

**ÇUKUROVA UNIVERSITY
INSTITUTE OF NATURAL AND APPLIED SCIENCES**

MSc THESIS

Atakan DURMAZ

**SINGULARITY ANALYSIS OF STEWART PLATFORM MECHANISMS
WITHIN A MEDICAL CONTEXT**

DEPARTMENT OF MECHANICAL ENGINEERING

ADANA, 2015

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ABSTRACT

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In recent years, use of parallel robots have increased in applications requiring high precision, loading capacity, rigidity and high velocity. One of the important groups of parallel manipulators is the Stewart platform. This platform has a wide range of applications. For example, it can be used as aircraft simulator, machine tools, telescopes etc. Also it is suitable for use in orthopaedic surgery. Limited working space, the complex kinematic-kinetic solutions and singularities within the workspace of the robot are some of the major problems encountered. In this thesis work, Stewart platform involved in singular cases are investigated within a medical context. During the process, singularity sources of robotic structure evaluated as fixators, devices used for orthopaedic purposes, have been investigated by numerical/analytical and geometrical methods. Main problem discussed here, calculation of displacement, should be guaranteed at all positions of the fixator during the treatment. Singularity analysis of the robotic frame is essential for safe use. If a robot is in a singular configuration it may show unstable behaviour and this is undesirable in practice. To prevent this situation, which will adversely affect the treatment process, a software has been developed to test the situation by three different methods leading to the reliability of the set-up of the robotic frame as applied by orthopaedist. Inputs of this software, which are the parameters of fixator, are selected by orthopaedist and its outputs can be easily reviewed. A proactive approach is aimed to prevent possible mishaps with the results obtained and testing is carried out before treatment.

Key Words: Parallel Manipulators, Gough-Stewart Platform, External Fixator, Singularity Analysis

ÖZ

YÜKSEK LİSANS TEZİ

STEWART MEKANİZMASININ TIBBİ BAĞLAMDA TEKİLLİK ANALİZİ

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Son yıllarda yüksek hassasiyet, taşıma kapasitesi, hız ve rijitlik gerektiren uygulamalarda paralel robotların kullanımı artmıştır. Paralel manipülatörlerin önemli gruplarından biri de Stewart platformudur. Bu platform uçuş simülatörlerinde, makine imalat teknolojisinde, su altı araştırmalarında, teleskoplarda ve ortopedik cerrahide kullanılmaya elverişli geniş bir uygulama alanına sahiptir. Kısıtlı çalışma uzayı, karmaşık kinematik-kinetik çözümler ve çalışma uzayı içerisindeki tekillikler bu robotlarda karşılaşılan önemli problemlerdendir. Bu çalışmada, Stewart platformunun tıbbi bağlamda tekil durumları incelenmiştir. Süreç içerisinde ortopedik amaçlarla kullanılan harici fiksator adı verilen robotik yapının tekillik nedenleri sayısal/analitik ve geometrik yöntemlerle araştırılmıştır. Buradaki temel problem, fiksatorün tüm konumlarında yer değiştirme hesaplarının güvenceye alınmasıdır. Robotik çerçevenin emniyetli kullanımı açısından tekillik analizi şarttır. Tekil konfigürasyonda olan bir robot kararsız davranış sergileyebilir bu durum tıbbi uygulamada istenmez. Tedavi sürecini olumsuz etkileyecek bu durumun önüne geçebilmek için ortopedist tarafından robotik çerçeveye uygulanan kurgunun güvenilirliğini üç farklı yöntemle test eden bir yazılım geliştirilmiştir. Girdileri ortopedist tarafından kurgulanan fiksatorün parametreleri olan yazılım, çıktıların ortopedist tarafından kolayca yorumlanabilmesine imkân sağlayacak şekilde geliştirilmiştir. Elde edilen sonuçlar ile tedavi öncesi gerçekleştirilen testlerle proaktif bir yaklaşım sergilenerek olası aksiliklerin önlenmesi amaçlanmıştır.

Anahtar Kelimeler: Paralel Manipülatörler, Gough-Stewart Platform, Harici Fiksator, Tekillik Analizi

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SYMBOLS AND ABBREVIATIONS

α	:Proximal joint angular position
ε	:Distal joint angular position
L	:Rod length
R	:Proximal ring diameter
R_1	:Distal ring diameter
A_i	:Proximal joint coordinate
B_i	:Distal joint coordinate
$\vec{e}_i$	:Rod unit vector
G	:Proximal ring center
G_1	:Distal ring center
x_1, y_1, z_1	:Motion parameters (Translation)
$\theta_x, \theta_y, \theta_z$	:Motion parameters (Rotation)
$\vec{F}$	:External force
$\vec{M}$	:External moment
$\vec{F}_i$	:Joint force
rF	:External force application point
s_{mn}	:Closest perpendicular distance
J	:Jacobian matrix
$[A_{xyz}^{uvw}]$	:Transformation matrix
δ_s	:Intersection threshold value
δ_p	:Singularity threshold limit value
Δ_p	:Intersection metrics
SP	:Stewart platform
GSP	:Gough-Stewart platform
PM	:Parallel manipulators
SM	:Serial manipulators
CPA	:Closest point approach
DOF	:Degrees of freedom

1. INTRODUCTION

1.1. Definition and Classification of Robots

Robot is defined as “a machine controlled by a computer that is used to perform jobs automatically” according to Cambridge Dictionaries Online, (Cambridge Dictionaries Online, 2015). Karel Capek, who is a Czech scriptwriter, used “robot” for the first time as a term in his play titles “Rossum’s Universal Robot” 1920, (Stone, 2005). The meaning of the word “robota” in Czech language is “forced labour”.

Robot is a machine that can perform a complicated series of tasks automatically. In daily life robots are used to do tasks which are dangerous, difficult and dirty for human health. Robots can perform these tasks efficiently. For this reason these machines are preferred in industry. Many companies with a leading position in the industry concentrated on robot development technologies. Also there are lots of universities and technological institutions working on this field.

Robots can be classified according to several criteria, such as kinematic structure, control type, degrees of freedom and motion characteristics (planar, spatial and spherical). Within the classification in this study, mechanical structure of robots can be viewed in two classes. One class referred to as serial type of manipulators is based on open loop mechanisms, Figure 1. 1. The second one called parallel type of manipulators is constructed from closed loop mechanisms, Figure 1. 2. This classification is made according to the type of the kinematic chains connecting the output link of the manipulator to the base link.

Serial manipulators (SM) have classical anthropomorphic architecture. In this form, each pair of adjacent links are connected by an active kinematic pair. Output link is connected to the base link by a single open loop kinematic chain. Advantages of serial manipulators are larger workspace with respect to the size of the robot, high velocity and accelerations. Also they have disadvantages. Cumulative joint errors and deflections in the links cause low precision. High inertia and low payload are another disadvantages of serial manipulators.

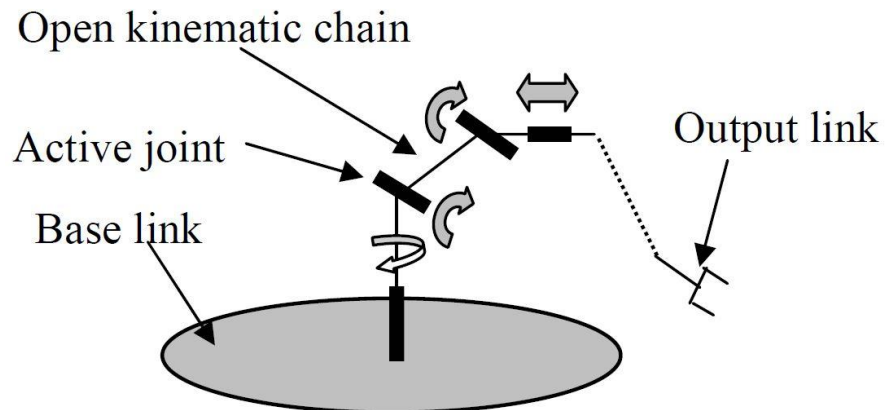


Figure 1. 1. Serial manipulator (Simaan, 1999)

Parallel manipulators (PM) have two platforms connected together by at least two independent kinematic chains. Because of low stiffness, low precision of positioning, low payload capacity characteristics of serial manipulators the parallel manipulators have gained popularity (Akçalı & Mutlu, 2006). Despite limited workspace property of parallel manipulators they provide high stiffness especially in the context of medical applications (Svinin et al., 2001), (Merlet, 2006). High accuracy which stems from sharing joint errors and low inertia are another advantages of these manipulators. Because all actuated links carry a part of payload.

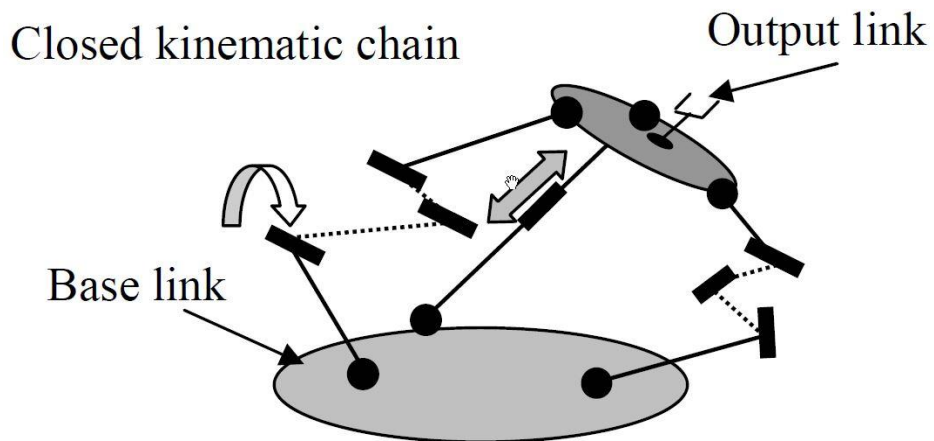


Figure 1. 2. Parallel manipulator (Simaan, 1999)

The location of the output link relative to the base link should be established for position analysis problem. This problem can be divided into two categories.

Location of the output link is determined according to the given joint variables in direct kinematics. However, location of the output link is given and the problem is to find the joint variables for desired location of the output link in inverse kinematics. This problem has found interesting applications different areas such as vibration simulator and machine tools (Fan et al., 2003; Hostens et al., 2005). Direct kinematics is quite simply done for serial manipulators, although inverse kinematics turns into extremely difficult job (Tsai, 1999). Many direct kinematic solution methods are developed by researchers (Akçalı & Mutlu, 2006; Innocenti et al., 1990; Husty, 1996; Der-Ming, 1999).

Comparison between properties of serial and parallel manipulators is given in Table 1. 1, (Zang, 2010). The following table is organized according to the information in the book.

Table 1. 1. Comparison between serial and parallel manipulator

	Serial Manipulator	Parallel Manipulator
Kinematic Chain	Open Loop	Closed Loop
Working Space	Larger	Limited
Direct Kinematics	Straightforward	Complicated
Inverse Kinematics	Complicated	Straightforward
Inertia Forces	High	Less
Stiffness	Less	High
Preferred Property	Dexterity	Stiffness
Payload/Weight Ratio	Low	High
Accuracy	Low	High
Position Errors	Cumulative	Averages

1.2. Applications of Parallel Manipulators and Gough - Stewart Platform

Parallel manipulators have been used for many industrial applications. The evolution of parallel manipulators can be traced back to 1960's when Gough and Whitehall, invented a system for use as a universal tyre testing machine which was used to discover properties of tires under the different loads, Figure 1. 3 (Gough et al., 1962). Later, Stewart developed a parallel manipulator which has six degrees of freedom for use as a flight simulator, Figure 1. 4 (Stewart, 1965). Gough's design is a milestone in the development of parallel manipulators, however Stewart was

responsible for popularizing his design in academia. This mechanism is called in the literature as Stewart platform (SP). However it was received fully by Gough's proposal so it is also called Gough-Stewart platform (GSP). These two men are considered pioneers of parallel robotics.

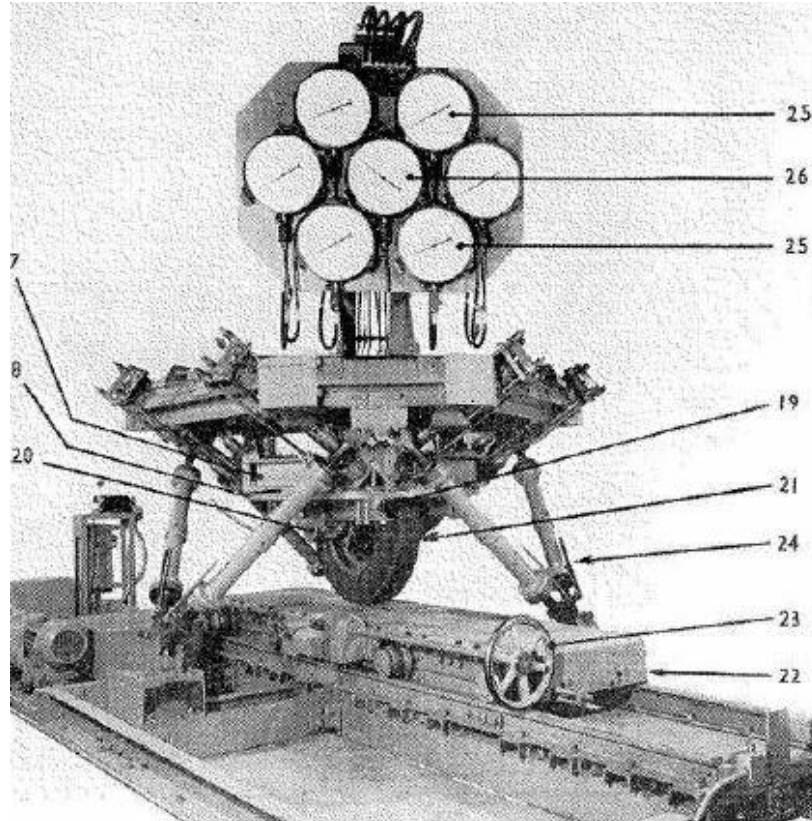


Figure 1. 3. Universal tyre test machine was designed by Gough and Whitehall (Gough et al., 1962)

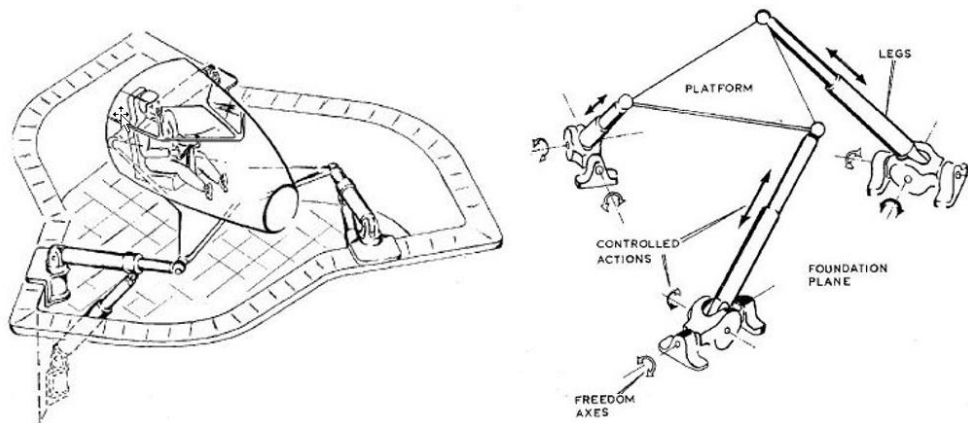


Figure 1. 4. Flight simulator concept was designed by D. Stewart (Stewart, 1965)

Parallel manipulators are preferred for precise positioning, high accuracy and rigid design in medical applications. Stewart platform application which can be used for precision microsurgery is given in Figure 1. 5 (Wapler et al., 2003). A Hexapod Robot which is produced by Physik Instrumente was used for setting up this robotic structure, (PhysikInstrumente, 2015). This can be modified with special tools and instruments for different medical application. According to paper, this robot can be used in neurosurgery, ophthalmology, spine surgery, orthopaedics and otolaryngologist (ENT), (Wapler et al., 2003). The mechanism which is shown in Figure 1. 6 originated from human elbow (Mendoza-Vázquez et al., 2009).

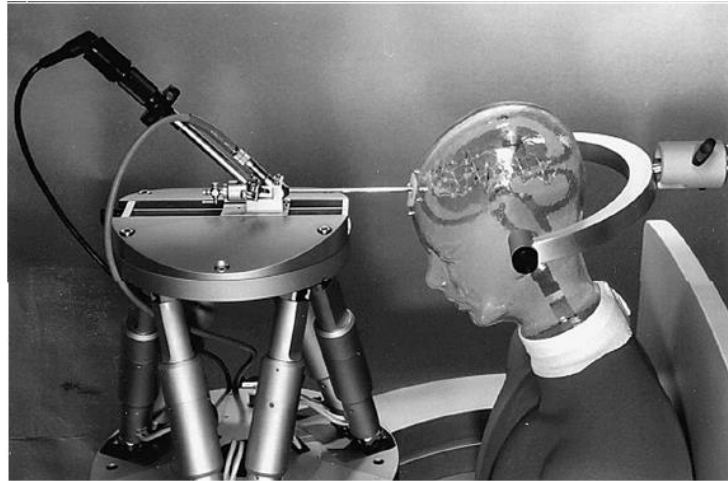


Figure 1. 5. Use of Stewart platform for precision surgery (Wapler et al., 2003)

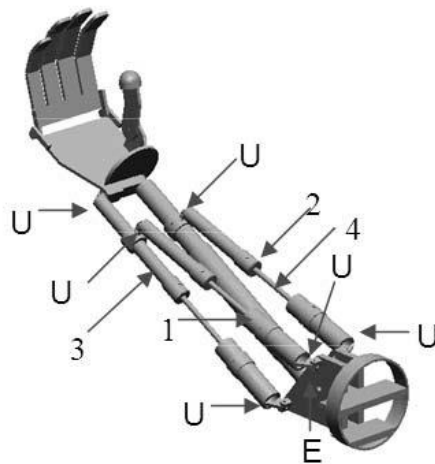


Figure 1. 6. A prosthetic elbow was developed with concept of parallel mechanism (Mendoza-Vázquez et al., 2009)

1.3. External Fixation and External Fixator

External fixator is defined as a comprehensive system which can be used to eliminate instability of bone fragments in the book titled “Concepts in External Fixation”. (Seligson & Pope, 1982; Akçalı et al., 2010c). External fixation is one of the Stewart Platform’s biomechanical applications. Stiffness and control accuracy make Stewart Platforms ideal structure for external fixator (Akçalı et al., 2010c). External fixators were used in the treatment of bone fractures by Hippocrates 2500 years ago then by Jean-Francois Malgaigne in 1840, by Claython Parkhill in 1894, and Albine Lambotte in 1902. However, fixator used without considering the principles of mechanical sciences caused unsatisfactory results. They have emerged again with successful results and conscious use under the light of the Burny and Bourgois’s studies. Ring fixator which was developed by Ilizarov is an important milestone for the practice of external fixation (Akçalı et al., 2010a; Akçalı et al., 2009). An example application of SP to external fixation is shown, Figure 1. 7.

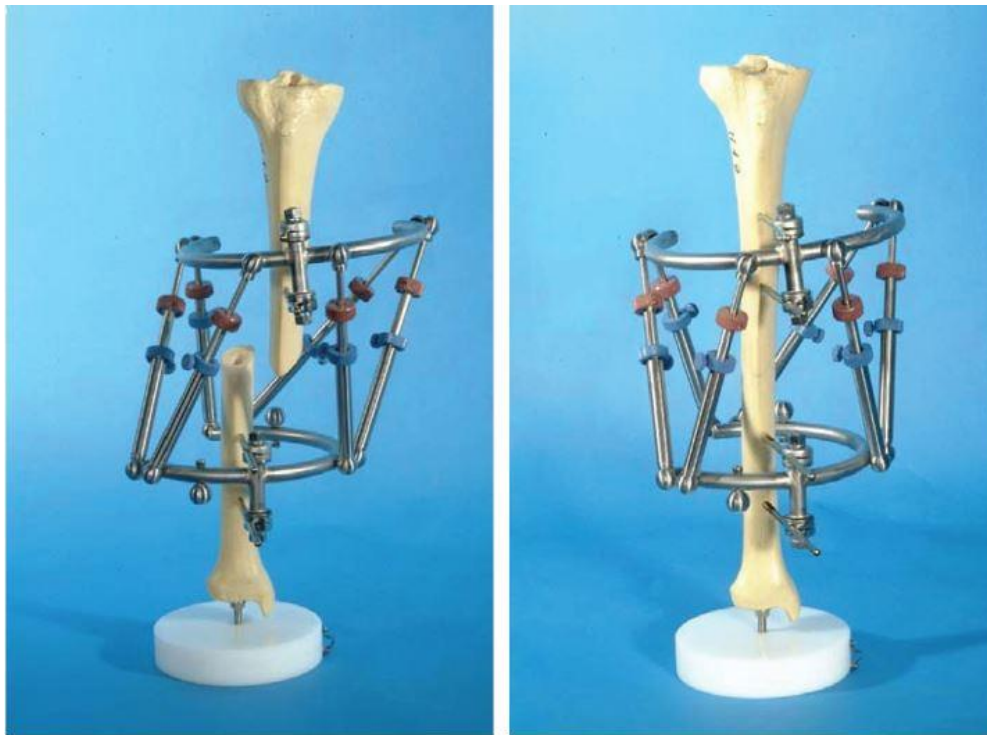


Figure 1. 7. An example of external fixation (Seide et al., 2004)

Spatial external fixator is used for extremity fractures and deformity-correction (Mutlu et al., 2006). It is a parallel mechanism that consists of two rings which are connected with six independent adjustable rods, Figure 1. 8. These two rings are called proximal ring and distal ring. Proximal ring, which is the upper part of the mechanism, is selected as the base platform. Distal ring has relative motion possibility with respect to the base. This motion is obtained by changing the rod lengths. This robotic structure provides six degrees of freedom, three translational and three rotational in space. Bone fragments can be controlled in all directions by this robotic structure.

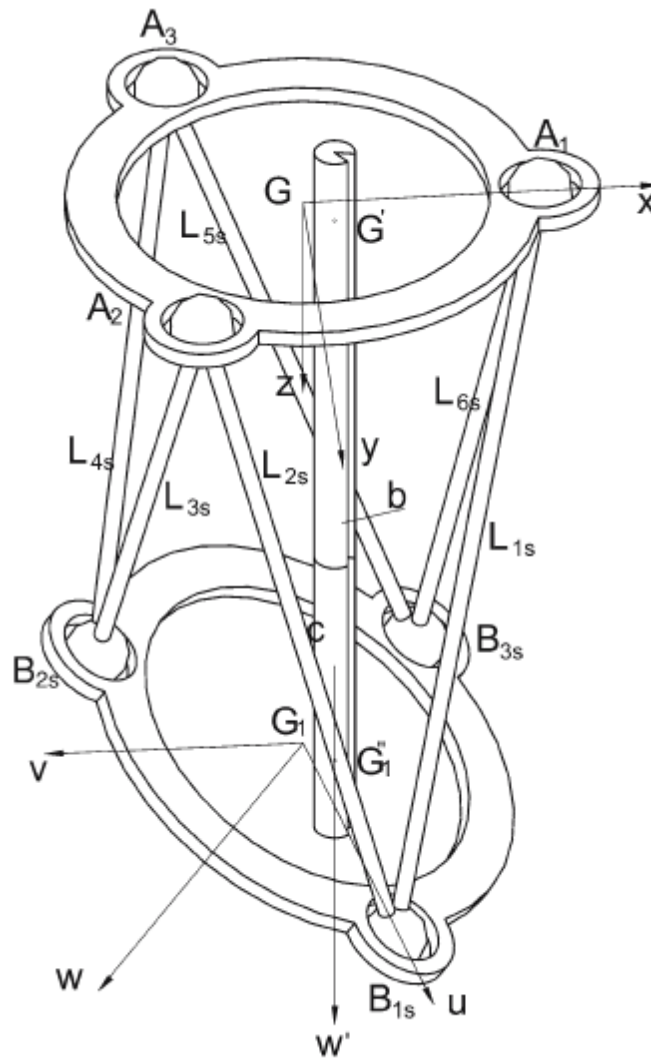


Figure 1. 8. Stewart Platform (Mutlu et al., 2006)

1.4. Definition of Singularity

The most important problem encountered in the analysis, design, motion planning and control of parallel mechanisms is singularity. Motion of the parallel mechanisms is restricted and there should be some singularities in workspace due to their closed loop kinematic chain. We can talk about singularities when joint axes become aligned or parallel, the boundaries of the workspace are reached or there is no solution of the direct or inverse kinematic problem (Anli et al., 2005).

In engineering, singularity is a configuration or position of a mechanism where the following action cannot be expected in the mechanical sense. One of the other source is forces on the system which become extremely large or infinite. When a mechanism is in a singular configuration, degrees of freedom of the mechanism changes instantaneously and the relation between the input parameters and output parameters cannot be fully defined. These cause the mechanism to be uncontrollable (Li et al., 2007). Serial manipulators lose one or more degrees of freedom. However, parallel manipulators gain one or more degrees of freedom in a singular configuration.

1.5. Statement of the Problem

The main objective of this thesis is to examine the case of singularity of Stewart platform within a medical context. During this process, causes of external fixator singularity will be investigated by using mathematical and geometrical methods. If there is a singular configuration on the robotic structure it shows unstable behaviour and this condition is inappropriate for application. This will adversely affect the treatment process. For this reason singularity analysis should be performed for a reliable use of the robotic frame. There are several areas in which singularity may arise in practice. However, those singularities which are possible to be encountered in medical applications will be handled in this thesis. For this purpose, it is necessary to do the motion and force analyses. Positions that make it impossible to make calculate the values of motion quantities and force must be detected by these analyses.

2. PREVIOUS STUDIES

Determination of the singularity point of the parallel mechanism is important for controlling the parallel manipulator, designing the mechanism, path planning in which specified points are not satisfied and determination of the workspace which does not contain singularity. Different approaches have been applied by researchers in the detection of singularities.

There are lots of studies for the classification of singularities. Gosselin and Angeles recognized singular configuration into three types based on the determinants of the Jacobian matrices (Gosselin & Angeles, 1990). D. Zlatanov et al. made the definition and classification of the kinematic singularities of a mechanism by means of the instantaneous kinematic knowledge (Zlatanov et al., 1994a; 1994b).

Some researchers have preferred geometric approach in their work for detecting singular configuration. All the information necessary for the solution can be obtained by information about the geometry of the mechanism such as size of the mobile platform, positions of the joints and link lengths. This approach not only provides intuitive understanding singular configuration but also can help to design mechanism and planning trajectories. Hunt indicated that singularity occurs when all the lines associated with the prismatic actuators intersect a common line (Hunt, 1978; St-Onge & Gosselin, 2000). Fichter showed that a singular configuration is attained when the platform is rotated around an axis orthogonal to the plane of the base by angle of $\pm 90^\circ$ (Fichter, 1986; St-Onge & Gosselin, 2000). Merlet used the Grassmann Geometry to find singular configuration (Merlet, 1989). Wen and O'Brian found singularities of 3-legged planar and spatial mechanism with geometric method (Wen & O'Brian, 2003).

There are a few researchers who choose analytical approach in their studies. In this approach, the resulting equations are general so that equation also be used after any modification on the platform. The resulting equations will be useful in the design of the platform. In 1993, Sefrioui and Gosselin derived an analytic expression of the singularity loci for a planar three-DOF parallel manipulator (Sefrioui & Gosselin, 1993; Kim & Chung, 1999). In 1996, Mayer St-Onge and Gosselin derived an analytic

equation directly from the property of the determinant, which is a fourth-degree polynomial of the position variables (Mayer St-Onge & Gosselin, 1996; Kim & Chung, 1999). Kim and Chung obtained third-degree polynomial by analytical study (Kim & Chung, 1999). Di Gregorio found singularities and workspace of the U-2PUS mechanism which can be used in biomechanical and aerospace applications (Di Gregorio, 2002).

The physical explanation of a singularity in kinematics refers to those configurations of parallel manipulators in which the number of degrees of freedom (DOF) of the mechanical structure changes immediately, either the manipulator gains some additional, uncontrollable DOF or loses some DOF. In this situation the singularity analysis can be carried out on the basis of the properties of the Jacobian matrices of the mechanical structure. Waldron et al. studied Jacobian matrix of serial manipulators in 1985, (Waldron et al., 1985). Rank of the Jacobian matrix which is related with the input velocity with the output velocity is reduced at singular points (Gosselin & Angeles, 1990). In 1991, Ma and Angeles classified singularities of parallel manipulators and focused on architecture singularities by investigating the properties of the Jacobian matrices (Ma & Angeles, 1991). Ottaviano et al. studied configuration singularities of CaPaMan (Cassino Parallel Manipulator) by using two different methods (Ottaviano et al., 2001). Wang et al. addressed the singularity analysis by using Jacobian matrices in their study, (Wang et al., 2009).

3. MATERIALS AND METHOD

In general, external fixator works on the basis of two main functions. First of these is to move the bones from one location to the other. Another task is to keep the bones under stable equilibrium by locking the system during the treatment. The singularity problem may occur in both cases. The question of singularity becomes of great importance for reliable use of these devices. Thus, methods and algorithms have been developed so that the fixator works safely in both standstill and mobile conditions.

Motion analysis is required for obtaining the displacement variables as input for other methods and as well as for smooth motion between initial and the final states in the treatment process. Geometric method and force analyses are needed for stable equilibrium conditions.

3.1. Methods

In this thesis work 3 theoretical approaches are used to investigate singularities within a medical context.

- i. Motion Analysis
- ii. Force Analysis
- iii. Geometric Method - Closest point approach

3.1.1. Motion Analysis

Singularity may be encountered in several areas. One of these areas is determined by examining the motion of the robotic frame. Relative motion can be conveniently expressed by a mathematical function called displacement function (f_i) involving the fixed parameters as well as the motion variables of the system. The fact that resulting displacement equation is a nonlinear one means that multiple configurations of the same robotic system can be obtained corresponding to same set

of input parameters. However, from a medical point of view, it is necessary to continue the treatment with one consistent configuration.

In order to have a unique configuration of the robotic frame throughout the treatment process, there is a need to linearize the nonlinear displacement function. This produces an iteration scheme involving a set of linear equations whereby the coefficient matrix turns out to be Jacobian (J). Jacobian matrix results from the differentiation of displacement function with respect to the motion variables. For the linear set of equation to have a unique solution, it is necessary that the Jacobian be non-zero. This means that singularity arises when the Jacobian determinant is equal to zero.

Inverse of the Jacobian must be calculated at each step of iteration, in order for the iteration scheme to converge to a solution. This operation can only be carried out if the Jacobian matrix is non-singular. This iteration process is repeated for each treatment step, the number of which is decided by orthopedist, depending on the medical conditions. Singularity check by means of Jacobian matrix is not only an essential requirement for the iteration to proceed with success but also solution found at the end should provide a stable configuration for immobilizing the bones in a particular situation.

Velocity analysis is another area in which singularities may evolve. Velocity equation can be obtained by differentiating the displacement function. It will be shown that the resulting coefficient matrix in the equation will be equal to Jacobian matrix. The crucial point that should be emphasized is that sign of the Jacobian determinant, which should not change during the treatment. Since the motion of the robotic frame is a continuous one, a change in the sign of Jacobian determinant is to indicate to the singularity of Jacobian matrix. It further means that velocities cannot be calculated.

In general, elements of the Jacobian matrix depend on fixed positions of joints, radii of rings and position variables. The abundance of these parameters makes the Jacobian determinant a complicated function. For this reason, Jacobian will have to be also examined under special circumstances, which may help to observe the several different cases of singularity.

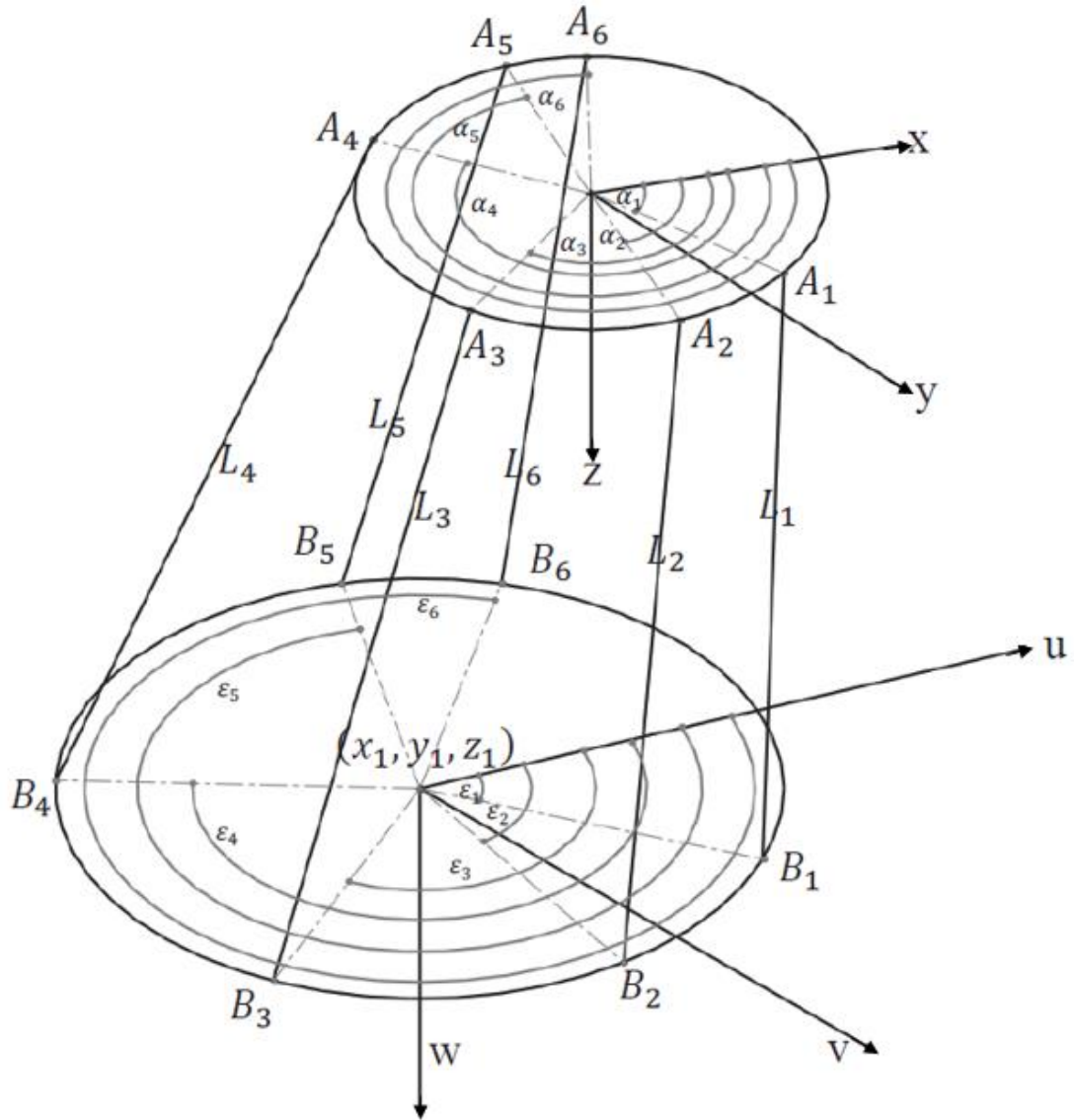


Figure 3. 1. The external fixator – SP (Akçali et al. 2014)

Referring to geometry of SP Figure 3. 1, the distance between the relevant proximal and distal joints (amounting to the rod lengths ($L_i, i = 1 - 6$)), is written to lead to the following displacement function ($f_i, i = 1 - 6$), (Akçali et al., 2010c).

$$f_i = f_{i0} - L_i^2 = 0 \quad i = 1 - 6 \quad (3.1.)$$

$$f_{i0} = x_1^2 + y_1^2 + z_1^2 + 2a_{xi}x_1 + 2a_{yi}y_1 + 2a_{zi}z_1 + a_{xi}^2 + a_{yi}^2 + a_{zi}^2 \quad (3.2.)$$

$$\left. \begin{aligned} u_{Bi} &= R_1 \cos \varepsilon_i \\ v_{Bi} &= R_1 \sin \varepsilon_i \\ x_{Ai} &= R \cos \alpha_i \\ y_{Ai} &= R \cos \alpha_i \end{aligned} \right\} i = 1 - 6 \quad (3.3.)$$

$$\begin{aligned} a_{xi} &= -x_{Ai} + u_{Bi} \cos \theta_z \cos \theta_y + v_{Bi} \cos \theta_z \sin \theta_y \sin \theta_x - v_{Bi} \sin \theta_z \cos \theta_x \\ a_{yi} &= -y_{Ai} + u_{Bi} \sin \theta_z \cos \theta_y + v_{Bi} \sin \theta_z \sin \theta_y \sin \theta_x + v_{Bi} \cos \theta_z \cos \theta_x \\ a_{zi} &= -u_{Bi} \sin \theta_y + v_{Bi} \cos \theta_y \sin \theta_x \end{aligned} \quad (3.4.)$$

A position vector $\vec{X}$, containing translation (x_1, y_1, z_1) and rotations $(\theta_x, \theta_y, \theta_z)$, and $\Delta\vec{X}$ which represent small changes are defined.

$$\vec{X} = \{p_j\}^T = (x_1, y_1, z_1, \theta_x, \theta_y, \theta_z)^T \quad (3.5.)$$

$$\Delta\vec{X} = \{\Delta p_j\}^T = (\Delta x_1, \Delta y_1, \Delta z_1, \Delta \theta_x, \Delta \theta_y, \Delta \theta_z)^T \quad j = 1 - 6 \quad (3.6.)$$

Displacement function f_i (3.1.) is expanded into a Taylor Series for $\vec{X} + \Delta\vec{X}$ vector.

$$f_i(\vec{X} + \Delta\vec{X}) = f_i(\vec{X}) + \frac{\partial f_i}{\partial p_j} \Delta\vec{X} + O^2(\Delta\vec{X}) = 0 \quad \begin{aligned} i &= 1 - 6 \\ j &= 1 - 6 \end{aligned} \quad (3.7.)$$

$O^2(\Delta\vec{X})$ contains two and higher order terms. So, following equation can be obtained when neglecting higher order terms $O^2(\Delta\vec{X})$ for small changes vector $\Delta\vec{X} = (\Delta x_1, \Delta y_1, \Delta z_1, \Delta \theta_x, \Delta \theta_y, \Delta \theta_z)^T$.

$$\frac{\partial f_i}{\partial p_j} \Delta\vec{X} = -f_i(\vec{X}) \quad \begin{aligned} i &= 1 - 6 \\ j &= 1 - 6 \end{aligned} \quad (3.8.)$$

Equation (3.8.) is nothing more than a set of linear equations. The above equation is written as follows in the form of matrix.

$$J\Delta\vec{X} = -\vec{f} \quad (3.9.)$$

Jacobian matrix (J) and elements of $\vec{f}$ vector are defined as follows.

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial y_1} & \frac{\partial f_1}{\partial z_1} & \frac{\partial f_1}{\partial \theta_x} & \frac{\partial f_1}{\partial \theta_y} & \frac{\partial f_1}{\partial \theta_z} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial y_1} & \frac{\partial f_2}{\partial z_1} & \frac{\partial f_2}{\partial \theta_x} & \frac{\partial f_2}{\partial \theta_y} & \frac{\partial f_2}{\partial \theta_z} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial y_1} & \frac{\partial f_3}{\partial z_1} & \frac{\partial f_3}{\partial \theta_x} & \frac{\partial f_3}{\partial \theta_y} & \frac{\partial f_3}{\partial \theta_z} \\ \frac{\partial f_4}{\partial x_1} & \frac{\partial f_4}{\partial y_1} & \frac{\partial f_4}{\partial z_1} & \frac{\partial f_4}{\partial \theta_x} & \frac{\partial f_4}{\partial \theta_y} & \frac{\partial f_4}{\partial \theta_z} \\ \frac{\partial f_5}{\partial x_1} & \frac{\partial f_5}{\partial y_1} & \frac{\partial f_5}{\partial z_1} & \frac{\partial f_5}{\partial \theta_x} & \frac{\partial f_5}{\partial \theta_y} & \frac{\partial f_5}{\partial \theta_z} \\ \frac{\partial f_6}{\partial x_1} & \frac{\partial f_6}{\partial y_1} & \frac{\partial f_6}{\partial z_1} & \frac{\partial f_6}{\partial \theta_x} & \frac{\partial f_6}{\partial \theta_y} & \frac{\partial f_6}{\partial \theta_z} \end{bmatrix} \quad (3.10.)$$

$$\vec{f} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{bmatrix} \quad (3.11.)$$

$$\frac{\partial f_i}{\partial x_1} = 2(x_1 + a_{xi}) \quad (3.12.)$$

$$\frac{\partial f_i}{\partial y_1} = 2(y_1 + a_{yi}) \quad (3.13.)$$

$$\frac{\partial f_i}{\partial z_1} = 2(z_1 + a_{zi}) \quad (3.14.)$$

$$\frac{\partial f_i}{\partial \theta_x} = 2x_1 \frac{\partial a_{xi}}{\partial \theta_x} + 2a_{xi} \frac{\partial a_{xi}}{\partial \theta_x} + 2y_1 \frac{\partial a_{yi}}{\partial \theta_x} + 2a_{yi} \frac{\partial a_{yi}}{\partial \theta_x} + 2z_1 \frac{\partial a_{zi}}{\partial \theta_x} + 2a_{zi} \frac{\partial a_{zi}}{\partial \theta_x} \quad (3.15.)$$

$$\frac{\partial a_{xi}}{\partial \theta_x} = v_{Bi} \cos \theta_z \sin \theta_y \cos \theta_x + v_{Bi} \sin \theta_z \sin \theta_x \quad (3.16.)$$

$$\frac{\partial a_{yi}}{\partial \theta_x} = v_{Bi} \sin \theta_z \sin \theta_y \cos \theta_x - v_{Bi} \cos \theta_z \sin \theta_x \quad (3.17.)$$

$$\frac{\partial a_{zi}}{\partial \theta_x} = v_{Bi} \cos \theta_y \cos \theta_x \quad (3.18.)$$

$$\frac{\partial f_i}{\partial \theta_y} = 2x_1 \frac{\partial a_{xi}}{\partial \theta_y} + 2a_{xi} \frac{\partial a_{xi}}{\partial \theta_y} + 2y_1 \frac{\partial a_{yi}}{\partial \theta_y} + 2a_{yi} \frac{\partial a_{yi}}{\partial \theta_y} + 2z_1 \frac{\partial a_{zi}}{\partial \theta_y} + 2a_{zi} \frac{\partial a_{zi}}{\partial \theta_y} \quad (3.19.)$$

$$\frac{\partial a_{xi}}{\partial \theta_y} = -u_{Bi} \cos \theta_z \sin \theta_y + v_{Bi} \cos \theta_z \cos \theta_y \sin \theta_x \quad (3.20.)$$

$$\frac{\partial a_{yi}}{\partial \theta_y} = -u_{Bi} \sin \theta_z \sin \theta_y + v_{Bi} \sin \theta_z \cos \theta_y \sin \theta_x \quad (3.21.)$$

$$\frac{\partial a_{zi}}{\partial \theta_y} = -u_{Bi} \cos \theta_y - v_{Bi} \sin \theta_y \sin \theta_x \quad (3.22.)$$

$$\frac{\partial f_i}{\partial \theta_z} = 2x_1 \frac{\partial a_{xi}}{\partial \theta_z} + 2a_{xi} \frac{\partial a_{xi}}{\partial \theta_z} + 2y_1 \frac{\partial a_{yi}}{\partial \theta_z} + 2a_{yi} \frac{\partial a_{yi}}{\partial \theta_z} + 2z_1 \frac{\partial a_{zi}}{\partial \theta_z} + 2a_{zi} \frac{\partial a_{zi}}{\partial \theta_z} \quad (3.23.)$$

$$\frac{\partial a_{xi}}{\partial \theta_z} = -u_{Bi} \sin \theta_z \cos \theta_y - v_{Bi} \sin \theta_z \sin \theta_y \sin \theta_x - v_{Bi} \cos \theta_z \cos \theta_x \quad (3.24.)$$

$$\frac{\partial a_{yi}}{\partial \theta_z} = u_{Bi} \cos \theta_z \cos \theta_y + v_{Bi} \cos \theta_z \sin \theta_y \sin \theta_x - v_{Bi} \sin \theta_z \cos \theta_x \quad (3.25.)$$

$$\frac{\partial a_{zi}}{\partial \theta_z} = 0 \quad (3.26.)$$

Main problem discussed here, calculation of displacement should be guaranteed at all position of fixator. Calculation of displacement which contains translation and rotation movement is important according to fixator parameters that are selected by orthopedist. For this reason, the set of linear equations must be solved for $\Delta \vec{X}$. Solution of the small changes vector is shown as follows.

$$\Delta \vec{X} = -J^{-1} \vec{f} \quad (3.27.)$$

A successive iteration scheme is needed for linearized displacement function necessary to provide non-linear displacement function (f_i). The following steps are carried out according to initial solution, where k represents the algorithm step running from 0 to m.

$$J^k \Delta \vec{X}^k = -\vec{f}^k (\vec{X}^k) \quad k = 0 - m \quad (3.28.)$$

$$\Delta \vec{X}^k = \begin{bmatrix} \Delta x_1 \\ \Delta y_1 \\ \Delta z_1 \\ \Delta \theta_x \\ \Delta \theta_y \\ \Delta \theta_z \end{bmatrix}^k \quad \vec{X}^k = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ \theta_x \\ \theta_y \\ \theta_z \end{bmatrix}^k \quad k = 0 - m \quad (3.29.)$$

$$\vec{f}^k(\vec{X}^k) = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{bmatrix}^k = \begin{bmatrix} f_1(\vec{X}^k) \\ f_2(\vec{X}^k) \\ f_3(\vec{X}^k) \\ f_4(\vec{X}^k) \\ f_5(\vec{X}^k) \\ f_6(\vec{X}^k) \end{bmatrix}^k \quad k = 0 - m \quad (3.30.)$$

$$J^k = \begin{bmatrix} \frac{\partial f_i}{\partial p_j} \end{bmatrix}^k \quad \begin{array}{l} i = 1 - 6 \\ j = 1 - 6 \\ k = 0 - m \end{array} \quad (3.31.)$$

$$\vec{X}^{k+1} = \vec{X}^k + \Delta \vec{X}^k \quad (3.32.)$$

The solution process above is terminated at m-th step if two successive error values (E^k, E^{k+1}) are below a previously determined threshold value (% *eps*).

$$E^k = \sqrt{\sum_{j=1}^6 (\Delta p_j^k)^2} \quad (3.33.)$$

$$(E^{k+1}/E^k) 100 < eps \quad (3.34.)$$

Convergence of iteration algorithm is always assured because the initial solution provided to the process at the beginning is obtained by inverse kinematics mentioned in (Akçalı et al., 2010b).

