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THE CONTROL OF ROBOT MANIPULATORS AT SINGULAR CONFIGURATIONS

Master Thesis

MARYAM DINI

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ABSTRACT

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AT SINGULARITY CONFIGURATIONS

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Singularities in manipulators are quite complex conditions where the end effector loses its one or more degrees of freedom along or about the singular direction(s). The control of robot manipulator in singular regions and configurations is considered as a remarkable problem since large torques in the joints lead the robot to breakdown. Hence, these singular configurations must normally be avoided, and defining the specified workspace and trajectory is essential for appropriate use of robots. The reduction in the end effector mobility affects its behavior to perform desired tasks in work space.

This study focused on the control of robot manipulator in singular region for performing its desired task. In this order, the dynamic consistency is used to avoid any acceleration and torque of joints which can influence the performance of the end effector. The regular control is applied to show how singularity can affect the manipulator behavior. The potential field controller and singular controller are used to lead the manipulator into the singular region. The potential field algorithm based on attractive forces drew the manipulator toward its singular configuration. The singular controller leads the manipulator into the singular boundary to follow its desired task. At the end, applying the switch control between regular control and singular control helps the manipulator to enter the singular region.

In purpose to design the controllers, it is important to observe the performance of manipulator during the simulation. MATLAB program and V-REP software are used to build the controllers and to monitor the manipulator in singular region. In order to develop the controllers, the essential definitions and the dynamic equations are introduced. Then, the kinematic and dynamic analysis is presented for the PHANTOM Omni manipulator as a RRR planar robot which used in this thesis.

Keywords: Manipulators, Singularity, Artificial Potential Fields.

ÖZET

ROBOT KOLLARININ TEKİL DURUMLARDA KONTROLÜ

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Robot kolarındaki tekillik, uç efektörün tekil yön(leri) boyunca ya da etrafında, bir ve ya daha fazla serbestlik derecesini kaybettiği durumlarda oldukça karmaşıktır. Eklemlere etkiyen yüksek torklar robotun arızalanmasına yol açabileceği için robot kollarının tekil bölgelerde ve yapılandırmalardaki kontrölü dikkate değer bir konudur. Bu nedenle, normal şartlarda tekil yapılandırmalardan kaçınılmalıdır ve belirlenen çalışma alanı ve yörüngenin tanımlanması, uygun robot kullanımı için çok önemlidir. Uç efektör hareketliliğindeki azalma, çalışma alanındaki istenen işleri yerine getirme davranışını etkiler. Bu çalışma, istenilen işi yerine getirmek için tekil bölgedeki robot kolunun kontrolü üzerine odaklanmıştır. Dinamik tutarlılık, eklemlerdeki uç efektörün performansını etkileyebilecek hızlanma ve torklardan kaçınmak için kullanılır. Normal kontrol, tekilliklerin robot kolu davranışını nasıl etkilediğini göstermek için uygulanır. Potansiyel alan kontrolörü ve tekil kontrolör, robot kolunu tekil bölgeye yönlendirmek için kullanılır. Çekici kuvvetler üzerine kurulu potansiyel alan algoritması robot kolunu tekil yapılandırmaya çeker.Tekil kontrolör, robot kolunu istenen işi takip etmesi için tekil sınırlara götürür. Sonunda, normal kontrol ve tekil kontrol arasında geçiş kontrolünün uygulanması, robot kolunun tekil bölgeye girmesine yardımcı olur. Kontrolör tasarımında, robot kolunun performansının simülasyon esnasında gözlenmesi önemlidir. MATLAB programı ve V-REP yazılımı, kontröler yapımında ve robot kolunun tekil alan gözleminde kullanılmıştır. Kontrolörleri geliştirmek için temel tanımlar ve dinamik denklemler oluşturulmuştur. Son olarak, bu tezde RRR düzlemsel robot olarak kullanılan Phantom Omni robot koluna kinematik ve dinamik analiz uygulanmıştır.

Anahtar Kelimeler: Robot Kolları, Tekillik, Yapay Potansiyel Alanlar.

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ABBREVIATIONS

DOF	:	Degree of Freedom
DH	:	Denavit-Hartenberg
RR	:	Revolute-Revolute
V-REP	:	Virtual Robot Experimentation Platform

SYMBOLS

Attractive Potential Field	:	U_{att}
Base Link	:	L_0
Distance between joint and obstacle	:	$q-c$
Distance of influence obstacle	:	Q^*
End effector command vector	:	F_m^*
Gain for motion control in joint space	:	Gp, Gv
Gain for motion control in operational space	:	Kp, Kv
Generalized inverse of Jacobian transpose matrix	:	$J^{T\#}$
Homogeneous transformation matrix	:	T
Identity matrix	:	I
Jacobian matrix	:	J
Jacobian transpose matrix	:	J^T
Joint and end effector positions	:	q, x
Joint torque vector	:	Γ_0
Kinematic energy of Joint space	:	$A(q), \Lambda(x)$
Linear and angular velocities of joints	:	v, ω
Last Link	:	L_n
Manipulator torque vector	:	Γ
Mass matrix	:	M
Moments of inertia of the links	:	I
Operational space	:	F
Parameter to scale the effects of attractive field	:	ξ
Parameter to scale the effects of repulsive field	:	η
Repulsive Potential Field	:	U_{rep}
Set of singular configuration	:	$S(q)$
The vector of centrifugal and Coriolis terms	:	b
The vector of gravitational terms	:	g

1. INTRODUCTION

The term of ″robot″ is used to describe a device that can perceive its environment, make decisions and after all take action. Robots have the potential to improve the safety and efficiency in performing a task compare to human. These abilities and potentials of robots can be provided with the vast investigations and researches in different disciplines. Motion is common for all kinds of robots, both fixed and mobile ones. So motion planning and robot controlling play the vital role in the function of the robot. While motion is considered as an important task in robots, the necessity of motion planning and control of the robot will be become more clear by the accomplishment of the desired tasks under various constrains. Every type of robots is designed for performing specific tasks or multi-task based to its abilities.

In motion control of the robot manipulator, there are one or more configurations which the end effector cannot move or rotate along or about them. It means that the end-effector losses its mobility in these directions, as a result, performing the task in those configurations would not be feasible. In this study the degree of freedom of the end effector becomes less than the manipulator's, and the robot acts as a redundant system in the singular configuration. For a serial arm robot the common singular configuration happens while at least two links are stretching along each other or folding completely, this configuration is known as kinematic singularity.

Kinematic singularity is one of the challenging problems in the manipulator controlling, the main reason for this is that the Jacobian matrix rank collapses along or about the singular direction(s). When this matrix loses its rank(s), it means that the Jacobian determinant for non-redundant manipulator is zero and then the inverse of this matrix will not be existed anymore. Conventionally the inverse of the Jacobian matrix is used for transforming the Cartesian space control to the joint space control. So this method is not sufficient in the neighborhood of singularity due to the high velocity of joints which have significantly affected the end effector performance.

Most of the studies were done to avoid the singular region and configuration; however, avoiding the singular region also needs to detect the singular-free region. Therefore, it is one of the important issues to know where the singular region starts, where the vicinity of the singular boundary is, and which posture is the singular configuration of the robot. The main purpose of this study is to go in the singular region with the aim of performing the desired task along singular direction. It should be considered that high joint velocities and joint torques in the singular boundary and configuration have a substantial impact on the behavior of the end effector for following the desired task.

Instead of avoiding singular configuration, its features and properties can be used in practical purposes. For instance, one of its advantages is that the generated joint torques, which can be used to pull or lift up and down the objects without knowing their weights. Urakobo, Yoshioka, Mashimo and Wan [10] experimentally used this advantage of the singular configuration. They described how the end effector can move the heavy object in the vicinity of the singular configurations since the joint torques produce the efficient kinetic energy.

This thesis describes the singular control of the PHANTOM Omni manipulator since it is not that much dynamically complicated for investigation.

1.1 MOTIVATION AND PROBLEM STATEMENT:

Motion control for manipulator performance is known as a key factor for working with robots. Beside of the robot motion control, force control brings another opportunity to control the robot. This force control gives a new development in robot control approaches such as lower mechanical stiffness and lower weighted robots. The obvious plus points of these improvements are power decreasing and safety enhancement.

Recognition of the manipulator singularities is the first step to control the robot in singular region. One of the unfavorable qualities of Singularity is that it leads to unattainable motion of the end-effector in certain directions. This problem limits joint velocities which may cause to unlimited end-effector forces and torques.

The singular control of the manipulator has been playing a certain role in the recent researches. Avoiding the singular configuration and region directs to separate the area into two regions in terms of regular and singular, in this order, the attempts would be made to maximize the singular-free region. Hence, the existence of a singular configuration of robot manipulators has considerable impact on the manipulator function and control. Eventually, the singular configuration is resulting in large torques or forces on the manipulator arms, then stiffness and regular control algorithms will not work anymore. For almost all robots manipulator, there are singularities in joint space.

To protect robots from singular configuration problems, many works and researches were done to handle the singularities by avoiding them. It means that instead of controlling and dealing with singularities, the desired tasks are planned to be performed out of the singular region. Moreover, the robot control is designed to perform the desired tasks in a regular region not in singular region. This is due to the fact that many limitations and depreciation are existed for robot components. The literatures and researches on the analysis of singularity is significant, but limited knowledge of the singularities consequences and its resulted phenomena are not being satisfied. Among all those different kinds of singularity investigations, the kinematic singularity avoidance is drawn many researchers' attention in this field.

Despite of all limitations included in singularity points, the main purpose of this study is that how to use those outcomes as a good result for reliable control of robot in different regions, and how the smoother transition between singular and regular regions will be held. The approximate boundary of each region can be detected by some related parameters such as condition number. This parameter can help to depict the variation in its amount, especially when the robot goes into the neighborhood or in the boundary of the singular region. But the objective idea is to know how the manipulator can detect these kinds of boundaries and regions through the inverse kinematic approach. It should be mentioned that in this study, the decomposing control of joint space torques and force of operational space is chosen according to Chang and Khatib's paper [2] since it is practical and useful for both redundant and non-redundant robot manipulator.

1.2 CONRIBUTION:

In this thesis the in-depth research is done on the control of the robot in the singular region with regard to the dynamically consistent. The null space motion is decoupled from the end effector motion and it is also based on potential function to generate the motion by the negative gradient of the attractive potential function.

Applying attractive force field leads the manipulator to its goal configuration and makes path planning a little more complicated. To design some objects as obstacles provides the opportunity to observe and analyze the behavior of the manipulator accurately. The manipulator passes through the repulsive potential field and avoids any collision with the obstacles to reach its final position. It helps to provide a better assumption of the potential field application in the null space motion.

Simulating in MATLAB is used in the aim of investigation into RRR planar robot manipulator's singularities. In this simulation decoupling the robot control in operational space and null space directs to better and stable manipulator controlling which is based on dynamic consistency. Detecting the kinematic singularity of the robot and also computing properly the Jacobian matrix are the important parts in recognizing the configurations as the singular configurations.

In the current study a broad overview has been done on joints, control motions in the joint space and force control of the manipulator in operational space. Finding the singular regions of the PHANTOM Omni robot manipulator is essential to apply the switch control between the regular and singular regions. This approach presents the method to enable the manipulator to enter the vicinity and also to go into the singular region with the reliable manner.

The control algorithm is performed in MATLAB in the aim of monitoring the behavior of RRR planar robot under the singular control and to guide the end effector through performing its desired task. The simulation is done in V-REP software according to the dynamic properties of the real robot manipulator.

1.3 RELATED PUBLICATIONS:

This section highlights the researches were carried out the relevant fields of this study, and also briefly explains and reviews some of the related articles.

Khatib [4] worked on performing the end effector task by involving the motion and active force control in operational space. He built the fundamental of the operational space equation regards to the end-effector motion control with respect to the joint force behavior in the null space.

Based on the operational space theory and formulation, Kyong-Sok Chang and Khatib [2] worked on two different types of kinematic singularities for identifying and dealing properly with those singularities. First of all, they decomposed the end-effector control in operational space from the joint control in the null space. The potential function was also used to control the null space motion. This potential field was applied to the calculation of the joint torques, and then this joint torques projected onto the null space.

The decouple control of end effector from joints was achieved by using dynamic consistency between operational space and joint space. They worked on two different types of kinematic singularities of PUMA560.

In Type 1 kinematic singularity, the end-effector motion in the singular direction was controlled directly by the motion in null space. The end-effector motion along the singular direction was controlled by the potential function in the null space. Type 2 of the singularity, joint motion in the null space was controlled the end-effector motion along or about the direction which was perpendicular to singular direction. The singularity problems which were occurred at kinematic singularity were handled by force and motion control in the operational space and the null space of the manipulator. Then this manipulator acted as a redundant mechanism in the neighborhood of singular configuration.

Denny Oetomo and Marcelo Ang Jr. [8] built on Chang and Khatib's work [4]. They began with identifying the singular directions and then eliminating the motion's degenerated components from Jacobian matrix, in order to control the robot in or about the singular direction(s). After that they applied the dynamically consistency to create the inverse Jacobian matrix. They also tried to move into non-feasible directions by tracking the position and orientation error in non-feasible direction with feasible directions. For this purpose, they used the inverse of Jacobian matrix and degeneration of the elements of Jacobian matrix, and then they performed the dynamical consistent inverse of Khatib. It was possible to control the force and motion of a manipulator in the singularity directions.

Caccavale, Chiaverini and Siciliano [5] tried to reach a smooth motion in the neighborhood of kinematic singularity, in this way, they used DLS (Damped Least-Squares inverse of Jacobian matrix) and CLIK (Closed-Loop Inverse Kinematic). Hence, they were able to calculate the joint position, velocity and acceleration as input for obtaining the joint torques. The base of closed loop inverse kinematics is in (pseudo) inverse of the Jacobian matrix of the manipulator. Damp least squares inverse of the Jacobian matrix was used for getting numerical robustness. A kinematic control presets for a six-joint industrial manipulator by transforming the end-effector position, velocity and acceleration into joint trajectories. This aim was gained from the

CLIK (second-order algorithm) and DLS (Damp least squares inverse) to control the stable motion in the vicinity of kinematic singularities.

In the other study [9], virtual joints were used to avoid collapsing the Jacobian rank in the neighborhood of singular configuration. This method handles the problems of switching control between two regions in and out of singularity by maintaining the rank of the Jacobian matrix. The benefit of using the extra joints in this method is that the workspace does not require to be divided. In addition, the motion and force control across the singularities is smoother. It should be considered that the end effector cannot do the task in the degenerated direction(s), it only prevents the excessive joint rates.

Urakubo, Yoshioka, Mashimo and Wan [10] used two-link robot arms in the vicinity of singular configuration to pull or lift up an unknown object regardless of its mass. They have a benefit from the relatively small joint torque. This method is not implemented on more than three-link manipulator based on dynamically consistent. The singular configuration of two-linked robot is used to make the unknown object to follow the desired trajectory. In the singular configuration the joint angle excessively accelerates regardless of the object weight. On the other side, the small joint torques on the singular configuration creates a large amount of kinetic energy. They used this excessive kinetic energy to pull and lift-up and down of heavy object by two-link manipulator. The joint torques depend on the joint acceleration, but their directions were related to the length of links.

In-depth study of the singularities of robot kinematics started in the1980s, especially for the serial manipulator. However, these studies could not compensate the lack of defining general framework for singularities. Until 1998 [13] the singular configuration was defined in general frame, FIKP (Forward Instantaneous Kinematic Problem) and IIKP (Inverse Instantaneous Kinematic Problem) which were used for computing the total configuration input or output velocities. The singularity definition was proposed by Zlatanov. This definition was more general than those earlier types of singularities, such as type I/II. He works to determine the singularity in specific configuration, not only in real robot, but also in theory. This computation needs to know all singular configurations for better controlling and path planning of the robot.

The difficult part of finding the exact set of configuration and singularity configurations is a great approach for singularity analyzing. One approach which is used by some researchers [6] is to give the specific configuration and then check and determine it is singular position or not. Maybe it is possible for determining some local singular configurations. But due to the lack of direct solution for the inverse kinematics, computing the global structure of singularities is not achievable.

There are other studies which are used geometric algebra for analyzing kinematic singularities of manipulators. It is a common concept that the singularity relates to some specific configurations of joints or links. For instance, when two jointed links become aligned (stretched out or folded), singularity will occur. This kind of situation can provide simple algebraic equations.

