

**FLEXURAL BEHAVIOR OF
LIGHTLY AND HEAVILY REINFORCED
STEEL FIBER CONCRETE BEAMS**

**A MASTER'S THESIS
IN
CIVIL ENGINEERING
ATILIM UNIVERSITY**

**BY
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AUGUST 2014**

**FLEXURAL BEHAVIOR OF
LIGHTLY AND HEAVILY REINFORCED
STEEL FIBER CONCRETE BEAMS**

**A THESIS SUBMITTED TO
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Approval of the Graduate School of Natural and Applied Sciences, Atılım University

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ABSTRACT

FLEXURAL BEHAVIOR OF LIGHTLY AND HEAVILY REINFORCED STEEL FIBER CONCRETE BEAMS

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Flexural behavior of lightly and heavily reinforced steel fiber concrete beams was investigated in this research. Test series consisted of 20 singly reinforced beams (10 pairs of beams) having 180×250×3500 mm dimensions. The main parameters in the testing program were the type of concrete and the amount of longitudinal reinforcement. Ten different longitudinal reinforcement ratios (with a minimum of 0.2 and a maximum of 2.5%) covering the range from under-reinforced to over-reinforced beam behavior were used in the testing program. Two specimens were tested for each longitudinal reinforcement ratio, one specimen using ordinary plain concrete (PC) and another specimen using steel fiber reinforced concrete (SFRC). Load-deflection behaviors were obtained and evaluated in terms of ultimate load, service stiffness, post-peak stiffness, and toughness. The results indicate that the use of SFRC increases the ultimate load and service stiffness of the beams slightly compared to that of PC specimens. As reinforcement ratio increases, the ultimate deflection of SFRC specimens becomes significantly greater than that of PC specimens. For over-reinforced sections the post-peak stiffness of the SFRC specimens is observed to be significantly lower than that of PC specimens. The toughness of SFRC specimens is greater than that of PC specimens with the difference being significantly larger for over-reinforced sections. Experimentally determined load-deflection relationships were compared to the load-deflection curves obtained from sectional analyses based on strain compatibility and best fit stress-strain relationships for SFRC in tension and compression in the literature are presented.

Keywords: Steel fiber reinforced concrete, under-reinforced beams, over-reinforced beams, load-deflection behavior, ultimate load, service stiffness, post-peak stiffness, toughness.

ÖZ

FLEXURAL BEHAVIOR OF LIGHTLY AND HEAVILY REINFORCED STEEL FIBER CONCRETE BEAMS

Bello, Hussain Jibril

Yüksek Lisans, İnşaat Mühendisliği Bölümü

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Bu çalışmada, çelik liflerle güçlendirilmiş ve farklı miktarlarda çekme donatısı içeren betonarme kirişlerin eğilme davranışı incelenmiştir. Toplam 20 adet 180x250x3500 mm ölçülerindeki kiriş numuneleri üzerinde yükleme deneyleri yapılmıştır. Çalışma kapsamında incelenen temel parametreler kirişlerde kullanılan beton içerisinde çelik liflerin bulunup bulunmaması ve kirişlerdeki çekme donatısı miktarıdır. En düşüğü %0.2, en yükseği %2.5 olmak üzere toplam on farklı çekme donatısı oranı kullanılmıştır. Bu oranlar hem dengealtı hem de dengeüstü betonarme kiriş davranışını kapsamaktadır. Herbir donatı oranı hem konvansiyonel beton ile hem de çelik liflerle güçlendirilmiş beton ile test edilmiştir. Yükleme deneyleri sonucunda kiriş numunelerinin yük-deformasyon davranışları elde edilmiş ve bu davranışlar yük taşıma kapasitesi, servis rijitliği, kapasite sonrası rijitlik ve tokluk parametreleri göz önüne alınarak değerlendirilmiştir. Çelik liflerin kullanılması durumunda kiriş numunelerinin yük taşıma kapasiteleri ve servis rijitliği seviyelerinde ufak artışlar gözlenmiştir. Çekme donatısı oranının artması ile çelik lif içeren kiriş numunelerinin maksimum sehim seviyeleri aynı miktarda donatıya sahip konvansiyonel beton kiriş numunelerine oranla önemli seviyede artmıştır. Çelik lif içeren kiriş numunelerinin konvansiyonel beton numunelere oranlar daha yüksek tokluk seviyelerine sahip oldukları da gözlenmiştir. İki seri numunelerin tokluk değerleri arasındaki fark, yüksek miktarda donatı içeren kiriş numunelerinde daha fazla gerçekleşmiştir. Doğrusal olmayan kesit analizleri yapılarak kiriş numunelerinin yük-deformasyon davranışları sayısal olarak da belirlenmiş ve deneysel olarak ölçülmüş davranışlarla karşılaştırılmıştır. Bu analizlerde, çelik liflerle güçlendirilmiş beton için

literatürde bulunan çekme ve basınç malzeme modelleri kullanılmış ve farklı modeller kullanılarak elde edilmiş kiriş davranışları arasında değerlendirmeler yapılmıştır.

Anahtar Kelimeler: Çelik lifli beton, denge-altı kirişler, denge üstü kirişler, yük-sehim davranışı, yük kapasitesi, servis rijitliği, kapasite sonrası rijitlik, tokluk.

DEDICATION

To my parents
For their endless support, love and prayers

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1 INTRODUCTION

1.1 General

Fibers have been used to reinforce and enhance the mechanical properties of construction materials for at least 3500 years. It is believed to start in Mesopotamia when straw was used to reinforce sun-baked bricks. Research in the late 1950's and early 1960's by Ramouldi and Baston (1963) and Ramouldi and Mandel (1964) on closely spaced random fibers, mainly steel fibers, triggered the era of using the fiber composite concretes that exist today. Steel and synthetic fibers have been used to enhance the properties of concrete in practice for many years. However, commercial use of fibers in concrete begun in 1970's particularly in Europe, Japan and the USA. In addition, Shah and Rangan (1971), Swamy (1975), and several other researchers in the United States, Japan, United Kingdom, and Russia performed extensive researches on the use of other types of fibers in addition to steel. Other developments using bundled fiberglass as the main reinforcement in concrete beams and slabs were introduced by Navy et al. (1971) and Navy and Neuwerth (1977). Nowadays, a wide range of engineering materials including ceramics, plastics, cements, and gypsum, is being used to enhance composite properties of concrete.

1.2 Motivation

Concrete is strong in compression but weak in tension. Micro cracks begin to form in a plain concrete matrix at about 10 to 15% of the ultimate strength, developing into macro cracks at about 25 to 30% of the ultimate strength. In structural applications, this deficiency can be minimized by providing tension reinforcement in form of steel bars to carry the tensile forces once the concrete has cracked. The reinforcement can also be in form of prestressing so that the concrete remains entirely in compression under load. The cracks formed, however, still cannot be arrested or slowed by the use of conventional reinforcement alone. The function of such reinforcements is to ensure that the load capacity of the cracked section exceeds the capacity of the plain concrete (Cement and Concrete Association of New Zealand, 2009). When steel fibers are present, a number of them intercept micro cracks as they form and continue

to provide reliable post-cracking strength and, by virtue of the post-cracking strength and deformation, provides energy absorption as well. The use of steel fibers in structural application has emerged as a result of extensive research in order to understand and quantify the material properties. From the 1970s to the present, the use of steel fibers has been well established as a complementary reinforcement to control cracking, enhance flexural and shear strengths and impact resistance of reinforced concrete elements both in situ cast and precast. Despite the high use of the steel fibers, however, the lack of universally accepted approach to the calculation and strength verification of SFRC and respective standard is one of the major obstacles in the development of the material in structural design (Andrzej, 2008). ACI 544.4R-88 and National Research Council (CNR-204/2006) have provided information on guidelines for the design and construction of steel fiber reinforced concrete (SFRC). Standard flexural test methods are available to determine the post-cracking flexural strength of SFRC in bending and tension and its toughness. Standard flexural test procedures have been proposed by organizations including RILEM TC 162-DF, the Japanese Concrete Institute (JCI-SF4), EFNARC (the European Federation of Producers and Applicators of Special Products for Structures), and ASTM among others. The ACI Committee ACI 544.1R-96 published a state-of-the-art report in 1996.

1.3 Objectives and Scope

The objective of the study is to investigate the flexural behavior of lightly and heavily reinforced steel fiber concrete beams by:

- Evaluating the previous research published by other researchers related to tensile, compressive and flexural behavior of SFRC.
- Conducting an experimental program consisting of load testing of ten pairs of lightly and heavily reinforced beams made of concrete with and without steel fibers.
- Investigating the effects of SFRC on the flexural behavior of lightly and heavily reinforced beams in terms of load carrying capacity, initial stiffness, and post-peak stiffness and toughness characteristics.

- Estimating the flexural load-deflection behavior of tested beams using proposed stress-strain relationships in the literature for tension and compression of SFRC.

1.4 Thesis Organization

The literature review for SFRC is presented in Chapter 2. Topics discussed in the chapter include definition of steel fiber; the types of steel fiber and its influence in concrete in terms of compression, tension, and flexure. Previous research conducted by some researchers is also discussed in this chapter. Other topics addressed in this chapter are some of the existing tension and compression models for SFRC.

Chapter 3 describes the materials and the specimens used in the research. It discusses also all steps through which the tests were conducted and the data obtained. Pictures are also presented for some of the specimens during fabrication, load testing, and after the test.

Results obtained from laboratory tests are presented and discussed in both graphical and tabular forms in Chapter 4. The results are interpreted based on the nature of curves.

Based on the load-deflection data for the beams tested, the analytical work was carried out using the program developed. The proposed models used in the analytical work for both tension and compression part of SFRC stress-strain relationship are in given in Chapter 5. All equations and assumptions used in developing the model utilized for the numerical part of the study are included and interpreted.

A summary of the testing program and analytical work is presented in Chapter 6. Conclusions are also presented in this chapter.

2 BACKGROUND

2.1 General

Unreinforced concrete is a brittle material with low tensile strength and low strain capacity at fracture. Conventional reinforcement in the form of either bars or mesh is used to enhance load carrying capacity at a cracked section at the ultimate limit state and also to control cracking, rotation and deflection at the serviceability limit state. As stated earlier, the ordinary reinforcing elements cannot prevent the formation and development of micro-cracks. Fibers, on the other hand, are discontinuous and randomly dispersed in the concrete matrix. They are able to improve the stiffness and crack-control performance by arresting and retarding the development of micro-cracks. Common applications of SFRC include overlays in bridge decks, highway and airport pavements, thin-shell structures, seismic, impact, and explosion-resisting structures, and shotcrete applications (Cement and Concrete Association of New Zealand - CCANZ, 2009).

2.2 Properties of Steel Fibers

Steel fibers are short, discrete lengths of steel wires having an aspect ratio (ratio of length to diameter) from about 20 to 100. They are manufactured in several different cross sections, and are sufficiently small to be randomly dispersed in an unhardened concrete mixture using the usual concrete mixing procedure. Steel fibers are produced by various processes, in various shapes, and geometries. Most of steel fibers however, are round in cross-section with an equivalent diameter ranging between 0.15 and 2 mm and lengths ranging between 7 and 75 mm. For fibers that are not circular the equivalent diameter is the diameter of a circle having the same area as that of the average cross sectional area of an actual fiber. The fibers used in the early times were round and smooth and obtained by cutting or chopping wires to the required lengths. Nowadays, steel fibers have either rough surfaces, can be crimped, hooked at their ends, or deformed along their length.

2.2.1 Types of Steel Fibers

Modern commercial production of steel fibers include those manufactured from drawn steel wire, from slit sheet steel, or by melt-extraction process which results in the production of fibers with crescent-shaped cross section. Four different classifications of steel fibers are presented in the literature based on the British Standards (BS), American Society for Testing and Materials (ASTM), Japanese Society for Civil Engineers (JSCE) and Cement and Concrete Association of New Zealand (CCANZ) definitions.

The BS EN 14889-1 classification is according to the method of production processes as follows:

- Group I - Cold-drawn wire
- Group II - Cut sheet
- Group III - Melt extract
- Group IV - Shaved cold-drawn wire
- Group V - Milled from blocks

Similarly, ASTM A820-96 categorizes steel fibers according to the product used in their manufacturing process. In addition, minimum requirements were set for tensile strength, dimensional tolerance, purity, etc. The four basic categories are as follows:

- Type 1 - Cold-drawn wire
- Type 2 - Slit sheets
- Type 3 - Melted extract
- Type 4 - Others

The JSCE defines steel fibers into three different types according to the shape of their cross sections.

- Type 1 - Square section
- Type 2 - Circular section and
- Type 3 - Crescent section

The CCANZ defines steel fibers based on their geometry as follows (Figure 2-1):

- Smooth surface
- Roughened surface
- Round with end paddles
- Round with end buttons
- Round with hooked ends
- Crimped
- Polygonal twisted

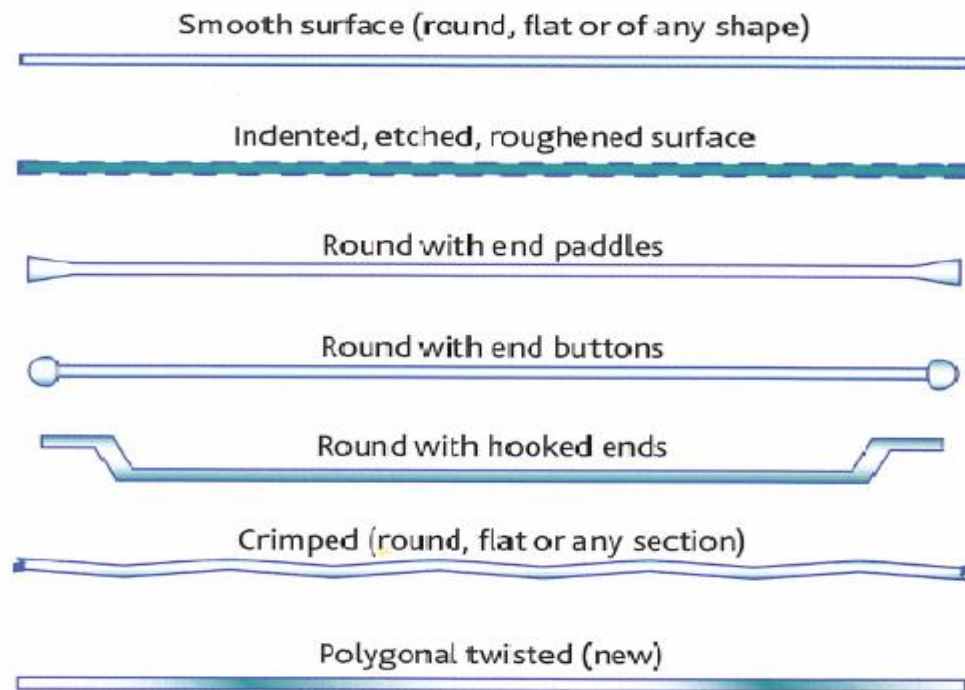


Figure 2-1 – Types of Steel Fibers (CCANZ, 2009)

2.2.2 Physical Properties of Steel Fibers

The fiber parameters related to geometry are an important tool in evaluating the efficiency of fiber in a given composite material. Individual fibers are produced in an almost limitless variety of geometric forms including prismatic, irregular cross-section, and collated geometry. The number of fibers within a unit volume of concrete, and the cross-sectional area of fibers across a given plane of SFRC volume are particularly the most relevant (Zollo, 1996).

2.2.3 Fiber Count and Fiber Specific Surface

The fiber count and fiber specific surface are the number of fiber in a unit volume of SFRC and the surface area of fiber in a unit volume of SFRC, respectively. The fiber count is directly related to the statistical probability of the fibers in arresting cracks. The fiber specific surface is directly related to the amount of energy that is absorbed in encounters between fibers and the forming cracks.

2.2.4 Fiber Length

According to the National Research Council (CNR-204/2006), the length of steel fibers l_f usually ranges from 6 to 70 mm. In the ACI 544.4R, steel fibers have lengths ranging between 25 and 60 mm. The definitions for length and the diameter of the fiber are shown in Figure 2-2.

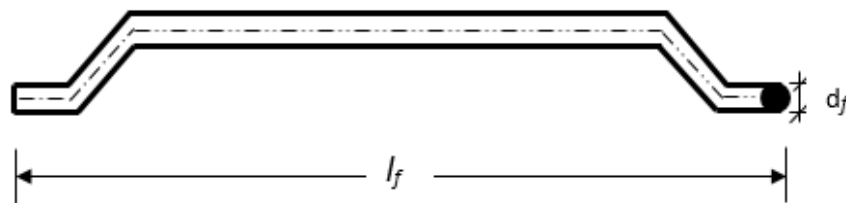


Figure 2-2 – Typical length and diameter for hooked end steel fiber

2.2.5 Fiber Equivalent Diameter

For fibers that are not circular in cross-section, the equivalent diameter is the diameter of a circle having the same area as that of the average cross-sectional area of the fiber.

2.2.6 Fiber Aspect Ratio

The fiber aspect ratio is the major characteristics of the slenderness of an individual fiber. It is defined as fiber length divided by the equivalent fiber diameter for a particular type of fiber. The aspect ratio is also a measure of fiber stiffness and bond characteristics.

2.2.7 Fiber Reinforcement Index (RI)

Fiber reinforcement index is defined as the weight or volume fraction of fiber multiplied by the fiber aspect ratio ($RI = W_f \times l_f / d_f$).

2.2.8 Requirements of Steel Fibers

For efficient and effective performance of steel fibers in hardened concrete, the following points are essential in achieving the expected results.

Fibers should be significantly stiffer than the matrix. This means that the fibers should have modulus of elasticity higher than that of the matrix itself (The Concrete Institute, 2007).

Additionally, following requirements should be met for effective performance:

- Fiber content in the matrix should be adequate.
- Fiber-matrix bond must be ensured.
- Length and diameter of fiber must be reasonably sufficient.
- Fibers must have high aspect ratio, i.e. they must be long relative to their diameter.
- The shape and material from which the fiber is made, i.e. whether hooked end, flat end, deformed or flat shape, and whether metallic fibers, glass fibers, polymeric or carbon fibers are used.

2.3 Properties of SFRC

SFRC is a concrete, the main components being Portland Cement, coarse and fine aggregates of different sizes, and discontinuous discrete steel fibers. Other components include chemical (superplasticizers, retarders, etc.) and mineral (silica fume, fly ash, etc.) admixtures. In tension, SFRC fails only when the steel fiber breaks or is pulled out of the concrete matrix. Previously, steel fibers were used as substitute for secondary reinforcement or for crack control in less critical concrete elements. Today, SFRC is being used at an increasing rate in various applications including mine and tunnel linings, slabs and floors, rock slope stabilization, repair mortars, shell domes, refractory linings, dam constructions, composite metal decks,

aqueduct rehabilitations, seismic retrofitting, repairs and rehabilitations of marine structures, fire protection coatings, concrete pipes, conventional reinforced frames, etc.

2.3.1 Effects of Steel Fibers in Concrete

In structural applications, concrete is a brittle material with low tensile strain capacity and with some softening post-peak behavior in compression due essentially to the transverse strain capacity. By adding steel fiber, the post-peak behavior can be significantly improved. The use of an adequate amount and appropriate shape of steel fibers increases the tensile strength and the ductility behavior of the concrete matrix. At normal dosages, however, it is observed that fibers do not affect the flexural strength of the concrete (The Concrete Society, 2007). When fibers are added to concrete, they tend to act in a variety of ways. They can reduce the formation and development of cracks due to early-age plastic settlement and drying shrinkage. They may also provide a degree of post-cracking load-carrying capacity (The Concrete Society, 2007).

The behavior of steel fibers in a concrete beam is shown in Figure 2-3 (Zollo, 1997). Each point can be defined as follows:

- Point 1 - Failure /breaking of fibers
- Point 2 - Pulling of fibers out of the matrix
- Point 3 - Bridging action by tension through the fibers
- Point 4 - Fiber/matrix debonding
- Point 5 - Point at which the matrix cracks

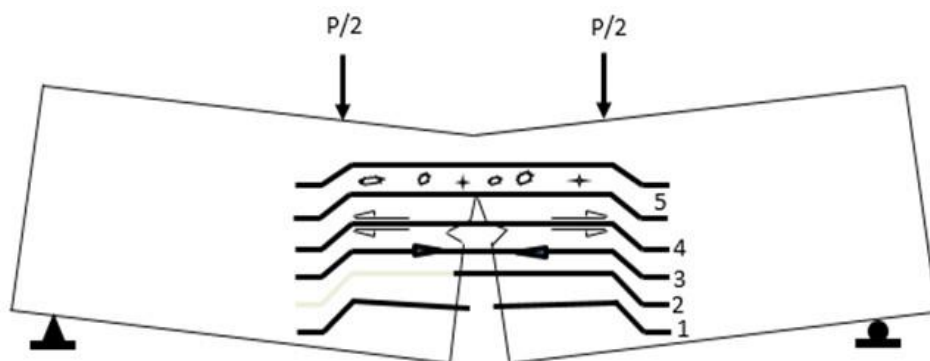


Figure 2-3 – Mechanical action of steel fiber in concrete (Zollo, 1997)

The mechanical properties of SFRC are influenced by the type of steel fibers, aspect ratio and the amount of fibers used, the strength of the matrix; the size, shape, and method of preparation of the specimen; and the size of the aggregate used. Fibers influence the mechanical properties of concrete and mortar in all failure modes, especially those that induce fatigue and tensile stresses such as direct tension, bending, impact, and shear (Gopalaratnam and Shah, 1987). The resistance of fibers to pull-out produces the toughness characteristics of SFRC during fracture. Comparisons between the engineering properties of SFRC and that of conventional concrete shows the superiority of material especially with regard to the flexural strength at higher fiber content, fatigue, toughness, energy absorption, and shock resistance as shown in Table 2-1 (Chi-Tso et al., 1999).

Table 2-1 – Comparison between the engineering properties of SFRC and that of conventional concrete (Chi-Tso et al., 1999).

Item	Engineering Properties	Conventional Concrete	SFRC(times better)
1	Compressive strength	1	1-1.3
2	Tensile strength	1	1.5-1.8
3	Initial cracking	1	1.5-2.0
4	Flexural strength	1	2-3
5	Shear strength	1	4-6
6	Fatigue resistance	1	1.4-2.0
7	Torsion resistance	1	1.5-2.0
8	Abrasive resistance	1	1.27
9	Skipping and rolling resistance	1	1.15
10	Ductility	1	70-100
11	Fatigue	1	100
12	Impact resistance	1	20
13	Shock resistance	1	20
14	Creep	1	0.8
15	Freezing and thawing resistance	1	Better than that of conventional concrete

2.3.2 SFRC in Compression

Steel fibers generally reduce the brittle nature of concrete matrix, but they don't have significant effect on the compressive behavior (Concrete Society, 2007; Baran et al., 2012). The influence of steel fibers in the compressive strength of concrete seems to be minor. However, the ductility and toughness are considerably improved as a

function of the increase in the volume fraction, aspect ratio, and type of the fiber used especially in the post-cracking region (Edward, 2008), as shown in Figure 2-4, Figure 2-5, and Figure 2-6 respectively. Toughness is a measure of the ability of the material to absorb energy during deformation. It can be calculated using the area under the load-deflection or stress-strain curves for the fibrous material.

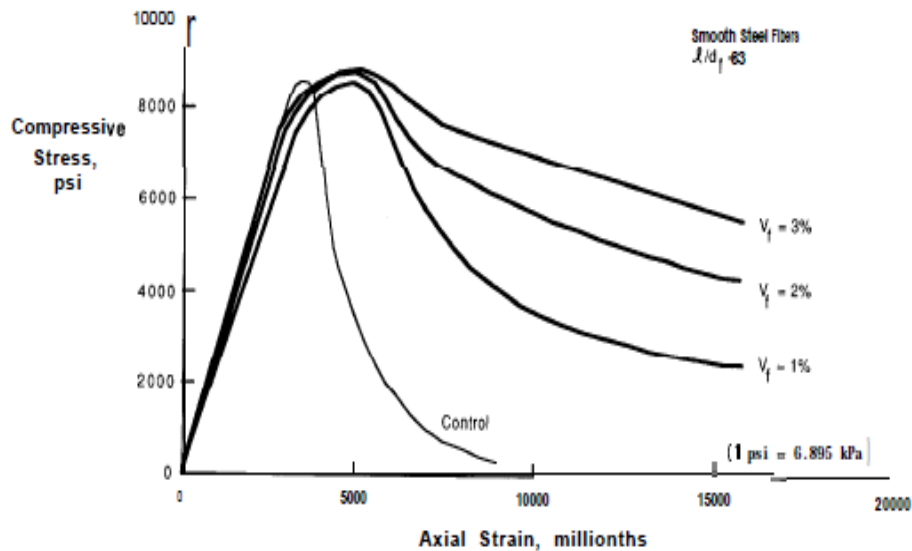


Figure 2-4 – Influence of fiber volume fraction on stress strain curve for concrete in compression (ACI 544-88)

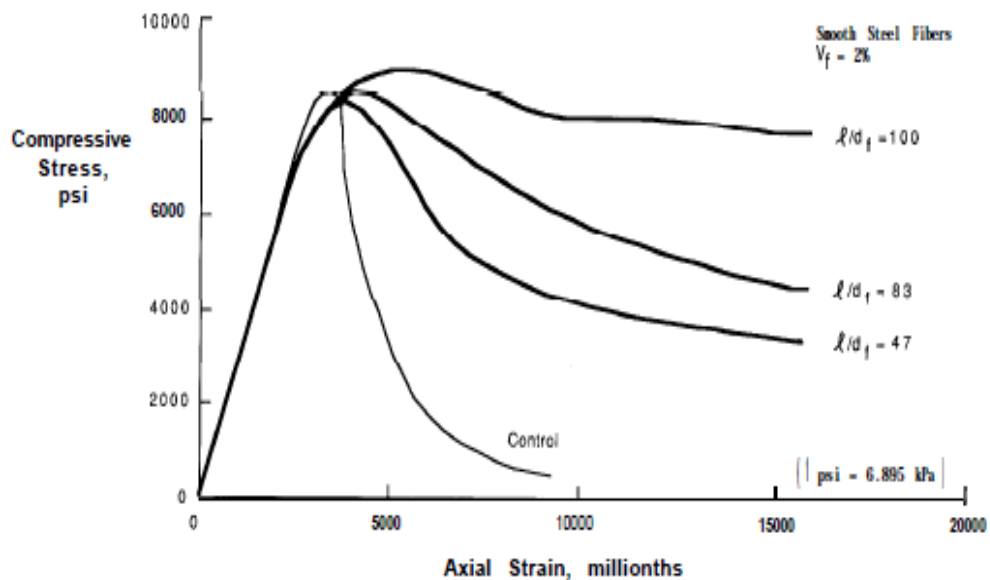


Figure 2-5 – Influence of fiber aspect ratio on stress strain curve for concrete in compression (ACI 544-88)

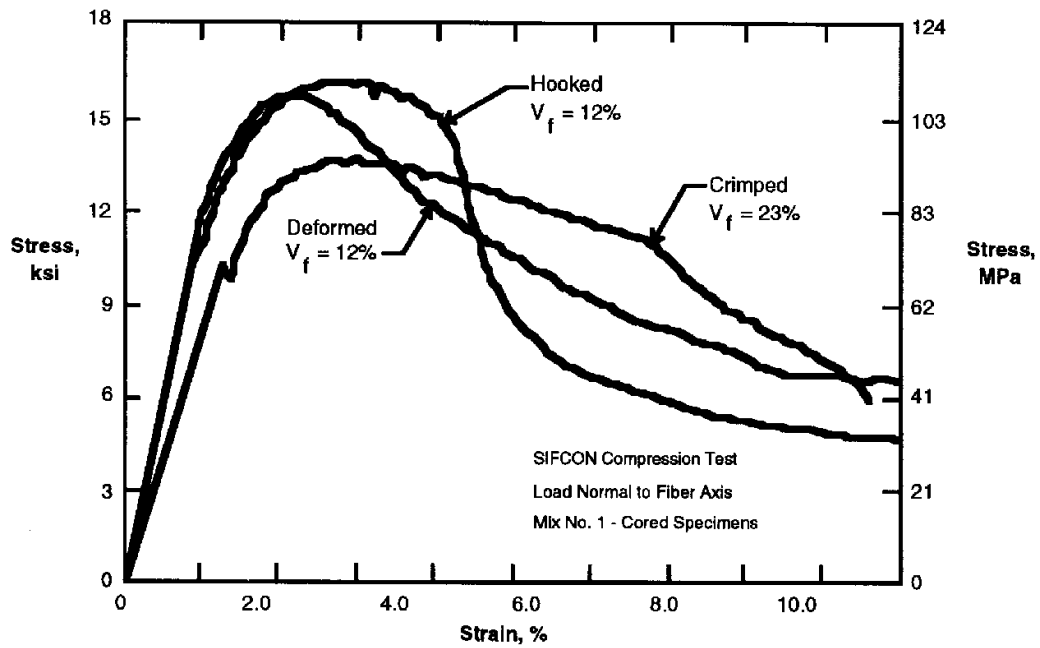


Figure 2-6 – Influence of fiber type on stress strain curve for concrete in compression (ACI 544-96)

Various analytical expressions have been proposed to model the compressive stress-strain behavior of plain concrete (Carreira and Chu 1985, Desayi et al. 1964, Popovics 1973, Hognestad 1951, Sargin 1971, CEB-FIP 1993, etc.). The proposed models, however, are in general not suitable for SFRC, because steel fiber reinforced composites have a less steep descending branch than does plain concrete, as reported in literatures. The compressive behavior of concrete is influenced by the proportions and properties of its constituent materials. The non-linear equation proposed by Popovics (1973) is as follows:

$$f_c = f_{ck} \frac{\varepsilon_c}{\varepsilon_{co}} \frac{n}{n-1+\left(\frac{\varepsilon_c}{\varepsilon_{co}}\right)^{nk}} \quad \text{Equation 2-1}$$

In which, f_c is the stress at any point on the curve, f_{ck} is the concrete compressive strength obtained from the cylinder test and ε_c is the strain corresponding to the stress f_c , and ε_{co} is the strain at peak stress. The ε_{co} , the constants n and k , and modulus of elasticity of concrete (E_c) is defined as follows:

$$\varepsilon_{co} = \frac{f_{ck}}{E_c} \frac{n}{n-1} \quad \text{Equation 2-2}$$

$$n = 0.8 + \frac{f_{ck}}{17} \quad \text{Equation 2-3}$$

$$k = 0.67 + \frac{f_{ck}}{62} \quad \text{Equation 2-4}$$

$$E_c = 3320\sqrt{f_{ck}} + 6900(\text{MPa}) \quad \text{Equation 2-5}$$

The above expressions however, cannot represent the compressive stress-strain behavior of SFRC owing to the fact that the effect of fibers has not been taken into consideration. Little research has been done on the effect of fiber on the descending portion of compressive stress- strain curve. One of the reasons is the difficulty encountered in obtaining a complete stress-strain curve (Shah, 1978). Only a few constitutive models are published for SFRC (Fanella and Naaman, 1985; Barros and Fiqueras, 1999; Ezeldin and Balaguru, 1992). Since the models of the compressive behavior of fiber reinforced concrete, in general, were developed from models for plain concrete, it is necessary to include some parameters in these models to consider the influence of steel fibers on the properties of the stress strain curve. Some guidelines such as RILEM TC 162-TDF (2003), CNR-DT 204 (2006), proposed the same shape of the stress-strain relationship in compression for ordinary concrete to be used in ultimate limit state design. The compression response is prescribed by a parabolic-rectangular stress-strain model with ultimate compressive strain of 0.0035 and a peak strain of 0.002. Fanella and Naaman (1985), proposed an analytical model to predict the complete stress-strain behavior of fiber-reinforced mortar taking into consideration the fiber shape, volume fraction and fiber geometry. Several authors proposed different analytical equations for modeling the actual stress-strain behavior of SFRC in compression by introducing some parameters relating to fiber properties.

2.3.2.1 Soroushian and Lee (1989)

The model proposed by Soroushian and Lee (1989) consists of a curvilinear ascending branch followed by a bi-linear descending branch. The expression for this model is as shown below:

$$f_c = f_{cf} \left(\frac{\varepsilon}{\varepsilon_{pf}} \right)^2 + 2f_{pf} \left(\frac{\varepsilon}{\varepsilon_{pf}} \right) \quad \text{for } \varepsilon \leq \varepsilon_{pf} \quad \text{Equation 2-6}$$

$$\sigma = z \left(\varepsilon - \varepsilon_{pf} \right) + f_{cf} > f_o \quad \text{for } \varepsilon > \varepsilon_{pf} \quad \text{Equation 2-7}$$

Where, $\varepsilon_{co} = 0.0021$,

$$f_{pf} = f_c + 3.6I_f \quad \text{Equation 2-8}$$

$$f_o = 0.12f_{cf} + 3.6I_f \quad \text{Equation 2-9}$$

$$z = -343f_c(1 - 0.66\sqrt{I_f}) \leq 0 \quad \text{Equation 2-10}$$

$$\varepsilon_{pf} = 0.0007I_f + 0.0021 \quad \text{Equation 2-11}$$

where, f_{pf} is the compressive strength of fiber concrete, f_c is the compressive strength for plain concrete, ε_{pf} is the strain at peak stress f_{pf} , z is the slope of the descending branch, I_f is the fiber reinforcement index, and f_o is the residual stress.

2.3.2.2 Ezeldin and Balaguru (1992)

Ezeldin and Balaguru (1992) developed an analytical equation to model the compressive stress-strain behavior of normal strength SFRC. The expression, which is based on the one proposed by Carreira and Chu (1985) for uniaxial compression of plain concrete, involves a material parameter β , and is the slope of the inflection point at the descending branch of the curve.

$$\frac{\sigma_c}{f_{cf}} = \beta \frac{\left(\frac{\varepsilon}{\varepsilon_{pf}} \right)}{\beta - 1 + \left(\frac{\varepsilon}{\varepsilon_{pf}} \right)^\beta} \quad \text{Equation 2-12}$$

In which, σ_c is the compressive stress, f_{cf} is the compressive strength for fibrous concrete, ε_c is the strain, ε_{pf} is the strain at peak stress and β is the factor related to the influence of fiber on the descending branch of the curve.

$$f_{cf} = f_c + 3.51(RI) \quad \text{Equation 2-13}$$

$$RI = W_f \times l_f / d_f \quad \text{Equation 2-14}$$

$$\varepsilon_{pf} = \varepsilon_{co} + 446 \times 10^{-6}(RI) \quad \text{Equation 2-15}$$

$$\varepsilon_{co} = 0.002 \quad \text{Equation 2-16}$$

For hooked end fibers:

$$\beta = 1.093 + 0.7132(RI)^{-0.926} \quad \text{Equation 2-17}$$

Based on the experimental results of Fanella and Naaman (1985) for straight steel fiber-reinforced mortar, the following equation was proposed:

$$\beta = 1.093 + 7.4818(RI)^{-1.387} \quad \text{Equation 2-18}$$

Where, RI is the fiber reinforcement index by weight of straight fibers.

