



MARMARA UNIVERSITY
INSTITUTE FOR GRADUATE STUDIES
IN PURE AND APPLIED SCIENCES



**DECISION MAKING APPROACHES FOR
FINANCIAL ACCOUNTING FRAUD: CASE
STUDY FOR SMALL AND MEDIUM SIZED
ENTERPRISES (SMEs) IN TURKEY**

SERHAN HAMAL

DOCTORAL THESIS

Department of Industrial Engineering

Thesis Supervisor

Assoc. Prof. Dr. Özlem ŞENVAR

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TABLE OF CONTENTS

	PAGE
ACKNOWLEDGEMENT	i
TABLE OF CONTENTS	ii
ÖZET	viii
ABSTRACT	ix
CLAIM FOR ORIGINALITY	x
SYMBOLS	xi
ABBREVIATIONS	xiii
LIST OF FIGURES	xvi
LIST OF TABLES	xviii
1.INTRODUCTION	1
2. THEORETICAL BACKGROUND	11
2.1. Financial Accounts in Detecting Financial Accounting Fraud	11
2.1.1. Cash	11
2.1.2. Accounts Receivable	11
2.1.3. Allowance for Doubtful Accounts	11
2.1.4. Due from Shareholders	11
2.1.5. Inventories	12
2.1.6. Due to Shareholders	12
2.2. Financial Ratios in Detecting Financial Accounting Fraud	12
2.2.1. Liquidity	15
2.2.1.1. Current assets to current liabilities (Current ratio)	15

2.2.1.2. Current assets-inventories to current liabilities (Quick ratio /Acid-test ratio)	15
2.2.1.3. Cash to current liabilities (Cash ratio)	15
2.2.2. Profitability	16
2.2.2.1. Net income to total assets (Return on assets)	16
2.2.2.2. Net Income to Net Sales (Net profit margin)	16
2.2.2.3. Earnings before interest and tax to total assets (EBIT to total assets)	16
2.2.2.4. Net Income to Shareholders' Equity (Return On Equity)	17
2.2.3. Leverage	17
2.2.3.1. Total liabilities to total assets (Debt ratio)	17
2.2.3.2. Total liabilities to total equity (Debt-to-equity ratio)	17
2.2.4. Efficiency	18
2.2.4.1. Accounts receivable to total sales	18
2.2.4.2. Accounts receivable to total assets	18
2.2.4.3. Inventory to total sales	19
2.2.4.4. Inventory to total assets	19
2.2.4.5. Inventory to current assets	19
2.2.4.6. Cost of good sold to average inventory (Inventory turnover)	19
2.2.4.7. Accounts payable to cost of good sold	20
2.2.4.8. Net sales to total assets (Total asset turnover)	20
2.2.4.9. Net sales to average accounts receivable (Accounts receivable turnover)	20
2.2.4.10. Cash conversion cycle	21
2.2.5. Growth	21
2.2.5.1. Growth of sales	21

2.2.5.2. Growth of accounts receivables	21
2.2.5.3. Growth of inventory	22
2.2.5.4. Growth of income from principal business operations	22
2.2.6. Other ratios	22
2.3. Qualitative Characteristics as Evaluation Criteria for Assessing the Performance of Financial Ratios	22
2.4. Turkish Accounting Standards	25
3. FUZZY SETS	29
3.1. Ordinary Fuzzy Sets	29
3.2. Spherical Fuzzy Sets	29
3.2.1. Single-Valued Spherical Fuzzy Sets	30
3.2.2. Interval-Valued Spherical Fuzzy Sets	33
3.3. Circular Intuitionistic Fuzzy Sets	36
4. FINANCIAL ACCOUNTING FRAUD DETECTION VIA MCDM	39
4.1. Integrated AHP and TOPSIS with Different Fuzzy Sets	40
4.1.1. Integrated Fuzzy AHP and Fuzzy TOPSIS	45
4.1.1.1. Fuzzy AHP	47
4.1.1.2. Fuzzy TOPSIS	50
4.1.2. Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets	52
4.1.2.1. Single-Valued Spherical Fuzzy AHP	53
4.1.2.2. Single-Valued Spherical Fuzzy TOPSIS	54
4.1.3. Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets	55
4.1.3.1. Interval-Valued Spherical Fuzzy AHP	55
4.1.3.2. Interval-Valued Spherical Fuzzy TOPSIS	56

4.1.4. Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets	59
4.1.4.1. Circular Intuitionistic Fuzzy AHP	60
4.1.4.2. Circular Intuitionistic Fuzzy TOPSIS	63
4.2. Integrated AHP and MULTIMOORA with Different Fuzzy Sets	66
4.2.1. Integrated Fuzzy AHP and Fuzzy MULTIMOORA	69
4.2.1.1. Fuzzy AHP	70
4.2.1.2. Fuzzy MULTIMOORA	70
4.2.2. Integrated AHP and MULTIMOORA with Single-Valued Spherical Fuzzy Sets	73
4.2.2.1. Single-Valued Spherical Fuzzy AHP	74
4.2.2.2. Single-Valued Spherical Fuzzy MULTIMOORA	74
4.2.3. Integrated AHP and MULTIMOORA with Interval-Valued Spherical Fuzzy Sets	76
4.2.3.1. Interval-Valued Spherical Fuzzy AHP	76
4.2.3.2. Interval-Valued Spherical Fuzzy MULTIMOORA	76
5. Financial Accounting Fraud Detection via Machine Learning	79
5.1. Machine Learning Classifiers	85
5.1.1. Support vector machines (SVM)	85
5.1.2. Naive bayes	86
5.1.3. Artificial neural network (ANN)	86
5.1.4. K-nearest neighbour (kNN)	87
5.1.5. Random forest	87
5.1.6. Logistic regression	88
5.1.7. Bagging	88
5.2. Sampling Methods	89
5.2.1. Oversampling	89

5.2.2. Undersampling	90
5.3. Feature Selection	90
5.3.1. T-test	90
5.3.2. Genetic search	90
5.4. Performance Metrics	91
6. APPLICATION STUDIES AND RESULTS	93
6.1. Data	93
6.1.1. Data collection	93
6.1.2. Descriptive statistics	93
6.1.3. Inferencial statistics	99
6.2. Financial Accounting Fraud Detection Via MCDM	101
6.2.1. Integrated AHP and TOPSIS with different fuzzy sets	101
6.2.1.1. Integrated Fuzzy AHP and Fuzzy TOPSIS	101
6.2.1.2. Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets	119
6.2.1.3. Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets	130
6.2.1.4. Integrated AHP and TOPSIS with with Circular Intuitionistic Fuzzy Sets	142
6.2.2. Integrated AHP and MULTIMOORA with different fuzzy sets	159
6.2.2.1. Integrated Fuzzy AHP and Fuzzy MULTIMOORA	159
6.2.2.2. Integrated AHP and MULTIMOORA with Single-Valued Spherical Fuzzy Sets	167
6.2.2.3. Integrated AHP and MULTIMOORA with Interval-Valued Spherical Fuzzy Sets	177
6.3. Financial Accounting Fraud Detection via Machine Learning	185
6.3.1. Algorithm of the Application	185
6.3.2. Steps of the Application	186

6.3.2.1. Phase 1	186
6.3.2.2. Phase 2	189
7. FINDINGS AND DISCUSSION	195
8. CONCLUSION	211
REFERENCES	215



ÖZET

FİNANSAL MUHASEBE DOLANDIRICILIĞI İÇİN KARAR VERME YAKLAŞIMLARI: TÜRKİYE'DEKİ KÜÇÜK VE ORTA ÖLÇEKLİ İŞLETMELER (KOBİ'LER) İÇİN VAKA ÇALIŞMASI

Finansal muhasebe dolandırıcılığı, finansal ekosistemdeki tüm paydaşlar için başa çıkılması gereken en büyük sorunlardan biridir. Paydaşlar arasında güven ve itibara zarar verir. Ayrıca finansal muhasebe dolandırıcılığının finansal ekosistemde neden olduğu yaralar, şirketlerin ve ülkelerin ekonomik büyümelerini olumsuz etkilemektedir. Bu tez çalışmasında, finansal muhasebe dolandırıcılığı sorununun üstesinden gelmek için karar verme yaklaşımları önerilmiştir. Hileli finansal tablolar üzerinde en büyük etkiye sahip finansal oranların belirlenmesinde özellik seçim yöntemleri olarak farklı bulanık kümeler ile entegre Çok Kriterli Karar Verme yöntemleri, T-testi ve Genetik Arama uygulanmıştır. Finansal oranların sıralamasına ilişkin karar verme sürecindeki belirsizliklerin üstesinden gelmek için Küresel Bulanık Kümeler (aralık değerli ve tek değerli bulanık sayılar dahil), Dairesel Sezgisel Bulanık Kümeler ve Tip-1 bulanık kümeler kullanılmaktadır. Finansal oranların önceliklendirilmesinin belirlenmesinden sonra, aşırı örnekleme ve alt örnekleme olmak üzere iki örnekleme yöntemi gerçekleştirilir. Daha sonra Destek Vektör Makineleri, Naive Bayes, Yapay Sinir Ağları, K-En Yakın Komşuluk, Rastgele Orman, Lojistik Regresyon ve Torbalama olmak üzere yedi farklı sınıflandırıcının performansları performans metrikleri kullanılarak karşılaştırılmıştır. Sınıflandırıcılar ayrıca herhangi bir özellik seçme yöntemi kullanılmadan karşılaştırılır. Rastgele Orman sınıflandırıcısının, dikkate alınan tüm performans ölçütleri boyunca yedi sınıflandırıcı arasında sürekli olarak en iyi veya ikinci en iyi performansı gösterdiği bulunmuştur. Naive Bayes ve Destek Vektör Makineleri de incelenen tüm sınıflandırıcılarda en kötü performansı gösteren iki sınıflandırıcı olarak bulunmuştur. Bu tezin bir başka bulgusu, aşırı örnekleme yönteminin Rastgele Orman sınıflandırıcısının genel doğruluğunu önemli ölçüde arttırmasıdır.

ABSTRACT

DECISION MAKING APPROACHES FOR FINANCIAL ACCOUNTING FRAUD: CASE STUDY FOR SMALL AND MEDIUM SIZED ENTERPRISES (SMES) IN TURKEY

Financial accounting fraud is one of the biggest problems to be overcome for all stakeholders in the financial ecosystem. It damages trust and reputation among stakeholders. Moreover, the wounds caused by financial accounting fraud on the financial ecosystem adversely affect the economic growth of companies and countries. In this dissertation, decision making approaches are proposed to deal with the problem of financial accounting fraud. Integrated Multi Criteria Decision Making methods with different fuzzy sets, T-test, and Genetic Search are applied as feature selection techniques to identify financial ratios having the greatest impact on fraudulent financial statements. Spherical Fuzzy Sets (including interval-valued and single-valued fuzzy numbers), Circular Intuitionistic Fuzzy Sets and Type1 fuzzy sets are employed to overcome uncertainties in decision-making process regarding the ranking of financial ratios. After determination of the prioritization of financial ratios, two sampling methods of oversampling and undersampling are performed. Then, the performances of seven different classifiers, which are Support Vector Machine, Naive Bayes, Artificial Neural Network, K-Nearest Neighbour, Random Forest, Logistic Regression and Bagging, are compared by employing performance measures. Classifiers are also compared without using any feature selection method. It is found that Random Forest classifier consistently performs the best or the second best among seven classifiers throughout all considered performance measures. Naive Bayes and Support Vector Machines are also found as two classifiers that perform the worst in all examined classifiers. Another finding of this dissertation is that oversampling method significantly enhances overall accuracy of the Random Forest classifier.

CLAIM FOR ORIGINALITY

Financial ratios have a critical importance in the success of detecting financial accounting fraud. Different financial ratios have been used in many studies and there is no common analytical method to identify which financial ratios should be utilized in financial accounting fraud detection. In this dissertation, Integrated AHP and TOPSIS and integrated AHP and MULTIMOORA approaches with different fuzzy sets, which are Type1, Spherical and Circular fuzzy sets, are proposed for determining financial ratios. This study is the first research which proposes integrated AHP and TOPSIS approach with the newly introduced Circular Intuitionistic Fuzzy Sets and Interval-Valued Spherical Fuzzy Sets. Moreover, this study proposes a novel integrated AHP and MULTIMOORA approach with Interval-Valued Spherical Fuzzy Sets and Single-Valued Spherical Fuzzy Sets to handle uncertainties in prioritization of financial ratios.

Prior studies have mostly focused on firms that are publicly listed or have a substantial market capitalization. There are fewer studies that have focused on financial accounting fraud for small and medium-sized enterprises (SMEs) in comparison to these studies. This study is the first research that focuses on financial accounting fraud detection for SMEs in Turkey.

Previous researches have also not yet adequately handled the problem of class imbalance. The majority of researches used a balanced sample or matched fraudulent and non-fraudulent companies at various ratios. Few researchers have employed sampling techniques in detecting financial accounting fraud. This study extends the existing literature by employing sampling methods to address the problem of class imbalance in financial accounting fraud detection.

SYMBOLS

A	: A set of the universe of discourse
A^-	: Circular intuitionistic fuzzy negative ideal solution
A^+	: Circular intuitionistic fuzzy positive ideal solution
C	: A subset of a fixed universe A
C^-	: Cost criteria
C^+	: Benefit criteria
C_C	: The cardinality of universe A
d_{ij}	: Circular intuitionistic fuzzy number of i^{th} alternative with respect to j^{th} criterion
$\mathcal{D}_i^{A^-}$	: The distance to the Circular intuitionistic fuzzy negative ideal solution
$\mathcal{D}_i^{A^+}$	: The distance to the Circular intuitionistic fuzzy positive ideal solution
$\mu_C(\alpha)$	: Membership degree of the element α for Circular Intuitionistic Fuzzy Sets
$\mu_S(x)$	: Membership parameter for Single-Valued Spherical Fuzzy Sets
$\mu_S^L(\alpha)$	: The lower degrees of membership for Interval-Valued Spherical Fuzzy Sets
$\mu_S^U(\alpha)$	: The upper degrees of membership for Interval-Valued Spherical Fuzzy Sets
$\pi_C(\alpha)$	: Indeterminacy (uncertainty) degree of the element α for Circular Intuitionistic Fuzzy Sets
$\pi_S(x)$	: Hesitancy parameter for Single-Valued Spherical Fuzzy Sets
$\pi_S^L(\alpha)$	: The lower degrees of hesitancy for Interval-Valued Spherical Fuzzy Sets
$\pi_S^U(\alpha)$	: The upper degrees of hesitancy for Interval-Valued Spherical Fuzzy Sets
$\nu_C(\alpha)$	: Non-membership degree of the element α for Circular Intuitionistic Fuzzy Sets
$\nu_S(x)$	: Non-membership parameter for Single-Valued Spherical Fuzzy Sets

$v_S^L(\alpha)$	: The lower degrees of non-membership for Interval-Valued Spherical Fuzzy Sets
$v_S^U(\alpha)$	: The upper degrees of non-membership for Interval-Valued Spherical Fuzzy Sets
M_i	: Convex fuzzy triangular numbers
M_{gi}^j	: The fuzzy addition operation of m extent analysis values for a particular matrix
$P(X)$	: The prior probability of class X
$P(X \setminus Y)$	: Conditional probability of a class for a given predictor
$P(Y)$	: The prior probability of predictor
$P(Y \setminus X)$	: The likelihood which is the probability of predictor given a class
$P(Y=1 \setminus X)$	: The likelihood of firm committing financial accounting fraud
r	: A radius of the circle around each element for Circular Intuitionistic Fuzzy Sets
$\tilde{U}_i'$	: Over all utility
w_j	: Importance weight
$\tilde{x}_{ij}$	: Linguistic variable
$x_{k,i}$	: An independent variable
$\tilde{Y}_i$	: ith alternative's objectives to be maximized
$\tilde{Z}_i$	: ith alternative's objectives to be minimized
α	: An element of A
β	: Forecasted parameters for independent variables

ABBREVIATIONS

ACFE	: Association of Certified Fraud Examiners
AHP	: Analytic Hierarchy Process
AICPA	: American Institute of Certified Public Accountants
ANN	: Artificial Neural Network
ANP	: Analytic Network Process
BBN	: Bayesian Belief Network
BNP	: Best Non-Fuzzy Performance
CART	: Classification and Regression Trees
CHAID	: Chi Squared Automatic Interaction Detector
C-IFN	: Circular Intuitionistic Fuzzy Number
C-IFS	: Circular Intuitionistic Fuzzy Set
CODAS	: Combinative Distance-Based Assessment
CR	: Consistency Ratio
DEMATEL	: Decision Making Trial and Evaluation Laboratory
DT	: Decision Tree
EBIT	: Earnings Before Interest and Tax
ELECTRE	: Elimination et Choice Translating Reality
FMF	: Full Multiplicative Form
FN	: False Negative
FP	: False Positive
GRP	: Grey Relational Projection
IAASB	: International Auditing and Assurance Standards Board
IFNs	: Intuitionistic Fuzzy numbers
IFS	: Intuitionistic Fuzzy Set

IFWA	: Intuitionistic Fuzzy Weighted Averaging
IVSF-NIS	: Interval-valued Spherical Fuzzy Negative Ideal Solution
IVSF-PIS	: Interval-valued Spherical Fuzzy Positive Ideal Solution
IVSFS	: Interval-Valued Spherical Fuzzy Sets
IVSWAM	: Interval-Valued Spherical Weighted Arithmetic Mean
IVSWGGM	: Interval-Valued Spherical Geometric Mean
kNN	: k-Nearest Neighbour
LFPP	: Logarithmic Fuzzy Preference Programming
MADM	: Multiple Attribute Decision Making
MCDM	: Multi Criteria Decision Making
MODM	: Multiple Objective Decision Making
MOORA	: Multi-Objective Optimization by Ratio Analysis
MULTIMOORA	: Multiple Multi-Objective Optimization by Ratio Analysis
NB	: Naïve Bayes
NLP	: Natural Language Processing
PROMETHEE	: Preference Ranking Organization Method for Enrichment Evaluation
QGA	: Queen Genetic Algorithm
ROC	: Receiver Operating Characteristic
RP	: Reference Point
RS	: Ratio System
RSF	: Relative Score Function
SFS	: Spherical Fuzzy Sets
SI	: Scoring Index
SMEs	: Small and Medium Sized Enterprises

SMOTE	: Synthetic Minority Oversampling Technique
SVM	: Support Vector Machine
SVM	: Support Vector Machine
SVSF-NIS	: Single-Valued Spherical Fuzzy Negative Ideal Solutions
SVSF-PIS	: Single-Valued Spherical Fuzzy Positive Ideal Solutions
SVSFS	: Single-Valued Spherical Fuzzy Sets
SWAM	: Single-Valued Spherical Weighted Arithmetic Mean
SWGM	: Single-Valued Spherical Weighted Geometric Mean
TN	: True Negative
TOPSIS	: Technique for Order of Preference by Similarity to Ideal Solution
TP	: True Positive
VIKOR	: Vise Kriterijumska Optimizacija I Kompromisno Resenje
WEKA	: Waikato Environment for Knowledge Analysis

LIST OF FIGURES

	PAGE
Figure 2.1. Financial reporting frameworks applied in Turkey	28
Figure 3.1. The membership functions of Intuitionistic Fuzzy Sets, Pythagorean Fuzzy Sets, Neutrosophic Sets and Spherical Fuzzy Sets	30
Figure 3.2. Fundamental geometric depiction of C-IFSs	37
Figure 4.1. The intersection between M1 and M2	49
Figure 6.1. SMEs' sectoral distribution	94
Figure 6.2. SMEs' relative frequency	94
Figure 6.3. Frequency distribution of fraudulent and non-fraudulent companies	95
Figure 6.4. Fraudulent firms' relative frequency on a sectoral basis	96
Figure 6.5. The distribution of fraudulent companies by the number of years	97
Figure 6.6. Cases' distribution	97
Figure 6.7. Fraud cases' distribution of by year	97
Figure 6.8. Frequency distribution of financial accounts utilized for fraudulent activities	98
Figure 6.9. The most commonly utilized financial accounts for financial accounting fraud on a sectoral basis	99
Figure 6.10. The flowchart of the proposed approach (Integrated Fuzzy AHP and Fuzzy TOPSIS)	103
Figure 6.11. The hierarchical structure of the problem (Integrated Fuzzy AHP and Fuzzy TOPSIS)	104
Figure 6.12. The flowchart of the proposed methodology (Integrated AHP and TOPSIS with Spherical Fuzzy Sets)	120
Figure 6.13. The hierarchical framework of the evaluation model (Integrated AHP and TOPSIS with Spherical Fuzzy Sets)	121

Figure 6.14. The flowchart of the proposed approach (Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets)	143
Figure 6.15. The hierarchical structure of the problem (Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets)	143
Figure 6.16. The flowchart of the proposed approach (Integrated Fuzzy AHP and Fuzzy MULTIMOORA)	160
Figure 6.17. The flowchart of the proposed methodology (Integrated AHP and MULTIMOORA with Spherical Fuzzy Sets)	168
Figure 6.18. A schematic diagram of the proposed methodology (T-test and Genetic search)	186

LIST OF TABLES

	PAGE
Table 2.1. Financial ratios which are examined for financial fraud detection	14
Table 2.2. Financial ratios which are commonly used in previous researches to detect financial accounting fraud	23
Table 2.3. Definition of qualitative characteristics (International Accounting Standards Board, 2018; Herath and Albarqi, 2017)	25
Table 2.4. Applied Turkish Accounting Standards adopted from International Accounting Standards (Public Oversight, Accounting and Auditing Standards Authority, 2022)	27
Table 2.5. Applied Turkish Financial Reporting Standards adopted from International Financial Reporting Standards (Public Oversight, Accounting and Auditing Standards Authority, 2022)	28
Table 4.1. Summary of studies that utilize AHP, TOPSIS, and Integrated AHP and TOPSIS approaches with different fuzzy sets	41
Table 4.2. Summary of studies that use integrated Fuzzy AHP and Fuzzy TOPSIS approach	46
Table 4.3. Linguistic variables for the importance weight of each criterion (Fuzzy AHP)	47
Table 4.4. Linguistic variables for alternatives (Fuzzy TOPSIS)	50
Table 4.5. SVSFS numbers relating to linguistic terms (Gündoğdu and Kahraman, 2019c)	53
Table 4.6. IVSFS numbers relating to linguistic terms (Gündoğdu and Kahraman, 2019a)	56
Table 4.7. Intuitionistic Fuzzy numbers relating to linguistic terms	60
Table 4.8. Decision matrix depending on intuitionistic fuzzy numbers for criteria	61
Table 4.9. C-IF weight vector of criteria	62

Table 4.10. Decision matrix depending on intuitionistic fuzzy numbers for alternatives	63
Table 4.11. Aggregated circular intuitionistic fuzzy decision matrix	64
Table 4.12. Some of researches that employed AHP, MULTIMOORA, Integrated AHP and MULTIMOORA approaches with different fuzzy sets	67
Table 4.13. Some of researches that utilize AHP, MOORA and MULTIMOORA methods with Ordinary Fuzzy Sets	70
Table 4.14. Linguistic terms for evaluating the alternatives (Fuzzy MULTIMOORA)	71
Table 5.1. Some of researches that utilize machine learning classifiers for financial accounting fraud detection	83
Table 5.2. Confusion matrix	92
Table 6.1. Relative frequency of fraudulent and non-fraudulent companies	95
Table 6.2. Frequency distribution of fraudulent and non-fraudulent companies	95
Table 6.3. Frequency distribution of financial accounts utilized for fraudulent activities	98
Table 6.4. Means, Standard deviations, T-values and P-values for manipulated financial accounts	100
Table 6.5. Sample pairwise comparison matrix of criteria in linguistic variables by a financial analyst	106
Table 6.6. Sample pairwise comparison matrix of criteria in triangular fuzzy numbers by a financial analyst	106
Table 6.7. Aggregated fuzzy evaluation matrix of criteria which is obtained from assessments of four financial analysts	106
Table 6.8. Fuzzy synthetic extent values (S_i), weights (W'_i), and normalized weights (W_i) of criteria	107
Table 6.9. Ratings of alternatives with respect to criteria in linguistic variables by a financial analyst	108

Table 6.10. Ratings of alternatives with respect to criteria in triangular fuzzy numbers by a financial analyst	109
Table 6.11. Aggregated fuzzy evaluation matrix of alternatives which is obtained from assessments of four financial analysts	110
Table 6.12. Fuzzy normalized decision matrix	111
Table 6.13. Weighted normalized fuzzy decision matrix (Fuzzy TOPSIS)	112
Table 6.14. The distance from fuzzy positive-ideal solution (d_i^*), distance from fuzzy negative-ideal solution (d_i^-) and closeness coefficients (CC_i)	113
Table 6.15. Final ranking of financial ratios (Integrated Fuzzy AHP and Fuzzy TOPSIS)	114
Table 6.16. Selected financial ratios for comparing the performances of classifiers (Integrated Fuzzy AHP and Fuzzy TOPSIS)	115
Table 6.17. Overall Accuracy of classifiers using Integrated Fuzzy AHP and Fuzzy TOPSIS and without using any feature selection method	116
Table 6.18. True Positive Rates of classifiers using Integrated Fuzzy AHP and Fuzzy TOPSIS and without using any feature selection method	116
Table 6.19. True Negative Rates of classifiers using Integrated Fuzzy AHP and Fuzzy TOPSIS and without using any feature selection method	117
Table 6.20. Precision of classifiers under Integrated Fuzzy AHP and Fuzzy TOPSIS and without using any feature selection method	117
Table 6.21. G-measure of classifiers using Integrated Fuzzy AHP and Fuzzy TOPSIS and without using any feature selection method	118
Table 6.22. Area Under ROC Curve of classifiers using Integrated Fuzzy AHP and Fuzzy TOPSIS and without using any feature selection method	118
Table 6.23. Pairwise comparison matrix in linguistic variables related to SVSFS scale by financial analyst 1	122
Table 6.24. Pairwise comparison matrix in linguistic variables related to SVSFS scale by financial analyst 2	122

Table 6.25. Pairwise comparison matrix in linguistic variables related to SVSFS scale by financial analyst 3	122
Table 6.26. Pairwise comparison matrix in linguistic variables related to SVSFS scale by financial analyst 4	122
Table 6.27. Aggregated evaluations of the single-valued spherical fuzzy weights of criteria	123
Table 6.28. Single-valued spherical fuzzy weights of criteria	123
Table 6.29. Weighted single-valued spherical fuzzy decision matrix (Single-Valued Spherical Fuzzy TOPSIS)	124
Table 6.30. Positive and negative single-valued spherical fuzzy ideal solutions	124
Table 6.31. The distance from SVSF-PIS $d(X_i, X^+)$, distance from SVSF-NIS $d(X_i, X^-)$, Closeness ratio and ranking of alternatives	125
Table 6.32. Selected financial ratios for comparing the performances of classifiers (Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets)	126
Table 6.33. Overall Accuracy of classifiers using Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets and without using any feature selection method	127
Table 6.34. True Positive Rates of classifiers using Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets and without using any feature selection method	127
Table 6.35. True Negative Rates of classifiers using Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets and without using any feature selection method	128
Table 6.36. Precision of classifiers using Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets and without using any feature selection method	128
Table 6.37. G-measure of classifiers using Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets and without using any feature selection method	129

Table 6.38. Area Under ROC Curve of classifiers using Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets and without using any feature selection method	129
Table 6.39. Aggregated evaluations of the interval-valued spherical fuzzy weights of criteria	131
Table 6.40. Interval-valued spherical fuzzy weights of criteria	131
Table 6.41. Aggregated interval-valued spherical fuzzy decision matrix (Interval-Valued Spherical Fuzzy TOPSIS)	132
Table 6.42 Weighted interval-valued spherical fuzzy decision matrix (Interval-Valued Spherical Fuzzy TOPSIS)	134
Table 6.43. Score function values	136
Table 6.44. Positive and negative interval-valued spherical fuzzy ideal solutions	136
Table 6.45. The distance from IVSF-PIS $d(X_i, X^+)$, distance from IVSF-NIS $d(X_i, X^-)$, Closeness ratio and ranking of alternatives	137
Table 6.46. Selected financial ratios for comparing the performances of classifiers (Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets)	138
Table 6.47. Overall Accuracy of classifiers using Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets and without using any feature selection method	139
Table 6.48. True Positive Rates of classifiers using Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets and without using any feature selection method	139
Table 6.49. True Negative Rates of classifiers using Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets and without using any feature selection method	140
Table 6.50. Precision of classifiers using Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets and without using any feature selection method	140

Table 6.51. G-measure of classifiers using Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets and without using any feature selection method	141
Table 6.52. Area Under ROC Curve of classifiers using Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets and without using any feature selection method	141
Table 6.53. Pairwise comparisons of all criteria by using linguistic phrases related to IFS scale	145
Table 6.54. Aggregated evaluations of the intuitionistic fuzzy weights of criteria	146
Table 6.55. Intuitionistic fuzzy weights of criteria	146
Table 6.56. The maximum radius lengths among all experts	146
Table 6.57. C-IF weights of criteria	147
Table 6.58. RSF values and weights of criteria	147
Table 6.59. Ratings of alternatives with respect to criteria in linguistic phrases related to IFS scale for all decision makers	149
Table 6.60. The aggregated intuitionistic fuzzy decision matrix	150
Table 6.61. The maximum radius lengths of each alternative	151
Table 6.62. The aggregated circular intuitionistic fuzzy decision matrix	152
Table 6.63. The weighted circular intuitionistic fuzzy decision matrix	154
Table 6.64. Positive and negative ideal C-IF solutions	155
Table 6.65. $\mathcal{D}_i^{A^+}$ and $\mathcal{D}_i^{A^-}$, Closeness ratio values and ranking of alternatives	155
Table 6.66. Selected financial ratios for comparing the performances of classifiers (Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets)	156
Table 6.67. Overall Accuracy of classifiers using Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets and without using any feature selection method	156

Table 6.68. True Positive Rates of classifiers using Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets and without using any feature selection method	157
Table 6.69. True Negative Rates of classifiers using Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets and without using any feature selection method	157
Table 6.70. Precision of classifiers using Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets and without using any feature selection method	158
Table 6.71. G-measure of classifiers using Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets and without using any feature selection method	158
Table 6.72. Area Under ROC Curve of classifiers using Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets and without using any feature selection method	159
Table 6.73. Weighted normalized fuzzy group decision matrix (Fuzzy MULTIMOORA)	162
Table 6.74. Ranking of the alternatives according to the fuzzy MULTIMOORA	163
Table 6.75. Final ranking of financial ratios (Integrated Fuzzy AHP and Fuzzy MULTIMOORA)	163
Table 6.76. Overall Accuracy of classifiers using Integrated Fuzzy AHP and Fuzzy MULTIMOORA and without using any feature selection method	164
Table 6.77. True Positive Rates of classifiers using Integrated Fuzzy AHP and Fuzzy MULTIMOORA and without using any feature selection method	164
Table 6.78. True Negative Rates of classifiers using Integrated Fuzzy AHP and Fuzzy MULTIMOORA and without using any feature selection method	165

Table 6.79. Precision of classifiers using Integrated Fuzzy AHP and Fuzzy MULTIMOORA and without using any feature selection method	165
Table 6.80. G-measure of classifiers using Integrated Fuzzy AHP and Fuzzy MULTIMOORA and without using any feature selection method	166
Table 6.81. Area Under ROC Curve of classifiers using Integrated Fuzzy AHP and Fuzzy MULTIMOORA and without using any feature selection method	166
Table 6.82. Weighted single-valued spherical fuzzy decision matrix (Single-Valued Spherical Fuzzy MULTIMOORA)	169
Table 6.83. The ranking of the alternatives by utilizing the RS approach	170
Table 6.84. Single-Valued Spherical Fuzzy reference points	170
Table 6.85. The distances ($d(X_{ij}, X^*)$) and the deviations ($\text{Max } d(X_{ij}, X_j^*)$) of the alternatives from the single-valued reference point, and the ranking of alternatives by utilizing the RP approach	171
Table 6.86. $\tilde{S}_i$ values, $\text{Score}(\tilde{S}_i)$ and the ranking of the alternatives by utilizing the FMF approach	172
Table 6.87. Rankings of the alternatives based on three approaches and the final rank (Integrated AHP and MULTIMOORA with Single-Valued Spherical Fuzzy Sets)	173
Table 6.88. Selected financial ratios for comparing the performances of classifiers (Integrated AHP and MULTIMOORA with Single-Valued Spherical Fuzzy Sets)	173
Table 6.89. Overall Accuracy of classifiers using Integrated AHP and MULTIMOORA with Spherical Fuzzy Sets (Single-Valued and Interval-Valued) and without using any feature selection method	174
Table 6.90. True Positive Rates of classifiers using Integrated AHP and MULTIMOORA with Spherical Fuzzy Sets (Single-Valued and Interval-Valued) and without using any feature selection method	175

Table 6.91. True Negative Rates of classifiers using Integrated AHP and MULTIMOORA with Spherical Fuzzy Sets (Single-Valued and Interval-Valued) and without using any feature selection method	175
Table 6.92. Precision of classifiers using Integrated AHP and MULTIMOORA with Spherical Fuzzy Sets (Single-Valued and Interval-Valued) and without using any feature selection method	176
Table 6.93. G-measure of classifiers using Integrated AHP and MULTIMOORA with Spherical Fuzzy Sets (Single-Valued and Interval-Valued) and without using any feature selection method	176
Table 6.94. Area Under ROC Curve of classifiers using Integrated AHP and MULTIMOORA with Spherical Fuzzy Sets (Single-Valued and Interval-Valued) and without using any feature selection method	177
Table 6.95. Aggregated interval-valued spherical fuzzy decision matrix (Interval-Valued Spherical Fuzzy MULTIMOORA)	178
Table 6.96. Weighted interval-valued spherical fuzzy decision matrix (Interval-Valued Spherical Fuzzy MULTIMOORA)	179
Table 6.97. The ranking of the alternatives by utilizing the RS approach	181
Table 6.98. Interval-Valued Spherical Fuzzy reference points	182
Table 6.99. The distances ($d(X_{ij}, X_j^*)$) and the deviations ($\text{Max } d(X_{ij}, X_j^*)$) of the alternatives from the interval-valued reference point, and the ranking of alternatives by utilizing the RP approach	182
Table 6.100. The ranking of alternatives by utilizing the FMF approach	183
Table 6.101. Rankings of the alternatives based on three approaches and the final rank (Integrated AHP and MULTIMOORA with Interval-Valued Spherical Fuzzy Sets)	184
Table 6.102. Selected financial ratios for comparing the performances of classifiers (Integrated AHP and MULTIMOORA with Interval-Valued Spherical Fuzzy Sets)	184

Table 6.103. Means, Standard Deviations, T-values and P-values for financial ratios	188
Table 6.104. Selected Financial Ratios by Examined Feature Selection Techniques (T-test and Genetic Search)	189
Table 6.105. Overall Accuracy of classifiers using two feature selection methods (T-test and Genetic Search) and without using any feature selection method	190
Table 6.106. True Positive Rates of classifiers using two feature selection methods (T-test and Genetic Search) and without using any feature selection method	190
Table 6.107. True Negative Rates of classifiers using two feature selection methods (T-test and Genetic Search) and without using any feature selection method	191
Table 6.108. Precision of classifiers using two feature selection methods (T-test and Genetic Search) and without using any feature selection method	191
Table 6.109. G-measure of classifiers using two feature selection methods (T-test and Genetic Search) and without using any feature selection method	192
Table 6.110. Area Under ROC Curve of classifiers using two feature selection methods (T-test and Genetic Search) and without using any feature selection method	192
Table 7.1. Final rankings of financial ratios by using MCDM methods with different extensions of fuzzy sets	197
Table 7.2. Selected financial ratios by using Integrated AHP and TOPSIS and Integrated AHP and MULTIMOORA approaches with different fuzzy sets	201
Table 7.3. Overall Accuracy of classifiers under different sampling methods and different integrated MCDM approaches using different fuzzy sets	202

Table 7.4. True Positive Rates of classifiers under different sampling methods and different integrated MCDM approaches using different fuzzy sets	203
Table 7.5. True Negative Rates of classifiers under different sampling methods and different integrated MCDM approaches using different fuzzy sets	204
Table 7.6. Precision of classifiers under different sampling methods and different integrated MCDM approaches using different fuzzy sets	205
Table 7.7. G-measure of classifiers under different sampling methods and different integrated MCDM approaches using different fuzzy sets	206
Table 7.8. Area Under ROC Curve of classifiers under different sampling methods and different integrated MCDM approaches using different fuzzy sets	207



1. INTRODUCTION

Financial statements reflect a company's financial situation and operating performance. Some decisions are made by creditor banks, investors, and governmental organizations based on the information presented in financial statements. Creditor banks, for example, decide whether or not to provide credit to businesses by evaluating their financial statements and checking any possible financial statement manipulations in order to avoid credit facilities that exceed real borrowing capacity.

Businesses may manipulate their financial statements for providing credit facilities beyond real borrowing capacities. Their common fraudulent techniques can be overstating assets, understating expenses or liabilities, misappropriating assets and other miscellaneous techniques. However, there is no commonly accepted definition of financial accounting fraud. International Auditing and Assurance Standards Board (IAASB) (2009) published International Standard on Auditing 240 (ISA 240) and defined fraud as “An intentional act by one or more individuals among management, those charged with governance, employees, or third parties, involving the use of deception to obtain an unjust or illegal advantage”. American Institute of Certified Public Accountants (AICPA), in its Statement on Auditing Standards (SAS) No.99 described fraud as "An intentional act that results in a material misstatement in financial statements that are the subject of an audit”.

Financial fraud may take numerous forms, including credit card fraud, management fraud, and money laundering. Financial fraud is divided into four categories by Ngai et al. (2011): Bank fraud, Securities and commodities fraud, Insurance fraud, and Other associated financial fraud. Financial fraud is also categorized into three groups by West and Bhattacharya (2016): Bank fraud, Corporate fraud, and Insurance fraud.

When financial reports contain misstatements or omissions of accounts, financial accounting fraud is referred to as corporate fraud (Kassem, 2016). Because accounting records, including expenses, invoices, sales figures, and revenues, may be falsified directly or indirectly by management (Kassem, 2016; Thiruvadi and Patel, 2011), financial accounting fraud can also be categorised as management fraud (Kassem, 2016; Thiruvadi and Patel, 2011). Financial accounting fraud, commonly known as management fraud or corporate fraud, is examined in this dissertation.

Financial fraud seriously damages the sustainable development of enterprises and financial markets (Jan, 2018). A research issued by Association of Certified Fraud Examiners (ACFE) in 2020 stated that organizations lost 5% of their annual revenues due to fraud. According to the same report, the overall cost from 2,504 cases was \$3.6 billion, with an average loss of \$1.5 million per case. Addition to economic losses, financial accounting fraud has far-reaching negative implications for the company's investors, employees, suppliers, and other stakeholders (Hess and Andiola, 2018). According to Amiram et al. (2018), financial accounting fraud weakens confidence between trade-related organizations and market players. It weakens the fundamental role of capital markets in the efficient allocation of resources. According to Ngai et al. (2011), financial accounting fraud detection is critical for avoiding financially damaging outcomes.

Previous studies (Spathis, 2002; Kotsiantis et al., 2006; Küçükkocaoğlu et al., 2007; Pai et al., 2011; Chen, 2016; Yao et al., 2018) have almost all focused on public listed companies or corporations with a substantial market capitalization. There have been fewer studies (Johnson and Rudesill, 2001; Shanmugam et al. 2012; Omar et al., 2017) that have focussed on fraud cases for SMEs in comparison to these studies. This study is noteworthy because it contributes to the existing literature, which is currently lacking, on financial accounting fraud detection for SMEs.

As a matter of fact, small businesses encounter specific fraud cases which are known to be different from the cases encountered by larger businesses. SMEs should be more concerned as businesses with fewer than 100 employees statistically experience more fraud cases than larger businesses (Aris et al., 2015). Cobo et al. (2017) emphasized that the role of auditing in terms of decision making in the banking sector should be investigated and attention should be paid to SMEs in particular.

Johnson and Rudesill (2001) investigated fraud prevention and detection for small companies in the United States. They recommended owners and managers of small companies to be proactive in avoiding fraud because small firms are more vulnerable than large enterprises and losses are more severe.

Internal controls, risk management, and fraud prevention regarding performances of SMEs were examined by Shanmugam et al. (2012). Effective internal control, risk management, and the existence of adequate fraud prevention measures were found to assist SMEs enhance their performance. Aris et al. (2015) examined the likelihood of manipulated financial statements in a Malaysian small-medium automotive firm. Mekic et al. (2017) studied SMEs' deceptive accounting in Bosnia and Herzegovina. They concluded that forensic accounting is vital and useful in pointing out incorrectly presented information.

According to Aladejebi and Oladimeji (2019), SMEs have fewer anti-fraud measures and resources to prevent and recover from financial accounting fraud. Thus, they are more open to fraud. In order to reduce fraud and manage risks proactively, SMEs have to identify factors that lead to fraudulent activities. The deterioration of cultural values and financial pressure are the primary causes of fraud. SMEs should employ fraud prevention methods, such as promoting social values (integrity), putting in place strong internal and external control procedures, and providing staff training. While anti-fraud procedures and controls, as well as relevant oversight functions, have positive and significant impact on fraud prevention mechanisms, the implementation of an effective fraud prevention mechanism requires the full commitment of top management to build a culture of honesty and high integrity (Sow et al., 2018). The European Federation of Accountants (2005) recommended the adoption of several steps to reduce fraud risk, such as building an ethical culture, identifying risk areas, establishing policies and principles, and providing a conduit for reporting suspected fraud. Hess and Cottrell (2016) provided the following seven useful suggestions for small firms dealing with fraud cases: (1) demonstrating the need of ethics, (2) making reporting simpler, (3) trusting and confirming, (4) beware of the slippery slope (5) searching for suspicious trends are all important (6) making decisions that yield positive results, and (7) providing personnel with the resources they require to achieve their objectives. These strategies enable them in strengthening their fraud defenses.

Weak internal and external controls, a deterioration of cultural values (honesty, trust, etc.), poor financial conditions, tax evasion, and gaps in government rules and legislation are all key reasons of fraudulent activities in Turkish SMEs. As a result, identifying fraud indicators that take into account the aforementioned reasons is critical for

developing an effective fraud risk management strategy.

ISA 240 defined fraud risk factors, which are based on the fraud triangle theory introduced by Cressey (1953). Omar et al. (2017) utilized this fraud triangle theory as a framework in order to predict fraudulent financial reporting in small market capitalization companies in Malaysia. The fraud triangle has three elements, which are pressure, opportunity and rationalization. Pressure is defined as motivation of an individual to commit a fraud crime because of financial problems. Opportunity is defined as lack of internal control that provides the chance for the fraudster to gain access to commit fraud in an organization. Rationalization, which occurs to justify the actions of the fraudster, is sort of a belief that fraudster's behaviors are not illegal and morally wrong.

Fraud risk factors are considered to be important for financial accounting fraud and are useful to detect the underlying causes of financial accounting fraud (Rezaee, 2002). The use of risk factors as potential fraud indicators should be considered within framework of fraud triangle. Many researchers (Skousen and Wright, 2006; Skousen et al., 2009; Lou and Wang, 2009) examined risk factors related to the fraud triangle theory. Omar et al.(2017) used several risk factors, such as weak internal control, company's rapid growth, the ownership status and etc. to detect fraudulent financial reporting.

Implementing audit processes may help to detect financial accounting fraud. Detecting financial accounting fraud, on the other hand, is a challenging task that takes more effort than regular audit processes (Chen et al., 2014). Notably, more than 98% of the enterprises operating in Turkey are Small and Medium Sized Enterprises (Halıcı and Erhan, 2013), and the implementation of independent auditing is not mandatory for these Turkish SMEs. The Cabinet Decree specifies companies, which are subject to audit. The vast majority of SMEs do not adhere to the Cabinet Decree's regulations (Karahan et al., 2017).

The Cabinet Decree issued in the Official Gazette on May 26, 2018, determines which firms are subject to audit. According to the criteria, companies that provide at least two of the three criteria for two consecutive accounting periods are subject to audit: total balance sheet of 35 million ₺; annual net turnover of 70 million ₺; employee headcount of 175.

In addition to these criteria, public interest entities including (but not limited to) Publicly held companies; Banks; Insurance reinsurance and pension companies; Factoring companies; Financing companies; Financial lease companies; Asset management companies are subject to audit (Public Oversight Accounting and Auditing Standards Authority, 2018).

For decision makers like auditors, financial accounting fraud detection requires more statistical tools and procedures than traditional techniques. Initially statistical models, including Altman Z-score (Altman, 1968), Beneish model (Beneish, 1999) and financial ratio (Kaminski et al., 2004) are utilized. Following that, the researchers employed data mining techniques to estimate the occurrence of financial statement fraud. Logistic regression, probit regression, decision trees, bayesian networks, and artificial neural networks are examples of utilized data mining algorithms (Kirkos et al., 2007; Ravisankar et al., 2011; Liou, 2008; Omar et al., 2017). West and Bhattacharya (2016) presented a comprehensive review of studies that use data mining methods in financial accounting fraud detection, with a particular focus on computational intelligence based techniques. Some studies in the field of financial accounting fraud detection have recently concentrated on hybrid approaches that combine data mining techniques. Chen et al. (2017) integrated natural language processing (NLP), queen genetic algorithm (QGA) and support vector machine (SVM) to propose a fraud detection methodology. Dutta et al. (2017) employed all widely used data mining techniques, such as Decision Tree (DT), Artificial Neural Network (ANN), Naïve Bayes (NB), Support Vector Machine (SVM), and Bayesian Belief Network (BBN) classifiers for developing the predictive models. Yao et al. (2018) suggested an approach that combines feature selection and machine learning methods for detecting financial fraud. Because financial ratios involve manipulated accounts that depart from normal behavior and indicate accounting fraud adequately, they are often utilized in all of these methods for detecting financial accounting fraud (Jofre, 2017). It is important to note that numerous financial ratios have been utilized for detecting financial accounting fraud. For instance, Persons (1995) utilized 10 financial ratios, Kirkos et al. (2007) compiled 27 financial ratios, Huang (2013) examined 19 financial ratios, An and Suh (2020) used 23 financial ratios for detecting financial accounting fraud. All of these researchers determined various significant financial ratios for financial accounting fraud detection.

Because of the sophisticated decision-making process that includes hesitation and ambiguity, there is no accepted approach for evaluating which financial ratios are important in financial accounting fraud detection approach in the literature. Furthermore, determining which financial ratios are important in financial accounting fraud detection necessitates a more complex approach than conventional methods for experts. Therefore, determining which financial ratios are important in financial accounting fraud detection is assessed as a MCDM (Multi Criteria Decision Making) problem in this dissertation.

MCDM has become a main research area for dealing with complex decision problems that include multiple objectives or criteria (Lertprapai, 2013; Alrababah et al., 2017). Hwang and Yoon (1981) classified MCDM problems into two main categories, which are Multiple Attribute Decision Making (MADM) and Multiple Objective Decision Making (MODM), based on different purposes and different data types. MADM deals with discrete and predetermined alternatives, which are described by determined discrete criteria set. MODM is not associated with the problem where the alternatives are predetermined. This classification is suitable for two aspects of problem solving: MADM is for selection/evaluation, MODM is for design.

Selection of an appropriate MCDM method, such as the Elimination et Choice Translating Reality (ELECTRE), the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), the Analytic Hierarchy Process (AHP), the Analytic Network Process (ANP) and etc. depends on various factors including the aim of the problem, available information, cost of the decision, and qualification of decision makers (Kersulienė and Turskis, 2014).

AHP and TOPSIS have mostly been used to evaluate alternatives with respect to a variety of criteria. AHP developed by Saaty (1980) has been widely utilized to estimate the relative importance weights of criteria by using pairwise comparisons. TOPSIS, developed by Hwang and Yoon (1981), is generally used to evaluate alternatives in terms of criteria that consider positive and negative ideal solutions. The optimal alternative is found to be the one with the smallest distance from the positive ideal solution and the largest distance from the negative ideal solution. The integration of AHP and TOPSIS methodology has been shown to be sufficient and successful for solving real-world issues in a variety of fields, including management science, engineering, education, and

public administration (Liu et al., 2019; Beskese et al.,2019; Chou et al.,2019; Samanlioglu et al.,2018). Integrated AHP and TOPSIS approach is utilized in this study for prioritizing financial ratios for financial accounting fraud detection due to the success of the integration of AHP and TOPSIS methodology.

Brauers and Zavadskas (2006) developed the Multi-Objective Optimization by Ratio Analysis (MOORA) approach, which they used to privatization in a developing economy. Brauers and Zavadskas (2010) later enhanced the MOORA method and the name given to this enhanced technique, which took into consideration the whole multiplicative form, was Multiple Multi-Objective Optimization by Ratio Analysis (MULTIMOORA). MULTIMOORA, one of the newest MCDM methods, is another important technique that considers the knowledge of the faults in previous methods.

AHP and MOORA techniques have been used severally or as the integration of AHP and MOORA methodology in various studies. Yildirim and Onay (2013) employed fuzzy AHP and MOORA techniques to evaluate various cloud-based firms. Vatansever and Ulukoy (2013) used fuzzy AHP and fuzzy MOORA methods for ERP software selection. However, there are few studies that use an integrated AHP and MULTIMOORA approach to handle MCDM problems in a variety of applications. Fattahi and Khalilzadeh (2018) proposed a new fuzzy hybrid methodology that combines fuzzy FMEA, extended fuzzy MULTIMOORA, and fuzzy AHP methods to evaluate various failure modes. This study proposes an integrated AHP and MULTIMOORA approach to prioritize financial ratios, as well as expanding the application of relevant approach.

Inconsistency of information can be caused by different views and decision makers' perceptions. Furthermore, the assessment process in MCDM problems frequently contains unclear facts, which can make the decision-making process more difficult (Ghorabae et al., 2018). To reduce these ambiguities, fuzzy logic approaches can be used. When there is ambiguity, fuzzy logic gives numerical information and aids in achieving a more realistic outcome in establishing the presence of the relationship (Guner et al., 2009). To reduce uncertainty in the decision-making process, it may be useful to apply fuzzy logic to MCDM techniques from these perspectives.

Fuzzy set theory, which was introduced by Zadeh in 1965, provides a framework to model the uncertainty of natural language. Fuzzy logic is derived from fuzzy set theory, and it provides a mechanism for decision making (Konrad and Philip, 1994). Decision making generally involves some “gray areas”. Fuzzy sets and fuzzy logic were developed for handling this gray areas and uncertainty in real-world applications (Lin et al., 2003). As a matter of fact, decision making in finance and business needs a combination of fuzzy logic and analytical models.

Few researchers applied fuzzy logic to detect financial fraud. Previously, Deshmukh et al. (1997) and Deshmukh and Talluru (1998) used fuzzy set models to assess the risk of managerial fraud. The results of studies showed that the fuzzy system can be used to measure the risk of management fraud. Moreover, Lin et al. (2003) developed an integrated system of neural networks and fuzzy logic for detecting fraudulent financial reporting. The performance of the developed Fuzzy neural network model was compared with logistic model. Fuzzy neural network model achieved significantly higher accuracy than the logistic model. Lenard et al. (2007) developed a model to detect fraudulent financial reporting in service based computer and technology organizations by using fuzzy logic. Two fraud detection models, which are Persons’ (1995) model and Lenard and Alam’s (2004) model, were used and 76.7% accuracy obtained in predicting financial fraud.

Several researchers have applied different fuzzy sets with integrated AHP and TOPSIS approach to solve real-world problems, such as assessing the performance of wind power coupling hydrogen storage projects using Type-2 fuzzy sets (Wu et al., 2019), evaluating outsource manufacturers using Intuitionistic fuzzy sets (Kahraman et al., 2020b), selecting an ERP software package using Hesitant fuzzy sets (Ayağ and Samanlıoğlu, 2021), selecting transportation firm using Pythagorean fuzzy sets (Sarkar and Biswas, 2021), evaluating urban development policies in the construction sector using Picture fuzzy sets (Kim and Van, 2021), selecting advanced manufacturing system using Spherical fuzzy sets (Mathew et al., 2020). However, there are few researches that utilize the integration of AHP and MULTIMOORA techniques with different extensions of fuzzy sets, such as selection of supplier using Type-1 fuzzy sets (Tavana et al., 2020), information management system selection for scientific research project using intuitionistic fuzzy sets (Göçer, 2020), selection of solar PV panel producer using

pythagorean fuzzy sets (Seker and Kahraman, 2020).

This study utilizes integrated AHP and TOPSIS and integrated AHP and MULTIMOORA approaches using several fuzzy sets, which are Type1, Spherical and Circular fuzzy sets, to rank financial ratios for detecting financial accounting fraud. Spherical fuzzy sets and Circular fuzzy sets are two newly proposed fuzzy sets for dealing with uncertainty in decision making process.

Spherical fuzzy sets (SFS) have been developed by Gündoğdu and Kahraman (2018, 2019c) as a novel extension of pythagorean, neutrosophic, and picture fuzzy sets. Due to its versatility, SFS have been applied in a variety of real-world problems, including optimal site selection for electric vehicle charging stations (Gündoğdu and Kahraman, 2020b), evaluation of suburban locations' livability indices (Karaşan et al., 2020), and supplier selection (Sharaf, 2020).

Atanassov (2020) introduced circular intuitionistic fuzzy sets (C-IFSs) as expansions of intuitionistic fuzzy sets (IFSs). C-IFS can be successfully implemented in MCDM techniques to help decision-makers cope with uncertainty more readily. However, there are few studies that use C-IFS to solve MCDM problems, such as selection of pandemic hospital location (Alkan and Kahraman, 2021), selection of a waste disposal location (Kahraman and Otay, 2021) selection of a supplier (Kahraman and Alkan, 2021).

This is the first study that proposes integrated AHP and MULTIMOORA approach using spherical fuzzy sets (comprising interval-valued and single-valued fuzzy numbers) to prioritize financial ratios for detecting financial accounting fraud. Furthermore, a novel integrated AHP and TOPSIS approach with interval-valued spherical fuzzy sets (IVSFS) and C-IFS is proposed to handle uncertainties in the decision-making process of ranking financial ratios for detecting financial accounting fraud.

This thesis includes eight main sections. In the following section, a detailed overview involving financial accounts, financial ratios and qualitative characteristics is given. Detailed explanations and formulas of financial accounts and financial ratios are presented. Section 3 provides theoretical background of fuzzy sets, which are used in this study. Definitions of used fuzzy sets, arithmetic operations, aggregation and defuzzification operations are given.

Section 4 presents steps of proposed methodologies, which are integrated AHP and TOPSIS and integrated AHP and MULTIMOORA approaches using different extensions of fuzzy set theory, for prioritization of financial ratios in detecting financial accounting fraud. Furthermore, the studies that use integrated AHP and TOPSIS and integrated AHP and MULTIMOORA approaches using different extensions of fuzzy set theory are presented in this section.

Section 5 explains machine learning classifiers, performance metrics, feature selection methods, and sampling methods used in this thesis. In section 6, the data obtained is introduced firstly, then application studies and results of analyses are provided. For the application studies, the algorithms and steps of proposed approaches are explained in detail.

Section 7 covers findings and discussion of the study. First, final rankings of financial ratios, which are obtained by using proposed approaches are given. Then, performances of classifiers using financial ratios selected through the proposed approaches are compared. Moreover, sectoral and financial accounts based results are discussed. Section 8 provides conclusion along with recommendations for further studies.

2. THEORETICAL BACKGROUND

In this section, the theoretical background of financial accounts and financial ratios is examined.

2.1. Financial Accounts in Detecting Financial Accounting Fraud

Financial ratios are widely used to detect financial accounting fraud. Many researchers have used different financial ratios for financial accounting fraud detection. Financial ratios associated with manipulated accounts will diverge from normal behavior and reveal signals of accounting fraud adequately if a fraudulent behavior is performed (Jofre, 2017). Therefore, it is important to examine the most used financial accounts for financial accounting fraud. These are as follows.

2.1.1. Cash (100)

Cash is a component of a company's net working capital and is included in the current assets part of the balance sheet. Legal tender, bills, coins, and savings accounts are all included. This account can be used to hide expenses and losses that are not transferred to the income statement.

2.1.2. Accounts receivable (120)

Management is more prone to manipulate accounts receivable (Loebbecke et al., 1989; Persons, 1995; Fanning and Cogger, 1998). Because of the differences in revenue recognition timing, sales might be recorded before they are earned, resulting in a rise in accounts receivable.

2.1.3. Allowance for doubtful accounts (129)

Because the allowance for doubtful accounts depends on assessments of management, it can be manipulated (Fanning and Cogger, 1998).

2.1.4. Due from shareholders (131)

Borrowing cash from the firm is a regular transaction in a company. A shareholder can take money out of the company when he or she needs it for personal reasons. This

account, however, is open to management manipulation.

2.1.5. Inventories (153)

Management may manipulate inventories, according to many researchers, including Persons (1995), Summers and Sweeney (1998), Fanning and Cogger (1998), and Spathis (2002). Summers and Sweeney (1998) claim that inventory is open to interpretation and preferences, and therefore manipulations on this account are easier. Such manipulations rely heavily on obsolete inventories. Companies eager to commit fraud do not properly record the amount of obsolete inventory (Fanning and Cogger, 1998). Reporting inventory at a lower cost, or market value, is another sort of manipulation. (Birol, 2017; Spathis, 2002).

2.1.6. Due to shareholders (331)

It is the account in which all payments, excluding capital, provided by the firm's shareholders are accounted for. This account can also be used to hide non-official receivables and sales (Turkmen, 2016).

There are certainly manipulated accounts that are not included in this section. Since there are many financial accounts, the most used financial items for financial accounting fraud are considered. To the best of our knowledge, financial ratios, which include "Cash" , "Due from Shareholders" and "Due to Shareholders" accounts have not been examined for detecting financial accounting fraud in previous studies.

2.2. Financial Ratios in Detecting Financial Accounting Fraud

Many studies emphasize that financial statement information is helpful for detecting financial accounting fraud. A detailed analysis of financial ratios, in particular, can indicate signals of fraud case. Therefore, financial ratios are calculated to measure the relationship between two financial accounts, and afterwards to define acceptable non fraudulent values.

Financial ratios are in widespread use as predictors of failure, which is defined as the inability of a firm to pay its financial obligations (Beaver, 1966). However, the use of financial ratios is not standard (Perols, 2008). For instance, Altman (1968) used 5

financial ratios, such as working capital to total assets, Retained Earnings to Total Assets, Earnings Before Interest and Taxes to Total Assets, Market Value of Equity to Book Value of Total Debt, Sales to Total Assets for the prediction of corporate bankruptcy. Beaver (1966) utilized 6 financial ratios including cash flow to total debt ratio, Net income to total assets, Total debt to total assets, Working capital to total assets, Current Ratio (current assets to current liabilities), No-credit interval (defensive assets minus current liabilities to fund expenditures for operations). Ohlson (1980) employed nine different financial ratios. Fanning and Cogger (1998) investigated 62 potential fraud indicators by using stepwise logistic regression method and found eight significant fraud predictors, such as debt to equity, sales to assets, whether CFO changed in the last three years etc. Kaminski et al. (2004) found that five out of 21 financial ratios tested were significant predictors of fraud during the period to prior the fraud year. These significant ratios are fixed assets to assets, sales to accounts receivable, inventory to current assets, inventory to sales, and sales to assets. Lin et al. (2011) examined a set of financial features that includes 21 commonly used financial ratios proposed in prior research, and 74 financial ratios from Taiwan Economic Journal database. They applied data mining techniques and found five significant financial ratios, three from the literature feature set and two from the Taiwan Economic Journal feature set. Tian and Yu (2017) applied a variable selection method, adaptive LASSO (least absolute shrinkage and selection operator), to select a set of significant predictor variables. They examined a total of 29 accounting-based financial ratios as the candidate predictor variable set and found different financial ratios for each international market. For instance, three predictor variables including Retained Earning/ Total Asset, Total Debt/Total Asset and Current Liability/Sales are selected for Japan market. On the other hand, Equity/Total Liability is selected as a predictor variable for some Europe countries including UK, Germany and France.

Table 2.1 indicates examined financial ratios for detecting financial accounting fraud.

Financial ratios are classified into five standard ratio categories including **liquidity**, **profitability**, **leverage**, **efficiency** and **growth ratios**. These financial ratios are explained in the following subsections:

Table 2.1. Financial ratios which are examined for financial fraud detection

Category	Financial Ratios	Formula
Liquidity Ratios	Current ratio	Current Assets/Current Liabilities
	Quick ratio	Current Assets-Inventories/Current Liabilities
	Cash ratio	Cash/Current Liabilities
Leverage Ratios	Debt ratio	Total Liabilities /Total Assets
	Debt-to-equity ratio	Total Liabilities /Total Equity
Efficiency Ratios	Inventory turnover	Cost of Goods Sold/Average Inventory
	Accounts Payable to Cost of Good Sold	Accounts Payable/ Cost of Good Sold
	Total asset turnover	Net Sales/ Total Assets
	Accounts receivable turnover	Net sales/Average Accounts Receivable
	Cash Conversion Cycle ratio	Days Inventory Outstanding + Days Sales Outstanding- Days Payable Outstanding
	Accounts Receivable to Sales	Accounts Receivable/ Sales
	Accounts Receivable to Total Assets	Accounts Receivable/ Total Assets
	Inventory to Sales	Inventory/Sales
	Inventory to Total Assets	Inventory/Total Assets
Profitability Ratios	Inventory to Current Assets	Inventory /Current Assets
	Return on Assets ratio	Net Income/Total Assets
	Net profit margin ratio	Net Income/ Net Sales
	EBIT to Total Assets	Earnings Before Interest and Tax / Total Assets
Growth Ratios	Return on Equity ratio	Net Income/ Shareholders' Equity
	Growth of accounts receivables	Growth of accounts receivables
	Growth of Inventory	Growth of Inventory
	Growth of income from principal	Growth of income from principal
Other Ratios	Growth of sales	Growth of sales
	Cash to Total Assets	Cash/ Total Assets
	Due from Shareholders to Total Assets	Due from Shareholders/ Total Assets
	Due to Shareholders to Total Liabilities	Due to Shareholders/ Total Liabilities
	Other Fixed Assets to Total Assets	Other Fixed Assets/ Total Assets
	Other Extraordinary Expenses and Losses to Net Sales	Other Extraordinary Expenses and Losses/ Net Sales
	Due to Shareholders to Average Inventory	Due to Shareholders/Average Inventory
	Doubtful Trade Receivables to Total Assets	Doubtful Trade Receivables/Total Assets
	Prepaid Expenses for Future Months to Total Assets	Prepaid Expenses for Future Months/Total Assets
	Prepaid Expenses for Future Years to Total Assets	Prepaid Expenses for Future Years/Total Assets

2.2.1. Liquidity

Liquidity ratios refer to the ability of an enterprise to meet its short term liabilities as they come due (Otieno,2016). Liquidity ratios are highly important for businesses, since liquid assets reduce some extent risks by ensuring the capacity of an enterprise to pay off debts as they come due (Jofre, 2017).

Lower liquidity may provide an incentive for managers to commit fraud by manipulating the accounting records. Companies, which are affected by fraud, tend to have low liquidity (Lari, 2009). Therefore, there is a need to investigate financial ratios related to the liquid composition of assets. There are three basic measures of liquidity which are current ratio, quick (acid test) ratio, and cash ratio.

2.2.1.1. Current assets to current liabilities (Current ratio)

It is also known as "Current Ratio". This ratio is used to determine financial health of the company, since it measures the ability to pay short term liabilities. Although the acceptable values of this ratio vary depending on the sector, it is believed that firms, which are affected by fraud, tend to present this ratio as high as possible to reflect a positive economic position (Jofre, 2017). Some studies (Kaminski et al., 2004; Kotsiantis et al., 2006; Kirkos et al.,2007; Lenard et al., 2007; Pai et al., 2011; Ravisankar et al., 2011; Song et al., 2014; Chen, 2016) used this ratio for detecting financial accounting fraud.

2.2.1.2. Current assets-inventories to current liabilities (Quick ratio/Acid-test ratio)

It is also known as "Acid-Test Ratio" or "Quick Ratio". This ratio is an indicator of a company's short term liquidity position and measures a company's ability to meet its short term obligations with its most liquid assets. Some studies (Kirkos et al.,2007; Song et al., 2014; Chen, 2016; Yao et al., 2018) used this ratio for detecting financial accounting fraud.

2.2.1.3. Cash to current liabilities (Cash ratio)

It is also called as "Cash Ratio". This ratio reflects a company's ability to pay current liabilities from the cash it has (Vasiu et al.,2015).

2.2.2. Profitability

Profitability ratios are used to assess the ability of a company to generate earnings compared to its costs. Managers may manipulate these ratios to resolve profitability issues when companies do not perform as expected (Jofre, 2017). According to Spathis (2002), companies with low profitability tend to have fraudulent financial statements. Therefore, it is useful to consider profitability ratios for detecting financial accounting fraud. Ratios associated with profitability are defined below.

2.2.2.1. Net income to total assets (Return on assets)

It measures efficiency of total assets. Higher values of this ratio indicate better efficiency (Lari, 2009). Spathis (2002) found that the ratio (Net Income / Total Assets) is the most important ratio for detecting financial accounting fraud. Spathis (2002) also noted that companies with low values of this ratio are more likely to have fraudulent financial reporting. Many studies (Persons, 1995; Spathis, 2002; Kaminski et al., 2004; Lenard et al., 2007; Pai et al., 2011; Ravisankar et al., 2011; Song et al., 2014; Chen, 2016; Omar et al., 2017; Yao et al., 2018) used this ratio for detecting financial accounting fraud.

2.2.2.2. Net income to net sales (Net profit margin)

Lower profit may provide an incentive for managers to overstate revenues or understate expenses (Persons, 1995). Spathis et al. (2002) found that firms with profitability problems try to manipulate financial statements either by increasing revenue or reducing expenditure so as to improve the profit and loss account. Spathis et al. (2002), Kaminski et al. (2004), Pai et al. (2011) and Omar et al. (2017) used net income to net sales ratio in their studies for detecting financial accounting fraud.

2.2.2.3. Earnings before interest and tax to total assets (EBIT to total assets)

It is one of the most important indicators of a company's profitability, and it is very likely to be manipulated in order to hide poor performance of the company (Kotsiantis et al., 2006). Kotsiantis et al. (2006) used this ratio for detecting financial accounting fraud, however he found that the developed model had not been influenced by this ratio.

2.2.2.4. Net income to shareholders' equity (Return on equity)

It measures the return on investments made by shareholders of a company. It is calculated as the ratio of net income to the total equity (Almaqtari et al. 2018). Ozcan (2016) used this ratio for financial accounting fraud detection, and he found that return on equity ratio is statistically significant. Ozcan (2016) also noted that non fraudulent firms have higher levels of return on equity ratio than fraudulent firms.

2.2.3. Leverage

Financial leverage represents the ability of a company to meet obligations both for short and long term. Analysis of this aspect is needed to measure the ability of a company to pay its debts (short and long term) (Sigit, 2008; Dance and Made, 2019). Companies which have trouble paying their debts may manipulate their financial statements in order to fulfill debt covenants. Therefore, high levels of debt increase the likelihood of accounting fraud (Jofre, 2017).

2.2.3.1. Total liabilities to total assets (Debt ratio)

Whether a high debt structure is associated with financial accounting fraud is an issue that needs to be investigated (Persons, 1995; Spathis, 2002). A firm with a high debt structure may be more likely to manipulate its financial statements. It can be measured through the ratio of total liabilities to total assets (Spathis et al, 2002), and a positive relation between this ratio and fraud can be expected (Jofre, 2017). Many researches (Persons, 1995; Spathis, 2002; Kaminski et al., 2004; Kotsiantis et al., 2006; Kirkos et al., 2007; Lenard et al., 2007; Pai et al., 2011; Song et al., 2014; Chen, 2016; Omar et al., 2017) used this ratio for detecting financial accounting fraud.

2.2.3.2. Total liabilities to total equity (Debt-to-equity ratio)

It gives information about the capital structure of a company. A high value of this ratio indicates that the firm uses debt financing aggressively (Taani and Banykhaled, 2011). High level of debt in a company increases the possibility of fraud risk in financial statements (Birol, 2017). Spathis (2002), Kaminski et al. (2004), Pai et al. (2011), Song et al. (2014) and Chen (2016) used Total Liabilities to Total Equity ratio for measuring

the effect of debt on financial accounting fraud.

2.2.4. Efficiency

Efficiency ratios can be described as a standard of measurement for the quality of an enterprise's receivables and the effectiveness of its assets (Das, 2019). Inefficiency arises from a firm's low performance, and it leads to higher costs (Jofre, 2017). Therefore, inefficiency may provide an incentive for managers to commit fraud by manipulating accounting records. Ratios associated with efficiency are defined below.

2.2.4.1. Accounts receivable to total sales

Accounts receivable is more likely to be manipulated by management (Loebbecke et al., 1989). Sales can be recorded before they are earned because of the differences of timing in revenue recognition, and this increase accounts receivable. When accounts receivable grow much faster than sales, it is necessary to evaluate the relationship between these two variables. The higher values of this ratio may increase the probability of financial accounting fraud (Birol, 2017). Many researchers (Spathis, 2002; Fanning and Cogger, 1998; Feroz et al., 2000; Kirkos et al., 2007; Pai et al., 2011; Chen, 2016; Omar et al., 2017) used this ratio for detecting financial accounting fraud.

2.2.4.2. Accounts receivable to total assets

Asset composition is a significant factor that influence the likelihood of financial accounting fraud (Kaminski et al. 2004), and asset composition is measured by current assets to total assets, accounts receivable to total assets and inventory to total assets (Persons, 1995). A positive relation is expected between this ratio and fraud occurrence since fraudulent firms have higher levels of accounts receivable to total assets than non fraudulent firms (Persons, 1995; Jofre, 2017). Persons (1995), Fanning and Cogger (1998), Kaminski et al. (2004), Kotsiantis et al. (2006), and Chen (2016) used accounts receivable to total assets ratio in their studies as a variable since this ratio is very open for manipulations.

2.2.4.3. Inventory to total sales

Many researchers, such as Persons (1995), Summers and Sweeney (1998), Fanning and Cogger (1998) and Spathis (2002) claim that management may manipulate inventories. Summers and Sweeney (1998) indicate that inventory is open for preferences and interpretation, and manipulations on this account can be performed more easily. Obsolete inventories are an important part of such manipulations. The amount of outdated inventory is not recorded accurately by organizations which are eager to commit fraud (Fanning and Cogger, 1998). Another type of manipulation involves reporting inventory at lower cost, or market value. (Spathis, 2002; Birol, 2017). Kaminski et al. (2004), Fanning and Cogger (1998), Kirkos et al. (2007), Pai et al. (2011), Omar et al. (2017), Yao et al. (2018) used inventory to total sales ratio in their studies for detecting financial accounting fraud.

2.2.4.4. Inventory to total assets

Firms can increase their assets through reporting higher inventory including obsolete stocks. Furthermore, the analysis of the change in inventory relative to total assets reduces bias from the size of company (Gepp, 2015). Therefore, it is important to investigate the use of this ratio as a possible indicator of financial accounting fraud (Persons, 1995; Fanning and Cogger, 1998; Spathis, 2002; Kaminski et al., 2004; Kirkos et al., 2007; Ravisankar et al., 2011; Pai et al., 2011; Song et al., 2014).

2.2.4.5. Inventory to current assets

Higher levels of inventories relative to current assets may be a significant indicator of financial accounting fraud (Jofre, 2017). Kaminski et al. (2004) found that this ratio was statistically significant for the fraud year (the first year for which the financial statements included fraudulent data).

2.2.4.6. Cost of good sold to average inventory (Inventory turnover)

It measures the average number of the inventory is sold during the period. Inventory converts into expense immediately after a sale has occurred. However, in some cases it is not such a simple process, and managers may utilize from this situation and intentionally

record excess and obsolete inventory. This may lead to artificial high values of inventory, moreover, it may lead to low values of cost of good sold. High values of this ratio in a company increase the possibility of fraud risk in financial statements (Jofre, 2017). Kaminski et al. (2004), Kotsiantis et al. (2006), Liou (2008) and Chen (2016) used Cost of Good Sold to Average Inventory ratio as a significant indicator for detecting financial accounting fraud.

2.2.4.7. Accounts payable to cost of good sold

It explains how much the company has pending payments in terms of operating costs. Lower values of this ratio may indicate a clear sign of accounting fraud (Jofre, 2017). Since the increment of operating cash flow through unsustainable activities, especially boosting cash flow from operations by artificially decreasing outstanding accounts is a common technique which is used to commit accounting fraud (Schilit and Perler, 2010), and this ratio may be used for detecting this kind of fraud technique.

2.2.4.8. Net sales to total assets (Total asset turnover)

It indicates the power of a firm's assets to generate sales. It also measures the ability of management to cope with competitive situations. Sale-related financial items can be manipulated to deal with competitive situations by managers of fraud firms. Low values of this ratio in a company increase the possibility of fraud risk in financial statements (Persons, 1995). Therefore, it is important to use this ratio as a possible indicator of financial accounting fraud. Persons (1995), Fanning and Cogger (1998), Spathis (2002), Kotsiantis et al. (2006), Lenard et al. (2007), Kirkos et al. (2007), Chen (2016) and Omar et al. (2017) used Net Sales to Total Assets ratio as a significant indicator for detecting financial accounting fraud.

2.2.4.9. Net sales to average accounts receivable (Accounts receivable turnover)

Accounts receivable turnover ratio measures the ability of a company to efficiently collect its receivables. A firm with low values of this ratio may be more likely to manipulate its financial statements. Ozcan (2016) found that fraudulent firms have lower levels of accounts receivable turnover ratio than non fraudulent firms.

2.2.4.10. Cash conversion cycle

This ratio measures how quickly the firm can turn around to its cash to fulfill its financial obligations (Arshad et al., 2015). Cash conversion cycle is calculated by adding accounts receivables turnover days to inventory turnover days and subtracting trade payables turnover days (Chang, 2018). The cash conversion cycle ratio is used to assess the liquidity and efficiency of a company (Attari and Raza, 2012; Arshad et al., 2015). Ferdinand and Santosa (2018) used the percentage of cash conversion cycle per year to examine the relationship between liquidity and financial accounting fraud. They found that the ratio may be a significant indicator of fraud for detecting financial accounting fraud.

