

The Bees Algorithm

Theory, Improvements and Applications

A thesis submitted to Cardiff University

For the degree of

Doctor of Philosophy

by

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Abstract

In this thesis, a new population-based search algorithm called the *Bees Algorithm (BA)* is presented. The algorithm mimics the food foraging behaviour of swarms of honey bees. In its basic version, the algorithm performs a kind of neighbourhood search combined with random search and can be used for both combinatorial and functional optimisation. In the context of this thesis both domains are considered. Following a description of the algorithm, the thesis gives the results obtained for a number of complex problems demonstrating the efficiency and robustness of the new algorithm.

Enhancements of the Bees Algorithm are also presented. Several additional features are considered to improve the efficiency of the algorithm. Dynamic recruitment, proportional shrinking and site abandonment strategies are presented. An additional feature is an evaluation of several different functions and of the performance of the algorithm compared with some other well-known algorithms, including genetic algorithms and simulated annealing.

The Bees Algorithm can be applied to many complex optimisations problems including multi-layer perceptrons, neural networks training for statistical process control and the identification of wood defects in wood veneer sheets. Also, the algorithm can be used to design 2D electronic recursive filters, to show its potential in electronics applications.

A new structure is proposed so that the algorithm can work in combinatorial domains. In addition, several applications are presented to show the robustness of the algorithm in various conditions. Also, some minor modifications are proposed for representations of the problems since it was originally developed for continuous domains.

In the final part, a new algorithm is introduced as a successor to the original algorithm. A new neighbourhood structure called Gaussian patch is proposed to reduce the complexity of the algorithm as well as increasing its efficiency. The performance of the algorithm is tested by use on several multi-model complex optimisation problems and this is compared to the performance of some well-known algorithms.

To my wife and daughter

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DECLARATION AND STATEMENTS

DECLARATION

This work has not previously been accepted in substance for any degree and is not concurrently submitted in candidature for any degree.

Signed  (Ebubekir Koç) Date 05/02/2010

STATEMENT 1

This thesis is being submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy (PhD).

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STATEMENT 2

This thesis is the result of my own independent work/investigation, except where otherwise stated. Other sources are acknowledged by explicit references.

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Abbreviations

AI	Artificial Intelligence
ML	Machine Learning
SI	Swarm Intelligence
BA	Bees Algorithm
BA-II	Bees Algorithm-II
EAs	Evolutionary Algorithms
EP	Evolutionary Programming
ES	Evolutionary strategies
GA	Genetic Algorithm
TS	Tabu Search
PSO	Particle Swarm Optimisation
DPSO	Discrete PSO
SOAs	Swarm-based optimisation algorithms
DE	Differential Evolution
AS	Ant System
ACO	Ant Colony Optimisation
SA	Simulated Annealing
SIMPSA	Simplex method
MLP	Multi-Layered Perceptrons
BP	Backpropagation
n	Number of scout bees
m	Number of patches selected out of n visited points
e	Number of best patches out of m selected patches

nep	Number of bees recruited for e best patches
nsp	Number of bees recruited for the other (m-e) selected patches
ngh	Size of patches
sc	Shrinking constant
pd	Patch density
ρ	Selection threshold (ρ defined as a percentage)
sat	Site abandonment threshold

List of Symbols

v_i^k	velocity of agent.
$\rho \in (0,1]$	the evaporation rate.
$\Delta\tau_{ij}^k$	the quantity of pheromone laid on edge (i,j) .
$N(s_k^p)$	the set of edges that do not belong to the partial solution s_k^p of ant .
s_i^k	current position of agent i at iteration k.
pbest _i	personal best of agent i.
gbest	the global best of the population.
$p(c_{ij} s_k^p)$	transition function
ρ_i^{k+1}	vector of random numbers
ngh(i)	Neighbouring function
$\varphi_{\mu\sigma}(\chi)$	Gaussian function
x_{imr}	Normal random variate generator

Chapter 1

INTRODUCTION

This chapter presents the motivation and research objectives, and the methods adopted. The chapter also outlines the general structure of the thesis.

1.1. Motivation

Many complex multi-variable optimisation problems cannot be solved exactly within polynomially bounded computation times. This has generated much interest in search algorithms that find near-optimal solutions in reasonable running times. Many intelligent swarm-based optimisation methods were developed. Evolutionary algorithms may be considered as one of the first of this class of algorithms. Evolutionary Strategies (Rechenberg, 1965), Evolutionary Programming (Fogel et al.,

1966) and Genetic Algorithms (Holland, 1975) were developed to deal with complex multi-variable optimisation problems. Although they are considered in this class because of their population-based structure, they may also be separated from swarm-based optimisation algorithms due to their centralised control mechanisms. Some of the first significant examples in this class were Ant System (Dorigo et al., 1991) and Particle Swarm Optimisation algorithms (Kennedy and Eberhart, 1995) which have no centralised control over their individuals.

In the context of optimisation and swarm intelligence, the first motivation for the research presented in this thesis was to develop a novel intelligent swarm-based optimisation method called the Bees Algorithm which would be capable of solving many complex multi-variable optimisation problems in more robust and efficient ways than existing algorithms. Secondly, to implement the algorithm for continuous domains and improve it with additional features which are necessary for complex industrial problems. Combinatorial problems are another domain for swarm intelligence algorithms. In this field, it is quite difficult to implement the current algorithm in this domain when it was proposed originally for continuous domains. Therefore, it is interesting to explore the opportunities and limitations of the improved algorithm to this challenging domain and this is the third motivation. Fourthly, to further develop the algorithm with the addition of a new organised structure as well as improved robustness and efficiency. The Final motivation for this research is to encourage the use of the Bees Algorithm for complex multi-variable optimisation problems as an alternative to the use of more popular intelligent swarm-based optimisation methods such as genetic algorithms, artificial ant colony algorithm and particle swarm optimisation algorithm.

1.2. Aims and Objectives

The overall research aim is to develop swarm-based optimisation algorithms which are inspired by honey-bees' foraging behaviours, for use in complex optimisation problems and with improved efficiency.

The main research objectives are:

- To develop a new intelligent swarm-based optimisation method inspired by the food foraging behaviours of honey-bees. Also to apply it to various industrial problems.
- To enhance the algorithm's neighbourhood search procedure so that it can perform well in combinatorial domains.
- To improve the original algorithm with a new neighbourhood strategy and to reduce the number of parameters as proposed in the original algorithm.

1.3. Methods

For the topics analysed in this thesis, each one will follow the same problem solving methods to reach the desired objectives. The methods used in this research may be summarised as follows:

- Literature review: the most relevant papers for each research topic will be reviewed to clarify the key points in the subject and to identify any shortcomings.

- A novel swarm-based optimisation procedure will be proposed along with improved versions of the algorithm. Innovation started with a informed criticism of existing methods. Studying Nature is a most important part of this thesis.
- Many experiments will be carried out to understand the behaviours of the algorithm under different conditions. To do this, some software needs to be written in a variety of programming languages and the results compared to existing works in the literature.

1.4. Outline of the thesis

Chapter 2 briefly reviews swarm intelligence and intelligent swarm-based optimisation algorithms. Behaviours of honey-bees in natural conditions, including food foraging used in the context of this thesis are explained in details. Computational simulations of honey-bee behaviours are reviewed to show the link between nature and optimisation algorithms. Honey-bees inspired algorithms are also reviewed in this chapter.

Chapter 3 introduces the Bees Algorithm as a novel intelligent swarm-based optimisation method in its simplest form. Then, it focuses on enhancements to the Bees Algorithm for local and global search. The algorithm is improved with the

addition of dynamic recruitment, proportional patch shrinking and site abandonment ideas. The performances of the basic and improved algorithms are also compared and the differences presented. Also, the improved algorithm is compared with some well-known algorithms to show its performance and robustness.

Chapter 4 focuses on implementations of the Bees Algorithm in continuous domains. The algorithm is applied first on training multi-layered perceptrons (MLP) neural networks for statistical process control. Details of this application and of MLP-Bees Algorithm training are explained. Then another application for similar MLP neural networks is presented. The application focuses on a real data set for wood defect identification. The performance of the algorithm is also evaluated for a 2D recursive filter design problem.

Chapter 5 describes improvements to the Bees Algorithm. These additions make the algorithm suitable for use in combinatorial domains. Since it was developed for continuous domains, some modifications were needed to apply the algorithm to neighbourhood search. Several local search algorithms are suggested for the algorithm as well as site abandonment. The performance of the modified algorithm is evaluated for several difficult applications including single machine job scheduling, flow shop scheduling and manufacturing cell formation problems. For all the problems, the algorithm is also modified in terms of suitable neighbourhood structure and representation.

Chapter 6 presents the Bees Algorithm-II as an improved version of the original algorithm. The chapter introduces a new structure that has more control over the randomness used for neighbourhood search. A Gaussian distribution is used in the form of a normal random variate generator as a new recruitment strategy. Also, the procedure to reduce the number of parameters, which were many and difficult to set, is explained. The performance of the proposed algorithm is evaluated for several functional optimisation problems and the results are compared both with the original Bees Algorithm and with other well-known algorithms.

Chapter 7 summarises the thesis and proposes directions for further research.

Appendix A presents the C++ Code for the Bees Algorithm.

Chapter 2

BACKGROUND

This chapter reviews the notation as well as the basic concepts of swarm intelligence theory. Several intelligent swarm-based optimisation algorithms are investigated. In this chapter, the individual and social behaviour of honey-bees is reviewed from a swarm intelligence point of view. Several computational models are also presented to explain the interactions between individuals that constitute the swarm intelligence. Finally, the most recent studies of algorithms inspired by the behaviour of honey-bees and their applications are reviewed.

2.1. Swarm intelligence

Swarm Intelligence (SI) is defined as the collective problem-solving capabilities of social animals (Bonabeau et al., 1999). SI is the direct result of self-organisation in

which the interactions of lower-level components create a global-level dynamic structure that may be regarded as intelligence (Bonabeau et al., 1999). These lower-level interactions are guided by a simple set of rules that individuals of the colony follow without any knowledge of its global effects. Individuals in the colony only have local-level information about their environment. Using direct and/or indirect methods of communication,, local-level interactions affect the global organization of the colony.

Self-organization is created by four elements as were suggested by Bonabeau et al., (1999). Positive feedback is defined as the first rule of self-organization. It is basically a set of simple rules that help to generate the complex structure. Recruitment of honey bees to a promising flower patch is one of the examples of this procedure. The second element of self-organization is negative feedback, which reduces the effects of positive feedback and helps to create a counterbalancing mechanism. The number of limited foragers is an example of negative feedback. Randomness is the third element in self-organisation. It adds an uncertainty factor to the system and enables the colonies to discover new solutions for their most challenging problems (food sources, nest sites etc.). Last but not least, there are multiple interactions between individuals. There should be a minimum number of individuals who are capable of interacting with each other to turn their independent local-level activities into one interconnected living organism. As a result of combination of these elements, a decentralised structure is created. In this structure there is no central control even though there seems to be one. A hierarchical structure is used only for dividing up the necessary duties; there is no control over individuals but over instincts. This creates dynamic and efficient structures that help the colony to survive despite many challenges.

There are many different species of animal that benefit from similar procedures that enable them to survive and to create new and better generations. Honey-bees, ants, flocks of birds and shoals of fish are some of the examples of this efficient system in which individuals find safety and food. Moreover, even some other complex life forms follow similar simple rules to benefit from each others' strength. To some extent, even the human body can be regarded as a self-organised system. All cells in the body benefit from each others' strength and share the duties of overcoming the challenges which are often lethal for an individual cell.

2.2. Intelligent swarm-based optimisation

Swarm-based optimisation algorithms (SOAs) mimic nature's methods to drive a search towards the optimal solution. A key difference between SOAs and direct search algorithms such as hill climbing and random walk is that SOAs use a population of solutions for every iteration instead of a single solution. As a population of solutions is processed as an iteration, the outcome of each iteration is also a population of solutions. If an optimisation problem has a single optimum, SOA population members can be expected to converge to that optimum solution. However, if an optimisation problem has multiple optimal solutions, an SOA can be used to capture them in its final population. SOAs include Evolutionary Algorithms (i.e. the Genetic Algorithm), the Ant Colony Optimisation (ACO) and the Particle Swarm Optimisation (PSO). Common to all population-based search methods is a strategy that generates variations of the solution being sought. Some search methods use a greedy criterion to decide

which generated solution to retain. Such a criterion would mean accepting a new solution if and only if it increases the value of the objective function.

2.2.1. Evolutionary algorithms

Inspired by the biological mechanisms of natural selection, mutation and recombination Evolutionary Algorithms (EAs) were first introduced in the forms of Evolutionary Strategies (Rechenberg, 1965), Evolutionary Programming (Fogel et al., 1966) and afterwards Genetic Algorithms (Holland, 1975). EAs were the first search methods to employ a population of agents. In EAs, using the stochastic search operators, the population is evolved to the optimal point(s) of the search space. Population, genome encoding and selections of good individuals are some of the most common features for all EAs. The selection procedure (deterministic or stochastic) is used to pick the good individuals that will produce the new population (Holland, 1975; Fogel, 2000). The crossover operator was introduced to create new individuals by randomly exchanging the genes. This operator provides a social interaction effect between individuals (Holland, 1975; Fogel, 2000). The mutation operator was introduced to generate small perturbations to enable exploration of the search space and avoid any premature convergence (Holland, 1975; Fogel, 2000). There are several different types of EAs in the literature including evolutionary strategies, evolutionary programming, genetic algorithm and differential evaluation.

Evolutionary strategies (ES) were first introduced by Rechenberg, (1965) as one of the first successful applications of EAs. Further improvements to these were also made by

Schwefel, (1981). ES was defined as an optimization technique based on ideas of adaptation and evolution (Rechenberg, 1965). ES uses primarily mutation and selection as its search operators. In EAs, solutions are represented in two (often three) n -dimensional real vectors as well as by standard deviations. The mutation operator is defined as the addition of a random value (normally distributed) to each vector. The selection operator is deterministic based on the fitness ranking. The population of basic ES consists of two individuals, parent and mutant of the parent. For the next generation, if the fitness of the mutant is better than or equal to that of its parent then the mutant becomes parent. Otherwise the mutant is ignored. This procedure is defined as a $(1 + 1)$ -ES. On the other hand, there is also a $(1 + \lambda)$ -ES in which λ mutants is generated and competes with the parent who was ignored and the best mutant becomes the parent of the next generation (Schwefel, 1981).

Evolutionary programming (EP) was first introduced by Fogel et al., (1966) to predict a binary time series. It was further developed by Fogel, (1995) and applied to several different problems, including optimisation and machine learning. The original model was based on the organic evolution of the species, thus recombination was not included in EP. The representations used for EP depended on the problem domains. The main difference between EP and other EAs is that there is no interaction between individuals in EP. This means that no crossover operator is used, only the mutation operator used to create offspring individuals. As a selection method, all individuals are selected to be parents and all parents mutated to create the same number of offspring. Gaussian mutation was used to generate an offspring from each parent and the next generation was constructed by better parents as well as selected offspring. EP was applied to many optimisation problems (Back, 1993; Fogel, 1995).

Holland, (1975) first introduced a schema theory as a basis for Genetic Algorithms (GAs). The GA is based on natural selection and genetic recombination. In GAs candidate solutions are encoded in the form of binary strings called chromosomes which are constructed by a number of sub-strings representing the features of candidate solutions. This binary representation, however, set aside the maximum number of schemata to be processed for all individuals (Holland, 1975). There were several binary encoding systems which were introduced in the literature making different representations available for different domains (Michalewicz, 1996). However, there are some shortcomings with these representations, including high computational cost and trapping to local optima (Fogel, 2000). The algorithm works by choosing solutions from the current population and then applying genetic operators – such as mutation and crossover – to create a new population. The algorithm efficiently exploits historical information to speculate on new search areas with improved performance (Fogel, 2000). When applied to optimisation problems, the GA has the advantage of performing a global search. The GA may be hybridised with domain-dependent heuristics for improved results. For example, Mathur et al., (2000) described a hybrid of the ACO algorithm and the GA for continuous function optimisation.

Differential Evolution (DE) was proposed by Stone and Price, (1997) as a population-based search strategy which was similar to standard EAs. The only difference was in the breeding stage where a different operator was used. In this stage, an arithmetic crossover operator was used to create an offspring out of three parents. An arithmetic operator was used to calculate the differences between randomly selected pairs of individuals (Price et al., 2005). A new generation was created by the offspring

population if one offspring was better than a parent, otherwise the parent would stay in the new generation list.

2.2.2. Ant colony optimisation

Ant Colony Optimisation (ACO) is a non-greedy population-based metaheuristic which emulates the behaviour of real ants. It can be used to find near-optimal solutions to difficult optimisation problems. Ants are capable of finding the shortest path from the food source to their nest using a chemical substance called pheromone which is used to guide the exploration. The pheromone is deposited on the ground as the ants move and the probability that a passing stray ant will follow this trail depends on the quantity of pheromone laid.

Ant-inspired algorithms were introduced by Dorigo et al., (1991). Ant System (AS) is one of very first versions of ant-inspired algorithms to be proposed in the literature (Dorigo et al., 1991; Dorigo, 1992; Dorigo et al., 1996). The first algorithm was aiming to search for an optimal path in a graph based on a probabilistic decision depending on positive and negative feedback (i.e. pheromone update and decay). Pheromone which is updated by all the ants after a complete tour is the key idea in this algorithm. Pheromone update (τ_{ij}) for the edges of the graph (c_{ij}) that are joining the cities i and j is calculated as follows (Dorigo et al., 1991):

$$\tau_{ij} \leftarrow (1 - \rho) \cdot \tau_{ij} + \sum_{k=1}^m \Delta \tau_{ij}^k \quad (2.1)$$

where m is the number of ants, $\rho \in (0,1]$ is the evaporation rate, and $\Delta\tau_{ij}^k$ is the quantity of pheromone laid on the edge (i,j) . The value of the quantity of pheromone laid on the edges is determined by the tour length (L_k) of an ant:

$$\Delta\tau_{ij}^k = \begin{cases} \frac{1}{L_k} & \text{if ant } k \text{ used edge } (i,j) \text{ in its tour,} \\ 0 & \text{otherwise,} \end{cases} \quad (2.2)$$

In AS, solutions are constructed according to a probabilistic decision made at each vertex. A transition function $p(c_{ij} | s_k^p)$ is used to calculate the probability of an ant moving from town i to town j :

$$p(c_{ij} | s_k^p) = \begin{cases} \frac{\tau_{ij}^\alpha \eta_{ij}^\beta}{\sum_{c_{il} \in N(s_k^p)} \tau_{il}^\alpha \eta_{il}^\beta} & \text{if } j \in N(s_k^p), \\ 0 & \text{otherwise,} \end{cases} \quad (2.3)$$

1. **Parameter Initialisation**
2. **WHILE (stopping criterion not met) do**
3. **ScheduleActivities**
4. **AntSolutionsConstruct()**
5. **PheromoneUpdate()**
6. **DeamonActions() optional**
7. **END ScheduleActivities**
8. **END WHILE**

Figure 2.1 The ant colony optimisation metaheuristic

where α and β are parameters that control the relative importance of the pheromone (τ_{ij}) versus the desirability of edge i,j (η_{ij}) which is determined using $\eta_{ij} = 1/d_{ij}$, where d_{ij} is the length of edge (c_{ij}) and $N(s_k^p)$ is the set of edges that do not belong to the partial solution s_k^p of ant k .

After AS was introduced as a basic method for ant-inspired algorithms, the ACO metaheuristic was developed to explain the behaviour of ants in more general ways (Dorigo and Di Caro, 1999) and was applied to TSP problems. ACO consists of three main functions (see Fig. 2.1) namely, AntSolutionConstruct(), PheromoneUpdate() and DeamonAction(). Equation 2.1 performs trail updates. Equation 2.3 is a procedure to construct the solution by iteratively moving through neighbouring positions using a transition rule. The DeamonAction() function is an optional procedure that updates the global best solution.

There are several improved versions of the ACO metaheuristic in the literature. The differences between the original idea and improved versions are made clear in this section. Further discussion can be found in references. Gambardella and Dorigo, (1996) proposed the Ant Colony System (ACS) which differs mainly by its pheromone update function. It was developed to be more in line with the natural behaviour of ants. A local pheromone update was employed including the update at the end of each epoch. Bullnheimer et al., (1996) presented a rank-based Ant System which includes the ranking concept into the pheromone update procedure. In this algorithm, ants are ranked according to the decreasing order of their fitness. The amount of pheromone deposited is distributed according to their order, meaning that the better fitness will receive more pheromone. Dorigo et al., (1996) proposed the Elitist Ant System, which

differs such that the global best solution deposits pheromone at every iteration along with all the other ants. Stützle and Hoos, (2000) proposed Max-Min Ant System with two improvements: namely, only the best ant updates the pheromone trails, and the pheromone update function is bound. There were also many hybrid versions of the ant-inspired algorithms which include Q-learning (Gambardella and Dorigo, 1995) and GA (Pilat and White, 2002).

2.2.3. Particle swarm optimisation

Kennedy and Eberhart, (1995) proposed the Particle Swarm Optimisation (PSO) which is a stochastic optimisation algorithm based on the social behaviour of groups of organisations, for example the flocking of birds or the schooling of fish. Pseudo-code of the PSO algorithm is given in Fig. 2.2. Similar to evolutionary algorithms, the PSO initialises with a population of random solutions. It searches for local optima by simply updating generations of individuals. However, PSO has no such operators such as crossover and mutation that EAs employ. Instead, individual solutions in a population are viewed as “particles” that evolve or change their positions with time. Each particle modifies its position in search space according to its own experience and also that of a neighbouring particle by remembering the best position visited by itself and its neighbours. Thus the PSO has a structure that combines local and global search methods (Eberhart and Kennedy, 2001).

1. Create particles (population) distributed over solution space (s_i^0, v_i^0) .
2. While (Stopping criterion not met) do
3. Evaluate each particle's position according to the objective function.
4. If $(s_i^{k+1}$ is better than s_i^k) (update pbest)

$$s_i^k = s_i^{k+1}$$

5. Determine the best particle (update gbest).
6. Update particles' velocities according to
7. Move particles to their new positions according to

$$v_i^{k+1} = v_i^k + c_1 rand_1 (pbest_i - s_i^k) + c_2 rand_2 (gbest - s_i^k)$$

$$s_i^{k+1} = s_i^k + v_i^{k+1}$$

8. Go to step 3 until stopping criteria are satisfied.

Figure 2.2 Pseudo-code of the PSO algorithm

In Fig. 2.2 the basic pseudo-code of the PSO algorithm is presented. The algorithm starts with creating particles that are uniformly distributed throughout the solution space by defining the initial conditions for each agent. Each agent is defined with an initial position (s_i^0) and an initial velocity (v_i^0). The pbest is set to current searching for each agent and the best pbest is set to gbest. After checking the stopping criterion in step 2, each particle's position is evaluated according to the objective function in step 3. If the existing position is better than the previous one, then pbest is updated in step 4 and followed by gbest update in step 5.

In step 6, particles' velocities are updated using the velocity vector given in equation 2.4. It has three main components: namely, particle's best performance so far (pbest), the best so far amongst all particles (gbest) and the inertia of particles (v). gbest represents the social interaction of particles in an indirect way.

$$v_i^{k+1} = v_i^k + c_1 rand_1 (pbest_i - s_i^k) + c_2 rand_2 (gbest - s_i^k) \quad (2.4)$$

where, v_i^k is velocity of agent i at iteration k , c_j is weighting factor, $rand_j$ is a random number between 0 and 1, s_i^k is current position of agent i at iteration k , pbest _{i} is personal best of agent i and gbest is the global best of the population.

In step 7, particles are moved to their new positions. The current position of a particle with a given velocity calculated by Equation 2.5 which updates the position as follows:

$$s_i^{k+1} = s_i^k + v_i^{k+1} \quad (2.5)$$

where, s_i^{k+1} is the position of agent i at iteration $k+1$, s_i^k is position of agent i at iteration k and v_i^{k+1} is the velocity of agent i at iteration $k+1$.

The PSO explained above was developed for continuous domains. Kennedy and Eberhart, (1997) also developed a discrete version of PSO for combinatorial domains. Instead of creating a continuous position, agents were determined as true or false as a result of a probability function (personal and social interactions) represented as follows:

$$P(s_i^{k+1} = 1) = f(s_i^k, v_i^k, pbest_i, gbest) \quad (2.6)$$

The probability threshold is calculated by an agent's predisposition (v) to be able to say true or false. The threshold is set in the range $[0, 1]$ and the agent is more likely to choose 1 if v is higher and 0 otherwise. A sigmoid function is often used to determine the value of v :

$$sig(v_i^k) = \frac{1}{1 + \exp(-v_i^k)} \quad (2.7)$$

The agent's disposition should be adjusted for its success and that of the group. In order to accomplish this, a formula for each v_i^k that will be some function of the difference between the agent's current position and the best positions found so far by

itself and by the group. Similar to the basic continuous version, the formula for the binary version of PSO can be described as follows:

$$v_i^{k+1} = v_i^k + rand(pbest_i - s_i^k) + rand(gbest - s_i^k) \quad (2.8)$$

$$\rho_i^{k+1} < sig(v_i^{k+1}) \text{ then } s_i^{k+1} = 1; \text{ otherwise } s_i^{k+1} = 0 \quad (2.9)$$

where, rand is a random number ($rand > 0$) drawn from a uniform distribution and ρ_i^{k+1} is a vector of random numbers between 0 and 1. The limit of rand is set so that two rand sum to no more than 4.0. Formulas are iteratively repeated for each dimension. The discrete PSO algorithm is almost identical to the basic PSO except the above decision equations 2.8 and 2.9. V_{max} is also set at beginning of an experiment, usually set to $[-4.0, +4.0]$. Several improved and hybrid versions of the PSO algorithm can also be found in the literature including (Kennedy, 2001; Kwang and Mohamed, 2008).

2.3. Bees in Nature: Food Foraging and Nest Site Selection

Behaviours

A colony of honey-bees can extend itself over long distances (more than 10 km) and in multiple directions simultaneously to exploit a large number of food sources (Von Frisch, 1967; Seeley, 1996). A colony prospers by deploying its foragers to good

fields. In principle, flower patches with plentiful amounts of nectar or pollen that can be collected with less effort should be visited by more bees, whereas patches with less nectar or pollen should receive fewer bees (Camazine et al., 2001).

The foraging process begins in a colony by scout bees being sent to search for promising flower patches. Scout bees move randomly from one patch to another. During the harvesting season, a colony continues its exploration, keeping a percentage of the population as scout bees (Seeley, 1996).

When they return to the hive, those scout bees that found a patch which is rated above a certain quality threshold (measured as a combination of some constituents, such as sugar content) deposit their nectar or pollen and go to the “dance floor” to perform a dance known as the “waggle dance” (Von Frisch, 1967). Source quality can be understood as simply the relation between gain and cost (see equation 2.10) from a specific nectar source (Von Frisch, 1976).

$$\text{Source Quality}[i] = (\text{gain}[i] - \text{costs}[i]) / \text{costs}[i] \quad (2.10)$$

This mysterious dance is essential for colony communication, and contains three pieces of information regarding a flower patch: the direction in which it will be found, its distance from the hive and its quality rating (or fitness) (Von Frisch, 1967; Camazine et al., 2001). This information helps the colony to send its bees to flower patches precisely, without using guides or maps. Each individual’s knowledge of the outside environment is gleaned solely from the waggle dance. This dance enables the colony to evaluate the relative merit of different patches according to both the quality

of the food they provide and the amount of energy needed to harvest it (Camazine et al., 2001). After waggle dancing on the dance floor, the dancer (i.e. the scout bee) goes back to the flower patch with follower bees that were waiting inside the hive. More follower bees are sent to more promising patches. This allows the colony to gather food quickly and efficiently.

While harvesting from a patch, the bees monitor its food level. This is necessary to decide upon the next waggle dance when they return to the hive (Camazine et al., 2001). If the patch is still good enough as a food source, then it will be advertised in the waggle dance and more bees will be recruited to that source.

Nectar source selection behaviour is one of the most challenging as well as vital tasks for honey-bee colonies (Camazine et al., 2001). When a honey-bee colony becomes overcrowded it needs to be divided for effective source management (Von Frisch, 1967; Camazine et al., 2001). This critical decision making process works without a central control mechanism. Nectar source selection behaviour mainly deals with the situation of a colony choosing between several nectar sources by simply measuring several factors at once and comparing them with other solutions. The decision is made when all the scout bees are dancing for the same site and it takes a couple of days before half of the colony moves to a new hive.

2.4. Computational Simulations of Honey-bee Behaviours

In this section, the computational simulation models of different honey-bee behaviours are presented as a bridging effort between nature and engineering to understand the

innovation path of swarm intelligence algorithms. The behaviours of honey-bees in nature have been studied thoroughly and several mathematical models were introduced. These models explain many aspects of the honey-bees in mathematical terms. There are several honey-bees related models introduced in the literature including nectar-source selection, nest-site selection, colony thermoregulation and comb pattern models (Camazine et al., 2001). Since the foraging behaviours of honey-bees is the scope of this thesis, nectar-source selection and nest-site selection models are presented in this section.

2.4.1. Nectar-Source Selection Models

Nectar source selection is one of the most challenging tasks for honey-bee colonies. This critical practice works without a central control mechanism. For nectar source selection, several mathematical models have been introduced. These models mainly deal with the situation of a colony choosing between several nectar sources. There are quite a few models developed to analyse the food source selection process of honey-bee colonies (Camazine and Sneyd, 1991; Bartholdi et al., 1993).

Camazine et al., (1991) and Camazine et al., (2001) presented a differential equations model to analyse the food-source selection process of honey-bees. Individual bees are represented in this model using a flow diagram for the nectar-source selection processes. Each forager bee needs to be in a compartment at any specific time. According to the model, there are five decision making branches for the situation of a colony choosing between two nectar sources. For every branch, there is a probability

function to calculate the probability of taking one or the other fork at each of the five branch points. Since the bees mostly depend on randomness, the probability of choosing one nectar source also depends on randomness related to number of dancers on the dance floor as well as on the time spent dancing. The results of the experiment show how a colony selectively exploits the richer food source for several hours. After altering the food sources, the model reacts promptly to adjust the population distribution and the exploitation process to a new environment.

To explain how the model works, a flow diagram given in Fig. 2.3 describes the foraging behaviour of a colony for every individual bee. In this model, each forager is in one of seven different compartments, represented by an activity (Camazine et al., 1991):

A: foraging at nectar source A

B: foraging at nectar source B

D_A : dancing for nectar source A

D_B : dancing for nectar source B

F: unemployed foragers observing a dancer

H_A : unloading nectar from source A

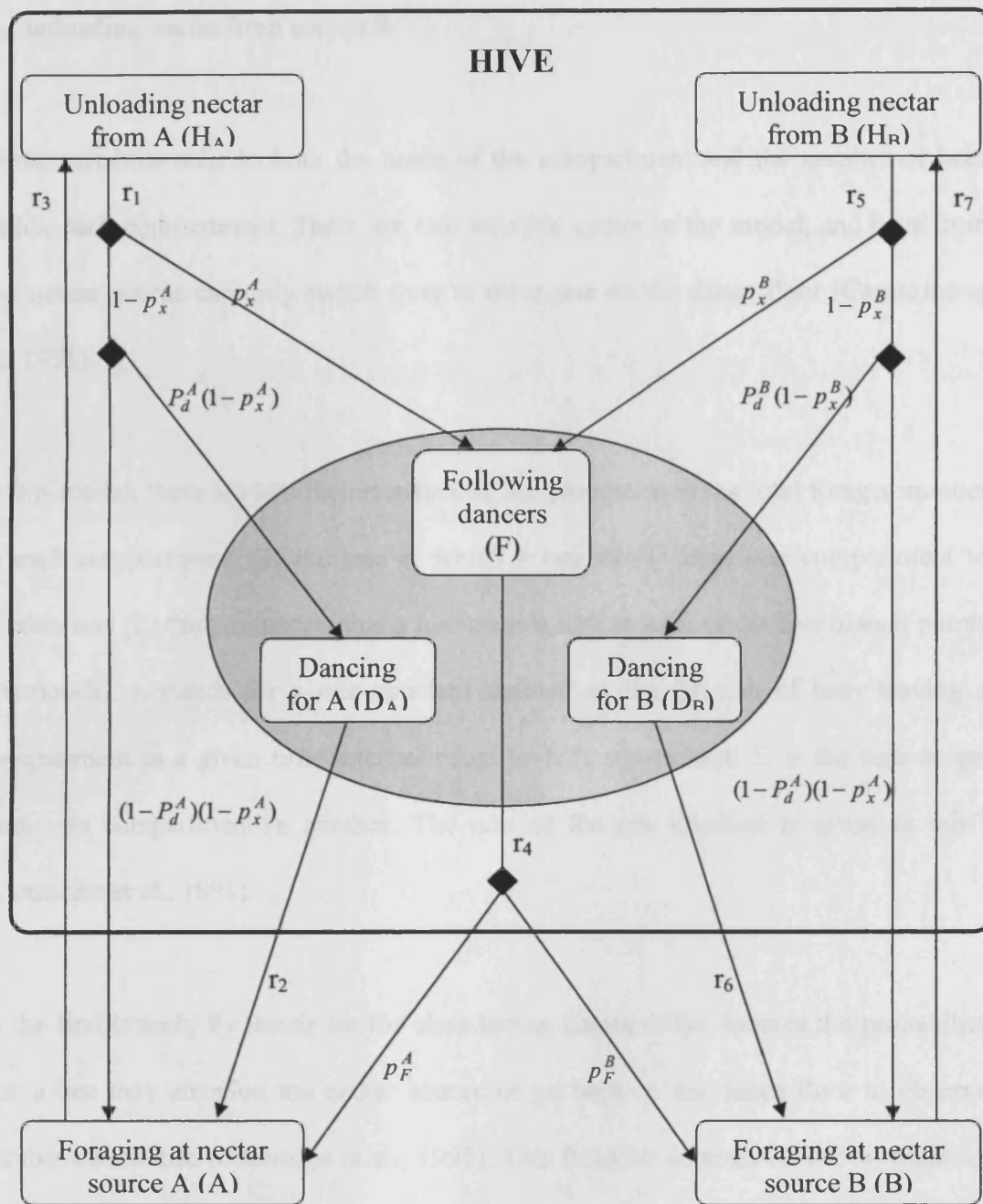


Figure 2.3 This mathematical model shows how honey-bee colonies allocate forager bees between two nectar sources (A and B). H_A , H_B , D_A , D_B , A , B , F are the compartments and the number of foragers in the compartments. r_1 - r_7 are the rates of leaving for each component. P_X^A , P_X^B , P_d^A , P_d^B , etc. denote the probability of choosing a fork in each branch. This flow diagram is drawn in accordance with the model figure given in (Camazine et al. 1991).

H_B : unloading nectar from source B

These variables refer to both the name of the compartment and the number of bees within each compartment. There are two separate cycles in the model, and bees from one nectar source can only switch over to other one on the dance floor (Camazine et al., 1991).

In this model, there are two factors affecting the proportion of the total forager number in each compartment: (1) the rate at which a bee moves from one compartment to another and (2) the probability that a bee takes a fork at each of the five branch points (diamonds). r_i stands for a rate constant defined as the fraction of bees leaving a compartment in a given time interval equal to $1/T_i$, where each T_i is the time to get from one compartment to another. The unit of the rate constant is given as min^{-1} (Camazine et al., 1991).

In the first branch, P_X stands for the abandoning function that denotes the probability that a bee may abandon the nectar source or go back to the dance floor to observe another dancer bee (Camazine et al., 1991). This function depends on the profitability of the source, so P_X^A represents the probability that a bee leaving H_A , abandoning the nectar source and becoming a follower bee (F) (Camazine et al., 1991).

The second branch point is for the bees that did not abandon their source (Camazine et al., 1991). At this point, a bee decides whether to dance for the nectar source or to fly back to the nectar source. P_d denotes the probability of performing a dance for the nectar source (Camazine et al., 1991). Its value also depends on the profitability of the

nectar source similar to the abandoning function. P_d^A denotes the probability of performing a dance for the nectar source.

The third branch occurs on the dance floor when a follower bee dances to decide for one of the nectar sources (Camazine et al., 1991). P_F^A , denotes the probability of a follower bee following dances for nectar source A and leaving for this nectar source (Camazine et al., 1991). Thus, the probability of following a dancer bee for A (P_F^A) can be calculated by $D_A/(D_A+D_B)$. The time limitation of D_A and D_B has been weighted and denoted as d_A and d_B . Therefore, each function (see equations 2.11 and 2.12) indicates the proportion of the total dancing for each nectar source by taking into account the number of dancers and the time spent dancing (Camazine et al., 1991).

$$P_F^A = \frac{D_A d_A}{D_A d_A + D_B d_B} \quad (2.11)$$

$$P_F^B = \frac{D_B d_B}{D_A d_A + D_B d_B} \quad (2.12)$$

Equations of the model, with some assumptions for simplicity, are written as the following set of differential equations (Camazine et al. 1991):

$$\frac{dA}{dt} = (1 - P_d^A)(1 - P_X^A)r_1 H_A + r_2 D_A + P_F^A r_4 F - r_3 A \quad (2.13)$$

$$\frac{dD_A}{dt} = P_d^A(1 - P_X^A)r_1 H_A - r_2 D_A \quad (2.14)$$

$$\frac{dH_A}{dt} = r_3 A - r_1 H_A \quad (2.15)$$

$$\frac{dB}{dt} = (1 - P_d^B)(1 - P_X^B)r_5 H_B + r_6 D_B + P_F^B r_4 F - r_7 B \quad (2.16)$$

$$\frac{dD_B}{dt} = P_d^B(1 - P_X^B)r_5 H_B - r_6 D_B \quad (2.17)$$

$$\frac{dH_B}{dt} = r_7 B - r_5 H_B \quad (2.18)$$

$$\frac{dF}{dt} = P_X^A r_1 H_A + P_X^B r_5 H_B - r_4 F \quad (2.19)$$

Yonezawa and Kikuchi, (1996) presented a model based on bee collective intelligence for honey collection (i.e. foraging). The model investigated the principle of intelligence generated by collective and cooperative behaviour in a complex environment. The model simulated one and three foraging bees. The results of the simulation showed that the three bees model produced more balanced results than those produced by the one bees model.

Cox et al., (2003) introduced a model of foraging in honey-bee colonies. This model addresses the missing factors of the model presented by Camazine et al., (1991). In this model, the effects of environmental and colony factors are investigated. The effects of the source (rate of nectar flow, distance from hive) and the consequences of forager behaviours are also implemented in the differential equations set.

Schmickl et al., (2004) presented a comprehensive model of nectar source selection in honey-bee colonies. Although the model is built on individual processes, it produces interesting results on the global-colony level. Another interesting feature of the model is that it is built to project the daily net honey gain of the hypothetical honey-bee colony. Thus, this gives the opportunity to explore the economic results of foraging decisions. The presented model is developed to examine the dynamics and efficiency of the decentralized decision making system of a honey-bee colony in a changing environment. However, as the most significant difference from previous models, target selection and workload balancing processes as well as the energy balance of each foraging bee have been implemented in the model. Also, the foragers are treated as individual agents who expend energy and show specific behaviours.