1.4 TERMINOLOGY:

Some terms have a specific meaning in the robotics context, they are briefly mentioned here:

DOF (Degree Of Freedom): motion freedom of the robot which make it able to move transitionally(x, y and z axis) and rotationally (pitch, yaw and roll).

Manipulator: An arm-like mechanism which its links are connected serially with sliding or joints connections.

End-effector: The robot device is connected to the last arm of the robot to perform specific task and also depends on the expected task and the function of the robot.

Configuration: A complete position specification of each point of physical robot.

Configuration space/ Joint space/ Null space: The space of all joints' possible position specification.

Cartesian Space: The normal Euclidean space which the end-effector operates there.

Operational space/ Task space/ Work space: The space robot's end-effector performs its task, with the same concepts of Cartesian space.

Forward kinematic: The mapping from joint space to operational space.

Inverse kinematic: The mapping from operational space to joint space.

Redundant manipulator: The manipulator has more degree of freedom than it's needed for executing the task.

1.5 OUTLINE:

This section includes a brief description of each chapter embodies in this study, with description of contributions and the related works. The structure of this thesis is as follows:

Chapter 2 ″ Operational Space Control of Manipulators″ presents a good vision of fundamental and theory concepts which are needed to tackle with the robot manipulator motion, operational forces and joint torques. In Chapter 3 ″ Potential Field Simulation″ the theoretical concepts and the related algorithms are used to perform potential function and the behavior of the manipulators on the effects of potential forces. In chapter 4 ″ Phantom Omni Manipulator″ reviews and demonstrates the kinematic and dynamic equations of the Phantom Omni manipulator which are used in this study. In chapter 5 ″ Simulation and Results″ the fundamental researches in this thesis presented by the simulation of manipulator in MATLAB coding and V-REP to depict the end effector behavior under the control algorithms which are applied to reach the desired tasks. Finally, in the chapter 6 ″ Conclusion″ the process and approaches which are took place in this study is briefly explained.

2. OPERATIONAL SPACE CONTROL OF MANIPULATOR

In this section before going through the problem some background materials which are used in the rest of this thesis, are presented briefly. The descriptions are more focused on serial manipulators.

2.1 KINEMATIC MODEL:

In the kinematic model, the manipulator has motion without including the effect of the force that causes it. The relationship between dimensions, joints, links position, velocity and acceleration in the manipulator is known as the robot kinematic. The kinematic is the function of joint position and orientation. Generally it is the relationship between the robot behavior in static space and the control parameters which defined the motion without the impact force on it [1].

In kinematic problem of all kinds of robots, there are both Forward Kinematic and Inverse Kinematic, which later will be explained them more. In the Forward (direct) Kinematic, the joint variables such as position and velocity and acceleration link to compute the end-effector variables respected to the base frame. But in the Inverse Kinematic the work space variables (end-effector's position, velocity and acceleration) leads to calculate the joint space variables.

The joint motion in the joint space causes the change at the end-effector motion which includes position/rotation in the task space. Every manipulator joint is numbered from 0 to n so it has related links:

$$X = f(q) = f(q_0, q_1, q_2, ..., q_n) \tag{2.1}$$

The set of configuration $q = [q_0, q_1, q_2, ...q_n]^T$ is expressed the joint space. X is defined the task space includes the rotation and position of the end-effector. Each robot manipulator with n joints has n+1 links, joint j_i connects two links together, its previous links l_{i-1} with its next link l_i, generally L_0 is named for base link and L_n defines the last link.

2.1.1 Forward and Inverse Kinematic:

Forward Kinematic equations help to access end-effector position and orientation from the joint angles.

$$X = f(q) = \begin{bmatrix} p \\ \Phi \end{bmatrix} \tag{2.2}$$

X is the tool frame position and orientation and f is a nonlinear function. p is the positional motion of the end effector and φ is the rotational motion of the end effector. The tool frame is defined by the reference frame in the base of the robot.

In the inverse kinematics, the reverse process is used to achieve the joint angle from a specific position of end-effector. In the manipulator kinematic system the configuration of the end-effector is the function of manipulator joint coordinate, x= f (q). The time derivative of end-effector configurations $\dot{X}$ is the function of $\dot{q}$ joint velocity multiplied J which is the Jacobian matrix.

$$\dot{X} = J(q)\dot{q} \tag{2.3}$$

In order to gain end-effector desired position, it requires determining the associated joint angles of manipulator through the inverse kinematic method. The forward kinematic is computed from the relevant equations (2.2) and (2.3) for required joint configurations $\theta_1, \theta_2, ..., \theta_n$, which makes it possible to get the desired result from the robot end-effector.

Many manipulators decompose the inverse kinematic problem into two simple one: For getting position variable, inverse kinematic used as: (x, y, z)→(q1,q2,q3) and for orientation: R→(q4,q5,q6).

2.1.2 Jacobian Matrix:

A robot Jacobian matrix computes the inter-relation between the velocities of task space and joints space. It means that joint velocities can calculate the linear and angular velocities of work space end-effector.

Generally Jacobian matrix columns individually can express the effect of each joint on the end-effector position derivative. So the end-effector linear or rotational velocity is the composition of joints angles' derivatives:

$$\dot{X} = \begin{bmatrix} v \\ \omega \end{bmatrix} = J(q)\dot{q} \tag{2.4}$$

By presenting n joints and their linear and angular velocities, the Jacobian can be calculated such as follows matrix:

$$J = \begin{bmatrix} \frac{\partial x}{\partial q_1}, \ldots, \frac{\partial x}{\partial q_n} \\ . \\ . \\ . \\ \frac{\partial \omega_z}{\partial q_1}, \ldots, \frac{\partial \omega_z}{\partial q_n} \end{bmatrix} \tag{2.5}$$

In the above matrix first three rows compute linear velocities in each joint and the last three rows express angular velocities.

Jacobian determines a relationship between the end-effector velocity (Cartesian velocity) and the joint velocity.

The higher derivative is calculated by the differentiation of the equation (2.3):

$$\ddot{X} = J(q)\ddot{q} + \dot{J}(q)q \tag{2.6}$$

The velocity of the Jacobian can be used in many cases such as the relationship between the applied force *F* and torque on the end-effector with joint torques τ :

$$\tau = J^T(q)F \tag{2.7}$$

2.1.3 Singular configuration:

The Jacobian is compiled by the joints variable and the rank of the Jacobian which is min (6, n); n is the number of joints. If the Jacobian loses its rank it is known as the manipulator is in singularities, hence these configurations are called Kinematic Singularities. The singularity configurations are important due to some main reasons:

The manipulator loses its capability of motions in some configurations and the end-effector is not able to move arbitrary in some directions. In the vicinity of the singular configuration a small velocity in operational space can generate a large velocity in joint space. The kinematic singularities may be without solution or with infinite solutions.

Generally the singularity is the configuration where the manipulator end-effector cannot move at or in the workspace boundary when two or more link is lined-up. The kinematic singularity is the set of singular configurations:

$$S(q) = s_1(q).s_2(q)...s_n(q) \tag{2.8}$$

At the singularity the Jacobian determinant for non-redundant manipulators becomes equal to zero: $|J| = 0$ so $J^{-1}(q)$ does not exist.

Those components of Jacobian matrix which are in the direction of singularity are eliminated so the rank of this matrix is less than its full rank. There are different types of singularities, maybe more than one, which the joint angles excessively accelerate and large kinematic energy generated from joint torques. When the Cartesian movement is near the singularity, it causes joint large velocity $\dot{q} = J^{-1}(q)\dot{X}$.

Most of the robot manipulators have one or more singularity position, so it is very important how to deal with singularity configuration. So depends on using the singularity or avoiding from it, it is important considering the accurate trajectory. Motion control should be planned whether to avoid from or go through singularities, because near this singular configuration possibly there are many solutions or no solution for the inverse kinematics problem.

Singularities occur inside the boundary of the workspace or on the border of the work space when the motions of two or more links are aligned in the workspace area. On the border of task space when the manipulator arms stretch out or fold, it can generate singularity. Generally, when there is a change in the expected or typical degree of freedom of a robot manipulator configuration it can be recalled as kinematic singularity. This definition will be explained in more details, especially for the rank of Jacobian matrix when there is a change in the joint variables as input and end-effector position as the output.

In purpose to denote the kinematic singularities characters in brief:

a. Sometimes a manipulator controlling in work space goes into difficulties since mapping from joint space to work space has a problem. This problem for robot positioning is named as singularities.

b. At the singular configuration the mobility of the manipulator is losing and the motion of the end-effector in work space lost. This problem is known as losing a degree of freedom.

c. A common term in singularities is singularity boundary which is the same with work space boundary. It is required to reach to maximum work space by stretching the manipulator links to move into the maximum area as work space area.

d. For the internal singularities may be infinite solutions. A small work space motion may need lots of joint velocities which cause problems.

e. By the determinant of the Jacobian matrix of the manipulator, singular position and configuration could be found.

2.2 DYNAMIC MODEL:

Dynamic models of the robot describe the relationship between the motion and associated force and torques of the end-effector. During the motion the kinematic forces of static status turn into inertial, centrifugal and Coriolis forces that have a large effect on the performance of the end-effector, especially when the motion speed and acceleration exceed.

In the operational space the end-effector equation of motion is:

$$\Lambda(x)\ddot{x} + \mu(x,\dot{x}) + p(x) = F \tag{2.9}$$

Where $\Lambda(x)$ is the kinetic energy matrix, $\mu(x, \dot{x})$ represents the end-effector centrifugal and Coriolis forces, p(x) is the gravity force vector and F is the vector of operational force [4].

The motion equation of manipulator in joint space is:

$$A(q)\ddot{q} + b(q,\dot{q}) + g(q) = \Gamma \tag{2.10}$$

Where A represents the kinetic energy of joint space, b is centrifugal and Coriolis and g is gravity matrix respectively.

From the motion equation of manipulator in joint space (2.10), the inverse dynamic equation can be computed as:

$$\ddot{q} = A^{-1}(q)(\Gamma - (b(q,q^{\bullet}) + g(q))) \tag{2.11}$$

The relationship between each similar component of motion equation in operational space and joint space can be found as followings:

$$\Lambda = (J(q)A^{-1}(q)J^{T}(q))^{-1} \tag{2.12}$$

This above equation shows the kinetic energy matrix of operational space and joint space.

The relationship between $b(q,\dot{q})$ and $\mu(x,\dot{x})$ in operational space and joint space respectively is:

$$\begin{aligned} \mu(x,\ddot{x}) &= J^{-T}(q)b(q,\ddot{q}) - \Lambda(q)h(q,\ddot{q}) \\ h(q,\ddot{q}) &= \dot{J}(q)\dot{q} \end{aligned} \tag{2.13}$$

The equation (2.13) depicts the relationship between the gravity forces in operational space and joint space one:

$$p(x) = J^{-T}(q)g(q) \tag{2.14}$$

By using all equations (2.9), (2.10), (2.11), (2.12) and (2.14), the established equation between F and Γ is written as:

$$J^{-T}(q)[A(q)\ddot{q} + b(q,\dot{q}) + g(q)] = F \tag{2.15}$$

And the main relationship between end-effector and the manipulator dynamic equation is represented in the following equation:

$$\Gamma = J^{T}(q)F \tag{2.16}$$

This equation is the basis of manipulator control in operational space. Where F is known as end-effector operational force and Γ is the manipulator joint force.

2.2.1 Motion Control in Operational Space:

The operational control of the end-effector is directly depends on the joint space control of the manipulator. In this regards there are many approaches to establish the safe and stable control between these two systems. One of these practical approaches which Khatib [4] used in his method is a nonlinear dynamic decoupling approach. The structure of this approach is decoupling the end-effector controlling as below:

$$F = F_m + F_{ccg} \tag{2.17}$$

$$\begin{aligned} F_m &= \hat{\Lambda}(x)F_m^* \\ F_{ccg} &= \hat{\mu}(x,\dot{x}) + \hat{p}(x) \end{aligned} \tag{2.18}$$

As could be seen, $\hat{\Lambda}(x), \hat{\mu}(x,\dot{x}), \hat{p}(x)$ are the estimation of kinetic energy matrices, centrifugal and Coriolis forces and also gravity force. F_m^* is the end-effector command vector in this approach. With this fully exploit of nonlinear dynamic approach, the end effector becomes like a unit mass.

For performing the desired task of end-effector in operational space, it is necessary to have the desired acceleration and velocity and position of it. So the command vector of the decoupled end-effector is obtained as:

$$F_m^* = I_{m0}\ddot{x}_d - k_p(x - x_d) - k_v(\dot{x} - \dot{x}_d) \tag{2.19}$$

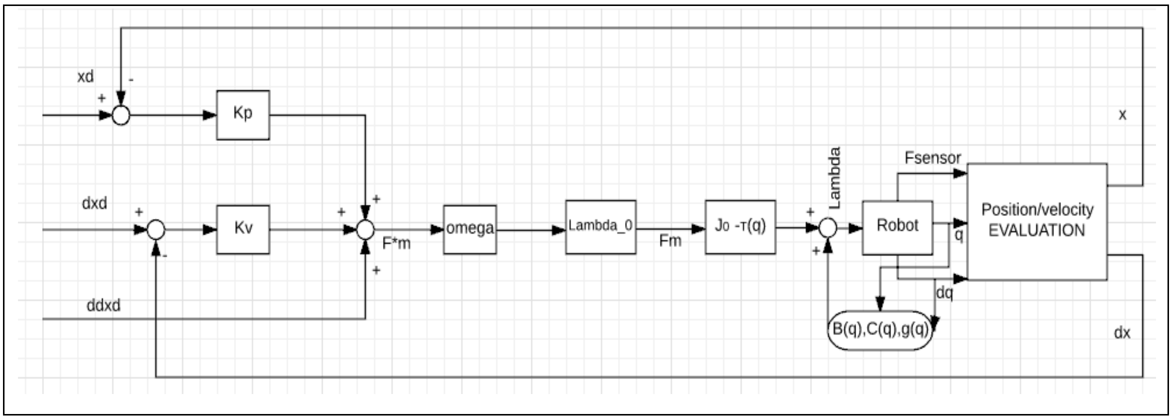
Where I_{m0} is a m_0 by m_0 Identity matrix, k_p and k_v are gain matrices.

If command vector equation (2.18) substituted into (2.16), the joint force control vector is computed as follows:

$$\Gamma = J^T(q)\Lambda(q)F_m^* + \tilde{B}(q)[q\dot{q}] + \tilde{C}(q)[\dot{q}^2] + g(q) \tag{2.20}$$

$\tilde{B}(q)$ & $\tilde{C}(q)$ are the dynamic coefficient of isolating end-effector. It means that they are joint forces under the mapping of end-effector Coriolis and centrifugal forces in the joint spaces [11]. In Figure (2.1) the motion controller diagram is illustrated clearly.

Figure 2.1: End – effector motion controller diagram



2.2.2 Dynamic Consistency:

Decoupling of the manipulator motion in the null space from motion in operational space is one of the approaches to obtain the dynamic consistency of manipulator [4]. It means that any null space torque changes should not create any acceleration in operational space.

In this achievement the relationship between joint torques and operational force is as follows:

$$\Gamma = J^T(q)F + [I - J^T(q)\bar{J}^T(q)]\Gamma_0 \tag{2.21}$$

$\bar{J}$ is dynamic consistency which generalized Jacobian matrix inverse:

$$\bar{J} = A^{-1}(q)J^T(q)\Lambda(q) \tag{2.22}$$

As explained in this chapter, q is joint coordinate vector, A is a kinetic energy matrix in joint space, J is Jacobian matrix, Λ is kinetic energy matrix in operational space and F is operational space control forces act to end-effector. $[I - J^T(q)\bar{J}^T(q)]$ is the projection on null space and Γ_0 is torques of joint control for desired movement in null space.

The (2.21) relationship shows the decomposition of joint torques into two decoupling dynamic controls. $J^T(q)F$ joint torque, which is related to active force on the end-effector and the second part $[I - J^T(q)\bar{J}^T(q)]\Gamma_0$ joint torques that has affected the joint motion in null space.

