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CALIBRATION OF THE UNCONSTRAINED DEMAND HEURISTIC

by

Nasrullah HAZIRBABA

February 2014

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Nasrullah HAZIRBABA

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M.S. Thesis – Industrial Engineering
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ABSTRACT

Forecasting and optimization are two important elements of the modern revenue management (RM) system and in order to have a successful optimization, demand forecasting is the key issue. An extended version of previously used heuristic that considers market recapture effects is proposed by using historical data. The heuristic generates unconstrained demand values for flight classes and it can be applied at carrier and market level. In this study, the heuristic is run for real life data which contains booking and availability information for 5 different markets and one day of week. The heuristic is compared with an airlines' unconstraining and the independent demand methods. It is seen that the proposed heuristic generates more satisfying results due to considering recapture and market share effects. The heuristic can easily integrated into demand forecast models and it is expected that it reduces forecast errors.

Keywords: Forecasting, optimization, recapture, heuristic, demand, unconstraining.

KISITSIZ TALEP TAHMİN SEZGİSEL METODUNUN KALİBRE EDİLMESİ

Nasrullah HAZIRBABA

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ÖZ

Tahminleme ve optimizasyon, modern gelir yonetimi sisteminin iki önemli alanıdır ve başarılı bir optimizasyonun gercekleşmesi icin talep tahmininin doğruluğu son derece önemlidir. Bu çalışmada, geçmiş verileri kullanarak, pazar recapture etkilerini dikkate alan sezgisel bir metodun farklı bir versiyonu önerilmektedir. Sezgisel metod, ücret sınıfları için kısıtsız talep değerleri üretmekte ve havayolu ve pazar bazında da uygulanabilmektedir. Çalışmada, 5 değişik pazar ve bir haftanın günü için gerçek rezervasyon ve sınıf statü durumları bilgisi kullanılmaktadır. Sezgisel metod, bir havayolunun kullanmış olduğu talep hesaplama metodu ve bağımsız talep hesaplama metodu ile karşılaştırılmaktadır. Bu karşılaştırmada, sezgisel metodun üretmiş olduğu değerlerin daha tatmin edici olduğu gözlemlenmiştir. Metod diğer talep tahmin modellerine kolay entegre olmakla beraber, tahmin hatalarını azaltması beklenmektedir.

Anahtar Kelimeler: Tahminleme, optimizasyon, recapture, sezgisel metod, talep, unconstraining.

To my wife

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I express my thanks and appreciation to my family for their understanding, motivation and patience. Lastly, but in no sense the least, I am thankful to all colleagues and friends who made my stay at the university a memorable and valuable experience.

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SYMBOL/ABBREVIATION

b_{ij}	Booking for flight i class j.
j	Class. 0 denotes competitor carrier.
a_{00}	Competitor or do not fly option.
d_{ij}	Demand for flight i class j.
i	Flight. 0 denotes competitor carrier.
Π_{ij}	Flight class share. (MNL customer selection probability.)
π^{H}	Host carrier market share. (Probability of choosing the host carrier.)
H	Host carrier`s flight classes.
M	Market. It contains all flight classes, the host and the competitor.
κ_{ij}	Percent availability of demand closed.
Φ_{ij}	Percent availability of demand open.
$\prod_{c}^{H}$	Probability of choosing a closed class from the host carrier.
$\prod_{o}^{H}$	Probability of choosing an open class from the host carrier.
π_{c}^{M}	Probability of choosing a closed class from market.
π_{o}^{M}	Probability of choosing an open class from market.
r_{ij}	Recapture for flight i class j.
a_{ij}	Specific flight class.
s_{ij}	Spill for flight i class j.
ρ	Total recapture rate for all open host flight classes

CHAPTER 1

INTRODUCTION

1.1 BACKGROUND

In all industries, sellers should give basic decisions about their products. For example, a furniture company has to decide on which season to increase or decrease the price, how much to produce, which offer to accept or which season to give campaign. All these questions involve uncertainty. Businesses have more complex situations on these decisions. They should also consider market segmentation process. Also, what price it should charge for the each segment is a challenging job. Generally, market conditions are taken into consideration, however there is still uncertainty because who can know the future. Revenue management deals with such kind of decisions by using a methodology with the objective of increasing revenues. In practice, there are other names of revenue management such as yield management, pricing and revenue management, pricing and revenue optimization, revenue process optimization, demand management, demand-chain management.

There are four steps of revenue management. These key components are:

1. **Data collection:** Collect and store relevant historical data (prices, demand, causal factors).
2. **Estimation and forecasting:** Estimate the parameters of the demand model; forecast demand based on these parameters; forecast other relevant quantities like no-show and cancellation rates, based on transaction data.
3. **Optimization:** Find the optimal set of controls (allocations, prices, markdowns, discounts, and overbooking limits) to apply until the next re-optimization.

4. **Control:** Control the sale of inventory using the optimized control. This is done either through the firm's own transaction-processing systems or through shared distribution systems (such as GDSs).

Talluri, K., van Ryzin, G.J. (2004b) say that optimization is the process of finding the optimal set of control for flights according to forecasted demand. In order to have a successful optimization step, forecasting should be done accurately.

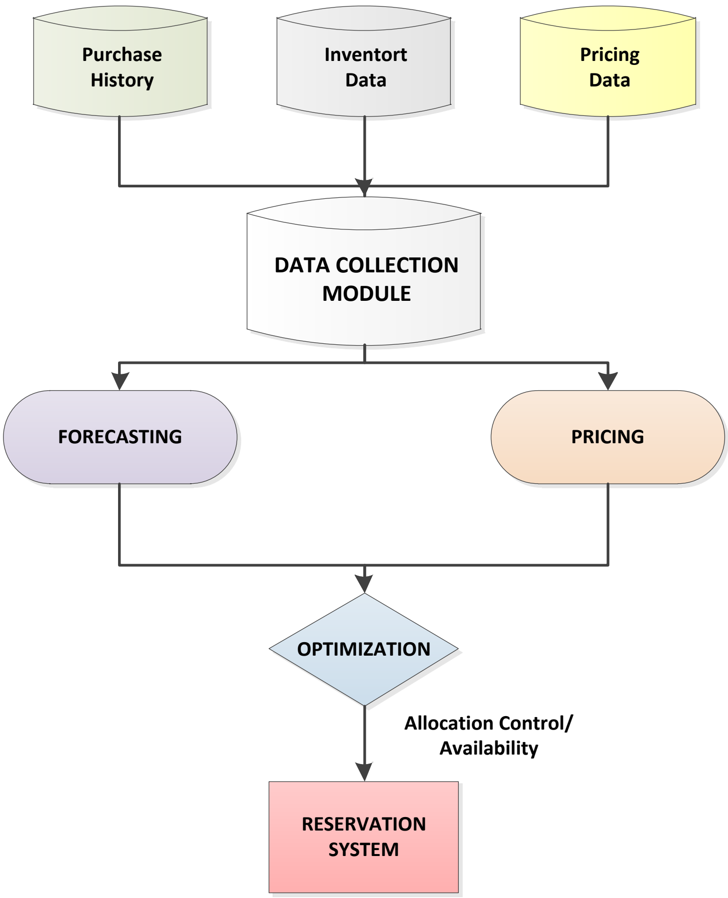

Figure 1.1 Revenue management flow.