According to the experiments performed by Luiz et al. (2010):

$$\beta = (0.0536 - 0.5754V_f)f_c \quad \text{Equation 2-19}$$

$$\varepsilon_{pf} = (0.00048 - 0.01886V_f)\ln f_c \quad \text{Equation 2-20}$$

Values of β apply to reinforcing index values ranging from 0.75 to 2.5 for hooked-end steel fibers, and from 2 to 5 for straight fibers. A particular case of the equation proposed by Carreira and Chu (1985) was proposed by Desayi and Krishnan (1964), in which, $\beta = 2$. Another particular case is the one proposed by Tulin and Gerstle (1964) for $\beta = 3$ as reported by Nataraja et al. (1999).

2.3.2.3 Nataraja et al. (1999)

Based on their extensive experimental works on the effect of steel fibers on the shape of compressive stress-strain curve of concrete, Nataraja et al. (1999), formulated an analytical model to generate both the ascending and descending branches of the

stress-strain curve. They adopted the equation of Carreira and Chu (1985) for uniaxial compression of plain concrete, the same expression used by Ezeldin and Balaguru (1992).

$$\frac{f_c}{f_{cf}} = \beta \frac{\left(\frac{\varepsilon_c}{\varepsilon_{of}}\right)}{\beta - 1 + \left(\frac{\varepsilon_c}{\varepsilon_{of}}\right)^\beta} \quad \text{Equation 2-21}$$

Where, f_{cf} is the compressive strength of fibrous concrete and ε_{of} the peak strain corresponding to the peak stress, f_c and ε_c are the stress and strain values at any point on the curve.

$$f_{cf} = f_c + 2.1604(RI) \quad \text{Equation 2-22}$$

$$\varepsilon_{of} = \varepsilon_c + 0.0006(RI) \quad \text{Equation 2-23}$$

$$\beta = 0.5811 + 1.93(RI)^{-0.7406} \quad \text{Equation 2-24}$$

$$\varepsilon_c = 0.0022 \quad \text{Equation 2-25}$$

$$RI = W_f \times l_f / d_f \quad \text{Equation 2-26}$$

2.3.2.4 Barros and Figueiras (1999)

The expression proposed by Barros and Figueiras (1999) is similar to that of Ezeldin and Balaguru (1992), which is based on one parameter only. The same equation was proposed by Vipulanandan and Paul (1990) mainly intended to predict polymer concrete behavior, and used by Mebarkia and Vipulanandan (1992) for glass-fiber reinforced polymer.

$$\sigma_c = f_{cm} \frac{\left(\frac{\varepsilon_c}{\varepsilon_{c1}}\right)}{(1-p-q) + q\left(\frac{\varepsilon_c}{\varepsilon_{c1}}\right) + p\left(\frac{\varepsilon_c}{\varepsilon_{c1}}\right)^{\frac{1-q}{p}}} \quad \text{Equation 2-27}$$

$$q = 1 - p - \frac{E_{pf}}{E_c}, \quad \text{Equation 2-28}$$

$$p + q = \epsilon] 0, 1[\quad \text{Equation 2-29}$$

$$\frac{1-q}{p} > 0 \quad \text{Equation 2-30}$$

$$E_{pf} = \frac{f_{cf}}{\varepsilon_{pf}} \quad \text{Equation 2-31}$$

$$E_c = 21500 \sqrt[3]{\frac{f_{cf}}{10}} \quad \text{Equation 2-32}$$

Where, f_{cm} is the average compressive strength of concrete determined from test on cylinder.

For hooked end fiber with $l_f = 30$ mm, $d_f = 0.5$ mm, $l_f/d_f = 60$:

$$\varepsilon_{pf} = \varepsilon_{co} + 0.0002W_f \quad \text{Equation 2-33}$$

$$p = 1 - 0.919^{-0.394W_f} \quad \text{Equation 2-34}$$

For hooked end fiber with $l_f = 60$ mm and $d_f = 0.8$ mm; $l_f/d_f = 75$:

$$\varepsilon_{pf} = \varepsilon_{co} + 0.00026 W_f \quad \text{Equation 2-35}$$

$$p = 1 - 0.722^{-0.144W_f} \quad \text{Equation 2-36}$$

Where, ε_{co} is the strain at peak for plain concrete and W_f is the fiber weight fraction in the matrix.

According to CEB-FIP Model Code, 1993:

$$\varepsilon_{co} = 0.002 \quad \text{Equation 2-37}$$

The values of ε_{pf} and p above are applicable for f_{cf} values ranging from 30 to 60 MPa and for fiber content ranging from 1 to 3%.

2.3.2.5 Wang (2006)

Wang (2006) proposed an expression to model the stress-strain behavior of SFRC in compression by modifying Popovic's (1973) expression (Equation 2-1) for plain concrete. The expression was modified by introducing a parameter relating to the volume percentage of steel fiber.

$$f_c = f_{ck} \frac{\varepsilon_c}{\varepsilon_{co}} \frac{n}{n-1+\left(\frac{\varepsilon_c}{\varepsilon_{co}}\right)^{nk}} \quad \text{Equation 2-38}$$

$$\alpha = 0.92 - 25000V_f^3 \quad \text{Equation 2-39}$$

Where, V_f is the volume percentage of steel fiber used.

To obtain new values of n and k , their corresponding values in the above expression are multiplied by the parameter of α .

$$n = \alpha \left(0.8 + \frac{f_{ck}}{17} \right) \quad \text{Equation 2-40}$$

$$k = \alpha \left(0.67 + \frac{\varepsilon_{ck}}{62} \right) \quad \text{Equation 2-41}$$

Where, f_{ck} and ε_{ck} are peak stress and peak strain, respectively.

2.3.3 SFRC in Tension

The tensile strength of normal concrete is 8 to 14% of the compressive strength as result of cracking at low stress levels. Steel fibers when added to concrete, are able to improve the tensile strength by preventing the micro cracks from propagating and widening (Edward, 2008). Ductility is also increased due to the fibers' energy absorption capacity. The main use of SFRC is in the construction of statically redundant structures, where the residual tensile strength can improve the load bearing capacity of the structures as well as its ductility. The influence of steel fibers in improving the tensile behavior of a cracked matrix is shown in Figure 2-7 to Figure 2-9. For low volume fraction of fibers the phenomena in the post cracking region is characterized by softening behavior and this implies that the post crack flexural strength provided by the steel fiber is typically less than the capacity of the non-cracked matrix. For higher volume ratio however (higher than 2%), the post-cracking behavior is termed strain-hardening and this means the strength can be higher than that of the non-cracked matrix due to the formation of multiple cracks (Cement and Concrete Association of Switzerland, 2009). In direct tension, the improvement of

strength is significant, with increases on the order of 30 to 40% reported for the addition of fiber in mortar or concrete.

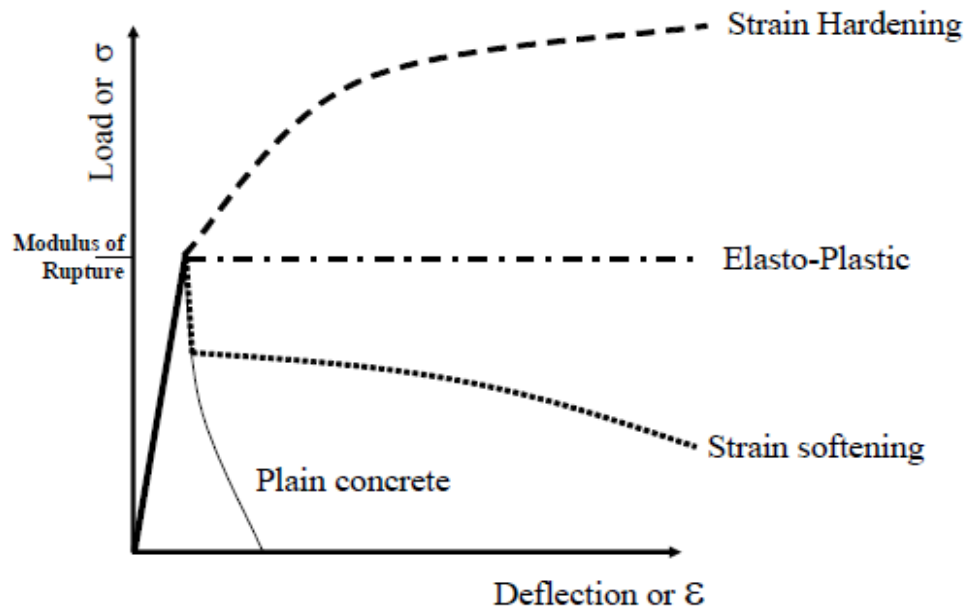


Figure 2-7 – Typical load-deflection (stress-strain) curve for SFRC (CCANZ, 2009)

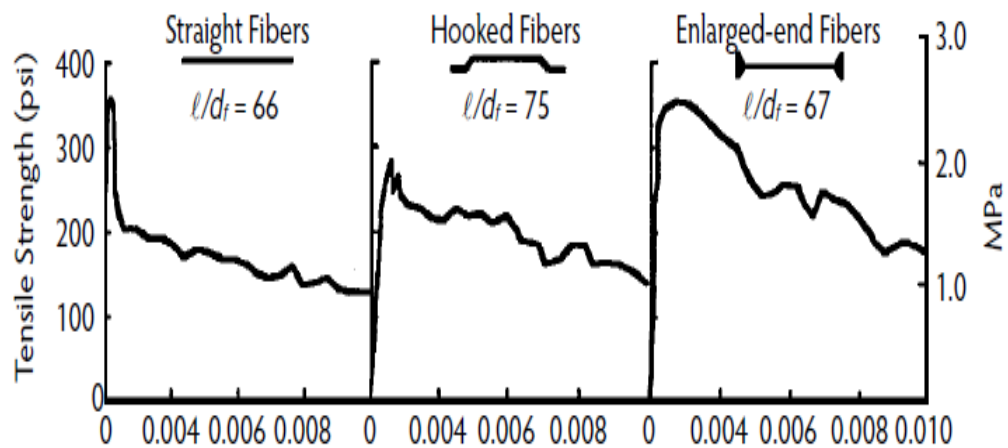


Figure 2-8 – Stress-strain relationships for steel fiber-reinforced mortar in tension (ACI 544-88)

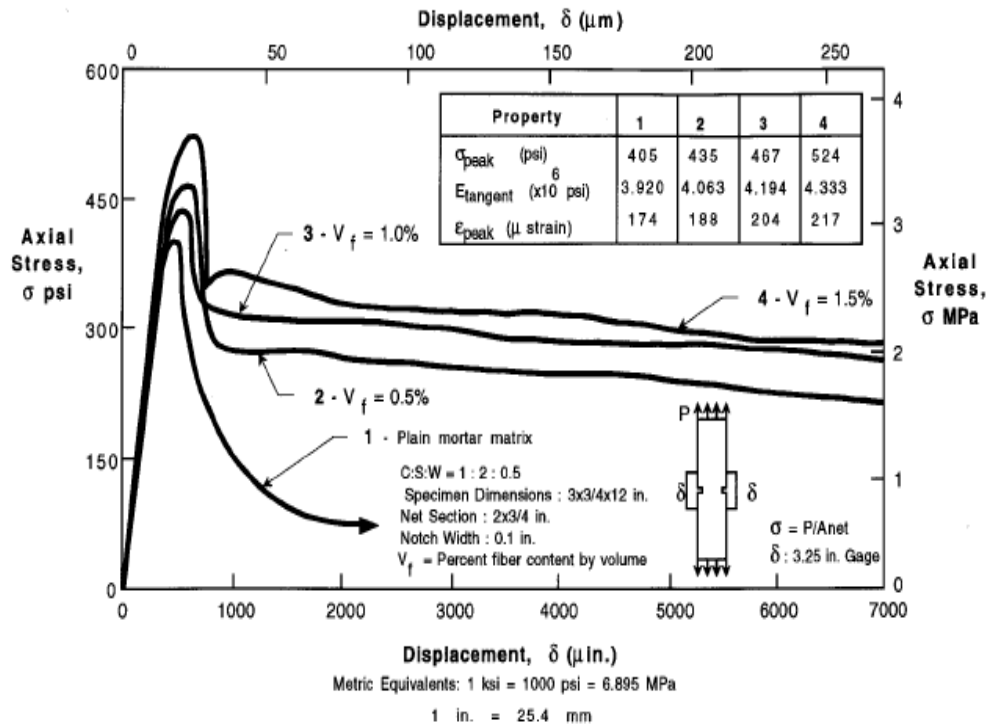


Figure 2-9 – Stress-displacement graphs under direct tension for plain and fiber-reinforced mortar (ACI 544-96)

Similar to models for predicting the behavior of SFRC in compression, there are also models for tension behavior. A few empirical expressions for direct tensile stress-strain curves for plain concrete have been proposed (Scanlon, 1971; Lin and Scordelis, 1975; Gopalaratnam and Shah, 1985; Dakhal, 2006). The model proposed by Scanlon (1971) involves a softening behavior in terms of a prescribed drop in the stress carrying-capacity followed by reduced secant stiffness. The model by Lin and Scordelis (1975) assumes a polynomial relationship between stress and strain. Recently, several models and expressions have been proposed to predict the tensile behavior of SFRC members subjected to uniaxial tension. The models of Abrishami and Mitchell (1997), Bentur and Mindes (1990), Frank et al. (2013), Henager and Doherty (1976), Lok and Xiao (1998), Sorankom and Mobasher (2009), and Chuang (2006) are some of the models discussed in the literature.

2.3.3.1 Henager and Doherty (1976)

Based on the concept of basic design assumptions, Henager and Doherty (1976) proposed a tensile stress block model for predicting the flexural strength of a singly

reinforced beams containing steel fiber (Figure 2-10). The model which is similar to the ACI 318-05 ultimate strength design method, was developed by computing the tensile strength of the fibrous concrete separately and then adding the corresponding strength contributed by the reinforcing bar to obtain the ultimate moment of the member.

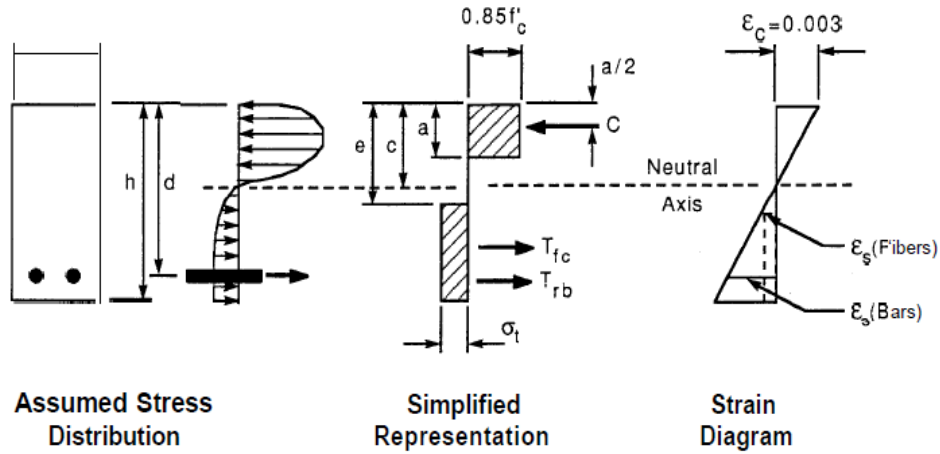


Figure 2-10 – Design assumptions for analysis of a singly-reinforced concrete beams containing steel fibers, (Henager and Doherty, 1976)

$$M_n = A_s f_y \left(d - \frac{a}{2} \right) + \sigma_t b (h - e) \left(\frac{h}{2} + \frac{e}{2} - \frac{a}{2} \right) \quad \text{Equation 2-42}$$

The parameter σ_t represents the tensile strength contributed by steel fibers and is given as:

$$\sigma_t = 0.00772 \frac{l}{d_f} \rho_f F_{be} \text{ (MPa)} \quad \text{Equation 2-43}$$

Where, M_n is the ultimate moment capacity of the beam, l is the length of fiber, d_f is the fiber diameter, ρ_f is the percent by volume of fiber, and F_{be} is the bond efficiency of the fiber and it varies from 1.0 to 1.2 depending upon fiber characteristics.

2.3.3.2 Bentur and Mindes (1990)

Bentur and Mindes (1990) proposed the empirical expression given in Eq. 2-44 for predicting the flexural strength of steel fiber-reinforced concrete based on a composite-material.

$$\sigma_c = A\sigma_m(1 - V_f) + BV_f \frac{l}{d} \quad \text{Equation 2-44}$$

where σ_c is the composite flexural strength, σ_m is the ultimate strength of the matrix, V_f is the volume fraction of fibers adjusted for randomness, l/d is the fiber aspect ratio, l is the fiber length, d is the diameter of fiber, A and B are the constants.

Analytical equations developed by Swamy et al (1974) have a form based on theoretical derivation with the above constants A and B obtained from regression analysis of the tested laboratory data. The test was performed on a 100×100×305 mm beam specimen. The obtained values of the constants A and B lead to the following expressions.

For first cracking composite flexural strength:

$$\sigma_c = 0.843f_r V_m + 425V_f l/d_f \quad (2.94 \text{ MPa}) \quad \text{Equation 2-45}$$

Where, f_r is the stress in the matrix (modulus of the plain mortar or concrete), V_m is the volume fraction of matrix ($= 1 - V_f$), V_f is the volume fraction of fibers ($= 1 - V_m$), $\frac{l}{d}$ is the ratio of length to diameter of fiber (i.e., aspect ratio).

For ultimate composite flexural strength:

$$\sigma_{cu} = 0.97f_r V_m + 494V_f l/d_f \quad (3.41 \text{ MPa}) \quad \text{Equation 2-46}$$

2.3.3.3 Lok and Xiao (1998)

Based on their parametric study, Lok and Xiao (1998) developed a constitutive idealized model for predicting the tensile behavior of steel fiber-reinforced concrete element (Figure 2-11). Expressions representing the proposed model focused entirely on a parameter τ_d , the fiber bond stress, which according to the developed model, contributed to the tensile behavior of SFRC.

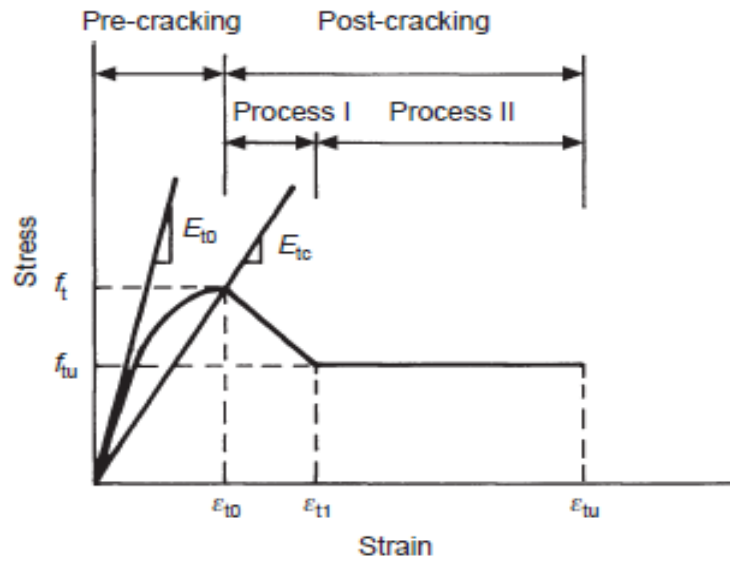


Figure 2-11 – Idealized stress-strain model (Lok and Xiao, 1998)

In the above figure, the tensile behavior before cracking is expressed by the following equation.

$$\sigma = f_t \left[2 \times \left(\frac{\varepsilon}{\varepsilon_{to}} \right) - \left(\frac{\varepsilon}{\varepsilon_{to}} \right)^2 \right] \quad \text{Equation 2-47}$$

Where, f_t is the ultimate tensile stress to determine from direct tensile test or from Equation 2-49.

$$f_t = f_m(1 - V_f) + \frac{\eta_o \eta_1 \tau_d V_f L}{d} \quad \text{Equation 2-48}$$

Where, η_o and η_1 are the fiber orientation and fiber length factors respectively, τ_d is the fiber bond stress, V_f is the fiber volume percentage, and $\frac{L}{d}$ is the fiber aspect ratio.

$$\varepsilon_{to} = \frac{2f_t}{E_{to}} \quad \text{Equation 2-49}$$

$$E_{to} = 2E_{tc} \quad \text{Equation 2-50}$$

Where, E_{to} is the tensile secant modulus, E_o is the initial tangent modulus of concrete in tension, and ε_{to} is the strain at stress f_t .

$$f_{tu} = \eta V_f \tau_d \frac{L}{d} \quad \text{Equation 2-51}$$

$$\varepsilon_{t1} = \tau_d \frac{L}{d} \frac{1}{E_s} \quad \text{Equation 2-52}$$

Where, η is the fiber orientation factor (= 0.405 for beams and 0.50 for slabs), E_s is the elastic modulus of steel fibers.

$$\tau_d = \frac{f_{tu}}{0.5V_f \left(\frac{L}{d}\right)} \quad \text{Equation 2-53}$$

2.3.3.4 Chuang (2006)

Based on experimental and analytical research, Chuang (2006) developed an expression to predict the tensile behavior of an SFRC member. The ascending branch of the model is based on a relationship between the tensile and compressive strength of the concrete, while the descending part is based on a stress versus crack width relationship.

$$\sigma = E_t \varepsilon \text{ for } \sigma \leq f_t \quad \text{Equation 2-54}$$

$$E_t = (4000 + 100000\alpha_f V_f) \sqrt{f_{c'}} \quad \text{Equation 2-55}$$

Where, σ is the tensile stress for a SFRC, ε is the tensile strain, E_t is the direct tensile elastic modulus (MPa), $f_{c'}$ is the compressive strength of plain concrete (MPa).

The descending branch was represented by the expression below:

$$\sigma = \frac{1+w}{1+w+\frac{w^2}{1000\alpha_f v_f}} \times f_t \quad \text{Equation 2-56}$$

Where, σ is the tensile stress, f_t is the tensile strength, w is the crack width (mm), V_f is the fiber volumetric ratio, α_f is an experimental constant which considers the fiber type and is 1 for Norvotex fiber and 1.6 for 1% Dramix fiber type.

2.3.3.5 Soranakom and Mobasher (2009)

A model has been developed by Soranakom and Mobasher (2009) for predicting the strain-softening behavior of fiber-reinforced concrete elements. The equations representing the developed model are based on tensile and compressive constitutive response. The model consists of a linear stress-strain behavior up to the point where cracking tensile strain occurred after which the post-cracking behavior is characterized by a decaying stress-strain relationship. The contribution of fibers in the post-cracking response is represented by an average constant post-crack tensile parameter, σ_p which is related to the fiber volume fraction and bond behavior of the matrix.

$$\sigma_{cr} = E \varepsilon_{cr} = 0.56 \sqrt{f_{c'}} \quad (\text{MPa}) \quad \text{Equation 2-57}$$

$$E = 4733 \sqrt{f_{c'}} \quad (\text{MPa}) \quad \text{Equation 2-58}$$

Where, f_c is the ultimate uniaxial cylinder compression strength. The first crack tensile strain of the SFRC model was obtained assuming Hooke's Law as:

$$\varepsilon_{cr} = \frac{\sigma_{cr}}{E} = \frac{0.56 \sqrt{f_{c'}}}{4733 \sqrt{f_{c'}}} = \frac{6.7 \sqrt{f_{c'}}}{57,000 \sqrt{f_{c'}}} = 118 \text{ microstrain} \quad \text{Equation 2-59}$$

$$\beta_{tu} = \frac{\varepsilon_{tu}}{\varepsilon_{cr}} = \frac{0.025}{118 \times 10^{-6}} \approx 212 \quad \text{Equation 2-60}$$

$$\sigma_p = \mu E \varepsilon_{cr} : \mu (0 \leq \mu \leq 1) \quad \text{Equation 2-61}$$

According to RILEM model 2003:

$$\varepsilon_{tu} = 0.025 \quad \text{Equation 2-62}$$

The parameter μ represent the post-crack strength as a fraction of the cracking tensile strength, σ_{cr} .

The idealized tension model proposed by Mobasher and Soranakom (2009) is shown in Figure 2-12.

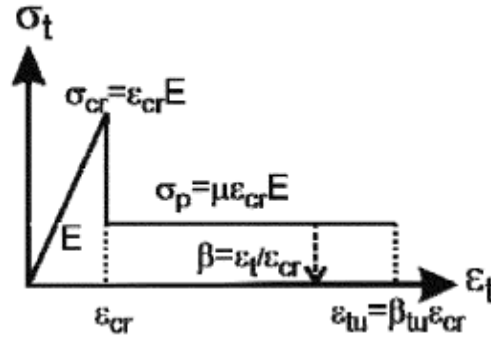


Figure 2-12 – Idealized tension model for strain- softening fiber-reinforced concrete (Mobasher and Soranakom, 2009)

2.3.3.6 Frank et al. (2013)

Frank et al. (2013) conducted an analytical study to develop a tension-stiffening model for SFRC. By utilizing the crack analysis procedure developed by Lee et al. (2011), Frank et al. (2013) modified the conventional tension-stiffening of Bentz (2005) to take into account the effect of steel fibers. The tensile stress due to tension stiffening is shown below:

$$f_{c, Ts} = \frac{f_{cr}}{1 + \sqrt{3.6 c_f M \epsilon_{t, avg}}} \quad \text{Equation 2-63}$$

$$c_f = 0.6 + \frac{1}{0.034} \left[\frac{l_f}{d_f} \right]^{1.5} \frac{100 V_f}{M^{0.8}} \text{ for hooked-end fiber} \quad \text{Equation 2-64}$$

Where, c_f is the coefficient to consider the effect of steel fibers, $\epsilon_{t, avg}$ is the average tensile strain of reinforcing bar of reinforced FRC member, V_f is the fiber volumetric ratio, l_f is the fiber length, f_{cr} is the cracking strength of concrete, M is the bond parameter ($= A_c / (\Sigma d_{bs} \pi)$), A_c is the cross sectional area of concrete matrix, and d_{bs} is the diameter of reinforcing bar.

In the above expressions:

$$E_c = 3300\sqrt{f_c'} + 6900 \text{ (MPa)} \quad \text{Equation 2-65}$$

$$f_{cr} = 0.33\sqrt{f_c'} \text{ (MPa)} \quad \text{Equation 2-66}$$

Where, E_c is the elastic modulus of concrete matrix and f_c' is the cylinder compressive strength of concrete.

2.3.4 SFRC in Flexure

Fibers seem to affect the magnitude of flexural strength in concrete and mortar elements to a much greater extent than they affect the strength of comparable elements subjected to direct tension or compression (ACI Committee 544, 1993). The flexural strength is the maximum flexural tensile stress attained in a beam test, while residual flexural strength is the flexural tensile strength retained by fiber-reinforced concrete after cracking, generally at a specified deflection in a test. Increases in flexural strength of SFRC are substantially greater than in tension and/or compression (ACI 544.1R-96). Oh et al. (1999), reported that the flexural strength of SFRC has been increased to about 55% with the addition of 2% by volume of steel fiber. Steel fibers have been proved to increase the ultimate moment and ultimate deflection of a conventionally reinforced beam: the higher the tensile stress due to fibers the higher the ultimate moment (Craig, 1984). Two types of flexural strength values are associated with SFRC members. The first one, termed the first-crack strength corresponds to the point at which load-deflection curve is no longer linear and it is the same point at which a plain concrete members attains its ultimate strength. The second strength, referred to as the ultimate flexural strength is the maximum strength attained by an SFRC member and is the function of the amount and type of fiber used. Although early studies gave the impression that the flexural strength can be more than doubled with about 4 percent by volume of fibers in a sand-cement mortar, it is now recognized that the presence of coarse aggregate coupled with normal mixing and placing considerations limits the maximum practical fiber volume in concrete to 1.5 to 2.0 percent (ACI 544-96). Typical load versus deflection and moment versus deflection curves of SFRC beams are shown in Figure 2-13 and Figure 2-14.

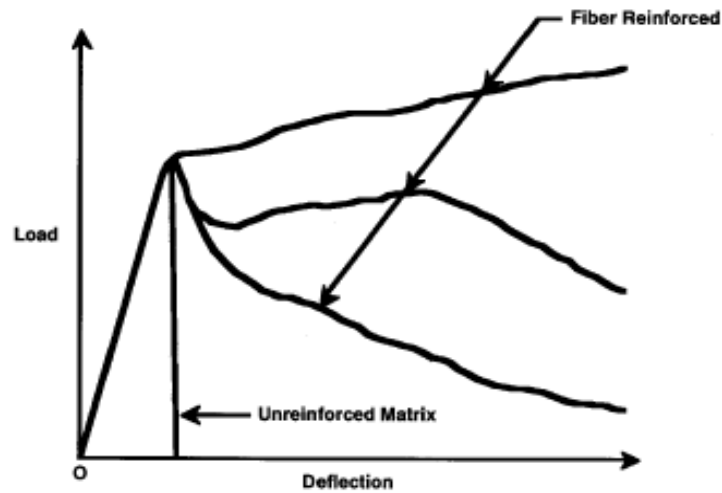


Figure 2-13 – Schematic illustration of load versus deflection curves for plain and fiber reinforced concrete (ACI 544-96)

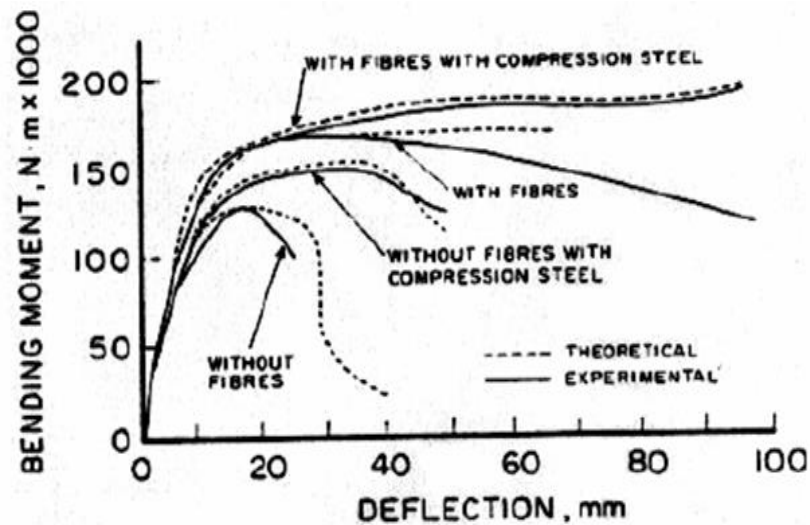


Figure 2-14 – Moment versus deflection curves for SFRC beams (Craig, 1984)

A lot of research has been conducted to evaluate the mechanical influences of steel fiber in concrete or mortar composites in flexure. Authors have used factors such as concrete of varying compressive strengths, and types and volume fractions of fiber in evaluating the influence of steel fibers.

2.3.4.1 Faisal (1990)

Faisal (1990) conducted a research to study the mechanical behavior of SFRC. The variables used were two different steel fiber of hooked-end and straight type, and fiber volume percentages of 1.0, 1.5 and 2.0%. The hooked-end fibers were glued together into bundles using a water soluble adhesive to create an artificial aspect ratio of 15 while, the straight fibers were made by cutting galvanized steel wires manually. Specimens used in the study included sixty concrete cylinders of 150×300 mm dimensions and sixty beam specimens of 100×100×350 mm dimensions. All the cylinders were tested in a compressive testing machine at the end of 7, 28, and 90 days to determine their compressive strength values. While for flexural test, all sixty beams were tested under four-point bending also at the end of 7, 28, and 90 days. Results presented by Faisal indicated that the fiber type and content have little influence on the compressive strength of the specimens. But there was transformation in the mode of failure of the SFRC beams from brittle to a more ductile one with increase in the fiber volume fraction. The results showed also that, the addition of 1.5% hooked-end steel fiber increased the flexural strength by 67%, the splitting tensile strength by 57%, toughness index up to 20 folds, and impact strength by 25 times. In the case of the straight fiber, the addition of 2% increased the flexural strength by 40% as compared to the specimen without steel fibers.

2.3.4.2 Vandewalle (1999)

Vandewalle (1999) conducted an analytical and theoretical study with the aim of studying the cracking behavior of SFRC in combination with conventional reinforcing bars. The variables used were the amount of steel fibers on one hand and the fiber aspect ratio on the other. The specimens used include five full-scale beams of 200×350×3600 mm dimensions and a four series of six SFRC prisms of 150×150×600 mm. All beams had the same amount of longitudinal tension reinforcement ratio of 0.0103. Two types of tests were conducted. In the first type of test, a total of five beams were tested under four-point bending to investigate their cracking behavior. In the second type of test, four series of concrete prisms were tested to determine their mechanical behavior under tension. All specimens were prepared to provide the same concrete compressive strength value of 40 MPa.

Findings of the study showed that the addition of steel fibers enhances the cracking behavior of the conventionally-reinforced SFRC beams. Large decrease in the crack width at higher bending moments due to the use of steel fibers with higher aspect ratio was also reported.

2.3.4.3 Ashour et al. (2000)

Ashour et al. (2000) conducted an experimental work to study the influence of concrete compressive strength and tensile reinforcement ratio on the flexural behavior of SFRC beams. The variables used were three different concrete compressive strengths of 49, 79, and 102 MPa, tensile reinforcement ratios of 1.18, 1.77, and 2.3%, and fiber volume percentages of 0.0, 0.5 and 1.0%. The specimens used included twenty seven beams of 200×3400×250 mm dimensions and cylinders having 150×300 mm dimensions. Deformed bars were used for the tensile reinforcement while hooked-end steel fibers having aspect ratio of 75 were used for fiber reinforcement. All beam specimens were tested for flexure under four-point bending test while cylinders were tested under compression testing machine to determine compressive strength values. Ashour et al. (2000) reported an improvement in the stiffness of the fiber reinforced concrete cylinders with larger increase as the compressive strength increases. It was also reported that the increase in the concrete compressive strength lead to an increase in the flexural rigidity of the tested beams with increase in the amount of fibers.

2.3.4.4 Soulioti et al. (2009)

Soulioti et al. (2009) carried out an experimental study to investigate the effect of steel fibers on mechanical behavior (compression strength, flexural strength and toughness) of SFRC. Two different fibers of waved and hook-end types were used in the study, each at three different volume fractions of 0.5, 1, and 1.5%. The specimens include fifteen concrete cubes of 150×150×150 mm dimensions and fifteen beams of 100×100×400 mm dimensions with each three cubes and three beams for a particular specimen. A total of seven different mixtures was prepared, one of which was plain concrete. All cubes were tested under compression to obtain the compressive strength values. For determining the flexural behavior, all beams were tested under

four-point bending with three beams for each concrete mixture. The findings of Soulioti et al. (2009) showed that the addition of 0.5 and 1.5% steel fiber in the plain concrete increased its compressive strength, with the wave type of fiber exhibiting increase in the compressive strength value higher than with the hooked-end type. The specimen having 1% fiber gave a compressive strength value lower than that of plain specimen. The result showed also that the increase in fiber volume lead to an increase in the first peak strength, peak strength, residual strength as well as flexural strength of the tested beams with hooked-end fiber exhibiting higher flexural toughness behavior.

