A STUDY ON FIFTH GRADE STUDENTS' MISTAKES, DIFFICULTIES AND MISCONCEPTIONS REGARDING BASIC FRACTIONAL CONCEPTS AND OPERATIONS

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Prof. Dr. Meliha ALTUNIŞIK
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

Prof. Dr. Jale ÇAKIROĞLU
Head of Department

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

Assoc. Prof. Dr. Mine IŞIKSAL
Supervisor

Examining Committee Members

Assoc. Prof. Dr. Erdinç ÇAKIROĞLU	(METU, ELE)	
Assoc. Prof. Dr. Mine IŞIKSAL	(METU, ELE)	
Assoc. Prof. Dr. Yezdan BOZ	(METU, SSME)	
Assist. Prof. Dr. Çiğdem HASER	(METU, ELE)	
Assist. Prof. Dr. Didem AKYÜZ	(METU, ELE)	

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Name, Last name: Nilgün Yurtsever

Signature

ABSTRACT

A STUDY ON FIFTH GRADE STUDENTS'
MISTAKES, DIFFICULTIES AND MISCONCEPTIONS
REGARDING BASIC FRACTIONAL CONCEPTS AND OPERATIONS

Yurtsever, Nilgün
M.S., Department of Elementary Science and Mathematics Education

Supervisor: Assoc. Prof. Dr. Mine IŞIKSAL

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The purpose of this study was to investigate mistakes made by elementary fifth grade students regarding basic fractional concepts and operations, and difficulties that they encounter. The other purpose was to investigate underlying misconceptions and reasons of those difficulties and mistakes. For this purpose, a mixed-method research combining quantitative and qualitative approach respectively was performed

Data were collected from elementary fifth grade students at the end of the spring semester of 2009-2010. Operation with Fraction Questionnaire (OFQ) was administered to 151 fifth grade students who were chosen from the two public elementary schools in Eskişehir province. By this way, difficulties that elementary fifth grade students encounter and mistakes they make regarding basic fractional concepts and operations was analyzed. Afterwards, sixteen of these students participated in a semi-structured interview which was designed to investigate underlying reasons and misconceptions behind those mistakes and difficulties.

Results were presented in two phases. In the first phase, common mistakes and difficulties of students were analyzed in detail and representative examples of these errors were introduced. In the second phase, students' mistakes were grouped under five categories as: algorithmically based mistakes, intuitively based mistakes, mistakes based on formal knowledge on fractions, misunderstanding on problem, and missing information in solution. In this phase, misconceptions and underlying reasons of those mistakes and difficulties which students may encounter while learning fractions were described. Results revealed that there was evidence that fifth grade students made various mistakes regarding fractional concepts and operations in the fifth grade elementary mathematics curriculum and they had many misconceptions regarding fraction concepts and operations.

Keywords: Fractions, misconception, mistake, difficulty, fifth grade students

ÖZ

BEŞİNCİ SINIF ÖĞRENCİLERİNİN
KESİRLER VE KESİRLERLE İŞLEMLER
KONUSU İLE İLGİLİ HATALARI, ZORLUKLARI VE
KAVRAM YANILGILARI ÜZERİNE BİR ÇALIŞMA

Yurtsever, Nilgün
Yüksek Lisans, İlköğretim Fen ve Matematik Eğitimi

Tez Danışmanı: Doç. Dr. Mine IŞIKSAL
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Bu çalışmanın amacı, beşinci sınıf öğrencilerinin temel kesir kavramları ve kesirlerle işlemler konusu ile ilgili yaptıkları hataları ile karşılaştıkları zorlukları belirlemektir. Diğer bir amaç ise bu hataların altında yatan sebeplerin ve kavram yanılgılarının araştırılmasıdır. Bu amaçla, nitel ve nicel araştırma yöntemlerinin birleştirildiği karma bir araştırma yöntemi kullanılmıştır.

Araştırma verileri, 2009-2010 öğretim yılı bahar dönemi sonunda Eskişehir ilinde iki devlet okulundan seçilen toplam 151 beşinci sınıf öğrencisine 'Kesirlerle İşlemler Anketi' uygulanarak elde edilmiştir. Daha sonra, bu öğrencilerden 16 tanesi ile yarı yapılandırılmış görüşme yapılmış ve öğrencilerin kesirler konusu ile ilgili yaptıkları hataların ve karşılaştıkları zorlukların altında yatan sebep ve kavram yanılgıları araştırılmıştır.

Bu çalışmada, sonuçlar iki kısım halinde sunulmuştur. Birinci kısımda, öğrencilerin Kesirlerle İşlemler Soru Formu'nu çözerken yaptıkları hatalar detaylı olarak analiz edilmiş ve öğrencilerin genel hatalarını en iyi temsil eden örnekler ve bunların frekansları verilmiştir. İkinci kısımda ise öğrencilerin görüşme sorularına verdikleri yanlış cevaplar: algoritmik temelli hatalar, sezgisel hatalar, kesir konusu bilgisine dayalı hatalar, problemi anlamama ve çözümde eksik bilgi şeklinde beş kategori altında gruplandırılmıştır. Bu kısımda, öğrencilerin kesirler konusu ile ilgili hatalarının altında yatan sebepler ve kavram yanılgıları tanımlanmıştır. Sonuçlar, beşinci sınıf öğrencilerinin, beşinci sınıf matematik dersi programında yer alan kesirler konusuna yönelik çeşitli hatalar yaptıklarını ve kesirler konusu ve işlemleri ile ilgili kavram yanılgıları olduğunu ortaya çıkarmıştır.

Anahtar Kelimeler: Kesirler, kavram yanılgısı, hata, zorluk, beşinci sınıf öğrencileri

This thesis is dedicated to my mom, who has been my inspiration and a great source of motivation during my life, to my husband who has supported me all the way since the beginning of my studies and to my baby for the participation of our lives just in time.

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TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURES

LIST OF ABBREVIATION

METU Middle East Technical University

MoNE Ministry of National Education

OFQ Operations of Fractions Questionnaire

CHAPTER I

INTRODUCTION

‘‘No area of elementary school mathematics is as mathematically rich, cognitively complicated, and difficult to teach as fractions, ratios, and proportionality” (Smith, 2002, p. 3).

The main goal of mathematics education is to enable all students to perform at the highest level of their learning. However, very few students are able to reach this level, majority of them have difficulties in learning and they take it for granted (Tall & Razali, 1993). Fractions play an undeniable role in this case; because, in the last decades, results of many researchers’ studies, which investigated mathematical conceptions, knowledge and students’ understanding on many elementary issues such as fractions, decimals and ratios, revealed that these issues are both complex and not straightforward; and also require a long time and great effort to learn (Bishop, Clements, Keitel, Kilpatrick & Laborde, 1996; Borasi, 1996; English, 2002; Grouws, 1992; Schoenfeld, Smith & Arcavi, 1993; Smith, DiSessa, & Roschelle, 1993). Since there are strong prerequisite relationships among fractions and other issues, a student who already has learning difficulties on fractions will find it difficult to succeed in later issues such as probably, percents even algebra (Kieren, 1993). In order to accomplish a teaching of such an important and problematic subject, two important steps should be taken. The first one is, to know the difficulties and misconceptions of students and reasons underlying them, the second one is to design teaching models to eliminate these misconceptions and difficulties (Yetkin, 2003).

As it is mentioned above, developing an understanding of mathematics is a difficult goal, and fraction domain, which is a part of mathematical curriculum in both primary and middle schools, is one of the main parts of it (Smith, 2002). Although students may have many opportunities to start constructing basic fraction concepts before they begin school, such as breaking a chocolate bar in half, cutting a birthday cake or sharing marbles with a friend, these situations do not avert having difficulty in learning fractions. It means learning and mastering fractions is still a major difficulty for students (Saxe, Taylor, McIntosh & Geahart, 2005). In addition, Davis, Hunting, and Pearn (1993) stated that ‘the teaching and learning of fractions is not only very hard, it is, in the broader scheme of things, a dismal failure’ (p. 63). This statement is supported by many studies which have reported that students have difficulties in understanding the fundamental fraction concepts, operations and solving fraction word problems in elementary grade level (Baturo, 2004; Behr, Lesh, Post & Silver, 1983; Booker, 1998; Corbitt, 1989; Hart, 1993; Haser & Ubuz, 2001; Kerslake, 1986; Leinhardt & Smith, 1984; Mack, 1990; Newstead & Murray, 1998; Tirosh, Fischbein, Graeber & Wilson, 1998). Although most students eventually learn the specific algorithms that they are taught, their general conceptual knowledge often remains remarkably deficient (Tirosh, 2000).

According to Ball (1993), fractions may be interpreted (a) in part-whole terms, where the whole unit may vary; (b) as a number on the number line; (c) as an operator (or scalar) that can shrink or stretch another quantity; (d) as a quotient of two integers; (e) as a rate and (f) as a ratio. All of these interpretations of fractions, fractional operations, probabilities, numeration, and so on, form the basis of complexity of fraction subject. Ohlsson (1988) expressed this situation as:

> The difficulty of the topic is . . . semantic in nature: How should fractions be understood? The complicated semantics of fractions is, in part, a consequence of the *composite nature* of fractions. How is the meaning of 2 combined with the meaning of 3 to generate a meaning for $\frac{2}{3}$? The difficulty of fractions is also . . . in part, a consequence of the bewildering array of *many related but only partially overlapping ideas* that surround fractions (cited in Ball, 1993, p. 53)

As a consequence, researchers agreed on the fact that understanding fractions requires comprehension and coordination of several powerful mathematical processes (e.g., unitizing, reunitising, and multiplicative relationships) (Baturo, 1997, 2000) therefore students have difficulties and misconceptions while learning fractions.

In Turkey, when children begin the first grade with an intuitive understanding of natural numbers, they are directly exposed to context of counting a collection of objects and this prolongs through the end of third grade. Fraction issues are introduced in third grade but in the fourth and fifth grades, issues related to fractions are much more complicated and most of time, students cannot develop an understanding of fractions in their mind (MoNE, 2005). In addition to fractions, notation and terminology are also problematic for students to perceive.

1.1. Purpose of the Study

As mentioned above, although fractions have been a large part of mathematics education in elementary schools in Turkey, students still have difficulties and misconceptions in understanding fraction concepts (Ardahan & Ersoy, 2002; Başgün & Ersoy, 2000; İşeri, 1997; Pesen, 2008). These difficulties experienced by students are listed as follows: students overgeneralize whole number or natural number properties to fraction concepts or to rational numbers (Behr & Bright, 1984; Behr et al., 1992; Mack, 1995); students have difficulty in relating fraction symbols and language to the various constructs of fractions (Mack, 1990); and they fail to consider fractions as numbers (Kerslake, 1986). Furthermore, large percentage of students lack basic fraction skills like identifying fractions as symbols, part-whole representations or partitioning (Ball, 1990a; Hecht, 1998; Rittle-Johnson, Siegler, & Alibali, 2000; Saenz-Ludlow, 1995; Smith, 1995).

This situation shows that, learners need help to construct fraction concepts because they have difficulties and misconceptions in perceiving fractions as numbers. After all, fractions involve a new number system based on a multiplicative relationship instead of the additive one; hence, students should not overgeneralize natural number properties to fractions. To provide this assist to elementary students, there is a need for more information, regarding how students think about fractions and how their thinking changes with development, which can be obtained by identifying students' difficulties, mistakes, reasons behind them and misconceptions. Because, misconceptions are substantial when students learn the connections amongst method, fraction number and oral expression (reading) in solution of oral and written problems related to fractions (Pesen, 2007). Such an approach may help the students and also teachers to deal with fractions in a more meaningful way.

Many research studies have been carried out on pre-service teachers' knowledge of fractional concepts including multiplication, division, equivalence, and number line in fractions (Işıksal, 2006; Pesen, 2008; Tirosh, 2000) and teachers' knowledge of fractions (Ball, 1990; Ma, 1999; Türnüklü, 2005). But there is still few research in elementary level related to fractions (Ball, 1993; Behr, Wachsmuth &

Post, 1985; Haser & Ubuz, 2001; Kocaoğlu & Yenilmez, 2010; Soylu & Soylu, 2005). Thus, the present study addressed to investigate mistakes made by elementary fifth grade students regarding basic fractional concepts and operations, difficulties that they encounter, underlying reasons and misconceptions behind those mistakes and difficulties.

Specifically, two research questions were posed to achieve the purpose of this study:

1) What are the mistakes that elementary fifth grade students make and difficulties they encounter regarding basic fractional concepts and operations?
2) What are the underlying reasons and misconceptions behind those mistakes and difficulties regarding basic fractional concepts and operations?

1.2. Definitions of the Important Terms

Operational and constitutive definitions that are related to the research question are given below:

Fraction: refers to 'the equal shares or equal-sized portions of a whole or unit' (Van de Walle, 2004, p.242).

In this study the term 'fraction' refers to the set of non-negative rationals as $\frac{a}{b}$, both a and b will be natural numbers and b will be nonzero number (Behr& Nichols, 1982).

Basic Fractional Concepts: refers to the concepts that include what fractions are, how they are represented, and how they are related to each other and whole numbers (NCTM, 2000).

In this study, basic fractional concepts include: fractional reading and writing, fractional representations, the meaning of fractional symbols, equivalency of fractions, and ordering fractions.

Fractional Operations: refers to addition, subtraction, multiplication and division of fractions (Behr & Post, 1992).

In this study, fractional operations include: addition, subtraction and multiplication of fractions, converting mixed numbers to improper fractions and vice versa, and ability to solve word problems that involve fractions.

Mistakes and Difficulties: mistake refers ‘a wrong idea or wrong action that often is the result of a misconception, but not always so’ (Koshy, 2000, p.172).

Difficulty refers to ‘the result of carelessness, misinterpretation of symbols or text; lack of relevant experience or knowledge related to that mathematical topic/ concept; a lack of awareness or inability to check the answer given; or the result of a misconception’ (Drew, 2005, p.14).

In this study the term ‘mistakes and difficulties’ was used for errors originating from operationally made mistakes due to the incorrect or inadequate use of algorithms; misapplication of basic rules; errors that result from intuitions about the operations, the inadequate or limited knowledge related to formal knowledge (Tirosh, 2000), errors that result from wrong perception of the problem and condition resulting from those mistakes, lack of relevant experience or knowledge related to fraction subject.

Misconception: refers to ‘a line of thinking that causes a series of errors all resulting from an incorrect underlying premise, rather than sporadic, unconnected and nonsystematic errors’ (Nesher, 1987, p.35).

In this study misconception refers to incomplete or inaccurate approaches to basic fraction concepts and operations, which cause students to produce serious of mistakes or inappropriate strategies resulting from an incorrect underlying premise.

1.3. Significance of the Study

Fraction is reminiscent of anxiety, discomfort and even fear to many children and adults (Bezuk, 1988). But at the same time, fraction is the first abstract concept in elementary school mathematics (Pesen, 2008). However, there are few studies done in Turkey in fraction domain (Haser & Ubuz, 2003; Toluk, 2002). The role of fractions in development of higher level mathematical concepts, such as ratio, proportion and probability, makes it one of the most important and essential conceptual subjects in the elementary mathematics curriculum (Post, 1989). The literature review on learning and teaching fractions in elementary grades in Turkey presents many unanswered questions about students’ difficulties, mistakes and misconceptions behind them (Kılcan, 2006). Based on all of these facts, fraction domain was chose as a subject of this study.

In many studies, misconceptions about fractions related to elementary grade levels have been handled collectively instead of researching separately and specifically (Ardahan & Ersoy, 2002; Başgün & Ersoy, 2000; Durmuş, 2004; Hart, 1981; Kerslake, 1986; Newstead & Murray, 1998; Olkun & Toluk, 2001). When the literature is reviewed, it is seen that significant findings that identify the current difficulties and misconceptions of fifth grade students related to fractions are limited. Moreover, in research studies that have been done in Turkey related to the fifth grade elementary students' misconceptions in fractions, only specific issues have been handled such as solving word problems about fractions (Haser & Ubuz, 2003; Kocaoğlu & Yenilmez, 2010), locating a fraction on a number line (Pesen, 2008) or division of fractions (Kılcan, 2006). In conclusion, considering fifth grade objectives, there are limited studies performed on misconceptions and difficulties concerning fractions (Haser & Ubuz, 2001; Soylu & Soylu, 2005). In the present study, all the objectives related to fraction concepts that fifth grade students are supposed to learn, were covered. More specifically, this research is important from the aspects of identifying fifth grade students' mistakes and difficulties related to addition, multiplication and subtraction of fractions, ordering fractions, equal fractions, solving and writing word problems about fractions, relationship between division operation and fraction, transition between types of fractions, and comparison of fractions and natural numbers. In this study, mistakes made by elementary fifth grade students regarding basic fractional concept and operations, difficulties that they encounter, and underlying reasons and misconceptions behind those mistakes and difficulties investigated from a broader perspective.

Studies of many researchers revealed that students have some difficulties in understanding basic concepts on fractions on any class level (Booker, 1998; Haser & Ubuz, 2003; Leinhardt & Smith, 1984; Newstead & Murray, 1998). This situation indicating that fractions topic should be handled and studied differently among classes. When students get to sixth year, they begin to learn rational numbers, ratios, decimals and probability subjects that are based on fractions. Moreover, teaching and learning these domains continue in both seventh and eighth grades. Thus, difficulties that unresolved and experienced with regard to fractions affect students' all mathematics education life. Therefore, to provide a high level of readiness for these domains, identification of mistakes elementary fifth grade students' make, reasons behind those mistakes and misconceptions they have about fractions comes into prominence. In conclusion, conducting of this study with elementary fifth grade students would provide significant information.

In addition, in order to accomplish effective teaching, it is necessary for teachers to foreknow difficulties of students when they learn certain issues and concepts, so that they can mention about frequent mistakes and misconceptions to their students and can organize their teaching process depending on these mistakes and misconceptions (Geddis, 1993; Grossman, 1990; Fernandez-Balboa & Stiehl, 1995; Hasweh, 2005; Loughran, 2006; Marks, 1990; Smith & Neale, 1989; Tamir, 1988). This study investigated mistakes made by elementary fifth grade students regarding basic fractional concepts and operations, difficulties that they encounter, underlying reasons and misconceptions behind those mistakes and difficulties.

In conclusion, teachers can help the students replace their misconceptions with scientifically correct knowledge and they can teach how to conceptually deal with fractions, through this study. Information obtained from this study may assist teachers in detecting and correcting common mistakes that students make while dealing with fractions.

1.4. My Motivation for the Study

In Turkey, national curriculum has been changed in 2005 by Turkish Ministry of National Education and mathematics curriculum was arranged appropriate to conditions of the age. Principle of the new mathematic program is every child can learn mathematics and it emphasized concepts of mathematics, relationship between these concepts, meanings of processes and gaining ability of making these operations (MoNE, 2008). As a primary school teacher, the researcher struggled to understand why her students have difficulties in understanding fractions. They seemed to remember what fractions are but they were not sure what to do with them and how. They tried to order fractions but were taking only the denominator into account. They tried to change denominators but they were not sure what they should do or they were on the wrong track. As a teacher, the researcher wanted to know what were the difficulties for these students and why did they make these mistakes. She realized that teachers should be careful not to assume that their students understand fractions merely because students in my class could memorize these procedures' in fact could produce correct answers but when the teacher was asking them to explain what they are doing while solving problems, they could not reason out even a very simple fraction problem or gave responds that contradicted with each other. Helping students internalize the concepts of fraction can be accomplished through careful and meaningful teaching (Post, 1981). For a meaningful teaching, teachers and the researcher as being a teacher herself, need to know what kind of mistakes are made by students, what the reasons behind those mistakes are and what kind of misconceptions students have. Because the researcher observed that teachers did not have sufficient information about their students' misconceptions and they were often ineffective to eliminate students' mistakes and misconceptions.

1.5. Organization of the Study

This chapter has introduced the purpose of the study, research question, definitions of important terms, and the importance of fractions, significance of the study and motivation of the researcher for the study. The literature review chapter aims to clarify meanings of mathematical mistake and misconception; the meaning of fraction; common difficulties, mistakes and misconceptions of students related to fractions and related studies about students' misconceptions and difficulties in fractions. The third chapter describes the method employed in the study, the participants, development of the Operations of Fractions Questionnaire, administrations and results of the pilot test, administration of OFQ, procedures of analysis, complementary interview procedures and reliability and validity issues, assumptions and limitations. The fourth chapter presents the findings of students' performance on the OFQ, analysis of mistake patterns, examples of students' results. The fifth chapter presents the discussion and the conclusions drawn from the analysis and recommendations.

CHAPTER II

LITERATURE REVIEW

The purpose of this study was to investigate mistakes made by elementary fifth grade students regarding basic fractional concept and operations, and difficulties that they encounter. The other purpose was to investigate underlying reasons and misconceptions behind those mistakes and difficulties. In accordance with the purpose, this chapter mainly consists of five sections: definition of mathematical mistake and misconception, literature review on meaning of fraction, literature review on transition from natural numbers to fractions, mistakes and misconceptions on fractions, common difficulties and misconceptions and related studies about students' misconceptions and difficulties in fractions.

2.1. Mathematical Mistakes and Misconceptions

Despite often the usage mistake and misconception terms together or occasionally instead of one another there is an obvious difference between the mistakes and misconceptions about mathematical ideas and procedure (Luneta and Makonye, 2010). In order to explain these terms, this section mentioned the definitions of mistake and misconception, and the relationship between them.

2.1.1. Mathematical Mistakes/ Errors

In many of the research studies related to errors, there are various definitions. For instance, according to Luneta and Makonye (2010), error refers a mistake, slip, blunder, or inaccuracy and a deviation from accuracy. Additionally, Riccomini (2005) divided errors into two categories: unsystematic errors and systematic errors. Unsystematic errors are unintended, non-recurring wrong answers which learners can readily correct by themselves. Systematic errors though, are recurrent wrong responses methodically constructed and produced across space and time. Systematic errors are symptomatic of a faulty line of thinking causing them referred to as a misconception (Green, Piel & Flowers, 2008; Nesher, 1987; Riccomini, 2005). According to Rouche (1988), an error reveals inadequacy of knowledge and is closely connected with imagination and creativity in a new situation, and is caused by an insufficient mastery of basic facts, concepts and skills. Also, Booker (1988) stated 'children do not make errors in mathematics thoughtlessly; they either believe that what they are doing is correct, or are not at all sure what they are doing' (p.100).

Another researcher who studied the term errors was Cox (1975); he divided errors in two types: systematic errors and random errors. Systematic error is defined as "an error that is repeatedly resulting in an incorrect response that is evident in a specific algorithmic computation (e.g., 24+3=9, child correctly knows addition fact, but he adds each digit separately 2+3+4=9)' Random error is defined as 'an error that gives no evidence of a recurring incorrect process of thinking' (p.203).

In another approach, Bouvier (1987) claimed that the term error does not carry the usual connotations of mistake term. Mistakes are not results of chance. According to Bouvier (1987), students who make mistake have used a particular logic, though not the appropriate one. Casey, Ernest and Koshy (2000) described the term mistake as 'a wrong idea or wrong action that often be the result of a misconception, but not always so' (p.172). That is why, analysis of mistakes helps to understand reasons of the errors and to focus on the possible misconceptions held by students. According to Casey et. al. (2000), possible reasons of children mistakes are careless mistakes (e.g., 7+5=11), reliance on rule (e.g., $\frac{2}{3}+\frac{3}{6}=\frac{5}{9}$), and problems with language and mathematical vocabulary (e.g., difficulty in remembering words such as mean, median, mode, numerator, denominator).

Another researcher, Radatz (1979) stated that students' mistakes in learning mathematics are a world-wide phenomenon, and he provided a definition:

> First, mistakes in the learning of mathematics are not simply the absence of correct answers or the result of unfortunate accidents. They are the consequence of definite processes whose nature must be discovered. Second, it seems to be possible to analyze the nature and the underlying causes of mistakes in terms of the individual's information processing mechanisms. (p. 170).

As mentioned before, one of the purposes of the research was to investigate underlying reasons of students' mistakes regarding fraction concepts. Radatz (1980) stated that various reasons of mistakes in mathematics can be determined by examining the mechanisms used in obtaining, processing, retaining, and reproducing the information in mathematical tasks. Based on this knowledge, he identified four mistake categories: mistakes due to processing iconic representations; mistakes due to deficiencies of mastery prerequisite skills and concepts; mistakes due to incorrect associations or rigidity of thinking leading to inadequate flexibility in decoding and encoding new information and the inhibition of processing new information; and mistakes due to the application of irrelevant rules or strategies.

Radatz (1980) also stated that student's mistakes in mathematics education are not simply a result of ignorance and situational accidents. Most student mistakes are not due to unsureness, carelessness or unique situational conditions. Rather, student mistakes are the product of previous experience in mathematics learning process. Student mistakes illustrate individual difficulties, concepts that were failed to be grasped and misunderstood, or problems. He presents term of mistake:

- is causally determined and very often systematic;
- is persistent and will last for several school years, unless the teacher intervenes pedagogically;
- can be analyzed and described as error techniques;
- can be derived from certain difficulties experienced by students while receiving and processing information in the mathematical learning process or from effects of the interaction of variables acting on mathematics education (p.16).

Tirosh (2000) carried out comprehensive studies on this field and determined three types of mathematical mistakes: algorithmically based mistakes, intuitively based mistakes, and mistakes based on formal knowledge. Algorithmically based mistakes involve mistakes originating from operationally made mistakes. Misapplication of basic rules, like inverting and multiplying the second multiplier in division or finding a common denominator for the given fractions while carrying out subtraction, are common examples for algorithmically based mistakes. Rote memorization of algorithms could be main source of algorithmically based mistakes. The second category of student mistakes was intuitively based mistakes, which were a result of intuitions about the operations in which students tend to overgeneralize the properties of operations of whole numbers to fractions (Tirosh 2000). Finally, the third category was mistakes based on formal knowledge, which referred to the axioms, definitions, theorems, and proofs (Fischbein, 1994). Also, mistakes resulted from inadequate knowledge were related to this group (Tirosh, 2000).

2.1.2. Mathematical Misconceptions

Misconceptions research generated a wide variety of terms to characterize students' conceptions, including: 'preconceptions' (Clement, 1982b; Glaser & Bassok, 1989), 'alternative conceptions' (Hewson & Hewson, 1984), 'naive beliefs' (McCloskey, Caramazza, & Green, 1980), 'alternative beliefs' (Wiser, 1989), 'alternative frameworks' (Driver, 1983; Driver & Easley, 1978), and 'naive theories' (McCloskey, 1983; Resnick, 1983)' (cited by Bingölbalı,& Özmantar, 2009, p. 3).

One of the definitions of misconception is 'a student conception that produces a systematic pattern of error' (Smith, diSessa & Roschelle, 1993, p.118). Nesher (1987) defined misconception as an 'idea or conception that is at variance with the accepted meaning in science' (p.35). In another approach, the term misconception is described as 'observed differences which may in fact be a natural stage of development between novice student ideas and corresponding expert, naive beliefs' (Swan, 2001, p.154). According to Swan (2001), a 'misconception' is not a wrong thinking but is a concept in embryo or a local generalization that the pupil has made. In fact, it may be a natural stage of development. Similarly, Fischbein et al. (1985) claimed that the misconceptions from previous instructions generally seem to become so deeply rooted in the learner's mind that they continue to exert an unconscious control over mental behavior even after the learner has acquired formal mathematical notions that are solid and correct. Accordingly, Askew and Wiliam (1995) claimed that, 'teaching in a way that avoids pupils creating any misconceptions is not possible, and that it has to be accepted that pupils will make some generalizations that are not correct and many of these misconceptions will remain hidden unless the teacher makes specific efforts to uncover them' (p.3).

Misconceptions are often caused when new information is added to an incompatible knowledge base, producing consecutive, synthetic models (Behr, Harel, Post & Lesh, 1992). Behr et. al. (1992) stated that some misconceptions may result from new concepts not being strongly connected with the student's previous concepts. On the other hand, some other misconceptions may result from the absence of some actually essential detail of the knowledge-scheme which has been overlooked in the design of the teaching material. Nesher (1987) expressed that in elementary mathematics, misconceptions usually originate in prior instruction as students incorrectly generalize prior knowledge to grapple with new tasks (p. 34). Moreover, Clement (1980) emphasized that misconceptions generally appear both before and after instruction, in substantial numbers of students, in a wide variety of subject-matter domains, and are often actively defended.

Characteristically, misconceptions are intuitively sensible to learners and can be resilient to instruction designed to correct them (Smith, DiSessa & Roschelle, 1993). Mistakes are visible in learners' artifacts such as written text or speech. However misconceptions are often hidden from the undiscerning observer. Sometimes misconceptions can even be hidden in correct answers, when correct answers are accidental (Smith, DiSessa & Roschelle, 1993) (cited by Luneta & Makonye, 2010).

The purpose of this study was to investigate mistakes made by elementary fifth grade students regarding basic fractional concept and operations, difficulties that they encounter, underlying reasons and misconceptions behind those mistakes and difficulties. In the previous section, the definitions of mistake and misconception were mentioned. In the next section, in accordance with the purpose of this study, the meaning of fraction was mentioned.

2.2. What is Fraction?

Fraction concepts can be described by teachers or students by using written symbols, spoken language, concrete materials, pictures, and real world examples (Lesh et al., 1983). But actually, fraction is a difficult concept to clearly define. Fraction knowledge forms a basis for understanding a wide range of related concepts, such as ratio, decimals, proportion, percents and rational numbers, and is essential to expertise in more advanced issues for example calculus and algebra (Kieren, 1993). As Niemi (1996) stated that, the concept of 'meaning' is far from conspicuous when applied to the term 'fraction', however which no doubt accounts for much of the difficulty of teaching and learning about fractions.

Although the increase of efforts to expound the concept of fraction, no single definition for fractions has been defined and used in the literature (Ball, 1990, 1993; Behr, Harel, Post & Lesh, 1988; Behr, Lesh, Post & Silver, 1983; Clarke, Roche & Mitchell, 2008; Kieren, 1988, 1992; Ohlsson, 1988; Post, Behr& Lesh, 1986; Pothier

& Sawada, 1983; Smith, 1991). Sometimes fractions are matching with rational numbers (Cianca, 2006; Clemets & Campo, 1989). Lamon (1999) mentioned that equating these two terms, both colloquially and mathematically is problematic, because, not all fractions are rational numbers (e.g. $\frac{e}{2}$). In some other studies, fractions are denoted as subconstruct of rational numbers (Behr, Lesh, Post& Silver, 1983; Kieren, 1988), applications (Ohlsson, 1988) or elements of equivalence classes of rational numbers (Behr, Harel, Post & Lesh, 1992).

Gould (2005) divided fractions into two forms in practice: partitioned fractions and quantity fractions. He described partitioned fractions as 'the fraction formed when partitioning objects into b equal parts and selecting a out of b parts to arrive at the partitioned fraction $\frac{a}{b}$. A partitioned fraction can be of either discrete or continuous objects but a partitioned fraction is always a fraction of something' (p.134). On the other hand he defined quantity fractions as 'mathematical objects defined as fractions that refer to a universal unit'.

In addition, according to Van de Walle (2004), fraction refers to 'the equal shares or equal-sized portions of a whole or unit' (Van de Walle, 2004, p.242). He stated that fractions are important because it forces students to look at numbers in a multiple of ways because it is not dealing with just whole 'natural' numbers (Van De Walle, 2007).

According to Lamon (1999), fractions have five interconnected, yet distinct interpretations. Using these interpretations, one can explore the various characteristics and manipulations of fractions (such as proper and improper fractions, mixed numerals, fraction equivalence, comparison, addition, multiplication and division). The concept of fractions is also linked to other mathematical concepts such as geometry, number-lines, and whole number multiplication and division.

Kieren (1993) defined fractions as part of a family of sub-constructs of rational numbers. In another study, Kieren (1988) identified five constructs associated with fractions: part-whole, quotient, operator, measure and ratio. She based her constructs on how children experienced fractions. Five sub-constructs of rational numbers and their examples were given in the Table 2.1 below:

Table 2.1 Different interpretations of the fraction $\frac{3}{8}$ according to Kieren (1988)

Interpretations	Examples of Interpretations
Part/ Whole relationship	 I II Part-whole representations of 3/8
Measure	3/8 means a distance of 3units from 0 on the number line (1/8 units)
Operator	3/8 of something, stretching or shrinking
Quotient	3 divided by 8, 3/8 is the amount of each person received
Ratio	3 parts water to 8 parts alcohol

Ohlsson (1988) tried to identify various meanings of the fractions used in the '$\frac{x}{y}$' symbol. Analysis of Ohlsson yielded four interpretations for $\frac{x}{y}$: quotient, rational number, binary vector, and composite function. Partitioning, extracting, shrinking, and reducing are applications associated with this interpretation of $\frac{x}{y}$. This is similar to what Kieren (1988) defined as the division construct. Ohlsson (1988) agreed with Lamon and Kieren about interpretations of fractions. He stated that list of interpretations is a necessary starting point, but a theory of meaning of fractions must also clarify the relationship between the interpretations (Ohlsson, 1988).

In the light of all these definitions, the researcher defined fractions as the set of non- negative rationals as $\frac{a}{b}$ both a and b will be natural numbers and b will be non zero number (Behr & Nichols, 1982). This definition was choosen because the participant of the study were fifth grade students and when they get to sixth grade, they will begin to learn rational numbers and this definition clearly shows the difference between the term fraction and rational number.

In the next section, differences encountered in the transition from natural numbers to fractions will be reviewed.

2.3. Transition from Natural Numbers to Fractions

Different theoretical approaches exist for explaining difficulties with fractions. One common aspect of several approaches is the emphasis on discontinuities between natural and fractional numbers (Prediger, 2006) since fractions are the first and important step in transition from natural numbers to rational numbers that students encounter in elementary grades (Behr, Harel, Post & Lesh, 1982). This major transition from natural numbers to rational numbers indicates some fundamental changes that cause learning difficulties, because this recent number concept requires new cognitive requirements. For example the fact that multiplication always makes bigger for natural numbers (apart from 0 and 1), but no more for fractions less than 1 (Streefland, 1984). According to Prediger (2006), children develop an intuitive understanding of natural numbers in the context of counting objects. Because fractions and the arithmetic of fractions are much more complicated, intuition cannot be counted on to develop an understanding in a learner's mind of what fraction is, much less how to calculate with them. Olive (1999) stated that due to the transfer of ideas about the natural numbers to the fractions, students work hard to learn the fractional numbers (Olive, 1999). But according to Stafulidou and Vasniadou (2005), students don't use their knowledge and experience on natural numbers while learning fractional numbers. It is known that, the main difference between fractions and natural numbers is, while the concept of natural numbers was derived from counting, the concept of fractional number was derived from measuring. Natural numbers and counting process support each other whereas fractions are dense, meaning between any two fractions; there are infinite numbers of other fractions. Besides, the uniqueness on visualization of natural numbers is not observed in fractions. All these situations make it difficult to understand the concept of fractions (Stafulidou & Vasniadou, 2005; Steencken, Maher, 2003).

Streefland (1993), Mack (1995) and Behr, Wachsmuth, Post and Lesh (1984) have argued that students' natural number concepts can interfere with their reasoning with fractions. Also, Saenz-Ludlow (2003) is one of the researchers who explored the transition from natural number to fraction conceptualizations. After an in-depth discussion he stated that the purpose of the study was to describe the transition via the theory of signs and chains of signification of how 4th graders reconceptualize natural numbers and their initial conceptions of fraction numbers. The researcher suggested that students need to conceptualize natural numbers as manifolds of units, or an awareness of the different and equivalent approaches that numbers can be composed or decomposed. Although students were initially successful with continuous models for fractions, they had some difficulties in transitioning to discrete models without a manifold of unit conception. Other difficulties noted by the researchers included interpretations of natural numbers where fractions were interpreted as multiples of numerators or denominators; and additive interpretations of the use of double-counting to establish multiplicative part-to-whole relations prior to fractional whole-to-part relations (Behr & Wachsmuth, Post, Lesh, 1984; Mack, 1995; Streefland, 1993).

When literature is reviewed, many studies show that when students encounter problems involving fractions, they often solve the problems by using their knowledge of natural numbers (Behr, Wachsmuth, Post & Lesh, 1984; Mack, 1995). Behr et al. (1984) brought clarity to this issue with a good example: with common fractions students may reason that one over eight is larger than one over seven, because eight is larger than seven, or they may believe that three over four is the same as four over five, because in both fractions the difference between numerator and denominator is one (Behr, Wachsmuth, Post et al., 1984). The reason for this was explained by Post and Cramer as: for all children, their previous natural-number schemas have influenced their ability to reason about the order relation for fractions (Post & Cramer, 1987).

Because of the differences between fractions and natural numbers such as counting and partitioning, notation of size, additive and multiplicative relationships; learning and teaching fraction is not easy task neither for students nor for teachers. Therefore it is suitable to continue to this chapter with students' difficulties in transition process from natural numbers to fractions.

2.3.1. Difficulties in Transition Process from Natural Numbers to Fractions

First of all, Behr and Hiebert (1988) stated that since fractions are more dense compare to natural numbers, they are more difficult to comprehend. They claimed, given that there are no consecutive fractions to any fraction and while value of a natural number increases the values of its numerals also increase and this is not the case for fractions, learning of fractions becomes more challenging. For instance, learners have difficulties in grasping while 60 is smaller than 150 in natural number domain, in fraction domain, $\frac{150}{800}$ is smaller than $\frac{55}{60}$ even if 150 and 800 are larger than 55 and 60 in the means of numeral values (Hiebert& Behr, 1988).

Secondly, as it is mentioned before, learning fractions require new cognitive requirements. Because in transition to fractions, symbolic representations, notion of representing a quantity by infinite equivalent forms and density of new number concepts require cognitive modification and reorganization (Hiebert& Behr, 1988).

Thirdly, Behr and Post (1988) mentioned that transition from counting scheme to partitioning scheme is also problematic for students; because, it also means transition from concrete objects to discreet objects. Partitioning concept is provided with part - whole interpretations (Behr & Post, 1988; Hiebert & Behr, 1988). On the other hand, arithmetic means that students used in early elementary grades required extensive counting (Bergeron& Herscovics, 1990). But when students encounter fractions, partitioning becomes main quantifying activity.

In addition, Behr and Post (1988) claimed that another difficulty of students in learning fractions is the transition of additive relationship to multiplicative one. In the fraction domain, the denominator and numerator are multiplicatively related, not additively. Therefore, it is confusing that $\frac{3}{7}$ is not equal to $\frac{5}{9}$ although differences between 3 and 5 and between 7 and 9 are 2 but then $\frac{3}{7}$ is equal to $\frac{6}{14}$. Because of all this complexity, students should construct this multiplicative relationship for deep understanding (Behr& Post, 1988).

In the next section, some of the common mistakes that students made and misconceptions they hold about fractions are highlighted.

2.4. Common Difficulties and Misconceptions Related to Fractions

The literature review on students' understanding of fractions shows that students typically have difficulties and misconceptions regarding fractions. As a matter of fact, there is sufficient evidence that students find the learning of fractions as a major difficulty (Bell, Harel, Lesh & Post, 1992; Hiebert & Behr, 1988; Kieren 1976; Mack, 1990; Olive, 1999; Saenz-Ludlow, 1994). In this sectioned, in accordance with the purpose of this study, literature was reviewed about difficulties regarding basic fraction concepts and fraction operations and reasons behind those mistakes and difficulties.

2.4.1. Difficulties Regarding Basic Fraction Concepts

The literature shows that students have difficulties in understanding basic concepts of fractions (Aksu, 1997; Ersoy & Ardahan, 2003). Interestingly, researches also show that although many students may be considerably capable of working with fractional computations, they do not really understand what fractions are (Kouba, Zawojewski & Strutchens, 1997). Correspondingly, Haser and Ubuz (2001) stated that even though students can easily carry out the algorithms with fractions, they do not understand the meanings of such algorithms. According to Haser and Ubuz (2001) elementary school students have difficulties in considering fractions as divisions of wholes in equal parts. Because of this they can easily forget these algorithms. Moreover, it was revealed that elementary grade students have difficulty relating fraction symbols and language to the various constructs of fractions (Mack, 1990) and fail to consider fractions as numbers (Kerslake, 1986).

When students' difficulties in fractions are approached in a more specific manner, partitioning becomes the very first difficulty. Accordingly, Peck and Jenks (1981) claimed that failure to partition continuous quantities, such as area model, leads to conceptual mistakes in comparing fractions. Moreover, Nik Pa (1989) stated that unless partitioning skills are well established, the learners will make mistakes in locate fractions on a number line and to construct the unit given a fractional part. On the other hand, when students begin to learn fractions, natural number conceptions no longer hold, partitioning becomes more difficult for them to comprehend (Ott et. al., 1991). Regarding this issue, many researchers stated although children can use a procedure for partitioning discrete quantities, they have difficulties in using continuous quantities. This means that the transition from continuous to discreet quantities in the use of partitioning is difficult for students; (Hiebert and Tonnessen, 1978; Novillis, 1976) because, partitioning continuous quantities requires the ability to concurrently foresight both the whole and the final solutions before partitioning (Behr & Post, 1992). Moreover, about the partitioning of continuous quantities, Hiebert and Tonnessen (1978) claimed that children often have difficulties in determining how many splits they should do to divide a whole in a certain number, and fail to cut the whole completely, leaving a part of the it un-divided. According to Lasher (2001), children believe that parts must 'look alike' or be adjacent and they also have difficulties when the total number of parts do not match the denominator of given fraction. Also she stated that, children often lack experiences with partitioning discrete sets and the number line (Lasher, 2001). In general, a fundamental notion to the ‘part/whole’ fraction subconstruct is ‘partitioning’ a whole, whatever its representation, into a number of ‘equal’ parts. Kieren (1983) stated that, partitioning experiences may be as important to the development of fraction concepts as counting experiences are to the development of whole number concepts. Partitioning, unitizing and reunitizing are often the reasons of difficulties in interpreting fraction representations (Baturo, 2000, 2004; Behr, Harel, Lesh & Post, 1992; Kieren, 1983; Lamon, 1996; Pothier & Sawada, 1983).

When the literature is reviewed, it is seen that another subject students have difficulties in is equivalence of fractions. Many studies have documented students’ various difficulties in both understanding equivalence and ordering fractions (Behr, Harel, Post and Lesh, 1992; Behr, Wachsmuth, Post and Lesh, 1984; Hart, 1986; Kamii and Clark, 1995; Kerslake, 1986). Behr, Wachsmuth, Post and Lesh (1984) asserted that, students find it difficult to order fractions with same numerators and different denominators and they have difficulties in deciding which strategy to use while ordering fractions. Moreover, Kerslake (1986) noted that when students are given diagrams, in which the same shapes are divided into different numbers of parts, and asked to compare two fractions, it is relatively simple because the possibility of using a perceptual comparison. However, for example, if students are given a diagram with six or nine divisions and asked to mark $\frac{2}{3}$ of the shape, a large proportion of them fail to mark the equivalent fractions, $\frac{4}{6}$ and $\frac{6}{9}$. Approaching from a different perspective, Lesh, Harel, Post, Cramer and Behr (1993) stated that conceptual understanding of equivalent fractions involves more than remembering a fact or applying a procedure. It is based on an intricate relationship between declarative and procedural knowledge; between fraction interpretation and representation. According to Lesh et. al. (1993), students have troubles in understanding equivalency, because they have difficulties in making connections between fraction models by understanding the sameness and distinctness within these interpretations and making connections between the different representations and showing that a fraction represents a number with many names (Lesh et al., 1983).