In the airline industry, there are fare products or fare classes and the RM system collects data related with them to forecast demand. This data can be booking, cancellation or availability. However, the airline can just collect data for open classes. If a class is closed or capacity is sold out, it is not possible to observe a demand for that class because most reservation systems record only bookings not attempted bookings. There might be a significant bias if this case is ignored. A fully closed class might have booking if it would be opened. Ignoring this case results 0 demands for that class which is not true. Considering this case is called `unconstraining`.

Demand unconstraining is an important element of RM systems. It helps to estimate more accurate forecasts for optimizer and optimizer determines optimal protection levels for fare products with the objective of maximizing the revenue. It is suggested that a 20 per cent reduction in forecast error equates to approximately 1 per cent revenue improvement for an airline (Talluri, K. and van Ryzin, G., 2004) and also said that use of constrained historical bookings (rather than unconstrained demand) for RM forecasting is a provably poor strategy that leads to a systematic degradation of RM controls over time known as ‘revenue spiral down’ (Zeni, R. H., 2001a).

Unconstraining is the statistical problem of estimating the underlying demand distribution given observed historical bookings and availability information. Uncensoring and untruncation are the similar terms used in the airline industry. In fact, all modern RM systems estimate demand and spill, however a few of them take `recaptured spill` into account. Recapture occurs when desired class is not available and the customer chooses a class of different flights at the same airline. Recapture is very common because customers usually purchase their second or third choice. Industry practitioners report recapture rates in the range of 15–55 per cent as typical values (Ja, S., Rao, B. V. and Chandler, S., 2001). Calculating recapture has a very complex function because it is related with attractiveness of flights or classes. In the literature, discrete choice models are used as tools to estimate spill and recapture rates for airline and market. Ignoring recapture can lead overestimating demand because the single actual demand is counted on both the original, closed flight (as spill) and the alternate, open flight as an observed booking.

1.2 PROBLEM DESCRIPTION

In airline industry, products are fare classes of flights. These classes have own features or specialties and each class has its own fare level. This level depends on elasticity of fare class rules such as maximum stay, minimum stay, Saturday night, advance purchase. There are also promotional fare classes that are available for a specific time range. Pricing unit of revenue management department gives decisions about these classes. The question of "what should rules or fare levels be?" is in the scope of pricing unit of RM.

After determination of rules and fare levels, demand management unit decides on protection levels of classes. This determination is very crucial because it affects how much customer will pay. These protection levels are determined by a revenue management system and it is output of an optimization process. Several optimization methods are available in the industry, but the most used optimizations are leg optimization and network optimization. A picture of class map is shown in Table 1.1.

Table 1.1 Fare classes and their attributes

Fare Class	**Price (€)**	**Adv. Purc.**	**Sat. Night**	**Refund**	**Round Trp**
Y	750	---	---	100%	---
M	600	3 days	Yes	25%	---
E	500	7 days	Yes	---	Yes
Q	350	14 days	Yes	---	Yes

It can be said that output of optimization is the amount that customer will pay. In order to have a successful optimization step, inputs of this step should be estimated well. Demand forecasting is the key issue for an optimization. Forecasting accuracy helps optimization to produce efficient protection levels. However, having an accurate forecasting is a challenging work. There are lots of biases that affect forecasting results. Availability of classes and capacity are the most important constraints for an airline.

Unconstraining is the method of estimating hidden demand by using related booking and availability data. If unconstraining is ignored, underforecast situation occurs and protection levels for lower classes might be high. As a result, spill down effect and revenue loss occur. The reverse case also can happen. In case of overforecast, protection levels might be low for lower classes and as a result spill effect and revenue loss occurs. Because of that each airline store historical data starting from the time the inventory was released to time to the departure. By using this historical data, forecaster of RM system estimates demand for the related flight class. As stated before, unconstraining is the key element of this process. For unconstraining, four inputs are required:

1. Observed historical bookings
2. Flight and fare class availability status
3. The airline`s estimated market share
4. The relative attractiveness of flight and fare class

The first three inputs are easy to collect, however the last one is difficult to estimate. Actually, this study tries to improve the 4th input which is attractiveness. By applying the propsed method, demand, spill and recapture values are estimated both flight and flight class level for a time period.

Some good heuristics related with unconstraining are present in the literature. In this study, multi-flight recapture heuristic of Richard M. Ratliff, B.Venkateshwara Rao (2008) is used. This heuristic estimates demand, spill and recapture rates both at flight and market level. They have used equations of Sven-Eric Andersson (1998) which he presented at the AGIFORS`s symposium. In this method, multinomial logit choice models (MNL) used to estimate spill rates and then these rates were incorporated into a deterministic network linear programming model.

MNL model is used extensively in travel demand forecasting and marketing. MNL models are flexible and easy to use, however calibrating accurately is difficult. There are a few methods used for calibration in the literature. One of them is expectation method of Talluri, K. and van Ryzin, G. (2004). Results show that by calibrating the choice model, more accurate demand, spill and recapture rates are achieved. In this study, a big airline`s own data is used for calibration. Flight and flight class shares are derived from historical data and they are used in a successful heuristic which was proposed by Richard M. Ratliff, B. Venkateshwara Rao (2008).

CHAPTER 2

LITERATURE REVIEW

2.1 UNCONSTRAINED DEMAND HEURISTICS

There are many papers about demand unconstraining, however for a general overview, Talluri and van Ryzin (2004) and Zeni (2001b) are the most important references. In the literature, unconstraining methods are divided into three parts (Andersson, S. E., 1998):

1. Single-class methods
2. Multi-class methods
3. Multi-flight methods

Single-class method is the widest category. This model thinks the demand as independent and works best within an independent framework. EMSR approach of Belobaba requires independent demand inputs. According to this method, there is no correlation between fare classes which is hard to accept and can be thought as an important disadvantage. Zeni (2001b) used Mean Imputation method to unconstrain demand for single-class. For historical pre-departure periods, classes which have been closed for sale are eliminated and instead of these values, mean demands are used. Zeni (2001b) and Talluri and van Ryzin (2004) used EM (Independent Demands) method for this subject. This method includes 2 iteratives; firstly censored observations are replaced by sample mean and then new mean and variance are computed for updated sample. This process continues until convergence. Poelt (2000) and Zeni (2001a) used booking profile method to unconstrain demand for single-class. This method requires using pre-departure availability status with historical demand buildup curves by days to departure. Crystal (2007) used Double Exponential Smoothing method which is similar to booking profile method. It uses a historical time series of incremental demands.

In 2002, Weatherford and Poelt did a study by using Projection Detruncation method. This method has similarities with EM method, but instead of replacing mean demand, the sample median is replaced by constrained observations. Among these papers, EM method of Talluri and van Ryzin (2004) is cited as most accurate. Disadvantage of this method is intensive to estimate.

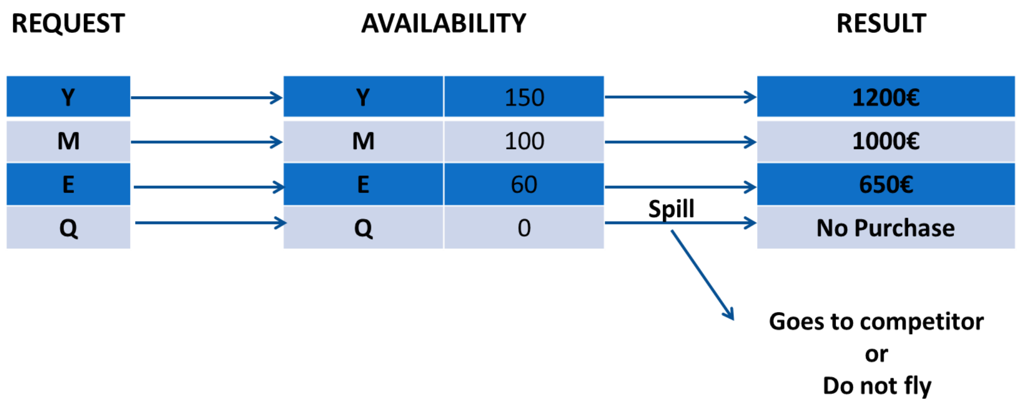

Figure 2.1 Single class demand method.