2.3.4.5 James et al. (2010)

James et al. (2010) performed a comprehensive investigation aimed at studying the behavior of SFRC beams in shear as well as the possibility of using steel fibers as minimum shear reinforcement. The variables used in the study include two different beams, one having a depth of 455 mm and the other 685 mm, hooked-end steel fibers having two different lengths of 30 mm and 60 mm, fiber volume fractions of 0.75, 1.0 and 1.5%, and longitudinal reinforcement ratios of 1.6, 2.0, and 2.7%. The compressive strength used was 41 MPa for all specimens. The specimens were two series of beams designated as series B18 and series B27, with eight pairs of beams for series B18 and four pairs of beams for series B27. The dimensions of series B18 specimens were 152×455×2440 mm and those of series B27 were 205×685×4320 mm. A pair of beams in series B18 specimens was prepared with neither steel fiber nor stirrup reinforcement for comparison with specimens containing steel fiber. In the other series, two beams, one made of plain concrete and the other containing stirrup reinforcement in minimum quantity were also prepared for comparison with beams containing steel fibers. A number of cylinders having dimensions of 100×200 mm were tested under compression to determine compressive strength values for the test specimens. All beams were tested under four-point bending to investigate flexural behavior of SFRC beams and that of beams with stirrup as minimum reinforcement. Results of the study showed that, for both series of the SFRC beams, a considerable increase in the ultimate shear strength of the specimens with an increase in the fiber volume fraction was observed. In the case of the effect of the

longitudinal reinforcement ratio, the SFRC beams having a reinforcement ratio of 2% showed an increase in the ultimate shear strength compared to the specimens without steel fiber and having 2.7% longitudinal reinforcement ratio. The results showed also that with an increase in the fiber aspect ratio the normalized shear strength of the SFRC beams was increased.

2.3.4.6 Olivito et al. (2010)

Olivito et al. (2010) conducted an experimental study with the aim of investigating the mechanical influence of steel fiber in concrete with respect to mix-design variations and different fiber volumes. The variables used in the research included six different mixtures of varying fiber volumes, fiber lengths, and aggregate fractions and cement content. Specimens used include twenty four concrete cubes of 150×150×150 mm dimensions, twenty four prisms having a dimension of 150×150×600 mm with four cubes and four prisms for each mixture type, and another eight prisms of 30×80×350 mm dimensions. All specimens were prepared to have a cube compressive strength higher than 30 MPa. Tests conducted included compressive strength tests on all cube specimens in order to establish compressive strength values, direct tensile test on the 30×80×350 mm prisms to determine the tensile strength of the SFRC specimens, and four-point bending test on the 150×150×600 mm notched prism specimens in order to classify and characterize the specimens. Olivito et al. (2010) reported that the compressive strengths of the specimens tested were slightly affected by the presence of steel fibers. However, the results showed a considerable change in the mode of failure from brittle to a ductile one. Results from the four-point bending tests showed also how the ductility of the SFRC specimens increased with an increase in the fiber volume, and at the same fiber amount, when fiber length was increased. Post-cracking behavior of specimens was also reported with specimens having short fibers showing softening-behavior while those with longer fibers exhibited hardening behavior as well as increased maximum load.

2.3.4.7 Mukesh (2011)

Mukesh (2011) performed an experimental study with the aim of investigating the behavior of SFRC beams in the presence of longitudinal reinforcement under flexural loading. The only variable used is straight type of steel-fiber in content of 1 and 2%. The specimens include a total of nine beams, all having the same dimension of 120×240×1900 mm and same amount of tension longitudinal reinforcement of 3 ϕ 10. Mild steel bars of 6 mm diameter were used for both hanger bars and stirrups. Three out of nine beams were prepared without inclusion of steel fibers, three with 1% fibers, and the remaining three with 2% fibers. All the beams were tested in flexure under four point bending test. Mukesh (2011) reported an increase of about 25% in the first cracking load, 23% increase in service load, and 15% increase in the ultimate load for the beams with 1% fiber content. In the case of beam with 2% fiber volume, a 40% increase in the first cracking load, 39% increase in the service load, and 29% increase in the ultimate load was reported. The results showed also that there was a reduction in mid-span deflection of around 16.4% at the ultimate stage for beams containing 1% steel fiber and 28% reduction in the case of beams containing 2% steel fiber.

2.3.4.8 Giovanni et al. (2012)

Giovanni et al. (2012) performed an experimental work to investigate the contribution of steel fiber to the flexural behavior of reinforced concrete elements. The variables used in the study were two steel fiber volume percentages of 0.37 and 0.76%, and two longitudinal tension reinforcement ratios of 0.75 and 1.50%. Specimens used in the experiment include seven full-scale beams all having same dimensions of 4000×3600×300 mm. Two of the seven beams were prepared using same amount of reinforcement ratio of 1.5%, one of which was reinforced with 0.37% steel fiber in addition to the longitudinal reinforcement. The remaining five beams were prepared using same amount of longitudinal reinforcement ratio of 0.75%. Two of the five beams were also reinforced with 0.37% steel fibers and one of them was bonded while the other was unbonded. One of the remaining three beams was reinforced with 0.76% steel fiber. The remaining two were not reinforced with steel fiber and out of these two, one was bonded and the other was unbonded. In the

unbonded specimens, the unbonding was released between the two load points of the beams through a plastic pipe with the aim of highlighting the bond effect on the flexural behavior. All beams were tested in bending under a displacement-controlled procedure using an electromechanical screw jack. Findings of the study indicated that the presence of steel fiber in the specimens provided a limited improvement in capacity with an increase of the ultimate moment of about 6.7% in specimen containing 0.75% reinforcement ratio and at 0.37% fiber volume percentage. It was also reported that the ductility of the specimen with 0.76% longitudinal reinforcement ratio and 0.37% of steel fiber was remarkably increased (73.1% in terms of displacement) compared to the specimen cast without steel fiber. Conversely, a significant decay in the ductility when fiber content was increased from 0.38% to 0.76% was observed in the specimen with same amount of reinforcement ratio of 0.75%. The increase in the ductility was only 21.2% larger than the one in the reference beam without steel fibers.

2.3.4.9 Hamid et al. (2012)

Hamid et al. (2012) conducted an experimental study with the aim of determining the optimum percentage of steel fibers and also to investigate the flexural behavior of steel-fiber-added reinforced concrete using the predetermined optimum fiber percentage. Variables used in the research include two different compressive strengths values of 30 and 50 MPa, and steel fiber volume percentages of 1%, 1.5%, and 2%. Specimens used include thirty concrete cubes of 150×150×150 mm dimensions and fifteen concrete prisms 150×150×170 mm dimensions for each concrete class, with every six cubes and three prisms for each fiber volume percentage. All cubes were tested in a compression testing machine to establish compressive strength values. All the prisms were tested for flexure under four-point loading in order to determine optimum fiber percentage. For each concrete grade of 30 and 50 MPa, a pair of beams with dimensions of 120×170×2400 mm was prepared and the predetermined optimum fiber volume was added to one of each pair. The two pair of beams were tested under four-point bending to determine the flexural behavior using the optimum amount of fiber. Findings of the study showed that the addition of 1% of steel fibers gave the higher compressive strength values for the two classes of concrete. It was also reported that the addition of 1% by volume of steel

fiber led to an increase of 13 and 25% in the first cracking load and an increase of 7 and 15% in ultimate load of the tested beams of the C30 and C50 classes of concrete respectively.

2.3.4.10 Qiaoyan et al. (2013)

Qiaoyan et al. (2013) performed a parametric experimental study to investigate the flexural behavior of high-strength SFRC beams. The variables used in the study included four different longitudinal reinforcement ratios, four different fiber volume percentages, and four beams of different heights. Specimens used in the study were a total of ten beams of 150×3000×300 mm dimensions. All beams specimens were tested under three-point bending in order to evaluate their flexural behavior. The design concrete compressive strength was 60 MPa and same for all beams. The findings of Qiaoyan et al. showed that there was a considerable increase in the ultimate flexural capacity of the high-strength SFRC beams with an increase in the fiber volume fraction. A gradual increase in the ultimate flexural capacity with increase in the longitudinal reinforcement ratio was also observed. Furthermore, a significant increase in the ultimate flexural capacity with an increase in the beam height was also reported in the results.

2.3.4.11 Baran et al. (2012)

Baran et al. conducted an experimental research with the aim of investigating the pull-out behavior of prestressing strands embedded in SFRC. The variables considered were two different steel fibers of 30mm and 60mm lengths each at five different contents of 0%, 0.2%, 0.4%, 0.6%, and 0.8%. Specimens used in the research included 150 x 300mm cylinder samples, 150 x 150 x 600mm beam specimens, and 70 x 50 x 20cm concrete blocks. Four types of tests were carried out which were categorized into two phases. In the first phase compressive strength test and split tensile strength test were conducted on the concrete cylinders, while flexural toughness test was conducted on the 150 x 150 x 600mm beams specimens. In the second phase, a total of 108 pull-out tests were conducted on 12.5mm and 15.2mm diameter untensioned prestressing strands embedded in 70 x 50 x 20cm concrete blocks. The types and different amount of fibers used in the previous tests were all the same for the pull-out tests. All the specimens were prepared to have the

same compressive strength of 40 MPa. Findings of Baran et al. showed that there was improvement in the tensile behavior and post-cracking flexural response of SFRC due to crack control by the fibers. No improvement was reported in the compressive strength of the SFRC cylinders. A considerable increase in the pull-out resistance of the strands was also reported in the findings.

3 EXPERIMENTAL PROGRAM

3.1 General

This chapter describes the test program performed at Structural Mechanics Laboratory of Civil Engineering Department at Atilim University. The concrete mixture proportions of test specimens, mixing and casting sequences, test set-up and test procedure are discussed in detail in this chapter. Photos taken during preparation, casting, and testing of specimens are also included.

3.2 Test Specimens

Test series consisted of 20 singly reinforced beams (10 pairs of beams) having 180×250×3500 mm dimensions. The main parameters in the testing program were the type of concrete (ordinary plain concrete - PC or SFRC) and the amount of longitudinal reinforcement. Ten different longitudinal reinforcement ratios (starting from 0.2 to 2.5%) covering the range from under-reinforced to over-reinforced beam behavior were used in the testing program. Two specimens were cast for each longitudinal reinforcement ratio, one specimen using ordinary plain concrete (PC) and another specimen using SFRC. Details of the test specimens are shown in Table 3-1. The number at the beginning of each designation indicates the number of longitudinal reinforcing bars used while the parameter at the end indicates the diameter of the reinforcing bars used.

Table 3-1 – Details of test specimens

Specimen	Concrete Type	Bar Diameter (mm)	Number of Bars	Reinf. Area (mm ²)	Reinf. Ratio (%)
1 ϕ 10*	PC	10	1	78.5	0.20
	SFRC				
1 ϕ 12*	PC	12	1	113.1	0.30
	SFRC				
1 ϕ 14*	PC	14	1	153.9	0.40
	SFRC				
1 ϕ 16*	PC	16	1	201.1	0.53
	SFRC				
2 ϕ 14*	PC	14	2	307.9	0.81
	SFRC				
2 ϕ 16*	PC	16	2	402.1	1.06
	SFRC				
3 ϕ 16*	PC	16	3	603.2	1.60
	SFRC				
3 ϕ 18	PC	18	3	763.4	2.02
	SFRC				
4 ϕ 16	PC	16	4	804.0	2.13
	SFRC				
3 ϕ 20	PC	20	3	942.5	2.50
	SFRC				

*Specimens fabricated and tested previously by a former graduate student (Baran and Arsava, 2013)

The casting and testing of both the PC and SFRC specimens of 1 ϕ 10, 1 ϕ 12, 1 ϕ 14, 1 ϕ 16, 2 ϕ 14, 2 ϕ 16, and 3 ϕ 16 reinforcement were performed previously by a former graduate student, Tuğba Demircioğlu Arsava. In this research only PC and SFRC specimens of 3 ϕ 18, 4 ϕ 16, and 3 ϕ 20 were cast and tested by the author of this thesis.

Eight millimeter diameter transverse reinforcement was provided only at the shear spans of the beams. Since the specimens were subjected to four point bending tests, the constant moment region is produced at the mid-span. This eliminated the confinement effect due to the transverse reinforcement at the compression zone of the beams. General and cross-sectional views of the test specimens are shown in Figure 3-1.

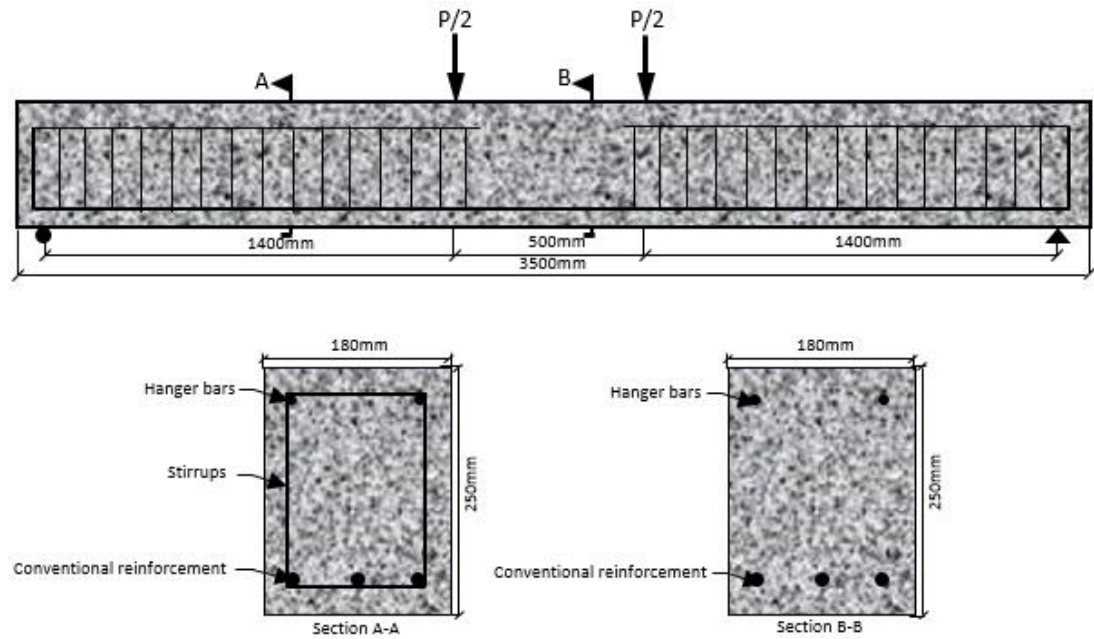


Figure 3-1 – General and cross-sectional views of the specimens

3.3 Materials

3.3.1 Concrete

The materials for concrete used in this research include ordinary Portland Cement (PC 42.5), clean tap water, superplasticizer, river sand of 0.44 mm size, fine aggregate of size ranging from 4 to 16 mm, and coarse aggregate of size ranging from 15 to 25 mm. The proportions of materials used in this research are presented in Table 3-2. Proportioning of materials was performed using a digital weighing scale where the exact quantity of each material was measured accurately before mixing (Figure 3-2).

Table 3-2 – Concrete mixture proportions

Materials	Quantity (kg)	
	PC	SFRC
Cement	40	40
Sand	90	90
Fine aggregate	44	44
Coarse aggregate	58	58
Steel fiber	-	7.7
Water	20	22
Superplasticizer	-	0.1



Figure 3-2 – Materials proportioning

3.3.2 Steel Fibers

Dramix ZP 305 type steel fibers were added in the concrete mix for the SFRC specimens used. The manufacturer specified mechanical properties of the steel fibers are shown in Table 3-3. A photograph of the fibers is shown in Figure 3-3.

Table 3-3 – Properties of steel fibers

Effective Length (mm)	Equivalent Diameter (mm)	Aspect Ratio	Young's Modulus (MPa)	Tensile Strength (MPa)	Density (kg/m ³)
30	0.55	60	210MPa	1345	7850



Figure 3-3 – Dramix ZP 305 steel fiber used in the research

3.3.3 Reinforcement

Deformed reinforcing steel bars with nominal yield strength of 420 MPa were used both for longitudinal and transverse reinforcement in the construction of the specimens. The stress-strain relationship of the reinforcement obtained from tension tests performed in the laboratory is shown in Figure 3-4.

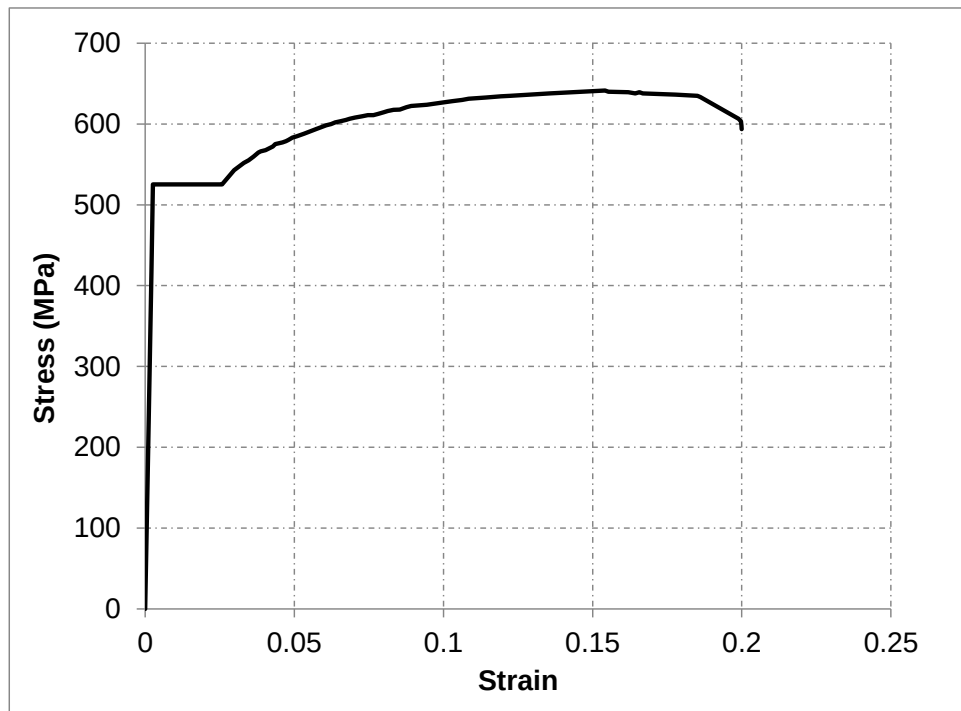


Figure 3-4 – Measured stress-strain relationship of steel reinforcing bars

3.4 Specimen Preparation

3.4.1 Reinforcement Cage

The reinforcement cage was prepared based on the specimen properties. A typical reinforcement cage placed inside the beam formwork is shown in Figure 3-5.



Figure 3-5 – Reinforcement cage in the formwork

3.4.2 Concrete Mixing

A stationary drum mixer was used to mix the materials. Aggregates and cement were placed into the mixer and mixed for about one minute. Steel fiber was simultaneously discharged into the mixer while it rotates at a considerable speed. About 80% of water was added to the materials while mixing was in progress. Finally, the remaining water and superplasticizer were added to the mixing constituents and the mixing became fully operational until a homogenous mixture with a desired workability was obtained. A photograph during mixing of constituents for one of the specimens is presented in Figure 3-6.



Figure 3-6 – Mixing of materials in progress

3.4.3 Casting and Curing

For each beam specimen, three 150×350 mm cylindrical samples were prepared to measure the concrete compressive strength at the time of testing of the beam specimens. All beam specimens and cylinder samples were cast using steel formwork and molds followed by curing twenty four hours after casting. Before casting, all formwork and molds were properly cleaned and then lubricating oil was applied on their inner surfaces for easy removal of formworks and molds. An external vibrator was used for the beam specimens to obtain a well compacted composite and also to prevent segregation of fibers. The concrete mix used for the specimens had a target compressive strength of 30 MPa, but some modifications had to be made on the amount of water and superplasticizer in some specimens in order to get proper workability. A photograph taken during casting is shown in Figure 3-7.



Figure 3-7 – Placing concrete in formwork

After casting, specimens were allowed to set and dry in formworks and molds prior to curing. Adequate curing was provided until testing day which was minimum 28 days after the day of casting. Proper curing was achieved by making the specimens sufficiently wet using water followed by covering them with wet burlap to minimize loss of moisture due to evaporation as shown in Figure 3-8. Same curing procedure is applied to both the beam specimens and cylinder samples.



Figure 3-8 – Curing of specimens

3.4.4 Test Set-Up and Procedure

Two tests were conducted in this research, material tests to establish the test day-compressive strengths of concrete cylinders and four-point bending tests to investigate the flexural behavior of the beam specimens.

3.4.4.1 Material Tests

In the compression tests all cylinders samples were tested to establish their compressive strength values with three cylinders for each beam specimen. The test was performed at least 28 days after the day of casting. The test was conducted on a hydraulic compression testing machine with 1500 kN capacity. Before testing, each cylinder was capped with a sulphur based composition both on the top and bottom face in order to provide parallel loading surfaces of the test specimens. This helps the specimens achieving proper contact with the testing machine as well as equal distribution of compression load on the cylinders. The load was applied slowly by lowering the top testing plate of the machine to bring it into contact with the specimen. Load was applied at the rate of approximately 0.25 MPa/sec by adjusting the lever attached to the testing machine. Each cylinder was subjected to compression load until failure occurred. The average values of the compressive loads obtained from three cylinders for each beam specimen divided by the initial cross sectional area of the cylinder were evaluated as the compressive strength for that particular beam. The photographs of capping and testing of concrete cylinder samples are shown in Figure 3-9.



Figure 3-9 – Capping and testing of cylinder samples

3.4.4.2 Flexural Tests

Four-point bending tests were conducted on beam specimens to investigate the effect of fibers on their flexural behavior. The beams were simply supported at their ends on steel rollers. The test set-up is shown in Figure 3-10. The rollers were placed 10 cm away from each end of the beam. The load was applied through a 300 kN hydraulic jack with a 200 kN load cell attached to it. A swivel head was attached to the load cell, and a load spreader beam was attached to the underside of the swivel head. Load was transferred from the spreader beam to the specimen through a steel roller at each loading point. The two loading points were spaced 500mm and were positioned symmetrically with respect to the mid-span cross section of specimens. Details related to the hydraulic jack and the spreader beam are shown in Figure 3-11.

During load testing, the mid-span deflection as well as the horizontal displacement at the bottom surface of beam specimens were measured and recorded continuously using two Linear Variable Displacement Transducers (LVDTs). One of these LVDTs was attached vertically at the bottom of the beam at the mid-span cross section, and the other one was attached horizontally at the bottom of the beam in the constant moment region. The applied load, mid-span deflection, and bottom fiber stretch readings were recorded continuously during the tests by a data acquisition system. Details related to the instrumentation are shown in Figure 3-11.

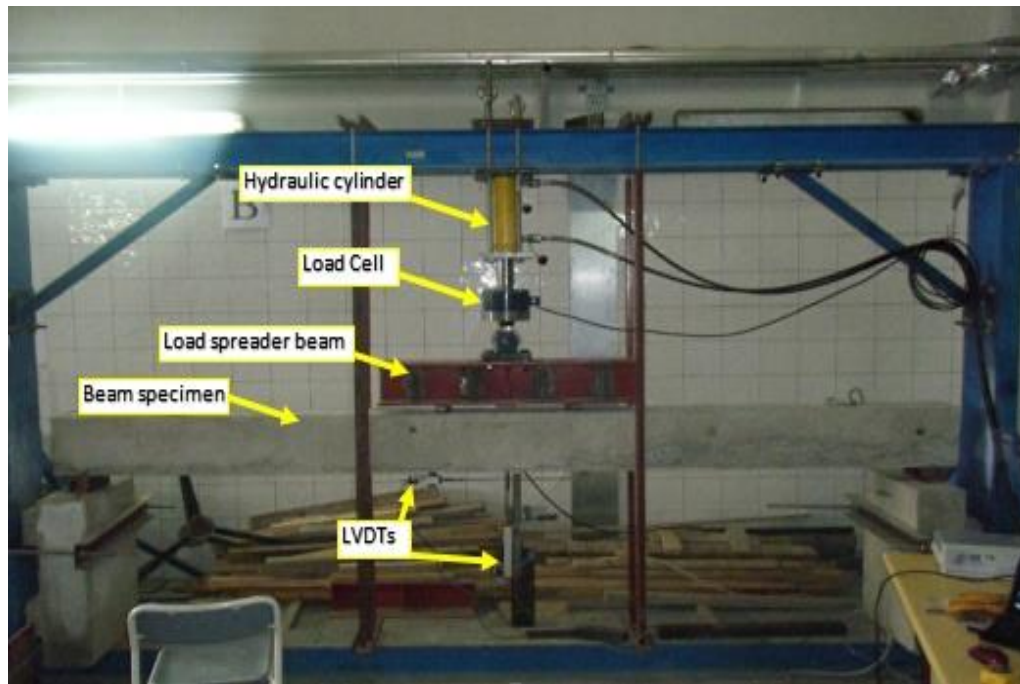


Figure 3-10 – Set-up used for flexural load tests



Figure 3-11 – Details of instrumentation and loading devices

4 TEST RESULTS AND DISCUSSIONS

4.1 Introduction

In this chapter the results obtained from concrete cylinder compression and four-point bending tests performed on the beams are presented in tabular and graphical forms. Comments were made based on visual observations made during the tests and comparisons made based on the test data collected throughout the tests.

4.2 Material Tests

The mean cylinder concrete compressive strength values of each tested specimen are shown in Table 4-1. Based on the concrete compressive strengths and the tensile strength of steel reinforcement used, the balanced reinforcement ratio for each specimen is calculated and the section behavior of each specimen is also presented in this table.

Table 4-1 – Compression test results and section behaviors for specimens

Specimen	Reinf. Ratio (%)	ρ_b (%)		Concrete Strength (MPa)		Section Behavior
		PC	SFRC	PC	SFRC	
1 ϕ 10	0.20	2.23	2.37	32	34.7	Under-reinforced
1 ϕ 12	0.30	2.34	2.65	34.2	40.6	Under-reinforced
1 ϕ 14	0.40	2.32	2.18	33.8	31.1	Under-reinforced
1 ϕ 16	0.53	2.52	2.70	37.8	41.7	Under-reinforced
2 ϕ 14	0.81	2.20	2.10	31.4	29.6	Under-reinforced
2 ϕ 16	1.06	2.38	2.79	35	43.9	Under-reinforced
3 ϕ 16	1.60	2.47	2.22	36.7	31.9	Under-reinforced
3 ϕ 18	2.02	1.57	1.83	21.5	25	Over-reinforced
4 ϕ 16	2.13	1.87	2.22	25.7	31.8	Over/Balanced reinforcement
3 ϕ 20	2.50	1.81	1.63	24.7	22.3	Over-reinforced

Typical failure modes of concrete cylinders are shown in Figure 4-1 and Figure 4-2. It is observed from the table that, although all samples were prepared using the same mix design with the same amount of steel fibers, different compressive strength values were obtained. Therefore, the influence of steel fibers on the compressive strength behavior has not been ascertained due to the fact that there was no variation in the fiber content in all cylinders. But based on the visual observation made during the tests, it was observed that cylinders made of plain concrete showed a sudden and brittle mode of failure immediately after reaching the maximum values which can be considered as their respective peak strength values. On the other hand, specimens containing steel fibers sustained a considerable amount of strain under significant level of compressive load.



Figure 4-1 – Failure mode of concrete cylinders for SFRC Beam 3 ϕ 20



Figure 4-2 – Failure mode of concrete cylinders for SFRC Beam 3φ18

4.3 Flexural Tests

Typical failure modes of flexural tests are shown in Figure 4-3 to Figure 4-10. All beams showed vertical flexural cracks mainly around their constant-moment regions prior to crushing. It is observed for beams containing steel fibers that the ultimate load and deflection increases considerably as the reinforcement ratio increases. The beams suffer damage by gradual development of multiple cracks with increasing deflection, but retain some degree of structural integrity and post – crack resistance even with considerable deflection. In the case of specimens without fibers the load at which crack occurs is similar to those of the SFRC specimens. In addition, a sudden rather than gradual crushing of concrete was observed in the specimens without fibers with few larger cracks prior to crushing.



Figure 4-3 – Failure and crack pattern in SFRC Beam 3φ20 ($\rho = 2.50\%$)



Figure 4-4 – Failure and crack pattern in SFRC Beam 3φ20 ($\rho = 2.50\%$)



Figure 4-5 – Failure and crack pattern in SFRC Beam 3φ18 ($\rho = 2.02\%$)



Figure 4-6 – Failure and crack pattern in SFRC Beam 3φ18 ($\rho = 2.02\%$)



Figure 4-7 – Failure and crack pattern in SFRC Beam 4 ϕ 16 ($\rho = 2.13\%$)



Figure 4-8 – Failure and crack pattern in SFRC Beam 4 ϕ 16 ($\rho = 2.13\%$)



Figure 4-9 – Failure and crack pattern in a PC Beam 3φ18 ($\rho = 2.02\%$)



Figure 4-10 – Failure and crack pattern in a PC Beam 4φ16 ($\rho = 2.13\%$)



Figure 4-11 – Failure and crack pattern in a PC Beam 3 ϕ 20 ($\rho = 2.50\%$)

4.3.1 Load-Deflection Relationships

The load-deflection curves of all beams obtained from four-point bending tests are presented in Figure 4-12 to Figure 4-23. The plots in Figure 4-12 and Figure 4-13 show the load-deflection graphs for PC and SFRC specimens respectively. It can be seen from the two plots that the ultimate load increases as the amount of reinforcement ratio increases with SFRC possessing higher ultimate load values than their counterparts due to the presence of steel fiber.

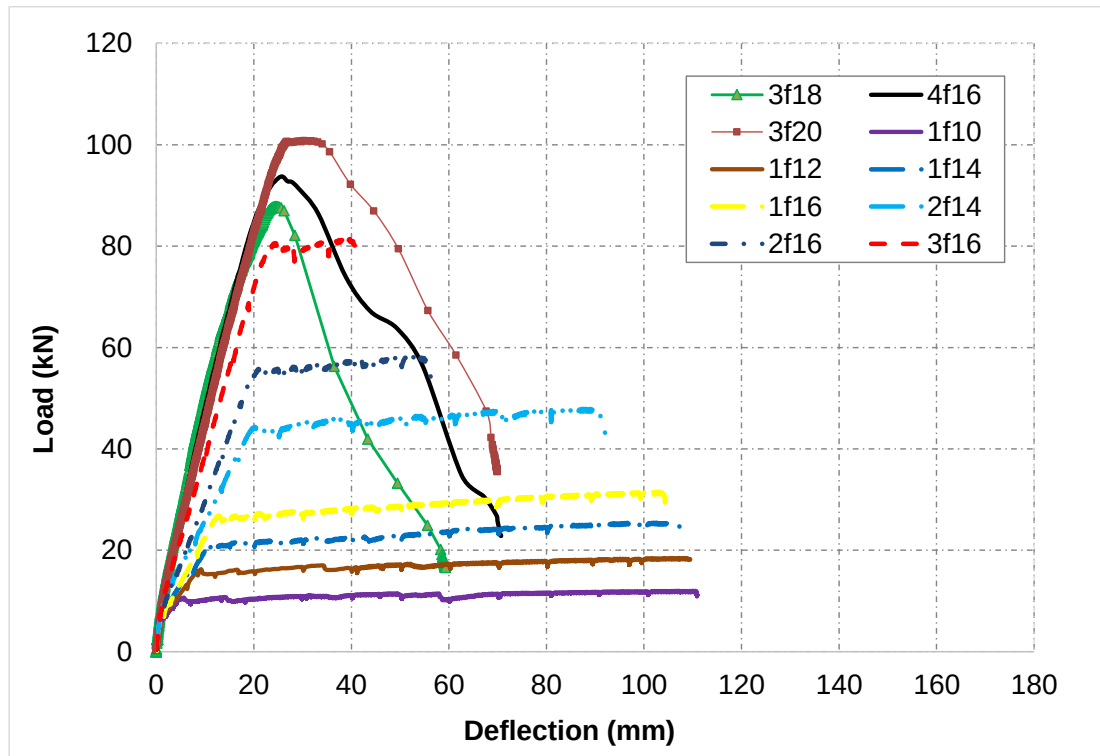


Figure 4-12 – Experimental load-deflection graphs for PC specimens

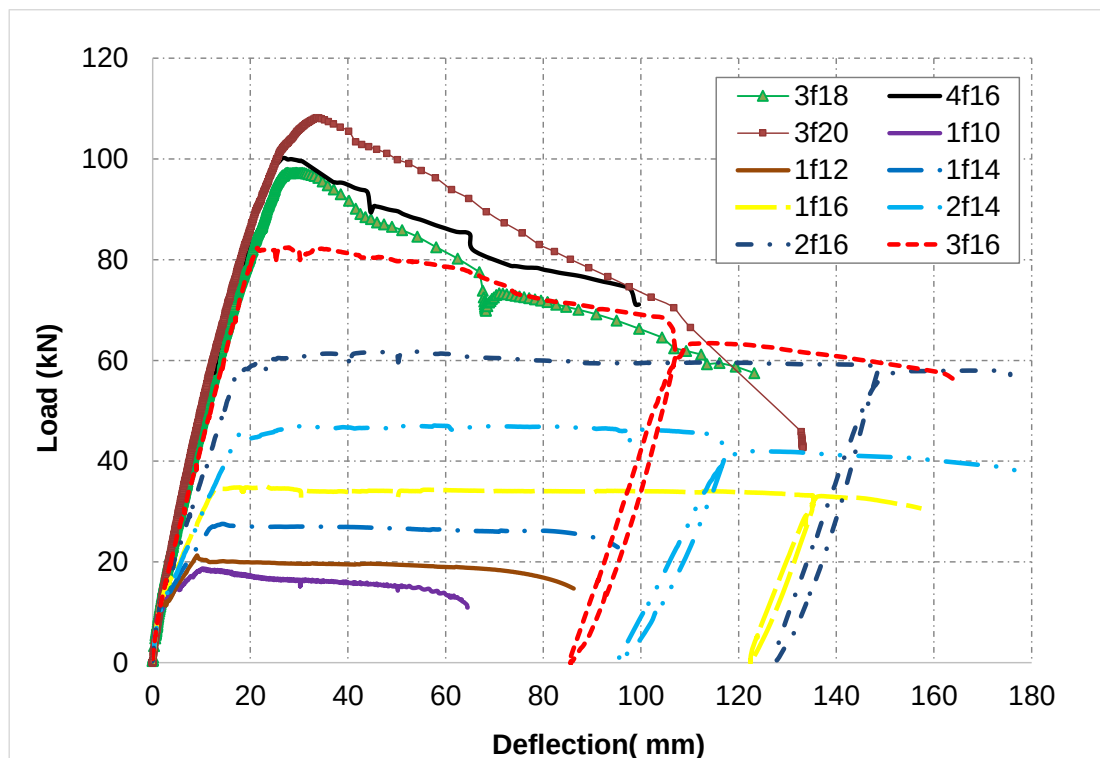


Figure 4-13 – Experimental load-deflection graphs for SFRC specimens

Comparisons between each pair of specimens are shown from Figure 4-15 to Figure 4-23. Graphs in Figure 4-14 present the comparison of specimens with 1 ϕ 10 reinforcement. It is observed that the presence of fibers increases the ultimate load by about 58% compared to the plain concrete specimen.

