# ESSAYS ON MODELING HOUSING MARKETS, INCOME DISTRIBUTION, AND WEALTH CONCENTRATION

by

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# ABSTRACT

**Essay 1: Modeling Housing Market Dynamics: A Survey Article**

This chapter provides an extended survey on different models commonly used in Economics and Finance which aim to explain the driving forces behind market behavior. Special emphasis is placed on the application of housing markets. Each model tries to provide a more realistic framework to represent market behaviors in order to explain the causal relationships which give rise to large market fluctuations such as boom and bust cycles mostly followed by crises. These models attempt to build their own framework without the assumptions of the rational expectations efficient market hypothesis (and the limitations attached to it) by introducing heterogeneity of individuals or house stocks, bounded rationality, and interactions among economic agents. Each of these approaches bring new ideas to the dynamics of markets, but also have their own limitations. The methodological and technical differences between these models are contrasted, and discussion is presented as to how successful they are to explain observed stylized facts in data such as, excess volatility and fat tails in price changes, and boom-bust cycles mostly followed by crashes.

**Essay 2: Quantal Response Statistical Equilibrium Model of the US Housing Market, 2000-2015**

In this chapter, it is argued that a quantal response statistical equilibrium approach to the US housing market with the help of maximum entropy method of modeling is a powerful way of revealing different characteristics of the housing market behavior before, during and after the recent housing market crash in the US. In this line, a maximum entropy approach to quantal response statistical equilibrium model (QRSE), introduced by Scharfenaker and Foley (2017a), is employed in order to model housing market dynamics in different phases of the most recent housing market cycle using the S&P Case Shiller housing price index for 20 largest- Metropolitan Regions, and Freddie Mac housing price index (FMHPI) for 367 Metropolitan Cities for the US between 2000 and 2015.

**Essay 3: Wealth Concentration, Income Distribution, and Alternatives for the USA**

US household wealth concentration is not likely to decline in response to fiscal interventions alone. Creation of an independent public wealth fund could lead to greater equality. Sim-

ilarly, once-off tax/transfer packages or wage increases will not reduce income inequality significantly; on-going wage increases in excess of productivity growth would be needed. These results come from the accounting in a simulation model based on national income and financial data. The theory behind the model borrows from ideas that originated in Cambridge UK (especially from Luigi Pasinetti and Richard Goodwin).

*To my family and Thomas*

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# CHAPTER I

# Modeling Housing Market Dynamics: A Survey Article

## 1.1 Introduction

One of the main concerns of Economics and Finance is to understand and explain the driving forces behind movements in asset prices. The Efficient Market Hypothesis (EMH), which was independently developed by Samuelson (1965) and Fama (1965), has been employed extensively in both theoretical and empirical studies in order to serve to this purpose.

According to Samuelson (1965), price changes are random and unpredictable as long as they incorporate all available information and expectations of all market participants. The random character of the price movements is explained by Fama (1965, 1969) as the result of competition between a large number of rational profit maximizing individuals who compete with each other, trying to predict future market values of individual assets where important current information is almost “freely" available to everyone. Thereby, at any time, competition leads to a situation where “actual prices of individual assets already reflect the effects of information based both on an event that has already occurred and on events which the market expects to takes place in the future" (Fama, 1965, 1969). EMH therefore, pays a special attention to informational efficiency and competition in justifying the price changes.

In this set up, behavior of individuals are driven by profit opportunities which can only be created by informational advantages brought about by the arrival of new information.

This in turn encourages all sophisticated individuals to incorporate the new information into market prices, and thus quickly eliminates profit opportunities. As a result of this process, prices reflect all available information and actual prices will be equal to the fundamental values of the assets (if the markets are perfectly frictionless, where there is no transaction costs).

Later, the neoclassical version of the EMH[1] started to gain power with the help of Lucas (1978) and followers of rational expectations hypothesis. They tied risk-averse investors' marginal expected utilities to price changes[2]. According to Lucas (1978), all investors have "rational expectations", and utility weighted prices (reflecting all available information) become unpredictable and follow martingales (Lo, 2004). Therefore, current price of an asset is the market's best forecast of the future price. In this set up, the aggregated optimization problem of rational individual buyers and sellers with perfect foresight subject to certain constraints allows one to build up demand and supply schedules, whose intersection determines the general equilibrium prices in the market. Hence, with the "rational expectations revolution", the main role of interrelation between the competition and arrival of new information on market efficiency in Fama (1965, 1969) was reduced to the role of perfectly rational representative agent's expectations in a general equilibrium set up[3].

Yet, this highly simplified approach to depict the real economies carries fundamental problems representing behaviors of highly complex economic systems. Introducing the rational representative agent with rational expectations into their analysis for the sake of providing a micro-foundational explanation for the individuals' behavior, and simplifying

[1]It will be called as "Rational Expectations Efficient Market Hypothesis" and represented as the "traditional approach" in the rest of the paper.

[2]Corresponding extensions of expected utilities and rational expectations in behaviors of individuals in finance are followed by portfolio optimization models, dynamic general equilibrium asset pricing models, and consumption capital asset pricing models (CCAPM).

[3]Fama (2013) accepts that testing market efficiency is problematic because if the efficiency is rejected, this could be either because the markets are inefficient or an incorrect equilibrium model are assumed. On the other hand, to overcome this problem, traditional approach specifies additional structures about individuals' preferences and expectations etc. such that market efficiency is given.

the aggregation problem, ignores the very existence of uncertainty, interactions among agents and the market psychology during the transaction process[4] which are some of the most important drivers of out of equilibrium behaviors observed in reality (i.e. boom-bust cycles and crashes).

According to the Rational Expectations Efficient Market Hypothesis, large price fluctuations observed in data could be explained as temporary movements, caused by unexpected random factors, which could be fixed by rational market participants sooner or later, and crises would only be caused by external shocks. Since this line of thinking ignores the reality that investors are not perfectly rational and exhibit catastrophic behaviors (e.g. imitation and herding) at the macro level, the models which take the Rational Expectations Efficient Market Hypothesis as given, are not capable of capturing and representing the out of equilibrium problems which plague economic systems in reality.

A large number of empirical studies investigate and model the price dynamics mostly by taking some form of efficient market hypothesis argument as granted in a general equilibrium framework. Contrary to the efficient market hypothesis based on the assumption of rational expectations, markets can only be “close" to martingale behavior[5]. Empirical observations from the data, such as excess volatility of prices, fat tails of distributions of price changes[6], bubbles and crashes provide enough reasons for researchers to deviate from the arguments of perfect market efficiency and complete randomness of prices to understand the real market behaviors.

Based on these facts, during early 1980’s, the Rational Expectations Efficient Market

[4]Assumption of rational expectations of individuals with a set of complete, transitive preferences, allows neoclassical theory to overshade the problem of individual’s uncertainty in their decision making process, because a rational agent given her fully defined set of preferences would have perfect foresight about her future asset value which would meet the actual price created in the real market in the equilibrium.

[5]The martingale property of prices assumes that price differences are uncorrelated. On the other hand, observed data show that higher powers of “absolute value" of price changes indicate higher and longer lasting auto-correlation which can be interpreted as high degree of predictability of volatility (Lux, 2008; Sornette, 2017)

[6]Deviation from normal distribution of price changes is accepted by Fama (1963) and Mandelbrot (1963). They claimed that price change distributions follow Levy distributions, which are the limiting distributions of sums of random variables with infinite second moment, due to arrival of new information.

Hypothesis started to be questioned. Shiller (1981) argued that stock prices exhibit excess volatility - that is, movements in prices are much larger than movements in underlying fundamentals. For example, the October 1987 Crash could not be explained by representative agent rational expectations theories (Hommes, 2006). Simon (1957) supported the idea of new modeling techniques developing the theory of bounded rationality based on the impossibility of individuals' ability to reach to perfect information. With the help of new developments in mathematics and physics, new modeling techniques started to be developed considering the dynamical, complex nature of the social systems.

The main purpose of this paper is to provide a methodological investigation for modeling housing market behaviors, given the improvements in modeling. Housing markets are highly underrepresented in the literature, even though they constitute a large portion of households' wealth[7], and consequently large deviations in house prices have significant effects on the rest of the economy[8]; thus motivating the argument that housing market behaviors should be better understood. Moreover, housing markets are proven to be highly inefficient (Case and Shiller, 1989a, 2003, 1988; Shiller, 1981, 1992, 2008a), hence, they provide an alternative venue to investigate the traditional modeling techniques which takes efficient market hypothesis as given. Compared to stock market prices, house prices follow a slower adjustment process[9] in which the presence of the arbitrage opportunities are easier to capture. This is firstly because of the heterogeneity of housing stocks and market participants which prevent standard arbitrage process from functioning properly; and secondly because of high sensitivity to inefficiencies compared to other markets (i.e. due to lack of effective

[7]According to Wolff (2017), owner-occupied housing is the most important component of household wealth. His decomposition of household wealth shows that the share of owner-occupied housing in total assets for the US was around 28-30% between 1983 and 2001, increased to 34% in 2004, and declined to 25% in 2016.

[8]Wolff (2017) argues that 24% and 26% declines in housing and stock prices, respectively, are the main causes behind a 44% decline in median wealth in the US between 2007 and 2010. Moreover, based on an analysis of a panel of 14 countries, and a panel of US states, Case et al. (2001) demonstrates that fluctuations in housing market wealth have a larger impact on the consumption behaviors of households than stock market fluctuations.

[9]It is a "thin market " (Ge, 2017), meaning the number of transactions in a given period can be low, and buyers and sellers often have to wait on the market to secure a deal. Moreover, they engage in negotiations and other strategic interactions.

short selling mechanism that prevent bearish investors from participating).

This paper serves to provide an extensive literature review for commonly used models in Economics and Finance based on their technical and methodological strengths and weaknesses, with special emphasis is placed on the the application of housing markets. All of these models' main objective is to capture and explain the observed price movements in the markets by overcoming the problems which efficient market hypothesis leaves out unexplained. The following sections will discuss Search and Matching Models, Log Periodic Power Law Models, Heterogeneous Agent Models (both simple and computational agent based models) and Quantal Response Statistical Equilibrium Models (QRSE).

## 1.2 Search and Matching Models of Housing Market

One of the earliest modeling techniques trying to explain the housing market behaviors, is called Search and Matching Models of Housing Markets. Search and matching models have been inspired by equilibrium search models of labor markets[10], in which workers search sequentially, drawing price offers from a known distribution while firms react optimally to individual's behavior so that price dispersion arises as an outcome of the equilibrium. Literature which adapted the labor market search model framework consists of one-sided buyer /seller models and random matching models (with some variety in their model set up) whose main focus is to explain the process of housing market clearing.

In these type of models, houses are considered to be heterogeneous in terms of their sizes and the neighborhood they are located in, while the heterogeneity of individuals are ignored. Housing transactions take place under uncertainty, in a sense that until they search for houses, buyers do not know which houses would be the best fit while sellers do not know which buyers would be the best match. Most of the models base their analysis on the existence of market frictions, such as cost of search and transaction costs (i.e. brokerage fees, taxes, moving costs etc.). Moreover, market participants (buyers, sellers, brokers etc.)

[10]See Albrecht and Axell (1994); Eckstein and Wolpin (1990) and Mortensen (1990).

are assumed to be rational, and their interactions through transaction process, and bargaining / matching between buyers and sellers are central in these models. They focus on explaining how the outcomes of matching process are affected by the market and strategic forces that impact the search/matching, bargaining, and intermediation, such as sellers' characteristics, house characteristics, market conditions, strategic choices made by buyers and sellers, and intermediation factors that affect brokers' performances.

Earlier theoretical frameworks (i.e. one-sided search models) are mostly followed by partial equilibrium analysis of the market, while more detailed models (i.e. two-sided random matching models) build their analysis based on competitive general equilibrium framework with microfoundations, i.e. in one-sided search models, the process of arrival of buyers and sellers is modeled exogenously (e.g. arbitrary arrival of buyers). Yet, more advanced models such as random matching and directed search models, the arrival is considered as an equilibrium outcome consistent with utility maximization and learning by involved parties.

One-sided search models of housing markets (Elder et al., 1999; Horowitz, 1992; Salant, 1991; Yinger, 1981) take the behavior of only one side of the market searches (sellers, buyers) into account, in order to make the model simpler. They generally assume identical agents who search sequentially, draw price offers from a known stationary distribution. In one sided "seller" search models, for example, there is generally one seller encountering a sequence of random buyers, and sets an asking price for the house. Sales occur if one of the buyers accepts the offer, meaning that there is a "take-it-or-leave-it" offer of the asking price to simplify the bargaining process[11]. Models in this tradition, in general, generate some version of price trade-off in which a higher asking price generates higher sale price resulting in an later expected sale(Han and Strange, 2014). In the case of the housing market, the time time spent on the market (TOM) helps explaining the liquidity of housing and the

[11]See Merlo and Ortalo-Magne (2004) for a more advanced method which they set up a Bayesian Nash Equilibrium, in which a Bayesian updating process is used for mapping priors of the buyer and seller about each other's valuation of the property in question. This iterative learning process of bargaining, in the end of the period, converges to a "temporary equilibrium configuration".

rate of vacancy of housing markets. Empirical studies in this genre deal with relationship between price and seller time on the market, housing heterogeneity (Haurin, 1988), seller motivation (Glower et al., 1998), homeowner equity on home (Genesove and Mayer, 1997), and asking price and its effects on selling time and price (Yavas and Yang, 1995). They also try to show how the characteristics of the market such as transaction costs affect search behavior.

