

Darwin College

TEACHING WITH TECHNOLOGY:
A Multiple-Case Study of Secondary Teachers’
Practices of GeoGebra use in Mathematics Teaching

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This thesis is submitted for the degree of *Doctor of Philosophy*

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Declaration

The following work was carried out at the Faculty of Education, University of Cambridge, during the years 2012 – 2015, under the supervision of Prof. Kenneth Ruthven.

I hereby declare that this thesis has not been submitted, in whole or in part, for any other degree, diploma or qualification at any other university. This thesis is the result of my own work and includes nothing that is the outcome of work done in collaboration. I have attempted to reference appropriately any idea or finding which is not my own.

This thesis does not exceed the limit 80,000 words in length.

Gülay Bozkurt

September 2015

Abstract

Although the influence of new technologies on education has been increasing over recent decades, the incorporation of technology, particularly into mathematics education, has been slow. It has become apparent that teachers have a central role in the integration of technology in mathematics classrooms, which needs more attention when it comes to researching this issue. In this regard, understanding the process that teachers go through to appropriate technology effectively into their instruction is of crucial importance to help facilitate the successful integration of technology into classrooms.

This research aimed at developing a more comprehensive understanding of technology integration in classrooms, by examining further a proposed model of key structuring features of classroom practice (Ruthven, 2009) which shape the use of technology in lessons and the kinds of professional knowledge required. Focusing on the use of GeoGebra (Dynamic Mathematics Software) in the context of English secondary school mathematics, this case study investigated the teaching practices, and craft knowledge of three teachers. In accordance with this conceptual framework, the classroom practices of these teachers, who were at different stages of using GeoGebra in their teaching, were analysed with the aim of shedding light on the character of the relevant expertise and the process through which it developed.

The case study analysis, using data triangulation of interviews and lesson observations, illustrated ways in which teachers adapted their classroom practices and provided some indications of the growth of their craft knowledge in the course of appropriating GeoGebra. The main conclusion was that although teachers' classroom practices with GeoGebra appeared consistent across the topics, the stage they were at in terms of learning to teach with this software indicated differences especially in regard to establishing a functioning resource system and appropriate activity formats, and to developing a script for handling those topics. It became evident that the most experienced teacher had been able to fine-tune his use of GeoGebra to his pedagogical approach: he was not only more fluent in shifting between lesson

segments and in managing his system of resources, but he had developed much richer curriculum scripts for the topics taught. This study also proved the usefulness of the Structuring Features of Classroom Practice framework for examining teachers' adaptation process with the use of technology and provided constructive contributions to the development of this framework.

Keywords: Classroom teaching, digital technology integration, mathematics teaching, teacher learning, craft knowledge, structuring features of classroom practice.



To the memory of my dearest uncle, Hüseyin Turhan...

Dayım Hüseyin Turhan'ın anısına...

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List of Abbreviations

BERA: British Education Research Association

CAS: Computer Algebra Systems

DGS: Dynamic Geometry Software

DME: Digital Mathematics Environment

GSP: The Geometer's Sketchpad

ICT: Information and Communication Technology

IT: Information Technology

IWB: Interactive Whiteboards

NCTM: National Council of Teachers of Mathematics

OFSTED: Office for Standards in Education

PGCE: Postgraduate Certificate in Education

SFCP: Structuring Features of Classroom Practice

TIMSS: Trends in International Mathematics and Science Study

TPACK: Technological Pedagogical and Content Knowledge

UK: United Kingdom

VLE: Virtual Learning Environment



1

INTRODUCTION

1.1 Research Context

In many countries, the use of new technologies has been seen as a way to improve teaching and learning in schools. In particular mathematics was one of the education domains where computers were first introduced as an assistive vehicle in the classroom (Papert, 1980), leading to numerous years of research focused on the integration and embedding of technology into pedagogical practice. The National Council of Teachers of Mathematics (NCTM) states that “Technology is an essential tool for learning mathematics in the 21st century, and all schools must ensure that all their students have access to technology” (NCTM, 2008, p. 1). There have been major investments in equipping schools with computers, creating suitable mathematics resources, as well as training and encouraging teachers to use these resources. In spite of this, the Trends in International Mathematics and Science Study (TIMSS) 2007 research (Mullis, Martin & Foy, 2008) has shown that use of technology remains relatively low even amongst teachers who have access to technology and have received training in its use.

The picture painted by Lagrange et al. (2003) of research on technology within mathematics education is “of a field where publications about innovative use or new tools and applications dominated” and “converged in a focus on the student and in an emphasis on epistemological and semiotic aspects” (p. 248). Therefore, not only the role of the teacher has been notably absent, but also issues of the integration of technology into ordinary classrooms have been largely neglected. In recent years,

therefore, researchers have turned their attention to the teacher dimension of technology integration.

Research projects focusing on teachers' use of technology in mathematics classrooms (Laborde, 2001; Monaghan, 2004) have recognised the complexity of introducing new technologies into teachers' practice in mathematics teaching. These studies have reached some conclusions regarding teachers' failure to change their practices with the use of new technologies contrary to their optimism that technology would support a greater focus on conceptual understanding. It has become apparent in these studies that teachers tended to utilise technology in the classroom with the intention of not moving far from their established routines. Furthermore, other studies (Lagrange, Dedeoglu & Erdogan, 2005; Ruthven, 2007) have indicated that teachers' plans might sometimes be overly optimistic regarding the potentials of technological tools. There is the additional chance that they might encounter unforeseen problems once in the classroom. In regard to ordinary teaching practices with technology, "a persistent observation is that unexpected phenomena and discrepancies between what was expected and what really happened frequently occur" (Lagrange & Ozdemir Erdogan, 2009, p. 66).

Researchers have suggested that teachers are faced with the need to consider a number of classroom management issues in technology-rich settings, which challenge their existing teaching practices (Lagrange & Ozdemir Erdogan, 2009; Ruthven, 2009). They do not need only to develop new types of interactions (Monaghan, 2004; Drijvers et al., 2010) but they must also manage different kinds of time in their classrooms (Assude, 2005). There is also a need to establish a relationship between a technological environment and a paper and pencil environment in order to "build connections with the official mathematics outside the microworld", which is considered an important aspect of the teacher's role by Guin and Trouche (1998, p. 200). Furthermore, some researchers (Monaghan, 2004; Lagrange et al., 2005) have argued how technology could affect the emergent goals of the teacher during the lesson. When a teacher integrates Dynamic Geometry Software (DGS) in order to make students engage with the task, s/he must consider not only a mathematical learning trajectory, but also the instrumental aspect of this including the operation of the tool

(Lagrange et al., 2005). In this light, it is not surprising that the process of orchestrating technology-integrated mathematics learning is not straightforward, in particular for teachers who are novice in technology use since the availability of digital resources leads to “change in teachers’ professional knowledge and development” (Gueudet & Trouche, 2009, p. 199). In respect to this, Tabach (2011) showed that classroom practices of teachers in relation to technology use influence their knowledge as well as their thinking, and vice versa. The two-way situation, involving both activity and knowledge, may change. Similarly, a number of studies (e.g. Monaghan, 2004; Drijvers, 2012) have shown that teachers’ practices with technology over time have generated professional growth as teachers can revise and adapt their teaching in relation to technology from feedback and their own personal experiences. In other words, teachers build up knowledge based on the reflection of their own classroom experiences and the strategies they develop when unexpected issues occur in the classroom as a result of applying techniques instrumented by the available technological tools. In this regard, teachers who are experienced in technology use, as well as in teaching, can provide a great deal of insight into pedagogical changes and how it can be achieved, which may still be vague for novice teachers. In line with this, Crisan, Lerman, and Winbourne (2007) found that it was important for teachers to have access to experienced people from whom they can learn more about the ways technology may improve their classroom practices in mathematics teaching. In addition, Meskill et al. (2002) concluded the relative merit of classroom experience over formal training:

Indeed, those novice teachers who had received “state of the art” training in classroom technologies were far less comfortable in their implementation than the more experienced teachers who had no formal training with computers but a great deal of classroom experience. (p. 9)

Based on the discussion above, as a starting point, two fundamental questions need to be addressed: **"What aspects of classroom teaching shape the ways in which teachers integrate new technologies?"** and **"What kind of professional knowledge do teachers need to teach with them?"**

The purpose of this study is to develop a more comprehensive understanding of technology integration in classrooms. The underlying goal of the inquiry is to provide a picture of how technology lessons are structured and what kind

of professional knowledge might be involved. Researchers, in particular Leinhardt and her colleagues (e.g. Leinhardt, 1989), have studied the novice-expert contrast in mathematics lessons and highlighted the nature of the competencies expert teachers possess. Similarly, I will study an expert technology-using teacher's classroom practices and contrast them with those of novice teachers, which has the potential to shed considerable light on the role of knowledge and experience in the development of pedagogical expertise.

1.2 Rationale and Significance of the Study

First of all, this proposed study aiming to re-focus on the teacher dimension (Lagrange et al., 2003) is essential, given the premium that schools are now placing on technology integration, to achieve an understanding of the teacher's crucial role. As a prospective teacher educator in Turkey, this study also provides an opportunity in particular for me to build an understanding of the conditions of technology in ordinary classrooms, and of how teachers adapt new technologies to their schooling. Thus, bringing new directions for teacher education with the aim of tackling teachers' problems by appreciating empirical realities is my long-term goal.

Secondly, the significance of these two rather mundane questions is perhaps most apparent in mathematics education since the type of teaching expertise for adaptation of new technologies into their classroom practices is still not well identified and analysed. Much of the literature is based on a series of factors influencing teachers' use of technology, which tend to consider managerial aspects of teaching with technology rather than exploring teachers understanding qualitatively. In addition, research on the use of instructional technology mostly focused on the type of technology but not the process of implementation or teachers' adoption of technology at a classroom level (Lagrange et al., 2003). For instance, although there is a considerable amount of research and literature on the innovative use of Computer Algebra Systems (CAS), in particular in a French context, teachers' use in ordinary classrooms is not common. This means that there is little reported about the nature of teachers' pedagogical understanding about the use of new technologies as part of their teaching, and the different ways teachers discover to include technology. When considering studies emphasising technology and the teacher, Monaghan (2004)

argued that these “have largely focused on teachers’ beliefs and forms of teacher knowledge” and they “by design, miss the wholeness of teachers’ practices” (p. 330).

The research to be reported here is therefore distinctive in studying teachers’ use of technology in classrooms and in seeking to understand associated expertise on the part of the teacher that is used in their everyday classroom practice. In terms of teachers’ professional development, this study can contribute to the field through making accessible diverse and successful approaches to other teachers, a ‘building on strength model’ (Brown & McIntyre, 1993). In particular, the importance in teacher education is that it can enable beginning teachers to gain access to such knowledge.

Additionally, this study offers an opportunity to observe and to find out about ordinary¹ teachers’ appropriation of new technologies into their practices, since previous research has paid insufficient attention to teachers’ use of technology in ordinary classrooms (Ruthven & Hennessy, 2002). Therefore, Ruthven's (2009) conceptual framework, which was developed by synthesising observations from recent studies of what teachers do with new technologies in ordinary classrooms, will have the potential to offer a tool for a deeper examination and interpretation of what happens in classrooms. However, this conceptual framework is tentative and Ruthven (2012) suggests that “further studies are now required in which data collection (as well as analysis) is guided by the conceptual framework so that it can be subjected to fuller testing and corresponding elaboration and refinement” (p. 100). This study has the potential to test regarding the suitability and value of Structuring Features of Classroom Practice (SFCP) framework as an interpretative lens for research into teaching with technological tools. In this regard, the second aim of this research is to contribute to the development of this model, its evaluation in terms of affordances and limitations since it is crucial to “[c]onduct research that makes use of these theories to test their operational value and domain of validity” in order to advance research on technology integration (Goos et al., 2010, p. 327).

1 The phrase ‘ordinary teachers’ refer to those teachers who use technology in ‘ordinary conditions’, that is to say in their everyday classrooms, rather than those who try to use technology innovatively in externally supported experimental projects.

1.3 Scope of the Study

I aimed to examine how the use of technology develops in general, but what is covered by the term “technology” in classroom use is very varied, and so it is desirable to focus on some specific type of widely used technology. In this regard, Dynamic Mathematics Software, GeoGebra will be the particular focus of my interest due to its easy accessibility and its relatively widespread use amongst teachers.

GeoGebra is open-source educational software package, which provides dynamic mathematical representations (Hohenwarter & Preiner, 2007) linking together the features of a dynamic geometry system (that works with points, vectors, segments, polygons, lines and conic sections) and a computer algebra system (in that equations and coordinates can be entered directly and functions can be defined algebraically and then changed dynamically). GeoGebra’s display is divided into two parts: an ‘Algebra window’ (left side) and a ‘Geometry and Graphics window’ (right side) (see Figure 1.1). The geometry toolbar contains tools organised into several main groups and each tool is briefly described which helps to orientate the users.

GeoGebra provides a simultaneous display of these two representational systems, hence permitting mathematical connections to be made between them. In this regard, one of the main affordances of GeoGebra is a bidirectional representation of every object: every input in the algebra window has a simultaneous and corresponding object in the geometry window and vice versa providing relationships between geometry and algebra. The drag mode of GeoGebra allows free movement between windows and automatically adjusts to any change in the algebraic representation (Chrysanthou, 2008).

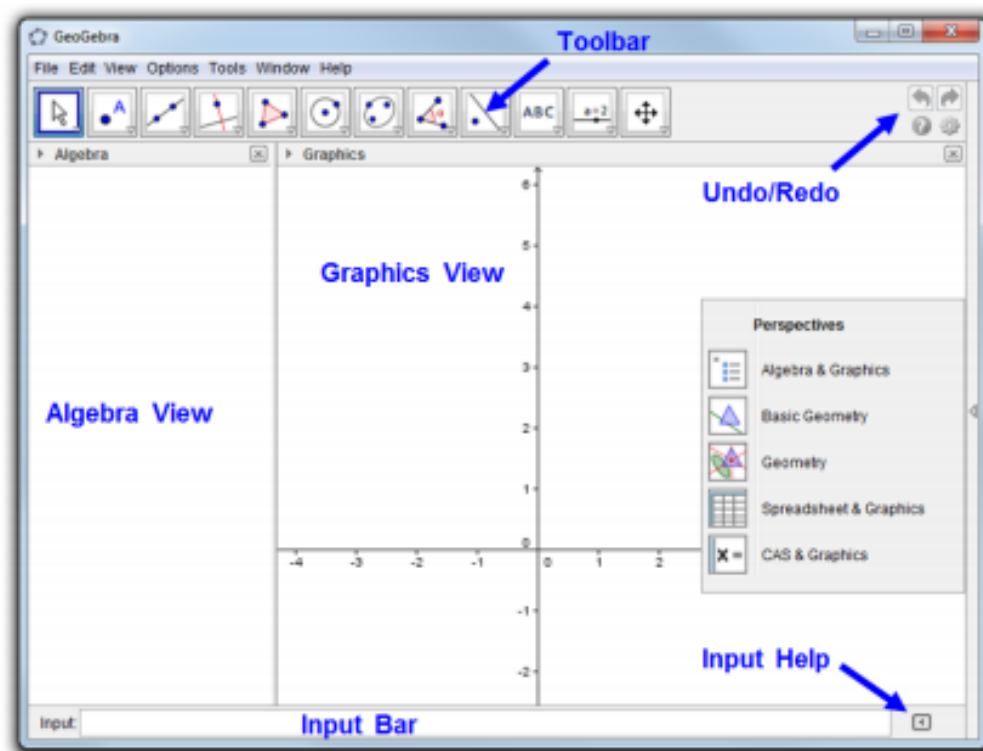


Figure 1.1 Default GeoGebra screen showing both the algebra and geometry windows

The uniqueness of GeoGebra is that it is free, multi-platform, open-source software. Therefore, there is no licensing issue associated with its usage. It is not only accessible on conventional desktop and laptop computers, but the GeoGebraMobile also affords its use in the modern web browsers available on tablet devices and mobile phones. This allows students and teachers, irrespective of location, to access the large pool of GeoGebra dynamic resources on virtually all devices with a web browser. This might be one of the reasons why teachers choose GeoGebra over commercial software that offers similar potentialities such as The Geometer's Sketchpad (GSP), Cabri-géomètre or Autograph (Hohenwarter and Lavicza, 2007). For instance, Erfjord (2008) in his Ph.D. thesis indicated that teachers regarded the fact that Cabri is not free of charge as limiting teachers' and students' access to Cabri at home and so as an issue for successful implementation.

Furthermore, the evolving development of GeoGebraTouch allows the adjustment of use with an Interactive Whiteboard (IWB) (Hanc et al., 2011; Ancsin, Hohenwarter & Kovac, 2011). An archive of online resources is available – GeoGebraWiki and

GeoGebraForum- which are free and available to all users as a collaborative platform and community of practice in over 50 languages (Hohenwarter, Jarvis & Lavicza, 2009; Lavicza et al., 2010). GeoGebra features easily web-exportable, so called dynamic worksheets that “have led many teachers to foster experimental and discovery learning for their students and to share thousands of such worksheets on the GeoGebraWiki.” (Hohenwarter & Lavicza, 2007, p. 51). Through these websites, teachers and students all over the world share a number of GeoGebra related worksheets and exercises. Additionally, the GeoGebraTube, a new online platform, which aimed to make it easier to share, rate and comment on interactive GeoGebra web pages and constructions, has been developed.

While there is evidence of considerable professional interest in the use of GeoGebra amongst secondary mathematics teachers in England, how best to develop such use and integrate it into ordinary teaching practice is not yet well understood (Jones et al., 2009). Therefore, it is of crucial importance to study the practices of teachers who are recognised as having successfully developed and integrated use of GeoGebra into their classroom practices as well as the ones who are technologically less experienced in the use of GeoGebra.

1.4 Outline of the Thesis

This thesis is organised in six chapters. The current chapter has introduced the problem addressed in this study, the need for the research study in the educational context, general contributions of the present work, and the scope of the study.

The second chapter presents a review of theoretical and research literature illuminating the use of technology in mathematics education. This is organised around the key themes of the Structuring Features of Classroom Practice framework, explaining the derivation of the model, describing its key components, and giving some examples of these concepts from the research where they originated, furthermore relating each component of the model in relation to technology use.

The third chapter states the philosophical stance and methodology that I aim to employ to conduct the study. It also gives an account of the rationale for the research

design, including the methods for data collection and analysis undertaken as well as the ethical considerations of the research.

The fourth chapter examines the individual teachers and their lessons, comparing the lessons given by each teacher in a within-case analysis. The fifth chapter then reports a further cross-case analysis in which different teachers' use of GeoGebra for the same topic is compared, and similarities/differences between their approaches are discussed. The sixth chapter discusses the findings, limitations and implications of the study, and makes recommendations for the future studies.





2

LITERATURE REVIEW

2.1 Introduction

The purpose of this chapter is to develop a conceptual framework for describing, exploring and analysing teachers' use of technology in classrooms. This chapter articulates the chosen conceptual model by structuring a literature review around the five components of the model and weaving in the literature pertaining to the relevant elements of technology integration.

2.2 Conceptual Framework

Researchers (e.g. Burns, 1984; Miles & Huberman, 1994; Yin, 2009) highlight that it is difficult to make sense of the large amount of data potentially available through classroom research without a conceptually based organising framework.

It is clear that a better conceptualization of the context of teaching and the classroom environment would result in better observational data in general, and better time-use data in particular. (Burns, 1984, p. 119)

In this regard, I aimed to have a list of initial ideas for conceptualisation of teachers' classroom practices prior to fieldwork rather than starting from scratch. In particular, considering that a number of research studies have already focused on the practices of secondary mathematics with the explicit aim to offer theories regarding technology uses within school settings (Artigue, 2002; Ruthven & Hennessy, 2002; Monaghan, 2004; Godwin & Sutherland, 2004; Ruthven, Hennessy & Deaney, 2005;

2008; Drijvers et al., 2010; Clark-Wilson, 2013).

In searching for a framework within which to conveniently locate the research on teachers' classroom practices with the use of new technologies, the recently conceived framework described by Ruthven (2009) as Structuring Features of Classroom Practice (SFCP) appeared a logical choice. He takes a naturalistic approach, and focuses on constructs that are directly related to teachers' classroom practices. By reviewing teaching and schooling in general, he calls our attention to five constructs, which have already been developed, defined and used to evaluate what happens within classrooms. These are working environment (e.g. Rivlin & Weinstein, 1984; Horne-Martin, 2002), resource system (e.g. Cohen, Raudenbush & Ball, 2002), activity format (e.g. Burns & Anderson, 1987; Burns & Lash, 1986), curriculum script (e.g. Putnam, 1987; Leinhardt, Putnam, Stein & Baxter, 1991), and time economy (e.g. Anderson, 1981; Assude, 2005). Although one or another of these concepts has been emphasised at different points in time by different researchers, their interrelated functions are already recognised as: "the activities teachers select and the way they allocate attention and use space, time, and resources in classrooms are influenced by the demands of a particular classroom environment." (Doyle, 1979, p. 140)

Recently, Ruthven has developed these concepts as a form of framework, which will be used here, for reviewing research and thinking about teachers in classroom environments. Ruthven's (2009) motive for developing an explicit framework was to attempt to describe teaching practices with new technologies and to explain the gap between expectations and classroom realities. He develops the SFCP framework through grounding it in his earlier work to create a 'Practitioner Model'. The Practitioner Model (Ruthven & Hennessy, 2002) was elicited through group discussions between teachers about what they saw as the contribution of new technologies; it focused on teaching aims, and the classroom states that support these. However, some studies show that the Practitioner Model is not adequate to describe contextualised teaching practices as it characterises mostly teachers' ideals related to potentials of new technologies (Lagrange et al., 2005; Ruthven, Hennessy & Deaney, 2008). Hence, the SFCP framework highlights the fact that structuring features of classroom practice

influence the way in which teachers actualise their idealistic use of technology described in Practitioner Model.

2.2.1 Structuring Features of Classroom Practice

I will now explore the SFCP model, not only in general terms explaining the derivation of the model, describing its key components, and giving some examples of these concepts from the research in which they originated, but also each component of this model in relation to technology use.

2.2.1.1 *Working Environment*

When one looks at the educational scene, it is impossible to deny the term ‘working environment’ (e.g. classroom) where teacher and students work (e.g. teach/learn/interact) during lessons. However, until the 1970s, educators tended to ignore the fact that physical aspects of classroom environments could affect learning and teaching (Weinstein, 1979; Rivlin & Weinstein, 1984). The emergence of ecological (environmental) psychology stimulated a recognition and awareness of the interrelationship between physical environment and human experience in the education context. As Williams, Armstrong, and Malcolm (1985) states,

Four elements functioning together- an organization, its members, its work and its physical environment- form an ecology, an ever-changing web of relationships that aims to accomplish whatever the organization exists for- to educate people, to make furniture, or to process insurance claims (p. 5).

In conjunction with this, researchers in education also indicated that physical features of a classroom milieu could have an impact not only on behaviour of students and teacher, but also on educational programmes (Rivlin & Weinstein, 1984). It has been recognised that a working environment can be an important instrument for teaching (Horne-Martin, 2002). Researchers indicated that successful classroom practices are related to teachers’ understanding and awareness of their classroom environment. In this way, teachers can use deliberately organised space to facilitate and support physical activity for learning (Horne-Martin, 2002).

Traditional classrooms have a typical physical surrounding; the students' desks are arranged in rows, a horseshoe, or perhaps clusters of four, while the teacher's desk is positioned in the front near the blackboard. In this sense, teachers might not have much choice in actively participating in the design process of classroom layout. This is another reason why researchers started paying attention to the effects of the working environment on learning/teaching when the term “open education classroom” was introduced, which encouraged the design possibilities for teachers (Weinstein, 1979).

Similarly, the entry of new technologies into education also stimulates the importance of the term ‘working environment’, as different arrangements in a working environment are possible depending on teachers’ pedagogical goals, in particular with the use of portable technologies. Even a small change in the classroom organisation can lead to a significant change in interaction between teacher-student and student-student (see Figure 2.1).

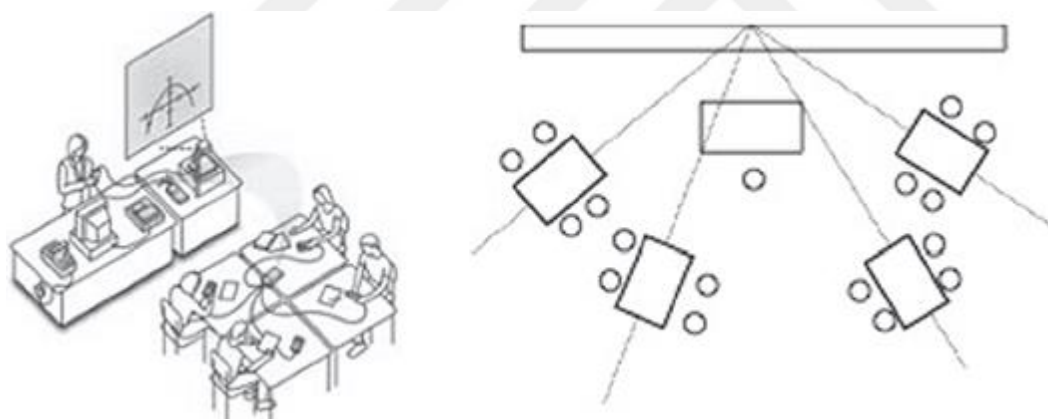


Figure 2.1 Different classroom organisations (Hoyles et al., 2010, p. 449)

Furthermore, use of new technologies may require changing room locations, perhaps moving into a computer suite where students could also use computers (which could require additional management for teachers). Re-locating lessons into the computer suite may require both teachers and students to adapt their routines from the regular classroom to a computer room.

The kinds of resources that exist in a working environment and the kind of activities that are stimulated in such a working environment are the important aspects that

could, to some extent, determine or affect teachers' technology utilisation according to their pedagogical goals. For instance, in Drijvers' et al. (2010) study, one of the teachers stated that the features of the working environment such as the arrangement of the tables, the way the computers were positioned, and availability of Internet access drove her choices of activity formats. In addition, teachers might prefer to teach in a normal classroom rather than a computer room because the computer room might not be appropriate for whole-class teaching (Drijvers et al., 2010).

In addition to objective features of a working environment, a teacher's perception and understanding of such an environment can have also an impact on the way in which they use technology. For instance, Tabach (2011) examined a mathematics teacher's practices during four lessons in a technological environment in order to find out if there was a relationship between her knowledge and practice. The teacher in all four lessons did not use technology during the initiation and summary phases. The physical setting of the classroom was considered one of the reasons for non-technology use during these phases. However, the teacher indicated in her diary that understanding of working environment had an influence on her activity structures for subsequent lessons.

During this lesson, just like in a previous lesson, during the summary phase I find myself asking the students to use their own computers to show what other students did. I wonder if this is a new development of me as a teacher, because I did not use it before. Maybe I understand better the environment in which I am working. For the next lesson, I will try to plan how to use this during the initiation phase as well. (p. 262)

Similar to Ruthven's concept of working environment, Drijvers et al. (2010) used one of the concepts of the Instrumental Orchestration framework called 'didactical configuration' in order to show an arrangement of technological tools in a classroom environment. The Instrumental Orchestration framework is applied as a theoretical tool to analyse teachers' classroom activities with new technologies and is defined as follows:

[...] the teacher's intentional and systematic organisation and use of the various artefacts available in a—in this case computerised— learning environment in a given mathematical task situation, in order to guide students' instrumental genesis. (p. 214)

The researchers focused on mainly whole-class teaching and showed that teachers' use of Digital Mathematical Environment (DME) software did not create many organisation issues. For instance, 'Technical-demo' or 'Explain-the-screen' orchestrations, namely how teachers organise the classroom activity, only required access to the DME software and equipment to project the computer screen and a classroom arrangement for students to see the screen for demonstrations. The Technical-demo was used to demonstrate techniques of a new tool such as what could be possible with the tool and how to use it. The Explain-the-screen orchestration was used to explain what happened on the computer screen to the whole class, and also sometimes used to provide students with a good starting position for new tasks.

2.2.1.2 Resource System

Resources play a key role in teachers' practices. Therefore, this concept has always received attention from researchers in education. Initially research focused on access to and allocation of resources in schools, assuming that the quantity of resources would influence students' performance (e.g. Miles & Darling-Hammond, 1998). When no direct link between these two variables was found, some researchers shifted their interest towards looking into how resources were used during classroom instructions and claimed "resources in practice in context need to become a focus of attention" (Adler, 2000, p. 221). The aim was to observe how resources came together and became active in teaching and learning through their use by teachers and students during instructions. As Cohen, Raudenbush, and Ball (2002, p. 91) write: "Even the best curriculum materials are of little consequence if teachers cannot turn them to advantage in framing tasks for students, or if students cannot understand or engage them."

Researchers have proposed specific conceptualisations of the term 'resource'. Adler (2000) suggests using the term "resource" as "the verb 're-source', to source again or differently" (p. 207) in a very broad sense which includes human resources, cultural resources and material resources. This concept also appears in the recent theoretical notion, Documentational Genesis, introduced by Gueudet and Trouche (2009) who emphasise the distinction between resources and documents. The

researchers extend the idea in the instrumental approach of the relation between artefact and instrument. They used the term of ‘resource’ instead of the term ‘artefact’ with the intention of emphasising a number of things that teachers could use such as a textbook, a piece of software, a student’s worksheet, an Internet resource, or a discussion with a colleague. The researchers point out that teachers develop their own documents (a special class of instruments for teaching) from available resources through the process of Documentational Genesis. However, the concept of resource system in the SFCP framework does not fully overlap with the above descriptions, which also include non-material elements, such as a discussion with a colleague. Ruthven (2009) suggests that the term resource system consists of those aspects of the artefacts in uses that are directly and deliberately related to teaching and learning of the specific topic/subject in question. Also, he points out the difference between a collection of resources and a coherent system, using the concept of ‘resource system’ to emphasise the coherent use of resources according to curricular materials and pedagogical goals –as Cohen, Raudenbush, and Ball (2002) write: “resources are not self enacting, and differences in their effects depend on differences in their use.” (p. 80)

With the influx of new technologies in schools, on the one hand classrooms are filled with resources, which can be considered as providing more opportunities for learning. On the other hand, the usability and adaptability of new technologies can present challenges for some teachers and students since “resources are not self explanatory objects with mathematics shining clearly through them” (Adler, 2000, p. 207). It is because the technology’s relative advantage, complexity, and compatibility with the teacher’s existing beliefs, pedagogy, and environment have an impact on the technology’s adoptability (Rogers, 2003). As Cohen, Raudenbush and Ball (2002) put it, “When conventional resources appear to affect learning accomplishments, it is because they were usable, because teachers and students knew how to use them effectively, and because environments enabled or did not impede use by these particular teachers and students” (p. 102). For instance, in a study by Lagrange et al. (2005), one of the teachers expected Dynamic Geometry Software (DGS) to facilitate routines and make students engage with the task. However, there were discrepancies between the teacher’s expectations and what actually happened in the classroom, as she had not thought through the complexity of DGS for students and the operation of

the resource system. When students used DGS in the classroom, the teacher ended up spending most of her time trouble-shooting the difficulties that they experienced because they were not sufficiently familiar with the DGS and proficient in using it. The teacher did not take into account the complexity of the operation system of the software and the fact that pupils were not sufficiently au fait with this software. However, Monaghan's (2004) study focusing on technologically less experienced mathematics teachers' use of technology suggested that teachers' design of worksheets initially concentrated on the technology itself such as technology specific commands, afterwards they changed the focus towards how to do mathematics with the technology such as how to input mathematical expressions. Accordingly, Monaghan (2004, p. 342) argues: "There is a sense in which this is quite a natural development: if you want to do mathematics on a computer package, you first have to learn how to use the package."

Researchers have shown that the challenge of adaptability of dynamic geometry software lead the majority of teachers to use them in the more conventional types as "a reduced and static use of the possibilities of the software", and "the absence of autonomous experimentation by students" (Laborde, 2001, p. 299). Additionally, Ruthven et al. (2008) argues that there is an 'interpretative flexibility' in the incorporation of dynamic geometry software into educational practices. The researchers highlight the difference between how mathematics educators interpret DGS to be used in classrooms (e.g. more open student exploration) and how more typical teachers use it in their teaching.

Where dynamic geometry has entered mainstream classrooms, it appears to be used to support more established forms of pedagogical practice, notably student activity directed towards empirical confirmation of standard curricular results, often through guided discovery, as already prevalent in the teaching of geometry in many educational systems. (Ruthven et al., 2008, p. 314)

In terms of the design of tasks, Laborde's (2001) study indicated that it was challenging for teachers to go beyond textbook tasks when using DGS. Development of task design scaled from almost traditional geometry tasks, to tasks that could only be approached in a dynamic geometry environment. The former indicated tasks typically applied in a paper-and-pencil environment often supported by a textbook and by applying available tools such as compasses and ruler. Using DGS on such

tasks only facilitated drawing the shapes more accurately and quickly. In this regard, Laborde emphasised the fact that integration of DGS into mathematics teaching is a lengthy process. On the other hand, Monaghan's (2004) study indicated that most of the teachers managed to leave aside their textbooks in their technology-based lessons. However, the tasks in the worksheets "emphasised students' management of the computer software per se" as mentioned before. Similarly, findings from Erfjord's (2011) study investigating teachers' initial use of DGS in Norway indicated that two teachers used DGS with prepared instructional material focusing on the technical aspects of using DGS and had little explicit focus on the mathematics by mostly demoing for students what they needed to do in technical terms. This offered students a good background to further develop their ability to use compasses. Dragging mode was utilised for checking the accurate use of DGS rather than for provoking mathematical interpretation with the aim of improving students' abilities of using compasses. Furthermore, Assude (2005) also reported that in terms of use of compasses and Cabri there was not "any major changes with regard to the broad types of tasks: construction, description and property identification." (p. 192). Similarly, Ruthven, Hennessy, and Deaney (2005) argue that the typical use of DGS proposed by teachers in England is to let students work with geometrical properties utilising the dynamic dragging-function; further that many teachers in their teaching tried to control and constrain student work in order to avoid students spending too much time on the exploratory affordances in the DGS.

2.2.1.3 Activity Structure

Activity structure is an approach, which ecological psychologists developed from the idea of the behavioural setting in order to conceptualise the environment of human behaviour (Barker, 1968; Barker & Gump, 1964). It has been applied in particular to the instructional environment of classrooms (e.g. Doyle, 1977). From this point of view, classroom lessons are considered as a sequence of activity segments (Gump, 1964). A segment has a temporal boundary with a specific goal. Researchers (e.g. Stodolsky, 1983; Burns & Anderson, 1987), who examined the features of lesson segments, highlighted the term of **activity format** (etc. seatwork, lectures, group work), which "structures the activity and provides the organizational means by which learning tasks will be accomplished." (Burns, 1984, p. 103) Grouping arrangements,

and delivery system define the activity format. There is an interrelationship between these two components as choosing one specific delivery system might require a specific grouping arrangement. Also, these two components of activity format determine and affect the role expectations of, and consequently interaction between teacher and students (e.g. Stodolsky, 1983; Burns, 1984; Burns & Anderson, 1987). For instance, a lecture activity format defines, to large extent, the role of the teacher and student (e.g. teacher presents information in a whole class setting, students listen passively), how they will interact (e.g. students can ask questions when teacher invites them), and who has primary responsibility for pacing (e.g. the teacher).

As a whole, the components of segments (e.g. activity formats, the purpose and content of the segment) as well as the teacher and student roles, and the behavioral interaction, define the activity structure of lesson segments. In this regard, this concept can characterise not only what the typical activity format is, but also who is present and what kind of interaction takes place. Burns and Lash (1986) used the activity structure concept as a methodological tool to describe problem-solving instruction and examine whether it was different from basic skills instruction. It appeared that whole class recitation and seatwork activity formats continued to mostly persist for both basic skills and problem-solving instruction.

In a similar vein, with the influx of new technologies into schools, researchers also started examining activity structures in relation to technology-use in order to make sense of the instructional environment. What kind of activity structures appear in technology based lessons? Are they different from non-technology based lessons? For instance, Monaghan (2004) studied activity structures with the introduction of new technologies compared to non-technology based lessons. He found that non-technology lessons have a similar general picture: teacher exposition and examples followed by students doing questions from a textbook. However, technology-based lessons tended to include more open tasks where teachers aimed to reduce teacher exposition. Monaghan (2004) found that when teachers used technology in mathematics lessons, they spent more time speaking to two students or small groups of students and less time speaking in plenary to all of the students. In this sense, he described a typical pattern of work observed: short introduction to the task, worksheets which directed student activity, teacher circulating around the class

ensuring the students were working and attending to mathematical or technical issues (Monaghan, 2004, p. 336).

In a more micro level, some researchers examined what specific activity formats look like in technology classrooms. The instrumental orchestration lens was used to describe observed teaching practices on the use of applets (Drijvers, 2012; Drijvers et al., 2010). Six orchestrations for whole-class teaching were identified: Technical-demo, Link-screen-board, Discuss-the-screen, Explain-the-screen, Spot-and-show and Sherpa-at-work. Additionally, Work-and-walk-by was observed for the setting in which students work individually or in pairs with technology in Drijvers' (2012) study. With the aim of developing the framework, Drijvers' et al. (2013) elaborated Work-and-walk-by and identified five types of individual orchestrations during teacher walking by in the classroom whilst the students sitting individually or in pairs in front of their technological devices (see Figure 2.2).

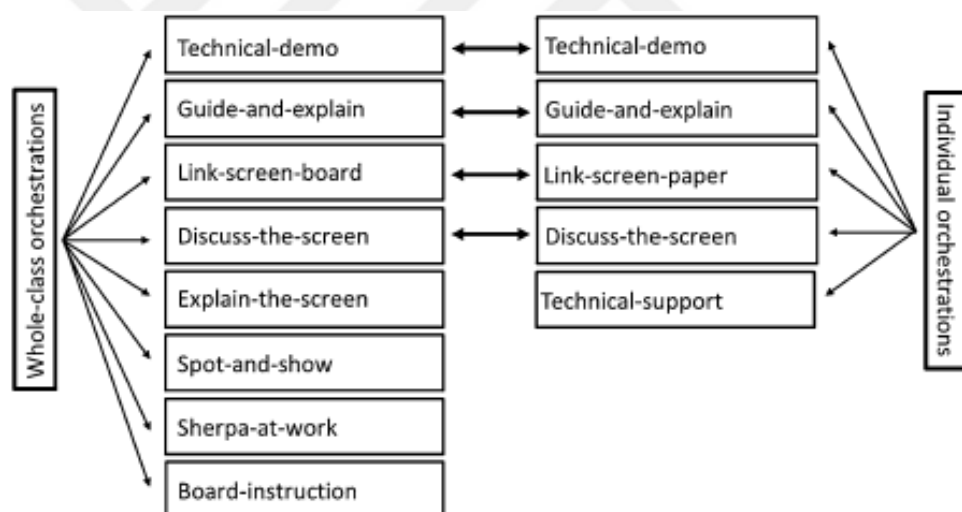


Figure 2.2 Overview of whole-class and individual orchestrations (Drijvers et al., 2013, p. 998)

However, these formats have different characteristics in terms of the structure of interaction, according to whether it focuses on students' inputs (Spot-and-show, Sherpa-at-work, Discuss-the-screen) or on teacher instructions (Technical-demo, Explain-the-screen, Link-screen-board). Additionally, not all of them are uniquely influenced by technology-use. For instance, Discuss-the-screen is a technological

alternative of a regular teaching practice as this can be considered Discuss-the-board in non-technology based lessons, where a teacher and students have a discussion about a problem on the board. On the other hand, there are also some that are more specific to technology-use. Link-screen-board is perhaps the most obvious example of this kind. Drijvers et al. (2010) showed that one teacher used this activity format with the intention of showing students the relationship between using a technological environment and a paper and pencil environment. She developed this idea independently, as the teacher-guide, which was developed by the designers of the computer environment, lacked such an activity format. Activity structures can also vary over time for each teacher as teachers develop more expertise from their own personal experiences. Drijvers (2012) showed that there was a gradual shift in the structure of interaction between teachers and students as well as student questions. The focus became more on algebra and less on technical issues since both teacher and students during the teaching sequence got more knowledgeable about the online module, its technical demands and its feedback. Consequently, the structure of these interactions switched from technical discussions into ‘Explain-the-screen’ or ‘Discuss-the-screen’ interactions. Furthermore, Drijvers’ et al. (2013) study concluded that mid-adopting teachers’ practices with technology (those who are less-experienced in technology-use) exploited activity formats that did not differ much from what was usual in their teaching. In particular, for whole class instruction, the researchers noticed that Board-instruction, the traditional whole-class orchestration of the teacher standing at the board without using technology, was relatively frequent.

2.2.1.4 Curriculum Script

Curriculum script as a concept was developed by Putnam (1987). This concept emerged from his research on how teachers structured and adjusted content for students during tutoring. Putnam (1987) showed that experienced teachers, while tutoring individual students, did not produce student specific instructional material from a remedial/diagnostic perspective. Rather, they moved through a curriculum script, which is “a loosely ordered but well defined set of skills and concepts students were expected to learn, along with the activities and strategies for teaching this material” (p. 13). The researcher concluded that the essential feature of the

curriculum script concept was the focus on the curriculum rather than idiosyncratic knowledge of individual students.

The term “script” was used “as a knowledge structure, similar to a schema, that provides expectancies and structures for interpreting situations and acting in them.” (p. 94). The concept of “script” was also defined by Abelson (1981) as follows:

In sum, a script is a hypothesized cognitive structure that when activated organizes comprehension of event-based situations. In its weak sense, it is a bundle of inferences about the potential occurrence of a set of events and may be structurally similar to other schemata that do not deal with events. In its strong sense, it involves expectations about the order as well as the occurrence of events. (Abelson, 1981, p. 717)

In this regard, the curriculum script provides the overall structure for the lesson agenda (see Figure 2.3).

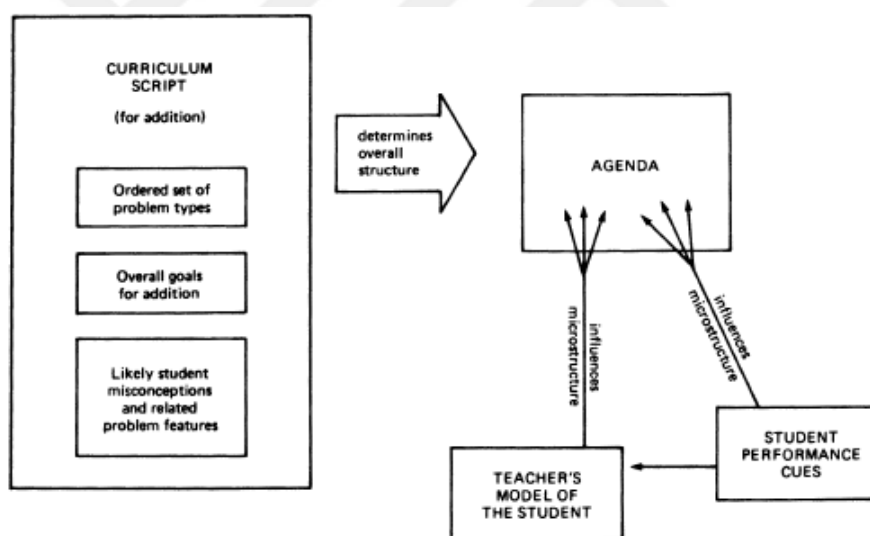


Figure 2.3 Curriculum script model (Putnam, 1987, p. 40)

In addition, the curriculum script also has *ad hoc* characteristics including the decisions the teacher must make during a lesson when unexpected issues and opportunities occur, what direction to take with the mathematical activity and which questions to pose to students (Ruthven, 2009).

The introduction of new technologies in teaching has the potential to alter the actions and representations used for a particular topic. In this regard, it requires a change in

the ways in which teachers transfer their knowledge of the content into the course of instruction. In this sense, teachers need to develop their curriculum scripts with the incorporation of new technologies into their classrooms. Having a well-developed curriculum script with new technologies requires not only an expertise in content, but also the knowledge of the technology used. Although a teacher can be experienced in teaching and have a well-developed curriculum script for a specific topic, s/he can be a novice in technology use and lack a structure for interpreting classroom situations involving use of a new form of technology, which they have not fitted into their existing resource system. As an example, in the aforementioned study by Lagrange et al. (2005), the teacher had not thought through the operation of the resource system and adapted her curriculum script accordingly. She had hypothesised a mathematical learning trajectory but underestimated the instrumental aspect of this; particularly the process through which her students would instrumentalise the DGS and their mathematical actions would become instrumented through the DGS. Furthermore, Gueudet and Trouche (2011) used the notion of operational invariants, which are “both driving forces and outcomes of the teacher’s activity, instrumented by a set of resources” (p. 205), to show how a teacher’s curriculum script was adapted to be consistent with her past experience. A teacher in Gueudet and Trouche's (2011) study had developed her curriculum script according to her experience with Geoplan, and brought this to bear on a lesson using GeoGebra for the first time. Since the teacher was already accustomed to using Geoplan, she did not use the Algebra window in her GeoGebra lesson, a feature of GeoGebra that differentiates it from Geoplan. In other words, she had developed operational invariants attached to the use of Geoplan, which had no algebra window or input field; accordingly, she followed her established curriculum script, treating GeoGebra simply as a substitute for Geoplan.

By tracing changes retrospectively in one elementary teacher's curriculum script for teaching mathematics, Leinhardt (1988) found concrete evidence of gradual evolution over the course of years; for example, vestiges of activities and formats that had been elaborated or transformed. After a number of years in the classroom, during which activities and formats are transmuted, a teacher's curriculum script for a particular lesson represents "a rich pentimento in that it contains many layers of understanding and experience" (Leinhardt, 1988, p. 166). Especially after encountering unexpected difficulties in the classroom, teachers become reflective of

their classroom practices either consciously or unconsciously in order to revise and adapt their curriculum scripts. In this sense, a curriculum script builds on the knowledge that teachers generate, as a result of their experience and there are differences between curriculum scripts of expert and novice teachers. As Leinhardt (1989) concludes: “beginning teachers need to craft simple but effective scripts for teaching key topics-these can and should be modified by the teacher over time” (p. 74). Along these lines, Drijvers (2012) emphasised how an experienced teacher’s professional learning and pedagogical experience shaped his curriculum script. The teacher was aware that entering formulae in the digital environment could be laborious, and that shortcut keystrokes and copy-paste options could help a great deal. He knew that students initially complained, saying that writing down formulae with paper and pencil was much faster than entering them in a digital environment. Therefore, his evolving script included a ‘Technical-demo’ activity format in which he demonstrated the main editing techniques and highlighted their importance.

2.2.1.5 Time Economy

Concern about time when it comes to learning and teaching is not new. Researchers (e.g. Anderson, 1981; Burns, 1984) have considered time as useful variable in analysing classrooms for many years. Consequently, a number of time constructs (see e.g. Berliner, 1990) have been developed in order to conceptualise the time-related issues in classrooms. By the distinction of different times, the importance in the research literature has been given to how teachers use the allocated classroom time in order to produce academic learning time. As Burns (1984) states: “It is the quality of learning time, not the quantity of time *per se*, that will be the primary determinant of student learning” (p. 92).

Reviewing the related literature, the time at classroom level with which researchers paid attention can be divided into four different types:

- Allocated classroom time is the amount of time spent in the classrooms within the school that is a fixed or stable time and not in control of teachers (i.e., excluding lunch, recess, time spent changing classes, etc.).

- Instructional time is a subcategory of the classroom time that is used for instruction (excluding instructional disruptions related to discipline, etc.). This is related to “teacher's ability to organize instructional activities and expedite non instructional ones such as transitions and discipline.” (Cotton, 1990)
- Engaged time/ Time-on-task (Anderson, 1976; Good & Beckerman, 1978): these labels are used to refer to the part of instructional time in which students are actively paying attention to or engaged with a learning task.
- Academic learning time (Wyne & Stuck, 1982): Academic learning time is that portion of time on task or engaged time during which the learner is experiencing a high rate of success.

The research extant on time and learning suggests that the time students spend on task is related to their improved academic performance. However, the amount of time on task per se is not enough to reflect positively on academic achievement. It is important for teachers to provide effective instruction, which includes high levels of student time on task so that increased academic learning time can be achieved.

Time-related issues in classrooms have been re-focused with the entry of new technologies into education as the available classroom time is widely recognised as an institutional barrier (e.g. Cuban et al., 2001). Researchers started investigating whether or not use of time in relation to new technologies has implications for teacher behaviour. For instance, Assude (2005) argues that time related issues in classrooms stem from the difficulties of time management in the classroom. Therefore, Assude investigated what kind of strategies teachers develop to manage different kind of times in a classroom when they use Dynamic Geometry Software, Cabri. She highlighted two main kinds of different time in a classroom: didactic time and time capital. While the former is defined “as a gauge of the advancement of knowledge” (p. 185) which is a similar concept to ‘academic learning time’, the latter represents the clock time available for a lesson, which is similar to ‘allocated classroom time’. Additionally, the concept ‘pace’ is used in order to show the

relationship between didactic time and time capital. For instance, a slow-paced lesson represents that not much progress is achieved in proportion to the time capital being spent. In discussing the use of Cabri, Assude also referred to the tool time as being part of the time capital. The tool time is the time needed for students to familiarise themselves with the tool.

We propose that the teacher views the time he has available indeed as a certain capital, which can be used (invested) in different ways. He makes decisions based on his assessment of the costs of relations such as the relation between tool time and didactic time... in view of saving as much time capital as possible, whilst promoting didactic time and pupils' learning time (Assude, 2005, p. 186).

Assude argues that time management, particularly control of the didactic time, is a challenge for teachers when they incorporate a new technological tool like Cabri for the first time in their teaching. She found that teachers initially experienced discomfort, as "this adaptation did not necessarily allow the teachers, in the first year of integration of Cabri, to have a global vision of the new didactic time" (p. 192). However, since the research was carried out in the second year, she noted "it proved to be an easier experience, since teachers could anticipate what pupils would be doing, and therefore they were able to manage their time more economically". She concluded that the teachers' ability to control the didactic time increased when they became more experienced compared to first year of use. In other words, teachers managed time saving strategies based on their first year experience. One of the time saving actions observed was to make "small" authoritative contributions. "In the first year, they dared not make comments during individual work done by pupils; however, in the second year they believed that certain contributions could help save time" (Assude, 2005, p. 196). Additionally, teachers started giving intermediate syntheses through showing the progress of some pupils' work (which would advance the work of others) to the whole class.

Following Assude (2005), Ruthven (2009) used the concept of time economy in order to denote "how teachers seek to manage the 'rate' at which the physical time available for classroom activity is converted into a 'didactic time' measured in terms of the advance of knowledge" (p. 14). Assude's (2005) research showed that teachers initially did not know how to fit a piece of new technology into their existing

resource system, hence, how to produce and manage their didactic time allowing them to have “a global view of how the teaching of certain content is progressing” (p. 200). Similarly, in the aforementioned study of Lagrange et al. (2005) the teacher could not sufficiently manage the different forms of time in the classroom. She planned her activity time without allocating time for tool learning referring to the time for students to get familiar with the software. Therefore, she spent a great deal of time capital on trouble shooting due to the students’ unfamiliarity with the DGS, while didactic time did not progress as expected. However, Drijvers' et al. (2010) study showed that teachers mostly used a ‘Technical-demo’ activity format in order to demonstrate techniques aimed at minimising the time spent on trouble-shooting the difficulties that students might experience.

2.2.2 The Relationship between the Concepts of the SFCP Framework

Resource systems and activity formats that teachers can develop are to a certain degree constrained or facilitated by the working environment, and through teachers’ perceptions of that working environment. Therefore, in a sense, resource system and activity format are fundamental to the working environment, but not necessarily determined by it. In other words, different working environments can offer different resources to be used and deem more convenient for certain activity formats. However, it is the teacher, who will create a coherent resource system and employ the most suitable activity formats according to their pedagogical goals in those working environments. Additionally, the resource system available to teachers in the classroom, their perceptions of how resources fit together, combined with the activity formats chosen by the teachers, all influence the development of their curriculum script. Ultimately though, time economy is the aspect that impacts all of these structuring features (Figure 2.3).

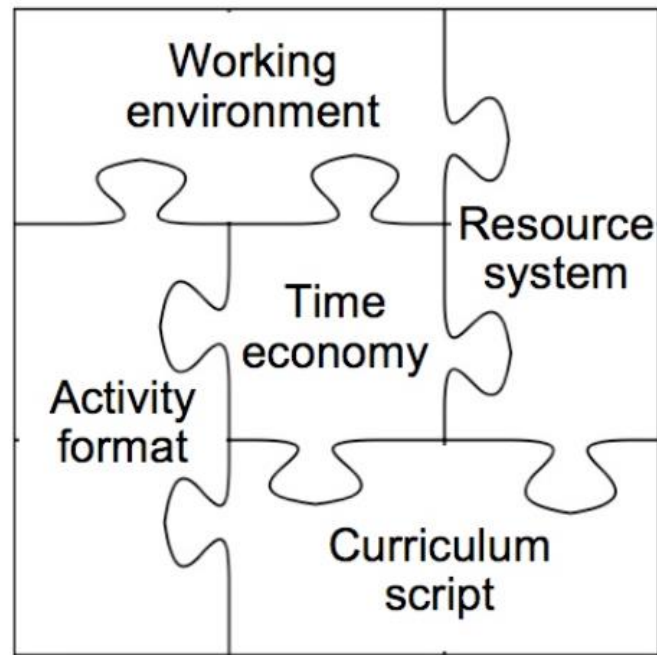


Figure 2.4 Overarching constructs and structure of the SFCP (Ruthven, 2014)

2.2.3 The Relationship between the SFCP Framework and Craft Knowledge

It is widely accepted that what teachers know is one of the most important influences on what is happening in the classroom in terms of teaching and learning. While there are different conceptions of teacher knowledge and learning, the SFCP framework is developed based on the assumption that “over a period of time experienced teachers have acquired substantial practical knowledge about teaching, largely through their classroom experience rather than their formal training” (Brown & McIntyre, 1993, p. 12). In other words, Ruthven (2009) in his framework emphasises the kind of professional learning that builds on the idea of teaching as a “craft” (e.g., Grimmer & Erickson, 1988; Grimmer & MacKinnon, 1992; Grimmer, MacKinnon, Erickson & Riecken, 1990; Leinhardt, 1990) in order for teachers to accomplish their ideal use of new technologies in classrooms. Craft knowledge and a number of similar labels such as practical knowledge, or knowing-in-action are derived from the perspective of knowledge in practice (Cochran-Smith & Lytle, 1999). With the SFCP framework, Ruthven points out five craft aspects of teaching (which are discussed

above) and the importance of understanding teachers' corresponding craft knowledge and thinking about these aspects for successful incorporation of new technologies.

Despite the differences in research purposes or definitions, craft knowledge refers broadly to the knowledge that teachers generate “as they deal with classroom situations that are inherently indeterminate, including how decisions are made, how strategies are selected, how disparate instances are connected to one another, how subject matter is conveyed, and how new occurrences are understood and framed” (Cochran-Smith & Lytle, 1999, p. 268). The craft view of teaching research has received criticism on the grounds that its “antiscientific epistemology”, “the conservatism inherent in reliance on the wisdom of past practice” as well as suggestions of codification of craft knowledge (Tom & Valli, 1990, 377). However, the SFCP defines craft knowledge not as “a knowledge base as such, but as a framework for helping prospective and experienced teachers develop their repertoire of responses, understandings, and magical tricks” (Grimmett & MacKinnon, 1992, p. 441).

In my research, the purpose is to explore “that part of (teachers’) professional knowledge which teachers acquire primarily through their practical experience in the classroom ... which guides their day-to-day actions in classrooms, which is for the most part not articulated in words and which is brought to bear spontaneously, routinely and sometimes unconsciously on their teaching” (Brown & McIntyre 1993, p. 17). Although this knowledge is context-specific and deeply rooted in teachers’ practical work, my aim is to explore some commonalities in how teachers’ expertise or lack of knowledge interacts with some aspects of classroom practice as a result of applying techniques instrumented by the available technological tools.