Research has also shown that students have difficulties in identifying a proper fraction in a number line (Baturo, 2004; Hannula, 2003; Kerslake, 1986; Pesen, 2008). When literature reviewed, it is seen that students have difficulties in dividing units on the number line into equal parts (Bright, Behr, Post & Wachsmuth, 1988), and about how many equal parts the whole was supposed to be divided into and how many of these equal parts were to be selected (Pesen, 2008). Moreover, the identification of a unit in number line seems to be difficult for some students as in part-whole diagrams.

2.4.2. Difficulties Regarding Fraction Operations

In addition to all of these issues given above, there are many researches in literature about students' difficulties regarding fraction operations. For instance, Soylu and Soylu (2005) denoted that elementary students have learning difficulties in ordering, addition, subtraction, multiplication of fractions. Also, Ball (1990) also expressed that students get confused with the meaning of operations and their rules. Furthermore, in their study, Şandır, Ubuz and Argün (2007) confirmed that students had difficulties while performing arithmetic operations of fractions.

Review of studies related to addition and subtraction of fractions show that, although it is assumed that the most important point in addition and subtraction of fractions is equality or inequality of the denominators, it is difficult for students to realize that denominators represent number of pieces in a whole in operations of fractions (Kerslake, 1986). In her study, Mack (1990) stated that students have difficulties in comprehending symbols and algorithms related to addition; associating addition and subtraction operations of fractions with daily life; and representing these operations via figures. Also, Mack (1990) and Aksu (1997) expressed that students having inadequate fundamental knowledge that would help them with these operations and trying to perform operations by focusing only on the algorithms, causes difficulties for them. In addition, Orhun (2007) stated that students find more difficulties in adding and subtracting fractions with different denominators compared to the ones with same denominators.

According to Ashlock (1994), when multiplication with fractions is introduced to students, they often have difficulty believing that correct answers make sense; throughout their previous experiences with multiplication; factors and products, the product was always at least as great as the smaller factor. In the mind of students, the concept of product had come to include the idea of a greater number because this was common throughout most of their experiences with products

(Ashlock, 1994). In addition, difficulties in models for the multiplication of fractions were studied and discussed by Fischbein et al. (1985), Prediger (2008) and Greer (1994). According to them, diversity of models for multiplication of fractions causes difficulties in understanding this operation (Prediger, 2011).

Another important issue is solving word problems that involve fractions. Carpenter et al. (1981) emphasized that solving fraction problems is much more difficult than performing simple computational algorithms. These results may indicate that students apply algorithms without any real understanding or comprehension of fraction concepts. In addition, according to Murray, Olivier and deBeer (1999), students have difficulties in understanding realistic word problems related to fractions. Aksu (1997) claimed that there are significant differences between students' abilities in solving fraction problems in different contexts. She also stated, students find difficult to solve problems involving fraction operations even though they could perform these operations (Aksu, 1997).

2.4.3. Reasons behind Mistakes and Difficulties Regarding Fractions

When it comes to addressing the reasons of those difficulties, Tirosh, Fishbein, Graeber and Wilson (1999) emphasized several reasons as 'children do not have the same everyday experience in using fractions as they do with natural numbers , many children find it difficult to accept a given fraction as a number and tend to view it as two natural numbers (Hart, 1981; Kerslake, 1986), children often incorrectly attribute observed properties of operations with natural numbers to those with fractions (Hiebert and Wearne, 1986; Nesher, 1988), the need to recognize and work with various interpretations of, and notations for, fractions such as part-whole comparison, decimal, ratio, operator, indicated division, and measure of continuous and discrete quantities' (Behr, Lesh, Post, and Silver, 1983) (cited by Tirosh, Fishbein, Graber &Wilson,1998). In another research study, Murray et al. (1998) stated that there appear to be three main possible reasons of students' difficulties of common fractions: First of all, the way and sequence in which the content is initially presented to the students, in particular exposure to a limited variety of fractions, for instance, only halves and quarters. Secondly, a classroom environment in which incorrect intuitions and informal conceptions of fractions affects meaningful learning, through lack of opportunity. The last one is the inappropriate application of whole-number schemes, based on the interpretation of the digits of a fraction at face value or seeing the numerator and denominator as separate whole numbers.Moreover, according to Greer (1994) children do not have the same everyday experience in using fractions as they do with natural numbers and this situation is the most important reason of students' difficulties regarding fractions (Greer, 1994). And lastly, in his research about difficulties and common mistakes of third grade students regarding fractions, Hanson (1995) explored that origin of the mistakes and difficulties of students in fractions is trying to memorize algorithms and procedures.

2.4.4. Misconceptions Regarding Fractions

Several misconceptions underlying all of these difficulties that were mentioned above have been summarized by many researchers as: According to Hart (1981) many children did not accept a given fraction as a number and tend to view it as two whole numbers and this is one of the most important misconception of learners (Hart, 1981; Kerslake, 1986), and children often incorrectly attribute observed properties of operations with natural numbers to those with fractions (Bell, Fischbein and Greer, 1984; Hiebert and Behr, 1982; Nesher, 1988).

When the misconceptions of students related to fractions are approached in a more detailed way, it is seen that learners have many misconceptions about fractions including believes like the fraction having the larger denominator is always larger; fractions are parts of shapes; fractions cannot be bigger than one and that fractions are not numbers on their own (Mack, 1990), multiplication always makes bigger, and division always makes smaller (Tirosh, 2000), denominator refers to the number of pieces, regardless of unequal sizes of the pieces (Reys et al, 1999). In addition, some researchers dealt with the misconceptions involved the vocabulary of fractions (Mack, 1990; Post, Behr & Lesh, 1986; Behr, Wachsmuth, Post & Lesh, 1985) such as mistaking 'enlarging' for amount becoming larger and mistaking 'reducing' for amount becoming smaller.

Another researcher, Lamon (1999) pointed out that in students' initial number theory, fractions are rejected as member of set of numbers, because they are not part of the counting sequence. This resistance to accept fractions as numbers leads students to conceptualize fractions as two distinct whole numbers, a misconception that often results in computational bugs, such as $\frac{3}{2}+\frac{5}{3}=\frac{3+5}{2+3}=\frac{8}{5}$.

To give an example about misconceptions related to division and multiplication operations, Prediger (2006) set off the study of Fischbein, Deri, Nello and Marino (1985) related to division of fractions. They refer to two other ‘primitive, intuitive beliefs’ about division; namely, the divisor is always less than the dividend, and the divisor must be a whole number. According to her, Fischbein et al. (1985) gave empirical evidence for an explanation situating the misconception: pertinacity of the intuitive rule ‘multiplication makes bigger’. These conceptions are in harmony with the operations of multiplication and division in the natural number domain but incompatible in the fraction domain. Based on this information, Prediger (2006) expressed that, ‘multiplication makes bigger’ and ‘division makes smaller’ is a very widespread misconception.

Mitchell and Horne (2008) descriptively analyzed three types of misconceptions related location of fractions to a number line: the first was conceptual, over-generalized part-whole unit-forming. They entitle this situation as instrumental part-whole knowledge. In this type of misconception, students use entire segment of a number line as a whole. For instance, they marked $\frac{2}{3}$ at 6 on a number line from 0 to 9. The second one was counting lines. It means, counting lines instead of spaces on a number line. And the third one was decimalizing and they entitled this as decimalizing the count. In this type of misconception, students read the mark as if the whole was divided into tenths and they count left to right).

In the light of these findings, it is notable that national and international studies consistently show that fractions are a stumbling block for many students (Carpenter, Corbitt, Kepner, Lindquist and Reys, 1981; Carpenter, Lindquist, Brown, Kouba, Silver, and Swafford, 1988; Hart, 1981; Hiebert, 1988; Kieren, 1988). In the next heading, related studies about students’ misconceptions and difficulties in fractions are mentioned.

2.5. Related Studies about Students’ Mistakes/difficulties and Misconceptions in Fractions

In recent years several studies were conducted both in abroad and in Turkey to investigate students’ mistakes, difficulties and misconceptions about specific fraction concepts. Some of these previous studies are mentioned in this part of the literature review.

While students may be familiar with fractions, many of them appear not to have fully developed an understanding that fractions are considered numbers (e.g., Hannula, 2003; Kerslake, 1986). Kerslake (1986) expressed that students of thirteen to fourteen years relied heavily on rote memory of previously learned techniques when working with fractions. She believes that this is mainly due to the fact that fractions do not form a normal part of a children's environment and the operations on fractions are abstractly defined. This abstraction and lack of feel for fractions lead students to have many misconceptions about them. In her study, Kerslake (1986) emphasized the need for students to understand fractions at least as an extension of the number system. Many examples can be seen in literature about this situation, for instance, in the fourth National Assessment of Educational Progress; only 44 percent of 11th graders chose the correct answer to the question as $5+\frac{1}{4}$ that is given below (Carpenter et al., 1988; Niemi, 1994). Results show that many students perceive fractions as purely symbolic and they could not able to link concepts or principles (Niemi, 1994).

$$5\frac{1}{4} \text{ is the same as} \quad \text{a) } 5+\frac{1}{4} \quad \text{b) } 5-\frac{1}{4} \quad \text{c) } 5\times\frac{1}{4} \quad \text{d) } 5\div\frac{1}{4}$$

Figure 2.1 Question of Niemi related to meaning of mixed numbers (1994)

In another study, Kerslake (1986) expected students to make a multi-selection of 'numbers' from a set of options, including natural numbers, fractions and decimals. In her study, 41 % of 13 year-old students did not select any fractions. Interview results of her study also confirmed that many children do not think of fractions as numbers. In conclusion, studies of Kerslake proved that children encountered difficulties and made mistakes in fraction subject because they refused to acknowledge that fractions are numbers (Kerslake, 1986).

Additionally, in another study related to the same subject, Newstead and Murray (1998) administered a test to 370 fourth and 386 sixth grade students, which was designed to evaluate their concepts of what a fraction is, comparison of the size of fractions with different denominators and their operations with fractions. After the test, they identified success rates and common misconceptions; in summary, only 15

% of sixth grade stated meaning of $\frac{4}{5}$ correctly; 7 % of students stated shaded geometric shapes meaning of $\frac{4}{5}$. Moreover, in their study, it was expected to show $\frac{4}{3}$ in at least 3 different ways. Only 11% of fourth grade students and 14 % of sixth grade students were able to show $\frac{4}{3}$ correctly. 8 % of fourth grades and 3 % of sixth grades represented this fraction as 12, 1, 7, three minus four, four minus three or three plus four. Also, 12 % of fourth grade and 10 % of sixth grade students showed $\frac{4}{3}$ as shaded geometric shapes incorrectly. According to Newstead and Murray, students did not consider the role of the whole and one of the most common general mistakes in this study was the students' inability to see a fraction as a quantity - a quotient relation between two numbers – rather than two separate whole numbers (Newstead and Murray, 1998).

Furthermore, some children hold misconceptions about the meaning of fraction symbols written in $\frac{a}{b}$ form. When Mack (1995) asked a third grader to interpret the meaning of $\frac{3}{5}$, the third grader responded, "Oh, three fifths, that's three whole pumpkin pies with five pieces in each" (p.431). Given that some children hold misconceptions about the meaning of fraction symbols written in $\frac{a}{b}$ form, it is not surprising that some children also misinterpret mixed fractions written in $A\frac{b}{c}$ form. For example, Mack (1995) found that some children interpreted $1\frac{1}{4}$ as "one whole with four pieces (referring to $\frac{1}{4}$) and one piece more (referring to the 1)" (p.432).

In another research study, even though Kieren (1988) described five fraction constructs, it is shown that many children view fractions only in terms of a geometric part-whole model. Kerslake (1986) found that all twenty of the middle-school students she interviewed accepted a model like the one shown in Figure 2.2, as representing three-fourths. However, 15 of them rejected a model like the one shown in Figure 2.3 as representing three-fourths. Although both figures represented the same fraction, the second figure did not depict three-fourths in the context of the part-whole construct like the first figure did. The second figure depicted three-fourths in terms of the ratio construct (e.g., the ratio of shaded circles to unshaded circles). Apparently, students did not that as a valid representation of three-fourths.

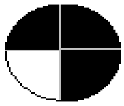


Figure 2.2 First representations of $\frac{4}{3}$ from study of Kerslake (1986)

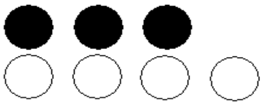


Figure 2.3 Second representation of $\frac{4}{3}$ from study of Kerslake (1986)

Misconceptions are dependent upon the developmental level of the child. Mack (1990) asked “Suppose you have two pizzas of the same size, and you cut one of them into six equal-sized pieces and you cut the other one into eight equal-sized pieces. If you get one piece from each pizza, which one do you get more from?” to students. All students responded that they would get more from the pizza cut into six pieces. Each student was also asked a question like “Tell me which fraction is bigger, $\frac{1}{6}$ or $\frac{1}{8}$” (p.22). Four of the students who first solved the real-world problem and three students who first worked with the symbolic representation responded, “One eighth is bigger, because eight is a bigger number I think” (p. 22).

Pesen (2007) identified the misconceptions of third grade students underlying mistakes about fraction concepts. 113 students from eleven elementary schools were administered a questionnaire consisting of 24 items. At the end of the study, Pesen maintained that, students had mistakes in dividing the whole into equal parts. Furthermore, he stated that students had difficulty in dividing the circular shapes into equal parts while comparing to the rectangular shapes. Besides, they were confused about the location of the numerator and denominator, and one of them was placed in the place of the other.

In another study, Ball (1990) used area and set models in one of her lessons to develop the part-whole interpretation of fractions. The main problem asked how much brownie each person has if there are four persons to share five brownies evenly. One of the students used partitioning by cutting each of four brownies into halves and shared these halves among four persons. He partitioned the remaining one brownie into quarters but called each of these pieces 'halves' instead of 'quarters'. After distributing the pieces, he asserted that each person got 'half'. In this case, the learner used partitioning successfully but, unfortunately, he did not have immediate access to the term 'fourths'. Ball also described some of her teaching techniques. For example, when she observed that a student had difficulty in partitioning circular cookies into fifths, she switched to rectangular cookies to see if the same problem ensued. Towards the end of the lesson, she posed transfer problems involving improper fractions like, "show $\frac{4}{2}$", and saw that students made partitioning better in rectangular cookies.

Clarke, Roche and Mitchell (2007) conducted a study with sixth grade students. They interviewed 323 sixth grade students using a set of tasks, at the end of the school year. It was expected to share three pizzas equally among five people, as shown in Figure 2.4. In this study, 30 % of students correctly divided the pizzas and calculated equal shares. Surprisingly, 12 % of the students were unable to make a start. Clarke et al. (2007) suggested that greater exposure to division problems and explicit discussion connecting division with their fractional answers, for example, $3 \div 5 = \frac{3}{5}$ may help lead students to the generalization that $a \div b = \frac{a}{b}$ (Clarke, Roche & Mitchell, 2007).

Figure 2.4 Pizza task (Clarke et. al., 2007)

Additionally, Clarke, et. al. (2007) were expected to figure out which fractions represent B and D parts of the circle. Question's fraction pie figure given below in Figure 2.5, represent.

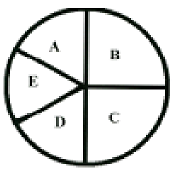


Figure 2.5 Fraction pie task (Clarke et. al., 2007)

While 83 % of students stated that part B represented $\frac{1}{4}$, 3.3% of them gave a correct equivalent fraction, decimal, or percentage as their answers. But 5.6% and 1.9% answered '$\frac{1}{5}$' and '$\frac{1}{2}$', respectively. Only 42.7% offered a correct answer, 13.6% answered as $\frac{1}{5}$. In the same study, students were shown the array in Figure 2.6 which is given below, and asked, "What fraction of the dots is black?", then they were then asked to state "another name for that fraction". 77 % gave a correct answer, with the three most common answers being $\frac{2}{3}$ (35 %), $\frac{12}{18}$ (31 %), and $\frac{4}{6}$ (9%). The most common wrong answer was $\frac{3}{4}$. Only 53.5% of students were able to offer another correct name for the fraction, with $\frac{4}{6}$ being the most common response with 17 % of participants (Clarke, Roche & Mitchell, 2007).

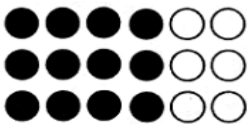


Figure 2.6 Dots array task (Clarke et. al., 2007)

In another research, Behr et al. (1983, 1984) studied on students' understanding of fraction size. Their tasks included comparing the sizes of a pair of fractions (selecting the smaller fraction from pair of $\frac{3}{5}$ and $\frac{6}{8}$) and filling in a missing denominator or numerator to make two fractions equivalent. Meanwhile, the study of Behr and Bright (1984) compared the knowledge and reasoning of students on various tasks involving the order and equivalence of fractions. Students were asked to compare fractions; select two addends for a given sum from a set of possibilities; create fractions between two given fractions; and evaluate correct and incorrect examples of comparison reasoning (Behr, et al., 1984; Hiebert, 1992). As a result of all these studies, they found that children used number facts in faulty ways to evaluate the validity of equivalence statements. For instance one third-grader considered $\frac{3}{4} = \frac{7}{8}$ because adding four to three equals seven and adding four to four equals eight. When the researchers asked another child if $\frac{3}{5}$ was equivalent to $\frac{6}{10}$, the child replied, 'They are not equivalent because six divided by three equals two, five plus two is seven, and seven is not equal to ten'. According to Vance (1986), difficulties of students about comparison of fractions may be caused by two reasons: each fraction has many different names and the fractions are dense. He interviewed six sixth grade and six seven grade students who used different strategies to compare fractions and one common strategy was to compare only the denominators: less parts means larger fraction.

When working with whole numbers, children learn that when they count, the numbers get larger. So they understand that $7 < 8 < 9$. This natural number knowledge sometimes seems to interfere with the concept of fraction order. Behr and Bright (1984) found supportive results in their study of five fourth graders. One child in their study claimed, "One third is less than one fourth because 3 is less than 4". A multiple-choice item on the 1990 NAEP was designed to assess children's understanding of fraction order. When asked to select the choice that best described why $\frac{4}{5}$ is greater than $\frac{2}{3}$, sixty-two percent of fourth graders could not select the correct answer "because $\frac{4}{5}$ is closer than $\frac{2}{3}$ to 1". Of the incorrect answers, 26% of fourth graders chose the reason "because 4 is greater than 2", 19 % of them chose the reason "because 4+5 is more than 2+3", and 17 % of them chose "because 5 is larger than 3".

The Rational Number Project (Behr, Wachsmuth & Post, 1984; Behr, Wachsmuth, Post et al., 1984) used various tasks to investigate students' perception of fraction size. Several different strategies to compare fractions were identified in their analysis of 12 fourth grade students' responses to ordering tasks. For fractions that have same numerators, they described five different strategies: only denominator, numerator and denominator, reference point, manipulative and whole number dominance. The denominator only strategy, which included most of the explanations, referred only to the denominators. For example, an explanation like 'the bigger the number is, the smaller the pieces get' would be recorded as denominator only. The explanations that referred to both of them, identified 'numerator and denominator' strategy, indicating that the same number of parts was present but the fraction with the larger denominator had the parts with smaller sized. The reference point strategy required the usage of a third number, such as one half, in fraction comparison. Explanations that made use of pictures or manipulative materials were identified by the manipulative strategy. Whole number dominance was used to define an ordering consistent with whole number arithmetic applied to fraction denominators. Some of these strategies, such as reference point, manipulative and whole number dominance, were also evident in comparing fractions that have different numerators and denominators.

Accordingly, Vinner (1997) described similar strategies used in fraction comparison, like 'the bigger the denominator the smaller the fraction', as pseudo-analytical behavior. Applying these false or incomplete strategies can result in correct answers for the wrong reasons. A question from the Third International Mathematics and Science Study (TIMSS, 1996) was "Which of the following numbers is the smallest? (a) $\frac{1}{6}$ (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$" and could be answered correctly by 'the bigger the denominator the smaller the fraction'. However the application of a similar strategy, which is 'the smaller the denominator the greater the fraction', to another TIMSS question as 'Which number is the greatest? (a) $\frac{4}{5}$ (b) $\frac{3}{4}$ (c) $\frac{5}{8}$ (d) $\frac{7}{10}$', would lead the students to selection of option (b). In addition, Vinner (1997) reported that 42.1% fewer 8th grade students correctly answered the second question compared to the first question. Approximately 39% of the students selected option (b), which may have been influenced by using 'the smaller the denominator the greater the fraction'. The difference in the correct responses' percentage to these similar fraction comparison tasks suggests that the expected method of using equivalent fractions may not have been the only substantial coherent method applied.

In another fraction comparison study, Hart (1981) found that only 66 % of fifteen year olds could recognize that $\frac{3}{10}$ was larger than $\frac{1}{5}$. In an American Survey (NAEP), only 3 % of thirteen year olds were able to find which of the fractions $\frac{1}{4}$, $\frac{5}{32}$, $\frac{5}{16}$, $\frac{3}{8}$ was nearest to $\frac{3}{16}$. These results clearly demonstrate that students either do not know how to find equivalent fractions or do not make the connection between equivalence and size. Equivalence of fractions can also be used to find a fraction between two given fractions, such as $\frac{1}{2}$ and $\frac{2}{3}$. Hart (1981) reported that only 21 % of fifteen year olds were able to find such a fraction and added students often do not realize between two fractions on the number line there are many (infinitely many) fractions.

In a study conducted by Hart, Brown, Kerslake, Küchermann and Ruddock (1985), the equivalency of $\frac{4}{6}$ and $\frac{6}{9}$ was questioned with a sample of 55 students in the age range 11 to 13 years. In the results, a huge proportion of them failed to mark the equivalent fractions. In general, about 60% of the 11 to 12 year olds and about 65 % of the 12 to 13 year olds were able to solve this task. The same item was given to 130 primary school students in Years four and five (meaning ages, respectively, 8.6 and 9.6 years) (Nunes, Bryant, Pretzlik & Hurry, 2006) more recently. The correct response rate across these items was 28 % for the children in year 4 and 49 % for the children in year five.

Most of the time, students have difficulty in ordering fractions according to their magnitude (Nunes et. al., 2006). Hart et al. (1985) asked students to compare two fractions that have the same denominator ($\frac{3}{7}$ and $\frac{5}{7}$) and another two that have the same numerator ($\frac{3}{5}$ and $\frac{3}{4}$). If denominators are the same, students can respond correctly by considering only the numerators and ordering them like natural numbers. In this case, the rate of correct answers is relatively high but it does not effectively test students' understanding of rational numbers. Hart et al. (1985) observed approximately 90% correct responses among their students who were between 11 and 13 years of age, and Nunes et al. (2006) reported that 94% of the students in Year 4 and 87% of the students in Year 5 gave correct responses. In contrast, when the only numerator was the same, and the students had to consider the value of the fractions without any help from natural number ordering strategy, the rate of correct responses was significantly lower: in the study by Hart et al. approximately 70% of the answers were correct, whereas in study of the Nunes et al. (2006) percent of correct responses were 25 % in Year 4 and 70 % in Year 5.

In another subject, solving fraction word problem, Niemi (1996) performed a study with 540 fifth grade students which indicated that students have significantly different levels of performance according to the types of fraction problems. It was designed to assess the students' understanding of fraction concepts and solving word problem abilities. The tasks involved the pictorial and symbolic representations of equivalent and non-equivalent fractions; also, students were to explain their solutions verbally or with pictorial representations. Results showed that there is an inconsistency in students' performance; namely, learners were more successful with problems that had direct representations than problems that required an understanding of equivalent fractions.

Addition operation is also a trouble for students from time to time: when some children add two fractions together, they often use a faulty counting strategy that involves adding numerators to numerators and denominators to denominators. In addition, according to Mack (1995), while most students will have no problems comprehending the meaning of $\frac{1}{4}+\frac{2}{3}$, often they will struggle to find the right result. Mack found that, by tapping students' informal knowledge, students were able to invent their own procedures for adding and subtracting fractions. She observed that students' informal solutions involved partitioning which they used to rewrite fraction like $1\frac{1}{4}$ as $\frac{4}{4}+\frac{1}{4}$. However, she also noted that previously learned symbolic procedures oftentimes interfered with their ability to use informal knowledge to solve problems presented in either symbolic or real-world context. For instance, when asked to estimate $\frac{5}{6}+\frac{7}{8}$, one student immediately wrote, $\frac{35}{48}+\frac{35}{48}=\frac{70}{96}$. When asked where he/she got "35" and "48" the students replied,"35 goes into 5 and 7 while 48 goes into 6 and 8". Mack also claimed that, when faced with the addition of fractions, students often choose the 'easiest way out'. Instead of looking for equivalent fractions having the same denominator, they simply add the numerators and the denominators, thus using the following "rule": $\frac{a}{b}+\frac{c}{d}=\frac{a+c}{b+d}$.

Tirosh (2000) found that when she asked the question to pre-service teachers given below in figure 2.7, they looked only at the denominators of the fractions and concluded that since 4 + 6 + 8 has a greater sum than 3 + 5 + 7, the sum of Problem B is greater.

Problem A: $\frac{1}{3} + \frac{1}{5} + \frac{1}{7} =$ Problem B: $\frac{1}{4} + \frac{1}{6} + \frac{1}{8} =$

Which result is greater that the other?

Figure 2.7 Addition of fractions task (Tirosh, 2000)

Furthermore, another fact she observed in this kind of problem was that learners looked at the denominators one pair at a time, and saw 4 as “3+1,” 6 as “5+1,” and 8 as “7+1.” They thought of each addend in Problem B as somehow “1 more” than the corresponding addend in Problem A, so they concluded Problem B’s sum was greater. It is seen that although students are able to correctly calculate the sum of Problem A and the sum of Problem B, they use faulty reasoning to determine which of these is larger.

Similarly, Post (1989) mentioned that when he asked the question “$\frac{12}{13}+\frac{7}{8}$” to students, 55 % of thirteen year old students selected either 19 or 21 as answer and to find the answer of $\frac{1}{2}+\frac{1}{3}$, 30 % of students added numerators and denominators (Post, 1989). Consider the following question on the TIMSS, 8th grade test: Find the approximate value, to the closest integer, of the sum: $\frac{19}{20}+\frac{23}{25}$ where possible answers were 1, 2, 42 and 45. Many students chose 42 or 45. They added only numerators or denominators. In conclusion, these students did not think of a fraction as numbers.

Students also have difficulties in solving problem with fractions. Haser and Ubuz (2003) investigated the conceptions of fifth grade students in solving word-problems by administering a ten word problem test to 122 ten year old students in a private school. According to this research, students have difficulty while solving problems related to fractions because of their inadequate conceptual understanding of ‘part’ and ‘quantity’, and students are not aware of the resultant unit of an operation. Moreover, when students are given a part of a whole, and required to calculate the whole, they commonly cannot under understand the problem clearly and choose incorrect operations, providing an evidence for them not understanding the problem clearly.

Additionally, research of Aksu (1997) indicated that student performance varies significantly according to the context of the solving fraction word problem. In her study, 155 sixth grade students were expected to solve test questions in three different formats; fraction concepts, operations involving fractions, and word problems including fractions. Aksu observed the lowest student performance was on the solving problem part of the test. However, she did not find any difference in the students' ability to accurately perform the basic four operations in computation problems. When those same operations were involved in word problems, interestingly, there was a difference in student performance. In general, she found significant differences between students' abilities to solve fraction problems in different contexts and she stated that student performance of the four operations declined significantly when the operations were presented in the form of word problems.

Brown and Quinn (2006) administered a twenty five item test to five ninth grade classes. They observed that most students used algorithms to solve fraction problems. Moreover, when students were unsure how to solve a problem, they resorted back to previously memorized algorithms, whole numbers, which they were more familiar with. Brown and Quinn claimed that, these students lacked experience with fractions and did not have conceptual knowledge about fractions. Based on this result, Brown and Quinn (2006) stated, 'Algorithms that are taught when the concept is beyond the learner's cognitive development force the learner to abandon his or her own thinking and resort to memorization' (p. 29). According to them, if a learner is not ready to learn fractions, he can memorize the producers for solving the problems, but he does not have the time or cognitive development to construct understanding of fraction concepts and therefore will not be able to develop a complete understanding that will stay with him (Brown, Quinn, 2006).

In this section, in order to obtain large-scale information about the research subject, literature was reviewed about fractions, student mistakes regarding fractions, their mistakes, misconceptions underlying those mistakes and mathematical definitions of these terms.

2.6. Summary of the Literature Review

There are numerous research and studies conducted in both abroad (Behr et al., 1982; Fischbein, 1984; Hanson, 1995; Hart, 1986; Kerslake, 1986; Lamon, 1999;

Mack, 1995; Tirosh, 2000; Wu, 1999; etc.) and Turkey (Aksu, 1997; Haser & Ubuz, 2001; Işıksal, 2006; Pesen, 2008; Soylu & Soylu, 2005; Toluk, 2002) about the subjects that are related to the purposes of this study.

In brief, students made mistakes, they have many difficulties and misconceptions about fractions, and it is not easy to identify all of them; nevertheless, these issues were tried to be highlighted as much as possible. Also, this review indicated that fractions is a rich and also the first abstract concept students learn; provides a basis for fifth grade elementary students regarding upcoming subjects like decimals, ratios, proportions; therefore, have an important role in elementary mathematics (Booker, 1996). However they are complex for learners so students have many difficulties and make mistakes related to the fractions. Previous studies conducted in many countries have reported that students have difficulties in understanding the fundamental fraction concepts, operations and solving fraction word problems in each elementary grade (Behr, Lesh, Post & Silver, 1983; Booker, 1998; Corbitt, 1989; Hart, 1993; Haser & Ubuz, 2001; Kerslake, 1986; Leinhardt & Smith, 1984; Mack, 1990; Newstead & Murray, 1998; Orton & Frobisher, 1996).

According to literature review, students have difficulties and they made mistakes regarding to both basic fraction concepts and fraction operations. For instance; elementary school students have difficulties in considering fractions as divisions of wholes in equal parts (Haser & Ubuz, 2001). In addition elementary grade students have difficulty relating fraction symbols and language to the various constructs of fractions (Mack, 1990). Also, fail to consider fractions as numbers (Kerslake, 1986). According to Hiebert and Tonnessen (1978), children often have difficulties in determining how many splits they should do to divide a whole in a certain number, and fail to cut the whole completely, leaving a part of the it un-divided. Moreover, many researchers claimed that partitioning, unitizing and reunitizing are often caused difficulties in interpreting fraction representations (Baturo, 2000, 2004; Behr, Harel, Lesh & Post, 1992; Kieren, 1983; Lamon, 1996; Pothier & Sawada, 1983). Furthermore, many studies have documented students' various difficulties in both understanding equivalence and ordering fractions (Behr, Harel, Post and Lesh, 1992; Behr, Wachsmuth, Post and Lesh, 1984; Hart, 1986; Kamii and Clark, 1995; Kerslake, 1986). Behr, Wachsmuth, Post and Lesh (1984) asserted that, students find it difficult to find equivalent fractions with a fraction, order fractions with same numerators and different denominators and they have difficulties in deciding which strategy to use while ordering fractions. Research has also shown that students have difficulties in identifying a proper fraction in a number line (Baturo, 2004; Hannula, 2003; Kerslake, 1986; Pesen, 2008). When literature reviewed, it is seen that students have difficulties in dividing units on the number line into equal parts (Bright, Behr, Post & Wachsmuth, 1988),

In addition to all of these issues given above, there are many researches in literature about students' difficulties regarding fraction operations. For instance, review of studies related to addition and subtraction of fractions show that, although it is assumed that the most important point in addition and subtraction of fractions is equality or inequality of the denominators, it is difficult for students to realize that denominators represent number of pieces in a whole in operations of fractions (Kerslake, 1986). Moreover, Mack (1990) stated that students have difficulties in comprehending symbols and algorithms related to addition; associating addition and subtraction operations of fractions with daily life; and representing these operations via figures. According to Ashlock (1994), when multiplication with fractions is introduced to students, they often have difficulty believing that correct answers make sense; throughout their previous experiences with multiplication; factors and products, the product was always at least as great as the smaller factor. In the mind of students, the concept of product had come to include the idea of a greater number because this was common throughout most of their experiences with products (Ashlock, 1994). Another important issue is solving word problems that involve fractions. Carpenter et al. (1981) emphasized that solving fraction problems is much more difficult than performing simple computational algorithms. These results may indicate that students apply algorithms without any real understanding or comprehension of fraction concepts. According to some researchers, students have Lack of understanding about the relationship between a part and a quantity (Haser & Ubuz, 2003; Pesen, 2007).

Several misconceptions underlying these mistakes and difficulties that were mentioned above, have been summarized by many researchers as: thinking that it is only the denominator or numerator that determines the size of the fractions, for instance, students believe that the fraction having the larger denominator is always larger (Baroody & Hume, 1991; Vinner, 1997); believing that fractions are parts of shapes (Mack, 1990); believing that fractions cannot be bigger than one and that fractions are not numbers on their own (Mack, 1990); using part - part strategy instead of part - whole ones (Moss and Case, 1998); considering that a fraction which has the larger numbers is larger (Hart et al., 1980); thinking that multiplication always makes bigger, and division always makes smaller (Tirosh, 2000); thinking that denominator refers to the number of pieces, regardless of unequal sizes of the pieces (Reys et al, 1999); thinking that the dividend should be always bigger than the divisor (Tirosh, Fischbein, Graeber, & Wilson, 1993) and thinking that the denominator refers to the number of pieces, regardless of unequal sizes of the pieces (Reys et al, 1999). In addition, some researchers dealt with the misconceptions involved the vocabulary of fractions (Behr, Wachsmuth, Post & Lesh, 1985; Mack, 1990; Post, Behr and Lesh, 1986) such as mistaking ‘enlarging’ for amount becoming larger and mistaking ‘reducing’ for amount becoming smaller,

As the literature review promotes the fact that, fractions subjects are almost always studied specifically; also, especially in Turkey, there are not many studies with fifth grade elementary students on fractions (Haser & Ubuz, 2003; Toluk, 2002). All in all, the literature review supports this study having the purpose of investigating general mistakes made by elementary fifth grade students regarding basic fractional concept and operations, difficulties that they encounter, underlying reasons and misconceptions behind those mistakes and difficulties.

CHAPTER III

METHODOLOGY

This chapter is devoted to information about the research design, population and sample, data collection instruments, internal and external validity of the study, data collection procedure, analyses of data and assumptions and limitations.

3.1. Design of the Study

The purpose of this study was to investigate mistakes made by elementary fifth grade students regarding basic fractional concept and operations, and difficulties that they encounter. The other purpose was to investigate underlying reasons and misconceptions behind those mistakes and difficulties. In this study, mixed method was performed since single methodology would not be sufficient to meet the essential information about research questions. Creswell and Plano Clark (2007) defined mixed method as:

'Mixed research is a research design with philosophical assumptions as well as methods of inquiry. As a methodology, it involves the philosophical assumptions that guide the direction of the collection and analysis of data and the mixture of qualitative and quantitative approaches in many phases in the research process. As a method, it focuses on collecting, analyzing, and mixing both quantitative and qualitative data in a single study or series of studies.' (p. 5)

In order to identify mistakes and difficulties of students along with their frequencies, survey method was used since the objective of a survey research is to obtain data from members of a population or a sample to determine current status of that population (Fraenkel & Wallen, 2005). More specifically, cross-sectional survey design was deemed appropriate for the study. Fraenkel and Wallen (2005) defined cross-sectional survey as 'a survey in which data collected at one point in time from the predetermined population or populations' (p.398). In addition in order to identify underlying reasons and misconceptions behind fifth grade students' mistakes and difficulties regarding fractions required a more detailed *in-depth* study, thus semi-structured interviews were conducted.

3.2. Participants of the Study

Sample selection procedure of this study consists of two phases. In the first phase, the research was carried out with 151 students who were chosen from 2 public elementary schools in Odunpazarı district, Eskişehir. Four classes of 5th grade students from one of the schools and two classes of 5th grade students from the other school, a total of six classes that were heterogeneously grouped, constitute the sample of the study. Fifth grade students were particularly chosen for this study as they had been completed all of the mathematics lessons about fractions in elementary school. The average age of these students was 10.85 and the boy to girl ratio was nearly equal with slight male dominance as seen in Table 3.1. Detailed demographics regarding gender, age and grade level of participants of the study were given in the table below.

Table 3.1 Participants' Demographic

Classes	**Sample Size (n)**	**Age (years)**	**Gender**	
		Average	**Boys**	**Girls**
5-A	25	10.88	13 (%52,0)	**12 (%48,0)**
5-B	24	10.33	14 (%58.3)	**10 (%41.7)**
5-C	24	10.94	11 (%45,8)	**12 (%54,2)**
5-D	25	11.08	13 (%52,0)	**12 (%48,0)**
5-A	26	11	12 (%46.1)	**14 (%53.9)**
5-B	27	10.84	13 (%48,1)	**14 (%51,9)**
Total (N)	**151**	**10.85**	**76 (%50,3)**	**75 (%49,7)**

In this study, all fifth grade students in Central Anatolian Region were identified as a target population. The accessible population of this study was determined as all fifth grade students in Eskişehir where the results of the study will be generalized.

The sampling method used in the first phase was convenience sampling which involves the sample being drawn from a part of the population which is close to hand. In this method, researchers collect data from the individuals who are readily available (Fraenkel & Wallen, 2006). Four occasions have conducted the researcher to choosing convenient sampling: usage of existing individuals, having easy contact, voluntary involvement, and being a master in the school environment. Since the researcher was an elementary school teacher in a public elementary school, the study was applied in this school and the neighbor school.

In the second phase, underlying reasons and misconceptions behind those mistakes and difficulties were examined through semi-structured interviews. In order to select the students to interview, first of all, participants were categorized into three groups, which represented their ability levels, based on their performance in OFQ: The first group was defined as "high", meaning they correctly answered most of the items and their scores were higher than 60 according to rubric; the second group was defined as "medium", meaning their scores were between 40 and 60; the third and last group was defined as "low", meaning their scores were lower than 40.

Each group of students was asked separately whether they were voluntary to participate in semi-structured interviews. Five students from "high" group, five from "medium" and six from "low", a total of 16 students (approximately 10 % of all participants) were willing to participate in the interview. The aim of categorizing students and asking for voluntariness separately was to provide participation in interviews from all performance levels. The sampling procedure and participants of the study is summarized in Figure 3.1.

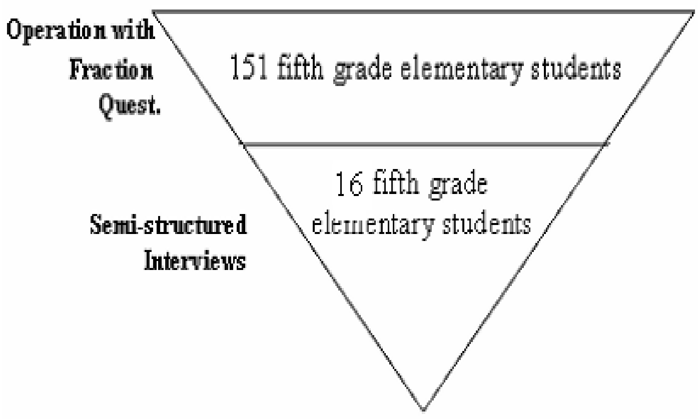


Figure 3.1 Sampling procedure and participants of the study

3.3. Data Sources

This study investigated mistakes made and difficulties encountered by elementary fifth grade students related to fractions. In addition, misconceptions and reasons behind of those mistakes and difficulties were examined. First part of the required data was gathered via Operation with Fraction Questionnaire (OFQ) and the second part was via interviews following the questionnaire on fraction concept.

3.3.1. Operations with Fraction Questionnaire

In accordance with the purpose of the present study, the researcher needed an instrument that took cognizance of the basic fraction concepts and fractional operations of fifth grade students. As the standardized tests were not sufficiently detailed in the domain of fractions to provide insights into fifth grade students' specific cognitive difficulties and because of the lack of a test that measures all of the objectives of mathematics education program of MoNE regarding fractions, Operation with Fraction Questionnaire (OFQ) was developed by the researcher to identify fifth grade students' mistakes and difficulties related to the basic fraction concepts and fractional operations, which were expressed in Chapter I.

In order to prepare the items of Operation with Fraction Questionnaire, first of all, the objectives of fifth grade Turkish National Elementary Mathematics Education Curriculum related to fractions were determined. Objectives and question numbers in the OFQ that correspond to these objectives. Secondly, the literature was reviewed about the difficulties, mistakes and misconceptions of students related to fractions. The OFQ, which was composed of 12 items, was developed using these information.

In order to ensure a better identification of mistakes and to cover as many aspects of the objectives as possible, some of the items, except 2, 5, 6 and 10, were presented in two parts. In addition, items 1-a, 1-b, 4-b, 5, 6, 7-a, 7-b and 8-b were adapted from other studies in the literature investigating the difficulties and misconceptions of students related to fractions (Lasher, 2001; Mack, 1993; Niemi, 1994; Taube, 1992; Willoughby, Bereiter, Hilton& Rubinstein, 1998b; Yiu, 1992). The remaining items were developed by the researcher. Moreover, to explore students' visual understanding, certain diagrams or illustrations were constructed for students to use as part of their explanations. Explanations and details of each item in OFQ are given below.

The first item in the OFQ evaluated the students' knowledge of conversion of an improper fraction to a mixed number and vice versa. The first part of item 1 addressed the conversion of a mixed number to an improper fraction and students were expected to show pictorial representation of this equation. The second part addressed the conversion of an improper fraction given in a pictorial representation, as shown in Figure 3.2, to a mixed fraction and an improper fraction. The first part was adapted from the study of Niemi (1994); the second part was adapted from the study of Mack (1993).

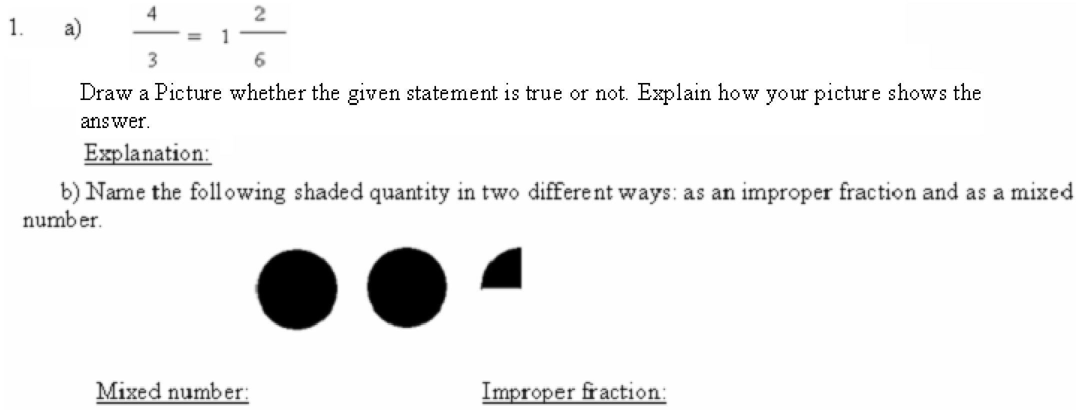


Figure 3.2 The first item of the OFQ

In item 2, students were required to locate $\frac{3}{4}$ on a number line as seen in Figure 3.2. Number line was used since it is suitable for representing the comparison of a fraction with a natural number. This item developed by the researcher.