The main purpose is the maintenance of the end effector motion in task space by operational forces which controlled the end-effector position, while the manipulator has motion in null space. The null space torque (Γ_0) is produced by the negative gradient of the attractive potential function (∇V_0); $\Gamma_0 = -A(q)(gradV_0)$, A is a weight to account for the manipulator dynamic. In the next chapter, potential field is applied to RR planar robot in MATLAB.

3. POTENTIAL FIELD SIMULATION

One of the approaches for path planning of manipulators is potential field, especially for robots with the representation of obstacles in their task spaces. The potential field is similar to the electrostatic field which is assigned to obstacles and the manipulator has motion in the field of forces. The desired position is attractive pole for the end-effector and obstacles surfaces are repulsive force for manipulator parts, links and joints [3].

3.1 INTRODUCTION:

Treating the configuration of the robot in potential field with the combination of attracting to the specific goal and repulsing obstacles is an approach with advantages and disadvantages. The less computing trajectory process would be considered as its advantage and the probability of getting stuck in local minima in this field and losing the path could be remarked as disadvantages of this approach. The goal position of the end-effector is considered as an attractive potential field and the objects or obstacles cause collisions with manipulator parts are known as repulsive fields. The tool or end-effector is drawn to the goal position or direction by attractive force and the joints avoid from collisions by repulsive forces which come from obstacles. If the attractive force and the repulsive forces become balanced, it means their resultant force equals zero, so no progress will be occurred in the manipulator motions. The desired result would be the end-effector motion toward the goal configuration while the manipulator parts try to avoid any collision with the objects scattered in the joint pace. The attractive force decreases while the end-effector approaches to its final configuration as a goal position, and repulsive forces increase when robot parts are near the collision.

U is known as a field which includes an additive field consisting of one component that attracts the manipulator to q final and the other component that repels the manipulator from the boundary of the configuration space obstacle.

$$U(q) = U_{att}(q) + U_{rep}(q)$$

$$F(q) = -\nabla U(q) = -(\nabla U_{att}(q) + \nabla U_{rep}(q)) \qquad \left(U_{rep}(q) = \sum_{i=1}^{n} U_{repi}(q) \right) \tag{3.1}$$

U_{att} is the attractive potential which moves toward the goal and U_{rep} is the repulsive potential which avoids obstacles. Energy is minimized by force as the negative gradient of the potential energy function is applying on the manipulator, $-\nabla U(q)$.

In the attractive field, the field grows linearly with the distance (conic well potential) when the goal position is near $(\|(d(q)-d^*(q_{goal})\|)$ and grows quadratically with the distance (parabolic well potential) while the end-effector is very far from its final configuration $(\frac{1}{2}\xi_i \|d(q)-d(q_{goal})\|^2)$.

$$U_{att}(q) = \begin{cases} \frac{1}{2}\xi d^2(q, q_{goal}), & d(q, q_{goal}) \le d^*_{goal} \\ d^*_{goal}\xi d(q, q_{goal}) - \frac{1}{2}\xi (d^*_{goal})^2, & d(q, q_{goal}) > d^*_{goal} \end{cases} \tag{3.2}$$

$$\nabla U_{att}(q) = \begin{cases} \xi(q - q_{goal}), & d(q, q_{goal}) \le d^*_{goal} \\ \dfrac{d^*_{goal}\xi(q - q_{goal})}{d(q, q_{goal})}, & d(q, q_{goal}) > d^*_{goal} \end{cases} \tag{3.3}$$

d is the distance while moves from conic to parabolic well potential, q is joint configuration, q_{goal} is the joint goal configuration and ξ is the parameter to scale the effects of attractive field.

When the obstacles are very far from the robot parts, they do not have an influence on the manipulator, as a result, the repulsive potential field will be equal to zero. In the definition of repulsive potential field boundary, if this condition $(d_i(q)) \le Qi^*$ is met, obstacles will influence the manipulator.

$$U_{repi}(q) = \begin{cases} \frac{1}{2}\eta\left(\frac{1}{d_i(q)} - \frac{1}{Q_i^*}\right)^2, & if \quad d_i(Q) \le Q_i^* \\ 0, & if \quad d_i(Q) > Q_i^* \end{cases} \tag{3.4}$$

$$\nabla U_{repi,j}(q)=\begin{cases}\eta_j(\frac{1}{Q_i^*}-\frac{1}{d_i(r_j(q))})\frac{1}{d_i^2(r_j(q))}, & d_i(r_j(q))\le Q_i^* \\ 0, \quad d_i(r_j(q))>Q_i^* & \end{cases} \tag{3.5}$$

$$\nabla d_i(q)=\frac{q-c}{d(q,c)} \tag{3.6}$$

Where q-c is the distance between joint and obstacles, c is the point in the boundary of the obstacle which is nearer to q. d(q) is the shortest distance between joint configuration and a configuration space obstacle boundary, Q* is the distance of influence of an obstacle. η is a scalar gain coefficient that depicts the repulsive filed influence. The gradient of the distance to the nearest obstacle is ▽d (q).

In this chapter, field of attractive force and repulsive forces are applied on planar two-link manipulators. In MATLAB programing the potential field control algorithm is applied to see the behavior of this planar manipulator. Moving towards the goal position and avoiding any obstacle collision generates a specific trajectory for manipulator, which affected by the artificial potential field U. Potential field should be computed for every q, as a manipulator configuration.

e.g., if $q(x,y)\in R^2$ and gradient of U at q is $\nabla U(q)=\begin{bmatrix}\frac{\partial U}{\partial x}\\ \frac{\partial U}{\partial y}\end{bmatrix}$ then:

$$|\nabla U|=\sqrt{(\frac{\partial U}{\partial x})^2+(\frac{\partial U}{\partial y})^2} \tag{3.7}$$

$$F(q)=-\nabla U_{att}(q)-\nabla U_{rep}(q) \tag{3.8}$$

When q goes far from its goal configuration the attractive potential force increases for drawing it to the desired position, while q is near the environment of an obstacle the repulsive force field from the surface of that obstacle repels q to avoid any collision.

3.2 RR PLANAR ROBOT SIMULATION:

In this section, the potential functions are applied on 2-arm planar manipulator. The behavior which is expected from this manipulator is to avoid any collision with the obstacle surface and keep moving to get the target configuration.

The concepts and equations which are used in this section are briefly described for better understanding of manipulator behavior. In this simulation both arm length and mass are equal to unit of values.

$$F_m^* = ddx_d + K_{Vm} dE_m + K_{pm} E_m \tag{3.9}$$

In the computing of the command vector F_m^* equation, ddx_d is the desired acceleration and K_{Vm}, K_{pm} are gains for velocity and position control respectively. Error in the position (E_m) and error in the velocity (dE_m) are as below:

$$\begin{aligned} E_m &= x_d - x \\ dE_m &= dx_d - dx \end{aligned} \tag{3.10}$$

The operational space dynamic equation includes the inverse of Jacobian matrix and kinetic energy matrix from the equation $J^{-1}(q) = \Lambda(x) J^T(q) A^{-1}(q)$.

The end effector speed is according to the equation (2.3), includes joint 1and 2 angle velocities.

The forward kinematic equation for this 2R-planar robot is:

$$\begin{aligned} x &= l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) \\ y &= l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) \end{aligned} \tag{3.11}$$

So the Jacobian matrix can be calculated as:

$$J = \begin{bmatrix} -l_1 s_1 \; - \; l_2 s_{12}, & -l_2 s_{12} \\ l_1 c_1 \; + \; l_2 c_{12}, & l_2 c_{12} \end{bmatrix} \tag{3.12}$$

And the mass matrix, centrifugal and Coriolis and also gravity matrix are all computed by:

$$M_q = \begin{bmatrix} m_2 l_2^2 + 2m_2 l_1 l_2 \mathrm{c}_2 + (\mathrm{m}_1 + \mathrm{m}_2) l_1^2, & m_2 l_2^2 + m_2 l_1 l_2 \mathrm{c}_2 \\ m_2 l_2^2 + m_2 l_1 l_2 \mathrm{c}_2, & m_2 l_2^2 \end{bmatrix} \tag{3.13}$$

$$C_q = \left[-m_2 l_1 l_2 s_2 dq_1^2 - 2m_2 l_1 l_2 s_2 dq_1 dq_2 ; m_2 l_1 l_2 s_2 dq_1^2 \right] \tag{3.14}$$

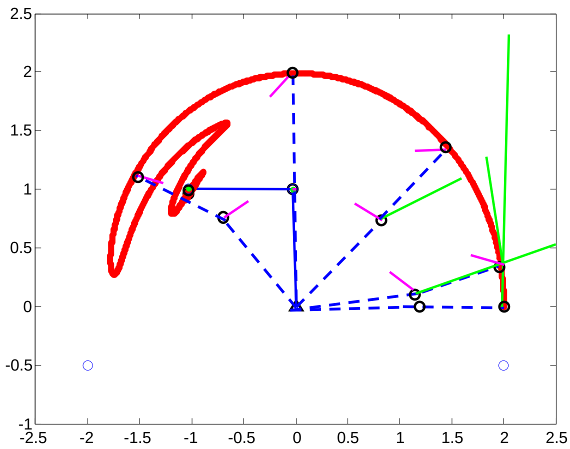
$$G_q = \left[m_2 l_2 g c_{12} + (\mathrm{m}_1 + \mathrm{m}_2) l_1 g c_1 ; m_2 l_2 g c_{12} \right] \tag{3.15}$$

Now the potential force for joints 1and 2 are computed individually. The joint space torque is calculated by the sum of both joints torques:

$$\Gamma = \Gamma_1 + \Gamma_2 \tag{3.16}$$

The following Figure (3.1) is depicted path and behavior of 2R planar robot for reaching target position, in the influence of repulsive and attractive potential field. It is clearly illustrated the repulsive forces how affect every individual joint for the purpose of avoiding the collision. The forces from the final configuration attract the joints to place in their desired positions.

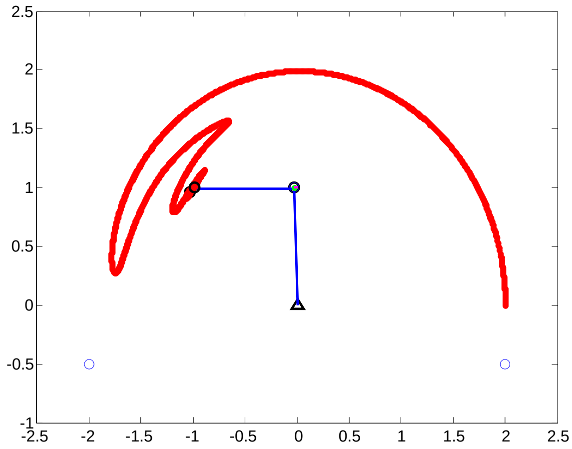
Figure 3.1: 2R Repulsive and Attractive Potential Field



In the color print, Figure (3.1) shows attractive forces in green line and repulsive forces in violet line and the red line shows the path which the end effector follows its desired position.

As it can be followed from Figure (3.2) that the RR manipulator starts to move from the initial position [0, 1] to a the goal position of joints [0, 1] for first joint, and for second joint moves from [0, 2] as the initial position to a goal position [-1,1]. The obstacles are located in [2, -0.5] and [-2, -0.5] positions. As depicted, the motion of each joint is influenced by the potential forces, in the way to lead the manipulator to its desired joint configuration.

Figure 3.2: 2R Planar Manipulator Final Position in Potential Field



4. PHANTOM OMNI MANIPULATOR

The PHANTOM Omni is a 6R haptic device as shown in Figure (4.1). It has three drive motors which are attached to its three first joints. These motors can only give the position of end effector which is set by the computer. The first three joints are used for positioning the end effector and the rest three joints are used for finding the orientations. The last three joints can be considered as spherical joints due to their intersection in one point. In this study the first three joints are used.

In this thesis, the PHANTOM robot is controlled in the purpose of following its desired task in the singular region. The end effector requires both motion and force control for representing its accurate task and performance in operational space.

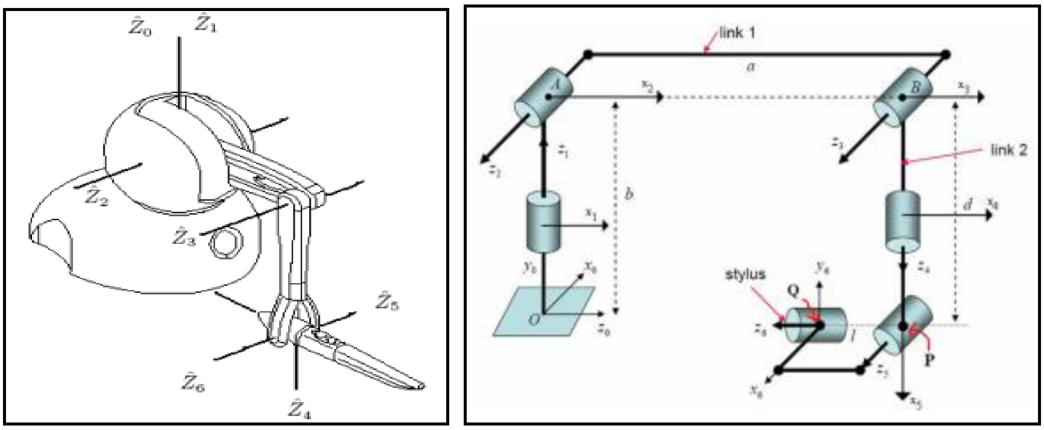
In order to achieve the dynamic consistency in PHANTOM Omni, decoupling the null space motion from operational space motion is necessary. This can be achieved by computing the kinematic and dynamic equations and the relationship between operational forces and joint torques.

Figure 4.1: PHANTOM Omni manipulator

4.1 FORWARD KINEMATICS

For computing the kinematics of PHANTOM as the others manipulator the frames should be defined at first. Figure (4.2) shows the frames fixed on PHANTOM.

Figure 4.2: PHANTOM with frames axes



Source: Beckman, John Albert, the phantom Omni, UMI Number: 1447474.Figure 3.2

Denavit–Hartenberg method can be used with four parameters of joint positions and orientations.

DH parameters express the position vector of an arbitrary point on the link or joint of manipulator respect to its base, it is a standard method to determine link and joint position. A brief expression of each parameter is defined as below:

a_i : Distance between (Z_i, Z_{i+1}) along X_i

α_i : Angle between (Z_i, Z_{i+1}) about X_i

d_i : Distance between (X_{i-1}, X_i) along Z_i

θ_i : Angle between (X_{i-1}, X_i) about Z_i

Denavit-Hartenberg parameters can be written in various ways for each robot. It depicts the position of each link regarding the position of previous link from the base frame to the end-effector frame.

The base frame is called frame 0 which is attached to the ground and the end effector frame is defined as frame 4. By using the DH parameters as Table (4.1), it is easy to describe the architecture of the PHANTOM Omni robot manipulator.