2.4.2. Nest-site Selection Models

Nest-site selection is another vital practice which requires an optimisation process as nectar source selection behaviour does in honey-bee colonies. Nest-site selection in honey-bee colonies can be summarised as a social decision making process. In this process, scout bees locate several potential nest sites, evaluate them, and select the best one on a competitive signalling basis (Passino and Seeley, 2006).

Several nest-site selection models have been introduced (Camazine et al., 1999; Britton, 2002; Passino and Visscher, 2003) and then a comprehensive one introduced by Passino and Seeley, (2006). It was developed using the bees' decision making processes extracted by early empirical studies. The effects of several features of the

nest-site selection processes in honey-bee colonies have been studied using this model (Passino and Seeley, 2006).

2.5. Honey-bees inspired algorithms

In this section, honey-bees inspired algorithms are reviewed, many developed recently. The main streams in this domain can be divided in three subgroups: (1) Foraging and nectar source selection behaviours related, (2) marriage behaviours related and (3) queen bee behaviours related studies. Because of their efficiency and robustness, the foraging and nectar source selection behaviours are the most studied field in terms of an optimisation approach.

Sato and Hagiwara, (1997) introduced the very first honey-bees inspired algorithm, called the bee system, as an improved version of genetic algorithms. This system claims to be inspired basically from *'finding a source and recruiting others to it'* behaviour. However, this idea was implemented as a hybrid genetic algorithm. In the algorithm, some of the chromosomes are considered as superior ones and others try to find solutions around them using multiple populations. Moreover, the algorithm uses some operators such as the concentrated crossover and pseudo simplex methods. The bee system applied to function optimisation and simulation results were presented in a normalised error form. As a result, this algorithm produces better results compared to GA with a high success rate and less normalised error values for nine different test functions.

Lucic and Teodorovic, (2001) presented another bee system, which is one of the early attempts to develop a direct bee-inspired algorithm in last decade. The algorithm was developed for combinatorial domains and applied to traveller salesman problems (TSP) that aim to find the minimum distance route between paths passing through each only once. In this algorithm, the hive is located in a solution space randomly and following a probabilistic selection similar to that used in the Ant Colony Optimisation. Partial solutions are constructed in stages using a probabilistic equation derived from the Logit model (see equation 2.20).

$$p_{\xi}(u, z) = \frac{e^{\rho\beta_{\xi}(u, z) - \theta\alpha_{\xi}(u, z)}}{\sum_{\tau \in Y(u, z)} e^{\rho\beta_{\tau}(u, z) - \theta\alpha_{\tau}(u, z)}} \quad \xi \in Y(u, z), \forall u, z$$

(2.20)

Then, bees recruited to these partial solutions are expanded further. After initial improvements, before relocating the hive, the solution produced in the current iteration is improved using the 2-opt and 3-opt heuristic algorithms. The results for the traveller salesman problem are also presented.

Yang, (2005) proposed a virtual bee algorithm (VBA) to solve the function optimisation in engineering problems. The VBA begins with deploying a troop of virtual bees in the phase space for random exploration. The main steps of the VBA for function optimisation are given as: “1) *creating a population of multi-agents or virtual bees, each bee is associated with a memory bank with several strings*; 2) *encoding of the objectives or optimization functions and converting into the virtual food*; 3) *defining a criterion for communicating the direction and distance in the similar*

fashion of the fitness function or selection criterion in the genetic algorithms; 4) marching or updating a population of individuals to new positions for virtual food searching, marking food and the direction with virtual waggle dance; 5) after certain time of evolution, the highest mode in the number of virtual bees or intensity/frequency of visiting bees corresponds to the best estimates; 6) decoding the results to obtain the solution to the problem". This procedure may be presented as the following pseudo-code:

```

1: // Create a initial population of virtual bees A(t)
2: // Encode the function f(x,y,...) into virtual food/nectar
3:     Initial Population A(t);
4:     Encode f(x,y) |-> F(x,y);
5: // Define the criterion for communicating food location with others
6:     Food F(x,y) |-> P(x,y)
7: // Evolution of virtual bees with time
8:     t=0;
9:     while (criterion)
10: // March all the virtual bees randomly to new positions
11:     t=t+1;
12:     Update A(t);
13: // Find food and communicate with neighbouring bees
14:     Update F(x,y), P(x,y);
15: // Evaluate the encoded intensity/locations of bees
16:     Evaluate A(t), F(x,y), P(x,y)
17:     end while

```

18: // Decode the results to obtain the solution

19: Decode S(x,y,t);

In terms of encoding the location of agents, the algorithm deals with the problem domain similar to Genetic Algorithms. The algorithm is applied to one and two dimensional functional optimisation problems and compared with GA. Results showed that the algorithm finds solutions to the problems (1-D and 2-D) better than GA.

Lemmens et al., (2006) introduced a non-pheromone-based algorithm inspired by the behaviour of honey-bees, called the Bee Foraging Algorithm. The algorithm uses two essential strategies; recruitment and navigation. The recruitment strategy is used to distribute information regarding a nectar source to other members of the colony. The navigation strategy is proposed for efficiency of navigation in an unknown environment. It is based on a strategy called Path Integration, which is actually used by natural bees to navigate back to hive while they are moving between far apart nectar sources. The general structure of the algorithm is similar to the structure of the ant colony optimisation. The algorithm consists of three main functions and internal states in these functions. The very first function is called *ManageBeesActivity()* which deals with the activity of agents based on their internal states. There are six internal states in which each agent performs a specific behaviour; *'AtHome'*, *'StayAtHome'*, *'Exploration'*, *'Exploration'*, *'HeadHome'*, *'CarryingFood'*. The agent internal state changes are called “Algorithm 1” and this process may be outlined as follows:

1: **If** State is StayAtHome **then**

2: **If** Vector exists **then**

3: Exploitation

```

4:      and if
5: else if Agent not AtHome then
6:      if Agent has food then
7:          CarryingFood
8:      else if Depending on chance then
9:          HeadHome, Exploration or Exploitation
10:     end if
11: else if exploit preference AND state is AtHome then
12:     if Vector exists then
13:         Exploitation
14:     else
15:         Exploration
16:     end if
17: else if StayAtHome preference AND state is AtHome then
18:     if Vector exists then
19:         Exploitation
20:     else
21:         StayAtHome
22:     end if
23: else
24:     Exploration
25: end if

```

The second function, which is called *CalculateVector()*, is used to calculate the path integration vector for each agent. The algorithm uses a third optional function called *DemonAction()* which can be used to implement the centralised actions such as global information, which is important for an agent to decide to dance (or not to dance).

Lemmens et al., (2007) introduced a hybrid swarm intelligence algorithm called the Bee System with inhibition pheromones (BSP). It combines the algorithm presented above (Lemmens et al., 2006) and the ant colony optimisation. In order to overcome

the shortcomings of the previous bee system, in which there are two procedures both of which are borrowed from the ant colony optimisation, new procedures are implemented in the algorithm. The first proposed procedure employs a rather simpler way to improve the obstacle avoidance capabilities. It helps an agent while following a path integration (PI) vector. When it bounces into an obstacle it simply selects a random direction (in this case left or right) and then follows the outlines of the obstacle in that direction until following the PI vector becomes possible again. The second procedure is proposed to enhance the learning capability of the algorithm. In this procedure agents can deposit inhibition pheromone at a certain location. Agents following a PI vector benefited from this enhanced learning mechanism just to find a better solution both in static and dynamic environments.

Karaboga et al., (2007), Karaboga et al., (2008) and Karaboga et al., (2009) presented an Artificial Bee Colony (ABC) algorithm for optimising numerical test functions. ABC is inspired by the foraging behaviour of honey-bees swarms. The algorithm uses three types of bees, called employed bees, onlooker bees and scout bees. The population is split equally into two parts, the first half as employed bees and the onlookers as the other half. This algorithm also employs a random scout bee for exploration of the search space. The algorithm has three main steps for each iteration; employed bees placed on food sources, onlooker bees placed on food sources depending on their nectar amount and scout bees sent to the search area for exploration. The detailed pseudo-code of the ABC algorithm is presented in Fig. 2.4 but the main steps of the algorithm are given below:

1: Initialize Population

- 2: Repeat
- 3: Place the employed bees on their food sources
- 4: Place the onlooker bees on the food sources depending on their nectar amounts
- 5: Send the scouts to the search area for discovering new food sources
- 6: Memorize the best food source found so far
- 7: Until (requirements are met)

For each flower patch, ABC uses proportional selection to recruit the onlooker bees to promising patches (see equation 2.21).

$$P_i = \frac{F(Q_i)}{\sum_{k=1}^S F(Q_k)} \quad (2.21)$$

Where P_i is the probability of selection for a patch by each onlooker bee; Q_i is the position of the i th food source; $F(Q_i)$ represents the nectar amount of the food source located at Q_i and S : the number of food sources around the hive. The neighbourhood search algorithm uses the extrapolation crossover method to create new solutions. In this phase, an employed bee randomly chooses another employed bee and generates a new solution. If this solution is better than the existing one, a new employed bee is selected as the representative bee for the patch. As presented in this thesis, ABC also uses site abandonment, which is simply leaving a patch if no more improvement is observed on the patch after certain number of iterations. This is defined as the *limit* in the ABC and can be calculated according to the formula:

$$\text{Limit} = D * SN \quad (2.22)$$

where SN is the number of employed bees and D is the dimension of the problem.

1. Initialize the population of solutions x_i , $i = 1 \dots SN$
2. Evaluate the population
3. cycle=1
4. repeat
5. Produce new solutions v_i for the employed bees and evaluate them
6. Apply the greedy selection process for the employed bees
7. Calculate the probability values P_i for the solutions x_i
8. Produce the new solutions v_i for the onlookers from the solutions x_i selected depending on P_i and evaluate them
9. Apply the greedy selection process for the onlookers
10. Determine the abandoned solution for the scout, if exists, and replace it with a new randomly produced solution x_i
11. Memorize the best solution achieved so far
12. cycle=cycle+1
13. until cycle=MCN

Figure 2.4 Pseudo-code of the ABC algorithm

Teodorovic et al., (2006) proposed a Bee Colony Optimisation (BCO) metaheuristic which is capable of solving combinatorial optimisation problems. A Fuzzy Bee System was also proposed in (Teodorovic et al., 2006). BCO has been developed for combinatorial problems and the pseudo-code of the algorithm is given in Fig. 2.5. Similar to the ant colony optimisation algorithm, it has a constructive way of building the solutions but the main difference is that the BCO algorithm builds the solutions partially. In each stage bees build a partial solution by flying a couple of nodes during a forward pass. In the backward pass stage, all bees are sent back to the hive and bees are allowed to exchange information about the quality of the partial solutions created and to decide whether to abandon the created partial solution and become again an uncommitted follower; continue to expand the same partial solution without recruiting the nest mates; or dance and thus recruit nest mates before returning to the created partial solution. On the other hand, there is no procedure for the selection of the best sites or becoming an uncommitted bee. The Fuzzy Bee System was developed to help bees during decision making process but instead of random selection, bees select patches using a roulette wheel approach. To be able to do this a verbal explanation of the partial solution is used as follows:

If the length of the advertised path is SHORT and the number of bees advertising the path is:

SMALL

Then the advertised partial solution attractiveness is:

MEDIUM

1. Initialization. Determine the number of bees B , and the number of iterations I .

Select the set of stages $ST = \{St_1, st_2, \dots, St_m\}$. Find any feasible solution x of the problem. This solution is the initial best solution.

2. Set $i: 1$. Until $i = I$, repeat the following steps:

3. Set $j: 1$. Until $j = m$, repeat the following steps:

Forward pass: Allow bees to fly from the hive and to choose B partial solutions from the set of partial solutions S_j at stage st_j .

Backward pass: Send all bees back to the hive. Allow bees to exchange information about quality of the partial solutions created and to decide whether to abandon the created partial solution and become again uncommitted follower, continue to expand the same partial solution without recruiting the nestmates, or dance and thus recruit the nestmates before returning to the created partial solution. Set $j: = j + 1$.

4. If the best solution x_i obtained during the i -th iteration is better than the best-known solution, update the best known solution ($x: = x_i$).

5. Set, $i: = i + 1$.

Figure 2.5 Pseudo-code of the BCO algorithm.

Wong et al., (2008) introduced a bee colony optimisation (BCO) algorithm for travelling salesman problem (TSP). The basic procedure for the BCO with a 2-opt local search for TSP is given in Fig. 2.6. It is similar to the bee colony optimisation algorithm discussed above. According to the authors, there are several alterations that differentiate this algorithm from the BCO algorithm developed by Teodorovic et al., (2006). Bees in the earlier model do not have the ability to remember the number of bees that have visited an arc (Wong et al., 2008). In this improved model, bees show the entire feasible path rather than partial tours using the waggle dance and the bee hive was proposed to have an equal distance from all cities. During the construction of solutions procedure, bees were influenced by both arc fitness and the distance between cities. With these differences, the improved algorithm may be applied to many well-known TSP data sets and the results shows that it performs well compared to other state of the art algorithms.

Wedde et al., (2004) presented a routing algorithm for fixed networks, called BeeHive, that was inspired by the communication activities of honey-bees. The algorithm employed the idea of a bee agent model in which the agents travel through network regions called foraging zones. These were designed as fixed partitions in a network containing representative and non-representative nodes which were capable of launching long or short bee agents to update routing. Zang, (2005) further developed the BeeHive algorithm based on a stochastic process. It works without saving the topology or other global information in the routing tables. The main procedures of the algorithm may be presented as follows:

```
1. procedure BCO
2.   Initialize_Population( )
3.   while all bees have not built a complete path do
4.     Observe_Dance( )
5.     Forage_ByTransRule( )
6.     Perform_Waggle_Dance ( )
7.   end while
8. end procedure BCO
```

Figure 2.6 BCO with 2-opt local search for TSP by Wong et al. (2008)

▪ main procedure

in parallel

- 1: send the received packet, bee agents, echo or ack to a corresponding procedure.
- 2: run procedure for routing decision.
- 3: run *explore* procedure for updating data.
- 4: run *update* procedure for updating module.
- 5: run *measure* procedure for measuring module.
- 6: run *echo* procedure for measuring procedure.
- 7: run *echo* procedure for measuring module.
- 8: run *ack* procedure for measuring module.

Three modules were introduced, namely the routing, the updating and the updating module. Further details of these modules can be found in (Zang, 2005). This algorithm performed several simulations on some well-known routing data and the results were compared to AntNet and Distributed Genetic Algorithm. These results show that the algorithm performed well against many benchmarks compared to other population-based algorithms.

Wedde et al., (2005a) and Wedde et al., (2005b) presented BeeAdHoc, a routing algorithm for energy efficient routing in mobile ad hoc networks. The algorithm was developed based on the BeeHive algorithm. Although they share some features, the BeeAdHoc routing algorithm uses several types of different agents, namely packers (used to receive and store data packets from the transport layer), scouts (used to discover new routes), foragers (used to receive and transport data packets) and swarms (used to help with unreliable transport protocols). Similar to BeeHive, the artificial

bee agents are used in packet switching networks to find suitable paths between nodes by updating the routing table. Two types of agents are used – short distance bee agents which disseminate routing information by travelling within a restricted number of hops and long distance bee agents which travel to all nodes of the network. The BeeAdHoc was also defined as a reactive source routing algorithm and it claimed to consume less energy compared to other Mobile Ad Hoc Networks algorithms. Results confirmed that the BeeAdHoc did indeed consume less energy compared to the DSR, AODV and DSDV algorithms.

Abbas, (2001) presented a marriage in the honey-bees optimisation algorithm (MBO). The algorithm simulates the evolution of honey-bees in several stages. It starts with a solitary colony (i.e. single queen bee without a colony) and goes all the way up to the emergence of eusocial colony (a full colony with one or two queens in the chamber). The algorithm is based on simulated annealing and in many ways it resembles the annealing procedure. The pseudo-code of the MBO algorithm is given in Fig. 2.7. The algorithm starts with the random initialisation of workers as well as the genotype of each queen. A set of mating-flights is made with a random initialisation of the values of energy, speed, and position of each queen. Then, each queen moves between states according to her speed and she mated with a drone using an equation similar to a simulated annealing procedure. A drone mates with a queen probabilistically using an annealing function:

$$Pr ob(Q, D) = e^{-\frac{\Delta(f)}{S(t)}} \quad (2.23)$$

1. The algorithm starts with the mating-flight, where a queen (best solution) selects drones probabilistically to form the spermatheca (list of drones). A drone is then selected from the list at random for the creation of broods.
2. Creation of new broods (trial solutions) by crossovering the drones' genotypes with the queen's.
3. Use of workers (heuristics) to conduct local search on broods (trial solutions).
4. Adaptation of workers' fitness based on the amount of improvement achieved on broods.
5. Replacement of weaker queens by fitter broods.

Figure 2.7 Basic steps of the MBO algorithm.

If the result of mating is successful, then drones' sperm is added to a list of partial solutions, the so called spermatheca. After turning back to the nest, the queen bee starts breeding by randomly selecting a sperm from the spermatheca. Then crossover and mutation operators are applied to produce different broods. Workers are also used to improve the broods. If any of these broods are better than the queen, the queen is replaced. The remaining broods are then killed and a new mating flight starts. The algorithm was applied to a fifty propositional satisfiability problems (SAT) with 50 variables and 215 constraints and results showed that the algorithm performed well for these specific types of problems. Moreover, Benetcha et al., (2005) adapted this algorithm to a Max-Sat problem and presented further simulation results. Haddad et al., (2006) applied the same procedure given in Fig. 2.7 for a water resource optimisation problem. Although the procedure is the same, it was called the Honey-Bees Mating Optimisation (HBMO) algorithm. Further functional optimisation tests also presented in the paper.

Jung, (2003) proposed a queen bee evolution algorithm for enhancing the optimisation capability of genetic algorithms. The algorithm was inspired by the role of queen bee in the reproduction process. In the algorithm, as the fittest individual in a generation, the queen bee crossbreeds with the other bees selected as parents by a selection algorithm. This proposed procedure increases the chance of premature convergence. An intensive mutation procedure is proposed to deal with this problem. Experimental results of one combinatorial and two continuous applications demonstrated that the proposed hybrid algorithm was able to converge in most cases. Azeem and Saad, (2004) and Qin et al., (2004) made some improvements to the algorithm with several different applications.

Gordon et al., (2003) proposed the application of a discrete bee dance algorithm to the problem of pattern formation on a grid for a group of identical autonomous robotic agents with limited communication capabilities. The algorithm was defined as a sequence of several coordinated waggle dances on a grid where the bee agents share their information, cooperate and solve their problems to be able to overcome their shortcomings.

2.6. Summary

This chapter has reviewed the theory and applications of swarm intelligence as well as the behaviours of honey-bees to provide general background information for the research reported in subsequent chapters of the thesis.

Chapter 3

THE BEES ALGORITHM: THEORY AND IMPROVEMENTS

3.1. Preliminaries

Many complex multi-variable optimisation problems cannot be solved exactly within polynomially bounded computation times. This generates much interest in search algorithms that find near-optimal solutions in reasonable running times. The swarm-based algorithm described in this thesis is a search algorithm capable of locating good solutions efficiently. The algorithm is inspired by the food foraging behaviour of honey bees and could be regarded as belonging to the category of “intelligent” optimisation tools.

In this chapter, a new population-based search algorithm called the Bees Algorithm (BA) is presented (Pham et al., 2006). The algorithm mimics the food foraging behaviour of swarms of honey bees. In its basic version, the algorithm performs a kind of neighbourhood search combined with random search and can be used for both combinatorial optimisation and functional optimisation. The Bees Algorithm is presented in this chapter, with benchmark results comparing the performance of the algorithm with some well-known algorithms in the literature. Further details are also given of the local and global search methods used in this algorithm. Moreover, in this chapter, details of the improvements made to local and global search methods are presented, including dynamic recruitment, proportional shrinking and abandonment strategies.

The chapter is organised as follows: section 3.2 presents a description the basic Bees Algorithm in its simplest form with a simple example of the algorithm procedure. In section 3.3, some key characteristics of the Bees Algorithm are discussed, including site selection, neighbourhood search and global search. Improvements to local and global search are presented in section 3.4, including dynamic recruitment, proportional shrinking for selected sites and site abandonment. Experimental results and benchmark tables are presented in section 3.5 to demonstrate the performance and the robustness of the algorithm compared to some well-known algorithms in the literature.

3.2. The basic Bees Algorithm

The Bees Algorithm is an optimisation algorithm inspired by the natural foraging behaviour of honey bees to find the optimal solution. Fig. 3.1 shows the pseudo-code and Fig. 3.2 presents the flowchart of the basic Bees Algorithm in its simplest form. The algorithm requires a number of parameters to be set, namely: number of scout bees (n), number of patches selected out of n visited points (m), number of best patches out of m selected patches (e), number of bees recruited for e best patches (n_{ep}), number of bees recruited for the other ($m-e$) selected patches (n_{sp}), size of patches (n_{gh}) and the stopping criterion. The algorithm starts with the n scout bees being placed randomly in the search space. The fitnesses of the points visited by the scout bees are evaluated in step 2. A simple demonstration is given in Fig. 3.3 which shows the basis steps of the algorithm.

In step 4, bees that have the highest fitnesses are chosen as “selected bees” and those sites that have been visited will be chosen for neighbourhood search. Then, in steps 5 and 6, the algorithm conducts searches in the neighbourhood of the selected bees in terms of more bees for the e best bees. The latter can be chosen directly according to the fitnesses associated with the points they are visiting. Alternatively, the fitness values are used to determine the probability of the bees being selected. Searches in the neighbourhood of the e best bees which represent more promising solutions are made more detailed by recruiting more bees to follow the e best bees than other selected bees. Also within scouting, differential recruitment is one of the key operations of the Bees Algorithm. Both scouting and differential recruitment are used in nature.

-
1. Initialise population with random solutions.
 2. Evaluate fitness of the population.
 3. While (stopping criterion not met)
 //Forming new population.
 4. Select sites for neighbourhood search.
 5. Recruit bees for selected sites (more bees for e best sites) and evaluate fitnesses.
 6. Select the fittest bee from each site.
 7. Assign remaining bees to search randomly and evaluate their fitnesses.
 8. End While.
-

Figure 3.1 Pseudo-code of the basic Bees Algorithm

However, in step 6, for each site only one bee with the highest fitness will be selected to form the next bee population. In nature, there is no such a restriction. This restriction is introduced here to reduce the number of points to be explored.

In step 7, the remaining bees in the population are assigned randomly around the search space scouting for new potential solutions. These steps are repeated until a stopping criterion is met. At the end of each iteration, the colony will have two parts to its new population – representatives from each selected patch and other scout bees assigned to conduct random searches.

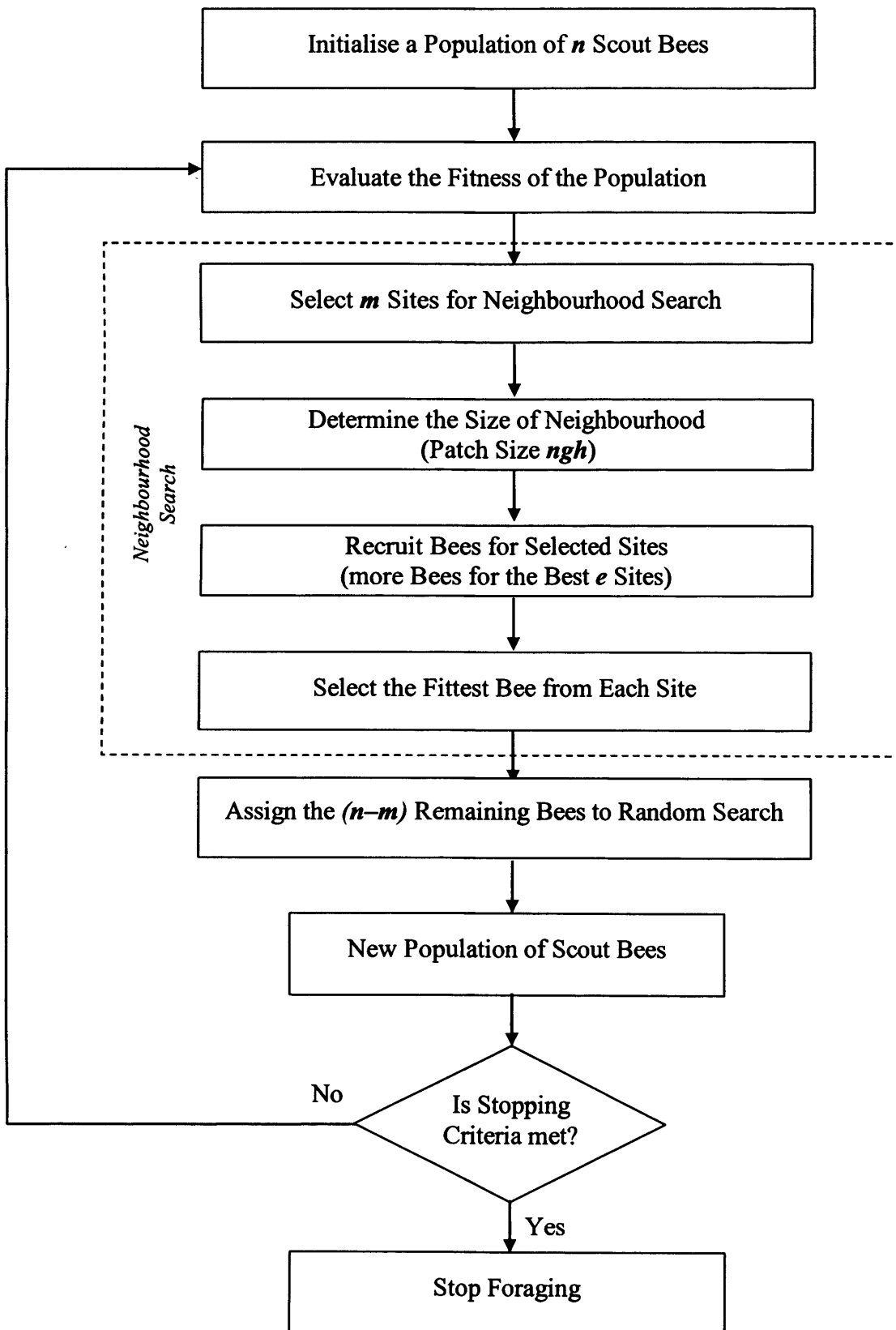


Figure 3.2 Flowchart of the basic Bees Algorithm

3.3. Characteristics of the proposed Bees Algorithm

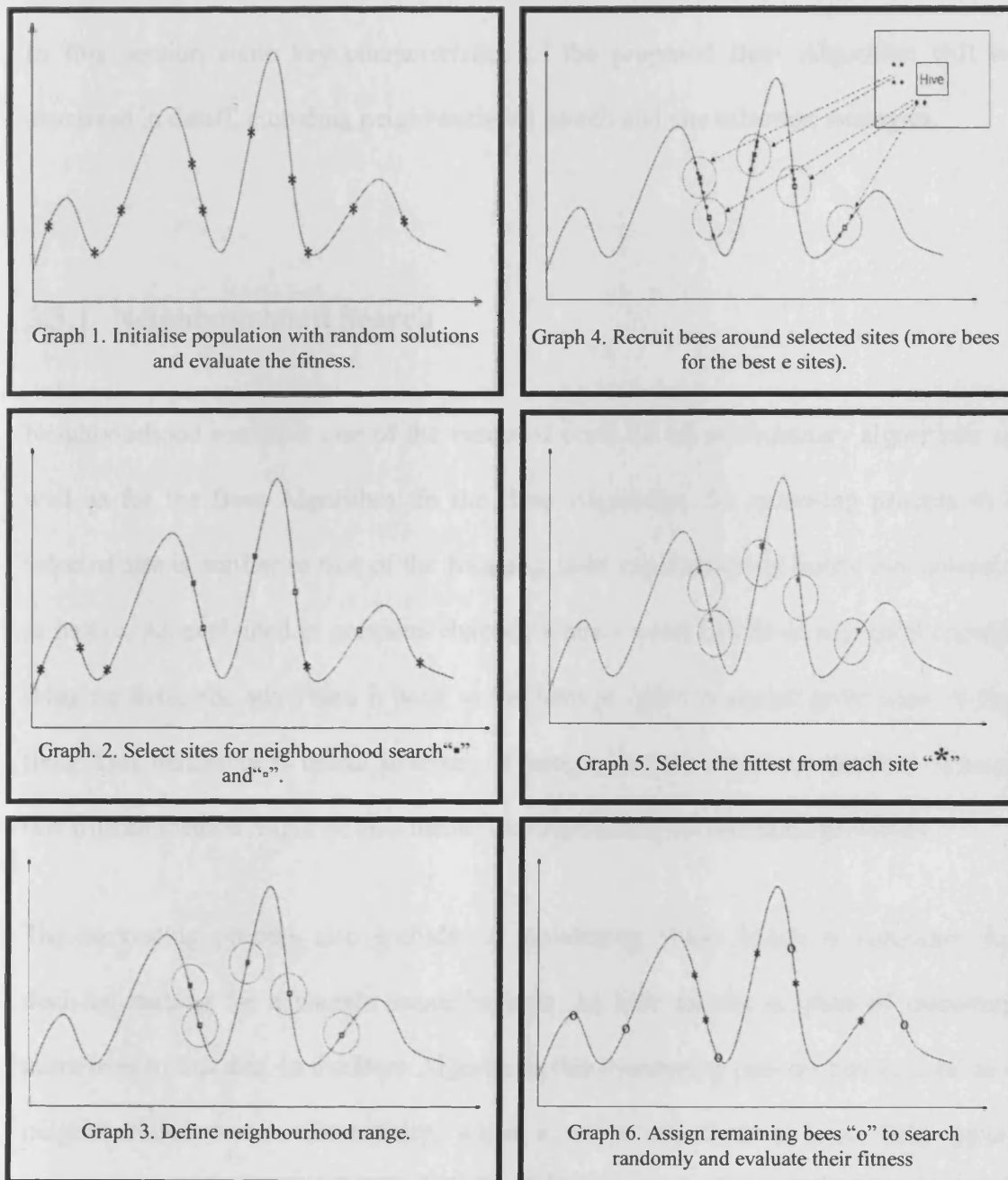


Figure 3.3 Simple example of the Bees Algorithm

3.3. Characteristics of the proposed Bees Algorithm

In this section some key characteristics of the proposed Bees Algorithm will be discussed in detail, including neighbourhood search and site selection strategies.

3.3.1 Neighbourhood Search

Neighbourhood search is one of the essential parts for all evolutionary algorithms as well as for the Bees Algorithm. In the Bees Algorithm, the searching process in a selected site is similar to that of the foraging field exploitation of honey bee colonies in nature. As explained in previous chapter, when a scout bee finds any good enough foraging field, she advertises it back to the hive in order to recruit more bees to that field. This behaviour is useful in terms of bringing more nectar into the hive. Hence, this fruitful method might be also useful for engineering optimization problems.

The harvesting process also includes a monitoring phase which is necessary for decision making for a waggle dance back in the hive for the purpose of recruiting more bees to that site. In the Bees Algorithm, this monitoring process can be used as a neighbourhood search. Essentially, when a scout bee finds a good field (good solution), she advertises her field to more bees. Subsequently, those bees fly to that source, take piece of nectar and return back to hive. Depending on the quality, this source can be advertised by some of the bees that are already aware of the source. In the proposed Bees Algorithm, this behaviour has been used as a neighbourhood search. As explained above, from each foraging site (or neighbourhood site) only one

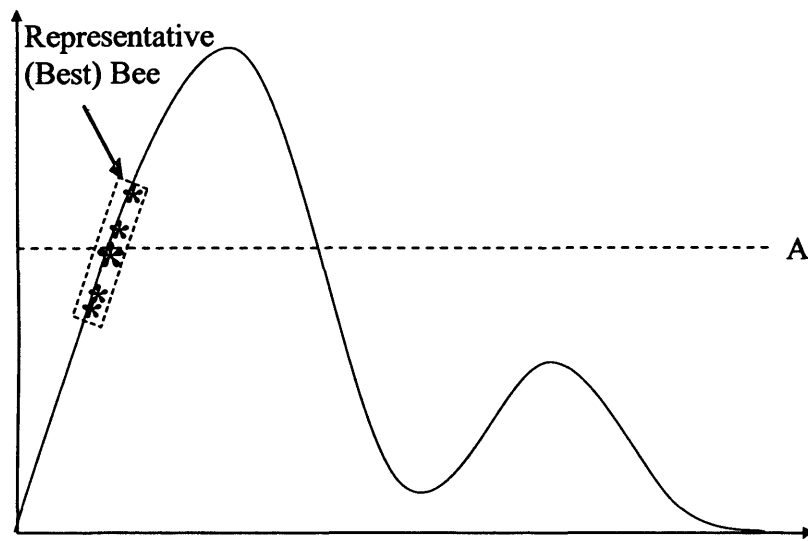
bee is chosen. This bee must have the best solution information about that field. Thus, the algorithm can create some solutions which are related to the previous ones.

Neighbourhood search is based on a random distribution of bees in a predefined neighbourhood range (patch size). For every selected site, bees are randomly distributed to find a better solution. As shown in Fig. 3.4, only the fittest (best) bee is chosen as a representative and the centre of the neighbourhood patch shifted up to best bees' position (from A to B).

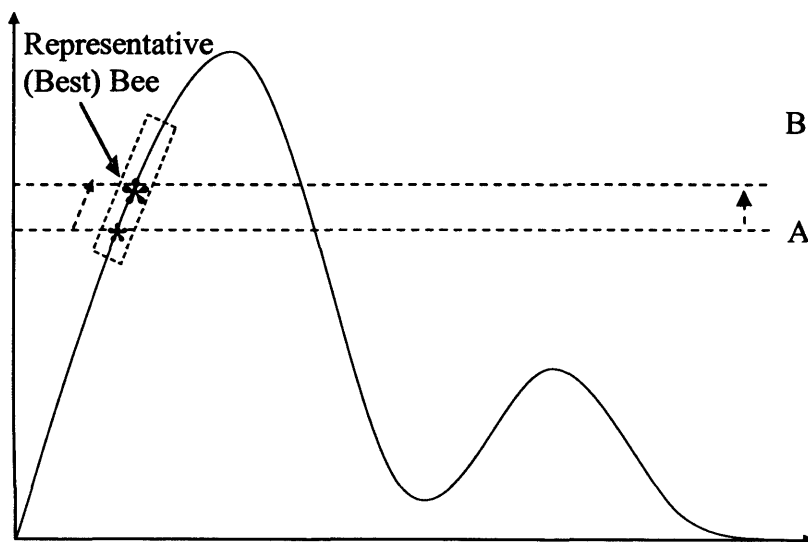
On the other hand, during the harvesting process other elements should also be taken into account for increasing efficiency, such as number of recruited bees in the neighbourhood patch and the patch size.

The number of recruited bees around selected sites should be defined properly. When the number is increased, then the number of function evaluations will also be increased and vice versa.

This problem also depends on the neighbourhood range. If the range can be arranged adequately, then the number of recruited bees will depend on the complexity of a solution space. This will be discussed later with more details.



(a)



(b)

Figure 3.4 Graphical explanation of basic neighbourhood search.

3.3.4 Site Selection

For site selection two different techniques have been implemented: probabilistic selection and best site selection.

In probabilistic selection, the roulette wheel method has been used and sites with better fitness have more chance of being selected, but in best site selection, the best sites according to fitness will be selected. In this section, different combinations of selection using the two methods from pure probabilistic selection ($q=0$) to pure best site selection ($q=1$) have been investigated and mean iterations required to enrich the answer. Results are shown in Fig. 3.5 and Fig. 3.6.

Regarding results, best selection will present better results and also is simpler, so in its basic form the best sites method is selected as the neighbourhood site selections method for the Bees Algorithm.

3.3.5. Global Search

In the first step, all scout bees (n) are placed randomly across the fitness landscape to explore for new flower patches. After neighbourhood search, $n-m$ bees are again placed randomly across the fitness landscape to explore new patches. The latter part is the main global search tool for the Bees Algorithm.

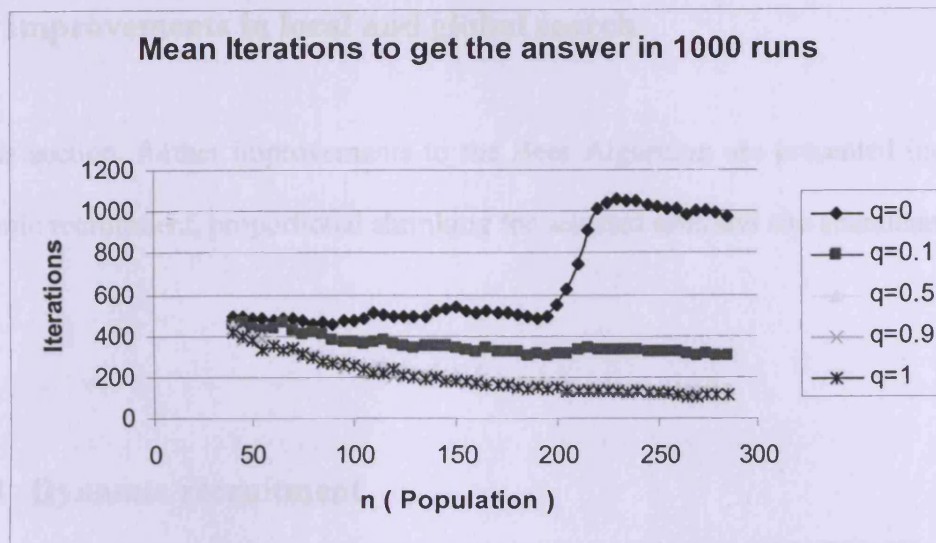


Figure 3.5 Mean iteration required for different combinations of selection

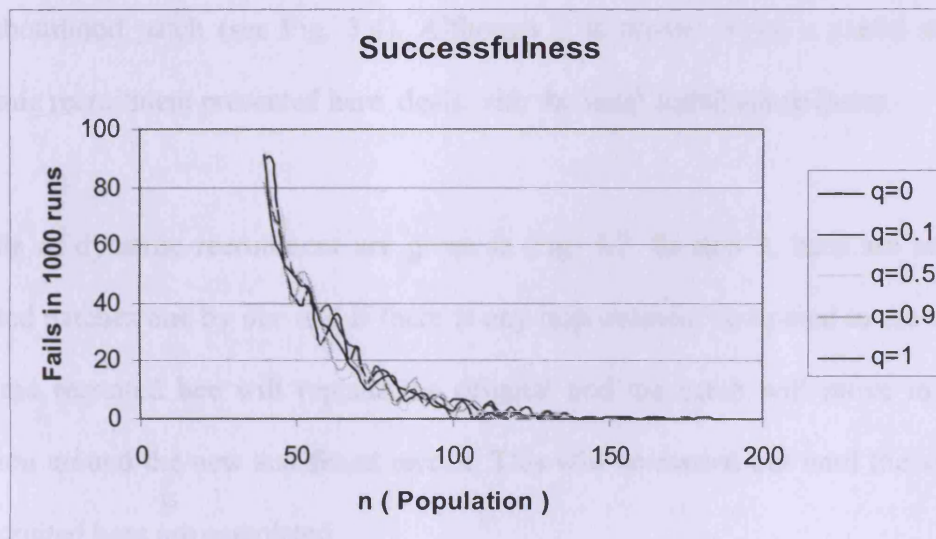


Figure 3.6 Successfulness of different combinations of selection methods

3.4. Improvements in local and global search

In this section, further improvements to the Bees Algorithm are presented including dynamic recruitment, proportional shrinking for selected sites and site abandonment.