The next process, how many intermediate treatment steps should be taken from initial position to final position occurs according to the treatment strategy which is determined by orthopaedist. In a linear model, difference between the rod lengths corresponding to these in the initial position and final positions is divided by number of treatment step (n). In this way, how to increase the rod lengths is determined. The above-mentioned iterative solution process is repeated at each treatment step.

When all of the processes are considered, inverse of the Jacobian matrix should be calculated $n \times m$ times. For this reason determinant of the Jacobian matrix must be non-zero. The important point to be considered in the calculation of the determinant is elements of the matrix must be calculated with high precision. When computing the determinant of the matrix too many calculations are required. So even in the range of 1% deviation in rounding on the elements of matrix leads to different results of the different determinant value

This analysis may show a similar result in terms of velocity. Following matrix equation can be obtained when the function (f_i) defined in (3.1.) is differentiated with respect to time.

$$\left[\frac{\partial f_{i0}}{\partial p_j} \right] [\vec{V}] = \left[2 L_i \frac{dL_i}{dt} \right] \quad (3.35.)$$

$$\left[\frac{\partial f_{i0}}{\partial p_j} \right] = \left[\frac{\partial f_i}{\partial p_j} \right] = J \quad \begin{matrix} i = 1 - 6 \\ j = 1 - 6 \end{matrix} \quad (3.36.)$$

$$[\vec{V}] = \{dp_j/dt\}^T = \begin{bmatrix} dx_1/dt \\ dy_1/dt \\ dz_1/dt \\ d\theta_x/dt \\ d\theta_y/dt \\ d\theta_z/dt \end{bmatrix} \left[2 L_i \frac{dL_i}{dt} \right] \quad (3.37.)$$

In equation (3.35.) $\frac{dL_i}{dt}$ is symbolized desired rate of change in rod length. $\vec{V}$ is velocity vector of displacement and rotation on the distal ring. Also coefficient matrix, which is located in equation (3.35.), is nothing more than a Jacobian matrix. If determinant of the Jacobian matrix has no sign change we can say there is no singularity in terms of velocity.

3.1.2. Force Analysis

Another area in which singularity can be detected is the force analysis of the robotic structure. Force analysis under stable equilibrium conditions reduces to a set of linear equations. In order to calculate the forces in the system, the coefficient matrix should have a well-defined inverse. If the determinant of the coefficient matrix happens to take on zero value, then it will not be possible to complete the force analysis. In view of the fact that determinant of the coefficient matrix is a continuous function (of fixed and position variables) a sign change in its value is a potential risk for singularity.

The robotic structure in question here is a (6-6) type Stewart platform (SP), in Figure 3. 2. One of the areas that will cause singularity in the structure is the continuity of forces.

In order to determine under what conditions discontinuity of forces occurs the following parameters must be known. These are proximal and distal ring diameter R, R_1 respectively. Proximal joint coordinates A_i , distal joint coordinates B_i and rod lengths $A_i B_i = L_i$; $i = 1 - 6$. External force and moment which are acting at r_u, r_v point of the distal ring according to local coordinate system uvw are given as input, Figure 3. 2. In this case, effects on the distal joints and rods are shown at Figure 3. 3 and Figure 3. 4. Joint forces occurring along the rod directions and force on a representative rod are shown with relevant sign convention in the corresponding free-body diagram, Figure 3. 3. The sign of rod force in the form of compression is accepted as (+).

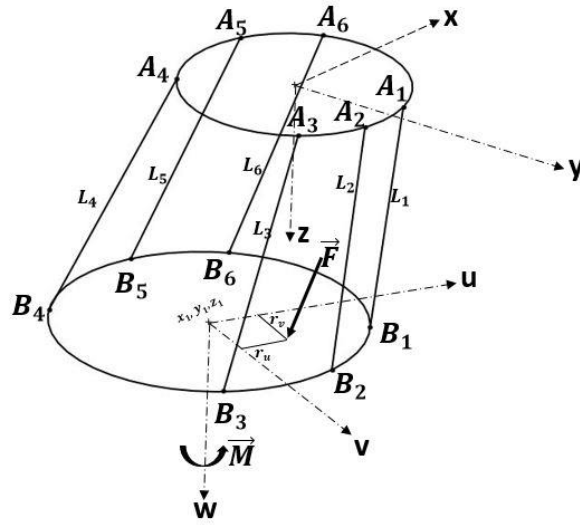


Figure 3. 2. The external fixator under the influence of moment and forces

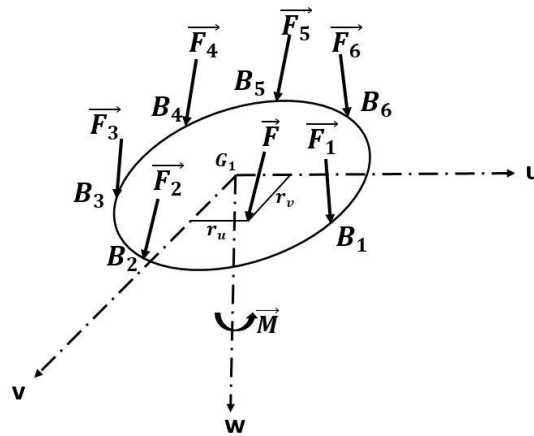


Figure 3. 3. Free-body diagram of the distal ring

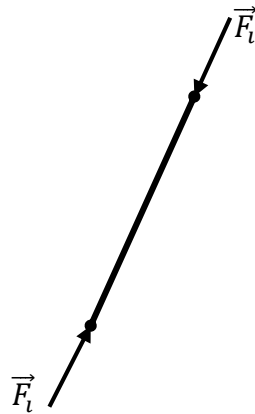


Figure 3. 4. Free-body diagram of the rod

Force vector $\vec{F}$ which is represented by components F_x, F_y, F_z according the xyz coordinate system at the distal ring center can be written depending on the respective unit vectors $\vec{i}, \vec{j}, \vec{k}$.

$$\vec{F} = F_x \vec{i} + F_y \vec{j} + F_z \vec{k} \quad (3.38.)$$

Alternatively, $\vec{F}$ is defined according to distal ring coordinate system (uvw) it can be written depending on the respective unit vectors ($\vec{e}_u, \vec{e}_v, \vec{e}_w$) as follows.

$$\vec{F} = F_u \vec{e}_u + F_v \vec{e}_v + F_w \vec{e}_w \quad (3.39.)$$

In this case, transformation matrix $[A_{xyz}^{uvw}]$ defined between the axes of xyz - uvw reference frames is used to transform forces into the world coordinate system xyz .

$$[A_{xyz}^{uvw}] = \text{rot}[\theta_x, \theta_y, \theta_z][e_u, e_v, e_w] = \begin{bmatrix} e_{ux} & e_{vx} & e_{wx} \\ e_{uy} & e_{vy} & e_{wy} \\ e_{uz} & e_{vz} & e_{wz} \end{bmatrix} \quad (3.40.)$$

Accordingly, transformation is done with the following calculation.

$$\begin{aligned} F_x &= F_u e_{ux} + F_v e_{vx} + F_w e_{wx} \\ F_y &= F_u e_{uy} + F_v e_{vy} + F_w e_{wy} \\ F_z &= F_u e_{uz} + F_v e_{vz} + F_w e_{wz} \end{aligned} \quad (3.41.)$$

Following equation is obtained when equilibrium of forces is written from the free-body diagram in Figure 3. 3 (Akçalı, 2012).

$$\sum_{m=1}^6 \vec{F}_m + \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \vec{F}_5 + \vec{F}_6 + \vec{F} = 0 \quad (3.42.)$$

Unit vector in the direction of the force $\vec{e}_m$ is to be calculated as follows.

$$\vec{e}_m = \frac{\overrightarrow{A_mB_m}}{L_m} = e_{mx} \vec{i} + e_{my} \vec{j} + e_{mz} \vec{k} \quad m = 1 - 6 \quad (3.43.)$$

Under these conditions, vector equation with number (3.42.) can be written in scalar form as follows:

$$\begin{aligned} F_1 e_{1x} + F_2 e_{2x} + F_3 e_{3x} + F_4 e_{4x} + F_5 e_{5x} + F_6 e_{6x} &= -F_x \\ F_1 e_{1y} + F_2 e_{2y} + F_3 e_{3y} + F_4 e_{4y} + F_5 e_{5y} + F_6 e_{6y} &= -F_y \\ F_1 e_{1z} + F_2 e_{2z} + F_3 e_{3z} + F_4 e_{4z} + F_5 e_{5z} + F_6 e_{6z} &= -F_z \end{aligned} \quad (3.44.)$$

F_{1-6} are magnitudes of the forces along the direction of rods $\vec{e}_1 - \vec{e}_6$.

Following equation can be obtained when equilibrium of moment is written relative to the distal ring center of the free-body diagram in Figure 3. 2.

$$\sum_{m=1}^6 \overrightarrow{G_1 B_m} \times \vec{F}_m + r\vec{F} \times \vec{F} + \vec{M} = 0 \quad (3.45.)$$

$r\vec{F}$ which is measured from G_1 point is moment arm vector of $\vec{F}$ force and it can be defined in two ways.

$$r\vec{F} = r_u \vec{e}_u + r_v \vec{e}_v \quad (3.46.)$$

Equation (3.46.) can be written using the transformation matrix $[A_{xyz}^{uvw}]$ elements as follows.

$$r\vec{F} = rF_x \vec{i} + rF_y \vec{j} + rF_z \vec{k} \quad (3.47.)$$

$$\begin{aligned}
rF_x &= r_u e_{ux} + r_v e_{vx} \\
rF_y &= r_u e_{uy} + r_v e_{vy} \\
rF_z &= r_u e_{uz} + r_v e_{vz}
\end{aligned} \tag{3.48.}$$

Moment $\vec{M}$ which is applied externally can be identified according to local coordinates (uvw) of distal ring or global coordinate system xyz as follows.

$$\vec{M} = M_u \vec{e}_u + M_v \vec{e}_v + M_w \vec{e}_w \tag{3.49.}$$

or

$$\vec{M} = M_x \vec{i} + M_y \vec{j} + M_z \vec{k} \tag{3.50.}$$

$$\begin{aligned}
M_x &= M_u e_{ux} + M_v e_{vx} + M_w e_{wx} \\
M_y &= M_u e_{uy} + M_v e_{vy} + M_w e_{wy} \\
M_z &= M_u e_{uz} + M_v e_{vz} + M_w e_{wz}
\end{aligned} \tag{3.51.}$$

Joint coordinates of the moment arm $\overrightarrow{G_1 B_m}$; $m = 1 - 6$ which is measured from distal ring center can be expressed as follows.

$$\overrightarrow{G_1 B_m} = G_1 B_{mx} \vec{i} + G_1 B_{my} \vec{j} + G_1 B_{mz} \vec{k} \tag{3.52.}$$

$$\left. \begin{aligned}
G_1 B_{mx} &= u_{Bm} \times e_{ux} + v_{Bm} \times e_{vx} \\
G_1 B_{my} &= u_{Bm} \times e_{uy} + v_{Bm} \times e_{vy} \\
G_1 B_{mz} &= u_{Bm} \times e_{uz} + v_{Bm} \times e_{vz}
\end{aligned} \right\} m = 1 - 6 \tag{3.53.}$$

Therefore, components of unit vectors e_{mx}, e_{my}, e_{mz} can be calculated as follows.

$$\left. \begin{aligned} e_{mx} &= \frac{1}{L_m} (x_1 + G_1 B_{mx} - x_{Am}) \\ e_{my} &= \frac{1}{L_m} (y_1 + G_1 B_{my} - y_{Am}) \\ e_{mz} &= \frac{1}{L_m} (z_1 + G_1 B_{mz} - z_{Am}) \end{aligned} \right\} m = 1 - 6 \quad (3.54.)$$

The measured coordinates of the center of the distal ring (G_1) according to the global coordinate system xyz is symbolized with (x_1, y_1, z_1) , proximal ring joint coordinates is symbolized with $(x_{Am}, y_{Am}, z_{Am}), m = 1 - 6$ again according to the xyz coordinate system. Rod lengths are represented by $L_m = |A_m B_m|, m = 1 - 6$. Also we need to remember joint force vector $\vec{F}_m$ as follows.

$$\vec{F}_m = F_m (e_{mx} \vec{i} + e_{my} \vec{j} + e_{mz} \vec{k}) \quad m = 1 - 6 \quad (3.55.)$$

The following scalar equations can be obtained when equations numbered (3.47.), (3.52.), (3.55.) are substituted into equation (3.45.).

$$\begin{aligned} a_{1x}F_1 + a_{2x}F_2 + a_{3x}F_3 + a_{4x}F_4 + a_{5x}F_5 + a_{6x}F_6 &= -M_x - m_x \\ a_{1y}F_1 + a_{2y}F_2 + a_{3y}F_3 + a_{4y}F_4 + a_{5y}F_5 + a_{6y}F_6 &= -M_y - m_y \\ a_{1z}F_1 + a_{2z}F_2 + a_{3z}F_3 + a_{4z}F_4 + a_{5z}F_5 + a_{6z}F_6 &= -M_z - m_z \end{aligned} \quad (3.56.)$$

$$\left. \begin{aligned} a_{mx} &= G_1 B_{my} e_{mz} - G_1 B_{mz} e_{my} \\ a_{my} &= G_1 B_{mz} e_{mx} - G_1 B_{mx} e_{mz} \\ a_{mz} &= G_1 B_{mx} e_{my} - G_1 B_{my} e_{mx} \end{aligned} \right\} m = 1 - 6 \quad (3.57.)$$

Components of moment $\vec{m}$ generated by the externally applied force are defined as follows:

$$\begin{aligned}
m_x &= rF_y F_z - rF_z F_y \\
m_y &= rF_z F_x - rF_x F_z \\
m_z &= rF_x F_y - rF_y F_x
\end{aligned} \tag{3.58}$$

The equilibrium equations (3.44.), (3.56.) are obtained above which contain magnitudes of the six joint forces with known directions lead to a set of linear equations as indicated below:

$$\begin{bmatrix} e_{1x} & e_{2x} & e_{3x} & e_{4x} & e_{5x} & e_{6x} \\ e_{1y} & e_{2y} & e_{3y} & e_{4y} & e_{5y} & e_{6y} \\ e_{1z} & e_{2z} & e_{3z} & e_{4z} & e_{5z} & e_{6z} \\ a_{1x} & a_{2x} & a_{3x} & a_{4x} & a_{5x} & a_{6x} \\ a_{1y} & a_{2y} & a_{3y} & a_{4y} & a_{5y} & a_{6y} \\ a_{1z} & a_{2z} & a_{3z} & a_{4z} & a_{5z} & a_{6z} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix} = \begin{bmatrix} -F_x \\ -F_y \\ -F_z \\ -M_x - m_x \\ -M_y - m_y \\ -M_z - m_z \end{bmatrix} \tag{3.59}$$

Determinant of the force coefficient matrix must be non-zero value at (3.59.) matrix equation for the calculation of forces. Determinant value must be checked at each position of the fixator to see there is a singular case or not. To this end, determinant of the force coefficient matrix should not change sign within the motion domain.

3.1.3. Geometric Method - Closest Point Approach (CPA)

Closest point approach (CPA) is a geometric method. The purpose is to research the intersection possibilities of lines in 3D space. First, the shortest distance between any two lines in 3D space and the coordinates of the closest points on the given lines are formulated. By examining whether the distance is zero or not, the intersection situation of two lines will be determined. Secondly, this method will be utilized to decide whether or not two, three, four etc. combinations of rods in the mechanical structure of the fixator intersect at common points.

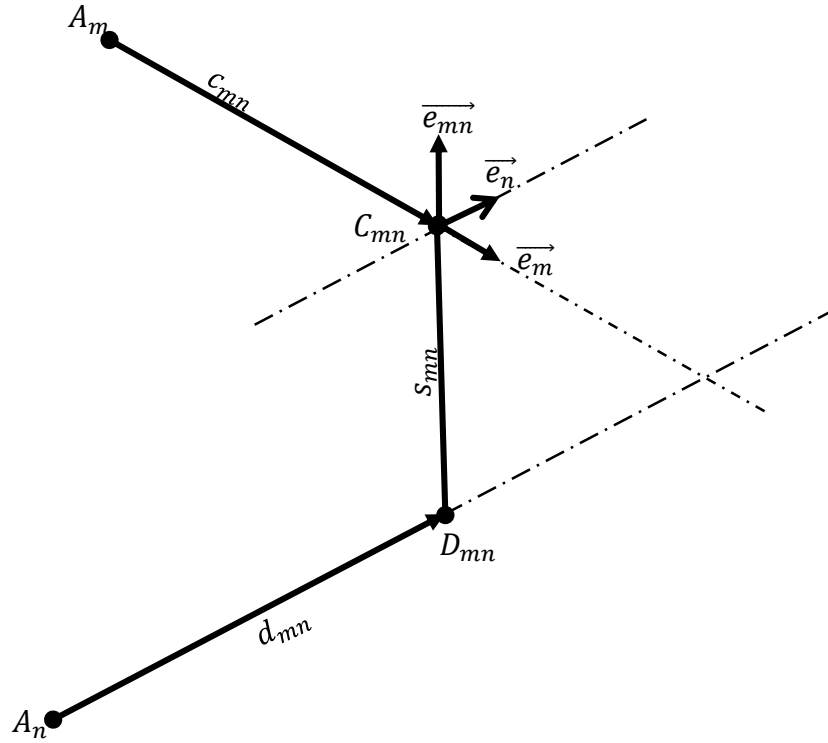


Figure 3. 5. Closest points on” two lines in 3D –Space

Let A_m and A_n be two points in 3D space, e_m and e_n unit vectors passing through the relevant points, respectively. In Figure 3. 5, C_{mn} and D_{mn} are two points where the two lines are closest to each other. c_{mn} designates distance between point A_m and point C_{mn} , s_{mn} denotes the distance between points C_{mn} and D_{mn} . d_{mn} is the distance between points A_n and D_{mn} . In this approach, s_{mn} distance is important for the analysis. e_{mn} is a unit vector along distance s_{mn} perpendicular to the unit vectors e_m and e_n simultaneously. (i.e $\overrightarrow{e_{mn}} \perp \overrightarrow{e_m}$ and $\overrightarrow{e_{mn}} \perp \overrightarrow{e_n}$)

$$\overrightarrow{e_{mn}} = \frac{\overrightarrow{e_m} \times \overrightarrow{e_n}}{|\overrightarrow{e_m} \times \overrightarrow{e_n}|} \quad (3.60.)$$

Due to the orthogonality conditions the following equations can be written:

$$\overrightarrow{e_{mn}} \cdot \overrightarrow{e_m} = 0 \quad (3.61.)$$

$$\overrightarrow{e_{mn}} \cdot \overrightarrow{e_n} = 0 \quad (3.62.)$$

$\overrightarrow{e_m}$ and $\overrightarrow{e_n}$ unit vectors can be written in terms of unit vector in the direction of the any xyz coordinate system axis $(\vec{i}, \vec{j}, \vec{k})$ and its components $(e_{mx}, e_{my}, e_{mz}; e_{nx}, e_{ny}, e_{nz})$.

$$\overrightarrow{e_m} = e_{mx} \vec{i} + e_{my} \vec{j} + e_{mz} \vec{k} \quad (3.63.)$$

$$\overrightarrow{e_n} = e_{nx} \vec{i} + e_{ny} \vec{j} + e_{nz} \vec{k} \quad (3.64.)$$

When (3.63.) and (3.64.) expressions are substituted into equation (3.60.) the components of $\overrightarrow{e_{mn}}$ unit vector can be written as follows.

$$\overrightarrow{e_{mn}} = e_{mnx} \vec{i} + e_{mny} \vec{j} + e_{mnz} \vec{k} \quad (3.65.)$$

$$e_{mnx} = (e_{my} e_{nz} - e_{mz} e_{ny}) / \left(\sqrt{e_{mnx}^2 + e_{mny}^2 + e_{mnz}^2} \right) \quad (3.66.)$$

$$e_{mny} = e_{mz} e_{nx} - e_{mx} e_{nz} / \left(\sqrt{e_{mnx}^2 + e_{mny}^2 + e_{mnz}^2} \right) \quad (3.67.)$$

$$e_{mnz} = e_{mx} e_{ny} - e_{my} e_{nx} / \left(\sqrt{e_{mnx}^2 + e_{mny}^2 + e_{mnz}^2} \right) \quad (3.68.)$$

The following vector equation is written from Figure 3. 5:

$$\overrightarrow{A_m A_n} = \overrightarrow{A_m C_{mn}} + \overrightarrow{C_{mn} D_{mn}} + \overrightarrow{D_{mn} A_n} \quad (3.69.)$$

(3.69.) vector equation is written as follows in terms of unit vectors:

$$\overrightarrow{A_m A_n} = c_{mn} \overrightarrow{e_m} - s_{mn} \overrightarrow{e_{mn}} - d_{mn} \overrightarrow{e_n} \quad (3.70.)$$

Unknown scalar quantities are c_{mn} , s_{mn} and d_{mn} in the equation (3.70.). Both sides of the equation (3.70.) is multiplied by $\overrightarrow{e_m}$ unit vector. Then we can obtain these values.

$$\overrightarrow{A_m A_n} \cdot \overrightarrow{e_m} = c_{mn} \overrightarrow{e_m} \cdot \overrightarrow{e_m} - s_{mn} \overrightarrow{e_{mn}} \cdot \overrightarrow{e_m} - d_{mn} \overrightarrow{e_n} \cdot \overrightarrow{e_m} \quad (3.71.)$$

If expressions (3.61.), (3.62.) and $\overrightarrow{e_m} \cdot \overrightarrow{e_m} = 1$ are used in (3.71.), the following is obtained:

$$\overrightarrow{A_m A_n} \cdot \overrightarrow{e_m} = c_{mn} - d_{mn} (\overrightarrow{e_n} \cdot \overrightarrow{e_m}) \quad (3.72.)$$

Then both sides of the equation (3.70.) is multiplied by $\overrightarrow{e_n}$ unit vector.

$$\overrightarrow{A_m A_n} \cdot \overrightarrow{e_n} = c_{mn} \overrightarrow{e_m} \cdot \overrightarrow{e_n} - s_{mn} \overrightarrow{e_{mn}} \cdot \overrightarrow{e_n} - d_{mn} \overrightarrow{e_n} \cdot \overrightarrow{e_n} \quad (3.73.)$$

If expressions (3.61.), (3.62.) and $\overrightarrow{e_n} \cdot \overrightarrow{e_n} = 1$ are used in (3.73.), the following is obtained:

$$\overrightarrow{A_m A_n} \cdot \overrightarrow{e_n} = c_{mn} (\overrightarrow{e_n} \cdot \overrightarrow{e_m}) - d_{mn} \quad (3.74.)$$

c_{mn} and d_{mn} are obtained as follows from common solution of the (3.72.) and (3.74.) linear equations:

$$d_{mn} = \frac{(\overrightarrow{A_m A_n} \cdot \overrightarrow{e_n}) - (\overrightarrow{A_m A_n} \cdot \overrightarrow{e_m})(\overrightarrow{e_m} \cdot \overrightarrow{e_n})}{(\overrightarrow{e_m} \cdot \overrightarrow{e_n})^2 - 1} \quad (3.75.)$$

$$c_{mn} = \frac{(\overrightarrow{A_m A_n} \cdot \overrightarrow{e_m}) - (\overrightarrow{A_m A_n} \cdot \overrightarrow{e_n})(\overrightarrow{e_m} \cdot \overrightarrow{e_n})}{1 - (\overrightarrow{e_m} \cdot \overrightarrow{e_n})^2} \quad (3.76.)$$

Both sides of the equation (3.70.) is multiplied by $\overrightarrow{e_{mn}}$ unit vector, the following is obtained:

$$\overrightarrow{A_m A_n} \cdot \overrightarrow{e_{mn}} = c_{mn} \overrightarrow{e_m} \cdot \overrightarrow{e_{mn}} - s_{mn} \overrightarrow{e_{mn}} \cdot \overrightarrow{e_{mn}} - d_{mn} \overrightarrow{e_n} \cdot \overrightarrow{e_{mn}} \quad (3.77.)$$

If expressions (3.61.), (3.62.) and $\overrightarrow{e_{mn}} \cdot \overrightarrow{e_{mn}} = 1$ are substituted in (3.77.), the closest perpendicular distance formula s_{mn} is obtained:

$$s_{mn} = -\overrightarrow{A_m A_n} \cdot \overrightarrow{e_{mn}} \quad (3.78.)$$

Closest point coordinates C_{mn} and D_{mn} can be calculated as follows when the starting point of any xyz coordinate system is “O”.

$$\overrightarrow{OC_{mn}} = \overrightarrow{A_m} + c_{mn} \overrightarrow{e_m} \quad (3.79.)$$

$$\overrightarrow{OD_{mn}} = \overrightarrow{A_n} + d_{mn} \overrightarrow{e_n} \quad (3.80.)$$

If the denominator turns out to be zero in equations (3.75.) and (3.76.) then lines become parallel to each other.

Closest point approach (CPA) will be implemented to external fixator. One way to determine whether random two lines intersect or not in space, the closest distance between two lines is to be calculated. If we consider that there are six rods which connect the two rings of the fixator, we should investigate all the binary combinations of the rods whether such is this case. Upon actuating the rods in the system, the external fixator makes a six-degree-of-freedom motion; but when the rod lengths are kept constant, then the robotic structure is supposed to behave like a truss with zero degree of freedom. However, it becomes moveable truss when more than three rods intersect at common points. This is not a desirable situation from a medical point of view. In order to detect this type of singularity, the so-called closest point approach is to be applied to (6-6) SP.

Closest point approach is to be applied to all binary combinations of rods in the fixator. There are 15 ($6 \times 5/2$) binary combinations. The distance between rod directions for all binary combinations are checked whether they happen to be zero or not. The coordinates of the intersection of binary combinations whereby distance is zero are compared for whether there is at least one such case.

The method described above is applied to all intermediate configurations from initial to final one. In this way, the fixator is checked against singularity.

In Figure 3. 6, rod directions are represented by $A_m B_m$. Proximal ring and distal ring centers are represented by G and G_1 , respectively. G_{xyz} reference system is located at G and G_{1uvw} reference system is fixed to point G_1 . Proximal joint coordinates A_m , distal joint coordinates B_m are calculated as follows, depending on primary input $(\alpha_i, \varepsilon_i, R, R_1; i = 1 - 6)$ together with $(x_1, y_1, z_1, \theta_x, \theta_y, \theta_z)$ motion variables.

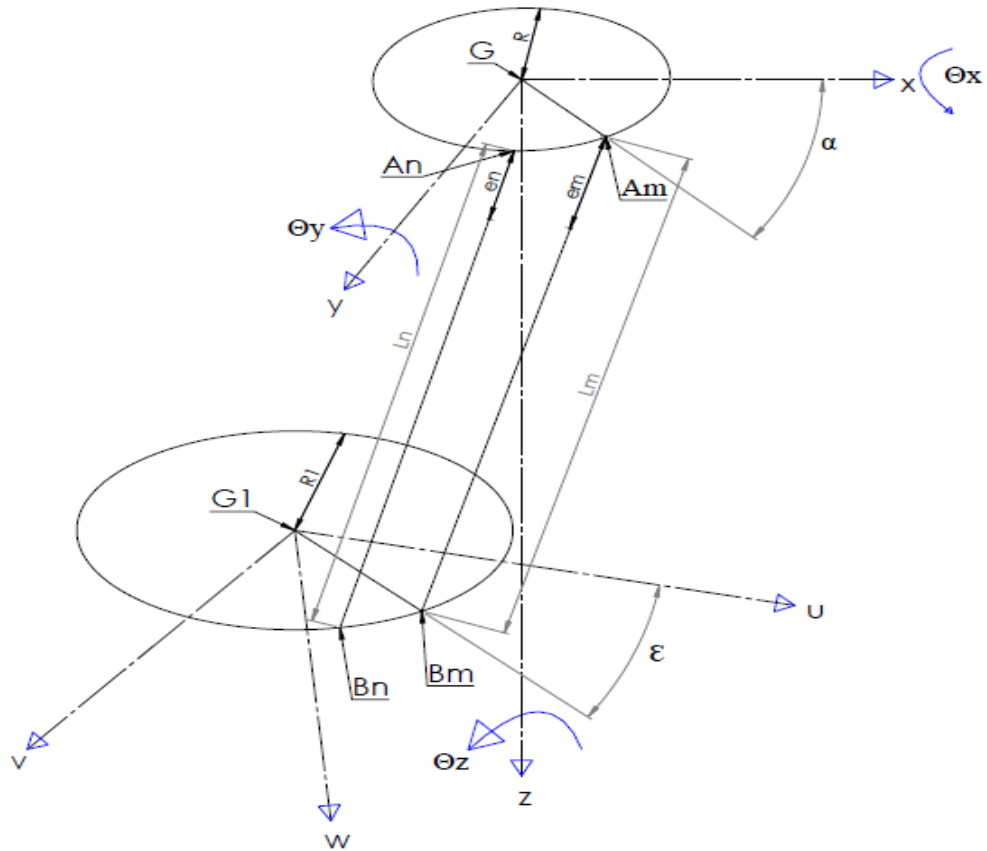


Figure 3. 6. Application of closest point approach

A_m is expressed in the global coordinate system, as follows:

$$\overrightarrow{GA_m} = A_{mx}\vec{i} + A_{my}\vec{j} \quad (3.81.)$$

$$A_{mx} = R \times \cos \alpha_m \quad (3.82.)$$

$$A_{my} = R \times \sin \alpha_m \quad (3.83.)$$

The following transformation matrix is used to calculate coordinates of B_m , in which $e_{ux}, e_{uy}, e_{uz}, e_{vx}, e_{vy}, e_{vz}, e_{wx}, e_{wy}, e_{wz}$ are the components of the unit vectors of u, v, w coordinate system $(\vec{e}_u, \vec{e}_v, \vec{e}_w)$ along the x, y, z axes.

$$[A_{xyz}^{uvw}] = \text{rot}[\theta_x, \theta_y, \theta_z][e_u, e_v, e_w] = \begin{bmatrix} e_{ux} & e_{vx} & e_{wx} \\ e_{uy} & e_{vy} & e_{wy} \\ e_{uz} & e_{vz} & e_{wz} \end{bmatrix} \quad (3.84.)$$

$$[A_{xyz}^{uvw}] = \begin{bmatrix} \cos \theta_z \cos \theta_y & \cos \theta_z \sin \theta_y \sin \theta_x - \sin \theta_z \cos \theta_x & \cos \theta_z \sin \theta_y \cos \theta_x + \sin \theta_z \sin \theta_x \\ \sin \theta_z \cos \theta_y & \sin \theta_z \sin \theta_y \sin \theta_x + \cos \theta_z \cos \theta_x & \sin \theta_z \sin \theta_y \cos \theta_x - \cos \theta_z \sin \theta_x \\ -\sin \theta_y & \cos \theta_y \sin \theta_x & \cos \theta_y \cos \theta_x \end{bmatrix} \quad (3.85.)$$

$$e_{ux} = \cos \theta_z \cos \theta_y \quad (3.86.)$$

$$e_{uy} = \sin \theta_z \cos \theta_y \quad (3.87.)$$

$$e_{uz} = -\sin \theta_y \quad (3.88.)$$

$$e_{vx} = \cos \theta_z \sin \theta_y \sin \theta_x - \sin \theta_z \cos \theta_x \quad (3.89.)$$

$$e_{vy} = \sin \theta_z \sin \theta_y \sin \theta_x + \cos \theta_z \cos \theta_x \quad (3.90.)$$

$$e_{vz} = \cos \theta_y \sin \theta_x \quad (3.91.)$$

$$e_{wx} = \cos \theta_z \sin \theta_y \cos \theta_x + \sin \theta_z \sin \theta_x \quad (3.92.)$$

$$e_{wy} = \sin \theta_z \sin \theta_y \cos \theta_x - \cos \theta_z \sin \theta_x \quad (3.93.)$$

$$e_{wz} = \cos \theta_y \cos \theta_x \quad (3.94.)$$

Unit vectors can be written as follows:

$$\vec{e}_u = e_{ux} \vec{i} + e_{uy} \vec{j} + e_{uz} \vec{k} \quad (3.95.)$$

$$\vec{e}_v = e_{vx} \vec{i} + e_{vy} \vec{j} + e_{vz} \vec{k} \quad (3.96.)$$

$$\vec{e}_w = e_{wx} \vec{i} + e_{wy} \vec{j} + e_{wz} \vec{k} \quad (3.97.)$$

Local coordinates of joints in the distal ring are represented by u_{Bm} and v_{Bm} . The distance vector between the distal joint and the distal ring center and its components are symbolized by $\overrightarrow{G_1 B_m}$, $G_1 B_{mx}$, $G_1 B_{my}$, $G_1 B_{mz}$, respectively.

$$\overrightarrow{G_1 B_m} = u_{Bm} \vec{e}_u + v_{Bm} \vec{e}_v \quad (3.98.)$$

$$u_{Bm} = R_1 \times \cos \varepsilon_m \quad (3.99.)$$

$$v_{Bm} = R_1 \times \sin \varepsilon_m \quad (3.100.)$$

$\overrightarrow{G_1 B_m}$ vector can be written as follows:

$$\overrightarrow{G_1 B_m} = G_1 B_{mx} \vec{i} + G_1 B_{my} \vec{j} + G_1 B_{mz} \vec{k} \quad (3.101.)$$

$$G_1 B_{mx} = u_{Bm} \times e_{ux} + v_{Bm} \times e_{vx} \quad (3.102.)$$

$$G_1 B_{my} = u_{Bm} \times e_{uy} + v_{Bm} \times e_{vy} \quad (3.103.)$$

$$G_1 B_{mz} = u_{Bm} \times e_{uz} + v_{Bm} \times e_{vz} \quad (3.104.)$$

B_m is expressed in the global coordinate system, as follows:

$$GB_{mx} = x_1 + G_1 B_{mx} \quad (3.105.)$$

$$GB_{my} = y_1 + G_1 B_{my} \quad (3.106.)$$

$$GB_{mz} = z_1 + G_1 B_{mz} \quad (3.107.)$$

To calculate rod unit vector $\overrightarrow{e_m}$, we need rod vector directed from proximal to distal ring $\overrightarrow{A_m B_m}$ and rod length L_m , $m = 1 - 6$.

$$\overrightarrow{A_m B_m} = \overrightarrow{GB_m} - \overrightarrow{GA_m} = (A_m B_m)_x \vec{i} + (A_m B_m)_y \vec{j} + (A_m B_m)_z \vec{k} \quad (3.108.)$$

$$(A_m B_m)_x = B_{mx} - A_{mx} \quad (3.109.)$$

$$(A_m B_m)_y = B_{my} - A_{my} \quad (3.110.)$$

$$(A_m B_m)_z = B_{mz} - A_{mz} \quad (3.111.)$$

$$L_m = \sqrt{\overrightarrow{A_m B_m} \cdot \overrightarrow{A_m B_m}} = \sqrt{(A_m B_m)_x^2 + (A_m B_m)_y^2 + (A_m B_m)_z^2} \quad (3.112.)$$

$$\overrightarrow{e_m} = \frac{\overrightarrow{A_m B_m}}{L_m} = e_{mx} \vec{i} + e_{my} \vec{j} + e_{mz} \vec{k} \quad (3.113.)$$

$$e_{mx} = \frac{(A_mB_m)_x}{L_m} \quad (3.114.)$$

$$e_{my} = \frac{(A_mB_m)_y}{L_m} \quad (3.115.)$$

$$e_{mz} = \frac{(A_mB_m)_z}{L_m} \quad (3.116.)$$

$\overrightarrow{A_mA_n}, \overrightarrow{e_m}, \overrightarrow{e_n}$ can be calculated such that $m = 1 - 6, n = 1 - 6, m \neq n, m < n$ at CPA. 15 s_{mn}, c_{mn}, d_{mn} values can be obtained using these vectors and (3.75.), (3.76.), (3.78.) equations. C_{mn} and D_{mn} point coordinates should calculate when s_{mn} is equal to zero.

$$\overrightarrow{A_mA_n} = \overrightarrow{GA_n} - \overrightarrow{GA_m} = (A_mB_m)_x \vec{i} + (A_mB_m)_y \vec{j} + (A_mB_m)_z \vec{k} \quad (3.117.)$$

$$(A_mB_m)_x = B_{mx} - A_{mx} \quad (3.118.)$$

$$(A_mB_m)_y = B_{my} - A_{my} \quad (3.119.)$$

$$(A_mB_m)_z = B_{mz} - A_{mz} \quad (3.120.)$$

$$\overrightarrow{GC_{mn}} = \overrightarrow{GA_m} + c_{mn} \cdot \overrightarrow{e_m} = (GC_{mn})_x \vec{i} + (GC_{mn})_y \vec{j} + (GC_{mn})_z \vec{k} \quad (3.121.)$$

$$\overrightarrow{GD_{mn}} = \overrightarrow{GA_n} + d_{mn} \cdot \overrightarrow{e_n} = (GD_{mn})_x \vec{i} + (GD_{mn})_y \vec{j} + (GD_{mn})_z \vec{k} \quad (3.122.)$$

By examining these coordinates, it is determined whether direction of the three or more rods intersect at a point or not. If there is no intersection like this we can say there is no singularity by the method described above. Otherwise, location of one of the joints $(\alpha_i, \varepsilon_i; i = 1 - 6)$ is changed and calculations are repeated to avoid singularity.

3.2. Developed Algorithms for Interfaces

Algorithms have been developed by the use of methods whose details are given in the previous section.

3.2.1. Algorithms for Singularity Detection under Motion

Algorithm whose aim is to detect singularity under motion contains a number of other auxiliary algorithms. It is assumed that the rod lengths of the known system are biomechanical both for initial and final configurations ($L_{10}, L_{20}, L_{30}, L_{40}, L_{50}, L_{60}, L_{1n}, L_{2n}, L_{3n}, L_{4n}, L_{5n}, L_{6n}$), initial solution vector which is related to initial configuration variables ($x_1, y_1, z_1, \theta_x, \theta_y, \theta_z$) is needed to start the calculation process derived from the work “A Mathematical Model in the Implementation of a Stewart-Gough Platform as an External Fixator” (Akçalı et al., 2010b). This section includes calculation of rod lengths and solution vector for all intermediate configurations, solution of the system of linear equation by Cramer method, evaluation algorithm of singularity.

3.2.1.1. Calculation of Rod Length for Intermediate Configurations

Rod lengths of the corresponding configuration is required for calculation of the motion variables ($x_1, y_1, z_1, \theta_x, \theta_y, \theta_z$) in all intermediate position. The algorithm steps are listed for determining the length of the rods based on a linear model which are used in the calculation of the motion variables.

- i. Rod lengths of the fixator both for initial (unhealthy) and final (healthy) configurations are calculated as output of the TUBİTAK project 106M466. These data are obtained by entering X-ray film and clinical data.
- ii. Length difference is calculated for each rod by the following formula.

$$\Delta L_i = L_{in} - L_{i0} ; i = 1 - 6$$

- iii. The length difference of the rod (ΔL_i) is divided by step number (n). So, the length difference of the amount of step (a_i) is calculated as follows.

$$a_i = \frac{\Delta L_{1i}}{n} ; i = 1 - 6$$

- iv. The rod lengths in the secondary configuration ($L_{i1}, i = 1 - 6$) is obtained when the length difference of the amount of step ($a_i, i = 1 - 6$) and the rod lengths in the initial configuration ($L_{i0}, i = 1 - 6$) are summed.
- v. The rod lengths in each intermediate configuration ($L_{ij}, i = 1 - 6, j = 3, \dots, n$) are calculated when previous rod length ($L_{i(j-1)}, i = 1 - 6, j = 2, 3, \dots, n$) and the length difference of the amount of step ($a_i, i = 1 - 6$) are collected.

3.2.1.2. Calculation of Solution Vector for Intermediate Configuration and Singularity Evaluation

Solution of the vector ($x_1, y_1, z_1, \theta_x, \theta_y, \theta_z$) which is created by motion variables between initial and final configurations are calculated by successive calculations during the treatment process. This solution is obtained by the following algorithm steps.

- i. Parameters of the fixator ($\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6$) (R, R_1), rod lengths of the corresponding configuration ($L_{ij}, i = 1 - 6, j = 0, 1, \dots, n$), threshold value (%e) and initial solution vector of the corresponding configuration ($x_1, y_1, z_1, \theta_x, \theta_y, \theta_z$) are inputs of this algorithm.