Table 4.1: PHANTOM Omni DH table

I	α_i	a_i	d_i	θ_i
0-1	*0*	*0*	*0*	θ_1
1-2	*-π/2*	*0*	*0*	θ_2
2-3	*0*	*l1*	*0*	θ_3
3-4	*0*	*l2*	*0*	*0*

The transformation matrices refer to each frame are calculated as follows:

$$
{}^0_1T = \begin{bmatrix} \cos\alpha_1 & -\sin\alpha_1 & 0 & 0 \\ \sin\alpha_1 & \cos\alpha_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^1_2T = \begin{bmatrix} \cos\alpha_2 & -\sin\alpha_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\alpha_2 & -\cos\alpha_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^2_3T = \begin{bmatrix} \cos\alpha_3 & -\sin\alpha_3 & 0 & l_1 \\ \sin\alpha_3 & \cos\alpha_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^3_4T = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.1}
$$

The end effector homogeneous transformation matrix with respect to base frame is given as:

$$
{}^0_4T = {}^0_1T\,{}^1_2T\,{}^2_3T\,{}^3_4T \tag{4.2}
$$

$$
{}^{0}_{4}T = \begin{bmatrix} \cos\alpha_1\cos(\alpha_1+\alpha_2) & -\cos\alpha_1\sin(\alpha_1+\alpha_2) & -\sin\alpha_1 & l_2\cos\alpha_1\cos(\alpha_2+\alpha_3)+l_1\cos\alpha_1\cos\alpha_2 \\ \sin\alpha_1\cos(\alpha_2+\alpha_3) & -\sin\alpha_1\sin(\alpha_2+\alpha_3) & \cos\alpha_1 & l_2\sin\alpha_1\cos(\alpha_2+\alpha_3)+l_1\sin\alpha_1\cos\alpha_2 \\ -\sin(\alpha_2+\alpha_3) & -\cos(\alpha_2+\alpha_3) & 1 & l_1\sin\alpha_2-l_2\sin(\alpha_2+\alpha_3) \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.3)
$$

x,y and z coordinate of the end-effector are as below:

$$
\begin{aligned}
x &= l_2\cos\alpha_1\cos(\alpha_2+\alpha_3)+l_1\cos\alpha_1\cos\alpha_2 \\
y &= l_2\sin\alpha_1\cos(\alpha_2+\alpha_3)+l_1\sin\alpha_1\cos\alpha_2 \\
z &= l_1\sin\alpha_2-l_2\sin(\alpha_2+\alpha_3)
\end{aligned} \quad (4.4)
$$

The Jacobian matrix of PHANTOM Omni computed as following:

$$
J = \begin{bmatrix} -\sin\alpha_1(l_1\cos\alpha_2+l_2\cos(\alpha_2+\alpha_3)) & -\cos\alpha_1(l_1\sin\alpha_2+l_2\sin(\alpha_2+\alpha_3)) & l_2\cos\alpha_1\sin(\alpha_2+\alpha_3) \\ -\cos\alpha_1(l_1\cos\alpha_2+l_2\cos(\alpha_2+\alpha_3)) & -\sin\alpha_1(l_1\sin\alpha_2+l_2\sin(\alpha_2+\alpha_3)) & l_2\sin\alpha_1\sin(\alpha_2+\alpha_3) \\ 0 & -l_1\cos\alpha_2-l_2\cos(\alpha_2+\alpha_3) & l_2\cos(\alpha_2+\alpha_3) \end{bmatrix} \quad (4.5)
$$

4.2 DYNAMICS

In this study, it is important to know the dynamic of the PHANTOM Omni manipulator in the purpose of applying the regular and singular control on RRR serial manipulator. In this section, a dynamic analysis for Omni is presented based on the Newton-Euler algorithm [12].

In the Newton-Euler formulation, the equations describe linear motion and angular motion for each link of the manipulator. Of course, since each link is coupled to other links and between them there are action and reaction forces. So these equations of each link contain coupling forces and torques. By computing the inward and outward iterations it is possible to determine all terms of each link as velocity, acceleration, force and torque.

The following equations are used for outward and inward iterations to compute the joint torques and forces and calculate the links velocities and accelerations. Outward iteration (joint 0 to 2) computes link velocities and accelerations, and by using inward iteration (joint 3 to 1), link forces and torques are determined.

Outward Iteration:

$$^{i+1}\omega_{i+1} = {}^{i+1}_{1}R\,{}^{i}\omega_i + \dot{q}_{i+1}\,{}^{i+1}z_{i+1} \tag{4.6}$$

$$^{i+1}\dot{\omega}_{i+1} = {}^{i+1}_{1}R\,{}^{i}\dot{\omega}_i + {}^{i+1}_{i}R\,{}^{i}\omega_i \times \dot{q}_{i+1}\,{}^{i+1}z_{i+1} + \ddot{q}_{i+1}\,{}^{i+1}z_{i+1} \tag{4.7}$$

$$^{i+1}\dot{v}_{i+1} = {}^{i+1}_{1}R({}^{i}\dot{\omega}_i \times {}^{i}P_{i+1} + {}^{i}\omega_i \times ({}^{i}\omega_i \times {}^{i}P_{i+1}) + {}^{i}\dot{v}_i) \tag{4.8}$$

$$^{i+1}\dot{v}_{ci+1} = {}^{i+1}\dot{\omega}_{i+1} \times {}^{i}P_{ci+1} + {}^{i+1}\omega_{i+1} \times ({}^{i+1}\omega_{i+1} \times {}^{i+1}P_{ci+1}) + {}^{i+1}\dot{v}_{i+1}) \tag{4.9}$$

$$^{i+1}F_{i+1} = m_{i+1} \times {}^{i+1}\dot{v}_{ci+1} \tag{4.10}$$

$$^{i+1}N_{i+1} = {}^{c_{i+1}}I_{i+1}\,{}^{i+1}\dot{\omega}_{i+1} + {}^{i+1}\omega_{i+1} \times {}^{c_{i+1}}I_{i+1}\,{}^{i+1}\omega_{i+1} \tag{4.11}$$

Inward Iteration:

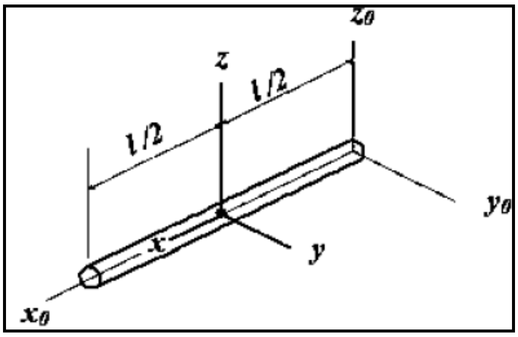
$$^{i}f_i = {}^{i}_{i+1}R^{i+1\,i+1}f_{i+1} + {}^{i}F_i \tag{4.12}$$

$$^{i}n_i = {}^{i}N_i + {}^{i}_{i+1}R^{i+1\,i+1}n_{i+1} + {}^{i}P_{c_{i+1}} \times {}^{i}F_i + {}^{i}P_{i+1} \times {}^{i}_{i+1}R^{i+1\,i+1}f_{i+1} \tag{4.13}$$

$$\tau_i = {}^{i}n^{T}{}_{i}\,{}^{i}z_i \tag{4.14}$$

The dynamic parameters of the PHANTOM Omni are calculated by considering each link in cylindrical shape which the diameter of every link is less than its length. The gravity center of each link is the distance between the origins of link frame to the link center of gravity.

Figure 4.3: Inertia Calculation



By assuming link 2 and 3 as slender rods with cylindrical diameter which is shown in Figure (4.3), their moments of inertia can be calculated by equation (4.15). Ixx is zero since the x axis is a symmetry axis.

$$I_{ci} = \begin{bmatrix} I_{xx_i} & 0 & 0 \\ 0 & I_{yy_i} & 0 \\ 0 & 0 & I_{zz_i} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & ml^2/3 & 0 \\ 0 & 0 & ml^2/3 \end{bmatrix} \tag{4.15}$$

For the first link, moment of inertia is computed by assuming it as a solid sphere. Center of mass coordinates is shown in equation (4.16), where *m* is the mass of the link and *l* is the length.

$$P_{C1} = \{0,0,0\}, P_{C2} = \{l_1/2,0,0\}, P_{C3} = \{l_2/2,0,0\} \tag{4.16}$$

In order to assume the center of mass is too close to the first link rotation axis, its mass location is considered as zero.

The dynamic equation of motion for the Omni robot is derived regarding to each link inertial parameters. The mass matrices (M), centrifugal and Coriolis matrix (b) and gravity (g) one are calculated in the form of the equation (4.17).

$$\begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} + \begin{bmatrix} 0 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \end{bmatrix} \tag{4.17}$$

Each element of mass, centrifugal – Coriolis vector and gravitational vector is calculated as followings. Assume that ci = cos(qi), si = sin(qi), cij = cos(qi + qj)and sij = sin(qi + qj),(i =1, 2, 3 and j =1, 2, 3) anddq(1) = q(4), dq(2) = q(5), dq(3) = q(6).

$$\begin{aligned}
m_{11} \;=\; & (1/8)(8\left(s_2^2\right)I_{xx2} \;+\; 8\left(s_{23}{}^2\right)I_{xx3} \;+\; 4I_{yy2}\left(1+c_{2_2}\right) \;+ \\
& 4I_{yy3}\left(1+c_{2_23}\right) \;+\; 8I_{zz1} \;+\left(l_1{}^2\right)m_2\left(1 \;+\; c_{2_2}\right) \;+ \\
& 4\left(l_1{}^2\right)m_3\left(1+c_{2_2}\right) \;+\; 4l_1l_2m_3\left(c_3+c_{22_3}\right) \;+\; \left(l_2{}^2\right)m_3\left(1+c_{2_23}\right)) \\
m_{22} \;=\; & (1/8)\left(2\left(4I_{zz2}+4I_{zz3}+l_1{}^2m_2+4m_3l_1{}^2+4c_3l_1l_2m_3 \;+\; \left(l_2{}^2\right)m_3\right)\right) \\
m_{23} \;=\; & (1/8)2\left(4I_{zz3} \;+\; l_2m_3\left(2c_3l_1 \;+\; l_2\right)\right) \\
m_{32} \;=\; & (1/4)\left(4I_{zz3} \;+\; l_2m_3\left(2c_3l_1+l_2\right)\right) \\
m_{33} \;=\; & (1/4)\left(4I_{zz3}+m_3\left(l_2{}^2\right)\right)
\end{aligned} \tag{4.18}$$

$$h = b + g$$

$$\begin{aligned}
h_1 \;=\; & 1/8(2q_{(4)}(q_{(5)}(4s_{2_2}\left(I_{xx2} \;-\; I_{yy2}\right) \;+\; 4s_{2_23}(I_{xx3} \;-I_{yy3}) \\
& -\; s_{2_2}\left(l_1^2\right)\left(m_2 \;+4m_3\right) \;-\; 4s_{22_3}l_1l_2m_3 \;-\; s_{2_23}\left(l_2^2\right)m_3))+ \\
& 4s_{23}q_{(4)}q_{(6)}\left(c_{23}\left(4I_{xx3} \;-\; 4I_{yy3} \;+l_2^2m_3\right)-2l_1l_2m_3c_2\right))
\end{aligned} \tag{4.19}$$

$$\begin{aligned}
h_2 \;=& 1/8(\left(q_{(4)}^2\right)(4s_{2_2}\left(I_{yy2} \;-\; I_{xx2}\right) \;+\; 4s_{2_23}\left(I_{yy3} \;-\; I_{xx3}\right) \;+ \\
& s_{2_2}\left(l_1^2\right)\left(m_2 \;+\; 4m_3\right) \;+\; 4s_{22_3}l_1l_2m_3 \;+\; s_{2_23}\left(l_2^2\right)m_3) \;- \\
& 4\left(gc_{23}l_2m_3 \;+l_1gc_2m_2 \;+\; 2l_1gm_3c_2\right) \;+\; 8q_{(5)}q_{(6)}s_3l_1l_2m_3 \;+\; 4l_1l_2m_3s_3q_{(6)}{}^2)
\end{aligned}$$

$$\begin{aligned}
h_3 \;=\; & 1/4(-2l_2m_3gc_{23} \;+\; 2s_3q_{(5)}^2l_1l_2m_3 \;-\; s_{23}\left(q_{(4)}^2\right)(4c_{23}\left(I_{xx3}-I_{yy3}\right) \;- \\
& -l_2m_3\left(2c_2l_1+c_{23}l_2\right)))
\end{aligned}$$

4.3 SINGULAR CONTROL OF OMNI MANIPULATOR

The robot manipulator in singular configuration behaves as a redundant robot which means that the rank of the Jacobian collapses by eliminating the row along the direction of the degenerated direction.

First of all, for applying the singular control, it is required to compute the singular Jacobian matrix of the PHANTOM Omni. This Singular Jacobian is built on the singular frame of the end effector respects to the base frame of the manipulator. It means that the singular frame could be different from the end effector frame. To be more direct, the singularity can be occurred not exactly along or about of end effector axis. Therefore, it is essential to calculate the rotation matrix of singular frame respects to the base frame. This rotation matrix is used to find the Jacobian matrix of the singular configuration.

Each link rotation matrix can be extracted from the transformation matrix of that link.

$$ {}^{0}_{S}R = {}^{0}_{1}R\,{}^{1}_{2}R\,{}^{2}_{3}R\,{}^{3}_{S}R \tag{4.20}$$

${}^{3}_{S}R$ is a rotation matrix where the singular frame is different from the end effector's frame. The singular frame could be found from the singular configurations by computing $|J| = 0$. In this study, the singular rotation matrix of PHANTOM Omni is an identity matrix, which means that the singular frame and end effector's frame are the same.

The singular rotation matrix is used to compute the singular Jacobian matrix J_S as below:

$$ \boldsymbol{J}_S = {}^{0}_{S}\boldsymbol{R}\boldsymbol{J} \tag{4.21}$$

In this study, J is the full-ranking Jacobian matrix of the robot, x direction is the singular direction and the components of the Jacobian matrix along this direction are eliminated. So $\boldsymbol{J}_{SR}$ is J_S with the components of the matrix along y and z directions.

The operational space inertia matrix from equation (2.12) is determined as:

$$\Lambda(x) = J_{SR}^{-T}(q)A(q)J_{SR}^{-1}$$

And then dynamically consistent inverse of the singularity Jacobian is computed as:

$$J^{-1}(q) = \Lambda(x)J_{SR}^{T}(q)A^{-1}(q)$$

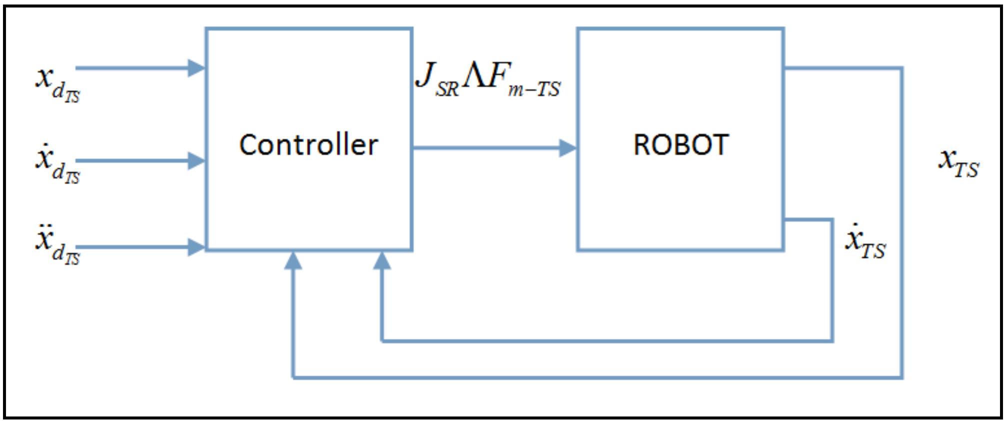
"y" and "z" coordinates are in task space which x is in null space. Task space's desired velocity and acceleration are zero, but null space's desired velocity and acceleration are derived from the desired motion equation along x direction.

Total torque is based on the dynamical consistency of the operational forces and null space torques for this robot in singularity region is $\Gamma = J_{SR}^{T}\Lambda F_{m-TS} + [I - J_{SR}^{T}\bar{J}^{T}]\Gamma_0$.

Figure (4.4) depicts the motion control of end effector along a desired direction in the task space. The controller compares the actual end effector position and velocity in task space with the desired end effector position and velocities. The components along degenerated direction(x) are controlled in null space and the components along y and z direction are controlled in operational space.

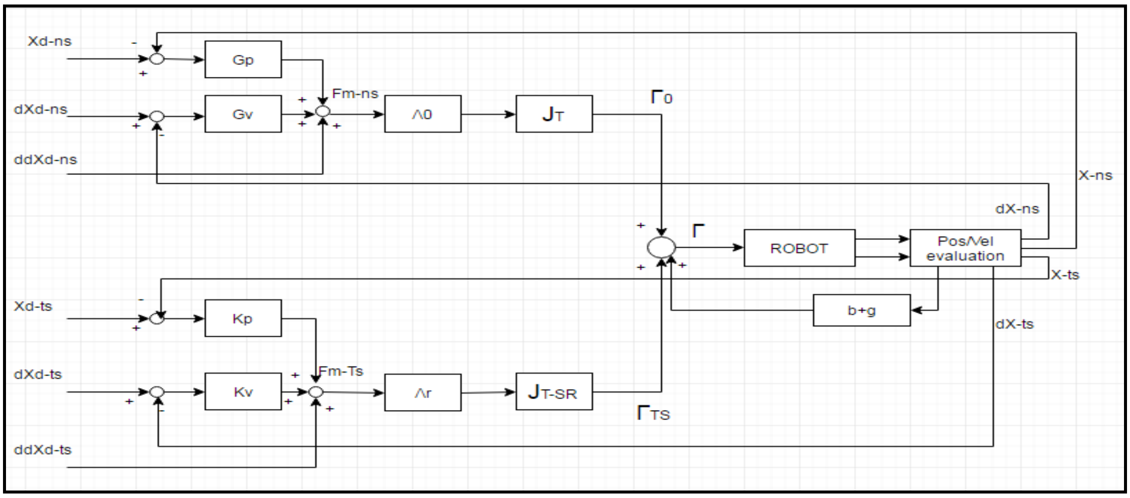
As explained, the Jacobian matrix in this controller is the singular Jacobian matrix which includes the rows along two task space directions y and z. It should be considered that the first row along x direction as the singular direction is eliminated.

