

CUKUROVA UNIVERSITY
INSTITUTE OF NATURAL AND APPLIED SCIENCES

MSc THESIS

**Thermal Stresses in Variable Cross-Sectional Composite
Annular Fins**

Orhun ÇAĞLAR

Mechanical Engineering Department

August, 2024

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This Master's Thesis was evaluated by the following Jury Members on 08/08/2024 and was approved by unanimity / majority of votes.

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Department of Mechanical Engineering

ABSTRACT

In this thesis, the temperature distribution and associated thermal stresses of variable cross-sectional annular fin have been numerically investigated using the Pseudospectral Chebyshev Method. It has been assumed that the annular fin is made of a particle-reinforced composite material with ceramic reinforcement uniformly distributed in a metal matrix. The mechanical properties of the composite material are constant throughout the annular fin and have been obtained using the Halpin-Tsai Homogenization method. The thickness of the annular fin varies along the radial direction. In this case, the energy equilibrium and thermal stress equilibrium equations became more complex, making analytical solutions challenging. Therefore, the validation of the Pseudospectral Chebyshev Method (PCM) has been demonstrated on an annular fin with a uniform cross-section. Based on the results obtained, the temperature distribution and thermal stresses of a variable cross-sectional annular fin that made of composite material have been discussed.

Keywords: Annular fin, thermal stresses, pseudospectral Chebyshev method, composite materials

Değişken Kesitli Kompozit Dairesel Kanatçıklardaki Isıl Gerilmeler

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ÖZ

Bu tezde, değişken kesitli dairesele kanatçığın sıcaklık dağılımı ve ilişkili ısı gerilmeleri Pseudospectral Chebyshev Yöntemi kullanılarak nümerik olarak incelenmiştir. Dairesel kanatçığın, metal matris içinde uniform olarak dağıtılmış seramik takviyeli bir parçacık kompozit malzemeden yapıldığı varsayılmıştır. Kompozit malzemenin mekanik özellikleri dairesele kanatçık boyunca sabit olup, Halpin-Tsai Homojenizasyon yöntemi kullanılarak elde edilmiştir. Dairesel kanatçığın kalınlığı, radyal yönde doğrusal olarak değişmektedir. Bu durumda, enerji dengesi ve ısı gerilme denklemleri daha karmaşık hale gelmiş ve analitik çözümler zorlaşmıştır. Bu nedenle, Pseudospectral Chebyshev Yöntemi (PCM) düzgün kesitli bir dairesele kanatçık üzerinde doğrulanmıştır. Elde edilen sonuçlara dayanarak, kompozit malzemeden yapılmış değişken kesitli dairesele kanatçığın sıcaklık dağılımı ve ısı gerilmeleri tartışılmıştır.

Anahtar Kelimeler: Dairesel kanatçık, termal gerilmeler, pseudospektral Chebyshev yöntemi, kompozit malzemeler

EXTENDED ABSTRACT

The phenomenon of heat transfer, defined as the flow of energy from a warmer environment to a cooler one, has become increasingly prominent in industry in recent years due to improvements in cost and thermal efficiency. Researchers have been studying how to improve this phenomenon and the required techniques since past decades. This topic is studied under 'active' techniques and 'passive' techniques. Active techniques require extra energy something like an oscillation or giving to flow some pulsation. Passive techniques do not require extra energy. Passive controls mainly focus on the surface geometry affects. Fins, one of the passive control techniques, are leading in this regard. Fin designs involve the selection of fin materials, profiles, and dimensions are critical for enhancing heat transfer rates. Annular fins are used for the heat transfer between two ambients with temperature difference. The design of fin which consists of selecting fin materials, profiles, and dimensions is critical to improving the heat transfer rates with temperature difference. In the design phase of annular fins, there are several important factors to be taken into account. One of the important design parameters is the geometry of annular fin (cross-sectional area) which directly influences the thermal stresses and temperature distributions, hence necessitating prior selection. Another important design parameter is the choice of material of annular fin. Different temperature values on the annular fin cause varying degrees of thermal expansion. These varying amounts of thermal expansion result in thermal stresses on the annular fin. These stresses will cause mechanical failures, for example, occurrence of creep, fatigue, crack and crack increment in materials. To prevent these issues and ensure appropriate fin design, it is necessary to predict the thermal stresses that will develop on the fin. One method to reduce thermal stresses in fins exposed to high temperature differences is to use composite materials instead of homogeneous ones. Composite annular fins are made from uniformly distributed ceramic and metal materials. The homogeneous composite annular fin is made with ceramic and metal. Composite materials are classified into fiber-reinforced, particle reinforced, and plate layered reinforced types. In this thesis, an annular fin made from particle reinforced composite has been utilized. Particle reinforcement composite have been selected due to the mechanical and thermal properties of particle reinforced composites can be distributed homogeneously throughout the annular fin. The material that used in this thesis as a ceramic is Silicium Carbide (SiC) and the material that used as a metal is Alluminium alloy 1100 (Al). Silicium Carbide finds widespread application across various fields. Fabrication of the Al-SiC composites are usually made using powder metallurgy techniques (Ling, 1995). After the powder metallurgy, samples are subjected to compacting processes. On account of these processes, particles reinforcements are homogeneously distributed along the annular fin. The mechanical properties of Alluminium alloy 1100 and Silicium Carbide composite are established for homogeneosation procedure. In annular fin made from composite material, the properties of the material are obtained using the Halpin-Tsai method. The Halpin-Tsai method is an empirical technique developed using experimental data and is highly

effective for determining material properties in particle reinforced composites. Using this method, the thermal and mechanical properties of a composite material are determined as follows.

$$\frac{p}{p_m} = \frac{1+\xi\eta V_f}{1-\eta V_m} \quad 0.1$$

η and ξ defined as follows:

$$\eta = \frac{\frac{p_f}{p_m} - 1}{\frac{p_f}{p_m} + \xi}, \quad \xi = 2 \left(\frac{l}{m} \right) \quad 0.2$$

Here, p represents any property of the particle reinforced composite material. p_f denotes the property of fiber and p_m represents of the matrix material. ξ is a parameter related to fiber geometry, l and m specify the length and width of the fiber geometry, respectively. V_f denotes the volume fraction of the fiber while V_m represents the volume fraction of the matrix and they are defined as follows:

$$V_f = \left(\frac{r-a}{b-a} \right)^n, V_m = 1 - V_f \quad 0.3$$

As established in this thesis, the value of n is taken to be zero in composite materials with homogeneous distribution.

The temperature distribution and associated thermal stresses of a variable cross-sectional annular fin have been numerically investigated using the Pseudospectral Chebyshev Method. The following assumptions are made while formulating the equations governing the temperature field and the corresponding boundary conditions.

1. The fin is axially symmetric
2. Thickness of the annular fin is very small compared to radius of the fin
3. There is only convection heat transfer on the fin surface, radiation heat transfer is neglected,
4. Ambient temperature and heat transfer coefficient are constant,
5. There is no time dependent temperature change at any point of the fin, the fin has reached a steady state situation,
6. There is no heat loss or heat production in the fin.
7. In addition to the mentioned assumptions in the literature, there is another assumption that utilized in this thesis.

8. The thickness of the fin depends on the diameter of the fin.

In order to determine the thermal stresses, in addition to the assumptions used in determining the temperature distributions, the following assumptions are made:

9. No internal forces in the fin,
10. There is no traction on the fin surfaces

The energy equilibrium Eq. which gives the temperature distribution on account of these assumptions:

$$\frac{d^2T}{dr^2} + \left(\frac{1}{r} + \frac{1}{A(r)} \frac{dA(r)}{dr} \right) \frac{dT}{dr} - \frac{2h}{A(r)k} (T - T_\infty) = 0 \quad 0.4$$

And boundary conditions:

$$T(a) = T_b \quad 0.5$$

$$T'(b) = 0 \quad 0.6$$

Where $A(r)$ is the thickness of the fin.

The equations that describe the relationship between stress, strain and temperature in annular fins, the equations that relate strain to displacement and the stress equilibrium equations are given as follows.

Relationship between stress, strain and temperature:

$$\sigma_r = \frac{E}{1-\nu^2} (\varepsilon_r + \nu\varepsilon_\theta - (1+\nu)aT) \quad 0.7$$

$$\sigma_\theta = \frac{E}{1-\nu^2} (\varepsilon_\theta + \nu\varepsilon_r - (1+\nu)aT) \quad 0.8$$

Relationship between strain and displacement:

$$\varepsilon_r = \frac{du}{dr} \quad 0.9$$

$$\varepsilon_\theta = \frac{u}{r} \quad 0.10$$

Stress equilibrium equation:

$$\frac{dA(r)_r}{dr} \frac{\sigma_r}{A(r)_r} + \frac{\sigma_r - \sigma_\theta}{r} + \frac{d\sigma_r}{dr} = 0 \quad 0.11$$

Using the equations that describe the stress-strain-temperature relationship, the strain-displacement relationship and the stress equilibrium equations, the equation governing the stress component in a variable cross-sectional annular fin is obtained as follows:

$$\frac{d^2u}{dr^2} + \frac{du}{dr} * \left(\frac{A'(r)_r}{A(r)_r} + \frac{1}{r} \right) + \frac{u}{r} * \left(\frac{A'(r)_r}{A(r)_r} * v - \frac{1}{r} \right) = \frac{A'(r)_r}{A(r)_r} * (1 + v)aT + (1 + v)a * \frac{dT}{dr} \quad 0.12$$

Boundary conditions for the thermal stresses are obtained as follows, using assumption (8):

$$\sigma_r(a) = 0 \quad 0.13$$

$$\sigma_r(b) = 0 \quad 0.14$$

The equations governing the temperature field and thermal stresses are numerically investigated using the Pseudospectral Chebyshev method under the given boundary conditions. In this method, which is based on the principle of creating a denser grid near the boundaries and a sparser mesh towards the center of the geometry being analyzed, the grid points are obtained from right to left using Gauss-Lobatto points as follows.

$$x_j = \cos\left(\frac{j\pi}{N}\right), \quad j = 0, 1, 2, 3 \dots N \quad 0.15$$

The distribution of grid points in the Chebyshev differentiation matrix given in Eq. (0.15) will be obtained by following the solution scheme outlined below. To establish the solution procedure, consider a vector v defined at the grid points. The discrete derivative ω of the vector v at the same grid points is obtained in two steps: (Trefethen 2000)

- If an interpolation polynomial of degree $p \leq N$ is selected as $p(x_j) = v_j$ for $0 \leq j \leq N$.
- Derivative at the grid points will be $\omega_j = p'(x_j)$ for $0 \leq j \leq N$.

Since this process is linear, the derivative ω at the grid points can be represented as the product of v and a matrix of dimensions $(N+1) \times (N+1)$ denoted as D_N :

$$\omega = D_N v \quad 0.16$$

The D_N differentiation matrix is obtained using Lagrange interpolation and represents the first-order derivative at the grid points. Second order and higher-order derivative are obtained by multiplying differentiation matrices together ($D_N^2 = D_N \times D_N$).

The equations governing the temperature are discretized as follows using the differentiation matrices.

$$L_1 T = RHS_1 \quad 0.17$$

In the linear system equation provided above, L_1 and RHS_1 are given as follows:

$$L_1 = D^2 + \left(\frac{1}{r} + \frac{1}{A(r)} \frac{dA(r)}{dr} \right) D - \frac{2h}{A(r)k} \quad 0.18$$

$$RHS_1 = \frac{2h}{A(r)k} T_\infty \quad 0.19$$

If the linear system of equations given in Eq. (0.17) is solved with the temperature boundary conditions (0.5-0.6), a non-trivial solution for the temperature is obtained. The same procedure is applied to the equations governing thermal stresses.

$$L_2 u = RHS_2 \quad 0.20$$

A linear system of equation in above form is obtained. In this equation, L_2 and RHS_2 are given as follows:

$$L_2 = \left[D^2 + D \left(\frac{A'(r)_r}{A(r)_r} + \frac{1}{r} \right) + \frac{1}{r} \left(\frac{A'(r)_r}{A(r)_r} v + \frac{1}{r} \right) \right] \quad 0.21$$

$$RHS_2 = \frac{A'(r)_r}{A(r)_r} (1 + v) a T + (1 + v) a D T \quad 0.22$$

If the linear system of equations given in Eq. (0.20) is solved with the stress boundary conditions (0.13-0.14), a non-trivial solution for the displacements is obtained.

The validation of the method is carried out by comparing it with an analytical solution for the temperature distribution and thermal stresses in a uniform cross-section annular fin.

The temperature distributions and thermal stresses obtained for variable cross-sectional annular fin made from a composite material are compared with those in a uniform cross-sectional fin made of the same material using the graphs.

GENİŞLETİLMİŞ ÖZET

Sıcak bir ortamdan, daha soğuk bir ortama enerji akışı olarak tanımlanan ısı transferi olgusu, son yıllarda, maliyet ve ısı verimlilik açısından iyileştirilmiş ve sanayide daha popüler bir konu haline gelmiştir. Araştırmacılar, bu olguyu ve gerekli teknikleri geliştirmeye yönelik çalışmalar yapmışlardır. Bu konu, 'aktif' ve 'pasif' teknikler olarak iki ana başlık altında incelenmiştir. Aktif teknikler ısı transferi için ilave enerji gerektirirken, örneğin bir salınım hareketi veya akışa pals verilmesi gibi, pasif teknikler ilave enerji gerektirmemektedir. Pasif kontrol teknikleri, genellikle yüzey geometrisinin etkilerine odaklanır. Bu bağlamda, pasif kontrol tekniklerinden biri olan kanatçıklar öne çıkmaktadır. Kanatçık tasarımı, kanatçık malzemelerinin, profillerinin ve boyutlarının seçilmesini içerir ve ısı transferi oranlarını artırmak için kritik öneme sahiptirler. Dairesel kanatçıklar, sıcaklık farkı bulunan iki ortam arasında ısı transferi sağlamak için kullanılır. Dairesel kanatçıkların tasarım aşamasında, dikkate alınması gereken birkaç önemli unsur bulunmaktadır. Bu unsurlardan biri, dairesele kanatçıkların geometrisi (kesit alanı) olup, bu doğrudan ısı verimi ve kanatçık üzerinde sıcaklık farklarından dolayı oluşan termal gerilmeleri etkiler, bu yüzden tasarım esnasında dikkate alınması kritik öneme sahiptir. Bir diğer önemli tasarım parametresi ise dairesele kanatçıkta kullanılan malzemedir. Kanatçık üzerindeki farklı sıcaklık değerleri, farklı ısı genleşmelere neden olur. Farklı miktardaki ısı genleşmeler ise kanatçık üzerinde ısı gerilmelere neden olur. Bu gerilmeler: sünme, yorgunluk, çatlak ve çatlakların ilerlemesi gibi mekanik kusurlara neden olabilir. Bu hataları önlemek ve uygun kanatçık tasarımı için kanatçık üzerinde oluşacak ısı gerilmelerin önceden belirlenmesi gerekir. Yüksek sıcaklık farklarına maruz kalan kanatçıklarda oluşacak olan ısı gerilmeleri azaltmanın bir yolu homojen malzeme kullanımı yerine kompozit malzeme kullanımıdır. Kompozit dairesele kanatçık homojen dağılımlı seramik ve metalden yapılmıştır. Kompozit malzemeler, lif takviyeli, parçacık takviyeli ve levha tabakalı takviye türleri olarak sınıflandırılır. Bu tezde, parçacık takviyeli kompozit malzemeden yapılmış bir dairesele kanatçık kullanılmıştır. Parçacık takviyeli kompozitlerin mekanik ve termal özelliklerinin dairesele kanatçık boyunca homojen bir şekilde dağıtılabilmesi nedeniyle tercih edilmiştir. Bu tezde seramik olarak kullanılan malzeme Silisyum Karbür (SiC), metal olarak kullanılan malzeme ise Alüminyum Alaşım 1100 (Al) dür. Silisyum Karbür, çeşitli alanlarda yaygın olarak kullanılmaktadır. Al-SiC kompozitlerinin üretimi genellikle toz metalürjisi teknikleri kullanılarak yapılır (Ling, 1995). Toz metalürjisinden sonra, presleme işlemlerine tabi tutulur. Bu işlemler nedeniyle, parçacık takviyeleri dairesele kanatçık boyunca homojen olarak dağılır. Alüminyum Alaşım 1100 ve Silisyum Karbür malzemelerinden oluşan kompozit malzemenin mekanik özellikleri, homojenizasyon prosedürü kullanılarak hesaplanmıştır.

Kompozit malzemeden oluşan dairesele kanatçıkta malzemenin özellikleri Halpin Tsai yöntemi kullanılarak elde edilmiştir. Halpin-Tsai deneysel verileri kullanarak üretilmiş olan amprik bir yöntem olup, parçacık takviyeli kompozitlerde malzeme özelliklerinin belirlenmesinde oldukça

etkili bir yöntemdir. Bu yöntem kullanılarak kompozit bir malzemedeki ısı ve mekanik özellikler şu şekilde belirlenir.

$$\frac{p}{p_m} = \frac{1+\xi\eta V_f}{1-\eta V_m} \quad 0.1$$

η ve ξ aşağıdaki gibi tanımlanır:

$$\eta = \frac{\frac{p_f}{p_m} - 1}{\frac{p_f}{p_m} + \xi}, \quad \xi = 2 \left(\frac{l}{m} \right) \quad 0.2$$

Burada, p parçacık takviyeli kompozit malzemenin her hangi bir özelliğini temsil eder. p_f fiberin özelliğini, p_m matris malzemenin özelliğini temsil eder. ξ fiber geometrisi ile ilişkili bir parametreyi ve l fiber geometrisinde uzunluğu m genişliği belirtir. V_f fiberin hacim oranını, V_m ise matrisin hacim oranını ifade eder ve aşağıdaki gibi tanımlanır: (Shariyat, 2020),(Eker ve ark., 2021)

$$V_f = \left(\frac{r-a}{b-a} \right)^n, V_m = 1 - V_f \quad 0.3$$

Bu tezde belirlendiği üzere, homojen dağılıma sahip kompozit malzemelerde n değeri sıfır alınır.

Değişken kesitli bir dairesel kanatçığın sıcaklık dağılımı ve buna bağlı termal gerilmeleri Pseudospectral Chebyshev Yöntemi kullanılarak nümerik olarak incelenmiştir. Sıcaklık kısmını yöneten denklemler ve ilgili sınır koşulları oluşturulurken şu varsayımlar yapılmıştır.

1. Dairesel kanatçık eksenel simetriktir.
2. Kanatçığın kalınlığı, yarıçapa kıyasla çok küçüktür.
3. Kanatçık yüzeyinde sadece iletimle ısı transferi vardır, radyasyon ısı transferi ihmal edilmiştir,
4. Çevre sıcaklığı ve ısı transferi katsayısı sabit kabul edilmiştir,
5. Kanatçık termal kararlılıktadır, kanatçık içerisinde sıcaklık zamana bağlı değişmemektedir,
6. Kanatçık içerisinde ısı kaybı veya ısı üretimi yoktur.
- a. Literatürde bahsedilen varsayımlara ek olarak, bu tezde kullanılan bir diğer varsayım şudur:
7. Kanatçığın kalınlığı, kanatçık yarıçapına bağlı olarak değişmektedir.

