert} = \Delta g_{FAA} + tc + \delta\Delta g \quad (3.17)$$

In the provided formula above, c signifies the terrain correction while $\delta\Delta g$ represents the indirect effect on gravity resulting from the transformation of gravity anomalies between geoid to co-geoid.

Using the reduction method described in this section, a gravimetric geoid determination method named Stokes-Helmert is applied, developed by University of New Brunswick (See Section 5.5 for further information).

3.4 Residual Terrain Model (RTM)

The Residual Terrain Model (RTM), also known as high-frequency gravity modeling, involves the replacement of Earth's topography with a smoothed reference topography. This process entails identifying a mean elevation surface through low-pass filtering, removing masses situated above this surface, and filling voids below it. The smooth gravity observations obtained through RTM reduction are suitable for interpolation and quasi-geoid modeling (Tziavos & Sideris, 2013).

As the second advantage of the model, spectral enhancement to GGMs can be achieved by incorporating high-frequency content beyond the resolution of the GGM alone (Hirt et al., 2019; Lin et al., 2023). Figure 3.5 below illustrates the RTM reduction scheme.

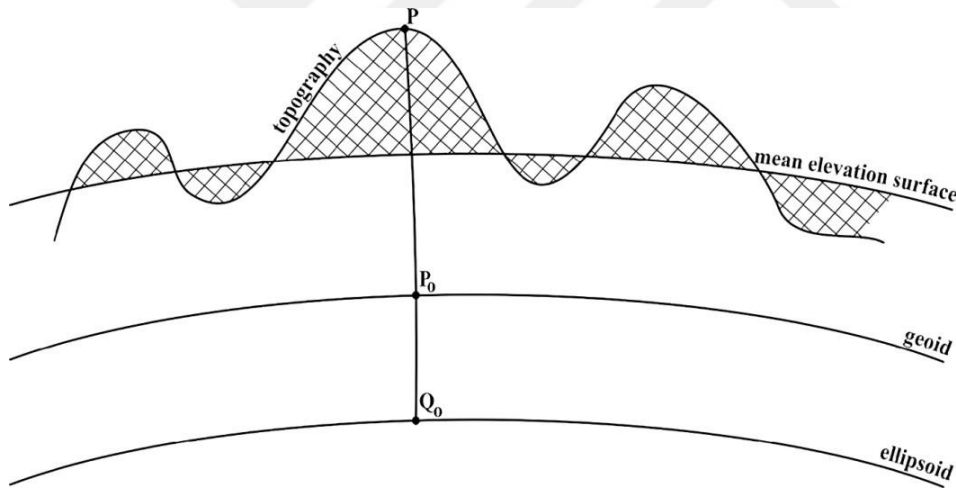


Figure 3.5 : Geometrical representation of the RTM reduction scheme (Tziavos & Sideris, 2013).

In applications of RTM, high resolution (is used for inner zone) and lower resolution (is used for far zone) are used together. The radius of the inner zone depends on the resolution of the available detailed DEM. In quasigeoid determination, RTM correction that is affected by DEM resolution and integration zones (r_1 , r_2) has an important role. Thus, The Digital Elevation Model (DEM) that represents the topography at the desired level of detail and integration radii that do not compromise processing speed should be selected. An exemplary case study illustrating the rationale behind these choices can be found here (Yang et al., 2020).

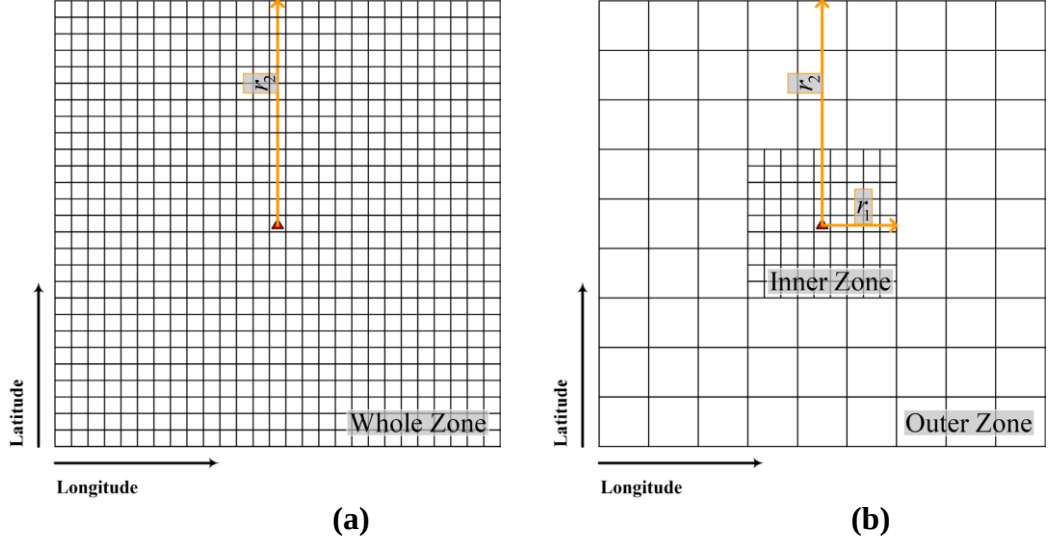


Figure 3.6 : Integration zones over topography: **(a)** Employing only high-resolution DEM, and **(b)** utilizing both high and lower resolution DEMs with r_1 and r_2 radii, respectively (Lin et al., 2023).

RTM gravity anomalies can be computed using Equation (3.18) (Erol, 2007):

$$\Delta g_{RTM} = g_p - \gamma_Q - \delta A_{RTM} \quad (3.18)$$

and RTM reduction δA_{RTM} can be computed as (Bajracharya, 2003):

$$\delta A_{RTM} = 2\pi G\rho(H - H_{ref}) - tc \quad (3.19)$$

where topographical height is denoted as H obtained from DEM, H_{ref} is height from reference surface, and c is terrain correction. It is important to note that RTM correction does not ensure harmonicity for masses filled below the reference surface. Consequently, a harmonic correction value is added to these calculations (Hirt et al., 2019; Lin et al., 2023).

3.5 Rudzki Inversion Method

This reduction technique, in which topographical masses situated above the geoid are inverted below it, deviates from conventional reduction methods by preserving the equipotential surface. The potentials of these masses, whether situated below or above the geoid, are equal, indicating the absence of any indirect effect to geoid heights (see Figure 3.7 below) (Bajracharya, 2003; (Tziavos & Sideris, 2013)).

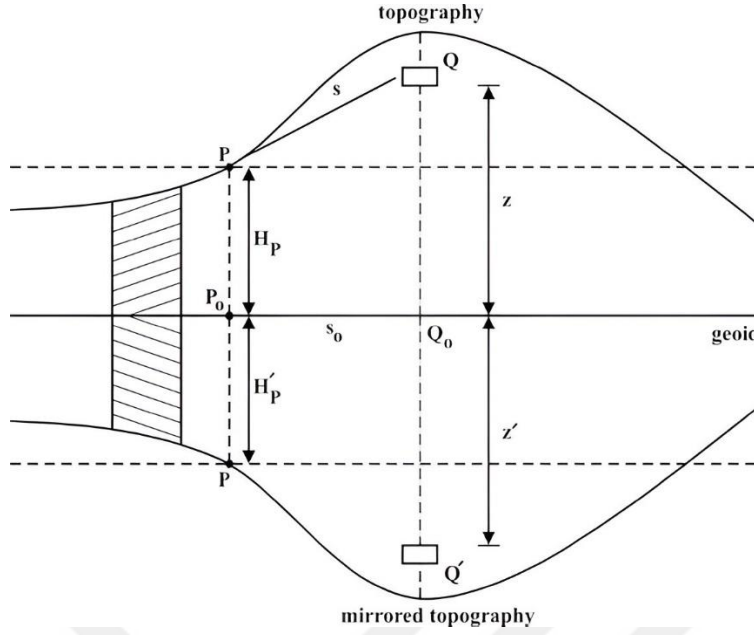


Figure 3.7 : Planar reduction scheme utilized in Rudzki's inversion method (Bajracharya, 2003).

In Figure 3.7, The single quoted terms (such as H_P' and Q') below the geoid represent inverted quantities. The geometrical illustration of spherical approximation can be found in Bajracharya (2003). Rudzki gravity anomalies can be formulated as follows (Tziavos & Sideris, 2013):

$$\Delta g_{Rudzki} = g_p - \gamma_{Q_0} + \delta g_F - \delta A_{Rudzki} \quad (3.20)$$

and the direct topographical effect on gravity (δA_{Rudzki}) of both original (non-inverted) and inverted masses is computed using (Bajracharya, 2003):

$$\delta A_{Rudzki} = G \iint_E \rho \left[\frac{1}{s_0} - \frac{1}{[s_0^2 + (H - H_P)^2]^{\frac{1}{2}}} + \frac{1}{[s_0^2 + (H_P - H')^2]^{\frac{1}{2}}} - \frac{1}{[s_0^2 + (2H_P)^2]^{\frac{1}{2}}} \right] dE \quad (3.21)$$

4. GRAVITY DATA INTERPOLATION AND GRIDDING

4.1 Methods of Interpolation

The majority of geoid computation algorithms, such as the LSMSA method employed in this investigation, require gridded free-air anomalies as essential input. These observed anomalies, distributed irregularly across the topography, are closely associated with terrain heights. Consequently, they prove unsuitable for direct gridding. Instead, the adoption of simple or complete Bouguer anomalies presents a more viable alternative (Janak et al., 2017; Märdla, 2017; Torge, 2001).

Simple or complete Bouguer anomalies demonstrate reduced sensitivity to variations in topography and yield smoother surfaces. In flat terrain, either simple or complete Bouguer anomalies are suitable for gridding; however, in rough areas, complete Bouguer anomalies are preferable. In Section 6.2, a comprehensive analysis is conducted to ascertain the appropriate Bouguer anomaly for application within the Auvergne study area, employing meticulous calculations and statistical scrutiny.

Interpolation algorithms, which are mathematical techniques that utilize known data to estimate missing values, are applied across various software platforms, including Surfer, MATLAB, ArcGIS, Python, Gravsoft, ENVI, GMT (Generic Mapping Tools), and others. In this study, we utilize the Surfer program, which offers 13 different methods for interpolating complete Bouguer anomalies, including Kriging, Cokriging, Inverse Distance to a Power, Triangulation with Linear Interpolation, Minimum Curvature, Natural Neighbor, Nearest Neighbor, Local Polynomial, Radial Basis Function, Polynomial Regression, Modified Shepard's Method, Data Metrics, and Moving Average.

The selection of an appropriate interpolation algorithm in gravity gridding is crucial, as it directly influences the accuracy of geoid model computations (Ellmann et al., 2019; Janak et al., 2017). This influence arises because the chosen algorithm impacts both the magnitude and spatial distribution of gravity values assigned to the grid nodes. Hence, after evaluating their algorithmic design, computational capabilities, data

accuracy, surface type, data density and size, four interpolation methods are selected that are Kriging, Nearest Neighbor, Inverse Distance to a Power (IDP), and Artificial Neural Network (ANN) (Golden Software, 2023; Karaca, 2016; Robeson, 1997).

These three well known and commonly used interpolation techniques (IDP, Nearest Neighbor and Kriging) are commonly used for many purposes including geodetic infrastructure establishment, estimating the GNSS-velocities, determination of local geoid models using GNSS/leveling method, gravimetric method, astrogeodetic method, regional gravimetric geoid determination of country geoid (a national project to the government), hydrogeology projects for determining the ground water trends, estimating the sea-level variations and mean sea level determination from altimetric and tide-gauge data. The ANN method is preferred due to its complex structure, reliance on optimization principles within soft computing paradigms, and significant relevance in contemporary applications.

4.2 Inverse Distance to a Power (IDP)

The IDP is a deterministic interpolation method wherein observation points in proximity to the point being interpolated are accorded greater weight. As the distance from the interpolation point increases, the rate of decrease in weight is determined by the power parameter (k). Equivalently, a higher weighting power strengthens the influence of nearby data points on the interpolated value at a grid node, while diminishing the impact of points located farther away (Babak & Deutsch, 2009; Golden Software, 2022; Li & Heap, 2008).

In applications, the power parameter is typically selected between “1” and “3” (Erol & Çelik, 2004; Li & Heap, 2008). Values equal to or greater than 4 tend to resemble the nearest neighbor method (Yiğit, 2003). In the study conducted in this thesis, the power parameter was chosen as 2, which is the default value of the program and commonly selected in similar studies.

Below is the equation for Inverse Distance to a Power (Li & Heap, 2008; Yilmaz, 2009):

$$\hat{Z}_j = \frac{\sum_{i=1}^n Z_i P_i}{\sum_{i=1}^n P_i} \quad \text{and} \quad h_{ij} = \sqrt{d_{ij}^2 + \delta^2} \quad (4.1)$$

$$P_i = \frac{1}{h_{ij}^k}$$

In the Equation (4.1) above, Z_i denotes the observed data, $\hat{Z}_j$ represents the interpolated data, P_i signifies the weight of the i^{th} sample data, n denotes the number of observations, k represents the power parameter, d_{ij} represents the distance between the interpolated and observed values, h_{ij} signifies the effective separation distance between the interpolated point (j) and the observed point (i), and δ denotes the smoothing parameter. The software's smoothing parameter enables the application of an uncertainty factor to input data, whereby increasing its value diminishes the disproportionate influence of individual observations when computing a neighboring grid node (Golden Software, 2022).

This exact/smooth interpolation method, which is employed when the distribution of points is irregular and rapid calculation is required, restricts the estimated values to remain within the observed data range, thereby avoiding extrapolation.

4.3 Nearest Neighbor

In contrast to the Inverse Distance to a Power (IDP) method, which incorporates a weighted average based on distance, this algorithm employs a nearest neighbor approach. Here, the value of the closest data point to the target grid node is directly assigned, disregarding the influence of other neighboring points (Golden Software, 2022; Knotters et al., 2010; Yang et al., 2004a).

In this exact interpolation method, wherein the unknown data to be interpolated adopts the value of the nearest point, extrapolation is absent, thereby precluding the possibility of the unknown value exceeding that of existing data.

This method is particularly recommended for data with consistent spacing and converts files from XYZ format to a grid format. It can address small gaps within the data by assigning the value of the closest point, but for larger gaps or complex patterns, the resulting surface might not be perfectly smooth. Additionally, it assigns null values to areas where no data exists, providing a clear distinction between measured points and empty regions (Golden Software, 2022).

4.4 Kriging

Kriging, an exact or smooth geostatistical interpolation method, resembles the Inverse Distance to a Power (IDP) technique in that points closer to the calculation point have more weight. However, it differs from IDP in that these weights are also based on a variogram model, revealing the changes in the difference between points depending on distance (Yaprak & Arslan, 2008a). The weights of the reference points are computed using this variogram model. Kriging is termed as the Best Linear Unbiased Estimator (BLUE) because the aforementioned weights are determined to minimize prediction error (Erol & Çelik, 2004; Golden Software, 2022; Yiğit, 2003). One other difference from the IDP method is Kriging's capacity for extrapolating grid values outside the boundaries of the data values.

The semi-variogram “ $\gamma(d)$ ” is half the variance of the differences between pairs of data points separated by a specific distance, and it is a graphical representation of spatial variability. It measures the degree of similarity or dissimilarity between data points as a function of distance or direction. It provides information about how the variance of the data changes with increasing separation distance. By analyzing the semi-variogram, Kriging determines the spatial dependence structure of the data, which is essential for making accurate predictions at unsampled locations. The computation of the empirical semi-variogram value is conducted through the utilization of the subsequent formula (Isaaks & Srivastava, 1989; Yaprak & Arslan, 2008b; Yiğit, 2003).

$$\gamma(d) = \frac{1}{2n(d)} \sum_{d_{ij}}^{n(d)} (Z(x_i, y_i) - Z(x_j, y_j))^2 \quad (4.2)$$
$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

where $Z(x_i, y_i)$ and $Z(x_j, y_j)$ are variables at i and j locations respectively, d_{ij} is horizontal distance between i and j , $n(d)$ is the number of pairs of sample points.

The reliability of theoretical variograms obtained using experimental variograms calculated with the Equation (4.2) is tested through cross-validation, which compares actual and predicted values (Yaprak & Arslan, 2008a, 2008b).

The programs used for gridding offer a variety of variogram models that mathematically capture spatial relationships. These models include exponential, Gaussian, linear, logarithmic, nugget effect, power, quadratic, rational quadratic, spherical, wave, cubic, and penta-spherical. Figure 4.1 illustrates variogram models, including those of the sphere, exponential, and linear types.

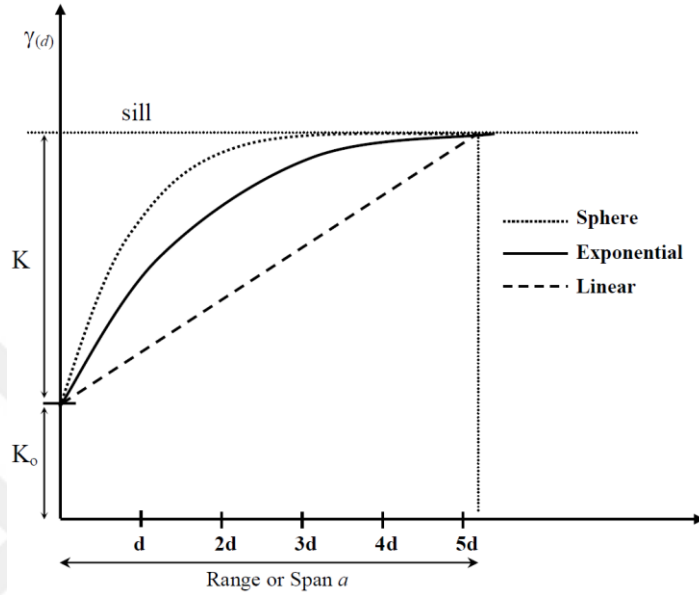


Figure 4.1 : Variogram models and their parameters (Erol, 2007)

where K is first order effect of the variogram, K_0 is nugget effect (the semi-variance value at the point where the theoretical curve intersects the vertical axis at $d=0$), d is the horizontal distance between sample data, α represents the range beyond which spatial autocorrelation or the relationship between the proximity of data points ceases to exist. The maximum value of the variogram at distance d is also referred to as “sill”. After all these detailed explanations, the formula for Kriging is provided below (Heuvelink, 2006; Yaprak & Arslan, 2008a).

$$\hat{Z}_j = \sum_{i=1}^n Z_i P_i \quad (4.3)$$

$\hat{Z}_j$ denotes the estimated data, Z_i represents the observed data, P_i stands for the weight assigned to the i^{th} observation, and n indicates the total number of observations.

In Surfer, the Kriging algorithm can be customized by incorporating drifts and adjusting the search radius. By default, Surfer employs point Kriging without drifts (Ordinary Kriging) and employs a circular search ellipse. Point Kriging, characterized by its simplicity, derives the value of a point from a set of neighboring values, whereas

Block Kriging computes the average value within rectangular blocks centered on the grid nodes (Golden Software, 2022; Isaaks & Srivastava, 1989; Karaca, 2016).

Within the domain of spatial interpolation techniques, ***Least Squares Collocation (LSC)*** emerges as another method analogous to Kriging, enjoying widespread application. The fundamental goal of these geostatistical techniques, which use mathematical models for the interpolation process, is to minimize the discrepancy between observed and predicted values. However, disparities arise in their handling of spatial variation and underlying assumptions (Erol & Erol, 2021).

LSC leverages a statistical model to estimate the underlying signal within spatial data, accounting for noise through distance-based weighting from surrounding observations. To explore the superiority of Kriging, which typically involves more complex mathematical computations than LSC, refer to Kiammehr (2007). For a more detailed explanation including the mathematical formulation of LSC and examples of covariance functions, refer to (Erol, 2007; Sjöberg & Bagherbandi, 2017; Yiğit, 2003).

4.5 Artificial Neural Network (ANN)

Artificial neural networks (ANNs), a prominent soft computing algorithm inspired by the structure and function of the human nervous system, are trained using sample data to estimate target values through network connections based on weights. Their versatility extends across numerous fields such as automotive engineering, banking, entertainment, and aerospace. Additionally, they find utility in tasks like predicting geodetic deformations, forecasting sea level changes, and determining Earth's orientation parameters (Demuth et al., 2018; Erol, 2011, Kavzoglu & Saka, 2005; Schuh et al., 2022; Seager et al., 1999; Stopar et al., 2006).

MATLAB stands out as a popular platform for implementing Artificial Neural Networks (ANNs). While the “nnstart” command provides a graphical interface for working with ANNs, users can also develop and execute ANN applications directly through MATLAB code, offering greater flexibility (Mathworks, 2021). The software features three primary learning algorithms: Levenberg-Marquardt, Scaled Conjugate Gradient, and Bayesian Regularization. Each method has its strengths and weaknesses, and the best choice depends on nature of the data, the complexity of model, computational considerations, and need to speed and accuracy.

For building Artificial Neural Networks (ANNs), MATLAB boasts a rich library of techniques. Among these are radial basis functions, generalized regression neural networks (GRNNs), NARX networks, Hopfield networks, and both feed-forward and Elman backpropagation architectures (Albayrak et al., 2020; Demuth et al., 2018; Yilmaz & Arslan, 2011). This study utilizes the feed-forward backpropagation technique. In MATLAB, the "trainlm" function is utilized to apply the Levenberg-Marquardt optimization algorithm for network training, with the network architecture featuring two hidden layers, each consisting of 200 neurons. Further customization is achieved by selecting the "tansig" transfer function and the "learngdm" learning function. The architecture of the artificial neural network algorithm used in this study is given in Figure 4.2.

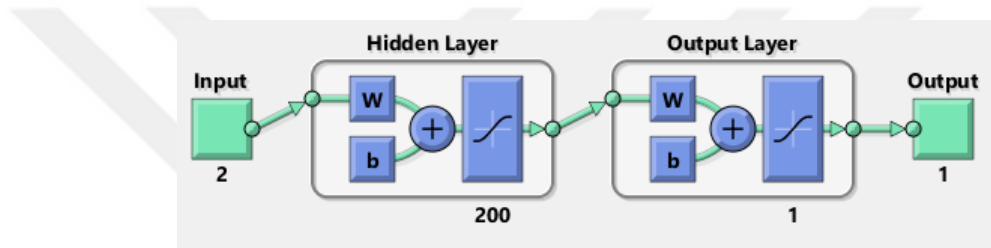


Figure 4.2 : Representation of the artificial neural network algorithm implemented in MATLAB, illustrating the notation for weights (w) and bias (b).

The architecture of a feed-forward backpropagation network revolves around interconnected layers of neurons. These layers are categorized as input, hidden, and output layers, each serving a specific purpose. In a feed-forward neural network, information flows in a specific direction: data is fed into the network via the input layer, undergoes computations within the hidden layer, which serves as the computational core, and then the transformed data is stored as desired output values in the output layer (Erol & Erol, 2013; Kavzoglu & Saka, 2005).