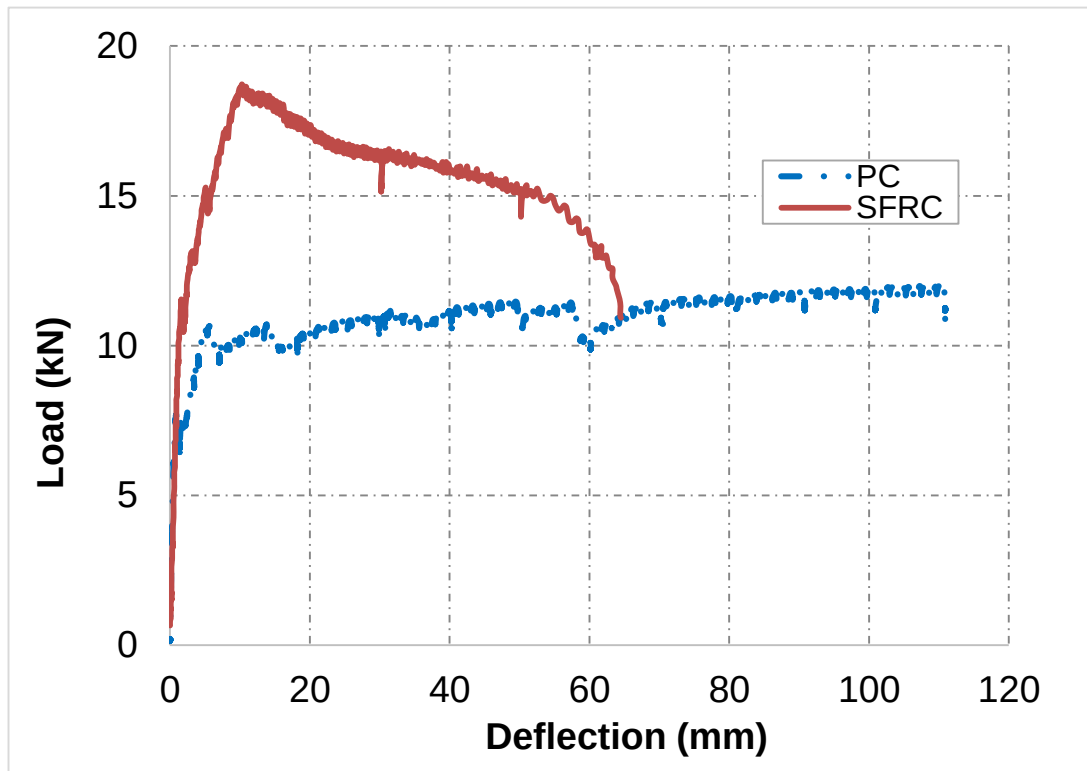


Figure 4-14 – Experimental load-deflection graphs for 1 ϕ 10 specimens ($\rho = 0.20\%$)

For 1 ϕ 12 specimens in Figure 4-15 the increase in load capacity with the addition steel fibers is also. The increase for this reinforcement amount is about 15%.

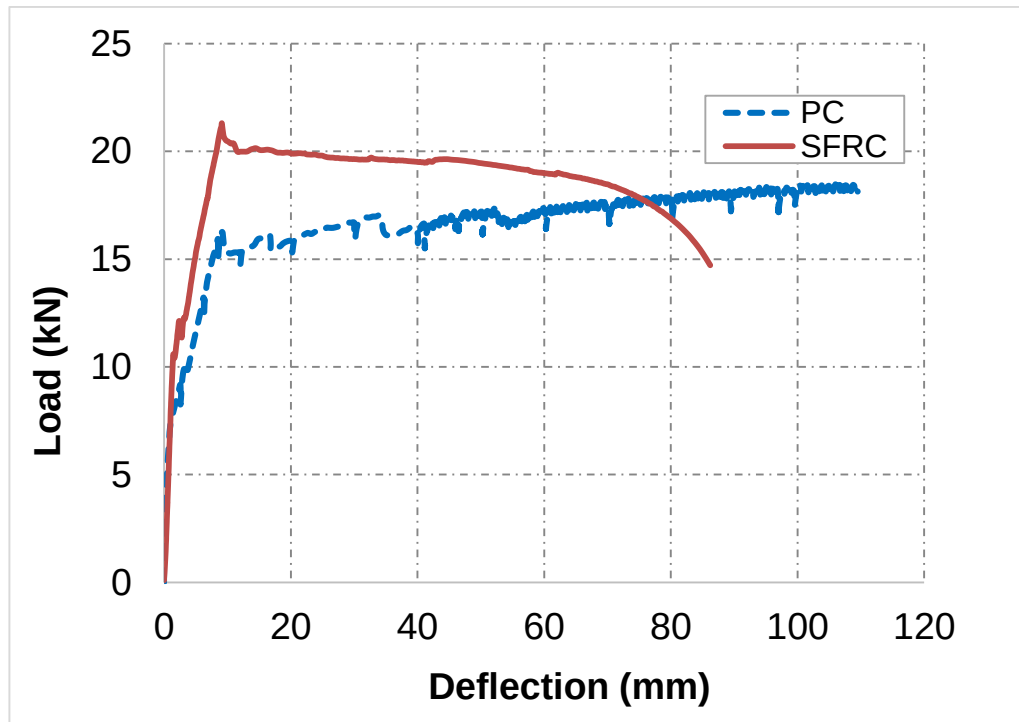


Figure 4-15 – Experimental load-deflection graphs for 1 ϕ 12 specimens ($\rho = 0.30\%$)

Figure 4-16 presents the relationship for 1 ϕ 14 specimens. The specimen containing fibers shows an increase in the ultimate load of 8% compared to the plain specimen.

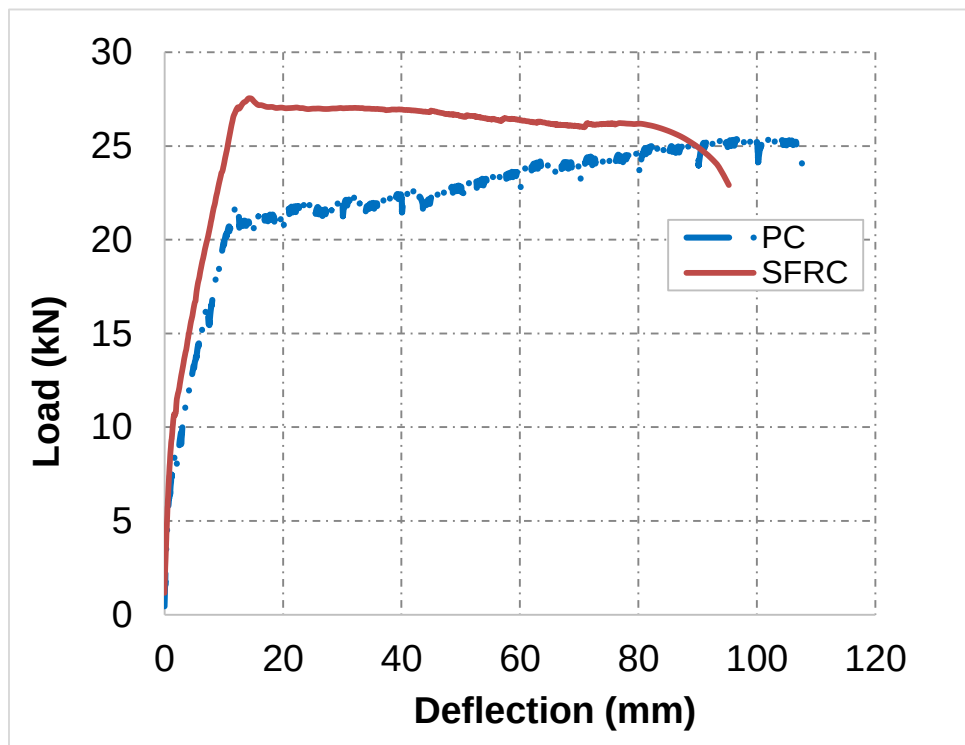


Figure 4-16 – Experimental load-deflection graphs for 1 ϕ 14 specimens ($\rho = 0.40\%$)

Graph of load-deflection behavior for 1 ϕ 16 specimens is presented in Figure 4-17. The increase in the ultimate load due to the presence of steel fibers was found to be 10%. The corresponding initial slope also appears to be larger in the SFRC specimen than in the PC specimen.

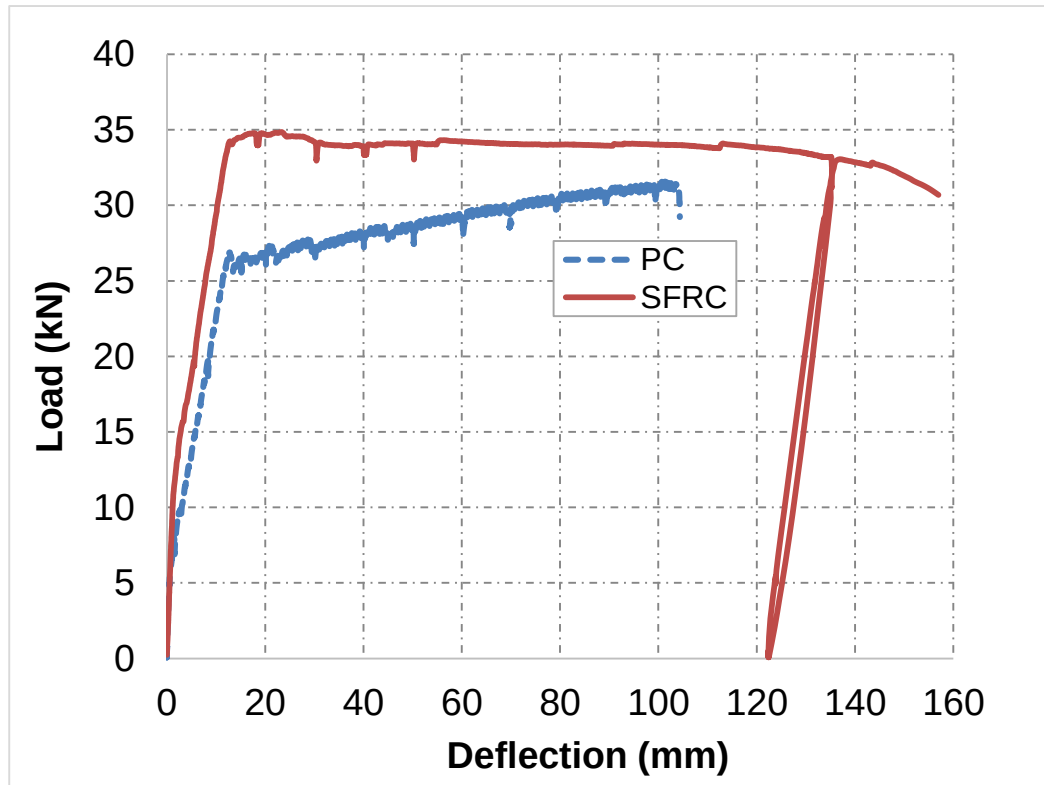


Figure 4-17 – Experimental load-deflection graphs for 1 ϕ 16 specimens ($\rho = 0.53\%$)

Unlike the previous specimens, where an increase in the ultimate load was observed with the addition of steel fibers, a slight decrease is observed in the SFRC specimen as compared to the companion PC specimens with 2 ϕ 14 reinforcement. A comparison of behaviors for these two specimens is shown in Figure 4-18. The plain specimen possesses load value of approximately 2% higher than the SFRC specimen but with the latter possessing higher yield load value.

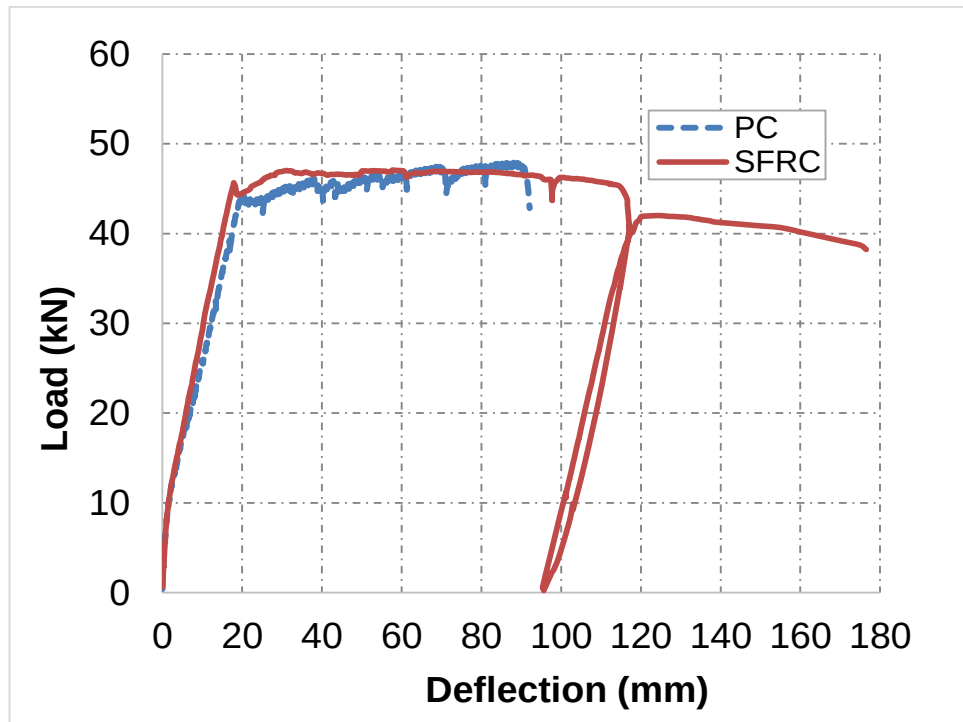


Figure 4-18 – Experimental load-deflection graphs for 2φ14 specimens ($\rho = 0.81\%$)

Figure 4-19 presents the load-deflection relationship for 2φ16 specimens. An increase of approximately 6% is observed in the specimen containing steel fibers. The initial slope is also observed to be higher in the SFRC specimen.

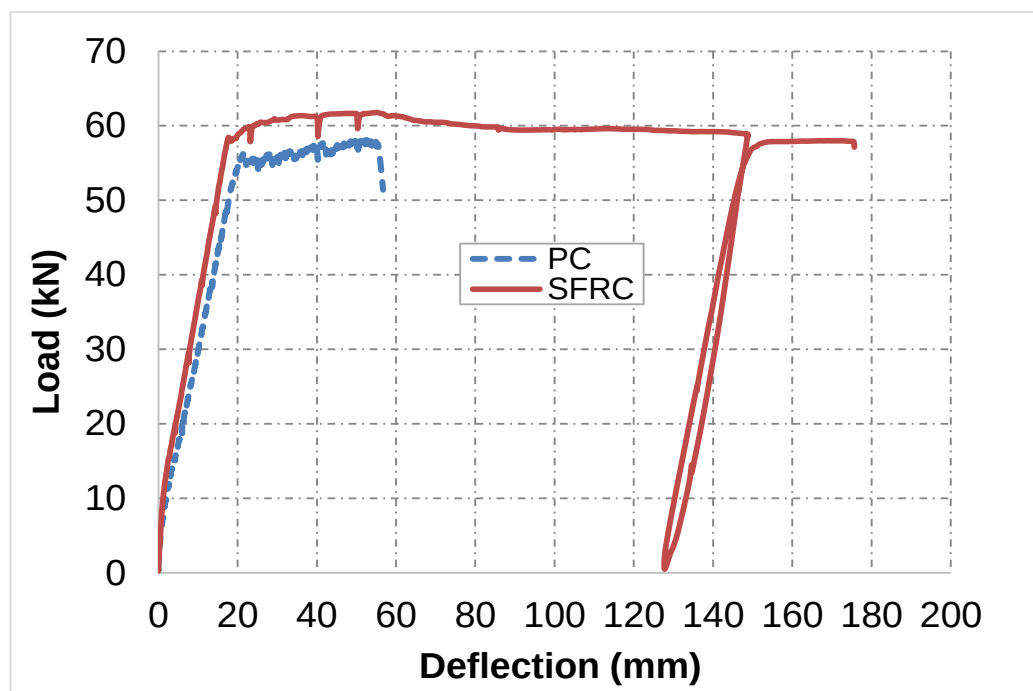


Figure 4-19 – Experimental load-deflection graphs for 2φ16 specimens ($\rho = 1.06\%$)

For 3 ϕ 18 specimens of Figure 4-20 the difference in the ultimate load values between the two specimens is relatively small. Specimen containing fibers has an ultimate load of about 1% higher than for the plain specimen. Initial slope is also observed to be higher in the former than in the latter.

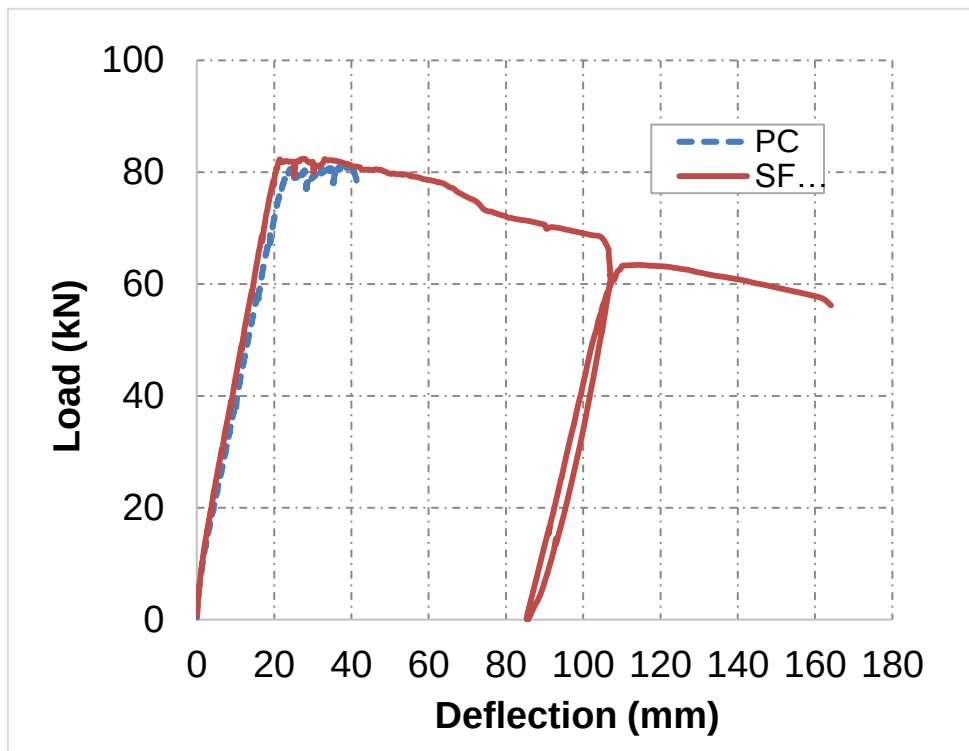


Figure 4-20 – Experimental load-deflection graphs for 3 ϕ 16 specimens ($\rho = 1.60\%$)

Figure 4-21, Figure 4-22, and Figure 4-23 present the load deflection relationships for the heavily reinforced specimens. An increase of 11%, 7%, and 7% respectively, was observed with the addition of steel fibers in the 3 ϕ 18, 4 ϕ 16, and 3 ϕ 20 specimens. For these three reinforcement amounts it is observed that specimens without fibers have their respective cracking load as their failure load, while for specimens containing steel fibers, a considerable increase in the load even after cracking is observed. Also, in the post-peak region of the load-deflection curve, specimens containing steel fibers undergo significantly higher amount of deflections than the plain specimens.

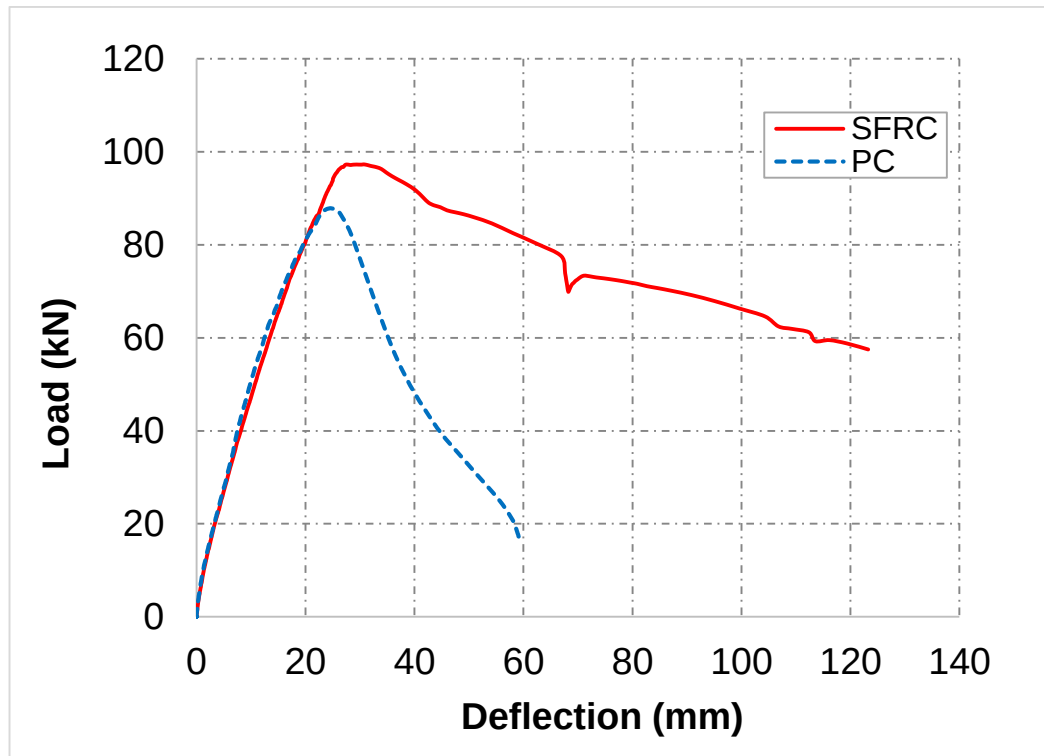


Figure 4-21 – Experimental load-deflection graphs for 3 ϕ 18 specimens ($\rho = 2.02\%$)

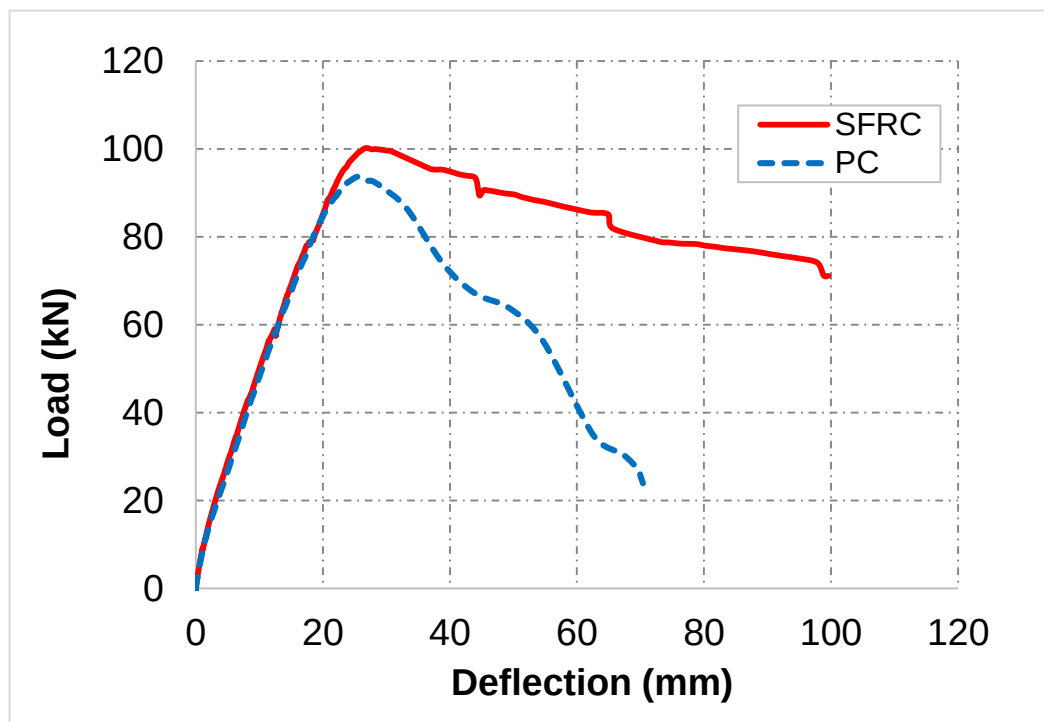


Figure 4-22 – Experimental load-deflection graphs for 4 ϕ 16 specimens ($\rho = 2.13\%$)

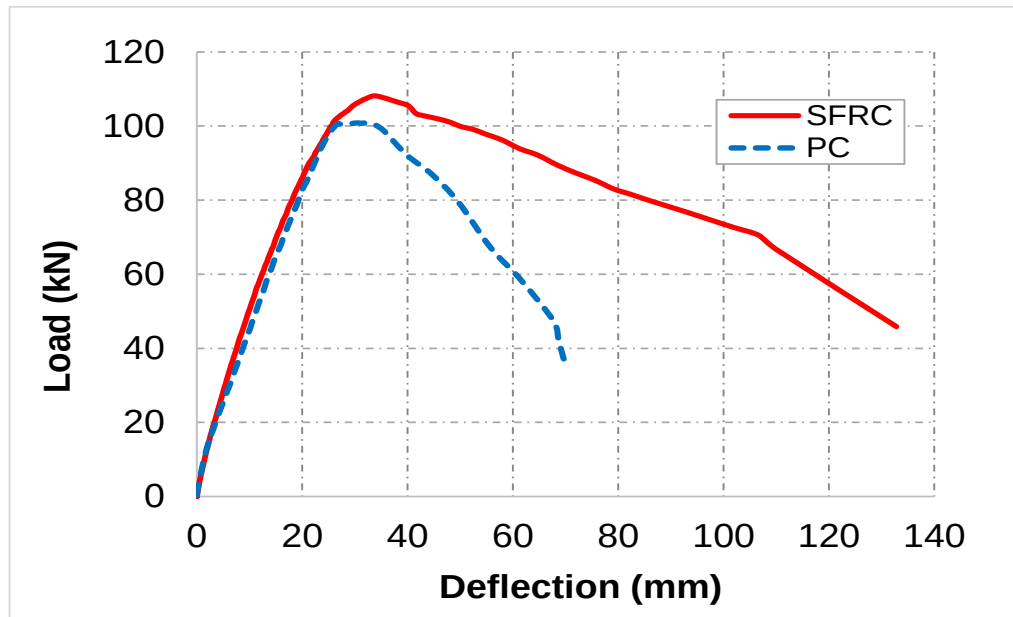


Figure 4-23 – Experimental load-deflection graphs for 3 ϕ 20 specimens ($\rho = 2.50\%$)

4.3.1.1 Ultimate Load

Ultimate load is defined as the maximum load the specimen reached during the load tests. The comparisons of the ultimate load values of PC and SFRC beams are given in Table 4-2 and Figure 4-24. It can be seen from the table and the figure that the ultimate load values are greater for SFRC specimens than the companion PC specimens for all reinforcement amounts except 2 ϕ 14. This increase does not have a clear trend with the increasing reinforcement ratio.

Table 4-2 – Experimental ultimate load values

Specimen	Reinf. Ratio (%)	Ultimate Load (kN)		Amount Change (kN)	% Change
		PC	SFRC		
1 ϕ 10	0.20	11.81	18.72	6.91	58.5
1 ϕ 12	0.30	18.48	21.29	2.81	15.2
1 ϕ 14	0.40	25.39	27.54	2.15	8.5
1 ϕ 16	0.53	31.56	34.85	3.29	10.4
2 ϕ 14	0.81	47.90	47.01	-0.89	-1.9
2 ϕ 16	1.06	58.11	61.77	3.66	6.3
3 ϕ 16	1.60	81.35	82.40	1.05	1.3
3 ϕ 18	2.02	87.85	97.30	9.45	10.8
4 ϕ 16	2.13	93.74	100.20	6.46	6.9
3 ϕ 20	2.50	100.82	108.15	7.33	7.3

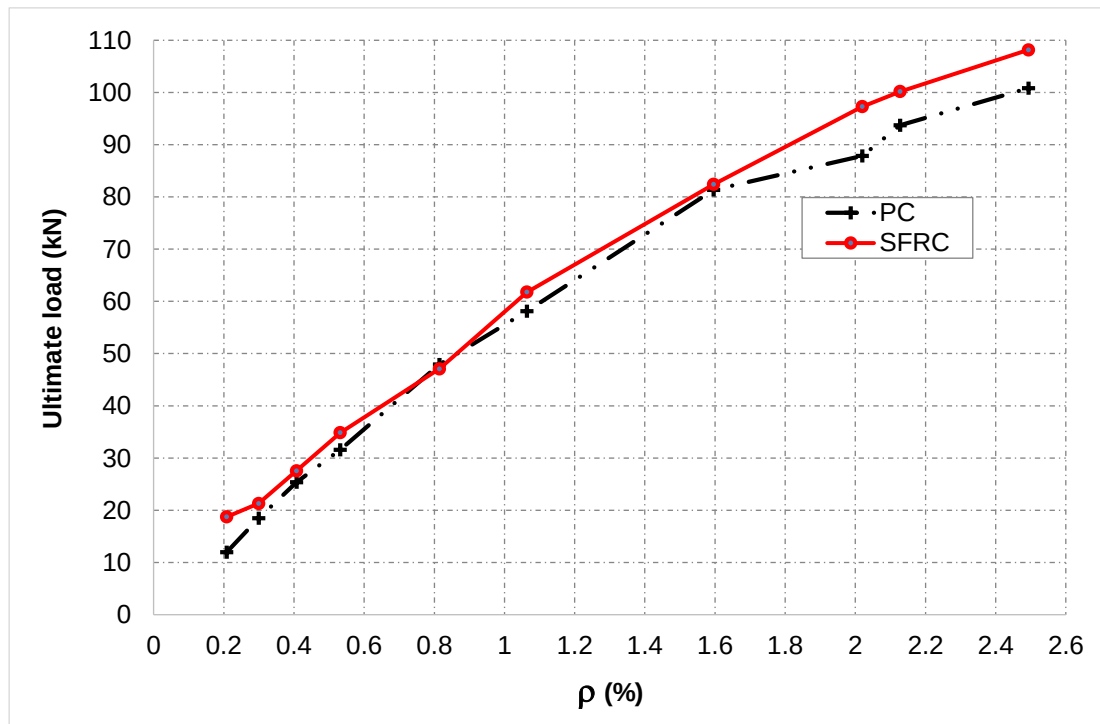


Figure 4-24 – Ultimate load versus reinforcement ratio

Since the concrete compressive strength of beam specimens had some variation, the effect of concrete compressive strength on the ultimate load values was tried to be eliminated by normalizing the values. In Table 4-3 and Table 4-4, and Figure 4-25, and Figure 4-26, the ultimate load values were normalized by the concrete compressive strength and square root of the concrete compressive strength of the beams. Based on these tables and figures, there is a considerable decrease in the maximum load values for the specimens considered over-reinforced, due to variation in the compressive strength values.