One-sided search models have tremendous methodological and theoretical defects which are defended as necessary simplifications for the sake of estimating the model results. As a result, they only focus on explaining the relationship between the asking prices, sale prices and the time that houses stay on the market. Sooner or later, market clearing conditions are satisfied, and equilibrium prices are reached such that they cannot provide any possible explanation for the boom-bust cycles which are one of the most important characteristics of the housing markets.

The main objective of random matching models[12] is to replicate the stylized facts of the housing market observed in the data which one-sided search models are not capable of capturing and explaining. The commonly accepted stylized facts in search and matching literature (Diaz and Jerez, 2013) are considered to be as follows: houses are illiquid assets so that they cannot be sold quickly at the market price, average time on the market is highly volatile and negatively correlated with prices so that adjustment in the housing market takes place through prices, quantities and the degree of liquidity. Moreover, boom-bust cycles are caused via some external shocks by affecting the frictions in the housing market, such as vacancy rate of houses.

**Basic Framework for Random Search and Matching Models**

Following the example by Lisi (2013), this section provides a standard theoretical matching framework with random search and prices determined by Nash bargaining. In the

[12]See Diaz and Jerez (2013); Krainer (2001); Lisi (2013); Wheaton (1990).

model, a seller can become a buyer at the same time, so that they own at least two houses. Price dispersion in the market exists only assuming a different number of houses per capita.

Sellers (**s**) hold $h \geq 2$ of houses which $(h-1)$ are on the market creating some vacancies (**v**) given by $v = (h-1)s$. Buyers (**b**), already hold a house, $h = 1$. They assume two different buyer states in the search process, $i \in \{n, o\}$ where $n$ = homeless-buyer and $o$ = homeowner-buyer.

The value of an occupied house is the selling price $(P)$, therefore expected values of a vacant house (**V**) and of finding/buying a house (**H**) are given by the following equations:

$$rV = -a + q(\theta) * [P - V] \tag{1.1}$$

$$rH = -e^i + g(\theta) * [x - H - P] \tag{1.2}$$

where $r$ is the interest rate, and $\theta = v/b$ is the overall market tightness from the sellers' point of view, while $q(\theta)$ and $g(\theta)$ are the probability of filling a vacant house and of finding/buying a home, respectively. Parameter "a" is the cost sustained by sellers for the advertisements of the vacancies; while "$e^i$" is the effort in monetary terms made by buyers to find and visit the largest number of houses with $i \in \{n, o\}$ with an assumption that $e^n > e^o$. The buyer obtains some benefit $x$ ("abondening the homesearch value") from the property if they decide on transaction. Therefore, the standard matching function with constant returns to scale is represented by[13]

$$m = m(v, b) \tag{1.3}$$

Following this framework of housing markets with search frictions, endogenous vari-

[13]The same matching function is also adopted by many others (Diaz and Jerez, 2013; Genesove and Han, 2012; Leung and Zhang, 2011; Novy-Marx, 2009; Peterson, 2012; Piazzesi and Scheneider, 2013). Properties of the matching function are as follows: $\frac{\partial q(\theta)}{\partial \theta} < 0$ and $\frac{\partial g(\theta)}{\partial \theta} > 0$. By definition, $\lim_{\theta \to 0} q(\theta) = \lim_{\theta \to \infty} g(\theta) = \infty$, and $\lim_{\theta \to 0} g(\theta) = \lim_{\theta \to \infty} q(\theta) = 0$. These explain how markets with frictions require positive and finite tightness, i.e. $0 < \theta < \infty$, since for $\theta = 0$ the vacancies are always filled, whereas for $\theta \to \infty$ the home-seekers immediately find a vacant house.

ables market tightness, $\theta$ and sale price, $P$ are determined simultaneously at the equilibrium. Long term equilibrium condition ("zero profit" or free entry condition) where $V = 0$ in Eq. 1.1, provides an explanation for the relationship between price (P) and time spent on the market (TOM):

$$\frac{a}{P} = q(\theta) \rightarrow q(\theta)^{-1} = \frac{P}{a} \tag{1.4}$$

(with $\frac{\partial \theta}{P} > 0$ since $q(\theta)^{-1}$ is increasing in $\theta$) which means that more vacant houses will be on the market if prices increase. Moreover, with an arrival rate of $q(\theta)$, the expected time on the market (TOM) is $q(\theta)^{-1}$ which means that a higher price causes longer waiting time to sell a house. The sum values of seller's and buyer's after transaction taking place gives the total surplus from the trade given as:

$$Surplus = (P - V) + (x - P - H) = x - H. \tag{1.5}$$

Solving for the optimization problem which takes bargaining power of seller's ($\gamma$) and buyer's ($1 - \gamma$) in to account[14] produce equilibrium selling price, $P$, where

$$P = \frac{\gamma(rx + e^i)}{r + g(\theta) * (1 - \gamma)} \tag{1.6}$$

Based on Eq. 1.6, model tells us that when market tightness increases, the probability of finding a house increases while the sale price declines. Following Eqns. 1.4 & 1.6, model shows the results of interrelation between the behavior of selling price and time spent on the market. Houses with a higher price require more time to spend on the market, while a longer time spent on the market decreases the sale price by mimicking the observed trade-off in the market. The model also demonstrates the relationship between trading volume and house prices via matching function which gives the number of matches in time. As can be seen from the matching function, a high number of vacancies and buyers increase

[14] $P = argmax[(P - V)^{\gamma} * (x - H - P)^{1-\gamma}]$ where $0 < \gamma < 1$.

the matching rate in the market, implying a positive effect on both matching probabilities $q(\theta) = m(v, b)/v$ and $g(\theta) = m(v, b)/b$; however, $v$ and $b$ have an indirect effect on both probabilities. Increasing vacancies reduces the probability of filling the house, while a rise in the number of buyers reduces the probability of finding a house. Therefore, using Eqns. 1.3 & 1.4 the model is able to show the positive correlation between trading volume and prices since an increase in fraction of buyers and sellers increase both the selling price and matching rate.

Basic random matching process has been modified in several ways. For example, Wheaton (1990) deals with housing turnover[15], search, and pricing in the owner occupied market in order to question the efficiency of the housing market in which he conducts a steady-state analysis of the determinants of the vacancy rate and the optimal intensity of search. His model tries to explain how prices and vacancy can coexist and how small changes in supply-demand schedule can have strong impact on the market outcome. Greater turnover (as an acceleration mechanism) creates higher housing prices.

Krainer (2001) develops a model of individual choice under uncertainty and frictions which causes “illiquidity" in the housing market and are considered as the main explanation to market imbalances and price fluctuations. He tries to provide an equilibrium explanation for the observed correlation between housing prices, liquidity, and the volume of sales after a demand shock. He shows how sellers’ high opportunity cost of failing to sell, and buyers’ opportunity cost of mismatch during the boom encourage more rapid sales and create a positive correlation between the illiquidity of houses and price appreciation.

Different from previously mentioned models which assume the rationality and homogeneity of buyers and sellers, Piazzesi and Scheneider (2013) show how a small fraction of irrational-optimistic agents can create a positive momentum in the housing market. Finally, by combining a random matching model with a simple heterogeneous agent model (SHAM), Burnside et al. (2016) focus on the power of social dynamics in affecting to change beliefs

[15]It is a model of turnover in a sense that home buyers are frequently home sellers.

of different heterogeneous buyers and sellers to create strong boom and bust cycles.

Overall, search and matching models of housing markets reduce the analysis of housing market dynamics to a simple supply and demand analysis of the equilibrium, in which an external demand/supply shock propagates the system by affecting the vacancies of housing and tightness of the sellers-buyers ratio. This process in turn, causes temporary delays in matching process by affecting the sellers' and buyers' time spent on the market and variation in prices. As a result, observed fluctuations in both volume of transactions and prices are caused exogeneously. Moreover, they ignore the heterogeneity and interactions between and within different groups of market participants, i.e. buyers and sellers, during their decision making process, which are proven to be one of the main reasons of out-of equilibrium dynamics in real world[16]. Finally, they neglect the existence of uncertainty which could result from different causes, such as lack of information about the behaviors of other market participants, cognitive inabilities or change in fundamental economic variables etc. Yet, the only uncertainty, if they are ever included in the model, stems from uncertainties about duration of stay in a house or lack of knowledge about the real characteristics of the house. Moreover, the uncertainty is introduced as an exogenous event.

## 1.3 Log Periodic Power Law Models of Market Crashes

The log periodic power law (LPPL) model, which is also known as Johansen-Ledoit-Sornette (JLS) model, carries the idea of critical point and phase transition from statistical physics[17]. The main purpose of these types of models is to predict the crises in different markets based on the "weaker efficient market hypothesis" (Sornette and Johansen, 1997)

[16]Some of the models try to introduce heterogeneity by including an idiosyncratic part in the housing dividend for the seller, such that it is independently and uniformly distributed. The independence assumption refers to independence across agents (See Krainer (2001) as an example).

[17]"Critical points are described in mathematical parlance as singularities associated with bifurcation and catastrophe theory. At critical points, scale invariance holds and its signature is the power law behavior of observables (Ohnishi et al., 2012; Zhou and Sornette, 2008).

[18].

In this model framework, a crash is not a critical (regime shift) point itself but the proximity of the system to the critical point, in which the hallmark of a crisis is believed to follow a power law acceleration of prices (Feigenbaum and Freund, 1996; Gluzman and Yukaloov, 1998; Ohnishi et al., 2012; Sornette and Johansen, 1997; Sornette et al., 1996; Tay et al., 2016). Furthermore, LPPL argues that additional properties and ingredients related to crash should be expected beyond the power law acceleration of prices[19].

The behavior of the system is modeled based on the positive feedback mechanism, which results from cooperation between and within imitative noise traders and rational traders by creating a network effect. According to Sornette (2017), market crashes are caused by the slow build up of long range correlations leading to a global cooperative behavior of the market and eventually ending in a collapse in a short, critical time interval. His key assumption, hence, is that a crash may be caused by a "local self-reinforcing imitation process". This process leads to the bubble if the tendency for traders to imitate their neighbors increases up to a certain point called the "critical point", where many traders may place the same order (i.e. sell) at the same time, thus causing a crash. A crash is not a certain outcome of the bubble but can be realized by its "Hazard rate", which is the probability per unit time that crash will happen in the next instant provided it has not happened yet (Johansen et al., 2000; Sornette, 2017; Sornette et al., 2013; Zhou and Sornette, 2006). Since the crash is not a certain deterministic outcome of a bubble, which could be price-driven or risk-driven, it is rational for investors to remain in the market given

[18]They assume the process of the "emergence of intelligent behavior at a macroscopic scale " that individuals at the "microscopic scale " do not have.

[19]Price has two components governed by different rules. The fundamental component, follows a random walk based on martingales, and JLS adds a log-periodic power law structure into it which is used to detect the bubbles. Dynamics of the bubble component, on the other hand, satisfies a simple stochastic equation: $\frac{dp/p}{\mu_{(t)}dt+\sigma dW-\kappa dj}$, where $\mu_{(t)}$ is the drift, $dW$ is the increment of the Wiener process, $dj$ is the jump process such that $j = 0$ before the crash and 1 after, and $\kappa$ is the loss amplitude related to crash. Dynamics of the jump is governed by a crash hazard rate,$h_{(t)}$ such that expectations of jumps is given by $E_t[dj] = h_{(t)}dt$ such that aggregate effect of noise traders' imitation can be followed by the dynamics of the hazard crash rate , $h_{(t)} = B'|t_c - t|^{m-1} + C'|t_c - t|^{m-1}cos(\omega ln(|t_c - t|) - \phi$. The cosine part captures the existence of possible "hierarchical cascades" of accelerating panic leading to bubble (Sornette et al., 2013).

that they are compensated by a higher rate of growth of bubble for taking the risk of crash, because there is a finite probability of landing smoothly.[20].

Two forces affect agents' opinions. First, they have idiosyncratic signals that they themselves receive, and second, agents tend to imitate the opinions of their neighbors. The first force creates disorder while the second creates order in the market. The crash occurs when order takes over (when everyone has the same opinion for which action to take). In normal times, disorder becomes dominant (when buyers and sellers disagree with each other and roughly balance each other out) (Johansen et al., 2000). Macro level coordination can arise from micro level imitation. To describe the imitation process they employ mean field theory through the hazard rate which is the collective result of the interaction between the agents. The higher the price the higher the hazard rate, and the probability of crash. This process is assumed to reflect the self fulfilling nature of the crises. The idea behind this is that when systems get closer to critical points, local changes propagate over longer distance and the average state of the system becomes more vulnerable to small perturbations because different parts of the system becomes highly correlated (just like the spread of infectious disease). At the vicinity of critical point, the system becomes scale invariant, which means different scales (from smallest scale to the largest) become alike. This is why when the imitation strength of the over all market becomes really strong, local imitation cascades through the scales towards global coordination.

