

FLUID-STRUCTURE INTERACTION ANALYSIS IN EXTERNAL FLOWS
USING LINEARIZED EULER SOLVER

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USING LINEARIZED EULER SOLVER**

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ABSTRACT

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Flutter is an aeroelastic instability that requires numerous fluid-structure interaction (FSI) analyses during the design process of an aircraft. Even with today's technology, numerical solution of FSI problems is a challenge as these of problems require a high amount of computational power. Most of the computational cost stems from the computational fluid dynamics analysis (CFD) part of the computation, and time-dependent nature of the problem. Harmonic methods, which are very cost-efficient compared to unsteady solvers, can be utilized to solve FSI problems by assuming the solution is time-periodic. In this thesis, the linearized Euler methodology, one of the harmonic methods, is used to detect the flutter speed of aircraft wings. For FSI capabilities and external flow applications, oscillating wall and characteristic far-field boundary conditions are implemented. NACA0012 airfoil with prescribed pitching and plunging motion cases are analyzed to verify the solver's accuracy. All test cases were compared against and verified with unsteady Euler solutions. Additionally, the moment damping coefficient of the basic-finner missile geometry was estimated using a prescribed pitching motion and compared to flight test data. The outcome dis-

played an excellent match, which confirms the solver's proficiency with 3D flows. After the validation studies, a 2D coupled structural solver algorithm was proposed to detect flutter speed. The algorithm benefits from the computational efficiency of linearized methods. Furthermore, aeroelastic analysis of the AGARD 445.6 wing was performed using its first mode shape in preparation for detecting flutter speed of 3D wings in the future.

Keywords: Fluid-Structure Interaction, flutter, harmonic methods, Linearized Euler Method, Computational Fluid Dynamics



ÖZ

DOĞRUSALLAŞTIRILMIŞ EULER ÇÖZÜCÜ KULLANARAK DIŞ AKIŞLARDA SIVI- YAPI ETKİLEŞİMİ ANALİZLERİ

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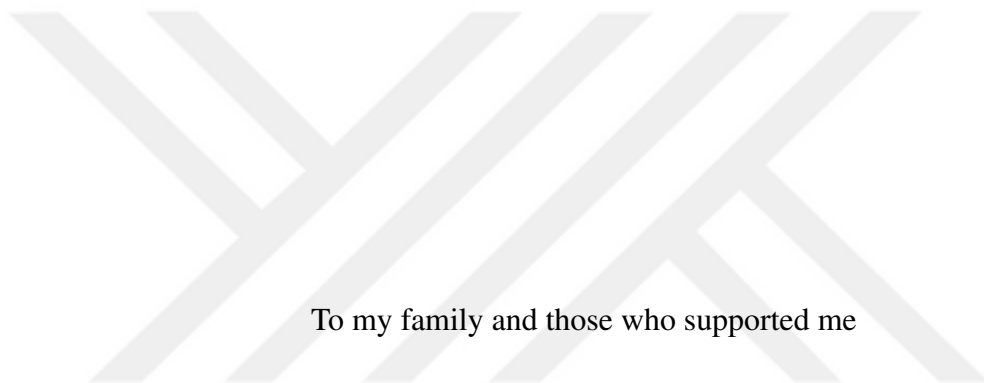
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Çırpıntı bir aeroelastik kararsızlıktır ve uçak tasarım sürecinde çok sayıda akış-yapı etkileşimi analizi gerektirir. Günümüz teknolojiyle bile akış-yapı etkileşimi problemlerini sayısal olarak çözmek yüksek miktarda hesaplama gücü gerektirmektedir. Bu analizlerin hesaplama maliyetinin çoğu, hesaplamanın hesaplamalı akışkanlar dinamiği (HAD) analizi kısmından ve problemin zamana bağlı doğasından gelir. Zamana bağlı HAD çözümlerine göre çok düşük maliyetli olan harmonik yöntemler, çözümün zamanda periyodik olduğu varsayılarak akış-yapı etkileşimi problemlerini çözmek için kullanılabilir. Bu tezde, uçak kanatlarının çırpıntı hızlarını tespit etmek için harmonik yöntemlerden biri olan doğrusallaştırılmış Euler metodolojisi kullanılmıştır. Akış-yapı etkileşimleri ve dış akış uygulamaları için salınımlı duvar ve karakteristik uzak alan sınır koşulları doğrusallaştırılmış Euler çözücüsünde uygulanmıştır. Önceden belirlenen yunuslama salınımı ve yukarı-aşağı salınım hareketi durumlarına sahip NACA0012 kanat profilleri, çözücünün doğruluğunu kontrol etmek için analiz edilmiştir. Tüm test durumları, kararsız Euler çözümleriyle karşılaştırılmış ve doğru-

lanmıştır. Ek olarak, basic finner roket geometrisinin moment sönümleme katsayısı, önceden belirlenmiş bir yunuslama hareketi kullanılarak tahmin edilmiş ve uçuş testi verileriyle karşılaştırılmıştır. Sonuçlar çözücünün 3B akışlarda olan yeterliliğini doğrulayan mükemmel bir eşleşme göstermiştir. Doğrulama çalışmalarından sonra, çırpıntı hızını tespit etmek için 2B birleştirilmiş yapısal çözücü algoritması önerilmiştir. Algoritmada doğrusallaştırılmış yöntemlerin hesaplama verimliliğinden yararlandığı gözlenmiştir. Ayrıca, AGARD 445.6 kanadının aeroelastik analizi, gelecekte 3B kanatların çırpıntı hızını tespit etmeye hazırlık olarak ilk mod şekli kullanılarak gerçekleştirilmiştir.

Anahtar Kelimeler: akış-yapı etkileşimleri, çırpıntı, harmonik yöntemler, Doğrusallaştırılmış Euler Yöntemi, Hesaplamalı Akışkanlar Dinamiği



To my family and those who supported me

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LIST OF SYMBOLS

Greeks

α	Angle of Attack
λ	Eigenvalue
ω	Oscillation Frequency
Φ	Mode Shape
ρ	Density
τ	Pseudo Time
φ	Phase Angle
Ω	Volume
ζ	Damping Ratio

Roman

A	Area
C_L	Lift Coefficient
C_M	Moment Coefficient
C_p	Pressure Coefficient
E	Total Energy
F	Flux
h	Plunging Motion
H	Total Enthalpy
i	Unit Imaginary Number

I_p	Moment of Inertia
$[J]$	Jacobian Matrix
k	Reduced Frequency
k_h	Linear Stiffness
k_α	Rotational Stiffness
K_{st}	Stiffness Matrix
L	Lift Force
m	Mass
M	Moment
M_{st}	Mass matrix
Ma	Mach Number
n	Unit Normal Vector of Face
N	Node Coordinate
$n - t$	Normal Tangential Coordinate System
P	Pressure
r	Position Vector of Wall Surface
R	Gas Constant
Rot_z	Rotation matrix around z-axis
t	Time/ tangential vector
T	Temperature

u	x-Velocity	$\tilde{X}$	Unsteady/ Perturbed Variable
U	Free-stream Velocity	$\hat{X}$	Complex Amplitude of Perturbed Variable
v	y-Velocity		
V	Velocity Vector	$\bar{X}$	Steady/ Mean Variable
w	z-Velocity	Operators	
$\mathbf{W}$	Vector of Conservative Variables	dX	Derivative of X
Subscripts		∂X	Partial Derivative of X
d	Domain	∇X	Gradient of X
f	Face	$\int_A X dA$	Integral of X
$imag$	Imaginary	Abbreviations	
max	maximum deflection position	2D	Two Dimensional
n	Normal	3D	Three Dimensional
$real$	Real	CFD	Computational Fluid Dynamics
$rotcenter$	Rotation Center	FSI	Fluid-Structure Interaction
st	Structure	LCO	Limit Cycle Oscillation
Overs		LH	Linearized Harmonic

CHAPTER 1

INTRODUCTION

1.1 Motivation and Problem Definition

Computational power for numeric analysis has increased over the decades. Computer simulations can now more than ever be used to complete the design process without the need for a significant number of testing. This fact leads to the ability to solve complex problems, reducing both costs and overall design process time. The number of assumptions made to solve a problem is reduced, allowing for more detailed models, where the pure form of the governing equations will be solved, resulting in more costly analyses. Therefore, computation time is an issue that stays relevant. This is even more true when one considers aerodynamic analysis, where both Euler and Navier-Stokes equations are nonlinear, which leads to an iterative solution process. Especially when fluid-structure interactions (FSI) that involve oscillations are concerned, computational cost increases substantially. The reasons for this increased cost stems from physics of the problem. The dynamic FSI problem is unsteady, and fluid-structure coupling information should be shared between two different solvers in each time step. The difference of the physics of the fluids and solids leads to very small time steps for a comprehensive research.

The FSI is a broad research field with a lot of applications. The substantial number of problems requires FSI solvers as in the aviation and the biomedical industry. Although such analyses are possible for academic research purposes, in most cases time dependent FSI problems are not affordable for industry standards. Therefore, there is an industrial demand for reducing the computational cost without significantly compromising from accuracy of the solution.

Like any other, the aviation industry has its challenges in the design process. Aeroelastic instability, causing changes in aerodynamic forces on the aircraft wings, is one of the challenges that stays relevant to this day. These instabilities could result in the failure of the aircraft and loss of human life. Thus, it is crucial to address these problems in the design process before manufacturing begins.

Flutter is one of the most dangerous dynamic instabilities. It occurs when fluid flow excites elastic structure due to flow disturbances created by the structure itself, where the net damping coefficient becomes zero. Old aircraft, in general, were not designed to avoid flutter due to the insufficient computational power in their time. F16 fighting falcon is a good example for this case and it is demonstrated by Denegri [1]. Depending on the weapons that F16 carries, the initial flutter onset could lead to a limit cycle oscillation which is a behavior observed in non-linear systems fed by changes in oscillatory amplitude and flight speed. As a result, the aircraft wing could fail. Considering these fighters are still on active duty, detection of flutter speed is of great importance. The number of experiments and test flights required for determining the allowable flight speed is very costly. A modern fighter aircraft has a vast number of external weapons, fuel tanks, and instrumentation of different sizes and weights. Therefore, flight tests need to be conducted to determine flutter speeds for external store configurations makes the process very expensive. Thus, there is a need for an affordable numerical analysis method for flutter velocity detection, both for designing new aircraft and the external store integration process of existing ones.

Although there are methods for FSI problems that give solutions by only solving structural parts and assuming pressure field due to fluid flow, such as piston theory [2], these methods have limited usage since they generally require the wing to be very thin and angle of attack to be small. These assumptions are not applicable to most aeroelastic problems. Thus, fully coupled FSI modeling at a reasonable computational cost is still a significant problem, which needs to be addressed. The primary computational cost of FSI solvers stems from unsteady solution of fluid flow. Therefore, decreasing the solution time of CFD solver has a massive effect on the total solution time of the fully coupled FSI solvers. Assuming the given problem (including FSI problems) is periodic over time, and perturbations in flow variables can be expressed harmonically, harmonic methods can be used for solving fluid equations in

frequency domain efficiently.

1.2 Literature Survey

1.2.1 General Overview

All harmonic methods are based on decomposing unsteady flow variables into the mean and perturbation flow parts. While linearized solvers assume the mean flow part is equivalent to steady-state (equilibrium point) solution, non-linear ones have considered the effect of flow disturbances in the mean (time average) flow solution. The linearized-harmonic (LH) methods are faster but less accurate compared to non-linear models. Therefore, choosing linearized methods over non-linear ones may be more advantageous since the computing cost would be lower without significantly compromising accuracy where perturbations do not significantly impact the mean solution. These methods were initially used for simple channel flows and turbo machinery applications. In both linear and non-linear methods, excitation could be given to any boundary and flow variable. The harmonic excitation used for FSI problems is the oscillating wall boundary condition, where the amplitude of perturbed normal velocity on the wall is prescribed. More detailed aspects of this boundary condition and how to make it fully coupled fluid-structure solver are discussed in the Methodology Subsection 2.2.1.

1.2.2 Development

The first implementation of linearized methods to Euler equations is reported by Hall [3]. This solver is capable to solve 2D harmonic problems. He focused on channel flow and turbomachinery problems. Holmes [4] implemented linearized method to 3D Navier-Stokes equations together with $k - \omega$ turbulence model. Their solver was applied to flutter and force response problems. Silkowski and Hall [5] later worked on a linearized method, which included the harmonic effects of other blades to achieve more accurate result finding force response of the blades in multistage flows.

The non-linear methods, which are more accurate and costly substitutes for linearized

methods, emerged. Giles [6] was the first researcher to implement the non-linear harmonic method to fluid equations and used his method in stator/rotor interactions. He and Ning [7] further improved the capabilities of Giles' method by proposing a novel non-linear harmonic method, which did not use steady-state solution at all. This method is further improved and used by several researchers [8][9][10].

The most commonly accepted non-linear harmonic method is the harmonic balance (HB) method introduced by Hall [11]. HB method is still an active research topic. The flow variables are stored in numerous sub time levels in one period and solved together in the HB solver. After the original HB paper, a number of studies by Hall and Ekici [12][13] improved and extended the HB methodology in multistage turbomachinery flows with multiple excitation frequencies. Gopinath [14] took one step forward and used sliding mesh in inter-row coupling of the turbomachinery blades. Another critical study is Ekici and Hall's work [15] on developing one-shot method. In this method, a fully coupled HB method is implemented with one degree of freedom structural solver to analyze limit cycle oscillations (LCO) on turbomachinery blades. Since the blades used in turbomachinery applications are very thin, one degree of freedom (DoF) assumption is considered to be valid.

There are numerous contributions to the harmonic methods, some of the selected ones related to FSI problems that made a high impact are given here. Yao and Marques [16] proposed an efficient implicit time integrator to increase the convergence rate for non-linear FSI problems such as limit cycle oscillations. Later, Yao and Jaiman [17] focused on vortex-induced vibrations on flow over a cylinder with elastic mounting at small Re numbers to test HB method in lock-in situations where natural frequency of flow and structure is synchronized. They also included a frequency updating feature to the solver so that frequency can be found iteratively.

For aeroelastic problems, Thomas and Dowell [18] proposed a new approximate fixed-point HB approach that enables the simultaneous solution of structural and fluid governing equations. Moreover, the inclusion of more structural degrees of freedom is handled without proportionally increasing computational cost. The one-shot approach, which was only created for LCO problems, was expanded by Li and Ekici [19] to evaluate flutter FSI simulations. Its novelty stems from eliminating the need

to go through a range of frequencies to find the flutter frequency. Their research includes a method for identifying initial conditions. They also proposed a stabilization technique that ensured numeric convergence on the HB one-shot method.

1.2.3 External Flows

Frequency domain methods started to be used on external flows only after the mid-2000s. One of the first examples is from the work of Thomas et al. [20]. They applied the HB method to model flutter offset and the LCO behavior in transonic AGARD 445.6 wing. Later, the study further expanded to F16 Fighter Falcon wing flutter problem [21, 22]. Dufour et al. [23] compared the accuracy of the linearized harmonic methods with the HB methods for the oscillating NACA 64A006 airfoil test case. They concluded that linearized methods were much more time efficient than HB in subsonic flow regions, whereas in transonic region and around shock waves accuracy of linearized solvers was limited. Howison et al. [24] analyzed 1.5MW WindPACT wind turbine blades using modal analysis in the structural part. The flow equations closed with Spalart-Almaras turbulence modeling. Since small amplitude assumption holds in flutter cases like this wind turbine, only one harmonic is needed in the solution. Their results predict first a few modes of the blades accurately, demonstrating that commonly used blade element theory (BEM) will become obsolete. Gong and Zhang [25] proposed a new time-spectral FSI method, in which the gradient method was used in inner iterations. This improvement provides lower computational costs and reduces the possibility of convergence stagnation. They also used unstructured mesh in their test case and obtain excellent results. Li and Ekici [26] later used HB based one-shot method for 3D AGARD 445.6 Wing to predict LCOs and flutter predictions. Their results show that the method can predict the first mode shapes as well as the LCO response of the structure.

1.3 Objective

As mentioned above, flutter phenomenon is very critical for aircraft wing, and needs to be addressed. However, time-dependent analysis are too costly and far away from

practical use for making such analysis. Harmonic methodologies could be the solution to these problems as their biggest advantage is cost-efficiency.

The main objective of this thesis is to develop a FSI-capable linearized Euler solver for external compressible flows with oscillating wings for fixed wing aircraft to detect flutter speed for aircraft wings.

Although these analyses can be conducted in the time domain with unsteady-capable CFD software, the computational cost of the numerical analysis is the main concern. Harmonic methodologies might be the answer to these issues because their main benefit is cost-effectiveness. Given the trade-offs associated with employing these solvers, another crucial goal is to identify the limitations of linearized solvers and determine the situations and applications where these methods are superior to more traditional ones. Hence, the following statements are set as the objectives of this thesis:

- Implementing the harmonic, oscillating wall and far-field boundary conditions to an existing linearized Euler solver to develop an external flow FSI solution capability
- Comparing the accuracy of the linearized Euler solutions to the unsteady Euler solutions for different free-stream Mach numbers, to assess the applicability of LH methods for external aerodynamics problems.
- Evaluate the performance of linearized Euler solvers at external flow problems where shock waves occur
- Investigating the efficiency of linearized Euler solver in computational costs
- Developing a fully coupled FSI solver to determine an aircraft wing's forced response and excitation frequency
- Detecting flutter speed for an aircraft wing by making use of fully coupled FSI solver

The overall strategy in this thesis begins with verifying and validating the linearized solver prescribed motion results against time-domain analysis and experiments respectively for simple test cases. The outcomes of these test cases make it possible

to continue working on actual engineering issues. One of these issues was detecting flutter speed. Flutter speed detection study utilizes a fully coupled FSI solution with two DoF structural model. The developed flutter prediction algorithm makes use of derivatives of lift and moment with regard to generalized coordinates and uses false-position approach to converge on flutter speed. Second, stability derivatives are determined for the fundamental finner situation, which is a problem with prescribed motion that can be efficiently solved using the linearized Euler solver. Finally, AGARD 445.6 wing is analyzed under its first mode shape to show 3D aeroelastic capabilities of the current solver. Thus, the main contribution of this thesis is modifying an existing linearized Euler solver, which is limited by internal flow applications, to develop a fully coupled FSI solver that can predict flutter speed of aircraft wing.