Isıl gerilemeleri yöneten denklemler ve sınır koşulları elde edilirken yukarıdaki varsayımlara ek olarak şu varsayımlar yapılmıştır.

8. Kanatçıkta iç kuvvetlerin olmadığı,
9. Kanatçık yüzeylerinde traksiyon olmadığı varsayılmıştır.

Bu varsayımlar dikkate alındığında sıcaklık dağılımını yöneten denklem :

$$\frac{d^2T}{dr^2} + \left(\frac{1}{r} + \frac{1}{A(r)} \frac{dA(r)}{dr} \right) \frac{dT}{dr} - \frac{2h}{A(r)k} (T - T_\infty) = 0 \quad 0.4$$

ve sınır koşulları

$$T(a) = T_b \quad 0.5$$

$$T'(b) = 0 \quad 0.6$$

şeklinde elde edilmiştir.

Dairesel kanatçıklarda gerilme, gerinme ve sıcaklık arasındaki ilişkiyi veren denklemler, gerinme ile deplasman arasındaki ilişkiyi veren denklemler ve gerilme denge denklemleri aşağıdaki gibi verilmiştir.

Gerilme gerinme ve sıcaklık arasındaki ilişkiyi veren denklemler:

$$\sigma_r = \frac{E}{1-\nu^2} (\varepsilon_r + \nu \varepsilon_\theta - (1 + \nu) \alpha T) \quad 0.7$$

$$\sigma_\theta = \frac{E}{1-\nu^2} (\varepsilon_\theta + \nu \varepsilon_r - (1 + \nu) \alpha T) \quad 0.8$$

Gerinme-deplasman arasındaki ilişkiyi veren denklem

$$\varepsilon_r = \frac{du}{dr} \quad 0.9$$

$$\varepsilon_\theta = \frac{u}{r} \quad 0.10$$

Gerilme denge denklemleri

$$\frac{dA(r)_r}{dr} \frac{\sigma_r}{A(r)_r} + \frac{\sigma_r - \sigma_\theta}{r} + \frac{d\sigma_r}{dr} = 0 \quad 0.11$$

şeklinde elde edilmiştir.

Gerilme-gerinme sıcaklık ilişkisini veren denklemler, gerinme deplasman ilişkisini veren denklemler ve gerilme denge denklemleri kullanılarak, değişken kesitli kanatçıkta gerilme kısmını yöneten denklem aşağıdaki gibi elde edilir.

$$\frac{d^2u}{dr^2} + \frac{du}{dr} \left(\frac{A'(r)_r}{A(r)_r} + \frac{1}{r} \right) + \frac{u}{r} \left(\frac{A'(r)_r}{A(r)_r} v - \frac{1}{r} \right) = \frac{A'(r)_r}{A(r)_r} (1 + v) a T + (1 + v) a \frac{dT}{dr} \quad 0.12$$

Isıl gerilmeler kısmındaki sınır koşulları ise (8) numaralı varsayım kullanılarak aşağıdaki gibi elde edilir.

$$\sigma_r(a) = 0 \quad 0.13$$

$$\sigma_r(b) = 0 \quad 0.14$$

Sıcaklık kısmını ve ısı gerilmeler kısmını yöneten denklemler verilen sınır koşulları altında Pseudospectral Chebyshev yöntemi kullanılarak nümerik olarak incelenmiştir. Çözüm yapılan parça geometrisinin tamamının göz önünde bulundurulduğu, sınırlara yakın bölgelerde daha yoğun, merkeze doğru ise daha seyrek bölüntü noktası oluşturarak çözümleme yapma mantığına dayanan yöntemde bölüntü noktaları, sağdan sola, Gauss-Lobatto noktaları kullanılarak şu şekilde elde edilir.

$$x_j = \cos\left(\frac{j\pi}{N}\right), \quad j = 0, 1, 2, 3 \dots N \quad 0.15$$

Denklem (0.15)'te verilen Chebyshev türevleme matrisindeki örgü noktalarının dağılımını veren ifade aşağıda belirtilen çözüm şeması izlenerek oluşturulacaktır.

Çözüm prosedürünü uygulamak için, örgü noktalarında tanımlı bir v vektörünü dikkate alalım. Aynı örgü noktaları için v vektörünün ayrık türevi ω iki adımda elde edilir: (Trefethen, 2000)

- Eğer $0 \leq j \leq N$ için $p(x_j) = v_j$ şeklinde $p \leq N$ derecesinde bir interpolasyon polinomu seçilirse,
- Örgü noktalarını veren türevler $0 \leq j \leq N$ için $\omega_j = \rho'(x_j)$ şeklinde olacaktır.

Örgü noktalarının türevini ifade eden ω , $(N+1) \times (N+1)$ boyutlarındaki D_N ile v matris çarpımı ile elde edilir.

$$\omega = D_N v \quad 0.16$$

D_N türevleme matrisi Lagrange interpolasyonu kullanılarak elde edilir ve bölüntü noktalarındaki birinci dereceden türevi temsil eder. İkinci ve daha yüksek dereceden türevler, türevleme matrislerinin (D_N) birbiri ile çarpılması sonucunda elde edilir ($D_N^2 = D_N \times D_N$).

Türevleme matrisleri kullanılarak sıcaklık kısmını yöneten denklemler aşağıdaki şekilde ayrıklaştırılır.

$$L_1 T = RHS_1 \quad 0.17$$

Yukarıda verilen lineer sistem denklemindeki L_1 ve RHS_1 aşağıdaki gibi verilmiştir

$$L_1 = D^2 + \left(\frac{1}{r} + \frac{1}{A(r)} \frac{dA(r)}{dr} \right) D - \frac{2h}{A(r)k} \quad 0.18$$

$$RHS_1 = \frac{2h}{A(r)k} T_\infty \quad 0.19$$

(0.17) numaralı lineer denklem sistemi, sıcaklık sınır koşullarında (0.5 ve 0.6) çözülür ise sıcaklık için aşikar olmayan çözüm elde edilmiş olur. Aynı prosedür ısıl gerilmeleri yöneten denklem için uygulanırsa

$$L_2 u = RHS_2 \quad 0.20$$

şeklinde bir lineer sistem denklemini elde edilir. Bu denklemde geçen L_2 ve RHS_2 aşağıdaki gibi verilmiştir.

$$L_2 = \left[D^2 + D \left(\frac{A'(r)_r}{A(r)_r} + \frac{1}{r} \right) + \frac{1}{r} \left(\frac{A'(r)_r}{A(r)_r} v + \frac{1}{r} \right) \right] \quad 0.21$$

$$RHS_2 = \frac{A'(r)_r}{A(r)_r} (1 + v) aT + (1 + v) aDT \quad 0.22$$

(0.20) numaralı lineer denklem sistemi, gerilme (0.13-0.14) sınır koşullarında çözülür ise deplasmanlar için aşikar olmayan çözüm elde edilmiş olur.

Yöntemin doğrulanması, literatürde bulunan uniform kesitli dairesel bir kanatçıktaki sıcaklık dağılımı ve ısı gerilmeleri için yapılmış olan bir analitik çözümle karşılaştırılarak yapılmıştır.

Değişken kesitli kompozit malzemeden yapılmış dairesel kanatçıkta elde edilen sıcaklık dağılımları ve ısı gerilmeleri, aynı malzemeden yapılmış düzgün kesitli kanatçıktaki sıcaklık dağılımları ve ısı gerilmeleri ile karşılaştırılmıştır.



ACKNOWLEDGEMENTS

For choosing my master's thesis topic and undertaking the execution of my thesis study, my supervisor, who provided valuable knowledge, experience and contributions throughout my studies, Dear Associate Professor Dr. Kerimcan ÇELEBİ, I offer my thanks.

I also express my appreciation to the esteemed Professor Dr. Naki TÜTÜNCÜ for his invaluable insights and constructive feedback on my dissertation.

I would like to express my appreciation to the esteemed Associate Professor Dr. Durmuş YARIMPABUÇ who generously shared their insights and comments on my dissertation.

And last but not least, I want to thank my mother Seyyare ÇAĞLAR for helping my life journey until today and forever with all my heart.



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SYMBOLS AND ABBREVIATIONS

$A(r)$	: Thickness (m^2)
$B(r)$	: Radial area (m^2)
$C(r)$	: Lateral surface area (m^2)
a, b	: Inner and outer radius of the fin (m)
y_b	: Thickness at the base of the fin (m)
M	: Thickness at the tip of the fin (m)
E	: Modulus of Elasticity (Gpa)
h	: Heat transfer coefficient ($\frac{W}{m^2.K}$)
k	: Thermal conductivity ($\frac{W}{m.K}$)
r, θ	: Polar coordinates
σ_r, σ_θ	: Radial and tangential stresses (Pa)
$\tau_{r\theta}, \tau_{\theta r}$	: Shear stresses (Pa)
T	: Fin temperature (K)
T_∞	: Ambient temperature (K)
U	: Radial displacement (m)
α	: Thermal expansion coefficient ($10^{-6}C^{-1}$)
R_r, R_θ	: Internal stresses (Pa)
$\varepsilon_r, \varepsilon_\theta$	: Radial and tangential strains (m)
F_r, F_θ	: Radial and tangential forces (N)
M_O	: Moment acting on point “O” (N.m)
ν	: Poisson ratio
r	: Radius (m)
Δt	: Per unit time
Δ_r	: Per unit radius
D	: Derivative operator of Pseudospectral Chebyshev Method

1. INTRODUCTION

In the past decades, heat transfer phenomenon has been intended to be cost effective, thermal efficient and became more popular in industry and researchers are paying more attention about this manner. Researchers have been studied how to improve this phenomenon and the required techniques since past decades.

Heat transfer has studied under ‘active’ techniques and ‘passive’ techniques. (Ajarostaghi, 2022) Active techniques require extra energy something like an oscillation or giving to flow some pulsation. Passive techniques do not require extra energy. Passive controls mainly focus on the surface geometry affects. Fins, one of the passive control techniques, are leading in this regard. Fins, a passive heat transfer enhancement technique, are often employed to increase heat transfer in various engineering applications. The design of fin which consists of selecting fin materials, profiles, and dimensions is critical to improving the heat transfer rates. Fins have variety geometries and have been analyzed recently. These types of fins are illustrated in the figure 1.

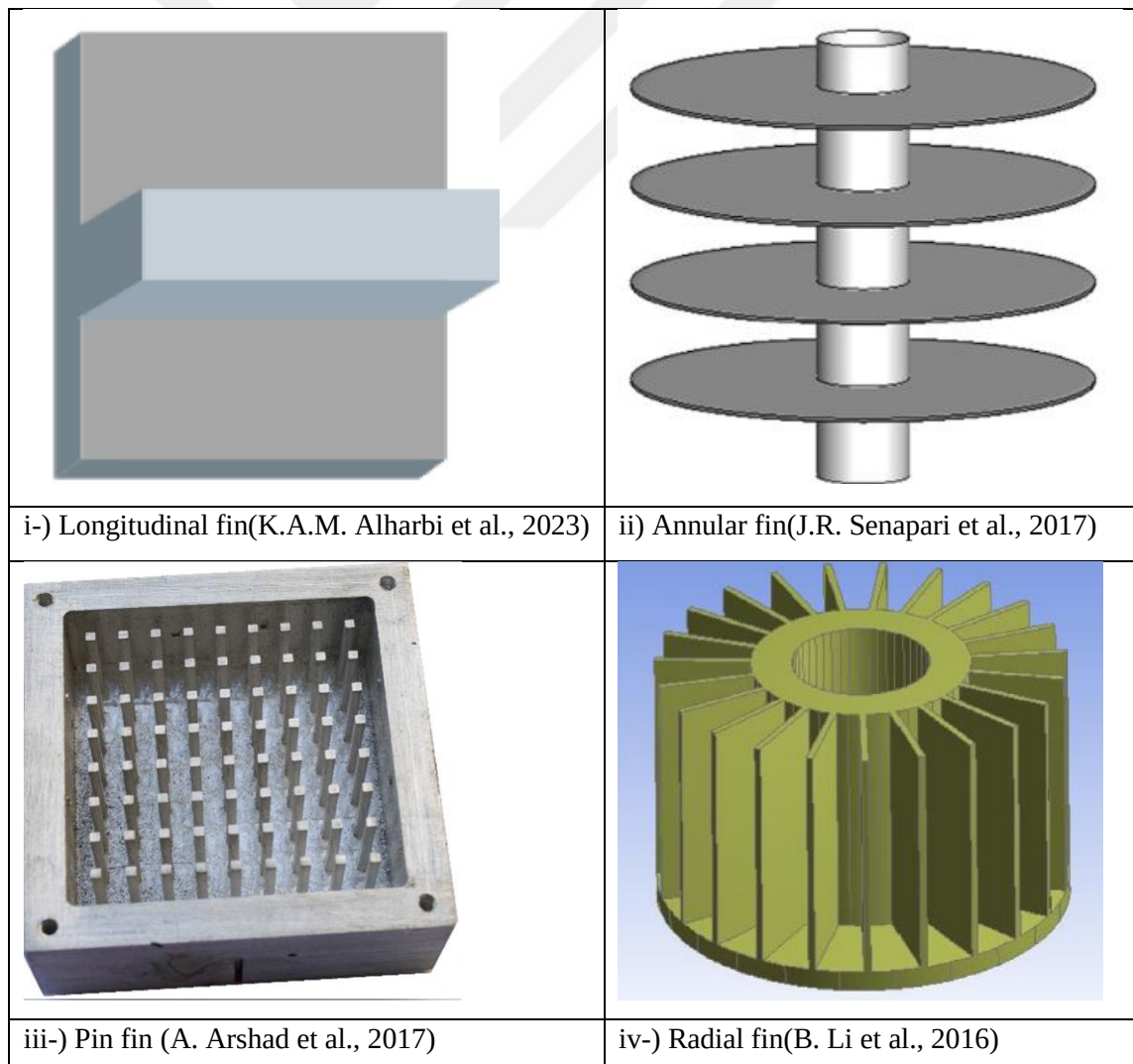


Figure 1.1. Types of fins

Cylindrical shapes fins commonly used in various engineering applications from heat exchangers to combustion engines (Tanmoy, 2019). Aim of enhancing heat transfer to cylindrical shapes, radial fins and annular fins are most suitable for enhancing the heat transfer phenomenon. In heat transfer applications, annular fins are much popular in use compared to radial fins because annular fins are much easier to fabricate and design, so in this thesis annular fins are studied in line with the needs of the industry.

Annular fins are used for the heat transfer between two ambients with temperature difference. Considering the design stage of annular fins, there are several crucial aspects that need to be taken into account. One of the important design parameters is the geometry of annular fin (cross-sectional area) which directly influences the thermal stresses and temperature distributions, hence necessitating prior selection. Another important design parameter is the choice of material of annular fin. Because of every material has a thermal expansion ratio for each temperature, these differences will cause internal stresses in the material. These stresses will cause mechanical failures, for example, occurrence of creep, fatigue, crack and crack increment in materials. To prevent these errors, it is essential to select the correct geometry and materials.

As observed in numerous studies conducted in the literature to date, various geometries are utilized in annular fin studies (Mokheimer, 2002). In order to improve the calculated temperature and thermal stress distributions in uniform cross-sectional fin geometries, variable cross-sectional geometries have been investigated (Kanthimathi and Ram,2016). Generally, i-) trapezoidal, ii-) parabolic, iii-) triangle and iv-) uniform cross-sectional area geometries are popular in the conducted research. Annular fins, which have variable cross-section, offer significant advantages for achieving better conduction of heat and uniform temperature distribution in annular fins. Therefore, in this thesis, an annular fin with variable cross-sectional area geometry has been chosen to be examined.

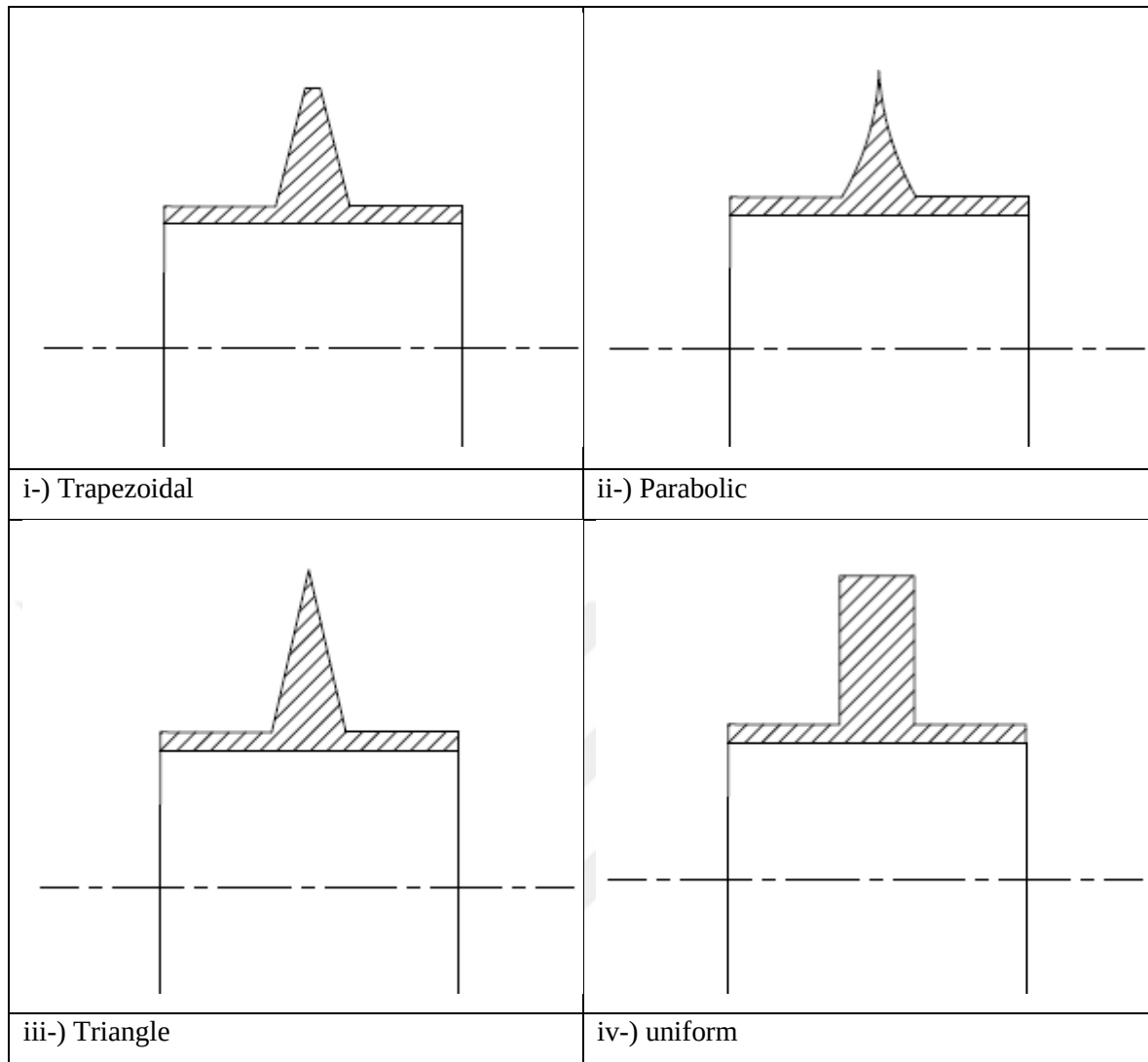


Figure 1.2. Different geometries of annular fin

High thermal conductivity materials commonly used for fins. However, homogeneous materials may not achieve the uniform temperature distribution along the fin. In contemporary technology, a single type of material can be used or composite materials comprising multiple substances can be employed, as well as functionally graded materials.