Table 4-3 – Normalized ultimate load values with respect to f_c

Specimen	Reinf. Ratio (%)	PC/ f_c (kN/MPa)	SFRC/ f_c (kN/MPa)	Amount Change (kN/MPa)	% Change
1 ϕ 10	0.20	0.37	0.54	0.16	44.0
1 ϕ 12	0.30	0.54	0.52	-0.02	-3.0
1 ϕ 14	0.40	0.75	0.89	0.13	17.9
1 ϕ 16	0.53	0.84	0.84	0.00	0.1
2 ϕ 14	0.81	1.53	1.59	0.06	4.3
2 ϕ 16	1.06	1.66	1.41	-0.25	-15.3
3 ϕ 16	1.60	2.22	2.58	0.37	16.5
3 ϕ 18	2.02	4.09	3.89	-0.19	-4.8
4 ϕ 16	2.13	3.65	3.15	-0.50	-13.6
3 ϕ 20	2.50	4.08	4.85	0.77	18.8

Table 4-4 – Normalized ultimate load values with respect to $\sqrt{f_c}$

Specimen	Reinf. Ratio (%)	PC/ $\sqrt{f_c}$ (kN/ $\sqrt{\text{MPa}}$)	SFRC/ $\sqrt{f_c}$ (kN/ $\sqrt{\text{MPa}}$)	Amount Change (kN/ $\sqrt{\text{MPa}}$)	% Change
1 ϕ 10	0.20	2.12	3.18	1.06	49.9
1 ϕ 12	0.30	3.16	3.34	0.18	5.7
1 ϕ 14	0.40	4.37	4.94	0.57	13.1
1 ϕ 16	0.53	5.13	5.40	0.26	5.1
2 ϕ 14	0.81	8.55	8.65	0.10	1.2
2 ϕ 16	1.06	9.82	9.32	-0.50	-5.1
3 ϕ 16	1.60	13.43	14.59	1.16	8.6
3 ϕ 18	2.02	18.95	19.46	0.51	2.7
4 ϕ 16	2.13	18.49	17.77	-0.73	-3.9
3 ϕ 20	2.50	20.29	22.90	2.62	12.9

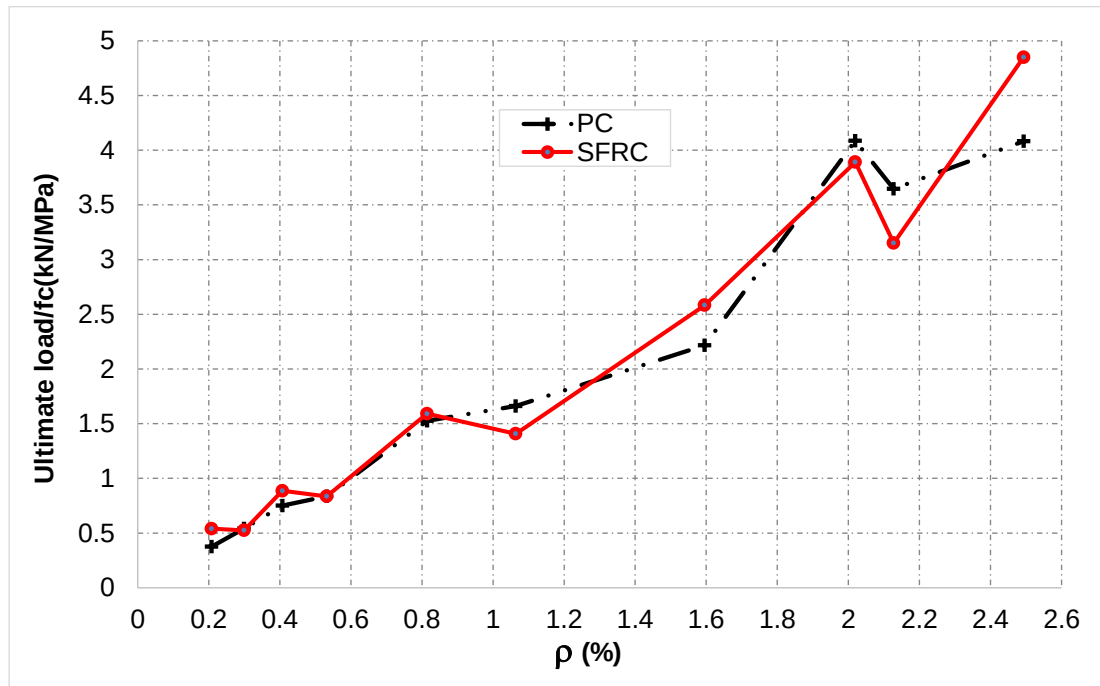


Figure 4-25 – Normalized ultimate load with respect to f_c versus reinforcement ratio

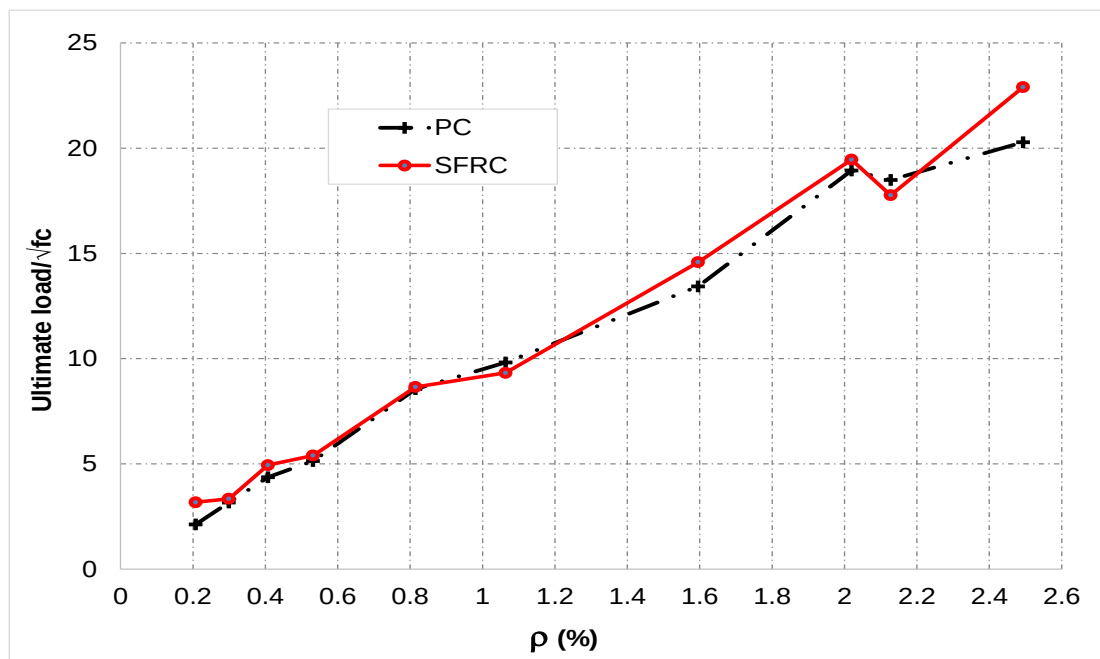


Figure 4-26 – Normalized ultimate load with respect to $\sqrt{f_c}$ versus reinforcement ratio

Based on the results of the ultimate load values of the load-deflection curves, it can be concluded that although there is a slight increase, there is no significant increase on the ultimate load capacity when PC and SFRC specimens are compared. Also there is no clear trend on the ratio of increase of the load carrying capacity of SFRC specimens with respect to PC specimens as the increasing reinforcement ratio.

4.3.1.2 Service Stiffness

Stiffness for each specimen was obtained by calculating the slope between two points, 50 and 80% of the ultimate load on the ascending branch of the load-deflection curve as shown in Figure 4-27. This stiffness is called as the “service stiffness” mainly because under realistic service loads, the behavior of properly designed reinforced concrete beams will be in the cracked state and prior to yielding of reinforcement. Considering the load and resistance factors (or material factors) specified in the current reinforced concrete design codes, the service load level for a reinforced concrete beam corresponds approximately to 60-70% of the ultimate load of the beam. Therefore, using 50 and 80% of the ultimate load is believed to be a realistic representation of the service stiffness for the test specimens.

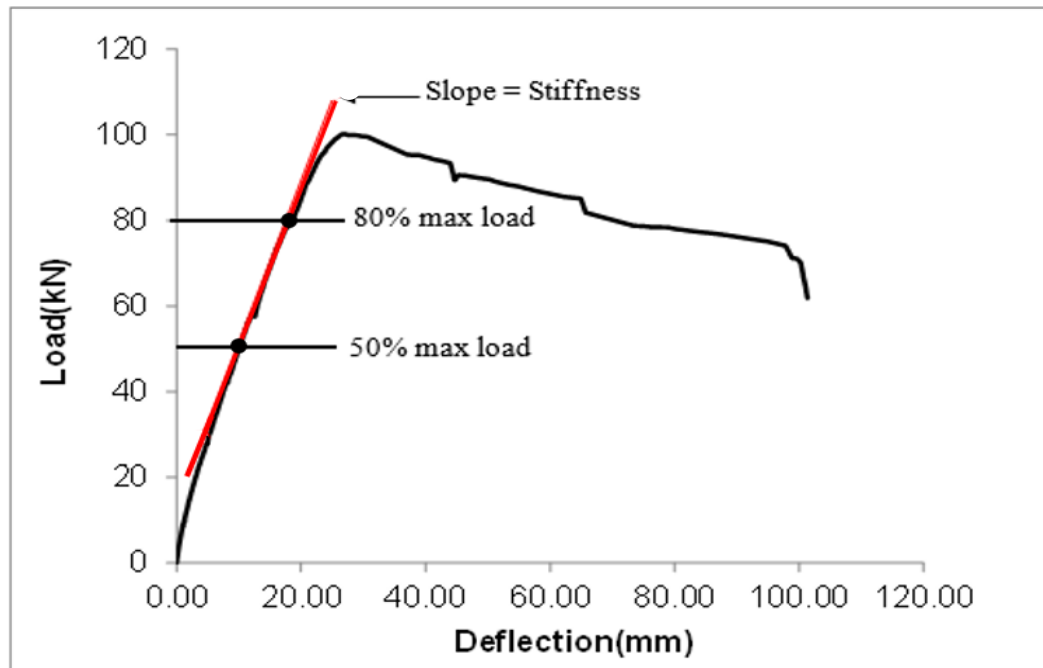


Figure 4-27 – Method of computing initial stiffness

The comparisons of the service stiffness values of PC and SFRC beams obtained from load-deflection curves are shown in Table 4-5 and Figure 4-28. Based on the table and the figure, an increase in the stiffness of SFRC specimens is observed up to the reinforcement ratio of 1.6%, after which the stiffness drops drastically and becomes even lower than for specimens without fibers. It should be noted here that the reinforcement ratio of 1.6% represents the limit between the under-reinforced and

over-reinforced behavior. In other words, specimens having reinforcement ratio between 0.2 and 1.6% have under-reinforced behavior, while those with reinforcement ratio between 2.02 and 2.5% have over-reinforced behavior. Based on this classification, Figure 4.28 clearly indicate that for under-reinforced behavior SFRC specimens have higher service stiffness than their PC counterparts, while for over-reinforced behavior stiffness of SFRC beams remained lower than the companion PC beams.

Table 4-5 – Experimental service stiffness values

Specimen	Reinf. Ratio (%)	PC (kN/mm)	SFRC (kN/mm)	Amount Change (kN/mm)	% Change
1 ϕ 10	0.20	1.28	1.50	0.22	17.5
1 ϕ 12	0.30	1.19	1.43	0.23	19.6
1 ϕ 14	0.40	1.28	1.60	0.32	24.6
1 ϕ 16	0.53	1.82	2.11	0.28	15.5
2 ϕ 14	0.81	1.96	2.21	0.26	13.2
2 ϕ 16	1.06	2.62	2.88	0.26	10.0
3 ϕ 16	1.60	3.42	3.75	0.33	9.7
3 ϕ 18	2.02	3.63	3.41	-0.22	-6.0
4 ϕ 16	2.13	3.80	3.48	-0.32	-8.4
3 ϕ 20	2.50	3.73	3.48	-0.25	-6.7

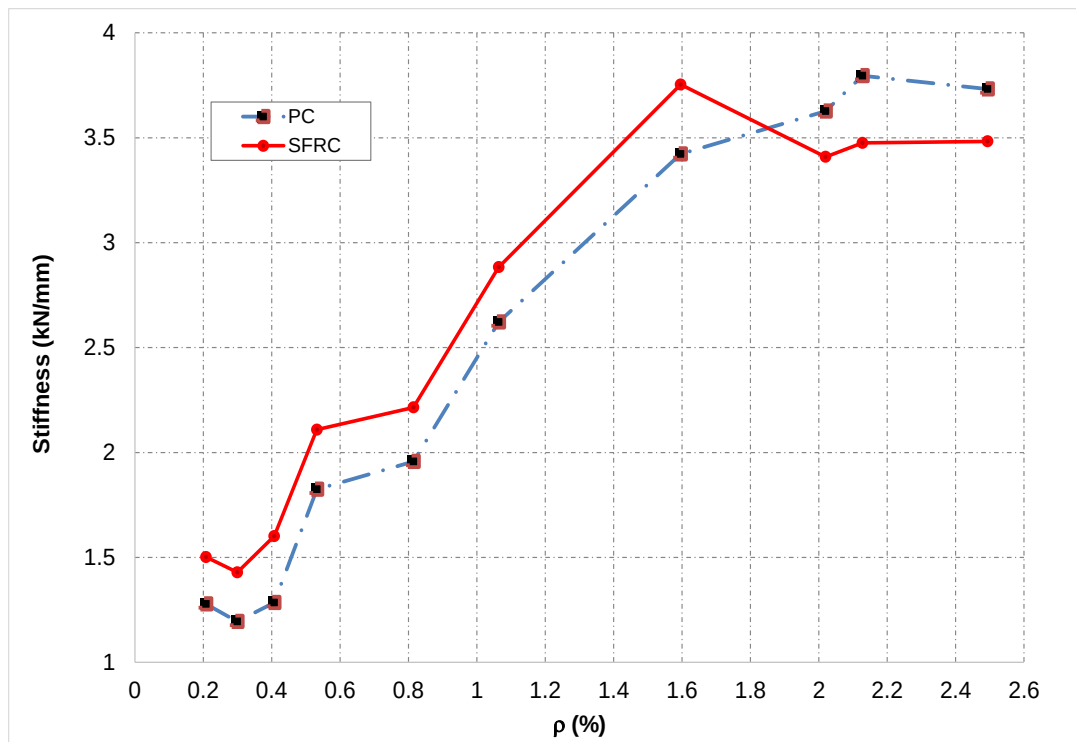


Figure 4-28 – Stiffness versus reinforcement ratio

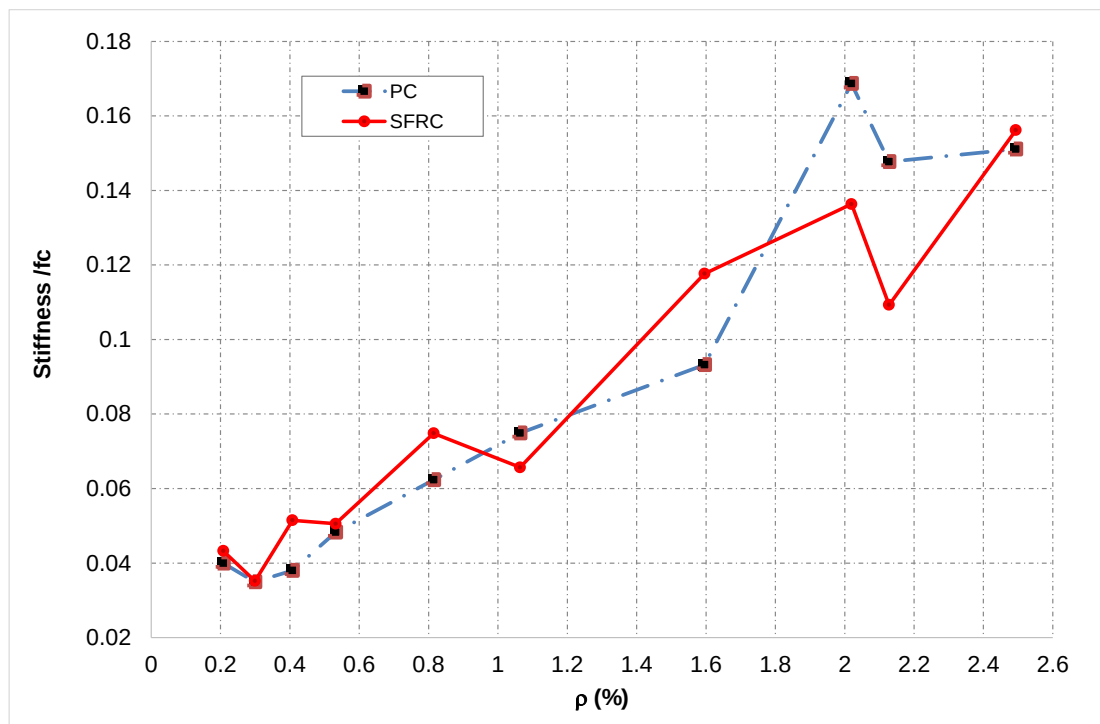
Since the concrete compressive strength of beam specimens had some variation, the effect of concrete compressive strength was tried to be eliminated by normalizing the curves. In Table 4-6, Table 4-7, Figure 4-29, and Figure 4-30, the service stiffness values were divided by the concrete compressive strength and square root of the concrete compressive strength of the beams. In the case of the normalized curves, the improvement is not as significant as in the previous case. The two curves follow almost same trend with the exception of some points and this reflects the variation in the compressive strength values which were used in normalizing the stiffness. Therefore when normalizing the stiffness is considered, all values decrease drastically by more than a hundred percent of their actual values with the influence of fibers disappearing in some of the specimens.

Table 4-6 – Normalized service stiffness values with respect to f_c

Specimen	Reinf. Ratio (%)	PC/ f_c (kN/mm/MPa)	SFRC/ f_c (kN/mm/MPa)	Amount Change (kN/mm/MPa)	% Change
1 ϕ 10	0.20	0.040	0.043	0.003	8.3
1 ϕ 12	0.30	0.035	0.035	0.000	0.7
1 ϕ 14	0.40	0.038	0.051	0.013	35.5
1 ϕ 16	0.53	0.048	0.051	0.002	4.7
2 ϕ 14	0.81	0.062	0.075	0.013	20.1
2 ϕ 16	1.06	0.075	0.066	-0.009	-12.3
3 ϕ 16	1.60	0.093	0.118	0.024	26.2
3 ϕ 18	2.02	0.169	0.136	-0.032	-19.2
4 ϕ 16	2.13	0.148	0.109	-0.038	-26.0
3 ϕ 20	2.50	0.151	0.156	0.005	3.4

Table 4-7 – Normalized service stiffness values with respect to $\sqrt{f_c}$

Specimen	Reinf. Ratio (%)	PC/ $\sqrt{f_c}$ (kN/mm/ $\sqrt{\text{MPa}}$)	SFRC/ $\sqrt{f_c}$ (kN/mm/ $\sqrt{\text{MPa}}$)	Amount Change (kN/mm/ $\sqrt{\text{MPa}}$)	% Change
1 ϕ 10	0.20	0.226	0.255	0.029	12.8
1 ϕ 12	0.30	0.204	0.224	0.020	9.8
1 ϕ 14	0.40	0.221	0.287	0.066	29.9
1 ϕ 16	0.53	0.297	0.327	0.030	10.0
2 ϕ 14	0.81	0.349	0.407	0.058	16.6
2 ϕ 16	1.06	0.443	0.435	-0.008	-1.8
3 ϕ 16	1.60	0.565	0.665	0.100	17.7
3 ϕ 18	2.02	0.782	0.682	-0.100	-12.8
4 ϕ 16	2.13	0.749	0.616	-0.132	-17.7
3 ϕ 20	2.50	0.751	0.737	-0.013	-1.8

Figure 4-29 – Normalized service stiffness with respect to f_c versus reinforcement ratio

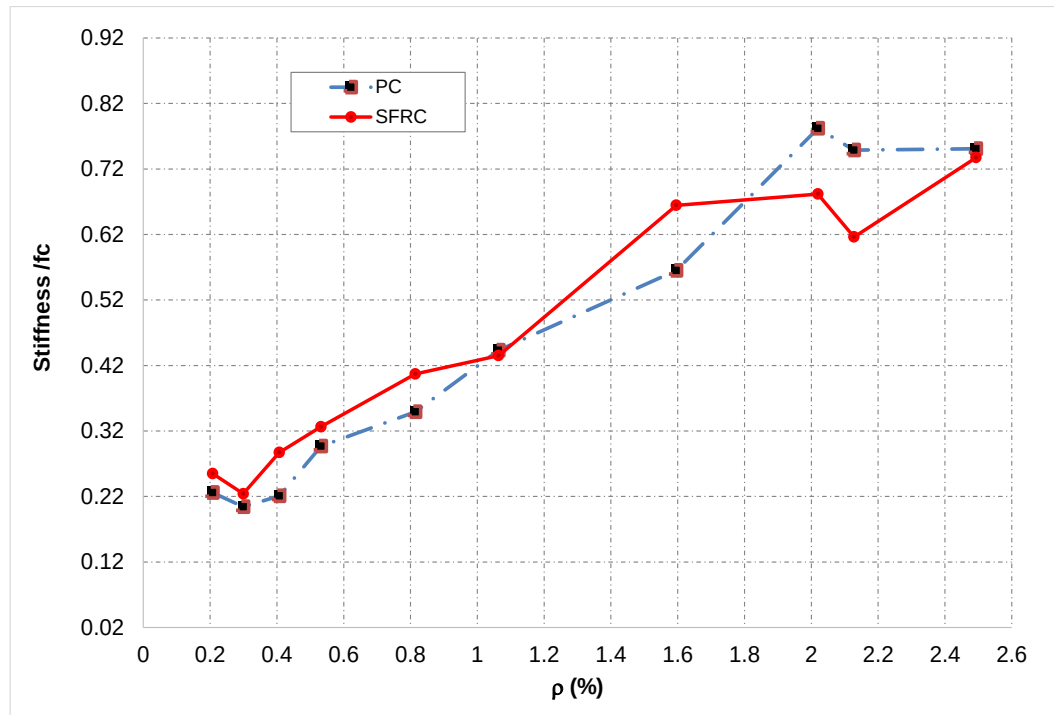


Figure 4-30 – Normalized service stiffness with respect to $\sqrt{f_c}$ versus reinforcement ratio

Based on the results of the service stiffness values of the load-deflection curves, it can be concluded that there is a slight increase in the service stiffness as a result of steel fiber addition. Also there is no clear trend on the ratio of increase of the service stiffness of SFRC specimens with respect to PC specimens with the increasing reinforcement ratio.

4.3.1.3 Toughness

The flexural toughness is the area under the load-deflection graph for a specified deflection value. The ultimate deflection value is defined as the maximum mid-span deflection attained by the specimen during the load test. In this research, toughness is calculated using the ultimate deflection value for each SFRC and PC specimen and the ultimate deflections obtained from the load tests for each specimen are shown in Table 4-8. The variation of ultimate deflection with respect to the reinforcement ratio is shown in Figure 4-31. The figure indicates that for the under-reinforced sections, the ultimate deflection values for PC specimens are greater than that of SFRC specimens. However, as reinforcement ratio increases above 0.40% the ultimate

deflection of SFRC specimens becomes approximately 50 to 300% larger than that of PC specimens.

Table 4-8 – Ultimate deflection values for specimens used to calculate toughness

Specimen	Reinf. Ratio (%)	PC Specimens (mm)	SFRC Specimens (mm)
1 ϕ 10	0.20	110.9	64.5
1 ϕ 12	0.30	109.5	86.2
1 ϕ 14	0.40	107.9	95.3
1 ϕ 16	0.53	104.3	157.0
2 ϕ 14	0.81	92.1	176.5
2 ϕ 16	1.06	56.7	175.7
3 ϕ 16	1.60	41.5	164.2
3 ϕ 18	2.02	59.4	126.0
4 ϕ 16	2.13	70.7	101.2
3 ϕ 20	2.50	70.0	133.0

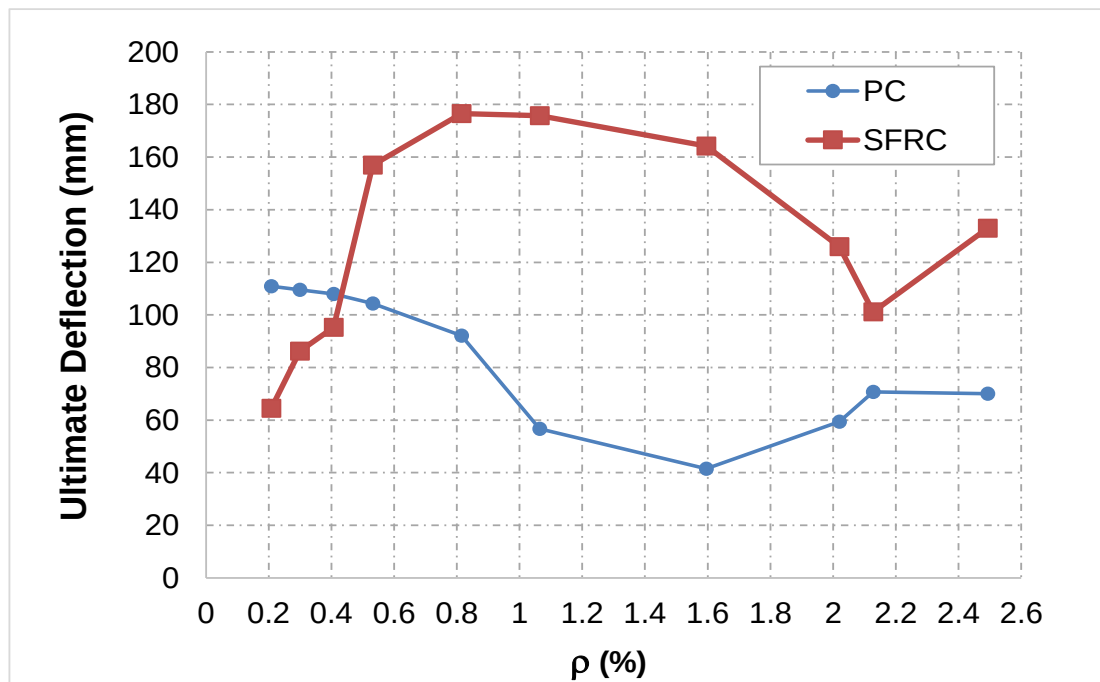


Figure 4-31 – Ultimate deflection values for specimens versus reinforcement ratio

The comparisons of the toughness values of PC and SFRC beams obtained from load-deflection curves are given in Table 4-9 and Figure 4-32. It can be seen from the table and the figure that the SFRC specimens have greater toughness than PC

specimens, and the enhancement is very significant when the reinforcement ratio is over 1.6%.

Table 4-9 – Experimental toughness values for ultimate deflection

Specimen	Reinf. Ratio (%)	PC (kN.mm)	SFRC (kN.mm)	Amount Change (kN.mm)	% Change
1 ϕ 10	0.20	1219.9	1003.7	-216.2	-17.7
1 ϕ 12	0.30	1820.0	1593.3	-226.7	-12.5
1 ϕ 14	0.40	2412.3	2408.5	-3.8	-0.2
1 ϕ 16	0.53	2868.5	5186.8	2318.3	80.8
2 ϕ 14	0.81	3827.8	7582.4	3754.6	98.1
2 ϕ 16	1.06	2674.8	10113.4	7438.6	278.1
3 ϕ 16	1.60	2481.9	11094.3	8612.4	347.0
3 ϕ 18	2.02	3053.6	7472.7	4419.1	144.7
4 ϕ 16	2.13	3854.9	7919.9	4065.0	105.4
3 ϕ 20	2.50	4343.23	8437.8	4094.6	94.3

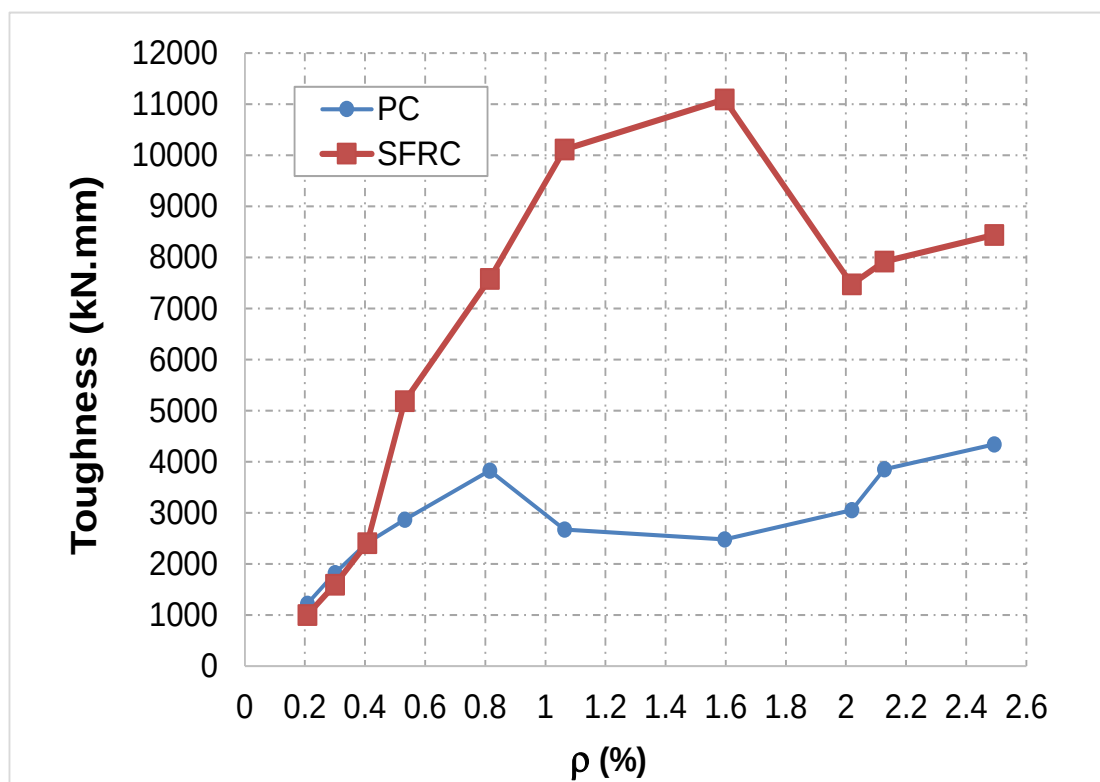


Figure 4-32 – Toughness versus reinforcement ratio for ultimate deflection

Since the ultimate deflection values in the calculation of toughness are different, the toughness values are normalized with respect to the ultimate deflection value used to calculate the toughness value for each specimen by dividing the toughness value to the ultimate deflection value. The results are shown in Figure 4-33. It can be concluded from the figure that SFRC specimens have normalized toughness values always greater than that of PC specimens. As stated above, this difference is very significant when the reinforcement ratio is over 1.6%.

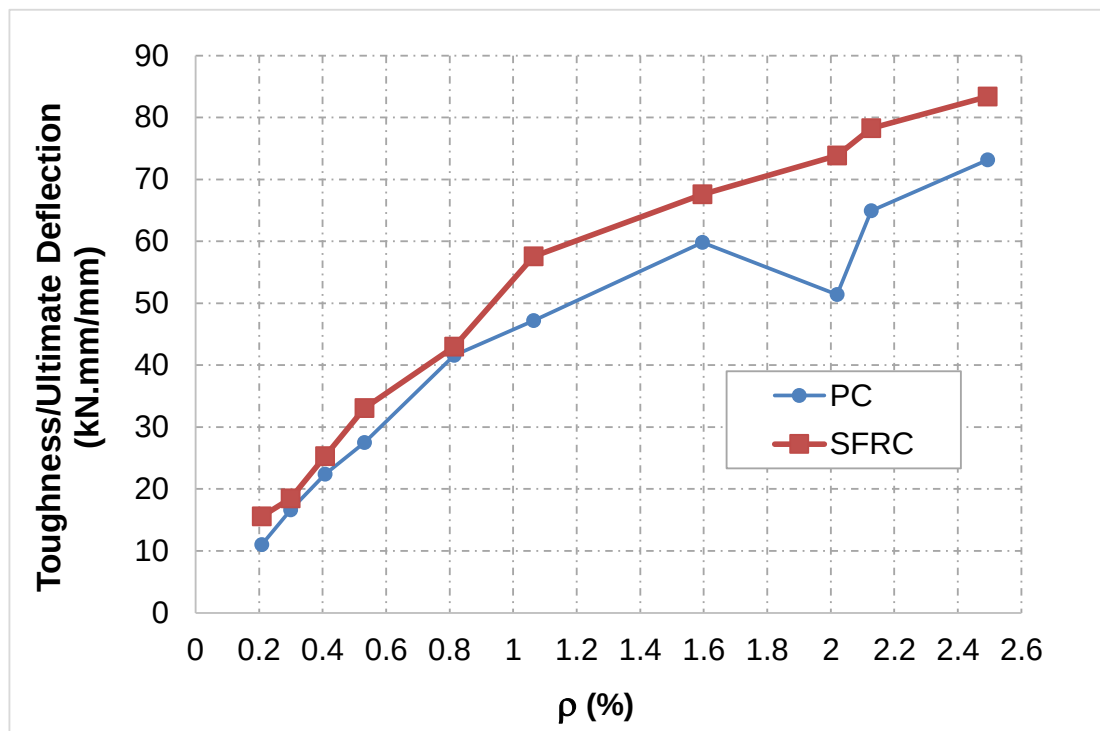


Figure 4-33 – Toughness/ultimate deflection versus reinforcement ratio for ultimate deflection

Since the concrete compressive strength of beam specimens had some variation, the effect of concrete compressive strength on the toughness values was tried to be eliminated by normalizing the curves. In Table 4-10, Table 4-11, Figure 4-34, and Figure 4-35, the toughness values were divided by the concrete compressive strength and square root of the concrete compressive strength of the beams.