Multi-class methods are more realistic than single-class methods because they take correlation between classes into account. Upsell and downsell situations are inherent in these methods. As fare restrictions disappear, these methods gain popularity. Low cost carriers usually apply this RM method. They make all classes equal except their prices. As a result, a customer easily changes his request if he sees lower class available. Multi-class methods work well, however there is a revenue dilution risk. Another disadvantage of these methods is not being able to recapture cross-flight. McGill (1995) uses EM and multivariate censored regression to calculate demand for two or more correlated classes. Belobaba and Farkas (1999) use demand densities by taking RM nested inventory capacity limits into account. Hopperstad (2006) and Boyd (2001) use a method called Q-Forecasting. The Q stands for lowest nested class. In this method, bookings, fares, price elasticity and availability data are used to estimate Q. Mishra and Viswanathan (2003) also use bookings, availability and fares, however they also use flight level demand curves in order to estimate dependent demand for closed classes.

Multi-flight methods are the most sophisticated methods among three methods because they consider interactions between flight and fare products and these features make multi-flight methods more popular. However, these methods are so difficult to calibrate. In the literature, discrete choice demand models are used to estimate attractiveness of flights or flight classes. The most popular discrete choice model is MNL choice model. The reason of using widely is that MNL models are so flexible and easy to estimate upsell, downsell and recapture. However, calibrating these models is a little bit difficult. By calibrating MNL models, more accurate results are derived. Stefanescu (2004) used correlated demand forecasting method which uses historic correlation between flights, classes. The parameter estimates are derived from EM. A successful study has done by Talluri and van Ryzin (2004), Vulcano (2008). The method is called as EM (Discrete Choice Model). This method estimates demand arrival and parameters of MNL discrete choice model and it is used in simulation based RM optimizer and gives successful results for a big US airline.

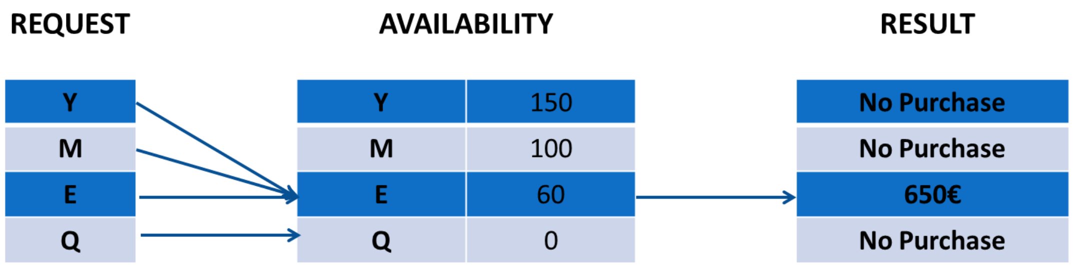

Figure 2.2 Multi class demand method.

Carried Crystal Queenan and Mark Ferguson (2007) introduced an unconstraining method to improve revenue management system. They compare their proposed method with EM and “no unconstraining” method. They use a hotel data instead of airline. This method is based on double exponential smoothing forecasting technique. Alwin Haensel and Ger Koole (2010) proposed a method including EM. In this paper, a general method is analyzed for estimating customer choice parameters from sales observations and comparison is made in terms of revenue. Sandeep Karmarkar and Dutta Goutham

(2010) analyzed revenue impacts of demand unconstraining in their papers. This paper considers both censorship and correlation of demands and compares four methods as:

- No unconstraining and independent demand
- Unconstraining, independent demand
- No unconstraining, dependent demand
- Unconstraining and dependent demand

Paper says that neglecting censorship causes 1% revenue decrease and neglecting correlation between classes also causes 2% revenue decrease. Olivier d'Huart and Peter P. Belobaba (2011) highlight the role of spill between airlines on RM seat allocations. They show that neglecting competitive RM effects causes a double-counting of demand. As a result, RM system generates higher forecasts and higher protection levels. In 2012, Peng Guo and Baichun Xiao (2012) highlighted unconstraining methods in revenue management. They survey the history of researches done in this area.

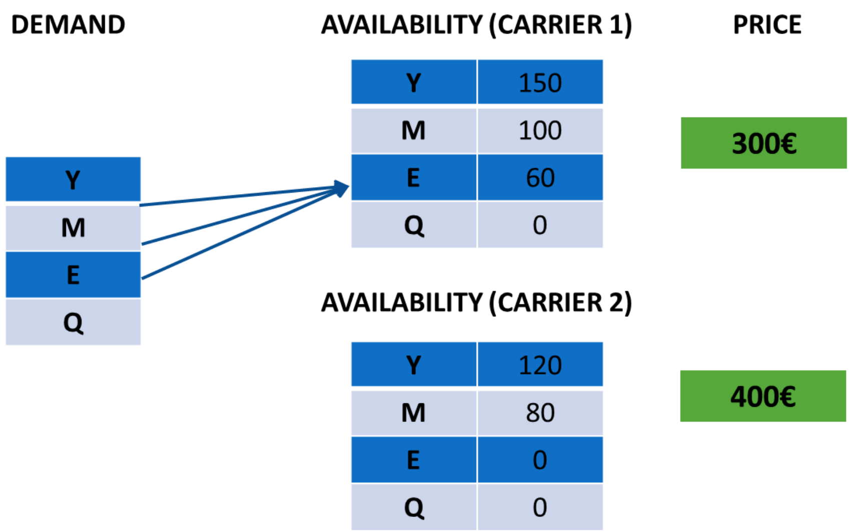

Figure 2.3 Multi flight demand method.

There are also different PODS (Passenger Origin-Destination Simulator) studies done in order to compare different RM strategies. This simulator is developed by Boeing in 1990s. The working methodology of PODS is shown below:

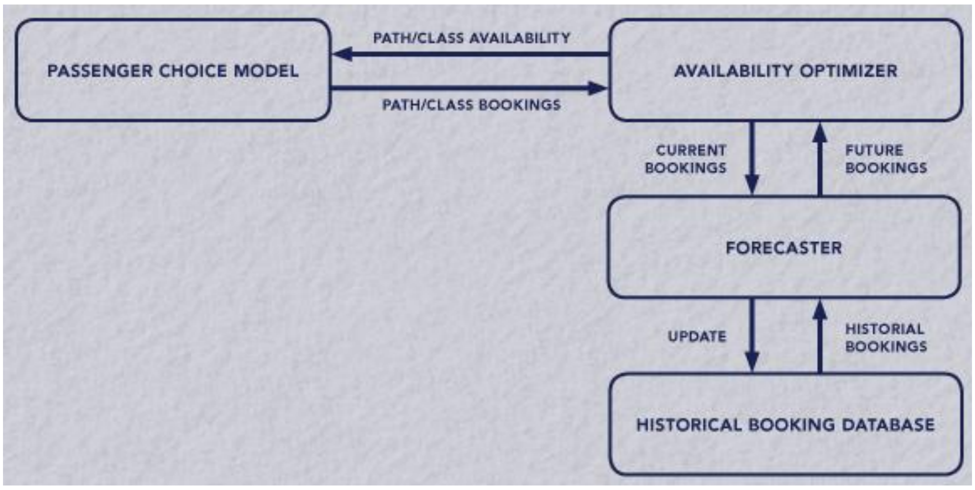

Figure 2.4 PODS work flow.