Figure 4.4: Task space motion control



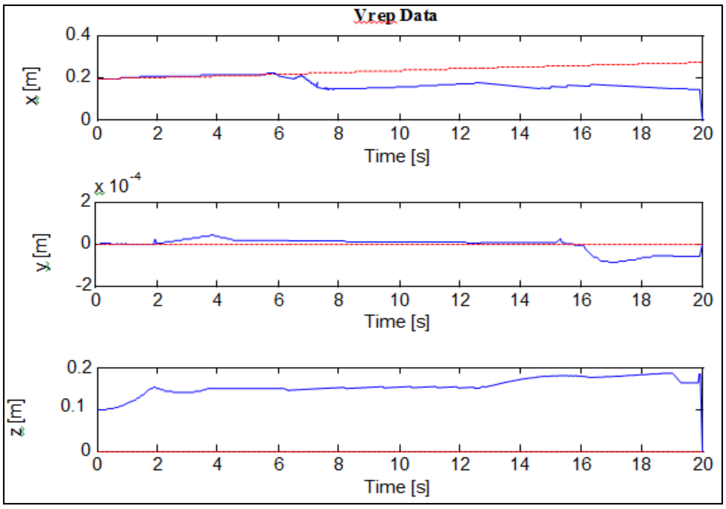
The control diagram as shown in Figure (4.5) demonstrates decomposition of motion control in the joint space from the operational space motion control. This decoupling is based on achieving the dynamically consistent behavior of the manipulator. It means that the null space motion control torques should not have any alteration in the end effector function.

Figure 4.5: Singular control of manipulator



The null space torque was eliminated from the control algorithm in order to investigate the effect of null space control on the singular controller. From the Figure (4.6) can be obviously seen that the robot was not able to follow its desired direction. It means that the robot was unable to enter the singular region, and before the singular boundary it started to have unstable motion. Therefore, the singular control is necessary to lead the robot to its desired task along the singular direction, from the vicinity of singular boundary into the singular region.

Figure 4.6: Eliminating Null Space Torque from Singular Control

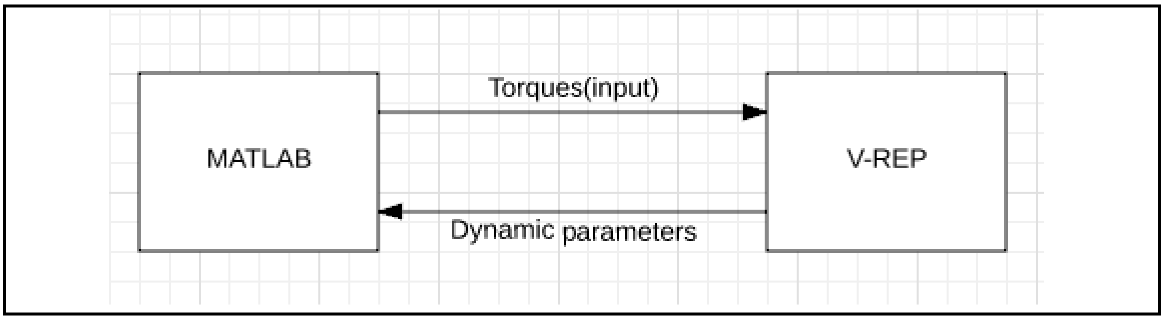


5. SIMULATION AND RESULTS

In order to observe the performance of controllers in the vicinity of singular boundary and also in the singular region, MATLAB and V-rep (Virtual Robot Experimentation Platform) are used for simulation. V-rep as a manipulator simulator [7] can be run and communicate with the other coding language via remote API programming (Application Interface). MATLAB sends and receives data to and from V-rep as shown in Figure (5.1).

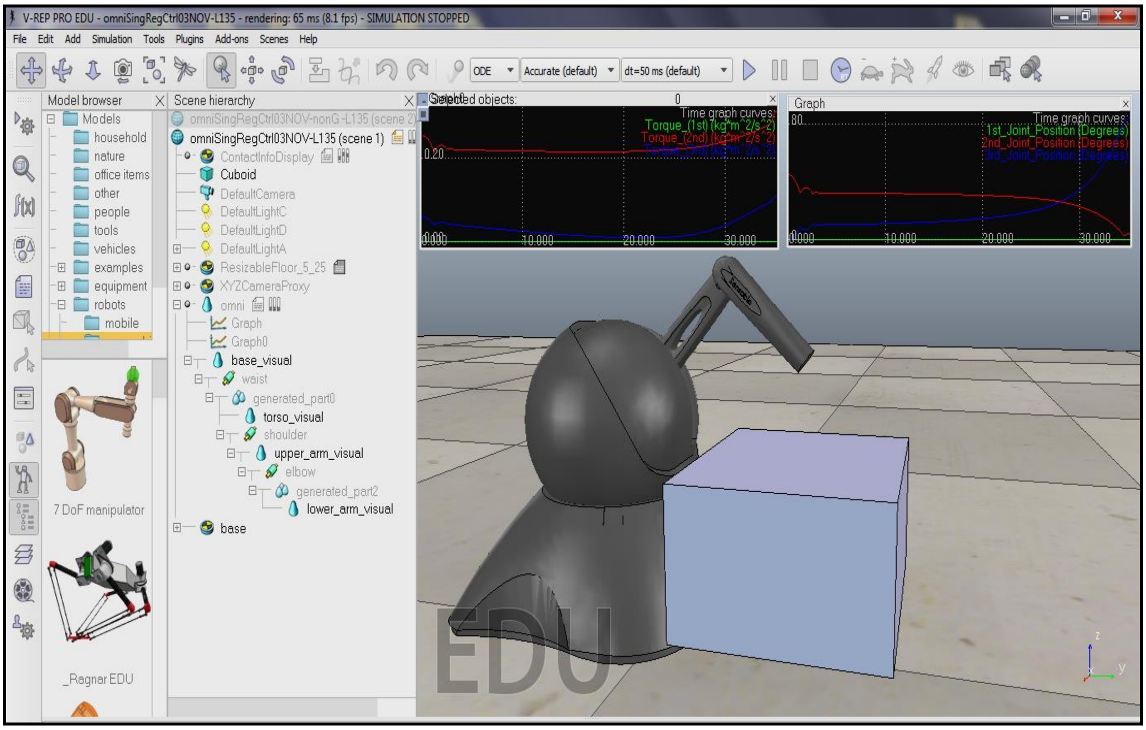
V-rep is used for different applications such as fast algorithm development, industrial automation simulator, quick prototyping and verification and remote controlling. This simulator allows controlling simulation remotely from a real robot or another PC. It includes four physics engines (Bullet Physics, ODE, Newton and Vortex Dynamic) for fast and customizable dynamics calculation, to simulate real-world physics and object interactions. V-rep can calculate forward /inverse Kinematics of any type of mechanism. It contains powerful and realistic and accurate sensor simulation which can calculate minimum distance within the customizable detection volume.

Figure 5.1: MATLAB and V-rep Communication



In this chapter, the PHANTOM Omni behavior is monitored in MATLAB and V-REP under regular control and singular control and switching between these two controllers in the vicinity and inside of singular boundary.

Figure 5.2: Phantom Omni Robot in V-rep environment



In V-REP simulation the Omni robot is imported as URDF (Universal Robot Description Format) as depicted in Figure (5.2). The behavior of this robot and the environment where is working, is very similar to a real robot. So it is preferred to investigate on the robot control near and also in singular boundary. Joint limitation of the real robot and the danger of damaging itself in singular boundary and configuration are other important reasons to simulate with V-REP in this study.

The controllers' algorithms are built in two different initial positions of PHANTOM Omni. The desired motion equations are changed regarding each initial position. The end effector is expected to have motion along x direction as a singular direction and to reach the singular configuration. For Omni manipulator, the purposed singular configuration is achievable when two adjacent links are stretched along each other.

5.1 POTENTIAL FILED CONTROL

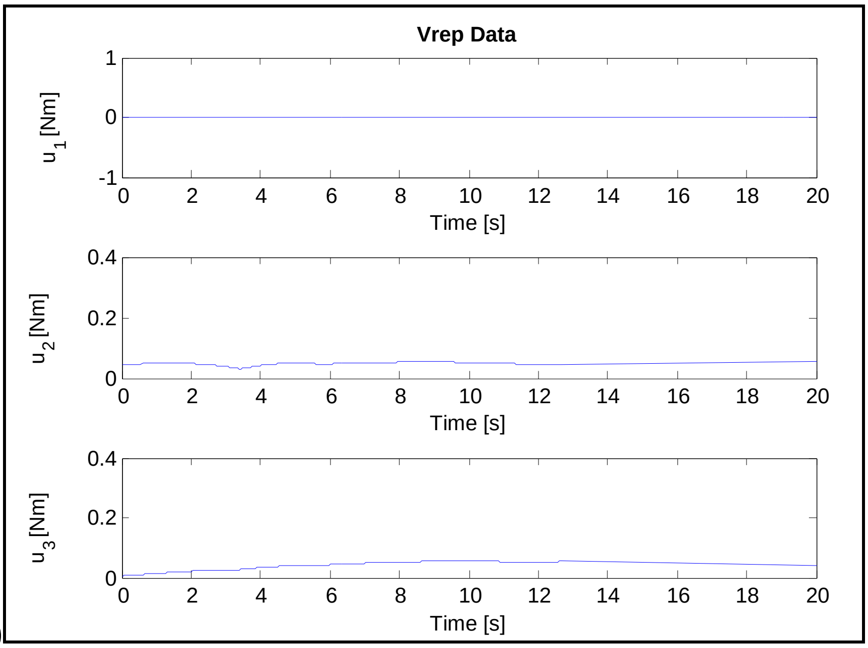
As explained in Chapter 3, one of approaches to control the manipulator for performing its desired task would be the attractive potential field. In this part of the simulation Omni robot manipulator is controlled to achieve its desired position and motion by applying the attractive potential filed.

First the initial positions of the joints are defined and the final position as a goal position is assigned in potential filed algorithms. The initial joints angles are q_0[0, 60,120] $\times\pi/180$ and the initial x0 of the end effector is computed as [0.135,0,0](m) and the desired motion of the end effector along x direction is defined as: xd = [(x0+ (0.135/20)*t), 0, z0(Nt)], where xd is desired direction along x axis and any movement along or about y axis is considered zero and z is computed regarding to the time iteration (Nt). x0 and z0 are representing the initial x and z position of the end effector. No motion and rotation should be done along or about y direction. This equation is derived from the condition that the end effector reaches its final position in t=20 sec.

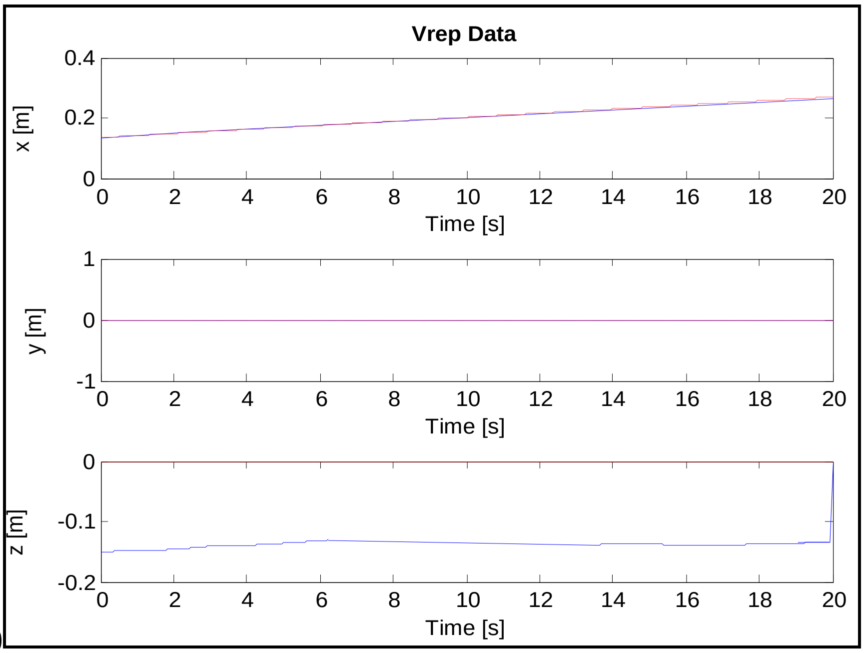
From the Figure (5.3 a) the applied torques on joint 1, 2 and 3 could be seen. The first joint torques are equal to zero, which means that there is no torques on this joint for moving and rotating so this joint is fixed without any motion.
Figure (5.3 b) illustrates the motion of end effector in x, y and z direction. This figure is also shown the actual motion of end effector in three directions (x, y and z). It also can be seen that the end effector follows the desired x and y with high accuracy, but there is considerable error between actual motion and desired motion along z direction. One of the main reasons for this difference could perhaps be because of the mismatch between the real Omni robot and URDF model in V-rep. It means that the length of the links in URDF is not exactly same as the real Omni. Therefore, dynamic properties of V-rep model such as mass, inertia matrix and Center Of Mass are different from those features of computed in MATLAB. These differences cause that the torques which sent from MATLAB to V-rep were not enough to move the end effector along the desired z direction. But the torques would be adequate for the motion of the end effector along x direction to follow its desired task.

Figure 5.3: Potential Field Control for Omni Robot in V-REP

(a) Joints Torques and (b) End effector motion



(a)



(b)

5.1.1 Potential Field in Singular Control

In this section the attractive potential field is applied in the singular control of Omni manipulator. The attractive field is used to calculate the null space torque. Regarding to the initial position which is defined as initial joints angles $q_0[0, 60,120] \times\pi/180$ and initial x0 of the end effector is computed as [0.135,0,0](m). The desired motion of the end effector along x direction is defined as: xd = [(x0+ (0.135/45)*t), 0, z0(Nt)], where xd is desired direction along x axis and any motion along or about y axis is considered zero and z is computed regarding to the time iteration (Nt). x0 and z0 are representing the initial x and z position of the end effector. No motion and rotation should be done along or about y direction. This equation is derived from the condition that the end effector reaches its final position in t=45 sec. The final position as a goal position is assigned in potential filed algorithms.

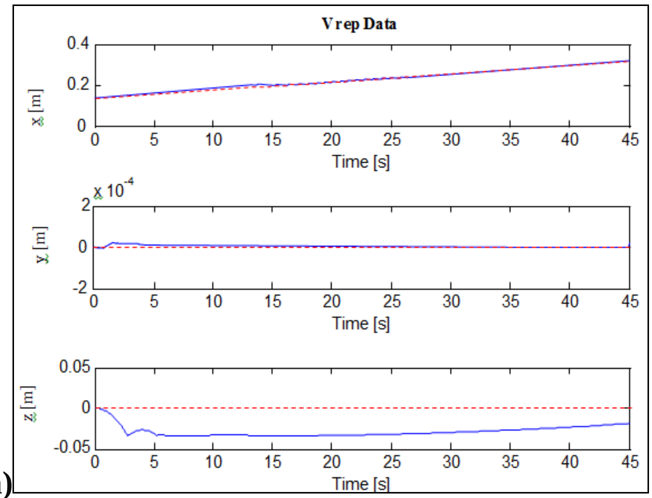
Figure (5.4 a) illustrates the motion of end effector in x, y and z direction. This figure is also shown the actual motion of end effector in three directions (x, y and z). It can be seen, the end effector follows the desired x and y with high accuracy.