3.4.1 Dynamic recruitment

Dynamic recruitment is introduced to improve the way that the bees are recruited into a selected site. In the basic Bees Algorithm, when a site is selected for neighbourhood search there will be a certain number of bees assigned for local search. In the previous strategy, bees are sent all at once to the same local search space defined as the neighbourhood patch (see Fig. 3.4). Although it is proven to be a useful strategy, dynamic recruitment presented here deals with the local search space faster.

Details of dynamic recruitment are given in Fig. 3.7. In step 5, bees are sent into selected patches one by one and if there is any improvement compared to the original bee, the recruited bee will replace the original and the patch will move to a new position around the new and fittest recruit. This will be carried out until the visits by all recruited bees are completed..

1. Initial population with n random solution.
2. Evaluate fitness of the population.
3. While (stopping criterion not met)
4. Select sites (m) for neighbourhood search.
5. Recruit bees for selected sites (more bees for best e sites), evaluate fitnesses, select the fittest bee from each site and shrink patches


```

for(k=1 ; k=e ; k++) // Elite Sites

    for(Bee=1 ; Bee= nep ; Bee++) // More Bees for Elite Sites

        BeesPositionInNgh() = GenerateRandomValueInNgh(from x+ngh to x-ngh);

        Evaluate Fitness = Bee(i); //Evaluate the fitnesses of recruited Bee(i)

        If (Bee(i) is better than Bee(i-1)) RepresentativeBee = Bee(i);

for(k=e ; k=m ; k++) // Other selected sites (m-e)

    for(Bee=1 ; Bee= nsp ; Bee++) // Less Bees for Other Selected Sites (m-e)

        BeesPositionInNgh() = GenerateRandomValueInNgh(from x+ngh to x-ngh);

        Evaluate Fitness = Bee(i); //Evaluate the fitnesses of recruited Bee(i)

        If (Bee(i) is better than Bee(i-1)) RepresentativeBee = Bee(i);

```
6. Assign remaining bees to search randomly and evaluate their fitnesses. // (n-m) assigned to search randomly into whole solution space
7. End While

Figure 3.7 The Bees Algorithm with dynamic recruitment.

3.4.2 Proportional shrinking for selected sites

The term proportional shrinking refers to a contraction of patch sizes of all selected sites (m) in every iteration of the algorithm proportional to a constant ratio called Shrinking Constant (sc). Equation 3.1 gives the definition of the proportional shrinking idea, in which the initial patch size is set as a starting patch size in the first iteration of the algorithm. Depending on the iteration (i), the patch size of the site m ($Ngh_m(i)$) is calculated as a contraction from the previous size ($Ngh_m(i-1)$) proportional to the value of sc . The value of sc can be defined by the user between 0 and 1 that represents the percentage by which the patch will shrink. However, the patch size must be a positive value all the time ($Ngh_m(i) > 0$).

$$Ngh_m(i) = \begin{cases} i=1 & Ngh_m(i) = InitialPatchSize, \\ i>1 & Ngh_m(i) = Ngh_m(i-1)(1-SC) \text{ and } Ngh_m(i) > 0. \end{cases} \quad (3.1)$$

This new strategy, implemented as step 6 in Fig. 3.8, is proposed to improve the solution quality and evaluation time. At the beginning, for the local search, a wide patch size increases the probability of finding a better solution. By shrinking it saves the time and increases the solution quality by fine tuning in relatively narrow local search space.

3.4.3 Site Abandonment

The site abandonment strategy is introduced to improve the efficiency of the local search. The term refers the abandonment of a site in which there is no more improvement of the fitness value of the fittest bee after a certain number of iterations. In many complex optimisation problems, there may be many local solutions in their solution spaces and it is not possible to escape from local optima without an efficient procedure.

Parallel scout bee search and better neighbourhood exploitation are two strong features of the Bees Algorithm that are capable of dealing with many complex optimisation problems. But for all other algorithms it may not be possible to escape from local optima. Site abandonment is here introduced to deal with this problem.

In step 8 (see Fig. 3.8), if the points visited near a selected site are all inferior to that site, after a certain number of iterations (i.e. *sat*: site abandonment threshold), then the location of the site is recorded and the site abandoned. Bees at the site are assigned to random search (i.e. made to scout for new potential solutions).

This step is directly inspired by honey-bees in nature. Depending on the solution, quality bees either continue to exploit a patch by sending more bees or will abandon the site after several visits.

```

1. Initial population with n random solution.
2. Evaluate fitness of the population.
3. While (stopping criterion not met)
4. Select sites (m) for neighbourhood search.
5. Recruit bees for selected sites (more bees for best e sites), evaluate fitnesses, select the fittest
   bee from each site and shrink patches
   for (k=1 ; k=e ; k++) // Elite Sites
       for (Bee=1 ; Bee= nep ; Bee++) // More Bees for Elite Sites
           BeesPositionInNgh() = GenerateRandomValueInNgh(from x+ngh to x-ngh);
           Evaluate Fitness = Bee(i); //Evaluate the fitnesses of recruited Bee(i)
           If (Bee(i) is better then Bee(i-1)) RepresentativeBee = Bee(i);
   for (k=e ; k=m ; k++) // Other selected sites (m-e)
       for (Bee=1 ; Bee= nsp ; Bee++) // Less Bees for Other Selected Sites (m-e)
           BeesPositionInNgh() = GenerateRandomValueInNgh(from x+ngh to x-ngh);
           Evaluate Fitness = Bee(i); //Evaluate the fitnesses of recruited Bee(i)
           If (Bee(i) is better then Bee(i-1)) RepresentativeBee = Bee(i);
6. for (patch=1; patch=m; patch++) //Shrink all patches (m) proportional to SC
   
$$Ngh_m(i) = Ngh_m(i-1) * ((1 - SC));$$

7. If (Iteration > sat)
   If (no improvement on the site)
       Save the Best Fitness;
       Abandon the Site;
       Bee(m) = GenerateRandomValue(All Search Space);
8. Assign remaining bees to search randomly and evaluate their fitnesses. // (n-m) assigned to
   search randomly into whole solution space

```

Figure 3.8 Pseudo-code of the Bees Algorithm with proportional shrinking and site abandonment.

3.5. Experimental Results

Clearly, the Bees Algorithm as described above is applicable to both combinatorial and functional optimisation problems. In this section, functional optimisation is presented to show the robustness of the algorithm.

Three standard functional optimisation problems were used to test the Bees Algorithm and establish the correct values of its parameters and seven problems for benchmarking the algorithm. As the Bees Algorithm searches for the maximum, functions to be minimised were inverted before the algorithm was applied.

The first test function (see equation 3.2) is the axis parallel hyper-ellipsoid which is similar to De Jong's function 1 (see Fig. 3.9). It is also known as the weighted sphere model. It is continuous, convex and unimodal.

$$f_{1a}(x) = \sum_{i=1}^n i x_i^2 \quad (3.2)$$

$$-5.12 \leq x_i \leq 5.12$$

Global Minimum for this function:

$$f(x) = 0; \quad x(i) = 0, \quad i = 1:n$$

The following parameter values were set for the axis parallel hyper-ellipsoid test function: scout bee population $n=10$, number of selected sites $m=3$, number of elite sites $e=1$, initial patch size $ngh=0.5$, number of bees around elite points $nep=2$, number of bees around other selected points $nsp=2$.

The following parameter values of the Bees Algorithm were set for this test: scout bee population $n=10$, number of selected sites $m=3$, number of elite sites $e=1$, initial patch size $ngh=2.75$, number bees around elite points $nep=2$, number of bees around other selected points $nsp=2$. And the following parameter values of the improved Bees Algorithm were set for this test: scout bee population $n=10$, number of selected sites $m=3$, number of elite sites $e=1$, initial patch size $ngh=5.12$, number of bees around elite points $nep=2$, number of bees around other selected points $nsp=2$, shrinking constant $sc=0.20$ (20%) and site abandonment threshold $sat=10$. The basic Bees Algorithm was set with exactly the same parameters excluding the shrinking constant and site abandonment threshold.

Fig. 3.10 shows the fitness values obtained as a function of the number of points visited for both original and improved algorithms. The results are averages for 100 independent runs. It can be seen that after approximately 500 visits, the improved algorithm was able to find solutions close to the optimum while the original algorithm needs more time to find the optimum. It is also important to emphasise that the initial patch size was set as the whole solution space for the improved Bees Algorithm.

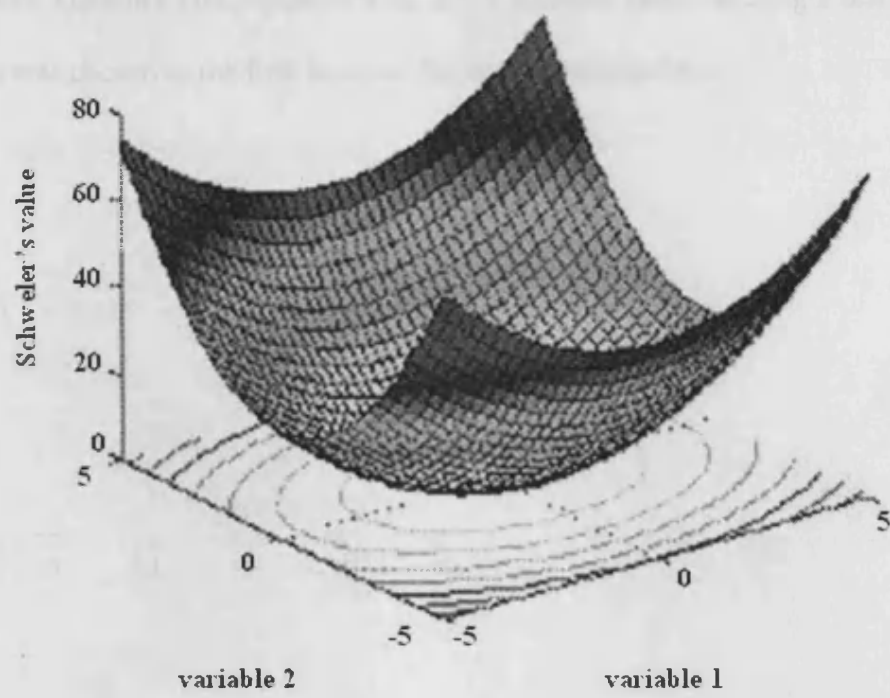


Figure 3.9 Visualization of 2D axis parallel hyper-ellipsoid function.

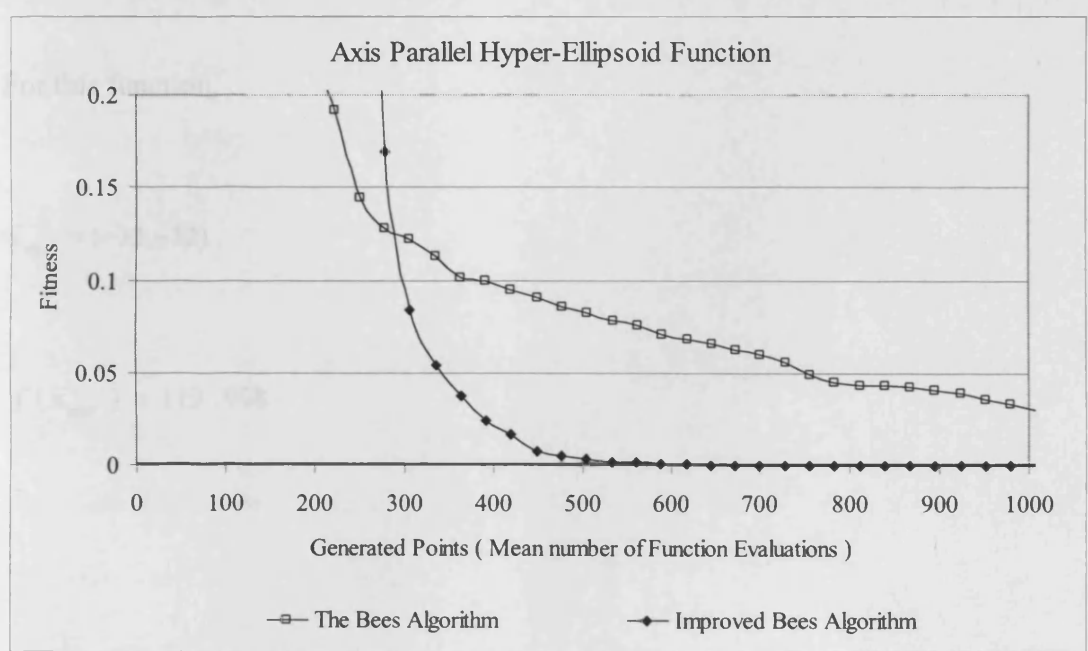


Figure 3.10 Evolution of fitness with the number of points visited (the axis parallel hyper-ellipsoid)

Shekel's Foxholes (see equation 3.3), a 2D function from De Jong's test suite (Fig. 3.11), was chosen as the first function for testing the algorithm.

$$f(\vec{x}) = 119.998 - \sum_{i=1}^{25} \frac{1}{j + \sum_{i=1}^2 \left(x_i - a_{ij} \right)^6} \quad (3.3)$$

$$a_{ij} = \begin{pmatrix} -32 & -16 & 0 & 16 & 32 & \dots & 0 & 16 & 32 \\ -32 & -32 & -32 & -32 & -32 & \dots & 32 & 32 & 32 \end{pmatrix}$$

$$-65.536 \leq x_i \leq 65.536$$

For this function,

$$\vec{x}_{\max} = (-32, -32)$$

$$f(\vec{x}_{\max}) = 119.998$$

The following parameter values of the Bees Algorithm were set for this test: scout bee population $n=45$, number of selected sites $m=3$, number of elite sites $e=1$, initial patch size $ngh=3$, number bees around elite points $nep=7$, number of bees around other selected points $nsp=2$. And the following parameter values for the improved Bees Algorithm were set for this test: scout bee population $n=10$, number of selected sites $m=3$, number of elite sites $e=1$, initial patch size $ngh=3$, number bees around elite points $nep=2$, number of bees around other selected points $nsp=2$, shrinking constant $sc=0.01$ (1%) and site abandonment threshold $sat=10$. The basic Bees Algorithm was set with exactly the same parameters excluding the shrinking constant and site abandonment threshold.

Note that ngh defines the initial size of the neighbourhood in which follower bees are placed. For example, if x is the position of an elite bee in the i th dimension, follower bees will be placed randomly in the interval $x_{ie} \pm ngh$ in that dimension at the beginning of the optimisation process.

Fig. 12 shows the fitness values obtained as a function of the number of points visited. The results are averages for 100 independent runs. It can be seen that after approximately 1200 visits, the Bees Algorithm was able to find solutions close to the optimum. However, the improved algorithm is able to find solutions close to the optimum faster than its predecessor

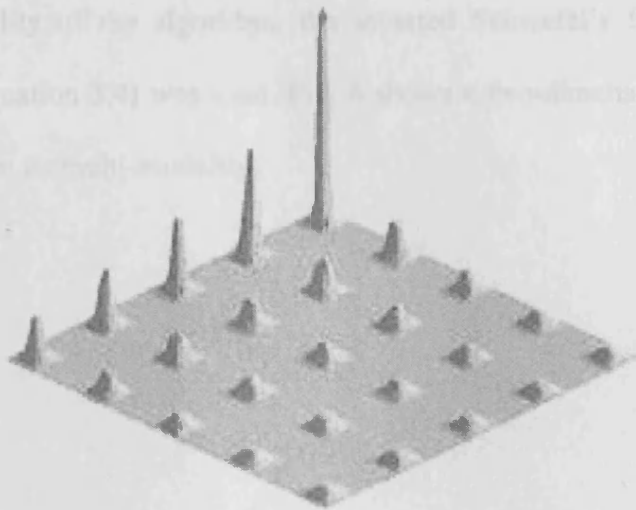


Figure 3.11 Inverted Shekel's Foxholes

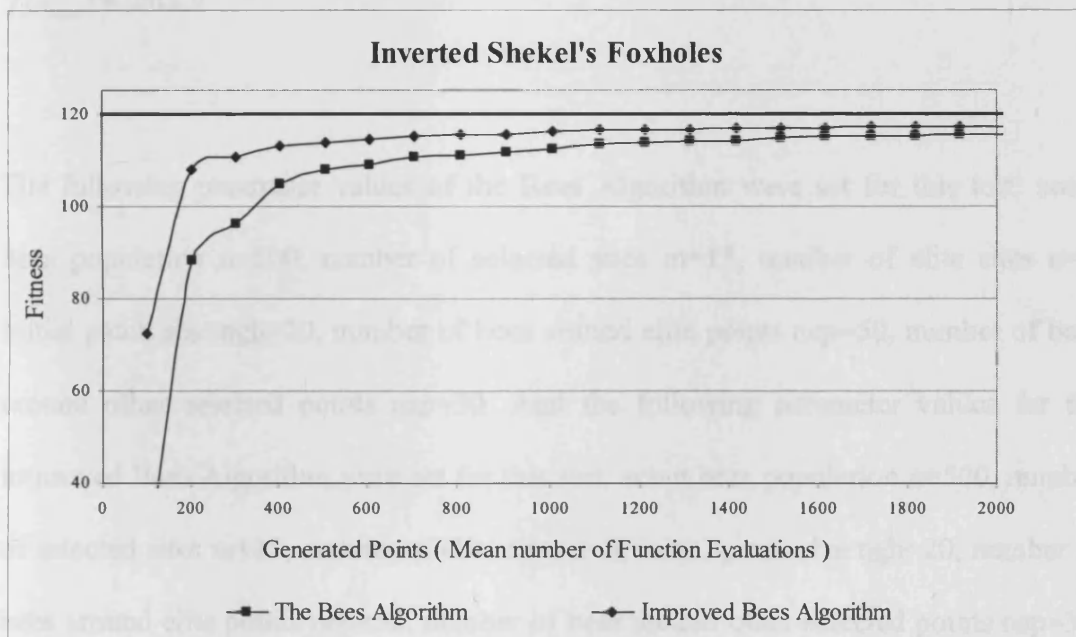


Figure 3.12 Evolution of fitness with the number of points visited (Inverted Shekel's Foxholes)

To test the reliability of the algorithm, the inverted Schwefel's function with six dimensions (see equation 3.4) was used. Fig. 4 shows a two-dimensional view of the function to highlight its multi-modality.

$$f(\vec{x}) = - \sum_{i=1}^6 x_i \sin(\sqrt{|x_i|}) \quad (3.4)$$

$$-500 \leq x_i \leq 500$$

For this function,

$$\vec{x}_{\max} = (420.9829, 420.9829, 420.9829, 420.9829, 420.9829, 420.9829)$$

$$f(\vec{x}_{\max}) \approx 2513.9$$

The following parameter values of the Bees Algorithm were set for this test: scout bees population $n=500$, number of selected sites $m=15$, number of elite sites $e=5$, initial patch size $ngh=20$, number of bees around elite points $nep=50$, number of bees around other selected points $nsp=30$. And the following parameter values for the improved Bees Algorithm were set for this test: scout bees population $n=500$, number of selected sites $m=15$, number of elite sites $e=5$, initial patch size $ngh=20$, number of bees around elite points $nep=50$, number of bees around other selected points $nsp=30$, shrinking constant $sc=0.05$ (5%) and site abandonment threshold $sat=20$. The basic Bees Algorithm was set with exactly the same parameters excluding the shrinking constant and site abandonment threshold.

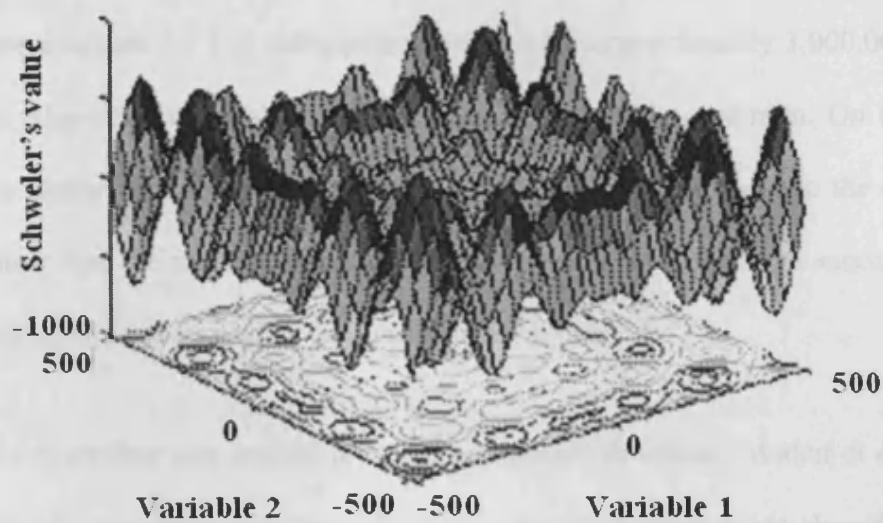


Figure 3.13 2D Schwefel's function

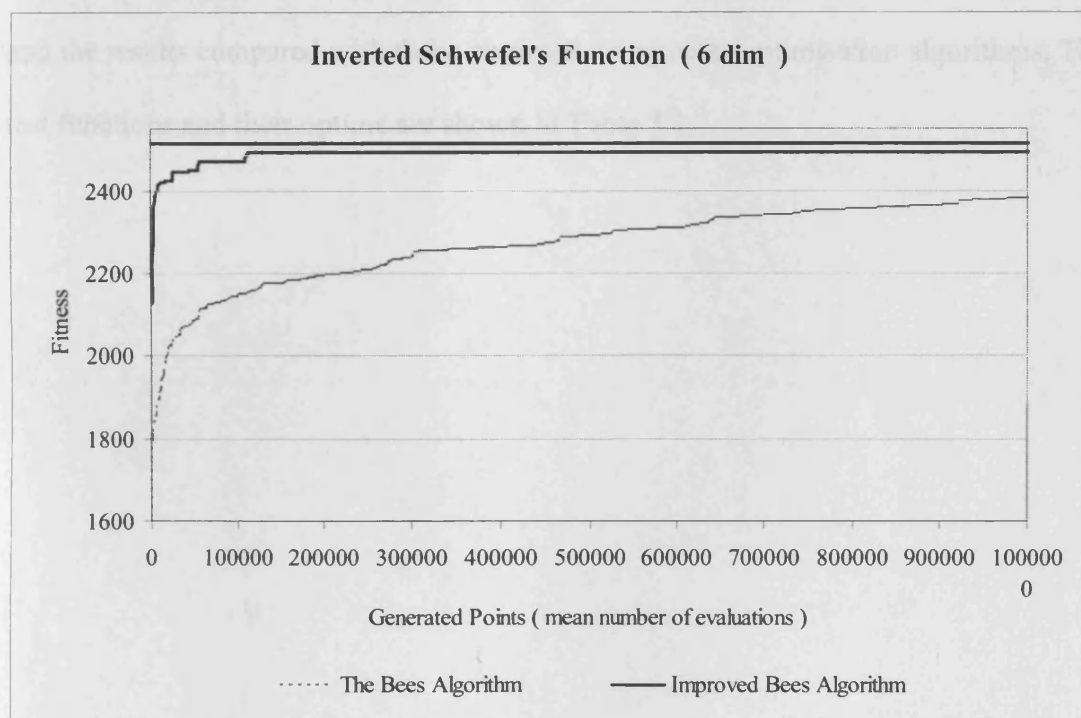


Figure 3.14 Evolution of fitness with the number of points visited (Inverted Schwefel's Function)

Fig. 3.14 shows how the fitness values evolve with the number of points visited. The results are averages for 100 independent runs. After approximately 3,000,000 visits, the Bees Algorithm was able to find solutions close to the optimum. On the other hand, the improved Bees Algorithm was able to find solutions close to the optimum much faster than the original algorithm. The main reason for this high success rate is the shrinking strategy as well as the dynamic recruitment.

The Bees Algorithm was applied to seven benchmark functions (Mathur et al., 2000) and the results compared with those obtained using other optimisation algorithms. The test functions and their optima are shown in Table 3.1.

The Bees Algorithm was applied to eight benchmark functions (Mathur et al., 2000) and the results compared with those obtained using other optimisation algorithms. The test functions and their optima are shown in Table 3.1.

Table 3.1 Test Functions (Mathur et al., 2000)

No	Function Name	Interval	Function	Global Optimum
1	De Jong	[-2.048, 2.048]	$\max F = (3905.93) - 100(x_1^2 - x_2^2) - (1 - x_1)^2$	X(1,1) F=3905.93
2	Goldstein & Price	[-2, 2]	$\min F = [1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)] \times [30 + (2x_1 - 3x_2)^2(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)]$	X(0,-1) F=3
3	Branin	[-5, 10]	$\min F = a(x_2 - b x_1^2 + c x_1 - d)^2 + e(1 - f) \cos(x_1) + e$ $a=1, b=\frac{5.1}{4} \left(\frac{7}{22}\right)^2, c=\frac{5}{22} \times 7, d=6, e=10, f=\frac{1}{8} \times \frac{7}{2}$	X(-22/7, 12.275) X(22/7, 2.275) X(66/7, 2.475) F=0.3977272
4	Martin & Gaddy	[0, 10]	$\min F = (x_1 - x_2)^2 + ((x_1 + x_2 - 10)/3)^2$	X(5,5) F=0
5	Rosenbrock	[-1.2, 1.2] [-10, 10]	$\min F = 100 (x_1^2 - x_2)^2 + (1 - x_1)^2$	X(1,1) F=0
6	Rosenbrock	[-1.2, 1.2]	$\min F = \sum_{i=1}^3 \{100 (x_i^2 - x_{i+1})^2 + (1 - x_i)^2\}$	X(1,1,1,1) F=0
7	Hyper sphere	[-5.12, 5.12]	$\min F = \sum_{i=1}^6 x_i^2$	X(0,0,0,0,0,0) F=0

Table 3.2. Parameter Settings for the Bees Algorithm

	Parameters							
Func No	n	m	e	nep	nsp	ngh	sc(%)	sat
1	10	3	1	2	4	0.1	1	10
2	20	3	1	1	5	2	20	10
3	10	3	1	2	4	0.8	5	10
4	20	3	1	2	4	0.5	5	10
5a	8	2	1	1	4	0.1	1	10
5b	10	3	1	2	4	0.1	0.5	10
6	20	5	1	4	10	0.01	0.5	10
7	10	3	1	2	10	0.3	1	10

Table 3.3 Results

func no	SIMPSA		NE SIMPSA		GA		ANTS		The Bees Algorithm	
	success %	mean no. of eval.	success %	mean no. of eval.	success %	mean no. of eval.	success %	mean no. of eval.	success %	mean no. of eval.
1	***	***	***	***	100	10160	100	6000	100	1210
2	***	***	***	***	100	5662	100	5330	100	999
3	***	***	***	***	100	7325	100	1936	100	1657
4	***	***	***	***	100	2844	100	1688	100	526
5a	100	10780	100	4508	100	10212	100	6842	100	898
5b	100	12500	100	5007	***	***	100	7505	100	2306
6	99	21177	94	3053	***	***	100	8471	100	29185
7	***	***	***	***	100	15468	100	22050	100	7113

*** Data not available

Table 3.3 presents the results obtained by the Bees Algorithm and those by the deterministic Simplex method (SIMPSA) (Mathur et al., 2000), the stochastic simulated annealing optimisation procedure (NE SIMPSA) (Mathur et al., 2000), the Genetic Algorithm (GA) (Mathur et al., 2000) and the Ant Colony System (ANTS) (Mathur et al., 2000). Again, the numbers of points visited shown are averages for 100 independent runs.

All the algorithms were run 100 times for each parameter setting on each benchmark problem. For each of the 100 trials, the optimisation procedure was run until either it located an exact solution or found a solution which was less than 0.001 (or %0.1, whichever was smaller).

The first test function was De Jong's, for which the Bees Algorithm could find the optimum 120 times faster than ANTS and 207 times faster than GA, with a success rate of 100%. The second function was Goldstein and Price's, for which the Bees Algorithm reached the optimum almost 5 times faster than ANTS and GA, again with 100% success. With Branin's function, there was a 15% improvement compared with ANTS and 77% improvement compared with GA, also with 100% success.

Functions 5 and 6 were Rosenbrock's functions in two and four dimensions respectively. In the two-dimensional function, the Bees Algorithm delivers 100% success and good improvement over the other methods (at least twice fewer evaluations than the other methods). In the four-dimensional case, the Bees Algorithm needed more function evaluations to reach the optimum with 100% success. NE SIMPSA could find the optimum with 10 times fewer function evaluations but the success rate was only 94% and ANTS found the optimum with 100% success and 3.5

times faster than the Bees Algorithm. Test function 7 was a Hyper Sphere model of six dimensions. The Bees Algorithm needed half the number of function evaluations compared with GA and one third of that required for ANTS.

3.6. Summary

A new swarm-based intelligent optimisation procedure called the Bees Algorithm is presented. The algorithm mimics the food foraging behaviour of swarms of honey bees. In its basic version, the algorithm performs a kind of neighbourhood search combined with random search. Further investigations are also given on details of the local and global search methods used in the algorithm. Also, details of the improvements made to local and global search methods are presented, including dynamic recruitment, proportional shrinking and abandonment strategies. The performance of the algorithm is evaluated on benchmark results, comparing it to some other well-known algorithms in the literature.

Chapter 3

THE BEES ALGORITHM: THEORY AND IMPROVEMENTS

3.1. Preliminaries

Many complex multi-variable optimisation problems cannot be solved exactly within polynomially bounded computation times. This generates much interest in search algorithms that find near-optimal solutions in reasonable running times. The swarm-based algorithm described in this thesis is a search algorithm capable of locating good solutions efficiently. The algorithm is inspired by the food foraging behaviour of honey bees and could be regarded as belonging to the category of “intelligent” optimisation tools.

In this chapter, a new population-based search algorithm called the Bees Algorithm (BA) is presented (Pham et al., 2006). The algorithm mimics the food foraging behaviour of swarms of honey bees. In its basic version, the algorithm performs a kind of neighbourhood search combined with random search and can be used for both combinatorial optimisation and functional optimisation. The Bees Algorithm is presented in this chapter, with benchmark results comparing the performance of the algorithm with some well-known algorithms in the literature. Further details are also given of the local and global search methods used in this algorithm. Moreover, in this chapter, details of the improvements made to local and global search methods are presented, including dynamic recruitment, proportional shrinking and abandonment strategies.

The chapter is organised as follows: section 3.2 presents a description the basic Bees Algorithm in its simplest form with a simple example of the algorithm procedure. In section 3.3, some key characteristics of the Bees Algorithm are discussed, including site selection, neighbourhood search and global search. Improvements to local and global search are presented in section 3.4, including dynamic recruitment, proportional shrinking for selected sites and site abandonment. Experimental results and benchmark tables are presented in section 3.5 to demonstrate the performance and the robustness of the algorithm compared to some well-known algorithms in the literature.

3.2. The basic Bees Algorithm

The Bees Algorithm is an optimisation algorithm inspired by the natural foraging behaviour of honey bees to find the optimal solution. Fig. 3.1 shows the pseudo-code and Fig. 3.2 presents the flowchart of the basic Bees Algorithm in its simplest form. The algorithm requires a number of parameters to be set, namely: number of scout bees (n), number of patches selected out of n visited points (m), number of best patches out of m selected patches (e), number of bees recruited for e best patches (n_{ep}), number of bees recruited for the other ($m-e$) selected patches (n_{sp}), size of patches (n_{gh}) and the stopping criterion. The algorithm starts with the n scout bees being placed randomly in the search space. The fitnesses of the points visited by the scout bees are evaluated in step 2. A simple demonstration is given in Fig. 3.3 which shows the basis steps of the algorithm.

In step 4, bees that have the highest fitnesses are chosen as “selected bees” and those sites that have been visited will be chosen for neighbourhood search. Then, in steps 5 and 6, the algorithm conducts searches in the neighbourhood of the selected bees in terms of more bees for the e best bees. The latter can be chosen directly according to the fitnesses associated with the points they are visiting. Alternatively, the fitness values are used to determine the probability of the bees being selected. Searches in the neighbourhood of the e best bees which represent more promising solutions are made more detailed by recruiting more bees to follow the e best bees than other selected bees. Also within scouting, differential recruitment is one of the key operations of the Bees Algorithm. Both scouting and differential recruitment are used in nature.

-
1. Initialise population with random solutions.
 2. Evaluate fitness of the population.
 3. While (stopping criterion not met)
 //Forming new population.
 4. Select sites for neighbourhood search.
 5. Recruit bees for selected sites (more bees for e best sites) and evaluate fitnesses.
 6. Select the fittest bee from each site.
 7. Assign remaining bees to search randomly and evaluate their fitnesses.
 8. End While.
-

Figure 3.1 Pseudo-code of the basic Bees Algorithm

However, in step 6, for each site only one bee with the highest fitness will be selected to form the next bee population. In nature, there is no such a restriction. This restriction is introduced here to reduce the number of points to be explored.

In step 7, the remaining bees in the population are assigned randomly around the search space scouting for new potential solutions. These steps are repeated until a stopping criterion is met. At the end of each iteration, the colony will have two parts to its new population – representatives from each selected patch and other scout bees assigned to conduct random searches.

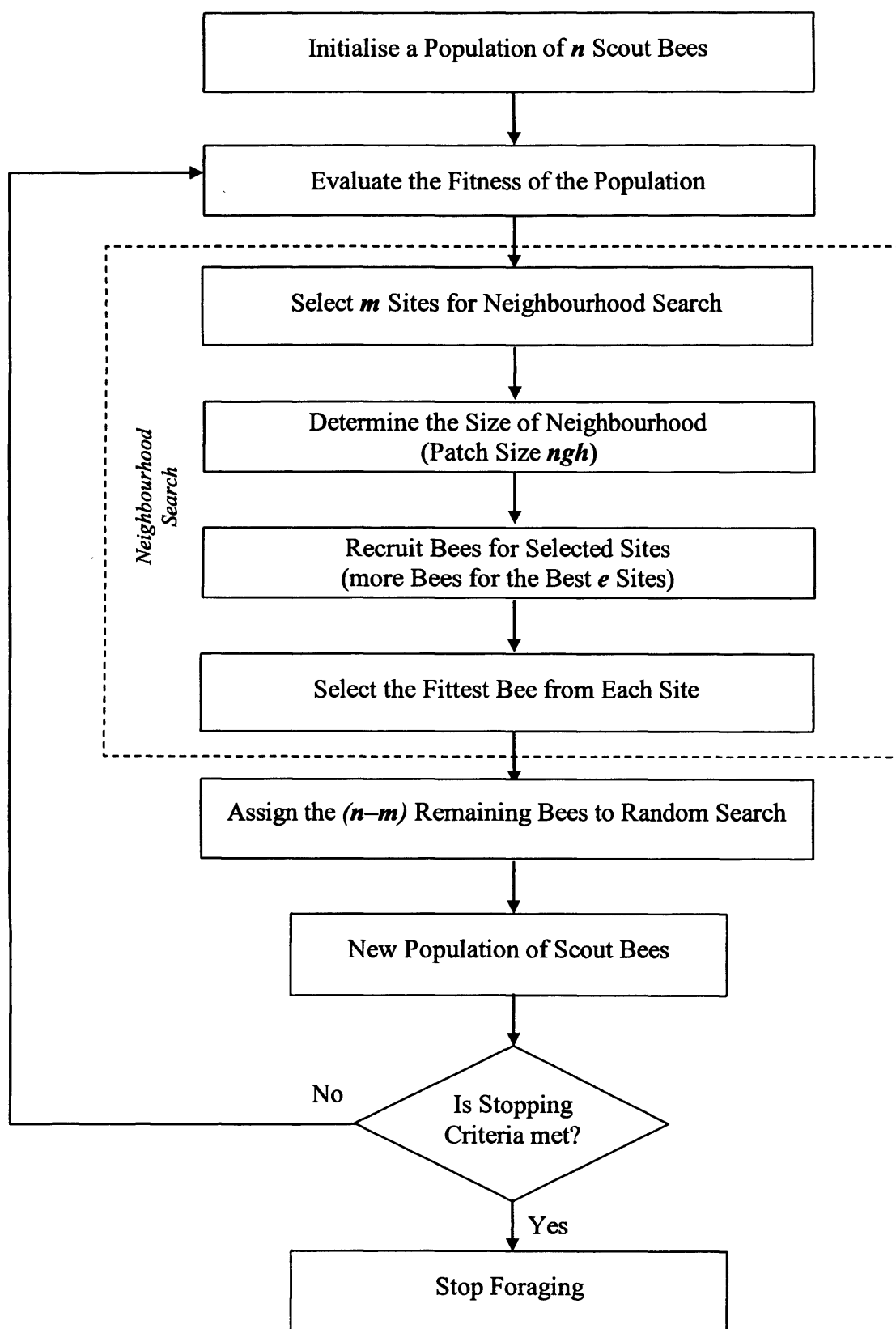
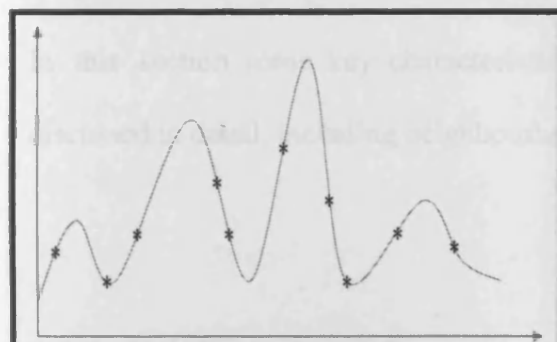
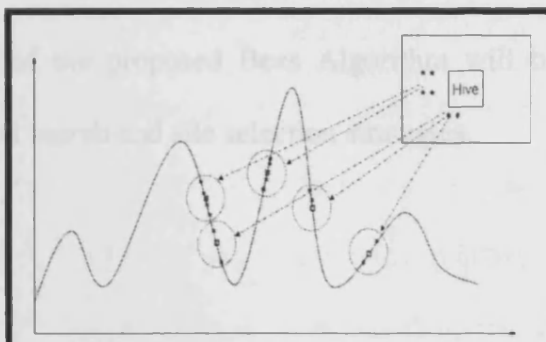


Figure 3.2 Flowchart of the basic Bees Algorithm

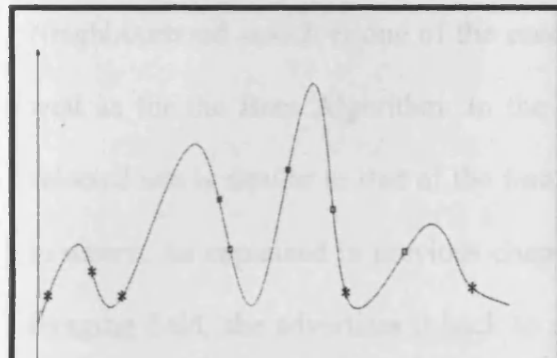
3.3. Characteristics of the proposed Bees Algorithm



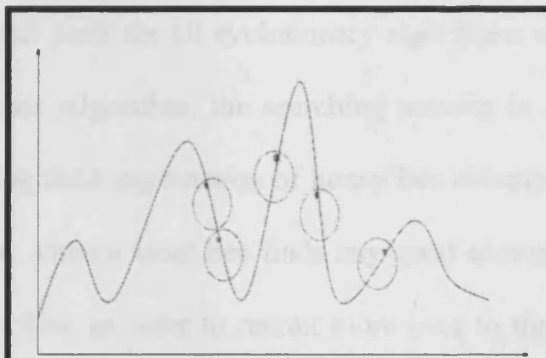
Graph 1. Initialise population with random solutions and evaluate the fitness.