This study utilizes an Artificial Neural Network (ANN) to process pointwise Bouguer anomalies. The goal is to generate gridded data suitable for calculating free air anomalies, a crucial component in determining the Earth's gravity field (geoid) through gravimetric methods. The process involves training the ANN on a dataset of 244,009 complete Bouguer anomalies (CBAs). Here, 15% of the data is designated for testing and validation purposes. Once trained, the network estimates CBAs for a grid containing 173,641 nodes by simulating its learned behavior. Figure 4.3 showcases the network's performance during training using the Levenberg-Marquardt method. Notably, the Figure 4.3. depicts the network's regression behavior, along with

additional details such as the network size of 200 neurons, maximum allowed failures of 6, and a total of 300 training epochs.

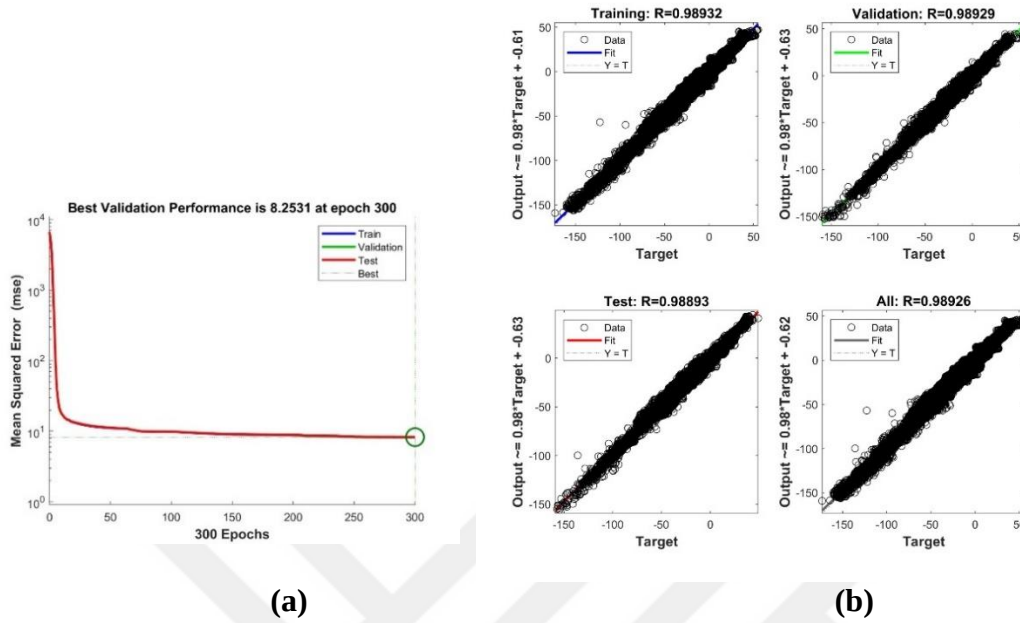


Figure 4.3 : Levenberg-Marquardt training Performance (a) and regression (b).

When compared with the other 3 interpolation methods used in the study, values interpolated with the ANN method create a smooth surface pattern. This result is due to a difference between their computational steps and algorithms: Other methods directly interpolate the value at a grid node using the data points (weighting their contribution) around it. However, the ANN method determines a function first (as a result of an iterative training process of its parameters) and then derives the values at grid nodes (1 arc minute resolution) by running this function, and this process generates a smooth surface pattern. The difference between the computation algorithms and the smooth surface pattern generated by the ANN is clarified in Section: Case Study-6.4.

5. GRAVIMETRIC GEOID DETERMINATION

5.1 A Brief Overview of Geoid Determination Methods

In earlier sections, it was mentioned that the geoid is a crucial tool in determining orthometric heights using ellipsoidal heights obtained from satellite positioning systems. The determination of the geoid surface relies on solving a boundary value problem. This problem is tackled using Stokes' formula (see Equation 5.1), introduced by G.G. Stokes in 1849. A key assumption of this formula is the absence of external mass beyond the geoid. To satisfy this condition, the theory behind geoid computation mathematically excludes the influence of topographical masses located outside the geoid (Abbak et al., 2012b; Heiskanen & Moritz, 1967; Işık et al., 2022).

$$N = \frac{T}{\gamma} = \frac{R}{4\pi\gamma} \iint_{\sigma} S(\Psi) \Delta g d\sigma \quad (5.1)$$

In the expression, $S(\Psi)$ represents the Stokes function, where Ψ denotes the geocentric angle, Δg stands for the gravity anomaly, $d\sigma$ signifies the infinitesimal surface element of the unit sphere, R represents the mean Earth radius, and γ denotes the normal gravity on the reference ellipsoid (Abbak et al., 2012a, 2012b; Ellmann et al., 2019; Janak et al., 2017; Sjöberg & Bagherbandi, 2017).

Beyond the requirement for a harmonic potential field outside the geoid (indicating an absence of mass above geoid), the theory behind geoid determination necessitates the computation of input gravity quantities, such as gravity anomalies, referenced to the geoid itself rather than the actual topography. Furthermore, when delineating the boundary conditions related to Earth's gravity potential, the classical Stokes' formula assumes a spherical Earth for boundary conditions. However, obtaining gravity information for the entire globe as required by the equation is not practically feasible. Consequently, this method is constrained to use within a restricted region by disregarding gravity data in remote areas. This limitation introduces a truncation error (Märdla, 2017; Vermeer, 2020).

While Stokes' formula ideally requires a comprehensive dataset of terrestrial gravity observations across the entire globe, obtaining such dense coverage is practically infeasible. To mitigate the consequences of this data limitation, researchers have integrated readily available, globally-observed satellite gravity data with local, high-density terrestrial measurements. This integration necessitates modifications to the original Stokes formula, and the scientific literature offers both deterministic and stochastic approaches for achieving this (Abbak & Ustun, 2015; Ellmann et al., 2019; Sakil et al., 2021). In the deterministic approach, truncation error caused by the lack of terrestrial gravity data in remote areas is minimized based on an assumption of having the global geopotential model (GGM) coefficients as error-free. On the other hand, stochastic approaches consider the errors belong to the GGM data and terrestrial gravity data in a stochastic model while minimizing the truncation error stem from the poor coverage of terrestrial data in computations (Abbak et al., 2012b; Sjöberg, 2003c). One of the stochastic modifications of Stokes integral formula utilizes the least squares method and it is formulated by the Royal Institute of Technology in Sweden as the least squares modification of Stokes integral with additive corrections (LSMSA) approach or shortly the KTH approach in literature. The LSMSA approach has been used and proved its success in the determination of regional geoid models in many studies so far.

Besides the KTH method, there are various algorithms for gravimetric geoid determination such as Stokes-Helmert, Least Squares Collocation (LSC) and Remove-Compute-Restore (RCR). The RCR method performs fast Fourier transformation on grid data, but the LSC method can use both grid and pointwise data in its process.

In ***Remove-Compute-Restore (RCR)*** approach, terrestrial gravity anomalies are reduced by removing long wavelength component represented by Global Geopotential Model (GGM) and short wavelength part (terrain effect).

$$\Delta g_{red.} = \Delta g_{FAA} - \Delta g_{GGM} - \Delta g_T \quad (5.2)$$

Here, Δg_{red} represents the reduced gravity anomaly, Δg_{FAA} stands for the free-air gravity anomaly, Δg_{GGM} denotes the contribution of long-wavelength gravity anomaly obtained from the Global Geopotential Model (GGM), and Δg_T is high-frequency component of terrain-induced gravity anomaly also known as the direct topographic effect (Sjöberg, 2018b; Vermeer, 2020).

Then, long and short wavelength components are restored to computed geoid undulation. Equation 5.3 formulizes the final geoid and Figure 5.1 illustrates the components used in RCR geoid computation.

$$N = N_{GGM} + N_{\Delta g_{red.}} + N_{ind.} \quad (5.3)$$

In this framework, the geoid undulation N is determined computationally, with its constituent elements delineated as follows, N_{GGM} denotes the long-wavelength components of geoid height derived from the Global Geopotential Model (GGM), $N_{ind.}$ represents the indirect effect of the terrain on geoid height, and $N_{\Delta g_{red.}}$ signifies the residual geoid height, which embodies the medium-wavelength component and computed utilizing the Stokes integral (Abbak et al., 2012a; Sjöberg & Bagherbandi, 2017).

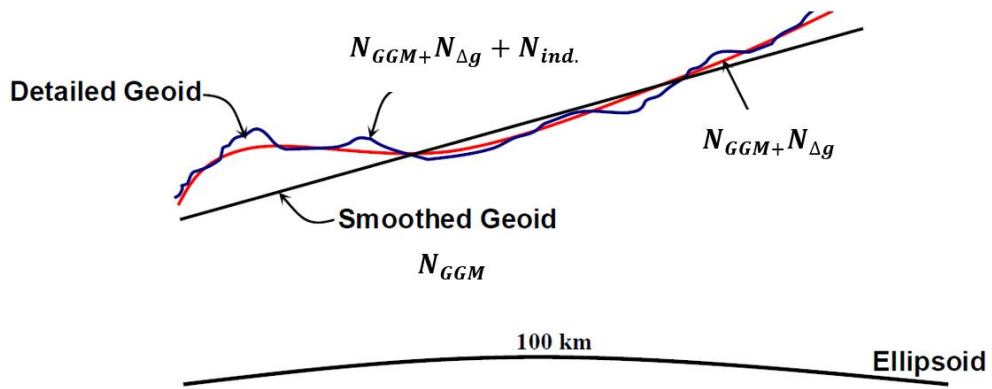


Figure 5.1 : Visual representation illustrating components of geoid undulation computation (Schwarz et al., 1987).

Unlike the LSMSA method, where preliminary geoid heights are corrected by additive corrections, in the RCR method, the Stokes kernel remains unaltered (unmodified), whereas gravity anomalies are reduced prior to the application of the Stokes integral (Abbak et al., 2012a). The comparison of the RCR and LSMSA methods can be seen in detail in (Sjöberg & Bagherbandi, 2017).

Besides the Stokes integral formula, the Hotine integral has recently gained popularity in several applications. In the case studies, the ***least squares modification of the Hotine integral with additive corrections (LSMHA)*** method is used as an alternative to the LSMSA method (Abbak et al., Işık et al., 2021; 2022; Märdla, 2017; Märdla et al., 2018; Sakil et al., 2021; Sjöberg & Bagherbandi, 2017). Since this formula employs the gravity disturbances on topography instead of the gravity anomalies, it

may be more practical in computation areas with well distributed and dense terrestrial gravity observation stations having known geodetic coordinates. Hence it may yield more rigorous results since it does not require gravity reduction and gridding process in such fields. However, on the other hand, gravity reduction is unavoidable for the geoid computation algorithms which work with grid gravity data in applications. If this alternative formula is run with pointwise data, it yields more rigorous results since the gravity reduction and gridding process are not applied. But this is not always possible especially in the regions with sparse and insufficient distribution of gravity observations. The computation of geoid undulation using the basic formula of the Hotine integral is given in Equation 5.4 below (Märdla, 2017; Märdla et al., 2018):

$$N = \frac{R}{4\pi\gamma} \iint_{\sigma} H(\Psi) \delta g d\sigma \quad (5.4)$$

where R represents the mean radius, $H(\Psi)$ signifies the Hotine function, Ψ denotes the spherical distance from computation point and integration point, γ refers to the normal gravity on the reference ellipsoid, δg is the gravity disturbance, and $d\sigma$ stands for the unit sphere.

The Stokes-Helmert regional geoid determination method, developed by the University of New Brunswick (UNB) Geodesy Group, utilizes Helmert's second condensation model for assessing the impact of topographical masses. These masses are condensed onto or beneath an extremely thin layer on the geoid, creating Helmert space. Within this space, the gravitational attraction of topographical and atmospheric masses has been excluded, simplifying the calculations for determining the geoid shape (Ellmann et al., 2006; Vaniček et al., 2012).

This geoid determination method utilizes Helmert gravity anomalies within Helmert space. These anomalies are derived from the observed free-air gravity anomalies by applying a series of corrections that account for various factors influencing the calculation of the geoid, including: direct topographical and atmospheric effects, secondary indirect topographical and atmospheric effects, the geoid-quasigeoid correction, and, if available, the direct topographical density effect using a density model (Kingdon, 2012; Vaniček & Martinec, 1994).

The Stokes formula is then employed within the framework of Helmert space to compute co-geoidal heights. These co-geoidal heights (denoted by N^h), when

combined with the primary indirect topographical and atmospheric effects, and the primary indirect topographical density effect (if density data is available), are used to compute the geoid undulations in real space (represented by N). Finally, a small correction, known as the Hörmander correction, is applied to account for the slight shift in the Earth's center of mass caused by the Helmert condensation process (Ellmann & Vaníček, 2007; Janak et al., 2017; Vaníček et al., 2013; Yang et al., 2004b).

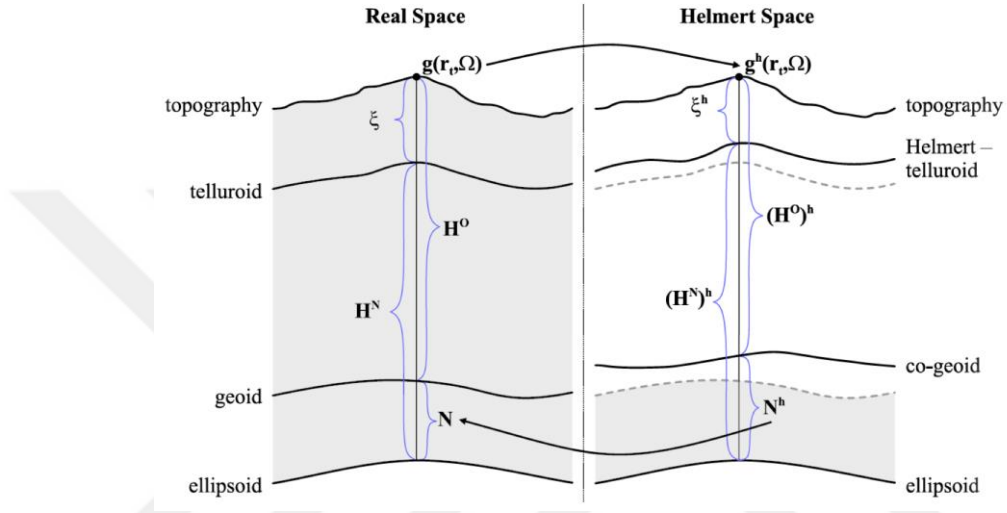


Figure 5.2 : The principle underlying the Stokes-Helmert method. Geocentric position of a point is represented by geocentric radius (r) and geocentric coordinates ($\Omega = \varphi, \lambda$) (Ellmann & Vaníček, 2007).

A comprehensive elaboration of the LSMSA theory and its computational formulas is provided in the subsequent section (Section 5.2).

5.2 Least Squares Modification of Stokes Formula with Additive Corrections

The limitations of the Stokes formula (Section 5.1, Equation 5.1) stem from the necessity for global gravity data, which is practically unattainable (Abbak et al., 2012a, 2012b; Ellmann et al., 2019). This constraint, particularly the omission of data from remote regions, results in a truncation error. To tackle this challenge, the modified Stokes formula with additive corrections (LSMSA), a stochastic solution, is employed. Developed by the Royal Institute of Technology (KTH) in Sweden, the LSMSA method (also known as the KTH method) utilizes the least squares concept to minimize the predicted global mean square error of the formula. This minimization incorporates contributions from both the Global Geopotential Model (GGM) and terrestrial gravity

data, as demonstrated in the equation below (Abbak et al., Abdalla, 2009; Abdalla & Tenzer, 2011, 2012b; Sjöberg, 1981, 1984, 1991, 2003a, 2003b, 2003c, 2005).

$$\hat{N} = \tilde{N} + \delta N_{com}^{top} + \delta N_{DWC} + \delta N_{com}^{atm} + \delta N_{ell} \quad (5.5)$$

In the equation, $\hat{N}$ represents the geoid undulation, while $\tilde{N}$ denotes the approximate geoid height. Within the equation, δN_{com}^{top} signifies the combined topographic correction, δN_{DWC} denotes the downward continuation effect, δN_{com}^{atm} represents the combined atmospheric correction, and δN_{ell} stands for the ellipsoidal correction. Collectively, these correction parameters are referred to as additive corrections (Abbak et al., 2012b; Ellmann & Sjöberg, 2004; Sjöberg, 2003c, 2007; Sjöberg & Nahavandchi, 2000).

The calculation of the approximate geoid height ($\tilde{N}$) proceeds as follows:

$$\tilde{N} = \frac{R}{4\pi\gamma} \iint_{\sigma_0} S^L(\Psi) \Delta g \, d\sigma + \frac{R}{2\gamma} \sum_{n=2}^M b_n \Delta g_n \quad (5.6)$$

The modified Stokes function $S^L(\Psi)$, crucial in determining the approximate geoid height $\tilde{N}$, integrates modified harmonics up to a maximum degree (L), aligned with the expansion degree of the employed Global Geopotential Model (GGM). The upper limit of the GGM (M) is also considered in the computation. Additionally, essential elements such as the spherical cap (σ_0), modification parameter (b_n), and gravity anomaly (Δg_n) obtained from the GGM are all incorporated into the computation of $S^L(\Psi)$. A formula provided below outlines the calculation process of this modified Stokes function.

$$S^L(\Psi) = \sum_{n=2}^{\infty} \frac{2n+1}{n-1} P_n(\cos\Psi) - \sum_{n=2}^L \frac{2n+1}{2} s_n P_n(\cos\Psi) \quad (5.7)$$

In the aforementioned context, $P_n(\cos\Psi)$ represents a Legendre polynomial, while s_n denotes the stochastic modification parameter. As discussed in prior studies by Abbak et al. (2012a), Abbak and Ustun (2015), and Sjöberg (2003c), the parameter b_n can be expressed in three different formulations: biased ($b_n = s_n$), unbiased ($b_n = s_n + Q_n^L$), and optimum ($b_n = \frac{(s_n + Q_n^L)c_n}{(c_n + dc_n)}$). Here, dc_n signifies the error degree variance,

Q_n^L represents the truncation coefficient, and c_n denotes the signal degree variance (Abbak et al. 2012b).

When it comes to describe additional corrections that is added to approximate geoid height, combined topographic effect, which consists of direct and indirect parts, can be computed as (Abbak et al., 2012a; Sjöberg, 2007):

$$\delta N_{com}^{top} = \delta N_{dir} + \delta N_{ind}^{top} = -\frac{2\pi G\rho H^2}{\gamma} \left(1 + \frac{2H}{3R}\right) \quad (5.8)$$

where H is orthometric height, G is Newton gravitational constant and ρ is density value. Another additive correction term named downward continuation (DWC), which consists of three components, is calculated with the formula below (Sjöberg, 2003a):

$$\delta N_{DWC} = \delta N_{dwc}^{(1)} + \delta N_{dwc}^{L1, Far} + \delta N_{dwc}^{L2} \quad (5.9)$$

where

$$\delta N_{dwc}^{(1)} = \frac{\Delta g_P}{\gamma} H_P + 3 \frac{\tilde{N}}{r_P} H_P - \frac{1}{2\gamma} \frac{\partial \Delta g}{\partial r} \Big|_P H_P^2 \quad (5.10)$$

and

$$\delta N_{dwc}^{L1, Far} = \frac{R}{2\gamma} \sum_{n=2}^M b_n \left[\left(\frac{R}{r_P} \right)^{n+2} - 1 \right] \Delta g_n \quad (5.11)$$

The right-hand term δN_{dwc}^{L2} is computed as:

$$\delta N_{dwc}^{L2} = \frac{R}{4\pi\gamma} \iint_{\sigma_0} S^L(\Psi) \left[\frac{\partial \Delta g}{\partial r} \Big|_P (H_P - H_Q) \right] d\sigma_0 \quad (5.12)$$

r_P is the spherical radius ($r_P = R + H_P$) and $\frac{\partial \Delta g}{\partial r} \Big|_P$ is gravity gradient of point P . Sjöberg & Nahavandchi (2000) suggests a formula to compute the combined atmospheric effect:

$$\delta N_{com}^{atm} = -\frac{2\pi GR\rho^a}{\gamma} \sum_{n=2}^M \left(\frac{2}{n-1} - b_n \right) H_n - \frac{2\pi GR\rho^a}{\gamma} \sum_{n=M+1}^{\infty} \left(\frac{2}{n-1} - \frac{n+2}{2n+1} Q_n^M \right) H_n \quad (5.13)$$

where ρ^a is atmospheric density. As a last additive correction, ellipsoidal correction is computed as (Abbak et al. 2012b; Ellmann & Sjöberg, 2004):

$$\delta N_{ell} \cong [(0.0036 - 0.0109 \sin^2 \varphi) \Delta g + 0.0050 \tilde{N} \cos^2 \varphi] Q_0^L \quad (5.14)$$

In the formula above, Q_0^L represents the Molodensky truncation coefficient.

This thesis utilizes the LSMSA method implemented through ITU-LSMSA software. Optimizing the selection of parameters for this computation is critical, particularly the integration cap size (Ψ_0), the global geopotential model expansion degree (M), and the upper bound of the Stokes function (L) (Abbak et al., 2012a, 2012b; Ellmann et al., 2019; Erol & Erol, 2021; Kiamehr, 2006; Sjöberg & Bagherbandi, 2017).

To address the lack of gravity data, high-degree Global Geopotential Models (GGMs) can be employed; however, this leads to increased errors in harmonic coefficients, affecting the resultant geoid height parameter. Hence, careful consideration is required when selecting the optimal expansion degree of the GGM (Abbal et al., 2012b; Abbak & Ustun, 2015). In this study area, an investigation is conducted on the optimal integration radii (Ψ_0), expansion degree of the GGM (M), error variance (C), and choices for the biased/unbiased/optimum b_n parameter. Determining the optimal computation parameters for geoid modeling involves a trial-and-error test procedure. Consequently, all geoid models computed with varying parameters are compared to each other.

Here, preferably chosen as a Global Geopotential Model (GGM) representing Earth's gravitational field through spherical harmonic coefficients, XGM2019e stands out (Hackney & Featherstone, 2003). The coefficients of the XGM2019e model are readily accessible from ICGEM (2023), and an application utilizing the XGM2019e model can be found in Tocho et al. (2022). The XGM2019e global geopotential model extends up to degree and order 2190. Zingerle et al. (2020) offer a comparison between XGM2019e and its high-resolution counterparts, namely the EIGEN-6C4 and EGM2008 models (Pail et al., 2018).

6. CASE STUDY AND NUMERICAL RESULTS

This section begins by outlining the study area and the data utilized for testing. Following this, it presents the findings from numerical studies.

Within the section dedicated to presenting the numerical test results, firstly, the effects of simple and complete Bouguer anomalies on the geoid computation are explored. Following this investigation, the ideal parameters utilized in LSMSA gravimetric geoid calculation are identified. As the third numerical assessment, an analysis is conducted to investigate the impacts of interpolation techniques on geoid determination while gridding complete Bouguer anomalies. Then, an examination is conducted to explore how the positioning of GPS/Leveling points affects the validation results. Finally, the standardization of the results obtained from the Artificial Neural Network (ANN) tool is being investigated.