Table 4-10 – Normalized toughness values for ultimate deflection with respect to f_c

Specimen	Reinf. Ratio (%)	PC/ f_c (kN.mm/MPa)	SFRC/ f_c (kN.mm/MPa)	Amount Change (kN.mm/MPa)	% Change
1 ϕ 10	0.20	38.1	28.9	-9.2	-24.1
1 ϕ 12	0.30	53.2	39.2	-14.0	-26.3
1 ϕ 14	0.40	71.4	77.4	6.1	8.5
1 ϕ 16	0.53	75.9	124.4	48.5	63.9
2 ϕ 14	0.81	121.9	256.2	134.3	110.1
2 ϕ 16	1.06	76.4	230.4	154.0	201.4
3 ϕ 16	1.60	67.6	347.8	280.2	414.3
3 ϕ 18	2.02	142.0	298.9	156.9	110.5
4 ϕ 16	2.13	150.0	249.1	99.1	66.0
3 ϕ 20	2.50	175.8	378.4	202.5	115.2

Table 4-11 – Normalized toughness values for ultimate deflection with respect to $\sqrt{f_c}$

Specimen	Reinf. Ratio (%)	PC/ $\sqrt{f_c}$ (kN.mm/ $\sqrt{\text{MPa}}$)	SFRC/ $\sqrt{f_c}$ (kN.mm/ $\sqrt{\text{MPa}}$)	Amount Change (kN.mm/ $\sqrt{\text{MPa}}$)	% Change
1 ϕ 10	0.20	215.6	170.4	-45.3	-21.0
1 ϕ 12	0.30	311.2	250.1	-61.2	-19.7
1 ϕ 14	0.40	414.9	431.9	17.0	4.1
1 ϕ 16	0.53	466.6	803.2	336.6	72.2
2 ϕ 14	0.81	683.1	1393.7	710.6	104.0
2 ϕ 16	1.06	452.1	1526.4	1074.3	237.6
3 ϕ 16	1.60	409.7	1964.3	1554.6	379.5
3 ϕ 18	2.02	658.5	1494.5	836.0	126.9
4 ϕ 16	2.13	760.4	1404.5	644.0	84.7
3 ϕ 20	2.50	873.9	1786.8	912.9	104.5

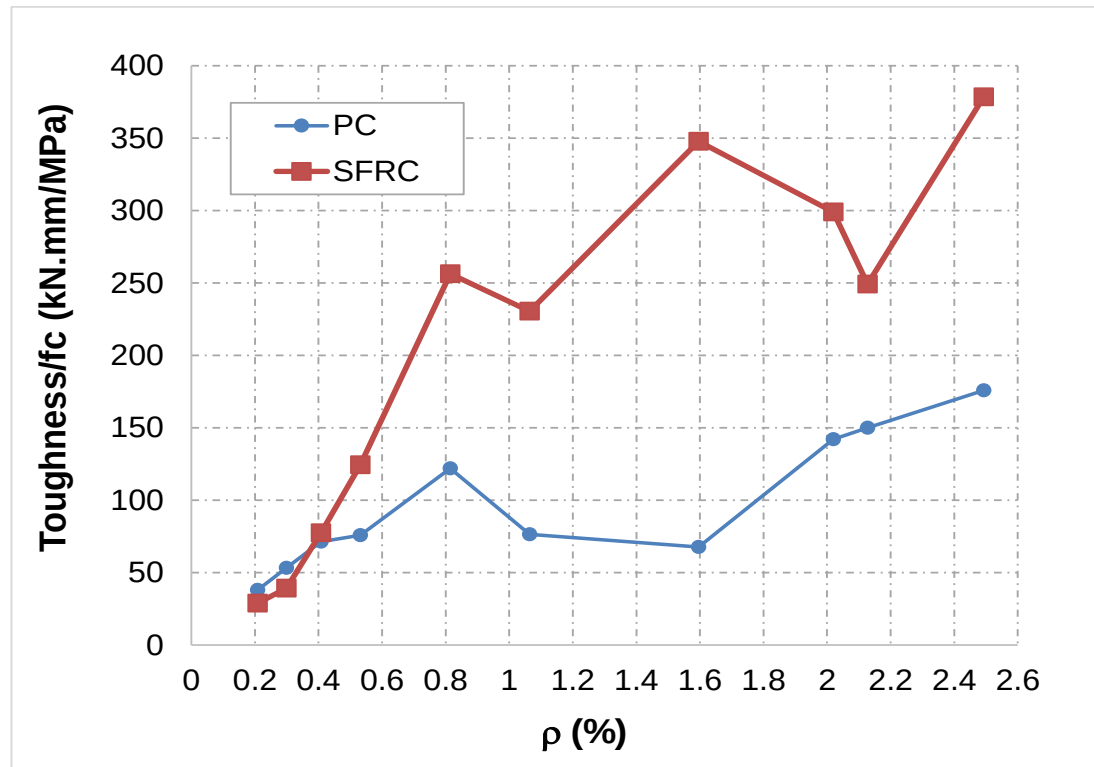


Figure 4-34 – Normalized toughness for ultimate deflection with respect to f_c versus reinforcement ratio

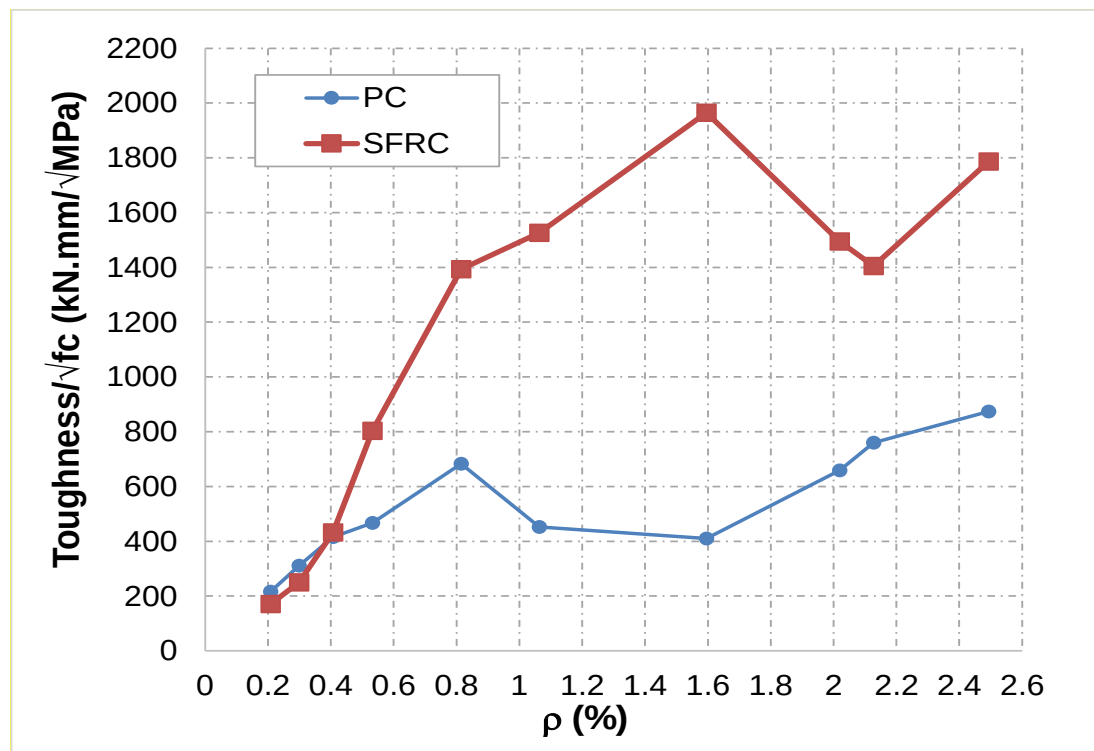


Figure 4-35 – Normalized toughness for ultimate deflection with respect to $\sqrt{f_c}$ versus reinforcement ratio

Since the ultimate deflection values in the calculation of toughness are different, the normalized toughness values with respect to concrete compressive strength are normalized again with respect to the ultimate deflection value used to calculate the toughness value for each specimen by dividing the toughness value to the ultimate deflection value. The results are shown in Figure 4-36 and Figure 4-37. It can be concluded from the figures that SFRC specimens have normalized toughness values always greater than that of PC specimens (except for one specimen).

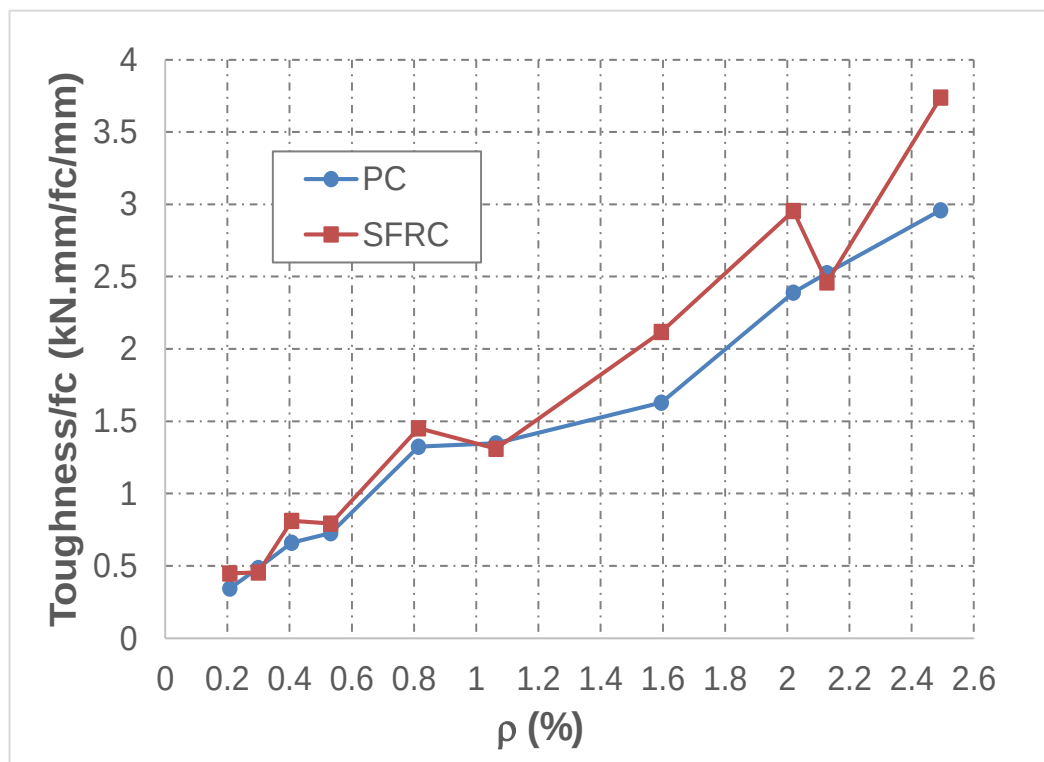


Figure 4-36 – Normalized toughness for concrete compressive strength and ultimate deflection with respect to f_c versus reinforcement ratio

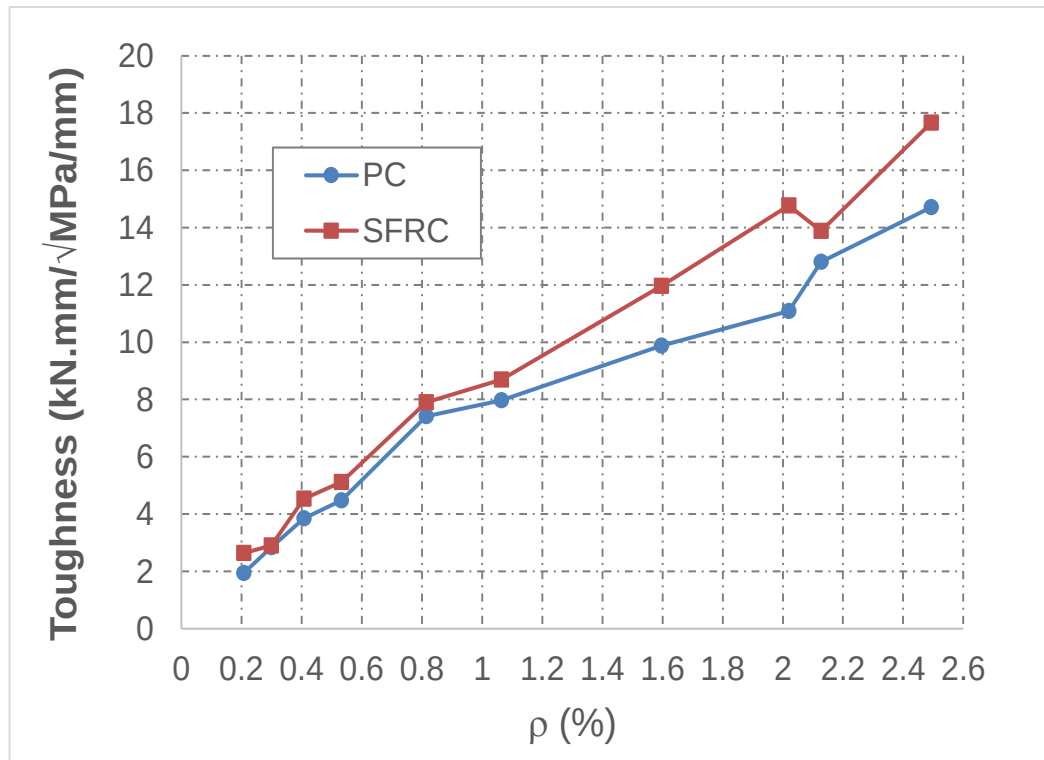


Figure 4-37 – Normalized toughness for concrete compressive strength and ultimate deflection with respect to $\sqrt{f_c}$ versus reinforcement ratio

The toughness values at the deflection corresponding to the ultimate load (defined as $\mu = 1$ in this research) and twice the deflection corresponding the ultimate load (defined as $\mu = 2$ in this research) are also calculated as shown in Table 4-12, Table 4-13, Figure 4-38, and Figure 4-39. It can be seen from the figures and tables that when the reinforcement ratio is over 1.6% (for over-reinforced sections), the difference in toughness is very significant both for $\mu = 1$ and $\mu = 2$. The PC Beam 3 ϕ 16 could not resist a deflection equal to twice the deflection corresponding to the ultimate load therefore, the values for this specimen is not included in Table 4-13 and Figure 4-39.

Table 4-12 – Experimental toughness values for $\mu = 1$

Specimen	Reinf. Ratio (%)	PC (kN.mm)	SFRC (kN.mm)	Amount Change (kN.mm)	% Change
1 ϕ 10	0.20	47.9	133.9	86.0	179.6
1 ϕ 12	0.30	104.6	125.8	21.3	20.3
1 ϕ 14	0.40	181.6	217.7	36.0	19.8
1 ϕ 16	0.53	205.9	290.1	84.1	40.9
2 ϕ 14	0.81	515.1	507.7	-7.4	-1.4
2 ϕ 16	1.06	721.4	706.1	-15.3	-2.1
3 ϕ 16	1.60	1159.9	1061.4	-98.4	-8.5
3 ϕ 18	2.02	1342.2	1603.0	260.8	19.4
4 ϕ 16	2.13	1402.2	1565.9	163.7	11.7
3 ϕ 20	2.50	1876.4	2258.5	382.1	20.4

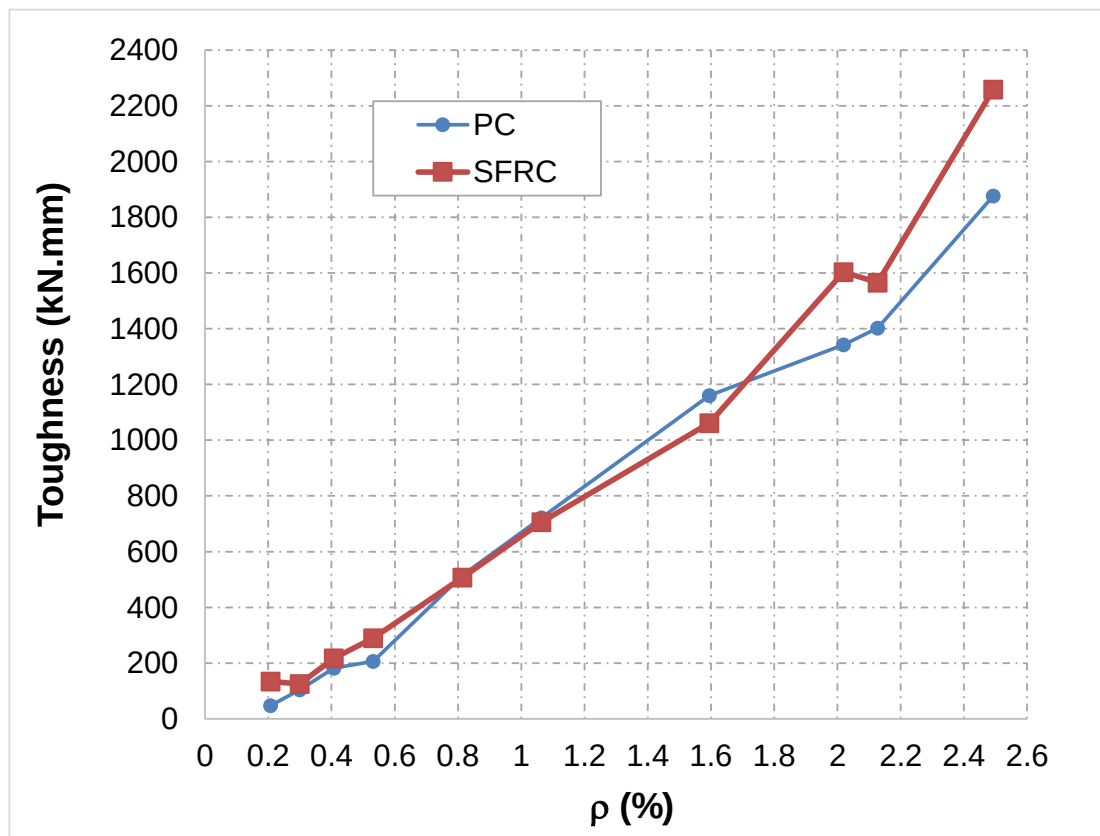
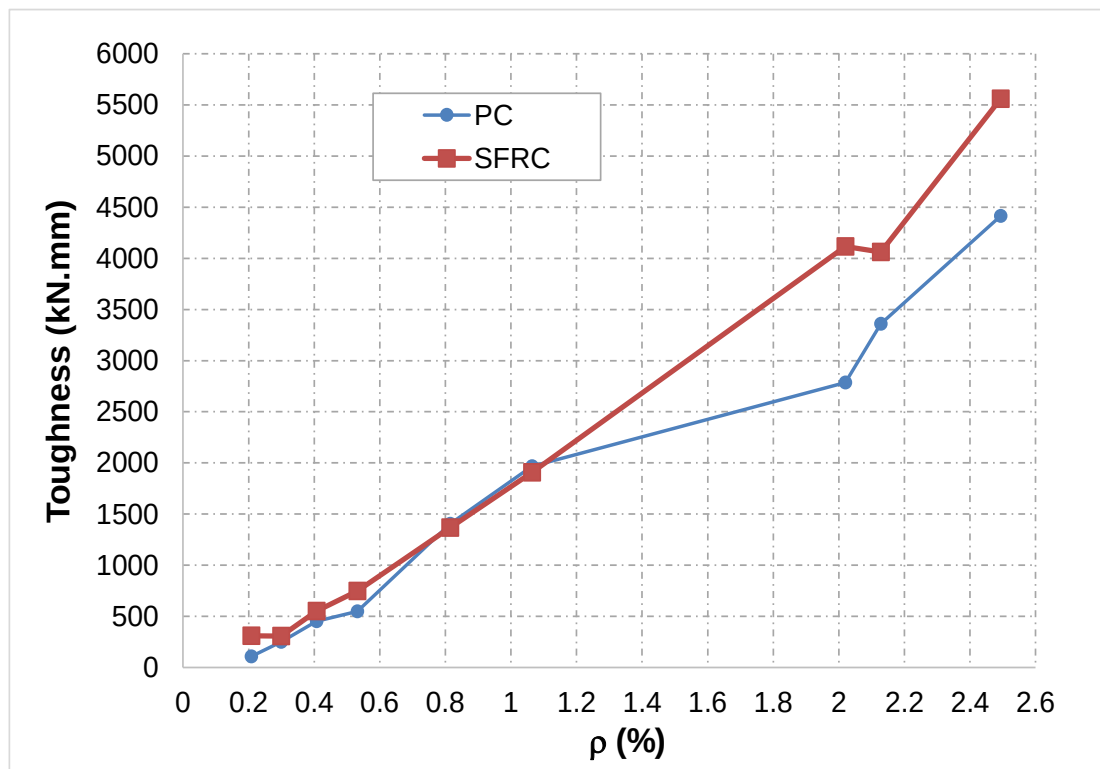
Figure 4-38 – Toughness versus reinforcement ratio for $\mu = 1$

Table 4-13 – Experimental toughness values for $\mu = 2$

Specimen	Reinf. Ratio (%)	PC (kN.mm)	SFRC (kN.mm)	Amount Change (kN.mm)	% Change
1 ϕ 10	0.20	106.7	310.5	203.9	191.1
1 ϕ 12	0.30	250.8	308.1	57.3	22.8
1 ϕ 14	0.40	451.1	550.2	99.1	22.0
1 ϕ 16	0.53	547.4	748.7	201.3	36.8
2 ϕ 14	0.81	1404.4	1366.0	-38.4	-2.7
2 ϕ 16	1.06	1968.0	1908.4	-59.6	-3.0
3 ϕ 16	1.60	-	-	-	-
3 ϕ 18	2.02	2785.8	4115.8	1330.0	47.7
4 ϕ 16	2.13	3359.7	4062.0	702.4	20.9
3 ϕ 20	2.50	4413.7	5560.3	1146.6	26.0

Figure 4-39 – Toughness versus reinforcement ratio for $\mu = 2$

Since the concrete compressive strength of beam specimens had some variation, the effect of concrete compressive strength on the toughness values for $\mu = 1$ and $\mu = 2$ was tried to be eliminated by normalizing the curves. In Table 4-14,

Table 4-15, Table 4-16,

Table 4-17, Figure 4-40, Figure 4-41, Figure 4-42, and Figure 4-43 the toughness values were divided by the concrete compressive strength and square root of the concrete compressive strength of the beams.

Table 4-14 – Normalized toughness values for $\mu = 1$ with respect to f_c

Specimen	Reinf. Ratio (%)	PC/ f_c (kN.mm/MPa)	SFRC/ f_c (kN.mm/MPa)	Amount Change (kN.mm/MPa)	% Change
1 ϕ 10	0.20	1.5	3.9	2.4	157.9
1 ϕ 12	0.30	3.1	3.1	0.0	1.4
1 ϕ 14	0.40	5.4	7.0	1.6	30.2
1 ϕ 16	0.53	5.4	7.0	1.5	27.7
2 ϕ 14	0.81	16.4	17.2	0.7	4.6
2 ϕ 16	1.06	20.6	16.1	-4.5	-22.0
3 ϕ 16	1.60	31.6	33.3	1.7	5.3
3 ϕ 18	2.02	62.4	64.1	1.7	2.7
4 ϕ 16	2.13	54.6	49.2	-5.3	-9.7
3 ϕ 20	2.50	76.0	101.3	25.3	33.3

Table 4-15 – Normalized toughness values for $\mu = 1$ with respect to $\sqrt{f_c}$

Specimen	Reinf. Ratio (%)	PC/ $\sqrt{f_c}$ (kN.mm/ $\sqrt{\text{MPa}}$)	SFRC/ $\sqrt{f_c}$ (kN.mm/ $\sqrt{\text{MPa}}$)	Amount Change (kN.mm/ $\sqrt{\text{MPa}}$)	% Change
1 ϕ 10	0.20	8.5	22.7	14.3	168.5
1 ϕ 12	0.30	17.9	19.7	1.9	10.5
1 ϕ 14	0.40	31.2	39.0	7.8	24.9
1 ϕ 16	0.53	33.5	44.9	11.4	34.1
2 ϕ 14	0.81	91.9	93.3	1.4	1.5
2 ϕ 16	1.06	121.9	106.6	-15.4	-12.6
3 ϕ 16	1.60	191.5	187.9	-3.5	-1.8
3 ϕ 18	2.02	289.5	320.6	31.1	10.8
4 ϕ 16	2.13	276.6	277.7	1.1	0.4
3 ϕ 20	2.50	377.6	478.3	100.7	26.7

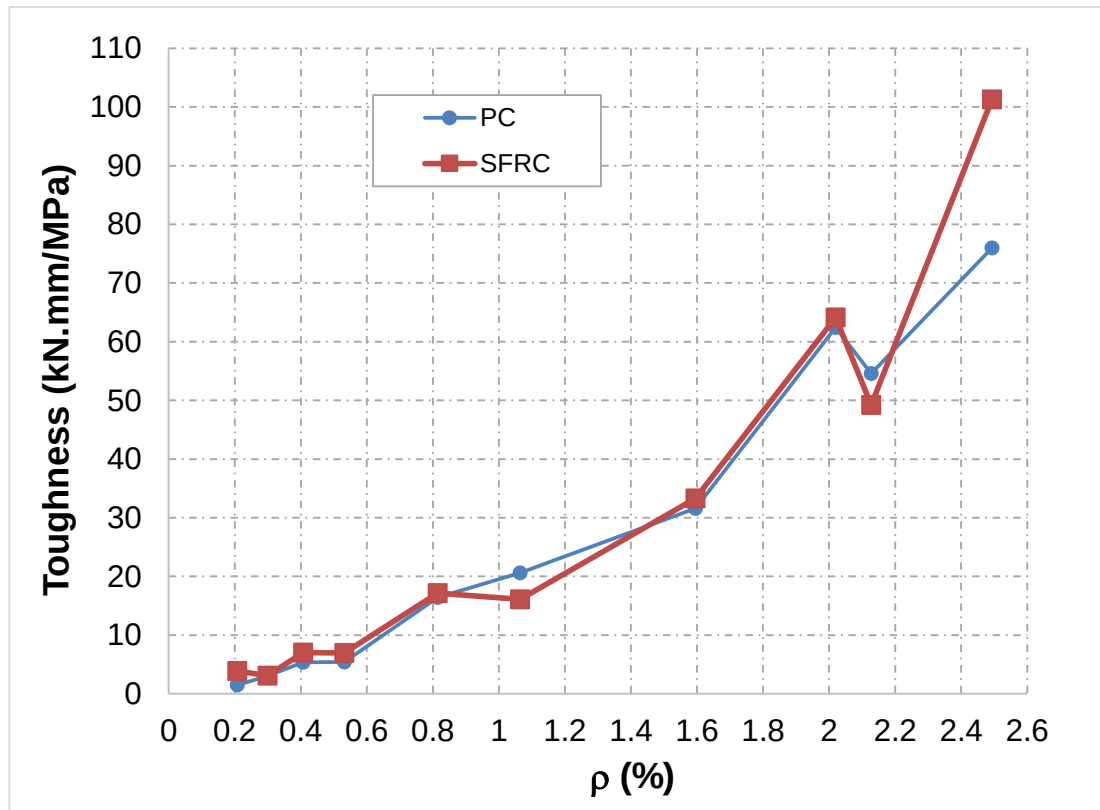


Figure 4-40 – Normalized toughness for $\mu = 1$ with respect to f_c versus reinforcement ratio

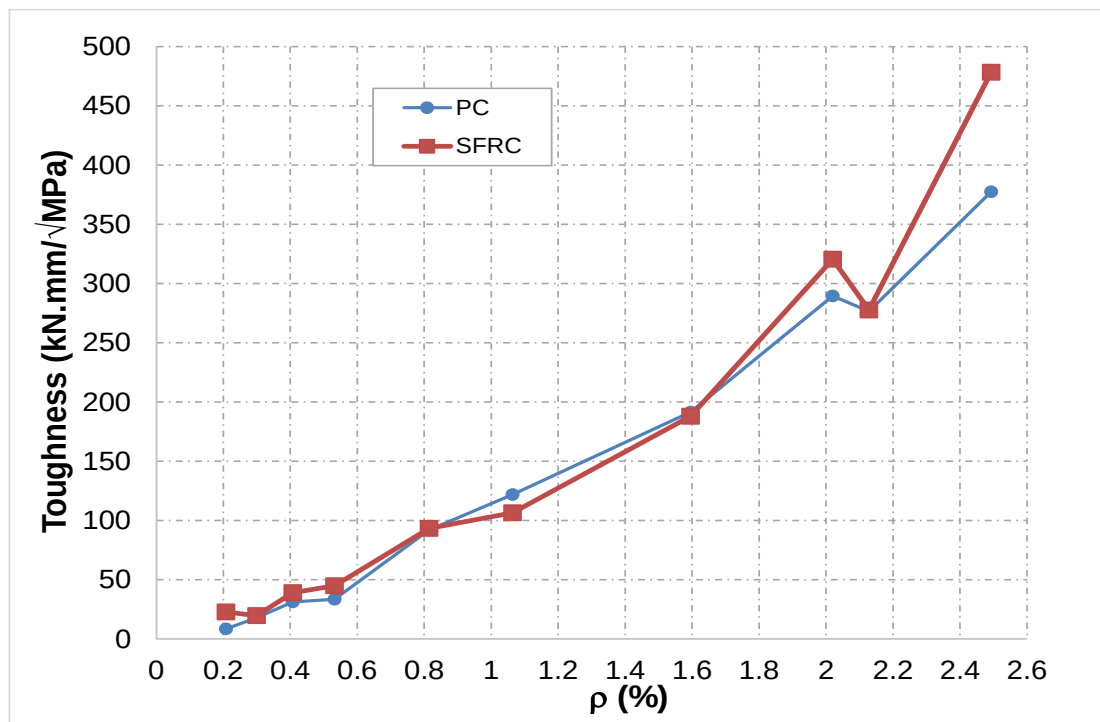


Figure 4-41 – Normalized toughness for $\mu = 1$ with respect to $\sqrt{f_c}$ versus reinforcement ratio

Table 4-16 – Normalized toughness values for $\mu = 2$ with respect to f_c

Specimen	Reinf. Ratio (%)	PC/ f_c (kN.mm/MPa)	SFRC/ f_c (kN.mm/MPa)	Amount Change (kN.mm/MPa)	% Change
1 ϕ 10	0.20	3.1	8.9	5.9	191.1
1 ϕ 12	0.30	6.2	7.6	1.4	22.8
1 ϕ 14	0.40	14.5	17.7	3.2	22.0
1 ϕ 16	0.53	13.1	18.0	4.8	36.8
2 ϕ 14	0.81	47.4	46.1	-1.3	-2.7
2 ϕ 16	1.06	44.8	43.5	-1.4	-3.0
3 ϕ 16	1.60	-	-	-	-
3 ϕ 18	2.02	111.4	164.6	53.2	47.7
4 ϕ 16	2.13	105.6	127.7	22.1	20.9
3 ϕ 20	2.50	197.9	249.3	51.4	26.0

Table 4-17 – Normalized toughness values for $\mu = 2$ with respect to $\sqrt{f_c}$

Specimen	Reinf. Ratio (%)	PC/ $\sqrt{f_c}$ (kN.mm/ $\sqrt{\text{MPa}}$)	SFRC/ $\sqrt{f_c}$ (kN.mm/ $\sqrt{\text{MPa}}$)	Amount Change (kN.mm/ $\sqrt{\text{MPa}}$)	% Change
1 ϕ 10	0.20	18.1	52.7	34.6	191.1
1 ϕ 12	0.30	39.4	48.4	9.0	22.8
1 ϕ 14	0.40	80.9	98.7	17.8	22.0
1 ϕ 16	0.53	84.8	115.9	31.2	36.8
2 ϕ 14	0.81	258.1	251.1	-7.1	-2.7
2 ϕ 16	1.06	297.0	288.0	-9.0	-3.0
3 ϕ 16	1.60	-	-	-	-
3 ϕ 18	2.02	557.2	823.2	266.0	47.7
4 ϕ 16	2.13	595.8	720.3	124.6	20.9
3 ϕ 20	2.50	934.7	1177.5	242.8	26.0

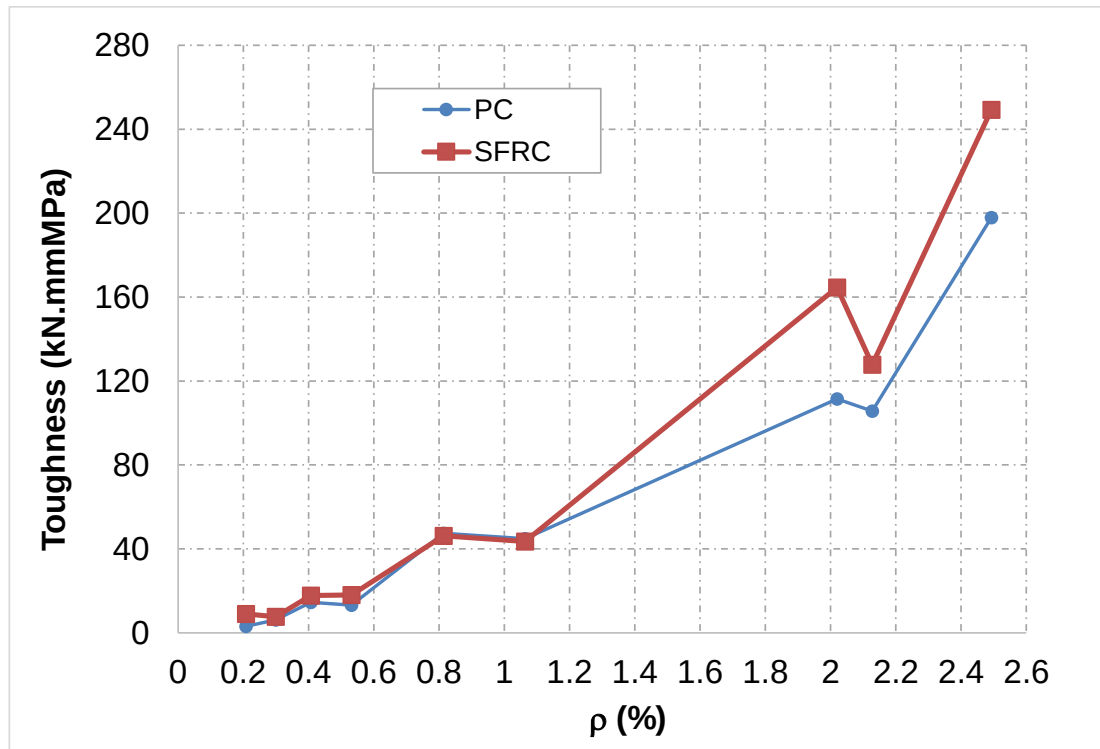


Figure 4-42 – Normalized toughness for $\mu = 2$ with respect to f_c versus reinforcement ratio

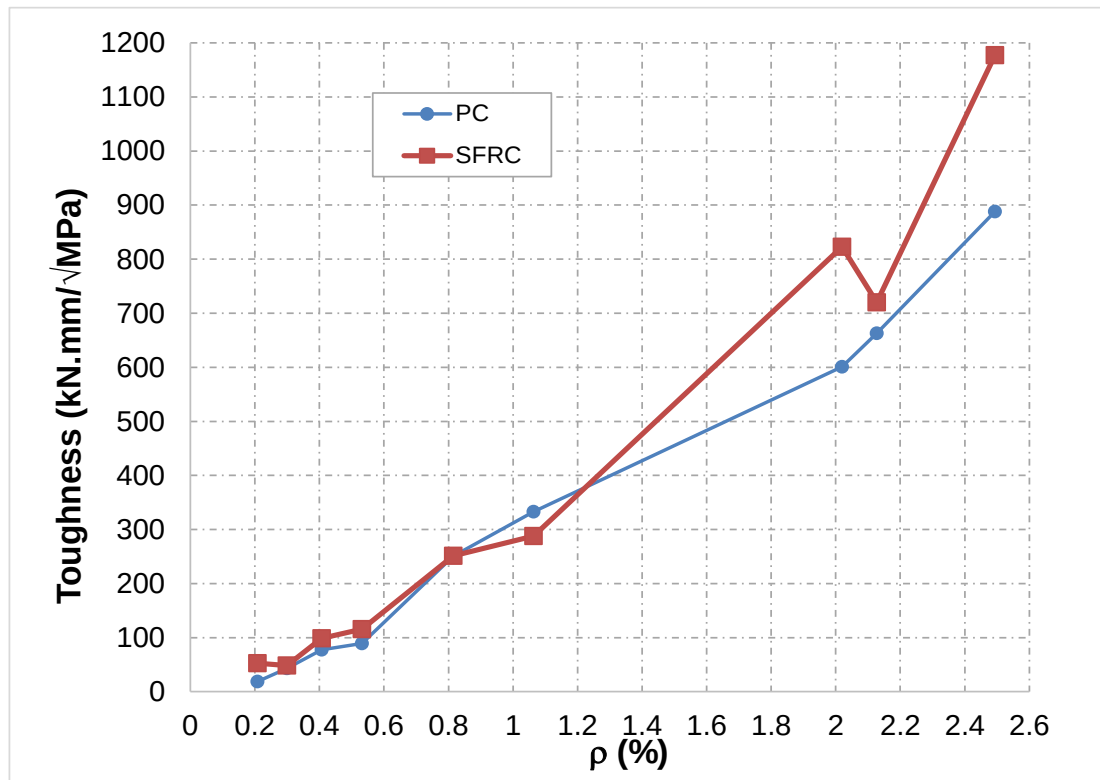


Figure 4-43 – Normalized toughness for $\mu = 2$ with respect to $\sqrt{f_c}$ versus reinforcement ratio

Based on the results of the toughness values of the load-deflection curves, it can be concluded that the toughness of the SFRC specimens are greater than that of PC specimens. Also there is a clear trend that as the reinforcement ratio increases over 1.6% (for over-reinforced sections) toughness increases significantly.

4.3.1.4 Post-Peak Stiffness

Post-peak stiffness for each specimen was obtained by calculating the slope between two points, 90 and 70% of the ultimate load on the descending branch of the load-deflection curve as shown in Figure 4-44. Although different post-peak stiffness values were computed between 50%, 60%, 70%, 80% and 100% of the ultimate load, it was found that the desirable values were those obtained using 70% and 90% of the ultimate load. Hence adopted in the research.