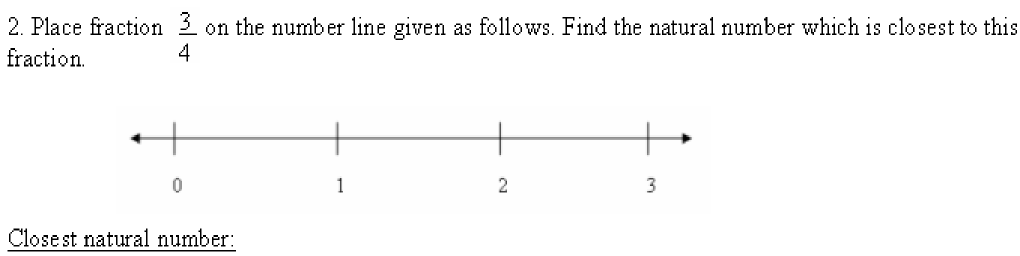


Figure 3.3 The second item of the OFQ

In item 3, students were asked to order five fractions while the denominator of each fraction is a multiple of a certain number. This item developed by the researcher. The first part evaluated the students' knowledge about ordering proper fractions from smallest to largest (Behr et al, 1984). In the second part, the ordering direction was reversed to largest to smallest; additionally, improper fractions and mixed numbers were included, as seen in Figure 3.4. In general, this item assessed their ability to order fractions represented by symbolic notation.

3. a) Please order fractions given below from smallest to largest. Explain your reasoning.

$$\frac{3}{4}, \frac{7}{8}, \frac{11}{12}, \frac{15}{16}, \frac{23}{24}$$

Your reasoning:

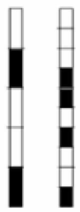
b) Please order fractions given below from largest to smallest. Explain your reasoning.

$$\frac{13}{16}, \frac{20}{16}, \frac{3}{4}, \frac{4}{2}, 1\frac{3}{16}$$

Figure 3.4 The third item of the OFQ

In item 4, certain area models were given and students were expected to interpret the equivalency of these models' shaded regions, as seen in Figure 3.5. In the first part, the required task was to interpret the equivalency of two fractions through their pictorial representations. The second part required showing the certain equivalency of two fractions by shading two blank areas respectively and comparing them to each other. This item examined whether participants had developed ideas of the equality of parts, the conservation of the relationship between these parts irrespective of the arrangement of the parts. The first part developed by the researcher. The second part was adapted from the study of Willoughby, Bereiter, Hilton and Rubinstein (1998).

4. a) Pictorial representations of two fractions given below. Are they equivalent to each other? Explain your reasoning.



Your reasoning:

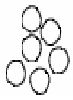
b) Sketch models using the figures below to show that $\frac{3}{4} = \frac{6}{8}$ is true. Explain your reasoning.

Your reasoning:

Figure 3.5 The fourth item of the OFQ

In item 5, students' ability to calculate a whole when some part of it is given as a proper fraction was examined. As shown in Figure 3.6, students were given a shape representing $\frac{3}{5}$ of the marbles and were required to calculate and draw the whole set of marbles. Item 5, which was adapted from the study of Lasher (2001), is as follows:

5. The figure given below represents 3/5 of marbles of Hakan. What is the total number of marbels of Hakan? Draw all of Hakan's marble. Explain how you solved the problem.



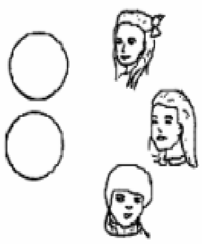
Total number of marbels :

Explanation:

Figure 3.6 The fifth item of the OFQ

Item 6 was related to the partitive interpretations of division (Behr et al., 1993; (Lamon, 1993). According to curricula, the important relation between fractions and division operation should be emphasized and partitioning a region into equal-size parts is fundamental to the fraction concepts so item 5 included finding "fair shares" of continuous quantity. In this item, the students were asked how much cake each person would get if three people were to share two cakes equally. The purpose of this item was to examine students' knowledge about the relation between fraction and division and accordingly to determine how successful respondents were with using fraction notation to represent "how much" a person would get. Item 6, which was adapted from the study of Yiu (1992), is as follows:

6. Nermin, Kaan and Gülsah want to share these two cakes. How much cakes each person has if there are three persons to share two cakes evenly? Represent amout of cake each person has pictorially and explain your resoning.



Your drawing:

Your reasoning:

Figure 3.7 The sixth item of the OFQ

The remaining items were associated with operations on fractions. These items examined students' procedural fluency in these operations, whereas some of them were related to conceptual understanding of fraction operations. Specifically, items 7 through 11 were used to evaluate students' abilities to add and subtract mixed numbers and proper fractions, and fractions with like and unlike denominators.

Item 7 was related to addition of proper fractions. The first part, which was asked in a multi-choice format, required addition of two fractions where one of their denominators could be equalized to the other one by enlarging. According to the objective that measured by this item, students should be able to add fractions which have same denominator or one of the denominators is multiple of the others. But the meetings that organized before the application of OFQ with the classroom teachers of the participants showed that teacherhas taught addition of fractions with finding least common denominator strategy. Thus, $\frac{1}{2}+\frac{1}{3}$ is added as one of the alternatives.

In the second part, students were asked to add fractions with common denominators and numerators, namely $\frac{34}{61}$ and $\frac{34}{61}$. The purpose of item 7 was to determine what kind of mistakes students were making while doing addition of fractions. The first part was adapted from the study of Niemi (1994); the second part was adapted from the study of Taube (1992). The seventh item is as follows:

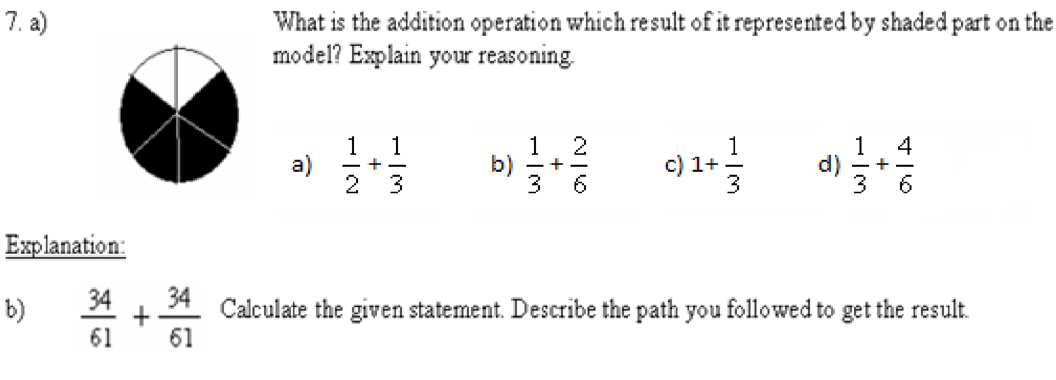


Figure 3.8 The seventh item of the OFQ

Item 8 involved addition of a fraction and a natural number. In the first part, the task was to simply apply the addition procedure. On the other hand, in the second item it was expected from the students to comprehend that mixed number is formed by addition of a proper fraction and a natural number, unlike multiplication of them. Students, who have only learned the shortcut algorithm for addition of a natural number and a proper fraction, may not know why addition of them results as a mixed number. The first part developed by the researcher. The second part of this item was adapted from the study of Lasher (2001). Item 8 is given below:

8. a) $2+\frac{2}{5}$ Calculate the given statement. Describe the path you followed to get the result.

b) Emrah said that $5\frac{1}{3}$ was the same as $5+\frac{1}{3}$ and Sinan said that $5\frac{1}{3}$ was the same as $5\times\frac{1}{3}$.

Who, if either, is right and why do you think so?

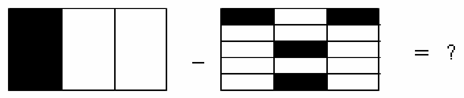
Figure 3.9 The eighth item of the OFQ

Items 9 and 10 included subtraction operations. Both two items developed by the researcher. In the first part of item 9, students were given two improper fractions as seen in Figure 3.10 and were expected to do a basic subtraction operation. In the second part, which was asked in a multi-choice format, again a subtraction operation was required; however, the second part could be solved via either transforming pictorial models into written format, namely, $\frac{a}{b}$ form to represent fractions, or making the operation directly on the figures, without any transformation. In both parts, it was expected to subtract a fraction that was large in numerical but small in value compared to the other fraction, from a fraction with opposite features. Because, it was presumed that students would switch the fractions and then subtract both the numerator and denominator from each other without finding lowest common denominator while doing subtraction operation.

Item 10 was similar to the second part of item 9 in most ways; however, item 10 additionally involved a natural number in the subtraction process. Items 9 and 10 are as follows:

9. a) Estimate the operation $\frac{5}{4} - \frac{14}{12}$ and explain your reasoning.

b)



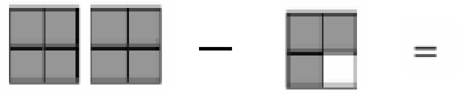
Each rectangle divided into equal parts in itself. Acording to this, what is the difference of these two models? Describe the path you follow in your solution.

a) $\frac{3}{15}$ b) $\frac{1}{3}$ c) $\frac{1}{15}$ d) 0 (zero)

Explanation:

Figure 3.10 The ninth item of the OFQ

10.



Name the operation given pictorially. What is the result of the operation?

Figure 3.11 The tenth item of the OFQ

In item 11, which is shown in Figure 3.12, the first part was composed of a word problem involving basic fraction concepts and simple fraction computations. The purpose of the first part was to determine the capability of an individual deciding upon the most useful operation to a given set of conditions (Lamon, 1999, p.42). The second part was opposite of the prior one, requiring creation of a word problem that described the given addition operation, and solving the problem by showing all of their work. Both two parts developed by the researcher. In general, the purpose of item 11 was to examine students' skills in both solving and creating problem about addition and subtraction of fractions. Item 11 is given below:

11. a) There is a jerry can which $\frac{2}{5}$ is filled. When 20 liters more water is filled to jerry can, it is filled of $\frac{4}{5}$. How many liters is the capacity of this jerry can?

b) Write a word problem for which $2\frac{2}{4} + \frac{3}{4}$ is a solution operation. Then, solve your problem showing all of their work.

Figure 3.12 The eleventh item of the OFQ

In item 12, which was the last item, the purpose was to examine students' ability to determine a part of another fraction. The first part was a word problem that required calculating one third of a half. The second part involved multiplication of two fractions that are shown in Figure 3.12. Both two parts developed by the researcher. It was presumed that students would apply shortcut algorithms without conceptual understanding of multiplication of fractions. Item 12 is as follows:

12. a) There is a half cake and you eat one third of it. How much of the cake are you eat? Represent your solution pictorially and explain your reasoning.

b) Estimate the operation $\frac{2}{3} \times \frac{5}{2}$ and explain your reasoning.

3.13 The twelfth item of the OFQ

A total of 12 items were asked in the OFQ that was followed by complementary interview study, which is explained in the following heading.

3.3.2. Interview Protocol

One of the purposes of this study was to investigate underlying reasons and misconceptions behind elementary fifth grade students' mistakes and difficulties. In order to reach this purpose, sixteen participants were selected for interviews that provided a better understanding of students' approaches to the OFQ items, thus, creating a chance to make in-depth exploration on mistakes and their reasons.

In this phase, sixteen of the students were interviewed by the researcher individually without any time limits, and in their own classrooms in order to provide a comfortable environment. They were asked to clarify their written answers; explain their strategies behind their solutions if needed; give explanations for the items they had left blank or their answers that were unreadable. An important benefit of the interviews was that they may have stimulated thought processes of students that actually existed in their minds but were not effective while answering the question in isolation; also, they helped to describe how students perceived fractions.

3.4. Pilot Study

The purposes of the pilot study were to get feedback about directions and statements of OFQ items; to determine the average testing time, the difficulty level for each item, the possible troubles of actual administration of the questionnaire; and to check the validity and reliability of OFQ.

In autumn 2010, the pilot study was conducted by the researcher in the neighbor elementary school at Odunpazarı district, Eskişehir. Sixty one fifth grade elementary school students who were similar to the main respondents in the aspects of demographics that are related to the study, were administered a fifteen item test (twenty-three sub items) (see appendix A). In the pilot study students were expected to complete the given questionnaire in fifty minutes however they had some difficulties; at the end of fifty minutes, they either returned the papers with many blank answers or requested more time. Due to this time issue, the common decision made with the mathematic educators who were mentioned in the beginning of this heading, was to remove some of the items (or sub items) evaluating the same objectives. Thus, number of items was reduced from fifteen to twelve and number of sub items was decreased from twenty three to twenty.

After the pilot study and these revisions, and a last evaluation with the mathematic educators, the final version of the questionnaire was obtained and OFQ was administered to 151 fifth grade students.

3.5. Reliability and Validity Issues

Validity is the appropriateness, meaningfulness and usefulness of the inferences researchers make based on the data they collect (Fraenkel & Wallen, 2006). Before the pilot study, OFQ was controlled by three experts who could be expected to render an intelligent judgment about the adequacy of the instrument. A mathematics educator in Elementary Mathematics Education Department, a primary school teacher and an elementary mathematics teacher check the questionnaire according to the table of specification to see whether the items and objectives supported each other; evaluated the compatibility of the items with the related target objectives; checked the clarity of the problem statements and illustrations; evaluated the items in the terms of difficulty levels; and checked appropriateness of the items' language for fifth grade level students. A separate discussion was made with the mathematics educator in Elementary Mathematics Education Department to judge whether the items could detect sources of students' mistakes and difficulties. Table 3.2 presents the table of specification of OFQ items.

Table 3.2 Table of specification for objectives and questionnaire items

Number of Objective	Time Devoted in Class	Objectives	Number of item	Item Placement	Percentage
1	2 (80 minutes)	Students are able to transform an improper fraction into a mixed number and vice-versa	1	1.a 1.b	10%
2	1 (40 minutes)	Students are able to compare a fraction and a natural number.	2	2	5%
3	2 (80 minutes)	Students are able to order at most five fractions while the denominator of each fraction is a multiple of a certain number.	3	3.a 3.b	10%
4	2 (80 minutes)	Students are able to draw and write equivalent fractions.	4	4.a 4.b	5%
5	1 (40 minutes)	Students are able to determine the whole which some part is given as proper fraction.	5	5	10%
6	1 (40 minutes)	Students are able to explain relation between fraction and division.	6	6	5%
7	2 (80 minutes)	Students are able to add fractions which have same denominator or one of the denominators is multiple of the others.	7	7.a 7.b	10%
8	2 (80 minutes)	Students are able to add a fraction with a natural number.	8	8.a 8.b	10%
9	2 (80 minutes)	Students are able to subtract fractions having same denominator or one of the denominator is multiple of the others.	9	9.a 9.b	10%
10	1 (40 minutes)	Students are able to subtract a fraction from a natural number.	10	10	5%
11	2 (80 minutes)	Students are able to solve and create problems about addition and subtraction of fractions.	11	11.a 11.b	10%
12	2 (80 minutes)	Students are able to determine a fraction of another fraction.	12	12.a 12.b	10%

The other concern was reliability that was defined as the degree to which scores obtained with an instrument are consistent measures of whatever the instrument measures (Fraenkel & Wallen, 2006), and it was an important property in educational measurement (Colton, Gao, Harris, Kolen, Martinovich-Barhite, Wang, & Welch, 1997). Many evaluation methods require raters to judge students’ behavior (Stemler, 2004), and Johnson, Penny, and Gordon (2000) challenge those who design and implement appraisement to effort to achieve high levels of inter-rater reliability. The more consistent the scores are between different raters and occasions, the more reliable the assessment is thought to be (Moskal & Leydens, 2000). Since the rubric, which was controlled by the mathematic educator was used in this study to score students' mistakes and difficulties related to the fractions, inter-rater reliability measures were worth investigating. Thus, the responses of 151 students were evaluated with a second rater and scoring agreement method was used in order to find an inter rater reliability. There was a %98 correlation between the scores of students.

3.6. Data Collection Procedure

The purpose of this study was to investigate mistakes made by elementary fifth grade students regarding basic fractional concept and operations, and difficulties that they encounter. The other purpose was to investigate underlying reasons and misconceptions behind those mistakes and difficulties. In data gathering process, the official permissions were taken up from the Middle East Technical University Human Subjects Ethics Committee. After this permission, the researcher asked for permissions from the head of the two elementary schools. The purpose and the procedure of the study were explained to four classroom teachers from one of the schools and two classroom teachers from the other school elementary classroom teachers and administrators. Towards the end of the spring semester of 2010, in May, after the necessary permissions were taken and OFQ was finalized, 151 elementary fifth grade students were administered the questionnaire. OFQ were particularly implemented and data gathered in this time period as teachers finished to cover all of the mathematics lessons about fractions. In total, 50 minutes was given to the students for completing the test. The questionnaire was administered in participants' own classrooms, which were similar in terms of equipment. In phase two, the interviews, which were mentioned in detail in Interview Protocol heading, were conducted and audio-recorded. Students individually interviewed in their own classrooms in order to provide a comfortable environment. A schedule indicating the order of data collection is given in the Table 3.3.

Table 3.3 Time schedule for data collection

Date	*Events*
February-April 2010	Development of measuring tool
April 2010	Pilot study-last revision of measuring tool
May- June 2010	Implementation of the instrument
May- June 2010	Conducting Interviews
June- October 2010	Analysis of the data

3.7. Analysis of Data

In the first phase of the study, in order to answer the research questions, item based analysis was conducted. More specifically, the rubric developed by the researcher to rate the variety of different types of mistakes that participants make and the answers of the participants were evaluated quantitatively with respect to their accuracy using this rubric. The twelve items of OFQ were scored using a five level scoring system (0 to 4 points).The rubric given below was adapted for each item in the OFQ by considering the responses. Each item measured different objective and because of this a single rubric could not be applied to all items. But in general, for methods that led to correct solutions and appropriate explanations of reasoning, four points were awarded; for responses containing minor mistakes, three points were awarded; for correct responses without explanation of reasoning, two points were awarded; for responses containing mistakes with incorrect explanations, one point was awarded; for items that were left blank or indicated no mathematical understanding, no points were awarded. When there was more than one mistake in a single answer, the major one was chosen. As a result, the answers of items were analyzed with respect to rubric and categorized by type and frequency. Table 3.4 shows the details of the rubric.

Table 3.4 Open-Ended Questions Scoring Rubric

Scores	Answer Types
0	• Blank • Completely irrelevant • No mathematical understanding
1	• Answer with unclear explanation or without explanation • Partial understanding without explanation • Minimal understanding of the task • Misunderstanding in the question and correct answer through that misunderstanding • Partially correct answer and/or drawings without explanation
2	• Correct answer and/or drawing without explanation
3	• Applied correct procedure with operational error • Correct answer and/or drawing with partial knowledge of mathematical concept • Insufficient and lacking in some minor way's of answer or explanation
4	• Correct answer with clear and understandable explanation of reasoning • A response demonstrating full and complete understanding

The second phase of the analysis dealt with misconceptions and reasons behind those mistakes and difficulties. In phase two, data (papers, audiotapes) obtained from sixteen interviews was read and transcribed. Namely, audiotapes of interviews were listened and papers of these sixteen students were checked over simultaneously and the answers were categorized. Mistakes that were revealed from the responses of the sixteen students, who participated in the interviews, were categorized under five headings for the better in-depth analysis of the underlying reasons and misconceptions behind students' mistakes and dificulties. All of the answers were separated into categories. More specifically, mistake classification system developed by Tirosh (2000) was adapted as a basis for this data analysis. She refers to the work of other researchers (Ashlock, 1990; Fischbein, Deri, Nello, & Marino, 1985; Graeber, Tirosh, & Glover, 1989; Tirosh, Fischbein, Graeber, & Wilson, 1993) who have studied elements of each of these categories. She classified student mistakes according to the following three categories: Algorithmically based mistakes, intuitively based mistakes and mistakes based on formal knowledge of fractions. A forth category, namely "misunderstanding of the problem", was adapted from the study of Işıksal (2006). The fifth category, which didn't have a good match in the literature, originated from the data, was "missing information (correct but not complete)", which occurs when a learner presents some of the information required by the task but not all of it. In final, a total of five categories were determined, and was checked by a second coder. In accordance with the purpose, in this section, the researcher described possible underlying reasons and misconceptions behind mistakes students made and difficulties students had while learning fractions.

3.8. Assumptions and Limitations

In this section, the main assumptions and limitations about the study were discussed. First of all, it was assumed that there were no differences among students regarding age, intelligence, belief, and socioeconomic background. Also it was assumed that all the students answered all the questions in OFQ and interview honestly, without cheating, seriously, intimately and carefully.

In this study, the sampling procedure could be the limitation of the study. Participants were selected by applying non-random sampling method. Besides, the sample includes only fifth grade public school students in Eskişehir. Therefore, the generalizability of the results of this study on a larger population would be limited.

3.9. Internal and External Validity of the Study

Both internal validity threats and external validity for this study were discussed in the last part of the methodology chapter.

3.9.1. Internal Validity

Internal validity of the study refers to the validity of the study refers to the degree to which observed differences on dependent variable affected by the independent variable directly (Fraenkel & Wallen, 2006). Namely, internal means that observed results are not related to dependent variable itself, but related to some unintended variables. There are three main threats to internal validity in survey research: location, mortality (loss of subject), and instrumentation. Thus, these threats were discussed.

Location: Fraenkel and Wallen (2006) stated that location threat is the possibility that results due to characteristics of the setting or location in which a study is conducted, thereby producing a threat to internal validity. A location threat can occur if the data collection is carried out in a place that may affect responses (Fraenkel & Wallen, 2006). In this study, the questionnaire was administered in participants' own classrooms, which were similar in terms of equipment. Therefore, location was not a possible threat for this study.

Mortality: Mortality threat is the possibility that results are due to the fact that subjects who are for whatever reason lost to a study may differ from those who remain so that their absence has an important effect on the results of the study (Fraenkel and Wallen, 2006). In this study, the questionnaire was administered to all participants at one time; hence, mortality apparently did not occur.

Instrumentation: Instrumentation could create a problem if the nature of the instrument or scoring procedure was changed in some way (Fraenkel & Wallen, 2006). Possible instrumentation threats include instrument decay, data collector characteristics and data collector bias.

In order to control instrumentation threats, standardizing the conditions under which the study occurs was required. In our case, measurement method kept constant; it was made certain that all measuring equipment was functioning properly; questionnaire was administered in the same format and under the same conditions. In this way, effect of instrumentation threat was minimized.

Problems about nature of instrument, scoring procedure and changes in instrumentation over time are referred as instrument decay (Fraenkel & Wallen, 2006). This usually occurs when instrument leads to different interpretations of results and researcher's standards or process of measurement changes. In order to control instrument decay, the basic method is scheduling data collection or scoring, in order to minimize the changes. The researcher scheduled both data collection and scoring period. In this study, since the questionnaire included open-ended items, two teachers scored the papers using the same rubric and they agreed on scoring. Thus instrumentation decay was minimized and taken under control. In addition, the second rater also scored the data in order to minimize the changes in analysis procedure.

Furthermore, since the questionnaire administered by only the researcher, the data collector characteristics were the same during the all administrations. In order to control data collector bias threat, the researcher did not communicate with students and there were no interaction between the researcher and students during the administration of the questionnaire. Thus, data collector bias was also not a threat for this study.

3.9.2. External Validity

The external validity 'is concerned with the extent to which the findings of one study can be applied to other situations' (Merriam, 1998). The degree how much research sample represents the population of interest is called population generalizability (Fraenkel & Wallen, 2006). Since the researcher was a primary teacher in a public school, she carried out this study in the same school. Convenience sampling method was used in order to select the sample. Fraenkel and Wallen (2006) stated that a recommended sample size is 100 for a descriptive study. Thus, owing to the sample size, representativeness of this sample is sufficient. Since the sampling method was the convenience sampling method, the generalizability of the study on a population was not very easy. However, according to Fraenkel and Wallen (2006) the results of a study can be generalized to conditions or settings other than those that prevailed in a particular study. Therefore, the results of this study could be generalized to the fifth grade elementary students under the same conditions with the participants of this study.

CHAPTER IV

FINDINGS

In this chapter, findings are organized under two parts based on purposes of the study. The first purpose of the study was to investigate mistakes made by elementary fifth grade students regarding basic fractional concept and operations and difficulties that they encounter. Therefore, the first part of the analysis dealt with the students' mistakes and difficulties based on individual items and frequency of these mistakes and difficulties they made attempting to answer OFQ are introduced based on individual questions.

The second phase of the analysis dealt with misconceptions and reasons behind those mistakes and difficulties. Those mistakes and difficulties that were revealed from the responses of the sixteen students, who participated in the interviews, were categorized under five headings for the better in-depth analysis of the underlying reasons and misconceptions behind students' mistakes and difficulties. These categories were: Algorithmically based mistakes/difficulties, intuitively based mistakes/difficulties, mistakes/difficulties based on formal knowledge of fractions, mistakes/difficulties due to the misunderstanding of the problem, mistakes / difficulties due to the missing information.

4.1. Mistakes and Difficulties Encountered in Students' Responses

In this phase of the analysis dealt with the students' mistakes and difficulties, item based analysis was conducted. Moreover, frequencies of these mistakes the participants made attempting to answer OFQ are introduced based on individual questions.

4.1.1. Item 1

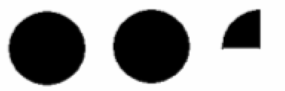
Both item 1-a and 1-b addressed the conversion of a mixed number to an improper fraction and vice versa related to pictorial representation of fractions.

1. a) $\frac{4}{3} = 1\frac{2}{6}$

Explain by drawing the given statement above is correct or not.

Explanation:

b) Name the following shaded quantity in two different ways: as an improper fraction and as a mixed number.



Mixed number: Improper fraction:

Figure 4.1 Item 1

Analysis of data revealed that students' mistakes could be grouped under five categories which were shown in Table 4.1 along with their frequencies. In particular, responses that were not relevant to fractions as they indicated no mathematical understanding or were left totally blank, grouped under the first category; responses that stated the expression was 'correct' or 'incorrect' but were not significant because of meaningless drawing and explanations, grouped under the second category; responses in which students correctly identified the equivalency of fractions but without acceptable pictorial representations, grouped under the third category; and responses in which students correctly identified equivalency of fractions but demonstrated limited mathematical knowledge about the subject, grouped under the forth category. In Table 4.1 number of students according to classification is given:

Table 4.1 Frequency of classification of item 1-a

Item 1-a: Converting an improper fraction to a mixed number.	
	Frequency
Had no mathematical understanding / left blank	27 (17.9 %)
Stated 'correct' or 'incorrect' with unreasonable pictorial representation and explanation	49 (32.4 %)
Stated 'correct' with unacceptable pictorial representation	38 (25.1 %)
Stated 'correct' but had limited mathematical knowledge	9 (6.0 %)
Stated 'correct', draw appropriate picture, made acceptable explanation	28 (18.5 %)
TOTAL	151

More specifically, analysis of the data revealed that twenty seven (17.9%) of the 151 students indicated no mathematical understanding or they did not answer to the question. Namely, they failed to demonstrate any cognitive evidence of converting an improper fraction to a mixed fraction. Additionally, forty nine (32.4 %) students asserted that the statement were 'correct' or 'incorrect' with unreasonable pictorial representation and explanation where twenty nine (19.2%) of them drew pictorial representations of fractions correctly but they stated that the equation is incorrect. For instance Participant 124 drew pictures which represent $\frac{4}{3}$ and $1\frac{2}{6}$ nevertheless she stated that the given expression was incorrect.

Participant 124:

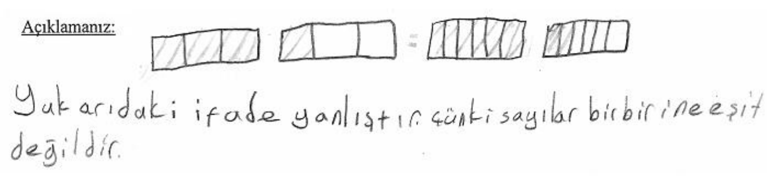


Figure 4.2 Answer of Participant 124

Twenty (13.2%) of those forty nine participants drew incorrect pictorial representations of fractions and stated that the expression is incorrect such as Participant 3 did as follows:

Participant 3:

Figure 4.3 Answer of Participant 3

According to thirty eight (25.1%) of respondents the statement was 'true' but they did not give an acceptable reason behind this answer. These students, for instance Participant 105 illustrated inappropriate pictorial model and his explanation was missing.

Participant 105:

Figure 4.4 Answer of Participant 105

Only twenty eight (18.5%) of students were able to justify that the equation was 'true', and drew appropriate picture with clear and understandable explanation of equivalent fractions.

In item 1-b, the students were given a figure and expected to write the given amount in improper fraction and mixed number. However, students applied various incorrect forms of the conversion algorithm and figure translation. In particular, responses that were not relevant to fractions as they indicated no mathematical understanding or were left totally blank, grouped under the first category; responses in which students correctly identified only one of two fractions without giving any reasoning, grouped under the second category; responses in which students correctly identified mixed number and converted it to improper fraction without explaining their reasoning, grouped under the third category; responses in which students correctly identified mixed number and converted it to improper fraction but demonstrated limited mathematical knowledge about the subject, grouped under the forth category. The results are given below:

Table 4.2 Frequency of classification of item 1-b

Item 1-b: Converting a mixed number to an improper fraction	
	Frequency
Had no mathematical understanding / left blank	32 (21.2%)
Correctly represented mixed number or improper fraction but without acceptable explanation.	29 (19.2%)
Correctly converted mixed number to improper fraction but without acceptable explanation.	24 (15.9%)
Correctly converted mixed number to improper fraction but had limited mathematical knowledge.	23 (15.2%)
Correctly converted mixed number to improper fraction, made acceptable explanation.	43 (28.5%)
TOTAL	151

As it seen above, many students were able to represent the figure neither as a mixed number nor as an improper and they were not able to transform them to each other. Thirty two (21.2%) of the students were able to write neither improper fraction represented by figure nor its mixed number equivalent. For instance, Participant 65's solution is as follows:

Participant 65:

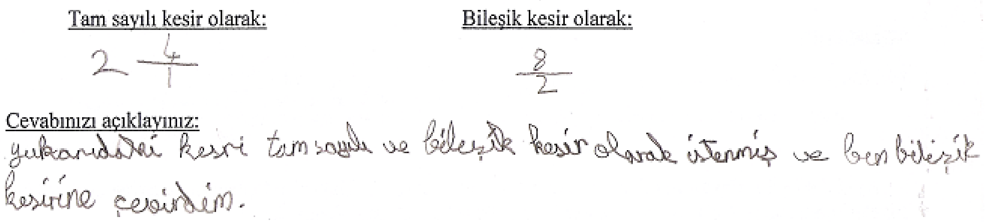


Figure 4.5 Answer of Participant 65

As shown in the figure, he did not find the mixed number correctly which represented by figure. Also he defined $\frac{8}{2}$ as an improper fraction because he multiplied whole part with numerator and the result was new numerator.

To give another example, Participant 47 was not able to represent the picture symbolically. When the answer is examined, it is seen that he knew the denominator should be number of equal parts in a whole but he divided the first whole into ten equal parts and then defined the denominator as 10.

Participant 47:

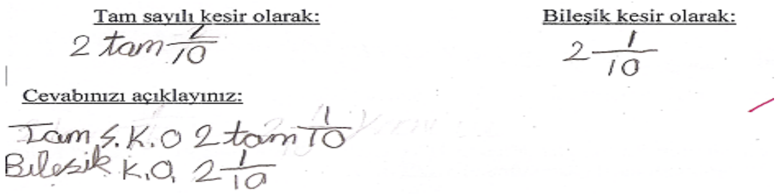


Figure 4.6 Answer of Participant 47

Additionally, seventeen (11.2 %) of thirty two students who wrote neither mixed number nor improper fraction, stated that they did not know the answer. Solutions of fifteen (9.9 %) students demonstrated a complete lack of understanding like Participant 48.

Participant 48:

Figure 4.7 Answer of Participant 48

Twenty nine (19 %) of participants did not correctly convert mixed number to improper fraction or vice versa. They applied different incorrect forms of the conversion algorithm. For instance, Participant 85 demonstrated her knowledge of using algorithmic procedures and formal notational processing to find mixed numbers and convert mixed numbers into improper fractions as 2+1= 3 and 3x4=12 (denominator) and 2x4=8 and 8+1=9 (numerator).

Participant 85:

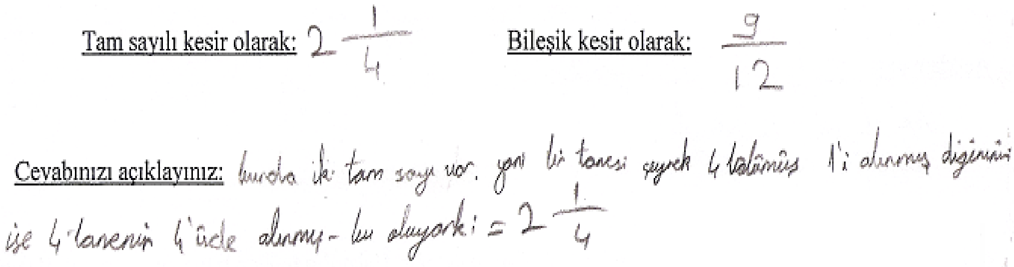


Figure 4.8 Answer of Participant 85

Twenty four (15.9%) respondents converted mixed fraction to improper fraction without explanation. Sixty six (43.7%) of the 151 students completed the conversion of the mixed number to an improper fraction task correctly and fourth three (28.5%) out of sixty six (43.7%) gave clear and correct explanation as shown in Table 4.2. Twenty three (15.2%) students correctly converted mixed number to improper fraction but their responds demonstrated limited mathematical knowledge which was considered ‘acceptable’, meaning that it was correct. For example Participant 42 expressed his explanations as follows:

Participant 42:

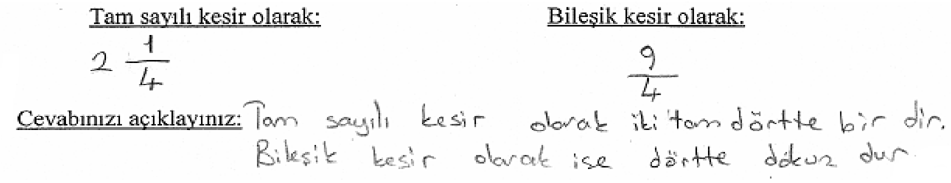


Figure 4.9 Answer of Participant 42

4.1.2. Item 2

In this question students need to explicate the fraction symbol, recognize the properties of the number-line model, and how fractional quantities are represented.

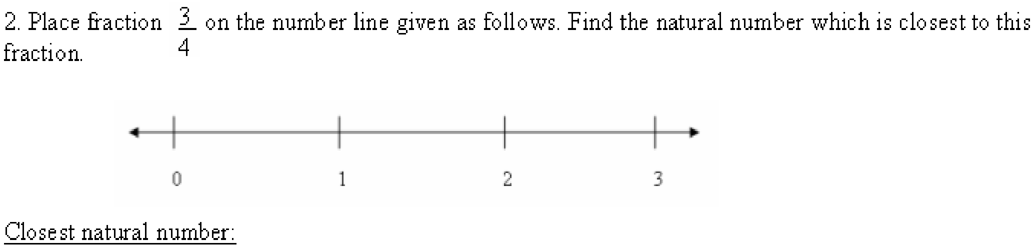


Figure 4.10 Item 2

The Table 4.3 represents the categories and frequency of each category. In particular, responses that were not relevant to fractions as they indicated no mathematical understanding or were left totally blank, grouped under the first category; responses in which students did not demonstrate enough knowledge about number line concept, namely, did not know what number line means and how to use it, grouped under the second category; responses in which students located the fraction between two incorrect natural numbers, thus , determined closest natural number incorrectly, grouped under the third category; responses in which students located the fraction between correct natural numbers, but demonstrated limited mathematical knowledge about the subject, grouped under the forth category. In Table 4.3, number of students according to classification is given:

Table 4.3 Frequency of classification of item 2

Item 2: Location of a proper fraction on the number line	
	Frequency
Had no mathematical understanding / left blank	21 (13.9%)
Had no knowledge of mathematical concept about number line	17 (11.2 %)
Located the fraction and determined closest natural number incorrectly	55 (36.4 %)
Located the fraction correctly but determined the closest natural number incorrectly	8 (5.2 %)
Located the fraction correctly and determined the closest natural number correctly	50 (33.1 %)
TOTAL	151

When the results are examined, it is seen that, of the ninety three (61,5%) students who were unsuccessful at locating $\frac{3}{4}$ on the given number line, twenty one(13.9 %) of them represented no mathematical understanding (15- 9.9%) or left the question blank (6-3.9%). As it seen above in Table 4.3, fifty five (36.4 %) students incorrectly located fraction $\frac{3}{4}$ on the number line. Results also revealed that twenty two (14.5 %) of these fifty five students located $\frac{3}{4}$ between another whole numbers instead of 0 and 1, such as between 2 and 3 or next to 3. For example Participant 48 located $\frac{3}{4}$ between 2 and 3 on the number line.

Participant 48:

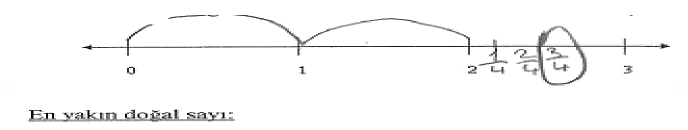


Figure 4.11 Answer of Participant 48

In addition, eight (5.3 %) of them did not consider the size of the whole, they placed division lines under whole numbers on the number line and wrote 4 to all denominators then put a mark on 3 as $\frac{3}{4}$ like Participant 128 did. For instance, Participant 128 ignored size of the unit, instead using entire segment as a whole. The answer is as follows:

Participant 128:

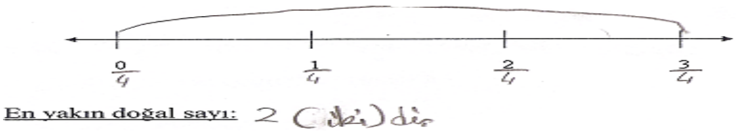


Figure 4.12 Answer of Participant 128

Six (4 %) of the students constructed four sets and three dots between 0 and 1 but unequally spaced, then indicated that the incorrect dot represented the location of three fourths. As it seen below, Participant 58 is one of them. She ignored that spaces between numbers (wholes) should be divided into equal parts on the number line.

Participant 58:

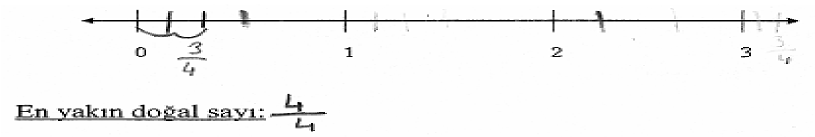


Figure 4.13 Answer of Participant 58

Thirteen (8.6 %) of these students divided the interval between 0 and 1 equally however they placed four dots and formed five sets. Namely, they marked $\frac{4}{4}$ before 1 or $\frac{0}{4}$ after 0. For instance, Participant 28 formed five sets and located $\frac{4}{4}$ before 1 and she did not determine closest natural number correctly. She expressed her result as follows:

Participant 28:

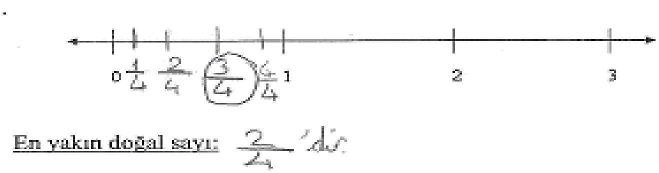


Figure 4.14 Answer of Participant 28

In item 2, three (2 %) participant formed four sets of three dots between 0 and 1 but they located fractions in the form of $\frac{3}{1}, \frac{3}{2}, \frac{3}{3}$ and $\frac{3}{4}$. Participant 23 is a good example of this case.

Participant 23:

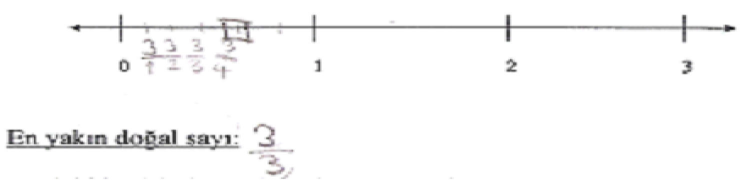


Figure 4.15 Answer of Participant 23

The other three (2 %) participant confused number line with measure of length and divided it into ten equal parts. For example, Participant 51 divided the intervals between natural numbers into ten sets and located fractions as $\frac{4}{1}, \frac{4}{2}, \frac{4}{3}$ between 0 and 1.

Participant 51:

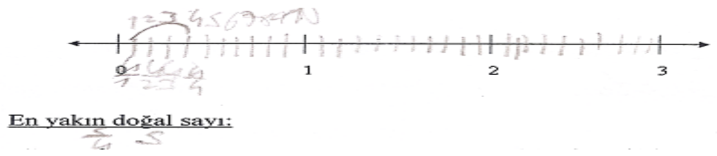


Figure 4.16 Answer of Participant 51

As a result, seventeen (11.2 %) students had no knowledge of number line and fifty five students located the fraction to the number line incorrectly. In total, seventy two (47.6 %) of the 151 respondents represented seven different incorrect strategies to locate the number $\frac{3}{4}$.

4.1.3. Item 3

Item 3 evaluated the students' knowledge about ordering fractions from smallest to largest and vice versa (Behr et al, 1984).

3. a) Please order fractions given below from smallest to largest. Explain your reasoning.

$$\frac{3}{4}, \frac{7}{8}, \frac{11}{12}, \frac{15}{16}, \frac{23}{24}$$

Your reasoning:

b) Please order fractions given below from largest to smallest. Explain your reasoning.

$$\frac{13}{16}, \frac{20}{16}, \frac{3}{4}, \frac{4}{2}, 1\frac{3}{16}$$

Figure 4.17 Item 3

Table 4.4 and Table 4.5 represent the categories and frequency of each category. In particular, responses that were not relevant to fractions as they indicated no mathematical understanding or were left totally blank, grouped under the first category; responses in which students ordered the fractions incorrectly, grouped under the second category; responses in which students ordered the fractions correctly but did not give any reason behind these arrangement, grouped under the third category, responses in which students ordered the fractions correctly but demonstrated limited mathematical knowledge about the subject, grouped under the forth category. Results were given below:

Table 4.4. Frequency of classification of item 3-a

Item 3-a: Ordering at most five fractions having one of the denominators is a multiple of others	
	Frequency
Had no mathematical understanding / left blank	16 (10.5 %)
Ordered the fractions incorrectly	69 (45.6 %)
Ordered the fractions correctly but without explanation or with inappropriate explanation	44 (29.1 %)
Ordered the fractions correctly but had limited mathematical knowledge	7 (4.6 %)
Ordered the fractions correctly with clear and understandable explanation	15 (10.0 %)
TOTAL	151

In this study, results revealed that a large portion (69 of the 151 participants) of students made mistakes on ordering fractions having one of the denominators are multiple of the others. Out of sixty nine (45.6 %) students who ordered the fractions incorrectly, forty eight (31.7 %) of them placed them in reverse order and stated that ‘fraction which has larger denominator is smaller than others’. Participant 147 is the one of the good examples of this situation.

