İSTANBUL TECHNICAL UNIVERSITY ★ INSTITUTE OF SCIENCE AND TECHNOLOGY

ONE-DIMENSIONAL VISCOPLASTICITY MODEL USING FRACTIONAL DERIVATIVE OPERATORS

M.Sc. Thesis by
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Department : Mechanical Engineering

Programme : Solid Mechanics

FEBRUARY 2011

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İSTANBUL TEKNİK ÜNİVERSİTESİ ★ FEN BİLİMLERİ ENSTİTÜSÜ

KESİRLİ TÜREV OPERATÖRÜ KULLANILARAK ELDE EDİLMİŞ BİR BOYUTLU VİSKOPLASTİSİTE MODELİ

YÜKSEK LİSANS TEZİ

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FOREWORD

The aim of this study is to obtain a new one-dimensional viscoplasticity model by using fractional calculus. In this study to introduce this model theoretical studies, numerical progrramming as well as experimental works in the laboratory were done.

I would like to express my deep appreciation and thanks to my supervisor in İstanbul Technical University Prof. Dr. Ata Muğan for encouraging me to work on this issue, and to achieved it abroad, being a perfect guide and having endless indulgence. I would like to thank my thesis supervisor in Ruhr University of Bochum Prof. Dr. rer. nat. Klaus Hackl who gave me opportunity to join his group for his support and immortal help. With the rewarding contribution to this thesis, I would like to Express my deep appreciation and thanks to my adviser in Ruhr University of Bochum Dr. Inz. Jerzy Makowski for not being spare his help, supporting me in every way whenever I abandon myself to despair about my thesis, being really good guidind and having endless indulgence. It was a pleasure and honor for me to be both of my supervisors's and advisor's student.

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ABBREVIATIONS

PA6 **:** Polyamide 6

HDPE : High Density Polythylene

GL **:** Grünwald Letnikov scheme

G **:** Gear scheme

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LIST OF SYMBOLS

σ	stress
E	elastic modulus
ε	strain
η	the viscosity parameter
τ_σ	the relaxation time for constant stress
τ_ε	the relaxation time for constant strain
E_R	elastic modulus
t	time
ε^e	elastic strain
ε^p	plastic strain
σ_y	yield stress
$\dot{\varepsilon}^p$	plastic strain rate
$\dot{\gamma}$	plastic multiplier
sign( )	signum function
μ	the viscosity_related parameter
$\in$	the non_dimensional rate sensitivity parameter
Φ	yield function
$\bar{\varepsilon}^p$	the accumulated plastic strain
GL	the Grünwald–Letnikov fractional differential operator
Δt	time step
δ^-	delay operator
α	fractional order
C	Grünwald–Letnikov scheme coefficient
Γ	gamma function
A	Grünwald–Letnikov scheme coefficient
B	Grünwald–Letnikov scheme coefficient

τ	relaxation time
E_0	the relaxed elastisite moduli
E_∞	the non_relaxed elastisite moduli
σ_0	initial stress
$\langle . \rangle$	the McCauley brackets
m	the direction of $\dot{\varepsilon}^p$
g	potential function

ONE-DIMENSIONAL VISCOPLASTICITY MODEL USING FRACTIONAL DERIVATIVE OPERATORS

SUMMARY

In this study, a new one-dimensional viscoplastic model was developed by using fractional calculus. This topic required theoretical studies, numerical programming at the computer as well as experimental work in the laboratory.

Mathematical concepts of creep and relaxation for fractional viscoplastic model was defined. These definitions were compared with creep and relaxation definitions for classical viscoplastic model using numerical programs. Creep-recovery and relaxation tests were performed using Polyamide 6 (PA6) dog-bone specimens in viscoplastic region. Experiments were tested on PA6 dog-bone specimens in mechanic laboratory of Ruhr University of Bochum in Germany. Fractional viscoplastic model parameters were defined for PA6 material by using test results and curve fitting method. Program codes were run again using new parameter values and material constants. Fractional viscoplastic model and classical viscoplastic model were compared with experimental tests results. Fractional viscoplastic model give closer results to the experimental tests results than classical viscoplastic model is deduced. However this new fractional viscoplastic models limitation for creep was determined.

KESİRLİ TÜREV OPERATÖRÜ KULLANILARAK ELDE EDİLMİŞ BİR BOYUTLU VİSKOPLASTİSİTE MODELİ

ÖZET

Bu çalışmada kesirli türev hesaplamaları kullanılarak yeni bir boyutlu viskoplastik model geliştirilmiştir. Bu konu, teorik çalışmaları, bilgisayarda sayısal programlamaların yanı sıra laboratuarda deneysel çalışmaları da içermektedir.

Kesirli türev hesaplamalı/Fraksiyonel viskoplastik model için sürünme ve gevşemenin matematiksel kavramları tanımlandı. Bu tanımlamalar klasik viskoplastik modelin mevcut sürünme ve gevşeme tanımlamaları ile nümerik programlar kullanılarak karşılaştırıldı. Sürünme-düzelme ve gevşeme testleri Polyamid6 (PA6) kemik şekilli numune ile viskoplastik bölgede gerçekleşirildi. Deneyler Almanya- Bochum Ruhr üniversitesi mekanik laboratuarında PA6 kemik şekil numuneleri ile gerçekleştirildi. Kesirli viskoplastik modelin parametreleri PA6 malzemesi için test sonuçları ve eğri-düzeltme metodu kullanılarak tanımlandı. Program kodları yeni parametre değerleri ve malzeme sabitlerini kullanarak yeniden çalıştırıldı. Kesirli viskoplastik model ve klasik viskoplastik model deney sonuçları ile karşılaştırıldı. Kesirli viskoplastik modelin klasik viskoplastik modele göre test sonuçlarına daha yakın sonuçlar verdiği tespit edildi. Ancak bu yeni fraksiyonel viskoplastik modelin sünme için sınırlamaları olduğu belirlendi.

1. INTRODUCTION

Nowadays there are many viscoelastic and viscoplastic material model exists. These material models applicability limitations are being developed day by day.

The observed behaviour of real materials is generally time dependent; that is, the stress response always depends on the rate of loading and/or the timescale considered. Rate-dependence effects are described by means of so-called viscoplasticity (or rate-dependent plasticity) models.

Many of the microscopic phenomena underlying the inelastic deformation of solids depend on time. Materials such as metals, rubbers, geomaterials in general, concrete and composites may all present substantially time-dependent mechanical behaviour under many practical circumstances.

The purpose of this master's dissertation, is to develop a new one-dimensional viscoplastic model by using fractional calculus. Accordingly, one-dimensional classical viscoplastic model introduced by Neto, Perìc, Owens [1] was used. This one-dimensional classical viscoplastic model was composed with one-dimensional viscoelastic model introduced by Bagley and Torvik [6,7] and fractional application.

In this context, classical one dimensional viscoplasticity model is modified by using fractional calculus. The notion of fractional calculus appears in various fields of science and engineering. A precise definition has been proposed by Riemann–Liouville and more recently by Caputo . We try to show the importance of fractional calculus for modelling viscoplastic materials.

1.1 Background and Objectives

In this work to describe the viscoplastic model, classical one dimensional viscoplastic constitutive model [1] is developed by using mathematical concepts of fractional calculus based on Gear scheme derivatives [2-5] and viscoelastic material model based on the one-dimensional constitutive equation introduced by Bagley and Torvik [6,7]. Curve-fitting aspects are focused, showing a good agreement with experimental data. Until the beginning of the 80s, the concept of fractional derivatives associated to viscoelasticity was regarded as a sort of curve-fitting method. Later, Bagley and Torvik [6,7] gave a physical justification of this concept in a thermodynamic framework. Their fractional model has become a reference in literature.

Here, in consequence of using Bagley and Torvik approximation which includes using fractional calculus definition in viscoelasticity, we also interested in viscoelastic material behaviour [12-17]. Some standard viscoelasticity models as Maxwell, Kelvin-Voigt and Standard Linear Solid are considered. Some experimental test results were determined which are done by Lai-Bakker [16].

For obtain the constitutive fractional viscoplastic model fractional calculus methods and definitions [8,9] were considered. Classical one-dimensional viscoplastic model introduced by Neto, Peric, Owen [1] was scrutinized. Neto, Peric, Owen were defined the mathematical constitutive equations of one-dimensional viscoplastic model considering phenomenological aspects of viscoplasticity as creep, relaxation and strain rate dependence. Particular definition of plastic multiplier introduced by Peric (1993) [10] and Perzyna (1963) [11] were used in classical one-dimensional viscoplastic model.

Constitutive model of creep and relaxation for this one-dimensional fractional viscoplastic model was defined. These definitions program codes were written by using Matlab. The macros were run for fractional and classical viscoplastic models and results were compared. In these comparisons for fractional viscoplastic model the codes were run for different fractional order derivative and the behaviour of fractional viscoplastic model for different fractional derivative values were considered.

To validate and analyze our viscoplastic approach, numerical examples are carried out for creep, relaxation, stress-strain dependence and also some experimental tests. To illustrate this modeling capability, the master curves of a viscoplastic material are presented, showing the agreement between the fractional model and experimental data. Furthermore, to identify the model parameters is tried by using the experimental tests data.

Experimental tests were performed using PA6 dog-bone specimens in viscoplastic region. After tests one-dimensional fractional viscoplastic model parameters were defined for PA6 material. These parameters and material constants were used in program codes and creep-recovery and relaxation curves were handled for one-dimensional fractional viscoplastic model. These creep-recovery and relaxation curves were compared with one-dimensional classical viscoplastic model creep-recovery and relaxation curves and experimental tests results. Fractional viscoplastic model give closer results to the experimental tests results than classical viscoplastic model is deduced. However this new fractional viscoplastic models limitation for creep was determined.

1.2 Outlines

The present master thesis is divided into eight chapters. A short outline of its organization is given in the following. In Chapter 2 Classical models of viscoelastic behavior is covered. Creep and relaxation viscoelastic phenomena are represented. Classical viscoelastic models, Maxwell, Kelvin-Voigt and Standard linear solid models were discussed and their creep and relaxation behaviors are examined.

Chapter 3 deals with viscoplasticity and viscoplastic model which is used in proposed new one-dimensional fractional viscoplastic model. This chapter presents a brief introduction to phenomenological aspects of viscoplasticity. The theoretical basis of one-dimensional classical viscoplastic model introduced by Neto, Peric and Owen, are represented and the establishment of mathematical model of it is introduced.

Mathematical concepts of fractional calculus is introduced in Chapter 4. Viscoelastic model based on the concept of fractional derivative introduced by Bagley and Torvik is represented. New one dimensional fractional viscoplastic model developed by using extension of fractional viscoelastic models include plastic deformation is defined.

In Chapter 5 this obtained new one-dimensional fractional viscoplastic models mathematical formulations and creep at constant stress, stress relaxation at constant strain definitions are determined. Some simple analytical solutions are presented to demonstrate the ability of the one dimensional model in capturing the fundamental phenomenological features of viscoplastic behaviour. By using these definitions and Matlab program, creep and relaxation graphics of our new viscoplastic model are obtained. These graphics are compared with one-dimensional classical viscoplastic model creep and relaxation plottings in Matlab.

In Chapter 6 the parameters of one-dimensional fractional viscoplastic model were identified. By using these parameters in Matlab simulink program codes for creep and relaxation same numerical simulations are performed. Then this simulation graphics were compared with one-dimensional classical viscoplastic model creep and relaxation graphics.

Chapter 7 deals with experimental tests results and verification of the one-dimensional fractional viscoplastic model. For this purpose failure, creep and relaxation tests were performed with PA6 dog-bone specimens. Tests results and its comparison with one-dimensional fractional and classical viscoplastic models are represented in this chapter.

Conclusion of this work is given in Chapter 8. The achievement and limits of this new one-dimensional fractional viscoplastic model are presented.

2. CLASSICAL MODELS OF VISCOELASTIC BEHAVIOR

2.1 Viscoelastic Phenomena

Most engineering materials are described, for small strains, by Hooke's law of linear elasticity: stress σ is proportional to strain ε. In one dimension, Hooke's law is as follows:

$$\sigma = E\varepsilon \,, \qquad \textbf{(2.1)}$$

with E as Young's modulus.

The theory of viscoelasticity is suitable on bodies which both exhibit elastic and viscous effects, i.e. simultaneous storage of interior energy and dissipation at deformation [12].

When the deformation is irreversible, the mechanical energy supplied to the material, is transformed (dissipated) into heat [13]. Normally, the transformation requires a certain amount of time and cannot occur instantaneous, as a time-independent plastic deformation.

If a material exhibits a combination of elastic and viscous behavior, the relation between stress and strain can not be prosperous described with material constants. Hence, the material behavior must be described by time-dependent material functions. At a general constitutive formulation, the stress becomes a functional of the strain, i.e. depending on the entire strain history. Thus, the stress at a given time is dependent on all previous moments [13].

At a certain time, the strain of a linear viscoelastic material increases linearly with the

stress, contrary to a non-linear viscoelastic material. When the strains exceeds the proportional limit, the behavior becomes non-linear viscoelastic.

Some phenomena in viscoelastic materials are

(i) if the stress is held constant, the strain increases with time (creep);

(ii) if the strain is held constant, the stress decreases with time (relaxation);

(iii) the effective stiffness depends on the rate of application of the load;

All materials exhibit some viscoelastic response. In common metals such as steel or aluminum as well as in quartz, at room temperature and at small strain, the behavior does not deviate much from linear elasticity. Synthetic polymers, wood, and human tissue as well as metals at high temperature display significant viscoelastic effects. In some applications, even a small viscoelastic response can be significant. To be complete, an analysis or design involving such materials must incorporate their viscoelastic behavior.

2.1.1 Creep

Creep can be defined as a progressive increase of strain in a material exposed to a constant load, observed over an extended period of time._Creep tests involve a sample with a set weight and observation of the strain change over time. Recovery tests look at how the material relaxes once the load is removed.

A typical time-dependent material at constant load obtains the appearance in Figure 2.1 [12]. The curve can be divided into three regions. In the primary region the initial strains appears instantaneous with the load and the strain rate $d\varepsilon / dt$ decreases. Even though the load is constant in the secondary region, the deformation continues to increase. The main part of the creep deformation occurs in the secondary region. Eventually, in the tertiary region, the strain rate increases until the material reaches the failure stress.

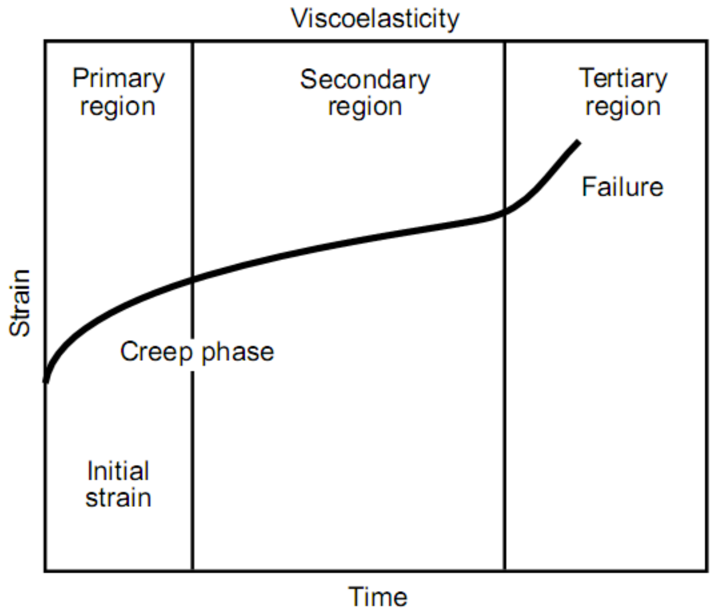

Figure 2.1 : A creep curve divided into three region.

2.1.2 Relaxation

Stress relaxation is the inverse of creep: a sample is held at a set length and the force it generates is measured [14].The sample is suddenly strained and then the strain is maintained constant afterward, the corresponding stresses induced in the body decrease with time. If the sample is subjected to a cyclic loading, the stress-strain relationship in the loading process, and the phenomenon is called hysteresis.

The features of hysteresis, relaxation, and creep are found in many materials. Collectively, they are called features of viscoelasticity.

2.2 Classical Viscoelastic Models

Classical linear viscoelastic material models can be represented by an arbitrary composition of linear springs and dashpots. These models, which include the Maxwell model, the Kelvin-Voigt model, and the Standard Linear Solid Model, are used to predict a material's response under different loading conditions.

The elastic components can be modeled as springs of elastic constant E, given the formula:

$$\sigma = E\varepsilon \tag{2.2}$$

where σ is the stress, E is the elastic modulus of the material, and ε is the strain that occurs under the given stress, similar to Hooke's Law, as previously mentioned.

The viscous components can be modeled as dashpots such that the stress-strain rate relationship can be given as,

$$\sigma = \eta \frac{d\varepsilon}{dt} \quad \textbf{(2.3)}$$

where σ is the stress, η is the viscosity of the material, and dε/dt is the time derivative of strain.

In figures (2.2), (2.3),(2.7) are shown three mechanical models of material behavior, namely, the Maxwell model, the Kelvin-Voigt model, and the standard linear solid model, all of which are composed of combinations of linear spring constant E and dashpots with coefficient of viscosity η. A linear spring is supposed to produce instantaneously a deformation proportional to the load. A dashpot is supposed to produce a velocity proportional to the load at any instant.

The relationship between stress and strain can be simplified for specific stress rates. For high stress states/short time periods, the time derivative components of the stress-strain relationship dominate. A dashpots resists changes in length, and in a high stress state it can be approximated as a rigid rod. Since a rigid rod cannot be stretched past its original length, no strain is added to the system [15].

Conversely, for low stress states/longer time periods, the time derivative components are negligible and the dashpot can be effectively removed from the system - an "open" circuit. As a result, only the spring connected in parallel to the dashpot will contribute to the total strain in the system [15].

2.2.1 Maxwell model

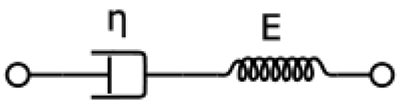

Figure 2.2 : Maxwell model.

The Maxwell model can be represented by a purely viscous dashpot and a purely elastic spring connected in series, as shown in the Figure (2.2) The model can be represented by the following equation:

$$\dot{\varepsilon} = \frac{\dot{\sigma}}{E} + \frac{\sigma}{\eta} \text{ (Maxwell model)} \quad \textbf{(2.4)}$$

When the material is put under a constant stress, the stress is transmitted from the spring to the dashpot and the strain has two components. First, an elastic component occurs instantaneously, corresponding to the spring, and relaxes immediately upon release of the stress. The second is a viscous component that grows with time as long as the stress is applied.

This stress produce a strain $\dot{\sigma}$/E in the spring and a velocity σ/η in the dashpot. The velocity of the spring extension is $\dot{\sigma}$/E if we denote a differentiation with respect to time by a dot. The total velocity is the sum of these two:

$$\dot{\varepsilon} = \frac{\dot{\sigma}}{E} + \frac{\sigma}{\eta} \quad \textbf{(2.5)}$$

Furthermore, if the stress is suddenly applies at the instant of time t=0, the spring will be suddenly deformed to $\varepsilon(0)= \sigma(0)$/E, but the initial dashpot deflection would be zero, because there is no time to deform. Thus the initial condition for the differential equation (2.4) is

$$\varepsilon(0) = \frac{\sigma(0)}{E} \quad \textbf{(2.6)}$$

The Maxwell model predicts that stress decays exponentially with time, which is accurate for most polymers. One limitation of this model is that it does not predict creep accurately. The Maxwell model for creep or constant-stress conditions postulates that strain will increase linearly with time. However, polymers for the most part show the strain rate to be decreasing with time.

2.2.2 Kelvin–Voigt model

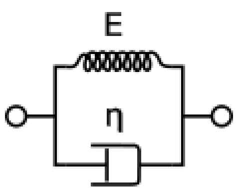

Figure 2.3 : Kelvin-Voigt model.

The Kelvin–Voigt model, also known as the Voigt model, consists of a Newtonian dashpot and Hookean elastic spring connected in parallel, as shown in the Figure (2.3) It is used to explain the creep behaviour of most materials.

For the Kelvin-Voigt model, the spring and the dashpot have the same strain. If the strain is ε, the velocity is $d\varepsilon/dt$, and the spring and dashpot will produce stress $E\varepsilon$ and $\eta\, d\varepsilon/dt$, respectively. The total force σ and so the constitutive relation is obtained as a linear first-order differential equation:

$$\sigma(t) = E\varepsilon(t) + \eta \frac{d\varepsilon}{dt} \text{(Kelvin-Voigt model)} \qquad \textbf{(2.7)}$$

If σ is suddenly applied, the appropriate initial condition is

$$\varepsilon(0) = 0 \qquad \textbf{(2.8)}$$

This model represents a solid undergoing reversible, viscoelastic strain. Upon application of a constant stress, the material deforms at a decreasing rate, asymptotically approaching the steady-state strain. When the stress is released, the material gradually relaxes to its undeformed state. At constant stress (creep), the Model is quite realistic as it predicts strain to tend to σ/E as time continues to infinity. Similar to the Maxwell model, the Kelvin–Voigt model also has limitations. The model is extremely good with modelling creep in materials, but with regards to relaxation the model is much less accurate.

To observe Kelvin-Voigt model's creep behaviour mathematical constitutive equation of this model was reduced to Matlab program codes (see Appendix B-1). On condition that to determine the parameter values of Kelvin-Voigt model, experimental creep test results (see Appendix B-2,3) for viscoelastic behaviour of High Density Polyethylene (HDPE) occurred in Lai-Bakker article [16] were used. The parameter values of Kelvin-Voigt model was determined using these test result values. Parameter identification was performed by using the method which was developed for Kelvin-Voigt model and presented in R.K.Makowska's lecture note [17] . According to this method parameter identification is obtained like below;

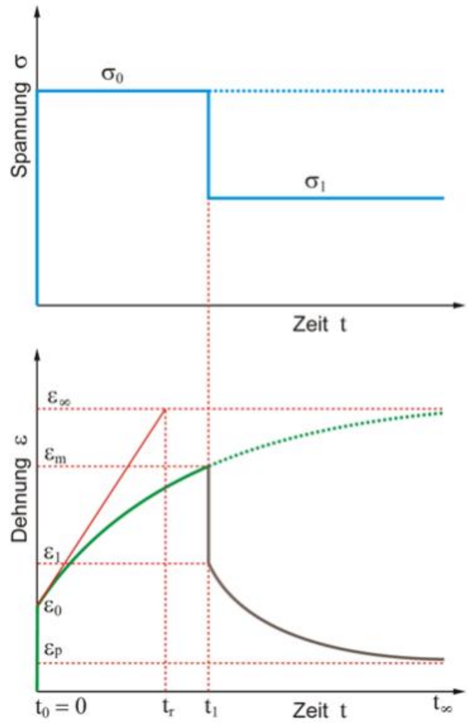

Figure 2.4 : Defining Kelvin-Voigt Parameters.

Kelvin-Voigt model mathematical definition;

$$\sigma(t) = E\varepsilon(t) + \eta\dot{\varepsilon}(t) \tag{2.7}$$

To define Kelvin-Voigt model parameters, E and η following formulas were used.

$$E = \frac{\sigma_0}{\varepsilon_\infty} \tag{2.9}$$

$$\varepsilon_{tr} = \frac{\sigma_0}{E}\left(1 - e^{-1}\right) \tag{2.10}$$

tr read from the graphics in Figure 2.4.

$$\eta = E.tr \tag{2.11}$$

For different σ values Kelvin-Voigt parameters were obtained like below by using Eqs.(2.9-11) and graphics values (Figure 2.4 and Figure 2.5).

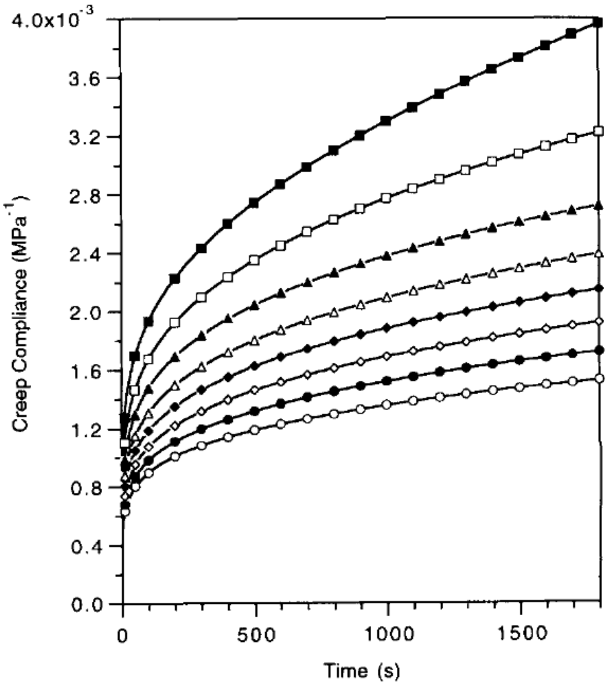

Figure 2.5 : The experimental data (markers) and model response of creep compliance at stress levels of 2MPa (O), 4MPa (●), 6MPa (◊), 8MPa (♦), 10MPa (Δ), 12MPa (Δ), 14MPa (⊙), 16MPa (☐).(Lai-Bakker-1995).

$$\sigma = 16 \rightarrow E = 250(Mpa); \eta = 27,5.10^3(Mpa.s)$$

$$\sigma = 14 \rightarrow E = 312,5(Mpa); \eta = 28,125.10^3(Mpa.s)$$

$$\sigma = 12 \rightarrow E = 370,37(Mpa); \eta = 37,037.10^3(Mpa.s)$$

$$\sigma = 10 \rightarrow E = 425,53(Mpa); \eta = 38,297.10^3(Mpa.s)$$

$$\sigma = 6 \rightarrow E = 526,316(Mpa); \eta = 52,631.10^3(Mpa.s)$$

$$\sigma = 2 \rightarrow E = 666,667(Mpa); \eta = 66,667.10^3(Mpa.s)$$

By using above σ,E and η values for Kelvin-Voigt model below creep compliance Figure 2.6 is obtained with Matlab (see Appendix B-1).