Larry Weatherford (2013) studies improved revenues from various unconstraining methods such as expectation maximization, projection detruncation, booking curve (BC) and pickup by using PODS. At the final part of the study they show that upgrading the unconstraining process can lead to revenue gains of 2-15 per cent. There are other PODS studies done by Larry Weatherford; *Combining hybrid forecasting and fare adjustment with various unconstraining methods* (2013), *Revenue maximization with implementation variations of unconstraining methods in a semi-restricted fare environment* (2013) and *Sophisticated unconstrainer revenue performance in a large airline network* (2013).

Our study is related with multi-flight recapture method which is proposed by Richard M. Ratliff, B. Venkateshwara Rao (2008). In this method, demand closure rates used to jointly estimate spill and recapture across numerous flight classes. This method is based on Sven-Eric Andersson`s paper which was presented at the AGIFORS symposium meeting (Algers, 1993). They give a solution to the problem of having `first-choice` demand or `undisturbed` demand. They use a concept called demand mass balance. This concept uses three key equations:

- Relation between bookings and spill to demand and recapture
- Linkage between demand and spill
- Linkage between spill and recapture

These equations are applied both at flight and flight class level and as a result, demand, spill and recapture rates are derived.

2.1.1 Multi Flight Demand Heuristic

In the airline industry, there are many competitors and customers have lots of alternatives to choose. In general, 3 possible outcomes are:

- Fly with an airline
- Fly with a competitor airline
- Do not fly

Most airlines can collect data for the first two options. They can easily see bookings made for their flights. By using software packages or reservation systems, they are also able to see competitor sales. However, it is not possible to estimate `do not fly` option because there is no record about this behavior. In MFRM, we can combine 2nd and 3rd options as `other airline, do not fly`. In fact, there is no so much difference between 2nd and 3rd options in the aspect of the reference airline However, in future papers, the 3rd option may be analyzed and estimated. For our study, we think other airline and do not fly options as a whole. Despite of accepting this assumption, there is still unknown value which is utility. According to study of Richard M. Ratliff, B. Venkateshwara Rao (2008), utility can be estimated by using complement of the host carrier`s market share.

Host carrier market share is:

$$\sum_{ij\varepsilon H} \pi_{ij} \tag{2.1}$$

By definition:

$$\pi_{00} = \sum_{ij\in M} \pi_{ij} - \sum_{ij\in H} \pi_{ij} = 1 - \sum_{ij\in M} \pi_{ij} \quad (2.2)$$

From here, utility can be estimates as follows:

$$u_{00} = (\sum_{ij\in M} u_{ij}) \times (\pi_{00}) \div (1 - \pi_{00}) \quad (2.3)$$

where *H* denotes the host carrier, *M* denotes market.

MFRM method uses probability of market level open and closure status of flights in aggregate level in order to estimate recapture rates. This is achieved by using two important inputs:

- Flight class availability
- Flight class share

Flight class availability status is derived by making observation. By making observation, we can easily estimate open or closure percentages. Combining this value with flight class share (which is derived in previous section) result market level open and closure rates. Equations are shown below:

$$\pi_o^M = \sum_{ij\in M} \emptyset_{ij}\pi_{ij} \quad (2.4)$$

where $\emptyset_{00}$ is defined as 1.

$$\pi_c^M = 1 - \pi_o^M \quad (2.5)$$

Then, recapture rate or probability of selecting alternative host carrier option is given by equation:

$$\rho = \frac{(\pi_o^M - \pi_{00})}{\pi_o^M} \quad (2.6)$$

In above equation, competitor market share is subtracted from all shares. This means that it eliminates a_{00} alternative (other option). Since the host carrier usually lacks specific information on competitor bookings, the MFRM bookings input data are limited to the host carrier only. For this reason, some steps in the MFRM require selection probabilities that have been renormalize with respect to the host carrier alternatives. To get rid of a_{00} alternatives, below equations are used (Richard M. Ratliff, B. Venkateshwara Rao, 2008):

$$\pi^H = 1 - \pi_{00} = \sum_{ij \in H} \pi_{ij} \tag{2.7}$$

$$\prod_c^H = \frac{\pi_c^M}{\pi^H} \tag{2.8}$$

$$\prod_o^H = 1 - \prod_c^H \tag{2.9}$$

2.1.2 Demand Mass Balance

Demand mass balance equations described by Andersson and Ja are key elements of MFRM. There are three equations which are:

1. Demand mass balance
2. Spill equation
3. Recapture equation

These equations can be applied both flight, class, market and other option level. Bookings are denoted by *b*, spill as *s*, and demand as *d* and recapture as *r*. (Andersson, S. E., 1998).

For flight level:

$$d^H - s^H + r^H = b^H \tag{2.10}$$

$$s^H = \prod_c^H d^H \tag{2.11}$$

$$r^H = \rho s^H \tag{2.12}$$

With minor adjustments, these equations can be fitted for flight class level as follows:

$$d_{ij} - s_{ij} + r_{ij} = b_{ij} \tag{2.13}$$

$$s_{ij} = d_{ij}\kappa_{ij} \tag{2.14}$$

$$r_{ij} = r^H(b_{ij} / b^H) \tag{2.15}$$

An alternate calculation for recapture of flight class level is using market open selection probability as follows:

$$r_{ij} = s^H(\pi_{ij} / \pi_o^M) \tag{2.16}$$

Both recapture equations give desired results, but the first version is better because instead of MNL flight class share, it uses observed bookings.

Above equations can be designed for market level estimations as follows:

$$d^M = d^H / \pi^H \tag{2.17}$$

$$s^M = \pi_c^M d^M \tag{2.18}$$

$$r^M = s^M \tag{2.19}$$

$$b^M = d^M \tag{2.20}$$

For competitor or `do not fly` option, equations are shown below:

$$d_{00} = (d^H)\,(1 - \pi^H) / (\pi^H) \tag{2.21}$$

$$s_{00} = 0 \tag{2.22}$$

$$r_{00} = s^H - r^H \tag{2.23}$$

$$b_{00} = d_{00} - s_{00} + r_{00} \tag{2.24}$$

2.2 DISCRETE CHOICE METHODS

Discrete choice analysis is the methodology used to analyze and predict travel decisions. These models use disaggregate travel behavior and it means that these models consider the choice behavior of individual travelers. These choices might be short-term or long-term. In the airline industry, choices are generally short-term such as which destination to fly, with which flight to fly or choosing class of the flight. There are some assumptions in these models which are stated below by Ben-Akiva, M., S. R. Lerman. (1985):

1. Decision-maker; defining the decision-making entity and its characteristics.

2. Alternatives; determining the options available to the decision-maker.

3. Attributes; measuring the benefits and costs of an alternative to the decision maker.

4. Decision rule; describing the process used by the decision-maker to choose alternative.

Decision maker is an individual in these models. This individual can be group or segment, however in this case we do not consider interactions within the group. Disaggregate or discrete choice models are capable of including characteristics of individual such as gender, education level, income, price or age. In these models, there are alternatives and an individual is supposed to choose one of these alternatives, but for these models, knowing what the individual has been chosen is not sufficient.