CHAPTER 2

METHODOLOGY

2.1 Derivation of Linearized Equations

To derive the Linearized Euler equations, we start with the integral form of 3D unsteady Euler equations. In Equation (2.1), $\mathbf{W}$ is the vector for conservation variables and $\mathbf{F}$ is the convective flux vector of time dependent Euler Equations [27]. The conservative form of the Euler equations without any source term is given as:.

$$\frac{\partial}{\partial t} \int_{\Omega} \mathbf{W} d\Omega + \oint_{\mathbf{A}} \mathbf{F} dA = 0 \quad (2.1)$$

where $\mathbf{W}$ and $\mathbf{F}$

$$\mathbf{W} = \begin{Bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho E \end{Bmatrix} \quad (2.2)$$

$$\mathbf{F} = \begin{Bmatrix} \rho V \\ \rho u V + n_x p \\ \rho v V + n_y p \\ \rho w V + n_z p \\ \rho H V \end{Bmatrix} \quad (2.3)$$

Later, the conservative variables are decomposed into mean, $\overline{\mathbf{W}}(x, y, z)$, and harmon-

ically fluctuating (perturbed) parts, $\widetilde{\mathbf{W}}(x, y, z, t)$ with predefined frequency ω .

$$\mathbf{W} = \overline{\mathbf{W}} + \widetilde{\mathbf{W}} \quad (2.4)$$

Note that mean flow is assumed to be the steady-state solution in the case of LH method. Substituting into (2.1) is rewritten as:

$$\frac{\partial}{\partial t} \int_{\Omega} (\overline{\mathbf{W}} + \widetilde{\mathbf{W}}) d\Omega = - \int_A \mathbf{F} dA \quad (2.5)$$

At this point we make another assumption such that unsteady convective fluxes can also be decomposed into steady-state and perturbation components similar to conservative variables.

$$\mathbf{F} = \overline{\mathbf{F}} + \widetilde{\mathbf{F}} \quad (2.6)$$

Applying Taylor series over $\mathbf{F}$ around $\overline{\mathbf{W}}$ yields to

$$\mathbf{F} \approx \overline{\mathbf{F}} + \left. \frac{\partial \mathbf{F}}{\partial \mathbf{W}} \right|_{\overline{\mathbf{W}}} (\mathbf{W} - \overline{\mathbf{W}}) + O^2 (\mathbf{W} - \overline{\mathbf{W}}) + \dots \quad (2.7)$$

From Equation (2.4), we substitute $(\mathbf{W} - \overline{\mathbf{W}})$ is equal to $\widetilde{\mathbf{W}}$. To linearize the problem, take only the first order terms. Equation (2.7) reduces to

$$\widetilde{\mathbf{F}} = [\overline{J}] \widetilde{\mathbf{W}} \quad (2.8)$$

Thus, this simplification allows us to use the Jacobian of the mean flow, $\overline{J}$, while writing Linearized Harmonic equations.

$$\frac{\partial}{\partial t} \int_{\Omega} (\overline{\mathbf{W}} + \widetilde{\mathbf{W}}) d\Omega = - \int_A (\overline{\mathbf{F}} + [\overline{J}] \widetilde{\mathbf{W}}) dA \quad (2.9)$$

Now, one can separate steady flow from the perturbed part since the steady state part of the equation alone should be satisfied (Equation 2.1). Then perturbation conservation equations can be written as the following

$$\frac{\partial}{\partial t} \int_{\Omega} \widetilde{\mathbf{W}} d\Omega = - \int_A [\bar{J}] \widetilde{\mathbf{W}} dA \quad (2.10)$$

Moreover, we assume that any perturbed flow variable should be harmonic in time with the excitation frequency of ω . Hence $\widetilde{\mathbf{W}}$ can be written as

$$\widetilde{\mathbf{W}} = \widehat{\mathbf{W}} e^{i\omega t} = \left| \widehat{\mathbf{W}} \right| e^{i(\omega t + \varphi)} \quad (2.11)$$

The new conservative variable denoted as $\widehat{\mathbf{W}}$ is the complex amplitude. As Equation 2.11 demonstrates complex amplitude can be replaced with a real amplitude if a phase angle, φ , is introduced. The complex equation does not have a clear physical meaning. If a perturbation is given as a sine wave as a boundary condition, this suggests that the boundary condition should be given to the imaginary part of the equation 2.11, and the real part of the boundary condition should be zero (it is vice versa for the cosine case). If the boundary condition is given with a phase, corresponding sine and cosine counter parts of the harmonic wave should be introduced in the real and the imaginary parts of the equations

With the introduction of the Equation (2.11), one can drop the time derivative at (2.10), and $e^{i\omega t}$ term in the equation will cancel out. Therefore, we end up with an equation with complex perturbed amplitudes of the primary variables.

$$i\omega \int_{\Omega} \widehat{\mathbf{W}} d\Omega + \oint \overbrace{[\bar{J}]}^{\widehat{\mathbf{F}}} \widehat{\mathbf{W}} dA = 0 \quad (2.12)$$

To solve (2.12) numerically, a pseudo-time derivative term, $\frac{\partial \widehat{\mathbf{W}}}{\partial \tau}$, is introduced at the left hand side of the equation:

$$\frac{\partial \widehat{\mathbf{W}}}{\partial \tau} + i\omega \int_{\Omega} \widehat{\mathbf{W}} d\Omega + \oint \overbrace{[\overline{J}]\widehat{\mathbf{W}}}^{\widehat{\mathbf{F}}} dA = 0 \quad (2.13)$$

This term enables marching in the frequency domain. The numerical scheme aims to eliminate the residual iteratively to satisfy the condition $\frac{\partial \widehat{\mathbf{W}}}{\partial \tau} = 0$. Here, the linearized Euler equation can also be discretized by using the Finite Volume Method with explicit time scheme.

$$\widehat{\mathbf{W}}_{new} = \widehat{\mathbf{W}}_{old} + \left(i\omega \widehat{\mathbf{W}}_{old} - \frac{\sum [\overline{J}]\widehat{\mathbf{W}}_{old} A}{\Omega} \right) \Delta \tau \quad (2.14)$$

Furthermore, since complex flux vectors could be characterized by the Jacobian of the mean flow as in 2.13, it is conceivable and used to introduce flux vector splitting to linearized Euler equations. Steger-Warming flux vector splitting [28] is utilized. The complex flux vector could be splitted into two part as in Equation 2.15. These parts could be expanded as in 2.16 by using Jacobian of the mean flow.

$$\widehat{\mathbf{F}} = \widehat{\mathbf{F}}^+ + \widehat{\mathbf{F}}^- \quad (2.15)$$

$$\widehat{\mathbf{F}}^+ = [\overline{J}]^+ \widehat{\mathbf{W}}_L \quad \widehat{\mathbf{F}}^- = [\overline{J}]^- \widehat{\mathbf{W}}_R \quad (2.16)$$

2.2 Boundary Conditions

In this thesis, reflecting boundary conditions for aircraft wings and far-field boundary conditions are used for the steady-state Euler solutions. These boundary conditions are well-known and can be found in Blazek's textbook [27]. The characteristic boundary conditions for the harmonic solver, however, is not straightforward. In this section, the harmonic boundary conditions for the Linearized Euler solver for the oscillating wings in external flow are discussed.

2.2.1 Harmonic oscillating wall boundary condition

The oscillating wall boundary condition for the stationary grid was introduced for 2D flows by Hall [3], and extended to 3 dimensions in his 1993 study [29].

Before describing the boundary condition, a few parameters need to be explained. Let $r(t)$ be the position vector of a given wall and it can be expanded to mean and perturbed parts with harmonic assumption as in Equation (2.17). The same expansion can be applied to velocity of the flow, as well as the face normal vector of the wall.

$$r(x, y, z, t) = \bar{r}(x, y, z) + \hat{r}(x, y, z)e^{i\omega t} \quad (2.17)$$

$$V(x, y, z, t) = \bar{V}(x, y, z) + \hat{V}(x, y, z)e^{i\omega t} \quad (2.18)$$

$$n(x, y, z, t) = \bar{n}(x, y, z) + \hat{n}(x, y, z)e^{i\omega t} \quad (2.19)$$

The logic of the boundary condition is straightforward; it simply suggests that no flow should pass through the moving wall. This condition is expressed by Equation 2.20 at the wall face.

$$V \cdot n - \frac{\partial r}{\partial t} \cdot n = 0 \quad (2.20)$$

Steady part of the equation alone should be satisfied, which is $\bar{V} \cdot \bar{n} = 0$ (since $\frac{\partial \bar{r}}{\partial t} = 0$). When all variables in Equation 2.20 are expanded as it is described in Equations 2.17, 2.18 and 2.19, and the steady part is extracted from the (2.20), remaining comparable terms will give the boundary condition in Equation 2.21.

$$\hat{V} \cdot \bar{n} = -\bar{V} \cdot \hat{n} + i\omega \hat{r} \cdot \bar{n} \quad (2.21)$$

Note that any term that includes the multiplication of two perturbed amplitudes is neglected in (2.21) due to the linearized assumptions. This assumption is valid if the vibration amplitude of the oscillating wall is relatively small. This is the major limitation of this method.

Finally, we need another term to compensate for the use of a fixed grid for the boundary that should normally move.

$$\hat{V} \cdot \bar{n} = -\bar{V} \cdot \hat{n} + i\omega \hat{r} \cdot \bar{n} - \hat{r} \cdot \nabla(\bar{V}_n) \quad (2.22)$$

The term $\hat{r} \cdot \nabla(\bar{V}_n)$ is an extrapolation from the mean (steady-state) position to the instantaneous position of the face center. The normal component of the steady state velocity gradient can be expanded as:

$$\nabla(\bar{V}_n) = \bar{n} \cdot \nabla(\bar{V}) + \bar{V} \cdot \nabla(\bar{n}) \quad (2.23)$$

The second term on the right-hand side of Equation (2.23) is also neglected, assuming $\nabla(\bar{n})$ will be small. The first term is easy to calculate in the boundary since both the velocity gradient in the mean position and the surface normal of the face is known from the steady-state solution. The harmonic perturbation of the position $\hat{r}$ is also known since this is a prescribed motion. Application of this boundary condition allows the user to impose any kind of harmonic motion on the wall surface.

2D Rigid Harmonic motion

For a 2D rigid harmonic motion, for example, an airfoil can be subjected to either pitching or plunging motion (or both). For the pitching motion, a rotation center input is required. Prescribed pitching motion of the airfoil can be defined as $\alpha(t) = \alpha_{max} \cos(\omega t)$ if there is no initial angle of attack. Then, our position vector at any point at the wall surface r starts from the rotation center and ends in instantaneous face center for a given face in the boundary. Later, using Equation (2.17) and setting $e^{i\omega t} = 1$ should give as $\hat{r}$ as in 2.25 which is the amplitude of perturbed position vector of the wall surface.

$$r_{max} = \bar{r} + \hat{r} \quad (2.24)$$

$$\hat{r} = r_{max} - \bar{r}. \quad (2.25)$$

where the wall surface's position vector at the maximum pitch is designated as r_{max} .

For a 2D rotational motion such as pitching r_{max} can be found as

$$Rot_z = \begin{bmatrix} \cos\alpha_{max} & -\sin\alpha_{max} & 0 \\ \sin\alpha_{max} & \cos\alpha_{max} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2.26)$$

$$r_{max} = Rot_z \cdot \bar{r} \quad (2.27)$$

Since the resultant $\hat{r}$ in Equation (2.26) is real, the imaginary part will be zero (since input motion is cosine). This motion can also be easily translated to 3D rigid motion where there are three rotational axes by changing the definition of the rotational matrix at (2.26). One other parameter required calculation is $\hat{n}$. $\hat{n}$ can be calculated similarly to $\hat{r}$. This time, however, each node's new position should be calculated at $\alpha(t_{max}) = \alpha_{max}$. If we describe node locations with N and new node locations as N_{new} , relation in Equation (2.28) can be established.

$$N_{new} = Rot_z \cdot (N - N_{rotcenter}) + N_{rotcenter} \quad (2.28)$$

After finding the new node coordinates at the maximum deflection state, one can use the cross product to find the new surface's normal direction as it is described in Equation (2.31).

$$t_1 = N_{new2} - N_{new1} \quad (2.29)$$

$$t_2 = N_{new3} - N_{new1} \quad (2.30)$$

$$n_{max} = \frac{t_2 \times t_1}{|t_2 \times t_1|} \quad (2.31)$$

Another issue to note here is to ensure that n_{max} is not pointing the reverse direction of $\bar{n}$. This can be achieved by checking $n \cdot n_{max} > 0$. If this is not the case, multiplying n_{max} with -1 should be sufficient.

Plunging motion only involves transitional motion. Therefore, for plunging motion, face normal vectors are constant. This fact makes the first term in the right-hand side of the (2.22) disappear. Plunging motion of an airfoil can be described as

$$\begin{aligned} h(t) &= h_{max} \cos(\omega t - \varphi) \\ \hat{r}_{real}(y) &= h_{max} \cos(\varphi) \\ \hat{r}_{imag}(y) &= h_{max} \sin(\varphi) \end{aligned}$$

This also enables to application of phase difference between pitching and plunging motions, if they were to be solved together. If that is the case, only operation required is the superposition $\hat{r}$ vector as

$$\hat{r}_{total} = \hat{r}_{pitching} + \hat{r}_{plunging}$$

Elastic Deformations

Until now the main focus was on rigid motions. However, the FSI solution of a 3D non-rigid wing involves elastic deformations. Let's consider dynamic elastic wall deformation, meaning all faces are subjected to different amplitudes. As long as we have the deformation information for each node on the moving surface (under single harmonic excitation) oscillating wall boundary condition can also handle this motion. For example, for a 3D airfoil, the finite element model solution provides the mode shape of the structure which means deformation data, $\hat{r}$, is available for a number of points. Since the FEM model will have a different grid configuration than harmonic CFD solver, the calculated deformations for each cell should be either interpolated to fluid grid node locations in the wall boundary or a curve-fitting should be used. After that, for each cell, the given deformation will be equated to $\hat{r}_{real}$ (since there will be no phase, there is no $\hat{r}_{imag}$). n_{max} could be found with the same method as given in

the equation (2.31). The only difference calculating new node locations will be easier as how much each node will move is given directly.

Other Conditions

For a 3D case, Linearized Euler Solver has 10 unknowns (five for real and five for imaginary part). Thus, five complex equations in the boundary are required. The boundary condition for the perturbed velocity is discussed above. Equation (2.22) is used to calculate perturbed velocity at the mean normal direction of the boundary face. From there, one can convert the perturbed velocity from normal tangential coordinate system, $n - t$ to the Cartesian coordinate system, which provides all three complex velocity components. Therefore, two more complex boundary conditions are needed. One can choose to get two out of pressure, temperature and density. It is physically more sense to take temperature and pressure for boundary conditions. The principal idea in both pressure and temperature is the same. The moving wall face should have the same pressure and temperature as the boundary cell to satisfy the solid wall with zero heat flux. However, since the boundary should reflect the effect of the movement, extrapolation from the mean (steady-state) position to the instantaneous position of the face center is made (Equation (2.32)). Subscript $_d$ refers to variables coming from the domain whereas the subscript $_f$ represents variables on the boundary face. Although this extrapolation in pressure and temperature may seem like it will create a considerable difference, it did not make any considerable change in results for the tried test cases.

$$\hat{P}_f = \hat{P}_d + \hat{r} \cdot \nabla(\bar{P}) \quad (2.32)$$

$$\hat{T}_f = \hat{T}_d + \hat{r} \cdot \nabla(\bar{T}) \quad (2.33)$$

Later, primitive variables are converted to conservative ones since conservative equations are used for the solution iterations. The perturbed density is also calculated from the ideal gas equation for the calculated perturbed pressure and temperature as in Equation (2.35).

$$\hat{P} = R[\bar{\rho}\hat{T} + \hat{\rho}\bar{T}] \quad (2.34)$$

$$\hat{\rho} = \frac{\hat{P} - R\bar{\rho}\hat{T}}{R\bar{T}} \quad (2.35)$$

2.2.2 Harmonic non-reflecting far-field boundary conditions

The far-field boundary condition is important, and how to extend it into linearized solver changes the solution significantly. These changes in the linearized solution arise most likely because the harmonic pressure waves reflect from the far-field boundaries. Therefore, non-reflecting boundary condition for the Euler equations proposed by Saxer and Giles [30] is used for subsonic flows.