2.2.4 Implications of the SFCP Framework

The SFCP framework seems to be a powerful way to organise and examine the various aspects of classroom practice associated with technology use in schools. This framework implies that the process of technology adoption depends on teachers’ professional knowledge as well as their experience in classroom. Therefore, it suggests that future researchers should pay closer attention to understanding

teachers' craft knowledge and thinking about certain aspects of classroom practice in relation to technology use. This has implications for teachers' professional development and teacher education programmes. This knowledge and thinking on certain features of classroom teaching with new technologies can contribute to other teachers' thinking in a similar context. In particular, the value it provides for teacher education is that it potentially offers good examples that can be used in training student teachers. It also provides insight on what the problems are, what kind of knowledge teachers require in solving these problems, and teaching these lessons in the future. In terms of teachers' professional development, this means that teachers can share their diverse and successful approaches. In addition, this framework, by pointing out certain craft aspects in classrooms, can guide student teachers on what to focus on in order to gain access to the craft knowledge of experienced teachers in their school based components of pre-service education programmes. It is important because "while we recognize that there are those with mastery of some aspects of teaching, we have no coherent account of what they are masters of and how they achieve what they achieve" (Brown & McIntyre, 1993, p. 13).

2.2.5 Limitations of the SFCP Framework

With regard to the generic constructs of the SFCP, the existing literature makes extensive use of them and has developed a variety of categories (and codes) in order to conceptualise classroom environments. These are not necessarily specific to a certain subject or to a specific artifact in use; rather, the ideas underpinning these concepts are more generic to teaching and classroom practice. Therefore, the SFCP framework is well developed at a generic level.

However, at a more specific level, the use of these constructs in relation to the use of technology has not yet been fully developed. In other words, the SFCP does not suggest specific descriptions for each construct with reference to technology-use. Ruthven (2009) analysed a case from earlier research where the unusually full interview report allowed him to apply the SFCP to this concrete example. Additionally, I conducted my pilot study with an English mathematics teacher guided by this framework in a systematic way; however, this particular teacher's pedagogy takes a form relatively unusual in England, when one considers the points made in

UK-based research studies (e.g. Bretscher, 2011). Another tool that also focuses on teachers' classroom practices is Instrumental Orchestration (e.g. Drijvers et al., 2010; Drijvers, 2012), which has provided characterisations, based on empirical data, for several orchestration types relating to particular activity formats and working environments. These studies, which show the significance of teachers' professional knowledge and development for successful technology incorporation, have made useful contributions to the development of the framework. The available evidence has confirmed that there are classroom effects and has suggested that systematic knowledge about these effects provides a useful framework for broadening research on teaching with new technologies. Nevertheless, the number of studies is small, and many of the propositions in this field are preliminary and tentative as they are only based on a small number of rather exceptional teachers.

In summary, the work accomplished by Ruthven on the SFCP provides a skeleton, in terms of framework, which needs to be filled out and developed. I am using this framework as a basis instead of starting at point zero. However, I am aware of the fact that relatively little research has been done, therefore, I am open-minded about what more can be found. Hence, I need more information for further elaboration and refinement of the framework in order to provide the quality of descriptions of each construct and the ways in which they relate to classroom practices.

2.3 Summary

Structuring Features of Classroom Practice (Ruthven, 2009) was the framework chosen for this research on teachers' classroom practices with the use of technology in classrooms. This framework identified key structuring features of classroom teaching that indicate the corresponding professional reasoning and craft knowledge (see Table 2.1) that teachers must develop about these aspects in order to successfully incorporate new technologies. While utilising this framework, the research seeks to understand the associated expertise that teachers use in their everyday classroom practice. Additionally, making use of this framework through data collection and analysis is essential to test its usefulness and how it contributes to advancing research on technology integration. As Ruthven (2014) states, "Studies are required in which data collection (rather than just *post hoc* data analysis) is guided by

the framework, so that it can be subjected to fuller testing and corresponding elaboration and refinement.”

Integrating the literature review and research questions, the next chapter examines the methodology used in this study.

Table 2.1 Components of the structuring features framework (Ruthven, 2014, p. 387)		
Structuring feature	Defining characterisation	Examples of associated craft knowledge related to incorporation of digital technologies
Working Environment	Physical surroundings where lessons take place, general technical infrastructure available, layout of facilities, and associated organization of people, tools and materials	Organising, displaying and annotating materials Capturing or converting student productions into suitable digital form Organising and managing student access to, and use of, equipment and other tools and materials Managing new types of transition between lesson stages (including movement of students)
Resource System	Collection of didactical tools and materials in use, and coordination of use towards subject activity and curricular goals	Establishing appropriate techniques and norms for use of new tools to support subject activity Managing the double instrumentation in which old technologies remain in use alongside new Coordinating the use and interpretation of tools

Activity Structure	Templates for classroom action and interaction which frame the contributions of teacher and students to particular types of lesson segment	Employing activity templates organised around predict test explain sequences to capitalise on the availability of rapid feedback Establishing new structures of interaction involving students, teacher and machine and the appropriate (re)specifications of role
Curriculum Script	Loosely ordered model of goals, resources, actions and expectancies for teaching a curricular topic including likely difficulties and alternative paths	Choosing or devising curricular tasks that exploit new tools, and developing ways of staging such tasks and managing patterns of student response Recognising and responding to ways in which technologies may help/ hinder specific processes and objectives involved in learning a topic
Time Economy	Frame within which the time available for class activity is managed so as to convert it into “didactic time” measured in terms of the advance of knowledge	Managing modes of use of tools so as to reduce the “time cost” of investment in student learning to use them or to increase the “rate of return” Fine-tuning working environment, resource system, activity structure and curriculum script to optimise the didactic return on time investment

3

RESEARCH METHODOLOGY

3.1 Introduction

I now outline and discuss the design of my research, which aims at developing a more comprehensive understanding of technology integration in classrooms, by providing a model of key structuring features of classroom practice (Ruthven, 2009) that shape the use of technology in lessons and the kinds of professional knowledge.

The aim of this chapter is to present the design, strategies and methods employed to address the research questions of the present study. I begin the chapter by highlighting the philosophical underpinnings of the research, followed by examining the chosen methodological structure, the case study, including its strengths and weaknesses, and the justification of chosen methods, e.g. observation and interview. To conclude, I clarify how I analysed the data, and provide a consideration of trustworthiness and ethics.

3.2 Philosophical Underpinnings

The basic structural and process elements of research should be discussed, taught and popularised so that we have more methodological independence, away from the crutch of established paradigms and the designs that go along with them. In order to achieve this, we ask researchers to consider moving away from methods approaches that are more historical than empirical and towards the rebirth of plain ‘research’ as a craft.
(Symonds & Gorard, 2010, p. 134)

From a scientific point of view, the five constructs of the SFCP framework can be seen as objective concepts. The aim would be to collect reliable data in an objective way by putting these concepts into operation and developing a protocol for systematically recording relevant information about the resulting lesson. In other words, one way of conducting the study would be to decide exactly what these constructs are, and how to observe classrooms in terms of them. In this objectivist model, what teachers think about these constructs would not be taken into account. On the other hand, from an interpretive point of view, the crucial importance lies with “immediate meanings of action from the actors’ point of view” (Erickson, 1986, p. 120). The teachers’ point of view becomes central, and features of classroom lessons relating to these constructs are thus salient. Investigation can occur through the teachers’ own accounts, for example, through interviews. Indeed, my role as a researcher in the interview, in particular the way in which I frame questions and respond to answers becomes part of the generation of the account.

In this research, it is not suitable to take a simple objectivist point of view since the perceptions of the participants will affect their thoughts and actions. A theory of education, in the realm of social sciences as opposed to physical sciences, needs to be connected with how teachers think in order to serve as a useful guide for action. However, a strong interpretive worldview tends to be over subjectivist. While there may be differences between the ways teachers conceptualise these constructs, there are also likely to be substantial commonalities. In terms of building a theory of a social phenomenon, it is crucial to understand how these two perspectives interact; with the aim of describing the range of ways of thinking that form the collective representation of the phenomenon. In other words, this research acknowledges “the importance of the subjective human creation of meaning, but doesn’t reject outright some notion of objectivity. Pluralism, not relativism, is stressed with focus on the circular dynamic tension of subject and object.” (Miller & Crabtree, 1999, p. 10)

In light of the above discussion, I aim to let my research questions drive the decision of which philosophical worldview to take. Therefore, I am adopting a practical perspective in which I am deliberately triangulating between a point of view suggesting that to a certain degree there are some objective features in these five

constructs and, equally, a point of view of teachers. This is because the teachers may actively produce changes in these constructs, which are not likely to be directly observable in order to understand the phenomenon more fully. Working environments are an illustrative example. The location of a lesson and the basic infrastructure of a classroom can be directly observed, and can be recorded in a more structured way. These could, of course, be said to be ‘constructions’ of the teachers in the sense that they have actively influenced the layout, or even simply decided not to change it. However, the teachers’ perception of the working environment cannot be directly observed. Therefore, to gain access to the teachers’ point of view, teachers’ accounts are of crucial importance for my study since I aim to explore their knowledge of teaching in the context of the use of technology.

In summary, I would argue that a more open-ended approach is needed to investigate teachers’ behaviour in a technology-rich environment where there are not already validated frameworks that could be used to underpin a more closed approach.

3.3 The Case Study

3.3.1 Why the Case Study Approach

Case study is my choice of research strategy for a number of reasons.

At the heart of this study is an examination of teachers’ classroom practices with the use of new technologies for mathematics teaching, as well as what kind of professional knowledge these practices might involve. As already discussed in the earlier sections, research has recognised the complexity and uncertainty of teachers’ activity involving classroom use of technology (e.g. Laborde, 2001; Monaghan, 2004; Lagrange et al., 2005; Ruthven, Hennessy & Deaney, 2008). These studies show that innovators’ expectations for a more open classroom management and for more emphasis on mathematics in teacher-student interactions were not fulfilled. It is arguable that optimistic expectations of designers/researchers in relation to teachers’ use of technology could be due to the fact that they did not take classroom realities sufficiently into account. In particular, some studies (Lagrange et al., 2005; Ruthven, Hennessy & Deaney, 2008) found that teachers’ ideals regarding technology use were not always fulfilled in the actual classroom environment. Therefore, I argue that

the classroom environment and the expectations of the classroom must be taken into account when researching the issues of technology integration. As Newman (1989) writes (cited in Savenye & Robinson, 1996), “How a new piece of educational technology gets used in a particular environment cannot always be anticipated ahead of time. It can be argued that what the environment does with the technology provides critical information to guide design process” (p. 1). He adds, “it is seldom the case that the technology can be inserted into a classroom without changing other aspects of the environment” (p. 3). This points to the importance of choosing a research design in which environment as well as technology is the focus of the study, and particularly the interaction between them. For this reason, I have chosen a case study design that extends to cover the classroom conditions, which can be expected to prove a highly pertinent component of the phenomenon of this study. This reflects Yin’s description of an essential feature of case study being enquiry in a real-life context. He writes that the case study is an empirical inquiry that

- investigates a contemporary phenomenon in depth and within its real-life context, especially when
- boundaries between phenomenon and context are not clearly evident (2009, p. 18)

In addition, these complexities and realities of technology integration require educational researchers to adopt a research approach that can capture the unique aspects of each situation. In this sense, my investigation must take into account of the argument that craft aspects of teaching are not “a set of standardized teaching behaviours; they will be personalized to the individual teacher but there are likely to be certain over-arching generalizable features which are common across teachers” (Brown & McIntyre, 1993, p. 19). The main interest of the study is to understand “how they function in their ordinary pursuits and milieus” (Stake, 1995, p. 1) through examining teachers “in a particular context shaped by organisational arrangements” (Cohen, Manion & Morrison, 2007, p. 254). This seems close to Stake’s (1995, p. xi) description of typical features of case studies: “Case study is the study of the particularity and complexity of a single case, coming to understand its activity within important circumstances.”

Furthermore, when researching technology integration, multiple views must be sought to understand the interactions of players with the environment as well as technology. One of the strengths of the case study approach is that “it encourages use of multiple methods” (Denscombe, 2003, p. 38). This is particularly essential for my project design since my research questions necessarily require different approaches in order to be fully addressed. The SFCP framework, which guides data collection and analysis of my research, focuses on for example teachers’ activity formats within their use of resource system and classroom arrangements. The best way to achieve this is to study what is the structure of interaction in their classroom when they use technology, to take note of what arrangements of working environment are needed for this specific activity and to listen to their voices about their perceptions, intentions, actions and reflections for the use of such activity. Doing so can be expected to help me to understand the structuring features of classroom practice and corresponding professional knowledge that might be associated with teachers’ use of new technologies for teaching mathematics. Furthermore, the multiple methods approach facilitates validation of data and integrity of the study through triangulation, i.e. “findings can be corroborated or questioned by comparing the data produced by different methods” (Denscombe, 2003, p. 133). In other words, I “attempt to map out, or explain more fully, the richness and complexity of human behaviour by studying it from more than one standpoint” (Cohen, Manion & Morrison, 2000, p. 112). Additionally, the need for triangulation is brought up to verify the trustworthiness of the research in section 3.6.5.

3.3.2 Motivation of the Case Study

Case study is an examination of either a specific case or a specific phenomenon in its real life context (Stake, 1995; Robson, 2011). The use of the case study in this research is to understand two linked phenomena, as follows:

- Classroom practice with the use of technology
- Professional knowledge involved for technology use

In this regard, my case study has an instrumental purpose in an effort to developing insight into these two phenomena without taking a special interest in the subject of

the case. Case study here is instrumental because: particular cases are examined “mainly to provide insight into an issue or to redraw a generalization. The case is of secondary interest, it plays a supportive role, and it facilitates our understanding of a wider issue.” (Stake, 2000, p. 437)

Several bounded instrumental cases (see the detail information in section 3.4.2) have been selected to develop a more in-depth understanding of the phenomena through examining variations and differences between cases. Such case studies are called a ‘collective case study’ or ‘multiple case study’, and aim to provide a general understanding using a number of cases that either occur on the same site or come from multiple sites (Stake, 1995; Yin, 2009).

3.3.3 Limitations of the Case Study Approach

Apart from being considered time consuming, expensive and labour intensive, case study research has received criticism on the grounds that its findings are not generalisable due to small sample sizes, especially when compared to survey research (Bell, 2005; Denscombe, 2010). However, Yin (2003) differentiates between statistical generalisation (generalising to populations) and analytic generalisation (generalising to theory), writing, “case studies, like experiments, are generalizable to theoretical propositions and not to populations or universes” (p. 10). In this regard, the SFCP might be described as a theory in making, and this case study as a form of testing. As discussed in the conceptual framework section, the application of the SFCP requires rather different approaches at two levels. At the top level of the generic constructs of this model, the use of the terms in the existing literature provides quite a strong definition (and exemplification) of them. On the other hand, at the more concrete level of the way in which these constructs interact with the use of new technologies, there is much sparser literature, which has not developed the same degree of definition. On this basis, while observation at the generic level might be explicit, it is not clear that this can be so at the technology-specific level. In this regard, the five concepts of the SFCP are far more tentative and less well formed than they are in the generic state. Therefore, there is room for testing and refining the SFCP model. By testing I mean to investigate:

- whether these pre-specified constructs serve well to capture influences on teachers' appropriation of new technologies;

in particular

- whether any ambiguity in these constructs, or overlapping between them arises in seeking to operationalise them in the classroom setting
- whether the SFCP framework misses any important structuring feature that are important to teachers

Accordingly, the use of the SFCP in data collection and data analysis of this research becomes the essential means for generalising the results of the case study, which are expected “in some way to be typical of something more general” (Bassey, 1999, p. 62). Under these circumstances, the mode of generalization is “analytic generalization,” in which a previously developed theory is used as a template with which to compare the empirical results of the case study (Yin, 2003, p. 33). In this regard, this multiple/collective case study, in an attempt to capture as many perspectives as possible, presumes that certain forms of generalisation are possible due to regularities across and within cases (Gomm et al., 2000). In this respect, multiple case designs are more powerful than single-case design. As Chmiliar (2010) writes, “Examining the issues across cases can help to provide a tougher test of a theory” (p. 582).

3.3.4 What the Case Study Offers

This case study's contribution is twofold. Firstly, it offers an immediate value as a study of teachers' use of GeoGebra at a classroom level. In this regard, this study contributes to the field of research on technology by underlining the significance of pedagogy and directs the attention of researchers, software designers as well as policy makers to the realities of classroom practice. In addition, by looking at expert practice, this study provides less experienced teachers with examples of technology-related professional knowledge, which can make them aware of some issues beforehand and help them to handle a variety of situations in the classroom. Therefore, I believe that these case studies examine the phenomenon in a way that

can be expected to lend itself to effective communication with mathematics teachers at secondary schools in England when they consider incorporating GeoGebra into their teaching.

Secondly, this study is valuable for the advancement of knowledge with regard to conceptual tools for classroom practice in relation to technology-use. In this regard, this research has the potential to test the suitability and value of the SFCP framework as an interpretative lens for research into teaching with technological tools. As Goos et al. (2010) write, it is crucial to “[c]onduct research that makes use of these theories to test their operational value and domain of validity” in order to advance research on technology integration (p. 327).

Drawing on the concepts of episteme and phronesis from Aristotle, Korthagen and Kessels (1999) define two meanings of theory in education. While the former is theoretical general knowledge that can be applied to a broad range of settings, the latter refers to situation-specific knowledge derived directly from experience within the context. With regard to the pedagogy of realistic teacher education, the researchers argue that in order to make connection between theory and practice, it is of crucial importance to build the needed episteme of a specific domain, which is a larger picture of educational knowledge, upon teachers’ phronesis, which is situation-specific and context-related knowledge. Through such an approach teachers are more likely to make sense of the theory and to be reflective in their practices. In this regard, I, as a prospective teacher educator, aim to explore teachers’ phronesis, a specific process that teachers are experiencing in the context of a number of lessons, with the hope of contributing to episteme, the SFCP as a conceptual tool for classroom practices with new technologies.

In this regard, the useful intellectual consequence of my research would be:

- an enlargement of the repertoire of concepts relevant to investigate organised within the SFCP framework.
- examples of technology integration framed in ways that are useful to novice teachers and supportive of their practice

In summary, my research positions itself in the intersection of research for immediate applications and research for the advancement of knowledge, in what is known as Pasteur's Quadrant (see Figure 3.1). The former refers to providing knowledge of GeoGebra-mediated teaching and the future pedagogical development whereas the latter concerns improving the level of richness of description possible within the framework.

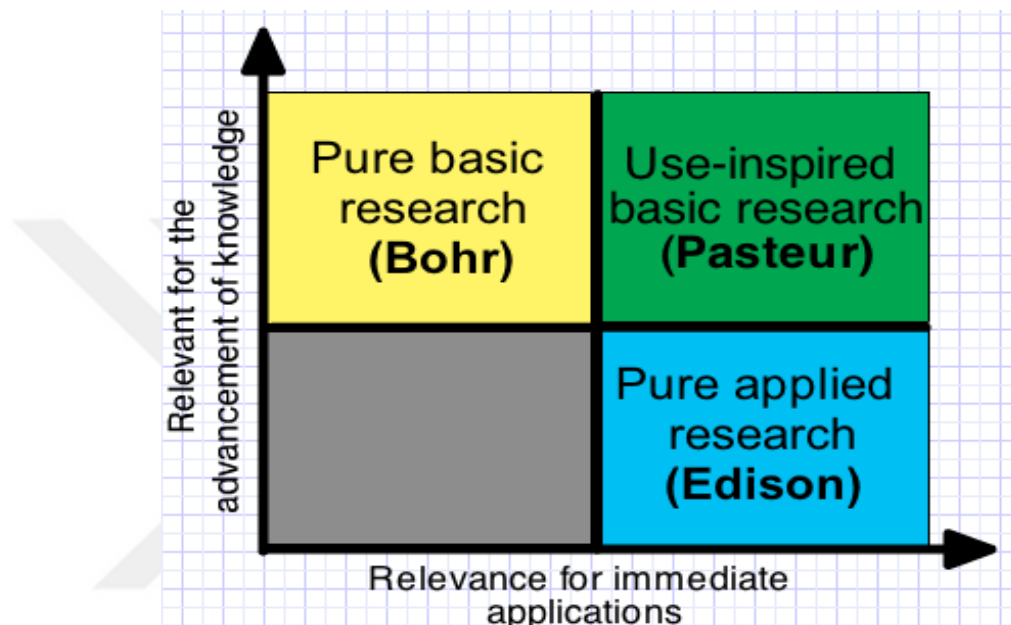


Figure 3.1 Pasteur's quadrant

3.4 The Fieldwork: Pilot Study, Participants and Procedure

3.4.1 Pilot Study

During my MPhil (Master of Philosophy) in Educational Research course (2011/2012) in University of Cambridge, I conducted a pilot study prior to the major Ph.D. level study, which was my ultimate goal. This one year MPhil course helped me to become familiar with an appropriate range of intellectual and methodological traditions within the field and to make an informed decision about relevant research design for my Ph.D. research. In addition, the course enabled me to translate this design practically into English settings for a small-scale study in order to gain experience of educational research. This pilot study focused on one mathematics

teacher's use of GeoGebra in teaching the topic of 'Transformations of shapes' to a Year 9 class in an English secondary school and served several objectives:

- to gain practical experience in the use of methods like interviews and observations during teaching in classrooms and computer labs.
- to examine the appropriateness and the feasibility of the initial design, instruments, and research procedures for my Ph.D. research.
- to develop more relevant research questions and instruments to address them.

The fieldwork related to the pilot study involved an advanced skills teacher who has around fifteen years of experience in teaching as well as in technology-use, and who utilises digital technologies in a progressive way in mathematics instruction. I conducted seven classroom observations and four post- lesson interviews lasting around 30 minutes with this teacher (see Table 3.1).

Table 3.1 Data collection schedule for the pilot study		
Date	Observation (50 min long lessons)	Post-Lesson Interview (around 30 minutes)
15th May 2012	Lesson 1	
16th May 2012	Lesson 2	Interview 1
18th May 2012	Lesson 3	Interview 2
21st May 2012	Lesson 4	
22nd May 2012	Lesson 5	Interview 3
23rd May 2012	Lesson 6	
25th May 2012	Lesson 7	Interview 4

The main strength of studying such a professional outlier lay in ensuring that the study would generate rich information and identify key issues, which could easily be overlooked by initially studying a more typical teacher, providing an in-depth analysis related to the research questions (Stake, 1995). This pilot study examining a number of lessons of one expert teacher using GeoGebra in an IWB-equipped classroom suggested that there had been some interesting shifts in his practice, and it

showed the ways in which that GeoGebra could be used. Although following only one case allowed me to look into the invariants in one teacher's practice between lessons, it did not provide a degree of contrast between different cases in approach, which limit the scope for reaching analytical conclusions. Hence, my doctoral study needs to follow more cases in order to understand how different teachers approached similar teaching situations and the potential differences between their GeoGebra uses, which is likely to enhance the external generalisability of my findings (Yin, 2003).

Analysis of these data contributed to the thesis submitted in part-fulfillment of the requirements for the degree of Master of Philosophy of University of Cambridge.

3.4.2 Context and The Participants

Researchers in the field (e.g. Stake, 1995; Yin, 2003) assert that case study research is not sampling research. However, the process of selecting cases is of paramount importance in order to maximise what can be learned in the period of time available for the study. This study focuses on teachers using GeoGebra for mathematics teaching, situated within the English secondary-school system. The selection of participants is through a purposive sampling strategy, one of the types of non-probability sampling method. Such sampling focuses on a relatively small number of people who have particular characteristics. This study aims to build a deliberate principle of variation into that sampling strategy, based on different levels of teaching experience and expertise using technology. In other words, I deliberately looked for variations to be able to describe what different approaches might look like through comparing the classroom practices of teachers that specifically differentiated dimensions of expertise in technology use and in mathematics teaching, seeking both commonalities and important differences in teaching across this range of teachers.

I continued following the expert teacher in my pilot study in order to study his practice further as I had only examined his teaching of one topic in the pilot study.

The number of cases examined in this study is three. This is sufficiently large and appropriately varied to make it possible for the study to achieve depth of

understanding and reach analytical generalisations, but not so large as to generate an unmanageable volume of data.

The three participating teachers in my research have rather different profiles:

- an experienced teacher and Advanced Skills Teacher (a recognised grade of classroom teacher within the English school system, also taking special responsibility for leading professional development), who utilises new technologies in a progressive way in mathematics instruction.
- an experienced teacher in terms of mathematics teaching but a novice technology user who is in her early stages in using GeoGebra. It was the first time she had integrated GeoGebra into her teaching to this extent.
- a less experienced teacher in terms of mathematics teaching but familiar with the use of GeoGebra in mathematics teaching. He started using GeoGebra regularly in his teaching from the beginning of his career.

All teachers volunteered for the study. In order to gain access to potential participants amongst secondary-school mathematics teachers in England, I used GeoGebra related websites, and particularly the GeoGebra community in the UK since GeoGebra is open-source software with online resources and networking on the GeoGebra User Forum. It must be acknowledged that, however, difficulties arose in finding suitable lessons to examine, in line with the indications from inspection reports and survey studies (e.g. Ofsted, 2008; Bretscher, 2011) that the use of available new technologies by mathematics teachers in English classrooms is limited. In this regard, unfortunately, it proved more time-consuming than expected to study the relevant practice of this group of teachers over a realistic timescale.

Table 3.2 below presents some basic information about the participating teachers. Pseudonyms are used for all teacher participants due to ethical considerations.

Table 3.2 Profile of the participated teachers

Teacher	School	Experience in teaching	Experience in GeoGebra use	Previous Knowledge and Skills About Technology use
Chris	State school	18 years	15 years	Sophisticated
Susan	State school	20 years	Rarely used, mostly for demonstration	Limited
Tom	Private school	4 years	4 years	Developing

3.4.3 The Procedure

After e-mail discussion with teachers, which led to their agreement to be involved in the research, I visited their schools to discuss their timetables and to find out when they were planning to make some significant use of GeoGebra. Observations then took place at a time agreed in advance with each teacher at his or her convenience. Since GeoGebra lessons did not occur on a regular basis in secondary mathematics classes, the complete dataset was collected over several school visits in different months during 2013/2014.

This study involved no attempt to control or influence teachers' lessons. In particular, I did not participate in the planning of lessons, although it is clearly possible that a teacher's planning was influenced by the knowledge that the lesson would be observed. To try to forestall this, the teachers were requested to teach any topic, using GeoGebra however they saw fit, and in any manner they wished. That was emphasised in the general email sent out at the beginning of the search process for teachers.

However, when it became clear that “Circle Theorems” and “Transformations of shapes” and “Straight Line Graphs” were the topics that some teachers chose to teach, I did make a request to other teachers as to whether they could also teach one of these topics if possible in order to make comparisons between teachers' use of GeoGebra for the same topic. In this sense, data in my pilot study (see Table 3.1) seemed to be helpful data to have alongside other cases for developing the framework since the teacher taught “Transformations of shapes” at the time. Hence, I included part of the data set (three lesson observations and related post lesson interviews) from him that were collected two years ago with the intention of analysing afresh without relying on the previous analysis that was presented in the MPhil thesis. In this regard, for the purpose of comparison, I aim to present three lessons (out of seven) of Chris covering basic transformations. However, I will also draw on episodes from other lessons in order to explain fully how the teacher thought about what he was doing. Additionally, Chris taught Circle Theorems lessons in a series of six lessons, which included two of non-GeoGebra lessons and four of GeoGebra lessons. I also included his non-GeoGebra lessons in my analysis to find out whether/how his technology lessons differed from his non-technology lessons. This was of crucial importance in the sense it provided an opportunity to make a type of comparison that was not possible for the other teachers.

In Susan's case, after teaching Transformations in two lessons to her class of year 7 she encouraged one of her colleagues (pseudonym James) to teach this topic with GeoGebra by using the same worksheet. She provided him with feedback from her own lessons, hence, initiated the activities to be used. Accordingly, Susan and James worked on and revised the “Transformations” worksheet that Susan had used for her lessons. James employed this revised worksheet with his Year 7 class to teach this topic over two lessons with Susan walking around the classroom and helping students. In this regard, this was a rather special case that revealed Susan's learning or expressing her learning from teaching the previous lessons through working with James. Therefore, James was not considered as a fourth case, but, analysed as a subcase of Susan, which provided information regarding Susan's teaching. James had seven years teaching experience but was also novice in teaching with GeoGebra to this extent where students were actively engaged with the use of software.

In total, data analysis consisted of 18 GeoGebra + 2 non-GeoGebra lesson observations alongside 11 post-lesson interviews (see Table 3.3).

Table 3.3 Data analysed in this study			
	Chris	Susan	Tom
Transformations	Data from pilot study (see Table 3.1) 3 Lesson observations 2 post-lesson interviews	(Feb-2014) 2 Lesson observations 1 post-lesson interview <u>James (with Susan)</u> (June-2014) 2 Lesson observations 1 post-lesson interview	(Feb-2014) 3 Lesson Observations 2 post-lesson interviews
Circle Theorems	(Oct-2013) 2 Lesson Observations (non-GeoGebra) 4 Lesson Observations (GeoGebra) 2 post-lesson interviews	(Feb-2014) 2 Lesson observations 1 post-lesson interview	
Straight Line Graphs		(Feb-2014) 1 Lesson observation 1 post-lesson interview	(Feb-2014) 1 Lesson observation 1 post-lesson interview

3.5 Data Collection

I argue that in order to answer the aforementioned research questions I needed a two-fold approach to data collection:

- observing the participant teachers teaching their lessons in the classroom and/or computer room
- listening to teachers' voices about their teaching to illuminate their perceptions, intentions, actions and reflections for using GeoGebra

3.5.1 Observations

Observing classroom events while teachers were teaching a technology lesson was valuable for several reasons. Simply, observation offered me an opportunity to “look directly at what is taking place in situ” (Cohen et al., 2007, p. 396) and to gather detailed information about what actually transpires during the course of a lesson when GeoGebra was used in a classroom/computer lab environment. This is of paramount importance as Eraut (2007) argues that “the advantage of observing professionals undertaking their practice, alongside interviews, is that it allows complexity to be appreciated and discourages informants from idealising practice.” (pp. 404–405)

Additionally, observations allowed me to see some aspects of their practice about which teachers were not articulate during interviews. For instance, the teachers did not bring up some of the activity formats in lessons during the interviews, as these were aspects of the lesson that they took for granted, and indeed did not have a well-developed language for expressing. Therefore, the observation records helped me to gather information about activity formats and so then to access teachers' thinking about these during interviews.

3.5.1.1 *The Specifications of my Observation Design*

I adopted a semi-structured, non-participant observation approach that allowed me ‘to have an agenda of issues and gather data to address those issues, and in a far less

predetermined or systematic manner than structured observation’ (Cohen et al., 2007, p. 397). The SFCP framework as an interpretative lens provided me with guidelines to shape my interpretation during observations. Being clear about an observational framework (see observation protocol in Appendix 1) in advance also helped to guard against the potential for lone researcher bias, which has been stated as a drawback of observation method (Cohen et al., 2007). In regard to the SFCP framework, observation data provided evidence about teachers’ classroom practices in relation to use of GeoGebra, with a focus on pre-specified aspects (see an observational record in Appendix 2) as follows:

- the settings in which the teachers were working and arrangements of these settings
- what resources were available in these settings and which ones were made use of
- coordination of different resources
- how and by whom these resources were used
- teaching strategies and methods used to incorporate the technology into the lesson
- types of interactions between teacher, students and technology during a lesson
- the sequence of tasks that students were set over the course of lessons
- how available time for a lesson was used for students’ learning
- any time-related issues with regard to using GeoGebra during lessons
- the relationships between these aspects

I initially requested teachers to video record their lessons, which would be an invaluable way of recording the classroom events in detail including non-verbal processes. Having a video recording of lessons would also allow researchers to watch those lessons repeatedly with an open-minded stance to see whether new aspects could be derived from the data (Edwards & Westgate, 1987). However, participating schools’ policies did not make it possible for me to video record teachers’ classroom practices. Instead, all of the lessons were audio recorded with the permission of participant teachers and students as this can help to reduce the selectivity of

researchers. This was also helpful to fill out my field notes (see an example in Appendix 3) on key incidents, and review my written record of them since recording by hand everything was impossible in a limited time. An audio-recorder on teachers was used with the aim of having a detailed record of oral interactions during independent student work since the lessons included paired/small group of students working at computers with the teacher circulating around the classroom and having discussions with each group. Although audio recording allowed me to capture discussions between teacher and students, it did not provide information regarding related screens that were part of the discussions. In this sense, it is essential to follow teachers with a video camera during such investigative lessons.

I had a digital camera to photograph particular examples of the teachers' activities/examples on the board, which were of importance to reporting the results. Additionally, I requested the relevant screen shots and files from teachers.

3.5.2 Interviews

The second method involves having interaction with teachers focusing on providing opportunities for them to describe and reflect on their practices, in particular how the use of GeoGebra impacted upon their decisions and actions in the classroom. Teachers may employ rather different constructs from those of the SFCP framework to describe and analyse their practice, and these are not likely to be directly observable. In this regard, listening to teachers helped me to go beyond my interpretation in order to investigate teachers' perceptions of classroom phenomena. Additionally, interviewing is a very powerful tool in this study in order to understand teachers' craft knowledge in the context of technology. As Hagger and McIntyre (2006, p. 28) emphasise, studies examining craft knowledge of experienced teachers must not only examine what teachers do in their classrooms, but also the thinking that underlies it. Similarly, Cochran-Smith and Lytle (1999) write: "To improve teaching, then, teachers need opportunities to enhance, make explicit, and articulate the tacit knowledge embedded in experience and in the wise action of very competent professionals." (p. 263)

In this light, interviews with the teachers formed the major opportunity to hold in-depth discussions about their thinking, intentions, actions, reflections and learning.

3.5.2.1 The Specifications of my Interview Design

Although there are a number of interview types, three - the structured interview, the semi-structured interview, and the unstructured interview- are commonly used in education (Robson, 2011). Structured interviews have pre-established questions that are asked in the same order and offer a pre-set range of responses for participants to choose, and hence, do not allow an interviewer much flexibility and freedom. In this regard, this type of interview for my research would have limited my freedom to explore particular themes and responses further. On the other hand, unstructured interviews generate questions based on the interviewee's responses, hence allowing an interviewer greater freedom and flexibility (Cohen et al., 2007). As I already have the SFCP framework as an interpretative lens providing five pre-set categories to start with, unstructured interviews are not particularly useful for my study. I chose to employ semi-structured interviews, which provided a degree of flexibility encouraging teachers to talk expansively on the topic. At the same time, it provided a degree of control, making sure that teachers remained reasonably on topic.

Semi-structured post-lesson interviews were conducted in order to clarify the observed lesson and thinking behind it according to the key themes on the SFCP framework as well as any other aspects which the framework did not cover. Interview questions focused on how using GeoGebra in the lesson made it rather different to organise and run from a similar lesson in which technology was not being used (see interview protocol in Appendix 4). As a starting point, I combined a pre-determined set of open questions that prompted discussion with the opportunity for me to explore particular themes or responses further. I firstly found out what teachers' intentions/expectations were while preparing their lessons in order to distinguish whether or not something unexpected happened during lessons. I could have asked these questions as pre-lesson interviews, however, I noticed difficulties with this approach during my pilot study, since the teacher tended to rush before lessons and did not have time to answer my questions.

Then I asked questions about the five aspects of the SFCP in order to triangulate observation data with the contextualised teachers' accounts. Additionally, at the end of the interview I invited teachers to make any contributions they found important in working with the technology they made use of as a follow up from their lessons. This was particularly important for investigating the aspects that the framework did not cover. The post lesson interviews lasted for approximately 30 minutes, although it depended on the participants and their willingness to express their thinking (see a part of an interview transcript in Appendix 5).

All the interviews were video-recorded with the permission of the participants in order for me to concentrate on the topic without needing to concern myself with note taking (Gillham, 2000). This was also useful for capturing the many aspects of non-verbal communication, such as gestures. This guarded against the danger of data distortion. Recordings were transcribed soon after the interviews were completed.

3.5.3 Lesson resources

In addition to the data from classroom observations and post-lesson interviews, lesson materials for each lessons such as software files and worksheets developed for students' use as well as students' software files captured during the activity were also used as supplementary data.

In Chris's case, it was easier to reach students' software files as he mostly used an activity format aiming to share and discuss students' examples with the whole class, therefore, he asked his students to save their files in a shared network. In particular, his lessons were investigative and included open-ended tasks, which led students to come up with various ideas for solving the same task.

3.6 Introduction to Data Analysis

3.6.1 Introduction

In this section, I give an introduction to the data analysis in Chapters 4 and 5. Data analysis informally started during the data collection with the aim of discussing any clarifications needed with participants during the process. As Stake (1995) writes, “There is no particular moment when data analysis begins. Analysis is a matter of giving meaning to first impressions as well as to final compilations” (p. 71).

In order to systematise the analysis process, I followed Miles and Huberman’s design (see Figure 3.2) that consists of “three concurrent flows of activity: data reduction, data display, and conclusion drawing/verification” (1994, p. 10). I focus on the gradually more systematic processing of data where data reductions such as transcriptions became helpful tools.

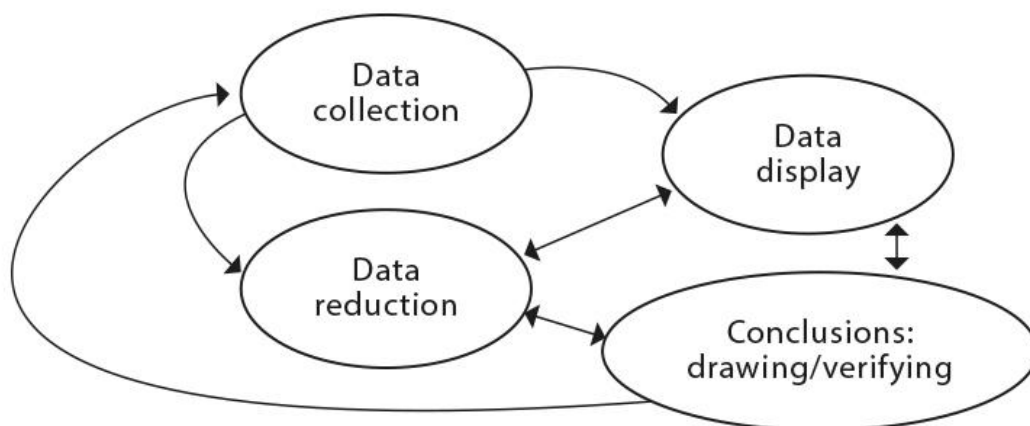


Figure 3.2 Interactive model of data analysis (Miles & Huberman, 1994, p. 12)

In each stage of data analysis, research questions were used as rationales of control and/or focus of the study. Working within the SFCP framework implies looking at characteristics of classroom practices linked to the five aspects of this framework, with a view to identifying variations and commonalities in these practices. In this sense, following sub-questions provided a framework for analysis:

1- Working Environment

What kind of working environment does the lesson take place in?

What strategies does the teacher employ to manage this working environment?

2- Resource System

What resources are used in this lesson by the teacher and by the students, and how do they relate to other resources used in lessons on the topic?

What strategies does the teacher employ to manage use of particular resources, and their coordinated use as a resource system?

3-Activity Structure

What is the activity structure of the lesson, broken down into a sequence of segments each involving a particular type of activity format?

What roles do teacher, students and technology play in each type of activity format, and what strategies does the teacher employ to manage this?

4-Curriculum Script

What components of an established curriculum script are indicated by the teacher's (smooth and confident) handling of the approach to studying the topic over the course of the lesson?

What enhancements to this curriculum script are suggested by the teacher's (responsive and improvised) handling of unplanned issues relating to the topic emerging during the course of the lesson?

5-Time Economy

What strategies does the teacher employ to manage the allocation and use of time, in planning the lesson, and during the teaching of it?

3.6.2 Data Reduction

"Data reduction refers to the process of selecting, focusing, simplifying, abstracting, and transforming the data that appear in written up field notes or transcriptions" (Miles & Huberman, 1994, p. 10). Data analysis as data reduction aims to make a large amount of raw data manageable. Data reduction is not a process isolated from the analysis rather it is a form of analysis that occurs during transcription, coding, and data displaying since it is a way to organise the data for drawing conclusions.

Data can be reduced and managed in a number of ways. First, I used conceptual reduction through a coding process, which involved thinking through the meaning of data chunks (see the detailed discussion in 3.6.2.2 Coding section) and writing supporting memos/interpretations about data chunks and codes. Furthermore, physical reduction of data was done through writing summaries and creating tables in order to use space efficiently in the body of the thesis (see Table 3.6 for Activity Formats in section 3.6.3.1 as an example).

I now discuss transcribing and coding processes when data reduction took place.

3.6.2.1 *Transcribing process*

The resulting evidence base consisted of audio recordings of observations, video recordings of interviews, and fieldwork notes. Just after collection of data, I started transcribing these data into written form for closer study that helped me to grow closer and more familiar with the data (Lapidat & Lindsey, 1999; Tilley, 2003; Wengraf, 2001).

Rapley (2007) wrote,

The actual process of making detailed transcripts enables you to become familiar with what you are observing. You have to listen/watch the recording again and again (and again...). Through this process you begin to notice the interesting and often subtle ways that people interact. These are the taken-for-granted features of people's talk and interaction that without recordings you would routinely fail to notice, fail to remember, or be unable to record in sufficient detail by taking hand-written notes as it happened. (p. 50)

However, transcribing is an interpretive process, which involves making decisions about what level of detail is required and how data are to be represented in written form, and is therefore the first step in analysing data. This process requires reduction, interpretation and representation to make the written text readable and meaningful, which rely upon what is given when listening to or watching recordings. In terms of level of detail, I adopted a denaturalised approach with the aim of focusing “the meanings and perceptions created and shared during a conversation” rather than “depicting accents or involuntary vocalization” for examining the conversation for patterns (Oliver, Serovich, & Mason, 2005, p. 1277). It is because understanding of

classroom practices with technology and professional learning of teachers can be captured in the content of the data rather than in the mechanics of the conversation. In other words, what I have in transcripts treated as data are not representations of sounds recorded but rather of words heard that are components of utterances that are taken to represent teachers' classroom practices in relation to technology use.

In terms of representations, I mostly used written form. However, I also employed visual information such as drawings or pictures as classroom layouts/interaction types are of crucial importance in my study for data interpretation (see Figure 3.3 in section 3.6.3.1 as an example). Furthermore, when talking about a problem/a question in lesson, a related file is used in transcriptions.

3.6.2.2 A coding process

Researchers (Yin, 2009; Miles and Huberman, 1994) have pointed out that having a conceptual framework to start with to some extent guards against data overload since it provides systematic and visible stages to the initial analysis without constant comparing and contrasting of large amounts of data. As discussed before, the SFCP model was chosen as a conceptual framework for data analysis when characterising teachers' classroom practices in a technological environment, which can be regarded as anticipatory data reduction (Miles & Huberman, 1994).

The data coding was organised to define patterns or themes in the transcriptions. This process involved reading transcriptions and listening to audio/ watching video recordings to get a sense of the whole. In line with Flick's (1998) description of theoretical coding, my interpretation of the data consisted of three stages: open coding, axial coding, and selective coding. I considered the five concepts of the SFCP framework as selective codes based on Flick's description in order to verify and also to look for other ideas that seemed to arise out of the data in this context. Accordingly, I started classifying expressions in the events based on these selective categories through open coding of data since the state of these constructs is still tentative concerning technology use. In this regard, the analysis was partly based on examining, categorising and tabulating evidence to assess whether the evidence in this multiple case study supports the generic constructs of Ruthven's framework.

Adjustment of codes in the selective categories, that is to say elaboration of subthemes seems close to what Flick denotes as the axial coding. In what I regard as my axial coding process, I continued to analyse the data by breaking down into distinct issues relating to each of the five constructs of the framework (which is at best only implicit in Ruthven's (2009) framework). This categorisation process with codes therefore has a potential to stimulate further development and refinement of the SFCP framework. I labeled any important information in the process. The name of the labels was decided either using the literature or using the content of the transcripts. While the former is called "constructed codes", the latter style is called "in vivo codes" (Glaser & Strauss, 1967).

During this stage, I reviewed already constructed concepts in the literature to confirm whether or not connections can be drawn between the literature on teachers' use of new technologies, and my empirical findings. In the case of activity structure, as mentioned before, this notion was first invented in 1980s and at the time researchers would not focus on the relationship or the interaction between the classroom participants and the technologies in place in the classroom. Therefore, research on activity structure in 1980s does not suggest specific axial codes that involved any use of technology. However, my research considers this concept as being not just about the interaction between the teacher and students but the interaction between the teacher, pupils and the mediating technologies. Hence, structural interaction multiplies between these and there is a need for new descriptors in order to specify what kind of activity structures appear in technology-based lessons. On the other hand, more recent research on Instrumental Orchestration (e.g. Trouche, 2004; Drijvers et al., 2010; Drijvers, 2012; Tabach, 2011; 2013) has particularly focused on how activity structures relate to particular ways of making use of the technological component of the resource system, and has provided operational descriptions based on a combination of data-driven and theory-driven analysis (see Table 3.4)

Table 3.4 Identified whole-class activity formats in the literature		
Label	Setting	Description
Technical-demo (Drijvers et al., 2010)	Whole class setting	Teacher demonstrates tool techniques.
Explain-the-screen (Drijvers et al., 2010)	Whole class setting Or Small groups of students at computers	Teacher explains what happens on the screen to the whole-class. The explanation goes beyond techniques, and involves mathematical content.
Discuss- the-screen (Drijvers et al., 2010)	Whole class setting Or Small groups of students at computers or on IWB	Teacher creates a whole-class discussion about what happens on the computer screen. The goal is to enhance collective instrumental genesis.
Spot-and-show (Drijvers et al., 2010)	Whole class setting	The teacher brings up student work that he/she had identified as relevant for further discussion and its deliberate use in a classroom discussion.
Sherpa-at-work (Trousche, 2004)	Whole class setting	A so-called Sherpa-student uses the technology to present his or her work, or to carry out actions the teacher requests.
Work-and-walk-by (Drijvers, 2012) Monitor-and-guidance (Tabach, 2013)	Students work individually or in pairs at computers	Teacher walk around the classroom while students work at computers in order to provide feedback.
Link-screen-and-board (Drijvers et al., 2010)	Whole-class setting, one central screen	The teacher connects representations on the screen to representations of the same mathematical objects that appear either in the book or on the board.
Not-use-tech (Tabach, 2011)	Whole-class setting, one central screen	The technology is available but the teacher chooses not to use it.

Based on the above orchestrations, Drijvers' et al. (2013) study identified more whole-class as well as individual orchestrations (see below Table 3.5). The researchers labeled individual orchestrations according to corresponding whole-class orchestrations:

Table 3.5 Identified individual activity formats in the literature

Label	Setting	Description
Guide-and-explain (Drijvers et al., 2013)	Whole class setting	The teacher provides a somewhat closed explanation based on what is on the screen. Also, teacher asks closed questions to students.
Board-instruction (Drijvers et al., 2013)	Whole class setting	The teacher makes no connections to the use of digital technology and uses the board for writing as a regular teaching for whole-class.
Technical-demo (Drijvers et al., 2013)	Students work individually or in pairs at computers and the teacher walking by in the classroom	The teacher demonstrates techniques for using the digital content to individual/pair students.
Guide-and-explain (Drijvers et al., 2013)	Students work individually or in pairs at computers and the teacher walking by in the classroom	The teacher provides guidance and instruction to the student, explains mathematical concepts or methods based on what happens on the screen, or raises questions to make the student reflect on his actions and results.
Link-screen-paper (Drijvers et al., 2013)	Students work individually or in pairs at computers and the teacher walking by in the classroom	The teacher connects the representations and techniques encountered in the digital environment and their conventional paper-and-pencil and textbook counterparts. The goal is to link the mathematics on the screen and the mathematics of the regular paper-and-pencil.
Discuss-the-screen (Drijvers et al., 2013)	Students work individually or in pairs at computers and the teacher walking by in the classroom	During student work, the teacher and student(s) discuss a problem on the screen.
Technical-support (Drijvers et al., 2013)	Students work individually or in pairs at computers and the teacher walking by in the classroom	The teacher supports the student in technical problems that go beyond the particular technology used, such as login difficulties, software bugs or hardware issues.

However, it became clear that these orchestration types identified so far do not fully correspond to activity structures suggested by my study. First of all, these labels mainly describe what the teacher does and work on the assumption that the teacher is always leading the activity or central to the activity in the classroom. Nevertheless, my data suggest that there are some activities that are not really about what the teacher is doing, but what the students are doing in particular when students are working at computers themselves. For instance, on one occasion one of the teachers (Chris) asked students to stand up and walk in a clockwise direction around the room to see different people's screens and then to implement some of the things they had seen. This was about students spotting interesting examples for themselves rather than the teacher spotting and showing. Thinking of in-vivo codes, this could be characterised 'Walk-and-spot', however, this label forms a format for students' activity. Another example to consider is that all the teachers in this study encouraged their students to test their ideas on the computer rather than teacher validating or invalidating their conjecture. I labelled this as 'Predict-and-test' but again this label works on the view that students are leading the activity. For both examples, teachers were circulating around the classroom to maintain students' involvement and engagement with the task. However, other research (Ruthven, Deaney & Hennessy, 2009) indicated a different element of Predict-and-test that was achieved through rather different activity format where the teacher was leading the activity through the whole-class exposition and questioning format. In this sense, having one single label for Predict-and-test would undermine the notion of activity structure since there are different interactional structures between teacher, students and the technology. Therefore, I included both formats as descriptors that provide information for what students are doing alongside what the teacher is doing (i.e Work-and-walk-by/Walk-and-spot and Work-and-walk-by/Predict-and-test).

Secondly, my data indicates that a set of descriptors is needed for lesson episodes that do not involve working with technologies. Although Tabach (2011) already developed a label (Not-use-tech) for non-technology segments in lessons, this label does not give information about the interactional structure between teacher and students during such segments. Additionally, Drijvers' et al. (2013) identified Board-instruction for no use of or reference to technology segments which again limits the interactional structure between teacher and students. For instance, one of the

participating teachers (Tom) in my study gave students an introduction to vectors at the beginning of the first lesson, which he thought was necessary for students to do translations. During this, he basically employed a questioning format, which involved addressing the whole class without using any technology. Similarly, another teacher (Susan) noticed that some students were making more progress on their worksheet than others so she showed their worksheet to the whole class as an example. The interactional structure here was Spot-and-show but the way she used this format was not instrumented by the technology. She held up those students' worksheet and showed their progress to other students rather than using GeoGebra on the IWB. In this sense, it is necessary to have equivalent labels for those that are about the pattern of interaction alongside with whether it is 'Not-use-tech'. Hence, I will refer to interactional structure after the label 'Not-use-tech' (i.e Not-use-tech/Spot-and-show).

In this respect, it became apparent that in order to describe the activity structures in technology lessons fully, three slots including teacher action, student action and whether the technology in use (or not) are of crucial importance.

3.6.3 Data Display

The second stage in Miles and Huberman's (1994) model of qualitative data analysis is data display. This process provides "an organized, compressed assembly of information that permits conclusion drawing..." (p. 11). In addition to narrative descriptions, I displayed data visually by using diagrams, tables, figures, pictures and matrices. Such visuals aimed to facilitate the process of making sense of the data for myself as the researcher and for readers.

3.6.3.1 Data Display for Intra-Case Analysis

Before seeking to understand the cross case patterns, I first made sure that I gained insight into each case by looking through the data for each case one at a time. As already discussed in the previous section, the data were transferred into the key themes through a coding process which helped me to reduce and organise the material in terms of the conceptual framework. However, it became clear that

I needed a more efficient and economical presentation in order to use space efficiently in the body of the thesis. Below, I give more systematic discussion about how data were incorporated into visual representations in various forms as well as narrative descriptions to summarise the lesson transcripts.

For the selective code of working environment, I drew figures (see an example below in Figure 3.3) that showed working environments where the lessons of teachers took place. This diagrammatic form of display helped me to note physical surroundings and organisation of the rooms in a more compact way, which also allowed comparisons between different arrangements that were used by different teachers. Later, these were useful to show the interrelationships between the different working environments and the other categories such as activity format and time economy.

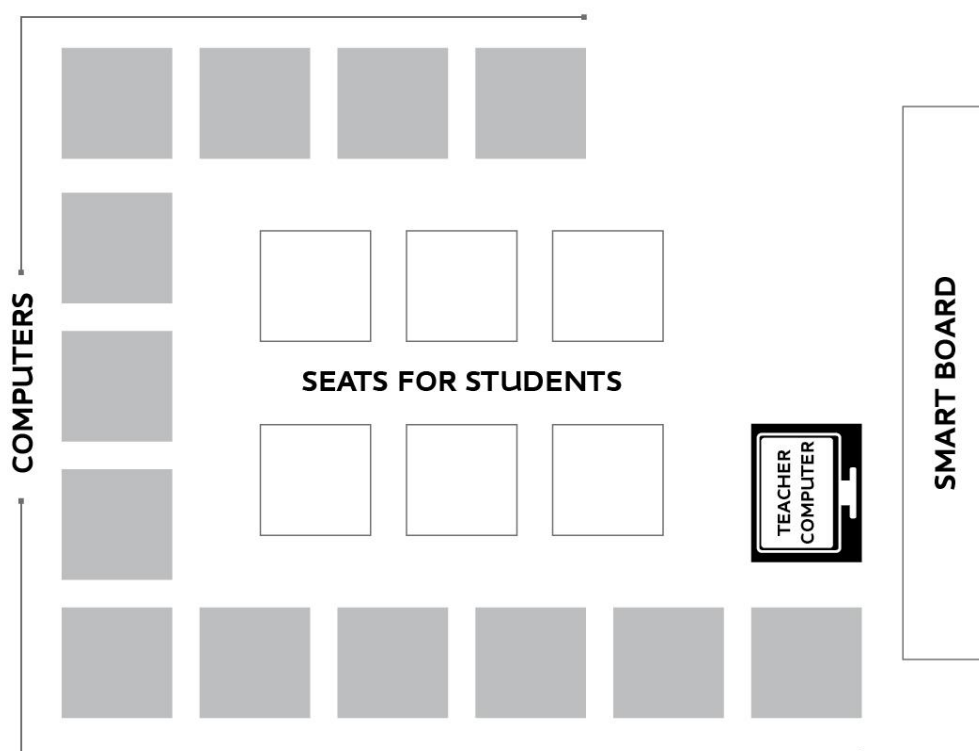


Figure 3.3 The room where Chris’s ‘Circle Theorems’ lessons took place

I also created tables in order to have a more compact and rich form of data display that captures a picture of what the lesson looks like in the terms that interest my research. For instance, Table 3.6 describes a teacher’s chosen activity formats alongside the particular curriculum pathway followed by the teacher in the lesson,

which can be taken as indicative of that part of his curriculum script for that topic which eventuated in the lesson. This compact table allowed me to make sense out of the data: to discern systematic patterns for activity formats and to show the coordination between activity format and lesson agenda. In addition, it made the differences and similarities accessible in terms of the use of activity formats amongst teachers as well as different pathways that teachers take in order to teach the same topic.