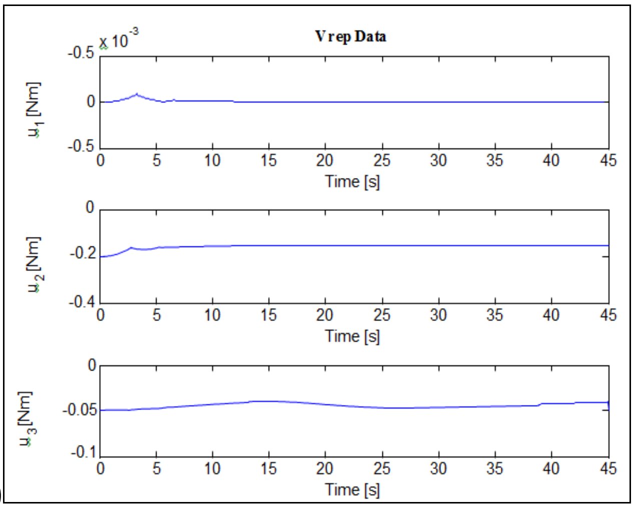
As expected, there is no motion in y direction due to lack of applied torques on the first joint names as U1.

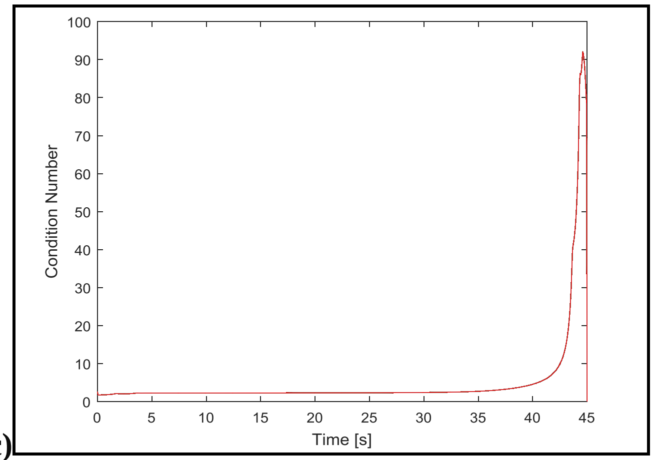
From the Figure (5.4 b) the applied torques on joint 1, 2 and 3 could be seen. The first joint torques are equal to zero, which means that there is no torques on this joint for moving and rotating so this joint is fixed without any motion. The torques on second and third joints is large enough to move links in the purpose of following the desired task.

In this simulation, the potential field control at the singularity can be seen. The end effector follows its desired motion, in x direction same as the singular control. This means that the potential field can lead the end effector to perform its task at the singular configuration. The error in z direction happened due to the mismatch problem which was explained in the previous section (5.1).

Figure 5.4: Potential Field in Singular Control for Omni Robot in V-rep
(a) End effector motion, (b) Joint Torque and (c) Condition Number

(a)


(b)


(c)


5.2 REGULAR CONTROL

In this controller of PHANTOM Omni, the last two links should have motion to lead the end effector along x direction. It is not expected to have any rotation and movement about or along y axis. If these conditions are correctly performed Omni manipulator acts like a RR planar robot. So for applying the regular control, at first it is required to perform regular controller in MATLAB for monitoring and observing the performance of the controller on this robot.

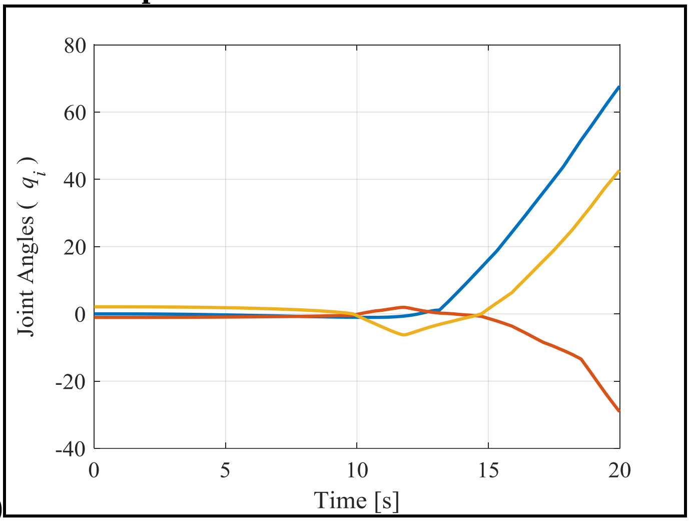
According to kinematic and dynamic of Omni robot, from Chapter 4, the regular control is performed for monitoring how this Omni manipulator behaves while it is approaching the vicinity of singular boundary.

Referring to Figure (5.5) the manipulator starts to move in x desired direction, but almost after couples of seconds it stops and will not be able to follow the defined direction and behaves in an unstable manner. It shows that in the regular control, the robot cannot enter the singular boundary. In this simulation, the initial position is defined as [0 -60 120] degree and this regular controller of the robot cannot establish the stable behavior of the robot to track the desired direction in the neighborhood of the singular boundary.

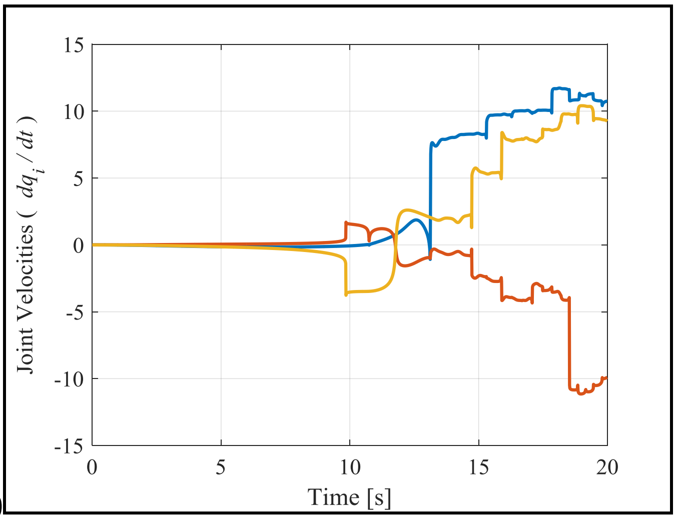
The desired task direction for end effector is defined along the x direction. The movement along or about y axis is expected zero. The manipulator acts as a planar robot:

xd = [(x0+ 0.00675*t), 0, z0 z0(Nt)], where xd is defined as a desired direction along x axis and any motion along or about y axis considered as zero and z is computed regarding to the time iteration (Nt).

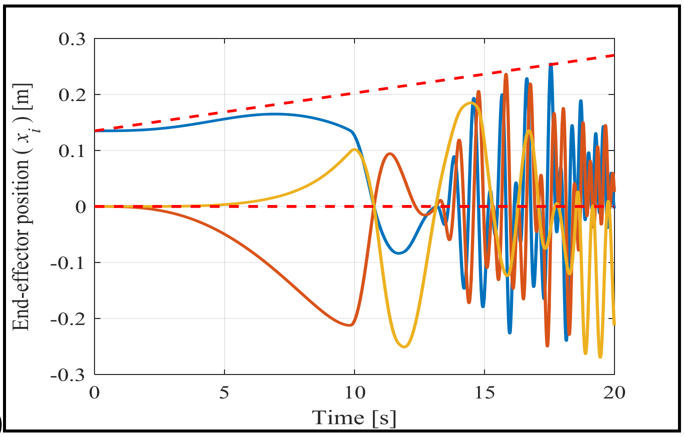
Figure 5.5: Regular Control for Omni Robot in MATLAB (a) Joint Angle, (b) Joint Velocity and (c) End effector position



(a)



(b)

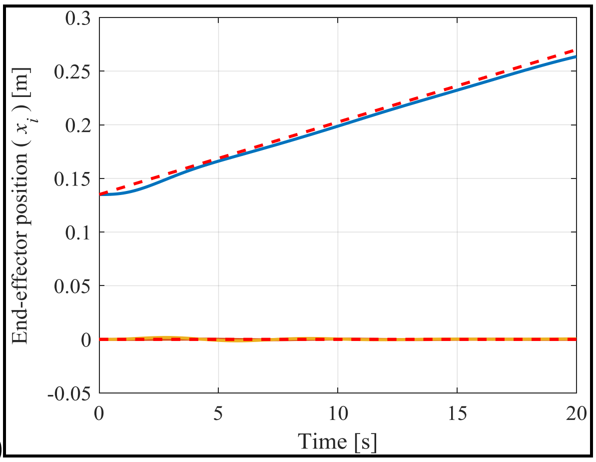


(c)

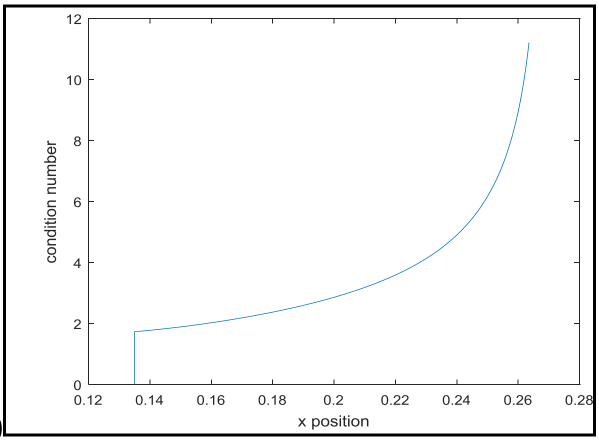
5.3 SINGULAR CONTROL

As explained in the previous chapter, in the singular control of Omni manipulator, the robot completely follows the desired direction outside and inside of the singular boundary. Finally manipulator reaches the singular configuration in a stable manner. The Condition number which is the largest singular value of the Jacobian matrix over the smallest value is used as a parameter to detect the singular boundary. When the manipulator approaches its singular position this number increases to the highest value. In Figure (5.6) for MATLAB simulation of singular control, it shows how the Omni follows accurately the desired direction and also depicts the enhancement of Condition number value in the vicinity of singularities.

Figure 5.6: Singular Control for Omni Robot in MATLAB (a) End effector Position and (b) Condition Number



(a)



(b)

5.4 SWITCH CONTROL

In this study, the parameter which is used for recognizing the singular boundary is known as Condition Number. It is chosen as the case for switching between regular control and singular control in V-REP simulation. This parameter is the ratio of the largest to the smallest singular value in the singular value decomposition of the Jacobian matrix. Depends on the robot manipulator structure-design the regions outside and inside of the singular boundary are different. In this simulation, it is possible to use the switching control regarding to dynamic of the robot and its singular boundary. In the other word, the normal region needs to be large enough to show how the switching control works between regular control and singular control, out of singular boundary and inside the singular region.

In this thesis, respect to various initial positions, the controller is performed under regular, singular and switching control of PHANTOM Omni. This manipulator behaves differently in the singular boundary under various the controllers. In this order, joints' positions and torques are monitored in V-REP simulation graph individually.

5.5 V-REP SIMULATION AND RESULTS

In this V-REP simulator, the torques are sent from MATLAB coding as input to Omni in V-REP, the joints' and end-effector's actual motion and also torques and dynamic parameters, all are obtained from V-REP and then sent to MATLAB.

First the manner of the manipulator is monitored under regular control and then singular and switch controls. It will show how the robot acts under these controllers, before and after passing singular boundary and region. MATLAB is used as a client to read data from V-REP as a server to receive/ send data.

The initial position which will be seen in the next section, is the first, second and last joint angles in V-REP: $\left[q_1, q_2, q_3\right]$. x, y and z are end effector motion along the singular direction , y axis and z axis in [m] unit respectively. U1, U2 and U3 are torques on joint 1 joint 2 and joint 3 in [Nm] unit respectively.

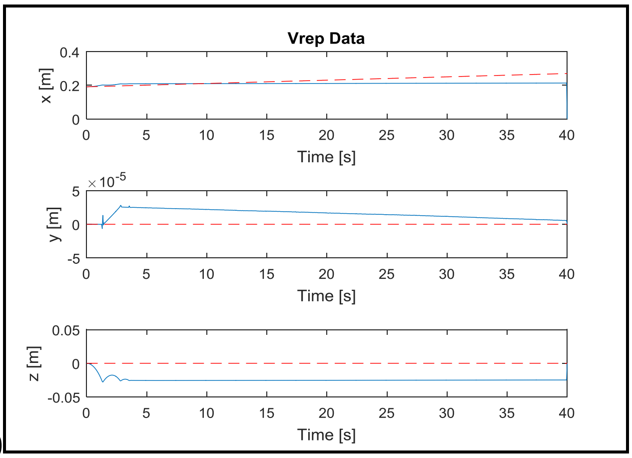
5.5.1 Case1: Initial Position [0, 45, 90] × π/180

Regarding to this defined initial position the desired task along x direction follows the equation, x-desired = x0+ (0.00175)*t. which the x0 is the initial x in meter regarding to the joint initial angles and t is time of simulation in second. The torque on the first joint will be zero in this simulation (u1) due to the purpose that the end effector follows the desired direction along x axis. If this manipulator has any rotation around z axis or any movement along y direction, it will not result in a positive approach to desired x direction.

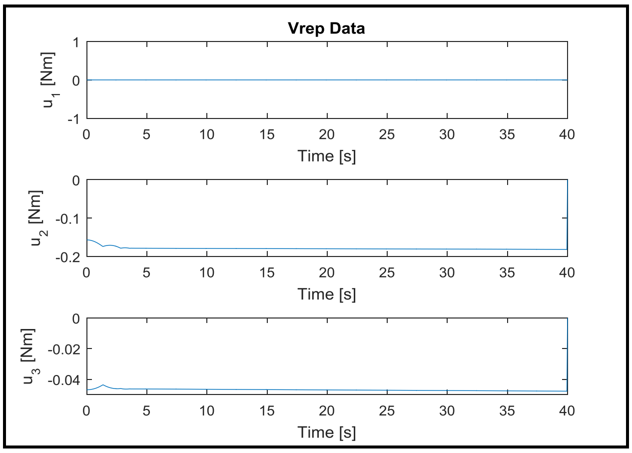
The following Figure (5.7 c), according to condition number, shows that the singular boundary is close to this initial position. The condition number after some fluctuations becomes almost fixed, but with a very slight increasing in amount. It shows that in the first seconds of the manipulator motions this singular boundary begins. So this robot was close to the singular boundary and then stopped and did not have any further motion (5.7 a). In Figure (5.7 b) the torque was sent to the first joint is zero (u1) so approximately there is no rotation in this joint (about y axis). It is clear to see from the torques on joint 2 and 3 that there is not any torque after a few seconds due to lack of the robot motion in the vicinity of singular boundary. The torque on joint 2 which causes the motion of link 1 is almost 3 times more than the torque on joint 3 for the motion of link2.

As explained, the condition number is an index in this simulation to show how much the end effector is near the singular boundary and singular configuration. This parameter remains on its peak when the robot is reaching the singular position. In this simulation, singular configuration is in the result of stretching of links 1 and 2 along x axis. In the regular control of Omni the condition number is approximately 3.3 near the singular region.

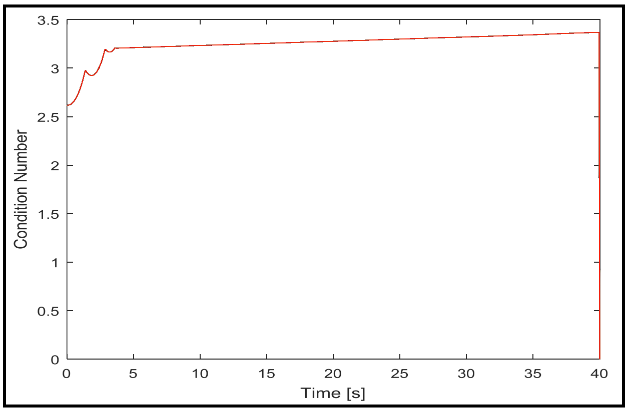
Figure 5.7: V-REP Regular Control (Case1), Kp=25, Kv=5 (a) End effector motion, (b) Joint Torque and (c) Condition Number



(a)



(b)



(c)

From Figure (5.8) it can be seen that the singular controller handles the robot motion along the singular direction. The end effector follows the desired Xd (5.8 a). By comparing the condition number graph in regular control with singular one, it is clear that at the first seconds of motion this parameter began to increase. The condition number of regular control is 3.3 which in the singular control this value is for the start point of motion. It can be explained that the singular region starts very soon after normal region.