Supersonic Flows

Supersonic inflow All the information is coming from upstream. Therefore no effects from the harmonic motion transferred to the boundary condition.

$$\tilde{P}_f = 0, \tilde{T}_f = 0, \tilde{u}_f = \vec{0} \quad (2.36)$$

Supersonic outflow All the information is coming from upstream. Therefore, all domain perturbations directly determine the boundary condition values.

$$\tilde{P}_f = \tilde{p}_d, \tilde{T}_f = \tilde{T}_d, \tilde{u}_f = \tilde{u}_d \quad (2.37)$$

Subsonic flows

The discussed boundary condition in Saxer and Giles' paper only provides one-dimensional non-reflecting cases. Meaning that although the equations are in three dimensions, the non-reflecting condition only applies to x -axis. If $n - t$ coordinate

system is used, velocity can be reduced to one component. Then, we can use the non-reflecting boundary condition in $n - t$ coordinate system to expand to all three dimensions.

$$\begin{Bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{Bmatrix} = \begin{bmatrix} -\bar{c}^2 & 0 & 1 \\ 0 & \bar{\rho}\bar{c} & 1 \\ 0 & -\bar{\rho}\bar{c} & \bar{c} \end{bmatrix} \cdot \begin{Bmatrix} \hat{\rho} \\ \hat{V}_n \\ \hat{P} \end{Bmatrix} \quad (2.38)$$

$$\begin{Bmatrix} \hat{\rho} \\ \hat{V}_n \\ \hat{P} \end{Bmatrix} = \begin{bmatrix} -1/\bar{c}^2 & 1/(2\bar{c}^2) & 1/(2\bar{c}^2) \\ 0 & 1/(2\bar{\rho}\bar{c}) & -1/(2\bar{\rho}\bar{c}) \\ 0 & 1/2 & 1/2 \end{bmatrix} \cdot \begin{Bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{Bmatrix} \quad (2.39)$$

Characteristic variables can be found using Equation (2.38). Also, equation (2.39) is the inverse operation of the (2.38), such that we can calculate primitive perturbed variables.

$$\phi_1 = -\bar{c}^2 \hat{\rho} + \hat{P} \quad (2.40)$$

$$\phi_2 = \bar{\rho}\bar{c} \hat{V}_n + \hat{P} \quad (2.41)$$

$$\phi_3 = -\bar{\rho}\bar{c} \hat{V}_n + \hat{P}\bar{c} \quad (2.42)$$

Subsonic inflow For the subsonic inflow cases, the characteristic variables ϕ_1 and ϕ_2 are taken as 0, and all ϕ are substituted back to Equation (2.39).

$$\hat{\rho} = \frac{1}{2\bar{c}^2} \phi_3 \quad (2.43)$$

$$\hat{V}_n = -\frac{1}{2\bar{\rho}\bar{c}} \phi_3 \quad (2.44)$$

$$\hat{P} = \frac{1}{2}\phi_3 \quad (2.45)$$

Subsonic outflow For the subsonic outflow cases, the characteristic variable ϕ_3 is taken 0 and substituted back to Equation (2.39).

$$\hat{\rho} = -\frac{1}{\bar{c}^2}\phi_1 + \frac{1}{2\bar{c}^2}\phi_2 \quad (2.46)$$

$$\hat{V}_n = \frac{1}{2\bar{\rho}\bar{c}}\phi_2 \quad (2.47)$$

$$\hat{P} = \frac{1}{2}\phi_2 \quad (2.48)$$

After all calculations are completed, $\hat{V}_n$ is transferred to the Cartesian coordinate system. Also, perturbed variable $\hat{T}$ is calculated using ideal gas equation, (2.34). Finally, primitive variables are converted to conservative ones as in the oscillating wall boundary condition.

2.3 Two Way Coupled Fluid-Structure Interaction Solver

In the above sections, it is described how oscillating walls with prescribed motion cases can be solved for external flows in the linearized Euler solver. If a two-way coupled fluid-structure solution is desired, this can be achieved with the addition of a structural solver.

Let's take a 2 DoF structural system that describes pitching and plunging motion of an airfoil. The structural system, which is demonstrated in Figure 2.1, is consist of a mass matrix, M_{st} , and a stiffness matrix, K_{st} . The system has no structural damping and its close form is described with Equation (2.49). This is a forced vibration problem since the oscillations of airfoil are fed by disturbances in the pressure field of the fluid flow. A detailed version of the systems and matrices can be found in the study of Yıldız [31]. There is also no viscous damping term since viscous damping effect

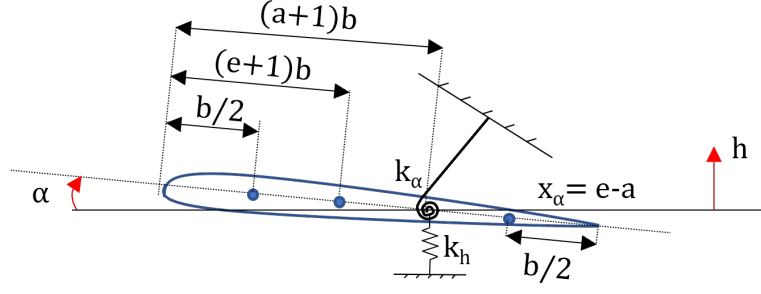


Figure 2.1: Structural modal of the airfoil

is coming from aerodynamic forcing. The forcing vector Q , in Equations (2.50) and (2.49), is consist of aerodynamic force and moment. These parameters are calculated from the Linearized Euler solver, and supplied to the structural system as input for a given mode shape q and free-stream velocity U . Later, the structural system is solved by the given forcing vector, mode shape vector q and new excitation frequency ω are found. At this point, h and α should be given back to the fluid solver as new harmonic motion input. This procedure leads to an iterative process. Once the difference in results (q and ω) of two sequential iteration drops below a user-defined tolerance, fluid-structure analysis is considered as converged. Therefore, the final solution provides a specific amplitude and excitation frequency for plunging and pitching motion for a given free-stream velocity.

$$[M_{st}] \{\ddot{q}\} + [K_{st}] \{q\} = \{Q\} \quad (2.49)$$

$$\{q\} = \begin{Bmatrix} h \\ \alpha \end{Bmatrix}, \{Q\} = \begin{Bmatrix} L \\ M \end{Bmatrix} \quad (2.50)$$

$$\{q\} = \{\hat{q}\} e^{i\omega t} \quad (2.51)$$

2.3.1 Solution Technique

Since the force and moment coming from the fluid side will be complex, the structural system could be solved in two different ways. Either 2×2 complex structural system

as in Equations (2.52) or expended equivalent 4×4 system with separated real and imaginary parts could be solved as in Equation (2.53). C_{st} is zero matrix as viscous damping is only modeled with aerodynamic forcing.

$$-\omega^2 [M_{st}] \{\widehat{q}\} + [K_{st}] \{\widehat{q}\} = \{\widehat{Q}\} \quad (2.52)$$

$$\begin{bmatrix} 0 & M_{st} \\ M_{st} & C_{st} \end{bmatrix} \begin{Bmatrix} \ddot{q} \\ \dot{q} \end{Bmatrix} + \begin{bmatrix} -M_{st} & 0 \\ 0 & K_{st} \end{bmatrix} \begin{Bmatrix} \dot{q} \\ q \end{Bmatrix} = \begin{Bmatrix} 0 \\ Q \end{Bmatrix} \quad (2.53)$$

In the case of equivalent 4×4 , one can also feed the forcing vector into damping and stiffness matrices as in Equation (2.54).

$$K_a + i\omega [C_a] = \frac{\partial \widehat{Q}}{\partial \widehat{q}} = J_a \quad (2.54)$$

$$J_a = \begin{bmatrix} \frac{\partial \widehat{L}}{\partial h} & \frac{\partial \widehat{L}}{\partial \alpha} \\ \frac{\partial \widehat{M}}{\partial h} & \frac{\partial \widehat{M}}{\partial \alpha} \end{bmatrix} \quad (2.55)$$

Therefore, we need to calculate Jacobian matrix shown in (2.55) and that requires three linearized Euler solutions for each iteration. Then, one can find force derivatives with respect to motion by using finite difference method as the following.

$$\frac{\partial \widehat{L}}{\partial h} = \frac{\widehat{L}(h + d_h, \alpha) - \widehat{L}(h, \alpha)}{d_h} \quad (2.56)$$

$$\frac{\partial \widehat{L}}{\partial \alpha} = \frac{\widehat{L}(h, \alpha + d_\alpha) - \widehat{L}(h, \alpha)}{d_\alpha} \quad (2.57)$$

$$\frac{\partial \widehat{M}}{\partial h} = \frac{\widehat{M}(h + d_h, \alpha) - \widehat{M}(h, \alpha)}{d_h} \quad (2.58)$$

$$\frac{\partial \widehat{M}}{\partial \alpha} = \frac{\widehat{M}(h, \alpha + d_\alpha) - \widehat{M}(h, \alpha)}{d_\alpha} \quad (2.59)$$

Thus, one can rewrite matrices in (2.53) as

$$[A] = \begin{bmatrix} 0 & M_{st} \\ M_{st} & C_{st} - C_a \end{bmatrix} \quad (2.60)$$

$$[B] = \begin{bmatrix} -M_{st} & 0 \\ 0 & K_{st} - K_a \end{bmatrix} \quad (2.61)$$

$$\{\phi\} = \begin{Bmatrix} \dot{q} \\ q \end{Bmatrix} = \{\hat{\phi}\} e^{\lambda t} \quad (2.62)$$

$$[A] \{\dot{\phi}\} + [B] \{\phi\} = \{0\} \quad (2.63)$$

This way, given forced vibration problem could be reduced to a free-vibration problem. Eigenvalue problem is solved for λ_j for (2.63).

$$(\lambda [A] + [B]) \{\hat{\phi}\} e^{\lambda t} = \{0\} \quad (2.64)$$

$$[I] \lambda = -[B] [A]^{-1} \quad (2.65)$$

$$\lambda_j = -\zeta_j \omega_j \pm i \omega_{dj} \quad (2.66)$$

$$\omega_{dj} = \omega_j \sqrt{1 - \zeta_j^2} \quad (2.67)$$

where ω_j and ω_{dj} are undamped and damped natural frequencies of the j^{th} mode respectively, and ζ_j is damping ratio. From the calculated eigenvalues, $\hat{\phi}$ and consequently h and α could be found. Hence, linearized solver inputs h , α and ω are established for the iterative process.

2.4 Flutter Speed Detection

Detecting flutter speed is one of the objectives of this thesis. It is merely an extension of the fully coupled fluid-structure interaction (FSI) analysis discussed above. The distinct part is to check the damping ratio for a given free-stream velocity.

For a given mode j that flutter is observed, the real part of λ_j should be just above zero for the corresponding excitation frequency ω_{dj} of the tracked mode. Since our only interest is to find in which free-stream speed zero damping value occurs, the actual amplitude of the system is not relevant. Only the mode shape of the system is required, meaning it is possible to scale the q or ϕ vector with any number as long as the ratio of h and α is the same. This is useful because the linearization assumption in harmonic CFD solver limits the magnitude of amplitudes to some extent.

The schematic for the whole flutter detection process can be seen in Figure 2.2. The inner loop is basically a fully coupled fluid-structure solver which is discussed in the above section. When the coupled solver is converged, algorithm checks for the damping ratio. If it is not close to zero, the flutter solver needs to try different free-stream speeds until it concludes the flutter. This try and error process can either be applied by an incremental free-stream speed search or more sophisticated root finding algorithms.

A new false-position based method for the external velocity loop is proposed here. Therefore, the algorithm starts with upper and lower speed brackets to converge to the flutter speed. Although using Newton's method is also possible and it has a better convergence rate, real concern is the CFD solution times. Since Newton's method needs more CFD solutions per velocity loop to calculate the Jacobian matrix, which will increase the overall solution time although the number of iterations are reduced.

2.4.1 Theodorsen Theory

It is crucial to compare the results of the flutter analysis with reliable sources. Theodorsen Theory is utilized for this purpose, and the theory is briefly explained here. For given reduced frequency

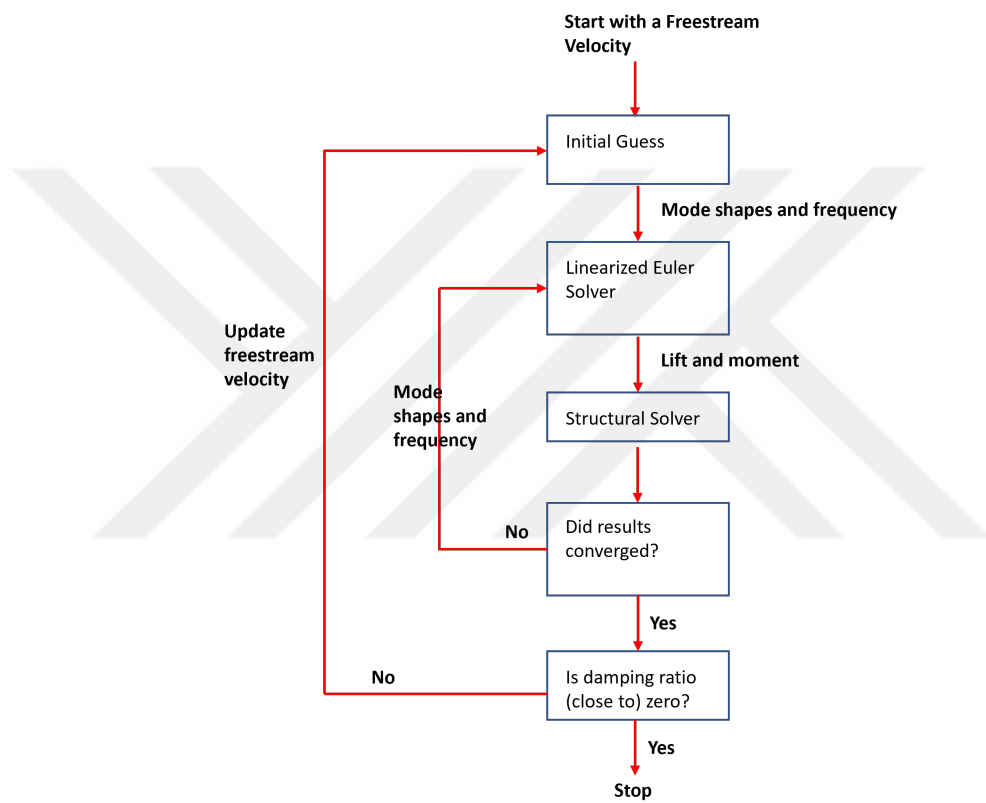


Figure 2.2: Flutter algorithm

$$k_{st} = \frac{\omega c}{U} \quad (2.68)$$

Theodorsen function given as follows

$$C_{theodorsen}(k_{st}) = \frac{H_1^{(2)}(k_{st})}{H_1^{(2)}(k_{st}) + iH_0^{(2)}(k_{st})} \quad (2.69)$$

where $H_v^{(2)}$ is the Hankel function of the second kind. Theodorsen solution for lift and moment forces for 2 degree of freedom airfoil can be found below

$$L_{theodorsen}/p_\infty = \pi b^2 (\ddot{h} + U\dot{\alpha} - ab\ddot{\alpha}) + 2\pi b U C_{theodorsen}(k_{st}) \left[\dot{h} + U\alpha + b(1/2 - a)\dot{\alpha} \right] \quad (2.70)$$

$$M_{theodorsen}/p_\infty = \pi b^2 \left[ba\ddot{h} - Ub(1/2 - a)\dot{\alpha} - b^2(1/8 + a^2) \right] + 2\pi b^2 U (a + 1/2) C_{theodorsen}(k_{st}) \left[\dot{h} + U\alpha + b(1/2 - a)\dot{\alpha} \right] \quad (2.71)$$

The geometric parameters a and b is shown in the Figure 2.1.

Now, that aerodynamic lift and moment forces are predicted, the system can be solved for any free-stream velocity U . Since the motion is harmonic, derivatives of α and h can be determined using Equation (2.51). Then, the same system (2.65) can be solved. With an incremental search, flutter point can be found where damping ratio is zero.

2.5 Lift and Moment Calculations in Linearized Solver

For a conventional unsteady CFD analysis, calculating lift is a straightforward process. It is simply the integration of the pressure over the surface area for a given time. The harmonic solver, however, does not provide immediate solutions in sub-times. There are two approaches that one can take for calculating time-dependent lift. First, it is possible to calculate sufficient amount of sub-time levels and calculating the lift

force separately for each sub-time. Since $\tilde{P} = \hat{P}e^{i\omega t} = |\hat{P}|e^{i\omega t + \varphi}$, each cells pressure amplitude and phase angle is known. $|\hat{P}|\cos(\omega t + \varphi)$ can be used to find pressures in each sub-time and integrate them over the wall surface. The second approach is to directly calculate harmonic lift by integrating complex pressure amplitudes. This method is applicable as the single harmonic excitation input. More than one excitation frequency would not make this possible as terms with $e^{i\omega t}$ could not sum up with terms with, for example, $e^{i2\omega t}$. The beauty of this approach is that obtaining unsteady the lift force as a function $\tilde{L} = \hat{L}_{real}\cos(\omega t) + \hat{L}_{imag}\sin(\omega t)$, not as a discrete set of values.

If the lift is nonzero in steady state position (angle of attack is excited in mean position or airfoil is not symmetric), the steady lift should also be added to the unsteady part of the lift as a constant value like $L(t) = \bar{L}(t) + \tilde{L}(t)$.

$$L(t) = \sum P_i(t) \cdot \vec{n}_i A_i \quad (2.72)$$

$$\hat{L} = \sum \hat{P}_i \cdot \vec{n}_i A_i \quad (2.73)$$

For the moment calculation, the same discussion is valid, and it can be found as

$$\hat{M} = \sum \hat{P}_i \cdot (\vec{n}_i \times \vec{r}_{rot}) A_i \quad (2.74)$$

where $\vec{r}_{rot}$ is the vector from rotation center to face center. If the moment is needed around another point, g , than rotation center, position can be shifted with lift force as $\hat{M}_a = \hat{L}(\vec{r}_{rotx} - g_x) + \hat{M}$.