Participant 147:

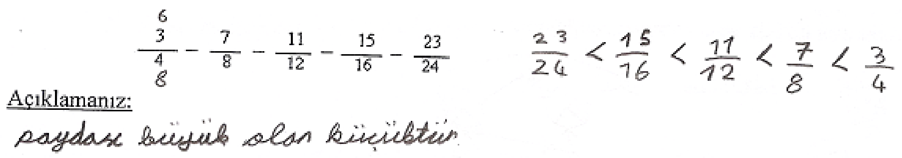


Figure 4.18 Answer of Participant 147

Furthermore, eleven (7.2 %) students reversed the order and stated that ‘fraction which has larger numerator is smaller than others’. For instance according to Participant 144, the largest fraction was $\frac{3}{4}$ and the smallest one was $\frac{23}{24}$ because he thought the fraction with largest numerator was the smallest fraction. He provided the following reverse order and explanation.

Participant 144:

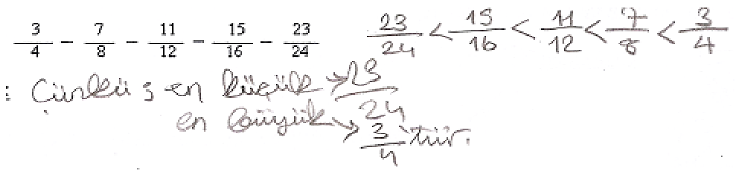


Figure 4.19 Answer of Participant 144

Five (3.31 %) of these sixty nine students who ordered fraction incorrectly, compared sizes of the equal pieces. They stated that ‘a fraction which is dividing into more pieces than other fractions are smaller’ like Participant 143 whose solution is given as follows or they stated ‘fraction which has larger numerator is smaller than others’.

Participant 143:

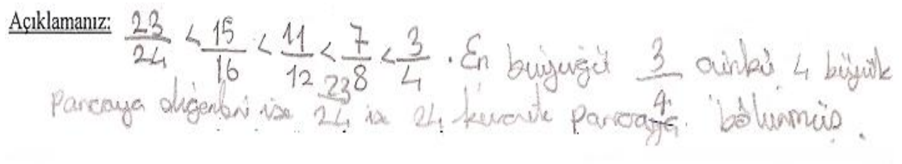


Figure 4.20 Answer of Participant 143

Twenty two (14.5 %) of forty four (29.1 %) students, who gave correct answer without explanation or gave answer with inappropriate explanation, did not explain their reasoning. Explanations of thirteen (8.6 %) out of forty four students were insufficient. Moreover, nine (5.9 %) of these forty four students gave correct answer but demonstrated incorrect reasoning. They stated that 'fraction which has larger numerator is larger than others'. As it seen below, Participant 56 was one of the students who made this mistake. She ordered correctly but her reason behind this process was invalid because she claimed that $\frac{23}{24}$ was the largest fraction because in $\frac{23}{24}$, the number of people who eat more than others.

Participant 56:

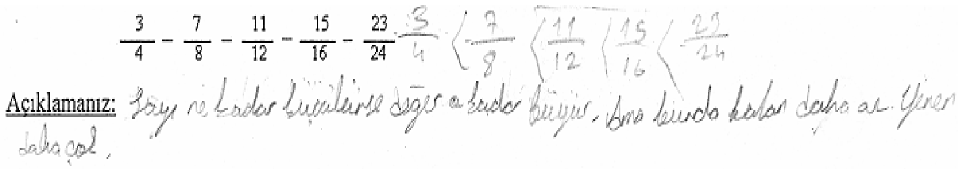


Figure 4.21 Answer of Participant 56

Additionally, five (3.31 %) of these nine students who gave correct answer but demonstrated incorrect reasoning, ordered fractions in this item according to the values of the whole numbers forming fractions. The following response of the Participant 81 exemplified the ways of students in this study. He ordered the fractions looking at the actual values of the numbers.

Participant 81:

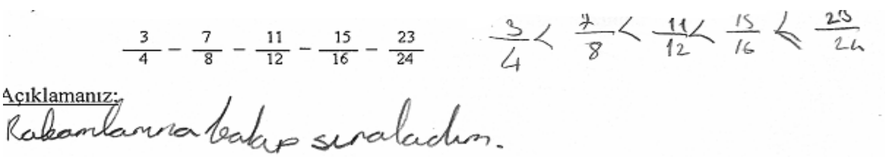


Figure 4.22 Answer of Participant 81

Some of them initially gave an incorrect answer but subsequently corrected themselves while they were explaining their reasoning on the task in interviews. They were deemed to have completed the task successfully. Only fifteen (10 %) of the respondents used a least common denominator and correctly ordered fractions with clear and understandable explanation but no one realized that both are only one ‘piece’ away from a whole.

In item 3-b, students were asked to order fractions $\frac{13}{16}, \frac{20}{16}, \frac{3}{4}, \frac{4}{2}, 1\frac{3}{16}$ from largest to smallest. They were expected to order fractions which were given as improper fractions, proper fractions and mixed numbers. In Table 4.5, number of students according to classification is given.

Table 4.5 Frequency of classification of item 3-b

Item 3-b: Ordering at most five mixed numbers and improper fractions having one of the denominator is multiple of others	
	Frequency
Had no mathematical understanding / left blank	13 (8.6 %)
Ordered the fractions incorrectly without explanation	113 (74.8 %)
Ordered the fractions correctly but without explanation or inappropriate explanation	4 (2.6 %)
Ordered the fractions correctly but had limited mathematical knowledge	9 (6.0 %)
Ordered the fractions correctly with clear and understandable explanation	12 (7.9 %)
TOTAL	151

One hundred and thirteen (74.8 %) students incorrectly ordered proper fractions, improper fractions and mixed numbers and they did not explain their reasoning. None of the students used benchmark numbers strategy like 0, $\frac{1}{2}$ and 1 to compare these fractions. Both similarities and differences were observed in the fifth grade students' standpoints to questions of order and equivalence. Data revealed that one of the two common explanations for this item was considering the fractions smaller or larger depending solely on numerators or denominators. Twenty seven (17.8 %) students argued for this idea like Participant 43 did.

Participant 43:

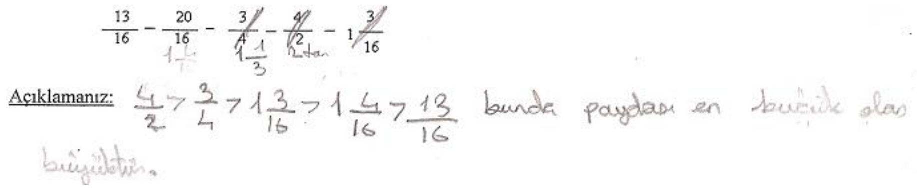


Figure 4.23 Answer of Participant 43

A second common idea for this item, which was supported by fifty eight (38.4 %) students, was considering the mixed number as the largest fraction among all the others because it had a whole. Participant 144 is one of the students who justified this idea by stating that $1\frac{3}{16}$ is the largest fraction but it is seen that he correctly ordered the rest of the fractions.

Participant 144:

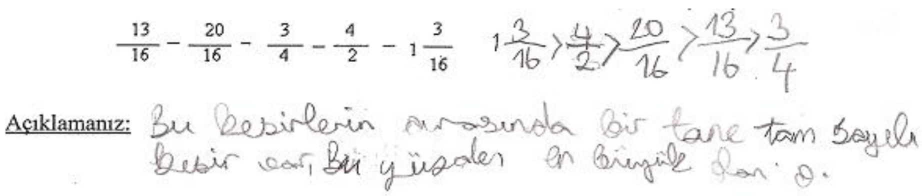


Figure 4.24 Answer of Participant 144

Since the numerator and denominator of $\frac{20}{16}$ are the largest natural numbers in this group of fractions, ten (6.6 %) students stated that the largest fraction was $\frac{20}{16}$. It is clearly seen that, Participant 44 also ordered fractions according to the size of natural numbers which had formed the fractions.

Participant 44:

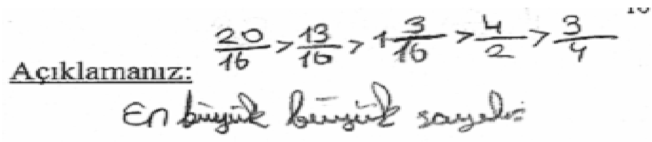


Figure 4.25 Answer of Participant 44

Ten (6.6 %) respondents tried to equalize denominators but only the denominators were multiplied, numerators were written exactly the same. These ten students could not perceive that multiplying both numerator and denominator with the same number doesn't change a fraction's value. Participant 72 was one of the students who made this mistake. She tried to find a common denominator but she did not multiply the numerators with the required natural numbers.

Participant 72:

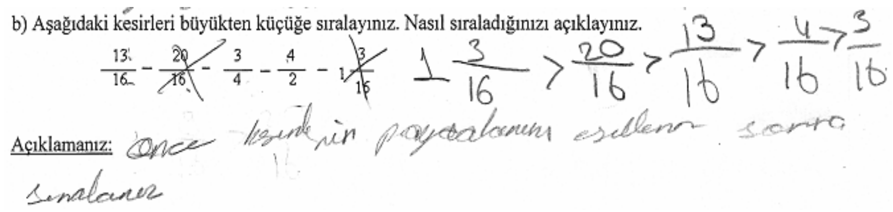


Figure 4.26 Answer of Participant 72

Eventually nine students ordered fractions correctly with limited fraction knowledge and twelve students ordered fractions correctly with clear and understandable explanation. In total, twenty one (13.9 %) of the fifth grade students were able to put the fractions in correct order from smallest to largest, which were given in the item 3-b.

4.1.4. Item 4

In items 4-a and 4-b, students were expected to decide the equivalency of shaded regions.

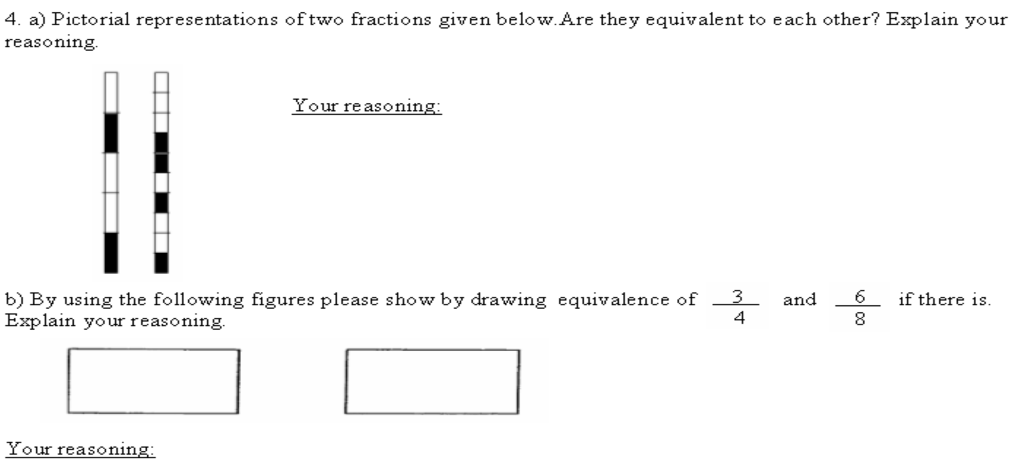


Figure 4.27 Item 4

The Table 4.6 represents the categories and frequencies of each category. In particular, responses that were not relevant to fractions as they indicated no mathematical understanding or were left totally blank, grouped under the first category; responses in which students did not determine the equivalence of fractions $\frac{2}{5}$ and $\frac{4}{10}$ and gave missing or unclear explanation of reasoning, grouped under the second category; responses in which students justified the equivalence of fractions $\frac{2}{5}$ and $\frac{4}{10}$ but did not give any reason behind these process, grouped under the third category; responses in which students justified the equivalence of fractions $\frac{2}{5}$ and $\frac{4}{10}$ but demonstrated limited mathematical knowledge about the subject, grouped under the forth category.

Table 4.6 Frequency of classification of item 4-a

Item 4-a: Picturizing and writing equivalent fractions	
	Frequency
Had no mathematical understanding / left blank	33 (21.9 %)
Could not justify equivalence of fractions and gave no explanation or inappropriate explanation	20 (13.2 %)
Justified equivalence of fractions but without explanation or inappropriate explanation	16 (10.6 %)
Justified equivalence of fraction but had limited mathematical knowledge	28 (18.5 %)
Had complete knowledge of mathematical concept demonstrated about equivalence of fractions	54 (35.8 %)
TOTAL	151

As it seen in the table, sixteen (10.6 %) participants justified equivalence of the fractions without explanation, twenty eight (18.5 %) participants justified equivalence of the fractions with limited mathematical knowledge and fifty four (35.8 %) participants justified equivalence of the fractions with complete knowledge of mathematical concept. Eventually, most of the students (64.9 %) wrote correct fractions for all images representing $\frac{2}{5}$ and $\frac{4}{10}$. On the other hand, fifteen (9.9 %) of the thirty three students who had no mathematical understanding related to the item had difficulties in expressing fraction numbers belonging to the models like Participant 94. He was not able to determine fractions which were represented by figures.

Participant 94:

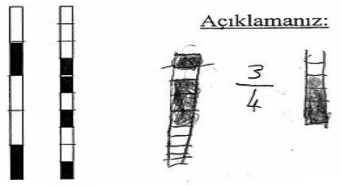


Figure 4.28 Answer of Participant 94

In item 4-a, the analysis of results showed that in total, sixty nine (45.6 %) students had difficulties in expressing equivalent fractions and suggested three invalid strategies, one of them, which is supported by eight (5.2 %) students, being stating the fractions that were represented by images were not equal since sizes of shaded pieces were not the same. For instance, Participant 9 stated that, $\frac{2}{5}$ and $\frac{4}{10}$ are not equivalent because pieces of the figure which represented $\frac{2}{5}$ were bigger and numbers of pieces were fewer than the other figure.

Participant 9:

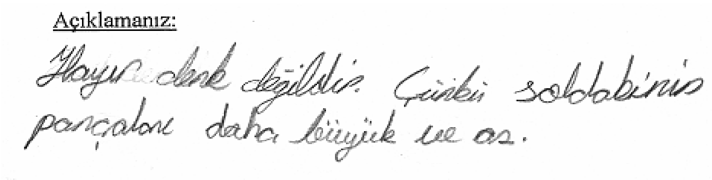


Figure 4.29 Answer of Participant 9

In a similar manner, three (2 %) students denoted that fractions which were represented by images were not equal but this time the reason of the idea was numbers of pieces being not equal. For example, Participant 133 realized that one of the images was divided into ten parts and the other was divided into five parts, thus, according to Participant 133 these two fractions were not equivalent.

Participant 133:

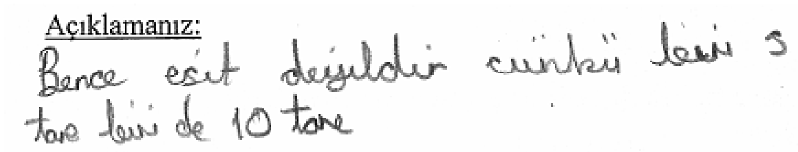


Figure 4.30 Answer of Participant 133

Third and last of the three invalid strategies was considering the illustrations and fractions as not equal because they were not divided in the same way and was supported by seven (4.6 %) of the one hundred and fifty one fifth grade students. Participant 26 looked at overall figure and stated that they were not equivalent because they were divided in the same way, as it can be seen below.

Participant 26:


Açıklamanız: Değildir çünkü denk olması için aynı şekilde bölünmesi gerekir.

Figure 4.31 Answer of Participant 26

Additionally, two (1.3 %) students made operational mistakes. Nearly all students (44 of 54 fifth graders) who stated ‘these two fractions are equivalent’ with clear explanation of the solution process, used symbolic procedures to confirm equivalence of fractions.

The objective of item 4-b which involved symbolic notation was to justify the equivalence of $\frac{3}{4}$ and $\frac{6}{8}$. In this item, students were expected to decide whether two fractions were equivalent or not by shading empty areas. Table 4.7 represents the categories and frequencies of each category. In particular, responses that were not relevant to fractions as they indicated no mathematical understanding or were left totally blank, grouped under the first category; responses lacking both figures that represented the fractions and information regarding the equivalence, grouped under the second category; responses including figures that represented the fractions but lacking information regarding the equivalence, grouped under the third category, responses including figures that represented the fractions and information regarding the equivalence but lacking clear reasons behind their decisions, grouped under the forth category.

Table 4.7 Frequency of classification of item 4-b

Item 4-b: Picturizing and writing equivalent fractions	
	Frequency
Had no mathematical understanding / left blank	8 (5.3 %)
Had inappropriate drawing / Gave unclear explanation	18 (11.9 %)
Had appropriate drawing / Gave missing or unclear explanation	39 (25.8 %)
Had partially appropriate drawing / Gave clear and correct explanation or vice versa	18 (11.9 %)
Had complete knowledge of mathematical concept demonstrated about equivalence of fractions	47 (31.1 %)
TOTAL	151

As it seen in table 4.7, forty seven (31.1 %) of the respondents correctly represented each fraction on rectangle area model with complete knowledge of mathematical concept; whereas, eight (5.3 %) of the participants had no mathematical understanding regarding question and eighteen (11.9 %) of one hundred and fifty one participants incorrectly represented fractions pictorially and their explanations of reasoning were unclear. These students misapplied additive ideas when finding equivalent fractions and demonstrated mistakes related to equivalent fractions. For instance, although Participant 131 drew the image which represented $\frac{6}{8}$ correctly, she made a mistake while drawing the image for $\frac{3}{4}$.

Participant 131:

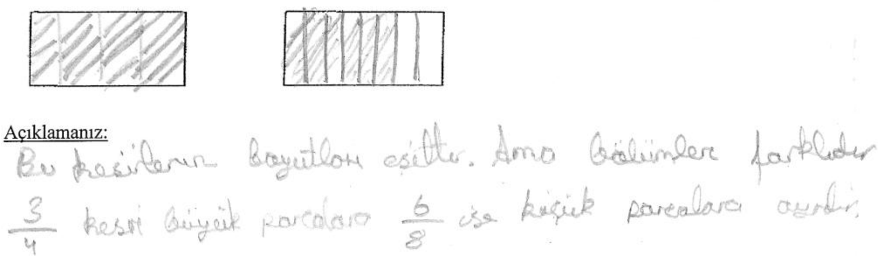


Figure 4.32 Answer of Participant 131

Moreover, thirty nine (25.8 %) of these students correctly represented fractions pictorially but their explanations of reasoning were missing or unclear. Four (2.6 %) of them concluded that $\frac{3}{4}$ was greater than $\frac{6}{8}$ because 'in $\frac{3}{4}$ size of the pieces was bigger than $\frac{6}{8}$ or vice versa'. For example Participant 8's answer was '$\frac{3}{4}$ is larger than $\frac{6}{8}$ because of the size of the pieces' which can be seen below, along with the drawings.

Participant 8:

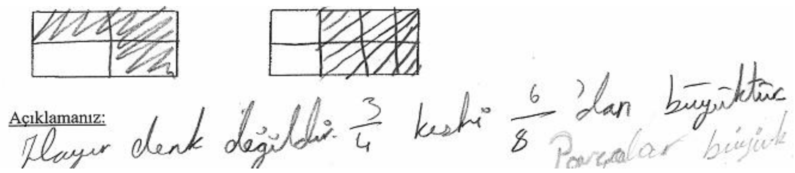


Figure 4.33 Answer of Participant 8

Participant 60, however, wrote exactly the opposite statement. According to him, $\frac{6}{8}$ was larger than $\frac{3}{4}$ because in $\frac{6}{8}$, number of the shaded pieces were much more than were in $\frac{3}{4}$.

Participant 60:

Figure 4.34 Answer of Participant 60

Three (2%) of those thirty nine participants who had appropriate drawing but gave missing or unclear explanation claimed that enlarging procedure changes the value of a fraction. Participant 148 was a good example of this situation. Because she stated that these fractions were not equivalent since second one was expanded by 2.

Participant 148:

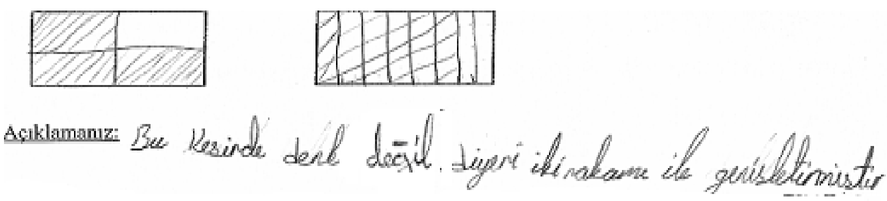


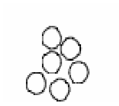
Figure 4.35 Answer of Participant 148

While almost all of the forty seven (31.1 %) students who answered item 4-b correctly and used symbolic procedures to confirm equivalence of fractions, none of them used their pictorial representations to explain the equivalency of the fractions (forty- 26.4 %). The researcher expected them to explain using conceptual strategies, but they stayed in the symbolic explanations.

4.1.5. Item 5

In item 5, it was aimed to examine students' ability to calculate a whole when some part of it is given as a proper fraction.

5. The figure given below represents 3/5 of marbles of Hakan. What is the total number of marbels of Hakan? Draw all of Hakan's marble. Explain how you solved the problem.



Total number of marbels :

Explanation:

Figure 4.36 Item 5

Table 4.8 represents the categories and frequencies of each category. In particular, responses that were not relevant to fractions as they indicated no mathematical understanding or were left totally blank, grouped under the first category; responses in which students calculated the number of marbles incorrectly and gave missing or unclear explanation of solution strategy, grouped under the second category; responses in which students calculated the number of marbles correctly but with missing or unclear explanation of solution strategy, grouped under the third category; responses in which students calculated the number of marbles incorrectly due to an operational mistake, grouped under the forth category.

Table 4.8 Frequency of classification of item 5

Item 5: Determining the whole when some part is given	
	Frequency
Had no mathematical understanding / left blank	37 (24.5 %)
Calculated the number of marbles incorrectly / Had missing or unclear explanation of solution strategy	37 (24.5 %)
Calculated the number of marbles correctly / Had missing or unclear explanation of solution strategy	42 (27.8 %)
Made operational mistake/ Had acceptable explanation of solution strategy	3 (2.0 %)
Calculated the number of marbles correctly / Had acceptable explanation of solution strategy	32 (21.1 %)
TOTAL	151

Analysis of data revealed that students have difficulties calculating the whole, when a fractional part is given. In item 5, out of thirty seven (24.5 %) students who failed to answer correctly, twenty two (14.5 %) of them did not attempt to solve the problem and fifteen of them did not represent mathematical understanding. Another thirty seven (24.5 %) students calculated the number of marbles incorrectly; moreover they had missing or unclear explanation of solution strategy. In total, seventy four (49 %) of the students were unable to solve this word problem related to fractions. Twelve (8 %) of them did not use the correct fraction operator. For instance, Participant 57 only added $\frac{3}{5}$ and $\frac{2}{5}$ to find total number of marbles.

Participant 57:

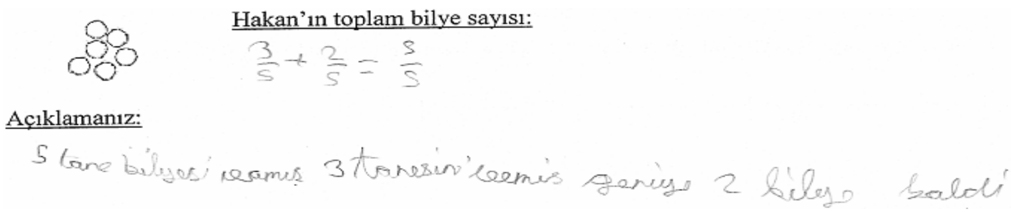


Figure 4.37 Answer of Participant 57

Four (2.6 %) of them students used the sum of numerator and denominator. For instance Participant 112 added 5 to 3 since $\frac{3}{5}$ were given in the question. He stated that total number of marbles was 8.

Participant 112:

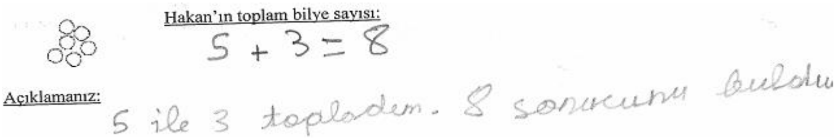


Figure 4.38 Answer of Participant 112

Eight (5.2 %) of the students used multiplication of numerator and denominator to tell how many marbles were in the whole set and the result was 15 marbles. For instance Participant 92 multiplied 5 and 3 since $\frac{3}{5}$ was given in the question. He stated that total number of marbles was 15.

Participant 92:

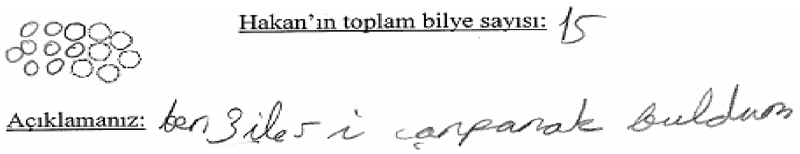


Figure 4.39 Answer of Participant 92

Thirteen (8.6 %) students applied disintegrated algorithms using different combinations of 3, 5, and 6. Three students wrote multiplied 3 and 6, then tried to divide 18 by 5 and they concluded that it is not possible to find total number of marbles. Also, two students, one of them was Participant 65 whose solution is shown below, found 3 in various incorrect ways (e.g., 6÷5=1, 1×3=3 or half of 6 is 3) and then they added 3 and number of marbles in figure, the result was 9 marbles. For example, as it seen in figure 4.40, Participant 65 stated that $\frac{3}{5}$ of marbles are 6 and 6+3 is the total number of marbles, for calculating $\frac{5}{5}$, it necessary to find the sum of 6 and 3 because 3 is the half of 6 marbles.

Participant 65:

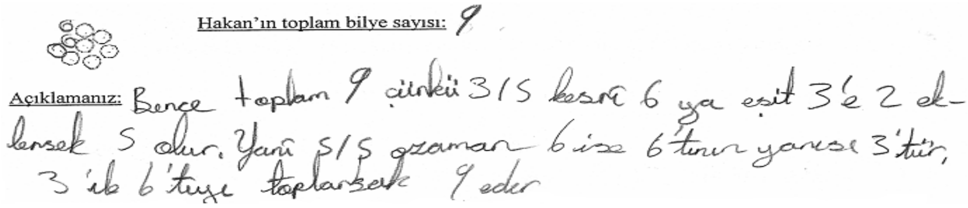


Figure 4.40 Answer of Participant 65

Six (4 %) participants directly multiplied 3 and 2 to find total number of marbles. For instance, as it seen in figure 4.41, Participant 60 multiplied 3 and 2 and drew 18 marbles.

Participant 60:

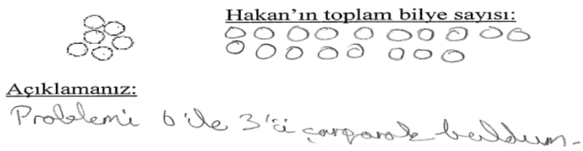


Figure 4.41 Answer of Participant 60

Data revealed that seventeen (11.2 %) students tried to write only the answer without explanation. Moreover, seven (4.6 %) students drew only the pictorial model which represents the fraction of $\frac{3}{5}$. Participant 4 was one of them and he only drew a figure which represent fraction $\frac{3}{5}$.

Participant 4:

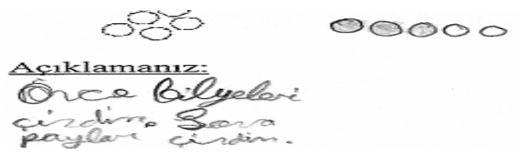


Figure 4.42 Answer of Participant 4

As it seen in table 4.8, forty two (27.8%) of the students correctly calculated number of marbles but their explanations of solution strategy were missing or unclear. For instance, Participant 135 stated that total number of marbles was 10 but he has not done any operation and his explanation was not satisfactory.

Participant 135:

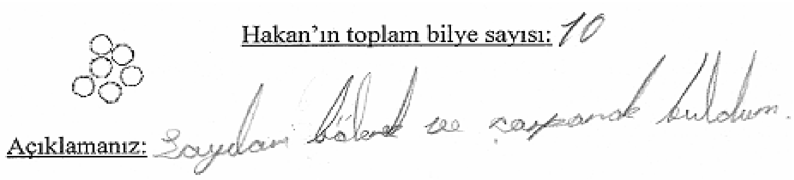
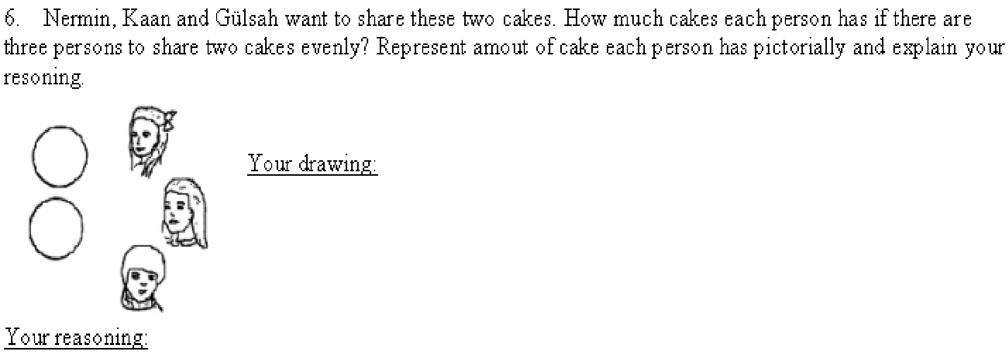


Figure 4.43 Answer of Participant 135

Additionally, three (2 %) students made operational mistakes. Only thirty two (21.1%) of one hundred and fifty one students calculated the number of marbles correctly and their explanations of solution strategy were acceptable.

4.1.6. Item 6

The purpose of this item was to find out how successful respondents were with partitioning exercises and using fraction notation to represent the 'how much' a person would get.



6. Nermin, Kaan and Gülsah want to share these two cakes. How much cakes each person has if there are three persons to share two cakes evenly? Represent amout of cake each person has pictorially and explain your resoning.

Your drawing:

Your reasoning:

Figure 4.44 Item 6

The Table 4.9 represents the categories and frequencies of each category. In particular, responses that were not relevant to fractions as they indicated no mathematical understanding or were left totally blank, grouped under the first category; responses without any appropriate answers or drawings, grouped under the second category; responses including appropriate drawings but with missing or unintelligible explanations of reasoning, grouped under the third category, responses including appropriate drawings with partially correct but unsatisfactory explanations, grouped under the forth category.

Table 4.9 Frequency of classification of item 6

Item 6: Finding the relation between a fraction and division operation	
	Frequency
Had no mathematical understanding /left blank	21 (13.9 %)
Had inappropriate drawing / Gave incorrect explanation	69 (45.6 %)
Had appropriate drawing / Gave missing or unclear explanation	10 (6.6 %)
Had appropriate drawing / but had limited mathematical knowledge or vice verse	9 (5.9 %)
Had appropriate drawing and complete knowledge of mathematical concept	42 (27.8 %)
TOTAL	151

Table 4.9 shows that, twenty one students had no mathematical understanding regarding question or they left the question blank. Sixty nine (45.6 %) students did not accurately partition the picture. Moreover, ten (6.6 %) of the participants drew appropriate picture but did not demonstrate clear quotient understanding because students failed to represent the amount each person would get as fraction. For instance, Participant 54 drew appropriate picture which demonstrated correct quotient understanding but he did not define any expression about the amount of cake that each person received.

Participant 54:

Figure 4.45 Answer of Participant 54

Six (4 %) of the participants who had inappropriate drawing and who gave incorrect explanation, used a rectangle area representation instead of a circular area which was given in the task such as Participant 63. Although a circular region model was given in the question she used rectangular region model. Besides, she divided the figures into 6 equal parts but her figure could be divided into 3 equal parts.

Participant 63:

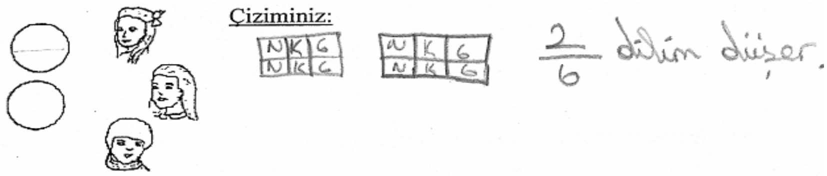


Figure 4.46 Answer of Participant 63

Representations of twenty four (15.8 %) of students who had inappropriate drawing and who gave incorrect explanation showed that they considered the number of parts rather than the area of the parts: they divide the cakes into three, six, nine or twelve pieces horizontally or vertically. For instance, Participant 143 divided each cake into six unequal parts.

Participant 143:

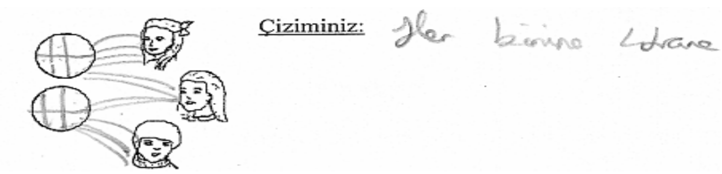


Figure 4.47 Answer of Participant 143

Five (3.3 %) of those twenty four fifth graders who considered the number of parts rather than the area of the parts, divided cakes into halves and quarters and then these pieces were shared among three person and they stated that a few pieces of cake outnumbered. Participant 107 was a good example for this mistake.

Participant 107:

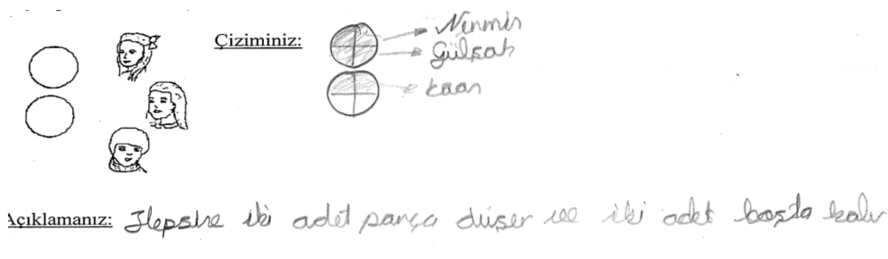


Figure 4.48 Answer of Participant 107

Furthermore, twelve (8 %) students reported that 3 is not a number that is divisible by 2. Therefore seven (4.6 %) of them stated that these two cakes could not be shared by three person evenly. According to Participant 2, two cannot be divided by three but number of cake should have divided by number of person.

Participant 2:

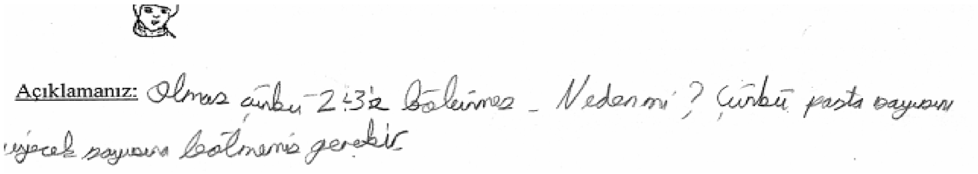


Figure 4.49 Answer of Participant 2

Five (3.3 %) of these twelve students who reported that 3 is not a number that is divisible by 2, added one more cake and partitioned three cakes. For instance Participant 26 expressed that she had noticed that one cake was missing. Consequently she added one more cake.

Participant 26:

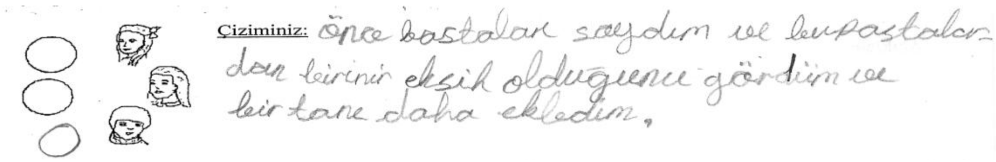


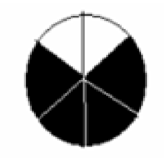
Figure 4.50 Answer of Participant 26

On the whole, one hundred and eleven (73.5 %) of these students gave no numerical answer. They only drew a correct picture and shaded or labeled the appropriate amount for each person. A small amount of students (forty two-27.8 %) were able to draw an accurate cake picture and label the parts for each person indicating that they understood how to ‘share’ equally.

4.1.7. Item 7

The purpose of item 7 was to identify what kind of mistakes students were making while doing addition of proper fractions.

7. a) What is the addition operation which result of it represented by shaded part on the model? Explain your reasoning.



a) $\frac{1}{2}+\frac{1}{3}$ b) $\frac{1}{3}+\frac{2}{6}$ c) $1+\frac{1}{3}$ d) $\frac{1}{3}+\frac{4}{6}$

Explanation:

b) $\frac{34}{61}+\frac{34}{61}$ Calculate the given statement. Describe the path you followed to get the result.

Figure 4.51 Item 7

Table 4.10 represents the categories and frequencies of each category. In particular, responses that were not relevant to fractions as they indicated no mathematical understanding or were left totally blank, grouped under the first category; responses in which students selected one of the incorrect answers with missing or incorrect reasons, grouped under the second category; responses in which students selected the correct answer but with missing or incorrect reasons, grouped under the third category, responses in which students selected the correct answer with correct reasons behind their decisions, grouped under the forth category.

Table 4.10 Frequency of classification of item 7-a

Item 7-a: Adding fractions	
	Frequency
Had no mathematical understanding / left blank	28 (18.5 %)
Selected an incorrect answer / Gave missing or incorrect explanation	61 (40.4 %)
Selected the correct answer / Gave missing or unreasonable explanation	24 (15.9 %)
Selected the correct answer / Had limited mathematical knowledge	9 (5.9 %)
Selected the correct answer / Had complete knowledge of mathematical concept	29 (19.2 %)
TOTAL	151

Eighty nine of the students (58.9 %) could not find the correct addition operation. In the first category, answers of eleven (7.28%) of the twenty eight (8.5%) students were irrelevant to the issue and item. Seventeen (4.6 %) students either left the problem. Twenty nine of the students (19.2 %) selected the correct answer and their explanations of reasoning were correct and understandable. Twenty students selected (a), sixty two students (b), fourteen (c), twenty seven (d), as the correct answer and seventeen did not select an answer. Eight (5.2 %) of the answers were unreasonable. Thirty two (21.1 %) of the sixty one students who selected incorrect answer and stated incorrect explanation, determined five different algorithmic variations. First one was selecting the correct answer but giving incorrect reason behind their decisions by twenty four (15.9%) students. For instance, Participant 68 selected the correct answer but when his explanation was considered it could be seen that the explanation is unreasonable.

Participant 68:

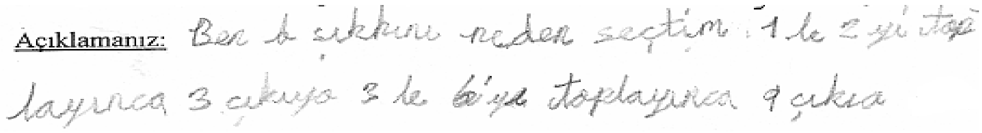


Figure 4.52 Answer of Participant 68

Second one, which was the most common mistake in item 7-a, was ticking (d) owing to seeing $\frac{4}{6}$ in (d) by 22 (14.5 %) students. Participant 21 stated that the figure was divided into 6 parts and 4 of them were shaded since the correct answer is (d).

Participant 21:

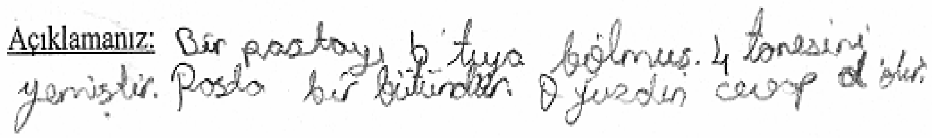


Figure 4.53 Answer of Participant 21

Third of the algorithmic variations was stating there is no correct answer because the addition operation should be $\frac{4}{6} + \frac{2}{6}$ by 8 (5.2 %) students. Participant 13 agreed with this idea.

Participant 13:

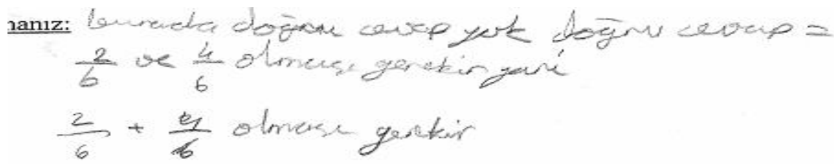


Figure 4.54 Answer of Participant 13

Forth and the last one were misconceiving the procedure of finding lowest common denominator by 8 (5.2 %) students. For instance, Participant 12 tried to find lowest common denominator but she did not multiply numerators with required numbers so she did not find the correct answer.

Participant 12:

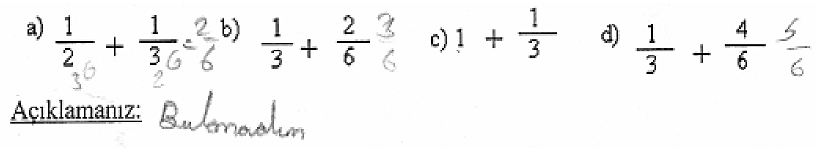


Figure 4.55 Answer of Participant 12

In fifth and also the last algorithmic variation, students did not know that fractions cannot be added unless they represented parts of a same-sized unit, and they had not to know how to create same units. For example Participant 116 added numerators and added denominators. She did not find common denominator.

Participant 116:

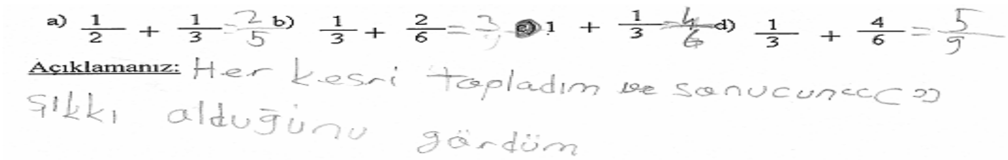


Figure 4.56 Answer of Participant 116

Specifically, items 7-a, 7-b, 8-a and 8-b were linked to additive operations. Table 4.11 represents the categories and frequencies of each category. In particular, responses that were not relevant to fractions as they indicated no mathematical understanding or were left totally blank, grouped under the first category; responses in which students gave incorrect answers without any explanation or operation, grouped under the second category; responses in which students gave correct answers but without acceptable explanations, grouped under the third category, responses in which students gave correct answers but explained their process only in accordance with the algorithms they had memorized, grouped under the forth category.

Table 4.11 Frequency of classification of item 7-b

Item 7-b: Adding fractions which have same denominator	
	Frequency
Had no mathematical understanding / left blank	17 (11.3 %)
Gave incorrect answer	30 (19.8 %)
Gave correct answer / Gave missing or unclear explanation	42 (27.8 %)
Gave correct answer/ Had limited mathematical knowledge	17 (11.3 %)
Gave correct answer / Had complete knowledge of mathematical concept	45 (29.8 %)
TOTAL	151

Ten (6.6 %) of the answers were irrelevant to the issue and item. Seven (4.6 %) students either left the problem blank or quit after a brief attempt. There were thirty (19.8 %) students who seemed to miss the point that fractions could simply be added when they have same denominator. These students applied four different incorrect algorithmic variations. Firstly, nineteen (12.5%) of the students added numerators and the denominators together to get the answer. Participant 116 opted this way as follows.