Table 3.6 Activity formats used and lesson agenda followed by Tom in the first ‘Transformations’ lesson		
Segment purpose	Activity Formats	Lesson Agenda
Translation by vectors	Not-use-tech/Questioning	The teacher reminded students of the definition of Vector by questioning the whole class.
	Use-tech/Technical-demo/Observe-demo	The teacher showed students some GeoGebra techniques and then wanted them to practice copy and pasting of a shape.
	Use-tech/Work-and-walk-by/Practice-with-technology	Students are practicing questions by using the copy-paste technique in GeoGebra
	Use-tech/Explain-the-screen	The teacher explained the screen for translating object by vector.
	Use-tech/Work-and-walk-by/Set-work-on-screen	Students are practicing a question from their textbook about translation by vector in GeoGebra

Furthermore, I produced narrative summaries for lesson transcripts. I will explain an example to show how I analysed lesson extracts from the perspectives of different components (i.e. activity structure, resource system and curriculum script) to produce

a summary of each particular component. Later chapters will then refer more to these summaries than to lengthy transcript extracts. This aims to help the reader to understand the pathway from what I observed in the classroom to my final model through analysis of what is going on. For instance, Table 3.7 provides information about the teacher's chosen activity format including interaction pattern and medium of communication associated with this activity format.

Table 3.7 A lesson verbatim for Technical-demo at the beginning of Tom's first lesson teaching 'Transformations'

(Students are simultaneously implementing tool techniques on their computers with the teacher)

You need to listen carefully to instructions guys because now you are not looking at me, you are purely listening so don't miss any instructions. Load up GeoGebra and first thing I would like you to do maximise the window. Fill your whole screen. It starts programmes, maths, GeoGebra. You have done it before. OK. Just try this. Hold down ctrl and drag the window around, try not to zoom in or zoom out. That is a tool. Could you all go to view at the top left and turn the grid on?

Do take a little look. You should have all the dashed gray lines filled the screen.

Once you got that there right click you should just get graphic menu. Right click graphics menu. If you then click the graphics dot dot dot. This is where we fix rather than giving you. We are going to fix it to 1. Click on the x axes tab and where it says distance turn the check box and set it to 1. Save settings and close. If you close GeoGebra and reopen GeoGebra again it should maintain this. OK. Check everyone is OK.

Next thing you need to copy this exactly as well. If you click and drag with the right button, it actually zooms in, just try and avoid doing that. You want this button on the toolbar. Polygon button, we have used it before. I'd like you draw a specific polygon with this coordinate please. $(-3,7)$ $(-7,6)$. There is a bit more to it. You can draw this in a minute if you are behind. $(-5,6)$ and $(-5,5)$ and $(-6,5)$, $(-6,3)$. You should have a kind of this shape. Try and generate this on your screen. If you drag a box, there you miss some of the points. Try again. It does not matter if it is slightly different.

(Students are watching the teacher's actions on the screen)

This bit, you definitely have to watch me. Follow this carefully. You have to click once on the polygon. What we can do then to copy and paste it. I am going to do it in old-fashioned way. Click once select the polygon edit copy, click somewhere else, and edit paste. I want you just to get used to the idea. Once you are happy with the idea, you can clear your screen then. You can get rid of everything. You have got 5 min just to glance through question 3.

The teacher's intentional action was to demonstrate some techniques instrumented by GeoGebra, which is characterised as Technical-demo (Drijvers et al., 2010). The grouping arrangement of this format consisted of whole class teaching through demonstration/explaining on the Smart-board where all the students were able to hear teacher's commands and directly practice on their computers. It is also clear in this particular example that the teacher employed different types of Technical-demo. First he asked his students to listen and apply instructions simultaneously. Later, he asked them to watch his actions carefully before practicing at the computer. Furthermore, at the beginning of the lesson transcript, the teacher reminded students how to find GeoGebra, how to maximise the window, how to turn grid on. This indicated that he introduced some routines on setting up to the students for GeoGebra lessons. Additionally, this lesson transcript presented description about the teacher's resource system in the sense what features of GeoGebra the teacher introduced the students to. These are also parts of the teacher's curriculum script (i.e. the crystallised sequence of topic-related ideas) as he took account of students' previous knowledge about that particular technology (GeoGebra) that was tied to this particular topic (i.e how to use polygon tool with the use of vectors, how to copy and paste a shape, how to translate by a vector)

Another example to consider is that Table 3.8 provides information about a particular activity structure where the teacher spotted students' examples during paired or group student work at computers and showed those examples to the whole class with the aim of giving students different ideas of what to do.

This activity format is characterised Spot-and-show (Drijvers et al., 2010). The teacher paused the class to look at the screen either on IWB or on their computer screen as he could control students' screens through specific software in the classroom. In this sense, grouping arrangement for this format consisted of a whole class exposition. The teacher aimed at sharing and discussing different examples from students, which was part of the teacher's curriculum script, in the sense that he pointed out the example that most of the students had not worked on. This activity format was delivered through explaining the screen and discussing the screen by questioning.

Table 3.8 Spot-and-show in the middle of Chris’s first ‘Circle Theorems’ lesson teaching with GeoGebra

Ladies and Gents I am going to stop you there briefly. Could you leave the stuff up on your screen and leave the mouse for a moment?

(Explain the screen)

Now on my screen and on yours, you can see James and Matt’s work. One of the things they have done here has been to draw some quadrilaterals.

(He chose a different example)

Rest of us has been looking at triangles. I can see on there a quadrilateral and I can see some angles and there is a nice relationship between some of the angles.

(Discuss the screen through question/answer)

What is the bit that isn’t a surprise about the angles in a quadrilateral? What do we know about angles in a quadrilateral? Yes, they added up 360 degrees. So we know about that in a quadrilateral.

(Explain the screen)

In a circle we call it a cyclic quadrilateral where all of the four vertices on the circumference of the circle. And there is a link between some of the angles in the quadrilateral.

(Suggesting different ideas from students’ work through explaining the screen)

You might want to have a look at the left hand circle. Have a look on your screen. Have a look at the angles in their quadrilateral so that might be something you could use to form a conjecture. So there is the first one.

Next one there are other people who made conjectures about the angle we get at the centre of the circle and about the angle we get at the circumference. And you could get that as a conjecture. There is some people who did all of those lines from a single point and calculated worked out as GeoGebra told them what those angles were.

Now because Sam’s computer is not connecting to the network properly I can’t show you what they (Sam and Ben) have done. They started with a chord they put a line, radius which cut that at 90 degrees. They did a perpendicular through that and they found something interesting happened they reckoned to the chord. All sorts of things that you could do.

So we have got 5 min. in that 5 min could you please have a conjecture of some kind? Could you make sure you have written it either in a GeoGebra file or in word? Could you save it to your bit of the network and then could you drag and copy across to student share and go.

This transcript also indicated that the teacher signaled transition times in order for students to focus on their work without losing learning time, which introduced time economy in teaching this topic with technology. Additionally, the teacher developed a closing routine for technology lessons as he instructed students to save their work and specifically explained how they should save their files. In summary, I used these examples to explain what lies underneath the more compact representations that I used for these activity formats and also how these formats intervened with other features of classroom practices. In later chapters, I am going to present more condensed accounts of what activity formats look like in order to conserve space.

3.6.3.2 Data Display for Cross-Case Analysis

I adopted the cross-case analysis as my study consists of multiple individual cases (Yin, 2009; Miles & Huberman, 1994). As has been mentioned before, one of the aims of using multiple cases is to increase the generalisability of the results. Most importantly, “the aim is to see processes and outcomes across many cases, to understand how they are qualified by local conditions, and thus develop more sophisticated descriptions and more powerful explanations” (Miles & Huberman, 1994, p. 172). The aim of the cross-case analysis is to seek what is similar and different about the cases in order to understand the phenomena better. In other words, the analytical frame of such analysis is provided by comparison through looking at the differences between cases (Stake, 2006).

In order to see the general picture of the data from multiple cases, I created a matrix for generating theme-based assertions (Stake, 2006), which helped me to see similarities and differences between cases.

3.6.4 Conclusion Drawing and Verification

Conclusion Drawing and Verification is the third stage of qualitative analysis. This process is basically about considering what the analysed data say and evaluating their implications for the research questions (Miles & Huberman, 1994). During coding and data displaying, I have already begun to explore conclusions, as analysis is an iterative and ongoing process. I refer back to the data displays and raw data

while descriptions of classroom practices and relationships are made. In order to generate meaning from the data, I made use of the following tactics (Miles & Huberman, 1994):

- noting patterns, themes (within-case analysis)
- making contrasts/comparisons (cross-case analysis)
- making conceptual/theoretical coherence

Verification simply involves cross checking the conclusions being drawn and contributes to the rigor of a study. "The meanings emerging from the data have to be tested for their plausibility, their sturdiness, their 'confirmability' - that is, their validity" (Miles and Huberman, 1994, p. 11). The meaning of validity is different in this context than in quantitative research, where "it was essentially a demonstration that a particular instrument in fact measures what it purports to measure" (Cohen et al., 2007, p. 133). However, validity in qualitative research refers to whether the conclusions being drawn from the data are "plausible, credible, trustworthy, and, therefore, defensible" (Johnson, 1997, p. 282).

I now discuss the verification strategies I employed for this study.

3.6.5 Verification Strategies

Maxwell (1992) identifies three types of validity that are relevant to qualitative research:

- Descriptive validity
- Interpretive validity
- Theoretical validity

These are important concepts particularly to my research since description of observed events and interpretation of teachers' accounts are the primary tools for data analysis. Descriptive validity encompasses whether the reported descriptions are accurate or not. In order to reduce the threat to descriptive validity, audio and video recordings were transcribed. Interpretive validity indicates, "accurately portraying the *meaning* attached by participants to what is being studied by the

researcher” (Johnson, 1997, p. 285). For achieving interpretive validity, I received participant feedback, which is also called ‘member checking’ (Stake, 1995). Member checks provide verification for the accuracy of the information transcribed and interpreted in the research study attempting to control for researcher bias. Therefore, participants were given the opportunity to review and edit the information representing their perspectives to verify its accuracy. As a second verification strategy for interpretive validity, description phrases that are close to the participants’ accounts and my field notes as well as low-inference descriptors such as direct quotations from participants were used.

Here is an example of a direct quotation from a teacher in my study that was talking about the classroom where the lessons took place:

In most cases because either you are in a classroom that does not have computers so it is not possible to use them, or you are in a computer room where everybody has to sit at a computer because there is not any other space. You have to do one or the other. Whereas the wonderful thing about that classroom and the reason why I was so excited about teaching in that classroom is that we can spend part of the lesson at the tables and part of the lesson at the computers. (Chris)

This quotation provided information about the teacher’s interpretations of the working environment. He expressed his excitement working in such a classroom. Hence, I aimed to achieve readers’ understanding of participants’ perspectives through these kinds of verbatim.

Furthermore, in order to cross check conclusions being drawn, triangulation, which is an important theoretical validity check, was employed. The data were triangulated among different methods used to collect data: teacher interviews and classroom observations. The main strength of observational data is that I saw the actual classroom events. A weakness of interviews is that what the teachers say and what they actually do may be different. However, through interviewing, it is possible probe the teachers’ thinking, and reasoning. With the intention of understanding the phenomena in whole, interview data were compared to what take place during observations in order to see how what was being said fit together with my observations in the classroom. Furthermore, classroom resources were used to supplement this data. A variety of instruments for data collection, such as audio

recording, field-notes and observations, were also used to enhance the trustworthiness of the findings.

I linked the set of data derived from a series of semi-structured teacher interviews together with classroom observations to support detailed, and accurate evidence representations for teachers' individual lessons with GeoGebra. This included on the whole:

- Field notes/transcripts of classroom observations (triangulated by the audio recording);
- Verbatim transcripts for related episodes in lessons ;
- Direct quotes from teacher interviews ;
- Screenshots of students' and/or the teacher's technological display for related episodes within the lesson ;
- Photographs of students' work.

I provide an example below from a participating teacher (Chris) to show how a direct quote of the teacher from an interview, a verbatim transcript of a lesson and a screenshot of a student's technological display were used in order to represent a new emergent activity format, Predict-and-test.

A direct quote from a post-lesson interview:

There was Ben with his excellent centre of enlargement but error with the scale factor. I deliberately let him make the mistake and then suggested him just check it. I deliberately left him to it. So he would first of all realise that checking was important. He would realise that he made a mistake and then wanted to sort it out; I did not tell him what he had done wrong. (Chris)

A Verbatim Transcript of a Lesson (An Example of Predict-And-Test Activity Structure)

Ben: I did the enlargement question.

T: How did you find the centre of enlargement?

Ben: Line-you connect those two up. When they all join together, obviously from that point they are all branching up. *(He was explaining the screen in Figure 3.4)*

T: What is the other bit of information that we need for an enlargement?

Ben: Scale factor

T: How are you going to work that out?

Ben: I need to measure how far that one (HD) and that one (DA') and then divide

(The teacher knows that he is making a mistake. Ben should measure (HD) and (HA') and then divide. But the teacher wants Ben to discover this mistake by self-testing)

T: You can do that on GeoGebra- somewhere on one of these things there is a measurement tool- keep going over to the right. It has something to do with measuring things and sizing things. Does it say measurement there? Go left a bit then-*(while Ben was looking for the icon)*

There we go. It says select two points. Be aware, it gives everything to one decimal place. It might have been rounded.

(Ben starts doing the calculations)

T: Do the enlargement yourself? And check that out if it works

(The teacher goes to other groups to see what they are doing while Ben is doing calculations. After 2 min, Ben calls the teacher and says that the scale factor is 1.5)

T: Could you just check it? Go to enlargement. You are saying 1.5, let's see

(Student is doing enlargement using the scale factor 1.5 he just found. But it shows that 1.5 is wrong.)

Ben: Ahhhhhhhh (with disappointment)

T: Ben, do some thinking- what has gone wrong?

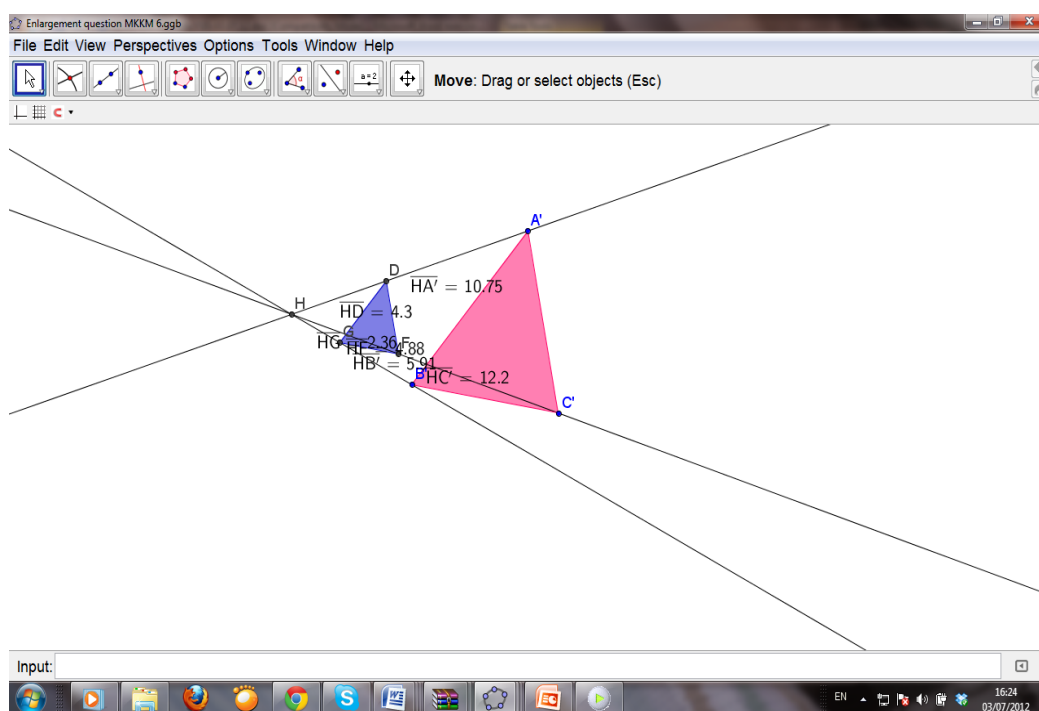


Figure 3.4 A screenshot of a student’s technological display from Chris’s ‘Transformations’ lessons

In the above example, the direct quote from a post-lesson interview provided some description (i.e. what the student did) but it also provided some information about the teacher’s intentions and personal meanings for his chosen activity format. From this quote in conjunction with the verbatim lesson transcript, it became apparent that the teacher’s intentional action is to have the students testing their predictions out on the computer rather than him validating or invalidating their conjecture. In this regard, the underlying structure of this activity format consisted of mainly interaction between students and computer; in this particular example it was afforded by the use of GeoGebra.

3.7 Ethical Considerations

As a first step, before the data collection begins, the purpose of the study including “why their participation is necessary, how it will be used, and how and to whom it will be reported” (BERA, British Education Research Association, 2011, section 11) were explained clearly to the participants in order for them to understand the nature

of the research as well as its probable impact on them. This also helped me to minimise the teachers' potential preconceptions towards my role as a researcher. So they knew I was not there to judge them but just to understand how GeoGebra was being used in classrooms. The participant teachers were also informed that participation in interviews and observations was voluntary, that they had the right to refuse to answer any questions and the right to withdraw from the research process at any time without penalty or prejudice. Permission for the documents such as teachers' lesson plans, student resources and software files was sought from each teacher orally.

In addition, I agreed with participating teachers how to approach the ethical issues involved in audio/video recording in a way that respects the policy on such matters within their school. I informed teachers and students that I was present and capturing the events through either video/audio recording or field notes. However, this could suffer from the weakness of direct observation of being reflexive, since the "event may proceed differently because it is being observed" (Yin, 2003, p. 86). During the interviews, interviewees were asked if they were comfortable being video-recorded. In addition, they had direct access to the recorder so that they could stop the recording at anytime during interviews. Furthermore, regarding privacy of the participants, in the ethical guidelines published by BERA (2011) the section on privacy states:

The confidential and anonymous treatment of participants' data is considered the norm for the conduct of research. Researchers must recognize the participants' entitlement to privacy and accord them their rights to confidentiality and anonymity. (p.7)

Researchers have a moral obligation not to reveal or divulge a respondent's identity (Hartas, 2010). A letter of consent (see Appendix 6) in accordance with the guidelines published by BERA (2011) was sent to participant teachers prior to the data collection. This was to ask if they wished their contributions to be identified, anonymised, or acknowledged in any written reports arising from the study (BERA, 2011, section 25). Thus, unless they wished their contributions to be identified, or acknowledged, all interview and observation transcripts were anonymous and confidential; their names and personal information were not used in reporting the outcomes of this research. Quotes within this study were expressed using

pseudonyms. In Section 3.4.2, I have introduced the pseudonyms I used for the teachers who featured in each case. Confidentiality of the respondents was maintained by not divulging information to other persons, except those directly involved within the study. Moreover, all participants were assured that their data would be kept fully secure and destroyed on completion of the study. At the conclusion of the study, each participant would be given a copy of the final summary of the research project.

3.8 Conclusion

This chapter detailed the research design and methodology focusing on instrumental case study used in this research. Multiple methods of data collection were utilised, namely, classroom observations, and teacher interviews. Having outlined the design and methodology, I now turn to the actual data that was produced through these methods. The next chapter will examine each of the three teachers' individual lessons and compare their lessons within teacher.

4

INTRA-CASE ANALYSIS

4.1 Introduction

This chapter will analyse the three secondary mathematics teachers' (Susan, Tom and Chris) use of GeoGebra in their teaching. Data will be analysed in accordance with the conceptual framework, which identifies structuring features of classroom practice including five aspects: working environment, resource system, activity format, curriculum script and time economy discussed in the second chapter.

Chapter 1 already stated the two research questions that need to be addressed in this thesis: **"What aspects of classroom teaching shape the ways in which teachers integrate new technologies?"** and **"What kind of professional knowledge do teachers need to teach with them?"**

As has been pointed out in Chapter 3, working within the SFCP framework implies looking at characteristics of classroom practices linked to the five aspects of this framework, with a view to identifying variations and commonalities in these practices. In this connection, following sub-questions provided a framework for the analysis:

1- Working Environment

What kind of working environment does the lesson take place in?

What strategies does the teacher employ to manage this working environment?

2- Resource System

What resources are used in this lesson by the teacher and by the students, and how do they relate to other resources used in lessons on the topic?

What strategies does the teacher employ to manage use of particular resources, and their coordinated use as a resource system?

3-Activity Structure

What is the activity structure of the lesson, broken down into a sequence of segments each involving a particular type of activity format?

What roles do teacher, students and technology play in each type of activity format, and what strategies does the teacher employ to manage this?

4-Curriculum Script

What components of an established curriculum script are indicated by the teacher's (smooth and confident) handling of the approach to studying the topic over the course of the lesson?

What enhancements to this curriculum script are suggested by the teacher's (responsive and improvised) handling of unplanned issues relating to the topic emerging during the course of the lesson?

5-Time Economy

What strategies does the teacher employ to manage the allocation and use of time, in planning the lesson, and during the teaching of it?

This chapter consists of three sections, one for each teacher, providing first basic descriptions of the teacher's lessons and a comparison between them (i.e. a within-teacher comparison). Firstly, I will analyse Susan's use of GeoGebra in teaching "Transformations", "Circle Theorems" and "Straight Line Graphs" in section 4.2. Following this, I will present my analysis regarding Tom's use of GeoGebra in teaching "Transformations" and "Straight Line Graphs" in section 4.3. Finally, section 4.4 will state the analysis of Chris's use of GeoGebra in teaching

“Transformations” and “Circle Theorems”. These sections are composed of sub-sections where I present analytical findings to address each of sub questions regarding the SFCP framework leading to an overall discussion of these teachers’ teaching of different topics with GeoGebra at the end of each section. In other words, I will make a summary at the end of each teacher’s lessons about to what degree there were consistent patterns or differences across their lessons. These sections aim to provide a good basis for making a comparison between lessons by the same teacher just as much as a comparison between two different teachers teaching the same topic (e.g. Circle Theorems). In this regard, I will present two main kinds of comparison to make in general, a comparison of the same teacher teaching different topics with GeoGebra (in this chapter), and a comparison of different teachers teaching the same topic with GeoGebra (in the next chapter). For this reason, my initial descriptions of the lessons aimed to be equally useful and unbiased as regards either type of comparison.

4.2 Susan

In this section, I will present my analysis of Susan’s implementation of GeoGebra in teaching the three different topics of Transformations, Circle Theorems, and Straight Line Graphs to three different classes. I will then make a comparison between those lessons across topics.

4.2.1 Transformations

Susan taught ‘Transformations’ to a lower set Year 7 class over two lessons. As has been already mentioned in Methodology section, after teaching these lessons, Susan encouraged one of her colleagues (pseudonym James) to teach this topic with GeoGebra. Consequently, they revised the worksheet (that Susan had used for her lessons) together considering the problems that she encountered during her lessons. James then made use of this revised worksheet with his Year 7 class to teach ‘Transformations’ over the course of two lessons. During James’s lessons, Susan was present in the classroom and helped James to guide students when they worked at computers. Thus, my analysis in this section consists of four lessons: two of which were taught by Susan and two of those were prepared by Susan and James, but taught

by James to his Year 7 class. However, James's lessons will not be discussed in greater detail in the section since the case of James will be mainly used to reveal Susan's learning or expressing her learning as has been already noted in Methodology chapter.

4.2.1.1 Working Environment

Susan's lessons occurred in an ordinary classroom where the seating consisted of rows of tables with chairs. There was a Smart-board as well as an ordinary white board in the front (see Figure 4.1). The teacher provided students with laptops on which GeoGebra was installed. Students used laptops at their normal tables in the classroom. In order to ensure that students would have a functioning computer system, she spent a considerable amount of time beforehand.

I have worked so hard to pre-empt them [*any technical problem with laptops*]. There are 30 laptops. Every single laptop was checked by that year 7 class. They had logged on everything and made sure that GeoGebra worked. That was hard work and to just get GeoGebra on the computers was very really difficult. For some reason, GeoGebra was not on laptops. I could not have done the lessons I have done with you without GeoGebra on laptops. So I had to get it sorted out. [S/1]

Seating position for students was within the traditional row and column classroom. Therefore, there was not any particular movement from students' side during lessons except collecting their laptops from a corner of the classroom at the beginning of each lesson. On the other hand, the teacher had two different forms of positioning in general. During the initiation and summary phases of each lesson she did whole class teaching from the front. During the middle phase of the lesson while students working in pairs on laptops she walked around the classroom in order to determine whether students had any difficulties during tasks and to encourage students' involvement and engagement with the task. In other words, she was less tied to the front and recognising the impact of proximity on involvement, interacted verbally with students when they worked on laptops.

If I am using computers, I got to be making sure they are on task. This is all me talking about scanning the room. I want to see what they are doing. Even with the remote, I would not be able to see exactly what they are up to as far as the maths point of view. So I want to walk around and glance and see what they are doing. [S/2]

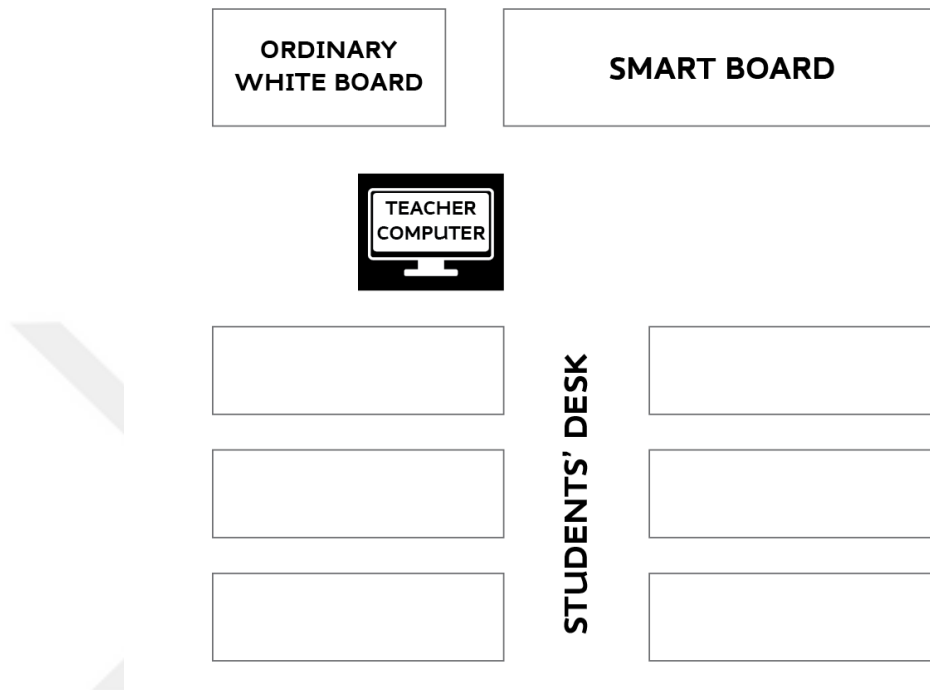


Figure 4.1 The room where Susan's 'Transformations' lessons took place

Working with laptops required Susan to create some routines. She introduced some opening routines including how to log on, how to find GeoGebra and how to load up files at the beginning of the first lesson. In terms of attracting students' attention, she made sure students turned their laptops' screen towards the front and closed them down in order for students to focus on the board while she was conducting a whole class teaching, which aimed at involving them in classroom discussions without getting distracted by computers in front of them. However, both Susan and James emphasised that since students had not developed routines with working at computers, it was more difficult to maintain their attention to the whole class teaching.

I had difficulties to get them started and lack of pace I suppose at the start, real focus at the start because they come in and faffing about I suppose. But I don't know if there is any choice to that. It is that faff, isn't it? My laptop is not charging etc etc. When they know they will use laptops they stop listening. "Oh good we use laptops today" and then they start a conversation. I say "stop talking, listen". It is hard to get their attention a little bit more at the start of the lesson. [J/1]

It is a little bit like you say because they are not necessarily used having laptops regularly; they can't cope with the excitement of having them. They almost carried away with that. I think the worksheet was good and I think if they do it more regularly they will get used to following the routine and they will get used to doing that. [S/3]

As a closing routine, Susan reminded students to save and upload their work on the Virtual Learning Environment (VLE) when they finished. At the beginning of the first lesson, she specifically explained how students should upload their work onto the VLE as they went along in order for the teacher to see those files. She considered this as a way of supervising students, which aimed to maintain the activity flow.

I think with the transformations the uploading was not essential. Again you see my obsession with classroom management. I was making them to upload it to put the pressure on them. The files are on Moodle. I could go through and look at them. What I could do is that next year I can show them to other students. But actually in reality it was more as you are doing something I am going to see. So you have got to get on. [S/4]

She also made sure that laptops were logged off and plugged in, which aimed at minimising potential technical problems for future lessons.

4.2.1.2 Resource System

Susan employed GeoGebra with a pre-developed (paper) worksheet including four tasks, accompanied by prepared dynamic versions of each task for students to experiment with in GeoGebra. She used the school's VLE for students to download/upload the tasks. Use of the VLE also aimed at sharing students' work with the whole class.

The purpose of these lessons was for students to learn a specific type of transformation, reflection. Susan's use of GeoGebra for this topic aimed at easing

students' learning by permitting them to observe and think about the processes involved.

The objective of that lesson was that they could do reflections on paper. I think having tried loads of reflections themselves on GeoGebra and then having to write them in the instructions helped them to see what was happening. They have done lots of reflections on GeoGebra and got them all right because GeoGebra was doing it for them. So they could see what was happening. And that would work with the diagonal lines, which they find it impossible to do. In this sense, they were seeing what happens and giving themselves confidence almost to do by hand. They were investigating how changing things makes the image different. [S/5]

In addition, Susan reported that GeoGebra speeded the learning process up by allowing the students to engage with more examples that were unusual to see in the traditional paper-and-pencil environment (See Figure 4.2 showing a student's screen who was trying out reflection task 2 shown in Figure 4.4)

They can see weird ones. They can change an easy into a weird and see straightaway what happens much quicker than a mirror. They can't see it only once but they can see another time and another time. [S/6]

Furthermore, she valued the use of GeoGebra for providing students with self-testing. In her words: "I like it when they were being able to quickly check their answers. So they are doing something on paper and then they can check their answers at computers" [S/7]

As has been noted before, Susan has had little experience in using technology. She used GeoGebra mostly for demonstration purposes, which required less preparation and fewer resources to organize the lesson in comparison with making students use technology themselves. When she decided to use GeoGebra for this topic, she adopted a pre-developed worksheet that was not prepared by her. Instead, she borrowed it from an online resource website due to lack of preparation time.

I basically brought in something. I went on Google. I typed in 'year 7', 'Transformations', 'GeoGebra'. That is what I got. I did not have time. I want to be realistic to you. I did not change it. I thought this looks good. It had some good reviews. People have used it before. I thought yeah I would go for it. [S/8]

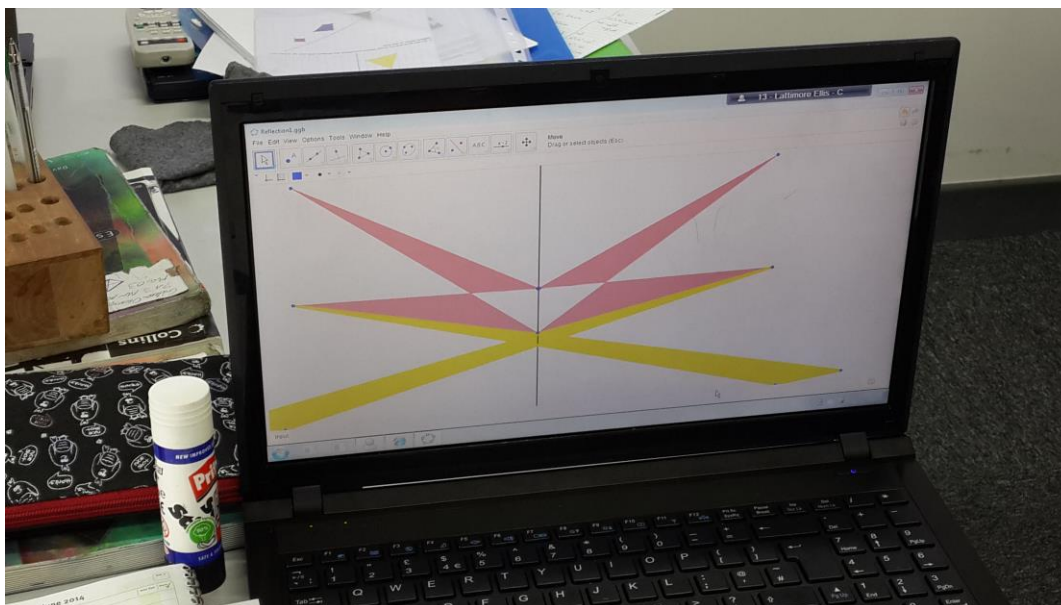


Figure 4.2 A student's screen about Reflection task 2

In addition to the timing aspect, she pointed out that having prepared files for students to engage with was also important for her confidence as a novice technology-user in conducting such lessons with technology.

I think if you are doing something like this for the first time as myself, worksheet almost helps the teacher because it means there is almost like a crutch for the teacher isn't it? You know that is effectively the lesson plan. It is there and it is ready and you can hopefully rely on them actually following those instructions without having to constantly tell them what to do next. I think as a somebody starting GeoGebra new, I would never want to go in without something like that, just even the fact that it has got the pictures on means I am not going to panic. [S/9]

Susan's class was also novice in GeoGebra use; therefore she allocated some lesson time to showing students the general set up of GeoGebra and the VLE, prior to the Transformations lessons, for students to get familiar with the technology. Additionally, as has been mentioned before in quote [S/1], this aimed at checking all the laptops to make sure GeoGebra was installed on the computers.

They opened GeoGebra, they saw how they had to download from VLE. They basically learned the very basic things. They had 5 min actually hands on a lesson before. [S/10]

Students experimented by themselves with four different dynamic GeoGebra files. The accompanying paper worksheet stepped them through a set of activities, which aimed to help them to understand and see reflections as well as gain familiarity with GeoGebra. The idea behind this was that they tried first to draw the image specified by the worksheet on paper, then followed by using the computer to check their results, after that to write instructions for drawing the image with the aim of helping them clarify the process they went through to do the reflection. During the period when students were working at the computer in this way, the teacher's role was to walk around and make sure that they engaged with their task [S/2].

In the first task of the worksheet, students were presented with a symmetric face (see Figure 4.3), where they could play with its shape and create new objects on the face (such as earrings, eyebrows). This aimed for students to become accustomed to the software such as how to draw a polygon, circle or line segment and how to use the reflection tool (see Table 4.1) as Susan pointed out: “I wanted them to practice GeoGebra with face activity and get familiar with GeoGebra. I also wanted them to become familiar with the construction and create new objects on the face.”[S/11]

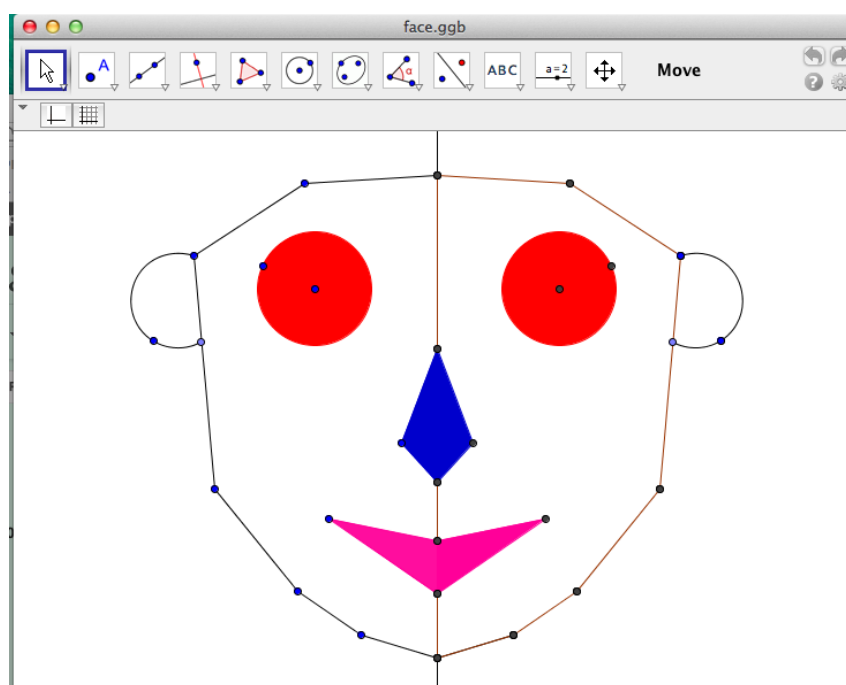


Figure 4.3 The face task as a GeoGebra file

Table 4.1 Instructions for the face activity (Reflection task 1)

Open this file to find the picture of a face! Make sure you have the Move tool selected and then start making the following changes:

- Reduce the size of his eyes.
- Make him look sad.
- Give him 2 earrings (chose a shape: polygon  or circle ).
- Make one on the left and then reflect it in the line using the reflect tool .
- Give him eyebrows (use line segment  and reflect it in the line)

All the reflection files required, first, students' predictions of where they thought the shapes would end up after reflection, second, students checking their predictions in GeoGebra to see if they were correct. In addition, with Reflection Task 2 and Reflection Task 3 files (see Figure 4.4 and Figure 4.5), students were told to move some of the points at the corner of the shapes and observe the changes (see Table 4.2 and Table 4.3).

Furthermore, Reflection Task 3 and Reflection Task 4 files (see Figure 4.5 and Figure 4.6) required students to write down how to do reflection on paper-and-pencil (see Table 4.3 and Table 4.4).

I did like the idea of the instruction thing where you count and you see how far because the girl at the back got that idea into her head. She was counting them wrong. But she knew exactly what she was trying to do. [S/12]

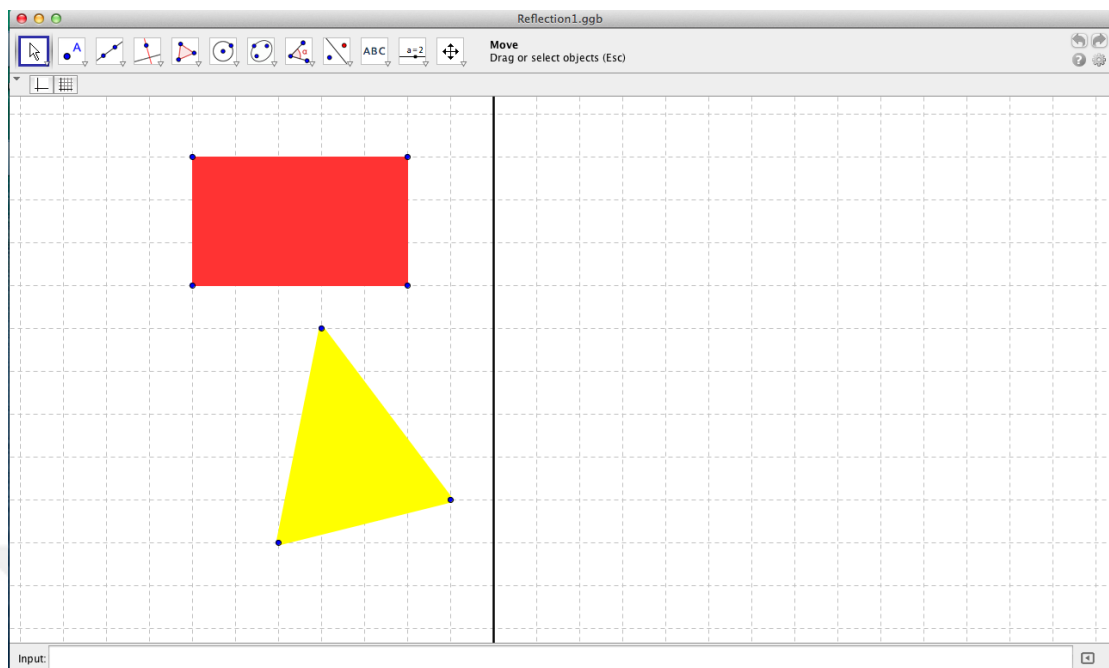


Figure 4.4 Reflection task 2

Table 4.2 Instructions for Reflection task 2

a, On the grid below, draw where you think the shapes will end up when you **reflect** them in the line.



b, Now use the computer to check if you are correct.

c, Have a go at moving some of the points at the corner of the shapes (don't forget to select the "Move" tool first).



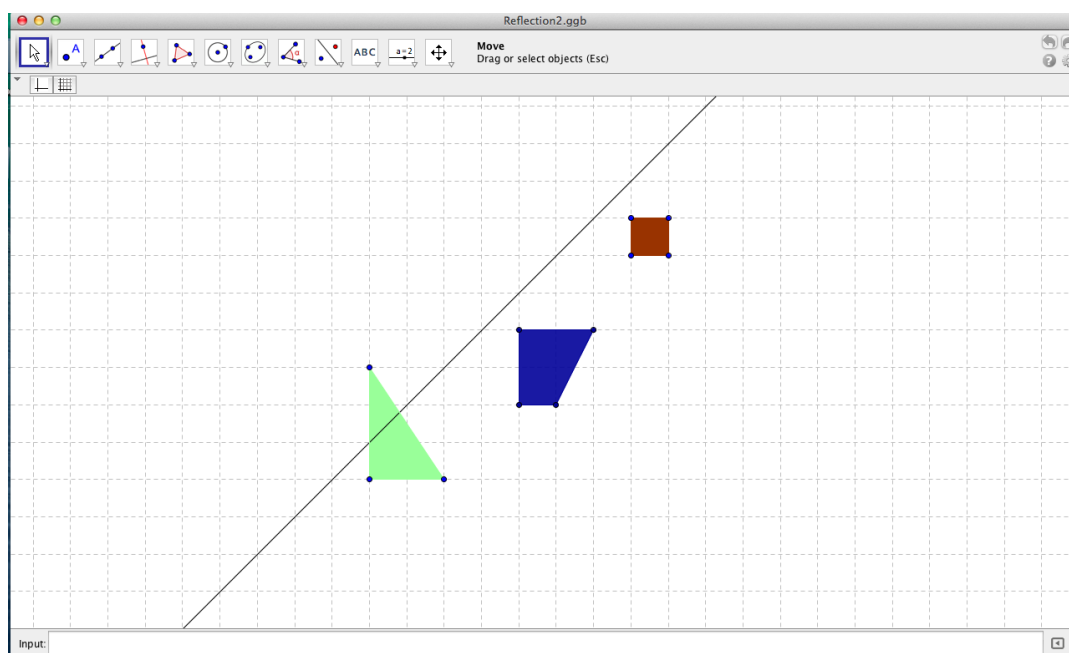


Figure 4.5 Reflection task 3

Table 4.3 Instructions for Reflection task 3

- a, On the grid below, draw where you think the shapes will end up when you **reflect** them in the line
- b, Now use the computer to check if you are correct.
- c, Have a go at moving some of the points at the corner of the shapes (don't forget to select the "Move" tool first).
- d, Imagine you need to explain to someone else in Year 7 who isn't allowed to use the computer how to do reflections. Use the space below to write step-by-step instructions for them

However, the difficulty of one of the tasks in the Reflection task 3 (shown in Figure 4.5) appeared to be important since students had difficulties in moving forward.

It is a bit weird what happens with that one. That has gone about three jumps too fast. It is a triangle which is harder anyways, it is a diagonal which is harder anyways, and it is an overlap which is harder anyways. They have gone straight from easy peasy to this one. That jump is too big. So I want to help them through those jumps. [S/13]

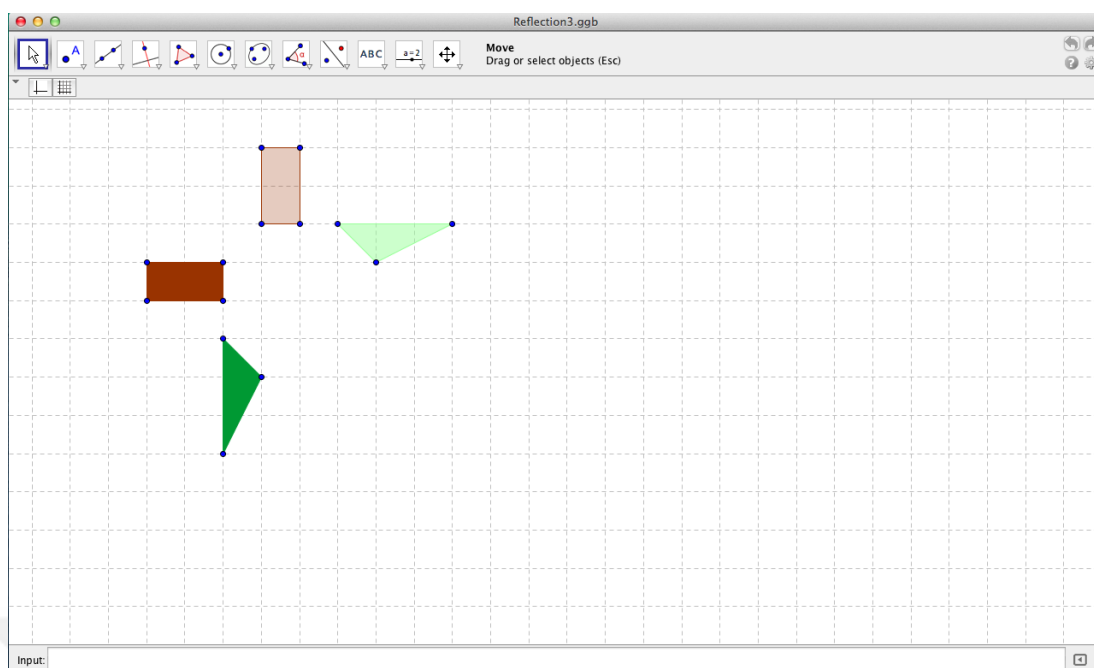


Figure 4.6 Reflection task 4

Table 4.4 Instructions for Reflection task 4

- a, On the grid below are 2 **objects** (dark) and their **images** (light). Can you draw exactly where you think the **reflection line** is?
- b, Now on the computer check to see if you were right: first draw the line then reflect the **objects** in the **reflection line** to see if they land on the same place as the **images** that are already there. If you didn't get it right first time, have a go at moving the reflection line to see what happens.
- c, Now try moving the reflection line so that it cuts through one of the shapes.
- e. Once again, you need to explain to someone else in Year 7 who isn't allowed to use the computer how to do reflections. Use the space below to write step-by-step instructions for how to find the line of reflection.

As noted already, James and Susan revised the worksheet and rewrote some of the instructions in accordance with Susan's reflections from her lessons. After her own 'Transformations' lessons, Susan considered that this worksheet needed to combine more pencil and paper activities. Accordingly, both teachers worked on this aspect and added two more worksheets (Figure 4.7 and Figure 4.8) that enabled students to combine computer activities with paper-and-pencil tasks.

We did go through the whole A3 worksheet step by step and interspersed the worksheet. I did not like the fact that they were doing on computer and they did not have enough practice on paper to put into practice what they had actually seen on the computer. That is why we basically added the pen and paper worksheets. We also changed the wording to tell them when to do the paper worksheets. [S/14]

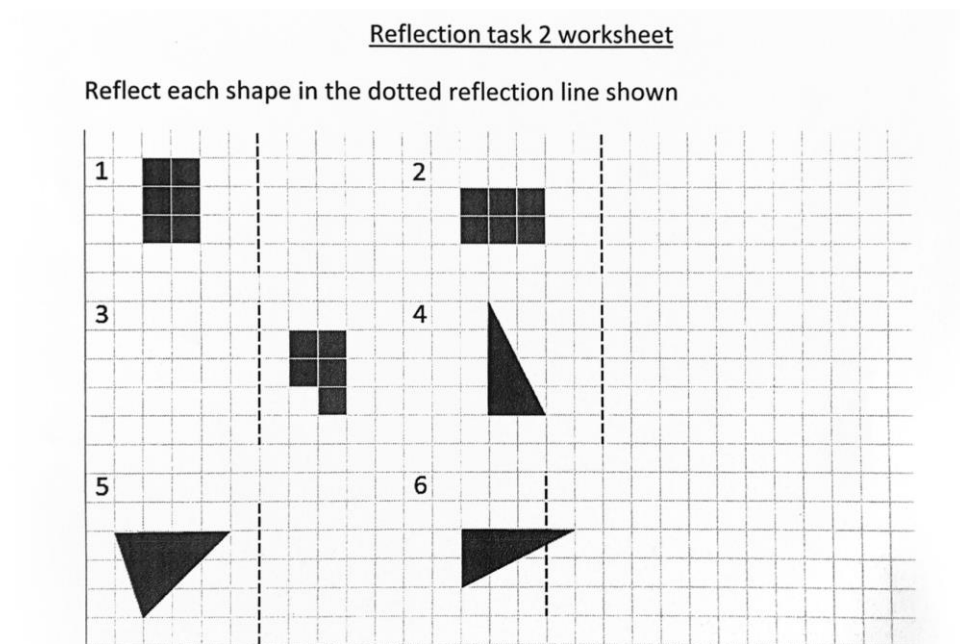


Figure 4.7 Additional paper worksheet 1

They maintained the GeoGebra files but only altered some of the wording of the instructions on the worksheet.

Susan: We were happy with the actual files themselves, won't we?

James: Yeah files were fine.

Susan: We added the pen and paper worksheets and we went through the A3 worksheet that was changed a bit. We changed the wording to tell them when to do the paper worksheets (see Table 4.5 and Table 4.7).

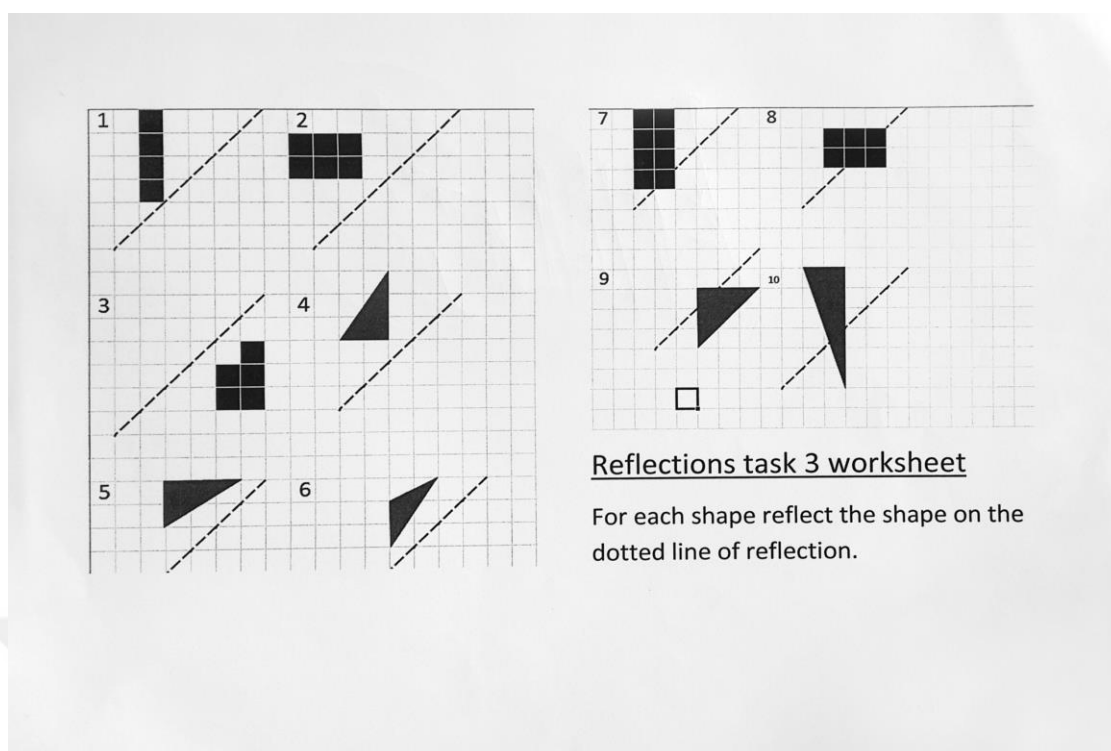


Figure 4.8 Additional paper worksheet 2

Table 4.5 Revised instructions for Reflection task 2

- b, Now use the computer to check if you are correct
- c, Have a go at moving some of the points at the corner of the shapes (don't forget to select the "Move" tool first.
- d, Have a go at moving the reflection line. Look at how the position of the image changes. Try making the reflection line pass through the original object and notice what happens to the reflected image.
- e, Complete the Reflection Task 2 worksheet.

For the Reflection Task 2, they altered the wording of the instruction (shown in Table 4.6) , where students are directed what specific things to look at since Susan noticed during her lessons that students did not know what to look at when changing parts of the diagram. As Susan commented after her lessons implementing the worksheet for the first time: "Some of these here where it says have a go, it is not really asking them to reflect on what happened when they had a go. They almost did not see the point of having a go." [S/15]

Table 4.6 Instruction for Reflection task 2

Original instruction	Revised instruction
Have a go at moving some of the points at the corner of the shapes (don't forget to select the "Move" tool first).	Have a go at moving the reflection line. Look at how the position of the image changes. Try making the reflection line pass through the original object and notice what happens to the reflected image.

Table 4.7 Revised instructions for Reflection task 3

b, Now use the computer to check if you are correct c, Have a go at moving some of the points at the corner of the shapes (don't forget to select the "Move" tool first). d, Complete the Reflection Task 3 worksheet. e, Read through your first draft of instructions for someone else in Year 7 who isn't allowed to use the computer and complete a final version to ensure that they could do all the reflections that you have now done.

Table 4.8 Instruction for Reflection task 3

Original instruction	Revised instruction
Imagine you need to explain to someone else in Year 7 who isn't allowed to use the computer how to do reflections. Use the space below to write step-by-step instructions for them	Read through your first draft of instructions for someone else in Year 7 who isn't allowed to use the computer and complete a final version to ensure that they could do all the reflections that you have now done.

Furthermore, as James encouraged his students to check their answers also with tracing paper in addition to computer checking, Susan also highlighted the importance of combining the use of tracing paper with GeoGebra use. In this way, she believed that students could link screen to other conventional resources.

GeoGebra is one tool to check and get them used to it, but tracing paper is a traditional tool for this topic and just introducing the tracing paper idea as well, which you started to do it *[telling James]*, would be good. [S/16]

4.2.1.3 Activity Structure

Susan first introduced the lessons from the front to the whole class and then allocated activity time for students to work at computers during which she walked around the classroom and provided students with guidance (see Table 4.9 for the detailed activity formats and related lesson agenda). As has been discussed in the Methodology chapter, I will mostly make use of those already identified activity formats (see Table 3.4, and Table 3.5) to label teachers' lesson segments.

Technical-demo

At the beginning of the first lesson, Susan did a technical demo in order to show students how to go onto the VLE and find the GeoGebra files for students to download and work on and then upload back the finished activity to the VLE. In addition, she explained the tasks on the given worksheet and the dynamic version of those tasks found through the VLE. This format was carried on in a whole class setting with the use of IWB and delivered by demonstration and explaining.

Table 4.9 Activity formats and lesson agenda followed by Susan in 'Transformations' lessons		
Lesson	Activity Formats	Lesson agenda
<u>Lesson 1</u>	Use-tech/Work-and-walk-by/Technical-support	Susan instructed students to take one laptop each, and on where to find prepared files for these two lessons. While students were settling down with their laptops, and logging into the VLE and finding GeoGebra files, the teacher monitored students and guided them when necessary.
	Use-tech/Technical-demo/Observe-demo	Once all the students settled, Susan showed some tool techniques to the whole class on the IWB such as which GeoGebra icon they need to use for reflection, and explained the tasks on the worksheet. She also demonstrated how to upload files to the VLE since she wanted students to upload each files to the VLE when they finished.
	Use-tech/Work-and-walk-by/ Practice-with-technology	The teacher walked amongst students to provide feedback whilst students were working on the face activity to familiarise

		themselves with the software techniques related to Transformations.
	Use-tech/Spot-and-show	She showed some examples of face activity, which students had finished and uploaded to the VLE, to the whole class to remind them of uploading files as they went through.
	Use-tech/Explain-the-screen Use-tech/Work-and-walk-by/Predict-and-test	She provided intermediate synthesis through discussing a diagram with the whole class regarding writing instructions for how to do reflection on paper, and then let students continue their work at computers.
	Not-use-tech/Review-student-work	Last 5 minutes, the teacher asked students to read out their descriptions regarding how to do reflection on paper.
<u>Lesson 2</u>	Use-tech/Explain-the-screen	Susan reviewed students' homework about reflection, in particular, their instructions about how to do reflection. She checked their answers in GeoGebra projecting on the IWB to the whole class. In addition, she encouraged them to download GeoGebra at home in order to check their homework.
	Not-use-tech/Whole-class-instruction	The teacher instructed students to carry on the tasks where they left off from yesterday.
	Use-tech/Work-and-walk-by / Predict-and-test	While students were working on diagonal line reflection, Susan circulated around the classroom. Since this task seemed more difficult for students and resulted in some losing motivation, she stopped the whole class twice to remind them what they were expected to do.
	Not-use-tech/Whole-class-instruction	Susan finished the lesson by reminding students about the closing routine.