- ii. Proximal joint coordinates $(x_{A_i}, y_{A_i}, i = 1 - 6)$ and distal joint coordinates $(u_{B_i}, v_{B_i}, i = 1 - 6)$ are calculated by equation (3.3.).
- iii. $\left(a_{x_i}, a_{y_i}, a_{z_i}, \frac{\partial a_{x_i}}{\partial \theta_x}, \frac{\partial a_{y_i}}{\partial \theta_x}, \frac{\partial a_{z_i}}{\partial \theta_x}, \frac{\partial a_{x_i}}{\partial \theta_y}, \frac{\partial a_{y_i}}{\partial \theta_y}, \frac{\partial a_{z_i}}{\partial \theta_y}, \frac{\partial a_{x_i}}{\partial \theta_z}, \frac{\partial a_{y_i}}{\partial \theta_z}, \frac{\partial a_{z_i}}{\partial \theta_z}, i = 1 - 6\right)$ are computed by equations (3.4.), (3.16.) - (3.18.), (3.20.) - (3.18.)(3.22.), (3.24.) - (3.26.) and $(\theta_x, \theta_y, \theta_z)$.
- iv. $\left(\frac{\partial f_i}{\partial \theta_x}, \frac{\partial f_i}{\partial \theta_y}, \frac{\partial f_i}{\partial \theta_z}, i = 1 - 6\right)$ are calculated by equations (3.15.), (3.19.), (3.23.).
- v. $\left(\frac{\partial f_i}{\partial x_1}, \frac{\partial f_i}{\partial y_1}, \frac{\partial f_i}{\partial z_1}, i = 1 - 6\right)$ are calculated by equations (3.12.) - (3.14.) depending on the displacement variables (x_1, y_1, z_1) .
- vi. Determinant of the Jacobian matrix (3.10.) is calculated.
- vii. $(f_i, i = 1 - 6)$ vector (3.11.) is generated by equation (3.2.).
- viii. Difference vector $\overrightarrow{\Delta X} = (\Delta x_1, \Delta y_1, \Delta z_1, \Delta \theta_x, \Delta \theta_y, \Delta \theta_z)$ is solved using equation (3.9.) or equation (3.27.) and Cramer's method that is given in the chapter (3.2.1.4).
- ix. The new solution vector $\vec{X}^{k+1}$ is obtained by summing the previous difference vector $\overrightarrow{\Delta X}^k$ and preliminary solution vector $\vec{X}^k$ according to equation (3.32.).
- x. Error value E^k is calculated using equation (3.33.). If percentage of two consecutive error values E^{k+1}, E^k exceeds the determined threshold value (eps) (3.34.) the solution process is terminated otherwise computation is repeated until it is less than the threshold value.
- xi. If sign of the determinant is different from the sign of the previous one during the treatment this condition is detected as singularity.

3.2.1.3. Singularity Analysis In Terms of Velocity

Singularity detection in terms of velocity is performed when the calculation of displacement and velocity of the distal ring is done at each configuration. Coefficient

matrix is equal to Jacobian matrix at calculation of both displacement and velocity (3.36.).

- i. Parameters of the fixator $(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6)$ (R, R_1) , rod lengths $(L_{ij}, i = 1 - 6, j = 0, 1, \dots, n)$ and solution vectors $((x_1, y_1, z_1, \theta_x, \theta_y, \theta_z)_j, j = 0, 1, \dots, n)$ of the corresponding configuration and actuation rates of the rods $((\frac{dL_i}{dt})_j, j = 0, 1, \dots, n)$ are inputs of this algorithm.
- ii. Jacobian matrix in the corresponding position is calculated by that is calculated as shown in the previous section.
- iii. Vector $[2 L_i \frac{dL_i}{dt}]_j, i = 1 - 6, j = 0, 1, \dots, n$ is generated using equation (3.35.).
- iv. Velocity vector $[\vec{V}]_j = [\frac{dx_1}{dt}, \frac{dy_1}{dt}, \frac{dz_1}{dt}, \frac{d\theta_x}{dt}, \frac{d\theta_y}{dt}, \frac{d\theta_z}{dt}]_j, j = 0, 1, \dots, n$ is computed by solving the system of linear equation (3.35.). Cramer method described in the next section is used in solution.
- v. The preceding process is repeated from $j = 0$ to $j = n$.

3.2.1.4. Solution of Systems of Linear Equations by Cramer's Method

Cramer's algorithm is used both in the iterative solution and in the velocity solution as well. It is an algorithm for solving square linear systems using determinants.

- i. Elements of the coefficient matrix and column vector which is located on the right side of the system of linear equations are inputs of this algorithm.
- ii. The determinant of the coefficient matrix is calculated.

- iii. Elements of the coefficient matrix column vector corresponding to the unknown vector are interchanged with vector elements on the right side. Then the determinant of the newly formed matrix is calculated.
- iv. Values are calculated as the ratio of the determinants found in (ii) and (iii) steps according to the order in which unknowns appear in the column vector.

3.2.2. Algorithms for Singularity Detection under Static Equilibrium

Singularity of the robotic structure was examined by two methods described in the chapter (3.1) (Force Analysis and Geometric Method - Closest point approach) mathematically under static equilibrium. The algorithms developed for these methods are discussed here.

3.2.2.1. Singularity Detection by Force Analysis

Mathematical theory for the calculation of the forces was given in the chapter (3.1.2). There are two main objectives of the force analysis. First one is to check the force continuity. In other words, effects of applying external forces and moments to distal ring should not be discontinuous throughout the treatment. If a discontinuity is encountered, the fixator configuration becomes unstable. This method is useful when the fixator is not moving, meaning that the treatment is in the stable state waiting for the time of movement to next treatment step. The second one is to calculate the forces at each treatment step when the first step is verified. Algorithm of the force analysis is described below.

- i. Parameters of the fixator ($\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6$) (R, R_1), components of external force and moment ($F_x, F_y, F_z, M_x, M_y, M_z$) which are defined according to the global coordinate system also application point of the external force (rF_x, rF_y, rF_z) are entered as input. If $r\vec{F}$ is defined according to local

coordinate system, components of this point (rF_u, rF_v) are entered as application point.

- ii. Displacement analysis result $(x_1, y_1, z_1, \theta_x, \theta_y, \theta_z)$ which is done all treatment step are entered as a text file (problem.txt).
- iii. Components of the unit vectors of u, v, w coordinate system ($\vec{e}_u, \vec{e}_v, \vec{e}_w$) along the x, y, z axes are $e_{ux}, e_{uy}, e_{uz}, e_{vx}, e_{vy}, e_{vz}, e_{wx}, e_{wy}, e_{wz}$. These values are calculated by using equations (3.86.) - (3.94.).
- iv. Using the transformation matrix, all the values of external force and moment components defined by $(F_u, F_v, F_w), (M_u, M_v, M_w)$ and the force application point $(r_u, r_v, 0)$ in the local u, v, w system are transformed into global coordinates resulting in (F_x, F_y, F_z) by eq. (3.41.), (M_x, M_y, M_z) by eq. (3.51.) and (rF_x, rF_y, rF_z) by eq. (3.48.), respectively.
- v. Components of moment $\vec{m}$ generated by the externally applied force (m_x, m_y, m_z) are calculated by using equation (3.58.).
- vi. Proximal joint coordinates $(x_{A_i}, y_{A_i}, i = 1 - 6)$ and distal joint coordinates $(u_{B_i}, v_{B_i}, i = 1 - 6)$ are calculated by equation (3.3.).
- vii. $\vec{G_1 B_m}$, $(m = 1 - 6)$ with components $(G_1 B_{mx}, G_1 B_{my}, G_1 B_{mz})$ symbolizes x, y, z components of the distance vector from the distal ring center (G_1) to the distal joint. These components are calculated by equation (3.53.).
- viii. Position data (x_1, y_1, z_1) are used to calculate distance from proximal ring center to distal joints $(GB_{mx}, GB_{my}, GB_{mz})$. This computation is done according to (3.105.) - (3.107.). Components of rod vector $((A_m B_m)_x, (A_m B_m)_y, (A_m B_m)_z; m = 1 - 6)$ are calculated by equations (3.109.) - (3.111.). Rod lengths $(L_m, m = 1 - 6)$ and rod unit vectors (e_{mx}, e_{my}, e_{mz}) are computed by equations (3.112.), (3.114.) - (3.116.), respectively.
- ix. Components of moment arm vector (a_{mx}, a_{my}, a_{mz}) belonging to joint forces $(F_m, m = 1 - 6)$ are calculated by (3.57.).

- x. The determinant of the force coefficient matrix in the equation (3.59.) is computed.
- xi. Matrix equation (3.59.) is solved for unknown forces.
- xii. This analysis is repeated from initial configuration to final configuration. The resulting force values and determinant values are plotted in a graph.
- xiii. If sign of the matrix determinant is different from previous one this case is considered as singular configuration. Otherwise, we can say there is no singularity.

3.2.2.2. Singularity Detection by Geometric Approach (CPA)

Closest point approach will be implemented in two stages. This algorithm contains fundamental case of CPA and application of this methodology to external fixator (6-6 SP).

3.2.2.2.(1). Determination of the Closest Points Between Two Lines in the Space

Closest points between two lines in space and outputs associated with this are calculated by the following steps.

- i. The coordinates of a pair of points (A_m, A_n) which are measured on the same axes in the space and unit vectors $(\vec{e}_m, \vec{e}_n)$ passing through these points that determine the directions are entered as input.
- ii. Vector $\overrightarrow{A_m A_n}$ is obtained by subtracting the coordinates of A_m from the coordinates of A_n .
- iii. Dot product of unit vectors $\vec{e}_m$ and $\vec{e}_n$ is obtained.
- iv. When $\vec{e}_m \cdot \vec{e}_n = \pm 1$, we can say these two lines are parallel. If this condition happens, next step is **viii**. Otherwise, next step is **v**.
- v. Closest point distance d_{mn} on the $\vec{e}_n$ is calculated by equation (3.75.).
- vi. Closest point distance c_{mn} on the $\vec{e}_m$ is calculated by equation (3.76.).

- vii. Unit vector $\overrightarrow{e_{mn}}$ which is perpendicular to the both of $\overrightarrow{e_m}$ and $\overrightarrow{e_n}$ is generated by equations (3.66.) - (3.68.).
- viii. Closest perpendicular distance s_{mn} which is between the direction of $\overrightarrow{e_m}$ and $\overrightarrow{e_n}$ is calculated by (3.78.).
- ix. The coordinates of the closest points C_{mn} and D_{mn} are computed by equations (3.79.) - (3.80.).

3.2.2.2.(2). Implementation of Closest Point Approach to SP Mechanism (Fixator)

The most important points in the implementation of this approach to SP mechanism are determination the corresponding closest point pair of robot geometry and calculation of the unit vectors that are passing through these points. This methodology is applied to all point pairs and the results are evaluated for the singularity decision.

- i. Parameters of the fixator $(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6)$ (R, R_1) and motion variables $((x_1, y_1, z_1, \theta_x, \theta_y, \theta_z)_j, j = 0, 1, \dots, n)$ of the corresponding configuration are inputs of this algorithm.
- ii. Joint pairs A_m, A_n ($m = 1 - 5, n = 2 - 6$) on the proximal ring is determined by equations (3.82.) and (3.83.). Expressions whose index of “n” are obtained by changing the index “m” in the same equations.
- iii. Joint pairs B_m, B_n ($m = 1 - 5, n = 2 - 6$) on the distal ring is determined by equations (3.99.) and (3.100.). Expressions whose index of “n” are obtained by changing the index “m” in the same equations.
- iv. Components of the unit vectors of u, v, w coordinate system $(\overrightarrow{e_u}, \overrightarrow{e_v}, \overrightarrow{e_w})$ along the x, y, z axes are $e_{ux}, e_{uy}, e_{uz}, e_{vx}, e_{vy}, e_{vz}, e_{wx}, e_{wy}, e_{wz}$. These values are calculated by using equations (3.86.) - (3.94.) according to corresponding displacement analysis result $(\theta_x, \theta_y, \theta_z)_j, j = 0, 1, \dots, n$ which is done for all treatment steps in the motion analysis.

- v. $\overrightarrow{G_1B_m}, \overrightarrow{G_1B_n}$; $(m = 1 - 5, n = 2 - 6)$ with components $(G_1B_{mx}, G_1B_{my}, G_1B_{mz}, G_1B_{nx}, G_1B_{ny}, G_1B_{nz})$ symbolizes x, y, z components of the distance vector from the distal ring center (G_1) to the distal joint. These are calculated by equations (3.101.) - (3.104.).
- vi. Position data $(x_1, y_1, z_1)_j, j = 0, 1, \dots, n$ are used to calculate distance from proximal ring center to distal joints $(GB_{mx}, GB_{my}, GB_{mz}, GB_{nx}, GB_{ny}, GB_{nz})$. This computation is done according to (3.105.) - (3.107.).
- vii. Rod vector pairs $(\overrightarrow{A_mB_m}, \overrightarrow{A_nB_n}; m = 1 - 5, n = 2 - 6)$ are calculated by equations (3.108.) - (3.111.).
- xiv. Length of the rod pairs $(L_m, L_n; m = 1 - 5, n = 2 - 6)$ and rod unit vectors $(\overrightarrow{e_m}, \overrightarrow{e_n}; m = 1 - 5, n = 2 - 6)$ are calculated by equations (3.112.), (3.114.) - (3.116.), respectively.
- viii. Fifteen closest perpendicular distance s_{mn} and all closest points are determined by implementing the determination of the closest points between two lines in the space algorithm, which is given in chapter (3.2.2.2.(1).), to pair of points (A_m, A_n) and pair of unit vectors $(\overrightarrow{e_m}, \overrightarrow{e_n})$.
- ix. Number of treatment step is symbolized by “ j ”. The preceding process is repeated from $j = 0$ to $j = n$.

3.2.2.2.(3). Singularity Evaluation Algorithm for Geometric Approach (CPA)

The following steps are executed, given the results obtained in the previous section, to evaluate singularity.

- i. Fifteen closest perpendicular distances s_{mn} and corresponding coordinates of closest points C_{mn}, D_{mn} , intersection threshold value δ_s and singularity threshold value δ_p are entered as input.

- ii. s_{mn} ($m = 1 - 5$, $n = 2 - 6$) values which are smaller than the intersection threshold value δ_s are identified by numbers (j_s) and point (C_k ; $1 < k < j_s$).
- iii. If j_s is equal to “0” or “1” we can say there is no singularity and this result is displayed to user on the interface.
- iv. If j_s is equal to “14” or “15” we can say there is a singular case and this result is displayed to user on the interface.
- v. If j_s value is between “2” and “13”, point pairs (C_k, C_l) are generated according to the following relationship.

$$C_k; 1 < k < j_s - 1$$

$$C_l; 2 < l < j_s$$

- vi. Following metric calculation Δ_p is performed j_p times for generated point pairs.

$$j_p = \frac{j_s \times (j_s - 1)}{2}$$

$$\Delta_p = \sqrt{[(x_{ck} - x_{cl})^2 + (y_{ck} - y_{cl})^2 + (z_{ck} - z_{cl})^2]} ; \quad 1 < p < j_p$$

- vii. Number of point pairs p_s is determined when $\Delta_p < \delta_p$.
- viii. If p_s is equal to “0” singularity case is not in question.
- ix. If p_s value is equal to “1” or more, singular case occurs. The result is displayed to user on the interface as “Singularity”.

3.3. Interfaces

Interfaces developed for singularity evaluation based on the theory and algorithms of previous sections will be explained here. Also, details in conjunction with the use of the interfaces are described in this part.

Microsoft Visual Studio Professional 2013 with free licence for university student is to be used for educational purposes as the design tool and code editor. C# programming language is used for programming interfaces.

3.3.1. Singularity Evaluation Interface by Motion Analysis

The main window of the developed interface is shown in Figure 3. 7. Data are entered from “Input” group on the left side of the interface. Program outputs are located on the right side of the interface. A dropdown menu “Treatment” which contains auxiliary outputs situated in the menu strip, Figure 3. 8.

There are 35 boxes for data entry. Twelve of them are reserved for initial rod lengths (denoted as “Unhealthy”) and final rod lengths (denoted as “Healthy”) in the “Rod Lengths” group. There are fourteen text box for ring radii (R, R_1), joint angles on the proximal ring and distal ring ($\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6$) in the “Angles and Ring Diameter” group. The proximal ring is considered stationary and assumed to be the reference frame. So, the configuration of the fixator frame is described in terms of the motion of the distal ring relative to the proximal ring. Three translation (x_1, y_1, z_1) and three rotation ($\theta_x, \theta_y, \theta_z$) parameters are located in the “Initial Parameters”. Also there are two text boxes for “Treatment Step” representing the duration of treatment prescribed by the orthopaedist and “Stopping Criteria (%)” to stop the iteration carried out at each treatment step. A combo box is used for entering the predetermined inputs quickly and easily into relevant boxes. Instant displacement values at each steps are written to a text file whose name is specified in “File Name.txt”. Naming is done by entering the file name into combo box. This output is used as input for the other interfaces. “Calculate” button is also located here.

[illegible]

Figure 3. 7. Main window of the Motion Analysis Interface

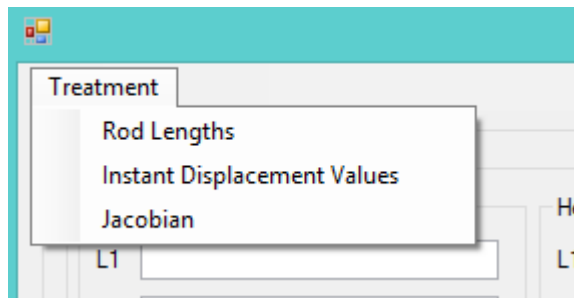


Figure 3. 8. Dropdown menu of the Motion Analysis Interface “Treatment”

There are two tables and a result text as output on the right side of the interface. Rod lengths and instant displacement values at each step are listed in these tables. A text message indicates if any singular position is encountered in the proposed treatment at the “Singularity Result” label.

Additional information can be gathered about the treatment process from dropdown menu located on top of the main window. The “Treatment” menu has three sub-menus, Figure 3. 8. “Rod Lengths” button shows the length of each rod for each of the treatment steps, Figure 3. 9. In many cases, this information is given to the patient for self-adjustment of the fixator. Variation of the rod lengths are shown at “Graphs of Rod Lengths” tab in the Rod Lengths form, Figure 3. 10. Lines colour and thickness in the graph can be changed by radio buttons. Form of the “Instant Displacement Values” displays a table of translation and rotation values of distal ring at each treatment step, Figure 3. 11. Jacobian button shows the variation of determinant values, Figure 3. 12. This chart can be saved by “Save Image” button as image file (filename.bmp) to desktop.

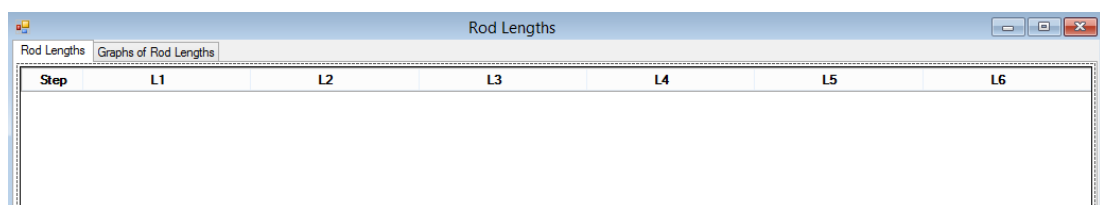


Figure 3.9. Rod lengths

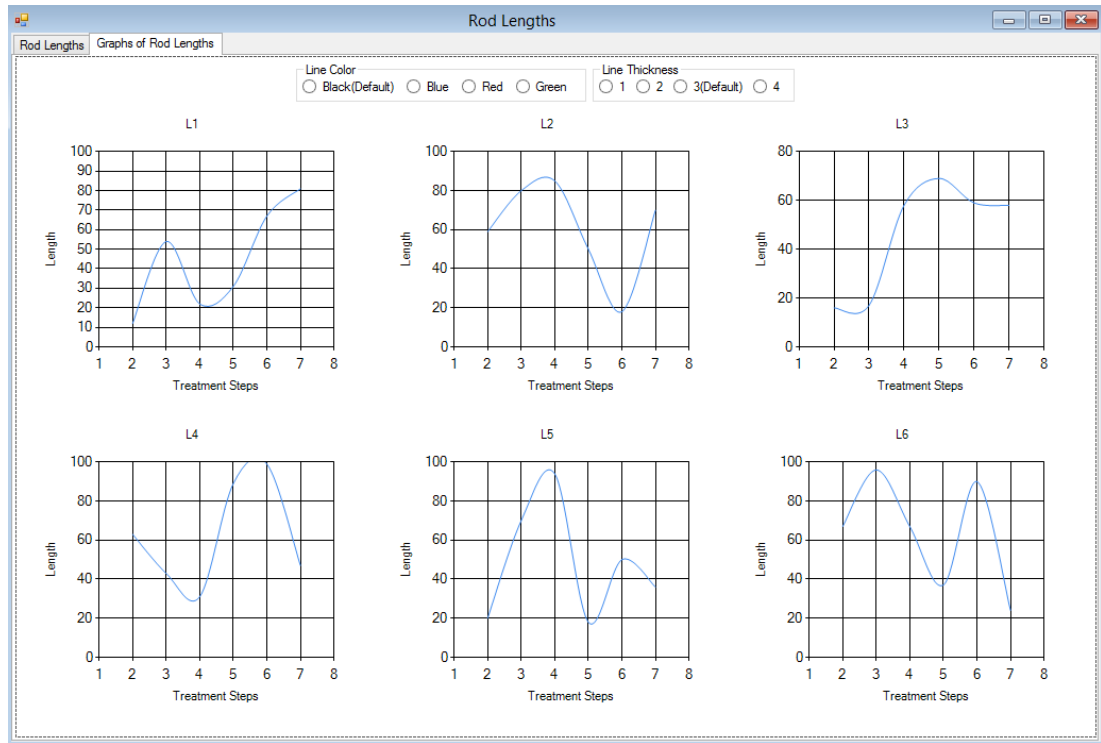


Figure 3. 10. Variation of the rod lengths

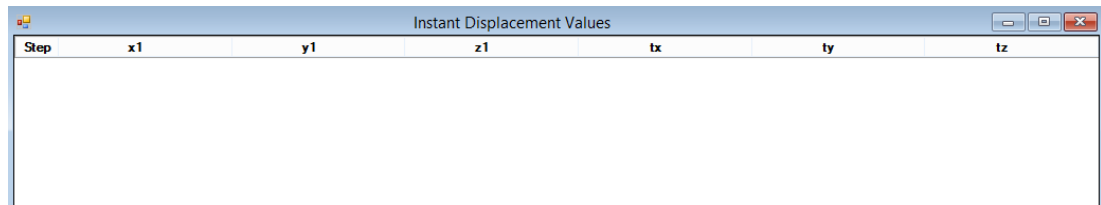


Figure 3. 11. Instant displacement values

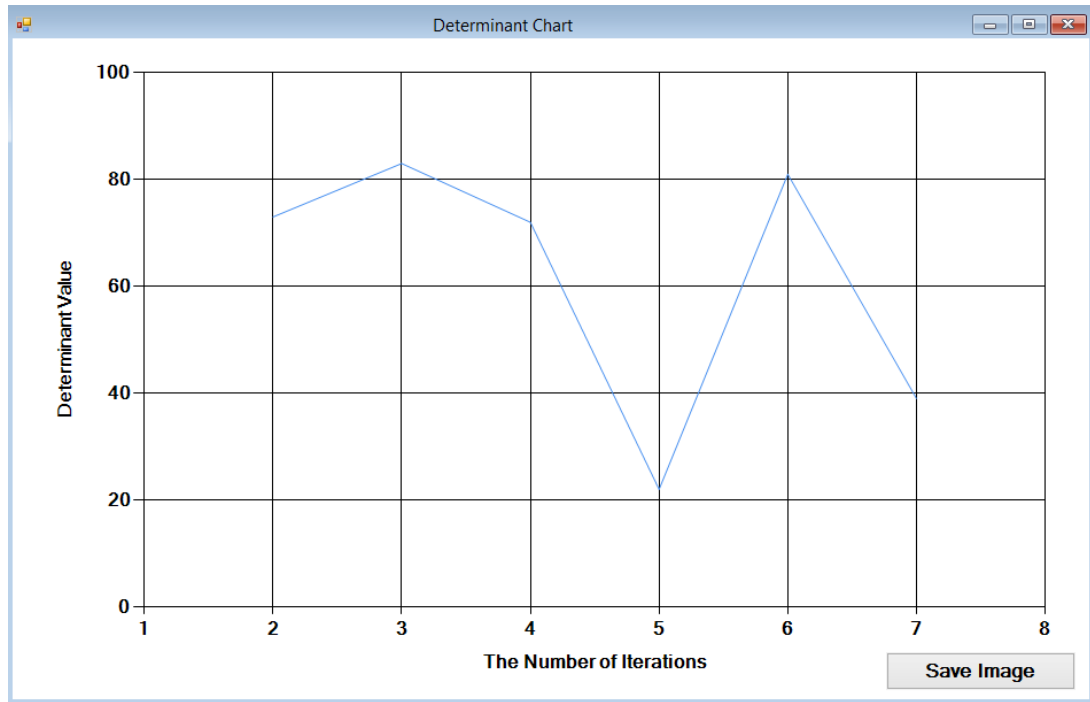


Figure 3. 12 Variation of the determinant values

The input data required for the operation of the interface is obtained by an interface developed under the TÜBİTAK Project 106M466, except for proposed treatment step, stopping criteria and file name, Figure 3. 13. Calculations are started by clicking “Calculate” button after entering the data into relevant boxes. Instant displacement values $(x_1, y_1, z_1, \theta_x, \theta_y, \theta_z)$ and rod lengths $(L_1 - L_6)$ at each treatment step, result which is related whether the singularities existed or not shown on the right side of the interface. Also a text file (filename.txt) which contains instant displacement values at each step is generated to the desktop with given file name. New forms which contain related output data can be opened from “Treatment” dropdown menu.

The screenshot displays the 'Fixator Otomasyonu' software interface. The window title is 'Fixator Otomasyonu'. The interface is organized into several sections with input fields and buttons:

- AP Data:** Includes fields for 'PsAp', 'BeAp', 'Be'Ap', 'cAP', and 'bAP'.
- Lateral Data:** Includes fields for 'PsL', 'BeL', 'Be'L', 'cL', and 'bL'.
- e Data:** Includes fields for 'ex', 'ey', and 'ez'.
- Alfa Data:** Includes fields for 'a1', 'a2', 'a3', 'a4', 'a5', and 'a6'.
- Axial Data:** Includes fields for 'Del0', 'DelAx', and 'Del'.
- Fixator Yarıcap:** Includes fields for 'R' and 'R1'.
- Epsilon Degerleri:** Includes fields for 'e1', 'e2', 'e3', 'e4', 'e5', and 'e6'.
- r Data:** Includes fields for 'rx', 'ry', and 'rz'.
- q Data:** Includes fields for 'qx', 'qy', and 'qz'.
- Sonuçlar (Results):** A section with two columns: 'Sağlıksız' (L1-L6) and 'Sağlıklı' (L1-L6).
- Uzunluklar (Lengths):** Includes fields for 'b' and 'c'.
- Kontrol:** Includes fields for 'ru' and 'rv'.
- Buttons:** 'Hesapla' (Calculate), 'Kaydet' (Save), 'Aç' (Open), and 'Çıkış' (Exit).
- Checkboxes:** A checkbox labeled 'AP için Hesapla' (Calculate for AP).
- Other Labels:** 'Auvw' and 'Axyz' are visible at the top right.

Figure 3. 13. One of the interfaces developed in the TÜBİTAK Project 106M466

3.3.2. Singularity Evaluation Interface by Force Analysis

Main window of the Force Analysis Interface can be categorized into two parts as inputs and outputs, Figure 3. 14. Control buttons are located at the bottom left of the interface. A dropdown menu “Outputs” contains auxiliary outputs situated in the menu strip, Figure 3. 15.

The interface is titled "Singularity Control - Force Analysis". It is divided into several sections for input and output.

Inputs:

- Ring Parameters:** Includes input fields for R and $R1$.
- Proximal Joint Parameters(Degree):** Includes input fields for $\alpha1, \alpha2, \alpha3, \alpha4, \alpha5, \alpha6$.
- Distal Joint Parameters(Degree):** Includes input fields for $\epsilon1, \epsilon2, \epsilon3, \epsilon4, \epsilon5, \epsilon6$.
- Force-Moment Inputs:** Includes input fields for $F_u, F_v, F_w, M_u, M_v, M_w$.
- Force Application Point:** Includes input fields for r_{Fu}, r_{Fv}, r_{Fw} and radio buttons for "Local Coordinate System" (selected) and "Global Coordinate System".
- Determinant Limit Value:** Set to $10^$ with a small input field.
- Buttons:** "Read Displacement Data", "Choose Problem No. ▾", and a large "CALCULATE" button.

Outputs:

- Primary Output:**
 - Unhealthy:** Fields for $F1, F2, F3, F4, F5, F6$ and "Determinant".
 - Healthy:** Fields for $F1, F2, F3, F4, F5, F6$ and "Determinant".
- Singularity Result:** Displayed in red text at the bottom right.

Figure 3. 14. Main window of the Force Analysis Interface

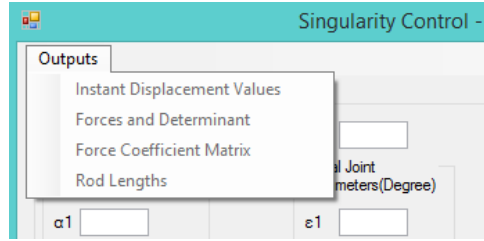


Figure 3. 15. Dropdown menu of the Force Analysis Interface

There are fourteen text box for ring radii (R, R_1), joint angles on the proximal ring and distal ring ($\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5, \epsilon_6$) in the “Ring Parameters” group. There is a button “Read Displacement Data” for reading displacement data ($x_1, y_1, z_1, \theta_x, \theta_y, \theta_z$) from text file at the bottom left of the interface. These data are obtained by using Motion Analysis Interface. Numerical definition of the robotic structure is made by these 20 parameters. Then, components of external force and moment ($F_x, F_y, F_z, M_x, M_y, M_z$) which are defined according to the global coordinate system are entered into “Force-Moment Inputs” group. Force application point can be defined in two ways. (rF_u, rF_v) text boxes are enabled when radio button “Local Coordinate System” is preferred. If radio button “Global Coordinate System” is selected labels (rF_u, rF_v, rF_w) are turned into (rF_x, rF_y, rF_z). This option allows the user to enter data according to the preferred coordinate system. A text box is located under the inputs for relative determinant limit value. This value is used for determining the singularity condition. A combo box is used for entering the predetermined inputs quickly and easily into relevant boxes.

Magnitudes of joint forces $F_m, (m = 1 - 6)$ along the rod directions and determinant of the force coefficient matrix are to be found in the “Primary Output” group. Singularity result is also located here. A dropdown menu “Output” is situated to access additional information. This menu has four sub-menus, Figure 3. 15. Table of translation and rotation values of distal ring ($x_1, y_1, z_1, \theta_x, \theta_y, \theta_z$) which is shown in Figure 3. 16 at each treatment step can be accessed by clicking the “Instant Displacement Values” button. Details of the calculated forces and determinant values are shown “Forces and Determinant” form with graphs which show changing of the values during the treatment, Figure 3. 17 - Figure 3. 19. Lines colour and thickness in

the graph can be changed by radio buttons. Matrix equation can be examined at each configuration in “Force Coefficient Matrix” form, Figure 3. 20. Length of the rods can be seen in “Rod Lengths” form, which is shown in Figure 3. 21. Variation of the rod lengths are shown in “Graphs of Rod Lengths” tab in this form, Figure 3. 22.

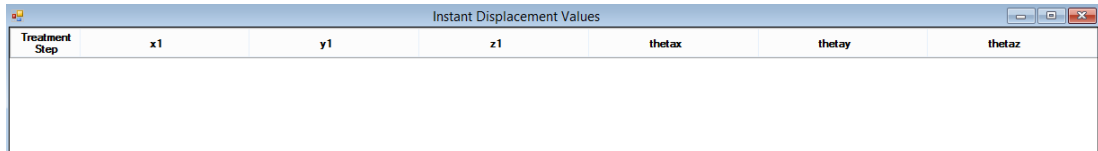


Figure 3. 16. Instant displacement values

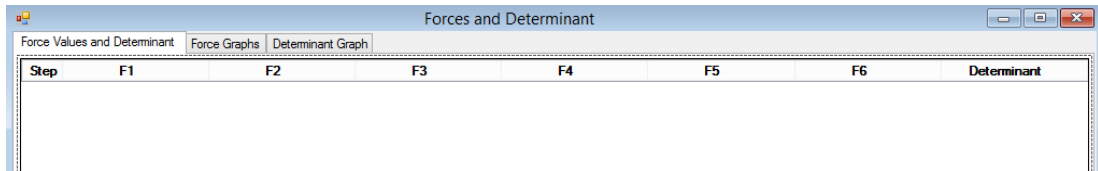


Figure 3. 17. Forces and determinants

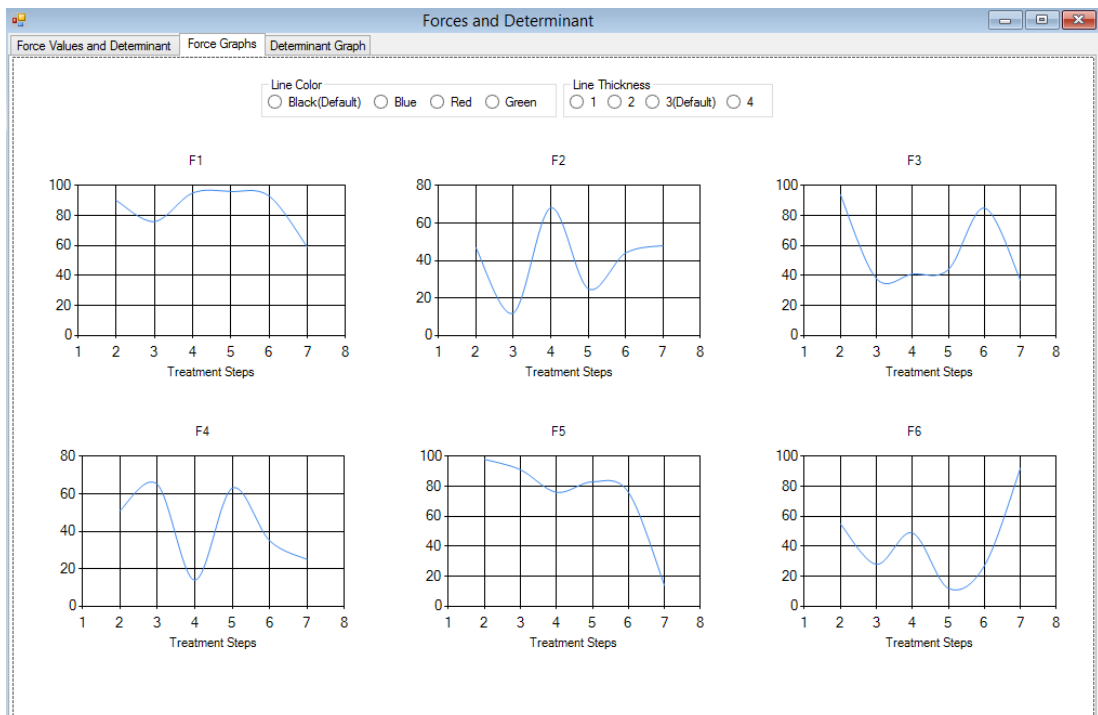


Figure 3. 18. Force graphs

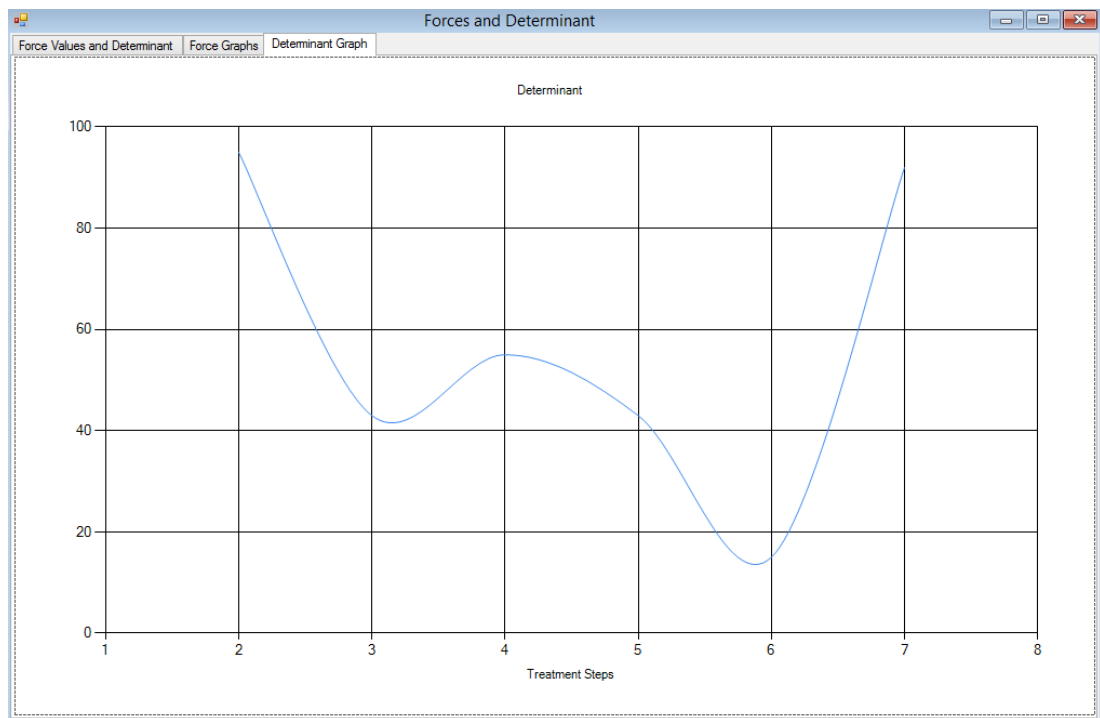


Figure 3. 19. Determinant graphs

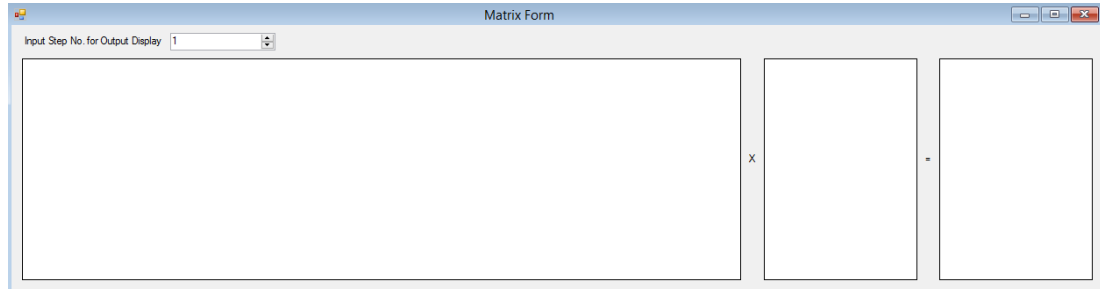


Figure 3. 20. Matrix form of the linear equation system (6x6)

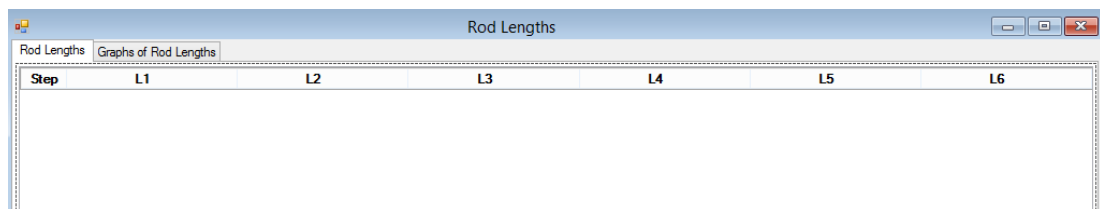


Figure 3. 21. Rod lengths

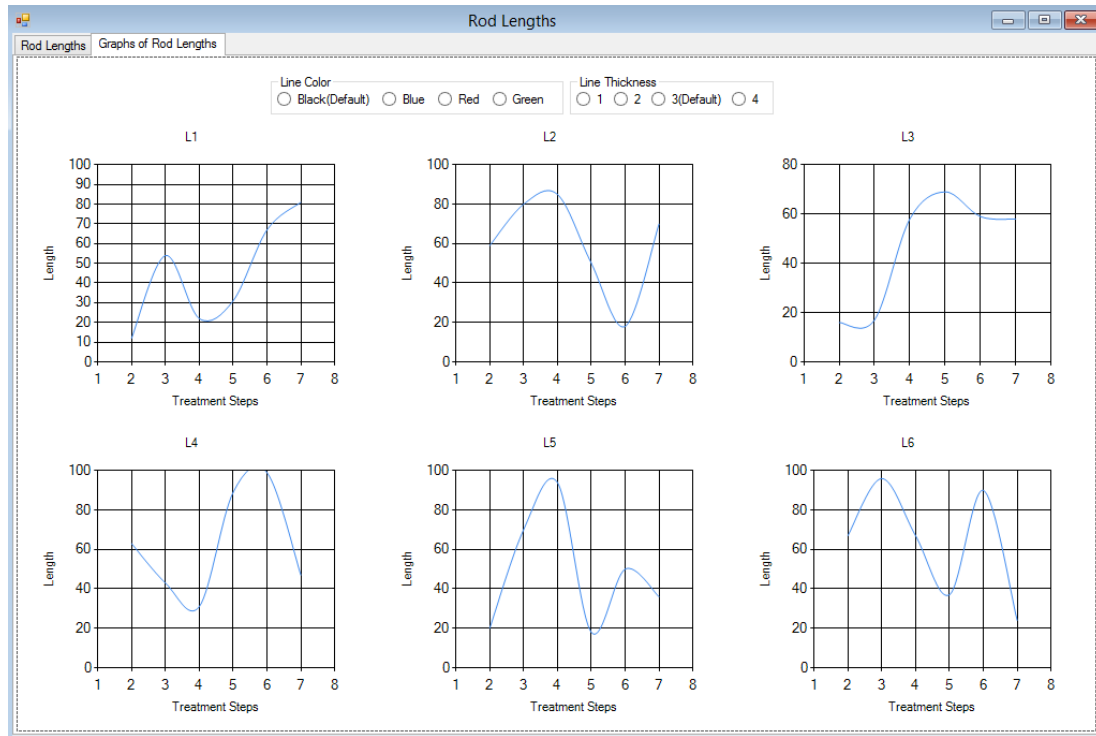


Figure 3.22 Variation of the rod lengths

Ring parameters ($R, R_1, \alpha_1 - \alpha_6, \varepsilon_1 - \varepsilon_6$) are entered into relevant text boxes in order to perform calculations. Text file (filename.txt) which contains instant displacement values ($x_1, y_1, z_1, \theta_x, \theta_y, \theta_z$) at each step is selected by “Read Displacement Data” button. Components of force (F_x, F_y, F_z) and moment (M_x, M_y, M_z) which are applied externally to the distal ring are entered. Force application point is determined then entered in the relevant coordinate systems by using radio button. Data entry is completed by entering the “Determinant Limit Value”. Computation is started by clicking on the “Calculate” button. A tick symbol will appear when there is no missing input. Magnitude of the forces and determinant values at the both healthy and unhealthy positions will be displayed in the “Primary Output” group. In addition to these outputs, singularity result will be written in bold and red font. The user can access detailed information via the “Outputs” menu.

3.3.3. Singularity Evaluation Interface by CPA (Geometric Method)

Singularity Evaluation Interface by CPA which is shown in Figure 3. 23 has been developed for detecting the singularities by geometric method.

There are fourteen text boxes for ring radii (R, R_1), joint angles on the proximal ring and distal ring ($\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6$) in the “Ring Parameters” group. There is a button “Read Displacement Data” for reading displacement data ($x_1, y_1, z_1, \theta_x, \theta_y, \theta_z$) from text file. These data are obtained by using Motion Analysis Interface. Two text boxes are located under the inputs for Intersection Threshold Value (δ_s) and Singularity Threshold Limit Value (δ_p). These values are used for determining the singularity condition. Also a combo box is used for entering the predetermined inputs quickly and easily into relevant boxes. Outputs which are calculated at each step can be seen by using counter “Input Step No. For Output Display”.