The knowledge of what has not been chosen is also important. Set of alternatives is called the choice set. Attributes are used to characterize each alternative. It does not have to be a measurable quantity. Examples of attributes might be fare class of a flight, departure time, elapsed time. In these models, individuals are utility maximizers and they choose the alternative that has the highest utility. This utility comes from random utility theory.

2.2.1 Random Utility Theory

Random utility models assume, as does the economic consumer theory, that the decision-maker has a perfect discrimination capability. However, the analyst is assumed to have incomplete information and, therefore, uncertainty must be taken into account. There are four different sources of uncertainty:

- Unobserved alternative attributes
- Unobserved individual characteristics
- Measurement errors
- Instrumental variables

The utility is random variable according to this theory and given by:

$$U_{in} = V_{in} + \varepsilon_{in} \tag{2.25}$$

where V_{in} is the deterministic part of the utility, and ε_{in} is the random term, capturing the uncertainty. The alternative with the highest utility is chosen. Therefore, the probability that alternative i is chosen by decision-maker n from choice set C_n is

$$P(i| C_n) = P[\, U_{in} \geq U_{jn} \;\; \forall j \in C_n] = P[U_{in} = \max_{j \in Cn} U_{jn}] \tag{2.26}$$

If we denote the mean of the error term of alternative i by m_i = E[ε_{in}], we can define a new random variable $e_{in} = \varepsilon_{in} - m_i + c$ such that E[e_{in}] = c. We have

$$\mathrm{P}[\, U_{in} \geq U_{jn} \; \forall \; j \in C_n] = \mathrm{P}[\, V_{in} + m_i + e_{in} \geq V_{jn} + m_j + e_{jn} \; \forall \; j \in C_n] \tag{2.27}$$

V_{in} is a function of the attributes of the alternative itself and the characteristics of the decision-maker. We can denote vector of attributes as z_{in} and vector of characteristics as S_n. So, we have

$$V_{in} = V(z_{in}, S_n) \tag{2.28}$$

This formulation can be simplified as:

$$x_{in} = h(z_{in}, S_n) \tag{2.29}$$

Then, we have,

$$V_{in} = V(x_{in}) \tag{2.30}$$

A linear in the parameters function is denoted as follows:

$$V_{in} = \sum \beta_k x_{ink} \tag{2.31}$$

The deterministic term of the utility is therefore fully specified by the vector of parameters β (Ben-Akiva, M., S. R. Lerman., 1985).

2.2.2 Multinomail Logit Model

In order to calculate flight class share for recapture, several techniques are available such as quality service index, MNL and NL. In fact, all models try to estimate the likelihood of selection across alternatives. For our case, alternatives are different flight classes. Multi flight recapture method is used as demand unconstraining method, but first MNL choice model is introduced to estimate flight class shares.

MNL is a discrete choice model that uses some parameters to estimate customer selection. In this model, there are different product offerings and according to defined attributes the model predicts the likelihood of the selection. MNL uses linear estimate of the utility in order to determine flight class shares. These steps are shown below:

$$u_{ij} = \sum \beta_g X_g \ , \qquad g = 1,2, .., n \tag{2.32}$$

β is the weight of n different flight quality and fare attributes.

U_{ij} is the utility of flight i class j.

Then, MNL customer selection probability for flight class share is given by:

$$\pi_{ij} = \frac{e^{uij}}{\sum_{kl\varepsilon M} e^{ukl}} \tag{2.33}$$

π_{ij} is the MNL customer selection probability or flight class share.

Use of an MNL model requires the independence from irrelevant attributes (IIA) assumption which holds that the ratio of the probabilities of any two alternatives is unaffected by the utilities of other alternatives. (Richard M. Ratliff, B. Venkateshwara Rao, 2008). From the equation it is understood that $\sum_{ij\varepsilon M} \pi_{ij} = 1$.

2.3 CALIBRATION METHODS

2.3.1 Using Fare Search Data

Customer choice model is the key element of MFRM, however as stated before, fitting the choice model to the data is not an easy work. Several techniques are available in this topic. Fare-search motor data is the most suitable one, but it is too difficult for airlines to collect this data because they can just observe bookings made for their flights via their own reservation system or web-site which are not sufficient. Even if it is possible to catch this data, there is still uncovered business segment. These business passengers generally use corporate travel agencies to book on a flight. Because of this difficulty, some alternative approaches might be preferable.

2.3.2 Expectation Maximization Method

In this part, EM method of Talluri and van Ryzin is introduced. The advantage of this method is that it does not require shopping data, despite of being computationally intensive.

As stated in MNL model part, choice probabilities are given by:

$$P_j(S) = \frac{e^{u_j}}{\sum_{i \in S} e^{u_j}} \tag{2.34}$$

It is assumed that the choice probabilities are given by below form:

$$P_j(S) = P_j(z, \beta, S) \qquad j = 0, 1, 2, \ldots, n \tag{2.35}$$

where z is a vector of known attributes (price, schedule quality, elapsed time) of choice j and β is the weight of attributes. The utility is:

$$u_j = \beta^T z_j \tag{2.36}$$

The EM method tries to estimate (β) the weight of the attributes from historical data. This method is also capable of estimating (λ) arrival rate. The EM method needs inputs such as purchase, no-purchase outcomes and then it estimates β by using maximum likelihood methods.

For each observation $t \in$ D (D, time intervals from many flight departures):

$$a(t) = \begin{cases} 1, \text{ if customer arrives in period t} \\ 0, \text{ otherwise} \end{cases}$$

The likelihood function is:

$$\prod_{t \in D} [\lambda P_{j(t)}(z, \beta, S(t))]^{a(t)} (1-\lambda)^{(1-a(t))} \tag{2.37}$$

Taking log of the above equation results:

$$\sum_{t \in D}[a(t)\ln(P_{j(t)}(z, \beta, S(t))) + a(t)\ln(\lambda) + (1 - a(t))\ln(1 - \lambda)] \tag{2.38}$$

Above logarithmic function is separable in β. So, maximizing it with respect to β gives:

$$\max_{\beta} \ln(P_{j(t)}(z, \beta, S(t))) \tag{2.39}$$

In order to solve above equations, all observations including non-purchase transactions should be taken into consideration. However, as stated before, only purchase records are available for the airlines. In order to solve this problem, the EM method of Dempster (1977) is used. It starts with arbitrary initial values of β and λ, then calculate conditional expected value. Finally it maximizes the log-likelihood function with respect to β to generate new estimates. This procedure continues until it converges. To apply EM method to above equations, we let P denotes the set of periods in which customers purchase and P^- denotes the periods with no purchase. Then the log function can be written as:

$$\sum_{t \in P}[\ln(\lambda) + \ln(P_{j(t)}(z, \beta, S(t)) + \sum_{t \in P-}[a(t)\ln(\lambda) + \ln(P_0(z, \beta, S(t)))] + (-a(t))\ln(1 - \lambda)] \tag{2.40}$$

With given observation, unknown values of above equations are a(t) and $t \epsilon P^-$. Given estimates of β\` and λ\`, their expected values can be calculated by Bayes\`s rule:

$$a`(t) = \frac{\lambda`P_0(z,\beta`,S(t))}{\lambda`P_0(z,\beta`,S(t)) + (1-\lambda)} \tag{2.41}$$

By substituting a\`(t) to the log function and maximizing with β, we have,

$$\max_{\beta}\{\sum_{t \epsilon P} \ln\left(P_{j(t)}(z, \beta, S(t))\right) + \sum_{t \epsilon P`} a`(t) \ln\left(P_0(z, \beta, S(t))\right)\} \tag{2.42}$$

If we summarize the algorithm:

1st step: initialize β\` and λ\`.