Annular fins, commonly using in various applications, are made from highly conductive homogeneous materials. However, these materials such as Al, don't achieve uniform temperature distribution along the fins. Recognizing the potential of composite materials to improve the thermal performance by combining two distinct materials, this thesis proposes their use as annular fin materials to achieve better thermal performance. The homogeneous composite annular fin is made with ceramic and metal. Composite materials are classified into fiber-reinforced, particle reinforced, and plate layered reinforced types. In this thesis, an annular fin made from particle reinforced composite has been utilized. Due to the fact that the mechanical and thermal properties of particle reinforced composites can be distributed homogeneously throughout the annular fin. The material that used as a ceramic is Silicium Carbide and the material that used as a metal is Aluminum alloy

1100. Silicium Carbide finds widespread application across various fields. Researchers are particularly motivated by its thermal and mechanical properties, making it a preferred ceramic material choice within this thesis (G. Roewer, (2002), Sharma et al. (2016), Mohan et al. (2002)). Aluminum alloy 1100 is widely used and commercially available in the industry. In the literature, Aluminum alloys show not only good thermal and mechanical properties but also simple to find and cost effective (Asif et al. (2011), Adetunla and Akinlabi (2018)). Fabrication of the Al-SiC composites are usually manufactured using powder metallurgy techniques (Ling et al., 1995). After the powder metallurgy, samples are subjected to compacting processes. On account of these processes, particles reinforcements are homogeneously distributed along the annular fin. The mechanical properties of Aluminum alloy 1100 and Silicium Carbide composite should be obtained for homogeneous procedure. In order to do that Halpin-Tsai Homogenization scheme is utilized to the mechanical properties. Halpin-Tsai model uses experimental data to fit on a curve and provides us to determine the mechanical properties at any point in the annular fin (Zhou et al. 2023).

Besides composite materials, functionally graded materials are rapidly gaining prominence by researchers. Unlike composite materials, the mechanical properties of the materials in the region are determined as a power function without sudden change. This allows to use any desired property at any point in the annular fin.

In designing to ensure safe operation against potential failures such as fatigue, fractures, crack propagation and wear, arising from stresses in the annular fins, it is crucial to predict and design for these issues in advance. However, often the equations governing temperature distribution and consequent stresses become more complex and challenging to calculate than initially anticipated. Therefore, expressing the annular fin as a variable cross-section, complicates the temperature distribution and thermal stress equations. Numerical methods are employed to alleviate the complexity and difficulty of solving these equations.

There are numerous amounts of numerical methods in the literature such as Finite Element Method, Finite Difference Method, Perturbation Methods, Spectral Methods, Runge-Kutta Methods, Complimentary Functions Method etc. and they have a wide using range in applications. The main purpose of all numerical methods is converting the problems into to the polynomial shapes in order to apply to and solve with computers. These polynomials can be separated into two groups. Some methods use local polynomials where the polynomials are utilized step by step. Mesh points are solvable after their neighbors are solved. On the other hand, some methods use global polynomials where the polynomials can fit and solve all points, and this feature provides more superior accurate results and decreased time cost compared to the methods that use local polynomials.

In Spectral Methods, error rates are almost non-existent since the exact values of the functions can be found. This is the difference from methods that use local polynomials such as Finite Element Method. Another difference is that less data and unknowns are needed to reach the solution in Spectral Methods. In the literature, there are two types of polynomials exist in Spectral Methods.

These are Legendre Polynomials and Chebyshev Polynomial, however, Chebyshev Polynomials used in this thesis and explained in future sections that how Chebyshev Polynomials work.

In this thesis, the general thermal solution to the composite annular fin with the variable cross-section is investigated. Annular fin material has assumed to be homogeneous by using Halpin-Tsai Homogenization scheme. Mechanical and thermal properties are presented as established with Halpin-Tsai Homogenization scheme. The Pseudospectral Chebyshev Method (PCM) is used to solve the general solution obtained for temperature and displacement. The numerical solution is checked with the results obtained with the uniform cross-sectional annular fin for the validation purpose.

The process of the current thermal analysis can be summarized as follows:

- Energy equilibrium equation and boundary values are obtained in order to investigate the temperature distribution
- Displacement equation and boundary values are obtained for thermal stresses
- The composite material is assumed to be homogeneous and its mechanical properties have scrutinized with Halpin-Tsai Homogenization Method
- Pseudospectral Chebyshev Method has been validated with the analytical results of uniform cross-sectional annular fin
- Pseudospectral Chebyshev Method is utilized on energy equilibrium and displacement equations



2. PRELIMINARY WORK

Nowadays, there are lots of works and researchers have been done for fins in the literature. Some of them are about efficiency of fin and its effectiveness, designing of fins for reaching optimum dimensions with respect to thermal stresses and temperature distributions.

First of all, while the thermal distribution and thermal stress analysis are being analyzed, some assumptions should have been applied to work in order to prevent more complex problems. These assumptions are used in this thesis and being held by Gardner (1945) first time. In Gardner (1945) initially proposed efficiency equations for fins, assuming calculations based on a single material. S. Lalot et al. (1999) demonstrated that this approach could also be applied to dual-material composite fins. Their study highlighted that the efficiency of fins made from low thermal conductivity composite materials could be enhanced by using a coating material with higher thermal conductivity to increase heat transfer rates. Rather than using a coating, T. Kanthimathi and Naga Sai R.G. (2016) utilized the second material as a coating. One material is chosen to be a base material where the whole annular fin is made of and the second material is implemented to tips of the annular fin which has a high thermal conductivity, for example brass, some steels and non-metallic materials. They inspected that utilizing the non-metallic materials give significant improvement on fin efficiency. With using the same methodology for the composite fins, ANSYS package program has been utilized for analyzing annular composite fins by A.Giri and S.A.K. Jilani (2012). The efficiency of the fin is assessed for different cross-sections of the coating materials. The effects of temperature variation and fluctuations in the convective heat transfer coefficient on fin efficiency are considered.

The calculation of expressions such as thermal stresses, temperature distributions and fin efficiency can often become daunting tasks. This is primarily due to the complexity of stress analysis in annular fins in polar coordinate systems, as well as the variation of thickness along the radial axis in annular fins, which complicates the simplification of thickness terms in stress equations. Furthermore, the formulation and solution of second-order differential equations pose challenges. Consequently, various numerical methods and assumptions have facilitated researchers in solving these problems more effectively.

Arslanturk (2009) has investigated annular fin's temperature dependent thermal conductivity. In this study, non-linear problem solved analytically with Adomian Decomposition Method (ADM) and furthermore, used an optimization technique to achieve optimum design equations of Biot Number, fin's volume and thermal conductivity. Certain results are maintained after optimization which is the thermal conductivity is decreasing with respect to increase of Biot Number.

An exercise has been devoted by Deka and Datta (2018) to formulate thermal stresses as a multiobjective optimization problem. Two functions are considered to reduce the total volume of the annular fin as much as possible and to make the fin surface that is effective in heat transfer as efficient

as possible. Differential equations are solved with Hybrid-Spline-Difference-Method and to make an optimization, Nondominated Sorting Generic Algorithm II is utilized. Problems and functions have been created to reduce the maximum thermal stress, increase the heat transfer, reduce the volume and maximize the surface efficiency of the annular fin. It has been seen that in order to bring one of the examined functions to the desired level, it is necessary to sacrifice its dimensions and other functions.

In study of Hosseinzadeh (2022), thermal analysis of three different porous fins with trapezoidal, concave-parabolic and convex cross-sectional areas is performed. Problem has analyzed analytically with Akbari-Ganji's Method (AGM). To validate AGM, the Differential Transformation Method is compared with numerical and Finite Element Methods. It has been shown to have a higher accuracy rate than other methods. The highest heat transfer and fin effect occurred in the fin with parabolic-concave cross-sectional area. It has been observed that the AGM method has more accuracy in solving heat transfer equations when compared to the results obtained from other methods.

In the study conducted by Mallick et al. (2016), the temperature equations and thermal stresses of an annular fin exposed to thermal load are examined. The problem is solved analytically with the Homotopy Perturbation Method. It is assumed that the thermal conductivity varies linearly, and the problem is in steady state. Homotopy Perturbation Method is a method with high accuracy. Thermal parameters obtained with the Inverse Solution give the same thermal stresses as those found with the Homotopy Perturbation Method.

Mallick et al. (2016) aimed to design an efficient annular fin by examining the thermal parameters of an annular fin in a non-linear manner. The problem is solved by Homotopy Perturbation Method in both linear and non-linear way. It has clearly shown that thermal conductivity affects thermal stresses, but the internal heat production does not effect on the radial stresses.

The temperature values and fin efficiency of an annular fin with variable thermal conductivity explicated by Deepak U. et al. The non-linear differential equation is solved with the Differential Transformation Method. Pade Approximant Method is used to convert polynomials into power series. As thermal conductivity increases, radial and tangential stresses also increase.

Deepak U. et al. evaluated the temperature distribution and thermal stresses of a hyperbolic annular fin with temperature-dependent thermal conductivity. Non-linear differential equations are analyzed analytically with the Differential Transformation Method and then numerically with the Runge-Kutta Fehlbergfourth-fifth Method.

Mallick et al. studied the thermal stresses of a hyperbolic cross-sectional annular fin with temperature-dependent thermal conductivity. Adomian Decomposition Method is used in this study and validated by comparison with analytical results in the literature. Stress values in the hyperbolic annular fin are checked, it is seen that it is safer against cracks and fatigue.

Venkitesh et al. (2020) have used Homotopy Perturbation Method to investigate the heat transfer characteristics of a porous annular fin. All parameters are written in forms of temperature. The governing equations were formulated with using Darcy's model.

Mokheimer (2002), appraised annular fins with different cross-sections. The equations are solved by finite element methods and the numerical solutions of annular fins with different cross-sections are compared graphically with the analytical results in literature.

Kang and Dwight (2013) tested and examined the trapezoidal cross-section on annular fins and aimed to optimize all thermal parameters. They introduced a new 2-Dimensional analytical method and compared with the results of generally analytical methods

Haghighi (2016) explored the heat flux of a functionally graded annular fin with variable cross-section. They performed the Differential Quadrature Method and validation with the literature proves that it is a reliable method with low error rate.

Hassanzadeh and Pekel (2016) tried a new analysis method on functionally graded materials in annular fins. They simplified the equation with Mean Value Theorem and solved it with Approximate Analytical Approach. As a result, functionally graded annular fin achieved better heat transfer than the homogeneous annular fin.

Aziz and Rahman (2009) considered an annular fin made of functionally graded material. They investigated the temperature performance and the fin efficiency of the functionally graded annular fin.

Gaba et al. (2014) investigated the temperature distributions and parameters of a functionally graded annular fin with a variable cross-section. They solved the problem in a non-linear way using the MATLAB package program. It has been shown that the lower the combination of the shape and thickness of the cross-section, it gives better performance and optimal values of thermal conductivity and efficiency.

Kundu and Das (2001), investigated complex geometries and stepped thickness with fins. They used the method of Lagrange multipliers to solve the annular fins with a sudden change in thickness. With the increase of variables and unknowns, the Lagrange Multiplier Technique still gives reliable values.

Peng and Chen (2011) assessed the variable thermal conductivity of the annular fin. They solved the problem numerically in a non-linear manner using both the Differential Transformation and Finite Difference Methods. It has been observed that the optimal parameters decrease with increasing thermal conductivity.

Ullman and Kalman (1989) evaluated the optimal parameters and efficiency of annular fins with different cross-sections. Numerical analysis is confirmed with analytical results. It has been observed that the tapering of the fin tip reduces the efficiency of the fin and that the parabolic cross-sectional annular fin gives the most optimal and efficient results among the four different cross-sectional fins.

Moitsheki (2011) examined the thermal diffusion in fins with both uniform and variable cross-sections. Bessel functions are used in the analytical solution of the fin with uniform cross-section while Airy functions are used for variable cross-section. It has been observed that fin efficiency and performance decrease with increasing thermal parameters and heat flux is important variable for thermal parameters.

Khani and Aziz (2010) analyzed the temperature distribution and thermal parameters with trapezoidal cross-sectional areas. They solved the problem analytically with the Homotopy Analysis Method. The accuracy and usability of the method has been proven. In their studies, it is checked that which thermal parameters affect the efficiency and performance of the fin.

Sudheer et al. (2012) investigated the heat transfer efficiency, performance and parameters of annular fins with various cross-sections. In this study, Finite Element Method is performed on problem with using ANSYS package program. It has been observed that the most cost-effective fins are the triangular cross-sectional fins and the highest stress values are in the fin with uniform cross-section.

It is observed that there are many numerical analysis methods applied on investigating fins. Unlike classical and generally used numerical methods, Spectral methods are deemed suitable for application to this thesis due to their high accuracy rate and ease of use.

Spectral Methods can be used in many application areas and one of them is climate studies. Ghil et al. (2002), investigated the usage of Spectral Methods also explained how to use and classified in climatic problems. In general, Spectral Methods are based on Fourier Series, in climatic problems Fourier Series transform time series. Spectral estimates vary into some spectrums, and these are called Periodogram and Correlogram.

Another application field of Spectral techniques is aerodynamic problems. Press and Tukey (1959), have investigated the problem at steady state. They performed power spectral techniques in order to calculate effects of disturbance, atmospheric turbulence, damping, fluctuations, wing flexibility and buffeting loads.

Shen and Tang (2006), stated some applications of Spectral methods. They showed how to use Spectral methods and its equations forms with wave equations; Shrodinger equations, Stokes equations and Navier-Stokes equations. Spectral methods solve the problems with polynomials as discussed before in the introduction section. These polynomials, most likely Chebyshev and Legendre, use grid points with different meshing procedures. Most used one is Chebyshev collocations points and differentiation matrix can be achieved using Chebyshev-Gauss-Lobatto mapping system in order to simplify some meshing problems.

Methods that Finite Element Method or Finite Difference Method and Spectral Methods are being part of a same family methods of Weighted Residual Methods. Vosse and Minev (1996) clearly explained Spectral approximation and Pseudospectral approximations (also known as Spectral-Collado). A combination of the Finite Element Method and Spectral method is introduced as Spectral

element method and its application on the stress-strain equilibrium has been shown and stated a possible problem of the usability of Spectral element method on the geometrically non-linear problems.

A literature review is made by Rai and Mondal (2021). They obtained a lot of data for Spectral Methods. Their advantages and disadvantages are illustrated in their study where Spectral Homotopy analysis method (SHAM), Spectral-Homotopy perturbation methods (SHPM), Spectral linearization method (SLM), Spectral Quasi Linearization method (SQLM), Bivariate Spectra Quasi Linearization method (BSQLM), Piecewise Successive Linearization method (PSLM), Spectral Relaxation Method (SRM), Spectral Homotopy perturbation method (SHPM) are considered.

These new methods have accuracy results on initial value problems and boundary value problems. They all decreased the computation time cost and proved their convergence criteria.

Literature review on hyperbolic problems is studied by Gottlieb and Heashaven (2001). Pseudospectral methods have become superefficient and super accurate with non-linear multi-domain problems. The authors noted that complex geometries may still create some difficulties to Spectral methods.

While Spectral methods may have some difficulties in complex geometries, Orszag (1980) studied some complicated geometries and problems with Spectral methods. Author discussed the difficulties with nontrivial boundary conditions, non-linear and non-constant coefficient problems. In order to overcome the difficulties caused by complex geometries, two major techniques are introduced. These are mapping and patching. Mapping is illustrated as Chebyshev polynomials where boundary regions of complex geometries turned into simple boundary conditions. Such as polar coordinates can be mapped like a straight, rectangular coordinated. Also patching, similar to mapping, is a technique that improved in order to simplify complex regions and divide to simple regions.

Between all these Spectral methods, Pseudospectral Chebyshev is most accurate and widely used by researchers nowadays. Porous fin has been analyzed by Oguntala et al. (2019) to investigate the inclination effect on the fin with the Pseudospectral Collocation method. They concluded enhancing thermal efficiency effect of porous fin.

Pseudospectral Chebyshev Method has been carried out by Alexandrecy et al. (2009) on Shrodinger type problems. Mapping is done with using Chebyshev polynomials in order to simplify the coordinate functions and for the grids, Gauss-Lobatto points are utilized to mapping procedure. For the physical domains, coupled beams are choosed to be analyzed. It has proven how easy to implement Chebyshev polynomials compared with the Decomposition methods and also numerical investigations that are compared with the analytical results show the reliability and good accuracy of the Pseudospectral Chebyshev Method.

Yildirim et al. (2018) studied the temperature distribution and thermal stresses in an annular fin. Isotropic homogeneous material is solved by using the Complementary Functions Method and

the Pseudospectral Chebyshev Method. Both numerical methods are compared with analytical results that obtained by using Bessel functions and thermoelastic theory. Error analyzes of these numerical methods for different division points are shown using Euclidean norm. It has been observed that Pseudospectral Chebyshev Method approaches the analytical result faster as the number of points increases.

Yildirim et al. (2020) also did a thermal wave propogation and efficiency test for the annular fin with fixed base. Annular fin is assumed to be made by functionally graded material and mechanical properties are obtained with using Mori-Tanaka homogenization scheme. Fixed boundary condition results in zero deformation at the base. Annular fin made by functionally graded material has a lower stress at the base section compared with the aluminium annular fin bu has a better fin efficiency compared with the steel annular fin.

Another mapping procedure and effects of using Chebyshev Collocation Methods are studied by Bayliss (1991). Difficulties of Pseudospectral Chebyshev Method has been decreased with since the Chebyshev Collocation approximation for mapping takes place. As the mapping area varies speedily, Collocation mapping greatly enhance the accuracy of the results. If the study region is not a single domain, this phenomenon is much more improved by Chebyshev Collocation approximation.

Don (1994) applied Pseudospectral Methods to shock wave applications. In order to compare and validate the numerical results, essentially non-oscillatory third order finite difference method is carried out and the comparison proved that Pseudospectral Methods have high accuracy on shock wave problems with Chebyshev Collocation mapping procedure. On the other hand, despite the better results of Pseudospectral Method that compared with other numerical results using different methods, future investigations are suggested to investigate the locations of shock waves.