2.6 Dynamic Stability Derivatives

In this section calculation of the stability derivatives using harmonic solvers is given. The stability derivatives could be directly calculated by harmonic solvers and calculation of these derivatives are used for validation purposes. The dynamic stability

derivatives, sometimes referred to as aerodynamic transfer functions are calculated to assess an aircraft's stability under excitation from the trim condition. The theory to find these derivatives are taken from Schmidt and Newman [32] and briefly demonstrated here.

Consider a pitching motion case with a non-zero initial angle of attack such as:

$$\tilde{\alpha}(t) = \alpha_1 \cos(\omega t) \quad (2.75)$$

The corresponding dynamic moment coefficient, C_m

$$\tilde{C}_m(t) = \left| \widehat{C}_m \right| \cos(\omega t + \varphi) \quad (2.76)$$

and derivative of $\alpha(t)$ will be given in the equation below, which is also equal to pitching rate q

$$\dot{\alpha}(t) = -\alpha_1 \omega \sin(\omega t) = q \quad (2.77)$$

Now, we can use Taylor series expansion to define $\tilde{C}_m$ with the combined effects of q and $\alpha(t)$

$$\tilde{C}_m(t) = C_{m_\alpha} \alpha(t) + C_{m_{\dot{\alpha}}} \dot{\alpha}(t) \frac{c}{2U} + C_{m_q} q(t) \frac{c}{2U} + \dots \quad (2.78)$$

As the perturbations are small other terms can be neglected, where

$$C_{m_\alpha} = \frac{\partial C_m}{\partial \alpha} \quad (2.79)$$

$$C_{m_q} = \frac{\partial C_m}{\partial \frac{qc}{2U}} \quad (2.80)$$

$$C_{m_{\dot{\alpha}}} = \frac{\partial C_m}{\partial \frac{\dot{\alpha}c}{2U}} \quad (2.81)$$

Modifying Equation (2.78) with (2.77), and equating to (2.76), one can find the unknown derivatives as the following (pitching moment stiffness and damping, respectively)

$$C_{m_\alpha} = \frac{|\widehat{C}_m|}{\alpha_1} \cos \varphi \quad (2.82)$$

$$C_{m_{\dot{\alpha}}} + C_{m_q} = -\frac{2U |\widehat{C}_m|}{\omega c \alpha_1} \sin \varphi \quad (2.83)$$

The current linearized solver takes pitching amplitude α_1 , excitation frequency ω and free-stream velocity U as input. c is the characteristic length of the aircraft. Finally, $\widehat{C}_m$ is one of the outputs and the complex form allows us to find phase difference φ . Hence, pitching moment stiffness and damping coefficient can be easily calculated. Usually this process is slow as unsteady CFD analysis is needed. The computations continue until the periodicity is reached. Harmonic solvers, however, are free from these problems. Thus, harmonic methods are advantageous to calculate stability derivatives.



CHAPTER 3

VERIFICATION AND VALIDATION OF THE LINEARIZED EULER SOLVER

Before tackling actual engineering situations, the linearized solver's potential for simulating flow around harmonically oscillating solid bodies must be validated. Thus, 2D and 3D studies of prescribed motion are performed in this chapter and compared with an experiment and unsteady Euler solutions.

3.1 2D Oscillating NACA0012 airfoil

The study is done for three different free-stream Mach numbers (0.1, 0.5, 0.8) with pitching and plunging motion cases. These test cases aim to observe the accuracy of the linearized Euler solver in subsonic and transonic regions in comparison to unsteady Euler solutions. The angle of attack is set to zero for steady-state parts. The selected airfoil profile is NACA0012. The structured grids are preferred for all NACA0012 test cases.

Operating conditions are as follows: Pressure and temperature in the far-field were taken as 1 atm and 300K, respectively. The excitation frequency for all test cases is set to 5.6608 Hz. The frequency is taken from an experiment [16] discussed in Solver Limitations Chapter 5. Prescribed motion is given as a basic sine wave for both pitching and plunging motion cases, as in Equations (3.1) and (3.2), respectively. For the corresponding pitching and plunging motions, the positions of an airfoil in each quarter of the period are illustrated in Figure 3.1, where T stands for the full period of the oscillation.

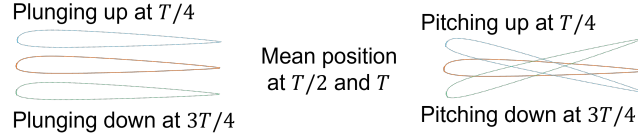


Figure 3.1: Plunging and pitching motion

$$\alpha(t) = \alpha_0 \sin(\omega t) \quad (3.1)$$

$$h(t) = h_0 \sin(\omega t) \quad (3.2)$$

All linearized Euler solutions are compared with unsteady solutions. Both solutions are obtained on the same grid. After a time step independence study, unsteady Euler solver time step dt is set to 3×10^{-4} s. All unsteady solutions were solved until the 10th cycle. It is observed that periodicity is reached totally after the third cycle. Pressure coefficient plots are given for comparison for all cases. The plots are taken from the quarter, half, three forth, and full cycles. As the harmonic pressure contours show physical behavior and not the correctness of the results, they are only shown only for 0.5 Mach cases. Residuals are also presented with the harmonic lift force in each iteration. Momentum residuals are defined as the average of the momentum x , y and z components.

3.1.1 Mesh independence study

The mesh independence study for a linearized solver is not as straightforward as a conventional CFD case. The quality of the mesh is relatively more important, especially near boundaries. The orthogonality of the cells close to the wall boundary is carefully considered throughout this work. This issue is further discussed in the Solver Limitations Chapter. The number of cells increased by a factor roughly $\sqrt{2}$ at each step, until harmonic forcing on the reflecting boundary converged to a constant value. While increasing the number of cells, it is prioritized to increase the number

Table 3.1: Mesh numbers and corresponding lift forces

Mesh	Number of cells	Real part of the Lift Force	Imaginary part of the Lift Force
M1	12800	1243.85	-158.58
M2	17600	1268.28	-168.45
M3	26400	1097.05	-133.1
M4	36708	1091.30	-130.61

of nodes on the airfoil surface, and grid density around the airfoil. Mesh dependency was conducted for all three free-stream cases for both plunging and pitching motions. Only 0.5 free stream Mach number pitching test case (other parameters for the test cases are demonstrated in the next section) is demonstrated in Table 3.1. The final mesh independent grid (M4) is also presented in Figure 3.2. Between M3 and M4, there is a 0.53% difference between real part of the lift forces and a %1.90 difference between imaginary parts. Increasing the mesh density even higher than the M4 case started to cause convergence issues in the linearized solver. Thus, M4 is chosen to be the final grid. Note that the airfoil is away from the upstream and downstream by 10 times the chord length.

The effect of the distance of the far-field was also investigated. As a result, it is observed that the linearized Euler solver can achieve the same results with a relatively small far-field. We have obtained good results up to five times the chord length distance to the airfoil. This is a feature of harmonic methods in general, and is especially useful in turbomachinery.

3.1.2 Results of the Harmonic Solver

Plunging motion test cases

Plunging motion is a relatively simple case with higher convergence rates than pitching motion. Due to the small effective angle of attack variation in plunging motion, pressure differences between upper and lower boundaries are subtle. Therefore, for all cases amplitude of the oscillation is kept at 0.1 times the chord length for a better

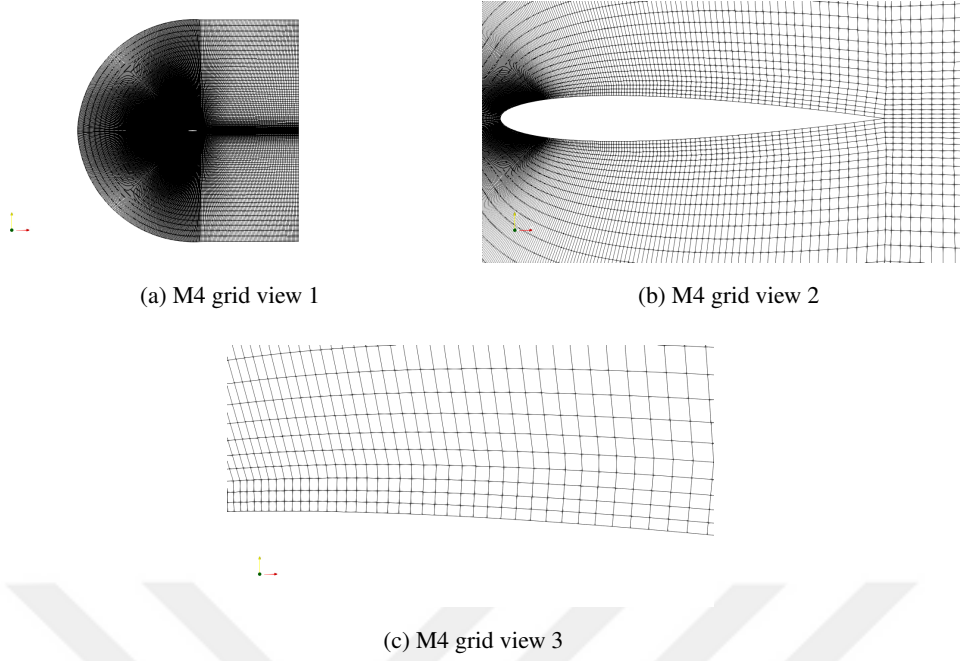
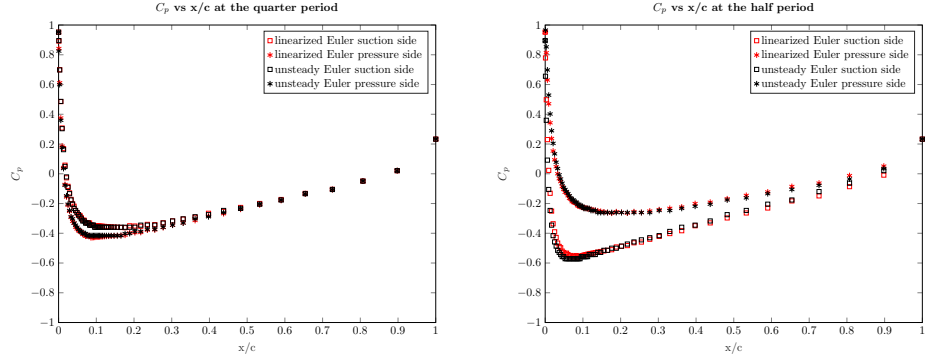


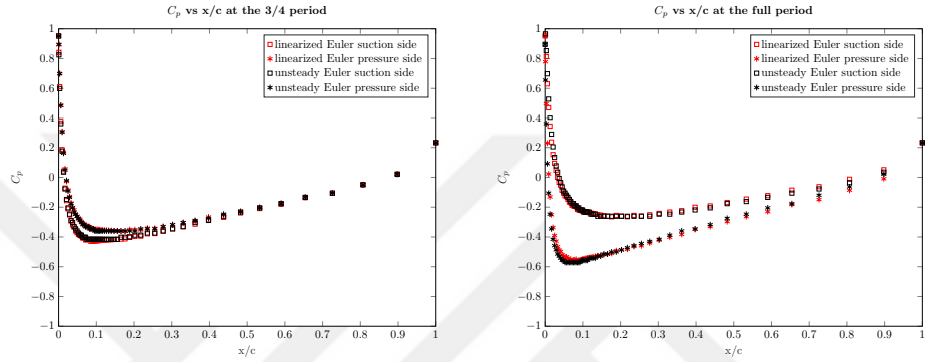
Figure 3.2: M4 grid in several views

comparison. This is also a good test of whether the small amplitude assumption of the linearized solver still holds in this amount of amplitude. Plunging motion results with harmonic solver are obtained without any grid motion compensation term as it is described in Subsection 2.2.1. It is observed that this term should be neglected for better results in translating motions.

Ma=0.1 For 0.1 Mach, an unsteady moving grid for validation simulations are conducted where Chorin-Turkel preconditioning is used for the steady part. Results can be seen in Figure 3.3. The pressure plots demonstrate a good match between linearized Euler and unsteady Euler results despite the magnitude of the oscillation amplitude. The airfoil has zero velocity at the quarter and three-fourth periods (plunging up and down), hence zero effective angle of attack. Thus, the pressure difference between the upper and lower surface is close to zero. However, when the airfoil is located in the mean position (during the half and full period), its velocity is maximum which leads to higher pressure differences between the upper and lower surface. Harmonic residuals are also shown in Figure 3.4. Real and imaginary parts of the lift are illustrated in the same figure as well. Both parts of the lift appeared to be unchanged



(a) Pressure coefficient plot at the quarter period (b) Pressure coefficient plot at the half period



(c) Pressure coefficient plot at the 3/4 period (d) Pressure coefficient plot at the full period

Figure 3.3: Pressure coefficient plots of plunging airfoil in (a,b,c,d shows sub times levels 0.25,0.5,0.75,1 times of the period respectively) for $Ma=0.1$ case

after 20,000 iterations. The lift can be considered a good indication of when to stop the analysis.

Ma=0.5 For 0.5 Mach, a first-order reconstruction is used in both steady and harmonic solvers. Harmonic pressure amplitude and phase contours can be seen in Figure 3.5. The pressure phase angle difference of any symmetrical cells in the upper and lower surfaces equals π . Figure 3.6 shows the pressure plots for four sub-time levels. Compared to the $Ma=0.1$ case, the pressure difference between the pressure and suction side is smaller for all sub-time levels as mainstream starts to dominate the effect of the oscillation. Residuals in Figure 3.7 exhibit the same behavior as earlier.

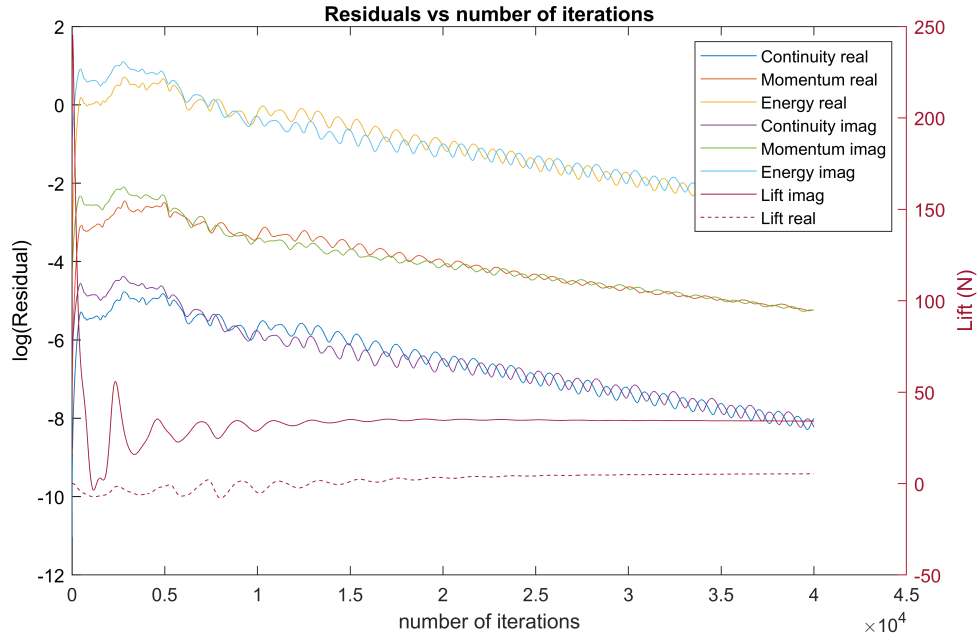


Figure 3.4: Residuals and lift force vs. iterations for $Ma = 0.1$ plunging case

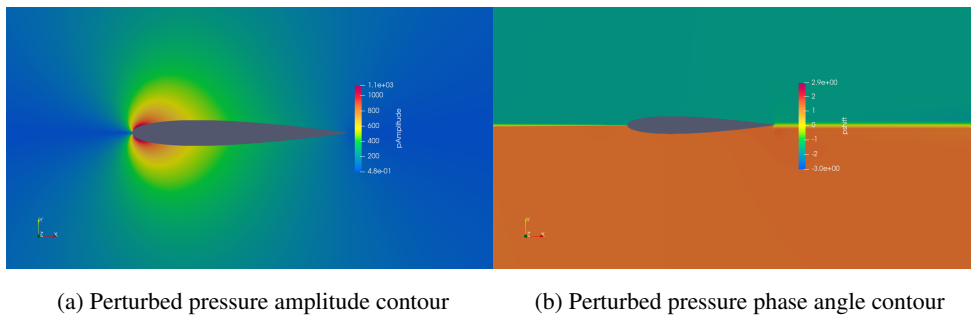
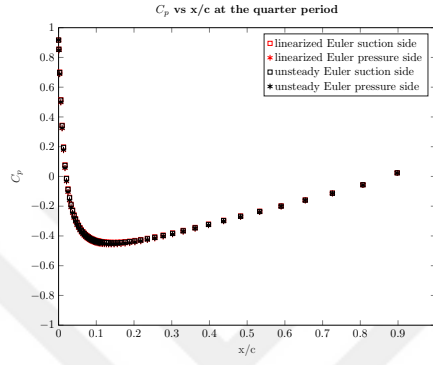
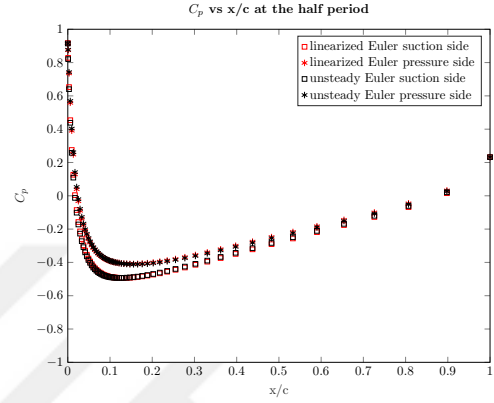