2nd step: for tϵP\`, by using β\` and λ\`, compute a\`(t) by using:

$$a`(t) = \frac{\lambda`P_0(z,\beta`,S(t))}{\lambda`P_0(z,\beta`,S(t)) + (1-\lambda)} \tag{2.43}$$

3rd step: compute β by using:

$$\max_{\beta} \sum_{t \epsilon P} \ln\left(P_{j(t)}\big(z,\beta,S(t)\big)\right) + \sum_{t \epsilon P`} \alpha`(t) \ln\left(P_0\big(z,\beta,S(t)\big)\right) \tag{2.44}$$

4th step: continue to this process until it converges. If it diverges, use the new estimated β* and λ* values. By this algorithm, the weights of selected attributes are improved and the choice model is calibrated.

CHAPTER 3

METHODOLOGY PROCES

3.1 CALIBRATION

There are a few methods in the literature for calibrating the choice model, however in this study airline`s own data is used for this purpose. Choice models are used for estimating flight and flight class shares among alternatives, but in today`s world, airlines have their own databases and they know these shares by making historical analysis even by class level. Another reason for using airline’s own data is that methods in the literature have some unrealistic assumptions which will be mentioned at the conclusion part of this study.

In order to estimate flight and flight class shares, historical data is used. It is very important to analyze historical data because of the booking curve. A flight or a class has peak, off-peak and shoulder periods. Each period has different characteristics in the aspect of demand. Because of this variability, it is better to use the same period in the historical data. For example, if pre-departure time period being unconstrained involves the range 24-27 days before departure, the same period is also used for the last year historical data. By this way, unconstraining gains ability of being dynamic across time periods.

Let $b_{ij}(l)$ denotes the bookings made for flight i and class j last year. We use the same notation with MNL choice model to show flight class share:

$$\pi_{ij} = \frac{b_{ij}(l)}{\sum b_{ij}(l)} \qquad i, j = 1, 2, \ldots, n \tag{3.1}$$

In the application part, three cases:

- Single class with two classes
- Two flights with single class
- Three flights with three classes

are analyzed with real data and then in the results part, market level comparison is done with independent demand model.

3.2 APPLICATION

Unconstrained demand heuristic of Richard M. Ratliff, B. Venkateshwara Rao (2008) is applied to three cases; single flight with two classes, two flight with single class and three flights with three classes. Instead of using MNL choice model with the weight of attributes, an airlines' historical data is used to estimate flight and flight class shares or attractiveness. Market share data is taken from MIDT software (Marketing Information Data Tapes) and for the privacy, we do not cite the name of the airline or the market.

3.2.1 Single Flight Two Classes Case

In this real case, a market with one flight and two classes is chosen and required data is collected for 18-21 days prior. It is shown below:

b_{11}= 5 (bookings)

b_{12}= 3 (bookings)

$b_{11}(l)$ = 4 (bookings)

$b_{12}(l)$ = 2 (bookings)

ϕ_{11}= 1 (open class)

ϕ_{12}= 0.5 (semi-closed class)

Airlines` market share is 52%

By given this information, demand, spill and recapture values are estimated jointly.

By using equation (2.25), flights class shares at airline level are estimated as:

$\prod_{11} = 4 / 6 = 0.6667 = 67\%$
$\prod_{12} = 2 / 6 = 0.3334 = 33\%$

From these values, we can estimate flight class shares at market level as:

$\pi_{11} = 0.6667 \text{ x } 0.52 = 0.3467 = 35\%$
$\pi_{12} = 0.3334 \text{ x } 0.52 = 0.1734 = 17\%$
$\pi_{00} = 0.48 = 48\%$

After calculating flights class shares at market level, closure (first two equations) and recapture rate (third equation) can be estimated by using equations (2.4) – (2.6):

$\pi_{o}^{M} = 1 \text{ x } 0.3467 + 0.5 \text{ x } 0.1734 \text{ x } 1 \text{ x } 0.48 = 0.9134 = 92\%$
$\pi_{c}^{M} = 1 – 0.9134 = 0.0866 = 9\%$
$\rho = ((0.9134\text{-}0.48)) / 0.9134 = 0.4745 = 47\%$

Above results are at market level. In order to convert them to the host level, equations (2.7) – (2.9) are used as:

$\pi^{H} = \sum_{ij \in H} \pi_{ij} = 1 – 0.48 = 0.52 = 52\%$
$\prod_{c}^{H} = 0.0866 / 0.52 = 0.1665$
$\prod_{o}^{H} = 1 – 0.1665 = 0.8335$

Now, demand mass balance equations (2.10) – (2.12) can be applied to find demand, spill and recapture:

$d^{H} - s^{H} + r^{H} = 8$
$s^{H} = \prod_{c}^{H} d^{H} = 0.1665 d^{H}$
$r^{H} = \rho s^{H} = 0.4745 s^{H}$

If we jointly solve above three equations we find:
d^{H} (demand) = 8.77

s^H (spill) = 1.46

r^H (recapture) = 0.69

If we apply demand mass balance equations (2.13) – (2.15) at flight class level for a_{11}:

$d_{ij} - s_{ij} + r_{ij} = 5$
$s_{ij} = d_{ij}\kappa_{ij}$
$r_{ij} = r^H(b_{ij} / b^H) = 0.4313$

If we jointly solve above three equations we find:

d_{11} (demand) = 4.57

s_{11} (spill) = 0

r_{11} (recapture) = 0.43

Similarly for a_{12}:

d_{12} (demand) = 8.77 – 4.57 = 4.20

s_{12} (spill) = 1.46 – 0 = 1.46

r_{12} (recapture) = 0.69 – 0.43 = 0.26

In order to find market level results, equations (2.17) – (2.20) are used as:

d^M = 8.77 / 0.52 = 16.87

s^M = 0.0866 x 16.87 = 1.46

$r^M = s^M$ = 1.46

$b^M = d^M$ = 16.87 (17)

For competitor results, equations (2.21) – (2.24) are used as:

d_{00} = 8.77 x 0.48 / 0.52 = 8.10

s_{00} = 0

r_{00} = 1.46 – 0.69 = 0.77

b_{00} = 8.10 – 0 + 0.77 = 8.87

After all these calculations, the summary table is constructed as shown in Table 3.1 in order to see the overall picture clearly:

Table 3.1 Summary of results for single flight two classes case.

Level	demand	spill	recapture	booking
$\mathbf{a_{11}}$	4.57	0	0.43	5
$\mathbf{a_{12}}$	4.2	1.46	0.26	3
H	8.77	1.46	0.69	8
$\mathbf{a_{00}}$	8.1	0	0.77	9
M	16.87	1.46	1.46	17

When we analyze above table, we see that the host carrier has more demand than the competitor. There are totally 1.46 spills for the host carrier, 0.69 of it is recaptured by the host carrier and 0.77 goes to the competitor. There is no spill for the flight class a_{11} because it was fully open. If we apply demand mass balance equation for flight class a_{11}: 4.57 – 0 + 0.43 = 5. This fits with our results. The same check can be also done for other levels. Because a_{12} was closed, 0.43 of spill was captured by a_{11} and 0.77 of it was captured by the competitor. As it is said before, this heuristic assumes that number of competitors is sufficient to capture remained spill. So, spill of the competitor is 0, naturally. By looking carrier level, we can say that the competitor airline is better in recapturing, but the host carrier has more demand.