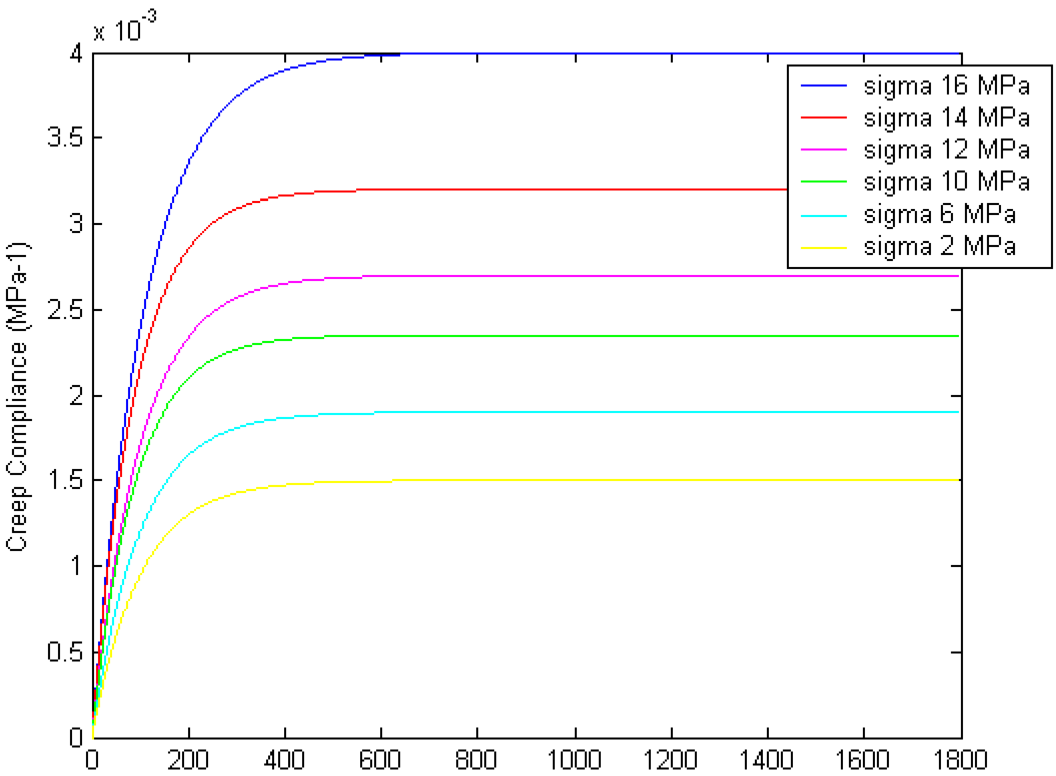

Figure 2.6 : Creep compliance of Kelvin-Voigt model.

2.2.3 Standard linear solid model

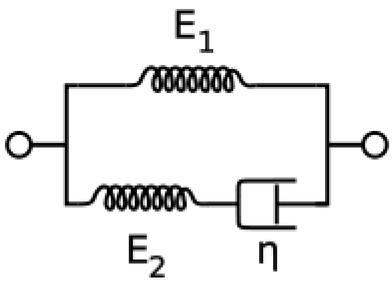

Figure 2.7 : Standard Linear Solid model.

The Standard Linear Solid Model effectively combines the Maxwell Model and a Hookean spring in parallel. A viscous material is modeled as a spring and a dashpot in series with each other, both of which are in parallel with a lone spring. For this model, the governing constitutive relation is:

$$\frac{d\varepsilon}{dt} = \frac{\frac{E_2}{\eta}\left(\frac{\eta}{E_2}\frac{d\sigma}{dt} + \sigma - E_1\varepsilon\right)}{E_1 + E_2} \qquad \textbf{(2.12)}$$

Under a constant stress, the modeled material will instantaneously deform to some strain, which is the elastic portion of the strain, and after that it will continue to deform and asymptotically approach a steady-state strain. This last portion is the viscous part of the strain.

For this model, let us break down the strain into ε_1 of the dashpot and $\dot{\varepsilon}_1$ for the spring, whereas the total stress σ is the sum of the stress σ_0 from the spring and σ_1 from the Maxwell element. Thus

$$\varepsilon = \varepsilon_1 + \varepsilon_1' \quad \textbf{(2.13)}$$

$$\sigma = \sigma_0 + \sigma_1 \quad \textbf{(2.14)}$$

$$\sigma_0 = E_0 \varepsilon \quad \textbf{(2.15)}$$

$$\sigma_1 = \eta_1 \dot{\varepsilon}_1 = E_1 \varepsilon_1' \quad \textbf{(2.16)}$$

From this we can verify by substitution that

$$\sigma = E_0 \varepsilon + E_1 \varepsilon_1' = (E_0 + E_1)\varepsilon - E_1 \varepsilon_1 \quad \textbf{(2.17)}$$

Hence

$$\sigma + \frac{\eta_1}{E_1}\dot{\sigma} = (E_0 + E_1)\varepsilon - E_1 \varepsilon_1 + \frac{\eta_1}{E_1}(E_0 + E_1)\dot{\varepsilon} - \eta_1 \dot{\varepsilon}_1 \quad \textbf{(2.18)}$$

Replacing the last term by $E_1 \varepsilon_1'$ and using Eq. (2.13), we obtain

$$\sigma + \frac{\eta_1}{E_1}\dot{\sigma} = E_0 \varepsilon + \left(1 + \frac{E_0}{E_1}\right)\dot{\varepsilon} \quad \textbf{(2.19)}$$

This equation can be written in the form

$$\sigma + \tau_\varepsilon \dot{\sigma} = E_R(\varepsilon + \tau_\sigma \dot{\varepsilon}) \text{(Standard linear solid model)} \quad \textbf{(2.20)}$$

Where

$$\tau_\varepsilon = \frac{\eta_1}{E_1}, \qquad \tau_\sigma = \frac{\eta_1}{E_0}\left(1 + \frac{E_0}{E_1}\right), \qquad E_R = E_0 . \quad \textbf{(2.21)}$$

For a suddenly applied stress $\sigma(0)$ and strain $\varepsilon(0)$, the initial condition is

$$\tau_\varepsilon \sigma(0) = E_R \tau_\sigma \varepsilon(0) \quad \textbf{(2.22)}$$

For reasons that will become clear below, the constant τ_ε is called the relaxation time for constant strain, whereas τ_σ is called the relaxation time for constant stress.

Although the Standard Linear Solid Model is more accurate than the Maxwell and Kelvin-Voigt models in predicting material responses, mathematically it returns inaccurate results for strain under specific loading conditions and is rather difficult to calculate.

2.2.4 Creep and Relaxation for three viscoelastic models

When we solve Eqs. (2.4), (2.7), and (2.20) for ε(t) when σ(t) is a unit-step function 1(t), the results are called creep functions, which represent the elongation produced by a sudden application at t=0 of a constant force of magnitude unity. They are:

Maxwell model:

$$c(t) = \left(\frac{1}{E} + \frac{1}{\eta} t \right) 1(t) \,, \tag{2.23}$$

Kelvin-Voigt model:

$$c(t) = \frac{1}{E} \left(1 - e^{-(E/\eta)t} \right) 1(t) \,, \tag{2.24}$$

Standart linear solid model:

$$c(t) = \frac{1}{E_R} \left[1 - \left(1 - \frac{\tau_\varepsilon}{\tau_\sigma} \right) e^{-t/\tau_\sigma} \right] 1(t) \,, \tag{2.25}$$

Where the unit-step function is defined as (see Figure 2.8)

$$1(t) = \begin{matrix} 1 & when & t > 0 \\ \frac{1}{2} & when & t = 0 \\ 0 & when & t < 0 \end{matrix}$$

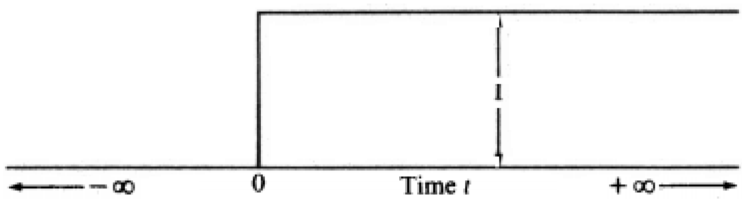

Figure 2.8 : A unit step function 1(t).

Interchanging the roles of σ and ε, we obtain the relaxation functions as a response σ(t)=k(t) corresponding to an elongation ε(t)=1(t). The relaxation function k(t) is the force that must be applied in order to produce an elongation that changes at t=0 from zero to unity and remains unity thereafter. They are

Maxwell solid:

$$k(t) = Ee^{-(E/\eta)t}1(t) \qquad \textbf{(2.26)}$$

Voigt solid:

$$k(t) = \eta\delta(t) + E1(t) \qquad \textbf{(2.27)}$$

Standart linear solid:

$$k(t) = E_R\left[1-\left(1-\frac{\tau_\sigma}{\tau_\varepsilon}\right)e^{-t/\tau_\varepsilon}\right]1(t) \qquad \textbf{(2.28)}$$

Here we have used the symbol δ(t) to indicate the unit-impulse function or Dirac-delta function, which is defined as a function with a singularity at the origin (see Figure 2.9)

$\delta(t) = 0$ (for t<0, and t>0),

$\int_{-\varepsilon}^{\varepsilon} f(t)\delta(t)dt = f(0)$ (ε>0)

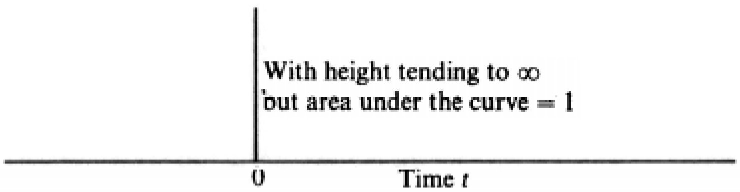

Figure 2.9 : A unit impulse function δ(t).

Where f(t) is an arbitrary function, continuous at t=0. These funtions, c(t) and k(t), are illustrated in Figures 2.10 and 2.11, respectively, for which we add the following comments.

For the Maxwell model, the sudden application of a load induces an immediate deflection by the elastic spring, which is followed by “creep” of the dashpot. On the other hand, a sudden deformation produces an immediate reaction by the spring, which is followed by stress relaxation according to an exponential law. The factor η / E, with the dimension of time, may be called the relaxation time: it characterizes the rate of decay of the force.

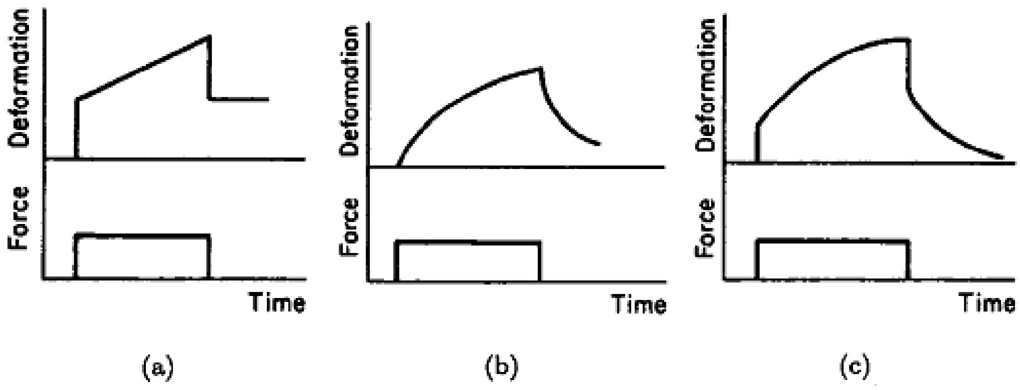

Figure 2.10 : Creep functions of (a) a Maxwell, (b) a Kelvin-Voigt, and (c) a Standard linear solid.

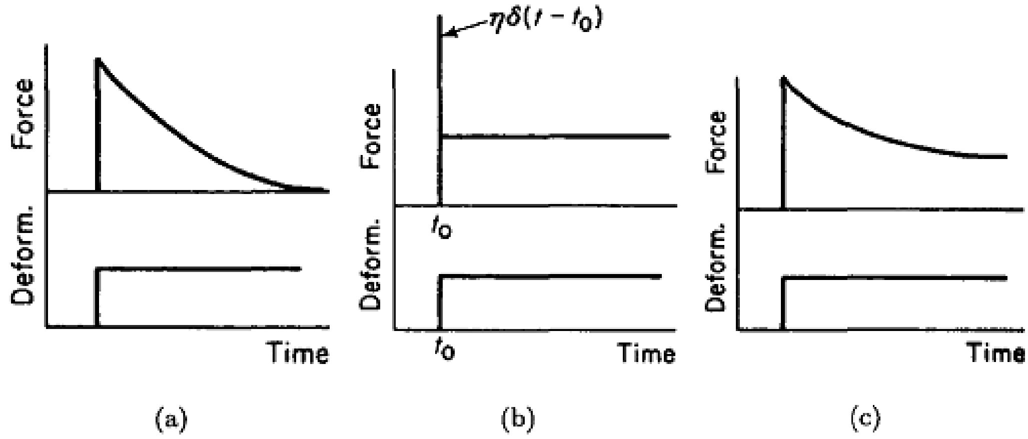

Figure 2.11 : Relaxation functions of (a) a Maxwell, (b) a Kelvin-Voigt, and (c) a Standard linear solid.

For the Kelvin-Voigt model, a sudden application of force will produce no immediate deflection, because the dashpot, arranged in parallel with the spring, will not move instantaneously. Instead, as shown by Eq. (2.24) and (2.25), a deformation will be gradually built up, while the spring takes a greater and greater share of the load. The dashpot displacement relaxes exponentially. Here the ratio η / E is again a relaxation time: it characterizes the rate of relaxation of the dashpot.

For the standard linear solid model, a similar interpretation is applicable. The constant τ_{ε} is the time of relaxation of load under the condition of constant deflection as shown in Eq.(2.28), whereas the constant τ_{σ} is the time of relaxation of deflection under the condition of constant load Eq. (2.25). As $t \to \infty$, the dashpot is completely relaxed, and the load-deflection relation becomes that of the springs, as is characterized by the constant E_R in Eq. (2.25) and (2.28). Therefore, E_R is called the relaxed elastic modulus.

Maxwell introduced the model represented by Eq. (2.7), with the idea that all fluids are elastic to some extent. Lord Kelvin showed the inadequacy of the Maxwell and Voigt models in accounting for the rate of dissipation of energy in various materials subjected to cyclic loading. Kelvin's model is commonly called the standard linear model because it is the most general relationship that includes the load, the deflection, and their first (commonly called "linear") derivatives.

3. VISCOPLASTICITY AND CLASSICAL VISCOPLASTIC MODEL

In contrast to elastic theories, plasticity is the behavior, where the material would undergo unrecoverable deformations due to the response of applied forces.

Viscoplasticity, is defined as a rate-dependent plasticity model. Rate dependent plasticity is important for (high-speed) transient plasticity calculations. It should be used, however, in combination with a plasticity law. In that aspect, viscoplastic solids exhibit permanent deformations after the application of loads but continue to undergo a creep flow as a function of time under the influence of the applied load (equilibrium is impossible).

A widely-used viscoplastic formulation is the *Perzyna model*. The main feature of this model is that the rate-independent yield function used for describing the viscoplastic strain can become larger than zero, which effect is known as 'overstress'. The characteristics of the Perzyna model as well as the numerical discretization have been addressed by various authors.

Perzyna model

In the Perzyna model the evolution of the viscoplastic strain rate is defined as (Perzyna, 1966)

$$\dot{\varepsilon}^{p} = \frac{\langle \Phi(f) \rangle}{\eta} m \quad \textbf{(3.1)}$$

$$m = \frac{\partial g}{\partial \sigma} \quad \textbf{(3.2)}$$

with η the viscosity parameter, Φ the overstress function that depends on the rate-independent yield surface $f(\sigma, \Phi)$, and m determines the direction of $\dot{\varepsilon}^{p}$ given by Eq. (3.2) and g is a potential function. An explicit expression for the consistency parameter is obtained,

$$\dot{\lambda} = \frac{\langle \Phi(f) \rangle}{\eta} \tag{3.3}$$

where "$\langle . \rangle$" are the McCauley brackets, such that

$$\langle \Phi(f) \rangle = \begin{cases} \Phi(f) & if \quad \Phi(f) \geq 0, \\ 0 & if \quad \Phi(f) < 0. \end{cases} \tag{3.4}$$

According to Simo (1989), the overstress function Φ must fulfill the following conditions:

$\Phi(f)$ is continuous in [0,∞),

$\Phi(f)$ is convex in [0,∞),

$\Phi(0) = 0$.

so that a rate-independent elasto–plastic model is recovered if η→0. The following, widely-used expression for Φ is employed (Desai and Zhang, 1987; Simo, 1989; Sluys, 1992; Wang et al., 1997; Simo and Hughes, 1998):

$$\Phi(f) = \left(\frac{f}{\alpha} \right)^N . \tag{3.5}$$

Herein, α is commonly chosen as the initial yield stress, and N is a calibration parameter that should satisfy $N \geq 1$ in order to meet condition "$\Phi(f)$ is convex in [0,∞)".

3.1 Classical one-dimensional viscoplastic constitutive model

In this section mathematical constitutive definitions of classical one-dimensional viscoplastic constitutive model introduced Neto, Peric, Owen [1] was shown.

The decomposition of the total axial strain into a sum of an elastic (recoverable) and a plastic (permanent) component is introduced,

$$\varepsilon = \varepsilon^e + \varepsilon^p \tag{3.6}$$

The axial stress is assumed to be related to the elastic component of the axial strain by means of the standard linear elastic constitutive relation

$$\sigma = E\varepsilon^e \tag{3.7}$$

The elastic domain can be conveniently defined by means of a yield function

$$\Phi\left(\sigma,\sigma_y\right)=|\sigma|-\sigma_y \tag{3.8}$$

where σ_y is the yield stress. The elastic domain is defined as the set

$$E=\left\{\sigma \mid \Phi(\sigma,\sigma_y)<0\right\} \tag{3.9}$$

The viscoplastic flow rule can be postulated with a format

$$\dot{\varepsilon}^p=\dot{\gamma}(\sigma,\sigma_y)sign(\sigma) \tag{3.10}$$

Where sign is the signum function

$$sign(a)=\begin{cases}+1 & if \quad a\geq 0\\ -1 & if \quad a<0\end{cases} \tag{3.11}$$

The plastic strain rate Eq. (3.10) is the actual time derivative of ε^p. $\dot{\gamma}$ is termed the plastic multiplier and is here a given explicit function of σ and σ_y. Essentially, the explicit function for $\dot{\gamma}$ should model how the rate of plastic straining varies with the level of stress.

The one-dimensional viscoplasticity model is adopted the following particular definition below

$$\dot{\gamma}(\sigma,\sigma_y)=\begin{cases}\dfrac{1}{\mu}\left[\left(\dfrac{|\sigma|}{\sigma_y}\right)^{1/\in}-1\right] & if \quad \Phi\left(\sigma,\sigma_y\right)\geq 0\\ 0 & if \quad \Phi\left(\sigma,\sigma_y\right)<0\end{cases} \tag{3.12}$$

where the material constants are the viscosity-related parameter μ, whose dimension is time, and the non-dimensional rate-sensitivity parameter, $\in$. Both parameters are strictly positive. This particular form has been introduced by Peric (1993) [10] similarly to the power law form of the viscoplastic potential proposed by Perzyna (1963) [11] which was shown in the beginning of this chapter. It is important to emphasise that the material parameters μ and $\in$ are temperature dependent.

As a general rule, as temperature increases (decreases) μ and $\in$ increase (decrease). For many metals, $\mu, \in \rightarrow 0$ for sufficiently low temperatures, when the material behaviour may be assumed rate-independent. In the viscoplastic model, hardening can be incorporated in the same manner as in the elastoplastic case by letting the yield stress, σ_y, be a given (experimentally determined) function

$$\sigma_y = \sigma_y(\bar{\varepsilon}^p) \quad \textbf{(3.13)}$$

of the accumulated plastic strain

$$\bar{\varepsilon}^p = \int_0^t \left|\dot{\varepsilon}^p\right| dt \quad \textbf{(3.14)}$$

It should be noted that Eq. (3.12) implies at a given constant applied stress σ, an increase (decrease) in σ_y will produce a decrease (increase) in the plastic strain rate $\dot{\varepsilon}^p$. As in the elastoplastic case, an increase of σ_y will be referred to as hardening whereas its reduction will be described as softening. If σ_y is a constant, the model is referred to as perfectly viscoplastic.

The one dimensional viscoplastic model with items like below cab be summarized.

1. Elastoplastic split of the axial strain

$$\varepsilon = \varepsilon^e + \varepsilon^p \quad \textbf{(3.15)}$$

2. Uniaxial elastic law

$$\sigma = E\varepsilon^e \quad \textbf{(3.16)}$$

3. Yield function

$$\Phi\left(\sigma, \sigma_y\right) = |\sigma| - \sigma_y \quad \textbf{(3.17)}$$

4. Plastic flow rule

$$\dot{\varepsilon}^p = \dot{\gamma} sign(\sigma) \quad \textbf{(3.18)}$$

$$\dot{\gamma}(\sigma,\sigma_y)=\begin{cases}\dfrac{1}{\mu}\left[\left(\dfrac{|\sigma|}{\sigma_y}\right)^{1/\epsilon}-1\right] & if \quad \Phi\left(\sigma,\sigma_y\right)\geq 0 \\ 0 & if \quad \Phi\left(\sigma,\sigma_y\right)<0\end{cases} \tag{3.19}$$

5. Hardening law

$$\sigma_y=\sigma_y(\bar{\varepsilon}^p) \tag{3.20}$$

$$\dot{\bar{\varepsilon}}^p=\dot{\gamma} \tag{3.21}$$

4. CONCEPTS OF FRACTIONAL CALCULUS AND ITS APPLICATIONS

Fractional Calculus is a field of applied mathematics that deals with derivatives and integrals of arbitrary orders (including complex orders), and their applications in science, engineering, mathematics, economics, and other fields. It is also known by several other names such as Generalized Integral and Differential Calculus and Calculus of Arbitrary Order. The name “Fractional Calculus” is holdover from the period when it meant calculus of ration order. The seeds of fractional derivatives were planted over 300 years ago. Since then many great mathematicians (pure and applied) of their times, such as N. H. Abel, M. Caputo, L. Euler, J. Fourier, A. K. Grünwald, J. Hadamard, G. H. Hardy, O. Heaviside, H. J. Holmgren, P. S. Laplace, G. W. Leibniz, A. V. Letnikov, J. Liouville, B. Riemann, M. Riesz, and H. Weyl, have contributed to this field. However, most scientists and engineers remain unaware of Fractional Calculus; it is not being taught in schools and colleges; and others remain skeptical of this field. There are several reasons for that: several of the definitions proposed for fractional derivatives were inconsistent, meaning they worked in some cases but not in others. The mathematics involved appeared very different from that of integer order calculus. There were almost no practical applications of this field, and it was considered by many as an abstract area containing only mathematical manipulations of little or no use.

The paradigm began to shift from pure mathematical formulations to applications in various fields. During the last decade Fractional Calculus has been applied to almost every field of science, has made a profound impact include viscoelasticity and rheology, electrical engineering, electrochemistry, biology, biophysics and bioengineering, signal and image processing, mechanics, mechatronics, physics, and control theory. Although some of the mathematical issues remain unsolved, most of the difficulties have been overcome, and most of the documented key mathematical issues in the field have been resolved to a point where many of the mathematical tools for both the integer- and fractional-order calculus are the same. The books and monographs of Oldham and Spanier, Oustaloup, Miller and Ross, Samko, Kilbas, and Marichev, Kiryakova, Carpinteri and Mainardi, Podlubny , and Hilfer have been helpful in introducing the field to engineering, science, economics and finance, pure and applied mathematics communities. The progress in this field continues. Three recent books in this field are by West, Grigolini, and Bologna, Kilbas, Srivastava, and Trujillo, and Magin.