Work-and-walk-by

Work-and-walk-by was highly predominant in Susan's lessons. She monitored students during their work at computers and provide them with feedback. During work-and-walk-by, when she noticed any common mistake or misconception she attracted students' attention to the front and talked to them as a whole class.

It is just the strategies to make sure because from the teaching point of view, it is often the most efficient way to do it bring them all back together and show them something, particularly when you are going round and you are saying the same thing over and over again. [S/17]

During this activity format, students were seated in front of their laptop computers in pair, with access to the GeoGebra tasks through the VLE and the printed worksheets on their desk. In addition, the VLE allowed the teacher to monitor students' tasks and share those tasks with the whole class on the IWB with the aim of helping their progress. As Susan pointed out that monitoring students in the classroom during work-and-walk-by was difficult for her. Hence, the idea of combining computer and paper-and-pencil activities was also useful to ease monitoring students for her during this activity format.

If I have a computer work and hand drawn work, it is going to be easier for me to see how far people are. It would be visually easier to be able to see when they do it on paper because the laptops would be gone away and then get the laptop again. It will be physically easier to tell where they are up to because I still find it very hard when you actually walking around the class you have got the child face to look at not what they are doing on their laptop. [S/18]

Predict-and-test

A Predict-and-test activity format was notable during paired student work at computers while moving through the sequence provided by the worksheet. All of the reflection tasks on the worksheet were formed according to a predict-and-test activity format. Students were expected to make their own estimates about the reflections on paper and then to test them out at the computer by using the reflection tool to find out if they were correct.

With GeoGebra, students can actually see correct reflections to check their hand drawn work without me telling them if they are right or wrong. [S/19]

In this respect, students were leading the activity as well as engaging in segment assignment. In the mean time, the teacher's task involved observing students' progress and encouraging the students to self-test with the use of GeoGebra and then to self-correct (see Figure 4.9 and Figure 4.10).

I was looking at what they have done on paper and then identifying where they had gone wrong and actually getting them to use GeoGebra to spot if it is right. That was nice when they were sort of “oh yes” and then they can see where it went wrong and went back and then correct some of the others. I think I was more getting them to use GeoGebra to check it but then asking them about what they have done, ask them trying to explain what it was wrong. And then explain so that one is wrong how are you going to correct that one? [S/20]

As believed by Susan, students learning to use technology for self-testing was one of the crucial aims of teaching mathematics.

I think where we are trying to go with maths teaching is that getting students to be able to use technology as a tool to test their own ideas. To get that real understanding to make them sort of think ‘what I know from before make me think of something so I can use this tool to see whether or not it is true.’ [S/21]

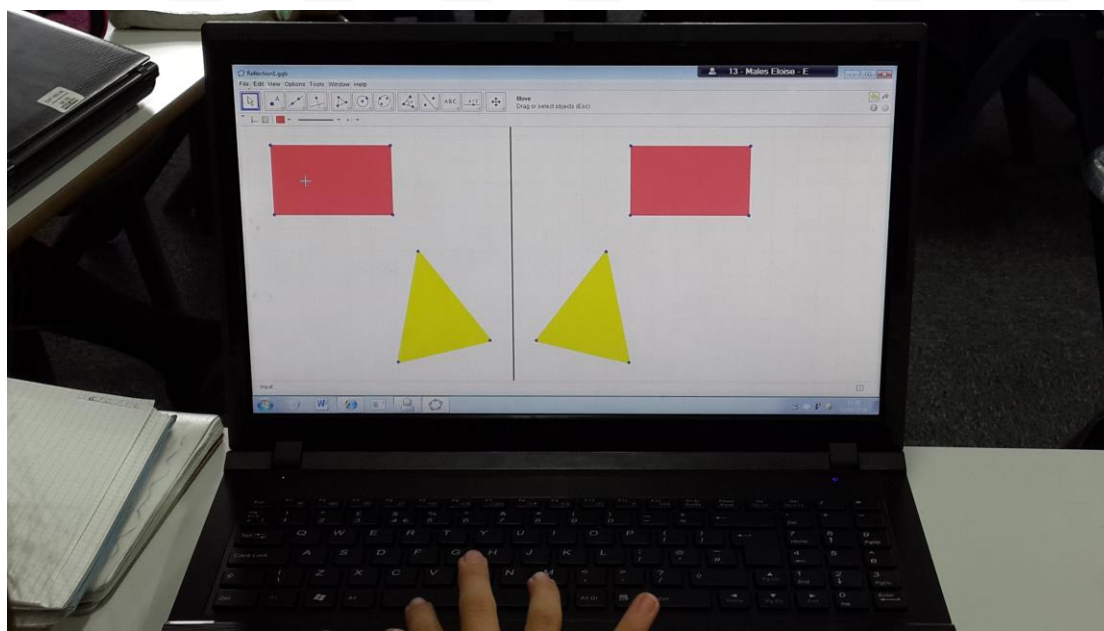


Figure 4.9 A screenshot of students checking their answer in GeoGebra

Furthermore, she stated that this specific activity format was particularly important for students, who were more reserved in participating in the lesson. In her words: “I think it was nicer that they could just check it themselves and also shy ones and the ones that don't want to make mistakes, they can see if they are right.” [S/22]

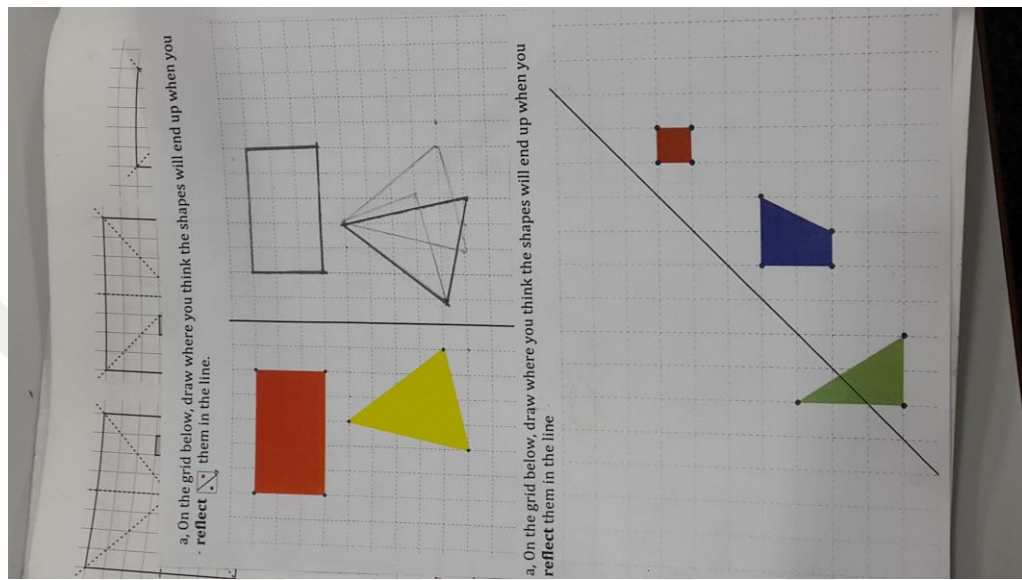


Figure 4.10 A photograph of students correcting their answer on the paper worksheet according to feedback from GeoGebra

4.2.1.4 Curriculum Script

Susan's curriculum script consisted of following the sequence of tasks on a pre-developed worksheet, which she took from a website. In this sense, she was not familiar with the tasks to be undertaken and representations to be employed. Therefore, she could not anticipate the difficulties that students might have encountered. For instance, students seemed to have difficulties in one of the tasks in Reflection task 3 especially because the mirror line was diagonal and one of the shapes was over the line [S/13]. Based on this issue, she indicated that she would add more paper-and-pencil activities for students to practice particularly for diagonal reflections (see Table 4.10). In this respect, her comments on unexpected occasions regarding teaching this topic with GeoGebra mostly focused on the way in which tasks were developed on the worksheet as has been detailed in the Resource System section.

Table 4.10 Reflections from Susan

Transcript from a post-lesson interview after Susan’s lessons	Transcript from a post-lesson interview after James’s lessons
What I would do next time is that I would do some work where they have got some actual reflection without technology because they need to be able to do this without technology.	We changed the worksheet because I felt when I did it there was not enough on pen and paper. So we brought in those additional worksheets in which we put quite a lot of time into scaffolding them up, and ramping them. And we matched them to the worksheets.

Furthermore, James suggested that preparing dynamic files of additional paper-and-pencil worksheets in GeoGebra would be helpful for students to check their answers in a more straightforward way. However, Susan thought that they could show those techniques (opening up a new page, drawing the shape and checking it) to students so that they can do it by themselves.

It would be good maybe having the paper worksheets in GeoGebra. They could have checked it. It would have been useful thing. Next time I would probably have those worksheets in GeoGebra as another file they could download so that they could then check their answers. [J/2]

If we just show them how to check it. What would have been nice was that if they could just open up a blank page. I did not get them to open up a blank page because I then have to get rid of the axes or that sort of stuff. But if we just show them how to open up a blank page and put the line on, put the shape on and then they could just go through and check it. So again this is where we are learning ourselves. It sort of contracts what you would be able to do, restricted in what you want to do because of your own knowledge of showing that. [S/23]

4.2.1.5 Time Economy

As it was the first time for Susan to teach this particular topic with GeoGebra and the worksheet was not prepared by her, she did not anticipate the pacing of particular situations in the lessons. However, she was aware of a degree of temporal instability

due to an initial lack of control over students' learning time. Therefore, she planned additional tasks for students, who might finish early.

"I sorted out some extra work already, if I had problems like things being finished off early, or things going wrong. So I had to have this sorted out in advance." [S/24]

At the same time, Susan used several strategies in order to achieve time saving through working with GeoGebra. During students' activity time, the time spent on the first task was expected to serve as a tool time and it aimed to make students become familiar with use of GeoGebra.

"I wanted them to practice GeoGebra with the face activity and get familiar with GeoGebra. I also wanted them to become familiar with the construction and create new objects on the face." [S/25]

She mostly provided intermediate syntheses (Assude, 2005) as a way to boost the work of individuals, who were lagging behind. For instance, during the first lesson, she shared some students' work with the whole class in order to exemplify how to upload their work onto the VLE.

OK look at the board. Who recognises whose face is this? That is the face Martina has just uploaded. Remember when you have done a task, save it and go into Moodle and upload it. 3 faces are sitting there and waiting for me. If I look at another one, Liz has got one in there. You need to save it in your area and then go into Moodle, and click that little hand in button. Is there a new one? Yeah there is next one. That is Tim's one. So you have to upload it. Ok they are coming in thick and fast now. [S/26]

Furthermore, during work-and-walk-by, she provided strong guidance so that students could use their activity time productively, especially considering students were new to Geogebra use. She also signaled time spans for transition between activities such as reminding students how long they had left to finish a certain activity. Moreover, she reported that making students upload their work onto the VLE as they went through was one of her strategies aiming at students' efficient use of activity time [S/4].

4.2.2 Circle Theorems

In this section, I will analyse Susan's GeoGebra use to teach 'Circle Theorems' over the course of two lessons to a Year 10 class of lower attaining students (Set 5 out of 8 sets).

4.2.2.1 *Working Environment*

Susan's Circle Theorems lessons took place in the same classroom with laptops in which she carried out her "Transformations" lessons (see Figure 4.1). In this regard, most of the information about her use of this classroom with laptops did not differ from what has already been discussed in section 4.2.1.1.

4.2.2.2 *Resource System*

The resource system for these two lessons consisted of a teacher-devised sequence of (paper) worksheets (that had been used in past years for teaching this topic in a non-technological environment) now to be completed through using GeoGebra (see Figure 4.11).

The big worksheet was the identical sheet of paper that I have used for ten plus years. So this is a genuine me finding out the difference between giving them a pencil, a ruler and a computer compared to giving them a pencil, a ruler and a protractor. [S/27]

With this worksheet, she wanted students to set up the diagrams (that were stated on the paper worksheet) in GeoGebra, to explore triangles inscribed into circles and to formulate five different Circle Theorems for further generalisation with the use of dragging techniques of GeoGebra. Susan suggested that the dynamic element of GeoGebra was particularly useful for students to change properties of a diagram and see what was happening on diagrams. In addition, she believed that such software enabled the students to see more unusual cases since they were in charge of what they were doing with the use of the software whereas in non-technology lessons they mostly considered the examples that the teacher gave them. Furthermore, she believed that the accuracy, and speed of GeoGebra would facilitate measuring angles

by reducing drudgery by hand (see Table 4.11 below for related interview quotes from the teacher).

Table 4.11 Susan’s interview quotes related to the affordances of GeoGebra for supporting learning

See what happens to a figure when points are moved

I expected them to discover the circle theorems. That was the aim of it. I wanted them to have that moment of awe and wonder, when they notice that special things happen in the diagrams. And I think the GeoGebra lent itself to that. [S/28]

Consider special cases

Some of them were actually looking where going inside out that sort of arrowhead things like that. You would not normally give them an example in when you actually teaching. If you give them diagrams and got them to copy out the rules they can't get the opportunity to see the odd cases. Now they can look at the odd cases and see how the situation would apply. [S/29]

Produce measurements more efficiently and from multiple figures

To do GCSE maths they don't need to be able to use a protractor. They would have got bogged down with measuring angles and working out which angle to measure, which bit I am doing whereas that they were fine with working out the angle on GeoGebra. [S/30]

Correct mistakes more easily, and so feel freer to take risks

When I got it wrong that many times, I was saying oh what is going on here. I was almost modeling the fact that “look things are going to go wrong”. And you got the edit, undo. The best thing about computers and then you could do it. [S/31]

Moreover, as Susan indicated during the post lesson interview, GeoGebra was used to add ‘fun’ into lessons and also to help students engage with more examples through using dragging techniques and to facilitate noticing of the relationships between angles in circles. In comparison with undertaking the same worksheet of Circle Theorems by hand, Susan stated that GeoGebra assisted investigating Circle Theorems through its accuracy, speed and manipulative ease.

It is more fun and more accurate and speed in itself. I mean once they sorted out the angle measuring there are no inaccuracies with angles. When I say speed, it is speed of seeing a lot more versions of it and a lot more imaginative versions they might not bother because they could not have gone out of their comfort zone. [S/32]

As noted in the previous section, Susan was a novice in using GeoGebra and did not have sophisticated knowledge of this software. Hence, she was learning some aspects of the software during these lessons. For instance, while she was doing a technical-demo at the beginning of the first lesson she wanted to show how to measure an angle in GeoGebra; however, she was not aware that GeoGebra measures anti-clockwise. In this case, she measured a reflex angle a number of times while wanting to measure an acute angle. During paired student work at laptops, a student informed her about clicking in an anti-clockwise direction.

I was learning on the hoof this bit where it measures it anti clockwise; I had not really twigged that. As I say Mike [*a student*] helped me with that one. I mean, I did learn during the lesson. So then I could show the students. [S/33]

Similarly, she did not know how to set up GeoGebra to measure to the nearest whole number, which she thought created some confusion for students since they had difficulties in seeing the relationships between angles.

If it measures it to the nearest degree, I think it will become a lot easier for them to see the rules. I think two decimal places confuse the students. But that is something where I am new to GeoGebra. I have learned that about GeoGebra now and I will get it sorted. The awe and wonder “oh look wow it is the same or wow it is doubled” would be more effective if that bit of GeoGebra I dealt with. [S/34]

In particular, as has been mentioned Susan’s class was a lower attaining group and their arithmetic skill was not good enough to notice what was going on in diagrams in terms of comparing angles.

The other thing, when they were doing centre and circumference there were issues where you and I can see straightaway that two numbers are doubles, but their mental arithmetic is not up to it. They are looking at 130 and 65 and they just “himmmm”, their maths, their numeracy is not up to it. So again that takes away from some of it. [S/35]

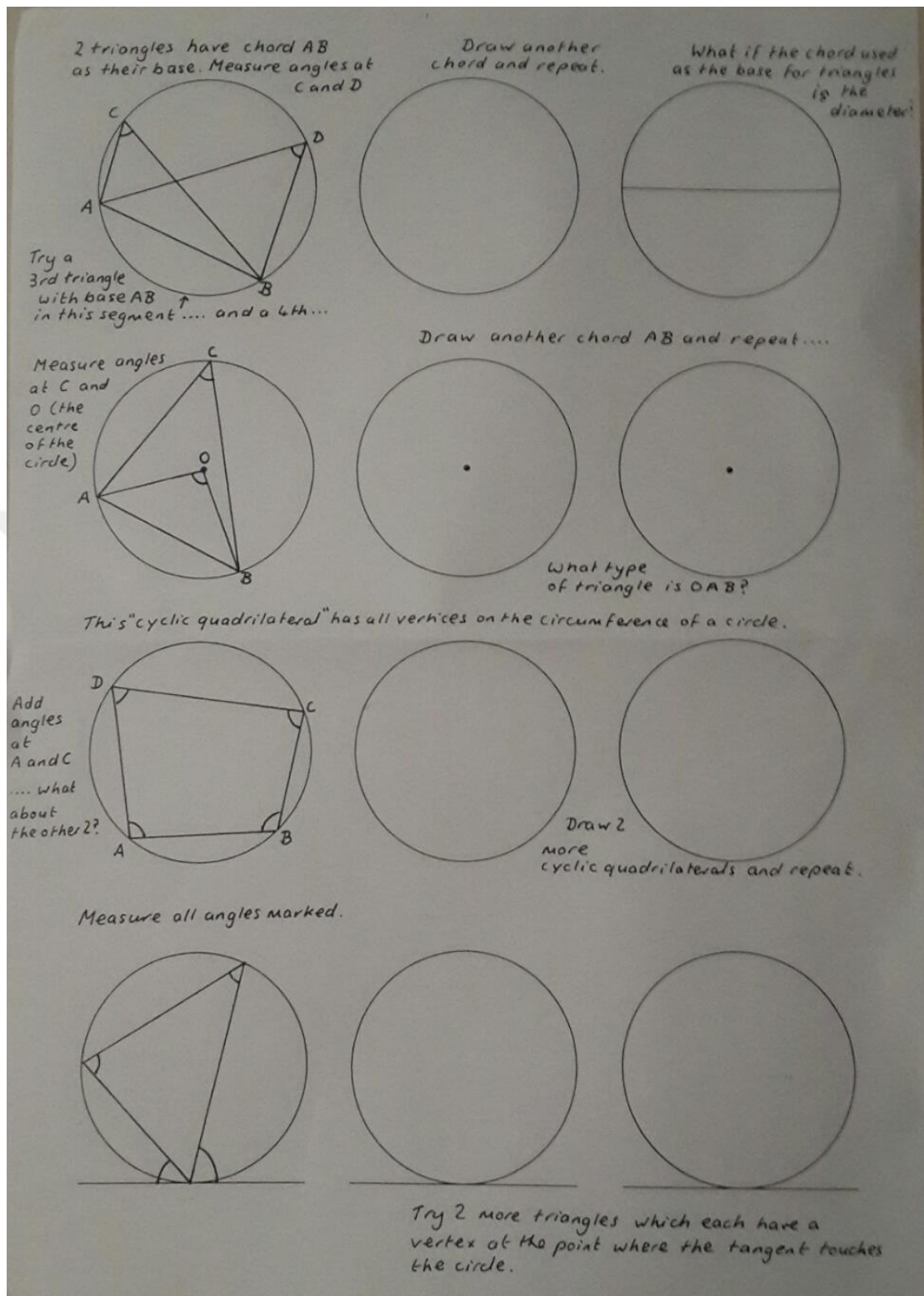


Figure 4.11 Susan's paper worksheet for Circle Theorems lessons

4.2.2.3 Activity Structure

Susan's activity formats for Circle Theorems lessons (see Table 4.12) consisted of whole class teaching as well as paired student work at computers with her moving around to monitor and provide support. Susan, first, revised some circle words as a whole class from the front by question/answer. After that, she outlined what the lesson was about and what she expected from students to do. In order to exemplify, she used GeoGebra and modelled the first question from the worksheet on the IWB. During this, she showed some of the tool techniques that students needed for constructing the shapes in GeoGebra and also they could use for angle properties such as angle measuring.

Table 4.12 Activity formats and lesson agenda followed by Susan in 'Circle Theorems' lessons		
Lesson	Activity Formats	Lesson agenda
<u>Lesson 1</u>	Not-use-tech/ Questioning	Susan revised seven circle words (radius, diameter, circumference, chord, sector, arc, tangent) by questioning students.
	Use-tech/Technical-demo/Observe-demo	After that, she showed how to find the software and some tool techniques through exemplifying the first question from the worksheet in GeoGebra including how to set up the questions in GeoGebra, and how to measure angles in GeoGebra. For this whole class demonstration, she made use of IWB.
	Use-tech/Work-and-Walk-by/Set-work-on-screen	The task for students was to notice relationships between angle properties of circle through using GeoGebra. Each task involved setting questions from the worksheet in GeoGebra and exploring with the use of dragging techniques. In the meantime, the teacher moved around to provide support and guidance.

	Not-use-tech/Spot-and-show	During work-and-walk-by, Susan spotted a pair, who had made more progress than the others did, so she stopped the whole class and held up their work to show the rest of the class. She discussed this example with the whole class to instruct them what they were expected to do with the worksheet.
	Use-tech/Work-and-walk-by/ Practice-with-technology	The teacher continued walking amongst students and providing feedback during their work at computers.
	Not-use-tech/ Review-students-work	For the last five minutes, she had a whole class exposition and asked students to read out the general statements (Circle Theorems) that they came to realisation with the use of dragging during the lesson.
<u>Lesson 2</u>	Not-use-tech/ Teacher-instruction	The teacher instructed students to carry on the tasks where they left off from yesterday.
	Use-tech/Work-and-walk-by /Practice-with-technology	While students were working at computers, the teacher moved around the classroom in order to make sure students were involved and engaged in the task.
	Use-tech/Discuss-the-screen	During work-and-walk-by, when a pair of students had a difficulty noticing a link between angles due to rounding, she took them to the IWB and discussed the screen on the big screen from the front.
	Use-tech/Technical-demo/Observe-demo	During work-and-walk-by, when she noticed that some students were having problems with angle measuring in GeoGebra, she stopped the whole class and showed this specific technique emphasising GeoGebra measures in an anticlockwise manner.
	Not-use-tech/Whole-class-discussion	The teacher addressed the whole class and reviewed the five Circle Theorems in order for students to compare the official wording with their own expressions.

The use of Technical-demo and Walk-and-work-by formats were not different from the Transformations lessons. Therefore, I will only detail the formats that have not already been talked about in the previous section.

Discuss-the-screen

For Discuss-the-screen, she occasionally used the IWB for a group of students during work-and-walk-by. This was particularly important to her from the classroom management point of view.

There were two struggling with the number work that was tricky because of the rounding. I said just go and change mine to look like it. And then we can actually look at it. We went there and talked about around the whiteboard because again from the classroom management point of view, if you get a couple of students up to the whiteboard you talk to them there it is easier to keep scanning whereas if you crunch down over here with a laptop you can't scan so easily. They know I am still there. I had not hidden myself away. That is pure classroom management. [S/36]

In this regard, Susan considered the way in which IWB offered easier small group discussion as being one of its key characteristics.

The way I could bring them up to the board and we could look at something that was nice. It is easier for three sets of eyes to look at the screen than something is on a piece of paper. [S/37]

Spot-and-show

During Work-and-walk-by, Susan noticed that some students made more progress on their worksheet than others so she showed this to the whole class at the end of the first lesson to instruct the other students what to do, which aimed to move activity time forward. But the way she used Spot-and-show was not instrumented by the technology. She held up those students' worksheet and showed their progress to the other students.

Table 4.13 Verbatim transcript for Spot-and-show activity format in Susan’s Circle Theorems lessons

(The teacher is asking for students’ attention)

Listen stop, all eyes are looking at me.

I am going to use these two as an example because they are pretty much done what I wanted people to do. Everyone is looking. I am about to hold Charlotte’s work up.

(The teachers is reminding students what the task is)

What they have done is on their first diagram they have actually shown what they have got on the screen. What I then asked you to do on the second one just draw a sketch what you got when you did it a bit moving around. I moved it around. They have first one labeled and second one labeled. And then you need to write down what you have noticed this happening on the diagram. There is something going on. That is a really important mathematical fact. What I want you to try and to do before you go on and have a go putting your chord as a diameter. I want you to write in your notebook what you actually notice.

(The teacher is explaining and questioning students about the spotted work)

They have actually written “if you move the triangles in the circle the angles will always stay the same”. But at the moment they haven’t told me something special about those triangles, what is special about those two triangles. Are they just any two triangles in a circle? What they have got in common? Do they overlap? What do they have the same?

Both triangles shared the bottom line.

What is the bottom line? They shared the same length, where did it go? It goes from the one edge to the other edge. Chord.

4.2.2.4 Curriculum Script

Susan has been teaching Circle Theorems for many years and therefore familiar with the content. However, as a novice technology user, it was the first time she was teaching this topic with GeoGebra and she did not have a structure for interpreting classroom situations with the use of GeoGebra. In that sense she did not have a well-developed aspect of her curriculum script for the topic, which combines her content knowledge with her technology knowledge. As mentioned, she used a worksheet that she has been using for ten years in a non-technological environment and tried to adapt this material with technology. In this regard, her curriculum script consisted of following the sequence of this worksheet and she was at the stage of learning and adapting. During Susan’s interview after the lessons on what were the issues came up

she did not expect, she made more explicit references to her lack of knowledge of GeoGebra such as measuring acute angles and also how to set GeoGebra to round angles. These issues have already been raised in the section on resource system as these are more about GeoGebra not yet being part of a functioning personal resource system for the teacher. Having said that, during lessons she noticed many students were having difficulties to measure acute angles properly, which was affecting their activity time, therefore she stopped the whole class and showed them how to do it properly since she learned this from a student during the lesson. This is where it started to become part of her curriculum script, as she registered that this was something that she needed to give attention to in teaching Circle Theorems with GeoGebra. In this sense, this aspect was not already a part of Susan's curriculum script, but she was in the process of developing her curriculum script to take account of it [S/33].

Susan's comment on rounding (discussed in the Resource System section) was again related to just the basic use of the instrument in a general way and that level of detail about the functioning of technology in a general mathematical sense rather than about nuances of teaching and developing Circle Theorems. On the other hand, during the post lesson interview, Susan indicated she would include GeoGebra files for each Circle Theorem from now on whenever she teaches this topic, even if it is only for teacher use in order to discuss the unusual cases, which suggested that she was developing her curriculum script teaching Circle Theorems drawing on GeoGebra. She believed that using GeoGebra in Circle Theorems provided students with more examples, which were not seen in a non-technology lesson and which she found important from a mathematical point of view.

I think what I do now is any time that I am teaching Circle Theorems I would like to have a GeoGebra file setup that is a basic file of each of the theorems. So if I am looking at angle at the centre and angle at the circumference, I can just get a basic one pinned up and then we can look at some of the odd cases. If I could have those as a fast resource like I can just get up on the board quickly and then I can bring the student on board and then stop them. "This is a bit weird one, isn't it?" "Can I make this weird one on this diagram?" because they have had the experience using GeoGebra, they know it is for real and I can just move things around of it. They would understand that I could use it as a tool to explain something that very hard to draw to them. [S/38]

4.2.2.5 Time Economy

It became clear in post lesson interviews, how Susan considered that the use of Dynamic Mathematics Software contributed to time efficiency in lessons. Susan indicated that GeoGebra, in particular the angle-measuring tool and undo option, enabled students to progress faster compared to measuring/calculating angles by hand. This speed helped the students to produce and engage with more examples [S/32] without spending as much time capital, therefore, maximised the didactic time. Furthermore, being able to use GeoGebra on the IWB made episodes of the whole class teaching easier in the sense that she would be able to demonstrate tool techniques and explain/discuss the diagrams with the use of dynamic geometry, which helped her to have an efficient instructional time.

“I mean I could not have done these lessons without whiteboard. I had to have an interactive one there as well. So I could interact with the whole class and show them what they could see.” [S/39]

Susan spent half of the time capital as introduction time at the beginning of the first lesson in order to show students some circle words and some specific tool features especially because it was the first time for students to use GeoGebra. She introduced the software and demonstrated to them what tools they would need for their task. This was aimed at minimising time spent trouble-shooting the difficulties that students might experience. However, as already discussed in Resource System section, when she wanted to show students how to measure an angle, she needed to try a couple of times for which she spent a full 55 seconds (quite a long time in a lesson) since she had not figured out the fact that GeoGebra measures in an anti clockwise way. As a result, she had lost valuable instructional time. However, during Work-and-walk-by, when she learned this from a student (Mike), and noticed a number of students were still having difficulties in measuring angles, she stopped the students and instructed the whole class about it. In this regard, in order to increase the activity time, she gave intermediate syntheses.

“I think Mike saved us massive amount of work by showing me GeoGebra measures acute angles all anti clockwise.” [S/40]

Similarly, when she noticed some students did not know how to undo their actions, she instructed the whole class saying: “What is the best thing about computers edit, undo. If you go wrong- edit undo.” [S/41]

As discussed in the Activity Structure section, she used Spot-and-show as an intermediate synthesis to change the pace of those students, who did not know what to do (see Table 4.13).

4.2.3 Straight Line Graphs

In this section, I present my analysis of Susan’s implementation of GeoGebra in teaching ‘Straight Line Graphs’ in one lesson to a high attaining Year 9 class. The purpose of this one-hour lesson was for students to practice this topic (which had already been taught in a paper-and-pencil environment) with the use of GeoGebra. Hence, this section does not consist of large amount of information.

4.2.3.1 *Working Environment*

This one-hour lesson took place in a small U-shaped computer room (see Figure 4.12). However, the introduction and conclusion of the lesson were carried out in Susan’s normal classroom (where Transformations and Circle Theorems took place).

Although Susan was happy with the layout of the room, she indicated in the post-lesson interview that she would prefer working with laptops in her own classroom where there was enough desk space for students.

It is too small for a big class. I prefer laptops because they have more space. U shape is actually probably the ideal, little bit of monitoring and they would be fine to get on. So it would be great if we had more space in there but the layout is fine. [S/42]



Figure 4.12 The room where Susan’s ‘Straight Line Graphs’ lesson took place

She was aware of the fact that teaching in a technological environment might result in off- lesson behaviour depending on a class.

I am just really strict if I do a computer lesson no chances of you are on anything else you should not be on. That class I did not want to say anything because I knew they would be fine. But that would be a rule. That would just be laid at the beginning. [S/43]

4.2.3.2 *Resource System*

The resource system for the lesson consisted of a pre-designed sequence of worksheets involving use of GeoGebra. With the use of GeoGebra, Susan aimed to make tasks more accessible and less laborious to students, which in return enabled them to see patterns in a more straightforward way.

The fact that they don't need to practice drawing graphs meant that there was none of the drudgery although they have very good graph drawing skills. So GeoGebra takes away the drudgery of drawing graphs to see the patterns. It is just the pure speed of it. [S/44]

She also pointed out the multiple representations as students could see the graph and equation on the algebra window simultaneously which help them to make links in between.

Also, the fact that they highlighted the equation, and at the same time highlighted the line, so they could see the links going through and they very quickly could spot the patterns. [S/45]

Susan made use of a pre-developed worksheet (see Figure 4.13) that was shared on a GeoGebra website. She particularly liked this worksheet, as it was very structured in terms of leading the students.

I think I always wanted them to know exactly what they are meant to be doing and the beauty of that worksheet was that there was no query and there was no uncertainty. They had something very specific to do. [S/46]

This particular class was new to GeoGebra, therefore Susan showed how to find the software and how to use input bar on GeoGebra to the whole class at the beginning of the lesson.

“All they needed from the beginning was that information about how to actually input it. Once they knew that they could do the worksheet.” [S/47]

Investigating Straight Lines in Geogebra	
1. Go to http://web.geogebra.org and use the input bar to draw: $y = 3x + 2$ $y = 3x + 3$ $y = 3x + 4$ $y = 3x + 5$ In the box below write down what the graphs have in common.	4. Open a new window and draw the graphs $y = 2x + 3$ $y = 2x + 4$ $y = 2x + 5$ $y = 2x + 6$ What happens as the number after the 'x' increases?
2. Open a new window and draw the graphs $y = 2x + 3$ $y = 3x + 3$ $y = 5x + 3$ $y = x + 3$ In the box below write down what all the graphs have in common.	5. Can you predict what the graph $y = 2x + 7$ will look like? How steep will it be? Where on the axes will it be?
3. Open a new window and draw the graphs $y = x + 3$ $y = 2x + 3$ $y = 3x + 3$ $y = 4x + 3$ What happens as the number before the 'x' increases?	6. Draw the line $y = 3x + 2$ Can you write below a line which will be parallel? Draw this graph, was your prediction correct?
	7. Draw the line $y = -2x + 3$, in the box below explain what is different about this line, why do you think this is? Can you draw another line to see if you were right?

Figure 4.13 Susan's paper worksheet for the Straight Line Graphs lesson

4.2.3.3 Activity Structure

Susan started the lesson with a teacher led demonstration addressing the whole class in her normal classroom (the one where she did her Circle Theorems and Transformations lessons). She started by demonstrating how to find the software, how to use the input bar in GeoGebra, and explained what the worksheet was about. After that, she asked students to move to the nearest computer suite in order for them to work on the questions on the worksheet with the use of GeoGebra. This was a more extended period of individual student work on worksheet tasks with the teacher circulating to provide guidance and support (See the detailed activity formats with the related lesson agenda in Table 4.14).

Table 4.14 Activity formats and lesson agenda followed by Susan in the ‘Straight Line Graphs’ lesson		
Segment purpose	Activity Formats	Lesson Agenda
Straight Line Graphs	Use-tech/Technical-demo/Observe-demo	Susan explained how to find the GeoGebra file and the tasks on the worksheet by modeling homework questions in GeoGebra on the IWB to the whole class.
	Use-tech/Work-and-walk-by/Practice-with-technology	Students used computers themselves to work on worksheet tasks. During this, Susan moved around the classroom in order to check what students were doing.
	Not-use-tech/ Board-instruction	Susan had a 5-min plenary back in her normal classroom reviewing findings through teacher-led presentation on the ordinary board.

4.2.3.4 Curriculum Script

The lesson agenda followed the sequence of worksheet tasks. Each task involved using the software to spot patterns for specified equations of straight lines, and then writing down the spotted patterns on the worksheet. The worksheet was structured in a step-by-step way since the class was new to GeoGebra.

You could maybe just give them $y=mx+c$ and ask them to investigate different values of m and c . I thought for students new to GeoGebra would have been too much at the same time but with a class who may be more used to using GeoGebra maybe practice with it before you could be more open with it. [S/48]

In this regard, the teacher’s curriculum script took account of students’ previous knowledge about that particular technology and anticipated that students would need a number of tool techniques such as how to input an equation in GeoGebra.

“They have got now sort of ability and sort of familiarity with it. They could not have done without instructions I had in that worksheet to begin with.” [S/49]

4.2.3.5 Time Economy

Susan's comments indicated that the use of Dynamic Mathematics Software contributed to the time efficiency for this particular topic through reducing the drudgery of drawing graphs that increased the speed and efficiency of this process. Therefore, it allowed students more learning time to spot the patterns [S/44, S/45].

4.2.4 Summary

In this section, I will compare Susan's practices teaching three different topics with the use of GeoGebra in the light of the SFCP framework.

In terms of working environment, Susan made use of two different rooms including an ordinary classroom where laptops were used by students and a small U-shaped computer room. For the latter, she needed to move students from her ordinary classroom to the computer room for individual student work: this required her to manage students' seating routines, to check non-functioning computers and to make sure students opened GeoGebra. Particularly in this small computer suite, as there were not enough computers for all the students she needed to arrange small groups of students to work together at one computer. She also offered her own laptop to a group of students for their use. Although size of the computer room/number of computers became a crucial aspect, Susan believed that students required more monitoring during technology lessons, and that U-shaped arrangements offered better scope for monitoring and interaction. Hence, she would rather teach technology lessons in a big U-shaped computer room where she could monitor the students better.

My ideal would have been a big U shaped computer room with big screens that they can see and that I can walk around easily... You can position yourself to see 3 quarters of the screen to scan the room. And I suppose this is where for me I am always thinking about the classroom management issues particularly in these lessons where I'm a bit out of my comfort zone. [S/50]

Another aspect to consider related to Susan's use of worksheets. For Transformations and Straight Line Graphs, Susan made use of pre-developed worksheets that were borrowed from a GeoGebra website. On the other hand, for Circle Theorems lessons she adopted her non-technology worksheet that she had used for around 10 years.

The salient difference was Susan's familiarity with the tasks. Particularly she was using the Transformations tasks for the first time and thought some of the tasks were difficult for students. All of the worksheets that she used provided a tightly structured form of investigation for individual student work in Geogebra. From post-lesson interviews with the teacher, her reasons [S/8,S/9] for using borrowed prepared worksheets came into the picture as follows: lack of preparation time, being appropriate in an early phase with GeoGebra for both herself as a teacher and for her students.

For all of the topics taught, Susan's curriculum script consisted of following the sequence of worksheets and she was at the stage of learning and adapting. The main observation from the analysed lessons of Susan is that her curriculum scripts for these three topics with the use of GeoGebra did not come over as being particularly rich because she was using borrowed worksheets with a new technology. Her focus tended to be about developing instrumental knowledge to make it part of the resource system, first for herself personally and then also for her students. Furthermore, she assisted her students by going around in the room typically repeating the same emphasis as in her plenaries. For instance, Susan's reflections on what unexpected issues came up in lessons focused mostly on her lack of knowledge about GeoGebra and the way in which tasks were developed on the worksheet.

Susan's comments on the contribution of GeoGebra for these three topics indicated 'Effecting working processes and improving production' (Ruthven & Hennessy, 2002), notably by increasing the speed and efficiency of measuring angles, doing reflections and drawing graphs, as well as through producing more examples that students would not see on paper-and-pencil. In terms of developing time saving strategies, many of the aspects did not differ according the topic taught but the class.

4.3 Tom

4.3.1 Transformations

In this section, I will offer my analysis of Tom's implementation of GeoGebra in teaching 'Transformations' over a period of three lessons to a Year 10 class (set 2 out of 4).

4.3.1.1 *Working Environment*

Tom's lessons took place in a computer room (see Figure 4.14), where there were a number of computers against both walls and two column rows of computers in the middle where students were facing each other. There was also a computer for teacher to use and a Smart-board at the front.

In this computer room, students were seated individually and faced computers at all times. In other words, there was no extra space for whole class teaching, hence, not any particular movement from students' side during the lessons. On the other hand, Tom employed two different positionings in general. He did a whole class teaching from the front when he needed to do demonstration or explanation. He then circulated around the classroom during individual student work at the computer in order to determine whether students had any difficulties with the tasks. His intention was to maintain students' involvement and engagement in the tasks. Therefore, he devoted more time to interaction with individual students when they worked at computers.

It is my typical approach. Usually give them some points, give them something to start with. That is just an introduction for me or to give them a task and say kick off with this. But I will always move around the room to check if everybody is on task and not getting stuck. [T/1]

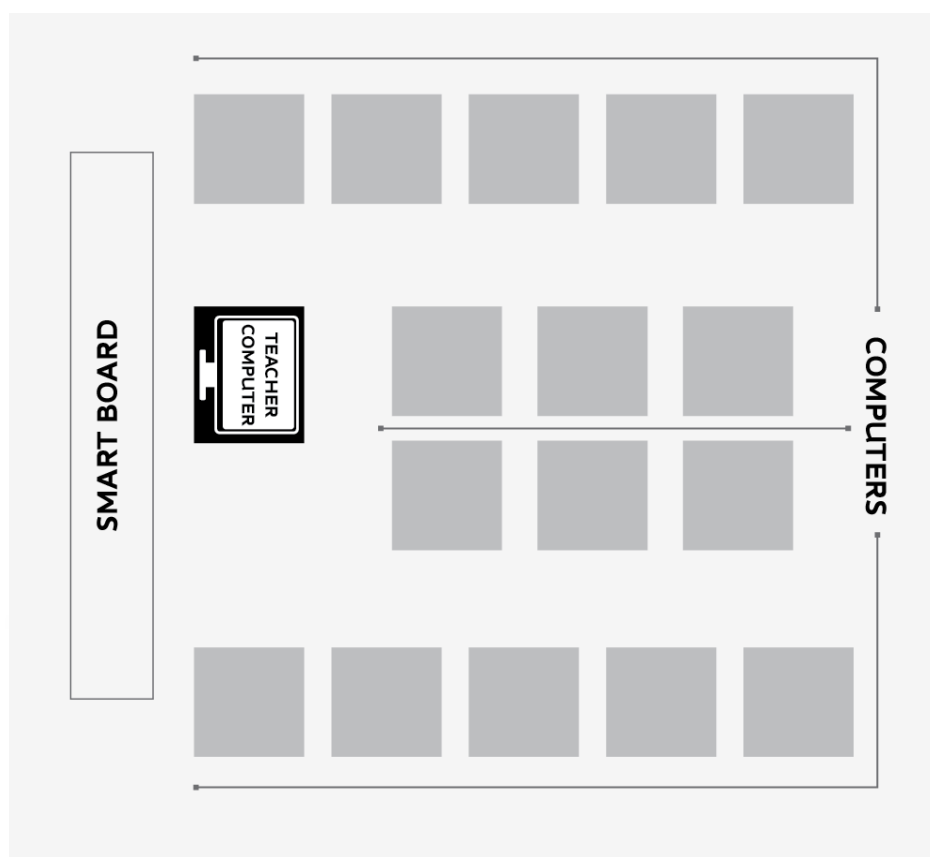


Figure 4.14 The room where Tom's 'Transformations' lessons took place

He was pleased with working in this room in comparison to his own classroom, in the sense that, in this room, all the computers were working reliably. In addition, the location of the room was close to the IT office, which was important in terms of receiving technical help when needed.

I find it quite frustrating in the past when I planned the lessons to use the computers and they don't work or some of them don't work it causes complete mayhem for me. And yeah the advantage of the other classroom is that all the computers work fine. And it is also right next to people who administer the network for the whole school. So if anything goes wrong, they would come and fix it whereas my classroom is a hike away. You don't get a quick response. [T/2]

The layout of the room was the same as his own classroom (consisting of rows of fixed computers with students facing each other) with which he was not satisfied due to the difficulty in attracting students' attention to the board. In particular, students tended to get distracted by the fixed computers in front of them. Furthermore, he pointed out, the fact that students faced each other tended to result in more off-task

behaviour. Hence, he had asked to rearrange his own classroom that has adjustable computer desks (that can become a flat desk when the computer is not in use) with the layout of students facing the front (see the requested computer room plan in Figure 4.15), something which was still in the process of being fixed at the time.

Nearly 2 years ago I made a request for rearrangement of the desks so that they are facing the front and to get those computers lift up so students can use it as a flat desk and/or a computer. Before it was just fixed computers in the same configuration as they are now. But it was difficult to teach maths in that environment to small Year 8 boys for example who just hide behind the monitors. They can mess around. I can't see them because there is a huge monitor in the way. [T/3]

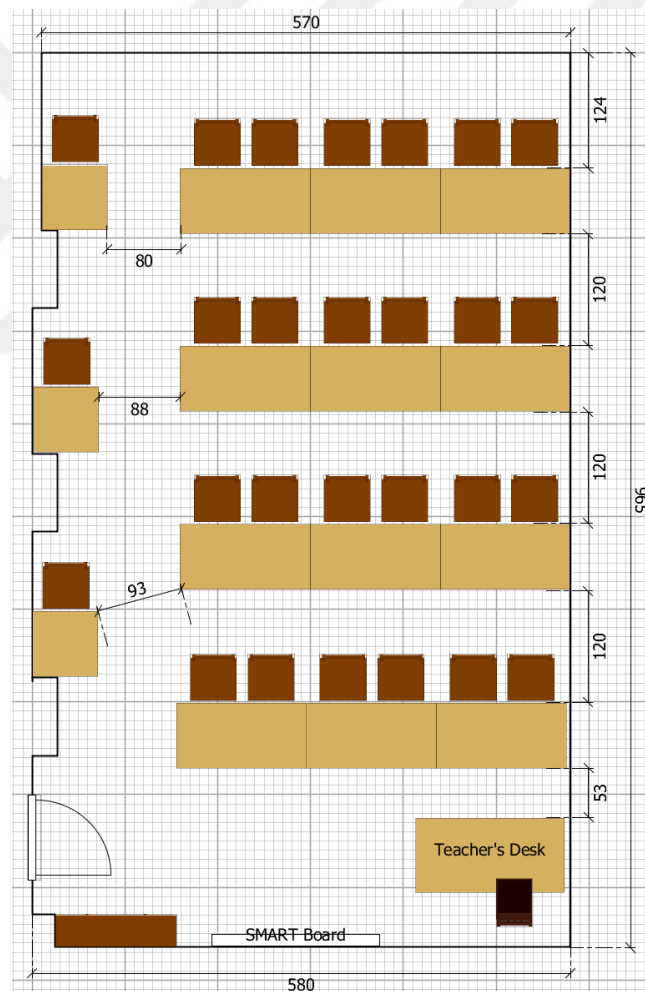


Figure 4.15 The requested room layout by Tom

Although it was more convenient for him to teach technology lessons in his own classroom, he indicated that moving to a new working environment for a lesson might motivate students academically. Considering the Practitioner Model (Ruthven & Hennessy, 2002) this idea is part of ‘Ambience enhanced’ that associates technology use with change, difference or variety in working ambience.

If you are changing the environment actually it is something different, something a bit more exciting for them. Oh we are going to another classroom today we are doing something different. They open up their mind to learning in a different way sometimes. It just depends on kids but I think this week it worked. It was like a different thing. So it can be actually quite good to go to different classroom. It worked quite well to be honest. [T/4]

He also indicated that technology lessons were conducive to off-task behaviour, which required him to be more attentive in such lessons.

You still get kids do something off task, distracting and doing something else. With non-technology that off topic behaviour would be only chatting with somebody or messing around with a book or staring out the window. With a computer they are still looking at the computer probably clicking on something else. If you are on the other side of the classroom, they can very easily disguise what they are doing. In that sense the experience for me is different. You have to be more kind of vigilant. [T/5]

Tom had introduced some routines to the students for working in a technological environment. As setting-up routines, at the beginning of the first lesson, he reminded students how to find GeoGebra, how to maximise the window, and how to turn the grid on. Work arrangements consisted of working individually at computers since Tom believed that being in charge of the computer was important for students to engage with the task.

Well if there is a computer each I'd like to give them chance to do it by themselves. But at the same rate I think some of the harder activities possible with year 10s today having two brains on one computer might have worked better. So I am not against pair work on computer. The danger with pair work with computers is that they love having their control of computer themselves if you are doing thinking. It is the same with everybody, it is the same with me, if I am paired with somebody and someone else having to click and you say “yeah yeah just set that value to 3” and they don't do that immediately, and it is incredibly frustrating, you just grab the mouse and set the value to 3 because they are thinking something else. So it can work or it can mess it up. [T/6]

In order to attract students' attention to the front board for whole class instructions, he asked students to turn off their computer screen so that they could focus on the board and not get distracted by their screen. His closing routine involved reminding students that they were not obliged to save their work and they should log off their computers.

4.3.1.2 *Resource System*

Teacher-created files involving use of GeoGebra formed the resource system for these lessons. Tom considered the dynamic and instant nature of GeoGebra as essential characteristics. With the aid of GeoGebra, students could be more curious and see what happens in a diagram, which Tom believed was distinctive to Dynamic Mathematics Software use in comparison with traditional methods.

The key advantage for me using GeoGebra or something like GeoGebra is that you can move stuff around. It is an exploration they have rather than doing set questions in a particular way. They are kind of following some instructions but they do click on some of the other things and see what happens if I do this whereas with pen and paper you can't. You can do what happens if I change this X to an A but they will not see the results until a few steps later. On a computer they see straight away, they see what happens. That is a great learning tool because you can't expect 14-15 year old kid would really properly investigate some maths on paper. They are just not interested enough. It is the dynamic and instant nature, the immediacy. It is the immediate nature of changing something and seeing the result instantly. [T/7]

In addition, he suggested that the visual element of GeoGebra would make a difference when it came to geometrical problems. This, in return, would help students develop dynamic thinking in a static environment.

It is the first thing that I think of when kids ask me a lot of the geometrical questions that they have. I can try and explain the answer to your problem or I could just show you a moving picture. It is way more fun and way more interesting, also way richer to give something they can interact with. It just enriches the learning. My theory is that the more you see something moving, the better you'll be able to imagine it moving even if it is not. [T/8]

In this connection, Tom's GeoGebra use in these lessons intended to promote dynamic thinking about transformations concepts.

I often do that by the way, use GeoGebra to get them just see something in action and understand what the idea of translation or rotation are, and be able to then visualise and use their imagination. So when they see an exam question, they could still see the thing twisting around the point in their mind, otherwise they think may be rotation is this, of course is not, it is that (*showing me*). Shape will rotate as well as going around in the circular. [T/9]

From the teaching point of view, he also valued the ease of adaptability of GeoGebra activities for future lessons.

You can design an activity with pieces of colour card and things where the same thing can happen on paper. But it is hard to organise something like that. You can make the resource once you have got it to use next time but you never know if that was also perfect for the next time when you need to teach this topic. You might have to change things depending on the class. GeoGebra makes it easy to set up an activity like that. I think you have to plan more but only once because then you have got an activity that works and you can just fine tune. [T/10]

As has been previously mentioned, Tom has been using GeoGebra since the beginning of his teaching (for four years at the time). During his PGCE, one of his course tutors introduced Tom and his peers to GeoGebra. Tom reflected on this experience thus:

I was excited when I saw somebody set something up, which I was familiar with, on GeoGebra, which was creating the inscribed on a circumcircle for a triangle. I have done that with compasses before. They set it up on a screen, which was fine, but then they started moving the point, everything they set up moved with it compensate. It was beautiful really really great. That is how I started. [T/11]

In this regard, he was not novice in teaching with GeoGebra and felt confident enough about how this software worked. Tom placed a particular emphasis on confidence in using the technology in order to be able to teach a topic with it.

You have to have confidence with software for what you are trying to do. You need to know how to respond when a kid does something wrong either by mistake or messes up there on purpose. You need to know how to fix it. You need to have competence and confidence to begin with. [T/12]

In addition, his class had used GeoGebra before and had the basic knowledge of the software, such as how to find and open GeoGebra and get access to the tool bar.

Furthermore, at the beginning of the first lesson, Tom demonstrated some new skills that students would need for this particular topic.

1- how to copy and paste, which was different in the new GeoGebra version.

2- how to use a vector from a point in GeoGebra

3- how to set a slider

In the first lesson, students used a blank GeoGebra page and focused on ‘Translation’ concept. The task was to draw a vector and a polygon, then, translate the shape by vector using a copy and paste technique. For the second and third lessons, Tom provided students with prepared dynamic files in comparison to getting them to construct one for themselves due to recognised complexities of starting with a blank page, which he took into account from his previous experience.

I have tried to get them start from scratch in the past. It did not work at all. They did not know where to start with it. I did a little bit demo, tried to show them a couple of tricks but actually I realised half way through it was just too much for them. It was not something I thought about until later. There is actually quite a lot going on at the background of the program. This is I learned this year from experience. [T/13]

One of the prepared files in the second lesson was the Reflection file shown in Figure 4.16. Students were asked to draw the reflections of those shapes by using the polygon tool, and then check their answers by using the reflection tool to see if they were right or wrong.

Use the polygon tool, which we used yesterday, and see if you can just draw the reflections of those shapes, just 2d symmetry. You have done things like this before. It is not completely straightforward; you might need to think about some of them a little bit. [T/14]

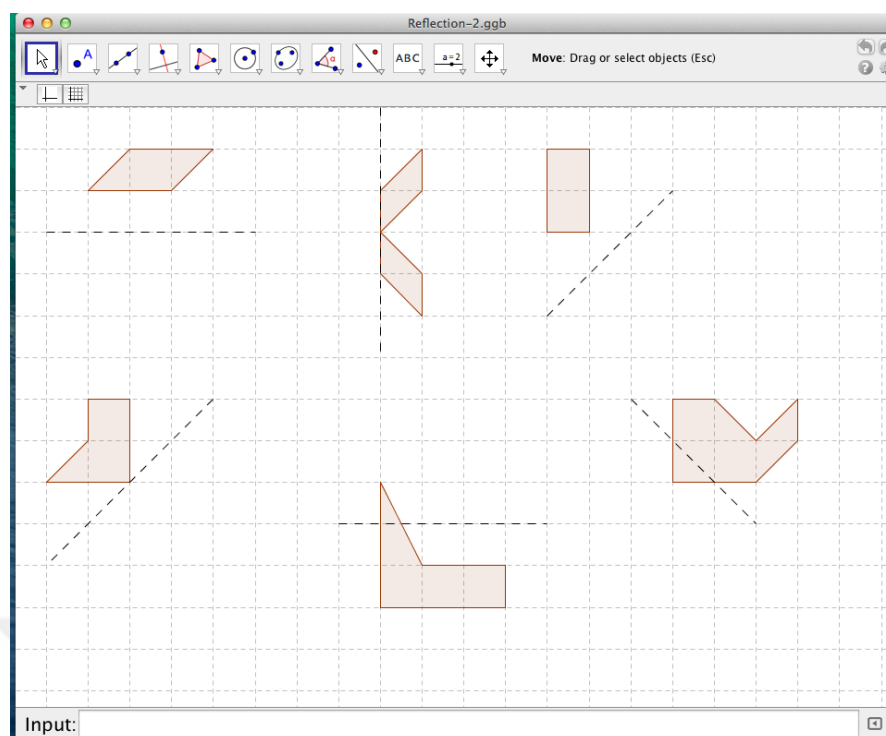


Figure 4.16 The reflection file that students worked on during Lesson 2

Similarly, for the Rotation file shown in Figure 4.17, Tom provided the angle of rotation and asked students to draw the shapes where they thought those were rotated. After that, he demonstrated how to set a slider in order for students to check their answers by using the rotation tool in GeoGebra.