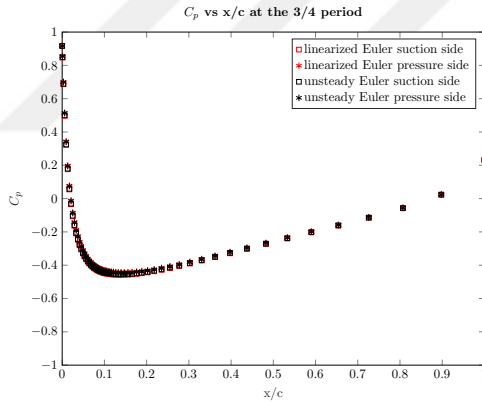
Figure 3.5: Pressure amplitude and phase contours respectively for $Ma = 0.5$ plunging motion case



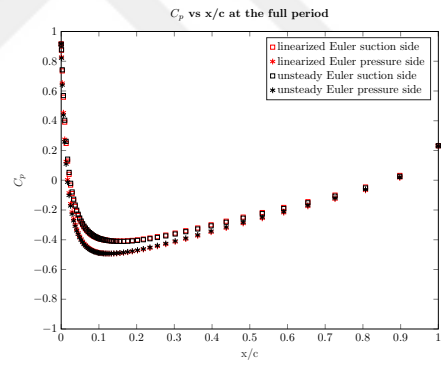
(a) Pressure coefficient plot at the quarter period



(b) Pressure coefficient plot at the half period



(c) Pressure coefficient plot at the 3/4 period



(d) Pressure coefficient plot at the full period

Figure 3.6: Pressure coefficient plots of plunging airfoil in (a,b,c,d shows sub times levels 0.25,0.5,0.75,1 times of the period respectively) for $Ma = 0.5$ case

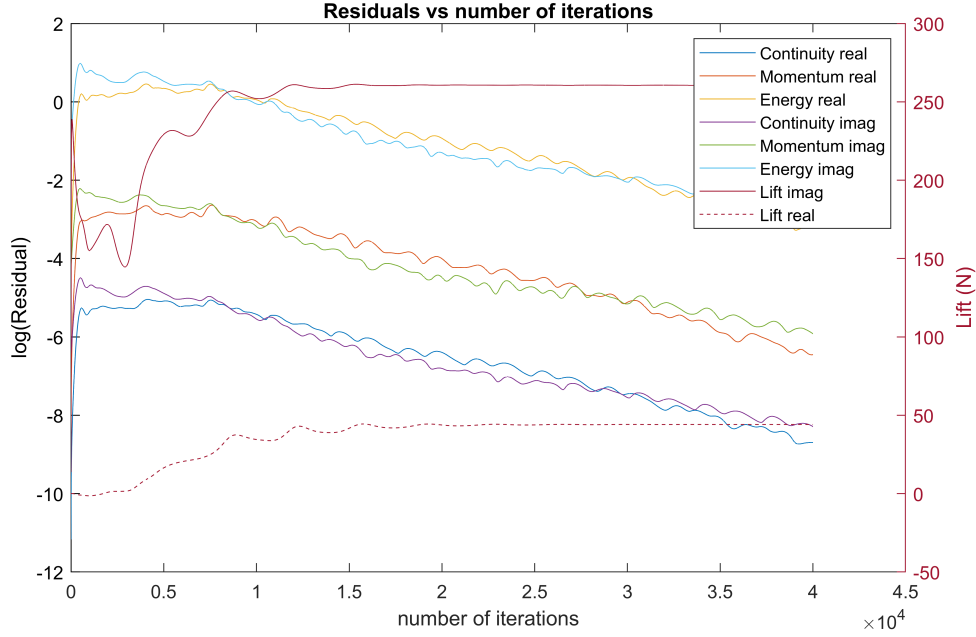
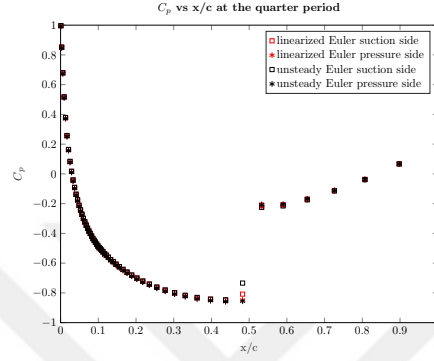


Figure 3.7: Residuals and lift force vs. iterations for $Ma = 0.5$ plunging case

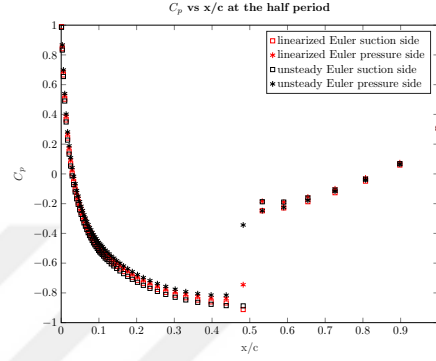
Ma=0.8 For 0.8 Mach, a first-order reconstruction is used in both steady and harmonic solvers. Results can be seen in Figure 3.8. This time test case is transonic, and a shock wave occurs on the surface of the airfoil. Free-stream Mach number is high enough to observe no changes between the upper and lower surfaces of the airfoil during the quarter and three-fourth period. there are some observable changes in the sub-time levels where the airfoil is in its equilibrium position. Especially, after the shock wave, the linearized solver predicted lower and upper surface pressures in reverse compared to unsteady Euler results. The case residuals are depicted in Figure 3.9, the discussion mentioned for $Ma = 0.1$ case is valid.

Pitching motion test case

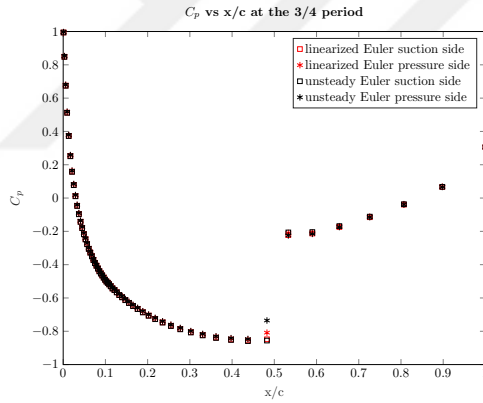
Due to linearized assumptions, the maximum angle of attack due to pitching motion needs to be small. Thus, the dynamic angle of attack amplitude is kept at 2.51 degrees for all pitching motion cases where the rotation center is taken as 0.25 times the chord length from the leading edge. The maximum pitching angle is taken from the same experiment where the excitation frequency is selected [16].



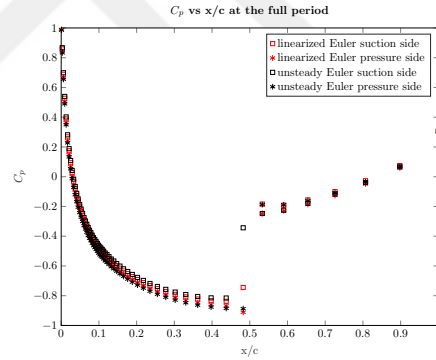
(a) Pressure coefficient plot at the quarter period



(b) Pressure coefficient plot at the half period



(c) Pressure coefficient plot at the 3/4 period



(d) Pressure coefficient plot at the full period

Figure 3.8: Pressure coefficient plots of plunging airfoil in (a,b,c,d shows sub times levels 0.25,0.5,0.75,1 times of the period respectively) for $Ma = 0.8$ case

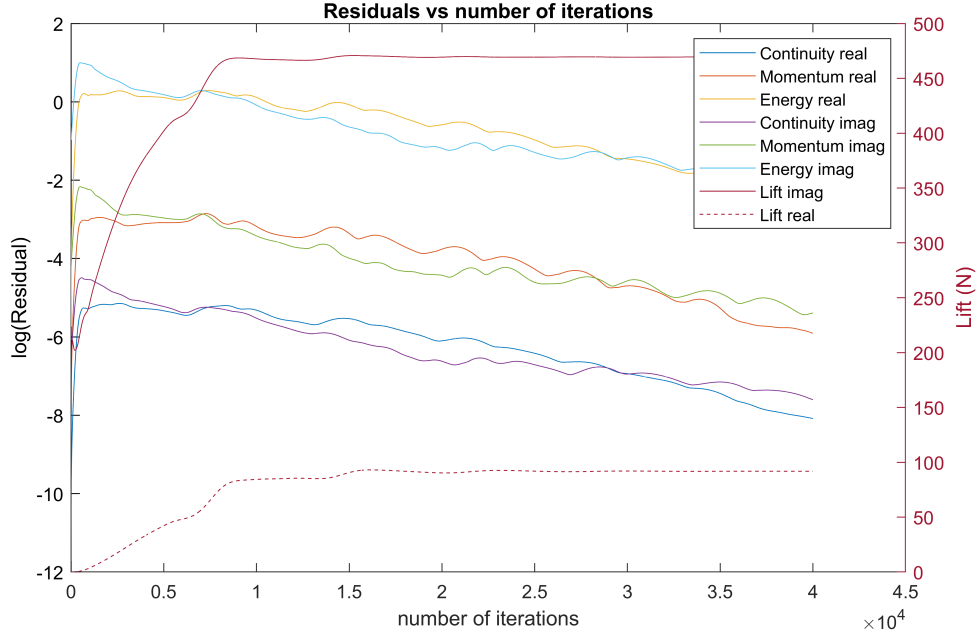
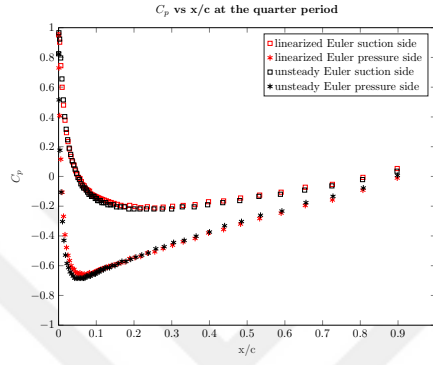


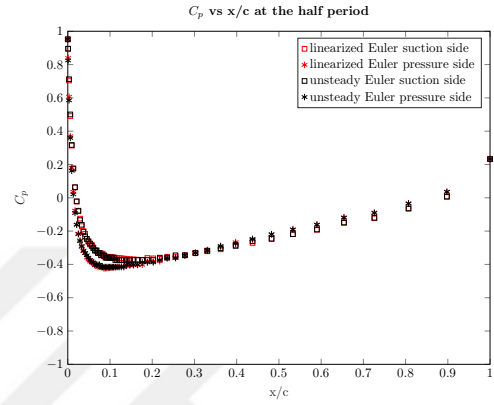
Figure 3.9: Residuals and lift force vs. iterations for $Ma = 0.8$ plunging case

Ma=0.1 For 0.1 Mach, the same steady and unsteady computational setup with plunging motion is utilized. Interestingly, $Ma = 0.1$ pitching motion provides better results without any grid motion compensation term. This term becomes vital in other demonstrated cases ($Ma = 0.5, 0.8$). Results can be seen in Figure 3.10. It is seen that the linearized Euler results of this case are in agreement with the unsteady Euler solver. Different than plunging motion cases, the pressure difference between two surfaces is highest when the airfoil is pitching up and pitching down (quarter and three-fourth of the period) since the angle of attack reaches its maximum and minimum values. Residuals are shown in Figure 3.11. Residuals dropped to a similar order of magnitudes with the plunging case, and force converged around 20,000 iterations.

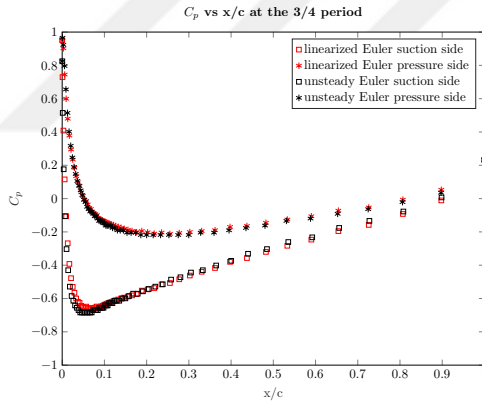
M=0.5 For 0.5 Mach, a first-order reconstruction is used in both harmonic and unsteady solvers. From contour plots in Figure 3.12, pressure distributions are very high on the leading edge as the rotation center is closer to the leading edge. The pressure phase contour visualizes the phase difference between upper and lower surfaces. This time, the pressure phase angle of the sum of any symmetrical cells in upper and lower boundaries equals π . Moreover, some distortions are observed in the trailing edge



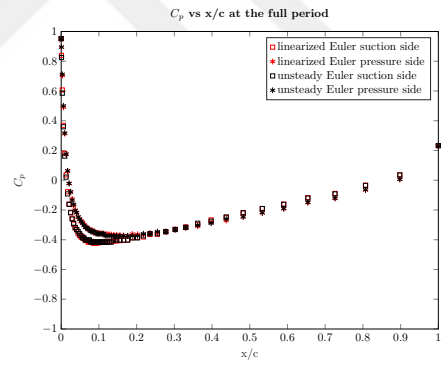
(a) Pressure coefficient at the quarter period



(b) Pressure coefficient at the half period



(c) Pressure coefficient at the 3/4 period



(d) Pressure coefficient at the full period

Figure 3.10: Pressure coefficient plots of pitching airfoil in (a,b,c,d shows sub times levels 0.25,0.5,0.75,1 times of the period respectively) for $Ma = 0.1$ case

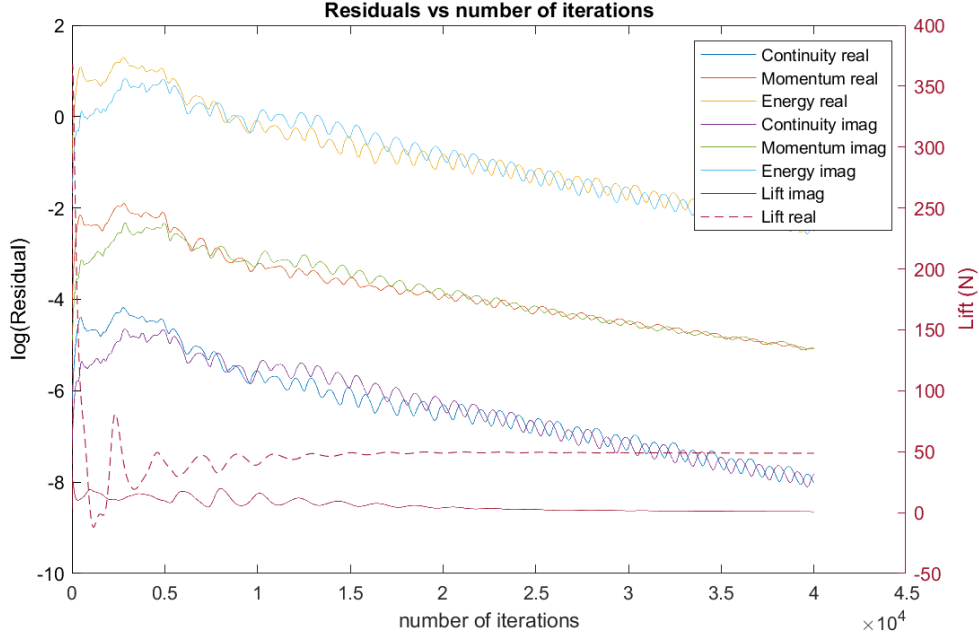
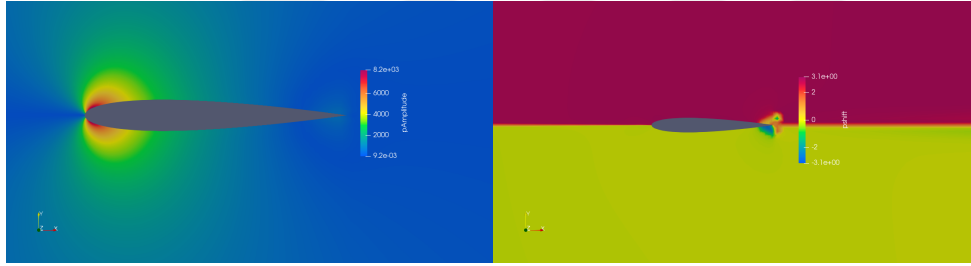


Figure 3.11: Residuals and lift force vs. iterations for $Ma = 0.1$ pitching case



(a) Perturbed pressure amplitude contour

(b) Perturbed pressure phase angle contour

Figure 3.12: Pressure amplitude and phase contours respectively

of the pressure phase contour. However, these deviations did not affect the C_p plots, as seen in Figure 3.13. Although the linearized solver was able to catch the main trend of unsteady Euler, this case showed some differences between results where C_p should be minimum (at the quarter and three-fourth of the period). The case residuals are depicted in Figure 3.14, and the discussion mentioned for $Ma = 0.1$ case is valid.