Participant 116:

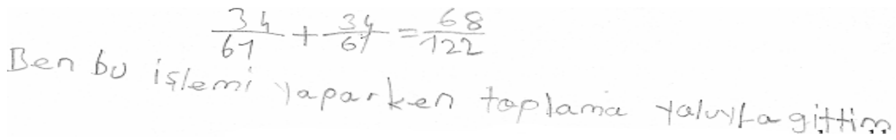


Figure 4.57 Answer of Participant 116

Secondly, two (1.3 %) students added all numbers to determine the new numerator and added denominators to determine the new denominator. For instance, Participant 123 calculated numerator as 196 and denominator as 122.

Participant 123:

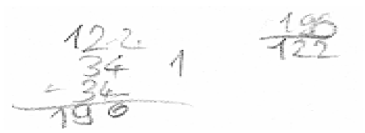


Figure 4.58 Answer of Participant 123

Thirdly, five (3.3 %) students made operational mistakes and Participant 140 was one of them. He made operational mistakes when calculating both numerator and denominator.

Participant 140:

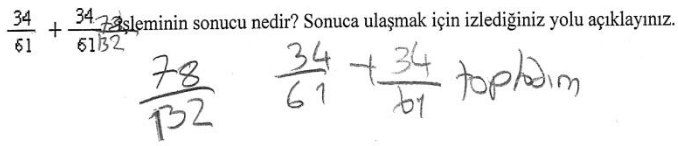


Figure 4.59 Answer of Participant 140

Forth and the last one   as it can be seen below. Four (2.6%) students made this mistake and Participant 150 was one of them.

Participant 150:

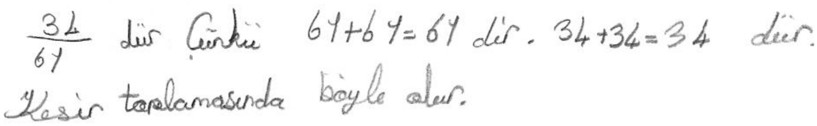


Figure 4.60 Answer of Participant 150

In general, one hundred and nine (72.1 %) students correctly added these fractions but only forty five (29.8 %) students were able to explain their strategy correctly.

4.1.8. Item 8

In item 8, it was required to perform an addition operation with a fraction and a natural number as it is shown if Figure 4.61. Additionally, in the second part, students were expected to understand adding a proper fraction and a natural number may create a mixed number.

8. a) $2+\frac{2}{5}$ Calculate the given statement. Describe the path you followed to get the result.

b) Emrah said that $5\frac{1}{3}$ was the same as $5+\frac{1}{3}$ and Sinan said that $5\frac{1}{3}$ was the same as $5\times\frac{1}{3}$.
Who, if either, is right and why do you think so?

Figure 4.61 Item 8

Table 4.12 represents the categories and frequencies of each category. In particular, responses that were not relevant to fractions as they indicated no mathematical understanding or were left totally blank, grouped under the first category; responses including incorrect answers and explanations, grouped under the second category; responses including correct answers but with missing or incorrect reasons, grouped under the third category, responses including correct answers with correct reasons behind their decisions, grouped under the forth category.

Table 4.12 Frequency of classification of item 8-a

Item 8-a: Adding a fraction and a natural number	**Frequency**
Had no mathematical understanding / left blank	20 (13.2 %)
Gave incorrect answer	32 (21.2 %)
Gave correct answer / Gave missing or unclear explanation	32 (21.2 %)
Gave correct answer but had limited mathematical knowledge	20 (13.2 %)
Gave correct answer / Had complete knowledge of mathematical concept	47 (31.1 %)
TOTAL	151

As it seen above, fifty two (34.4 %) students failed to reduce $2+\frac{2}{5}$ correctly. In the first category, fourteen students (9.2 %) of those twenty students left the item blank. Six (4 %) of them indicated no mathematical understanding. In this item, there were five different algorithmic variations that yielded twelve different results. The first group included thirteen (8.6 %) students who decided to use an algorithm of adding whole number and numerator. For example Participant 128 kept the denominator constant but she added whole part with the numerator.

Participant 128:

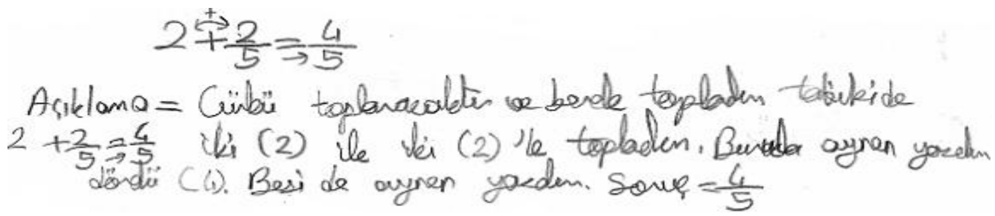


Figure 4.62 Answer of Participant 128

The second group included nine (5.9 %) students who, in a similar way to the first group, decided to use an algorithm of adding whole number with both numerator and denominator. For instance Participant 116, added whole part with both numerator and denominator and she indicated that the result was $\frac{4}{7}$.

Participant 116:

Figure 4.63 Answer of Participant 116

The third group included three (2 %) students who multiplied whole number with denominator and added whole number with numerator. As it seen below, Participant 150 added whole part with numerator and multiplied whole part with denominator and she indicated that the result was $\frac{4}{10}$.

Participant 150:

Figure 4.64 Answer of Participant 150

Moreover, the forth group included six (4 %) participants who tried to convert the whole number to improper fraction but have failed. For instance, Participant 126 tried to convert natural number to an improper fraction but he put 1 up to the place of numerator instead of bottom. Then he found the lowest common denominator.

Participant 126:

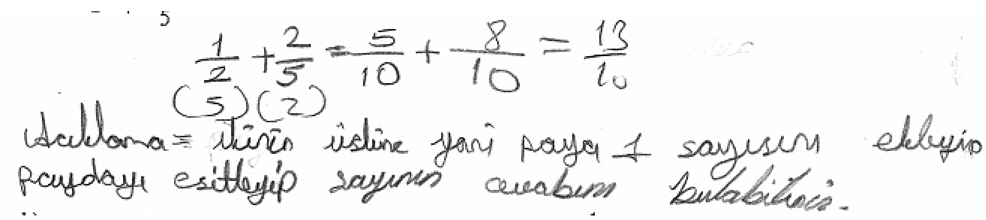


Figure 4.65 Answer of Participant 126

For instance, Participant 52 also tried to convert natural number to an improper fraction but he stated that 2 is equivalent to $\frac{5}{5}$. He did not comprehend that $\frac{5}{5}$ is equivalent to only one whole.

Participant 52:

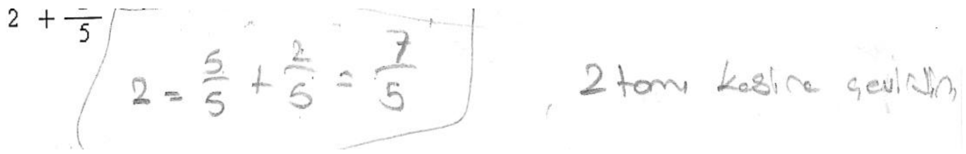


Figure 4.66 Answer of Participant 52

Lastly, the fifth group included two (1.3 %) students who added numerator, denominator and whole number and stated the result as the new numerator Participant 89 whose answer is given below could be given as example.

Participant 89:

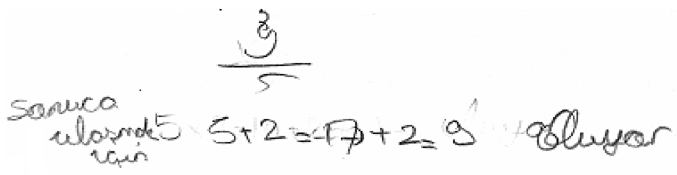


Figure 4.67 Answer of Participant 89

The general analysis of item 8-b showed that, students have difficulty interpreting mixed numbers as addition of a fraction and a whole number.

Table 4.13 represents the categories and frequencies of each category. In particular, responses that were not relevant to fractions as they indicated no mathematical understanding or were left totally blank, grouped under the first category; responses in which students indicated only Sinan or both Sinan and Emrah were right because of incorrect reasoning, grouped under the second category; responses in which students indicated Emrah was right but without acceptable explanation or because of incorrect reasoning, grouped under the third category, responses in which students indicated Emrah was right with acceptable explanation but didn’t clarify Sinan’s mistake grouped under the forth category.

Table 4.13 Frequency of classification of item 8-b

Item 8-b: Adding a fraction and a natural number	
	Frequency
Had no mathematical understanding / left blank	18 (11.9 %)
Chose “Sinan is correct” or “Both of them are correct”	23 (15.2 %)
Chose “Emrah is correct” / Gave missing or inappropriate explanation	64 (42.4 %)
Chose “Emrah is correct” / Had limited mathematical knowledge	24 (15.9 %)
Chose “Emrah is correct” / Had complete knowledge of mathematical concept	22 (14.6 %)
TOTAL	151

As it can be seen in the table, eighteen (11.9 %) had no mathematical understanding regarding question (eleven- 7.2 %) or they left the question blank (seven- 4.6 %). According to the data and table 4.13, sixty four (42.4 %) students stated that ‘Emrah is correct’ but their explanations were missing or inappropriate. Six (3.9 %) of them claimed that $5\frac{1}{3}$ should be interpreted as $5+\frac{1}{3}$ but rather as $5x\frac{1}{3}$ because $5x\frac{1}{3}$ is a multiplication operation and they approved the statement ‘multiplication makes bigger’ in this item. Namely, according to them, $5\ x\frac{1}{3}$ is greater than $5\frac{1}{3}$. For instance Participant 134 stated that the given expression is equal to $5+\frac{1}{3}$ because $5\ x\frac{1}{3}$ is equal to a larger natural number.

Participant 134:

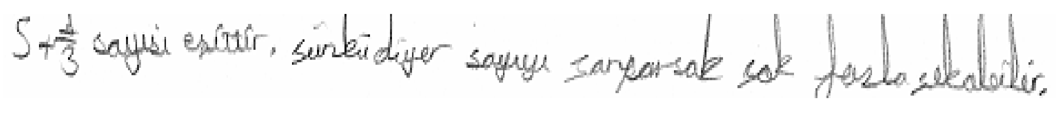


Figure 4.68 Answer of Participant 134

Moreover, 23 (15.2 %) of the participants did not interpret the mixed number $5\ \frac{1}{3}$ to mean $5+\frac{1}{3}$ and they stated ‘Sinan’ or ‘both of them’ are correct. Three (2 %) of them wrote ‘Sinan is right’ and they used a faulty procedure ($5x\frac{1}{3}$ = $5x3+1=\frac{16}{3}$)

to confirm the second interpretation and six (3.9 %) of them wrote ‘both are right’ such as Participant 129 did as follows.

Participant 129:

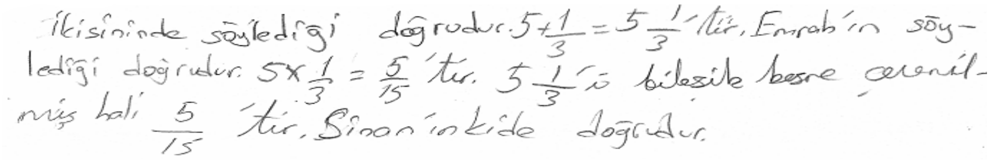


Figure 4.69 Answer of Participant 129

In addition to the information in the table 4.13, nineteen (12.5 %) of those stated that ‘Emrah is correct’ and nine (5.9 %) students stated that ‘Sinan is correct’ without explanation like Participant 105.

Participant 105:

Figure 4.70 Answer of Participant 105

Moreover, twenty one (13.9 %) of one hundred and fifty one students incorrectly calculated the multiplication operation in item 8. For instance Participant 3 equalized denominators as 3. In other words, she used common denominator strategy when she has multiplied 5 and $\frac{1}{3}$.

Participant 3:

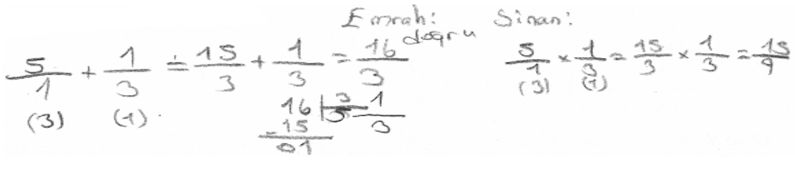


Figure 4.71 Answer of Participant 3

Students had difficulty interpreting mixed numbers correctly. In this study, fifth grade students did not interpret $5\frac{1}{3}$ to mean $5+\frac{1}{3}$, but rather as $5x\frac{1}{3}$, $5\ \frac{1}{3}$ or $\frac{5}{3}$.

4.1.9. Item 9

Item 9 required performing subtraction operations. The first part included improper fractions whereas the second part included proper fractions represented by shaded box areas as seen in Figure 4.72.

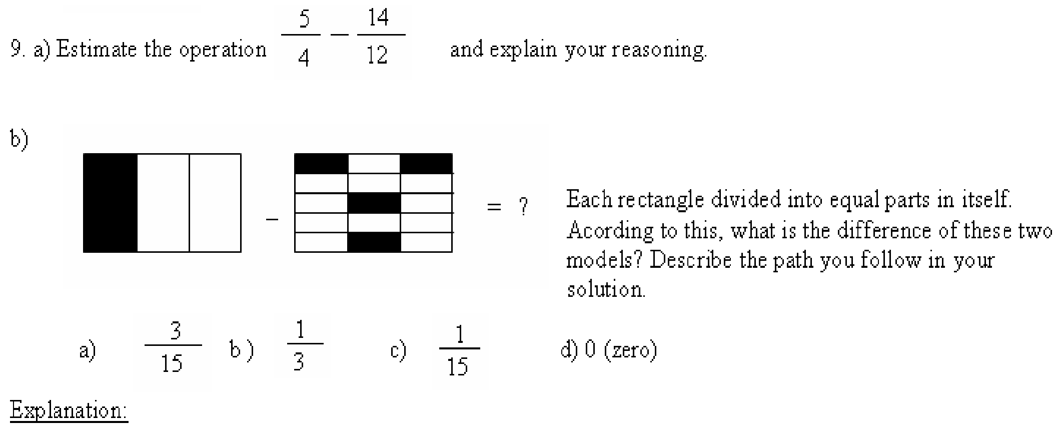


Figure 4.72 Item 9

Table 4.14 represents the categories and frequencies of each category. In particular, responses that were not relevant to fractions as they indicated no mathematical understanding or were left totally blank, grouped under the first category; responses including incorrect answers and explanations, grouped under the second category; responses including correct answers but with missing or incorrect reasons, grouped under the third category, responses including correct answers with correct reasons behind their decisions, grouped under the forth category.

Table 4.14 Frequency of classification of item 9-a

Item 9-a: Subtracting fractions from each other	
	Frequency
Had no mathematical understanding / left blank	33 (21.9 %)
Performed incorrect subtraction	32 (21.1 %)
Had the correct result/ Gave missing or unclear explanation	5 (3.3 %)
Had the correct result but had limited mathematical knowledge	16 (10.6 %)
Had the correct result / Had complete knowledge of mathematical concept	65 (43.0 %)
TOTAL	151

In item 9-a, out of thirty three students left the question blank or indicated no mathematical understanding. Twenty five (16.5 %) students left the task blank and eight (5.2 %) of them indicated no mathematical understanding about the item. Thirty (19.8 %) students applied 8 different incorrect strategies (e.g., just using the larger of the 2 denominators in the answer, subtracting numerator and dividing denominator, transforming improper fraction to mixed number then trying to subtract). Nineteen (12.5 %) students rewrote the example as $\frac{14}{12}-\frac{5}{4}$ and got answers of $\frac{9}{8}$ (subtracted the numerators and subtracted the denominators) or they said it is not possible to subtract.

For instance Participant 86 has changed the location of fractions in the opposite way. Then he subtracted numerator from numerator and denominator from denominator without finding lowest common denominator.

Participant 86:

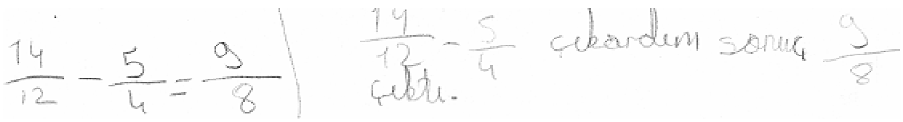


Figure 4.73 Answer of Participant 86

In addition to this, Participant 60 stated that $\frac{14}{12}$ could not subtracted from $\frac{5}{4}$ because $\frac{5}{4}$ is smaller than $\frac{14}{12}$. She thought that the bigger denominator or numerator cannot be subtracted from smaller ones.

Participant 60:

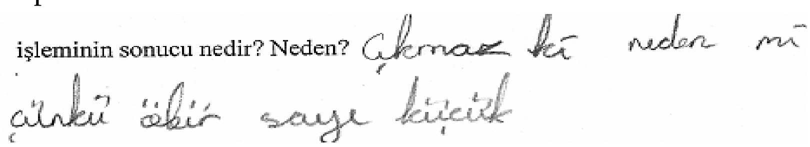


Figure 4.74 Answer of Participant 60

Eight (5.2 %) students had difficulty in finding the least common denominator. Accordingly, they were unable to solve problem. For instance, Participant 135 tried to find lowest common denominator but she multiplied only denominators with required number. Thus she was not able to do subtraction operation and changed the location of fractions.

Participant 135:

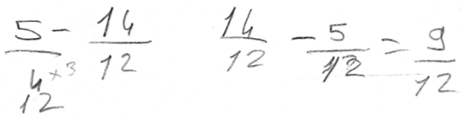


Figure 4.75 Answer of Participant 135

Five (3.3 %) students found the sum of fraction instead of the difference. For example, Participant 15 correctly determined lowest common denominator but she added the fractions.

Participant 15:

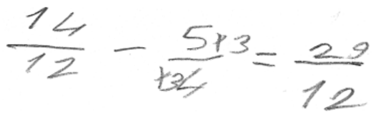


Figure 4.76 Answer of Participant 15

Sixty five (43 %) students subtracted fractions correctly and their solution strategies were understandable and correct.

In item 9-b and 10, firstly students were expected to be able to use pictorial model to represent symbolic model ($\frac{a}{b}$ form), then calculate subtraction operation. Table 4.15 represents the categories and frequencies of each category. In particular, responses that were not relevant to fractions as they indicated no mathematical understanding or were left totally blank, grouped under the first category; responses in which students selected one of the incorrect answers with missing or incorrect reasons, grouped under the second category; responses in which students selected the correct answer but with missing or incorrect reasons, grouped under the third category, responses in which students selected the correct answer but explained their process only in accordance with the algorithms they had memorized, grouped under the forth category.

Table 4.15 Frequency of classification of item 9-b

Item 9-b: Subtracting fractions from each other	
	Frequency
Had no mathematical understanding / blank	22 (14.5 %)
Selected incorrect answer	51 (33.7 %)
Selected the correct answer / Gave missing or unclear explanation	18 (11.9 %)
Selected the correct answer but had limited mathematical knowledge	6 (4.0 %)
Selected the correct answer / Had complete knowledge of mathematical concept	54 (35.7 %)
TOTAL	151

When viewed as the overall in the first category, while nineteen (12.5 %) students did not select an answer, the rest three (2 %) did not presented mathematical understanding. Eighteen students chose (c), which was actually the correct answer, but without making any operation or explanation. While fifty one students gave incorrect answers, thirteen selected (a), six (b), eight (d), again, without any operation and explanation. In short, twenty seven of these fifty one students selected the wrong option without making any operations or explanations. The rest twenty four (15.8 %) students gave incorrect answers and either gave invalid explanations or developed wrong strategies. These incorrect answers represented eight different algorithmic variations: seven (4.6 %) students ticked (a) owing to saw $\frac{3}{15}$ in (a). For instance Participant 20 selected $\frac{3}{5}$ because first figure was divided into 3 and second figure was divided into 15 equal parts.

Participant 20:

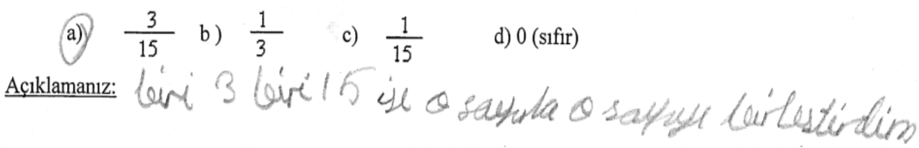


Figure 4.77 Answer of Participant 20

Six (4 %) students stated that the correct answer is zero because both figures represented the same fraction like Participant 80.

Participant 80:

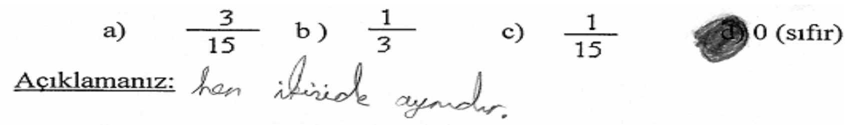


Figure 4.78 Answer of Participant 80

Furthermore, eleven (7.2 %) students were not able to correctly read pictorial representations involving fractions. For instance, Participant 48 confused numerators with denominators. Therefore she could not make the subtraction operation.

Participant 48:

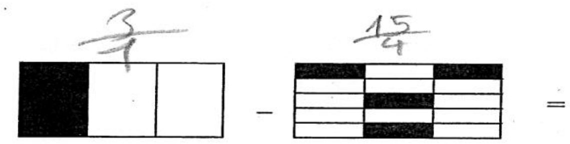


Figure 4.79 Answer of Participant 48

In the fourth category in the table 4.15, eighteen (11.9 %) students selected the correct answer but their explanations of reasoning were missing or their explanations were unclear. As it seen below, Participant 69 selected correct answer but he did not give reason behind this decision.

Participant 69:

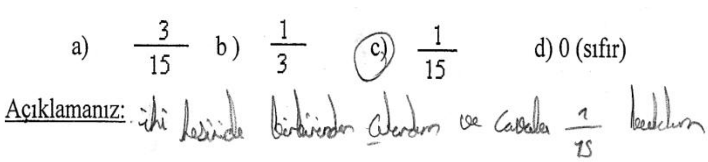


Figure 4.80 Answer of Participant 69

In total, seventy three (48.3 %) of the students (47.6 %) could not find the correct subtraction operation and answer of it. In many cases, the students did not show their work with their representations and explanations, making it impossible to ascertain the type of mistake that was made. Consequently, fifty four (35.7 %) of the students selected the correct answer with correct and understandable explanation.

4.1.10. Item 10

Item 10 was almost similar to the second part of item 9 but additionally involved a natural number in the subtraction operation, also, was not a multi-choice item.

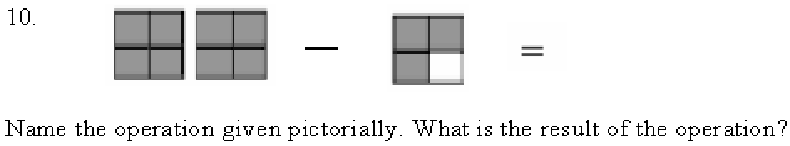


Figure 4.81 Item 10

Table 4.16 represents the categories and frequencies of each category. In particular, responses that were not relevant to fractions as they indicated no mathematical understanding or were left totally blank, grouped under the first category; responses in which students performed incorrect subtraction operations thus found incorrect results, grouped under the second category; responses in which students performed correct subtraction operations but found incorrect results, grouped under the third category, responses in which students found the result correctly but demonstrated limited mathematical knowledge, grouped under the forth category.

Table 4.16 Frequency of classification of item 10

Item 10: Subtracting a fraction from a natural number	
	Frequency
Had no mathematical understanding / left blank	29 (19.2 %)
Determined the subtraction operation incorrect / Found an incorrect result	34 (22.5 %)
Determined the subtraction operation correct / Found an incorrect result	37 (24.5 %)
Had the correct result but had limited mathematical knowledge	20 (13.2%)
Had the correct result/ Had complete knowledge of mathematical concept	31 (20.5 %)
TOTAL	151

In item 10, twenty nine (19.2 %) students did not attempt to solve the item or they did not show any indication about solution of the problem. Twenty (13.2 %) of them left the problem blank, nine (6 %) of them indicated no mathematical understanding. Thirty four (22.5 %) students have identified incorrect subtraction operation. These students did not correctly read symbolic representations involving fractions. Nearly all of these 34 students made their operations without considering how many whole parts were in the figure which represented the first fraction. Instead of considering them as whole numbers, they counted the parts separately, or they thought of it as either one whole, or eight wholes. Eight (5.2 %) students of these students determined subtraction operation as ‘8-3’. For instance, Participant 34 identified subtraction operation as 8-3. He used part/part relationship instead of part/whole relationship.

Participant 34:

Figure 4.82 Answer of Participant 34

Eleven (7.2 %) students wrote $\frac{8}{8}$ and they calculated the results as $\frac{2}{8}$ or they subtracted second numerator from first numerator and second denominator from first denominator and they calculated the result as $\frac{5}{4}$. For example, Participant 121 found lowest common denominator and applied subtraction algorithm but she defined first figure as $\frac{8}{8}$.

Participant 121:

Figure 4.83 Answer of Participant 121

Three (2 %) students identified first figure as $\frac{4}{4}$. For instance, Participant 61 applied subtraction algorithm correctly but she stated that the first figure represents $\frac{4}{4}$.

Participant 61:

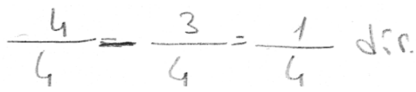


Figure 4.84 Answer of Participant 61

In addition, three (2 %) students stated that the first fraction was $\frac{0}{4}$ and second fraction was $\frac{1}{4}$. According to these students the numerator is the number of unshaded parts and the denominator is the number total parts. According to Participant 127, the first figure represents 4 and equivalence of 4 is $\frac{0}{4}$ as improper fraction. Also she did not apply subtraction algorithm correctly.

Participant 127:

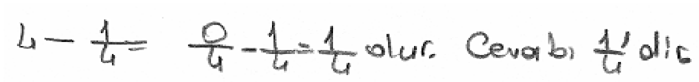


Figure 4.85 Answer of Participant 127

Even thought thirty seven (24.5 %) students performed subtraction operation correctly, their results were incorrect, including twenty one (18.5 %) of them who did not find the lowest common denominator. For instance, Participant 89 identified subtraction operation correctly but she did not find common denominator and he used an algorithm that produced by him.

Participant 89:

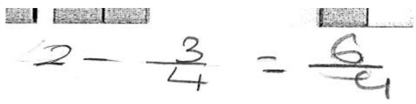


Figure 4.86 Answer of Participant 89

Seven (4.6 %) students applied addition rather than subtraction and found the result as $2\frac{3}{4}$. For instance, Participant 52 identified subtraction operation correctly and used subtraction sign but as it seen below, she added 2 and $\frac{3}{4}$ and she stated that the result was $\frac{11}{4}$.

Participant 52:

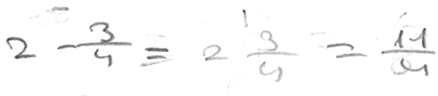


Figure 4.87 Answer of Participant 52

Nine (6 %) fifth graders read fractions which belong to the models but they subtracted the numerators and subtracted the denominators separately. Participant 72 is an example of these students who made this mistake. He correctly identified subtraction operation but he did not find common denominator. Therefore, he was unable to subtract fractions and he interchanged fractions and subtracted only the numerators.

Participant 72:

Figure 4.88 Answer of Participant 72

As a result, in this study, many students performed the addition and subtraction operation of fractions incorrectly.

4.1.11. Item11

The general purpose of item 11 was to examine students' mistakes in both solving and creating word problem about addition and subtraction of fractions.

11. a) There is a jerry can which $\frac{2}{5}$ is filled. When 20 liters more water is filled to jerry can, it is filled of $\frac{4}{5}$. How many liters is the capacity of this jerry can?

b) Write a word problem for which $2\frac{2}{4}+\frac{3}{4}$ is a solution operation. Then, solve your problem showing all of their work.

Figure 4.89 Item 11

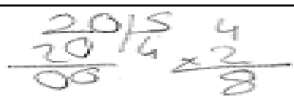
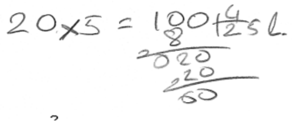
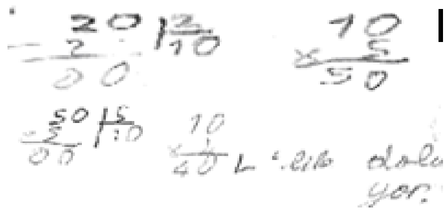
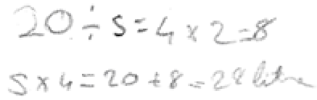
The objective of item 11-a was to determine if the students could recognize how a fraction should be used as an operator and then correctly apply the operator. Table 4.17 represents the categories and frequencies of each category. In particular, responses that were not relevant to fractions as they indicated no mathematical understanding or were left totally blank, grouped under the first category; responses in which students used faulty strategies to solve the problem thus found incorrect results, grouped under the second category; responses in which students calculated the result correctly but without explanation of reasoning, grouped under the third category, responses in which students used the right strategy with explanation of reasoning but made an operational mistake, grouped under the forth category.

Table 4.17 Frequency of classification of item 11-a

Item 11-a: Solving a word problem	
	Frequency
Had no mathematical understanding / left blank	42 (27.8 %)
Had incorrect result	58 (38.4 %)
Had correct result / Gave missing explanation	11 (7.2 %)
Had correct solution strategy and operations but made operational mistakes	15 (9.9 %)
Had correct result / Had complete knowledge of mathematical concept	25 (16.5 %)
TOTAL	151

Close examination of the data also revealed that only forty (26.4 %) of the students gave the required response to the question but fifteen of them made operational mistakes. Fifteen (10 %) of the forty two students in the first category, made mistakes resulting in answers that did not make sense. Twenty seven (17.8 %) of them did not attempt to solve the problem. While eleven (7.2 %) students simply gave only an answer of 50 lt., fifty eight (38.4 %) students gave several answers to this question. These students applied disintegrated algorithms using different combinations of 20, 4, 5, 2, $\frac{2}{5}$ or $\frac{4}{5}$. Table 4.18 shows examples of 58 incorrect results and solution strategies of students.

Table 4.18 Examples of incorrect results and solution strategies of students in item 11-a

Frequencies	***Participant***	***Examples***
11	P 11	
6	P 81	
21	P 100	
2	P 112	
3	P 4	

In item 11-b, fifth grade students were given an addition operation and they were asked to create a word problem describing the given addition operation and solve that problem showing all of their work.

Table 4.19 represents the categories and frequencies of each category. In particular, responses that were not relevant to fractions as they indicated no mathematical understanding or were left totally blank, grouped under the first category; responses in which students did not create correct word problems describing the given addition operation and found incorrect results, grouped under the second category; responses in which students created correct word problems describing the given addition operation but found incorrect results, grouped under the third category, responses in which students created correct word problems describing the given addition operation and found correct results, grouped under the forth category.

Table 4.19 Frequency of classification of item 11-b

Item 11-b: Creating a word problem for which an operation is a solution	
	Frequency
Had no mathematical understanding / left blank	35 (23.1 %)
Created an incorrect problem / Had incorrect response	71 (47.0 %)
Created a correct problem / Had incorrect response	15 (9.9 %)
Created an incorrect problem / Had correct response	17 (11.2 %)
Created a correct problem / Had complete knowledge of mathematical concept	13 (8.6 %)
TOTAL	151

When it came to applying fractions to a real world setting, one hundred and six (70.1 %) students struggled with trying to create story problem regarding addition of two fractions. Twenty eight (18.5 %) of the 35 students in the first category did not attempt to solve the item. Twenty two (14.5 %) students created a word problem which was mathematically correct while being incorrect in the terms of fraction concept. For instance, Participant 119, created a fraction word problem which required addition of $2\frac{2}{4}$ tablecloth and $\frac{3}{4}$ tablecloth.

Participant 119:

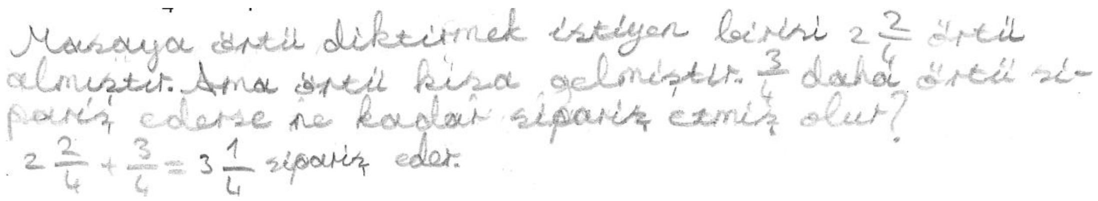

Masaya örtü diktirmek isteyen birisi $2\frac{2}{4}$ örtü almıştır. Ama örtü kısa gelmiştir. $\frac{3}{4}$ daha örtü sipariş ederse ne kadar sipariş etmiş olur?
$2\frac{2}{4} + \frac{3}{4} = 3\frac{1}{4}$ sipariş eder.

Figure 4.90 Answer of Participant 119

Ten (6.6 %) of the students had difficulty in defining the unknown target in their problems and their word problems were not phrased in terms of the same whole. For example, Participant 36 created a word problem as ‘a greengrocer has $2\frac{2}{4}$ apples and $\frac{3}{4}$ banana. How much fruit has he got?’ First of all she mentioned two different whole: an apple and a banana. Second, she calculated the result correctly. Nevertheless, she added apples and bananas but she stated that ‘there are $\frac{13}{4}$ apples.’

Participant 36:

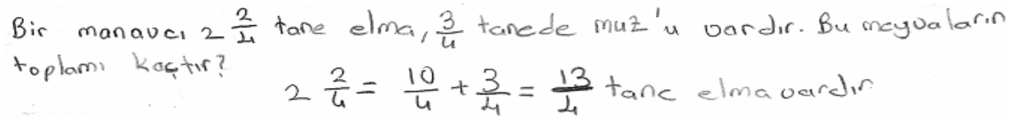


Figure 4.91 Answer of Participant 36

Twenty (13.2 %) students wrote mathematical exercises rather than a problem, like Participant 107. She only asked the sum of these two fractions.

Participant 107:

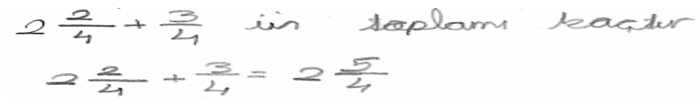


Figure 4.92 Answer of Participant 107

Problems created by six (4 %) students were not relevant to the addition operation which was given in item 11-b. For instance, Participant 43 created a word problem as follows: 'my mother made 2 $\frac{2}{4}$ cakes. I ate $\frac{3}{4}$ of it. How much cake left?' In item 11-b, an addition operation was given and it was asked to create a word problem which describes the given addition operation yet she created a word problem which describes a subtraction operation as in figure 4.93

Participant 43:

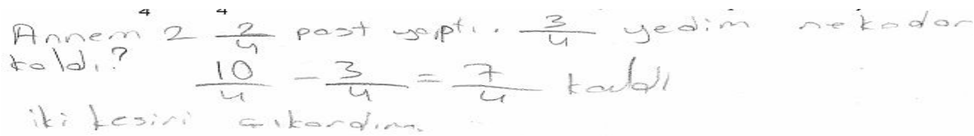


Figure 4.93 Answer of Participant 43

Forty one (27.1 %) students calculated the answer of operation as 2 $\frac{5}{4}$ and twenty two (14.5 %) of the students calculated as $2\frac{5}{8}$. Thus, analysis of fifth grade students' verbalization of the given addition operation revealed that most of these students are not able to construct a real life problem by using fraction operation correctly.

4.1.12. Item 12

In item 12, it was aimed to examine students' capability to calculate a part of another fraction. The first part was a word problem that required calculating one third of a half.

12. a) There is a half cake and you eat one third of it. How much of the cake are you eat? Represent your solution pictorially and explain your reasoning.

b) Estimate the operation $\frac{2}{3} \times \frac{5}{2}$ and explain your reasoning.

Figure 4.94 Item 12

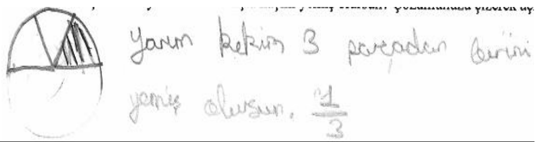
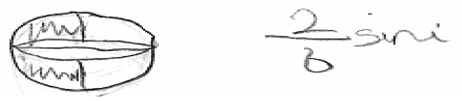
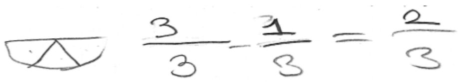
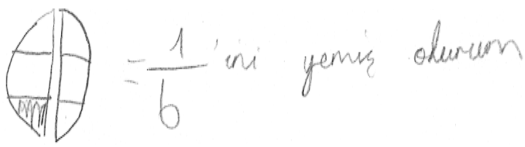
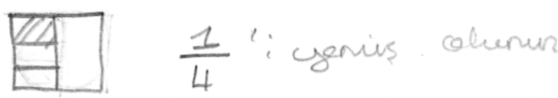
Table 4.20 represents the categories and frequencies of each category. In particular, responses that were not relevant to fractions as they indicated no mathematical understanding or were left totally blank, grouped under the first category; responses in which students drew the figure representing one third of the half cake incorrectly thus achieved a wrong result, grouped under the second category; responses in which students drew a correct figure but incorrectly represented the eaten part as a fraction, grouped under the third category, responses in which students did not divided the cake equally thus drew the figure incorrectly but correctly represented the eaten part as a fraction , grouped under the forth category.

Table 4.20 Frequency of classification of item 12-a

Item 12-a: Determining a fraction to the amount of another fraction	
	Frequency
Had no mathematical understanding / left blank	33 (21.8 %)
Had incorrect drawing and incorrect response	60 (39.7 %)
Had correct drawing but incorrect response	18 (11.9 %)
Had incorrect drawing but correct response	18 (11.9 %)
Had correct drawing and complete knowledge of mathematical concept	22 (14.5 %)
TOTAL	151

Analysis of the data illustrates that ninety three (61.5 %) of the students did not answer this question correctly. These students drew inappropriate figures. Thus, their models did not represent $\frac{1}{3}$ of the $\frac{1}{2}$ cake. Fifty one (33.7 %) of these ninety three students did not show any individual model. Twenty nine (19.2 %) students of thirty three students in the first category left the question blank. Eighteen (11.9 %) students drew appropriate pictures but amount of the eaten cake was incorrect. Only two students realized they needed a multiplication operation in order to find the answer. In total, thirty seven (24.5 %) of the students were able to complete the task without any mistakes. Table 4.21 shows some incorrect solutions strategies, examples of each strategy and frequency.

Table 4.21 Examples of incorrect solutions strategies and frequency of students in item 12-a

Frequency	Pictorial representation	Result	Example
16	Correct /Missing	$\frac{1}{3}$ was eaten	P 131 
3	Not equally divided	$\frac{2}{6}$ was eaten	P 7 
7	Correct	$\frac{2}{3}$ was eaten	P 72 
18	Not equally divided /Missing	$\frac{1}{6}$ was eaten	P 101 
15	Equally divided/Missing	No relevant ($\frac{5}{6}$, $\frac{1}{4}$ quarter)	P 17 

Item 12-b was about multiplication of fractions. Table 4.22 represents the categories and frequencies of each category. In particular, responses that were not relevant to fractions as they indicated no mathematical understanding or were left totally blank, grouped under the first category; responses in which students performed multiplication of the fractions incorrectly, grouped under the second category; responses in which students wrote the correct result but did not give acceptable reasons behind their calculations, grouped under the third category, responses in which students wrote the correct result but demonstrated limited mathematical knowledge, grouped under the forth category.

Table 4.22 Frequency of classification of item 12-b

Item 12-b: Determining a fraction to the amount of another fraction	
	Frequency
Had no mathematical understanding / left blank	23 (15.2 %)
Had incorrect response	18 (11.9 %)
Had correct result / Gave missing explanation	58 (38.4 %)
Had correct result / Had limited mathematical knowledge	17 (11.2 %)
Had correct result / Had complete knowledge of mathematical concept	35 (23.1 %)
TOTAL	151

In item 12-b, eighteen (11.9 %) students calculated multiplication incorrectly. Five (3.3 %) of them mistakenly transferred the rule of addition when performing multiplication with fractions and solved $\frac{2}{3} \times \frac{5}{2}$ by finding a common denominator and then they kept that denominator in the product. Namely, they wrote $\frac{2}{6} \text{ x } \frac{5}{6} = \frac{10}{6}$ or $\frac{4}{6} \times \frac{15}{6} = \frac{60}{6}$ like Participant 31. Many of them simplified this answer to "10", apparently not noticing that "10" was an unreasonable answer to the original problem.

Participant 31:

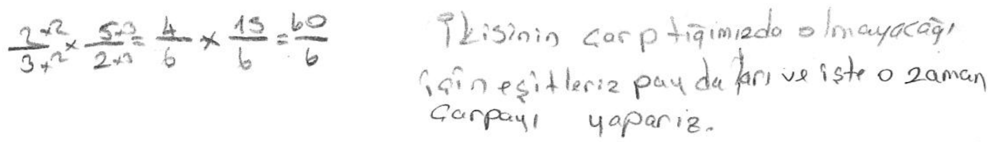


Figure 4.95 Answer of Participant 31

Besides, five (3.3 %) students mistakenly used cross-multiplying, which is a technique that can be used to compare fractions, and found the result as $\frac{15}{4}$, or they used a strategy that was similar to addition and found the result as $\frac{19}{4}$. For instance, Participant 119 multiplied 2 by 2 so as to calculate the denominator of multiplication; and used a similar strategy for the numerator by multiplying 3 and 5.

Participant 119:

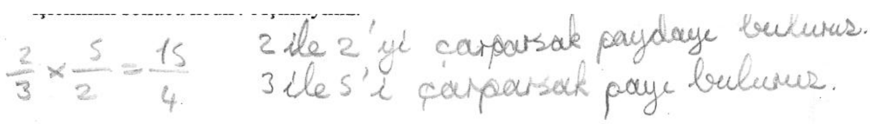


Figure 4.96 Answer of Participant 119

Three (2 %) of the students found that the result via a strategy involving the addition of denominators and the multiplication of numerators. As seen in figure 4. 97, Participant 8 found the result as $\frac{10}{5}$ by explaining the solution in this way.

Participant 8:

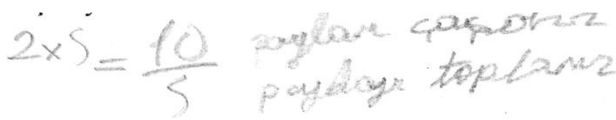


Figure 4.97 Answer of Participant 8

Thirty six (23.8 %) students misapplied the standard multiplication algorithm. As an example, Participant 90 applied standard multiplication algorithm correctly but before that she find common denominator even though there was no need.