I want to show you a way of checking your answers. This is a chance for you to learn something new about this software. So please look up at the board so you can see this. You will be able to use it to check your working and you will be able to finish this off. I am going to make a slider. [T/15]

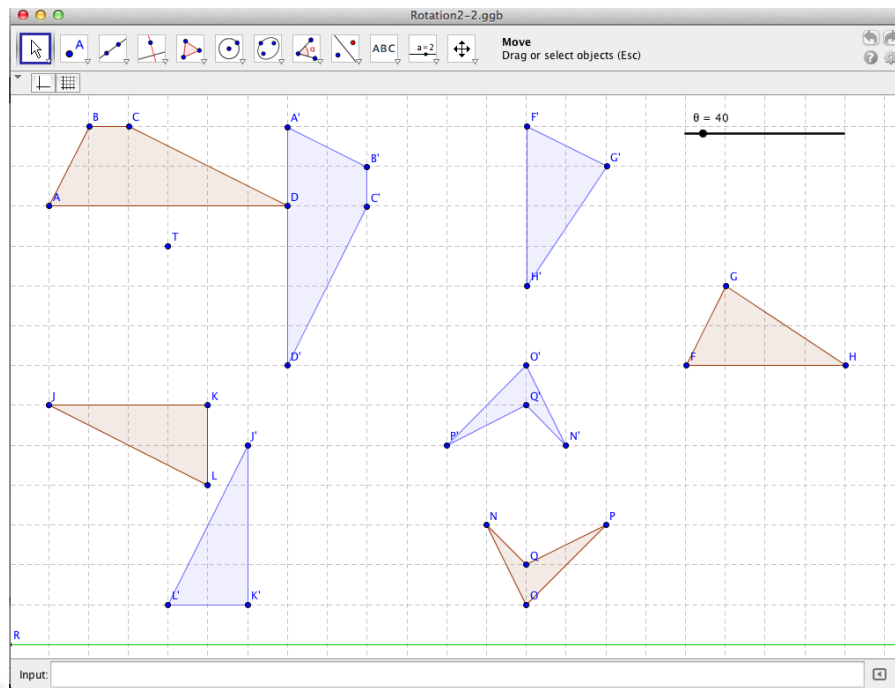


Figure 4.17 The rotation file that students worked on during Lesson 2

4.3.1.3 Activity Structure

Tom's lessons consisted of a sequence of activities regarding three types of transformations: translation, reflection, and rotation. During each activity segment, the teacher addressed the whole class and gave instructions about how to set things up. He also provided an introduction to explain what students were expected to do, which was driven by questions in a textbook and also prepared dynamic files on the computer. Afterwards, students worked individually at computers while the teacher was moving around and observing what students were doing (see Table 4.15 for the activity formats used by Tom during these three lessons).

Table 4.15 Activity formats and lesson agenda followed by Tom in ‘Transformations’ lessons

Lesson	Activity Formats	Lesson agenda
Lesson 1 Translation by vectors	Not-use-tech/Questioning	Tom reminded students what a vector was through questioning the whole class. Then he asked students to practice some vector questions in their course book.
	Use-tech/Technical-demo/Replicate-demo	After that, he showed students some GeoGebra techniques such as how to find GeoGebra, how to maximize a GeoGebra window, how to turn the grid window on, how to fix the x y-axes to a number, how to use the polygon tool with the use of vectors, how to copy and paste a shape. He asked students to practice these techniques simultaneously with his demonstration except copy and pasting of a shape. For this, he asked them to watch and practice afterwards.
	Use-tech/Work-and-walk-by/Practice-with-technology	Students were practicing the copy-paste function in GeoGebra while the teacher walked around the classroom to check if they were on track.
	Use-tech/Technical-demo/Observe-demo Use-tech /Work-and-walk-by/ Set-work-on-screen	The teacher demonstrated how to translate an object by a vector in GeoGebra. Students set some questions from their course book in GeoGebra in order to practice translation by a vector
	Not-use-tech/Teacher-instruction	Tom instructed students orally how to find the prepared file (Figure 4.16) and what they were expected to do with this file. The task was doing reflections by only using the polygon tool. So Tom wanted students to predict what those shapes’ reflection would be without using the specific reflection tool.

<u>Lesson 2</u> Reflection and Rotation	Use-tech/Work-and-walk-by/Predict-in-advance	In the meantime, he walked around and checked if students were engaging in the task. During this, he noticed some students were actually using the reflection tool to do reflections rather than guessing and he instructed the whole class by saying: “Please don't use the reflection tool until had a go at it. That is the point.”
	Not-use-tech/Board-instruction	Tom refreshed students' memories about equations of vertical and horizontal lines. And he asked students to practice some questions from their course book about reflecting shapes over vertical/horizontal lines again through setting them up in GeoGebra.
	Use-tech/Work-and-walk-by/Set-work-on-screen	In the meantime, he walked around the classroom and provided feedback.
	Use-tech/Discuss-the-screen	Tom discussed the answers of some questions with the whole class using GeoGebra on the IWB.
	Not-use-tech/Teacher-instruction	He provided instructions for how to find the file (Figure 4.17) and what to do. Students were asked to guess the correct answers for rotation questions.
	Use-tech/Work-and-walk-by/Predict-in-advance	Tom walked around the classroom and provided feedback while students were drawing the rotated shapes in GeoGebra by using the polygon tool.
	Use-tech/Technical demo/Observe-demo	He showed how to set up rotation by slider parameter 0-360 to the whole class in order for them to test their predictions of rotated shapes.

	Use-tech/Work-and-walk-by/Test-your-prediction	Students were asked to test their answers about rotation in GeoGebra using the specific rotation tool. During this, Tom walked around the classroom and provided feedback.
<u>Lesson 3</u> Rotation Finding the centre of rotation through GeoGebra and with the use of tracing paper	Use-tech/Technical-demo /Observe-demo	He showed how to use the new point tool to the whole class and asked students to guess centre of rotation by using this tool for four different problems.
	Use-tech/Work-and-walk-by/Predict-in-advance	Tom guided students by walking amongst them while students were working on finding centre of rotation.
	Use-tech/Technical-demo /Observe-demo	He showed students how to check their answers by using the specific rotation tool in GeoGebra.
	Use-tech/Work-and-walk-by/Test-your-prediction	He monitored students while they were checking their answers by using the rotation tool.
	Not-use-tech/Board-instruction	To sum up, Tom reviewed the three types of transformation that they covered during these lessons and asked students to take notes in order to formalise the topic. After that, he handed out tracing papers for students to practice finding the centre of rotation with tracing paper, as they needed to use it in exams.
	Use-tech/Work-and-walk-by/Link-screen-paper	The task was ‘draw a simple shape and rotate it 90, 180, 270 degrees and draw the new shape but don't mark the centre of rotation on the paper. And then give it to the next person along and they will try to work out where the centre of rotation is.’ He walked in the classroom and monitored students’ progress.

Technical-demo

Tom demonstrated and explained a number of GeoGebra techniques to students during different segments of his lessons when necessary. He was sitting in front of the classroom and using his computer, which was projected onto the Smart-board, while all the students were able to hear the teacher's commands and simultaneously practice at their computers.

"You need to listen carefully to instructions guys because now you are not looking at me you are purely listening. So don't miss any instructions." [T/16]

Applying the instructions simultaneously on the computer guarded against students forgetting the instructions, which could have required the teacher becoming a technical assistant at a later point. Furthermore, at the end of the second lesson, he also demonstrated how to set a slider (see Figure 4.18) to students in order for them to check their answers on GeoGebra. However, in this case, he asked students to watch his actions first and implement afterwards [T/15].

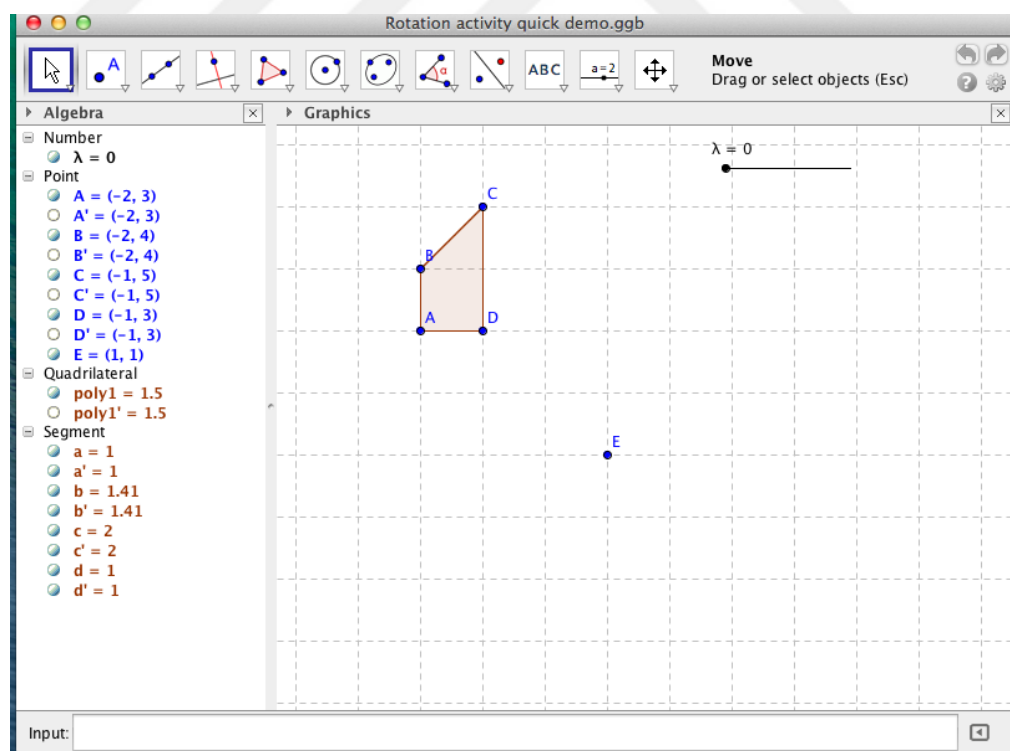


Figure 4.18 Rotation file that was used by Tom for Technical-demo

Work-and-walk-by

The Work-and-walk-by activity format comprised about more than 50% of Tom's lessons. Students were working at computers individually and Tom was circulating around the classroom. With this activity format, he concentrated on monitoring students' progress and engagement in the task, thus, maintaining activity flow. He also provided feedback and guided students when they needed help.

It is my typical approach. Usually give them some points, give them something to start with. That is just an introduction for me or to give them a task and say kick off with this. But I will always move around the room to check if everybody is on task and not getting stuck. Some people put their hand up as soon as they got stuck. If you are long way away they don't bother. But if you are nearby they will ask for your help. Some people never ask for help so you have to check everybody is doing OK. [T/17]

Predict-and-test

Tom wanted students to make predictions on reflection and rotation questions by using only the polygon tool in GeoGebra and then to check their answers by using the specific reflection/rotation tool. In this sense, Predict-and-test was part of the logic of the lesson agenda for handling the topic.

For instance, in Lesson 2, he wanted students to work on the aforementioned prepared Reflection file (see Figure 4.16) by first guessing where the reflections of the shapes in given lines would be, and then using the polygon tool to draw the images on the computer screen. However, as he did not spend much time on introduction from the front, some students did not understand his point and started using the reflection tool straight away.

Today the computer activities were I suppose more autonomous for them. I did very very quick recap something on the board. Just give them a file and 'right figure out what you got to do'. Although I made a mistake actually some of them... I suppose in a way it was good, it was a reflection; they had to reflect each object in the mirror line. But some of them have found before actually hunting through the toolbar, there was actually a tool that does that for them. And they did that and I had not told them not to but I did make them do it again. So it was beside the point of the activity. [T/18]

For the following activity in Lesson 2, he asked students to work on the similar type of rotation activity (shown in Figure 4.17). However, this time he instructed all the

students not to use the rotation tool straightaway since he wanted them first to think what it would be and then check their answers by using the software.

Can you draw them? Do not go hunting and packing for a rotation tool! You are not allowed to use the rotation tool on that. I want you to draw it and I want you to think about it. [T/19]

As believed by Tom, technology made it more conducive to adopt this activity format since checking answers could only come from the teacher in ordinary lessons.

There is a greater potential of the activity like today's for them to complete that cycle which most bookwork does not do. They are able to explore themselves, in fact they are able to make mistakes and look at the real answer and then adjust their mistakes to make it correct. That was an important thing. It is easier to include with technology. [T/20]

Link-screen-and-paper

In the Link-screen-and-paper, Tom stressed the relationship between what happened in the technological environment and how this was represented in conventional mathematics resources. In this case, he enabled students to make links between finding the centre of rotation combining GeoGebra and pencil-and-paper activities through using tracing papers during Lesson 3.

I wanted them to practice playing with transformations and understanding what happens particularly the rotation and the centre of rotation. The centre of rotation can be quite hard to find. I wanted to give them a chance to do a bit of it on paper and a bit of it on a computer just trying to take it from a wide different bunch of perspectives. That is the objective. [T/21]

He asked all of the students to practice rotation on paper individually but students interacted with people next to them since they needed to draw a polygon, rotate it and give it to the next person for them to guess where the centre of rotation would be (see the related lesson verbatim in Table 4.16).

I wanted to engage them with the idea straight away, allow them to use the computer to easily see what they are trying to do and see what happens whether the rotation is done correctly because the computer will get right. And then emulate them themselves on paper rather than trying to teach them something on paper. I think it is better to get this hook. They have to do it on paper anyway for test and exam. [T/22]

Table 4.16 Verbatim transcript for the tracing paper activity at the end of lesson 3

Could you draw a small polygon maybe a hexagons size? Not a regular one. You might want to draw something like just a simple polygon. Just a few dots. I would rather you do irregular polygon.

(Handing out the tracing papers)

Everybody got a piece of tracing paper. Can you maybe trace it maybe with your corner of your tracing paper? I ' d like you to do then use your pencil as you can pin down your tracing paper on the centre of rotation, turn tracing paper to see where it ends up. 90-degree clockwise rotation. It should be emulating this kind of thing (showing on the computer to whole class). Jab it down in the centre of rotation to see where it is going to end up after 90-degree rotation. Draw where it is going to end up. Now you access to that.

You are going to draw a shape. You practice that one first. Draw little polygon not too hard. And I want you to rotate it. Do exactly what you just did. Don't mark the centre of rotation on the paper. And then you are going to give it to the next person. And they are going to work it out where the centre of rotation is using the tracing paper by guessing

Step 1- Draw a simple shape, not too complicated. Try to make it fairly irregular.

Step 2- rotate it any number of degrees you want. Let's stick to 90, 180, 270 and draw the new shape but don't mark the centre of rotation on the paper. And then you are going to give it to the next person along and they will try to work out where the centre of rotation is.

4.3.1.4 Curriculum Script

In Tom's curriculum script, GeoGebra became an important tool in that provided students with dynamic representations to explore a number of transformations concepts as he emphasised at the beginning of the first lesson:

The key thing is you are just thinking about the concepts in today's lesson. I want you to be thinking lots today and using the visual representations of what is going on to help you understand. [T/23]

The tasks to be undertaken involved the use of GeoGebra and consisted of small problems for students to work on and focused on each concept individually with the aim of developing students dynamic thinking [T/9]. In addition, Tom's curriculum script anticipated that students would need a number of tool techniques such as how

to copy and paste a shape in GeoGebra, how to fix the x y axes to a number, how to use polygon tool with the use of vectors, and how to construct a slider [T/15] for carrying out transformations with this particular technology.

I showed some functionality, which only I found when I was preparing the lesson, which is how to copy paste, that is a new thing. It used to not be able to do it in the old version of GeoGebra. They were going to need that in order to be able to achieve what I wanted them to do. [T/24]

Equally, his curriculum script included the use of prepared files for this topic that was shaped by his previous experiences [T/13]. Furthermore, his script incorporated a specific activity format, Predict-and-test, the way in which students progressed by seeking validation from computers. However, he noticed during lessons that he had not thought to change the colour of the predicted reflected shapes from the original shapes, which had potential to create confusion amongst students.

I set up the original activity but I haven't changed the colour of the original shapes. So the new polygons they drew was the same colour of the original ones which got to be confusing when they went to check their answers because there was the original one, their answer and the reflected real answer they were all same colour and also the rotation one. [T/25]

Moreover, his curriculum script to teach this topic with GeoGebra aimed more self-reliant lessons for which Tom provided extended period of activity time for students and did short introductions from the front. However, this resulted in an unanticipated problem that affected the way in which Predict-and-test was used by students.

Today an unforeseen problem was that they cheated in a way by using the tool I had not forbidden. I was actually going to tell them later to use the tool to check their answers. But I did make them do it again. So it was beside the point of the activity. [T/26]

Similarly, his curriculum script did not anticipate that students might need thorough instructions in particular for rotation.

For rotation, I think I let them hear that a bit too soon without showing them some examples of rotation. It was also a bit of an experiment just to give them something. So right now do rotation. Off you go. And I don't think that worked very well. I think a bit more formal introduction for rotation would help. But I think I overestimated how much they remember from year 8, well it is 2 years. I made a mistake. But it is fixable. [T/27]

4.3.1.5 *Time Economy*

As has been noted before, Tom made use of GeoGebra for short time span activities to teach translation, reflection, and rotation. Before each activity, he utilised some introduction time when he instructed/demonstrated students about the activity in order for students to spend their activity time efficiently. However, Tom purposefully kept his introduction time at minimum at the beginning of each lesson with the aim of maximising the activity time.

I tried to not wobble it at the front for too long at the beginning. You need to get kids doing the activity quickly. Yeah just so quick introduction, don't speak more than about 3 min really if you have to speak really. Sometimes I just throw an activity and say, "right read the instructions and do that". And then you could do a proper introduction after they started to work. If I stand and talk at the front for too long they were not interested, anybody would. You don't have to be a kid. Adults are the same. [T/28]

In addition, during student activity time at computers, Tom circulated in the classroom, which enabled him to make 'authorative contributions' [T/17]. Furthermore, when he noticed a common mistake among students he stopped the class and instructed the whole class. These intermediate instructions aimed to support the timely progression of all students in the way that he wished.

"If three or four people asking me the same question then I will stop the whole class and go through it. You do need to go through some of the key points." [T/29]

He also signaled the time spans for each activity in order for students not to dissipate the activity time and give them a sense of timing.

I always remind them the time. It can be a complete lie as well. It is just to give them some feel that they have to do this in certain time. It just means that they know that there is a time limit. You can't mess around and wait 10 min and then get on with it. You have to get on with it now. [T/30]

The use of prepared GeoGebra files in the second and third lessons also introduced some time economy in comparison to the first lesson where students needed to start with a blank page.

Some of them struggled to get going yesterday [*first lesson*], stuck with the blank sheet. They did not know where to start even if they have been told

of where to start. Yesterday first thing I was going to do was to set to scaling of axes to 1. I was sort of giving them instructions at the beginning of the lesson. But I could have easily just sent them a file that was already set up to do that and it would save time. So I have learned something. [T/31]

4.3.2 Straight Line Graphs

This section analyses Tom's teaching 'Straight Line Graphs' involving use of GeoGebra to a Year 8 class (set 3 out of 4). For this topic, Tom prepared a finishing activity, which took 20 minutes of the lesson. The aim of this activity was for students to practice Straight Line Graphs that they had previously worked on with paper-and-pencil activities.

4.3.2.1 *Working Environment*

Tom's lesson took place in a similar classroom as the one in which he carried out his 'Transformations' lessons (see Figure 4.14). Therefore, most of the information about his use of this classroom has already been discussed in the section 4.3.1.1.

That was absolutely fine I thought the classrooms, I could have swapped two lessons for the two GeoGebra lessons, it would have been the same lessons I think. We have got a quite a few classrooms like that, 8 or 10 at least with fixed computers. [T/32]

4.3.2.2 *Resource System*

The resource system for this lesson consisted of a teacher-devised sequence of worksheets (see Figure 4.19), which were used in conjunction with the dynamic GeoGebra file shown in Figure 4.20.

As has been mentioned, this part of the lesson was planned as a finishing activity on the topic of Straight-Line Graphs. Tom prepared a dynamic file involving use of GeoGebra with the aim of advancing students' insight into the topic.

First part of the lesson [*Straight Line Graphs*] was end of a topic. That was GeoGebra to finish the topic to give them some puzzles to do on the computer after they understood the material they needed for it and to fill in the worksheet, which was to boost and improve their understanding. So it is trying

to boost their understanding of what the y intercept is and what the gradient is. Even just playing with the software I hope would deepen their understanding of gradient and y intercept. That was the key reason for using that. [T/33]

With this task, students were asked to move the black line on top of other coloured lines (they needed to tick in order to see those fixed lines) and write down what it said the gradient was and use that to write down the equations of coloured lines.

Since the class had not used GeoGebra before, Tom let them familiarise themselves with the dynamic file at the beginning of the lesson, which he thought was something intuitive for them.

I don't think I ever used it with this class. The first thing I said load it up file and spend a few minutes just play with it. And they did. They are used to these kind of things on smart phones, games, dragging etc. [T/34]

He instructed students that some of wording on the worksheet was out of date and when it said slider it meant the tool that controlled m for the gradient.

The accompanying worksheet for the activity, some of the text needs changing, it says drag the slider, it is not slider anymore, and it is this tool. The original version was slider but I changed it just to get them closer to the geometry. [T/35]

Task

1. Locate the Yellow slider; drag the black dot to the right to “switch on” the yellow line.
2. By dragging the sliders for m and c , transform the black line so that it covers the yellow line. Complete the boxes in the table with the values for m and c .
Ignore the last column at this point!
3. Do the same for the other coloured lines, filling in the table with all the correct information.

Straight line	Value of m	Value of c	$y = mx + c$
Yellow			
Cyan			
Red			
Green			
Purple			
Grey			

4. Now use your values for m and c to complete the last column, by substituting your values of m and c into the general equation for a straight line: $y = mx + c$
- 5a. Write a short sentence to describe the effect of adjusting the value of m .

- 5b. Write a short sentence to describe the effect of adjusting the value of c .

Figure 4.19 Tom’s paper worksheet for the ‘Straight Line Graphs’ Lesson

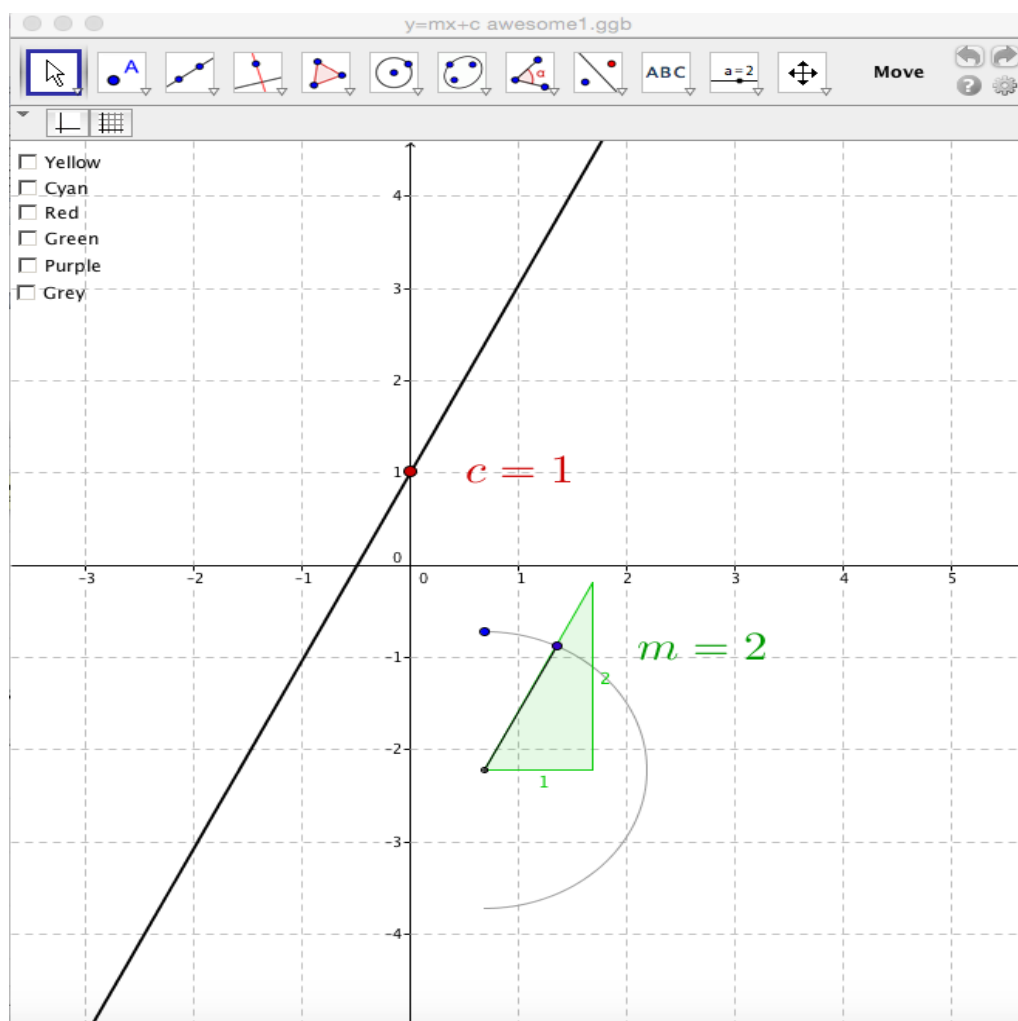


Figure 4.20 The GeoGebra file that was used by Tom’s class for the ‘Straight Line Graphs’ lesson

4.3.2.3 Activity Structure

The activity formats within this part of the lesson consisted of teacher-led exposition and questioning to elaborate or review this investigation task as well as individual student work on the task with the teacher walking around the classroom to provide guidance (see Table 4.17).

Table 4.17 Activity formats and lesson agenda followed by Tom in the ‘Straight Line Graphs’ lesson		
Segment purpose	Activity Formats	Lesson Agenda
Straight Line Graphs	Not-use-tech/Explain-the-screen	Tom explained to students how to find the GeoGebra file that he emailed to them and wanted them to have a play with it first by dragging the red point. While students were playing with it, the teacher handed out the worksheets and then explained the task to students.
	Use-tech/Work-and-walk-by/Practice-with-technology	He let students use the computers themselves so that they could work on the dynamic GeoGebra file (see Figure 4.20) and fill in the paper worksheet (see Figure 4.19). During this, Tom moved around the classroom in order to check what students were doing.
	Not-use-tech/ Discuss-the-worksheet	To conclude, he discussed the task for the last column by asking questions “How do we define a particular straight line?” and “What piece of information do we need to know for a straight line?”

4.3.2.4 Curriculum Script

This 20-minute activity aimed to review ‘Straight Line Graphs’ with GeoGebra in order to complement the development of understanding of this topic on paper. He believed that after learning the basic algebra, practicing with a GeoGebra based activity would reinforce students’ learning. In this regard, Tom’s curriculum script envisaged this part of the lesson as building on previous work that had been carried out on the topic in a paper-and-pencil environment.

I wanted them to try a deeper understanding how gradients, and intercepts work, particular GeoGebra file that I gave to them with the coloured line. I set it up rather than having a slide of a C, and slide of a M for gradient. I created a tool where they could drag it is like a one fixed point and another point they can drag it to change the gradient of the line rather than just changing a number to change the gradient they physically change the gradient of the line. Also y intercept was a point on the y-axes, which they drag up and down. So they click on the actual y intercept itself and moving that. [T/36]

His curriculum script took account of students' knowledge of technology; thus, he preferred to use a prepared file considering the complexity of the software, which he had experienced before with another class.

In fact, the activity I did at the beginning of the year 8 lesson I tried to get that in my year 10 class to make that from scratch. I gave them a few points but they did not know how to start. I did a little bit demo, tried to show them a couple of tricks but actually I realised half way through it was just too much for them. It was not something I thought about until later. There is actually quite a lot going on at the background of the program. [T/37]

He had also used this prepared file before with another class that led him to rethink about how to present variables m and c. Accordingly, he decided to attach them to each other with a tool rather than changing them with separate sliders and also arranged the line to be moved up and down by students as he wanted them to be in control.

The original version of that had a simple slider for M for the gradient and simple slider for C for the y intercept. But actually I thought I am not attaching a meaning dragging a slider like this and line goes like that [*showing me with his hand, line is going up and down*]. What they are doing now is much closer to the way that the line behaves. And also not just dragging a slider to make the line go up like this but grab the line itself on the y axis and drag it up and down. And again you are making them much more in control. I don't know they become the line is like an extension of their hand rather than something they control it from far. I would truly believe that is the way really to use technology to make the kids understand maths to make them part of the action not controlling it from the far from a kind of radio control in a helicopter, being actually in the driving seat. Being much closer that is the key. [T/38]

4.3.2.5 Time Economy

As Tom had used this activity before, he was aware of that how much timing this activity would take. Therefore, he did not have a temporal instability.

“I knew that this activity would not take the whole lesson because I have done it before. That does not take that long, 20 min or so.” [T/39]

In addition, Tom reported that use of GeoGebra improved the rate of learning return from classroom time by virtue of ‘Focusing on overarching issues and accentuating important features’ identified in the practitioner model (Ruthven& Hennessy, 2002).

Well the first activity with the year 8 is something that lent itself very well. That was purely GeoGebra based, computer based exercise that boost their learning $y=mx+c$. [T/40]

4.3.3 Summary

In terms of Tom’s teaching with GeoGebra, there seems not to be a salient difference between his approach to the two topics. The use of GeoGebra for both topics aimed to focus on individual concepts by working with dynamic files prioritising the dynamic and instant nature of GeoGebra. However, the Straight Line Graphs lesson (20 min-finishing activity) focused only on a single activity in order to reinforce students’ learning as they had already done this topic on paper.

In terms of activity structures used to teach these topics with GeoGebra, Tom’s lessons regardless of the topic consisted of extended periods of individual student work at computers with him circulating around the classroom to guide students. His curriculum script for both topics contained goals for individual students work with GeoGebra, and consisted of dynamic tasks focusing on a single concept at a time. Transformation tasks attempted to stimulate students’ use of a trial and refinement process through interactivity and immediacy of feedback of GeoGebra. However, with the Straight Line Graphs task, students, assisted by the dynamic dragging- function, were asked to find the value of gradients and intercepts for given lines that aimed to practice their knowledge about this topic, which they built up with paper-and-pencil activities. Hence, GeoGebra was made use to review the topic

through ‘Focusing on overarching issues and accentuating important features’. In this respect, how GeoGebra featured in Tom’s curriculum scripts for these two topics were different. In the sense that, for Transformations, GeoGebra provided students with dynamic representations to explore concepts for a series of three lessons whereas, for Straight Line Graphs, it was used as a 20 min-finishing activity for students to practice what they had learned before in a paper-and-pencil environment. Tom stated in a post lesson interview that how he decided to use GeoGebra in a lesson depended on what that topic depended on. In particular, for Straight Line Graphs, he reported that the finishing activity involving use of GeoGebra would help students’ learning better after having carried out the algebra on paper.

It does very much depend on the topic. We had to do a lot of algebra work. We could not really, gotten there with software without giving them a decent grounding in algebra and then do $y=mx+c$ and boost it up with GeoGebra. [T/41]

Time saving strategies that Tom developed to include technology for these topics did not differ. He managed to save time capital in working with GeoGebra by using:

- Technical-demo to demonstrate necessary tool techniques which allowed students more learning time focusing on mathematical content. In particular, demonstration of how to make a slider during Transformations lesson to check their work was intended to speed up the trial and improvement process.
- Prepared files so that students could focus on mathematical content rather than dealing with complexity of software
- Intermediate syntheses and small contributions during work-and-walk-by
- Short introductions from the front in order to give students more activity time

4.4 Chris

4.4.1 Transformations

In this section, I will analyse Chris's implementation of GeoGebra to teach 'Transformations' in a series of three lessons to a high attaining Year 9 class. To reiterate the data, I am using here is the data from the pilot study that I conducted for my MPhil thesis. In this section, I am focusing on part of the data set from this teacher since it is helpful data to have alongside other cases for developing the framework. However, I am analysing this data set afresh without relying on the previous analysis that was presented in the MPhil thesis. For the purpose of comparison, I am going to present three lesson observations (out of seven lessons) and related post-lesson interviews. In addition, I will draw on episodes from other lessons in order to explain fully how the teacher thought about what he was doing (see a summary of the sequence of tasks that students were set over the course of the seven lessons in Appendix 7).

4.4.1.1 *Working Environment*

All the observed lessons took place in a room (see Figure 4.21), where there were a number of computers against one wall, a computer connected to a data projector for the teacher, and a Smart-board in the front. There was also sufficient seating in the centre to accommodate the entire class being taught as a group.

This computer room was essentially a combination of a classroom and a computer suite, which allowed students to move between computers and their seats in the centre of the room depending on what the teacher required them to do. In the observed lessons, Chris usually made use of two main arrangements: students working as small groups at computers during the middle segments, and a whole-class discussion during the initiation and summary segments of each lesson. For the latter, the teacher gathered all students in the centre, where they stayed closely around him; this allowed the students to see the board in a very similar manner to the teacher's normal classroom.

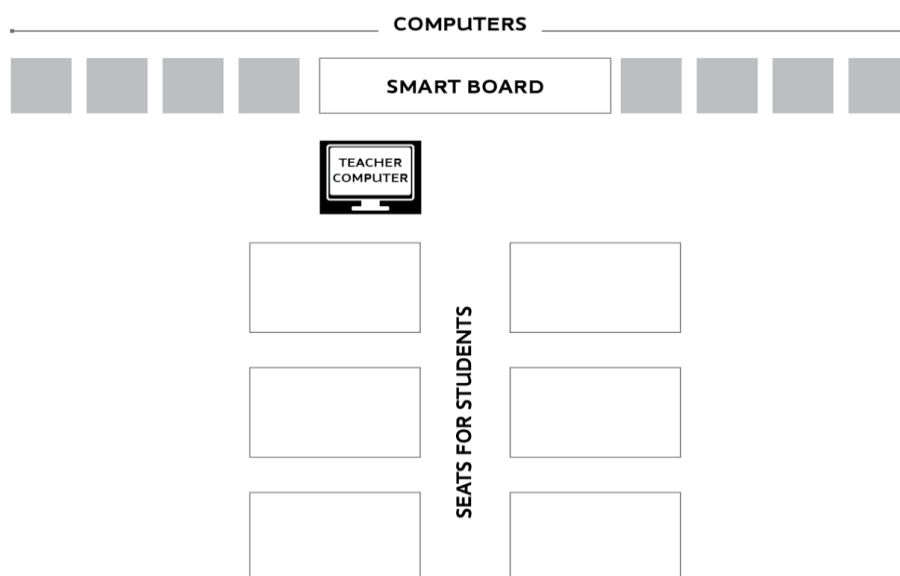


Figure 4.21 The room where Chris's 'Transformations' lessons took place

This layout made the teacher more positive about using the computer room as a teaching space since it gave him the flexibility of combining both independent student work at computers and whole class teaching.

In most cases, either you are in a classroom that does not have computers so it is not possible to use them, or you are in a computer room where everybody has to sit at a computer because there is not any other space. You have to do one or the other. Whereas the wonderful thing about that classroom and the reason why I was so excited about teaching in that classroom is that we can spend part of the lesson at the tables and part of the lesson at the computers. [C/1]

In particular, during the third lesson, students were preparing posters about the things they learned in the last two lessons and making use of GeoGebra when needed. The flexibility of the working environment enabled students to work at the tables as well as to use the software about which Chris expressed his appreciation.

For me that really shows a great way of using ICT where the flexibility allows it to be used properly as a tool rather than it being maths IT lesson. Technology is a tool just to support maths that pupils were doing which is absolutely as it should be. So even though they were not all at the computers for the whole lesson they got lots out of it. Technology helped. Either it helped them explicitly because they were using technology in the lesson or they were referring back to the things we had done in the diagrams we had drawn. [C/2]

Some routines were introduced to the students working in a technological environment. Chris launched the first lesson by introducing setting up routines including how to find GeoGebra, how to log on, how to load up files. Organisation of switching working arrangements depended on the teacher instructions. For instance, when he wanted the students to gather in the centre, he said that “Could you please save your work and then could you come and join me? You have got 2 minutes”. This showed how he had customised more generic routines for switching activity and moving place to this particular working environment. Reminding students of how much time they had encouraged efficient task switching, as it might be possible for students to waste time during changing places. There was also a closing routine developed, which was “save your work in my documents then log off and we will follow up on this tomorrow”. This end of the lesson routine helped students to get ready for the next lesson, in which they could just find the files easily and continue where they left off. It also helped the teacher to have all the files that students worked on, which enabled him to spot examples and share them in the coming lesson.

4.4.1.2 *Resource System*

The resource system for these lessons consisted of prepared dynamic files involving use of GeoGebra. In detailing why the teacher chose to use this software over more conventional methods, he pointed out that the accuracy and speed of GeoGebra allowed students to explore the topic without drudgery of using compasses in a more straightforward way.

GeoGebra allows the exploration. It lets them properly explore rather than just learning rules. It is very quick. Finding a point, doing that in reverse, finding the mirror line, finding the centre of rotation, scale factor of enlargement is more awkward on paper. Finding centre of the rotation for example is much more difficult on paper. I think pupils get stuck and get bogged down with mechanics of using a compass and using it really accurately whereas GeoGebra let us do that so much more straightforwardly, does not take away from the thinking they need to do but allows them to think. And the fact that we can drag and make changes as well adds an extra dimension to it. [C/3]

Thus, the teacher placed a premium on time economy for use of GeoGebra since it gave the students more learning time to focus on mathematics and their thinking by

allowing easier access to mathematics. As Chris emphasised:

Without the software they will have to draw perpendicular line using compasses and pencils and rulers. That is so much less convenient for them. I did not want them to get hooked down in the detail of where to put your compass point. I wanted them just to know if we got a perpendicular bisector that works and GeoGebra will do that immediately. For me the big difference here is that GeoGebra let us get at mathematics more quickly. Pupils still have to think about what they are doing. [C/4]

He also considered the fact that students could undo what they have done in GeoGebra was of crucial importance in terms of allowing them easy procedure.

And the idea of if you make a mistake, you don't have to start again with your diagram or rub it all or use tippex. Instead, you can use control+Z and this is very freeing for the pupil. [C/5]

Furthermore, he believed that such dynamic mathematics software enabled students to be in charge of their learning. Hence, this technology allowed him to use the kind of activity format (see further discussion in the Activity Structure section 4.4.1.3), where he encouraged the students to self-test and then to self-correct.

Pupils having ideas and then testing them are very easy with GeoGebra. For students to be able to think, try it and for them to be deciding what it should be without me needing put a cross in his book and say 'this is wrong, try again' and it gives them a chance to be in control and I hope gives them a chance to behave like mathematician to try things out, to explore things and then 'that did not work, try this', 'you solved the problem and then it works'. [C/6]

In terms of the operation of the software, Chris allocated the first lesson for students to explore the software since the students were using GeoGebra for the first time for this particular topic. Accordingly, one of the aims of the first lesson was for students to become accustomed to the software.

What I particularly wanted for them was to know the software to do the transformations and also I wanted them to do some thinking about mathematics. And they did, they worked. It was great. That was what I wanted. They are above average. [C/7]

To accomplish this, Chris began the lesson by demonstrating how to operate the software: how they could open the software, where to find the prepared files for transformations, where the related icons were located, as well as how to use the

dragging tool. In addition, recognising the complexity of the software, he designed dynamic files for students to work on to ease their exploration of the topic with GeoGebra. For instance, he prepared a dynamic file for Enlargement including a slider as well as an object and its image: “I wanted them to be able to have an accurate slider and just to move the slider and to see the changes immediately. It meant they were focused on the enlargement and on the slider.” [C/8]

In order to explain the nature of pre-designed GeoGebra files, two examples (see Figure 4.22 and Figure 4.23) will be illustrated more fully. Later examples were similar to these. In the first lesson, one of the three prepared files (Reflection, Rotation and Enlargement) was the Reflection file shown in Figure 4.22. Students were told to reflect the shape in line and then to find out what they could change directly, and what they could not change, by using the dragging function of GeoGebra.

I also wanted them in the first lesson to be doing some thinking, that is why I asked them to make notes and I asked them to look for interesting things. Things like they can move the object but not the image, you can drag certain points but not others. [C/9]

As a further example, in the second lesson the teacher set up a file shown in Figure 4.23, where there was an object and its image locked so that the students were not able to move anything. The assignment was to locate the line of symmetry.

Working with these prepared dynamic files, Chris encouraged his students to explore different paths to reach the same mathematical solution and placed less emphasis on finding the exact answer. In this case, he considered the use of GeoGebra for this topic as a tool to help students’ thinking.

I want them to understand properly how transformations work. The questions are not important as I mentioned to them. I don’t have a burning desire to know where the line of symmetry is. What I want them to know if you join up two related points and then do the perpendicular bisector that would be where it is. That is the sort of thing I am hoping we will move towards. [C/10]

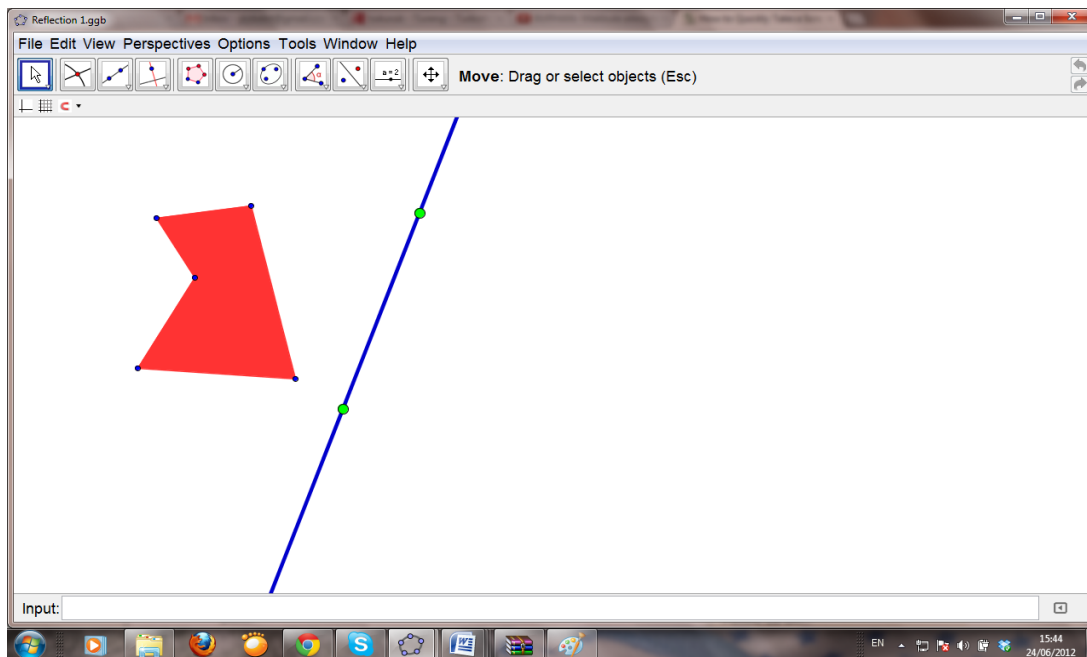


Figure 4.22 Reflection file that students worked on during Lesson 1

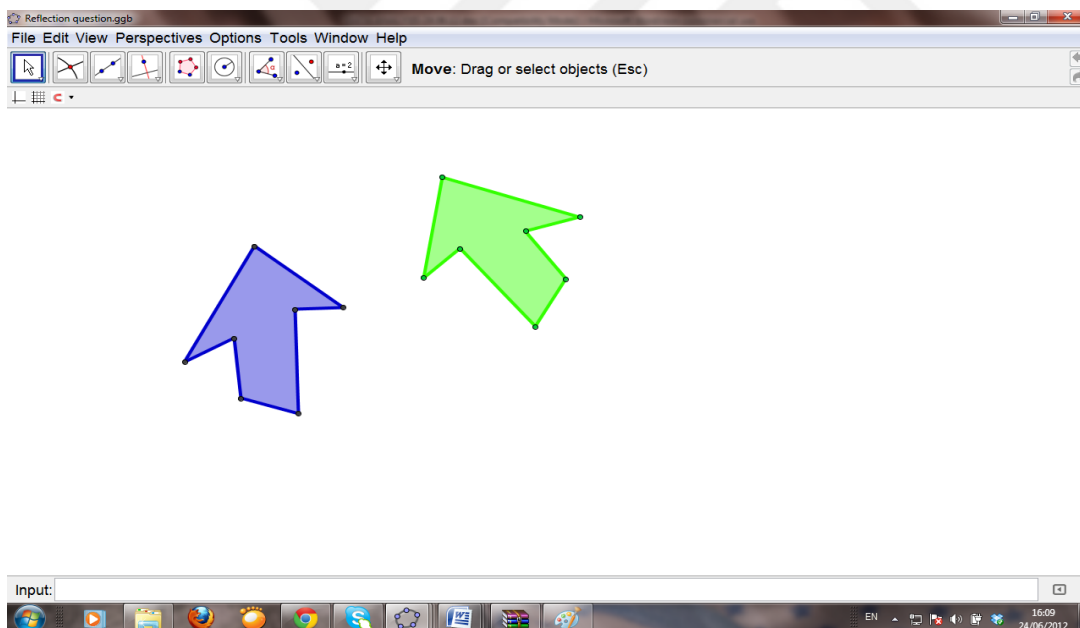


Figure 4.23 Reflection file that students worked on during Lesson 2

He also made this clear to the students in the Lesson 2 by articulating: “The questions are not really important for answering that particular question but they are important in the insights they will give you and the ideas they will give you about that we are looking at.” [C/11]

In addition to students' use of GeoGebra, the software was also used by Chris on the IWB for whole class teaching. In particular, at the beginning and end of the lessons, Chris addressed the whole class either to instruct them what to expect or share other pupils' work to give and discuss ideas.

4.4.1.3 *Activity Structure*

All of the observed lessons broadly broke down into 3 segments. Initially, Chris introduced the lessons in a whole class segment. During introduction segments, he demonstrated the related tool techniques and gave information about what students were expected to do in the lesson. In addition, he sometimes asked a question to help the students to get started or projected students' work from previous lessons or their homework for the whole class to discuss. In the second segment, students went onto the computers and worked with the prepared files to discover for themselves in pairs or small groups while Chris was walking around to guide and make sure that they engaged with their tasks. Finally, Chris gathered the students in the middle of the room for whole class teaching in order to discuss and connect the important/main ideas of the lesson mostly to create collective knowledge. During these three broad lesson segments, different kinds of more specific activity formats were observed (see Table 4.18).

Most of the lessons I will start the lesson all together talking to them from my computer then they will go to their computers and they will work together in pair and then we come back together afterwards. [C/12]

Table 4.18 Activity formats and lesson agenda followed by Chris in ‘Transformations’ lessons		
Lesson	Activity Format	Lesson agenda
<u>Lesson 1</u>	Use-tech/Technical-demo/Observe-demo	Chris started with a whole class teaching by using the IWB and showed students some tool techniques such as how to open GeoGebra, where to find prepared files, which icons they need to use for reflection, rotation and enlargement, how to drag the object, and how to use a slider.
	Use-tech/Work-and-walk-by/Discuss-the-screen	To start with, he asked students to work on a file that has got an object and an image under reflection and to find out about the link between the object and the image. While students were working at computers as groups, the teacher moved around the classroom in order to make sure students were involved and engaged with the task.
	Use-tech/Spot-and-show	Last five minutes Chris gathered students in the middle of the classroom. He showed some students’ examples, which he picked up during Work-and-walk-by. He projected these examples on the Smart-board for whole class discussion to make sure all students hear about the ideas different people had.
<u>Lesson 2</u>	Not-use-tech/ Questioning	He started the lesson by asking a question to the whole class for students to use their reasoning skills. The question was “yesterday we have one visitor with us. Today we have got two visitors with us. Tomorrow if we had a maths lesson, what possible sensible answer might have been of how many visitors might be with us tomorrow? Think and tell me with reasons.”

	Use-tech/Explain-the-screen	Then, he reviewed the yesterday's lesson by going through all the examples that they worked on and showed students how to find files for today's lesson. He also instructed students that they could go back to yesterday's files if they wanted to.
	Use-tech/Work-and-walk-by/Predict-and-test	Chris asked students to start with a reflection file where students needed to find the line of symmetry for locked image and object. In the meantime, he moved around the classroom to monitor students' progress. During Work-and-walk-by, he purposefully encouraged the students to test their ideas on the computer rather than him validating or invalidating their answer.
	Use-tech/Spot-and-show	Chris finished the lesson by showing and discussing some examples from students' work, which he spotted during Work-and-walk-by, with the whole class.
<u>Lesson 3</u>	Use-tech/Spot-and-show	He spotted a homework while he was walking past a table and directly took a photo of it with a smart phone and started the lesson with this picture projected on Smart-board. He said he had not intended this start but when he saw the homework he thought that would be a very good example for students to set off.
	Use-tech/Work-and-walk-by/Make-a-poster	The task for students was to draw things they have learned in the last two lessons together and to make a poster. While students were engaging with their task, Chris circulated in the classroom and provided feedback.

Below, I will give more systematic discussion of each of the activity format types.

Technical-demo

At the beginning of the first lesson, Chris focused on a Technical-demo format concerning the demonstration of tool techniques since the class was using GeoGebra for the first time for this topic. Chris demonstrated the tool techniques and explained the prepared GeoGebra files to the whole class on the IWB (see Table 4.19). As the IWB has touch screen functionality, Chris illustrated points using his finger.

Table 4.19 Chris’s instructions at the beginning of Lesson 1

- **Where to find prepared files** (go to My Computer- Maths- Mr. X- Yr 9- Transformations)

- **There are five files- start with Reflection 1**

“If it happens not to load properly you might need to go to start menu to find the software called GeoGebra which is in a sensible place and then open it up again.”

- **Which icon they need to use for reflection**

“If you click on the bottom right hand corner of this icon there are other transformation possibilities. And rotate is on there and enlarge is on there as well.”

Use ‘Reflect object in line’- tap on the middle of the object, then tap on the line and it does the reflection

- **How to drag the object**

“If you go to the pointer arrow, we can make some changes. I can make changes to the original one and it stays as a reflected picture.”

- **After Reflection, work with file called Rotation 1**

“Let’s go to rotation 1. I have got this all set up. To do a rotation we need to find the one it says ‘rotate object around point by angle’. There are three bits of information we need. We need the original object, we then click on the centre of the rotation, which is the point, and then it wants an angle. We got to have the degree symbol. If you don’t want 45-degree rotation, then you have to use the cursor and keep the degree symbol. You can have anti clockwise or clockwise you click okay and it will do it.”

- **How to use a slider**

“Perhaps a more sophisticated way of doing rotation is not to set it up by particular single one angle but to set it up by using a slider. Go to the file called Rotation 2. We click on the object, we click on the centre of rotation and then for the angle I

want to use this slider. So I can't click on it but I can find the Alpha here. I need to delete that and I delete the degree symbol. I click on alpha there and this now rotated by 45 degrees. If I go back to the pointer I can now move this (Students say 'wow'). And I can get it to do any sort of rotation that I want."

- **Enlargement**

"Final two files to mention there is enlargement. Enlargement is here under the same icon as well. Click on 'enlarge object from point by factor'. We click on the object and we click on the point. We do need number to enlarge it by, so scale factor 2 and we have got that."

I can make some changes; I can't make some changes so you can find out what is going on

Work-and-walk-by

In the second segment of each lesson, Chris allowed the students to engage with pre-made GeoGebra files by themselves for around half of each lesson as his aim was for students to be more self-reliant and to be in charge of their learning. While students were working at computers as small groups (consisted of at least two students), Work-and-walk-by activity format (See Figure 4.24) comprised about 50% of a lesson where Chris was moving around the classroom and guiding students. In other words, his role was to monitor students' progress, provide feedback and maintain activity flow.

So, my role in the lesson was very much making sure that the pupils were all engaged and involved and try and make sure they could include the interesting things they could think of and interesting ideas they had. [C/13]

During this format, Chris communicated with students by joining their group discussion through question/answer, where students explained/discussed what they were doing. Thus, this activity format allowed the teacher to gauge students' involvement and engagement with the task and also to determine whether they had any difficulties during tasks. In addition, he guided the students to think in various ways regarding problem solving rather than focusing only on the result itself.

One of the things I did not want pupils to do was to draw the line and go ‘yeah that is right’, I will move on to the next thing. I did not want them just to work out the answer and move on. I wanted them to work out the answer by thinking very deeply about it. That was the big concern that I had. That is why I moved so quickly amongst the groups to start with to make sure they were thinking about the different methods and not just finding an immediate answer and then moving on. [C/14]



Figure 4.24 A photograph of Chris walking around the classroom to guide students

During Work-and-walk-by, Discuss-the-screen was apparent in teacher-student interaction since the teacher was encouraging students to focus on ways of solving a problem [C/10]. For instance, he encountered five different ways of students finding mirror lines in Lesson 2 during Walk-and-work-by. One of them was by using the perpendicular bisector (see Figure 4.25). The other one was to join up two related points and to join up the other two related points and then to find the mid points of those and join them (see Figure 4.26).

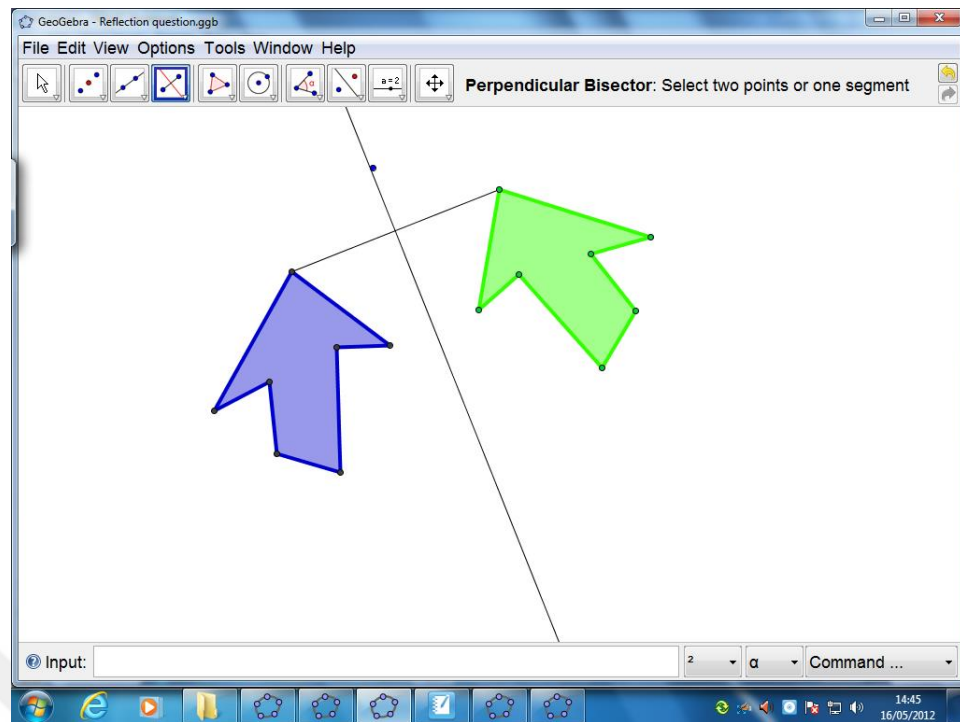


Figure 4.25 A screenshot of students finding the mirror line by using perpendicular bisector

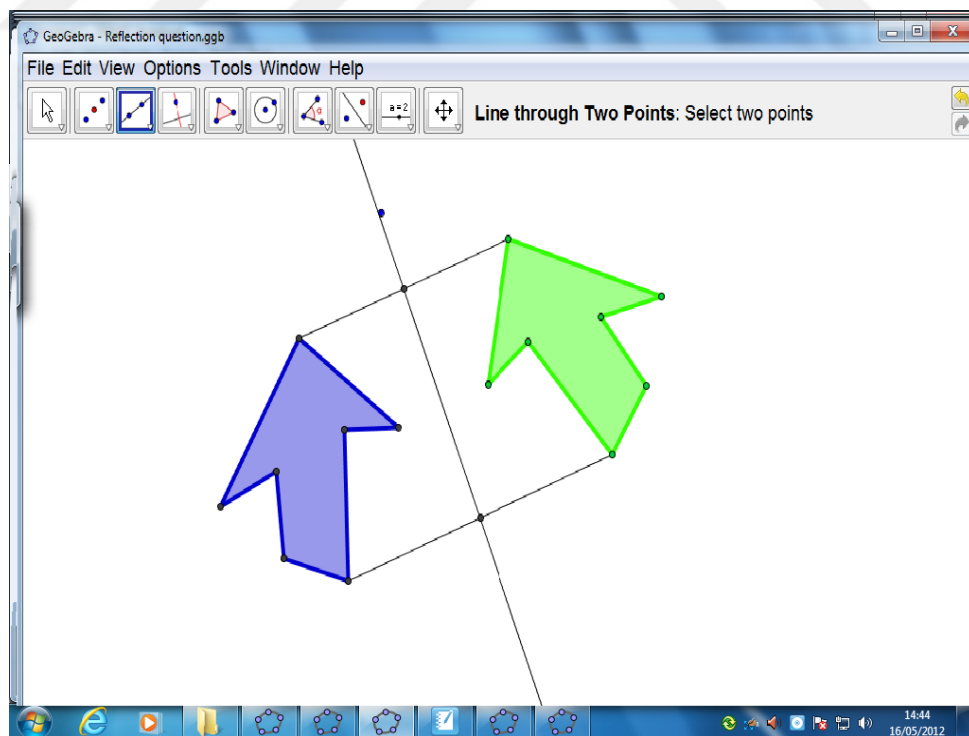


Figure 4.26 A screenshot of students finding the mirror line by using mid points

Predict-and-test

During Work-and-walk-by, students were also encouraged to use Predict-and-test activity format, where they tested their ideas at the computer themselves and progressed through seeking validation from computers (see Table 4.20).