The model tries to capture these characteristics of the complex systems (mostly financial markets) by a power law with an oscillatory expansion, which is called LPPL for the logarithm of price. The simplest version of the LPPL function is

$$ln[p_{(t)}] = A + B|t_c - t|^m + C|t_c - t|^m cos[\omega * log(t_c - t) - \phi] \tag{1.7}$$

where $t_c$ is an estimate of the end of a bubble so that $t < t_c$, $A, B, m$ are coefficients, $\phi$ is a phase constant and $\omega$ is the angular log-frequency. If the exponent $m$ is negative, $ln[p_{(t)}]$

[20]Main building block of LPPL is"Rational Bubbles" model introduced by Blanchard and Watson (1982)

is singular when $t \to t^{-}$ and $B > 0$ ensuring that $ln[p_{(t)}]$ increases. If $0 < m < 1$, $ln[p_{(t)}]$ is finite but its first derivative $dln[p_{(t)}]/dt$ is singular at $t_c$ and $B < 0$ ensuring that $ln[p_{(t)}]$ increases (Zhou and Sornette, 2006).

This equation shows that potential crash at the end of the bubble is proportional to price[21]. The key part of the model is that oscillations become visible in the price dynamics right before the critical date.

Using this framework, they state that bubbles are characterized not by an exponential increase in prices but rather by a faster than exponential growth of price. This phenomenon is basically generated by behaviors of investors and traders that create positive feedback in the valuation of assets leading to unbounded growth ending with a critical shift at a finite time $t_c$. This positive feedback mechanism could be created first, via technical strategies of investors such as option hedging and insurance portfolio strategies etc., or second, they could be caused by imitative behaviors of individuals which leads to a network effect and herding in the system. Critical time $t_c$ is interpreted as the end of the bubble, which is often (but not necessarily) the time of the crash. During the phase of the bubble build-up, the competition between noise traders and rational investors creates deviations around the "hyperbolic power law growth" in the form of oscillations that are periodic in the logarithm of the time of $t_c$. Therefore, prices, during the bubble process, are described as log periodic (hyperbolic) power law (LPPL). The main objective of the model is to predict the most probable time of the crash.

LPPL has been employed to detect different types of market crashes in different regions (i.e. 2006-2008 oil bubble by D. Sornette (2009), Las Vegas real estate bubble by Zhou and Sornette (2008), UK stock market bubble by Zhou and Sornette (2003). For example, Zhou and Sornette (2006) shows that 22 US states (mostly the North East and the West) exhibit faster-than-exponential growth signaling a fast growing real estate bubble between

[21]Sornette and Johansen (1997) propose a more general and complex formula with additional degrees of freedom to better capture behavior away from the critical point based on a finer analysis of the "renormalisation group theory" but it is not discussed here. For further information please see Johansen et al. (2000); Sornette (2017); Sornette and Johansen (1997).

1992 and 2006. Using LPPL model parameter estimations they argue that mid-2006 signals the critical point of the system. Similarly they found two different turning points for the UK real estate market which are end of 2003 and mid-2004 (Zhou and Sornette, 2003).

These models fit the time series data to an LPPL function in order to calibrate the model parameters for the detection of the critical point, $t_c$ . In order to check the robustness of calibration process, they first (generally) employ a GARCH (1,1) model estimated from the time series of log prices. From this GARCH(1,1) model, a large number of "independent data sets" (varies from 2000 to 10000) are generated. After, they employ a Tabu-Search[22] to find the initial values for non-linear parameters[23]. Using the obtained initial values, they find an ensemble of local minima of parameters[24] by using Levenberg-Marquardt algorithm. During the fitting process, they truncate the time interval used in the fitting by removing points and re-starting the fitting process. The procedure is repeated until the full time period is recovered. They found that a year (or more) from the crash is not enough to give any reliable results. After that, fit starts to signal the best fitted date for the crash, $t_c$.

Although the theoretical argument behind LPPL framewok, which analyzes complex dynamical systems as evolutionary, self-correcting mechanisms by taking into account the interaction between heterogeneous rational and noise traders, is promising, the estimation techniques of LPPL could be criticized in several grounds. First, parameter values heavily depend on the time scale and starting date, and they do not provide a convincing argument about their method of choosing the start date for the fitting process. Second, finding the local minima to approximate the global minima in a high dimensional parameter space of LPPL is highly problematic given the high complexity of the systems under consideration. As a result, randomness in the calibration and fitting process, causes inconsistency in the model applications. Moreover, even though LPPL models seem to be successful at approximating

[22]See Cvijovic and Klinowski (1995)

[23]Parameters ; A, B and C are expressed as functions of non linear parameters to reduce the dimensionality of search space.

[24]Sornette et al. (2013) claims that this method is more robust than searching for a single minimum.

critical times for some crises, they failed to approximate the timing for others[25].

## 1.4 Heterogeneous Agent Models (HAMs

Heterogenous agent models consist of two branches, one of which is called “Simple or Stylized Heterogenous Agent Models (SHAMs), while the other is called “Agent Based Models (ABMs)". The main objective of both models is to provide an alternative way of representing the real market dynamics, such as highly volatile prices, boom-bust cycles, and crashes, which are believed to be caused by the interaction among boundedly rational heterogeneous agents. Agent-Based Models (ABMs) are computationally more ambitious HAMs, which are complex, dynamical systems comprising autonomously interacting heterogeneous individuals. While SHAMs are analytically tractable to some extend, ABMs are impossible to follow analytically; therefore, they are constructed as simulations of real world dynamics like markets in small scale.

### 1.4.1 Simple Heterogeneous Agent Models (SHAMs)

Simple heterogeneous agent models (SHAMs) are a new branch of simple stochastic models of interacting traders which are inspired from the models of “multi-particle" interaction in physics (Lux, 1998). The idea is to explain the observed regularities in complex systems such as financial markets through the microscopic interactions of the individuals. Since the emergent macroscopic behavior of complex systems are different from each microscopic component, SHAMs support the least detailed determination of individuals’ behaviors by focusing only on studying a few plausible rules to define microscopic behavior of a large body of interacting individuals, and the emergence of macroscopic behavior of the overall market with a large number of traders. Their main purpose is to explain the “stylized facts" of financial markets such as fat tailed distributions of prices and their time-variations

[25]See Sornette (2017) for more detailed explanation for the results of LPPL application to different crises periods .

as a response to incapabilities of existing models which take efficient market hypothesis as their starting point.

SHAMs do not have a foundation for utility maximization or any other mechanism for decision making process of individuals. Instead, they employ certain behavioral rules in their models where individuals are assumed to be boundedly rational. The starting point is to set a finite number of rules to distinguish the types of agents and to bring the heterogeneity in to the model. The authors generally choose two types of rules to define the forecasting behavior of the agents, such as fundamentalists vs chartists[26]. Fundamentalists set their future expectations for assets and trading strategies based on market fundamentals such as dividends, interest rate, GDP growth, etc. They tend to invest in assets that are undervalued, whose values are below a benchmark fundamental value. Chartists[27] on the other hand, do not take market fundamentals into account, instead, they base their expectations about future asset prices and their trading strategies upon historical patterns in past prices such as trends, and exploit these patterns. Once the set of rules whether fundamentalist, chartist, or some combination of the two is established, how agents decide amongst the rules is specified. Their demands and the set of market prices or equilibrium prices are determined, depending on what type of equilibrium concept they are using. The choice of forecasting rule is based on the success of such rules in the past, but that success is determined by how many people follow the rules. The interaction between these different types of individuals and switching between different rules (Hommes, 2006), therefore, could set mechanisms such as herding behaviors, and positive feedback loops, which can cause endogenous fluctuations in prices, returns and market crashes.

Estimating an heterogeneous agent model with endogenous switching of behaviors

[26]Some other SHAM models prefer Rational Arbitrageurs vs Noise Traders (DeLong et al., 1990; Shleifer and Summers, 1990), Informed Arbitrageur vs Positive feedback traders or Rational optimizer vs simple imitator (Brock and Hommes, 1998; Lux and Marchesi, 1999; Sethi and Franke, 1995; Zeeman, 1974)

[27]Chartists in SHAMs are represented as Noise traders in LPPL models of previous section but imitative behavior of trend following noise traders in LPPL is epidemiological where agents infect each other and cause network clusters. On the other hand, in SHAMs, they are introduced as an alternative behavioral rules for the switching mechanism based on relative performances of different strategies.

introduced by Brock and Hommes (1998), Boswijk et al. (2007) explains the dot.com bubble as being triggered by economic fundamentals (i.e. optimistic news about the markets) amplified by a switch to trend following behaviors of the market participants. Similarly, Bolt et al. (2014) applies this framework to housing markets[28] by introducing heterogeneous expectations in to a standard asset pricing model[29] linking housing rentals to buying prices. They show that data for eight different countries support the heterogeneity in expectations with temporary endogenous switching between mean-reverting behaviors of fundamentalists and trend following chartists based on their relative past performances.

SHAMs are often highly non-linear. For example, weights of the fractions of the different traders are time dependent so that they can generate complicated fluctuations for a range of parameters (Hommes, 2006). Chaotic SHAMs with chaotic asset price fluctuations around a fundamental, may explain excess volatility in the market. A disequilibrium model with speculators by Beja and Goldman (1980) allows for limit cycles. Zeeman (1974), is able to demonstrate the possibility of a sudden market crash in a model similar to Beja and Goldman (1980) with a nonlinear reaction function for chartists. Again, based on Beja and Goldman (1980), Chiarella (1996) shows the possibility of periodic oscillations around a fundamental price if chartists excess demand gets sufficiently far from equilibrium case.

**Basic Framework for SHAMs**

Following the study by Kouwenberg and Zwinkels (2014)[30] this section presents a simple heterogeneous agent model. The model introduces three types of agents; consumers, investors and constructors[31], and provides demand functions for consumers ($D_t^C$), fundamentalist investors ($D_t^f$) and chartist investors ($D_t^c$) where total investors demand function is

[28]For more housing market applications, see Dieci and Westerhoff (2012); Eichholtz et al. (2015).

[29]In the model the annual cost of home ownership (“imputed rent") is equal to the housing rent, and future excess return on investing in housing is the the “sum of capital gains minus mortgage/maintenance costs and saving on rent" (Bolt et al., 2014).

[30]This is a multi-agent complex SHAM to capture the boom-bust cycle in the US housing market where individuals are heterogeneous, adaptive and boundedly rational such that market efficiency is not given.

[31]Consumers and investors are the demand side while constructors are the suppliers.

($D_t^I = D_t^f + D_t^c$), supply function for constructors on the other hand, is $S_t$. Investors choose between two rules to estimate the future expected returns, $E[R_{t+1}]$. First rule representing fundamentalist's behavior is set based on mean reversion of market log price ($P_t$) to log fundamental price ($F_t$ such) that :

$$E_t^f(R_{t+1}) = \alpha(P_t - F_t), \alpha < 0 \tag{1.8}$$

where $\alpha$ is the speed of mean reversion. Since the fundamentalists are assumed to be mean variance maximizers with the same level of risk aversion $\eta$ and have same level of beliefs about the conditional variance of housing returns $\sigma^2$, speculative demand of fundamentalists is represented as a linear function of their expected returns as: $D_{t+1}^f = \frac{1}{\eta\sigma^2} E_t^f(R_{t+1})$.

The second behavioral rule for chartists takes auto-correlation in returns into account such that their expected returns take the form

$$E_t^c(R_{t+1}) = \beta\Big(\sum_{l=1}^{L} R_{t-l+1}\Big), \beta > 0 \tag{1.9}$$

where $\beta$ is extrapolating factor, and $L$ is the number of lags since chartists expect past price changes to continue. Their speculative demand function hence, becomes a linear function of past housing returns as: $D_{t+1}^c = \frac{1}{\eta\sigma^2} E_t^c(R_{t+1})$.

The model also sets a rule for the switching between two forecasting rules (Eqns. 1.8, 1.9) depending on past performances such that weight of the fundamentalists is[32] :

$$W_t = \left(1 + e^{\gamma\left(\frac{\pi_t^f - \pi_t^c}{\pi_t^f + \pi_t^c}\right)}\right)^{-1} \tag{1.10}$$

(and the weight of chartists is $1 - W_t$) where $\pi_t^f$ and $\pi_t^c$ are the observed errors over the recent past of the fundamentalists and chartists and $\gamma > 0$ is the sensitivity of investors to differences in forecast errors.