3.2.2 Two Flights Single Class Case

In this real case, a market with 2 flights and one class is chosen and required data is collected for 18-21 days prior. It is shown below:

b_{11}= 6 (bookings)

b_{21}= 3 (bookings)

$b_{11}(l)$ = 5 (bookings)

$b_{21}(l)$ = 2 (bookings)

ϕ_{11}= 0.75 (open class)

ϕ_{21}= 0.5 (semi-closed class)

Airlines` market share is 67%

By given this information, demand, spill and recapture values are estimated jointly.

By using equation (2.25), flights class shares at airline level are estimated as:

$\prod_{11} = 5 / 7 = 0.7143 = 71\%$
$\prod_{21} = 2 / 7 = 0.2857 = 29\%$

From these values, we can estimate flight class shares at market level as:

$\pi_{11} = 0.7143 \text{ x } 0.67 = 0.4786 = 48\%$
$\pi_{21} = 0.2857 \text{ x } 0.67 = 0.1914 = 19\%$
$\pi_{00} = 0.33 = 33\%$

After calculating flights class shares at market level, closure (first two equations) and recapture rate (third equation) can be estimated by using equations (2.4) – (2.6):

$\pi_o^M = 0.75 \text{ x } 0.4786 + 0.5 \text{ x } 0.1914 + 1 \text{ x } 0.33 = 0.7847 = 78\%$
$\pi_c^M = 1 – 0.7847 = 0.2153 = 22\%$
$\rho = ((0.7847 – 0.33)) / 0.7847 = 0.5795$

Above results are at market level. In order to convert them to the host level, equations (2.7) – (2.9) are used as:

$\pi^H = 1 – 0.33 = 0.67 = 67\%$
$\prod_c^H = 0.2153 / 0.67 = 0.3213$
$\prod_o^H = 1 – 0.3213 = 0.6787$

Now, demand mass balance equations (2.10) – (2.12) can be applied to find demand, spill and recapture:

$d^H - s^H + r^H = 9$
$s^H = \prod_c^H d^H = 0.3213 d^H$
$r^H = \rho s^H = 0.5795 s^H$

If we jointly solve above three equations we find:

d^H (demand) = 10.40

s^H (spill) = 3.34

r^H (recapture) = 1.94

If we apply demand mass balance equations (2.13) – (2.15) at flight class level for a_{11}:

$d_{ij} - s_{ij} + r_{ij} = 6$
$s_{ij} = d_{ij}\kappa_{ij}$
$r_{ij} = r^H(b_{ij} / b^H) = 1.29$

If we jointly solve above three equations we find:

d_{11} (demand) = 6.28

s_{11} (spill) = 1.57

r_{11} (recapture) = 1.29

Similarly for $a_{21:}$

d_{21} (demand) = 10.4 – 6.28 = 4.12

s_{21} (spill) = 3.34 – 1.57 = 1.77

r_{21} (recapture) = 1.94 – 1.29 = 0.65

In order to find market level results, equations (2.17) – (2.20) are used as:

d^M = 10.4 / 0.67 = 15.52

s^M = 0.2153 x 15.52 = 3.34

$r^M = s^M$ = 3.34

$b^M = d^M$ = 15.52 (16)

For competitor results, equations (2.21) – (2.24) are used as:

d_{00} = 10.4 x 0.33 / 0.67 = 5.12

s_{00} = 0

r_{00} = 3.34 – 1.94 = 1.4

b_{00} = 5.12 – 0 + 1.4 = 6.52

After all these calculations, the summary table is constructed as shown in Table 3.2 order to see the overall picture clearly:

Table 3.2 Summary of results for two flights single class case.

Level	demand	spill	recapture	booking
a_{11}	6.28	1.57	1.29	6
a_{21}	4.12	1.77	0.65	3
H	10.4	3.34	1.94	9
a_{00}	5.12	0	1.4	7
M	15.52	3.34	3.34	16

Applying demand mass balance equations at carrier level, we see that the results are correct. 10.4 – 3.34 + 1.94 = 9. By looking at Table 3.2, we can easily say that the host carrier has much more demand than the competitor, it is more than doubled. In recapture side, the host carrier is again in front of the competitor, 1.94 of total 3.34 spills is recaptured by the host carrier. This is a good performance. The flight class a_{11} recaptures more than the flight class a_{12}, long duration of being available affects this result. When we look at demand of the flight classes, we see that a_{11} has more demand even than the competitor. This shows performance of the flight class. We see again no spill for the competitor because of the assumption.

3.2.3 Three Flights Three Classes Case

In this real case, a market with three flights and three classes is chosen and required data is collected for 18-21 days prior. Input values and required estimates are shown in Table 3.3.

Table 3.3 Booking, closure, class share data for three flights three classes case.

a_{ij}	b_{ij}	$b_{ij(l)}$	κ_{ij}	$\prod_{ij}$	π_{ij}	π_o^M	$\prod_C^H$	ρ_{ij}
a_{11}	3	2	0	0.09	0.04	0.04	0	0.05
a_{12}	4	4	0	0.17	0.09	0.09	0	0.1
a13	0	1	0.5	0.04	0.02	0.01	0.02	0.01
a_{21}	5	6	0	0.26	0.13	0.13	0	0.14
a_{22}	0	1	0.75	0.04	0.02	0.01	0.03	0.01
a_{23}	0	2	1	0.09	0.04	0	0.09	0
a_{31}	0	1	0	0.04	0.02	0.02	0	0.02
a_{32}	4	3	0	0.13	0.07	0.07	0	0.07
a_{33}	5	3	0.25	0.13	0.07	0.05	0.03	0.05
a_{00}			0		0.5	0.5		0.55

Airlines` market share is 50%

By given this information, demand, spill and recapture values are estimated jointly as:

d^H (demand) = 26.48

s^H (spill) = 4.50

r^H (recapture) = 2.03

In order to find market level results, equations (2.17) – (2.20) are used as:

$d^M = 26.48 / 0.50 = 52.96$

$s^M = 0.085 \times 52.96 = 4.50$

$r^M = s^M = 4.77$

$b^M = d^M = 52.96$ (53)

For competitor results, equations (2.21) – (2.24) are used as:

$d_{00} = 26.48 \times 0.33 / 0.67 = 26.48$

$s_{00} = 0$

$r_{00} = 4.50 – 2.03 = 2.47$

b_{00} = 26.48 – 0 + 2.47 = 28.95

The summary of results is shown below in Table 3.4 at airline level:

Table 3.4 Summary of results for three flights three classes case.

Level	demand	spill	recapture	booking
H	26.48	4.5	2.03	24
a_{00}	26.48	0	2.47	29
M	52.96	4.5	4.5	53

For multi-flight and multi-class case, comparison is done at carrier level. There is total 4.5 spills in the market –this is also the host carrier`s spill- and 2.03 of it was recaptured by the host carrier and 2.47 of it was captured by the competitor. So, we can say that the competitor is better than the host carrier in recapturing. Because of the equality of the market share, we see that demand of the carriers is the same which is 26.48. Spill of the competitor is again 0 because of the assumption.