6.1 Study Area and Data

The designated testing region within Auvergne encompasses a geographical expanse of $6^{\circ} \times 8^{\circ}$. The Institut Geographique National (IGN) shares the dense and highly accurate gravity observations obtained in Auvergne as the test dataset for investigations into gravity field modeling, accessible through the Bureau Gravimetrique International (BGI) database. Researchers have unrestricted access to these data to evaluate geoid computation techniques. The dataset contains 244,009 gravity points situated within the latitude coordinates of $43^{\circ} \text{ N} - 49^{\circ} \text{ N}$ and longitude coordinates of $1^{\circ} \text{ W} - 7^{\circ} \text{ E}$. The estimated spatial density of gravity observations equates to one point per 1.3 kilometers, a factor taken into consideration when establishing the grid spatial resolution in the process of data gridding. The reported accuracy of the gravity data stands at approximately 2 milliGals ($\sim 2 \text{ mGal}$), as cited in reference (Duquenne, 2006).

Complementing the comprehensive dataset of gravity observations within the Auvergne test area, 75 strategically positioned GPS/leveling points are available to facilitate the validation of computed geoid models. Nonetheless, the GPS/leveling

points are concentrated within a restricted area at the center of Auvergne, covering 45° N to 47° N latitudes and 1.5° E to 4.5° E longitudes. The dataset incorporates both orthometric heights referenced to the National Leveling Network (NGF-IGN69) datum and ellipsoidal heights referenced to the ITRF datum, demonstrating an accuracy level of approximately 2 cm. Notably, the topographical heights of these GPS/leveling benchmarks exhibit significant variation, ranging from 206.8 m to 1235.5 m, as documented in references Janak et al. (2017), Valty et al. (2012) and Yildiz et al. (2012).

The spatial distribution of both the topography within the Auvergne test area and the 75 GPS/leveling benchmarks deployed for validation purposes are visualized in Figure 6.1a (Jarvis et al., 2006; Reguzzoni et al., 2021). Figure 6.1b illustrates the distribution of gravity points within the same area.

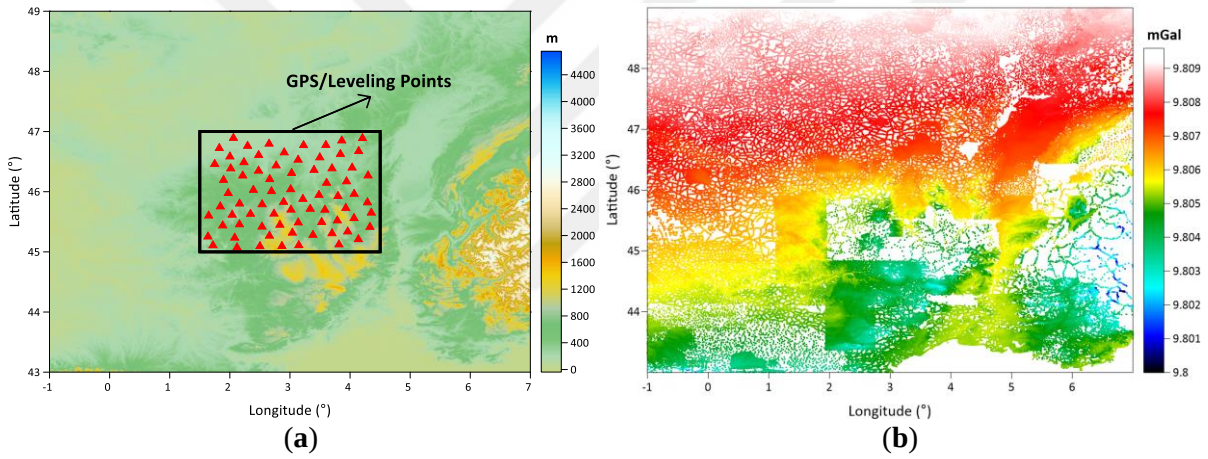


Figure 6.1 : The Topography of the Auvergne test region and the positions of the 75 GPS/leveling benchmarks denoted by red triangles **(a)** and the spatial distribution of the gravity observation points within the area **(b)**.

Workflow that depicts the sequential steps undertaken during the numerical tests conducted in the present study is given in Figure 6.2.

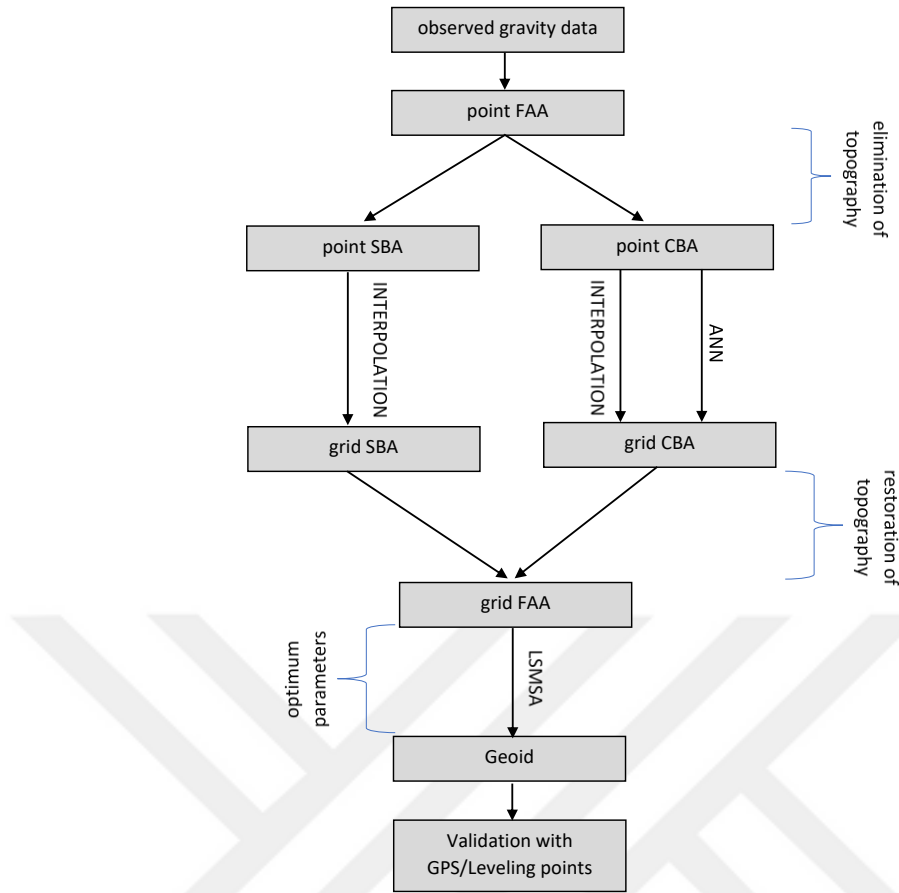


Figure 6.2 : Data processing steps followed in the numerical tests.

6.2 Simple and Complete Bouguer Anomalies in Gravity Gridding

In the initial numerical test, the impact of Bouguer anomaly type (simple or complete) on gravity gridding is examined. Bouguer anomaly datasets in grid format are analyzed for geoid determination in the test area and a comparative analysis is facilitated between the geoid models derived from these gridded gravity datasets. A meticulous interpretation of the discrepancies observed between these two approaches is conducted by utilizing statistical computations and visualizations (maps).

Complete Bouguer anomalies (CBAs) are determined by incorporating the terrain correction into the simple Bouguer anomalies (SBAs) as per Equation (3.6). The terrain corrections (TCs) are calculated using Fortran-based GRAVSOFTE geodetic gravity field computation software, employing 3" and 30" resolution SRTM DEMs. Furthermore, the integration radii adopted for the computations are $r_1 = 40$ km and $r_2 = 100$ km for the inner and outer radii, respectively.

The most laborious phase in the acquisition of complete Bouguer anomalies (CBAs) lies within the computation of terrain corrections. This stage requires approximately 8

hours for completion on a Razer Blade 14 Laptop equipped with an Intel (R) Core™ i7-6700HQ processor at 2.60 GHz, 16 GB of RAM, and a 64-bit Windows 10 operating system. To generate gridded free-air anomalies (FAAs) essential for geoid computation, terrain corrections are re-estimated at grid nodes. Notably, the parameters used in this step are identical to those employed during the prior Bouguer reduction process ($\Delta g_{FAA}^{GRID} = \Delta g_{CBA}^{GRID} + \delta g_B^{GRID} - \delta g_{TC}^{GRID}$).

Figure 6.3a illustrates terrain corrections at gravity points employed in the computation of Complete Bouguer Anomalies (CBAs) before data gridding. Subsequently, the computed CBAs are gridded with a spatial resolution of 1 arc-minute. The chosen grid resolution of 1 arc-minute aligns with the density of the original gravity observations, which are spaced approximately 1.3 km apart, corresponding roughly to 1 arc-minute intervals on the Earth's surface. This approach ensures an accurate representation of the spatial variations in the data by capturing the detail present within the observation spacing. Following the gridding of the complete Bouguer anomalies (CBAs), the free-air anomalies are subsequently calculated in gridded form. To recover the free-air anomalies from the gridded CBAs, terrain corrections are then determined for each grid node. Figure 6.3b visually depicts these gridded terrain corrections with a spatial resolution of 1 arc-minute. Additionally, statistical summaries of the terrain corrections for both the gravity points (Point TC) and the grid nodes (Grid TC), along with the alteration in topographic heights in the area based on the SRTM 3 DEM data are provided in Table 6.1.

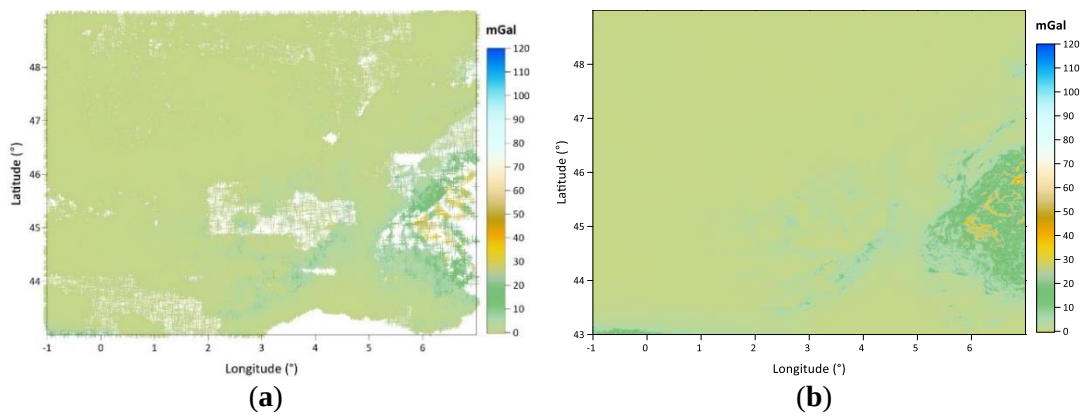


Figure 6.3 : Terrain corrections at gravity points utilized in computing the complete Bouguer anomalies prior to gridding the gravity data (a), and terrain corrections at grid nodes with 1 arcminute grid spacing utilized to restore free air anomalies from the complete Bouguer anomalies (b).

Table 6.1 : Statistical summary of terrain corrections at both the gravity points (Point TC) and grid nodes (grid TC), and topographic heights derived from SRTM 3" DEM data

	Max	Min	Mean	Median	Std	RMSE
Point TC (mGal)	56.533	-0.317	1.043	0.476	2.153	2.392
Grid TC (mGal)	118.221	-0.327	1.718	0.371	4.086	4.432
3" DEM (m)	4745.649	-33.506	378.900	230.454	433.149	575.485

An examination of the statistical data presented in Table 6.1 reveals that the topographic heights within the study area reach approximately ~4750 m meters above sea level. The terrain corrections applied in this region exhibit variations ranging from -0.317 mGal to 56.533 mGal for the pointwise gravity data with a standard deviation of 2.153 mGal. While the average value of pointwise data is 1.043 mGal, the median value is 0.476 mGal. Terrain correction (TC) values calculated on the grid nodes exhibit a spread between -0.327 mGal and 118.221 mGal. The standard deviation of these values is 4.086 mGal while their mean value is 1.718 mGal and median value is 0.371 mGal. In the table above, the root mean square error (RMSE) values are computed using the formula $RMSE = \sqrt{\mu^2 + \sigma^2}$, where μ is mean, and σ is standard deviation.

Figures 6.4a, b illustrate the maps of Simple Bouguer Anomalies (SBAs) and Complete Bouguer Anomalies (CBAs), respectively. These anomalies are computed at the gravity observation points and subsequently interpolated using the Kriging interpolation algorithm with a spacing of 1 arc-minute.

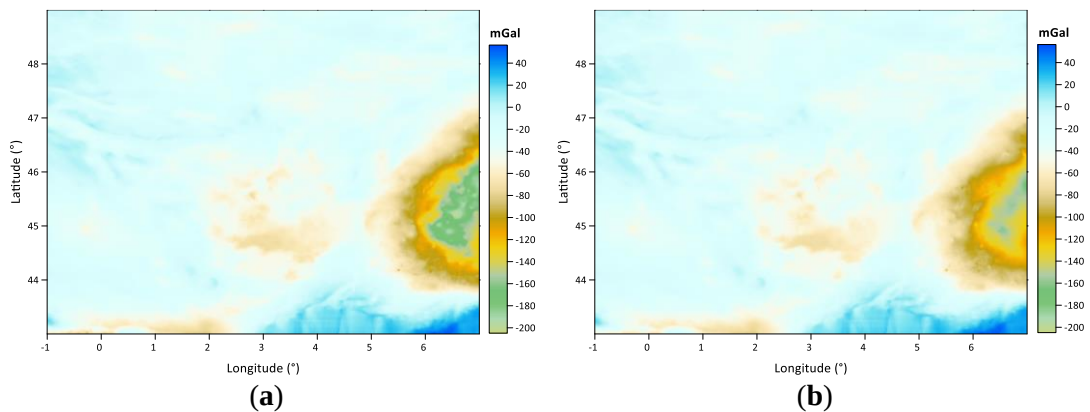


Figure 6.4 : Simple Bouguer anomalies (SBAs) computed at gravity points and interpolated at grid nodes spaced at 1 arc-minute intervals using Kriging (a). Complete Bouguer anomalies (CBAs) determined at gravity points and interpolated at grid nodes spaced at 1 arc-minute intervals using Kriging (b) (unit: mGal).

Figure 6.5 includes a difference map illustrating the disparity between the SBA and CBA values, representing the terrain corrections between them as outlined in Equation (3.6). The spatial distribution of these residual corrections closely mirrors the terrain correction maps presented in Figure 6.3, as expected.

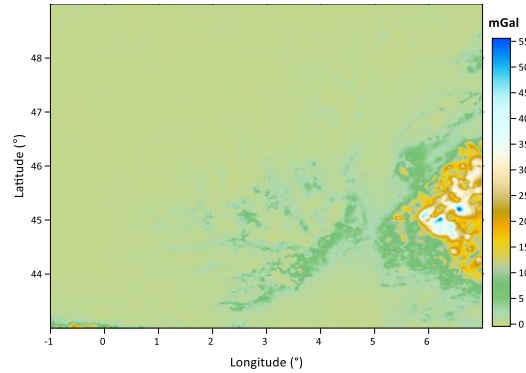


Figure 6.5 : Map showing the difference between SBAs and CBAs at grid nodes spaced at 1 arc-minute intervals (unit: mGal).

Table 6.2 summarizes the statistical properties of the Simple Bouguer Anomalies (SBAs), Complete Bouguer Anomalies (CBAs), and the difference between these two values. An examination of this table reveals that the SBAs range from -204.830 mGal to 56.373 mGal, with a standard deviation of 31.640 mGal. The CBAs exhibit a range of -172.656 mGal to 57.271 mGal, with a standard deviation of 27.905 mGal. An analysis of the difference between SBA and CBA values reveals a range of -0.439 mGal to 55.590 mGal, with a standard deviation of 4.879 mGal. Significantly, these statistics exhibit a high degree of consistency with the terrain correction statistics presented in Table 6.1, which strengthens the validity of the applied terrain corrections.

Table 6.2 : Statistical properties of the gridded Simple Bouguer Anomalies (SBAs) and Complete Bouguer Anomalies (CBAs), alongside their difference, all generated using Kriging interpolation with a 1 arc-minute grid spacing.

Unit: mGal	Max	Min	Mean	Median	Std	RMSE
SBA	56.373	-204.830	-34.170	-28.652	31.640	46.569
CBA	57.271	-172.656	-32.238	-28.097	27.905	42.638
Difference	55.590	-0.439	1.932	0.396	4.879	5.247

Based on the comparative statistics given above in Table 6.2, a notable disparity between the SBAs and CBAs is evident. Furthermore, examination of the difference map reveals distinct variations between SBAs and CBAs in the mountainous regions of the study area, while discrepancies appear negligible in flat terrain. The significant increase in the terrain correction parameter with increasing topographic heights is the

primary factor underlying this result. By calculating the difference between the terrain corrections directly computed at each grid node and those obtained by subtracting the interpolated SBAs from the CBAs, the impact of the interpolation process on gravity field mapping is revealed. This approach highlights the potential influence of interpolation on the accuracy of terrain corrections across the study area.

Then, to investigate the influence of Bouguer anomaly selection on geoid modeling, free-air anomalies are subsequently computed from both the gridded SBA and CBA data. These gridded free-air anomaly datasets then serve as inputs for geoid computations employing the Least Squares Modification of Stokes Formula with Additive Corrections (LSMSA) approach.

Figures 6.6a,b present the free-air anomalies calculated on a 1 arc-minute grid using the SBA (Figure 6.4a) and CBA grids (Figure 6.4b), respectively. Figure 6.7 illustrates a difference map of these two free-air anomaly grids. This map visually emphasizes the spatial variations in free-air anomalies that arise from the choice of Bouguer anomaly type (SBA or CBA). Table 6.3 provides the descriptive statistics of these grid-based free air anomaly datasets and their variances. The table reveals that the extremes of the difference grid range from -86.721 mGal to 37.167 mGal, with a standard deviation of 2.103 mGal. The locations where these extreme values occur within the study area can be readily identified by examining Figure 6.7.

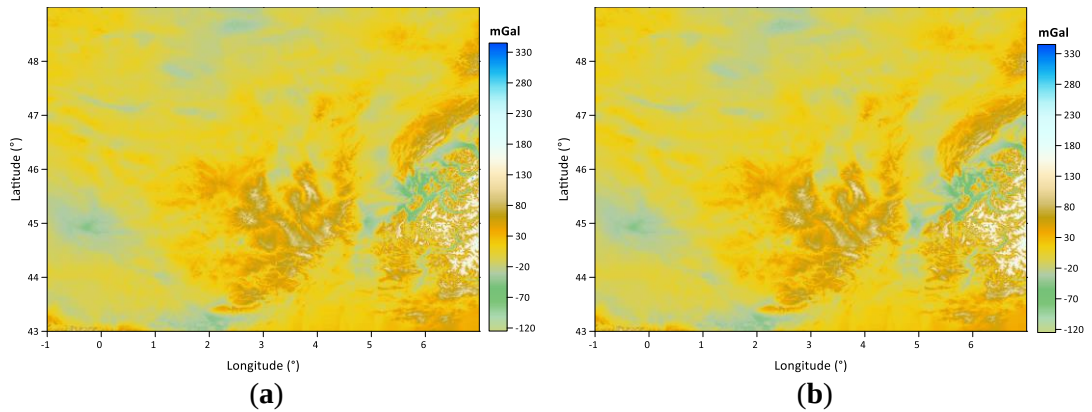


Figure 6.6 : Gridded free-air anomalies with a spatial resolution of 1 arc-minute, derived from the grids of Simple Bouguer Anomalies (SBA) (a) and Complete Bouguer Anomalies (CBA) (b) (unit: mGal).

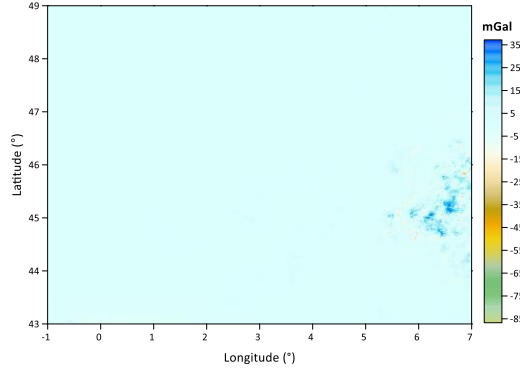


Figure 6.7 : Difference map depicting the variations between the free-air anomaly grids derived from the grids of Simple Bouguer Anomalies (SBAs) and Complete Bouguer Anomalies (CBAs) (unit: mGal).

Table 6.3 : Statistics of the gridded Free Air Anomalies (FAAs) generated from both simple and complete Bouguer anomalies, as well as their discrepancies.

Unit: mGal	Max	Min	Mean	Median	Std	RMSE
FAA (via SBAs)	345.683	-126.313	8.245	1.888	29.463	30.595
FAA (via CBAs)	258.962	-129.942	8.458	1.948	29.867	31.042
Difference	37.167	-86.721	0.213	0.016	2.103	2.114

Subsequently, two distinct gravimetric geoid models are generated utilizing these free air anomaly grids. In the geoid model computations employing the LSMSA technique, the designated computation parameters are specified as follows: the spherical harmonic expansion degree of the geopotential model is $M = 780$, the error variance value is $C = 4 \text{ mGal}^2$, the type of modification is $b_n = \text{biased}$, and the integration radius is $\Psi_0 = 0.25^\circ$. The selection process of these parameters for the LSMSA geoid model calculation is explained in detail in Section 6.3.

Building upon the free-air anomaly grids derived from the SBAs and CBAs (refer to Figure 6.6), Figure 6.8 presents the corresponding geoid models computed using the LSMSA method on a 1 arc-minute grid spacing. Figure 6.8a depicts the geoid model derived from the SBA-based free-air anomalies, while Figure 6.8b illustrates the geoid model obtained from the CBA-based free-air anomalies. Figure 6.9 further complements the analysis by presenting a difference map of these two geoid models. This map visually emphasizes the spatial variations in geoid heights arising from the choice of Bouguer anomaly type (SBA or CBA). A comparison of Figures 6.7 (free-air anomaly difference) and 6.9 (geoid height difference) reveals a similar spatial distribution pattern for both differences. Furthermore, both figures suggest that the magnitude of these differences tends to increase with increasing topographic elevation.