2.2.5. Growth

In order to maintain continuous growth may become a motivation for the managers of a company to commit fraud, and companies that grow rapidly are likely to engage in fraudulent activities (Stice, 1991; Song et al, 2014). Ratios associated with growth are defined below.

2.2.5.1. Growth of sales

It is measured as the percentage change in sales from period $(t-1)$ to period t , where period t is the financial year that the fraudulent activities occurred. This ratio may be a significant indicator of fraud that can provide utility for financial accounting fraud detection. Fanning and Cogger (1998), Kirkos et al. (2007), Chen (2016), and Yao et al. (2018) used Growth of Sales ratio as a significant indicator for detecting financial accounting fraud.

2.2.5.2. Growth of accounts receivables

It is measured as the percentage change in accounts receivables from period $(t-1)$ to period t , where period t is the financial year that fraudulent activities occurred. It should be tested for financial accounting fraud detection, since accounts receivable is more likely to be manipulated by management. Song et al. (2014) used Growth of Accounts Receivables ratio as a significant indicator for detecting financial accounting fraud.

2.2.5.3. Growth of inventory

It is measured as the percentage change in inventory from period $(t-1)$ to period t , where period t is the financial year that the fraudulent activities occurred. Song et al. (2014) used Growth of Inventory ratio as a significant indicator for detecting financial accounting fraud.

2.2.5.4. Growth of income from principal business operations

It is measured as the percentage change in income from principal business operations from period $(t-1)$ to period t , where period t is the financial year that the fraudulent activities occurred. Song et al. (2014) used this ratio as a significant indicator for detecting financial accounting fraud.

2.2.6. Other ratios

Financial ratios, which are classified in this group, are mostly manipulated by companies in practice, however, these ratios have not been considered in the literature. In this study, the importance of these ratios for detecting financial accounting fraud is examined.

Table 2.2 presents the financial ratios which are commonly used in previous researches for detecting financial accounting fraud. In the literature, however, cash ratio and cash conversion cycle ratio have not been examined for detecting financial accounting fraud.

2.3. Qualitative Characteristics as Evaluation Criteria for Assessing the Performance of Financial Ratios

Because low quality financial reporting comprises inaccurate, deceptive, or incomplete information, it may lead to financial accounting fraud. The manipulation of financial accounts by overstating assets, income, and profit or understating liabilities, costs, or losses is referred to as financial accounting fraud (Spathis et al., 2002; Omar et al., 2017). Deliberation of accounting records, falsification of transactions, or misapplication of accounting rules are all examples of financial accounting fraud (Pai et al., 2011). Low-quality financial reporting is related to financial accounting fraud for these reasons, and the measurement of the financial reporting quality is important for detecting financial accounting fraud.

Table 2.2. Financial ratios which are commonly used in previous researches to detect financial accounting fraud

Category	Financial Ratios	Authors
Liquidity	Current ratio	Kaminski et al., 2004; Kotsiantis et al., 2006; Kirkos et al., 2007; Lenard et al., 2007; Pai et al., 2011; Ravisankar et al., 2011; Song et al., 2014; Chen, 2016
	Quick ratio	Kirkos et al., 2007; Song et al., 2014; Chen, 2016; Yao et al., 2018
	Cash ratio	NA in literature
Leverage	Debt ratio	Persons, 1995; Spathis, 2002; Kaminski et al., 2004; Kotsiantis et al., 2006; Kirkos et al., 2007; Lenard et al., 2007; Pai et al., 2011; Song et al., 2014; Omar et al., 2017; Chen, 2016
	Debt-to-equity ratio	Spathis, 2002; Kaminski et al., 2004; Pai et al., 2011; Song et al., 2014; Chen, 2016
Efficiency	Inventory turnover	Kaminski et al., 2004; Kotsiantis et al., 2006; Liou, 2008; Chen, 2016
	Accounts Payable to Cost of Good Sold	Jofre, 2017
	Total asset turnover	Persons, 1995; Fanning and Cogger, 1998; Spathis, 2002; Kotsiantis et al., 2006; Lenard et al., 2007; Kirkos et al., 2007; Chen, 2016; Omar et al., 2017
	Accounts receivable turnover	Chen, 2016; Ozcan, 2016
	Cash Conversion Cycle ratio	NA in literature
	Accounts Receivable to Sales	Fanning and Cogger, 1998; Feroz et al., 2000; Spathis, 2002; Kirkos et al., 2007; Pai et al., 2011; Chen, 2016; Omar et al., 2017
	Accounts Receivable to Total Assets	Persons, 1995; Fanning and Cogger, 1998; Kaminski et al., 2004; Kotsiantis et al., 2006; Chen, 2016
	Inventory to Sales	Fanning and Cogger, 1998; Kaminski et al., 2004; Kirkos et al., 2007; Pai et al., 2011; Omar et al., 2017; Yao et al., 2018
	Inventory to Total Assets	Persons, 1995; Fanning and Cogger, 1998; Spathis, 2002; Kaminski et al., 2004; Kirkos et al., 2007; Pai et al., 2011; Ravisankar et al., 2011; Song et al., 2014
	Inventory to Current Assets	Kaminski et al., 2004; Jofre, 2017

Table 2.2. (Continuation)

Profitability	Return on Assets ratio	Persons, 1995; Spathis, 2002; Kaminski et al., 2004; Lenard et al., 2007; Pai et al., 2011; Song et al., 2014; Ravisankar et al., 2011; Omar et al., 2017; Chen, 2016; Yao et al., 2018
	Net profit margin ratio	Spathis et al., 2002; Kaminski et al., 2004; Pai et al., 2011; Omar et al., 2017
	EBIT to Total Assets	Kotsiantis et al., 2006
	Return on Equity ratio	Ozcan, 2016
Growth	Growth of accounts receivables	Song et al., 2014
	Growth of Inventory	Song et al., 2014
	Growth of income from principal business operations	Song et al., 2014
	Growth of sales	Fanning and Cogger, 1998; Kirkos et al., 2007; Chen, 2016; Yao et al., 2018
Other Ratios	Cash to Total Assets	
	Due from Shareholders to Total Assets	
	Due to Shareholders to Total Liabilities	
	Other Fixed Assets to Total Assets	
	Other Extraordinary Expenses and Losses to Net Sales	NA in literature
	Due to Shareholders to Average Inventory	
	Doubtful Trade Receivables to Total Assets	
	Prepaid Expenses for Future Months to Total Assets	
	Prepaid Expenses for Future Years to Total Assets	

The quality of financial reports can be measured by using qualitative characteristics of financial information embedded within the revised Conceptual Framework for Financial Reporting issued by International Accounting Standards Board (IASB, 2018). These qualitative characteristics are relevance, faithful representation, comparability, verifiability, timeliness and understandability. The Framework states that the qualitative characteristics are the attributes that make the information provided in financial statements useful to users.

Beest et al. (2009) constructed a measurement tool to comprehensively assess the quality of financial reporting by using these qualitative characteristics. Jonas and Blanchet (2000), Lee et al. (2002) and McDaniel et al. (2002) also utilized from qualitative

characteristics in order to assess the quality of financial reporting. Their researches indicate that qualitative characteristics can be used for measuring the quality of financial reporting. Due to the close relationship between fraudulent financial reporting and financial reporting quality, the qualitative characteristics, which are used for the measurement of the financial reporting quality, can be used as criteria for the selection of financial ratios. Table 2.3 indicates definition of qualitative characteristics which are used as criteria for assessing the performance of financial ratios.

Table 2.3. Definition of qualitative characteristics (International Accounting Standards Board, 2018; Herath and Albarqi, 2017)

Criteria	Definitions
Relevance	Relevance refers to the ability to influence how people make decisions.
Faithful Representation	The capability of financial statements to reflect and depict the company's actual economic status is referred to as faithful representation.
Comparability	Comparability refers to comparing a company's cash flow and performance with its financial statements in order to identify its financial status.
Verifiability	Verifiability refers to the accuracy with which financial statements reflect the company's actual financial status.
Timeliness	The provision of timely information to decision makers in order to affect their decisions is referred to as timeliness.
Understandability	Understandability refers to how well information is presented and categorised.

2.4. Turkish Accounting Standards

Due to the different accounting standards applied in the countries, the evaluation of the financial statements of the enterprises becomes difficult, and there are difficulties in international mergers and acquisitions. All these negativities have revealed the necessity of international accounting harmonization and international accounting standards to be developed and applied worldwide. Therefore, the International Accounting Standards Committee (IASC) was established and International Accounting Standards (IAS) were developed and published in 1973. It was foreseen that these published international accounting standards will be applied as common accounting standards around the world.

International Accounting Standards were published by the IASC from 1973 to 2001. In 2001, the International Accounting Standards Board (IASB) was established as an extension of IASC and has been publishing International Financial Reporting Standards

(IFRS) since 2001. In this context, the name of IAS was changed to IFRS. It was decided that the old standards will remain as IAS until they are abolished by another standard and the new standards will be issued under the name of IFRS (Yalkın et al., 2006; Korkmaz et al., 2007).

In our country, Turkish Accounting Standards (TAS), which are compatible with International Accounting Standards, are published by the Public Oversight, Accounting and Auditing Standards Authority. Within the framework of the license agreement made with the IASB, TAS refers to Turkish translations of IAS, whereas Turkish Financial Reporting Standards (TFRS) refers to IFRS. Therefore, there is no difference between TAS and TFRS in terms of binding (KPMG, 2022). The TAS/ IAS and the TFRS/IFRS issued in Turkey, as well as their subjects are presented in Table 2.4 and Table 2.5, respectively.

Financial Reporting Standard for Large and Medium Sized Entities (FRS for LMEs) was issued in 2018 to regulate the accounting rules to be applied in the preparation of the financial statements of companies that are subject to independent audit other than public interest organizations in Turkey. It was published to provide truthful, relevant and comparable information, in line with international financial reporting practices and EU regulations (Doğan, 2018).

Turkish Accounting Standards consist of TAS, TFRS, FRS for LMEs, and other decisions and regulations taken by the Board (KPMG, 2022). Financial reporting frameworks applied in Turkey are given in Figure 2.1.

The IASB published ‘‘IFRS for SMEs’’ in 2009 as a simplified form of the complete IFRS that addressed SMEs' financial reporting needs. The ‘‘IFRS for SMEs’’ was issued in Turkey in 2010 under the title ‘‘TFRS for SMEs’’. Turkey was one of the first countries to take significant efforts toward implementing ‘‘IFRS for SMEs’’, since SMEs constitute 99.8 percent of all enterprises, 72 percent of employment, 49.2 percent of personnel expenses, 49.4 percent of turnover, and 42.7 percent of total output value in Turkey (TSI, 2020). However, ‘‘TFRS for SMEs’’ is still not mandatory in Turkey. Financial reporting is often performed by SMEs for tax evasion. Furthermore, economic pressures on companies and a shortage of educated employees are substantial barriers to IFRS implementation (Kiliç and Uyar, 2017).

Table 2.4. Applied Turkish Accounting Standards adopted from International Accounting Standards (Public Oversight, Accounting and Auditing Standards Authority, 2022)

Name of the Standard	International Accounting Standards	Turkish Accounting Standards
Presentation of Financial Statements	IAS 1	TAS 1
Inventories	IAS 2	TAS 2
Statement of Cash Flows	IAS 7	TAS 7
Accounting Policies, Changes in Accounting Estimates and Errors	IAS 8	TAS 8
Events After the Reporting Period	IAS 10	TAS 10
Construction Contracts	IAS 11	TAS 11
Income Taxes	IAS 12	TAS 12
Property, Plant and Equipment	IAS 16	TAS 16
Leases	IAS 17	TAS 17
Revenue	IAS 18	TAS 18
Employee Benefits	IAS 19	TAS 19
Accounting for Government Grants and Disclosure of Government Assistance	IAS 20	TAS 20
The Effects of Changes in Foreign Exchange Rates	IAS 21	TAS 21
Borrowing Costs	IAS 23	TAS 23
Related Party Disclosures	IAS 24	TAS 24
Accounting and Reporting by Retirement Benefit Plans	IAS 26	TAS 26
Separate Financial Statements	IAS 27	TAS 27
Investments in Associates and Joint Ventures	IAS 28	TAS 28
Financial Reporting in Hyperinflationary Economies	IAS 29	TAS 29
Interests in Joint Ventures	IAS 31	TAS 31
Financial Instruments: Presentation	IAS 32	TAS 32
Earnings Per Share	IAS 33	TAS 33
Interim Financial Reporting	IAS 34	TAS 34
Impairment of Assets	IAS 36	TAS 36
Provisions, Contingent Liabilities and Contingent Assets	IAS 37	TAS 37
Intangible Assets	IAS 38	TAS 38
Financial Instruments: Recognition and Measurement	IAS 39	TAS 39
Investment Property	IAS 40	TAS 40
Agriculture	IAS 41	TAS 41

Table 2.5. Applied Turkish Financial Reporting Standards adopted from International Financial Reporting Standards (Public Oversight, Accounting and Auditing Standards Authority, 2022)

Name of the Standard	International Financial Reporting Standards	Turkish Financial Reporting Standards
First-time Adoption of International Financial Reporting Standards	IFRS 1	TFRS 1
Share-based Payment	IFRS 2	TFRS 2
Business Combinations	IFRS 3	TFRS 3
Non-current Assets Held for Sale and Discontinued Operations	IFRS 5	TFRS 5
Exploration for and Evaluation of Mineral Resources	IFRS 6	TFRS 6
Financial Instruments : Disclosures	IFRS 7	TFRS 7
Operating Segments	IFRS 8	TFRS 8
Financial Instruments	IFRS 9	TFRS 9
Consolidated Financial Statements	IFRS 10	TFRS 10
Joint Arrangements	IFRS 11	TFRS 11
Disclosure of Interests in Other	IFRS 12	TFRS 12
Insurance Contracts	IFRS 17	TFRS 17

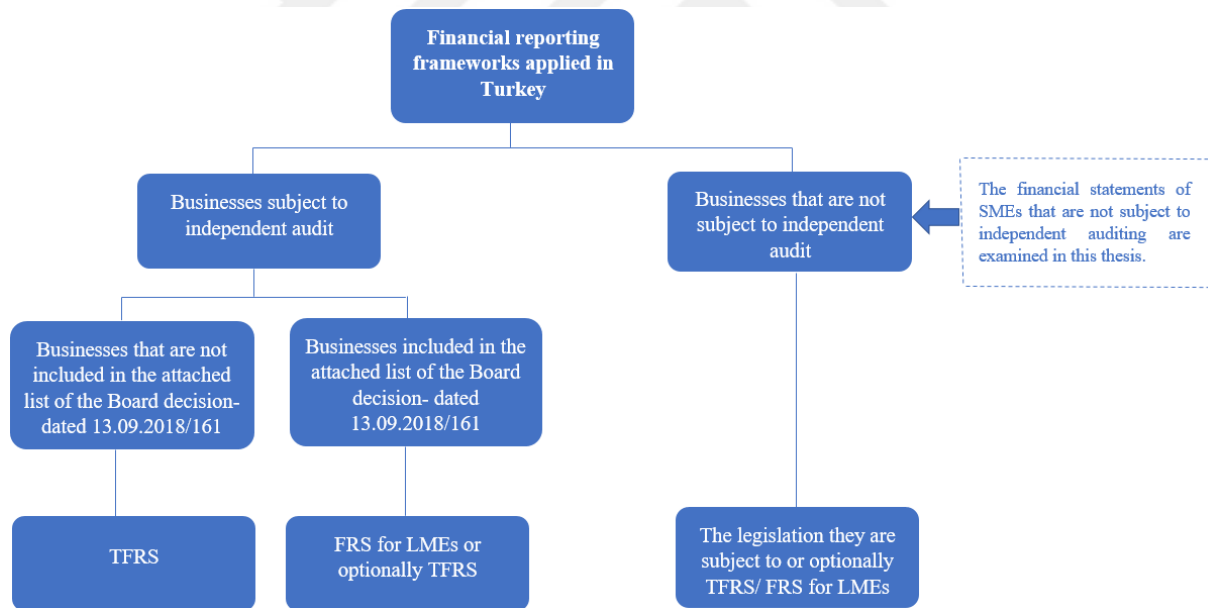


Figure 2.1. Financial reporting frameworks applied in Turkey (KPMG, 2022)

In this thesis, financial accounting fraud in the financial statements of SMEs that are not subject to independent auditing are examined. The majority of SMEs in Turkey do not have to apply “TFRS for SMEs” or “FRS for LMEs”. Thus, it is very possible to detect financial accounting fraud in the financial statements of SMEs that do not apply “TFRS for SMEs” or “FRS for LMEs”.

3. FUZZY SETS

To deal with uncertainty, ambiguity, and imprecision in decision-making process, Zadeh (1965) proposed fuzzy set theory. After Zadeh introduced the fuzzy set theory in 1965, numerous expansions including Type-2 fuzzy sets (Zadeh, 1975), Intuitionistic fuzzy sets (Atanassov, 1986), Neutrosophic sets (Smarandache, 1998), Hesitant fuzzy sets (Torra, 2010), Pythagorean fuzzy sets (Yager, 2013), Picture fuzzy sets (Cuong, 2014), Spherical fuzzy sets (Gündoğdu and Kahraman, 2018), and Circular intuitionistic fuzzy sets (Atanassov, 2020) have been developed to better cope with uncertainty.

In this study, three fuzzy sets, which are ordinary fuzzy sets (Type 1 fuzzy sets), spherical fuzzy sets and circular intuitionistic fuzzy sets, are applied for ranking financial ratios in financial accounting fraud detection.

3.1. Ordinary Fuzzy Sets

Zadeh (1965) introduced ordinary fuzzy sets as the first type of fuzzy sets to the literature. Such fuzzy sets are defined by a membership function which assigns to each object a membership degree ranging between zero and one. Let a fuzzy subset A of a universe of the domain U is defined by a membership function $\mu_{A(x)}:U \rightarrow [0,1]$, where $\mu_{A(x)}$ represents the membership function of U in A .

The simplest and commonly used membership function is represented by a triangular shape. Here, the fuzzy number is defined as triplet of values (l, m, u) , which represent the lower value, the medium value, and the upper value.

3.2. Spherical Fuzzy Sets

Spherical fuzzy sets (SFS) have been developed by Gündoğdu and Kahraman (2018, 2019c) as a novel extension of pythagorean, neutrosophic, and intuitionistic fuzzy sets. A spherical surface is used to define a membership function in SFS, and the membership function's parameters are allocated to a wider domain. The squared sum of membership, non-membership, and hesitation degrees should be between zero and one. Decision makers can define membership, non-membership, and hesitant degrees independently, thus it gives them additional flexibility and accuracy. The following

subsections provide detailed explanations of the Spherical fuzzy sets used in this study.

3.2.1. Single-Valued Spherical Fuzzy Sets

Gündoğdu and Kahraman (2018,2019c) introduced Single-Valued Spherical Fuzzy Sets (SVSFS), which incorporates three-dimensional fuzzy sets built as extensions of Intuitionistic fuzzy sets, Pythagorean fuzzy sets, and Neutrosophic sets. SVSFS comprises membership $(\mu_S(x))$, non-membership $(v_S(x))$, and hesitancy parameters $(\pi_S(x))$. Each of membership, non-membership and hesitancy parameters is defined as a value between 0 and 1, and the squared sum of these parameters can be calculated between 0 and 1. Figure 3.1 depicts the membership functions of Intuitionistic Fuzzy Sets, Pythagorean Fuzzy Sets, Neutrosophic Sets, and Spherical Fuzzy Sets in a three-dimensional graphical form.

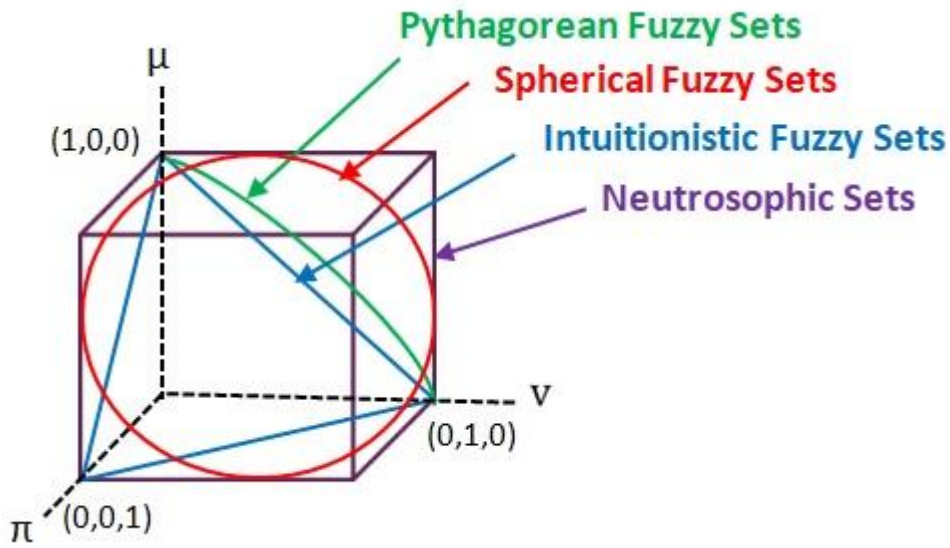


Figure 3.1. The membership functions of Intuitionistic Fuzzy Sets, Pythagorean Fuzzy Sets, Neutrosophic Sets and Spherical Fuzzy Sets

Different SVSFS arithmetic operations are needed when using SVSFS on a decision-making problem. The definition of SVSFS, arithmetic, aggregation and defuzzification operations are provided in the following.

Definition 1. Let A be a set of the universe of discourse. SVSFS $\tilde{S}$ is given by:

$$\tilde{S} = \{ \langle \alpha, (\mu_{\tilde{S}}(\alpha), v_{\tilde{S}}(\alpha), \pi_{\tilde{S}}(\alpha)) \mid \alpha \in A \rangle \} \quad (3.1)$$

where

$$\mu_{\tilde{S}}:A \rightarrow [0,1], \quad v_{\tilde{S}}:A \rightarrow [0,1], \quad \text{and} \quad \pi_{\tilde{S}}:A \rightarrow [0,1]$$

satisfy the following condition

$$0 \leq \mu_{\tilde{S}}^2(\alpha) + v_{\tilde{S}}^2(\alpha) + \pi_{\tilde{S}}^2(\alpha) \leq 1 \quad \forall \alpha \in A \quad (3.2)$$

The values $\mu_{\tilde{S}}(\alpha)$, $v_{\tilde{S}}(\alpha)$ and $\pi_{\tilde{S}}(\alpha)$ denote membership, non-membership and hesitancy degrees of α to $\tilde{S}$, respectively.

Definition 2. Basic operations of SVSFS are given as follows:

Union;

$$\tilde{S}_1 \cup \tilde{S}_2 = \left\{ \max \{ \mu_{\tilde{S}_1}, \mu_{\tilde{S}_2} \}, \min \{ v_{\tilde{S}_1}, v_{\tilde{S}_2} \}, \min \left\{ 1 - \left(\left(\max \{ \mu_{\tilde{S}_1}, \mu_{\tilde{S}_2} \} \right)^2 + \left(\min \{ v_{\tilde{S}_1}, v_{\tilde{S}_2} \} \right)^2 \right), \max \{ \pi_{\tilde{S}_1}, \pi_{\tilde{S}_2} \} \right\} \right\} \quad (3.3)$$

Intersection;

$$\tilde{S}_1 \cap \tilde{S}_2 = \left\{ \min \{ \mu_{\tilde{S}_1}, \mu_{\tilde{S}_2} \}, \max \{ v_{\tilde{S}_1}, v_{\tilde{S}_2} \}, \min \left\{ 1 - \left(\left(\min \{ \mu_{\tilde{S}_1}, \mu_{\tilde{S}_2} \} \right)^2 + \left(\max \{ v_{\tilde{S}_1}, v_{\tilde{S}_2} \} \right)^2 \right), \max \{ \pi_{\tilde{S}_1}, \pi_{\tilde{S}_2} \} \right\} \right\} \quad (3.4)$$

Addition;

$$\tilde{S}_1 \oplus \tilde{S}_2 = \left\{ \left(\mu_{\tilde{S}_1}^2 + \mu_{\tilde{S}_2}^2 - \mu_{\tilde{S}_1}^2 \mu_{\tilde{S}_2}^2 \right)^{1/2}, v_{\tilde{S}_1} v_{\tilde{S}_2}, \left(\left(1 - \mu_{\tilde{S}_1}^2 \right) \pi_{\tilde{S}_1}^2 + \left(1 - \mu_{\tilde{S}_2}^2 \right) \pi_{\tilde{S}_2}^2 - \pi_{\tilde{S}_1}^2 \pi_{\tilde{S}_2}^2 \right)^{1/2} \right\} \quad (3.5)$$

Multiplication;

$$\tilde{S}_1 \otimes \tilde{S}_2 = \left\{ \mu_{\tilde{S}_1} \mu_{\tilde{S}_2}, \left(v_{\tilde{S}_1}^2 + v_{\tilde{S}_2}^2 - v_{\tilde{S}_1}^2 v_{\tilde{S}_2}^2 \right)^{1/2}, \left(\left(1 - v_{\tilde{S}_1}^2 \right) \pi_{\tilde{S}_1}^2 + \left(1 - v_{\tilde{S}_2}^2 \right) \pi_{\tilde{S}_2}^2 - \pi_{\tilde{S}_1}^2 \pi_{\tilde{S}_2}^2 \right)^{1/2} \right\} \quad (3.6)$$

Multiplication by a scalar; $\lambda > 0$

$$\lambda \tilde{S} = \left\{ \left(1 - \left(1 - \mu_{\tilde{S}}^2 \right)^\lambda \right)^{1/2}, v_{\tilde{S}}^\lambda, \left(\left(1 - \mu_{\tilde{S}}^2 \right)^\lambda - \left(1 - \mu_{\tilde{S}}^2 - \pi_{\tilde{S}}^2 \right)^\lambda \right)^{1/2} \right\} \quad (3.7)$$

λ **Power of $\tilde{S}$** ; $\lambda > 0$

$$\tilde{S}^\lambda = \left\{ \mu_{\tilde{S}}^\lambda, \left(1 - (1 - v_{\tilde{S}}^2)^\lambda\right)^{1/2}, \left((1 - v_{\tilde{S}}^2)^\lambda - (1 - v_{\tilde{S}}^2 - \pi_{\tilde{S}}^2)^\lambda\right)^{1/2} \right\} \quad (3.8)$$

Definition 3. Considering the condition $\lambda, \lambda_1, \lambda_2 > 0$, some operations are given in the following for these SVSFS $\tilde{S}_1 = (\mu_{\tilde{S}_1}, v_{\tilde{S}_1}, \pi_{\tilde{S}_1})$ and $\tilde{S}_2 = (\mu_{\tilde{S}_2}, v_{\tilde{S}_2}, \pi_{\tilde{S}_2})$.

$$\text{i.} \quad \tilde{S}_1 \oplus \tilde{S}_2 = \tilde{S}_2 \oplus \tilde{S}_1 \quad (3.9)$$

$$\text{ii.} \quad \tilde{S}_1 \otimes \tilde{S}_2 = \tilde{S}_2 \otimes \tilde{S}_1 \quad (3.10)$$

$$\text{iii.} \quad \lambda(\tilde{S}_1 \oplus \tilde{S}_2) = \lambda \tilde{S}_1 \oplus \lambda \tilde{S}_2 \quad (3.11)$$

$$\text{iv.} \quad \lambda_1 \tilde{S} \oplus \lambda_2 \tilde{S} = (\lambda_1 + \lambda_2) \tilde{S} \quad (3.12)$$

$$\text{v.} \quad (\tilde{S}_1 \otimes \tilde{S}_2)^\lambda = \tilde{S}_1^\lambda \otimes \tilde{S}_2^\lambda \quad (3.13)$$

$$\text{vi.} \quad \tilde{S}^{\lambda_1} \otimes \tilde{S}^{\lambda_2} = \tilde{S}^{\lambda_1 + \lambda_2} \quad (3.14)$$

Definition 4. Single-valued Spherical Weighted Arithmetic Mean (SWAM),

where $w = (w_1, w_2, \dots, w_n)$; $w_i \in [0, 1]$; $\sum_{i=1}^n w_i = 1$,

SWAM is identified as;

$$\begin{aligned} \text{SWAM}_w(\tilde{S}_1, \dots, \tilde{S}_n) &= w_1 \tilde{S}_1 + w_2 \tilde{S}_2 + \dots + w_n \tilde{S}_n \\ &= \left\{ \left[1 - \prod_{i=1}^n (1 - \mu_{\tilde{S}_i}^2)^{w_i} \right]^{1/2}, \prod_{i=1}^n v_{\tilde{S}_i}^{w_i}, \left[\prod_{i=1}^n (1 - \mu_{\tilde{S}_i}^2)^{w_i} - \prod_{i=1}^n (1 - \mu_{\tilde{S}_i}^2 - \pi_{\tilde{S}_i}^2)^{w_i} \right]^{1/2} \right\} \end{aligned} \quad (3.15)$$

Definition 5. Single-valued Spherical Weighted Geometric Mean (SWGM),

where $w = (w_1, w_2, \dots, w_n)$; $w_i \in [0, 1]$; $\sum_{i=1}^n w_i = 1$,

SWGM is defined as;

$$\begin{aligned} \text{SWGM}_w(\tilde{S}_1, \dots, \tilde{S}_n) &= \tilde{S}_1^{w_1} + \tilde{S}_2^{w_2} + \dots + \tilde{S}_n^{w_n} \\ &= \left\{ \prod_{i=1}^n \mu_{\tilde{S}_i}^{w_i}, \left[1 - \prod_{i=1}^n (1 - v_{\tilde{S}_i}^2)^{w_i} \right]^{1/2}, \left[\prod_{i=1}^n (1 - v_{\tilde{S}_i}^2)^{w_i} - \prod_{i=1}^n (1 - v_{\tilde{S}_i}^2 - \pi_{\tilde{S}_i}^2)^{w_i} \right]^{1/2} \right\} \end{aligned} \quad (3.16)$$

Definition 6. Score function and Accuracy function of SVSFS are given in the following.

$$\text{Score}(\tilde{S}) = (\mu_{\tilde{S}} - \pi_{\tilde{S}})^2 - (v_{\tilde{S}} - \pi_{\tilde{S}})^2 \quad (3.17)$$

$$\text{Accuracy}(\tilde{S}) = \mu_{\tilde{S}}^2 + v_{\tilde{S}}^2 + \pi_{\tilde{S}}^2 \quad (3.18)$$

Note that: $\tilde{S}_1 < \tilde{S}_2$ if and only if

$$\text{Score}(\tilde{S}_1) < \text{Score}(\tilde{S}_2) \quad \text{or}$$

$$\text{Score}(\tilde{S}_1) = \text{Score}(\tilde{S}_2) \quad \text{and} \quad \text{Accuracy}(\tilde{S}_1) < \text{Accuracy}(\tilde{S}_2).$$

3.2.2. Interval-Valued Spherical Fuzzy Sets

Interval-valued spherical fuzzy sets have been proposed by Gündoğdu and Kahraman (2019a) to consider opinions of decision makers on parameters into the model with an interval rather than a single point. The definition of IVSFS, arithmetic, aggregation and defuzzification operations are provided in the following.

Definition 1. Let A be a set of the universe of discourse. IVSFS $\tilde{S}$ is given by:

$$\tilde{S} = \{ \langle \alpha, ([\mu_{\tilde{S}}^L(\alpha), \mu_{\tilde{S}}^U(\alpha)], [v_{\tilde{S}}^L(\alpha), v_{\tilde{S}}^U(\alpha)], [\pi_{\tilde{S}}^L(\alpha), \pi_{\tilde{S}}^U(\alpha)]) \rangle \mid \alpha \in A \} \quad (3.19)$$

$$\text{where } 0 \leq \mu_{\tilde{S}}^L(\alpha) \leq \mu_{\tilde{S}}^U(\alpha) \leq 1, 0 \leq v_{\tilde{S}}^L(\alpha) \leq v_{\tilde{S}}^U(\alpha) \leq 1 \text{ and}$$

$$0 \leq (\mu_{\tilde{S}}^U(\alpha))^2 + (v_{\tilde{S}}^U(\alpha))^2 + (\pi_{\tilde{S}}^U(\alpha))^2 \leq 1.$$

For each $\alpha \in A$, the values $\mu_{\tilde{S}}^U(\alpha)$, $v_{\tilde{S}}^U(\alpha)$ and $\pi_{\tilde{S}}^U(\alpha)$ denote the upper degrees of membership, the upper degrees of non-membership and the upper degrees of hesitancy of α to $\tilde{S}$, respectively. If $\mu_{\tilde{S}}^L(\alpha) = \mu_{\tilde{S}}^U(\alpha)$, $v_{\tilde{S}}^L(\alpha) = v_{\tilde{S}}^U(\alpha)$ and $\pi_{\tilde{S}}^L(\alpha) = \pi_{\tilde{S}}^U(\alpha)$ then, IVSFS $\tilde{S}$ reduces to a SVSFS.

For an IVSFS $\tilde{S}$, the pair $\langle [\mu_{\tilde{S}}^L(\alpha), \mu_{\tilde{S}}^U(\alpha)], [v_{\tilde{S}}^L(\alpha), v_{\tilde{S}}^U(\alpha)], [\pi_{\tilde{S}}^L(\alpha), \pi_{\tilde{S}}^U(\alpha)] \rangle$ is called an IVSFS number. For simplicity, an IVSFS number is represented by $\tilde{S} = \langle [\tilde{x}_1, \tilde{x}_2], [\tilde{x}_3, \tilde{x}_4], [\tilde{x}_5, \tilde{x}_6] \rangle$ where $[\tilde{x}_1, \tilde{x}_2] \subset [0, 1]$, $[\tilde{x}_3, \tilde{x}_4] \subset [0, 1]$, $[\tilde{x}_5, \tilde{x}_6] \subset [0, 1]$ and $\tilde{x}_2^2 + \tilde{x}_4^2 + \tilde{x}_6^2 \leq 1$.

Definition 2. Basic operations of IVSFS are given as follows:

$$\text{Let } \tilde{S}_1 = \langle [\tilde{x}_1, \tilde{x}_2], [\tilde{x}_3, \tilde{x}_4], [\tilde{x}_5, \tilde{x}_6] \rangle \text{ and } \tilde{S}_2 = \langle [\tilde{y}_1, \tilde{y}_2], [\tilde{y}_3, \tilde{y}_4], [\tilde{y}_5, \tilde{y}_6] \rangle$$

Union;

$$\tilde{S}_1 \cup \tilde{S}_2 = \{[\max\{\tilde{x}_1, \tilde{y}_1\}, \max\{\tilde{x}_2, \tilde{y}_2\}], [\min\{\tilde{x}_3, \tilde{y}_3\}, \min\{\tilde{x}_4, \tilde{y}_4\}], [\min\{\tilde{x}_5, \tilde{y}_5\}, \min\{\tilde{x}_6, \tilde{y}_6\}]\} \quad (3.20)$$

Intersection;

$$\tilde{S}_1 \cap \tilde{S}_2 = \{[\min\{\tilde{x}_1, \tilde{y}_1\}, \min\{\tilde{x}_2, \tilde{y}_2\}], [\max\{\tilde{x}_3, \tilde{y}_3\}, \max\{\tilde{x}_4, \tilde{y}_4\}], [\min\{\tilde{x}_5, \tilde{y}_5\}, \min\{\tilde{x}_6, \tilde{y}_6\}]\} \quad (3.21)$$

Addition;

$$\begin{aligned} \tilde{S}_1 \oplus \tilde{S}_2 = & [((\tilde{x}_1)^2 + (\tilde{y}_1)^2 - (\tilde{x}_1)^2(\tilde{y}_1)^2)^{1/2}, ((\tilde{x}_2)^2 + (\tilde{y}_2)^2 - (\tilde{x}_2)^2(\tilde{y}_2)^2)^{1/2}], [\tilde{x}_3\tilde{y}_3, \tilde{x}_4\tilde{y}_4], \\ & \left[\left((1 - (\tilde{y}_1)^2)(\tilde{x}_5)^2 + (1 - (\tilde{x}_1)^2)(\tilde{y}_5)^2 - (\tilde{x}_5)^2(\tilde{y}_5)^2 \right)^{1/2}, \left((1 - (\tilde{y}_2)^2)(\tilde{x}_6)^2 + (1 - (\tilde{x}_2)^2)(\tilde{y}_6)^2 - \right. \right. \\ & \left. \left. (\tilde{x}_6)^2(\tilde{y}_6)^2 \right)^{1/2} \right] \end{aligned} \quad (3.22)$$

Multiplication;

$$\begin{aligned} \tilde{S}_1 \otimes \tilde{S}_2 = & [\tilde{x}_1\tilde{y}_1, \tilde{x}_2\tilde{y}_2], [((\tilde{x}_3)^2 + (\tilde{y}_3)^2 - (\tilde{x}_3)^2(\tilde{y}_3)^2)^{1/2}, ((\tilde{x}_4)^2 + (\tilde{y}_4)^2 - (\tilde{x}_4)^2(\tilde{y}_4)^2)^{1/2}], \\ & \left[\left((1 - (\tilde{y}_3)^2)(\tilde{x}_5)^2 + (1 - (\tilde{x}_3)^2)(\tilde{y}_5)^2 - (\tilde{x}_5)^2(\tilde{y}_5)^2 \right)^{1/2}, \left((1 - (\tilde{y}_4)^2)(\tilde{x}_6)^2 + (1 - (\tilde{x}_4)^2)(\tilde{y}_6)^2 - \right. \right. \\ & \left. \left. (\tilde{x}_6)^2(\tilde{y}_6)^2 \right)^{1/2} \right] \end{aligned} \quad (3.23)$$

Multiplication by a scalar; $\lambda > 0$

$$\begin{aligned} \lambda \cdot \tilde{S} = & \left\{ \left[\left(1 - (1 - \tilde{x}_1^2)^\lambda \right)^{1/2}, \left(1 - (1 - \tilde{x}_2^2)^\lambda \right)^{1/2} \right], [\tilde{x}_3^\lambda, \tilde{x}_4^\lambda], \left[\left((1 - \tilde{x}_1^2)^\lambda - (1 - \tilde{x}_1^2 - \right. \right. \right. \\ & \left. \left. \tilde{x}_5^2)^\lambda \right)^{1/2}, \left((1 - \tilde{x}_2^2)^\lambda - (1 - \tilde{x}_2^2 - \tilde{x}_6^2)^\lambda \right)^{1/2} \right] \} \end{aligned} \quad (3.24)$$

λ th Power of $\tilde{S}$; $\lambda > 0$

$$\begin{aligned} \tilde{S}^\lambda = & \left\{ [\tilde{x}_1^\lambda, \tilde{x}_2^\lambda], \left[\left(1 - (1 - \tilde{x}_3^2)^\lambda \right)^{1/2}, \left(1 - (1 - \tilde{x}_4^2)^\lambda \right)^{1/2} \right], \left[\left((1 - \tilde{x}_3^2)^\lambda - (1 - \tilde{x}_3^2 - \right. \right. \right. \\ & \left. \left. \tilde{x}_5^2)^\lambda \right)^{1/2}, \left((1 - \tilde{x}_4^2)^\lambda - (1 - \tilde{x}_4^2 - \tilde{x}_6^2)^\lambda \right)^{1/2} \right] \} \end{aligned} \quad (3.25)$$

Definition 3. Let $\lambda, \lambda_1, \lambda_2 \geq 0$, then

$$\text{i.} \quad \tilde{S}_1 \oplus \tilde{S}_2 = \tilde{S}_2 \oplus \tilde{S}_1 \quad (3.26)$$

$$\text{ii.} \quad \tilde{S}_1 \otimes \tilde{S}_2 = \tilde{S}_2 \otimes \tilde{S}_1 \quad (3.27)$$

$$\text{iii.} \quad \lambda(\tilde{S}_1 \oplus \tilde{S}_2) = \lambda\tilde{S}_1 \oplus \lambda\tilde{S}_2 \quad (3.28)$$

$$\text{iv.} \quad \lambda_1.\tilde{S} \oplus \lambda_2.\tilde{S} = (\lambda_1 + \lambda_2).\tilde{S} \quad (3.29)$$

$$\text{v.} \quad (\tilde{S}_1 \otimes \tilde{S}_2)^\lambda = \tilde{S}_1^\lambda \otimes \tilde{S}_2^\lambda \quad (3.30)$$

$$\text{vi.} \quad \tilde{S}^{\lambda_1} \otimes \tilde{S}^{\lambda_2} = \tilde{S}^{\lambda_1 + \lambda_2} \quad (3.31)$$

Definition 4. Let $\tilde{S}_j = \langle [\tilde{x}_{1j}, \tilde{x}_{2j}], [\tilde{x}_{3j}, \tilde{x}_{4j}], [\tilde{x}_{5j}, \tilde{x}_{6j}] \rangle$ be a IVSFS number. Interval-valued Spherical Weighted Arithmetic Mean (IVSWAM) with respect to, $w_j = (w_1, w_2, \dots, w_n)$; $w_j \in [0, 1]$; $\sum_{j=1}^n w_j = 1$, is identified as;

$$\begin{aligned} \text{IVSWAM}_w(\tilde{S}_1, \dots, \tilde{S}_n) &= w_1\tilde{S}_1 \oplus w_2\tilde{S}_2 \oplus \dots \oplus w_n\tilde{S}_n \\ &= \left\{ \left[\left((1 - \prod_{j=1}^n (1 - \tilde{x}_{1j}^2)^{w_j}) \right)^{1/2}, \left((1 - \prod_{j=1}^n (1 - \tilde{x}_{2j}^2)^{w_j}) \right)^{1/2} \right], \left[\prod_{j=1}^n \tilde{x}_{3j}^{w_j}, \prod_{j=1}^n \tilde{x}_{4j}^{w_j} \right], \left[\left(\prod_{j=1}^n (1 - \tilde{x}_{1j}^2)^{w_j} - \prod_{j=1}^n (1 - \tilde{x}_{1j}^2 - \tilde{x}_{5j}^2)^{w_j} \right)^{1/2}, \left(\prod_{j=1}^n (1 - \tilde{x}_{2j}^2)^{w_j} - \prod_{j=1}^n (1 - \tilde{x}_{2j}^2 - \tilde{x}_{6j}^2)^{w_j} \right)^{1/2} \right] \right\} \end{aligned} \quad (3.32)$$

Definition 5. Let $\tilde{S}_j = \langle [\tilde{x}_{1j}, \tilde{x}_{2j}], [\tilde{x}_{3j}, \tilde{x}_{4j}], [\tilde{x}_{5j}, \tilde{x}_{6j}] \rangle$ be a IVSFS number. Interval-valued Spherical Geometric Mean (IVSWGGM) with respect to, $w_j = (w_1, w_2, \dots, w_n)$; $w_j \in [0, 1]$; $\sum_{j=1}^n w_j = 1$, IVSWGGM is identified as;

$$\begin{aligned} \text{IVSWGGM}_w(\tilde{S}_1, \dots, \tilde{S}_n) &= \tilde{S}_1^{w_1} \otimes \tilde{S}_2^{w_2} \otimes \dots \otimes \tilde{S}_n^{w_n} \\ &= \left\{ \left[\prod_{j=1}^n \tilde{x}_{1j}^{w_j}, \prod_{j=1}^n \tilde{x}_{2j}^{w_j} \right], \left[\left((1 - \prod_{j=1}^n (1 - \tilde{x}_{3j}^2)^{w_j} \right)^{1/2}, \left((1 - \prod_{j=1}^n (1 - \tilde{x}_{4j}^2)^{w_j} \right)^{1/2} \right], \left[\left(\prod_{j=1}^n (1 - \tilde{x}_{3j}^2)^{w_j} - \prod_{j=1}^n (1 - \tilde{x}_{3j}^2 - \tilde{x}_{5j}^2)^{w_j} \right)^{1/2}, \left(\prod_{j=1}^n (1 - \tilde{x}_{4j}^2)^{w_j} - \prod_{j=1}^n (1 - \tilde{x}_{4j}^2 - \tilde{x}_{6j}^2)^{w_j} \right)^{1/2} \right] \right\} \end{aligned} \quad (3.33)$$

Definition 6. The score function of IVSFS number is identified in the following.

$$\text{Score}(\tilde{S}) = S(\tilde{S}) = \frac{\tilde{x}_1^2 + \tilde{x}_2^2 - \tilde{x}_3^2 - \tilde{x}_4^2 - (\tilde{x}_5/2)^2 - (\tilde{x}_6/2)^2}{2} \quad (3.34)$$

where $\text{Score}(\tilde{S}) = S(\tilde{S}) \in [-1, +1]$.

Definition 7. The accuracy function of IVSFS number is identified in the following.

$$\text{Accuracy}(\tilde{S}) = H(\tilde{S}) = \frac{\tilde{x}_1^2 + \tilde{x}_2^2 + \tilde{x}_3^2 + \tilde{x}_4^2 + \tilde{x}_5^2 + \tilde{x}_6^2}{2} \quad (3.35)$$

where $H(\tilde{S}) \in [0, 1]$.

Note that: $\tilde{S}_1 < \tilde{S}_2$ if and only if

$$S(\tilde{S}_1) < S(\tilde{S}_2) \text{ or } S(\tilde{S}_1) = S(\tilde{S}_2) \text{ and } H(\tilde{S}_1) < H(\tilde{S}_2).$$

3.3. Circular Intuitionistic Fuzzy Sets

Circular intuitionistic fuzzy set (C-IFS), which was introduced by Atanassov (2020) as an extension of intuitionistic fuzzy sets, is represented by a circle of each element specified with membership degree and non-membership degree.

A C-IFS is defined as in the following.

Definition 1: Let C be a subset of a fixed universe called A , and α is an element of A .

$$C = \{ \langle \alpha, \mu_C(\alpha), \nu_C(\alpha); r \rangle \mid \alpha \in A \} \quad \text{where}$$

$$0 \leq \mu_C(\alpha) + \nu_C(\alpha) \leq 1 \text{ and } r \in [0, 1] \text{ for } \forall \alpha \in A \quad (3.36)$$

$\mu_C: A \rightarrow [0, 1]$ denote membership degree and $\nu_C: A \rightarrow [0, 1]$ indicate non-membership degree of the element $\alpha \in A$ to the set C . $r: A \rightarrow [0, 1]$ is named as a radius of the circle around each element.

The degree of indeterminacy (uncertainty) is described in the following.

$$\pi_C(\alpha) = 1 - \mu_C(\alpha) - \nu_C(\alpha) \quad (3.37)$$

Each element in regular IFSs is represented by a point in the intuitionistic fuzzy interpretation triangle, however in C-IFSs, each element is defined by a circle with center $\langle \mu_C(x), \nu_C(x) \rangle$, and radius r . Figure 3.2 illustrates the fundamental geometric depiction of C-IFS. Figure 3.2 also shows five different shapes the circles can be formed.

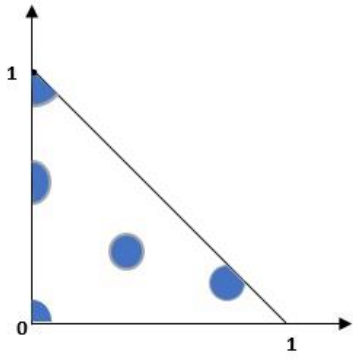


Figure 3.2. Fundamental geometric depiction of C-IFSs

$$\text{Let } L^* = \{\langle x, y \rangle | x, y \in [0, 1] \text{ \& } x + y \leq 1\} \quad (3.38)$$

Thus, C_r can be represented in the following.

$$C_r^* = \{\langle \alpha, O_r(\mu_C(\alpha), \nu_C(\alpha)); r \rangle | \alpha \in A\} \quad (3.39)$$

where

$$O_r(\mu_C(\alpha), \nu_C(\alpha)) = \{\langle x, y \rangle | x, y \in [0, 1], \sqrt{(\mu_C(\alpha) - x)^2 + (\nu_C(\alpha) - y)^2} \leq r\} \cap L^* = \{\langle x, y \rangle | x, y \in [0, 1], \sqrt{(\mu_C(\alpha) - x)^2 + (\nu_C(\alpha) - y)^2} \leq r, x + y \leq 1\}.$$

Each regular IFS can be denoted as $C = C_0 = \{\langle \alpha, O_r(\mu_C(\alpha), \nu_C(\alpha)); 0 \rangle | \alpha \in A\}$, however C-IFS with $r > 0$ can't be represented by a regular IFS.

Definition 2: Let a set of intuitionistic fuzzy pairs have the form $\{\langle \mu_{i,1}, \nu_{i,1} \rangle, \langle \mu_{i,2}, \nu_{i,2} \rangle, \dots\}$.

To get C-IFS, first compute the arithmetic average of the intuitionistic fuzzy pairs in the following.

$$\langle \mu(C_i), \nu(C_i) \rangle = \left\langle \frac{\sum_{j=1}^{k_i} \mu_{i,j}}{n_i}, \frac{\sum_{j=1}^{k_i} \nu_{i,j}}{n_i} \right\rangle \quad (3.40)$$

where n_i is the number of intuitionistic fuzzy pairs.

Then radius of the $\langle \mu(C_i), \nu(C_i) \rangle$ is computed as follows:

$$r_i = \max_{1 \leq j \leq n_i} \sqrt{(\mu(C_i) - \mu_{i,j})^2 + (\nu(C_i) - \nu_{i,j})^2} \quad (3.41)$$

For universe $C = \{C_1, C_2, \dots\}$, the C-IFS can be defined as follows:

$$C_r = \{\langle C_i, \mu(C_i), \nu(C_i); r \rangle \mid C_i \in C\} = \{\langle C_i, O_r(\mu(C_i), \nu(C_i)) \rangle \mid C_i \in C\} \quad (3.42)$$

Definition 3: Let $C_a = \langle \mu_{C_a}(\alpha), \nu_{C_a}(\alpha); r_a \rangle$ and $C_b = \langle \mu_{C_b}(\alpha), \nu_{C_b}(\alpha); r_b \rangle$ be two circular intuitionistic fuzzy numbers (C-IFNs). For the C-IFNs, some arithmetic operations are presented in the following.

$$C_a \cap_{\min} C_b =$$

$$\{\langle \alpha, \min(\mu_{C_a}(\alpha), \mu_{C_b}(\alpha)), \max(\nu_{C_a}(\alpha), \nu_{C_b}(\alpha)); \min(r_a, r_b) \rangle \mid \alpha \in A\} \quad (3.43)$$

$$C_a \cap_{\max} C_b =$$

$$\{\langle \alpha, \min(\mu_{C_a}(\alpha), \mu_{C_b}(\alpha)), \max(\nu_{C_a}(\alpha), \nu_{C_b}(\alpha)); \max(r_a, r_b) \rangle \mid \alpha \in A\} \quad (3.44)$$

$$C_a \cup_{\min} C_b =$$

$$\{\langle \alpha, \max(\mu_{C_a}(\alpha), \mu_{C_b}(\alpha)), \min(\nu_{C_a}(\alpha), \nu_{C_b}(\alpha)); \min(r_a, r_b) \rangle \mid \alpha \in A\} \quad (3.45)$$

$$C_a \cup_{\max} C_b =$$

$$\{\langle \alpha, \max(\mu_{C_a}(\alpha), \mu_{C_b}(\alpha)), \min(\nu_{C_a}(\alpha), \nu_{C_b}(\alpha)); \max(r_a, r_b) \rangle \mid \alpha \in A\} \quad (3.46)$$

$$C_a \oplus_{\min} C_b = \{\langle \alpha, \mu_{C_a}(\alpha) + \mu_{C_b}(\alpha) - \mu_{C_a}(\alpha) * \mu_{C_b}(\alpha), \nu_{C_a}(\alpha) * \nu_{C_b}(\alpha); \min(r_a, r_b) \rangle \mid \alpha \in A\} \quad (3.47)$$

$$C_a \oplus_{\max} C_b = \{\langle \alpha, \mu_{C_a}(\alpha) + \mu_{C_b}(\alpha) - \mu_{C_a}(\alpha) * \mu_{C_b}(\alpha), \nu_{C_a}(\alpha) * \nu_{C_b}(\alpha); \max(r_a, r_b) \rangle \mid \alpha \in A\} \quad (3.48)$$

$$C_a \otimes_{\min} C_b = \{\langle \alpha, \mu_{C_a}(\alpha) * \mu_{C_b}(\alpha), \nu_{C_a}(\alpha) + \nu_{C_b}(\alpha) - \nu_{C_a}(\alpha) * \nu_{C_b}(\alpha); \min(r_a, r_b) \rangle \mid \alpha \in A\} \quad (3.49)$$

$$C_a \otimes_{\max} C_b = \{\langle \alpha, \mu_{C_a}(\alpha) * \mu_{C_b}(\alpha), \nu_{C_a}(\alpha) + \nu_{C_b}(\alpha) - \nu_{C_a}(\alpha) * \nu_{C_b}(\alpha); \max(r_a, r_b) \rangle \mid \alpha \in A\} \quad (3.50)$$

$$C_a @_{\min} C_b = \left\{ \left\langle \alpha, \frac{\mu_{C_a}(\alpha) + \mu_{C_b}(\alpha)}{2}, \frac{\nu_{C_a}(\alpha) + \nu_{C_b}(\alpha)}{2}; \min(r_a, r_b) \right\rangle \mid \alpha \in A \right\} \quad (3.51)$$

$$C_a @_{\max} C_b = \left\{ \left\langle \alpha, \frac{\mu_{C_a}(\alpha) + \mu_{C_b}(\alpha)}{2}, \frac{\nu_{C_a}(\alpha) + \nu_{C_b}(\alpha)}{2}; \max(r_a, r_b) \right\rangle \mid \alpha \in A \right\} \quad (3.52)$$

4. FINANCIAL ACCOUNTING FRAUD DETECTION VIA MCDM

In this dissertation, decision making approaches are proposed for financial accounting fraud detection. Identifying which financial ratios are critical to detect financial accounting fraud can be regarded as a MCDM problem.

MCDM is defined as a process that selects the best solution from all considered possible alternatives under evaluation criteria (Kung et al., 2011). A typical MCDM problem includes several alternatives to be evaluated and a set of criteria to evaluate these alternatives. Each alternative has a value that indicates its performance on these criteria, and these values are used to evaluate and rank the alternatives.

Different perspectives along with perceptions of decision makers may cause inconsistency of information. In addition, evaluation process in MCDM problems often involves ambiguous data that can complicate this process for decision makers (Ghorabae et al., 2018). It is possible to use fuzzy logic approaches to eliminate these uncertainties. Fuzzy logic provides numerical information when there exists uncertainty situation and it helps to achieve a more realistic result in defining the existence of the relationship (Guner et al., 2009). For this reason, it can be reasonable to adapt fuzzy logic to MCDM methods.

Many studies have been conducted on applications of MCDM approaches in different areas, such as management science, engineering, education, public administration. However, few researchers have applied MCDM approaches for financial fraud detection. Peng et al. (2011) developed a model to evaluate the performance of classification algorithms for financial risk prediction. A performance score was defined to measure the performance of classification algorithms and three MCDM methods (TOPSIS, Preference Ranking Organization Method for Enrichment Evaluation(PROMETHEE), and Vise Kriterijumska Optimizacija I Kompromisno Resenje(VIKOR)) were used to provide a final ranking of classifiers. There is a need for improving fraud detection models by using MCDM approaches, especially adapting fuzzy logic, and in this study fuzzy MCDM approaches are proposed for feature selection in financial accounting fraud detection.

For this purpose, two integrated MCDM approaches are employed. These are as follows;

1. *Integrated AHP and TOPSIS*
2. *Integrated AHP and MULTIMOORA*

Detailed explanations of the integrated MCDM approaches utilized in this thesis are given in the following subsections.

4.1. Integrated AHP and TOPSIS with Different Fuzzy Sets

Two of the most commonly utilized MCDM methods in the literature are AHP proposed by Saaty (1980) and TOPSIS proposed by Hwang and Yoon (1981). Pairwise comparisons are used to evaluate several alternatives using the AHP technique, which takes into account both qualitative and quantitative criteria. Alternatives are evaluated in respect of many criteria in the TOPSIS technique, and both positive and negative ideal solutions are found. The best option is then chosen based on the shortest distance between the positive ideal solution and the largest distance between the negative ideal solution.

Because of the usefulness of AHP and TOPSIS, numerous research have used and also combined these two techniques to solve problems in a variety of fields. Furthermore, fuzzy set theory, which can handle ambiguity and vagueness in decision-making, is widely used in conjunction with these two MCDM techniques. Table 4.1 illustrates recent researches that utilized AHP, TOPSIS, and integrated AHP and TOPSIS approaches to solve real-world problems using various extensions of fuzzy set theory, including type-1 fuzzy sets, interval type 2 fuzzy sets, neutrosophic sets, Pythagorean Fuzzy Sets, Spherical fuzzy sets, and Interval-valued Spherical fuzzy sets.

Dozic et al. (2018) used fuzzy AHP to solve the problem of aircraft type selection. To derive crisp priorities from fuzzy pairwise comparison matrices, they employed a logarithmic fuzzy preference programming (LFPP) technique and an enhanced LFPP method. When compared to the enhanced LFPP approach for the FAHP, the final ranking of the LFPP-based fuzzy AHP varies.

Table 4.1. Summary of studies that utilize AHP, TOPSIS, and Integrated AHP and TOPSIS approaches with different fuzzy sets

Utilized Fuzzy	Utilized Methods	Research	Application Areas
Type-1 fuzzy sets Zadeh (1965)	AHP TOPSIS Integrated AHP-TOPSIS	Dozic et al. (2018) Carnero (2020) Azari et al. (2020)	Passenger aircraft model selection Evaluation of the environmental responsibility of health care Selection of the best color removal process utilizing carbon-based adsorbent materials
Type-2 fuzzy sets Zadeh (1975)	AHP TOPSIS Integrated AHP-TOPSIS	Ecer (2020) Pourmand et al. (2020) Wu et al. (2019)	Green supplier selection Ranking of allocating water and reclaimed wastewater scenarios Assessment of wind power coupling hydrogen storage projects
Intuitionistic fuzzy sets Atanassov (1986)	AHP TOPSIS Integrated AHP-TOPSIS	Wu et al. (2018) Büyüközkan and Güteryüz (2016) Dogan et al. (2020)	E-commerce firm selection Smart phone selection A corridor selection for locating autonomous vehicles
Neutrosophic sets Smarandache (1998)	AHP TOPSIS Integrated AHP-TOPSIS	Kahraman et al. (2020a) Biswas et al. (2016) Abdel-Basset et al. (2019)	Assessment of outsourcing law firms Most appropriate tablet selection Measurement of risks in supply chain
Hesitant fuzzy sets Torra (2010)	AHP TOPSIS Integrated AHP-TOPSIS	Xian et al. (2020) Amin et al. (2019) Kumar et al. (2020)	Talent selection Dealer selection Evaluation of the security durability of web application
Pythagorean fuzzy sets Yager (2013)	AHP TOPSIS Integrated AHP-TOPSIS	Shete et al. (2020) Rani et al. (2020) Yucesan and Gul (2020)	Ranking of sustainable supply chain innovation enablers Sustainable recycling partner selection Evaluation of hospital service quality
Picture fuzzy sets Cuong (2014)	AHP TOPSIS	Ju et al. (2019) Zeng et al. (2019)	Site selection for electric vehicle charging station Enterprise resource planning system selection

Table 4.1. (Continuation)

Spherical fuzzy sets Gündoğdu and Kahraman (2018)	AHP	Sharaf (2020)	A global supplier selection
	TOPSIS	Gündoğdu and Kahraman (2020b)	Optimal site selection for electric vehicle charging station
	Integrated AHP-TOPSIS	Mathew et al. (2020)	Manufacturing system selection
IV Spherical fuzzy sets Gündoğdu and Kahraman (2019a)	AHP	Gündoğdu and Kahraman (2020a)	Assessment of hospital performance
	TOPSIS	Gündoğdu and Kahraman (2019a)	3D printer selection
	<i>Integrated AHP-TOPSIS</i>	<i>This study</i>	<i>Ranking of financial ratios for detecting financial accounting fraud</i>

Ecer (2020) used interval type 2 fuzzy AHP to choose the appropriate green supplier. Previous works, such as interval type 2 fuzzy TOPSIS (Wu et al., 2018), interval type 2 fuzzy VIKOR (Ghorabae, 2016), and interval type 2 fuzzy Decision Making Trial and Evaluation Laboratory (DEMATEL) (Abdullah and Zulkifli, 2019) are compared to the proposed approach. The findings revealed that the proposed method is in accordance with the previous researches.

Pourmand et al. (2020) employed an interval type 2 fuzzy TOPSIS for determining the optimal scenario for distributing water and recovered wastewater to the household, agricultural, and industrial water sectors in Tehran, Iran. According to the study's findings, the chosen scenario leads to an adequate amount of groundwater conservation in Iran's Tehran metropolitan region.

Wu et al. (2019) used interval type-2 fuzzy AHP-TOPSIS to evaluate the performance of wind power coupling hydrogen storage facilities from a sustainability standpoint. To verify the usefulness of the suggested technique, they did a comparison with type-1 fuzzy AHP-TOPSIS. They discovered that the ranking provided by type-1 fuzzy AHP-TOPSIS is the same as the ranking obtained by the proposed approach.

Dogan et al. (2020) employed interval-valued intuitionistic fuzzy AHP and TOPSIS to find the optimal corridor for locating autonomous cars. They compared the suggested technique to the AHP-weighted scoring system and integrated AHP and TOPSIS approaches. All of the approaches for comparison analysis came up with the same result.

Kahraman et al. (2020a) compared the performance of law firms using neutrosophic AHP, which is based on Buckley's ordinary fuzzy AHP approach. They used five criteria to choose the best outsourced law company out of four alternatives.

Abdel-Basset et al. (2019) used integrated AHP and TOPSIS with neutrosophic set to measure supply chain risks. The findings demonstrated that the proposed approach can handle the uncertainty and inconsistency in the supply chain risk estimate process.

Amin et al. (2019) suggested a novel trapezoidal cubic hesitant fuzzy TOPSIS approach to choose an acceptable dealer. In order to verify the validity and efficiency of the suggested methodology, a comparison is made between the approaches of hesitant Fuzzy set (Torra, 2010) and Trapezoidal fuzzy TOPSIS (Pathinathan and Johnson, 2015), which are special examples of trapezoidal cubic hesitant fuzzy numbers. The trapezoidal

cubic hesitant fuzzy TOPSIS technique was shown to be more flexible in dealing with doubts and fuzziness.

Shete et al. (2020) employed AHP with pythagorean fuzzy set to assess the enhancers of sustainable supply chain innovation in a manufacturing industry case study in India. They also used sensitivity analysis to demonstrate the reliability of the prioritized weights for sustainable supply chain innovation enablers.

Rani et al. (2020) applied Pythagorean Fuzzy TOPSIS to select a sustainable recycling partner. They also compared the suggested Pythagorean Fuzzy TOPSIS to other earlier techniques like Fuzzy-VIKOR (Shemshadi et al., 2011), Intuitionistic Fuzzy TOPSIS (Mishra, 2016), Pythagorean Fuzzy MABAC (Peng and Yang, 2016b), and Fuzzy DEMATEL-AEW-VIKOR (Zhou et al., 2018). They discovered that the suggested Pythagorean Fuzzy TOPSIS is quite similar to the Fuzzy-VIKOR (Shemshadi et al., 2011), Intuitionistic Fuzzy TOPSIS (Mishra, 2016), and Fuzzy DEMATEL-AEW-VIKOR (Zhou et al., 2018).

Ju et al. (2019) integrated AHP and GRP (grey relational projection) methods with picture fuzzy sets to find the best location for an electric car charging station. To calculate the weights of criteria, the picture fuzzy AHP approach is employed first, and then extended GRP method is used to provide the final ranking.

Sharaf (2020) used spherical fuzzy AHP approach for solving global supplier selection problem adapted from Liao and Xu (2014). The findings of the used approach are compared to those of the intuitionistic fuzzy AHP method. The ranking found using the suggested spherical fuzzy AHP approach is the same as the ranking obtained using the intuitionistic fuzzy AHP method by Liao and Xu (2014).

Gündoğdu and Kahraman (2020b) employed spherical fuzzy TOPSIS to find the best location for an electric car charging station in İzmir. They compared the suggested spherical fuzzy TOPSIS to the spherical fuzzy Combinative Distance-Based Assessment (CODAS) technique and discovered that both methods provide the same ranking.

Mathew et al. (2020) used integrated AHP and TOPSIS with SVSFS to solve the problem of advanced manufacturing system selection. They calculated the weights of the criteria using spherical fuzzy AHP, and then used spherical fuzzy TOPSIS to rank the alternatives. They also compared the suggested approach's results to those of other prior

studies, including Rao and Parnichkun (2009), Maniya and Bhatt (2011), Karande and Chakraborty (2013), Mathew and Thomas (2019), and discovered that the rankings are similar.

Gündoğdu and Kahraman (2019a) developed IVSFS with their score and accuracy functions, as well as arithmetic and aggregation operations. They used IVSFS to develop a TOPSIS extension that allows users to choose a 3D printer. In addition, a comparison with SVSF-TOPSIS was performed, and both approaches found the same ranking.

Gündoğdu and Kahraman (2020a) developed IVSF-AHP approach to compare the service performances of different hospitals. They employed a comparison study using neutrosophic AHP to show that the suggested approach is valid. In both approaches, the ranks were determined to be similar.

4.1.1. Integrated Fuzzy AHP and Fuzzy TOPSIS

In this study, integrated Fuzzy AHP and Fuzzy TOPSIS is used for prioritizing financial ratios based on the fraud detection performance. First, the weight of the evaluation criteria is calculated using the Fuzzy AHP method. Next, fuzzy TOPSIS technique is used to find out the best prioritize of financial ratios. These methods are based on fuzzy logic, since the ordinary fuzzy set theory is a great tool for dealing with uncertainties of the prioritization of financial ratios.

The integration of Fuzzy AHP and Fuzzy TOPSIS has been proved to be sufficient and effective for solving real problems in different areas, such as management science, engineering, education, public administration (Liu et al.,2019; Beskese et al., 2019, Chou et al.,2019; Samanlioglu et al.,2018). Table 4.2 presents a summary of studies that use integrated Fuzzy AHP and Fuzzy TOPSIS approach for solving real problems in different areas. Prakash and Barua (2015) proposed a methodology based on fuzzy AHP and fuzzy TOPSIS to identify and rank the solutions of Reverse logistics adoption in electronics industry to overcome its barriers. Dincer et al. (2016) developed a model based on fuzzy AHP and fuzzy TOPSIS to determine the ranking of the industry alternatives for portfolio investments based on individual investors' perceptions. Çolak and Kaya (2017) used an integrated MCDM model based on fuzzy AHP and fuzzy TOPSIS for prioritization of renewable energy alternatives in Turkey. Dooki et al. (2017)

utilized an integrated approach of fuzzy AHP and fuzzy TOPSIS for ranking and selecting the chief inspectors of bank. Singh et al. (2018) proposed integrated fuzzy AHP and fuzzy TOPSIS for the selection of an appropriate third party logistics in order to outsource logistics activities of perishable products. Beskese et al. (2019) used integrated fuzzy AHP and fuzzy TOPSIS for ERP system selection in automotive industry. Wang et al. (2019) utilized integrated fuzzy AHP and fuzzy TOPSIS to plant selection for phytoremediation of petroleum-contaminated soils in shale gas and oil fields. Liu et al. (2019) used integrated fuzzy AHP and fuzzy TOPSIS to evaluate the sustainable performance of suppliers in an agrifood value chain.

Table 4.2. Summary of studies that use integrated Fuzzy AHP and Fuzzy TOPSIS approach

Research	Application	Title
Liu et al. (2019)	Supplier selection	A fuzzy decision tool to evaluate the sustainable
Beskese et al. (2019)	ERP software selection	A Model Proposal For Erp System Selection In
Chou et al. (2019)	Evaluation of the human resource performance in	Assessing the Human Resource in Science and
Wang et al. (2019)	Plant selection for the phytoremediation of	Application of AHP, TOPSIS, and TFNs to plant
Samanlioglu et al.(2018)	Personnel selection	A Fuzzy AHP–TOPSIS-Based Group Decision-
Singh et al. (2018)	Third party logistics selection for cold chain	Third party logistics (3PL) selection for cold chain
Anand and Vinodh (2018)	The selection of appropriate additive manufacturing process for microfabrication	Application of fuzzy AHP – TOPSIS for ranking additive manufacturing processes for
Belhadi et al. (2017)	Lean production in small manufacturing enterprises	Prioritizing the solutions of lean implementation in
Dooki et al. (2017)	The selection of the best chief inspector of bank	An Integrated Fuzzy AHP and Fuzzy TOPSIS
Rahmati and Noorbehbahani (2017)	Employee performance evaluation	A New Hybrid Method Based on Fuzzy AHP and
Dincer et al. (2016)	Industry selection	A fuzzy-hybrid analytic model to assess investors' perceptions for industry
Boutkhoul et al. (2016)	Green supply chain management	Multi-criteria decision support framework for
Prakash and Barua (2015)	Reverse logistics	Integration of AHP-TOPSIS method for prioritizing the solutions of reverse logistics

However, to the best of our knowledge, integrated Fuzzy AHP and Fuzzy TOPSIS approach has not been applied for ranking of financial ratios in financial statement fraud detection yet.

4.1.1.1. Fuzzy AHP

Firstly, fuzzy AHP method is used to evaluate the importance of qualitative characteristics (criteria) for financial accounting fraud detection. Chang's extent analysis is used for fuzzy pairwise comparisons of the qualitative characteristics. The following steps are need to be done for obtaining the weight of the criteria by using fuzzy AHP method.

Step 1 The importance weight of each criterion can be obtained by using pairwise comparisons. In this study, the importance weights of qualitative criteria are considered as linguistic variables. These linguistic variables can be converted by using triangular fuzzy numbers as shown in Tables 4.3. A triangular fuzzy number is represented as (l, m, u).

Table 4.3. Linguistic variables for the importance weight of each criterion (Fuzzy AHP)

The linguistic variable	Triangular fuzzy scale	Triangular fuzzy reciprocal scale
Equally important	(1, 1, 1)	(1, 1, 1)
Intermediate	(1, 2, 3)	(1/3, 1/2, 1)
Weakly important	(2, 3, 4)	(1/4, 1/3, 1/2)
Intermediate	(3, 4, 5)	(1/5, 1/4, 1/3)
Essentially important	(4, 5, 6)	(1/6, 1/5, 1/4)
Intermediate	(5, 6, 7)	(1/7, 1/6, 1/5)
Very strongly important	(6, 7, 8)	(1/8, 1/7, 1/6)
Intermediate	(7, 8, 9)	(1/9, 1/8, 1/7)
Absolutely important	(9, 9, 9)	(1/9, 1/9, 1/9)

Then, the comparison matrix will be as follows:

$$X = \begin{pmatrix} (1,1,1) & x_{12} & \cdot & \cdot & \cdot & x_{1n} \\ x_{21} & (1,1,1) & \cdot & \cdot & \cdot & x_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ x_{n1} & x_{n2} & \cdot & \cdot & \cdot & (1,1,1) \end{pmatrix}$$

where $x_{ji} = \frac{1}{x_{ij}} = (\frac{1}{u_{ij}}, \frac{1}{m_{ij}}, \frac{1}{l_{ij}})$

Supposed that a decision group has K persons, then calculate the weight of each criterion using the geometric mean.

$$\widetilde{X}_{ij} = (\prod_{k=1}^K \tilde{x}_{ij}^K)^{1/K} \quad (4.1)$$

where $\tilde{x}_{ij}^K$ is the importance weight of the Kth decision maker.

Step 2 The value of fuzzy synthetic extent with respect to the ith object is defined as follows:

$$S_i = \sum_{j=1}^n M_{gi}^j(x) [\sum_{i=1}^m \sum_{j=1}^n M_{gi}^j]^{-1} \quad (4.2)$$

where (x) denotes the extended multiplication of two fuzzy numbers. For obtaining $\sum_{j=1}^n M_{gi}^j$, the fuzzy addition operation of m extent analysis values for a particular matrix is performed as follows:

$$\sum_{j=1}^n M_{gi}^j = (\sum_{j=1}^n l_j, \sum_{j=1}^n m_j, \sum_{j=1}^n u_j), \quad \text{for } i=1, 2, \dots, m \quad (4.3)$$

and for obtaining $[\sum_{i=1}^m \sum_{j=1}^n M_{gi}^j]^{-1}$, the fuzzy addition operation is performed as follows: M_{gi}^j for $j=1, 2, \dots, n$ values such that

$$\sum_{i=1}^m \sum_{j=1}^n M_{gi}^j = (\sum_{i=1}^m l_i, \sum_{i=1}^m m_i, \sum_{i=1}^m u_i) \quad (4.4)$$

And, the inverse of the vector is computed as follows:

$$[\sum_{i=1}^m \sum_{j=1}^n M_{gi}^j]^{-1} = ((\sum_{i=1}^m u_i)^{-1}, (\sum_{i=1}^m m_i)^{-1}, (\sum_{i=1}^m l_i)^{-1}) \quad (4.5)$$

Step 3 The degree of possibility of $M_2 = (l_2, m_2, u_2) \geq M_1 = (l_1, m_1, u_1)$ is defined as follows:

$$V(M_2 \geq M_1) = \sup_{y \geq x} [\min(M_1(x), M_2(y))] \quad (4.6)$$

which can be equivalently expressed as follows:

$$V(M_2 \geq M_1) = \text{hgt}(M_1 \cap M_2) = M_2(d) \quad (4.7)$$

where d is the ordinate of the highest intersection point D between uM_1 and uM_2 , as shown in Figure 4.1.