The Figure (5.8 b) depicts that after the manipulator entered the singular region the joint torques became stable but not zeros. Figure (5.8 a) shows the end effector tracks the desired x direction with high accuracy and with very low errors in Y axis around 0.05 mm at the beginning of the movement until zeros in singular configuration. Again in the same figure the torque on joint 2 is almost 4 times bigger than the torque on joint 3. The condition number trend can be seen clearly from Figure (5.8 c) which starts with minimum amount such as regular control, but it keeps on increasing in the singular region until maximum number in its singular configuration, while both of link 2 and 3 get aligned in x direction.

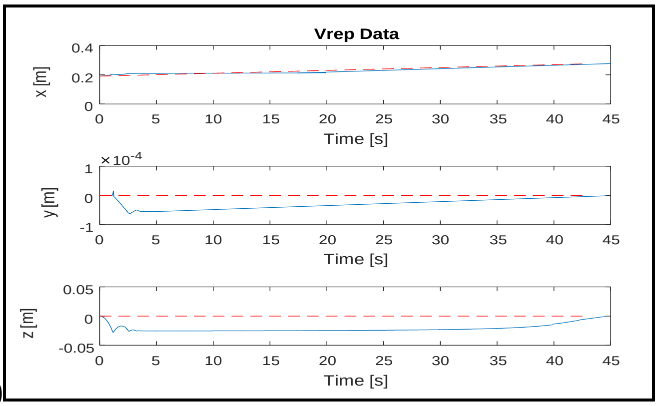
The small movement of the robot in y axis is very tiny and can be considered as zero. At the beginning of the motion the robot has a small fluctuation. These fluctuations mean that the end effector comes near the surface of its work-table and follows with a small moving up, after that tries to keep its movement in a reliable behavior. This small moving up and down can be tracked in torques of joint 2 and 3 as small quick changes at the beginning of their trend.

As illustrated in Figure (5.8), even with all small or slight altering in z-motion of the end effector, it follows the desired task with high accuracy. In defined initial positions of the joints, the singular control algorithm would be able to control the manipulator with the reliable manner and with high performance in singular boundary and singular configuration.

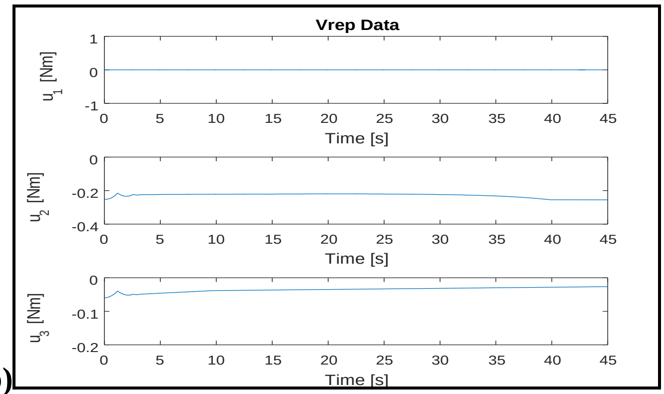
Figure 5.8: V-REP Singular Control (Case1), Kp=25, Kv=5

(a) End effector motion, (b) Joint Torque and

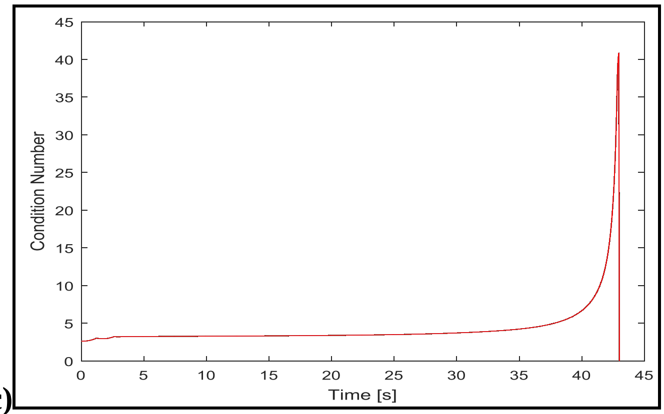
(c) Condition Number



(a)



(b)



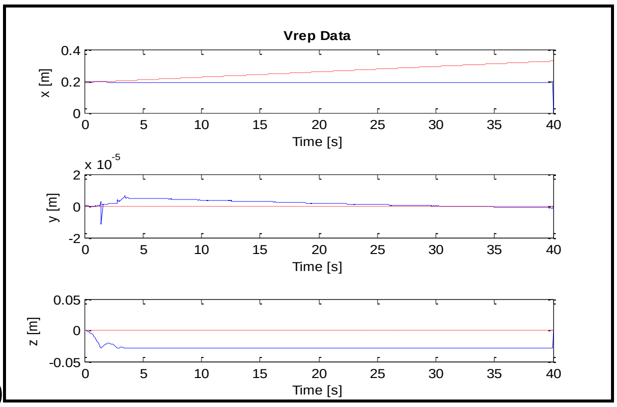
(c)

If the torques in singular control are compared with in regular control, it can be seen that, with the same gains for position and velocity Kp= 25 and Kv= 5 the torques on joint 2 in regular control is less than 0.2 N.m. but this amount is more than 0.2 for singular one. Even the gains are increased to 10 times bigger, no considerable changes in the values of the torques are depicted in Figure (5.9). Only for 100 times bigger in gains values, the unstable motion of manipulator will occur before the singular boundary.

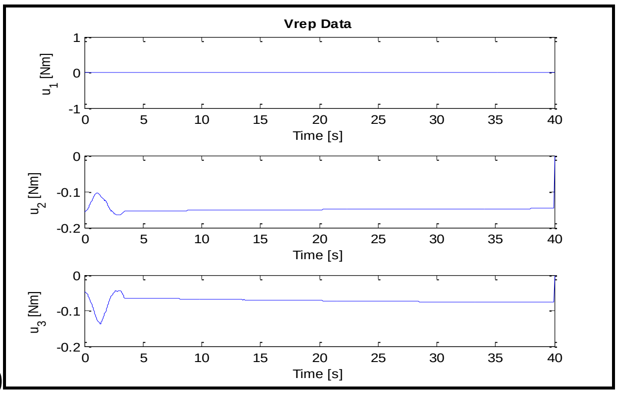
If the gains Kp and Kv were increasing, the only consequence would be the dangerous behavior of the robot in the vicinity of singular boundary. It means that the robot cannot enter the singular boundary with the stable manner to follow the desired task of the end effector. It may damage itself and its environment.

Figure 5.9: V-REP Regular Control (Case1), Kp=2500, Kv=500, (a) End effector motion, (b) Joint Torque

(a)



(b)



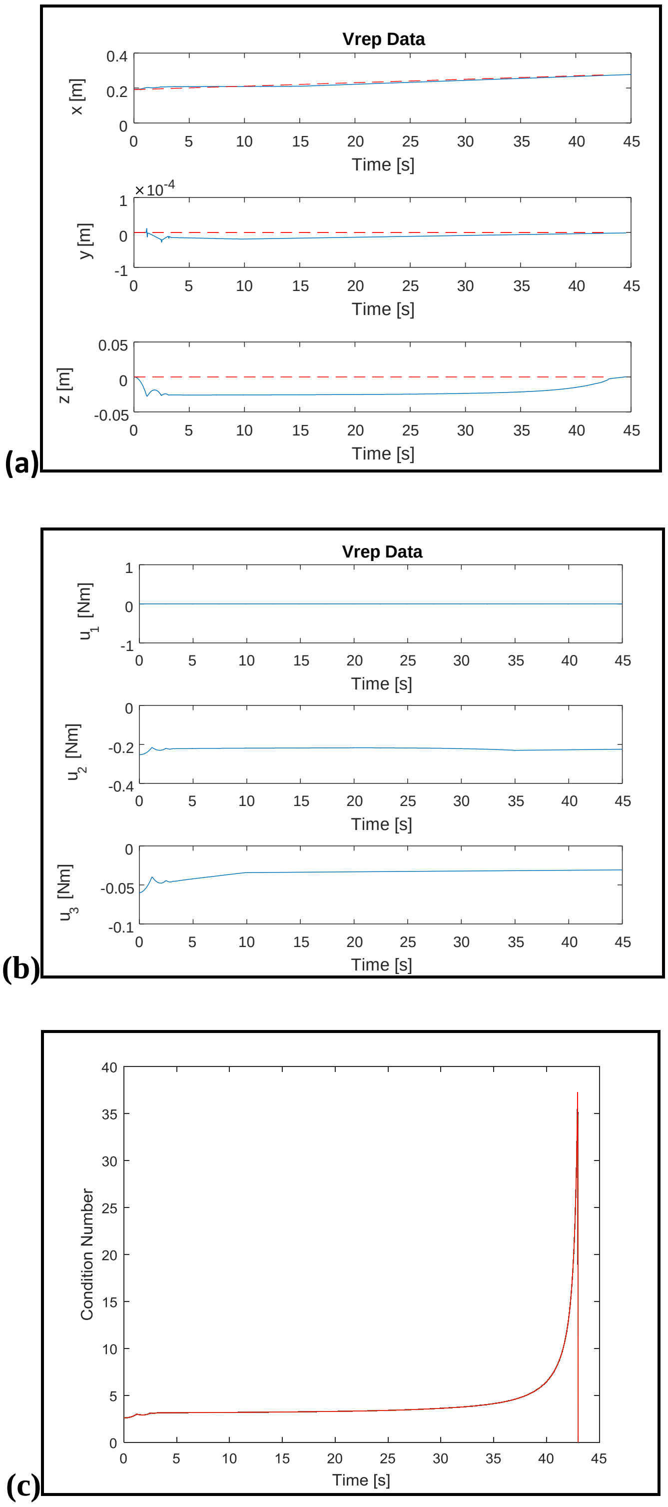
In the defined initial position: [0, 45, 90] × π/180, due to being closed to singular boundary a very short motion can be seen in normal boundary. So the switching control does not have opportunity to demonstrate how acts between two regular and singular controls. The behavior of switching and singular controls is similar as shown in Figure (5.10).

In this configuration the switching control is based on the condition number parameter which obtained from the Condition Number of regular control. This parameter is defined ≥3.3 in the controller that makes the switching from the regular control to singular control. In this state condition number starts to increase from the first seconds, so switching happens very soon and turns immediately into the singular control.

As x-desired = x0+ (0.00175)*t is defined for this initial position of the joints angle, the simulator calculates the initial x0 respect to its kinematic equation of the end effector. In this simulation, the initial position is almost.0.1909 (m) and the end effector of Omni manipulator is expected to move from this initial position to the singular configuration. The stretching out both of the links along each other provides the singular position of the end effector.

Respect to the link length of PHANTOM Omni, the singular configuration of Omni manipulator is 0.270 (m) in x direction. The Omni robot manipulator should move from x0 to the singular configuration in 45 seconds with the assigned gains. These gains weight the position and velocity errors which are used in computing of the force and the joint torques.

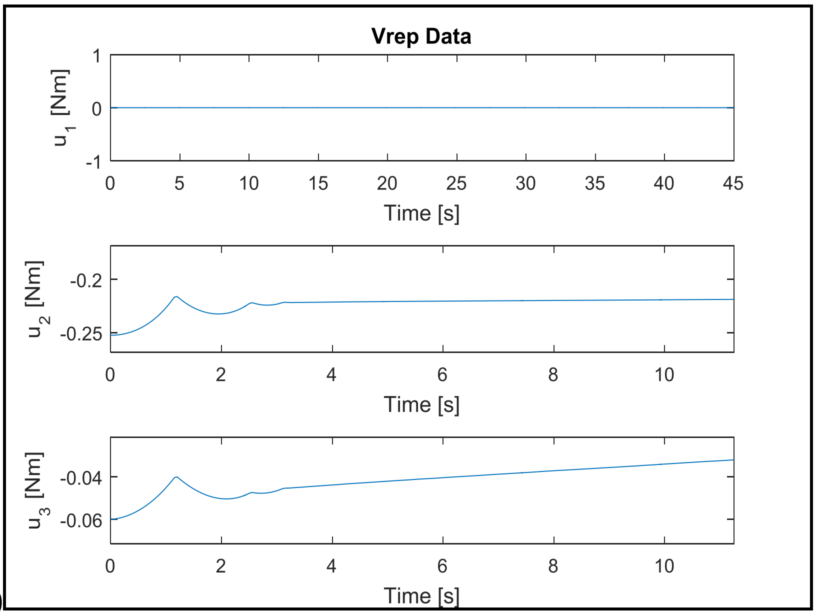
Figure 5.10: V-REP Switching Control (Case1), Kp=25, Kv=5 (a) End effector motion, (b) Joint Torque and (c) Condition Number



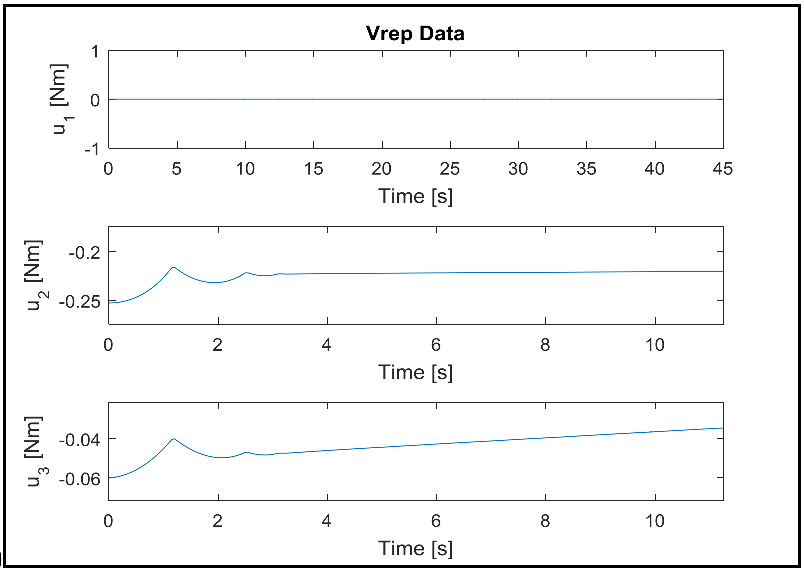
The torque graph of singular control and switch control is illustrated in Figure (5.11). At the first seconds of simulation of the current initial position, the end effector becomes very close to singular boundary, so there is not any remarkable difference in torques of singular and switching controls.

Figure 5.11: Joint Torques for Omni Robot in Case 1

(a) Singular Control and (b) Switching Control



(a)



(b)

5.5.2 Case 2: Initial Position [0, 60, 120] × π/180

In this defined initial position the desired task along x direction follows the equation x-desired = x0+ (0.003)*t, which the x0 is the initial x in meter regarding to the joint initial angles and t is time of simulation in second unit. The torque on the first joint becomes zero in this simulation (u1) so the end effector can follow the desired direction along x axis. If this manipulator has rotation around z axis or any movement along y direction, it is not possible to approach the desired task direction.

In the current initial configuration with regular control, as could be seen in Figure (5.12), the behavior of Omni manipulator near the singular boundary is similar to previous configuration in section 5.5.1. The manipulator after a little fluctuation is not able to go through the singular region and it stops out of this boundary.

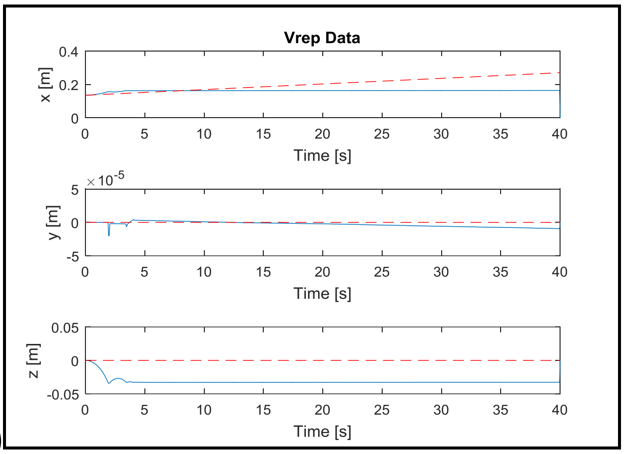
The torques on joints can be observed in Figure (5.12 b). After the small change in the amount of torques of joint 2 and 3, the torques will be stable for the rest of the simulation. The torques on joints 1, 2 and 3 are zeros, 0.15 and 0.027 N.m. respectively. The condition number has some variations in amount, but after a couple of second it remains around 2.3 in the neighborhood of the singular boundary.