There was Ben with his excellent centre of enlargement but error with the scale factor. I deliberately let him make the mistake and then suggested him just check it. I deliberately left him to it. So he would first of all realise that checking was important. He would realise that he made a mistake and then wanted to sort it out; I did not tell him what he had done wrong. [C/15]

Chris pointed out that it was more difficult to use this activity design in a non-technology based lesson in a way that enabled students to be more self-dependent. It was because in a traditional design the teacher needed to be the one who confirmed or rejected students' assumptions.

GeoGebra promotes the sorts of skills that I think it is important for being a mathematician. Rather than them saying 'tell me how to do this' or 'is this right?' they are able to say 'I will try this', 'oh no I discovered that is wrong'. When Ben made a mistake, he was able to self-test and he was able to then self correct. If we would have done this in the classroom, I or somebody else needed to say 'that is wrong'. It would not have been as useful for them in terms of their development as a mathematician. [C/16]

In this connection, he considered this to be one of the key characteristic features of working with GeoGebra, which uniquely influenced the interaction between students, mathematics and the teacher.

It is different pupils can be a bit more self-reliant, a bit more self-efficient. The thing we did at the very beginning of the lesson making use of the fact that 'yesterday there was one visitor today there are two visitors' it still involved then having ideas, being more than one way of doing things, explanation, trying things out. But I had to be conducive to that only comes through me. I then asked follow up questions. That is different from working with GeoGebra. In that sort of scenario if you have an idea there is only limited number of things you could try out. They did try things out. After six days there will be as many visitors as the students. But then they needed to seek validation from me if that is right. I think that is the difference. [C/17]

Table 4.20 An example of Predict-And-Test activity format

Ben: I did the enlargement question.

T: How did you find the centre of enlargement?

Ben: Line-you connect those two up. When they all join together, obviously from that point they are all branching up. **(he was explaining Figure 4.27)**

T: What is the other bit of information that we need for an enlargement?

Ben: Scale factor

T: How are you going to work that out?

Ben: I need to measure how far that one (HD) and that one (DA') and then divide
(The teacher knows that Ben is making a mistake. Ben should measure (HD) and (HA') and then divide. But the teacher wants Ben to discover this mistake by self testing)

T: You can do that on GeoGebra- somewhere on one of these things there is a measurement tool- keep going over to the right. Something to do with measuring things and sizing things. Does it say measurement there? Go left a bit then- **(while Ben was looking for the icon)** There we go. It says select two points. Be aware it gives everything to one decimal place, it might have been rounded.**(Ben starts doing the calculations)**

T: Do the enlargement yourself? And check that out if it works **(The teacher goes to other groups to see what they are doing while Ben is doing calculations. After 2 min, Ben calls the teacher and says that the scale factor is 1.5)**

T: Could you just check it? Go to enlargement. You are saying 1.5, let's see **(Student is doing enlargement using the scale factor 1.5 he just found. But it shows that 1.5 is wrong.)**

Ben: Ahhhhhhhh (with disappointment)

T: Ben, do some thinking- what has gone wrong?

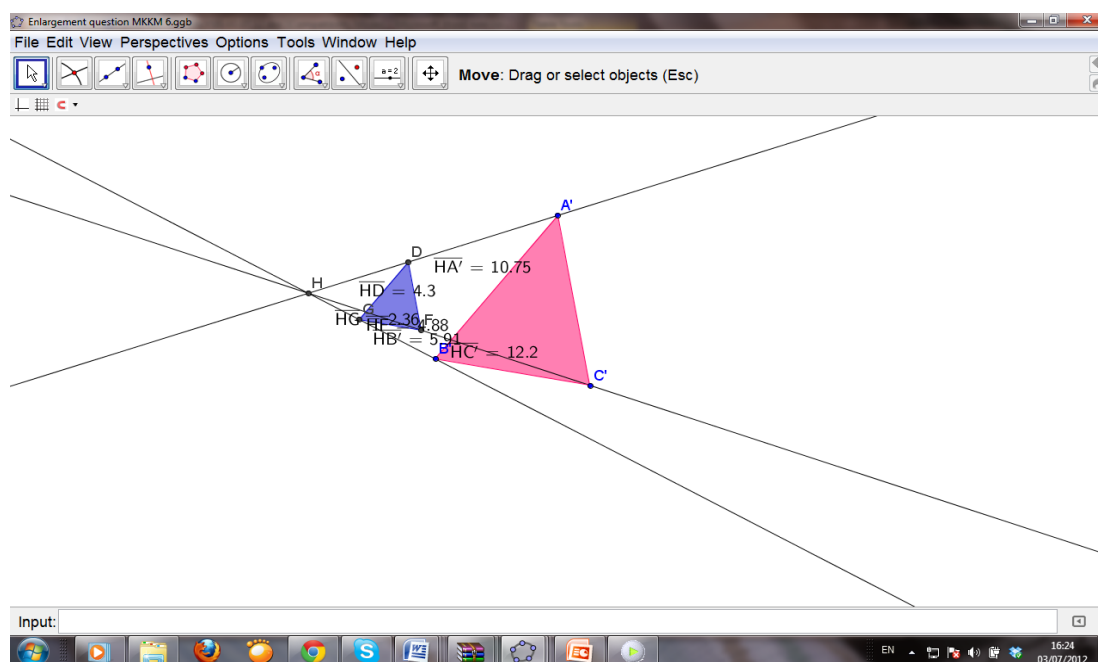


Figure 4.27 Enlargement file that students worked on during Lesson 2

He spotted another example (see table 4.21) and showed it to the whole class in Lesson 2 with the intention of encouraging students to test their ideas at computers in future lessons so that they could be sure of whether they had done it right.

Table 4.21 A verbatim transcript of an example where Chris encouraged students to check their answers

In Lesson 2, a group of three students were working on finding the mirror line (see Figure 4.23) and while the teacher was circulating in the classroom they showed their solution to him.

T: Does it work?

S: I think so

T: You say I think so. ‘Can you try out for me?’

S: Student goes to the reflection icon and does the reflect object in line of symmetry. And they get the **Figure 4.28**

Students: oooooooooohhhhhh (with disappointment)

T: Oh nearly.

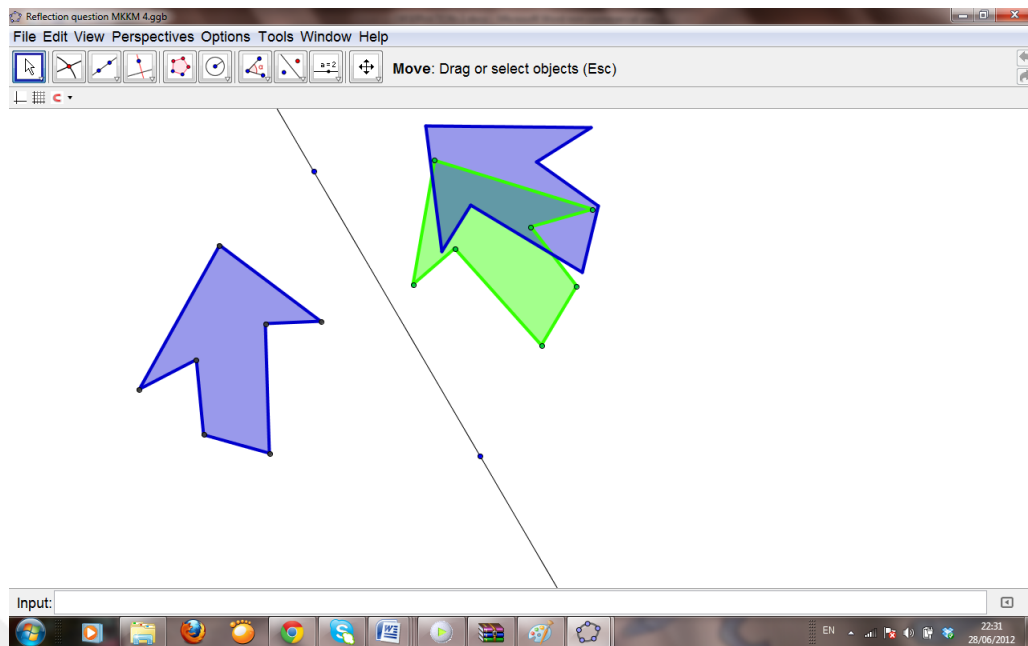


Figure 4.28 A screenshot of Predict-and-test example

Spot-and-show

Chris used the Spot-and-show activity format quite frequently, perceiving it as a means to enhance student involvement and discussion.

That always feels really powerful way of doing things because they have all focused on slightly different things. And learning from each other is really important way of doing things particularly in mathematics. [C/18]

In particular, during the last segment of the lessons, he mostly used the Spot-and-show. He brought up students' work, in particular original approaches, that he picked up during Work-and-walk-by and projected them onto the IWB for whole class discussion aimed at making pupils think of different perspectives. For him this part of the lesson was of crucial importance to make sure that all students heard about the ideas that different people had.

We need to sort out and put together some of the things people have noticed, some of the things people have discovered because there are some big mathematical ideas here that we need. Next time we will take this further but we really do need these ideas, we need to be able to nail this down right now. [C/19]

During this format, Chris gathered students in the middle of the classroom, where

students were seated at their tables and he was in front of the classroom that created an ordinary classroom environment. Chris involved the whole class in discussion of student work through open-ended questioning and stimulated students to discuss what they saw on the screen.

Some people decided that ‘one’ was not very interesting. Jack decided that ‘one’ is an interesting one. And I agree with Jack on this one. I think ‘one’ is an interesting one. What is going on with scale factor one? [C/20]

In this regard, this activity format aimed to give students the opportunity to react and have more input. Even though Chris was leading the activities during whole class teaching, he always allowed students to have more interaction either with him or with each other.

Walk-and-spot

Chris also used a variant of Spot-and-show in Lesson 7 (that was not discussed in this section fully), which could be characterised as Walk-and-spot (see the related lesson verbatim in Table 4.22). He asked students to stand up and walk in a clockwise direction around the room to see different people’s screens and then to implement some aspects of their work they had considered useful for their examples. Hence, this was that students spotting interesting examples for themselves rather than the teacher spotting and showing. In this case, students were leading this activity format rather than the teacher.

Table 4.22 Verbatim transcript for Walk-and-spot activity format

Okay Ladies and Gents. Could you please stop and listen and turn so that you are facing me but stay where you are just at the moment?

Now so far you have done some really impressive things. There are things going on here where some people have drawn all of the enlargements and are looking for the scale factor of enlargement between the two images comparing the two. Some people have found the centre of enlargement comparing those two images. Some people are looking at the areas of the enlargements. Some people are looking at what happens when they changed two scale factors using the slider.

Here is what we are going to do in the next 30 seconds you are going to –after I finish talking- have 30 seconds to get your file so that each all visible so you are not zoomed in you are not zoomed out it is looking- everything is visible.

Then we are going to walk in a clockwise direction that is if you are looking down we are just going to walk quite slowly so that you can just have a brief look at other people's screens. Please don't touch anybody else's computers, don't use the mouse, and don't make any changes to them. We just have a brief look at what other people have done. You might see something where you think 'I will steal that'. You can then when you get back to your computer, you could then implement some of the things you have seen. So we go around slowly in a clockwise direction. When we are going along the computers nice and slow when you get back to your computer you can then continue. So 30 seconds to get it looking good and useful.

4.4.1.4 Curriculum Script

Chris's curriculum script for this topic considered students' previous knowledge about GeoGebra. Since the students were novice in GeoGebra use, their use of technology was graduated by using the teacher's prepared files in order for them to become familiar with doing mathematics using GeoGebra. Similarly, the difficulty level of the problems the teacher prepared for the students to work on was also graduated. His curriculum script had clear overall goals - a list of important concepts to be taught such as reflection, rotation, and enlargement with GeoGebra in order to complement the development of real understanding of transformations with classical tools such as paper, ruler mirror, and tracing paper that this class used when they were in Year 7. He pointed out that after the pupils' initial understanding of what reflection was with a mirror, and what rotation was with a tracing paper in Year 7, when they got to Year 9, "GeoGebra offers the opportunity to do things that would be very difficult without" such as "to make changes to a diagram very quickly... to create a diagram quickly... to make mistakes and click Control Z and undo very quickly, which on paper is very awkward and very difficult". In addition, he believed that GeoGebra brought new diversity to ideas to be developed and representations to be employed when teaching this topic to Year 9. Therefore, his intention was to give the students an extra level of insight into what symmetry was. In other words, Chris envisaged these GeoGebra lessons as building on previous work that had been carried out on the topic using classical tools: "Technology takes this on a stage from there. And the facts that we can drag and make changes as well add an extra dimension to it." [C/21]

As previously discussed, his specific choice of activity format for teaching this topic with technology consisted of students being more self-reliant by having ideas and trying these ideas out themselves through seeking validation from computers; the Predict-and-Test activity format [C/16, C/17].

They could have used compasses, rulers, mirrors and tracing paper. But GeoGebra allows them to access some ideas that they are just so much more powerful and important. It lets them properly explore rather than just learning rules. [C/22]

Furthermore, Chris's frequent use of Spot-and-show offered the students the opportunity to see different ways of solving problems. The use of such activity format also enabled Chris to undertake loosely ordered actions. In particular, at the beginning of Lesson 3 while he was passing by the students, he spotted an example that he thought would be useful to show it to the whole class in order to give them some ideas to start. This was not a planned activity on his curriculum script – see the related verbatim from the lesson below:

I want to show you first of all a couple of bits of homework now these I just happened to see as people opened their book when I was walking past. I had not intended to show you these; these are so good and so useful that I have just taken a photo of them now and I am showing you this from my phone. This is being a little bit spur of the moment. [C/23]

During his post-lesson reflections, Chris signalled how his script had been developing in response to using technology to teach this topic. In particular, with the use of GeoGebra, he realised the importance of having discussions about enlargement by scale factor 1 and scale factor 0, which he did not have before. In this regard, Chris recognised and responded to the ways in which GeoGebra helped specific objectives involve in learning this topic.

It never occurred to me give them a question where it says enlarge it by scale factor 1 because the idea in a textbook or on a worksheet enlarge it scale factor 1, that is just waste of space, it doesn't do anything, nothing changes. Whereas it was using GeoGebra made me realize that really is important. It is the identity. It is the multiplicative identity. I now make a big deal with my pupils about it. Beyond 0, lower than zero there is a rotation happening as well and then enlargement some kind. That followed on nicely. But the real breakthrough for me was scale factor 1. Scale factor 0 then fell into place quite quickly whereas the one that surprised me was scale factor 1. [C/24]

4.4.1.5 Time Economy

It emerged in Chris's comments that GeoGebra contributed the time efficiency in the lessons by providing the students with more learning time as has been noted in the Resource System section. Considering the Practitioner Model (Ruthven & Hennessy, 2002), these comments related using GeoGebra to the theme of 'Effecting working processes and improving production', which made a change in time economy. For instance, being able to use a slider for scale factors of enlargement enabled the students to see a number of different diagrams without drudgery of doing it by hand. Furthermore, the fact that the students can use all the files retrospectively made a difference in pacing of the lessons.

Today (Lesson 2) at the start of the lesson I had a choice about whether to let them finish off yesterday's exploration, whether to explain them the things I wanted them to find out yesterday. I decided not to do those things and I decided I would again make them self-reliant. I would give them questions and tell them they could go back and look at the other materials if they wanted to. And lots of them did. That was the decision I had to make was about the pacing. [C/25]

As an experienced teacher both in teaching and in GeoGebra use, Chris had developed a number of time saving strategies that allowed him to change the pace of some activities and to save time capital. First of all, he divided the lesson time into small parts, introduction time (whole class), activity time (student independent work at computers), collective knowledge time (whole class), in order to produce overall didactic time [C/12]. In the first lesson, the activity time was spent as a tool time during which students focused on learning the software. At the same time, he asked the students to focus on thinking about diagrams with the intention of also progressing their learning time. In order to use the tool time economically, he first did a Technical-demo to show students how to find the software and the topic-related tools in GeoGebra.

In addition, he provided the students with prepared files to work on, which helped him to have control over didactic time, since the files were designed by him according to the content that he wanted his pupils to learn. Furthermore, during independent student work at computers, he joined and contributed in their discussions in order to increase their learning time. It was because students could

have spent a large amount of time capital on less didactic time. Another strategy to support the timely progression of all students in the manner that he desired included giving intermediate syntheses. During work-and-walk-by, he spotted and showed some students' work that he thought were useful for whole-class discussion. This intended to guide the students through offering them other ideas to consider. Moreover, he made use of students' examples (see Figure 4.29) to start a lesson and explain what they were expected to do in that lesson.

Then at the very beginning of the lesson there were two girls showed me their homework, just as I walked past their table. It was so lovely they have done very different things. What they done was very impressive. I adapted my ideas for the lesson and took a photo of their work and then projected that to show everybody else. And I think that was very helpful because it showed quite high quality the things they do could be. [C/26]

As indicated by Chris, sharing students' work at the start of the lesson helped him to use his instructional time efficiently.

Again one of the difficult things about a lesson like this one was making sure pupils have time to be able to stop and think and they are not being pushed the whole time. There is time to stop and to think and to reflect and to put ideas together. Also there is not time being wasted and so I decided I would have a brief introduction and in fact having their pictures to show helped with that. [C/27]

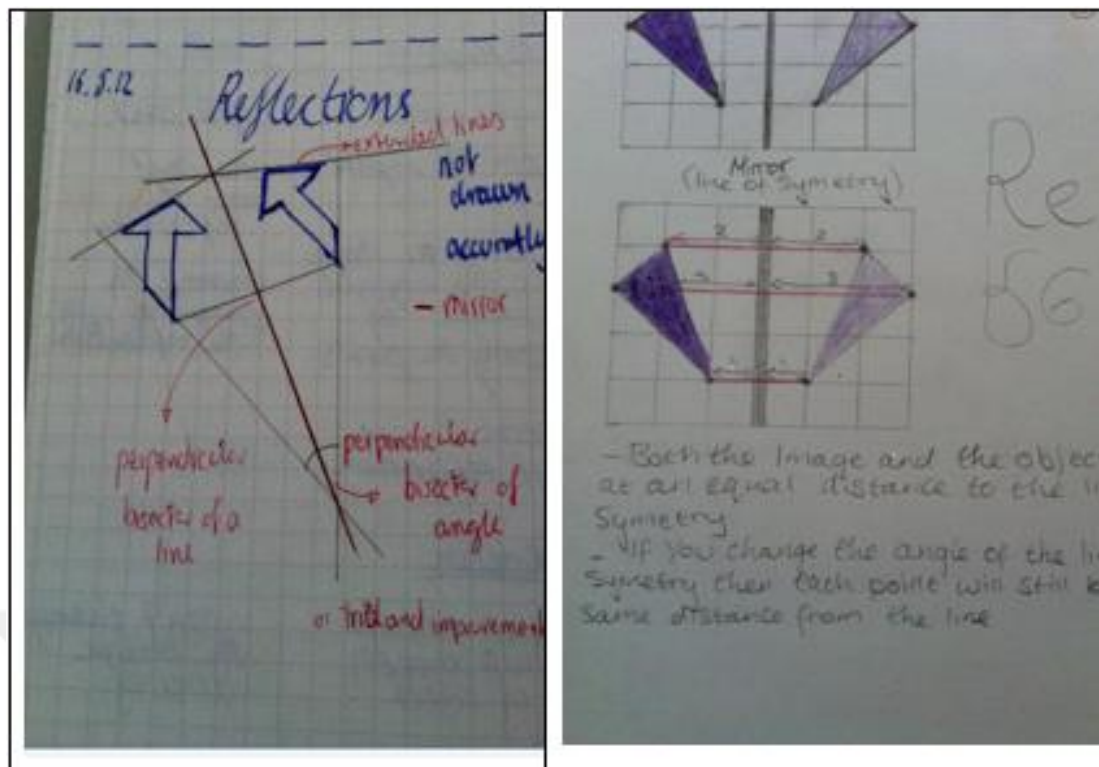


Figure 4.29 Projected examples in Lesson 3

Another strategy was that in Lesson 3 Chris required the students to prepare posters that covered the main points of the previous two lessons through freezing or referring to the dynamic diagrams of GeoGebra (see an example in Figure 4.30). This strategy (resembling a time saving strategy already noted in Assude, 2005) aimed to improve students' understanding of the concepts by synthesizing/condensing information that had learned during the previous two lessons with the intention of maximising their learning time.

I thought very hard about the lesson in advance because we have spent two lessons before now working on different aspects and for them to explore different things. What I did not want to happen was for them just to find out more new things and just to go “ooh that is interesting, that is interesting” and my anxiety was that if it turned out to be something like that, the pupils would just keep seeing new interesting things but would not stop and put them altogether and would not then remember any of them in future. They would be noticing interesting things but not combining them with their previous knowledge and not really remembering things long term. [C/28]

Furthermore, the fact that the students focused on different aspects and they were asked to put them together enabled them to have a faster paced course as their learning time was moving forward without spending large amount of time capital.

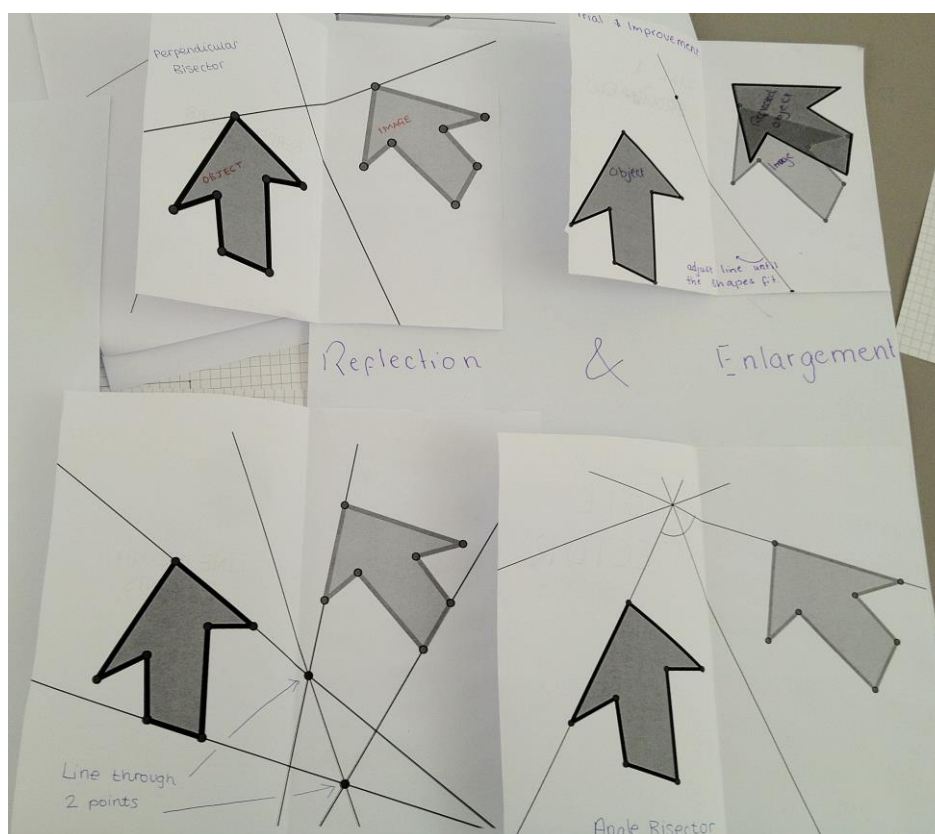


Figure 4.30 A photograph of a group of students' poster about different ways of finding a mirror line

4.4.2 Circle Theorems

After 2 years, Chris taught "Circle Theorems" to the same class, when they were Year 11. In this section, I will analyse his teaching of this topic over a series of six lessons consisting of two non-GeoGebra lessons and four GeoGebra lessons.

4.4.2.1 Working Environment

Chris's lessons occurred in a Computer Lab (see Figure 4.31), where the computers for students were arranged around the back and side walls in a U shape so that students working at them were facing away from the front of the room. There were also a number of tables/seats in the centre for students being taught as the entire class facing the front of the room. At the front of the room, there was a computer connected to a Smart-board for the teacher.

Students were seated in the centre of the room and worked at the tables during the first two non-GeoGebra lessons, whereas in GeoGebra lessons they moved between computers and their seats depending on whether the teacher required individual working or whole class teaching. In this case, the fusion of a traditional classroom layout with a modern computer classroom layout allowed for a more involved and interactive lesson, while still enabling Chris to remain the central focus of the lesson when necessary. This flexibility made the teacher more positive about using this computer room as a teaching space as he pointed out during a post-lesson interview.

I liked the layout of that room because it is clear that it is a classroom and it has computers. So it is not like going to a computer room for a computer lesson. It is our classroom and you turn around and you use computers. [C/29]

Furthermore, this layout permitted him the most convenient method of monitoring students during GeoGebra lessons, which in turn offered flowing engagement between the teacher and the students.

Another thing useful about it is that I can see all of their screens. So if I stand in the middle and turn my head I can see everybody's screen which means that I know immediately if somebody isn't doing what they should. That is straightforward. [C/30]

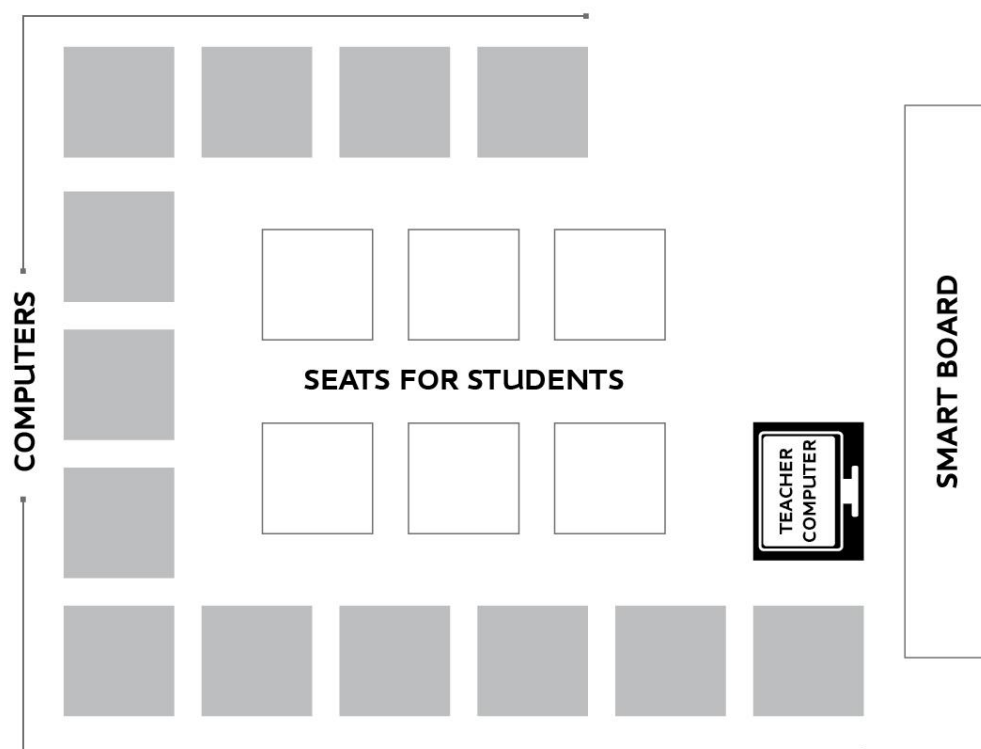


Figure 4.31 The room where Chris's 'Circle Theorems' took place

Another reason why he was happy with this room was the fact that it had controlling software that enabled Chris to manage students' screens. Thanks to this software, he could display a student's screen on the board for the whole class to see without the drudgery of saving the work and then loading it up. Displaying students' screens with less effort also allowed him to implement Spot-and-show activity format in a more straightforward way, which again introduced time economy in the lessons.

I also liked the fact that it has got the software that'll let me spotlight what pupils had done. And again in the other computer room children had to save their work, I then had to load it up. That made it very awkward whereas here I can just click and I can spotlight one pupil's work which is wonderful. [C/31]

Moreover, with this control software, he could blank everybody's screen and put the spotted example on their screen. This was advantageous in getting students to pay attention to whole class discussion when necessary without moving them to the tables in the centre of the classroom, which saved around two minutes of lesson time.

4.4.2.2 Resource System

The resource system consisted of a pencil-and-paper worksheet as well as dynamic files involving use of GeoGebra.

Chris started this topic with dotted circles on a paper-and-pencil worksheet (see Figure 4.32) as he argued that students had to learn to interpret the diagrams through building them up by themselves.

I liked to begin with dotted circles that partly because I think it helps with problem solving, it helps with self-reliance, and it helps with some of the skills that we want, things like putting an extra line. So when we got a triangle in a circle where we got a number of dots on the circumference joined up to make a triangle join them all to the centre of the circle that is actually a skill/ and idea that we need for some of the proofs later on. And if you are doing that already, that feels natural thing to do. [C/32]

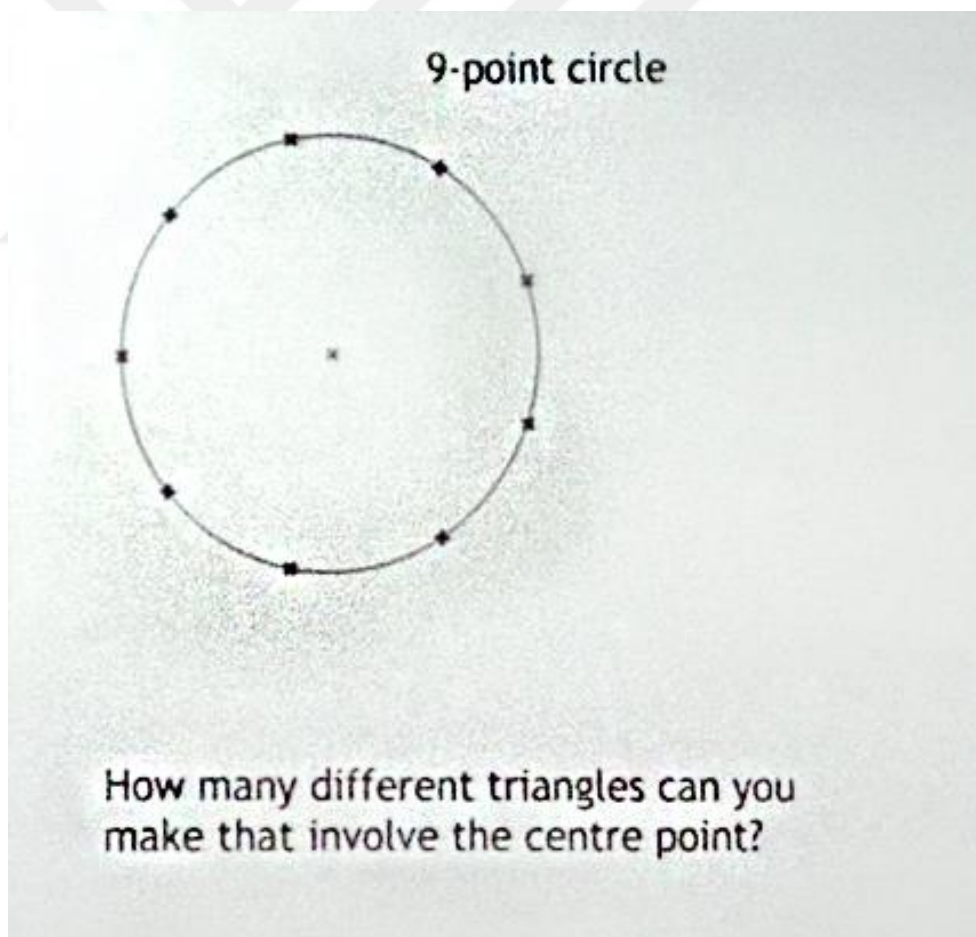


Figure 4.32 The paper worksheet for students to work on without GeoGebra

In addition, he placed emphasis on the fact that students first having a concrete understanding before the process of abstraction: “I think it is very important that we don't immediately jump to abstract representations and that we do allow pupils to create their own things in the medium that they are comfortable with.” [C/33]

After the non-technology lessons, students started working in GeoGebra with the aim of producing more examples, thus, making conjectures about the relationships between angles in a circle.

Last couple of lessons we have been doing this on paper. We have been drawing triangles. We started with triangles that always involve centre of the circle because they have equally spaced points so we can calculate all of the angles. We did some, it did not have centre points involved. It is not equilateral. We use GeoGebra to gather more information so they can start making conjectures alongside learning the circle theorems stuff. We are also learning about conjecturing, proof, and evidence. That is the idea. [C/34]

The teacher was clear in the purpose behind using GeoGebra for this topic and as to the support it can offer students (see Table 4.23 below for related interview quotes from the teacher). He pointed out that the dynamic element of GeoGebra was particularly useful for students since it enabled them to change the vertices of a diagram, accordingly, see what was happening on the diagram. In addition, the accuracy, and speed of GeoGebra would facilitate measuring angles by reducing the drudgery of doing it by hand. Furthermore, he valued that the fact that you could undo what you had done in GeoGebra rendered it easier to make mistakes.

Table 4.23 Chris’s interview quotes related to the affordances of GeoGebra for supporting learning
<u>See what happens to a figure when points are moved</u> It will let us do the dynamic part of it. It will let us move points and see what happens. [C/35] <u>Produce measurements more efficiently and from multiple figures</u> It will let us do lots of measuring without needing to do measuring or calculating so it will measure without us needing to do those by hand. That is a huge time saver. That saves a lot of time over drawing them out. [C/36]

Correct mistakes more easily, and so feel freer to take risks

Beyond that I think GeoGebra also allows us to be freer with things. If you drag something and it does not work, you can do ctrl z for undo and it is back. On a piece paper if you got a diagram and you put an extra line on it and it is the wrong line you are going to try to rub that out or start again. That takes a lot of time. I think it is easier to make mistakes. [C/37]

Two third of the students in Chris's class were already familiar with the software because, as has been mentioned before, this was the same class that he taught 'Transformations' lessons two years ago. Nevertheless, Chris was aware of the difficulties that might arise with students' use of GeoGebra for this specific topic. Hence, he allocated a tool time at the beginning of the first lesson during which students had hands-on activity with the software to figure out specific tools for angle properties, in particular, how to measure angles in GeoGebra. This was because, over the years, he had noticed that students often "measure" without clicking in the anti-clockwise order, which is about the technico-mathematical convention embedded in GeoGebra. During this tool time, he monitored students and provided feedback when needed.

For example, measuring angles, you can click on the three points and depending on which order you click. If you click one two three, then it will measure the interior angle (he shows on the paper). If you click them the other way round it measures the reverse angle. That was just a matter of try clicking in a different order. [C/38]

In addition, he specifically reminded students of the fact that they could undo an operation that had not produced the desired effect by using Control+Z, and also how to measure angles of two intersected segments.

The only two things I mentioned, one was Control Z to undo. And that feels important to know that works. And you can do that. The other one was just to intersection of two segments. If you have got two intersected segments you can't directly measure that angle but if you put the point of intersection in, then you can measure that angle. It seemed worth saying that because otherwise that is not a common one. I don't think that is something we used before. But it would seem natural that if you got two segments that crossed that you ought to be able to measure that angle. So yeah it seemed important to say that. But those were the only bits. [C/39]

Chris used open-ended tasks, which aimed to encourage students using GeoGebra as tools or representations to help their thinking. With this particular class, which consisted of a high attaining group of students, he pointed out that they had established this way of working over the years. The purpose was to show the students that there were many paths to reach the same mathematical solution. In particular, he regarded “Circle Theorems” as a vehicle for developing ideas about mathematical proof.

I could have taught this differently and asked pupils to sit down. I could have told them the rules and asked them to answer some questions. I wanted them to experience proof because I think it is important particularly for these children, who are considering A level maths, to know about proof. I did not want to do separately so they were not doing other parts of the curriculum as well. The other bit is the multiple perspectives. We had two or three different ways of proving the same thing. To make those links feels to me is really important.
[C/40]

4.4.2.3 *Activity Structure*

Chris introduced his lessons in a whole class segment followed by paired or group student work at computers with him circulating to provide guidance and support. At the end, he gathered the students to their seats in the middle of the room for whole class teaching in order to discuss various ideas that students came up with during the lesson. Table 4.24 shows the kinds of activity formats and the lesson agenda that were used by Chris during these four lessons.

Table 4.24 Activity formats and lesson agenda followed by Chris in ‘Circle Theorems’ lessons

Lesson	Activity Formats	Lesson agenda
<u>Lesson 1</u>	Use-tech/Spot-and-show/Discuss-the-screen	Chris showed some pictures from students’ previous work, which they did on paper using equally spaced circles in order to precede and prepare for a phase of individual student work at computers. He asked students to spot some links between some angles and make some conjectures with the use of GeoGebra.
	Use-tech/Work-and-walk-by/Discuss-the-screen	He let students use the computers themselves so that they could explore some relationships between angles through using a 9-point circle on GeoGebra. During this, Chris moved around the classroom in order to check what students were doing.
	Use-tech/Spot-and-show/Discuss-the-screen	Chris spotted different examples while students were working at computers. Then he stopped the whole class to show those examples since he thought it would give them some ideas of what to do.
	Use-tech/Work-and-walk-by/Discuss-the-screen	After showing students different examples, he asked students to look for a conjecture in the last five minutes and circulated amongst students to provide feedback. He finished by reminding the class of closing routines.
	Use-tech/Technical-demo/Observe-demo	He showed the whole class several slides that got some circles on. “I marked the centre of circles. I deliberately haven’t put dots around the edge because what we are looking for now things are true for all of the circles you try.” Then he asked them to explore a conjecture at computers and draw their conjectures on those slides with a sentence of what was going on next to the shape.

<u>Lesson 2</u>	Use-tech/Work-and-walk-by	He monitored and talked to students while they were exploring their conjectures and writing them on the board.
	Use-tech/Explain the Screen	After students wrote their conjectures on the board, Chris stopped the whole class and explained written conjectures to students and asked them to choose one and prove it whether it was right or wrong.
	Use-tech/Work-and-walk-by/Predict-and-test	He again circulated in the classroom and talked to students while they were exploring ways to prove their chosen conjecture.
	Use-tech/ Sherpa-at-work/ Discuss-the-screen	He spotted a student's example during work-and-walk-by and stopped the whole class to show them. He asked the student to move the diagram and he explained/discussed what was going on. So the student was in charge of her diagram, and she dragged points on the diagram according to the teacher's commands. "We can see when it works and we can see when it does not work. Could you please now try and do something similar with whatever conjecture you are working on."
	Use-tech/Work-and-walk-by/ Predict-and-test	Chris let students continue with their exploration considering the techniques that they discussed during the spotted example.
<u>Lesson 3</u>	Use-tech/Explain-the-screen	He introduced the notion of proof to the whole class and explained the previously discussed students' examples to show the difference between proving a conjecture and gathering lots of evidence by dragging a point in GeoGebra. "Pick one of them, computer if you want to but we are trying to find either more evidence or an explanation or we are looking at special cases."

	Use-tech/Work-and-walk-by/Predict-and-test	He walked around to monitor students' progress while students were trying to gather evidence for their conjecture.
	Use-tech/Spot-and-show/Discuss-the-screen	He spotted two students' examples, which got two different ways of proving conjectures, first one included algebra, and the second one did not have algebra. He stopped the whole class and showed them these two examples.
<u>Lesson 4</u>	Use-tech/Spot-and-show	He again showed the two examples from yesterday to remind students of different ways of proving and asked students to come up with proofs for their chosen conjectures.
	Use-tech/Work-and-walk-by/Predict-and-test	While students were working on proving their conjectures, Chris circulated in the classroom and provided feedback.
	Use-tech/Spot-and-show/Discuss-the-screen	He finished the lesson by showing three examples that he spotted during students work. He particularly chose different ways of proving a conjecture and asked students to make links between these examples and their way.

Most of the activity structures showed similarities with the ones that Chris used in the 'Transformations' lessons. Here, I give a more detailed discussion on Sherpa-at-work, which was different to the one used during the 'Circle Theorems' lessons.

Sherpa-at-work

Chris made use of Sherpa-at-work (to employ the term found in the literature (Trouche, 2004)). Using a similar process to Spot-and-show, he spotted an example while circulating in the classroom during students work at computers, and projected this example on the IWB for whole class discussion. However, its difference from the Spot-and-show format was the fact that Chris let the student, whose example he spotted, run the example from her computer (see Figure 4.33). In other words, the Sherpa-student was the owner of the spotted example and she was in charge of the technology. Concurrently, Chris was explaining the screen and questioning the whole class (see the related lesson transcript below in Table 4.25).

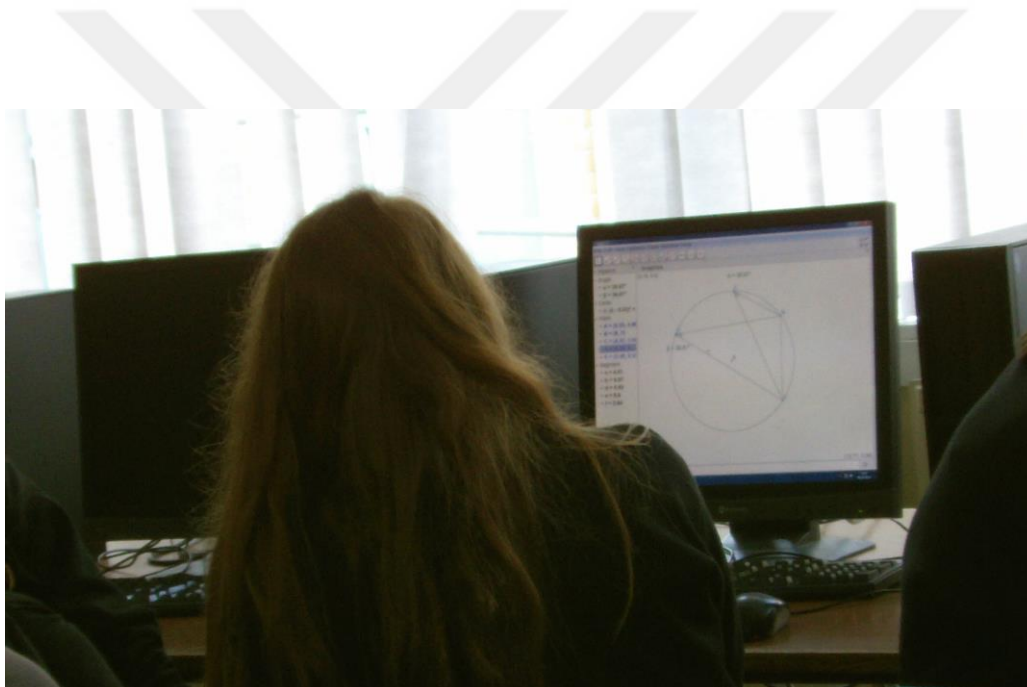


Figure 4.33 A photograph of sherpa student operating the diagram from her computer

Table 4.25 Verbatim transcript for Sherpa-at-work activity format

(The teacher is attracting students' attention to the board for whole class discussion)

Ladies and gents if are you ready please. Everybody is looking please and ready. I remind you earlier on I said that we needed to make some progress, get to the next stage. There are some people did brilliantly. On your screen and on mine, you can look at here or at yours. This one is absolutely brilliant. It is very very important and useful.

(The teacher is explaining Figure 4.34)

We have got a chord. The important chord is chord BC. We have got two sets of line coming of chord BC. Emma has measured the angles 36.87 degrees from both of them.

(The teacher is asking for specific actions to Sherpa student)

Could you do what you have showed me Emma?

(The teacher is asking questions to the whole class whilst the Sherpa student is moving the diagram)

As she moves that around what is going on could somebody explain what is going on there?

The angle is the same wherever it goes. One of the big ideas we need. That angle is always the same. Did anyone find a situation where is not always the same.

(The teacher is asking for specific actions to Sherpa student)

Now Emma could you move the point show us where it is not the same? There is couple of issues there, it measuring the reflex angle, next to that? Obtuse angle, I can gesture at you. This one here the inside version of this. Is that one in there?

Emma has found where it does not work. Could you take it back where it does work? Now when does not it show 36.87 degrees?

(The teacher is asking questions to the whole class whilst the Sherpa student is moving the diagram)

When did Emma break it? Rosa? If Emma goes past point C, Rosa reckons it breaks it.

When is it going to snap back?

(The teacher is asking for specific actions to Sherpa student)

Could you keep on going anti clockwise?

We are back. Well done Rosa. If you just drop it there. I am going to ask Emma to make one more change. Does it have to be all the time? If I want a bigger angle. What Emma makes that means than bigger angle? What change could she make so I have a bigger angle there? Move point b or point c further apart.

Could you drag d again and see what happens? It snaps into something different.

(The teacher is explaining why this spotted example is a good example)

Now then ladies and gents. Second reason Emma come up with brilliantly useful stuff. We can see when it works and we can see when it does not work.

(The teacher is reminding the other students what they should look for)

Could you please now try and do something similar with whatever conjecture you are working on?

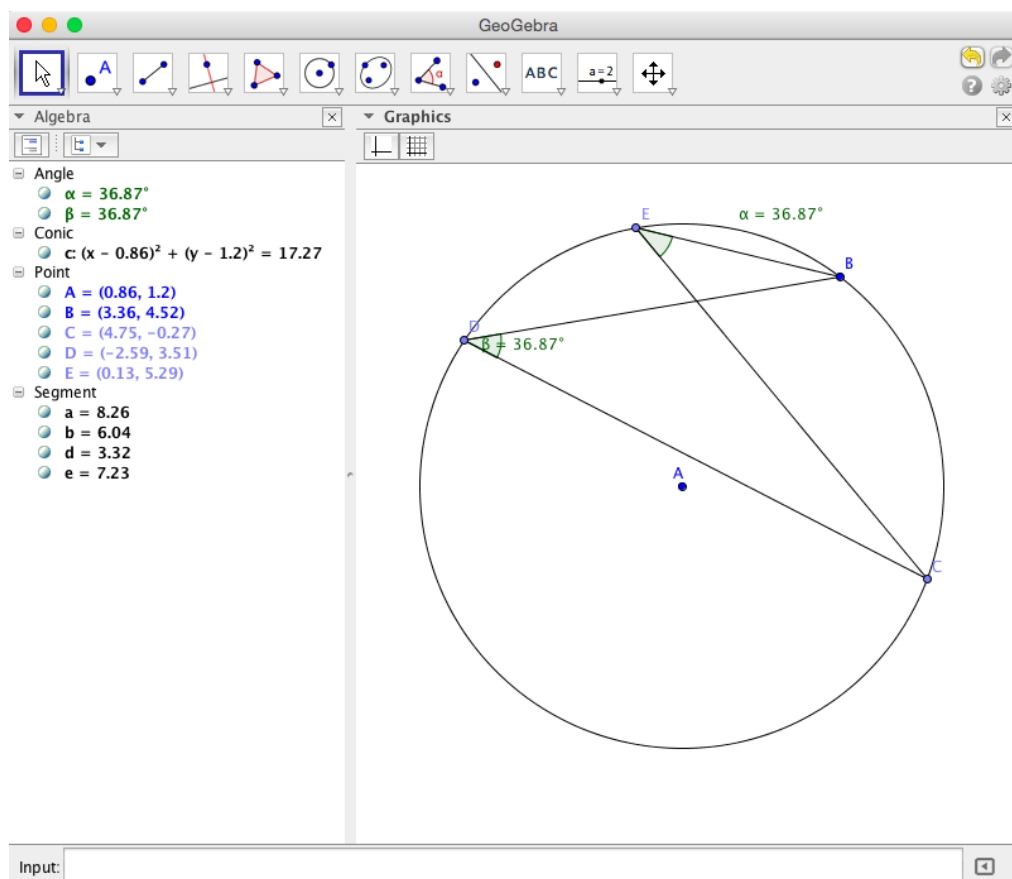


Figure 4.34 Spotted example for Sherpa-at-work activity format

During this activity, the sherpa student sat at her computer and was in control of using the technology for her example. In the meantime, the teacher was at the front running the explanation for the projected example and asking the Sherpa-student to carry out specific actions in GeoGebra, with all students able to follow the actions of both the Sherpa-student and Chris easily. While involving the Sherpa-student in the action, he also asked questions to the whole class about the Sherpa-student's diagram

It was lovely that Emma was able to move the point. And so it was very interesting that she was moving the point and she was doing the demonstrating but I was pushing on what was going on. I was making sure that I was emphasising the things that I found very important. [C/41]

He argued that this specific activity format was distinctive to technology use.

That was a really nice way of doing that because she (Sherpa-student) was able to be absolutely fully involved. I was asking her to do particular things I knew they would be helpful. We could then talk about as a class. Yeah that was particularly exciting and that of course can't happen without the technology. [C/42]

This Sherpa-at-work example described above was also an example of Predict-and-test since it showed how a student tested their predicted conjecture by using the dragging function of GeoGebra through which she found out when this particular conjecture worked and when it did not work.

4.4.2.4 Curriculum Script

Examining the sequences of tasks used in Chris's lessons (see Table 4.24) provided a starting point for looking at how technology featured in Chris's curriculum script, and how his script was developing in response to using technology. Chris's curriculum script already included a number of tool techniques such as measuring angles, Control+Z to undo, and intersection of two segments. At the beginning of the first lesson, he thought that those were important for students to know for angle properties. In addition, his curriculum script for teaching Circle Theorems had a number of concepts to be taught such as Conjecture, Evidence, Investigation and Proof. Although these goals were not dependent on use of any particular technology, GeoGebra became an important tool providing students with effective production and a great deal of representations to explore these concepts. In particular, GeoGebra enabled students to collect evidence about each Circle Theorem as a conjecture and try them out.

I also wanted them to be able to see things in a dynamic way, which pencil and paper can't do. I wanted them to see and to do calculations where they haven't got nice equally spaced number of points. I wanted them to do those sorts of things and to gather lots and lots of evidence. GeoGebra lends itself to doing that as well. [C/43]

While moving through the sequence provided by the curriculum script, Chris continually monitored students' performance to see whether the target concepts or understandings were being developed, as they were needed for a particular point in the lesson. Again, as previously discussed, he was also spotting students' work to share with the whole class. In this sense, Sherpa-at-work and Spot-and-show activity formats were specific adaptations to exploit potential of GeoGebra and aimed at sharing and discussing different examples from students, which also allowed the other students to see different representations of the same problem. In particular, he mostly chose the examples that most of the students had not worked on (see a verbatim from Lesson 1).

Now on my screen and on yours, you can see James and Matt's work. One of the things they have done here has been to draw some quadrilaterals. Rest of us has been looking at triangles. I can see on there a quadrilateral and I can see some angles and there is a nice relationship between some of the angles. [C/44]

Again, the example that he spotted for Sherpa-at-work activity was the one that not many students had worked on.

It was also nice to be able to choose one was different from the one that most people looked at. Most people explored the angle at the centre and the angle at the circumference. And it was nice to pick one that was different from that. So that was another reason why I liked her version because it was the one that some of them had not explored, had not spent time on. So it was going to be new for them as well. [C/45]

During a post-lesson interview, he mentioned how his appreciation about teaching this topic had changed over the years. Considering his previous experiences, it became clear to him that using prepared files (see an example in Figure 4.35) for each Circle Theorem with step-by-step instructions did not succeed in making students fully engage in the activity and interact with diagrams.

I have taught it in the past using GeoGebra where with one class I had drawn lots of diagrams that already exemplified a particular theorem and I asked the pupils to interact with the diagram to see what was going on. But I found that much less powerful because to start with there were lots of instructions. They were treating it as a bit like what shape is this, or what colour is that? Or they were giving a one-word answer that was forgotten immediately because it was not important and then move to the next one. What I have planned to be a week of work, they did in about ten minutes because they went through every single thing and then said I have finished. [C/46]

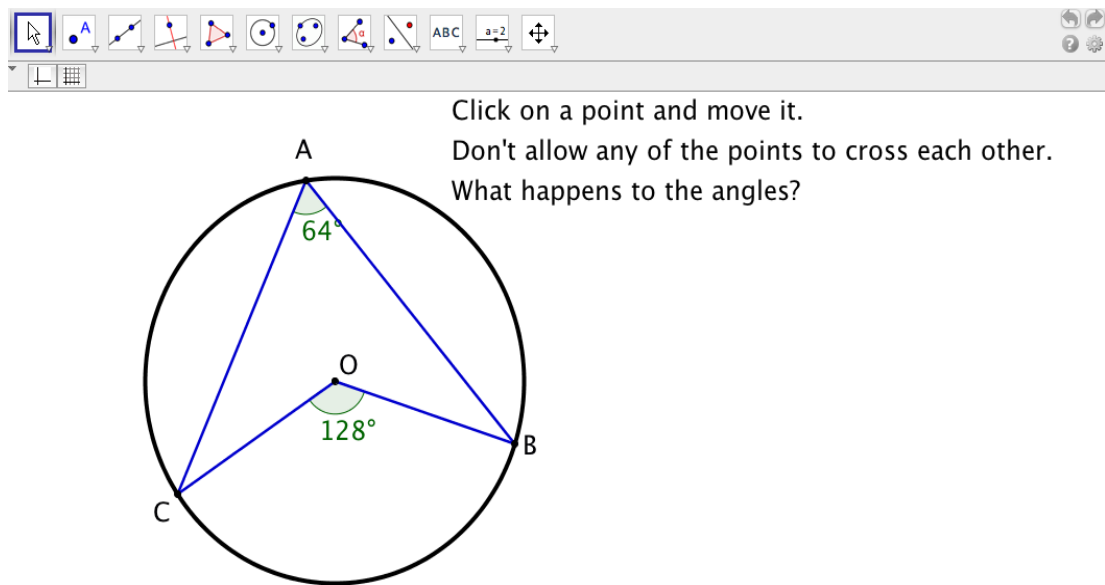


Figure 4.35 A structured dynamic file that were used by Chris in his previous ‘Circle Theorems’ lessons

Moreover, he noticed that students were generally having difficulties in making links between the dynamic nature of GeoGebra diagrams and static diagrams on paper. For these reasons, he required students first to start with paper-and-pencil activities to build up a concrete understanding of angle properties of circle, after that, continue with GeoGebra work to have numerous examples to interact with. In this connection, from his previous experience, Chris anticipated the crucial features or dimensions of what was important or difficult about learning this topic with GeoGebra and revised his curriculum script accordingly.

The other problem was that I hadn't appreciated that pupils would have some difficulty in making links between using the technology and the dynamic diagrams and the static diagrams they ought to see about having single angle dragging it and showing that the angle is always the same. And I expected that they would then see a static diagram with two angles on has been the same as each other because we could drag this mentally but they didn't see that. And some of the things I did with them in this sequence of lessons were to make sure we have already covered those things. And that is one of the reasons I now start on paper. We start building from ground up rather than me giving them diagrams to interact with. [C/47]

Similarly, Chris's comments about a file with the rounding problem indicated the logic of developing the topic with a class. When he talked about his previous use of GeoGebra to teach Circle Theorems, he indicated that his reaction to the ways of working with files had changed over the years.