[32]Some other SHAMs models set the rule for switching based on the distance between fundamental value and market price (Dieci and Westerhoff, 2012)

Total demand for housing becomes weighted average demand of fundamentalists and chartists $D^I_{t+1} = W_t D^f_{t+1} + (1 - W_t) D^c_{t+1}$

Performance of different strategies are therefore, calculated based on $\pi^f_t$ and $\pi^c_t$ as:

$$\pi^f_t = \sum_{k=1}^{K} |E^f_{t-k}(R_{t-k+1} - R_{t-k+1})| \tag{1.11}$$

$$\pi^c_t = \sum_{k=1}^{K} |E^c_{t-k}(R_{t-k+1} - R_{t-k+1})| \tag{1.12}$$

where $K$ represents previous periods. This tells us that higher values of observed forecast errors and positive sensitivity $\gamma$ lead investors to assign more weight to better performing rule.

Based on this set up, log price change, which depends on excess demand and some random error $\epsilon_t$, represents impact of pure noise traders as:

$$P_{t+1} - P_t = f(D^I_{t+1} - S_{t+1}) + \epsilon_{t+1} \tag{1.13}$$

After setting up forecasting rules and solving for the price changes, the empirical model is represented by a discrete time dynamical system of four equations which consist of excess returns $R_{t+1}$, fundamentalists' weight $W_t$, and observed forecast errors of fundamentalists and chartists $\pi^f_{t+1}, \pi^c_{t+1}$, and parameters are estimated by applying maximum likelihood estimation using the US housing market data between 1960-2014. Results of their model simulations show how the demand side of the economy pushes the prices upwards and then dominating weight of noise traders extrapolates the price movements further away from the fundamental value between 1980-2007 while fundamentalists take over the market between 2008-2009 causing prices to fall back down.

Compared to search and matching models of the housing markets, SHAMs provide a more realistic explanation for price fluctuations in the market by endogenizing demand and supply dynamics through a switching mechanism between different behaviors of two

types of heterogeneous agents. This way, they are also able to bring the interaction among individuals into the picture. However, introducing a restricted amount of stationary ad-hoc behavioral rules - generally not more than two-, to impose only two types of agents to represent a more complex interactions in the real markets, still fails to provide an adequate framework to capture and understand the dynamics of social systems.

### 1.4.2 Agent Based Models (ABMs)

Agent based models[33] are known as more complex computational heterogeneous agent models, which are analytically intractable compared to SHAMs. Since ABMs are hard to follow analytically, they are constructed as simulations of real world dynamics like markets in small scale (Lebaron, 2006). The main purpose of the ABMs is to explain the macrodynamics of a system via modeling detailed interactions between a large number of individuals, and capture how the observed patterns are formed when the economy is out-of-equilibrium (Arthur, 2006) [34]. Imitating the real world, ABM of a market includes a large number of heterogeneous buyers and seller who act independently, making decisions of buying and selling based on their information and behavioral traits. For example, if an ABM has 20000 agents, there will be 20000 of actual separate entities engaging in trade with one another. This means that equilibrium in ABMs is not imposed or assumed, but arises by itself. Agents in these models are equipped with a set of properties and algorithmic behavioral rules. The properties reflect the state they are in, and rules determine their actions. For example, an economic agent may have income and consumption as properties, while her search strategy on the market is a behavioral rule. Agents do not necessarily know others' behavioral rules or properties. Based on the setting, behavioral rules let individuals to be goal-seeking and adaptive to changes in the environment of the model.

ABM researchers criticize SHAMs for their representation of noise traders vs rational

[33] See (Alfaranso et al., 2005; Arthur et al., 1997; Erlingsson et al., 2013; Ge, 2017; Geanakoplos et al., 2012; Palmer et al., 1994).

[34] Out-of Equilibrium markets may converge or shows patterns of equilibrium but this could be a special case (Arthur, 2006).

traders (Arthur, 2006; Arthur et al., 1997; Palmer et al., 1994). They argue that the stationary decision making process of noise traders in which they do not learn from their previous mistakes, is not a realistic representation of the real world. They also believe that the assumption of rational agents' perfect insight about behaviors of noise traders and other rational agents, is far from reality. Instead, ABMs aim to bring an evolutionary framework to explain how heterogeneous individuals' set their expectations by adapting the changes in the market formed mutually by others' heterogeneous expectations. Hence, dynamic heterogeneity which is represented by "a distribution of agents across a fixed or changing set of strategies" plays a critical part in ABMs (Lebaron, 2006). Individuals try to optimize their actions by taking into account others' decisions but the state space of the decisions are too complicated to find an optimal solution most of the time, which in turn, brings the existence of bounded rationality of individuals and the modelers into ABM framework. These heterogeneity of individuals' expectations may give rise to either homogeneous rational expectations and efficient markets or a collective behavior, such as herding, can occur causing boom-bust cycles and following crashes.

Designing an ABM necessitates certain steps (Lebaron, 2006): They first choose what types of preferences will be available to agents (e.g. simple mean, variance preferences, myopic vs inter-temporal etc.) with certain behavioral components (e.g. loss aversion). Second, they decide the mechanism for price determination[35]. Then they choose a learning process for individual decision making. The most commonly used tool is called Genetic Algorithm (GA) with evolutionary strategies which is an optimization technique[36]. After setting up a learning mechanism for agents, the most delicate issue is how to present a large data set into a trading plan for individuals. They generally prefer to define different variables to convey the information as trading plans in the models. Besides, how to

[35]i.e. Slow price adjustment which market is never in equilibrium (Day and Huang, 1990); market clearing price mechanism (Arthur et al., 1997); a mechanism in which individuals declare their offers for transaction (Farmer et al., 2005); mechanism with a randomly existing and trading agents of trade is beneficial (Albin and Foley, 1992).

[36]See Alfaranso et al. (1992) for more detailed explanation.

signal informational changes to which agents and how frequent it should be, cause other complexities in the model[37]. Another model question is how to introduce social learning to the model. In some cases, agents only interact with each-other via prices but some other cases some form of communication rules are introduced. After these steps, model parameters are calibrated based on real data, so that the model would match the observed historical patterns. Finally, in order to control and validate the evolution of the model, modelers set up a baseline model (Alfaranso et al., 2002), which converges to a well-defined path (i.e. rational expectations, market clearing equilibrium), to compare more realistic scenarios for the market dynamics.

**A Basic Framework for ABMs : Artificial Market Models**

Palmer et al. (1994) and Arthur et al. (1997) employ an evolutionary approach[38], which agents generalized patterns from their past to be guided for their future decisions by taking into account others' expectations and adopt to new conditions in their environment using their inductive reasoning[39]. Here, expectations endogenously co-evolve and are co-created based on individuals adaptations to the changes in market environment. As a result, the market may either converge to the rational expectations efficient market outcome or more complex self-organizing patterns such as bubbles.

In this example, following a neoclassical asset pricing theory, market price is first declared by a market maker to all participants such that participants can find a matching rule for their decision to sell or buy. It is then set to adjust market state reflecting the excess demand-supply conditions based on

$$p_{t+1} = p_t + \lambda(B_t - S_t) \tag{1.14}$$

[37]To overcome these difficulties some uses artificial intelligence technologies such as "classifiers" (Arthur et al., 1997)

[38]Evolutionary approach cannot be solved analytically, therefore, only way to reach a solution is computational with possible algorithms for learning and adaptation.

[39]In contrast to deductive reasoning of normative orthodox models which they dictate how the markets should behave, ABMs take inductive reasoning approach (bottom-up) to show how markets behave in reality.

where $\lambda$ measures the sensitivity of agents to market depth (i.e. how easy to find a match), $B_t$ and $S_t$ represent buyers and sellers.

There are $N$ heterogeneous agents who decide on their portfolio independently based of a utility function. There are only two assets, risk free bond with a constant interest rate $r_f$ and infinite supply and risky stock paying a stochastic dividend following an AR(1) process as

$$d_t = \bar{d} + \rho(d_{t-1} - \bar{d}) + \mu_t + \epsilon_t \tag{1.15}$$

where $\mu_t = N(0, \sigma_\mu^2)$ such that price of a stock $p_t$ is endogenously determined. Individuals demand for a stock based on

$$x_t^i = \frac{\hat{E}_t^i(p_{t+1} + d_{t+1}) - p_t(1 + r_f)}{\gamma \hat{\sigma}_{p+d,i}^2} \tag{1.16}$$

where $\gamma$ represents the risk aversion and agents forecast[40] their future returns based on a linear rule,

$$\hat{E}_t^i(p_{t+1} + d_{t+1}) = a_j(p_t + dt) + b_j \tag{1.17}$$

where $j$ is the rule chosen by individual $i$. They get a temporary price equilibrium by setting the total number of shares to a fixed value; therefore, after the price is set agents are allowed to change their portfolio and trading volumes, which are recorded at each time.

After the rules for preferences and forecasting are set, they introduce a learning mechanism for agents using “Genetic Algorithm (GA)", which is implemented with a certain probability by agents . This parameter tells us about the learning speed of the agents who update their rules by estimating the strength of each rules. This produces an evolutionary dynamic which result in mutation or extinction of the rules. Simulations completed by

[40]Agents use classifier systems to predict mean and variance of the stock returns. A classifier rule is represented as bit-string and a parameter vector such as $(\#, 0, 1\#; a_j, b_j, \sigma_j^2)$ where the first part represent the current condition in the market , 1 would match the true condition, 0 represents the false, # means no match ( neither false nor true) (Alfaranso et al., 2002; Palmer et al., 1994). Given the rule, model searches to find an actual match to the system. Through time the matched rules are updated based on the forecast accuracy.

Arthur et al. (1997) runs for 250000 periods with N=25 agents and M=100 predictors based on J=12 market descriptors. Their results show that inductively rational agents can exist both in two different regimes. In the first regime, the market settles into rational expectations equilibrium if the rate of exploration of different forecast is low. In the second regime, with a more realistic rate of exploration market dynamics become more complex such that psychological components play a significant role, and bubbles and crashes are observed indicating more realistic features of price movements.

Agent based models mostly focus on simulating stock market behavior, but there are some other applications whose main focus is to explain housing market behaviors. Axtell et al. (2014) develop an ABM of the housing market for Washington DC metropolitan area by simulating a one-to-one map of the actual housing market to understand the housing market boom and crash of 1997-2009 by encompassing the empirical heterogeneity in households' expenditures on housing, borrowing conditions, and demographic variables. They initialize their model by making an initial guess at matching households to houses and running the model for a number of periods using 1997 data until it settles down into a more or less steady state. Then they simulate the period of 1997 to 2009. Their results show that the main factor of 1997-2010 boom and bust was the leverage in the market. Erlingsson et al. (2013) develop a macroeconomic ABM model including a housing market which is susceptible to endogenous crashes due to over borrowing by households. A paper by Ge (2017) proposes an agent-based model of housing markets, which centers on the behavior of speculators and leniency of the mortgage system.

ABMs have been criticized in several aspects. First, they are too ambitious in a sense that they cannot be analyzed analytically. For example, they create strong non-linearity and stochasticity in the individuals' behaviors which are made of multiple components connected through complex interactive networks, such that it is hard to capture causality between the behavior and the outcome (Sornette, 2014). Moreover, due to a large number of parameters used to define the rules for individuals and the states of the model, calibration

process suffers from high dimensionality (Foley, 2017), which in turn, causes amplification of simple data errors in calibrated parameters (Alfaranso et al., 2002). Additionally, since the results of these models are generated by the rules and structures, which are highly depended on the modeler's personal taste, different ABMs generate different results for the same problem. This, as a result, causes biases and lack of robustness in the model.

## 1.5 Quantal Response Statistical Equilibrium Models (QRSE)

Quantal Response Statistical Equilibrium models (QRSE), recently introduced by Scharfenaker and Foley (2017b), provide a method to explain the observed regularities in highly complex macroeconomic systems by taking into account social interactions between a large number of heterogeneous market participants and their reactions to changes in macroeconomic market variable(s). Their main objective is to analyze the market dynamics in a simple but realistic way without relying on ad-hoc normative assumptions. They aim at capturing and representing the reality based on available information gained from the data and related economic theory. In order to achieve this goal, they employ the Maximum Entropy Program (MEP) introduced by Jaynes (1957, 1978a, 2003), in which the concept of statistical equilibrium from the field of Statistical Mechanics (Boltzman, Gibbs and Maxwell) along with Information theory (Shannon, 1948) are combined in order to explain the observed market behavior of the system. QRSE, therefore, suggests an alternative framework to modeling real market dynamics, which traditional models based on the standard notion of equilibrium often are not capable of capturing (Scharfenaker and Foley, 2017b; Yang, 2017).

QRSE models are built upon three main concepts: Statistical Equilibrium of targeted economic variable(s) representing the macroeconomic state of the system, the assumption of Quantal response behavior of the market participants , and the maximum entropy principle as a method of inference.

Deviating from the traditional concept of general equilibrium, information theoretic

*statistical equilibrium* [41] takes equilibrium as a probability distribution of a macroeconomic variable (i.e. prices, profit rates) over all combinations of the possible states of the system. This probabilistic approach is thereby, able to capture the "endogenous fluctuations" around a central tendency simultaneously, as the result of the equalization process. The statistical equilibrium distribution of the macroeconomic variables therefore, takes the form of a single peaked (laplace-like) distribution. The fluctuations around the peak (i.e. mode of the distribution) are argued to be resulted from the transactions taken place at the out of equilibrium prices. This means, total gains and losses from market transactions do not cancel each other out[42]. The main goal of information entropic statistical equilibrium approach is to offer a general framework to model macroscopic (macroeconomic) behavior of a system without relying on strong assumptions about the detailed microscopic behavior of the individual components (i.e. market participants)[43].