Participant 90:

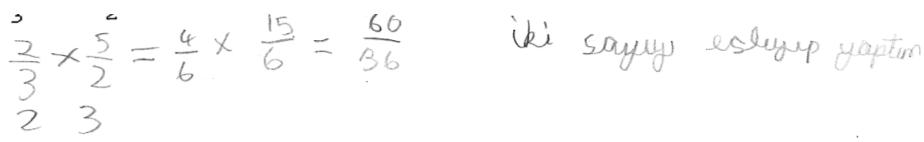


Figure 4.98 Answer of Participant 90

All of the students failed to reduce the fraction before multiplying. Most of the students multiplied the numerators together, multiplied the denominators together but then they did not reduce the fraction to lowest terms. One hundred and ten (72.8 %) of one hundred and fifty one students, calculated the multiplication in item 12-b correctly. But only thirty five (23.1 %) of them proved the mastery of basic algorithmic multiplication skills because fifty eight (38.4 %) of them had correct result but they gave missing explanation and wrote only $\frac{10}{6}$ as an answer.

4.2. Interview Findings

The purpose of this study was to investigate mistakes made by elementary fifth grade students regarding basic fractional concept and operations, and difficulties that they encounter. The other purpose was to investigate underlying reasons and misconceptions behind those mistakes and difficulties. Before identifying underlying reasons and misconceptions behind participants' mistakes and difficulties, the information about mistakes and difficulties regarding basic fractional concepts and operations was analyzed from the questionnaire that students completed before the interviews and they were presented in the previous section.

In this section, through reducing the number of participants, mistakes and difficulties that were revealed from the responses of the sixteen interview participants were categorized under five headings for the better in-depth analysis of the underlying reasons and misconceptions behind students' mistakes and difficulties. Students' responses in interviews showed that their mistakes and difficulties can be grouped under five categories as: algorithmically based mistakes/difficulties, intuitively based mistakes/difficulties, mistakes/difficulties based on formal knowledge on fractions, mistakes/difficulties due to the misunderstanding of the problem, and mistakes/difficulties due to the missing information in solution. In accordance with the purpose, in this section, the researcher described possible underlying reasons and misconceptions behind mistakes students made and difficulties students had while learning fractions. Table 24 represents frequencies of mistakes and difficulties of the students who participated in the interviews, regarding each item on category basis.

Table 4.23 Frequencies of interview students' mistakes/difficulties

CATEGORIES / *ITEMS*	*Algorithmically Based Mistakes/ Difficulties*	*Mistakes/ Difficulties Based on Formal Knowledge*	*Intuitively Based Mistakes/ Difficulties*	*Mistakes/Difficulties due to the Misunderstanding of the Problem*	*Mistakes/ Difficulties due to Missing Information*	*Correct Solution*	*TOTAL*
Item 1-a	2	5	-	2	1	6	16
Item 1-b	1	2	-	-	1	12	16
Item 2	-	3	3	3	-	7	16
Item 3-a	2	11	1	-	-	2	16
Item 3-b	-	8	2	1	-	5	16
Item 4-a	-	4	-	-	-	12	16
Item 4-b	-	2	-	1	-	13	16
Item 5	-	1	-	5	-	10	16
Item 6	-	10	1	-	-	5	16
Item 7-a	1	1	-	3	-	11	16
Item 7-b	4	1	-	-	1	10	16
Item 8-a	3	-	-	-	1	12	16
Item 8-b	2	2	2	2	-	8	16
Item 9-a	3	-	1	2	-	10	16
Item 9-b	4	-	1	4	-	7	16
Item 10	2	3	1	-	1	9	16
Item 11-a	-	-	-	10	-	6	16
Item 11-b	7	4	-	3	-	2	16
Item 12-a	1	10	-	1	-	4	16
Item 12-b	3	-	-	-	-	13	16

As it is seen in the table, especially students have difficulties and made mistakes in ordering the fractions in a set of fractions, partitioning and creating fraction word problem and solving their problems. In other words, almost all students were not able to show correct expressions for the ordering fractions and they were not able to create a fraction word problem. According to the Table 4.24, mistakes/difficulties based on formal knowledge of fractions were the most popular mistake that students made. Algorithmically based mistakes/difficulties were intensified on items which required solving fraction operations to find the solutions. While these students make algorithmically based mistakes/difficulties more in questions that need to be solved with arithmetic operations with fractions, their mistakes/difficulties based on formal knowledge were mostly in the items which involved knowledge about meaning of fraction, such as ordering, equivalence of fractions or converting mixed numbers to improper fractions. Although they are not too frequent, students made intuitively based mistakes/difficulties in questions that required ordering, operations of addition and subtraction. Mistakes due to the misunders$tanding of the problem show an increase in items related to solving and creating fraction word problems. Examples of students' responses are given below under the category headings given in Table 4.23

4.2.1. Underlying Reasons and Misconceptions behind Algorithmically Based Mistakes/Difficulties

Algorithmically based mistakes are mistakes generally made in the arithmetical operations (Ashlock, 1990; Barash & Klein, 1996). In this study, many students failed to remember or got confused about the steps in an algorithmic procedure especially in the items that required arithmetic operations with fractions, such as item 7, 8, 9, 10 and 11.

Algorithmically based mistakes/difficulties were also seen in the tasks that included conversion of mixed numbers to improper fractions or vice versa. In item 1, three (18. 75 %) of the participants stated that the expression is incorrect and by this way they made mistakes because of the rote memorization of the conversion algorithm. The following response of P$$articipant 143 is $an example as:$

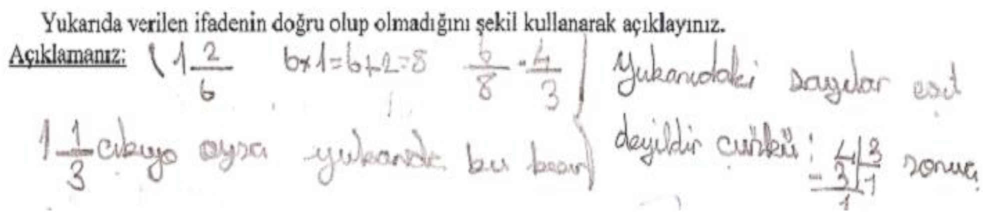


Figure 4.99 Answer of Participant 143 to item 1-a

> Participant 143: "First of all I converted $\frac{4}{3}$ to mixed number and I found $1\frac{1}{3}$. I see that $1\frac{1}{3}$ is not an equivalent of $1\ \frac{2}{6}$ because when it is converted, it is equal to $\frac{6}{8}$.We should multiply whole part and denominator, add numerator to result for the new denominator. "

[Önce $\frac{4}{3}$ ’ü tam sayılı kesre çevirdim ve $1\frac{1}{3}$ buldum. Sonra $1\frac{1}{3}$ ’ün $1\frac{2}{6}$ ’ye eşit olmadığını gördüm çünkü bu çevrildiğinde $\frac{6}{8}$ oluyor. Tam kısım ile paydayı çarpmalıyız sonra onunla payı toplarız, yeni payda olur.]

Clearly be seen in explanation of student, Participant 143 performed long division of 4 by 3 and got remainder of 1and placed it as the new numerator in $1\frac{1}{3}$. She had memorized the procedure of converting a mixed number to an improper fraction but at the end of her process, she confused the places of numerator and denominator. Moreover, she did not realize that $\frac{6}{8}$ was not an improper fraction.

Participant 150’s explanation shows that she was one of the interviewees who noticed the differences between operations with common denominators and different denominators but her solution demonstrated that she was one of four (25 %) interview students who misapplied the rule for addition of fractions with same denominator as follows:

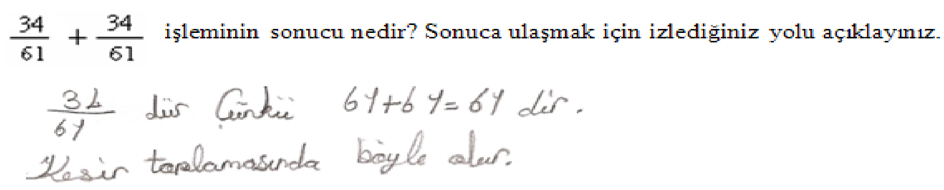


Figure 4.100 Answer of Participant 150 to item 7-b

Participant 150: “Now, both numerators are 34, isn’t adding 34 and 34 results in 34 in fractions? Both fractions are exactly the same, adding 61 and 61 results in 61. If numbers are the same, it is done like that. Our teacher taught us like that. She said that if number are same they are written exactly same. If the denominators are different, the procedure is different too. But if we change one of the denominators, please change the other one, both get the same. Because, if it is like that I can do it easily”

[Şimdi, ikisinin de payı 34, 34 ile 34’ü toplayınca sonuç 34 olmuyor mu kesirlerde? İki kesir de tıpatıp aynı, 61 ile 61’i toplarsak da 61 olur. Eğer sayılar aynı ise böyle yapılır. Öğretmenimiz bize böyle öğretti. Sayılar aynı ise aynen yazılır dedi. Eğer paydalar farklı olsaydı o zaman farklı bir yolu var. Ama eğer paydalardan birini değiştirirsek, lütfen diğerini de değiştirelim, ikisi aynı olsun. Öyle olunca daha kolay yapabiliyorum çünkü.]

Although her response suggested that she viewed her algorithm meaningful, she did not successfully answer the question that expected to find sum of fractions which have common denominators. She kept both numerators and denominators constant. She memorized the procedure by rote learning.

Many students in this study invented an alternative algorithm for converting mixed numbers and improper fractions to each other, based on their existing knowledge of equivalency of fractions. Participant 142 was one of them. When an algorithm is viewed as a meaningless series of steps, some students forgot these steps or change them like Participant 142 explained as given below.

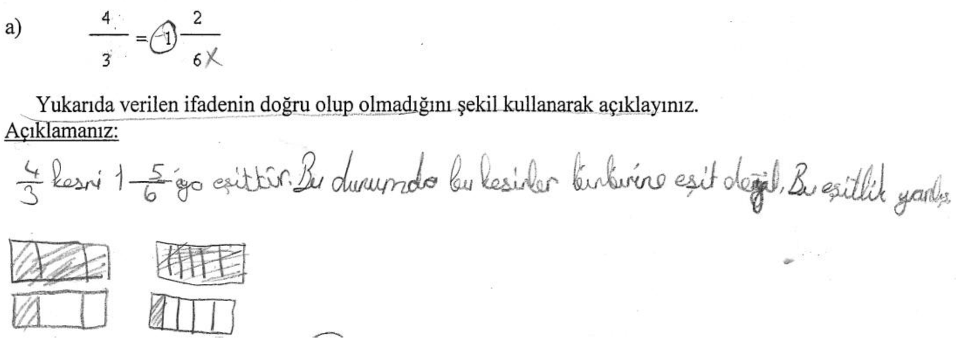


Figure 4.101 Answer of Participant 142 to item 1-a

Participant 142: “There was a rule for it……. but instead I solved it like this: Four minus three is one, so the other side should be 1 $\frac{5}{6}$ because basically numerator of the first side is one larger than the numerator of $\frac{3}{3}$, thus numerator of the other side should be one smaller than the numerator of 1 $\frac{6}{6}$.”

[Bununla ilgili bir kural vardı…… ama onun yerine ben şöyle çözdüm: Dörtten üç çıkartırsak bir kalır, o yüzden diğer taraf 1 $\frac{5}{6}$ olması lazım çünkü bir tarafın payı $\frac{3}{3}$ ‘ün payından bir fazla olduğuna göre diğer tarafın payının 1 $\frac{6}{6}$ ’nın payından bir eksik olması gerekir.]

In her explanation, Participant 142 could justify equivalency of fractions, neither geometrically nor procedurally through simplification and enlarging. Her explanation illustrates that she aware of the existence of an algorithm but her faulty procedure initially interfered in item 1-a.

This study shows that 9 (56.25 %) interview students made algorithmically based mistakes in operation of subtraction of fractions. The following response reflected Participant 136's rote application of the symbolic algorithm on subtraction of fractions in item 9-b.

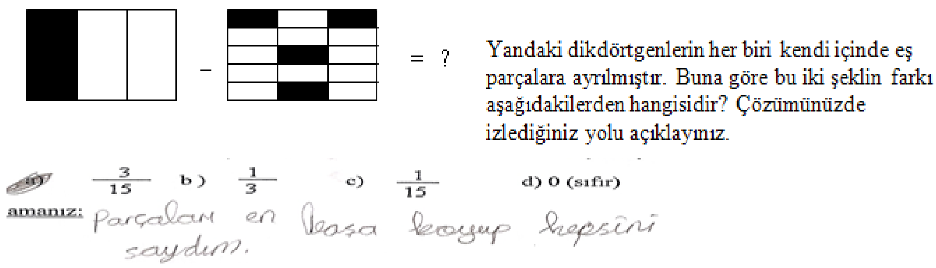


Figure 4.102 Answer of Participant 136 to item 9-b

Participant 136: "First fraction was divided into three. A single piece is taken, namely $\frac{1}{3}$. In the second figure, it is divided into fifteen and four pieces are taken, namely $\frac{4}{15}$. I have to subtract $\frac{4}{15}$ from $\frac{1}{3}$. We will subtract $\frac{1}{3}$ from $\frac{4}{15}$. The result is $\frac{3}{15}$. We do not touch the denominator. We subtract only on the nominators. I subtracted 1 from 4, the result is 3. I directly wrote the same denominator."

[İlk kesir üçe bölünmüş, bir parçası alınmış, yani $\frac{1}{3}$. İkinci şekilde, on beş parçaya bölünmüş, dört parçası alınmış, yani $\frac{4}{15}$. $\frac{4}{15}$'ü $\frac{1}{3}$'den çıkarmalıyım. $\frac{1}{3}$'ü $\frac{4}{15}$'den çıkaracağız, sonuç $\frac{3}{15}$. Paydaya hiç dokunmayız, sadece payları çıkarırız. Ben 4'den 1'i çıkardım, 3. Direk aynı paydaya yazdım.]

Participant 136 interpreted the fraction as a part of a quantity where the numerator indicates the parts ‘‘that we take’’ and the denominator indicates how many parts the unit is divided. Her common algorithmic mistake was subtracting the numerators from each other than keeping the larger denominator constant while performing subtraction operation and passing the larger denominator to result directly. She applied an incorrect rule without considering the size of fractions.

As seen in examples above, students make algorithmically based mistakes because of rote memorization; however, the research suggests that another reason for students to make this kind of mistake was inadequate knowledge of operations.

In item 8-b, which required addition and multiplication procedures, Participant 143 was one of the two students (12.5 %) who made mistake in item 8-b because of the inadequate knowledge of addition and multiplication operations and she explained her solution as follows:

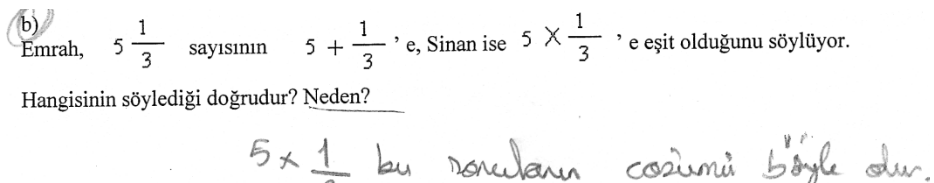


Figure 4.103 Answer of Participant 143 to item 8-b

Participant 143: “I thought Sinan was correct. In here, uh….Multiplication came to my mind and I did like that. Actually, I think both of them would not be correct. Because, Emrah’s result, when we put unreal 1 under 5, the result is $\frac{6}{4}$. Sinan’s result………the result will be $\frac{5}{15}$ because we multiply by 5. I learned like that.”

[Sinan'ın doğru söylediğini düşündüm. Ben yani burada ıııııııı, burada aklıma bu çarpma geldi ve öyle yaptım yani. İkisi de olmaz aslında bence. Çünkü Emrah'ınkinde 5'in altına yalancı 1 koyunca $\frac{6}{4}$ olur. Sinan'ınki de……… $\frac{5}{15}$ olur bence 5 ile çarpıyoruz. Ben böyle öğrendim.]

Participant 143 believed that both were incorrect interpretations. She explained both Sinan's and Emrah's ways incorrectly and her solution on Sinan's way was multiplying five with both the numerator and the denominator, finding $\frac{5}{15}$ as the result. She used incorrect procedures to confirm the interpretations. Actually, she used the enlarging algorithm as the multiplication algorithm. She had the misconception that over the use of the word 'multiplication' by 'enlarging'. She chose inadequate algorithm because of the inadequate knowledge of multiplication operation. Also, as it can be seen in Participant 143's answer and explanation, she told the correct result of item 12-a was $\frac{2}{6}$.

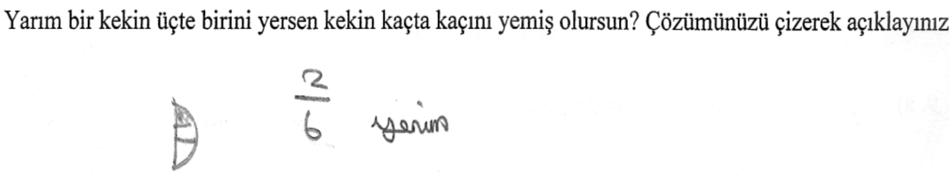


Figure 4.104 Answer of Participant 143 to item 12-a

Participant 143: "The half cake was divided into 3 pieces. If it is made a whole cake, it is divided into six pieces. We ate $\frac{1}{3}$ of the half cake. Actually, we ate $\frac{1}{3}$ of the other half cake too….It is one whole, so I thought like that. If we join both halves and get a whole cake, $\frac{1}{3}$ of cakes would be eaten in both two sides, the answer becomes $\frac{2}{6}$.

[Kekin yarısı 3' e bölünmüş, bunu bütün yaparsak, tamamı 6'ya bölünmüş olur. Yarım bir kekten yani $\frac{1}{3}$'ini yemiş oluyoruz. Aslında, diğerinden de bir tane $\frac{1}{3}$ yemiş oluyoruz o yüzden…..Bu bir tam olduğu için ben öyle düşündüm. Yarımları bir araya toplarsak, tam bir kek yaptığımızda iki tarafından $\frac{1}{3}$ yenmiş olur. Yani $\frac{2}{6}$'sı yenmiş olur.]

Analysis of Participant 143's response presents that she couldn't understand that the question was referring to multiplication and she performed an enlarging operation instead of a multiplication because of her inadequate knowledge of multiplication operation of fractions. Also she had the misconception that the denominator refers to the number of pieces, regardless of unequal sizes of the pieces. In addition, in this study, one (6.25 %) interview students overgeneralized finding common denominator rule on multiplication operation. For example,

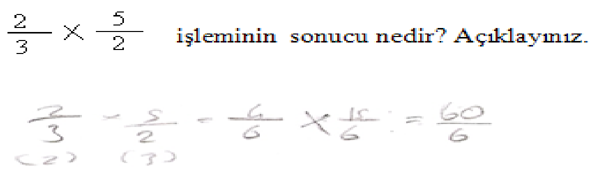


Figure 4.105 Answer of Participant 137 to item 12-b

Participant 137: "I multiplied them but I am not sure if the result is supposed to be 60. I solved this using the method I know. I found common denominators, multiplied the numerators, but I didn't change the denominators because when they are the same we write is just as it is to the result."

[Ben onları çarptım ama sonuç 60 mı olacaktı emin değilim. Bildiğim yöntemle yaptım. Paydalarını eşitledim, paylarını çarptım ama paydaları değiştirmedim çünkü paydalara aynı olduğunda onu aynen sonuca yazarız.]

Participant 137 unnecessarily found a common denominator when multiplying fractions. She multiplied numerators but kept denominator constant as in an addition operation. Moreover, she ignored simplification of fractions and it caused the misinterpretation of the result of multiplication. Similar to the response that is given above, one (6.25 %) interview participant overgeneralized addition algorithm to multiplication operation. Answer and explanation in the interview of Participant 142 is given below;

Figure 4.106 Answer of Participant 142 to item 12-b

> Participant 142: "I did this by switching the denominators. I multiplied 3 and 2. Then I multiplied 2 and 3. The result was 6. I wrote 6 as denominator. Afterwards, I multiplied 2 and 2. I multiplied 3 and 5, the result was 15. I added 4 to 15 and the result was 19."
>
> [Ben paydaları değiştirerek yaptım. 3 ile 2'yi çarptım. Sonra 2 ile 3'ü çarptım. Sonuç 6 çıktı. 6'yı paydayı yazdım. Sonra 2 ile 2'yi çarptım. 3 ile de 5'i çarptım, sonuç 15 çıktı. 15 ile 4'ü topladım ve sonuç 19 çıktı.]

When multiplying fractions, Participant 142, unnecessarily found a common denominator when multiplying fractions but unlike Participant 137, she added the product of the denominator of the first fraction and the numerator of the second one and the product of the denominator of the second fraction and the numerator of the first one. As a result, two (12.5 %) of the interview students mistakenly transferred some of the rules of addition when performing multiplication with fractions.

Investigation of fifth grade students' mistakes/difficulties on fractional concept revealed another dimension of mistakes/difficulties that were resulted from students' intuitively based mistakes/difficulties on fractions.

4.2.2. Underlying Reasons and Misconceptions behind Intuitively Based Mistakes/Difficulties

In addition to the algorithmically based mistakes/difficulties, five (31.25 %) participants chose an intuitive rule, which is only valid for natural numbers. Four (25 %) interview students overgeneralized fractional or non-fractional concepts to fraction concepts. Moreover, basic intuitions held about operation of fractions such as 'the dividend is always bigger than the divisor' (Tirosh, Fischbein, Graeber, &

Wilson, 1993, p. 18) or ‘multiplication makes bigger’ (Tirosh, 2000, p.12) performed by three (18.75 %) interview participants. The interview with Participant 148 can be presented as the first example for this matter:

Figure 4.107 Answer of Participant 148 to item 8-b

> Participant 148: “If this result were $5+\frac{1}{3}$ it would be the same with $5\frac{1}{3}$. If Sinan multiplied these by 5, the result would be larger. Emrah’s is the same with this result. Both of them are $5\frac{3}{1}$.
>
> [Eğer sonuç $5+\frac{1}{3}$ olsaydı $5\frac{1}{3}$ ile aynı olurdu. Sinan bu kesri 5 ile çarparsa sonucun daha büyük olması lazım. Emrah’ın cevabı bununla aynı, her ikisi de 5 tam 3 bölü bir.]

Two (12.5 %) interview participants, and Participant 148 was one of them, gave a correct answer to this item, but an incorrect intuition underlying this correct answer as ‘multiplication makes bigger’. It means, the misconception behind this mistake was thinking that multiplication makes the fraction larger than addition. Thus, they chose an intuitive rule, which is only valid for natural numbers, while expressing opinion about the multiplication of fractions. Moreover this Participant failed in naming fractions.

Participant 137 was the only (6.75 %) participant who made a similar mistake not in multiplication but in division operation in item 6 as follows:

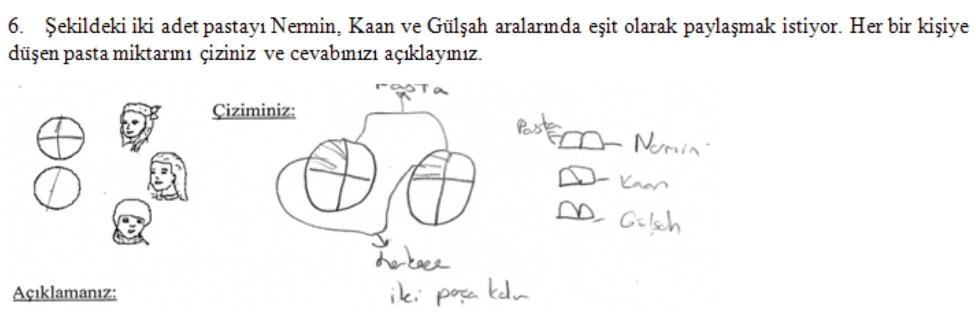


Figure 4.108 Answer of Participant 137 to item 6

Participant 137: “I cut them in half and then in half again and so there's four. I thought I would divide it like this and give everyone 2 pieces but then I realized there are excess pieces. So I think they cannot share the cake equally because it cannot be divided precisely. The number of cakes is less than the number of people to share the cake.2 cannot be equally divided by 3.”

[Şekilleri yarıya böldüm sonra yine yarıya böldüm ve dörder tane oldular. Ben bu şekilde bölmeyi ve herkese 2 parça vermeyi düşündüm ama sonra fark ettim ki fazla parçalar var. O yüzden kekleri eşit paylaşamazlar diye düşündüm çünkü tam bölünemez. Paylaşmak için kek sayısı kişi sayısından az. 2, 3’e eşit bölünemez.]

Participant 137 represented fractional quantities as only the number of parts she took into account the comparison of areas rather than the comparison of the number of parts. She did not know what to do with a remainder and what a fraction actually represents. More interestingly, she argued that it is impossible to divide 2 by 3. She stated that one cannot divide a smaller number by a larger number because it is impossible to share cakes which are less than number of people. As a result, Participant 137’s originated in the misconception that the dividend is always bigger than the divisor.

One (6.75 %) interview student made intuitively based mistake because of the transferring of natural number strategies to fractions. The following response of Participant 148 was another example to the intuitively based mistake/difficulty category:

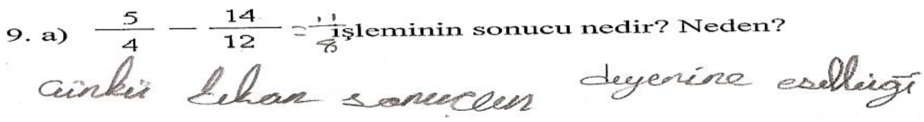


Figure 4.109 Answer of Participant 148 to item 9-a

Participant 148: “My answer was $\frac{11}{8}$, because it is not possible to subtract 5 from 4, when I subtracted 4 from 12 the result was 8 and when I subtracted 5 from 14 and found 11.”

[Benim cevabım $\frac{11}{8}$ çünkü 5, 4’ten çıkarılamaz, 4’ü 12’den çıkardığımda sonuç 8’di ve 5’i 14’den çıkardığımda da 11 buldum.]

As it is seen above, in this example, Participant 148 made a mistake while performing subtraction operation as subtracting first numerator and denominator from the second one. Her mistake was an attempt to extend subtraction algorithm for natural numbers to fractions. According to the primitive model of subtraction, minuend is always bigger than subtrahend; therefore, Participant 148’s intuitions about subtraction lead them to erroneous results. The misconception behind this mistake is ‘thinking that the bigger denominator or numerator cannot be subtracted from smaller ones’. Moreover, she had another misconception as seeing the numbers in a fraction as two unrelated natural numbers. Because, as it was seen, she considered numerators and denominators as separate entities.

In item 3, three (18.75 %) interview students made intuitively based mistakes which reason is transferring of natural number strategies to fractions. Participant 151 answered and explained item 3-a as follows:

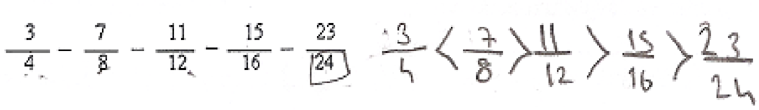


Figure 4.110 Answer of Participant 151 to item 3-a

Participant 151: “Actually I had some difficulties in this question. There was a rule for ordering fractions but I forgot. But I think $\frac{7}{8}$ is larger than $\frac{3}{4}$ because 3 is smaller than 7 and 4 is smaller than 8. I put a larger sign instead of smaller sign accidentally.”

[Bu soru ile ilgili sıkıntım var aslında. Sıralama ile ilgili bir kural vardı ama unuttum. Ama bence $\frac{7}{8}$, $\frac{3}{4}$’den daha büyük çünkü 3, 7’den küçük ve 4 de 8’den küçük. Yalnız yanlışlıkla küçüktür işareti yerine büyüktür işareti koymuşum.]

Students' mistakes are occasionally hidden behind correct answers. For instance, Participant 151 ordered fraction correctly but her explanation suggested a strategy of making separate comparisons of numerators and denominators using the ordering method for natural numbers. The rule provided an ordering consistent with natural number arithmetic but failed to incorporate with the inverse relation between numerator and denominator. It is obvious that Participant 151 influenced by appearances of fractions and she relied upon her intuitions and she saw the numbers in fractions as two unrelated natural numbers rather than as connected in any way. In addition, her explanation clearly showed that, the misconception as 'a fraction which has the larger numbers is larger' explained this kind of mistake.

Likewise, Participant 150 transferred natural number strategies to fractions. Her answer and explanation is given below:

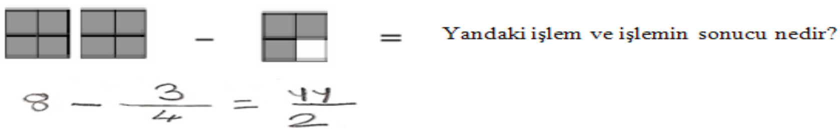


Figure 4.111 Answer of Participant 150 to item 10

> Participant 150: "Because, uh ... here is a subtraction operation with figures. On this one it is four there and on this one it's got four more. It's easier to say that there are eight shaded square there. How did I find the 2 as the denominator…..? If I multiply 8 by the result is not 2, no it is not. There are squares here, I counted them and figured out the first one as 8. Actually the denominator of the result should be 36. Uh…. I multiplied 8 by 4, the result is 36. Was it 36? Yes it is. I added 3 to 8 and the result was 11. I know doing it like what I did for addition operation."
>
> [Çünkü, ıııı… Burada da bir çıkarma işlemi var şekillerle. Bu tarafta 4 var, bu tarafta da başka bir 4 var. Burası kolay, 8 tane kare var taranmış. Paydadaki 2'yi nasıl bulmuşum….? 8 ile çarpsam 2 çıkmaz sonuç, hayır değil. Kareler var burada, onları saydım ve birincinin 8 olduğunu buldum. Aslında payda 36 olmalı. Iıııı, 8 ile 4 ü çarptım, 36. 36 mıydı? Evet 36. 8 ile de 3 ü topladım 11. Toplama işleminde yaptığım gibi yapmayı biliyorum ben.]

Her explanation shows that, Participant 150 made both intuitively based mistake and algorithmically based mistake. She used natural number counting scheme. She considered each shaded parts as individual wholes and she counted only these shaded parts without considering the fraction which represented by the first figure. She has not understood the relationship between the fractional part and the unit.

Based on the answers given in the questionnaire and interview results, it was seen that some students overgeneralized fraction concepts. It means they misapplied some of the rules about fractions that are proper for particular conditions to other unsuitable conditions.

For example, in item 3-b, Participant 137:

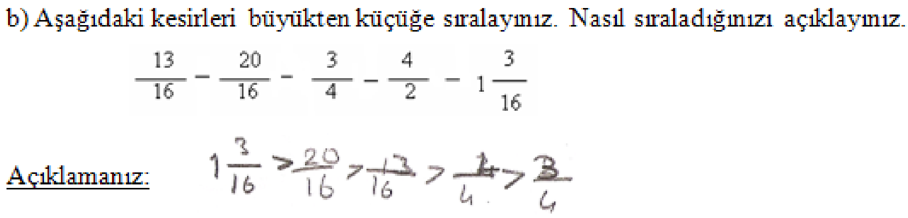


Figure 4.112 Answer of Participant 137 to item 3-b

> Participant 137: “1 $\frac{3}{16}$ has a whole; wholes are always larger than a fraction, so it is the largest one. I ordered the other fractions from largest to smallest according to their numerators, such as 20-13-4-3.”
>
> [1 $\frac{3}{16}$’ün bir tamı var; tamlar her zaman kesirden büyüktür o yüzden en büyük bu. Geri kalanı da paylarına bakarak büyükten küçüğe dizdim, 20-13-4-3 diye gidiyor.]

When the answer of Participant 137 is examined, it is seen that there was two misconceptions that could explain this kind of mistake. The first one is thinking that mixed numbers are larger than improper fractions. Because mixed numbers contain a whole number part and whole numbers are always larger than fractions. The second one is ‘the bigger the numerator, the bigger the amount of parts’ by ignoring denominators when comparing fractions. Participant 137 ordered the fractions according to sizes of the numerators and she did not refer to the denominators being the same. Similarly, answer and explanation of Participant 147 as:

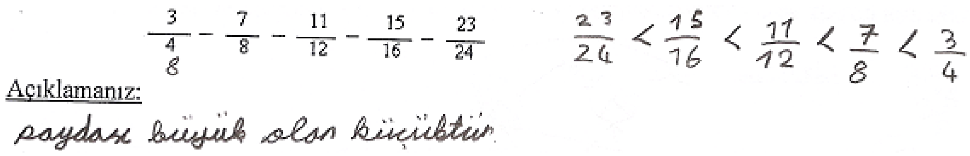


Figure 4.113 Answer of Participant 147 to item 3-a

> Participant 147: "The largest fraction is $\frac{3}{4}$. $\frac{23}{24}$ is less than $\frac{15}{16}$ because you have to look at the denominators. If we order the denominators that one is small, but in fractions that makes them big, the lesser number of pieces, the bigger the fractions are."
>
> [En büyük olan $\frac{3}{4}$, $\frac{23}{24}$, $\frac{15}{16}$'dan daha küçük çünkü paydalarına bakmamız lazım. Paydası küçükse o kesri büyük yapar, ne kadar az parçası varsa o büyüktür yani.]

The explanation of Participant 147 was associated with the misconception that referred 'thinking that it is only the denominator or numerator that determines the size of the fraction.' He did not consider the amounts represented by fractions. As in this example, ignorance of numerators when comparing fraction is considered overgeneralization of the idea that the bigger the denominator, the smaller the fraction.

In this study, it was seen that some students overgeneralized non-fractional concepts to fraction concepts such as measurement or decimals. For example, two (12.5 %) interview students, Participant 151 and 140 overgeneralized their experiences about measurement concepts to the fraction concepts.

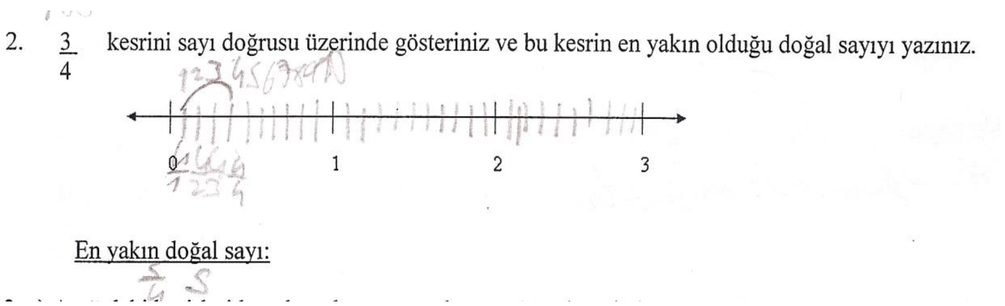


Figure 4.114 Answer of Participant 151 to item 2

Participant 151: "Here I divided the spaces between these numbers into 10 just like it is on a ruler. After, I counted and marked them like $\frac{4}{1}$, $\frac{4}{2}$, $\frac{4}{3}$ because $\frac{3}{4}$ was being asked."

[Bu sayıların arasındaki boşlukları 10 parçaya böldüm, hani cetvelde de öyle oluyor ya. Sonra saydım ve işaretlerdim $\frac{4}{1}$, $\frac{4}{2}$, $\frac{4}{3}$ diye çünkü $\frac{3}{4}$ soruluyordu.]

Similarly, Participant 140:

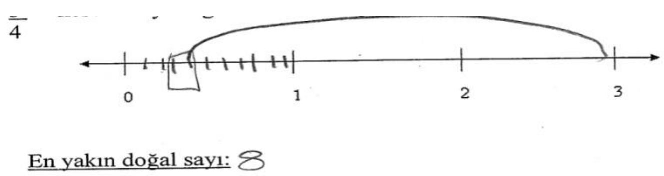


Figure 4.115 Answer of Participant 140 to item 2

Participant 140: "I divided the space between zero and one into ten parts, then I marked both 3 and 4 because it was expected to locate $\frac{3}{4}$.If we add 3 and 4, the result will be 7 and the closest number to 7 is 8."

[Birle sıfır arasını on parçaya ayırdım, sonar hem 3'ü hem 4'ü işaretledim çünkü $\frac{3}{4}$'ü işaretle demiş. 3 ile 4'ü toplarsak 7 eder, ona en yakın sayı da 8 olur.]

As it is seen in explanation of Participant 140, the reasons behind her answer were both overgeneralization of measurement concepts and transferring natural number counting scheme to fractions.

Moreover, the only interview student, Participant 137 overgeneralized decimal notation to fractions. Her response and explanation is given below:

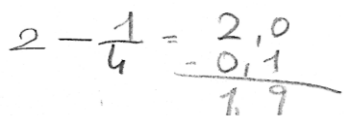


Figure 4.116 Answer of Participant 137 to item 10

Participant 137: "Here, I converted the fractions to decimals, 2 is already a whole and there are no decimals for it. For $\frac{1}{4}$, because we just take one part and it is 0, 1. I subtracted them."

[Burada kesirleri ondalık sayıya çevirdim ben. 2 zaten tam, o yüzden ondalık kesri yok. $\frac{1}{4}$ için de 1 parça alırız, o da 0,1 olur. Bunları da çıkardım.]

4.2.3. Underlying Reasons and Misconceptions behind Mistakes/difficulties Based on Formal Knowledge of Fractions

These mistakes are based on misconceptions about the nature of fractions and operations (Tirosh, 2000). Tirosh (2000) emphasized that the underlying reasons of mistakes under this category are due to both limited conceptions of the notion of fractions and inadequate knowledge related to the properties of the operations.

For item 1-a, as shown in Table 4.24, five (31.25 %) interview students made mistakes based on formal knowledge of fraction. Two (12.5 %) interview students had lack of knowledge about invariability of the whole and Participant 136 made this kind of mistake because of this reason. Participant 136 explained her reasoning as:

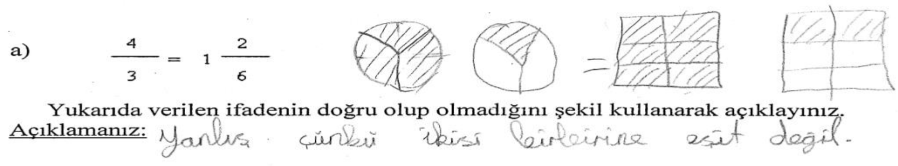


Figure 4.117 Answer of Participant 136 to item 1-a

Participant 136: "$\frac{4}{3}$ comprises a whole and $\frac{1}{3}$ but other side is $1\frac{2}{6}$, one whole and two sixths were taken. So these are not equal. I draw these fractions too. Amount of shaded parts are not equal. We shaded two parts in the second drawing."

[$\frac{4}{3}$, bir tama giriyor. Sonra da $\frac{1}{3}$' e dönüşüyor ama diğer taraf $1\frac{2}{6}$. Onda da 1 tam 6 da 2 alınmış. İkisi eşit değil, şekilde de çizdim. Taranan yerler eşit değil. İkincide 2 parça tarıyoruz.]

Participant 136 comprehended the importance of equal parts but she had no idea about the importance of equal wholes. In other words, when the student's explanation is considered, it is noticed that she actually knew how to convert an improper fraction to a mixed number but she did not know that fractions cannot represent the same amount unless they represent parts of a same-sized and shape unit.

In item 1, two (12.5 %) interview students made mistake based on formal knowledge of fraction concepts. Explanation of Participant 150 provided an example of the inadequacy of improper fraction knowledge. The following response of Participant 150 is given below:

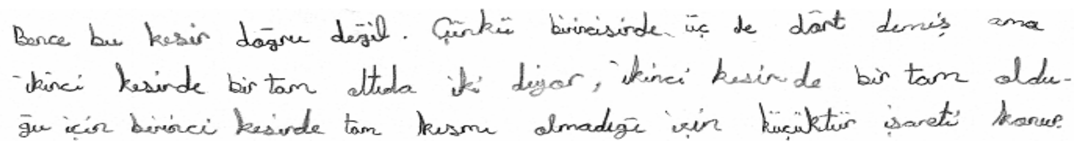

Bence bu kesir doğru değil. Çünkü birincisinde üç de dört demiş ama
ikinci kesirde bir tam altıda iki diyor, ikinci kesirde bir tam oldu-
ğu için birinci kesirde tam kısmı olmadığı için küçüktür işareti konur

Figure 4.118 Answer of Participant 150 to item 1-a

Participant 150: "I think this equivalence is not correct. One of them is $\frac{4}{3}$. The other one is not a real fraction because it has a whole. That is both number and fraction. Therefore, they are not equivalents."

[Bence bu eşitlik doğru değil. Bir tarafta $\frac{4}{3}$ var. Zaten diğeri tam bir kesir değil yani tamı olduğu için. Hem tam sayı hem kesir ya. O yüzden eşit değil.]

When confronted with a mixed fraction, Participant 150 said that it was not a fraction because in a fraction there should be only numerator and denominator. She did not know that a mixed number and an improper fraction both represent more than one whole.

In addition seven (43.75 %) interview students made mistakes in item 3-a and all eight (50 %) students shown in the Table 4.24 made mistakes in item 3-b because of inadequate knowledge of fraction ordering. The following response demonstrated that Participant 148 ordered left parts of the fractions:

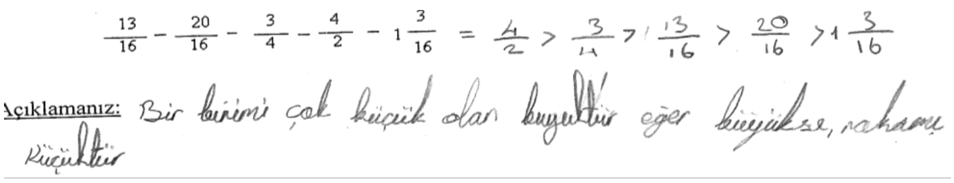


Figure 4.119 Answer of Participant 148 to item 3-b

Participant 148: "$\frac{4}{2}$ is the largest fraction and $1\frac{3}{16}$ is the smallest fraction. The biggest pieces are in $\frac{4}{2}$ because in $1\frac{3}{16}$, we ate one whole and $\frac{3}{16}$ of other whole. The amounts of remaining pieces in $\frac{4}{2}$ are less than remaining pieces in $1\frac{3}{16}$."

[$\frac{4}{2}$ en büyük kesir ve $1\frac{3}{16}$ en küçük kesir. En büyük parçalar $\frac{4}{2}$’de çünkü $1\frac{3}{16}$ ’da bir bütün yemişiz bir de diğer bütünün $\frac{3}{16}$’ ünü. $1\frac{3}{16}$ ’nin kalan kısmı $\frac{4}{2}$’nin kalan kısmından daha az.]

Participant 148 made an appropriate start and said $\frac{4}{2}$ was bigger than $1\frac{3}{16}$ then it appeared that she used an incorrect strategy and compared leftover parts instead of the number of parts taken under consideration when deciding which fraction is bigger.

Apart from the previous finding, in item 3-a, Participant 136 stated that:

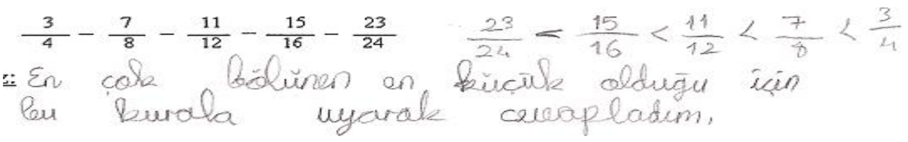


Figure 4.120 Answer of Participant 136 to item 3-a

Participant 136: “I ordered them like $\frac{23}{24} < \frac{15}{16} < \frac{11}{12}$…Because in $\frac{23}{24}$, it was divided into 24 pieces and 23 of them were taken, in $\frac{3}{4}$, it was divided into 4 pieces and 3 of them were taken. Three pieces are bigger; twenty three pieces would have to be smaller to fit the whole. I think $\frac{23}{24}$ is the smallest fraction because you have to cut it in smaller pieces. Besides, the fraction having the larger denominator is always smaller.”