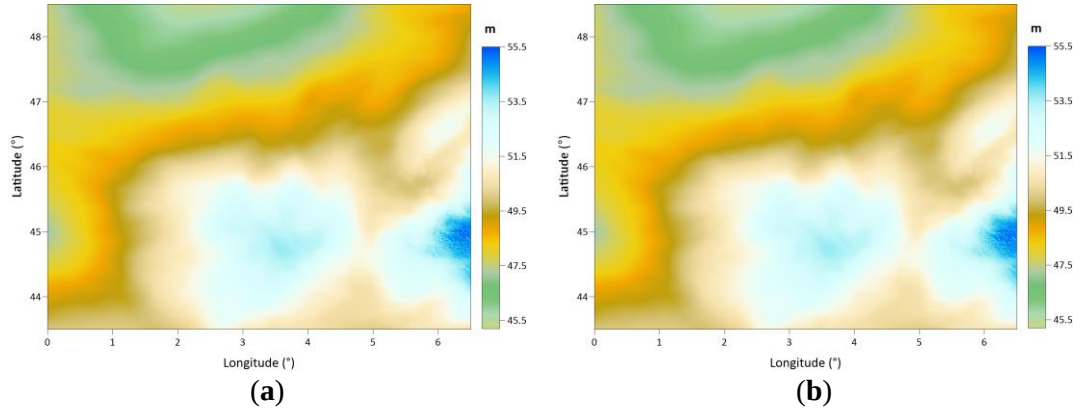


Figure 6.8 : Geoid models computed with free-air anomalies obtained from the gridded SBAs (a) and CBAs (b) with a resolution of 1 arc-minute (unit: meter).

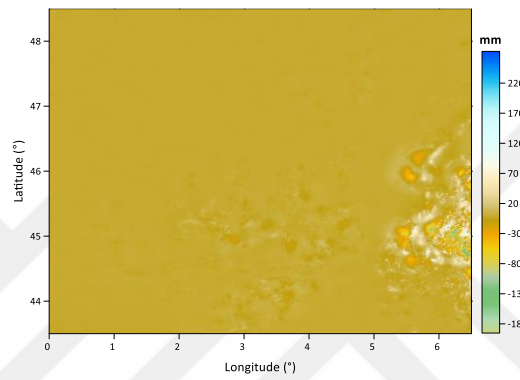


Figure 6.9 : The difference between the geoid models depicted in Figures 6.8a,b (unit: millimeter).

Table 6.4 provides the statistical analysis of the computed geoid models along with their discrepancies. The initial inspection of the table suggests that the two models share similar characteristics, evident from their matching mean, median and standard deviation values. However, upon closer examination, it becomes apparent that there are discrepancies between the two models, as indicated by the minimum and maximum difference values of -19.5 cm and 27.3 cm, respectively.

Table 6.4 : Geoid Model Statistics (computed using SBA & CBA-based FAA).

Unit: m	Max	Min	Mean	Median	Std	Rmse
SBA based GEOID	55.743	45.268	49.616	49.567	2.021	49.657
CBA based GEOID	55.470	45.267	49.616	49.567	2.021	49.657
Difference	0.273	-0.195	0.000	0.000	0.008	0.008

Beyond the direct comparison of the two geoid models, their individual accuracy is further assessed through validation at GPS/leveling benchmarks established within the study area. This validation process relies on Equation (6.1):

$$\Delta N = N^{GPS/Lev} - N^{Geoid} \quad (6.1)$$

Here, the geoid height ($N^{GPS/Lev}$), derived from GPS height (h) and orthometric height (H) at a benchmark ($N^{GPS/Lev} = h - H$), is compared with the geoid height obtained from the geoid model (N^{Geoid}). Throughout the validation process, the N^{Geoid} parameter is interpolated from the geoid model using the inverse distance to a power algorithm.

Table 6.5 provides validation statistics for the geoid models, indicating identical absolute accuracy for both, with a standard deviation of 4.1 cm for the geoid height variances at the benchmarks. The discrepancy between the two geoid models is not discernible in the results of the GPS/leveling validation, as the control benchmarks are situated within a limited area in the region, insufficient to capture topographical variations adequately. As a result of this limitation of the independent validation dataset, discrepancies in higher topographic regions between the two geoid models are not evident in the validation statistics. The examination of the effect of GPS/Leveling benchmark positions on the geoid validation process is conducted in Section 6.5.

Table 6.5 : Validation statistics for geoid models computed using SBA & CBA-based FAA.

Unit: m	Max	Min	Mean	Median	Std	RMSE
ΔN SBA	1.161	0.973	1.059	1.064	0.041	1.060
ΔN CBA	1.162	0.972	1.059	1.064	0.041	1.060

6.3 Optimum Parameters for the LSMSA Geoid Calculation Method

The accuracy of geoid models computed using the Least Squares Modification of Stokes' Formula with Additive Corrections (LSMSA) method is demonstrably influenced by the selection of computational parameters. This study investigates the identification of an optimal parameter set that yields the most accurate geoid model. We achieve this through a systematic trial-and-error procedure utilizing free-air anomaly grids derived from complete Bouguer anomalies via Kriging interpolation. To ensure the selection of the most accurate geoid model, each iteration utilizes a distinct parameter set. The resulting models are then rigorously evaluated against GPS/leveling benchmark data. This meticulous comparison of absolute accuracies ultimately leads to the identification of the optimal parameter set for the present study.

Initially, the computation of geoid models involves the utilization of varying degrees of harmonic expansion of the geopotential model (M), ranging from 180° to 780° ,

alongside alterations in the cap size parameter denoted by Ψ_0 , set at values of 0.10° , 0.25° , 0.50° , 0.75° , and 1.00° , respectively. During this preliminary phase, the error variance ($C = 4 \text{ mGal}^2$) and the b_n parameter ($b_n = \text{optimum}$) remain constant. The validation outcomes of the computed geoid models are presented in Table 6.6.

Table 6.6 : Standard deviations of the discrepancies in geoid heights resulting from the validation process of geoid models computed with diverse sets of M and Ψ_0 computational parameters.

Unit: m	M = 180	M = 240	M = 300	M = 360	M = 420	M = 480	M = 540	M = 600	M = 660	M = 720	M = 780
$\Psi = 0.10^\circ$	0.261	0.109	0.096	0.083	0.078	0.076	0.076	0.076	0.076	0.076	0.075
$\Psi = 0.25^\circ$	0.174	0.146	0.142	0.139	0.131	0.112	0.102	0.085	0.072	0.067	0.065
$\Psi = 0.50^\circ$	0.183	0.188	0.134	0.100	0.092	0.091	0.091	0.091	0.087	0.082	0.079
$\Psi = 0.75^\circ$	0.197	0.142	0.117	0.107	0.121	0.115	0.110	0.106	0.109	0.111	0.112
$\Psi = 1.00^\circ$	0.197	0.158	0.153	0.172	0.175	0.166	0.167	0.183	0.188	0.195	0.192

Building upon the findings in Table 6.6, which identified $M = 780^\circ$ and $\Psi_0 = 0.25^\circ$ as optimal parameters, Table 6.7 delves into the selection of the most suitable error variance (C) value. This table presents the means and standard deviations of geoid height discrepancies at GPS/leveling points for various C values, allowing for the determination of the C that yields the most accurate geoid model.

Table 6.7 : Standard deviations of the disparities in geoid heights resulting from the validation process of geoid models computed with different error variance (C) parameters.

Unit: m	Max	Min	Mean	Std	RMSE
$C = 1 \text{ mGal}^2$	1.113	0.803	0.987	0.065	0.989
$C = 4 \text{ mGal}^2$	1.113	0.802	0.987	0.065	0.989
$C = 9 \text{ mGal}^2$	1.113	0.801	0.986	0.066	0.989
$C = 16 \text{ mGal}^2$	1.113	0.801	0.986	0.066	0.988

The validation metrics indicate that the impact of the error variance parameter on the accuracy of the geoid model is less pronounced compared to the preceding two parameters. Given that all solutions exhibit similar performance levels, an error variance of $C = 4 \text{ mGal}^2$ is deemed appropriate, considering the accuracy reported for the terrestrial gravity data.

The final step involves determining the optimal modification type (b_n) for the geoid model, which can be biased, unbiased, or optimum. The validation results for geoid models calculated using these different b_n values are presented in Table 6.8. Interestingly, the biased solution yielded the most favorable outcome and was chosen

for further analysis. Consequently, all subsequent tests investigating the influence of the interpolation algorithm on gravity data gridding will employ a fixed set of computational parameters: a harmonic expansion degree of $M = 780^\circ$, a cap size of $\Psi_0 = 0.25^\circ$, an error variance of $C = 4 \text{ mGal}^2$, and a biased b_n solution.

Table 6.8 : Validation statistics demonstrating the impact of modification type (b_n) on geoid height differences.

Unit: m	Max	Min	Mean	Std	RMSE
biased	1.162	0.972	1.059	0.041	1.060
unbiased	1.113	0.815	0.991	0.063	0.993
optimum	1.113	0.802	0.987	0.065	0.989

6.4 Effects of Gravity Data Interpolation Methods in Geoid Model Computation

In this section, the aim of the numerical tests is to explain the role of the interpolation algorithm in gravity gridding for preparing the Bouguer gravity anomaly maps and input gravity grid for geoid model computations. Building upon previous assessments, a consensus has been reached regarding the choice of Bouguer gravity anomalies for this section. Based on the findings outlined in Section 6.2, complete Bouguer gravity anomalies are selected for use in this segment, given the diverse topographical terrain within the study area, where significant discrepancies between Simple Bouguer Anomalies (SBAs) and Complete Bouguer Anomalies (CBAs) are evident, particularly in mountainous regions. Figure 6.10a visualizes the complete Bouguer gravity anomalies at the gravity points, reduced to the geoid surface. This map represents the CBAs before interpolation with any specific algorithm. This depiction represents the CBAs before undergoing any gridding process with interpolation algorithms. In the experiments, these datasets are gridded at an arc-minute spacing (corresponding to the approximate spatial density of the gravity observations on the topography) using four distinct algorithms: Ordinary Kriging, Inverse Distance to a Power (IDP), Nearest Neighbor, and Artificial Neural Network (ANN).

Within these methods, Kriging is a commonly employed method in gravity field and geoid determination, often integrated into geoid model computation software. Given that gravity field and geoid modeling rely on measurements, errors are inevitable in these measurements. Consequently, minimizing these errors in the modeling process requires the implementation of an appropriate stochastic strategy. Kriging predicts the value of a function at a given point by computing the weighted average of the data in

the neighborhood of that point. However, the Kriging algorithm utilizes the Gauss–Markov theorem to independently consider the estimated value and its error, providing a best linear unbiased estimator at an unsampled location (interpolation point) based on adopted covariance assumptions. This advantageous feature of the Kriging algorithm contributes to its widespread adoption in studies related to gravity field prediction.

Inverse Distance to a Power (IDP) and Nearest Neighbor algorithms are popular choices in spatial data analysis due to their practicality and computational efficiency. These methods often deliver adequate accuracy for various applications. While conceptually similar, IDP offers a more advanced approach compared to Nearest Neighbor. IDP incorporates a wider range of observations beyond the single nearest neighbor. It calculates the value at a grid node using a weighted average of neighboring stations, where the weight of each data point is inversely related to its distance, potentially following a non-linear relationship determined by the chosen power parameter. This ability to consider a broader spatial context differentiates IDP from the Nearest Neighbor method.

The artificial neural network (ANN) algorithm constitutes the final method evaluated in this section. This algorithm operates on a mathematical framework and employs a "feed forward–back propagation" optimization-based processing strategy. Consequently, it differs significantly from the preceding three methods examined here. The inherent mathematical complexity of the ANN algorithm involves numerous computational parameters that necessitate determination for achieving optimal outcomes. In the ANN interpolation conducted in this study, the Levenberg–Marquardt training method is employed, as it is widely endorsed in neural network literature. Through trial-and-error experimentation, the optimal numbers of neurons and iterations for training are determined. Consequently, it is established that 200 neurons and 300 iterations yield optimal results in these computations. Figure 6.10b below presents the Complete Bouguer Anomaly (CBA) map generated using Kriging interpolation on a 1 arc-minute grid.

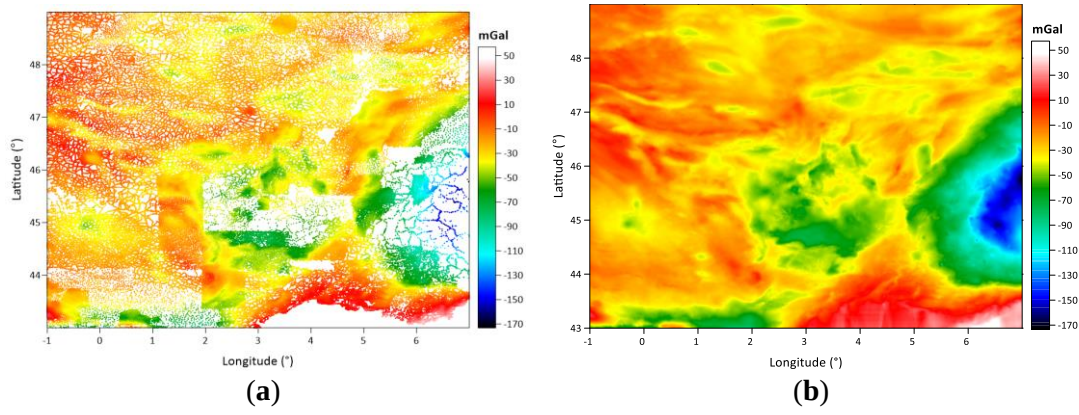


Figure 6.10 : Complete Bouguer gravity anomalies at gravity points reduced to the geoid surface prior to gridding are depicted in **(a)**, while **(b)** illustrates a map generated using a 1 arc-minute resolution CBA grid calculated via the Kriging interpolation algorithm.

Figure 6.11a-c presents Bouguer gravity anomaly maps generated using the complete Bouguer anomaly (CBA) grid data interpolated with three distinct algorithms: **(a)** inverse distance to a power, **(b)** nearest neighbor, and **(c)** artificial neural network.

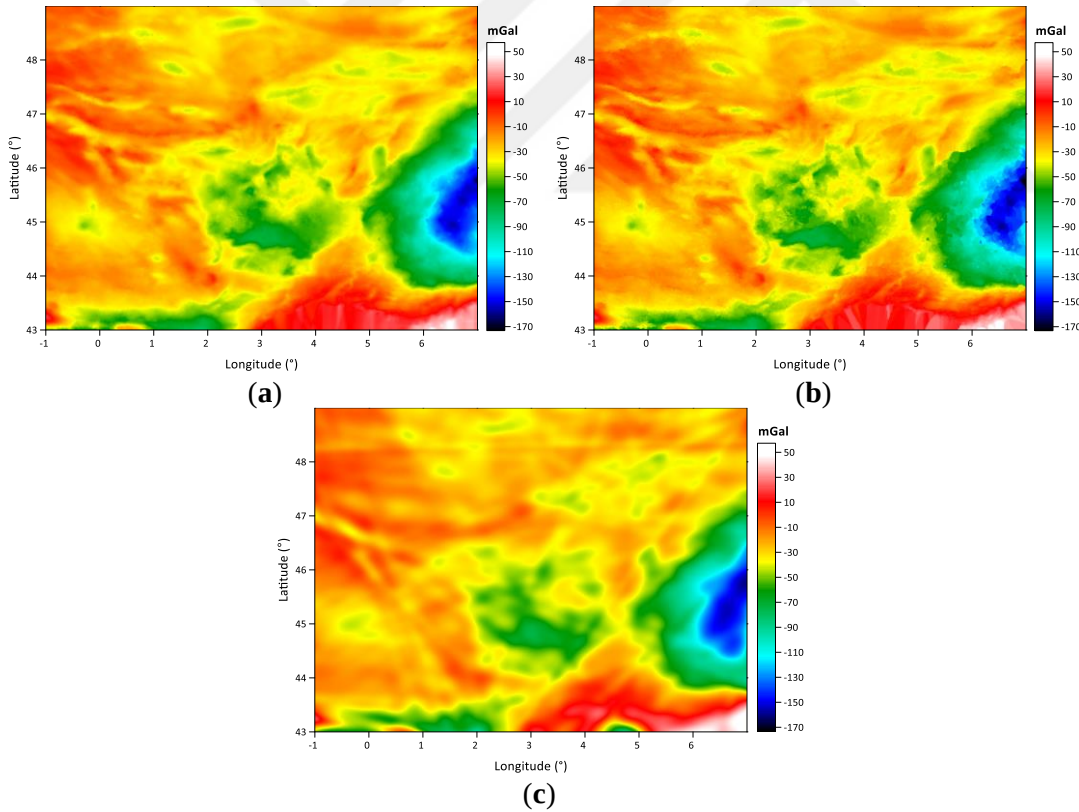


Figure 6.11 : Bouguer gravity anomaly maps derived from 1 arc-minute CBA grids interpolated with: **(a)** inverse distance to a power, **(b)** nearest neighbor, and **(c)** artificial neural network.

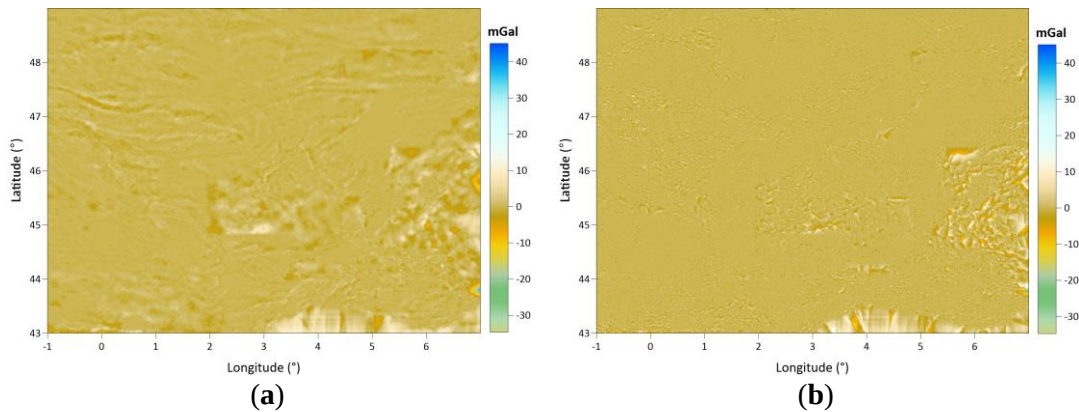
Table 6.9 summarizes the statistical properties of the CBA grid datasets generated using each interpolation algorithm. While the basic statistics suggest minimal variation

between the grids, a more rigorous assessment is necessary. To evaluate potential discrepancies beyond basic measures, we computed the differences between Inverse Distance to a Power (IDP), Nearest Neighbor, and Artificial Neural Network (ANN) grids from Kriging grid. The Kriging grid serves as the reference dataset due to its widespread adoption in gravity field calculations (see Tables 6.10, 6.11 and Figures 6.12a-c, 6.13a-c).

Table 6.9 : Statistics of the complete Bouguer anomaly (CBA) grids generated using various interpolation algorithms.

Unit: mGal	Max	Min	Mean	Median	Std	RMSE
Kriging	57.282	-172.616	-32.234	-28.095	27.901	42.632
IDP	55.503	-172.575	-32.310	-28.028	27.676	42.543
Nearest Neighbor	56.650	-173.173	-32.290	-28.087	27.866	42.652
ANN	55.196	-159.404	-32.538	-27.853	27.779	42.783

Figures 6.12a-c depict the variations between the Kriging grid and the grids interpolated using Inverse Distance to a Power (IDP), Nearest Neighbor, and Artificial Neural Network (ANN) algorithms, respectively. Upon examining the grid difference maps, it becomes evident that although the basic statistical measures are close, the grids generated by each algorithm depict distinct surface patterns. Furthermore, the distribution of discrepancies between the datasets varies. In Figure 6.12a, grid differences between the Kriging and IDP methods display a seemingly homogeneous distribution across the area, with differences escalating in mountainous terrain. Conversely, in Figure 6.12c, grid differences between the Kriging and ANN methods demonstrate a wavy pattern and nearly uniform distribution across the entire area. Based on this observation, no correlation between the distribution of grid differences and topography is discernible. Consequently, in terms of distribution pattern, the Kriging grid displays the least consistency with the ANN grid among the datasets.



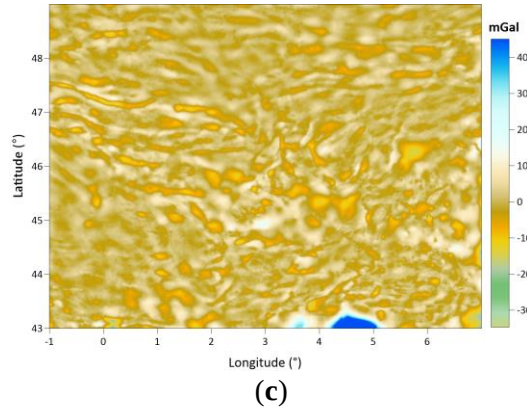


Figure 6.12 : Difference maps comparing the Kriging grid with grids interpolated using (a) Inverse Distance to a Power, (b) Nearest Neighbor, and (c) the Artificial Neural Network algorithm, representing complete Bouguer anomaly differences at 1 arc-minute resolution grid nodes.

Table 6.10 presents the statistical analysis of grid differences. Notably, the Kriging grid and IDP grid exhibit maximum consistency, fitting better in plain topography while their disparities increase in mountainous regions. As per these statistics, the Kriging grid closely matches the IDP and nearest neighbor grids, exhibiting standard deviations of 1.3 mGal and 1.6 mGal, respectively, while showing no significant offset in terms of the mean values of grid differences between these datasets. Nevertheless, upon comparing the grid differences between Kriging and ANN, it becomes apparent that these datasets diverge, displaying a standard deviation of 5.3 mGal and a 0.3 mGal offset. Consequently, these statistical findings corroborate the visual interpretation of the grid difference maps. In each evaluation, the grid difference maps highlight significant disparities, notably concentrated in the southeast corner of the map sheets. This localized discrepancy may arise from the Kriging algorithm's limited effectiveness in that specific area. This region, which aligns with a coastline and lacks data in the sea portion (the southeastern region of the Auvergne test area), might result in inadequate handling by the Kriging algorithm.

Table 6.10 : Statistical summary of the CBA grid differences between the Kriging grid and the grids interpolated using Inverse Distance to a Power (IDP), Nearest Neighbor, and Artificial Neural Network (ANN) algorithm.