CHAPTER 4

RESULTS AND CONCLUSION

4.1 RESULTS

Simulation of unconstraining demand heuristic requires a complex and sophisticated framework. Instead of doing simulation, recapture-independent approach is used in order to compare with the proposed heuristic. This method estimates the demand as:

$$d^{H}_{indep} = b^{H} / (1-\kappa^{H}) \tag{4.1}$$

In other words, the independent demand method does not consider recapture effect and it is expected that by ignoring recapture effect, estimated demand might be high because of double count problem as explained previous parts. As it is seen above equation, only closure rate is used to estimate the demand. In fact, majority of airlines use this method and they observe overestimated demand values. This problem causes lower classes to be generally closed which is undesirable for customers.

Comparison between the proposed heuristic and the independent demand method is basically based on estimated demand values. In order to make a healthier comparison, demand estimation method of the host airline is also added into the comparison part. In order to do that, five different types of market are selected. These markets differ from each other in aspect of availability, frequency and geographical regions. Moreover, market share of carriers differ from market to market. Different from previous cases, ten snapshots of the availability are taken for each market and they are recorded. The number of bookings in each snapshot is also included into input part of the heuristic.

Table 4.1 Spill and recapture rate estimation.

Market	b (hist)	κ (cls rt)	s (spill)	ρ (recap)
ABC	581	7,00%	35,63	64,00%
DEF	403	11,79%	32,71	82,00%
JKL	398	50,00%	72,86	58,00%
PRS	62	79,23%	44,27	76,00%
TUV	406	45,64%	200,44	70,00%

The heuristic needs four input values which are historical bookings, availability of classes, market share of airline and flight class shares. Last year's bookings are used in this method and as it is said before, ten snapshots of the availability are collected. Market share data is taken from MIDT (Marketing Information Data Tapes). Flight class shares are estimated by using airlines` own historical data.

Estimated spill and recapture rates are shown in Table 4.1. As it is seen in Table 4.1, each market has different bookings and closure rates. Based on these values, spill and recapture rates differ. ABC and DEF markets have very low closure rate, in other words, classes are generally open. Because of that, estimated spill values are small in these markets. TUV market has 45.64% closure rate and spill is 200.44. This is a high value as compare to bookings, however recapture rate is not bad. 70% of the spill is recaptured by the host airline. PRS market is a smaller market as compare to other markets, however there is a high spill in this market, too. The reason is availability. As it is seen in Table 4.1, this market is generally closed by having 79.23% closure rate. This causes many passengers to be rejected and generates lots of spill. As a result, by looking at Table 4.1 it can be said that spill is generally based on availability status of market.

The host airline is very good at recapturing. In all markets, recapture rates are high for the host airline. There may be many reasons for having high recapture rate. Market share is the most important one. There are some essential attributes of airline that attract customers. Price, schedule quality and aircraft type are some of them. By having good schedule, it is easy to attract customers because schedule means alternatives according to customers. For example if the price of the morning flight is high, customer may attend to make reservation for evening flight if he sees lower price.

Table 4.2 Comparison of different demand estimation methods.

Market	b (hist)	κ (cls rt)	ρ (recap)	d (indep)	d (host)	d (heur)	Δ (indep)	Δ (host)
ABC	581	7,00%	64,0%	624,73	611,72	593,83	-5,20%	-3,0%
DEF	403	11,79%	82,0%	456,86	452,64	408,90	-11,73%	-10,7%
JKL	398	50,00%	58,0%	796,00	545,68	428,60	-85,72%	-27,3%
PRS	62	79,23%	76,0%	298,51	168,30	100,63	-196,64%	-67,2%
TUV	406	45,64%	70,0%	746,87	532,20	466,13	-60,23%	-14,2%

As it is seen in Table 4.2, relative to observed bookings, demand estimations of the independent demand method basically depend on closure rates and it has the highest demand estimation because of double counting problem. In PRS market, the independent demand method estimates 298,51 demand which is too high. Difference regarding to the proposed method is also computed in order to see gap between methods. In PRS market, there is 196,64% difference between proposed and the independent demand methods. In this market, the proposed method estimates 100,63 demand and 76% recapture rate. At the host airlines` side, 298,51 demand is estimated for this market and it is between the proposed and the independent methods. The picture is the same for other markets. It can be said that by ignoring recapture effect, there is a risk of overforecast. The proposed heuristic involves both recapture rate and market share and it produces more satisfying results.

The host airlines` own model produces results close to the proposed heuristic, however ignoring market or flight level recapture may cause low performance on demand estimation. The advantage of this method is having a dynamic forecasting methodology. This feature provides an accuracy test and being able to adjust demand forecasts. Table 4.2 shows demand estimation of three methods for only one departure date. However, it is a good way to look different dates of the same day of week in order to make a healthier comparison. In order to do that, data of different dates is collected for JKL market and the same day of week which is DOW5 is preserved. Three models are run for this market and the result is shown in Table 4.3.

Table 4.3 Demand estimation methods for JKL market and the same day of week (5).

Market	b (hist)	κ (cls rt)	d (indep)	d (host)	d (heur)	Δ (indep)	Δ (host)
JKLDOW5	469	38,46%	762,11	609,62	505,06	-50,89%	-20,70%
JKLDOW5	398	50,00%	796,00	539,71	428,6	-85,72%	-25,92%
JKLDOW5	486	52,34%	1019,72	682,3	523,36	-94,84%	-30,37%
JKLDOW5	471	58,77%	1142,37	622,15	507,21	-125,23%	-22,66%

Different closure rates cause having different demand estimations. By keeping day of week dimension constant, above results are computed. The independent demand method has the highest demand estimations and the host airlines` demand model produces lower demand estimations as compare to it. The picture generally has not changed. The percentage of difference between the proposed heuristic and the host airlines` demand model is in the range of 20%-30%, while it is 50%-125% for the independent demand model. The host airlines` own demand estimation method is very dynamic and this helps the method to be updated. This is achieved by adjusting demand forecast of pre-departure flights, according to current forecast accuracy. In other words, the method corrects forecast error itself.

4.2 CONCLUSION

In this study, an extended version of a heuristic is proposed by estimating the new flight class share derived by using last year's booking data. The heuristic is applied to 3 different cases; single flight with two classes, two flights with single class and three flights with three classes. In order to make comparison between demand estimation methods, 5 different markets are selected and real booking and availability data are collected for one year reservation horizon. However, since it is not possible to see first choice demand, an exact comparison requires simulation and this is a topic of a future paper. The demand estimation of proposed heuristic is compared with the host airline's demand estimation and the independent demand methods. The proposed heuristic seems to have low overforecast risk due to considering recapture and market share effects. It is seen that by using recapture rate, double counting problem is eliminated and so is demand overestimation. The host airlines` own demand estimation method also produces demand estimations close to the proposed heuristic as compare to the independent demand method. The advantage of this method is being very dynamic and it helps the method to be updated. This is achieved by adjusting demand forecast of pre-departure flights, according to current forecast accuracy. In other words, the method corrects forecast error itself. Unconstraining is a key issue for a successful revenue management system. It calibrates the base forecasts by reducing forecast errors and helps the optimization system to give optimum availability.

MNL discrete choice models can also be successfully integrated into demand the heuristic in order to estimate flight class share, however it is difficult to calibrate these models. Expectation maximization method of Talluri, K. and van Ryzin, G (2004) is one of the most used methods.

Estimating dynamic market share can be another future research topic. By this way, demand estimations might be more accurate and fit to the market conditions. In our study, we assume 0 spill for market. It is known that there are many passengers who think to choose another transportation option, if they see high price. Therefore, a statistical value can be used for estimating "do not fly" option. Realistic simulations can be developed based on real inputs in order to measure the effects of different demand estimation methods.

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