There is a table that is used for presenting the calculated intermediate values to user at the bottom of the window. This table consists of the ($s_{mn}, c_{mn}, d_{mn}, GC_{mnx}, GC_{mny}, GC_{mnz}, GD_{mnx}, GD_{mny}, GD_{mnz}, E_{mn}$) values. Details of the output and main result are located at the top right corner of the interface on two tabs. First tab which contains Intersection Metrics (Δ_p), Result Box and Singularity Result is the default tab when the interface is opened. Rod unit vectors at each step are listed in the second tab. Singularity result is written in bold and red font after the calculation.

Ring parameters ($R, R_1, \alpha_1 - \alpha_6, \varepsilon_1 - \varepsilon_6$) are entered into relevant text boxes in order to perform calculations. Text file (filename.txt) which contains instant displacement values ($x_1, y_1, z_1, \theta_x, \theta_y, \theta_z$) at each step is selected by “Read Displacement Data” button. Threshold values (δ_s, δ_p) which are mentioned at the “Singularity Evaluation Algorithm” are entered into related boxes. Computation is started by clicking on the “Calculate” button. Outputs can be examined from the described part of the interface.

Fixator Singularity

Input Data

Ring Parameters
R1

Proximal Joint Parameters(Degree)
 α_1 α_2 α_3 α_4 α_5 α_6

Distal Joint Parameters(Degree)
 ε_1 ε_2 ε_3 ε_4 ε_5 ε_6

Intersection Threshold Value (Is)

Singularity Threshold Limit Value (Sp)

Read Displacement Data

Calculate

Input Step No. for Output Display

Intersection Metrics (Ap) & Result Rod Unit Vectors

Intersection Metrics (Ap)

Result Box

Singularity Result

Output	Rod No	Pairs	smin	cmn	dmin	EMN	GCmnx	GCmny	GCmnz	GDmnx	GDmny	GDmnz

Figure 3. 23. Main window of the Singularity Evaluation Interface by CPA

3.3.4. Singularity Evaluation Interface with Three Methods

Motion Analysis Method, Force Analysis Method and Geometric Method by CPA was used for detecting the singularities in the previous interfaces. An interface which is used to observe the output of these methods together has been developed, Figure 3. 24.

Ring radii (R, R_1), angular position parameters of the joints on the proximal and distal ring ($\alpha_1 - \alpha_6, \varepsilon_1 - \varepsilon_6$) can be entered into related boxes in the “Ring Parameters” group which is located on top of the left column of the interface. Six text boxes are reserved for displacement values ($x_1, y_1, z_1, \theta_x, \theta_y, \theta_z$) at the bottom of the left column. A button “Calculate” that allows the computation is given in the left column.

Output and some important parameters of fixator configuration are located in the interface. Thus, the observation can be made easily. There are 6 tabs “Determinant”, “Variation of the Determinant Values”, “Kinematic Diagram”, “Rod Length”, “Matrix” and “Geometric Method” at the top of this field. Determinant of the force coefficient matrix and Jacobian are listed in the columns “Force” and “Jacobian” respectively. The other twelve columns are reserved for angular position parameters of the joints. “Determinant” tab is the default tab when the interface is opened. Both determinants are recalculated by changing one of the input parameters. So exchange of determinant values can be observed from Figure 3. 25. Fixator configurations in some selected problems are drawn by using Solidworks. These drawings are shown on “Kinematic Diagram” for a better understanding of the configuration. Predetermined inputs and drawings can be filled into related boxes and tab by selecting the problem from combo box. Rod lengths ($L_1 - L_6$) can be seen in the fourth tab which is shown in Figure 3. 26. Calculated force coefficient matrix and Jacobian matrix are shown in “Matrix” tab, Figure 3. 27. “Geometric Method” tab contains two sub tabs. These tabs allow detailed examination of geometric method. The shortest distance between the rod pairs, relative distance on the related joint and unit vector on the direction of the rods are located in the “Primary Output” tab, Figure 3. 28. Closest point coordinates can be viewed from the second tab, Figure 3. 29.

[illegible]

Figure 3. 24. Singularity Evaluation Interface with Three Methods

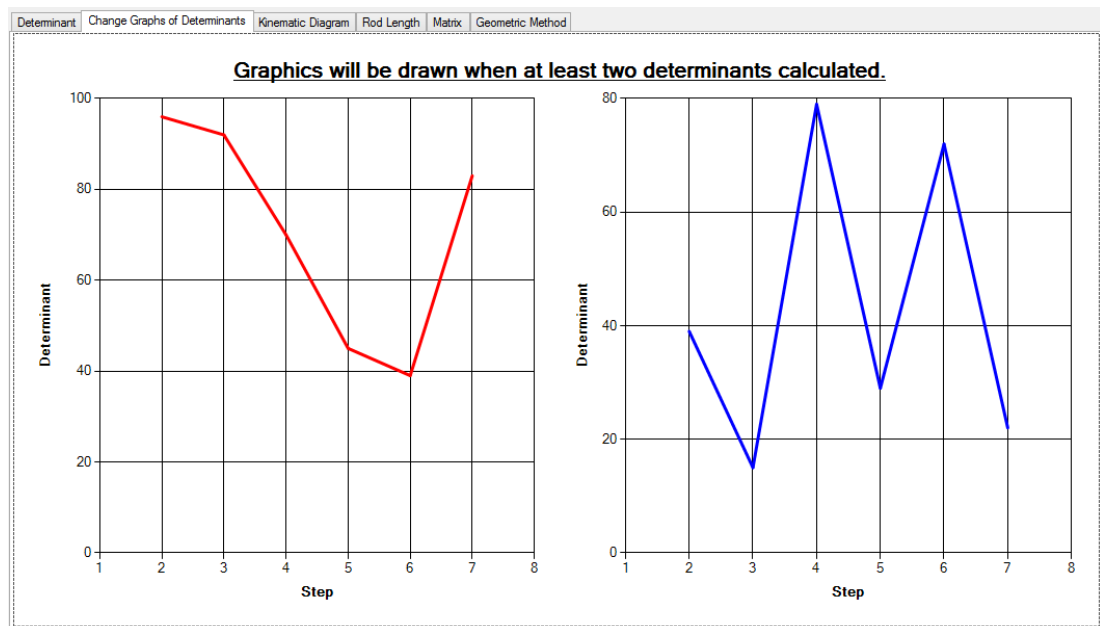


Figure 3. 25. Variation of the determinant values

Determinant Change Graphs of Determinants Kinematic Diagram Rod Length Matrix Geometric Method

L1	L2	L3	L4	L5	L6

Figure 3. 26. Rod lengths

Determinant Change Graphs of Determinants Kinematic Diagram Rod Length Matrix Geometric Method

Force Matrix

Jacobian Matrix

Figure 3. 27. Matrix tab

4. RESULTS AND DISCUSSION

There are two main objectives in this thesis. Situations that lead to singularity were researched with reference to three different methods. After analyzing the problem, algorithms were set up and on this basis, interfaces were developed. Then, these interfaces are to provide one to do numerical experiments. In a way these interfaces, can be viewed as parts of a virtual laboratory in which many hypotheses about singular cases can be tested. The test results will be made available to the service of the orthopedists, in order for them to avoid such situations in their orthopedical practice. Examples will be given to embody this information after the description of the structure and operation of the interfaces.

Firstly, nine examples are formed for the singularity of the external fixator under the static conditions to be tested by three methods formulated previously. These problems will be analysed by using “Motion Analysis Interface”, “Force Analysis Interface” and “Singularity Evaluation Interface by CPA”. Then singularity analysis for the known singular configurations will be performed by using “Singularity Evaluation Interface with Three Methods”. A search for the sign change in the determinant will be done when one of the fixed variables is varied between allowable limits by using “Singularity Evaluation Interface with Three Methods”. These examples demonstrate the effectiveness of the detection methodology explained above.

4.1. Applications of the Motion Analysis Interface

Three examples are given in order to analyse the result. “Singularity Evaluation Interface with Three Methods” is also utilized in the investigation of singular case.

4.1.1. Example1 (Non-Singular Case)

In this example, a non-singular case is presented.

- The input data “Rod Lengths” and “Angles and Ring Diameter” which are given in Table 4. 1 and Table 4. 2 required for the operation of the “Motion Analysis Interface” is obtained by an interface “Fixator Otomasyonu” which is shown in Figure 4. 1 developed under the TÜBİTAK Project 106M466.

Table 4. 1. Rod lengths of the fixator for unhealthy and healthy configurations (Example1)

Unhealthy Configuration		Healthy Configuration	
Rod Length	Value (cm)	Rod Length	Value (cm)
L_1	14,723862747508	L_1	15,5048490012119
L_2	16,1896631778258	L_2	15,4775959485369
L_3	15,5407931321066	L_3	13,7599082346094
L_4	15,630757022756	L_4	12,0980388722459
L_5	15,0878274233577	L_5	14,8296904304936
L_6	16,9947910009944	L_6	14,970565578973

Table 4. 2. Ring parameters (Example1)

Ring Parameters			
Proximal Ring	Diameter(cm)	Distal Ring	Diameter(cm)
R	7,25	R_1	7,25
Proximal Joints	Value (Degree)	Proximal Joints	Value (Degree)
α_1	90	ε_1	0
α_2	110	ε_2	100
α_3	210	ε_3	120
α_4	230	ε_4	220
α_5	330	ε_5	240
α_6	350	ε_6	340

Fixator Otomasyonu

AP Data

PsAp 0.3

BeAp -2.48

Be'Ap -6.21

cAP 0

bAP 0

Lateral Data

PsL -0.43

BeL -6.90

Be'L 5.1

cL 9.82165

bL 3.4564

e Data

ex 0.7927

ey 1.6361

ez 1.47395

Sonuçlar

Sağlıksız

L1 14.72386

L2 16.18966

L3 15.54079

L4 15.63075

L5 15.08782

L6 16.99479

Sağlıklı

L1 15.50484

L2 15.47759

L3 13.75990

L4 12.09803

L5 14.82969

L6 14.97056

Auvw

0.7234

-0.690

0.0024

0.685

0.718

-0.122

0.0824

0.0904

0.9924

Axial Data

Del0 65.64

DelAx 22

Del 0

Alfa Data

a1 90

a2 110

a3 210

a4 230

a5 330

a6 350

q Data

qx -1.042

qy -0.678

qz 0

r Data

rx 0.2789

ry 0.2653

rz 0.0015

Fixator Yarıcap

R 7.25

R1 7.25

Epsilon Degerleri

e1 0

e2 100

e3 120

e4 220

e5 240

e6 340

Uzunluklar

b 3.476640

c 9.830724

Kontrol

nu 0.356690

nv -0.144634

☐ AP için Hesapla

Figure 4. 1. Primary Inputs from “Fixator Otomasyonu” interface (Example1)

- Initial translational and rotational parameters $(x_1, y_1, z_1, \theta_x, \theta_y, \theta_z)$ are entered as input. These are given in Table 4. 3.

Table 4. 3. Initial motion parameters (Example1)

Initial Motion Parameters			
Translations	Value (cm)	Rotations	Value (Degree)
x_1	-1,325113325201732	θ_x	0,0959240540209131136
y_1	1,5654877354511432	θ_y	0,515453035808925953
z_1	14,66568148584712	θ_z	65,64

- The other parameters to be entered which are given in Table 4. 4 are “Treatment Step”, “Stopping Criteria (%)” and “File Name.txt”.

Table 4. 4. Basic input (Example1)

Basic Input					
Input1	Value (Day)	Input2	Value (%)	Input3	Textual Input
Treatment Step	10	Stopping Criteria	5	File Name.txt	Example1.txt

“Calculate” button is clicked after the entering all data into relevant boxes. Then following output are obtained.

- Rod lengths and instant displacement values $(x_1, y_1, z_1, \theta_x, \theta_y, \theta_z)$ at each step can be seen at the right side of the main window which is shown in Figure 4.
- 2. The result that no singularity exist can be observed directly.

Singularity Control - Motion Analysis

No Singularity.

Treatment

Inputs

Rod Lengths

Unhealthy

L1 14.723862747508
L2 16.1896631778258
L3 15.5407331321066
L4 15.630757022756
L5 15.0878274233577
L6 16.9947910009944

Healthy

L1 15.5048490012119
L2 15.4775959485369
L3 13.7599082346094
L4 12.0980388722459
L5 14.8296904304936
L6 14.970565578973

Angles and Ring Diameter

Proximal Joint Parameters(Degree)

α_1 90 α_2 110 α_3 210 α_4 230 α_5 330 α_6 350

Distal Joint Parameters(Degree)

ϵ_1 0 ϵ_2 100 ϵ_3 120 ϵ_4 220 ϵ_5 240 ϵ_6 340

R 7.25 R1 7.25

Initial Parameters

Translations

X1 -1.32511332520173
Y1 1.56548773545114
Z1 14.6656814858471

Rotations

Θ_x 0.0959240540209131
 Θ_y 0.51545303580829536
 Θ_z 65.64

Treatment Step 10

Stopping Criteria (%) 5

File Name: Example1 .txt

Calculate

Rod Lengths

Step	L1	L2	L3	L4	L5	L6
1	14.723862747508	16.1896631778258	15.5407331321066	15.630757022756	15.0878274233577	16.9947910009944
2	14.8019613728784	16.1184564548969	15.3627046423569	15.277485207705	15.0620137240713	16.7923684587923
3	14.8800599982488	16.047249731968	15.1846161526072	14.924213392654	15.0362000247849	16.5899459165901
4	14.9581586236192	15.9760430090391	15.0065276628574	14.570941577603	15.0103863254985	16.387523374388
5	15.0362572489896	15.9048362861102	14.8284391731077	14.217669762552	14.9845726262121	16.1851008321858
6	15.11435587436	15.8336295631814	14.650350683358	13.864397947501	14.9587589269257	15.9826782898837
7	15.1924544997303	15.7624228402525	14.4722621936083	13.5111261324499	14.9329452276392	15.7802557477816
8	15.2705531251007	15.6912161173236	14.2941737038586	13.1578543173989	14.9071315283528	15.5778332055794
9	15.3486517504711	15.620093943947	14.1160852141088	12.8045825023479	14.8813178290664	15.3754106633773
10	15.4267503758415	15.5488026714658	13.9379967243591	12.4513106872969	14.85550412978	15.1729881211751
11	15.5048490012119	15.4775959485369	13.7599082346094	12.0980388722459	14.8296904304936	14.970565578973

Instant Displacement Values

Step	x1	y1	z1	tx	ty	tz
1	-1.32511332520173	1.56548773545114	14.6656814858471	0.095924054020...	0.515453035808...	65.64
2	-1.49749166874539	1.187543366930801	14.5757016394603	-0.039122954897...	-0.911949340550...	63.2930152528535
3	-1.66509336369411	0.813129801841...	14.4700293499899	-0.033322189647...	-2.330216752530...	61.0103735575505
4	-1.82792194119776	0.442808198450...	14.3491403773672	0.107646496859...	-3.733208365712...	58.7796956279927
5	-1.98594570447879	0.077113126060...	14.2134691226334	0.379103988426...	-5.1155601502976	56.5898731337949
6	-2.13910721885304	-0.283445844956...	14.0634202861746	0.777198210729...	-6.472482693888...	54.4308622556873
7	-2.28732733954761	-0.638379637740...	13.8993743185273	1.29879951720775	-7.799604731443...	52.2935163821903
8	-2.43050825670227	-0.987215836706...	13.7216914368609	1.94140820198094	-9.092844733405...	50.1694489116916
9	-2.56853577177351	-1.329495144976...	13.5307145611076	2.70307176579322	-10.34830341765...	48.0509204875428
10	-2.70128095924188	-1.664768066204...	13.3267714011828	3.58230990836995	-11.56217178036...	45.9307463420722
11	-2.82860132425523	-1.992591754458...	13.1101758370694	4.57804552768313	-12.73065045163...	43.8022203482782

Figure 4. 2. Main window of the “Motion Analysis Interface” (Example1)

- Rod lengths and their variation plots which are shown in Figure 4. 3 and Figure 4. 4 can be accessed from “Rod Lengths” tab under the “Treatment” menu.

Step	L1	L2	L3	L4	L5	L6
1	14,723862747508	16,1896631778258	15,5407931321066	15,630757022756	15,0878274233577	16,9947910009944
2	14,8019613728784	16,1184564548969	15,3627046423569	15,277485207705	15,0620137240713	16,7923684587923
3	14,880059982488	16,047249731968	15,1846161526072	14,924213392654	15,0362000247849	16,5899459165901
4	14,9581586236192	15,9760430090391	15,0065276628574	14,570941577603	15,0103863254985	16,387523374388
5	15,0362572489896	15,9048362861102	14,8284391731077	14,217669762552	14,9845726262121	16,1851008321858
6	15,11435587436	15,8336295631814	14,650350683358	13,864397947501	14,9587589269257	15,9826782899837
7	15,1924544997303	15,7624228402525	14,4722621936083	13,5111261324499	14,9329452276392	15,7802557477816
8	15,2705531251007	15,6912161173236	14,2941737038586	13,1578543173989	14,9071315283528	15,5778332055794
9	15,3486517504711	15,6200093943947	14,1160852141088	12,8045825023479	14,8813178290664	15,3754106633773
10	15,4267503758415	15,5488026714658	13,9379967243591	12,4513106872969	14,85550412978	15,1729881211751
11	15,5048490012119	15,4775959485369	13,7599082346094	12,0980388722459	14,8296904304936	14,970565578973

Figure 4. 3. Rod lengths at each treatment step (Example1)

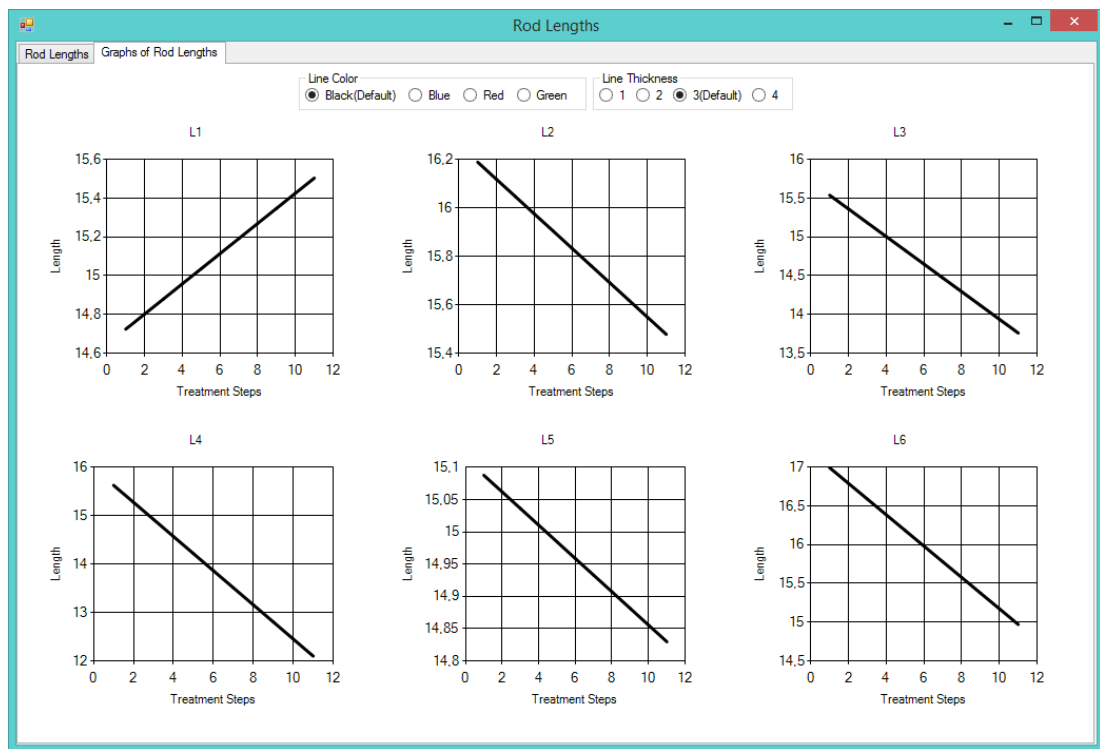


Figure 4. 4. Variation of the rod lengths at each treatment step (Example1)

- The contents of the file which is created to desktop by Motion Analysis Interface with the extension .txt can be examined from “Instant Displacement Values” tab under the “Treatment” menu, Figure 4. 5.

Step	x1	y1	z1	tx	ty	tz
1	-1,32511332520173	1,56548773545114	14,6656814858471	0,0959240540209131	0,515453035808926	65,64
2	-1,49749166874539	1,18754366930801	14,5757016394603	-0,0391229548974432	-0,911949340550393	63,2930152528535
3	-1,66509336369411	0,813129801841632	14,4700293499899	-0,0333221896477591	-2,33021675253023	61,0103735575505
4	-1,82792194119776	0,442808198450976	14,3491403773672	0,107646496859676	-3,73320836571275	58,7796956279927
5	-1,98594570447879	0,0771131260600958	14,2134691226334	0,379103988426985	-5,1156601502976	56,5898731337949
6	-2,13910721885304	-0,283445844956775	14,0634202861746	0,777198210729641	-6,47248269388804	54,4308622556873
7	-2,28732733954761	-0,63837963774094	13,8993743185273	1,29879951720775	-7,79960473144362	52,2935163821903
8	-2,43050825670227	-0,987215836706042	13,7216914368609	1,94140820198094	-9,09284473340542	50,1694489116916
9	-2,56853577177351	-1,32949514497693	13,5307145611076	2,70307176579322	-10,3483034176506	48,0509204875428
10	-2,70128095924188	-1,66476806620453	13,3267714011828	3,58230990836995	-11,5621717803683	45,9307463420722
11	-2,82860132425523	-1,99259175445802	13,1101758370694	4,57804552768313	-12,7306504516399	43,8022203482782

Figure 4. 5. Instant displacement values at each treatment step (Example1)

- Variation of the determinant values are plotted. This plot can be accessed from “Jacobian” tab under the “Treatment” menu, Figure 4. 6. It can be saved to desktop by using “Save Image” button.

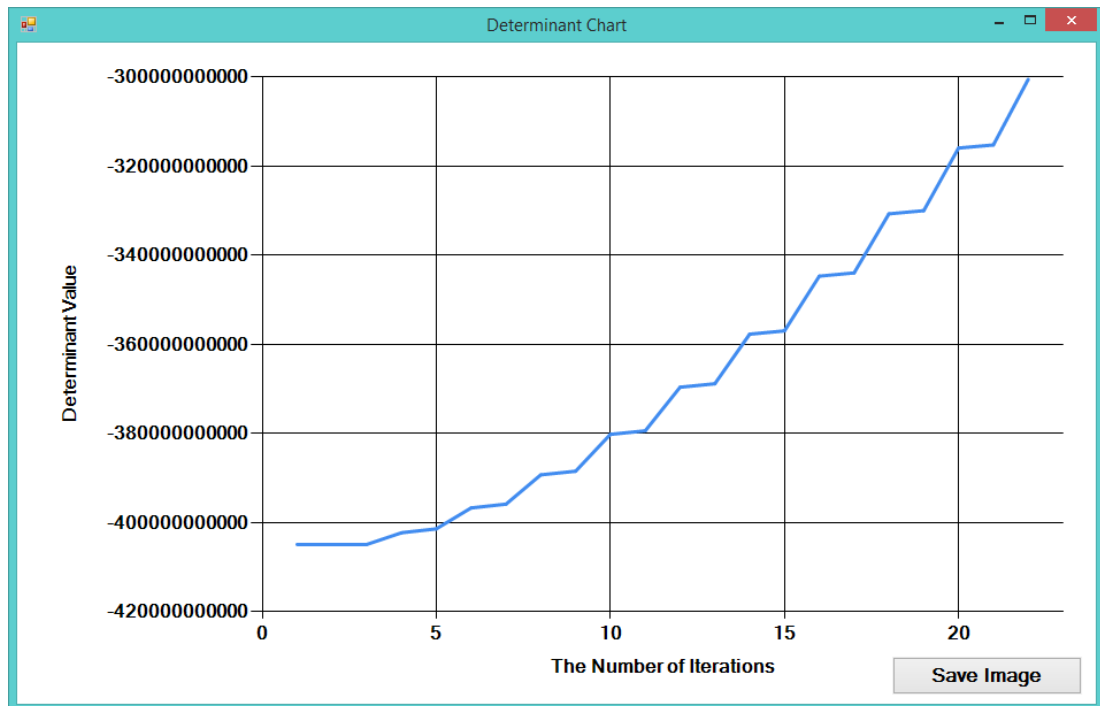


Figure 4. 6. Number of iteration vs Determinant Value (Example1)

4.1.2. Example2 (Non-Singular Case)

In this example, a non-singular case is presented.

- The input data “Rod Lengths” and “Angles and Ring Diameter” which are given in Table 4. 5 and Table 4. 6 required for the operation of the “Motion

Analysis Interface” is obtained by an interface “Fixator Otomasyonu” which is shown in Figure 4. 7 developed under the TÜBİTAK Project 106M466.

Table 4. 5. Rod lengths of the fixator for unhealthy and healthy configurations (Example2)

Unhealthy Configuration		Healthy Configuration	
Rod Length	Value (cm)	Rod Length	Value (cm)
L_1	16,7491533141476	L_1	15,2795854824229
L_2	16,8966869311011	L_2	15,3141043717684
L_3	16,0313504161473	L_3	13,9461123260481
L_4	16,811519743068	L_4	12,2163997481278
L_5	15,446261504557	L_5	14,9107384976952
L_6	18,4377900740517	L_6	14,5969838303337

Table 4. 6. Ring parameters (Example2)

Ring Parameters			
Proximal Ring	Diameter(cm)	Distal Ring	Diameter(cm)
R	7,25	R_1	7,25
Proximal Joints	Value (Degree)	Proximal Joints	Value (Degree)
α_1	90	ε_1	0
α_2	110	ε_2	100
α_3	210	ε_3	120
α_4	230	ε_4	220
α_5	330	ε_5	240
α_6	350	ε_6	340

Fixator Otomasyonu

AP Data

PsAp: -1,21

BeAp: -4,73

Be'Ap: -4,70

cAP: 0

bAP: 0

Lateral Data

PsL: 5,48

BeL: 0,030

Be'L: 4,41

cL: 9,643

bL: 3,55

e Data

ex: 2,619

ey: 2,714

ez: 2,619

Sonuçlar

Sağlıksız

L1: 16,73546

L2: 16,91930

L3: 16,03535

L4: 16,77546

L5: 15,45495

L6: 18,44822

Sağlıklı

L1: 15,26803

L2: 15,38061

L3: 14,01142

L4: 12,20522

L5: 14,88619

L6: 14,56626

Auvw

0,744

-0,667

0,011

0,663

0,739

-0,112

0,066

0,091

0,993

Axial Data

Del0: 69

DelAx: 27

Del: 0

Alfa Data

a1: 90

a2: 110

a3: 210

a4: 230

a5: 330

a6: 350

q Data

qx: -1,071

qy: -0,738

qz: 0

r Data

rx: 0,381

ry: 0,32

rz: 0,003

Uzunluklar

b: 3,561906

c: 9,675952

Kontrol

ru: 0,433534

rv: -0,24156

Fixator Yarıcap

R: 7,25

R1: 7,25

Epsilon Değerleri

e1: 0

e2: 100

e3: 120

e4: 220

e5: 240

e6: 340

☐ AP için Hesapla

Hesapla

Kaydet

Aç

Çıkış

Figure 4. 7. Primary Inputs from “Fixator Otomasyonu” interface (Example2)

- Initial translational and rotational parameters $(x_1, y_1, z_1, \theta_x, \theta_y, \theta_z)$ are entered as input. These are given in Table 4. 7.

Table 4. 7. Initial motion parameters (Example2)

Initial Motion Parameters			
Translations	Value (cm)	Rotations	Value (Degree)
x_1	0,162786933025028611	θ_x	0,8360717036284952906
y_1	1,3779806257450509077	θ_y	-5,547963126351450882
z_1	15,9848836016679156990	θ_z	69

- The other parameters to be entered which are given in Table 4. 8 are “Treatment Step”, “Stopping Criteria (%)” and “File Name.txt”.

Table 4. 8. Basic input (Example2)

Basic Input					
Input1	Value (Day)	Input2	Value (%)	Input3	Textual Input
Treatment Step	10	Stopping Criteria	5	File Name.txt	Example2.txt

“Calculate” button is clicked after the entering all data into relevant boxes. Then following output are obtained.

- Rod lengths and instant displacement values $(x_1, y_1, z_1, \theta_x, \theta_y, \theta_z)$ at each step can be seen at the right side of the main window which is shown in Figure 4. 8. The result that no singularity exists can be observed directly.

Singularity Control - Motion Analysis

No Singularity.

Rod Lengths

Step	L1	L2	L3	L4	L5	L6
1	16.7491533141476	16.8966869311011	16.0313504161473	16.811519743068	15.446261504557	18.4377900740517
2	16.6021965309751	16.7384286751678	15.8228266071374	16.352007743574	15.3927092038708	18.0537094496799
3	16.4552397478027	16.5801704192346	15.6143027981275	15.89249574408	15.3391569031846	17.6696288253081
4	16.3082829646302	16.4219121633013	15.4057789891175	15.4329837445859	15.2856046024985	17.2855482009363
5	16.1613261814577	16.263653907368	15.1972551801076	14.9734717450919	15.2320523018123	16.9014675765645
6	16.0143693982852	16.1053956514348	14.9887313710977	14.5139597455979	15.1785000011261	16.5173869521927
7	15.8674126151128	15.94717373955015	14.7802075620878	14.0544477461039	15.1249477004399	16.1333063278209
8	15.7204558319403	15.7888791395682	14.5716837530779	13.5949357466099	15.0713953997537	15.7492257034491
9	15.5734990487678	15.6306208836349	14.3631599440679	13.1354237471158	15.0178430990676	15.3651450790773
10	15.4265422655954	15.4723626277017	14.154636135058	12.6759117476218	14.9642907983814	14.9810644547055
11	15.2795854824229	15.3141043717684	13.9461123260481	12.2163997481278	14.9107384976952	14.5969838303337

Instant Displacement Values

Step	x1	y1	z1	tx	ty	tz
1	0.162786933025...	1.37798062574505	15.9848836016679	0.836071703628...	-5.547963126351...	69
2	-0.160354101286...	1.00328179263757	15.6152051801658	0.493993201468...	-5.916909306719...	65.8440950911296
3	-0.473393361793...	0.639324829252...	15.4092380802896	0.342610728373...	-6.348507338803...	62.8202876270809
4	-0.776673324654...	0.286217662998...	15.1821224918732	0.370480818422...	-6.824644531193...	59.912417175368
5	-1.07039627004784	-0.055852115840...	14.9351398856628	0.568010917535...	-7.330515599468...	57.1070579806806
6	-1.35466196566812	-0.386643217085...	14.6693927948329	0.926976408507...	-7.853823796764...	54.3929150300035
7	-1.62949024234441	-0.705863949257...	14.38584962353	1.4402779398023	-8.384259926700...	51.7603791824315
8	-1.89483753760529	-1.013207595966...	14.0853766501439	2.10177404260852	-8.913126056053...	49.2011991578445
9	-2.15060942857418	-1.308294970136...	13.7687612743807	2.90616734048401	-9.433059320397...	46.708276148581
10	-2.39667047816972	-1.5907347590893	13.4367290173677	3.84893200762094	-9.937827448895...	44.2752138897291
11	-2.63285231778958	-1.860091950183...	13.0899558261729	4.9262760147077	-10.42217759095...	41.896246527943

Inputs

Unhealthy

Healthy

Angles and Ring Diameter

Proximal Joint Parameters(Degree)

Distal Joint Parameters(Degree)

Initial Parameters

Translations

Rotations

Treatment Step: 10

Stopping Criteria (%): 5

File Name: Example2

CALCULATE

Figure 4. 8. Main window of the “Motion Analysis Interface” (Example2)

- Rod lengths and their variation plots which are shown in Figure 4. 9 and Figure 4. 10 can be accessed from “Rod Lengths” tab under the “Treatment” menu.

Rod Lengths						
Graphs of Rod Lengths						
Step	L1	L2	L3	L4	L5	L6
1	16,7491533141476	16,8966869311011	16,0313504161473	16,811519743068	15,446261504557	18,4377900740517
2	16,6021965309751	16,7384286751678	15,8228266071374	16,352007743574	15,3927092038708	18,0537094496799
3	16,4552397478027	16,5801704192346	15,6143027981275	15,89249574408	15,3391569031846	17,6696288253081
4	16,3082829646302	16,4219121633013	15,4057789891175	15,4329837445859	15,2856046024985	17,2855482009363
5	16,1613261814577	16,263653907368	15,1972551801076	14,9734717450919	15,2320523018123	16,9014675765645
6	16,0143693982852	16,1053956514348	14,9887313710977	14,5139597455979	15,1785000011261	16,5173869521927
7	15,8674126151128	15,9471373955015	14,7802075620878	14,0544477461039	15,1249477004399	16,1333063278209
8	15,7204558319403	15,7888791395682	14,5716837530779	13,5949357466099	15,0713953997537	15,7492257034491
9	15,5734990487678	15,6306208836349	14,3631599440679	13,1354237471158	15,0178430990676	15,3651450790773
10	15,4265422655954	15,4723626277017	14,154636135058	12,6759117476218	14,9642907983814	14,9810644547055
11	15,2795854824229	15,3141043717684	13,9461123260481	12,2163997481278	14,9107384976952	14,5969838303337

Figure 4. 9. Rod lengths at each treatment step (Example2)

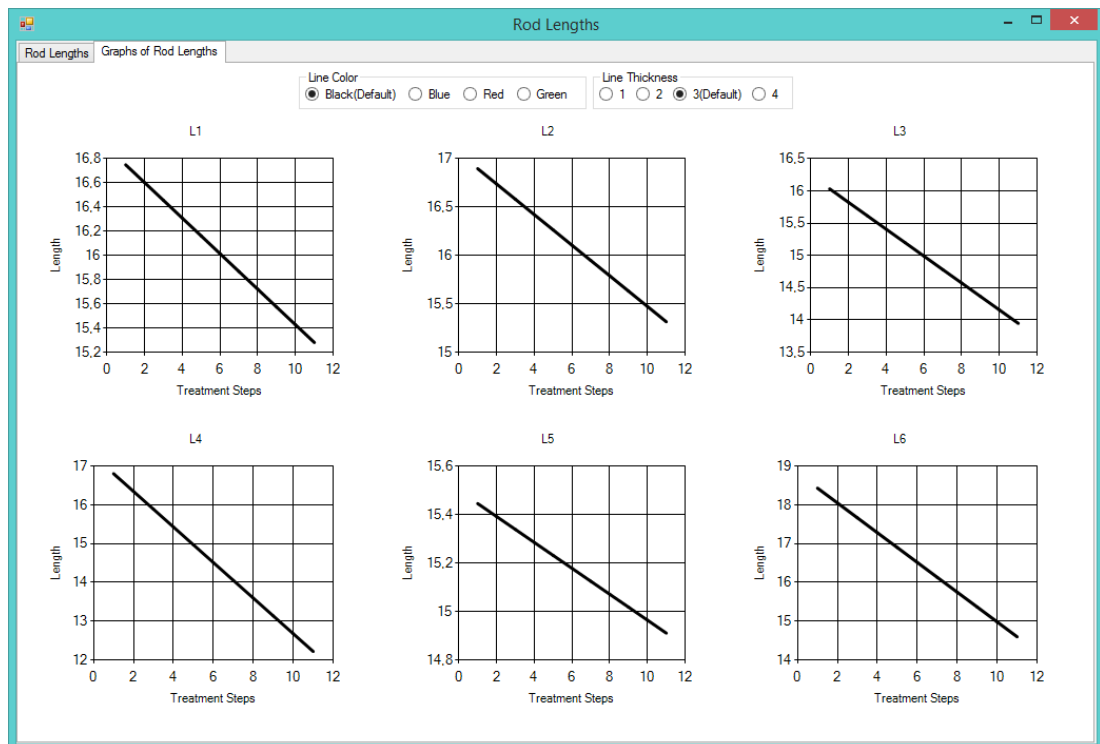


Figure 4. 10. Variation of the rod lengths at each treatment step (Example2)

- The contents of the file which is created to desktop by Motion Analysis Interface with the extension .txt can be examined from “Instant Displacement Values” tab under the “Treatment” menu, Figure 4. 11.

Step	x1	y1	z1	tx	ty	tz
1	0,162786933025029	1,37798062574505	15,9848836016679	0,836071703628495	-5,54796312635145	69
2	-0,160354101286841	1,00328179263757	15,6152051801658	0,493993201468321	-5,91690930671914	65,8440950911296
3	-0,473393361793519	0,639324829252035	15,4092380802896	0,342610728373169	-6,34850733880393	62,8202876270809
4	-0,776673324654513	0,286217662998882	15,1821224918732	0,370480818422465	-6,82464453119352	59,912417175368
5	-1,07039627004784	-0,0558521158401463	14,9351398856628	0,568010917535237	-7,33051559946809	57,1070579806806
6	-1,35466196566812	-0,386643217085592	14,6693927948329	0,926976408507812	-7,85382379676438	54,3929150300035
7	-1,62949024234441	-0,705869949257307	14,38584962353	1,4402779398023	-8,38425992670056	51,7603791824315
8	-1,89483753760529	-1,01320759596652	14,0853766501439	2,10177404260852	-8,91312605605364	49,2011991578445
9	-2,15060942857418	-1,30829497013662	13,7687612743807	2,90616734048401	-9,43305932039795	46,7082276148581
10	-2,39667047816972	-1,5907347590893	13,4367290173677	3,84893200762094	-9,93782744889522	44,2752138897291
11	-2,63285231778958	-1,86009195018372	13,0899558261729	4,9262760147077	-10,4221775909564	41,8966246527943

Figure 4. 11. Instant displacement values at each treatment step (Example2)

- Variation of the determinant values are plotted. This plot can be accessed from “Jacobian” tab under the “Treatment” menu, Figure 4. 12. It can be saved to desktop by using “Save Image” button.

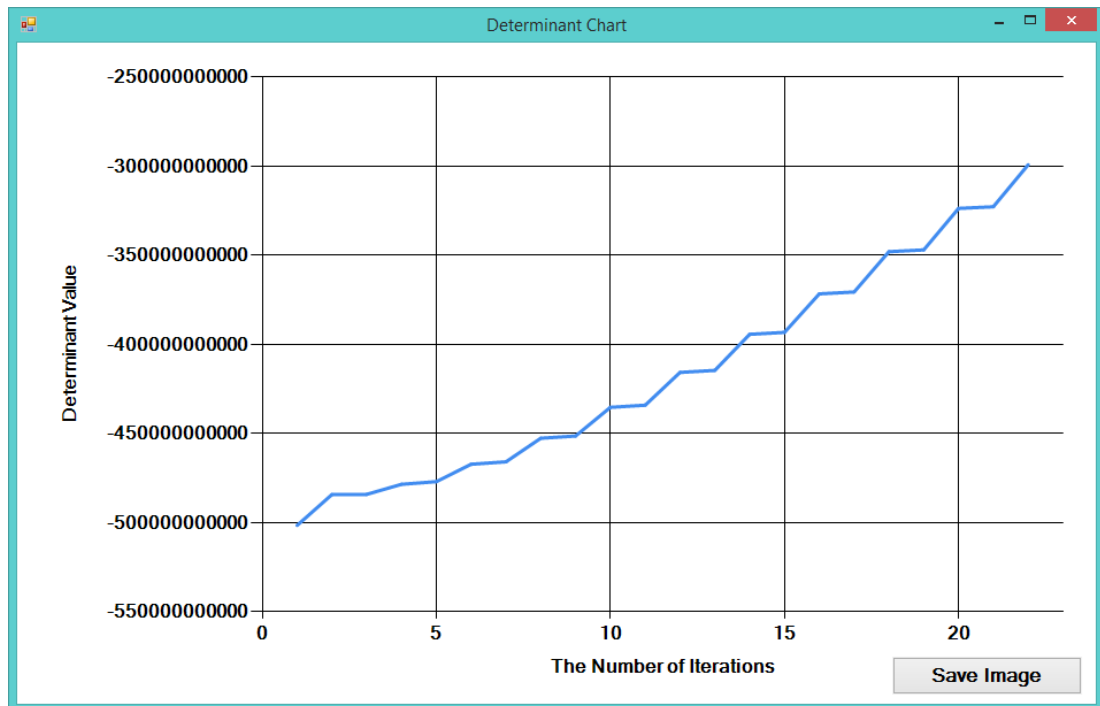


Figure 4. 12. Number of iteration vs Determinant Value (Example2)

4.1.3. Example3 (Singular Case)

In this example, a singular case is presented.

- The input data “Rod Lengths” and “Angles and Ring Diameter” which are given in Table 4. 9 and Table 4. 10 required for the operation of the “Motion

Analysis Interface” is obtained by an interface “Fixator Otomasyonu” which is shown in Figure 4. 13 developed under the TÜBİTAK Project 106M466.

Table 4. 9. Rod lengths of the fixator for unhealthy and healthy configurations (Example3)

Unhealthy Configuration		Healthy Configuration	
Rod Length	Value (cm)	Rod Length	Value (cm)
L_1	28,6757958946081	L_1	25,4950975679639
L_2	27,1237668809253	L_2	25,4950975679639
L_3	28,6757958946081	L_3	25,4950975679639
L_4	27,1237668809253	L_4	25,4950975679639
L_5	28,6757958946081	L_5	25,4950975679639
L_6	27,1237668809253	L_6	25,4950975679639

Table 4. 10. Ring parameters (Example3)

Ring Parameters			
Proximal Ring	Diameter(cm)	Distal Ring	Diameter(cm)
R	5	R_1	5
Proximal Joints	Value (Degree)	Proximal Joints	Value (Degree)
α_1	60	ε_1	120
α_2	180	ε_2	120
α_3	180	ε_3	240
α_4	300	ε_4	240
α_5	300	ε_5	0
α_6	60	ε_6	0

Fixator Otomasyonu

AP Data

PsAp 0

BeAp 0

Be'Ap 0

cAP 10

bAP 15

Lateral Data

PsL 0

BeL 0

Be'L 0

cL 10

bL 15

e Data

ex -1

ey 1

ez 2

Sonuçlar

Sağlıksız

L1 28,67579

L2 27,12376

L3 28,67579

L4 27,12376

L5 28,67579

L6 27,12376

Sağlıklı

L1 25,49509

L2 25,49509

L3 25,49509

L4 25,49509

L5 25,49509

L6 25,49509

Avvw

1 0 0

0 1 0

0 0 1

Axyz

1 0 0

0 1 0

0 0 1

AP Data

Del0 90

DelAx 90

Del 0

Alfa Data

a1 60

a2 180

a3 180

a4 300

a5 300

a6 60

q Data

qx 1

qy 0

qz 0

Fixator Yarıcap

R 5

R1 5

Epsilon Degerleri

e1 120

e2 120

e3 240

e4 240

e5 0

e6 0

r Data

rx 0

ry 1

rz 0

Uzunluklar

b 15

c 10

Kontrol

nu 1

rv 6,123031

☐ AP için Hesapla

Figure 4. 13. Primary Inputs from “Fixator Otomasyonu” interface (Example3)

- Initial translational and rotational parameters $(x_1, y_1, z_1, \theta_x, \theta_y, \theta_z)$ are entered as input. These are given in Table 4. 11.

Table 4. 11. Initial motion values (Example3)

Initial Motion Values			
Translations	Value (cm)	Rotations	Value (Degree)
x_1	0	θ_x	0
y_1	0	θ_y	0
z_1	27	θ_z	90

- The other parameters to be entered which are given in Table 4. 12 are “Treatment Step”, “Stopping Criteria (%)” and “File Name.txt”.

Table 4. 12. Basic input (Example3)

Basic Input					
Input1	Value (Day)	Input2	Value (%)	Input3	Textual Input
Treatment Step	10	Stopping Criteria	5	File Name.txt	Example3.txt

“Calculate” button is clicked after the entering all data into relevant boxes. The results obtained can be seen in Figure 4. 14.