	Unit: mGal	Max	Min	Mean	Median	Std	RMSE
KRIGING vs.	IDP	13.646	-26.880	0.076	0.009	1.314	1.316
	Nearest Neighbor	45.066	-34.711	0.056	0.006	1.618	1.619
	ANN	100.999	-37.245	0.304	0.082	5.310	5.318

To assess their performance from an alternative perspective, grids generated by interpolation methods other than Kriging are compared with each other. For a visual

and statistical representation of the 1 arc-minute grid differences of CBAs computed via Nearest Neighbor, IDP, and ANN algorithms, refer to Figures 6.13a-c and Table 6.11 below:

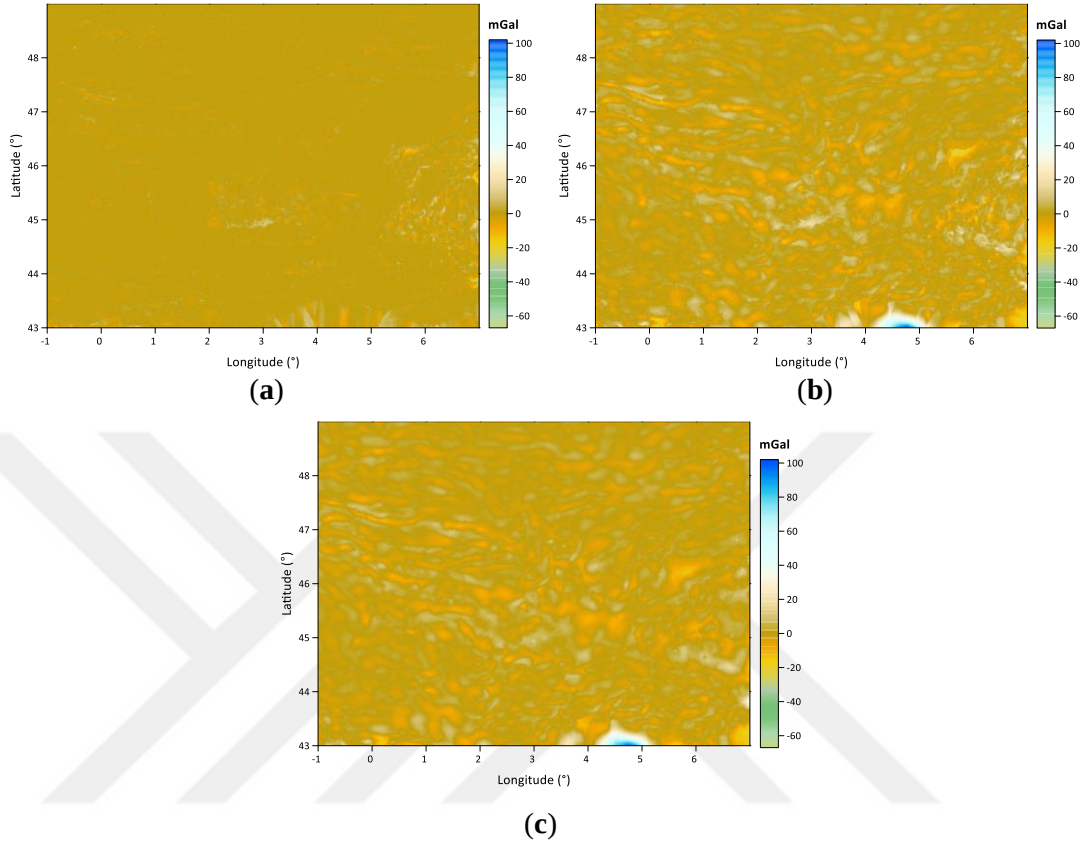


Figure 6.13 : Difference maps depicting comparisons between grids interpolated using Nearest Neighbor and Inverse Distance to a Power **(a)**, Nearest Neighbor grid and Artificial Neural Network algorithm **(b)**, and Inverse Distance to a Power and Artificial Neural Network algorithm **(c)**, illustrating differences in complete Bouguer anomalies at 1 arc-minute resolution grid nodes.

In the difference maps in Figure 6.13 above, given that the minimum and maximum values of the differences exceed those in Figure 6.12, the color scale is restricted to -67 mGal and 102 mGal, unlike Figure 6.12. As seen in the Figure 6.13, there is no correlation with topography in the difference grids calculated with the ANN algorithm. Grid differences of Nearest Neighbor and IDP illustrates homogeneous distribution.

Table 6.11 : Statistics of the CBA differences between Nearest Neighbor, Inverse Distance to a Power (IDP) and Artificial Neural Network (ANN) grids.

	Unit: mGal	Max	Min	Mean	Median	Std	RMSE
Nearest vs.	IDP	20.658	-62.672	0.020	0.003	1.788	1.788
Nearest vs.	ANN	102.417	-67.108	0.248	0.0618	5.381	5.387
IDP vs.	ANN	99.306	-24.959	0.228	0.045	4.995	5.000

When reviewing Table 6.11 above, the Nearest Neighbor grid aligns closely with the IDP grid, displaying a standard deviation of 1.788 mGal, while the differences between the Nearest Neighbor - ANN and IDP - ANN grids reach 5.381 mGal and 4.995 mGal, respectively. It is noteworthy that the average differences obtained with the ANN algorithm are approximately 0.2 mGal.

In the subsequent analyses, the impact of variations in Complete Bouguer Anomaly (CBA) grids on the determination of geoid models using the LSMSA technique is investigated. To generate free air anomaly grids for geoid computations with the LSMSA method, the CBA grids derived via Kriging, Inverse Distance to a Power (IDP), Nearest Neighbor, and Artificial Neural Network (ANN) algorithms are upward continued. As a result, the free air anomaly grids, obtained respectively from the Kriging, IDP, Nearest Neighbor, and ANN complete Bouguer anomaly grids, are presented in Figures 6.14b and 6.15a–c. For comparative purposes, the un-gridded free air anomaly values at the gravity points are also shown in Figure 6.14a. Additionally, Table 6.12 presents statistical information regarding the free air anomaly grid datasets.

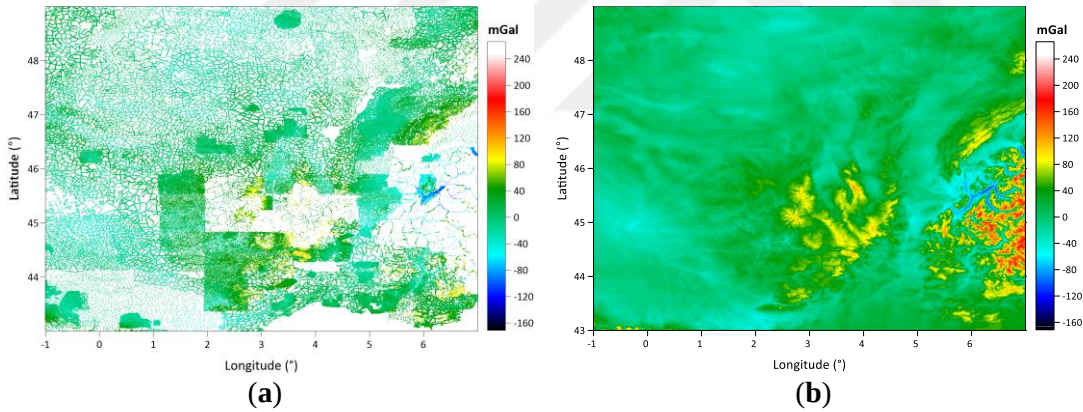
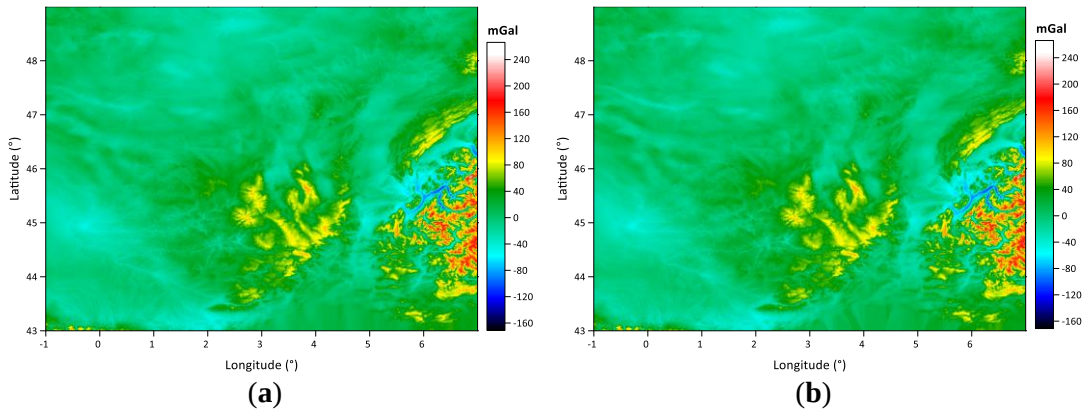


Figure 6.14 : Un-gridded free air anomalies at gravity points (a), and free air anomaly grid derived from the Kriging Complete Bouguer Anomaly (CBA) grid at 1 arc-minute resolution (b).



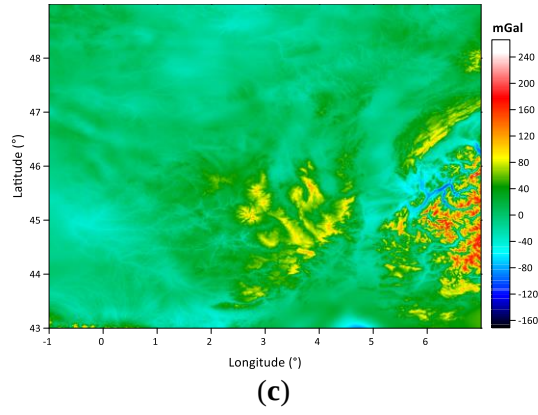
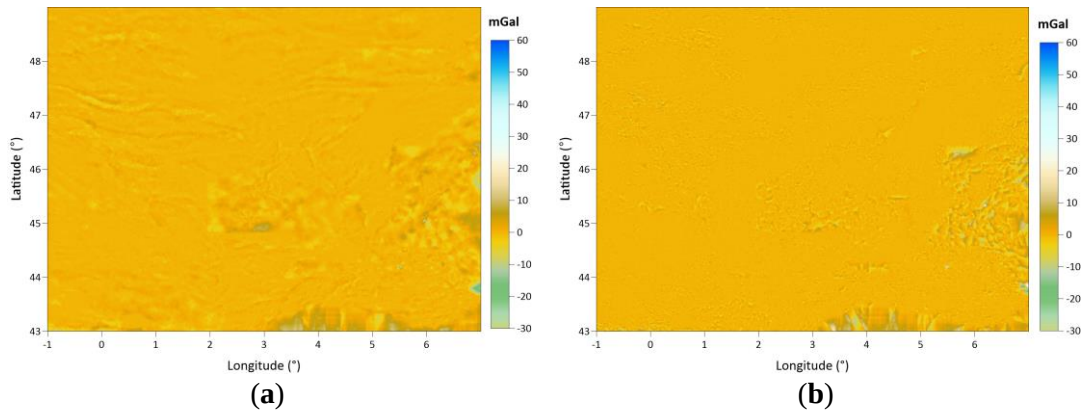


Figure 6.15 : Free air anomaly grids derived from: (a) Inverse Distance to a Power (IDP) CBA grid, (b) Nearest Neighbor CBA grid, and (c) Artificial Neural Network (ANN) CBA grid with 1 arc-minute grid spacing.

Table 6.12 : Statistical properties of free air anomaly grids derived from Kriging, the Inverse Distance to a Power (IDP), Nearest Neighbor, and Artificial Neural Network (ANN) CBAs grids.

Unit: mGal	Max	Min	Mean	Std	RMSE
Kriging	258.962	-129.942	8.458	29.867	31.042
IDP	266.128	-128.788	8.382	29.822	30.978
Nearest Neighbor	266.037	-132.073	8.402	29.884	31.042
ANN	256.155	-128.766	8.154	30.018	31.106

To delve deeper into the relationships between the free air anomaly grid datasets, their pairwise differences are computed. To streamline the analysis, the Kriging-derived grid serves as the reference, with differences calculated relative to the other datasets. Figure 6.16 displays the resulting free air anomaly difference maps. Visual inspection reveals a clear resemblance between these maps and their corresponding CBA grid difference maps presented in Figure 6.13.



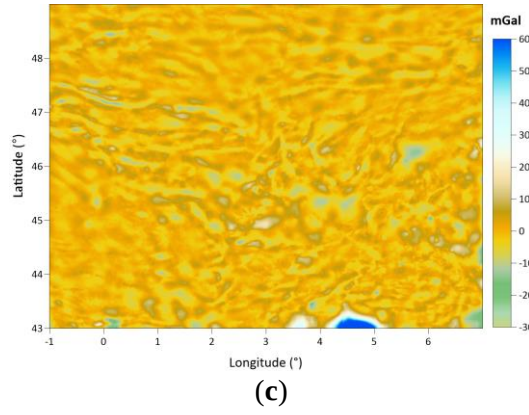


Figure 6.16 : Free air anomaly differences. Figures depict the free air anomaly variations between the Kriging-based grid and: (a) the Inverse Distance to a Power (IDP) grid, (b) the Nearest Neighbor grid, and (c) the Artificial Neural Network (ANN) grid. All figures display 1 arc-minute resolution.

Table 6.13 summarizes the statistics of these free air anomaly differences, which are notably identical to those presented in Table 6.11. This equivalence can be explained by Equation (2.6). When calculating free air anomaly differences, the restored Bouguer reduction (δg_B) and terrain correction (δg_{TC}) terms cancel out due to subtraction. Consequently, in this comparison, the free air anomaly difference at any grid node becomes equivalent to the complete Bouguer anomaly difference. This finding implies that while the interpolation process is applied directly to the CBAs, the resulting variations propagate to the derived free air anomaly grids. Therefore, we anticipate observing the effects of differences in free air anomaly grids on the results of geoid model determination.

Table 6.13 : Statistical analysis of the disparities in free air anomalies between the Kriging-based grid and the IDP-based grid, Nearest Neighbor-based grid, and the ANN-based grid, respectively.

	Unit: mGal	Max	Min	Mean	Median	Std	RMSE
KRIGING vs.	IDP	13.646	-26.880	0.076	0.009	1.314	1.316
	Nearest neighbor	45.066	-34.711	0.056	0.006	1.618	1.619
	ANN	100.999	-37.245	0.304	0.082	5.310	5.318

The differences between grids other than the Kriging grid are also given visually and statistically (see Figure 6.17 and Table 6.14).

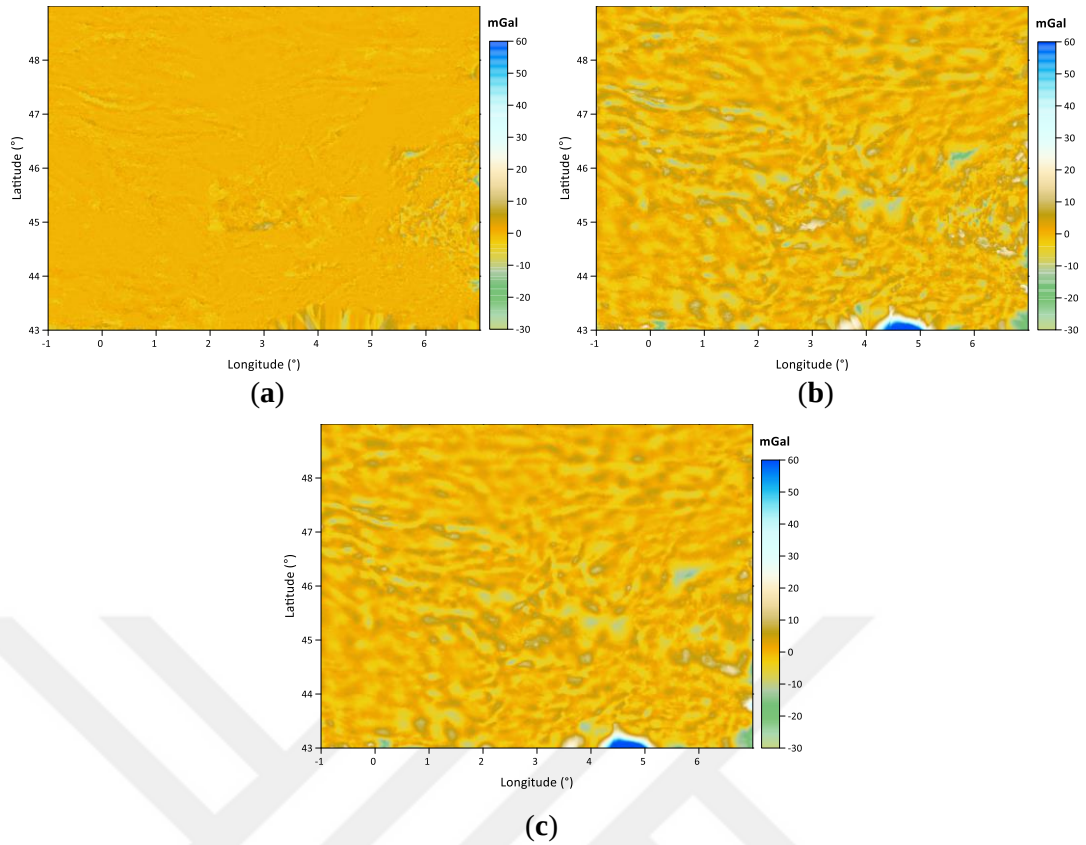


Figure 6.17 : Difference maps depicting comparisons between grids interpolated using Nearest Neighbor and Inverse Distance to a Power (a), Nearest Neighbor grid and Artificial Neural Network algorithm (b), and Inverse Distance to a Power and Artificial Neural Network algorithm (c), illustrating differences in free air anomalies at 1 arc-minute resolution grid nodes.

Table 6.14 : Statistics of the FAA differences between Nearest Neighbor, Inverse Distance to a Power (IDP) and Artificial Neural Network (ANN) grids.

	Unit: mGal	Max	Min	Mean	Median	Std	RMSE
Nearest vs.	IDP	20.658	-62.672	0.020	0.003	1.788	1.788
Nearest vs.	ANN	102.417	-67.108	0.248	0.062	5.381	5.387
IDP vs.	ANN	99.306	-24.959	0.228	0.045	4.995	5.000

As can be seen from the statistics given in table 6.14, FAA grid differences are the same as CBA grid differences in Table 6.11 (The rationale for this is elucidated in the preceding paragraphs).

In the concluding phase of this numerical assessment, gravimetric geoid models are generated with a resolution of 1 arc-minute. Employing the free air anomaly grid datasets as input and the computational parameters established in Section 6.3, the Least Squares Modification of Stokes Integral with Additive Corrections (LSMSA) method is utilized for geoid computation. Figure 6.18 presents a map visualization of each geoid model, depicting geoid heights ranging from 45.2 m to 55.4 m, with a mean

value of 49.6 m and a standard deviation of 2.0 m. The decision to present these statistics with decimeter precision is deliberate. While the calculated model statistics in Table 6.15 differ at the centimeter level, such a level of detail is not considered practically significant for this analysis.

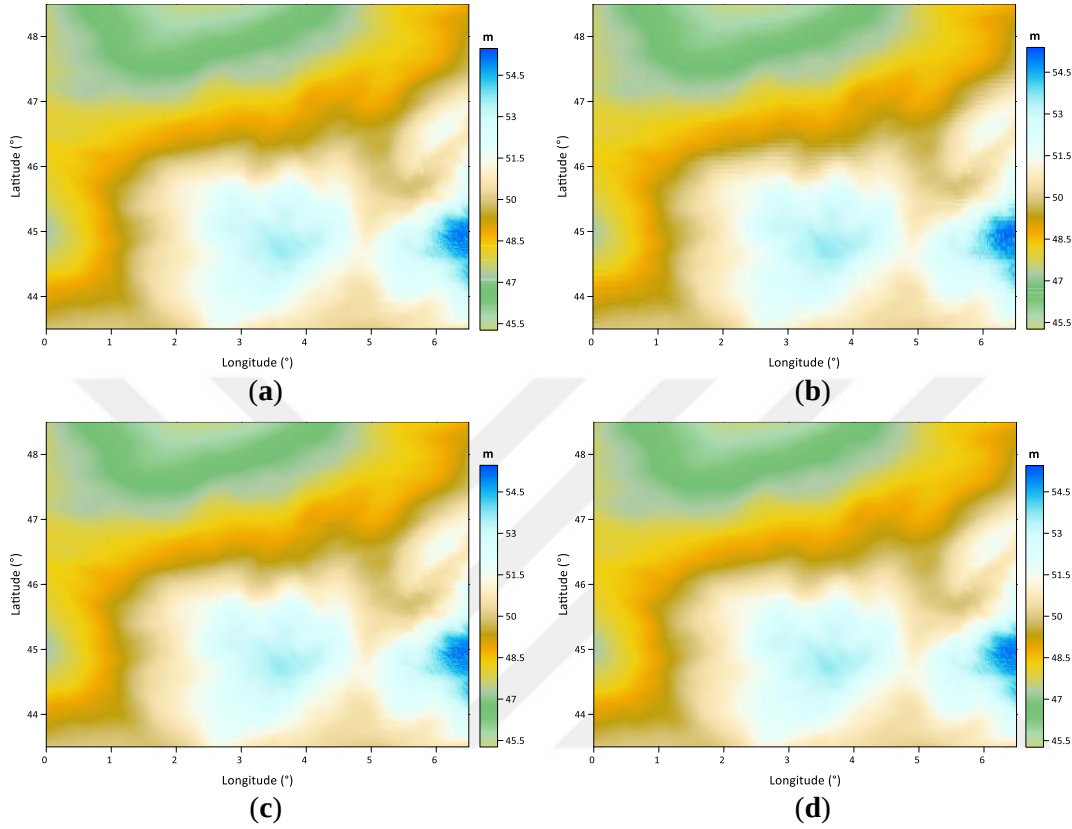


Figure 6.18 : Geoid models generated using free air anomaly grids obtained through Kriging (a), IDP (b), Nearest Neighbor (c), and ANN (d) interpolation methods, with a grid spacing of 1 arc-minute.

Table 6.15 : Statistical summary of geoid models derived from Free Air Anomaly grids.

Unit: m	Max	Min	Mean	Std	RMSE
Kriging	55.470	45.267	49.616	2.021	49.657
IDP	55.450	45.266	49.616	2.021	49.657
Nearest neighbor	55.462	45.267	49.616	2.021	49.657
ANN	55.451	45.264	49.616	2.021	49.657

Figure 6.19 depicts the differential patterns between the computed geoid models. Consistent with the approach adopted in prior stages of the numerical analysis, where results from each interpolation technique were compared to a reference Kriging-based solution, the current analysis focuses on geoid model differences. Here, each geoid model derived from a free air anomaly dataset is compared to the model generated using the Kriging-based free air anomaly grid.