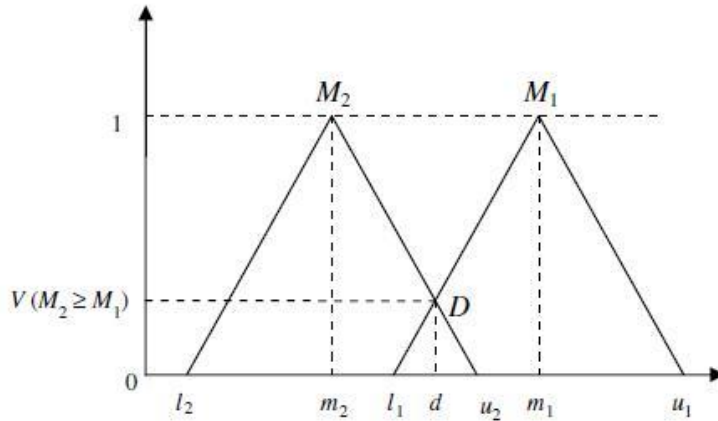


Figure 4.1. The intersection between M_1 and M_2

The ordinate of D is defined as follows:

$$\begin{aligned} V(M_2 \geq M_1) &= \text{hgt}(M_1 \cap M_2) = M_2(d) \\ &= (l_1 - u_2) / (m_2 - u_2) - (m_1 - l_1) \end{aligned} \quad (4.8)$$

To compare U_{M_1} and U_{M_2} ; we should need both the values of $V(M_1 \geq M_2)$ and $V(M_2 \geq M_1)$.

Step 4 The degree of possibility for a convex fuzzy number to be greater than k convex fuzzy numbers M_i for $i=1, 2, \dots, k$ can be defined as follows:

$$\begin{aligned} V(M \geq M_1, M_2, \dots, M_k) &= V[(M \geq M_1) \text{ and } (M \geq M_2) \text{ and } \dots \text{ and } (M \geq M_k)] = \min \\ &V(M \geq M_i), \text{ for } i=1, 2, \dots, k \end{aligned} \quad (4.9)$$

Assume that

$$d'(A_i) = \min V(S_i \geq S_k) \quad (4.10)$$

for $k=1, 2, \dots, n$; $k \neq i$. Then the weight vector is defined as in the following way:

$$W' = (d'(A1), d'(A2), \dots, d'(An))^T \quad (4.11)$$

where A_i for $i= 1, 2, \dots, n$ corresponding to n elements.

Step 5 Via normalization, the normalized weight vectors are evaluated as follows:

$$W=(d(A1), d(A2), \dots, d(An))^T \quad (4.12)$$

where W is a non fuzzy number.

4.1.1.2. Fuzzy TOPSIS

After determining the preference weights of qualitative characteristics for financial fraud detection, fuzzy TOPSIS technique is used to find out the best prioritize of financial ratios. Steps of fuzzy TOPSIS (Chen, 2000) are as follow, assuming there are m alternatives to rank and n criteria.

Step 6 Linguistic variables are used to evaluate the ratings of alternatives with respect to various criteria. These linguistic variables can be stated in triangular fuzzy numbers as seen in Table 4.4. Then, fuzzy matrix of alternatives is constructed as follows:

$$\tilde{X} = \begin{pmatrix} x_{11} & x_{12} & . & . & . & x_{1n} \\ x_{21} & x_{22} & . & . & . & x_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ x_{m1} & x_{m2} & . & . & . & x_{mn} \end{pmatrix} \quad (4.13)$$

Table 4.4. Linguistic variables for alternatives (Fuzzy TOPSIS)

Linguistic variables	Fuzzy ranks
Very poor(VP)	(0,1,1)
poor(P)	(0,1,3)
Medium poor(MP)	(1,3,5)
Fair(F)	(3,5,7)
Medium good(MG)	(5,7,9)
good(G)	(7,9,10)
Very good(VG)	(9,9,10)

Supposed that a decision group has K persons, then the rating of alternatives with respect to each criterion can be calculated as

$$\tilde{X} = \frac{1}{K} [\tilde{x}_{ij}^1 (+) \tilde{x}_{ij}^2 (+) \dots (+) \tilde{x}_{ij}^K] \quad (4.14)$$

where $\tilde{x}_{ij}^K$ is the rating of the K th decision maker.

Then, fuzzy matrix of multicriteria group decision making problem can be constructed as follows:

$$\tilde{D} = \begin{pmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdot & \cdot & \cdot & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdot & \cdot & \cdot & \tilde{x}_{2n} \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdot & \cdot & \cdot & \tilde{x}_{mn} \end{pmatrix} \quad (4.15)$$

where $\tilde{x}_{ij}$ is linguistic variable $\forall i, j$. Linguistic variables can be described by triangular fuzzy numbers, $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$.

Step 7 Construct the normalized matrix

A linear scale transformation is used to have comparable scale for positive and negative indicators, respectively:

$$\tilde{R} = [\tilde{r}_{ij}]_{m \times n} \quad (4.16)$$

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*} \right), c_j^* = \max c_{ij} \quad i=1,2,\dots,m, \quad j=1,2,\dots,n \quad (4.17)$$

$$\tilde{r}_{ij} = \left(\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}} \right), a_j^- = \min a_{ij} \quad i=1,2,\dots,m, \quad j=1,2,\dots,n \quad (4.18)$$

Step 8 Construct the weighted normalized matrix with the weights obtained by the fuzzy AHP method.

Assume that $\tilde{W}_j = (\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_n)$ is the weight importance of decision maker and $\sum_{j=1}^n \tilde{W}_j = 1$.

$\tilde{V} = [\tilde{v}_{ij}]_{m \times n}$ is the weighted normalized matrix where $i=1,2,\dots,m, \quad j=1,2,\dots,n$ and it can be calculated by the following equation.

$$\tilde{v}_{ij} = \tilde{r}_{ij} * \tilde{W}_j \quad (4.19)$$

Step 9 Calculate the fuzzy positive-ideal solution (FIPS, A^*) and fuzzy negative-ideal solution (FNIS, A^-) as

$$A^* = (\tilde{v}_1^*, \tilde{v}_2^*, \dots, \tilde{v}_n^*) \quad (4.20)$$

$$A^- = (\tilde{v}_1^-, \tilde{v}_2^-, \dots, \tilde{v}_n^-) \quad (4.21)$$

where $\tilde{v}_j^* = (1, 1, 1)$ and $\tilde{v}_j^- = (0, 0, 0)$, $j = 1, 2, \dots, n$.

Step 10 Calculate the distance of each alternative from A^* and A^- as

$$d_i^* = \sum_{j=1}^n d(\tilde{v}_{ij}, \tilde{v}_j^*), \quad i = 1, 2, \dots, m, \quad (4.22)$$

$$d_i^- = \sum_{j=1}^n d(\tilde{v}_{ij}, \tilde{v}_j^-), \quad i = 1, 2, \dots, m, \quad (4.23)$$

where $d(.,.)$ is the distance measurement between two fuzzy numbers.

Step 11 Each alternative closeness coefficient (CC_i) is calculated as

$$CC_i = \frac{d_i^-}{d_i^* + d_i^-}, \quad i = 1, 2, \dots, m. \quad (4.24)$$

Step 12 According to the closeness coefficient, all alternatives are sorted in descending order.

4.1.2. Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets

SFS has recently been used in a variety of applications, including the optimal location selection of electric vehicle charging stations (Gündoğdu and Kahraman, 2020b), the evaluation of suburban places' livability indices (Karaşan et al., 2020), global supplier selection (Sharaf, 2020), personnel selection (Gündoğdu, 2020), and storage location selection (Gündoğdu and Kahraman, 2019b).

To reduce uncertainties in financial ratio prioritization, integrated AHP and TOPSIS approach using SVSFS is presented in this study. The proposed approach includes two phases: obtaining criteria weights using AHP technique with SVSFS (SVSF-AHP) and then prioritizing the alternatives using TOPSIS technique with SVSFS (SVSF-TOPSIS).

4.1.2.1. Single-Valued Spherical Fuzzy AHP

The proposed methodology has multiple steps, which are outlined in this section. Here are the steps to follow:

Step 1 Build a hierarchical framework.

The decision-making problem is divided into various hierarchical levels in this step, with the objective (financial ratio prioritization) at the top, criteria (qualitative characteristics) at the second level, and alternatives (financial ratios) at the third level.

Step 2 Build a pairwise comparison matrix

The pairwise comparison matrix is built in the second step using an SVSFS scale of linguistic terms as shown in Table 4.5.

Table 4.5. SVSFS numbers relating to linguistic terms (Gündoğdu and Kahraman, 2019c)

Linguistic terms	Single-valued Spherical Fuzzy Numbers	Score Index
Absolutely More Importance	(0.9, 0.1, 0.1)	9
Very High Importance (VHI)	(0.8, 0.2, 0.2)	7
High Importance (HI)	(0.7, 0.3, 0.3)	5
Slightly More Importance (SMI)	(0.6, 0.4, 0.4)	3
Equally Importance (EI)	(0.5, 0.5, 0.5)	1
Slightly Low Importance (SLI)	(0.4, 0.6, 0.4)	1/3
Low Importance (LI)	(0.3, 0.7, 0.3)	1/5
Very Low Importance (VLI)	(0.2, 0.8, 0.2)	1/7
Absolutely Low Importance	(0.1, 0.9, 0.1)	1/9

Step 3 Checking the consistency

The consistency of each pairwise comparison matrix is checked utilizing the scoring index (SI) in Table 4.5. The score indices for the linguistic terms in the pairwise comparison matrix are swapped. Then, the regular consistency check is used. When the consistency ratio (CR) of the spherical fuzzy pairwise comparison matrix is less than 0.1, it is deemed consistent.

Step 4 Aggregate each decision maker's evaluations.

When there are several experts, pairwise comparison matrices for each expert are first calculated, and then the decision matrices are aggregated using SWAM by Eq. (3.15).

Depending on their expertise, each decision maker may have a different importance weight. $w_j, j = 1, 2, \dots, n$

Step 5 Aggregate weights of criteria.

Aggregate weights of criteria employing SWAM operator by Eq. (3.15).

4.1.2.2. Single-Valued Spherical Fuzzy TOPSIS

The single-valued spherical fuzzy TOPSIS approach is used to rank the alternatives after computing the weights of criteria using single-valued spherical fuzzy AHP. The following are the steps in the proposed approach:

Steps 6-7 First, decision makers use the linguistic terms in Table 4.5 to assess each alternative based on each criterion. The ratings of the alternatives are then combined with the SWAM operator. The weighted single-valued spherical fuzzy decision vector is computed using Eq. (3.6) once the weights of the criteria and the ratings of the alternatives have been calculated.

Steps 8-9 Eq. (3.17) is used to compute single-valued spherical fuzzy positive and negative ideal solutions (SVSF-PIS and SVSF-NIS). The corresponding SVSF-PIS and SVSF-NIS are identified based on the highest and lowest values.

Steps 10-12 Using Zhang and Xu's (2014) distance formulas, the distances of the alternatives from SVSF-PIS and SVSF-NIS are computed:

Eq. (4.25) is used to compute the distance to SVSF-PIS:

$$D_{ZX} (X_{ij}, X_j^*) = \frac{1}{2n} \sum_{j=1}^n \left(\left| \mu_{X_{ij}}^2 - \mu_{X_j^*}^2 \right| + \left| v_{X_{ij}}^2 - v_{X_j^*}^2 \right| + \left| \pi_{X_{ij}}^2 - \pi_{X_j^*}^2 \right| \right) \forall i \quad (4.25)$$

Eq. (4.26) is used to compute the distance to SVSF-NIS:

$$D_{ZX} (X_{ij}, X_j^-) = \frac{1}{2n} \sum_{j=1}^n \left(\left| \mu_{X_{ij}}^2 - \mu_{X_j^-}^2 \right| + \left| v_{X_{ij}}^2 - v_{X_j^-}^2 \right| + \left| \pi_{X_{ij}}^2 - \pi_{X_j^-}^2 \right| \right) \forall i \quad (4.26)$$

Then, using Eq. (4.34), closeness ratios are computed, and the ranking of the alternatives is determined based on the decreasing closeness ratio values.

4.1.3. Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets

Gündoğdu and Kahraman (2019a) proposed interval-valued spherical fuzzy sets, which include decision makers' judgments on the parameters of a fuzzy set into a model with an interval rather than a single point, and used TOPSIS method to apply IVSFS theory to the 3D printer selection problem. There are just two researches that we are aware of that have combined MCDM techniques with IVSFS to solve real-world problems: IVSF-AHP for evaluating hospital performance (Gündoğdu and Kahraman (2020a); IVSF-MULTIMOORA for assessing companies which utilize Industry 4.0 technologies (Aydın and Gündoğdu, 2020). There has not been a research that combines AHP and TOPSIS techniques with IVSFS. Therefore, to minimize uncertainties in financial ratio prioritization, this study presents an integrated AHP and TOPSIS methodology with IVSFS. The proposed approach includes two phases: calculating the criteria weights by employing AHP technique with IVSFS (IVSF-AHP) and then prioritizing the alternatives by using TOPSIS technique with IVSFS (IVSF-TOPSIS).

4.1.3.1. Interval-Valued Spherical Fuzzy AHP

The proposed methodology has multiple steps, which are outlined in this section. Here are the steps to follow:

Step 1 Build a hierarchical framework

The decision-making problem is divided into various hierarchical levels in this step, with the objective (financial ratio prioritization) at the top, criteria (qualitative characteristics) at the second level, and alternatives (financial ratios) at the third level.

Step 2 Build a pairwise comparison matrix

The pairwise comparison matrix is built in the second step using an IVSFS scale of linguistic terms as presented in Table 4.6.

Step 3 Checking the consistency

The consistency of each pairwise comparison matrix is checked utilizing scoring index (SI) in Table 4.6.

Table 4.6. IVSFS numbers relating to linguistic terms (Gündoğdu and Kahraman, 2019a)

Linguistic terms	IVSFS Numbers	Score Index
Absolutely More Importance (AMI)	$([0.85, 0.95], [0.10, 0.15], [0.05, 0.15])$	9
Very High Importance (VHI)	$([0.75, 0.85], [0.15, 0.20], [0.15, 0.20])$	7
High Importance (HI)	$([0.65, 0.75], [0.20, 0.25], [0.20, 0.25])$	5
Slightly More Importance (SMI)	$([0.55, 0.65], [0.25, 0.30], [0.25, 0.30])$	3
Equally Importance (EI)	$([0.50, 0.55], [0.45, 0.55], [0.30, 0.40])$	1
Slightly Low Importance (SLI)	$([0.25, 0.30], [0.55, 0.65], [0.25, 0.30])$	1/3
Low Importance (LI)	$([0.20, 0.25], [0.65, 0.75], [0.20, 0.25])$	1/5
Very Low Importance (VLI)	$([0.15, 0.20], [0.75, 0.85], [0.15, 0.20])$	1/7
Absolutely Low Importance (ALI)	$([0.10, 0.15], [0.85, 0.95], [0.05, 0.15])$	1/9

The score indices for the linguistic terms in the pairwise comparison matrix are swapped. Then, regular consistency check is applied. When the consistency ratio (CR) of the spherical fuzzy pairwise comparison matrix is less than 0.1, it is deemed consistent.

Step 4 Aggregate each decision maker's evaluations

When there are several experts, the pairwise comparison matrices for each expert are first constructed, and then the decision matrices are aggregated using IVSWAM by Eq (3.32). Depending on their expertise, each decision maker may have a different importance weight. $w_j, j = 1, 2, \dots, n$

Step 5 Aggregate weights of criteria.

Aggregate weights of criteria utilizing IVSWAM operator by Eq. (3.32).

4.1.3.2. Interval-Valued Spherical Fuzzy TOPSIS

The interval-valued spherical fuzzy TOPSIS approach is used to rank the alternatives after computing the weights of criteria using interval-valued spherical fuzzy AHP. The following are the steps in the proposed approach:

Step 6 Evaluate the alternatives with respect to the criteria

Decision makers use the linguistic terms in Table 4.6 to assess each alternative based on each criterion. The ratings of the alternatives are then combined with the IVSWAM operator regarding decision makers' importance levels. Aggregated decision matrix is

constructed in the following:

$$D = (C_j(X_i))_{m \times n} = \begin{pmatrix} [\mu_{11}^L(\alpha), \mu_{11}^U(\alpha)], [v_{11}^L(\alpha), v_{11}^U(\alpha)], [\pi_{11}^L(\alpha), \pi_{11}^U(\alpha)] & \cdots & [\mu_{1n}^L(\alpha), \mu_{1n}^U(\alpha)], [v_{1n}^L(\alpha), v_{1n}^U(\alpha)], [\pi_{1n}^L(\alpha), \pi_{1n}^U(\alpha)] \\ \vdots & \ddots & \vdots \\ [\mu_{m1}^L(\alpha), \mu_{m1}^U(\alpha)], [v_{m1}^L(\alpha), v_{m1}^U(\alpha)], [\pi_{m1}^L(\alpha), \pi_{m1}^U(\alpha)] & \cdots & [\mu_{mn}^L(\alpha), \mu_{mn}^U(\alpha)], [v_{mn}^L(\alpha), v_{mn}^U(\alpha)], [\pi_{mn}^L(\alpha), \pi_{mn}^U(\alpha)] \end{pmatrix}$$

Step 7 Construct the weighted interval-valued spherical fuzzy decision matrix.

The weighted interval-valued spherical fuzzy decision matrix is constituted using Eq. (3.23) once the weights of the criteria and the ratings of the alternatives have been calculated. Then, the weighted interval-valued spherical fuzzy decision matrix is constructed as follows:

$$D = (C_j(X_{iw}))_{m \times n} = \begin{pmatrix} [\mu_{11w}^L(\alpha), \mu_{11w}^U(\alpha)], [v_{11w}^L(\alpha), v_{11w}^U(\alpha)], [\pi_{11w}^L(\alpha), \pi_{11w}^U(\alpha)] & \cdots & [\mu_{1nw}^L(\alpha), \mu_{1nw}^U(\alpha)], [v_{1nw}^L(\alpha), v_{1nw}^U(\alpha)], [\pi_{1nw}^L(\alpha), \pi_{1nw}^U(\alpha)] \\ \vdots & \ddots & \vdots \\ [\mu_{m1w}^L(\alpha), \mu_{m1w}^U(\alpha)], [v_{m1w}^L(\alpha), v_{m1w}^U(\alpha)], [\pi_{m1w}^L(\alpha), \pi_{m1w}^U(\alpha)] & \cdots & [\mu_{mnw}^L(\alpha), \mu_{mnw}^U(\alpha)], [v_{mnw}^L(\alpha), v_{mnw}^U(\alpha)], [\pi_{mnw}^L(\alpha), \pi_{mnw}^U(\alpha)] \end{pmatrix}$$

Step 8 Using Eq. (4.27), the weighted interval-valued spherical fuzzy decision matrix is defuzzified.

$$S(C_j(X_{iw})) = \frac{(\mu_{ijw}^L(\alpha))^2 + (\mu_{ijw}^U(\alpha))^2 - (v_{ijw}^L(\alpha))^2 - (v_{ijw}^U(\alpha))^2 - \left(\frac{\pi_{ijw}^L(\alpha)}{2}\right)^2 - \left(\frac{\pi_{ijw}^U(\alpha)}{2}\right)^2}{2} \quad (4.27)$$

Step 9 The score values computed in Step 8 are used to determine the Interval-valued Spherical Fuzzy Positive Ideal Solution (IVSF-PIS) and the Interval-valued Spherical Fuzzy Negative Ideal Solution (IVSF-NIS).

The Interval-valued Spherical Fuzzy Positive Ideal Solution is defined as follows:

$$X^* = \{C_j, \max_i S(C_j(X_{iw})) \mid j=1, 2, \dots, n\} \quad (4.28)$$

or

$$X^* = \left\{ \langle C_1, ([\mu_1^{L*}, \mu_1^{U*}], [v_1^{L*}, v_1^{U*}], [\pi_1^{L*}, \pi_1^{U*}]) \rangle, \langle C_2, ([\mu_2^{L*}, \mu_2^{U*}], [v_2^{L*}, v_2^{U*}], [\pi_2^{L*}, \pi_2^{U*}]) \rangle \right. \\ \left. \dots \dots \langle C_n, ([\mu_n^{L*}, \mu_n^{U*}], [v_n^{L*}, v_n^{U*}], [\pi_n^{L*}, \pi_n^{U*}]) \rangle \right\} \quad (4.29)$$

The Interval-valued Spherical Fuzzy Negative Ideal Solution is defined as follows:

$$X^- = \{C_j, \min_i < S(C_j(X_{iw})) > | j=1,2,\dots,n\} \quad (4.30)$$

or

$$X^- = \left\{ \langle C_1, ([\mu_1^L, \mu_1^U], [v_1^L, v_1^U], [\pi_1^L, \pi_1^U]) \rangle, \langle C_2, ([\mu_2^L, \mu_2^U], [v_2^L, v_2^U], [\pi_2^L, \pi_2^U]) \rangle, \dots, \langle C_n, ([\mu_n^L, \mu_n^U], [v_n^L, v_n^U], [\pi_n^L, \pi_n^U]) \rangle \right\} \quad (4.31)$$

Step 10 Using Peng and Yang's (2016a,2016b) normalized distance formula, the distances of the alternatives from the IVSF-PIS and IVSF-NIS are computed.

Eq. (4.32) is used to compute the distance to IVSF-PIS:

$$d(X_{ij}, X_j^+) = \frac{1}{4n} \sum_{j=1}^n \left(\left| (\mu_{ij}^L)^2 - (\mu_j^+)^2 \right| + \left| (\mu_{ij}^U)^2 - (\mu_j^+)^2 \right| + \left| (v_{ij}^L)^2 - (v_j^+)^2 \right| + \left| (v_{ij}^U)^2 - (v_j^+)^2 \right| + \left| (\pi_{ij}^L)^2 - (\pi_j^+)^2 \right| + \left| (\pi_{ij}^U)^2 - (\pi_j^+)^2 \right| \right) \forall i \quad (4.32)$$

Eq. (4.33) is used to compute the distance to IVSF-NIS:

$$d(X_{ij}, X_j^-) = \frac{1}{4n} \sum_{j=1}^n \left(\left| (\mu_{ij}^L)^2 - (\mu_j^-)^2 \right| + \left| (\mu_{ij}^U)^2 - (\mu_j^-)^2 \right| + \left| (v_{ij}^L)^2 - (v_j^-)^2 \right| + \left| (v_{ij}^U)^2 - (v_j^-)^2 \right| + \left| (\pi_{ij}^L)^2 - (\pi_j^-)^2 \right| + \left| (\pi_{ij}^U)^2 - (\pi_j^-)^2 \right| \right) \forall i \quad (4.33)$$

Step 11 Use Eq. (4.34) for computing the relative closeness to the positive and negative ideal solutions.

$$\text{Closeness Ratio}_i = \frac{d(X_{ij}, X_j^-)}{d(X_{ij}, X_j^-) + d(X_{ij}, X_j^+)} \quad (4.34)$$

Step 12 The ranking of the alternatives is determined based on the decreasing closeness ratio values.

4.1.4. Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets

Atanassov (2020) proposed Circular intuitionistic fuzzy sets as expansions of intuitionistic fuzzy sets. C-IFS utilizes a circle with a radius of r to determine membership and non-membership degrees. Hence, membership functions are more adaptable than intuitionistic fuzzy sets. Furthermore, C-IFS may be successfully incorporated in MCDM approaches to deal with uncertainty in decision-making processes more effectively.

Since C-IFS have been proposed recently, there have been few researches that apply it to MCDM approaches. Alkan and Kahraman (2021) used Circular intuitionistic fuzzy TOPSIS approach for selecting the best pandemic hospital location. They compared the proposed C-IF TOPSIS approach to single-valued intuitionistic fuzzy TOPSIS, Pythagorean fuzzy TOPSIS, and picture fuzzy TOPSIS methods for evaluating its efficacy. The findings of the study supported the proposed methodology's validity.

Kahraman and Alkan (2021) employed C-IF TOPSIS approach to a problem of supplier selection. To validate their proposed approach, they applied single-valued intuitionistic fuzzy TOPSIS method. Kahraman and Otay (2021) integrated C-IFS with another MCDM technique, VIKOR, to determine a waste disposal location. Otay and Kahraman (2021) also proposed integrated C-IF AHP and C-IF VIKOR approach to handle an engineering company's multi-expert supplier assessment and selection challenge. The results of the proposed approach are compared to crisp AHP-VIKOR and ordinary fuzzy AHP-VIKOR approaches. It is found that the findings of the integrated C-IF AHP and C-IF VIKOR techniques are consistent with the other approaches' results.

C-IFS will be increasingly widely used to enhance MCDM approaches for solving real-world problems. This is the first study to propose an integrated AHP and TOPSIS approach for managing uncertainties in the decision-making process, as well as to utilize the proposed methodology to deal with the problem of financial ratio prioritization for financial accounting fraud detection. A new preference scale of measurement is also developed, which allows each decision maker's judgments to be checked for consistency. The proposed approach comprises two phases: (i) calculating criterion weights utilizing C-IF AHP technique, and (ii) ranking the alternatives using C-IF TOPSIS technique.

4.1.4.1. Circular Intuitionistic Fuzzy AHP

The following are the steps in the proposed approach:

Step 1 Build the hierarchical framework of the considered problem

The hierarchical framework of the considered problem is built in this step. The decision-making problem is divided into various hierarchical levels with the objective (financial ratio prioritization) at the top, criteria (qualitative characteristics) at the second level, and alternatives (financial ratios) at the third level.

Step 2 Gather individual views on criteria under intuitionistic environment

For describing the individual preferences of experts, IFS scale of linguistic concepts in terms of C-IFSs is developed. The IFS scale of linguistic terms is presented in Table 4.7.

Table 4.7. Intuitionistic Fuzzy numbers (IFNs) relating to linguistic terms

Linguistic terms	IFNs (μ , ν)	Score Index
Absolutely High Importance (AHI)	(0.9, 0)	9
Very High Importance (VHI)	(0.8, 0.1)	7
High Importance (HI)	(0.7, 0.2)	5
Slightly More Importance (SMI)	(0.6, 0.3)	3
Equally Importance (EI)	(0.5, 0.5)	1
Slightly Low Importance (SLI)	(0.3, 0.6)	1/3
Low Importance (LI)	(0.2, 0.7)	1/5
Very Low Importance (VLI)	(0.1, 0.8)	1/7
Absolutely Low Importance (ALI)	(0, 0.9)	1/9

The central linguistic expression "Equally Importance" is described as a fuzzy number with an intuitionistic value of (0.5, 0.5). The μ variable, which stands for membership degree, is identified between 0.9 and 0. The non-membership degree variable, ν , is scaled from 0 to 0.9 for the most significant linguistic term to the least important linguistic term, respectively.

Here, μ and ν can be treated as independent variables, while their score indices can be treated as dependent variables. Regression modeling is used to determine the influence of membership and non-membership degrees on the dependent variable based on this information. The following Regression model is constructed using the values in Table 4.7.

$$Y = 26.25 - 20.90 \mu_C(\alpha) - 30.65 v_C(\alpha)$$

It is revealed that the values of membership and non-membership degrees are related to the values of score indices in a meaningful and negative way. Adjusted R-sq is determined to be 89.83%.

Subsequently, using the scale in Table 4.7, decision matrices consisting of linguistic terms in relation to expert judgments are created. Decision matrix is built as follows:

Table 4.8. Decision matrix depending on intuitionistic fuzzy numbers for criteria

<i>Criteria</i>	<i>Criteria</i>			
	C_1	C_2		C_j
C_1	$\langle(\mu_{11k}, v_{11n})\rangle$	$\langle(\mu_{12n}, v_{12n})\rangle$		$\langle(\mu_{1jn}, v_{1jn})\rangle$
C_2	$\langle(\mu_{21k}, v_{21n})\rangle$	$\langle(\mu_{22n}, v_{22n})\rangle$		$\langle(\mu_{2jn}, v_{2jn})\rangle$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
C_j	$\langle(\mu_{j1k}, v_{j1n})\rangle$	$\langle(\mu_{j2n}, v_{j2n})\rangle$		$\langle(\mu_{jjn}, v_{jjn})\rangle$

Step 3 Checking the consistency

The consistency ratio is used to check that each expert's assessments are consistent. When the consistency ratio value is less than 0.1, expert assessments are acceptable; otherwise, expert assessments should be changed. The regular consistency check is done using the scoring index (SI) in Table 4.7.

Step 4 Construct the aggregated IF decision matrix by aggregating individual views.

The Intuitionistic Fuzzy Weighted Averaging (IFWA) operator in Eq. (4.35) is used to construct the aggregated intuitionistic fuzzy decision matrix. Depending on their expertise, each decision maker may have a different importance weight. w_j , $j = 1, 2, \dots, n$

Let $C_j = (\mu_{C_j}, v_{C_j})$ ($j = 1, 2, \dots, n$) be a intuitionistic fuzzy number. The following formula is used to calculate the IFWA operator:

$$IFWA(C_1, C_2, \dots, C_n) = w_1 C_1 \oplus \dots \oplus w_n C_n = \left(1 - \prod_{j=1}^n (1 - \mu_{C_j})^{w_j}, \prod_{j=1}^n v_{C_j}^{w_j}\right) \quad (4.35)$$

Next, using the IFWA operator in Eq. (4.35), intuitionistic fuzzy weights of criteria are computed.

Step 5 Compute each criterion's maximum radius lengths

The maximum radius lengths depending on the individual decision matrices of all experts are computed by using Eqs. (3.40) and (3.41). The radius lengths of each criterion are then computed depends on the maximum radius lengths of all experts.

Step 6 Construct a weight vector for the C-IFS.

Using the weights of criteria acquired in Step 4 and the maximum radius lengths of each criterion determined in Step 5, the circular intuitionistic fuzzy weight vector matrix is built. Table 4.9 shows the C-IF weight vector. The circular intuitionistic fuzzy number of j^{th} criterion is denoted by $D = (d_j)$, where $d_j = (\mu_j, v_j; r_j)$.

Table 4.9. C-IF weight vector of criteria

Criteria	$D = (d_j)$
C_1	$\langle(\mu_1, v_1; r_1)\rangle$
C_2	$\langle(\mu_2, v_2; r_2)\rangle$
$\vdots$	$\vdots$
C_j	$\langle(\mu_j, v_j; r_j)\rangle$

Step 7 Calculate the criteria's weights

In order to calculate the criteria's weights, Relative Score Function (RSF) equation introduced by Kahraman and Otay (2021) is modified. Eq. (4.36) shows RSF formula based on vector normalization.

$$RSF_j = \frac{(1-v_j)(1+\mu_j)+\mu_j}{6} \times \left(\frac{\frac{1}{r_j}}{\sqrt{\frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} + \frac{1}{r_4^2} + \frac{1}{r_5^2} + \frac{1}{r_6^2}}} \right)^\tau \quad (4.36)$$

in which τ is a small value like 0.1 or 0.01.

The weights of criteria are then calculated by normalizing the values of the Relative Score Function.

4.1.4.2. Circular Intuitionistic Fuzzy TOPSIS

The circular intuitionistic fuzzy TOPSIS approach is employed to rank the alternatives after calculating the criteria' weights using circular intuitionistic fuzzy AHP. The following are the steps in the proposed method:

Step 8 Gather individual views on alternatives based on the criteria under intuitionistic environment

To assess the ratings of alternatives with regard to each criterion, the linguistic terms presented in Table 4.7 are used. Then, with respect to opinions of experts, decision matrices comprising linguistic terms are formed as follows:

Table 4.10. Decision matrix depending on intuitionistic fuzzy numbers for alternatives

<i>Alternatives</i>	<i>Criteria</i>			
	C_1	C_2		C_j
A_1	$\langle(\mu_{11n}, v_{11n})\rangle$	$\langle(\mu_{12n}, v_{12n})\rangle$		$\langle(\mu_{1jn}, v_{1jn})\rangle$
A_2	$\langle(\mu_{21n}, v_{21n})\rangle$	$\langle(\mu_{22n}, v_{22n})\rangle$		$\langle(\mu_{2jn}, v_{2jn})\rangle$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
A_i	$\langle(\mu_{i1n}, v_{i1n})\rangle$	$\langle(\mu_{i2n}, v_{i2n})\rangle$		$\langle(\mu_{ijn}, v_{ijn})\rangle$

Step 9 Construct the aggregated IF decision matrix by aggregating individual views

To construct intuitionistic fuzzy decision matrix, the ratings of the alternatives are aggregated using the IFWA operator while taking into account the significance levels of decision makers.

Step 10 Compute each alternative's maximum radius lengths

Each alternative's maximum radius lengths depending on the individual decision matrices of all experts are computed by using Eqs. (3.40) and (3.41).

Step 11 Build the aggregated decision matrix for C-IFS

Using the aggregated intuitionistic fuzzy numbers acquired in Step 9 and the maximum radius lengths of each alternative determined in Step 10, the aggregated circular intuitionistic fuzzy decision matrix is built. Table 6 shows the aggregated circular intuitionistic fuzzy decision matrix. The circular intuitionistic fuzzy number of i^{th} alternative with respect to j^{th} criterion is denoted by $D = (d_{ij})$ where $d_{ij} = (\mu_{ij}, v_{ij}; r_{ij})$.

Table 4.11. Aggregated circular intuitionistic fuzzy decision matrix

<i>Alternatives</i>	<i>Criteria</i>			
	C_1	C_2		C_j
A_1	$\langle(\mu_{11}, v_{11}; r_{11})\rangle$	$\langle(\mu_{12}, v_{12}; r_{12})\rangle$		$\langle(\mu_{1j}, v_{1j}; r_{1j})\rangle$
A_2	$\langle(\mu_{21}, v_{21}; r_{21})\rangle$	$\langle(\mu_{22}, v_{22}; r_{22})\rangle$		$\langle(\mu_{2j}, v_{2j}; r_{2j})\rangle$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
A_i	$\langle(\mu_{i1}, v_{i1}; r_{i1})\rangle$	$\langle(\mu_{i2}, v_{i2}; r_{i2})\rangle$		$\langle(\mu_{ij}, v_{ij}; r_{ij})\rangle$

Step 12 Build the weighted decision matrix for C-IFS

The weighted circular intuitionistic fuzzy decision matrix is built using Eq. (4.37) which is adapted from Atanassov's (2020) research after the weights of the criteria and the ratings of the alternatives have been calculated.

Here, w_j stands for the weights of criteria determined in Step 7. The aggregated circular intuitionistic fuzzy number of i^{th} alternative with regard to j^{th} criterion is $d_{ij} = (\mu_{ij}, v_{ij}, r_{ij})$, where π_{ij} represents the degree of uncertainty, which is determined using Eq. (3.37).

$$w_j * (d_{ij}) = \{\langle \mu_{ij} + w_j \cdot \pi_{ij}, v_{ij} + (1 - w_j) \cdot \pi_{ij}; r_{ij} \rangle\} \quad (4.37)$$

Step 13 Obtain the positive and negative ideal C-IF solutions

The Circular intuitionistic fuzzy positive ideal solution A^+ and the Circular intuitionistic fuzzy negative ideal solution A^- are identified based on the weighted circular intuitionistic fuzzy decision matrix using Eqs. (4.38)-(4.43). C^+ denotes the benefit criteria, whereas C^- denotes the cost criteria.

For benefit criteria:

$$A^+ = \{\langle \alpha, \mu_{C^+}(\alpha), v_{C^+}(\alpha); r^+ \rangle \mid \alpha \in C^+\} \text{ and } A^- = \{\langle \alpha, \mu_{C^-}(\alpha), v_{C^-}(\alpha); r^- \rangle \mid \alpha \in C^+\} \quad (4.38)$$

where

$$\mu_{C^+}(\alpha) = (\max_i \mu_{c_i}(\alpha) \mid \alpha \in C^+), v_{C^+}(\alpha) = (\min_i v_{c_i}(\alpha) \mid \alpha \in C^+) \quad (4.39)$$

$$\mu_{C^-}(\alpha) = (\min_i \mu_{c_i}(\alpha) \mid \alpha \in C^+), v_{C^-}(\alpha) = (\max_i v_{c_i}(\alpha) \mid \alpha \in C^+) \quad (4.40)$$

For cost criteria:

$$A^+ = \{\langle \alpha, \mu_{C^+}(\alpha), \nu_{C^+}(\alpha); r^+ \rangle \mid \alpha \in C^-\} \text{ and } A^- = \{\langle \alpha, \mu_{C^-}(\alpha), \nu_{C^-}(\alpha); r^- \rangle \mid \alpha \in C^-\} \quad (4.41)$$

where

$$\mu_{C^+}(\alpha) = (\min_i \mu_{c_i}(\alpha) \mid \alpha \in C^-), \nu_{C^+}(\alpha) = (\max_i \nu_{c_i}(\alpha) \mid \alpha \in C^-) \quad (4.42)$$

$$\mu_{C^-}(\alpha) = (\max_i \mu_{c_i}(\alpha) \mid \alpha \in C^-), \nu_{C^-}(\alpha) = (\min_i \nu_{c_i}(\alpha) \mid \alpha \in C^-) \quad (4.43)$$

Step 14 Compute the alternatives' distances from C-IF positive and negative ideal solutions

Using Atanassov and Marinov's (2021) intuitionistic fuzzy Euclidean distance formula, the alternatives' distances from the C-IF positive ideal solution and the C-IF negative ideal solution are calculated.

Assume that the cardinality of universe A is C_C .

Eq. (4.44) is used to compute the distance to the C-IF positive ideal solution (A^+):

$$\mathcal{D}_i^{A^+} = \frac{1}{2} \left(\frac{r_i - r^+}{\sqrt{2}} + \sqrt{\frac{1}{2C_C} \cdot \left(\sum_{\alpha \in A} \left((\mu_{c_i}(\alpha) - \mu_{C^+}(\alpha))^2 + (\nu_{c_i}(\alpha) - \nu_{C^+}(\alpha))^2 \right) \right)} \right) \quad (4.44)$$

Eq. (4.45) is used to compute the distance to the C-IF negative ideal solution (A^-):

$$\mathcal{D}_i^{A^-} = \frac{1}{2} \left(\frac{r_i - r^-}{\sqrt{2}} + \sqrt{\frac{1}{2C_C} \cdot \left(\sum_{\alpha \in A} \left((\mu_{c_i}(\alpha) - \mu_{C^-}(\alpha))^2 + (\nu_{c_i}(\alpha) - \nu_{C^-}(\alpha))^2 \right) \right)} \right) \quad (4.45)$$

Step 15 Compute the closeness ratio

Eq. (4.46) is used to compute alternatives' relative closeness to the ideal solutions.

$$Closeness\ Ratio_i = \frac{\mathcal{D}_i^{A^-}}{\mathcal{D}_i^{A^-} + \mathcal{D}_i^{A^+}} \quad (4.46)$$

Step 16 Determine ranking of alternatives

The alternatives is ranked in descending order in terms of their closeness ratio values.

4.2. Integrated AHP and MULTIMOORA with Different Fuzzy Sets

The AHP technique, developed by Saaty (1980), is one of the most widely used MCDM methods for calculating criteria weights and ranking alternatives based on pairwise comparisons. Many researches in the literature have proposed various approaches for calculating weights/priorities/importances using the AHP technique (Liu et al., 2020). The use of fuzzy set theory to AHP aids in the capture of uncertainty and ambiguity in decision-making (Mathew et al., 2020). AHP has been widely used in a number of applications in distinct fields with different extensions of fuzzy set theory, including type-1 fuzzy sets (Averill, 2020), type 2 fuzzy sets (Turan et al., 2019), intuitionistic fuzzy sets (Wu et al., 2013), neutrosophic sets (Kahraman et al., 2020a), hesitant fuzzy sets (Samanlioglu and Kaya, 2020), pythagorean fuzzy sets (Kaya et al., 2020), picture fuzzy sets (Ju et al., 2019), spherical fuzzy sets (Sharaf, 2020), and interval-valued spherical fuzzy sets (Gündoğdu and Kahraman, 2020a).

MOORA, developed by Brauers and Zavadskas (2006), is another excellent MCDM technique. This technique combines the use of a ratio system with the use of a reference point. Following that, Brauers and Zavadskas (2010) proposed MULTIMOORA by including the full multiplicative form and dominance theory into the MOORA steps. The MULTIMOORA technique does not necessitate complex mathematical calculations, and it also does not necessitate normalization procedure utilizing full multiplicative form (Aydın and Gündoğdu, 2020). Because of MULTIMOORA's effectiveness, it has been used in a number of studies to solve real-world problems using various extensions of fuzzy set theory, including type-1 fuzzy sets (Brauers et al., 2011), type 2 fuzzy sets (Baležentis and Zeng, 2013), intuitionistic fuzzy sets (Zhang et al., 2019), neutrosophic sets (Siksnelyte et al., 2019), hesitant fuzzy sets (Liao et al., 2019), pythagorean fuzzy sets (Huang et al., 2020), picture fuzzy sets (Lin et al., 2020), spherical fuzzy sets (Gündoğdu, 2020), and interval-valued spherical fuzzy sets (Aydın and Gündoğdu, 2020).

Table 4.12 summarizes researches that employed AHP, MULTIMOORA, and integrated AHP-MULTIMOORA approaches using various extensions of fuzzy set theory to solve real-world problems in various fields.

Table 4.12. Some of researches that employed AHP, MULTIMOORA, Integrated AHP and MULTIMOORA approaches with different fuzzy sets

Used Fuzzy	Used Tehniques	Researches	Application Fields
Type-1 fuzzy sets Zadeh (1965)	AHP MULTIMOORA Integrated AHP- MULTIMOORA	Averill (2020) Brauers et al. (2011) Tavana et al. (2020)	Metal cleaning agent selection Prioritization of the EU Member States Supplier selection
Type-2 fuzzy sets Zadeh (1975)	AHP MULTIMOORA Integrated AHP- MULTIMOORA	Turan et al. (2019) Baležentis and Zeng (2013) -	Evaluation of soil quality index Staff selection -
Intuitionistic fuzzy sets Atanassov (1986)	AHP MULTIMOORA Integrated AHP- MULTIMOORA	Wu et al. (2013) Zhang et al. (2019) Göçer (2020)	E-commerce firm selection Evaluation of the energy storage technologies Information Management System selection
Neutrosophic sets Smarandache (1998)	AHP MULTIMOORA Integrated AHP- MULTIMOORA	Kahraman et al. (2020a) Siksnylyte et al. (2019) -	Assessment of outsourcing law firms Evaluation of energy policy sustainability -
Hesitant fuzzy sets Torra (2010)	AHP MULTIMOORA Integrated AHP- MULTIMOORA	Samanlioglu and Kaya (2020) Liao et al. (2019) -	Assesment of the COVID-19 pandemic intervention strategies Global talent selection -
Pythagorean fuzzy sets Yager (2013)	AHP MULTIMOORA	Kaya et al. (2020) Huang et al.(2020)	Location selection for waste electric and electronic equipment recycling facility Selection of solid-state disk productions

Table 4.12. (Continuation)

	Integrated AHP-MULTIMOORA	Seker and Kahraman (2020)	Solar PV panel manufacturer selection
Picture fuzzy sets Cuong (2014)	AHP MULTIMOORA Integrated AHP-MULTIMOORA	Ju et al. (2019) Lin et al. (2020) -	Site selection for electric vehicle charging station Site selection of car sharing stations -
Single-Valued Spherical fuzzy sets Gündoğdu and Kahraman	AHP MULTIMOORA <i>Integrated AHP-MULTIMOORA</i>	Sharaf (2020)) Gündoğdu (2020) <i>This study</i>	Selection of a global supplier Staff selection <i>Ranking of financial ratios for detecting financial accounting fraud</i>
Interval-Valued Spherical fuzzy sets Gündoğdu and Kahraman (2019a)	AHP MULTIMOORA <i>Integrated AHP-MULTIMOORA</i>	Gündoğdu and Kahraman (2020a) Aydın and Gündoğdu (2020) <i>This study</i>	Evaluation of hospital performance Selection of Industry 4.0 strategy <i>Ranking of financial ratios for detecting financial accounting fraud</i>

In this study, an integrated AHP and MULTIMOORA approach with different fuzzy sets, which are ordinary fuzzy sets and spherical fuzzy sets(single-valued and interval-valued), are proposed for prioritizing financial ratios based on the fraud detection performance.

4.2.1. Integrated Fuzzy AHP and Fuzzy MULTIMOORA

A number of researches have used the fuzzy AHP, MOORA, and fuzzy MOORA approaches separately or in combination. However, only a few researches have used the integration of Fuzzy AHP and Fuzzy MULTIMOORA methods to solve real-world problems in various fields. To assess numerous failure modes, Fattahi and Khalilzadeh (2018) developed a new fuzzy hybrid approach based on fuzzy FMEA, extended fuzzy MULTIMOORA, and fuzzy AHP methodologies. Our study proposes the use of integrated Fuzzy AHP and Fuzzy MULTIMOORA approach for prioritizing financial ratios, as well as expanding the application of related approach.

The following are some examples of recent researches that employed fuzzy AHP, MOORA, fuzzy MOORA, MULTIMOORA, and fuzzy MULTIMOORA approaches:

In order to assess the elements determining the performance of university business incubators in Thailand, Somsuk and Laosirihongthong (2014) suggested the fuzzy AHP technique. For the problem of selection in process engineering, Tan et al. (2014) applied the fuzzy AHP technique. The multi-objective optimization by ratio analysis (MOORA) approach was developed by Brauers and Zavadskas (2006), and it was used to privatization in a transition economy. The MOORA approach was later enhanced by Brauers and Zavadskas (2010). Multiple multi-objective optimization by ratio analysis (MULTIMOORA) was the term given to this enlarged approach, which took into account the complete multiplicative form. In the process of selecting staff, Baležentis et al. (2012) used the fuzzy MULTIMOORA approach, which targets the best employee specified for a firm. Table 4.13 shows some of researches that utilize Fuzzy AHP, Fuzzy MOORA and Fuzzy MULTIMOORA approaches to solve real-world problems in various fields.

An integrated fuzzy AHP and fuzzy MULTIMOORA approach for the prioritizing of financial ratios in financial accounting fraud detection is proposed in this dissertation.

First, the weight of the evaluation criteria is calculated using the Fuzzy AHP method. Next, fuzzy MULTIMOORA technique is used to find out the best prioritize of financial ratios. These methods are based on fuzzy logic, since the ordinary fuzzy set theory is a great tool for dealing with uncertainties of the prioritization of financial ratios.

Table 4.13. Some of researches that utilize AHP, MOORA and MULTIMOORA methods with Ordinary Fuzzy Sets

Researches	Application fields	Used Approaches
Somsuk and Laosirihongthong (2014)	Assessment of the university business incubators	Fuzzy AHP
Tan et al. (2014)	Selection of process engineering	Fuzzy AHP
Brauers and Zavadskas (2006)	Privatization in a transition economy	MOORA
Gadakh (2010)	Production	MOORA
Brauers and Zavadskas (2010)	Project management	MULTIMOORA
Balezientiene et al. (2013)	Energy	MULTIMOORA
Yildirim and Onay (2013)	Evaluation of cloud-based enterprises	Fuzzy AHP and Fuzzy MOORA
Vatansever and Ulukoy (2013)	Selection of ERP software system	Fuzzy AHP and Fuzzy MOORA
Akkaya et al. (2015)	Selection of Industrial engineering sector	Fuzzy AHP and Fuzzy MOORA
Kazancoglu and Ozturkoglu (2018)	Production	Fuzzy AHP and Fuzzy MOORA
Baležentis et al. (2012)	Selection of staff	Fuzzy MULTIMOORA
Fattahi and Khalilzadeh (2018)	Assesment of various failure modes	Fuzzy FMEA, extended Fuzzy MULTIMOORA, and Fuzzy AHP
This study	Ranking of financial ratios	Fuzzy AHP and Fuzzy MULTIMOORA

4.2.1.1. Fuzzy AHP

The steps of the fuzzy AHP method are given in section 4.1.1.1.

4.2.1.2. Fuzzy MULTIMOORA

The fuzzy MULTIMOORA approach is used to determine the best prioritization of financial ratios after calculating the preference weights of qualitative characteristics (criteria) for financial accounting fraud detection. The fuzzy MULTIMOORA approach has the following steps:

Step 6 The views of the decision-makers are aggregated.

Using Eqs. (4.47) and (4.48), the aggregated fuzzy ratings $\tilde{x}_{ij}^K = (\tilde{a}_{ij}^K, \tilde{b}_{ij}^K, \tilde{c}_{ij}^K)$ of alternatives for each criterion can be calculated to build a fuzzy group decision matrix $\tilde{X} = [\tilde{x}_{ij}]_{m \times n}$.

$$\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij}) \quad (4.47)$$

$$x_{ij1} = \frac{1}{l} \sum_{k=1}^l a_{ij}^K, \quad x_{ij2} = \frac{1}{l} \sum_{k=1}^l b_{ij}^K, \quad x_{ij3} = \frac{1}{l} \sum_{k=1}^l c_{ij}^K \quad (4.48)$$

It's worth noting that the decision makers use Table 4.14 to construct fuzzy ratings $\tilde{x}_{ij}^K = (a_{ij}^K, b_{ij}^K, c_{ij}^K)$ of alternatives for each criterion.

Table 4.14. Linguistic terms for evaluating the alternatives (Fuzzy MULTIMOORA)

Linguistic variable	Triangular fuzzy number
Very low (VL)	(0, 0, 0.16)
Low (L)	(0, 0.16, 0.34)
Medium low (ML)	(0.16, 0.34, 0.5)
Moderate (M)	(0.34, 0.5, 0.66)
Medium high (MH)	(0.5, 0.66, 0.84)
High (H)	(0.66, 0.84, 1)
Very high (VH)	(0.84, 1, 1)

Step 7 The fuzzy group decision matrix is normalized.

The vector normalization approach (Liu et al. 2015) can convert the fuzzy group decision matrix $\tilde{X}$ into a normalized fuzzy group decision matrix $\tilde{R} = [\tilde{r}_{ij}]_{m \times n}$:

$$\tilde{r}_{ij} = (r_{ij1}, r_{ij2}, r_{ij3}) = \left(\frac{a_{ij}}{\tilde{c}_{ij}^*}, \frac{b_{ij}}{\tilde{c}_{ij}^*}, \frac{c_{ij}}{\tilde{c}_{ij}^*} \right) \quad (4.49)$$

$$\tilde{c}_{ij}^* = \sqrt{\sum_{i=1}^m c_{ij}^2} \quad (4.50)$$

Step 8 The weighted normalized fuzzy group decision matrix is built.

The weighted normalized fuzzy group decision matrix is constructed by using W criteria weights derived in fuzzy AHP.

$$\tilde{r}_{ij}' = (\tilde{r}_{ij1}', \tilde{r}_{ij2}', \tilde{r}_{ij3}') = w_j \otimes \tilde{r}_{ij} = (w_j r_{ij1}, w_j r_{ij2} w_j, r_{ij3}) \quad (4.51)$$

where w_j denotes criteria weights derived in fuzzy AHP method.

Step 9 The fuzzy ratio system is implemented.

Normalized performance values are calculated by subtracting the worthless criteria from the total of useful criteria described according to the type of problem (Baležentis et al., 2012).

$$\tilde{v}_i = \sum_{j=1}^g \tilde{r}_{ij}' \ominus \sum_{j=g+1}^n \tilde{r}_{ij}' \quad (4.52)$$

where $i = 1, 2, \dots, g$ denote the criteria to be maximized; $i = g + 1, g + 2, \dots, n$ denote the criteria to be minimized. The following equation is used to defuzzify normalized performance values:

$$BNP_i = \frac{(v_{i3} - v_{i1}) + (v_{i2} - v_{i1})}{3} + v_{i1}, \quad (4.53)$$

where BNP_i denotes i th alternative's best non-fuzzy performance value. As a consequence, the alternatives with larger BNP values are sorted from the most significant to the least significant.

Step 10 The fuzzy reference point methodology is implemented.

The fuzzy reference point theory is based on $\tilde{R}' = [\tilde{r}_{ij}']_{m \times n}$, the weighted normalized fuzzy decision matrix constructed using Eq (4.51). According to the ratios determined in Step 3, the Maximal Objective Reference Point is computed. The reference point's j th coordinate is reminiscent of the fuzzy maximum or minimum of the j th criterion $\tilde{X}_j^+$, where

$$\begin{cases} \tilde{X}_j^+ = (\max_i \tilde{r}_{ij1}', \max_i \tilde{r}_{ij2}', \max_i \tilde{r}_{ij3}'), & j \leq g; \\ \tilde{X}_j^+ = (\min_i \tilde{r}_{ij1}', \min_i \tilde{r}_{ij2}', \min_i \tilde{r}_{ij3}'), & j > g. \end{cases} \quad (4.54)$$

The deviation from the reference point and the Min–Max Metric of Tchebycheff are then used to determine the final rank (Balezentienė et al., 2013):

$$\min_i \left(\max_j d(\tilde{r}_j, \tilde{r}_{ij}') \right). \quad (4.55)$$

Step 11 The fuzzy full multiplicative form is implemented.

The following equation can be used to describe the overall utility of the i th alternative as a triangular fuzzy number:

$$\tilde{U}_i' = \tilde{Y}_i \oslash \tilde{Z}_i, \quad (4.56)$$

where $\tilde{Y}_i = (Y_{i1}, Y_{i2}, Y_{i3}) = \prod_{j=1}^g \tilde{r}_{ij}'$, $i = 1, 2, \dots, m$ denotes i th alternative's objectives to be maximized with $g = 1, \dots, n$ being the number of objectives to be maximized and where $\tilde{Z}_i = (Z_{i1}, Z_{i2}, Z_{i3}) = \prod_{j=g+1}^n \tilde{r}_{ij}'$ denotes i th alternative's objectives to be minimized with $n - g$ being the number of objectives to be minimized.

Because over all utility $\tilde{U}_i'$ is a fuzzy number, the alternatives can be ranked using Equation (4.53). As a result, the alternatives with larger BNP values are prioritized from largest to smallest.

4.2.2. Integrated AHP and MULTIMOORA with Single-Valued Spherical Fuzzy Sets

Few studies have integrated AHP and MULTIMOORA techniques with various extensions of fuzzy sets, such as Type-1 fuzzy sets, intuitionistic fuzzy sets, and pythagorean fuzzy sets. Tavana et al. (2020) employed integrated fuzzy AHP and fuzzy MULTIMOORA approach for selecting supplier. They used fuzzy AHP to assess the relevance of supply chain risks, as well as fuzzy MULTIMOORA to prioritize vendors. Göçer (2020) advocated combining AHP with MULTIMOORA under intuitionistic fuzzy sets environment to choose information management system for a scientific research project. Seker and Kahraman (2020) used integrated AHP and MULTIMOORA with interval-valued pythagorean fuzzy sets to choose the best acceptable solar PV panel manufacturer for solar power plants in Turkey's Southeastern Anatolia Region.

To the best of our knowledge, there is no study that integrates AHP and MULTIMOORA techniques with single-valued spherical fuzzy sets. Therefore, for the first time in the literature, this study proposes integrated AHP and MULTIMOORA

approach using single-valued spherical fuzzy sets for prioritization of financial ratios in financial accounting fraud detection.

4.2.2.1. Single-Valued Spherical Fuzzy AHP

The steps of Single-Valued Spherical Fuzzy AHP method are given in section 4.1.2.1.

4.2.2.2. Single-Valued Spherical Fuzzy MULTIMOORA

The ranking of the alternatives is obtained using the single-valued spherical fuzzy MULTIMOORA approach. The single-valued spherical fuzzy MULTIMOORA approach has the following steps:

Step 6 To begin, decision makers evaluate each alternative with respect to each criterion using the linguistic terms displayed in Table 4.5. The SWAM operator is then used to aggregate the evaluations of the alternatives by taking into account the decision makers' importance levels.

Step 7 Eq. (3.6) is used to build weighted single-valued spherical fuzzy decision matrix after getting the criteria' weights and the alternatives' ratings.

Step 8 Employ Ratio System (RS) approach.

Y_i^+ values are obtained using Eq. (4.57) in the RS approach. Y_i^+ values are then defuzzified utilizing the scoring function provided in Eq (4.58).

Finally, the alternatives are ordered in descending order based on Score (Y_i^+) values.

$$Y_i^+ = \left\{ \left[1 - \prod_{j=1}^n \left(1 - \mu_{\tilde{s}_j}^2 \right) \right]^{1/2}, \prod_{j=1}^n v_{\tilde{s}_j}, \left[\prod_{j=1}^n \left(1 - \mu_{\tilde{s}_j}^2 \right) - \prod_{j=1}^n \left(1 - \mu_{\tilde{s}_j}^2 - \pi_{\tilde{s}_j}^2 \right) \right]^{1/2} \right\} \quad (4.57)$$

$$\text{Score}(Y_i^+) = \left(2\mu_{Y_i^+} - \pi_{Y_i^+} \right)^2 - \left(v_{Y_i^+} - \pi_{Y_i^+} \right)^2 \quad (4.58)$$

Step 9 Employ Reference Point (RP) approach.

Eqs. (4.59) and (4.60) are used in the RP approach to determine single-valued reference points based on the score function values.

$$X_j^* = \left\{ C_j, \max_i \left\{ \text{Score} \left(C_j(X_{i_w}) \right) \right\} \mid j=1,2,\dots,n \right\} \quad (4.59)$$

or

$$X_j^* = \{ \langle C_1, (\mu_1^*, v_1^*, \pi_1^*) \rangle, \dots, \langle C_n, (\mu_n^*, v_n^*, \pi_n^*) \rangle \} \quad (4.60)$$

The alternatives' distances from a single-valued reference point are then computed utilizing the following equation:

$$d(X_{ij}, X_j^*) = \frac{1}{2} \left(\left| \mu_{X_{ij}}^2 - \mu_{X_j^*}^2 \right| + \left| v_{X_{ij}}^2 - v_{X_j^*}^2 \right| + \left| \pi_{X_{ij}}^2 - \pi_{X_j^*}^2 \right| \right) \quad (4.61)$$

The alternatives' deviations from the single-valued reference point are computed utilizing Eq. (4.62) after the alternatives' distances from the single-valued reference point have been calculated. The alternatives are then ordered in ascending order based on the deviation values. The best alternative is defined as the one with the lowest deviation value.

$$\min_i \left\{ \max_j d(X_{ij}, X_j^*) \right\} \quad (4.62)$$

Step 10 Employ Fuzzy Full Multiplicative Form (FMF) approach.

The values of $\tilde{S}_i$ and $\text{Score}(\tilde{S}_i)$ are computed utilizing Eqs. (4.63) and (4.64), respectively.

$$\tilde{S}_i = \left\{ \prod_{j=1}^m \mu_{\tilde{S}_{ij}}, \left[1 - \prod_{j=1}^m (1 - v_{\tilde{S}_{ij}}^2) \right]^{1/2}, \left[\prod_{j=1}^m (1 - v_{\tilde{S}_{ij}}^2) - \prod_{j=1}^m (1 - v_{\tilde{S}_{ij}}^2 - \pi_{\tilde{S}_{ij}}^2) \right]^{1/2} \right\} \quad (4.63)$$

$$\text{Score}(\tilde{S}_i) = \left(2\mu_{\tilde{S}_i} - \pi_{\tilde{S}_i} \right)^2 - \left(v_{\tilde{S}_i} - \pi_{\tilde{S}_i} \right)^2 \quad (4.64)$$

The alternatives are then ordered in descending order based on the values of $\text{Score}(\tilde{S}_i)$.

Step 11 Use the dominance theory to obtain final rank.

The alternatives are ordered based on all three approaches employed, and the final ranking is determined using the dominance theory.

4.2.3. Integrated AHP and MULTIMOORA with Interval-Valued Spherical Fuzzy Sets

Interval-valued spherical fuzzy sets, as discussed in previous sections, is newly suggested fuzzy set that incorporates decision makers' judgments on the parameters of a fuzzy set into the model using an interval rather than a single point. To the best of our knowledge, there is no study that combines AHP and MULTIMOORA techniques with IVSFS. Therefore, this study proposes integrated AHP and MULTIMOORA approach with IVSFS to evaluate financial ratios in detecting financial accounting fraud. The proposed methodology has two phases: first, calculating criteria weights using the AHP method with IVSFS (IVSF-AHP), and then prioritizing the alternatives using the MULTIMOORA technique with IVSFS (IVSF- MULTIMOORA).

4.2.3.1. Interval-Valued Spherical Fuzzy AHP

The steps of Interval-Valued Spherical Fuzzy AHP method are given in section 4.1.3.1.

4.2.3.2. Interval-Valued Spherical Fuzzy MULTIMOORA

The final ranking of the alternatives is obtained using IVSF-MULTIMOORA approach. The IVSF-MULTIMOORA approach has the following steps:

Step 6 Each alternative is evaluated with respect to each criterion.

Using the linguistic phrases displayed in Table 4.6., decision makers evaluate each alternative with respect to each criterion. The IVSWAM operator is then used to aggregate the evaluations of the alternatives by taking into account the decision makers' importance levels.

Step 7 Build the weighted interval-valued spherical fuzzy decision matrix.

Eq. (3.23) is used to build the interval-valued spherical fuzzy decision matrix after getting criteria' weights and alternatives' ratings.

Step 8 Employ Ratio System approach.

Initially, the values of Y_i^+ are obtained using Eq. (3.32). The Y_i^+ values are then defuzzified using the scoring function provided in Eq (3.34). Finally, the alternatives are ordered in descending order based on Score (Y_i^+) values.

Step 9 Employ Reference Point approach.

First, Eqs. (4.28) and (4.29) are used to calculate interval-valued reference points, as shown in Table 12.. Second, Eq. (4.32) is used to calculate the alternatives' distances from the interval-valued reference point. Third, Eq. (4.62) is used to compute the alternatives' deviations from the reference point, and then the alternatives are sorted in ascending order based on the deviation values.

Step 10 Employ Fuzzy Full Multiplicative Form approach.

Firstly, Eq. (3.33) is used to compute the values of $\tilde{S}_i$. Then, $\tilde{S}_i$ values are defuzzified utilizing the scoring function presented in Eq. (3.34). Finally, the alternatives are sorted in descending order based on the values of $\text{Score}(\tilde{S}_i)$.

Step 11 Employ the dominance theory to obtain final rank.

The alternatives are ordered based on all three approaches employed, and the final ranking is determined using the dominance theory.



5. FINANCIAL ACCOUNTING FRAUD DETECTION VIA MACHINE LEARNING

Instead of using traditional procedures for decision makers like auditors, advanced analytical tools and methodologies are needed to detect financial accounting fraud. In the literature, however, there is no common technique for financial accounting fraud detection. Altman Z-score (Altman, 1968), Beneish model (Beneish, 1999), and financial ratio (Kaminski et al., 2004) were previously utilized statistical models. Following that, researchers employed data mining techniques to detect financial accounting fraud. Logistic regression, decision trees, and artificial neural networks are examples of used data mining techniques (Persons, 1995; Spathis, 2002; Özdağoğlu et al., 2017; Omar et al., 2017). In recent years, various studies in the field of detecting financial accounting fraud have focused on hybrid approaches that combine data mining and other methodologies (Pai et al. 2011, Chen, 2016; Dutta et al., 2017; Yao et al., 2018). Table 5.1 summarizes some of researches on financial accounting fraud detection.

Spathis (2002) employed Logistic Regression technique to detect financial accounting fraud. The study did not utilize any feature selection technique to select financial ratios. Ten financial ratios based on prior studies were utilized. A balanced sample of 38 fraud firms and 38 non-fraud firms was employed, resulting in accuracy rate of 84%.

Kaminski et al. (2004) investigated the effectiveness of financial ratios in detecting financial accounting fraud. Fraudulent and nonfraudulent companies were matched, and the two groups' 21 financial ratios were analyzed using Univariate analysis (paired sample T-test) for seven years. The study's findings revealed that throughout the seven-year period, 16 of the 21 financial ratios were statistically significant at a level of 0.10 or less. Nine of the ratios were significant for only one time period, whereas four were significant for two. For three time periods, three ratios were significant. Financial accounting fraud was also detected using Discriminant analysis, with misclassifications for fraud companies ranging from 58 percent to 98 percent. Selected financial ratios were shown to have poor ability for financial accounting fraud detection.

Kotsiantis et al. (2006) employed machine learning methods, including Decision Trees, ANNs, Bayesian Networks, Logistic Regression, k-Nearest Neighbor, and SVM to

construct a hybrid decision approach. They utilized imbalanced sample of 164 Greek industrial companies that are publicly traded on the Athens Stock Exchange. There were 41 fraudulent companies and 123 non-fraudulent companies among them. Fraudulent companies were matched with non-fraudulent companies. For feature selection, a statistical metric called ReliefF was applied. The classification results are mostly determined by a narrow set of financial ratios, according to the findings.

Kirkos et al. (2007) assessed the performances of three data mining techniques, which are Decision Trees, Neural Networks, and Bayesian Belief Networks for financial accounting fraud detection. They examined balanced sample of 76 Greek industrial companies (38 fraud and 38 non-fraud companies). In terms of performance, the results revealed that Bayesian Belief Networks outperform Decision Trees and Neural Networks.

Liou (2008) examined three data mining techniques comprising Logistic Regression, ANN, and Decision Tree model in financial accounting fraud detection and company failure prediction. From previous studies, a total of 52 financial ratios were selected. Using stepwise logistic regression, 25 of the 52 were found to have substantial detection power in detecting financial accounting fraud. 19 out of 52 financial ratios were found to have substantial predictive potential in predicting company failure using the same strategy. The study's findings show that Logistic Regression surpasses the other two algorithms in terms of overall accuracy for financial accounting fraud detection, and a decision tree predicts company failure with a larger proportion.

Persons (1995) employed Stepwise-Logistic model to predict fraudulent financial reporting. The study examined a sample of 103 fraud businesses for the current fraud year and 100 fraud firms from the previous fraud year. On the basis of industry and time period, each fraud firm was paired with a non-fraud firm. The study's findings demonstrate that the model properly detects a big number of fraud companies while incorrectly classifying a small percentage of non-fraud companies.

Ravisankar et al. (2011) used data mining methods including Multilayer Feed Forward Neural Networks, Support Vector Machines, Genetic Programming, Group Method of Data Handling, Logistic Regression, and Probabilistic Neural Network for financial accounting fraud detection. These methods are compared with and without feature

selection. As a feature selection approach, T-test is utilized. The dataset for this study was compiled from 202 firms listed in various Chinese stock markets, 101 of which were fraudulent and 101 of which were non-fraudulent. Results revealed that Genetic Programming and Probabilistic Neural Network outperform all other approaches with feature selection, whereas Probabilistic Neural Network outperforms others without feature selection.

Huang (2013) employed Support Vector Machine and Logistic Regression to construct fraud detection model. The instances for this study were taken from public offering firms, and they were chosen in a "1:4" ratio of those in the same industry who were prosecuted in the same year with those who were not. The variables were chosen using T-test. The findings revealed that Logistic Regression using chosen indicators can accurately predict financial accounting fraud one year before the fraud year, with an accuracy rate of 91 percent. Furthermore, Support Vector Machine using all variables has an 82 percent accurate rate for the two years before the fraud year.

Chen (2016) developed a two-stage methodology for financial accounting fraud detection. For variable selection in the first stage, two decision tree methods were used: Classification and Regression Trees (CART) and Chi Squared Automatic Interaction Detector (CHAID). In the second stage, models for detecting financial accounting fraud were built using CART, CHAID, Bayesian Belief Networks, Support Vector Machines, and Artificial Neural Networks. The findings revealed that the CHAID–CART model has the best detection performance, with an overall accuracy of 87.97 percent.

Özdağoğlu et al. (2017) used Decision Trees, Logistic Regression, and Artificial Neural Networks to construct a decision support model to classify enterprises. The financial statements of Turkish companies listed on Borsa Istanbul were used and a balanced sample was employed in the study, with 110 instances classified as fraudulent and 114 instances classified as non-fraudulent. According to the findings, Artificial Neural Network has the best prediction accuracy.

Omar et al. (2017) employed Artificial Neural Network to predict financial accounting fraud in Malaysian small market value enterprises. To predict financial accounting fraud, ten financial ratios were employed as fraud risk indicators. Additionally, each fraud firm was paired with about six non-fraud firms. The findings of this study showed

that the suggested ANN approach predicts financial accounting fraud with a greater accuracy (94.87%) than classical statistics, Linear Regression (92.4%), and other techniques used in prior studies.

Dutta et al. (2017) used Decision Tree, Artificial Neural Network, Naive Bayes, Support Vector Machine and Bayesian Belief Network to construct prediction models based on intentional and unintentional financial restatements. Their dataset includes 3513 restatement cases from 2001 to 2014. There are 3513 cases in the restatement group and 60720 cases in the non-restatement group. Because the two classes in the dataset are substantially imbalanced, an oversampling approach (i.e. SMOTE) was utilized to mitigate the problem of class imbalance. A feature selection technique based on Stepwise Forward Selection was also utilized in the study to exclude less important or unnecessary attributes. In terms of accuracy and area under the ROC curve, the results showed that ANN outperforms other data mining techniques.

Yao et al. (2018) suggested an approach that integrates feature selection and machine learning classification methods for detecting financial fraud. As classification techniques, Random Forest, ANN, Logistic Regression, SVM, and Decision Tree were employed. The feature selection techniques were Principal Component Analysis and Xgboost. Random Forest outperforms the other four techniques, according to the findings. Xgboost has better performance in terms of feature selection.

Ye et al. (2019) examined several classification methods including Random Forest, Artificial Neural Network, Logistics Regression, Support Vector Machines, CART, Decision Trees, Bayesian Networks, Bagging, Stacking, and Adaboost in financial accounting fraud detection. The utilized dataset comprises publicly available financial statements from 2011 Chinese enterprises, of which 1314 were fraudulent and 10412 were not. SMOTE was used because of the dataset's high-class imbalance. To minimize dimensionality, the Chi-Square test was utilized as a feature selection approach in the study. Random forest outperformed the other nine approaches, according to the findings.

Table 5.1. Some of researches that utilize machine learning classifiers for financial accounting fraud detection

Study	Considered Data	Utilized Feature Selection Techniques	Used Sampling Techniques	Utilized Machine Learning Classifiers	Performance Measure
Spathis (2002)	Athens Stock Exchange	-	-	Logistic Regression	Accuracy
Kaminski et al. (2004)	U.S SEC's listed companies (AAERs)	-	-	Resubstitution, Cross-validation method	Accuracy
Kotsiantis et al. (2006)	Athens Stock Exchange manufacturing firms	ReliefF	-	Decision Tree, ANN, Bayesian Networks, kNN, Rule-learners, SVM, Stacking	Accuracy
Kirkos (2007)	Athens Stock Exchange	ANOVA	-	Decision Trees, Neural Networks, Bayesian Belief Networks	Accuracy, Type I and Type II error
Liou (2008)	Taiwan Economic Journal data bank and Taiwan Stock Exchange Corporation	Stepwise Logistic Regression	-	Logistic Regression, ANN, Decision Tree	Accuracy
Persons (1995)	U.S SEC's listed companies (AAERs) and COMPUSTAT database	-	-	Logistic Regression	Type I and Type II error
Ravisankar et al. (2011)	Chinese listed companies	T-test	-	Genetic Programming, SVM, Group Method of Data Handling, Multi-layer Feedforward Neural Network , Probabilistic Neural Network, Logistic regression	Accuracy, Sensitivity, Specificity, AUC

Table 5.1. (Continuation)

Huang (2013)	Taiwan Economic Journal data bank, Financial Supervisory Commission, the Securities and Futures Investors Protection Center	T-test	-	SVM, Logistic Regression	Accuracy, Type I and Type II error
Chen (2016)	Taiwan's listed and OTC companies	DT- CART and CHAID algorithm	-	CART,CHAID,BBN,SVM,ANN	Accuracy, Type I and Type II error
Özdağoğlu et al. (2017)	Turkish listed companies	-	-	Decision Tree,Logistic Regression,ANN	Accuracy,Precision,Recall,F-Measure,AUC
Omar et al. (2017)	Malaysian small market capitalization companies	-	-	ANN	Accuracy
Dutta et al. (2017)	Audit Analytics and COMPUSTAT database	Stepwise Forward Selection	Over Sampling (SMOTE)	Decision Tree ,ANN, Naïve Bayes, SVM, Bayesian Belief Network	Accuracy, False positive rate,Precision, Recall, Specificity, F-Measure, AUC
Yao et al. (2018)	Chinese listed companies	Principal component analysis, Xgboost	-	Random Forest, ANN, Logistic Regression, SVM, Decision Tree	Accuracy
Ye et al. (2019)	Chinese listed companies	Chi-square	Over Sampling (SMOTE)	Random Forest,ANN,Logistic Regression, SVM, CART, Decision Tree, BBN, Bagging, Stacking, Adaboost	Precision,Recall, F-Measure, AUC, Kappa statistic
<i>This study</i>	<i>Turkish SMEs</i>	<i>T-test and Genetic Search</i>	<i>Over Sampling (SMOTE), Under Sampling</i>	<i>Random Forest, ANN, Logistic Regression, SVM, kNN, Naive Bayes, Bagging</i>	<i>Accuracy, True positive rate, Precision, G-Measure, AUC</i>

To the best of our knowledge, prior researches have not yet adequately handled the problem of class imbalance. Most studies used a balanced sample (Spathis, 2002; Kirkos et al., 2007; Ravisankar et al., 2011; Özdağoğlu et al., 2017; Yao et al., 2018) or paired fraudulent and non-fraudulent enterprises at various ratios (Kaminski et al., 2004; Kotsiantis et al., 2006; Chen, 2016; Omar et al., 2017). Few researchers have employed sampling techniques in financial fraud detection models, as shown in Table 4.1. (Dutta et al., 2017 and Ye et al., 2019). This study extends the existing literature by employing sampling techniques to handle the problem of class imbalance in financial accounting fraud detection.

5.1. Machine Learning Classifiers

In a comprehensive literature study on machine learning approaches, Ngai et al. (2011) found that Logistic Regression and Neural Networks are the most often utilized data mining techniques in detecting financial accounting fraud. In addition to these frequently used data mining techniques, Support Vector Machine, Naive Bayes, K-Nearest Neighbour, Random Forest, and Bagging were utilized in this study to detect financial account fraud. The following subsections outline the theoretical basis of the classifiers employed in this study.