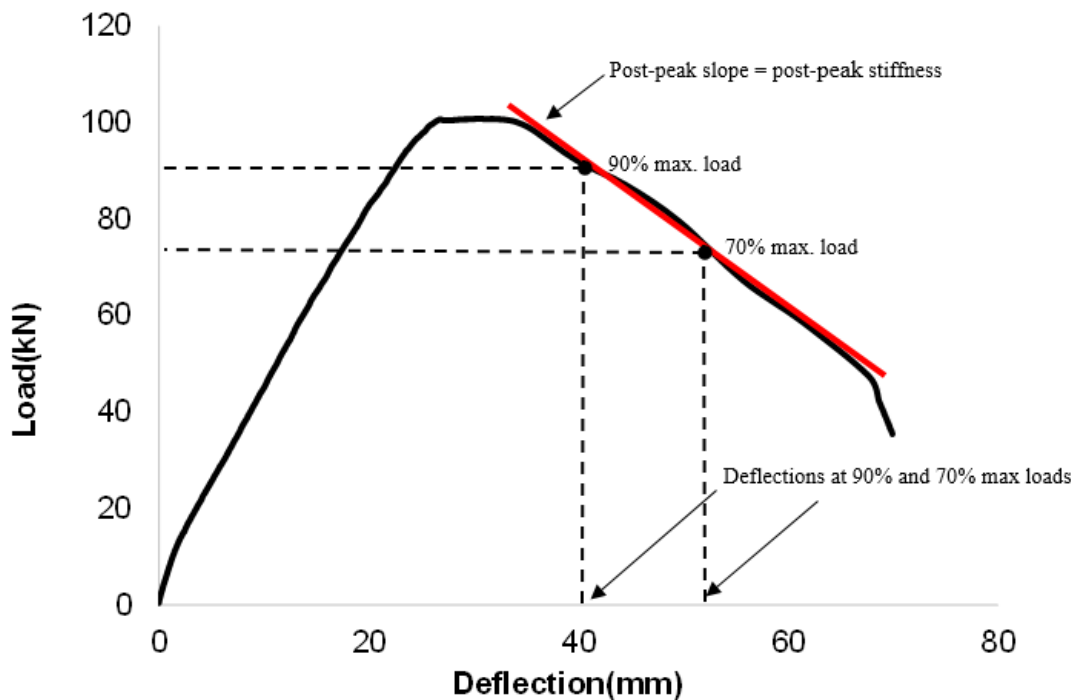


Figure 4-44 – Method of computing post-peak stiffness

In this research, post-peak stiffness values can only be calculated for Beams 3 ϕ 18, 4 ϕ 16, and 3 ϕ 20 (i.e., beams with over-reinforced behavior) due to the shape of the load-deflection curves. The comparisons of the post-peak stiffness values of PC and

SFRC beams obtained from load-deflection curves are shown in Table 4-18 and Figure 4-45. Based on the table and the figure, specimens containing steel fibers possess lower post-peak slope values. In the case of specimens without fibers the specimen having reinforcement ratio of 2.02% possess highest post-peak stiffness value. The post-peak stiffness then decreases for the reinforcement ratio of 2.13% and then it remains almost constant for the reinforcement ratio of 2.50%.

Table 4-18 – Experimental post-peak stiffness values

Specimen	Reinf. Ratio (%)	PC (kN/mm)	SFRC (kN/mm)	Amount Change (kN/mm)	% Change
3 ϕ 18	2.02	3.20	0.40	-2.80	-87.5
4 ϕ 16	2.13	1.53	0.36	-1.17	-76.5
3 ϕ 20	2.50	1.56	0.55	-1.01	-64.7

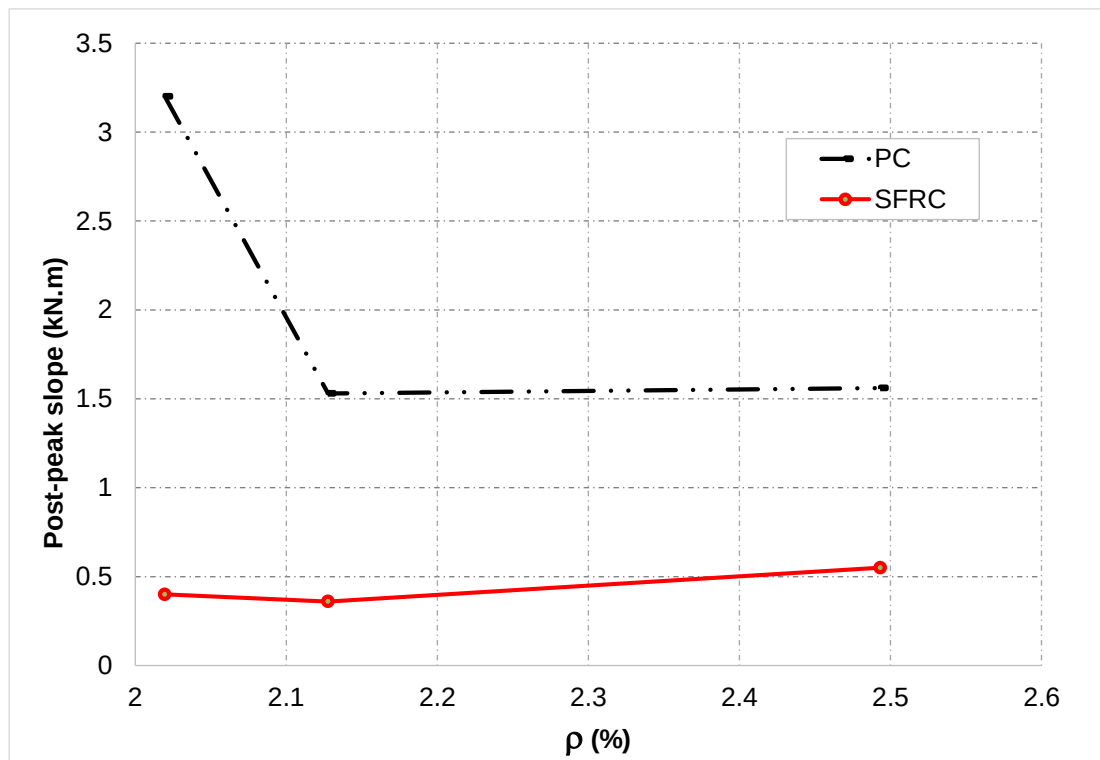


Figure 4-45 – Post-peak stiffness versus reinforcement ratio

Since the concrete compressive strength of beam specimens had some variation, the effect of concrete compressive strength was tried to be eliminated by normalizing the curves. In Table 4-19, Table 4-20, Figure 4-46, and Figure 4-47, the post-peak stiffness values were divided by the concrete compressive strength and square root of

the concrete compressive strength of the beams. For the two normalized graphs the behavior follows the same trend as in the unnormalized case without any significant change in the shape of the curves.

Table 4-19 – Normalized post-peak stiffness values with respect to f_c

Specimen	Reinf. Ratio (%)	PC/ f_c (kN/mm/MPa)	SFRC/ f_c (kN/mm/MPa)	Amount Change (kN/mm/MPa)	% Change
3 ϕ 18	2.02	0.15	0.018	-0.13	-88.0
4 ϕ 16	2.13	0.06	0.014	-0.05	-76.6
3 ϕ 20	2.50	0.063	0.022	-0.04	-65.1

Table 4-20 – Normalized post-peak stiffness values with respect to $\sqrt{f_c}$

Specimen	Reinf. Ratio (%)	PC/ $\sqrt{f_c}$ (kN/mm/ $\sqrt{\text{MPa}}$)	SFRC/ $\sqrt{f_c}$ (kN/mm/ $\sqrt{\text{MPa}}$)	Amount Change (kN/mm/ $\sqrt{\text{MPa}}$)	% Change
3 ϕ 18	2.02	0.69	0.086	-0.604	-87.5
4 ϕ 16	2.13	0.30	0.071	-0.23	-76.3
3 ϕ 20	2.50	0.31	0.110	-0.20	-65.5

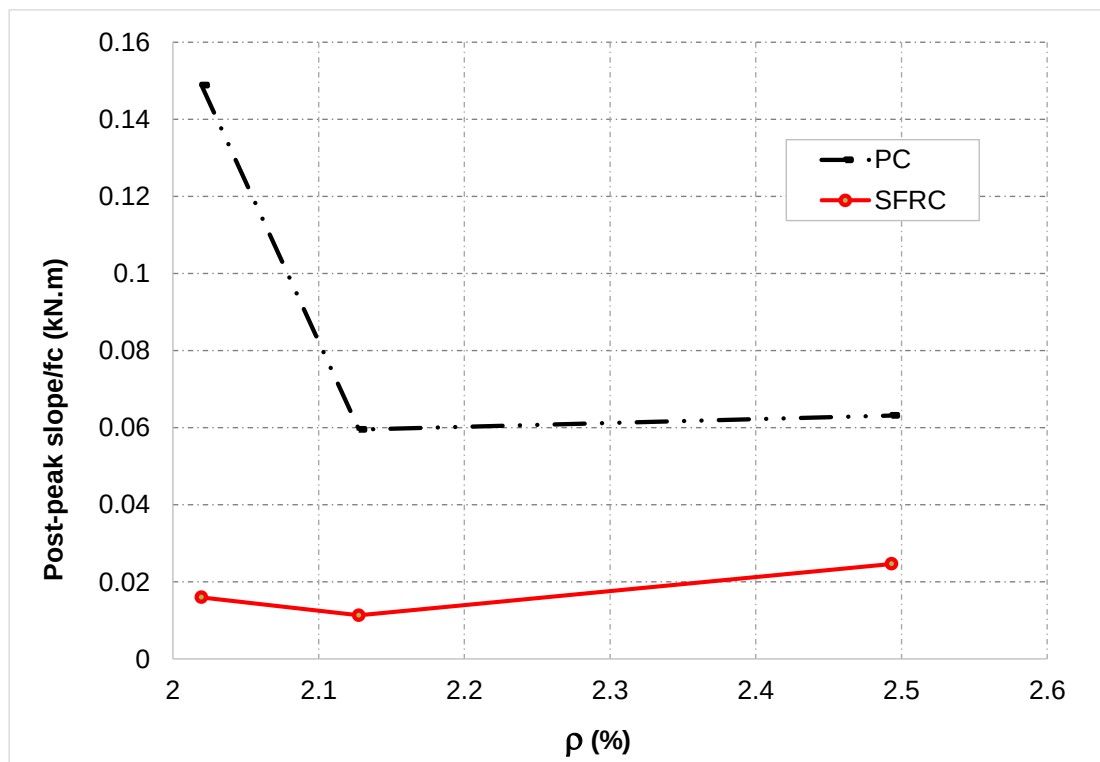


Figure 4-46 – Normalized post-peak stiffness with respect to f_c versus reinforcement ratio

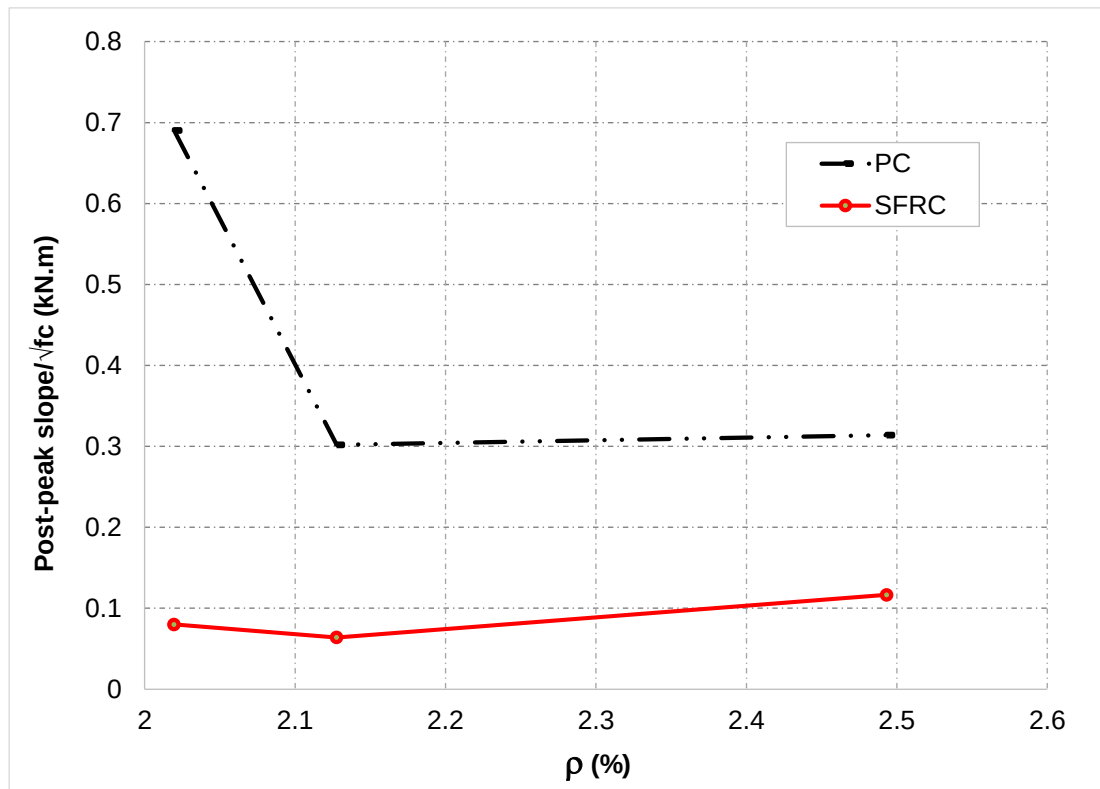


Figure 4-47 – Normalized post-peak stiffness with respect to $\sqrt{f_c}$ versus reinforcement ratio

Based on the results of the post-peak stiffness values of the load-deflection curves, it can be concluded that the post-peak stiffness of the SFRC specimens are significantly lower than that of PC specimens. Higher post-peak slope of PC specimens indicate the higher rate of decrease in load in the post-peak region. When the toughness and post-peak slope diagrams are evaluated together, it can be concluded that the reason for higher toughness values of SFRC specimens is associated with the lower post-peak slope of these specimens.

4.3.2 Bottom Fiber Stretch (Elongation)

The relationships between load and stretch at the bottom fiber of beams for Beams 3 ϕ 18, 4 ϕ 16, and 3 ϕ 20 are presented in Figure 4-48, Figure 4-49, and Figure 4-50. In all of these graphs it is noticed that SFRC specimens developed greater bottom fiber extension for the same load value compared to PC specimens.

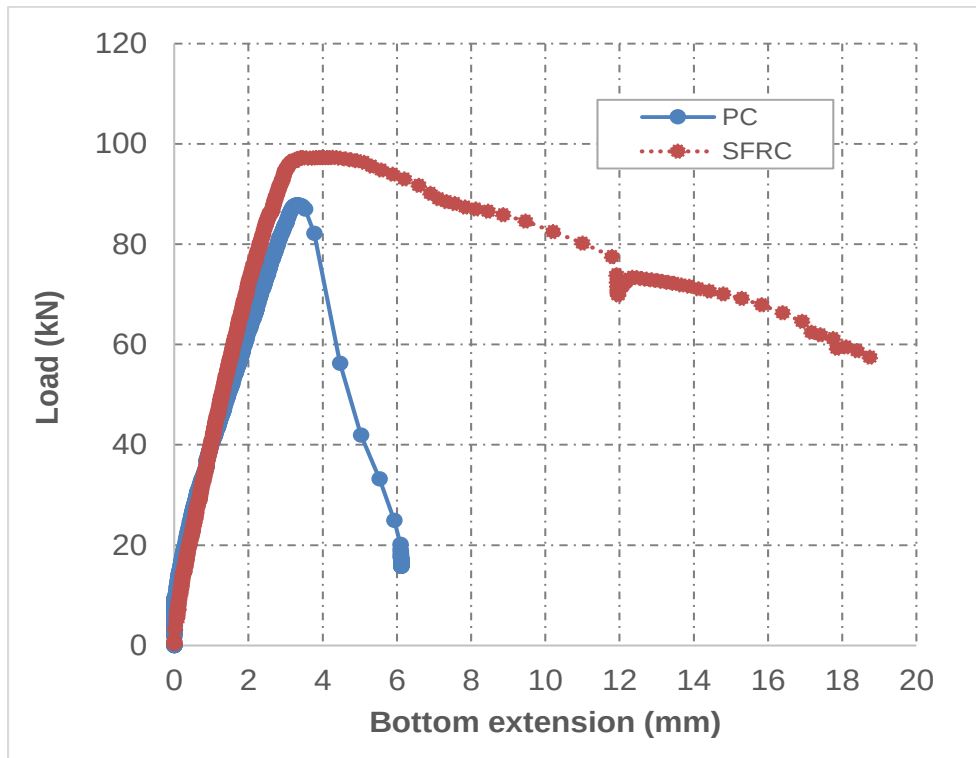


Figure 4-48 – Load versus bottom fiber extension for 3 ϕ 18 specimens ($\rho = 2.02$)

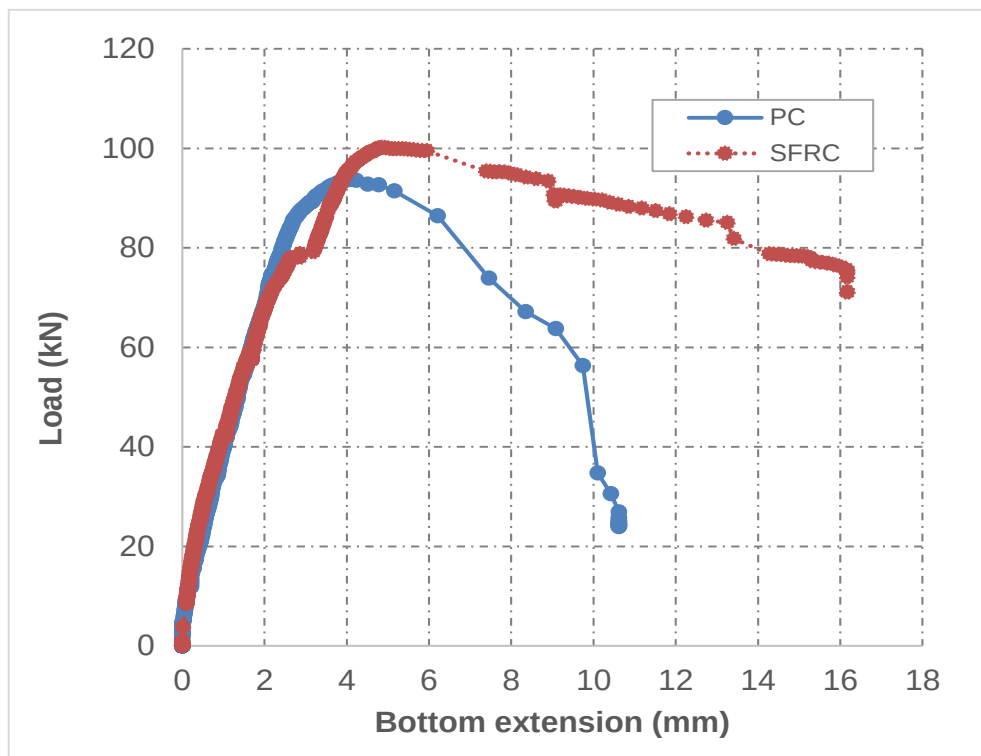


Figure 4-49 – Load versus bottom fiber extension for 4 ϕ 16 specimens ($\rho = 2.13$)

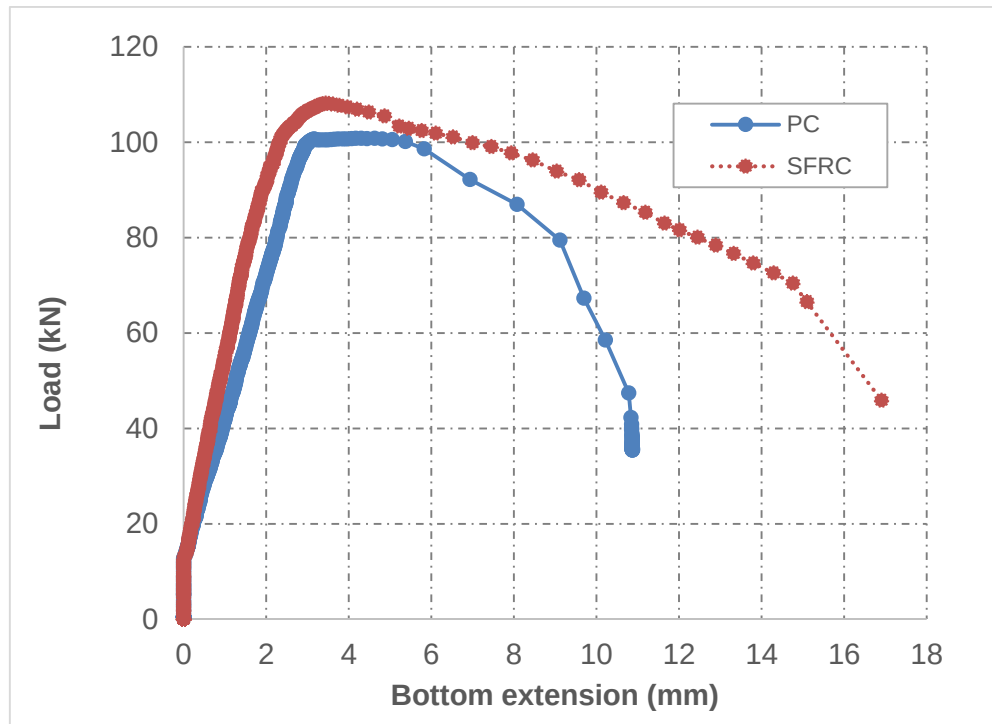


Figure 4-50 – Load versus bottom fiber extension for 3 ϕ 20 specimens ($\rho = 2.50$)

5 ANALYTICAL WORK

5.1 Introduction

In this chapter, analytical work performed to estimate the load-deflection behavior of the tested specimens is presented. For this purpose, a program was written in Microsoft Excel Spreadsheets. Using various proposed stress-strain relationships for SFRC in tension and compression in the literature, the moment-curvature relationship is produced and using these relationships the load deflection curves were obtained for the beam specimens.

5.2 Sectional Analysis Program

The program performs sectional analysis of the beam, produces moment-curvature relationship, and develops the load-deflection curves based on moment-curvature relationship.

In the program, the cross-sectional properties of specimens are defined as follows (Table 5-1):

Table 5-1 – Section properties

Section Properties		
$h=$	250	mm
$d=$	210	mm
$b_w=$	180	mm
$A_s=$	942	mm ²

Where, h is the height of beam, d is the depth of reinforcement, b_w is the width of beam and A_s is the area of reinforcing steel. Area for the reinforcing steel is changed for every different reinforcement ratio.

After defining the section properties, concrete cross-section is divided in to an arbitrary number of layers with equal height throughout the beam height " h ". For a given top strain and a depth of neutral axis, concrete strains and corresponding concrete stresses are determined at the middle height of each layer. The corresponding concrete stress values are calculated using the selected stress-strain

relationship from the literature. For each strain value at the middle height of each layer, the program searches for the stress value from the selected stress-strain relationship. If the strain value is positive, meaning that the concrete is in compression, selected stress-strain relationship for concrete in compression is used. If the strain value is negative, meaning that the concrete is in tension, selected stress-strain relationship for concrete in tension is used. By multiplying the area with the stress, the force at each layer is calculated and the moments of these forces with respect to plastic centroid of the section (mid-height) is calculated for each layer. In this research, the beam height was divided into one hundred and twenty four discrete layers, that is, "n" is equal to 124. The table filled to calculate concrete effect on the cross-section is shown in Table 5-2.

Table 5-2 – Concrete section discretization

Layer No	Height (mm)	Strain	Stress (MPa)	Force (kN)	Moment (kN.m)
1					
2					
...					
...					
...					
n th					

For steel stress, force and moment, a similar calculation is performed. For a given top strain and a depth of neutral axis, steel strain and corresponding steel stress are determined. The corresponding steel stress value is calculated using the selected stress-strain relationship. For the calculated steel strain value, the program searches for the stress value from the selected stress-strain relationship of steel. By multiplying the area of the reinforcement with the stress, the steel force is calculated and then the moment of this force with respect to plastic centroid of the section (mid-height) is calculated. Table 5-3 is filled using this procedure.

Table 5-3 – Total forces and moments

Calculations		
$\varepsilon_c =$		
$c =$		mm
$F_c =$		kN
$M_c =$		kN.m
$\varepsilon_s =$		
$\sigma_s =$		MPa
$F_s =$		kN
$M_s =$		kN.m
Total Force =		KN
Total Moment =		kN.m
Curvature =		1/mm

where ε_c is the concrete strain at the top fiber, c is the depth of neutral axis, F_c is the force in concrete, M_c is the moment due concrete, ε_s is the strain in steel reinforcing bar, σ_s is the stress in reinforcing bar, M_s is the moment due to steel reinforcement, Total Force and Total Moment is the summation of total forces and moments acting on beam section due to concrete and steel reinforcing bar. For a given concrete strain at the top fiber, the program calculates the depth of neutral axis by equating the total forces to zero using the "Goalseek" procedure in the Excel program.

The strain, ε_s , in the reinforcing steel was obtained from strain distribution on the beam section using the concept of similar triangles.

$$\varepsilon_s = \frac{\varepsilon_c(d-c)}{c} \quad \text{Equation 5-1}$$

The stress in steel, σ_s , was obtained from Young's Modulus ($E_s = 200000$ MPa) as:

$$\sigma_s = \varepsilon_s E_s \quad \text{Equation 5-2}$$

The force due to concrete alone F_c , was obtained by summing all forces acting at the middle of each discrete layer. The force due to steel reinforcement F_s was obtained from the expression:

$$F_s = \sigma_s A_s$$

Equation 5-3

Moment due to steel bar (M_s) was obtained by multiplying the force due to steel bar by the distance at which it is acting. While moments due to concrete was obtained by summing all moments acting on each layer of the beam.

$$M_s = F_s(d - \frac{h}{2})$$

Equation 5-4

Actual moment on the beam is that due to steel bar and those due to concrete. At the value of the actual moment obtained, the corresponding curvature is determined.

$$\text{Total Moment} = M_c + M_s$$

Equation 5-5

$$\text{Curvature: } \phi = \varepsilon_c / c$$

Equation 5-6

By changing top strain at the top concrete compression fiber starting from 0 to maximum top concrete strain in increments and running the program to find the depth of neutral axis for each top concrete strain, total moment and corresponding curvature is obtain for each top concrete strain. Table 5-4 is filled using this procedure to get the moment curvature relationship.

Table 5-4– Moment and curvature values

Top Concrete Strain	Moment (kN.m)	K (1/mm)
0	0	0
0.0001		
0.0002		
...		
...		
maximum top concrete strain		

Using the moment-curvature curves generated above, the load-deflection graphs for each load increment are plotted using the curvature-area method.

5.3 Stress Strain Models used in Analysis

5.3.1 Reinforcing Steel Model

The selected stress-strain relationship for steel was obtained from coupon tests performed in our laboratory (Figure 3-4).

5.3.2 SFRC Model in Tension

The model used for SFRC in tension was developed by Soranakom and Mobasher (2009) as shown in Figure 2-12. In their research, the parameter μ which is the post-crack strength as a fraction of the cracking tensile strength is offered to be taken between 0 and 1.0. A parametric study was performed to determine the value of the parameter μ , that fits best to our test data. The results of the analysis for a typical under-reinforced section and over-reinforced section are shown in Figure 5-1 and Figure 5-2. It was observed that the effect of tension model used in the analysis is very significant in under-reinforced sections. This effect becomes insignificant when the reinforcement ratio of the section increases. Based on this parametric study on under-reinforced sections using various values for the parameter μ , the value 0.2 is selected. This value is used throughout the section analysis in the next section.

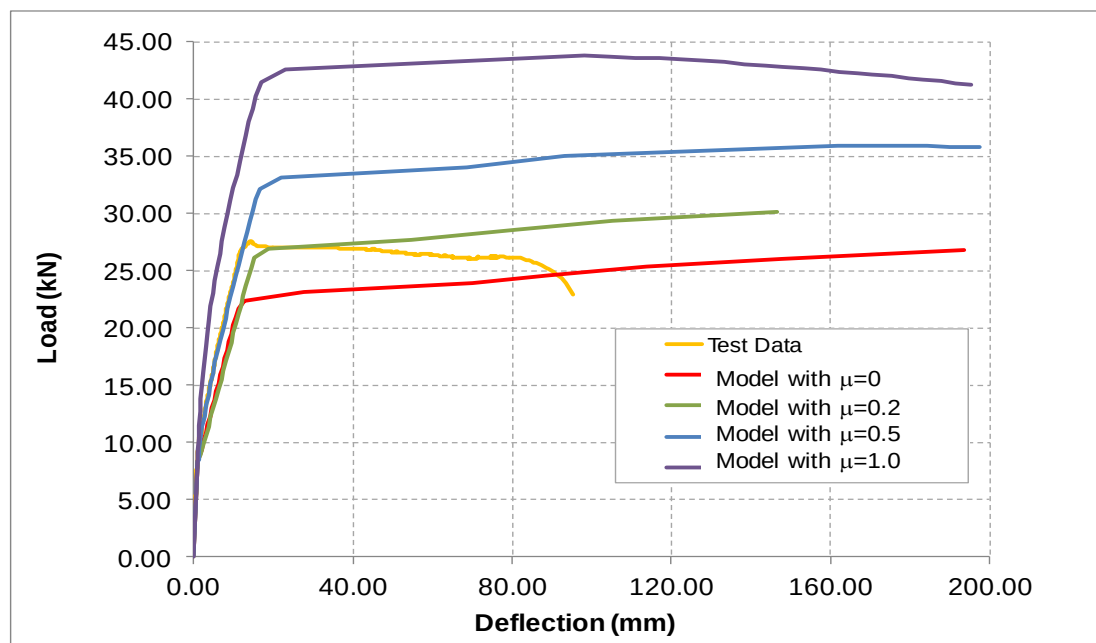


Figure 5-1 – Parametric study on μ for the tension model by Soranakom and Mobasher (2009) (Beam SFRC 1 ϕ 14)

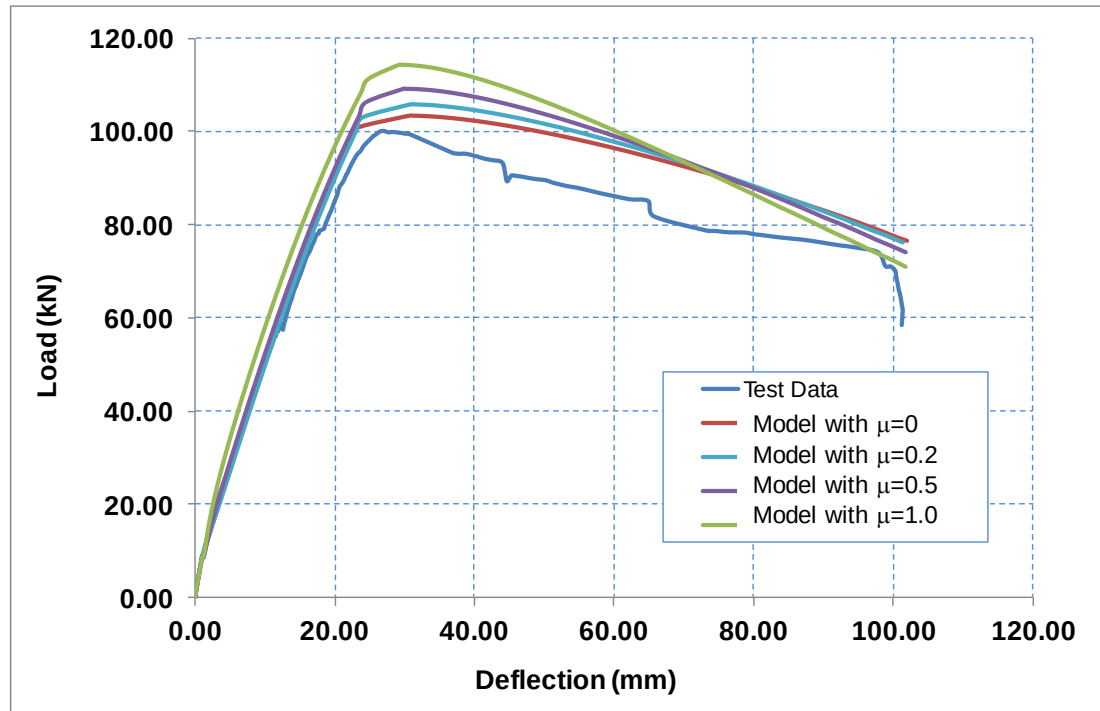


Figure 5-2 – Parametric study on μ for the tension model by Soranakom and Mobasher (2009) (Beam SFRC 4 ϕ 16)

5.3.3 SFRC Model in Compression

The expression suggested by Ezeldin and Balaguru (1992) as shown in Equation 2-12, Nataraja et al. (1999) as shown in Equation 2-21, and Wang (2006) as shown in Equation 2-38 was used as the SFRC models in compression.

5.4 Analytical Work on the SFRC Models in Compression

The load-deflection relationships of the tested SFRC specimens and the estimated load-deflection curves calculated using the models explained previously are shown in Figure 5-3 to Figure 5-12.