One particular example is with a file showing the angle at the centre and the angle at the circumference. I had a problem with it where it will show something like 51 and 103. And I decided this was very bad because the students might think that actually it is not double. After that it was about a year later I suddenly realized this was really useful because why it does not appear to work because of the rounding errors. I then used this file I deliberately made it there was a problem like this. I asked the pupils what has happened. And they said things like well maybe this one 51, 3 if you double that 102, 6 and it was wonderful because the class told me (just a computer with the projector...) maybe is that. I then set GeoGebra one decimal place and it showed one of these. They said ah we solved the problem. If I set it in one decimal place it will be okay, there would not be a problem. I should have done, should not I? Now when I drag this point it always works. [C/48]

4.4.2.5 Time Economy

Chris commented that GeoGebra, in particular the angle-measuring tool and Undo option, enabled students to progress faster compared to measuring/calculating angles by hand.

It will let us do lots of measuring without needing to do measuring or calculating so it will measure without us needing to do those by hand. That is a huge time saver. That saves a lot of time over drawing them out. [C/49]

In addition, IWB use made whole class teaching easier in the sense it allowed him to demonstrate a number of tool techniques and explain/discuss the diagrams with the use of GeoGebra. This, in return, helped him to have an efficient instructional time. Furthermore, IWB use enabled him to display all the conjectures that students had discovered. This was particularly useful in terms of space issue, and IWB permitted him to save those conjectures for the next lessons that again was efficient working process in terms of time.

There is also an issue of space. We got five different conjectures and could scroll up and down whereas on an ordinary board you couldn't do that. But I have saved those and I now got those and get back to them tomorrow and that

is useful as well. If they are written on an ordinary board, we have to re write them again tomorrow. [C/50]

As he was carrying out an investigative lesson where students were making conjectures, and then collecting evidence to prove those conjectures, he allowed students to have enough activity time in order for them to think about these concepts.

I think if I was being pushy they would not have seen the things some of the things they did. And I think it was not the pace was slow, there was wasted time and I was tolerating that. It was that in order to make sure that people had time to think about it I had to be careful not to appear in a hurry. I wanted them to be focused. But I did not want to hurry them as I was standing next to them, as I was talking to them. I wanted them to have the opportunity to explore properly what was going on. They had times where they were set off the work, they were struggling 20 min what they were doing and trying things out. So I think you could say that pace was slower but things were still working hard. [C/51]

Time saving strategies, which were used during these lessons, did not differ much from the ones in the ‘Transformations’ lessons. For instance, at the beginning of each lesson, he spent some introduction time on instructing students about their task or showing students work from previous lessons as an opening synthesis in order for students to use their activity time productively. The use of Spot-and-show also helped him to use the instructional time efficiently. Furthermore, this format was used in order to give intermediate syntheses during paired or group student work at computers, which again intended to offer students some ideas in case their learning time had not moved forward.

Moreover, the kinds of resouces in the working environment allowed him to use his time capital in a more productive way. In particular, Spot-and-show was implemented in a quick and straightforward way for ‘Circle Theorems’ lessons with the use of control software in the classroom, which enabled him to get students’ attention up to the board/and or on their screens when needed. This contributed to time economy, since first, students did not need to move to the middle tables, which could cause a lost of time around two minutes. Second, they did not get distracted by the work on their screen as the teacher was able to display the spotted example on their screen as well with this software.

What is really lovely is that with the software. I can get to put one person's screen on everybody else's computer. So they have to focus because their computers were not working any more. But also what is lovely is that rather than having to look at the board they can look at the screen just in front of them. [C/52]

4.4.3 Summary

In terms of the five constructs of the SFCP, Chris's teaching with GeoGebra did not show much difference according to the particular topic taught (Transformations and Circle Theorems), in the sense that the ideas and patterns in his teaching were consistent except a few aspects.

One of the differences was the classroom layouts that Chris taught those two topics in, which had an effect on associated craft knowledge related to incorporation of GeoGebra. U-shaped layout of the room (with additional control software), where Circle Theorems lessons were carried out, enabled him to manage straightforward transition between segments during independent student work at computers. In particular, when he wanted to make important points to the whole class, he used the control software for securing students' attention without movement of the students due to U-shaped layout. Additionally, such software allowed an easy procedure capturing or converting students' screens for whole class discussion [C/31]. Another important difference was about establishing a new structure of interaction involving students, teacher, and GeoGebra. Differently, during Circle Theorems lessons, Chris made use of the Sherpa-at-work activity structure in which the involvement of the computer uniquely changed the nature of the interaction taking place [C/41, C/42].

In terms of the tasks to be undertaken, he indicated that they were affected by establishing patterns for technology lessons with the class rather than by the topic he taught. With this particular class, he made a comparison between teaching 'Transformations' lessons when they were Year 9s and teaching 'Circle Theorems' lessons when they were Year 11s:

There are different ways working with GeoGebra. Certainly when you came to observe me two years ago we were working on smaller problems. In most cases it was a single lesson working on a single problem. Whereas here it was, I structured mentally for the students what were going to do. And so each

lesson has its own shape. But we were still working on the same thing for all lessons. 2 years ago the class was new to me and there it was more of a case of here is the problem let's work on that for a lesson rather than for a longer period time. [C/53]

4.5 Conclusion

This chapter analysed the three teachers' use of GeoGebra in their teaching in accordance with the SFCP model and compared their lessons structured within teacher. The evidence presented has shown that these teachers' practices of GeoGebra use did not differ across topics, whereas their level of experience in teaching with GeoGebra suggested some variations in their teaching. Hence, a cross-case analysis will be conducted in the next chapter to discuss similarities/differences between the three teachers' teaching in regard to their use of GeoGebra.



5

CROSS-CASE ANALYSIS

5.1 Introduction

The previous chapter examined each of the individual teachers and their individual lessons grounded in the teacher accounts and teacher actions organised in terms of the SFCP constructs. It also provided comparisons across different topics taught within teacher. The conclusion that has been drawn was that the way each teacher operated appeared consistent across the topics. However, the stages they were at in their learning to teach with GeoGebra indicated differences. In this sense, it is of crucial importance to examine carefully the situations where the teachers are teaching the same topic in order to sharpen up comparisons between teachers. I, therefore, conduct a further analysis of teachers' use of GeoGebra for the same topic and discuss similarities/differences between their teaching in this chapter.

I will first look at the topic, which all three of them taught- Transformations, in order to get my base line model of what different/similar aspects appear between each of these teachers. I will take a systematic approach. Such approach consists of looking into emergent axial categories considering the three-teacher comparison (Transformations) and then doing a systematic traversal for the other two topics considering two-teacher comparison based on these axial categories. By systematic traversal I mean that all of the axial categories will be discussed for each teacher's practices even if those categories did not emerge as a theme or were not considered as an issue.

In terms of the development of the axial categories, while carrying out the within case analysis, I also made notes about the connections I was noticing among the teachers in anticipation of the later cross case analysis. I then discussed these connections with my supervisor so as not to misinterpret and/or overlook the complexity of the data. Once I started conducting cross case analysis, I reread the within case analysis to gain a stronger understanding of what features might be distinctive to one case or common to two or more. In this light, I returned to the data, looking for patterns and themes, commonalities and inconsistencies between the cases. As teacher actions were organised in terms of the SFCP constructs in the within case analysis, the themes from cross case analysis cropped up as subheadings of these five overarching constructs and therefore themes emerged based on these groupings of the SFCP framework. I labeled subsets of data into commonalities/differences of teacher actions with a descriptive title or code. For instance, the codes “choice of tasks” and “prepared files involving use of GeoGebra” are considered as sub-themes of Resource System concept (see the below table 5.1 for the relationship of the axial categories with the SFCP constructs).

Table 5.1 Axial categories and the SFCP Framework	
Working Environment	Promoting different structures of interaction through the use of working environment
Resource System	Prepared files involving use of GeoGebra Choice of tasks Promoting independent learning through the use of software Making connections between computer tasks and paper-and-pencil tasks
Activity Structure	The pedagogical functioning of activity formats Emerging variants of the same activity formats

Curriculum Script	Questions during Work-and-walk-by Promoting whole-class interaction through the use of IWB
Time Economy	Contributing to the pace and productivity of lessons through the use of software Promoting activity time through managing the physical time

5.2 Transformations (Susan-Tom-Chris)

Promoting different structures of interaction through the use of working environment

All of the teachers had two main forms of arrangements in lessons: independent student work at computers and whole class teaching from the front that took place mostly during the initiation and summary phases of their lesson. How the teachers shifted from student work at computers to whole class teaching reflected particular organisations of their working environment.

In Susan's (laptops in a traditional classroom layout) and Tom's cases (a computer suite without a separate teaching space), organisation of switching arrangements depended on the teacher asking students to turn off their screen and look up to the board without any change of seating by students. Both teachers' comments indicated that use of such an arrangement (that students sit at computers all the time) created difficulties for the teacher in attracting and maintaining the attention of students for whole class teaching. For this reason, Tom suggested that it would be desirable to redesign his own classroom so that students did not face each other and computers were adjustable to create a flat desk. Likewise, in the wider literature, Drijvers et al. (2010) reported a teacher in their study chose to teach in a normal classroom rather than in a computer lab because she thought that the computer lab was not appropriate for whole class teaching.

On the other hand, Chris's class (working in a room designed to combine classroom with computer suite) moved between computers and their seats in the centre of the room depending on whether the teacher required student work at computers or whole class teaching. In other words, this movement was related to the type of activity structure sought at the time. In this respect, the fact that this type of computer room did not take away from his whole class activities was a crucial aspect that affected Chris's technology utilisation according to his pedagogical goals.

Prepared files involving use of GeoGebra

All teachers provided students with pre-designed dynamic files so that students could work on transformations concepts. In Susan's case, she borrowed those dynamic files (accompanied by a worksheet) from a GeoGebra website. According to Susan, the use of such worksheet "helps the teacher" by acting as a "crutch" especially for those new to learning to teach with GeoGebra.

On the basis of his experience, Tom found starting from a blank page "too much" for students as "there is quite a lot going on at the background of the program". He prepared dynamic files for students "to get them just see something in action, understand what the idea of translation or rotation is and to be able to then visualise their imagination."

Chris graded students' use of technology by providing prepared files for the first five lessons and then asking them to start from blank pages to create their own files with the guide of instructions for the last two lessons. This implied that Chris saw value in eventually having the students create their own files. As believed by Chris, working with prepared files to start with gave the students more time to focus on the mathematics and their thinking, thus, increased their learning time.

These case studies illustrated a relationship between the technology experience level of teachers and the use of ready-made resources. Susan as a novice technology-using teacher found it more appropriate to start with files and worksheets already prepared by someone else: this gave her more confidence and also alleviated the time pressures she felt. On the other hand, Tom, who has made more use of GeoGebra in teaching than Susan, created the dynamic files himself taking into account of his previous experience. Chris, who is the most experienced of the three in teaching with

GeoGebra, also managed students use of GeoGebra gradually by allowing them to gain confidence in working from scratch with GeoGebra. This development of activities appeared very similar to a study carried out by Laborde (2001). She followed four teachers for three years and tried to understand how novice teachers integrated technology into their teaching. Laborde found that teachers gradually granted control of technology and the learning environment to students.

Choice of tasks

As stated above, Tom and Chris designed the tasks for students themselves unlike Susan. The tasks designed by Chris were more open ended in the sense that allowed the students to discover different ways of solving a problem. Furthermore, Chris encouraged the students to take various directions and focused on students' thinking rather than the result itself, which in turn enabled him to create a whole class discussion of those different ideas. However, the tasks prepared by Tom were more structured in that there was not room for students to consider from multiple perspectives. In addition, he mostly relied on questions from a textbook as he pointed out: "That particular textbook is the best textbook I think I have ever used. It is very close to the current GCSE specifications." This implied that he wanted his students to practice with questions that could also prepare them for the exam environment.

Again, Laborde (2001) emphasised that teachers experienced the design of tasks to be employed with computer software as a challenging process (exemplified in her study by the Dynamic Geometry Software, Cabri). Both Laborde (2001) and Monaghan (2004) found that most teachers used and designed worksheets characterised as having elements of control or guidance, which for these teachers usually were in accordance with their use of tasks from textbooks in non-technology lessons. Thus, Susan's and Tom's choices to use files/questions supposed to guide and support their teaching of the lessons and individual student work in GeoGebra, are not perplexing. However, Chris as an experienced teacher and technology user, moved beyond that and came up with his own resources, which were not inspired by paper-and-pencil activities. This suggested that he came to the point that those tasks that replicated textbooks were not actually learning productive.

Promoting independent learning through the use of software

The tasks to be utilised with GeoGebra for Transformations lessons attempted to stimulate students' use of a trial and refinement process. All the teachers pointed to the ways in which use of the software could facilitate the process of self-testing and self-correcting. Hence, the main new aspect that these case studies provided evidence for was that activity formats depended more on the fact that the students made a prediction and the technology provided the feedback rather than the teacher or other students validating – or invalidating – it. As Chris stated: “if we would have done this in the classroom, I or somebody else needed to say ‘that is wrong’”. That shifted the role of the teacher towards becoming an organizer/observer of this process. In addition, this activity format capitalising on the availability of interactive and immediate feedback appeared to be conducive to technology use, depending crucially on its use. Tom noted that such a format is “easier to include with technology”

In this connection, Predict-and-test was part of the logic of teachers' lesson agenda for handling this topic. In Tom's and Susan's cases, choice of the tasks with the use of GeoGebra aimed for pupils to check the results of Transformations already done 'by hand'. However, as has already been discussed, Chris provided students with more open-ended tasks that allowed students to have different ideas and encouraged them to test their ideas at computers. The aim was to promote students' skills “in terms of their development as a mathematician”.

Making connections between computer tasks and paper-and-pencil tasks

Tom provided a dynamic task as well as a paper-and-pencil activity for students to work on particularly finding the centre of rotation as he thought, “the centre of rotation can be quite hard to find”. In addition, he noted that linking screen with paper would be useful for students, since “they have to do it on paper anyway for test and exam”. In particular, using GeoGebra in parallel with paper-and-pencil tasks would develop their dynamic thinking, so then “when they see an exam question they could still see the thing twisting around the point in their mind, shape will rotate as well as going around in the circular.”

In Chris's case, although he did not provide a task, which specifically linked GeoGebra with conventional resources, there were some elements of linking screen to other representational media (mirror, tracing paper) through appealing to students' recollections of using them in earlier years. Notably during Work-and-walk-by, he helped students to make connections between computer and conventional resources: "Alex, that means if you did that here with tracing paper, you traced it and then rotated the tracing paper. What is going on with those points that are rotated?"

Susan did not use a task, which promoted making connections between computer tasks, and paper-and-pencil tasks. However, in the post lesson interview she mentioned the importance of providing tracing papers in addition to computer tasks for students to make links. In this respect, all the teachers were aware of the fact that making connections between computer tasks and paper-and-pencil tasks were of crucial importance for students.

The Pedagogical Functioning of Activity Formats

All the teachers adopted a three-part lesson structure: introductory whole-class sessions with the teacher introducing the software/the task, extended periods of student independent work at computers with the teacher circulating around the classroom to monitor and provide support, and summary whole-class sessions to review and/or elaborate the lesson.

There were differences in the ways in which these teachers implemented similar activity formats. For instance, all of them extended their practices by including a technical-demo aiming to familiarise students with core techniques of the software for Transformations. However, in comparison to Susan and Tom, Chris, even when he wanted students to learn about the software, made an explicit reference to mathematics by providing them with a dynamic file, where they could also discover "things like they can move the object but not the image". He required students not only to learn the software but also "to look for interesting things" in the diagram and "explore the mathematics". As Brown et al. (2009) distinguished between four modes of technology integration, this was regarded as instrumental exploration. Although students were not familiar with the use of a particular software package, Chris wanted them to increase their mathematical knowledge while

exploring the technological tool. That is, students “explore the technology through mathematical tasks” (Brown et al., 2009, p. 5).

On the contrary, Susan’s introductions were typically related to work with the worksheets and mostly focused on technical aspects of the software, which was regarded as instrumental initiation (Brown et al., 2009). As her students were novices in using GeoGebra, Susan’s main objective was to introduce them to the software and taught them the basic skills needed. Thereby, tasks focused on the technology, and the level of instrumental integration of the tool into learning of mathematical content was minimal. Similarly, in the wider literature, Erfjord (2011) investigated teachers’ initial use of Cabri (Dynamic Geometry Software) in Norway. Two teachers in his study used Cabri with prepared instructional material focusing on the technical aspects of using the software, which had little explicit focus on the mathematics but mostly demonstrated students what they needed to do technically.

In Tom’s case, he introduced the students to some new techniques with the software that they needed for learning Transformations. In addition, he gave them some time to practice those techniques with the focus on mathematics. However, in comparison with Chris, the tasks designed by Tom were close-ended and mostly from a textbook as has been mentioned before.

Furthermore, Chris and Susan employed the Spot-and-show activity format, where they spotted a student work and shared it with the whole class. Chris used this activity design to discuss different mathematical ideas from students’ work that he had spotted during Work-and-walk-by, whereas Susan’s use of Spot-and-show concentrated on technical aspects of using the VLE. In general, she tended to focus on advancing students’ learning of tool techniques. Tom did not make use of this activity format as the closed nature of his tasks meant that all the students had effectively followed the same path. This activity format, then, is possibly better fitted to open-ended tasks, where students can come up with numerous ideas, which can be shown to and discussed with the others.

Emerging variants of the same activity formats

As has been mentioned above, all the teachers used Technical-demo at the beginning of their lessons to show a number of GeoGebra techniques to the students. In Tom's lessons, he had two variants of 'Technical-demo' in that the teacher action slot was the same, but the student action slot was different: Technical-demo/Observe-demo (while the teacher was demonstrating, students were first watching and then implementing), Technical-demo/Replicate-demo (while the teacher was demonstrating with instructions, students were implementing simultaneously).

Both Susan and Chris utilised only Technical-demo/Observe-demo.

Questions during Work-and-walk-by

All the teachers circulated around the classroom while students were working at computers as they believed the importance of observing and guiding students closely particularly during technology based lessons. As has been already discussed in the Literature Review chapter, Drijvers et al. (2013) focused on the templates for individual interaction between teacher and students during Work-and-walk-by and identified five different interaction types including Technical-demo, Guide-and-explain, Link-screen-paper, Discuss-the-screen, and Technical-support. In Susan's case, most of the questions that she dealt with were related to technical aspects of the software such as how to do a reflection, how to download tasks from the VLE since the students were learning to use the tool. Therefore, she utilised Technical-demo and Technical-support formats based on Drijvers' et al. (2013) taxonomy. However, Chris aimed also to make sure "the pupils could include the interesting things they could think of and interesting ideas they had". In particular, his guidance was mostly related to mathematics with less emphasis on technical aspect of the software. Hence, his individual templates for interaction during Work-and-walk-by mostly centralised Discuss-the-screen. In Tom's case, he also aimed for his students to focus on mathematical concepts by using the visualisation affordance of GeoGebra. Most of the questions he handled were related to mathematics except some minor technical questions but the individual interaction type between him and students during Work-and-walk-by was mostly related to Guide-and-explain.

The evidence presented has shown that the main difference between Chris and the other two teachers was related to the design of their tasks. Both Tom and Susan primarily showed interest in students finishing the tasks whereas Chris was more interested in students' multiple perspectives in solving the problems. Hence, his focus was more in the process by asking open-ended questions and discussing about what students were doing. In particular, he made use of students' examples that did not reach the correct result just to show their thinking trajectory. In other words, while the questions, in Tom's and Susan's cases, were "restricted to numerical relations between elements", Chris posed more "qualitative questions about shapes, dynamic behaviours of the geometric elements" (Laborde, 2001, p.299).

Promoting whole-class interaction through the use of IWB

In Chris's case, the use of IWB reinforced the whole class interactivity especially in the case of sharing and discussing students' work. The joint use of GeoGebra and IWB supported the development of collective understanding of the class and a shared mathematical working space, where students collectively engaged in mathematics. Otherwise, there was a potential that individual students could develop their own idiosyncratic mathematical knowings; in particular, there was more scope for experimentation with the use of GeoGebra (Godwin & Sutherland, 2004). During the whole class sessions, students' contributions became the platform for encouraging classroom community discussion. In Chris's lessons, "classroom demonstrations and discussions played an essential role in capturing the students' results and in provoking a collective convergence of mathematical thinking" (Drijvers et al., 2007). In other words, Chris aimed at creating a learning community and making visible the mathematical thinking of the class (Sutherland, 2007).

However, Tom and Susan did not use the full interactive potential of the IWB. Tom utilised it only to project the dynamic files to the whole class while using the keyboard and mouse at his computer when he demonstrated tool techniques. As he put it, "the lessons I did this week would have been the same if I had a projector". Similarly, Susan tended to use the IWB as a projection device to demonstrate the tool techniques and to explain the tasks.

Contributing to the pace and productivity of lessons through the use of software

It emerged in all Chris's and Susan's comments that GeoGebra contributed to the pace and productivity of their lessons. For instance, Chris's remarks revealed how the use of a slider for different scale factors in enlargement introduced time economy by allowing students to carry out in a more straightforward way. As he noted, "If we had to keep on drawing that would be very time consuming". Susan also pointed out that GeoGebra increased the speed of the process of constructing reflections, hence, allowed the students to produce more examples and to witness "straightaway what happens much quicker than a mirror". This, in return, provided students with more time to observe and think about processes and eased their learning process. On the other hand, Tom did not mention the pace and productivity of GeoGebra. Instead, he attached importance to the visual, dynamic and instant natures of GeoGebra that permitted students to notice what happens in a diagram.

Promoting activity time through managing the physical time

All the teachers aimed to increase students' learning time by providing them with an extended period of activity time. In order to maximise the didactic time during students' activity time, they all circulated amongst students to make small contributions and intermediate syntheses (Assude, 2005). As has been previously mentioned, Susan's focus during these syntheses was mostly the technical aspects of the resources such as how to open up GeoGebra files and download them from the VLE. Likewise, Tom had intermediate technical demonstrations to ease students' use of the software. For instance, he showed how to make a slider in order for students to check their answers. Additionally, Tom purposefully shortened his introduction time from the front in order to give students more activity time. On the other hand, Chris's intermediate syntheses consisted of mostly spotting students' examples, in particular different approaches, and showing them to the whole class for a further discussion. The purpose of such an approach was to show different perspectives that pupils were taking and give them some ideas for proceeding their task, hence, to support the timely progression of all students.

5.3 Testing and Refining the Axial Codes from Transformations Lessons

On the basis of comparing the three teachers across Transformations tasks, I have identified some preliminary axial codes in the previous section. Now I will elaborate and refine these categories further by looking at the other two topics (which each involved two teachers) to find out to what degree they were different. Based on such two-teacher comparison I will first look at the Straight Line Graphs lessons of Susan and Tom and then examine the Circle Theorems lessons of Susan and Chris.

5.3.1 Straight Line Graphs (Susan-Tom)

As has been noted in the previous chapter, both Susan and Tom used GeoGebra to teach Straight Line Graphs in order to complement the development of understanding of this topic on paper. Although more extended introduction of graphs would have provided scope for more in depth analysis, a constraint on this study was that both teachers reviewed the topic with GeoGebra as a finishing activity. In this regard, this section provides limited analysis of this topic.

In this section, I will compare Susan and Tom teaching Straight Line Graphs to find out about similarities and differences between their teaching in terms of the axial categories identified above. In addition, I will state a sentence under each axial category heading which makes it explicit whether or not (or to what degree) this comparison confirmed the patterns reported for the Transformations comparison.

Promoting different types of interaction through the use of working environment

Susan's lesson took place in an open plan small U-shaped computer room where there were only computers for students and was not a place for whole class teaching. Therefore, she did the introduction of the lesson in her normal classroom and then made students move to the computer room to practice with GeoGebra. The location of the computer room was convenient in terms of proximity. Compared to Transformations lessons, Susan needed to introduce some routines in organising and managing student access to, and use of, computers and managing new

type of transition between lesson stages including movement of students from ordinary classroom to the computer suite.

Tom's lesson took place in the same computer room above so there was not any movement from students' side. As Tom indicated, he was used to working in this kind of room arrangement as most of the rooms had the same layout in his school.

Prepared files involving use of GeoGebra

For this axial category, similar patterns to the ones recorded in the Transformations comparison became apparent. Susan again made use of a prepared worksheet that she borrowed from a GeoGebra website. She preferred this worksheet as "there was no query and there was no uncertainty so they had something very specific to do". Similar to the Transformations worksheet, this also provided a tightly structured form of investigation for individual student work in GeoGebra since the students were novice in GeoGebra. As Susan emphasised "with a class who may be more used to using GeoGebra you could be more open with it"

Tom prepared a dynamic file accompanied by a worksheet for students to utilise the dragging function of the software. As he had used this file before with another class, he redesigned it by changing a slider to a tool which aimed for students to experience "much closer to the way that the line behaves by grabbing the line itself on the y axes and drag it up and down". Compared to Susan, he had more confidence in operating the software and was at the stage where he prepared his own construction file and dynamic figures.

Choice of tasks

The patterns in this axial category confirmed the ones reported for the Transformations comparison. Both teachers provided students with step-by-step instructions on the worksheets. Therefore, students were expected to fill in some blanks of a worksheet with answers. The tasks in Susan's worksheet provided students with four straight line equations for each question asking them to draw those in GeoGebra and see the patterns in terms of figuring out the ' $y = mx + c$ ' formula and the meaning of the constants ' m ' and ' c '. Tom's dynamic file also aimed to make students think of straight-line equations. In particular, he wanted his students to have

“a deeper understanding how gradients, and intercepts work”. Since he had used this file before, he rethought and revised some aspects of it. Rather than having two different sliders for ‘m’ and ‘c’ he created a tool, which controlled these two variables at the same time. He thought changing this would make students “much more in control” and “much closer to the way that the line behaves”.

Promoting independent learning through the use of software

As has been mentioned before, both teachers used structured worksheets to guide students’ progress for this topic. The aim of these lessons were review/practice of what students had learned before from paper-and-pencil activities. In this sense, it did not promote independent learning.

Making connections between computer tasks and paper-and-pencil tasks

Both teachers taught this topic first in a paper-and-pencil environment. Afterwards, with the use of a GeoGebra-based activity, they wanted to complement the development of students’ understanding of this topic on paper and envisaged this part of the lesson as building on previous work. In Tom’s words: “that was GeoGebra to finish the topic to give them some puzzles to do on the computer after they understood the material they needed for it.”

The Pedagogical Functioning of Activity Formats

As both lessons were designed as a finishing activity for graphs the main activity format for both teachers was individual student work on these tasks with the teacher walking around the classroom to provide guidance. At the beginning of the lessons, both teachers did a short teacher led presentation explaining the worksheet and related tool techniques. In this sense, the activity formats that Susan and Tom employed did not differ from the ones they used in Transformations lessons but the time span for each format became shorter.

Emerging different variants of the same activity formats

No different variants of the same activity format appeared.

The questions during work-and-walk-by

Both teachers observed students' progress by walking around the classroom. Compared to her Transformations lessons, Susan's focus was not on the technical aspects since, first, students only needed to know how to use the input field (that is placed at the bottom of the GeoGebra window) which permits the input of algebraic expressions directly by using the keyboard. Second, this group of students were high attaining and therefore more focused and motivated about mathematics. Some students finished earlier and Susan asked them some open-ended questions to work on with GeoGebra. In Tom's case, the patterns were similar to his Transformations lessons. He observed students closely and guided them when needed. Most of the questions were related to the worksheet.

Promoting whole-class interaction through the use of IWB

Both teachers did not do whole-class teaching apart from at the beginning of the lessons when they demonstrated the related tool techniques and explained what students were expected to do. Susan only used the IWB to show students how to use the input bar in GeoGebra by modeling some questions at the front to the whole class. Tom again used it to project the worksheet and dynamic file. This was probably due to the revision/practice goal of the lesson.

Contributing to the pace and productivity of lessons through the use of software

Susan again emphasised that GeoGebra increased the speed and the efficiency of the lesson by reducing "the drudgery of drawing graphs to see the patterns." Tom did not specifically mention the pace and productivity aspect using GeoGebra for this topic. But he noted that this GeoGebra based activity "lent itself very well" and "boost their learning".

In this regard, the comparison under this axial category were similar as the patterns already noted in Transformations section.

Promoting activity time through managing the physical time

As has been mentioned before, both Susan's and Tom's lesson on Straight Line Graphs aimed to finish the topic by providing students with a GeoGebra based activity. Therefore, in these lessons, students worked at computers going through their worksheets while teachers were walking around to provide guidance when needed.

5.3.2 Circle Theorems (Susan - Chris)

In this section, comparing Susan and Chris teaching Circle Theorems, I will continue testing out and elaborating those axial categories, which have been identified in the section compared three teachers teaching Transformations, to find out if there are interesting ways to discuss more about similarities and differences between Susan and Chris. I will also report at the end of this section if there are other aspects to become evident from the analysis of Circle theorems lessons that point more contrast and similarities between Chris and Susan.

Promoting different types of interaction through the use of working environment

Similar to their Transformations lessons, Susan's Circle Theorems lessons were with laptops in a normal classroom and Chris's lessons took place in a room where there were computers and separate seats for students. The main difference in Chris's case was the layout of the room, which was U-shaped, and also the additional software (control software) that existed in that specific room, which was of great help in shifting from paired student work at computers to whole class teaching. With the help of this software, Chris gave intermediate syntheses without moving them to the seats. As the room was U shaped, students were able to see the board from the place they sat at computers. In this sense, control software was useful to attract students' attention to the board by blocking their computer screen and displaying the example (that teacher wanted to show) on every screen. As Chris emphasised, "the fact that I can blank their screens so they can't do anything is brilliant". Furthermore, the U-shaped layout allowed him to monitor all students with less movement in the room with Chris's words "if I stand in the middle and turn my head I can see everybody's

screen”. For the same reason, Susan noted that she would prefer teaching in a big U-shaped room since students need more monitoring during technology lessons.

Comparing the patterns noted in the Transformations comparison, Susan’s use of working environment did not show any difference. In Chris’s case, both rooms provided the teacher the opportunity to move between two main arrangements: students sitting together in the centre for whole class discussion similar to a normal classroom condition, and students working in pairs at the computers. This flexibility was a prominent affordance of these working environments for the teacher since this arrangement enabled student involvement in classroom discussions without getting distracted by computers in front of them. Moreover, both rooms offered flexibility not only to the teacher in terms of structuring his activity, but also it allowed students to establish an independent way of working, which Chris valued generally for mathematics learning.

When I am working on something at home I had to decide ‘Do I need to do this at the computer?’ ‘Should I do this with pencil and paper?’ ‘Do I sit at the table?’ I have to make those decisions and move between them. The students were doing the same thing. I think it is a very adult way of working.

Prepared files involving use of GeoGebra

Susan adopted her own non-GeoGebra worksheet that she had used for more than ten years for Circle Theorems lessons and was familiar with the tasks. In this case, she returned to the tasks that she was accustomed to from her non-technology lessons unlike the Transformations worksheet.

Chris did not use structured files for this topic, as from what he had experienced before, he believed that this would bring a rather too structured and didactic form to the investigation, in particular with this class, consisting of high-attaining group of students. Starting with paper-and-pencil activities, students used dotted circle files in GeoGebra to find conjectures, which was followed by working with circles without dots in order to prove their conjectures for all circles. In this sense, again, Chris’s use of GeoGebra aimed at more open-ended investigative lessons. Compared to his Transformations tasks, the main difference was that the way in which Chris was more ambitious about what he wanted his students to learn that was not just the result

itself that should interest them rather it was actually a place for learning about proving. Having said that, for both topics, Chris placed far more emphasis on the development of high-level mathematical reasoning/thinking.

Design of tasks

The two teachers had prepared rather different sets of tasks for students. Chris aimed to promote students' concrete understanding before the process of abstraction and designed tasks through which students built up and interpreted the diagrams themselves. Through accumulating from his experience teaching with GeoGebra, he came to realisation that students could not make links between dynamic and static diagrams. Therefore, he wanted them to work on paper-and-pencil activities to start with. On the other hand, Susan asked students to formulate five different Circle Theorems (which were stated in the worksheet) for further generalisation through using dragging techniques with GeoGebra. In this regard, Chris was more open ended in terms of allowing the students to come up with conjectures and ideas, which they then test out in GeoGebra rather than as with Susan the ideas to test being already there. In other words, in Susan's lessons, "the elements to be compared and to be changed were given to the students". (Laborde, 2001, p. 299)

These patterns showed similarities with the Transformations lessons and related to Laborde's (2001) typology of level of task innovation.

Promoting independent learning through the use of software

Chris encouraged pupils to become more self-reliant through the use of software as he believed that GeoGebra enabled students to generate more examples efficiently compared to doing it by hand whilst affording them a chance to create their own conjectures before testing them out. For both topics, Chris promoted independent learning of students. The Transformations lessons were more about having ideas and testing them in GeoGebra. The Circle Theorems lessons were more about working towards proving. But the common strand was encouraging students to be more self-reliant and directing them towards higher-level mathematical processes, in particular

with this high-attaining group of students. Additionally, students' familiarity with the software was a crucial factor that influenced Chris's practices.

In Susan's case, her aim was also to encourage students' independent learning by allowing them to use GeoGebra by themselves. However, since students were novice in such use of GeoGebra, Susan needed to provide constant explicit guidance for students to solve tasks in GeoGebra and also "questions were restricted to numerical relations between elements" (Laborde, 2001, p. 299). The ideas for students to try out were already stated in the worksheet so students did not need to conjecture an idea. They made use of the dragging function of the software to see more examples of the Circle Theorems stated in the worksheet "to facilitate a conjecture about a general rule that students were asked to generalise by induction, without proving it" (Laborde, 2001, p. 299). In other words, *autonomous experimentation* by students was limited in Susan's lessons, which was the case also in her Transformations lessons. This was probably because not only the students were novice to the software but also the teacher.

The richness of the repertoire of activity formats

Although both teachers used mostly similar types of activity format such as Technical-demo, Walk-and-work-by, Spot-and-show, the quality and the breadth of activity formats were different. By quality I mean the interaction between technology, students, teacher and mathematics. Taking the example of the use of Spot-and-show, Chris highly instrumented this format with the use of GeoGebra, which in return offered more interaction potential through using it as more straightforward and quick way, which also contributed to time economy in the lessons. However, when Susan made use of Spot-and-show, she held up a student's worksheet in order to show others how this student had done the question. In this case, technology did not make any difference for her use of this particular format. In the wider literature, this is consistent with the results of Drijvers et al. (2013) study where the researchers concluded that mid-adopting teachers' practices with technology (those who are less-experienced in technology-use), exploited activity formats that did not differ much from what was usual in their classes.

In addition, Chris's repertoire consisted of some technology-specific formats e.g. Sherpa-at-work unlike Susan, who tended to customize existing (and more generic) formats for implementing activity. Chris considered Sherpa-at-work as distinctive to technology use where the involvement of the computer uniquely influenced the nature of the interaction taking place.

These patterns did not show much difference compared to the ones noted in the Transformations section.

Emerging different variants of the same activity formats

Comparing Susan and Chris, there were not any variants of the same activity formats identified.

The questions during work-and-walk-by

In comparison to Chris, Susan mainly focused on technical points when she was walking around the classroom while students worked at computers. This was because students as novice GeoGebra users needed to set the tasks from the paper worksheet on GeoGebra, which made them focus first on the tool techniques rather than on mathematics. In this sense, the fact that Susan did not provide prepared dynamic files for each task directed students' focus towards learning the use of technology rather than mathematics. The observed emphasis on techniques in GeoGebra is similar to what has been argued by Monaghan (2004) regarding teachers' focus on students' learning to use the tool. Such use of the tool is, according to Monaghan, typical and natural in an early phase of work with a computer software tool in order to be sure that technical problems do not obstruct students' utilisation of the tool. In this regard, students' learning of the software influenced the help the teacher provided during Walk-and-work-by.

However, Chris concentrated mostly on the mathematical points and guided students to think about the process rather than the result itself by asking reflexive questions and leaving them to think about what they did and why they did. In this regard, he did not dominate students' discussion but only channeled their thinking by raising questions. On the other hand, Susan mostly took over the control of the computer and

showed students what to do or where to focus. However, I must acknowledge the difference between their classes in terms of achievement level: as Susan specified during the post-lesson interview, what she wanted her students to notice was not as obvious for them since the class was lower-set.

Again Susan's and Chris's focuses during Work-and-walk-by in the Circle Theorems lessons appeared to be similar to the patterns presented in the Transformations lessons.

Promoting whole-class interaction through the use of IWB

Using GeoGebra on the IWB made episodes of whole class teaching easier for teachers particularly to demonstrate tool techniques. Both teachers used the IWB to show students what to do with GeoGebra software: in particular, it was important to do Technical-demo at the beginning of the lesson. Susan demonstrated briefly what she expected students to do, and in order to exemplify she used GeoGebra on the IWB to model how to tackle the first question from the printed worksheet. During this, she showed the specific tools in GeoGebra that students would need for constructing the shapes shown on the worksheet.

In Chris's case, he used GeoGebra on the IWB to explain and discuss the diagrams to the whole class, which helped him to have an efficient instructional time. He made use of IWB mainly to promote whole class involvement and discussion in particular through implementing Spot-and-show. As has been discussed before, he projected students' examples of particularly original approaches, which he picked up during Work-and-walk-by, onto the IWB for whole class discussion aimed at making pupils think of different ways of solving a problem. For him this part of the lesson was of importance to make sure that all students heard about the ideas that different people had. Furthermore, he made students part of the process and used the IWB more interactively by inviting students to write their conjectures on the IWB and then discussing them with the whole class. This intended to broaden the whole-class communication and vitalise mathematics conversations.

Considering the wider literature, Knight, Pennant and Piggott (2004) examined the IWB use in daily numeracy lessons with six mathematics teachers. The researchers

identified five different contexts for the IWB use: Teacher as demonstrator, Teacher as modeler, Teacher in control - inviting the pupils (shared), Pupils in control with the "teacher" advising (guided), and Pupils working independently. In this regard, while Susan tended to use IWB as only Teacher as demonstrator, Chris had variety of use of IWB including Teacher as demonstrator, Teacher as modeler, Teacher in control-inviting the pupils (shared) and Pupils in control with the teacher advising (guided). These results confirmed the patterns put forward in the Transformations comparison.

Contributing to the pace and productivity of lessons through the use of software

Similar to the patterns noted in the Transformations comparison under this axial category, Susan and Chris considered that the use of dynamic geometry contributed to time efficiency in Circle Theorems lessons. Both teachers indicated that GeoGebra – in particular the angle-measuring tool and the edit/undo option – enabled students to progress faster compared to measuring/calculating angles by hand. As Chris noted, “that saves a lot of time over drawing them out”. This speed helped the students to produce and engage with more examples without spending as much time capital, therefore, enabled to maximise their didactic time. In Susan’s words: “The speed is that they can see lots and lots of versions of it and all the weird versions.”

Promoting activity time through managing the physical time

Similar to the Transformations lessons, both teachers provided an extended period of activity time for students to work on GeoGebra tasks at computers. In particular Chris, carrying out an investigative lesson, emphasised the importance of providing enough time for students to think and focus by saying the following:

I wanted them to be focused but I did not want to hurry them as I was standing next to them, as I was talking to them. I wanted them to have the opportunity to explore properly what was going on.

Both teachers again circulated around the classroom to provide guidance as they might lose substantial amount of time on small problems. Chris, similar to his Transformations lessons, provided intermediate syntheses to give students ideas by

using Spot-and-show activity format in case they were stuck with their task. Furthermore, both teachers extended their practice with a technical-demo in order to familiarise their students with the tool techniques. However, Susan's lack of knowledge about the software introduced time diseconomy.

5.3.2.1 More axial categories emerged in comparison between Circle Theorems lessons

In addition to all of these axial categories which emerged in the Transformations comparison, the Circle Theorems lessons pointed out further contrasts between Chris and Susan, which will now be elaborated in this section.

Handling technico-mathematical conventions of software operation

Susan experienced difficulties in handling technico-mathematical conventions of the software such as the fact that GeoGebra measures angles anti-clock wise. She “was learning on the hoof this bit where it measures it anticlockwise”, which indicated that Susan had not thought through the complexity of GeoGebra for students and the operation of the resource system. This was not very obvious in Transformations lessons as Susan used externally prepared dynamic files so that students did not need to use the software from scratch. However, during Circle Theorems lessons, she asked students to set the diagrams (that were stated on the paper worksheet) in GeoGebra by themselves, which directed students' focus on being *au fait* with the software.

In Chris's case, he handled technico-mathematical conventions of GeoGebra confidently as he was knowledgeable about the software. Over the years he had gathered what difficulties students generally faced with the software; therefore, he already knew that students might have had difficulties in measuring angles in GeoGebra. He also specifically showed them how to measure angles for intersecting segments, as he thought “that is not a common one”. Again similar to the Transformations lessons, he provided students with tool time so that they could figure out the tool techniques for this specific topic in addition to doing a Technical-demo at the beginning of the first lesson.

Privileging mathematical proof over having numerous of examples through dragging

Chris considered Circle Theorems as being an instrumental topic for teaching about mathematical proof and focused on the fact that proving a conjecture was different than having a number of examples of a conjecture by using the dragging action of the software. He required students to conjecture properties from what they could see in GeoGebra. In addition, he concentrated on the possibility of using GeoGebra to find a reason or to elaborate a proof. His aim was to show students that there were multiple ways of proving the same mathematical conjecture. Therefore, use of Spot-and-show was deliberate and aimed to discuss different conjectures and their proofs. It must be stated that this particular class of students consisted of high-attaining students; this links with Evans' concern that "proof is often reserved for 'high ability' students" (2007, p. 38).

On the other hand, Susan treated this topic in an intrinsic manner in the sense that the aim for students was simply to come to know and understand the results through a tightly structured form of investigation. The purpose of GeoGebra use for this topic was to help students engage with more examples to facilitate their noticing the relationships between angles in circles. In this case, Susan asked students to set up the diagrams shown on the worksheet in GeoGebra and then to explore triangles inscribed into circles and try to formulate four different Circle Theorems rules by using the dragging function. In other words, Susan aimed for the technology to be used as facilitating the formulation of conjectures. However, technology's role did not go beyond that. As Hölzl (2001, p. 65) noted, the drag mode was "used only in a verifying manner" and that "learners are just supposed to vary geometric configurations and confirm empirically more or less explicitly stated facts".

5.4 Development of GeoGebra related Craft Knowledge

Looking at the comparisons between teachers at different stages in terms of learning to teach with GeoGebra, some indications of the growth of craft knowledge have become evident. The case study analysis illustrates that Chris (expert in technology

and in teaching) possessed rich and effective curriculum scripts for teaching these two topics, which had been modified/refined through his accumulated experience. The development of his curriculum scripts for these topics interlinked with similar adaptation as regards other aspects of the SFCP framework such as working environment, resource system, and activity format.

Chris's comments indicated evidence of gradual evolution regarding the network of ideas about teaching both topics with technology over the course of years. For instance, the teacher had revised the GeoGebra files for Circle Theorems because he thought that other versions brought a rather too structured and didactic form to the investigation considering the achievement level of class. In addition, he had come to include more discussion about enlargements by scale factor 1 and scale factor 0 in response to using technology. As he emphasised: "using GeoGebra made me realize that enlargement by scale factor 1 is really important. It is the multiplicative identity."

On the other hand, Susan (novice technology using teacher), did not only need to craft curriculum scripts teaching these topics with technology but also needed to design routines for actions both managerial and instructional that would introduce time economy. During the post-lesson interviews, she pointed out that she noticed some unexpected issues in her lessons that were mainly about developing instrumental knowledge to make it part of the resource system, first for herself personally and then also for her students. There were also some indications that she was learning and adapting. She suggested that after having taught Circle Theorems lessons with GeoGebra, she would from now on incorporate dynamic GeoGebra files for each circle theorem into lesson agenda, even if it was only for teacher use, in order to discuss the unusual cases. She indicated that using GeoGebra for Circle Theorems provided students with more examples than were seen in a non-technology lesson, which she found important from a mathematical point of view. Additionally, working with her colleague to teach similar Transformations lessons, she drew his attention to issues related to the worksheet as an important part of planning; accordingly, they re-designed the worksheet, and revised some of the explanations.

In this regard, while Chris provided retrospective information regarding how his approach had changed, which in effect showed how his curriculum script had

been modified and adapted over the years, Susan displayed initial stages of developing/building up her curriculum script, particularly in subsequently planning a similar lesson with her colleague.

Including Tom, who was technologically more experienced than Susan but less experienced than Chris, the comparisons of the three teachers provided a clear picture of the fact that Resource System was an issue for teachers and they were at different stages facing that. Susan was at the stage where consideration of the resource system almost overwhelmed everything else as she and the students needed to learn how to use the particular software. On the other hand, Tom was more confident in operating the software and as has been discussed he constructed his own files for students to work on. In Chris's case, he was at the stage where a coherent set of resources was already developed as a functioning resource system and knew how to prepare his students in order for them to be able to use those. In this sense, it is possible to see the transition to actually creating geometric constructions and other dynamic figures from scratch as teachers gain more confidence in operating the software.

In terms of design of technology-based tasks, these case studies illustrated that the experience level also suggests that there is a change from regular forms of extension activity to things that need high-level thinking. Considering the most experienced teacher in the study, it is clear that for teaching both topics Chris placed far more emphasis on the development of high-level mathematical reasoning/thinking in comparison with Susan and Tom. Furthermore, between Tom and Susan, the evidence highlights that Tom, drawing on his experience, put more thought into designing his task for Straight Line Graphs by focusing on students' thinking.

Moreover, there is an indication that the expert teacher, Chris, developed his craft knowledge in establishing new structures of interaction involving students, teacher and the software. He had more fluent and stabilised activity structures that were instrumented by technology. In comparison to Susan and Tom, Chris was more efficient in shifting between different segments with the use of technologically instrumented activity structures, most of which were student-centered. One of Chris's specific choices of activity formats for teaching these two topics with technology consisted of spotting students' examples, in particular different

approaches, and showing them to the students for a whole class discussion. In this sense, through utilising GeoGebra and IWB, as an expert teacher, he made “more explicit references to student actions and to test their points within lessons” (Leinhardt et al., 1991, p. 91).

In the table below, I summarise some characterisations of Novice/Expert teachers that relate SFCP constructs to the stages that teachers are at.

Table 5.2 Characterisation of Novice/Expert teachers that relates to SFCP constructs			
Structuring feature	Teacher		
	Susan (limited)	Tom (developing)	Chris (sophisticated)
Working Environment	Lacked confidence with classroom management strategies in particular getting students attention for whole class activity in technology-based lessons. Preferred room arrangements that offered better scope for monitoring and interaction i.e. big U-shaped classroom layout.	Preferred adjustable computer desks (not fixed computers) with the layout of students facing the front (rather than to each other), which aimed at getting students attraction easily to the board and minimizing the off-task behaviour.	Appreciated room layout combining space for student-pair work at computer with central seated area for whole-class activity. Had developed fluent techniques for moving students and switching between these modes. Had also developed techniques for capturing student productions in suitable digital form for later use and for using software to access and control student computers during whole-class sessions.
Resource System	Just embarked on establishing a functioning resource system with GeoGebra for her classes. Prioritized mainly developing instrumental knowledge to make it part of the resource system, first for	Had confidence in operating the software since he previously made use of GeoGebra in teaching. Created dynamic files himself taking into account of his previous	Had already established a set of resources as a functioning resource system. Managed students use of GeoGebra gradually by allowing them to gain confidence in working from scratch with GeoGebra. Designed more open ended tasks

	herself personally and then also for her students. Started with files and worksheets already prepared by someone else: this gave her more confidence and also alleviated the time pressures she felt.	experience so that students could focus on mathematical content rather than dealing with complexity of software.	that allowed students to discover different ways of solving a problem.
Activity Structure	Mostly customised the existing and more generic formats for implementing activity.	Customised the existing and more generic formats for implementing activity. Focused on extended periods of individual student work at computers with him circulating around the classroom to guide students.	Had developed his craft knowledge in establishing new structures of interaction involving students, teacher and the software. Had more fluent and stabilised activity structures that were instrumented by technology, most of which were student-centered. Had a rich repertoire of activity formats.
Curriculum Script	Her existing curriculum scripts made minimal reference to dynamic software. This lead to her either using from her files the same worksheet as previously, substituting use of dynamic software for classical tools on paper, or borrowing from elsewhere a worksheet designed for dynamic software. Her immediate priority was not on elaborating her	Had developing curriculum scripts. Refined his dynamic files according to previous experience with his classes.	Developed his craft knowledge in building up his curriculum scripts for teaching those topics with technology and had rich and flexible instructional scripts. This richness was evidenced in the sheer amount of talk Chris produced in describing lessons in terms of how technology changed the way in which his lessons progressed.

	curriculum script, but on developing instrumental knowledge to make it part of the resource system. Focused on students' learning to use the tool in a more general mathematical sense.		
Time Economy	Did a technical-demo in order to familiarise her students with the tool techniques however her lack of knowledge in operating the software introduced time diseconomy.	Shortened his introductions from the front in order to give students more activity time. Demonstrated necessary tool techniques which allowed students more learning time focusing on mathematical content. Provided intermediate syntheses and small contributions during Work-and-walk-by.	Extended his practice with a technical-demo in order to familiarise students with the tool techniques. Provided intermediate syntheses to give students ideas by using Spot-and-show activity format in case they were stuck with their task. Provided enough time for students to think and focus on tasks especially as he was carrying out investigative lessons.



6

DISCUSSION

6.1 Introduction

Drawing on the responses provided in Chapters 4 and 5 to the subsidiary questions posed in Chapter 3, this chapter provides a summary of the main findings that emerged in this study relating to the more fundamental research questions posed in Chapter 1. It also discusses the elaboration and refinement of the SFCP framework (which guided the framing of the subsidiary research questions) through connecting it with other theoretical approaches such as Instrumental Orchestration and Technological Pedagogical and Content Knowledge (TPACK).

This will be followed by a discussion on the limitations of the study. Finally, recommendations for future research will be presented, together with some thoughts concerning implications and the overall conclusions of the study.

6.2 Summary of Findings

The purpose of this study was to investigate teachers' classroom practices and professional knowledge that support the integration of technology. This study aimed to build a holistic understanding of the conditions of technology use in ordinary classrooms.

The first research question of the study regards **"What aspects of classroom teaching shape the ways in which teachers integrate new technologies?"**. Guided by the general SFCP framework, and triangulating evidence from interviews and lesson observations of three mathematics teachers using GeoGebra, this study

developed a detailed picture of the important aspects of classroom teaching for teachers' adaptation of technology. Looking across lessons taught by the three teachers on similar topics it became evident that the five aspects of the SFCP (e.g. working environment, resource system, activity formats, curriculum script and time economy) featured in their lessons regardless of the topic and proved to be crucial aspects, which should be taken into consideration for the incorporation of technology into classroom practice. Teachers' practices with GeoGebra did not vary greatly from topic to topic, in the sense that teachers' general use of GeoGebra stayed the same. However, comparisons across teachers within topic elucidated differences between teachers' practices: in particular, how they established a functioning resource system, what kind of activity formats they employed, and how their curriculum scripts had been developed with the use of technology for handling a topic.

For Susan (the least technologically experienced teacher amongst the three) GeoGebra acted as a direct tool substitute, but one that provided functional improvement in her lessons. For instance, students were able to use the dragging function of GeoGebra, which improved the speed of seeing a number of versions of a diagram. In particular, for the Circle Theorems lessons, Susan used an existing worksheet (that was designed to be used with a protractor) with GeoGebra as a substitute for the protractor, with the aim of measuring angles more effectively. However, her lack of technical knowledge with GeoGebra created some difficulties in running her lessons fluently, which in return caused her not managing time efficiently. In addition, her post lesson interviews suggested that her focus was more on developing instrumental knowledge of GeoGebra to make it part of the resource system for herself personally and for her students. In this regard, the evidence highlights that developing a repertoire of suitable GeoGebra techniques for related topics is important for teachers to establish their resource system coherently. Furthermore, Susan was at the stage where she needed pre-made worksheets/tasks (either borrowed from a website or a sequence of activities that were strongly influenced by the existing curriculum script) to incorporate GeoGebra into her lessons so that she could follow those worksheets as a lesson agenda. This suggested that she had a minimal curriculum script for those topics with the use of GeoGebra.

On the other hand, Tom had suitable GeoGebra-related instrumental knowledge for the topics taught. Evidently, Tom was managing to use GeoGebra in his teaching in a way that functioned for his pedagogical approach. For Tom, technology allowed for task redesign (this was not the case for Susan). He designed his own tasks involving GeoGebra use and redesigned them in response to his experience. Notably, he prepared a dynamic file for Straight Line Graphs, which included a tool that controlled two variables at a time with the help of the dragging function of GeoGebra. However, Tom's lessons were more structured towards guiding students to work on closed-ended tasks, which showed similarities with Susan's choice of tasks.

Chris, as the technologically most experienced and accomplished teacher, had well established teaching repertoires with the use of GeoGebra. He planned investigative lessons and focused on students' becoming an explorer by using the GeoGebra software. For both topics, he wanted students to engage in multiple approaches to problem solving, hence, encouraged them to come up with their own ideas and test them in GeoGebra. This was apparent especially in the lessons, where students were guided to explore and prove conjectures that lead to different Circle theorems. He was clear about the crucial qualities of GeoGebra in supporting the discovery-oriented style of task sequence that he used with students. Consequently, a major difference was evident with Chris in comparison to Susan and Tom. The emphases in Chris's lessons were related to students taking control and being in charge of their own work with the focus on higher-order thinking. In this connection, Predict-and-test was the main format of interaction that promoted independent work of students with GeoGebra. Furthermore, with the use of his working environment (including computers as well as a separate teaching space), he promoted different interactions to take advantage of the interactivity of GeoGebra and IWB. In this case, Spot-and-show was the main format of interaction that promoted whole class discussion through the joint use of GeoGebra and IWB. The shifts between these segments were smooth and fluent.

The second research question regards **"What kind of professional knowledge do teachers need to teach with technology?"**. The SFCP framework also guided me to make some further observations about the patterns of behaviour of teachers, which were related to their degree of experience. The analysis of the classroom

practices of Chris as an expert technology-using teacher in contrast to those of comparably technologically novice teachers, Susan and Tom, gave insight on the relevant expertise and on the process through which it developed as a result of his use of GeoGebra.

Chris, providing evidence retrospectively, indicated that GeoGebra brought about transformative change to his teaching these topics, which led him to redefine his curriculum scripts by including new tasks (i.e. the discussion of scale factor 1 and 0 in enlargement) and new activity formats (i.e. Sherpa-at-work), which were previously inconceivable. He had developed much richer curriculum scripts for the topics taught through fine-tuning his use of GeoGebra over the years to his discovery oriented pedagogical approach. He favoured a different pedagogical approach than Tom and Susan as they both tended to use closed-ended tasks with the focus on students getting the questions correct. In contrast, Chris's focus was on the process and even when students went wrong, he used this to stimulate their thinking. The difference in teachers' pedagogical approaches might be related to their philosophy of learning mathematics, which could affect the mathematical processes and products that would receive emphasis in their curriculum script. Indeed, Drijvers et al. (2010) show how such philosophies of learning mathematics affect activity structures in demonstrating links between teachers' choices for orchestrations and their expressed views on mathematical learning and teaching. However, the evidence from this study suggests that this might be related to the experience level of the teacher in the appropriation of technology, which played a pivotal role in the development of their craft knowledge. In Chris's case, such experience made him reasonably confident regarding lesson 'hiccups', which were defined by Clark-Wilson, (2013) as "the perturbations experienced by the teachers during the lesson, triggered by the use of the technology that seemed to illuminate discontinuities in their knowledge" (p.2548). His post lesson interviews provided evidence that he possessed a recollection of previous hiccups that he encountered, and different ways to overcome these so that he could turn unexpected things into productive learning in lessons. Furthermore, looking also at Chris's non-technology lessons for Circle Theorems, it has become apparent that the main features of his approach for technology and non-technology lessons were the same.