5.1.1. Support vector machines (SVM)

Vapnik (1995) proposed SVM as a machine learning approach based on statistical learning theory and structural risk reduction. Through the learning process, SVM develops an ideal separating hyperplane to categorize two or more classes of data. SVM is characterized as linear divisibility when the dataset is linearly separable or linear non-divisibility when there is no linear hyperplane that clearly separates the dataset (Chen, 2016). The linear non-divisibility category encompasses the majority of real-world situations. The non-linear SVM algorithm is kernelized in such circumstances to describe a separating hyperplane (Dutta et al., 2017). Polynomial learning, radial basis functions, and two-layer neural networks are the three types of learning machines considered by Vapnik (1995). The polynomial kernel, a kernel function mostly used with Support Vector Machines, is utilized to detect financial accounting fraud in this study.

5.1.2. Naive bayes

Bayes' theorem is used to build Naive Bayes classifier. The conditional probability $P(X|Y)$ can be calculated using the Bayes theorem in terms of $P(X)$, $P(Y)$, and $P(Y|X)$.

$$P(X|Y) = \frac{P(Y|X) * P(X)}{P(Y)} \quad (5.1)$$

$$P(X|Y) = (\prod_{i=1}^n P(Y_i|X)) * P(X) \quad (5.2)$$

where, $P(X|Y)$ denotes the conditional probability of a class for a given predictor; $P(X)$ denotes the prior probability of class X ; $P(Y|X)$ denotes the likelihood which is the probability of predictor given a class; and $P(Y)$ denotes the prior probability of predictor (Dutta et al., 2017).

Because of flaws in assumptions made for its usage, such as class-conditional independence, and a lack of sufficient probability data, the Naive Bayes classifier has a disadvantage. However, in order to estimate the parameters, this technique only requires a small amount of training data. When the input dimensionality is large, this classification algorithm also provides an advantage (Han et al., 2012). For financial accounting fraud detection, the Naive Bayes classifier is commonly employed (Deng, 2010; Moepya et al., 2014; Dutta et al., 2017; Krishna and Ravi, 2019).

5.1.3. Artificial neural network (ANN)

ANN algorithm is a supervised machine learning technique based on biological neural networks. Through the learning method, the ANN obtains significant computing capabilities. A neural network is made up of a large number of artificial neurons that are linked together. Each of these neurons has its own memory and is capable of communicating with other neurons via one-way signal channels (Omar et al., 2017). Layers of neurons have been created. The input layer and output layer are the two primary layers. Between the input and output layers, there may be one or more hidden layers. By training examples, neural networks build their own rules. Firstly, a collection of paired data is used to train the network. The network is then used to categorize a new dataset once the connections' weights between the neurons are fixed (Kotsiantis et al.,

2006). ANN has been utilized in a number of researches in recent years to construct models for detecting financial accounting fraud (Özdağoğlu et al., 2017; Omar et al., 2017; Yao et al., 2018; Ye et al., 2019).

5.1.4. K-nearest neighbour (kNN)

The K-Nearest Neighbour technique predicts a sample's categorization by looking at its geographic vicinity. This approach expects that the instances in a dataset will be quite intimate to those in the same class. Hence, the k-closest instances from the training set are used to predict a new sample. Euclidean distance is a distance metric that is commonly employed to determine the measure of proximity. Manhattan, Minkowski, and Mahalanobis distances can be utilized instead of the Euclidean distance. The measure chosen is determined on the features of the variables (Moepya et al., 2014). A new instance is allocated to the appropriate class when the measure of proximity has been determined. The kNN approach is commonly used to detect financial accounting fraud (Kotsiantis et al., 2006; Moepya et al., 2014; Tarjo and Herawati, 2017; Yao et al., 2019).

5.1.5. Random forest

A random forest is a collection of numerous decision trees used in an ensemble approach. Each decision tree is generated using a random choice of attributes at each node to compute the split. The values of a random vector sampled independently and with the same distribution for all of the forest's trees are dependent on each tree (Breiman, 2001; Han et al., 2012). Following the construction of each tree, the categorized sample is input, and each tree provides a classification, which is referred to as voting. The majority rule is then used to determine the classification result for all classifiers (Yao et al., 2018).

Some studies in the field of financial accounting fraud detection have utilized this approach in recent years (Liu et al., 2015; Yao et al., 2018; Ye et al., 2019), and these researches have noticed that the Random Forest has some benefits: it is useful for processing high-dimensional data; it may prevent overfitting to some extent; it has higher durability and reliable results; and it has good accuracy performance.

5.1.6. Logistic regression

Logistic Regression is a frequent approach in binary classification circumstances, where the classifying variable can only have two possible values, such as true/false or success/failure. It learns a set of parameters that maximizes the probability of class labels for a given batch of training data (Williams, 2009; Ravisankar et al, 2011). These parameters are utilized to predict the class' values in the testing dataset once they have been learned.

The formula for the Logistic Regression model is as follows:

$$P(Y=1|X)=\frac{1}{1+e^{-(\beta_1x_{1,i}+\dots+\beta_kx_{k,i}+\dots+\beta_nx_{n,i})}}, \quad (5.3)$$

where $P(Y=1|X)$ denotes the likelihood that firm commits financial accounting fraud; $x_{k,i}$ (for $k=1,\dots,n$) denotes an independent variable; β coefficients denote forecasted parameters for independent variables (Papik and Papikova, 2020).

Logistic Regression has a straightforward and understandable concept. It has also comparatively low computational cost (Yao et al.,2018). Hence, Logistic Regression is used in the majority of existing studies on financial accounting fraud detection (Persons, 1995; Spathis, 2002; Liou, 2008; Ravisankar et al., 2011; Özdağoğlu et al., 2017; Yao et al., 2018; Ye et al.; 2019).

5.1.7. Bagging

Bagging is an ensemble approach that uses a number of different classifiers. Breiman(1996) proposed this composite approach for improving classification accuracy of class-imbalanced data.

The following is how Bagging works. A training set, A_i , is sampled with replacement from the original dataset (A) for each iteration ($i = 1, 2, \dots, n$), and each training set is a bootstrap sample. Because replacement sampling is employed, some of the original samples may not be contained in A_i , while others may appear many times. For each training set, A_i learns a classifier model. Each classifier produces a prediction, which counts as one vote, in order to categorize an unknown sample. The majority rule is then used to determine the classification result.

Because the composite model decreases the variance of the separate classifiers, Bagging is more accurate than the basic classifiers. It is also less susceptible to the impacts of noisy data and overfitting (Han et al., 2012). The ensemble model in this study is built using a bagging classifier with the decision tree method J48, which is based on the C4.5 model. Because our dataset is highly imbalanced, we used decision tree as a single classifier for our ensemble model. By weighting trees's outcomes and lowering dataset variance, the decision tree algorithm has a very strong performance (Zareapoor and Shamsolmoali, 2015).

5.2. Sampling Methods

For seven distinct machine learning classifiers, two sampling techniques were used in this study. Oversampling is the first technique. As an oversampling technique, Synthetic Minority Oversampling Technique (SMOTE), which creates new synthetics data for the minority class was utilized. The second technique is undersampling, which involves removing data from the majority class at random.

5.2.1. Oversampling

Chawla et al. (2002) introduced SMOTE as an oversampling technique. Rather than replicating the existing instances of the minority sample, this technique creates random synthetic examples of the minority class. To generate these synthetic minority instances, the SMOTE approach employs the k-nearest neighbor technique. Depending on the quantity of over-sampling necessary, neighbors are selected at random from the k nearest neighbors. The difference between the feature vector (minority class sample) and its closest neighbor is then calculated. This difference is then multiplied by a random value between 0 and 1 before being added to the feature vector. As a result, a new feature vector is generated (Chawla et al., 2002).

To construct a balanced dataset, different amounts of synthetic oversampling may be required. Because the previous imbalance ratio is 1:4.31, the proportion of SMOTE instances to be generated in this research is 300.

5.2.2. Undersampling

Random undersampling was used as one of the undersampling approaches in this study. This technique chooses a collection of majority class instances at random and removes them to mitigate the problem of class imbalance (Hasanin and Khoshgoftaar, 2018). Randomization, on the other hand, might be trouble when undersampling datasets to balance them. The split is fully random with this method, and only a small portion of the data is kept. Due to this randomness, extremely excellent (clean) sampled examples, which can improve model performance or noisy examples, which may reduce model performance can be generated (Hasanin et al., 2019). As a result, there is no clear agreement on which approach produces superior outcomes (Drummond and Holte, 2003; Bajer et al., 2019). It should be stated that while comparing the success of classifiers, the "without sampling" approach is also used in this study.

5.3. Feature Selection

Two selection techniques were employed to minimize dimensionality: T-test and Genetic Algorithm.

5.3.1. T-test

In several researches, the T-test has been employed to select features (Liu et al., 2002; Fu et al. 2006; Ravisankar et al., 2011; Wang et al., 2012; Huang, 2013). The T-test is also used as the first feature selection technique in this study.

The T-test is used to determine if the means of fraudulent and non-fraudulent businesses are statistically different. For each financial ratio, the t-statistic values are calculated.

5.3.2. Genetic search

The second used feature selection technique is Genetic Search, which is a search based on a genetic algorithm. This technique can successfully explore huge search areas, which is generally necessary for selecting attributes.

5.4. Performance Metrics

Seven machine learning classifiers were examined using performance measures (metrics) after preprocessing data with feature selection and sampling methods. Overall accuracy, TP rate (Sensitivity-Recall), TN rate (Specificity), Precision, G-measure, and ROC (Receiver operating characteristic) curve are the performance measures employed in this study. These performance measures are computed as follows:

$$\text{Overall Accuracy} = \frac{\text{True Positive} + \text{True Negative}}{\text{True Positive} + \text{False Negative} + \text{False Positive} + \text{True Negative}} \quad (5.4)$$

$$\text{TP Rate (Sensitivity-Recall)} = \frac{\text{True Positive}}{\text{True Positive} + \text{False Negative}} \quad (5.5)$$

$$\text{TN Rate (Specificity)} = \frac{\text{True Negative}}{\text{True Negative} + \text{False Positive}} \quad (5.6)$$

$$\text{Precision} = \frac{\text{True Positive}}{\text{True Positive} + \text{False Positive}} \quad (5.7)$$

$$\text{G-measure} = \sqrt{\frac{\text{True Positive}}{\text{True Positive} + \text{False Negative}} \cdot \frac{\text{True Negative}}{\text{True Negative} + \text{False Positive}}} \quad (5.8)$$

The confusion matrix, which defines variables in formulas (5.4)- (5.8), is shown in Table 5.2. The quantity of instances that are genuinely positive and accurately classified as positive is referred to as True Positive (TP). The quantity of instances that are genuinely negative and accurately classified as negative is referred to as True Negative (TN). The quantity of instances that are misclassified as positive when they are actually negative is referred to as False Positive (FP). The quantity of instances that are truly positive but are misclassified as negative is represented by False Negative (FN) (Sun et al., 2018).

Overall accuracy is the percentage of correctly predicted cases, TP rate is the percentage of correctly detected fraud cases, TN rate is the percentage of accurately detected non-fraud cases, and Precision is the percentage of accurately predicted fraud cases that are correct. The geometric mean of true positive and true negative rates is referred to as G-measure.

Plotting the TP rate against the FP rate at various threshold levels yields the Receiver Operating Characteristic (ROC) curve. The ROC curve has been commonly utilized to assess classifiers' performances in skewed and overlapping datasets (Sisodia et al., 2017).

Table 5.2. Confusion matrix

		Predicted Class	
		Positive	Negative
Actual Class	Positive	True Positive(TP)	False Negative (FN)
	Negative	False Positive(FP)	True Negative(TN)

6. APPLICATION STUDIES AND RESULTS

In this section, the obtained data is introduced firstly. Descriptive and inferencial statistics are presented. Following that, application studies and the results of the proposed approaches are given. The algorithms and phases of the proposed approaches are detailed in the application studies.

6.1. Data

341 Turkish SMEs have been evaluated in this study using their own financial statements for a five-year period from 2013 to 2017. There are 321 fraud cases and 1384 non-fraud cases in the sample. Because two classes in the dataset are severely imbalanced, sampling methods are especially regarded to be used in order to reduce the problem of class imbalance. This data is collected from one of the sector's main banks.

6.1.1. Data collection

This study takes into consideration the codes and related accounts in Turkey's Uniform Chart of Accounts, which was published in the Official Gazette on December 26, 1992. The Uniform Chart of Accounts governs accounting's fundamental concepts and principles. Enterprises in Turkey are required to follow the accounting procedures and principles outlined in this communication.

The financial statements of 341 Turkish SMEs from various industries (manufacturing, construction, transportation, agriculture, and so on) were examined using Microsoft Excel and Minitab Statistical Software 17. After examining all financial accounts of these companies, manipulated financial accounts are detected and identified.

6.1.2. Descriptive statistics

According to the European Union official classification of economic activities, called NACE (Nomenclature statistique des) (Eurostat, 2008), the firms that were analyzed for fraud detection in this study were classified into six sectors: Wholesale and Retail Trade, Agricultural, Transportation, Manufacturing, Construction, and Accomodation and Food Service. Then, the percentage of each sector in the sample was computed. For instance, the Wholesale and Retail Trade sector has been calculated to account for 40% of the

sample. Similar calculations are carried out for the other industries. Figure 6.1 depicts the proportion of each sector in the sample.

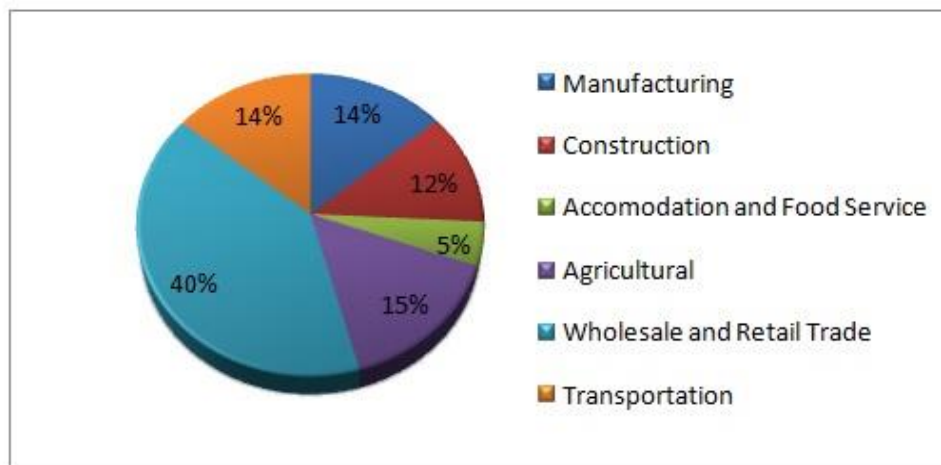


Figure 6.1. SMEs' sectoral distribution

Following the foregoing computations, more in-depth analyses have been carried out utilizing Turkish SMEs' financial statements. In total, 64% of companies are determined to be non-fraudulent, whereas 36% are found to be fraudulent (Figure 6.2).



Figure 6.2. SMEs' relative frequency

The analyses have been then expanded on a sectoral basis. In the Wholesale and Retail Trade sector, 65% of companies are determined to be non-fraudulent, while 35% are found to be fraudulent. Furthermore, for each industry, the relative frequencies of both fraudulent and non-fraudulent financial statements have been assessed. In the Agricultural sector, 85% of the companies have non-fraudulent financial statements, whereas 15% have fraudulent financial statements. In the Transportation sector, 54% of companies have non-fraudulent financial statements, whereas 46% have fraudulent

financial statements. In the Manufacturing sector, 72% of companies have non-fraudulent financial statements, whereas 28% have fraudulent financial statements. In the Construction sector, 44% of companies have non-fraudulent financial statements, whereas 56% have fraudulent financial statements. In the Accommodation and Food Service sector, 50% of companies have non-fraudulent financial statements and 50% have fraudulent financial statements. Table 6.1 shows the relative frequency of fraudulent and non-fraudulent companies.

Table 6.1. Relative frequency of fraudulent and non-fraudulent companies

Sectors	Non Fraud	Fraud	Total
Manufacturing	72%	28%	100%
Construction	44%	56%	100%
Accomodation and Food Service	50%	50%	100%
Agricultural	85%	15%	100%
Wholesale and Retail Trade	65%	35%	100%
Transportation	54%	46%	100%

Table 6.2 shows the frequency distribution of fraudulent and non-fraudulent companies. Figure 6.3 also illustrates the frequency distribution of fraudulent and non-fraudulent companies.

Table 6.2. Frequency distribution of fraudulent and non-fraudulent companies

Sectors	Non Fraud	Fraud	Total
Manufacturing	34	13	47
Construction	18	23	41
Accomodation and Food Service	8	8	16
Agricultural	44	8	52
Wholesale and Retail Trade	89	48	137
Transportation	26	22	48
Total	219	122	341

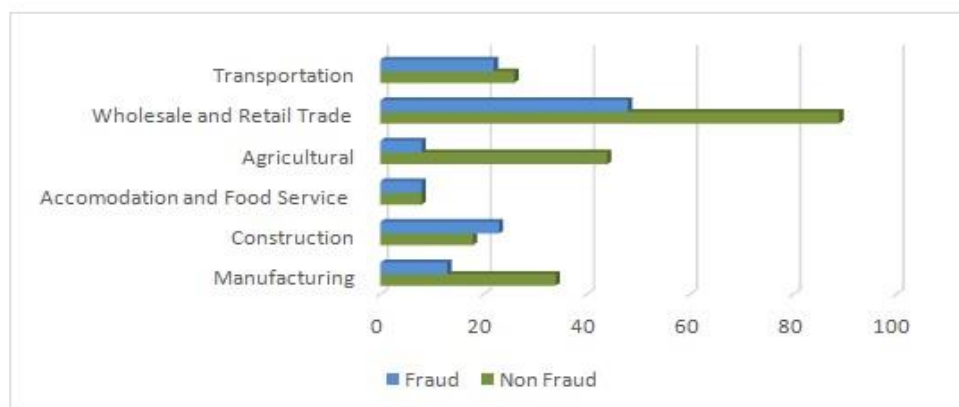


Figure 6.3. Frequency distribution of fraudulent and non-fraudulent companies

Non-fraudulent companies are in the majority in all sectors except Construction and Accommodation and Food Service, as seen in the graph above. The number of non-fraudulent and fraudulent companies in the Accommodation and Food Service sector is equal. Fraudulent companies are predominant in the Construction sector. Figure 6.4 also displays the relative frequency of fraudulent companies by sector.

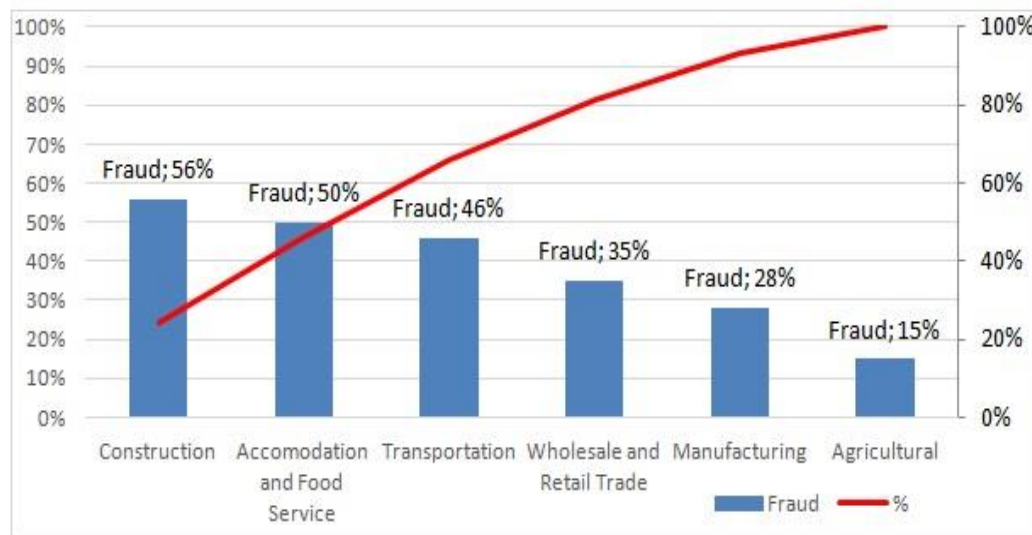


Figure 6.4. Fraudulent firms' relative frequency on a sectoral basis

A company may commit fraud for one year, non-consecutive years, or consecutive years. Figure 6.5. depicts the distribution of fraudulent companies by the number of years they have been committed fraud. Only 39 out of 122 companies committed fraud for a year. For two years, 22 firms committed fraud, and for three years, 23 firms committed fraud. For the five years from 2013 to 2017, a total of 321 fraudulent cases are found among 122 fraudulent companies. This dataset contains 1384 non-fraudulent cases and 321 fraudulent cases in total (Figure 6.6). The distribution of fraudulent cases by year is shown in Figure 6.7. Figure 6.7 also shows that the number of fraudulent cases rose considerably from 2013 to 2017, peaking in 2017.

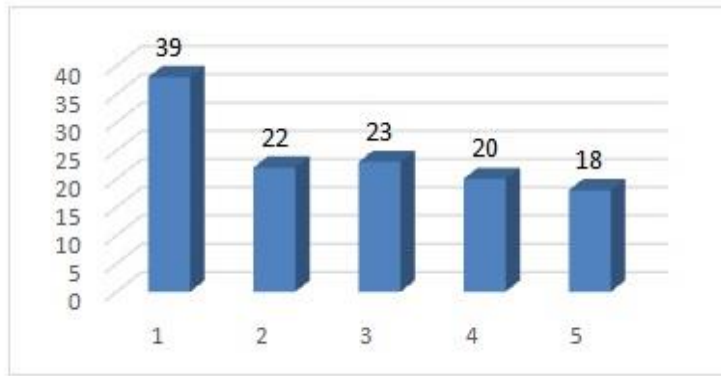


Figure 6.5. The distribution of fraudulent companies by the number of years

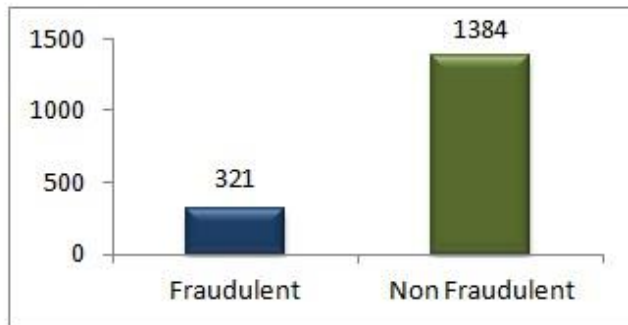


Figure 6.6. Cases' distribution

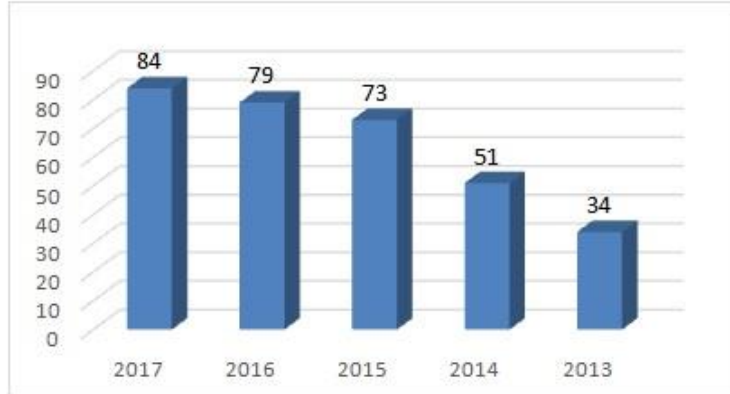


Figure 6.7. Fraud cases' distribution of by year

Financial accounts used for fraudulent financial reporting activities are depicted in Figure 6.8. Cash (100), Due from Shareholders (131), Inventories (15), Due to Shareholders (331), Prepaid Expenses for Future Months (180), and Prepaid Expenses for Future Years (280) are the most commonly utilized financial accounts for financial accounting fraud. Table 6.3 also shows the frequency distribution of financial accounts utilized for fraudulent financial reporting activities.

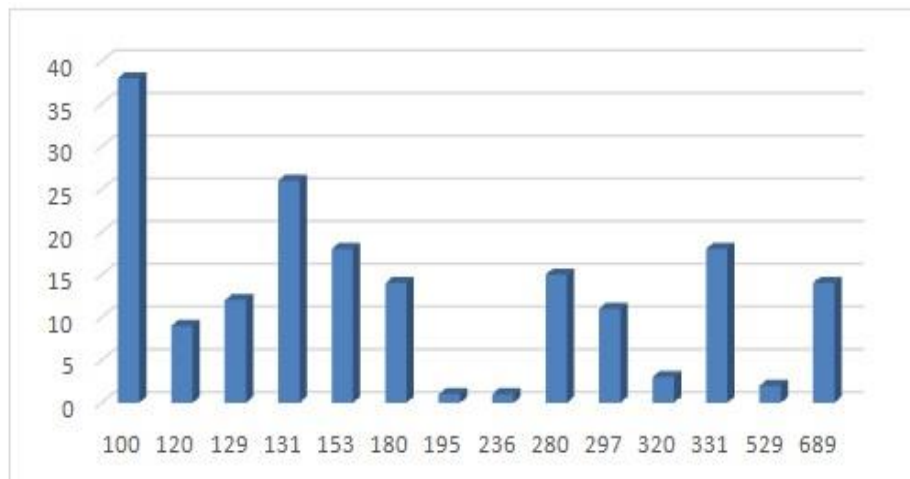


Figure 6.8. Frequency distribution of financial accounts utilized for fraudulent activities

Table 6.3. Frequency distribution of financial accounts utilized for fraudulent activities

Sectors	*100	120	128	131	15	180	195	236	280	297	320	331	529	689
Manufacturing	4		2	5	1			1	1	1		1	1	1
Construction	11	1	1	6		2			2	4		1		4
Accomodation and Food Service	3			2	2				2			4		
Agricultural			3	3	3	2	1				1	2		
Wholesale and Retail Trade	17	8	4	6	10	1			4	5	2	8		6
Transportation	3		2	4	2	9			6	1		2	1	3
Total	38	9	12	26	18	14	1	1	15	11	3	18	2	14

Figure 6.9 shows the most commonly utilized financial accounts for financial accounting fraud on a sectoral basis. Cash (100), Doubtful Trade Receivables (128) and Due from Shareholders (131) were the most commonly manipulated financial accounts by fraudulent companies in the manufacturing sector. Cash (100) and Due to Shareholders (331) financial accounts were the most commonly utilized for fraudulent activities in the Accomodation and Food Service sector. The most common financial accounts manipulated by fraudulent construction companies were Cash (100) and Due from Shareholders (131). Doubtful Trade Receivables (128), Due from Shareholders (131), and Inventories (15) financial accounts were the most commonly utilized for fraudulent activities in the Agricultural sector. The most commonly utilized financial accounts for fraudulent activities in the Wholesale and Retail Trade sector were Cash (100) and Inventories (153). Prepaid Expenses for Future Months (180) and Prepaid Expenses for Future Years (280) financial accounts were mostly manipulated by fraudulent transportation companies.

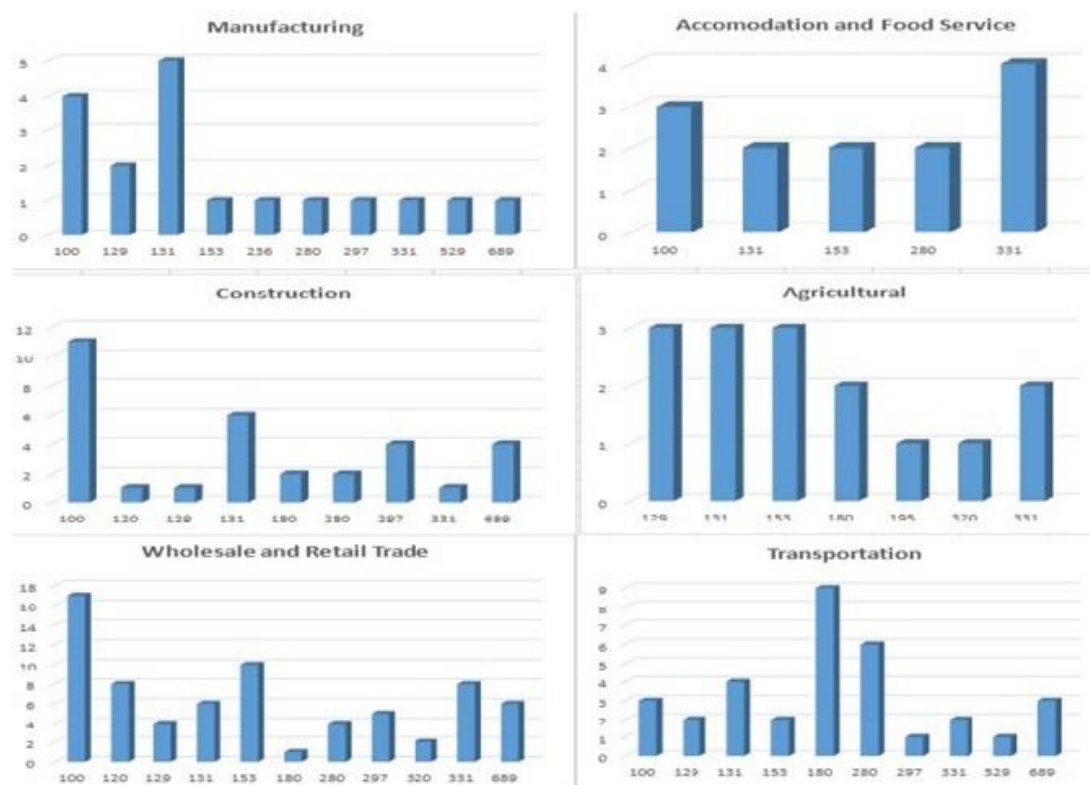


Figure 6.9. The most commonly utilized financial accounts for financial accounting fraud on a sectoral basis

6.1.3. Inferencial statistics

To determine if the difference in averages of manipulated financial accounts is statistically significant, T-test for independent samples has been applied between fraudulent and non-fraudulent companies. The means, standard deviations, t-values, and p-values for each financial account manipulated in this dataset are shown in Table 6.4. All of the manipulated financial accounts have low p-values ($p \leq 0.05$), as shown in Table 6.4. In other words, the difference in averages of manipulated financial accounts between fraudulent and non-fraudulent companies is statistically significant. There is enough proof that the manipulated accounts are out of the ordinary and exhibit indicators of accounting fraud.

Table 6.4. The Means, Standard deviations, T-values and P-values for manipulated financial accounts

			Fraudulent Companies		Non-Fraudulent Companies		t-statistic	
Category		Financial Accounts	Mean	Standard deviation	Mean	Standard deviation	t-value	p-value
Balance Sheet	Current Assets	Cash (100)	1,621,813	1,401,232	50,211	99,946	10.03	0.000
		Accounts Receivable (120)	9,394,563	2,770,517	2,158,464	2,183,268	13.94	0.000
		Doubtful Trade Receivables (128)	1,738,804	841,926	12,985	59,228	10.25	0.000
		Due from Shareholders (131)	2,471,783	1,473,028	18,973	108,666	13.73	0.000
		Inventories (15)	6,412,806	3,110,386	1,140,014	2,030,495	11.91	0.000
		Prepaid Expenses for Future Months (180)	1,747,432	1,020,907	34,271	126,828	7.87	0.000
	Long-Term Assets	Work Advances (195)	2,509,569	977,684	26,769	230,808	5.68	0.005
		Other Miscellaneous Receivables (236)	3,162,690	803,045	1,865	21,832	7.87	0.004
		Prepaid Expenses for Future Years (280)	1,453,295	1,044,036	64,148	172,72	7.52	0.000
		Other Fixed Assets (297)	2,407,272	1,432,269	8,339	99,537	6.04	0.000
	Liabilities	Accounts Payable (320)	9,922,304	2,254,619	1,646,648	2,257,685	6.34	0.000
		Due to Shareholders (331)	3,680,307	2,922,639	445,250	1,369,503	7.64	0.000
	Shareholder's Equity	Other Capital Reserves (529)	7,671,716	3,241,836	155,724	661,706	6.13	0.001
		Other Extraordinary Expenses and Losses (689)	2,372,291	1,296,676	73,166	271,850	6.87	0.000
Income Statement	Expenses							

6.2. Financial Accounting Fraud Detection Via MCDM

This study focuses on MCDM approaches, which are integrated AHP and TOPSIS and integrated AHP and MULTIMOORA for ranking financial ratios in detecting financial accounting fraud. These two approaches are applied with different extensions of fuzzy set theory which are Ordinary fuzzy sets, Single-valued spherical fuzzy sets and interval-valued spherical fuzzy sets. Moreover, integrated AHP and TOPSIS approach is extended with Circular intuitionistic fuzzy sets. Detailed steps and algorithms of MCDM approaches with different fuzzy sets used in this study are given in the following subsections.

6.2.1. Integrated AHP and TOPSIS with different fuzzy sets

In this study, integrated AHP and TOPSIS with different fuzzy sets have been proposed for ranking of financial ratios. In the proposed approaches, the performances of financial ratios for financial accounting fraud detection have been assessed by financial analysts. Then, integrated AHP and TOPSIS with different fuzzy sets, which are Ordinary Fuzzy Sets (Type-1), Spherical Fuzzy Sets (including interval-valued and single-valued fuzzy numbers) and Circular Intuitionistic Fuzzy Sets, have been used to find out the best prioritization of financial ratios. After finding the prioritization of financial ratios for detecting financial accounting fraud, seven different machine learning classifiers are compared by using performance metrics.

6.2.1.1. Integrated Fuzzy AHP and Fuzzy TOPSIS

In this section, integrated Fuzzy AHP and Fuzzy TOPSIS is employed for prioritizing financial ratios based on the fraud detection performance.

Algorithm of the application

This study consists of two phases. In the first phase, feature selection is employed. In the second phase, sampling methods and seven different classifiers are executed and compared by using performance metrics.

The details of the phases are given below.

Phase 1: In this phase, integrated Fuzzy AHP and Fuzzy TOPSIS is proposed as a

feature selection method. First, the performance of financial ratios for financial accounting fraud detection is assessed by financial analysts. Four decision makers are selected to evaluate the importance of qualitative characteristics for financial fraud detection and then determining the preference weights of this evaluation is done by fuzzy AHP method. Next, fuzzy TOPSIS technique is used to find out the best prioritization of financial ratios. These methods are based on fuzzy logic, since the ordinary fuzzy set theory is a great tool for dealing with uncertainties of the prioritization of financial ratios.

Phase 2: After finding ranking of financial ratios for detecting financial accounting fraud, two sampling methods of SMOTE as oversampling and undersampling are performed. Then, seven different machine learning classifiers, which are Support Vector Machine, Naive Bayes, Artificial Neural Network, K-Nearest Neighbour, Random Forest, Logistic Regression and Bagging, are compared by utilizing performance measures. Without employing any feature selection or sample methods, classifiers are also compared. Such, the success of integrated Fuzzy AHP and Fuzzy TOPSIS-MCDM methodology proposed as a feature selection method for improving the effectiveness of fraud detection model is measured.

On the basis of algorithm of proposed approach for improving financial accounting fraud detection model, the implemented steps are shown in Figure 6.10.

Steps of the application

The proposed approach is composed of several steps as given in this section. These steps are as follows;

Phase 1: In this phase, a case study is presented to provide prioritization of financial ratios which are used for detecting financial fraud. The proposed approach proves there may be different perspectives along with perceptions of decision makers to make prioritization of financial ratios. Four financial analysts are selected to demonstrate the applicability and validity of the proposed approach.

A collection of financial ratios is examined in this study, which comprises 20 financial ratios that have been extensively utilized in prior studies and 5 financial ratios that are commonly manipulated by enterprises in practice. These alternatives can be seen from Figure 6.11.

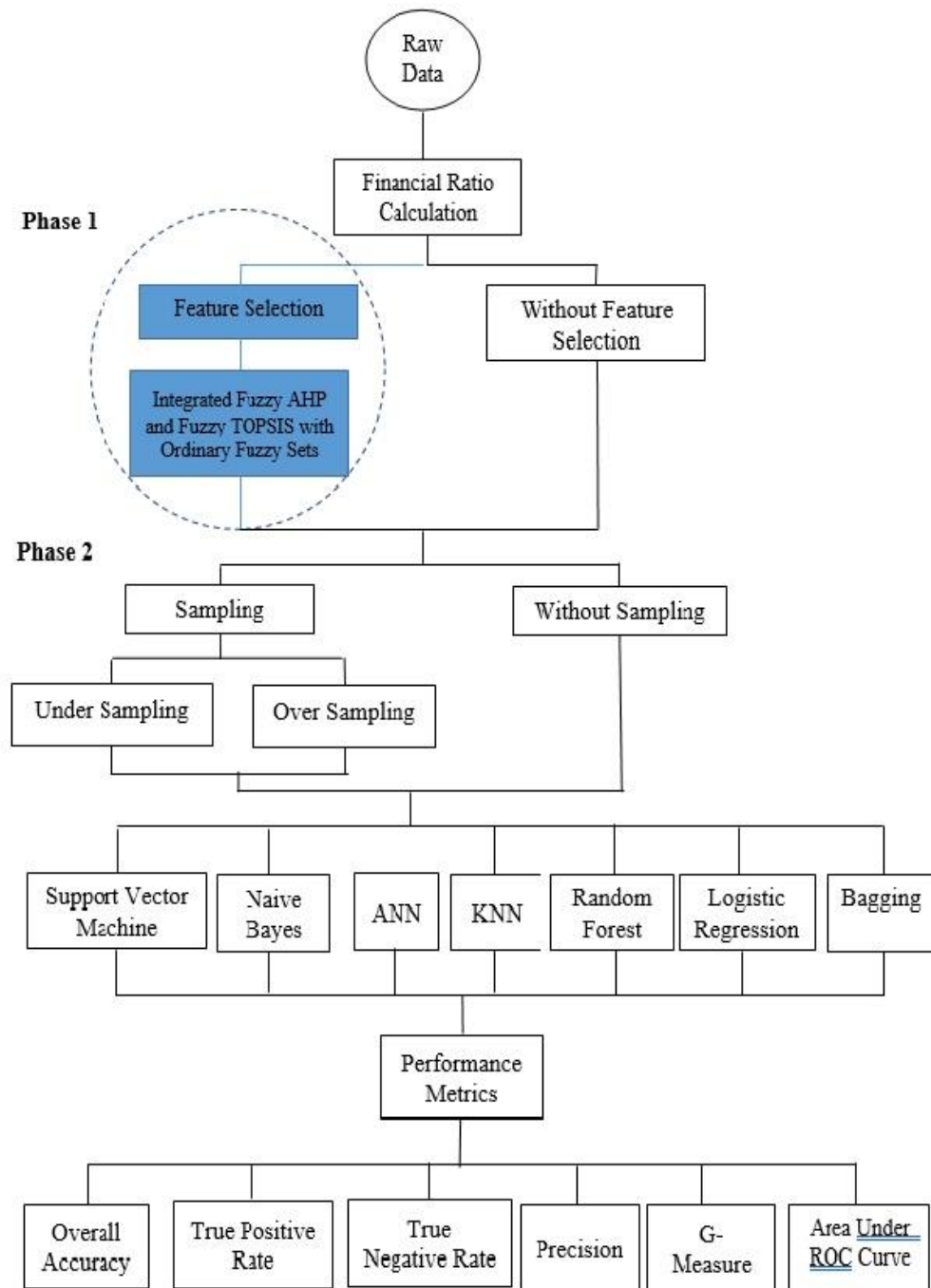


Figure 6.10. The flowchart of the proposed approach (Integrated Fuzzy AHP and Fuzzy TOPSIS)

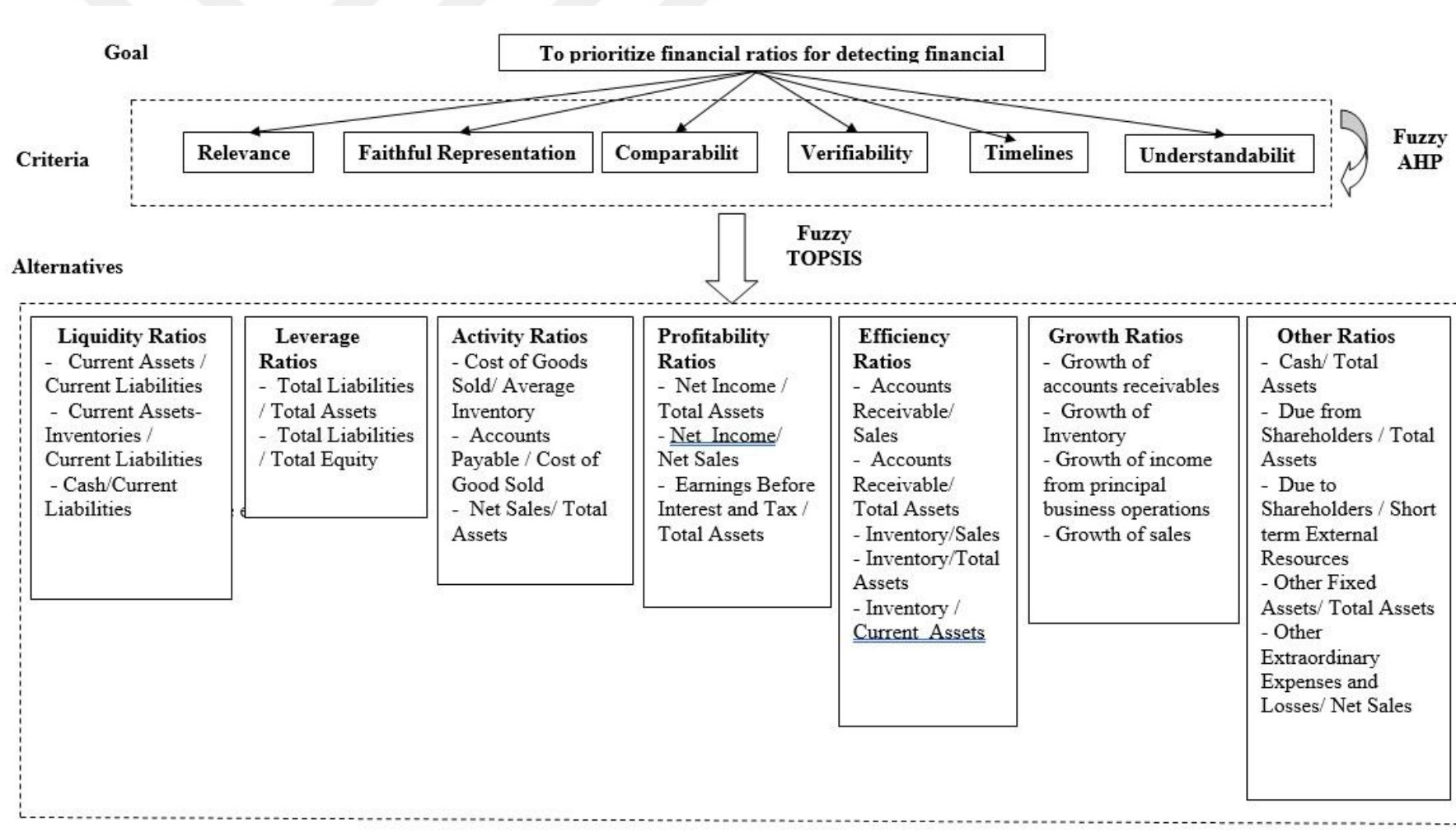


Figure 6.11. The hierarchical structure of the problem (Integrated Fuzzy AHP and Fuzzy TOPSIS)

Because of the close relationship between fraudulent financial reporting and financial reporting quality, the qualitative characteristics are selected as criteria for prioritization of financial ratios. Definitions of the qualitative characteristics are shown in Table 2.3.

After the determination of experts, criteria and alternatives, proposed approach is employed. First, fuzzy AHP is used to determine the weights of criteria and then fuzzy TOPSIS is used to rank the financial ratios through the implementation of these weights.

Integrated Fuzzy AHP and Fuzzy TOPSIS approach, which is implemented to assess the performance of financial ratios for detecting financial accounting fraud, is composed of following steps.

Step 1 6 Criteria and 25 financial ratios are identified from the literature review and experts' opinions. The evaluation model is shown in Figure 6.11. First, fuzzy AHP is used to determine the weights of criteria.

Pairwise comparisons were carried out according to survey conducted considering preferences of experts. Pairwise comparisons of all criteria were carried out by using the linguistic terms and corresponding membership functions of triangular fuzzy numbers which are shown in Table 4.3. Each pairwise comparison rating is based on Saaty's nine-point priority scale. The linguistic scales include "equally important," "weakly important," "essentially important," "very strongly important," and "absolutely important" with respect to fuzzy level scale. A sample of this pairwise comparison matrix of criteria is shown in Table 6.5. The corresponding sample pairwise comparison matrix in triangular fuzzy numbers is shown in Table 6.6.

After obtaining this pairwise comparison matrix of criteria in triangular fuzzy numbers, the weighted geometric mean (shown in Equation 4.1) is used for the aggregation of individual judgments of K decision makers. The aggregated fuzzy evaluation matrix of criteria which is obtained from assessments of four financial analysts is given in Table 6.7.

Table 6.5. Sample pairwise comparison matrix of criteria in linguistic variables by a financial analyst

CRITERIA		Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
Fuzzy AHP Matrix	Relevance			Weakly important	Equally important	Essentially important	
	Faithful Representation	Weakly important		Very strongly important	Equally important	Very strongly important	Weakly important
	Comparability					Weakly important	
	Verifiability			Essentially important		Weakly important	
	Timeliness						
	Understandability	Essentially important		Essentially important	Equally important	Very strongly important	

Table 6.6. Sample pairwise comparison matrix of criteria in triangular fuzzy numbers by a financial analyst

CRITERIA		Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
Fuzzy AHP Matrix	Relevance	(1,1,1)	(1/4, 1/3, 1/2)	(2, 3, 4)	(1, 1, 1)	(4, 5, 6)	(1/6, 1/5, 1/4)
	Faithful Representation	(2,3,4)	(1,1,1)	(6,7,8)	(1,1,1)	(6,7,8)	(2,3,4)
	Comparability	(1/4,1/3,1/2)	(1/8, 1/7, 1/6)	(1,1,1)	(1/6,1/5,1/4)	(2,3,4)	(1/6,1/5,1/4)
	Verifiability	(1,1,1)	(1,1,1)	(4,5,6)	(1,1,1)	(6,7,8)	(1,1,1)
	Timeliness	(1/6,1/5,1/4)	(1/8, 1/7, 1/6)	(1/4, 1/3, 1/2)	(1/8, 1/7, 1/6)	(1,1,1)	(1/8, 1/7, 1/6)
	Understandability	(4,5,6)	(1/4, 1/3, 1/2)	(4,5,6)	(1,1,1)	(6,7,8)	(1,1,1)

Table 6.7. Aggregated fuzzy evaluation matrix of criteria which is obtained from assessments of four financial analysts

CRITERIA		Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
Fuzzy AHP Matrix	Relevance	(1,1,1)	(1/4, 1/3, 1/2)	(1/3, 1/2, 1)	(1/4, 1/3, 1/2)	(1/3, 1/2, 1)	(1/4, 1/3, 1/2)
	Faithful Representation	(2,3,4)	(1,1,1)	(1,2,3)	(1,1,1)	(1,2,3)	(1,2,3)
	Comparability	(1,2,3)	(1/3, 1/2, 1)	(1,1,1)	(1,1,1)	(1,2,3)	(1,1,1)
	Verifiability	(2,3,4)	(1,1,1)	(1,1,1)	(1,1,1)	(2,3,4)	(1,1,1)
	Timeliness	(1,2,3)	(1/3, 1/2, 1)	(1/3, 1/2, 1)	(1/4, 1/3, 1/2)	(1,1,1)	(1/3, 1/2, 1)
	Understandability	(2,3,4)	(1/3, 1/2, 1)	(1,1,1)	(1,1,1)	(1,2,3)	(1,1,1)

Steps 2–5 are applied to obtain the importance weights of criteria. Using the matrix presented in Table 6.7, fuzzy synthetic extent values (S_i) are calculated. Then, the probabilities of preference are found. The combination of probabilities introduces the weight vector. The normalized version of weight vectors are calculated and these weights are then used in fuzzy TOPSIS calculations. The importance weights of qualitative criteria are given in Table 6.8.

Table 6.8. Fuzzy synthetic extent values (S_i), weights (W'_i), and normalized weights (W_i) of criteria

Criteria	S_i	Normalized Weights			
			Weights (W'_i)		(W_i)
Relevance	0.040	0.067	0.139	0.036	0.009
Faithful Representation	0.117	0.245	0.464	1.000	0.255
Comparability	0.089	0.167	0.309	0.712	0.181
Verifiability	0.133	0.223	0.371	0.919	0.234
Timeliness	0.054	0.108	0.232	0.456	0.116
Understandability	0.106	0.190	0.340	0.800	0.204

After calculating the preference weights of qualitative characteristics, fuzzy TOPSIS method is employed to determine the best prioritization of financial ratios. The procedure is as follows.

Step 6 Financial Analysts rate each alternative based on each criterion by using the linguistic terms in Table 4.4. The ratings of alternatives with respect to criteria in linguistic variables by a financial analyst are given in Table 6.9. These linguistic terms are converted to corresponding triangular fuzzy numbers which are presented in Table 4.4. The fuzzy matrix of alternatives in triangular fuzzy numbers is shown in Table 6.10.

After obtaining the fuzzy matrix of alternatives, the arithmetic mean (shown in Equation 4.14) is used for the aggregation of individual judgments of K decision makers. The aggregated fuzzy decision matrix which is obtained by four financial analysts is presented in Table 6.11.

Table 6.9. Ratings of alternatives with respect to criteria in linguistic variables by a financial analyst

		Criteria	Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
		Alternatives						
Liquidity Ratios	Current Assets/Current Liabilities		MP	VG	VG	G	MG	MG
	Current Assets-Inventories/Current Liabilities		MP	G	VG	G	MG	MG
	Cash/Current Liabilities		MP	G	MP	MP	MP	MP
Leverage Ratios	Total Liabilities /Total Assets		MP	G	MP	MP	G	MP
	Total Liabilities /Total Equity		MP	G	MP	MP	MP	MP
Activity Ratios	Cost of Goods Sold/Average Inventory		F	G	G	G	G	G
	Accounts Payable/ Cost of Good Sold		F	G	G	MP	F	F
	Net Sales/ Total Assets		F	G	G	F	F	F
Profitability Ratios	Net Income/Total Assets		F	G	F	F	F	F
	Net Income/ Net Sales		MP	G	F	MP	MP	MP
	Earnings Before Interest and Tax / Total Assets		P	G	P	P	P	P
Efficiency Ratios	Accounts Receivable/ Sales		MG	G	G	G	G	G
	Accounts Receivable/ Total Assets		F	G	G	G	G	G
	Inventory/Sales		G	G	G	G	G	G
	Inventory/Total Assets		F	VG	G	G	G	G
	Inventory /Current Assets		G	VG	G	G	G	G
Growth Ratios	Growth of Accounts Receivables		F	G	F	F	F	G
	Growth of Inventory		F	VG	G	G	G	G
	Growth of income from principal business operations		F	G	G	G	G	G
	Growth of Sales		P	VG	P	P	G	P
Other Ratios	Cash/Total Assets		F	G	G	G	F	G
	Due from Shareholders/ Total Assets		G	VG	G	G	G	G
	Due to Shareholders/Total Liabilities		G	VG	G	G	G	G
	Other fixed assets/ Total Assets		MP	G	MP	G	G	MG
	Other Extraordinary Expenses and Losses/ Net Sales		F	G	F	F	G	F

Table 6.10. Ratings of alternatives with respect to criteria in triangular fuzzy numbers by a financial analyst

Alternatives \ Criteria		Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
Liquidity Ratios	Current Assets/Current Liabilities	(1,3,5)	(9,9,10)	(9,9,10)	(7,9,10)	(5,7,9)	(5,7,9)
	Current Assets-Inventories/Current Liabilities	(1,3,5)	(7,9,10)	(9,9,10)	(7,9,10)	(5,7,9)	(5,7,9)
	Cash/Current Liabilities	(1,3,5)	(7,9,10)	(1,3,5)	(1,3,5)	(1,3,5)	(1,3,5)
Leverage Ratios	Total Liabilities /Total Assets	(1,3,5)	(7,9,10)	(1,3,5)	(1,3,5)	(7,9,10)	(1,3,5)
	Total Liabilities /Total Equity	(1,3,5)	(7,9,10)	(1,3,5)	(1,3,5)	(1,3,5)	(1,3,5)
Activity Ratios	Cost of Goods Sold/Average Inventory	(3,5,7)	(7,9,10)	(7,9,10)	(7,9,10)	(7,9,10)	(7,9,10)
	Accounts Payable/ Cost of Good Sold	(3,5,7)	(7,9,10)	(7,9,10)	(1,3,5)	(3,5,7)	(3,5,7)
	Net Sales/ Total Assets	(3,5,7)	(7,9,10)	(7,9,10)	(3,5,7)	(3,5,7)	(3,5,7)
Profitability Ratios	Net Income/Total Assets	(3,5,7)	(7,9,10)	(3,5,7)	(3,5,7)	(3,5,7)	(3,5,7)
	Net Income/ Net Sales	(1,3,5)	(7,9,10)	(3,5,7)	(1,3,5)	(1,3,5)	(1,3,5)
	Earnings Before Interest and Tax / Total Assets	(0,1,3)	(7,9,10)	(0,1,3)	(0,1,3)	(0,1,3)	(0,1,3)
Efficiency Ratios	Accounts Receivable/ Sales	(5,7,9)	(7,9,10)	(7,9,10)	(7,9,10)	(7,9,10)	(7,9,10)
	Accounts Receivable/ Total Assets	(3,5,7)	(7,9,10)	(7,9,10)	(7,9,10)	(7,9,10)	(7,9,10)
	Inventory/Sales	(7,9,10)	(7,9,10)	(7,9,10)	(7,9,10)	(7,9,10)	(7,9,10)
	Inventory/Total Assets	(3,5,7)	(9,9,10)	(7,9,10)	(7,9,10)	(7,9,10)	(7,9,10)
	Inventory /Current Assets	(7,9,10)	(9,9,10)	(7,9,10)	(7,9,10)	(7,9,10)	(7,9,10)
Growth Ratios	Growth of Accounts Receivables	(3,5,7)	(7,9,10)	(3,5,7)	(3,5,7)	(3,5,7)	(7,9,10)
	Growth of Inventory	(3,5,7)	(9,9,10)	(7,9,10)	(7,9,10)	(7,9,10)	(7,9,10)
	Growth of income from principal business operations	(3,5,7)	(7,9,10)	(7,9,10)	(7,9,10)	(7,9,10)	(7,9,10)
	Growth of Sales	(0,1,3)	(9,9,10)	(0,1,3)	(0,1,3)	(7,9,10)	(0,1,3)
Other Ratios	Cash/ Total Assets	(3,5,7)	(7,9,10)	(7,9,10)	(7,9,10)	(3,5,7)	(7,9,10)
	Due from Shareholders/ Total Assets	(7,9,10)	(9,9,10)	(7,9,10)	(7,9,10)	(7,9,10)	(7,9,10)
	Due to Shareholders/ Total Liabilities	(7,9,10)	(9,9,10)	(7,9,10)	(7,9,10)	(7,9,10)	(7,9,10)
	Other fixed assets/ Total Assets	(1,3,5)	(7,9,10)	(1,3,5)	(7,9,10)	(7,9,10)	(5,7,9)
	Other Extraordinary Expenses and Losses/ Net Sales	(3,5,7)	(7,9,10)	(3,5,7)	(3,5,7)	(7,9,10)	(3,5,7)

Table 6.11. Aggregated fuzzy evaluation matrix of alternatives which is obtained from assessments of four financial analysts

	Alternatives	Criteria					
		Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
Liquidity Ratios	Current Assets/Current Liabilities	(5, 6.5, 8)	(7, 8, 9.3)	(8, 8.5, 9.8)	(7, 8.5, 9.8)	(7, 8.5, 9.8)	(7, 8, 9.5)
	Current Assets-Inventories/Current Liabilities	(5, 6.5, 8)	(8, 9, 10)	(8, 8.5, 9.8)	(8, 9, 10)	(7, 8.5, 9.8)	(8, 8.5, 9.8)
	Cash/Current Liabilities	(2.8, 4.5, 6.3)	(5.5, 7, 8.5)	(3.8, 5, 6.3)	(4.3, 5.5, 7)	(3.8, 5.5, 7)	(5, 6.5, 8.3)
Leverage Ratios	Total Liabilities /Total Assets	(4.5, 6.5, 8)	(7, 8.5, 9.8)	(3.5, 5, 6.8)	(6, 7.5, 8.8)	(7, 9, 10)	(6, 7, 8.5)
	Total Liabilities /Total Equity	(5.5, 7, 8.5)	(7, 8.5, 9.8)	(4, 5.5, 7.3)	(5.5, 6.5, 8)	(3.5, 5.5, 7.3)	(5, 6.5, 8)
Activity Ratios	Cost of Goods Sold/Average Inventory	(4.5, 6.5, 8)	(8.5, 9, 10)	(7.5, 9, 10)	(8, 9, 10)	(7.5, 9, 10)	(7.5, 8.5, 9.8)
	Accounts Payable/ Cost of Good Sold	(4.3, 6, 7.5)	(7, 8, 9.3)	(6, 7.5, 8.8)	(4.5, 6, 7.5)	(5, 6.5, 8)	(4.5, 6, 7.8)
	Net Sales/ Total Assets	(5.5, 7.5, 9)	(7.5, 8.5, 9.8)	(7.5, 9, 10)	(7, 8, 9.3)	(6.5, 8, 9.3)	(6.5, 7.5, 9)
Profitability Ratios	Net Income/Total Assets	(4, 6, 7.8)	(6, 7.5, 9)	(5.5, 7, 8.5)	(7, 8, 9.3)	(4.5, 6.5, 8.3)	(6.5, 7.5, 9)
	Net Income/ Net Sales	(3.5, 5.5, 7.3)	(6.5, 8, 9.3)	(5.5, 7, 8.5)	(6.5, 7.5, 8.8)	(4.5, 6.5, 8)	(6.5, 7.5, 8.8)
	Earnings Before Interest and Tax / Total Assets	(3.3, 5, 6.8)	(6.5, 8, 9.3)	(4.8, 6, 7.5)	(6.3, 7, 8.3)	(4.3, 6, 7.5)	(5.8, 6.5, 8)
Efficiency Ratios	Accounts Receivable/ Sales	(7, 8.5, 9.8)	(8, 9, 10)	(8, 9, 10)	(8, 9, 10)	(8, 9, 10)	(7.5, 8.5, 9.8)
	Accounts Receivable/ Total Assets	(4.5, 6.5, 8.3)	(6, 7.5, 9)	(6.5, 8, 9.3)	(6, 7.5, 9)	(6, 8, 9.3)	(6, 7.5, 9)
	Inventory/Sales	(7.5, 9, 10)	(8, 9, 10)	(8, 9, 10)	(8, 9, 10)	(8, 9, 10)	(8, 9, 10)
	Inventory/Total Assets	(5, 7, 8.8)	(7, 8, 9.5)	(7, 8.5, 9.8)	(6.5, 8, 9.5)	(6.5, 8.5, 9.8)	(6.5, 8, 9.5)
	Inventory /Current Assets	(6, 8, 9.5)	(7.5, 8.5, 9.8)	(7, 8.5, 9.8)	(7, 8.5, 9.8)	(6.5, 8.5, 9.8)	(6.5, 8, 9.5)
Growth Ratios	Growth of Accounts Receivables	(6.5, 8, 9.3)	(8.5, 9, 10)	(6.5, 7.5, 9)	(7.5, 8, 9.3)	(6.5, 8, 9.3)	(8.5, 9, 10)
	Growth of Inventory	(6.5, 8, 9.3)	(9, 9, 10)	(8, 9, 10)	(8.5, 9, 10)	(7.5, 9, 10)	(8.5, 9, 10)
	Growth of income from principal business operations	(4.5, 6.5, 8)	(6.3, 7, 8.3)	(5.8, 7, 8.3)	(8, 9, 10)	(5.5, 7.5, 8.8)	(8, 9, 10)
	Growth of Sales	(3.8, 5.5, 7)	(6.3, 7, 8.3)	(4.3, 5.5, 7)	(6.3, 7, 8.3)	(6, 8, 9.3)	(6.3, 7, 8.3)
Other Ratios	Cash/Total Assets	(3.8, 5.5, 6.8)	(7.7, 9, 10)	(7, 8.3, 9.7)	(7.7, 9, 10)	(5, 7, 8.7)	(7.7, 9, 10)
	Due from Shareholders/ Total Assets	(4.8, 6.5, 7.5)	(9, 9, 10)	(5.5, 7, 8.5)	(8.5, 9, 10)	(6.5, 8.5, 9.8)	(8, 9, 10)
	Due to Shareholders/ Total Liabilities	(5, 7, 8.5)	(8.5, 9, 10)	(5.5, 7, 8.5)	(8, 9, 10)	(7, 9, 10)	(8, 9, 10)
	Other fixed assets/ Total Assets	(3.8, 5.5, 6.5)	(5.8, 7, 8.3)	(4.3, 5.5, 6.5)	(5.8, 7, 7.8)	(5.3, 7, 7.8)	(5.3, 6.5, 7.5)
	Other Extraordinary Expenses and Losses/ Net Sales	(3.8, 5.5, 6.8)	(6.3, 7, 8.3)	(3.8, 5, 6.3)	(4.8, 6, 7)	(5.8, 7, 7.8)	(5.3, 6, 7)

Steps 7-8 All criteria which are presented in this study are assumed to be benefit criteria. So, the fuzzy normalized decision matrix is obtained by dividing the value in each decision alternative with the largest value in the entire decision alternative matrix using Eq. (4.17). The fuzzy normalized decision matrix is shown in Table 6.12. After obtaining the fuzzy normalized decision matrix, considering the different importance of each criterion, the fuzzy weighted normalized decision matrix is constructed by using Eq. (4.19). Table 6.13. shows the fuzzy weighted normalized decision matrix.

Table 6.12. Fuzzy normalized decision matrix

Criteria Weights		0.009	0.255	0.181	0.234	0.116	0.204
Alternatives		Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
Liquidity Ratios	Current Assets/Current Liabilities	(0.50, 0.65, 0.80)	(0.70, 0.80, 0.93)	(0.80, 0.85, 0.98)	(0.70, 0.85, 0.98)	(0.70, 0.85, 0.98)	(0.70, 0.80, 0.95)
	Current Assets-Inventories/Current Liabilities	(0.50, 0.65, 0.80)	(0.80, 0.90, 1.00)	(0.80, 0.85, 0.98)	(0.80, 0.90, 1.00)	(0.70, 0.85, 0.98)	(0.80, 0.85, 0.98)
	Cash/Current Liabilities	(0.28, 0.45, 0.63)	(0.55, 0.70, 0.85)	(0.38, 0.50, 0.63)	(0.43, 0.55, 0.70)	(0.38, 0.55, 0.70)	(0.50, 0.65, 0.83)
Leverage Ratios	Total Liabilities /Total Assets	(0.45, 0.65, 0.80)	(0.70, 0.85, 0.98)	(0.35, 0.50, 0.68)	(0.60, 0.75, 0.88)	(0.70, 0.90, 1.00)	(0.60, 0.70, 0.85)
	Total Liabilities /Total Equity	(0.55, 0.70, 0.85)	(0.70, 0.85, 0.98)	(0.40, 0.55, 0.73)	(0.55, 0.65, 0.80)	(0.35, 0.55, 0.73)	(0.50, 0.65, 0.80)
Activity Ratios	Cost of Goods Sold/Average Inventory	(0.45, 0.65, 0.80)	(0.85, 0.90, 1.00)	(0.75, 0.90, 1.00)	(0.80, 0.90, 1.00)	(0.75, 0.90, 1.00)	(0.75, 0.85, 0.98)
	Accounts Payable/ Cost of Good Sold	(0.43, 0.60, 0.75)	(0.70, 0.80, 0.93)	(0.60, 0.75, 0.88)	(0.45, 0.60, 0.75)	(0.50, 0.65, 0.80)	(0.45, 0.60, 0.78)
	Net Sales/ Total Assets	(0.55, 0.75, 0.90)	(0.75, 0.85, 0.98)	(0.75, 0.90, 1.00)	(0.70, 0.80, 0.93)	(0.65, 0.80, 0.93)	(0.65, 0.75, 0.90)
Profitability Ratios	Net Income/Total Assets	(0.40, 0.60, 0.78)	(0.60, 0.75, 0.90)	(0.55, 0.70, 0.85)	(0.70, 0.80, 0.93)	(0.45, 0.65, 0.83)	(0.65, 0.75, 0.90)
	Net Income/ Net Sales	(0.35, 0.55, 0.73)	(0.65, 0.80, 0.93)	(0.55, 0.70, 0.85)	(0.65, 0.75, 0.88)	(0.45, 0.65, 0.80)	(0.65, 0.75, 0.88)
	Earnings Before Interest and Tax / Total Assets	(0.33, 0.50, 0.68)	(0.65, 0.80, 0.93)	(0.48, 0.60, 0.75)	(0.63, 0.70, 0.83)	(0.43, 0.60, 0.75)	(0.58, 0.65, 0.80)
Efficiency Ratios	Accounts Receivable/ Sales	(0.70, 0.85, 0.98)	(0.80, 0.90, 1.00)	(0.80, 0.90, 1.00)	(0.80, 0.90, 1.00)	(0.80, 0.90, 1.00)	(0.75, 0.85, 0.98)
	Accounts Receivable/ Total Assets	(0.45, 0.65, 0.83)	(0.60, 0.75, 0.90)	(0.65, 0.80, 0.93)	(0.60, 0.75, 0.90)	(0.60, 0.80, 0.93)	(0.60, 0.75, 0.90)
	Inventory/Sales	(0.75, 0.90, 1.00)	(0.80, 0.90, 1.00)	(0.80, 0.90, 1.00)	(0.80, 0.90, 1.00)	(0.80, 0.90, 1.00)	(0.80, 0.90, 1.00)
	Inventory/Total Assets	(0.50, 0.70, 0.88)	(0.70, 0.80, 0.95)	(0.70, 0.85, 0.98)	(0.65, 0.80, 0.95)	(0.65, 0.85, 0.98)	(0.65, 0.80, 0.95)
	Inventory /Current Assets	(0.60, 0.80, 0.95)	(0.75, 0.85, 0.98)	(0.70, 0.85, 0.98)	(0.70, 0.85, 0.98)	(0.65, 0.85, 0.98)	(0.65, 0.80, 0.95)
Growth Ratios	Growth of accounts receivables	(0.65, 0.80, 0.93)	(0.85, 0.90, 1.00)	(0.65, 0.75, 0.90)	(0.75, 0.80, 0.93)	(0.65, 0.80, 0.93)	(0.85, 0.90, 1.00)
	Growth of Inventory	(0.65, 0.80, 0.93)	(0.90, 0.90, 1.00)	(0.80, 0.90, 1.00)	(0.85, 0.90, 1.00)	(0.75, 0.90, 1.00)	(0.85, 0.90, 1.00)
	Growth of income from principal business operations	(0.45, 0.65, 0.80)	(0.63, 0.70, 0.83)	(0.58, 0.70, 0.83)	(0.80, 0.90, 1.00)	(0.55, 0.75, 0.88)	(0.80, 0.90, 1.00)
	Growth of sales	(0.38, 0.55, 0.70)	(0.68, 0.70, 0.83)	(0.43, 0.55, 0.70)	(0.63, 0.70, 0.83)	(0.60, 0.80, 0.93)	(0.63, 0.70, 0.83)
Other Ratios	Cash/Total Assets	(0.38, 0.55, 0.68)	(0.77, 0.90, 1.00)	(0.70, 0.83, 0.97)	(0.77, 0.90, 1.00)	(0.50, 0.70, 0.87)	0.77, 0.90, 1.00)
	Due from Shareholders/ Total Assets	(0.48, 0.65, 0.75)	(0.90, 0.90, 1.00)	(0.55, 0.70, 0.85)	(0.85, 0.90, 1.00)	(0.65, 0.85, 0.98)	(0.80, 0.90, 1.00)
	Due to Shareholders/ Total Liabilities	(0.50, 0.70, 0.85)	(0.85, 0.90, 1.00)	(0.55, 0.70, 0.85)	(0.80, 0.90, 1.00)	(0.70, 0.90, 1.00)	(0.80, 0.90, 1.00)
	Other Fixed Assets/ Total Assets	(0.38, 0.55, 0.65)	(0.58, 0.70, 0.83)	(0.43, 0.55, 0.65)	(0.58, 0.70, 0.78)	(0.53, 0.70, 0.78)	(0.53, 0.65, 0.75)
	Other Extraordinary Expenses and Losses/ Net Sales	(0.38, 0.55, 0.68)	(0.63, 0.70, 0.83)	(0.38, 0.50, 0.63)	(0.48, 0.60, 0.70)	(0.58, 0.70, 0.78)	(0.53, 0.60, 0.70)

Steps 9-12 The distance from fuzzy positive-ideal solution (d_i^*), distance from fuzzy negative-ideal solution (d_i^-) and closeness coefficients (CC_i) are calculated as shown in Table 6.14. According to the closeness coefficient, the ranking order of the alternatives is shown in Table 6.15.