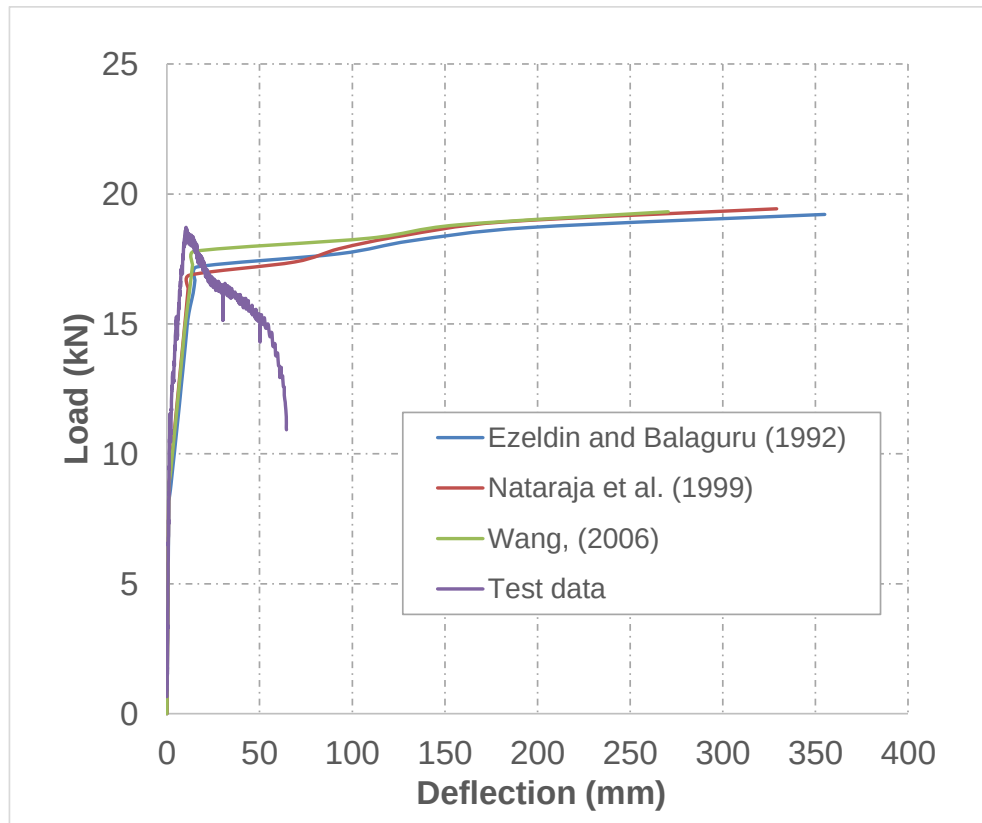


Figure 5-3 – Estimations of load-deflection behavior of SFRC Beam 1φ10

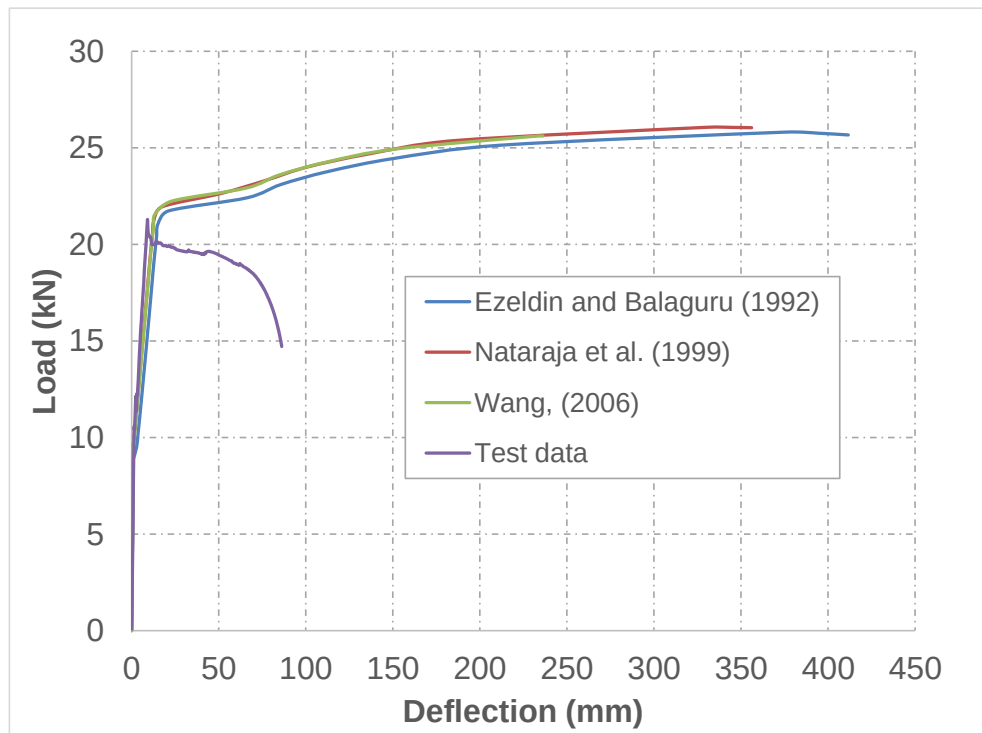


Figure 5-4 – Estimations of load-deflection behavior of SFRC Beam 1φ12

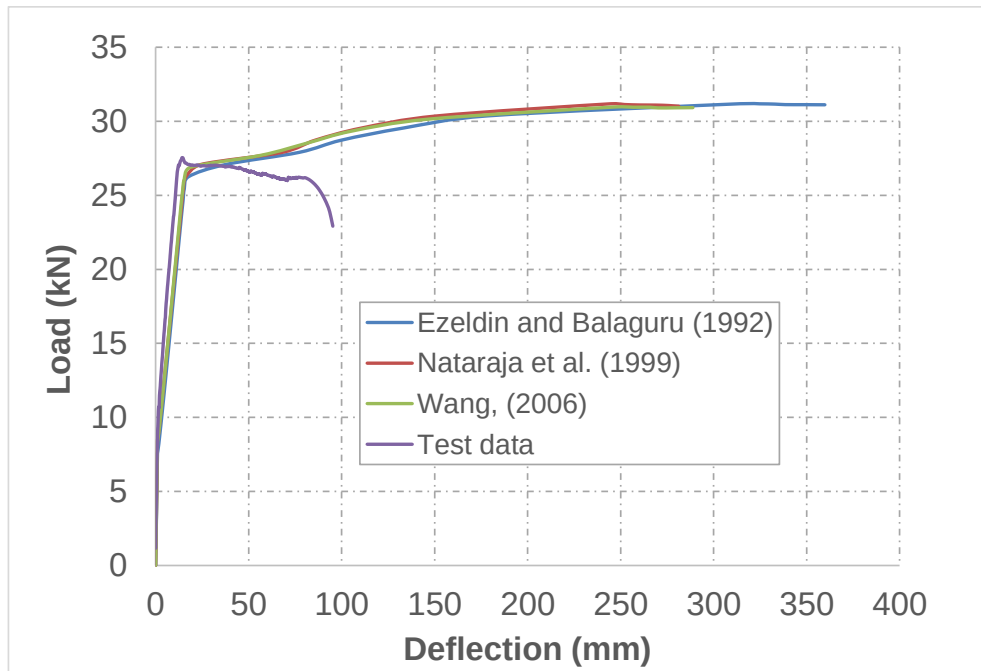


Figure 5-5 – Estimations of load-deflection behavior of SFRC Beam 1φ14

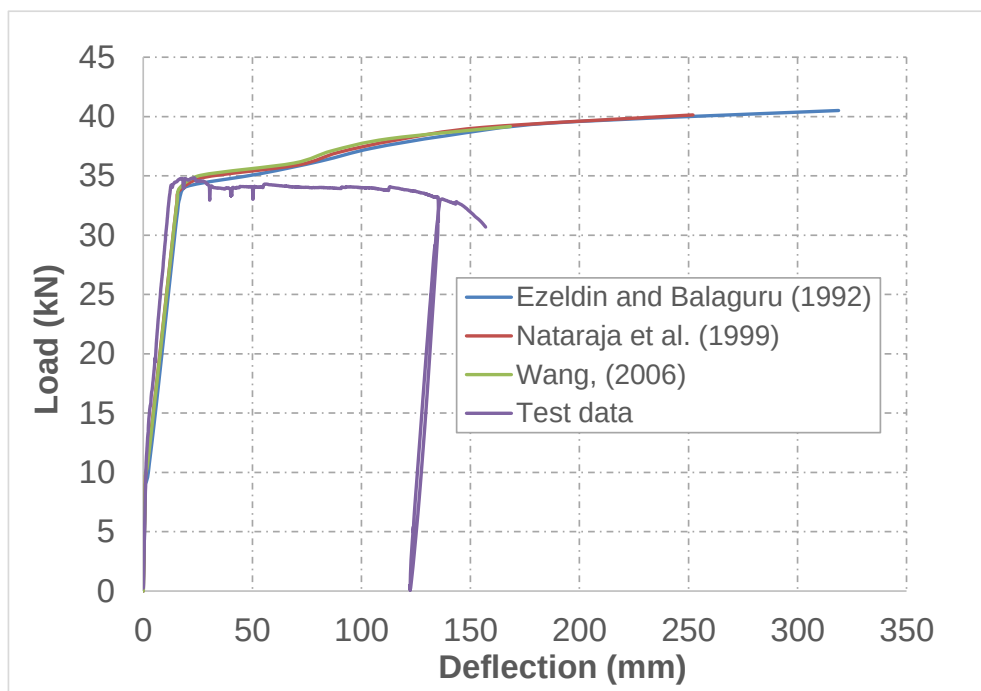


Figure 5-6 – Estimations of load-deflection behavior of SFRC Beam 1φ16

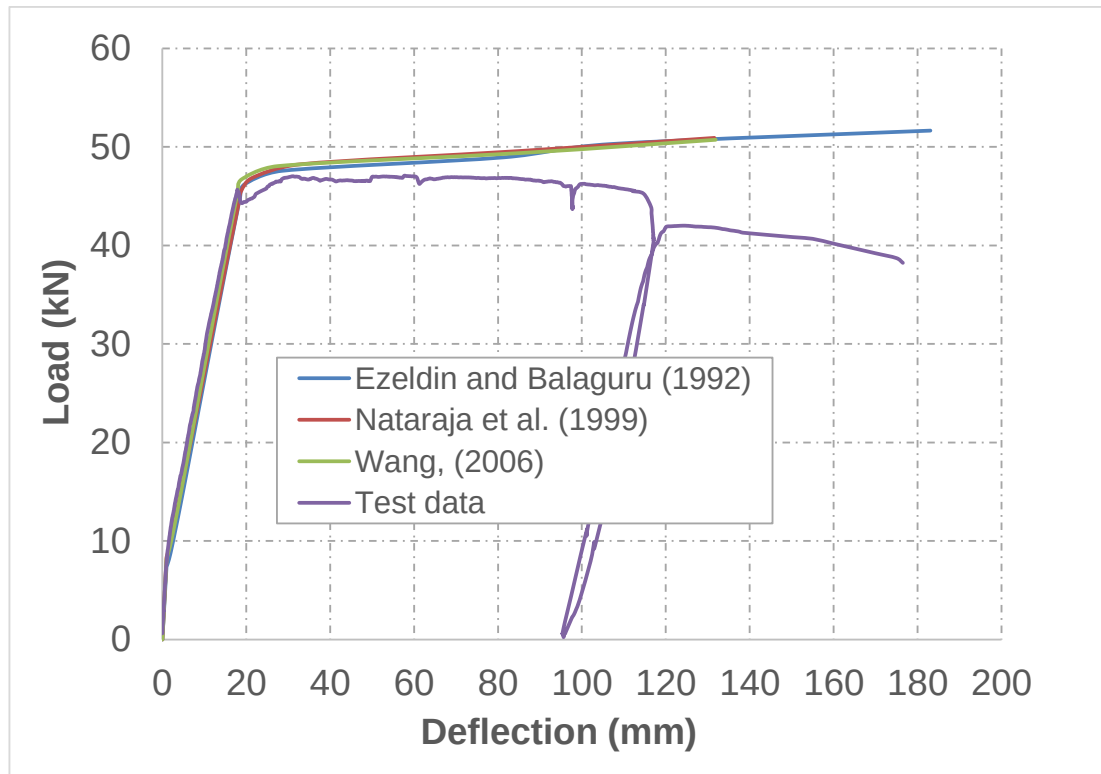


Figure 5-7 – Estimations of load-deflection behavior of SFRC Beam 2φ14

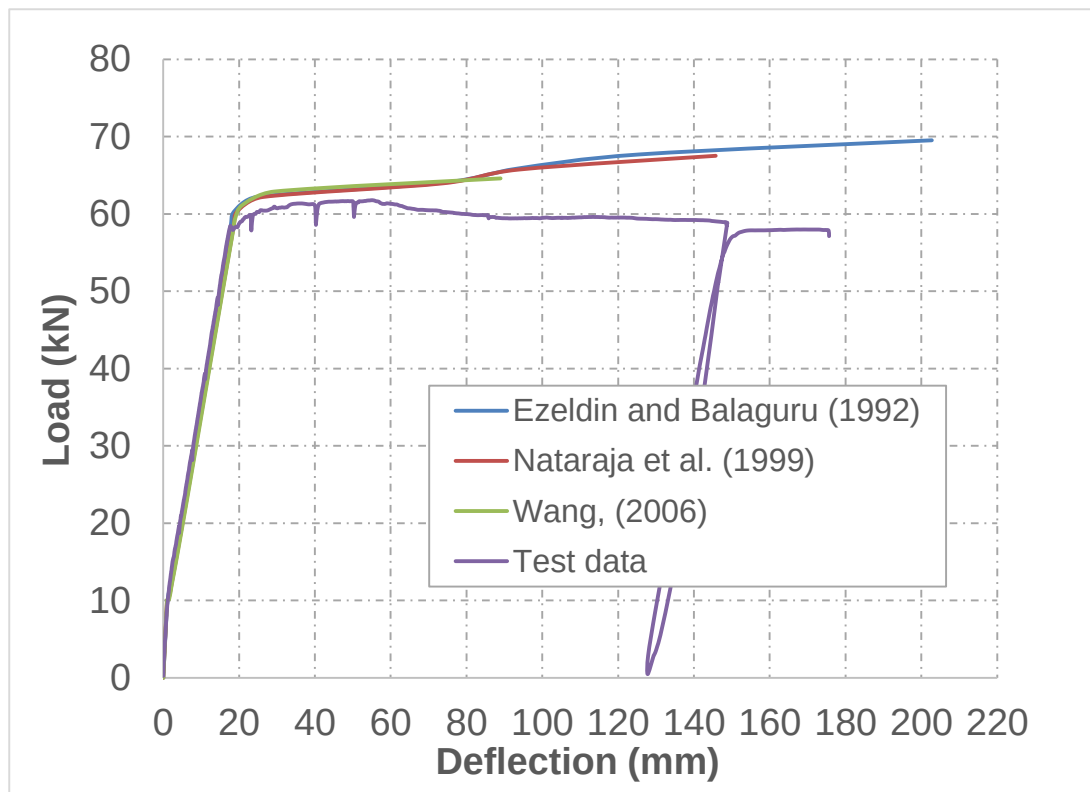


Figure 5-8 – Estimations of load-deflection behavior of SFRC Beam 2φ16

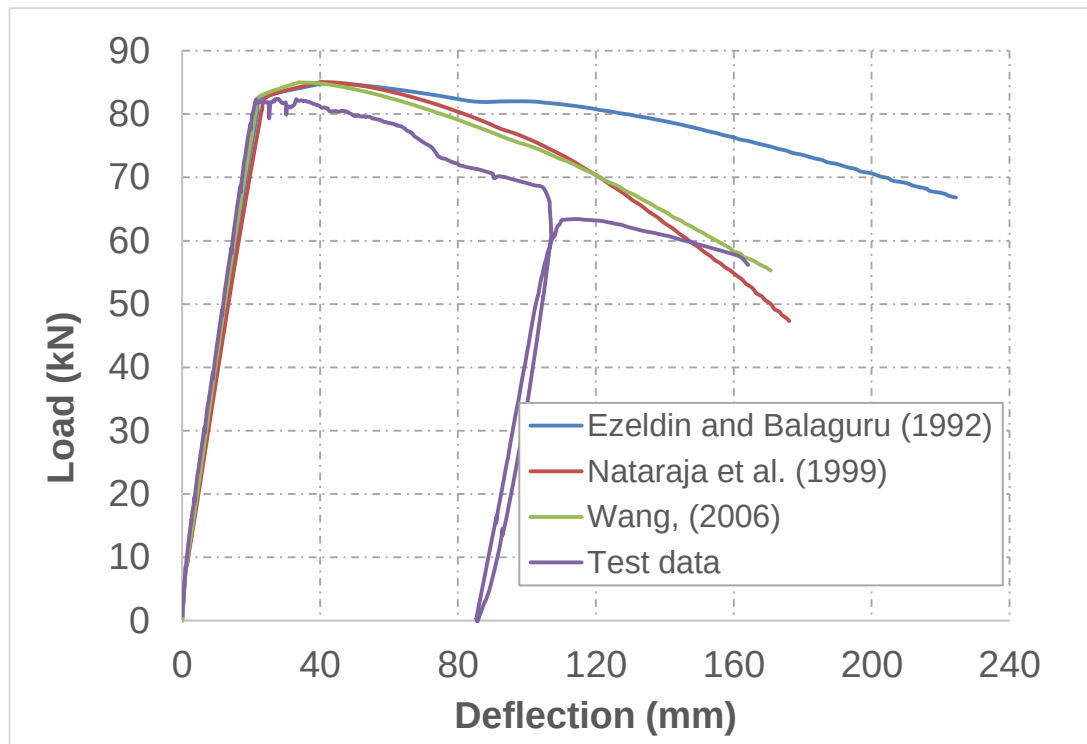


Figure 5-9 – Estimations of load-deflection behavior of SFRC Beam 3φ16

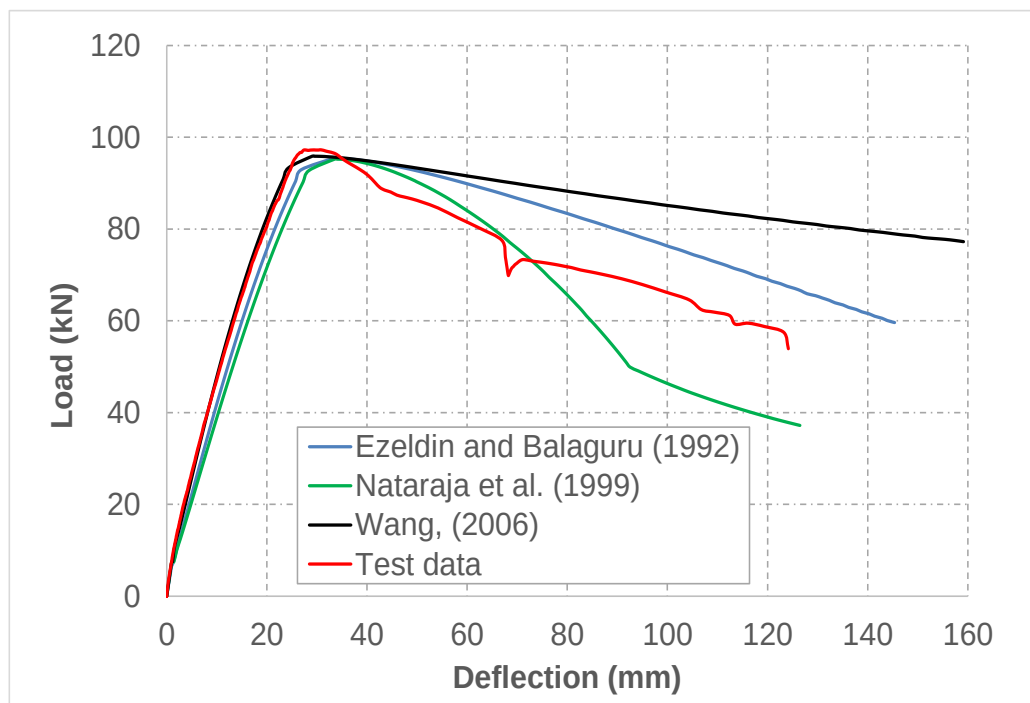


Figure 5-10 – Estimations of load-deflection behavior of SFRC Beam 3φ18

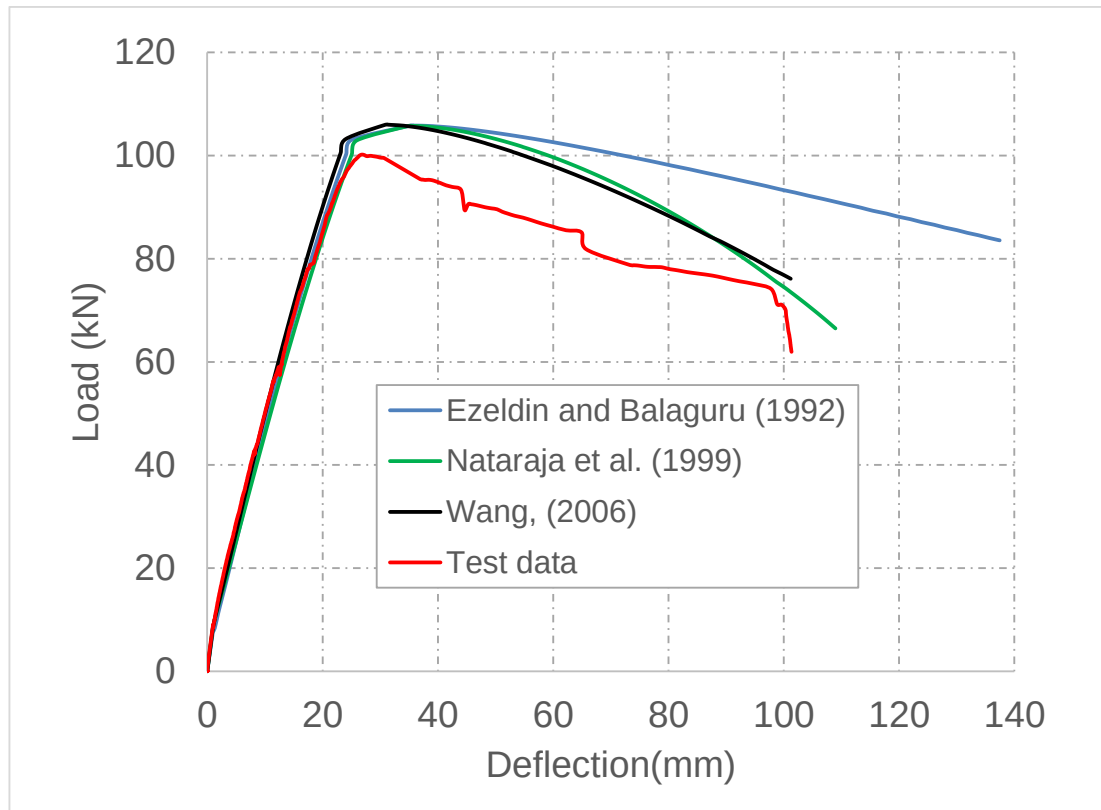


Figure 5-11 – Estimations of load-deflection behavior of SFRC Beam 4φ16

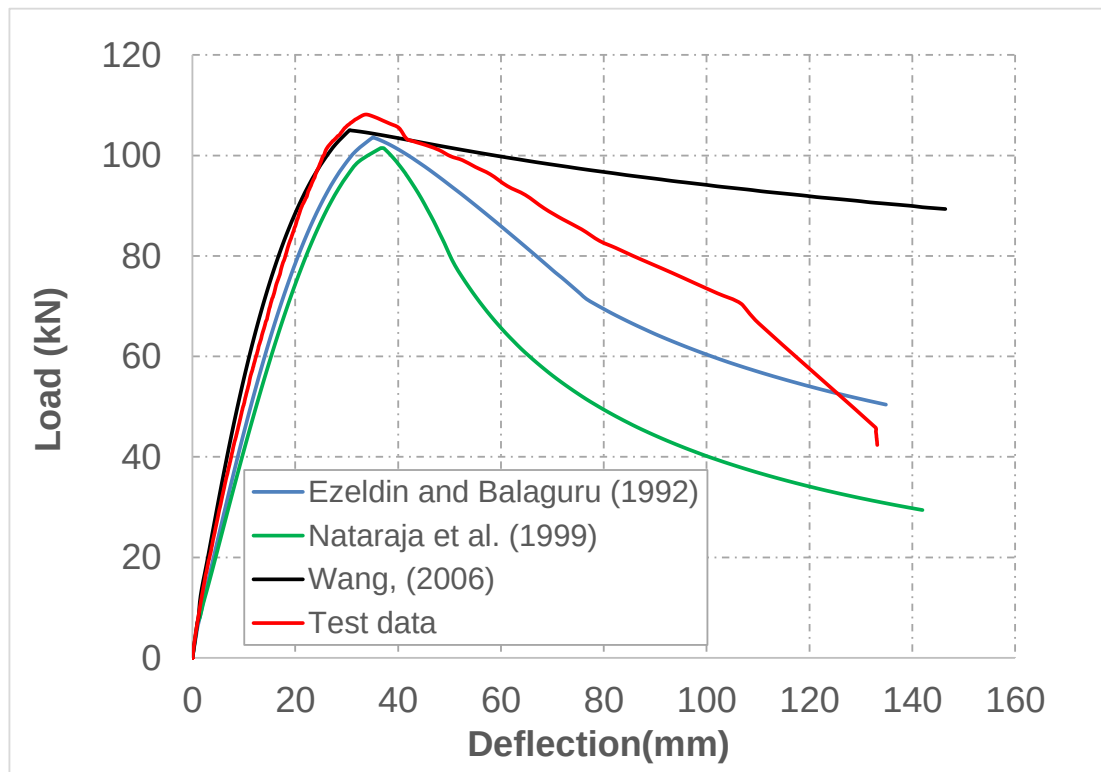


Figure 5-12 – Estimations of load-deflection behavior of SFRC Beam 3φ20

The experimental and the estimated load deflection behaviors are compared to each other in terms of ultimate load, initial stiffness, post-peak stiffness and toughness characteristics.

The estimation of ultimate load values for under- and over-reinforced sections are shown in Table 5-5 and Table 5-6. For under-reinforced sections, the ultimate load is best estimated by Wang (2006) with an average of 9.2% estimation difference. For over-reinforced sections, Ezeldin and Balaguru (1992) estimated the ultimate load behavior with an average of 0.2% estimation difference.

Table 5-5 – Estimation of ultimate load values for under-reinforced sections

Specimen	Test data	Nataraja et al. (1999)	%	Wang (2006)	%	Ezeldin and Balaguru (1992)	%
1 ϕ 10	18.7	19.4	3.9	19.3	3.3	19.2	2.7
1 ϕ 12	21.3	26.1	22.4	25.6	20.3	25.8	21.2
1 ϕ 14	27.5	31.2	13.4	31.0	12.6	31.2	13.4
1 ϕ 16	34.8	40.1	15.4	39.2	12.6	40.5	16.4
2 ϕ 14	47.1	50.9	8.1	50.7	7.7	51.7	9.7
2 ϕ 16	61.8	67.5	9.2	64.6	4.5	69.5	12.5
3 ϕ 16	82.4	85.1	3.2	85.0	3.1	84.9	3.0
		Maximum Value=	22.4	Maximum Value=	20.3	Maximum Value=	21.2
		Minimum Value=	3.2	Minimum Value=	3.1	Minimum Value=	2.7
		Average=	10.8	Average=	9.2	Average=	11.3

Table 5-6 – Estimation of ultimate load values for over-reinforced sections

Specimen	Test data	Nataraja et al. (1999)	%	Wang (2006)	%	Ezeldin and Balaguru (1992)	%
3 ϕ 18	97.3	95.3	-2.1	95.9	-1.4	95.5	-1.9
4 ϕ 16	100.2	105.8	5.6	106.0	5.8	105.8	5.6
3 ϕ 20	108.2	101.5	-6.2	105.0	-2.9	103.6	-4.2
		Maximum Value=	5.6	Maximum Value=	5.8	Maximum Value=	5.6
		Minimum Value=	-6.2	Minimum Value=	-2.9	Minimum Value=	-4.2
		Average=	-0.9	Average=	0.5	Average=	-0.2

The estimation of initial stiffness values for under- and over-reinforced sections are shown in Table 5-7 and Table 5-8. For both under- and over-reinforced sections, the initial stiffness is best estimated by Ezeldin and Balaguru (1992) with an average of 13 and 1.5% estimation difference, respectively.

Table 5-7 – Estimation of initial stiffness for under-reinforced sections

Specimen	Test data	Nataraja et al. (1999)	%	Wang (2006)	%	Ezeldin and Balaguru (1992)	%
1φ10	1.50	0.91	-39.5	0.89	-41.0	0.91	-39.3
1φ12	1.43	1.17	-18.0	1.11	-22.5	1.19	-16.8
1φ14	1.60	1.34	-16.5	1.42	-11.6	1.37	-14.5
1φ16	2.11	1.73	-17.7	1.76	-16.6	1.78	-15.5
2φ14	2.21	2.12	-4.1	2.30	3.7	2.21	-0.4
2φ16	2.88	2.88	-0.2	2.90	0.6	2.99	3.8
3φ16	3.75	3.26	-13.1	3.26	-13.1	3.46	-7.9
		Maximum Value=	-0.2	Maximum Value=	3.7	Maximum Value=	3.8
		Minimum Value=	-39.5	Minimum Value=	-41.0	Minimum Value=	-39.3
		Average=	-15.6	Average=	-14.4	Average=	-13.0

Table 5-8 – Estimation of initial stiffness for SFRC over-reinforced sections

Specimen	Test data	Nataraja et al. (1999)	%	Wang (2006)	%	Ezeldin and Balaguru (1992)	%
3φ18	3.41	3.10	-9.0	3.55	4.2	3.50	2.5
4φ16	3.48	3.71	6.7	4.02	15.7	3.90	12.1
3φ20	3.48	3.05	-12.4	3.50	0.5	3.13	-10.0
		Maximum Value=	6.7	Maximum Value=	15.7	Maximum Value=	12.1
		Minimum Value=	-12.4	Minimum Value=	0.5	Minimum Value=	-10.0
		Average=	-4.9	Average=	6.8	Average=	1.5

The estimation of toughness for under- and over-reinforced sections are shown in Table 5-9 and Table 5-10. The ultimate deflection values for SFRC specimens shown in Table 4-8 is used in the calculations. For both under- and over-reinforced sections, the toughness is best estimated by Nataraja et al. (1992) with an average of 3.1% and 5.2% estimation difference, respectively.

Table 5-9 – Estimation of toughness at maximum deflection for under-reinforced sections

Specimen	Test data	Nataraja et al. (1999)	%	Wang (2006)	%	Ezeldin and Balaguru (1992)	%
1φ10	1004	1411	40.6	1418	41.3	1359	35.4
1φ12	1593	2419	51.8	2415	51.6	2361	48.2
1φ14	2408	2976	23.5	2976	23.5	2906	20.7
1φ16	5187	3349	-35.4	6491	25.1	6467	24.7
2φ14	7582	5038	-33.6	8788	15.9	9046	19.3
2φ16	10113	6367	-37.0	6367	-37.0	11869	17.4
3φ16	11094	7548	-32.0	11739	5.8	13027	17.4
		Maximum Value=	51.8	Maximum Value=	51.6	Maximum Value=	48.2
		Minimum Value=	-37.0	Minimum Value=	-37.0	Minimum Value=	17.4
		Average=	-3.1	Average=	18.0	Average=	26.1

Table 5-10 – Estimation of toughness at maximum deflection for over-reinforced sections

Specimen	Test data	Nataraja et al. (1999)	%	Wang (2006)	%	Ezeldin and Balaguru (1992)	%
3φ18	8958	7918	-11.6	10558	17.9	9787	9.3
4φ16	7920	8641	9.1	11200	41.4	9311	17.6
3φ20	10358	7475	-27.8	12178	17.6	9332	-9.9
		Maximum Value=	51.80	Maximum Value=	51.56	Maximum Value=	48.15
		Minimum Value=	-37.04	Minimum Value=	-37.04	Minimum Value=	-9.90
		Average=	-5.24	Average=	20.30	Average=	19.99

The estimation of post-peak stiffness values for over-reinforced sections are shown in Table 5-11. For over-reinforced sections, the post-peak stiffness is best estimated by Ezeldin and Balaguru (1992) with an average of 1.2% estimation difference.

Table 5-11 – Estimation of post-peak stiffness for SFRC specimens

Specimen	Test data	Nataraja et al. (1999)	%	Wang (2006)	%	Ezeldin and Balaguru (1992)	%
3 ϕ 18	0.40	0.95	137.7	0.40	0.0	0.25	-36.8
4 ϕ 16	0.36	0.74	104.6	0.60	65.8	0.29	-20.3
3 ϕ 20	0.55	1.80	228.7	0.30	-44.4	0.88	60.9
		Maximum Value=	228.7	Maximum Value=	65.8	Maximum Value=	60.9
		Minimum Value=	104.6	Minimum Value=	-44.4	Minimum Value=	-36.8
		Average=	157.0	Average=	7.1	Average=	1.2

Summary of the SFRC compression models that fits best to the test data is shown in Table 5-12.

Table 5-12 – SFRC Compression Models that Fits Best to Test Data

	Under-reinforced Sections	Over-reinforced Sections
Ultimate Load	Wang (2006)	Ezeldin and Balaguru (1992)
Initial Stiffness	Ezeldin and Balaguru (1992)	Ezeldin and Balaguru (1992)
Toughness	Nataraja et al. (1992)	Nataraja et al. (1992)
Post-peak Stiffness	-	Ezeldin and Balaguru (1992)

6 CONCLUSIONS

6.1 Summary

Flexural behavior of lightly and heavily reinforced steel fiber concrete beams was investigated in this research. Test series consisted of 20 singly reinforced beams (10 pairs of beams) having 180×250×3500 mm dimensions. The main parameters in the testing program were the type of concrete and the amount of longitudinal reinforcement. Ten different longitudinal reinforcement ratios (from 0.2 to 2.5%) covering the range from under-reinforced to over-reinforced beam behavior were used in the testing program. Two specimens were cast for each longitudinal reinforcement ratio, one specimen using ordinary plain concrete and another specimen using SFRC. Load deflection behaviors were obtained and evaluated in terms of ultimate load, initial stiffness, post-peak stiffness, and toughness. Experimental load deflection relationships obtained in this research were compared to the load-deflection curves obtained using the written Excel program. The stress-strain relationships available in the literature for SFRC in tension and compression were utilized in the numerical analyses. The proposed stress-strain relationships in the literature were compared and evaluated based on how accurately they predict the test data obtained in this research.

6.2 Observations and Conclusions

The observations made and conclusions drawn from this study are based on the test results of this research. These observations and conclusions are as follows:

- There is a slight increase in the ultimate load capacity with the addition of steel fibers. There is no clear trend on the ratio of increase in the load carrying capacity of SFRC specimens with respect to PC specimens with increasing reinforcement ratio.
- There is a slight increase in the service stiffness of beam specimens with the addition of steel fibers. There is no clear trend on the ratio of increase in the

service stiffness of SFRC specimens with respect to PC specimens with increasing reinforcement ratio.

- For the under-reinforced sections, the ultimate deflection values for PC specimens are greater than that of SFRC specimens. However, as reinforcement ratio increases the ultimate deflection of SFRC specimens are approximately 50 to 300% larger than that of PC specimens.
- Toughness of the SFRC specimens is greater than that of companion PC specimens. Also there is a clear trend that as the reinforcement ratio increases over 1.6% (i.e., when section behavior changes to over-reinforced), toughness of SFRC specimen becomes significantly larger than the companion PC specimen.
- Post-peak stiffness of the SFRC specimens is significantly lower than that of PC specimens. This corresponds to smaller rate for load decrease in SFRC specimens in the post-peak region. When the toughness and post-peak slope diagrams are evaluated together, it can be concluded that the reason for higher toughness values of SFRC specimens is associated with the lower post-peak slope of these specimens.
- It is noticed that SFRC specimens developed greater bottom fiber extension for the same load value compared to PC specimens.
- Based on the analytical study to estimate the load-deflection behavior of SFRC specimens, Soranakom and Mobasher (2009) SFRC tension model with a μ value equal to 0.2 can be used to estimate the tension behavior of SFRC members with a reasonable accuracy.
- Based on the analytical study performed to estimate the load-deflection behavior of SFRC specimens, SFRC compression models presented in Table 5-12 can be used to estimate the ultimate load, initial stiffness, toughness, and post-peak stiffness with a reasonable accuracy.

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