[illegible]

Figure 4. 14. Main window of the “Motion Analysis Interface” (Example3)

Singularity was detected at initial configuration of the fixator by “Motion Analysis Interface”. Elements of the Jacobian matrix which can be seen in (4.1) are calculated as follows according to the equation (3.10.).

$$J = \begin{bmatrix} -13.66 & -13.66 & 54 & 233.83 & 135 & 25 \\ 1.34 & -5 & 54 & 233.83 & 135 & 25 \\ 18.66 & -5 & 54 & -233.83 & 135 & 25 \\ 3.66 & 3.66 & 54 & -233.83 & 135 & 25 \\ -5 & 18.66 & 54 & 0 & -270 & 25 \\ -5 & 1.34 & 54 & 0 & -270 & 25 \end{bmatrix} \quad (4.1)$$

It is obvious that the determinant of this matrix is zero because the third and sixth columns are linearly dependent. So, inverse of this matrix cannot be calculated which will be used in equation (3.27.). This means that initial (unhealthy) configuration of the biomechanical system is a singular position with which the treatment process should not begin.

4.2. Applications of the Force Analysis Interface

Three examples are given in order to analyse the result.

4.2.1. Example4 (Non-Singular Case)

In this example, a non-singular case is presented.

- The input data “Ring Parameters” and “Force-Moment Inputs” which are required for the operation of the “Force Analysis Interface” are given in Table 4. 13 and Table 4. 14.

Table 4. 13. Ring parameters (Example4)

Ring Parameters			
Proximal Ring	Diameter(cm)	Distal Ring	Diameter(cm)
R	7,25	R_1	7,25
Proximal Joints	Value (Degree)	Proximal Joints	Value (Degree)
α_1	90	ε_1	0
α_2	110	ε_2	100
α_3	210	ε_3	120
α_4	230	ε_4	220
α_5	330	ε_5	240
α_6	350	ε_6	340

Table 4. 14. Force-Moment Parameters (Example4)

Force-Moment Parameters					
Force	Value	Moment	Value	Force Application Point (Local Coordinate System)	Value(cm)
F_x	0	M_x	0	rF_u	0,3713007
F_y	0	M_y	0	rF_v	0,0195003
F_z	-100	M_z	0	—	-

- The other parameters to be entered which are given in Table 4. 15 are “Determinant Limit Value” and “File Name.txt”.

Table 4. 15. Basic input (Example4)

Basic Input			
Input1	Value	Input2 (Instant Displacement Values)	Textual Input
Determinant Limit Value	10^{-6}	File Name.txt	Example4.txt

- Instant displacement values which are obtained by using “Motion Analysis Interface” at each step are read from “Example4.txt” by using “Read Displacement Data”.

Instant Displacement Values						
Treatment Step	x1	y1	z1	thetax	thetay	thetaz
1	0.47691668605336700000	0.32625357209351300000	14.10066830308810000000	4.17959274555139000000	-4.02926424111713000000	49.000000000000000000
2	0.183985909896300000	0.05100144093129940000	14.05608919992450000000	4.13159139922409000000	-4.74339680924913000000	48.30132838693670000000
3	-0.10699681159834300000	-0.22286338344574600000	14.00064972424500000000	4.12229568115342000000	-5.45719406960976000000	47.60576919743430000000
4	-0.39612203984379500000	-0.49524350713623900000	13.93267227323060000000	4.15147712475385000000	-6.17093853649109000000	46.91231452397060000000
5	-0.68342914672018600000	-0.76604102002347400000	13.85395865314690000000	4.21907321208203000000	-6.88496783455013000000	46.21995080702960000000
6	-0.96897079014303200000	-1.03515695584007000000	13.76394369294780000000	4.32518146072797000000	-7.59967490015341000000	45.52764967423880000000
7	-1.25280108038074000000	-1.30249032950796000000	13.66261823356910000000	4.47006598530572000000	-8.31551191425633000000	44.83435655138930000000
8	-1.53497648654779000000	-1.56793708072468000000	13.54994315418430000000	4.65416712382530000000	-9.03299552329944000000	44.13897805073660000000
9	-1.81555683196568000000	-1.83138889231076000000	13.42584764827760000000	4.87811457231001000000	-9.75271352673674000000	43.44036772397600000000
10	-2.09460641387468000000	-2.09273184452727000000	13.29022692612910000000	5.14274463679686000000	-10.47533327592460000000	42.73709666289600000000
11	-2.37219529115250000000	-2.35184485661736000000	13.14293924499970000000	5.44912242550655000000	-11.20161211303820000000	42.02849931964450000000

Figure 4. 15. Instant displacement values as input (Example4)

“Calculate” button is clicked after the entering all data into relevant boxes. Then following output are obtained.

- Instant displacement values $(x_1, y_1, z_1, \theta_x, \theta_y, \theta_z)$ at each step can be seen by using “Instant Displacement Values” tab under the “Outputs” menu, Figure 4. 15.
- Magnitudes of joint forces and determinant values for both initial and final configurations of the robotic structure can be seen under the “Primary Output” group on the right hand side of the main window, Figure 4. 16. The result that no singularity exists can be observed directly.

Singularity Control - Force Analysis

Outputs

Ring Parameters

R: 7,25 R1: 7,25

Proximal Joint Parameters(Degree)

α_1 : 90 α_2 : 110 α_3 : 210 α_4 : 230 α_5 : 330 α_6 : 350

Distal Joint Parameters(Degree)

ϵ_1 : 0 ϵ_2 : 100 ϵ_3 : 120 ϵ_4 : 220 ϵ_5 : 240 ϵ_6 : 340

Force-Moment Inputs

Fx: 0 Fy: 0 Fz: -100 Mx: 0 My: 0 Mz: 0

Force Application Point

rFu: 0,3713007 rFv: 0,0195003 rFw:

☒ Local Coordinate System ☐ Global Coordinate System

Determinant Limit Value= 10^{-6}

Read Displacement Data

Example4

CALCULATE ✓

Primary Output

Unhealthy

F1: 25,9844373014223
F2: 30,2784331269586
F3: 2,56911028128042
F4: 13,4643544306757
F5: 23,6142703586417
F6: 9,26605366916176
Determinant: 517,413810128521

Healthy

F1: 25,9042422510619
F2: 18,2792424026363
F3: 13,3404894799266
F4: 26,7005008331134
F5: 7,4931627273547
F6: 14,0890784691638
Determinant: 541,08699592049

No Singularity

Figure 4. 16. Main windows of the “Force Analysis Interface” (Example4)

- Magnitudes of joint forces and determinant values at each treatment step can be observed from “Force Values and Determinant” tab under the “Outputs” menu, Figure 4. 17. Also variation of the joint forces and determinant can be seen by clicking “Force Graphs” and “Determinant Graph” tabs, Figure 4. 18 and Figure 4. 19.

Forces and Determinant							
Force Values and Determinant							
Step	F1	F2	F3	F4	F5	F6	Determinant
1	25,9844373014223	30,2784331269586	2,56911028128042	13,4643544306757	23,6142703586417	9,26605366916176	517,413810128521
2	25,7874706725444	29,0652755380949	3,72183739036551	14,6948090134761	22,1506157187449	9,9236194589788	523,449250501489
3	25,6297112116231	27,8601630292823	4,8527396607133	15,9383916886804	20,6662878985517	10,5408616024853	528,756583397287
4	25,511941215725	26,6612975318053	5,9635092681008	17,197630893042	19,1569698520836	11,1186549579436	533,305918900607
5	25,4350752363927	25,4668463001914	7,05591709125042	18,4751617405579	17,6182528657159	11,6576663441195	537,066003513653
6	25,4001620122694	24,2749137457866	8,13184301212358	19,7737756285539	16,0455752751078	12,1583704098654	540,004008333349
7	25,4084071491304	23,0835202328578	9,1932946645705	21,0964835822353	14,4341548696944	12,6210420487397	542,085412547319
8	25,4612000533946	21,8905769462794	10,2424311000553	22,4465875065756	12,7789140345321	13,045744233102	543,273866098043
9	25,5601461727128	20,6938557468187	11,2815924842346	23,8277626212423	11,0743944003612	13,4323107076024	543,531028450425
10	25,7071059366997	19,490952567549	12,3133373484056	25,2441551866135	9,31465699230701	13,7803227017415	542,816379330694
11	25,9042422510619	18,2792424026363	13,3404894799266	26,7005008331134	7,4931627273547	14,0890784691638	541,08699592049

Figure 4. 17. Magnitudes of joint forces and determinant values at each step (Example4)

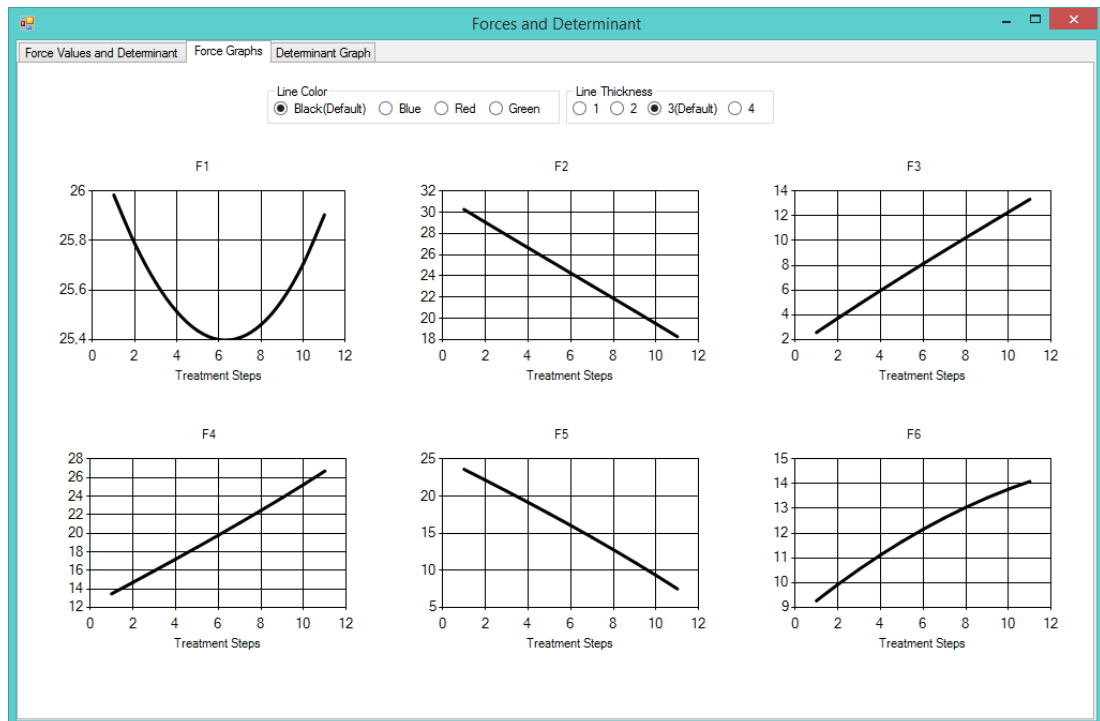


Figure 4. 18. Variation of the joint forces during the treatment (Example4)

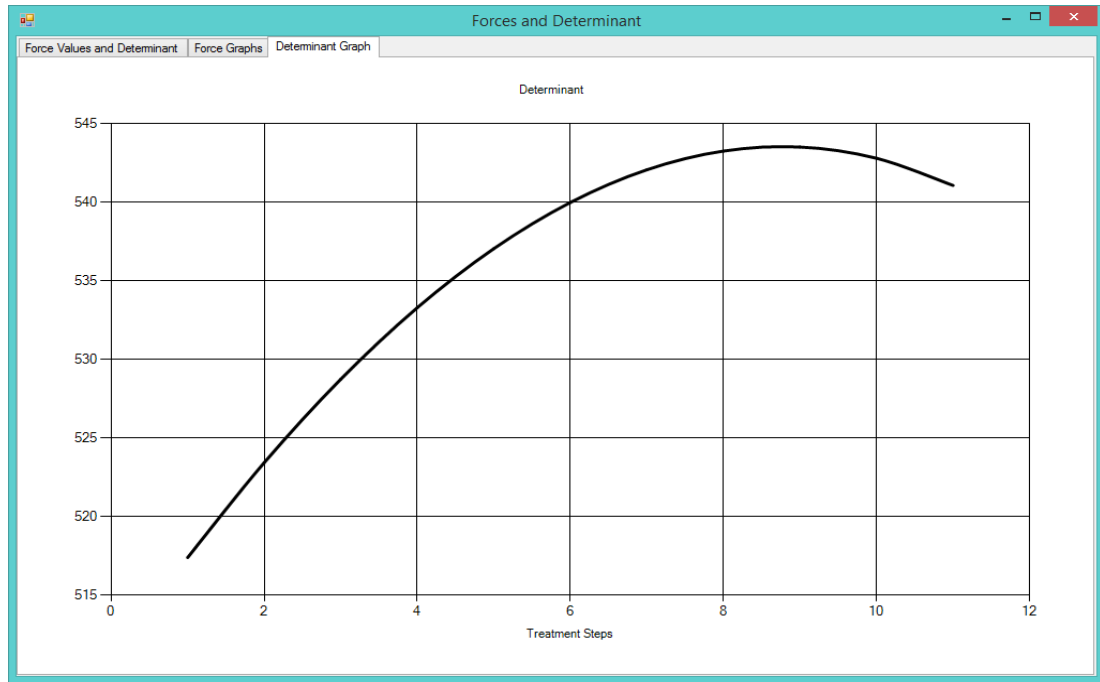


Figure 4. 19. Variation of the determinant values during the treatment (Example4)

- Rod lengths and their variation plots which are shown in Figure 4. 20 and Figure 4. 21 can be accessed from “Rod Lengths” under the “Outputs” menu.

Rod Lengths						
Step	L1	L2	L3	L4	L5	L6
1	15,5842217330173	15,152799148148	15,2593331155484	14,3469453821305	14,1142595101498	15,2795830393268
2	15,58670674404	15,1838519627193	15,1139926036797	14,1449128942915	14,1927971887	15,2121915179971
3	15,5891907765478	15,2149043183464	14,9686516802294	13,9428780697571	14,2713322059299	15,1447985733075
4	15,5916748129544	15,2459566760815	14,8233107586193	13,7408432611455	14,3498672378773	15,0774056352989
5	15,5941588532571	15,2770090360241	14,6779698388602	13,5388084696845	14,4284022845344	15,010012704119
6	15,5966428975319	15,3080613983357	14,5326289210072	13,3367736970522	14,5069373462293	14,9426197800681
7	15,5991269459325	15,3391137632448	14,3872880051569	13,1347389454587	14,5854724236438	14,8752268636089
8	15,6016109986989	15,3701661310582	14,2419470914496	12,9327042177548	14,664007517849	14,8078339553914
9	15,604095056172	15,4012185021772	14,0966061800744	12,7306695175827	14,7425426303611	14,7404410562899
10	15,6065791188126	15,4322708771207	13,9512652712748	12,5286348495812	14,8210777632238	14,6730481674528
11	15,6090631872289	15,463323256557	13,805924365359	12,3266002196661	14,8996129191256	14,6056552903737

Figure 4. 20. Rod lengths at each treatment step (Example4)

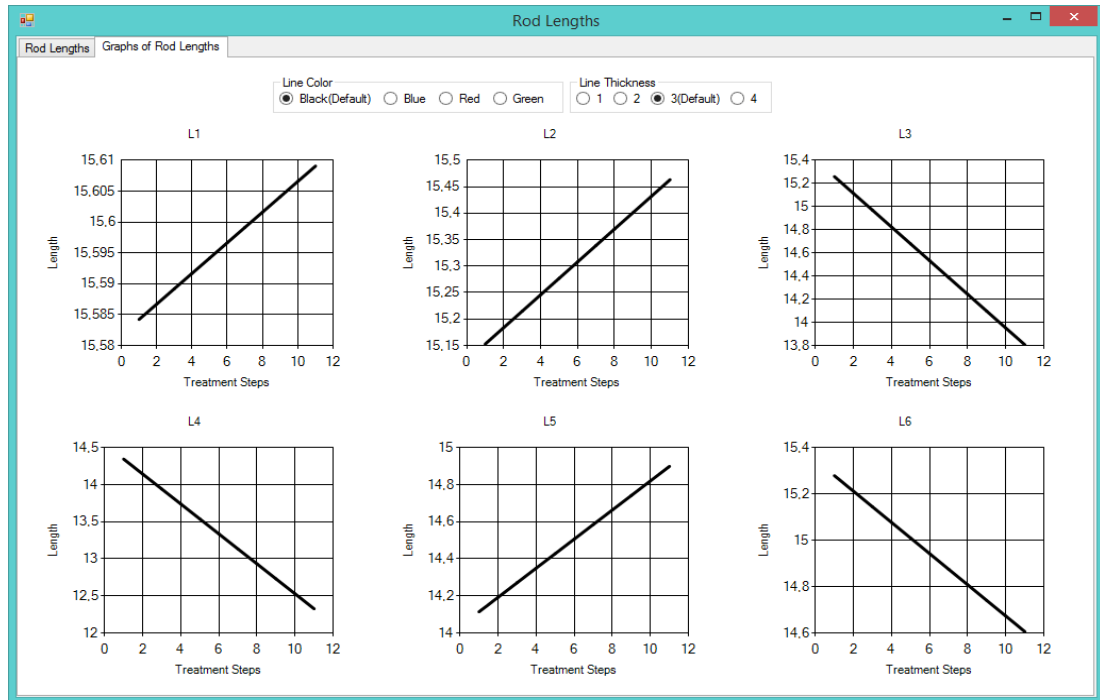


Figure 4. 21. Variation of the rod lengths during the treatment (Example4)

- Matrix form of the force equation at each treatment step can be examined by using counter in the “Matrix Form”. Matrices which are calculated with values of the first and last configuration are shown in Figure 4. 22.



Figure 4. 22. Matrix form of the force equation at initial and final step (Example4)

4.2.2. Example5 (Non-Singular Case)

In this example, a non-singular case is presented.

- The input data “Ring Parameters” and “Force-Moment Inputs” which are required for the operation of the “Force Analysis Interface” are given in Table 4. 16 and Table 4. 17.

Table 4. 16. Ring parameters (Example5)

Ring Parameters			
Proximal Ring	Diameter(cm)	Distal Ring	Diameter(cm)
R	7,25	R_1	7,25
Proximal Joints	Value (Degree)	Proximal Joints	Value (Degree)
α_1	80	ε_1	0
α_2	100	ε_2	100
α_3	190	ε_3	130
α_4	210	ε_4	200
α_5	320	ε_5	220
α_6	340	ε_6	340

Table 4. 17. Force-Moment Parameters (Example5)

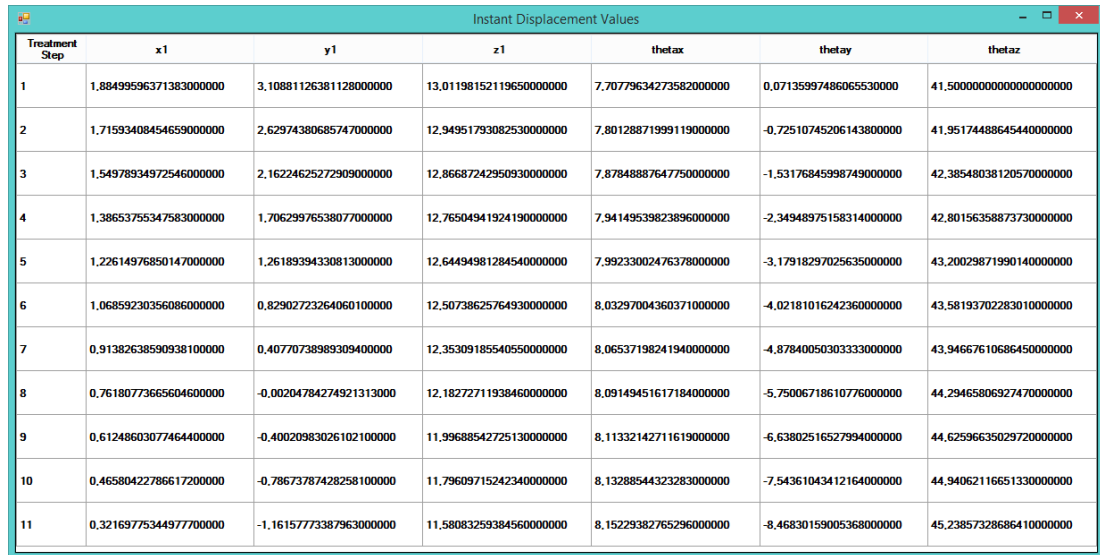
Force-Moment Parameters					
Force	Value	Moment	Value	Force Application Point (Local Coordinate System)	Value(cm)
F_x	0	M_x	0	rF_u	0,1214679
F_y	0	M_y	0	rF_v	0,8258311
F_z	-100	M_z	0	—	-

- The other parameters to be entered which are given in Table 4. 18 are “Determinant Limit Value” and “File Name.txt”.

Table 4. 18. Basic input (Example5)

Basic Input			
Input1	Value	Input2 (Instant Displacement Values)	Textual Input
Determinant Limit Value	10^{-6}	File Name.txt	Example5.txt

- Instant displacement values which are obtained by using “Motion Analysis Interface” at each step are read from “Example5.txt” by using “Read Displacement Data”.



Treatment Step	x1	y1	z1	thetax	thetay	thetaz
1	1.88499596371383000000	3.10881126381128000000	13.01198152119650000000	7.70779634273582000000	0.07135997486065530000	41.500000000000000000
2	1.71593408454659000000	2.62974380685747000000	12.94951793082530000000	7.80128871999119000000	-0.72510745206143800000	41.951744886454000000
3	1.54978934972546000000	2.16224625272909000000	12.86687242950930000000	7.87848887647750000000	-1.53176845998749000000	42.38548038120570000000
4	1.38653755347583000000	1.70629976538077000000	12.76504941924190000000	7.94149539823896000000	-2.34948975158314000000	42.80156358873730000000
5	1.22614976850147000000	1.26189394330813000000	12.64494981284540000000	7.99233002476378000000	-3.17918297025635000000	43.20029871990140000000
6	1.06859230356086000000	0.82902723264060100000	12.50738625764930000000	8.03297004360371000000	-4.02181016242360000000	43.58193702283010000000
7	0.91382638590938100000	0.40770738989309400000	12.35309185540550000000	8.06537198241940000000	-4.87840050303330000000	43.94667610686450000000
8	0.76180773665604600000	-0.00204784274921313000	12.18272711938460000000	8.09149451617184000000	-5.75006718610776000000	44.29465806927470000000
9	0.61248603077464400000	-0.40020983026102100000	11.99688542725130000000	8.11332142711619000000	-6.63802516527994000000	44.62596635029720000000
10	0.46580422786617200000	-0.78673787428258100000	11.79609715242340000000	8.13288544323283000000	-7.54361043412164000000	44.94062116651330000000
11	0.32169775344977700000	-1.16157773387963000000	11.58083259384560000000	8.15229382765296000000	-8.46830159005368000000	45.23857328686410000000

Figure 4. 23. Instant displacement values as input (Example5)

“Calculate” button is clicked after the entering all data into relevant boxes. Then following output are obtained.

- Instant displacement values ($x_1, y_1, z_1, \theta_x, \theta_y, \theta_z$) at each step can be seen by using “Instant Displacement Values” tab under the “Outputs” menu, Figure 4. 23.
- Magnitudes of joint forces and determinant values for both initial and final configurations of the robotic structure can be seen under the “Primary Output” group on the right hand side of the main window, Figure 4. 24. The result that no singularity exists can be observed directly.

Singularity Control - Force Analysis

Outputs

Ring Parameters

R: 7,25 R1: 7,25

Proximal Joint Parameters(Degree)

α_1 : 80 α_2 : 100 α_3 : 190 α_4 : 210 α_5 : 320 α_6 : 340

Distal Joint Parameters(Degree)

ϵ_1 : 0 ϵ_2 : 100 ϵ_3 : 130 ϵ_4 : 200 ϵ_5 : 220 ϵ_6 : 340

Force-Moment Inputs

Fx: 0 Fy: 0 Fz: -100 Mx: 0 My: 0 Mz: 0

Force Application Point

rFu: 0,1214679 rFv: 0,8258311 rFw:
☒ Local Coordinate System ☐ Global Coordinate System

Determinant Limit Value= 10^{-6}

Read Displacement Data

Example5

CALCULATE ✓

Primary Output

Unhealthy

F1: 44,5913866397376
 F2: 48,8570582752517
 F3: -26,0364940287442
 F4: 28,7175630163369
 F5: 18,3600403818601
 F6: -11,2382387191829
 Determinant: 428,806419095829

Healthy

F1: 30,1744766255518
 F2: 22,6838969717153
 F3: 10,8985597172391
 F4: 16,9041557346487
 F5: 16,0264554161359
 F6: 11,2453924334233
 Determinant: 624,969963495994

No Singularity

Figure 4. 24. Main windows of the “Force Analysis Interface” (Example5)

- Magnitudes of joint forces and determinant values at each treatment step can be observed from “Force Values and Determinant” tab under the “Outputs” menu, Figure 4. 25. Also variation of the joint forces and determinant graphs can be seen by clicking “Force Graphs” and “Determinant Graph” tabs, Figure 4. 26 and Figure 4. 27.

Forces and Determinant							
Force Values and Determinant							
Step	F1	F2	F3	F4	F5	F6	Determinant
1	44,5913866397376	48,8570582752517	-26,0364940287442	28,7175630163369	18,3600403818601	-11,2382387191829	428,806419095829
2	42,8958097645686	45,6433429638687	-21,3691969985113	27,2887869366308	17,9817223635779	-8,52684269400699	450,348957481098
3	41,263390541614	42,6122343743192	-16,9826519323187	25,9255540976306	17,6462487668805	-5,93586888637566	471,647726009822
4	39,690310648471	39,7413314609205	-12,8478883227144	24,6227746052034	17,3490808915363	-3,4569131297134	492,641267191567
5	38,1735899622008	37,0108813474905	-8,93939172442265	23,3761468812881	17,0860748604998	-1,08282703002285	513,264696590811
6	36,7109862104624	34,4032210906364	-5,23443653230463	22,182026439808	16,8533991342523	1,19248516730425	533,44883279018
7	35,3009135909936	31,9023320087725	-1,71254961490205	21,0373064202863	16,6474818724239	3,37423274359334	553,119563132217
8	33,9423845181024	29,4934644392616	1,64493964176249	19,9393211814459	16,4649632287105	5,46687028475609	572,197121074814
9	32,634971291534	27,1628118729248	4,85528731884832	18,8857670437761	16,3026508636342	7,47420220933708	590,595257474041
10	31,3787858107117	24,8972173125462	7,93468082223962	17,8746355841996	16,1574770163484	9,39946522862202	608,220284190024
11	30,1744766255518	22,6838969717153	10,8985597172391	16,9041557346487	16,0264554161359	11,2453924334233	624,969963495994

Figure 4. 25. Magnitudes of joint forces and determinant values at each step (Example5)

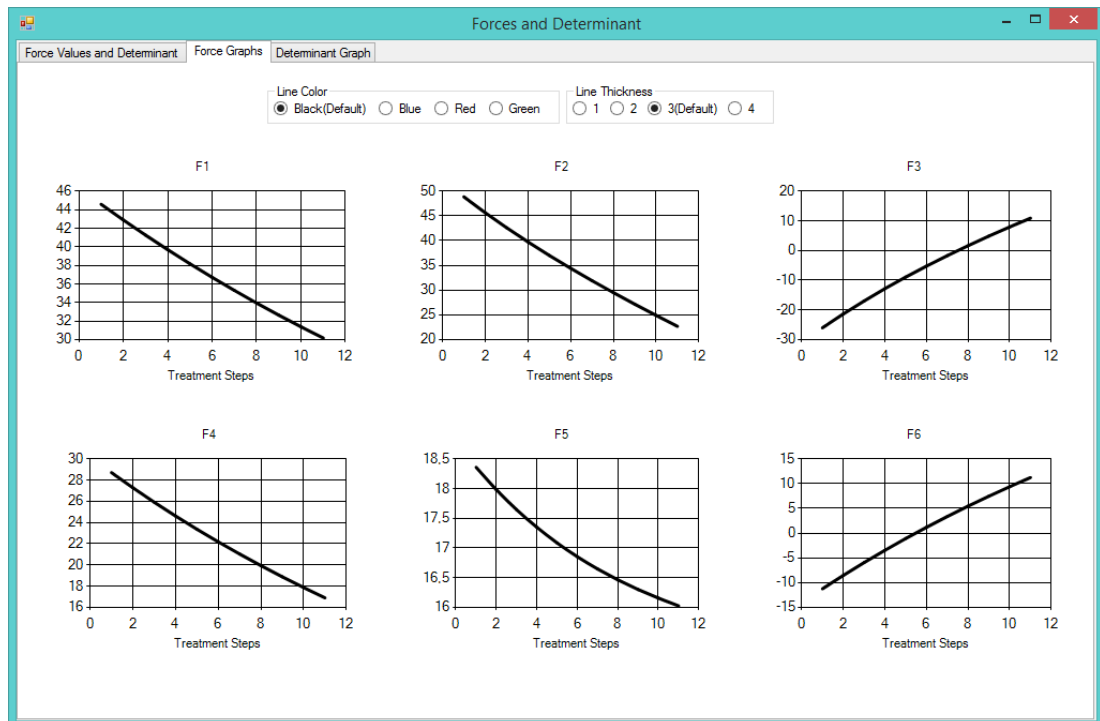


Figure 4. 26. Variation of the joint forces during the treatment (Example5)

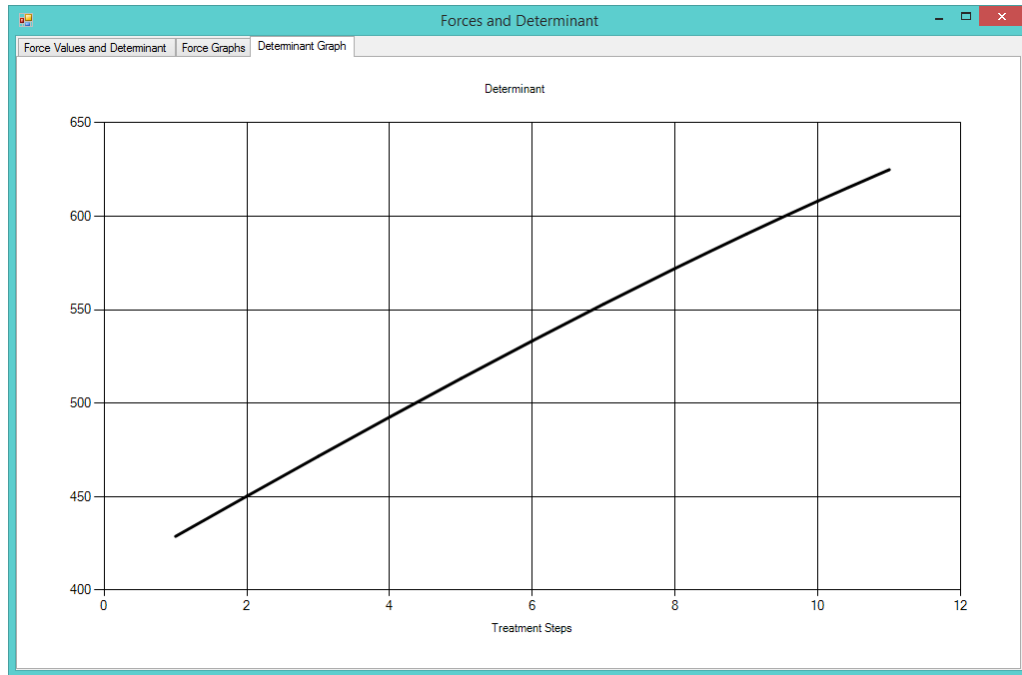


Figure 4. 27. Variation of the determinant values during the treatment (Example5)

- Rod lengths and their variation plots which are shown in Figure 4. 28 and Figure 4. 29 can be accessed from “Rod Lengths” under the “Outputs” menu.

Step	L1	L2	L3	L4	L5	L6
1	14,3648527502285	14,1973193696612	14,9051444654682	13,5319673112311	13,2946255481056	15,2336413818047
2	14,2965689730004	14,1626240464982	14,5838257857998	13,3361981680448	13,1823629795757	15,0109930869208
3	14,2282825267373	14,1279234339478	14,2625014609012	13,1404239105619	13,0700958012697	14,7883425459155
4	14,1599961158897	14,0932228749751	13,9411771856154	12,9446496997108	12,9578286646547	14,5656920324344
5	14,0917097341079	14,0585223614025	13,6198529558866	12,7488755332309	12,8455615665749	14,3430415417136
6	14,023423376531	14,0238218870011	13,298528769234	12,5531014103405	12,7332945051979	14,1203910701348
7	13,9551370394142	13,9891214470898	12,9772046245968	12,3573273316901	12,6210274799051	13,8977406149472
8	13,8868507198747	13,9544210382332	12,6558805222257	12,1615532993507	12,5087604912161	13,6750901740838
9	13,8185644157115	13,9197206580375	12,3345564636633	11,9657793168966	12,3964935407896	13,4524397460358
10	13,750278125277	13,8850203050219	12,0132324518092	11,7700053895888	12,2842266314979	13,2297893297703
11	13,6819918473887	13,850319978551	11,6919084910729	11,5742315246747	12,1719597675812	13,0071389246838

Figure 4. 28. Rod lengths at each treatment step (Example5)

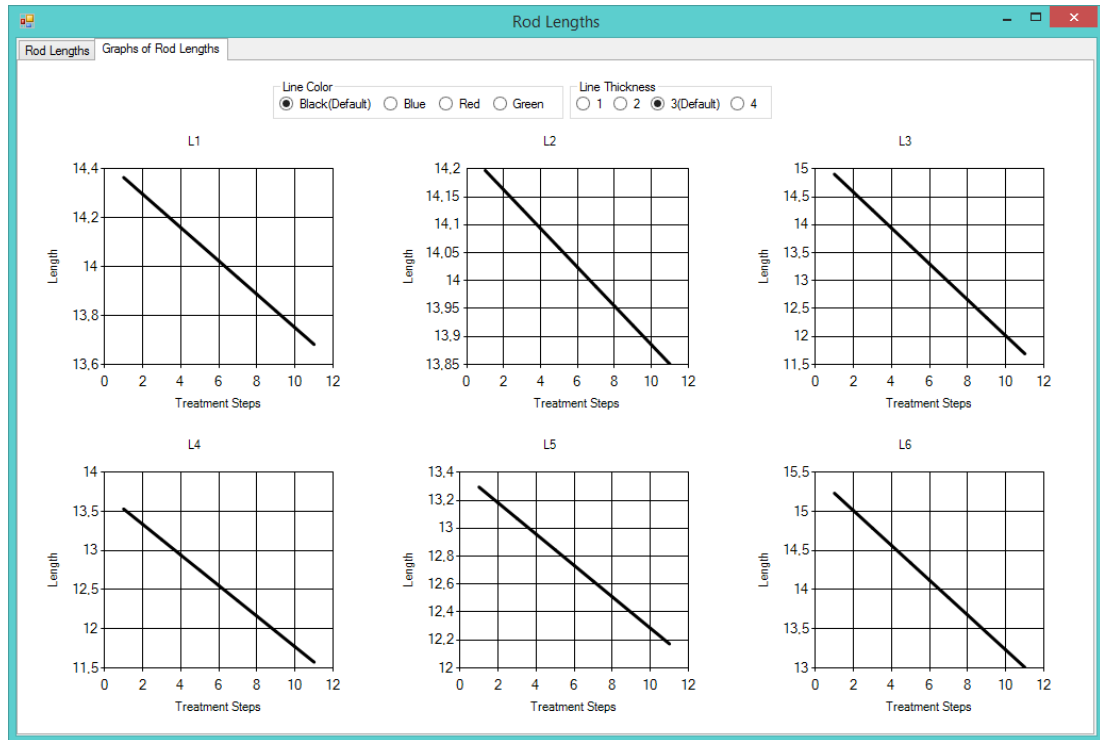


Figure 4. 29. Variation of the rod lengths during the treatment (Example5)

- Matrix form of the force equation at each treatment step can be examined by using counter in the “Matrix Form”. Matrices which are calculated with values of the first and last configuration are shown in Figure 4. 30.



Figure 4. 30. Matrix form of the force equation at initial and final step (Example5)

4.2.3. Example6 (Singular Case)

In this example, a singular case is presented. Data of the Example6 are the same as Example3.

- The input data “Ring Parameters” and “Force-Moment Inputs” which are required for the operation of the “Force Analysis Interface” are given in Table 4. 19 and Table 4. 20.

Table 4. 19. Ring parameters (Example6)

Ring Parameters			
Proximal Ring	Diameter(cm)	Distal Ring	Diameter(cm)
R	5	R_1	5
Proximal Joints	Value (Degree)	Proximal Joints	Value (Degree)
α_1	60	ε_1	120
α_2	180	ε_2	120
α_3	180	ε_3	240
α_4	300	ε_4	240
α_5	300	ε_5	0
α_6	60	ε_6	0

Table 4. 20. Force-Moment Parameters (Example6)

Force-Moment Parameters					
Force	Value	Moment	Value	Force Application Point (Local Coordinate System)	Value(cm)
F_x	0	M_x	0	rF_u	1
F_y	0	M_y	0	rF_v	0
F_z	-100	M_z	0	—	-

- The other parameters to be entered which are given in Table 4. 21 are “Determinant Limit Value” and “File Name.txt”.

Table 4. 21. Basic input (Example6)

Basic Input			
Input1	Value	Input2 (Instant Displacement Values)	Textual Input
Determinant Limit Value	10^{-6}	File Name.txt	Example6.txt

- Instant displacement values at the intermediate steps couldn't be obtained by using "Motion Analysis Interface". Calculation process was interrupted because singularity detected at first step of the Example3. "Example6.txt" which contains only initial and final displacement values is read as input by using "Read Displacement Data". The motion parameters data in the "Example6.txt" are given in Table 4. 22.

Table 4. 22. Initial and final motion parameters (Example6)

Initial Motion Parameters			
Translations	Value (cm)	Rotations	Value (Degree)
x_1	0	θ_x	0
y_1	0	θ_y	0
z_1	27	θ_z	90
Final Motion Parameters			
Translations	Value (cm)	Rotations	Value (Degree)
x_1	0	θ_x	0
y_1	0	θ_y	0
z_1	25	θ_z	0

Fixator Otomasyonu

AP Data PsAp 0 BeAp 0 Be'Ap 0 cAP 10 bAP 15	Lateral Data PsL 0 BeL 0 Be'L 0 cL 10 bL 15	e Data ex -1 ey 1 ez 2	Sonuçlar Sağlıksız L1 28,67579 L2 27,12376 L3 28,67579 L4 27,12376 L5 28,67579 L6 27,12376 Sağlıklı L1 25,49509 L2 25,49509 L3 25,49509 L4 25,49509 L5 25,49509 L6 25,49509	Auvw 1 0 0 0 1 0 0 0 1
Axial Data Del0 90 DelAx 90 Del 0	Alfa Data a1 60 a2 180 a3 180 a4 300 a5 300 a6 60	q Data qx 1 qy 0 qz 0	Axyz 1 0 0 0 1 0 0 0 1	
Fixator Yarıcap R 5 R1 5	Epsilon Değerleri e1 120 e2 120 e3 240 e4 240 e5 0 e6 0	r Data rx 0 ry 1 rz 0	Uzunluklar b 15 c 10 Kontrol ru 1 rv 6.123031	☐ AP için Hesapla Hesapla Kaydet Aç Çıkış

Figure 4. 31. Calculation of rod lengths both initial and final step (Example6)

“Calculate” button is clicked after the entering all data into relevant boxes. The results obtained can be seen in Figure 4. 32. Singularity is detected at initial configuration according to these results.

Singularity Control - Force Analysis

Outputs

Ring Parameters

R 5 R1 5

Proximal Joint Parameters(Degree)

α_1 60 α_2 180 α_3 180 α_4 300 α_5 300 α_6 60

Distal Joint Parameters(Degree)

ϵ_1 120 ϵ_2 120 ϵ_3 240 ϵ_4 240 ϵ_5 0 ϵ_6 0

Force-Moment Inputs

Fx 0 Fy 0 Fz -100 Mx 0 My 0 Mz 0

Force Application Point

rFu 1 rFv 0 rFw

☒ Local Coordinate System ☐ Global Coordinate System

Determinant Limit Value= 10^{-6}

Read Displacement Data

Example6

CALCULATE ✓

Primary Output

Unhealthy

F1 -1,00016649442836E+17
F2 9,46034172397214E+16
F3 -1,00016649442836E+17
F4 9,46034172397219E+16
F5 -1,00016649442836E+17
F6 9,46034172397215E+16
Determinant 2,34243775576406E-15

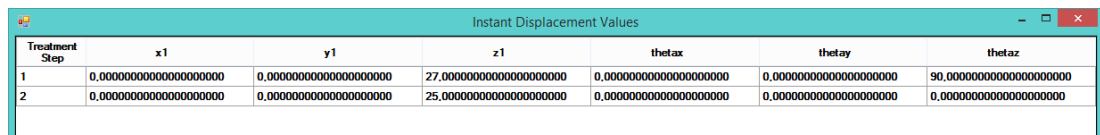
Healthy

F1 16,996731711976
F2 10,1980390271855
F3 10,1980390271856
F4 16,996731711976
F5 23,7954243967663
F6 23,7954243967663
Determinant 31,1806839344754

Singularity

Figure 4. 32. Main windows of the “Force Analysis Interface” (Example6)

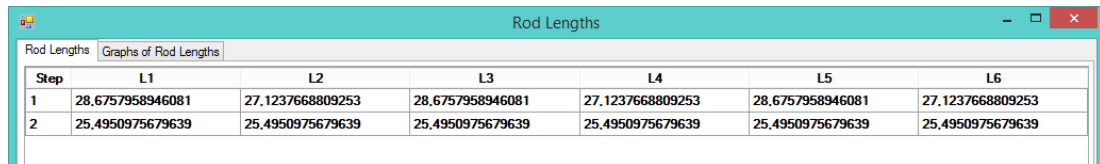
- Magnitudes of joint forces and determinant values for both initial and final configurations of the robotic structure can be seen under the “Primary Output” group on the right hand side of the main window, Figure 4. 32. The result that singularity exists can be observed directly.
- Instant displacement values $(x_1, y_1, z_1, \theta_x, \theta_y, \theta_z)$ both initial and final step can be seen by using “Instant Displacement Values” tab under the “Outputs” menu, Figure 4. 33.



Treatment Step	x1	y1	z1	thetax	thetay	thetaz
1	0,000000000000000000	0,000000000000000000	27,000000000000000000	0,000000000000000000	0,000000000000000000	90,000000000000000000
2	0,000000000000000000	0,000000000000000000	25,000000000000000000	0,000000000000000000	0,000000000000000000	0,000000000000000000

Figure 4. 33. Instant displacement values as input (Example6)

- Rod lengths are shown in Figure 4. 28 can be accessed from “Rod Lengths” under the “Outputs” menu.



Step	L1	L2	L3	L4	L5	L6
1	28,6757958946081	27,1237668809253	28,6757958946081	27,1237668809253	28,6757958946081	27,1237668809253
2	25,4950975679639	25,4950975679639	25,4950975679639	25,4950975679639	25,4950975679639	25,4950975679639

Figure 4. 34. Rod lengths at each treatment step (Example6)

- Matrix form of the force equation at initial and final treatment step can be examined by using counter in the “Matrix Form”. Matrices which are calculated with values of the first and last configuration are shown in Figure 4. 35.



Figure 4. 35. Matrix form of the force equation at initial and final step (Example6)

Singularity was detected at initial configuration of the fixator by “Force Analysis Interface”, Figure 4. 32. Determinant of the force coefficient matrix is calculated as zero. Equation (3.59.) cannot be solved under this condition.

4.3. Applications of the Singularity Evaluation Interface by CPA

Three examples are given in order to analyse the result.

4.3.1. Example7 (Non-Singular Case)

In this example, a non-singular case is presented. Data of the Example7 are the same as Example4.

- The input data “Ring Parameters” which are required for the operation of the “Singularity Evaluation Interface by CPA” given in Table 4. 23.