Table 6.13. Weighted normalized fuzzy decision matrix (Fuzzy TOPSIS)

	Criteria Alternatives	Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
Leverage Ratios	Current Assets/Current Liabilities	(0.005, 0.006, 0.007)	(0.178, 0.204, 0.236)	(0.145, 0.154, 0.177)	(0.164, 0.199, 0.228)	(0.081, 0.099, 0.113)	(0.143, 0.163, 0.194)
	Current Assets- Inventories/ Current Liabilities	(0.005, 0.006, 0.007)	(0.204, 0.229, 0.255)	(0.145, 0.154, 0.177)	(0.187, 0.211, 0.234)	(0.081, 0.099, 0.113)	(0.163, 0.173, 0.199)
	Cash/Current Liabilities	(0.003, 0.004, 0.006)	(0.140, 0.178, 0.217)	(0.068, 0.091, 0.113)	(0.100, 0.129, 0.164)	(0.044, 0.064, 0.081)	(0.102, 0.133, 0.168)
	Total Liabilities /Total Assets	(0.004, 0.006, 0.007)	(0.178, 0.217, 0.249)	(0.063, 0.091, 0.122)	(0.141, 0.176, 0.205)	(0.081, 0.105, 0.116)	(0.122, 0.143, 0.173)
	Total Liabilities /Total Equity	(0.005, 0.006, 0.008)	(0.178, 0.217, 0.249)	(0.073, 0.100, 0.131)	(0.129, 0.152, 0.187)	(0.041, 0.064, 0.084)	(0.102, 0.133, 0.163)
	Cost of Goods Sold/Average Inventory	(0.004, 0.006, 0.007)	(0.217, 0.229, 0.255)	(0.136, 0.163, 0.181)	(0.187, 0.211, 0.234)	(0.087, 0.105, 0.116)	(0.153, 0.173, 0.199)
	Accounts Payable/ Cost of Good Sold	(0.004, 0.006, 0.007)	(0.178, 0.204, 0.236)	(0.109, 0.136, 0.159)	(0.105, 0.141, 0.176)	(0.058, 0.076, 0.093)	(0.092, 0.122, 0.158)
	Net Sales/ Total Assets	(0.005, 0.007, 0.008)	(0.191, 0.217, 0.249)	(0.136, 0.163, 0.181)	(0.164, 0.187, 0.217)	(0.076, 0.093, 0.108)	(0.133, 0.153, 0.184)
	Net Income/Total Assets	(0.004, 0.006, 0.007)	(0.153, 0.191, 0.229)	(0.100, 0.127, 0.154)	(0.164, 0.187, 0.217)	(0.052, 0.076, 0.096)	(0.133, 0.153, 0.184)
	Net Income/ Net Sales	(0.003, 0.005, 0.007)	(0.166, 0.204, 0.236)	(0.100, 0.127, 0.154)	(0.152, 0.176, 0.205)	(0.052, 0.076, 0.093)	(0.133, 0.153, 0.178)
	Earnings Before Interest and Tax / Total Assets	(0.003, 0.005, 0.006)	(0.166, 0.204, 0.236)	(0.086, 0.109, 0.136)	(0.146, 0.164, 0.193)	(0.049, 0.070, 0.087)	(0.117, 0.133, 0.163)
	Accounts Receivable/ Sales	(0.006, 0.008, 0.009)	(0.204, 0.229, 0.255)	(0.145, 0.163, 0.181)	(0.187, 0.211, 0.234)	(0.093, 0.105, 0.116)	(0.153, 0.173, 0.199)
	Accounts Receivable/ Total Assets	(0.004, 0.006, 0.008)	(0.153, 0.191, 0.229)	(0.118, 0.145, 0.168)	(0.141, 0.176, 0.211)	(0.070, 0.093, 0.108)	(0.122, 0.153, 0.184)
	Inventory/Sales	(0.007, 0.008, 0.009)	(0.204, 0.229, 0.255)	(0.145, 0.163, 0.181)	(0.187, 0.211, 0.234)	(0.093, 0.105, 0.116)	(0.163, 0.184, 0.204)
	Inventory/Total Assets	(0.005, 0.006, 0.008)	(0.178, 0.204, 0.242)	(0.127, 0.154, 0.177)	(0.152, 0.187, 0.223)	(0.076, 0.099, 0.113)	(0.133, 0.163, 0.194)
	Inventory /Current Assets	(0.006, 0.007, 0.009)	(0.191, 0.217, 0.249)	(0.127, 0.154, 0.177)	(0.164, 0.199, 0.228)	(0.076, 0.099, 0.113)	(0.133, 0.163, 0.194)
	Growth of accounts receivables	(0.006, 0.007, 0.009)	(0.217, 0.229, 0.255)	(0.118, 0.136, 0.163)	(0.176, 0.187, 0.217)	(0.076, 0.093, 0.108)	(0.173, 0.184, 0.204)
	Growth of Inventory	(0.006, 0.007, 0.009)	(0.229, 0.229, 0.255)	(0.145, 0.163, 0.181)	(0.199, 0.211, 0.234)	(0.087, 0.105, 0.116)	(0.173, 0.184, 0.204)
Growth Ratios	Growth of income from principal business operations	(0.004, 0.006, 0.007)	(0.159, 0.178, 0.210)	(0.104, 0.127, 0.150)	(0.187, 0.211, 0.234)	(0.064, 0.087, 0.102)	(0.163, 0.184, 0.204)
	Growth of sales	(0.003, 0.005, 0.006)	(0.172, 0.178, 0.210)	(0.077, 0.100, 0.127)	(0.146, 0.164, 0.193)	(0.070, 0.093, 0.108)	(0.127, 0.143, 0.168)
	Cash/ Total Assets	(0.003, 0.005, 0.006)	(0.195, 0.229, 0.255)	(0.127, 0.151, 0.175)	(0.180, 0.211, 0.234)	(0.058, 0.081, 0.101)	(0.156, 0.184, 0.204)
	Due from Shareholders/ Total Assets	(0.004, 0.006, 0.007)	(0.229, 0.229, 0.255)	(0.100, 0.127, 0.154)	(0.199, 0.211, 0.234)	(0.076, 0.099, 0.113)	(0.163, 0.184, 0.204)
	Due to Shareholders/ Total Liabilities	(0.005, 0.006, 0.008)	(0.217, 0.229, 0.255)	(0.100, 0.127, 0.154)	(0.187, 0.211, 0.234)	(0.081, 0.105, 0.116)	(0.163, 0.184, 0.204)
	Other Fixed Assets/ Total Assets	(0.003, 0.005, 0.006)	(0.147, 0.178, 0.210)	(0.077, 0.100, 0.118)	(0.135, 0.164, 0.182)	(0.061, 0.081, 0.090)	(0.107, 0.133, 0.153)
Other Ratios	Other Extraordinary Expenses and Losses/ Net Sales	(0.003, 0.005, 0.006)	(0.159, 0.178, 0.210)	(0.068, 0.091, 0.113)	(0.111, 0.141, 0.164)	(0.067, 0.081, 0.090)	(0.107, 0.122, 0.143)

Table 6.14. The distance from fuzzy positive-ideal solution (d_i^+), distance from fuzzy negative-ideal solution (d_i^-) and closeness coefficients (CC_i)

	Alternatives (Financial Ratios)	d_i^+	d_i^-	CC_i
Liquidity Ratios	Current Assets/Current Liabilities	5.1689	0.8383	0.1395
	Current Assets-Inventories/ Current Liabilities	5.1196	0.8853	0.1474
	Cash/Current Liabilities	5.4004	0.6133	0.1020
Leverage Ratio	Total Liabilities /Total Assets	5.2684	0.7429	0.1236
	Total Liabilities /Total Equity	5.3277	0.6855	0.1140
Activity Ratios	Cost of Goods Sold/Average Inventory	5.1126	0.8927	0.1486
	Accounts Payable/ Cost of Good Sold	5.3154	0.6964	0.1158
	Net Sales/ Total Assets	5.1776	0.8296	0.1381
Profitability Ratios	Net Income/Total Assets	5.2576	0.7533	0.1253
	Net Income/ Net Sales	5.2617	0.7482	0.1245
	Earnings Before Interest and Tax / Total Assets	5.3102	0.6993	0.1164
Efficiency Ratios	Accounts Receivable/ Sales	5.1099	0.8949	0.1490
	Accounts Receivable/ Total Assets	5.2424	0.7693	0.1280
	Inventory/Sales	5.1010	0.9036	0.1505
	Inventory/Total Assets	5.1880	0.8222	0.1368
	Inventory /Current Assets	5.1666	0.8426	0.1402
Growth Ratios	Growth of accounts receivables	5.1484	0.8563	0.1426
	Growth of Inventory	5.0877	0.9155	0.1525
	Growth of income from principal business operations	5.2068	0.7999	0.1332
	Growth of sales	5.3036	0.7037	0.1171
Other Ratios	Cash/Total Assets	5.1490	0.8591	0.1430
	Due from Shareholders/ Total Assets	5.1359	0.8697	0.1448
	Due to Shareholders/ Short term External Resources	5.1389	0.8674	0.1444
	Other Fixed Assets/ Total Assets	5.3511	0.6569	0.1093
	Other Extraordinary Expenses and Losses/ Net Sales	5.3805	0.6267	0.1043

Table 6.15. Final ranking of financial ratios (Integrated Fuzzy AHP and Fuzzy TOPSIS)

Alternatives (Financial Ratios)	CC_i	Rank
Growth of Inventory	0.1525	1
Inventory/Sales	0.1505	2
Accounts Receivable/ Sales	0.1490	3
Cost of Goods Sold/Average Inventory	0.1486	4
Current Assets-Inventories/ Current Liabilities	0.1474	5
Due from Shareholders/ Total Assets	0.1448	6
Due to Shareholders/ Total Liabilities	0.1444	7
Cash/ Total Assets	0.1430	8
Growth of accounts receivables	0.1426	9
Inventory /Current Assets	0.1402	10
Current Assets/Current Liabilities	0.1395	11
Net Sales/ Total Assets	0.1381	12
Inventory/Total Assets	0.1368	13
Growth of income from principal business operations	0.1332	14
Accounts Receivable/ Total Assets	0.1280	15
Net Income/Total Assets	0.1253	16
Net Income/ Net Sales	0.1245	17
Total Liabilities /Total Assets	0.1236	18
Growth of sales	0.1171	19
Earnings Before Interest and Tax / Total Assets	0.1164	20
Accounts Payable/ Cost of Good Sold	0.1158	21
Total Liabilities /Total Equity	0.1140	22
Other Fixed Assets/ Total Assets	0.1093	23
Other Extraordinary Expenses and Losses/ Net Sales	0.1043	24
Cash/Current Liabilities	0.1020	25

Table 6.15 shows the ranking of financial ratios according to the aggregated opinion of the individual judgments of financial analysts by utilizing an integrated AHP and TOPSIS with Ordinary Fuzzy Sets. The top fifteen financial ratios are selected as shown in Table 6.16 for comparing the performances of classifiers in detecting financial accounting fraud. Without employing any feature selection method, classifiers are also compared.

Table 6.16. Selected financial ratios for comparing the performances of classifiers (Integrated Fuzzy AHP and Fuzzy TOPSIS)

Alternatives (Financial Ratios)	Rank
Growth of Inventory	1
Inventory/Sales	2
Accounts Receivable/ Sales	3
Cost of Goods Sold/Average Inventory	4
Current Assets-Inventories/ Current Liabilities	5
Due from Shareholders/ Total Assets	6
Due to Shareholders/ Total Liabilities	7
Cash/ Total Assets	8
Growth of accounts receivables	9
Inventory /Current Assets	10
Current Assets/Current Liabilities	11
Net Sales/ Total Assets	12
Inventory/Total Assets	13
Growth of income from principal business operations	14
Accounts Receivable/ Total Assets	15

Phase 2: After determining the best prioritization of financial ratios for detecting financial accounting fraud, seven different machine learning classifiers are compared by utilizing performance measures. Without employing any feature selection method, classifiers are also compared. The study was carried out using the Waikato Environment for Knowledge Analysis (WEKA) software package. Ten-fold cross validation was employed to analyze the classification algorithms. The default values were chosen for most of the functions' parameters. Tables 6.17-6.22 show the performance metrics of classifiers using the proposed feature selection method and without using any feature selection method. Moreover, different sampling techniques are employed. Over all, 42 model comprising 7 classifiers were constructed and compared according to their performance metrics. Each performance measure's greatest value is highlighted in bold, while the worst value is underlined. According to performance measures, comparisons of the results are given below.

In terms of Overall Accuracy, Random Forest classifier always performs the best among the seven classifiers (Table 6.17). The Random Forest-without-feature-selection-oversampling model has the best overall accuracy at 93.74 percent. Naive Bayes-Integrated AHP and TOPSIS with Ordinary Fuzzy Sets -Over Sampling model performs the worst at 53.90% among all other models. As shown in Table 6.17, Integrated Fuzzy

AHP and Fuzzy Topsis, which is used as feature selection method, and sampling methods, which are oversampling and undersampling methods, usually can not improve performances of classifiers in terms of Overall Accuracy. In terms of Overall Accuracy, oversampling method has only improved performances of Random Forest .

Table 6.17. Overall Accuracy of classifiers using Integrated Fuzzy AHP and Fuzzy TOPSIS and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and TOPSIS with Ordinary Fuzzy Sets					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.8628	0.7879	0.7477	0.8704	0.8587	0.8131
Naive Bayes	0.6657	<u>0.5390</u>	0.7134	0.8211	0.5881	0.7695
ANN	0.8774	0.8268	0.7539	0.8979	0.8834	0.8287
KNN	0.8592	0.8343	0.7352	0.8944	0.8992	0.7882
Random Forest	0.8968	0.8999	0.7741	0.9179	0.9374	0.8707
Logistic Regression	0.8680	0.8010	0.7555	0.8956	0.8808	0.8536
Bagging	0.8862	0.8782	0.7555	0.9138	0.9127	0.8614

With respect to True Positive Rate, Naive Bayes classifier outperforms all other classifiers (Table 6.18). Especially, Naive Bayes- Integrated AHP and TOPSIS with Ordinary Fuzzy Sets -Over Sampling model outperforms all other models. Support Vector Machines- Integrated AHP and TOPSIS with Ordinary Fuzzy Sets -Without Sampling model perform the worst at 30.80% among all other models. Oversampling method always improve performances of all classifiers in terms of True Positive Rate.

Table 6.18. True Positive Rates of classifiers using Integrated Fuzzy AHP and Fuzzy TOPSIS and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and TOPSIS with Ordinary Fuzzy Sets					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.3080	0.6250	0.5550	0.3710	0.7830	0.7040
Naive Bayes	0.7980	0.9630	0.6700	0.7880	0.9380	0.6950
ANN	0.5360	0.7870	0.7320	0.6230	0.8790	0.8350
KNN	0.5080	0.8090	0.7100	0.6570	0.8930	0.7690
Random Forest	0.5450	0.8910	0.7320	0.6600	0.9470	0.8820
Logistic Regression	0.3890	0.7060	0.6450	0.5860	0.8440	0.8100
Bagging	0.5110	0.8590	0.6920	0.7200	0.9170	0.8540

Considering True Negative Rate, it is found that SVM classifier outperforms all other classifiers (Table 6.19). It is also obtained that Support Vector Machines- Integrated AHP and TOPSIS with Ordinary Fuzzy Sets -Without Sampling model is the best at 99.10% in terms of True Negative rate. Naive Bayes- Integrated AHP and TOPSIS with Ordinary Fuzzy Sets -Over Sampling model perform the worst at 14.60% among all other models. Oversampling method significantly reduces the performance of Naive Bayes in terms of True Negative Rate.

Table 6.19. True Negative Rates of classifiers using Integrated Fuzzy AHP and Fuzzy TOPSIS and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and TOPSIS with Ordinary Fuzzy Sets					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.9910	0.9390	0.9410	0.9860	0.9280	0.9220
Naive Bayes	0.6350	<u>0.1460</u>	0.7570	0.8290	0.2640	0.8440
ANN	0.9570	0.8640	0.7760	0.9620	0.8880	0.8220
KNN	0.9410	0.8580	0.7600	0.9490	0.9050	0.8070
Random Forest	0.9780	0.9080	0.8160	0.9780	0.9280	0.8600
Logistic Regression	0.9790	0.8890	0.8660	0.9670	0.9150	0.8970
Bagging	0.9730	0.8960	0.8190	0.9590	0.9080	0.8690

In terms of Precision, Random Forest-without feature selection-oversampling model is the highest at 92.50% (Table 6.20). Naive Bayes- Integrated AHP and TOPSIS with Ordinary Fuzzy Sets -Without Sampling model performs the worst at 33.60% among all other models.

Table 6.20. Precision of classifiers using Integrated Fuzzy AHP and Fuzzy TOPSIS and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and TOPSIS with Ordinary Fuzzy Sets					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.8920	0.9040	0.9040	0.8620	0.9100	0.9000
Naive Bayes	<u>0.3360</u>	0.5110	0.7340	0.5160	0.5420	0.8170
ANN	0.7410	0.8430	0.7650	0.7910	0.8790	0.8250
KNN	0.6650	0.8410	0.7480	0.7510	0.8970	0.7990
Random Forest	0.8540	0.9000	0.7990	0.8720	0.9250	0.8630
Logistic Regression	0.8120	0.8550	0.8280	0.8070	0.9020	0.8870
Bagging	0.8160	0.8850	0.7930	0.8020	0.9030	0.8670

With regard to G-measure, Random Forest classifier outperforms all other classifiers (Table 6.21). Especially, all other models are outperformed by the Random Forest-without feature selection-oversampling model. Naive Bayes- Integrated AHP and TOPSIS with Ordinary Fuzzy Sets -Over Sampling model perform the worst at 37.50% among all other models. Oversampling method significantly reduces the performance of Naive Bayes in terms of G-measure.

Table 6.21. G-measure of classifiers using Integrated Fuzzy AHP and Fuzzy TOPSIS and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and TOPSIS with Ordinary Fuzzy Sets					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.5525	0.7661	0.7227	0.6048	0.8524	0.8057
Naive Bayes	0.7118	<u>0.3750</u>	0.7122	0.8082	0.4976	0.7659
ANN	0.7162	0.8246	0.7537	0.7742	0.8835	0.8285
KNN	0.6914	0.8331	0.7346	0.7896	0.8990	0.7878
Random Forest	0.7301	0.8995	0.7729	0.8034	0.9375	0.8709
Logistic Regression	0.6171	0.7922	0.7474	0.7528	0.8788	0.8524
Bagging	0.7051	0.8773	0.7528	0.8310	0.9125	0.8615

In terms of Area Under ROC Curve, Random Forest classifier performs the best among the seven classifiers (Table 6.22). Random Forest-without feature selection-oversampling model has highest values at 98.40%. Support Vector Machines - Integrated AHP and TOPSIS with Ordinary Fuzzy Sets - Without Sampling model perform the worst at 65% among all other models. Oversampling method always improve performances of all classifiers in terms of Area Under ROC Curve.

Table 6.22. Area Under ROC Curve of classifiers using Integrated Fuzzy AHP and Fuzzy TOPSIS and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and TOPSIS with Ordinary Fuzzy Sets					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	<u>0.6500</u>	0.7820	0.7480	0.6780	0.8560	0.8130
Naive Bayes	0.8000	0.8410	0.7670	0.8390	0.8580	0.8110
ANN	0.8400	0.8910	0.8340	0.8890	0.9390	0.8980
KNN	0.7920	0.8940	0.7730	0.7980	0.8980	0.7890
Random Forest	0.8750	0.9620	0.8330	0.9490	0.9840	0.9190
Logistic Regression	0.8330	0.8820	0.8200	0.9130	0.9380	0.9050
Bagging	0.8640	0.9370	0.8250	0.9280	0.9690	0.9080

In terms of overall accuracy, precision, G-measure, and ROC curve, the Random Forest classifier is found to be the best among all other classifiers. Naive Bayes classifier performs the worst among all other classifiers in terms of overall accuracy, true negative rate, precision and G-measure.

Integrated AHP and TOPSIS with Ordinary Fuzzy Sets, which is proposed as a feature selection method, is generally failed to improve performances of classifiers. However, proposed approach increases the performance of Naive Bayes classifier in terms of True Positive rate. Besides, Integrated AHP and TOPSIS with Ordinary Fuzzy Sets increases performances of SVM classifiers in terms of True Negative rate.

The results show that the proposed feature selection method can not improve the machine learning classifiers' performances for this dataset. The reason is that each financial ratio has a certain impact in detecting financial accounting fraud for this dataset.

Another finding of the study is that Oversampling technique (SMOTE) usually produce better Overall Accuracy, TP rates, Precision, G-measure and ROC curve than other sampling methods ('without sampling' and 'undersampling') for all the classifiers except Naive Bayes. SMOTE reduces the performances of Naive Bayes in terms of Overall Accuracy, TN rates and G-measure.

6.2.1.2. Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets

In this section, Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets is used for ranking financial ratios.

Algorithm of the application

As stated in previous sections, proposed methodology comprises two phases. In the first phase, Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets is used as feature selection technique. In the second phase, sampling methods and seven different classifiers are employed.

The flowchart of the methodology is shown in Figure 6.12. First, SVSF-AHP method is used to determine the weights of criteria and then SVSF-TOPSIS is used to rank the financial ratios through the implementation of these weights. After determination of the

best prioritization of financial ratios for detecting financial accounting fraud, seven different machine learning classifiers are employed.

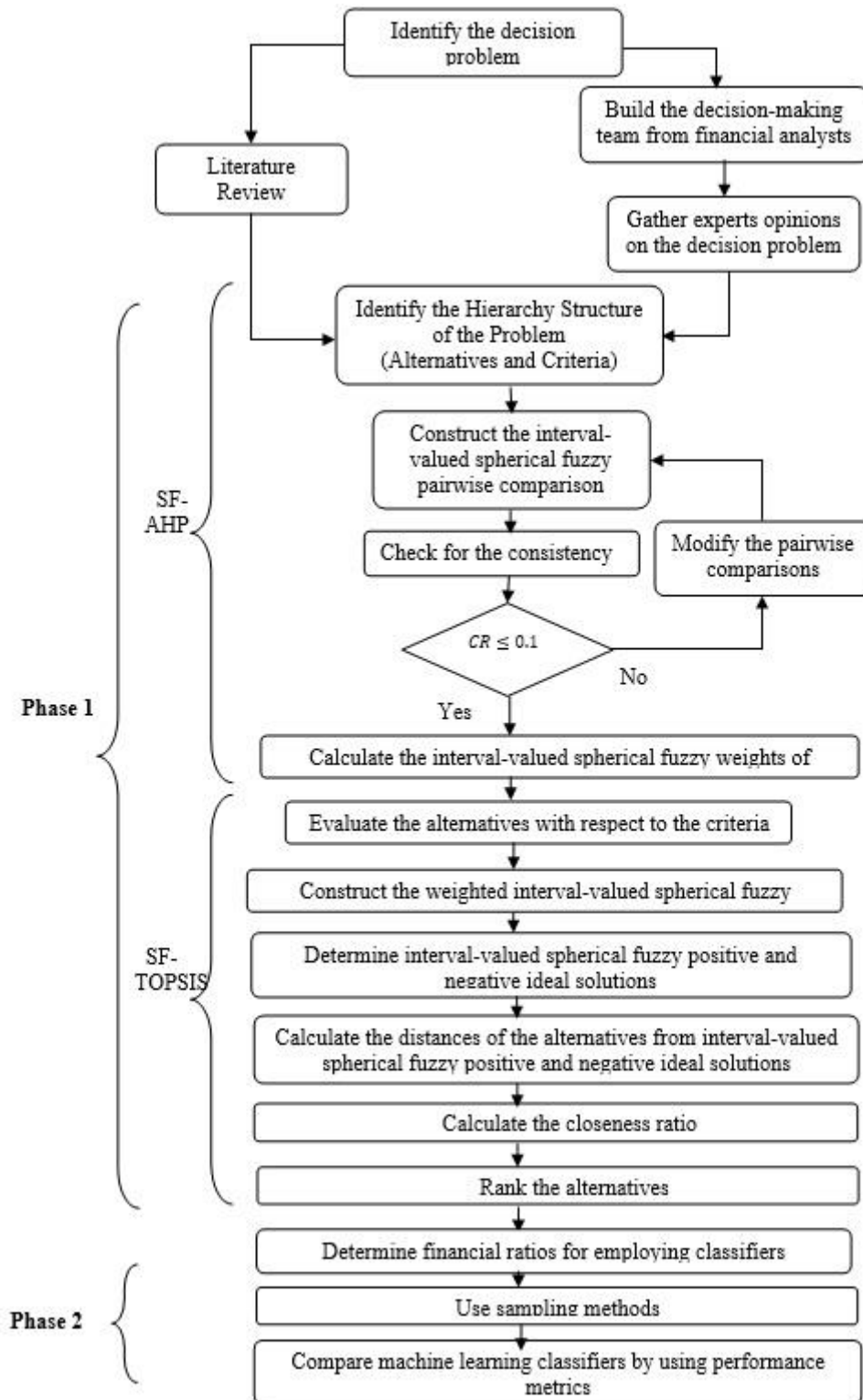


Figure 6.12. The flowchart of the proposed methodology (Integrated AHP and TOPSIS with Spherical Fuzzy Sets)

Steps of the application

The proposed approach is composed of several steps as given in this section. These steps are as follows;

Phase 1: First, SVSF-AHP method is utilized to calculate the weights of criteria. The steps are given in the following:

Step 1 Build a hierarchical framework

The hierarchical framework of the evaluation model is built after determining decision makers, criteria and alternatives. Figure 6.13 illustrates the hierarchical framework of the considered problem.

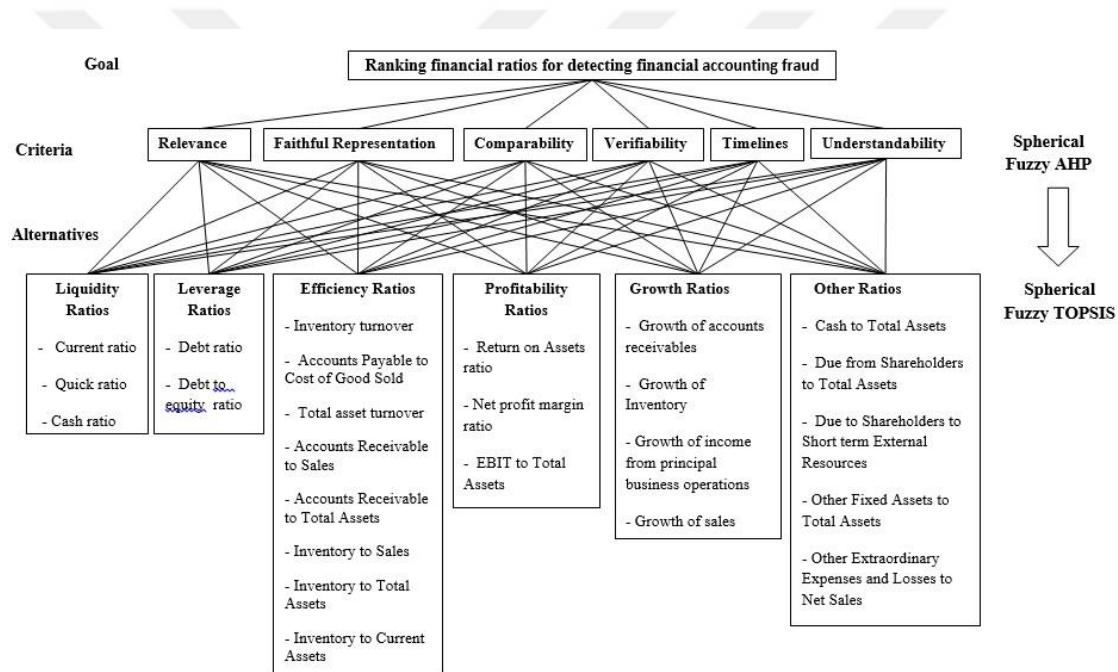


Figure 6.13. The hierarchical framework of the evaluation model (Integrated AHP and TOPSIS with Spherical Fuzzy Sets)

Step 2 Build a pairwise comparison matrix

Using the linguistic phrases listed in Table 4.5, pairwise comparisons of all criteria are conducted with respect to the goal. The SVSFS scale is used to rate each pairwise comparison. Tables 6.23, 6.24, 6.25, and 6.26 present all of the evaluations.

Table 6.23. Pairwise comparison matrix in linguistic variables related to SVSFS scale by financial analyst 1

Criteria		Faithful					
		Relevance	Representation	Comparability	Verifiability	Timeliness	Understandability
Fuzzy AHP Matrix	Relevance	EI	SLI	SMI	EI	HI	LI
	Faithful Representation	SMI	EI	VHI	EI	VHI	SMI
	Comparability	SLI	VLI	EI	LI	SMI	LI
	Verifiability	EI	EI	HI	EI	VHI	EI
	Timeliness	LI	VLI	SLI	VLI	EI	VLI
	Understandability	HI	SLI	HI	EI	VHI	EI

*CR= 0.076

Table 6.24. Pairwise comparison matrix in linguistic variables related to SVSFS scale by financial analyst 2

Criteria		Faithful					
		Relevance	Representation	Comparability	Verifiability	Timeliness	Understandability
Fuzzy AHP Matrix	Relevance	EI	EI	SLI	VLI	LI	EI
	Faithful Representation	EI	EI	SLI	LI	LI	EI
	Comparability	SMI	SMI	EI	EI	SLI	EI
	Verifiability	VHI	HI	EI	EI	EI	SMI
	Timeliness	HI	HI	SMI	EI	EI	SMI
	Understandability	EI	EI	EI	SLI	SLI	EI

*CR= 0.044

Table 6.25. Pairwise comparison matrix in linguistic variables related to SVSFS scale by financial analyst 3

Criteria		Faithful					
		Relevance	Representation	Comparability	Verifiability	Timeliness	Understandability
Fuzzy AHP Matrix	Relevance	EI	LI	LI	LI	SLI	LI
	Faithful Representation	HI	EI	EI	EI	EI	EI
	Comparability	HI	EI	EI	EI	SMI	EI
	Verifiability	HI	EI	EI	EI	SMI	EI
	Timeliness	SMI	EI	SLI	SLI	EI	SLI
	Understandability	HI	EI	EI	EI	SMI	EI

*CR= 0.028

Table 6.26. Pairwise comparison matrix in linguistic variables related to SVSFS scale by financial analyst 4

Criteria		Faithful					
		Relevance	Representation	Comparability	Verifiability	Timeliness	Understandability
Fuzzy AHP Matrix	Relevance	EI	VLI	EI	LI	SLI	SLI
	Faithful Representation	VHI	EI	VHI	SMI	SMI	VHI
	Comparability	EI	VLI	EI	SLI	LI	SLI
	Verifiability	HI	SLI	SMI	EI	EI	EI
	Timeliness	SMI	SLI	HI	EI	EI	SMI
	Understandability	SMI	VLI	SMI	EI	SLI	EI

*CR= 0.051

Step 3 Checking the consistency

The consistency of each pairwise comparison matrix is checked using the scoring index (SI) in Table 4.5. Tables 6.23, 6.24, 6.25, and 6.26 also indicate the consistency ratios of the pairwise comparison matrices, respectively.

Step 4 Aggregate each decision maker's evaluations

The assessments, which are shown in Tables 6.23, 6.24, 6.25, and 6.26, are aggregated using SWAM operator given in Eq. (3.15). Table 6.27 shows the results of the aggregated evaluations of the criteria weights.

Table 6.27. Aggregated evaluations of the single-valued spherical fuzzy weights of criteria

Criteria	Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
Relevance	(0.50, 0.50, 0.50)	(0.37, 0.64, 0.39)	(0.47, 0.54, 0.42)	(0.35, 0.67, 0.36)	(0.49, 0.52, 0.35)	(0.39, 0.62, 0.40)
Faithful Representation	(0.67, 0.33, 0.35)	(0.50, 0.50, 0.50)	(0.68, 0.33, 0.32)	(0.49, 0.51, 0.44)	(0.61, 0.41, 0.36)	(0.63, 0.38, 0.40)
Comparability	(0.57, 0.44, 0.40)	(0.43, 0.60, 0.38)	(0.50, 0.50, 0.50)	(0.44, 0.57, 0.45)	(0.50, 0.51, 0.39)	(0.44, 0.57, 0.45)
Verifiability	(0.70, 0.31, 0.32)	(0.55, 0.46, 0.43)	(0.59, 0.42, 0.43)	(0.50, 0.50, 0.50)	(0.63, 0.38, 0.40)	(0.53, 0.47, 0.48)
Timeliness	(0.58, 0.43, 0.36)	(0.50, 0.52, 0.38)	(0.55, 0.46, 0.37)	(0.42, 0.59, 0.44)	(0.50, 0.50, 0.50)	(0.49, 0.53, 0.38)
Understandability	(0.64, 0.37, 0.38)	(0.42, 0.59, 0.44)	(0.59, 0.42, 0.43)	(0.48, 0.52, 0.48)	(0.60, 0.41, 0.34)	(0.50, 0.50, 0.50)

Step 5 Aggregate weights of criteria

SWAM is also used for determining the single-valued spherical fuzzy weights of criteria, as shown in Table 6.28.

Table 6.28. Single-valued spherical fuzzy weights of criteria

Criteria	Weight
Relevance	(0.44, 0.58, 0.41)
Faithful Representation	(0.61, 0.40, 0.39)
Comparability	(0.48, 0.53, 0.43)
Verifiability	(0.59, 0.42, 0.42)
Timeliness	(0.51, 0.50, 0.41)
Understandability	(0.55, 0.46, 0.43)

After determining the single-valued spherical fuzzy weights of criteria, single-valued spherical fuzzy TOPSIS method is used for ranking the alternatives. The steps of the proposed method are as follows:

Step 6-7 First, the assessments of decision makers on alternatives with respect to the criteria are used and aggregated using SWAM operator. After the weights of the criteria and the ratings of the alternatives have been determined, the weighted single-valued

spherical fuzzy decision matrix is constructed by using Eq. (3.6). The weighted single-valued spherical fuzzy decision matrix is presented in Table 6.29.

Table 6.29. Weighted single-valued spherical fuzzy decision matrix (Single-Valued Spherical Fuzzy TOPSIS)

Alternatives	Relevance	Faithful Representa	Comparability	Verifiability	Timeliness	Understandability
Current Ratio	(0.29, 0.64, 0.46)	(0.45, 0.48, 0.46)	(0.39, 0.56, 0.46)	(0.37, 0.54, 0.48)	(0.37, 0.56, 0.45)	(0.39, 0.54, 0.47)
Quick ratio	(0.30, 0.64, 0.45)	(0.46, 0.46, 0.44)	(0.39, 0.55, 0.46)	(0.44, 0.48, 0.46)	(0.38, 0.56, 0.46)	(0.44, 0.50, 0.46)
Cash ratio	(0.28, 0.66, 0.44)	(0.36, 0.55, 0.47)	(0.24, 0.70, 0.45)	(0.29, 0.63, 0.46)	(0.30, 0.63, 0.44)	(0.31, 0.61, 0.50)
Debt ratio	(0.22, 0.71, 0.45)	(0.39, 0.52, 0.48)	(0.22, 0.70, 0.49)	(0.34, 0.58, 0.46)	(0.33, 0.59, 0.44)	(0.37, 0.56, 0.46)
Debt-to-equity ratio	(0.29, 0.65, 0.43)	(0.39, 0.52, 0.48)	(0.24, 0.69, 0.48)	(0.38, 0.54, 0.47)	(0.23, 0.70, 0.45)	(0.28, 0.64, 0.48)
Inventory turnover	(0.26, 0.67, 0.46)	(0.50, 0.43, 0.42)	(0.33, 0.59, 0.47)	(0.44, 0.48, 0.46)	(0.38, 0.55, 0.44)	(0.40, 0.53, 0.47)
Accounts Payable to Cost of Good Sold	(0.22, 0.72, 0.44)	(0.45, 0.48, 0.46)	(0.29, 0.63, 0.48)	(0.30, 0.62, 0.47)	(0.34, 0.59, 0.46)	(0.27, 0.65, 0.50)
Total asset turnover	(0.26, 0.66, 0.47)	(0.50, 0.44, 0.43)	(0.32, 0.60, 0.48)	(0.42, 0.50, 0.49)	(0.38, 0.56, 0.46)	(0.38, 0.55, 0.48)
Return on Assets ratio	(0.25, 0.68, 0.47)	(0.39, 0.52, 0.48)	(0.30, 0.62, 0.49)	(0.43, 0.49, 0.48)	(0.28, 0.63, 0.50)	(0.38, 0.55, 0.48)
Net profit margin ratio	(0.24, 0.69, 0.45)	(0.46, 0.47, 0.45)	(0.35, 0.58, 0.48)	(0.44, 0.48, 0.46)	(0.31, 0.61, 0.47)	(0.40, 0.53, 0.47)
EBIT to Total Assets	(0.24, 0.70, 0.44)	(0.40, 0.51, 0.47)	(0.27, 0.65, 0.48)	(0.42, 0.50, 0.46)	(0.29, 0.64, 0.47)	(0.38, 0.54, 0.47)
Accounts Receivable to Sales	(0.36, 0.60, 0.43)	(0.50, 0.43, 0.42)	(0.38, 0.56, 0.46)	(0.49, 0.45, 0.45)	(0.43, 0.52, 0.43)	(0.42, 0.51, 0.46)
Accounts Receivable to Total Assets	(0.30, 0.63, 0.46)	(0.37, 0.54, 0.49)	(0.32, 0.60, 0.48)	(0.38, 0.53, 0.50)	(0.34, 0.58, 0.47)	(0.35, 0.56, 0.49)
Inventory to Sales	(0.36, 0.59, 0.43)	(0.50, 0.43, 0.42)	(0.38, 0.56, 0.46)	(0.49, 0.45, 0.45)	(0.45, 0.51, 0.42)	(0.43, 0.50, 0.46)
Inventory to Total Assets	(0.31, 0.63, 0.46)	(0.45, 0.47, 0.45)	(0.33, 0.59, 0.47)	(0.44, 0.48, 0.47)	(0.35, 0.57, 0.46)	(0.36, 0.55, 0.49)
Inventory to Current Assets	(0.32, 0.62, 0.45)	(0.46, 0.46, 0.44)	(0.37, 0.57, 0.46)	(0.40, 0.51, 0.48)	(0.35, 0.57, 0.46)	(0.41, 0.52, 0.47)
Growth of accounts receivables	(0.33, 0.62, 0.45)	(0.53, 0.42, 0.41)	(0.34, 0.59, 0.48)	(0.47, 0.46, 0.46)	(0.38, 0.55, 0.45)	(0.45, 0.49, 0.45)
Growth of Inventory	(0.33, 0.62, 0.45)	(0.55, 0.41, 0.40)	(0.38, 0.56, 0.46)	(0.49, 0.45, 0.45)	(0.40, 0.54, 0.44)	(0.45, 0.49, 0.45)
Growth of income from principal business	(0.24, 0.69, 0.47)	(0.45, 0.48, 0.43)	(0.30, 0.63, 0.46)	(0.45, 0.47, 0.46)	(0.32, 0.60, 0.46)	(0.42, 0.51, 0.46)
Growth of sales	(0.22, 0.71, 0.45)	(0.48, 0.46, 0.42)	(0.25, 0.68, 0.47)	(0.41, 0.51, 0.47)	(0.32, 0.60, 0.48)	(0.38, 0.54, 0.47)
Cash to Total Assets	(0.29, 0.65, 0.44)	(0.44, 0.40, 0.44)	(0.34, 0.53, 0.47)	(0.43, 0.42, 0.46)	(0.28, 0.50, 0.50)	(0.40, 0.46, 0.46)
Due from Shareholders to Total Assets	(0.31, 0.63, 0.43)	(0.55, 0.41, 0.40)	(0.35, 0.58, 0.46)	(0.51, 0.43, 0.44)	(0.35, 0.57, 0.46)	(0.45, 0.49, 0.45)
Due to Shareholders to Total Liabilities	(0.27, 0.66, 0.45)	(0.49, 0.44, 0.43)	(0.30, 0.62, 0.47)	(0.46, 0.47, 0.46)	(0.35, 0.57, 0.46)	(0.42, 0.51, 0.46)
Other Fixed Assets to Fixed Assets	(0.25, 0.69, 0.42)	(0.37, 0.54, 0.45)	(0.27, 0.66, 0.44)	(0.43, 0.50, 0.44)	(0.31, 0.62, 0.44)	(0.33, 0.60, 0.46)
Other Extraordinary Expenses and Losses to Net Sales	(0.29, 0.65, 0.44)	(0.49, 0.45, 0.41)	(0.32, 0.62, 0.47)	(0.34, 0.58, 0.47)	(0.37, 0.56, 0.43)	(0.38, 0.55, 0.46)

Steps 8-9 Single-valued spherical fuzzy positive and negative ideal solutions (SVSF-PIS and SVSF -NIS) are calculated by using Eq. (3.17). According to the highest and the lowest values, the corresponding SVSF-PIS and SVSF -NIS are given in Table 6.30.

Table 6.30. Positive and negative single-valued spherical fuzzy ideal solutions

	Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
SVSF-PIS	(0.24, 0.69, 0.47)	(0.55, 0.41, 0.40)	(0.22, 0.70, 0.49)	(0.38, 0.53, 0.50)	(0.28, 0.50, 0.50)	(0.27, 0.65, 0.50)
SVSF-NIS	(0.25, 0.69, 0.42)	(0.45, 0.48, 0.43)	(0.27, 0.66, 0.44)	(0.43, 0.50, 0.44)	(0.37, 0.56, 0.43)	(0.38, 0.55, 0.46)

Steps 10-12 The distances of the alternatives from SVSF-PIS and SVSF -NIS are calculated by using Equations (4.25) and (4.26), respectively. Next, closeness ratios are calculated using Eq. (4.34) and the ranking of the alternatives is determined with respect to the decreasing values of closeness ratios. The distance from SVSF-PIS $d(X_i, X^+)$, distance from SVSF -NIS $d(X_i, X^-)$, closeness ratios and the ranking of the alternatives are shown in Table 6.31.

Table 6.31. The distance from SVSF-PIS $d(X_i, X^+)$, distance from SVSF-NIS $d(X_i, X^-)$, Closeness ratio and ranking of alternatives

Alternatives	$d(X_i, X^+)$	$d(X_i, X^-)$	Closeness ratio	Rank
Current ratio	0.088	0.047	0.345	15
Quick ratio	0.103	0.048	0.318	21
Cash ratio	0.084	0.076	0.475	3
Debt ratio	0.073	0.050	0.407	6
Debt-to-equity ratio	0.073	0.075	0.507	1
Inventory turnover	0.077	0.036	0.320	20
Accounts Payable to Cost of Good Sold	0.066	0.062	0.487	2
Total asset turnover	0.067	0.037	0.353	13
Return on Assets ratio	0.082	0.050	0.381	10
Net profit margin ratio	0.084	0.039	0.316	22
EBIT to Total Assets	0.080	0.033	0.290	23
Accounts Receivable to Sales	0.111	0.064	0.367	12
Accounts Receivable to Total Assets	0.083	0.063	0.430	4
Inventory to Sales	0.115	0.068	0.372	11
Inventory to Total Assets	0.085	0.040	0.322	19
Inventory to Current Assets	0.093	0.050	0.347	14
Growth of accounts receivables	0.097	0.064	0.399	8
Growth of Inventory	0.104	0.070	0.403	7
Growth of income from principal business operations	0.081	0.032	0.282	24
Growth of sales	0.060	0.031	0.340	16
Cash to Total Assets	0.090	0.066	0.423	5
Due from Shareholders to Total Assets	0.101	0.067	0.398	9
Due to Shareholders to Total Liabilities	0.083	0.039	0.322	18
Other Fixed Assets to Fixed Assets	0.083	0.030	0.268	25
Other Extraordinary Expenses and Losses to Net Sales	0.078	0.038	0.328	17

The top fifteen financial ratios are selected as shown in Table 6.32 for comparing the performances of classifiers in detecting financial accounting fraud. Without employing any feature selection method, classifiers are also compared.

Table 6.32. Selected financial ratios for comparing the performances of classifiers (Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets)

Alternatives (Financial Ratios)	Rank
Debt-to-equity ratio	1
Accounts Payable to Cost of Good Sold	2
Cash ratio	3
Accounts Receivable to Total Assets	4
Cash to Total Assets	5
Debt ratio	6
Growth of Inventory	7
Growth of accounts receivables	8
Due from Shareholders to Total Assets	9
Return on Assets ratio	10
Inventory to Sales	11
Accounts Receivable to Sales	12
Total asset turnover	13
Inventory to Current Assets	14
Current ratio	15

Phase 2: After determining financial ratios for comparing the performances of classifiers in detecting financial accounting fraud, seven different machine learning classifiers are executed in this phase. Classifiers are also compared without using any feature selection method.

Tables 6.33-6.38 summarize the performance measures of classifiers using the proposed feature selection method and without using any feature selection method. Furthermore, different sampling techniques are employed.

As shown in Table 6.33, the Random Forest-without feature selection-oversampling model has the best overall accuracy at 93.74 percent. Naive Bayes- without feature selection-oversampling model performs the worst at 58.81% among all other models.

In terms of True Positive Rate, Random Forest-without feature selection-oversampling model outperforms all other models (Table 6.34). Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets, which is proposed as feature selection method, can not improve performances of classifiers in terms of True Positive Rate.

Table 6.33. Overall Accuracy of classifiers using Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.8639	0.7755	0.7368	0.8704	0.8587	0.8131
Naive Bayes	0.7977	0.5952	0.7290	0.8211	<u>0.5881</u>	0.7695
ANN	0.8674	0.8186	0.7632	0.8979	0.8834	0.8287
KNN	0.8534	0.8400	0.7274	0.8944	0.8992	0.7882
Random Forest	0.8850	0.9055	0.7679	0.9179	0.9374	0.8707
Logistic Regression	0.8716	0.7961	0.7508	0.8956	0.8808	0.8536
Bagging	0.8839	0.8572	0.7570	0.9138	0.9127	0.8614

100

Table 6.34. True Positive Rates of classifiers using Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.3150	0.6030	0.5360	0.3710	0.7830	0.7040
Naive Bayes	0.6670	0.9210	0.6040	0.7880	0.9380	0.6950
ANN	0.4980	0.7660	0.7790	0.6230	0.8790	0.8350
KNN	0.4610	0.8250	0.6880	0.6570	0.8930	0.7690
Random Forest	0.5020	0.8980	0.7350	0.6600	0.9470	0.8820
Logistic Regression	0.4050	0.6860	0.6510	0.5860	0.8440	0.8100
Bagging	0.5050	0.8260	0.6790	0.7200	0.9170	0.8540

As shown in Table 6.35, Support Vector Machines- Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets -Without Sampling model is the highest at 99.10% in terms of True Negative rate. Naive Bayes- Without Feature Selection -Over Sampling model perform the worst at 26.40% among all other models. Oversampling method significantly reduces the performance of Naive Bayes in terms of True Negative Rate.

Table 6.35. True Negative Rates of classifiers using Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.9910	0.9360	0.9380	0.9860	0.9280	0.9220
Naive Bayes	0.8280	0.2930	0.8540	0.8290	0.2640	0.8440
ANN	0.9530	0.8670	0.7480	0.9620	0.8880	0.8220
KNN	0.9440	0.8540	0.7660	0.9490	0.9050	0.8070
Random Forest	0.9740	0.9130	0.8010	0.9780	0.9280	0.8600
Logistic Regression	0.9800	0.8980	0.8500	0.9670	0.9150	0.8970
Bagging	0.9720	0.8870	0.8350	0.9590	0.9080	0.8690

As shown in Table 6.36, it is found that precision of the Random Forest-without feature selection-oversampling model is the highest at 92.50%. Naive Bayes- Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets - Without Sampling model performs the worst at 47.30% among all other models.

Table 6.36. Precision of classifiers using Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.8940	0.8970	0.8960	0.8620	0.9100	0.9000
Naive Bayes	<u>0.4730</u>	0.5470	0.8050	0.5160	0.5420	0.8170
ANN	0.7110	0.8420	0.7550	0.7910	0.8790	0.8250
KNN	0.6580	0.8400	0.7470	0.7510	0.8970	0.7990
Random Forest	0.8170	0.9050	0.7870	0.8720	0.9250	0.8630
Logistic Regression	0.8230	0.8620	0.8130	0.8070	0.9020	0.8870
Bagging	0.8060	0.8710	0.8040	0.8020	0.9030	0.8670

In terms of G-measure, Random Forest classifier outperforms all other classifiers (Table 6.37). Especially, all other models are outperformed by the Random Forest-without feature selection-oversampling model. Naive Bayes- Without Feature Selection -Over Sampling model perform the worst at 49.76% among all other models. Oversampling

method significantly reduces the performance of Naive Bayes in terms of G-measure.

Table 6.37. G-measure of classifiers using Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.5587	0.7513	0.7091	0.6048	0.8524	0.8057
Naive Bayes	0.7432	0.5195	0.7182	0.8082	<u>0.4976</u>	0.7659
ANN	0.6889	0.8149	0.7633	0.7742	0.8835	0.8285
KNN	0.6597	0.8394	0.7260	0.7896	0.8990	0.7878
Random Forest	0.6992	0.9055	0.7673	0.8034	0.9375	0.8709
Logistic Regression	0.6300	0.7849	0.7439	0.7528	0.8788	0.8524
Bagging	0.7006	0.8560	0.7530	0.8310	0.9125	0.8615

With regard to Area Under ROC Curve, Random Forest classifier performs the best among the seven classifiers (Table 6.38). Random Forest-without feature selection-oversampling model is the highest at 98.40%. Support Vector Machines- Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets- Without Sampling model performs the worst at 65.30% among all other models. Oversampling method always improve the performances of all classifiers in terms of Area Under ROC Curve.

Table 6.38. Area Under ROC Curve of classifiers using Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	<u>0.6530</u>	0.7690	0.7370	0.6780	0.8560	0.8130
Naive Bayes	0.7940	0.8240	0.7640	0.8390	0.8580	0.8110
ANN	0.8340	0.8900	0.8280	0.8890	0.9390	0.8980
KNN	0.8170	0.8920	0.7730	0.7980	0.8980	0.7890
Random Forest	0.8820	0.9650	0.8390	0.9490	0.9840	0.9190
Logistic Regression	0.8370	0.8750	0.8200	0.9130	0.9380	0.9050
Bagging	0.8690	0.9270	0.8220	0.9280	0.9690	0.9080

6.2.1.3. Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets

In this section, Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets is utilized for prioritization of financial ratios.

Algorithm of the application

The flowchart of the proposed methodology is presented in Figure 6.12. Notably, the suggested methodology's steps are same and should be performed in both IVSFS and SVSFS. First, IVSF-AHP method is performed for determining the weights of criteria. Then IVSF-TOPSIS is utilized for prioritization of the financial ratios through the implementation of these weights. After finding prioritization of financial ratios for detecting financial accounting fraud, seven different machine learning classifiers are compared by using performance metrics.

Steps of the application

The proposed approach is constituted of various steps, which are outlined in this section. The following are the steps to be taken:

Phase 1: First, IVSF-AHP method is employed to obtain the weights of criteria. The steps are given in the following:

Step 1 Build a hierarchical framework

The hierarchical framework of the evaluation model is built after determining decision makers, criteria and alternatives. The evaluation model is same in both IVSFS and SVSFS and shown in Figure 6.13.

Step 2 Build a pairwise comparison matrix

Using the linguistic phrases listed in Table 4.6, pairwise comparisons of all criteria are conducted with respect to the goal. The IVSFS scale is utilized for rating each pairwise comparison. The same assessments as given in Tables 6.23, 6.24, 6.25, and 6.26 are used.

Step 3 Checking the consistency

In the pairwise comparison matrix, the linguistic phrases are converted to their corresponding score indices. The pairwise comparison matrices' consistency ratios are

then computed. The consistency ratios of all pairwise comparison matrices are determined to be smaller than 0.1, indicating that pairwise comparison matrices are consistent. Tables 6.23, 6.24, 6.25, and 6.26 also present consistency ratios of the pairwise comparison matrices, respectively.

Step 4 Aggregate each decision maker's evaluations

By considering the importance levels of decision makers, all judgments on criteria weights are aggregated using IVSWAM. Due to equal degrees of expertise, each decision maker's importance weight is 0.25 in this study. Table 6.39 shows the aggregated evaluations of the interval-valued spherical fuzzy weights of criteria.

Table 6.39. Aggregated evaluations of the interval-valued spherical fuzzy weights of criteria

Criteria	Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
Relevance	([0.50, 0.55], [0.45, 0.55], [0.30, 0.40])	([0.31, 0.36], [0.59, 0.69], [0.24, 0.31])	([0.41, 0.48], [0.45, 0.53], [0.26, 0.33])	([0.31, 0.35], [0.61, 0.72], [0.23, 0.30])	([0.41, 0.49], [0.45, 0.53], [0.22, 0.28])	([0.32, 0.37], [0.57, 0.67], [0.25, 0.32])
Faithful Representation	([0.63, 0.73], [0.24, 0.30], [0.22, 0.28])	([0.50, 0.55], [0.45, 0.55], [0.30, 0.40])	([0.63, 0.73], [0.27, 0.35], [0.21, 0.27])	([0.47, 0.53], [0.43, 0.51], [0.27, 0.35])	([0.56, 0.66], [0.32, 0.40], [0.23, 0.29])	([0.60, 0.69], [0.30, 0.37], [0.25, 0.32])
Comparability	([0.52, 0.61], [0.33, 0.40], [0.25, 0.32])	([0.40, 0.47], [0.50, 0.59], [0.23, 0.31])	([0.50, 0.55], [0.45, 0.55], [0.30, 0.40])	([0.40, 0.44], [0.52, 0.62], [0.27, 0.36])	([0.43, 0.52], [0.39, 0.46], [0.24, 0.30])	([0.40, 0.44], [0.52, 0.62], [0.27, 0.36])
Verifiability	([0.65, 0.75], [0.23, 0.29], [0.21, 0.27])	([0.50, 0.58], [0.39, 0.47], [0.26, 0.34])	([0.56, 0.64], [0.32, 0.39], [0.26, 0.34])	([0.50, 0.55], [0.45, 0.55], [0.30, 0.40])	([0.60, 0.69], [0.30, 0.37], [0.25, 0.32])	([0.51, 0.58], [0.39, 0.47], [0.29, 0.38])
Timeliness	([0.53, 0.63], [0.30, 0.36], [0.23, 0.28])	([0.45, 0.53], [0.44, 0.52], [0.24, 0.31])	([0.48, 0.57], [0.35, 0.42], [0.24, 0.29])	([0.39, 0.44], [0.54, 0.64], [0.27, 0.35])	([0.50, 0.55], [0.45, 0.55], [0.30, 0.40])	([0.43, 0.52], [0.40, 0.47], [0.24, 0.29])
Understandability	([0.59, 0.69], [0.26, 0.32], [0.24, 0.30])	([0.39, 0.44], [0.54, 0.64], [0.27, 0.35])	([0.56, 0.64], [0.32, 0.39], [0.26, 0.34])	([0.45, 0.50], [0.47, 0.57], [0.29, 0.38])	([0.53, 0.63], [0.33, 0.40], [0.22, 0.27])	([0.50, 0.55], [0.45, 0.55], [0.30, 0.40])

Step 5 Aggregate weights of criteria

IVSWAM is used to determine the interval-valued spherical fuzzy weights of criteria, as shown in Table 6.40.

Table 6.40. Interval-valued spherical fuzzy weights of criteria

Criteria	Weight
Relevance	([0.39, 0.44], [0.51, 0.61], [0.25, 0.33])
Faithful Representation	([0.57, 0.66], [0.33, 0.40], [0.25, 0.32])
Comparability	([0.44, 0.51], [0.45, 0.53], [0.26, 0.34])
Verifiability	([0.56, 0.64], [0.34, 0.41], [0.26, 0.34])
Timeliness	([0.47, 0.54], [0.41, 0.49], [0.25, 0.32])
Understandability	([0.51, 0.59], [0.38, 0.46], [0.26, 0.34])

After determining the interval-valued spherical fuzzy weights of criteria, interval-valued spherical fuzzy TOPSIS method is used for ranking the alternatives. The steps are given in the following:

Step 6 Evaluate the alternatives with respect to the criteria

First, decision makers use the linguistic phrases in Table 4.6 to evaluate each alternative based on each criterion. The evaluations of the alternatives are then aggregated using IVSWAM, taking into account the decision makers' importance levels. Aggregated interval-valued spherical fuzzy decision matrix is given in Table 6.41.

Table 6.41. Aggregated interval-valued spherical fuzzy decision matrix (Interval-Valued Spherical Fuzzy TOPSIS)

Alternatives	Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
Current Ratio	[[0.62, 0.74], [0.32, 0.41], [0.22, 0.30]]	[[0.68, 0.80], [0.22, 0.28], [0.20, 0.26]]	[[0.74, 0.87], [0.18, 0.25], [0.16, 0.23]]	[[0.56, 0.66], [0.27, 0.33], [0.22, 0.28]]	[[0.66, 0.78], [0.23, 0.29], [0.18, 0.25]]	[[0.64, 0.77], [0.24, 0.31], [0.20, 0.26]]
Quick ratio	[[0.63, 0.75], [0.28, 0.36], [0.21, 0.28]]	[[0.70, 0.82], [0.18, 0.23], [0.17, 0.23]]	[[0.75, 0.87], [0.16, 0.21], [0.15, 0.21]]	[[0.68, 0.80], [0.22, 0.28], [0.17, 0.24]]	[[0.68, 0.80], [0.22, 0.28], [0.20, 0.26]]	[[0.74, 0.87], [0.18, 0.25], [0.16, 0.23]]
Cash ratio	[[0.58, 0.71], [0.36, 0.45], [0.20, 0.28]]	[[0.53, 0.63], [0.29, 0.35], [0.24, 0.29]]	[[0.42, 0.51], [0.41, 0.49], [0.23, 0.28]]	[[0.41, 0.49], [0.45, 0.53], [0.23, 0.28]]	[[0.51, 0.60], [0.35, 0.42], [0.21, 0.27]]	[[0.50, 0.59], [0.30, 0.36], [0.25, 0.30]]
Debt ratio	[[0.46, 0.53], [0.42, 0.51], [0.24, 0.31]]	[[0.59, 0.69], [0.26, 0.32], [0.24, 0.30]]	[[0.40, 0.45], [0.50, 0.60], [0.28, 0.36]]	[[0.51, 0.60], [0.35, 0.42], [0.21, 0.27]]	[[0.59, 0.69], [0.27, 0.33], [0.20, 0.26]]	[[0.59, 0.73], [0.31, 0.38], [0.18, 0.26]]
Debt-to-equity ratio	[[0.59, 0.73], [0.31, 0.38], [0.18, 0.26]]	[[0.59, 0.69], [0.26, 0.32], [0.24, 0.30]]	[[0.42, 0.49], [0.43, 0.51], [0.27, 0.33]]	[[0.58, 0.71], [0.36, 0.45], [0.20, 0.28]]	[[0.35, 0.43], [0.47, 0.56], [0.24, 0.29]]	[[0.46, 0.53], [0.42, 0.51], [0.24, 0.31]]
Inventory turnover	[[0.55, 0.64], [0.32, 0.39], [0.24, 0.30]]	[[0.77, 0.89], [0.14, 0.19], [0.13, 0.20]]	[[0.63, 0.73], [0.21, 0.26], [0.21, 0.26]]	[[0.68, 0.80], [0.22, 0.28], [0.17, 0.24]]	[[0.68, 0.80], [0.22, 0.28], [0.17, 0.24]]	[[0.66, 0.78], [0.23, 0.29], [0.18, 0.25]]
Accounts Payable to Cost of Good Sold	[[0.45, 0.53], [0.44, 0.53], [0.23, 0.30]]	[[0.68, 0.80], [0.22, 0.28], [0.20, 0.26]]	[[0.55, 0.64], [0.32, 0.39], [0.24, 0.30]]	[[0.41, 0.49], [0.43, 0.51], [0.24, 0.29]]	[[0.59, 0.72], [0.34, 0.43], [0.21, 0.29]]	[[0.42, 0.49], [0.43, 0.51], [0.27, 0.33]]
Total asset turnover	[[0.57, 0.66], [0.27, 0.33], [0.25, 0.31]]	[[0.76, 0.88], [0.15, 0.20], [0.14, 0.21]]	[[0.62, 0.71], [0.24, 0.30], [0.23, 0.29]]	[[0.67, 0.79], [0.25, 0.33], [0.21, 0.28]]	[[0.68, 0.80], [0.22, 0.28], [0.20, 0.26]]	[[0.63, 0.75], [0.28, 0.36], [0.21, 0.28]]
Return on Assets ratio	[[0.52, 0.61], [0.33, 0.40], [0.25, 0.32]]	[[0.59, 0.69], [0.26, 0.32], [0.24, 0.30]]	[[0.58, 0.67], [0.30, 0.37], [0.25, 0.32]]	[[0.68, 0.80], [0.22, 0.28], [0.20, 0.26]]	[[0.53, 0.60], [0.34, 0.41], [0.28, 0.35]]	[[0.63, 0.75], [0.28, 0.36], [0.21, 0.28]]
Net profit margin ratio	[[0.48, 0.57], [0.35, 0.42], [0.24, 0.29]]	[[0.70, 0.81], [0.21, 0.27], [0.19, 0.25]]	[[0.67, 0.79], [0.25, 0.33], [0.21, 0.28]]	[[0.68, 0.80], [0.22, 0.28], [0.17, 0.24]]	[[0.55, 0.64], [0.32, 0.39], [0.24, 0.30]]	[[0.66, 0.78], [0.23, 0.29], [0.18, 0.25]]
EBIT to Total Assets	[[0.47, 0.56], [0.37, 0.44], [0.23, 0.28]]	[[0.62, 0.71], [0.24, 0.30], [0.23, 0.29]]	[[0.52, 0.60], [0.35, 0.42], [0.24, 0.31]]	[[0.65, 0.78], [0.24, 0.30], [0.17, 0.24]]	[[0.52, 0.60], [0.35, 0.42], [0.24, 0.31]]	[[0.63, 0.76], [0.25, 0.32], [0.19, 0.25]]
Accounts Receivable to Sales	[[0.76, 0.88], [0.15, 0.20], [0.14, 0.21]]	[[0.77, 0.89], [0.14, 0.19], [0.13, 0.20]]	[[0.72, 0.84], [0.17, 0.22], [0.16, 0.22]]	[[0.77, 0.89], [0.14, 0.19], [0.13, 0.20]]	[[0.77, 0.89], [0.14, 0.19], [0.13, 0.20]]	[[0.70, 0.82], [0.18, 0.23], [0.17, 0.23]]

Table 6.41. (Continuation)

Accounts	[[0.65, 0.77], [0.57, 0.66],	[[0.62, 0.71], [0.59, 0.69],	[[0.62, 0.71], [0.59, 0.69],	[[0.62, 0.71], [0.59, 0.69],
Receivable to	[0.27, 0.34], [0.27, 0.33],	[0.24, 0.30], [0.26, 0.32],	[0.24, 0.30], [0.26, 0.32],	[0.24, 0.30], [0.26, 0.32],
Total Assets	[0.22, 0.29], [0.25, 0.31],	[0.23, 0.29], [0.24, 0.30],	[0.23, 0.29], [0.24, 0.30],	[0.23, 0.29], [0.24, 0.30],
Inventory to	[[0.77, 0.89], [0.77, 0.89],	[[0.72, 0.84], [0.77, 0.89],	[[0.82, 0.93], [0.72, 0.84],	[[0.72, 0.84], [0.77, 0.89],
Sales	[0.14, 0.19], [0.14, 0.19],	[0.17, 0.22], [0.14, 0.19],	[0.12, 0.17], [0.17, 0.22],	[0.17, 0.22], [0.14, 0.19],
	[0.13, 0.20], [0.13, 0.20],	[0.16, 0.22], [0.13, 0.20],	[0.09, 0.17], [0.16, 0.22],	[0.16, 0.22], [0.13, 0.20],
Inventory to	[[0.66, 0.78], [0.69, 0.81],	[[0.63, 0.73], [0.69, 0.81],	[[0.63, 0.73], [0.69, 0.81],	[[0.63, 0.73], [0.69, 0.81],
Total Assets	[0.23, 0.29], [0.19, 0.24],	[0.21, 0.26], [0.19, 0.24],	[0.21, 0.26], [0.19, 0.24],	[0.21, 0.26], [0.19, 0.24],
	[0.21, 0.27], [0.19, 0.24],	[0.21, 0.26], [0.19, 0.24],	[0.21, 0.26], [0.19, 0.24],	[0.21, 0.26], [0.19, 0.24],
Inventory to	[[0.69, 0.81], [0.70, 0.82],	[[0.70, 0.82], [0.63, 0.73],	[[0.63, 0.73], [0.69, 0.81],	[[0.69, 0.81], [0.70, 0.82],
Current Assets	[0.19, 0.24], [0.18, 0.23],	[0.18, 0.23], [0.21, 0.26],	[0.21, 0.26], [0.19, 0.24],	[0.19, 0.24], [0.18, 0.23],
	[0.19, 0.24], [0.17, 0.23],	[0.17, 0.23], [0.21, 0.26],	[0.21, 0.26], [0.19, 0.24],	[0.19, 0.24], [0.17, 0.23],
Growth of	[[0.70, 0.81], [0.82, 0.93],	[[0.66, 0.78], [0.74, 0.87],	[[0.70, 0.81], [0.77, 0.89],	[[0.77, 0.89], [0.82, 0.93],
accounts	[0.21, 0.27], [0.12, 0.17],	[0.23, 0.29], [0.18, 0.25],	[0.21, 0.27], [0.14, 0.19],	[0.14, 0.19], [0.12, 0.17],
receivables	[0.19, 0.25], [0.09, 0.17],	[0.21, 0.27], [0.16, 0.23],	[0.19, 0.25], [0.13, 0.20],	[0.13, 0.20], [0.09, 0.17],
Growth of	[[0.70, 0.81], [0.85, 0.95],	[[0.72, 0.84], [0.77, 0.89],	[[0.72, 0.84], [0.77, 0.89],	[[0.76, 0.88], [0.85, 0.95],
Inventory	[0.21, 0.27], [0.10, 0.15],	[0.17, 0.22], [0.14, 0.19],	[0.17, 0.22], [0.14, 0.19],	[0.15, 0.20], [0.10, 0.15],
	[0.19, 0.25], [0.05, 0.15],	[0.16, 0.22], [0.13, 0.20],	[0.16, 0.22], [0.13, 0.20],	[0.14, 0.21], [0.05, 0.15],
Growth of income	[[0.51, 0.58], [0.67, 0.80],	[[0.56, 0.66], [0.70, 0.82],	[[0.56, 0.66], [0.70, 0.82],	[[0.70, 0.82], [0.67, 0.80],
from principal	[0.39, 0.47], [0.23, 0.29],	[0.28, 0.34], [0.18, 0.23],	[0.27, 0.33], [0.18, 0.23],	[0.18, 0.23], [0.23, 0.29],
business	[0.26, 0.34], [0.16, 0.23],	[0.22, 0.27], [0.17, 0.23],	[0.22, 0.28], [0.17, 0.23],	[0.17, 0.23], [0.16, 0.23],
operations				
Growth of sales	[[0.46, 0.53], [0.72, 0.86],	[[0.46, 0.53], [0.65, 0.77],	[[0.58, 0.67], [0.65, 0.77],	[[0.65, 0.77], [0.72, 0.86],
	[0.42, 0.51], [0.20, 0.27],	[0.42, 0.51], [0.28, 0.35],	[0.30, 0.37], [0.28, 0.35],	[0.28, 0.35], [0.20, 0.27],
	[0.24, 0.31], [0.13, 0.22],	[0.24, 0.31], [0.19, 0.26],	[0.25, 0.32], [0.19, 0.26],	[0.19, 0.26], [0.13, 0.22],
Cash to Total	[[0.62, 0.75], [0.67, 0.79],	[[0.65, 0.78], [0.67, 0.79],	[[0.51, 0.59], [0.67, 0.79],	[[0.67, 0.79], [0.65, 0.78],
Assets	[0.31, 0.39], [0.00, 0.00],	[0.00, 0.00], [0.00, 0.00],	[0.00, 0.00], [0.00, 0.00],	[0.00, 0.00], [0.00, 0.00],
	[0.19, 0.27], [0.14, 0.22],	[0.16, 0.23], [0.14, 0.22],	[0.23, 0.30], [0.14, 0.22],	[0.14, 0.22], [0.14, 0.22],
Due from	[[0.65, 0.78], [0.85, 0.95],	[[0.66, 0.78], [0.82, 0.93],	[[0.63, 0.73], [0.77, 0.89],	[[0.77, 0.89], [0.85, 0.95],
Shareholders to	[0.26, 0.32], [0.10, 0.15],	[0.23, 0.29], [0.12, 0.17],	[0.21, 0.26], [0.14, 0.19],	[0.14, 0.19], [0.10, 0.15],
Total Assets	[0.16, 0.24], [0.05, 0.15],	[0.18, 0.25], [0.09, 0.17],	[0.21, 0.26], [0.13, 0.20],	[0.13, 0.20], [0.05, 0.15],
Due to	[[0.56, 0.66], [0.76, 0.88],	[[0.56, 0.66], [0.72, 0.84],	[[0.63, 0.73], [0.70, 0.82],	[[0.70, 0.82], [0.76, 0.88],
Shareholders to	[0.27, 0.33], [0.17, 0.24],	[0.27, 0.33], [0.17, 0.22],	[0.21, 0.26], [0.18, 0.23],	[0.18, 0.23], [0.17, 0.24],
Total Liabilities	[0.22, 0.28], [0.15, 0.22],	[0.22, 0.28], [0.16, 0.22],	[0.21, 0.26], [0.17, 0.23],	[0.17, 0.23], [0.15, 0.22],
Other Fixed	[[0.50, 0.60], [0.56, 0.66],	[[0.50, 0.60], [0.67, 0.80],	[[0.55, 0.65], [0.55, 0.65],	[[0.55, 0.65], [0.56, 0.66],
Assets to Fixed	[0.37, 0.44], [0.28, 0.34],	[0.37, 0.44], [0.24, 0.31],	[0.30, 0.37], [0.30, 0.37],	[0.30, 0.37], [0.28, 0.34],
Assets	[0.20, 0.26], [0.22, 0.27],	[0.20, 0.26], [0.14, 0.23],	[0.20, 0.26], [0.20, 0.26],	[0.20, 0.26], [0.22, 0.27],
Other				
Extraordinary	[[0.62, 0.75], [0.74, 0.87],	[[0.61, 0.73], [0.54, 0.63],	[[0.67, 0.80], [0.64, 0.77],	[[0.64, 0.77], [0.74, 0.87],
Expenses and	[0.31, 0.39], [0.19, 0.25],	[0.36, 0.46], [0.35, 0.43],	[0.24, 0.31], [0.30, 0.37],	[0.30, 0.37], [0.19, 0.25],
Losses to Net	[0.19, 0.27], [0.12, 0.21],	[0.20, 0.29], [0.22, 0.29],	[0.14, 0.23], [0.17, 0.26],	[0.17, 0.26], [0.12, 0.21],
Sales				

Step 7 Constitute the weighted interval-valued spherical fuzzy decision matrix

The weighted interval-valued spherical fuzzy decision matrix is formed using Eq. (3.23) once the weights of the criteria and the ratings of the alternatives have been calculated.

Table 6.42 shows the weighted interval-valued spherical fuzzy decision matrix.

Table 6.42. Weighted interval-valued spherical fuzzy decision matrix (Interval-Valued Spherical Fuzzy TOPSIS)

Alternatives	Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
Current Ratio	[[0.24, 0.33], [0.58, 0.69], [0.30, 0.37]]	[[0.39, 0.53], [0.39, 0.48], [0.30, 0.38]]	[[0.33, 0.44], [0.47, 0.57], [0.29, 0.38]]	[[0.31, 0.42], [0.42, 0.51], [0.32, 0.40]]	[[0.31, 0.43], [0.46, 0.55], [0.29, 0.37]]	[[0.33, 0.45], [0.44, 0.54], [0.31, 0.39]]
Quick ratio	[[0.24, 0.33], [0.57, 0.67], [0.30, 0.37]]	[[0.40, 0.54], [0.37, 0.45], [0.29, 0.37]]	[[0.33, 0.45], [0.47, 0.56], [0.29, 0.37]]	[[0.38, 0.51], [0.39, 0.49], [0.30, 0.38]]	[[0.32, 0.43], [0.45, 0.54], [0.30, 0.38]]	[[0.38, 0.51], [0.42, 0.51], [0.30, 0.38]]
Cash ratio	[[0.23, 0.32], [0.60, 0.71], [0.29, 0.36]]	[[0.30, 0.41], [0.42, 0.51], [0.32, 0.39]]	[[0.19, 0.26], [0.58, 0.67], [0.31, 0.37]]	[[0.23, 0.31], [0.54, 0.64], [0.31, 0.37]]	[[0.24, 0.33], [0.51, 0.61], [0.30, 0.37]]	[[0.26, 0.35], [0.47, 0.56], [0.34, 0.40]]
Debt ratio	[[0.18, 0.24], [0.63, 0.73], [0.30, 0.36]]	[[0.34, 0.45], [0.41, 0.50], [0.32, 0.40]]	[[0.18, 0.23], [0.63, 0.73], [0.33, 0.39]]	[[0.29, 0.39], [0.47, 0.56], [0.31, 0.38]]	[[0.27, 0.37], [0.47, 0.57], [0.30, 0.37]]	[[0.30, 0.43], [0.48, 0.58], [0.30, 0.38]]
Debt-to-equity ratio	[[0.23, 0.32], [0.58, 0.68], [0.28, 0.36]]	[[0.34, 0.45], [0.41, 0.50], [0.32, 0.40]]	[[0.19, 0.25], [0.59, 0.69], [0.33, 0.39]]	[[0.33, 0.46], [0.47, 0.58], [0.30, 0.38]]	[[0.16, 0.23], [0.59, 0.69], [0.31, 0.36]]	[[0.23, 0.31], [0.55, 0.65], [0.32, 0.39]]
Inventory turnover	[[0.21, 0.28], [0.58, 0.68], [0.31, 0.37]]	[[0.44, 0.59], [0.35, 0.44], [0.27, 0.35]]	[[0.28, 0.37], [0.48, 0.58], [0.32, 0.39]]	[[0.38, 0.51], [0.39, 0.49], [0.30, 0.38]]	[[0.32, 0.43], [0.45, 0.54], [0.29, 0.36]]	[[0.34, 0.46], [0.44, 0.53], [0.30, 0.38]]
Accounts Payable to Cost of Good Sold	[[0.17, 0.23], [0.64, 0.74], [0.30, 0.36]]	[[0.39, 0.53], [0.39, 0.48], [0.30, 0.38]]	[[0.25, 0.33], [0.53, 0.63], [0.32, 0.39]]	[[0.23, 0.32], [0.52, 0.62], [0.32, 0.38]]	[[0.27, 0.39], [0.51, 0.62], [0.30, 0.37]]	[[0.22, 0.29], [0.55, 0.65], [0.34, 0.40]]
Total asset turnover	[[0.22, 0.29], [0.57, 0.66], [0.32, 0.38]]	[[0.43, 0.58], [0.36, 0.44], [0.27, 0.36]]	[[0.28, 0.36], [0.50, 0.59], [0.32, 0.39]]	[[0.38, 0.50], [0.41, 0.51], [0.32, 0.40]]	[[0.32, 0.43], [0.45, 0.54], [0.30, 0.38]]	[[0.32, 0.44], [0.46, 0.56], [0.31, 0.39]]
Return on Assets ratio	[[0.20, 0.27], [0.59, 0.69], [0.32, 0.38]]	[[0.34, 0.45], [0.41, 0.50], [0.32, 0.40]]	[[0.26, 0.34], [0.52, 0.62], [0.33, 0.40]]	[[0.38, 0.51], [0.39, 0.49], [0.31, 0.39]]	[[0.25, 0.33], [0.51, 0.60], [0.34, 0.41]]	[[0.32, 0.44], [0.46, 0.56], [0.31, 0.39]]
Net profit margin ratio	[[0.18, 0.25], [0.60, 0.70], [0.31, 0.36]]	[[0.40, 0.54], [0.38, 0.47], [0.30, 0.37]]	[[0.30, 0.40], [0.50, 0.60], [0.31, 0.39]]	[[0.38, 0.51], [0.39, 0.49], [0.30, 0.38]]	[[0.26, 0.35], [0.50, 0.59], [0.32, 0.39]]	[[0.34, 0.46], [0.44, 0.53], [0.30, 0.38]]
EBIT to Total Assets	[[0.18, 0.25], [0.60, 0.70], [0.30, 0.36]]	[[0.35, 0.47], [0.40, 0.49], [0.32, 0.39]]	[[0.23, 0.31], [0.54, 0.64], [0.32, 0.39]]	[[0.37, 0.50], [0.40, 0.50], [0.30, 0.38]]	[[0.24, 0.33], [0.52, 0.61], [0.32, 0.39]]	[[0.32, 0.45], [0.45, 0.54], [0.30, 0.38]]
Accounts Receivable to Sales	[[0.29, 0.39], [0.53, 0.63], [0.27, 0.36]]	[[0.44, 0.59], [0.35, 0.44], [0.27, 0.35]]	[[0.32, 0.43], [0.47, 0.56], [0.29, 0.38]]	[[0.43, 0.57], [0.36, 0.45], [0.28, 0.37]]	[[0.36, 0.48], [0.43, 0.52], [0.27, 0.36]]	[[0.36, 0.48], [0.42, 0.51], [0.30, 0.38]]
Accounts Receivable to Total Assets	[[0.25, 0.34], [0.56, 0.67], [0.30, 0.38]]	[[0.32, 0.43], [0.42, 0.51], [0.33, 0.40]]	[[0.28, 0.36], [0.50, 0.59], [0.32, 0.39]]	[[0.33, 0.44], [0.42, 0.51], [0.33, 0.41]]	[[0.29, 0.39], [0.46, 0.55], [0.31, 0.39]]	[[0.30, 0.40], [0.45, 0.54], [0.33, 0.41]]
Inventory to Sales	[[0.30, 0.40], [0.53, 0.63], [0.27, 0.35]]	[[0.44, 0.59], [0.35, 0.44], [0.27, 0.35]]	[[0.32, 0.43], [0.47, 0.56], [0.29, 0.38]]	[[0.43, 0.57], [0.36, 0.45], [0.28, 0.37]]	[[0.38, 0.50], [0.42, 0.51], [0.26, 0.35]]	[[0.37, 0.49], [0.41, 0.50], [0.30, 0.38]]

Table 6.42. (Continuation)

Inventory to Total Assets	([0.25, 0.35], [0.55, 0.65], [0.30, 0.37])	([0.39, 0.53], [0.37, 0.46], [0.30, 0.37])	([0.28, 0.37], [0.48, 0.58], [0.32, 0.39])	([0.38, 0.52], [0.38, 0.47], [0.31, 0.39])	([0.29, 0.40], [0.45, 0.54], [0.31, 0.38])	([0.31, 0.41], [0.43, 0.52], [0.33, 0.40])
Inventory to Current Assets	([0.26, 0.36], [0.54, 0.64], [0.29, 0.37])	([0.40, 0.54], [0.37, 0.45], [0.29, 0.37])	([0.31, 0.42], [0.47, 0.57], [0.30, 0.38])	([0.35, 0.47], [0.39, 0.48], [0.32, 0.39])	([0.29, 0.40], [0.45, 0.54], [0.31, 0.38])	([0.35, 0.48], [0.42, 0.51], [0.31, 0.39])
Growth of accounts receivables	([0.27, 0.36], [0.54, 0.65], [0.29, 0.37])	([0.47, 0.61], [0.35, 0.43], [0.26, 0.35])	([0.29, 0.40], [0.49, 0.59], [0.31, 0.39])	([0.42, 0.55], [0.38, 0.47], [0.30, 0.38])	([0.33, 0.44], [0.45, 0.54], [0.30, 0.37])	([0.40, 0.52], [0.40, 0.49], [0.28, 0.37])
Growth of Inventory	([0.27, 0.36], [0.54, 0.65], [0.29, 0.37])	([0.48, 0.63], [0.34, 0.43], [0.25, 0.34])	([0.32, 0.43], [0.47, 0.56], [0.29, 0.38])	([0.43, 0.57], [0.36, 0.45], [0.28, 0.37])	([0.34, 0.45], [0.43, 0.52], [0.29, 0.36])	([0.39, 0.52], [0.41, 0.50], [0.29, 0.37])
Growth of income from principal business operations	([0.20, 0.26], [0.61, 0.72], [0.32, 0.38])	([0.38, 0.52], [0.39, 0.48], [0.28, 0.36])	([0.25, 0.34], [0.51, 0.61], [0.31, 0.38])	([0.39, 0.53], [0.38, 0.46], [0.30, 0.38])	([0.26, 0.36], [0.48, 0.57], [0.31, 0.38])	([0.36, 0.48], [0.42, 0.51], [0.30, 0.38])
Growth of sales	([0.18, 0.24], [0.63, 0.73], [0.30, 0.36])	([0.41, 0.56], [0.38, 0.47], [0.27, 0.36])	([0.20, 0.27], [0.59, 0.69], [0.32, 0.38])	([0.36, 0.49], [0.43, 0.52], [0.30, 0.39])	([0.27, 0.36], [0.49, 0.58], [0.33, 0.40])	([0.33, 0.45], [0.46, 0.56], [0.30, 0.39])
Cash to Total Assets	([0.24, 0.33], [0.58, 0.68], [0.29, 0.36])	([0.38, 0.52], [0.33, 0.40], [0.28, 0.37])	([0.29, 0.40], [0.45, 0.53], [0.30, 0.39])	([0.37, 0.51], [0.34, 0.41], [0.29, 0.39])	([0.24, 0.32], [0.41, 0.49], [0.32, 0.40])	([0.34, 0.47], [0.38, 0.46], [0.29, 0.39])
Due from Shareholders to Total Assets	([0.25, 0.35], [0.56, 0.66], [0.28, 0.36])	([0.48, 0.63], [0.34, 0.43], [0.25, 0.34])	([0.29, 0.40], [0.49, 0.59], [0.30, 0.38])	([0.46, 0.59], [0.35, 0.44], [0.27, 0.36])	([0.29, 0.40], [0.45, 0.54], [0.31, 0.38])	([0.40, 0.52], [0.40, 0.49], [0.28, 0.37])
Due to Shareholders to Total Liabilities	([0.22, 0.29], [0.56, 0.66], [0.31, 0.37])	([0.43, 0.58], [0.37, 0.46], [0.28, 0.36])	([0.25, 0.34], [0.51, 0.60], [0.32, 0.39])	([0.40, 0.54], [0.37, 0.46], [0.30, 0.38])	([0.29, 0.40], [0.45, 0.54], [0.31, 0.38])	([0.36, 0.48], [0.42, 0.51], [0.30, 0.38])
Other Fixed Assets to Fixed Assets	([0.19, 0.27], [0.60, 0.70], [0.29, 0.35])	([0.32, 0.43], [0.42, 0.51], [0.31, 0.38])	([0.22, 0.31], [0.56, 0.65], [0.30, 0.37])	([0.38, 0.51], [0.41, 0.50], [0.29, 0.37])	([0.26, 0.36], [0.49, 0.58], [0.30, 0.37])	([0.28, 0.38], [0.47, 0.57], [0.31, 0.38])
Other Extraordinary Expenses and Losses to Net Sales	([0.24, 0.33], [0.58, 0.68], [0.29, 0.36])	([0.42, 0.57], [0.37, 0.47], [0.27, 0.35])	([0.27, 0.37], [0.55, 0.66], [0.30, 0.38])	([0.30, 0.40], [0.47, 0.57], [0.31, 0.39])	([0.31, 0.43], [0.46, 0.56], [0.28, 0.36])	([0.33, 0.45], [0.47, 0.57], [0.30, 0.38])

Step 8 To define IVSF-PIS and IVSF-NIS, the weighted interval-valued spherical fuzzy decision matrix is defuzzified using Eq. (4.27). Table 6.43 shows the derived score function values.