One of the major advantages of fractional calculus is that it can be considered as a super set of integer-order calculus. Thus, fractional calculus has the potential to accomplish what integer-order calculus cannot. It is believed that many of the great future developments will come from the applications of fractional calculus to different fields and researchers, new and old, would realize that it cannot remained within the boundaries of integral order calculus, that fractional calculus is indeed a viable mathematical tool that will accomplish far more than what integer calculus promises, and that fractional calculus is the calculus for the future.

4.1 Mathematical Concepts of Fractional Calculus

4.1.1 The Liouville-Riemann Scheme

As a starting point, we recall the hierarchy of formal limiting expressions defining standard whole integer derivatives [4]

$$\frac{d^1}{dt^1}(Y(t)) = \lim_{\Delta t\to 0}(\Delta t)^{-1}(Y(t)-Y(t-\Delta t)), \tag{4.1}$$

$$\frac{d^2}{dt^2}(Y(t)) = \lim_{\Delta t\to 0}(\Delta t)^{-2}(Y(t)-2Y(t-\Delta t)+Y(t-2\Delta t)), \tag{4.2}$$

$$\frac{d^3}{dt^3}(Y(t)) = \lim_{\Delta t\to 0}(\Delta t)^{-3}(Y(t)-3Y(t-\Delta t)+3Y(t-2\Delta t)-Y(t-3\Delta t)) \tag{4.3}$$

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Continuing the sequence of operators defined by Eqs. (4.1), (4.2), (4.3), we see, using mathematical induction, that

$$\frac{d^n}{dt^n}(Y(t)) = \lim_{\Delta t \to 0}\left\{(\Delta t)^{-n}\sum_{j=0}^{n}(-1)^j \binom{n}{j} Y(t - j\Delta t)\right\}, \qquad \textbf{(4.4)}$$

where n is an integer and

$$\binom{n}{j} = \frac{n!}{j!(n-j)!} \qquad \textbf{(4.5)}$$

Letting

$$\Delta t = t / N, \qquad N \in [1, \infty), \qquad \textbf{(4.6)}$$

Eq. (4.4) can be recast in the form

$$\frac{d^n}{dt^n}(Y(t)) = \lim_{\frac{t}{N} \to 0}\left\{\left(\frac{t}{N}\right)^{-n}\sum_{j=0}^{N-1}(-1)^j \binom{n}{j} Y(t(1 - j/N))\right\}. \qquad \textbf{(4.7)}$$

Now, noting that

$$\binom{n}{j} = \begin{cases} 0, & j > n \\ \neq 0, & j \leq n \end{cases}, \qquad \textbf{(4.8)}$$

we can generalize Eq. (4.7) to yield the expression

$$\frac{d^n}{dt^n}(Y(t)) = \lim_{N \to \infty}\left\{\left(\frac{t}{N}\right)^{-n}\sum_{j=0}^{N-1}(-1)^j \binom{n}{j} Y(t(1 - j/N))\right\} \qquad \textbf{(4.9)}$$

Since

$$(-1)^j \binom{n}{j} = \binom{j-n-1}{j} \qquad \textbf{(4.10)}$$

Wherein

$$\binom{j-n-1}{j} = \frac{\Gamma(j-n)}{\Gamma(-n)\Gamma(j+1)} \qquad \textbf{(4.11)}$$

we see that Eq. (4.9) reduces to the form

$$\frac{d^n}{dt^n}(Y) = \lim_{N \to \infty}\left\{\left(\frac{t}{N}\right)^{-n}\sum_{j=0}^{N-1}\frac{\Gamma(j-n)}{\Gamma(-n)\Gamma(j+1)} Y(t(1 - j/N))\right\}. \qquad \textbf{(4.12)}$$

where as per Artin (1964), $\Gamma(\,)$ denotes the gamma function.

Equation (4.12) represents a formal definition of the nth order derivative of any well behaved function for all $t \in [0,\infty)$. Based on Eq. (4.12), if we reinterpret n to be an irrational number q, then Grünwald's (1867)[8] definition of the 'so-called' fractional derivative is obtained

$$\frac{d^q}{dt^q}(Y) = \lim_{N\to\infty}\left\{\left(\frac{t}{N}\right)^{-q}\sum_{j=0}^{N-1} A_{j+1} Y(t(1-j/N))\right\}, \qquad \textbf{(4.13)}$$

Where

$$A_{j+1} = \Gamma(j-q)/\left(\Gamma(-q)\Gamma(j+1)\right). \qquad \textbf{(4.14)}$$

Note, Eq. (4.12) can be reinterpreted to define the 1st, 2nd . . . . , nth order integrals of the function Y(t). This follows directly from the Riemannian definition of integration. For such situations n ranges the interval $n \in [-\infty, 0)$. In this context, Eq. (4.13) can be used to define either fractional integrals or derivatives. This is obtained by letting q range the rational and irrational numbers contained in the interval $q \in [-\infty,\infty)$.

Note, since q may be an irrational number, it follows that

$$\frac{\Gamma(j-q)}{\Gamma(-q)\Gamma(j+1)} \neq 0 \quad for \quad j > q \quad and \quad j < q. \qquad \textbf{(4.15)}$$

To simplify the calculation of the gamma function, based on coefficients appearing in (4.15), the following recursive expression can be used

$$\frac{\Gamma(j-q)}{\Gamma(-q)\Gamma(j+1)} = \frac{(j-1-q)}{j}\frac{\Gamma(j-1-q)}{\Gamma(-q)\Gamma(j)}. \qquad \textbf{(4.16)}$$

An alternate but equivalent definition of the fractional integrodifferential operator is obtained through the use of Cauchy's formula for repeated integrals

$$\frac{d^{-n}}{dt^{-n}}(Y(t)) = \frac{1}{(n-1)!}\int_0^t (t-\tau)^{n-1} Y(\tau)d\tau. \qquad \textbf{(4.17)}$$

This expression follows directly from the continued use of Leibnitz's theorem. To motivate the so-called Liouville-Riemann [4] formalism, n in Eq. (4.17) is replaced by the rational/irrational variable q, wherein $q \in (-\infty,\infty)$. This yields the generalization (Ross 1975)

$$\frac{d^q}{dt^q}(Y(t)) = \frac{1}{\Gamma(-q)}\int_0^t (t-\tau)^{-q-1}Y(\tau)d\tau. \quad \textbf{(4.18)}$$

The equivalence of Eqs. (4.13) and (4.18) has been proven by Riesz (1949), Knopp (1945) and Duff (1956). This is achieved by letting q be a complex variable so that analytic continuation can be used to establish convergence for both Re (q) < 0 and Re (q) ≥ 0.

4.1.2 The Grünwald-Letnikov Scheme

The derivative of a function f (t) of an arbitrary order α, $0 \le \alpha \le 1$, may be given by the Grünwald–Letnikov definition [2],

$$GL = E^{\alpha} = \frac{1}{\Delta t^{\alpha}}(I-\delta^{-})^{\alpha} \quad \textbf{(4.19)}$$

In Eq. (4.19), the term in brackets can be computed by using the Newton binomial formula

$$(1+x)^{\alpha} = \sum_{j=0}^{\infty} C_{\alpha}^{j} x^{j} \quad \textbf{(4.20)}$$

By applying this expression to Eq. (4.19), one obtains the following fractional derivative operator:

$$GL = \frac{1}{\Delta t^{\alpha}} \sum_{j=0}^{\infty} (-1)^{j} C_{\alpha}^{j} (\delta^{-})^{j} \quad \textbf{(4.21)}$$

with the coefficients $(-1)^{j} C_{\alpha}^{j}$ in terms of the Gamma function

$$(-1)^{j} C_{\alpha}^{j} = \frac{\Gamma(j-\alpha)}{\Gamma(-\alpha)\Gamma(j+1)} = A_{j+1}^{\alpha} \quad \textbf{(4.22)}$$

where A_{j+1}^{α} are the so-called GL-coefficients, with $A_{1}^{\alpha} = 1$ for any α. Then, the α_derivative of function u evaluated at time t^{n} is approximated by the Grünwald_Letnikov scheme [8,9]:

$$(GLu)^{n} = \frac{1}{\Delta t^{\alpha}} \sum_{j=0}^{\infty} A_{j+1}^{\alpha} u^{n-j} \quad \textbf{(4.23)}$$

Using the property $n\Gamma(n)=\Gamma(n+1)$, the GL-coefficients in Eq. (4.22) can be computed by the recurrence formula

$$A_{j+1}^{\alpha}=\frac{j-\alpha-1}{j}A_{j}^{\alpha} \qquad \textbf{(4.24)}$$

Based on the previous procedure used to achieve the Grünwald–Letnikov fractional differential operator GL from the Euler backward formula, one introduces the fractional differential operator G^{α} as

$$G^{\alpha}=\frac{1}{\Delta t^{\alpha}}\left(\frac{3}{2}\right)^{\alpha}\left[I-\frac{4}{3}\delta^{-}+\frac{1}{3}\left(\delta^{-}\right)^{2}\right]^{\alpha} \qquad \textbf{(4.25)}$$

using Eqs. (4.20) and (4.22), the Gear operator for fractional derivatives is written as

$$G^{\alpha}=\frac{1}{\Delta t^{\alpha}}\left(\frac{3}{2}\right)^{\alpha}\sum_{j=0}^{\infty}\sum_{l=0}^{j}\left(\frac{4}{3}\right)^{j}\left(\frac{1}{4}\right)^{l}(-1)^{j}C_{\alpha}^{j}(-1)^{l}C_{j}^{l}(\delta^{-})^{j+l} \qquad \textbf{(4.26)}$$

Thus, the α -derivative of u at time t^{n} can be approximated by

$$\left(G^{\alpha}u\right)^{n}=\frac{1}{\Delta t^{\alpha}}\left(\frac{3}{2}\right)^{\alpha}\sum_{j=0}^{\infty}\sum_{l=0}^{j}\left(\frac{4}{3}\right)^{j}\left(\frac{1}{4}\right)^{l}A_{j+1}^{\alpha}B_{l+1}^{j}u^{n-j-l} \qquad \textbf{(4.27)}$$

where the coefficients A_{j+1}^{α} are given in Eq. (4.24) and the coefficients B_{l+1}^{j} are calculated using the recurrence formula

$$B_{l+1}^{j}=\frac{l-j-1}{l}B_{l}^{j} \qquad \textbf{(4.28)}$$

one notes that $B_{1}^{j}=1$ for any j.

4.2 Viscoelastic model based on the concept of fractional derivative

Many investigation about fractional viscoelastic and viscoplastic models take place in literature. Besides most of these studies are about viscoelasticity, most of fractional viscoelastic theories based on the one-dimensional constitutive equation introduced by Bagley and Torvik [6,7]. In this study Bagley and Torvik's equation about viscoelastic material behaviour was adopted into the one-dimensional constitutive equation explain viscoplastic material behaviour to overcome the disadvantages of classical models.

The one-dimensional constitutive equation introduced by Bagley and Torvik [6,7]

$$\sigma_{(t)} + \tau^{\alpha} \frac{d^{\alpha}\sigma_{(t)}}{dt^{\alpha}} = E_0 \varepsilon_{(t)} + \tau^{\alpha} E_{\infty} \frac{d^{\alpha}\varepsilon_{(t)}}{dt^{\alpha}} \tag{4.29}$$

where σ and ε are the stress and the strain, E_0 and E_{∞} are the relaxed and non-relaxed elastic moduli, and τ is the relaxation time.

4.3 Extension of "fractional" viscoelastic models to include plastic deformation

In this section, to obtain one-dimensional fractional viscoplastic material behaviour, The one-dimensional constitutive equation (4.29) introduced by Bagley and Torvik was redefined by using Gear operator equation (4.27) to calculate fractional derivative. It was obtained like below;

$$\sigma_{(t)} + \tau^{\alpha} \frac{d^{\alpha}\sigma_{(t)}}{dt^{\alpha}} = E_0 \varepsilon_{(t)} + \tau^{\alpha} E_{\infty} \frac{1}{\Delta t^{\alpha}} \left(\frac{3}{2}\right)^{\alpha} \sum_{j=0}^{\infty} \sum_{l=0}^{j} \left(\frac{4}{3}\right)^{j} \left(\frac{1}{4}\right)^{l} A_{j+1}^{\alpha} B_{l+1}^{j} \varepsilon^{n-j-l} \tag{4.30}$$

To calculate plastic multiplier definition for viscoplastic material model, Eq. (4.30) was used instead of stress in classical viscoplastic model. It was expected, the viscoelastic fractional definition can be used in viscoplastic definition also.

When the Eq. (4.30) was written into the classical viscoplastic model`s Eq. (3.12) instead of σ, $\dot{\gamma}$ was obtained in our new fractional viscoplastic model like below;

$$\dot{\gamma} = \frac{1}{\mu} \left[\left(\frac{\left| E_0 \varepsilon_{(t)} + \tau^{\alpha} E_{\infty} \frac{1}{\Delta t^{\alpha}} \left(\frac{3}{2}\right)^{\alpha} \sum_{j=0}^{\infty} \sum_{l=0}^{j} \left(\frac{4}{3}\right)^{j} \left(\frac{1}{4}\right)^{l} A_{j+1}^{\alpha} B_{l+1}^{j} \varepsilon^{n-j-l} \right|}{\sigma_y} \right)^{\frac{1}{\epsilon}} - 1 \right] \quad if \quad \Phi\left(\sigma, \sigma_y\right) \geq 0 \tag{4.31}$$

5. THE PROPOSED CONSTITUTIVE MODEL

Accordance with the new strain rate definition Eq. (4.31) in the previous section our new model can be summarized as follows.

1. Elastoplastic split of the axial strain

$$\varepsilon = \varepsilon^{e} + \varepsilon^{p} \tag{5.1}$$

2. Uniaxial elastic law

$$\sigma = E\varepsilon^{e} \tag{5.2}$$

3. Yield function

$$\Phi\left(\sigma, \sigma_y\right) = |\sigma| - \sigma_y \tag{5.3}$$

4. Plastic flow rule

$$\dot{\varepsilon}^{p} = \dot{\gamma}(\sigma, \sigma_y) sign(\sigma)$$

$$\dot{\gamma}(\sigma, \sigma_y) = \begin{cases} \dfrac{1}{\mu}\left[\left(\dfrac{\left| E_0 \varepsilon_{(t)} + \tau^{\alpha} E_{\infty} \dfrac{1}{\Delta t^{\alpha}} \left(\dfrac{3}{2}\right)^{\alpha} \displaystyle\sum_{j=0}^{\infty} \sum_{l=0}^{j} \left(\dfrac{4}{3}\right)^{j} \left(\dfrac{1}{4}\right)^{l} A_{j+1}^{\alpha} B_{l+1}^{j} \varepsilon^{n-j-l} \right|}{\sigma_y}\right)^{\frac{1}{\epsilon}} - 1\right] & if \quad \Phi\left(\sigma, \sigma_y\right) \geq 0 \\ 0 & if \quad \Phi\left(\sigma, \sigma_y\right) < 0 \end{cases} \tag{5.4}$$

5. Hardening law

$$\sigma_y = \sigma_y(\bar{\varepsilon}^{p}) \quad , \tag{5.5}$$

$$\dot{\bar{\varepsilon}}^{p} = \dot{\gamma}$$

5.1 Methodology of experimental determination of relevant material parameters

In this section, the basic properties of viscoelastic material creep and stress relaxation were reexamined by the proposed one-diensional fractional viscoplastic constitutive model. By using these definitions and Matlab program, creep and relaxation graphics of our new viscoplastic model are obtained. These graphics are compared with classical viscoplastic model creep and relaxation plottings in Matlab.

5.1.1 Creeping at constant stress

In this section, the case of a bar subjected to an axial load that produces a uniform stress $\sigma > \sigma_y$ over its cross-section was considered. The load is applied instantaneously and remained constant in time.

With the instantaneous loading (at time t = 0), the bar is initially deform (also instantaneously) in a purely elastic manner. Even without a formal proof, it makes sense to accept that, as there is no time for plastic strains to develop over an (idealised) instantaneous loading event, the behaviour must be purely elastic under such a condition [1].

Assuming the codition mentioned above as zero initial plastic strain and the elastoplastic split of the total strain together with the elastic law that the total strain in the bar at t = 0, immediately after the instantaneous application of load, is

$$\varepsilon_0 = \varepsilon_0^e = \frac{\sigma}{E} \tag{5.6}$$

where the zero subscript denotes quantities at t = 0. Then, the bar is loaded with a stress remained constant in time above the yield limit. Under constant stress, the elastic law implies that the elastic strain is also remain constant. Hence, after the instantaneous loading the straining of the bar proceeds from purely viscoplastic flow and can be modelled by constitutive equations in section 5, item 4. Assuming that σ is positive (tensile), it is obtained

$$\dot{\varepsilon}^p = \frac{1}{\mu}\left[\left(\frac{\sigma_{(t)} + \tau^{\alpha}\dfrac{d^{\alpha}\sigma_{(t)}}{dt^{\alpha}}}{\sigma_y}\right)^{1/\in} - 1\right] \tag{5.7}$$

For a perfectly viscoplastic material (constant σ_y), the integration of the above equation, in relation to the elastoplastic decomposition of the total strain and the initial condition Eq. (5.6), gives the following solution for the straining of the bar

$$\varepsilon(t) = \frac{\sigma}{E} + \frac{1}{\mu}\left[\left(\frac{\sigma + \tau^{\alpha}\dfrac{d^{\alpha}\sigma_{(t)}}{dt^{\alpha}}}{\sigma_y}\right)^{1/\in} - 1\right]t \tag{5.8}$$

The creep rate in this case is constant.

5.1.2 Stress relaxation at constant strain

In this section, the case of a bar which is instantaneously stretched (stretched at an infinitely fast strain rate) to a total strain ε and then kept stretched indefinitely at that constant strain was considered. The instantaneous stretching to ε is assumed to produce a stress above the yield limit of the material [1].

Through the instantaneous stretching (at time t = 0), the bar deforms purely elastically. Hence, assuming that the plastic strain is zero at t = 0 (immediately after the instantaneous stretching), it is obtained that $\varepsilon_0^e = \varepsilon$. Bearing in mind the elastic law the corresponding stress is given by

$$\sigma_0 = E\varepsilon_0^e = E\varepsilon\,. \tag{5.9}$$

Than, the stress in the bar is expressed by the equation below,

$$\sigma = E\left(\varepsilon - \varepsilon^p\right) = \sigma_0 - E\varepsilon^p \tag{5.10}$$

where ε^p evolves in time according to the differential equation (5.7) which, in view of the above expression can be equivalently written as

$$\dot{\varepsilon}^p = \frac{1}{\mu}\left[\left(\frac{\sigma_0 - \tau^\alpha E_\infty \frac{1}{\Delta t^\alpha}\left(\frac{3}{2}\right)^\alpha \sum_{j=0}^{\infty}\sum_{l=0}^{j}\left(\frac{4}{3}\right)^j\left(\frac{1}{4}\right)^l A_{j+1}^\alpha B_{l+1}^j \varepsilon^{n-j-l}}{\sigma_y}\right)^{1/\in} - 1\right] \quad \textbf{(5.11)}$$

To simplify the problem, as in the previous section, it was assumed that the material is perfectly viscoplastic (constant σ_y) and $\in=1$. In this case, the initial value problem is to find a function $\varepsilon^p(t)$ like below

$$\dot{\varepsilon}^p(t) = c_1 - c_2\varepsilon^p(t) \quad \textbf{(5.12)}$$

with initial condition

$$\varepsilon^p(0) = 0 \quad \textbf{(5.13)}$$

where the constants c_1 and c_2 are defined as

$$c_1 = \frac{1}{\mu}\left(\frac{\sigma_0}{\sigma_y} - 1\right) \;, c_2 = \frac{\tau^\alpha E_\infty}{\mu\sigma_y} \quad \textbf{(5.14)}$$

The analytical solution to Eqs. (5.12)-(5.13) can be trivially obtained as

$$\varepsilon^p(t) = \frac{c_1}{c_2}\left(1 - e^{-c_2 t}\right) \quad \textbf{(5.15)}$$

Finally, by placing the above solution into Eq.(5.10), and taking into account the definition of c_1 and c_2, the stress as a function of time was obtained.

$$\sigma(t) = \sigma_0 - \left(\sigma_0 - \sigma_y\right)\left(1 - e^{-c_2 t}\right) \quad \textbf{(5.16)}$$

Clearly, the above function describes the stress relaxation process of the bar, with the stress taking the value $\sigma = \sigma_0 > \sigma_y$ at t = 0 and subsequently relaxing asymptotically to σ_y as t→∞. This is illustrated in Figure 5.1 where a graph of the analytical function $\sigma(t)$ (with $\sigma_0 = 2\sigma_y$) is plotted for different alpha values and also classical viscoplastic model relaxation curve with red color is exist in that figure.

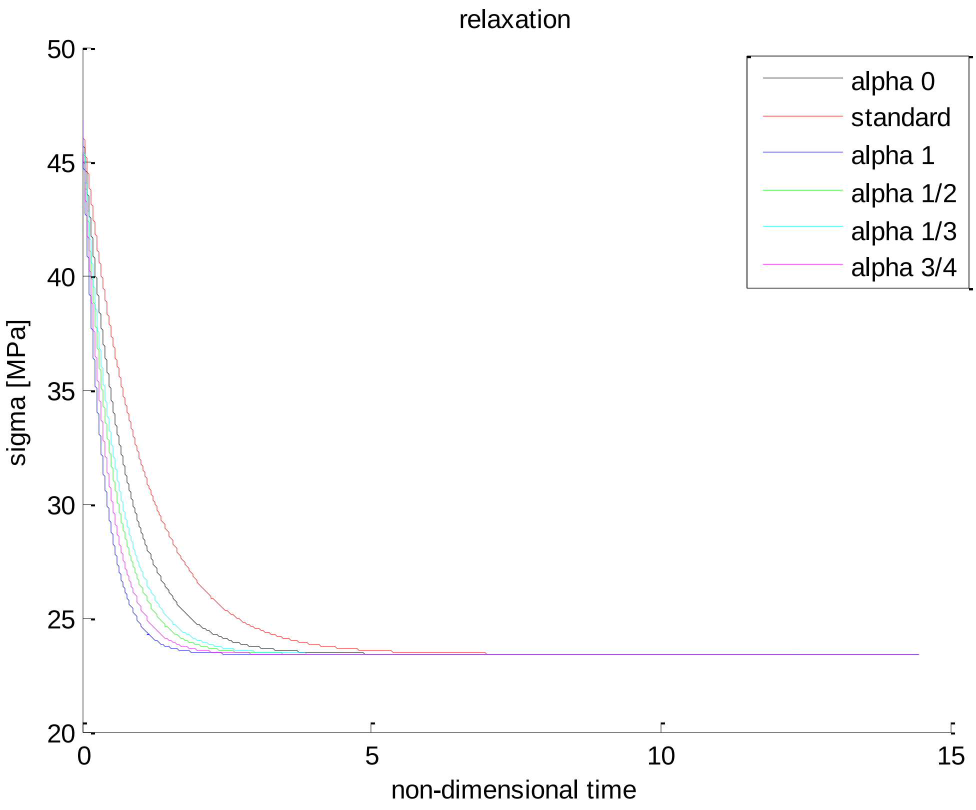

Figure 5. 1 : One-dimensional viscoplastic model with fractional application. Analytical solution to the stress relaxation for different alpha values.

6. NUMERICAL SIMULATION OF THE VISCOPLASTİC MODEL WITH FRACTIONAL CALCULUS APPLICATION

In this section the mathematical constitutive equations of creep and relaxation definitions for fractional viscoplastic model, showed previous section, were programmed by using Matlab. For getting a better results with, program codes (see Appendix B-4,5) were run for a real material HDPE with material parameter values.

Elastic young modulus, yield stress and viscosity releated parameter values of HDPE are taken from Lai-Bakker's article [16] (see Appendix C, Table C-1,2) and MatWeb online database. In line with these obtained material constants the optimum values for τ and E_∞ were determined by using curve fitting methods. Creep and relaxation graphs obtained by using these values and creep Eq. (5.3) ,relaxation Eq.(5.11) ,are shown in Appendix A, Figures A.1-13.

Then this charts for different alpha values were compared with the creep and relaxation graphics of classical viscoplastic model. Classical viscoplastic model was represented in the figures with red color curves and it compared with the new fractional viscoplastic model for different alpha values.

Relaxation's codes were calculated for different value of tau, E_∞ and μ as mentioned above. Figures of the most appropriate three combinations of this values are presented Appendix A, Figures. A.1-9.

The material constants and determined parameter values of HDPE which were used in program codes with different combination are shown in Appendix C, Table C.3,4.

7. EXPERIMENTAL VERIFICATION OF THE PROPOSED MODEL

It was not content with comparisons for HDPE and some experimental tests for creep and relaxation were done in the laboratory to see the real viscoplastic behaviour of a material. PA6 dog-bone specimen was choosen as a test sample material which can be seen in Figure 7.1 and with a gauge length of 60 mm and 4*10 mm sizes.