Regardless of the lesson being technology based or non-technology based, this teacher was eager to collect/share students' achievements and to stimulate their thinking. In other words, his curriculum script already included more open-ended tasks and student-centred activity formats, therefore, they (e.g. Discuss-the-board, Spot-and-show and Work-and-walk-by) were easily adapted to his technology based-lessons. For instance, the Spot-and-show format was used in non-technology lessons by asking pupils during the lesson to go and write things on the board and then discussing it with the rest of the class. The key difference in technology-lessons for him was about the new relationship between teacher, student and technology rather than between teacher and student. As he pointed out, the use of Sherpa-at-work in particular was distinctive to technology use as the involvement of the computer uniquely influenced the nature of the interaction taking place.

On the other hand, Susan was at the stage where she was preoccupied with learning to make use of the tool in its own terms, both for herself and her students. The difficulties she encountered in lessons such as instrumentation issues experienced by her and her students also made her think in terms of establishing her resource system with the inclusion of technology. In particular, post lesson interviews provided evidence of her interrogation process for the unexpected instrumentation issues and the borrowed tasks designed by someone else. Additionally, for the Transformations lessons, redesigning the worksheet with her colleague provided strong evidence of the development of her knowledge/thinking. Although she chose somebody else's sheets to use in Transformations and Straight Line Graphs lessons, she chose her own non-technology sheet for Circle Theorems lessons, which implies that she was seeking to have a similar style of lesson to her non-technology lesson. It became clear that she did not develop a new repertoire of teaching tasks instrumented by GeoGebra, and likewise her activity formats were related to already existing ones.

The evidence presented has shown that Chris managed a number of aspects of classroom teaching related to using GeoGebra such as including GeoGebra-mediated tasks aligned with his pedagogical goals, preparing his students to use the technology efficiently, adapting his activity formats that were instrumented by GeoGebra, and extending his curriculum scripts in response to GeoGebra use. I argue

that finding solutions to all these challenges helped him to run his lessons smoothly and to shift between segments fluently, which in return fed into his time economy. In this connection, this study provides evidence that there are a number of classroom aspects that teachers need to learn about and develop their knowledge relating to technology integration depending on their stages in teaching with technology. In addition, the evidence in this study highlights that innovators' expectations for a more open classroom management and for more emphasis on mathematics in teacher-student interactions requires the teacher not only to be able to integrate technology but to reorient their lessons through crafting effective curriculum scripts.

To sum up, the table 6.1 below reconceptualises the features of the SFCP in relation to the findings showing new aspects of the associated craft knowledge, specific examples that index these aspects to individual cases, and sources of information. Focusing on “activity structure” concept, particularly in Chris’s lessons new types of activity structures began to emerge. Some of these may be considered variants of already identified types, while others were new (see Table 6.2 and Table 6.3).

Table 6.1 Components of the SFCP framework in relation to the findings of this study

Structuring feature	New aspects of craft knowledge related to incorporation of digital technologies	Specific examples	Sources of information
Working environment	Attracting students' attention to the board for whole class teaching. Capturing or converting student productions into suitable digital form. Observing and monitoring students' work. Managing new types of transition between lesson stages (including movement of students).	Chris' use of control software allowed him to secure students' attention, and an easy procedure capturing or converting students' screens for whole class discussion. U-shaped classroom layout enabled Chris to manage straightforward monitoring and transition between segments during independent student work at computers	Sourced mostly from observation records, but extended by elucidating information from the interview transcripts
Resource system	Establishing appropriate techniques for use of new tools to support subject activity Graduating demands on students of working with dynamic software	All three teachers demonstrated related tool techniques to students. In practice, however, Susan's lack of knowledge in operating software created difficulties in lesson flow. Chris graduated students' use of technology by first providing prepared dynamic files for students' exploration and only later asking them to start from blank pages to create their own files.	Sourced mostly from the observation records extended by elucidating information from the interview transcripts

Activity structure	Employing activity templates organised around predict-test-correct sequences to capitalise on the availability of rapid feedback Establishing new structures of interaction involving students, teacher and the software.	All three teachers pointed to the ways in which use of the software could facilitate the process of self-testing and self-correcting. Chris' use of Sherpa at work was particularly distinctive to technology use and the involvement of the software subtly changed the nature of the interaction that took place.	Sourced mostly from the observation records extended by elucidating information from the interview transcripts
Curriculum script	Recognising and responding to ways in which technologies may help/ hinder specific processes and objectives involved in learning a topic.	Chris added a discussion about the scale factor 1 and 0, which came to his realisation by the application of GeoGebra.	Sourced mostly from the interview transcripts extended by elucidating information from the observation records
Time economy	Developing time saving strategies, which aim to help increase students' activity time.	All three teachers used prepared files so that students could move immediately to focus on mathematical content rather than spending initial time setting up an appropriate figure with the software. In addition, they all did a Technical-demo to show students how to find the software and the topic-related tools in GeoGebra in order to use the tool time economically. Alternatively, Chris used a strategy involving use of posters that make it possible to condense students' work information by freezing the dynamic properties of GeoGebra. (Assude, 2005)	Sourced mostly from the interview transcripts extended by elucidating information from observation records

Table 6.2 New activity formats emerging in this study

Label	Setting	Description
Predict-and-test	Students work individually or in pairs at computers and the teacher walking by in the classroom	Students were encouraged to test their ideas at the computer themselves and progress through seeking validation from computers.
Walk-and-spot	Students work individually or in pairs at computers and the teacher walking by in the classroom	This was about students spotting interesting examples for themselves rather than the teacher spotting and showing. On one occasion Chris asked students to stand up and walk in a clockwise direction around the room to see different people's screens and then to implement some of the things they had seen.

Table 6.3 Variants of already identified activity formats in this study

Label	Setting	Description
Sherpa-at-work	Whole class setting	Chris's use of this format seemed to be a variant of Spot-and-show since he spotted this example while walking around the classroom. In that sense, this was a combination of Spot-and-show and Sherpa-at-work.
Spot-and-show	Whole class setting	Chris in this study spotted student work during lessons and showed them to the whole class on the spot. This had more <i>ad hoc</i> elements than through browsing students' work in advance as teachers did in the literature.
Technical-demo/Replicate-demo	Whole class setting	Occasionally, Tom used a variant of Technical-demo in that while he was demonstrating a number of tool techniques with instructions, students were implementing these simultaneously rather than after the end of his demonstration.

All the teachers planned lessons where students were active in dynamic exploration by using GeoGebra themselves. In this sense, this study has indicated that the SFCP framework was usable and useful as a tool to offer a window into teachers' classroom practices in student-centred activity settings. Considering the components of SFCP framework in relation to the findings, it is especially the emerging activity structures which relate to more student centred lessons, particularly those taught by Chris. His use of working environment (transition between peripheral computers and central seats) and use of resources depended on what activity structure he sought to use, such as one centred student exploration (at the computers), such as Predict-and-test, or discussing student work as a whole class (in the seats), such as Spot-and-show. He also used Spot-and-show as a strategy to support the timely progression of all students in the manner that he desired. For instance, he used this format to give intermediate syntheses to the whole class, which aimed to offer students some ideas, which could help them to progress if they had not yet identified a way forward, so helping to maintain a suitable pace within their learning time. Accordingly, his curriculum scripts consisted of undertaking loosely ordered actions, which were depended on student work in lessons.

Considering the Figure 2.4 presented in the Literature Review chapter, the jigsaw model conveys the idea that these five concepts of the SFCP framework are interlocking and have to be considered as a system. For this reason, how each structuring feature interacts with other elements needs to receive attention. While the jigsaw image is useful in the initial stage it is not a helpful way of going more in depth. Thus Table 6.4 focuses on the interactions between different constructs for each case to attend to the locking together implied by the jigsaw image. It pinpoints and discusses the most important things in the jigsaw diagram for each case in more depth, which is each of the five elements of course but also the interlocking between them.

Table 6.4 The interactions between different constructs for each case

Teacher	Structuring feature	Interlocking between the structuring features
SUSAN	For Susan working environment was fairly steady state in the background. She was mainly preoccupied with resource system, especially developing instrumental knowledge for herself personally and for her students. Therefore, she thought very little about the other structuring features in the sense she mostly used existing activity formats and had almost no curriculum script with the use of dynamic software.	Susan as a novice technology-using teacher was in the stage of learning and adapting how to teach with dynamic software. Her focus on developing a functioning resource system also influenced to a certain degree the development of her curriculum script. For instance, during Circle Theorems lessons when she learned how to measure angles in GeoGebra, she demonstrated this to her students as she registered that this was something that she needed to give attention to in teaching Circle Theorems with GeoGebra. Therefore, it started to become part of her curriculum script.
TOM	Tom was more focused on resource system and time economy. The working environment he used was the available computer room in the school at the time. Also, the activity formats he employed did not show much difference than his existing activity formats in non-technology lessons. In this sense, these aspects were hardly changed for this teacher.	Tom was developing his resource system in the sense that he was revising and adapting his previously used dynamic files according to his own experience with his students, which was to some extent related to his development of curriculum script. In respect of time economy, he also focused on extending students' activity time working on dynamic files. For this, he specifically shortened his instructions from the front, by quickly demonstrating the required tool techniques.

CHRIS	For Chris, all the structuring features were of crucial importance. He had adapted the working environment and resources available in his school to meet his pedagogical goals and optimise his time economy. These factors also actively influenced his curriculum script and activity structure.	In Chris's case, there were quite important interactions between all the structuring features. His use of particular working environment (designed to combine classroom with computer suite) and the resources in this working environment facilitated the implementation of distinctive activity formats – in particular Spot-and-show, Sherpa-at-work and Predict-and-test – by offering flexibility in whole class teaching and independent student work at computers. For instance, the U-shaped layout allowed him to monitor all students with less movement in the room. In addition, the control software in this working environment was useful to attract students' attention to the board by blocking their computer screen and displaying the example (that he wanted to show) on every screen. With the help of this software, Chris employed Spot-and-show and also Sherpa-at-work in a more straightforward way without moving students to the seats. In another connection, this allowed him to give intermediate syntheses with less effort, therefore – in terms of time economy – supported the timely progression of all students in the way that he wished. These (the resource system available to Chris in the classroom, GeoGebra-mediated tasks aligned with his pedagogical goals, preparing his students to use the technology combined with the activity formats that were instrumented by GeoGebra) all extended his curriculum script in response to GeoGebra use.
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6.3 Discussion

As has been discussed in Chapter 3, the second aim of this study was to refine the SFCP framework in order to provide the quality of descriptions of each construct and the ways in which they relate to classroom practices in relation to the use of technology. As for the SFCP model, it offered a useful lens to elucidate the difficulties encountered by teachers using technology in the classroom, while giving insight on how their activity and craft knowledge evolve during these uses. In this sense, the SFCP framework provided a number of researchable units of analysis, which serve as tools indicating certain aspects of teachers' professional knowledge. In terms of developing a fuller operational description of the types of data relating to each of the five constructs of SFCP framework, the idea of connecting different approaches is of crucial importance. In recent years, developing strategies to establish connections amongst emergent theories is gaining attention as researchers came to the realisation that "it is possible to connect theories" rather than "to decide between competing theories" (Prediger et. al., 2008, p. 170). This study aimed to connect theories for better communication and understanding and more coherence at the global level of the field.

In accordance with a view of connecting theories (Prediger et al., 2008, see Figure 6.1), I compared and combined the SFCP with Instrumental Orchestration (also aimed at understanding teacher practice with the use of technology) in order to describe and explain some constructs of the SFCP further, and to explain how these theories work together.

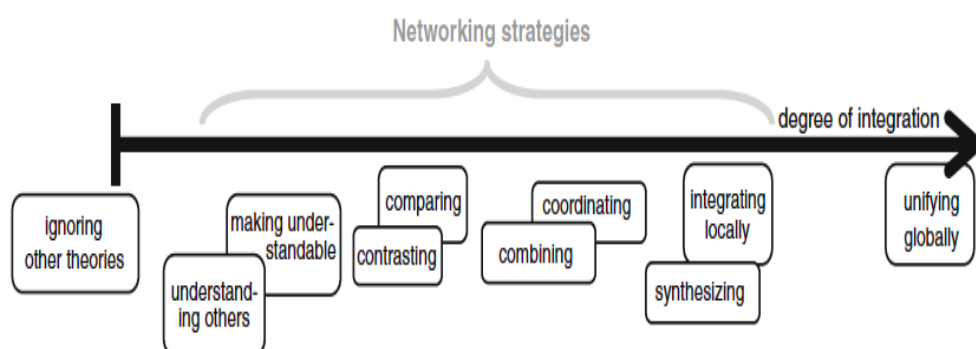


Figure 6.1 Networking strategies to connect theoretical approaches
(Prediger et al. 2008, p. 170)

The ‘Instrumental Orchestration’ model introduced by Trouche (2004) is derived from the instrumental approach issuing from cognitive ergonomics (Verillon & Rabardel, 1995). The instrumental approach, which offers a model to describe the process of learning to use technological tools, has been used by various French researchers (e.g. Artigue, 2002). An important concept in this approach is ‘instrumental genesis’ which is a twofold process including instrumentation and instrumentalisation. The former concerns the way in which the artefact shapes the actions and thinking of the user. The latter describes the way in which the user shapes the artefact by developing mental schemes, thus turning the artefact into an instrument (Hoyles, Noss & Kent, 2004). In other words, the instrument is used in a psychological sense, which is developed by the user through mental schemas in order to use it for specific tasks. For instance, in this study, Chris in his Transformations lessons focused on students learning how to drag points considering the drag mode as an instrument in the first lesson. This was an evidence of instrumentation process. However, in Lesson 5 the students used drag mode in order to explore important scale factors on their own for a mathematical reason, which was an evidence of the instrumentalisation process.

Instrumental Orchestration points out the necessity for teachers to guide their students’ instrumental genesis by systematic organisation of their tasks and use of the various artefacts available. This framework uses three concepts: a didactical configuration, an exploitation mode (characterised by Trouche (2004)) and a didactical performance (added by Drijvers et al. (2010)). As Drijvers et al. (2010) and Ruthven (2012) pointed out, SFCP and Instrumental Orchestration are complementary. Most of the concepts that are used in these frameworks have the potential to investigate similar aspects of teachers’ practices with new technologies. A didactical configuration is essentially an arrangement of technological tools in the environment similar to the working environment in SFCP terms. An exploitation mode is concerned with how teachers plan to take advantage of technology in order to achieve their lesson aim, which is related to interaction between the resource system and curriculum script. A didactical performance emphasises teachers’ *ad hoc* strategies when an unexpected aspect of the mathematical task or the technological tool occurs in classroom teaching, with regard to the chosen didactic configuration and exploitation mode. From a SFCP point of view, this might be associated with

how teachers' curriculum script guides teaching pathways, and *ad hoc* decisions could result in change or extend their curriculum script.

With the Instrumental Orchestration model as a point of departure, case studies in this study suggested further descriptions and explanations for activity structures with the use of technology. Research on the Instrumental Orchestration model identified a number of orchestrations (which have been already discussed in the Methodology Chapter) and provided labels for each of them to describe the interactions between teacher and students in a technological environment. Although describing the interaction between teacher, student and technology with one label might seem economical and practical, it became evident in this study that these labels provided insufficient information to characterise teachers' practices with technology. The evidence in this study indicates that three pieces of information need to be taken into consideration in order to describe teachers' practices fully in particular when the students are more active in the dynamic exploration. These are as follows: the technology in use (or not in use), teacher action and student action. In particular, this study highlights that the main aspect of the technology that shaped the interaction in classrooms is about the students making a prediction and then testing it out at the computer rather than teacher validating (or invalidating) it, which is called Predict-and-test. This shifts the role of the teacher towards becoming an organizer/observer of this process. In this sense, a label describing student action in addition to teacher action is needed. The descriptors for both are important in particular because different teachers might use the same formats differently. For instance, Predict-and-test in my study has resemblance to an activity format already noted in Ruthven, Deane and Hennessy (2009), which was used in whole class teaching during the introductory phase. The teacher set up a situation and then invited students to make a prediction and asked them to put the prediction in the computer and see the feedback as a whole class. However, in this study Predict-and-test provided a structure for the independent student work. Therefore, the interactional structure of teacher could be different. Furthermore, having a slot for whether the format included technology or not is also important because in particular the technologically novice teacher, Susan, tended to exploit her existing activity formats that were not instrumented by technology.

With the identification of this additional format, Predict-and-test, the results of this study are extending the Instrumental Orchestration framework. In addition, most of the orchestration types characterised by Drijvers et al. (2010) and Drijvers (2012) featured in this study but with more variants. For instance, the use of Sherpa-at-work has shown some differences. In this study, Chris's use of this format seemed to be a variant of Spot-and-show since he spotted this example while walking around the classroom. In that sense, this was a combination of Spot-and-show and Sherpa-at-work. However, the potential overlap between these two was not identified in Drijvers' (2010) study. Furthermore, the Spot-and-show activity format used by Chris in the current study was different from the Spot-and-show example in Drijvers' et al. (2010) study. Whereas in their study this format depended on the opportunity for the teacher to access student work while preparing the lesson, Chris in this study spotted students' work during lessons and showed them to the whole class on the spot. Evidently, the Spot-and-show examples that Chris used had more *ad hoc* elements than through browsing students' work in advance, which teachers in Drijvers' et al. (2010) study did. Moreover, Chris had another variant of Spot-and-show, which was more student-centred. On one occasion in Transformations lessons, he asked students to stand up and walk in a clockwise direction around the room to see different people's screens and then to implement some of the things they thought were useful for them. This was about students spotting interesting examples for themselves rather than the teacher spotting and showing, which could be characterised as Walk-and-spot that did not feature in Drijvers' et al. (2010) study (possibly because their study examined only whole class teaching rather than individual student work). However, considering Drijvers' system of labels, which tended to focus on what the teacher does, this might be labelled differently, e.g. Circus-of-ideas.

Activity structure also interacts with curriculum script. As has been discussed in Chapter 2, curriculum script is a mental scheme/branching structure about the knowledge of teaching a topic, which teachers build up that is mostly organised around the sequence of the activity they would engage with students. However, it is not only the decisions that the teacher makes for a particular lesson in the classroom but also a crucial component "in preactive planning of a lesson agenda, and in post-active reflection on (and learning from) a lesson" (Ruthven, 2012, p. 99). Although,

for a specific lesson, teachers firstly draw on their lesson agenda, when ‘hiccups’ come up in their lesson, they draw on their curriculum script in order to recognise what they could do in that kind of situation. It is because curriculum script incorporates alternative ways of teaching the same topic. Therefore, this concept is more difficult to discover in the sense that it requires a longitudinal research through observing the same teacher teaching the same topic with technology to different classes in order to find out how technology features in their curriculum script.

This study showed that observing teachers teaching a topic with one class provided information about their lesson agenda that was the particular curriculum pathway followed by the teachers in those lessons. This can be taken as indicative of that part of their curriculum script for that topic, which eventuated in the lessons taught to that particular class. As Putnam (1987, p. 40) stated, “the curriculum script can be thought of as the foundation upon which the agenda for a particular lesson is built”. The analysis of this concept focused on lesson ‘hiccups’ (potentially critical incidents during the lesson), which “offer opportunities for the teachers’ epistemological development within the domain of the study” (Clark-Wilson, 2013, p. 2548). Nevertheless, observing lesson hiccups per se did not provide certain information particularly on teachers who had made use of technology before as there were two options for them to respond to a lesson hiccup. Firstly, they might be extending their curriculum script in the light of this issue and adapt the lesson to the situation since what they were dealing with had not been part of their curriculum script before. Secondly, teachers might be using their existing knowledge within their curriculum script because those hiccups could be already encountered before. Hence, the post lesson interviews played a pivotal role in terms of finding out about those critical incidents by giving teachers an opportunity to reflect themselves on the lesson agenda.

The general observation was that it was much easier to elucidate information from more experienced teachers (in teaching) on their curriculum script. In comparison with Tom, Susan and Chris were more expressive about their lessons. In addition, the reflections of Susan and Chris indicated that their level of experience in technology use produced differences in what they focused on. In particular, the degree of emphasis on curriculum script against resource system varied depending on the stage of development of the teacher in terms of technology use. Susan's case

suggested that development of her curriculum script and resource system might be overlapping to some degree. As a novice technology user, she was calling on some knowledge in her lessons (i.e how to measure angles in GeoGebra) and the question was whether this knowledge should be regarded as part of her curriculum script or as being part of her instrumental knowledge about the software. I decided this was regarded as her instrumental knowledge because it was about understanding the way the artefact works (how the order she clicked affected whether she measured acute angle or obtuse angle). In other words, this was not about developing the topic in response to use of technology, but rather her focus was on the level of detail about the functioning of technology in a general mathematical sense. In this respect, the evidence highlights that development of curriculum script is related to teachers' engagement in a process of double instrumental genesis. This includes, first, personal instrumental genesis that leads to "the construction and appropriation of a tool into an instrument for mathematical work", second, professional instrumental genesis that leads to "the construction and the appropriation of the previous instrument into a didactical instrument for mathematics teaching" (Haspekian, 2011, p.2300). As Haspekian (2014) indicates, this double instrumental genesis of the teacher can also restrict their capacity to support students' development, which became evident in Susan's remarks in this study: "So again this is where we are learning ourselves. It sorts of contracts what you would be able to do, restricted in what you want to do because of your own knowledge of showing that."

Putnam (1987) pointed out that curriculum scripts contain "loosely ordered sets of goals for what students are to learn, activities and examples to be used in teaching for those goals, and likely student misconceptions or difficulties in learning a particular domain. These are elements of teacher knowledge that fall within pedagogical subject-matter knowledge" (p.40). In addition, with the use of technology, it is not only pedagogical subject matter knowledge that was crucial within teachers' curriculum scripts but also it is technological knowledge including knowledge of operating systems and computer hardware, and the ability to use standard sets of software tools. In this case, connecting the SFCP with the TPACK model would be useful to describe and explain further particularly 'curriculum script' construct of the SFCP. TPACK model was proposed by Mishra and his colleagues (2006) in an attempt to define what types of knowledge teachers need for technology integration.

They employed Shulman's (1986) concept of Pedagogical Content Knowledge (PCK) and developed it further by introducing a new component, Technological Knowledge (see Figure 6.2).

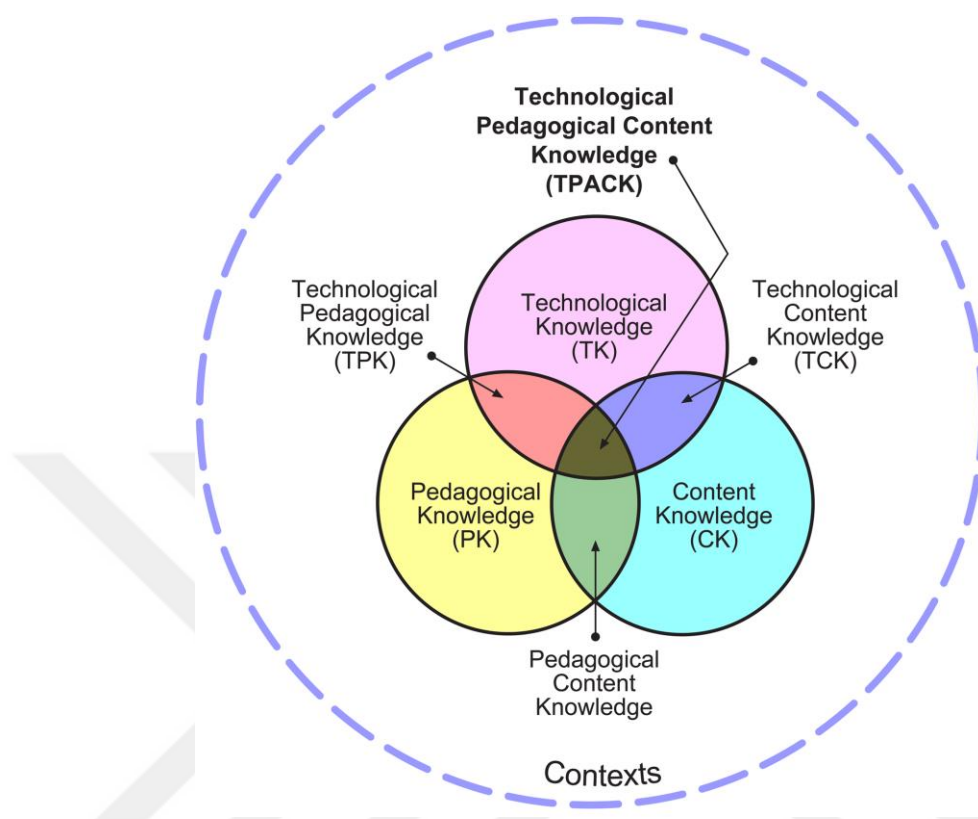


Figure 6.2 The TPACK framework and its knowledge components (Harris, Mishra & Koehler, 2009, p. 396)

Considering the knowledge components of TPACK, Susan's focus was on developing students and her technological knowledge rather than developing the topics with technology i.e. her Technological Content Knowledge (TCK). On the other hand, Chris provided information regarding how technology had changed his knowledge related to the logic of the development of the topic (i.e. discussion about the scale factor 1 and 0), which is an example of Chris's development of Technological Content Knowledge (TCK). It is because he came to realise that the focus of subject matter can be changed by the application of GeoGebra.

It never occurred to me give them a question where it says enlarge it by scale factor 1 because the idea in a textbook or on a worksheet enlarge it scale factor 1, that is just waste of space, it doesn't do anything, nothing changes. Whereas it was using GeoGebra made me realize that really is important. It is the identity. It is the multiplicative identity. (Chris)

Furthermore, teaching Circle Theorems with GeoGebra, Susan encountered a problem where GeoGebra was not set to show whole number values (instead it was set to show values to two decimal place), which she thought was confusing for students to see the relationship between angles. Susan's response on this issue (discussed in the Resource System section) was again related to developing her Technological Knowledge i.e learning to set rounding to show whole number values in GeoGebra.

That is something where I am new to GeoGebra. I have learned that about GeoGebra now and I will get it sorted. The awe and wonder “oh look wow it is the same or wow it is doubled” would be more effective if that bit of GeoGebra I dealt with. (Susan)

In contrast, Chris mentioned a Circle Theorems file that he had used before in which the rounding was set to show whole number values. He thought that working with this file created confusion for students due to rounding errors. Although initially he wanted to change this file, but after some time he purposefully made use of it for whole class discussion to make students think about rounding. This example relates to Chris developing his knowledge that is within Technological Pedagogical Content Knowledge (TPACK).

I decided this was very bad because the students might think that actually it is not double. After that, it was about a year later I suddenly realized this was really useful because why it does not appear to work because of the rounding errors. I then used this file I deliberately made it there was a problem like this. I asked the pupils what has happened. (Chris)

Moreover, whole class lesson segments, where Chris used GeoGebra on IWB to discuss different ways of solving mathematical problems, could be placed in TPACK, since during such lesson segments his content, pedagogy and technology knowledge interacted with each other. In particular, in Circle Theorems lessons, when he discussed aforementioned Sherpa student's example with the whole class, his instrumental knowledge of GeoGebra and IWB allowed him to represent and discuss the content and particularly involving sherpa student in the process supported his pedagogical approach and also choosing a specific example aimed to connect students with subject matter as a whole class.

That was a really nice way of doing that because she (Sherpa-student) was able to be absolutely fully involved. I was asking her to do particular things I knew they would be helpful. We could then talk about as a class. Yeah that was particularly exciting and that of course can't happen without the technology. (Chris)

In terms of Tom, as mentioned with being the least experienced teacher in mathematics teaching, his curriculum scripts were less accessible comparing to the Chris and Susan. Having said that, it was clear that he was confident in his technological knowledge and there were indications of him developing his TCK (i.e. revising his worksheet for Straight Line Graphs).

In summary, it became apparent that technological and teaching experience level of teachers showed differences in the elements of teacher knowledge within their curriculum scripts. Similar to the results of Drijvers et al. (2013) study, this study suggests that the most experienced teacher in teaching with technology is able to integrate the TPACK skills “at stake in a satisfying and effective way” (p.994).

6.4 Limitations

This study included three mathematics teachers with different level of GeoGebra experience in teaching, which made it possible to compare and contrast practices of different teachers. However, it became clear through setting up this study that mathematics lessons with GeoGebra are not a frequent occurrence, even for an advanced skills teacher with a particular interest in the software. In this regard, difficulties arose in obtaining consecutive sessions for observation and interviews, and also in observing teachers teaching the same topic with GeoGebra to different classes. This placed a limitation on analysing teachers' curriculum scripts fully as discussed in the previous section. In addition, particularly for curriculum scripts, this study indicated that teachers have to have some experience and to have changed the way they think about teaching the topic so as to incorporate the technology in an innovative way in order to get to the stage where they really think about their curriculum script. Therefore, only observing one experienced teacher (Chris) did not provide a variety of information especially when this particular experienced teacher's pedagogy is relatively unusual even in England, when one considers the points made

in UK research studies (e.g. Bretscher, 2011). Thus, some of the findings presented in this study might not translate into other settings inside (and outside) England.

Furthermore, this study showed that Ruthven's (2009) conceptual framework including pre-specified constructs served well to capture influences on teachers' appropriation of new technologies in ordinary classrooms. However, it did not provide enough information about the framework's completeness to evaluate whether there are any missing aspects from the framework. An explicit question was included in the interview protocol to ask the teachers whether there were any important issues involved in working with this technology that they had not had an opportunity to talk about so far during the interview. Therefore, the interview responses to this specific question were the main determiner as to whether aspects were missing from the framework. Including an explicit question in the interview protocol guarded to some extent against a tension between the theory-driven coding and the more inductive investigation of possible gaps in the framework. All the teachers brought up the confidence level of the teacher at using technology. This has been reported by other studies (Crisan et al., 2007; Laborde, 2001) as a factor affecting teachers' use of technology in their classroom, but from a SFCP perspective, confidence in teaching with technology would be a result of overcoming the types of practical issues which SFCP highlights rather than referring to it as one of the features itself.

There was also a methodological limitation to consider. During classroom observations, teachers wore an audio recorder in order for me to have a full record of oral interactions between teacher and students; however, such a data collection tool did not capture the related computer screens that they were having discussions about. Therefore, audio recording only was not enough to fully present classroom data during students' individual working at computers with the teacher circulating around the classroom and having discussions with each group. In this sense, it is essential to follow teachers with a video camera during such investigative lessons where students are active in the dynamic exploration.

6.5 Implications

Despite the small number of participants, this study illustrated how teachers adapt their classroom practices and developed their craft knowledge through incorporating

GeoGebra software into their mathematics teaching. The current study also suggests a number of ways in which research on technology integration focusing on structuring features of classroom practice, can contribute to research and practice. This section addresses a few of these contributions more fully.

6.5.1 Implications for Teacher Professional Development

The SFCP framework proved to be a powerful way to organise and examine the various craft aspects of classroom practice associated with technology use in schools. This framework implies that the process of technology adoption depends on teachers' professional knowledge and learning from their experience in classrooms. Therefore, it suggests that future research should pay more attention to understanding teachers' craft knowledge and thinking about certain aspects of classroom practice in relation to technology use. This knowledge and thinking on certain features of classroom teaching with new technologies can contribute to other teachers' thinking in a similar context.

Therefore, comparing the experienced and novice teachers' classroom practices with the use of technology through considering the five craft aspects of the SFCP would reveal important insights into what other teachers might do to integrate technology in their own lessons. This could be achieved through presenting a report of the teachers' teaching with technology in ways that could be accessible and useful to other teachers. It is because this study reports not just problems that a teacher commencing integrating technology would encounter, but also more practical solutions to such problems. Indeed, even for a more experienced and proficient teacher, making the results of this study accessible might be illuminating: one of the participant teachers in the current study reported that it had allowed him to have a greater insight into his own teaching. In his own words: "I learned so much about my own teaching from reading it" (Chris).

6.5.2 Implications for Teacher Education

This study also has implications for teacher education programmes. It is of value for teacher education in that it potentially provides useful examples, which can be used in training student teachers and thinking about the potential problems and the kinds

of knowledge teachers require in delivering such lessons. In addition, by pointing out certain craft aspects in classrooms, the SFCP framework can guide student teachers on what to focus in order to gain access to the craft knowledge of experienced teachers in their school based components of pre-service education programmes. This is important because “while we recognize that there are those with mastery of some aspects of teaching, we have no coherent account of what they are masters of and how they achieve what they achieve” (Brown & McIntyre, 1993, p. 13). Hence, the analysis of five structuring features could provide student teachers with a powerful framework to observe experienced teachers in classrooms. For student teachers, examining in particular an experienced teacher's curriculum script could be an insightful way to portray teacher knowledge.

6.5.3 Bridging the Research-Practice Gap

I was guided in this study by concerns that the findings of research into technology integration must be made useful and relevant for teachers. Ruthven et al. (2008) elaborated the idea of interpretative flexibility in technology integration particularly in the way in which conceptions of dynamic geometry differ between its pioneering advocates and more typical classroom practitioners.

Mathematics educators have tended to interpret geometry software as a means of supporting approaches to school mathematics based on relatively open student exploration, experimentation and investigation. However, pioneering projects have shown that teachers find it challenging to conceive appropriate tasks, and to manage student traversal of a mathematics curriculum through tackling them. (p. 314)

McIntyre (2005) described this ‘acknowledged gap between research and practice [as] primarily a gap between two sharply contrasting kinds of knowledge’ about teaching and learning (p. 357): placing ‘craft knowledge for classroom teaching’ at one extreme and ‘research findings and conclusions’ at the other (p. 361). In addition, McIntyre argued that members of the academic community must take responsibility to bridge that gap by seeking out and working with the craft knowledge of teachers, and should not simply expect practitioners to engage with research knowledge. In this sense, the SFCP framework’s use of relatively accessible terminology might encourage practitioners, who are not involved in academic

research to read research studies and help to enhance their knowledge, thus fulfilling the aim of education research to directly impact upon teachers' practices. As one of my informal conversations with a participant teacher showed, while he had not come across the idea of an “activity format” or “activity structure”, he found it easy to recognise that this was expressing something that underpinned his practice. Therefore, it seems that the value of using these ordinary terms is to make these concepts accessible and to encourage teachers to think about them as they are important part of their use of technology in teaching.

6.6 Recommendations for Future Research

This study not only investigated the teaching practices and craft knowledge of three teachers with the use of GeoGebra but also showed that the SFCP framework offers researchers and educators tools of language with which they can describe, discuss, and understand teachers' classroom practices and knowledge for technology integration. This study also suggests areas for further research.

First, as has been discussed before, in order to reveal teachers' curriculum scripts more fully, their teaching the same topic to different groups needs to be observed for a period of time, which would provide an opportunity to examine the development or reorganization of scripts over time. Second, non-technology lessons of teachers alongside technology lessons (teaching the same topic) need to be studied and described in order to find out to what extent technology has changed their practice and knowledge, focusing on the five aspects of the SFCP. This study was only able to show a small amount of evidence regarding the comparison between Chris's technology and non-technology lessons, in which the main aspects of his teaching did not differ much.

Third, further studies should focus on what kinds of initiative in teacher education might contribute to developing beginning teachers' knowledge for technology integration. Identifying the professional knowledge expert teachers are using might help guide the development of teachers' craft knowledge. In addition, student mathematics teachers' possess classroom scripts, which need to be revealed and examined in order to change their focus from a student's to a teacher's. The knowledge that could be gathered through these lines of inquiry has the potential

to provide teacher educators with a framework for working with practicing teachers and preparing new teachers.

In terms of the scope, while the focus of this study was on the dynamic mathematics software, GeoGebra, there was some evidence that it was the joint use of IWB and GeoGebra that promoted whole class dialogical interaction. In this sense, a further study should focus on teachers' use of GeoGebra, especially in IWB-equipped classrooms to investigate ways in which the joint use of GeoGebra and IWB enhances interactivity in the mathematics classroom (Lavicza & Papp-Varga, 2010). It is because IWB is a presentational medium that has been found to introduce additional pedagogical challenges as well as opportunities for the classroom use of technologies (Miller, Averis, Door & Glover, 2004).

6.7 Concluding Remarks

Education change depends on what teachers do and think– it's as simple and complex as that. . . If educational change is to happen, it will require that teachers understand themselves and be understood by others.

(Fullan, 1982, p. 107)

This study, in re-focusing on the teacher dimension (Lagrange et al., 2003), provides insights to help those engaged in professional development on the issue of the integration of technological tools in teaching – mathematics teachers, teacher educators and researchers – to build a holistic understanding of the conditions of technology use in ordinary classrooms. By comparing teachers with different levels of experience, this study showed that, as teachers developed their experience in the use of technology, they counted on first establishing a functioning resource system, then proceeded to instrument their activity formats with the use of technology, and eventually reoriented their curriculum scripts as a response to use of technology for the specific topic taught. They also developed time management strategies, which fed into their fluency in technology lessons. In addition, this study showed how listening to experienced teachers and contrasting them with novice teachers within examples of technology integration was crucial in enabling researchers to examine, refine and extend their conceptualisation of technology integration.

Furthermore, this study provided a further empirically-based test of the usability and usefulness of the SFCP conceptual framework (Ruthven, 2009) as a tool for advancing research on technology integration. Applying the SFCP framework in this study proved to have potential as one means to move us towards more refined descriptions of the pathway from novice thinking to expert understanding of technology integration. This study confirmed that five aspects of classroom practice including working environment, resource system, activity format, curriculum script and time economy provided the tools of language to help students teachers, practicing teachers, teacher educators, and researchers to benefit from the practical knowledge about technology integration. Moreover, connecting the SFCP framework with other frameworks provided richer descriptions and understanding for some constructs of the SFCP. In respect to activity structure concept, the findings of this study also contributed to extending the Instrumental Orchestration framework to some extent.



7

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APPENDICES

Appendix 1 - Observation Protocol

Key Themes	Questions guide observation structure
Theme 1: Working Environment	I will pay attention to physical surroundings, technical infrastructure and social organization by answering following questions Where is the lesson taking place? What is the basic infrastructure of computer room? Is there any projection devices or IWB? Is a computer also available to the teacher? Is there any additional management required? Are students seated individually or in pairs at a machine? At any point is there a movement around the classroom? Is there a need for the students to use resources? Does the teacher have to set up procedures for anything? How is teacher doing the whole class in computer room, for instance? Does that reflect any particular organisation of the computer room?

Theme 2: Resource System	Specific attention will be paid to available resources and who uses them Does the teacher use any mathematical specific resources to handle of mathematics? Do students also use software? What connections are there between students and teachers? Is there any investment in helping the students understand how to operate technology?
Theme 3: Activity Format	I will look at what the structures of interaction are What do students do? What does the teacher do? Is there exposition, questioning or any discussion between teacher-students or among students? Is it mostly whole class teaching? Are students exploring themselves? Are there any way and which the involvement of the computer is actually subtly changing the nature of the interaction that takes place?
Theme 4: Curriculum Script	What is the lesson agenda? Is it only improving existing approaches or involving more dynamic presentations? Is it the adaptation and extension of established curriculum scripts?
Theme 5: Time Economy	Is there any issue related to timing and pacing?

Appendix 2 - A Transcript of a Recorded Lesson

Verbatim transcript of Lesson 2 of Chris teaching Circle Theorems with GeoGebra

Turn your face in. where we got... We are aiming for a particular mathematical topic. There are some things that you need to know and some things that we need to be able to do with the stuff you are finding out rather than me telling you here are some rules, you are exploring, finding out things yourselves. This is the other thing that we are doing. We are also exploring how there are bits of maths not just here is the topic but way we come up with mathematical ideas. What we then do after we conjecture, what I want you to do during this lesson, this is the dangerous lesson. The reason why it is dangerous is that what we are going to do with those. We really need to do a lot.

First of all I have several slides up here just got some circles on (*he is showing slides on the IWB*). You can use the arrows to move to the next page. It has got more of those. I marked the centre of the circle. I deliberately haven't put dots around the edge because what we are looking for now are things that true for all of the circles you try. We had a word last week start with C; it meant an idea, mathematical idea. You think it might be true you then try it out, starting with C. It might be true you need to try out. *A Student:* Conjecture. *The Teacher:* Yes, Conjecture. Our Conjecture is our idea. Hypothesis often gets used with statistics, physics. Conjecture is often gets used with maths.

Over the next five minutes. I will release your screens. So you can get properly logged on and loaded up stuff from last time. While you are doing that can you make sure you are only using GeoGebra? You are not on the Internet, checking emails or anything else. Can you also make sure you are also thinking what things do you think might be true? What conjectures are there? if you got a conjecture, you could then come up and draw it on here. There aren't any dots. You could just draw your shape to any points on the circumference; you can just jot something down next to it. What is going on? If you write a conjecture that turn out not to be true, that is fine.

Make sure you logged on, found your stuff and then come up and write some things.

(5.30 min) Students are working at the computers to find out their conjectures.

The Teacher is circulating in the classroom and joining in students' discussion by asking questions.

What have you found? What is your conjecture?

This one is 15 degrees. This one is 15. Could you load up a fresh page? Which has got one of my circles on. Joined them up one bit. Measure the angle. And see what happens? Could you go and put your conjecture up?

What is the deal with those angles? We need that up on the board.

That is beautiful. Tell me what you found.

There is a link between those numbers you are telling me. Have a look. You will know what is going on then.

(20.35 min) Whole class teaching

Turn and face me please! Now so far we have got some conjectures, where conjectures are ideas. We are not yet sure if they are right. Maybe they are not true. What do we do then as mathematicians with that conjectures? We want to try and prove it if it is right or wrong. How might we do that? How might we decide whether particular conjecture is right or wrong? We could test out some things. We could test out some of the ideas. We could draw more diagrams and we could see what happens. Are there any other things we could do?

Here is what I liked to do please

Have you come across counterexample? A counterexample is an example that does not work. You might look for one of those.

Could you please save the work you have done, start a new geogebra file? I want you to choose one of those conjectures. I want you to start testing it out without using dots on the circle anymore.

Now here are our conjectures. First conjecture is that radius bisects the chord. When does the radius bisect the chord when does not it? If you are using geogebra, if we got a chord, if we have got radius cutting it, we might need to do something with that.

Second conjecture says angle in a semicircle is always 90 degrees. They have drawn a diameter. That angle they reckon is 90 degrees.

Next one - year 11 there are couple of people are not focusing here.

The angle at the centre and angle at the circumference one of them is doubled the other. We could check that out as well.

Opposite angles in a cyclic quadrilateral that means all of his vertices on a circle. Opposite angles added up 180 degrees. Prove that it is true; prove that it is not true.

They haven't used the centre of the tool this time. These three angles they reckon are the same and you draw three different angles. All the same is that true. Take your pick one of those conjectures. Could you explore it what information can you get?

(29 min) Students are picking up a conjecture and trying to prove it at the computers.

(36 min) Whole class Teaching (Sherpa-at-work)

Ladies and gents if you are ready please everybody is looking please and ready. I remind you earlier on I said that we needed to make some progress, get to the next stage. There are some people did brilliantly. On your screen and on mine you can look at here or at yours. This one is absolutely brilliant. It is very very important and useful. We have got a chord. The important chord is chord BC. We have got two sets of line coming of chord BC. Emma has measured the angles 36.7 degrees from both of them. Could you do what you have showed me Emma? As she moves that around what is going on could somebody explain what is going on there?

A student: The angle is the same wherever it goes.

One of the big idea we need: that angle is always the same. Did anyone find a situation where is not always the same. Now Emma could you move the point show

us where it is not the same? There is couple of issues there, it measuring the reflex angle, next to that? Obtuse angle, I can gesture at you. This one here is the inside version of this. Is that one in there? Emma has found where it does not work. Could you take it back where it does work? 36.7 degrees. Now when does not it show 36.7 degrees. When did Emma break it? Rosa? If Emma goes past point C, Rosa reckons it breaks it. When is it going to snap back? Could you keep on going anti clockwise? We are back, well-done Rosa! If you just drop it there. I am going to ask Emma to make one more change. Does it have to be all the time? If I want a bigger angle, what Emma makes that means than bigger angle? What change could she make so I have a bigger angle there. Move point B or point C further apart. Could you drag D again and see what happens? It snaps into something different.

Now then ladies and gents, second reason why Emma come up with brilliantly useful stuff is that we can see when it works and we can see when it does not work. Could you please now try and do something similar with whatever conjecture you are working on.

Appendix 3 – An Example of Field Notes of a Lesson based on the Observation Protocol

First Lesson of Chris teaching Circle Theorems with GeoGebra

This one-hour lesson involved a Year 11 class of higher attaining students.

Working Environment

Lesson took place in a computer room where there were a number of computers with the software downloaded in U shaped and a computer for the teacher with an Interactive Whiteboard as well as tables/seats in the centre of the room where students can seat for whole class teaching. At the beginning of the lesson, teacher used IWB in order to give instructions about the software while the students were seated in the centre of the room. He showed some examples from previous lessons where students worked on drawing triangles and calculating angles by using a 9-point circle with paper-and-pencil. By building up on previous lessons, he wanted students to use Geogebra to make some conjectures, which are true with every circle (which don't have 9 point on the circumference). Students moved to the computers to work on their task as small groups. In this sense there was a movement from the students' side and the teacher was walking around the classroom to check their work.

At the end of the lesson, the teacher instructed students save their work and log off as they were going to continue the next day.

Resource System

The resource system for this lesson consisted of GeoGebra. He wanted his students to use GeoGebra to quickly measure angles and to utilise dragging function so that they could make conjectures and gather more evidence in a more straightforward way. During this, I observed most of the students seemed quite excited and motivated. Additionally, Chris used GeoGebra himself to show some tool techniques on the IWB for whole class teaching.

Activity Structure

The lesson broke down into 3 segments. Initially the teacher introduced the lesson in a whole class segment. The teacher showed the software techniques specifically related to Circle Theorems such as how to measure angles in GeoGebra. In the second segment, students went onto the computers and worked with the software themselves as small groups (at least two students) while the teacher was walking around to see how they were doing (Walk-and-work-by). Finally, the teacher gathered the students in the middle of the room for whole class teaching in order to show and discuss students' examples (that he spotted during work-and-walk-by), which aimed to guide other students what type of conjectures they could make and also to show them different ways of thinking and involve in discussion by questioning.

Curriculum Script

He saw Circle Theorems as a topic in which students can work on conjectures and proof of a set of related theorems. Also, his curriculum script for this topic included for students first to work on paper-and-pencil activities to build up the diagrams.

Time Economy

After showing some tool techniques, Chris provided students with some tool time for them to get accustomed to the software before they started exploring conjectures related to Circle Theorems. He quickly circulated around the classroom to check on students and made sure that all of them got on with their work. Also, by spotting and sharing students' examples, he showed some ideas of what to do to the students in case they were stuck.

Appendix 4 - Interview Questions

Post-Lesson Interview (around 30 min) – after two observed session

I'm particularly interested in how using technology in this lesson might make it rather different to organise and run from a similar lesson in which you were not using technology. I'd like to ask you about several aspects of this.

[WORKING ENVIRONMENT]

Does working in the computer room with this technology affect what is involved in organising the work and managing students?

Are there any particular aspects of organising work or managing students that you associate with teaching in the computer room?

[RESOURCE SYSTEM]

Today you were using GeoGebra as a resource for teaching the topic X (Transformations, Circle Theorems or Straight Line Graphs)

First, what's different about the way in which this technology allows this topic to be treated compared to using more traditional classroom mathematical resources?

Second, how does today's work on the topic with this technology fit together with other work on the topic that the class has done or will do without this particular technology?

Third, was there anything that you had to, before or during today's lesson, to make sure that the pupils would be able to cope with the particular way in which the technology operates?

[ACTIVITY STRUCTURE]

Thinking now about the interactions between you, the students and the technology. Were there any types of interaction in the lesson today that you would see as particularly associated with working with this specific technology, or with working with ICT more generally?

[If so] What's distinctive about that type of interaction?

[CURRICULUM SCRIPT]

Before you access to this technology, did you teach a similar type of lessons?

If Yes- how has using this technology changed the way in which the lesson progresses?

Are there other new activities in the lesson? How have the activities you had before changed? Has sequence changed? Has pace changed? Other new possibilities they were not before that you include in the lesson?

If No- with this technology what is different about the kind of lesson you will teach? As you taught this lesson using the technology a number of times has anything changed? Or you changed your approach to it or found new ways of doing it?

[TIME ECONOMY]

What issues of timing or pacing came to mind when you were preparing today's lesson or teaching it?

Did the fact that you were using the technology affect the timing or pacing of the lesson today, or your thinking about these issues?

[MISSING ASPECTS]

Are there any important issues involved in working with this technology that you haven't had an opportunity to talk about so far in the interview?

[If so] Tell me more about that.

Appendix 5 - A Part of an Interview Transcript

A Part of Chris's first interview after Circle Theorems lessons with GeoGebra

Working Environment (Computer room where there are also seats and tables for students in the middle of the room)

Does working in the computer room with this technology affect what is involved in organising the work and managing students?

Yes, but in a good way. I liked the layout of that room because it is a clear that it is a classroom and it has computers. So it is not like going to a computer room for a computer lesson and there are... so that is one thing. Another thing useful about it is that I can see all of their screens. So if I stand in the middle and turn my head I can see everybody's screen, which means that I know immediately if somebody isn't doing what they should if somebody is just checking email or if somebody using a different program. I can see that, that is easy. That is straightforward (in the other computer room two years ago I could not). I also liked the fact that it has got the software that'll let me spotlight what pupils had done and again in the other computer room children had to save their work, I then had to load it up that made it very awkward whereas here I can just click and I can spotlight one pupil's work which is wonderful. There is some software that is networked in that computer room that'll let me do that. That is really useful. The bits that aren't as nice is that it is quite a dark room it does not get cleaned as often as it should. It is just the environment is not quite as nice as some of the other rooms. Many of other computer rooms have carpet on the floor that one does not so that is the only bit. But the rest of it let me do exactly what I want to do. The fact that it has got control software it is wonderful. The fact that I can blank their screens so they can't do anything is brilliant. The fact that I can spotlight what a single pupil is doing is brilliant. There are so many good things about it. The fact that it is not we are going to have a computer room lesson. It is our classroom and you turn around and you use computers.

Resource system (GeoGebra and IWB)

Today you were using GeoGebra as a resource for teaching the topic ‘Circle Theorems’. First, what’s different about the way in which GeoGebra allows this topic to be treated compared to using more traditional classroom mathematical resources?

The thing is the geogebra will let us do so many different extra things. It will let us do the dynamic part of it. It ll let us move points and see what happens. It will let us do lots of measuring without needing to do measuring or calculating so it ll measure without us needing to do those by hand. That is a huge time saver. That saves a lot of time over drawing them out. Beyond that I think geogebra also allows us to be freer with things. If you drag something and it does not work, you can do ctrl z for undo and it is back. On a piece paper if you got a diagram and you put an extra line on it and it is the wrong line you are going to try to rub that out or start again. That takes a lot of time. I think it is easier to make mistakes and it is easier to try things out if you have got GeoGebra and so I think it helps like that as well. So I think it helps pupils to take more risks. So that is geogebra. And then the IWB, I like that because the pupils could come up and write things without being the main focus. So there is one girl who says, “I don't want to write” I don't want to put up my ideas in case they are wrong. Actually I got her and a friend to go up and do it but I didn't refer to whose work it was. So it wasn't this is Lucie’s work lets have a look at. It was just we have now got some whole class work and what started off as her work had then turned into public ownership. You could say you could still do that with chalk on a blackboard except I think that is much more difficult. Partly it is more difficult to write and to do the diagrams but there is also an issue of space. We got 5 different conjectures and could scroll up and down whereas on an ordinary board you couldn't do that. But I have saved those and I now got those and get back to them tomorrow and that is useful as well. If they are written on an ordinary board, we have to re write them again tomorrow. I think IWB adds extra bit to it as well and I think also the IWB is nice because it shows the pupils who writing on it actually this is not as good for mathematics as GeoGebra and whereas with GeoGebra they could draw lines an angle, they would be straight line, it would attach itself nicely to points. On Smart-board file it will over shoot, it wouldn't fit nicely, it did not look as good.

So just a reminder that GeoGebra is a mathematical tool and IWB is a just presentational tool. They have been designed to do different jobs and I think it is quite useful as well.

What do you think about using the combination of the two?

Let me have some flexibility to do the things that we were doing, lets me have the flexibility to get the pupils to investigate and explore in a mathematical environment and then for us to share and present those ideas using a presentational tool. As I say next time I am very keen we haven't explored the proof we haven't proved any of the things we have done. That we would do the next time.

So you did the first two lessons without technology. Could you please tell me more about this choice and links between technology and non-technology lessons to teach this topic?

We haven't studied circle theorems at all before. That is where as you say links into the lessons that you saw (first two lessons) where there I wanted them to get their hands dirty, I wanted them to draw a triangle and discover they could work out all of the angles, they could join radiator to the centre. We have got three radia, that we have got isosceles triangle. I wanted them to do that. That felt important but then I also after they have done that I also wanted them to be able to see things in a dynamic way, which pencil and paper can't do. I wanted them to see and to do calculations where they haven't got nice equally spaced number of points. I wanted them to do those sorts of things and to gather lots of lots of evidence. GeoGebra lends itself to doing that as well. So both of them seem to me to be a really important way of doing things. And I absolutely see a place still for pencil and paper activities. I absolutely see a place for manipulatives and things for pupils to interact with. I also see a place for GeoGebra and other digital tools, which will let us access maths.



Appendix 6- Letter of Consent

Informed Consent

Gülay Bozkurt
PhD in Education
Faculty of Education
University of Cambridge, UK

Dear teachers,

For my doctoral degree at University of Cambridge, I am carrying out a substantial piece of research to learn more about how secondary mathematics teachers develop their understanding of the use of GeoGebra in their classrooms. Therefore, I hope to interview teachers, and also observe these same teachers giving a number of mathematics lessons in which GeoGebra is being used in their classrooms.

Participation in the interview and the observation is voluntary, and participants have the rights to refuse taking part in the study, either for interviews or observations as well as the rights to review materials in respect of the transcriptions of interviews and observations. For the purpose of data preservation, the interviews wish to be audio-recorded; however, if the participant is uncomfortable being audio-recorded I will take notes. Therefore, I will be present and be recording the events through either audio recording or field notes.

All the information you provide will remain anonymous and confidential. The data and results of the data will be used for academic purposes only. Participants' names and personal information will not be used in reporting the outcomes of this research unless they wish their contributions to be identified, or acknowledged.

If you have any questions or concerns about how the information I collect will be used, or you would just like to know some more about the research, please ask me or email me (gb408@cam.ac.uk). It is my greatest wish that this study would facilitate the improvement of and contribution to educational development in mathematics teaching with the use of digital technologies. I do sincerely hope you will be willing to be involved and thank you very much in advance for participating in this study.

With kind regards and many thanks,

Yours sincerely,
Gulay Bozkurt



Appendix 7 - A Summary of the Tasks during Seven Lessons of Chris Teaching Transformations

LESSON	TASK	AIM
Lesson 1	To carry out transformations (Reflection, Rotation, and Enlargement) using prepared GeoGebra files.	To get students to be familiar with the software.
	To explore what points they can drag directly in files and what points they cannot.	To get them starting to think about some of the reasoning.
Lesson 2	To solve some problems about Reflection, Rotation and Enlargement such as finding the mirror line and finding the centre of enlargement.	To begin to solve problems and to expand their knowledge of transformations.
Lesson 3	Following from their homework, which had been to write down what they thought were important ideas from the last two lessons, to draw things different people used in homework together in groups and make a poster.	To get them engaged with the topic by comparing different methods and ideas.
Lesson 4	To work out the scale factor of enlargements.	To fix for the students the idea that they can see

	The question was ‘Where on the diagram you can see evidence of that scale factor?’	‘scale factor’ and to deal with the misconceptions about the area and about where the scale factor starts from.
Lesson 5	By working with an Enlargement file, to find out what interesting, important scale factors there are.	To engage with the different scale factors of enlargement.
Lesson 6	By working with a blank GeoGebra page, to explore what combination of reflections will have the same effects as the rotation.	To get them started with some geometrical problems.
Lesson 7	By working with a blank GeoGebra page, to explore the relationship between the two enlarged triangles with two different centres of enlargement and two different scale factors on the sliders.	To help them become competent at problem solving.