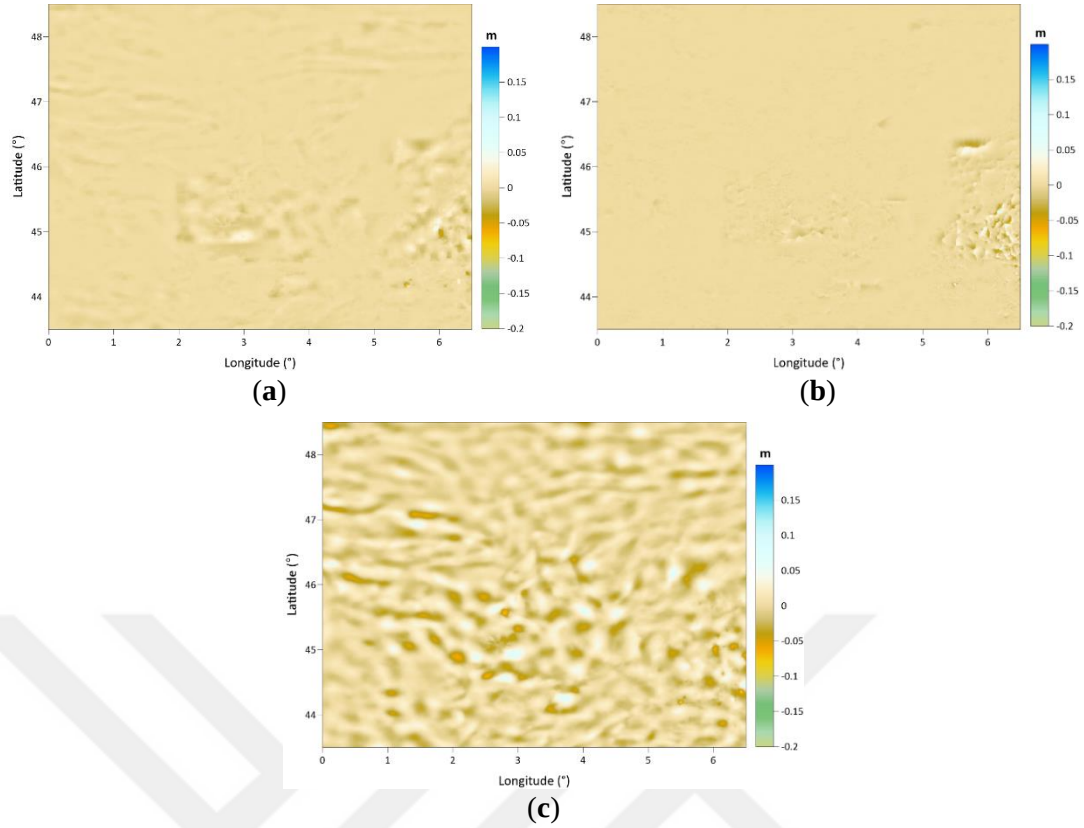


Figure 6.19 : Geoid height difference maps between the Kriging-derived geoid model and those based on: the IDP **(a)**, Nearest Neighbor **(b)**, and the ANN **(c)** methods, with a 1 arc-minute grid resolution.

Analysis of geoid model differences reveals a close correspondence between the models derived using Kriging and Inverse Distance to a Power (IDP) free air anomaly grids (Figure 6.19a, Table 6.16). These models exhibit a near-zero mean difference and a low standard deviation of 0.4 cm in geoid heights. The observed variations range from -8.4 cm to 5.8 cm, indicating good overall agreement.

Table 6.16 : Statistics depicting the differences in geoid height between the Kriging-based model and those derived from the IDP, Nearest Neighbor, and ANN methods.

	Unit: cm	Max	Min	Mean	Std	RMSE
	IDP	5.810	-8.420	-0.006	0.384	0.384
KRIGING vs.	Nearest neighbor	14.240	-10.050	-0.001	0.408	0.408
	ANN	7.830	-11.940	-0.001	1.580	1.580

Figure 6.19b and Table 6.16 illustrate the differences between the Kriging and Nearest Neighbor geoid models. Similar to the IDP case, the standard deviation of geoid height differences remains low at 0.4 cm, with variations between -10.1 cm and 14.2 cm. These results suggest that both IDP and Nearest Neighbor methods produce geoid models that closely align with the Kriging model (standard deviation < 1 cm). However, the Artificial Neural Network (ANN)-based geoid model deviates more

significantly from the Kriging model in Figure 6.19c. The geoid height differences in this case range from -11.9 cm to 7.8 cm, with a higher standard deviation of 1.58 cm.

The areal comparisons of geoid models reveal spatial disparities and offer insights into the spatial coherence among the compared models, serving as a relative measure of the accuracy of a computed geoid model compared to a reference model. In addition to the Kriging-based comparisons in Figure 6.19, geoids calculated with the Nearest Neighbor, IDP and ANN algorithms are also compared to each other (see Figure 6.20 and Table 6.17 below).

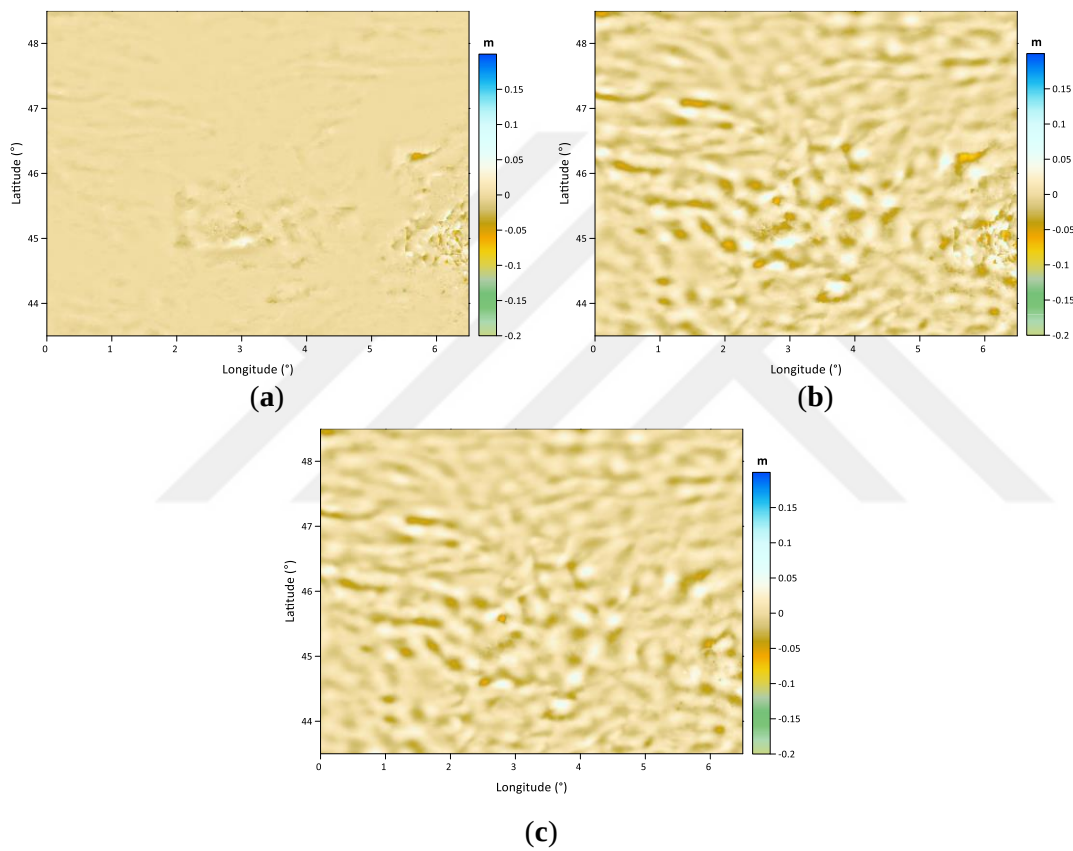


Figure 6.20 : 1 arc-minute geoid height difference maps depicting comparisons between grids interpolated using Nearest Neighbor and Inverse Distance to a Power (a), Nearest Neighbor grid and Artificial Neural Network algorithm (b), and Inverse Distance to a Power and Artificial Neural Network algorithm (c).

Table 6.17 : Statistics of the geoid differences between Nearest Neighbor, Inverse Distance to a Power (IDP) and Artificial Neural Network (ANN) grids.

	Unit: cm	Max	Min	Mean	Std	RMSE
Nearest vs.	IDP	11.840	-16.040	-0.005	0.512	0.512
Nearest vs.	ANN	9.960	-19.480	0.000	1.644	1.644
IDP vs.	ANN	7.190	-7.240	0.005	1.430	1.430

According to Table 6.17, it can be observed that there is a strong agreement in geoid models computed using the Nearest Neighbor and IDP methods. These two models, with differences ranging between 11.8 cm and -16.0 cm, exhibit a standard deviation of 0.5 cm and an almost zero mean value. When examining the ANN geoid, it becomes apparent that the standard deviations of the geoid height differences reach 1.6 cm for the ANN-Nearest Neighbor geoids and 1.4 cm for the ANN-IDP geoids.

Furthermore, we conducted a validation of the geoid models using a set of 75 independent GPS/leveling benchmarks situated within the area (see Figure 6.1a). The validation statistics of these models are compared in Table 6.18.

Table 6.18 : Geoid model validation statistics.

Unit: m	Max	Min	Mean	Std	RMSE
Kriging	1.162	0.972	1.059	0.041	1.060
IDP	1.164	0.974	1.059	0.041	1.060
Nearest neighbor	1.163	0.972	1.059	0.041	1.059
ANN	1.165	0.967	1.060	0.045	1.061

The accuracy of the computed geoid models, as determined by the standard deviations of the geoid height differences observed at the GPS/leveling benchmarks, stands at 4.1 cm, increasing to 4.5 cm for models incorporating ANN-based free air anomalies. The observed mean difference of 1.059 m in geoid heights likely corresponds to a systematic offset, reflecting the datum difference between the reference surface defined by the GPS/leveling data and the geoid model surface. The distribution and number of benchmarks within the GPS/leveling control dataset present a critical limitation for validation. With only 75 data points concentrated in a limited area, the dataset lacks sufficient representation of the rough terrain where geoid model accuracy is expected to deteriorate. The restricted geographic representation of the control dataset hinders a comprehensive evaluation of the models' performance across the spectrum of topographical features within the study area. This limitation introduces uncertainty regarding the models' accuracy in regions with complex terrain where geoid determination is known to be more challenging. The effects of the GPS/Leveling benchmarks locations on geoid validation using the Auvergne test data are examined in detail in Section 6.5.

6.5 Effects of GPS/Leveling Data on Geoid Model Validations

This section investigates the effect of the distribution and number of independent GPS/levelling points used in validation on the validation results. To investigate the aforementioned effect, 75 existing GPS/leveling points were grouped based on their orthometric heights and homogeneously distributed, and then two separate tests were carried out. For the selection of GPS/Leveling points based on orthometric heights, points with orthometric heights ranging from 150 to 500 meters are selected for plain terrain, while those ranging from 500 to 1200 meters are chosen for rough terrain to validate the produced geoids. Meanwhile, the orthometric heights of all GPS points used in the study are between 160.0 meters and 1183.9 meters. In determining these boundaries, classifications of plain and rough terrains made in similar studies are considered. In this classification, there are 36 GPS/leveling points allocated for plain terrain, and 39 points allocated for rough terrain. Figure 6.21 shows the GPS/Leveling points selected to validate geoids produced in both plain and rough terrains on a 3 arcsec DEM. Statistics of both orthometric and ellipsoidal heights of all GPS/leveling points in the study area are summarized in the Table 6.19 below.

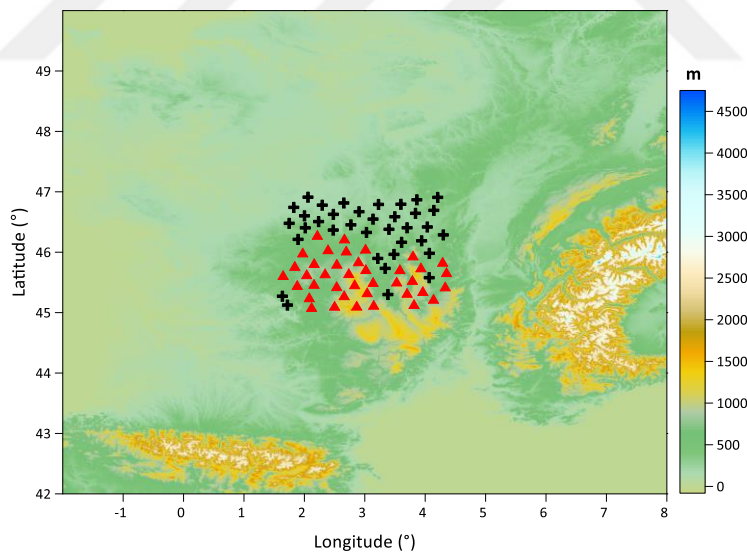


Figure 6.21 : GPS/Leveling points. The red triangle symbol represents points used to validate geoids produced in the rough terrain, while the black plus symbol represents points used to validate geoids produced in the plain terrain.

Table 6.19 : Statistics of orthometric and ellipsoidal heights for plain and rough terrain, determined based on orthometric heights.

	unit: m	Max	Min	Mean	Std	Rmse
Ellipsoidal Height	Plain Topo	531.89	206.81	363.82	89.96	374.47
	Rough Topo	1235.49	589.66	806.09	170.79	823.53
Orthometric Height	Plain Topo	481.05	160.03	315.41	89.30	327.47
	Rough Topo	1183.93	538.81	755.56	170.14	774.00

Taking into account the elevation of the topography, the plain area is confined between latitudes 46.4 and 48.4 and longitudes 1.5 and 3.5, while the rough area spans latitudes 44 to 46 and longitudes 2 to 4, ensuring both areas are of similar size. In this grouping, 14 GPS/Leveling points are allocated to the plain area, while 26 points are allocated to the rough area, as shown in Figures 6.22a,b below.

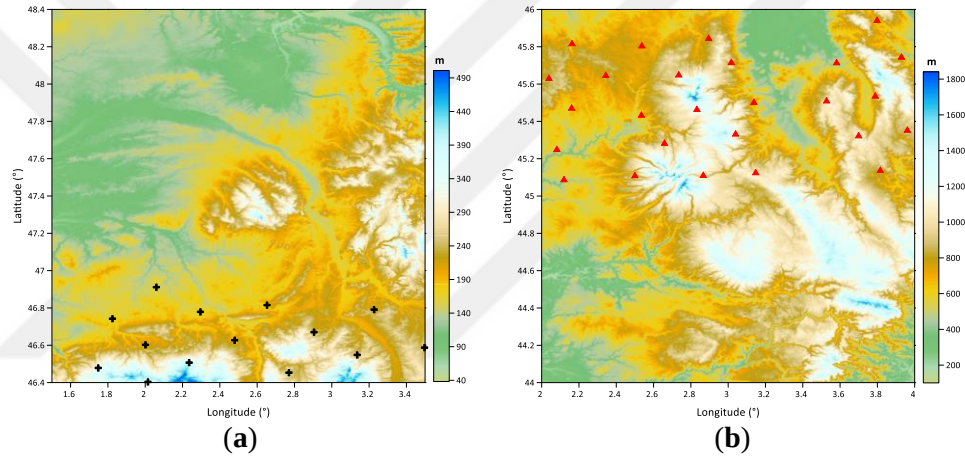
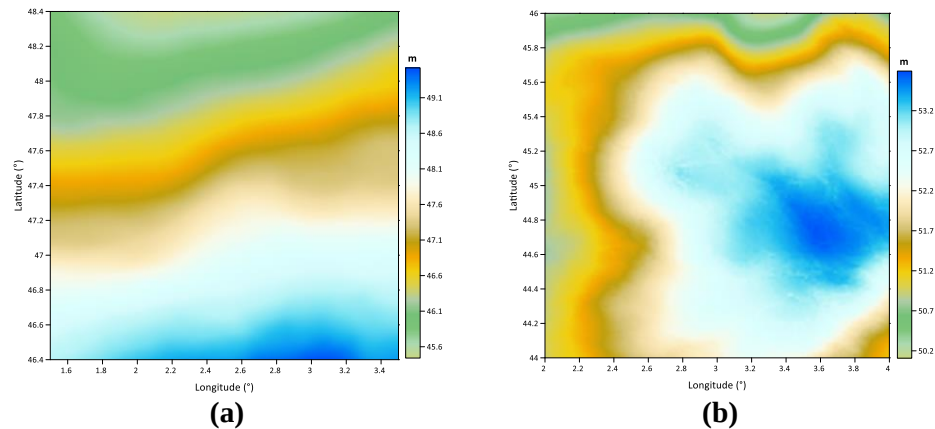


Figure 6.22 : Distribution of GPS/Leveling points: points in plain (a) and rough (b) terrains.

As seen in Figure 6.23, geoid models representing plain and rough terrain in the study areas are created using the 1 arc-minute resolution geoid interpolated with CBAs using Kriging, Nearest Neighbor, and IDP methods (Figure 6.10b and Figure 6.11a, b). Their statistics are also shared in Table 6.20.



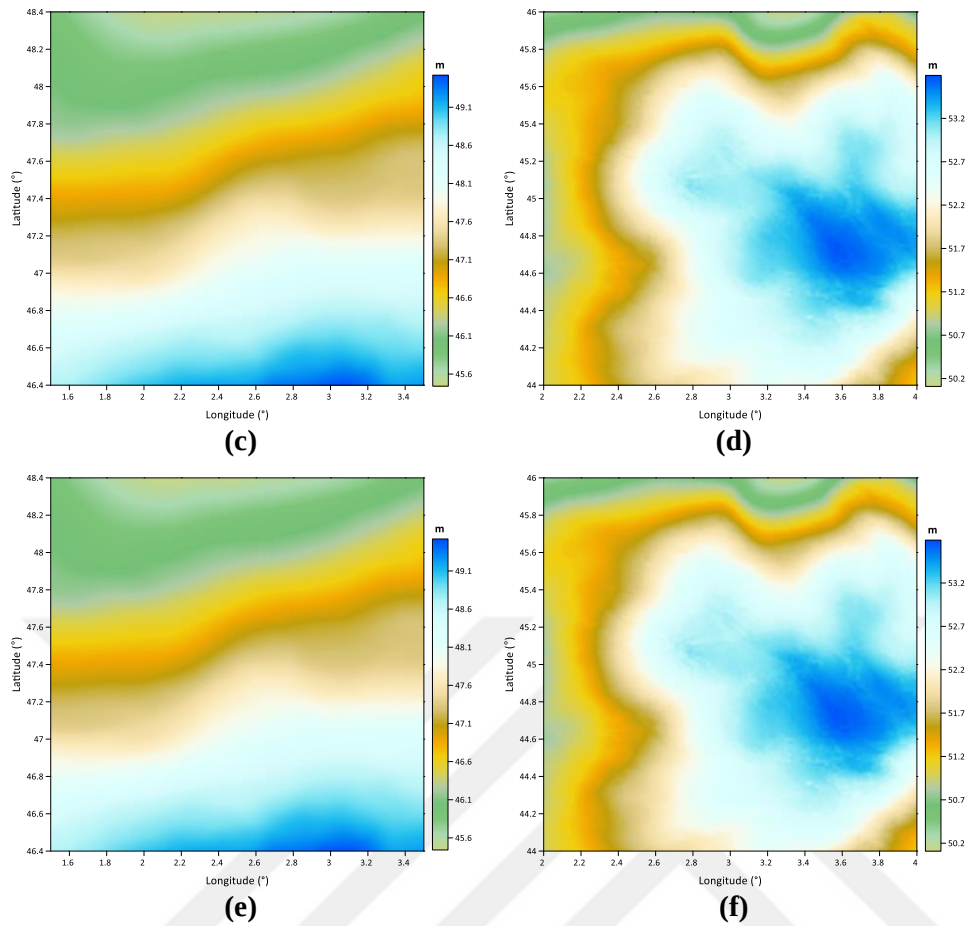


Figure 6.23 : 1 arc-minute Kriging geoid model generated in plain (a) and rough (b) terrains, IDP geoid in plain (c) and rough (d), Nearest Neighbor geoid in plain (e) and (f).

Table 6.20 : Statistics of generated geoids in plain and rough areas.

	unit: m	Max	Min	Mean	Std	Rmse
Kriging	Plain	49.520	45.441	47.298	1.085	47.310
	Rough	53.695	50.108	52.134	0.854	52.141
IDP	Plain	49.521	45.441	47.298	1.085	47.310
	Rough	53.695	50.109	52.134	0.854	52.141
Nearest	Plain	49.520	45.441	47.298	1.085	47.310
	Rough	53.694	50.108	52.134	0.854	52.141

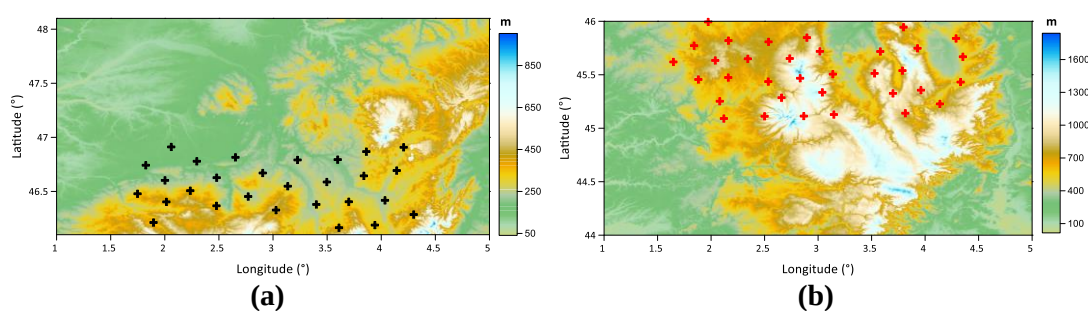
Kriging, IDP, and Nearest Neighbor geoids generated for plain and rough areas are validated with selected GPS/Leveling points for these areas, and the statistics for this process are shared in Table 6.21.

Table 6.21 : Statistics regarding the validation results.

	unit: m	Max	Min	Mean	Std	Rmse
Kriging	Plain	1.111	1.034	1.073	0.022	1.073
	Rough	1.162	0.972	1.058	0.042	1.058
IDP	Plain	1.111	1.036	1.073	0.022	1.073
	Rough	1.164	0.974	1.060	0.042	1.060
Nearest	Plain	1.112	1.033	1.072	0.022	1.073
	Rough	1.163	0.972	1.058	0.043	1.059

As seen in the table above, validation values conducted for rough terrain in all gridded geoids produced using three different interpolation methods have approximately twice the standard deviation values compared to tests conducted on plain terrain. These disparities stem from the challenges associated with establishing GPS/Leveling benchmarks in rough terrain, coupled with the insufficient representation of these areas due to a constrained number of points. As understood from this test, the reason for the nearly identical geoid validation results, calculated with different interpolation methods and whose statistics are provided in Table 6.18, is the insufficient number and distribution of GPS/Leveling points used in validation, particularly in mountainous areas.

To investigate the significance of the number and distribution of GPS/Leveling points for geoid validation, an additional test is conducted. Similar to the previous study, GPS/Leveling benchmarks with orthometric heights between $150 \leq H_{\text{orthometric}} \leq 500$ are grouped for plain terrain, while those within the range of $500 \leq H_{\text{orthometric}} \leq 1200$ are grouped for use in the validation of the geoid in rough terrain. However, this time, the study area was expanded to include latitudes 44 - 46 and longitudes 1 - 5 for plain terrain, and latitudes 46.1 - 48.1 and longitudes 1 - 5 for rough terrain. The GPS/Leveling benchmarks within the study area are depicted on the 3 arcsecond SRTM in Figure 6.24.

**Figure 6.24** : Distribution of GPS/Leveling benchmarks in plain (a) and rough (b) terrains.

In this classification, there are 28 GPS/Leveling points in plain terrain, while 34 points are allocated to rough terrain. The Kriging geoids calculated in these areas (CBAs gridded with Kriging) are provided in Figure 6.25 below.