Ma=0.8 For 0.8 Mach, a first-order reconstruction is used in both solvers. Results can be seen in Figure 3.15. Unlike the plunging motion case, shock waves move through the airfoil while pitching, as seen in Figure 3.15. In this region, over and

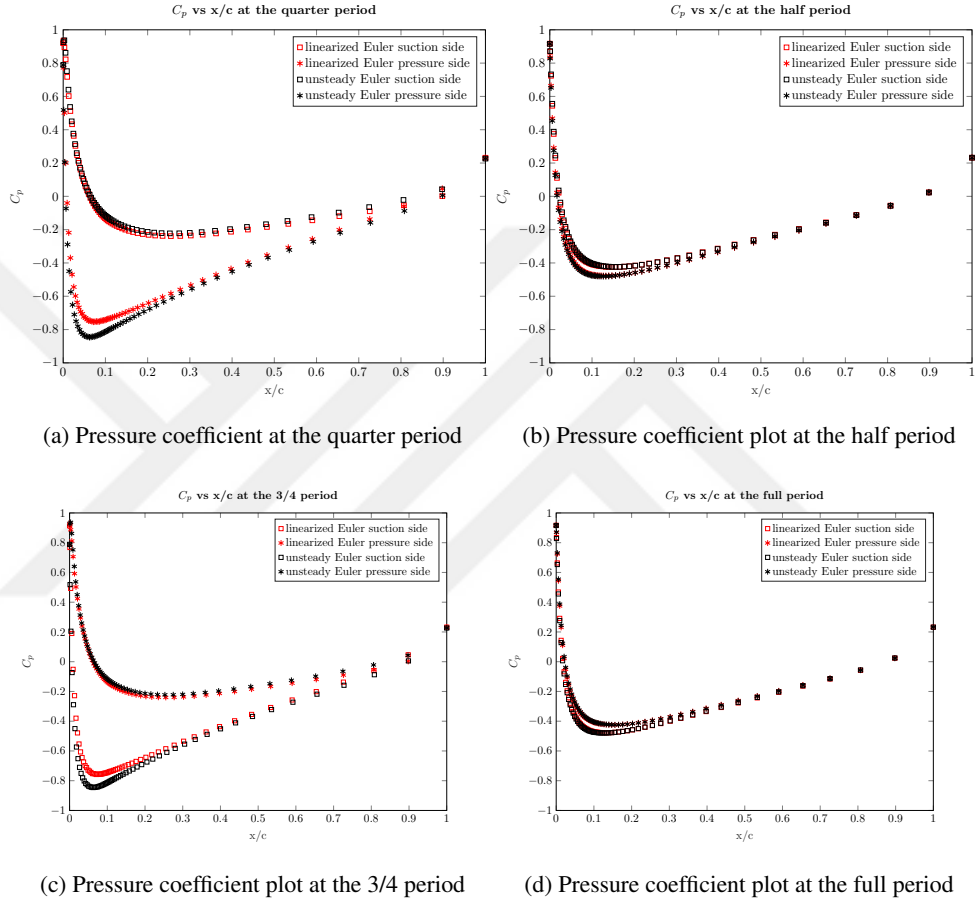


Figure 3.13: Pressure coefficient plots of pitching airfoil in (a,b,c,d shows sub times levels 0.25,0.5,0.75,1 times of the period respectively) for $Ma = 0.5$ case

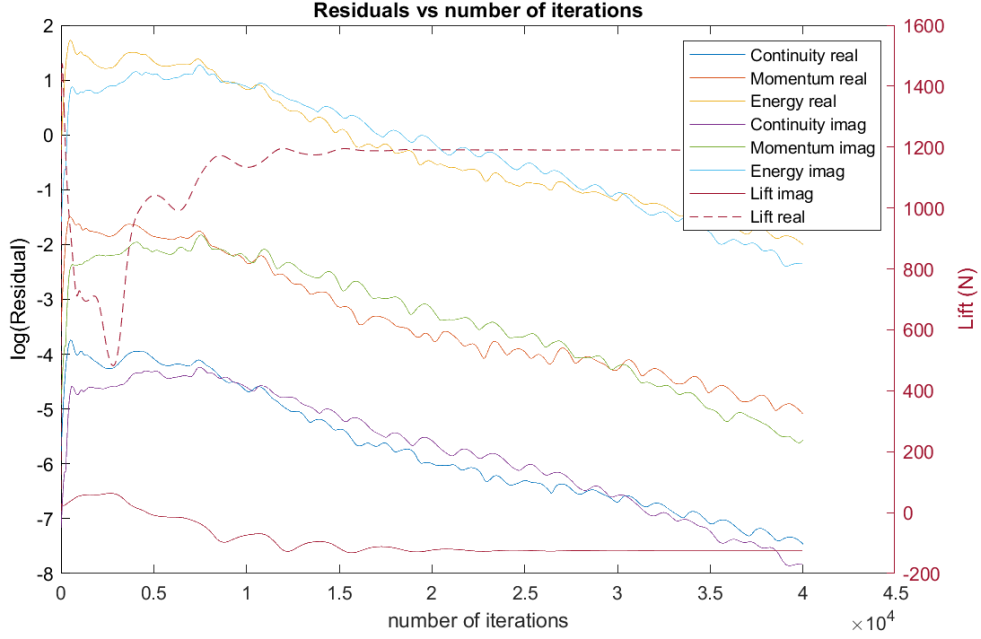
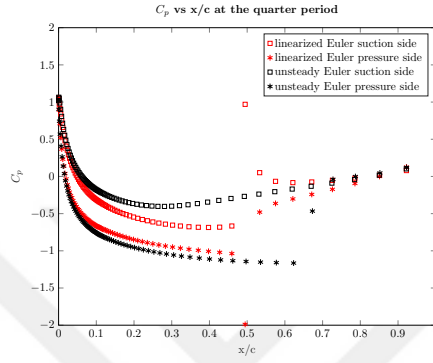


Figure 3.14: Residuals and lift force vs. iterations for $Ma = 0.5$ pitching case

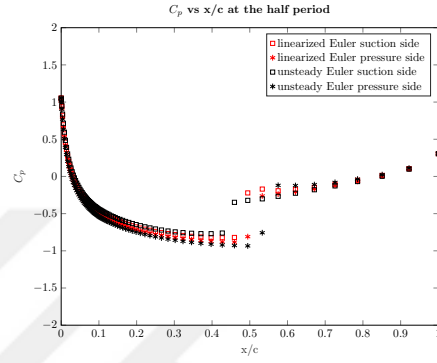
undershoots were observed in the pressure near the shock wave in linearized Euler results. These deviations are due to the fact that linearized methods rely on steady solutions. The linearization operation froze the shock location at the steady state case. Then, the harmonic solver could not update the shock location. Figure 3.16 shows the case's residuals, and the discussions in the above cases for residuals are still relevant.

3.1.3 Time Efficiency

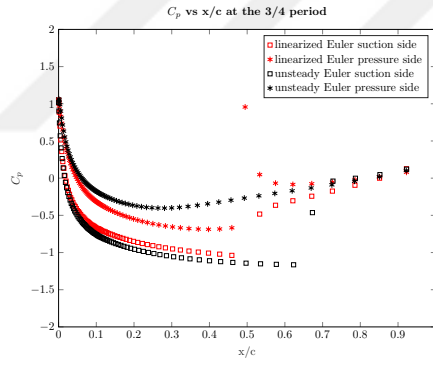
The harmonic method's most significant advantage is computational cost efficiency. While comparing the time efficiency, it is assumed that linearized Euler results converged in 30,000 iterations, whereas force converged at around 20,000 iterations. For unsteady Euler analysis, convergence is assumed when the third cycle is completed. Considering these facts, the current linearized Euler solver is 3 to 4 times faster than the Euler analysis for these simple 2D test cases. Moreover, one should note that the current solver is explicit. An implicit solver could decrease the solution times significantly.



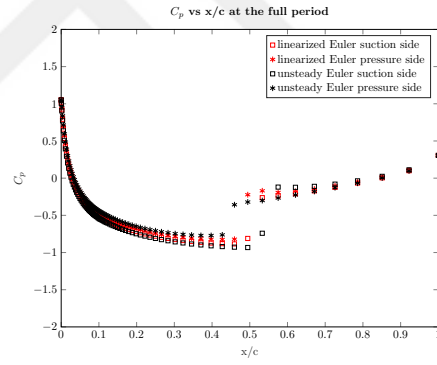
(a) Pressure coefficient plot at the quarter period



(b) Pressure coefficient plot at the half period



(c) Pressure coefficient plot at the 3/4 period



(d) Pressure coefficient plot at the full period

Figure 3.15: Pressure coefficient plots of pitching airfoil in (a,b,c,d shows sub times levels 0.25,0.5,0.75,1 times of the period respectively) for $Ma = 0.8$ case

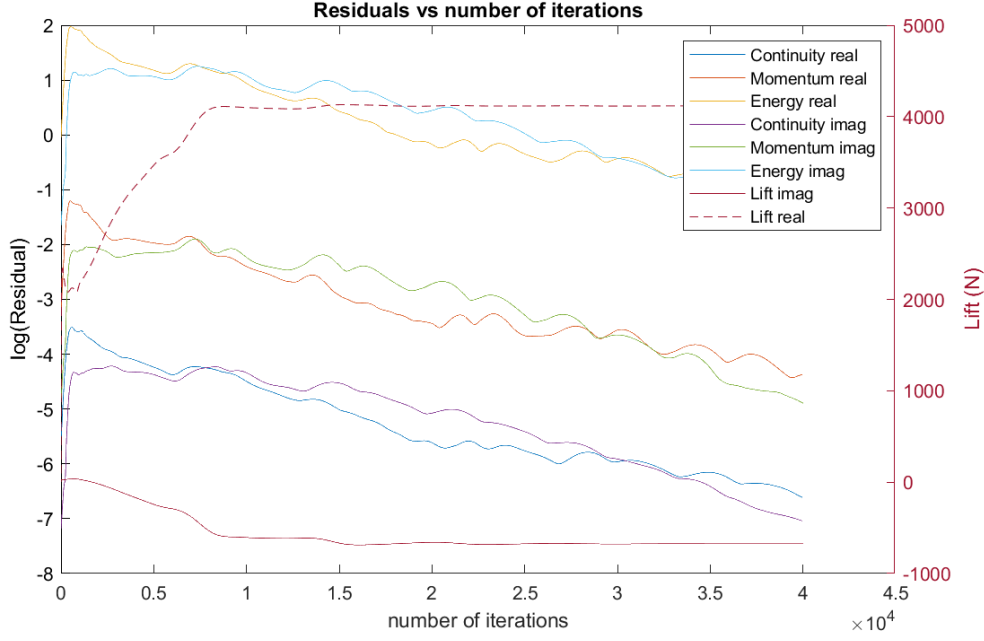


Figure 3.16: Residuals and lift force vs. iterations for $Ma = 0.8$ pitching case

3.2 Validation of the Harmonic Solver

As the validation of the harmonic solver, the NACA0012 pitching test case results are compared with an experimental study of Landon [33] and also Gong and Zhang's nonlinear harmonic solver [25]. For this purpose, dynamic lift coefficient vs. angle of attack results is compared. Dynamic lift is defined as the lift force, where the steady part of the lift is not included. Reduced frequency is defined in Equation (3.3), where c is the characteristic length (chord length for this case), and U is the free-stream speed. M4 mesh is utilized for this analysis. The case parameters can be seen in Table 3.2.

$$k = \frac{\omega c}{2U} \quad (3.3)$$

As it is discussed in the previous chapters, one of the advantages of linearized methods is that lift coefficient can be found as a function such as $\widetilde{C}_L(t) = \hat{C}_{Lreal} \cos(\omega t) + \hat{C}_{Limag} \sin(\omega t)$. Since prescribing the pitching motion in the form of $\alpha(t) = \alpha_0 + \hat{\alpha} \cos(\omega t)$, plotting the lift coefficient against the angle of attack is very straightforward. This feature provides non-discrete data for the plotted results (Figure 3.17).

Table 3.2: Parameters for the validation case

Free-stream Mach Number	Static angle of attack (deg)	Pitching Amplitude (deg)	k (Reduced Frequency)	x_c (Rotation Center)
0.6	2.89	2.41	0.0808	0.273c

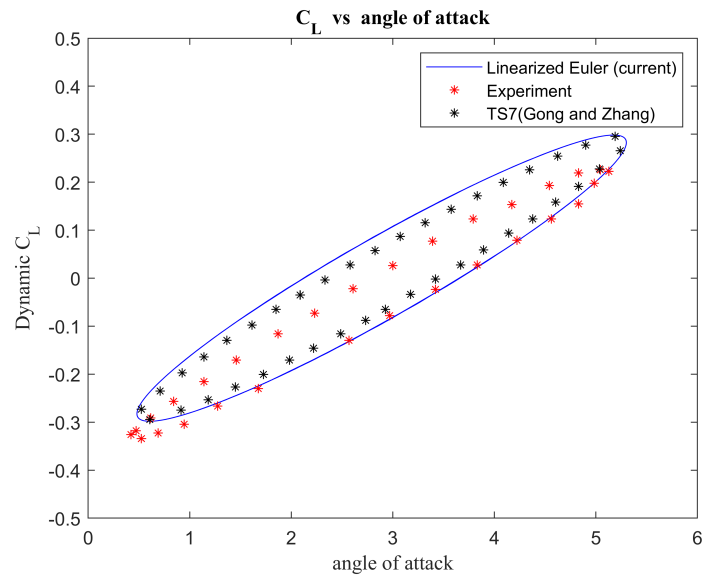


Figure 3.17: Lift coefficient vs. angle of attack comparison

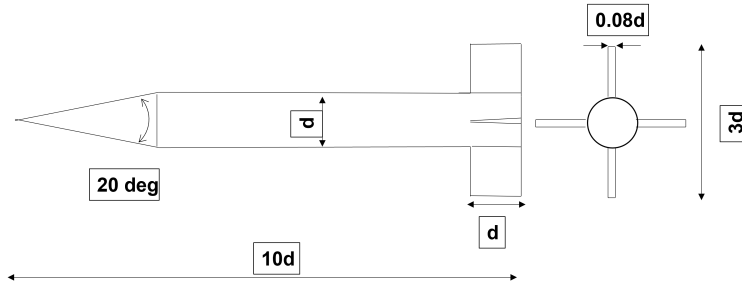


Figure 3.18: Basic Finner Geometry

Results match well with Gong's nonlinear harmonic solver results, where 7 harmonics are used to capture perturbed flow variables (Figure 3.17). Our solver predicts a higher amount of hysteresis in the lift coefficient compared to both reference results. Both harmonic results over predict the lift coefficient with a higher phase difference in lift. The linearized solvers are more cost-efficient than nonlinear ones. Therefore, of linear methods provide comparable results with higher computational efficiency for the determination of lift coefficient amplitude.

3.3 Pitch damping moment of Basic Finner

Designing ballistic and missile weapons requires aerodynamic prediction of dynamic stability derivatives. The dynamics of the forces and moments acting on a flight body as a result of changes in flight states are quantified by the dynamic stability derivatives. This section aims to determine the basic finner's pitch damping moment. A vast amount of data is available for the basic-finner test case. Therefore, this test case enables validation of harmonic implementation on 3D cases with a single harmonic.

3.3.1 Geometry

3.3.2 Computational Mesh

The current mesh is shown in Figures (3.19) and (3.20). The grid has 4,121,375 cells in total. Here, the farfield diameter was taken as 200d.