Several experimental tests were performed on PA6 dog-bone samples to collect data to determine elasticity constant E_0, yield stress and also the creep and stress relaxation responses of the PA6. The experimental tests were performed in mechanic laboratory of Ruhr University of Bochum in Germany. All experiments were taken at room temperature. The specimens were mounted to a tensile- strength machine Zwick / Roell Z100 with a load cell of 100KN, displayed in Figure 7.2. The strain was measured by a tensile extensometer. The experimental data of strain and stress vs. time were recorded simultaneously by a computer.

The tests were begun with tension test up to failure to define the viscoplastic region in which is used . The yield stress is determine by using failure test results and stress values were chosen for creep tests.

In pursuit of failure test relaxation tests were performed with 0.02 mm/mm strain value. The graphics which were attained from this tests were compared with classical viscoplastic model and our new fractional order viscoplastic model by using different fractional order values. This comparison graphics (see Appendix A, Figures. A.14-18), programme codes (see Appendix B-5) and the parameter values (see Appendix C,Table C.5) can be seen in appendicies.

Finally two different type of creep tests were performed. In the first type of creep tests, the specimen was loaded instantaneously to 25 MPa as stress values, which is higher than yield stress value and a creep time of 1800s. Then for recovery part the specimen instantaneously unloaded to 0MPa with a recovery time of 2500s. The same comparison repeated for creep tests as in relaxation tests. This comparison results graphics (see Appendix A, Figures A.19-22), programme codes (see Appendix B-6) and the parameter values (see Appendix C,Table C.6) can be seen in appendicies.

In the second type of creep tests two step creep were performed. The first step was conducted at 25MPa. After a creep time of 1800s, the stress level was changed to 23,21,18,15 and 0MPa respectively for the second step creep for a time of 2500s. The two step loading tests results can be seen in Appendix A, Figure A.23.

After experimental tests, parameter identification for fractional viscoplastic model was realized by using data obtained from the experiments. By using PA6 material constants and parameter values for fractional viscoplastic constitutive creep and relaxation definitions were programmed in Matlab (see Appendix B-7). Program codes were run to obtain creep and relaxation graphic charts to show how PA6 behave in real tests according to fractional viscoplastic and classical viscoplastic models.It was studied to illustrate a better insight into the theory.

Figure 7. 1 : PA6 dog-bone specimen.

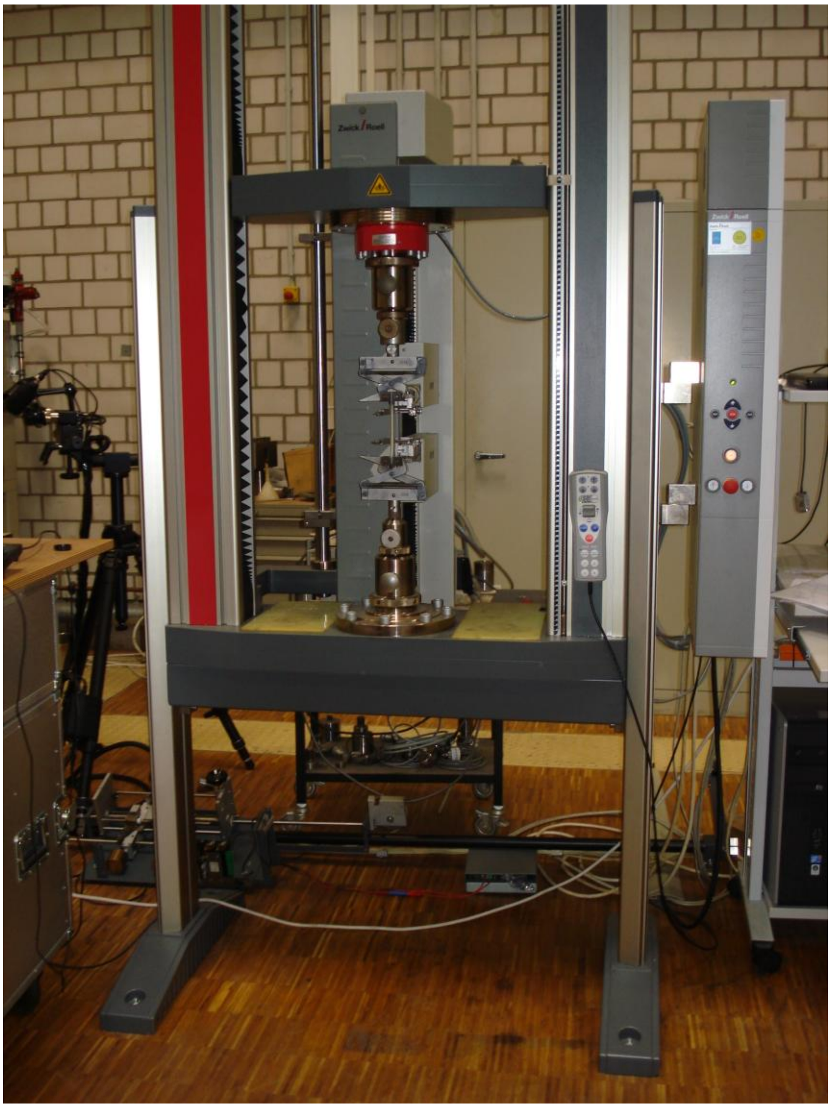

Figure 7. 2 : Test machine.

8. CONCLUSION

In this study, to obtain a new fractional viscoplastic model, viscoplastic model introduced by EA de Souza Neto,D. Peric, and DRT. Owen [1] was modified by using one-dimensional viscoelastic model introduced by Bagley and Torvik [6,7] and Gear operator scheme introduced by Grünwald and Letnikov [2] was used for fractional calculus in viscoelastic definition. Aim of this study is to get a new viscoplastic model which results are closer to the experimental results and so minimize the error in theoretical calculations with this new one-dimensional fractional viscoplastic model.

At first theoretical calculations was completed for one-dimensional fractional viscoplastic model. Then this calculations were compared with one-dimensional classical viscoplastic model. In comparison for the fractional viscoplastic model what looked like if fractional order derivative are taken instead of the first order derivative of the model were checked. As can be seen in Appendix A, Figures A.1-13 when fractional order derivative are taken instead of the first order derivative of the model relaxation and creep curves are closer to the one-dimensional classical model creep and relaxation curves.

Then experimental tests were carried out. After tension test up to failure, creep-recovery and relaxation tests, some material constants and parameters were determined for PA6 material. These values were used in program codes which were written for new one-dimensional fractional viscoplastic model and one-dimensional classical viscoplastic model's creep and relaxation calculus by using Matlab.

Experimental tests creep and relaxation curves were compared with one-dimensional fractional and classical viscoplastic models creep and relaxation curves and for fractional model first order derivative and fractional order derivatives were calculated to get creep and relaxation curves. μ and ϵ parameters were investigated with curve fitting method. As can be seen in Appendix A, Figures A.14-22 for different combinasions of μ and ϵ different solutions can be obtained. This indicate our new method can give different solutions by using different values for a few parameters.

For relaxation tests it can be seen theoretical results are in accord with experimental results. Besides, for creep tests new fractional model results are closer to the experimental results than the classical model results. But the fractional viscoplastic model also has a limitation for creep.

Creep and recovery tests were performed for PA6 dog-bone specimen. It was shown in Appendix A, Figure A.7. In these tests the specimen subjected to an axial load equal to 25 MPa over its cross section. The load is applied instantaneously and, after being applied, remains constant for 1800 seconds and then it was unloaded to 0 MPa instantaneously. In the following tests the same conditions were repeated but the specimen was unloaded to respectively 23, 21, 18, and 15 MPa. 15 and 18 MPa were chosen because of they are under the specimen material yield stress(19.7 MPa) . 21 and 23 MPa were chosen especially because of they are above the specimen material yield stress and we are interested in viscoplastic behavior in this study.

Although it is good with modeling relaxation in materials, but with regards to creep the model is much less accurate still. Because the fractional viscoplastic model for creep or constant-stress conditions postulates that strain increase linearly with time. However as can be seen from the creep curves of the material PA6 which is used in experimental tests shows the strain rate to be decreasing with time. Our fractional viscoplastic model can be redefined later for creep curves.

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APPENDICES

APPENDIX A : Figures

APPENDIX B : Matlab Codes

APPENDIX C : Tables

APPENDIX A

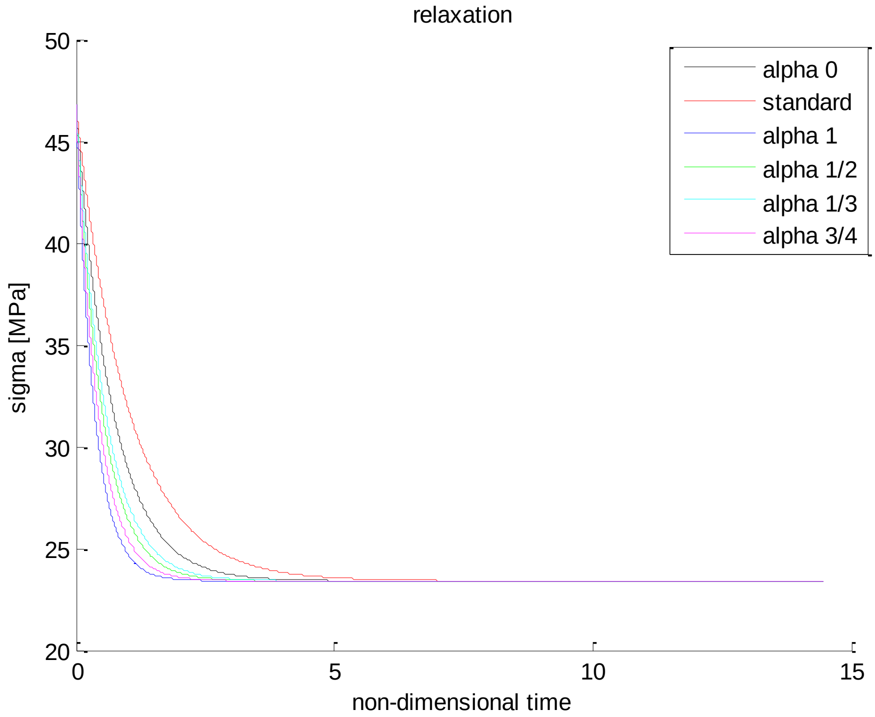

Figure A.1 : HDPE relaxation graphics with non-dimensional time.

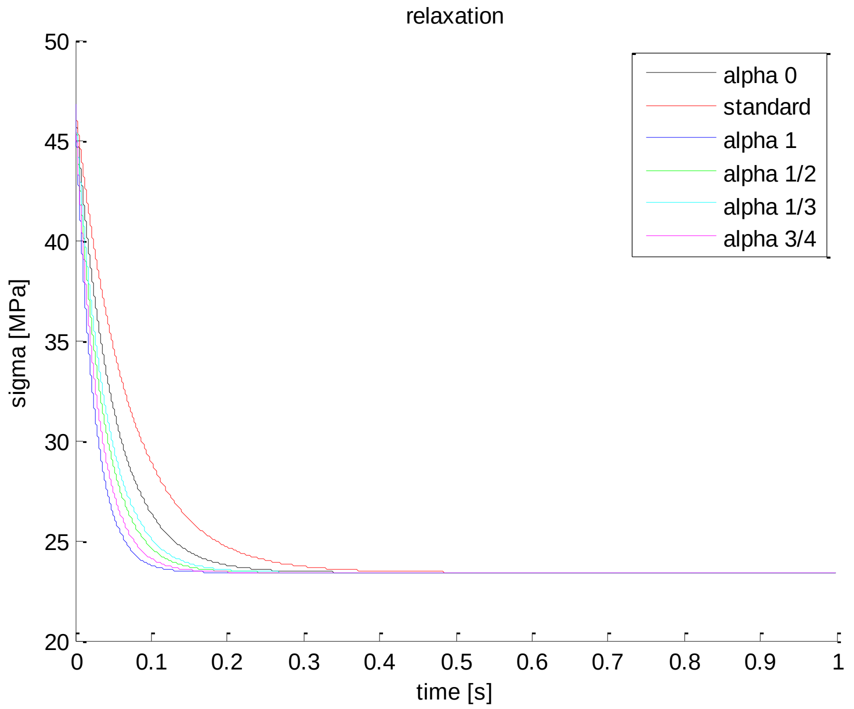

Figure A.2 : HDPE relaxation graphics with time.

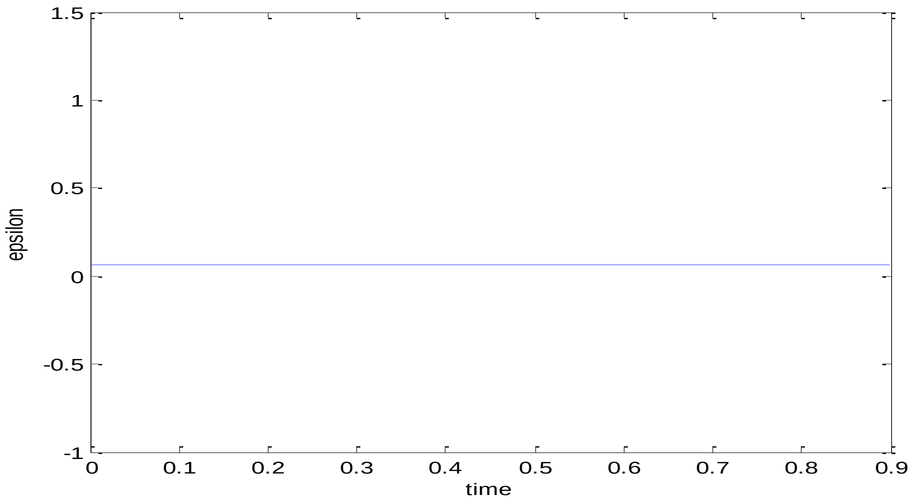

Figure A.3 : HDPE strain-time graphics.

Figure A. (1),(2),(3) : Relaxation graphics one dimensional viscoplastic model with fractional application for HDPE (mu=2.2; tau=2; E_inf=1065)

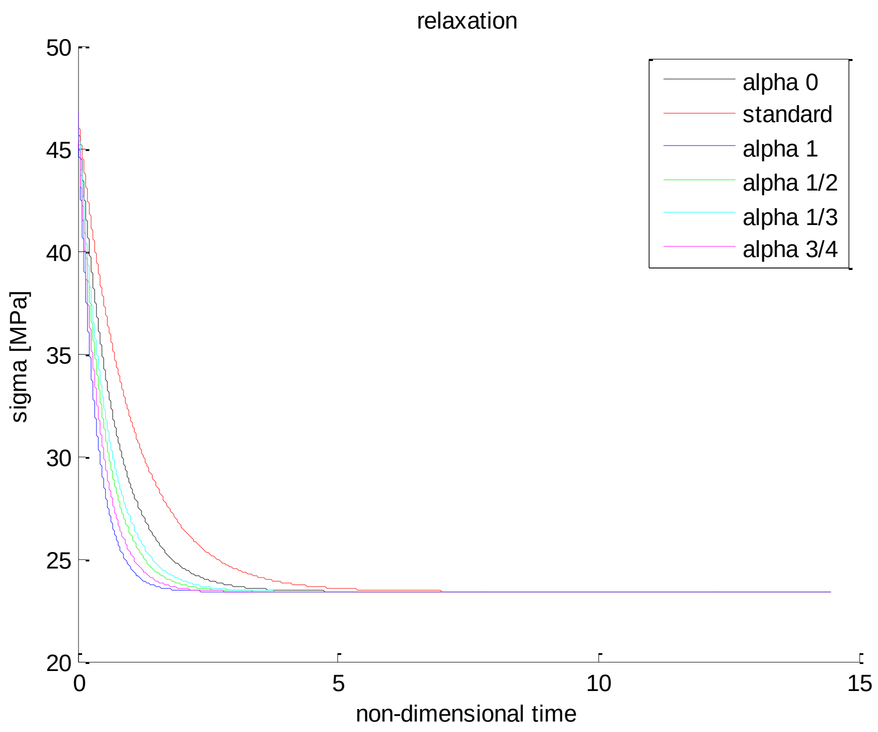

Figure A.4 : HDPE relaxation graphics with non-dimensional time.

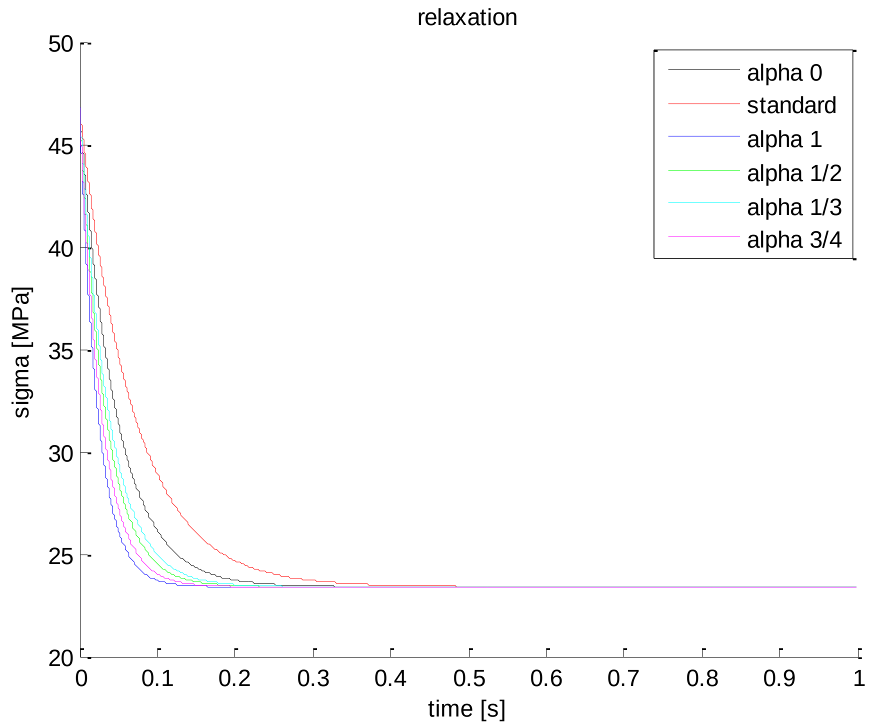

Figure A.5 : HDPE relaxation graphics with time.

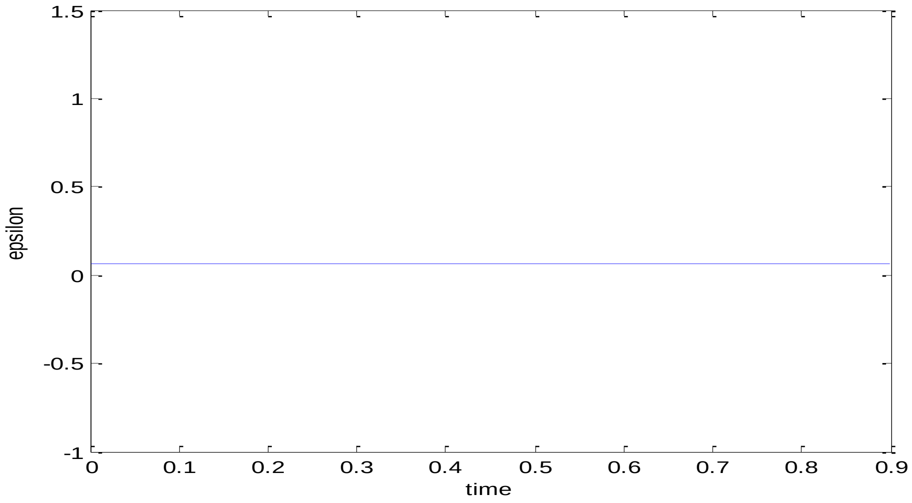

Figure A.6 : HDPE strain-time graphics.

Fig.A.(4),(5),(6) : Relaxation graphics one dimensional viscoplastic model with fractional application for HDPE (mu=2.2; tau=2; E_inf=1100).

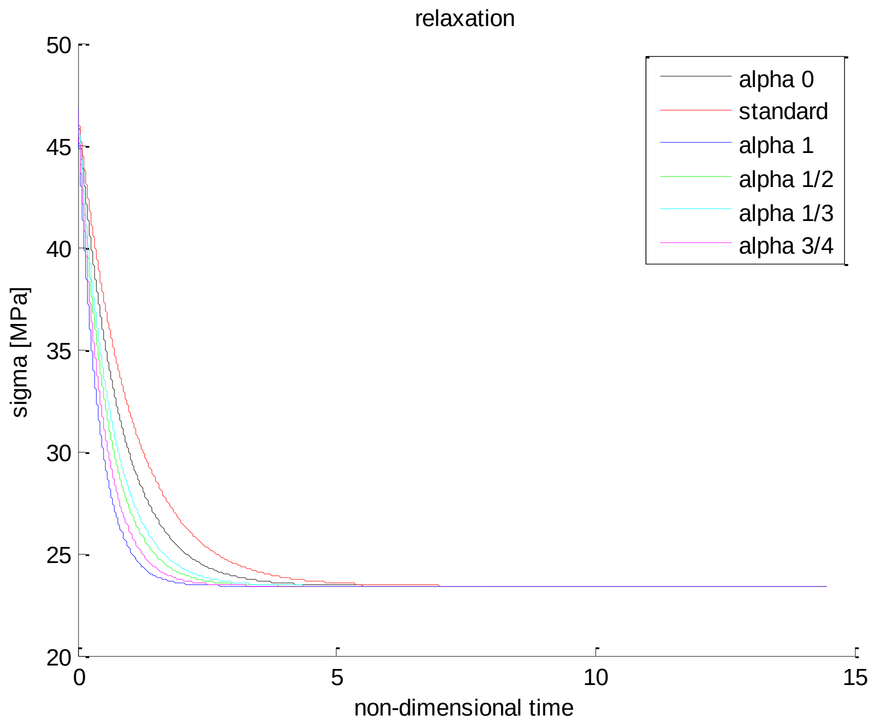

Figure A.7 : HDPE relaxation graphics with non-dimensional time.

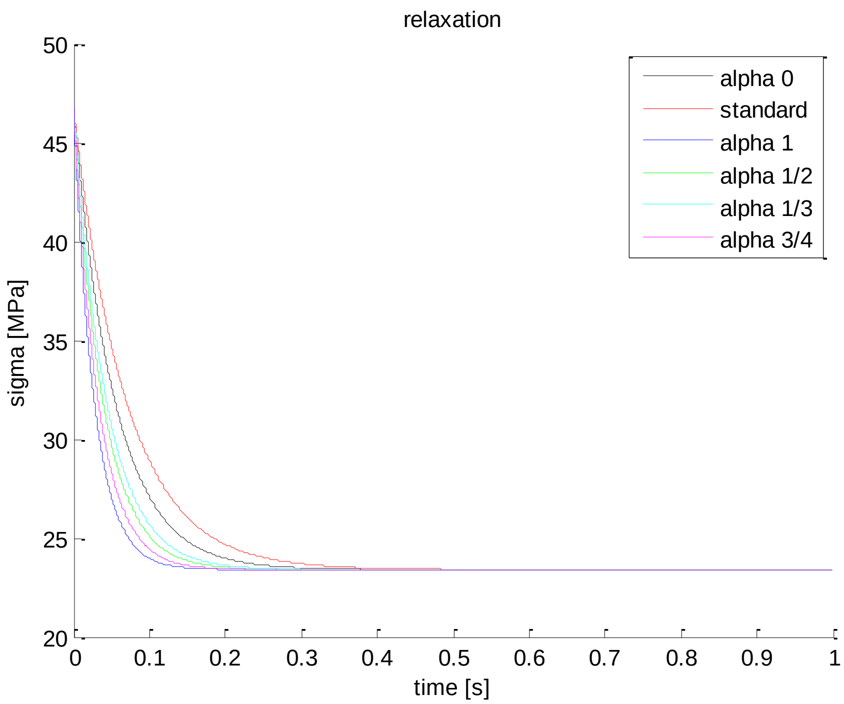

Figure A.8 : HDPE relaxation graphics with time.

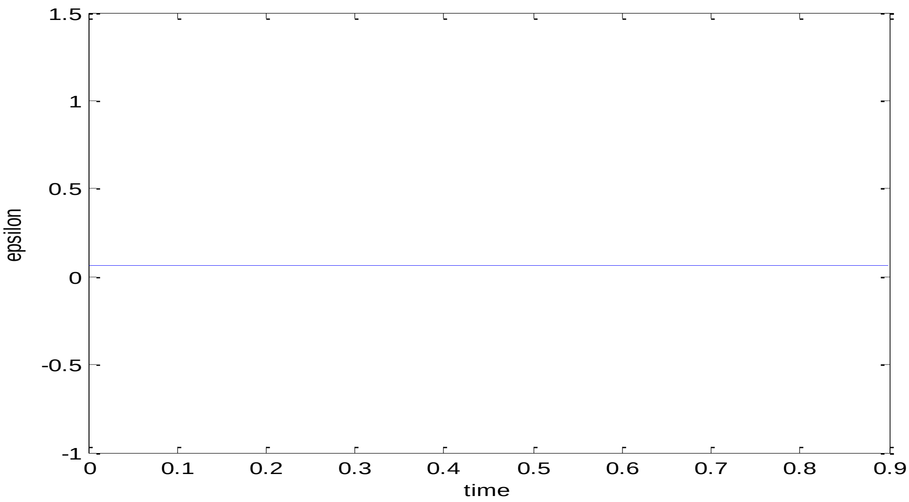

Figure A.9 : HDPE strain-time graphics.