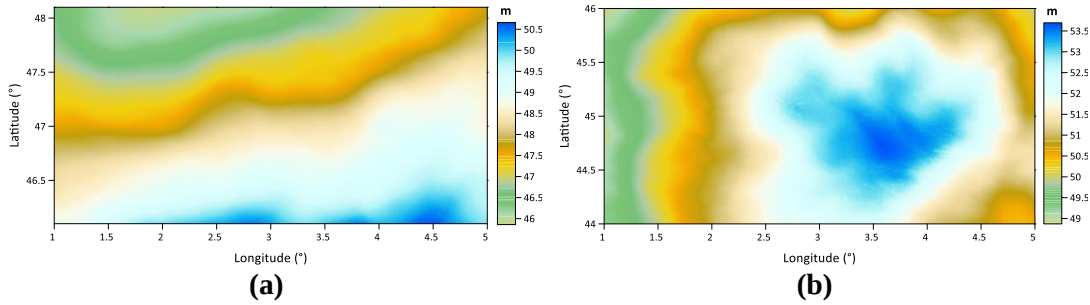


Figure 6.25 : Performed geoids in plain (a) and rough (b) topography.

The statistics of the generated geoids in plain and rough areas are given in Table 6.22 together with the validation statistics.

Table 6.22 : Statistics of geoids and validation results.

	Unit: m	Max	Min	Mean	Std	Rmse
GEOID	Plain	50.6581	45.859	48.094	1.130	48.108
	Rough	53.695	48.871	51.448	1.160	51.462
VALIDATION	Plain	1.145	0.988	1.077	0.037	1.077
	Rough	1.162	0.972	1.055	0.042	1.056

Upon evaluation of the statistics in Table 6.22, the validation results of the geoids produced for plain and rough areas do not indicate significant differences as shown in Table 6.21. In this implementation, despite the increase in the number of geoid points and the calculation area compared to previous tests, this result has emerged due to the GPS/Leveling points not being densely and homogeneously distributed to effectively represent this study area.

6.6 Investigating the Standardization of Artificial Neural Network (ANN) Algorithm Results

In Chapter 6.4 of the thesis, the Complete Bouguer Anomaly (CBA) grid generated using the Artificial Neural Network (ANN) algorithm, recognized as a machine learning technique is specifically selected for comparisons against the Kriging, IDP, and Nearest Neighbor interpolation methods. The representation of this grid, created with the ANN algorithm, is given in Figure 6.11c. In the studies conducted with ANN, the Levenberg-Marquardt algorithm is applied, and parameters of 200 neurons and 300

epochs are selected. This section investigates the impact of parameter selections (epochs, neurons) on gridding results and assesses the reliability of the ANN tool for CBA gridding applications. Within this scope, repeated tests are being conducted with both different and identical parameters, and the generated CBA grids are being compared. The statistics related to this comparison are presented in Table 6.23.

Table 6.23 : Statistics of CBAs on grid nodes.

Unit: mGal	Max	Min	Mean	Std	RMSE
100 neurons - 300 epochs	149.323	-157.674	-32.007	28.647	42.955
100 neurons - 300 epochs (2nd test)	87.879	-157.051	-32.054	29.105	43.297
100 neurons - 300 epochs (3rd test)	56.427	-163.818	-32.170	28.658	43.084
100 neurons - 400 epochs	143.218	-172.657	-32.051	29.037	43.248
100 neurons - 400 epochs (2nd test)	46.877	-155.536	-32.066	28.225	42.718
150 neurons - 500 epochs	140.305	-194.920	-31.593	29.961	43.541
200 neurons - 300 epochs	186.649	-181.645	-31.167	31.224	44.117
*200 neurons - 300 epochs (2nd test)	55.196	-159.404	-32.538	27.779	42.783
200 neurons - 400 epochs	179.592	-165.579	-32.087	28.438	42.875
200 neurons - 400 epochs (2nd test)	56.627	-163.878	-32.210	28.232	42.831
300 neurons - 300 epochs	119.027	-176.331	-31.768	29.347	43.249
500 neurons - 400 epochs	133.247	-166.176	-31.625	29.625	43.333
700 neurons - 300 epochs	114.756	-169.377	-32.495	28.940	43.514
KRIGING	57.282	-172.616	-32.234	27.901	42.632

The "*200 neurons - 300 epochs" results written in green in the figure above belong to the ANN application that is finalized in the thesis study. "Kriging" results marked in yellow are given so that ANN grids can be compared with the Kriging grid used in the thesis study. Upon examining the statistics above, it is observed that the "*200 neurons - 300 epochs" ANN grid is the closest grid to the Kriging grid. Hence, in the thesis study, the ANN grid with these parameters is utilized for gridding CBAs.

CBAs at the desired grid nodes give different results each time, even if they are calculated with the same parameters due to the nature of the method. Additionally, although the ratio of test and validation points corresponds to 15% each, equivalent to 36601 points, the same number of points with different distributions (and hence different grid values) are chosen in each test due to the algorithm's random selection. It turns out that the standardization of the method is not sufficient for the gravity gridding process since different results are obtained each time. For this reason, it is not recommended for use in physical geodesy applications such as gravity gridding.

The performance and regression visuals for the tests provided in Table 6.23, along with the distributions of validation and test points, and the 1 arc-minute final CBA grid, are shared in Appendix A. It should be noted that visuals from the first of the repeated calculations conducted with the same parameters are being shared..



7. CONCLUSIONS

Gravimetric geoid determination methods like the LSMSA method used in this study take gridded free-air anomalies as input. However, free-air anomalies located on Earth's surface cannot be directly gridded due to their high-frequency nature. This high-frequency character arises from the gravitational effects of topographical masses underneath the Earth's surface, which cannot be directly captured through surface observations alone.

To address this challenge, the observed free-air anomalies are first transformed into Bouguer anomalies, which are known to be less influenced by topographic variations. Exhibiting a smoother character, Bouguer anomalies, less susceptible to the aliasing effect compared to FAA, yield more accurate results during the gridding process. Complete Bouguer anomalies, generating a smoother surface compared to simple Bouguer anomalies with terrain corrections, are particularly employed in areas with significant topographic variations. However, for plain terrains, the use of simple Bouguer anomalies (SBAs) instead of computationally intensive complete Bouguer anomalies (CBAs) is also feasible. The selection of the appropriate Bouguer anomaly type for this study was determined based on the tests conducted in Section 6.2. The study area encompasses a complex terrain characterized by both plain and mountainous regions. Topographic elevations within the study area vary significantly, ranging from as high as 4746 meters above sea level to depths of 33.5 meters below sea level. The average point-wise terrain correction (TC) value across the region is 1 mGal, while the gridded TC value averages 1.7 mGal. Examination of the SBA and CBA difference map in Figure 6.5 reveals that the choice of Bouguer anomaly is not particularly critical in plain terrain. However, within the mountainous portions of the study area, particularly in the southeastern region, differences are found to reach up to 55.6 mGal. Comparison of the geoids obtained using LSMSA calculations with FAAs derived from both SBA and CBA grids indicates that the discrepancies can rise to 27.3 cm. These findings highlight the importance of employing CBA grids for gridding in regions with diverse topography, particularly in mountainous areas like the Auvergne study area utilized in this investigation.

In this study, optimal parameters for the LSMSA computation are determined through tests conducted in Section 6.3. These parameters are then employed in all numerical implementations. To determine the optimal parameters, a series of tests are conducted where the expansion degrees (M) ranged from 180 to 780 degrees, and the cap sizes vary from 0.10 to 1.00 degrees. Additionally, different b_n and error variance parameters were also tested. Based on the results of these tests, it was determined that the optimal parameters for LSMSA gravimetric geoid computation in the study area are $M = 780$, a cap size of $\Psi_0 = 0.25^\circ$, an error variance $C = 4 \text{ mGal}^2$, and a biased solution.

The influence of interpolation methods on gravity gridding is investigated in the third numerical experiment (Section 6.4). To achieve this, the complete Bouguer anomalies are subjected to gridding using three commonly employed interpolation methods: Kriging, Inverse Distance to a Power (IDP), and Nearest Neighbor. Additionally, a well-established soft-computing algorithm from the latest generation, the Artificial Neural Network (ANN), is also applied for gridding. This study is not about introducing or evaluating the performance of the latest "state-of-the-art" data evaluation approaches. Instead, it aims to address the extent to which the interpolation algorithm used in gravity anomaly gridding impacts the mapping of the gravity field and geoid modeling.

Common interpolation methods like Kriging, Inverse Distance Weighting (IDP), and Nearest Neighbor estimate unknown values in spatial data by considering weighted contributions from nearby data points. These methods directly utilize the data points themselves for interpolation. In contrast, Artificial Neural Networks (ANNs), a more recent approach, employ a function with parameters determined through a training process. Here, the dataset is divided into training and testing data. Iterative training optimizes the function's parameters, which are then used to predict values at grid nodes. Unlike the aforementioned methods that directly use data points, ANNs interpolate values by applying the trained function. This often results in a smoother appearance for the generated grid. Furthermore, ANNs introduce an element of randomness as each computation utilizes a randomly chosen training dataset and weights, leading to potentially different grid outputs for each run. Crucially, selecting the optimal number of neurons and iterations within the ANN architecture requires careful consideration through trial-and-error procedures. It is worth noting that the

suitability of learning-based "state-of-the-art" data interpolation techniques for modeling in geodesy and surveying applications is still subject to debate. Modeling processes that utilize observations (data) are naturally influenced by measurement errors. Learning-based techniques often struggle to effectively manage these errors in the measurement data, as they incorporate them into the model. Consequently, they may yield overly optimistic results regarding modeling performance (as seen in overfitted systems) if not executed with carefully selected computational parameters.

In the tests conducted here, the differences between complete Bouguer anomaly grids generated using various interpolation methods and a control grid obtained through Kriging are examined. The IDP and Nearest Neighbor grids, with standard deviations of 1.3 mGal and 1.6 mGal respectively, most closely resemble the Kriging grid based on Table 6.10. Conversely, the ANN grid exhibits the greatest difference, with a standard deviation of 5.3 mGal, exceeding 100 mGal in some areas compared to Kriging. Similarly, an evaluation of geoid height differences between geoids generated from three distinct grids and the Kriging geoid reveals a low standard deviation of approximately 4 mm for IDP- and Nearest Neighbor-derived geoids. This finding indicates that IDP and Nearest Neighbor methods produce geoids that closely align with the Kriging geoid. This similarity arises from the similarities in the mathematical backgrounds of these methods. The ANN-based geoid, in contrast, exhibits a wider range of geoid height differences compared to the Kriging model, with values varying from -11.9 cm to 7.8 cm and a higher standard deviation of 1.6 cm.

The generated grids are compared not only with the Kriging grid selected as the control method but also with each other. Comparisons based on the ANN-generated grid reveal higher standard deviation values compared to the others. As seen in Table 6.9, the standard deviations of the differences between the Complete Bouguer Anomalies (CBAs) gridded with Nearest Neighbor and IDP interpolation methods and those gridded with ANN are 5.4 and 5.0 mGal, respectively, while the difference between Nearest Neighbor and IDP grids is determined to be 1.8 mGal. When examining the differences in geoid heights, it is observed that the differences between geoids produced based on Nearest Neighbor and IDP techniques range from -16 cm to 11.8 cm, with a standard deviation of 0.5 cm. Examining the absolute validation statistics in Table 6.16, obtained through 75 GPS/Leveling benchmarks, reveals that the standard deviations of the validation results for geoids obtained with conventional

interpolation methods (Kriging, IDP, Nearest Neighbor) are 4.1 cm, while the standard deviation for the validation of the ANN geoid is 4.5 cm.

Contrary to expectations, the learning-based ANN method does not outperform conventional interpolation methods (Kriging, IDP, Nearest Neighbor) in this study. As evidenced by the statistics, grid datasets obtained using these conventional methods achieve the highest compatibility.

In light of recent advancements in data acquisition techniques and sensor technologies, opportunities have emerged for sub-centimeter accuracy in determining geoid models. However, meticulous attention must be paid to every stage within the modeling process, as even minor variations can have an impact on the final result. This study highlights the critical role played by interpolation algorithms. It was demonstrated that the selection of an interpolation algorithm significantly impacts both the gridding of gravity data and the accuracy of the geoid model generated using the LSMSA method with the interpolated grid. Consequently, for the precise determination of the Earth's gravity field and geoid, it is recommended that complete Bouguer anomalies be utilized in conjunction with a carefully chosen interpolation algorithm for gravity gridding.

The importance of selecting appropriate GPS/Leveling benchmarks for the absolute validation process is investigated through the tests conducted in Section 6.5. Due to the inherent challenges of performing leveling measurements in rugged terrain, as mentioned in the introduction, the GPS/Leveling benchmarks are strategically placed within the low-topography region of the study area. Consequently, the test statistics primarily reflect the geoid model accuracy within these flat regions. In this application, where the topography is categorized as either plain or rugged, the validation of geoid models generated for these two distinct regions is conducted using separate control points. The results presented in Tables 6.21 and 6.22 for these computations demonstrate that the validation outcomes are influenced by the selection of GPS/Leveling points with appropriate density and distribution, effectively representing the topography. Regarding the potential disadvantage arising from the spatial distribution of GPS/Leveling control data in geoid determination studies, it's important to consider that geoid models are generally employed for height transformation in mapping and engineering projects, which are primarily conducted in settlements or industrialized regions. Therefore, from a geodesy and surveying

applications perspective, this perceived disadvantage of insufficient control data in mountainous areas may be somewhat alleviated.

In the final test conducted within the scope of the thesis (Section 6.6), the reliability of the ANN tool for CBA gridding applications is evaluated. This involved performing computations using different epoch-neuron parameter combinations with the Levenberg-Marquardt method and repeating the computations with the same parameters to assess the standardization of the results obtained using the ANN algorithm. Upon examining the CBA statistics presented in Table 6.23, the performance and regression analyses shared in the Appendix, and the distributions of test and validation points, it has been observed that the ANN algorithm does not perform well in gravity gridding—a crucial component of high-precision gravimetric geoid computation—due to its tendency to produce divergent results even with identical parameters. Although the ANN technique did not yield satisfactory results in this thesis study, it should be noted that there are various alternative soft-computing approaches available, and comprehensive studies solely utilizing deep learning methods should be conducted to assess the algorithm's adequacy in its application domain.

The rapid advancement of artificial intelligence (AI) tools has opened up exciting possibilities for their integration into geodetic computations. Given AI algorithms' capacity to handle large datasets, identify complex patterns, and make accurate predictions, challenges faced in geoid modeling can be addressed easier and efficiently using AI tools.

In the context of height system modernization, centimeter-level (or even sub-centimeter) accuracy is essential for computing regional geoids. While data quality plays a crucial role in achieving this precision, it is not the sole factor. The preparation of data from reduction to gridding, the selection of parameters used in computations, and the rigorous testing of generated results all directly influence the accuracy of the generated geoid. Consequently, these intricate computations should be conducted meticulously and carefully by experts in the field.



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APPENDICES

APPENDIX A: Regression and performance visualizations of Complete Bouguer Anomaly (CBA) grids calculated with the ANN algorithm, along with the distribution of test/validation points and the 1 arc-minute final CBA grid.



APPENDIX A: Regression and performance visualizations of Complete Bouguer Anomaly (CBA) grids calculated with the ANN algorithm, along with the distribution of test/validation points and the 1 arc-minute final CBA grid.

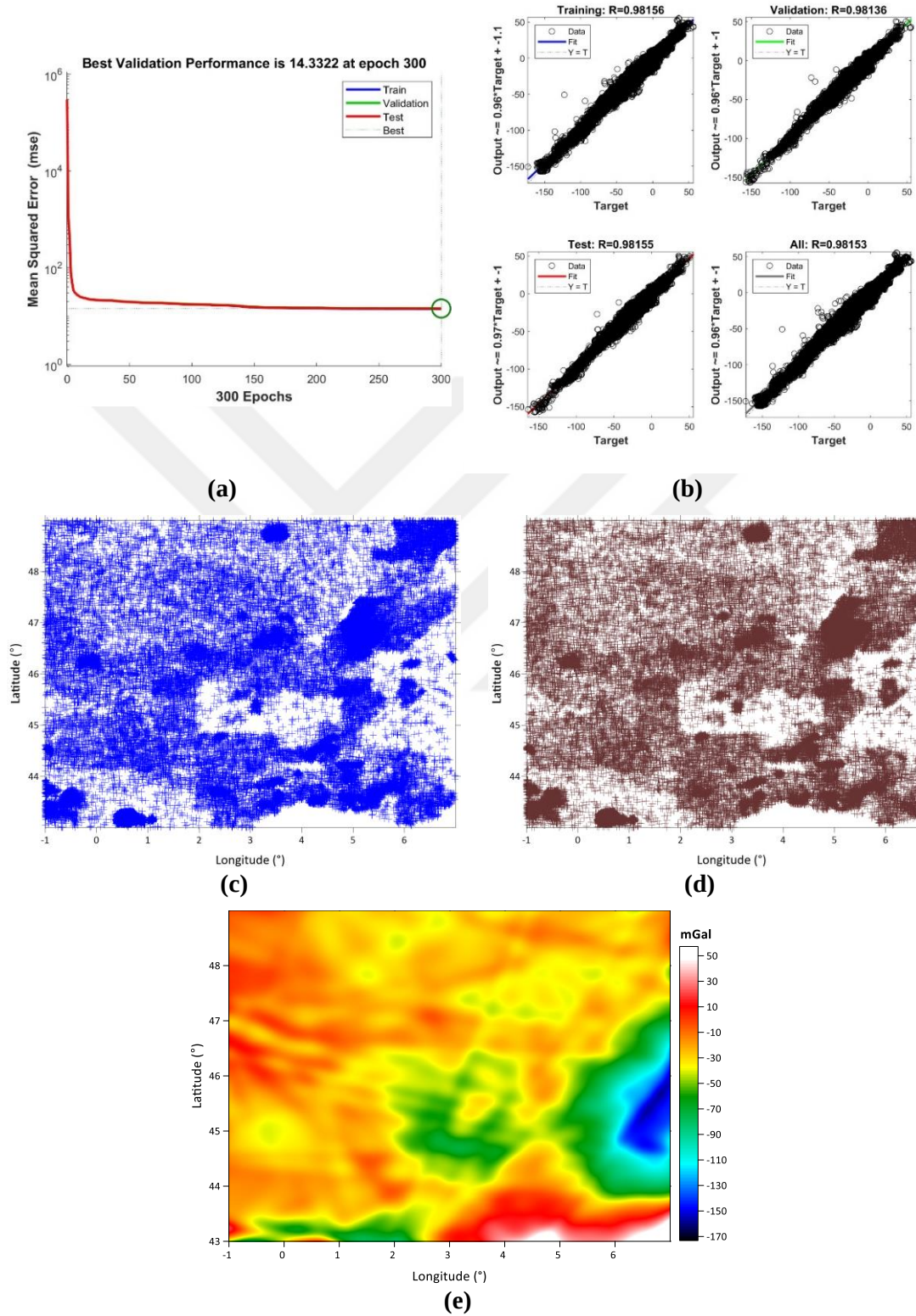


Figure A.1 : Performance (a) and regression (b), distribution of validation (c) and test (d) points, and final CBA grid computed using 100 neurons - 300 epochs (e).

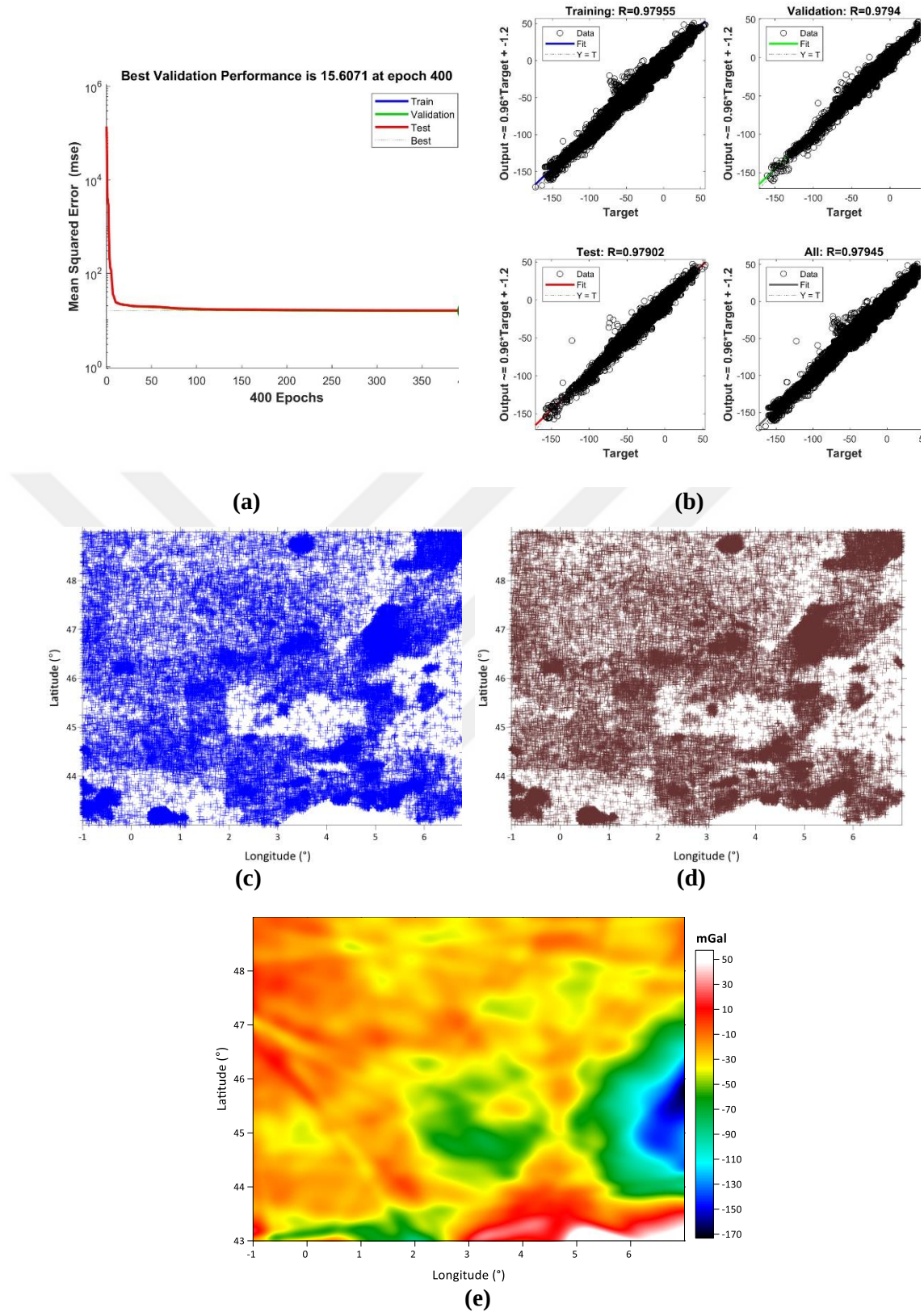


Figure A.2 : Performance (a) and regression (b), distribution of validation (c) and test (d) points, and final CBA grid computed using 100 neurons - 400 epochs (e).