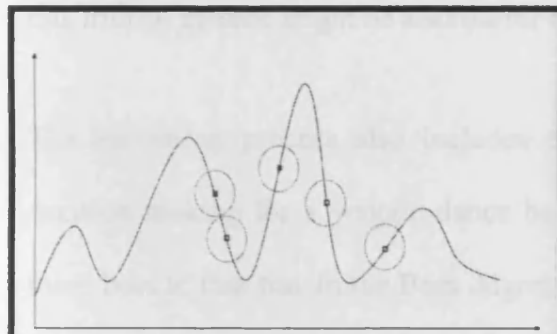
Graph 4. Recruit bees around selected sites (more bees for the best sites).



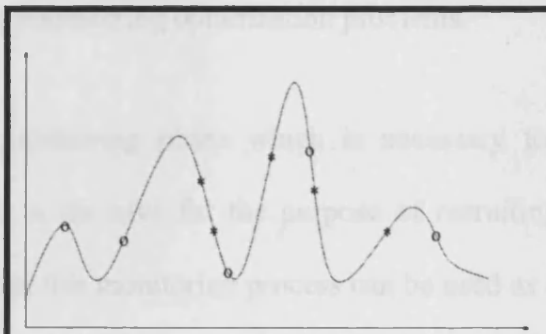
Graph 2. Select sites for neighbourhood search "*" and "o".



Graph 5. Select the fittest from each site "*".



Graph 3. Define neighbourhood range.



Graph 6. Assign remaining bees "o" to search randomly and evaluate their fitness

Figure 3.3 Simple example of the Bees Algorithm

3.3. Characteristics of the proposed Bees Algorithm

In this section some key characteristics of the proposed Bees Algorithm will be discussed in detail, including neighbourhood search and site selection strategies.

3.3.1 Neighbourhood Search

Neighbourhood search is one of the essential parts for all evolutionary algorithms as well as for the Bees Algorithm. In the Bees Algorithm, the searching process in a selected site is similar to that of the foraging field exploitation of honey bee colonies in nature. As explained in previous chapter, when a scout bee finds any good enough foraging field, she advertises it back to the hive in order to recruit more bees to that field. This behaviour is useful in terms of bringing more nectar into the hive. Hence, this fruitful method might be also useful for engineering optimization problems.

The harvesting process also includes a monitoring phase which is necessary for decision making for a waggle dance back in the hive for the purpose of recruiting more bees to that site. In the Bees Algorithm, this monitoring process can be used as a neighbourhood search. Essentially, when a scout bee finds a good field (good solution), she advertises her field to more bees. Subsequently, those bees fly to that source, take piece of nectar and return back to hive. Depending on the quality, this source can be advertised by some of the bees that are already aware of the source. In the proposed Bees Algorithm, this behaviour has been used as a neighbourhood search. As explained above, from each foraging site (or neighbourhood site) only one

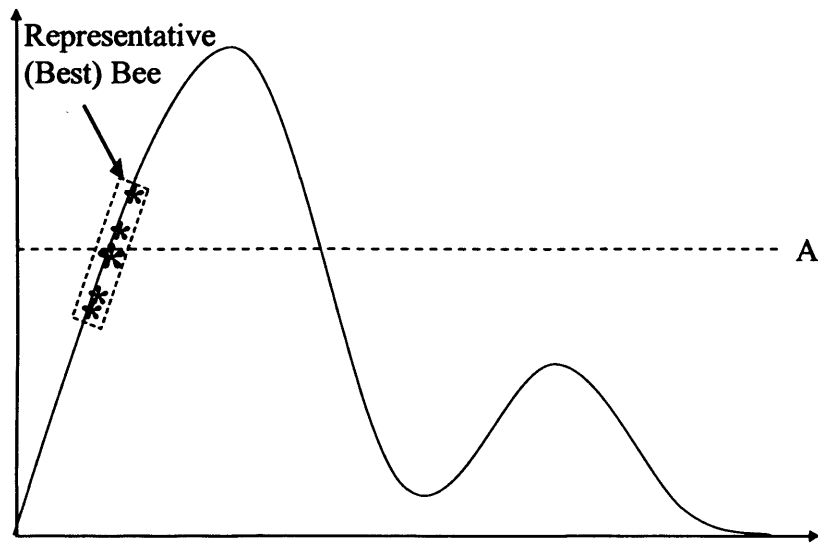
bee is chosen. This bee must have the best solution information about that field. Thus, the algorithm can create some solutions which are related to the previous ones.

Neighbourhood search is based on a random distribution of bees in a predefined neighbourhood range (patch size). For every selected site, bees are randomly distributed to find a better solution. As shown in Fig. 3.4, only the fittest (best) bee is chosen as a representative and the centre of the neighbourhood patch shifted up to best bees' position (from A to B).

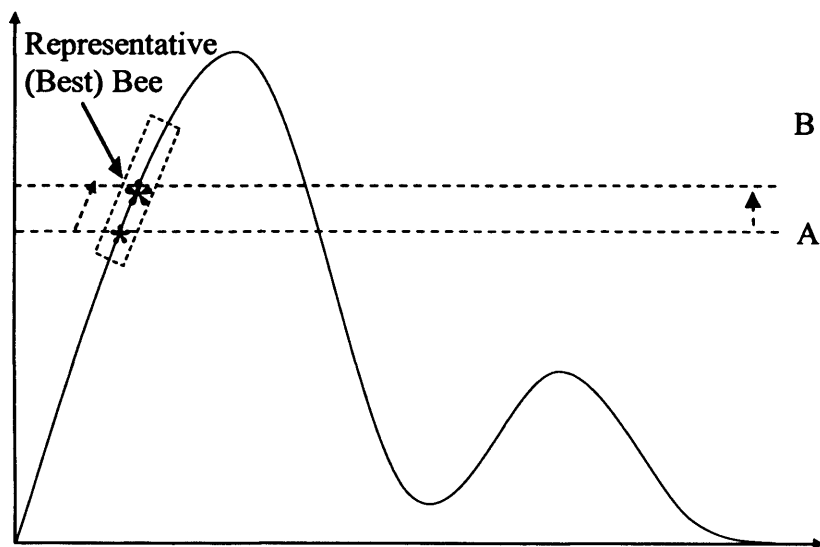
On the other hand, during the harvesting process other elements should also be taken into account for increasing efficiency, such as number of recruited bees in the neighbourhood patch and the patch size.

The number of recruited bees around selected sites should be defined properly. When the number is increased, then the number of function evaluations will also be increased and vice versa.

This problem also depends on the neighbourhood range. If the range can be arranged adequately, then the number of recruited bees will depend on the complexity of a solution space. This will be discussed later with more details.



(a)



(b)

Figure 3.4 Graphical explanation of basic neighbourhood search.

3.3.4 Site Selection

For site selection two different techniques have been implemented: probabilistic selection and best site selection.

In probabilistic selection, the roulette wheel method has been used and sites with better fitness have more chance of being selected, but in best site selection, the best sites according to fitness will be selected. In this section, different combinations of selection using the two methods from pure probabilistic selection ($q=0$) to pure best site selection ($q=1$) have been investigated and mean iterations required to enrich the answer. Results are shown in Fig. 3.5 and Fig. 3.6.

Regarding results, best selection will present better results and also is simpler, so in its basic form the best sites method is selected as the neighbourhood site selections method for the Bees Algorithm.

3.3.5. Global Search

In the first step, all scout bees (n) are placed randomly across the fitness landscape to explore for new flower patches. After neighbourhood search, $n-m$ bees are again placed randomly across the fitness landscape to explore new patches. The latter part is the main global search tool for the Bees Algorithm.

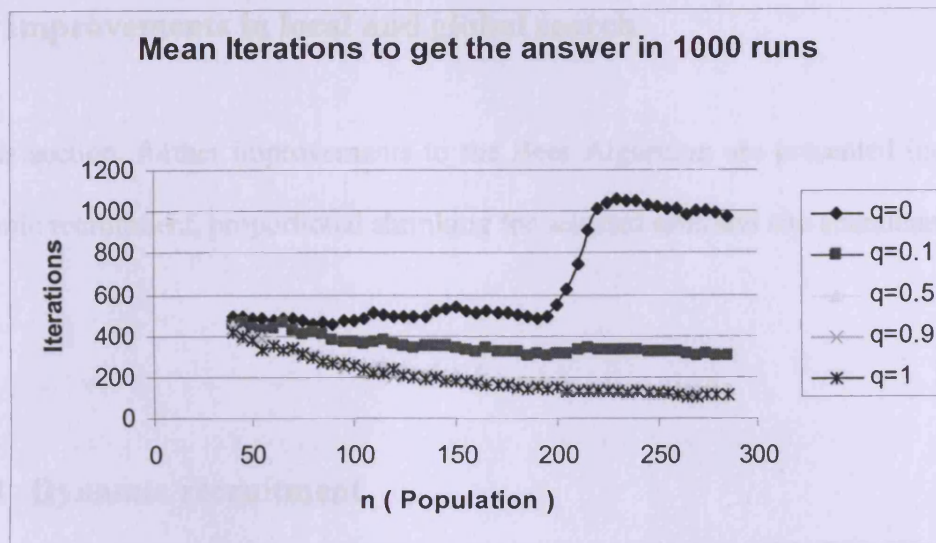


Figure 3.5 Mean iteration required for different combinations of selection

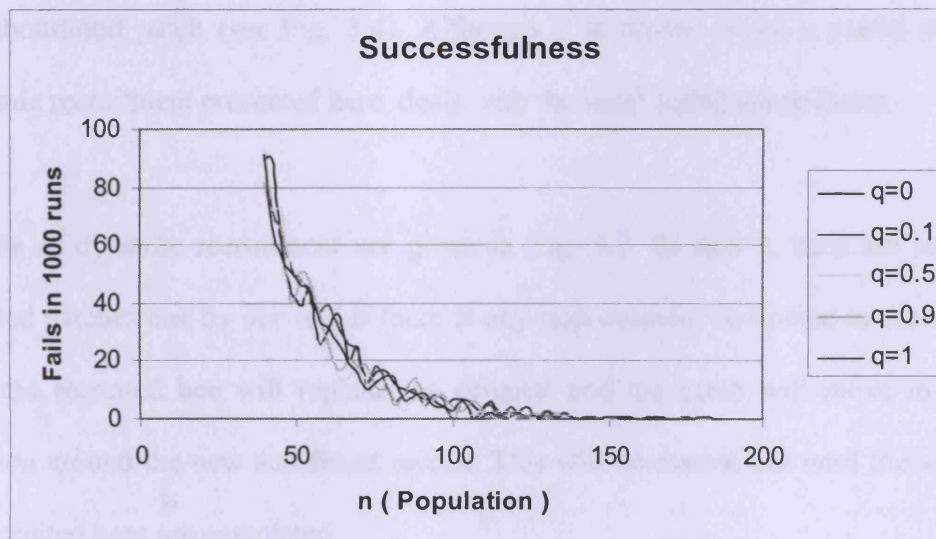


Figure 3.6 Successfulness of different combinations of selection methods

3.4. Improvements in local and global search

In this section, further improvements to the Bees Algorithm are presented including dynamic recruitment, proportional shrinking for selected sites and site abandonment.

3.4.1 Dynamic recruitment

Dynamic recruitment is introduced to improve the way that the bees are recruited into a selected site. In the basic Bees Algorithm, when a site is selected for neighbourhood search there will be a certain number of bees assigned for local search. In the previous strategy, bees are sent all at once to the same local search space defined as the neighbourhood patch (see Fig. 3.4). Although it is proven to be a useful strategy, dynamic recruitment presented here deals with the local search space faster.

Details of dynamic recruitment are given in Fig. 3.7. In step 5, bees are sent into selected patches one by one and if there is any improvement compared to the original bee, the recruited bee will replace the original and the patch will move to a new position around the new and fittest recruit. This will be carried out until the visits by all recruited bees are completed..

1. Initial population with n random solution.
2. Evaluate fitness of the population.
3. While (stopping criterion not met)
4. Select sites (m) for neighbourhood search.
5. Recruit bees for selected sites (more bees for best e sites), evaluate fitnesses, select the fittest bee from each site and shrink patches


```

for(k=1 ; k=e ; k++) // Elite Sites
    for(Bee=1 ; Bee= nep ; Bee++) // More Bees for Elite Sites
        BeesPositionInNgh() = GenerateRandomValueInNgh(from x+ngh to x-ngh);
        Evaluate Fitness = Bee(i); //Evaluate the fitnesses of recruited Bee(i)
        If (Bee(i) is better than Bee(i-1)) RepresentativeBee = Bee(i);
for(k=e ; k=m ; k++) // Other selected sites (m-e)
    for(Bee=1 ; Bee= nsp ; Bee++) // Less Bees for Other Selected Sites (m-e)
        BeesPositionInNgh() = GenerateRandomValueInNgh(from x+ngh to x-ngh);
        Evaluate Fitness = Bee(i); //Evaluate the fitnesses of recruited Bee(i)
        If (Bee(i) is better than Bee(i-1)) RepresentativeBee = Bee(i);

```
6. Assign remaining bees to search randomly and evaluate their fitnesses. // (n-m) assigned to search randomly into whole solution space
7. End While

Figure 3.7 The Bees Algorithm with dynamic recruitment.

3.4.2 Proportional shrinking for selected sites

The term proportional shrinking refers to a contraction of patch sizes of all selected sites (m) in every iteration of the algorithm proportional to a constant ratio called Shrinking Constant (sc). Equation 3.1 gives the definition of the proportional shrinking idea, in which the initial patch size is set as a starting patch size in the first iteration of the algorithm. Depending on the iteration (i), the patch size of the site m ($Ngh_m(i)$) is calculated as a contraction from the previous size ($Ngh_m(i-1)$) proportional to the value of sc . The value of sc can be defined by the user between 0 and 1 that represents the percentage by which the patch will shrink. However, the patch size must be a positive value all the time ($Ngh_m(i) > 0$).

$$Ngh_m(i) = \begin{cases} i = 1 & Ngh_m(i) = InitialPatchSize, \\ i > 1 & Ngh_m(i) = Ngh_m(i-1)(1 - SC) \text{ and } Ngh_m(i) > 0. \end{cases} \quad (3.1)$$

This new strategy, implemented as step 6 in Fig. 3.8, is proposed to improve the solution quality and evaluation time. At the beginning, for the local search, a wide patch size increases the probability of finding a better solution. By shrinking it saves the time and increases the solution quality by fine tuning in relatively narrow local search space.

3.4.3 Site Abandonment

The site abandonment strategy is introduced to improve the efficiency of the local search. The term refers the abandonment of a site in which there is no more improvement of the fitness value of the fittest bee after a certain number of iterations. In many complex optimisation problems, there may be many local solutions in their solution spaces and it is not possible to escape from local optima without an efficient procedure.

Parallel scout bee search and better neighbourhood exploitation are two strong features of the Bees Algorithm that are capable of dealing with many complex optimisation problems. But for all other algorithms it may not be possible to escape from local optima. Site abandonment is here introduced to deal with this problem.

In step 8 (see Fig. 3.8), if the points visited near a selected site are all inferior to that site, after a certain number of iterations (i.e. *sat*: site abandonment threshold), then the location of the site is recorded and the site abandoned. Bees at the site are assigned to random search (i.e. made to scout for new potential solutions).

This step is directly inspired by honey-bees in nature. Depending on the solution, quality bees either continue to exploit a patch by sending more bees or will abandon the site after several visits.

```

1. Initial population with n random solution.

2. Evaluate fitness of the population.

3. While (stopping criterion not met)

4. Select sites (m) for neighbourhood search.

5. Recruit bees for selected sites (more bees for best e sites), evaluate fitnesses, select the fittest
   bee from each site and shrink patches

   for (k=1 ; k=e ; k++) // Elite Sites

       for (Bee=1 ; Bee= nep ; Bee++) // More Bees for Elite Sites

           BeesPositionInNgh() = GenerateRandomValueInNgh(from x+ngh to x-ngh);

           Evaluate Fitness = Bee(i); //Evaluate the fitnesses of recruited Bee(i)

           If (Bee(i) is better then Bee(i-1)) RepresentativeBee = Bee(i);

   for (k=e ; k=m ; k++) // Other selected sites (m-e)

       for (Bee=1 ; Bee= nsp ; Bee++) // Less Bees for Other Selected Sites (m-e)

           BeesPositionInNgh() = GenerateRandomValueInNgh(from x+ngh to x-ngh);

           Evaluate Fitness = Bee(i); //Evaluate the fitnesses of recruited Bee(i)

           If (Bee(i) is better then Bee(i-1)) RepresentativeBee = Bee(i);

6. for (patch=1; patch=m; patch++) //Shrink all patches (m) proportional to SC


$$Ngh_m(i) = Ngh_m(i-1) * ((1 - SC));$$


7. If (Iteration > sat)

   If (no improvement on the site)

       Save the Best Fitness;

       Abandon the Site;

       Bee(m) = GenerateRandomValue(All Search Space);

8. Assign remaining bees to search randomly and evaluate their fitnesses. // (n-m) assigned to
   search randomly into whole solution space

```

Figure 3.8 Pseudo-code of the Bees Algorithm with proportional shrinking and site abandonment.

3.5. Experimental Results

Clearly, the Bees Algorithm as described above is applicable to both combinatorial and functional optimisation problems. In this section, functional optimisation is presented to show the robustness of the algorithm.

Three standard functional optimisation problems were used to test the Bees Algorithm and establish the correct values of its parameters and seven problems for benchmarking the algorithm. As the Bees Algorithm searches for the maximum, functions to be minimised were inverted before the algorithm was applied.

The first test function (see equation 3.2) is the axis parallel hyper-ellipsoid which is similar to De Jong's function 1 (see Fig. 3.9). It is also known as the weighted sphere model. It is continuous, convex and unimodal.

$$f_{1a}(x) = \sum_{i=1}^n i x_i^2 \quad (3.2)$$

$$-5.12 \leq x_i \leq 5.12$$

Global Minimum for this function:

$$f(x) = 0; \quad x(i) = 0, \quad i = 1:n$$

The following parameter values were set for the axis parallel hyper-ellipsoid test function: scout bee population $n=10$, number of selected sites $m=3$, number of elite sites $e=1$, initial patch size $ngh=0.5$, number of bees around elite points $nep=2$, number of bees around other selected points $nsp=2$.

The following parameter values of the Bees Algorithm were set for this test: scout bee population $n=10$, number of selected sites $m=3$, number of elite sites $e=1$, initial patch size $ngh=2.75$, number bees around elite points $nep=2$, number of bees around other selected points $nsp=2$. And the following parameter values of the improved Bees Algorithm were set for this test: scout bee population $n=10$, number of selected sites $m=3$, number of elite sites $e=1$, initial patch size $ngh=5.12$, number of bees around elite points $nep=2$, number of bees around other selected points $nsp=2$, shrinking constant $sc=0.20$ (20%) and site abandonment threshold $sat=10$. The basic Bees Algorithm was set with exactly the same parameters excluding the shrinking constant and site abandonment threshold.

Fig. 3.10 shows the fitness values obtained as a function of the number of points visited for both original and improved algorithms. The results are averages for 100 independent runs. It can be seen that after approximately 500 visits, the improved algorithm was able to find solutions close to the optimum while the original algorithm needs more time to find the optimum. It is also important to emphasise that the initial patch size was set as the whole solution space for the improved Bees Algorithm.

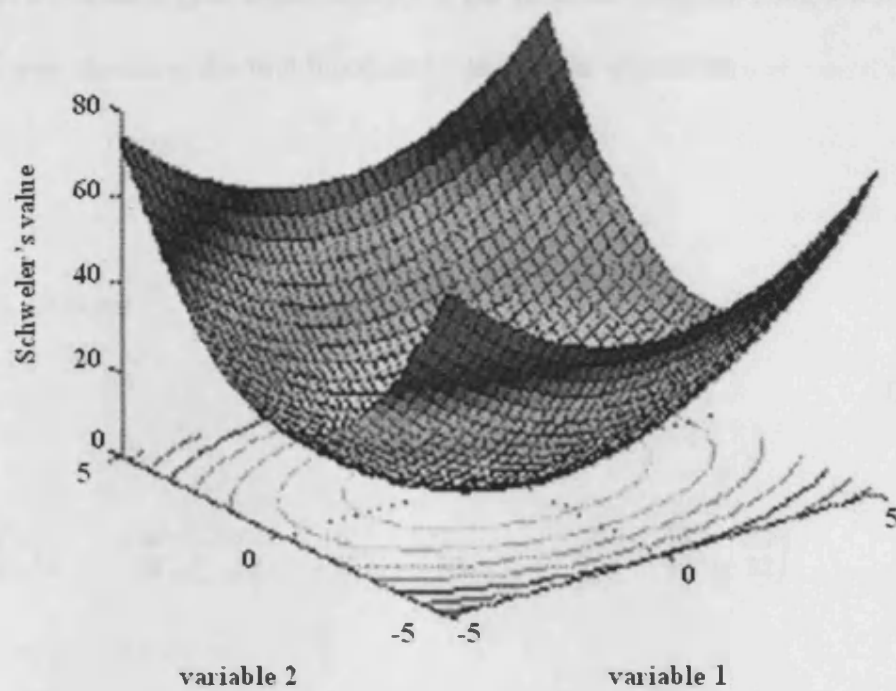


Figure 3.9 Visualization of 2D axis parallel hyper-ellipsoid function.

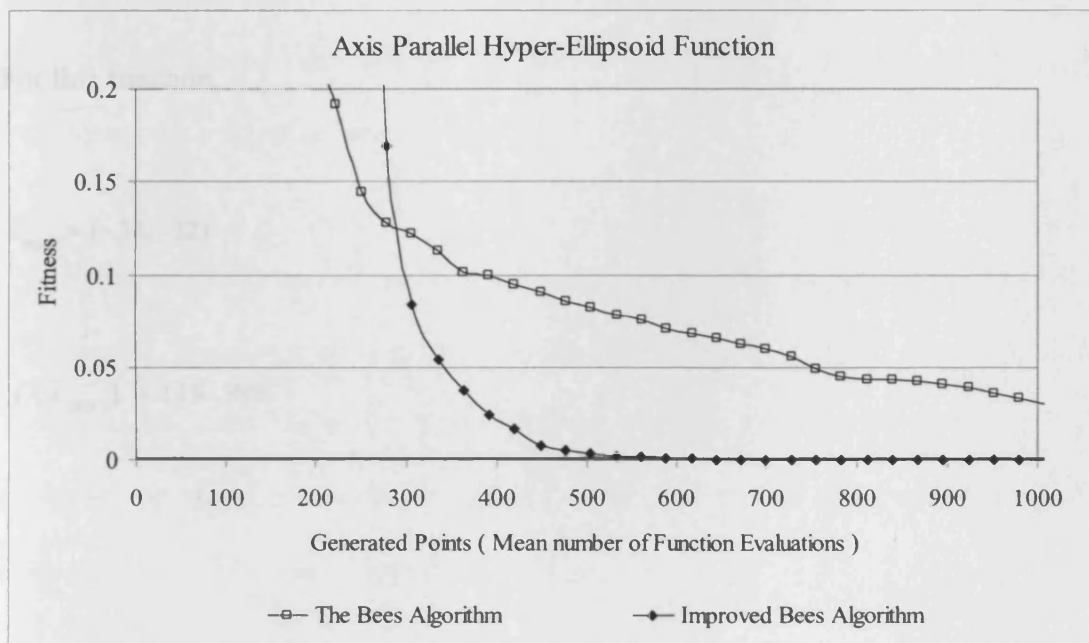


Figure 3.10 Evolution of fitness with the number of points visited (the axis parallel hyper-ellipsoid)

Shekel's Foxholes (see equation 3.3), a 2D function from De Jong's test suite (Fig. 3.11), was chosen as the first function for testing the algorithm.

$$f(\vec{x}) = 119.998 - \sum_{i=1}^{25} \frac{1}{j + \sum_{i=1}^2 \left(x_i - a_{ij} \right)^6} \quad (3.3)$$

$$a_{ij} = \begin{pmatrix} -32 & -16 & 0 & 16 & 32 & \dots & 0 & 16 & 32 \\ -32 & -32 & -32 & -32 & -32 & \dots & 32 & 32 & 32 \end{pmatrix}$$

$$-65.536 \leq x_i \leq 65.536$$

For this function,

$$\vec{x}_{\max} = (-32, -32)$$

$$f(\vec{x}_{\max}) = 119.998$$

The following parameter values of the Bees Algorithm were set for this test: scout bee population $n=45$, number of selected sites $m=3$, number of elite sites $e=1$, initial patch size $ngh=3$, number bees around elite points $nep=7$, number of bees around other selected points $nsp=2$. And the following parameter values for the improved Bees Algorithm were set for this test: scout bee population $n=10$, number of selected sites $m=3$, number of elite sites $e=1$, initial patch size $ngh=3$, number bees around elite points $nep=2$, number of bees around other selected points $nsp=2$, shrinking constant $sc=0.01$ (1%) and site abandonment threshold $sat=10$. The basic Bees Algorithm was set with exactly the same parameters excluding the shrinking constant and site abandonment threshold.

Note that ngh defines the initial size of the neighbourhood in which follower bees are placed. For example, if x is the position of an elite bee in the i th dimension, follower bees will be placed randomly in the interval $x_{ie} \pm ngh$ in that dimension at the beginning of the optimisation process.

Fig. 12 shows the fitness values obtained as a function of the number of points visited. The results are averages for 100 independent runs. It can be seen that after approximately 1200 visits, the Bees Algorithm was able to find solutions close to the optimum. However, the improved algorithm is able to find solutions close to the optimum faster than its predecessor

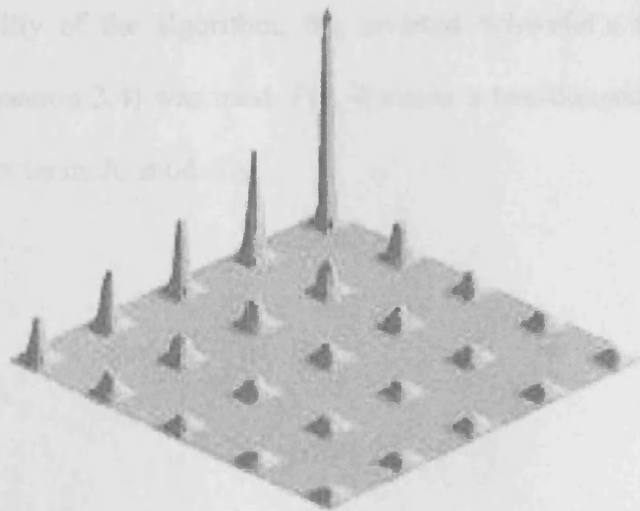


Figure 3.11 Inverted Shekel's Foxholes

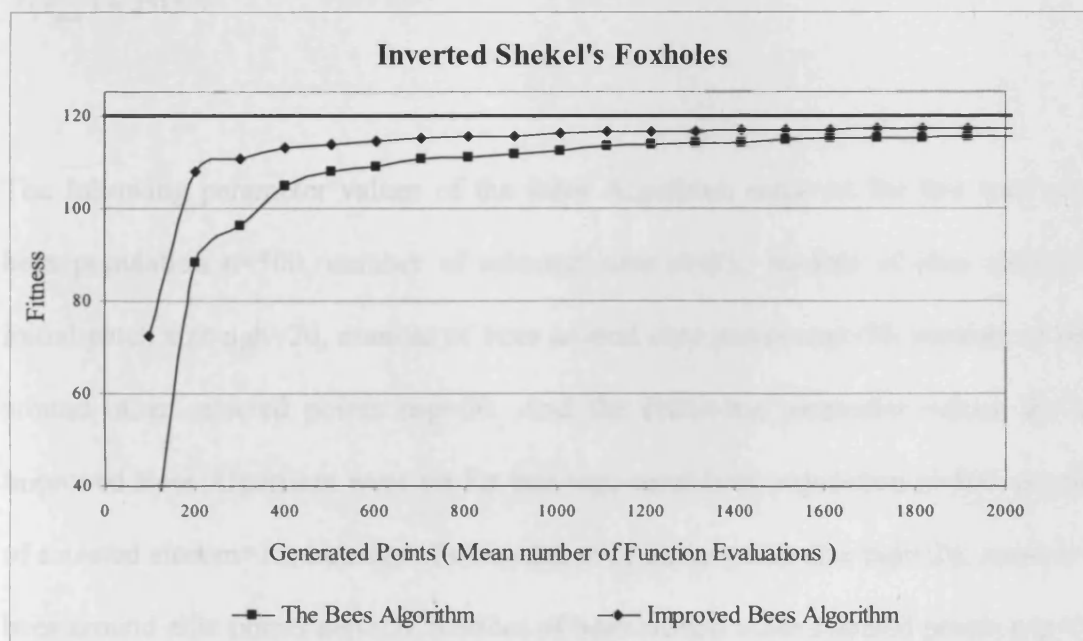


Figure 3.12 Evolution of fitness with the number of points visited (Inverted Shekel's Foxholes)

To test the reliability of the algorithm, the inverted Schwefel's function with six dimensions (see equation 3.4) was used. Fig. 4 shows a two-dimensional view of the function to highlight its multi-modality.

$$f(\vec{x}) = - \sum_{i=1}^6 x_i \sin(\sqrt{|x_i|}) \quad (3.4)$$

$$-500 \leq x_i \leq 500$$

For this function,

$$\vec{x}_{\max} = (420.9829, 420.9829, 420.9829, 420.9829, 420.9829, 420.9829)$$

$$f(\vec{x}_{\max}) \approx 2513.9$$

The following parameter values of the Bees Algorithm were set for this test: scout bees population $n=500$, number of selected sites $m=15$, number of elite sites $e=5$, initial patch size $ngh=20$, number of bees around elite points $nep=50$, number of bees around other selected points $nsp=30$. And the following parameter values for the improved Bees Algorithm were set for this test: scout bees population $n=500$, number of selected sites $m=15$, number of elite sites $e=5$, initial patch size $ngh=20$, number of bees around elite points $nep=50$, number of bees around other selected points $nsp=30$, shrinking constant $sc=0.05$ (5%) and site abandonment threshold $sat=20$. The basic Bees Algorithm was set with exactly the same parameters excluding the shrinking constant and site abandonment threshold.

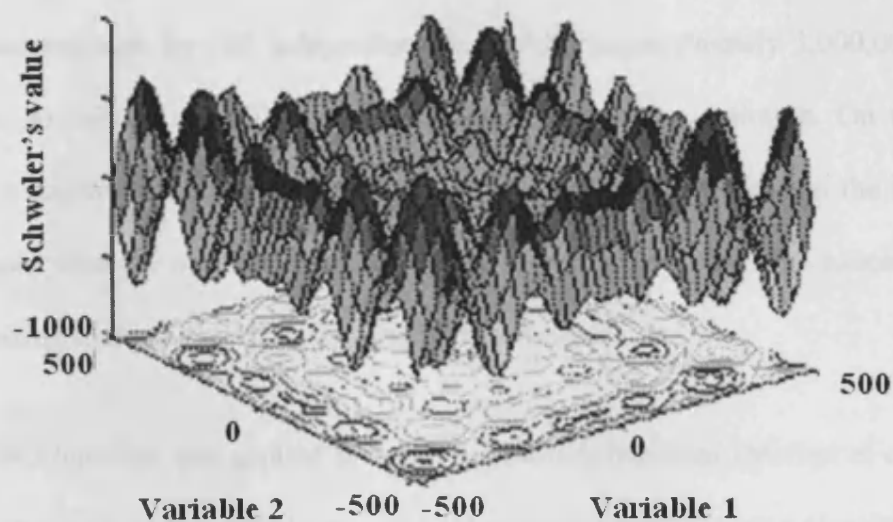


Figure 3.13 2D Schwefel's function

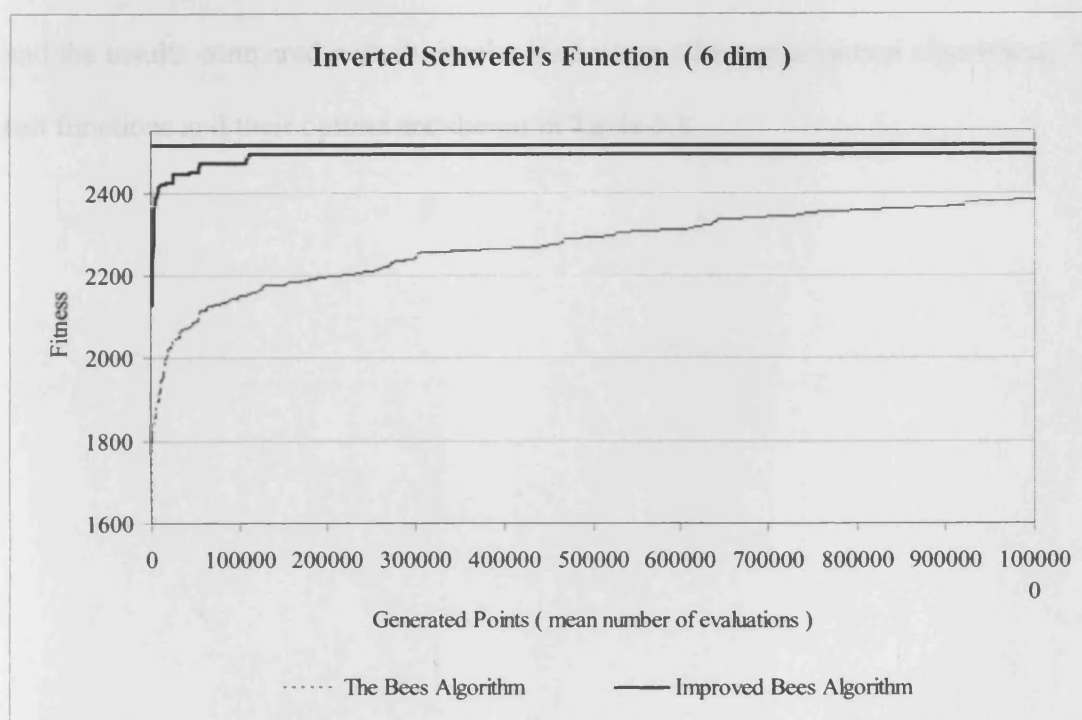


Figure 3.14 Evolution of fitness with the number of points visited (Inverted Schwefel's Fuction)

Fig. 3.14 shows how the fitness values evolve with the number of points visited. The results are averages for 100 independent runs. After approximately 3,000,000 visits, the Bees Algorithm was able to find solutions close to the optimum. On the other hand, the improved Bees Algorithm was able to find solutions close to the optimum much faster than the original algorithm. The main reason for this high success rate is the shrinking strategy as well as the dynamic recruitment.

The Bees Algorithm was applied to seven benchmark functions (Mathur et al., 2000) and the results compared with those obtained using other optimisation algorithms. The test functions and their optima are shown in Table 3.1.

The Bees Algorithm was applied to eight benchmark functions (Mathur et al., 2000) and the results compared with those obtained using other optimisation algorithms. The test functions and their optima are shown in Table 3.1.

Table 3.1 Test Functions (Mathur et al., 2000)

No	Function Name	Interval	Function	Global Optimum
1	De Jong	[-2.048, 2.048]	$\max F = (3905.93) - 100(x_1^2 - x_2^2) - (1 - x_1)^2$	X(1,1) F=3905.93
2	Goldstein & Price	[-2, 2]	$\min F = [1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)] \times [30 + (2x_1 - 3x_2)^2(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)]$	X(0,-1) F=3
3	Branin	[-5, 10]	$\min F = a(x_2 - b x_1^2 + c x_1 - d)^2 + e(1 - f) \cos(x_1) + e$ $a=1, b=\frac{5.1}{4} \left(\frac{7}{22}\right)^2, c=\frac{5}{22} X 7, d=6, e=10, f=\frac{1}{8} X \frac{7}{2}$	X(-22/7,12.275) X(22/7,2.275) X(66/7,2.475) F=0.3977272
4	Martin & Gaddy	[0, 10]	$\min F = (x_1 - x_2)^2 + ((x_1 + x_2 - 10)/3)^2$	X(5,5) F=0
5	Rosenbrock	[-1.2, 1.2] [-10, 10]	$\min F = 100 (x_1^2 - x_2)^2 + (1 - x_1)^2$	X(1,1) F=0
6	Rosenbrock	[-1.2, 1.2]	$\min F = \sum_{i=1}^3 \{100 (x_i^2 - x_{i+1})^2 + (1 - x_i)^2\}$	X(1,1,1,1) F=0
7	Hyper sphere	[-5.12, 5.12]	$\min F = \sum_{i=1}^6 x_i^2$	X(0,0,0,0,0,0) F=0

Table 3.2. Parameter Settings for the Bees Algorithm

	Parameters							
Func No	n	m	e	nep	nsp	ngh	sc(%)	sat
1	10	3	1	2	4	0.1	1	10
2	20	3	1	1	5	2	20	10
3	10	3	1	2	4	0.8	5	10
4	20	3	1	2	4	0.5	5	10
5a	8	2	1	1	4	0.1	1	10
5b	10	3	1	2	4	0.1	0.5	10
6	20	5	1	4	10	0.01	0.5	10
7	10	3	1	2	10	0.3	1	10

Table 3.3 Results

func no	SIMPSA		NE SIMPSA		GA		ANTS		The Bees Algorithm	
	success %	mean no. of eval.	success %	mean no. of eval.	success %	mean no. of eval.	success %	mean no. of eval.	success %	mean no. of eval.
1	***	***	***	***	100	10160	100	6000	100	1210
2	***	***	***	***	100	5662	100	5330	100	999
3	***	***	***	***	100	7325	100	1936	100	1657
4	***	***	***	***	100	2844	100	1688	100	526
5a	100	10780	100	4508	100	10212	100	6842	100	898
5b	100	12500	100	5007	***	***	100	7505	100	2306
6	99	21177	94	3053	***	***	100	8471	100	29185
7	***	***	***	***	100	15468	100	22050	100	7113

*** Data not available

Table 3.3 presents the results obtained by the Bees Algorithm and those by the deterministic Simplex method (SIMPSA) (Mathur et al., 2000), the stochastic simulated annealing optimisation procedure (NE SIMPSA) (Mathur et al., 2000), the Genetic Algorithm (GA) (Mathur et al., 2000) and the Ant Colony System (ANTS) (Mathur et al., 2000). Again, the numbers of points visited shown are averages for 100 independent runs.