Fig.A (7),(8),(9) : Relaxation graphics one dimensional viscoplastic model with fractional application for HDPE (mu=2.2; tau=2; E_inf=950).

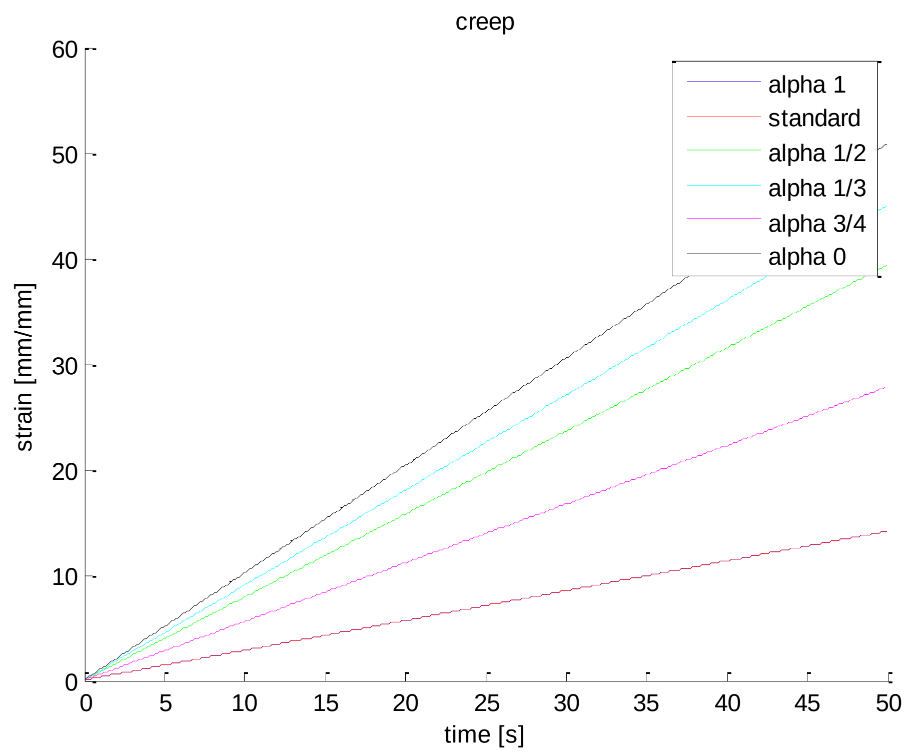

Figure A.10 : Creep graphics for HDPE.

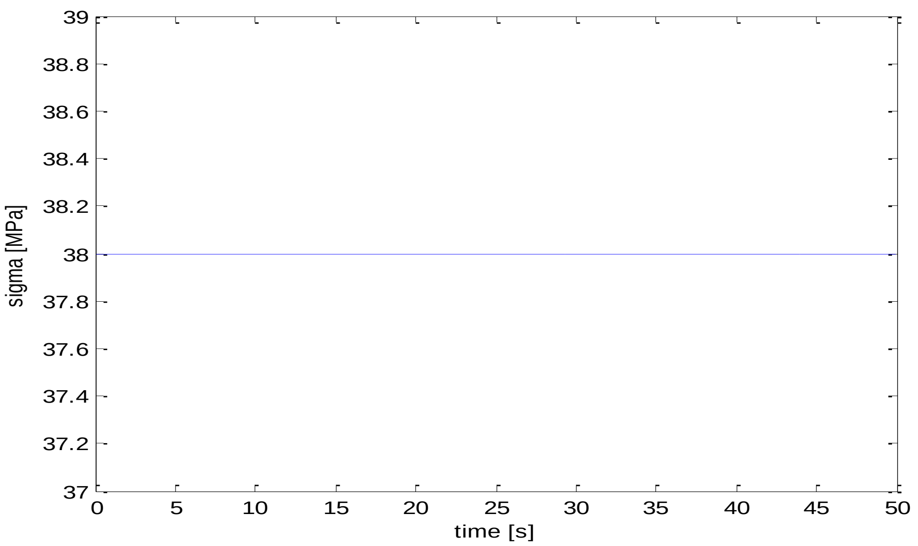

Figure A.11 : HDPE stress-time graphics.

Fig.A (10), (11) : Creep graphics one dimensional viscoplastic model with fractional application for HDPE (mu=2.2; tau=2; E_inf=1065).

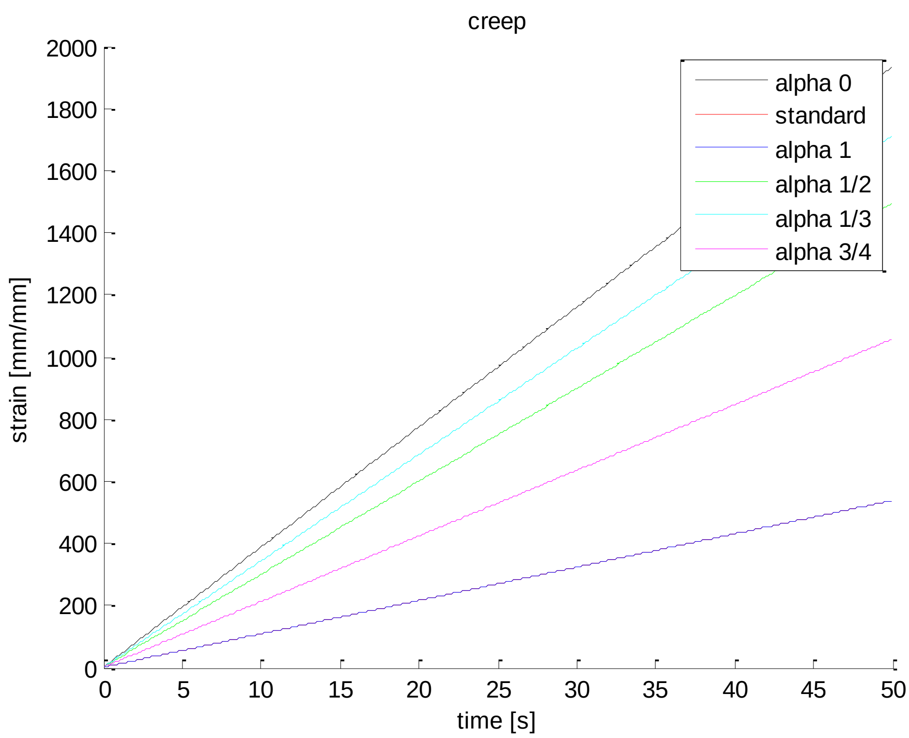

Figure A.12 : Creep graphics for HDPE.

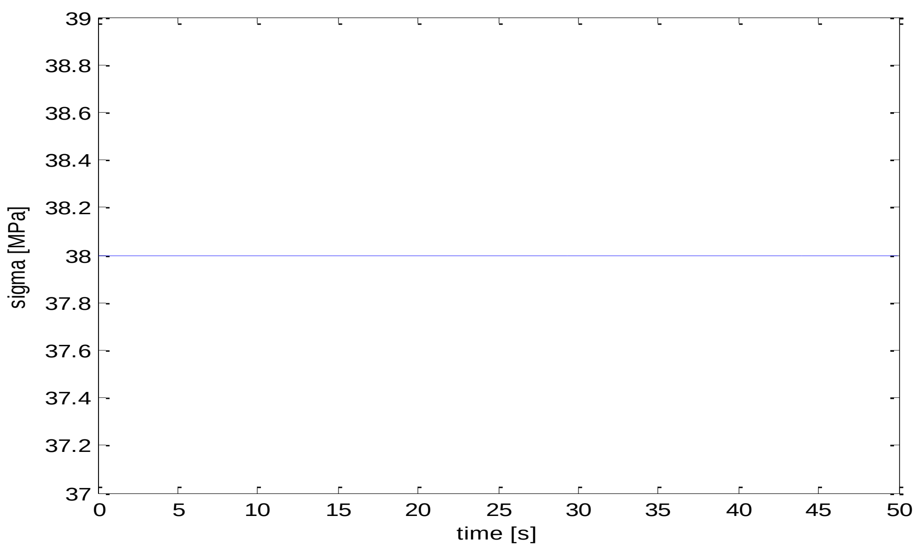

Figure A.13 : HDPE stress-time graphics.

Fig.A (12), (13) : Creep graphics one dimensional viscoplastic model with fractional application for HDPE (mu=2.2; tau=2; E_inf=950).

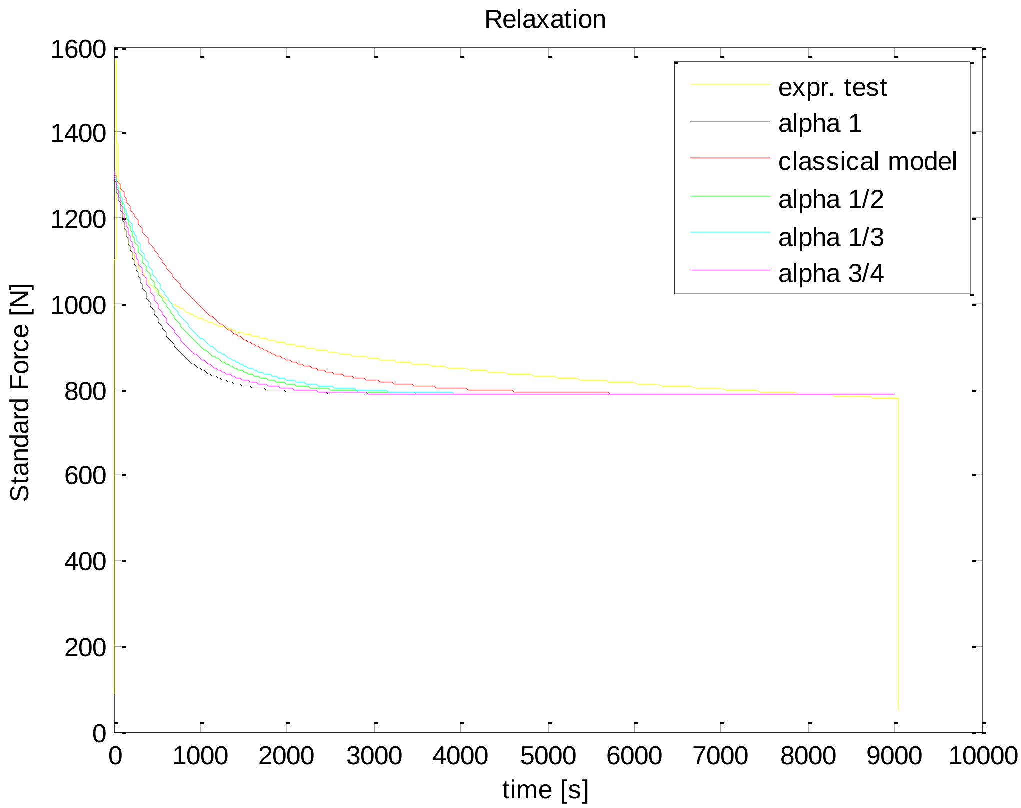

Figure A.14 : Stress relaxation response of PA6 and its comparison with fractional and classical viscoplastic models.

sig_y = 19.7[MPa]	yield stress,
e = 1	the non-dimensional rate sensitivity parameter
E_0 = 1640 [MPa]	relaxed elastisite moduli, young modulus
tau = 2 [s]	relaxation time
E_inf = 1900 [MPa]	non-relaxed elastisite moduli, dynamic modulus
alpha = 1,1/2,1/3,3/4,1/7	fractional derivative order
eps=0.02 [mm/mm]	epsilon for relaxation
mu = 90007.2	the viscosity-related parameter/ material constant

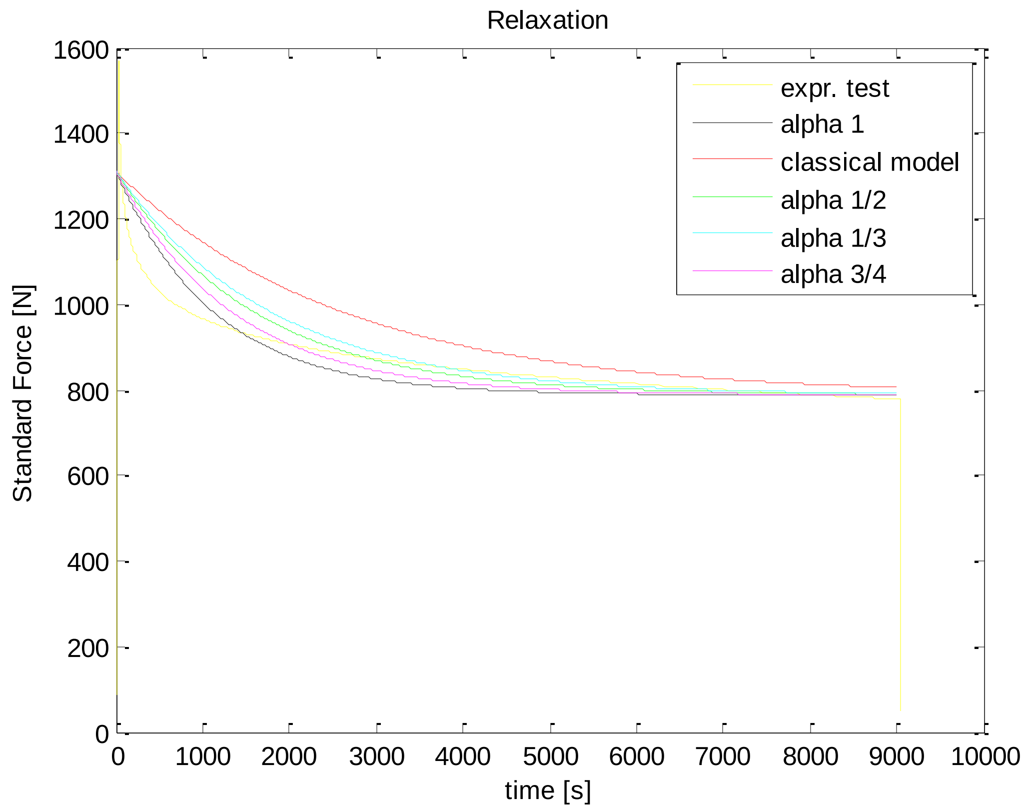

Figure A.15 : Stress relaxation response of PA6 and its comparison with fractional and classical viscoplastic models.

sig_y = 19.7[MPa]	yield stress,
e = 1	the non-dimensional rate sensitivity parameter
E_0 = 1640 [MPa]	relaxed elastisite moduli, young modulus
tau = 2 [s]	relaxation time
E_inf = 1900 [MPa]	non-relaxed elastisite moduli, dynamic modulus
alpha = 1,1/2,1/3,3/4	fractional derivative order
eps=0.02 [mm/mm]	epsilon for relaxation
mu = 220000.0	the viscosity-related parameter/ material constant

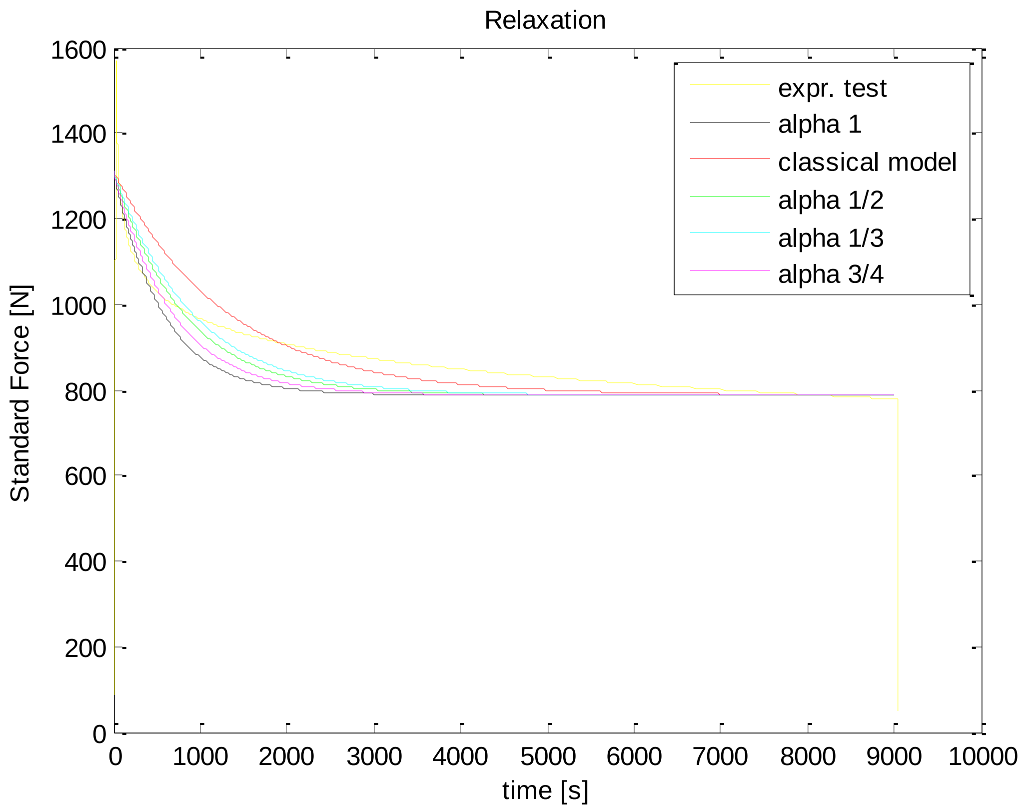

Figure A.16 : Stress relaxation response of PA6 and its comparison with fractional and classical viscoplastic models.

sig_y = 19.7[MPa]	yield stress,
e = 1	the non-dimensional rate sensitivity parameter
E_0 = 1640 [MPa]	relaxed elastisite moduli, young modulus
tau = 2 [s]	relaxation time
E_inf = 1900 [MPa]	non-relaxed elastisite moduli, dynamic modulus
alpha = 1,1/2,1/3,3/4	fractional derivative order
eps=0.02 [mm/mm]	epsilon for relaxation
mu = 110000.0	the viscosity-related parameter/ material constant

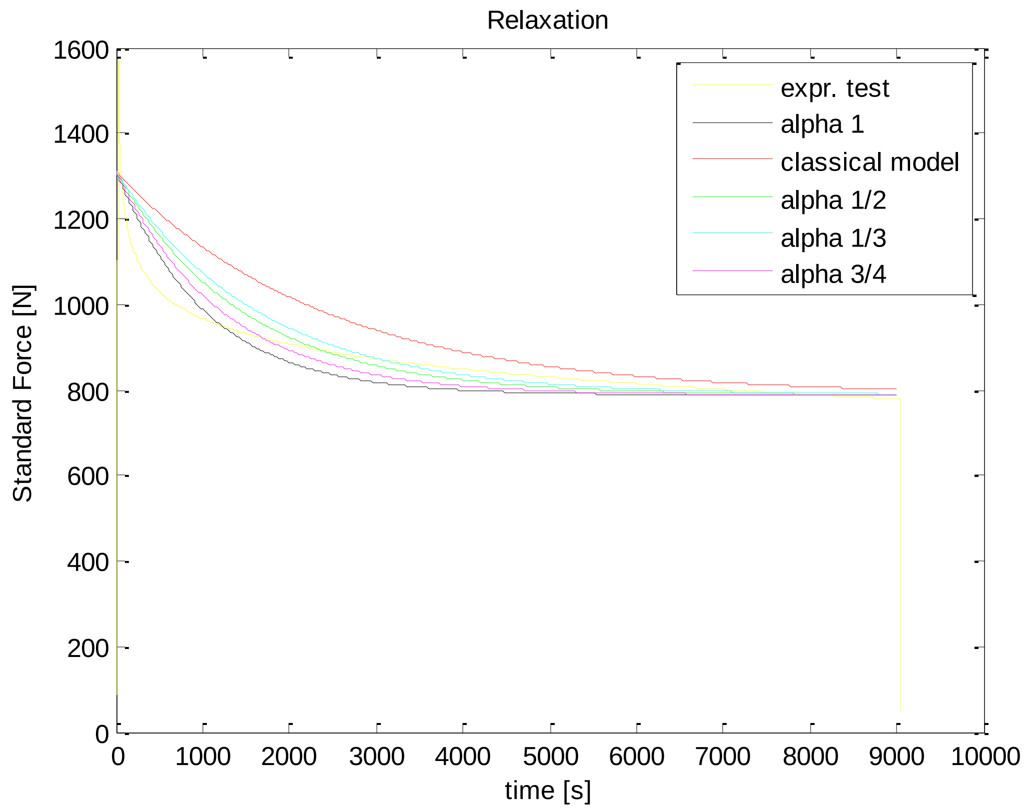

Figure A.17 : Stress relaxation response of PA6 and its comparison with fractional and classical viscoplastic models.

sig_y = 19.7[MPa]	yield stress,
e = 1	the non-dimensional rate sensitivity parameter
E_0 = 1640 [MPa]	relaxed elastisite moduli, young modulus
tau = 2 [s]	relaxation time
E_inf = 1900 [MPa]	non-relaxed elastisite moduli, dynamic modulus
alpha = 1,1/2,1/3,3/4	fractional derivative order
eps=0.02 [mm/mm]	epsilon for relaxation
mu = 90007.2	the viscosity-related parameter/ material constant

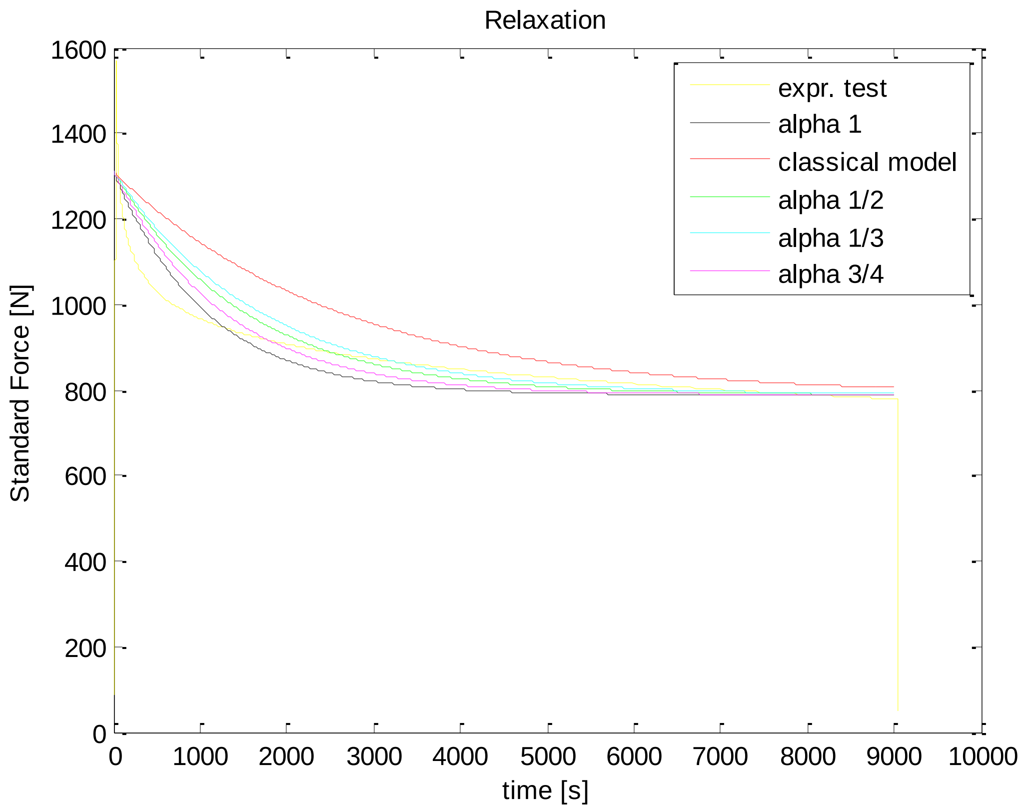

Figure A.18 : Stress relaxation response of PA6 and its comparison with fractional and classical viscoplastic models.

sig_y = 19.7[MPa]	yield stress,
e = 1	the non-dimensional rate sensitivity parameter
E_0 = 1640 [MPa]	relaxed elastisite moduli, young modulus
tau = 2 [s]	relaxation time
E_inf = 1900 [MPa]	non-relaxed elastisite moduli, dynamic modulus
alpha = 1,1/2,1/3,3/4	fractional derivative order
eps=0.02 [mm/mm]	epsilon for relaxation
mu = 218507.2	the viscosity-related parameter/ material constant