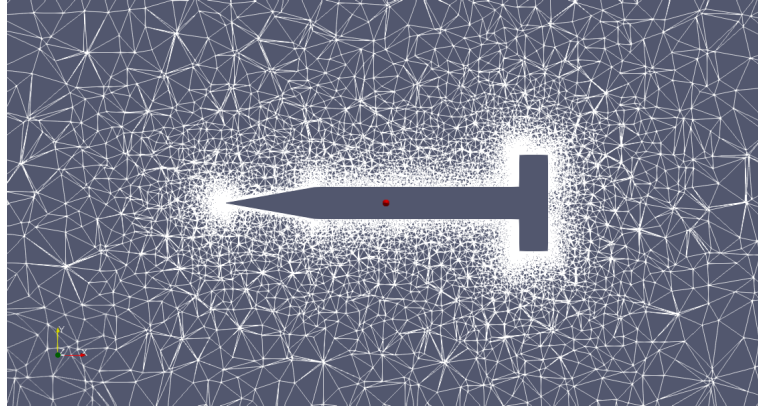


Figure 3.19: Basic Finner Mesh View 1

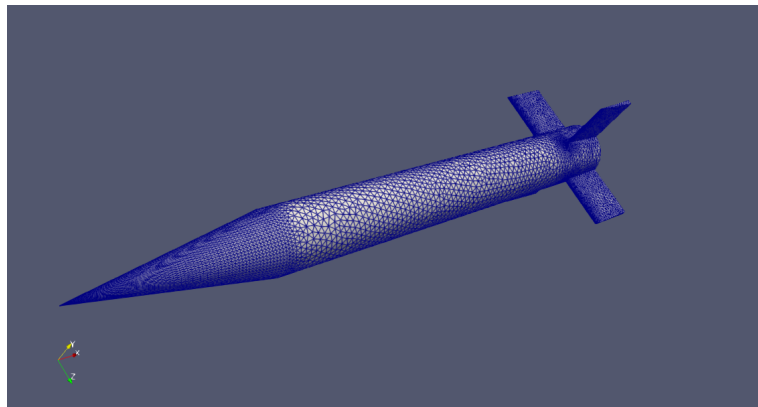


Figure 3.20: Basic Finner Mesh View 2

Table 3.3: Basic finner pitching motion parameters

Static angle of attack	Pitching Amplitude	Reduced Frequency	Rotation Center
0 deg	0.25 deg	0.1	5.5d

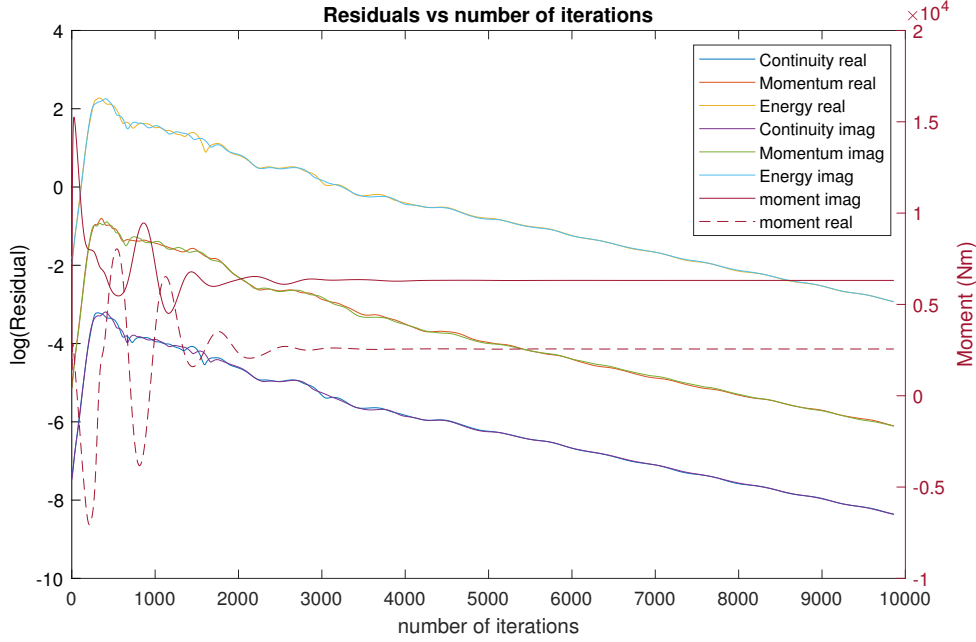


Figure 3.21: Residuals and moment vs. iterations for basic finner at $Ma = 0.63$

3.3.3 Parameters and results

Basic finner is a reference test case for calculating dynamic stability derivatives, and there is a significant number of experiments in the literature. However, most of these cases are for supersonic flows. In linearized Euler solver test case results, it has already been established that transonic region results for pitching are less accurate. Two subsonic test cases are utilized in this thesis. The first is an CFD analysis conducted by Bhagwandin and Sahu's [34]. The second one is an experiment conducted by Dupuis [35]. Parameters for the test case can be found in Table 3.3.

Characteristic length for non-dimensional parameters such as moment coefficient and reduced frequency is taken as the diameter of the missile body (Figure 3.18).

The 0.5 Mach simulation results are not in agreement. In fact current calculated value is half of Bhagwandin and Sahu's results as in Table 3.4. However, Bhagwandin and

Table 3.4: Damping derivatives for given Mach number

	$C_{m_{\dot{\alpha}}} + C_{m_q}$	Mach number
Current Study	-208	0.5
Bhagwandin and Sahu's work (CFD)	-430	0.5
Experiment[35]	-228±65	0.63
Current Study	-259	0.63

Sahu also analyzed these derivatives for several Mach numbers between 0.5-4.5, and their results does not match well with the flight test results conducted by Defense Research and Development Canada (DRDC) [36, 35] for Mach numbers less than 1.5. Note that, in some of the cases the error was more than 100%. Thus, it is concluded that their CFD solutions in this range is not good for comparison. Bhagwandin and Sahu also share one subsonic case DRDC flight test data for the pitching case. The experimentation was conducted at 0.63 free-stream Mach number. Using the same parameters as before the case is analyzed, and the results are demonstrated in the Table 3.4. In this free-stream Mach number, Bhagwandin and Sahu have not performed a CFD analysis. However, it is safe to assume that result would not be anticipated to line up with the current study considering the difference between results in the 0.5 Mach case. The experimental findings, however, concur with the current analysis when the experimental uncertainty is taken into account. This is a really good validation for the current linearized solver for 3D cases.

One other highlight of our result is the convergence rate. Only after 4000 iterations (Figure 3.21), a converged harmonic moment was achieved. Considering calculation of these derivatives is very challenging, the current linearized solver's ability to find matching results with the experiment is remarkable.



CHAPTER 4

EXTERNAL FLOW FSI SOLUTIONS

4.1 The Flutter Analysis for a thin airfoil

In this section, we aim to detect the flutter speed of a symmetrical and thin airfoil which is a limitation of Theodorsen's theory. The current linearized Euler results are compared with Theodorsen's theory [37]. The current linearized solver is expected to provide accurate results even without these limitations unless shock waves are not present. Structural and geometric parameters of the airfoil are given in Table 4.1. The airfoil profile is NACA0012. The test case is the same as the verification and validation cases in the previous chapter. Therefore, the mesh independence study is already completed.

Table 4.1 shows structural parameters including I_p as the moment of inertia, k_h as the linear stiffness, k_α as the rotational stiffness, and m as the mass of airfoil. What geometrical parameters represents is shown in Figure 4.1.

Table 4.1: Parameters for the flutter case

b	0.15 m
a	-0.5
x_α	0.03
I_p	0.13 kg · m ²
k_h	31,604 N/m
k_α	78.33 N · m/rad
m	24.80 kg

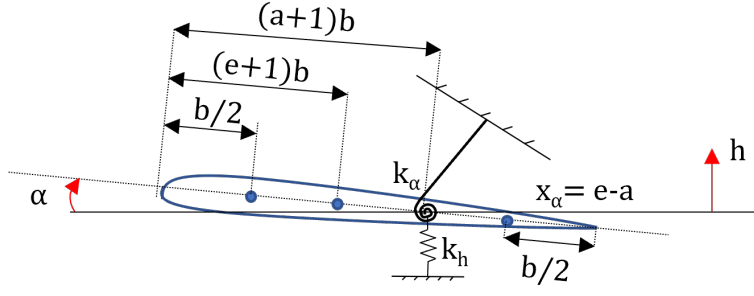


Figure 4.1: Structural model for airfoil

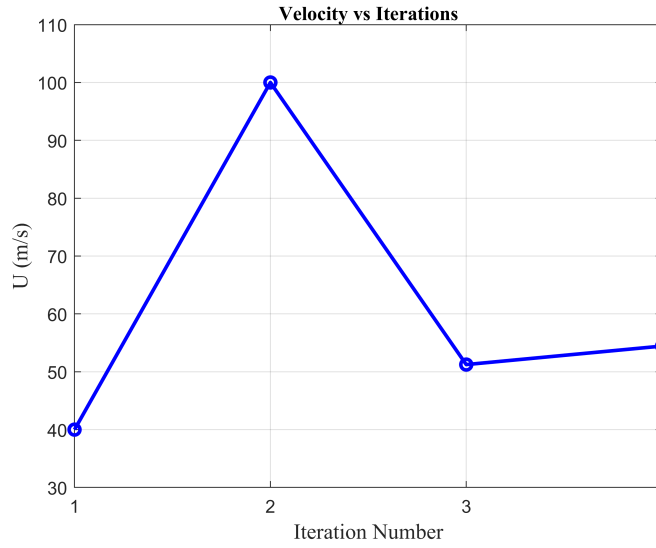


Figure 4.2: Speed convergence for the flutter case

The flutter algorithm proposed here has already been discussed in the Chapter 2. The linearized Euler solver receives the mode shapes and frequency information from the structural solver and gives complex lift and moment as output. The structural solver then takes the lift and moment for calculating the new mode shape and frequency. This iterative process continues until relative error reaches the given tolerance level. At this moment, the convergence of the fully coupled solver is achieved for a certain free-stream velocity, and the damping ratio is found. Finally, the false position method is utilized to find to flutter velocity where the damping ratio is zero. Results and convergence for an example run can be found in Figures 4.2 and 4.3.

The far-field boundary velocity is very small for a compressible-based solver, so a preconditioner is used in the fluid solver as in the $Ma = 0.1$ test cases. The upper

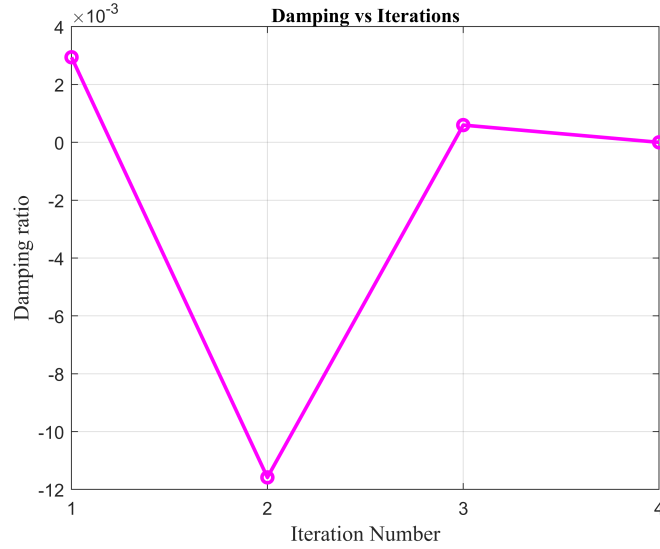


Figure 4.3: Damping convergence for the flutter case

and lower speed bracket limits are selected as 40 and 100 m/s, respectively. During this solution process, each converged solution takes 5-7 internal iterations for a certain free-stream velocity. Moreover, each internal iteration requires 3 different solutions to evaluate lift and moment derivatives with respect to h and α . After only 4 external iterations, the linearized Euler-based flutter algorithm is converged by estimating zero damping (Figure 4.3). Therefore, 72 linearized Euler solver analyses are required in total. Of course, the number of iterations depends on upper and lower velocity bracket limits and changes for every case and bracket range. The convergence rate is linear as the false position method is preferred for external iterations. The flutter speed calculated from the current algorithm is 54.44 m/s (Figure 4.2), whereas the Theodorsen method detects the system flutter at 57.97 m/s free-stream velocity, which provides 6.04% difference with the harmonic solver. Theodorsen's theory is valid for thin plates, and the NACA0012 airfoil is not particularly flat. Thus, this factor can account for the variations in the results.

4.2 Aeroelastic Analysis of AGARD 445.6 Wing

After basic finner case validation in the previous chapter, There is no question regarding the linearized solver's capacity to solve 3D flows. Thus, a more in-depth

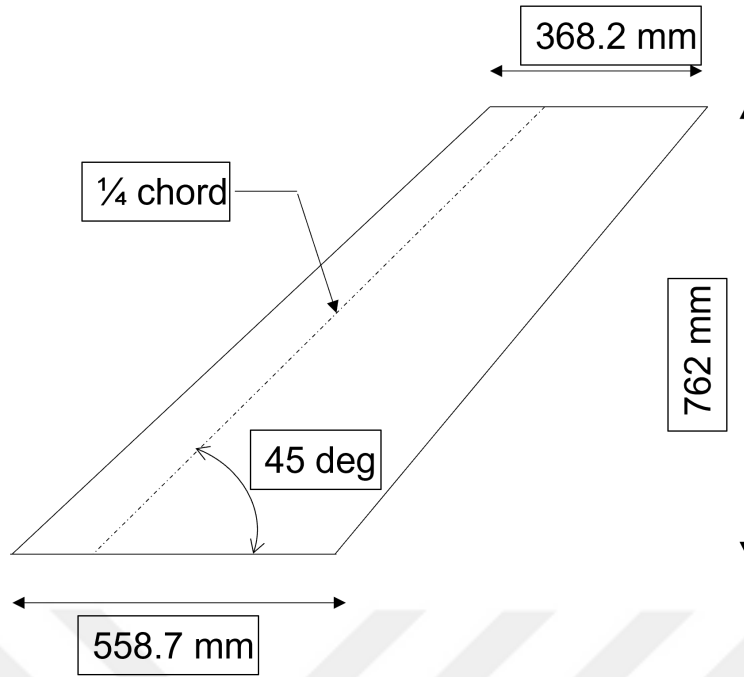


Figure 4.4: AGARD 445.6 Wing geometry

aeroelastic analysis could be performed. AGARD 445.6 wing is a common test case for flutter analysis. In this section, the Linearized Euler Solver's capabilities are being evaluated in preparation for future work to identify the flutter speed of 3D elastic wings.

4.2.1 Geometry

AGARD 445.6 wing is based on the NACA65A004, airfoil profile and other geometric parameters is given in Figure 4.4.

4.2.2 Computational Mesh

There are 2,476,989 total cells in the final grid. The computational grid is shown in Figures 4.5, 4.6 and 4.7. The domain measures 10 m in width, 5 m in depth, and 10 m in height. The symmetry boundary condition is employed in the plane at wing root.

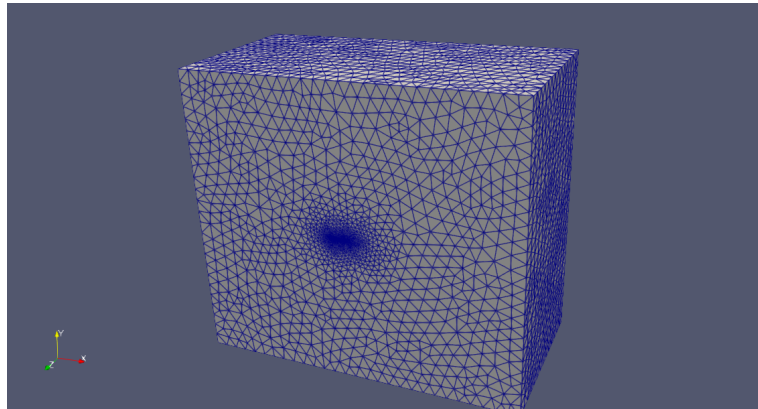


Figure 4.5: AGARD 445.6 Wing computational domain

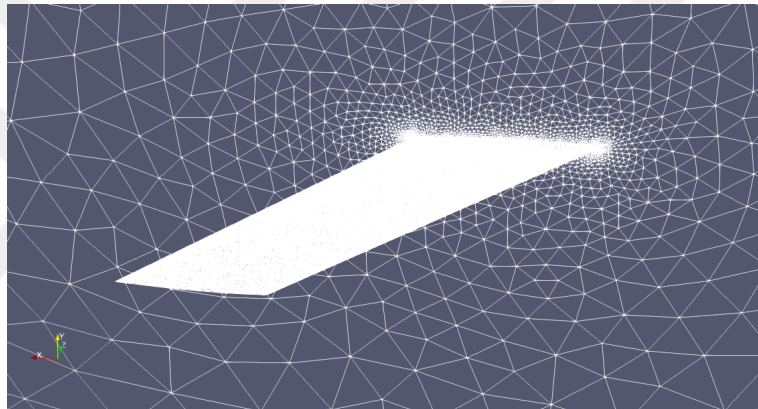


Figure 4.6: AGARD 445.6 Wing surface mesh

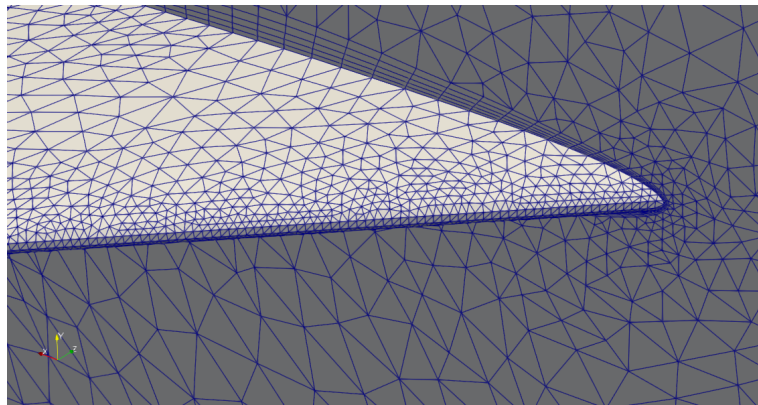


Figure 4.7: AGARD 445.6 Wing detail for mesh density

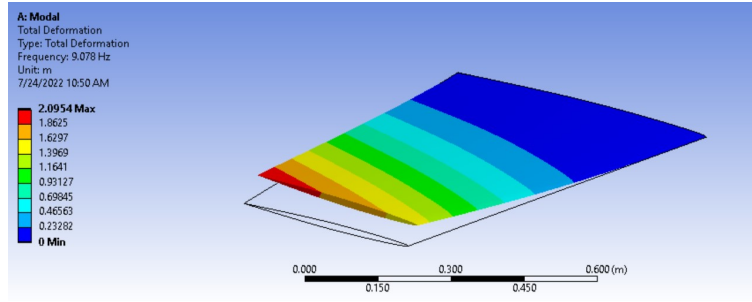


Figure 4.8: AGARD 445.6 Wing First Mode Shape

4.2.3 Structural model

For the airfoil, a finite element analysis is conducted. Elasticity information is sourced to Howison et al. [24]. The wing is fixed from its root, and all other surfaces are free. All mode shapes were found in the experimental campaign. The first mode shape was used and visualized in Figure 4.8.

The mode shape data is used for curve fitting with fifth-degree polynomials to provide prescribed oscillation amplitude on the wing surface in the harmonic solver. The AGARD 445.6 is a thin wing. Curve fitting may then be reduced to 2D, and deflection could be applied to the wing in the transverse direction.