Optimal control techniques are mostly focused on Pseudospectral Legendre Method and Gong (2010) investigated Pseudospectral Chebyshev Method for the purpose of optimal controlling. Chebyshev polynomials are more suitable for optimal control and provides better mapping procedure. Lastly, thanks to Pseudospectral Chebyshev method computational time is greatly decreased. On the other hand, optimal control techniques are also solved with Pseudospectral Chebyshev-Gauss Method by Tang et al. (2015). Rather than Gauss-Lobatto points, Chebyshev-Gauss points are utilized during mapping procedure. Method that used in this paper has better stability and decreased computational time than other Pseudospectral methods.

Guo et al. (2002) utilized Pseudospectral Chebyshev Method on semi-infinite interval problems. Due to Chebyshev's weighted results are much smaller than Legendre's weighted results, the convergence criteria of Chebyshev method are much smaller than Legendre method's convergence criteria. Compared Chebyshev results with theoretical results proves the reliability and accuracy of the Pseudospectral Chebyshev method.

Guo and Zhu (2013) utilized Pseudospectral Chebyshev Method for Trajectory optimization. Chebyshev points are rebuilt with a new approximation. Pseudospectral Chebyshev Method enhances the stability and provides accuracy on Trajectory optimization.

Darvishi et al. (2016) examined the temperature distribution of an annular fin with variable cross-section by taking the thermal conductivity as a variable. Numerical analysis of the problem is made with a Pseudospectral method. In this method, called Pseudospectral Collocation Method, the solution is provided by Chebyshev-Gauss-Lobatto points.

Im et al. (2015) aimed to solve unsteady flows with using new approaches. Unsteady flow is another application area of Pseudospectral Chebyshev Method. Chebyshev Collocation is utilized for mapping procedure with both periodic and non-periodic problems. Im et al. (2015) compared the Chebyshev method solutions with other solution approaches and agreed on the reliability and usability of the Chebyshev method. Conclusions show that the number of grid points have no effect on the convergence criteria.

Pseudospectral Chebyshev Method has also implemented on incompressible flow equations. Pulicani and Ouzzani (1991) solved 3-D Navier-Stokes equations with using Pseudospectral Chebyshev Method. Chebyshev Collocation mapping and Chebyshev method eliminates the disadvantage and difficulties of the cylindrical coordinates and also proves that Chebyshev method provides better results and more reliable than general methods.

Despite the application of numerical methods such as Spectral Methods, Perturbation Methods, Finite Methods, etc., analytical methods are gaining more importance in academic fields and applications. Because numerical solutions must be validated with analytical results; otherwise, the reliability and usability of numerical methods could become a significant issue. Speaking for the analytical methods, Bessel Functions are considered in this thesis and their applications.

Charters (1989) studied annular fins with uniform cross-section and calculated fin efficiency with using Bessel Functions. The efficiency of fin relations is considered in this paper for determining every shape and geometries. Fin efficiency and temperature distribution equations are modified to Bessel Functions and evaluated with the computers.

Bessel functions have various fields of application because they aim to solve second-order differential equations. One of the application fields is extended surfaces, so called fins. Irvan et al. (2022) investigated two types of geometries on fins and modified formulas for Bessel Functions to simplify while studying extended surfaces. Formulas that modified are less accurate for calculation of thermal problems on fins.

In the extended surface applications, fin efficiency can be modified as a Bessel's function. Hong and Webb (1996) calculated fin efficiency on wet and dry fins. Wet fin has a lower efficiency comparing dry fin. Hashizume et al. (1999), investigated fin efficiency of serrated fins. They built an experimental setup to obtain exact fin efficiency results. Later on, analytical solution of fin efficiency is compared with the experimental results.

In the literature, composite materials and functionally graded materials are popular for designing a certain fin. When the composite materials are utilized, certain materials are thought to be homogenous. Considering homogeneous materials, properties of materials must be calculated in a certain way. Halpin-Tsai method is one of the homogenization schemes among the literature.

Halpin-Tsai method is used for obtaining mechanical properties. The main reason to usage of Halpin-Tsai method is laying down in its simplicity and accuracy. In the study made by Giner et al. (2015), the reinforcement factor is the main property in order to investigate stiffness. This reinforcement factor is given as an equation obtained from the Halpin-Tsai method. Later on, this Halpin-Tsai equation of reinforcement factor is solved with Finite Element method.

Vibration is another important parameter for mechanical design, Ebrahimi and Dabbagh (2019) has investigated vibration of a nano-composite material. Rule of Mixture is used for same purpose as Halpin-Tsai Method. The combination of both homogenization scheme equations is solved with Galerkin's method numerically. The maximum frequency is obtained when the carbon fiber is implemented to homogenization matrix.

Halpin-Tsai method is also utilized for an aim of calculating elastic modulus of a composite material. Zhou et al. (2024), used Halpin-Tsai model with a modified scheme. Purpose of the modification is using more than two materials with Halpin-Tsai model.

Silicium Carbide (SiC) finds widespread application across various fields, driven significantly by its exceptional thermal performance. Researchers are particularly motivated by its thermomechanical properties, making it a preferred ceramic material choice (G. Roewer, 2002).

SiC composites have the ability of maintaining its thermomechanical properties at elevator temperature. Buet et al. (2011) identified the properties of the SiC fibres that affect the Fibre/Matrix interface bonding.

Sharma et al. (2016) studied the wear characteristics of SiC and its composites. They found that SiC composites with clean and smooth grain boundaries exhibit higher wear resistance compared to those with irregular grain boundaries.

Machining of the Al-SiC composites are investigated by Mohan et al. (2002). They obtained the optimal machining process parameters by investigating different Material Removal Rate (MRR) and Tool Wear Rate (TWR). Due to the decrease of volumetric ratio of SiC in the composite, TWR is decreased as observed. Also, increase of the SiC volume ratio results a decrease in MRR.

Due to their exceptionally good mechanical properties, aluminum and aluminum alloys find applications across various industrial sectors. (Asif et al., 2011), due to their machinability, weldability and good thermal conductivity, researchers prefer aluminum and its alloys in composite materials. Adetunla and Akinlabi (2018) studied to enhance the 1100 aluminum alloy's mechanical properties. Reinforcement technique can be a significant enhancement method as observed.

3. MATERIAL AND METHOD

There are numerous amounts of methods in the literature in order to obtain desired material property in desired direction. Such approaches are influenced by diverse assumptions depend on the particle geometry. In order to obtain the exact mechanical behavior, material properties must be achieved with proper predictions and assumptions. Mechanical properties can depend on many conditions such as volume fractions, spatial distribution and orientation. Empirical data remains limited since mechanical properties need to be measured frequently (Yang, 2016).

Techniques such as Mori-Tanaka (Mori and Tanaka 1973; Benveniste 1987), Halpin-Tsai (Afdl and Halpin 1976), power functions, exponential and linear mixtures, Voigt (Voigt 1889) and Reuss (Reuss 1929) models, Chen and Lin's sinusoidal rule (Chen and Lin 2010) can be utilized for obtaining material properties in a desired direction. In this thesis, Halpin-Tsai Homogenization Method is used in order to describe mechanical properties.

3.1. Halpin-Tsai Homogenization Method

Halpin-Tsai Homogenization Method has been used for composite materials to describe material's mechanical properties. Halpin-Tsai scheme is a material model based on empirical relationship developed by curve fitting to experimental data. It is used to describe the thermal expansion ratio, heat transfer coefficient, Poisson's ratio and Modulus of Elasticity of composite annular fin. (Halpin, 1969; Halpin, 1992; Gibson, 2016).

$$\frac{p}{p_m} = \frac{1+\xi\eta V_f}{1-\eta V_m} \quad 3.1$$

η and ξ can be represent as follows:

$$\eta = \frac{\frac{p_f}{p_m} - 1}{\frac{p_f}{p_m} + \xi}, \quad \xi = 2 \left(\frac{l}{m} \right) \quad 3.2$$

Here, p represents a property of the particle reinforced composite material. p_f describes the property of fibre while p_m describes the property of matrix material. ξ is a parameter related to the fiber geometry and l is length, m is the width of the fiber geometry. V_f expresses volumetric ratio of the fiber while V_m is the volumetric ratio of the matrix and they are defined as: (Shariyat, 2020; Eker et al., 2021)

$$V_f = \left(\frac{r-a}{b-a} \right)^n, V_m = 1 - V_f \quad 3.3$$

In the case of homogeneous distribution composite materials, as discussed in this thesis, the value of n is equal to zero.

3.2. Numerical Method

The term “differential equations”, denotes to equations that establish the relationship between the derivatives of functions and variables. In various academic disciplines, many problems are analyzed by examining how certain variables depend on other variables change. In some cases, the changes mentioned are so small that such problems are described by differential equations. Such problems may involve both the variables and their derivatives. Such differential problems appear in a wide range of fields, for example, engineering, statistics, aerodynamics, computational fluid dynamic. In some differential equations, the differentiated expression may not depend on a single variable. In such cases, the differential equations are denoted as partial differential equations (Çengel and Palm, 2013).

The solution to a differential equation expresses to finding a function that ensures the equation identically. However, there are cases where an analytical solution of differential equations is either impossible or extremely difficult. Such situations necessitate the use of numerical methods (Çengel and Palm, 2013). In contemporary literature, the use of numerical methods has become highly prevalent, and researchers have led to the development of methods that yield results very close to analytical solutions. Commonly used numerical methods in the literature, such as Finite element and Finite difference methods, are referred to as local methods since they use local polynomials. One of the major disadvantages of such numerical methods is that achieving highly accurate results necessitates a considerable number of grid points. On the other hand, methods such as spectral methods, which are referred to as global methods due to their use of global polynomials, can provide highly accurate results with significantly less grid points (Kırköse, 2019). Spectral methods vary depending on the distribution and structure of the grid points. This situation is because of the development of different spectral methods, such as Fourier Galerkin collocation, Legendre Galerkin collocation and Pseudospectral Chebyshev method (Voigt and Gottlieb, 1984; Trefethen, 2000). Pseudospectral methods, which are a type of spectral methods, provide elemental solutions. The fundamental difference between pseudospectral methods and finite element or finite difference methods is that pseudospectral methods utilize a high-degree polynomial covers the entire solution domain. Since the degree of polynomial can be selected in a desired way, the accuracy of the material is relatively high (Altunalan, 2016).

3.2.1. Chebyshev Gauss-Lobatto- Grid Points

In differential equations, some spectral methods are used for problems in periodic intervals; these include Fourier Spectral and Fourier Pseudospectral methods. However, for some differential equations, solutions are obtained for non-periodic intervals and methods such as Legendre

Pseudospectral, Chebyshev Spectral and Chebyshev Pseudospectral methods (Guo 1998). Chebyshev polynomials are defined on the interval $[-1,1]$ for solving differential equations in non-periodic intervals. The defined interval can be converted with very minor adjustments to arbitrary interval. In the selected solution interval, the grid points distribution that keep the error at the lowest rate is selected and the interpolation polynomial corresponding to these grid points is sought. These grid points are known as Chebyshev Gauss-Lobatto points which have a denser point distribution at the boundaries compared to midpoints are preferred.

Grid points are defined as (Trefethen, 2000):

$$x_j = \cos\left(\frac{j\pi}{N}\right), \quad j = 0, 1, 2, 3 \dots N \quad 3.4$$

Grid points are placed on a semicircle as the vertical projections of the grid points are considered, density of the grid points can be seen clearly on Figure 3.1. Therefore, the differential matrix is obtained with fewer grid points provides precision results (Trefethen 2000). At these points, the differentiation matrix operator is created by utilizing the Lagrange Interpolation polynomials. The obtained differentiation matrix transforms the differentiated expression into a linear equation system which can be explained in study of Kırköse, (2019).

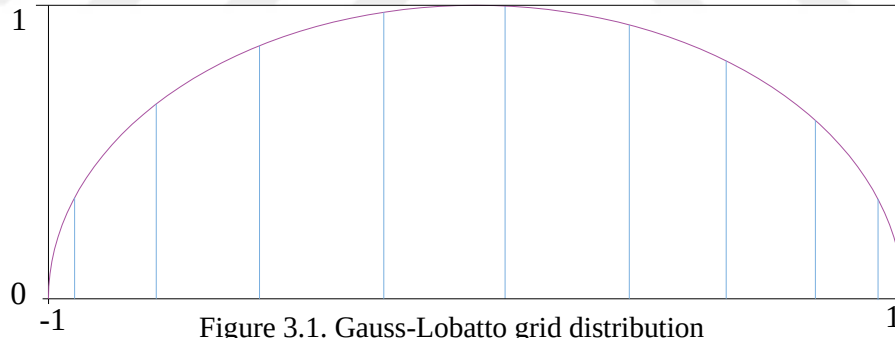


Figure 3.1. Gauss-Lobatto grid distribution

3.2.2. Differentiation Matrix

The expression that obtained the distribution of grid points in the Chebyshev differentiation matrix that given in Eq. (3.4) will be created by following the solution scheme outlined below.

To establish the solution procedure, consider a vector v defined at the grid points. For the same grid points the discrete derivative ω of the vector v can be obtained in two steps (Trefethen, 2000):

- If an interpolation polynomial of degree $p \leq N$ is selected as $p(x_j) = v_j$ for $0 \leq j \leq N$.
- Derivative at the grid points will be $\omega_j = p'(x_j)$ for $0 \leq j \leq N$.

Since this process is linear, the derivative ω at the grid points can be represented as the product of v and a matrix of dimensions $(N+1) \times (N+1)$ denoted as D_N :

$$\omega = D_N v \quad 3.5$$

Here, N is a positive integer. There are no limitations on the integer N to be odd or even for the PCM. Grid points and differentiation matrix are established as discussed above and to understand the obtaining differentiation matrix, first order and second order Lagrange Interpolations are explained (Kırköse, 2019).

In order to obtain first order Lagrange interpolation, two points are required, and the Eq. (3.5) is utilized to find $x_0 = -1$ and $x_1 = 1$. Suppose these two points and their corresponding values are namely (x_0, v_0) and (x_1, v_1) . The first order Lagrange interpolation polynomial (Eker, 2020):

$$p(x) = \frac{(x-x_1)}{(x_0-x_1)} v_0 + \frac{(x-x_0)}{(x_1-x_0)} v_1 \quad 3.6$$

The grid points $x_0 = -1$ and $x_1 = 1$ for $N=1$ have substituted into the Lagrange interpolation polynomial:

$$p(x) = \frac{1}{2}(1+x)v_0 + \frac{1}{2}(1-x)v_1 \quad 3.7$$

Taking the derivative of the $p(x)$ interpolation polynomial:

$$w_j = p'(x) = \frac{1}{2}v_0 + \frac{1}{2}v_1 \quad 3.8$$

Derivative values for the obtained two points $x_0 = -1$ and $x_1 = 1$:

$$\begin{aligned} j = 0; w_0 &= w'(x_0) = p'(1) = \frac{1}{2}v_0 - \frac{1}{2}v_1 \\ j = 1; w_1 &= w'(x_1) = p'(-1) = \frac{1}{2}v_0 + \frac{1}{2}v_1 \end{aligned} \quad 3.9$$

Hence, differentiation of the first order Lagrange interpolation polynomial can be substituted into matrix formation:

$$\begin{bmatrix} w_0 \\ w_1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} v_0 \\ v_1 \end{bmatrix} \quad 3.10$$

In the case of obtaining the unknown vector, the coefficient matrix is 2 x 2 Chebyshev differentiation matrix:

$$D_1 = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix} \quad 3.11$$

In order to obtain second order Lagrange interpolation, three points are required, and the Eq. (3.5) is utilized to find $x_0 = -1$, $x_1 = 0$ and $x_2 = 1$. Suppose these three points and their corresponding values are namely (x_0, v_0) , (x_1, v_1) and (x_2, v_2) . The second order Lagrange interpolation polynomial (Eker, 2020):

$$p(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} v_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_2-x_0)} v_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} v_2 \quad 3.12$$

The grid points $x_0 = -1$, $x_1 = 0$ and $x_2 = 1$ for $N=2$ have substituted into the Lagrange interpolation polynomial:

$$p(x) = \frac{x(x+1)}{2} v_0 + \frac{(x-1)(x+1)}{-1} v_1 + \frac{x(x-1)}{2} v_2 \quad 3.13$$

Taking the derivative of the $p(x)$ interpolation polynomial:

$$w_j = p'(x) = \frac{2x+1}{2} v_0 + 2v_1 + \frac{2x-1}{2} v_2 \quad 3.14$$

Derivative values for the obtained three points $x_0 = -1$, $x_1 = 0$ and $x_2 = 1$:

$$j = 0; w_0 = p'(x_0) = p'(-1) = \frac{3}{2} v_0 - 2v_1 + \frac{1}{2} v_2 \quad 3.15$$

$$j = 1; w_1 = p'(x_1) = p'(0) = \frac{1}{2} v_0 - \frac{1}{2} v_2 \quad 3.16$$

$$j = 2; w_2 = p'(x_2) = p'(-1) = \frac{1}{2}v_0 + 2v_1 - \frac{3}{2}v_2 \quad 3.17$$

Hence, differentiation of the first order Lagrange interpolation polynomial can be substituted into matrix formation:

$$\begin{bmatrix} w_0 \\ w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & -2 & \frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{1}{2} & 2 & -\frac{3}{2} \end{bmatrix} \begin{bmatrix} v_0 \\ v_1 \\ v_2 \end{bmatrix} \quad 3.18$$

In the case of obtaining the unknown vector, the coefficient matrix is 3 x 3 Chebyshev differentiation matrix:

$$D_2 = \begin{bmatrix} \frac{3}{2} & -2 & \frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{1}{2} & 2 & -\frac{3}{2} \end{bmatrix} \quad 3.19$$

Henceforth, differentiation matrix can be obtained generally for any number of N points.

For $N \geq 1$ interpolation point $p(x_0, v_0)$, $p(x_1, v_1)$ and $p(x_N, v_N)$ and the Chebyshev differentiation matrix D_N in its most general form of size $(N+1) \times (N+1)$. Consider the rows and columns be indexed from 0 to N.

$$(D_N)_{00} = \frac{2N^2+1}{6} \quad 3.20$$

$$(D_N)_{jj} = \frac{-x_j}{2(1-x_j^2)}, \quad j = 1, 2, \dots, N-1 \quad 3.21$$

$$(D_N)_{ij} = \frac{c_i(-1)^{i+j}}{c_j(x_i-x_j)}, \quad i \neq j, \quad i, j = 1, 2, \dots, N-1 \quad 3.22$$

$$(D_N)_{NN} = -\frac{2N^2+1}{6} \quad 3.23$$

And c_i can be defined as:

$$c_i = \begin{cases} 2, & i = 0 \text{ or } i = N \\ 1, & \text{other situations} \end{cases} \quad 3.24$$

With this approach, a general relation for the differentiation matrix defined at any number N points of discrete points has been demonstrated. The differentiation matrix that applied to the discrete unknown vector $v = [v_0, v_1, \dots, v_N]^T$, results in $v''(x_j) = (Dv_j)$. Applying the matrix to $v' = [v'_0, v'_1, \dots, v'_N]$ produces $v''(x_j) = (D(D(v_j)v_j))$. This process provides a precise approximation of derivatives to the desired degree.