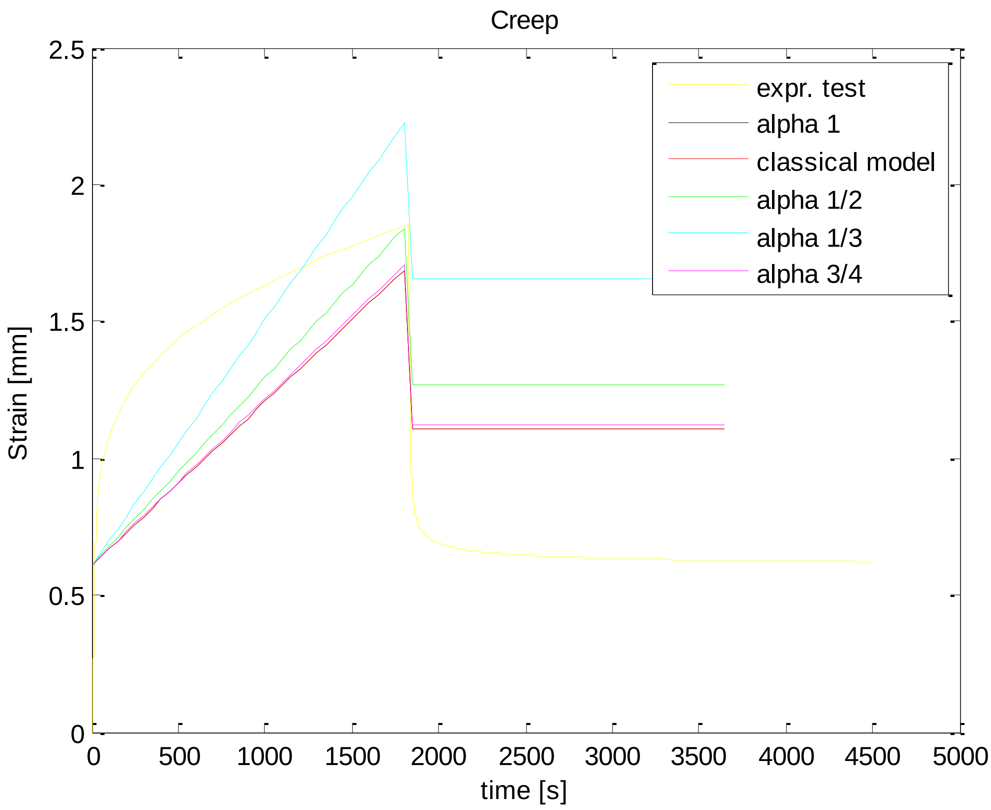

Figure A.19 : PA6 creep and recovery experimental result and its comparison with fractional and classical viscoplastic models.

sig_y = 19.7[MPa]	yield stress,
e = 1	the non-dimensional rate sensitivity parameter
E_0 = 1640 [MPa]	relaxed elastisite moduli, young modulus
tau = 2 [s]	relaxation time
E_inf = 1900 [MPa]	non-relaxed elastisite moduli, dynamic modulus
alpha = 1,1/2,1/3,3/4	fractional derivative order
sigma=25 [MPa]	stress for creep
mu = 18000.0	the viscosity-related parameter/ material constant

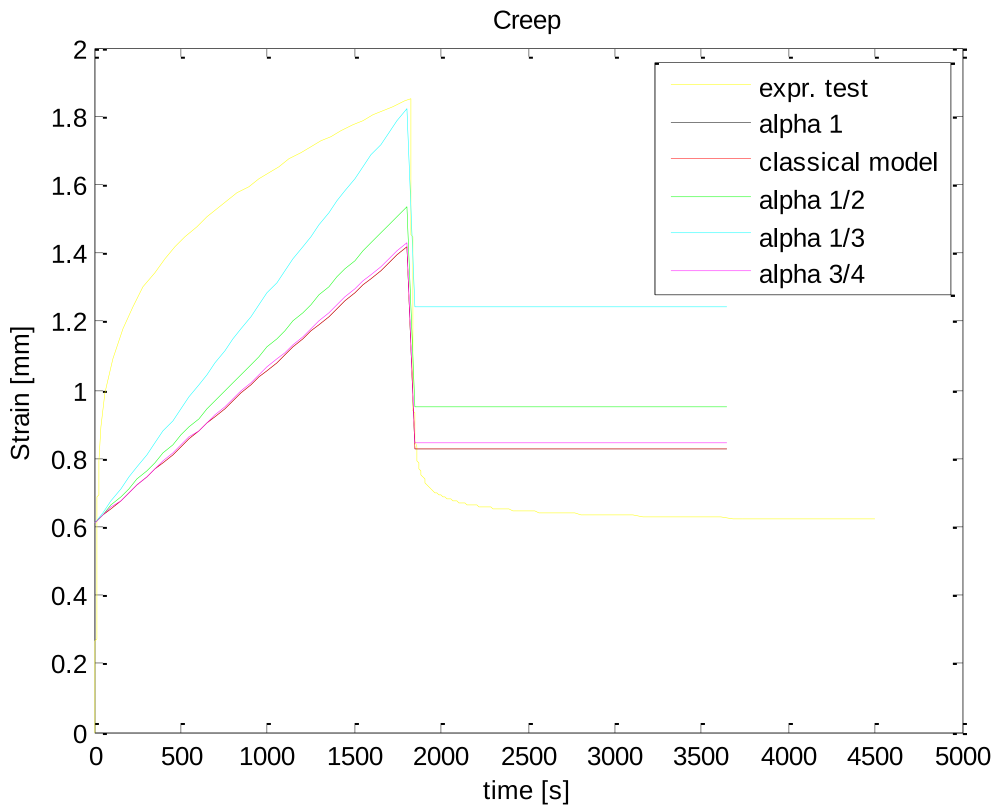

Figure A.20 : PA6 creep and recovery experimental result and its comparison with fractional and classical viscoplastic models.

sig_y = 19.7[MPa]	yield stress,
e = 1	the non-dimensional rate sensitivity parameter
E_0 = 1640 [MPa]	relaxed elastisite moduli, young modulus
tau = 2 [s]	relaxation time
E_inf = 1900 [MPa]	non-relaxed elastisite moduli, dynamic modulus
alpha = 1,1/2,1/3,3/4	fractional derivative order
sigma=25 [MPa]	stress for creep
mu = 24000.0	the viscosity-related parameter/ material constant

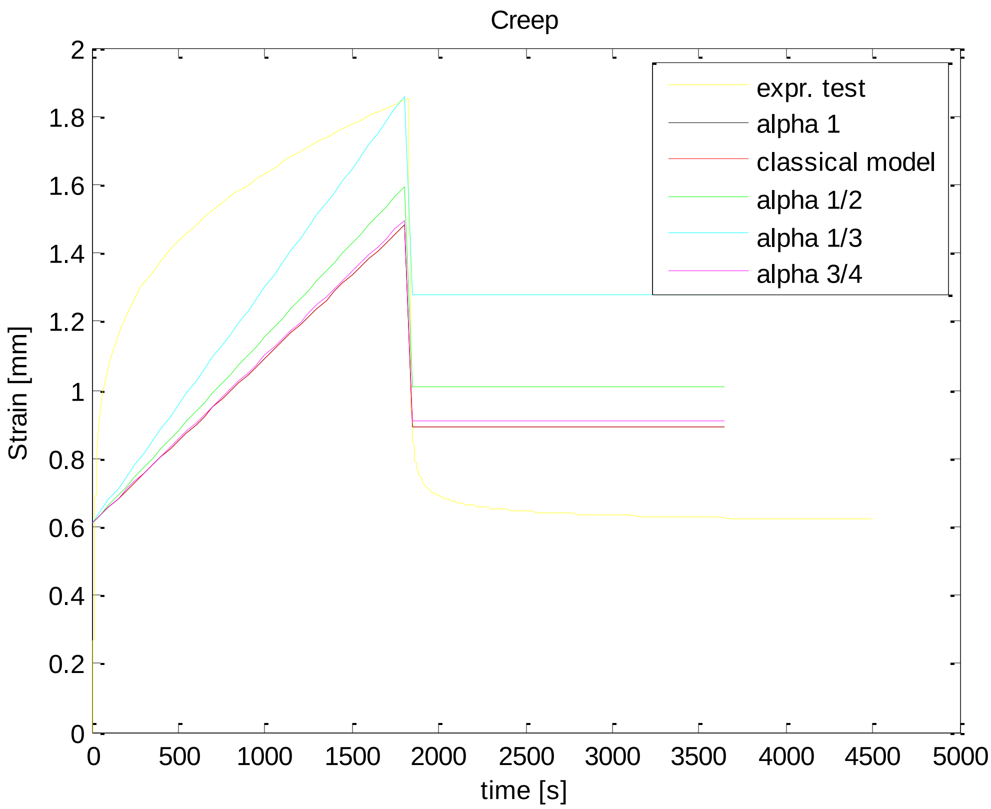

Figure A.21 : PA6 creep and recovery experimental result and its comparison with fractional and classical viscoplastic models.

sig_y = 19.7[MPa]	yield stress,
e = 8	the non-dimensional rate sensitivity parameter
E_0 = 1640 [MPa]	relaxed elastisite moduli, young modulus
tau = 2 [s]	relaxation time
E_inf = 1900 [MPa]	non-relaxed elastisite moduli, dynamic modulus
alpha = 1,1/2,1/3,3/4	fractional derivative order
sigma=25 [MPa]	stress for creep
mu = 2500.0	the viscosity-related parameter/ material constant

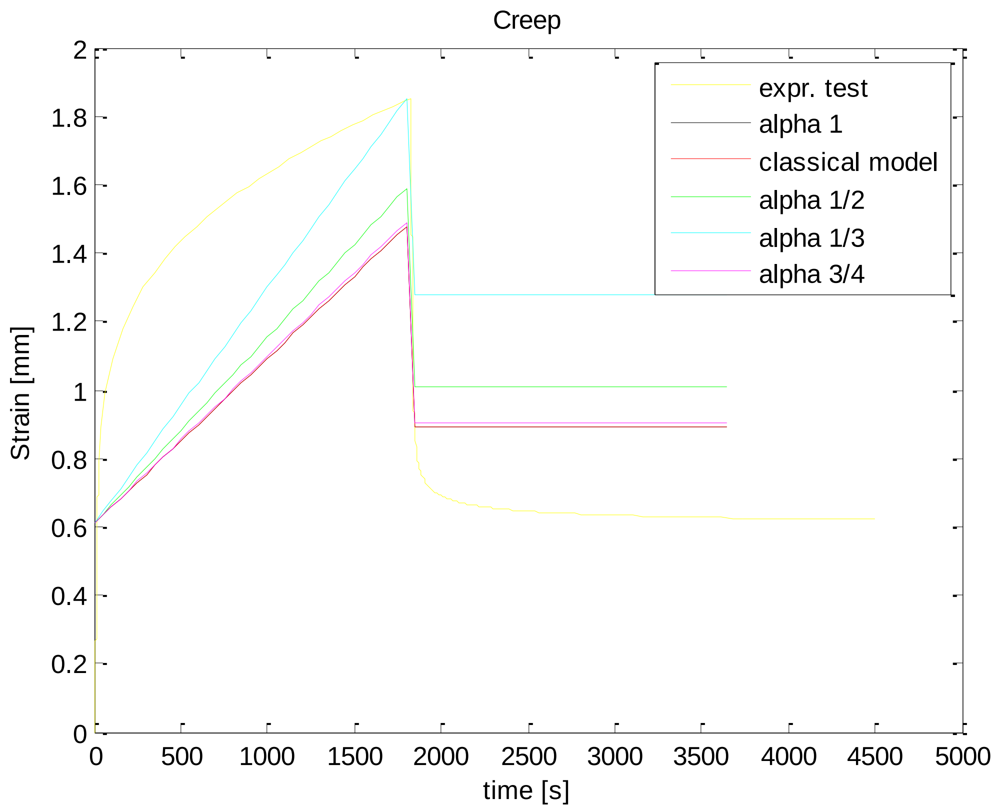

Figure A.22 : PA6 creep and recovery experimental result and its comparison with fractional and classical viscoplastic models.

sig_y = 19.7[MPa]	yield stress,
e =10	the non-dimensional rate sensitivity parameter
E_0 = 1640 [MPa]	relaxed elastisite moduli, young modulus
tau = 2 [s]	relaxation time
E_inf = 1900 [MPa]	non-relaxed elastisite moduli, dynamic modulus
alpha = 1,1/2,1/3,3/4	fractional derivative order
sigma=25 [MPa]	stress for creep
mu = 2000.0	the viscosity-related parameter/ material constant

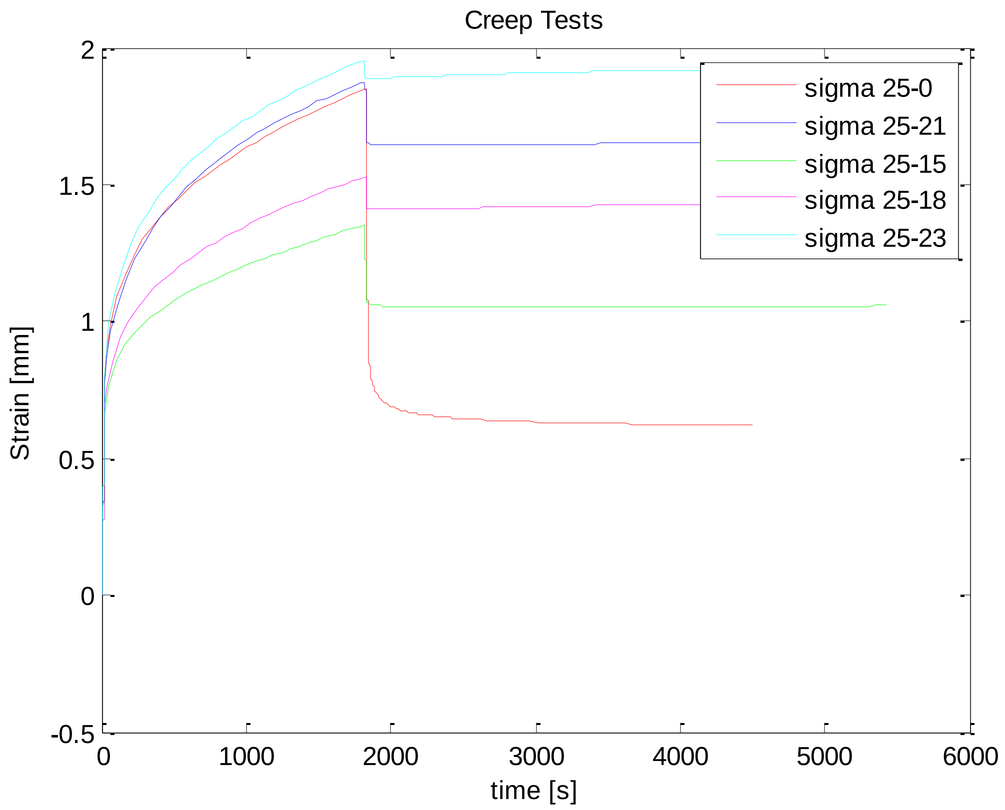

Figure A.23 : Strain response under two step loads for PA6 specimen.

APPENDIX B

APPENDIX B.1 : Definition of Kelvin-Voigt Model's mathematical constitutive equation and creep compliance graphics programme codes are in below.

```
% Kelvin Model
clear all;
close all;
clc;
E = 1.3081;             % MPa
nu = 0.14128;           % MPa/s
tau = nu/E;
alpha = 1;              % fractional derivative order
deltaT = 0.001;         % time step
tFinal = 3.6;           % total time
n=tFinal/deltaT;        % time increment
sigma0=2;
k1=0.001;
k2=n/2;
% Time
t(1)=0;
for i=1:n

   if (i>1)
       t(i) = t(i-1) + deltaT ;

   end
end

A(1) = 0;
for ii=1:n
  if (ii>k1)
      f(ii)=1;
  else
      f(ii)=0;
  end
  if (ii>k2)
      fb(ii)=1;
  else
      fb(ii)=0;
  end
  J1 = ((sigma0*f(ii))/E)*(1-exp((-t(ii)+k1*deltaT)/tau));
  J0 = ((sigma0*fb(ii))/E)*(1-exp((-t(ii)+k2*deltaT)/tau));
  A(ii) = J1-J0;
end
figure
%subplot(2, 1, 1);
plot(t, A,'Color','red');
ylabel('Strain [mm/mm]');
xlabel('Time [s]') % xlabel('Time [s]')
title('Creep') % Title of the plot
```

APPENDIX B.2 : The programme codes which reduce Lai-Bakker first experimental test results (see Table C.1) for HDPE to creep graphics are placed in below.

```
clear
fid = fopen('crp7.txt', 'r');
An = fscanf(fid, '%g %g %g %g %g', [5 inf]);    % It has two rows
now.
An = An';
fclose(fid);
data2Plot = [
     0.01E+00    0.633072E+00    0.676073E+00    0.738186E+00    0.802688E+00
0.864800E+00    0.974692E+00    0.110131E+01    0.127331E+01;
    0.05E+00    0.800299E+00    0.876745E+00    0.953192E+00    0.104875E+01
0.114670E+01    0.128526E+01    0.146204E+01    0.169854E+01;
    0.10E+00    0.894662E+00    0.980664E+00    0.107503E+01    0.118253E+01
0.129839E+01    0.147040E+01    0.167346E+01    0.193505E+01;
    0.20E+00    0.100694E+01    0.110967E+01    0.121717E+01    0.134737E+01
0.148951E+01    0.168541E+01    0.192430E+01    0.222770E+01;
    0.30E+00    0.108219E+01    0.119448E+01    0.131870E+01    0.146443E+01
0.161732E+01    0.183233E+01    0.209750E+01    0.243076E+01;
    0.40E+00    0.113834E+01    0.126017E+01    0.139634E+01    0.155162E+01
0.171646E+01    0.195058E+01    0.223486E+01    0.259918E+01;
    0.50E+00    0.118731E+01    0.131631E+01    0.145965E+01    0.162688E+01
0.179649E+01    0.204016E+01    0.235073E+01    0.274252E+01;
    0.60E+00    0.122673E+01    0.136887E+01    0.151579E+01    0.168779E+01
0.186697E+01    0.212019E+01    0.244987E+01    0.287032E+01;
    0.70E+00    0.126137E+01    0.141068E+01    0.156357E+01    0.174274E+01
0.193266E+01    0.219664E+01    0.254423E+01    0.298619E+01;
    0.80E+00    0.129481E+01    0.145009E+01    0.160776E+01    0.179410E+01
0.199000E+01    0.226234E+01    0.262546E+01    0.309728E+01;
    0.90E+00    0.132706E+01    0.148473E+01    0.165196E+01    0.183830E+01
0.204136E+01    0.232326E+01    0.270310E+01    0.320239E+01;
    0.10E+01    0.135692E+01    0.152176E+01    0.168899E+01    0.188249E+01
0.208794E+01    0.237462E+01    0.277357E+01    0.330153E+01;
    0.11E+01    0.138320E+01    0.154923E+01    0.172602E+01    0.192191E+01
0.213453E+01    0.242598E+01    0.283688E+01    0.339112E+01;
    0.12E+01    0.140590E+01    0.158268E+01    0.175707E+01    0.195775E+01
0.217633E+01    0.247256E+01    0.290138E+01    0.348309E+01;
    0.13E+01    0.143098E+01    0.160776E+01    0.178694E+01    0.199239E+01
0.221456E+01    0.251915E+01    0.296230E+01    0.357029E+01;
    0.14E+01    0.145129E+01    0.163285E+01    0.181919E+01    0.202702E+01
0.225159E+01    0.256215E+01    0.301844E+01    0.365271E+01;
    0.15E+01    0.147159E+01    0.165554E+01    0.184427E+01    0.205928E+01
0.228862E+01    0.260635E+01    0.307219E+01    0.373393E+01;
    0.16E+01    0.149309E+01    0.167824E+01    0.187174E+01    0.208914E+01
0.232564E+01    0.264099E+01    0.312594E+01    0.381396E+01;
    0.17E+01    0.151101E+01    0.170213E+01    0.189683E+01    0.212019E+01
0.235431E+01    0.268279E+01    0.317492E+01    0.389160E+01;
    0.18E+01    0.152893E+01    0.172482E+01    0.192072E+01    0.215006E+01
0.238656E+01    0.271624E+01    0.322508E+01    0.396924E+01;
    0.18E+01    0.851379E+00    0.980498E+00    0.112273E+01    0.130430E+01
0.147680E+01    0.168258E+01    0.199933E+01    0.246940E+01;
    0.19E+01    0.607263E+00    0.707128E+00    0.828178E+00    0.995629E+00
0.115098E+01    0.131641E+01    0.159886E+01    0.200437E+01;
    0.20E+01    0.522529E+00    0.613315E+00    0.724277E+00    0.881641E+00
0.102690E+01    0.118023E+01    0.144855E+01    0.182986E+01;
    0.21E+01    0.470074E+00    0.555817E+00    0.664761E+00    0.816073E+00
0.954270E+00    0.109953E+01    0.135272E+01    0.172091E+01;
    0.22E+01    0.439812E+00    0.519502E+00    0.620377E+00    0.765635E+00
0.898790E+00    0.104001E+01    0.128413E+01    0.164324E+01;
    0.23E+01    0.409549E+00    0.492266E+00    0.591123E+00    0.730330E+00
0.859449E+00    0.996638E+00    0.123067E+01    0.158171E+01;
    0.24E+01    0.391392E+00    0.471083E+00    0.567922E+00    0.705111E+00
0.824143E+00    0.957297E+00    0.118830E+01    0.152925E+01;
    0.25E+01    0.377270E+00    0.451917E+00    0.545730E+00    0.678884E+00
0.799933E+00    0.926026E+00    0.115098E+01    0.148588E+01;
    0.26E+01    0.361130E+00    0.437794E+00    0.526564E+00    0.655683E+00
0.776732E+00    0.897781E+00    0.111970E+01    0.145057E+01;
```

```
      0.27E+01    0.349025E+00    0.426698E+00    0.513450E+00    0.636516E+00
0.753531E+00    0.874580E+00    0.109449E+01    0.141930E+01;
      0.28E+01    0.338937E+00    0.414593E+00    0.502354E+00    0.622394E+00
0.734365E+00    0.854405E+00    0.107028E+01    0.138702E+01;
      0.29E+01    0.332885E+00    0.405514E+00    0.490249E+00    0.601210E+00
0.718225E+00    0.839274E+00    0.104707E+01    0.136079E+01;
      0.30E+01    0.321789E+00    0.397445E+00    0.483188E+00    0.596167E+00
0.706120E+00    0.809011E+00    0.102690E+01    0.133860E+01;
      0.31E+01    0.315736E+00    0.390383E+00    0.473100E+00    0.586079E+00
0.695024E+00    0.808003E+00    0.100975E+01    0.131843E+01;
      0.32E+01    0.308675E+00    0.385340E+00    0.466039E+00    0.574983E+00
0.679892E+00    0.792872E+00    0.994620E+00    0.129825E+01;
      0.33E+01    0.303631E+00    0.379287E+00    0.458978E+00    0.567922E+00
0.672831E+00    0.781775E+00    0.980498E+00    0.128110E+01;
      0.34E+01    0.297579E+00    0.373235E+00    0.452925E+00    0.559852E+00
0.660726E+00    0.769670E+00    0.965366E+00    0.126496E+01;
      0.35E+01    0.294553E+00    0.369200E+00    0.443847E+00    0.552791E+00
0.653665E+00    0.758574E+00    0.956288E+00    0.125084E+01;
      0.36E+01    0.288500E+00    0.364156E+00    0.440820E+00    0.545730E+00
0.642569E+00    0.747478E+00    0.942165E+00    0.123672E+01;
      0.37E+01    0.284465E+00    0.358104E+00    0.436785E+00    0.539677E+00
0.634499E+00    0.739408E+00    0.931069E+00    0.122360E+01;
      0.38E+01    0.280430E+00    0.357095E+00    0.433759E+00    0.532616E+00
0.627438E+00    0.732347E+00    0.924008E+00    0.121049E+01;
      0.39E+01    0.279422E+00    0.351042E+00    0.426698E+00    0.527572E+00
0.624412E+00    0.724277E+00    0.911903E+00    0.119839E+01;
      0.40E+01    0.273369E+00    0.350034E+00    0.424681E+00    0.521520E+00
0.616342E+00    0.717216E+00    0.904842E+00    0.118830E+01;
      0.41E+01    0.272360E+00    0.347007E+00    0.419637E+00    0.516476E+00
0.609280E+00    0.710155E+00    0.897781E+00    0.118023E+01;
      0.42E+01    0.269334E+00    0.343981E+00    0.416611E+00    0.511432E+00
0.605245E+00    0.703093E+00    0.886685E+00    0.116812E+01;
      0.43E+01    0.265299E+00    0.339946E+00    0.414593E+00    0.509415E+00
0.598184E+00    0.695024E+00    0.878615E+00    0.115905E+01;
      0.44E+01    0.262273E+00    0.340955E+00    0.413584E+00    0.504371E+00
0.594149E+00    0.691997E+00    0.873571E+00    0.115198E+01;
      0.45E+01    0.263282E+00    0.335911E+00    0.410558E+00    0.503362E+00
0.588097E+00    0.683927E+00    0.868527E+00    0.114391E+01;
      0.46E+01    0.260256E+00    0.333894E+00    0.406523E+00    0.495293E+00
0.584062E+00    0.680901E+00    0.860457E+00    0.113484E+01;
      0.47E+01    0.256221E+00    0.332885E+00    0.403497E+00    0.494284E+00
0.581036E+00    0.673840E+00    0.855414E+00    0.112979E+01;
      0.48E+01    0.254203E+00    0.327841E+00    0.400471E+00    0.489240E+00
0.578009E+00    0.670814E+00    0.845326E+00    0.112172E+01;
      0.49E+01    0.253194E+00    0.328850E+00    0.398453E+00    0.487223E+00
0.571957E+00    0.666779E+00    0.842300E+00    0.111466E+01;
      0.50E+01    0.252186E+00    0.328850E+00    0.395427E+00    0.486214E+00
0.570948E+00    0.659718E+00    0.838265E+00    0.110760E+01;
      0.51E+01    0.249159E+00    0.325824E+00    0.394418E+00    0.483188E+00
0.567922E+00    0.660726E+00    0.832213E+00    0.110256E+01;
      0.52E+01    0.251177E+00    0.324815E+00    0.395427E+00    0.477135E+00
0.563887E+00    0.657700E+00    0.828178E+00    0.109449E+01;
      0.53E+01    0.248151E+00    0.320780E+00    0.391392E+00    0.476126E+00
0.560861E+00    0.649630E+00    0.822125E+00    0.109146E+01;
      0.54E+01    0.245124E+00    0.323806E+00    0.389375E+00    0.475118E+00
0.556826E+00    0.645595E+00    0.820108E+00    0.108843E+01  ];


figure(1)
plot(data2Plot(:,1),data2Plot(:,2),data2Plot(:,1),data2Plot(:,3),..
.
    data2Plot(:,1),data2Plot(:,4),data2Plot(:,1),data2Plot(:,5),...
    data2Plot(:,1),data2Plot(:,6),data2Plot(:,1),data2Plot(:,6),...
    data2Plot(:,1),data2Plot(:,8),data2Plot(:,1),data2Plot(:,9))
xlabel('Time [s]') % xlabel('Time [s]')
ylabel('Strain [mm/mm]') % ylabel('Magnitude [-]')
legend('sigma 2','sigma 4','sigma 6','sigma 8','sigma 10',...
    'sigma 12','sigma 14','sigma 16')
title('Creep') % Title of the plot
```