The second important concept in QRSE is the maximum entropy principle (MEP)[44] (Jaynes, 1957, 1978a, 2003), which serves as a method of inference, allows modeler to estimate the statistical equilibrium distribution of the system. In implementation, MEP can be expressed as a constrained maximization problem. For the discrete case, where the observed discrete variables are $x_k$, $x = 1, 2, ..X$, the maximum entropy program can be

[41]It is called information theoretic statistical equilibrium because, as it is first realized by (Jaynes, 1957, 1978a, 2003), information entropy, provides an alternative way to interpret entropy concept from the statistical mechanics. Entropy in statistical mechanics represents a measure of the number of micro-states of the components of a system (e.g. gas molecules) consistent with a macro-state described by the distribution of a variable, while it is interpreted as a measure of uncertainty in information theory. Shannon (Information) Entropy of a probability measure, $p_i \geq 0$, on a finite set of variable $X = x_1, x_2....x_n$ is defined as the "expected value of information content" , $I[p] = log[1/p]$: $H(p) = -\sum_{i=1}^{n} p_{(x_i)} log[p_{(x_i)}]$ , where $\sum p_i = 1$. As a result, the higher the entropy is, the higher the uncertainty in the system. For more detailed information see Yang (2017).

[42]Walrasian equilibrium process with an auctioneer, on the other hand, is the case where gains and losses from the market transactions cancel each other out such that market clears. Therefore, fluctuations can only be caused by random shocks (i.e. policy changes, or arrival of news) to the system, but they would not be lasting long since the market instantaneously corrects and clears itself.

[43]For example, models based on traditional economic theory assume complete and transitive set of preferences for "homogeneous" individuals so that they can reduce the problem of providing a micro-foundation for the macroeconomic system to a simple aggregation of all "representative agents' " decisions.

[44]MEP program is argued to be the least biased method among all available techniques, because it only allows the use of available information about the system from the theory and observed data in the form of constraints to the maximization problem. See Golan (2018) for further details, and applications of maximum entropy principle to different economic problems.

set by maximizing the entropy of the distribution of the variables subject to $m$ different constraints on the expectations of the functions $f_m(x_k)$ as:

$$\begin{aligned} \underset{p_k}{\text{Maximize}} \quad & -\sum p_{(k)} Log(p_{(k)}) \\ \text{subject to} \quad & \sum_{k}^{K} p_{(k)} = 1 \\ & \sum_{k=1}^{K} p_{(k)} f_m(x_k) = F_m, m = 1, 2, ...M; M < K \end{aligned} \tag{1.18}$$

where $K$ (such that $k = 1, 2, ..K$) represents all possible states of the system, $M$ (such that $m = 1, 2, ..M$) represents the the constraints of the maximization problem, $p_{(k)}$ is all probability distributions on the finite variables $x_k$, and $F_m$ is the expected values of $f_m(X)$.

The solution to this entropy maximization gives the inferred statistical equilibrium frequency distribution with $M$ parameters is

$$p_{(k)} = \frac{e^{\lambda_1 f_1(x_1)+...+\lambda_m f_m(x_k)}}{\sum_{k=1}^{K} e^{\lambda_1 f_1(x_1)+...+\lambda_m f_m(x_k)}} \tag{1.19}$$

where $\lambda_m$ represents the Lagrangian multipliers. The most crucial part of setting up an entropy maximization program is to come up with a set of constraints that are relevant to the system under consideration. The constraints could be obtained from the available data series representing the system (e.g. introducing mean or variance of the data as a constraint), or the theory that helps to explain the dynamics observed in real world (e.g. theory of competition can be introduced as a constraint). This, in turn, helps researchers to deal with the problems related to data (i.e. lack of high quality data)[45].

The maximum entropy statistical equilibrium approach has been used to model different markets. A maximum entropy-statistical equilibrium model of commodity markets as an alternative to Walrasian equilibrium models is introduced by Foley (1994), in which the

[45]For example, the main difficulty of modeling the effect of individuals' actions in the market, is to get a detailed data representing individual's actions, e.g. as buying and selling assets. Moreover, economic phenomena are unique in their nature most of the time, such that it is not possible to have repetitive events to have a large enough data sets to get robust results.

assumption of market auctioneer is removed from the picture. By doing so, he is able to show how complex interactions of economic agents in a "decentralized" commodity market, where transactions can take place at different price ratios than market clearing prices, give rise to observed regularities in transactions[46]. Stutzer (1996) develops a statistical equilibrium model of asset pricing to find the most likely "Gibbs Canonical distribution" of risk neutral asset prices. Assuming no arbitrage opportunities, he first estimates empirical distribution of risk neutral asset prices based on the data to satisfy the martingale measure of the prices. Then, by minimizing the KL divergence[47] subject to the risk-neutral (martingale) measure of the asset price, he estimates the statistical equilibrium distribution of the asset prices, which allows him to replicate the results from Black-Scholes tests in a simpler way.

Finally, the last concept used in QRSE models is the assumption of quantal response behavior of individuals represented as a logit function[48], obtained from maximizing individual's expected payoff function given a mixed strategy over the choices (i.e. buying or selling) subject to a a lower bound on the informational entropy of the frequency distribution describing the mixed strategy by using the maximum entropy program introduced above.[49]. By doing so, QRSE introduces bounded-rationality into the model as a characteristic of economic agents without over-imposing it.

As a result of combining these three concepts, QRSE models introduce a new way of modeling the endogenous dynamics of complex social systems which result from the

[46]See Foley (1996) for the labor market application of his general theory of statistical market equilibrium, Farjoun and Machover (1983) and Scharfenaker and Semieniuk (2017) for an analysis of firm level profit rate, and dos Santos and Scharfenaker (2016) for the application on the Tobin's q.

[47]It minimizes the "entropy deficiency" between the empirical distribution and the estimated probabilities(Golan, 2018).

[48]If the agent chooses a mixed strategy, $f[a|x] : AxX \rightarrow (0, 1)$ over binary actions $a \in A$ (i.e. buying and selling) to maximize her expected payoff, $\sum_a f[a|x]u[a, x]$, subject to a constraint on the entropy of the frequency distribution describing the mixed strategy which represents the uncertainty in their decisions, the result gives the Logit (Gibbs) function; $f[a|x] \propto e^{\left[\frac{u[a,x]}{T}\right]}$ where $u[.]$ is the payoff function of the agent, $f[a, x]$ is the joint frequency distribution, $f[a|x]$ is the conditional action frequency distribution and $T$ is the responsiveness parameter of the individual which tells us how responsive the agent is to reacting the any changes in $x$, outcome variable( i.e. price changes).

[49]According to Foley, this maximization problem can be explained as a process of "satisfising" (Simon, 1957) (rather than optimizing) the expected payoff in which the individual gains some level of satisfaction from her action.

interactions among a large number of heterogeneous, goal oriented individuals' behaviors without using any ad-hoc assumptions. Earlier applications of maximum entropy statistical equilibrium approach mentioned above, have been criticized due to their lack of providing economically meaningful reasoning for the choice of the constraints. The main motive behind this criticism is the necessity of emphasizing the difference between modeling the behavior of physical components of the physical world (i.e. gas molecules) and the behavior of the goal oriented purposeful socially interacting agents of the social systems. QRSE aims to overcome this deficiency by introducing economically/socially meaningful constraints in the form of conditional action and outcome frequencies representing the real world dynamics of observed systems based on the theory and data which in turn, provides meaningful interpretation of model parameters.

**Basic Framework for QRSE Models**

First QRSE model is introduced and applied by Scharfenaker and Foley (2017b). In their application, they show how the social interactions between the US firms based on profit rate differentials give rise to observed patterns of profit rate by employing the QRSE model given the data, and the theory of competition by Adam Smith. They set up their model based on the relationship between exit/enter decisions of the firms in each sector, $A \rightarrow \{a = enter, \bar{a} = exit\}$ as the binary action variable, and the aggregate profit rate, $x$ as the outcome variable. They infer the maximum entropy-statistical equilibrium joint distribution of the $x$ and $A$, $\hat{f}[A, x]$ such that they also derive the marginal distribution of the profit rate $(x)$ and corresponding conditional outcome and conditional action distributions $f[A|x]$ and $f[x|A]$ by maximizing the the entropy of the joint distribution of the system, $f[A, x]$.

To set up their model, they first, assume that during their decision making process, firms have some uncertainty about assessing the profit rate in a particular region, and they express this as a constraint on their action, which is represented by a quantal response logit function

as; $f[a|x] = \frac{1}{1+e^{-\frac{x-\mu}{T}}}$. Parameter $\mu$ determines the point where the firms are indifferent between entering and exiting (50-50% probability of enter and exit). $T$, on the other hand, signals how responsive are the firms to changes in profit rate to enter-exit in a particular sector.

They introduce another constraint based on the theory of competition of Adam Smith, which they argue that firm's enter and exit decisions create a negative feedback mechanism in the sector such that if a firm enters a sector to maximize its expected profit rate, it causes a decline in the profit rate in that sector. The opposite is true when the firm exits the sector. This constraint is represented as the difference between the expected conditional expectations of profit rate weighted by the marginal probability of enter and exit actions by setting up a bound on the degree of negative feedback (effectiveness of competition), $\delta$

$$E[x|a]f[a] - E[x|\bar{a}]f[\bar{a}] = \delta \tag{1.20}$$

Combining these two constraints and including an additional mean constraint to the maximum entropy program, they inferred following statistical equilibrium (marginal) distribution of the profit rate:

$$\hat{f[x]} = \frac{e^{H_{\mu,T}[x]} e^{-\beta Tanh\left[\frac{x-\mu}{2T}\right]_x} e^{\gamma}}{\sum_x e^{H_{x,\mu,T}} e^{-\beta Tanh\left[\frac{x-\mu}{2T}\right]_x} e^{\gamma}} \tag{1.21}$$

where $H_{\mu,T}[x] = -\sum_{A=\{a,\bar{a}\}} f[A|x] log[f[A|x]$ is the binary entropy function, and $Tanh[.]$ is the hyperbolic tangent function. The resulting recovered statistical equilibrium distribution (Eq. 1.21) fits the observed data pretty well[50]. Their estimation results prove that the observed patterns in the profit rate data can be explained as the result of the relationship between mechanisms produced by the interaction of a large number of competitive firms' quantal actions and the resulting feedback effect of those actions on the profit rates in the

[50]They use the "information distinguishability (*ID*) statistics introduced by Soofi and Retzer (2002), and calculate how well the fitted frequency distribution, $\hat{f}[x]$ captures the observed frequency distribution, $\bar{f}[x]$. They are able to capture approximately 97% of the information content of the observed distribution.

sectors.

Different applications of QRSE model have been studied for different economic problems (Omer, 2018; Yang, 2018) where it seems to produce promising results to explain complex dynamics of different social phenomena. For example, applying the same model framework to the US housing market between 2000-2015 based on the relationship between buying/selling actions of agents and regional house price changes, Omer (2018), is able to capture different characteristics of the most recent US housing market boom-bust cycles and following market crash[51]. The results indicate the existence of a statistical equilibrium for before (2000-2006) and after the crash (2010-2015) periods, while a drastic shift in the statistical equilibrium is captured for the crisis period (2007-2009). Model fits and the estimated parameters prove the existence of the housing bubble before the crash of 2007 as a result of the interrelation between increasing price appreciation due to wrong expectations of individuals' about the expected price increase in their houses and worsening negative feedback mechanism from their selling/buying actions on house price increase. Based on the estimated parameters, which explain the catastrophic shifts in individuals' behaviors and market reaction to them, the crisis period is shown to be an example against the "Rational Expectations Efficient Market Hypothesis" and " Perfect Foresight of Representative Agents" argument by explaining the causes of existing negative fat-tails in the data.

The problem with QRSE models is that they require discretization of the data in the form of coarse-grained bins in order to recover statistical equilibrium joint distribution of action $A$, and outcome $x$. This issue raises a criticism since the estimation of the results are sensitive to bin size chosen by the modeler.

## 1.6 Conclusion

Modeling highly complex social systems such as housing markets is difficult because their dynamics evolve through the interactions among a large number of heterogeneous

[51]This paper will be presented in the next chapter.

individuals under the influence of each others' decisions and possible external impacts (e.g. policy changes, natural events etc.). This is mostly because market interactions and individuals actions are often are not observable to the researcher and the parameter space necessary to model them has high dimensionality, which is not possible to fully account. Moreover, factors such as the lack of high quality data to represent unique non-repetitive social events and the computational inabilities make it even harder. Conventional theory and techniques used to model market behavior seem to ignore the reality, and focus on picturing an idealized world. Therefore, they mostly suffer from over simplifications which are over-imposed to the models through unrealistic ad-hoc assumptions (i.e. perfect rationality of individuals, perfect efficiency of market clearance ).