3.3. Temperature Distribution and Thermal Stresses of The Annular Fin

To determine the expression that describes heat conduction and temperature distribution in a fin, it is crucial first to identify the type of heat transfer occurring within the fin. Additionally, the energy generation or consumption within the fin must also be taken into account.

The energy accumulated within the annular fin is calculated based on the energy entering the annular fin, the energy exiting the annular fin, the energy generated within the annular fin and the energy consumed within the annular fin.

The most general way to approach energy equilibrium equation based on the first rule of thermodynamics is shown in the below (Çengel, 2017):

$$\begin{aligned} \left(\begin{array}{c} \text{The change of the} \\ \text{energy of the fin} \end{array} \right) &= \left(\begin{array}{c} \text{Heat transfer} \\ \text{into the fin} \end{array} \right) - \left(\begin{array}{c} \text{Heat transfer} \\ \text{out of the fin} \end{array} \right) \\ + \left(\begin{array}{c} \text{Heat generation} \\ \text{inside the fin} \end{array} \right) &- \left(\begin{array}{c} \text{Heat loss} \\ \text{inside the fin} \end{array} \right) \end{aligned} \quad 3.25$$

Energy entering the fin is transferred with convection, while energy leaving the fin occurs through conduction and radiation from the fin surface. Consequently, the energy equilibrium Eq. (3.25) can be reformulated per a unit time using symbolic terms.

$$\dot{Q}_{conduction} + \dot{Q}_u - \dot{Q}_{convection} - \dot{Q}_{radition} - \dot{Q}_t = \frac{\Delta E}{\Delta t} \quad 3.26$$

Here, $\dot{Q}_u$ represents rate of heat generation inside the fin per unit time while $\dot{Q}_t$ rate of heat loss inside the fin per unit time.

In order to simplify the energy equilibrium Eq. (3.26) for the annular fin, some assumptions are made (Gardner, 1945; Ullmann and Kalman, 1989):

1. The fin is axially symmetric
2. Thickness of the annular fin is very small compared to radius of the fin
3. There is only convection heat transfer on the fin surface, radiation heat transfer is neglected,
4. Ambient temperature and heat transfer coefficient are constant,
5. There is no time dependent temperature change at any point of the fin, the fin has reached a steady state situation,
6. There is no heat loss or heat production in the fin.

In addition to the mentioned assumptions in the literature, there is another assumption that utilized in this thesis.

7. The thickness of the fin depends on the diameter of the fin.

According to assumptions (3), (5) and (6), the Eq. (3.26) is rearranged:

$$\dot{Q}_{conduction} - \dot{Q}_{convection} = 0 \quad 3.27$$

Upon the application of Eq. (3.27) to a 3-D annular fin:

$$\dot{Q}_{conduction} = \dot{Q}_r - \dot{Q}_{r+\Delta r} + \dot{Q}_\phi - \dot{Q}_{\phi+\Delta\phi} + \dot{Q}_z - \dot{Q}_{z+\Delta z} \quad 3.28$$

According to assumption (1), the axial symmetry of the fin, leads the Eq. (3.28) independent from tangential axis and according to assumption (2), heat conduction in the z direction is negligible:

$$\dot{Q}_{conduction} = \dot{Q}_r - \dot{Q}_{r+\Delta r} \quad 3.29$$

Assumption (2) leads heat transfer through the tip of the fin is negligible due to the area of the fin's tip is too small. Therefore, heat conduction only occurs at the lateral faces of the fin:

$$\dot{Q}_{convection} = h A_{convection} (T - T_\infty) \quad 3.30$$

Where h is the heat transfer ratio. Eq. (3.27) can be rearranged with using Eq. (3.28 - 3.30):

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} - h A_{convection} (T - T_\infty) = 0 \quad 3.31$$

In the figure 3.2, variable cross-sectional annular fin that studied in this thesis has been shown and here, $A(r)$ is the thickness of the annular fin.

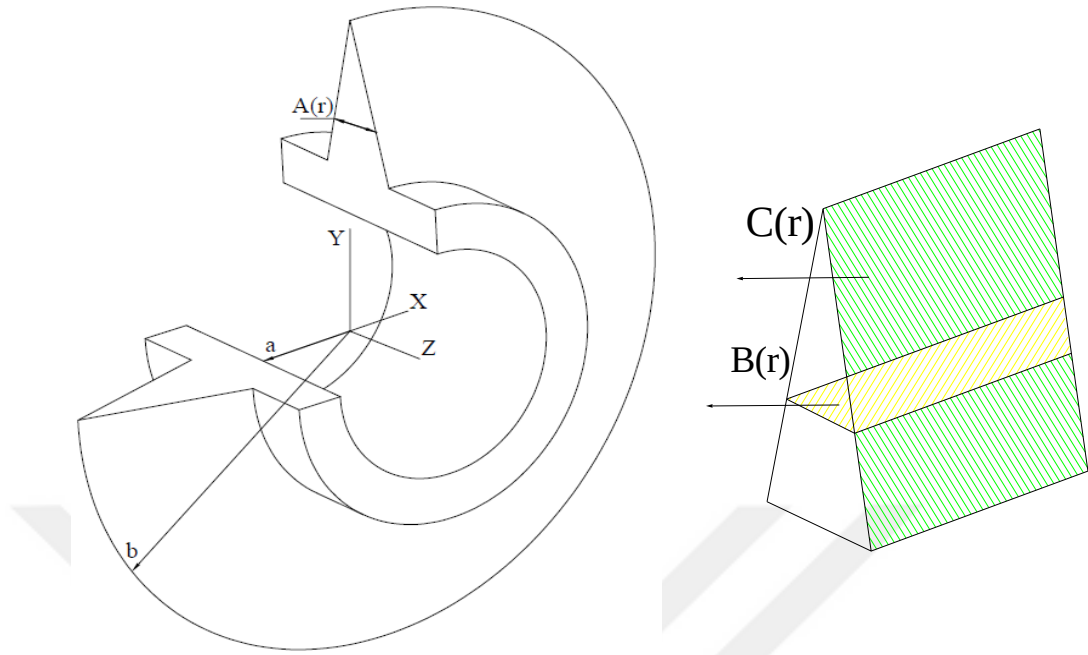


Figure 3.2. Annular fin with various cross-section

The lateral areas are the regions where convection heat transfer takes place:

$$A_{convection} = 2C(r), \quad C(r) = 2\pi \left(r + \frac{\Delta r}{2} \right) \quad 3.32$$

Since the heat conduction occurs on the two lateral surfaces, the heat convection area is the twice the value of lateral surface area and lateral surface can be written in terms of radial surface area and thickness:

$$C(r) = \frac{B(r)}{A(r)} \Delta r \quad 3.33$$

Hence, the heat conduction area can be rewritten in terms of radial surface area and thickness:

$$A_{convection} = \frac{2B(r)\Delta r}{A(r)} \quad 3.34$$

Thickness is varying along the radial axis. $A(r)$ represents the thickness and is a function of the radius of the fin. Since, thickness of the tip of the fin can't be zero, M has been appointed as a thickness of the tip, and y_b is the thickness at the base of the fin:

$$\begin{aligned} r = a & \rightarrow A(r) = y_b \\ r = b & \rightarrow A(r) = M \end{aligned} \quad 3.35$$

$$A(r) = \frac{(r-b)(y_b-M)}{(a-b)} + M \quad 3.36$$

Substituting Eq. (3.28) into the Eq. (3.25):

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} - h \frac{2*B(r)*\Delta r}{A(r)} (T - T_\infty) = 0 \quad 3.37$$

Both sides of the Eq. (3.31) are multiplied by $\frac{1}{B(r)\Delta r}$ for simplicity:

$$\frac{1}{B(r)} \frac{\dot{Q}_r - \dot{Q}_{r+\Delta r}}{\Delta r} - h \frac{2}{A(r)} (T - T_\infty) = 0 \quad 3.38$$

When Δr is considered to be infinitesimally small, subsequently the Eq. (3.38) becomes:

$$\frac{1}{B(r)} \frac{d\dot{Q}_r}{dr} - h \frac{2}{A(r)} (T - T_\infty) = 0 \quad 3.39$$

According to Fourier's law of heat conduction, the heat conduction in radial direction:

$$\dot{Q}_r = kB(r) \frac{dT}{dr} \quad 3.40$$

Where k is the thermal conductivity.

Thus, energy Eq. (3.39) is rearranged by incorporating inner radius “a” and outer radius “b”:

$$\frac{k}{rA(r)} \frac{d}{dr} \left(rA(r) \frac{dT}{dr} \right) - \frac{2h}{A(r)} (T - T_\infty) = 0 \quad 3.41$$

Upon expansion of the derivatives in Eq. (3.41):

$$\frac{k}{rA(r)} \left[[r'A(r) + rA'(r)] \frac{dT}{dr} + rA(r) \frac{d^2T}{dr^2} \right] - \frac{2h}{A(r)} (T - T_\infty) = 0 \quad 3.42$$

$$\left(\frac{k}{r} + \frac{k}{A(r)} \frac{dA(r)}{dr} \right) \frac{dT}{dr} + \frac{k*d^2T}{dr^2} - \frac{2h}{A(r)} (T - T_\infty) = 0 \quad 3.43$$

By multiplying with $\frac{1}{k}$, Eq. (3.43) derived as:

$$\frac{d^2T}{dr^2} + \left(\frac{1}{r} + \frac{1}{A(r)} \frac{dA(r)}{dr} \right) \frac{dT}{dr} - \frac{2h}{A(r)k} (T - T_\infty) = 0 \quad 3.44$$

The analysis of energy equations in an annular fin provides values related to the temperature distribution. Due to the variation in temperature distributions on the surface of the fin, thermal stresses develop in the fin. These thermal stresses have significant importance for the prediction of potential stress-related errors during the design of the fin.

In order to determine the thermal stresses, in addition to the assumptions used in determining the temperature distributions, the following assumptions are made (Yıldırım, 2018):

8. No internal forces in the fin,
9. There is no traction on the fin surfaces

The strains that occurs in the axis of a three-dimensional annular fin depend on the stresses in these axes and the temperature difference (Timoshenko, 1970):

$$\begin{aligned} \varepsilon_r &= \varepsilon_r^e + \varepsilon_r^T \\ \varepsilon_\theta &= \varepsilon_\theta^e + \varepsilon_\theta^T \\ \varepsilon_z &= \varepsilon_z^e + \varepsilon_\theta^T \end{aligned} \quad 3.45$$

Here, the expressions ε_r^T , ε_θ^T , ε_z^T indicate the strains that occur with the effect of temperature. The expressions ε_r^e , ε_θ^e , ε_z^e indicate the strains that occur as a result of the stresses in the specified directions. Thermal stresses resulting from the temperature changes are expressed as the multiplication of thermal expansion coefficient and the temperature.

$$\varepsilon_r^T = \varepsilon_\theta^T = \varepsilon_z^T = \alpha T \quad 3.46$$

Stresses in the radial direction lead to elongation in the radial direction while in the tangential direction contraction occurs related to the poisson's ratio. These are described with Hooke's Law:

$$\varepsilon_r^e = \frac{\sigma_r}{E} \quad 3.47$$

$$\varepsilon_\theta^e = -\nu \frac{\sigma_r}{E} \quad 3.48$$

Similarly, stresses in the tangential direction lead to elongation in the tangential direction while in the radial direction contraction occurs related to the poisson's ratio. These are described with Hooke's Law:

$$\varepsilon_{\phi}^e = \frac{\sigma_{\phi}}{E} \quad 3.49$$

$$\varepsilon_r^e = -\nu \frac{\sigma_{\phi}}{E} \quad 3.50$$

Hence, strains along the radial and tangential directions related to stresses along these directions:

$$\varepsilon_r^e = \frac{\sigma_r}{E} - \nu \frac{\sigma_{\phi}}{E} = \frac{1}{E} (\sigma_r - \nu \sigma_{\phi}) \quad 3.51$$

$$\varepsilon_{\phi}^e = \frac{\sigma_{\phi}}{E} - \nu \frac{\sigma_r}{E} = \frac{1}{E} (\sigma_{\phi} - \nu \sigma_r) \quad 3.52$$

The equations modelling the stress-strain-temperature relationship, stress equilibrium equation, strain and boundary conditions for an annular fin with axial symmetry which is assumed to have no internal forces on the fin and no traction on its surfaces are given below (Yayla, 2014):

Stress-strain-temperature relationship:

$$\varepsilon_r = \frac{1}{E} (\sigma_r - \nu \sigma_{\phi}) + \alpha T \quad 3.53$$

$$\varepsilon_{\phi} = \frac{1}{E} (\sigma_{\phi} - \nu \sigma_r) + \alpha T \quad 3.54$$

Or

$$\sigma_r = \frac{E}{1-\nu^2} (\varepsilon_r + \nu \varepsilon_{\phi} - (1 + \nu) \alpha T) \quad 3.55$$

$$\sigma_{\phi} = \frac{E}{1-\nu^2} (\varepsilon_{\phi} + \nu \varepsilon_r - (1 + \nu) \alpha T) \quad 3.56$$

The correlations radial and tangential stresses (σ_r, σ_{ϕ}), shear stresses ($\tau_{r\phi}, \tau_{\phi r}$) and the internal forces (R_r, R_{ϕ}) are determined by analyzing a differential element that extracted from the annular fin between the r and $r + \Delta r$ as illustrated in the figure 3.3. Since the thickness varies along the radial axis, it changes along the r -direction in the examined differential element (Yayla, 2014).

At the starting point of the element, thickness represented as $A(r)_r$ while at a distance Δr from the starting point it is represented as $A(r)_{r+\Delta r}$. In the region between these two boundary points, the thickness is assumed to be the average of these two boundary values. The correlations are illustrated below:

$$A(r)_r = \frac{(r-b)(y_b-M)}{(a-b)} + M \quad 3.57$$

$$A(r)_{r+\Delta r} = \frac{(r+\Delta r-b)(y_b-M)}{(a-b)} + M \quad 3.58$$

$$A(r)_{avg} = \frac{A(r) + A(r+\Delta r)}{2} \quad 3.59$$

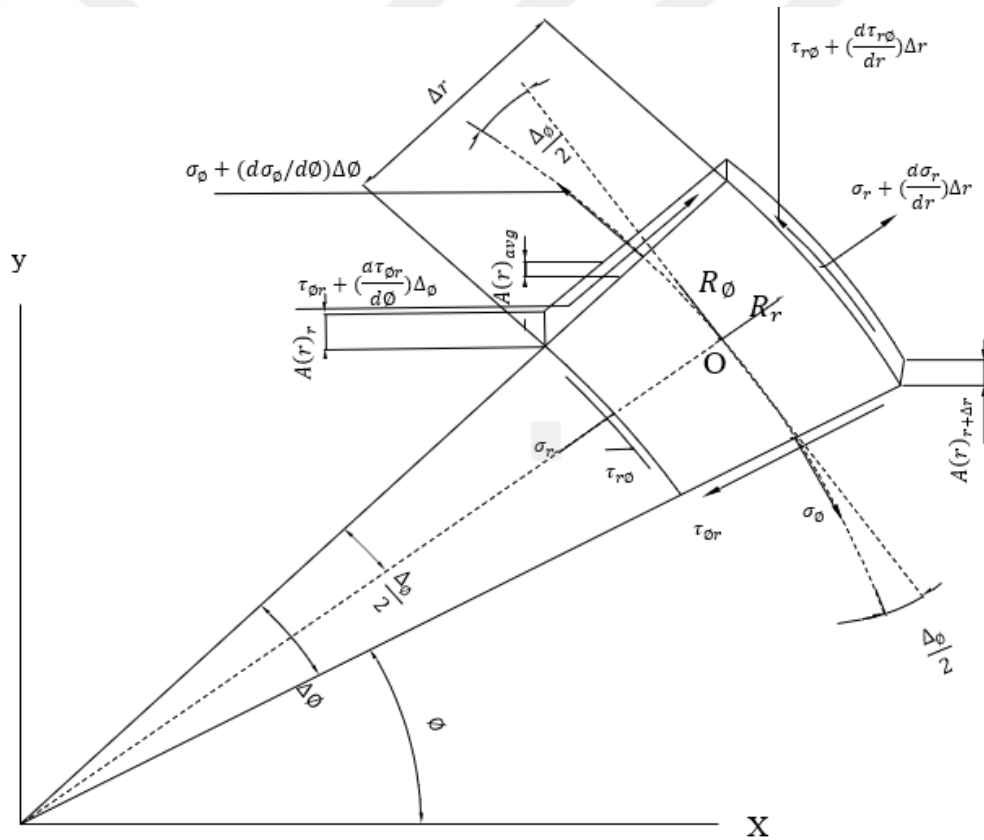


Figure 3.3. Radial and shear stresses of a differential element

The radial stress σ_r that occurred at any distance of r from the center is equal to the stress $\sigma_{r+\Delta r}$ at an infinitesimally distance Δr away from this point. The amount of change in the stress in the r direction is as follows (Yayla, 2014):

$$\lim_{\Delta r \rightarrow 0} \frac{\sigma_{r+\Delta r} - \sigma_r}{\Delta r} = \frac{d\sigma_r}{dr} \quad 3.60$$

Hence, radial stress at a distance of Δr , defined as:

$$\sigma_{r+\Delta r} = \sigma_r + \frac{d\sigma_r}{dr} \Delta r \quad 3.61$$

Similarly, shear stress at a distance of Δr , defined as:

$$\tau_{r\phi+\Delta r} = \tau_{r\phi} + \frac{d\tau_{r\phi}}{dr} \Delta r \quad 3.62$$

In the tangential direction according to explanation above, radial and shear stresses after an infinitesimally change of angle $\Delta\phi$ are defined as follows:

$$\sigma_{\phi+\Delta\phi} = \sigma_{\phi} + \frac{d\sigma_{\phi}}{d\phi} \Delta\phi \quad 3.63$$

And

$$\tau_{\phi r+\Delta\phi} = \tau_{\phi r} + \frac{d\tau_{\phi r}}{d\phi} \Delta\phi \quad 3.64$$