All the algorithms were run 100 times for each parameter setting on each benchmark problem. For each of the 100 trials, the optimisation procedure was run until either it located an exact solution or found a solution which was less than 0.001 (or %0.1, whichever was smaller).

The first test function was De Jong's, for which the Bees Algorithm could find the optimum 120 times faster than ANTS and 207 times faster than GA, with a success rate of 100%. The second function was Goldstein and Price's, for which the Bees Algorithm reached the optimum almost 5 times faster than ANTS and GA, again with 100% success. With Branin's function, there was a 15% improvement compared with ANTS and 77% improvement compared with GA, also with 100% success.

Functions 5 and 6 were Rosenbrock's functions in two and four dimensions respectively. In the two-dimensional function, the Bees Algorithm delivers 100% success and good improvement over the other methods (at least twice fewer evaluations than the other methods). In the four-dimensional case, the Bees Algorithm needed more function evaluations to reach the optimum with 100% success. NE SIMPSA could find the optimum with 10 times fewer function evaluations but the success rate was only 94% and ANTS found the optimum with 100% success and 3.5

times faster than the Bees Algorithm. Test function 7 was a Hyper Sphere model of six dimensions. The Bees Algorithm needed half the number of function evaluations compared with GA and one third of that required for ANTS.

3.6. Summary

A new swarm-based intelligent optimisation procedure called the Bees Algorithm is presented. The algorithm mimics the food foraging behaviour of swarms of honey bees. In its basic version, the algorithm performs a kind of neighbourhood search combined with random search. Further investigations are also given on details of the local and global search methods used in the algorithm. Also, details of the improvements made to local and global search methods are presented, including dynamic recruitment, proportional shrinking and abandonment strategies. The performance of the algorithm is evaluated on benchmark results, comparing it to some other well-known algorithms in the literature.

Chapter 4

BEES ALGORITHM FOR CONTINUOUS DOMAINS

4.1. Preliminaries

In this chapter, several continuous applications of the Bees Algorithm are presented, including neural network training for a variety of industrial applications and recursive filter design. Neural networks are computational models of the biological brain. Like the brain, a neural network comprises a large number of interconnected neurons. Each neuron is capable of performing only simple computation. However, as an assembly of neurons, a neural network can learn to perform complex tasks including pattern recognition, system identification, trend prediction and process control.

One of the best known types of neural network is the Multi-Layered Perceptron (MLP). MLP networks are usually trained; that is the weights of the connections

between the neurons are adjusted, by employing the backpropagation (BP) algorithm, which is a gradient-based optimisation algorithm. Because of its reliance on gradient information, the BP algorithm sometimes has difficulties handling local optima and cannot develop MLP networks with optimally adjusted weights.

The chapter is organised as follows: section 4.2 presents a detailed discussion on the optimisation of the weights of multi-layered perceptrons (MLPs) using the Bees Algorithm for pattern recognition in statistical process control charts. Section 4.3 presents the identification of defects in wood veneer sheets using the neural networks for several different features. Section 4.4 presents an application of the Bees Algorithm to electronic recursive filter design. The general summary of the proposed method, applications and possible future research are analysed in section 4.5.

4.2. Optimisation of the Weights of Multi-Layered Perceptrons

Using the Bees Algorithm for Pattern Recognition in Statistical Process Control Charts

In this section, an implementation of the Bees Algorithm is presented for training the weights of Multi-Layered Perceptrons (MLP) for pattern recognition in statistical process control charts. The results of control chart pattern recognition experiments using the Bees Algorithm are compared with the standard backpropagation algorithm.

4.2.1. Control Chart Patterns

Control charts enable a manufacturing engineer to compare the actual performance of a process with customer specifications and provide a process capability index to guide and assess quality improvement efforts (Montgomery, 2000). By means of simple rules, it is possible to determine if a process is out of control and needs corrective action. However, it is possible to detect incipient problems and prevent the process from going out of control by identifying the type of patterns displayed by the control charts. These patterns can indicate if the process being monitored is operating normally or if it shows gradual changes (trends), sudden changes (shifts) or periodic changes (cycles), (see Fig. 4.1). Various techniques that have been applied to this control chart pattern recognition task can be seen in (Pham and Oztemel, 1992; Pham and Oztemel, 1995; Pham and Chan, 2001).

Training and testing data are produced using equation (4.1) for normal patterns, equation (4.2) for cyclic patterns, equation (4.3) for increasing or decreasing trend patterns, and equation (4.4) for upwards or downwards shift patterns. A total of 1500 patterns, 250 patterns in each of the six classes, were generated using the following equations:

1. Normal patterns:

$$y(t) = \mu + r(t) \sigma \quad (4.1)$$

2. Cyclic patterns:

$$y(t) = \mu + r(t) \sigma + a \sin(2\pi / T) \quad (4.2)$$

3. Increasing or decreasing trends:

$$y(t) = \mu + r(t) \sigma \pm g t \quad (4.3)$$

4. Upwards or downwards shifts:

$$y(t) = \mu + r(t) \sigma \pm k s \quad (4.4)$$

where

μ = mean value of the process variable being monitored (taken as 80 in this work)

σ = standard deviation of the process (taken as 5)

a = amplitude of cyclic variations (taken as 15 or less)

g = magnitude of the gradient of the trend (taken as being in the range 0.2 to 0.5)

k = parameter determining the shift position (= 0 before the shift position; = 1 at the shift position and thereafter)

r = normally distributed random number (between – 3 and +3)

s = magnitude of the shift (taken as being in the range 7.5 to 20)

t = discrete time at which the pattern is sampled (taken as being within the range 0 to 59)

T = period of a cycle (taken as being in the range 4 to 12 sampling intervals)

$y(t)$ = sample value at time t

498 patterns (83 in each class) were used for training an MLP classifier and 1002 patterns (167 in each class) were employed for testing the trained classifier.

Each pattern used in the experiments was a time series comprising 60 points. The value $\bar{y}(t)$ at each point t was normalised to fall in the range $[0, +1]$ according to the following equation (Pham and Oztemel, 1995):

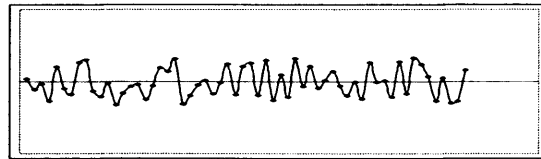
$$\bar{y}(t) = \frac{y(t) - y_{\min}}{y_{\max} - y_{\min}} \quad (4.5)$$

where,

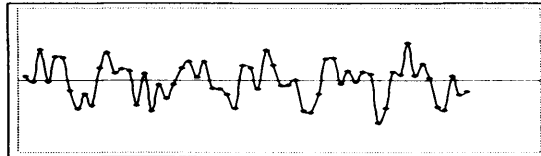
$\bar{y}(t)$ = scaled pattern value (in the range 0 to 1)

$y_{\min}$ = minimum allowed value (taken as 35)

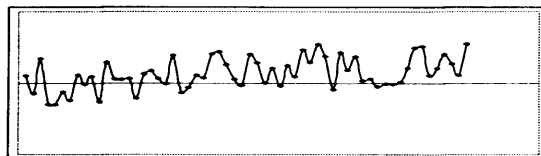
$y_{\max}$ = maximum allowed value (taken as 125)



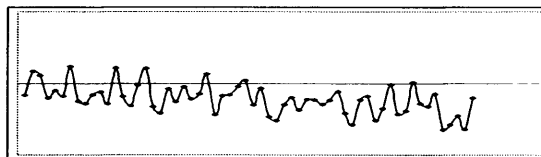
Normal Pattern



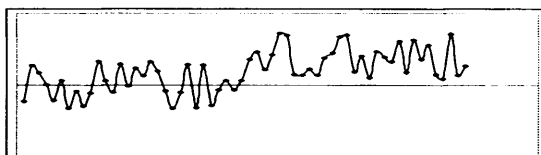
Cycle



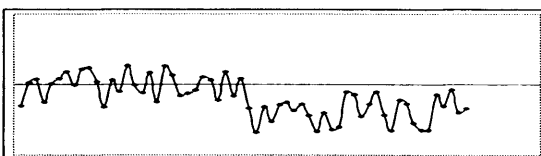
Increasing Trend



Decreasing Trend



Upward Shift



Downward Shift

Figure 4.1. Examples of the control chart patterns

4.2.2. Proposed Bees Algorithm for MLP weight optimisation

This section summarises the main steps of the Bees Algorithm applied to MLP weight optimizations. It is based on the pseudo-code of the algorithm given in Fig. 3.8.

The algorithm requires a number of parameters to be set, namely: number of scout bees (n), number of sites selected for exploitation out of n visited sites (m), number of top-rated (elite) sites among the m selected sites (e), number of bees recruited for the best e sites (n_{ep}), number of bees recruited for the other ($m-e$) selected sites (n_{sp}), initial size of each patch (n_{gh} ; a patch is a region in search space that includes a visited site and its neighbourhood), shrinking constant (sc) and stopping criterion. The algorithm starts with the n scout bees being placed randomly in the search space. The fitnesses of the sites visited by the scout bees are evaluated in step 2.

In step 4, the m sites with the highest fitnesses are designated as “selected sites” and chosen for neighbourhood search. In step 5, the algorithm conducts searches around the selected sites, assigning more bees to search in the vicinity of the best e sites. Selection of the best sites can be made directly according to the fitnesses associated with them. Alternatively, the fitness values are used to determine the probability of the sites being selected. Searches in the neighbourhood of the best e sites – those which represent the most promising solutions - are made more detailed. As already mentioned, this is done by recruiting more bees for the best e sites than for the other selected sites. Together with scouting, this differential recruitment is a key operation of the Bees Algorithm. For each patch, only the bee that has found the site with the highest fitness (the “fittest” bee in the patch) will be selected to form part of the next bee population. In nature, there is no such a restriction. This restriction is introduced

here to reduce the number of points to be explored. In step 6, after the recruitment, all patches will shrink down evenly, proportional to shrinking constant (sc). Also in step 6, if the points visited near a selected site are all inferior to that site, after a certain number of iterations (i.e. Abandon threshold), then the location of the site is recorded and the site abandoned. Bees at the site are assigned to random search (i.e. made to scout for new potential solutions).

In step 8, the remaining bees in the population are assigned randomly around the search space to scout for new potential solutions.

At the end of each iteration, the colony will have two parts to its new population: representatives from the selected patches, and scout bees assigned to conduct random searches. These steps are repeated until a stopping criterion is met.

The MLP network training procedure using the Bees Algorithm thus comprises the following steps given in Fig. 4.2. The training of an MLP network can be regarded as the minimisation of an error function. The error function defines the total difference between the actual output and the desired output of the network over a set of training patterns (Pham and Liu, 1995). Training proceeds by presenting to the network a pattern of known class taken randomly from the training set. The error component associated with that pattern is the sum of the squared differences between the desired and actual outputs of the network corresponding to the presented pattern. The procedure is repeated for all the patterns in the training set and the error components for all the patterns are summed to yield the value of the error function for an MLP network with a given set of connection weights.

1. Generate an initial population of bees.
2. Apply the training data set to determine the value of the error function associated with each bee.
3. Based on the error value obtained in step 2, create a new population of bees comprising the best bees in the selected neighbourhoods and randomly placed scout bees.
4. Stop if the value of the error function has fallen below a predetermined threshold.
5. Else, return to step 2.

Figure 4.2 The MLP network training procedure using the Bees Algorithm.

In terms of the Bees Algorithm, each bee represents an MLP network with a particular set of weight vectors. The aim of the algorithm is to find the bee with the set of weight vectors producing the smallest value of the error function.

The configuration of the multi-layer perceptrons (MLPs) involves three layers: an input layer, a hidden layer and an output layer (see Fig. 4.3). The input layer has 60 neurons, one for each point in a pattern. The hidden layer consists of 35 neurons. This number of hidden neurons is the same as adopted in previous experiments with BP-trained networks. The output layer comprises 6 neurons, one for each of the six classes (Pham and Oztemel, 1992). The input neurons perform no processing roles, acting only as buffers for the input signals. Both hidden and output layers' biases are used for each neuron. Processing is carried out by the hidden and output neurons, the activation functions for which were chosen to be of the sigmoidal type (Pham and Liu, 1995).

4.2.3. Experimental results

Table 4.1 shows the parameter values adopted for the Bees Algorithm. The values were decided empirically. In addition, the algorithm was initialised with all weight values set randomly within the range -1 to 1. However, a square root error function is deployed to determine the difference between the ideal and actual outputs for each bee. Table 4.2 presents the training and test results for ten separate runs of the Bees Algorithm. A typical plot of how classification accuracy evolves during training is shown in Fig. 4.4. For comparison, the average for the ten runs is given in Table 4.3 against the classification results for an MLP network trained using backpropagation (Pham and Oztemel, 1992).

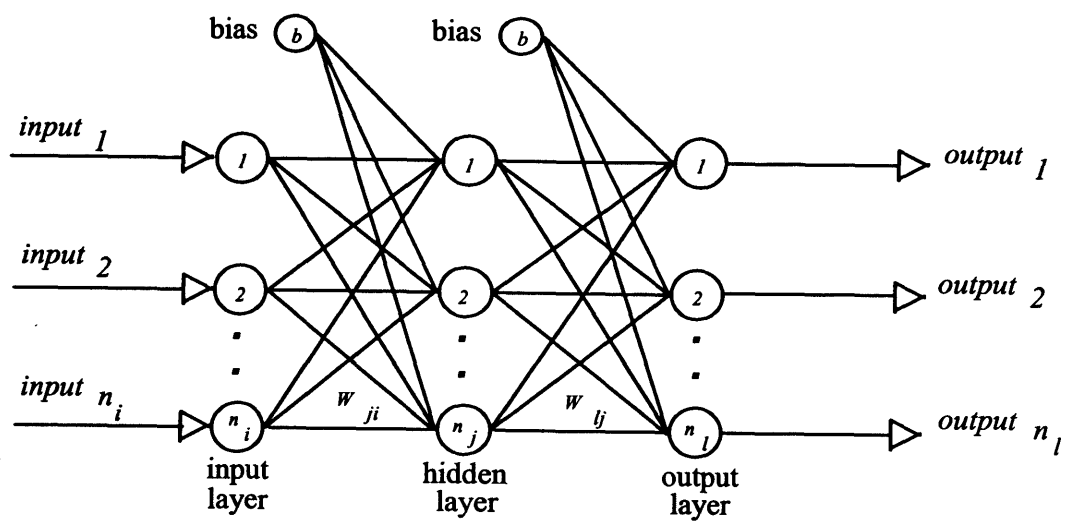


Figure 4.3. Structure of a multi-layered perceptron network

Table 4.1 The parameters of the Bees Algorithm for SPC-MLP weight training

Bees Algorithm parameters	Symbol	Value
Population	n	200
Number of selected sites	m	20
Number of elite sites	e	2
Initial patch size	ngh	0.1
Number bees around elite points	nep	50
Number of bees around other selected points	nsp	20
Shrinking constant	sc	0.01 (1%)
Site abandonment threshold	sat	50

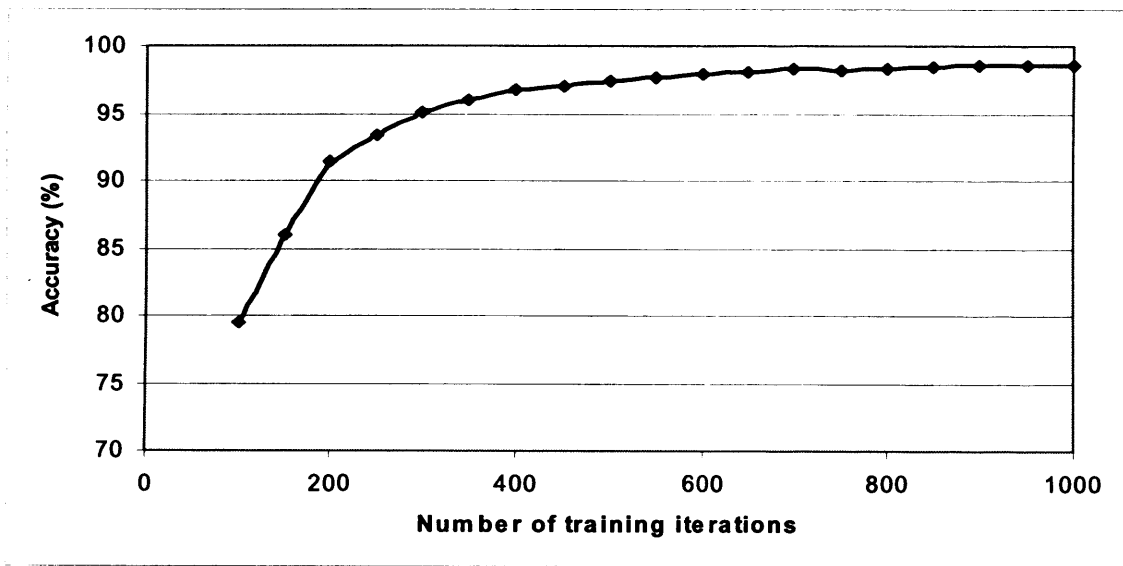


Figure 4.4 Performance of the system

Table 4.2. MLP classification results

Number of runs	Training accuracy	Test accuracy
1	98.59%	97.10%
2	98.59%	96.70%
3	98.39%	96.50%
4	98.85%	97.20%
5	98.19%	96.30%
6	96.39%	97.30%
7	97.78%	97.80%
8	98.99%	96.50%
9	98.91%	97.30%
10	97.79%	96.60%
Maximum	98.99%	97.80%
Minimum	97.78%	96.30%
Mean	98.25%	96.93%

Table 4.3 Results for different pattern recognisers

Pattern recogniser	Learning accuracy	Test accuracy
MLP (Backpropagation)	96.0%	95.2%
MLP (Bees Algorithm)	98.2%	96.9%

4.3. Optimising Neural Networks for Identification of Wood Defects

Using the Bees Algorithm

This section presents an application of the new algorithm to the problem of identifying defects in plywood veneer. An example of a sheet of wood veneer is shown in Fig. 4.5. It can be seen that the sheet contains several defects. These could create quality problems when the sheets are bonded together. Researchers have developed systems for automatically detecting and identifying defects in plywood veneer. Such systems generally involve the use of image processing techniques, feature extraction to capture the essential characteristics of all defects and a classifier to recognise these defects. The following sections provide an introduction to the algorithm and an explanation of the wood defect problem and the neural network used to identify the defects.

4.3.1. Wood veneer defects

In this study, using a charge-coupled device (CCD) matrix camera, the wood veneer defects were captured and stored on a digital computer. The wood veneer data acquisition rig is shown in Fig. 4.6. These images were converted into grey level histograms after applying segmentation and image processing algorithms. From the first and second order statistical features extracted from the histogram, 17 features were selected for training the neural network. These are shown in Table 4.4. Altogether 12 wood veneer defects and clear wood as shown in Fig. 4.5 were included in the examples used for training and testing the neural networks. Automated Visual Inspection (AVI) systems for identifying defects using neural networks have been

proposed by Pham and Alcock, (1996) and Packianather and Drake, (2005). The generic process for the visual inspection of wood defects is given in Fig. 4.7.

The wood panels are automatically moved to the image capture area by a conveyor belt. The system uses a Hamamatsu monochrome CCD matrix camera (resolution 739 x 575 pixels) to take images of the wood veneer. Uniform illumination is provided by a back light (58W fluorescent lamp) and front lighting system (halogen lamps: edges 500W and middle 300W). Basic image processing functions (e.g. thresholding and filtering) are implemented in hardware. Image segmentation algorithms are used to detect the boundaries of the sheet and open defects and defect detection algorithms are used to find potential defect areas.

For the particular application studied here, 232 examples (both defects and clear wood) were employed. This represents the complete set of examples available to the authors. Each example is a vector containing 17 features. Table 4.5 shows thirteen different classes of vectors and the number of examples in class. The initial classification of these examples had been performed by a human inspector. For subsequent neural network classification experiments, for each class, 80% (185 in total) of the examples were selected at random to form the training set and the remaining 20% (47 in total) formed the test set.

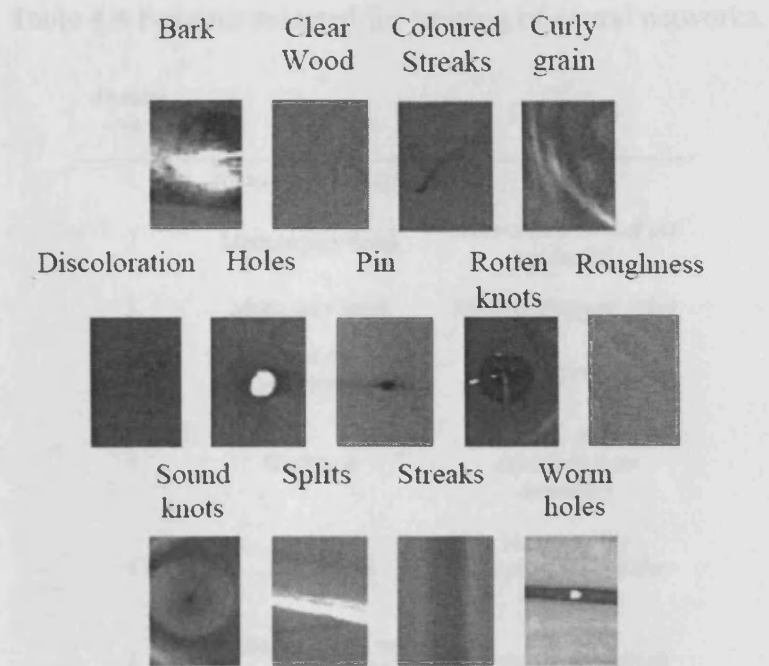


Figure 4.6 Wood veneer defect types. (There are 12 distinct types of defect that need to be identified by the neural network plus clear wood)

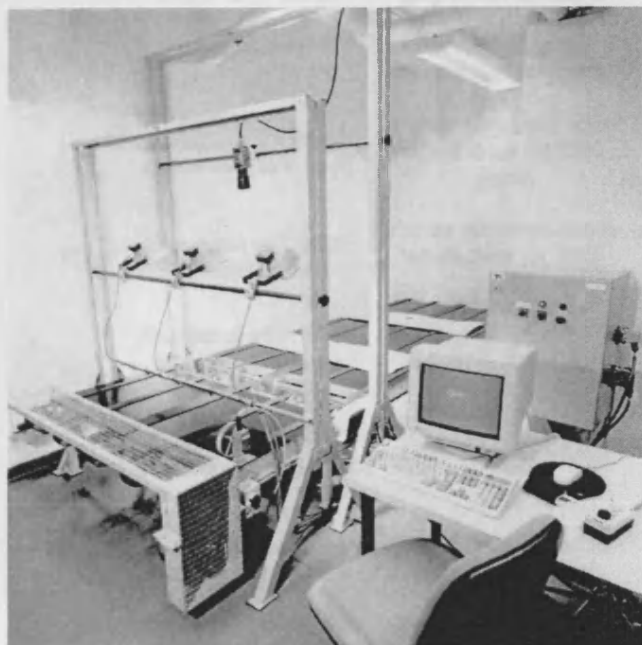


Figure 4.5 The inspection rig for wood defect detection.

Table 4.4 Features selected for training of neural networks.

Feature No.	Feature	Description
1	Mean grey level (μ)	
2	Median grey level	<i>below which 50% of the values fall</i>
3	Mode grey level	<i>the most frequent value</i>
4	Standard deviation of the grey levels (σ)	<i>the spread</i>
5	Skewness	<i>direction, extent of departure from symmetry</i>
6	Kurtosis	<i>measures the "peakedness" of the histogram</i>
7	Number of pixels with a grey level ≤ 80	<i>number of dark pixels</i>
8	Number of pixels with a grey level ≥ 220	<i>number of bright pixels</i>
9	Grey level (p) for which there are 20 pixels below	<i>lowest grey level - The grey level p is used as the lowest grey level to accommodate for potential noise pixels</i>
10	Grey level (s) for which there are 20 pixels above	<i>highest grey level - The grey level s is used as the highest grey level to accommodate for potential noise pixels</i>
11	Histogram tail length on the dark side (q-p)	<i>q is the grey level below which there are 2000 pixels</i>
12	Histogram tail length on the bright side (s-r)	<i>r is the grey level above which there are 2000 pixels</i>
13	Number of edge pixels after thresholding a segmented window at mean value	<i>defined to detect dark and bright defects</i>
14	Number of pixels after thresholding at $\mu - 2\sigma$	
15	The number of edge pixels for feature 14	<i>f14 and f15 defined to detect dark defects</i>
16	Number of pixels after thresholding at $\mu + 2\sigma$	
17	Calculate the number of edge pixels for feature 16	<i>f16 and f17 defined to detect bright defects</i>

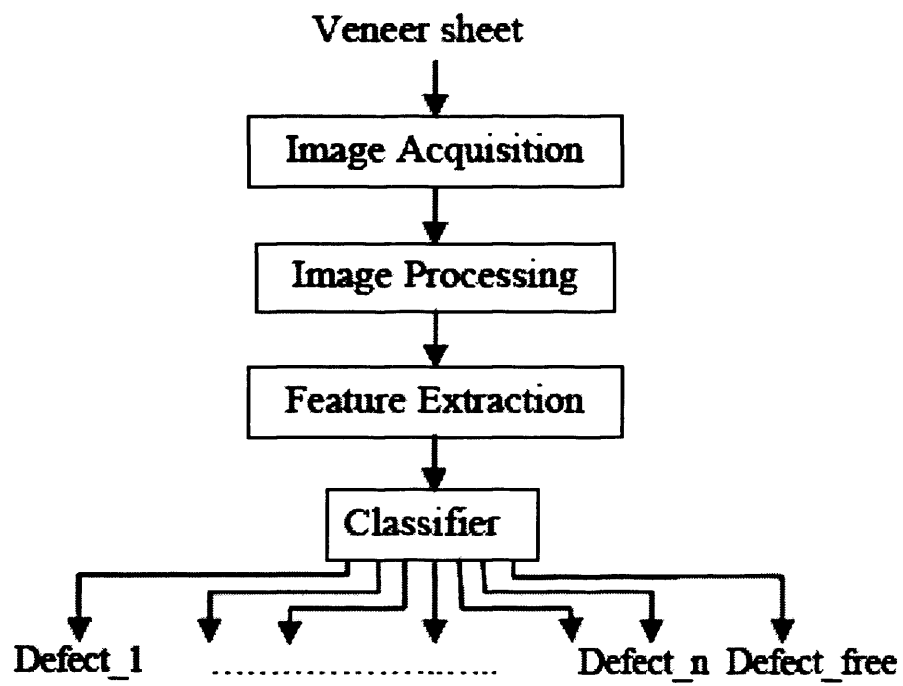


Figure 4.7 Generic automated visual inspection system for wood defect identification.

Table 4.5 Pattern classes and the number of examples used for training and testing.

Pattern Class	Total	Used for training	Used for Testing
Bark	20	16	4
Clear wood	20	16	4
Colored streaks	20	16	4
Curly grain	16	13	3
Discoloration	20	16	4
Holes	8	6	2
Pin knots	20	16	4
Rotten knots	20	16	4
Roughness	20	16	4
Sound knots	20	16	4
Splits	20	16	4
Streaks	20	16	4
Wormholes	20	16	4
Total	232	185	47

4.3.2. Neural networks and their optimisation

The architecture of a feedforward neural network with one hidden layer is shown in Fig. 4.8. Each layer is made up of processing elements called neurons. Every neuron has a number of inputs, each of which must store a connection weight to indicate the strength of the connection. Connections are initially made with random weights. The neuron sums the weighted inputs and computes a single output using an activation function. A number of different activation functions can be used. In this study, a hyperbolic tangent function is used in order to increase the difference between the outputs and further details are provided in (Packianather and Drake, 2005). Each neuron in a layer is fully connected to every neuron in the subsequent layer, forming a fully connected feedforward neural network. In a feedforward neural network, information flows from the input layer to the output layer without any feedback. There is one bias neuron for each hidden layer and the output layer, as illustrated in Fig. 4.8, and they are connected to each neuron in their respective layer. These connections are treated as weights. During the training process these weights are adjusted to achieve optimal accuracy and coverage.

The standard method for refining such a neural network is using an error backpropagation algorithm. During the training phase, the feedforward calculation is combined with backward error propagation to adjust the weights. The error term for a given pattern is the difference between the desired output and the actual output (the output from feed forward calculation). In this study, the Bees Algorithm is used to optimise the weights the neural network in place of the backpropagation. Feedforward neural networks have one hidden layer.

In this section, the Bees Algorithm is used to optimise the weights the neural network uses in place of the backpropagation algorithm described above and utilised in the work described in (Packianather and Drake, 2005). The algorithm follows the exact same procedure given in section 4.2.2 but with different configurations for the MLP.

In order to maintain comparability, the neural network structure (number of hidden layers) remained the same in both neural networks, with 17 input neurons, 51 hidden neurons and 13 output neurons. The optimisation using the Bees Algorithm will involve the “Bees” searching for the optimal values of the weights assigned to the connections between the neurons within the network, where each bee represents a neural network with a particular set of weights. The aim of the Bees Algorithm is to find the bee producing the smallest value of the error function.

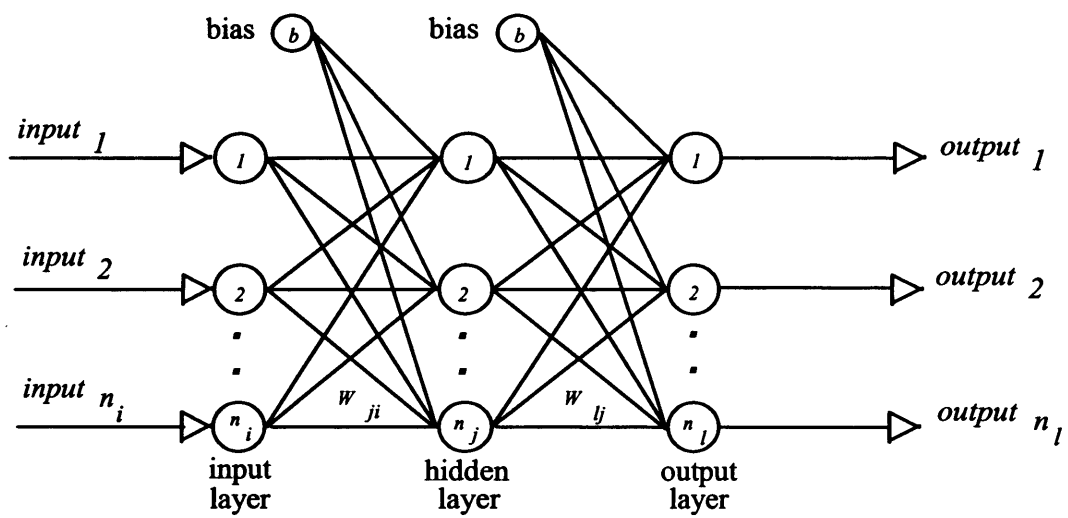


Figure 4.8 Feedforward neural network with one hidden layer.

4.3.3. Experimental results

The parameters used by the Bees Algorithm are given in Table 4.6. Table 4.7 shows the mean accuracies achieved in thirty experiments using the conventional backpropagation method (Packianather and Drake, 2005) with 17 features, the Bees Algorithm and the results using a non-neural network approach, in this case, a Minimum Distance Classifier (MDC).

The results show that the Bees Algorithm is able to achieve accuracies comparable to the backpropagation method. As the two mean accuracies are identical, this suggests that for this particular neural network structure and for the data set being used for training, accuracies in the region of 86% are the highest that can be achieved. The results given in Table 4.8 for data with 11 features show that for the reduced feature set the accuracy of both methods increases and the Bees Algorithm achieves better accuracy than the backpropagation method.

Table 4.6 Parameters of the bees algorithm for identification of wood defects

Bees Algorithm parameters	Symbol	Value
Population	n	100
Number of selected sites	m	20
Number of elite sites	e	1
Initial patch size	ng	0.1
Number bees around elite points	nep	50
Number of bees around other selected points	nsp	10
Shrinking constant	sc	0.01 (1%)
Site abandonment threshold	sat	50

Table 4.7 Results for defect identification – 17 features

Method	Mean Accuracy (%)
NN - Backpropagation	86.52
NN - Bees Algorithm	86.52
MDC (Non-NN)	63.12

Table 4.8 Results for defect identification – 11 features

Method	Mean Accuracy (%)
NN - Backpropagation	87.97
NN - Bees Algorithm	88.65

4.4. Design of a Two-dimensional Recursive Filter Using the Bees

Algorithm

Two-dimensional (2-D) digital filters have many applications in fields such as digital image processing, medical data processing, radar and sonar data processing, pattern recognition, robotics and mechanical engineering (Mladenov and Mastorakis, 2001; Mastorakis and Gonos, 2003). An overview of the area of 2-D digital filter design is given in (Kaczorek, 1985; Tzafestas, 1986)

It is essential that the designed filters are stable (Mladenov and Mastorakis, 2001). Several optimisation methods have been proposed for determining the values of the parameters of 2-D digital filters to produce stable designs. Due to the complex search spaces involved, the methods used tend to be search methods that look for near-optimal solutions in order to be able to complete the optimisation in finite times (Mladenov and Mastorakis, 2001; Mastorakis, Gonos, 2003).

The swarm-based algorithm described in this thesis is a search algorithm capable of locating good solutions efficiently. Fig. 3.8 shows the pseudo-code for the algorithm in detail. The algorithm requires a number of parameters to be set, namely: number of scout bees (n), number of sites selected for exploitation out of n visited sites (m), number of top-rated (elite) sites among the m selected sites (e), number of bees recruited for the best e sites (n_{ep}), number of bees recruited for the other ($m-e$) selected sites (n_{sp}), initial size of each patch (n_{gh} ; a patch is a region in search space that includes a visited site and its neighbourhood), shrinking constant (sc) and

stopping criterion. The algorithm adapted for this application aims to find an optimum design for two-dimensional digital filters.

4.4.1. Recursive filter design problem

The filter has the following transfer function (Mastorakis and Gonos, 2003):

$$H(z_1, z_2) = H_0 \frac{\sum_{i=0}^2 \sum_{j=0}^2 a_{ij} z_1^i z_2^j}{\prod_{k=1}^2 (1 + b_k z_1 + c_k z_2 + d_k z_1 z_2)}, \quad a_{00} = 1 \quad (4.6)$$

The desired amplitude response of the filter is (see also Fig. 4.10):

$$M_d(\omega_1, \omega_2) = \begin{cases} 1 & \text{if } \sqrt{\omega_1^2 + \omega_2^2} \leq 0.08\pi \\ 0.5 & \text{if } 0.08\pi < \sqrt{\omega_1^2 + \omega_2^2} < 0.12\pi \\ 0 & \text{otherwise} . \end{cases} \quad (4.7)$$

The aim is to minimise J,

$$J(a_{ij}, b_k, c_k, H_0) = \sum_{n_1=0}^{N_1} \sum_{n_2=0}^{N_2} \left[|M(\omega_1, \omega_2)| - |M_d(\omega_1, \omega_2)| \right]^2 \quad (4.8)$$

where

$$M(\omega_1, \omega_2) = H(z_1, z_2) \left| \begin{array}{l} z_1 = e^{-j\omega_1} \\ z_2 = e^{-j\omega_2} \end{array} \right. \quad (4.9)$$

For this design problem, let $\omega_1 = (\pi/50)n_1$, $\omega_2 = (\pi/50)n_2$ and $n_1=50$ and $n_2=50$. Function J becomes:

$$J = \sum_{n_1=0}^{50} \sum_{n_2=0}^{50} \left[\left| M\left(\left(\frac{\pi n_1}{50}\right), \left(\frac{\pi n_2}{50}\right)\right) \right| - \left| M_d\left(\left(\frac{\pi n_1}{50}\right), \left(\frac{\pi n_2}{50}\right)\right) \right| \right]^2 \quad (4.10)$$

The design constraints are given as (Mastorakis and Gonos, 2003):

$$-(1 + d_k) < (b_k + c_k) < (1 + d_k)$$

$$-(1 - d_k) < (b_k - c_k) < (1 - d_k)$$

$$(1 + d_k) > 0 \quad (4.11)$$

$$(1 - d_k) > 0$$

where $k = 1, 2$.

In this problem, the aim is to determine vector $x = (a_{01}, a_{02}, a_{10}, a_{20}, a_{11}, a_{12}, a_{21}, a_{22}, b_1, b_2, c_1, c_2, d_1, d_2, H_0)^T$ of unknown parameters to minimise J , subject to the design constraints.

4.4.2. Experimental results

The pseudo-code of the Bees Algorithm used for this experiment is given in Fig. 3.8 in the previous chapter. The following parameter values for the Bees Algorithm were set empirically for this exercise: the scout bee population $n = 1000$, number of selected sites $m = 20$, number of elite sites $e = 2$, initial patch size $ngh = 0.1$, number of bees around elite points $nep = 50$, number of bees around other selected points $nsp = 20$ and shrinking constant $sc = 1\%$ and site abandonment threshold $sat = 50$ iterations.

The scout bee population (n) is usually in the range 100 to 1500 bees. Because of the complexity of this problem, n was set to a high value. Number of selected sites is one of the most important parameter, especially if there are many local optima, so it was set to a relatively high number. According to experimental studies on the Bees Algorithm, number of elite sites (e) usually does not need to set carefully, thus it was set to a small value. Initial patch size (ngh) was set as 0.1 and shrank down to 0.01 after several iterations due to the value of the shrinking constant.

The Bees Algorithm was run for 10,000 iterations to find a minimum value for the function (5) with the settings given for the algorithm. The performance of the algorithm for a two-dimensional (2D) recursive digital filter design is presented in Fig. 4.9.