[Ben $\frac{23}{24} < \frac{15}{16} < \frac{11}{12}$ şeklinde sıraladım. Çünkü $\frac{23}{24}$’de 24 parçaya ayrılmış, 23’ü alınmış. $\frac{3}{4}$’de dört parçaya bölünmüş, 3’ü alınmış. Üç parça daha büyük, bütün oluşturmak için 23 parçaya ayrılanlar daha küçük. Bence en küçük $\frac{23}{24}$ çünkü daha küçük parçalara ayırmanız gerekir. Zaten paydası büyük olan küçük oluyor.]

Participant 136's response shows she made an incorrect comparison of the sizes of parts, inverting the relation between denominator and size of a fraction. As she explained above, she ordered size of the pieces instead of the size of fractions. She also thought that denominator determines the size of the fraction. She claimed that the fraction having the larger denominator is always smaller.

In addition, six (37.5 %) interview students made mistakes based on formal knowledge of fraction comparison and equivalency. For instance, Participant 144:

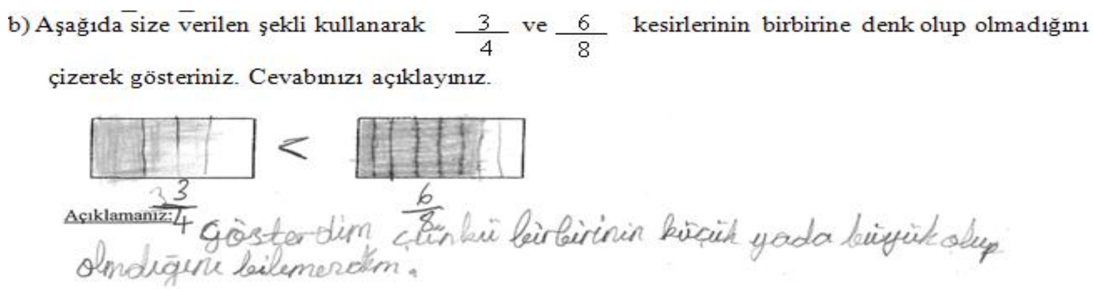


Figure 4.121 Answer of Participant 144 to item 4-b

> Participant 144: "They are not equal because, in my first drawing, 3 parts out of 4 are shaded but in my second drawing 6 parts out of 8 are shaded. I shaded 6 parts and it is more than 3 parts. $\frac{6}{8}$ is larger than $\frac{3}{4}$. If we look at like this, in $\frac{3}{4}$, if we multiply 3 by 2, the result will be 6 and if we multiply 4 by 2, the result will be 8. One of them is double of other one."
>
> [Eşit değiller çünkü ilk çizimimde, şekilde 4 parçanın 3'ü taranmış ama ikinci çizimimde 8 parçanın 6'sı taranmış. 6 parça taradım ve bu 3 parça daha fazla. $\frac{6}{8}$, $\frac{3}{4}$'den daha büyük. Şöyle bakarsak bir de $\frac{3}{4}$'te 3'ü iki ile çarparsak 6, 4'ü iki ile çarparsak 8 eder. Biri diğerinin iki katı.]

Participant 144 correctly represented the fractions on the rectangle area models as shaded parts but she could not find the connection between equivalency of the fractions and her area models. She did justify the equivalence neither by simplifying the fractions nor by matching the areas (visual spatial). She just took the increased number of shaded part into consideration. Her mistake originated in the misconception that enlarging a fraction makes it larger.

In another item, 4-a, the solution strategy of Participant 138 in the interview group is representative of the general incorrect approach of the students. As an example, Participant 138' response is given below:

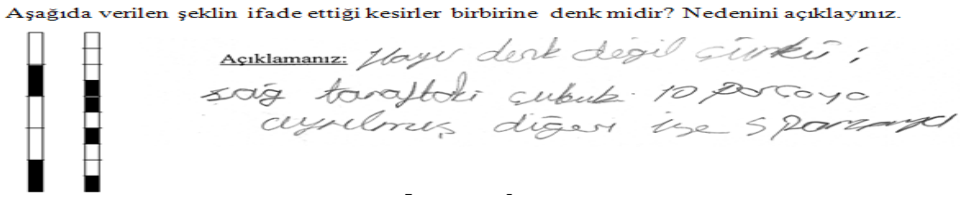


Figure 4.122 Answer of Participant 138 to item 4-a

> Participant 138: "They are not equal because the stick on the right side is divided into 10 pieces and the stick on the left side divided into 5 pieces. They divided in different ways and sizes of the pieces are not same. In $\frac{4}{10}$ more pieces was taken, it is larger than $\frac{2}{5}$."

> [Eşit değiller çünkü sağdaki çubuk 10 parçaya bölünmüş, soldaki çubuk 5 parçaya bölünmüş. Farklı bölünmüşler ve parçaların büyüklükleri aynı değil. $\frac{4}{10}$'da daha fazla parça alınmış, bu $\frac{2}{5}$'dan daha büyük.]

Participant 138 used part-part relationship to compare equivalent fractions. As it is seen in her explanation, when asked to prove her answer she said that $\frac{2}{5}$ is not equal to $\frac{4}{10}$ because they have different numbers of pieces. Participant 138 and other students who made the same mistake displayed an inadequate knowledge of equivalent fractions.

The last example is related to equal partitioning. Item 6 and item 12 were the questions which the most mistakes were made based on formal knowledge of fraction owing to lack of knowledge of partitioning. These items required equal partitioning and ten (62.5 %) interview students made this kind of mistake. Participant 151 explained her reasoning as follows:

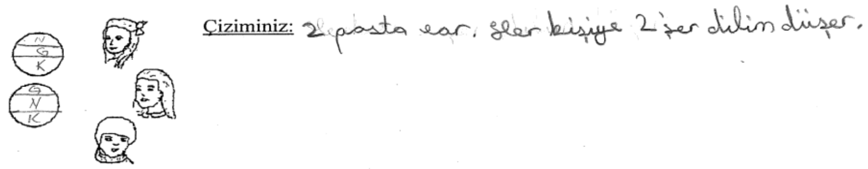


Figure 4.123 Answer of Participant 151 to item 6

> Participant 151: "I divided these by 3. First, I divided one of the cakes into three. Each person got one slice. I divided the second cake into three pieces too. Each person got one more slice. I think it is distributed evenly now because I gave Nermin one slice from the top and one slice from the middle. From my other point of view they are not even but again from another, they are even. Because it seems small (bottom pieces), I mean the middle part is bigger than others. That's why from another point of view it is bigger. What was her name? Nermin? They are not even if we add some of them."

> [Ben bunları 3'e böldüm. Önce bir tanesini 3'e böldüm. Herkes bir dilim aldı. İkinci keki de 3 parçaya böldüm, herkes birer dilim daha aldı. Bence şimdi eşit dağıtıldı çünkü Nermin'e bir dilim yukarıdakinden, bir dilim ortadan verdim. Ama bir baktığımda eşit olmadı , bir baktığımda da eşit oldu. Çünkü bu parça küçük gibi (en üst parça), demek istediğim ortadaki parça diğerlerinden daha büyük. Yani böyle bakınca o büyük. Adı neydi? Nermin? Bunların bazılarını eklerseniz eşit olmadılar.]

For, Participant 151, fractions appeared to be defined solely by the number of parts without attention to the equality of all of the parts. She had lack of knowledge about the fractions should always consist of equal portions of a whole as she drew lines in the circles horizontally. The misconception as thinking that the denominator refers to the number of pieces, regardless of unequal sizes of the pieces explains this mistake.

Analysis of students' responses revealed that, another reason of the students' mistakes/difficulties based on formal knowledge of fraction is failing to form connections between symbols, procedures and representations. Mistakes of three (18.75 %) interview students in item 1-a and two (12.5 %) interview students in item 1-b made mistakes based on formal knowledge of fractions because of initial disconnection between knowledge of fraction symbols and representations.

For instance, in item 1-a, Participant 138 claimed that:

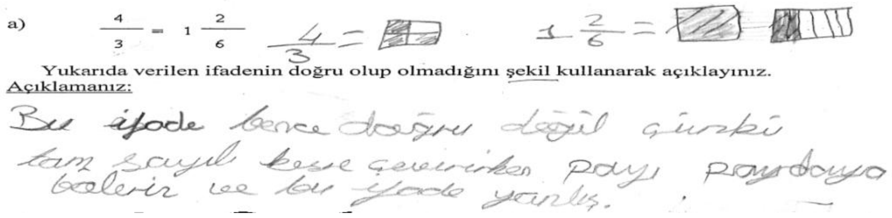


Figure 4.124 Answer of Participant 138 to item 1-a

> Participant 138: "I think this statement is not correct. I tried to make these fractions equivalent, I multiplied 3 and 2, and I found 6. Then I multiplied 4 and 2, I found 8 but in here there is 2 instead of 6 and 6 instead of 8."
>
> [Bence bu ifade doğru değil. Ben bunları denk yapmaya çalıştım, 3 ile 2'yi çarptım, 6 buldum. Sonra 4 ile 2'yi çarptım, 8 buldum ama burada 6'nın yerinde 2, 8'in yerinde 6 var.]

Participant 138 had difficulty seeing how $\frac{4}{3}$ relates to the whole. She misconstrued the meaning of $\frac{4}{3}$ and perceived it as $\frac{3}{4}$ as she divided the square area model into four parts, shaded three of them for representing $\frac{4}{3}$. She also tried to enlarge $\frac{4}{3}$ to equalize the denominators but she was unaware of whole part of the improper fraction. As a result, in the answer of Participant 138 there is no transition from symbols to her procedure or visual representations. Another student, Participant 150 responded item 1-b as:

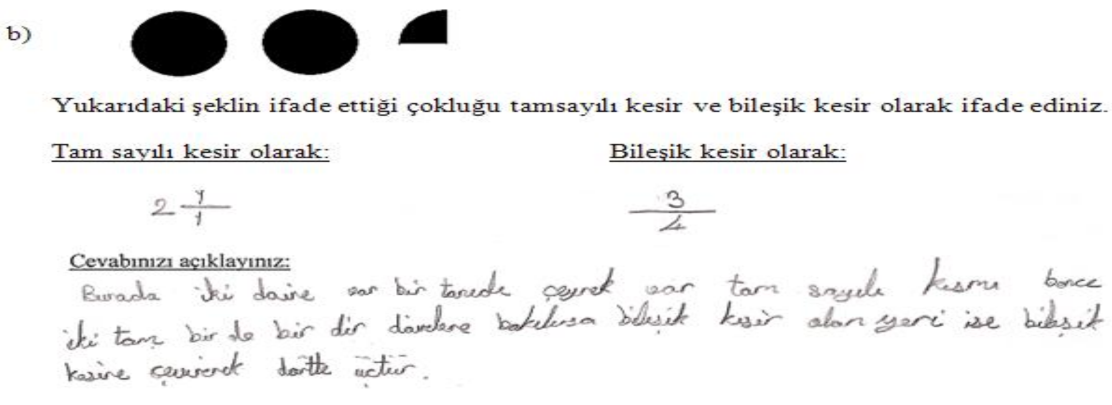


Figure 4.125 Answer of Participant 150 to item 1-b

Participant 150: "There are two circles. These are wholes, namely they are unpartitioned. There is also a half, one piece and it was shaded. Therefore the picture represents $2\frac{1}{1}$. If we divide all figures into four parts, it is seen that, in the last figure, three of them are not taken; only one part is taken. That is why I wrote $\frac{3}{4}$ as improper fraction. I have not considered whole images."

[İki tane daire var. Bunlar bütün, yani bölünmemiş. Bir tane de yarım var, bir parça ve taranmış. Yani şekil $2\frac{1}{1}$ gösteriyor. Eğer bütün şekilleri dörde bölersek, son şekilde 3 tanesinin alınmadığını gösterir, sadece bir parça alınmış. Bu yüzden bileşik kesir olarak $\frac{3}{4}$ yazdım. Bütün olanları katmadım buna.]

She indicated that there were two wholes so she wrote 2, after, she noticed that the third part was not a whole but there was one piece and it was painted .Thus she concluded that it was $2\frac{1}{1}$. Based on her answer and explanation, she made mistake caused by not only translating the picture to the symbol but also translating the symbol to the picture. She also erroneously used a proper fraction and a mixed number to represent the same figure.

Lastly, in this study, two (12.5 %) interview students ignored size of the unit and by this way they made mistake based on formal knowledge of fraction in item 2. Participant 136's answered and explained item 2 as:

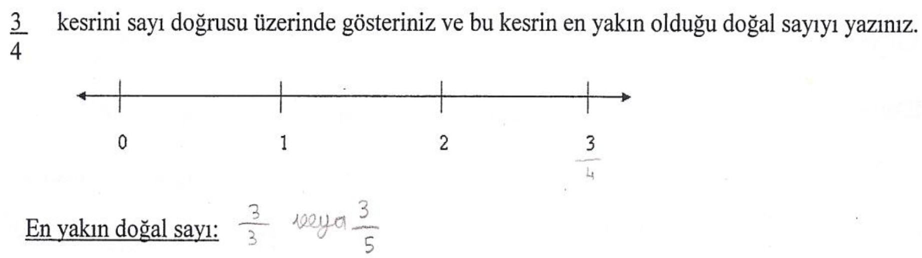


Figure 4.126 Answer of Participant 136 to item 2

Participant 136: “I placed the fraction at the end of the number line because $\frac{3}{4}$ coincide with here. In here, fractions continues in the way as $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$. So, $\frac{3}{4}$ coincide with the place of 3.

[Ben kesri sayı doğrusunun sonuna koydum çünkü $\frac{3}{4}$ oraya denk geliyor. Burada kesirler $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$ şeklinde devam ediyor. O yüzden $\frac{3}{4}$, 3’ün olduğu yere denk geliyor.]

Participant 136 made mistake because of the lack of knowledge of meaning of whole. As mentioned above, she ignored size of the unit, instead using entire segment as an equivalent partitioned whole.

4.2.4. Underlying Reasons and Misconceptions behind Mistakes/difficulties Due to the Misunderstanding of the Problem

Some students made mistakes due to misunderstanding of the problem or misunderstanding what is being given and what is being asked to them especially in fraction word problems (e.g., item 5 and item 12). Hence they performed the operations incorrectly. As shown more detailed in Table 4.24, some students inadvertently made mistakes while solving some items. As a result, these students’ procedures were correct however their final answers were incorrect because of lack of attention. Examples of their explanations and answers are given below:

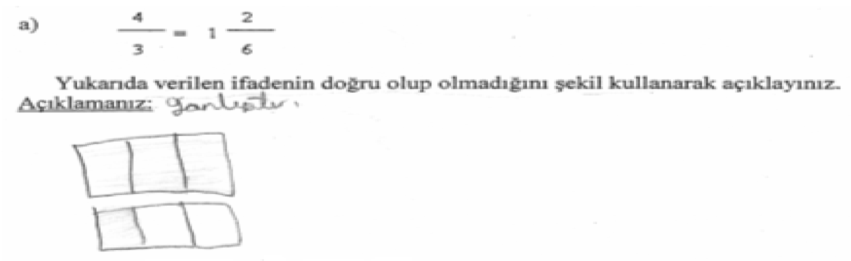


Figure 4.127 Answer of Participant 149 to item 1-a

Participant 149: “Here, when we multiply 6 by 1 it’s 6. When we add 2 to that, it is 7… it is 8. Then, is it right? Yes, one minute, this is actually correct. If we multiply 3 by 2 it gives us 6, multiplying 4 by 2 gives us 8. When we do it this way, they are equal. I made a mistake.”

[Burada, 6 ile 1’I çarparsak 6 olur. 2 eklediğimizde, 7…, bu 8. O zaman, doğru mu? Evet, bir dakika, bu aslında doğru. Eğer 3’ü 2 ile çarparsak 6 verir bu bize, 4 ile 2’yi çarparsak da 8 gelir. Bu yolla yaptığımızda, bunlar eşit. Hata yapmışım.]

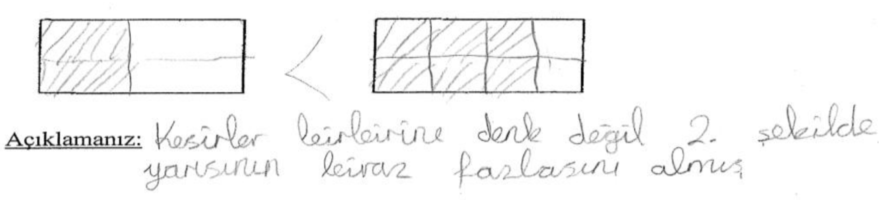


Figure 4.128 Answer of Participant 136 to item 4-b

Participant 136: “These are not equivalent. As I also drew in the figures, one of them is half. But one minute, it says $\frac{3}{4}$ here, I thought it was $\frac{2}{4}$, that’s why I said they were not equivalent. They may be equivalent if we do it this way.”

[Bunlar denk değil. Şekilde de çizdiğim gibi bunlardan biri yarıma eşit. Ama bir dakika, burada $\frac{3}{4}$ diyor, ben $\frac{2}{4}$ diyor zannettim, o yüzden denk değiller dedim. Ama böyle yaptığımızda denk olabilirler.]

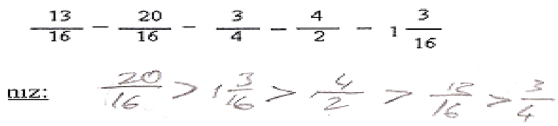


Figure 4.129 Answer of Participant 146 to item 3-b

Participant 146: "$\frac{20}{16}$ becomes $1\frac{4}{16}$. And there is $1\frac{3}{16}$. The first one is larger. Now I'm thinking about how I can enlarge $\frac{4}{2}$'s numerator to 16. We have to multiply it by 8. 2 times 8 is 16, 4 times 8 is 32, I subtract 16 from 32, 16 plus 16 is 32 anyway. Therefore, this one is the largest one."

[$\frac{20}{16}$'yi çevirdiğimizde $1\frac{4}{16}$ olur ve burada $1\frac{3}{16}$ var. Birincisi daha büyük. Şimdi $\frac{4}{2}$'yi paydası 16 olacak şekilde nasıl genişletebilirim diye düşünüyorum. 8 ile çarpmalıyız. 2 kere 8, 16, 4 kere 8, 32. 32'den 16'yı çıkarırım, 16. 16 ile 16'yı toplarsam 32, neyse. Sonuç olarak bu en büyük.]

Additionally, in this study, students made mistakes because they did not understand the problem. Some of the students were not competent in using mathematical language effectively. They could not understand what was being given or what was being asked correctly and performed faulty operations. For example, in item 11-b, Participant 146 explained her reasoning as:

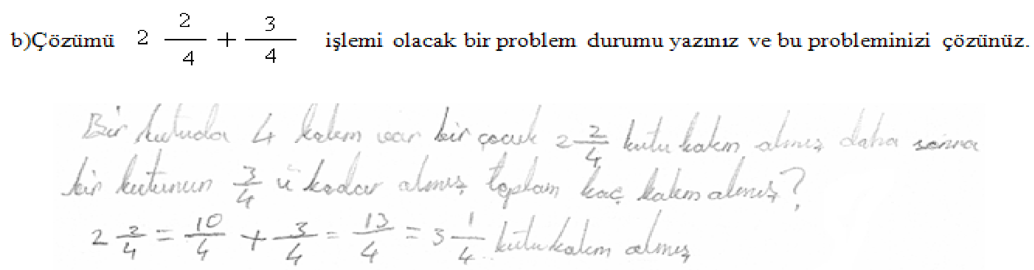


Figure 4.130 Answer of Participant 146 to item 11-b

Participant 146: "In this operation… Actually I firstly do the operations because I got confused. There were 4 pencils in a box. 4 times 2 is 8. And this child had $\frac{2}{4}$ more, it means 2 more. He took a total of 10 pencils; this was the idea that popped up in my mind at that time."

[Bu işlemde…. Aslında ben önce işlemi yaptım çünkü kafam karıştı. Bir kutuda 4 kalem varmış. 4 kere 2, 8 eder. Sonra bu çocukta $\frac{2}{4}$ kutu daha varmış, 2 daha var demek. Toplamda 10 kalem almış; o anda aklıma böyle bir fikir geldi.]

Seven (43.75 %) interview students tried to create a word problem appropriate to the operation in item 11-a however they couldn't determine what was being given and asked correctly like Participant 146. She not only gave unnecessary information in her question (number of pencils in the box) but also asked the number of total pencils despite giving number of the boxes.

In addition to explanation of reasoning above; five (31.25 %) interview students made similar mistakes in item 5 and one of them explained her response to item 5 as follows:

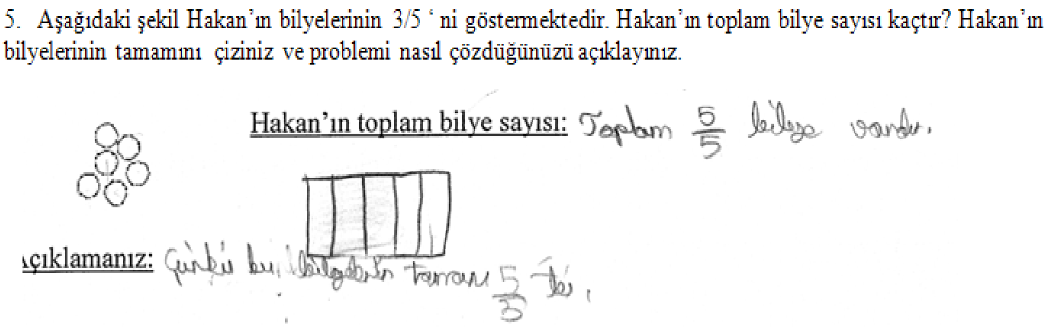


Figure 4.131 Answer of Participant 149 to item 5

Participant 149: "It gives us $\frac{3}{5}$ of it in the question and asks for the whole. The total is $\frac{5}{5}$ and it forms a whole."

[Soruda bize $\frac{3}{5}$ vermiş ve tamamını soruyor. Tamamı $\frac{5}{5}$ dir ve bir bütün olur.]

Participant 149 used a model in her solution. But she did not make any reference to sizes parts shown in pictures or materials. In the marble problem students did not see six marbles was $\frac{3}{5}$ of the whole. Participant 149 did not understand what was being given and what was being asked.

Similarly, Participant 148:

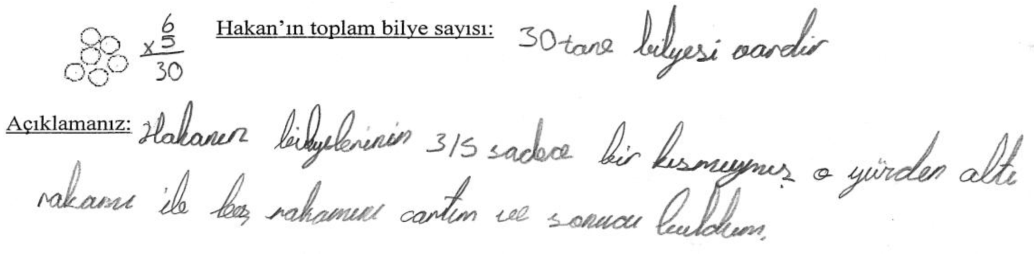


Figure 4.132 Answer of Participant 148 to item 5

> Participant 148: "It said three fifths so I multiplied 6 by 5. If it were two fifths, I would multiply 5 by 6… But the result would be the same and it wouldn't be right because $\frac{2}{5}$ and $\frac{3}{5}$ are not the same. Well, then I would divide it by 2. I did it like this but I should have divided it by three, otherwise the result is always thirty."
>
> [Burada beşte üçü diyor bu yüzden ben de 6 ile 5'i çarptım. Eğer beşte iki deseydi 5 ile 6'yı çarpardım … Ama sonuç aynı çıkardı ve doğru olmazdı çünkü $\frac{2}{5}$ ile $\frac{3}{5}$ aynı değil. Öyleyse, o zaman 2'ye bölerdim. Ben böyle yaptım ama 3'e bölmeliydim, aksi halde cevap hep 30 çıkar.]

Participant 148 picked two numbers from the problem and applied some procedures to them. She misconstrued the 5 number in the denominator. Participant 148 and 149 had lack of understanding about the relationship between a part and a quantity. They did not understand relationship between a part and a quantity.

In another example, Participant 136 seemed to make a single operation but when her explanation is considered, it has been understood that there were many other operations behind this single operation. Her explanation of reasoning is as follows:

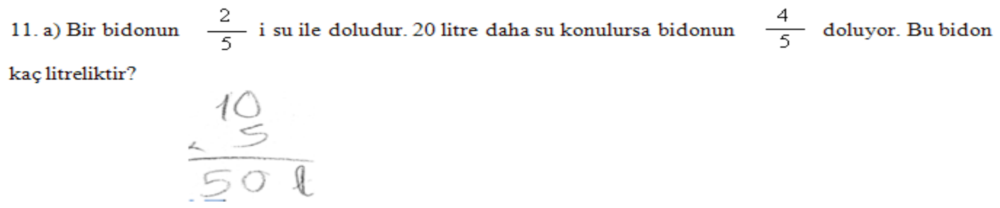


Figure 4.133 Answer of Participant 136 to item 11-a

Participant 136: “The water was filling with 5 jerry cans, that’s why $\frac{2}{5}$ of it was 20 liters. When another 20 liters of water is added it is $\frac{4}{5}$. I divided 20 by 4, it is 5. And I found 10 by multiplying this 5 and 2. I multiplied 2 and 5 in the $\frac{2}{5}$ fraction. And I multiplied it with 5 to find the total.”

[5 bidon ile su doluyormuş, o yüzden 20 litre oluyormuş $\frac{2}{5}$’i. 20 litre konulursa $\frac{4}{5}$ oluyormuş. O yüzden 20’yi 4’e böldüm ve 5 buldum. 10’u 5 ile 2’yi çarparak buldum, $\frac{2}{5}$’teki 5 ve 2’yi. 10 ile de 5’i çarptım tamamı için.]

Ten (62.5 %) interview students wrote down a mathematical statement involving one or more operations on the numbers given in the problem as did Participant 136.Since she did not have a basic understanding of mathematical language she stated that the water filled with 5 jerry cans but in the problem there is no information like that and this statement was also meaningless.

4.2.5. Underlying Reasons and Misconceptions behind Mistakes/difficulties Due to the Missing Information

In this study, in the interview group, five interview participants gave correct but incomplete information required by the tasks to various items. In all the examples given below, although participants started with correct strategy and they showed a part of correct solution, they left their answers incomplete. They likely had memorized the strategies but they did not remember steps of the algorithms.

For instance, Participant 148;

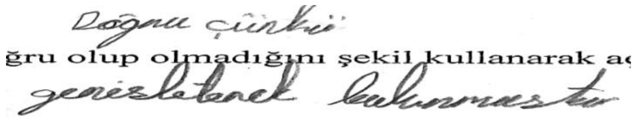


Figure 4.134 Answer of Participant 148 to item 1-a

> Participant 148: “We always do enlarging in this kind of questions. 2 by 4... Actually it should be the opposite, enlarging 4 by 2... When we divide it by 2, the result is 2. Anyway it is kind of clear if we look at the numbers. I thought like that.”
>
> [Bu tip sorularda hep genişletme yaparız. 2, 4 ile… Aslında tam tersi olmalıydı, 4’ü 2 ile genişletme… Biz onu 2’ye böldüğümüzde, sonuç 2 olur. Neyse, sayılara bakarsanız açıkça görülüyor zaten, ben böyle düşündüm.]

Similarly, Participant 146:

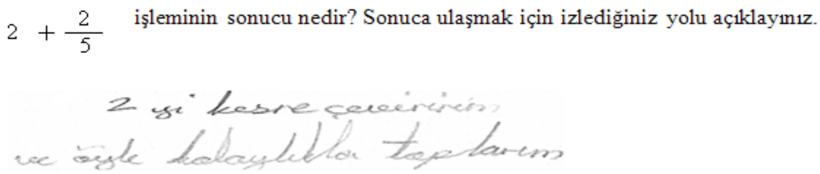


Figure 4.135 Answer of Participant 146 to item 8-a

> Participant 146: “Now what is being asked is to add 2 and a fraction but since 2 is a number we have to convert it. I learned that we have to convert it in our lessons but I forgot how to make an addition operation.”
>
> [Şimdi 2 ile kesrin toplanmasını sormuş ama 2 sayı olduğu için çevirmemiz lazım. Ben çevirmemiz gerektiğini öğrenmiştim derste ama toplama işlemi nasıl yapılıyordu unuttum.]

Participant 146 remembered and stated that 2 have to convert to a fraction. But as she expressed she forgot how convert and then add fractions.

Finally, the last example was Participant 138:

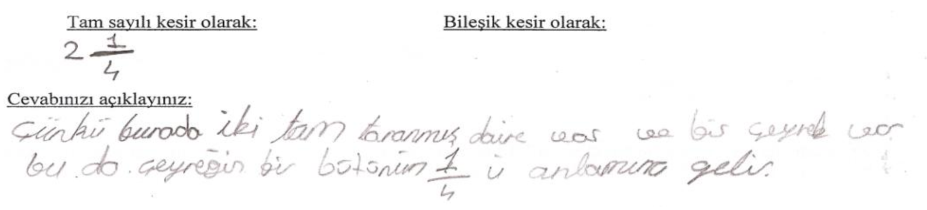


Figure 4.136 Answer of Participant 138 to item 1-b

Participant 138: "There are 2 wholes and a quarter in the figure so I wrote $2\frac{1}{4}$ but I can't find out now how I would write this figure as a mixed number. The denominator is 4 but I couldn't find the numerator."

[İki tane bütün ve bir çeyrek var burada, o yüzden $2\frac{1}{4}$ yazdım ama şekli tam sayılı kesir olarak nasıl yazacağımı bulamıyorum şimdi. Payda 4 de payı bulamıyorum.]

Participant 138 stated that the figure represents $2\frac{1}{4}$ correctly but it was seen that she could not find out how she would write this figure as a mixed number or how she would convert this mixed number to the improper fraction.

As a consequence, this study showed that fifth grade students made mistakes and they had difficulties regarding basic fractional concept and operations. In addition, there were various reasons and misconceptions behind those mistakes and difficulties.

4.2.6. Summary of Findings

The purpose of this study was to investigate mistakes made by elementary fifth grade students regarding basic fractional concept and operations, and difficulties that they encounter. The other purpose was to investigate underlying reasons and misconceptions behind those mistakes and difficulties. The sample of the research is composed of 151 students from fifth grade who are enrolled in a public school in Eskişehir city center in the 2009-2010 academic years. In line with the expert opinion, the OFQ consisting of 12 questions has been prepared to determine students' mistakes. Moreover, 16 of those 151 students selected for interviews to determine students' misconceptions and reasons behind those mistakes and difficulties.

As mentioned before, the first part of the analysis dealt with the students' mistakes and difficulties based on individual items and frequency of these mistakes they made attempting to answer OFQ are introduced based on individual questions. If it is need to summarize findings of this part, first of all, many students failed to demonstrate any cognitive evidence of converting an improper fraction to a mixed fraction. It means, they were not able to represent the figure neither as a mixed number nor as an improper and they were not able to transform them to each other. They also drew incorrect pictorial representations of improper fractions and mixed numbers.

Moreover, findings revealed that students who were unsuccessful at locating a proper fraction on the given number line did not consider the size of the whole; some students placed division lines under natural numbers on the number line and wrote denominator of the fraction to all denominators then put a mark on 3 because the numerator was three. Moreover, many students ignored that spaces between numbers (wholes) should be divided into equal parts on the number line. In addition, some participants confused number line with measure of length and divided it into ten equal parts.

Furthermore, it was seen that a large portion of students made mistakes on ordering fractions. They placed fractions in reverse order and stated that 'fraction which has larger denominator is smaller than others or they reversed the order and stated that 'fraction which has larger numerator is smaller than others'. Moreover, many students ordered fraction incorrectly, compared sizes of the equal pieces. In addition, though not many but some students gave correct answer but demonstrated incorrect reasoning. They stated that 'fraction which has larger numerator is larger than others'. Some students considered the mixed number as the largest fraction among all the others because it had a whole.

Additionally, findings of this study showed that students had difficulties in expressing equivalent fractions and suggested three invalid strategies; the first one was stating the fractions that were represented by images were not equal since sizes of shaded pieces were not the same. The second one was denoting fractions which were represented by images were not equal but this time the reason of the idea was numbers of pieces being not equal and the last one was stating the fractions that were represented by images were not equal since they were not divided in the same way. Analysis of data revealed that students have difficulties calculating the whole when a fractional part is given.

Moreover, nearly half of the students did not accurately partitioned two cakes among three people. It was seen that, some of those students used a rectangle area representation instead of a circular area and some students considered the number of parts rather than the area of the parts. Moreover, some of the participants drew appropriate picture but did not demonstrate clear quotient understanding because they failed to represent the amount each person would get as fraction. Substantial number of students stated that two cakes could not be shared by three people evenly and some of them added one more cake or claimed that a few pieces of cake outnumbered.

Specifically, in items that linked to additive operations, some students tried to find lowest common denominator but they did not multiply numerators with required. Moreover, many students added numerators and added denominators. They did not know that fractions cannot be added unless they represented parts of a same-sized unit. Also, some of the participants added all numbers to determine the new numerator and added denominators to determine the new denominator. Lastly, a few students were leaving the same numerator and denominator. In the item that required performing addition operation with a fraction and a natural number, many students decided to use an algorithm of adding natural number and numerator or adding naturalnumber with both numerator and denominator. Besides some of the participants who tried to convert the whole number to improper fraction but have failed. Moreover, item 8-b showed that students had difficulty interpreting mixed numbers correctly. In this study, fifth grade students did not interpret $5\frac{1}{3}$ to mean $5+\frac{1}{3}$, but rather as $5x\frac{1}{3}$, $5\ \frac{1}{3}$ or $\frac{5}{3}$.

In the items that required performing subtraction operations, many students rewrote the example as $\frac{14}{12}-\frac{5}{4}$ and got answers of $\frac{9}{8}$ (subtracted the numerators and subtracted the denominators) or they said it is not possible to subtract. They thought that the bigger denominator or numerator cannot be subtracted from smaller ones. It was seen that, students had difficulty in finding the least common denominator in subtraction operation. Differently, when it was expected to transform figures to symbolic fraction representations and subtract a fraction from a natural number, students counted the parts separately, or they thought of it as either one whole, or eight wholes. In addition, some students wrote $\frac{8}{8}$ or $\frac{4}{4}$ instead of 2.

Moreover, students made mistakes in both solving and creating word problem about addition and subtraction of fractions. They applied disintegrated algorithms using different combinations of numbers which was given in the problem statement or they used multiplication or addition of numerator and denominator of the fraction given in the problem. Also, many students created a word problem which was mathematically correct while being incorrect in the terms of fraction concept and it was seen that they had difficulty in defining the unknown target in their problems and their word problems were not phrased in terms of the same whole.

One of the aims of the OFQ was to examine students' capability to calculate a part of another fraction. The findings revealed that, many students did not show any individual model or they drew inappropriate figures that represent $\frac{1}{3}$ of a half cake. In contrast, some students drew appropriate pictures but amount of the eaten cake was incorrect. In the item related to multiplication of proper fractions, some students mistakenly transferred the rule of addition when performing multiplication with fractions and some other students mistakenly used cross-multiplying. Moreover, many participants misapplied the standard multiplication algorithm. All of the students failed to reduce the fraction before multiplying.

In the second phase, according to the data analysis of interviews of 16 students, students' mistakes and difficulties grouped under five categories as: algorithmically based mistakes/difficulties, intuitively based mistakes/difficulties, mistakes/difficulties based on formal knowledge on fractions, mistakes/difficulties due to the misunderstanding of the problem, and mistakes/difficulties due to the missing information in solution. It was determined that the findings of this study regarding underlying reasons of students' algorithmically based mistakes/difficulties were: rote memorization of the algorithm and inadequate knowledge of operations. The reasons behind intuitively based mistakes/difficulties were: overgeneralization of a fraction operation in another operation, transferring of natural number strategies to fractions and overgeneralization of fraction and non-fractional concepts. The reasons behind mistakes/difficulties based on formal knowledge of fractions were: inadequate knowledge related to the fraction and initial disconnection between knowledge of symbols and representations. And lastly, the underlying reasons of mistakes/difficulties due to the misunderstanding of the problem were: lack of attention, lack of adequacy in mathematical language and unconnected learning (lack of memorizing isolated facts)

Moreover, according to data analysis of interviews, students' misconceptions regarding basic fractional concepts and operations were identified. First of all, item 1 revealed that students conceptualized that all fractions are less than one. Because they stated that fraction given in the item was not a real fraction because it has a whole. Moreover, in item 1 and 4, many students thought that reducing a fraction makes it smaller and many of them use the word 'multiplication and division instead of enlarging and reducing. Item 2 showed that, many students ignored size of the unit, instead using entire segment as a whole. Item 3 revealed that students had the misconception that thinking that it is only the denominator or numerator that determines the size of the fraction. In the same item, they also believed that the fraction having the larger denominator is always larger/ smaller. As it was seen in items 3- 4- 7 and 9, students saw the numbers in a fraction as two unrelated whole numbers: considering numerators and denominators as separate entities rather than as connected in any way. As for the misconceptions regarding operations, item 8 revealed that, many students thought that multiplication always makes the fraction larger than addition and item 9 revealed that students had the misconception that thinking that the bigger denominator or numerator cannot be subtracted from smaller ones. In items which fractions represented with figures, such as item 9 and 10, students used part - part strategy instead of part - whole ones: When interpreting the shaded parts of a whole figure, thinking that the numerator is the number of shaded parts and the denominator is the number of unshaded parts or vice versa. Lastly, items which required partitioning knowledge such as item 6 and 12 revealed that students had the misconception that thinking that the denominator refers to the number of pieces, regardless of unequal sizes of the pieces and thinking that one cannot divide a smaller number by a larger number (item 6).

CHAPTER V

DISCUSSION, IMPLICATIONS AND RECOMMENDATIONS

The purpose of this study was to investigate mistakes made by elementary fifth grade students regarding basic fractional concept and operations, and difficulties that they encounter. The other purpose was to investigate underlying reasons and misconceptions behind those mistakes and difficulties. In this chapter, findings of quantitative and qualitative analyses were discussed under two main sections based on the research questions. In the first section, mistakes and difficulties of fifth grade students regarding fractions, in the second section, reasons behind those mistakes, difficulties, and misconceptions were discussed with references to the previous studies. This chapter also addressed a discussion of major findings, recommendations for further studies and educational implications.

As it was expected, close examination of the students' answers of the fraction questionnaire demonstrated mistakes and difficulties regarding fraction concepts and operations that exist among the students who participated in the study. Accordingly, in this part, examples of common mistakes and difficulties, underlying reasons and misconceptions behind those mistakes and difficulties were discussed.

5.1. Fifth Grade Students' Mistakes and Difficulties Regarding Fractions

The purpose of this study was to investigate mistakes made by elementary fifth grade students regarding basic fractional concept and operations, and difficulties that they encounter. The results of the study revealed an overall lack of experience of students with basic fraction concepts. In other words, research findings that were reported in findings chapter showed that fifth grade students have mistakes and difficulties with fractions. Considering the students' mistakes and difficulties, based on OFQ items, several general specific mistake patterns were identified: First of all, students made mistakes regarding translation of symbolic representations to pictorial and verbal representations or vice versa, and translating real life problems to symbolic representations. For instance, many students couldn't determine the fractions that belong to the fraction model (e.g. item 1, 4, 7 and 10) and only a few students used pictorial representations to help them answer regarding questions (e.g item 1, 4, 5 and 11). It was also seen that, students were able to solve symbolic problems but did not to use any representations to validate their answers or vice versa (e.g. although they could form equivalent fractions pictorially, they failed to interpret them as being the same symbolically). These findings were supported by Mack (1990) since in her study with sixth-grade students; she stated that students have initial disconnection between knowledge of symbols, procedures, and visual representations.

Secondly, students should comprehend meaning of the fraction and fractional operations. But the results showed that, fifth grade students have no idea about what fraction is and many of the students chose to use an algorithm but they were not really sure of the correct process, demonstrating several of the mistake patterns. They have a tendency for using procedures even in items that required pictorial representations but they were unable to validate these procedures. So, students need to understand the meaning of each operation not just do the computations and they should decide which operation is needed in particular situations. But many of the mistakes produced unreasonable answers, indicating that doing an operation on fractions is not connected to understanding the operation. This approach yielded a host of illogical answers. Such inconsistent results demonstrate a lack of knowledge of fraction and computation. In this context, these findings again concur with the findings of Mack (1990) because in her study, she stated that students mostly trusted answers obtained from applying incorrect procedures more than those obtained from drawing on knowledge. The main reason behind them might be the insufficient attention to the meaning of fraction and fraction operations given in the elementary mathematic program. Because, in the third, fourth and fifth grade mathematics student textbooks, the definition of fraction or any fraction operations have not been mentioned. Only in the third grade teacher guide book, teachers were advised that teaching fractions should be taught as numbers corresponding to equivalent parts of a whole.

And the other important finding was students had difficulties in understanding basic fractions concepts such and they made mistakes about reading fractions (e.g. they read $\frac{4}{3}$ as three fourths), modeling fractions with equal parts (e.g.

some students seemed not to understand that the parts in fractions must be equal), and defining the whole with location of numerator and denominator. Those findings were consistent with results of Pesen (2008) where he stated that students had common mistakes in dividing the whole into equal parts. Furthermore, he stated that students had more difficulties in dividing the circular shapes into equal parts compared to rectangular shapes and this statement supports one of the findings of this study. Besides, the participants were confused about the location of the numerator and denominator, and one of them was placed instead of the other. These findings were parallel with results of the study of Newstead and Murray (1998). In their study, which was done on fourth and sixth grade students, many students stated meaning of $\frac{4}{5}$ incorrectly and drew and shaded $\frac{4}{3}$ incorrectly.

When fifth grade mathematics objectives taken into consideration, it is seen that the first objective is related to conversion of a mixed number to an improper fraction and vice versa. The other objectives are related to comparing of a fraction with a natural number, ordering and equivalence of fractions, determination of a whole when some part of it is given as a proper fraction, operations with fractions and solving word problems with fractions. Besides, as mentioned before, items in OFQ corresponded to these objectives. Therefore, in this section, to continue the discussion under these headings was more favorable.