Table 4. 23. Ring parameters (Example7)

Ring Parameters			
Proximal Ring	Diameter(cm)	Distal Ring	Diameter(cm)
R	7,25	R_1	7,25
Proximal Joints	Value (Degree)	Proximal Joints	Value (Degree)
α_1	90	ε_1	0
α_2	110	ε_2	100
α_3	210	ε_3	120
α_4	230	ε_4	220
α_5	330	ε_5	240
α_6	350	ε_6	340

- The other parameters to be entered which are given in Table 4. 24 are “Intersection Threshold Value (δ_s)” and “Singularity Threshold Value (δ_p)”.

Table 4. 24. Basic input (Example7)

Basic Input			
Input1	Value	Input2	Value
Intersection Threshold Value (δ_s)	0,03	Singularity Threshold Value (δ_p)	1,5

- Instant displacement values which are obtained by using “Motion Analysis Interface” at each step are read from “Example7.txt” by using “Read Displacement Data”, Figure 4. 36.

Treatment Step	Instant Displacement Values					
	x1	y1	z1	thetax	thetay	thetaz
1	0.47691668605336700000	0.32625357209351300000	14.10066830308810000000	4.17959274555139000000	-4.02926424111713000000	49.00000000000000000000
2	0.18399859098963000000	0.05100144093129940000	14.05608919992450000000	4.13159139922409000000	-4.74339680924913000000	48.30132838693670000000
3	-0.10699681159834300000	-0.22286338344574600000	14.00006497242450000000	4.12229568115342000000	-5.45719406960976000000	47.60576919743430000000
4	-0.39612203984379500000	-0.49524350713623900000	13.93267227323060000000	4.15147712475385000000	-6.17093853649109000000	46.91231452397060000000
5	-0.68342914672018600000	-0.76604102002347400000	13.85395865314690000000	4.21907321208203000000	-6.88496783455013000000	46.21995080702960000000
6	-0.96897079014303200000	-1.03515695584007000000	13.76394369294780000000	4.32518146072797000000	-7.59967490015341000000	45.52764967423880000000
7	-1.25280108038074000000	-1.30249032950796000000	13.66261823356910000000	4.47006598530572000000	-8.31551191425633000000	44.83435655138930000000
8	-1.53497648654779000000	-1.56793708072468000000	13.54994315418430000000	4.65416712382530000000	-9.03299552329944000000	44.13897805073660000000
9	-1.81555683196568000000	-1.83138889231076000000	13.42584764827760000000	4.87811457231001000000	-9.75271352673674000000	43.44036772397600000000
10	-2.09460641387468000000	-2.09273184452727000000	13.29022692612910000000	5.14274463679686000000	-10.47533327592460000000	42.73730966628960000000
11	-2.37219529115250000000	-2.35184485661736000000	13.14293924499970000000	5.44912242550655000000	-11.20161211303820000000	42.02849931964450000000

Figure 4. 36. Instant displacement values as input (Example7)

“Calculate” button is clicked after the entering all data into relevant boxes. Then following output are obtained.

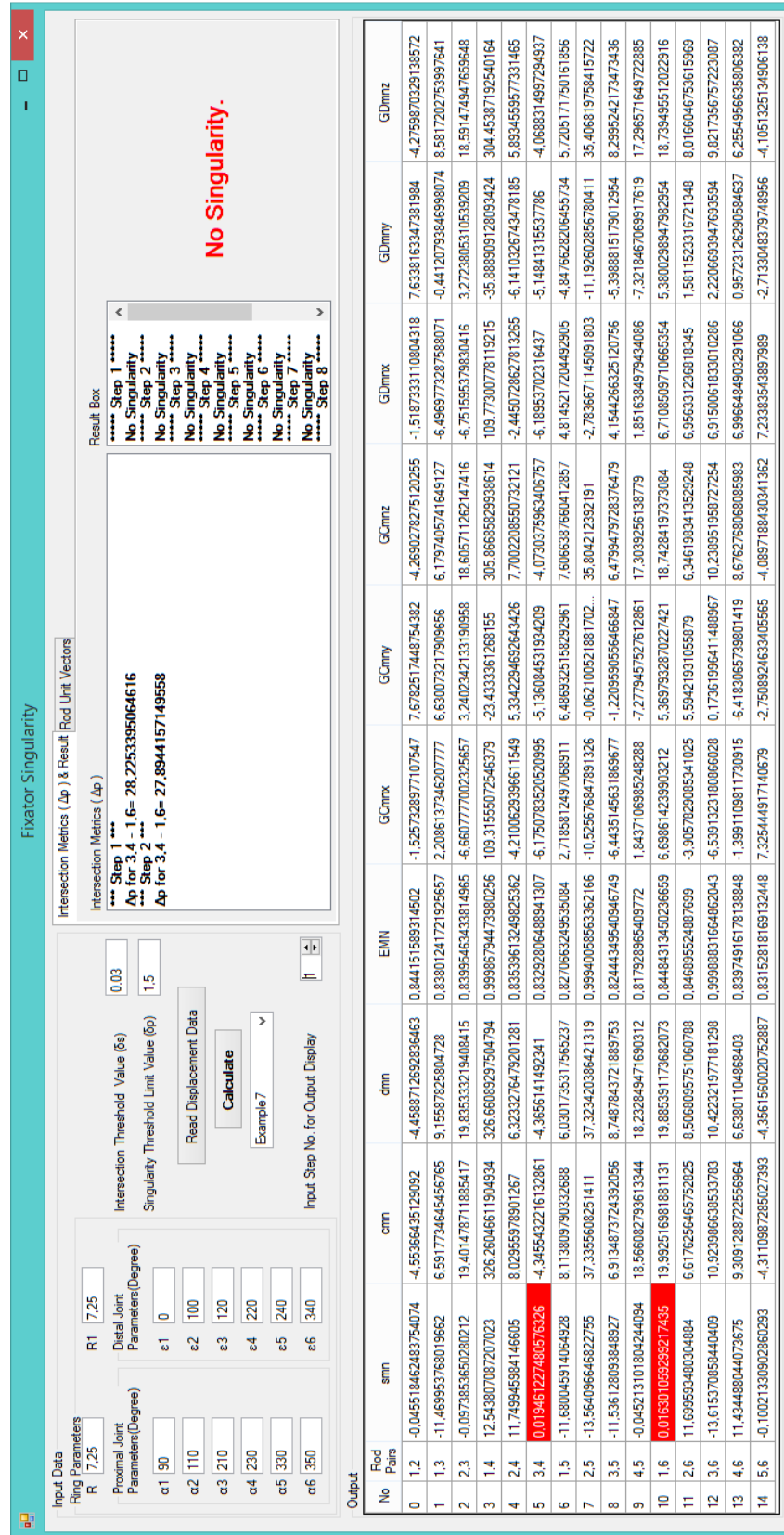


Figure 4. 37. “Singularity Evaluation Interface by CPA” – Step1 (Example7)

Fixator Singularity

Input Data

Ring Parameters
R 7.25

Proximal Joint Parameters(Degree)
 $\alpha 1$ 90 $\alpha 2$ 110 $\alpha 3$ 210 $\alpha 4$ 230 $\alpha 5$ 330 $\alpha 6$ 350

Distal Joint Parameters(Degree)
 $\epsilon 1$ 0 $\epsilon 2$ 100 $\epsilon 3$ 120 $\epsilon 4$ 220 $\epsilon 5$ 240 $\epsilon 6$ 340

Intersection Threshold Value (Gs) 0.03
 Singularity Threshold Limit Value (Gp) 1.5

Read Displacement Data

 Example 7

Input Step No. for Output Display 2

Intersection Metrics (Δp) & Result Rod Unit Vectors

Intersection Metrics (Δp)

Result Box

No Singularity.

Output

No	Rod Pairs	smn	cmn	dmm	EMN	Gcmnx	Gcmny	Gcmnz	Gdmnx	Gdmny	Gdmnz
15	1,2	-0.0791526645812416	-4.5330991245123311	-4.4881898655061558	0.84439902184137651	-1.4513272662966494	7.7747465752939764	-4.262308983459036	-1.4402348889515424	7.69750935513947	-4.2755948894480369
16	1,3	-11.4866564913158837	6.7470780453332591	9.02250055246779	0.83107755676035	2.1601597640062873	6.4689656567656257	6.3440332043956733	-6.6614323464173131	-0.57061871322728885	8.4811423177209218
17	2,3	-0.14447200959720341	19.424241459494354	19.660820291006537	0.8385650086959542	-6.9780682898179656	2.983753070549227	18.504160921064944	-7.1127260179746088	3.0307647993744435	18.481153201542792
18	1,4	11.075309831134627	837.77073546650013	837.81819672175266	0.9999551284224758	268.22257309864875	-89.729420092618426	787.7254908139068	270.84577128922433	-100.2643328539165	785.53529788285618
19	2,4	11.758901779213403	8.0231467204704714	6.0530164177448684	0.83931374714163676	-4.3377109629492985	5.2312025769110164	7.6431091634617335	-2.6597518160329814	-6.2380807721027924	5.6752862057757678
20	3,4	-0.022826657880606393	-4.29896091603953	-4.3146165129690566	0.82912116125474178	-6.0963157298474213	-5.0803244633414835	-4.0410193532522163	-6.079017569825437	-5.0660797258646593	-4.0453687697726757
21	1,5	-11.634465723587868	8.18606330233310054	5.9846938388853906	0.82159316142084493	2.6208684485056777	6.302390336358302	7.6970586799729857	4.6775463970093192	-4.957155537663505	5.61055557662559
22	2,5	-13.58424267245047	22.59546050444685	22.590748602724716	0.998914729113156	-7.7124847454272931	2.586240837159771	21.525166847206648	0.23478707404047593	-8.6535560017244784	21.178452605716558
23	3,5	-11.474223968276913	6.65242735055627	8.8043061980436921	0.81722850336611819	-6.5613144587610313	-1.3695727705213781	6.2626756532687713	3.9231924819846484	-5.5847821901241828	8.253891219833033
24	4,5	-0.078327786179563	18.324647113180514	18.311897300645345	0.81616728151375	1.365619668053333	-7.6253177730860671	17.181122569281076	1.379544932457951	-7.701111097222169	17.167100387974347
25	1,6	-0.0263218293962888	19.991838811422333	19.80581011884253	0.844244167693162	6.4006323209502352	4.935766699122941	18.797602218980252	6.808309976741606	4.9190699913581541	18.802289035501197
26	2,6	11.71598104895637	6.6367317009885829	8.539293832966046	0.85175022979997161	-4.0166338086441	5.504500696396741	6.3223653619173392	6.8126015940870079	1.4047110109940688	8.1066250942670953
27	3,6	-13.620783438449607	9.9050920300905041	9.6772755379069366	0.99959427877658724	-6.6988732166121254	-0.27183559014080005	9.3107775043963574	6.7689904851182208	1.7596798155431077	9.1863471962071483
28	4,6	11.5233467972730152	9.1081661950543946	6.7836564116124194	0.84222622000089986	-1.6851042192192338	-6.5834478158002652	8.5397835391904078	6.8798896955768117	0.85707423453704012	6.4399420070842135
29	5,6	-0.1475470375923071	-4.3349059302469639	-4.30648880268035	0.83080244726438823	7.4384388122241285	-2.660776941572808	-4.0639025031203255	7.3048953344927385	-2.6022708028338339	-4.08828756361305

Figure 4. 38. “Singularity Evaluation Interface by CPA” – Step2 (Example7)

- Singularity condition at each step is written in the “Result Box”. Also singularity result is printed as the main output at the end of the whole process. Result is written in bold and red font after the calculation, Figure 4. 37 and Figure 4. 38.
- Rod unit vectors at each step can be examined from second tab on the right side of the interface as an additional information, Figure 4. 39.

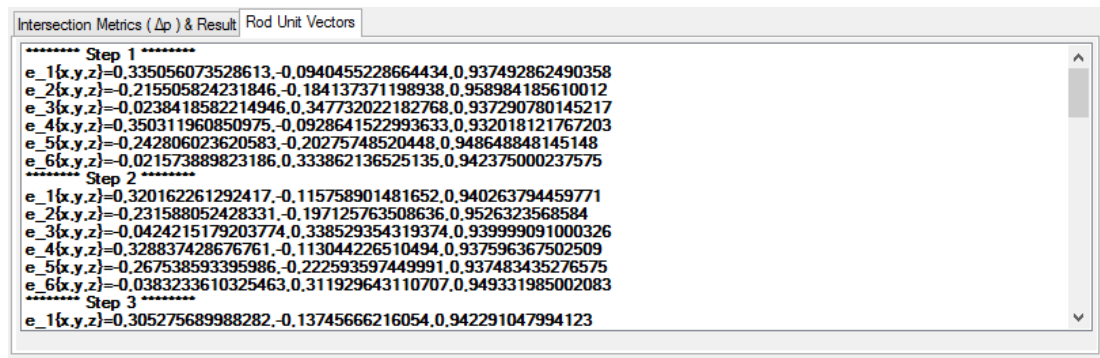


Figure 4. 39. Rod unit vectors at each step (Example7)

4.3.2. Example8 (Non-Singular Case)

In this example, a non-singular case is presented. Data of the Example8 are the same as Example5.

- The input data “Ring Parameters” which are required for the operation of the “Singularity Evaluation Interface by CPA” given in Table 4. 25.

Table 4. 25. Ring parameters (Example8)

Ring Parameters			
Proximal Ring	Diameter(cm)	Distal Ring	Diameter(cm)
R	7,25	R_1	7,25
Proximal Joints	Value (Degree)	Proximal Joints	Value (Degree)
α_1	80	ε_1	0
α_2	100	ε_2	100
α_3	190	ε_3	130
α_4	210	ε_4	200
α_5	320	ε_5	220
α_6	340	ε_6	340

- The other parameters to be entered which are given in Table 4. 26 are “Intersection Threshold Value (δ_s)” and “Singularity Threshold Value (δ_p)”.

Table 4. 26. Basic input (Example8)

Basic Input			
Input1	Value	Input2	Value
Intersection Threshold Value (δ_s)	0,02	Singularity Threshold Value (δ_p)	1,4

- Instant displacement values which are obtained by using “Motion Analysis Interface” at each step are read from “Example8.txt” by using “Read Displacement Data”, Figure 4. 40.

Instant Displacement Values							
Treatment Step	x1	y1	z1	thetax	thetay	thetaz	
1	1.88499596371383000000	3.10881126381128000000	13.01198152119650000000	7.70779634273582000000	0.07135997486065530000	41.50000000000000000000	
2	1.71593408454659000000	2.62974380685747000000	12.94951793082530000000	7.80128871999119000000	-0.72510745206143800000	41.95174488645440000000	
3	1.54978934972546000000	2.16224625272909000000	12.86687242950930000000	7.87848887647750000000	-1.53176845998749000000	42.38548038120570000000	
4	1.38653755347583000000	1.70629976538077000000	12.76504941924190000000	7.94149539823896000000	-2.34948975158314000000	42.80156358873730000000	
5	1.22614976850147000000	1.26189394330813000000	12.64494981284540000000	7.99233002476378000000	-3.17918297025635000000	43.20029871990140000000	
6	1.06859230356086000000	0.82902723264060100000	12.50738625764930000000	8.03297004360371000000	-4.02181016242360000000	43.58193702283010000000	
7	0.91382538590938100000	0.40770738989309400000	12.35309185540550000000	8.06537198241940000000	-4.87840050303333000000	43.94667610686450000000	
8	0.76180773665604600000	-0.00204784274921313000	12.18272771938460000000	8.09149451617184000000	-5.75006718610776000000	44.29465806927470000000	
9	0.61248603077464400000	-0.40020983026102100000	11.99688542725130000000	8.11332142711619000000	-6.63802516527994000000	44.62596635029720000000	
10	0.46580422786617200000	-0.78673787428258100000	11.79609715242340000000	8.13288544323283000000	-7.54361043412164000000	44.94062116651330000000	
11	0.32169775344977700000	-1.16157773387963000000	11.58083259384560000000	8.15229382765296000000	-8.46830159005368000000	45.23857328686410000000	

Figure 4. 40. Instant displacement values as input (Example8)

“Calculate” button is clicked after the entering all data into relevant boxes. Then following output are obtained.



Figure 4. 41. “Singularity Evaluation Interface by CPA” – Step1 (Example8)

- Singularity condition at each step is written in the “Result Box”. Also singularity result is printed as the main output at the end of the whole process. Result is written in bold and red font after the calculation, Figure 4. 41.
- Rod unit vectors at each step can be examined from second tab on the right side of the interface as an additional information, Figure 4. 42.

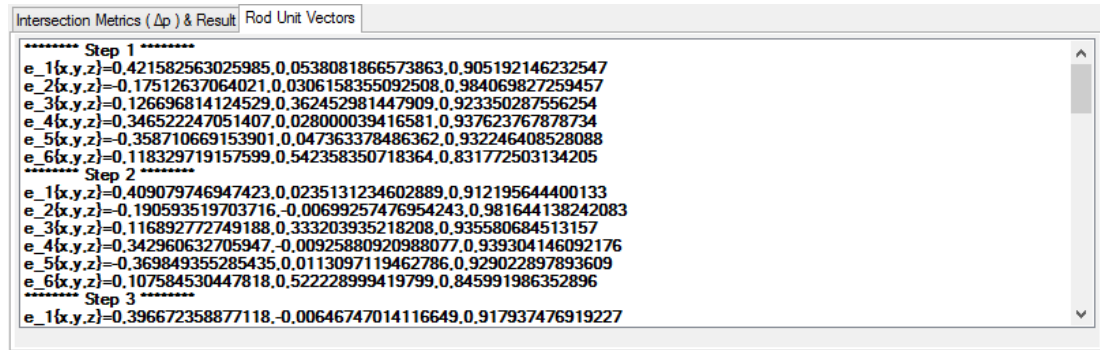


Figure 4. 42. Rod unit vectors at each step (Example8)

4.3.3. Example9 (Singular Case)

In this example, a singular case is presented. Data of the Example9 are the same as Example6 and Example3.

- The input data “Ring Parameters” which are required for the operation of the “Singularity Evaluation Interface by CPA” given in Table 4. 27.

Table 4. 27. Ring parameters (Example9)

Ring Parameters			
Proximal Ring	Diameter(cm)	Distal Ring	Diameter(cm)
R	5	R_1	5
Proximal Joints	Value (Degree)	Proximal Joints	Value (Degree)
α_1	60	ε_1	120
α_2	180	ε_2	120
α_3	180	ε_3	240
α_4	300	ε_4	240
α_5	300	ε_5	0
α_6	60	ε_6	0

- The other parameters to be entered which are given in Table 4. 28 are “Intersection Threshold Value (δ_s)” and “Singularity Threshold Value (δ_p)”.

Table 4. 28. Basic input (Example9)

Basic Input			
Input1	Value	Input2	Value
Intersection Threshold Value (δ_s)	0,02	Singularity Threshold Value (δ_p)	8,66025403784436

- Instant displacement values at the intermediate steps couldn't be obtained by using “Motion Analysis Interface”. Calculation process was interrupted because singularity detected at first step of the Example3. “Example9.txt” which contains only initial and final displacement values is read as input by using “Read Displacement Data”. The motion parameters data in the “Example9.txt” are given in Table 4. 29.

Table 4. 29. Initial and final motion parameters (Example9)

Initial Motion Parameters			
Translations	Value (cm)	Rotations	Value (Degree)
x_1	0	θ_x	0
y_1	0	θ_y	0
z_1	27	θ_z	90
Final Motion Parameters			
Translations	Value (cm)	Rotations	Value (Degree)
x_1	0	θ_x	0
y_1	0	θ_y	0
z_1	25	θ_z	0

“Calculate” button is clicked after the entering all data into relevant boxes. The results obtained can be seen in Figure 4. 44, Figure 4. 45. Singularity is detected at initial configuration according to these results.

Fixator Otomasyonu

AP Data PsAp 0 BeAp 0 Be'Ap 0 cAP 10 bAP 15	Lateral Data PsL 0 BeL 0 Be'L 0 cL 10 bL 15	e Data ex -1 ey 1 ez 2	Sonuçlar Sağlıksız L1 28,67579 L2 27,12376 L3 28,67579 L4 27,12376 L5 28,67579 L6 27,12376 Sağlıklı L1 25,49509 L2 25,49509 L3 25,49509 L4 25,49509 L5 25,49509 L6 25,49509	Auvw 1 0 0 0 1 0 0 0 1
Axial Data Del0 90 DelAx 90 Del 0	Alfa Data a1 60 a2 180 a3 180 a4 300 a5 300 a6 60	q Data qx 1 qy 0 qz 0	Axyz 1 0 0 0 1 0 0 0 1	
Fixator Yarıcap R 5 R1 5	Epsilon Değerleri e1 120 e2 120 e3 240 e4 240 e5 0 e6 0	r Data rx 0 ry 1 rz 0	Uzunluklar b 15 c 10 Kontrol ru 1 rv 6.123031	☐ AP için Hesapla Hesapla Kaydet Aç Çıkış

Figure 4. 43. Calculation of rod lengths both initial and final step (Example9)

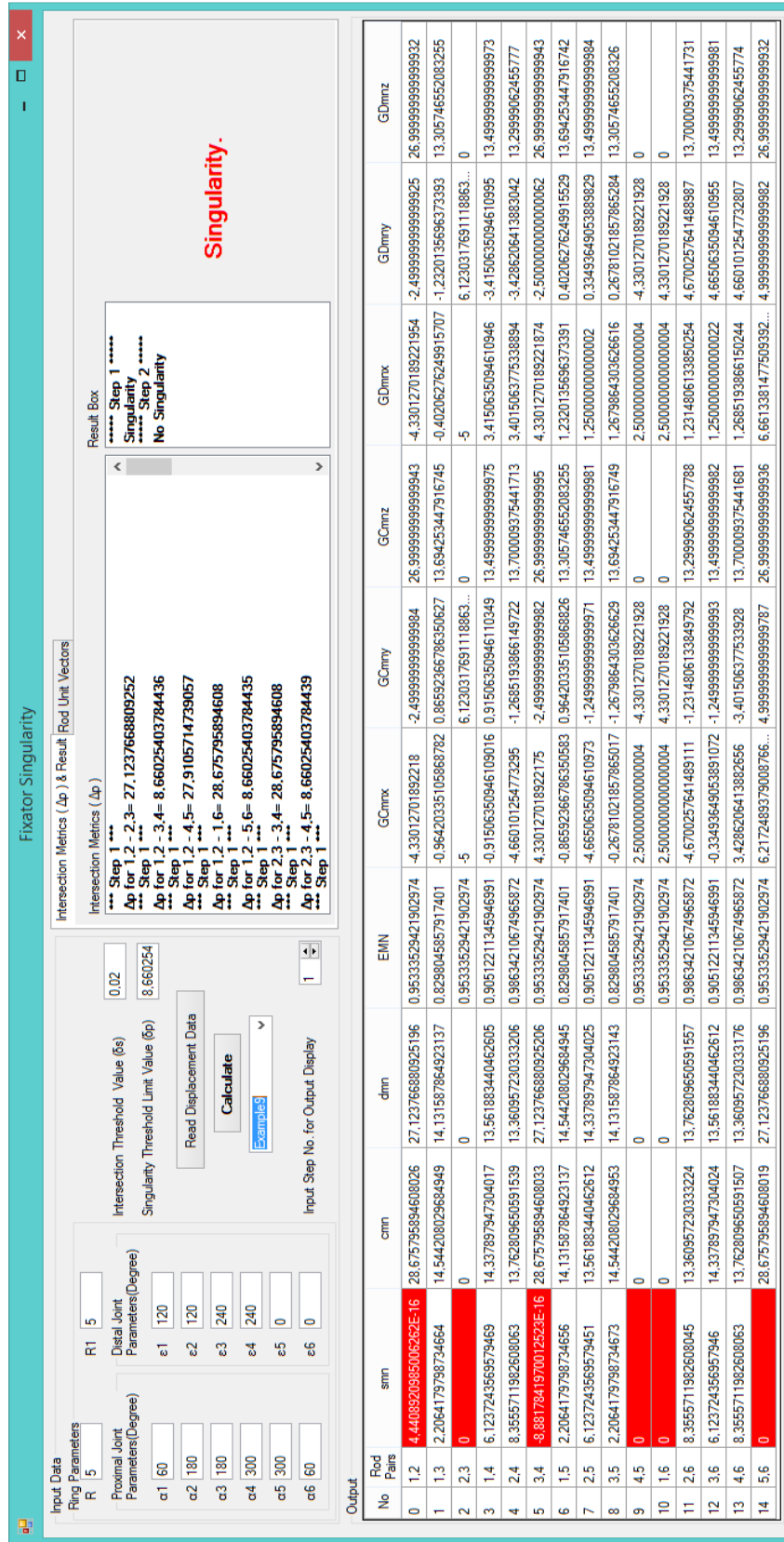


Figure 4. 44. “Singularity Evaluation Interface by CPA” – Step1 (Example9)

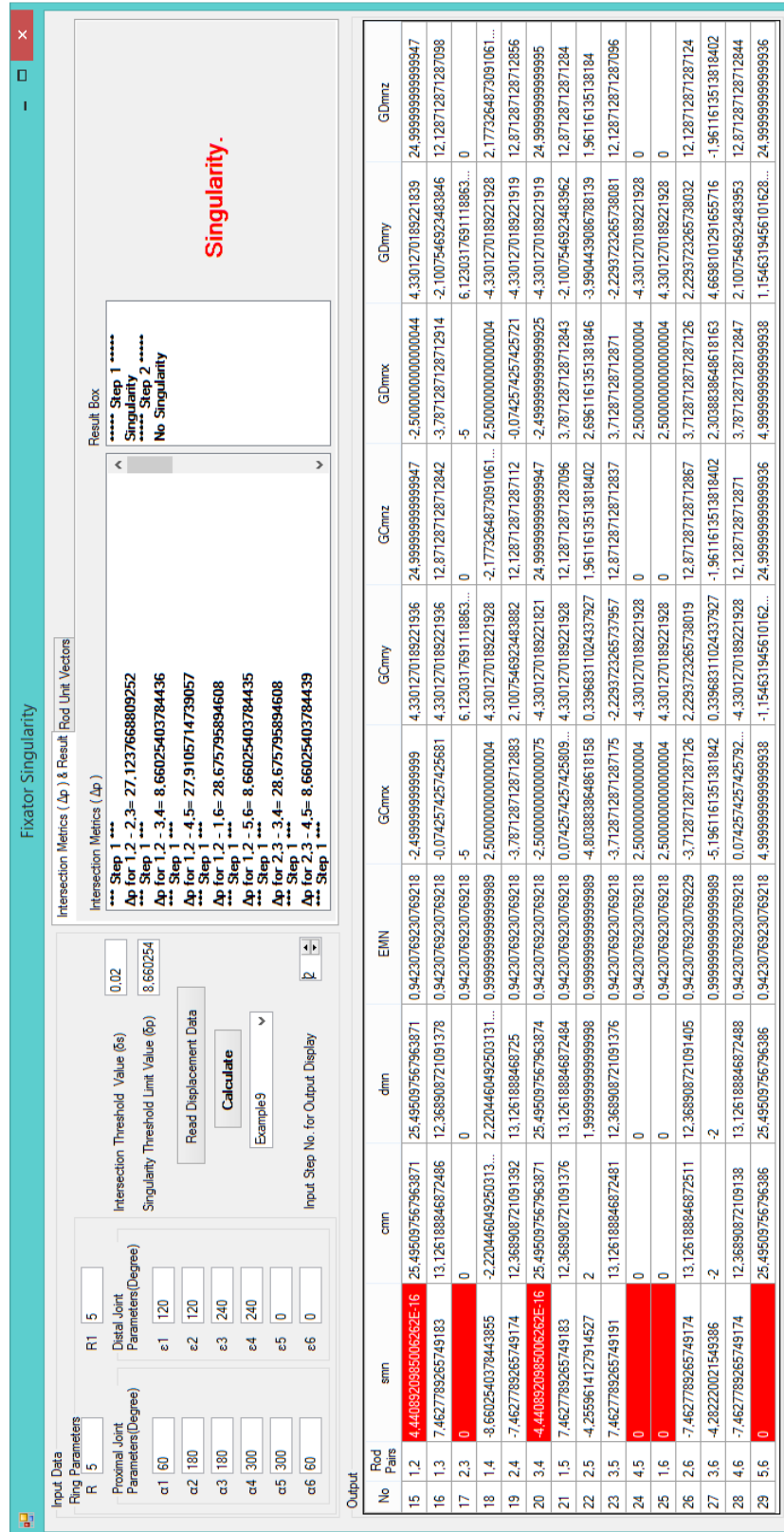


Figure 4. 45. “Singularity Evaluation Interface by CPA” – Step2 (Example9)

- Singularity condition at each step is written in the “Result Box”. Also singularity result is printed as the main output at the end of the whole process. Result is written in bold and red font after the calculation, Figure 4. 44.
- Rod unit vectors at each step can be examined from second tab on the right side of the interface as an additional information, Figure 4. 46.

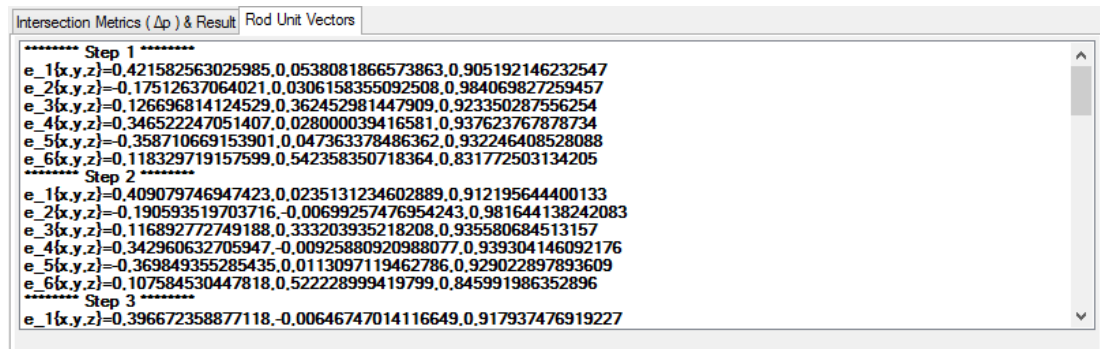


Figure 4. 46. Rod unit vectors at each step (Example9)

4.4. Applications of the Singularity Evaluation Interface with Three Methods

Four examples are given in order to analyse the result.

4.4.1. Example10 (Singular Case)

In this example, a singular case is presented.

It is proposed that extensions of the 4 rods pass through a specified point “P” whose coordinates are (0, 0, -10) according to the global coordinate system at the center of the proximal ring, Figure 4. 47. The stability condition of the (6-6) SP is to be investigated when an external force of 100N is applied in (-z) direction to the center of the distal ring, whose coordinates are (0, 0, 10). Singularity condition will be demonstrated by using “Singularity Evaluation Interface”.

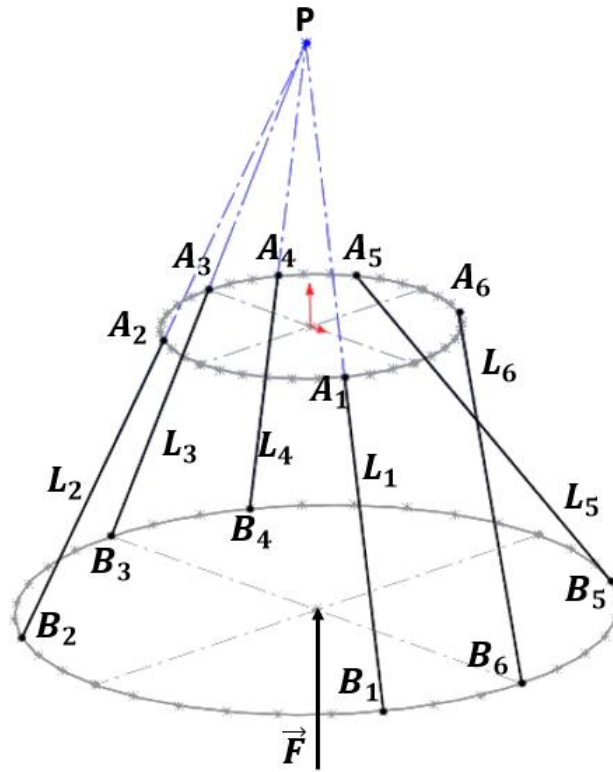


Figure 4. 47. Kinematic diagram (Example10)

- The input data “Ring Parameters” and “Displacement Data” which are required for the operation of the “Singularity Evaluation Interface with Three Methods” are given in Table 4. 30 and Table 4. 31.

Table 4. 30. Ring parameters (Example10)

Ring Parameters			
Proximal Ring	Diameter(cm)	Distal Ring	Diameter(cm)
R	5	R_1	10
Proximal Joints	Value (Degree)	Proximal Joints	Value (Degree)
α_1	30	ε_1	30
α_2	120	ε_2	120
α_3	180	ε_3	180
α_4	210	ε_4	210
α_5	240	ε_5	300
α_6	300	ε_6	360

Table 4. 31. Displacement data (Example10)

Displacement Data			
Translations	Value (cm)	Rotations	Value (Degree)
x_1	0	θ_x	0
y_1	0	θ_y	0
z_1	10	θ_z	0

- “Force-Moment Inputs” are given in Table 4. 32.

Table 4. 32. Force-Moment Parameters (Example10)

Force-Moment Parameters					
Force	Value	Moment	Value	Force Application Point (Global Coordinate System)	Value(cm)
F_x	0	M_x	0	rF_x	0
F_y	0	M_y	0	rF_y	0
F_z	-100	M_z	0	rF_z	10

Force analysis method is used for calculation of the joint forces. Matrix equation (3.59.) is obtained by the use of “Force Analysis Method” as follows after using these inputs for solution.

$$\begin{bmatrix}
 \frac{\sqrt{3}}{2} & -\frac{1}{2\sqrt{5}} & -\frac{1}{\sqrt{5}} & -\frac{\sqrt{3}}{2} & \frac{3}{2\sqrt{7}} & \frac{3}{2\sqrt{7}} \\
 \frac{1}{2\sqrt{5}} & \frac{\sqrt{3}}{2} & 0 & -\frac{1}{2\sqrt{5}} & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\
 \frac{2}{\sqrt{5}} & \frac{2}{\sqrt{5}} & \frac{2}{\sqrt{5}} & \frac{2}{\sqrt{5}} & \frac{2}{\sqrt{7}} & \frac{2}{\sqrt{7}} \\
 2\sqrt{5} & 2\sqrt{15} & 0 & -2\sqrt{5} & -10\sqrt{\frac{3}{7}} & 0 \\
 -2\sqrt{15} & 2\sqrt{5} & 4\sqrt{5} & 2\sqrt{15} & -\frac{10}{\sqrt{7}} & -\frac{20}{\sqrt{7}} \\
 0 & 0 & 0 & 0 & 5\sqrt{\frac{3}{7}} & 5\sqrt{\frac{3}{7}}
 \end{bmatrix} \times \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 100 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (4.2)$$

Determinant of the force coefficient matrix is equal to zero. This means that unstable equilibrium state prevails under the given conditions. Same result can be obtained by using data which are given in Table 4. 30 and Table 4. 31 in the evaluation interface, Figure 4. 48. Also determinant of the Jacobian matrix is calculated as zero with the given data. Force coefficient matrix and Jacobian matrix can be seen by clicking the “Matrix” tab of the interface, Figure 4. 49.

Rod lengths are more useful parameters in configuring the parallel robot structure. Data which are given in Table 4. 30 and Table 4. 31 are converted to rod lengths by calculations. This output can be examined from “Rod Lengths” tab, Figure 4. 50. Fixator configuration is drawn for a better understanding of the structure. This drawing can be seen from “Kinematic Diagram” tab, Figure 4. 51. Output of the geometric method can be examined from Figure 4. 52 and Figure 4. 53.

As a result, singularity occurs when extension of any four out of six rods intersect within a sphere diameter of which represent tolerance value. This situation can be happen when 15 combinations of four rods (1-2-3-4, 1-2-3-5, 1-2-3-6, 1-2-4-5, 1-2-4-6, 1-2-5-6, 1-3-4-5, 1-3-4-6, 1-3-5-6, 1-4-5-6, 2-3-4-5, 2-3-4-6, 2-3-5-6, 2-4-5-6, 3-4-5-6) meet at a common point. This result can easily be extended to cover the cases of 5 or 6 rod directions intersecting within a sphere of tolerance.

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Ring Parameters

R 5
R1 10

Proximal Joint Parameters(Degree)

$\alpha 1$ 30
 $\alpha 2$ 120
 $\alpha 3$ 180
 $\alpha 4$ 210
 $\alpha 5$ 240
 $\alpha 6$ 300

Distal Joint Parameters(Degree)

$\epsilon 1$ 30
 $\epsilon 2$ 120
 $\epsilon 3$ 180
 $\epsilon 4$ 210
 $\epsilon 5$ 300
 $\epsilon 6$ 360

Displacement Data

Translations

X1 0
Y1 0
Z1 10

Rotations

θx 0
 θy 0
 θz 0

Example10

Calculate

Change Graphs of Determinants

Force

-1.4210854715202E-14

Jacobian

9.5367431640625E-06

Kinematic Diagram

$\alpha 1$

30

Rod Length

$\alpha 1$

120

Matrix

$\alpha 2$

120

Geometric Method

$\alpha 3$

180

$\alpha 4$

210

$\alpha 5$

240

$\alpha 6$

300

$\epsilon 1$

30

$\epsilon 2$

120

$\epsilon 3$

180

$\epsilon 4$

210

$\epsilon 5$

300

$\epsilon 6$

360

Figure 4. 48. Main window of the “Singularity Evaluation Interface” (Example10)

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Determinant | Change Graphs of Determinants | Kinematic Diagram | Rod Length | Matrix | Geometric Method

Force Matrix

0.387298334620742	-0.223606797749979	-0.447213595499958	-0.387298334620742	0.566946709513841	0.566946709513841
0.223606797749979	0.387298334620742	5.47660610564999E-17	-0.223606797749979	-0.327326835353988	0.327326835353988
0.894427190999916	0.894427190999916	0.894427190999916	0.894427190999916	0.755928946018454	0.755928946018455
4.47213595499958	7.74596669241483	1.09532122113E-15	-4.47213595499958	-6.54653670707977	-1.8514307806649E-15
-7.74596669241483	4.47213595499958	8.94427190999916	7.74596669241483	-3.77964473009227	-7.55928946018455
0	0	0	0	3.27326835353989	3.27326835353988

Jacobian Matrix

8.66025403784439	5	20	100	-173.205080756888	0
-5	8.66025403784439	20	173.205080756888	100	0
-10	1.22460635382238E-15	20	2.44921270764475E-14	200	0
-8.66025403784439	-5	20	-100	173.205080756888	0
15	-8.66025403784439	20	-173.205080756888	-100	86.6025403784439
15	8.66025403784438	20	-4.89842541528951E-14	-200	86.6025403784438

Ring Parameters

R 5 R1 10

Proximal Joint Parameters(Degree)

α1 30 α2 120 α3 180 α4 210 α5 300 α6 360

Distal Joint Parameters(Degree)

ε1 30 ε2 120 ε3 180 ε4 210 ε5 300 ε6 360

Displacement Data

Translations

X1 0 Y1 0 Z1 10

Rotations

Ex 0 Ey 0 Ez 0

Example10

Figure 4. 49. “Matrix” tab of the “Singularity Evaluation Interface” (Example10)

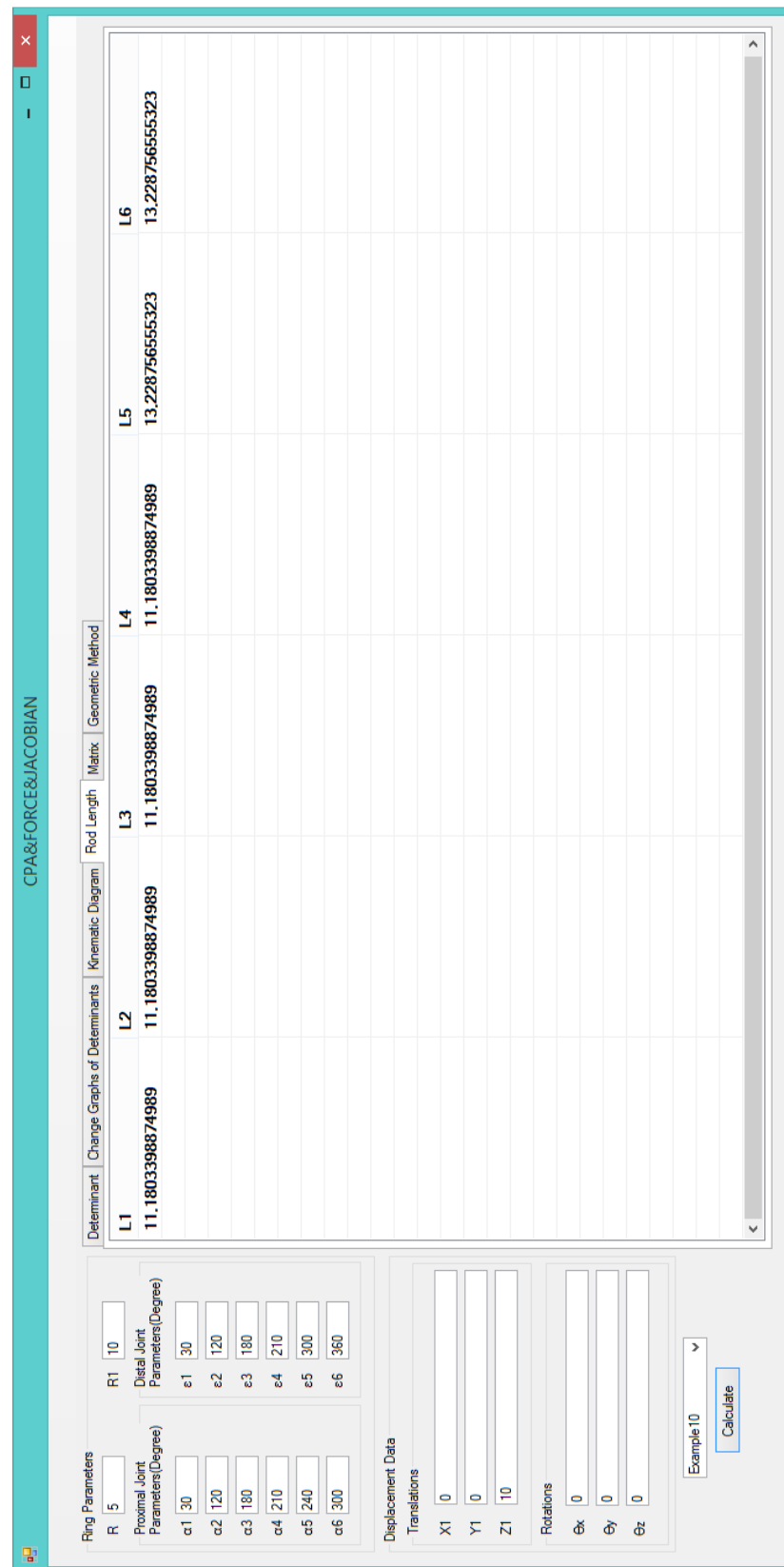


Figure 4. 50. “Rod Length” tab of the “Singularity Evaluation Interface” (Example10)