Stresses acting on any point of the annular fin are represented as the force per unit area and can be obtained as follows:

$$\sigma = \frac{F}{A}, \quad \tau = \frac{F}{A} \quad 3.65$$

And the forces that caused stresses are derived as:

$$F = \sigma A, \quad F = \tau A \quad 3.66$$

The element depicted in the figure 3.3 is in a steady state situation where the resultant force acting on that element in the radial direction equals zero:

$$\sum F_r = 0 \quad 3.67$$

$$\begin{aligned} \sum F_r = & \left(\sigma_r + \frac{d\sigma_r}{dr} * \Delta r \right) (r + \Delta r) \Delta\phi A(r)_{r+\Delta r} + \left(\tau_{\phi r} + \frac{d\tau_{\phi r}}{d\phi} \Delta\phi \right) \Delta r A(r)_{avg} \cos \frac{\Delta\phi}{2} + \\ & R_r \frac{(r+r+\Delta r) \Delta\phi}{2} * \Delta r A(r)_{avg} - \sigma_r r \Delta\phi A(r)_r - \sigma_{\phi} \Delta r A(r)_{avg} \sin \frac{\Delta\phi}{2} - \left(\sigma_{\phi} + \right. \\ & \left. \frac{d\sigma_{\phi}}{d\phi} \Delta\phi \right) \Delta r A(r)_{avg} \sin \frac{\Delta\phi}{2} - \tau_{\phi r} \Delta r A(r)_{avg} \cos \frac{\Delta\phi}{2} = 0 \end{aligned} \quad 3.68$$

Expanding the parentheses in the Eq. (3.68) yields:

$$\begin{aligned}\Sigma F_r = & \sigma_r(r + \Delta r)\Delta\phi A(r)_{r+\Delta r} + \frac{d\sigma_r}{dr}\Delta r(r + \Delta r)\Delta\phi A(r)_{r+\Delta r} + \tau_{\phi r}\Delta r A(r)_{avg} \cos \frac{\Delta\phi}{2} + \\ & \frac{d\tau_{\phi r}}{d\phi}\Delta\phi\Delta r A(r)_{avg} \cos \frac{\Delta\phi}{2} + R_r \frac{(r+\Delta r)\Delta\phi}{2}\Delta r A(r)_{avg} - \sigma_r r \Delta\phi A(r)_r - \\ & \sigma_{\phi}\Delta r A(r)_{avg} \sin \frac{\Delta\phi}{2} - \sigma_{\phi}\Delta r A(r)_{avg} \sin \frac{\Delta\phi}{2} - \frac{d\sigma_{\phi}}{d\phi}\Delta\phi\Delta r A(r)_{avg} \sin \frac{\Delta\phi}{2} - \\ & \tau_{\phi r}\Delta r A(r)_{avg} \cos \frac{\Delta\phi}{2} = 0\end{aligned}\quad 3.69$$

Since $\Delta\phi$ is infinitesimally small, $\sin \frac{\Delta\phi}{2}$ is approximately equals to $\frac{\Delta\phi}{2}$ and $\cos \frac{\Delta\phi}{2}$ is approximately equals to 1, and thus the Eq. (3.69) yields as follows:

$$\begin{aligned}\Sigma F_r = & \sigma_r(r + \Delta r)\Delta\phi A(r)_{r+\Delta r} + \frac{d\sigma_r}{dr}\Delta r(r + \Delta r)\Delta\phi A(r)_{r+\Delta r} + \frac{d\tau_{\phi r}}{d\phi}\Delta\phi\Delta r A(r)_{avg} + \\ & R_r \frac{(2r+\Delta r)\Delta\phi}{2}\Delta r A(r)_{avg} - \sigma_r r \Delta\phi A(r)_r - \sigma_{\phi}\Delta r A(r)_{avg}\Delta\phi - \frac{d\sigma_{\phi}}{d\phi}\Delta\phi\Delta r A(r)_{avg} \frac{\Delta\phi}{2} = 0\end{aligned}\quad 3.70$$

Following the rearrangement of the Eq. (3.70):

$$\begin{aligned}\Sigma F_r = & \sigma_r r \Delta\phi (A(r)_{r+\Delta r} - A(r)_r) + \sigma_r \Delta\phi \Delta r A(r)_{r+\Delta r} + \frac{d\sigma_r}{dr}\Delta r(r + \Delta r)\Delta\phi A(r)_{r+\Delta r} + \\ & \frac{d\tau_{\phi r}}{d\phi}\Delta\phi\Delta r \left(\frac{A(r)_{r+\Delta r} + A(r)_r}{2} \right) + R_r \frac{(2r+\Delta r)\Delta\phi}{2}\Delta r \left(\frac{A(r)_{r+\Delta r} + A(r)_r}{2} \right) - \\ & \Delta\phi\Delta r \left(\frac{A(r)_{r+\Delta r} + A(r)_r}{2} \right) \Delta\phi - \frac{d\sigma_{\phi}}{d\phi}\Delta\phi\Delta r \left(\frac{A(r)_{r+\Delta r} + A(r)_r}{2} \right) \frac{\Delta\phi}{2} = 0\end{aligned}\quad 3.71$$

Multiplying with $\frac{1}{\Delta r \Delta\phi}$ both sides of the Eq. (3.71) yields

$$\begin{aligned}\Sigma F_r = & \sigma_r r \left(\frac{A(r)_{r+\Delta r} - A(r)_r}{\Delta r} \right) + \sigma_r A(r)_{r+\Delta r} + \frac{d\sigma_r}{dr}(r + \Delta r)A(r)_{r+\Delta r} + \\ & \frac{d\tau_{\phi r}}{d\phi} \left(\frac{A(r)_{r+\Delta r} + A(r)_r}{2} \right) + R_r \frac{(2r+\Delta r)}{2} \left(\frac{A(r)_{r+\Delta r} + A(r)_r}{2} \right) - \sigma_{\phi} \left(\frac{A(r)_{r+\Delta r} + A(r)_r}{2} \right) - \\ & \frac{d\sigma_{\phi}}{d\phi} \left(\frac{A(r)_{r+\Delta r} + A(r)_r}{2} \right) \frac{\Delta\phi}{2} = 0\end{aligned}\quad 3.72$$

Since $\Delta\phi$ and Δr are infinitesimally small, the limits of $\Delta\phi$ and Δr are approximately zero, consequently Eq. (3.72) is expressed as:

$$\Sigma F_r = \sigma_r r \frac{dA(r)_r}{dr} + \sigma_r A(r)_r + \frac{d\sigma_r}{dr} r A(r)_r + \frac{d\tau_{\phi r}}{d\phi} A(r)_r + R_r r A(r)_r - \sigma_{\phi} A(r)_r = 0 \quad 3.73$$

By multiplying with $\frac{1}{r A(r)_r}$ of both sides of the Eq. (3.73) derived as:

$$\sum F_r = \sigma_r \frac{dA(r)_r}{dr} \frac{1}{A(r)_r} + \frac{\sigma_r}{r} + \frac{d\sigma_r}{dr} + \frac{d\tau_{\emptyset r}}{d\emptyset} \frac{1}{r} + R_r - \frac{\sigma_{\emptyset}}{r} = 0 \quad 3.74$$

$$\sum F_r = \frac{d\tau_{\emptyset r}}{d\emptyset} \frac{1}{r} + \sigma_r \frac{dA(r)_r}{dr} \frac{1}{A(r)_r} + \frac{\sigma_r - \sigma_{\emptyset}}{r} + \frac{d\sigma_r}{dr} + R_r = 0 \quad 3.75$$

At the steady state situation, the resultant force on a differential element in the tangential direction equals zero.

$$\sum F_{\emptyset} = 0 \quad 3.76$$

$$\begin{aligned} \sum F_{\emptyset} = & \left(\sigma_{\emptyset} + \frac{d\sigma_{\emptyset}}{d\emptyset} \Delta\emptyset \right) \Delta r A(r)_{avg} \cos \frac{\Delta\emptyset}{2} + \left(\tau_{\emptyset r} + \frac{d\tau_{\emptyset r}}{d\emptyset} \Delta\emptyset \right) \Delta r A(r)_{avg} \sin \frac{\Delta\emptyset}{2} + \\ & \tau_{\emptyset r} \Delta r A(r)_{avg} \sin \frac{\Delta\emptyset}{2} + R_{\emptyset} \frac{(r+r+\Delta r)\Delta\emptyset}{2} \Delta r A(r)_{avg} + \left(\tau_{\emptyset r} + \frac{d\tau_{\emptyset r}}{dr} \Delta r \right) (r + \\ & \Delta r) \Delta\emptyset A(r)_{r+\Delta r} - \sigma_{\emptyset} \Delta r A(r)_{avg} \cos \frac{\Delta\emptyset}{2} - \tau_{\emptyset r} \Delta\emptyset r A(r)_r = 0 \end{aligned} \quad 3.77$$

Upon expanding the parentheses in Eq. (3.77) and given that $\Delta\emptyset$ is infinitesimally small, $\cos \frac{\Delta\emptyset}{2}$ and $\sin \frac{\Delta\emptyset}{2}$ can be approximated as 1 and $\frac{\Delta\emptyset}{2}$.

$$\begin{aligned} \sum F_{\emptyset} = & \sigma_{\emptyset} \Delta r \left(\frac{A(r)_{r+\Delta r} + A(r)_r}{2} \right) + \frac{d\sigma_{\emptyset}}{d\emptyset} \Delta\emptyset \Delta r \left(\frac{A(r)_{r+\Delta r} + A(r)_r}{2} \right) + \tau_{\emptyset r} \Delta r \left(\frac{A(r)_{r+\Delta r} + A(r)_r}{2} \right) \frac{\Delta\emptyset}{2} + \\ & \frac{d\tau_{\emptyset r}}{d\emptyset} \Delta\emptyset \Delta r \left(\frac{A(r)_{r+\Delta r} + A(r)_r}{2} \right) \frac{\Delta\emptyset}{2} + \tau_{\emptyset r} \Delta r \left(\frac{A(r)_{r+\Delta r} + A(r)_r}{2} \right) \frac{\Delta\emptyset}{2} + \\ & R_{\emptyset} \frac{(r+r+\Delta r)\Delta\emptyset}{2} \Delta r \left(\frac{A(r)_{r+\Delta r} + A(r)_r}{2} \right) + \tau_{\emptyset r} r \Delta\emptyset (A(r)_{r+\Delta r} - A(r)_r) + \frac{d\tau_{\emptyset r}}{dr} \Delta r (r + \\ & \Delta r) \Delta\emptyset A(r)_{r+\Delta r} - \sigma_{\emptyset} \Delta r \left(\frac{A(r)_{r+\Delta r} + A(r)_r}{2} \right) = 0 \end{aligned} \quad 3.78$$

Multiplying with $\frac{1}{\Delta r \Delta\emptyset}$ both sides of the Eq. (3.78) yields:

$$\begin{aligned} \sum F_{\emptyset} = & \frac{d\sigma_{\emptyset}}{d\emptyset} \left(\frac{A(r)_{r+\Delta r} + A(r)_r}{2} \right) + \tau_{\emptyset r} \left(\frac{A(r)_{r+\Delta r} + A(r)_r}{2} \right) + \frac{d\tau_{\emptyset r}}{d\emptyset} \left(\frac{A(r)_{r+\Delta r} + A(r)_r}{2} \right) \frac{\Delta\emptyset}{2} + \\ & R_{\emptyset} \frac{(r+r+\Delta r)}{2} \left(\frac{A(r)_{r+\Delta r} + A(r)_r}{2} \right) + \tau_{\emptyset r} r \left(\frac{A(r)_{r+\Delta r} - A(r)_r}{\Delta r} \right) + \frac{d\tau_{\emptyset r}}{dr} (r + \Delta r) A(r)_{r+\Delta r} = 0 \end{aligned} \quad 3.79$$

Since $\Delta\emptyset$ and Δr are infinitesimally, the limits of $\Delta\emptyset$ and Δr are approximately zero, consequently Eq. (3.79) is expressed as:

$$\sum F_{\emptyset} = \frac{d\sigma_{\emptyset}}{d\emptyset} A(r)_r + \tau_{\emptyset r} A(r)_r + R_{\emptyset} r A(r)_r + \tau_{\emptyset r} r \frac{dA(r)}{dr} + \frac{d\tau_{\emptyset r}}{dr} r A(r)_r = 0 \quad 3.80$$

Multiplying with $\frac{1}{r A(r)_r}$ both sides of the Eq. (3.80) yields:

$$\sum F_{\emptyset} = \frac{d\sigma_{\emptyset}}{d\emptyset} \frac{1}{r} + \tau_{\emptyset r} \frac{1}{r} + R_{\emptyset} + \tau_{r\emptyset} \frac{dA(r)}{dr} \frac{1}{A(r)_r} + \frac{d\tau_{r\emptyset}}{dr} = 0 \quad 3.81$$

At the steady state situation, the resultant moment acting on point “O” which illustrated in the figure (3.81) equals zero:

$$\begin{aligned} \sum M_O &= \left(\tau_{r\emptyset} + \frac{d\tau_{r\emptyset}}{dr} \Delta r \right) (r + \Delta r) \Delta \emptyset A(r)_{r+\Delta r} \frac{\Delta r}{2} + \tau_{r\emptyset} r \Delta \emptyset A(r)_r \frac{\Delta r}{2} \\ &- \left(\tau_{\emptyset r} + \frac{d\tau_{\emptyset r}}{d\emptyset} \Delta \emptyset \right) \Delta r A(r)_{avg} \left(r + \frac{\Delta r}{2} \right) \frac{\Delta \emptyset}{2} \\ &- \tau_{\emptyset r} \Delta r A(r)_{avg} \left(r + \frac{\Delta r}{2} \right) \frac{\Delta \emptyset}{2} = 0 \end{aligned} \quad 3.82$$

Subsequently expanding the parentheses in the Eq. (3.82) yields following the rearrangement of Eq. (3.82):

$$\begin{aligned} \sum M_O &= \tau_{r\emptyset} (r + \Delta r) \Delta \emptyset A(r)_{r+\Delta r} \frac{\Delta r}{2} + \frac{d\tau_{r\emptyset}}{dr} \Delta r (r + \Delta r) \Delta \emptyset A(r)_{r+\Delta r} \frac{\Delta r}{2} + \\ &\tau_{r\emptyset} r \Delta \emptyset A(r)_r \frac{\Delta r}{2} - \tau_{\emptyset r} \Delta r \left(\frac{A(r)_{r+\Delta r} + A(r)_r}{2} \right) \left(r + \frac{\Delta r}{2} \right) \frac{\Delta \emptyset}{2} - \frac{d\tau_{\emptyset r}}{d\emptyset} \Delta \emptyset \Delta r \left(\frac{A(r)_{r+\Delta r} + A(r)_r}{2} \right) \left(r + \right. \\ &\left. \frac{\Delta r}{2} \right) \frac{\Delta \emptyset}{2} - \tau_{\emptyset r} \Delta r \left(\frac{A(r)_{r+\Delta r} + A(r)_r}{2} \right) \left(r + \frac{\Delta r}{2} \right) \frac{\Delta \emptyset}{2} = 0 \end{aligned} \quad 3.83$$

Multiplying with $\frac{1}{\Delta r \Delta \emptyset}$ both sides of the Eq. (3.83) yields:

$$\begin{aligned} \sum M_O &= \tau_{r\emptyset} (r + \Delta r) \frac{A(r)_{r+\Delta r}}{2} + \frac{d\tau_{r\emptyset}}{dr} (r + \Delta r) A(r)_{r+\Delta r} \frac{\Delta r}{2} + \tau_{r\emptyset} r \frac{A(r)_r}{2} \\ &- \tau_{\emptyset r} \left(\frac{A(r)_{r+\Delta r} + A(r)_r}{4} \right) \left(r + \frac{\Delta r}{2} \right) \\ &- \frac{d\tau_{\emptyset r}}{d\emptyset} \left(\frac{A(r)_{r+\Delta r} + A(r)_r}{2} \right) \left(r + \frac{\Delta r}{2} \right) \frac{\Delta \emptyset}{2} - \tau_{\emptyset r} \left(\frac{A(r)_{r+\Delta r} + A(r)_r}{4} \right) \\ &\left(r + \frac{\Delta r}{2} \right) = 0 \end{aligned} \quad 3.84$$

After accompanied by upon the $\Delta \emptyset$ and Δr are infinitesimally small, the limits of $\Delta \emptyset$ and Δr are approximately zero and thus Eq. (3.84) rearranged:

$$\sum M_O = \tau_{r\emptyset} r \frac{A(r)_r}{2} + \tau_{r\emptyset} r \frac{A(r)_r}{2} - \tau_{\emptyset r} \left(\frac{A(r)_r}{2} \right) r - \tau_{\emptyset r} \left(\frac{A(r)_r}{2} \right) r = 0 \quad 3.85$$

With the rearrangement of the Eq. (3.85), the equality of shear stresses has achieved, respectively:

$$\tau_{r\phi} = \tau_{\phi r} \quad 3.86$$

Thus, these two shear stresses are equal each other, $\tau_{\phi r}$ can be replaced with $\tau_{r\phi}$ in the equilibrium Eq. in radial direction and tangential direction:

$$\frac{d\sigma_\phi}{d\phi} \frac{1}{r} + \tau_{r\phi} \frac{1}{r} + R_\phi + \tau_{r\phi} \frac{dA(r)}{dr} \frac{1}{A(r)_r} + \frac{d\tau_{r\phi}}{dr} = 0 \quad 3.87$$

$$\frac{d\tau_{r\phi}}{d\phi} \frac{1}{r} + \sigma_r \frac{dA(r)_r}{dr} \frac{1}{A(r)_r} + \frac{\sigma_r - \sigma_\phi}{r} + \frac{d\sigma_r}{dr} + R_r = 0 \quad 3.88$$

In line with the assumptions that made, the annular fin is symmetrical, and thus, stresses are not related to ϕ and also not a function of radius. Therefore, $\frac{d\sigma_\phi}{d\phi}$ and $\tau_{r\phi}$ are zero. Also, according to assumption (1), there are no internal stresses in the annular fin, R_r and R_ϕ are equal zero. So, all the terms of the equilibrium equation in tangential direction are equal zero, and thus, equilibrium equations yield only in radial direction.

$$\frac{dA(r)_r}{dr} \frac{\sigma_r}{A(r)_r} + \frac{\sigma_r - \sigma_\phi}{r} + \frac{d\sigma_r}{dr} = 0 \quad 3.89$$

The deformations in the element between r and Δ_r that subjected to stresses are illustrated in the figure 3.4. $A'B'C'D'$ element represents the shape of the deformed $ABCD$ element after exposed the stresses (Yayla, 2014).

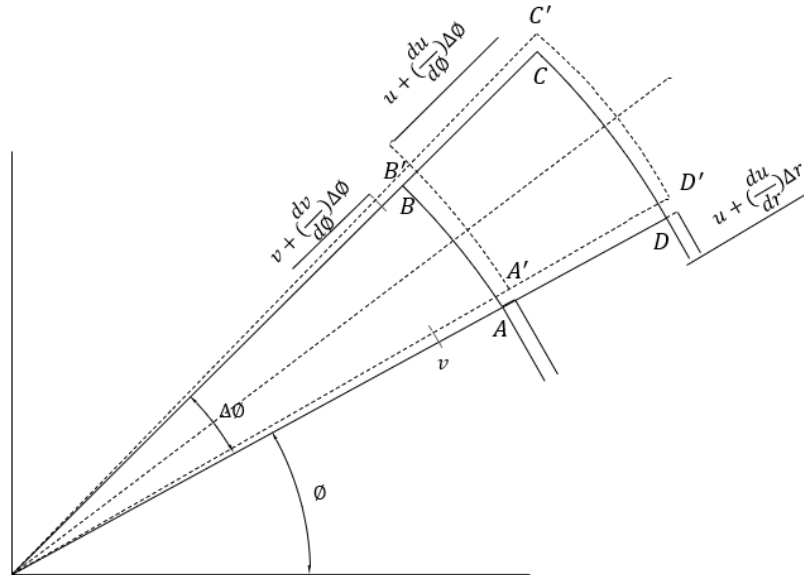


Figure 3.4. Radial and tangential strains of a differential element

As seen in the figure 4.3 u_r is the deformation that results from the stresses acting on the surface AB while $u_{r+\Delta r}$ is deformation at the surface DC .