APPENDIX B.3 : The programme codes which reduce Lai-Bakker second experimental test results (see Table C.2) for HDPE to creep graphics are placed in below.

```
clear all
close all
clc
data2Plot = [
     0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00
0.000000E+00    0.000000E+00    0.000000E+00    0.000000E+00;
    0.100000E+02    0.126614E-02    0.270429E-02    0.442912E-02    0.642150E-02
0.864800E-02    0.116963E-01    0.154183E-01    0.203730E-01;
    0.500000E+02    0.160060E-02    0.350698E-02    0.571915E-02    0.839000E-02
0.114670E-01    0.154231E-01    0.204686E-01    0.271766E-01;
    0.100000E+03    0.178932E-02    0.392266E-02    0.645018E-02    0.946024E-02
0.129839E-01    0.176448E-01    0.234284E-01    0.309608E-01;
    0.200000E+03    0.201388E-02    0.443868E-02    0.730302E-02    0.107790E-01
0.148951E-01    0.202249E-01    0.269402E-01    0.356432E-01;
    0.300000E+03    0.216438E-02    0.477792E-02    0.791220E-02    0.117154E-01
0.161732E-01    0.219880E-01    0.293650E-01    0.388922E-01;
    0.400000E+03    0.227668E-02    0.504068E-02    0.837804E-02    0.124130E-01
0.171646E-01    0.234070E-01    0.312880E-01    0.415869E-01;
    0.500000E+03    0.237462E-02    0.526524E-02    0.875790E-02    0.130150E-01
0.179649E-01    0.244819E-01    0.329102E-01    0.438803E-01;
    0.600000E+03    0.245346E-02    0.547548E-02    0.909474E-02    0.135023E-01
0.186697E-01    0.254423E-01    0.342982E-01    0.459251E-01;
    0.700000E+03    0.252274E-02    0.564272E-02    0.938142E-02    0.139419E-01
0.193266E-01    0.263597E-01    0.356192E-01    0.477790E-01;
    0.800000E+03    0.258962E-02    0.580036E-02    0.964656E-02    0.143528E-01
0.199000E-01    0.271481E-01    0.367564E-01    0.495565E-01;
    0.900000E+03    0.265412E-02    0.593892E-02    0.991176E-02    0.147064E-01
0.204136E-01    0.278791E-01    0.378434E-01    0.512382E-01;
    0.100000E+04    0.271384E-02    0.608704E-02    0.101339E-01    0.150599E-01
0.208794E-01    0.284954E-01    0.388300E-01    0.528245E-01;
    0.110000E+04    0.276640E-02    0.619692E-02    0.103561E-01    0.153753E-01
0.213453E-01    0.291118E-01    0.397163E-01    0.542579E-01;
    0.120000E+04    0.281180E-02    0.633072E-02    0.105424E-01    0.156620E-01
0.217633E-01    0.296707E-01    0.406193E-01    0.557294E-01;
    0.130000E+04    0.286196E-02    0.643104E-02    0.107216E-01    0.159391E-01
0.221456E-01    0.302298E-01    0.414722E-01    0.571246E-01;
    0.140000E+04    0.290258E-02    0.653140E-02    0.109151E-01    0.162162E-01
0.225159E-01    0.307458E-01    0.422582E-01    0.584434E-01;
    0.150000E+04    0.294318E-02    0.662216E-02    0.110656E-01    0.164742E-01
0.228862E-01    0.312762E-01    0.430107E-01    0.597429E-01;
    0.160000E+04    0.298618E-02    0.671296E-02    0.112304E-01    0.167131E-01
0.232564E-01    0.316919E-01    0.437632E-01    0.610234E-01;
    0.170000E+04    0.302202E-02    0.680852E-02    0.113810E-01    0.169615E-01
0.235431E-01    0.321935E-01    0.444489E-01    0.622656E-01;
    0.180000E+04    0.305786E-02    0.689928E-02    0.115243E-01    0.172005E-01
0.238656E-01    0.325949E-01    0.451511E-01    0.635078E-01;
    0.180000E+04    0.170276E-02    0.392199E-02    0.673638E-02    0.104344E-01
0.147680E-01    0.201910E-01    0.279906E-01    0.395104E-01;
    0.190000E+04    0.121453E-02    0.282851E-02    0.496907E-02    0.796503E-02
0.115098E-01    0.157969E-01    0.223840E-01    0.320699E-01;
    0.200000E+04    0.104506E-02    0.245326E-02    0.434566E-02    0.705313E-02
0.102690E-01    0.141628E-01    0.202797E-01    0.292778E-01;
    0.210000E+04    0.940148E-03    0.222327E-02    0.398857E-02    0.652858E-02
0.954270E-02    0.131944E-01    0.189381E-01    0.275346E-01;
    0.220000E+04    0.879624E-03    0.207801E-02    0.372226E-02    0.612508E-02
0.898790E-02    0.124801E-01    0.179778E-01    0.262918E-01;
    0.230000E+04    0.819098E-03    0.196906E-02    0.354674E-02    0.584264E-02
0.859449E-02    0.119597E-01    0.172294E-01    0.253074E-01;
    0.240000E+04    0.782784E-03    0.188433E-02    0.340753E-02    0.564089E-02
0.824143E-02    0.114876E-01    0.166362E-01    0.244680E-01;
    0.250000E+04    0.754540E-03    0.180767E-02    0.327438E-02    0.543107E-02
0.799933E-02    0.111123E-01    0.161137E-01    0.237741E-01;
    0.260000E+04    0.722260E-03    0.175118E-02    0.315938E-02    0.524546E-02
0.776732E-02    0.107734E-01    0.156758E-01    0.232091E-01;
    0.270000E+04    0.698050E-03    0.170679E-02    0.308070E-02    0.509213E-02
0.753531E-02    0.104950E-01    0.153229E-01    0.227088E-01;
    0.280000E+04    0.677874E-03    0.165837E-02    0.301412E-02    0.497915E-02
0.734365E-02    0.102529E-01    0.149839E-01    0.221923E-01;
```

```
    0.290000E+04    0.665770E-03    0.162206E-02    0.294149E-02    0.480968E-02
0.718225E-02    0.100713E-01    0.146590E-01    0.217726E-01;
    0.300000E+04    0.643578E-03    0.158978E-02    0.289913E-02    0.476934E-02
0.706120E-02    0.970813E-02    0.143766E-01    0.214176E-01;
    0.310000E+04    0.631472E-03    0.156153E-02    0.283860E-02    0.468863E-02
0.695024E-02    0.969604E-02    0.141365E-01    0.210949E-01;
    0.320000E+04    0.617350E-03    0.154136E-02    0.279623E-02    0.459986E-02
0.679892E-02    0.951446E-02    0.139247E-01    0.207720E-01;
    0.330000E+04    0.607262E-03    0.151715E-02    0.275387E-02    0.454338E-02
0.672831E-02    0.938130E-02    0.137270E-01    0.204976E-01;
    0.340000E+04    0.595158E-03    0.149294E-02    0.271755E-02    0.447882E-02
0.660726E-02    0.923604E-02    0.135151E-01    0.202394E-01;
    0.350000E+04    0.589106E-03    0.147680E-02    0.266308E-02    0.442233E-02
0.653665E-02    0.910289E-02    0.133880E-01    0.200134E-01;
    0.360000E+04    0.577000E-03    0.145662E-02    0.264492E-02    0.436584E-02
0.642569E-02    0.896974E-02    0.131903E-01    0.197875E-01;
    0.370000E+04    0.568930E-03    0.143242E-02    0.262071E-02    0.431742E-02
0.634499E-02    0.887290E-02    0.130350E-01    0.195776E-01;
    0.380000E+04    0.560860E-03    0.142838E-02    0.260255E-02    0.426093E-02
0.627438E-02    0.878816E-02    0.129361E-01    0.193678E-01;
    0.390000E+04    0.558844E-03    0.140417E-02    0.256019E-02    0.422058E-02
0.624412E-02    0.869132E-02    0.127666E-01    0.191742E-01;
    0.400000E+04    0.546738E-03    0.140014E-02    0.254809E-02    0.417216E-02
0.616342E-02    0.860659E-02    0.126678E-01    0.190128E-01;
    0.410000E+04    0.544720E-03    0.138803E-02    0.251782E-02    0.413181E-02
0.609280E-02    0.852186E-02    0.125689E-01    0.188837E-01;
    0.420000E+04    0.538668E-03    0.137592E-02    0.249967E-02    0.409146E-02
0.605245E-02    0.843712E-02    0.124136E-01    0.186899E-01;
    0.430000E+04    0.530598E-03    0.135978E-02    0.248756E-02    0.407532E-02
0.598184E-02    0.834029E-02    0.123006E-01    0.185448E-01;
    0.440000E+04    0.524546E-03    0.136382E-02    0.248150E-02    0.403497E-02
0.594149E-02    0.830396E-02    0.122300E-01    0.184317E-01;
    0.450000E+04    0.526564E-03    0.134364E-02    0.246335E-02    0.402690E-02
0.588097E-02    0.820712E-02    0.121594E-01    0.183026E-01;
    0.460000E+04    0.520512E-03    0.133558E-02    0.243914E-02    0.396234E-02
0.584062E-02    0.817081E-02    0.120464E-01    0.181574E-01;
    0.470000E+04    0.512442E-03    0.133154E-02    0.242098E-02    0.395427E-02
0.581036E-02    0.808608E-02    0.119758E-01    0.180766E-01;
    0.480000E+04    0.508406E-03    0.131136E-02    0.240283E-02    0.391392E-02
0.578009E-02    0.804977E-02    0.118346E-01    0.179475E-01;
    0.490000E+04    0.506388E-03    0.131540E-02    0.239072E-02    0.389778E-02
0.571957E-02    0.800135E-02    0.117922E-01    0.178346E-01;
    0.500000E+04    0.504372E-03    0.131540E-02    0.237256E-02    0.388971E-02
0.570948E-02    0.791662E-02    0.117357E-01    0.177216E-01;
    0.510000E+04    0.498318E-03    0.130330E-02    0.236651E-02    0.386550E-02
0.567922E-02    0.792871E-02    0.116510E-01    0.176410E-01;
    0.520000E+04    0.502354E-03    0.129926E-02    0.237256E-02    0.381708E-02
0.563887E-02    0.789240E-02    0.115945E-01    0.175118E-01;
    0.530000E+04    0.496302E-03    0.128312E-02    0.234835E-02    0.380901E-02
0.560861E-02    0.779556E-02    0.115098E-01    0.174634E-01;
    0.540000E+04    0.490248E-03    0.129522E-02    0.233625E-02    0.380094E-02
0.556826E-02    0.774714E-02    0.114815E-01    0.174149E-01  ];
figure(2)
plot(data2Plot(:,1),data2Plot(:,2),data2Plot(:,1),data2Plot(:,3),..
.
    data2Plot(:,1),data2Plot(:,4),data2Plot(:,1),data2Plot(:,5),...
    data2Plot(:,1),data2Plot(:,6),data2Plot(:,1),data2Plot(:,6),...
    data2Plot(:,1),data2Plot(:,8),data2Plot(:,1),data2Plot(:,9))
xlabel('Time [s]') % xlabel('Time [s]')
ylabel('Strain [mm/mm]') % ylabel('Magnitude [-]')
legend('sigma 2','sigma 4','sigma 6','sigma 8','sigma 10',...
    'sigma 12','sigma 14','sigma 16')
title('Creep') % Title of the plot
```

APPENDIX B.4 : The programme codes which attribute relaxation graphics (see section 5.1.2, Figure 5.1) were presented.

```
clear all;
close all;
%==================================================================
===
mu = 110000.0;     % the viscosity-related parameter/material
constant
sig_y = 19.7;      % yield stress, constant (MPa)
e = 1;             % the non-dimensional rate sensitivity
parameter/material constant
E_0 = 1640;        % relaxed elastisite moduli  (MPa)?????????????
tau = 2;           % relaxation time (constant)(ms)????????????
E_inf = 1900;      % non-relaxed elastisite moduli(MPa)?????????
alpha = 3/4;       % fractional derivative order
deltaT = 1;        % time step
tFinal = 9000;     % total time
n=tFinal/deltaT;   % time increment
eps=0.02;          % constant sigma for creep
%m=20;
sig_0 = E_0*eps;
% Time
t(1)=0;
for i=1:n
   if (i>1)
       t(i) = t(i-1) + deltaT ;

   end
end

k21 = E_0/(mu*sig_y);                                  %standart
constants

Gs(1)=0;
for ii=1:n
   kigma1 = sig_0-[(sig_0-sig_y)*(1-exp(-k21*t(ii)))];
%standart sigma formula
   sum = 0;
   c2 = ((tau^alpha)*E_inf)/(mu*sig_y);
   %c2 = sum/(mu*sig_y);                                %fractional
constants

   sigma = sig_0-[(sig_0-sig_y)*(1-exp(-c2*t(ii)))];
%fractional sigma formula
   Gs(ii) = sigma*40;          %fractional
   Gk1(ii) = kigma1*40;        %standart

end
for iii=1:n
    if (0.001 < iii*deltaT) && (iii*deltaT < n )
        f(iii) = eps;
    elseif (iii==n)
        f(iii) = 0;
    else
        f(iii) = 0;
    end
end
figure;
```

```
%subplot(2, 1, 1);
hold on
%plot(E_0*t/mu/sig_y,Gs,'Color','magenta');
plot(t,Gs,'Color','magenta');       %sacma ama daha sonra burdan
zamani bul
%plot(E_0*t/mu/sig_y,Gk1,'Color','red');
plot(t,Gk1,'Color','red');
xlabel('time'); ylabel('sigma');
title('relaxation');
hold off
```

APPENDIX B.5 : Matlab program codes of relaxation graphics (see Appendix A, Figures A.1-9,14-18) mentioned in section 6 and 7 are below. These codes were used in comparison of classical and fractional viscoplastic models for HDPE material in section 6, as to in section 7 they were used in comparison of classical, fractional models and experimental test results for PA6 material.

```
clear all;
close all;

mu = 110000.0;            % the viscosity-related
parameter/material constant
sig_y = 19.7;       % yield stress, constant (MPa)
e = 1;               % the non-dimensional rate sensitivity
parameter/material constant
E_0 = 1640;      % relaxed elastisite moduli (MPa)?????????????
tau = 2;            % relaxation time (constant)(ms)????????????
E_inf = 1900;          % non-relaxed elastisite
moduli(MPa)?????????
alpha = 3/4;           % fractional derivative order
deltaT = 1;        % time step
tFinal = 9000;         % total time
n=tFinal/deltaT;     % time increment
eps=0.02;            % constant sigma for creep
%m=20;

sig_0 = E_0*eps;
% Time
t(1)=0;
for i=1:n
   if (i>1)
       t(i) = t(i-1) + deltaT ;

   end
end
%tmp = E_inf*tau^(alpha)*1/deltaT^(alpha)*(3/2)^(alpha);
k21 = E_0/(mu*sig_y);                              %standart
constants
Gs(1)=0;
for ii=1:n
   kigma1 = sig_0-[(sig_0-sig_y)*(1-exp(-k21*t(ii)))];
%standart sigma formula
   sum = 0;
   c2 = ((tau^alpha)*E_inf)/(mu*sig_y);
   %c2 = sum/(mu*sig_y);                           %fractional
constants
   sigma = sig_0-[(sig_0-sig_y)*(1-exp(-c2*t(ii)))];
%fractional sigma formula
   Gs(ii) = sigma*40;         %fractional
   Gk1(ii) = kigma1*40;       %standart

end
for iii=1:n
    if (0.001 < iii*deltaT) && (iii*deltaT < n )
        f(iii) = eps;
    elseif (iii==n)
        f(iii) = 0;
    else
```

```
        f(iii) = 0;
    end
end

figure;
%subplot(2, 1, 1);
hold on
%plot(E_0*t/mu/sig_y,Gs,'Color','magenta');
plot(t,Gs,'Color','magenta');
%plot(E_0*t/mu/sig_y,Gk1,'Color','red');
plot(t,Gk1,'Color','red');
xlabel('time'); ylabel('sigma');
title('relaxation');
hold off
```

APPENDIX B.6 : Matlab program codes of creep graphics (see Appendix A, Figures A.10-13,19-22) mentioned in section 6 and 7 are below. These codes were used in comparison of classical and fractional viscoplastic models for HDPE material in section 6, as to in section 7 they were used in comparison of classical, fractional models and experimental test results for PA6 material.

```
clear all
close all
mu = 2.21;              % the viscosity-related parameter/material
constant
sig_y = 23.4;          % yield stress, constant (MPa)
e = 1;                 % the non-dimensional rate sensitivity
parameter/material constant
E_0 = 745.0;           % relaxed elastisite moduli (MPa)?????????????
tau = 2;                % relaxation time (constant)(ms)????????????
E_inf = 1065.0;              % non-relaxed elastisite
moduli(MPa)?????????
alpha = 1;              % fractional derivative order
deltaT = 0.01;           % time step
tFinal = 25;            % total time
n=tFinal/deltaT;        % time increment
sigma=38;              % constant sigma for creep
k1=20;
k2=0.001;
% Time
t(1)=0;
for i=1:n
   if (i>1)
       t(i) = t(i-1) + deltaT ;
       %epsT(i) = sin(t(i));
   end
end

for ii=1:n
  if (ii>k1)
      f(ii)=1;
  else
      f(ii)=0;
  end
  if (ii>k2)
      fb(ii)=1;
  else
      fb(ii)=0;
  end

  sigmaFa = sigma + (tau^alpha)*topal(sigma,alpha,deltaT,tFinal-
k1*deltaT);
  x1(ii)=topal(sigma*f(ii),alpha,deltaT,tFinal-k1*deltaT);
  strnRatea = (1/mu) * [(sigmaFa/sig_y)^(1/e)-1];
  epsa = (sigma)/E_0+ strnRatea*(t(ii)-k1*deltaT) ;

  sigmaFb = sigma + (tau^alpha)*topal(sigma,alpha,deltaT,tFinal-
k2*deltaT);
  x2(ii)=topal(sigma*fb(ii),alpha,deltaT,tFinal-k2*deltaT);
  strnRateb = (1/mu) * [(sigmaFb/sig_y)^(1/e)-1];
  epsb = (sigma)/E_0+ strnRateb*( t(ii) - k2*deltaT );
```

```
  Gs(ii)=f(ii)*epsa-fb(ii)*epsb;

  eps1 = f(ii)*((sigma/E_0)+ (1/mu)* ( (sigma/sig_y)^(1/e) -
1)*(t(ii)-k1*deltaT) );
  eps0 = fb(ii)*((sigma/E_0)+ (1/mu)* ( (sigma/sig_y)^(1/e) -1)*(
t(ii) - k2*deltaT ));
  Gs1(ii)=eps1-eps0;
end

figure;
%subplot(2, 1, 1);
hold on
plot(t,Gs,'Color','magenta');
plot(t,Gs1,'Color','red');
%plot(t,Gs0,'Color','blue');
xlabel('time [s]'); ylabel('strain [mm/mm]');
title('creep');
hold off
%subplot(2, 1, 2);
%plot(t, f-fb,'color','blue');
%xlabel('time'); ylabel('epsilon');
```

The programme codes called as “topal” doesn’t mean anything alone but it calculates the function topal presented inside above programme codes.It is used as a subroutine while above programme was being run.

```
function sum = topal(sigma,alpha,deltaT,tFinal)
n=tFinal/deltaT;
m=20;
% Time
t(1)=0;
for i=1:n
   if (i>1)
       t(i) = t(i-1) + deltaT ;
   end
end
%t1(1)=0;
%for i=1:n
   %if (i>1)
       %t1(i) = t1(i-1) + deltaT ;
       %u2(i) = cos(t1(i));

   %end
%end
tmp=(1/deltaT^(alpha))*((3/2)^(alpha));
Gu(1) = 0;

%for ii=1:n
for i=1:length(t)
    sum = 0;
     for j=0:m

         if (j==0)
             A(j+1) = 1 ;
         else
             A(j+1) = (j-alpha-1)/j * A(j);
```

```
        end

        for l=0:j
            if (l==0)
                B(l+1) = 1 ;
            else
                B(l+1) = (l-j-1)/l * B(l);
            end

            %u(l+1) = sin( deltaT* (ii-j-l));
            t(i) = deltaT* (i-j-l);
            sum = sum + tmp*(4/3)^(j) * (1/4)^(l) * A(j+1) *
B(l+1)*sigma;
        end
    end
    Gu(i)=sum;
end
   %end

%'derivative value'
%sum
%'exact solution'
%u2
%===================================================================
===
%figure;
%hold on
%plot(t,Gu,'Color','blue');
%plot(t1,u2,'Color','red');
%xlabel('Time'); ylabel('u function derivative')
%title('alpha=1,u=sin(tFinal)');
%hold off
```

APPENDIX B.7 : The programme codes reduced experimental creep and relaxation test results (Appendix D in CD see Table D.1-D.2)to creep and relaxation graphics for PA6 are presented below.

```
clear
fid = fopen('crp7.txt', 'r');
An = fscanf(fid, '%g %g %g %g %g', [5 inf]);     % It has two rows
now.
An = An';
fclose(fid);

A=An;
figure;
%plot(A(:,1),A(:,2),'Color','yellow')
%figure;
%plot(A(:,1),A(:,3),'Color','cyan')
%figure;
%plot(A(:,1),A(:,4),'Color','cyan')
%figure;
plot(A(:,1),A(:,5),'Color','cyan')
```