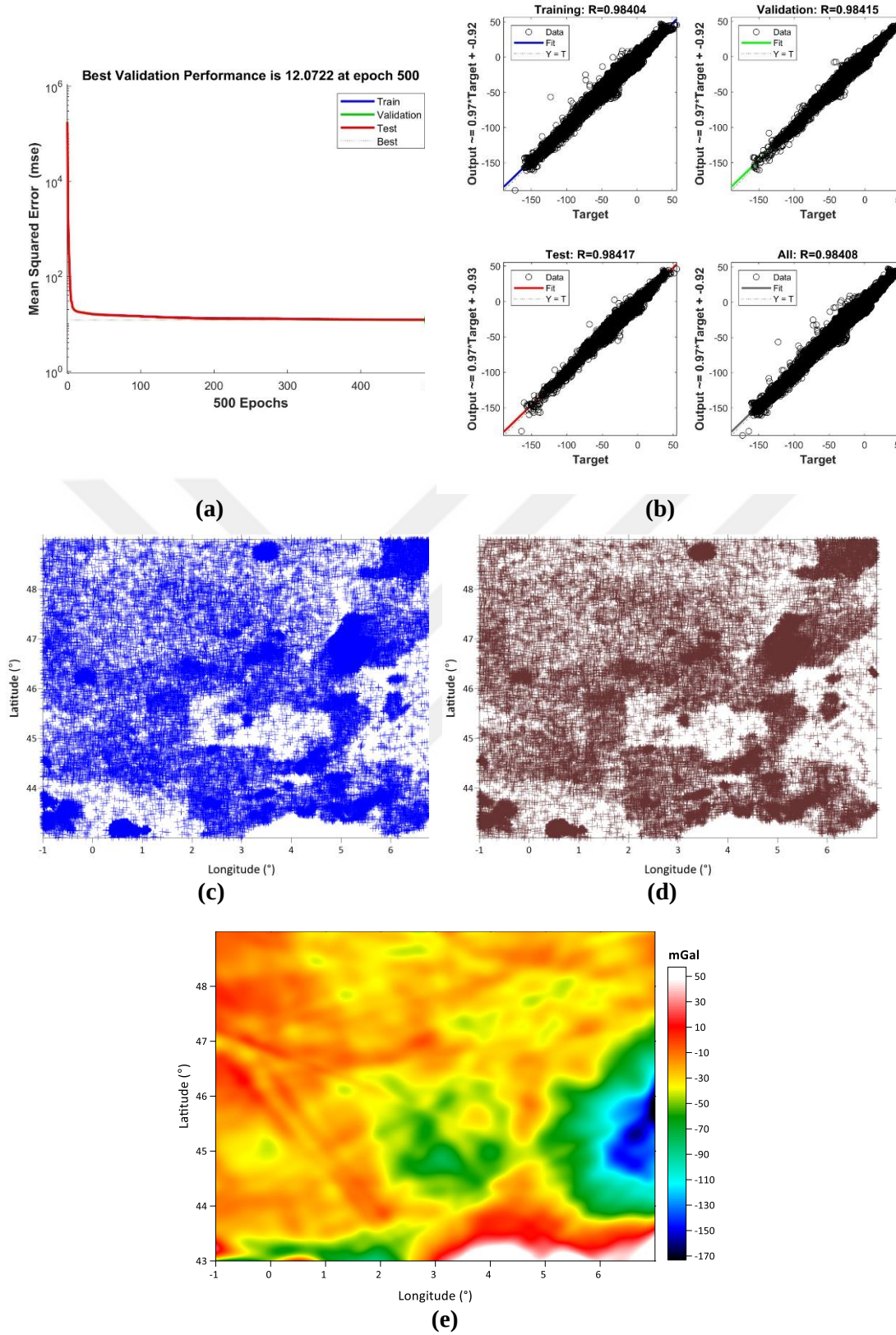


Figure A.3 : Performance (a) and regression (b), distribution of validation (c) and test (d) points, and final CBA grid computed using 150 neurons - 500 epochs (e).

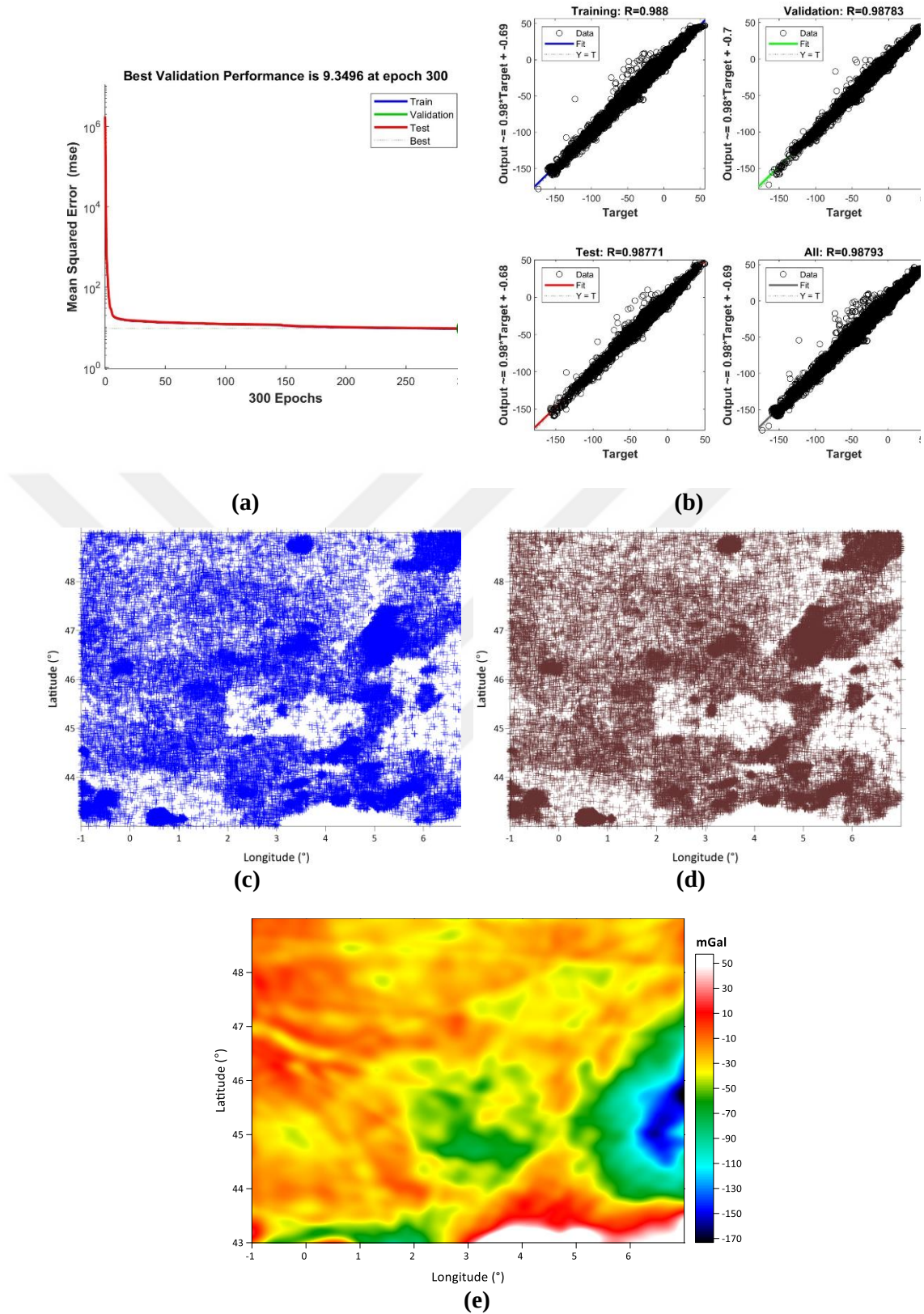


Figure A.4 : Performance (a) and regression (b), distribution of validation (c) and test (d) points, and final CBA grid computed using 200 neurons - 300 epochs (e).

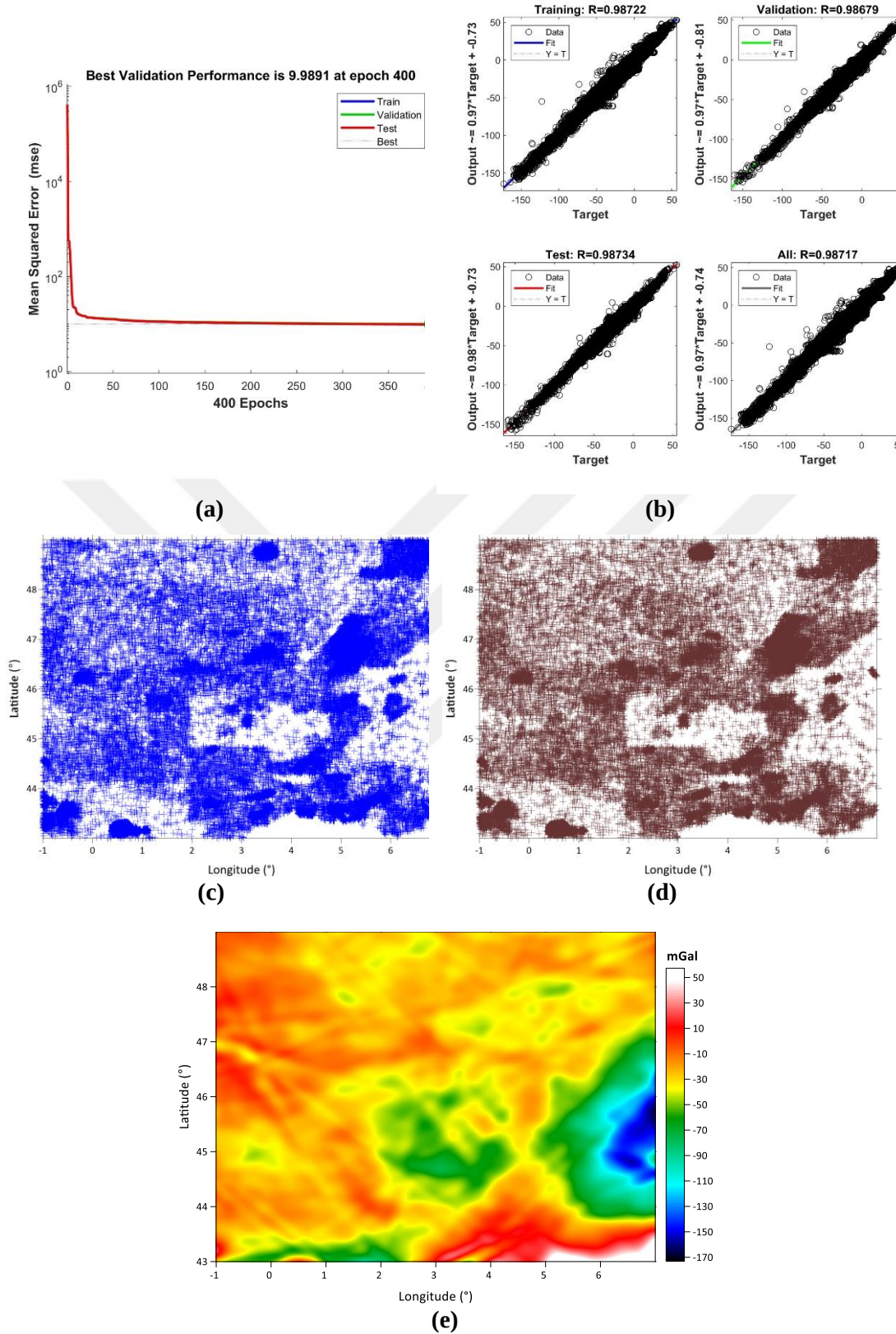


Figure A.5 : Performance (a) and regression (b), distribution of validation (c) and test (d) points, and final CBA grid computed using 200 neurons - 400 epochs (e).

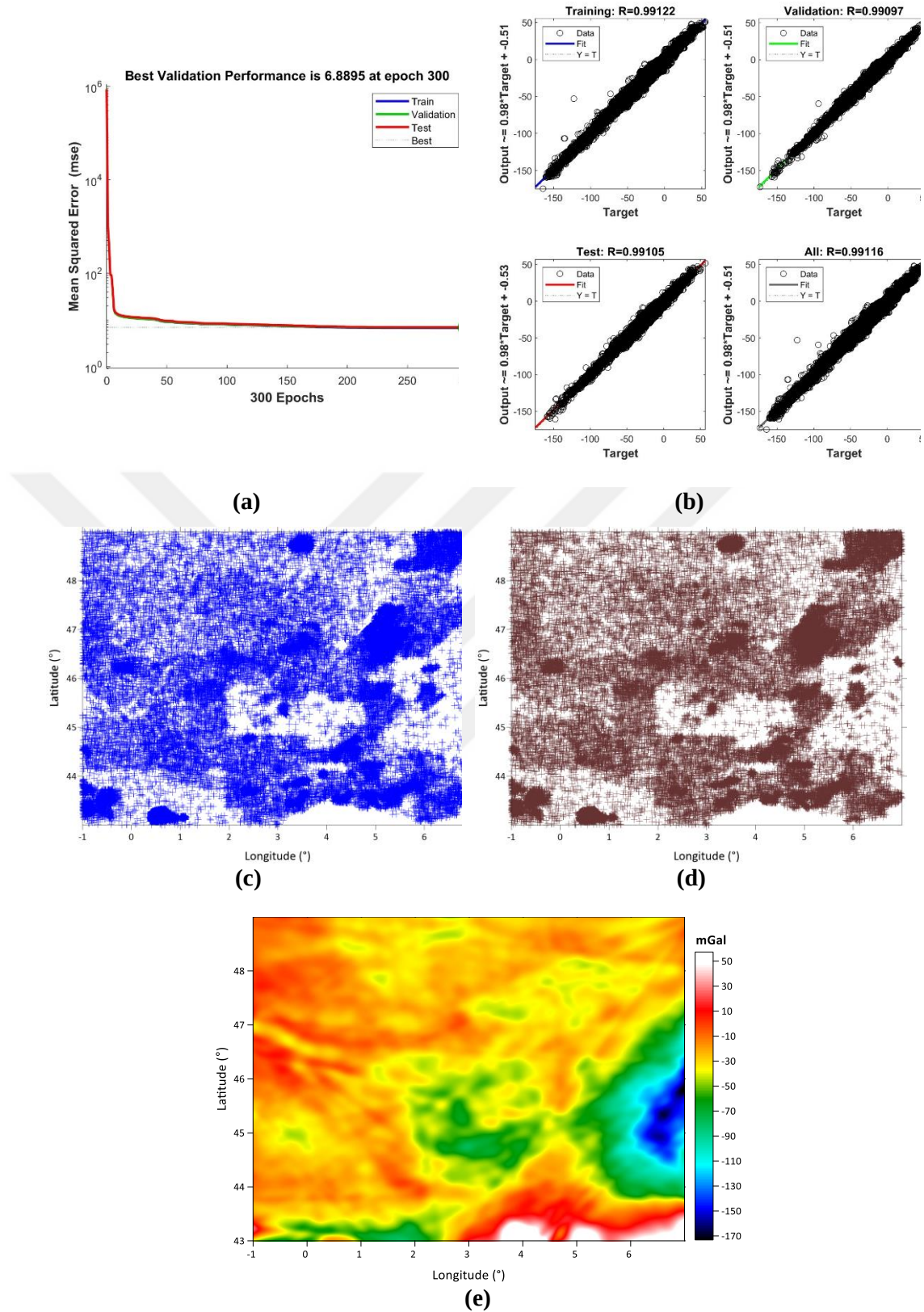


Figure A.6 : Performance (a) and regression (b), distribution of validation (c) and test (d) points, and final CBA grid computed using 300 neurons - 300 epochs (e).

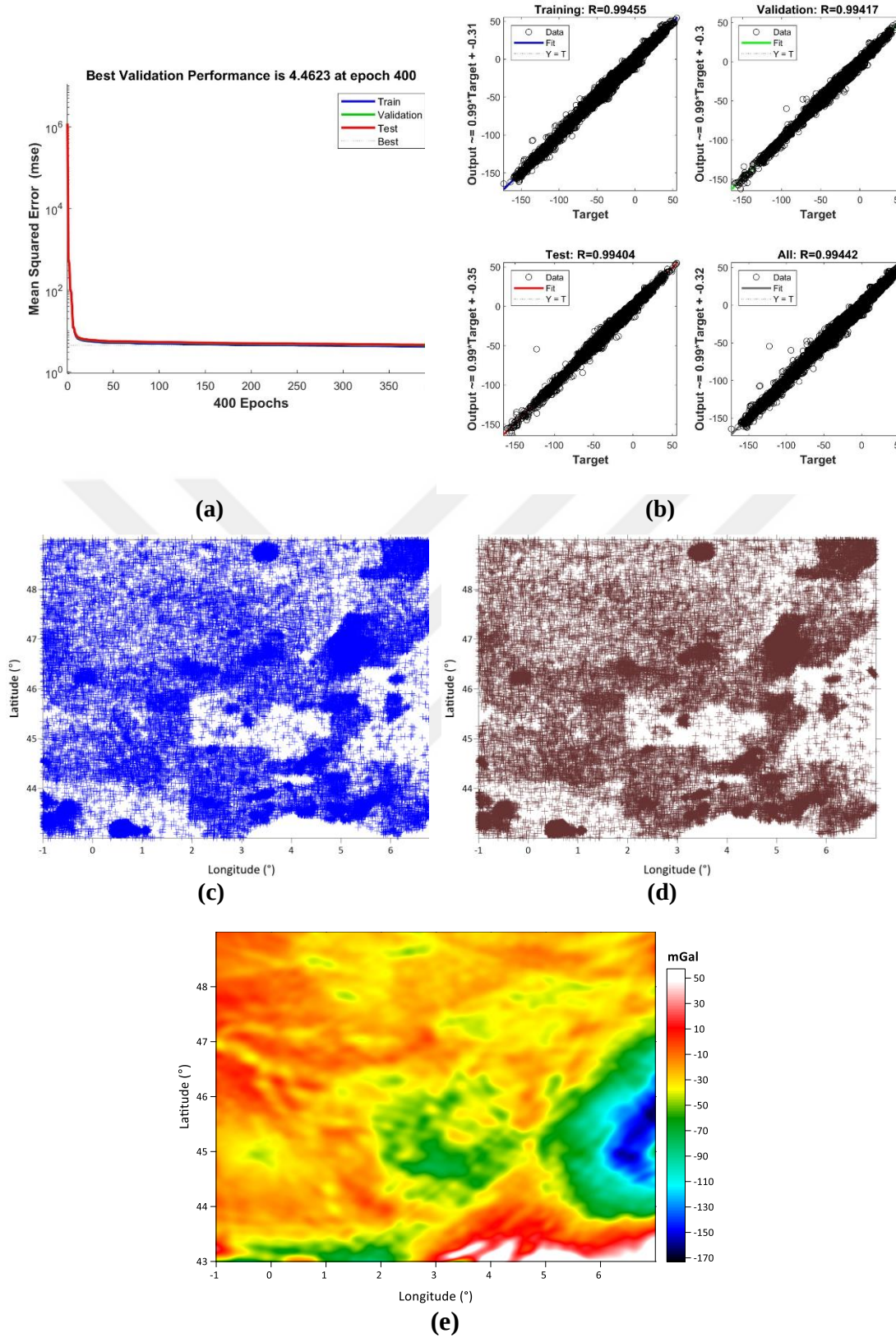


Figure A.7 : Performance **(a)** and regression **(b)**, distribution of validation **(c)** and test **(d)** points, and final CBA grid computed using 500 neurons - 400 epochs **(e)**.

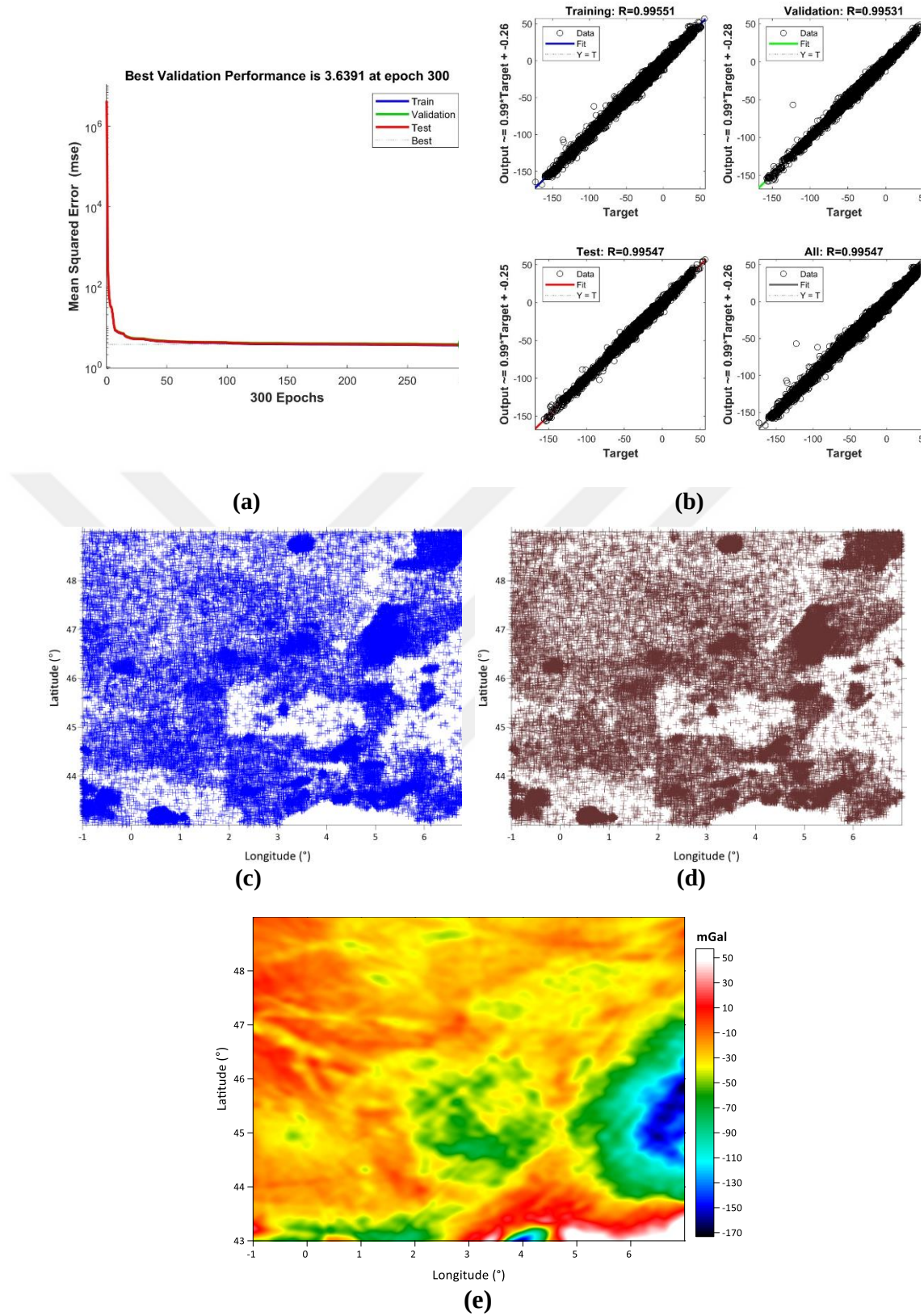


Figure A.8 : Performance (a) and regression (b), distribution of validation (c) and test (d) points, and final CBA grid computed using 700 neurons - 300 epochs (e).



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