Step 9 The IVSF-PIS and IVSF-NIS are defined based on Table 6.42. Table 6.44 shows the corresponding IVSF-PIS and IVSF-NIS based on the highest and lowest values.

Steps 10-12 Eqs. (4.32) and (4.33) are used to compute the distances of the alternatives from IVSF-PIS and IVSF-NIS. Then, using Eq. (4.34), closeness ratios are computed, and the ranking of the alternatives is obtained based on the decreasing closeness ratio values. Table 6.45 shows the distance from IVSF-PIS $d(X_i, X^+)$, distance from IVSF NIS $d(X_i, X^-)$, closeness ratios and the ranking of the alternatives.

Table 6.43. Score function values

Alternatives	Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
Current ratio	-0.357	-0.005	-0.152	-0.114	-0.146	-0.119
Quick ratio	-0.330	0.029	-0.140	-0.022	-0.135	-0.046
Cash ratio	-0.379	-0.122	-0.371	-0.302	-0.263	-0.213
Debt ratio	-0.450	-0.079	-0.459	-0.183	-0.194	-0.170
Debt-to-equity ratio	-0.347	-0.079	-0.394	-0.154	-0.399	-0.314
Inventory turnover	-0.368	0.085	-0.207	-0.022	-0.133	-0.104
Accounts Payable to Cost of Good Sold	-0.461	-0.005	-0.283	-0.286	-0.236	-0.332
Total asset turnover	-0.345	0.076	-0.226	-0.049	-0.135	-0.146
Return on Assets ratio	-0.385	-0.079	-0.268	-0.024	-0.262	-0.146
Net profit margin ratio	-0.399	0.011	-0.210	-0.022	-0.237	-0.104
EBIT to Total Assets	-0.407	-0.059	-0.311	-0.043	-0.267	-0.124
Accounts Receivable to Sales	-0.245	0.085	-0.156	0.063	-0.066	-0.062
Accounts Receivable to Total Assets	-0.320	-0.101	-0.226	-0.096	-0.175	-0.155
Inventory to Sales	-0.239	0.085	-0.156	0.063	-0.042	-0.052
Inventory to Total Assets	-0.301	0.016	-0.207	-0.004	-0.154	-0.130
Inventory to Current Assets	-0.277	0.029	-0.164	-0.052	-0.154	-0.074
Growth of accounts receivables	-0.282	0.118	-0.202	0.029	-0.123	-0.016
Growth of Inventory	-0.282	0.142	-0.156	0.063	-0.098	-0.024
Growth of income from principal business operations	-0.421	-0.007	-0.259	0.009	-0.205	-0.062
Growth of sales	-0.450	0.036	-0.379	-0.071	-0.221	-0.135
Cash to Total Assets	-0.345	0.048	-0.150	0.028	-0.154	-0.042
Due from Shareholders to Total Assets	-0.309	0.142	-0.200	0.094	-0.154	-0.016
Due to Shareholders to Total Liabilities	-0.342	0.062	-0.254	0.021	-0.154	-0.062
Other Fixed Assets to Fixed Assets	-0.402	-0.105	-0.322	-0.035	-0.222	-0.188
Other Extraordinary Expenses and Losses to Net Sales	-0.345	0.049	-0.291	-0.175	-0.144	-0.146

Table 6.44. Positive and negative interval-valued spherical fuzzy ideal solutions

	Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
IVSF-PIS	([0.30, 0.40], [0.53, 0.63], [0.27, 0.35])	([0.48, 0.63], [0.34, 0.43], [0.25, 0.34])	([0.33, 0.45], [0.47, 0.56], [0.29, 0.37])	([0.46, 0.59], [0.35, 0.44], [0.27, 0.36])	([0.38, 0.50], [0.42, 0.51], [0.26, 0.35])	([0.40, 0.52], [0.40, 0.49], [0.28, 0.37])
IVSF-NIS	([0.17, 0.23], [0.64, 0.74], [0.30, 0.36])	([0.30, 0.41], [0.42, 0.51], [0.32, 0.39])	([0.18, 0.23], [0.63, 0.73], [0.33, 0.39])	([0.23, 0.31], [0.54, 0.64], [0.31, 0.37])	([0.16, 0.23], [0.59, 0.69], [0.31, 0.36])	([0.22, 0.29], [0.55, 0.65], [0.34, 0.40])

Table 6.45. The distance from IVSF-PIS $d(X_i, X^+)$, distance from IVSF-NIS $d(X_i, X^-)$, Closeness ratio and ranking of alternatives

Alternatives	$d(X_i, X^+)$	$d(X_i, X^-)$	Closeness ratio	Rank
Current ratio	0.064	0.104	0.618	13
Quick ratio	0.042	0.125	0.749	6
Cash ratio	0.128	0.037	0.225	24
Debt ratio	0.118	0.048	0.289	22
Debt-to-equity ratio	0.131	0.034	0.207	25
Inventory turnover	0.051	0.116	0.696	9
Accounts Payable to Cost of Good Sold	0.124	0.041	0.249	23
Total asset turnover	0.060	0.110	0.649	12
Return on Assets ratio	0.091	0.083	0.476	18
Net profit margin ratio	0.070	0.098	0.582	15
EBIT to Total Assets	0.092	0.076	0.452	20
Accounts Receivable to Sales	0.017	0.147	0.895	3
Accounts Receivable to Total Assets	0.083	0.089	0.516	17
Inventory to Sales	0.013	0.151	0.919	1
Inventory to Total Assets	0.056	0.112	0.667	11
Inventory to Current Assets	0.047	0.121	0.719	7
Growth of accounts receivables	0.027	0.139	0.840	5
Growth of Inventory	0.015	0.150	0.910	2
Growth of income from principal business operations	0.069	0.100	0.594	14
Growth of sales	0.091	0.078	0.461	19
Cash to Total Assets	0.057	0.128	0.693	10
Due from Shareholders to Total Assets	0.022	0.143	0.867	4
Due to Shareholders to Total Liabilities	0.050	0.117	0.702	8
Other Fixed Assets to Fixed Assets	0.094	0.071	0.429	21
Other Extraordinary Expenses and Losses to Net Sales	0.075	0.091	0.550	16

Table 6.46 shows the top fifteen financial ratios which are selected for comparing the performances of classifiers in detecting financial accounting fraud. Without employing any feature selection method, classifiers are also compared.

Table 6.46. Selected financial ratios for comparing the performances of classifiers (Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets)

Alternatives (Financial Ratios)	Rank
Inventory to Sales	1
Growth of Inventory	2
Accounts Receivable to Sales	3
Due from Shareholders to Total Assets	4
Growth of accounts receivables	5
Quick ratio	6
Inventory to Current Assets	7
Due to Shareholders to Short term External Resources	8
Inventory turnover	9
Cash to Total Assets	10
Inventory to Total Assets	11
Total asset turnover	12
Current ratio	13
Growth of income from principal business operations	14
Net profit margin ratio	15

Phase 2: In the first phase, financial ratios are determined for comparing the performances of classifiers in detecting financial accounting fraud, then in this phase, sampling methods and machine learning classifiers are employed, respectively. Seven different machine learning classifiers are compared by using performance metrics. Moreover, the performances of classifiers are compared without using any feature selection method. Tables 6.47-6.52 present performance measures of classifiers in detecting financial accounting fraud.

Table 6.47 shows overall accuracy of classifiers using different sampling and feature selection techniques. The Random Forest-without feature selection-oversampling model has the best overall accuracy at 93.74 percent.

As shown in Table 6.48, Random Forest-without feature selection-oversampling model is the highest at 94.70% in terms of True Positive rate. Support Vector Machines-Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets- Without sampling model performs the worst at 30.20% among all other models.

Table 6.47. Overall Accuracy of classifiers using Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.8616	0.7777	0.7274	0.8704	0.8587	0.8131
Naive Bayes	0.7290	<u>0.5405</u>	0.7368	0.8211	0.5881	0.7695
ANN	0.8739	0.8220	0.7850	0.8979	0.8834	0.8287
KNN	0.8622	0.8381	0.7383	0.8944	0.8992	0.7882
Random Forest	0.8915	0.9025	0.7835	0.9179	0.9374	0.8707
Logistic Regression	0.8716	0.8025	0.7726	0.8956	0.8808	0.8536
Bagging	0.8903	0.8718	0.7539	0.9138	0.9127	0.8614

100

Table 6.48. True Positive Rates of classifiers using Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	<u>0.3020</u>	0.5920	0.5050	0.3710	0.7830	0.7040
Naive Bayes	0.7350	0.9570	0.6790	0.7880	0.9380	0.6950
ANN	0.5080	0.7760	0.7630	0.6230	0.8790	0.8350
KNN	0.4770	0.8130	0.6760	0.6570	0.8930	0.7690
Random Forest	0.5330	0.8950	0.7480	0.6600	0.9470	0.8820
Logistic Regression	0.4020	0.7000	0.6760	0.5860	0.8440	0.8100
Bagging	0.5260	0.8500	0.6820	0.7200	0.9170	0.8540

In terms of True Negative Rate, SVM classifier outperforms all other classifiers (Table 6.49). It is also found that Support Vector Machines- Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets -Without Sampling model is the highest at 99.10% in terms of True Negative rate.

Table 6.49. True Negative Rates of classifiers using Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.9910	0.9500	0.9500	0.9860	0.9280	0.9220
Naive Bayes	0.7280	0.1540	0.7940	0.8290	0.2640	0.8440
ANN	0.9590	0.8640	0.8070	0.9620	0.8880	0.8220
KNN	0.9520	0.8610	0.8010	0.9490	0.9050	0.8070
Random Forest	0.9750	0.9100	0.8190	0.9780	0.9280	0.8600
Logistic Regression	0.9800	0.8970	0.8690	0.9670	0.9150	0.8970
Bagging	0.9750	0.8920	0.8260	0.9590	0.9080	0.8690

With respect to Precision, Random Forest-Without Feature Selection- Oversampling model is found as the highest at 92.50% (Table 6.50). Naive Bayes- Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets -Without Sampling model performs the worst at 38.50% among all other models.

In terms of G-measure, the Random Forest-without feature selection-oversampling model surpasses all other models, as shown in Table 6.51. Naive Bayes- Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets -Over Sampling model performs the worst at 38.39% among all other models.

Table 6.50. Precision of classifiers using Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.8900	0.9170	0.9100	0.8620	0.9100	0.9000
Naive Bayes	<u>0.3850</u>	0.5120	0.7680	0.5160	0.5420	0.8170
ANN	0.7410	0.8410	0.7980	0.7910	0.8790	0.8250
KNN	0.6950	0.8450	0.7720	0.7510	0.8970	0.7990
Random Forest	0.8300	0.9020	0.8050	0.8720	0.9250	0.8630
Logistic Regression	0.8270	0.8640	0.8380	0.8070	0.9020	0.8870
Bagging	0.8280	0.8800	0.7960	0.8020	0.9030	0.8670

Table 6.51. G-measure of classifiers using Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
	SVM	0.5471	0.7499	0.6926	0.6048	0.8524
Naive Bayes	0.7315	<u>0.3839</u>	0.7343	0.8082	0.4976	0.7659
ANN	0.6980	0.8188	0.7847	0.7742	0.8835	0.8285
KNN	0.6739	0.8367	0.7359	0.7896	0.8990	0.7878
Random Forest	0.7209	0.9025	0.7827	0.8034	0.9375	0.8709
Logistic Regression	0.6277	0.7924	0.7664	0.7528	0.8788	0.8524
Bagging	0.7161	0.8707	0.7506	0.8310	0.9125	0.8615

With respect to Area Under ROC Curve, it is found that Random Forest classifier performs the best among the seven classifiers (Table 6.52). Random Forest-without feature selection-oversampling model is the highest at 98.40%. Support Vector Machines- Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets - Without Sampling model performs the worst at 64.70% among all other models.

The results indicate that the proposed approach as a feature selection method can not improve the machine learning classifiers' performances for this dataset. It is found that the proposed feature selection method can only improve the performance of the SVM classifier in terms of True Negative Rates.

Table 6.52. Area Under ROC Curve of classifiers using Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and TOPSIS with Interval-Valued Spherical Fuzzy Sets					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
	SVM	0.6470	0.7710	0.7270	0.6780	0.8560
Naive Bayes	0.8040	0.8420	0.7720	0.8390	0.8580	0.8110
ANN	0.8470	0.8940	0.8450	0.8890	0.9390	0.8980
KNN	0.7760	0.8960	0.7940	0.7980	0.8980	0.7890
Random Forest	0.8860	0.9630	0.8490	0.9490	0.9840	0.9190
Logistic Regression	0.8270	0.8750	0.8120	0.9130	0.9380	0.9050
Bagging	0.8670	0.9330	0.8280	0.9280	0.9690	0.9080

6.2.1.4. Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets

A novel integrated AHP and TOPSIS approach using circular intuitionistic fuzzy sets is applied in this section.

Algorithm of the application

The proposed approach consists of two stages: (i) ranking and determining financial ratios by employing integrated AHP and TOPSIS with circular intuitionistic fuzzy sets and then (ii) employing sampling methods and seven different classifiers. The flowchart of the proposed approach is given in Figure 6.14.

Steps of the application

The steps of proposed approach are given as follows;

Phase 1: In this phase, integrated AHP and TOPSIS with circular intuitionistic fuzzy sets is applied to determine financial ratios for financial accounting fraud detection.

Step 1 Build the hierarchical framework of the considered problem

In order to build a hierarchical structure, determined criteria and alternatives with respect to the goal are restructured. Figure 6.15 depicts the problem's hierarchical structure.

Step 2 Apply C-IF AHP method

C-IF AHP method is applied to calculate the weights of criteria in this step. The following are the sub-steps of the proposed method:

Step 2.1 Gather individual views on criteria under intuitionistic environment

Pairwise comparisons can be used to determine importance weight of each criterion. Using our proposed IFS scale of linguistic phrases (Table 4.7), decision makers make pairwise comparisons. Table 6.53 displays decision makers' linguistic term-based judgements.

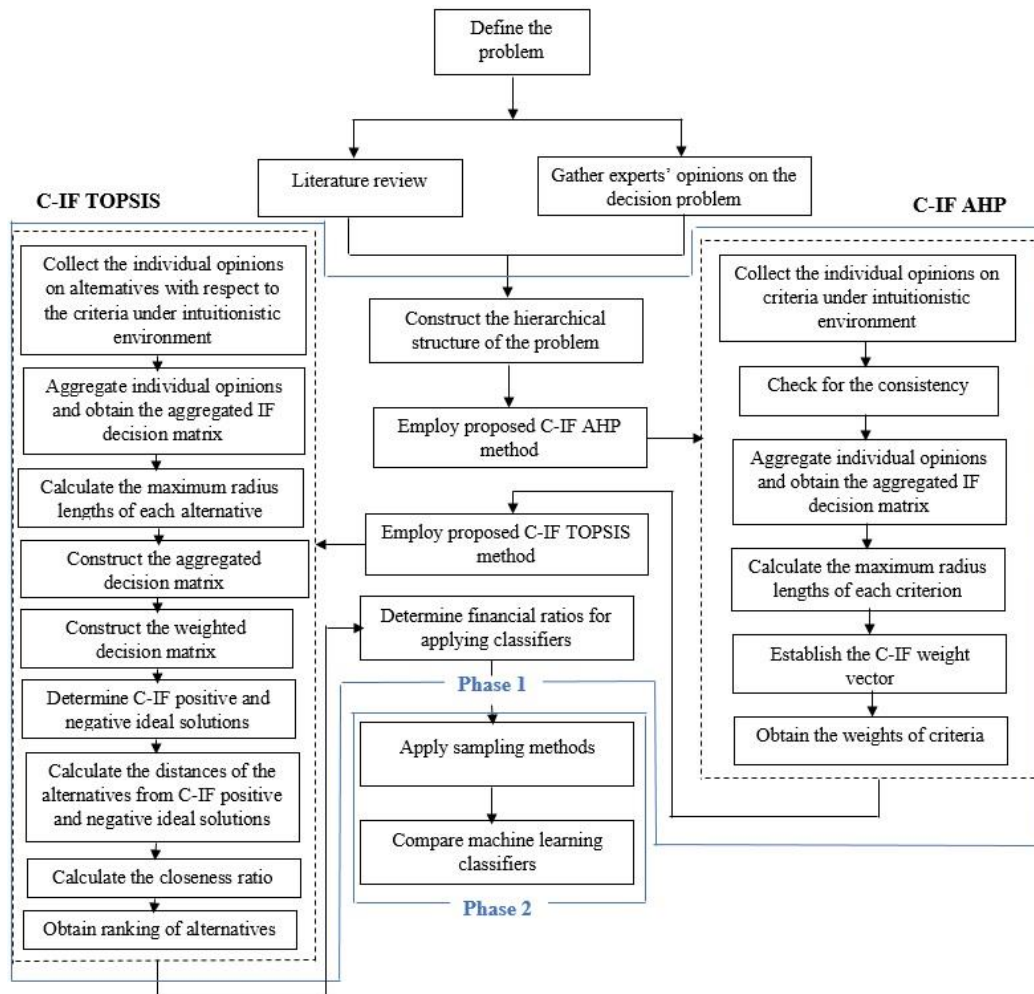


Figure 6.14. The flowchart of the proposed approach (Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets)

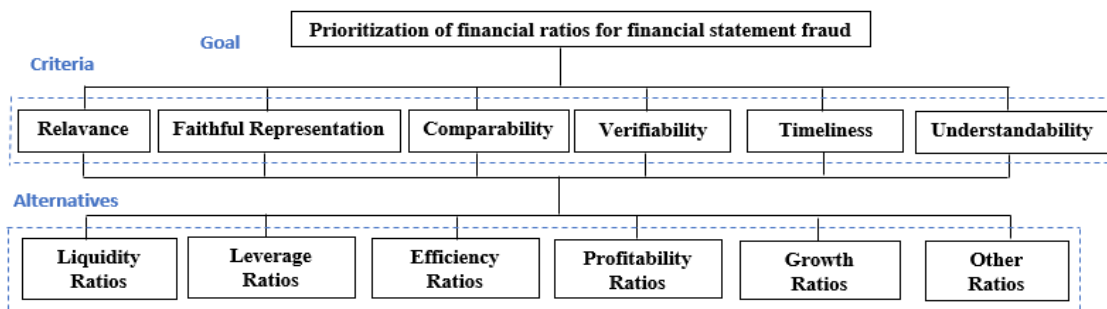


Figure 6.15. The hierarchical structure of the problem (Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets)

Step 2.2 Checking the consistency

The decision makers' linguistic term-based judgements are converted into their corresponding score indices. The consistency ratios of each decision maker's judgments are then computed and presented in Table 6.53. It is found that each decision maker's consistency ratios are smaller than 0.1, indicating that pairwise comparison matrices are consistent.

Step 2.3 Construct the aggregated IF decision matrix by aggregating individual views

The scale in Table 4.7 is used to transform linguistic phrases to their corresponding IFNs. Then, using the IFWA operator in Eq. (4.35), all judgements of criteria are aggregated. The significance weights of each decision maker are taken into account here. Because of their similar degrees of expertise, each decision maker in this study is given equal weight. Table 6.54 shows the aggregated intuitionistic fuzzy decision matrix.

Then, using the IFWA operator, intuitionistic fuzzy weights of criteria are computed, as shown in Table 6.55.

Step 2.4 Compute each criterion's maximum radius lengths

First, the radius lengths based on each decision maker's judgments are computed, and then the maximum radius lengths among all experts are calculated using Eqs. (3.40) and (3.41), as shown in Table 6.56.

Table 6.53. Pairwise comparisons of all criteria by using linguistic phrases related to IFS scale

DM ₁							DM ₂						
Criteria	Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability	Criteria	Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
Relevance	EI	SLI	SMI	EI	HI	LI	Relevance	EI	EI	SLI	VLI	LI	EI
Faithful Representation	SMI	EI	VHI	EI	VHI	SMI	Faithful Representation	EI	EI	SLI	LI	LI	EI
Comparability	SLI	VLI	EI	LI	SMI	LI	Comparability	SMI	SMI	EI	EI	SLI	EI
Verifiability	EI	EI	HI	EI	VHI	EI	Verifiability	VHI	HI	EI	EI	EI	SMI
Timeliness	LI	VLI	SLI	VLI	EI	VLI	Timeliness	HI	HI	SMI	EI	EI	SMI
Understandability	HI	SLI	HI	EI	VHI	EI	Understandability	EI	EI	EI	SLI	SLI	EI
CR ₁ = 0.076							CR ₂ = 0.044						
DM ₃							DM ₄						
Criteria	Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability	Criteria	Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
Relevance	EI	LI	LI	LI	SLI	LI	Relevance	EI	VLI	EI	LI	SLI	SLI
Faithful Representation	HI	EI	EI	EI	EI	EI	Faithful Representation	VHI	EI	VHI	SMI	SMI	VHI
Comparability	HI	EI	EI	EI	SMI	EI	Comparability	EI	VLI	EI	SLI	LI	SLI
Verifiability	HI	EI	EI	EI	SMI	EI	Verifiability	HI	SLI	SMI	EI	EI	EI
Timeliness	SMI	EI	SLI	SLI	EI	SLI	Timeliness	SMI	SLI	HI	EI	EI	SMI
Understandability	HI	EI	EI	EI	SMI	EI	Understandability	SMI	VLI	SMI	EI	SLI	EI
CR ₃ = 0.028							CR ₄ = 0.051						

Table 6.54. Aggregated evaluations of the intuitionistic fuzzy weights of criteria

Criteria	Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
Relevance	(0.500, 0.500)	(0.291, 0.640)	(0.421, 0.501)	(0.267, 0.665)	(0.414, 0.474)	(0.312, 0.619)
Faithful Representation	(0.669, 0.234)	(0.500, 0.500)	(0.656, 0.234)	(0.468, 0.479)	(0.577, 0.320)	(0.624, 0.294)
Comparability	(0.547, 0.366)	(0.366, 0.557)	(0.500, 0.500)	(0.388, 0.569)	(0.453, 0.441)	(0.388, 0.569)
Verifiability	(0.692, 0.211)	(0.521, 0.416)	(0.584, 0.350)	(0.500, 0.500)	(0.624, 0.294)	(0.527, 0.440)
Timeliness	(0.557, 0.335)	(0.446, 0.468)	(0.508, 0.383)	(0.370, 0.589)	(0.500, 0.500)	(0.437, 0.456)
Understandability	(0.634, 0.278)	(0.370, 0.589)	(0.584, 0.350)	(0.456, 0.523)	(0.555, 0.322)	(0.500, 0.500)

Table 6.55. Intuitionistic fuzzy weights of criteria

Criteria	$\tilde{W}$
Relevance	(0.373, 0.561)
Faithful Representation	(0.589, 0.328)
Comparability	(0.444, 0.494)
Verifiability	(0.580, 0.355)
Timeliness	(0.473, 0.448)
Understandability	(0.524, 0.411)

Table 6.56. The maximum radius lengths among all experts

Max $r(i)$	Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
Relevance	0.000	0.270	0.301	0.305	0.460	0.236
Faithful Representation	0.270	0.000	0.407	0.320	0.442	0.320
Comparability	0.301	0.407	0.000	0.215	0.318	0.215
Verifiability	0.305	0.320	0.215	0.000	0.320	0.168
Timeliness	0.460	0.442	0.318	0.320	0.000	0.424
Understandability	0.236	0.320	0.215	0.168	0.424	0.000

The radius lengths of each criterion are determined as follows using the values in Table 6.56:

$$r_{Relevance} = \max(0.000, 0.270, 0.301, 0.305, 0.460, 0.236) = 0.460$$

$r_{Faithful Representation}$, $r_{Comparability}$, $r_{Verifiability}$, $r_{Timeliness}$ and $r_{Understandability}$ are computed by using the same way and obtained as 0.442, 0.407, 0.320, 0.460 and 0.424 respectively.

Step 2.5 Construct a weight vector for the C-IFS

Using the criteria weights acquired in Step 2.3 and the maximum radius lengths of each criterion determined in Step 2.4, the circular intuitionistic fuzzy weight vector matrix is formed as shown in Table 6.57.

Table 6.57. C-IF weights of criteria

Criteria	$D = \langle (\mu_j, \nu_j; r_j) \rangle$
Relevance	$\langle (0.373, 0.561; 0.460) \rangle$
Faithful Representation	$\langle (0.589, 0.328; 0.442) \rangle$
Comparability	$\langle (0.444, 0.494; 0.407) \rangle$
Verifiability	$\langle (0.580, 0.355; 0.320) \rangle$
Timeliness	$\langle (0.473, 0.448; 0.460) \rangle$
Understandability	$\langle (0.524, 0.411; 0.424) \rangle$

Step 2.6 Calculate the criteria's weights

Relative Score Function (RSF) values are derived using the values in Table 6.57 as follows:

$$RSF_{\text{Relevance}} = \frac{(1-0.561)(1+0.373)+0.373}{6} \times \left(\frac{\frac{1}{0.460}}{\sqrt{\frac{1}{0.460^2} + \frac{1}{0.442^2} + \frac{1}{0.407^2} + \frac{1}{0.320^2} + \frac{1}{0.460^2} + \frac{1}{0.424^2}}} \right) 0.1$$

$$= 0.147$$

Other criteria' RSF values are determined in the same way. The weights of criteria are then obtained by normalizing RSF values. Table 6.58 shows the RSF values and weights of the criteria.

Table 6.58. RSF values and weights of criteria

Criteria	RSF values	Criteria weights
Relevance	0.147	0.119
Faithful Representation	0.251	0.203
Comparability	0.179	0.145
Verifiability	0.250	0.202
Timeliness	0.194	0.157
Understandability	0.216	0.175

After determining the weights of criteria, C-IF TOPSIS method is executed for ranking the alternatives. The steps of the proposed method are given as follows:

Step 3. Apply C-IF TOPSIS method

C-IF TOPSIS method is applied to rank the alternatives in this step. The following are the sub-steps of the proposed method:

Step 3.1 Gather individual views on alternatives based on the criteria under intuitionistic environment

Using the linguistic phrases listed in Table 4.7, decision makers rate each alternative with respect to each of the criteria. Table 6.59 shows decision makers' linguistic preferences.

Steps 3.2 Construct the aggregated IF decision matrix by aggregating individual views

The linguistic preferences of decision makers on the alternatives shown in Table 6.59 are converted into intuitionistic fuzzy numbers, which are then aggregated using the IFWA operator given in Eq (4.35). Table 6.60 shows the aggregated intuitionistic fuzzy decision matrix.

Step 3.3 Compute each alternative's maximum radius lengths

Eqs. (3.40) and (3.41) are used to calculate the maximum radius lengths of each alternative, as shown in Table 6.61.

Step 3.4 Build the aggregated decision matrix for C-IFS

The aggregated circular intuitionistic fuzzy decision matrix is built as shown in Table 6.62 after getting the aggregated intuitionistic fuzzy decision matrix in Table 6.60 and the maximum radius lengths of alternatives in Table 6.61.

Step 3.5 Build the weighted decision matrix for C-IFS

Eqs. (3.37) and (4.37) are used to compute the weighted circular intuitionistic fuzzy numbers using the aggregated circular intuitionistic fuzzy numbers in Table 6.62 and the weights of criteria in Table 6.58.

Table 6.59. Ratings of alternatives with respect to criteria in linguistic phrases related to IFS scale for all decision makers

Alternatives	Relevance				Faithful Representation				Comparability				Verifiability				Timeliness				Understandability			
	DM1	DM2	DM3	DM4	DM1	DM2	DM3	DM4	DM1	DM2	DM3	DM4	DM1	DM2	DM3	DM4	DM1	DM2	DM3	DM4	DM1	DM2	DM3	DM4
Current Ratio	AHI	EI	EI	LI	EI	VHI	HI	AHI	AHI	HI	EI	AHI	HI	VHI	LI	VHI	AHI	VHI	LI	HI	AHI	HI	LI	HI
Quick ratio	AHI	EI	HI	LI	AHI	VHI	HI	VHI	AHI	HI	HI	AHI	AHI	VHI	LI	VHI	AHI	VHI	EI	HI	AHI	AHI	EI	HI
Cash ratio	VLI	EI	ALI	LI	LI	HI	HI	VHI	ALI	HI	HI	LI	VLI	VHI	LI	LI	VLI	VHI	VHI	LI	HI	HI	HI	LI
Debt ratio	VHI	EI	VLI	LI	VHI	HI	EI	VHI	LI	EI	EI	LI	VHI	VHI	VLI	LI	VHI	VHI	VLI	VHI	AHI	HI	VLI	LI
Debt-to-equity ratio	AHI	HI	VLI	LI	HI	VHI	EI	VHI	LI	HI	EI	LI	EI	AHI	VLI	LI	LI	HI	VLI	LI	EI	VHI	VLI	LI
Inventory turnover	LI	VHI	VHI	EI	AHI	AHI	VHI	VHI	VHI	VHI	HI	VHI	AHI	VHI	LI	VHI	VHI	AHI	LI	VHI	AHI	HI	LI	VHI
Accounts Payable to Cost of Good Sold	VLI	VHI	VLI	EI	EI	AHI	HI	VHI	LI	VHI	EI	VHI	LI	VHI	LI	LI	LI	AHI	LI	EI	LI	HI	LI	EI
Total asset turnover	VHI	HI	HI	EI	AHI	HI	AHI	VHI	VHI	VHI	EI	VHI	AHI	VHI	EI	EI	AHI	VHI	HI	EI	AHI	HI	LI	EI
Return on Assets ratio	LI	HI	VHI	EI	EI	HI	VHI	VHI	EI	VHI	VHI	EI	AHI	VHI	HI	EI	EI	HI	HI	EI	AHI	HI	LI	EI
Net profit margin ratio	LI	HI	VHI	LI	EI	VHI	AHI	VHI	EI	VHI	AHI	EI	AHI	VHI	VHI	LI	EI	VHI	VHI	LI	AHI	VHI	HI	LI
EBIT to Total Assets	LI	HI	VHI	VLI	EI	VHI	VHI	VHI	EI	VHI	HI	VLI	AHI	VHI	HI	VLI	EI	VHI	HI	VLI	AHI	HI	HI	VLI
Accounts Receivable to Sales	AHI	VHI	AHI	HI	AHI	VHI	AHI	VHI	AHI	VHI	VHI	VHI	AHI	VHI	AHI	VHI	AHI	AHI	VHI	VHI	AHI	HI	VHI	VHI
Accounts Receivable to Total Assets	EI	HI	AHI	EI	EI	HI	HI	VHI	EI	VHI	VHI	VHI	EI	HI	VHI	VHI	EI	VHI	VHI	VHI	EI	HI	VHI	VHI
Inventory to Sales	AHI	VHI	AHI	VHI	AHI	VHI	AHI	VHI	AHI	VHI	VHI	VHI	AHI	VHI	AHI	VHI	AHI	AHI	AHI	VHI	AHI	VHI	VHI	VHI
Inventory to Total Assets	HI	HI	AHI	EI	HI	HI	VHI	AHI	HI	VHI	VHI	VHI	HI	HI	AHI	VHI	HI	VHI	VHI	VHI	HI	HI	VHI	VHI
Inventory to Current Assets	HI	HI	AHI	VHI	HI	VHI	VHI	AHI	HI	VHI	AHI	VHI	HI	VHI	VHI	VHI	HI	VHI	VHI	VHI	HI	HI	AHI	VHI
Growth of accounts receivables	AHI	VHI	VHI	EI	AHI	AHI	AHI	VHI	AHI	HI	HI	EI	AHI	AHI	HI	EI	AHI	VHI	VHI	EI	AHI	AHI	VHI	VHI
Growth of Inventory	AHI	VHI	VHI	EI	AHI	AHI	AHI	AHI	AHI	VHI	VHI	VHI	AHI	AHI	VHI	VHI	AHI	VHI	VHI	VHI	AHI	AHI	HI	VHI
Growth of income from principal business operations	LI	VHI	EI	EI	VLI	AHI	VHI	VHI	VLI	VHI	HI	VHI	VHI	AHI	HI	VHI	LI	VHI	HI	VHI	VHI	AHI	HI	VHI
Growth of sales	LI	VHI	EI	VLI	VLI	AHI	HI	AHI	LI	VHI	EI	VLI	VHI	AHI	EI	VLI	EI	VHI	EI	VHI	VHI	AHI	EI	VLI
Cash to Total Assets	ALI	HI	AHI	EI		VHI	AHI	VHI		HI	AHI	VHI		VHI	AHI	VHI		HI	VHI	EI		VHI	AHI	VHI
Due from Shareholders to Total Assets	ALI	HI	AHI	VHI	AHI	AHI	AHI	AHI	LI	HI	AHI	VHI	AHI	AHI	AHI	VHI	VHI	HI	VHI	VHI	AHI	VHI	AHI	VHI
Due to Shareholders to Total Liabilities	LI	HI	VHI	VHI	AHI	VHI	EI	AHI	LI	HI	VHI	VHI	AHI	VHI	VHI	VHI	VHI	VHI	HI	VHI	AHI	VHI	HI	VHI
Other Fixed Assets to Fixed Assets	ALI	VHI	VHI	LI	VLI	VHI	HI	VHI	ALI	VHI	VHI	LI	ALI	VHI	AHI	VHI	ALI	VHI	HI	VHI	ALI	VHI	VHI	HI
Other Extraordinary Expenses and Losses to Net Sales	ALI	HI	AHI	EI	VLI	AHI	AHI	VHI	ALI	EI	AHI	EI	ALI	VHI	VHI	EI	ALI	AHI	VHI	VHI	ALI	AHI	VHI	EI

Table 6.60. The aggregated intuitionistic fuzzy decision matrix

Alternatives	Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
Current Ratio	(0.624, 0)	(0.650, 0)	(0.803, 0)	(0.687, 0.193)	(0.737, 0)	(0.709, 0)
Quick ratio	(0.669, 0)	(0.843, 0)	(0.827, 0)	(0.762, 0)	(0.766, 0)	(0.803, 0)
Cash ratio	(0.564, 0)	(0.510, 0.230)	(0.482, 0.398)	(0.417, 0.445)	(0.588, 0.274)	(0.617, 0.274)
Debt ratio	(0.482, 0.409)	(0.722, 0.178)	(0.368, 0.592)	(0.588, 0.274)	(0.709, 0.168)	(0.617, 0)
Debt-to-equity ratio	(0.617, 0)	(0.692, 0.178)	(0.443, 0.470)	(0.564, 0)	(0.355, 0.529)	(0.482, 0.409)
Inventory turnover	(0.644, 0.243)	(0.881, 0)	(0.779, 0.119)	(0.762, 0)	(0.762, 0)	(0.737, 0)
Accounts Payable to Cost of Good Sold	(0.467, 0.423)	(0.706, 0)	(0.644, 0.243)	(0.434, 0.430)	(0.577, 0)	(0.443, 0.470)
Total asset turnover	(0.692, 0.211)	(0.868, 0)	(0.749, 0.150)	(0.734, 0)	(0.766, 0)	(0.669, 0)
Return on Assets ratio	(0.606, 0.289)	(0.650, 0.178)	(0.684, 0.224)	(0.766, 0)	(0.613, 0.316)	(0.669, 0)
Net profit margin ratio	(0.557, 0.315)	(0.734, 0)	(0.734, 0)	(0.762, 0)	(0.644, 0.243)	(0.737, 0)
EBIT to Total Assets	(0.544, 0.325)	(0.684, 0.150)	(0.595, 0.299)	(0.729, 0)	(0.595, 0.299)	(0.700, 0)
Accounts Receivable to Sales	(0.843, 0)	(0.881, 0)	(0.832, 0)	(0.859, 0)	(0.859, 0)	(0.814, 0)
Accounts Receivable to Total Assets	(0.706, 0)	(0.613, 0.211)	(0.749, 0.150)	(0.722, 0.178)	(0.749, 0.150)	(0.722, 0.178)
Inventory to Sales	(0.859, 0)	(0.881, 0)	(0.832, 0)	(0.859, 0)	(0.881, 0)	(0.832, 0)
Inventory to Total Assets	(0.741, 0)	(0.729, 0)	(0.779, 0.119)	(0.794, 0)	(0.779, 0.119)	(0.755, 0.141)
Inventory to Current Assets	(0.794, 0)	(0.755, 0)	(0.814, 0)	(0.779, 0.119)	(0.779, 0.119)	(0.794, 0)
Growth of accounts receivables	(0.789, 0)	(0.900, 0)	(0.741, 0)	(0.803, 0)	(0.789, 0)	(0.859, 0)
Growth of Inventory	(0.789, 0)	(0.900, 0)	(0.832, 0)	(0.859, 0)	(0.832, 0)	(0.843, 0)
Growth of income from principal business operations	(0.553, 0.364)	(0.643, 0)	(0.678, 0.200)	(0.814, 0)	(0.687, 0.193)	(0.814, 0)
Growth of sales	(0.482, 0.409)	(0.605, 0)	(0.482, 0.409)	(0.692, 0)	(0.684, 0.224)	(0.692, 0)
Cash to Total Assets	(0.650, 0)	(0.624, 0)	(0.722, 0)	(0.749, 0)	(0.584, 0)	(0.749, 0)
Due from Shareholders to Total Assets	(0.722, 0)	(0.900, 0)	(0.737, 0)	(0.881, 0)	(0.779, 0.119)	(0.859, 0)
Due to Shareholders to Total Liabilities	(0.687, 0.193)	(0.822, 0)	(0.687, 0.193)	(0.832, 0)	(0.779, 0.119)	(0.814, 0)
Other Fixed Assets to Fixed Assets	(0.577, 0.282)	(0.530, 0.200)	(0.577, 0.282)	(0.749, 0)	(0.669, 0.206)	(0.669, 0.206)
Other Extraordinary Expenses and Losses to Net Sales	(0.650, 0)	(0.700, 0)	(0.602, 0)	(0.624, 0.259)	(0.749, 0)	(0.684, 0)

Table 6.61. The maximum radius lengths of each alternative

Alternatives	Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
Current Ratio	0.567	0.375	0.410	0.601	0.636	0.601
Quick ratio	0.513	0.141	0.141	0.672	0.375	0.410
Cash ratio	0.690	0.566	0.566	0.672	0.530	0.530
Debt ratio	0.584	0.340	0.180	0.530	0.742	0.601
Debt-to-equity ratio	0.601	0.340	0.442	0.690	0.566	0.584
Inventory turnover	0.513	0.071	0.106	0.672	0.672	0.636
Accounts Payable to Cost of Good Sold	0.619	0.375	0.513	0.636	0.654	0.442
Total asset turnover	0.305	0.177	0.375	0.355	0.375	0.513
Return on Assets ratio	0.478	0.340	0.250	0.375	0.180	0.513
Net profit margin ratio	0.460	0.410	0.355	0.672	0.513	0.636
EBIT to Total Assets	0.495	0.375	0.584	0.742	0.584	0.707
Accounts Receivable to Sales	0.177	0.071	0.106	0.071	0.071	0.141
Accounts Receivable to Total Assets	0.391	0.305	0.375	0.340	0.375	0.340
Inventory to Sales	0.071	0.071	0.106	0.071	0.106	0.106
Inventory to Total Assets	0.340	0.177	0.106	0.177	0.106	0.071
Inventory to Current Assets	0.177	0.141	0.141	0.106	0.106	0.177
Growth of accounts receivables	0.410	0.106	0.340	0.410	0.410	0.071
Growth of Inventory	0.410	0.000	0.106	0.071	0.106	0.177
Growth of income from principal business operations	0.461	0.778	0.707	0.141	0.601	0.141
Growth of sales	0.584	0.778	0.584	0.654	0.250	0.654
Cash to Total Assets	0.725	0.836	0.806	0.836	0.718	0.836
Due from Shareholders to Total Assets	0.849	0.000	0.636	0.106	0.106	0.071
Due to Shareholders to Total Liabilities	0.601	0.445	0.601	0.106	0.106	0.141
Other Fixed Assets to Fixed Assets	0.636	0.707	0.636	0.884	0.813	0.813
Other Extraordinary Expenses and Losses to Net Sales	0.725	0.813	0.637	0.725	0.884	0.760

Table 6.62. The aggregated circular intuitionistic fuzzy decision matrix

Alternatives	Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
Current Ratio	(0.624, 0; 0.567)	(0.650, 0; 0.375)	(0.803, 0; 0.410)	(0.687, 0.193; 0.601)	(0.737, 0; 0.636)	(0.709, 0; 0.601)
Quick ratio	(0.669, 0; 0.513)	(0.843, 0; 0.141)	(0.827, 0; 0.141)	(0.762, 0; 0.672)	(0.766, 0; 0.375)	(0.803, 0; 0.410)
Cash ratio	(0.564, 0; 0.690)	(0.510, 0.230; 0.566)	(0.482, 0.398; 0.566)	(0.417, 0.445; 0.672)	(0.588, 0.274; 0.530)	(0.617, 0.274; 0.530)
Debt ratio	(0.482, 0.409; 0.584)	(0.722, 0.178; 0.340)	(0.368, 0.592; 0.180)	(0.588, 0.274; 0.530)	(0.709, 0.168; 0.742)	(0.617, 0; 0.601)
Debt-to-equity ratio	(0.617, 0; 0.601)	(0.692, 0.178; 0.340)	(0.443, 0.470; 0.442)	(0.564, 0; 0.690)	(0.355, 0.529; 0.566)	(0.482, 0.409; 0.584)
Inventory turnover	(0.644, 0.243; 0.513)	(0.881, 0; 0.071)	(0.779, 0.119; 0.106)	(0.762, 0; 0.672)	(0.762, 0; 0.672)	(0.737, 0; 0.636)
Accounts Payable to Cost of Good Sold	(0.467, 0.423; 0.619)	(0.706, 0; 0.375)	(0.644, 0.243; 0.513)	(0.434, 0.430; 0.636)	(0.577, 0; 0.654)	(0.443, 0.470; 0.442)
Total asset turnover	(0.692, 0.211; 0.305)	(0.868, 0; 0.177)	(0.749, 0.150; 0.375)	(0.734, 0; 0.355)	(0.766, 0; 0.375)	(0.669, 0; 0.513)
Return on Assets ratio	(0.606, 0.289; 0.478)	(0.650, 0.178; 0.340)	(0.684, 0.224; 0.250)	(0.766, 0; 0.375)	(0.613, 0.316; 0.180)	(0.669, 0; 0.513)
Net profit margin ratio	(0.557, 0.315; 0.460)	(0.734, 0; 0.410)	(0.734, 0; 0.355)	(0.762, 0; 0.672)	(0.644, 0.243; 0.513)	(0.737, 0; 0.636)
EBIT to Total Assets	(0.544, 0.325; 0.495)	(0.684, 0.150; 0.375)	(0.595, 0.299; 0.584)	(0.729, 0; 0.742)	(0.595, 0.299; 0.584)	(0.700, 0; 0.707)
Accounts Receivable to Sales	(0.843, 0; 0.177)	(0.881, 0; 0.071)	(0.832, 0; 0.106)	(0.859, 0; 0.071)	(0.859, 0; 0.071)	(0.814, 0; 0.141)
Accounts Receivable to Total Assets	(0.706, 0; 0.391)	(0.613, 0.211; 0.305)	(0.749, 0.150; 0.375)	(0.722, 0.178; 0.340)	(0.749, 0.150; 0.375)	(0.722, 0.178; 0.340)
Inventory to Sales	(0.859, 0; 0.071)	(0.881, 0; 0.071)	(0.832, 0; 0.106)	(0.859, 0; 0.071)	(0.881, 0; 0.106)	(0.832, 0; 0.106)
Inventory to Total Assets	(0.741, 0; 0.340)	(0.729, 0; 0.177)	(0.779, 0.119; 0.106)	(0.794, 0; 0.177)	(0.779, 0.119; 0.106)	(0.755, 0.141; 0.071)
Inventory to Current Assets	(0.794, 0; 0.177)	(0.755, 0; 0.141)	(0.814, 0; 0.141)	(0.779, 0.119; 0.106)	(0.779, 0.119; 0.106)	(0.794, 0; 0.177)
Growth of accounts receivables	(0.789, 0; 0.410)	(0.900, 0; 0.106)	(0.741, 0; 0.340)	(0.803, 0; 0.410)	(0.789, 0; 0.410)	(0.859, 0; 0.071)
Growth of Inventory	(0.789, 0; 0.410)	(0.900, 0; 0)	(0.832, 0; 0.106)	(0.859, 0; 0.071)	(0.832, 0; 0.106)	(0.843, 0; 0.177)
Growth of income from principal business operations	(0.553, 0.364; 0.461)	(0.643, 0; 0.778)	(0.678, 0.200; 0.707)	(0.814, 0; 0.141)	(0.687, 0.193; 0.601)	(0.814, 0; 0.141)
Growth of sales	(0.482, 0.409; 0.584)	(0.605, 0; 0.778)	(0.482, 0.409; 0.584)	(0.692, 0; 0.654)	(0.684, 0.224; 0.250)	(0.692, 0; 0.654)
Cash to Total Assets	(0.650, 0; 0.725)	(0.624, 0; 0.836)	(0.722, 0; 0.806)	(0.749, 0; 0.836)	(0.584, 0; 0.718)	(0.749, 0; 0.836)
Due from Shareholders to Total Assets	(0.722, 0; 0.849)	(0.900, 0; 0)	(0.737, 0; 0.636)	(0.881, 0; 0.106)	(0.779, 0.119; 0.106)	(0.859, 0; 0.071)
Due to Shareholders to Total Liabilities	(0.687, 0.193; 0.601)	(0.822, 0; 0.445)	(0.687, 0.193; 0.601)	(0.832, 0; 0.106)	(0.779, 0.119; 0.106)	(0.814, 0; 0.141)
Other Fixed Assets to Fixed Assets	(0.577, 0.282; 0.636)	(0.530, 0.200; 0.707)	(0.577, 0.282; 0.636)	(0.749, 0; 0.884)	(0.669, 0.206; 0.813)	(0.669, 0.206; 0.813)
Other Extraordinary Expenses and Losses to Net Sales	(0.650, 0; 0.725)	(0.700, 0; 0.813)	(0.602, 0; 0.637)	(0.624, 0.259; 0.725)	(0.749, 0; 0.884)	(0.684, 0; 0.760)

The following example is shown to demonstrate how to calculate weighted circular intuitionistic fuzzy numbers:

Let $\alpha_{Relavance} = 0.119$ be relavance criterion' weight , which is derived in Step 2.6.

The aggregated circular intuitionistic fuzzy number of the 'Current ratio' alternative with respect to the Relavance' criterion, which is derived in Step 3.4., is $d_{11} = (0.624, 0; 0.567)$.

π_{11} is computed using Eq.(3.37) in the following way:

$$\pi_{11} = 1 - 0.624 - 0 = 0.376$$

Then, using Eq.(4.37), the weighted circular intuitionistic fuzzy number of the Current ratio' alternative with respect to 'Relavance' criterion is computed as follows:

$$\begin{aligned} & 0.119 * (0.624, 0; 0.567) \\ &= \{ \langle 0.624 + 0.119 * 0.376, 0 + (1 - 0.119) * 0.376; 0.567 \rangle \} \\ &= \langle (0.669, 0.331; 0.567) \rangle \end{aligned}$$

The same way is used to compute other weighted circular intuitionistic fuzzy numbers. Table 6.63 shows the weighted circular intuitionistic fuzzy decision matrix.

Step 3.6 Obtain the positive and negative ideal C-IF solutions

Eqs. (4.38)-(4.43) are used to determine the positive and negative ideal C-IF solutions based on the weighted circular intuitionistic fuzzy decision matrix. Table 6.64 shows the derived C-IF positive and C-IF negative ideal solutions.

Step 3.7 Compute the alternatives' distances from positive and negative ideal C-IF solutions

Eqs. (4.44) and (4.45) are used to compute the alternatives' distances from the C-IF positive ideal solution and the C-IF negative ideal solution, and the results are given in Table 6.65.

Step 3.8 Compute the closeness ratio

The relative closeness of alternatives to the ideal solutions are determined using Eq. (4.46) after calculating the distances of the alternatives from the C-IF positive and negative ideal solutions.

Table 6.63. The weighted circular intuitionistic fuzzy decision matrix

Alternatives	Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
Current Ratio	(0.669, 0.331; 0.567)	(0.721, 0.279; 0.375)	(0.832, 0.168; 0.410)	(0.711, 0.289; 0.601)	(0.778, 0.222; 0.636)	(0.760, 0.240; 0.601)
Quick ratio	(0.708, 0.292; 0.513)	(0.875, 0.125; 0.141)	(0.852, 0.148; 0.141)	(0.810, 0.190; 0.672)	(0.803, 0.197; 0.375)	(0.838, 0.162; 0.410)
Cash ratio	(0.616, 0.384; 0.690)	(0.563, 0.437; 0.566)	(0.499, 0.501; 0.566)	(0.445, 0.555; 0.672)	(0.610, 0.390; 0.530)	(0.636, 0.364; 0.530)
Debt ratio	(0.495, 0.505; 0.584)	(0.742, 0.258; 0.340)	(0.373, 0.627; 0.180)	(0.616, 0.384; 0.530)	(0.728, 0.272; 0.742)	(0.684, 0.316; 0.601)
Debt-to-equity ratio	(0.662, 0.338; 0.601)	(0.718, 0.282; 0.340)	(0.456, 0.544; 0.442)	(0.652, 0.348; 0.690)	(0.373, 0.627; 0.566)	(0.501, 0.499; 0.584)
Inventory turnover	(0.658, 0.342; 0.513)	(0.905, 0.095; 0.071)	(0.794, 0.206; 0.106)	(0.810, 0.190; 0.672)	(0.799, 0.201; 0.672)	(0.783, 0.217; 0.636)
Accounts Payable to Cost of Good Sold	(0.480, 0.520; 0.619)	(0.765, 0.235; 0.375)	(0.661, 0.339; 0.513)	(0.462, 0.538; 0.636)	(0.643, 0.357; 0.654)	(0.458, 0.542; 0.442)
Total asset turnover	(0.703, 0.297; 0.305)	(0.895, 0.105; 0.177)	(0.763, 0.237; 0.375)	(0.788, 0.212; 0.355)	(0.803, 0.197; 0.375)	(0.727, 0.273; 0.513)
Return on Assets ratio	(0.619, 0.381; 0.478)	(0.685, 0.315; 0.340)	(0.697, 0.303; 0.250)	(0.813, 0.187; 0.375)	(0.624, 0.376; 0.180)	(0.727, 0.273; 0.513)
Net profit margin ratio	(0.573, 0.427; 0.460)	(0.788, 0.212; 0.410)	(0.773, 0.227; 0.355)	(0.810, 0.190; 0.672)	(0.662, 0.338; 0.513)	(0.783, 0.217; 0.636)
EBIT to Total Assets	(0.560, 0.440; 0.495)	(0.718, 0.282; 0.375)	(0.610, 0.390; 0.584)	(0.784, 0.216; 0.742)	(0.611, 0.389; 0.584)	(0.752, 0.248; 0.707)
Accounts Receivable to Sales	(0.862, 0.138; 0.177)	(0.905, 0.095; 0.071)	(0.856, 0.144; 0.106)	(0.887, 0.113; 0.071)	(0.881, 0.119; 0.071)	(0.846, 0.154; 0.141)
Accounts Receivable to Total Assets	(0.741, 0.259; 0.391)	(0.648, 0.352; 0.305)	(0.763, 0.237; 0.375)	(0.742, 0.258; 0.340)	(0.764, 0.236; 0.375)	(0.739, 0.261; 0.340)
Inventory to Sales	(0.875, 0.125; 0.071)	(0.905, 0.095; 0.071)	(0.856, 0.144; 0.106)	(0.887, 0.113; 0.071)	(0.900, 0.100; 0.106)	(0.861, 0.139; 0.106)
Inventory to Total Assets	(0.772, 0.228; 0.340)	(0.784, 0.216; 0.177)	(0.794, 0.206; 0.106)	(0.836, 0.164; 0.177)	(0.795, 0.205; 0.106)	(0.773, 0.227; 0.071)
Inventory to Current Assets	(0.819, 0.181; 0.177)	(0.805, 0.195; 0.141)	(0.841, 0.159; 0.141)	(0.799, 0.201; 0.106)	(0.795, 0.205; 0.106)	(0.830, 0.170; 0.177)
Growth of accounts receivables	(0.814, 0.186; 0.410)	(0.920, 0.080; 0.106)	(0.779, 0.221; 0.340)	(0.843, 0.157; 0.410)	(0.822, 0.178; 0.410)	(0.883, 0.117; 0.071)
Growth of Inventory	(0.814, 0.186; 0.410)	(0.920, 0.080; 0)	(0.856, 0.144; 0.106)	(0.887, 0.113; 0.071)	(0.858, 0.142; 0.106)	(0.871, 0.129; 0.177)
Growth of income from principal business operations	(0.563, 0.437; 0.461)	(0.716, 0.284; 0.778)	(0.695, 0.305; 0.707)	(0.852, 0.148; 0.141)	(0.706, 0.294; 0.601)	(0.846, 0.154; 0.141)
Growth of sales	(0.495, 0.505; 0.584)	(0.685, 0.315; 0.778)	(0.498, 0.502; 0.584)	(0.754, 0.246; 0.654)	(0.698, 0.302; 0.250)	(0.746, 0.254; 0.654)
Cash to Total Assets	(0.692, 0.308; 0.725)	(0.700, 0.300; 0.836)	(0.762, 0.238; 0.806)	(0.799, 0.201; 0.836)	(0.649, 0.351; 0.718)	(0.792, 0.208; 0.836)
Due from Shareholders to Total Assets	(0.755, 0.245; 0.849)	(0.920, 0.080; 0)	(0.775, 0.225; 0.636)	(0.905, 0.095; 0.106)	(0.795, 0.205; 0.106)	(0.883, 0.117; 0.071)
Due to Shareholders to Total Liabilities	(0.701, 0.299; 0.601)	(0.858, 0.142; 0.445)	(0.704, 0.296; 0.601)	(0.866, 0.134; 0.106)	(0.795, 0.205; 0.106)	(0.846, 0.154; 0.141)
Other Fixed Assets to Fixed Assets	(0.594, 0.406; 0.636)	(0.585, 0.415; 0.707)	(0.598, 0.402; 0.636)	(0.799, 0.201; 0.884)	(0.689, 0.311; 0.813)	(0.691, 0.309; 0.813)
Other Extraordinary Expenses and	(0.692, 0.308; 0.725)	(0.761, 0.239; 0.813)	(0.660, 0.340; 0.637)	(0.648, 0.352; 0.725)	(0.788, 0.212; 0.884)	(0.739, 0.261; 0.760)

Table 6.64. Positive and negative ideal C-IF solutions

	Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
C-IF PIS	(0.875, 0.125; 0.071)	(0.920, 0.080; 0.000)	(0.856, 0.144; 0.106)	(0.905, 0.095; 0.106)	(0.900, 0.100; 0.106)	(0.883, 0.117; 0.071)
C-IF NIS	(0.480, 0.520; 0.619)	(0.563, 0.437; 0.566)	(0.373, 0.627; 0.180)	(0.445, 0.555; 0.672)	(0.373, 0.627; 0.566)	(0.458, 0.542; 0.442)

Step 3.9 Determine ranking of alternatives

The ranking of the alternatives is determined by the decreasing closeness ratio values, which are calculated in Step 3.8. The first alternative is determined to be the one with the highest value of closeness ratio. The closeness ratio values and the ranking of the alternatives are shown in Table 6.65.

Table 6.65. $\mathcal{D}_i^{A^+}$ and $\mathcal{D}_i^{A^-}$, Closeness ratio values and ranking of alternatives

Alternatives	$\mathcal{D}_i^{A^+}$	$\mathcal{D}_i^{A^-}$	Closeness ratio	Rank
Current ratio	0.171	0.075	0.306	21
Quick ratio	0.127	0.083	0.395	14
Cash ratio	0.224	0.045	0.169	25
Debt ratio	0.147	0.059	0.287	22
Debt-to-equity ratio	0.177	0.050	0.221	24
Inventory turnover	0.108	0.092	0.460	12
Accounts Payable to Cost of Good Sold	0.168	0.051	0.233	23
Total asset turnover	0.086	0.120	0.582	8
Return on Assets ratio	0.090	0.095	0.513	10
Net profit margin ratio	0.140	0.072	0.339	19
EBIT to Total Assets	0.165	0.086	0.341	18
Accounts Receivable to Sales	0.052	0.184	0.779	4
Accounts Receivable to Total Assets	0.142	0.106	0.426	13
Inventory to Sales	0.007	0.191	0.966	1
Inventory to Total Assets	0.043	0.156	0.783	3
Inventory to Current Assets	0.070	0.166	0.703	6
Growth of accounts receivables	0.124	0.135	0.522	9
Growth of Inventory	0.024	0.170	0.876	2
Growth of income from principal business operations	0.151	0.132	0.467	11
Growth of sales	0.167	0.090	0.351	16
Cash to Total Assets	0.222	0.129	0.368	15
Due from Shareholders to Total Assets	0.082	0.195	0.704	5
Due to Shareholders to Short term External Resources	0.096	0.146	0.604	7
Other Fixed Assets to Fixed Assets	0.214	0.106	0.332	20
Other Extraordinary Expenses and Losses to Net Sales	0.215	0.115	0.348	17

The top fifteen financial ratios are determined as presented in Table 6.66 for comparing the performances of classifiers in financial accounting fraud detection. Without employing any feature selection method, classifiers are also compared.

Table 6.66. Selected financial ratios for comparing the performances of classifiers (Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets)

Alternatives (Financial Ratios)	Rank
Inventory to Sales	1
Growth of Inventory	2
Inventory to Total Assets	3
Accounts Receivable to Sales	4
Due from Shareholders to Total Assets	5
Inventory to Current Assets	6
Due to Shareholders to Total Liabilities	7
Total asset turnover	8
Growth of accounts receivables	9
Return on Assets ratio	10
Growth of income from principal business operations	11
Inventory turnover	12
Accounts Receivable to Total Assets	13
Quick ratio	14
Cash to Total Assets	15

Phase 2: In this phase, the performances and effectiveness of seven different machine learning classifiers in financial accounting fraud detection are compared. Tables 6.67-6.72 present the performance measures of classifiers using the proposed feature selection method and without using any feature selection method.

Random Forest-without feature selection-oversampling model is the most effective with an overall accuracy of 93.74% as shown in Table 6.67. Naive Bayes- Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets -oversampling model is found as the worst at 53.34% among all other models.

Table 6.67. Overall Accuracy of classifiers using Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.8639	0.7856	0.7523	0.8704	0.8587	0.8131
Naive Bayes	0.6985	<u>0.5334</u>	0.7352	0.8211	0.5881	0.7695
ANN	0.8745	0.8373	0.7710	0.8979	0.8834	0.8287
KNN	0.8563	0.8523	0.7259	0.8944	0.8992	0.7882
Random Forest	0.8938	0.8999	0.7695	0.9179	0.9374	0.8707
Logistic Regression	0.8680	0.8085	0.7601	0.8956	0.8808	0.8536
Bagging	0.8856	0.8771	0.7617	0.9138	0.9127	0.8614

Naive Bayes-Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets-oversampling model has the highest True Positive rate of 96% as shown in Table 6.68. Support Vector Machines - Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets - Without Sampling model performs the worst at 31.50% among all other models. Oversampling method significantly increases performances of classifiers in terms of True Positive Rate.

Table 6.68. True Positive Rates of classifiers using Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.3150	0.6170	0.5640	0.3710	0.7830	0.7040
Naive Bayes	0.7820	0.9600	0.6760	0.7880	0.9380	0.6950
ANN	0.5510	0.8030	0.7200	0.6230	0.8790	0.8350
KNN	0.4830	0.8290	0.6790	0.6570	0.8930	0.7690
Random Forest	0.5330	0.8960	0.7320	0.6600	0.9470	0.8820
Logistic Regression	0.3930	0.7200	0.6640	0.5860	0.8440	0.8100
Bagging	0.5110	0.8610	0.6980	0.7200	0.9170	0.8540

In terms of True Negative rate, Support Vector Machines- Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets -Without Sampling model is the highest at 99.10% as shown in Table 6.69. Naive Bayes- Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets-Over Sampling model performs the worst at 13.70% among all other models. Oversampling method significantly reduces the performance of Naive Bayes in terms of True Negative Rate.

Table 6.69. True Negative Rates of classifiers using Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.9910	0.9420	0.9410	0.9860	0.9280	0.9220
Naive Bayes	0.6790	0.1370	0.7940	0.8290	0.2640	0.8440
ANN	0.9490	0.8690	0.8220	0.9620	0.8880	0.8220
KNN	0.9430	0.8740	0.7730	0.9490	0.9050	0.8070
Random Forest	0.9780	0.9040	0.8070	0.9780	0.9280	0.8600
Logistic Regression	0.9780	0.8900	0.8570	0.9670	0.9150	0.8970
Bagging	0.9730	0.8920	0.8260	0.9590	0.9080	0.8690

In terms of precision, Random Forest-without feature selection-oversampling model is the highest at 92.50% as shown in Table 6.70. Naive Bayes- Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets - Without Sampling model is found as the worst at 36.10% among all other models.

Table 6.70. Precision of classifiers using Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.8940	0.9080	0.9050	0.8620	0.9100	0.9000
Naive Bayes	<u>0.3610</u>	0.5080	0.7670	0.5160	0.5420	0.8170
ANN	0.7170	0.8510	0.8020	0.7910	0.8790	0.8250
KNN	0.6620	0.8590	0.7490	0.7510	0.8970	0.7990
Random Forest	0.8470	0.8960	0.7910	0.8720	0.9250	0.8630
Logistic Regression	0.8080	0.8590	0.8220	0.8070	0.9020	0.8870
Bagging	0.8120	0.8810	0.8000	0.8020	0.9030	0.8670

As shown in Table 6.71, Random Forest-Without Feature Selection-Over Sampling model is the highest at 93.75% in terms of G-measure rate. Naive Bayes- Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets -Over Sampling model has the lowest G-measure rate at 36.27% among all other models. Oversampling method significantly reduces the performance of Naive Bayes in terms of G-measure.

Table 6.71. G-measure of classifiers using Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.5587	0.7624	0.7285	0.6048	0.8524	0.8057
Naive Bayes	0.7287	<u>0.3627</u>	0.7326	0.8082	0.4976	0.7659
ANN	0.7231	0.8353	0.7693	0.7742	0.8835	0.8285
KNN	0.6749	0.8512	0.7245	0.7896	0.8990	0.7878
Random Forest	0.7220	0.9000	0.7686	0.8034	0.9375	0.8709
Logistic Regression	0.6200	0.8005	0.7544	0.7528	0.8788	0.8524
Bagging	0.7051	0.8764	0.7593	0.8310	0.9125	0.8615

With regard to Area Under ROC Curve, Random Forest-without feature selection-oversampling model is the highest at 98.40% as presented in Table 6.72. Support Vector Machines- Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets- Without Sampling model performs the worst at 65.30% among all other models. Oversampling method improves performances of all classifiers in terms of Area Under ROC Curve.

Table 6.72. Area Under ROC Curve of classifiers using Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and TOPSIS with Circular Intuitionistic Fuzzy Sets					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.6530	0.7800	0.7520	0.6780	0.8560	0.8130
Naive Bayes	0.8060	0.8430	0.7850	0.8390	0.8580	0.8110
ANN	0.8410	0.8970	0.8330	0.8890	0.9390	0.8980
KNN	0.7960	0.9030	0.7730	0.7980	0.8980	0.7890
Random Forest	0.8840	0.9640	0.8420	0.9490	0.9840	0.9190
Logistic Regression	0.8450	0.8840	0.8290	0.9130	0.9380	0.9050
Bagging	0.8750	0.9360	0.8230	0.9280	0.9690	0.9080

6.2.2. Integrated AHP and MULTIMOORA with different fuzzy sets

Integrated AHP and MULTIMOORA approach with different fuzzy sets have been applied to obtain the best prioritization of financial ratios. Three fuzzy sets, which are ordinary fuzzy sets, single-valued spherical fuzzy sets and interval-valued spherical fuzzy sets, have been employed. After obtaining the prioritization of financial ratios for financial accounting fraud detection, seven different machine learning classifiers are compared by utilizing performance metrics.

6.2.2.1. Integrated Fuzzy AHP and Fuzzy MULTIMOORA

In this section, integrated Fuzzy AHP and Fuzzy MULTIMOORA is employed to prioritize financial ratios.

Algorithm of the application

The proposed methodology consists of two phases which are determining financial ratios by using integrated Fuzzy AHP and Fuzzy MULTIMOORA and employing machine

learning classifiers. The flowchart of the methodology is presented in Figure 6.16. In the first phase, Integrated Fuzzy AHP and Fuzzy MULTIMOORA is used to rank the financial ratios. In the second phase, sampling methods and seven different classifiers are employed for financial accounting fraud detection.

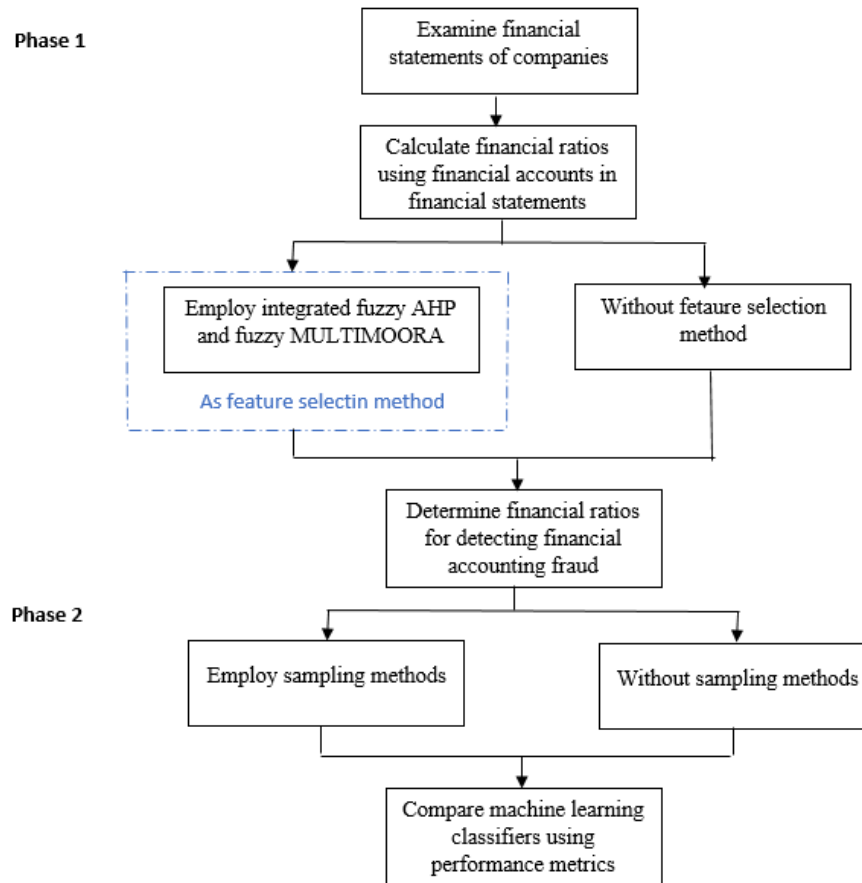


Figure 6.16. The flowchart of the proposed approach (Integrated Fuzzy AHP and Fuzzy MULTIMOORA)

Steps of the Application

The steps of the proposed approach are given as follows;

Phase 1: The steps and obtained results of fuzzy AHP method are given in Section 6.2.1.1. After having found the importance weights of criteria as shown in Table 6.8, fuzzy MULTIMOORA method is used to obtain the ranking of financial ratios. The proposed method is applied as follows:

Steps 6-8 Financial analysts use the language phrases in Table 4.14 to rate each alternative based on each criterion. The equations (4.47) and (4.48) are used to aggregate individual judgements of decision makers after getting the fuzzy matrix of alternatives. The equations (4.49) and (4.50) are used to convert the fuzzy group decision matrix into the normalized fuzzy group decision matrix. Then, using the equation (4.51), a weighted normalized fuzzy group decision matrix is constructed. Table 6.73 shows the weighted normalized fuzzy group decision matrix.

Steps 9-11 are employed to determine the final ranking of financial ratios. The alternatives are ranked according to the fuzzy Ratio System by employing the equations (4.52) and (4.53). The maximal objective reference point is defined according to the equation (4.54). Given the previously defined maximal objective reference point, deviations of each alternative from the reference point are found by applying the equation (4.55). The alternatives, then, are sorted in order of the fuzzy Reference Point. Finally, the fuzzy Full Multiplicative Form is employed to determine ranking of the alternatives. Table 6.74 summarizes the findings.

Then, the theory of dominance (Brauers and Zavadskas, 2011) is used to determine the final ranking using the ranks provided by the different parts of fuzzy MULTIMOORA. Finally, the final ranking of financial ratios is obtained as given in Table 6.75.

The top fifteen financial ratios are selected to compare the performances of classifiers in financial accounting fraud detection. Without employing any feature selection method, classifiers are also compared.