APPENDIX C

Table C. 1: Lai-Bakker first results of experimental test for HDPE

Time [s]	Strain [mm/mm]							
	σ = 2 Mpa	**σ = 4 Mpa**	**σ = 6 Mpa**	**σ = 8 Mpa**	**σ = 10 Mpa**	**σ = 12 Mpa**	**σ = 14 Mpa**	**σ = 16 Mpa**
0.54D+55	0.245124D+56	0.323806D+56	0.389375D+56	0.475118D+56	0.556826D+56	0.645595D+56	0.820108D+56	0.108843D+55
0.54D+54	0.245124D+55	0.323806D+55	0.389375D+55	0.475118D+55	0.556826D+55	0.645595D+55	0.820108D+55	0.108843D+54
0.54D+53	0.245124D+54	0.323806D+54	0.389375D+54	0.475118D+54	0.556826D+54	0.645595D+54	0.820108D+54	0.108843D+53
0.54D+52	0.245124D+53	0.323806D+53	0.389375D+53	0.475118D+53	0.556826D+53	0.645595D+53	0.820108D+53	0.108843D+52
0.54D+51	0.245124D+52	0.323806D+52	0.389375D+52	0.475118D+52	0.556826D+52	0.645595D+52	0.820108D+52	0.108843D+51
0.54D+50	0.245124D+51	0.323806D+51	0.389375D+51	0.475118D+51	0.556826D+51	0.645595D+51	0.820108D+51	0.108843D+50
0.54D+49	0.245124D+50	0.323806D+50	0.389375D+50	0.475118D+50	0.556826D+50	0.645595D+50	0.820108D+50	0.108843D+49
0.54D+48	0.245124D+49	0.323806D+49	0.389375D+49	0.475118D+49	0.556826D+49	0.645595D+49	0.820108D+49	0.108843D+48
0.54D+47	0.245124D+48	0.323806D+48	0.389375D+48	0.475118D+48	0.556826D+48	0.645595D+48	0.820108D+48	0.108843D+47
0.54D+46	0.245124D+47	0.323806D+47	0.389375D+47	0.475118D+47	0.556826D+47	0.645595D+47	0.820108D+47	0.108843D+46
0.54D+45	0.245124D+46	0.323806D+46	0.389375D+46	0.475118D+46	0.556826D+46	0.645595D+46	0.820108D+46	0.108843D+45
0.54D+44	0.245124D+45	0.323806D+45	0.389375D+45	0.475118D+45	0.556826D+45	0.645595D+45	0.820108D+45	0.108843D+44
0.54D+43	0.245124D+44	0.323806D+44	0.389375D+44	0.475118D+44	0.556826D+44	0.645595D+44	0.820108D+44	0.108843D+43
0.54D+42	0.245124D+43	0.323806D+43	0.389375D+43	0.475118D+43	0.556826D+43	0.645595D+43	0.820108D+43	0.108843D+42
0.54D+41	0.245124D+42	0.323806D+42	0.389375D+42	0.475118D+42	0.556826D+42	0.645595D+42	0.820108D+42	0.108843D+41
0.54D+40	0.245124D+41	0.323806D+41	0.389375D+41	0.475118D+41	0.556826D+41	0.645595D+41	0.820108D+41	0.108843D+40
0.54D+39	0.245124D+40	0.323806D+40	0.389375D+40	0.475118D+40	0.556826D+40	0.645595D+40	0.820108D+40	0.108843D+39
0.54D+38	0.245124D+39	0.323806D+39	0.389375D+39	0.475118D+39	0.556826D+39	0.645595D+39	0.820108D+39	0.108843D+38
0.54D+37	0.245124D+38	0.323806D+38	0.389375D+38	0.475118D+38	0.556826D+38	0.645595D+38	0.820108D+38	0.108843D+37
0.54D+36	0.245124D+37	0.323806D+37	0.389375D+37	0.475118D+37	0.556826D+37	0.645595D+37	0.820108D+37	0.108843D+36
0.54D+35	0.245124D+36	0.323806D+36	0.389375D+36	0.475118D+36	0.556826D+36	0.645595D+36	0.820108D+36	0.108843D+35
0.54D+34	0.245124D+35	0.323806D+35	0.389375D+35	0.475118D+35	0.556826D+35	0.645595D+35	0.820108D+35	0.108843D+34
0.54D+33	0.245124D+34	0.323806D+34	0.389375D+34	0.475118D+34	0.556826D+34	0.645595D+34	0.820108D+34	0.108843D+33
0.54D+32	0.245124D+33	0.323806D+33	0.389375D+33	0.475118D+33	0.556826D+33	0.645595D+33	0.820108D+33	0.108843D+32
0.54D+31	0.245124D+32	0.323806D+32	0.389375D+32	0.475118D+32	0.556826D+32	0.645595D+32	0.820108D+32	0.108843D+31

Table C. 1 (continued) : Lai-Bakker first results of experimental test for HDPE

Time [s]	Strain [mm/mm]							
	σ = 2 Mpa	σ = 4 Mpa	σ = 6 Mpa	σ = 8 Mpa	σ = 10 Mpa	σ = 12 Mpa	σ = 14 Mpa	σ = 16 Mpa
0.54D+30	0.245124D+31	0.323806D+31	0.389375D+31	0.475118D+31	0.556826D+31	0.645595D+31	0.820108D+31	0.108843D+30
0.54D+29	0.245124D+30	0.323806D+30	0.389375D+30	0.475118D+30	0.556826D+30	0.645595D+30	0.820108D+30	0.108843D+29
0.54D+28	0.245124D+29	0.323806D+29	0.389375D+29	0.475118D+29	0.556826D+29	0.645595D+29	0.820108D+29	0.108843D+28
0.54D+27	0.245124D+28	0.323806D+28	0.389375D+28	0.475118D+28	0.556826D+28	0.645595D+28	0.820108D+28	0.108843D+27
0.54D+26	0.245124D+27	0.323806D+27	0.389375D+27	0.475118D+27	0.556826D+27	0.645595D+27	0.820108D+27	0.108843D+26
0.54D+25	0.245124D+26	0.323806D+26	0.389375D+26	0.475118D+26	0.556826D+26	0.645595D+26	0.820108D+26	0.108843D+25
0.54D+24	0.245124D+25	0.323806D+25	0.389375D+25	0.475118D+25	0.556826D+25	0.645595D+25	0.820108D+25	0.108843D+24
0.54D+23	0.245124D+24	0.323806D+24	0.389375D+24	0.475118D+24	0.556826D+24	0.645595D+24	0.820108D+24	0.108843D+23
0.54D+22	0.245124D+23	0.323806D+23	0.389375D+23	0.475118D+23	0.556826D+23	0.645595D+23	0.820108D+23	0.108843D+22
0.54D+21	0.245124D+22	0.323806D+22	0.389375D+22	0.475118D+22	0.556826D+22	0.645595D+22	0.820108D+22	0.108843D+21
0.54D+20	0.245124D+21	0.323806D+21	0.389375D+21	0.475118D+21	0.556826D+21	0.645595D+21	0.820108D+21	0.108843D+20
0.54D+19	0.245124D+20	0.323806D+20	0.389375D+20	0.475118D+20	0.556826D+20	0.645595D+20	0.820108D+20	0.108843D+19
0.54D+18	0.245124D+19	0.323806D+19	0.389375D+19	0.475118D+19	0.556826D+19	0.645595D+19	0.820108D+19	0.108843D+18
0.54D+17	0.245124D+18	0.323806D+18	0.389375D+18	0.475118D+18	0.556826D+18	0.645595D+18	0.820108D+18	0.108843D+17
0.54D+16	0.245124D+17	0.323806D+17	0.389375D+17	0.475118D+17	0.556826D+17	0.645595D+17	0.820108D+17	0.108843D+16
0.54D+15	0.245124D+16	0.323806D+16	0.389375D+16	0.475118D+16	0.556826D+16	0.645595D+16	0.820108D+16	0.108843D+15
0.54D+14	0.245124D+15	0.323806D+15	0.389375D+15	0.475118D+15	0.556826D+15	0.645595D+15	0.820108D+15	0.108843D+14
0.54D+13	0.245124D+14	0.323806D+14	0.389375D+14	0.475118D+14	0.556826D+14	0.645595D+14	0.820108D+14	0.108843D+13
0.54D+12	0.245124D+13	0.323806D+13	0.389375D+13	0.475118D+13	0.556826D+13	0.645595D+13	0.820108D+13	0.108843D+12
0.54D+11	0.245124D+12	0.323806D+12	0.389375D+12	0.475118D+12	0.556826D+12	0.645595D+12	0.820108D+12	0.108843D+11
0.54D+10	0.245124D+11	0.323806D+11	0.389375D+11	0.475118D+11	0.556826D+11	0.645595D+11	0.820108D+11	0.108843D+10
0.54D+09	0.245124D+10	0.323806D+10	0.389375D+10	0.475118D+10	0.556826D+10	0.645595D+10	0.820108D+10	0.108843D+09
0.54D+08	0.245124D+09	0.323806D+09	0.389375D+09	0.475118D+09	0.556826D+09	0.645595D+09	0.820108D+09	0.108843D+08
0.54D+07	0.245124D+08	0.323806D+08	0.389375D+08	0.475118D+08	0.556826D+08	0.645595D+08	0.820108D+08	0.108843D+07
0.54D+06	0.245124D+07	0.323806D+07	0.389375D+07	0.475118D+07	0.556826D+07	0.645595D+07	0.820108D+07	0.108843D+06
0.54D+05	0.245124D+06	0.323806D+06	0.389375D+06	0.475118D+06	0.556826D+06	0.645595D+06	0.820108D+06	0.108843D+05
0.54D+04	0.245124D+05	0.323806D+05	0.389375D+05	0.475118D+05	0.556826D+05	0.645595D+05	0.820108D+05	0.108843D+04

Table C. 1 (continued) : Lai-Bakker first results of experimental test for HDPE

Time [s]	Strain [mm/mm]							
	σ = 2 Mpa	**σ = 4 Mpa**	**σ = 6 Mpa**	**σ = 8 Mpa**	**σ = 10 Mpa**	**σ = 12 Mpa**	**σ = 14 Mpa**	**σ = 16 Mpa**
0.54D+03	0.245124D+04	0.323806D+04	0.389375D+04	0.475118D+04	0.556826D+04	0.645595D+04	0.820108D+04	0.108843D+03
0.54D+02	0.245124D+03	0.323806D+03	0.389375D+03	0.475118D+03	0.556826D+03	0.645595D+03	0.820108D+03	0.108843D+02
0.54D+01	0.245124D+02	0.323806D+02	0.389375D+02	0.475118D+02	0.556826D+02	0.645595D+02	0.820108D+02	0.108843D+01
0.54D+00	0.245124D+01	0.323806D+01	0.389375D+01	0.475118D+01	0.556826D+01	0.645595D+01	0.820108D+01	0.108843D+00
0.54D+01	0.245124D+00	0.323806D+00	0.389375D+00	0.475118D+00	0.556826D+00	0.645595D+00	0.820108D+00	0.108843D+01

Table C. 2: Lai-Bakker first results of experimental test for HDPE

Time [s]	Strain [mm/mm]							
	σ = 2 Mpa	**σ = 4 Mpa**	**σ = 6 Mpa**	**σ = 8 Mpa**	**σ = 10 Mpa**	**σ = 12 Mpa**	**σ = 14 Mpa**	**σ = 16 Mpa**
0.000000D+00	0.000000D+00	0.000000D+00	0.000000D+00	0.000000D+00	0.000000D+00	0.000000D+00	0.000000D+00	0.000000D+00
0.100000D+02	0.126614D-02	0.270429D-02	0.442912D-02	0.642150D-02	0.864800D-02	0.116963D-01	0.154183D-01	0.203730D-01
0.500000D+02	0.160060D-02	0.350698D-02	0.571915D-02	0.839000D-02	0.114670D-01	0.154231D-01	0.204686D-01	0.271766D-01
0.100000D+03	0.178932D-02	0.392266D-02	0.645018D-02	0.946024D-02	0.129839D-01	0.176448D-01	0.234284D-01	0.309608D-01
0.200000D+03	0.201388D-02	0.443868D-02	0.730302D-02	0.107790D-01	0.148951D-01	0.202249D-01	0.269402D-01	0.356432D-01
0.300000D+03	0.216438D-02	0.477792D-02	0.791220D-02	0.117154D-01	0.161732D-01	0.219880D-01	0.293650D-01	0.388922D-01
0.400000D+03	0.227668D-02	0.504068D-02	0.837804D-02	0.124130D-01	0.171646D-01	0.234070D-01	0.312880D-01	0.415869D-01
0.500000D+03	0.237462D-02	0.526524D-02	0.875790D-02	0.130150D-01	0.179649D-01	0.244819D-01	0.329102D-01	0.438803D-01
0.600000D+03	0.245346D-02	0.547548D-02	0.909474D-02	0.135023D-01	0.186697D-01	0.254423D-01	0.342982D-01	0.459251D-01
0.700000D+03	0.252274D-02	0.564272D-02	0.938142D-02	0.139419D-01	0.193266D-01	0.263597D-01	0.356192D-01	0.477790D-01
0.800000D+03	0.258962D-02	0.580036D-02	0.964656D-02	0.143528D-01	0.199000D-01	0.271481D-01	0.367564D-01	0.495565D-01
0.900000D+03	0.265412D-02	0.593892D-02	0.991176D-02	0.147064D-01	0.204136D-01	0.278791D-01	0.378434D-01	0.512382D-01
0.100000D+04	0.271384D-02	0.608704D-02	0.101339D-01	0.150599D-01	0.208794D-01	0.284954D-01	0.388300D-01	0.528245D-01
0.110000D+04	0.276640D-02	0.619692D-02	0.103561D-01	0.153753D-01	0.213453D-01	0.291118D-01	0.397163D-01	0.542579D-01
0.120000D+04	0.281180D-02	0.633072D-02	0.105424D-01	0.156620D-01	0.217633D-01	0.296707D-01	0.406193D-01	0.557294D-01
0.130000D+04	0.286196D-02	0.643104D-02	0.107216D-01	0.159391D-01	0.221456D-01	0.302298D-01	0.414722D-01	0.571246D-01
0.140000D+04	0.290258D-02	0.653140D-02	0.109151D-01	0.162162D-01	0.225159D-01	0.307458D-01	0.422582D-01	0.584434D-01
0.150000D+04	0.294318D-02	0.662216D-02	0.110656D-01	0.164742D-01	0.228862D-01	0.312762D-01	0.430107D-01	0.597429D-01
0.160000D+04	0.298618D-02	0.671296D-02	0.112304D-01	0.167131D-01	0.232564D-01	0.316919D-01	0.437632D-01	0.610234D-01
0.170000D+04	0.302202D-02	0.680852D-02	0.113810D-01	0.169615D-01	0.235431D-01	0.321935D-01	0.444489D-01	0.622656D-01
0.180000D+04	0.305786D-02	0.689928D-02	0.115243D-01	0.172005D-01	0.238656D-01	0.325949D-01	0.451511D-01	0.635078D-01
0.180000D+04	0.170276D-02	0.392199D-02	0.673638D-02	0.104344D-01	0.147680D-01	0.201910D-01	0.279906D-01	0.395104D-01
0.190000D+04	0.121453D-02	0.282851D-02	0.496907D-02	0.796503D-02	0.115098D-01	0.157969D-01	0.223840D-01	0.320699D-01
0.200000D+04	0.104506D-02	0.245326D-02	0.434566D-02	0.705313D-02	0.102690D-01	0.141628D-01	0.202797D-01	0.292778D-01
0.210000D+04	0.940148D-03	0.222327D-02	0.398857D-02	0.652858D-02	0.954270D-02	0.131944D-01	0.189381D-01	0.275346D-01
0.220000D+04	0.879624D-03	0.207801D-02	0.372226D-02	0.612508D-02	0.898790D-02	0.124801D-01	0.179778D-01	0.262918D-01
0.230000D+04	0.819098D-03	0.196906D-02	0.354674D-02	0.584264D-02	0.859449D-02	0.119597D-01	0.172294D-01	0.253074D-01

Table C. 2(continued) : Lai-Bakker first results of experimental test for HDPE

Time [s]	Strain [mm/mm]							
	σ = 2 Mpa	**σ = 4 Mpa**	**σ = 6 Mpa**	**σ = 8 Mpa**	**σ = 10 Mpa**	**σ = 12 Mpa**	**σ = 14 Mpa**	**σ = 16 Mpa**
0.240000D+04	0.782784D-03	0.188433D-02	0.340753D-02	0.564089D-02	0.824143D-02	0.114876D-01	0.166362D-01	0.244680D-01
0.250000D+04	0.754540D-03	0.180767D-02	0.327438D-02	0.543107D-02	0.799933D-02	0.111123D-01	0.161137D-01	0.237741D-01
0.260000D+04	0.722260D-03	0.175118D-02	0.315938D-02	0.524546D-02	0.776732D-02	0.107734D-01	0.156758D-01	0.232091D-01
0.270000D+04	0.698050D-03	0.170679D-02	0.308070D-02	0.509213D-02	0.753531D-02	0.104950D-01	0.153229D-01	0.227088D-01
0.280000D+04	0.677874D-03	0.165837D-02	0.301412D-02	0.497915D-02	0.734365D-02	0.102529D-01	0.149839D-01	0.221923D-01
0.290000D+04	0.665770D-03	0.162206D-02	0.294149D-02	0.480968D-02	0.718225D-02	0.100713D-01	0.146590D-01	0.217726D-01
0.300000D+04	0.643578D-03	0.158978D-02	0.289913D-02	0.476934D-02	0.706120D-02	0.970813D-02	0.143766D-01	0.214176D-01
0.310000D+04	0.631472D-03	0.156153D-02	0.283860D-02	0.468863D-02	0.695024D-02	0.969604D-02	0.141365D-01	0.210949D-01
0.320000D+04	0.617350D-03	0.154136D-02	0.279623D-02	0.459986D-02	0.679892D-02	0.951446D-02	0.139247D-01	0.207720D-01
0.330000D+04	0.607262D-03	0.151715D-02	0.275387D-02	0.454338D-02	0.672831D-02	0.938130D-02	0.137270D-01	0.204976D-01
0.340000D+04	0.595158D-03	0.149294D-02	0.271755D-02	0.447882D-02	0.660726D-02	0.923604D-02	0.135151D-01	0.202394D-01
0.350000D+04	0.589106D-03	0.147680D-02	0.266308D-02	0.442233D-02	0.653665D-02	0.910289D-02	0.133880D-01	0.200134D-01
0.360000D+04	0.577000D-03	0.145662D-02	0.264492D-02	0.436584D-02	0.642569D-02	0.896974D-02	0.131903D-01	0.197875D-01
0.370000D+04	0.568930D-03	0.143242D-02	0.262071D-02	0.431742D-02	0.634499D-02	0.887290D-02	0.130350D-01	0.195776D-01
0.380000D+04	0.560860D-03	0.142838D-02	0.260255D-02	0.426093D-02	0.627438D-02	0.878816D-02	0.129361D-01	0.193678D-01
0.390000D+04	0.558844D-03	0.140417D-02	0.256019D-02	0.422058D-02	0.624412D-02	0.869132D-02	0.127666D-01	0.191742D-01
0.400000D+04	0.546738D-03	0.140014D-02	0.254809D-02	0.417216D-02	0.616342D-02	0.860659D-02	0.126678D-01	0.190128D-01
0.410000D+04	0.544720D-03	0.138803D-02	0.251782D-02	0.413181D-02	0.609280D-02	0.852186D-02	0.125689D-01	0.188837D-01
0.420000D+04	0.538668D-03	0.137592D-02	0.249967D-02	0.409146D-02	0.605245D-02	0.843712D-02	0.124136D-01	0.186899D-01
0.430000D+04	0.530598D-03	0.135978D-02	0.248756D-02	0.407532D-02	0.598184D-02	0.834029D-02	0.123006D-01	0.185448D-01
0.440000D+04	0.524546D-03	0.136382D-02	0.248150D-02	0.403497D-02	0.594149D-02	0.830396D-02	0.122300D-01	0.184317D-01
0.450000D+04	0.526564D-03	0.134364D-02	0.246335D-02	0.402690D-02	0.588097D-02	0.820712D-02	0.121594D-01	0.183026D-01
0.460000D+04	0.520512D-03	0.133558D-02	0.243914D-02	0.396234D-02	0.584062D-02	0.817081D-02	0.120464D-01	0.181574D-01
0.470000D+04	0.512442D-03	0.133154D-02	0.242098D-02	0.395427D-02	0.581036D-02	0.808608D-02	0.119758D-01	0.180766D-01
0.480000D+04	0.508406D-03	0.131136D-02	0.240283D-02	0.391392D-02	0.578009D-02	0.804977D-02	0.118346D-01	0.179475D-01
0.490000D+04	0.506388D-03	0.131540D-02	0.239072D-02	0.389778D-02	0.571957D-02	0.800135D-02	0.117922D-01	0.178346D-01
0.500000D+04	0.504372D-03	0.131540D-02	0.237256D-02	0.388971D-02	0.570948D-02	0.791662D-02	0.117357D-01	0.177216D-01

Table C. 2(continued) : Lai-Bakker first results of experimental test for HDPE

Time [s]	Strain [mm/mm]							
	σ = 2 Mpa	σ = 4 Mpa	σ = 6 Mpa	σ = 8 Mpa	σ = 10 Mpa	σ = 12 Mpa	σ = 14 Mpa	σ = 16 Mpa
0.510000D+04	0.498318D-03	0.130330D-02	0.236651D-02	0.386550D-02	0.567922D-02	0.792871D-02	0.116510D-01	0.176410D-01
0.520000D+04	0.502354D-03	0.129926D-02	0.237256D-02	0.381708D-02	0.563887D-02	0.789240D-02	0.115945D-01	0.175118D-01
0.530000D+04	0.496302D-03	0.128312D-02	0.234835D-02	0.380901D-02	0.560861D-02	0.779556D-02	0.115098D-01	0.174634D-01
0.540000D+04	0.490248D-03	0.129522D-02	0.233625D-02	0.380094D-02	0.556826D-02	0.774714D-02	0.114815D-01	0.174149D-01

HDPE material's parameter values and material constants combinations which are used in Matlab program codes(see Appendix B-5) of relaxation graphics (see Appendix A, Figures A.1-9) mentioned in section 6 are shown in below.

Table C. 3 : HDPE material constant and parameter values

	Figure A.1-3	Figure A.4-6	Figure A.7-9
σ_y :Sig_y	23.4	23.4	23.4
$\in$: e	1	1	1
E_0 : E_0 [MPa]	745	745	745
α : Alpha	1 , 1/2 , 1/3 , 3/4	1 , 1/2 , 1/3 , 3/4	1 , 1/2 , 1/3 , 3/4
ε : Eps [mm/mm]	0.0628	0.0628	0.0628
μ : mu	2.2	2.2	2.2
τ :Tau [s]	2	2	2
E_∞ : E_inf [MPa]	1065	1100	950

HDPE material's parameter values and material constants combinations which are used in Matlab program codes(see Appendix B-6) of creep graphics (see Appendix A, Figures A.10-13) mentioned in section 6 are shown in below.

Table C. 4: HDPE material constant and parameter values

	Figure A.10-11	Figure A.12-13
σ_y : Sig_y	23.4	23.4
$\in$: e	1	1
E_0 : E_0 [MPa]	745	745
α : alpha	1 , 1/2 , 1/3 , 3/4	1 , 1/2 , 1/3 , 3/4
σ : Sigma [MPa]	38	38
μ : mu	2.2	2.2
τ : Tau [s]	2	2
E_∞ : E_inf [MPa]	1065	950

PA6 material's parameter values and material constants combinations which are used in Matlab program codes(see Appendix B-5) of relaxation graphics (see Appendix A, Figures A.14-18) mentioned in section 7 are shown in below.

Table C. 5 : PA6 material constant and parameter values

	Figure A.14	Figure A.15	Figure A.16	Figure A.17	Figure A.18
σ_y :Sig_y [MPa]	19.7	19.7	19.7	19.7	19.7
$\in$: e	1	1	1	1	1
E_0 : E_0 [MPa]	1640	1640	1640	1640	1640
τ :Tau [s]	2	2	2	2	2
E_∞ : E_inf [MPa]	1900	1900	1900	1900	1900
ε : eps [mm/mm]	0.02	0.02	0.02	0.02	0.02
μ : mu	90007.2	222000.0	110000.0	90007.2	218507.2
α : alpha	1,1/2 ,1/3, 3/4	1,1/2,1/3,3/4	1,1/2,1/3,3/4	1,1/2,1/3,3/4	1,1/2,1/3,3/4

PA6 material's parameter values and material constants combinations which are used in Matlab program codes(see Appendix B-6) of creep graphics (see Appendix A, Figures A.19-22) mentioned in section 7 are shown in below.

Table C. 6 : PA6 material constant and parameter values

	Figure A.19	Figure A.20	Figure A.21	Figure A.22
σ_y :Sig_y [MPa]	19.7	19.7	19.7	19.7
$\in$: e	1	1	8	10
E_0 : E_0 [MPa]	1640	1640	1640	1640
τ :Tau [s]	2	2	2	2
E_∞ : E_inf [MPa]	1900	1900	1900	1900
σ : Sigma [MPa]	25	25	25	25
μ : mu	18000	24000	2500	2000
α : alpha	1 , 1/2 , 1/3 , 3/4	1 , 1/2 , 1/3 , 3/4	1 , 1/2 , 1/3 , 3/4	1 , 1/2 , 1/3 , 3/4

CURRICULUM VITA

Sevda TOPAL ARSLAN was born in 19.12.1984 in Izmir. In 2002 she graduated from Teğmen Ali Rıza Akıncı high school. In the same year, she started bachelor study in Balıkesir University, department of Mechanical Engineering and graduated as a Mechanical Engineer in August 2006. She was excepted to Istanbul Technical University,Institute of Science and Technology, Mechanical Engineering department, Solid Mechanics Programme in January 2007. She started her master study in İstanbul Technical University in January 2008. In October 2009 she went to Germany as a Erasmus Mundus master student to write and complete her M.Sc. thesis in Ruhr University of Bochum. She accomplished her thesis with 1.5/1 degree in July 2010 and came back to Turkey. She got married in August 2010.