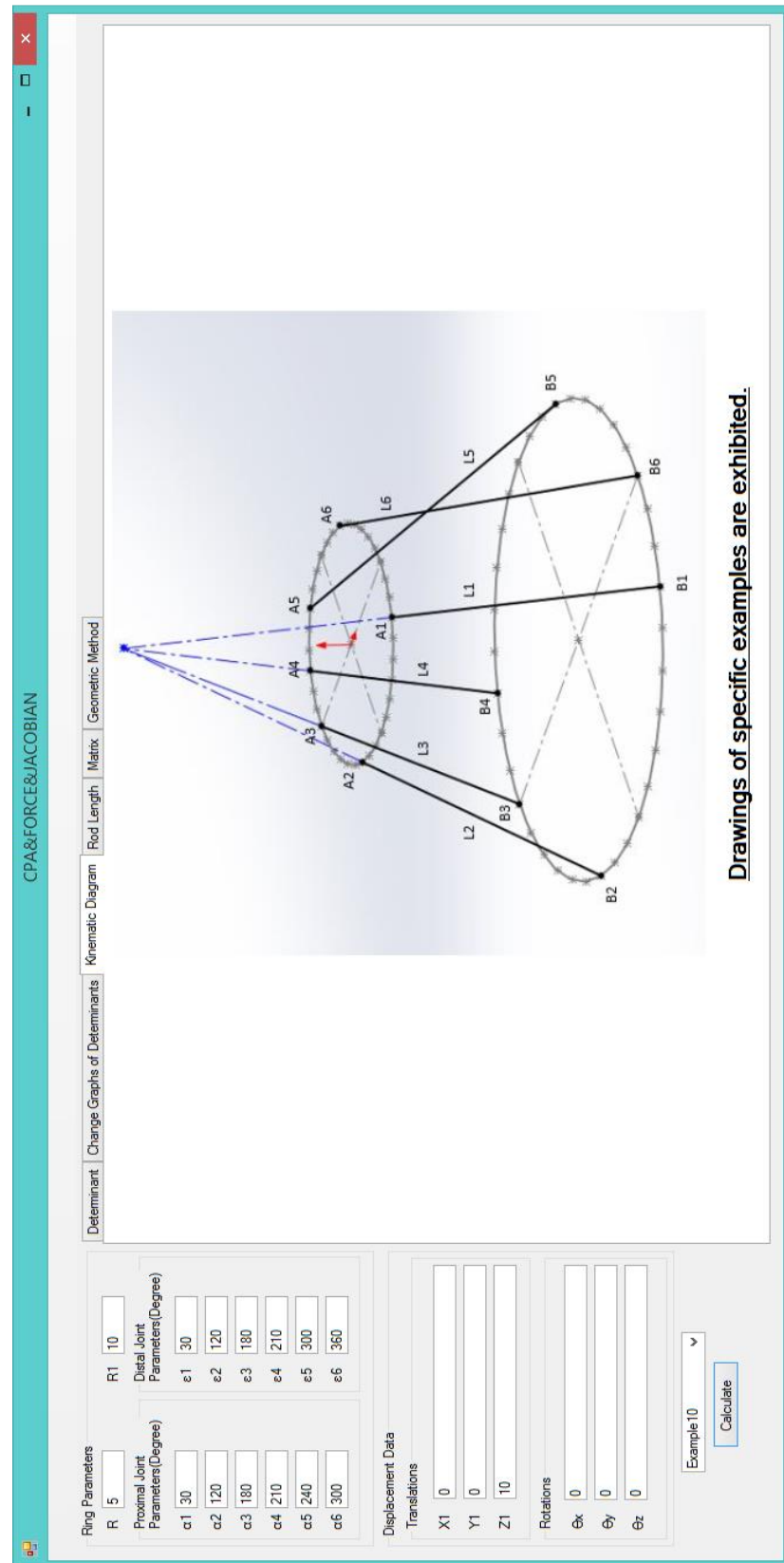


Figure 4. 51. “Kinematic Diagram” tab (Example10)

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Determinant Change Graphs of Determinants Kinematic Diagram Rod Length Matrix Geometric Method

Primary Output Closest Point Coordinates

Ring Parameters

R 5 R1 10

Proximal Joint Parameters(Degree)

α_1 30 α_2 120 α_3 180 α_4 210 α_5 300 α_6 360

Distal Joint Parameters(Degree)

ϵ_1 30 ϵ_2 120 ϵ_3 180 ϵ_4 210 ϵ_5 300 ϵ_6 360

Displacement Data

Translations

X1 0 Y1 0 Z1 10

Rotations

θ_x 0 θ_y 0 θ_z 0

Example 10 Calculate

Pair	smn	cmn	dmm	EMN
1,2	0	-11.1803398874989	-11.1803398874989	0.8
1,3	0	-11.1803398874989	-11.1803398874989	0.626794919243112
2,3	4.44089209850063E-16	-11.1803398874989	-11.1803398874989	0.9
1,4	-1.77635683940025E-15	-11.1803398874989	-11.1803398874989	0.6
2,4	0	-11.1803398874989	-11.1803398874989	0.8
3,4	-2.22044604925031E-16	-11.1803398874989	-11.1803398874989	0.973205080756888
1,5	8.11953380238779	-8.73752650797947	-5.55005489992672	0.822508414725093
2,5	6.46058282469799	-5.54155977032556	-5.17646995643072	0.422577127364258
3,5	3.23029141234899	-2.81939005858669	-4.02614329944611	0.422577127364258
4,5	1.45891119445989	-1.62163077432059	-2.49567425693928	0.529738392840533
1,6	5	15.272628309268	18.070817515051	0.968893425667373
2,6	7.94719414239026	-8.23814518026238	-5.7000276013598	0.676123403782813
3,6	6.46058282469799	-5.54155977032557	-5.17646995643072	0.422577127364258
4,6	5	-4.09228842176907	-4.84206095972813	0.383353381898253
5,6	4	1.58745078663876	-1.58745078663875	0.785714285714286

Rod	el.x	el.y	el.z
1	0.387298334620742	0.223606797749979	0.894427190999916
2	-0.223606797749979	0.387298334620742	0.894427190999916
3	-0.447213595499958	5.47660610564999E-17	0.894427190999916
4	-0.387298334620742	-0.223606797749979	0.894427190999916
5	0.566946709513841	-0.327326835353989	0.755928946018454
6	0.566946709513841	0.327326835353988	0.755928946018455

Figure 4. 52. “Primary Output” sub-tab of the “Geometric Method” tab (Example10)

CPA&FORCE&JACOBIAN

Determinant | Change Graphs of Determinants | Kinematic Diagram | Rod Length | Matrix | Geometric Method

Ring Parameters: R 5 R1 10
 Proximal Joint Parameters(Degree): α_1 30 α_2 120 α_3 180 α_4 210 α_5 300 α_6 360
 Distal Joint Parameters(Degree): ϵ_1 30 ϵ_2 120 ϵ_3 180 ϵ_4 210 ϵ_5 300 ϵ_6 360

Displacement Data:
 Translations: X1 0 Y1 0 Z1 10
 Rotations: Ex 0 Ey 0 Ez 0

Primary Output: Closest Point Coordinates

Pair	GCmnx	GCmny	GCmnz	GDmnx	GDmny	GDmnz
1,2	2.66453525910038E-15	1.77635683940025E-15	-9.99999999999999	-1.77635683940025E-15	2.66453525910038E-15	-9.99999999999999
1,3	8.88178419700125E-16	4.44089209850063E-16	-10	-8.88178419700125E-16	9.86076131526265E-32	-10
2,3	-2.66453525910038E-15	4.44089209850063E-15	-9.99999999999999	-5.32907051820075E-15	6.90253292068385E-31	-9.99999999999999
1,4	8.88178419700125E-16	4.44089209850063E-16	-10	-8.88178419700125E-16	-4.44089209850063E-16	-10
2,4	-1.77635683940025E-15	3.5527136788005E-15	-9.99999999999999	-3.5527136788005E-15	-2.22044604925031E-15	-9.99999999999999
3,4	-1.4210854715202E-14	1.77493703674728E-30	-9.99999999999997	-1.24344978758018E-14	-7.105427357601E-15	-9.99999999999997
1,5	0.946097553677162	0.546229677295156	-7.81508129081938	-5.645658536313463	-2.51344511248828	-4.19544715084617
2,5	-1.26086956521739	2.1838901486738	-4.95652173913043	-5.43478260869565	-2.63572948977873	-3.91304347826087
3,5	-3.73913043478261	4.57896288820541E-16	-2.52173913043478	-4.78260869565218	-3.01226227403283	-3.04347826086956
4,5	-3.70207212065809	-2.13739233542136	-1.45043065831458	-3.91491430799013	-3.51322586232384	-1.8865524106535
1,6	10.2451905283833	5.91506350946108	13.6602540378443	12.7451905283833	1.58493649053888	13.6602540378443
2,6	-0.657894736842105	1.13950711024268	-7.36842105263158	-0.657894736842106	-6.15333839531048	-4.21052631578948
3,6	-2.52173913043478	3.08813776181295E-16	-4.95652173913044	-0.434782608695653	-6.02452454806566	-3.91304347826087
4,6	-2.74519052838329	-1.5849364905389	-3.66025403784439	-0.24519052838329	-5.9150635094611	-3.66025403784439
5,6	-1.6	-4.84974226119286	1.2	1.6	-4.84974226119285	-1.2

Example10

Figure 4. 53. “Closest Point Coordinates” sub-tab (Example10)

4.4.2. Example11 (Singular Case)

In this example, a singular case is presented.

It is proposed that approximately a parallelogram is formed ($A_1A_2B_1B_2$) in the robotic structure. A (6-6) type Stewart Platform whose ring planes are parallel to each other is considered in this example. The stability condition of the SP is to be investigated when an external force of 100N is applied in (-z) direction to the center of the distal ring, whose coordinates are (0, 0, 25). Singularity condition will be demonstrated by using “Singularity Evaluation Interface”.

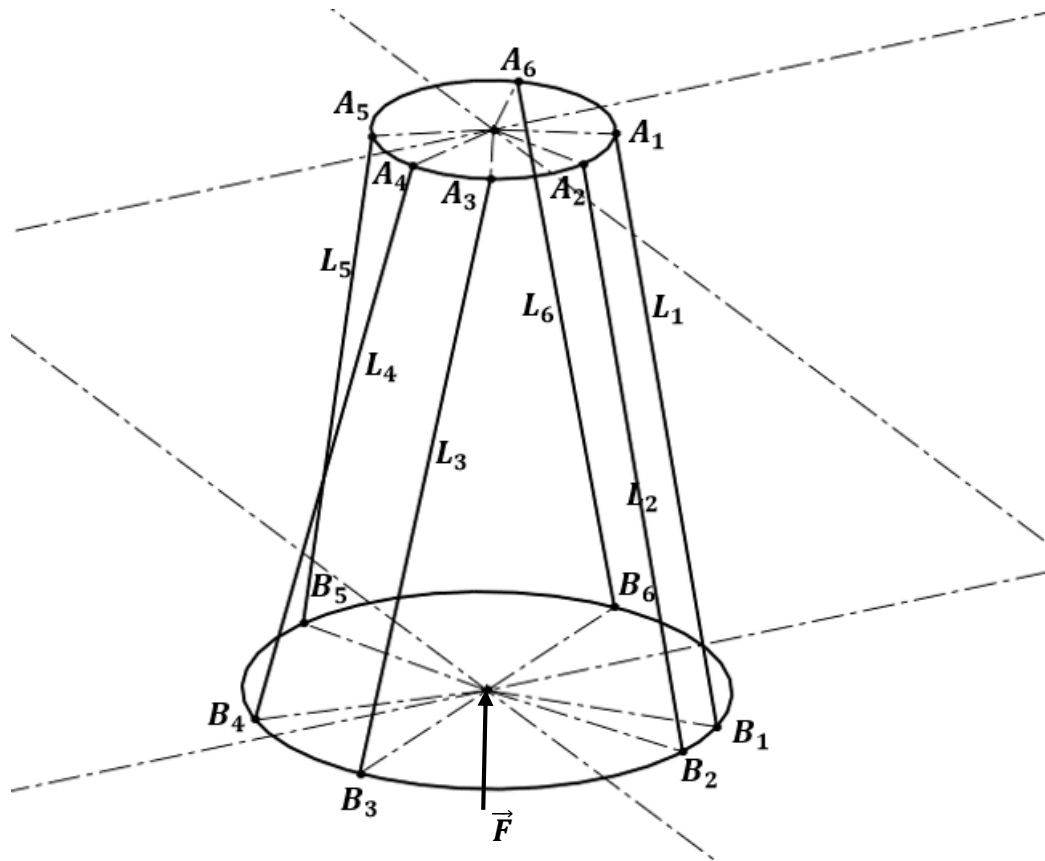


Figure 4. 54. Kinematic diagram (Example11)

- The input data “Ring Parameters” and “Displacement Data” which are required for the operation of the “Singularity Evaluation Interface with Three Methods” are given in Table 4. 33 and Table 4. 34.

Table 4. 33. Ring parameters (Example11)

Ring Parameters			
Proximal Ring	Diameter(cm)	Distal Ring	Diameter(cm)
R	5	R_1	11
Proximal Joints	Value (Degree)	Proximal Joints	Value (Degree)
α_1	32,38449944	ε_1	48,999378920738
α_2	71,50849895	ε_2	65,78358102
α_3	120	ε_3	150
α_4	160	ε_4	190
α_5	200	ε_5	250
α_6	310	ε_6	330

Table 4. 34. Displacement data (Example11)

Displacement Data			
Translations	Value (cm)	Rotations	Value (Degree)
x_1	0	θ_x	0
y_1	0	θ_y	0
z_1	25	θ_z	0

- “Force-Moment Inputs” are given in Table 4. 35.

Table 4. 35. Force-Moment Parameters (Example11)

Force-Moment Parameters					
Force	Value	Moment	Value	Force Application Point (Global Coordinate System)	Value(cm)
F_x	0	M_x	0	rF_x	0
F_y	0	M_y	0	rF_y	0
F_z	-100	M_z	0	rF_z	25

Outputs of this example can be seen from Figure 4. 55 when data which are given in Table 4. 37 and Table 4. 38 are entered into the “Singularity Evaluation Interface with 3 Methods”. It is observed that determinants of the force coefficient matrix and Jacobian matrix values are approximately equal to “0”.

Force coefficient matrix and Jacobian matrix can be seen by clicking the “Matrix” tab of the interface, Figure 4. 56. Data which are given in Table 4. 37 and Table 4. 38 are converted to rod lengths by calculations. This output can be examined from “Rod Lengths” tab, Figure 4. 57. Fixator configuration is drawn for a better understanding of the structure. This drawing can be seen from “Kinematic Diagram”

tab, Figure 4. 58. Output of the geometric method can be examined from Figure 4. 59 and Figure 4. 60.

Table 4. 36. Dimensions of the spatial quadrilateral which is illustrated in Figure 4. 54

Dimensions	
Line Segment	Value(cm)
$ A_1A_2 $	3,34826695827
$ B_1B_2 $	3,21082615542
$ A_1B_1 = L_1$	25,7990829584143
$ A_2B_2 = L_2$	25,7205880378329
$ A_1B_2 $	25,85907180
$ B_1A_2 $	26,07589059

Following result is calculated by using data which is given in Table 4. 36 according to parallelogram law:

$$1348,6616267184259 \cong 1348,6436644713576 \quad (4.3)$$

As a result, singularity risk is very high when there is an approximate parallelogram in the structure.

Ring Parameters

R	5	R1	11
Proximal Joint Parameters(Degree)		Distal Joint Parameters(Degree)	
a1	32.38449944	e1	48.999378920
a2	71.50849895	e2	65.78358102
a3	120	e3	150
a4	160	e4	190
a5	200	e5	250
a6	310	e6	330

Displacement Data

Translations

X1	0
Y1	0
Z1	25

Rotations

E _x	0
E _y	0
E _z	0

Example11

Calculate

Determinant

Change Graphs of Determinants

Kinematic Diagram

Rod Length

Matrix

Geometric Method

Force

-8.88178419700125E-15

Jacobian

9.1552734375E-05 32.38449944 71.50849895 120 160 200 310 48.999378920738 65.78358102 150 190 250 330

Figure 4.55. Main window of the “Singularity Evaluation Interface” (Example11)

CPA&FORCE&JACOBIAN

Determinant | Change Graphs of Determinants | Kinematic Diagram | Rod Length | Matrix | Geometric Method

Force Matrix

0.116065173292034	0.113769112349252	-0.270294060384061	-0.235985188562741	0.0353791907950049	0.24429879086185
0.217981987625297	0.205678623334563	0.0450038631136228	-0.139266713729662	-0.325983436182732	-0.0646233573770008
0.969026691386576	0.971983998313996	0.961725414671957	0.961725414671957	0.944713243367102	0.967544274162564
8.04459519718773	9.7509713662823	5.28948978069576	-1.83702052241134	-9.76514069905866	-5.32149350789411
-6.99321301522823	-4.38561939346502	9.16166504628142	10.4182610910161	3.55422054787882	-9.21709712782051
0.609577777961372	-0.213307212801132	1.05789795613915	1.05789795613915	1.59212312692901	0.728023188910622

Jacobian Matrix

5.98875006868784	11.2474707643702	50	415.086357718215	-360.836965451471	31.4530953264626
5.85241694033007	10.580350279538	50	501.601434961704	-225.601419420089	-10.9727738919126
-14.0525588832577	2.33974596215561	50	275	476.313972081441	55
-12.2688443584095	-7.24046134192916	50	-95.5064977168118	541.644264156714	55
1.87248305469436	-17.2530362240333	50	-516.83094143225	188.111078829118	84.2648887430876
12.6246827863923	-3.33955556881023	50	-275	-476.313972081441	37.6222157658235

Ring Parameters

R: 5 R1: 11

Proximal Joint Parameters (Degree)

α_1 : 32.38449944 α_2 : 71.50849895 α_3 : 120 α_4 : 160 α_5 : 200 α_6 : 310

Distal Joint Parameters (Degree)

ϵ_1 : 48.999378920 ϵ_2 : 65.78358102 ϵ_3 : 150 ϵ_4 : 190 ϵ_5 : 250 ϵ_6 : 330

Displacement Data

Translations

X1: 0 Y1: 0 Z1: 25

Rotations

θ_x : 0 θ_y : 0 θ_z : 0

Example 11

Figure 4. 56. “Matrix” tab of the “Singularity Evaluation Interface” (Example11)

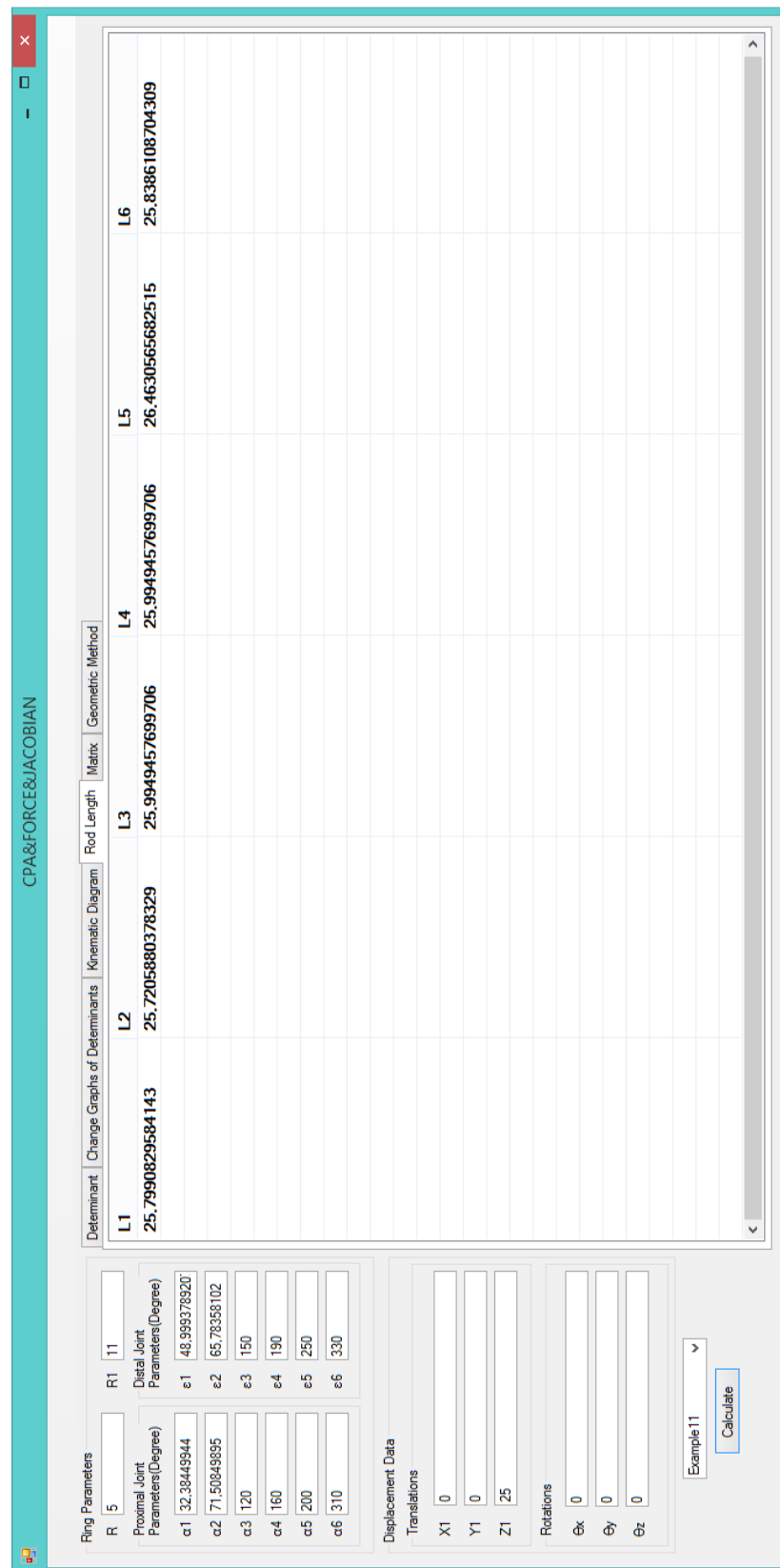


Figure 4. 57. “Rod Length” tab of the “Singularity Evaluation Interface” (Example11)

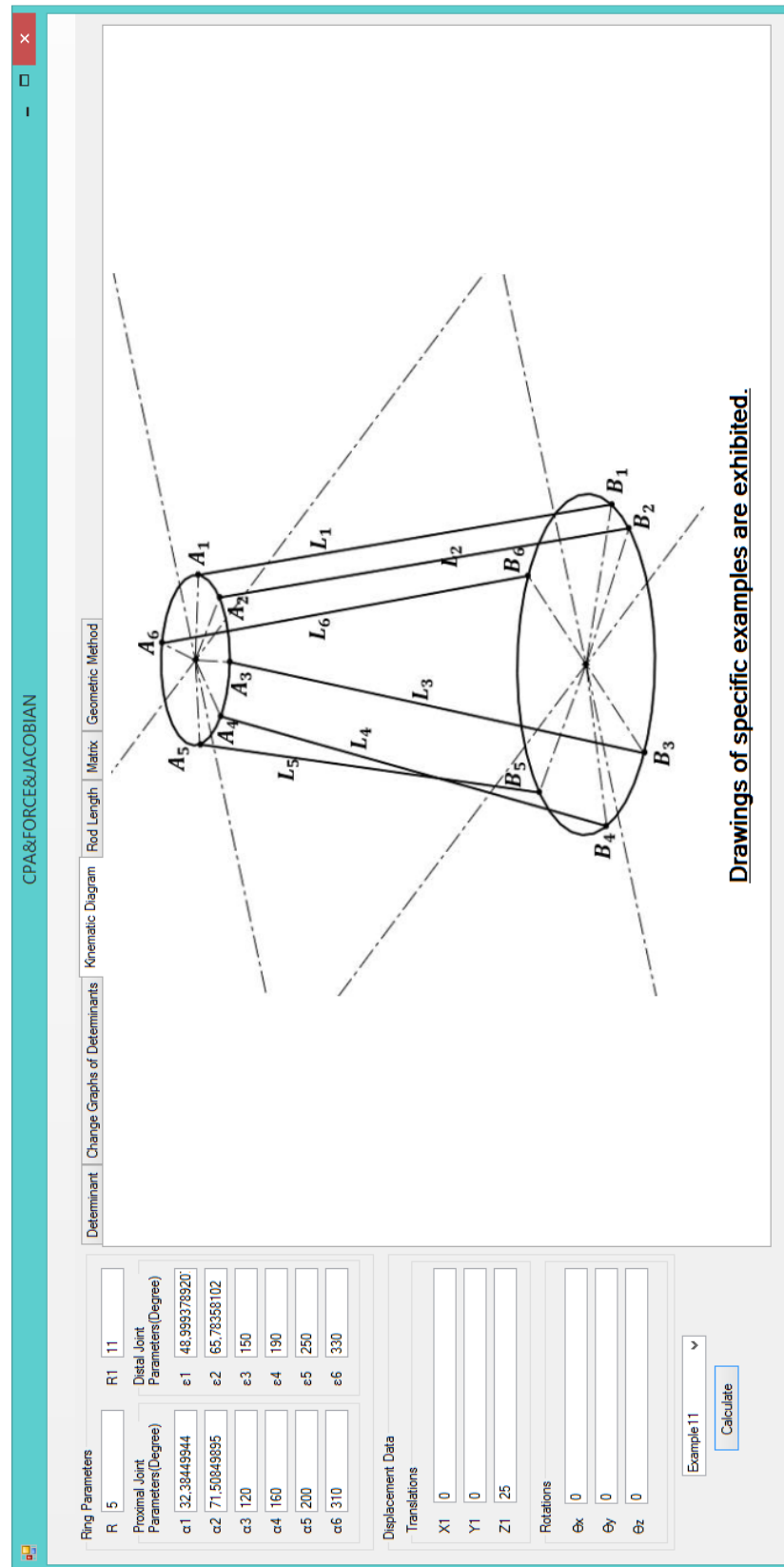


Figure 4. 58. “Kinematic Diagram” tab (Example11)



Figure 4. 59. “Primary Output” sub-tab of the “Geometric Method” tab (Example11)

CPA&FORCE&JACOBIAN

Determinant | Change Graphs of Determinants | Kinematic Diagram | Rod Length | Matrix | Geometric Method

Primary Output | Closest Point Coordinates

Ring Parameters
 R 5 R1 11
 Proximal Joint Parameters(Degree)
 α1 32.38449944 α2 71.50849895 α3 120 α4 160 α5 200 α6 310
 Distal Joint Parameters(Degree)
 e1 48.99378920 e2 65.78358102 e3 150 e4 190 e5 250 e6 330

Displacement Data
 Translations
 X1 0 Y1 0 Z1 25
 Rotations
 Ex 0 Ey 0 Ez 0

Example11
 Calculate

Pair	GCmnx	GCmny	GCmnz	GDmnx	GDmny	GDmnz
1,2	17.8014326658407	28.1808384405433	113.371473474517	14.8809912569719	28.7776615355681	113.587014342021
1,3	2.77035947064853	-0.049018270954746	-12.1227700887768	1.09961361977068	3.73245765191718	-12.7720995499524
2,3	0.528949891360652	2.83118055171568	-9.02934665841232	0.0866919142255083	3.89944375248448	-9.20363307392832
1,4	2.64496802506367	-0.284515929421366	-13.1696617117923	-1.36617392037459	3.6766519506996	-13.5802895782555
2,4	0.074363577945771	2.00935215559634	-12.9130951853008	-1.49216916279621	3.60229582917985	-13.0668131711021
3,4	0.541366364108785	3.82374062132898	-10.8213969760778	-1.88124966409903	3.37267988115044	-11.4811687129257
1,5	3.0047741210139	0.39123585494539	-10.1656450820829	-5.05282412695825	1.55497746465201	-9.46232923551199
2,5	0.0395301240975736	1.94637814898291	-13.2106940594356	-5.17624643854279	2.69218915117896	-12.7580149100799
3,5	-0.825897429324298	4.05138954751124	-5.95657554109324	-4.95915106095365	0.69187496717315	-6.96102312836846
4,5	-3.40972252319331	2.47065126906894	-5.25208627270946	-4.92060822059054	0.336741702124469	-5.93183249653642
1,6	2.12123714379866	-1.26813483284252	-17.5422843397037	-1.03819306579707	-2.70542340820282	-16.8405463573826
2,6	-1.62573991463639	-1.06419688297874	-27.4378933149969	-3.6040634917499	-2.02668422601343	-27.0026647621258
3,6	1.24318963161898	3.70688730159229	-13.3185338795437	-0.170771855318897	-2.93487878407007	-13.4051285130105
4,6	-1.39096294677664	3.66202270102254	-13.4792652858369	-0.365266854101715	-2.8834238195867	-14.1754250902538
5,6	-5.01645240062179	1.21984881799201	-8.49111280059496	0.635961730950686	-3.14828071156062	-10.2100637342775

Figure 4. 60. “Closest Point Coordinates” sub-tab (Example11)

4.4.3. Example12 (Singular Case)

In this example, a singular case is presented.

This example will verify the hypothesis which was given in Example11. Verification is done by searching for the sign change in the determinant when one of the fixed variables is varied between allowable limits. A (6-6) type Stewart Platform whose ring planes are nonparallel is considered in this example. The stability condition of the SP is to be investigated when an external force of 100N is applied in (-z) direction to the center of the distal ring, whose coordinates are (0, 0, 25). Singularity condition will be demonstrated by using “Singularity Evaluation Interface”.

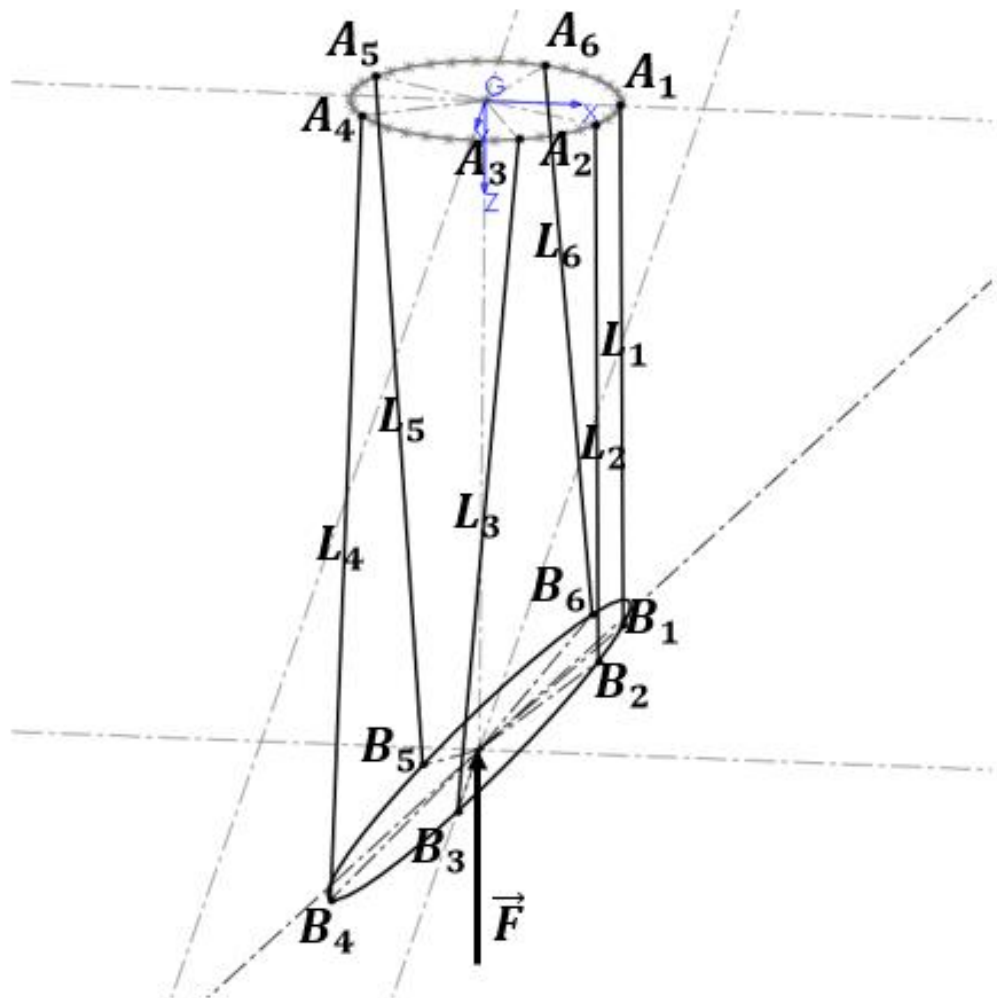


Figure 4. 61. Kinematic diagram (Example12)

- The input data “Ring Parameters” and “Displacement Data” which are required for the operation of the “Singularity Evaluation Interface with Three Methods” are given in Table 4. 37 and Table 4. 38.

Table 4. 37. Ring parameters (Example12)

Ring Parameters			
Proximal Ring	Diameter(cm)	Distal Ring	Diameter(cm)
R	5	R_1	7,794369626
Proximal Joints	Value (Degree)	Proximal Joints	Value (Degree)
α_1	0	ε_1	10,5102876262765
α_2	30	ε_2	30
α_3	70	ε_3	90
α_4	150	ε_4	160
α_5	210	ε_5	240
α_6	290	ε_6	310

Table 4. 38. Displacement data (Example12)

Displacement Data			
Translations	Value (cm)	Rotations	Value (Degree)
x_1	0	θ_x	0
y_1	0	θ_y	45
z_1	25	θ_z	0

- “Force-Moment Inputs” are given in Table 4. 39.

Table 4. 39. Force-Moment Parameters (Example12)

Force-Moment Parameters					
Force	Value	Moment	Value	Force Application Point (Global Coordinate System)	Value(cm)
F_x	0	M_x	0	rF_x	0
F_y	0	M_y	0	rF_y	0
F_z	-100	M_z	0	rF_z	25

Outputs of this example can be seen from Figure 4. 62 when data which are given in Table 4. 37 and Table 4. 38 are entered into the “Singularity Evaluation Interface with 3 Methods”. It is observed that determinants of the force coefficient matrix and Jacobian matrix values are approximately equal to “0”. This result is obtained when first joint angular position parameter (ε_1) is changed between 320°

and $10,52^\circ$. Also sign of both determinants changed when going from $\varepsilon_1 = 10,51^\circ$ to $\varepsilon_1 = 10,52^\circ$, Figure 4. 62 and Figure 4. 63. This means that the (ε_1) value making the determinants zero is between $\varepsilon_1 = 10,51^\circ$ and $\varepsilon_1 = 10,52^\circ$. In order to get zero value for the determinants, one should enter ε_1 value with a precision level of at least 13, which reveals the sensitivity of computational procedure.

Force coefficient matrix and Jacobian matrix can be seen by clicking the “Matrix” tab of the interface when $\varepsilon_1 = 10,5102876262765^\circ$, Figure 4. 64. Data which are given in Table 4. 37 and Table 4. 38 are converted to rod lengths by calculations. This output can be examined from “Rod Lengths” tab, Figure 4. 65. Fixator configuration is drawn for a better understanding of the structure. This drawing can be seen from “Kinematic Diagram” tab, Figure 4. 66. Output of the geometric method can be examined from Figure 4. 67 and Figure 4. 68.

Table 4. 40. Dimensions of the spatial quadrilateral which is illustrated in Figure 4. 61

Dimensions	
Line Segment	Value(cm)
$ A_1A_2 $	2,58819045
$ B_1B_2 $	2,63856596
$ A_1B_1 = L_1$	19,63703930
$ A_2B_2 = L_2$	20,27997858
$ A_1B_2 $	20,60021289
$ B_1A_2 $	19,64101478

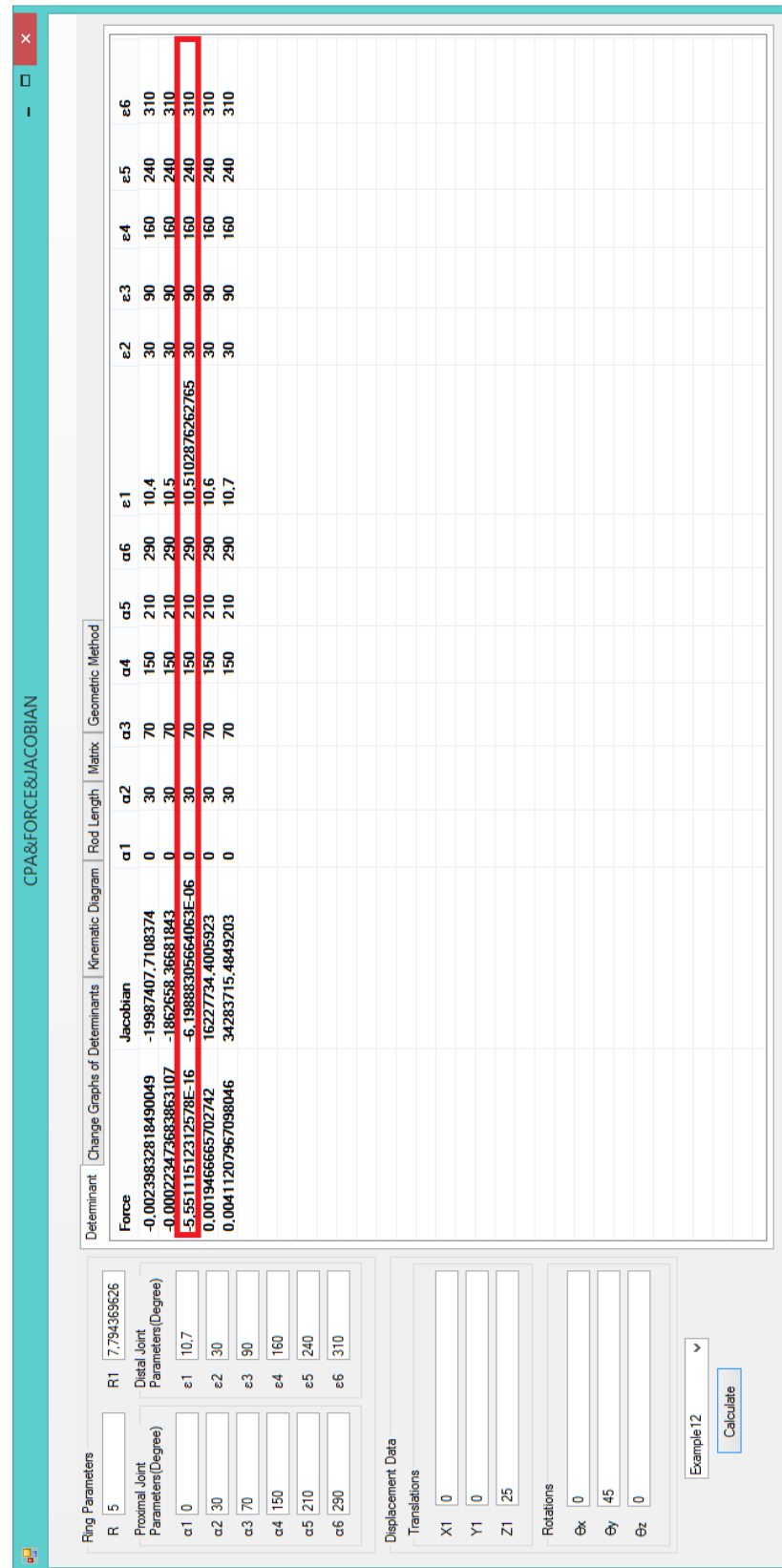
Following result is calculated by using data which is given in Table 4. 40 according to parallelogram law:

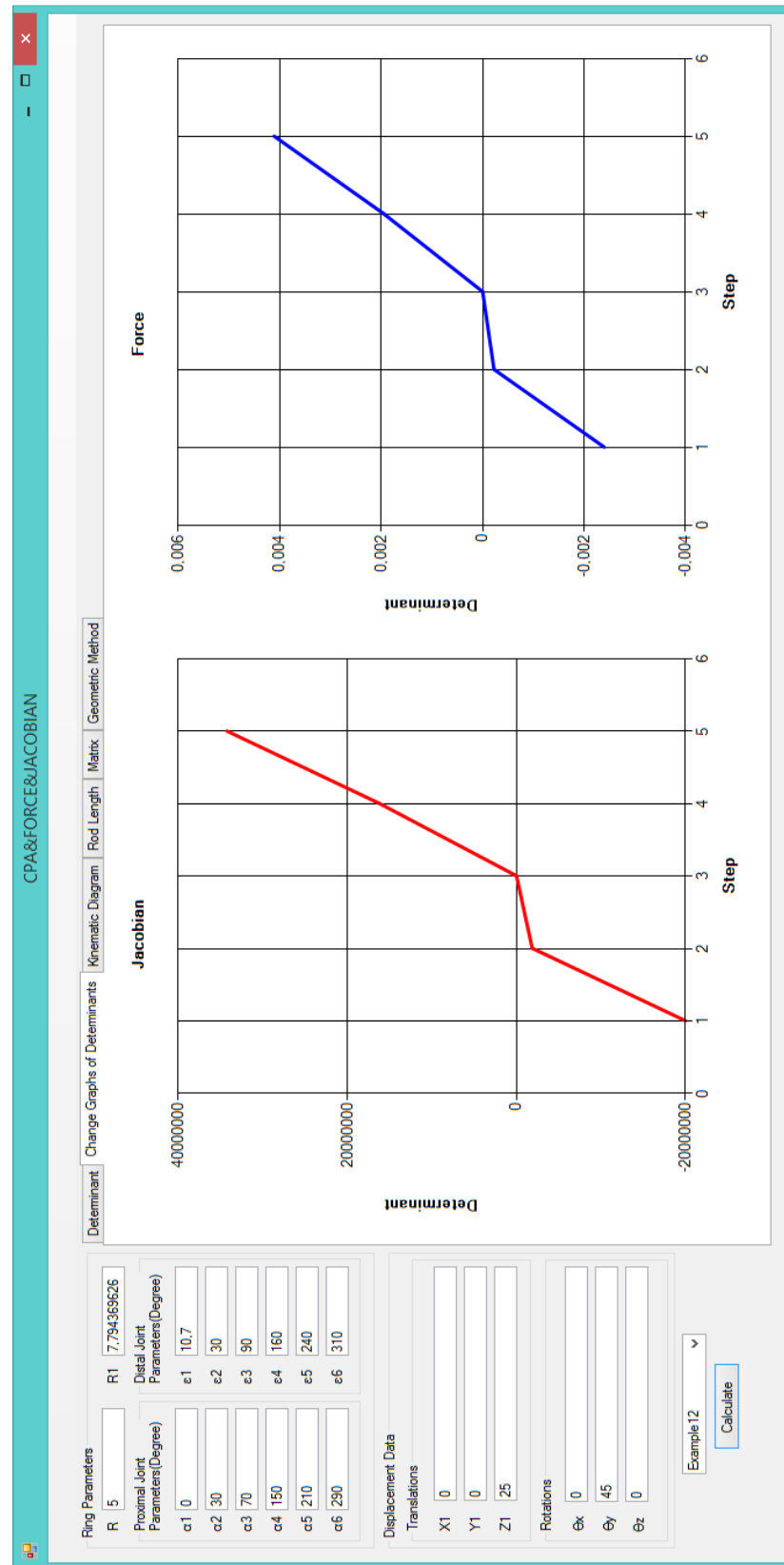
$$810,551603769566 \cong 810,1416958129989 \quad (4.4)$$

As a result, singularity risk is very high when there is an approximate parallelogram in the structure. Numerical results indicate that $(A_1A_2B_1B_2)$ spatial quadrilateral appears to be a planar parallelogram, Figure 4. 61.

Another result obtained from our observation when $EMN = \vec{e}_m \cdot \vec{e}_n \approx \pm 1$ these two rods are parallel each other. So c_{mn} and d_{mn} distances are calculated far

from the value which should be, Figure 4. 67. Geometric mean of this situation is there are many s_{mn} values when two rods parallel to each other. But it must be a unique solution. So we can say if two rods are parallel in the robotic structure the formulation of geometric methods has a singularity.