The optimal vector x obtained using the Bees Algorithm for 10,000 iterations is:

$$x = (-0.3057, -1.1113, 0.2214, 0.2410, -0.9693, 2.0700, -1.2764, 0.5772, -0.9303, -0.0986, -0.9190, 0.0214, 0.8711, -0.6785, 0.00007)^T.$$

Thus, the optimum filter transfer function is (Mastorakis and Gonos, 2003):

$$\begin{aligned} H(z_1, z_2) = & \left(\frac{0.00007}{(1 - 0.9303z_1 - 0.9190z_2 + 0.8711z_1z_2)} \right) \\ & * \left(\frac{1}{(1 - 0.0986z_1 + 0.0214z_2 - 0.6785z_1z_2)} \right) \\ & * (1 + 0.2214z_1 + 0.2410z_1^2 - 0.9693z_1z_2 + 2.0700z_1z_2^2 \\ & - 0.3057z_2 - 1.2764z_1^2z_2 + 0.5772z_1^2z_2^2 - 1.1113z_2^2) \end{aligned} \quad (4.12)$$

The amplitude response $|M(\omega_1, \omega_2)|$ of the obtained filter is shown in Fig. 4.11. For comparison, Fig. 4.12 presents the amplitude response $|M(\omega_1, \omega_2)|$ of a filter optimised using a GA (Mastorakis and Gonos, 2003).

Table 4.9 Parameters of the bees algorithm for 2-d recursive filter design

Bees Algorithm parameters	Symbol	Value
Population	n	1000
Number of selected sites	m	20
Number of elite sites	e	2
Initial patch size	ngh	0.1
Number bees around elite points	nep	50
Number of bees around other selected points	nsp	10
Shrinking constant	sc	0.01 (1%)
Site abandonment threshold	sat	50

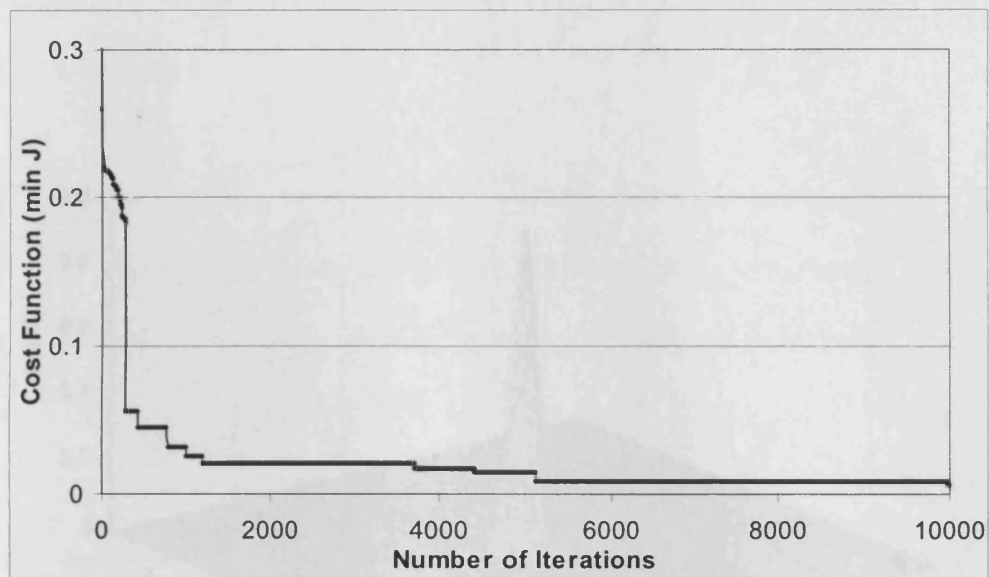


Figure 4.9 Performance of the Bees Algorithm.

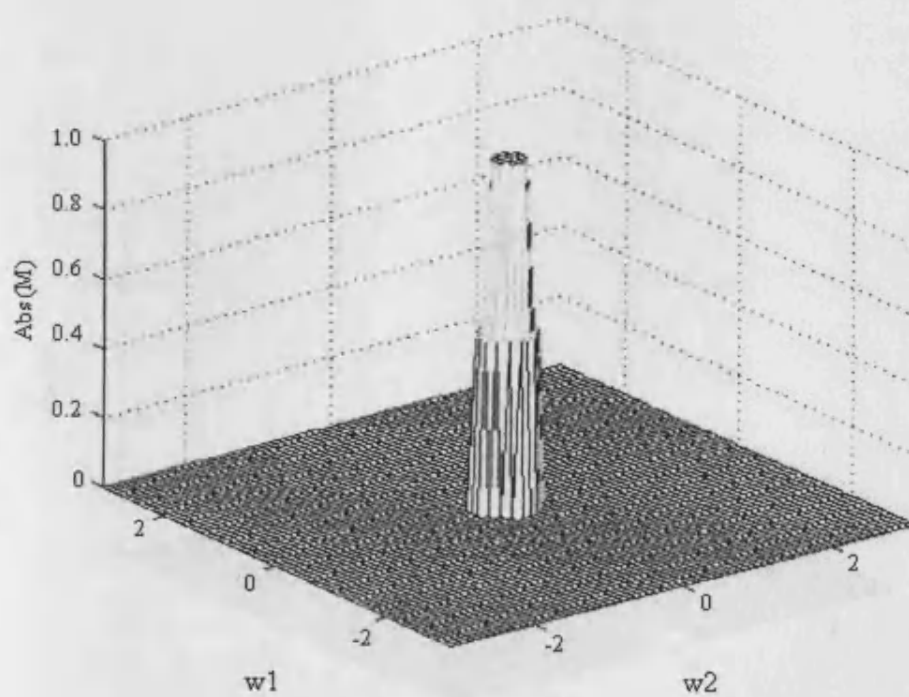


Figure 4.10 Desired amplitude response $|M_d(\omega_1, \omega_2)|$ of the 2-D filter (Mastorakis and Gonos, 2003).

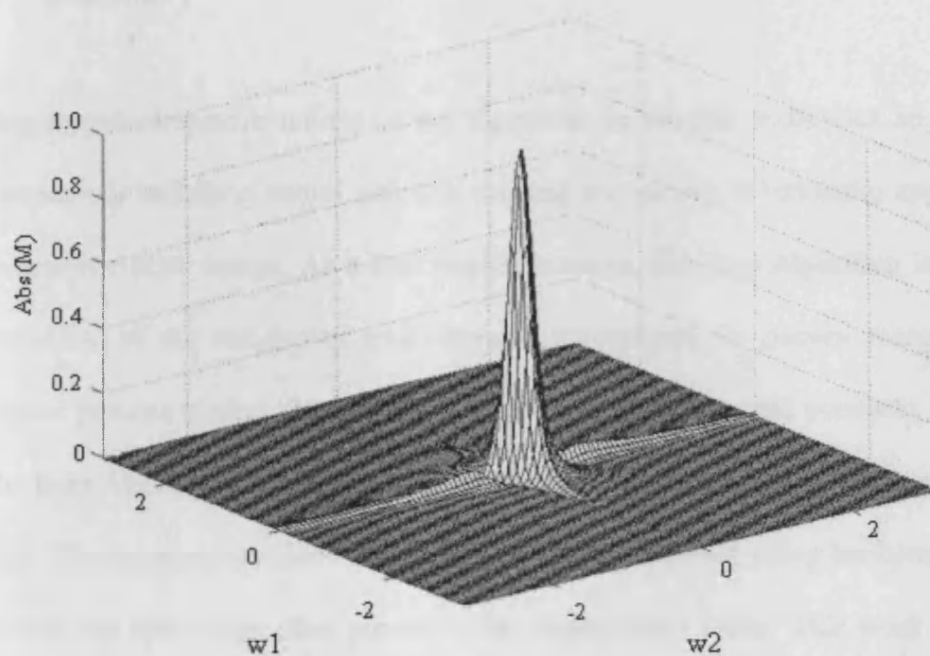


Figure 4.11 Amplitude response $|M(\omega_1, \omega_2)|$ obtained using the Bees Algorithm.

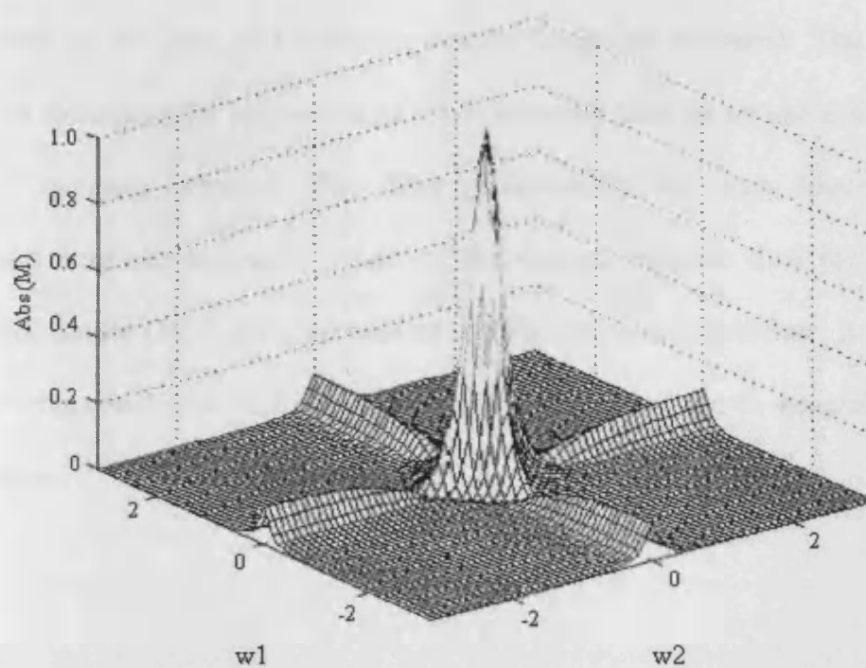


Figure 4.12 Amplitude response $|M(\omega_1, \omega_2)|$ obtained using a GA (Mastorakis and Gonos, 2003).

4.5. Summary

In this chapter, implementations of the algorithm on several continuous applications are presented, including neural network training for variety of industrial applications and recursive filter design. As a first implementation, the Bees Algorithm is used for optimisation of the weights of multi-layered perceptrons for pattern recognition in statistical process control charts. And a similar structure of neural networks is trained by the Bees Algorithm for identification of defects in wood veneer sheets in a plywood factory. The accuracy obtained is comparable to that achieved using backpropagation. However, the Bees Algorithm proved to be considerably faster. This work therefore confirms the usefulness of the algorithm as an optimisation tool, particularly when considering that it has produced even higher accuracies than backpropagation in other applications. Finally in this chapter, the first of many potential applications of the Bees Algorithm in the area of electronics circuit design is presented. The application involved optimising the parameters of a 2-D recursive filter to try and achieve a given desired frequency response. The filter produced by the Bees Algorithm has a frequency response noticeably closer to the desired response than that by a filter designed using a GA. Taking account of the No Free Lunch principle, it is important to limit this conclusion to the actual tests conducted and not to generalize it to all conditions.

Chapter 5

BEES ALGORITHM FOR COMBINATORIAL DOMAINS

5.1. Preliminaries

Combinatorial optimisation problems have attracted much attention over the years. Many of them are NP-hard (Garey and Johnson, 1979; Aarts E and Lenstra, 1997). It is generally believed that NP-hard problems cannot be solved to optimality within polynomially bounded computation times. Several algorithms that can find near-optimal solutions within reasonable running times have been developed. A population-based algorithm is one example.

In this chapter, the Bees Algorithm is presented for combinatorial domains. The algorithm mimics the food foraging behaviour of swarms of honey bees. In its basic

version, the algorithm performs a kind of neighbourhood search combined with random search and can be used for both combinatorial optimisation and functional optimisation. The functional optimisation application is discussed in previous chapters where it is proven that the algorithm works well in continuous domains. This is mainly due to its balanced local and global search architecture. However, combinatorial domains need a completely different approach when it comes to a mathematical definition of the distance. This raises many other challenges for an algorithm which was originally developed to work in continuous domains.

The neighbourhood concept defined in combinatorial domains is completely different of those defined in continuous domains. One of the aims of this chapter is to define a new neighbourhood structure which can be functional for local search. To be able to achieve that, several local search algorithms combined with the Bees Algorithm and the best combinations used in several applications are presented in this chapter.

The chapter is organised as follows: section 5.2 presents a description the Bees Algorithm for discrete problems including local and global search strategies used for the algorithm. A Bees Algorithm is presented for scheduling jobs for a machine and the results are presented and discussed in section 5.3. In section 5.4, a permutation flowshop sequencing problem is studied in many job/machine combinations using the Bees Algorithm and computational results compared with some other well-known algorithms. In section 5.5, an application of the Bees Algorithm to form machine-part cells is presented. A general summary of the proposed method is given and possible future research are analysed in section 5.6.

5.2. A proposed Bees Algorithm for the combinatorial domain

In this section, details of the Bees Algorithm for combinatorial domains are presented. The Bees Algorithm basically consists of two parts: neighbourhood search and global search. The pseudo-code of the Bees Algorithm for combinatorial domains is given in Fig. 5.2. In essence, the algorithm is very similar to those presented in chapter 3. The main differences here are: in step 5, the patch idea is replaced by a local search operator to be able to perform a local search and the, shrinking procedure is also removed from the algorithm. However, the abandonment procedure is kept to help the algorithm to improve the global search part. Improvements and modifications are discussed in the following sections.

5.2.1. Neighbourhood search strategies

As mentioned, the main feature of combinatorial domains, unlike continuous domains, is that there is no mathematical distance definition for the neighbourhood search. Since the Bees Algorithm was developed for continuous domains, it is necessary to modify the neighbourhood part by simply replacing the patch with a local search operator.

There are several exchange neighbourhood strategies and local search algorithms in the literature. Swap operators (simple, double, insert ect.) are considered as exchange neighbourhood strategies (Aarts and Lenstra, 1997). They simply change the position of a randomly selected city to create an altered path. By contrast, 2-Opt and 3-Opt are simple local search algorithms that delete two or three edges, thus breaking the tour into two paths and then reconnecting those paths later.

```

1. Initial population with n random solution; random(Sequence(n)).
2. Evaluate fitness of the population.
3. While (stopping criterion not met)
4. Select sites (m) for neighbourhood search.
5. Recruit bees for selected sites (more bees for best e sites), evaluate fitnesses, select the fittest
   bee from each site and shrink patches
   for (k=1 ; k=e ; k++) // Elite Sites
       for (i=1 ; i= nep ; i++) // More Bees for Elite Sites
           RecruitedBee(k)(i) = NeighbourhoodOperator(Sequence(k));
           Evaluate Fitness = RecruitedBee(k)(i); //Evaluate the fitnesses of recruited Bee(i)
           If (Bee(i) is better than Bee(i-1)) RepresentativeBee = RecruitedBee(k)(i);
   for (k=e ; k=m ; k++) // Other selected sites (m-e)
       for (Bee=1 ; Bee= nsp ; Bee++) // Less Bees for Other Selected Sites (m-e)
           RecruitedBee(k)(i) = NghOperator(Sequence(k));
           Evaluate Fitness = RecruitedBee(k)(i); //Evaluate the fitnesses of recruited Bee(i)
           If (Bee(i) is better than Bee(i-1)) RepresentativeBee = RecruitedBee(k)(i);
6. If (Iteration > sat)
   If (no improvement on the site)
       Save the Best Fitness;
       Abandon the Site;
       Bee(m) = GenerateRandomValue(All Search Space);
7. Assign remaining bees to search randomly and evaluate their fitnesses. // (n-m) assigned to
   search randomly into whole solution space
8. End While

```

Figure 5.1 Pseudo-code of the Bees Algorithm for combinatorial domains.

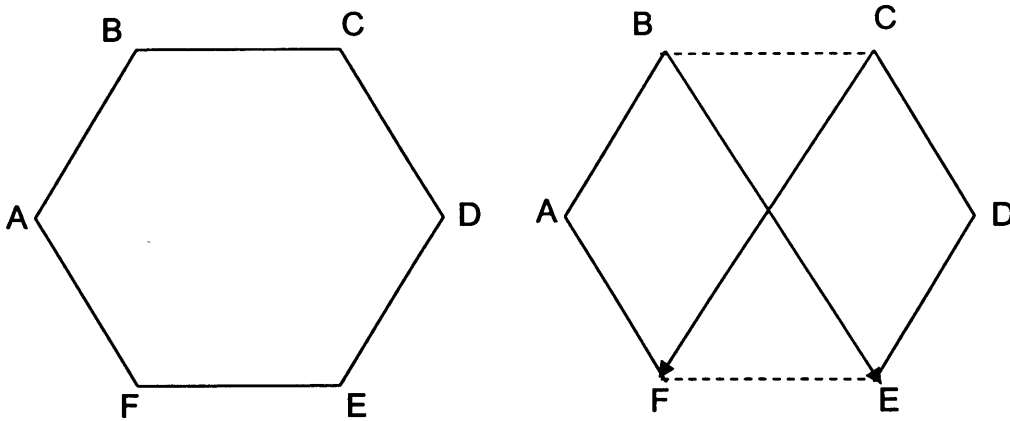


Figure 5.2 2-opt operator (a) Original tour (A, B, C, D, E, F, A),

(b) Tour after 2-opt (A, B, E, D, C, F, A)

There are also the k-Opt ($k > 3$) approach, which is basically a cut of several points and reconnect strategy. Tabu search, simulated annealing and GA other well-known local search operators in the literature (Aarts and Lenstra, 1997). In this study, only the exchange and 2/3-Opt operators are used to modify the Bees Algorithm.

2-opt and 3-opt heuristics are frequently applied to problems that deal with combinatorial domains. They are a simple and efficient local search method which finds near optimal solutions. Fig. 5.2(a) shows the original tour and Fig. 5.2(b) the tour after the 2-opt procedure (Aarts and Lenstra, 1997). It is based on eliminating two (or three) arcs in R in order to obtain two different paths. These eliminated two or three paths are then reconnected in the only other possible way. Let us consider a feasible solution, R, with the permutation of A, B, C, D, E, F, A as shown in Fig. 5.2(a). Two arcs are randomly picked i.e. (B, C) and (E, F) then the path between them is

eliminated to create two separate paths, B, A, F and E, D, C. In the next step these two separate tours are reconnected as A, B, E, D, C, F, A. 3-opt adopts the same procedure but in this version three arcs are randomly picked to create a new tour.

Fig. 5.3 shows the performance of the Bees Algorithm with several local search operators including simple (2 point) swap, double (4 point) swap, insert, 3 point swap, 2-Opt and 3-Opt. These strategies are adopted in this chapter as a new neighbourhood search method instead of the original patch idea. One or two procedures are used depending on the complexity of the problem. Further details are given in the experiment sections.

5.2.2. Random search and site abandonment

In the first step, all scout bees (n) are represented by a randomly created sequence (1,2,...,n) across the combinatorial domain to explore for new flower patches.

Site abandonment is introduced to improve the efficiency of the local search. In the case of combinatorial (NP-hard) problems, the algorithm is very likely to get trapped in local optima. In step 6 (see Fig. 5.1), if the points visited near a selected site are all inferior to that site, after a certain number of iterations (i.e. sat: site abandonment threshold), then the location of the site is recorded and the site abandoned. Bees at the site are assigned to random search (i.e. made to scout for new potential solutions). After neighbourhood search, in step 7 $n-m$ bees are again placed randomly across the combinatorial domain to explore for new patches.

5.3. Using the Bees Algorithm to schedule jobs for a machine

Now scheduling for a machine with a fixed due date involves finding an optimal schedule that minimises the number of tardy jobs. Each job has a due date, a release time and a processing time. The due date is the time by which the job must be completed. The release time is the time at which the job becomes available for processing. The processing time is the time required to complete the job. The objective is to find a schedule that minimises the number of tardy jobs. The Bees Algorithm is a metaheuristic algorithm that can be used to solve this problem. It is based on the foraging behaviour of bees. The algorithm starts with a population of bees. Each bee represents a schedule. The fitness of a bee is the number of tardy jobs in its schedule. The algorithm iterates until a stopping criterion is reached. In each iteration, the fitness of each bee is calculated. The best bee is selected. The other bees are updated based on the best bee. The algorithm is simple and easy to implement. It can be used to solve a wide range of scheduling problems.

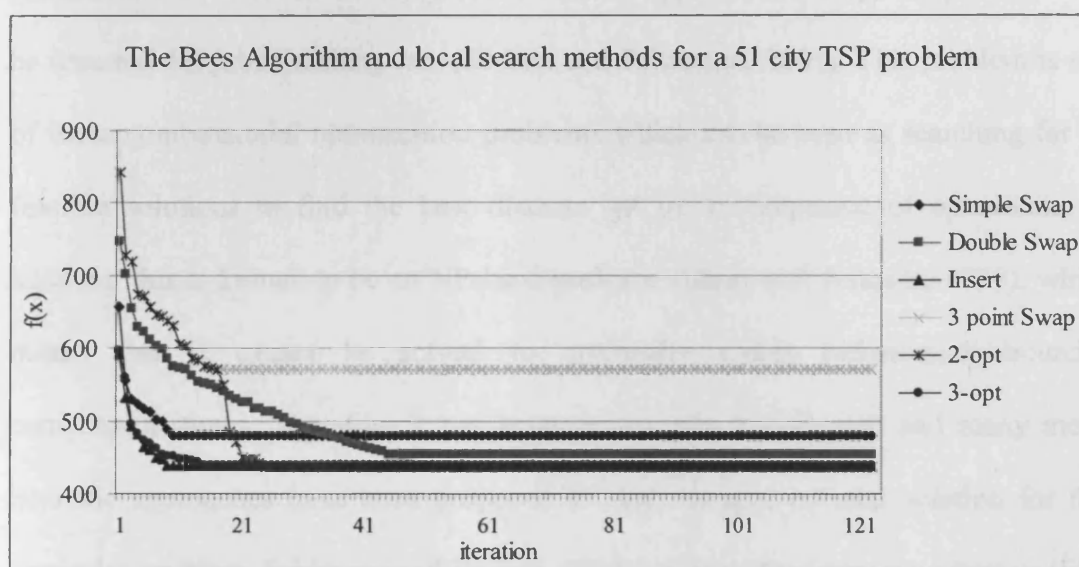


Figure 5.3 Performance of the Bees Algorithm with different local search methods.

5.3. Using the Bees Algorithm to schedule jobs for a machine

Job scheduling for a machine with a firm due date involves finding an optimal schedule that minimise the sum of early and late penalties. Each job has a different processing time and also early and late penalties in respect of the due date. All jobs must be completed before the due date. However, only one job can be completed on the exact due date and the others can be completed either before or after.. If some jobs finish before the due date, early penalties will be applied. Similarly, late penalties will be incurred for jobs finishing late (Biskup and Feldmann, 2001). This problem is one of those combinatorial optimization problems which can be seen as searching for all feasible solutions to find the best discrete set of the sequence of operations. In addition, this is known to be an NP-hard problem (Garey and Johnson, 1979), which means that it cannot be solved to optimality within polynomially-bounded computation times. Therefore it has been extensively investigated and many meta-heuristic approaches have been proposed to find the near optimal solution for this particular problem. Feldmann and Biskup, (2003) applied Evolutionary Strategy (ES), Simulated Annealing (SA) and Threshold Accepting (TA). Hino et al., (2005) applied Genetic Algorithm (GA) and Tabu Search (TS) to the same problem. Hino et al also proposed hybrid meta-heuristics such as HTG (Tabu search + Genetic Algorithm) and HGT (Genetic Algorithm + Tabu Search). Recently, population-based algorithms have been applied to this problem. Nearchou applied Differential Evolution (DE) (Nearchou, 2006) and Pan et al applied Discrete Particle Swarm Optimisation (DPSO) Algorithm (Pan et al., 2006). More recently, Lee et al applied Ant Colony Optimisation (ACO) (2007) but they only tested it on 200 jobs, while the above researchers tested on from 10 to 1000 jobs.

5.3.1. Single machine scheduling problem

Biskup and Feldmann, (2001) developed single machine scheduling benchmarks and also proposed two new heuristics to solve this problem. The characteristics of this benchmark set are explained in the following section. The survey revealed that many approaches have been applied to solve this data set recently.

In this problem, a number of jobs will be processed without interruption on a single machine. All jobs are available at time zero, each of which has its own processing time (p_j) and needs exactly one operation. If the completion time (C_j) of job j is smaller than or equal to the due date (d), the job's earliness is $E_j = d - C_j$. If it is greater than the due date, the job's tardiness is $T_j = C_j - d$. The goal of this problem is to find a sequence S of n jobs that minimises the total of the earliness and tardiness penalties:

$$f(S) = \sum_{j=1}^n (\alpha_j \cdot E_j + \beta_j \cdot T_j) \quad (5.1)$$

where α_j and β_j are the earliness and tardiness penalties per time unit respectively.

Three well-known properties (Biskup and Feldmann, 2001; Feldmann and Biskup, 2003) which are essential for an optimal schedule are as follows:

There are no idle times between consecutive jobs; a general proof is given by Cheng and Kahlbacher, (1991).

An optimal schedule has the so-called V-shape property, that is, jobs finished before the due date are ordered according to non-increasing ratios p_j/α_j and jobs finished after

the due date are ordered according to non-decreasing ratios p_j/β_j ; the proof can be made by the interchange argument, see, for example (Baker and Scudder, 1990).

There is an optimal schedule in which either the processing time of the first job starts at time zero or one job is finished at the due date; the proof is similar to that of Hooogeveen and van de Velde, (1991).

All potential optimal schedules can be divided into three cases:

- 1) The first job starts at time zero and the last early job is finished exactly at time d ,
- 2) The first job starts at time zero and the last early job is finished before d . Here a straddling job exists and
- 3) The first job does not necessarily start at time zero.

5.3.2. The Bees Algorithm for single machine scheduling problem

In this study, a solution set for the jobs scheduling problem consists of both continuous and combinatorial domains as shown in Fig. 5.4. Due to this characteristic the Bees Algorithm requires modification.

Fig. 5.1 presents the pseudo-code of the Bees Algorithm for a single machine scheduling problem. During initialization, the idle time is randomly generated in the

continuous domain and the jobs sequence in the combinatorial domain respectively. During the conduct of a neighbourhood search in the continuous domain for the idle time, the basic Bees Algorithm was adopted.. On the other hand, the combinatorial domain for establishing jobs sequences requires the other proper neighbourhood search. In Fig. 5.5, the neighbourhood operators used for the single machine scheduling problem is presented. Fig. 5.6 presents the neighbourhood search methods which were adopted in this work.

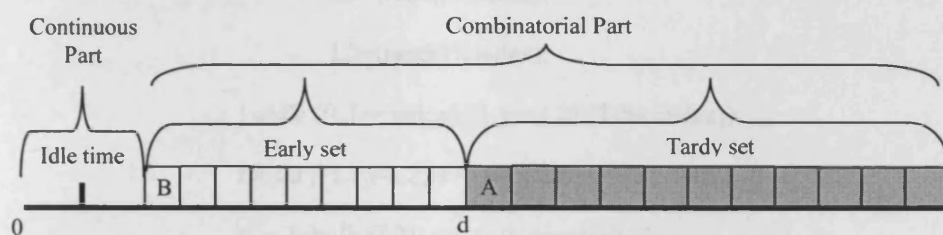


Figure 5.4 Illustration of the solution set

NeighbourhoodOperator (Sequence(k));

//Simple Swap operator

```
do{  
    L1=(rand()%index);  
    L2=(rand()%index);  
} while ((L1==index)||((L1==L2)||((L2==index)));  
t1 = Sequence [k][L1];  
Sequence [k][L1] = Sequence [k][L2];  
Sequence [k][L2] = t1;
```

//Insert Operator

```
do{  
    L1=(rand()%index);  
    L2=(rand()%index);  
} while ((L1==index)||((L1==L2)||((L2==index)));  
for(int j=L1;j<L2;j++) temparray1[j]=Tabu[k][j];  
t1 = Tabu[k][L2]; temp=0; temp=L1+1;  
for(j=L1;j<L2;j++){  
    Tabu[k][temp]=temparray1[j];  
    Temp=temp+1;}  
Tabu[k][L1]=t1;  
temp=0;
```

Figure 5.5 Neighbourhood operators for single machine scheduling problem

Neighbourhood search is an important element of all optimisation algorithms. The Bees Algorithm is no exception. It was originally developed for use in continuous domains, and proved to work well compared to other optimisation algorithms. For the first part of the problem investigated in this section, the original Bees Algorithm has been proposed without any modification. On the other hand, as mentioned in the previous chapter, combinatorial domains need a completely different approach when it comes to a mathematical definition of the distance. This raises many other challenges for an algorithm which was originally developed to function in the continuous domain.

In the neighbourhood search step, the Bees Algorithm benefits from some of the original ideas to explore towards a good solution. However, because of the issue of the definition of the distance, it is not possible to use the idea of the patch size in this study. Instead, combinations of several methods have been deployed to perform the neighbourhood search. Simple-swap and insert methods have been selected from the literature because of the partitioned structure of the problem (see Fig. 5.5). Given the equal chance to be chosen, each of these methods has performed for each recruited bee (nep and nsp) for every iteration.

Simple-swap is a well known local search method for combinatorial problems (Aarts and Lenstra, 1997). Because of the partitioned structure of the problem domain, the simple-swap method has been slightly modified for this implementation. Swapping can only occur between early and tardy sets (see Fig 5.7(b)). This means that it is unnecessary to perform any changes in the sets because of the early and tardy time evaluations. The insert method is another efficient means of creating new solutions in combinatorial domains. It is similar to simple-swap, but insertion does not work vice

versa. A randomly picked job order is simply inserted in a randomly defined position between job orders. It has also been slightly modified for this problem. One job order can only be inserted into a position in another set (see Fig. 5.7(c)). In this way, redundant evaluations have been reduced significantly.

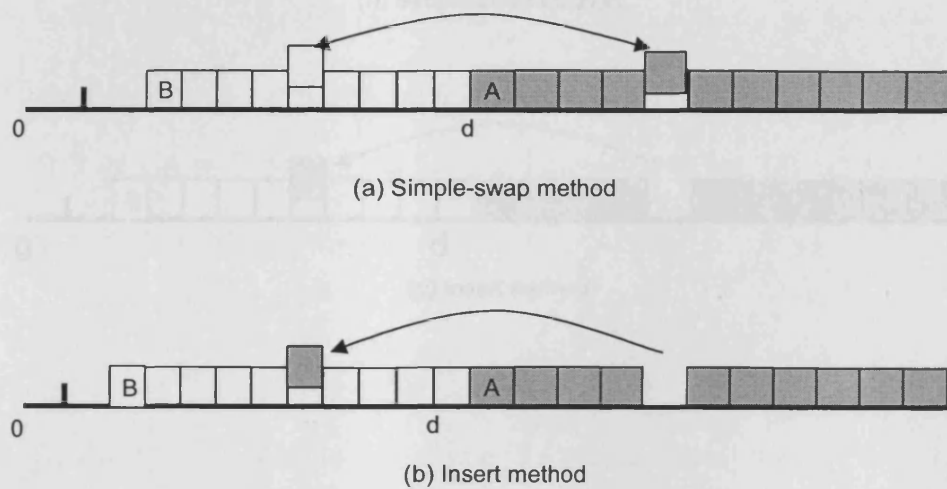
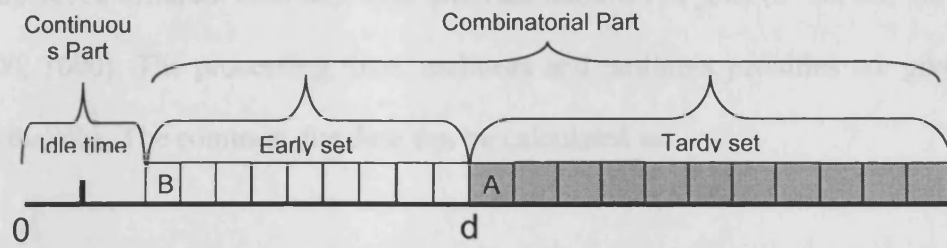


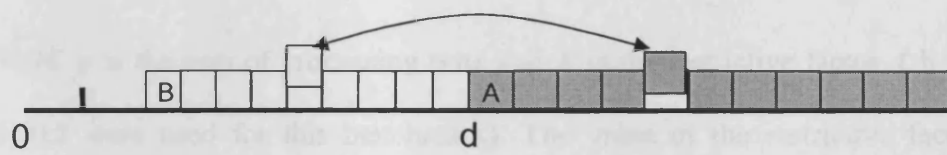
Figure 5.6 Illustration of the neighbourhood search methods

5.3.3. Experimental results

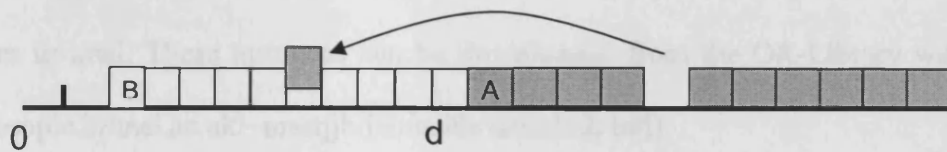
There are seven different data sets with different numbers of jobs ($n = 10, 20, 30, 40, 50$).



(a) Solution Set



(b) Simple-swap method



(c) Insert method

Figure 5.7 Solution set and neighbourhood methods

5.3.3. Experimental results

There are seven different data sets with different numbers of jobs (n=10, 20, 50, 100, 200, 500, 1000). The processing time, earliness and tardiness penalties are given to each of the jobs. The common due date can be calculated as:

$$\text{Common due date (d)} = \text{round} [\text{SUM_p} * h] \quad (5.2)$$

where SUM_p is the sum of processing time and h is the restrictive factor ($h=0.2, 0.4, 0.6, 0.8$ were used for this benchmark). The value of the restrictive factor h classifies the problems as less or more restricted against a common due date. Each data set contains 10 instances (from $K=1$ to $K=10$). Therefore the problem has 280 instances in total. These instances can be downloaded from the OR-Library website (<http://people.brunel.ac.uk/~mastjjb/jeb/orlib/schinfo.html>).

The performance of the algorithm was quantified by two indices: 1) percentage of relative deviations (Δ), 2) standard deviation. To obtain the average performance of the algorithm, 10 runs were carried out for each problem instance to report the statistics based on the percentage of relative deviations (Δ) from the upper bounds in Biskup and Feldmann, (2003). To be more specific, Δ_{avg} was computed as follows:

$$\Delta_{avg} = \frac{\sum_{i=1}^R \left(\frac{(F_{BA} - F_{ref})}{F_{ref}} \times 100 \right)}{R} \quad (5.3)$$

where, F_{BA} , F_{ref} and R are the fitness function values generated by the Bees Algorithm in each run, the reference fitness function value generated by Biskup and Feldmann,

(2003), and the total number of runs, respectively. For convenience, $\Delta_{\min}$, $\Delta_{\max}$ and Δ_{std} denote the minimum, maximum and standard deviation of the percentage of relative deviation in fitness function value over R runs, respectively. Table 5.1 shows the parameter values used for this experiment.

The results obtained by the Bees Algorithm were compared with the results from (Biskup and Feldmann, 2003; Feldmann and Biskup, 2003; Hino et al., 2005; Pan et al., 2006; Nearchou, 2006). Note that in Biskup and Feldmann (2003), the average percentage improvements and their standard deviations are given using the best solution among all the heuristics; namely evolution search (ES), simulated annealing (SA), threshold accepting (TA) and TA with a back step (TAR). Since the Bees Algorithm is stochastic, its minimum, maximum, average and standard deviation of runs should be given to evaluate its performance. However, Hino et al., (2005) conducted 10 runs and selected the best out of 10 runs even updating the idle time. For this reason, the minimum percentage of relative deviation ($\Delta_{\min}$) of the Bees Algorithm was compared to Hino et al., (2005) and Pan et al. (2006). Note that the best results so far in the literature are reported in bold in all tables given in this section.

Table 5.2 summarises $\Delta_{\min}$ of the computational results to be compared to Hino et al., (2005) and Pan et al. (2006) with regard to h . As seen in Table 5.2 ($h = 0.2$ and $h = 0.4$), there is not a large difference, but for $h = 0.6$ and $h = 0.8$ there is a great deal of difference, especially with the larger size problems (ranging from 100 to 1000 jobs). The Bees Algorithm, discrete particle swarm optimisation (DPSO) and GA have a similar tendency to yield a negative percentage of relative deviations ($\Delta_{\min}$), which means that they overperform (Biskup and Feldmann 2003). However, Tabu Search

(TS), HTG (TS+GA) and HGT (GA+TS) show a tendency to diverge after 100 jobs and give positive percentage of relative deviations ($\Delta_{\min}$), which means they are inferior to Biskup and Feldmann, (2003).

Table 5.3 shows maximum percentage of relative deviations ($\Delta_{\max}$) between the Bees Algorithm and DPSO with regard to h . When h is 0.2, 0.4 and 0.6, $\Delta_{\max}$ of the Bees Algorithm is superior to the DPSO and the total average is also much better than the DPSO. In particular, the Bees Algorithm is superior to the DPSO, when h is 0.6. It is also interesting to note that, as seen in Pan et al. (2006), even the average of maximum percentage of relative deviation ($\Delta_{\max}$) of the Bees Algorithm is much better than $\Delta_{\min}$ of TS, GA, HTG and HGT.

Table 5.4 shows comparative results for the Bees Algorithm and DPSO in terms of minimum, maximum and average percentage of relative deviations and standard deviations. The average percentage of relative deviation (Δ_{avg}) of the Bees Algorithm was compared to the DPSO (Pan et al., 2006) and differential evolution (DE) (Nearchou, 2006). It was found that the Bees Algorithm outperforms these two algorithms. As seen from the total averages in Table 5.4, the Bees Algorithm is slightly better than the DPSO at -2.15. For 200, 500 and 1,000 jobs, when h equals 0.6 or 0.8, the Bees Algorithm and DPSO performs better than the DE. As can be seen, the standard deviation for the Bees Algorithm is nearly zero, which means that it is slightly more robust than DPSO. All the statistics obtained show that the performance of the Bees Algorithm is superior to all other existing approaches.

Table 5.1 The parameters of the Bees Algorithm

Parameters	Value
p : Population	$2n^*$
m : Number of selected sites	200
e : Number of elite sites	100
ngh : Initial patch size	6
nep : Number of bees around elite points	50
nsp : Number of bees around other selected points	30
Sat: Site abandonment threshold	50

* : When the number of jobs n is less than 100, $p = 2n$. Otherwise, $p = 400$.