Deformation rate in the radial direction:

$$\frac{u_{r+\Delta r}-u_r}{\Delta r} \quad 3.90$$

Since Δr is infinitesimally small, strain in the radial direction yields accompanied by limitation of Eq. (3.90):

$$\lim_{\Delta r \rightarrow 0} \frac{u_{r+\Delta r}-u_r}{\Delta r} = \frac{du}{dr} \quad 3.91$$

$$\varepsilon_r = \frac{du}{dr} \quad 3.92$$

Radial deformation also yields on deformation in the tangential direction. Referring to figure 3.4, if it is considered that only radial deformation occurs along the AB segment, the length of the spring AB equals $r\Delta\phi$ while $A'B'$ segment results as $(r+u)\Delta\phi$.

$$\frac{(r+u)\Delta\phi-r\Delta\phi}{r\Delta\phi} = \frac{r\Delta\phi+u\Delta\phi-r\Delta\phi}{r\Delta\phi} = \frac{u\Delta\phi}{\Delta\phi} = \frac{u}{r} \quad 3.93$$

Strain in the tangential direction can be described in a manner analogous to that used for calculating strain in the radial direction:

$$\varepsilon_\phi = \frac{dv}{d\phi} \quad 3.94$$

Thus, total deformation in the tangential direction:

$$\varepsilon_\phi = \frac{dv}{d\phi} + \frac{u}{r} \quad 3.95$$

With accompanied by assumption (1) the described while achieving the energy equilibrium equations., the annular fin is axially symmetric. Therefore, strain is not related to angle ϕ :

$$\varepsilon_\phi = \frac{u}{r} \quad 3.96$$

As respect to assumption (2) which there is no traction on the fin surface that stated while achieving the equilibrium Eq. (3.89). Boundary conditions of the annular fin has become:

$$\begin{aligned}
\sigma_r &= 0 & r &= a \\
\sigma_r &= 0 & r &= b
\end{aligned} \tag{3.97}$$

Stress-strain-temperature relationships are modified by incorporating Eq. (3.55 - 3.56) into Eq. (3.89).

$$\begin{aligned}
& \frac{1}{A(r)_r} \left(\frac{E}{1-v^2} (\varepsilon_r + v\varepsilon_\theta - (1+v)aT) A'(r)_r \right. \\
& + \frac{\frac{E}{1-v^2} (\varepsilon_r + v\varepsilon_\theta - (1+v)aT) - \frac{E}{1-v^2} (\varepsilon_\theta + v\varepsilon_r - (1+v)aT)}{r} \\
& \left. + \frac{d}{dr} \left[\frac{E}{1-v^2} (\varepsilon_r + v\varepsilon_\theta - (1+v)aT) \right] \right) = 0
\end{aligned} \tag{3.98}$$

Upon grouped into a common parenthesis same expression in the Eq. (3.98):

$$\begin{aligned}
& \frac{A'(r)_r}{A(r)_r} \frac{E}{1-v^2} \left(\frac{du}{dr} + v \frac{u}{r} - (1+v)aT \right) \\
& + \frac{1}{r} \frac{E}{1-v^2} \left[\left(\frac{du}{dr} + v \frac{u}{r} - (1+v)aT \right) - \left(\frac{u}{r} + v \frac{du}{dr} - (1+v)aT \right) \right] \\
& + \frac{d}{dr} \frac{E}{1-v^2} \left(\frac{du}{dr} + v \frac{u}{r} - (1+v)aT \right) = 0
\end{aligned} \tag{3.99}$$

As inserting the expression $\frac{E}{1-v^2}$ into the parenthesis:

$$\begin{aligned}
& \frac{A'(r)_r}{A(r)_r} \frac{E}{1-v^2} \left(\frac{du}{dr} + v \frac{u}{r} - (1+v)aT \right) \\
& + \frac{1}{r} \frac{E}{1-v^2} \left[\left(\frac{du}{dr} - v \frac{du}{dr} + v \frac{u}{r} - \frac{u}{r} \right) \right] \\
& + \frac{d}{dr} \left[\frac{E}{1-v^2} * \frac{du}{dr} + \frac{E}{1-v^2} v \frac{u}{r} - \frac{E}{1-v^2} (1+v)aT \right] = 0
\end{aligned} \tag{3.100}$$

Upon grouped into a common parenthesis same expression in the Eq. (3.100):

$$\begin{aligned}
& \frac{A'(r)_r}{A(r)_r} \frac{E}{1-v^2} \left(\frac{du}{dr} + v \frac{u}{r} - (1+v)aT \right) \\
& + \frac{1}{r} \frac{E}{1-v^2} \left(\frac{du}{dr} - \frac{u}{r} \right) (1-v) \\
& + \frac{d}{dr} \left[\frac{E}{1-v^2} \frac{du}{dr} + \frac{E}{1-v^2} v \frac{u}{r} - \frac{E}{1-v^2} (1+v)aT \right] = 0
\end{aligned} \tag{3.101}$$

Instead of using $\frac{1}{r} \left(\frac{du}{dr} - \frac{u}{r} \right)$, $\frac{d}{dr} \left(\frac{u}{r} \right)$ utilized as its derivative form:

$$\begin{aligned}
& \frac{A'(r)_r}{A(r)_r} \frac{E}{1-v^2} \left(\frac{du}{dr} + v \frac{u}{r} - (1+v)aT \right) \\
& + \frac{E}{1-v^2} \frac{d}{dr} \left(\frac{u}{r} \right) * (1-v) \\
& + \frac{d}{dr} \left[\frac{E}{1-v^2} \frac{du}{dr} + \frac{E}{1-v^2} v \frac{u}{r} - \frac{E}{1-v^2} (1+v)aT \right] = 0
\end{aligned} \tag{3.102}$$

Derivative operation is performed and inserted the expression $\frac{E}{1-v^2} \frac{d}{dr} \left(\frac{u}{r} \right)$ inside the paranthesis:

$$\begin{aligned}
& \frac{A'(r)_r}{A(r)_r} \frac{E}{1-v^2} \left(\frac{du}{dr} + v \frac{u}{r} - (1+v)aT \right) \\
& + \frac{E}{1-v^2} \frac{d}{dr} \left(\frac{u}{r} \right) - \left(\frac{E}{1-v^2} \frac{d}{dr} \left(\frac{u}{r} \right) v \right) \\
& + \frac{d^2 u}{dr^2} \frac{E}{1-v^2} + \left[\frac{E}{1-v^2} v \frac{u}{r} \frac{d}{dr} \right] = 0
\end{aligned} \tag{3.103}$$

To rework the Eq. (3.103) derivative operation of $\frac{d}{dr} \left(\frac{u}{r} \right)$ may be written in differentially expanded form:

$$\begin{aligned}
& \frac{A'(r)_r}{A(r)_r} \frac{E}{1-v^2} \left(\frac{du}{dr} + v \frac{u}{r} - (1+v)aT \right) \\
& + \frac{E}{1-v^2} \frac{1}{r} \left(\frac{du}{dr} - \frac{u}{r} \right) + \frac{d^2 u}{dr^2} \frac{E}{1-v^2} = 0
\end{aligned} \tag{3.104}$$

Subsequently the expression $\frac{A'(r)_r}{A(r)_r} \frac{E}{1-v^2}$ distributed inside parentheses in the Eq. (3.104) yields following the rearrangement of Eq. (3.104):

$$\begin{aligned}
& \frac{A'(r)_r}{A(r)_r} \frac{E}{1-v^2} \frac{du}{dr} + \frac{A'(r)_r}{A(r)_r} \frac{E}{1-v^2} v \frac{u}{r} \\
& - \frac{A'(r)_r}{A(r)_r} \frac{E}{1-v^2} (1+v)aT + \frac{E}{1-v^2} \frac{1}{r} \left(\frac{du}{dr} - \frac{u}{r} \right) + \frac{d^2 u}{dr^2} \frac{E}{1-v^2} = 0
\end{aligned} \tag{3.105}$$

$\frac{E}{1-v^2} \frac{1}{r}$ is inserted inside parenthesis same as outlined above:

$$\begin{aligned}
& \frac{A'(r)_r}{A(r)_r} \frac{E}{1-v^2} \frac{du}{dr} + \frac{A'(r)_r}{A(r)_r} \frac{E}{1-v^2} v \frac{u}{r} \\
& - \frac{A'(r)_r}{A(r)_r} \frac{E}{1-v^2} (1+v)aT + \frac{E}{1-v^2} \frac{1}{r} \frac{du}{dr} - \frac{E}{1-v^2} \frac{1}{r} \frac{u}{r} + \frac{d^2 u}{dr^2} \frac{E}{1-v^2} = 0
\end{aligned} \tag{3.106}$$

Upon grouped into a common parenthesis same expression in the Eq. (3.106):

$$\begin{aligned}
& \frac{d^2 u}{dr^2} \frac{E}{1-v^2} + \frac{du}{dr} \left(\frac{A'(r)_r}{A(r)_r} \frac{E}{1-v^2} + \frac{E}{1-v^2} * \frac{1}{r} \right) \\
& - \frac{u}{r} \left(\frac{A'(r)_r}{A(r)_r} \frac{E}{1-v^2} v - \frac{E}{1-v^2} * \frac{1}{r} \right) \\
& - \frac{A'(r)_r}{A(r)_r} \frac{E}{1-v^2} (1+v) aT = 0
\end{aligned} \tag{3.107}$$

By relocating the temperature-dependent term to the right-hand side of the Eq. (3.107) and subsequently multiplying both sides of the Eq. (3.107) by $\frac{1-v^2}{E}$, the resulting equation can be reformulated. As a result of the final adjustments, the Eq. (3.107) is expressed in the form shown below, a second-order linear non-homogeneous differential equation is obtained:

$$\frac{d^2 u}{dr^2} + \frac{du}{dr} \left(\frac{A'(r)_r}{A(r)_r} + \frac{1}{r} \right) + \frac{u}{r} \left(\frac{A'(r)_r}{A(r)_r} v - \frac{1}{r} \right) = \frac{A'(r)_r}{A(r)_r} (1+v) aT + (1+v) a \frac{dT}{dr} \tag{3.108}$$

3.4. Numerical Solution

To obtain the temperature distribution and stresses for the variable cross-sectional composite annular fin, the Pseudospectral Chebyshev Method which is explained in the previous section has been employed. The Eq. (3.23) and (3.68), temperature distribution and displacement equations have been transformed into a linear system of equations using the Pseudospectral Chebyshev Method. These linear system equations are utilized in matrix forms and written in the MATLAB software.

The process of converting second-order inhomogeneous differential equations into linear systems equations is described as follows.

3.4.1. Temperature distribution

Based on the assumptions and the geometry of the annular fin, the energy equilibrium Eq. (3.20) has been formulated to provide the following Eq. (3.38) for the temperature distribution:

$$\frac{d^2 T}{dr^2} + \left(\frac{1}{r} + \frac{1}{A(r)} \frac{dA(r)}{dr} \right) \frac{dT}{dr} - \frac{2h}{A(r)k} (T - T_\infty) = 0 \tag{3.109}$$

To solve the Eq. (3.109) using Pseudospectral Chebyshev Method, the discretization process has been performed as follows:

$$D^2 + \left(\frac{1}{r} + \frac{1}{A(r)} \frac{dA(r)}{dr} \right) D - \frac{2h}{A(r)k} = L_1 \tag{3.110}$$

$$\frac{2h}{A(r)k} T_\infty = RHS_1 \tag{3.111}$$

Here, with these approaches, linear equation system has achieved:

$$L_1 T = RHS_1 \quad 3.112$$

Pseudospectral Chebyshev Method uses matrixes to solve the whole linear equation system at the same time, in the above linear equation system is written with matrix format:

$$\begin{bmatrix} (L_1)_1 \\ (L_1)_2 \\ (L_1)_3 \\ \vdots \\ \vdots \\ (L_1)_8 \\ (L_1)_9 \\ (L_1)_{10} \end{bmatrix} T = \begin{bmatrix} (RHS_1)_1 \\ (RHS_1)_2 \\ (RHS_1)_3 \\ \vdots \\ \vdots \\ (RHS_1)_8 \\ (RHS_1)_9 \\ (RHS_1)_{10} \end{bmatrix}$$

Boundary values:

$$\begin{aligned} T(a) &= T_b \\ T'(b) &= 0 \end{aligned} \quad 3.113$$

3.4.2. Thermal Stress distribution

Based on the assumptions and the geometry of the annular fin, the equilibrium Eq. (3.78) has been achieved. To investigate the thermal stress distributions, Eq. (3.97) has been formulated as displacement second-order non-homogenous differential equation.

To solve the Eq. (3.108) using Pseudospectral Chebyshev Method, the discretization process has been performed as follows:

$$\left[D^2 + D \left(\frac{A'(r)_r}{A(r)_r} + \frac{1}{r} \right) + \frac{1}{r} \left(\frac{A'(r)_r}{A(r)_r} v + \frac{1}{r} \right) \right] = L_2 \quad 3.114$$

$$\frac{A'(r)_r}{A(r)_r} (1 + \nu) a T + (1 + \nu) a D T = RHS_2 \quad 3.115$$

Now, with these approaches, linear equation system has achieved:

$$L_2 U = RHS_2 \quad 3.116$$

Pseudospectral Chebyshev Method use matrixes to solve the whole linear equation system at the same time, in the above linear equation system is written with matrix format:

$$\begin{bmatrix} (L_2)_1 \\ (L_2)_2 \\ (L_2)_3 \\ \vdots \\ \vdots \\ (L_2)_8 \\ (L_2)_9 \\ (L_2)_{10} \end{bmatrix} U = \begin{bmatrix} (RHS_2)_1 \\ (RHS_2)_2 \\ (RHS_2)_3 \\ \vdots \\ \vdots \\ (RHS_2)_8 \\ (RHS_2)_9 \\ (RHS_2)_{10} \end{bmatrix} \quad 3.117$$

Boundary values:

$$\begin{aligned} \sigma_a &= 0 \\ \sigma_b &= 0 \end{aligned} \quad 3.118$$

Boundary conditions should be applied to the Eq. (3.55-3.56):

$$\begin{aligned} \sigma_a &= 0 ; \frac{du}{dr} + v \frac{u(a)}{a} = (1 + \nu) a T(a) \\ \sigma_b &= 0 ; \frac{u(b)}{b} + v \frac{du}{dr} = (1 + \nu) a T(b) \end{aligned} \quad 3.119$$

4. RESULTS AND DISCUSSIONS

The general thermal analysis of the composite annular fin with variable cross-section is investigated with using Pseudospectra Chebyshev Method (PCM). In the analysis, inner radius $a = 0.01$ (m), outer radius $b = 0.02$ (m), the thickness of the annular fin is $y_b = 0.005$ (m) at the base and $M = 0.0005$ (m) at the tip are choosen. The heat convection coefficient is taken $h = 2.560165 \times 10^3$ ($\frac{W}{m^2 K}$), and the ambient and base temperature as $T_\infty = 273$ (K) and $T_b = 373$ (K), respectively. The cross-section of the fin is chosen as variable cross-section and illustrated in Figure (3.2). It is assumed that the composite fin is thought to be made of Silicium Carbide and Aluminum. The mechanical and thermal properties of the material pair considered for analysis are given in Table-4.1.

Table 4.1. Mechanical and Thermal properties of the Silicium Carbide and Aluminum

	E (Pa)	k (W/mK)	α (1/K)	ν
Silicium Carbide	350×10^9	80	$4,6 \times 10^{-6}$	0.27
Aliminum	69×10^9	220	$23,6 \times 10^{-6}$	0.33

It is thought that SiC reinforcements are distributed homogeneously within the Al matrix. For this reason, the Halpin-Tsai homogenization scheme is used to determine the mechanical properties of the composite annular fin. According to Eq. (3.1), the mechanical properties of the composite material has been obtained and given in Table 4.2. Note that, a suitable value has been chosen for the volume fraction of reinforcement material $V_f = 0.3$.

Table 4.2. Mechanical and Thermal properties of the SiC-Al Composite

	E (Pa)	k (W/mK)	α (1/K)	ν
SiC-Al Composite	$1,1223 \times 10^{11}$	170,6762	$1,6582 \times 10^{-5}$,	0,3112

To verify the validity of the introduced numerical technique, a comparison study is performed. The accuracy of the pseudospectral Chebyshev collocation method is determined by an analytical study (Yıldırım, 2018). In this analytical study, the cross-section of the composite fin is uniform. This comparison is available in Table 4.3. The compared results show that this solution procedure provides eight-digit accuracy by picking only *ten* collocation points. The largest relative error between analytical and numerical results are calculated as $0.19101726 \times 10^{-3}$ for radial stress, $0.49715404 \times 10^{-5}$ for tangential stress and $0.81352758 \times 10^{-8}$ for temperature.

Table 4.3. Comparison of the numerical method with the analytical study

r	T (K)			$\sigma_r \times 10^6$ (Pa)			$\sigma_\theta \times 10^6$ (Pa)		
	Analytical	PCM	Error $\times 10^{-8}$	Analytical	PCM	Error $\times 10^{-3}$	Analytical	PCM	Error $\times 10^{-5}$
0.010000	373.000000	372.999999	0.00000013	0,00000000	0,00000000	0,00000000	-40.0079587	-40.0079587	0.06282991
0.010244	371.309552	371.309552	0.03662082	-0.90670874	-0.90670874	0.06912914	-35.9553306	-35.9553306	0.01012795
0.010954	366.815964	366.815964	0.09261584	-2.83426246	-2.83426246	0.00028606	-25.6652201	-25.6652201	0.07623740
0.012061	360.904549	360.904548	0.19691170	-4.32633675	-4.32633675	0.01188002	-13.1720138	-13.1720138	0.01447886
0.013454	355.053252	355.053251	0.31241889	-4.62587336	-4.62587336	0.00031646	-01.9832269	-1.9832269	0.49715404
0.015000	350.295940	350.295938	0.43369227	-3.90228207	-3.90228207	0.01041034	06.1465300	6.1465300	0.11547695
0.016545	347.060492	347.060490	0.55623001	-2.71596667	-2.71596667	0.00037147	10.9813763	10.9813763	0.02626142
0.017938	345.265629	345.265627	0.65457545	-1.55508863	-1.55508863	0.02212730	13.1607337	13.1607337	0.07704613
0.019045	344.519249	344.519246	0.74296706	-0.68063951	-0.68063951	0.00044122	13.6752970	13.6752970	0.00610759
0.019755	344.333698	344.333695	0.78815199	-0.16647095	-0.16647095	0.19101726	13.5064382	13.5064382	0.08524002
0.020000	344.320831	344.320828	0.81352758	0,00000000	0,00000000	0,00000000	13.3639122	13.3639122	0.01494372

Thermal stresses and Temperature distribution in Composite fin with variable cross section using SiC as reinforcement and Al as matrix are examined under constant temperature boundary condition. In this case, the energy equilibrium and stress equilibrium equations became more complex, making analytical solutions challenging Temperature distributions and thermal stresses are calculated numerically using 10 collocation points and presented in figure (4.1-4.3).