Phase 2: After selection of financial ratios, seven different machine learning classifiers are compared by using performance metrics. Classifiers are also compared without using any feature selection method.

Tables 6.76-6.81 indicate the performance measures of classifiers using integrated Fuzzy AHP and Fuzzy MULTIMOORA and without using any feature selection method. Moreover, different sampling techniques are applied.

Table 6.73.Weighted normalized fuzzy group decision matrix (Fuzzy MULTIMOORA)

Criteria Alternatives	Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
Current Assets/Current Liabilities	(0.001, 0.001, 0.001)	(0.022, 0.027, 0.033)	(0.019, 0.021, 0.026)	(0.018, 0.025, 0.031)	(0.010, 0.013, 0.016)	(0.017, 0.021, 0.026)
Current Assets- Inventories/ Current Liabilities	(0.001, 0.001, 0.001)	(0.025, 0.030, 0.036)	(0.020, 0.023, 0.028)	(0.022, 0.027, 0.033)	(0.011, 0.013, 0.017)	(0.021, 0.024, 0.029)
Cash/Current Liabilities	(0.001, 0.001, 0.001)	(0.017, 0.024, 0.030)	(0.009, 0.013, 0.018)	(0.009, 0.016, 0.022)	(0.007, 0.010, 0.013)	(0.014, 0.019, 0.025)
Total Liabilities /Total Assets	(0.001, 0.001, 0.001)	(0.020, 0.027, 0.033)	(0.008, 0.013, 0.018)	(0.014, 0.020, 0.027)	(0.009, 0.013, 0.016)	(0.012, 0.017, 0.022)
Total Liabilities /Total Equity	(0.001, 0.001, 0.001)	(0.020, 0.027, 0.033)	(0.009, 0.014, 0.019)	(0.013, 0.017, 0.023)	(0.004, 0.007, 0.010)	(0.010, 0.015, 0.021)
Cost of Goods Sold/Average Inventory	(0.001, 0.001, 0.001)	(0.028, 0.032, 0.038)	(0.019, 0.024, 0.029)	(0.022, 0.027, 0.033)	(0.011, 0.014, 0.017)	(0.018, 0.022, 0.028)
Accounts Payable/ Cost of Good Sold	(0.001, 0.001, 0.001)	(0.022, 0.027, 0.033)	(0.014, 0.019, 0.024)	(0.011, 0.017, 0.023)	(0.007, 0.010, 0.013)	(0.010, 0.015, 0.021)
Net Sales/ Total Assets	(0.001, 0.001, 0.001)	(0.027, 0.030, 0.036)	(0.018, 0.023, 0.028)	(0.020, 0.025, 0.031)	(0.011, 0.013, 0.017)	(0.015, 0.019, 0.025)
Net Income/Total Assets	(0.001, 0.001, 0.001)	(0.020, 0.027, 0.033)	(0.015, 0.020, 0.025)	(0.022, 0.027, 0.033)	(0.008, 0.011, 0.014)	(0.015, 0.019, 0.025)
Net Income/ Net Sales	(0.001, 0.001, 0.001)	(0.024, 0.029, 0.035)	(0.016, 0.020, 0.025)	(0.022, 0.027, 0.033)	(0.009, 0.012, 0.015)	(0.018, 0.022, 0.028)
Earnings Before Interest and Tax / Total Assets	(0.001, 0.001, 0.001)	(0.022, 0.029, 0.035)	(0.011, 0.016, 0.021)	(0.019, 0.023, 0.030)	(0.007, 0.010, 0.013)	(0.015, 0.019, 0.025)
Accounts Receivable/ Sales	(0.001, 0.001, 0.002)	(0.028, 0.032, 0.038)	(0.021, 0.025, 0.030)	(0.028, 0.031, 0.037)	(0.014, 0.016, 0.019)	(0.022, 0.026, 0.032)
Accounts Receivable/ Total Assets	(0.001, 0.001, 0.001)	(0.019, 0.025, 0.032)	(0.018, 0.023, 0.028)	(0.020, 0.027, 0.033)	(0.011, 0.014, 0.017)	(0.018, 0.024, 0.029)
Inventory/Sales	(0.001, 0.001, 0.002)	(0.028, 0.032, 0.038)	(0.021, 0.025, 0.030)	(0.028, 0.031, 0.037)	(0.015, 0.016, 0.019)	(0.023, 0.028, 0.033)
Inventory/Total Assets	(0.001, 0.001, 0.001)	(0.024, 0.028, 0.035)	(0.019, 0.024, 0.029)	(0.023, 0.028, 0.034)	(0.012, 0.015, 0.018)	(0.019, 0.025, 0.031)
Inventory /Current Assets	(0.001, 0.001, 0.002)	(0.025, 0.030, 0.036)	(0.020, 0.024, 0.029)	(0.023, 0.030, 0.036)	(0.012, 0.015, 0.018)	(0.021, 0.025, 0.031)
Growth of accounts receivables	(0.001, 0.001, 0.002)	(0.030, 0.032, 0.038)	(0.016, 0.020, 0.025)	(0.024, 0.027, 0.033)	(0.012, 0.014, 0.017)	(0.025, 0.028, 0.033)
Growth of Inventory	(0.001, 0.001, 0.002)	(0.032, 0.032, 0.038)	(0.021, 0.025, 0.030)	(0.028, 0.031, 0.037)	(0.013, 0.016, 0.019)	(0.024, 0.026, 0.032)
Growth of income from principal business operations	(0.001, 0.001, 0.001)	(0.020, 0.025, 0.032)	(0.014, 0.019, 0.024)	(0.025, 0.030, 0.036)	(0.009, 0.013, 0.016)	(0.022, 0.026, 0.032)
Growth of sales	(0.001, 0.001, 0.001)	(0.021, 0.024, 0.030)	(0.009, 0.014, 0.019)	(0.017, 0.022, 0.028)	(0.009, 0.013, 0.016)	(0.015, 0.019, 0.025)
Cash/Total Assets	(0.001, 0.001, 0.001)	(0.027, 0.032, 0.038)	(0.020, 0.024, 0.029)	(0.027, 0.031, 0.037)	(0.009, 0.013, 0.016)	(0.024, 0.028, 0.033)
Due from Shareholders/ Total Assets	(0.001, 0.001, 0.001)	(0.032, 0.032, 0.038)	(0.016, 0.020, 0.025)	(0.030, 0.031, 0.037)	(0.012, 0.015, 0.018)	(0.025, 0.028, 0.033)
Due to Shareholders/ Total Liabilities	(0.001, 0.001, 0.001)	(0.025, 0.029, 0.035)	(0.015, 0.020, 0.025)	(0.026, 0.031, 0.037)	(0.012, 0.015, 0.018)	(0.022, 0.026, 0.032)
Other Fixed Assets/ Fixed Assets	(0.001, 0.001, 0.001)	(0.017, 0.024, 0.030)	(0.011, 0.015, 0.020)	(0.020, 0.024, 0.029)	(0.009, 0.011, 0.014)	(0.015, 0.019, 0.025)
Other Extraordinary Expenses and Losses/ Net Sales	(0.001, 0.001, 0.001)	(0.022, 0.025, 0.032)	(0.011, 0.014, 0.019)	(0.015, 0.020, 0.026)	(0.010, 0.012, 0.015)	(0.015, 0.018, 0.023)

Table 6.74. Ranking of the alternatives according to the fuzzy MULTIMOORA

	The Fuzzy Ratio System		The Fuzzy Full Multiplicative		The fuzzy Reference Point	
	BNPi	Rank	BNPi	Rank	maxjd(rj,xij)	Rank
Current Assets/Current Liabilities	0.083846	16	0.001399	18	0.008240	13
Current Assets-Inventories/ Current Liabilities	0.092917	10	0.001738	13	0.006000	10
Cash/Current Liabilities	0.062102	25	0.001069	22	0.017394	25
Total Liabilities /Total Assets	0.064864	24	0.001008	25	0.013050	22
Total Liabilities /Total Equity	0.067296	23	0.001789	11	0.015191	23
Cost of Goods Sold/Average Inventory	0.093577	9	0.001728	14	0.006064	11
Accounts Payable/ Cost of Good Sold	0.069528	20	0.001062	23	0.015860	24
Net Sales/ Total Assets	0.086493	14	0.001375	19	0.008812	15
Net Income/Total Assets	0.083749	17	0.001710	15	0.008812	15
Net Income/ Net Sales	0.088252	13	0.001894	9	0.006064	11
Earnings Before Interest and Tax / Total Assets	0.079083	18	0.001785	12	0.009279	18
Accounts Receivable/ Sales	0.102352	4	0.001919	8	0.001995	3
Accounts Receivable/ Total Assets	0.084728	15	0.001410	17	0.009092	17
Inventory/Sales	0.103521	3	0.001951	7	0.001965	2
Inventory/Total Assets	0.092487	11	0.001672	16	0.005380	8
Inventory /Current Assets	0.096150	7	0.001845	10	0.004040	4
Growth of accounts receivables	0.096858	6	0.002045	4	0.005269	7
Growth of Inventory	0.104171	2	0.002047	3	0.001385	1
Growth of income from principal business operations	0.089861	12	0.002044	5	0.008345	14
Growth of sales	0.068986	22	0.001118	21	0.011794	20
Cash/Total Assets	0.104840	1	0.002758	1	0.004151	5
Due from Shareholders/Total Assets	0.101889	5	0.002381	2	0.005036	6
Due to Shareholders/ Total Liabilities	0.094244	8	0.001966	6	0.005522	9
Other Fixed Assets/ Total Assets	0.072927	19	0.001304	20	0.010616	19
Other Extraordinary Expenses and Losses/ Net Sales	0.069230	21	0.001013	24	0.012164	21

Table 6.75. Final ranking of financial ratios (Integrated Fuzzy AHP and Fuzzy MULTIMOORA)

Alternatives (Financial Ratios)	Final Rank
Cash/Total Assets	1
Growth of Inventory	2
Inventory/Sales	3
Accounts Receivable/ Sales	4
Due from Shareholders/ Total Assets	5
Growth of accounts receivables	6
Inventory /Current Assets	7
Due to Shareholders/ Total Liabilities	8
Cost of Goods Sold/Average Inventory	9
Current Assets-Inventories/ Current Liabilities	10
Inventory/Total Assets	11
Growth of income from principal business operations	12
Net Income/ Net Sales	13
Current Assets/Current Liabilities	14
Net Income/Total Assets	15
Net Sales/ Total Assets	16
Accounts Receivable/ Total Assets	17
Earnings Before Interest and Tax / Total Assets	18
Other Fixed Assets/ Total Assets	19
Growth of sales	20
Other Extraordinary Expenses and Losses/ Net Sales	21
Accounts Payable/ Cost of Good Sold	22
Total Liabilities /Total Equity	23
Total Liabilities /Total Assets	24
Cash/Current Liabilities	25

The Random Forest-without feature selection-oversampling model has the best overall accuracy at 93.74 percent as shown in Table 6.76. Naive Bayes-Integrated Fuzzy AHP and Fuzzy MULTIMOORA-oversampling model performs the worst at 51.69% among all other models.

Table 6.76. Overall Accuracy of classifiers using Integrated Fuzzy AHP and Fuzzy MULTIMOORA and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated Fuzzy AHP and Fuzzy MULTIMOORA					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.8622	0.7912	0.7336	0.8704	0.8587	0.8131
Naive Bayes	0.7631	<u>0.5169</u>	0.7352	0.8211	0.5881	0.7695
ANN	0.8780	0.8280	0.7710	0.8979	0.8834	0.8287
KNN	0.8205	0.8310	0.7087	0.8944	0.8992	0.7882
Random Forest	0.8950	0.8999	0.7741	0.9179	0.9374	0.8707
Logistic Regression	0.8710	0.8115	0.7664	0.8956	0.8808	0.8536
Bagging	0.8921	0.8748	0.7617	0.9138	0.9127	0.8614

Naive Bayes-Integrated Fuzzy AHP and Fuzzy MULTIMOORA-oversampling model has the highest True Positive Rate of 95.40% among all other models as shown in Table 6.77. Support Vector Machines- Integrated Fuzzy AHP and Fuzzy MULTIMOORA-Without Sampling model has the lowest True Positive Rate of 30.80%.

Table 6.77. True Positive Rates of classifiers using Integrated Fuzzy AHP and Fuzzy MULTIMOORA and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated Fuzzy AHP and Fuzzy MULTIMOORA					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	<u>0.3080</u>	0.6250	0.5140	0.3710	0.7830	0.7040
Naive Bayes	0.7040	0.9540	0.6670	0.7880	0.9380	0.6950
ANN	0.5300	0.7540	0.7040	0.6230	0.8790	0.8350
KNN	0.4800	0.8230	0.7010	0.6570	0.8930	0.7690
Random Forest	0.5420	0.8890	0.7230	0.6600	0.9470	0.8820
Logistic Regression	0.4050	0.7080	0.6540	0.5860	0.8440	0.810
Bagging	0.5260	0.8480	0.7040	0.7200	0.9170	0.8540

As presented in Table 6.78, Support Vector Machines- Integrated Fuzzy AHP and Fuzzy MULTIMOORA -Without Sampling model outperforms all other models in terms of True Negative rate. Naive Bayes-Integrated Fuzzy AHP and Fuzzy MULTIMOORA- Oversampling Sampling model performs the worst at 11.10% among all other models.

Table 6.78. True Negative Rates of classifiers using Integrated Fuzzy AHP and Fuzzy MULTIMOORA and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated Fuzzy AHP and Fuzzy MULTIMOORA					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.9910	0.9460	0.9530	0.9860	0.9280	0.9220
Naive Bayes	0.7770	0.1110	0.8040	0.8290	0.2640	0.8440
ANN	0.9590	0.8970	0.8380	0.9620	0.8880	0.8220
KNN	0.9000	0.8380	0.7170	0.9490	0.9050	0.8070
Random Forest	0.9770	0.9100	0.8260	0.9780	0.9280	0.8600
Logistic Regression	0.9790	0.9080	0.8790	0.9670	0.9150	0.8970
Bagging	0.9770	0.9000	0.8190	0.9590	0.9080	0.8690

Table 6.79 indicates that Random Forest-without feature selection-oversampling model has the highest precision rate of 92.50% among all models. Naive Bayes-Integrated Fuzzy AHP and Fuzzy MULTIMOORA- Without Sampling model has the lowest precision rate of 42.20%.

Table 6.79. Precision of classifiers using Integrated Fuzzy AHP and Fuzzy MULTIMOORA and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated Fuzzy AHP and Fuzzy MULTIMOORA					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.8840	0.9140	0.9170	0.8620	0.9100	0.9000
Naive Bayes	<u>0.4220</u>	0.4990	0.7730	0.5160	0.5420	0.8170
ANN	0.7490	0.8710	0.8130	0.7910	0.8790	0.8250
KNN	0.5260	0.8250	0.7120	0.7510	0.8970	0.7990
Random Forest	0.8450	0.9010	0.8060	0.8720	0.9250	0.8630
Logistic Regression	0.8180	0.8770	0.8430	0.8070	0.9020	0.8870
Bagging	0.8410	0.8870	0.7960	0.8020	0.9030	0.8670

In terms of G-measure, Random Forest-without feature selection-oversampling model is the highest at 93.75% as shown in Table 6.80. Naive Bayes-Integrated Fuzzy AHP and Fuzzy MULTIMOORA -Over Sampling model performs the worst at 32.54% among all other models. Oversampling method significantly reduces the performance of Naive Bayes in terms of G-measure.

Table 6.80. G-measure of classifiers using Integrated Fuzzy AHP and Fuzzy MULTIMOORA and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated Fuzzy AHP and Fuzzy MULTIMOORA					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.5525	0.7689	0.6999	0.6048	0.8524	0.8057
Naive Bayes	0.7396	<u>0.3254</u>	0.7323	0.8082	0.4976	0.7659
ANN	0.7129	0.8224	0.7681	0.7742	0.8835	0.8285
KNN	0.6573	0.8305	0.7090	0.7896	0.8990	0.7878
Random Forest	0.7277	0.8994	0.7728	0.8034	0.9375	0.8709
Logistic Regression	0.6297	0.8018	0.7582	0.7528	0.8788	0.8524
Bagging	0.7169	0.8736	0.7593	0.8310	0.9125	0.8615

Table 6.81 indicates that Random Forest outperforms other machine learning classifiers in terms of Area Under ROC Curve. Random Forest-without feature selection-oversampling model has the highest rate of 98.40%. Support Vector Machines-Integrated Fuzzy AHP and Fuzzy MULTIMOORA- Without Sampling model performs the worst at 65% among all other models. It is also inferred that oversampling method generally improve performances of all classifiers in terms of Area Under ROC Curve.

Table 6.81. Area Under ROC Curve of classifiers using Integrated Fuzzy AHP and Fuzzy MULTIMOORA and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated Fuzzy AHP and Fuzzy MULTIMOORA					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.6500	0.7850	0.7340	0.6780	0.8560	0.8130
Naive Bayes	0.7920	0.8260	0.7610	0.8390	0.8580	0.8110
ANN	0.8440	0.8900	0.8230	0.8890	0.9390	0.8980
KNN	0.6930	0.8320	0.7140	0.7980	0.8980	0.7890
Random Forest	0.8830	0.9600	0.8420	0.9490	0.9840	0.9190
Logistic Regression	0.8220	0.8730	0.8060	0.9130	0.9380	0.9050
Bagging	0.8650	0.9370	0.8300	0.9280	0.9690	0.9080

6.2.2.2. Integrated AHP and MULTIMOORA with Single-Valued Spherical Fuzzy Sets

In this section, integrated AHP and MULTIMOORA with Single-Valued Spherical Fuzzy Sets is employed to assess financial ratios as fraud risk indicators in financial statement fraud detection.

Algorithm of the application

The proposed methodology includes two phases. First, Integrated AHP and MULTIMOORA with Single-Valued Spherical Fuzzy Sets is employed to rank the financial ratios. Then, seven different machine learning classifiers are compared by using performance metrics. Figure 6.17 presents the flowchart of the methodology.

Steps of the application

The steps of proposed approach are as follows:

Phase 1: The steps and obtained results of single-valued spherical fuzzy AHP method are given in Section 6.2.1.2.

After obtaining the single-valued spherical fuzzy weights of criteria as shown in Table 6.28, single-valued spherical fuzzy MULTIMOORA method is utilized to obtain the ranking of financial ratios. The proposed method is implemented as follows:

Steps 6-7 Firstly, the same assessments on alternatives with respect to the criteria are used, and the SWAM operator is used to aggregate the evaluations of the alternatives by taking decision makers' importance levels into account. Eq. (3.6) is utilized to build the weighted single-valued spherical fuzzy decision matrix after getting the criteria's weights and the alternatives' ratings. Table 6.82 shows the weighted single-valued spherical fuzzy decision matrix

Step 8 Employ RS approach.

The values of Y_i^+ are computed in the RS approach by utilizing Eq (4.57). Afterwards, the values of Y_i^+ are defuzzified utilizing the score function provided in Eq (4.58). Finally, the alternatives are sorted in decreasing order based on the values of Score (Y_i^+). Table 6.83 shows Y_i^+ values, Score (Y_i^+) values, and the ranking of alternatives.

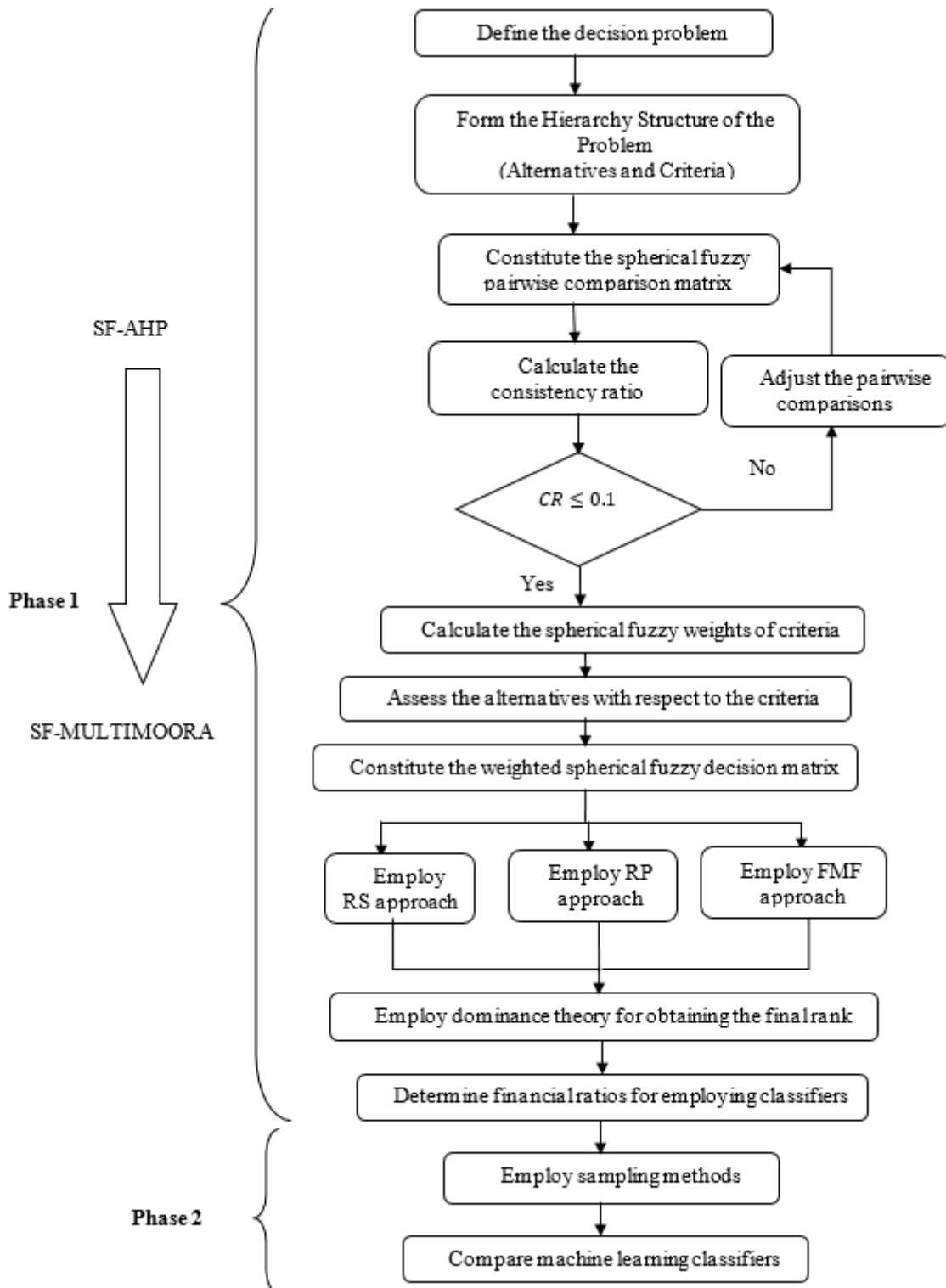


Figure 6.17. The flowchart of the proposed methodology (Integrated AHP and MULTIMOORA with Spherical Fuzzy Sets)

Table 6.82. Weighted single-valued spherical fuzzy decision matrix (Single-Valued Spherical Fuzzy MULTIMOORA)

Alternatives	Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
Current Ratio	(0.29, 0.64, 0.46)	(0.45, 0.48, 0.46)	(0.39, 0.56, 0.46)	(0.37, 0.54, 0.48)	(0.37, 0.56, 0.45)	(0.39, 0.54, 0.47)
Quick ratio	(0.30, 0.64, 0.45)	(0.46, 0.46, 0.44)	(0.39, 0.55, 0.46)	(0.44, 0.48, 0.46)	(0.38, 0.56, 0.46)	(0.44, 0.50, 0.46)
Cash ratio	(0.28, 0.66, 0.44)	(0.36, 0.55, 0.47)	(0.24, 0.70, 0.45)	(0.29, 0.63, 0.46)	(0.30, 0.63, 0.44)	(0.31, 0.61, 0.50)
Debt ratio	(0.22, 0.71, 0.45)	(0.39, 0.52, 0.48)	(0.22, 0.70, 0.49)	(0.34, 0.58, 0.46)	(0.33, 0.59, 0.44)	(0.37, 0.56, 0.46)
Debt-to-equity ratio	(0.29, 0.65, 0.43)	(0.39, 0.52, 0.48)	(0.24, 0.69, 0.48)	(0.38, 0.54, 0.47)	(0.23, 0.70, 0.45)	(0.28, 0.64, 0.48)
Inventory turnover	(0.26, 0.67, 0.46)	(0.50, 0.43, 0.42)	(0.33, 0.59, 0.47)	(0.44, 0.48, 0.46)	(0.38, 0.55, 0.44)	(0.40, 0.53, 0.47)
Accounts Payable to Cost of Good Sold	(0.22, 0.72, 0.44)	(0.45, 0.48, 0.46)	(0.29, 0.63, 0.48)	(0.30, 0.62, 0.47)	(0.34, 0.59, 0.46)	(0.27, 0.65, 0.50)
Total asset turnover	(0.26, 0.66, 0.47)	(0.50, 0.44, 0.43)	(0.32, 0.60, 0.48)	(0.42, 0.50, 0.49)	(0.38, 0.56, 0.46)	(0.38, 0.55, 0.48)
Return on Assets ratio	(0.25, 0.68, 0.47)	(0.39, 0.52, 0.48)	(0.30, 0.62, 0.49)	(0.43, 0.49, 0.48)	(0.28, 0.63, 0.50)	(0.38, 0.55, 0.48)
Net profit margin ratio	(0.24, 0.69, 0.45)	(0.46, 0.47, 0.45)	(0.35, 0.58, 0.48)	(0.44, 0.48, 0.46)	(0.31, 0.61, 0.47)	(0.40, 0.53, 0.47)
EBIT to Total Assets	(0.24, 0.70, 0.44)	(0.40, 0.51, 0.47)	(0.27, 0.65, 0.48)	(0.42, 0.50, 0.46)	(0.29, 0.64, 0.47)	(0.38, 0.54, 0.47)
Accounts Receivable to Sales	(0.36, 0.60, 0.43)	(0.50, 0.43, 0.42)	(0.38, 0.56, 0.46)	(0.49, 0.45, 0.45)	(0.43, 0.52, 0.43)	(0.42, 0.51, 0.46)
Accounts Receivable to Total	(0.30, 0.63, 0.46)	(0.37, 0.54, 0.49)	(0.32, 0.60, 0.48)	(0.38, 0.53, 0.50)	(0.34, 0.58, 0.47)	(0.35, 0.56, 0.49)
Inventory to Sales	(0.36, 0.59, 0.43)	(0.50, 0.43, 0.42)	(0.38, 0.56, 0.46)	(0.49, 0.45, 0.45)	(0.45, 0.51, 0.42)	(0.43, 0.50, 0.46)
Inventory to Total Assets	(0.31, 0.63, 0.46)	(0.45, 0.47, 0.45)	(0.33, 0.59, 0.47)	(0.44, 0.48, 0.47)	(0.35, 0.57, 0.46)	(0.36, 0.55, 0.49)
Inventory to Current Assets	(0.32, 0.62, 0.45)	(0.46, 0.46, 0.44)	(0.37, 0.57, 0.46)	(0.40, 0.51, 0.48)	(0.35, 0.57, 0.46)	(0.41, 0.52, 0.47)
Growth of accounts receivables	(0.33, 0.62, 0.45)	(0.53, 0.42, 0.41)	(0.34, 0.59, 0.48)	(0.47, 0.46, 0.46)	(0.38, 0.55, 0.45)	(0.45, 0.49, 0.45)
Growth of Inventory	(0.33, 0.62, 0.45)	(0.55, 0.41, 0.40)	(0.38, 0.56, 0.46)	(0.49, 0.45, 0.45)	(0.40, 0.54, 0.44)	(0.45, 0.49, 0.45)
Growth of income from principal business operations	(0.24, 0.69, 0.47)	(0.45, 0.48, 0.43)	(0.30, 0.63, 0.46)	(0.45, 0.47, 0.46)	(0.32, 0.60, 0.46)	(0.42, 0.51, 0.46)
Growth of sales	(0.22, 0.71, 0.45)	(0.48, 0.46, 0.42)	(0.25, 0.68, 0.47)	(0.41, 0.51, 0.47)	(0.32, 0.60, 0.48)	(0.38, 0.54, 0.47)
Cash to Total Assets	(0.29, 0.65, 0.44)	(0.44, 0.40, 0.44)	(0.34, 0.53, 0.47)	(0.43, 0.42, 0.46)	(0.28, 0.50, 0.50)	(0.40, 0.46, 0.46)
Due from Shareholders to Total Assets	(0.31, 0.63, 0.43)	(0.55, 0.41, 0.40)	(0.35, 0.58, 0.46)	(0.51, 0.43, 0.44)	(0.35, 0.57, 0.46)	(0.45, 0.49, 0.45)
Due to Shareholders to Total Liabilities	(0.27, 0.66, 0.45)	(0.49, 0.44, 0.43)	(0.30, 0.62, 0.47)	(0.46, 0.47, 0.46)	(0.35, 0.57, 0.46)	(0.42, 0.51, 0.46)
Other Fixed Assets to Fixed Assets	(0.25, 0.69, 0.42)	(0.37, 0.54, 0.45)	(0.27, 0.66, 0.44)	(0.43, 0.50, 0.44)	(0.31, 0.62, 0.44)	(0.33, 0.60, 0.46)
Other Extraordinary Expenses and Losses to Net Sales	(0.29, 0.65, 0.44)	(0.49, 0.45, 0.41)	(0.32, 0.62, 0.47)	(0.34, 0.58, 0.47)	(0.37, 0.56, 0.43)	(0.38, 0.55, 0.46)

Table 6.83. The ranking of the alternatives by utilizing the RS approach

Alternatives	Y_i^+ values	Score(Y_i^+)	Rank
Current Ratio	(0.78, 0.03, 0.57)	0.672	11
Quick ratio	(0.81, 0.02, 0.53)	0.944	6
Cash ratio	(0.66, 0.06, 0.68)	0.026	25
Debt ratio	(0.69, 0.05, 0.65)	0.169	23
Debt-to-equity ratio	(0.67, 0.06, 0.66)	0.098	24
Inventory turnover	(0.80, 0.02, 0.54)	0.850	7
Accounts Payable to Cost of Good Sold	(0.69, 0.05, 0.65)	0.173	22
Total asset turnover	(0.79, 0.03, 0.56)	0.744	10
Return on Assets ratio	(0.73, 0.04, 0.63)	0.351	19
Net profit margin ratio	(0.77, 0.03, 0.58)	0.636	14
EBIT to Total Assets	(0.73, 0.04, 0.62)	0.357	18
Accounts Receivable to Sales	(0.84, 0.02, 0.48)	1.242	4
Accounts Receivable to Total Assets	(0.73, 0.04, 0.63)	0.338	20
Inventory to Sales	(0.85, 0.02, 0.47)	1.301	2
Inventory to Total Assets	(0.78, 0.03, 0.57)	0.667	12
Inventory to Current Assets	(0.79, 0.02, 0.55)	0.776	9
Growth of accounts receivables	(0.84, 0.02, 0.49)	1.171	5
Growth of Inventory	(0.85, 0.02, 0.47)	1.313	1
Growth of income from principal business operations	(0.77, 0.03, 0.58)	0.628	15
Growth of sales	(0.75, 0.04, 0.59)	0.511	17
Cash to Total Assets	(0.77, 0.01, 0.58)	0.593	16
Due from Shareholders to Total Assets	(0.85, 0.02, 0.48)	1.256	3
Due to Shareholders to Total Liabilities	(0.80, 0.02, 0.55)	0.822	8
Other Fixed Assets to Fixed Assets	(0.71, 0.05, 0.62)	0.321	21
Other Extraordinary Expenses and Losses to Net Sales	(0.77, 0.03, 0.57)	0.655	13

Step 9 Employ RP approach.

Eqs. (4.59) and (4.60) are used in the RP approach to determine single-valued reference points. Table 6.84 displays Single-Valued Spherical Fuzzy reference points.

Table 6.84. Single-Valued Spherical Fuzzy reference points

	Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
Reference point	(0.33, 0.62, 0.45)	(0.55, 0.41, 0.40)	(0.38, 0.56, 0.46)	(0.49, 0.45, 0.45)	(0.40, 0.54, 0.44)	(0.45, 0.49, 0.45)

The alternatives' distances from the single-valued reference point are then determined using Eq (4.61). The alternatives' deviations from the single-valued reference point are determined using Eq. (4.62) after the alternatives' distances from the single-valued fuzzy reference point have been calculated. The alternatives are then ordered in ascending order based on the deviation values. Table 6.85 shows the distances ($d(X_{ij}, X^*)$) and deviations ($\text{Max } d(X_{ij}, X_j^*)$) of the alternatives, as well as the ranking of alternatives using the RP approach.

Table 6.85. The distances ($d(X_{ij}, X_j^*)$) and the deviations ($\text{Max } d(X_{ij}, X_j^*)$) of the alternatives from the single-valued reference point, and the ranking of alternatives by utilizing the RP approach

$d(X_{ij}, X_j^*)$	Relavance	Faithful Representa tion	Comparabi lity	Verifiabilit y	Timeliness	Understan dability	Max $d(X_{ij}, X_j^*)$	Rank
Financial Ratios								
Current ratio	0.035	0.104	0.009	0.118	0.027	0.061	0.118	16
Quick ratio	0.026	0.080	0.010	0.049	0.027	0.014	0.080	9
Cash ratio	0.041	0.184	0.131	0.185	0.091	0.137	0.185	25
Debt ratio	0.090	0.162	0.154	0.138	0.060	0.074	0.162	21
Debt-to-equity ratio	0.037	0.162	0.135	0.104	0.156	0.158	0.162	20
Inventory turnover	0.057	0.038	0.043	0.049	0.016	0.048	0.057	6
Accounts Payable to Cost of Good Sold	0.101	0.104	0.080	0.181	0.061	0.178	0.181	23
Total asset turnover	0.059	0.046	0.054	0.073	0.027	0.072	0.073	8
Return on Assets ratio	0.072	0.162	0.079	0.062	0.124	0.072	0.162	19
Net profit margin ratio	0.073	0.091	0.034	0.049	0.088	0.048	0.091	13
EBIT to Total Assets	0.080	0.143	0.100	0.062	0.110	0.062	0.143	17
Accounts Receivable to Sales	0.028	0.038	0.000	0.000	0.024	0.029	0.038	3
Accounts Receivable to Total Assets	0.026	0.182	0.054	0.117	0.059	0.099	0.182	24
Inventory to Sales	0.033	0.038	0.000	0.000	0.042	0.020	0.042	4
Inventory to Total Assets	0.018	0.093	0.043	0.051	0.047	0.085	0.093	14
Inventory to Current Assets	0.002	0.080	0.008	0.086	0.047	0.039	0.086	10
Growth of accounts receivables	0.000	0.016	0.035	0.023	0.017	0.007	0.035	2
Growth of Inventory	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
Growth of income from principal business operations	0.086	0.090	0.069	0.040	0.074	0.029	0.090	12
Growth of sales	0.090	0.062	0.117	0.074	0.087	0.060	0.117	15
Cash to Total Assets	0.036	0.070	0.036	0.047	0.088	0.039	0.088	11
Due from Shareholders to Total Assets	0.026	0.000	0.024	0.021	0.047	0.007	0.047	5
Due to Shareholders to Total Liabilities	0.046	0.054	0.067	0.029	0.047	0.029	0.067	7
Other Fixed Assets to Fixed Assets	0.084	0.165	0.099	0.053	0.081	0.107	0.165	22
Other Extraordinary Expenses and Losses to Net Sales	0.036	0.052	0.061	0.144	0.029	0.063	0.144	18

Step 10 Employ fuzzy FMF approach

$\tilde{S}_i$ values and $\text{Score}(\tilde{S}_i)$ are determined utilizing Eqs. (4.63) and (4.64), respectively, in the fuzzy FMF approach. The alternatives are then ordered in descending order based on the values of $\text{Score}(\tilde{S}_i)$. Table 6.86 shows the $\tilde{S}_i$ values, $\text{Score}(\tilde{S}_i)$, and the ranking of alternatives using the FMF approach.

Table 6.86. $\tilde{S}_i$ values, $\text{Score}(\tilde{S}_i)$ and the ranking of the alternatives by utilizing the FMF approach

Alternatives	$\tilde{S}_i$ values	$\text{Score}(\tilde{S}_i)$	Ranking
Current Ratio	(0.00, 0.95, 0.31)	-0.312	13
Quick ratio	(0.00, 0.93, 0.34)	-0.250	7
Cash ratio	(0.00, 0.98, 0.21)	-0.550	25
Debt ratio	(0.00, 0.97, 0.22)	-0.511	22
Debt-to-equity ratio	(0.00, 0.98, 0.21)	-0.543	24
Inventory turnover	(0.00, 0.94, 0.31)	-0.299	9
Accounts Payable to Cost of Good Sold	(0.00, 0.97, 0.22)	-0.519	23
Total asset turnover	(0.00, 0.95, 0.31)	-0.311	12
Return on Assets ratio	(0.00, 0.96, 0.27)	-0.410	18
Net profit margin ratio	(0.00, 0.95, 0.29)	-0.354	14
EBIT to Total Assets	(0.00, 0.96, 0.25)	-0.447	20
Accounts Receivable to Sales	(0.01, 0.92, 0.36)	-0.191	3
Accounts Receivable to Total Assets	(0.00, 0.96, 0.28)	-0.372	16
Inventory to Sales	(0.01, 0.92, 0.36)	-0.183	2
Inventory to Total Assets	(0.00, 0.94, 0.31)	-0.303	10
Inventory to Current Assets	(0.00, 0.94, 0.33)	-0.271	8
Growth of accounts receivables	(0.00, 0.93, 0.35)	-0.216	5
Growth of Inventory	(0.01, 0.92, 0.36)	-0.194	4
Growth of income from principal business operations	(0.00, 0.95, 0.29)	-0.367	15
Growth of sales	(0.00, 0.96, 0.26)	-0.437	19
Cash to Total Assets	(0.00, 0.91, 0.38)	-0.133	1
Due from Shareholders to Total Assets	(0.00, 0.93, 0.34)	-0.237	6
Due to Shareholders to Total Liabilities	(0.00, 0.94, 0.31)	-0.305	11
Other Fixed Assets to Fixed Assets	(0.00, 0.97, 0.23)	-0.485	21
Other Extraordinary Expenses and Losses to Net Sales	(0.00, 0.95, 0.28)	-0.379	17

Step 11: Use dominance theory to obtain the final rank

The alternatives are ranked based on all three approaches applied, and the final rank is determined using the dominance theory. Table 6.87 shows the rankings of the alternatives based on three approaches, as well as the final rank.

According to the final ranking, the top fifteen financial ratios are selected as shown in Table 6.88 for comparing the performances of classifiers in financial accounting fraud detection.

Table 6.87. Rankings of the alternatives based on three approaches and the final rank (Integrated AHP and MULTIMOORA with Single-Valued Spherical Fuzzy Sets)

Alternatives	RS Ranking	RP Ranking	FMF Ranking	Final Ranking
Current Ratio	11	16	13	13
Quick ratio	6	9	7	6
Cash ratio	25	25	25	25
Debt ratio	23	21	22	22
Debt-to-equity ratio	24	20	24	24
Inventory turnover	7	6	9	7
Accounts Payable to Cost of Good Sold	22	23	23	23
Total asset turnover	10	8	12	10
Return on Assets ratio	19	19	18	19
Net profit margin ratio	14	13	14	14
EBIT to Total Assets	18	17	20	18
Accounts Receivable to Sales	4	3	3	3
Accounts Receivable to Total Assets	20	24	16	20
Inventory to Sales	2	4	2	2
Inventory to Total Assets	12	14	10	12
Inventory to Current Assets	9	10	8	9
Growth of accounts receivables	5	2	5	5
Growth of Inventory	1	1	4	1
Growth of income from principal business operations	15	12	15	15
Growth of sales	17	15	19	17
Cash to Total Assets	16	11	1	11
Due from Shareholders to Total Assets	3	5	6	4
Due to Shareholders to Total Liabilities	8	7	11	8
Other Fixed Assets to Fixed Assets	21	22	21	21
Other Extraordinary Expenses and Losses to Net Sales	13	18	17	16

Table 6.88. Selected financial ratios for comparing the performances of classifiers (Integrated AHP and MULTIMOORA with Single-Valued Spherical Fuzzy Sets)

Alternatives (Financial Ratios)	Rank
Growth of Inventory	1
Inventory to Sales	2
Accounts Receivable to Sales	3
Due from Shareholders to Total Assets	4
Growth of accounts receivables	5
Quick ratio	6
Inventory turnover	7
Due to Shareholders to Total Liabilities	8
Inventory to Current Assets	9
Total asset turnover	10
Cash to Total Assets	11
Inventory to Total Assets	12
Current Ratio	13
Net profit margin ratio	14
Growth of income from principal business operations	15

Phase 2: After determining financial ratios for financial accounting fraud detection, seven different machine learning classifiers are compared by using performance metrics. Without employing any feature selection method, classifiers are also compared.

Tables 6.89-6.94 present the performance measures of classifiers using integrated AHP and MULTIMOORA with Spherical Fuzzy Sets (Single-Valued and Interval-Valued fuzzy numbers) and without using any feature selection method. Moreover, different sampling methods are employed.

Random Forest-without feature selection-oversampling model has the highest overall accuracy of 93.74% as shown in Table 6.89. Naive Bayes-Integrated AHP and MULTIMOORA with Spherical Fuzzy Sets -oversampling model has the lowest overall accuracy of 54.05% among all other models.

Table 6.89. Overall Accuracy of classifiers using Integrated AHP and MULTIMOORA with Spherical Fuzzy Sets (Single-Valued and Interval-Valued) and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and MULTIMOORA with Spherical Fuzzy Sets (Single-Valued and Interval-Valued)					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.8616	0.7777	0.7274	0.8704	0.8587	0.8131
Naive Bayes	0.7290	<u>0.5405</u>	0.7368	0.8211	0.5881	0.7695
ANN	0.8739	0.8220	0.7850	0.8979	0.8834	0.8287
KNN	0.8622	0.8381	0.7383	0.8944	0.8992	0.7882
Random Forest	0.8915	0.9025	0.7835	0.9179	0.9374	0.8707
Logistic Regression	0.8716	0.8025	0.7726	0.8956	0.8808	0.8536
Bagging	0.8903	0.8718	0.7539	0.9138	0.9127	0.8614

As shown in Table 6.90, Naive Bayes-Integrated AHP and MULTIMOORA with Spherical Fuzzy Sets-oversampling model is the highest at 95.70% in terms of True Positive Rate. Support Vector Machines- Integrated AHP and MULTIMOORA with Spherical Fuzzy Sets -Without Sampling model is found as the worst at 30.20% among all other models.

Table 6.90. True Positive Rates of classifiers using Integrated AHP and MULTIMOORA with Spherical Fuzzy Sets (Single-Valued and Interval-Valued) and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and MULTIMOORA with Spherical Fuzzy Sets (Single-Valued and Interval-Valued)					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.3020	0.5920	0.5050	0.3710	0.7830	0.7040
Naive Bayes	0.7350	0.9570	0.6790	0.7880	0.9380	0.6950
ANN	0.5080	0.7760	0.7630	0.6230	0.8790	0.8350
KNN	0.4770	0.8130	0.6760	0.6570	0.8930	0.7690
Random Forest	0.5330	0.8950	0.7480	0.6600	0.9470	0.8820
Logistic Regression	0.4020	0.7000	0.6760	0.5860	0.8440	0.8100
Bagging	0.5260	0.8500	0.6820	0.7200	0.9170	0.8540

Table 6.91 shows that Support Vector Machines- Integrated AHP and MULTIMOORA with Spherical Fuzzy Sets-Without Sampling model has the highest True Negative rate of 99.10%. Naive Bayes-Integrated AHP and MULTIMOORA with Spherical Fuzzy Sets - Oversampling model has the lowest True Negative rate of 15.40%.

Table 6.91. True Negative Rates of classifiers using Integrated AHP and MULTIMOORA with Spherical Fuzzy Sets (Single-Valued and Interval-Valued) and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and MULTIMOORA with Spherical Fuzzy Sets (Single-Valued and Interval-Valued)					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.9910	0.9500	0.9500	0.9860	0.9280	0.9220
Naive Bayes	0.7280	0.1540	0.7940	0.8290	0.2640	0.8440
ANN	0.9590	0.8640	0.8070	0.9620	0.8880	0.8220
KNN	0.9520	0.8610	0.8010	0.9490	0.9050	0.8070
Random Forest	0.9750	0.9100	0.8190	0.9780	0.9280	0.8600
Logistic Regression	0.9800	0.8970	0.8690	0.9670	0.9150	0.8970
Bagging	0.9750	0.8920	0.8260	0.9590	0.9080	0.8690

Table 6.92 indicates that Random Forest-without feature selection-oversampling model outperforms all other models in terms of precision. Naive Bayes-Integrated AHP and

MULTIMOORA with Spherical Fuzzy Sets - Without Sampling model performs the worst at 38.50%.

Table 6.92. Precision of classifiers using Integrated AHP and MULTIMOORA with Spherical Fuzzy Sets (Single-Valued and Interval-Valued) and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and MULTIMOORA with Spherical Fuzzy Sets (Single-Valued and Interval-Valued)					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.8900	0.9170	0.9100	0.8620	0.9100	0.9000
Naive Bayes	<u>0.3850</u>	0.5120	0.7680	0.5160	0.5420	0.8170
ANN	0.7410	0.8410	0.7980	0.7910	0.8790	0.8250
KNN	0.6950	0.8450	0.7720	0.7510	0.8970	0.7990
Random Forest	0.8300	0.9020	0.8050	0.8720	0.9250	0.8630
Logistic Regression	0.8270	0.8640	0.8380	0.8070	0.9020	0.8870
Bagging	0.8280	0.8800	0.7960	0.8020	0.9030	0.8670

As shown in Table 6.93, Random Forest-without feature selection-oversampling model is the highest at 93.75% in terms of G-measure. Naive Bayes-Integrated AHP and MULTIMOORA with Spherical Fuzzy Sets -Over Sampling model is found as the worst at 38.39% among all other models. Oversampling method significantly reduces the performance of Naive Bayes in terms of G-measure.

Table 6.93. G-measure of classifiers using Integrated AHP and MULTIMOORA with Spherical Fuzzy Sets (Single-Valued and Interval-Valued) and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and MULTIMOORA with Spherical Fuzzy Sets (Single-Valued and Interval-Valued)					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.5471	0.7499	0.6926	0.6048	0.8524	0.8057
Naive Bayes	0.7315	<u>0.3839</u>	0.7343	0.8082	0.4976	0.7659
ANN	0.6980	0.8188	0.7847	0.7742	0.8835	0.8285
KNN	0.6739	0.8367	0.7359	0.7896	0.8990	0.7878
Random Forest	0.7209	0.9025	0.7827	0.8034	0.9375	0.8709
Logistic Regression	0.6277	0.7924	0.7664	0.7528	0.8788	0.8524
Bagging	0.7161	0.8707	0.7506	0.8310	0.9125	0.8615

Random Forest outperforms other machine learning classifiers in terms of Area Under ROC Curve as shown in Table 6.94. Random Forest-without feature selection-oversampling model is the highest at 98.40%. It is also found that Support Vector Machines- Integrated AHP and MULTIMOORA with Spherical Fuzzy Sets - Without Sampling model performs the worst at 64.70% among all other models.

Table 6.94. Area Under ROC Curve of classifiers using Integrated AHP and MULTIMOORA with Spherical Fuzzy Sets (Single-Valued and Interval-Valued) and without using any feature selection method

	Feature Selection Method			Without Feature Selection		
	Integrated AHP and MULTIMOORA with Spherical Fuzzy Sets (Single-Valued and Interval-Valued)					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
	SVM	0.6470	0.7710	0.7270	0.6780	0.8560
Naive Bayes	0.8040	0.8420	0.7720	0.8390	0.8580	0.8110
ANN	0.8470	0.8940	0.8450	0.8890	0.9390	0.8980
KNN	0.7760	0.8960	0.7940	0.7980	0.8980	0.7890
Random Forest	0.8860	0.9630	0.8490	0.9490	0.9840	0.9190
Logistic Regression	0.8270	0.8750	0.8120	0.9130	0.9380	0.9050
Bagging	0.8670	0.9330	0.8280	0.9280	0.9690	0.9080

6.2.2.3. Integrated AHP and MULTIMOORA with Interval-Valued Spherical Fuzzy Sets

In this section, integrated AHP and MULTIMOORA with Interval-Valued Spherical Fuzzy Sets is used to evaluate financial ratios as fraud risk indicators in financial accounting fraud detection.

Algorithm of the application

The proposed methodology includes two phases as shown in Figure 6.17. In the first phase, Integrated AHP and MULTIMOORA with Interval-Valued Spherical Fuzzy Sets is used to prioritise the financial ratios. In the second phase, seven different machine learning classifiers are compared by using performance metrics.

Steps of the application

The steps of proposed approach are given as follows:

Phase 1: First, IVSF-AHP method is used to obtain the criteria weights. The steps and obtained results of interval-valued spherical fuzzy AHP method are given in Section 6.2.1.3.

After obtaining the interval-valued spherical fuzzy weights of criteria as shown in Table 6.40, interval-valued spherical fuzzy MULTIMOORA method is applied to obtain final ranking of financial ratios. The interval-valued spherical fuzzy MULTIMOORA method is applied as follows:

Step 6 The same assessments on alternatives in terms of criteria are used, and the IVSWAM operator is utilized to aggregate assessments of the alternatives by taking decision makers' importance levels into account. Table 6.95 shows the aggregated interval-valued spherical fuzzy decision matrix.

Table 6.95. Aggregated interval-valued spherical fuzzy decision matrix (Interval-Valued Spherical Fuzzy MULTIMOORA)

Alternatives	Relevance	Faithful Representatio n	Comparability	Verifiability	Timeliness	Understandabilit y
Current Ratio	[(0.62, 0.74), [0.32, 0.41], [0.22, 0.30]]	[(0.68, 0.80), [0.22, 0.28], [0.20, 0.26]]	[(0.74, 0.87), [0.18, 0.25], [0.16, 0.23]]	[(0.56, 0.66], [0.27, 0.33], [0.22, 0.28]]	[(0.66, 0.78], [0.23, 0.29], [0.18, 0.25]]	[(0.64, 0.77], [0.24, 0.31], [0.20, 0.26]]
Quick ratio	[(0.63, 0.75], [0.28, 0.36], [0.21, 0.28]]	[(0.70, 0.82], [0.18, 0.23], [0.17, 0.23]]	[(0.75, 0.87], [0.16, 0.21], [0.15, 0.21]]	[(0.68, 0.80], [0.22, 0.28], [0.17, 0.24]]	[(0.68, 0.80], [0.22, 0.28], [0.20, 0.26]]	[(0.74, 0.87], [0.18, 0.25], [0.16, 0.23]]
Cash ratio	[(0.58, 0.71], [0.36, 0.45], [0.20, 0.28]]	[(0.53, 0.63], [0.29, 0.35], [0.24, 0.29]]	[(0.42, 0.51], [0.41, 0.49], [0.23, 0.28]]	[(0.41, 0.49], [0.45, 0.53], [0.23, 0.28]]	[(0.51, 0.60], [0.35, 0.42], [0.21, 0.27]]	[(0.50, 0.59], [0.30, 0.36], [0.25, 0.30]]
Debt ratio	[(0.46, 0.53], [0.42, 0.51], [0.24, 0.31]]	[(0.59, 0.69], [0.26, 0.32], [0.24, 0.30]]	[(0.40, 0.45], [0.50, 0.60], [0.28, 0.36]]	[(0.51, 0.60], [0.35, 0.42], [0.21, 0.27]]	[(0.59, 0.69], [0.27, 0.33], [0.20, 0.26]]	[(0.59, 0.73], [0.31, 0.38], [0.18, 0.26]]
Debt-to-equity ratio	[(0.59, 0.73], [0.31, 0.38], [0.18, 0.26]]	[(0.59, 0.69], [0.26, 0.32], [0.24, 0.30]]	[(0.42, 0.49], [0.43, 0.51], [0.27, 0.33]]	[(0.58, 0.71], [0.36, 0.45], [0.20, 0.28]]	[(0.35, 0.43], [0.47, 0.56], [0.24, 0.29]]	[(0.46, 0.53], [0.42, 0.51], [0.24, 0.31]]
Inventory turnover	[(0.55, 0.64], [0.32, 0.39], [0.24, 0.30]]	[(0.77, 0.89], [0.14, 0.19], [0.13, 0.20]]	[(0.63, 0.73], [0.21, 0.26], [0.21, 0.26]]	[(0.68, 0.80], [0.22, 0.28], [0.17, 0.24]]	[(0.68, 0.80], [0.22, 0.28], [0.17, 0.24]]	[(0.66, 0.78], [0.23, 0.29], [0.18, 0.25]]
Accounts Payable to Cost of Good Sold	[(0.45, 0.53], [0.44, 0.53], [0.23, 0.30]]	[(0.68, 0.80], [0.22, 0.28], [0.20, 0.26]]	[(0.55, 0.64], [0.32, 0.39], [0.24, 0.30]]	[(0.41, 0.49], [0.43, 0.51], [0.24, 0.29]]	[(0.59, 0.72], [0.34, 0.43], [0.21, 0.29]]	[(0.42, 0.49], [0.43, 0.51], [0.27, 0.33]]
Total asset turnover	[(0.57, 0.66], [0.27, 0.33], [0.25, 0.31]]	[(0.76, 0.88], [0.15, 0.20], [0.14, 0.21]]	[(0.62, 0.71], [0.24, 0.30], [0.23, 0.29]]	[(0.67, 0.79], [0.25, 0.33], [0.21, 0.28]]	[(0.68, 0.80], [0.22, 0.28], [0.20, 0.26]]	[(0.63, 0.75], [0.28, 0.36], [0.21, 0.28]]
Return on Assets ratio	[(0.52, 0.61], [0.33, 0.40], [0.25, 0.32]]	[(0.59, 0.69], [0.26, 0.32], [0.24, 0.30]]	[(0.58, 0.67], [0.30, 0.37], [0.25, 0.32]]	[(0.68, 0.80], [0.22, 0.28], [0.20, 0.26]]	[(0.53, 0.60], [0.34, 0.41], [0.28, 0.35]]	[(0.63, 0.75], [0.28, 0.36], [0.21, 0.28]]
Net profit margin ratio	[(0.48, 0.57], [0.35, 0.42], [0.24, 0.29]]	[(0.70, 0.81], [0.21, 0.27], [0.19, 0.25]]	[(0.67, 0.79], [0.25, 0.33], [0.21, 0.28]]	[(0.68, 0.80], [0.22, 0.28], [0.17, 0.24]]	[(0.55, 0.64], [0.32, 0.39], [0.24, 0.30]]	[(0.66, 0.78], [0.23, 0.29], [0.18, 0.25]]
EBIT to Total Assets	[(0.47, 0.56], [0.37, 0.44], [0.23, 0.28]]	[(0.62, 0.71], [0.24, 0.30], [0.23, 0.29]]	[(0.52, 0.60], [0.35, 0.42], [0.24, 0.31]]	[(0.65, 0.78], [0.24, 0.30], [0.17, 0.24]]	[(0.52, 0.60], [0.35, 0.42], [0.24, 0.31]]	[(0.63, 0.76], [0.25, 0.32], [0.19, 0.25]]

Table 6.95. (Continuation)

Accounts Receivable to Sales	([0.76, 0.88], [0.15, 0.20], [0.14, 0.21])	([0.77, 0.89], [0.14, 0.19], [0.13, 0.20])	([0.72, 0.84], [0.17, 0.22], [0.16, 0.22])	([0.77, 0.89], [0.14, 0.19], [0.13, 0.20])	([0.77, 0.89], [0.14, 0.19], [0.13, 0.20])	([0.70, 0.82], [0.18, 0.23], [0.17, 0.23])
Accounts Receivable to Total Assets	([0.65, 0.77], [0.27, 0.34], [0.22, 0.29])	([0.57, 0.66], [0.27, 0.33], [0.25, 0.31])	([0.62, 0.71], [0.24, 0.30], [0.23, 0.29])	([0.59, 0.69], [0.26, 0.32], [0.24, 0.30])	([0.62, 0.71], [0.24, 0.30], [0.23, 0.29])	([0.59, 0.69], [0.26, 0.32], [0.24, 0.30])
Inventory to Sales	([0.77, 0.89], [0.14, 0.19], [0.13, 0.20])	([0.77, 0.89], [0.14, 0.19], [0.13, 0.20])	([0.72, 0.84], [0.17, 0.22], [0.16, 0.22])	([0.77, 0.89], [0.14, 0.19], [0.13, 0.20])	([0.77, 0.89], [0.12, 0.17], [0.09, 0.17])	([0.72, 0.84], [0.17, 0.22], [0.16, 0.22])
Inventory to Total Assets	([0.66, 0.78], [0.23, 0.29], [0.21, 0.27])	([0.69, 0.81], [0.19, 0.24], [0.19, 0.24])	([0.63, 0.73], [0.21, 0.26], [0.21, 0.26])	([0.69, 0.81], [0.19, 0.24], [0.19, 0.24])	([0.63, 0.73], [0.21, 0.26], [0.21, 0.26])	([0.60, 0.71], [0.22, 0.27], [0.22, 0.27])
Inventory to Current Assets	([0.69, 0.81], [0.19, 0.24], [0.19, 0.24])	([0.70, 0.82], [0.18, 0.23], [0.17, 0.23])	([0.70, 0.82], [0.18, 0.23], [0.17, 0.23])	([0.63, 0.73], [0.21, 0.26], [0.21, 0.26])	([0.63, 0.73], [0.21, 0.26], [0.21, 0.26])	([0.69, 0.81], [0.19, 0.24], [0.19, 0.24])
Growth of accounts receivables	([0.70, 0.81], [0.21, 0.27], [0.19, 0.25])	([0.82, 0.93], [0.12, 0.17], [0.09, 0.17])	([0.66, 0.78], [0.23, 0.29], [0.21, 0.27])	([0.74, 0.87], [0.18, 0.25], [0.16, 0.23])	([0.70, 0.81], [0.21, 0.27], [0.19, 0.25])	([0.77, 0.89], [0.14, 0.19], [0.13, 0.20])
Growth of Inventory	([0.70, 0.81], [0.21, 0.27], [0.19, 0.25])	([0.85, 0.95], [0.10, 0.15], [0.05, 0.15])	([0.72, 0.84], [0.17, 0.22], [0.16, 0.22])	([0.77, 0.89], [0.14, 0.19], [0.13, 0.20])	([0.72, 0.84], [0.17, 0.22], [0.16, 0.22])	([0.76, 0.88], [0.15, 0.20], [0.14, 0.21])
Growth of income from principal business operations	([0.51, 0.58], [0.39, 0.47], [0.26, 0.34])	([0.67, 0.80], [0.23, 0.29], [0.16, 0.23])	([0.56, 0.66], [0.28, 0.34], [0.22, 0.27])	([0.70, 0.82], [0.18, 0.23], [0.17, 0.23])	([0.56, 0.66], [0.27, 0.33], [0.22, 0.28])	([0.70, 0.82], [0.18, 0.23], [0.17, 0.23])
Growth of sales	([0.46, 0.53], [0.42, 0.51], [0.24, 0.31])	([0.72, 0.86], [0.20, 0.27], [0.13, 0.22])	([0.46, 0.53], [0.42, 0.51], [0.24, 0.31])	([0.65, 0.77], [0.28, 0.35], [0.19, 0.26])	([0.58, 0.67], [0.30, 0.37], [0.25, 0.32])	([0.65, 0.77], [0.28, 0.35], [0.19, 0.26])
Cash to Total Assets	([0.62, 0.75], [0.31, 0.39], [0.19, 0.27])	([0.67, 0.79], [0.00, 0.00], [0.14, 0.22])	([0.65, 0.78], [0.00, 0.00], [0.16, 0.23])	([0.67, 0.79], [0.00, 0.00], [0.14, 0.22])	([0.51, 0.59], [0.00, 0.00], [0.23, 0.30])	([0.67, 0.79], [0.00, 0.00], [0.14, 0.22])
Due from Shareholders to Total Assets	([0.65, 0.78], [0.26, 0.32], [0.16, 0.24])	([0.85, 0.95], [0.10, 0.15], [0.05, 0.15])	([0.66, 0.78], [0.23, 0.29], [0.18, 0.25])	([0.82, 0.93], [0.12, 0.17], [0.09, 0.17])	([0.63, 0.73], [0.21, 0.26], [0.21, 0.26])	([0.77, 0.89], [0.14, 0.19], [0.13, 0.20])
Due to Shareholders to Total Liabilities	([0.56, 0.66], [0.27, 0.33], [0.22, 0.28])	([0.76, 0.88], [0.17, 0.24], [0.15, 0.22])	([0.56, 0.66], [0.27, 0.33], [0.22, 0.28])	([0.72, 0.84], [0.17, 0.22], [0.16, 0.22])	([0.63, 0.73], [0.21, 0.26], [0.21, 0.26])	([0.70, 0.82], [0.18, 0.23], [0.17, 0.23])
Other Fixed Assets to Fixed Assets	([0.50, 0.60], [0.37, 0.44], [0.20, 0.26])	([0.56, 0.66], [0.28, 0.34], [0.22, 0.27])	([0.50, 0.60], [0.37, 0.44], [0.20, 0.26])	([0.67, 0.80], [0.24, 0.31], [0.14, 0.23])	([0.55, 0.65], [0.30, 0.37], [0.20, 0.26])	([0.55, 0.65], [0.30, 0.37], [0.20, 0.26])
Other Extraordinary Expenses and Losses to Net Sales	([0.62, 0.75], [0.31, 0.39], [0.19, 0.27])	([0.74, 0.87], [0.19, 0.25], [0.12, 0.21])	([0.61, 0.73], [0.36, 0.46], [0.20, 0.29])	([0.54, 0.63], [0.35, 0.43], [0.22, 0.29])	([0.67, 0.80], [0.24, 0.31], [0.14, 0.23])	([0.64, 0.77], [0.30, 0.37], [0.17, 0.26])

Step 7 Build the weighted interval-valued spherical fuzzy decision matrix

Eq. (3.23) is used to build the weighted interval-valued spherical fuzzy decision matrix after calculating the criteria's weights and the alternatives' ratings. Table 6.96 shows the weighted interval-valued spherical fuzzy decision matrix.

Table 6.96. Weighted interval-valued spherical fuzzy decision matrix (Interval-Valued Spherical Fuzzy MULTIMOORA)

Alternatives	Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
Current Ratio	([0.24, 0.33], [0.58, 0.69], [0.30, 0.37])	([0.39, 0.53], [0.39, 0.48], [0.30, 0.38])	([0.33, 0.44], [0.47, 0.57], [0.29, 0.38])	([0.31, 0.42], [0.42, 0.51], [0.32, 0.40])	([0.31, 0.43], [0.46, 0.55], [0.29, 0.37])	([0.33, 0.45], [0.44, 0.54], [0.31, 0.39])
Quick ratio	([0.24, 0.33], [0.57, 0.67], [0.30, 0.37])	([0.40, 0.54], [0.37, 0.45], [0.29, 0.37])	([0.33, 0.45], [0.47, 0.56], [0.29, 0.37])	([0.38, 0.51], [0.39, 0.49], [0.30, 0.38])	([0.32, 0.43], [0.45, 0.54], [0.30, 0.38])	([0.38, 0.51], [0.42, 0.51], [0.30, 0.38])
Cash ratio	([0.23, 0.32], [0.60, 0.71], [0.29, 0.36])	([0.30, 0.41], [0.42, 0.51], [0.32, 0.39])	([0.19, 0.26], [0.58, 0.67], [0.31, 0.37])	([0.23, 0.31], [0.54, 0.64], [0.31, 0.37])	([0.24, 0.33], [0.51, 0.61], [0.30, 0.37])	([0.31, 0.35], [0.47, 0.56], [0.34, 0.40])

Table 6.96. (Continuation)

Debt ratio	([0.18, 0.24], [0.63, 0.73], [0.30, 0.36])	([0.34, 0.45], [0.41, 0.50], [0.32, 0.40])	([0.18, 0.23], [0.63, 0.73], [0.33, 0.39])	([0.29, 0.39], [0.47, 0.56], [0.31, 0.38])	([0.27, 0.37], [0.47, 0.57], [0.30, 0.37])	([0.30, 0.43], [0.48, 0.58], [0.30, 0.38])
Debt-to-equity ratio	([0.23, 0.32], [0.58, 0.68], [0.28, 0.36])	([0.34, 0.45], [0.41, 0.50], [0.32, 0.40])	([0.19, 0.25], [0.59, 0.69], [0.33, 0.39])	([0.33, 0.46], [0.47, 0.58], [0.30, 0.38])	([0.16, 0.23], [0.59, 0.69], [0.31, 0.36])	([0.23, 0.31], [0.55, 0.65], [0.32, 0.39])
Inventory turnover	([0.21, 0.28], [0.58, 0.68], [0.31, 0.37])	([0.44, 0.59], [0.35, 0.44], [0.27, 0.35])	([0.28, 0.37], [0.48, 0.58], [0.32, 0.39])	([0.38, 0.51], [0.39, 0.49], [0.30, 0.38])	([0.32, 0.43], [0.45, 0.54], [0.29, 0.36])	([0.34, 0.46], [0.44, 0.53], [0.30, 0.38])
Accounts Payable to Cost of Good Sold	([0.17, 0.23], [0.64, 0.74], [0.30, 0.36])	([0.39, 0.53], [0.39, 0.48], [0.30, 0.38])	([0.25, 0.33], [0.53, 0.63], [0.32, 0.39])	([0.23, 0.32], [0.52, 0.62], [0.32, 0.38])	([0.27, 0.39], [0.51, 0.62], [0.30, 0.37])	([0.22, 0.29], [0.55, 0.65], [0.34, 0.40])
Total asset turnover	([0.22, 0.29], [0.57, 0.66], [0.32, 0.38])	([0.43, 0.58], [0.36, 0.44], [0.27, 0.36])	([0.28, 0.36], [0.50, 0.59], [0.32, 0.39])	([0.38, 0.50], [0.41, 0.51], [0.32, 0.40])	([0.32, 0.43], [0.45, 0.54], [0.30, 0.38])	([0.32, 0.44], [0.46, 0.56], [0.31, 0.39])
Return on Assets ratio	([0.20, 0.27], [0.59, 0.69], [0.32, 0.38])	([0.34, 0.45], [0.41, 0.50], [0.32, 0.40])	([0.26, 0.34], [0.52, 0.62], [0.33, 0.40])	([0.38, 0.51], [0.39, 0.49], [0.31, 0.39])	([0.25, 0.33], [0.51, 0.60], [0.34, 0.41])	([0.32, 0.44], [0.46, 0.56], [0.31, 0.39])
Net profit margin ratio	([0.18, 0.25], [0.60, 0.70], [0.31, 0.36])	([0.40, 0.54], [0.38, 0.47], [0.30, 0.37])	([0.30, 0.40], [0.50, 0.60], [0.31, 0.39])	([0.38, 0.51], [0.39, 0.49], [0.30, 0.38])	([0.26, 0.35], [0.50, 0.59], [0.32, 0.39])	([0.34, 0.46], [0.44, 0.53], [0.30, 0.38])
EBIT to Total Assets	([0.18, 0.25], [0.60, 0.70], [0.30, 0.36])	([0.35, 0.47], [0.40, 0.49], [0.32, 0.39])	([0.23, 0.31], [0.54, 0.64], [0.32, 0.39])	([0.37, 0.50], [0.40, 0.50], [0.30, 0.38])	([0.24, 0.33], [0.52, 0.61], [0.32, 0.39])	([0.32, 0.45], [0.45, 0.54], [0.30, 0.38])
Accounts Receivable to Sales	([0.29, 0.39], [0.53, 0.63], [0.27, 0.36])	([0.44, 0.59], [0.35, 0.44], [0.27, 0.35])	([0.32, 0.43], [0.47, 0.56], [0.29, 0.38])	([0.43, 0.57], [0.36, 0.45], [0.28, 0.37])	([0.36, 0.48], [0.43, 0.52], [0.27, 0.36])	([0.36, 0.48], [0.42, 0.51], [0.30, 0.38])
Accounts Receivable to Total Assets	([0.25, 0.34], [0.56, 0.67], [0.30, 0.38])	([0.32, 0.43], [0.42, 0.51], [0.33, 0.40])	([0.28, 0.36], [0.50, 0.59], [0.32, 0.39])	([0.33, 0.44], [0.42, 0.51], [0.33, 0.41])	([0.29, 0.39], [0.46, 0.55], [0.31, 0.39])	([0.30, 0.40], [0.45, 0.54], [0.33, 0.41])
Inventory to Sales	([0.30, 0.40], [0.53, 0.63], [0.27, 0.35])	([0.44, 0.59], [0.35, 0.44], [0.27, 0.35])	([0.32, 0.43], [0.47, 0.56], [0.29, 0.38])	([0.43, 0.57], [0.36, 0.45], [0.28, 0.37])	([0.38, 0.50], [0.42, 0.51], [0.26, 0.35])	([0.37, 0.49], [0.41, 0.50], [0.30, 0.38])
Inventory to Total Assets	([0.25, 0.35], [0.55, 0.65], [0.30, 0.37])	([0.39, 0.53], [0.37, 0.46], [0.30, 0.37])	([0.28, 0.37], [0.48, 0.58], [0.32, 0.39])	([0.38, 0.52], [0.38, 0.47], [0.31, 0.39])	([0.29, 0.40], [0.45, 0.54], [0.31, 0.38])	([0.31, 0.41], [0.43, 0.52], [0.33, 0.40])
Inventory to Current Assets	([0.26, 0.36], [0.54, 0.64], [0.29, 0.37])	([0.40, 0.54], [0.37, 0.45], [0.29, 0.37])	([0.31, 0.42], [0.47, 0.57], [0.30, 0.38])	([0.35, 0.47], [0.39, 0.48], [0.32, 0.39])	([0.29, 0.40], [0.45, 0.54], [0.31, 0.38])	([0.35, 0.48], [0.42, 0.51], [0.31, 0.39])
Growth of accounts receivables	([0.27, 0.36], [0.54, 0.65], [0.29, 0.37])	([0.47, 0.61], [0.35, 0.43], [0.26, 0.35])	([0.29, 0.40], [0.49, 0.59], [0.31, 0.39])	([0.42, 0.55], [0.38, 0.47], [0.30, 0.38])	([0.33, 0.44], [0.45, 0.54], [0.30, 0.37])	([0.40, 0.52], [0.40, 0.49], [0.28, 0.37])
Growth of Inventory	([0.27, 0.36], [0.54, 0.65], [0.29, 0.37])	([0.48, 0.63], [0.34, 0.43], [0.25, 0.34])	([0.32, 0.43], [0.47, 0.56], [0.29, 0.38])	([0.43, 0.57], [0.36, 0.45], [0.28, 0.37])	([0.34, 0.45], [0.43, 0.52], [0.29, 0.36])	([0.39, 0.52], [0.41, 0.50], [0.29, 0.37])
Growth of income from principal business operations	([0.20, 0.26], [0.61, 0.72], [0.32, 0.38])	([0.38, 0.52], [0.39, 0.48], [0.28, 0.36])	([0.25, 0.34], [0.51, 0.61], [0.31, 0.38])	([0.39, 0.53], [0.38, 0.46], [0.30, 0.38])	([0.26, 0.36], [0.48, 0.57], [0.31, 0.38])	([0.36, 0.48], [0.42, 0.51], [0.30, 0.38])
Growth of sales	([0.18, 0.24], [0.63, 0.73], [0.30, 0.36])	([0.41, 0.56], [0.38, 0.47], [0.27, 0.36])	([0.20, 0.27], [0.59, 0.69], [0.32, 0.38])	([0.36, 0.49], [0.43, 0.52], [0.30, 0.39])	([0.27, 0.36], [0.49, 0.58], [0.33, 0.40])	([0.33, 0.45], [0.46, 0.56], [0.30, 0.39])
Cash to Total Assets	([0.24, 0.33], [0.58, 0.68], [0.29, 0.36])	([0.38, 0.52], [0.33, 0.40], [0.28, 0.37])	([0.29, 0.40], [0.45, 0.53], [0.30, 0.39])	([0.37, 0.51], [0.34, 0.41], [0.29, 0.39])	([0.24, 0.32], [0.41, 0.49], [0.32, 0.40])	([0.34, 0.47], [0.38, 0.46], [0.29, 0.39])
Due from Shareholders to Total Assets	([0.25, 0.35], [0.56, 0.66], [0.28, 0.36])	([0.48, 0.63], [0.34, 0.43], [0.25, 0.34])	([0.29, 0.40], [0.49, 0.59], [0.30, 0.38])	([0.46, 0.59], [0.35, 0.44], [0.27, 0.36])	([0.29, 0.40], [0.45, 0.54], [0.31, 0.38])	([0.40, 0.52], [0.40, 0.49], [0.28, 0.37])
Due to Shareholders to Total Liabilities	([0.22, 0.29], [0.56, 0.66], [0.31, 0.37])	([0.43, 0.58], [0.37, 0.46], [0.28, 0.36])	([0.25, 0.34], [0.51, 0.60], [0.32, 0.39])	([0.40, 0.54], [0.37, 0.46], [0.30, 0.38])	([0.29, 0.40], [0.45, 0.54], [0.31, 0.38])	([0.36, 0.48], [0.42, 0.51], [0.30, 0.38])
Other Fixed Assets to Fixed Assets	([0.19, 0.27], [0.60, 0.70], [0.29, 0.35])	([0.32, 0.43], [0.42, 0.51], [0.31, 0.38])	([0.22, 0.31], [0.56, 0.65], [0.30, 0.37])	([0.38, 0.51], [0.41, 0.50], [0.29, 0.37])	([0.26, 0.36], [0.49, 0.58], [0.30, 0.37])	([0.28, 0.38], [0.47, 0.57], [0.31, 0.38])
Other Extraordinary Expenses and Losses to Net Sales	([0.24, 0.33], [0.58, 0.68], [0.29, 0.36])	([0.42, 0.57], [0.37, 0.47], [0.27, 0.35])	([0.27, 0.37], [0.55, 0.66], [0.30, 0.38])	([0.30, 0.40], [0.47, 0.57], [0.31, 0.39])	([0.31, 0.43], [0.46, 0.56], [0.28, 0.36])	([0.33, 0.45], [0.47, 0.57], [0.30, 0.38])

Step 8 Employ RS approach

First, Eq. (3.32) is utilized to compute Y_i^+ values. The Y_i^+ values are then defuzzified utilizing the score function provided in Eq (3.34). Finally, the alternatives are sorted in descending order based on the values of Score (Y_i^+). Table 6.97 presents Y_i^+ values,

Score (Y_i^+) values, and the ranking of alternatives.