Table 5.2 Minimum deviation of the computational results

n/h	0.2					
	DPSO	TS	GA	HTG	HGT	Bees Algorithm
10	0.00	0.25	0.12	0.12	0.12	0.00
20	-3.84	-3.84	-3.84	-3.84	-3.84	-3.84
50	-5.70	-5.70	-5.68	-5.70	-5.70	-5.70
100	-6.19	-6.19	-6.17	-6.19	-6.19	-6.19
200	-5.78	-5.76	-5.74	-5.76	-5.76	-5.78
500	-6.42	-6.41	-6.41	-6.41	-6.41	-6.43
1,000	-6.76	-6.73	-6.75	-6.74	-6.74	-6.76
AVG	-4.96	-4.91	-4.92	-4.93	-4.93	-4.96

n/h	0.4					
	DPSO	TS	GA	HTG	HGT	Bees Algorithm
10	0.00	0.24	0.19	0.19	0.19	0.00
20	-1.63	-1.62	-1.62	-1.62	-1.62	-1.63
50	-4.66	-4.66	-4.60	-4.66	-4.66	-4.66
100	-4.94	-4.93	-4.91	-4.93	-4.93	-4.94
200	-3.75	-3.74	-3.75	-3.75	-3.75	-3.75
500	-3.56	-3.57	-3.58	-3.58	-3.58	-3.57
1,000	-4.37	-4.39	-4.40	-4.39	-4.39	-4.35
AVG	-3.27	-3.24	-3.24	-3.25	-3.25	-3.27

Table 5.2 Minimum deviation of the computational results (Continues)

n/h	0.6					
	DPSO	TS	GA	HTG	HGT	Bees Algorithm
10	0.00	0.10	0.03	0.03	0.01	0.00
20	-0.72	-0.71	-0.68	-0.71	-0.71	-0.72
50	-0.34	-0.32	-0.31	-0.27	-0.31	-0.34
100	-0.15	-0.01	-0.12	0.08	0.04	-0.15
200	-0.15	-0.01	-0.13	0.37	0.07	-0.15
500	-0.11	0.25	-0.11	0.73	0.15	-0.11
1,000	-0.06	1.01	-0.05	1.28	0.42	-0.05
AVG	-0.22	0.04	-0.20	0.22	-0.05	-0.22

n/h	0.8					
	DPSO	TS	GA	HTG	HGT	Bees Algorithm
10	0.00	0.00	0.00	0.00	0.00	0.00
20	-0.41	-0.41	-0.28	-0.41	-0.41	-0.41
50	-0.24	-0.24	-0.19	-0.23	-0.23	-0.24
100	-0.18	-0.15	-0.12	-0.08	-0.11	-0.18
200	-0.15	-0.04	-0.14	0.26	0.07	-0.15
500	-0.11	0.21	-0.11	0.73	0.13	-0.11
1,000	-0.06	1.13	-0.05	1.28	0.40	-0.05
AVG	-0.16	0.07	-0.13	0.22	-0.02	-0.16

Table 5.3 Comparison of maximum deviations between the BA and DPSO

$\Delta_{\max}$		
h	DPSO	Bees Algorithm
0.2	-4.90	-4.95
0.4	-3.18	-3.26
0.6	-0.03	-0.22
0.8	-0.16	-0.16
Avg	-2.07	-2.15

Table 5.4 Comparison between the Bees Algorithm (BA), DPSO and DE

n	h	$\Delta_{\min}$		$\Delta_{\max}$		Δ_{avg}			Δ_{std}	
		DPSO	BA	DPSO	BA	DPSO	BA	DE	DPSO	BA
10	0.2	0.00	0.00	0.11	0.00	0.01	0.00	0.00	0.03	0.00
	0.4	0.00	0.00	0.15	0.00	0.02	0.00	0.00	0.05	0.00
	0.6	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	0.8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20	0.2	-3.84	-3.84	-3.79	-3.83	-3.83	-3.84	-3.84	0.02	0.00
	0.4	-1.63	-1.63	-1.57	-1.63	-1.62	-1.63	-1.63	0.02	0.00
	0.6	-0.72	-0.72	-0.66	-0.72	-0.71	-0.72	-0.72	0.03	0.00
	0.8	-0.41	-0.41	-0.41	-0.41	-0.41	-0.41	-0.41	0.00	0.00
50	0.2	-5.70	-5.70	-5.61	-5.69	-5.68	-5.70	-5.69	0.03	0.00
	0.4	-4.66	-4.66	-4.52	-4.66	-4.63	-4.66	-4.66	0.05	0.00
	0.6	-0.34	-0.34	-0.23	-0.34	-0.31	-0.34	-0.32	0.04	0.00
	0.8	-0.24	-0.24	-0.24	-0.22	-0.24	-0.24	-0.24	0.00	0.01
100	0.2	-6.19	-6.19	-6.15	-6.19	-6.18	-6.19	-6.17	0.02	0.00
	0.4	-4.94	-4.94	-4.82	-4.93	-4.90	-4.94	-4.89	0.04	0.00
	0.6	-0.15	-0.15	0.26	-0.14	-0.09	-0.14	-0.13	0.14	0.00
	0.8	-0.18	-0.18	-0.18	-0.17	-0.18	-0.18	-0.17	0.00	0.00
200	0.2	-5.78	-5.78	-5.74	-5.77	-5.77	-5.78	-5.77	0.01	0.00
	0.4	-3.75	-3.75	-3.68	-3.74	-3.72	-3.75	-3.72	0.02	0.01
	0.6	-0.15	-0.15	0.56	-0.15	-0.03	-0.15	0.23	0.27	0.00
	0.8	-0.15	-0.15	-0.15	-0.15	-0.15	-0.15	0.20	0.00	0.00
500	0.2	-6.42	-6.43	-6.40	-6.42	-6.41	-6.43	-6.43	0.01	0.00
	0.4	-3.56	-3.57	-3.51	-3.56	-3.54	-3.57	-3.57	0.01	0.00
	0.6	-0.11	-0.11	-0.11	-0.11	-0.11	-0.11	1.72	0.00	0.00
	0.8	-0.11	-0.11	-0.11	-0.11	-0.11	-0.11	1.01	0.00	0.00
1000	0.2	-6.76	-6.76	-6.73	-6.74	-6.75	-6.75	-6.72	0.01	0.01
	0.4	-4.37	-4.35	-4.32	-4.33	-4.35	-4.34	-4.38	0.01	0.01
	0.6	-0.06	-0.05	-0.03	-0.05	-0.04	-0.05	1.29	0.01	0.00
	0.8	-0.06	-0.05	-0.06	-0.05	-0.06	-0.05	2.79	0.00	0.00
Avg		-2.15	-2.15	-2.07	-2.15	-2.14	-2.15	-1.87	0.03	0.00

5.4. The Bees Algorithm for Permutation Flowshop Sequencing

Problem

For permutation flowshop sequencing, the problem to which the Bees Algorithm was applied in this investigation, m different machines are set up in series and each job must be processed on every machine (in the order to $1 \dots m$). The processing order of the n jobs is the same for every machine. Permutation of the n jobs gives a makespan (the time taken to complete the job sequence) and the aim is to find the permutation that gives the minimum makespan (Taillard, 1993).

The permutation flowshop sequencing problem (PFSP) was first introduced by Johnson, (1954) and has attracted the attention of many researchers since then. It has been proved that permutation flowshop sequencing with makespan minimisation is an NP-hard problem (Kan, 1976). Therefore, using different heuristic optimisation techniques, researchers have attempted to find high-quality solutions in a reasonable computational time instead of looking for the optimal solution (Campbell et al., 1970; Dannenbring, 1977; Framinan, 2003). To obtain high-quality solutions, many meta-heuristic optimisation techniques have been tried. They include the Genetic Algorithm (Reeves, 1993; Reeves, 1995), Tabu Search (Taillard, 1993; Nowicki, 1996), Simulated Annealing (Osman, 1989; Ogbu 2004), Particle Swarm Optimisation (Tasgetiren, et al., 2004) and Ant Colony Optimisation (Rajendran and Ziegler, 2004; Stützle, 1998).

5.4.1. Formulation of the permutation flowshop sequencing problem

A formulation of the PFSP is provided in (Tasgetiren, et al., 2004). Given the processing times p_{jk} for job j on machine k , and a job permutation $\pi = \{\pi_1, \pi_2, \dots, \pi_n\}$, the n jobs ($j = 1, 2, \dots, n$) will be sequenced through m machines ($k = 1, 2, \dots, m$) using the same permutation π . Let $C(\pi_j, m)$ denote the completion time of job π_j on machine m . The completion time of an n -job- m -machine problem can be calculated as follows (Tasgetiren, et al., 2004):

$$C(\pi_1, 1) = p_{\pi_1, 1} \quad (5.4)$$

$$C(\pi_j, 1) = C(\pi_{j-1}, 1) + p_{\pi_j, 1} \quad j = 2, \dots, n \quad (5.5)$$

$$C(\pi_1, k) = C(\pi_1, k-1) + p_{\pi_1, k} \quad k = 2, \dots, m \quad (5.6)$$

$$C(\pi_j, k) = \max\{C(\pi_{j-1}, k), C(\pi_j, k-1)\} + p_{\pi_j, k} \quad j = 2, \dots, n \quad k = 2, \dots, m \quad (5.7)$$

Then, among the set Π of all permutations, a permutation π^* can be found such that the makespan $C_{\max}(\pi^*)$ is:

$$C_{\max}(\pi^*) \leq C(\pi_n, m) \quad \pi \in \Pi \quad (5.8)$$

5.4.2. The Bees Algorithm for PFSP

Fig. 5.1 shows the pseudo-code for the algorithm. For a PFSP, the algorithm will try to find a permutation with minimum makespan $C_{\max}$. As explained in previous chapters, the algorithm requires the following parameters to be set: number of scout bees (n), number of sites selected out of n visited sites (m), number of the top sites among the m selected sites (e), number of bees recruited for the top e sites (n_{ep}), number of bees recruited for the other ($m-e$) selected sites (n_{sp}), site abandonment threshold (sat) and stopping criterion.

The algorithm starts with an initial population of n scout bees. Each bee is a symbolic string representing a sequence of machines and jobs. Details of the completion times for the jobs on the different machines are given in a machine-job matrix (see Fig. 5.8). For an $m \times n$ machine-job matrix, a string with a length of $m + n$ is needed to encode each candidate solution. The first m bits of the string represent the sequence of machines that appear in the rows of the matrix and the last n bits of the string represent the sequence of jobs appearing in the columns of the matrix.

In step 2, the fitness computation process is carried out for each site visited by a bee by calculating the corresponding makespan (see equations 5.4, 5.5, 5.6, 5.7 and 5.8).

In step 4, the m sites with the highest fitnesses (the shortest makespans) are designated as “selected sites” and chosen for neighbourhood search.

In steps 5 and 6, the algorithm conducts searches around the selected sites, assigning more bees to search in the vicinity of the best e sites. The neighbourhood operator for

the PFS problem is given in Fig. 5.8. Selection of the best sites can be made directly according to the fitnesses associated with them. Alternatively, the fitness values can be used to determine the probability of the sites being selected. Neighbourhood operators for the algorithm are presented in Fig. 5.9. Given the equal chance to be chosen, each of these methods is performed for each recruited bee (nep and nsp) for every iteration. Searches in the neighbourhood of the best e sites, which represent the most promising solutions, are made more detailed by recruiting more bees for the best e sites than for the other selected sites. Only the bee with the highest fitness will be selected to form the next bee population.

In step 6, if the points visited near a selected site are all inferior to that site, after a certain number of iterations (i.e. Abandon threshold), then the location of the site is recorded and the site abandoned. Bees at the site are assigned to random search (i.e. made to scout for new potential solutions).

In step 7, the remaining bees in the population are assigned randomly around the search space to scout for new potential solutions.

At the end of each iteration, the colony will have two parts to its new population: representatives from the selected sites, and scout bees assigned to conduct random searches. Steps 4-7 are repeated until either the best fitness value has stabilised or the specified maximum number of iterations has been reached.

m/n	1	...	n
1	$C(\pi_1,1)$	...	$C(\pi_k,1)$
$\vdots$	$\vdots$	...	$\vdots$
m	$C(\pi_1,k)$	...	$C(\pi_j,k)$

Figure 5.8 A machine-job matrix and the makespan.

NeighbourhoodOperator (Sequence(k));

//Simple Swap operator

```
do{  
    L1=(rand()%index);  
    L2=(rand()%index);  
} while ((L1==index)||(L1==L2)||(L2==index));  
t1 = Sequence [k][L1];  
Sequence [k][L1] = Sequence [k][L2];  
Sequence [k][L2] = t1;
```

//Insert Operator

```
do{  
    L1=(rand()%index);  
    L2=(rand()%index);  
} while ((L1==index)||(L1==L2)||(L2==index));  
for(int j=L1;j<L2;j++) temparray1[j]=Tabu[k][j];  
t1 = Tabu[k][L2]; temp=0; temp=L1+1;  
for(j=L1;j<L2;j++){  
    Tabu[k][temp]=temparray1[j];  
    Temp=temp+1;}  
Tabu[k][L1]=t1;  
temp=0;
```

Figure 5.9 Neighbourhood operators for PSFP

5.4.3. Experimental Results

The Bees Algorithm for solving PFSPs was implemented in C++ and run on an Intel P4 2.4 GHz PC with 1GB memory. For the Taillard benchmark data set (Taillard, 1993), the Bees Algorithm, a genetic algorithm (GA) (Tasgetiren, et al., 2004) and a particle swarm optimisation (PSO) algorithm were employed (Tasgetiren, et al., 2004). The parameters of the Bees Algorithm are shown in Table 5.5 for each data set. The population size was made equal to twice the number of jobs as was the case for the other algorithms (Tasgetiren, et al., 2004). The Bees Algorithm used the 'insert and exchange' operator to perform neighbourhood search.

For the GA, permutation representation was adopted and the crossover and mutation probabilities were taken as 1.0 and 0.05 percent respectively (Tasgetiren, et al., 2004). To perform two-cut crossover, one individual was selected randomly and the other by tournament selection with a size of 2. Tournament selection with a size of 2 was also used for constructing the population for the next generation. The insert operator was used as the mutation operator (Tasgetiren, et al., 2004).

The PSO parameters, c_1 , c_2 , w_0 , and α , were set as 2, 2, 0.9, and 0.975, respectively (Tasgetiren, et al., 2004). The performance of the algorithm was expressed by two indices: 1) percentage deviations (Δ) and 2) standard deviation. To obtain the average performance of the algorithm, ten runs were carried out for each problem instance. Δ is the percentage deviation from the reference value reported by Taillard, (1993). The average Δ_{avg} for the ten runs was computed as follows:

$$\Delta_{avg} = \sum_{i=1}^R \left(\frac{(F_{BA} - F_{ref})}{F_{ref}} \times 100 \right) / R \quad (5.9)$$

where F_{BA} is the fitness value generated by the Bees Algorithm in each run, F_{ref} is the reference fitness value generated by Taillard, (1993) and $R = 10$ is the total number of runs, respectively. Table 5.6 summarises the results obtained. In the table, Δ_{std} denotes the standard deviation in Δ over the R runs.

As can be seen in Table 5.6, the Bees Algorithm outperformed the GA, PSOSPv and PSOVns. Compared to the GA and PSOSPv, the Bees Algorithm gave results that were more stable and closer to those presented by Taillard, (1993). However, the algorithm produced only slightly better results compared to PSOVns. This similarity in the results may be due to the closeness between the local search method adopted by the Bees Algorithm and the variable neighbourhood search (VNS) technique implemented in PSO for conducting local search. However, although VNS employs the insert-and-exchange operator, unlike the Bees Algorithm, it performs this operation only on the best solution. For the Bees Algorithm, the selection of several sites (m) for local search provides more information than VNS can. This property can help the Bees Algorithm to escape from local optima.

Table 5.5 Bees Algorithm parameters for PFSP

Data Set	Parameters*				
	n	m	e	nep	nsp
20x5	40	20	5	200	100
20x10	40	20	5	400	100
20x20	40	20	5	600	100
50x5	100	50	30	400	200
50x10	100	70	10	600	300
50x20	100	80	10	1000	500
100x5	200	120	10	500	200
100x10	200	120	10	1000	300
100x20	200	120	20	1500	400

*Site abandonment threshold sat=50 for all data sets

Table 5.6 Benchmark results for the permutation flowshop sequencing problem

	GA[25]		PSO _{spv} [25]		PSO _{vns} [25]		The Bees Algorithm	
DATA SETS	Δ_{avg}	Δ_{std}	Δ_{avg}	Δ_{std}	Δ_{avg}	Δ_{std}	Δ_{avg}	Δ_{std}
20x5	3.13	1.86	1.71	1.25	0.28	0.49	0.28	0.3
20x10	5.42	1.72	3.28	1.19	0.7	0.46	0.13	0.12
20x20	4.22	1.31	2.84	1.15	0.56	0.34	0.17	0.41
50x5	1.69	0.79	1.15	0.70	0.18	0.22	0.1	0.35
50x10	5.61	1.41	4.83	1.16	1.04	0.64	1.03	1.07
50x20	6.95	1.09	6.68	1.35	1.71	0.48	1.48	0.57
100x5	0.81	0.39	0.59	0.34	0.11	0.17	0.1	0.06
100x10	3.12	0.95	3.26	1.04	0.67	0.33	0.58	0.13
100x20	6.32	0.89	7.19	0.99	1.28	0.39	2.55	0.45
MEAN	4.04	1.156	3.734	1.018	0.725	0.391	0.720	0.384

5.5. Manufacturing Cell Formation Using The Bees Algorithm

Manufacturing industry is under intense pressure from the increasingly competitive global marketplace. Shorter product life-cycles, unpredictable demands, and diverse customer needs have forced manufacturing firms to operate more efficiently and effectively in order to adapt to changing requirements. Traditional manufacturing systems, such as job shops and flow lines, cannot handle such environments. Cellular Manufacturing (CM), which incorporates the flexibility of job shops and the high production rate of flow lines, has emerged as a promising alternative for such cases (Mungwattana, 2000).

CM is the application of the concept of group technology (GT) in manufacturing systems. GT is a manufacturing philosophy that exploits similarities in product design and production processes. A fundamental issue in CM is the determination of part families and machine cells. This issue is known as the cell formation (CF) problem. The CF problem involves the decomposition of a manufacturing system into cells. Part families are identified such that they are fully processed within a cell. The cells are formed to capture the advantages of GT such as reduced setup times, reduced in-process inventories, improved product quality, shorter lead times, reduced tool requirements, improved productivity, and better overall control of operations (Wemmerlöv and Hyer, 1987).

The CF problem has long been recognised as the most challenging problem in realising the concept of cellular manufacturing. It belongs to the class of NP-hard problems, which means that an increase in the problem size will cause an exponential increase in the computational time for all prevalent optimisation techniques. Many

methods to solve this problem have been developed (Miltenburg and Zhang, 1991; Jeffrey et al., 1996), including array-based methods, clustering methods, mathematical programming-based methods, graph theoretic methods, and artificial intelligence-based methods.

5.5.1. The Cell Formation problem

The CF problem solved here is to simultaneously group machines and their corresponding part families into cells so that intercellular movements are minimised. It can be formulated by using an $M \times N$ machine-part incidence matrix, $A = [a_{ij}]$, where a_{ij} is a binary variable that takes the value of 1 if part j requires processing on machine i , and 0 otherwise. The problem is equivalent to decomposing A into a number of diagonal blocks of submatrices, where each diagonal block represents a manufacturing cell. The effectiveness of the decomposition can be determined by a normalised bond energy measure denoted as α in equation 5.10 (Mak et al., 2000).

$$\alpha = \frac{\sum_{i=1}^M \sum_{j=1}^{N-1} a_{ij} a_{i,j+1} + \sum_{i=1}^{M-1} \sum_{j=1}^N a_{ij} a_{i+1,j}}{\sum_{i=1}^M \sum_{j=1}^N a_{ij}} \quad (5.10)$$

The objective is to group parts and machines into clusters by sequencing the rows and columns of a machine-part incidence matrix, so as to maximise the bond energy measure of the incidence matrix. In the next section, a new method to solve the CF optimisation problem is described.

5.5.2. Cell Formation using the Bees Algorithm

The proposed CF algorithm utilises the ability of the Bees Algorithm to search for the appropriate groups of part families and machine cells such that the bond energy metric α (equation 5.10) is maximised. Fig. 5.1 shows the steps of the Bees Algorithm used for CF, which are also followed in the Bees Algorithm. These steps are described in detail below.

The proposed algorithm requires a number of parameters to be set, namely, number of scout bees (n), number of sites selected for neighbourhood search (out of n visited sites) (m), number of top-rated (elite) sites among m selected sites (e), number of bees recruited for the best e sites (nep), number of bees recruited for the other ($m-e$) selected sites (nsp), site abandonment threshold (sat) and the stopping criterion.

The algorithm starts with an initial population of n scout bees. Each bee is a symbolic string representing the sequence of machines and parts that appear in a machine-part incidence matrix (see Fig. 5.10). For an $M \times N$ machine-part incidence matrix, a string with a length of $M + N$ is needed to encode each candidate solution. The first M bits of the string represent the sequence of machines that appear in the rows of the incidence matrix, while the last N bits of the string represent the sequence of parts appearing in the columns of the matrix.

m/n	1	2	3	4	5	6
1	0	1	1	0	0	1
2	0	1	0	1	1	0
3	1	0	0	1	1	1
4	1	1	1	0	0	0
5	0	0	1	0	1	1
6	1	0	0	1	1	0

Figure 5.10 Representation of a machine-part incidence matrix.

In step 2, the fitness computation process is carried out for each site visited by a bee by calculating the bond energy measure α (see equation 5.10).

In step 4, the m sites with the highest fitnesses are designated as “selected sites” and chosen for neighbourhood search.

In steps 5, the algorithm conducts searches around the selected sites, assigning more bees to search in the vicinity of the best e sites. Neighbourhood operators for the algorithm are presented in Fig. 5.11. Selection of the best sites can be made directly according to the fitnesses associated with them. Alternatively, the fitness values are used to determine the probability of the sites being selected. Searches in the neighbourhood of the best e sites which represent the most promising solutions are made more detailed by recruiting more bees for the best e sites than for the other selected sites. Together with scouting, this differential recruitment is a key operation of the Bees Algorithm.

NeighbourhoodOperator (Sequence(k));

//Simple Swap operator

```
do{L1=(rand()%index); L2=(rand()%index);}
while ((L1==index)||(L1==L2)||(L2==index));
t1 = Sequence [k][L1];
Sequence[k][L1] = Sequence[k][L2];
Sequence[k][L2] = t1;
```

//2-Opt Operator...

```
do{L1=(rand()%index); L2=(rand()%(index-L1)+L1);}
while ((L1==index)||(L1==L2));
double deg=ceil((L2-L1)/2);
for(int i=0;i<deg;i++){
    int temp1=L1; int temp2=L2;
    t1 = Sequence[k][temp1];
    Sequence[k][temp1] = Sequence[k][temp2];
    Sequence[k][temp2] = t1;
    L1=L1+1; L2=L2-1;}
```

//Insert Operator

```
do{L1=(rand()%index); L2=(rand()%index);}
while ((L1==index)||(L1==L2)||(L2==index));
for(int j=L1;j<L2;j++) temparray1[j]= Sequence[k][j];
t1 = Sequence[k][L2]; temp=0; temp=L1+1;
for(j=L1;j<L2;j++){ Sequence[k][temp]=temparray1[j]; Temp=temp+1;}
Tabu[k][L1]=t1;
temp=0;
```

Figure 5.11 Neighbourhood operators for cell formation problem

In step 6, for each patch only the bee with the highest fitness will be selected to form the next bee population. In nature, there is no such a restriction. This restriction is introduced here to reduce the number of points to be explored.

In step 7, the remaining bees in the population are assigned randomly around the search space to scout for new potential solutions.

At the end of each iteration, the colony will have two parts to its new population: representatives from the selected patches, and scout bees assigned to conduct random searches. Steps 4-7 are repeated until either the best fitness value has stabilised or the specified maximum number of iterations has been reached.

5.5.3. Experimental Results

In this section, two examples are first used to illustrate the operation of the proposed CF algorithm. Then, eight benchmark CF problems with different sizes are used to test the effectiveness of the algorithm. The results obtained are compared to the best-known solutions reported in the literature. The grouping efficiency measure (Miltenburg and Zhang, 1991), ε , is adopted to assess the quality of the solutions. The ε measure is defined as follows:

$$\varepsilon = \frac{n_1}{\sum_{k=1}^K M_k N_k} - \left(1 - \frac{n_1}{n_1 + n_2} \right) \quad (5.11)$$

where n_1 is the number of non-zero entries within the manufacturing cells in the machine-part incidence matrix; K is the number of manufacturing cells formed; M_k and N_k ($k = 1, 2, \dots, K$) are the number of the machines and parts allocated to the manufacturing cell k ; n_2 is the number of exceptional elements in the machine-part incidence matrix.

In equation 5.11, the first term represents the cell density and can be written as:

$$\varepsilon_1 = \frac{n_1}{\sum_{k=1}^K M_k N_k} \quad (5.12)$$

A high value of ε_1 indicates that the machines and parts in each manufacturing cell are very similar to one another. The second term represents the intercellular material flows and can be given as:

$$\varepsilon_2 = 1 - \frac{n_1}{n_1 + n_2} \quad (5.13)$$

A low value of ε_2 will result if less exceptional elements exist in the incidence matrix. According to equation 5.11, the value of the grouping efficiency measure, ε , ranges from -1 to 1. The higher this value, the better the formed machines and parts groups.

In the first illustrative example (Srinivasan et al., 1990; Mak et al., 2000), a 16 x 30 machines-parts incidence matrix is utilised. The initial configuration of the matrix is shown in Fig. 5.12. The parameters of the proposed CF algorithm are set as follows: $n = 100$, $m = 40$, $e = 20$, $nep = 200$, $nsp = 100$, $sat = 50$ and maximum number of

iterations = 1000. By sequencing the order of rows (machines) and columns (parts) of the incidence matrix, the resulting configuration of the matrix is shown in Fig. 5.13. In order to maximise the bond energy of the matrix, the machines and parts are grouped into 4 manufacturing cells. The bond energy measure, α , of the final solution is 1.301. The cell density measure, ε_1 , is 0.816 which indicates that the machines and parts in the manufacturing cells are very similar. The measure of intercellular material flows, ε_2 , is 0.155. The corresponding grouping efficiency of the final solution, ε , is 0.661, which is better than the best solution given in (Srinivasan et al., 1990; Mak et al., 2000).

In the second illustrative example (Srinivasan et al., 1990; Mak et al., 2000), a 10 x 20 machines-parts incidence matrix (see in Fig. 5.14) is employed. The parameters of the proposed CF algorithm are set as follows: $n = 100$, $m = 40$, $e = 20$, $nep = 200$, $nsp = 100$, $sat = 50$ and maximum number of iterations = 1000. The final configuration of the incidence matrix is shown in Fig. 5.15. The machines and parts are grouped into 4 manufacturing cells. The bond energy of the final solution is 1.388. The cell values of ε_1 , ε_2 , and ε of this solution are 1.000, 0.000, and 1.000 respectively. This solution is exactly the same as that suggested in (Srinivasan et al., 1990; Mak et al., 2000).

In order to further test its effectiveness, the proposed CF algorithm is applied to 8 test problems. The results of the CF algorithm are compared against those of the best-known solutions. All the problems are formulated by 0-1 machine-part incidence matrices. The parameters of the CF algorithm are set to $n = 100$, $m = 40$, $e = 20$, $nep = 200$, $nsp = 100$, $sat=50$ and maximum number of iterations = 100.

Figure 5.14 The initial configuration of the machine-part incidence matrix of the second illustrative example (10x20).

Figure 5.15 The composition of the manufacturing cells for the second illustrative example (10x20).

Table 5.7 Results of solving 8 well-known benchmark CF problems from the literature.

No	Literature references	Size	K	Best-known solutions			Results from the CF-Bees Algorithm			
				ε_1	ε_2	ε	α	ε_1	ε_2	ε
1	Boctor, (1991)	7x11	3	0.760	0.095	0.665	1.095	0.760	0.095	0.665
2	Boctor, (1991)	7x11	3	0.760	0.000	0.760	1.053	0.760	0.000	0.760
3	Srinivasan et al., 1990	10x20	4	1.000	0.000	1.000	1.388	1.000	0.000	1.000
4	Carrie, (1973)	20x35	4	0.760	0.015	0.745	1.555	0.794	0.029	0.760
5	Chandrasekharan et al., (1989)	24x40	7	1.000	0.000	1.000	1.515	1.000	0.000	1.000
6	Chandrasekharan et al., (1989)	24x40	7	0.939	0.075	0.864	1.423	0.925	0.061	0.864
7	Chandrasekharan et al., (1989)	24x40	7	0.855	0.138	0.717	1.192	0.860	0.153	0.707
8	Mak et al., (2000)	40x100	10	0.910	0.086	0.824	1.471	0.910	0.077	0.833

Table 5.7 summarises the results obtained. As can be seen from the table, the proposed CF algorithm has produced similar results to those of the best-known solutions for problems 1, 2, 3, 5 and 6. In problem 7, the grouping efficiency measure, ε , is slightly reduced from 0.717 to 0.707. However, the CF algorithm achieved better results for problems 4 and 8. In problem 4, the cell density measure, ε_1 , has been increased from 0.760 to 0.794, while the measure of intercellular flows, ε_2 , has been increased from 0.015 to 0.029. This has led to an increase of the grouping efficiency, ε , from 0.745 to 0.760. In problem 8, which involves a 40 x 100 machine-part incidence matrix, the value of ε_2 has been reduced from 0.086 to 0.077. The value of ε_1 has also been increased from 0.903 to 0.910. Therefore, the grouping efficiency has been improved (increased) from 0.815 to 0.833.

5.6. Summary

In this chapter a new procedure is suggested for the Bees Algorithm to deal with combinatorial domains and the algorithm is applied to several complex optimisation problems with specific modifications. The algorithm is first applied to a single job scheduling problem. The results are compared to those obtained by some other well-known algorithms to be found in the literature, including evolution search (ES), simulated annealing (SA), threshold accepting (TA) and TA with a back step (TAR). The results obtained suggest that the modified Bees Algorithm performs better than or as well as the others. The second application introduced is of a permutation flow-shop sequencing problem. The modified Bees Algorithm used a slightly different neighbourhood strategy to deal with this complex problem. It performed well

compared to other well-established algorithms, including a genetic algorithm, PSO_{spv} and PSO_{vns} . In the last example, the modified Bees Algorithm also employed a different neighbourhood search strategy due to nature of the cell formation problem. The results show that the modified Bees Algorithm also performs better than the others in this domain.

Chapter 6

THE BEES ALGORITHM-II

6.1. Preliminaries

In this chapter, an improved version of the Bees Algorithm, so called The Bees Algorithm-II is presented. As mentioned in previous chapter, the Bees Algorithm is a new population-based search algorithm and it mimics the food foraging behaviour of swarms of honey bees. In its basic version, the algorithm performs a kind of neighbourhood search combined with random global search. The Bees Algorithm-II, however, is a more efficient and robust version of the original algorithm. The

enhanced version can be defined as a more compact version of its predecessor with additional improvements in terms of neighbourhood search and lesser parameters.

The aims of this chapter are: to lessen the complexity by reducing the user defined parameters of the Bees Algorithm, to create a single population leading to a more comprehensive algorithm and to introduce a new patch structure to improve efficiency and control over randomness throughout the neighbourhood search.

The chapter is organised as follows: section 6.2 presents a description of the Bees Algorithm-II in detail, including the new ideas of a Gaussian patch and reduced parameters. In section 6.3, to demonstrate the performance of the new improved algorithm, experimental results are presented, including comparison with the original version and other well-known algorithms. A general summary of the proposed method and of possible future research work are given in section 6.4.

6.2. The Bees Algorithm-II

As mentioned in previous chapters, the Bees Algorithm is an optimisation algorithm inspired by the natural foraging behaviour of honey bees to find the optimal solution. The basic Bees Algorithm in its simplest form is presented in Fig. 3.1 as well as the improved version in Fig. 3.8. Although the Bees Algorithm is exceptionally successful, there are a few concerns over its relatively high parameter numbers and the related difficulty of setting them for a specific application. In this section, several new ideas are introduced to address these issues, including Gaussian neighbourhood search

and reducing parameters by the new patch structure as well as by reorganisation of the parameter groups.

6.2.1 Gaussian patch structure

Gaussian distribution or normal distribution is defined as a theoretical continuous probability distribution with finite mean and variance (Simon, 2006). The graph of the associated probability density function is bell-shaped, with a peak at the mean, and is known as the Gaussian function or bell curve (Simon, 2006). The continuous probability function is given in equation 6.1:

$$\varphi_{\mu\sigma}(\chi) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \quad (6.1)$$

where,

$$X \sim N(\mu, \sigma^2)$$

μ : Candidate solution

σ : Standard deviation

In the Bees Algorithm, neighbourhood search is performed randomly with a predefined initial patch size for all selected sites. In the original version of the Bees Algorithm a uniform random distribution is used to send bees to a selected site for local search. An initial patch size (ngh) is set to define the boundaries of the local search and bees are sent into this search space. To improve the solution quality and performance, a proportional shrinking method was introduced. This new method adds another user defined parameter to the algorithm. In fact, the shrinking constant is one of the most critical parameters in the set and it needs careful tuning. The Gaussian distribution is introduced to overcome these vulnerabilities of the algorithm by eliminating the shrinking procedure.

A new parameter, so called the patch density (pd), is introduced in place of the initial patch size (ngh) parameter. In the Gaussian distribution this term refers to the standard deviation of a set σ . By adjusting patch density (pd) the shape of a patch can be modified, as illustrated in Fig. 6.1. Different patch density values can create bigger or lesser size bells and thus the size of the patch can be adjusted using this parameter.

In the original algorithm, ngh defines the initial size of the neighbourhood in which follower bees are placed. For example, if x is the position of an elite bee in the i^{th} dimension, follower bees will be placed randomly in the interval $x_{ie} \pm ngh$ in that dimension at the beginning of the optimisation process. In this case, x_{ie} is regarded as mean (μ) in a Gaussian distribution. Around this value a bell-shaped distribution is produced with patch density (pd). After the modification of equation 6.1, the normal random variate generator will be as follows in equations 6.2 and 6.3:

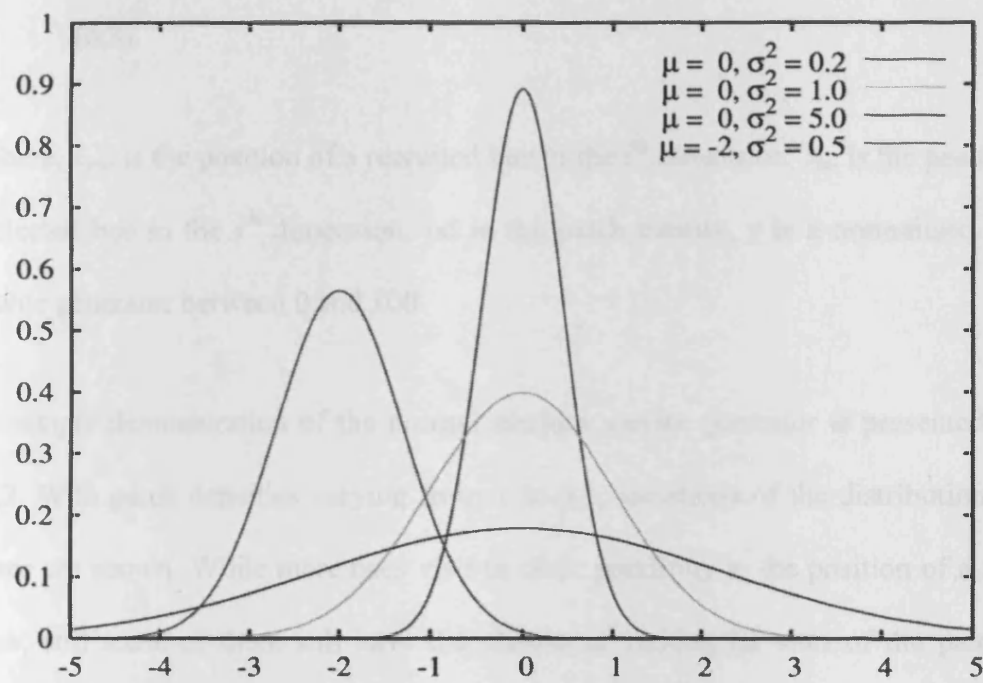


Figure 6.1 Bell shape of the Gaussian distribution with given mean and standard deviation.

$$y = (rand() \% 100) / 100$$

(6.2)

$$x_{imr} = x_{im} + pd \left((2y - 1) \frac{\sqrt{-2 \log(4y - 2)^2}}{2(2y - 1)^2} \right)$$

(6.3)

where, x_{imr} is the position of a recruited bee in the i^{th} dimension, x_{im} is the position of a selected bee in the i^{th} dimension, pd is the patch density, y is a normalised random value generator between 0 and 100.

A simple demonstration of the normal random variate generator is presented in Fig. 6.2. With patch densities varying from 1 to 0.1, variations of the distribution of 100 bees are shown. While more bees visit in close proximity to the position of a selected bee, still some of them still have the chance of visiting far sites of the patch. This guided search reduces the necessity of having an initial patch size and a shrinking procedure, which is difficult to adjust.

6.2.2 Parameters

There are eight parameters of the original Bees Algorithm to be set, namely:

- n : Number of scout bees,
- m : Number of patches selected out of n visited points,
- e : Number of best patches out of m selected patches (elite),

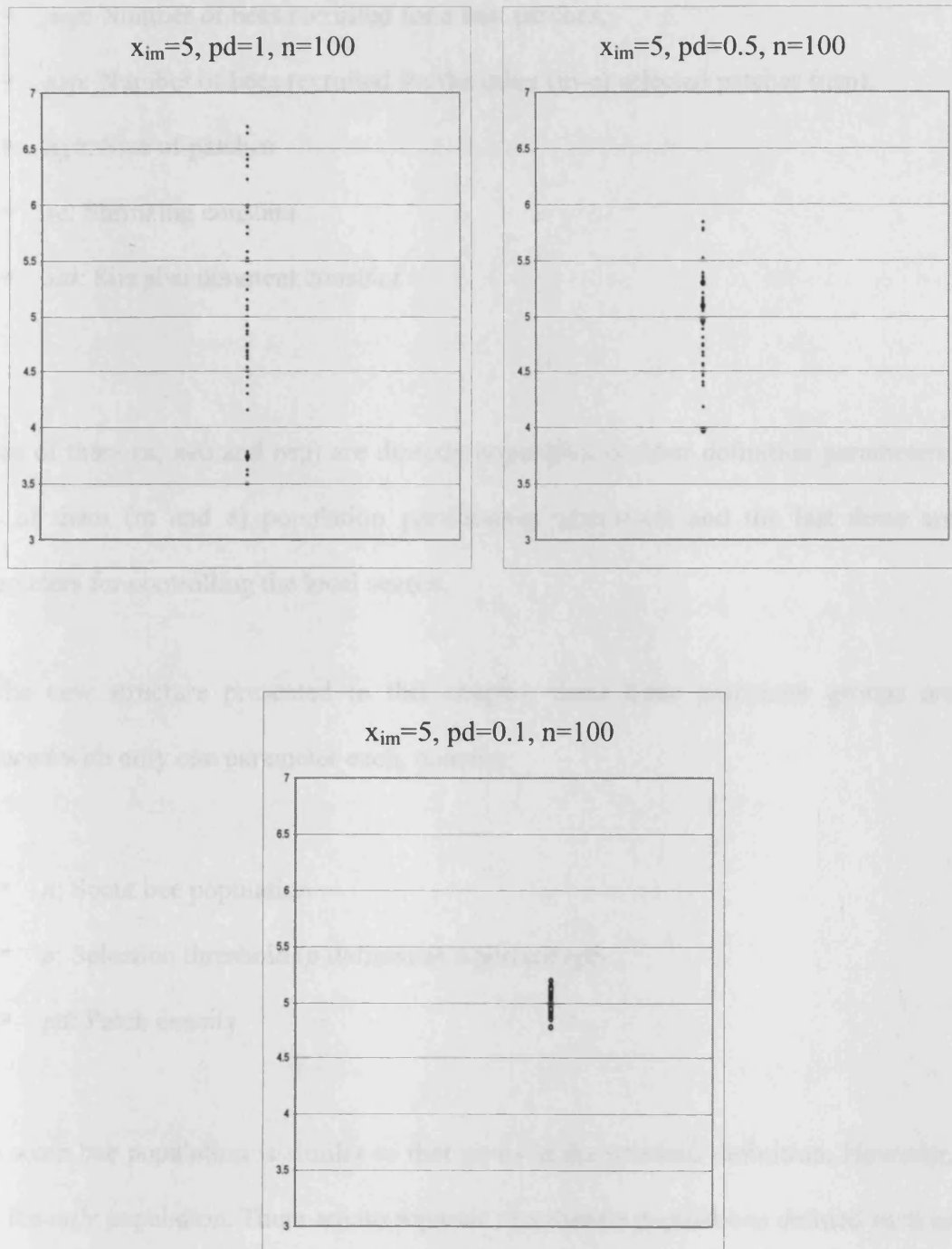


Figure 6.2. A simple demonstration of the normal random variate generator.