4.2.4 Parameters and Results

Similar to the test cases, the AGARD wing is examined at 0.5 free-stream Mach number and 9.078Hz excitation frequency, which is the first natural frequency of the wing. The harmonic motion is given the first mode form of the wing is scaled by 1/10,000 to satisfy the linearization criteria. The scaling factor is also taken from Howison et al. After 40,000 iterations, all residuals dropped under the order of magnitude of 10^{-5} as in Figure 4.9.

The contour plot shown in Figure 4.10 demonstrates the harmonic pressure amplitude field on the wing. The harmonic pressure amplitude is near zero on the fixed surface and keeps increasing through the tip of the wing, where the given deflection increases. However, near leading and trailing edges, harmonic pressure amplitude is increasing nonphysically comparing the corresponding mode shape and causing unfavorable re-

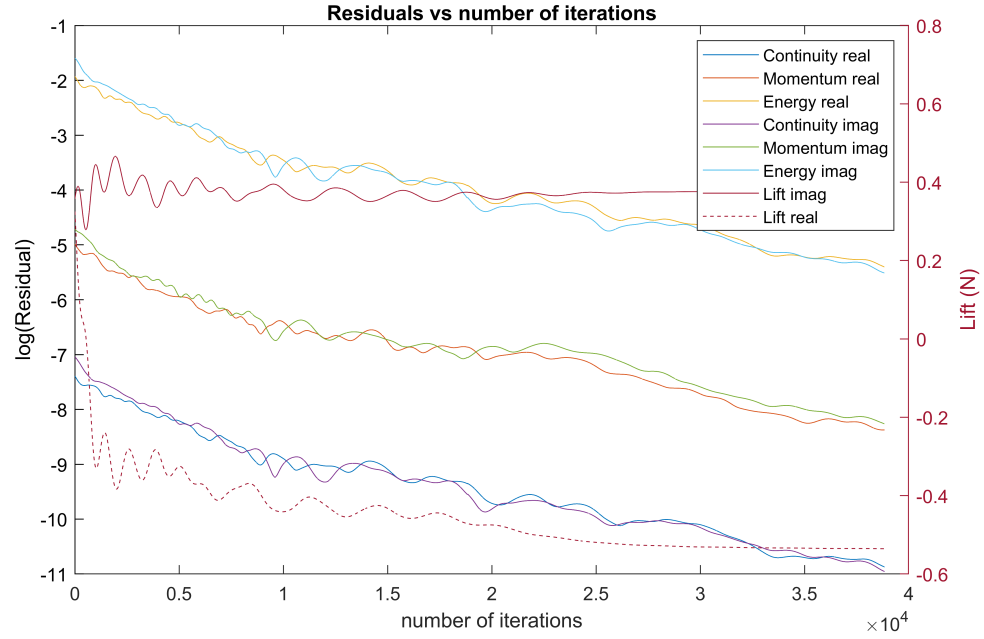


Figure 4.9: Residuals and lift force vs. iterations

sults. Mesh was refined, and quality was increased to avoid that problem. However, this process only reduced the effect, not totally eliminate it.

AGARD 445.6 is generally used for flutter analysis. Thus, no data is available to compare the current case without a comprehensive flutter analysis. The estimated lift force is not physically relevant in terms of amplitude either because the mode shape information is scaled.

This study is only an argument to support the solver's capacity to solve 3D aeroe-

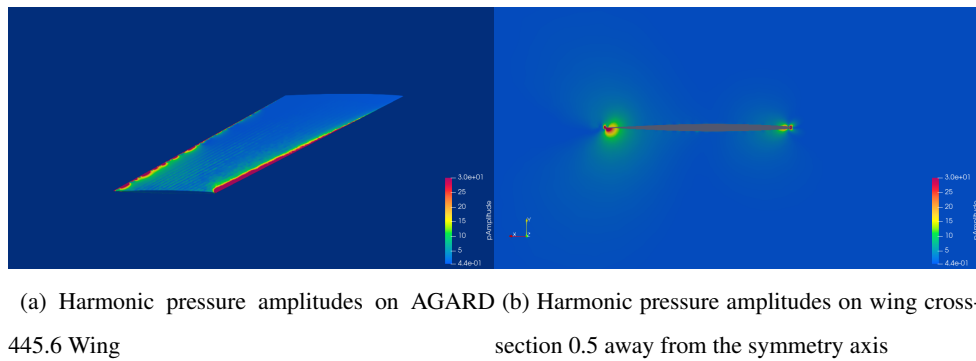


Figure 4.10: Harmonic pressure amplitudes

lastic problems. This analysis is crucial for 3D flutter analysis because every cell's oscillation amplitude varies. The expectation was to see higher pressure amplitudes where oscillation amplitudes increase, which is satisfied except for leading and trailing edges where the skewness of the cells is higher. For flutter analysis, further study is required, particularly with the structured mesh, to avoid nonphysical increases in the harmonic pressure amplitudes.



CHAPTER 5

SOLVER LIMITATIONS

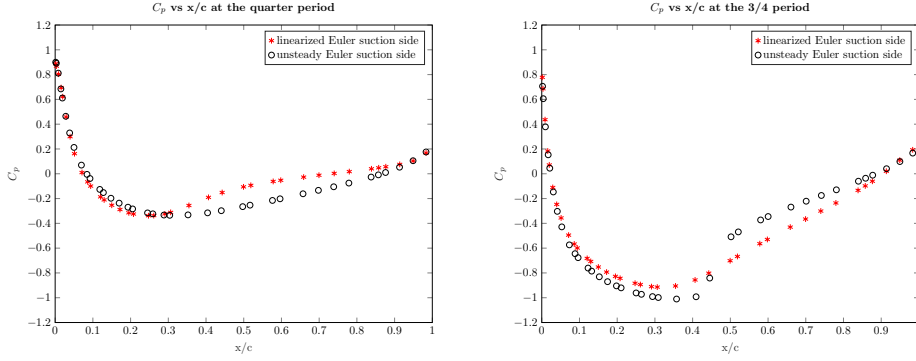
5.1 Moving Shock Waves

Results presented in Subsection 3.1.2, already showed that moving shock waves could deviate results drastically. However, the pitching case in that section was specifically chosen so that the steady-state solution had shock waves. In this section, this discussion is taken one step further.

Since the Linearized Euler Solver separates the solution into mean (assumed steady) and perturbed parts, the solver is unable to capture any shock wave unless the shock wave occurs in the steady solution. Therefore, if a shock is present on the moving boundary only when the given solid boundary is not in its equilibrium position, the obtained pressure field will not be valid.

As an example, one can check the following test case with the Naca0012 airfoil profile. The free stream Mach number is set to 0.755, which gives a maximum steady-state Mach number of 0.97 on the surface of the airfoil. The airfoil is set to make a prescribed pitching motion with a dynamic angle of attack amplitude of 2.51 degrees, and the frequency of the oscillation is 5.66 Hz. This case is taken from Yao [16]. From the unsteady Euler solution, one can see that the maximum Mach number on the upper surface of the airfoil reaches 1.1 when the angle of attack is 2.51. This phenomenon results in the following pressure plots (Figure 5.1).

This example shows that linearized solver is unable follow the shock motion unlike the unsteady solver. Also, the pressure plots of the other sub-time are not in complete coherency with the unsteady solution (though the trend is similar). However, this in-



(a) Pressure coefficient plot of upper surface of pitching airfoil at 1/4 period (b) Pressure coefficient plot of upper surface of pitching airfoil at 3/4 period

Figure 5.1: Pressure coefficient plots of upper surface of pitching airfoil in for $M=0.755$ case

accuracy in the pressure field does not affect the lift coefficient significantly, as shown in Figure 5.2. The current linearized Euler result for lift is close to what Yao's solver predicted with a higher slope and amplitude in the hysteric lift coefficient. Hence, our solver still provides acceptable results since the shocks are weak. A similar conclusion can be made for cases where shocks already exist in the steady solution. In this case, if the shock strength did not change significantly, our solver provides good results in terms of lift force. However, excessive frequencies and amplitudes should be avoided, as they elevate the difference in shock strengths.

5.2 Mesh Quality

Mesh quality is considered to be an crucial factor in every CFD analysis. However, it is a bigger challenge in harmonic solvers with oscillating boundary conditions. Since normal velocity is fed from airfoil to domain, the orthogonality of the cells on the airfoil surface is significant (Figure 3.2). In literature, most harmonic solver results (not necessarily linearized solvers, but nonlinear solvers as well as the harmonic balance method) are obtained with structural grids. The reason for this is to keep the spatial resolution as high as possible. The trial and error process demonstrated that convergence rates either dropped or the solution diverged with low-quality grids. Even for structured grids, meshing is a challenge for the discussed reasons.

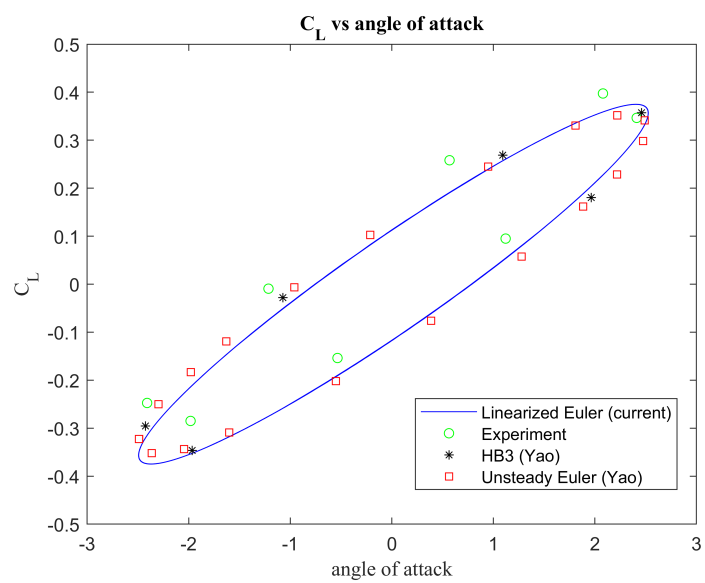


Figure 5.2: C_L vs. angle of attack for $M=0.755$ case

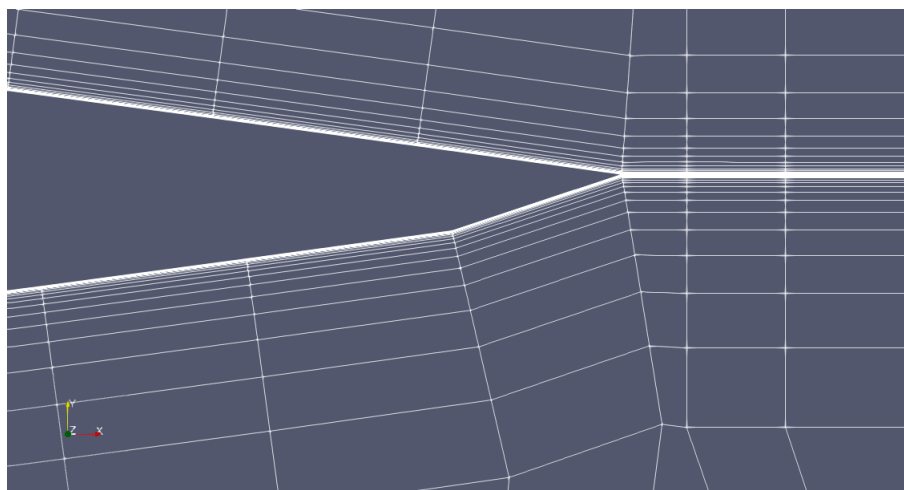


Figure 5.3: Bad quality mesh in the trailing edge

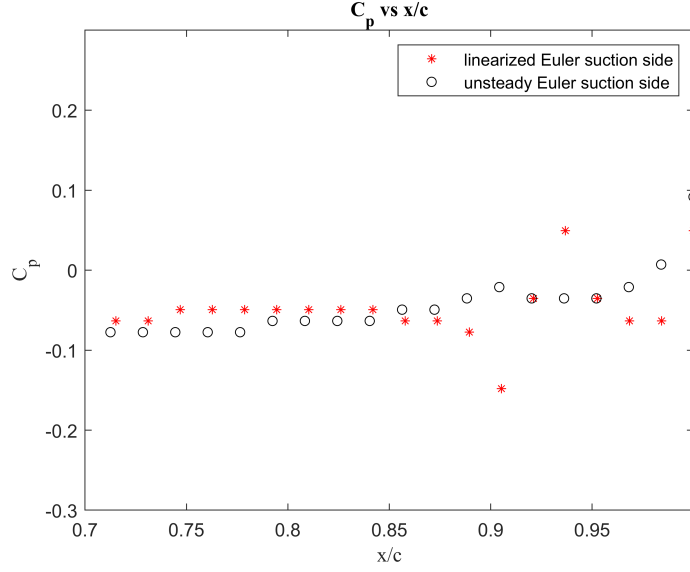


Figure 5.4: C_p vs. x/c for $M=0.1$ pitching case with bad quality meshing in trailing edge

As an example, results of one of the attempts with low-orthogonality mesh is given in Figure 5.3. The meshing of the NACA0012 airfoil is far from decent in the trailing edge, yet it is structured. The solution of this grid resulted C_p plots given in Figure 5.4. It is observed that perturbed pressure amplitudes behave non-physically at the trailing edge. The same trend can be observed in the unsteady result as well. However, the Linearized Euler solver seems to amplify any imperfections in the solution. As a result, the airfoil's meshing must be done carefully.

Moreover, the type of the grid has great importance as well. Hall and Lorence [29] discusses this problem, and they conclude that some grid types, such as C and O-type grids, do not have the sufficient resolution in the far-field flow to resolve the waves that carry information (pressure, entropy etc.) for linearized solvers. Although in this study C type meshes are used, and results are verified with unsteady Euler solutions, this inaccuracy issue is observed as some of the O-type grids tested.

CHAPTER 6

SUMMARY AND CONCLUSION

6.1 Summary and Conclusion

The objective of this thesis is to develop a solver for harmonically excited FSI problems to determine flutter speed with affordable computational cost. A linearized Euler solver was utilized for this purpose. Some necessary developments were made for flutter analysis. For FSI capabilities, oscillating wall boundary condition was implemented in Linearized Euler Solver. This boundary condition imposes the effect of the structure oscillation by feeding the problem domain with perturbed variables without any grid movement required. A characteristic far-field boundary condition for perturbed flow variables was also implemented for external flow solutions. The solver can resolve any harmonic motions of the airfoil. This feature and solver accuracy are tested with some 2D prescribed motion cases. The solver provides consistent and accurate harmonic pressure and the lift force estimations, as has been proven by both verification and validation studies for most cases. The improvement in the computational cost, which is the main advantage of harmonic methods, was demonstrated. Only one cycle of solution unsteady Euler analysis takes more time than linearized Euler results considering a steady solution is known. Even with the steady part included, the solution is more than three times faster comparing the time it takes for an unsteady Euler solution with three cycles of progress for the test cases.

The current solver's capability to find stability derivatives for aircraft was also demonstrated. This kind of analysis is important in missile studies. A subsonic analysis was performed on the well-known basic finner case to calculate damping derivatives. The results are matched with the experimental data. Thus, this solver could be used for

an affordable initial estimation at aircraft design process. The C_m calculation is not tested at transonic regime where shock waves occur. The reliability of the existing solver for 3D problems has been confirmed by this analysis.

An aeroelastic solver was developed to predict flutter speed within the scope of this thesis. There are two versions of this solver. The first one is a 2D fully coupled solver with 2 degrees of freedom structural part. The proposed method uses the false-position methodology for flutter speed estimation. The 2D solver was observed to give accurate results at low speeds flow when compared with Theodorsen methodology where the phase difference between pitching and plunging motions is small. The pitching and plunging frequencies need to be given as the same, whereas in reality, they can have different frequencies. As the flutter occurs in the pitching mode of the structure, it is uncertain if this affects the results or not. The solver is very time efficient because of the linearized Euler method, where the solver did not suffer from two-way information transfer process between structural and CFD parts during time steps. This is the case for conventional unsteady coupled solvers.

The second version is for 3D problems and takes the mode shape of the wing from finite element analysis. Then, the mode shape is given to the linearized Euler solver as input. Although no flutter analysis is conducted with the new solver, AGARD 445.6 wing is tested with prescribed motion. The first mode shape of the AGARD wing is used for the aeroelastic analysis. This scenario is significant because it demonstrates that this kind of analysis is possible using the existing linearized solver, even if there is nothing to compare our conclusions.

Some limitations of the method were also observed during the validation studies. The orthogonality of the cells adjacent to the wall boundary and the deviation of the findings in the absence of orthogonality are of the greatest general significance. These deviations were not high enough to affect the harmonic forces or alter the actual pressure trend around the airfoil. However, this fact made the solution process not user-friendly since mesh generation must be done with great care. Another big downside is the analysis of transonic flows, specially for prescribed pitching motion cases. Due to the linearization assumptions, the current solver can not resolve the moving shock waves and results in over and undershoots in C_p graphs. However,

considering C_L is the main only concern, the solver was able to provide acceptable results with less than 10% error. Thus, this analysis is still considered to be valuable.

In conclusion, this thesis accomplished its role in terms of delivering a cost-efficient FSI tool for external flows with harmonic motions.

6.2 Recommendations for Future Work

The current solver's biggest weakness is shock waves due to its linearized assumptions. A solver upgrade with nonlinear harmonic methodologies would eliminate this limitation, and more complicated cases would be solved accurately including transonic flow problems.

Predicting drag force on airfoils with an Euler solver is not possible as most of the drag comes from friction drag. With turbulence model inclusion, the proposed solver will become a powerful tool for any external flow application.

An implicit time scheme addition for the current linearized solver is also a good idea since this will decrease the solution times significantly.



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