This paper reviewed different models commonly used in explaining observed price movements with a particular focus on the housing markets. Each tries to provide a more realistic framework to represent market behaviors in order to explain the causal relationships which give rise to large market fluctuations such as boom and bust cycles mostly followed by crises. They question the unrealistic rational expectations hypothesis and the applicability of efficient market hypothesis to the real markets. In order to do so, they introduce some forms of heterogeneity of individuals or house stocks and interactions among economic agents to their models to varying degrees of success.

Search and matching models, for example, reduce the problem of interactions in the markets to the bargaining process between "homogeneous" sellers and buyers. This way, they represent temporary house price movements away from the competitive equilibrium as a result of "exogenous" shocks affecting the demand supply schedules in the market via market frictions, i.e. vacancy rate in the market. Therefore, they do not go far from repeating the results from the conventional theory.

Simple heterogeneous agent models (SHAMs) carry the analysis one step ahead by introducing two different types of individuals and their interactions with each other into the model. They are able to drop the assumption of utility maximizing, perfectly rational

representative agents . As a result, they are able to reproduce endogenous fluctuations caused by the interactions between heterogeneous individuals. However, they use some other unrealistic simplifications, such as stationary rules to represent the behaviors of individuals and their interactions, and the rules to represent the law of price motions which also raise questions about how realistic the model results are in explaining the real causes behind the market behaviors. Computational Agent Based Models (ABMs) solve the problems of SHAMs by providing an evolutionary process for the determination of behaviors of heterogeneous agents. They are able to model a larger scale of more complex interactions between the agents as the main cause of endogenous out-of-equilibrium dynamics of the markets. However, their highly complex and ambitious agenda complicates the estimation process of the model which weakens the robustness of the estimation results. Additionally, since the ABMs are structurally complex, they blur the causal relationships behind the market behaviors.

Different from the previous models, main focus of Log Periodic Power Law (LPPL) models is to predict the timing of the market crash as a critical/regime shift point. They provide a plausible behavioral explanation for the causes of crises, and the main motives behind the model choice, i.e. imitation between noise and rational traders and resulting network effect producing scale free power law distribution of price changes. Yet, their modeling technique is not well established, and the choice of parameterization and the estimations of the parameters are quite random, thus, they weaken the explanatory power of the model.

Finally, Quantal Response Statistical Equilibrium models seem to provide a powerful alternative to deal with the problems that exist in the models previously mentioned by including innovative approaches from the information theory and statistical mechanics. Using the statistical equilibrium approach, in which the equilibrium takes the form of a distribution representing all possible states of the system, QRSE models are able to predict the central tendency of the systems and the fluctuations around it. Therefore, they account

for the fluctuations of a system by the information about the interactions among the market participants, which the researchers can learn about the macroeconomic behavior of the observed system. As a result, QRSE models introduce a general framework for equilibrium where the conventional market clearing general equilibrium exists just as a special and highly unlikely state of the economic systems. The maximum entropy principle as a method of inference, is the least biased method among all, because it maximizes the uncertainty of the system and gets the least informative state with no additional assumptions other than the existing knowledge of the researcher and observed data. Therefore it is a useful tool for dealing with the hardship of modeling complex systems, such as housing markets. Additionally, by utilizing the economic theories in the form of model constraints, it manages to overcomes the problem of data availability that all models studied above suffer from. Finally, QRSE model deal with modeling heterogeneity of interacting agents by introducing an assumption on the behavior of “purposeful" individuals represented as quantal response logit functions in the form of an additional constraint. Therefore, individuals maximize their expected payoffs subject to a lower bound on the entropy of their mixed strategies such that their information processing capacities are subject to some degree of uncertainty. By doing so, QRSE rejects the assumption of rational representative agent, and replace it with boundedly rational typical agent whose preferences do not have to be well-defined, complete, and transitive. QRSE models can capture large fluctuations, such as boom-bust cycles and out-of-equilibrium tendencies of the real economies endogenously, and produce economically meaningful parameter estimation results in a simple but comprehensive way. As a result, they introduce a platform to model social interactions among heterogeneous individuals, uncertainties in individuals' decision making processes, and endogeneity of the system dynamics without imposing any ad-hoc assumptions.

Overall, the main take on from this paper is that both over-simplified and over-complicated representation of the real dynamics in a model can produce unrealistic results about the real complex behaviors of the economic systems that in turn, could be misleading.

In this case, information theoretic QRSE framework seem to be more advantageous than its competitors for providing a simpler but more elegant way of dealing with different economic problems.

# CHAPTER II

# Quantal Response Statistical Equilibrium Model of the US Housing Market, 2000-2015

## 2.1 Introduction

The US housing market experienced one of its key periods of boom-bust cycle between 2000 and 2015, urging researchers to understand and explain the movements of house price changes throughout the cycle. As can be seen from Figure 2.1, between 2000 and 2005, average annual house price growth rates (compound annual price growth rate) based on national and composite-20 Case-Shiller indices, follow an increasing pattern reaching their maximum in the end of 2005. Beginning in 2006, the rapid house price growth reverses with slowing appreciation, followed by a dramatic depreciation between 2007 and 2008. Decline in house price growth rates alleviates in 2009, but did not experience positive growth again until 2012. [1]

As stated by many authors (Gleaser and Nathanson, 2014; Shiller, 2008b) and evident from Figure 2.1, house prices between 2000-2005, seems to enter a positive price feedback

[1] Average annual growth rate, (CAGR) is calculated as:

$$CAGR = \left(\frac{Last\ Month\ House\ Price\ Index}{First\ Month\ House\ Price\ Index}\right)^{\left(\frac{1}{Number\ Of\ Years}\right)} - 1$$

employing inflation adjusted Case-Shiller Monthly National and Case-Shiller Monthly Composite-20 House Price Indices. National Index represents the US housing market as a whole, while Composite-20 Case-Shiller index covers the largest 20 Metropolitan areas in the US.

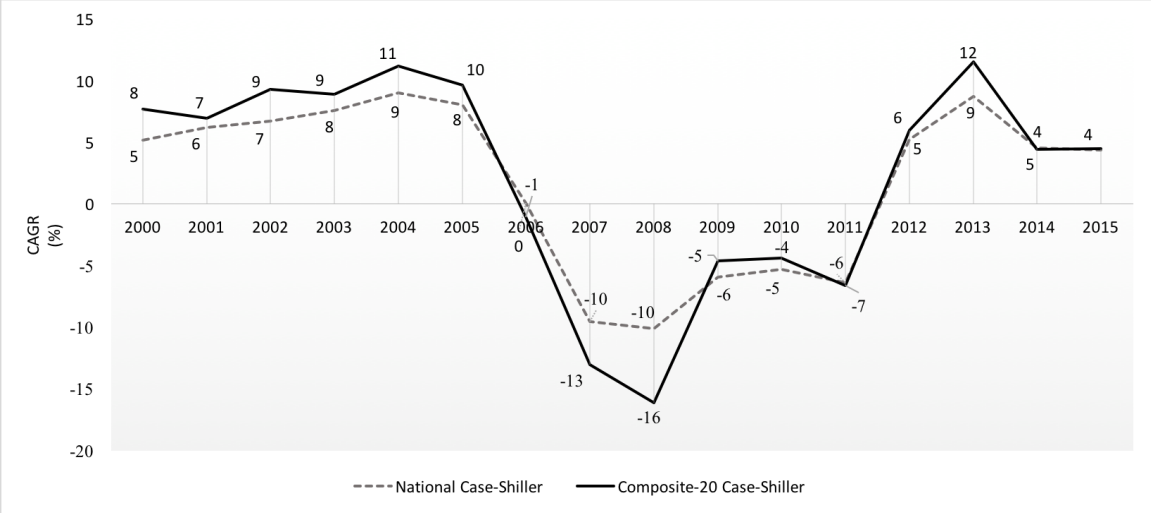


Figure 2.1: **Average Annual House Price Growth Rate (CAGR (%)** Based on Monthly Inflation Adjusted Case-Shiller National and Composite-20 Indices .

loop –in which any house price increase brings about a further rise in house price growth rates, signaling a boom period. After that, between 2007 and 2009, the system catastrophically collapses, in which house price growth rates simultaneously drops nationwide. Finally, recovery begins after 2009, and the system approaches to early 2000's values in 2012. The period 2007-2009, displays a distinct divergence from the periods of 2000-2006 and 2010-2015.

In order to capture the behavior of the US housing market for several sub-periods of before, during and after the market collapse, Figure 2.1 represents the log density distributions of monthly house price growth rates for all available metropolitan areas represented in two different monthly house price indices, (Case-Shiller Housing price index and Freddie Mac Housing Price Index (FMHPI). Distributions have a striking single peaked shape throughout the cycle indicating a central tendency of the system around an average monthly growth rate. Some distortions in the distribution appears to arise at the end of the boom (2000-2006) and during the crash (2007-2009) by pointing out a change in the characteristics of the market behavior.

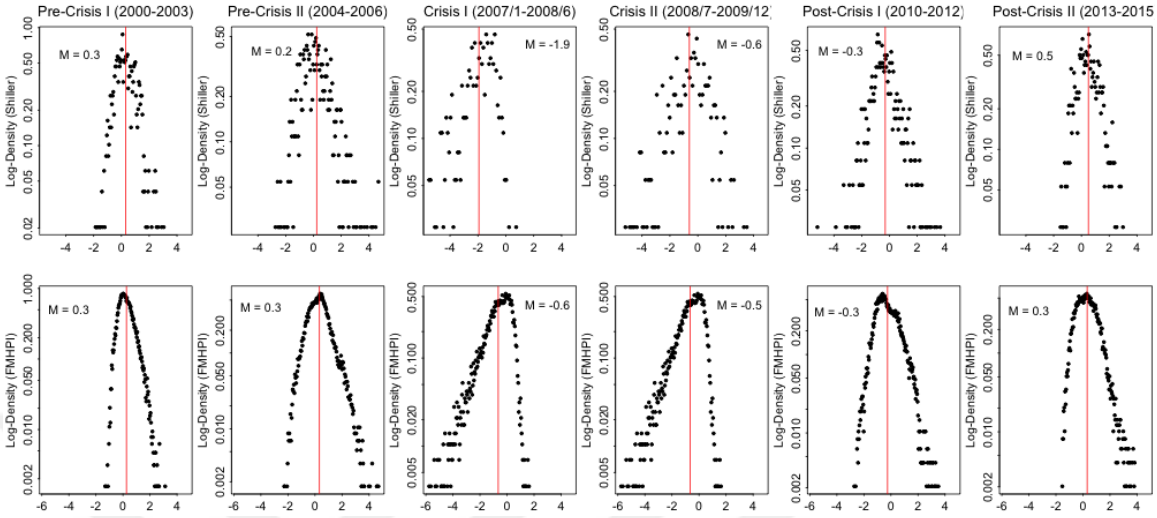


Figure 2.2: **Log Density Plots of Monthly House Price Growth Rates:** . Calculated as the histogram plots on a log density scale of house price growth rates. The first row shows the histogram plots on a log density scale of monthly house price growth rates based on Case-Shiller Price Index growth rates (%) for each 20 largest metropolitan areas while the second row is based on FMHPI, including monthly price growth rates for the DC region and 367 metropolitan areas in the US. “M" represents the average growth rate for the panel data in each window. Each plot includes all the cross sectional data represented by the specified index, and for the specified time period. Pooled house price growth rate data is then used to calculate log density distributions. More detailed explanation about the data sources will be provided in Section 2.2

In this chapter, these observations of the US housing market between 2000 and 2015 are used to study the question;“what forces behind the market dynamics give rise to the boom-bust cycles and the collapse, and how to capture them if they exist?". Here, the concept of equilibrium plays a substantial role distinguishing market behavior of the crash periods from the normal times, both qualitatively and quantitatively. Log density distributions of monthly house price growth rates in Figure 2.2 suggest that a statistical framework may capture the features of the market tendencies which could provide new insights into explaining sub-periods of the most recent US housing market cycle. The concept of statistical equilibrium from statistical mechanics (pioneered by , Boltzmann (1871), Gibbs (1902), Maxwell (1860)

and Jaynes (1957)) allows for relaxation of traditional assumptions of equilibrium.

Statistical equilibrium is a probabilistic approach to analyzing complex systems , which arise from the interaction of large number of individual agents. It allows one to capture the central tendency of the system and fluctuations around that tendency. Based on this approach, this study introduces a Quantal Response Statistical Equilibrium Model (QRSE) of two systemic forces which govern equilibrium/ out-of equilibrium dynamics in the housing market.

To define these market forces, the model sets a behavioral assumption that typical agents, based on the house price changes in the market, try to maximize the expected returns from their transactions subject to a constraint on the entropy of their mixed strategy over transactions. Moreover, the actions of buying/selling in the housing market create a negative feedback effect on the house price growth rates themselves. Therefore, both the central tendencies in the distribution of house price growth rates and the deviations around these tendencies, are the natural results of this process governed by the relationship between the changes in house price growth rates and the actions taken by the individuals in the system.