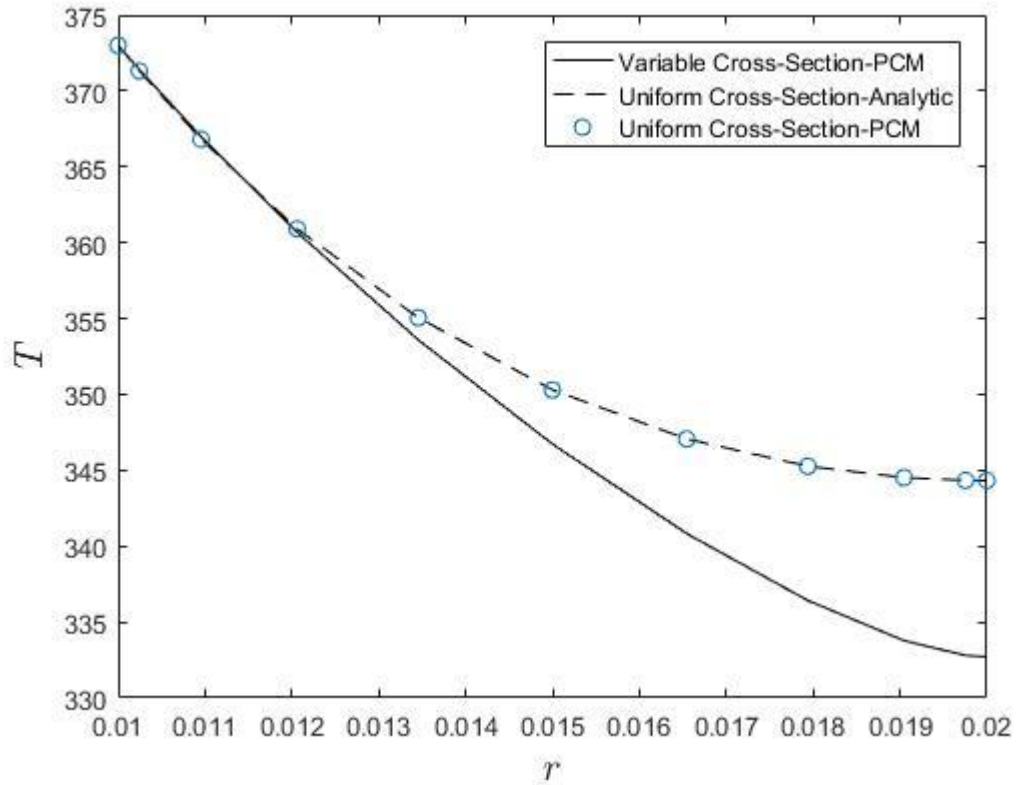


Figure 4.1. Temperature distribution of the annular fin

In Figure 4.1, we have plotted the temperature distributions in a composite annular fin for r ranging from 0.01 (m) to 0.02 (m). As we move along the radial axis, the thickness of the fin cross-section decreases. The lateral surface area increases and therefore the heat transfer rate increases compared to uniform cross-sectional fin. The temperature values of the tip of the fins indicate the effect of geometries. The temperature of the tip of variable cross-sectional fin is lower than the uniform cross-sectional fin. It can be discussed that variable cross-sectional fin has a better heat transfer by convection than uniform cross-sectional fin. Also, for the same heat transfer duty, variable cross-sectional fin offers considerable saving of material compared with the uniform cross-sectional fin.

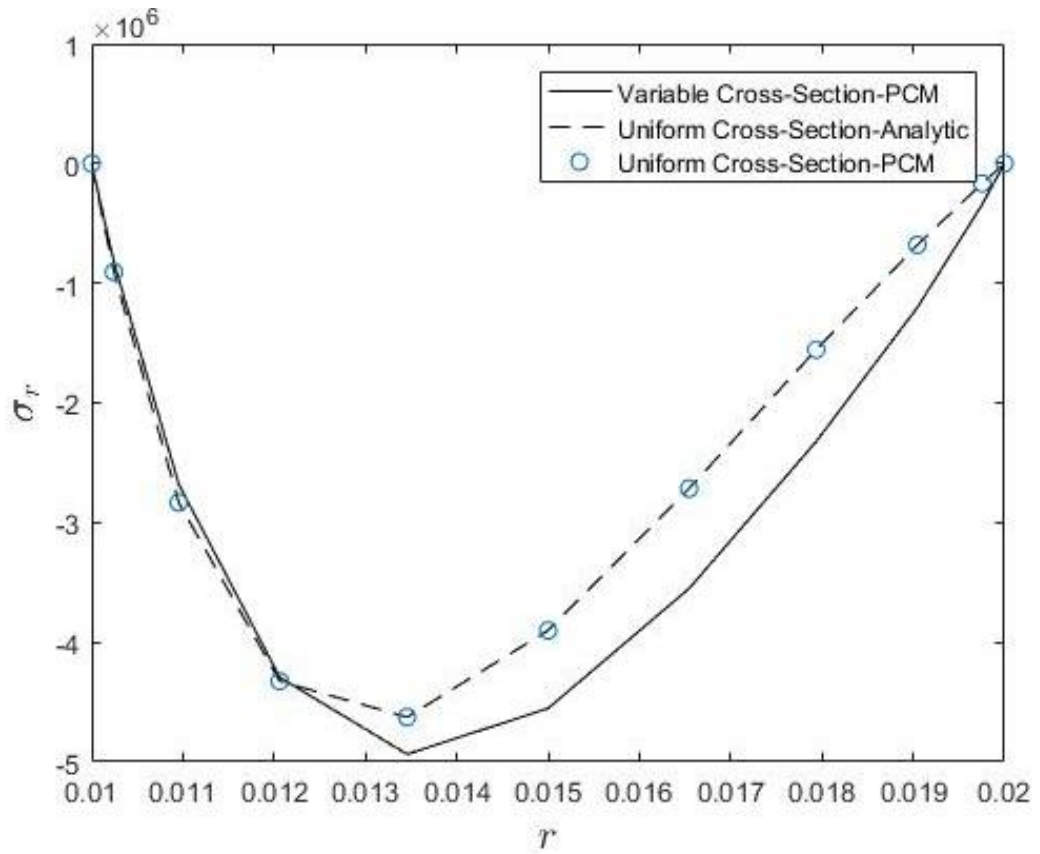


Figure 4.2. Radial stresses of the annular fin

Figure 4.2 depict the behavior of the radial stress for annular fin. In figure 4.2, the impact of the geometry on radial stress is discussed. The trend shows a decline pattern compared to the uniform cross-sectional fin. In each case stresses occur in compression direction. The amplitude of the radial stresses at the base is nearly equal to each other in variable cross-sectional and uniform cross-sectional fins. Maximum radial stress is obtained 4.9356×10^6 (Pa) as compressive stress at $r = 0.013454$ (m).

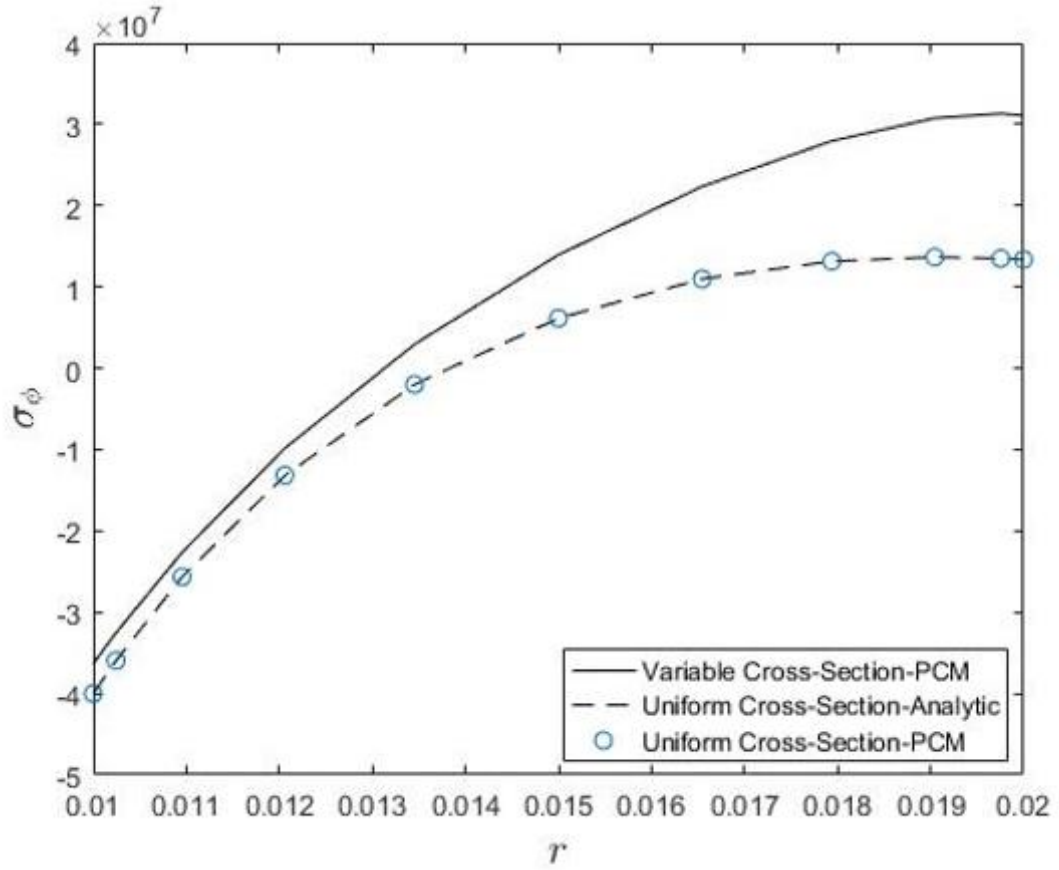


Figure 4.3. Tangential stresses of the annular fin

In Figure 4.3, the impact of the geometry on tangential stress is illustrated. Here, tangential stress changes very rapidly for variable cross-sectional fin and a very slow change observed for uniform cross-sectional fin, respectively. The maximum tangential stress is achieved 3.132204×10^7 (Pa) as tensile stress at $r = 0.019755$ (m).

In order to investigate the variable cross-sectional annular fins, various tip thicknesses have been employed. The geometry of annular fins which have different tip thicknesses (M) are given in Figure 4.4, all have the same base thickness (y_b). The values of base thickness of the annular fins are appointed as 0.005 (m). Figure 4.5 depict the behavior of thermal distribution and stresses for various values of M.

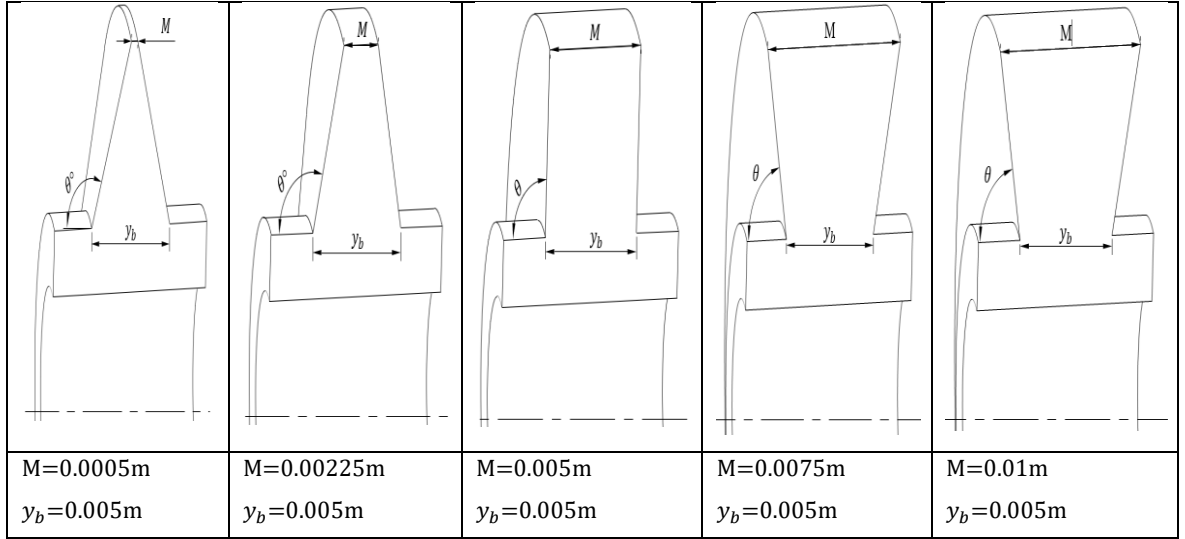


Figure 4.4. Annular fins with various tip thicknesses

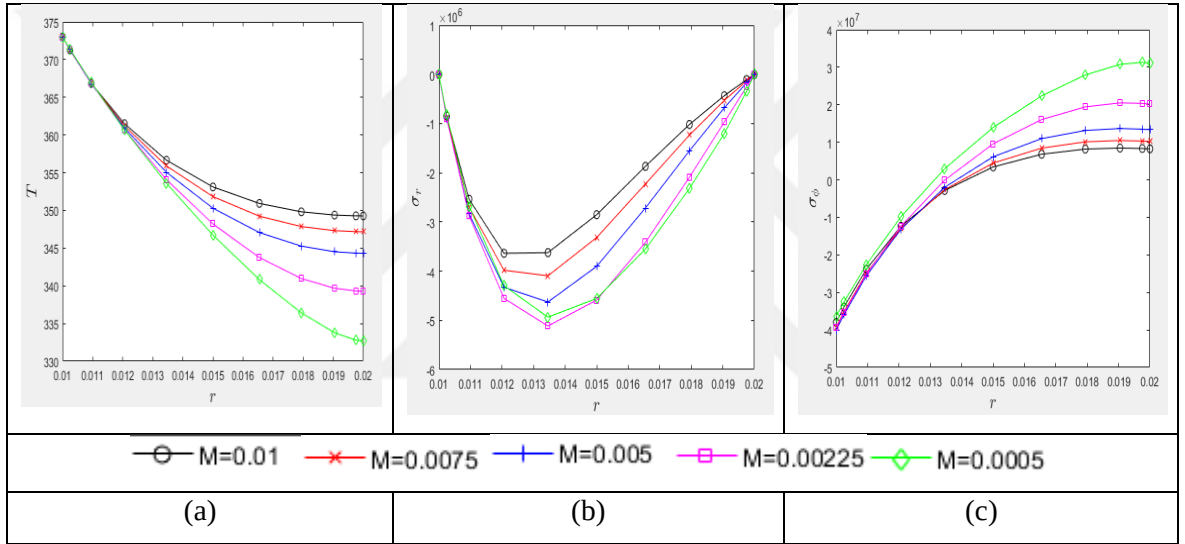


Figure 4.5. Temperature distributions, radial stresses and tangential stresses of the annular fins with various tip thickness

One can be seen in the Figure 4.5(a), temperature at the tip of the fin increase while the tip thickness increases. On account of the Figure 4.5(a), increasing the heat transfer area, also the lateral surface areas, does not necessarily increase the heat transfer rate. Considering the variable cross-sectional annular fin's lateral surface has made an angle θ from the vertical axis. The force that acting on a unit volume fluid in the heat transfer boundary layer can be rearranged into two components, $F_y = F \cos\theta$ and $F_x = F \sin\theta$ (Çengel, 2017). The force acting on the motion which is from hot environment to cold ambient is reduced in the fin with thicker tips. As a result of the reduced motion effect, the convection heat transfer current is reduced. Hence, heat transfer is more difficult to reach ambient compared with the uniform cross-sectional annular fin. As opposite of the fins with thicker tips, the tip temperature is relatively lower. The reason for this behavior is F_y tends to move upward while moving parallel to the lateral surface and as a result, heat transfer boundary layer breaks up to the upward. This behaviour leads an increment in the heat transfer to the ambient.

Figure 4.5(b) explores the influence of various tip thicknesses (M) on radial stress. The trend shows a decline pattern as the value of the tip thickness M is enhanced. Figure 4.5(c) illustrates the relationship between the tip thickness (M) and tangential stress. As the value of M decreases, the magnitudes of tangential stresses increase. As the convection increases by the impact of the tip thickness it causes an increase in heat transfer from the fin's surface to ambient resulting in the temperature gradient. Thermal stresses arise as the temperature gradient is developed on the fin's surface.





5. CONCLUSIONS

The general thermal analysis of the composite annular fin with variable cross-section is investigated with using Pseudospectra Chebyshev Method (PCM). It has been assumed that the annular fin is made of a particle-reinforced composite material with ceramic reinforcement uniformly distributed in a metal matrix. For this reason, the Halpin-Tsai homogenization model was used to determine the mechanical properties of the composite annular fin. The thickness of the annular fin varies linearly in the radial direction as triangular profile. In this case, the energy equilibrium and thermal stress equilibrium equations became more complex, making analytical solutions challenging. Therefore, to verify the validity of the introduced numerical technique, a comparison study has been performed for an annular fin with uniform cross-section. For a further comparison, various tip thicknesses have been employed and compared with each other for temperature distributions and stress values.

As a result the following conclusion are made:

- This study provides a general numerical solution to obtain temperature and stress distributions in annular fins made of variable cross-sectional profile.
- It is obvious that using the variable cross-sectional profile on the annular fin is an effective way to decrease the thermal stresses.
- For the same heat transfer duty, variable cross-sectional fin offers considerable saving of material compared with the uniform cross-sectional fin.
- The results indicate a clear superiority of variable cross-sectional fin over uniform cross-sectional fin in terms of convection heat transfer based on the temperature difference on the tips.
- The variable geometry on the fin effects the convection heat transfer better than uniform cross-sectional profile.
- Variable cross-sectional fin has superiority on uniform cross-sectional fin by 132,64639% when comparing the tangential stress results, 30.76886 % when comparing the radial stress.
- The increased thickness of the tip results in the reduced heat transfer by convection. On account of this behaviour the increased heat transfer does not always increase the heat transfer rate to the ambient.
- The results show a decent decrement in the both radial and tangential stress values as the thickness of the fin thickens.

- If a general comment is to be made, it seems almost impossible to increase the heat transfer rate and reduce thermal stresses by changing the geometry of the annular fins. For this reason, an optimum design parameter must be provided.



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