Table 6.97. The ranking of the alternatives by utilizing the RS approach

Alternatives	Y_i^+ values	Score(Y_i^+)	Rank
Current Ratio	([0.32, 0.44], [0.46, 0.55], [0.30, 0.38])	-0.140	13
Quick ratio	([0.35, 0.47], [0.44, 0.53], [0.30, 0.38])	-0.097	7
Cash ratio	([0.24, 0.33], [0.52, 0.61], [0.31, 0.38])	-0.267	24
Debt ratio	([0.27, 0.36], [0.51, 0.60], [0.31, 0.38])	-0.240	22
Debt-to-equity ratio	([0.26, 0.35], [0.53, 0.63], [0.31, 0.38])	-0.270	25
Inventory turnover	([0.34, 0.46], [0.44, 0.54], [0.30, 0.38])	-0.111	10
Accounts Payable to Cost of Good Sold	([0.26, 0.36], [0.52, 0.62], [0.31, 0.38])	-0.254	23
Total asset turnover	([0.33, 0.45], [0.45, 0.55], [0.31, 0.38])	-0.126	12
Return on Assets ratio	([0.30, 0.40], [0.48, 0.57], [0.32, 0.40])	-0.183	18
Net profit margin ratio	([0.32, 0.43], [0.46, 0.56], [0.31, 0.38])	-0.147	15
EBIT to Total Assets	([0.29, 0.40], [0.48, 0.58], [0.31, 0.38])	-0.189	20
Accounts Receivable to Sales	([0.37, 0.50], [0.42, 0.51], [0.28, 0.37])	-0.054	3
Accounts Receivable to Total Assets	([0.30, 0.40], [0.46, 0.56], [0.32, 0.40])	-0.173	17
Inventory to Sales	([0.38, 0.50], [0.42, 0.51], [0.28, 0.37])	-0.047	2
Inventory to Total Assets	([0.32, 0.44], [0.44, 0.53], [0.31, 0.38])	-0.121	11
Inventory to Current Assets	([0.33, 0.45], [0.44, 0.53], [0.30, 0.38])	-0.108	8
Growth of accounts receivables	([0.37, 0.49], [0.43, 0.52], [0.29, 0.37])	-0.066	5
Growth of Inventory	([0.38, 0.51], [0.42, 0.51], [0.28, 0.37])	-0.046	1
Growth of income from principal business operations	([0.32, 0.43], [0.46, 0.55], [0.31, 0.38])	-0.141	14
Growth of sales	([0.31, 0.42], [0.49, 0.59], [0.30, 0.38])	-0.184	19
Cash to Total Assets	([0.32, 0.44], [0.40, 0.49], [0.30, 0.38])	-0.086	6
Due from Shareholders to Total Assets	([0.38, 0.50], [0.43, 0.52], [0.28, 0.37])	-0.056	4
Due to Shareholders to Total Liabilities	([0.34, 0.45], [0.44, 0.53], [0.30, 0.38])	-0.108	9
Other Fixed Assets to Fixed Assets	([0.28, 0.39], [0.49, 0.58], [0.30, 0.37])	-0.201	21
Other Extraordinary Expenses and Losses to Net Sales	([0.32, 0.44], [0.48, 0.58], [0.29, 0.37])	-0.164	16

Step 9 Employ RP approach

First, Eqs. (4.28) and (4.29) are used to calculate interval-valued reference points, as shown in Table 6.98. Second, Eq. (4.32) is used to calculate the alternatives' distances from the interval-valued reference point. Third, using Eq. (4.62), the alternatives' deviations from the reference point are computed, and the alternatives are then sorted in ascending order based on the deviation values. Table 6.99 presents the distances ($d(X_{ij}, X^*)$), the deviations ($\text{Max } d(X_{ij}, X_j^*)$), and the ranking of alternatives.

Table 6.98. Interval-Valued Spherical Fuzzy reference points

	Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability
Reference point	([0.27, 0.36], [0.54, 0.65], [0.29, 0.37])	([0.48, 0.63], [0.34, 0.43], [0.25, 0.34])	([0.32, 0.43], [0.47, 0.56], [0.29, 0.38])	([0.43, 0.57], [0.36, 0.45], [0.28, 0.37])	([0.34, 0.45], [0.43, 0.52], [0.29, 0.36])	([0.39, 0.52], [0.41, 0.50], [0.29, 0.37])

Table 6.99. The distances ($d(X_{ij}, X_j^*)$) and the deviations ($\text{Max } d(X_{ij}, X_j^*)$) of the alternatives from the interval-valued reference point, and the ranking of alternatives by utilizing the RP approach

$d(X_{ij}, X_j^*)$	Relevance	Faithful Representation	Comparability	Verifiability	Timeliness	Understandability	Max $d(X_{ij}, X_j^*)$	Rank
Financial Ratios								
Current ratio	0.039	0.084	0.009	0.097	0.025	0.052	0.097	15
Quick ratio	0.025	0.064	0.009	0.046	0.022	0.013	0.064	8
Cash ratio	0.051	0.146	0.110	0.185	0.085	0.104	0.185	25
Debt ratio	0.086	0.126	0.158	0.127	0.050	0.075	0.158	23
Debt-to-equity ratio	0.036	0.126	0.125	0.112	0.154	0.151	0.154	22
Inventory turnover	0.046	0.032	0.029	0.046	0.018	0.043	0.046	6
Accounts Payable to Cost of Good Sold	0.092	0.084	0.070	0.180	0.072	0.164	0.180	24
Total asset turnover	0.037	0.038	0.041	0.064	0.022	0.067	0.067	10
Return on Assets ratio	0.056	0.126	0.065	0.050	0.095	0.067	0.126	19
Net profit margin ratio	0.060	0.075	0.031	0.046	0.076	0.043	0.076	13
EBIT to Total Assets	0.065	0.114	0.084	0.056	0.091	0.054	0.114	16
Accounts Receivable to Sales	0.022	0.032	0.000	0.000	0.019	0.022	0.032	4
Accounts Receivable to Total Assets	0.022	0.139	0.041	0.090	0.045	0.075	0.139	21
Inventory to Sales	0.025	0.032	0.000	0.000	0.032	0.016	0.032	5
Inventory to Total Assets	0.011	0.072	0.029	0.039	0.032	0.061	0.072	12
Inventory to Current Assets	0.005	0.064	0.005	0.065	0.032	0.029	0.065	9
Growth of accounts receivables	0.000	0.014	0.027	0.020	0.015	0.005	0.027	2
Growth of Inventory	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
Growth of income from principal business operations	0.075	0.081	0.055	0.031	0.058	0.022	0.081	14
Growth of sales	0.086	0.058	0.115	0.072	0.071	0.059	0.115	17
Cash to Total Assets	0.034	0.068	0.028	0.045	0.068	0.037	0.068	11
Due from Shareholders to Total Assets	0.018	0.000	0.023	0.018	0.032	0.005	0.032	3
Due to Shareholders to Total Liabilities	0.032	0.046	0.053	0.024	0.032	0.022	0.053	7
Other Fixed Assets to Fixed Assets	0.064	0.135	0.086	0.049	0.064	0.086	0.135	20
Other Extraordinary Expenses and Losses to Net Sales	0.034	0.050	0.068	0.125	0.026	0.063	0.125	18

Step 10 Employ fuzzy FMF approach

Firstly, Eq (3.33) is used to compute $\tilde{S}_i$ values. Then, using the score function given in Eq. (3.34), the $\tilde{S}_i$ values are defuzzified. Finally, the alternatives are ordered in descending order based on the $\text{Score}(\tilde{S}_i)$ values. Table 6.100 presents the values of $\tilde{S}_i$, the values of $\text{Score}(\tilde{S}_i)$, and the ranking of alternatives utilizing the FMF approach.

Table 6.100. The ranking of alternatives by utilizing the FMF approach

Alternatives	$\tilde{S}_i$ values	Score($\tilde{S}_i$)	Ranking
Current Ratio	([0.31, 0.43], [0.47, 0.57], [0.30, 0.38])	-0.158	13
Quick ratio	([0.34, 0.46], [0.45, 0.55], [0.30, 0.38])	-0.119	7
Cash ratio	([0.24, 0.33], [0.53, 0.62], [0.31, 0.38])	-0.282	23
Debt ratio	([0.25, 0.34], [0.53, 0.63], [0.31, 0.39])	-0.277	22
Debt-to-equity ratio	([0.24, 0.33], [0.54, 0.64], [0.31, 0.38])	-0.297	25
Inventory turnover	([0.32, 0.43], [0.46, 0.55], [0.30, 0.38])	-0.144	11
Accounts Payable to Cost of Good Sold	([0.25, 0.34], [0.53, 0.63], [0.31, 0.38])	-0.285	24
Total asset turnover	([0.32, 0.43], [0.46, 0.56], [0.31, 0.39])	-0.155	12
Return on Assets ratio	([0.28, 0.38], [0.49, 0.58], [0.32, 0.40])	-0.209	18
Net profit margin ratio	([0.30, 0.41], [0.48, 0.57], [0.31, 0.38])	-0.181	14
EBIT to Total Assets	([0.27, 0.37], [0.49, 0.59], [0.31, 0.38])	-0.221	19
Accounts Receivable to Sales	([0.36, 0.49], [0.43, 0.53], [0.28, 0.37])	-0.074	2
Accounts Receivable to Total Assets	([0.29, 0.39], [0.47, 0.57], [0.32, 0.40])	-0.184	16
Inventory to Sales	([0.37, 0.49], [0.43, 0.52], [0.28, 0.37])	-0.067	1
Inventory to Total Assets	([0.31, 0.42], [0.45, 0.54], [0.31, 0.38])	-0.141	9
Inventory to Current Assets	([0.33, 0.44], [0.45, 0.54], [0.30, 0.38])	-0.123	8
Growth of accounts receivables	([0.35, 0.47], [0.44, 0.54], [0.29, 0.37])	-0.096	4
Growth of Inventory	([0.37, 0.48], [0.43, 0.53], [0.28, 0.37])	-0.076	3
Growth of income from principal business operations	([0.30, 0.40], [0.48, 0.57], [0.31, 0.38])	-0.181	15
Growth of sales	([0.28, 0.38], [0.51, 0.61], [0.31, 0.38])	-0.232	21
Cash to Total Assets	([0.30, 0.42], [0.43, 0.51], [0.30, 0.38])	-0.119	6
Due from Shareholders to Total Assets	([0.35, 0.47], [0.44, 0.54], [0.28, 0.37])	-0.097	5
Due to Shareholders to Total Liabilities	([0.32, 0.42], [0.45, 0.55], [0.30, 0.38])	-0.143	10
Other Fixed Assets to Fixed Assets	([0.27, 0.37], [0.50, 0.60], [0.30, 0.37])	-0.227	20
Other Extraordinary Expenses and Losses to Net Sales	([0.31, 0.42], [0.49, 0.59], [0.29, 0.37])	-0.188	17

Step 11 Use dominance theory to obtain the final rank

The alternatives are ordered based on the three approaches examined, and the final rank is determined utilizing dominance theory. Table 6.101 depicts the rankings of the alternatives based on three approaches, and also the final rank.

Table 6.102 indicates that selected top fifteen financial ratios for comparing the performances of classifiers in financial accounting fraud detection. According to the final rankings (shown in Table 6.88 and 6.102), the top fifteen financial ratios are found to be identical for two spherical fuzzy sets, which are Interval-Valued Spherical Fuzzy Sets and Single-Valued Spherical Fuzzy Sets.

Table 6.101. Rankings of the alternatives based on three approaches and the final rank (Integrated AHP and MULTIMOORA with Interval-Valued Spherical Fuzzy Sets)

Alternatives	RS Ranking	RP Ranking	FMF Ranking	Final Ranking
Current Ratio	13	15	13	13
Quick ratio	7	8	7	7
Cash ratio	24	25	23	24
Debt ratio	22	23	22	22
Debt-to-equity ratio	25	22	25	25
Inventory turnover	10	6	11	10
Accounts Payable to Cost of Good Sold	23	24	24	23
Total asset turnover	12	10	12	12
Return on Assets ratio	18	19	18	18
Net profit margin ratio	15	13	14	14
EBIT to Total Assets	20	16	19	19
Accounts Receivable to Sales	3	4	2	3
Accounts Receivable to Total Assets	17	21	16	17
Inventory to Sales	2	5	1	2
Inventory to Total Assets	11	12	9	11
Inventory to Current Assets	8	9	8	8
Growth of accounts receivables	5	2	4	4
Growth of Inventory	1	1	3	1
Growth of income from principal business operations	14	14	15	15
Growth of sales	19	17	21	20
Cash to Total Assets	6	11	6	6
Due from Shareholders to Total Assets	4	3	5	5
Due to Shareholders to Total Liabilities	9	7	10	9
Other Fixed Assets to Fixed Assets	21	20	20	21
Other Extraordinary Expenses and Losses to Net Sales	16	18	17	16

Table 6.102. Selected financial ratios for comparing the performances of classifiers (Integrated AHP and MULTIMOORA with Interval-Valued Spherical Fuzzy Sets)

Alternatives (Financial Ratios)	Rank
Growth of Inventory	1
Inventory to Sales	2
Accounts Receivable to Sales	3
Growth of accounts receivables	4
Due from Shareholders to Total Assets	5
Cash to Total Assets	6
Quick ratio	7
Inventory to Current Assets	8
Due to Shareholders to Total Liabilities	9
Inventory turnover	10
Inventory to Total Assets	11
Total asset turnover	12
Current Ratio	13
Net profit margin ratio	14
Growth of income from principal business operations	15

Phase 2: The results of this approach are given in Section 6.2.2.2, since the selected financial ratios are same for two spherical fuzzy sets, which are Interval-Valued Spherical Fuzzy Sets and Single-Valued Spherical Fuzzy Sets.

6.3. Financial Accounting Fraud Detection via Machine Learning

The purpose of this study is to examine how good machine learning classifiers are in detecting financial accounting fraud. The methodology contains two phases. The first phase is called data preparation. The second phase is called evaluating and comparing the performances of classifiers. The first phase involves the calculation of financial ratios, feature selection techniques, and sampling techniques. T-test and Genetic Search are used as feature selection techniques to define financial ratios that have the biggest influence on financial accounting fraud. Following the selection of the best financial ratios, two sampling methods are used: oversampling and undersampling.

In the second phase, seven machine learning classifiers, including Support, Naive Bayes, Artificial Neural Networks, K-Nearest Neighbour, Random Forest, Logistic Regression, and Bagging are performed and compared using performance measures. Without employing any feature selection or sampling techniques, classifiers are also compared.

6.3.1. Algorithm of the application

There are two phases of this study. Data preparation is included in the first phase. Financial ratio computation, feature selection techniques, and sampling techniques are all performed during the data preparation phase. The performances of classifiers are evaluated and compared in the second phase. Performance measures, which are Overall accuracy, TP rate (Sensitivity-Recall), TN rate (Specificity), Precision, G-measure, and ROC curve are calculated to compare the performances of classifiers. Figure 6.18 depicts a schematic diagram of this study methodology.

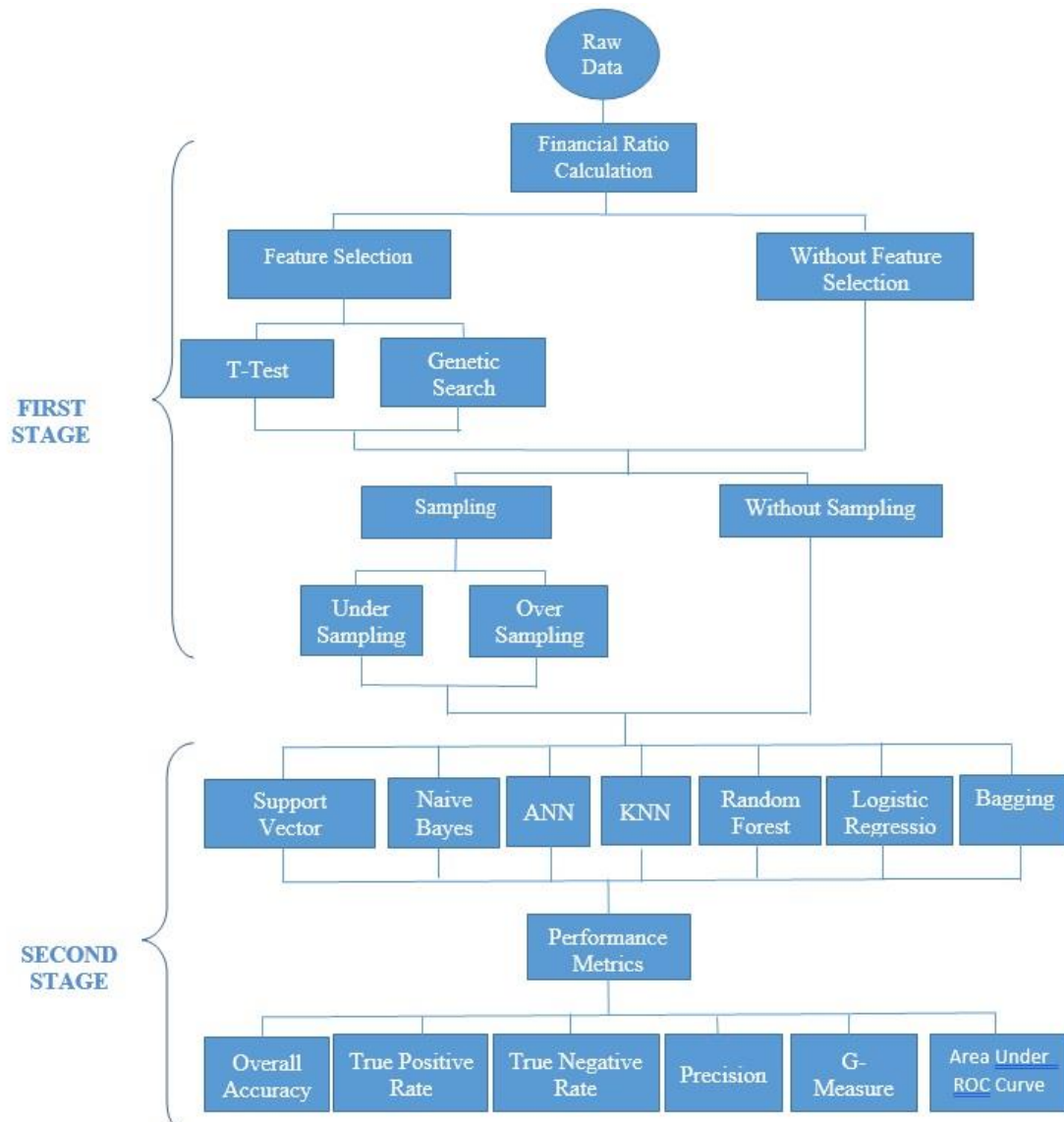


Figure 6.18. A schematic diagram of the proposed methodology (T-test and Genetic search)

6.3.2. Steps of the Application

The steps of proposed approach are as follows:

6.3.2.1. Phase 1

Two selection techniques were employed to decrease dimensionality: T-test and Genetic Algorithm.

T-test

T-test is used as the initial feature selection technique in this section. The T-test is used to determine if the averages of fraudulent and non-fraudulent companies are statistically different. For each financial ratio, the t-statistic values are calculated. The averages, standard deviations, t-values, and p-values for each financial ratio utilized in this study are shown in Table 6.103. As shown in Table 6.103, 15 variables have p-values of less than 0.05 ($p \leq 0.05$).

Debt ratio, "Accounts Payable to Cost of Good Sold", "Total asset turnover", "Cash Conversion Cycle ratio", "Accounts Receivable to Total Assets", "Inventory to Sales", "Inventory to Current Assets", "Return on Assets ratio", "Cash to Total Assets", "Due from Shareholders to Total Assets", "Other Fixed Assets to Total Assets", "Other Extraordinary Expenses and Losses to Net Sales", "Doubtful Trade Receivables to Total Assets", "Prepaid Expenses for Future Months to Total Assets" and "Prepaid Expenses for Future Years to Total Assets" are selected for comparing the performances of classifiers.

Genetic search

"Accounts receivable turnover", "Inventory to Total Assets", "Cash to Total Assets", "Due from Shareholders to Total Assets", "Other Fixed Assets to Total Assets", "Other Extraordinary Expenses and Losses to Net Sales", "Doubtful Trade Receivables to Total Assets", "Prepaid Expenses for Future Months to Total Assets", and "Prepaid Expenses for Future Years to Total Assets" are selected financial ratios by Genetic Search. Table 6.104 presents the selected financial ratios by two examined feature selection techniques.

Table 6.103. Means, Standard Deviations, T-values and P-values for financial ratios

Category	Financial Ratios	Fraudulent Firms		NonFraudulent Firms		t-statistic	
		Mean	Standard deviation	Mean	Standard deviation	t- value	p- value
Liquidity Ratios	Current ratio (V1)	2.43	6.6	2.23	6.37	-0.53	0.599
	Quick ratio (V2)	1.78	5.92	1.37	3.06	-1.73	0.085
	Cash ratio (V3)	0.62	4.34	0.26	1.14	-1.5	0.135
Leverage Ratios	Debt ratio (V4)	0.723	0.373	0.669	0.221	-2.48	0.013
	Debt-to-equity ratio (V5)	4.69	9.63	3.5	24.3	-0.87	0.386
Efficiency Ratios	Inventory turnover (V6)	104	912	214	1955	0.98	0.325
	Accounts Payable to Cost of Good Sold (V7)	0.319	0.413	0.243	0.235	-3.2	0.002
	Total asset turnover (V8)	1.205	0.814	1.57	1.02	6.84	0.000
	Accounts receivable turnover (V9)	61	260	36	604	-0.73	0.466
	Cash Conversion Cycle ratio (V10)	126	217	93	124	-2.63	0.009
	Accounts Receivable to Sales (V11)	0.277	0.301	0.258	0.209	-1.12	0.262
	Accounts Receivable to Total Assets (V12)	0.264	0.223	0.333	0.221	5.04	0.000
	Inventory to Sales (V13)	0.294	0.424	0.224	0.277	-2.79	0.006
	Inventory to Total Assets (V14)	0.225	0.238	0.244	0.200	1.35	0.178
	Inventory to Current Assets (V15)	0.300	0.290	0.345	0.250	2.58	0.010
	Return on Assets ratio (V16)	0.0215	0.0925	0.0346	0.0605	2.42	0.016
	Net profit margin ratio (V17)	0.012	0.133	0.023	0.072	1.43	0.154
	EBIT to Total Assets (V18)	0.076	0.105	0.0835	0.0804	1.27	0.203
Profitability Ratios	Return on Equity ratio (V19)	0.031	0.886	0.098	0.700	1.46	0.144
	Growth of accounts receivables (V20)	1.3	10.6	2.7	65.5	0.38	0.702
	Growth of Inventory (V21)	0.63	2.48	4	55.7	1.07	0.286
Growth Ratios	Growth of income from principal business operations	0.2	21.3	-0.7	19.6	-0.72	0.472
	Growth of sales (V23)	0.181	0.500	0.59	9.00	0.82	0.415
	Growth of Total Assets (V24)	0.0405	0.0765	0.0061	0.0217	-7.96	0.000
Other Ratios	Due from Shareholders to Total Assets (V25)	0.0447	0.0978	0.0021	0.0153	-7.79	0.000
	Due to Shareholders to Total Liabilities (V26)	0.097	0.191	0.083	0.166	-1.26	0.209
	Other Fixed Assets to Total Assets (V27)	0.0076	0.0410	0.0004	0.0062	-3.14	0.002
	Other Extraordinary Expenses and Losses to Net Sales	0.0198	0.0574	0.0050	0.0160	-4.58	0.000
	Due to Shareholders to Average Inventory (V29)	2.9	10.5	17	300	0.84	0.403
	Doubtful Trade Receivables to Total Assets (V30)	0.0112	0.0768	0.0009	0.0037	-2.43	0.016
	Prepaid Expenses for Future Months to Total Assets	0.011	0.030	0.0027	0.0108	-4.92	0.000
	Prepaid Expenses for Future years to Total Assets (V32)	0.0190	0.0422	0.0056	0.0133	-5.62	0.00

Table 6.104. Selected Financial Ratios by Examined Feature Selection Techniques (T-test and Genetic Search)

Methods	Selected Financial Ratios
T-test	V4, V7, V8, V10, V12, V13, V15, V16, V24, V25, V27, V28, V30, V31, V32
Genetic Search	V9, V14, V24, V25, V27, V28, V30, V31, V32

6.3.2.2. Phase 2

The performance metrics of classifiers under various sampling and feature selection methods are summarized in Table 6.105-6.110. A total of 63 models, including 7 classifiers, were formed and compared considering the performance metrics. The best value for each performance measure is highlighted in bold, while the worst value is underlined.

In terms of all performance measures, Random Forest classifier consistently performs the best or the second best among the seven machine learning classifiers, as shown in Tables 6.105-6.110. Particularly, all other models are outperformed by the Random Forest-without feature selection-oversampling model. In terms of True Negative Rate, however, Support Vector Machine has better performance than the Random Forest classifier. In terms of True Negative Rate, Support Vector Machine- Genetic Search - Without Sampling model is the best, with a value of 98.80 percent (Table 6.107).

Random Forest-without feature selection-oversampling model has the best overall accuracy (93.74%), as shown in Table 6.105. Among all other models, Naive Bayes- Genetic Search -Under Sampling model and KNN- Genetic Search -Under Sampling model have the worst performance at 76.48 percent.

Random Forest-without feature selection-oversampling model has the greatest True Positive Rate (94.70%), as shown in Table 6.106. As shown in Table 6.108, Random Forest-without feature selection-oversampling model and the Random Forest-T test-oversampling model have the highest values of 92.50 percent in terms of Precision. In terms of G-measure and ROC curve, Random Forest-without feature selection-oversampling model outperforms all other models, as shown in Tables 6.109 and 6.110.

Table 6.105. Overall Accuracy of classifiers using two feature selection methods (T-test and Genetic Search) and without using any feature selection method

	Feature Selection Methods						Without Feature Selection		
	T-Test			Genetic Search					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.8710	0.8579	0.8022	0.8721	0.8111	0.7726	0.8704	0.8587	0.8131
Naive Bayes	0.8856	0.7999	0.7726	0.8868	0.7882	<u>0.7648</u>	0.8211	0.5881	0.7695
ANN	0.8897	0.8726	0.8364	0.8938	0.8546	0.8069	0.8979	0.8834	0.8287
KNN	0.8938	0.8849	0.7928	0.8774	0.8370	<u>0.7648</u>	0.8944	0.8992	0.7882
Random Forest	0.9196	0.9292	0.8707	0.9161	0.9284	0.8723	0.9179	0.9374	0.8707
Logistic Regression	0.9003	0.8733	0.8380	0.8944	0.8489	0.8411	0.8956	0.8808	0.8536
Bagging	0.9150	0.9037	0.8442	0.9079	0.9055	0.8364	0.9138	0.9127	0.8614

Table 6.106. True Positive Rates of classifiers using two feature selection methods (T-test and Genetic Search) and without using any feature selection method

	Feature Selection Methods						Without Feature Selection		
	T-Test			Genetic Search					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	<u>0.3710</u>	0.7840	0.6730	0.3740	0.6750	0.6010	<u>0.3710</u>	0.7830	0.7040
Naive Bayes	0.6320	0.6560	0.6200	0.6320	0.6280	0.6110	0.7880	0.9380	0.6950
ANN	0.6140	0.8290	0.8040	0.5950	0.7990	0.7320	0.6230	0.8790	0.8350
KNN	0.6570	0.8700	0.7910	0.6290	0.8200	0.7660	0.6570	0.8930	0.7690
Random Forest	0.6920	0.9280	0.8720	0.7230	0.9390	0.8690	0.6600	0.9470	0.8820
Logistic Regression	0.5950	0.8240	0.7730	0.5610	0.7810	0.7760	0.5860	0.8440	0.810
Bagging	0.7130	0.9060	0.8440	0.7070	0.9140	0.8260	0.7200	0.9170	0.8540

Table 6.107. True Negative Rates of classifiers using two feature selection methods (T-test and Genetic Search) and without using any feature selection method

	Feature Selection Methods						Without Feature Selection		
	T-Test			Genetic Search					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.9870	0.9260	0.9310	0.9880	0.9370	0.9440	0.9860	0.9280	0.9220
Naive Bayes	0.9440	0.9340	0.9250	0.9460	0.9370	0.9190	0.8290	<u>0.2640</u>	0.8440
ANN	0.9540	0.9130	0.8690	0.9630	0.9060	0.8820	0.9620	0.8880	0.8220
KNN	0.9490	0.8990	0.7940	0.9350	0.8530	0.7630	0.9490	0.9050	0.8070
Random Forest	0.9730	0.9300	0.8690	0.9610	0.9180	0.8750	0.9780	0.9280	0.8600
Logistic Regression	0.9710	0.9190	0.9030	0.9720	0.9120	0.9070	0.9670	0.9150	0.8970
Bagging	0.9620	0.9020	0.8440	0.9540	0.8980	0.8470	0.9590	0.9080	0.8690

Table 6.108. Precision of classifiers using two feature selection methods (T-test and Genetic Search) and without using any feature election method

	Feature Selection Methods						Without Feature Selection		
	T-Test			Genetic Search					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.8690	0.9080	0.9080	0.8760	0.9090	0.9150	0.8620	0.9100	0.9000
Naive Bayes	0.7250	0.9010	0.8920	0.7300	0.9030	0.8830	<u>0.5160</u>	0.5420	0.8170
ANN	0.7550	0.8980	0.8600	0.7890	0.8880	0.8610	0.7910	0.8790	0.8250
KNN	0.7480	0.8890	0.7940	0.6920	0.8380	0.7640	0.7510	0.8970	0.7990
Random Forest	0.8540	0.9250	0.8700	0.8110	0.9140	0.8750	0.8720	0.9250	0.8630
Logistic Regression	0.8270	0.9040	0.8890	0.8220	0.8920	0.8920	0.8070	0.9020	0.8870
Bagging	0.8120	0.8950	0.8440	0.7830	0.8930	0.8440	0.8020	0.9030	0.8670

Table 6.109. G-measure of classifiers using two feature selection methods (T-test and Genetic Search) and without using any feature selection method

	Feature Selection Methods						Without Feature Selection		
	T-Test			Genetic Search					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.6051	0.8520	0.7916	0.6079	0.7953	0.7532	0.6048	0.8524	0.8057
Naive Bayes	0.7724	0.7828	0.7573	0.7732	0.7671	0.7493	0.8082	<u>0.4976</u>	0.7659
ANN	0.7653	0.8700	0.8359	0.7570	0.8508	0.8035	0.7742	0.8835	0.8285
KNN	0.7896	0.8844	0.7925	0.7669	0.8363	0.7645	0.7896	0.8990	0.7878
Random Forest	0.8206	0.9290	0.8705	0.8335	0.9284	0.8720	0.8034	0.9375	0.8709
Logistic Regression	0.7601	0.8702	0.8355	0.7384	0.8440	0.8389	0.7528	0.8788	0.8524
Bagging	0.8282	0.9040	0.8440	0.8213	0.9060	0.8364	0.8310	0.9125	0.8615

Table 6.110. Area Under ROC Curve of classifiers using two feature selection methods (T-test and Genetic Search) and without using any feature selection method

	Feature Selection Methods						Without Feature Selection		
	T-Test			Genetic Search					
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling
SVM	0.6790	0.8550	0.8020	0.6810	0.8060	0.7730	<u>0.6780</u>	0.8560	0.8130
Naive Bayes	0.8910	0.9080	0.8790	0.8470	0.8660	0.8420	0.8390	0.8580	0.8110
ANN	0.8630	0.9290	0.9060	0.8570	0.9070	0.8680	0.8890	0.9390	0.8980
KNN	0.8050	0.8880	0.8050	0.7850	0.8380	0.7620	0.7980	0.8980	0.7890
Random Forest	0.9470	0.9780	0.9230	0.9300	0.9700	0.9240	0.9490	0.9840	0.9190
Logistic Regression	0.9040	0.9400	0.9040	0.8850	0.9110	0.8810	0.9130	0.9380	0.9050
Bagging	0.9360	0.9610	0.9060	0.9280	0.9510	0.8970	0.9280	0.9690	0.9080

The study also revealed that the Bagging classifier is the second best classifier in terms of overall accuracy, G-measure, and ROC curve among all other classifiers. Random Forest and Bagging classifiers, which are known as ensemble approaches, clearly outperform other classifiers in detecting financial accounting fraud.

Each financial ratio may have a distinct influence on financial accounting fraud detection. In order to find the financial ratios with the highest influence on financial accounting fraud, two feature selection techniques have been employed in this section. As shown in Table 6.104, various financial ratios have been selected by using these two feature selection techniques. The results, however, reveal that feature selection techniques are unable to increase machine learning classifiers' performances for the dataset studied. Because the features of various datasets are taken into account, each financial ratio has a varied influence on financial accounting fraud detection.

In terms of TP rate, Precision, G-measure, and ROC curve, oversampling method (SMOTE) increases the performances of all machine learning classifiers compared to other sampling approaches ('without sampling' and 'undersampling'). SMOTE also improves the Random Forest classifier's overall accuracy. In terms of overall accuracy and TN rate, however, SMOTE and undersampling approaches decrease the performances of other machine learning classifiers.



7. FINDINGS AND DISCUSSION

Prioritization of financial ratios by using MCDM methods with different extensions of fuzzy sets

In this study, Integrated Fuzzy AHP and Fuzzy TOPSIS and Integrated Fuzzy AHP and Fuzzy MULTIMOORA approaches with different extensions of fuzzy sets have been used to prioritize financial ratios.

Table 7.1 indicates final rankings of financial ratios by using integrated MCDM approaches, which are integrated AHP and TOPSIS and integrated AHP and MULTIMOORA, with different extensions of fuzzy sets. Type 1 fuzzy sets and Spherical fuzzy sets (including interval-valued and single-valued fuzzy numbers) have been used with integrated AHP and TOPSIS and integrated AHP and MULTIMOORA approaches. In addition to these two fuzzy sets, Circular intuitionistic fuzzy sets have been used with integrated AHP and TOPSIS approach.

In comparison to integrated AHP and TOPSIS approach, integrated AHP and MULTIMOORA approach gives similar rankings when using different fuzzy sets. For instance, 'Inventory to Sales' is found as the second best financial ratio in financial accounting fraud detection for integrated AHP and MULTIMOORA with SVSFS and IVSFS. This ratio is also ranked third alternative for integrated AHP and MULTIMOORA with Type 1 fuzzy sets. 'Accounts Receivable to Sales' financial ratio is assessed as the third alternative for integrated AHP and MULTIMOORA with SVSFS and IVSFS. This ratio is ranked 4th alternative for integrated AHP and MULTIMOORA with Type 1 fuzzy sets.

It is also concluded that the ranking of alternatives obtained by using integrated AHP and TOPSIS with SVSFS differs considerably from that obtained by other approaches. For instance, 'Growth of Inventory' is ranked 7th for integrated AHP and TOPSIS with SVSFS, but this ratio is ranked the first or the second alternative in financial accounting fraud detection for other approaches including different MCDM method and different fuzzy sets. 'Inventory to Sales' is ranked 11th for integrated AHP and TOPSIS with SVSFS, but this ratio is ranked the first or the second alternative for other approaches.

Another finding of this study is that the ranking of alternatives is very sensitive to the changing fuzzy numbers. Single-valued spherical fuzzy sets and interval-valued spherical fuzzy sets have been employed in order to compare flexibility of the integrated MCDM methods. For this comparison, the same steps of proposed methods have been applied. Results indicate that changing fuzzy numbers may affect the final rankings. For example, the best alternative for integrated AHP and TOPSIS with IVSFS is 'Inventory to Sales', however this ratio is ranked 11th for integrated AHP and TOPSIS with SVSFS. 'Debt-to-equity ratio' is assessed as the first ratio for integrated AHP and TOPSIS with SVSFS, but this ratio is ranked 25th for for integrated AHP and TOPSIS with IVSFS.

Due to the use of different distance measures and defuzzification formulas, different rankings of alternatives may arise. It's also conceivable to argue that as the number of alternatives increases, different fuzzy sets may provide different results. Previous studies have found that both of these spherical fuzzy sets (single-valued and interval-valued) have the same ranking for the identical problem. For example, Gündoğdu and Kahraman (2019a) evaluated the IVSF-TOPSIS and the SVSF-TOPSIS, both of which include four alternatives, and found that the ranking is the same in both approaches.

Similarly, Aydın and Gündoğdu (2020) used SVSF-MULTIMOORA and IVSF-MULTIMOORA to perform a comparative research study for the Industry 4.0 strategy selection problem, in which the number of alternatives is 5, and discovered that both techniques yielded the same ranking for the same problem. Gündoğdu and Kahraman (2019a), as well as Aydın and Gündoğdu (2020), employed the same aggregation operations, score functions, and distance formulas in their studies for spherical fuzzy sets. Our study, on the other hand, gives a different ranking to the same problem. It's reasonable to say that as the number of alternatives increases, different fuzzy sets may provide different results. Furthermore, it is possible to underline that different distance measurement calculations may lead to different ranks. Chen and Tsao (2008) compared interval-valued fuzzy TOPSIS rankings based on different distance measures and number of alternatives. When the number of alternatives is more than 5, the interval-valued fuzzy TOPSIS approach using the different distance measures may provide different preference orders, according to their research. Oztaysi (2014) also showed that the distance measure employed in the computations can affect rankings. Due to

information loss caused by defuzzification steps, different MCDM approaches with different adoption of fuzzy sets may provide different ranking of alternatives.

Table 7.1. Final rankings of financial ratios by using MCDM methods with different extensions of fuzzy sets

Alternatives	AHP+ MULTIMOORA			AHP+TOPSIS			
	IVSFS	SVSFS	TYPE 1	IVSFS	SVSFS	TYPE 1	C-IFS
	Final Ranking	Final Ranking	Final Ranking	Final Ranking	Final Ranking	Final Ranking	Final Ranking
Growth of Inventory	1	4	2	2	7	1	2
Inventory to Sales	2	2	3	1	11	2	1
Accounts Receivable to Sales	3	3	4	3	12	3	4
Growth of accounts receivables	4	5	6	5	8	9	9
Due from Shareholders to Total Assets	5	6	5	4	9	6	5
Cash to Total Assets	6	13	1	10	5	8	15
Quick ratio	7	1	10	6	21	5	14
Inventory to Current Assets	8	7	7	7	14	10	6
Due to Shareholders to Total Liabilities	9	11	8	8	18	7	7
Inventory turnover	10	8	9	9	20	4	12
Inventory to Total Assets	11	10	11	11	19	13	3
Total asset turnover	12	12	16	12	13	12	8
Current Ratio	13	9	14	13	15	11	21
Net profit margin ratio	14	14	13	15	22	17	19
Growth of income from principal business operations	15	15	12	14	24	14	11
Other Extraordinary Expenses and Losses to Net Sales	16	16	21	16	17	24	17
Accounts Receivable to Total Assets	17	17	17	17	4	15	13
Return on Assets ratio	18	18	15	18	10	16	10
EBIT to Total Assets	19	20	18	20	23	20	18
Growth of sales	20	19	20	19	16	19	16
Other Fixed Assets to Total Assets	21	21	19	21	25	23	20
Debt ratio	22	22	24	22	6	18	22
Accounts Payable to Cost of Good Sold	23	23	22	23	2	21	23
Cash ratio	24	25	25	24	3	25	25
Debt-to-equity ratio	25	24	23	25	1	22	24

Comparing performances of classifiers by using financial ratios which are selected through MCDM methods with different extensions of fuzzy sets

The top fifteen financial ratios are determined by using integrated AHP and TOPSIS and integrated AHP and MULTIMOORA approaches with different fuzzy sets for comparing the performances of classifiers in detecting financial accounting fraud. The selected financial ratios are shown in Table 7.2. Classifiers are also compared by utilizing all of 25 financial ratios.

As seen in Table 7.2, 7 financial ratios including Growth of Inventory, Inventory to Sales, Accounts Receivable to Sales, Growth of accounts receivables, Due from Shareholders to Total Assets, Cash to Total Assets, and Inventory to Current Assets are found common when considering all proposed MCDM approaches using different fuzzy sets. Accordingly, Inventory and Accounts Receivable are identified as two financial accounts that may be manipulated the most, as they are included in seven financial ratios. Accounts receivable and Inventory are also found more likely to be manipulated in previous studies (Loebbecke et al., 1989; Persons, 1995; Fanning and Cogger, 1998; Summers and Sweeney, 1998; Spathis, 2002; Birol, 2017).

The top fifteen financial ratios are found to be common in integrated AHP and TOPSIS with IVSFS, integrated AHP and MULTIMOORA with SVSFS, and integrated AHP and MULTIMOORA with IVSFS. However, the rankings of financial ratios differ slightly in these approaches.

Tables 7.3-7.8 present performance measures of classifiers under different sampling methods and different integrated MCDM approaches using different fuzzy sets as feature selection techniques. In all, 147 models comprising 7 classifiers have been developed and performance measures of these models have been compared. The best value for each performance metric is highlighted in bold, while the worst value is underlined.

In terms of all performance metrics, Random Forest classifier consistently performs the best or the second best among the seven classifiers, as shown in Tables 7.3-7.8. Naive Bayes and Support Vector Machines are two of classifiers that perform the worst in all classifiers. When considering the performances of 147 models, it can be concluded that Random Forest-without feature selection-oversampling model outperforms all other models. It means that proposed feature selection methods (integrated MCDM methods

using different fuzzy sets) can't improve the performance of classifiers. However, it may not always be possible to use a large number of financial ratios for detecting fraudulent financial statement. Using a large number of financial ratios may also require long computation time. As a matter of fact, it can be seen that the performance of the models constructed using proposed MCDM methods with different fuzzy sets in detecting fraudulent financial statement is generally good, as shown in Tables 7.3-7.8. For instance, the Random Forest- Integrated AHP and TOPSIS with Single-Valued Spherical Fuzzy Sets-oversampling model is the second best at 90.55 % in terms of overall accuracy (Table 7.3).

Table 7.3 indicates overall accuracy of models, and it is inferred that Random Forest-without-feature-selection-oversampling model has the highest overall accuracy at 93.74 %. Among all other models, Naive Bayes- Integrated AHP and MULTIMOORA with Type 1- Over Sampling model performs the worst at 51.69 %.

Naive Bayes- Integrated AHP and TOPSIS with Type 1-oversampling model has the greatest True Positive Rate (96.30%), as shown in Table 7.4. Support Vector Machines - Integrated AHP and TOPSIS with IVSFS-Without sampling model is the worst at 30.20% in terms of True Positive Rate.

In terms of True Negative Rate, Support Vector Machines outperforms all other classifiers, as shown in Table 7.5. Using proposed MCDM methods with different fuzzy sets improves the performance of Support Vector Machines. It is also found that used sampling methods reduce the performance of all classifiers except Naive Bayes. However, undersampling method improves the performance of Naive Bayes in terms of True Negative Rate, while oversampling method reduces the performance of Naive Bayes.

Table 7.6 shows precision of the models, and it is concluded that Random Forest-without feature selection-oversampling model has the highest values of 92.50 %. Naive Bayes - Integrated AHP and TOPSIS with Type 1-Without sampling model has the worst value of 33.60 %.

In terms of the G-measure, it is shown that proposed feature selection methods (integrated MCDM methods using different fuzzy sets) can't improve the performance of classifiers (Table 7.7). Random Forest-without feature selection-oversampling model is

the best at 93.75 %. Oversampling method improves the performance of classifiers except Naive Bayes classifier. Naive Bayes - Integrated AHP and MULTIMOORA with Type 1-Without sampling model has the worst value of 32.54 %.

Random Forest-without-feature-selection-oversampling model outperforms all other models in terms of Area under ROC curve, as shown in Table 7.8. Oversampling method improves the performance of all classifiers. Support Vector Machines classifier performs the worst with respect to Area under ROC curve.

Each financial ratio may have a distinct influence on financial statements that are manipulated. This study employed two MCDM techniques with different fuzzy sets to identify the financial ratios with the greatest influence on fraudulent financial statements. Integrated AHP and TOPSIS and integrated AHP and MULTIMOORA approaches with different fuzzy sets have been used to determine financial ratios for comparing the performances of classifiers in detecting financial accounting fraud (As can be seen in Table 7.2). The results, however, reveal that used feature selection methods are unable to increase the performance of machine learning classifiers for the dataset studied. Because the properties of various datasets are taken into account, each financial ratio has a distinct impact on the detection of financial accounting fraud.

For all classifiers, oversampling method (SMOTE) produces better TP rates, Precision, G-measure, and ROC curve than the other sampling techniques ('without sampling' and 'undersampling'). SMOTE also improves Random Forest classifier's overall accuracy. In terms of overall accuracy and TN rate, however, SMOTE and undersampling method reduce the performance of other classifiers.

Table 7.2. Selected financial ratios by using Integrated AHP and TOPSIS and Integrated AHP and MULTIMOORA approaches with different fuzzy sets

Final Ranking	AHP+ MULTIMOORA				AHP + TOPSIS			
	IVSFS	SVSFS	TYPE 1	Selected Financial Ratios	IVSFS	SVSFS	TYPE 1	C-IFS
1	Growth of Inventory	Growth of Inventory	Cash to Total Assets	Inventory to Sales	Debt-to-equity ratio	Growth of Inventory	Inventory to Sales	Inventory to Sales
2	Inventory to Sales	Inventory to Sales	Growth of Inventory	Growth of Inventory	Accounts Payable to Cost of Good Sold	Inventory to Sales	Growth of Inventory	Growth of Inventory
3	Accounts Receivable to Sales	Accounts Receivable to Sales	Inventory to Sales	Accounts Receivable to Sales	Cash ratio	Accounts Receivable/ Sales	Inventory to Total Assets	Inventory to Total Assets
4	Growth of accounts receivables	Due from Shareholders to Total Assets	Accounts Receivable/ Sales	Due from Shareholders to Total Assets	Accounts Receivable to Total Assets	Cost of Goods Sold/Average Inventory	Accounts Receivable to Sales	Accounts Receivable to Sales
5	Due from Shareholders to Total Assets	Growth of accounts receivables	Due from Shareholders/ Total Assets	Growth of accounts receivables	Cash to Total Assets	Quick ratio	Due from Shareholders to Total Assets	Due from Shareholders to Total Assets
6	Cash to Total Assets	Quick ratio	Growth of accounts receivables	Quick ratio	Debt ratio	Due from Shareholders/ Total Assets	Inventory to Current Assets	Inventory to Current Assets
7	Quick ratio	Inventory turnover	Inventory to Current Assets	Inventory to Current Assets	Growth of Inventory	Due to Shareholders to Total Liabilities	Due to Shareholders to Total Liabilities	Due to Shareholders to Total Liabilities
8	Inventory to Current Assets	Due to Shareholders to Total Liabilities	Due to Shareholders to Total Liabilities	Due to Shareholders to Total Liabilities	Growth of accounts receivables	Cash to Total Assets	Total asset turnover	Total asset turnover
9	Due to Shareholders to Total Liabilities	Inventory to Current Assets	Cost of Goods Sold/Average Inventory	Inventory turnover	Due from Shareholders to Total Assets	Growth of accounts receivables	Growth of accounts receivables	Growth of accounts receivables
10	Inventory turnover	Total asset turnover	Quick ratio	Cash to Total Assets	Return on Assets ratio	Inventory to Current Assets	Return on Assets ratio	Return on Assets ratio
11	Inventory to Total Assets	Cash to Total Assets	Inventory to Total Assets	Inventory to Total Assets	Inventory to Sales	Current Ratio	Growth of income from principal business operations	Growth of income from principal business operations
12	Total asset turnover	Inventory to Total Assets	Growth of income from principal business operations	Total asset turnover	Accounts Receivable to Sales	Net Sales/ Total Assets	Inventory turnover	Inventory turnover
13	Current Ratio	Current Ratio	Net profit margin ratio	Current ratio	Total asset turnover	Inventory/Total Assets	Accounts Receivable to Total Assets	Accounts Receivable to Total Assets
14	Net profit margin ratio	Net profit margin ratio	Current Ratio	Growth of income from principal business operations	Inventory to Current Assets	Growth of income from principal business operations	Quick ratio	Quick ratio
15	Growth of income from principal business operations	Growth of income from principal business operations	Net Income/Total Assets	Net profit margin ratio	Current ratio	Accounts Receivable/ Total Assets	Cash to Total Assets	Cash to Total Assets

Table 7.3. Overall Accuracy of classifiers under different sampling methods and different integrated MCDM approaches using different fuzzy sets

Overall Accuracy	AHP + MULTIMOORA						AHP + TOPSIS												Without Feature Selection		
	IVSFS + SVSFS			TYPE 1			IVSFS			SVSFS			TYPE 1			C-IFS			Without Sampling	Over Sampling	Under Sampling
	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling	Without Sampling	Over Sampling	Under Sampling			
SVM	86.16 %	77.77 %	72.74 %	86.22 %	79.12 %	73.36 %	86.16 %	77.77 %	72.74 %	86.39 %	77.55 %	73.68 %	86.28 %	78.79 %	74.77 %	86.39 %	78.56 %	75.23 %	87.04 %	85.87 %	81.31 %
Naive Bayes	72.90 %	54.05 %	73.68 %	76.31 %	51.69 %	73.52 %	72.90 %	54.05 %	73.68 %	79.77 %	59.52 %	72.90 %	66.57 %	53.90 %	71.34 %	69.85 %	53.34 %	73.52 %	82.11 %	58.81 %	76.95 %
ANNs	87.39 %	82.20 %	78.50 %	87.80 %	82.80 %	77.10 %	87.39 %	82.20 %	78.50 %	86.74 %	81.86 %	76.32 %	87.74 %	82.68 %	75.39 %	87.45 %	83.73 %	77.10 %	89.79 %	88.34 %	82.87 %
KNN	86.22 %	83.81 %	73.83 %	82.05 %	83.10 %	70.87 %	86.22 %	83.81 %	73.83 %	85.34 %	84.00 %	72.74 %	85.92 %	83.43 %	73.52 %	85.63 %	85.23 %	72.59 %	89.44 %	89.92 %	78.82 %
Random Forest	89.15 %	90.25 %	78.35 %	89.50 %	89.99 %	77.41 %	89.15 %	90.25 %	78.35 %	88.50 %	90.55 %	76.79 %	89.68 %	89.99 %	77.41 %	89.38 %	89.99 %	76.95 %	91.79 %	93.74 %	87.07 %
Logistic Regression	87.16 %	80.25 %	77.26 %	87.10 %	81.15 %	76.64 %	87.16 %	80.25 %	77.26 %	87.16 %	79.61 %	75.08 %	86.80 %	80.10 %	75.55 %	86.80 %	80.85 %	76.01 %	89.56 %	88.08 %	85.36 %
Bagging	89.03 %	87.18 %	75.39 %	89.21 %	87.48 %	76.17 %	89.03 %	87.18 %	75.39 %	88.39 %	85.72 %	75.70 %	88.62 %	87.82 %	75.55 %	88.56 %	87.71 %	76.17 %	91.38 %	91.27 %	86.14 %

Table 7.4. True Positive Rates of classifiers under different sampling methods and different integrated MCDM approaches using different fuzzy sets

True Positive Rate	AHP + MULTIMOORA						AHP + TOPSIS												Without Feature Selection		
	IVSFS + SVSFS			TYPE 1			IVSFS			SVSFS			TYPE 1			C-IFS			Witho ut Sampl ing	Over Sampl ing	Under Sampl ing
	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing			
SVM	30.20 %	59.20 %	50.50 %	30.80 %	62.50 %	51.40 %	30.20 %	59.20 %	50.50 %	31.50 %	60.30 %	53.60 %	30.80 %	62.50 %	55.50 %	31.50 %	61.70 %	56.40 %	37.10 %	78.30 %	70.40 %
Naive Bayes	73.50 %	95.70 %	67.90 %	70.40 %	95.40 %	66.70 %	73.50 %	95.70 %	67.90 %	66.70 %	92.10 %	60.40 %	79.80 %	96.30 %	67.00 %	78.20 %	96.00 %	67.60 %	78.80 %	93.80 %	69.50 %
ANNs	50.80 %	77.60 %	76.30 %	53.00 %	75.40 %	70.40 %	50.80 %	77.60 %	76.30 %	49.80 %	76.60 %	77.90 %	53.60 %	78.70 %	73.20 %	55.10 %	80.30 %	72.00 %	62.30 %	87.90 %	83.50 %
KNN	47.70 %	81.30 %	67.60 %	48.00 %	82.30 %	70.10 %	47.70 %	81.30 %	67.60 %	46.10 %	82.50 %	68.80 %	50.80 %	80.90 %	71.00 %	48.30 %	82.90 %	67.90 %	65.70 %	89.30 %	76.90 %
Random Forest	53.30 %	89.50 %	74.80 %	54.20 %	88.90 %	72.30 %	53.30 %	89.50 %	74.80 %	50.20 %	89.80 %	73.50 %	54.50 %	89.10 %	73.20 %	53.30 %	89.60 %	73.20 %	66.00 %	94.70 %	88.20 %
Logistic Regression	40.20 %	70.00 %	67.60 %	40.50 %	70.80 %	65.40 %	40.20 %	70.00 %	67.60 %	40.50 %	68.60 %	65.10 %	38.90 %	70.60 %	64.50 %	39.30 %	72.00 %	66.40 %	58.60 %	84.40 %	81.00 %
Bagging	52.60 %	85.00 %	68.20 %	52.60 %	84.80 %	70.40 %	52.60 %	85.00 %	68.20 %	50.50 %	82.60 %	67.90 %	51.10 %	85.90 %	69.20 %	51.10 %	86.10 %	69.80 %	72.00 %	91.70 %	85.40 %

Table 7.5. True Negative Rates of classifiers under different sampling methods and different integrated MCDM approaches using different fuzzy sets

True Negative Rate	AHP + MULTIMOORA						AHP + TOPSIS												Without Feature Selection		
	IVSFS + SVSFS			TYPE 1			IVSFS			SVSFS			TYPE 1			C-IFS					
	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing
SVM	99.10 %	95.00 %	95.00 %	99.10 %	94.60 %	95.30 %	99.10 %	95.00 %	95.00 %	99.10 %	93.60 %	93.80 %	99.10 %	93.90 %	94.10 %	99.10 %	94.20 %	94.10 %	98.60 %	92.80 %	92.20 %
Naive Bayes	72.80 %	15.40 %	79.40 %	77.70 %	11.10 %	80.40 %	72.80 %	15.40 %	79.40 %	82.80 %	29.30 %	85.40 %	63.50 %	14.60 %	75.70 %	67.90 %	13.70 %	79.40 %	82.90 %	26.40 %	84.40 %
ANNs	95.90 %	86.40 %	80.70 %	95.90 %	89.70 %	83.80 %	95.90 %	86.40 %	80.70 %	95.30 %	86.70 %	74.80 %	95.70 %	86.40 %	77.60 %	94.90 %	86.90 %	82.20 %	96.20 %	88.80 %	82.20 %
KNN	95.20 %	86.10 %	80.10 %	90.00 %	83.80 %	71.70 %	95.20 %	86.10 %	80.10 %	94.40 %	85.40 %	76.60 %	94.10 %	85.80 %	76.00 %	94.30 %	87.40 %	77.30 %	94.90 %	90.50 %	80.70 %
Random Forest	97.50 %	91.00 %	81.90 %	97.70 %	91.00 %	82.60 %	97.50 %	91.00 %	81.90 %	97.40 %	91.30 %	80.10 %	97.80 %	90.80 %	81.60 %	97.80 %	90.40 %	80.70 %	97.80 %	92.80 %	86.00 %
Logistic Regression	98.00 %	89.70 %	86.90 %	97.90 %	90.80 %	87.90 %	98.00 %	89.70 %	86.90 %	98.00 %	89.80 %	85.00 %	97.90 %	88.90 %	86.60 %	97.80 %	89.00 %	85.70 %	96.70 %	91.50 %	89.70 %
Bagging	97.50 %	89.20 %	82.60 %	97.70 %	90.00 %	81.90 %	97.50 %	89.20 %	82.60 %	97.20 %	88.70 %	83.50 %	97.30 %	89.60 %	81.90 %	97.30 %	89.20 %	82.60 %	95.90 %	90.80 %	86.90 %

Table 7.6. Precision of classifiers under different sampling methods and different integrated MCDM approaches using different fuzzy sets

Precision	AHP + MULTIMOORA						AHP + TOPSIS												Without Feature Selection		
	IVSFS + SVSFS			TYPE 1			IVSFS			SVSFS			TYPE 1			C-IFS			Witho ut Sampl ing	Over Sampl ing	Under Sampl ing
	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing			
SVM	89.00 %	91.70 %	91.00 %	88.40 %	91.40 %	91.70 %	89.00 %	91.70 %	91.00 %	89.40 %	89.70 %	89.60 %	89.20 %	90.40 %	90.40 %	89.40 %	90.80 %	90.50 %	86.20 %	91.00 %	90.00 %
Naive Bayes	38.50 %	51.20 %	76.80 %	42.20 %	49.90 %	77.30 %	38.50 %	51.20 %	76.80 %	47.30 %	54.70 %	80.50 %	33.60 %	51.10 %	73.40 %	36.10 %	50.80 %	76.70 %	51.60 %	54.20 %	81.70 %
ANNs	74.10 %	84.10 %	79.80 %	74.90 %	87.10 %	81.30 %	74.10 %	84.10 %	79.80 %	71.10 %	84.20 %	75.50 %	74.10 %	84.30 %	76.50 %	71.70 %	85.10 %	80.20 %	79.10 %	87.90 %	82.50 %
KNN	69.50 %	84.50 %	77.20 %	52.60 %	82.50 %	71.20 %	69.50 %	84.50 %	77.20 %	65.80 %	84.00 %	74.70 %	66.50 %	84.10 %	74.80 %	66.20 %	85.90 %	74.90 %	75.10 %	89.70 %	79.90 %
Random Forest	83.00 %	90.20 %	80.50 %	84.50 %	90.10 %	80.60 %	83.00 %	90.20 %	80.50 %	81.70 %	90.50 %	78.70 %	85.40 %	90.00 %	79.90 %	84.70 %	89.60 %	79.10 %	87.20 %	92.50 %	86.30 %
Logistic Regression	82.70 %	86.40 %	83.80 %	81.80 %	87.70 %	84.30 %	82.70 %	86.40 %	83.80 %	82.30 %	86.20 %	81.30 %	81.20 %	85.50 %	82.80 %	80.80 %	85.90 %	82.20 %	80.70 %	90.20 %	88.70 %
Bagging	82.80 %	88.00 %	79.60 %	84.10 %	88.70 %	79.60 %	82.80 %	88.00 %	79.60 %	80.60 %	87.10 %	80.40 %	81.60 %	88.50 %	79.30 %	81.20 %	88.10 %	80.00 %	80.20 %	90.30 %	86.70 %

Table 7.7. G-measure of classifiers under different sampling methods and different integrated MCDM approaches using different fuzzy sets

G-measure	AHP + MULTIMOORA						AHP + TOPSIS												Without Feature Selection		
	IVSFS + SVSFS			TYPE 1			IVSFS			SVSFS			TYPE 1			C-IFS			Witho ut Sampl ing	Over Sampl ing	Under Sampl ing
	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing			
SVM	54.71	74.99	69.26	55.25	76.89	69.99	54.71	74.99	69.26	55.87	75.13	70.91	55.25	76.61	72.27	55.87	76.24	72.85	60.48	85.24	80.57
Naive Bayes	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%
	73.15	38.39	73.43	73.96	32.54	73.23	73.15	38.39	73.43	74.32	51.95	71.82	71.18	37.50	71.22	72.87	36.27	73.26	80.82	49.76	76.59
ANNs	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%
	69.80	81.88	78.47	71.29	82.24	76.81	69.80	81.88	78.47	68.89	81.49	76.33	71.62	82.46	75.37	72.31	83.53	76.93	77.42	88.35	82.85
KNN	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%
	67.39	83.67	73.59	65.73	83.05	70.90	67.39	83.67	73.59	65.97	83.94	72.60	69.14	83.31	73.46	67.49	85.12	72.45	78.96	89.90	78.78
Random Forest	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%
	72.09	90.25	78.27	72.77	89.94	77.28	72.09	90.25	78.27	69.92	90.55	76.73	73.01	89.95	77.29	72.20	90.00	76.86	80.34	93.75	87.09
Logistic Regression	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%
	62.77	79.24	76.64	62.97	80.18	75.82	62.77	79.24	76.64	63.00	78.49	74.39	61.71	79.22	74.74	62.00	80.05	75.44	75.28	87.88	85.24
Bagging	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%
	71.61	87.07	75.06	71.69	87.36	75.93	71.61	87.07	75.06	70.06	85.60	75.30	70.51	87.73	75.28	70.51	87.64	75.93	83.10	91.25	86.15

Table 7.8. Area Under ROC Curve of classifiers under different sampling methods and different integrated MCDM approaches using different fuzzy sets

Area Under ROC Curve	AHP + MULTIMOORA						AHP + TOPSIS												Without Feature Selection		
	IVSFS + SVSFS			TYPE 1			IVSFS			SVSFS			TYPE 1			C-IFS			Without Feature Selection		
	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing	Witho ut Sampl ing	Over Sampl ing	Under Sampl ing			
SVM	64.70	77.10	72.70	65.00	78.50	73.40	64.70	77.10	72.70	65.30	76.90	73.70	65.00	78.20	74.80	65.30	78.00	75.20	67.80	85.60	81.30
	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%
Naive Bayes	80.40	84.20	77.20	79.20	82.60	76.10	80.40	84.20	77.20	79.40	82.40	76.40	80.00	84.10	76.70	80.60	84.30	78.50	83.90	85.80	81.10
	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%
ANNs	84.70	89.40	84.50	84.40	89.00	82.30	84.70	89.40	84.50	83.40	89.00	82.80	84.00	89.10	83.40	84.10	89.70	83.30	88.90	93.90	89.80
	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%
KNN	77.60	89.60	79.40	69.30	83.20	71.40	77.60	89.60	79.40	81.70	89.20	77.30	79.20	89.40	77.30	79.60	90.30	77.30	79.80	89.80	78.90
	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%
Random Forest	88.60	96.30	84.90	88.30	96.00	84.20	88.60	96.30	84.90	88.20	96.50	83.90	87.50	96.20	83.30	88.40	96.40	84.20	94.90	98.40	91.90
	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%
Logistic Regression	82.70	87.50	81.20	82.20	87.30	80.60	82.70	87.50	81.20	83.70	87.50	82.00	83.30	88.20	82.00	84.50	88.40	82.90	91.30	93.80	90.50
	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%
Bagging	86.70	93.30	82.80	86.50	93.70	83.00	86.70	93.30	82.80	86.90	92.70	82.20	86.40	93.70	82.50	87.50	93.60	82.30	92.80	96.90	90.80
	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%

Sectoral basis results

It has been revealed that SMEs operating in the Construction sector, manipulate their financial accounts considerably more than firms operating in other sectors (with a rate of 56%). Understatement of expenses is a common fraudulent technique used by firms operating especially in the construction sector. Profitability can be overstated by using this technique, and so these firms may provide credit facilities beyond real borrowing capacities from banks and financial institutions. Based on this finding, it is possible to recommend that there would be reduction in fraudulent activities if governmental regulations and laws would be provided and imposed for auditing of operations in this sector and hence enhancement of efficiencies and effectiveness can be achieved, accordingly.

Accommodation and Food Service is the second sector that mostly manipulate financial statements. Unrecorded sales is a common fraudulent technique used in this sector. This fraudulent activity occurs when an employee sells goods or services to a customer, and collects the customer's payment, but makes no record of the sale. Firms use this method for tax avoidance, and this method is commonly used by Turkish SMEs.

Fraudulent financial statements are also common in the transportation sector (with a rate of 46%). In this sector, prepaid expenses are generally used for overstatement of assets. Prepaid expenses are classified as an asset on a company's balance sheet since the payment represents a benefit that will be used in future periods. However, the company may delay the recognition of some expenses, and this violates the expense recognition principle.

According to our dataset, it is found that fraudulent activities in wholesale and retail trade, manufacturing and agricultural sectors are less than other sectors (construction, accommodation and food service, transportation). However, it should be noted here that this situation may vary according to different datasets.

Financial accounts based results

The findings of this study revealed that manipulated financial accounts may change on a sectoral basis. For instances, Cash(100) and Due from Shareholders (131) are found as the most manipulated financial accounts in the construction sector. It is found that fraudulent firms in the transportation sector mostly manipulated financial accounts of Prepaid Expenses for Future Months (180) and Prepaid Expenses for Future Years(280). In all sectors, the most commonly used financial accounts for financial accounting fraud are found as Cash (100), Due from Shareholders (131), Inventories (15), Due to Shareholders (331), Prepaid Expenses for Future Months (180) and Prepaid Expenses for Future Years (280).

T-test for independent samples was conducted in order to find out whether there is significant difference in averages of manipulated financial accounts between fraudulent firms and non fraudulent firms. The results of the study demonstrated that the difference in averages of manipulated financial accounts is statistically significant between fraudulent firms and non fraudulent firms. This situation shows that the manipulated accounts differ from the normal behaviour and show signs of accounting fraud.

Cash (100) is a part of the current assets section of the balance sheet and contributes to company's net working capital. It has been found that this account is the most manipulated financial account by Turkish SMEs. Expenses and losses, which are not recognized in the period that appear on the income statement, can be hidden in this account. It is likely to say that this account is more susceptible for recording expenses improperly to overstate profitability.

According to our data, it is found that Due from Shareholders (131) is the second most manipulated account by Turkish SMEs. This account is used to record transactions of borrowing funds from the company by a shareholder. However, this account is liable to management manipulation.

Other financial accounts of Inventories (15), Due to Shareholders (331), Prepaid Expenses for Future Months (180) and Prepaid Expenses for Future Years (280) are found as mostly manipulated for fraudulent activities. Fraudulent firms utilize these financial accounts for capitalizing the expenses, understating expenses, overstating assets or other fraudulent techniques.



8. CONCLUSION

SMEs are critical to a country's economic growth. Because SMEs play such an important role in the global economy, many studies on financial accounting fraud detection for SMEs are needed. Furthermore, it should be noted that SMEs rely heavily on the banking sector to bridge credit gaps and get finance. The purpose of this research is to examine financial accounting fraud for SMEs. For a five-year period from 2013 to 2017, 341 Turkish SMEs from different industries (manufacturing, construction, transportation, agriculture, and so on) have been evaluated and analyzed using their specific financial statements. This data are collected from one of the industry's main banks. There are 321 fraud cases and 1384 non-fraudulent cases in the sample. Because two classes in the dataset are significantly imbalanced, sampling methods are especially regarded to be used in order to mitigate the problem of class imbalance. For identifying financial accounting fraud, sampling methods are highly suggested since they provide more insights.

The proposed research methodology of this study consists of two stages. In the first stage, financial ratio calculation and feature selection methods for financial ratios carried out, respectively. To identify the financial ratios having the biggest influence on fraudulent financial statements, integrated MCDM methods with different fuzzy sets, T-test, and Genetic Search are used as feature selection methods. Integrated AHP and TOPSIS and integrated AHP and MULTIMOORA approaches with different fuzzy sets, which are Type1, Spherical and Circular fuzzy sets, are employed as feature selection methods for comparing the performances of classifiers in detecting financial accounting fraud. SFS (including interval-valued and single-valued fuzzy numbers), C-IFS and Type1 fuzzy sets are used to overcome uncertainties in decision-making process of prioritization of financial ratios for financial accounting fraud detection.

This study is the first research which proposes integrated AHP and TOPSIS approach with the newly introduced C-IFS and IVSFS, and integrated AHP and MULTIMOORA approach with IVSFS and SVSFS for handling uncertainties in the decision-making process, and implements proposed approaches to solve the problem of prioritization of financial ratios for financial accounting fraud detection. AHP method using different fuzzy sets is performed to calculate the weights of criteria, and then TOPSIS or

MULTIMOORA method that use different fuzzy sets is applied to obtain the final ranking of financial ratios.

After determination of the best prioritization of financial ratios for detecting financial accounting fraud, two sampling methods of oversampling and undersampling are performed in the second stage. SMOTE is used as oversampling method. Then, seven different classifiers, which are Support Vector Machine, Naive Bayes, Artificial Neural Network, k-Nearest Neighbour, Random Forest, Logistic Regression and Bagging, are executed and compared by using performance metrics. Moreover, classifiers are also compared without using any feature selection and/or sampling techniques.

When considering the rankings of financial ratios, it can be noted that integrated AHP and MULTIMOORA approach using different fuzzy sets gives similar rankings in comparison with integrated AHP and TOPSIS approach using different fuzzy sets. It is found that the ranking of alternatives, which is obtained by using integrated AHP and TOPSIS with SVSFS, is significantly different from that obtained using other approaches. The research also revealed that the ranking of alternatives is quite sensitive to changing fuzzy numbers. To compare the flexibility of the integrated MCDM methods, single-valued spherical fuzzy sets and interval-valued spherical fuzzy sets have been used. The results show that changing fuzzy numbers can have an impact on the final rankings. Different rankings of alternatives may arise from the use of different distance measures and defuzzification formulas. Different MCDM methods using varied adoptions of fuzzy sets may provide different ranking of alternatives due to information loss caused by defuzzification steps. It is also possible to argue that as the number of alternatives increases, different fuzzy sets may provide inconsistent results. Future studies might consider the impact of increasing the number of alternatives on the flexibility of fuzzy sets.

Overall accuracy, TP rate , TN rate, Precision, G-measure and ROC curve have been used as performance metrics for comparing performances of machine learning classifiers. Random Forest classifier, also known as an ensemble method, consistently performs the best or the second best among seven classifiers throughout all considered performance measures. The classification accuracy of class imbalanced data is increased by utilizing ensemble methods. Naive Bayes and Support Vector Machines are also

found as two classifiers that perform the worst in all examined classifiers.

To compare performances of machine learning classifiers in detection of financial statement fraud, 147 models including proposed feature selection methods and sampling methods have been constructed. When 147 models are compared, it is determined that Random Forest-without-feature-selection-oversampling model has the best performance in detection of financial accounting fraud. It means that the proposed feature selection approaches (integrated MCDM methods utilizing different fuzzy sets) can not be able to improve the performance of machine learning classifiers on the dataset under consideration. Because the properties of various datasets are taken into consideration, each financial ratio has a unique impact on financial accounting fraud detection. However, using a large number of financial ratios to detect fraudulent financial statements may not always be viable. Using a large number of financial ratios might take a long time to calculate.

Oversampling method (SMOTE) provides better TP rates, Precision, G-measure, and ROC curve than other sampling techniques ('without sampling' and 'undersampling') for all classifiers. SMOTE significantly enhances the overall accuracy of the Random Forest classifier. As a result, in future researches, SMOTE method for dealing with imbalanced problems is highly recommended. It should be noted that different classifiers may produce better results for different datasets. Because the performance of all classifiers is dependent on the dataset at hand, it is worth noting that random forest classifier may or may not be effective for various datasets. Other machine learning classifiers, rather than random forest, may be more effective in detecting financial accounting fraud for different datasets. Contributions of various financial ratios can also be studied for future directions. Furthermore, several feature selection techniques can be used for identifying the financial ratios that have the most impact on fraudulent financial statements.

Further studies can utilize different integrated MCDM methods (ELECTRE, VIKOR, or PROMETHEE) using different extensions of fuzzy sets for prioritization of financial ratios for financial accounting fraud detection. Moreover, Analytic Network Process (ANP) method using different fuzzy sets can be implemented for identification of the interactions between criteria and sub-criteria. Pythagorean Fuzzy Sets, Neutrosophic Sets and other newly proposed fuzzy sets can be employed to solve this problem. The

findings of these studies can be compared with the results of this study. This research might be expanded by focusing on different occupational groups, such as chartered accountants and independent auditors, to demonstrate the applicability and validity of the proposed approach. Furthermore, for adaptations of different MCDM methods using different fuzzy sets, different defuzzification and distance measures can be applied. In addition to financial accounting fraud detection, proposed methodology can be used as a guideline for practical and theoretical research into other sorts of fraud, such as credit card, insurance, and etc. Furthermore, many businesses with difficulty identifying defects and/or outliers might use proposed approach of MCDM methods with different fuzzy sets.

Overall, the findings show that Turkish SMEs are extremely vulnerable to fraud. In all, 64% of the businesses are determined to be non-fraudulent, whereas 36% are found to be fraudulent. Furthermore, the findings revealed that construction companies manipulate their financial statements more than companies in other industries (with a rate of 56 percent). Cash(100) and Due from Shareholders(131) are identified as the most commonly used financial accounts for fraudulent activities in the construction industry. Cash (100), Due from Shareholders (131), Inventories (15), Due to Shareholders (331), Prepaid Expenses for Future Months (180), and Prepaid Expenses for Future Years (280) are the most commonly used financial accounts for financial accounting fraud in all industries. Financial ratios are mostly used to detect financial accounting fraud. Financial ratios, which include the most widely used financial accounts for financial accounting fraud, can be utilized to determine and select fraud risk indicators. Because this study has not focused on any particular industries, future studies may examine fraudulent companies in certain sectors, such as the construction industry.

To mitigate fraud risks for Turkish SMEs, independent internal and external audit procedures should be adopted. It's crucial to assess how effective the controls are. In the accounting department, effective reporting procedures should be implemented. Employees should be educated on fraud risks, internal controls, the company's code of conduct, and ethical rules. For auditing the effectiveness and efficiency of operations, government regulations and laws can be enhanced.

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