In order to compare the regular control of Omni in those two different configurations are shown in Figures (5.7) and (5.12), obviously it is clear that none of them can pass the singular boundary so they stop before this boundary. It shows that regular control is not able to lead the end effector of the manipulator to pass the singular boundary and reach its desired task in a singular direction.

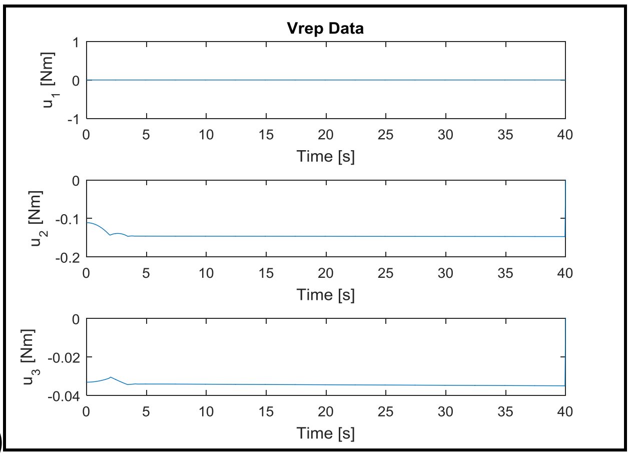
By assigning the same position and velocity gains Kp=25 and Kv=5 in both defined configurations, the joints 2 and 3 torques are bigger in the configuration [0, 45, 90] × π/180 than the configuration [0, 60, 120] × π/180.

Figure 5.12: V-REP Regular Control (Case 2), Kp=25, Kv=5 (a) End effector motion, (b) Joint Torque and (c) Condition Number

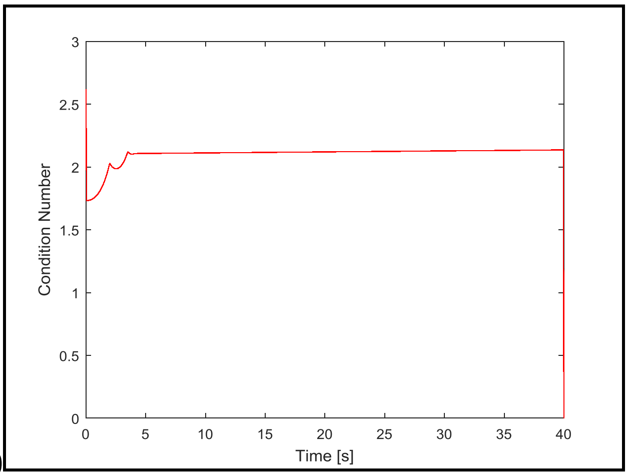
(a)



(b)



(c)



From Figure (5.13), it can be seen that the singular controller guides the robot towards moving in x direction. The end effector follows the desired Xd (5.13 a) and passes the singular boundary. After this, it can be seen that the joint torques is increasing slightly. In Figure (5.13 a) the end effector tracks the desired direction with high accuracy and with almost zero error in y axis, except a firm errors in the first seconds of the motion. After those changes in y axis, the motion in y direction is stable at zero amounts to reach the singular configuration.

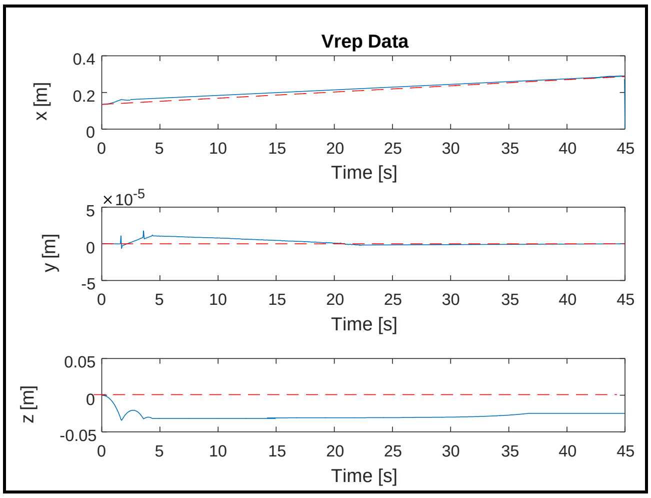
In Figure (5.13 b) the torque on joint 2 is almost 4 times bigger than the torque on joint 3 but both of them have approximately stable trends inside the singular boundary. The first fluctuation could be seen in z graph and also happened for torques u2 and u3, due to up and down motion of the end effector at its beginning.

The condition number trend can be observed clearly from Figure (5.13 c). It starts with minimum amount such as in regular control, but it keeps enhancement in singular region, until achieving the maximum number in its singular configuration.

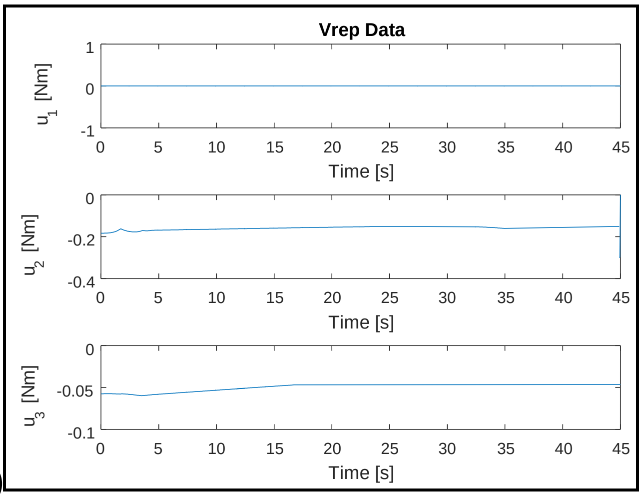
As explained in the previous initial configuration(5.5.1) and can be seen in Figure (5.13), even with small and slight altering in motion of the end effector along z axis and in the amount of applied torques on links' joints, the end effector follows its desired task with high accuracy.

In this initial position of the joints, the singular control algorithm would be able to make the reliable control of the end effector behavior with high performance in singular boundary and singular configuration.

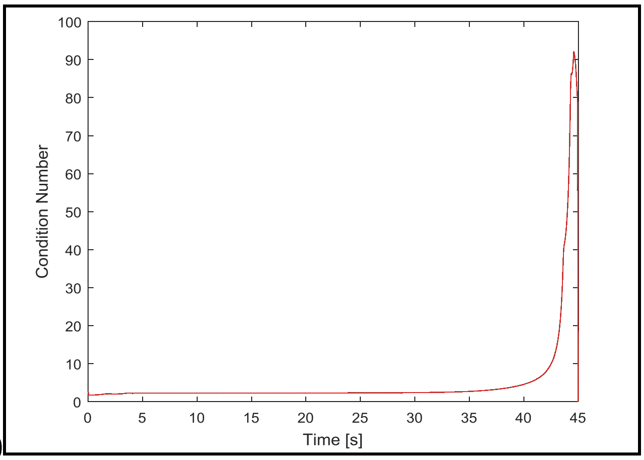
Figure 5.13: V-REP Singular Control (Case 2), Kp=25, Kv=5 (a) End effector motion, (b) Joint Torque and (c) Condition Number



(a)



(b)



(c)

In this initial position: [0, 60, 120] × π/180 the end effector is close to a singular boundary so there is a very small motion in normal boundary. It can be seen that the switching control behavior is similar to singular control as shown in Figure (5.14).

In this configuration the switching control works based on condition number while Condition Number is ≥2.3, so then the controller switches from regular control to singular control. In this situation, condition number starts increasing from the first seconds and then the controller switches very quickly into the singular control.

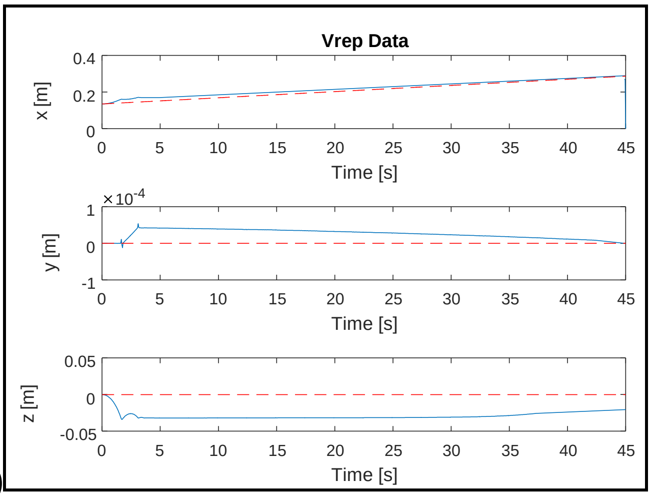
This switching control condition number is obtained from the Condition Number of regular control. This parameter is defined ≥2.3 in the controller that switches from regular control into singular control. In this situation, condition number starts to increase very quickly then the switching happened shortly afterwards.

For the current initial configuration the x-desired is defined as Xd = x0+ (0.003)*t, the simulator computes the initial x0, respects to kinematic equations of the end effector. In this simulation, the initial position is almost.0.135 (m) and the end effector of Omni manipulator is expected to move from this initial position to the singular configuration.

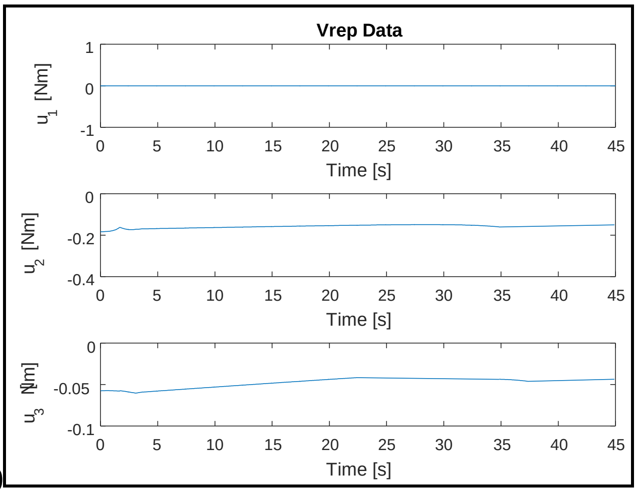
In regard to the link length of the manipulator, the singular configuration of Omni manipulator is 0.270 (m) along x direction. The Omni robot manipulator should move from x0 to the singular configuration in 45 seconds. The assigned gains weight the position and velocity errors, which are used in computing the force and joint torques.

Figure 5.14: V-REP Switch Control (Case 2)
(a) End effector motion, (b) Joint Torque and
(c) Condition Number

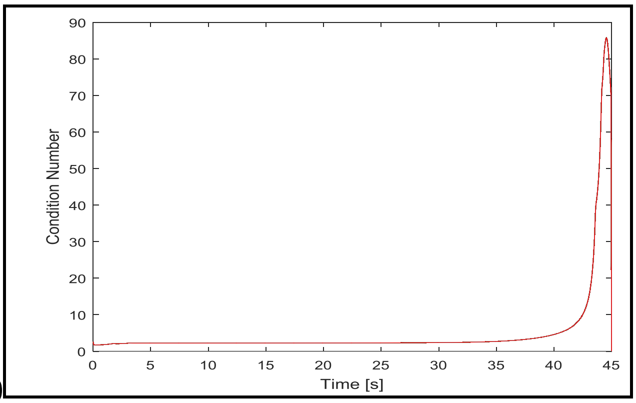
(a)



(b)



(c)

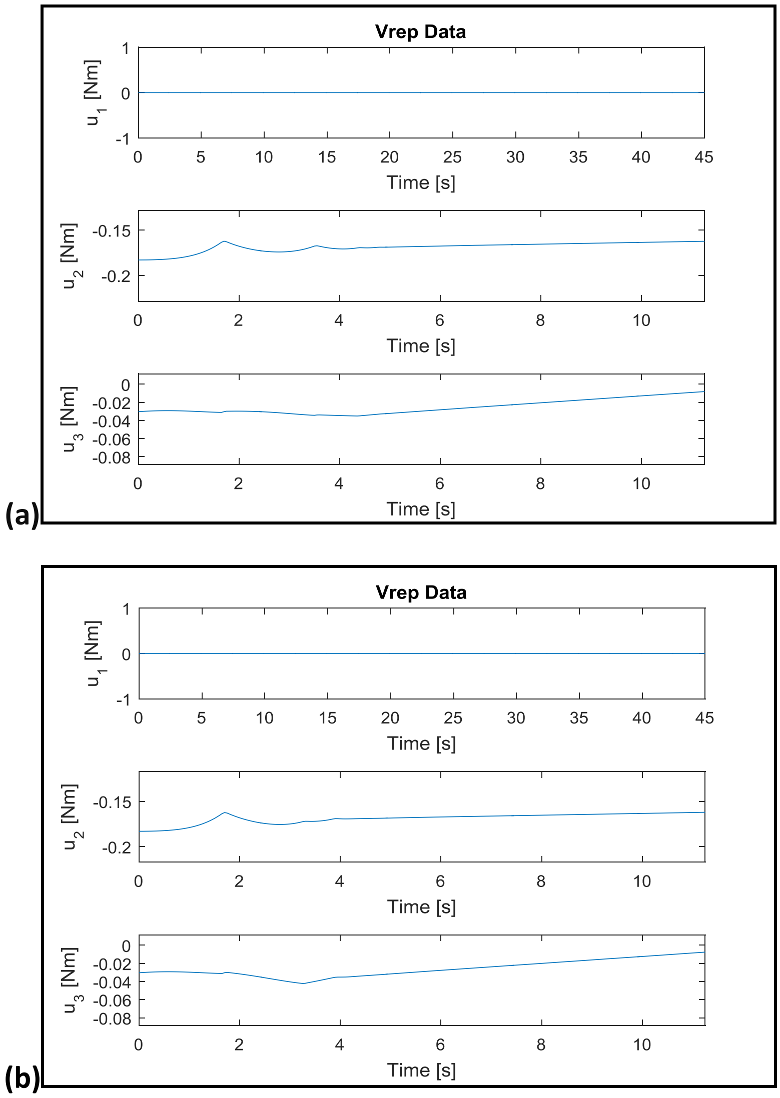


For comparing the torque graph of singular control and switch control of Initial configuration ([0, 60, 120] × π/180), a small difference can be seen before singular boundary at the beginning of the simulation, which is shown in Figure (5.15).

In the switch control, u2 torque of joint 2 is a little smoother than in singular control. For u3 torque of joint 3 is illustrated that the switching happened between second 2 and 4 in Figure (5.15 b).

Figure 5.15: Joint Torques for Omni Robot in Case 2

(a) Singular Control and (b) Switching Control



6. CONCLUSION

In this dissertation the kinematic and dynamic calculation of the Phantom Omni robot was described and then the simulation and dynamic analysis was done in MATLAB and V-REP in order to monitor the performance of the robot manipulator under different controllers, such as regular, singular, and switch control of robot in singular region.

This thesis presents the fundamental mathematic equations of forward and inverse kinematics and describes the singular configuration and dynamic equation of the manipulator and also explains the operational space and null space.

The potential field simulation in MATLAB was described for RR planar robot with the purpose of investigating the performance of this approach to control the manipulator in the singular region under the attractive forces.

PHANTOM Omni manipulator was described according to its kinematic and dynamic equations. Mathematical equations were used for regular and singular control of simulation part.

In the simulation part the PHANTOM Omni robot in V-REP, was analyzed according to its kinematic and dynamic architecture. Regular and singular control individually was applied for this robot outside of singular boundary and inside of singular region. Switch controller was applied between regular and singular control. Two different initial positions were defined for this manipulator to observe its behavior under the various conditions.

The results clearly show that PAHNTOM Omni robot could not enter the singular boundary with the regular control. Potential field handled the singularity problem with the attractive field and led the end effector to perform its desired task. In this study singular control guided the robot to pass the singular boundary and followed the desired task in the smooth motion of end effector in the singular direction which is defined along x axis. The switch control was designed based on the Condition number which is a parameter to show when the singular boundary starts. This control was applied in order to switch between two controllers regular and singular ones.

The more practical way to detect the singularity would perhaps be to monitor the rate of changes of the condition number. This rate has definitely various values when the robot moves from regular region to singular region. It means that this changing rate is increasing constantly until the robot reaches to its singular configuration, where the changing rate is at its peak. The rate of condition number can help the control algorithm to detect the singularity automatically instead of defining the exact value of condition number for switching from regular control to singular control. This method could be considered as the future work to apply for various degrees of freedom of robots.

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