5.1.1. Mistakes and Difficulties Regarding Improper Fractions/ Mixed Numbers

It was seen that the concepts of mixed numbers and improper fractions were really confusing for students because the participants had many misconstructions on improper fractions and mixed numbers. Item 1 showed that improper fractions were considered to be more difficult than proper fractions. This finding is in agreement with Kamii and Clark’s (1995) findings which showed learning improper fractions are more difficult than learning proper fractions. The present study indicated that many fifth graders understand $\frac{a}{b}$ as a representation of a part and a whole relationship; for example $\frac{3}{4}$ means ‘three out of four’. But this was problematic when they attempted to interpret $\frac{9}{4}$. This was consistent with findings of Brown (1993), Ball (1990) and Neumer (2007) since they stated that students do not understand that a numerator can be larger than the denominator, as well as the fact that a whole number can be written next to a fraction. Accordingly, in item 1-a, Participant 126 stated that '$1\frac{2}{6}$ is both whole number and fraction and also $\frac{4}{3}$ is smaller than a whole. 1 $\frac{2}{6}$ has a whole. Thus, they are not equal'. Participant 126's lack of knowledge about invariability of the whole could be the underlying reason of her mistake. Also, this response was like an evidence of missing the whole point and this finding of the current study is consistent with this of Hackenberg (2007) who elaborated on this problem by describing the construction of improper fractions. Her detailed analysis of sixth-grade students' work with improper fractions showed that participants of the study had confusion over the construction and meaning of the improper fractions. Moreover, the present study shows that fifth grade students understood neither the visual representation of $2\frac{1}{4}$ nor its equivalency to $2+\frac{1}{4}$. Changing an improper fraction into a mixed number was generally known as dividing the numerator by the denominator and placing the remainder as the numerator of the fractional part by students. But in this study, many students merely applied the rule without reasoning and explicated this procedure inaccurately. For instance, in item 1-b, Participant 150 stated that 'this figure was 2 $\frac{1}{4}$ and I translated it to improper fraction as $\frac{6}{8}$ because four times two is eight as denominator and 2+1=3, 3x2=6 is the numerator'. She tried to apply a 'procedure' but she could not justify her procedure to translate a mixed number to an improper fraction. Participant 150 did not recall the procedure of translating an improper fraction to a mixed number. This showed that construction of improper fractions and mixed numbers are no easy accomplishment and students havent necessarily established such fractions as numbers.

As another subject, it was seen that many students' weakest areas were measurement model that discussed below.

5.1.2. Mistakes and Difficulties Regarding Location of Fractions on a Number Line

In alignment with previous studies, it is suggested that the number line comprises a difficult model for students to manipulate (Baturo & Cooper, 1999). The participants of this study had difficulties with various models, especially with the number lines that are mathematical models used to illustrate fractions as quantities and the measure interpretation. As mentioned in the previous chapter, item 2 required students to locate $\frac{3}{4}$ on a number line and many students failed as stated by Mitchell and Horne (2008). They studied on location of fractions on a number line and claimed that many students could not be able to locate a proper fraction to a number line. Findings of this study and of Mitchell and Horne (2008) demonstrate striking similarities. In both studies the participants perceived the whole number line as a single whole, they perceived the numerator and denominator as two separate natural numbers and tried to locate these separate numbers to the number line, and they had difficulties in locating a fraction between two natural numbers. Kerslake (1986) studied the difficulties in perceiving fractions quantitatively and stated that children encountered great difficulties when they were asked to name any fraction between a pair of whole numbers, and also in locating fractions on a number line. So, findings of this study related to locating fractions to the number line were consistent with findings of Kerslake (1986). The reason of this kind of mistake might be the lack of knowledge of meaning of whole or meaning of fraction.

In addition to this, more than half of the participants picked a point between 3 and 4 as the location for $\frac{3}{4}$. Students' responses showed that the reason of this kind of mistake might be the deficiency of the concepts of fraction size. Because they may not know actually what $\frac{3}{4}$ is. Additionally, while many of them counted number of lines instead of number of parts, some students did not divide equal parts between natural numbers. Pesen (2008) identified same findings in his study. Consequently to his studies, he ascertained that some of the students had difficulties in dividing a whole on a number line; also, while locating $\frac{a}{b}$ on a number line, some of them could not perceive that $\frac{a}{b}$, which the symbolic representation of fractions is, is a single number, or refers to a single value. The present study also showed that the participants of the study were incapable of conceptualizing a fraction as a point on a number line. These results are consistent with findings of Novillis and Larson (1980) that suggest that number line interpretations are especially difficult for children. According to Ball (1993) and Kieren (1980), a number-line representation requires a clear understanding of the referent unit and the peculiarities of each fraction representation. Thus, the reason of student mistakes may be their lack of knowledge about these topics. Besides, there is a difference between area model (item 1, item 4), discreet set of objects (item 5, item 6) and a number line. While the number line was intended to be a part-whole task, it was still a task involving ‘length’ and therefore a ‘measure’ task as well. This particular number line task was an attempt to ‘relate’ part-whole and measure subconstruct. This might be the cause why some students divided the space between two natural numbers to ten equal parts. It may be harder to grasp when measure concept is involved.

Also, it was seen that another reason of failing to locate a fraction on a number line is having difficulty in partitioning wholes on number line. This result was supported by Peck and Jenks (1981) and they stated that, unless partitioning skills are well established, students will be deficient in their ability to locate fractions on a number line and that also leads to conceptual mistakes in comparing fractions with natural numbers. It was also seen in the participants of the present study that they were unable to detect a fraction as a number when they use symbolic representation of a fraction on the number line. Interview findings showed that these students influenced by appearances of fractions and the underlying reason was their misconception as seeing numerator and denominator as different two numbers. Consequently, this study shows that many students had difficulty in partitioning wholes on number line and locating a fraction on a number line; also, they were unable to detect a fraction as a number when they use symbolic representation of a fraction on the number line. They might be using a whole number counting schema and used other different incorrect strategies to locate the fraction $\frac{3}{4}$ in this case.

5.1.3. Mistakes and Difficulties Regarding Ordering and Equivalence of Fractions

Findings of this study revealed that students made various mistakes in the questions involving fraction ordering. Many students misapplied rules for ordering natural numbers to fraction situations. Some of them tended to look only to denominators and argued ‘the larger the denominator, the smaller/ larger the fraction or the larger the numerator, the smaller/larger the fraction’ and ignored numerators or vice versa. This result was consistent with the findings of Vinner (1997) and Baroody and Hume (1991) since concerning the ordering of fractions. They described this ignorance of numerators as the bigger the denominator the smaller the fraction; and the second finding is the application of a similar strategy, the smaller the numerator the greater the fraction. Moreover, some students used the relationship between the number of pieces and the size of the pieces incorrectly. They ordered the size of equal pieces instead of size of fractions. This result was consistent with results of Behr, Wachsmuth and Post et al. (1984) where they used a range of tasks to investigate students’ perception of the size, comparing and ordering of fractions. It is the researchers’ belief that many students used the ‘denominator only’ strategy, referred only to the denominators of the fractions and thought the fraction with the larger denominator had the smaller sized parts, so the larger the denominator the smaller the fraction was. In the present study, also, many students were unable to coordinate the information from denominator with numerator, therefore could not make correct judgment toward fraction ordering. These students took numerator and denominator into consideration separately while ordering the fractions. It means the performance of these students seemed to be dominated by their knowledge of the ordering of whole numbers. For instance, they stated $\frac{23}{24}$ was larger than $\frac{15}{16}$ because 23 was larger than 15, and 24 was larger than 16. Again, this finding of the current study is consistent with those of Behr, Wachsmuth, Post and Lesh (1984) who found some children claimed, one third is less than one fourth because three is less than four. Behr et. al. (1984) suggests that children's schemas for ordering whole numbers are very strong and they are overgeneralized. Kerslake (1986) found similar results in her study with sixth graders. When given the tasks of choosing the largest fraction from $\frac{3}{8}$, $\frac{3}{9}$ and $\frac{3}{7}$, the children apparently thought that since nine was larger than seven, $\frac{3}{9}$ was larger than $\frac{3}{7}$.

Another mistake pattern in this study is ‘ordering leftover parts’. In item 3-a, some students consistently explained their strategies; they thought that all fractions have one piece leftover; because of this they are equal. They had mistakenly viewed the fraction size as the leftover parts. Additionally, in the second part of the item that required ordering, when it was asked to order fractions that involved both a mixed number and improper fractions, it was seen that many students considered the mixed number as the largest choice since it included a natural number. This may indicate that students have misconceptions regarding improper fractions and that they always consider them to be smaller than natural numbers. This finding demonstrates similarities with findings of Tzur (1999). According to him, coordinating children for making sense of improper fractions did create a conflict. Because iteration of an unit fraction a number of times to form a fraction that exceeded the reference whole creates a complication because of the children’s conceptions of a fraction as a part of that whole (Tzur, 1999).

Equivalence of fractions was another complex concept for fifth grade students because fractions have infinitely many equivalent representations (Vance, 1992). This subject is probably more difficult with fractions than natural numbers since they may not understand that when the two quantities which form a fraction are multiplied by the same nonzero number but the value of the factions remains the same (Marshall, 1993). In this study, many participants were able to identify equivalent fractions when they were presented geometrically (better at circle models compared to rectangular ones). However mistakes and difficulties occurred when students were presented with equivalent fractions that were presented numerically (e.g. $\frac{3}{4}$ and $\frac{6}{8}$). Most students responded that $\frac{6}{8}$ and $\frac{3}{4}$ are not equivalent and that $\frac{6}{8}$ is the double of $\frac{3}{4}$. As a result, if the equivalence of fractions is considered procedurally, it can be said that students were not able to simplify fractions correctly, convert a fraction to one with a different denominator (Troutman & Lichtenberg, 1991). Furthermore, some students shaded $\frac{3}{4}$ of one rectangle and $\frac{6}{8}$ of the other, which had the same size as the first rectangle, correctly. Then they concluded that $\frac{3}{4}$ was less than $\frac{6}{8}$ which showed although these students were able to draw images, the decisions they made did not reflect the images they drew. When asked to confirm their answer many of them said that in $\frac{6}{8}$ more pieces were taken so it was greater than $\frac{3}{4}$. Vance (1986) claimed that students think that the fraction which has more pieces greater than others. Thus, findings of Vance (1986) support this finding. One of the probable reasons of this incorrect reasoning might be multiplicative nature of equivalent fractions (Kamii & Clark, 1995). It means, students might not understood that relation between the numerators and the denominators of equivalent fractions. They might not understood that all equivalent fractions share the same within-fraction multiplicative relation.

In item 4-b, many students thought that $\frac{4}{10}$ was larger than $\frac{2}{5}$ because there were more pieces in it, or thought $\frac{4}{10}$ and $\frac{2}{5}$ were not equivalent because pieces of $\frac{4}{10}$ were smaller than pieces of $\frac{2}{5}$. Ball (1993) observed consistent results in her study, she asked the equivalence of $\frac{2}{5}$ and $\frac{1}{2}$ to the students by using a figure with two shaded pieces for $\frac{2}{5}$ and only one shaded piece for $\frac{1}{2}$. She found that some students thought that $\frac{2}{5}$ was larger than $\frac{1}{2}$ because more pieces were shaded. One of the probable reasons of this mistake might be designation of equivalent fractions by different words such as; three fourths, six eights and also different written signs such as: $\frac{2}{5}$ and $\frac{4}{10}$. Students may not understand the concept that, 'a fraction represents a number that has many names (Larson, 1980, p.427). Moreover, nearly all students used symbolic procedures to confirm equivalence of fractions but only few of them used their pictorial representations to explain it. In this study, it was expected participants to explain using conceptual strategies, but they stayed in the symbolic world. The possible reasons of this avoiding the use of pictorial representations might be rote memorization of fraction operations and lack of knowledge of fraction concepts. Because, this study showed that students tended to memorize procedures instead of understand the fraction concepts.

5.1.4. Mistakes and Difficulties Regarding Partitive Interpretations of Division

When students encounter fractions, partitive interpretations becomes the primary quantifying activity (Hiebert& Behr, 1988; Kieren, 1980, 1983). It has been suggested that the act of partitioning a region into equal-size parts (item 6) or separating a set of discrete objects into equivalent subsets (item 5) is fundamental to the part-whole interpretation of fractions (Behr & Post, 1988). In item 6, some of the students stated that two cakes couldn't be possibly shared to three people equally; they either added a third cake, or distributed one half to everyone and said one half were left over. This is similar to the findings of Tirosh, Fischbein, Graeber and Wilson (1993). In their study, students stated that 'one cannot divide a small number by a large number because it is impossible to share less among more' (p. 18). The findings also showed that, students had difficulties using pictorial models representing equal partitioning. It was seen that the participants did not recognize that the parts which a pictorial unit is separated into must be the same size. Moreover, they did not associate their own experiences to even familiar models of fractions, such as a cake in item 6. In other words, this study demonstrated that participants of this study had misconceptions and they made mistakes in problems include daily life situations. This finding corroborates the ideas of Mack (1990) and Khoury and Zazkis (1994) who suggested that many students cannot able to relate their real life experiences to partitioning models and they cannot associate models which encountered in real life, such as cake or pizza. The possible reason might be students' lack of awareness of that fractions are nothing but an extension of their daily experience and is not an abstraction. The other possible reason might be related to teachers approaches. They might not support environment related to daily life whereas tasks related to real life can enable children to use their conceptual understanding of a familiar situation to develop procedures for solving the tasks.

Hoffre and Hoffer (1995) claimed that a large proportion of the population never develops proportional reasoning and it was corroborated with this study because in this study most of the students partitioned models incorrectly. This situation showed that they had lack of proportional reasoning but researchers identified the ability of partition as a basic requirement for understanding fractional concepts (Lamon, 1999). Moreover, based on the findings of the present study, it could be said that nearly none of the students determined the amount of cake that each person take as fraction number. This finding is in agreement with Mack's (1995) findings which showed when fair sharing or equally dividing 8 yards of ribbon between 3 children, many students do not recognize $\frac{8}{3}$ yards is the length of ribbon each would receive (Mack, 1995). This representation was clearly about the number of parts rather than the area of the parts.

5.1.5. Mistakes and Difficulties Regarding Fraction Operations

Another topic that students made mistakes was addition and subtraction operations with fractions. It was observed that many students applied algorithms to solve fraction addition and subtraction without being aware of the reasonability of the results. Algorithms are tools for understanding and doing mathematics. In order to effectively use these tools students need to have meanings for the numbers they are operating upon and have a sense of what happens to the numbers when a particular operation is used. From the interview findings, it could be understood that student could not understand neither addition nor subtraction concept of fractions. Also, instead of comprehending algorithms as tools, the participants supposed that the purpose was to memorize right algorithms; however, some of the students could not successfully remember these algorithms. So, they often applied a procedure or method that was not meaningful to them. For instance, most of them didn't use common denominators in addition and subtraction operation; and the ones that used them were unable to find and verbalize why they got common denominators to add or subtract fractions when the standard algorithm was used. This result concurs with the findings of Watanbe (2008) because he stated that stated participants of his study knew the standard subtraction algorithm to solve the problem, but he could not fully conceptualize how they could be subtracted together (Watanbe, 2008).

Moreover, it was clearly seen that participants of this study had difficulty in applying appropriate uses of a common denominator and they applied the existing natural number concept. These students added the denominators and numerators together or subtracted smaller denominator from larger and smaller numerator from larger one to get the answer without finding the least common denominator. This finding is supported by many researchers (e.g, Morton, 1923; Hasemann, 1981; Mack, 1995; Troutman & Lichtenberg, 1991; Wu, 2001) because of their finding as students have difficulty applying appropriate uses of common denominator and they may not be able to add or subtract fractions correctly. According to them, students generally choose incorrect strategies such as adding numerators and denominator or making cross cancelling. Also they failed to convert fractions to a common, equivalent denominator before adding or subtracting them, and instead just use the larger of the two denominators in the task (e.g. $\frac{5}{4}-\frac{14}{12}$ converted to $\frac{5}{12}-\frac{14}{12}=\frac{9}{12}$). In other words, they changed the denominator of the fraction without making a corresponding change to the numerator. Students did not understand that different denominators reflect different-sized unit fractions and that adding and subtracting fractions requires a common denominator (unit). In this context, this is consistent with findings of Hasemann (1981) because he claimed that students fail to change the fractions to a common denominator and change only the numerators or denominators of the fractions to make them accord with the common denominator. As a result, dealing with the addition and subtraction of fractions could be problematic for fifth graders and participants of this study tried to add and subtract fractions by memorizing the steps and merely applying them. These mistakes may be caused by traditional instruction that focuses on rote memorization of fractional operations. Teachers may not help their students make connections between conceptual and procedural knowledge.

The results of the study revealed another common mistake that is incorrect reasoning about addition of a natural number and a proper fraction. In item 8, many students made mistakes while adding a natural number and a fraction, such as adding natural number with only numerator or with both numerator and denominator. In other words, in this study, fifth grade students could not consider $5\frac{1}{3}$ as $5+\frac{1}{3}$, but rather as $5\times\frac{1}{3}$, $5\ \frac{1}{3}$ or $\frac{5}{3}$. This is supported by Ball (1990) because at the end of her study, she claimed that students have difficulties on interpreting mixed numbers and addition of a whole number and a fraction. According to her, students may not interpret the number 3 $\frac{1}{2}$ to mean 3+$\frac{1}{2}$, but rather as 3 x$\frac{1}{2}$, $\frac{3}{2}$ or even$\frac{32}{2}$. The present finding seem to be consistent with study of Mack (1990) which found that fifth grade students had difficulties with addition and subtraction operations of a whole number and a fraction. In addition, the participants who were unable to write the mixed number as a sum do not understand the meaning of mixed number notation. In other words, the participants of this study, who have only learned the shortcut algorithm for renaming mixed numbers as improper fractions, may not know that a mixed number is actually a sum. Accordingly, Rotman (1991) stressed that the reason of deficiency in writing mixed number as a sum is related to meaning of symbols because students who have difficulties regarding meaning of addition symbol also have difficulties related to meaning of mixed number.

In item 10, which was required subtraction a proper fraction from a natural number using models, students made numerous mistakes finding out the fractions that were represented by models. For instance, they used only the number of unshaded parts as numerator and only the number of shaded parts as denominator; or, while some used only the number of shaded parts and wrote 8 as the first number instead of 2, some others wrote $\frac{8}{8}$ since all 8 parts were shaded. These are consistent with findings of Moss and Case (1999) because in their study they stated that students count only the number of shaded parts in a figure or the total number of parts. They stated that, students regard each part as an independent entity or amount. The reason of why students wrote $\frac{8}{8}$considering two separate wholes in the question as one, or why they wrote 8 considering each shaded part as a whole, might be predominantly description of fraction as 'part of a whole'. Moreover, they considered only the number of shaded parts in the figure or number of unshaded parts so that each part is regarded as an independent entity or amount. Because, Moss and Case (1999) stated that this misconception might lead students to translate the $\frac{3}{4}$ into $\frac{4}{4}-\frac{3}{4}$and find an answer of$\frac{1}{4}$. In general, students have difficulties in operations of fractions because these operations are often taught using procedures instead of allowing students to experience multiple ways of the meanings of operations (Tirosh, 2000). This could be the reason why students tended to memorize operation algorithms and failed to remember or got confused about the steps in those algorithmic procedures.

Lastly, besides all, the findings of the study demonstrated that students had difficulties and they made mistakes related to multiplication of fractions. In item 12-a many students did not recognize that the solution was a product. Almost all of them tried to reach the result solely with drawings but some couldn't find the correct result. They showed similar mistakes in items that required partitioning. Similar findings were also stated in the literature by Mack (1998). She stressed the importance of drawing on students' informal knowledge. She used equal sharing situations in which parts of another part can be used to develop a basis for understanding multiplication of fractions; such as, sharing half a pizza equally among three children results in a child getting one third of one half. Both results of Mack and this study showed that students did not think taking a part in terms of multiplication (Mack, 1998), which means understanding multiplication of fractions requires re-conceptualization of multiplication to accommodate working with fractional quantities (Graeber and Tirosh 1988). The second part of the item only required multiplication of two proper fractions and it was seen that majority of the students applied the algorithm correctly; few of them misapplied the standard multiplication algorithm by trying to find common denominator or by applying cross multiplication. Moreover, all of the students failed to reduce the fractions before multiplying.

All of these mistakes and difficulties indicated a strict algorithmic approach to operations on fractions. These findings showed that many of these fifth grade students might learn multiplication operation with fractions through procedure-oriented, memory-based instruction, attributing little meaning to multiplication of fractions operation (Cramer & Bezuk, 1991; Kennedy & Steve, 1997). These are also consistent with study of Parmar (2003) because she claimed that students who made mistakes in multiplication of fractions frequently utilize less mature strategies and they also have more problems retrieving an effective strategy, even when it has been taught to them. She also stressed that a specific example of an incorrectly applied algorithm appears when students find the common denominator in multiplication operation of fractions or make other related mistakes appear to be based on students' rote learning of an algorithm without corresponding understanding (Parmar, 2003). This supports the findings of this study since the most frequent misconception in this study was either the overgeneralization of a cross-multiplication algorithm or the overgeneralization of the addition algorithm. For instance, some of them used flawed algorithms, such as calculating common denominator and multiplying numerators to multiply fractions. These findings are also consistent with determination of Soylu and Soylu (2005) because they identified that students mistakenly transfer the rule of addition (e.g. finding common denominator) to multiplication.

5.1.6. Mistakes and Difficulties Regarding Solving Fraction Word Problems

Solving and creating fraction word problems was another problematic area for the students. The study of Aksu (1997) demonstrates that when students are presented with problems involving multiple concepts and steps, they face several difficulties. More than half of the interview students who participated in this study used different ways for solving the word problems (item 11) but they made mistakes by choosing the wrong operator; they applied disjointed algorithms using different combinations of the numbers in the question, such as $\frac{2}{5}$, $\frac{4}{5}$ and 20, or they simply gave an answer of 50 lt. Participants of this study had difficulties in defining the unknown target in their problems. Fong and Hsui (1999) expected students to solve a fraction word problem but they write down a mathematical statement involving one or more operations on the numbers given in the problem. This might allowed them to cope with the confusion of fractions and at least to get an answer. As a result, it was observed consistent results with Fong and Hsui (1999) and similar results were also stated by Haser and Ubuz (2003) who investigated the conception of fifth grade students in solving word problems and found that students tried various operations with the given fractions and quantities in fraction word problems to reach the solution. Moreover, the participants of this study were not able to employ drawings which can help to construct a relationship between the visual and the verbal items. The mistakes that were made and the failed attempts to solve the problems by so many students may indicate a lack of experience with word problems. Many of the incorrect solutions may be resulted from not understanding the problems and not understanding the relationship between a part and a quantity (Haser & Ubuz, 2003). Besides, students' difficulties while solving fraction word problem may be caused by their misunderstanding of the semantics of mathematical text since translation from natural language to a more formal mathematical language might be difficult for students. In other words, they might not be able to transfer the given information into symbolic representation (Lesh & Zawojewski, 2007). Also, none of the students identified the given information and the goals and few of them drew a model for identifying solution strategy even though these could have helped the students in figuring out the correct procedures needed (Lesh & Zawojewski, 2007). Participants of this study also struggled with trying to create story problems to add fractions together; many of them ignored the story writing phase and tried to write the answer directly. Some students' created problems were not relevant to the addition operation which was given in the item. Some of them created a word problem which was mathematically correct while being incorrect in the terms of fraction concept. Unfortunately, these students had no mentality for checking the reasonability of their word problems and answers.

As a result, students were used to using natural number concepts since beginning of their educational lives. This could be the reason why they were highly successful when dealing with questions in which natural number concepts could be applied, such as multiplication of fractions. However, when they were required performing on more open ended items such as solving and creating fraction word problem or giving written explanations, they showed less consistent success. They tried to create a word problem appropriate to the operation however they couldn't determine what was being given and asked correctly. The main reason behind it could be wrong perception of the problem because lack of connection between known and unknown, lack of adequacy in mathematical knowledge and lack of adequacy in mathematical language.

5.2. Underlying Reasons and Misconceptions

One of the purpose of this study was investigating underlying reasons and misconceptions behind students' mistakes and difficulties. This study contributes to current research and literature by describing as thoroughly as possible what fifth grade students know or do not know about fractions, and was based on a need to identify students' misconceptions and reasons behind their mistakes that create obstacles in their learning process. It enabled to assess students' current levels of understanding in order to determine the next steps of learning rational numbers. Identifying students' mistakes, difficulties and underlying reasons and misconceptions behind them regarding fractions required a careful analysis beyond looking at percentage of incorrect answers on a written test and looking at tasks in interviews. The results of this study indicate that students are not in the same place in terms of the development of fractional knowledge. Moreover, looking more closely at the subconstruct tasks and various explanations in the interviews indicates that many students still have fundamental mistakes and misconceptions.

In general perspective, both OFQ and interviews identified several themes appeared to be underlying fifth grade students' mistakes in understanding of fractions. The first category of students' mistakes/difficulties was 'mistakes/difficulties based on formal knowledge of fractions' which means that students' inadequate knowledge of the properties of operations with fractions leads them to perform the operation incorrectly. In this study, the reason of this kind of mistakes was lack of understanding of the nature of fraction and fraction concept (Tirosh, 2000). Foremost, it can be said that participants of this study lacked the ability to form connections between representations. In other words, these participants' informal knowledge of fractions might be initially disconnected from their knowledge of fraction symbols, procedures, and concrete representations (Mack, 1990). These mistakes involved translating the visual model into a faulty symbolic model or vice versa; and they revealed misconceptions involving lack of knowledge of meaning of fraction (Ball, 1993). Moreover, these explanations show that, for these students, the concepts of fraction, mixed number and improper fraction are deficient and also many students do not think of fractions as numbers. This finding is in agreement with Kerslake (1986) and Hannula's (2003) findings which showed while students may have some facility with fractions, many of them appear not to have fully developed an understanding that fractions are numbers. Consequently, fractions may be confusing as one of the interviewees said that they do not behave like 'normal' numbers. Additionally, although it will be mentioned later in more detail, it can be briefly said that participants' knowledge of fractions for part/whole representations was also lacking. This was challenging since students needed to identify the referent whole (Smith, 2002) and understand that fractions refer to relationship of the equal parts to the whole unit (English & Halford, 1995).

The second category of students' mistakes/difficulties was intuitively based mistakes/difficulties. The most common reason of these mistakes was strong interference from natural number concepts (Tirosh, 2000). In other words, the underlying problem was inappropriate use of natural number ideas which subsequently led to misconceptions when comparing area and numerical representations of equivalent fractions and fraction operations. Although learning to use fractions demands a good portion of the fifth grades mathematics curriculum, some natural number topics are also taught in this grade. May be with the impact of this, they viewed numerator and denominator as separate natural numbers, which need to be combined in some way or other to give a natural number and failed to identify the importance of relationships between numerator and denominator (Newstead & Murray, 1998; Tirosh, 2000). Also Mack (1995) supported the findings of this study since she pointed out that students often operate on fractions in the symbolic representation mode as though they represent independent natural numbers rather than specific quantities. Moreover, participants of this study tended to apply a familiar algorithm from natural number arithmetic in fraction arithmetic (Niemi, 1995). For instance, they added numerators to find new numerator and added denominators to find new denominator instead of finding common least denominator.

The third category of students' mistakes/difficulties regarding basic fraction concepts and operations was algorithmically based mistakes/difficulties. Especially when students struggled with fraction operations, and attempted to solve items which were symbolically represented, strong interference from rote algorithms was observed. But unfortunately, without a deep conceptual understanding of fractions, students incorrectly applied rote operational algorithms in items, mostly the ones included operations with fractions. This study produced results which corroborate the findings of a great deal of the previous research in literature; for instance, Ashlock (1990), Mack (1990) and Tirosh (2000) stated that algorithmically based mistakes/difficulties arouses because of rote memorization of the algorithm since students perceived algorithm as a meaningless series of steps and they might forget or change steps which leads them to make mistakes. Mack (1990) stated that, rote procedures on students' attempts to construct meaningful algorithms for operations on fractions caused the mistakes. Also, according to Aksu (1997), even if their answers were correct, many students have instrumental understanding which indicates that they likely had memorized algorithms without understanding it (Aksu, 1997). Moreover, Kerslake (1986) found that when working with fractions, students relied heavily on rote memory of previously learned techniques which are not always compatible with fractions. The reason of this kind of mistake might be that fraction notations do not form a normal part of a children's environment and the operations on fractions are defined abstractly. As it seen in this study, many students may lack some essential conceptual knowledge, may have memorized procedures and they apply them inappropriately.

Based on this research, it can be said that, when students who make mistakes and have misconceptions face a problem involving fractions, there were two main incorrect tendencies: the first one was their existing schemas for natural number operations. Then, they approach the problem using the natural number mathematical resources they have. They struggle on the symbols without an adequate, quantitative basis for their thinking, and, in particular, without understanding the differences between natural number and fraction symbols. Secondly, based on the interviews and the evidence from written work on student tests, they appear to rely heavily on rules and procedural knowledge to compute fractions, and often times; they misapply the rules without realizing it. For most of the students, however, the strategy of choice is to select and use an algorithm. Even when students are able to apply the rule correctly, they cannot explain why the rule is correctly applied and why the answer is correct. Most students seem to lack the conceptual basis for fractions and many cannot extend their generalized concept of fractions to operations on fractions (Jencks, 1981). Such inconsistent results demonstrate a lack of fluency with fraction computation.

In addition to determining students' mistakes and difficulties, the other purpose of this study was to investigate underlying reasons and misconceptions behind those mistakes and difficulties of the participants regarding fractions. Participants' performance on the OFQ and interview findings also revealed several misconceptions that cause mistakes in fifth grade students' understanding of fraction concept as well. These misconceptions may have been developed from previous learning or when there was an over-emphasis on skill development without conceptual understanding (Hiebert & Wearne, 1986). The findings from this study related to misconceptions regarding fractions revealed that the participants have various common misconceptions. Interview participants' responses to the ordering item revealed that students have misconceptions as only the denominator or numerator that determines the size of the fraction. They believe that the fraction having the larger denominator is always larger and this finding is in agreement with Vinner (1997) and Baroody and Hume (1991). Also, in the same item some students thought that a fraction which has the larger numbers is larger. This finding supports previous research of Hart et al. (1980). In item 4, many participants stated that $\frac{3}{4}$ is not equivalent with $\frac{6}{8}$ because they had a misconception as reducing a fraction makes it smaller. In item 8, many students chose an intuitive rule, which is only valid for natural numbers, while expressing opinion about the multiplication of fractions. According to them multiplication makes fraction larger. The present finding seems to be consistent with research of Tirosh (2000) and Stafylidou and Vosniadou (2004) which found that in whole number operations, multiplication comes to be understood to mean making the fraction larger. In addition, numerous students used idea of primitive model of subtraction. They stated that minuend should be always bigger than subtrahend. They had a misconception as the smaller number is always subtracted from the larger number. So the bigger denominator or numerator cannot be subtracted from smaller ones. Similar transfer of natural number strategy was observed in item 6. Many students claimed that 2 cakes cannot be shared among three people because they had a misconception as the dividend should be always bigger than the divisor. This finding is in agreement with Tirosh, Fischbein, Graeber and Wilson's (1993) findings which showed that students make intuitively based mistakes/difficulties caused from transferring natural number strategies to fractions, including basic intuitions held about operation of fractions such as the dividend is always bigger than the divisor. Same item and item 12 revealed another misconception that found by Reys et. al (1999) before as the denominator refers to the number of pieces, regardless of unequal sizes of the pieces. Because nearly half of interview students, divided cakes into unequal pieces but according to number given as the denominator. Lastly, some of the participants rejected the idea of mixed numbers are fractions. According to them all fractions are less than one and mixed numbers are not fractions.

5.3. Recommendations and Implications

This study offered significant information to mathematics educators, regarding common difficulties that elementary fifth grade students encounter, mistakes they make regarding basic fractional concept and operations, and underlying reasons and misconceptions behind those mistakes and difficulties. This analysis will provide a source that can assist teachers in detecting and correcting common mistakes and misconceptions students make when manipulating fractions.

From the findings of this study, it is clearly seen that, students in elementary schools have difficulties when they begin to learn fractions after natural numbers. Moreover, understanding fractions requires comprehension and coordination of several powerful mathematical processes, such as unitizing, reunitising, and multiplicative relationships (Baturo, 1997, 2000). These situations may negatively affect the students' success in mathematics academically and their cognitive development. Also, as mentioned before, difficulties in learning fractions unfortunately lead to failure in other mathematics topics. Implications are given below for instruction and curriculum, which might prove beneficial to students.

5.3.1. Implications

By the end of fifth grade, students should have a good grasp of basic fraction concepts, operations used with natural numbers and fractions, however, not all students do. The standard algorithms that most of people have learned were developed to be efficient; however, complete descriptions may be lengthy and difficult to memorize without understanding. These algorithms are a challenge to teach and teachers fail to analyze that why particular mistakes were made regarding fraction algorithms (Troutman& Lichtenberg, 1991). Thus, findings of this study are critical to appropriate remediation. Because of these, teachers and pre-service teachers should be informed about specific mistakes, difficulties and underlying reasons and misconceptions behind those mistakes through the organization of seminars or in-service training programs to create awareness. Furthermore, in this study, the difficulties that student encountered and reasons behind those difficulties with fractions were investigated; that is, these findings should be shared with both primary teachers and mathematics teachers.

Therefore, the crucial parts of these findings could be added to the books that teachers use as guides, regarding to teaching of fractions. Since unit fraction concept that is taught at fourth grade is not remembered when students move on to fifth grade, they have difficulties in operations and other certain topics; thus, unit fraction concept and its relation with operations should be emphasized also in fifth grade by curriculum makers. As it is mentioned before, almost all of the students gave either incorrect answers or correct answers but with wrong underlying reasoning, to the question that required ordering fractions. Considering that these students don't think about concepts like a whole, smaller than, and bigger than; and they confused concepts like a quarter and a half, in the second part of item 1, the comparison of the common fractions like half, quarter or whole; using these as benchmarks, could be added to fifth grade elementary mathematics curriculum.

In addition, locating fractions on the number line may be an abstract subject for fourth grade students; however, since most of the participants of this study were not able to locate a proper fraction on the number line, ordering fractions and locating them on the number line could be emphasized further on fifth grade mathematics curriculum. Moreover, the conception 'multiplication makes the number bigger' should be revised as 'multiplication could make the numbers smaller, equal, or greater' in the curriculum (Stafylidou and Vosniadou 2004). Lastly, in this study, it was seen that students who made mistakes regarding ordering fractions also made mistakes regarding fraction operations. Because of this operations on fractions should be delayed until students have a solid understanding of order and equivalence of fractions.

5.3.2. Recommendations for Further Studies

As mentioned above, the concept of fraction is complex and cannot be comprehended all at once by children; it has to be acquired through a long process of sequential development through carefully organized sequences of teaching. There is still much scope in this area for further research. The researcher investigated the mistakes and difficulties, misconceptions and possible reasons of fifth graders' incorrect responses through application of OFQ and interview. For further research could be designed an intervention about how misconceptions that students have can be remedied. In other words, in further work, an experimental teaching design considering precise nature of these mistakes and reasons behind them; also, testing of these strategies in schools could be determined if appropriate instruction can overcome these mistakes. The research in this study may prove useful both in the area of research of teachers' professional development and practice. The identification of these specific mistakes related to fractions along with their reasons can be a powerful tool to pinpoint the knowledge about weak areas in instruction that need to be developed further.

Another further research area could be related to the different grades; that is, this study was performed solely on fifth grade students. A similar study might be conducted with other grades in both public and private schools. Further studies could be conducted on several grades simultaneously regarding same fraction concepts, or a smaller group can be observed over a few years, observing the changes.

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APPENDICES

Appendix A: Turkish Version of OFQ

5. SINIF KESİRLER KAZANIM ÖLÇME TESTİ

Sevgili öğrenciler:
Bu testin amacı sizlerin kesirler temasıyla ilgili kavrama düzeyinizi ölçmektir. Testin sonuçları sadece bilimsel bilgi edinmek amacıyla kullanılacaktır. Herhangi bir şekilde KESİNLİKLE not ile değerlendirme amacıyla <u>kullanılmayacaktır</u>. Bu amaçla:

1. Sayfayı çevirdikten sonra göreceğiniz soruları okuyarak soruları dürüst ve duyarlı bir şekilde cevaplandırınız.
2. Çözümünüzü açıklamanız istenen sorulara yapacağınız açıklamalar, elde etmeye çalıştığımız bilimsel bilgiler açısından çok önemlidir. Bu nedenle lütfen sorulara cevap vermek ya da çözmekle yetinmeyip 'neden o yolu izlediğinizi' ya da 'nasıl bir çözüm yolu belirlediğinizi' açıklayınız.
3. Testi tamamlamak için süreniz 50 dakikadır.

Size ve öğretmeninize teşekkürlerimi sunarım.

Nilgün TARKAN
Sınıf Öğretmeni
Yüksek Lisans Öğrencisi
ODTÜ Eğitim Fakültesi

Adınız:
Soyadınız:
Sınıfınız:

1. a) $$\frac{4}{3} = 1\frac{2}{6}$$

Yukarıda verilen ifadenin doğru olup olmadığını şekil kullanarak açıklayınız.

<u>Açıklamanız:</u>

b) 

Yukarıdaki şeklin ifade ettiği çokluğu tamsayılı kesir ve bileşik kesir olarak ifade ediniz.

<u>Tam sayılı kesir olarak:</u> <u>Bileşik kesir olarak:</u>

<u>Cevabınızı açıklayınız:</u>

2. $\frac{3}{4}$ kesrini sayı doğrusu üzerinde gösteriniz ve bu kesrin en yakın olduğu doğal sayıyı yazınız.

<u>En yakın doğal sayı:</u>

3. a) Aşağıdaki kesirleri küçükten büyüğe sıralayınız. Nasıl sıraladığınızı açıklayınız.

$$\frac{3}{4}, \frac{7}{8}, \frac{11}{12}, \frac{15}{16}, \frac{23}{24}$$

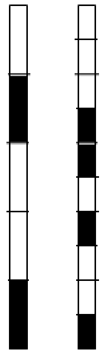
<u>Açıklamanız:</u>

b) Aşağıdaki kesirleri büyükten küçüğe sıralayınız. Nasıl sıraladığınızı açıklayınız.

$$\frac{13}{16}, \frac{20}{16}, \frac{3}{4}, \frac{4}{2}, 1\frac{3}{16}$$

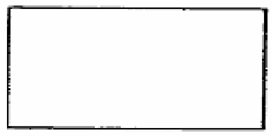
<u>Açıklamanız:</u>

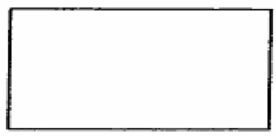
4. a) Aşağıda verilen şeklin ifade ettiği kesirler birbirine denk midir? Nedenini açıklayınız.



Açıklamanız:

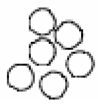
b) Aşağıda size verilen şekli kullanarak $\frac{3}{4}$ ve $\frac{6}{8}$ kesirlerinin birbirine denk olup olmadığını çizerek gösteriniz. Cevabınızı açıklayınız.





Açıklamanız:

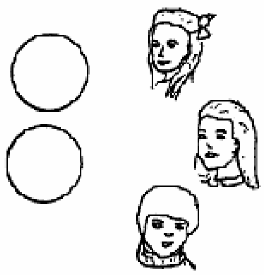
5. Aşağıdaki şekil Hakan'ın bilyelerinin 3/5 ' ni göstermektedir. Hakan'ın toplam bilye sayısı kaçtır? Hakan'ın bilyelerinin tamamını çiziniz ve problemi nasıl çözdüğünüzü açıklayınız.



Hakan'ın toplam bilye sayısı:

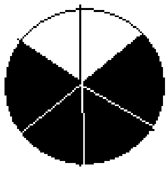
Açıklamanız:

6. Şekildeki iki adet pastayı Nermin, Kaan ve Gülşah aralarında eşit olarak paylaşmak istiyor. Her bir kişiye düşen pasta miktarını çiziniz ve cevabınızı açıklayınız.



Çiziminiz:

Açıklamanız:

7. a) 

Yanda verilen şekilde, taralı alanın gösterdiği toplama işlemi aşağıdakilerden hangisidir? Nedenini açıklayınız.

a) $\frac{1}{2}+\frac{1}{3}$ b) $\frac{1}{3}+\frac{2}{6}$ c) $1+\frac{1}{3}$ d) $\frac{1}{3}+\frac{4}{6}$

Açıklamanız:

b) $\frac{34}{61}+\frac{34}{61}$ işleminin sonucu nedir? Sonuca ulaşmak için izlediğiniz yolu açıklayınız.

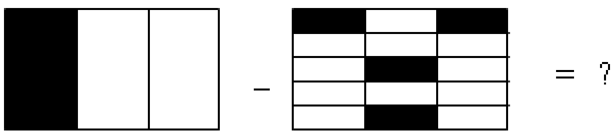
8. a) $2+\frac{2}{5}$ işleminin sonucu nedir? Sonuca ulaşmak için izlediğiniz yolu açıklayınız.

b) Emrah, $5\frac{1}{3}$ sayısının $5+\frac{1}{3}$ ’ e, Sinan ise $5x\frac{1}{3}$ ’ e eşit olduğunu söylüyor.

Hangisinin söylediği doğrudur? Neden?

9. a) $\frac{5}{4}-\frac{14}{12}$ işleminin sonucu nedir? Neden?

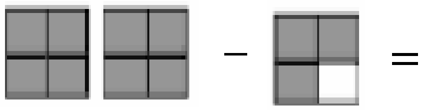
b)



Yandaki dikdörtgenlerin her biri kendi içinde eş parçalara ayrılmıştır. Buna göre bu iki şeklin farkı aşağıdakilerden hangisidir? Çözümünüzde izlediğiniz yolu açıklayınız.

a) $\frac{3}{15}$ b) $\frac{1}{3}$ c) $\frac{1}{15}$ d) 0 (sıfır)

Açıklamanız:

10.  Yandaki işlem ve işlemin sonucu nedir?

11. a) Bir bidonun $\frac{2}{5}$ i su ile doludur. 20 litre daha su konulursa bidonun $\frac{4}{5}$ doluyor. Bu bidon kaç litreliktir?

b)Çözümü $2\frac{2}{4} + \frac{3}{4}$ işlemi olacak bir problem durumu yazınız ve bu probleminizi çözünüz.

12. a) Yarım bir kekin üçte birini yersen kekin kaçta kaçını yemiş olursun? Çözümünüzü çizerek açıklayınız.

b) $\frac{2}{3}$ x $\frac{5}{2}$ işleminin sonucu nedir? Açıklayın

TEZ FOTOKOPİ İZİN FORMU

ENSTİTÜ

Fen Bilimleri Enstitüsü ☐

Sosyal Bilimler Enstitüsü ☐

Uygulamalı Matematik Enstitüsü ☐

Enformatik Enstitüsü ☐

Deniz Bilimleri Enstitüsü ☐

YAZARIN

Soyadı : YURTSEVER
Adı : Nilgün
Bölümü : İlköğretim Fen ve Matematik Eğitimi

TEZİN ADI (İngilizce) : A Study on Fifth Grade Students'Mistakes, Difficulties and Misconceptions Regarding Basic Fractional Concepts and Operations:

TEZİN TÜRÜ : Yüksek Lisans ☐ Doktora ☐

1. Tezimin tamamı dünya çapında erişime açılsın ve kaynak gösterilmek şartıyla tezimin bir kısmı veya tamamının fotokopisi alınsın. ☐
2. Tezimin tamamı yalnızca Orta Doğu Teknik Üniversitesi kullanıcılarının erişimine açılsın. (Bu seçenekle tezinizin fotokopisi ya da elektronik kopyası Kütüphane aracılığı ile ODTÜ dışına dağıtılmayacaktır.) ☐
3. Tezim bir (1) yıl süreyle erişime kapalı olsun. (Bu seçenekle tezinizin fotokopisi ya da elektronik kopyası Kütüphane aracılığı ile ODTÜ dışına dağıtılmayacaktır.) ☐

Yazarın imzası

Tarih 09.07.2012