Based on Figure 2.2, one can argue that periods before and after the market collapse of 2007 indicate a strong statistical equilibrium while the distortion in the equilibrium state starts to rise around late boom period of 2004-2006, and crisis of 2007-2009 signal a strong divergence from the equilibrium. After the crash, the system seems to recover towards a new statistical equilibrium between 2010-2015.

This chapter sets out , (i) analyze the observed regularities in the housing market during the most recent housing cycle based on the statistical equilibrium approach, (ii) provide a model to explain the systemic forces behind the behavior of the housing market for the consequent periods of the cycle, and (iii) combine the model results with a solid economic interpretation. Section 2.2 will present the data sources employed in our analysis, and discuss observed regularities and the divergence from these regularities in the US housing

market based on house price growth rates for a large number of metropolitan areas between 2000 and 2015. In Section 2.3, a maximum entropy method to quantal response statistical equilibrium model by Scherfenaker& Foley (2017) will be applied to the housing market dynamics. Section 2.4 will focus on the results of the model fit estimations for each sub periods and robustness of parameter estimations.

## 2.2 Observed Regularities in the House Price Growth Rates

### 2.2.1 Data

Sampling and modeling techniques of repeat sales house price indices have been largely criticized in the literature (Cho, 1996). In order to set an argument on a sound basis and check the robustness of the results, two different monthly house price index data for several metropolitan areas were used[2]. Monthly Freddie Mac House Price index (FMHPI) and the S&P Case-Shiller Index are employed in this chapter, to understand the housing market price dynamics.

In order to capture the regularities in the data, both data sets are deflated using the consumer price index (CPI). Corresponding real house price indices are then, used to calculate monthly real house price growth rates for each region in both data sets. Hence, inflation adjusted monthly house price growth rate, hence, is calculated as $x_t = \frac{100(g_t - g_{t-1})}{g_{t-1}}$, where $g_t$ is the house price index for time $t$. Growth rates of price indices rather than the

[2]Freddie Mac House Price monthly index, FMHPI, which is available for DC area and 367 Metropolitan statistical areas (MSAs) throughout the US, and the second one is monthly S&P Case-Shiller Index for 20 largest US metropolitan cities called as “Superstar" cities in the literature (Gyourko et al., 2013). Both data sets employ the repeat sales method of index calculation. Compared with S&P Case-Shiller Home Price index, FMHPI has a larger sample size, and covers more geographic areas. FMHPI is based on transactions involving on single family detached and town-home properties purchased by Fannie Mae or Freddie Mac, and is a weighted index, meaning that it measures average price changes in repeat sales or refinancing on the same properties. The S&P index is calculated monthly using a 3 month-moving average algorithm for the data on single-family housing stocks that have sold at least twice in order to capture the appreciation in the values of each sales unit. The most important difference between the two is that the S&P Case-Shiller index is value-weighted which means that price trends for more expensive houses have greater influence on estimated price changes than other homes, whereas FMHPI’s purchase only index weights price trends equally for all properties.

levels, are used in this analysis because the rate of change of housing prices play a more significant role than the levels of prices during the decision making process of economic agents because it provides information about the possible returns on the investment, and since we are dealing with cross sectional data, which each region carries different characteristics across time (i.e. peak levels of house prices, and timing to reach the peak differ among different regions), it is more convenient to use house price growth rates as a metric. Also, the rate of change of prices in logarithmic scale is able to capture the exponential or power law types of distributions from the data since house prices do not follow random walk[3]. Instead, they seem to persist in the short run while there exists a mean reversion in the long run (Case and Shiller, 1989b; Gleaser et al., 2014), and it is important to capture extreme values especially during the market crash.

### 2.2.2 Visualizing Statistical Equilibrium: Patterns of House Price Growth Rates (2000-2015)

As argued by Farjoun and Machover (1983); Foley (1999); Jaynes (1957), and Scharfenaker and Semieniuk (2017), the statistical equilibrium framework (from statistical mechanics) to describe macroeconomic systems, is a highly suitable approach, because laws governing macroeconomic variables such as price changes and profit rates carry a statistical character, so that one must employ probabilistic considerations in order to capture such laws. Statistical mechanics was pioneered by Boltzman(1871), Maxwell (1860) and Gibbs (1902), in order to explain the physical macroscopic properties of matter from the microscopic properties of atoms and molecules which comprise the system as a statistical

[3]Random walk theory pre-supposes the efficient market hypothesis, in which profit maximizing agents are fully rational, and can reach all available information freely in the market. Therefore, on the average, competition will cause the full effects of new information on intrinsic values of assets to be reflected "instantaneously" in actual prices. According to (Fama, 1965), this means:

> "...in a stock market, successive prices in individual securities will be independent. A market where successive price changes in individual assets are independent is by definition, a random walk market... Simply, the theory of random walk implies that a series of stock price changes has no memory. The past history of the series cannot be used to predict the future in any meaningful way. . . "

large population. In developing the kinetic theory of gasses, Boltzmann (1871) derived that pressure and temperature of an ideal gas are a function of the average kinetic energy of its particles. Since determination of each micro-state requires specification of the position and momentum of each gas molecule, which is a cumbersome task due to high degree of freedom for the entire system, the theory of statistical mechanics reduces the problem to analysis of macroscopically observable parameters via a statistical analysis of the molecules of which gas is made up. Probabilistic reasoning, hence allows us to reach informative results concerning the equilibrium behavior of the complex systems without relying on strong assumptions about the detailed microscopic behavior of the individual components of the system (Farjoun and Machover, 1983; Jaynes, 1957).

Based on the same logic, statistical equilibrium in social systems; such as housing markets, also result from the interaction of a large number of economic agents with a certain level of uncertainty in their actions which in turn, collectively have an impact on the outcome variable, such as house price growth rates. Following classical political economy tradition of Smith (1776), and Marx (1894), we argue that individual economic agents seeking for a higher return than the economy-wide average from their transactions in a market would generate a “tendential gravitation" of each sub-market’s house price growth rate as an unintentional result of many individual buyers and sellers. Therefore, their decisions in the process of competing for higher expected returns determine the change in house prices at any point in time creating an average rate of price change in the market as a whole. To illustrate, in a housing market, both buyers and sellers search for the best outcome from their transactions. Buyers tend to buy when a particular sub-market house price growth rate is less than the average growth rate in the market expecting higher returns in the future while the system evolves towards its central tendency. On the other hand, sellers are more prone to sell if the house price growth rate in a region is higher than the market average. The collective result of the actions, in turn, is strong enough to impact the house price growth rate in the market. This means, when sellers sell their properties in a

particular market, their actions set forces for a negative feedback mechanism on the house price change, and house price growth rate declines in the market. The opposite happens when the buyers buy houses so that upward price changes tend to increase. As a result of this statistical process, actual transactions among typical agents give rise to a statistical equilibrium distribution of house price growth rates around an average which the system tends to evolve towards. Concept of statistical equilibrium as a remarkable representation of economic-laws based on classical political economy tradition, hence, can help us to capture the governing forces behind the system's central tendency and deviations from it. This way we can detect and interpret empirical regularities in the housing market.

As briefly mentioned in introduction, Figure 2.2 shows the log density distributions of monthly house price growth rates of the panel data for "pre-crisis periods (2000-2003) and (2004-2006)", "Crisis periods (2007-2008/6) and (2008/7-2009/12)", and finally 'Post-Crisis periods (2010-2012) and (2013-2015)" for two different cross sectional house price index data mentioned above in order to determine statistical equilibrium dynamics in the US housing market. Each plot includes monthly real house price growth rates for each metropolitan area represented in the data for the specified periods[4]. This way, one will be able to observe the evolution of the distribution of house price growth rates; regularities during the statistical equilibrium periods, and the divergence from the equilibrium during the crisis, if they exist.

For both data sets[5], log density distributions of real house price growth rates for the US housing market seem to follow a symmetric Laplace-like single peaked distribution between

[4]Plots for each data set for specified time periods demonstrate the histogram plots on the log scale of monthly real house price index growth rates on the vertical axis. Each plot consists of the pooled data for real monthly house price index growth rates for each and every metropolitan area represented in the data sets. From left to right, first plot called "Pre-Crisis I 2000-2003" covers the time period between 2000/1 and 2003/12 (Year/Month), second plot called "Pre-Crisis II (2004-2006)" includes the period between 2004/1 and 2006/12; third plot called "Crisis I (2007/1-2008/6)" covers 2007/1 and 2008/6; fourth plot called "Crisis II (2008/7-2009/12) includes the data for 2008/7 and 2009/12, the last two plots represent the Post-Crisis periods.

[5]Note: Although each data sets are used for the same purpose in the analysis, some variation between the two is expected because S&P Case-Shiller house price index represents 20-largest metropolitan cities such as Chicago, Las Vegas and New York, while FMHPI consist of DC region and 367 metropolitan areas throughout the US representing a finer grained data. But, results follow similar patterns through time.

2000-2003 and 2010-2015 suggesting a strong statistical equilibrium in the housing market before and after the collapse. Results from FMHPI data emphasizes changes in skewness among the sample periods, while results from the Case-Shiller index data emphasizes the loss of definition of the distribution in the late pre-crisis II and crisis periods. According to results based on FMHPI index , between 2004 and 2006, symmetry in the system starts to deteriorate although the market forces still try to push the system back to the equilibrium. Symmetry in the distribution is not a necessary criterion for the statistical equilibrium, however it can be interpreted as an indicator of rapid appreciation/ depreciation of house prices in some sub markets before/after the crash. When the house market collapses, during 2007-2009 distribution becomes more dispersed for the Case-Shiller House price index signaling a deterioration of market forces whose roles are to push the system to the statistical equilibrium, while FMHPI distributions remains highly skewed to the left implying the increasing house price depreciation in some metropolitan areas.

Another way of examining the central tendencies for house price growth rates is to look at the average price growth rate movements in the system as a whole. This is because when the system is at or near the statistical equilibrium, in the sense that macroscopic conditions and macroscopic parameters, such as average change in prices, appear to be constant or vary comparatively slowly (Farjoun and Machover, 1983). Yet, this stable conditions in equilibrium is still dynamic and statistical, allowing microscopic parameters such as house price growth rate in any sub-market to change rapidly. The system as a whole evolves through time so do the macroscopic parameters. When the system is disrupted by a period of crisis, macroscopic parameters in hand can change drastically.

As Figure 2.3 points out, average monthly price growth rates[6] are quite stable between 2000-2003 around 0.2%-0.34% with a further increase between (2003-2005), while the crisis period shows meaningful changes, in which stability in house price appreciation is

[6]Calculated as the average price growth rate of the pooled cross sectional data for each year for each data sets.

replaced by a drastic decline in growth rates around –1.9% for Shiller house price index[7]. One can argue that the stability of average monthly house price growth rates during the statistical equilibrium of the pre and post-crisis periods indicates some systemic forces in the housing market, which are dysfunctional during the crisis, and also the purpose of this paper to explain.

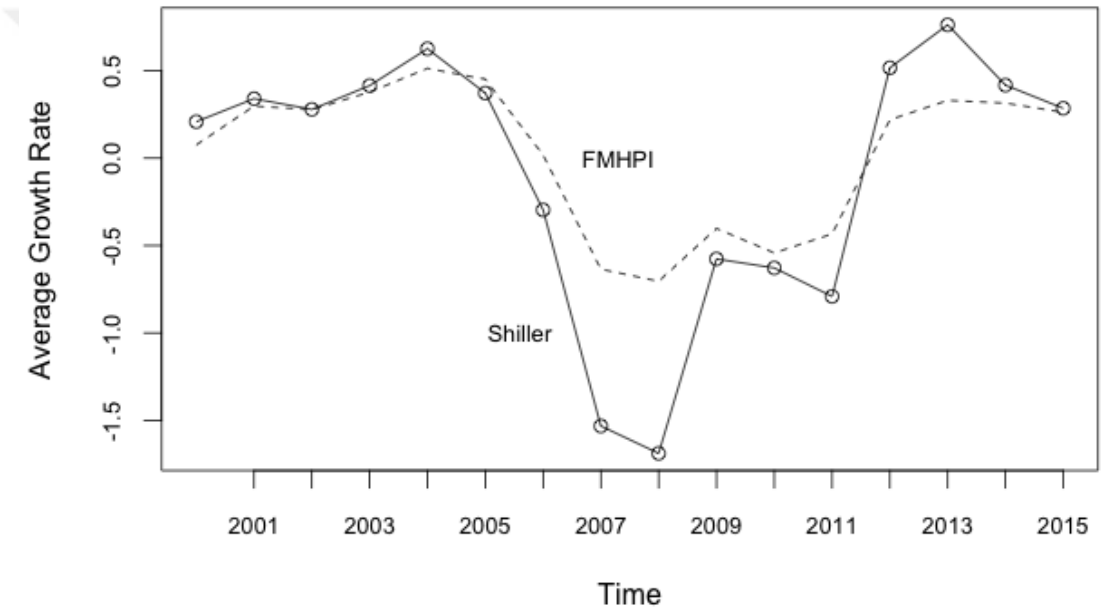


Figure 2.3: **Average Monthly Housing Price Growth Rates (%)**Calculated as the average price growth rate of the pooled cross sectional data for each year.