Figure 4. 62. Variation of the determinant values depending on " ϵ_1 " (Example12)

Figure 4. 63. Variation of the determinant values depending on " ϵ_1 " (Example12)

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Determinant | Change Graphs of Determinants | Kinematic Diagram | Rod Length | Matrix | Geometric Method

Force Matrix

0.0213362827909367	0.0218407574744892	-0.0677295984853382	-0.0281186562233031	0.0559820018616037	0.0849485012717304
0.0724033326697581	0.0688947874263648	0.122615296075303	0.00549265901238782	-0.151124339800826	-0.0589798252946477
0.997147190967802	0.997384825219642	0.990140490363555	0.999589501680149	0.98692828989108	0.994638191675697
1.81008331674395	4.21583174870822	7.71752096356244	2.63629022951007	-6.24542921974835	-6.14776647771365
-5.51914302461243	-4.86482191680879	-3.11284097507162E-16	5.03131591463876	2.87397480725595	-3.82464401616379
0.362016663348791	0.24372128681158	0.527909525215296	0.0465127293629907	0.794342591642344	0.298265957456346

Jacobian Matrix

0.8379628473379	2.8435741780886	39.1620371526621	40.2142116826682	-216.759256946758	14.217870890443
0.885860187329492	2.794369626	40.4538857748261	113.921004877542	-197.316968497204	9.88532495007438
-3.42020143325669	6.19181304414092	50	256.722306159047	-1.57191883645151E-14	26.6583141661776
-1.69788679194598	0.331662833235521	60.3581408297904	110.576190387014	303.805586217636	2.80857478449953
3.14880242022533	-8.50024420520362	55.5114516176191	-279.98874276149	161.651576003411	44.6791431509523
3.66518418912804	-2.54474087136621	42.9146143776153	-196.660696057827	-165.01791752919	12.8689694939672

Ring Parameters

R 5 R1 7.794363626

Proximal Joint Parameters(Degree)

α1 0 α2 30 α3 70 α4 150 α5 210 α6 290

Distal Joint Parameters(Degree)

e1 10.510287626 e2 30 e3 90 e4 160 e5 240 e6 310

Displacement Data

Translations

X1 0 Y1 0 Z1 25

Rotations

Ex 0 Ey 45 Ez 0

Example12 Calculate

Figure 4. 64. “Matrix” tab of the “Singularity Evaluation Interface” (Example12)



Figure 4. 65. “Rod Length” tab of the “Singularity Evaluation Interface” (Example12)

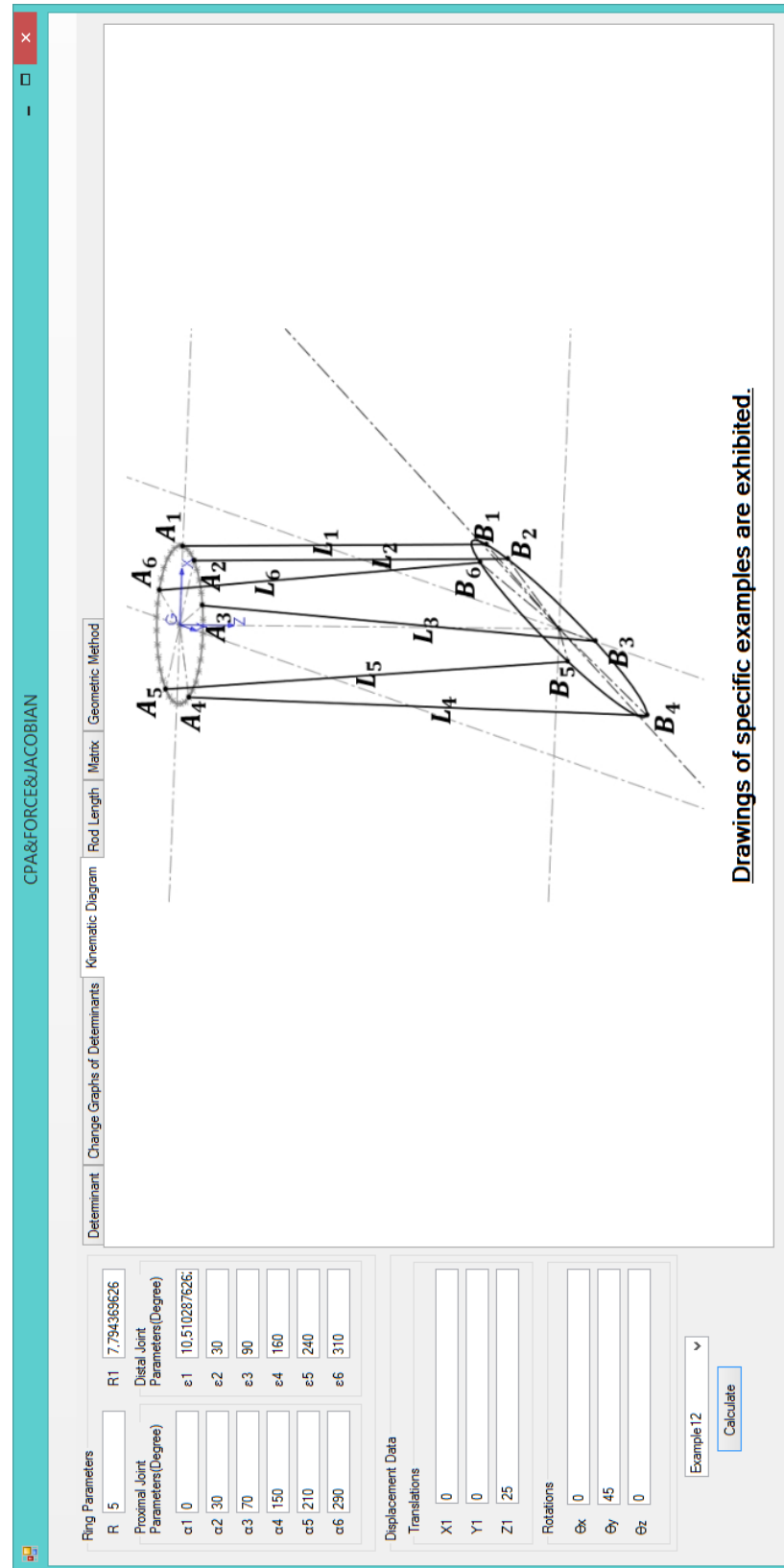


Figure 4. 66. “Kinematic Diagram” tab (Example12)

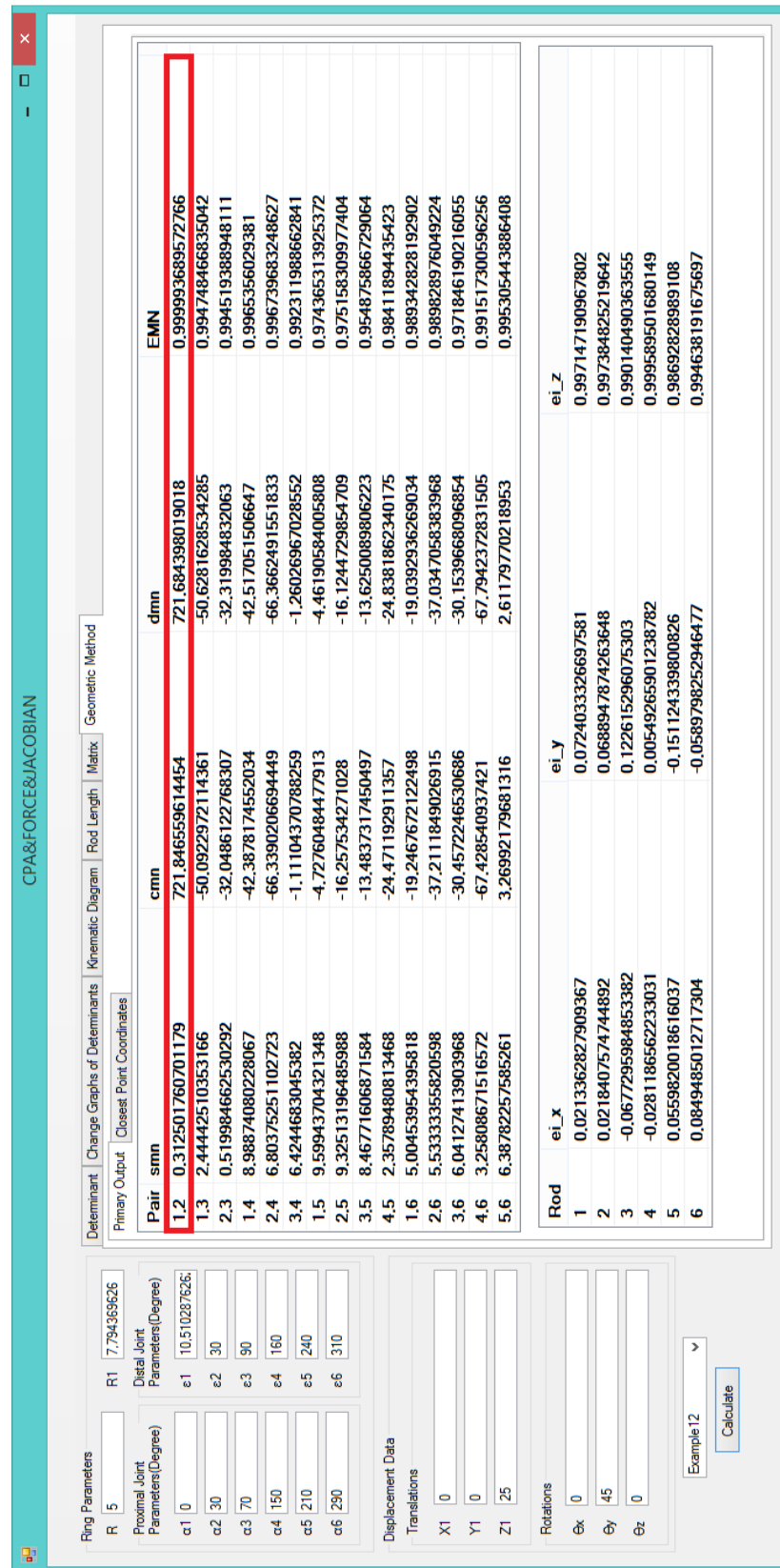


Figure 4. 67. “Primary Output” sub-tab of the “Geometric Method” tab (Example12)

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Determinant | Change Graphs of Determinants | Kinematic Diagram | Rod Length | Matrix | Geometric Method

Primary Output | Closest Point Coordinates

Ring Parameters: R 5, R1 7.794363626

Proximal Joint Parameters(Degree): α_1 0, α_2 30, α_3 70, α_4 150, α_5 210, α_6 290

Distal Joint Parameters(Degree): ϵ_1 10.510287626, ϵ_2 30, ϵ_3 90, ϵ_4 160, ϵ_5 240, ϵ_6 310

Displacement Data: Translations X1 0, Y1 0, Z1 25; Rotations Ex 0, Ey 45, Ez 0

Example12

Pair	GCmnx	GCmny	GCmnz	GDmnx	GDmny	GDmnz
1,2	20.4015223275987	52.2640965922857	719.7876929324	20.0922609291783	52.2202931904443	719.797067181941
1,3	3.93121658104915	-3.62684925919201	-49.94393934535078	5.13912585874137	-1.50932407409224	-50.128893938996
2,3	3.63016105079	0.292017669877761	-31.9647995542589	3.89912031235619	0.735538594596843	-32.0013256301615
1,4	4.09560153988518	-3.06901924835407	-42.266893106712	-3.13460466397832	2.26646833386186	-42.4995983284385
2,4	2.88123255738572	-2.07041272709463	-66.1655325356366	-2.46399727409751	2.13547282345941	-66.3390059214103
3,4	1.78535126086289	4.56223215073492	-1.10008936173821	-4.29468992931478	2.49307776843747	-1.25975233170331
1,5	4.89913048610799	-0.342294346307703	-4.71411789097728	-4.57991343996663	-1.82569742566777	-4.40358110038355
2,5	3.97505015577548	1.37994063232069	-16.2150179774116	-5.2328072956122	-0.0631996654344551	-15.9136985489457
3,5	2.62334845380457	3.04515134381032	-13.3507887619741	-5.09288229703976	-0.44029513023125	-13.4469068129962
4,5	-3.64202995807362	2.36558808171155	-24.4611475277821	-5.72061840691382	1.25365449646585	-24.513508663935
1,6	4.58934553194811	-1.39353008928592	-19.1918598609061	0.0927412577504882	-3.57552889208128	-18.9372085838458
2,6	3.51740655412414	-0.0636566737540827	-37.1138711503867	-1.43594203938287	-2.51416262374223	-36.8361328443443
3,6	3.77295631335842	0.963941485461527	-30.1569313531023	-0.951433571251937	-2.91998740955369	-29.9922870194345
4,6	-2.43412705666393	2.12963801692791	-67.4008616346562	-4.04891813543536	-0.699570832985439	-67.4307375773459
5,6	-4.1470702508057	-2.99416477274372	3.22717832700638	1.93196901705429	-4.85250647610964	2.59779374352853

Figure 4. 68. “Closest Point Coordinates” sub-tab (Example12)

4.4.4. Example13 (Singular Case)

In this example, a singular case is presented.

A search for the sign change in the determinant will be done when one of the fixed variables is varied between allowable limits. Then we will comment the result why determinants are equal to approximately zero. Singularity condition will be demonstrated by using “Singularity Evaluation Interface”.

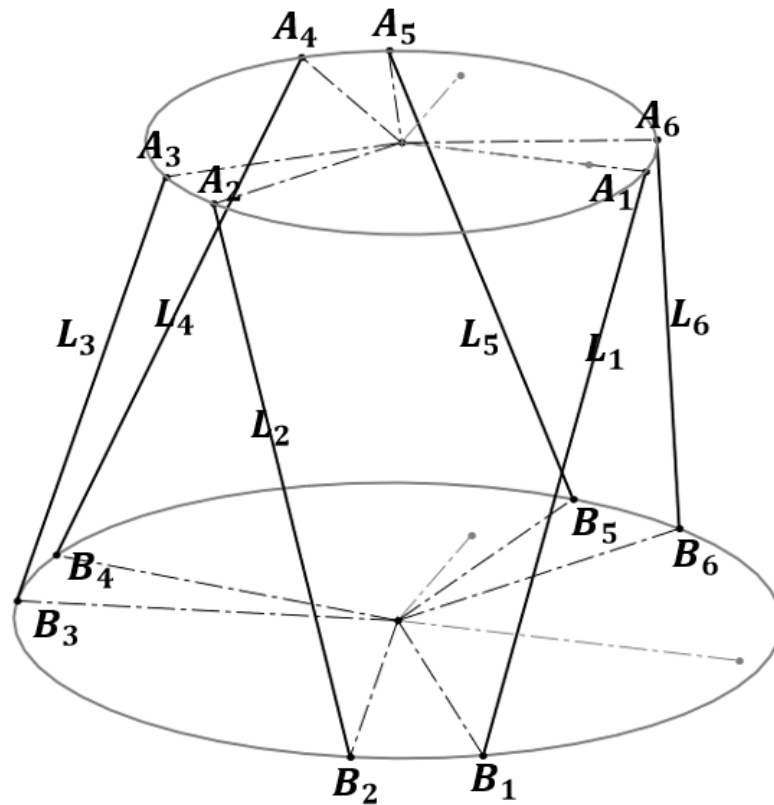


Figure 4. 69. Kinematic diagram at initial position (Example13)

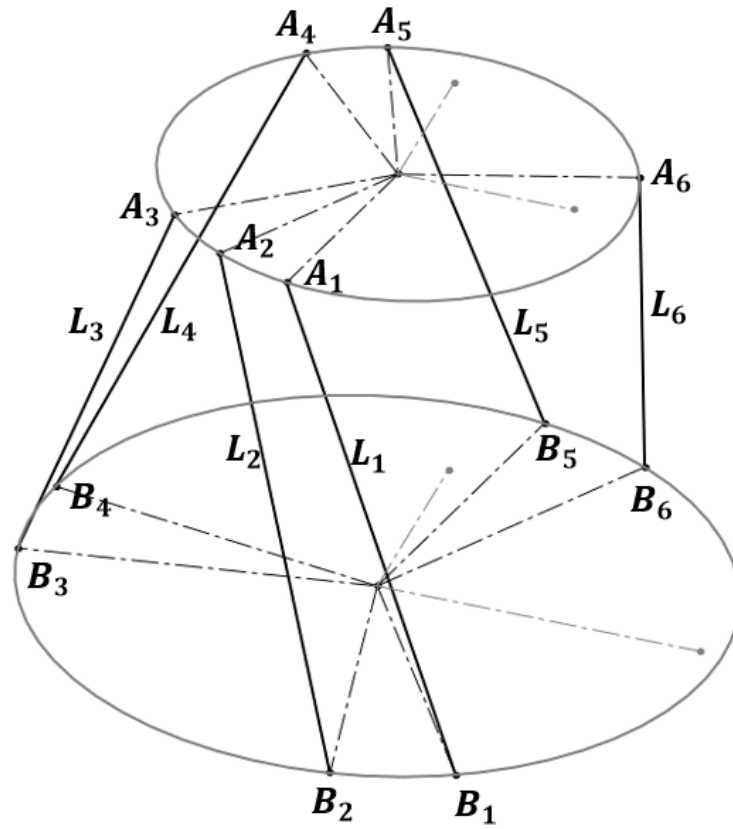


Figure 4. 70. Kinematic diagram at final position (Example13)

- The input data “Ring Parameters” and “Displacement Data” which are required for the operation of the “Singularity Evaluation Interface with Three Methods” are given in Table 4. 41 and Table 4. 42.

Table 4. 41. Ring parameters (Example13)

Ring Parameters			
Proximal Ring	Diameter(cm)	Distal Ring	Diameter(cm)
R	10	R_1	15
Proximal Joints	Value (Degree)	Proximal Joints	Value (Degree)
α_1	100	ε_1	60
α_2	120	ε_2	80
α_3	140	ε_3	170
α_4	230	ε_4	190
α_5	250	ε_5	280
α_6	340	ε_6	300

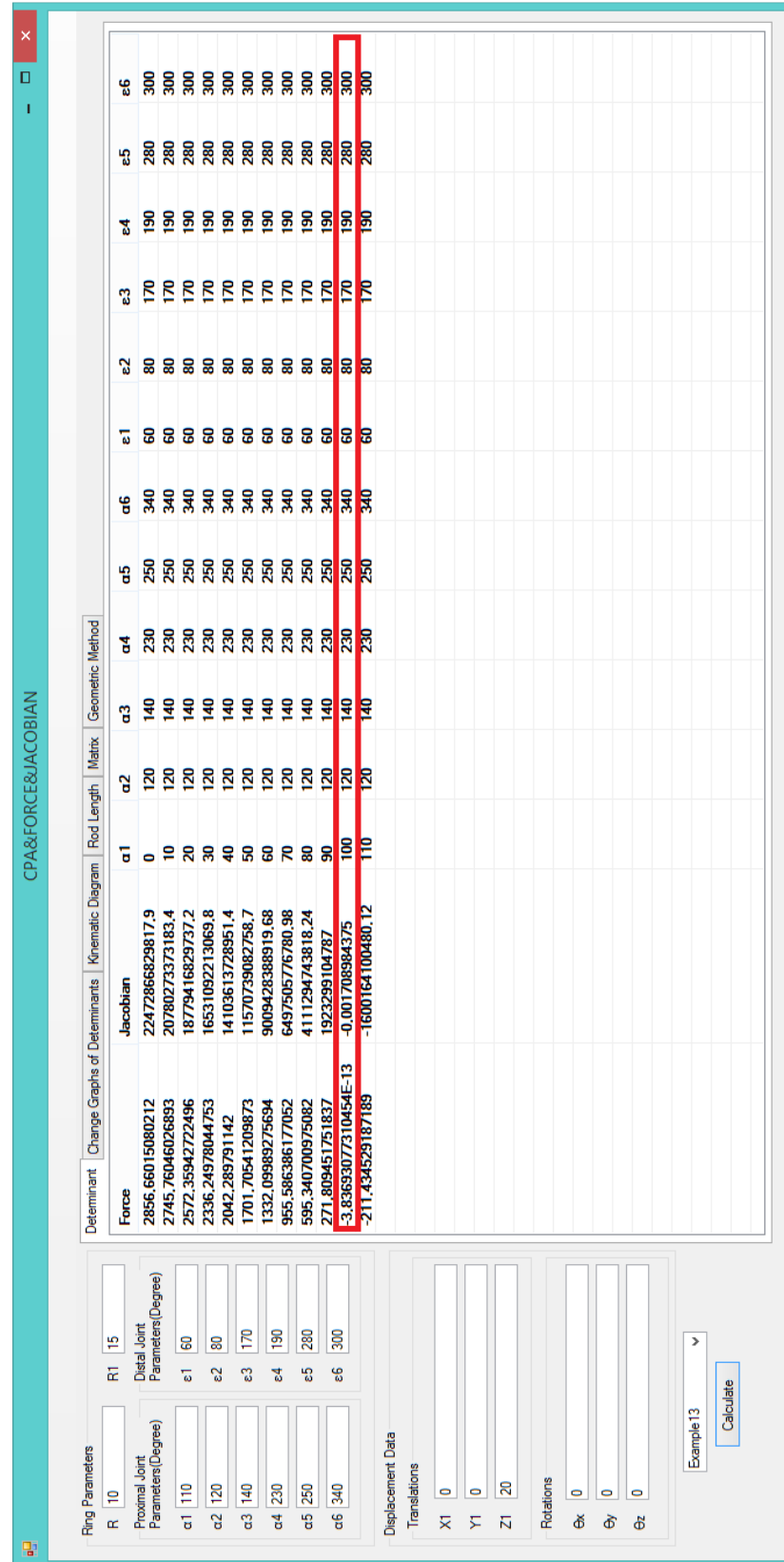
Table 4. 42. Displacement data (Example13)

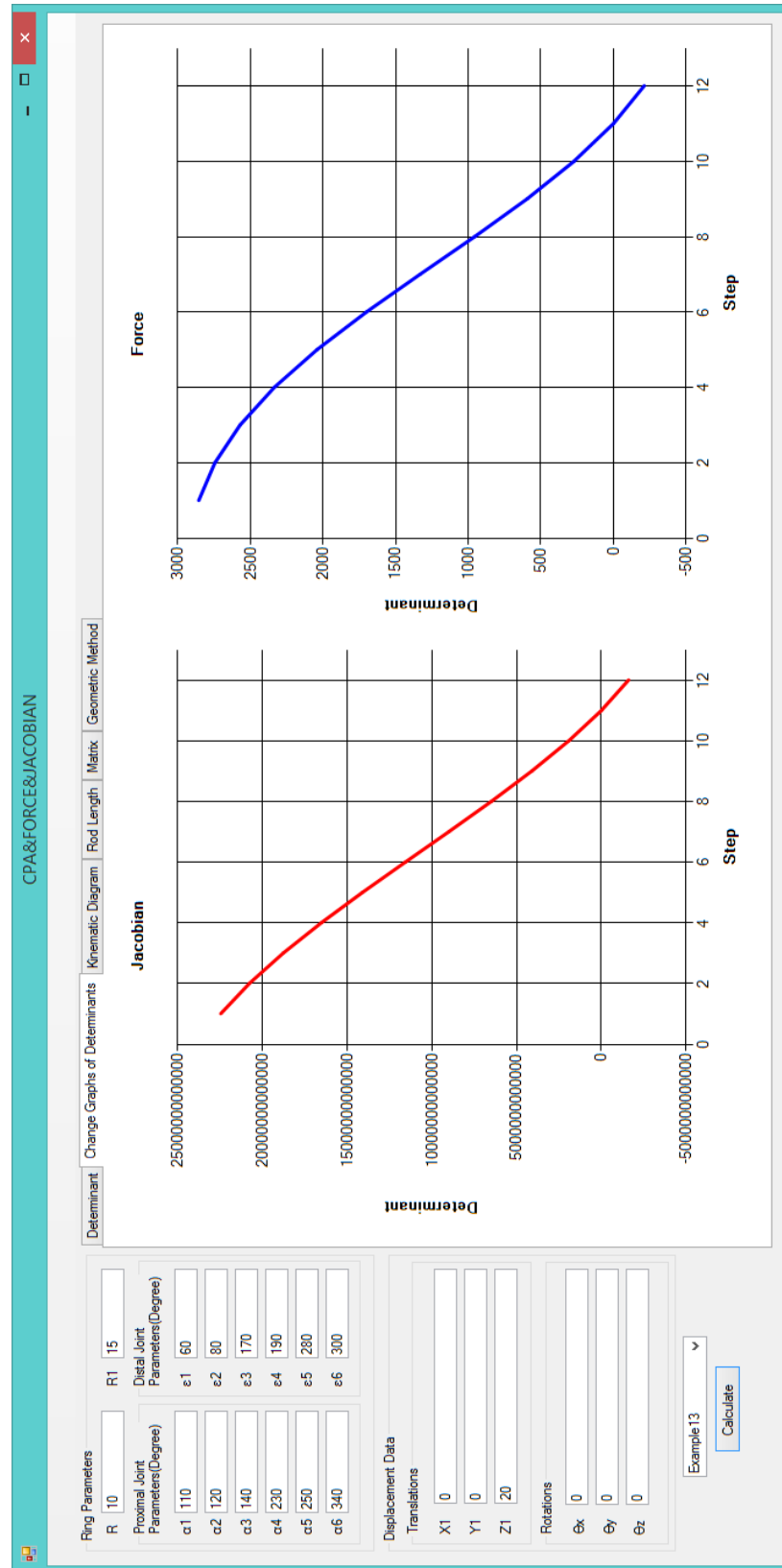
Displacement Data			
Translations	Value (cm)	Rotations	Value (Degree)
x_1	0	θ_x	0
y_1	0	θ_y	0
z_1	20	θ_z	0

Variation of the determinant values depending on " α_1 " can be seen from Figure 4. 71. It is observed that determinants of the force coefficient matrix and Jacobian matrix values are approximately equal to "0" when $\alpha_1 = 100^\circ$. Also sign of both determinants changed when going from $\alpha_1 = 90^\circ$ to $\alpha_1 = 110^\circ$, Figure 4. 71. This means that the (α_1) value making the determinants zero is between $\alpha_1 = 90^\circ$ and $\alpha_1 = 110^\circ$, Figure 4. 72.

Force coefficient matrix and Jacobian matrix can be seen by clicking the "Matrix" tab of the interface when $\alpha_1 = 100^\circ$, Figure 4. 73. Data which are given in Table 4. 41 and Table 4. 42 are converted to rod lengths by calculations. This output can be examined from "Rod Lengths" tab, Figure 4. 74. Fixator configuration is drawn for a better understanding of the structure. This drawing can be seen from "Kinematic Diagram" tab, Figure 4. 75. Output of the geometric method can be examined from Figure 4. 76 and Figure 4. 77.

Following result was observed when we examine the output of the geometric method. If triple rod group (1,2-1,3-2,3), pairs of rods (4,5) and other pairs of rods (5,6) intersect at a common point which can be seen from Figure 4. 76 this may cause a singular condition.

Figure 4. 71. Variation of determinant values depending on " α_1 " (Example13)

Figure 4. 72. Variation of determinant values depending on " α_1 " (Example13)

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Determinant | Change Graphs of Determinants | Kinematic Diagram | Rod Length | Matrix | Geometric Method

Force Matrix

0.415070727614424	0.341742434640231	-0.329727685637431	-0.374975010425906	0.279341386533709	-0.0852444210214057
0.141209417473677	0.274655997334465	-0.177257831496267	0.227194960467247	-0.249216924128359	-0.43006650321822
0.898763701700496	0.898763701700496	0.927285993907718	0.898763701700496	0.927285993907718	0.898763701700496
11.6752829650795	13.276641923409	2.41532284527215	-2.34103018430214	-13.6979765408993	-11.6752829650795
-6.74072776275372	-2.34103018430214	13.6979765408993	13.276641923409	-2.41532284527215	-6.74072776275372
-4.33285628616816	-4.33285628616816	3.47732247715394	-4.33285628616816	3.47732247715394	-4.33285628616816

Jacobian Matrix

18.4729635533386	6.284607053289	40	519.615242270663	-300	-192.836282905962
15.2094453300079	12.2237245146775	40	590.884651807325	-104.188906600158	-192.836282905962
-14.2233437279867	-7.64630686372288	40	104.188906600158	590.884651807325	150
-16.6884803966355	10.1114435323716	40	-104.188906600158	590.884651807325	-192.836282905962
12.0498481965213	-10.7503801746481	40	-590.884651807325	-104.188906600158	150
-3.79385241571817	-19.1403592470198	40	-519.615242270663	-300	-192.836282905962

Ring Parameters

R 10 R1 15

Proximal Joint Parameters(Degree)

α1 100 α2 120 α3 140 α4 230 α5 250 α6 340

Distal Joint Parameters(Degree)

ε1 60 ε2 80 ε3 170 ε4 190 ε5 280 ε6 300

Displacement Data

Translations

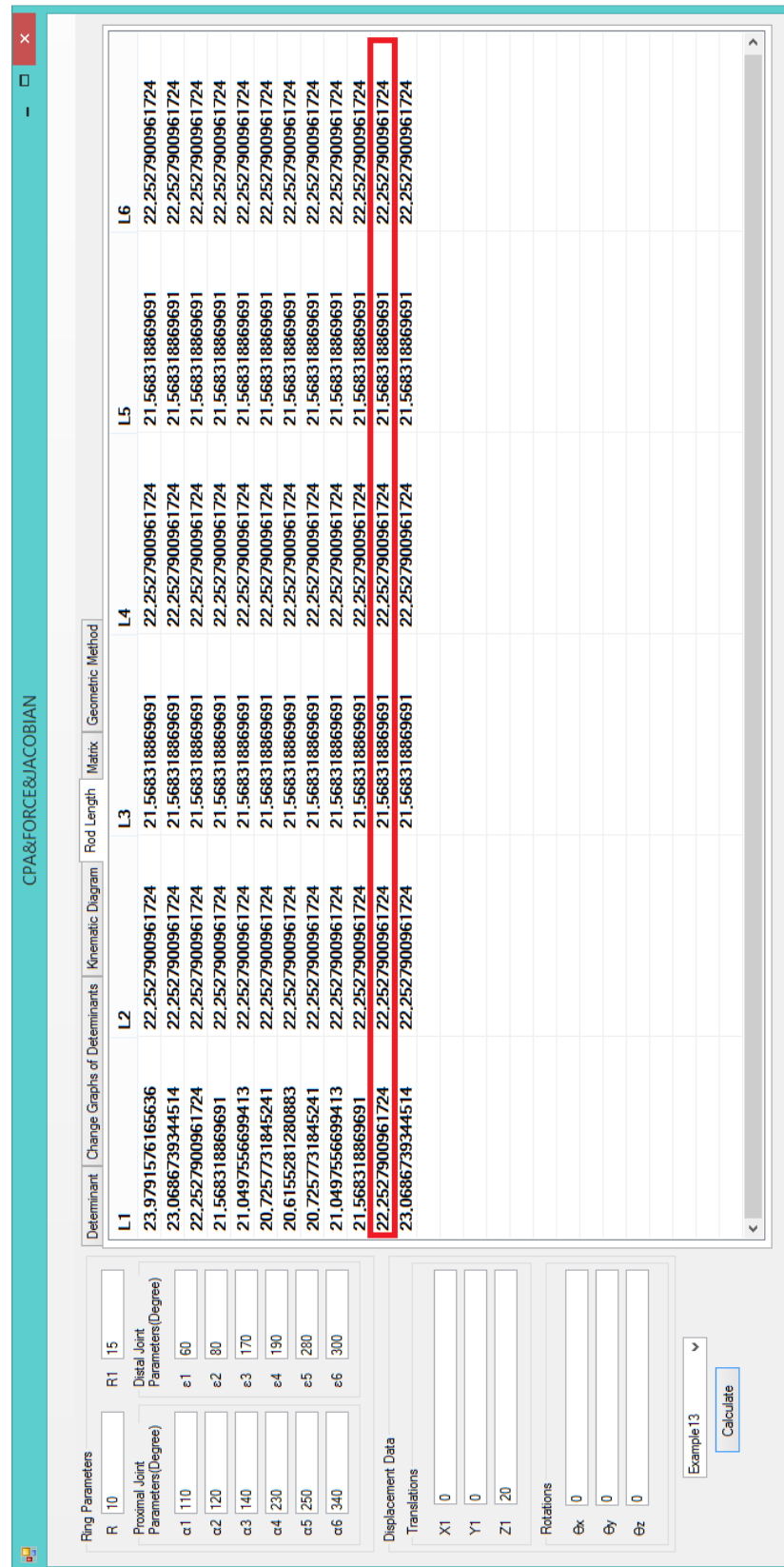
X1 0 Y1 0 Z1 20

Rotations

Ex 0 Ey 0 Ez 0

Example13

Figure 4. 73. “Matrix” tab of the “Singularity Evaluation Interface” (Example13)



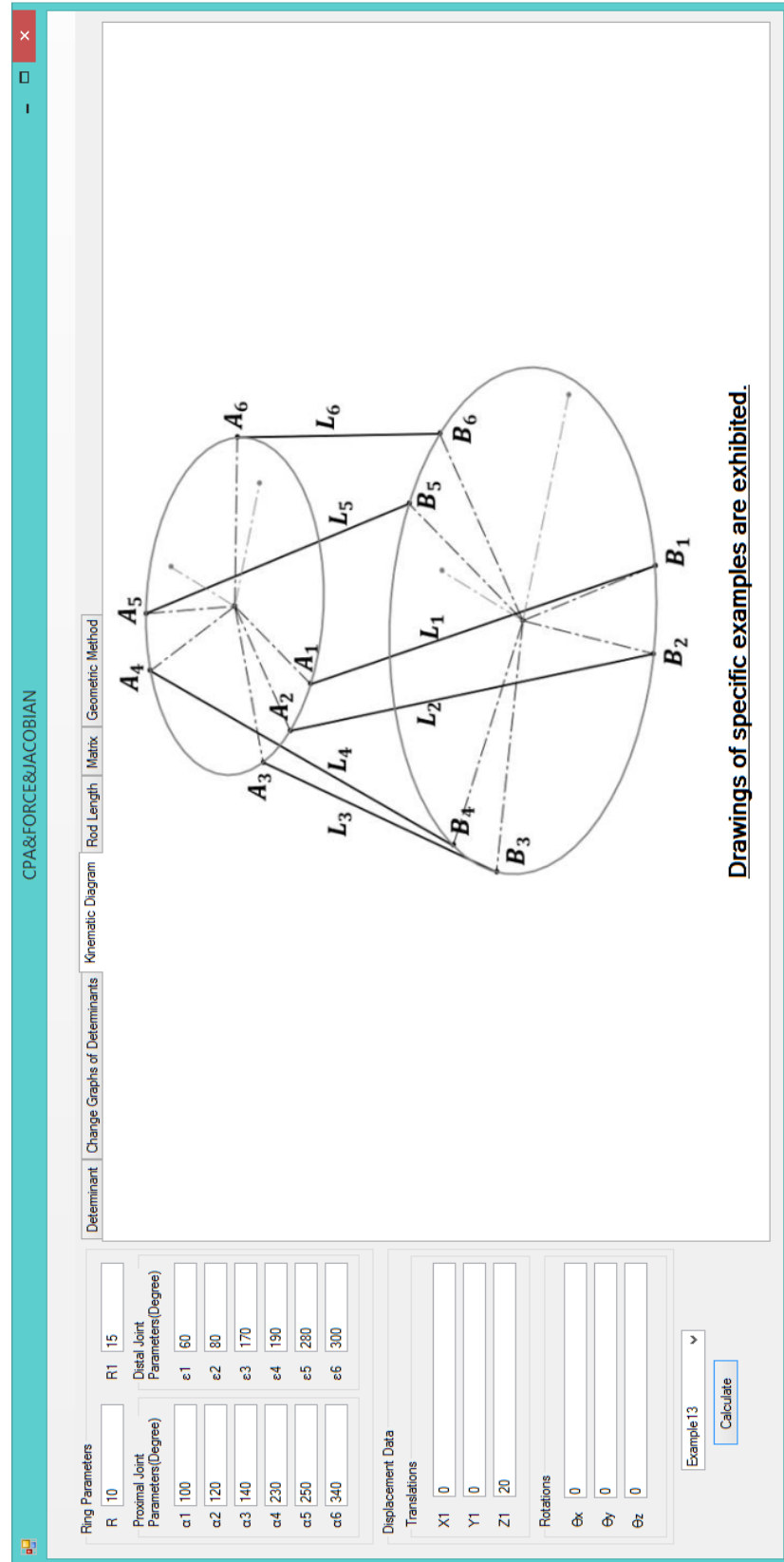


Figure 4. 75. “Kinematic Diagram” tab (Example13)

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Determinant | Change Graphs of Determinants | Kinematic Diagram | Rod Length | Matrix | Geometric Method

Primary Output | Closest Point Coordinates

Ring Parameters: R 10, R1 15
 Proximal Joint Parameters(Degree): α1 100, α2 120, α3 140, α4 230, α5 250, α6 340
 Distal Joint Parameters(Degree): e1 60, e2 80, e3 170, e4 190, e5 280, e6 300

Displacement Data:
 Translations: X1 0, Y1 0, Z1 20
 Rotations: Ex 0, Ey 0, Ez 0

Example13

Pair	smn	cmn	dmm	EMN
1,2	-3.0937039275164	-4.23017039415912	-2.73960657509958	0.988407485886475
1,3	-0.823915260082789	-8.48835068579288	-8.25965099397321	0.671520206899141
2,3	-0.361326774535964	-4.33620486594312	-4.18704965799984	0.672044123866548
1,4	-17.5443744318215	-5.45563231742738	-1.51414465183121	0.684217109100206
2,4	-15.591851736188	-5.82213234716455	-1.14764462209404	0.742031776958679
3,4	-0.669927964832998	34.1593028099602	34.9794938061981	0.916778548759825
1,5	-4.87961724260462	-44.862616093979	-45.3377105790061	0.914165648300146
2,5	-3.81674878883463	-33.3939781541971	-33.6744675237684	0.860424875100229
3,5	14.8257116354299	-8.09001941044613	-11.69680024469	0.811928377156076
4,5	-0.361326774535964	-4.33620486594311	-4.18704965799983	0.672044123866548
1,6	-16.6296286150462	-1.29265742463805	-5.67711954462052	0.711664287241567
2,6	-18.3199558939161	-1.8076432408718	-5.16213372838677	0.660524211153479
3,6	-5.60000997120717	-53.7774337451286	-53.2111299007188	0.937751093836059
4,6	-15.5918517361881	-5.82213234716454	-1.14764462209404	0.742031776958679
5,6	-0.669927964833001	34.1593028099602	34.9794938061981	0.916778548759825

Rod	ei_x	ei_y	ei_z
1	0.415070727614424	0.141209417473677	0.898763701700496
2	0.341742434640231	0.274655997334465	0.898763701700496
3	-0.329727685637431	-0.177257831496267	0.927285993907718
4	-0.374975010425906	0.227194960467247	0.898763701700496
5	0.279341386533709	-0.249216924128359	0.927285993907718
6	-0.0852444210214057	-0.43006650321822	0.898763701700496

Figure 4. 76. “Primary Output” sub-tab of the “Geometric Method” tab (Example13)

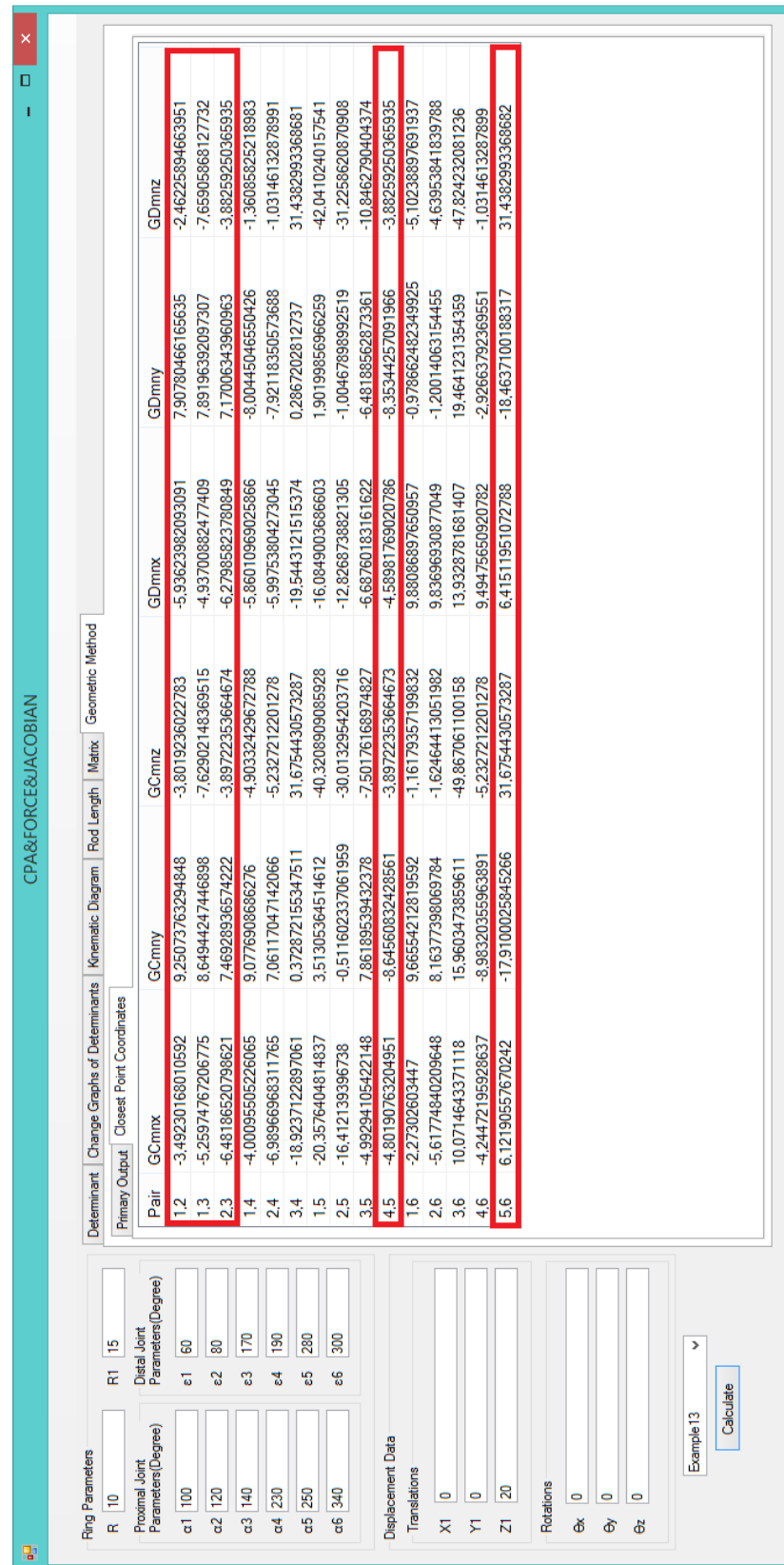


Figure 4. 77. “Closest Point Coordinates” sub-tab (Example13)

5. CONCLUSION

External fixation is used in treatment of bone disorders. External fixators are used in this treatment method. The treatment process is managed well by orthopaedist. Singularities which are important for the medical application have been investigated and formulated in this thesis work. Obtained results are guide for orthopaedist.

Firstly, the definition and classification of robots were made. Then external fixation process and external fixators were described. Statement of the problem was made after explaining the concept of singularity in the medical context. Conducted studies on singularity in robotics were explained briefly. Methodology that relates singularities to displacement, force and geometric condition of the fixator has been described. Algorithms have been developed in this context. Interfaces used for singularity detection have been developed utilizing these algorithms. Miscellaneous examples are given regarding the use of interfaces involving the singular and non-singular cases. The following guidelines should be considered to avoid the singularity during the use of the (6-6) Stewart type fixator by orthopaedists:

- Four or more out of six rods should never have directions intersecting at a common point in any phase of the treatment procedure.
- Two joints on the proximal and two joints on the distal ring should never form a parallelogram.
- Proximal ring radius should be selected different than distal ring radius.

If singularity is detected, it is necessary and sufficient to change the value of one of the fixed joint position parameters to yield non-singularity for all phases of the treatment process.

Another conclusion was obtained from the observations:

- Numerical precision must be at least 10 or more significant digits for robotic calculation procedure.

There is need for the singularity-free robotic structures. Future studies can be performed on this basis.

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BIOGRAPHY

Atakan DURMAZ was born on August 20th, 1989 in Tekirdağ-Malkara, Turkey. He graduated from İbrahim Atalı High School in 2007, and enrolled at Erciyes University to study mechanical engineering. He graduated with a B.Sc. degree in mechanical engineering from the Department of Mechanical Engineering in 2012. He began his M.Sc. studies at the Department of Mechanical Engineering of Çukurova University in 2012. He worked as a project assistant in a TUBITAK project from 04/2013 to 01/2015.