- *nep*: Number of bees recruited for e best patches,
- *nsp*: Number of bees recruited for the other (m-e) selected patches (nsp),
- *ngh*: Size of patches
- *sc*: Shrinking constant
- *sat*: Site abandonment constant

Three of them (*n*, *nep* and *nsp*) are directly population number definition parameters, two of them (*m* and *e*) population partitioning parameters and the last three are parameters for controlling the local search.

In the new structure presented in this chapter, these three parameter groups are replaced with only one parameter each, namely:

- *n*: Scout bee population
- ρ : Selection threshold (ρ defined as a percentage)
- *pd*: Patch density

The scout bee population is similar to that given in the previous definition. However, it is the *only* population. There are no separate recruitment populations defined such as *nep* and *nsp*. The selection threshold (ρ) replaces the partitioning group in the previous structure. The number of best sites is equal to $n\rho$ and the number of recruited bees is equal to $n - n\rho$. The patch density concept has been discussed above in detail.

The pseudo-code of the Bees Algorithm-II is presented in Fig. 6.3. There are four parameters of the Bees Algorithm-II to be set, namely: scout bee population (*n*),

selection threshold (ρ), patch density (pd) and site abandonment threshold (sat). The algorithm starts with the n scout bees being placed randomly in the search space. The fitnesses of the points visited by the scout bees are evaluated in step 2.

In step 4, bees (np) that have the highest fitnesses are chosen as “selected bees” and those sites that have been visited by them will be chosen for neighbourhood search.

Then, in step 5, the algorithm conducts searches in the neighbourhood of the selected bees. The latter can be chosen directly according to the fitnesses associated with the points they are visiting. Alternatively, the fitness values are used to determine the probability of the bees being selected.

In step 5, for each site only one bee with the highest fitness will be selected to form the next bee population.

In step 6, if the points visited near a selected site are all inferior to that site, after a certain number of iterations (i.e. *sat*: site abandonment threshold), then the location of the site is recorded and the site abandoned. Bees at the site are assigned to random search (i.e. made to scout for new potential solutions).

In step 7, the remaining bees in the population are assigned randomly around the search space scouting for new potential solutions. These steps are repeated until a stopping criterion is met.

1. Initial population with n random solution.
2. Evaluate fitness of the population.
3. While (stopping criterion not met)
4. Select sites (np) for neighbourhood search.
5. Recruit bees $n - np$ for selected sites, evaluate fitnesses and select the fittest bee from each site

for ($k=1$; $k=m$; $k++$)

for ($Bee=1$; $Bee= (n - np)$; $Bee++$)

$BeesPositionInNgh() = x_{imr};$
 $y = (rand()\%100)/100$

$$x_{imr} = x_{im} + pd \left((2y - 1) \sqrt{-2\log(4y - 2)^2} / 2(2y - 1)^2 \right)$$
 $Evaluate\ Fitness = Bee(i);$ //Evaluate the fitnesses of recruited Bee(i)
 $If\ (Bee(i)\ is\ better\ than\ Bee(i-1))\ RepresentativeBee = Bee(i);$
6. If (Iteration > sat) //Checking site abandonment threshold

If (no improvement on the site)

Save the Best Fitness;
 Abandon the Site;
 $Bee(m) = GenerateRandomValue(All\ Search\ Space);$
7. Assign remaining bees to search randomly and evaluate their fitnesses. // ($n - np$) assigned to search randomly into whole solution space
8. End While

Figure 6.3 Pseudo-code of the Bees Algorithm-II

6.3. Experimental Results

Clearly, the Bees Algorithm as described above is applicable to both combinatorial and functional optimisation problems. In this section, functional optimisation is presented to show the robustness of the algorithm.

Three standard functional optimisation problems were used to test the Bees Algorithm and to establish the correct values of its parameters and seven for benchmarking the algorithm. As the Bees Algorithm searches for the maximum, the functions to be minimised were inverted before the algorithm was applied.

The first test function is the axis parallel hyper-ellipsoid which is similar to De Jong's function 1 (see Fig. 6.4). It is also known as the weighted sphere model. It is continuous, convex and unimodal.

$$f_{1a}(x) = \sum_{i=1}^n i x_i^2$$

(6.4)

$$-5.12 \leq x_i \leq 5.12$$

Global Minimum for this function:

$$f(x) = 0; \quad x(i) = 0, \quad i = 1:n$$

The following parameter values were set for the axis parallel hyper-ellipsoid test function: scout bee population $n = 10$, number of selected sites $m = 3$, number of elite

sites $e=1$, initial patch size $ngh=0.5$, number bees around elite points $nep=2$, number of bees around other selected points $nsp=2$.

The following parameter values of the Bees Algorithm were set for this test: scout bee population $n=10$, number of selected sites $m=3$, number of elite sites $e=1$, initial patch size $ngh=2.75$, number of bees around elite points $nep=2$, number of bees around other selected points $nsp=2$. The following parameter values for the improved Bees Algorithm were set for this test: scout bee population $n=10$, number of selected sites $m=3$, number of elite sites $e=1$, initial patch size $ngh=5.12$, number bees around elite points $nep=2$, number of bees around other selected points $nsp=2$, shrinking constant $sc=0.20$ (%20) and threshold for site abandonment $sat=10$. The parameters for the Bees Algorithm-II were set for this test: scout bee population $n=8$, selection threshold $p=0.1$ (%10), patch density $pd=0.1$ and site abandonment threshold $sat=10$.

Fig. 6.5 shows the fitness values obtained as a function of the number of points visited for both original and improved algorithms. The results are averages for 100 independent runs. After approximately 500 visits, the Bees Algorithm was able to find solutions close to the optimum while the Bees Algorithm-II was able to find the optimum twice as fast as the original. The main reason behind this speed is the value of patch density, which is the only parameter in the set needs careful tuning. The scout bee population was set at 8 and 10 percent of these (rounded up to 1) are allowed to perform the waggle dance and the rest ($n-np$) sent for neighbourhood search.

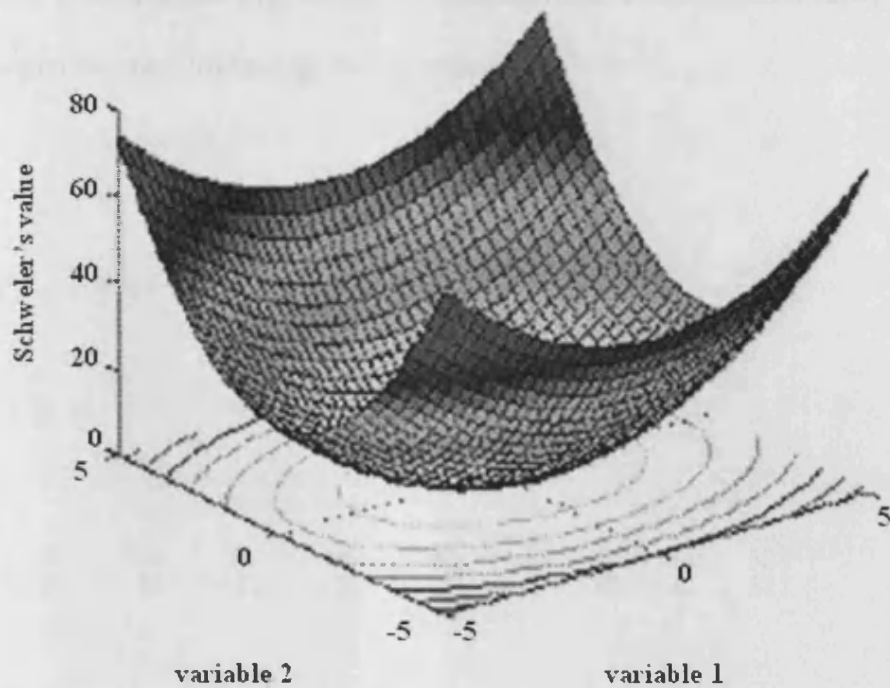


Figure 6.4 Visualization of 2D axis parallel hyper-ellipsoid function.

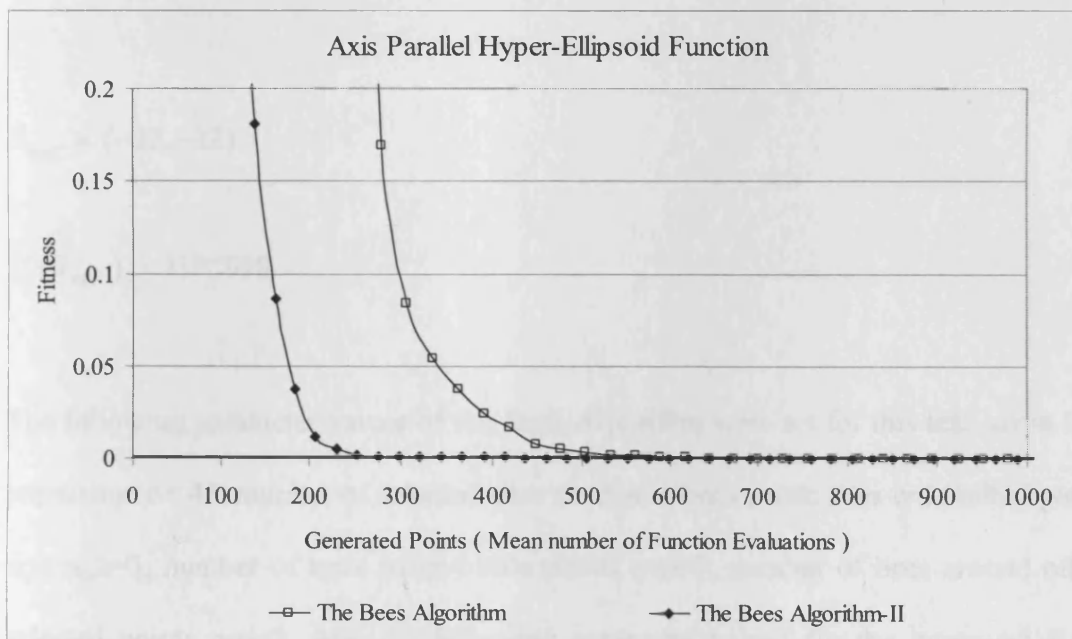


Figure 6.5 Evolution of fitness with the number of points visited (the axis parallel hyper-ellipsoid)

Shekel's Foxholes (see Fig. 6.6), a 2D function from De Jong's test suite, was chosen as the first function for testing the algorithm.

$$f(\vec{x}) = 119.998 - \sum_{i=1}^{25} \frac{1}{j + \sum_{i=1}^2 \left(x_i - a_{ij} \right)^6} \quad (6.5)$$

$$a_{ij} = \begin{pmatrix} -32 & -16 & 0 & 16 & 32 & \dots & 0 & 16 & 32 \\ -32 & -32 & -32 & -32 & -32 & \dots & 32 & 32 & 32 \end{pmatrix}$$

$$-65.536 \leq x_i \leq 65.536$$

For this function,

$$\vec{x}_{\max} = (-32, -32)$$

$$f(\vec{x}_{\max}) = 119.998$$

The following parameter values of the Bees Algorithm were set for this test: scout bee population $n=45$, number of selected sites $m=3$, number of elite sites $e=1$, initial patch size $ngh=3$, number of bees around elite points $nep=7$, number of bees around other selected points $nsp=2$. And the following parameter values for the improved Bees Algorithm were set for this test: scout bee population $n=10$, number of selected sites $m=3$, number of elite sites $e=1$, initial patch size $ngh=3$, number of bees around elite points $nep=2$, number of bees around other selected points $nsp=2$, shrinking constant

sc=0.01 (%1) and threshold for site abandonment sat=10. The parameters for the Bees Algorithm-II were set for this test as follows: scout bee population n=20, selection threshold $\rho=0.1$ (%10), patch density pd=9 and site abandonment threshold sat=10.

Fig. 6.7 shows the fitness values obtained as a function of the number of points visited. The results are averages for 100 independent runs. It can be seen that after approximately 1200 visits, the Bees Algorithm was able to find solutions close to the optimum. However, the Bees Algorithm-II was able to find a solution better than the original algorithm, twice as fast with less than half of the population set for the original algorithm.

To test the reliability of the algorithm, the inverted Schwefel's function with six dimensions (see equation 6.6) was used. Fig. 6.8 shows a two-dimensional view of the function to highlight its multi-modality.

$$f(\vec{x}) = - \sum_{i=1}^6 x_i \sin(\sqrt{|x_i|}) \quad (6.6)$$

$$-500 \leq x_i \leq 500$$

For this function,

$$\vec{x}_{\max} = (420.9829, 420.9829, 420.9829, 420.9829, 420.9829, 420.9829)$$

$$f(\vec{x}_{\max}) \approx 2513.9$$

The following parameter values for the Bees Algorithm were set for this test: scout bees population $n=500$, number of selected sites $m=15$, number of elite sites $e=5$, initial patch size $ngh=20$, number of bees around elite points $nep=50$, number of bees around other selected points $nsp=30$. And the following parameter values for the improved Bees Algorithm were set for this test: scout bees population $n=500$, number of selected sites $m=15$, number of elite sites $e=5$, initial patch size $ngh=20$, number of bees around elite points $nep=50$, number of bees around other selected points $nsp=30$, shrinking constant $sc=0.05$ (%5) and threshold for site abandonment $sat=20$. The parameters for the Bees Algorithm-II were set for this test as follows: scout bee population $n=100$, selection threshold $p=0.05$ (%5), patch density $pd=1$ and site abandonment threshold $sat=10$.

Fig. 6.9 shows how the fitness values evolve with the number of points visited. The results are averages for 100 independent runs. After approximately 30,000 visits, the Bees Algorithm was able to find solutions close to the optimum. However, the Bees Algorithm-II was able to find solutions very close to the optimum (approached less than 0.03 after 10,000 mean number of function evaluations) much faster than the original algorithm. The main reasons for this high success rate are again patch density as well as a high population number.

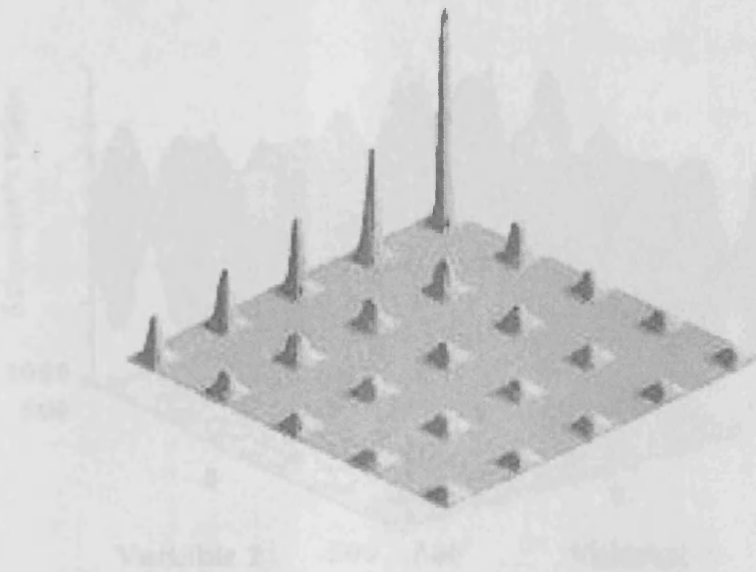


Figure 6.6 Inverted Shekel's Foxholes

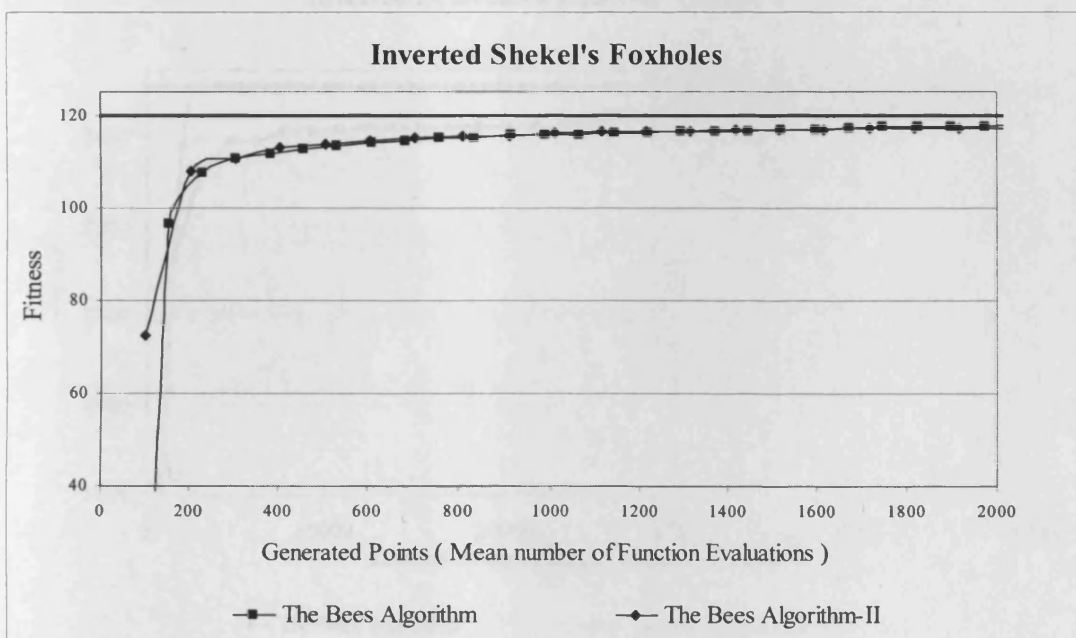


Figure 6.7 Evolution of fitness with the number of points visited (Inverted Shekel's Foxholes)

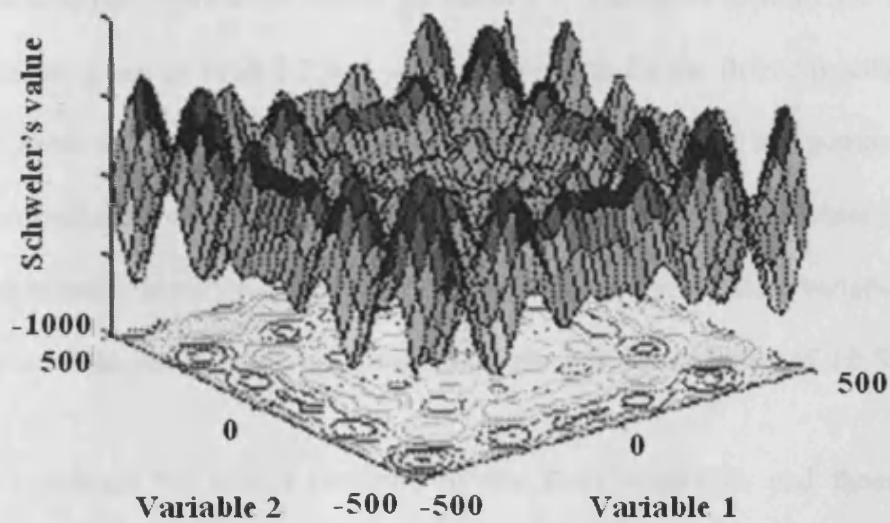


Figure 6.8 2D Schwefel's function

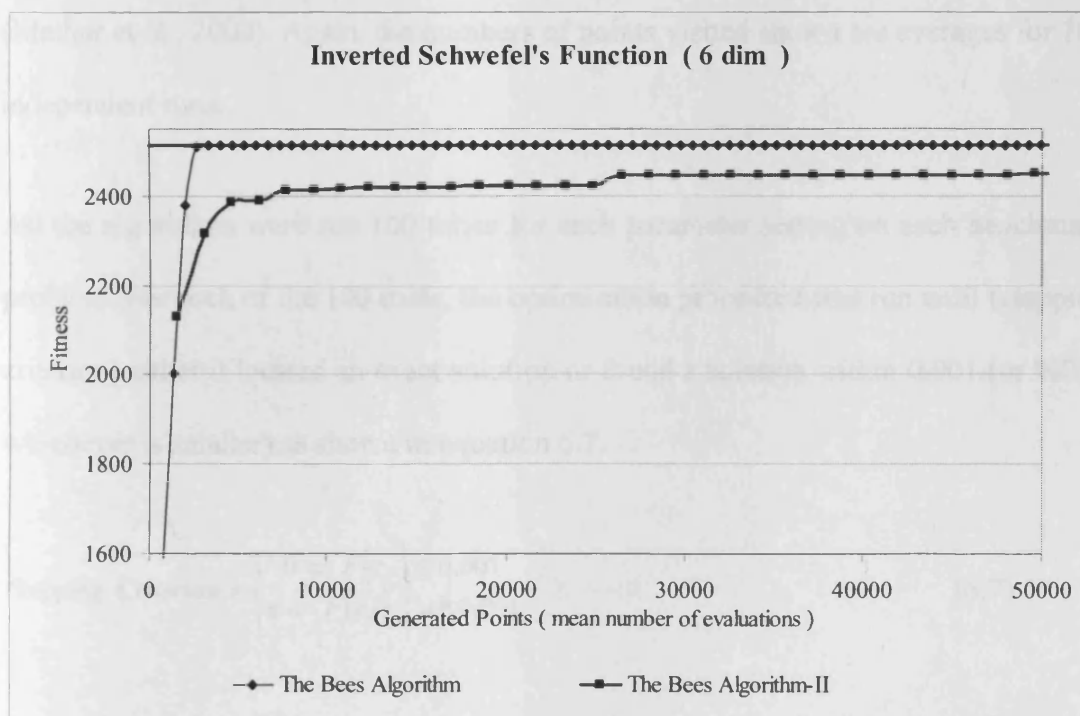


Fig 6.9 Evolution of fitness with the number of points visited (Inverted Schwefel's Function)

The algorithms were applied to seven benchmark functions (Mathur et al., 2000) and the results compared with those obtained using other optimisation algorithms. The test functions and their optima are shown in Table 6.1. Parameter settings for the Bees Algorithm are given in Table 3.2 and parameter settings for the Bees Algorithm-II are given in Table 6.2. As can be seen in the parameter table, scout bee population and selection threshold were almost identical for all problems with little fluctuation, while variations of patch density were greater. The main reason behind these variations is the dependence of the patch density on the solution space and complexity of the functions.

Table 6.3 presents the results obtained by the Bees Algorithm and those by the deterministic Simplex method (SIMPSA) (Mathur et al., 2000), the stochastic simulated annealing optimisation procedure (NE SIMPSA) (Mathur et al., 2000), the Genetic Algorithm (GA) (Mathur et al., 2000) and the Ant Colony System (ANTS) (Mathur et al., 2000). Again, the numbers of points visited shown are averages for 100 independent runs.

All the algorithms were run 100 times for each parameter setting on each benchmark problem. For each of the 100 trials, the optimisation procedure was run until (stopping criterion) either it located an exact solution or found a solution within 0.001 (or %0.1, whichever is smaller) as shown in equation 6.7.

$$\text{Stopping Criterion} = \begin{cases} 0 \Leftarrow F(x_f) \leq 0.001 \\ n \Leftarrow F(x_f) \leq n * 0.001 \end{cases} \quad (6.7)$$

The first test function was De Jong's, for which the Bees Algorithm-II found the optimum slightly faster than the original algorithm, 120 times faster than ANTS and 200 times faster than GA, with a success rate of 100%.

The second function was Goldstein and Price's, for which the Bees Algorithm-II reached the optimum much faster than the original algorithm and almost 7 times faster than ANTS and GA, again with 100% success.

With Branin's function, there was a big improvement with the Bees Algorithm-II. The algorithm performed almost three times better than the Bees Algorithm. There was a 30% improvement compared with ANTS and 120% improvement compared with GA, also with 100% success.

Functions 5 and 6 were Rosenbrock's functions in two and four dimensions respectively. In the two-dimensional function (function 5), the Bees Algorithm-II delivers a good improvement over the other methods (at least twice fewer evaluations than the original algorithm) with a 100% success rate. In the four-dimensional case (function 6), the Bees Algorithm needed more function evaluations (but less function evaluations compared to the original algorithm) to reach the optimum with 100% success. NE SIMPSA could find the optimum with almost 10 times fewer function evaluations but the success rate was only 94% and ANTS found the optimum with 100% success and was 3.5 times faster than the Bees Algorithm.

Test function 7 was a Hyper Sphere model of six dimensions. The Bees Algorithm-II needed almost one third of the number of function evaluations compared with the original algorithm and was much faster than GA and ANTS.

Table 6.1 Test Functions

No	Function Name	Interval	Function	Global Optimum
1	De Jong	[-2.048, 2.048]	$\max F = (3905.93) - 100(x_1^2 - x_2^2) - (1 - x_1)^2$	X(1,1) F=3905.93
2	Goldstein & Price	[-2, 2]	$\min F = [1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)] \times [30 + (2x_1 - 3x_2)^2(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)]$	X(0,-1) F=3
3	Branin	[-5, 10]	$\min F = a(x_2 - b x_1^2 + c x_1 - d)^2 + e(1 - f) \cos(x_1) + e$ $a = 1, b = \frac{5.1}{4} \left(\frac{7}{22} \right)^2, c = \frac{5}{22} \times 7, d = 6, e = 10, f = \frac{1}{8} \times \frac{1}{2}$	X(-22/7, 12.275) X(22/7, 2.275) X(66/7, 2.475) F=0.3977272
4	Martin & Gaddy	[0, 10]	$\min F = (x_1 - x_2)^2 + ((x_1 + x_2 - 10)/3)^2$	X(5,5) F=0
5	Rosenbrock	[-1.2, 1.2] [-10, 10]	$\min F = 100 (x_1^2 - x_2)^2 + (1 - x_1)^2$	X(1,1) F=0
6	Rosenbrock	[-1.2, 1.2]	$\min F = \sum_{i=1}^3 \{100 (x_i^2 - x_{i+1})^2 + (1 - x_i)^2\}$	X(1,1,1,1) F=0
7	Hyper sphere	[-5.12, 5.12]	$\min F = \sum_{i=1}^6 x_i^2$	X(0,0,0,0,0,0) F=0

Table 6.2 Parameter Settings for the Bees Algorithm-II

Function no	Parameters			
	n	ρ	pd	sat
1	10	0.3 (%30)	0.05	10
2	10	0.2 (%20)	0.005	10
3	10	0.2 (%20)	0.1	10
4	10	0.2 (%20)	0.1	10
5a	15	0.2 (%20)	0.05	10
5b	10	0.2 (%20)	0.1	10
6	15	0.4 (%40)	0.07	10
7	10	0.2 (%20)	0.3	10

Table 6.3 The performance of the Bees Algorithm-II

Func no	SIMPSA		NE SIMPSA		GA		ANT		The Bees Algorithm		The Bees Algorithm - II	
	success %	mean no of func evals	success %	mean no of func evals	success %	mean no of func evals	success %	mean no of func evals	success %	mean no of func evals	success %	mean no of func evals
1	***	**	***	***	100	10160	100	6000	100	868	100	853.82
2	***	***	***	***	100	5662	100	5330	100	999	100	771.97
3	***	***	***	***	100	7325	100	1936	100	1657	100	448.97
4	***	***	***	***	100	2844	100	1688	100	526	100	243.82
5a	100	10780	100	4508	100	10212	100	6842	100	631	100	633.422
5b	100	12500	100	5007	***	***	100	7505	100	2306	100	2016.56
6	99	21177	94	3053	***	***	100	8471	100	28529	100	28293.3
7	***	***	***	***	100	15468	100	22050	100	7113	100	2106.9

6.4. Summary

In this chapter, enhancements to neighbourhood search and parameter numbers are presented. A new local search method is introduced to reduce the need for patch

shrinking as well as guiding the randomness. Also, the number of user defined parameters is reduced to a reasonable number by a new theoretical approach. The simulation results compared with the original algorithm, as well as to other well-known algorithms are presented. The results showed that the algorithm performed much better than its original version and other algorithms.

Chapter 7

Conclusion

In this chapter, the contributions and conclusions of this thesis are presented and suggestions for future work provided.

7.1 Contributions

The main contributions of this thesis are:

1. A new intelligent swarm-based optimisation algorithm called the Bees Algorithm, which is inspired by the food foraging behaviour of honey-bees, is presented in this thesis.

2. Enhancements to the algorithm are also presented, with proofs to show that the algorithm is both robust and efficient. The provision of a dynamic neighbourhood helped the algorithm to perform procedures faster. Proportional shrinking improved the performance of the algorithm in terms of solution quality and speed as well as better source usage. Site abandonment was introduced to increase the overall effect of random global search as well as to enable better source allocation.
3. Local search was introduced to deal with combinatorial domains more efficiently. Implementations to several different types of problems were suggested.
4. A new neighbourhood procedure was developed to deal with local search without the need for use of the shrinking method but with even better performance.
5. A reduction in the number of parameters was suggested, achieving real improvements to the ease of use when setting the parameters and running the algorithm.

7.2. Conclusions

In this thesis the swarm intelligence and swarm-based optimisation algorithms are discussed. Swarm intelligence is considered as the collective problem-solving capabilities of social animals. In the context of developing an algorithm, first the biological and morphological features of the honey-bees are presented. Some basic rules which create intelligence are presented, including individual and social level interactions. Mathematical simulation models are also presented as a bridging effort

between nature and engineering. These simulation models were helpful in understanding the interactions in honey-bee colonies at various levels. Food foraging models are reviewed in detail as one of the most successful and efficient part of the social life of honey-bee colonies. The key conclusions for each topic analysed are:

- A new swarm-based intelligent optimisation procedure called the Bees Algorithm is presented. The algorithm mimics the food foraging behaviour of swarms of honey bees. In its basic version, the algorithm performs a kind of neighbourhood search combined with random search. Also given are further details of the local and global search methods used in the algorithm. Details of the improvements made to local and global search methods are presented, including dynamic recruitment, proportional shrinking and abandonment strategies. The performance of the algorithm is evaluated on benchmark results, comparing them to those results achieved by some other well-known algorithms in the literature.
- The implementations of the algorithm to several continuous applications are also presented, including neural network training for a variety of industrial applications and recursive filter design. As a first implementation, the Bees Algorithm was used for the optimisation of the weights of multi-layered perceptrons for pattern recognition in statistical process control charts. A similar structure of neural networks was trained with the Bees Algorithm for the identification of defects in wood veneer sheets in a plywood factory. Lastly, a 2-d electronic recursive filter was designed using the Bees Algorithm to show its robustness.

- A new procedure is suggested as an addition to the Bees Algorithm to deal with combinatorial domains. The algorithm is applied to several complex optimisation problems with specific modifications for each. The algorithm is first implemented in a single job scheduling problem. The results are compared to some other well-known algorithms in the literature and it is shown that the modified algorithm performs better than or as well the others. The second application is to a permutation flow-shop sequencing problem. The algorithm used a slightly different neighbourhood strategy to deal with this complex problem. The algorithm performed well compared to other well-established algorithms. In the last example, the algorithm also uses a different neighbourhood search strategy due to the nature of the cell formation problem. The results show that the algorithm performs better for this domain.

- Lastly, enhancements to neighbourhood search and parameter numbers are presented. A new local search method is introduced to reduce the need for patch shrinking and for guiding the randomness. Also, the number of user defined parameters is reduced to a reasonable number by a new theoretical approach. The simulation results compared with those obtained by the original algorithm as well as by other well-known algorithms, are presented. The results show that the modified algorithm performs much better than its original version and than the other algorithms.

7.3 Suggestions for future research

Possible extensions that can be made to the work presented in this thesis include:

- Developing a mathematical model for the algorithm to improve the theoretical base and to explain the convergence behaviour.
- Developing an enhanced algorithm which improves on the balanced local and global search structure of the Bees Algorithm.
- Developing a new local search algorithm for combinatorial domains to increase the efficiency of the Bees Algorithm.
- Developing a discretization method for the algorithm to improve the existing local search procedure.
- Using both the original and improved versions for several different industrial applications to extend the scope of the algorithm.
- Creating hybrid algorithms to benefit from each others' strength.

APPENDIX A

C++ Code for the Bees Algorithm

```
#include "stdafx.h"
#include <stdlib.h>
#include <time.h>
#include <math.h>
#include <stdio.h>
#include <conio.h>
#include <iostream.h>
#include <limits.h>
#include <fstream.h>
#include <iomanip.h>
#include "params.h"
#include "func.h"

-----
// Parameters used in main.cpp
#define pop 1000          // max num of population

//Test parameters:
int R=1;                 // Number of runs (Number of different tests)
int imax=5000;           // Number of iterations

//The Bees Algorithm parameters:
int n= 10;               // Number of Scout Bees
int m= 5;                // Number of selected Locations
int e= 1;                // Elite point/s
int nsp= 4;              // Number of Bees around each selected locations
int nep= 10;             // Number of Bees around each elite locations
double ngh=0.1;          // Neighbourhood initial patch size
double sc=0.01;          // Shrinking constant; defined as percentage (%) and
range is between 0-1
-----
```

```

#define dim 2    // Dimensions of the test function
double start_x[]={-5.12,-512};
double end_x[]={5.12,5.12, };
double ans=0.001;

int NumOfEvalCounter=0;

double func(double x[][pop], int i ) // Definition of Fitness Functions
{
    NumOfEvalCounter++;
    double y;
    //Axis parallel hyper-ellipsoid function
    y=0;
    for(int j=0;j<2;j++)
        y=y+((j+1)*pow(x[j][i],2));
    y=-y;
}

-----

double randfunc( double xs, double xe ) // Definition of Randon number Generator
Function
{
    double randnum;

    randnum=rand() ;
    randnum=xs+randnum*(xe-xs) / RAND_MAX;

    return randnum;
}

-----

double myrandom() //Random generator
{
    double r;
    int M,x;
    M = 10000;
    x= M-2;
    r = (1.0+(rand()%x))/M;
    return r;
}

-----

void funcSort(double inP1[],double oP1[],double inP2[][pop],double oP2[][pop],int
size) // Sorting function
{
    //
    double temp1=inP1[0];
    int temp2;
    double temp1=-INT_MAX;

```

```

for( int j=0;j<size;j++)      //sort
{
    for(int k=0;k<size;k++)
        if(inP1[k]> temp1 )
        {
            temp1=inP1[k];
            temp2=k;
        }

    oP1[j]=temp1;

    for(int d=0;d<dim;d++)
        oP2[d][j]=inP2[d][temp2];

    temp1=-INT_MAX;

    inP1[temp2]=-INT_MAX;

}      //end sort

```

```

}

```

```

void main()

```

```

{

```

```

    cout<<"Program started...\n";

```

```

    int i,d,j,k,aa[100],ranSearchBees,counter,runs,fail,iter;
    double nghx, temp1, bPos[dim][pop], bNghPos[dim][pop], fit[pop],
    bNghFit[pop], sortedFit[pop], candidx[dim][pop], bPosSort[dim][pop];

```

```

    ofstream Result; //Opening a file to report the results...
    Result.open("Result.xls");

```

```

    srand( (unsigned)time( NULL ) );// Different random number each time

```

```

    fail=0;

```

```

    for(runs=0; runs<R; runs++) //R is the number of runs
    {
        NumOfEvalCounter=0;
        nghx=ngh; //define patch size

```

```

    //Initial Random distribution

```

```

    for(i=0;i<n;i++)

```

```

    {

```

```

        for(d=0;d<dim;d++)
            bPos[d][i]=randfunc(start_x[d],end_x[d]);
        fit[i]=func(bPos,i);
    }//End of random distribution

//Run until maximum number of iteration met
for(iter=0; iter<imax ;iter++)
{
    // Sorting fitnesses & positions
    funcSort(fit, sortedFit, bPos, bPosSort, n);

    counter=0;
    // Choosing best m
    for(i=0;i<m;i++)
        for(d=0;d<dim;d++)
            candidx[d][i] = bPosSort[d][i];

    //Recruitment stage
    for(i=0;i<m;i++)
    {
        if(i<e)
            aa[i]=nep; // Number of bees around each elite sites
        else
            aa[i]=nsp; // Number of bees around other selected sites
    }

    // Search in the neighbourhood
    temp1=-INT_MAX;
    for(k=0;k<m;k++)//k site
    {
        for(j=0;j<aa[k];j++) //j recruited bee
        {
            for(d=0;d<dim;d++)//d dimension
            {
                do
                {
                    bNghPos[d][j] = randfunc(candidx[d][k]-nghx , candidx[d][k]+nghx);

                    //bNghPos[d][j]=NRVG(candidx[d][k],StdDev);

                }
                while(bNghPos[d][j]<start_x[d] || bNghPos[d][j]>end_x[d]);
            }
        }

        bNghFit[j]=func(bNghPos,j);

        if(bNghFit[j]>= sortedFit[k])
        {

```

```

for(d=0;d<dim;d++){ bPos[d][counter]=bNghPos[d][j];
    candidx[d][k]=bNghPos[d][j];}
sortedFit[k]=bNghFit[j];
}

} // end of recruitment

counter++; // next member of the new list

temp1=-INT_MAX;          //

} // end of Neighbourhood Search

//Shrink all the patches using the shrinking constant (sc) variable
nghx=nghx*(1-sc);

// Send rest of the bees for random search...
ranSearchBees=n-m; // Number of bees for random search
for(k=0;k<ranSearchBees;k++)
{
    for(d=0;d<dim;d++)

bPos[d][counter]=randfunc(start_x[d],end_x[d]);

    counter++;
}

//Evaluate the fitness values of the new list
for(j=0;j<n;j++) fit[j]=func(bPos,j);

//Stopping Criteria
//    if (NumOfEvalCounter>50000) break;
if (fit[0] >= ans) break;

} //end iter = imax

cout<<"\n "<<iter+1<<"    "<<NumOfEvalCounter<<"    "<<fit[0];

Result<<iter+1<<"    "<<NumOfEvalCounter<<"    "<<fit[0]<<endl;
} //End of Runs
Result.close();

cout<<"\n Program finished...\n";
}

```

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