For the sake of understanding the forces that push the housing market to the statistical equilibrium, it is crucial to keep in mind that housing market dynamics involve social

[7]The decline in average price growth rate is more moderate in FMHPI, which can also be interpreted as the indicator of severity of crisis experienced by different metropolitan areas. This may be because Case-Shiller price index include 20 largest metropolitan cities which experienced the largest price appreciation during the boom, and the largest depreciation during the crisis, while FMHPI covers DC region and 367 US cities causing finer results on average. This might be signaling one of the stylized facts of US housing market, which is that different metropolitan areas experienced different price appreciation (depreciation) before (after) the crisis. This is why focusing on only the national data would leave out some of the most important characteristics of boom and bust cycles in the housing markets.

interactions of a large number of economic agents through socially determined structures in the market, from which the statistical distribution of housing price growth rates is determined. As a result of these interactions, based on individuals' quantal actions (buy and sell), and their engagement with the resulting outcomes of these actions in a highly "competitive" environment, system generates complex dynamics, which are hard to capture from the data directly. As Scharfenaker and Foley (2017) argues, the difficulty of capturing the dynamics of the economic systems from the data arises due to the dependency of observable variables on unobserved ones, which in turn, causes incomplete information about the problem in hand, raising the importance of the method of estimating the parameters in the model.

In the next section, the maximum entropy-quantal response model of statistical equilibrium, called "QRSE", introduced by Scharfenaker and Foley (2017), will be applied to housing market to overcome incomplete information problem in modeling, and to explain the dynamics/ behaviors of the system during the most recent housing market cycle (2000-2015).

## 2.3 A Quantal Response Statistical Equilibrium Model of the Housing market

In this section, the Quantal Response Statistical Equilibrium (QRSE) model predicts the equilibrium as the most probable state of the system in the form of the statistical equilibrium probability distribution of monthly house price growth rates, $f[x]$, which is derived by maximizing the entropy of the unknown distribution, $H[f[x]] = -\sum f[x]Log[f[x]]$ subject to constraints including all information from the observations or theory related issues (Jaynes, 1957, 2003). It includes two variables, an action variable, $A$, and an outcome variable $x : X \rightarrow R$. In this case, the action variable is a binary variable, $A = \{a, \bar{a}\}$ where $a$ and $\bar{a}$ represent selling and buying actions respectively, and the outcome variable,

$x$ represents the monthly real house price growth rate (%)[8]. The heart of the model lies on the interaction between the action and outcome variable.

The model, which takes the interactions among buyers and sellers into account, is a joint distribution of actions $A = \{sell, buy\}$ and house price growth rates, $x$ which will be written as $f[a, x]$ with the marginal and conditional frequencies $f[x]$, $f[A]$, $f[A|x]$, $f[x|A]$:

$$f[x] = \sum f[A, x]; f[A] = \int f[A, x]dx \tag{2.1}$$

$$f[A|x] = \frac{f[A, x]}{f[x]} \quad \text{if} \quad f[x] > 0, f[x|A] = \frac{f[A, x]}{f[A]} \quad \text{if} \quad f[A] > 0 \tag{2.2}$$

In this set up, conditional distributions represent the causal relationships and hence, include the explanatory power for the model to overcome the problem of incomplete information. Quantal response of individuals' actions assumes that individuals make their decisions based on the outcome variable, in this case, house price growth rates. Therefore, not only do individuals' actions (selling/buying) affect the house price changes ($x$), but the change in house prices also affects individuals' selling/buying decisions.

The interdependence between selling/buying actions of individuals and the change in house prices are therefore, represented by the conditional action distributions:

$$f[A|x] = \{f[a|x] = f[selling|x], f[\bar{a}|x] = f[buying|x]\} \tag{2.3}$$

The causal relation that links the buying-selling behavior of individuals to the house price growth rate, $x$, symmetrically determines the conditional outcome frequencies;

$$f[x|A] = \{f[x|a] = f[x|selling], f[x|\bar{a}] = f[x|buying]\} \tag{2.4}$$

Overall, conditional frequency distributions Eqns. 2.3 & 2.4 allow us pursuing a theory

[8]In real applications, measurement based on $x$ variable will be coarse-grained in finite number of bins.

of interaction through the inter-dependency of actions and the outcome variable in the case of housing market representation. Therefore, statistical equilibrium in the housing market model is a joint distribution, which determines two conditional distributions, $f[A|x]$ and $f[x|A]$.

### 2.3.1 Problems in Modeling Housing Market Behavior

In general, if one knows one of the conditional distributions via observations or theoretical considerations, and also the corresponding marginal distributions from observations, such as $f[A|x]$ and $f[x]$, then they would be enough to determine the joint distribution $f[A, x] = f[A|x]f[x]$ from which one can derive the other marginal and conditional distributions. In this case, observed data would provide full information to estimate the statistical equilibrium. Yet, most of the time this is not the case. For example, in our case, we only have the marginal distribution of house price changes $f[x]$, and need to learn about the conditional distributions $f[A|x]$ and $f[x|A]$.

The problem of modeling the housing market occurs because we end up having house price changes as outcome variable, which is determined by individual actions in the housing markets. Buying-selling actions of individuals could be easily observable for some data sets, but for our data sets it is not the case. Individual actions of house buyers and sellers cannot be observed simultaneously apart from house price growth rate data. This means, when we are observing house price changes, we cannot measure the intensity of buying and selling actions of individual agents in the housing market from the same data directly. Here, we want to estimate the joint distribution in order to infer the causal links, but we only have a partial information about the joint distribution, such as observations on house price growth rates that determine the marginal distribution of housing price changes $f[x]$.

This type of a quantal response social interaction problem with only a partial information about the joint distribution of quantal actions and the outcome variable that is determined by these actions, such as an observed marginal distribution, necessitates using a theory that

offers a prior over one of the conditional distribution using Shannon's information theory (Shannon, 1948), and Jaynes' (1957) principle of maximum entropy inference.

### 2.3.2 Model Constraints

As emphasized by Jaynes (1978b, 2003), it is important to choose the right constraints either using the observable data or theoretical frameworks, when employing the maximum entropy program to generate the most likely statistical equilibrium distribution of the system. Here, there are two important constraints, which are needed to represent the housing market dynamics. The first constraint represents the behavior of the "typical individual" in the housing market based on the quantal response, and the second one is based on the impact of individuals' actions on the house price changes.

In the Classical Theory of Prices, equilibrium is explained as a process of equalization, which is shaped by competition. Social interaction among individuals through competition plays a major role to drive the equalization process in the market. When we look at the housing market equalization of housing price changes, statistical regularities occur due to buying and selling decisions that are conditional on the expected changes in the housing prices for maximizing their returns from their transactions. The process of deciding when and where to sell/buy works as a competitive process, and creates a negative feedback mechanism while payoff maximizing sellers are seeking for markets with above- average price change (increase) to sell, and buyers are looking for markets below average price change to buy. Unintentionally, actions of sellers cause a decline in housing price increase in sub-markets, while behavior of buyers causes an increase in housing price change. In this setting, hence, quantal actions (buying or selling) of economic participants are conditional on the change of house prices. The frequency of an action depends on the conditional distribution i.e. $f[A|x] = \frac{f[A,x]}{f[x]}$ and represents the typical behavior of the economic agents in the housing market, while the effect of buying/selling actions on the observed house price growth rates $x$, reflects the dependency of house price changes on buying and selling

behaviors of agents such that $f[x|A] = \frac{f[A,x]}{f[A]}$ given that the payoff maximizing individuals create a competitive environment in the market overall.

**Conditional Action Constraint: The impact of house price changes on the actions of economic agents**

Applying maximum entropy approach to the social systems is more challenging than the applications of physical systems due to the fact that social systems evolve through the purposeful behaviors of individuals who seek for a better outcome from their actions. Taking this obstacle into account, the statistical equilibrium model in this chapter, describes the behaviors of the typical individuals as a binary quantal response in the market. This way, statistical equilibrium idea of physical systems takes a more applicable form for more complex social systems.

Modeling quantal response of market participants in this framework assumes that the agent may respond to a payoff function $u[A, x]$ . In the case of housing market, it represent the payoff to the typical individuals who buy/ sell houses in the market. If the typical agent chooses a mixed strategy $f[A|x] : AxX \rightarrow (0, 1)$ over actions $\{a, \bar{a}\} = \{sell, buy\} \in A$ to maximize the expected payoff, $\sum_A f[A, x]u[A, x]$, subject to a constraint on the entropy of the mixed strategy, the resulting Gibbs function is the quantal response function $f[A|x] \propto e^{u[A,x]/T}$. When, the behavior temperature is $T > 0$, recovered link functions will assign probabilities to different responses.

Scharfenaker & Foley (2017) presents the difference between the payoffs of selling and buying as a linear function by introducing a shift parameter $\mu$, defined as $\mathbf{u}(\mathbf{a}|\mathbf{x}) - \mathbf{u}(\bar{\mathbf{a}}|\mathbf{x}) = \mathbf{x} - \mu$. This tells us that higher house price growth rate increases the payoff of selling action while payoff of buying diminishes, thus the gap between the payoffs increases. Moreover, if the gap between the two is zero individuals become indifferent between buying and selling. In this case, $\mu$ determines the indifference point of the house price growth rate in which the probability of selling is 50%. It provides useful information about the willingness of market

participant's to sell at a certain house price growth rate, which will be discussed in more detail in following sections.

With this new specification, the impact of house price changes on the actions of selling and buying will be modeled as:

$$\begin{aligned} f[sell|x] &= \frac{1}{1+e^{-\frac{(x-\mu)}{T}}}, \\ f[buy|x] &= \frac{e^{-\frac{(x-\mu)}{T}}}{1+e^{-\frac{(x-\mu)}{T}}}. \end{aligned} \tag{2.5}$$

**Conditional Outcome Constraint: The impact of actions on the house price growth rates**

Another constraint that will help us to model the impact of selling/buying actions on house price changes in the market is the conditional outcome constraint. As mentioned before, the act of selling houses itself tends to lower the house price growth rate in the sub-market relative to what it would have been if selling behavior had no downward pressure on the price change in the market. The effectiveness of buying and selling process can be represented in a maxent formalism by a constraint on the difference between the weighted expected house price growth rates conditional on selling and buying actions of the individuals which can be written as:

$$\begin{aligned} f[sell]E[x|sell] - f[buy]E[x|buy] &\leq \delta \\ \int (f[sell,x] - f[buy,x])xdx &\leq \delta \end{aligned} \tag{2.6}$$

where $\delta$ represents the "effectiveness" of the competition in the market. When $\delta$ gets larger, the effect of selling and buying actions on the house price growth rate worsens indicating a deterioration in negative feedback mechanism from actions to house price change.

### 2.3.3 Solution to Maximum Entropy Program of the QRSE Model for the Housing Market

Given the observed frequency data for monthly house price growth rates $\bar{f}[x]$, the theory of dependency of buying/selling actions on house price growth rates, and dependency of house price growth rates on actions, we aim to infer a quantitative model of housing market behavior in the form of a joint probability distribution $f[A, x]$ over the action, $A$, and observed house price growth rates by using the maximum entropy method.

As the result of maximizing entropy of the the joint distribution subject to three constraints [9] –the constraint on the difference between the weighted conditional expected outcomes, Eq 2.6, a constraint on the mean value of the outcome, $\int f[x]xdx = \xi$, and the assumption that the conditional action is a logit function with parameters $\mu$ and $T$ – the maximum entropy (recovered) marginal distribution $\hat{f}[x]$ is inferred as the most probable distribution representing the statistical equilibrium in the housing market as:

$$\hat{f}[x] = \frac{e^{H_{\mu,T}[x]} e^{-\gamma x} e^{\beta Tanh[\frac{x-\mu}{2T}]x}}{\int e^{H_{\mu,T}[x]} e^{-\gamma x} e^{\beta Tanh[\frac{x-\mu}{2T}]x} dx} \tag{2.7}$$

$\hat{f}[x]$ is predicted as a Kernel to the maximum entropy program, and together with $T, \mu, \beta$ and $\gamma$ provides us with multinomial model of the house price growth rates. The hyperbolic Tangent function, $(Tanh)$ [10], which also is a symmetric function, represents the kurtosis of the system, and determines how spread out the marginal distribution can get. $T$ is called "behavioral temperature" parameter, and gives information about how the change in house prices affect the behavior of the individuals. Therefore, it is useful to understand how strong is the relationship between the outcome and the action, i.e. how much the change in house price growth rate affects the decisions of individuals' action of selling houses. It provides useful information about the fluctuations and uncertainties in

[9] Appendix A shows the derivation of maximum entropy program of the QRSE model for the housing market in detail and Appendix B explains the posterior probability estimates of the parameters, which provides the credible intervals for the estimated parameters.

[10] Please see the Appendix A for the derivation.

agents' decision making process. The impact parameter $\beta$, which is the Lagrange multiplier for $\delta$, measures the impact of intensity of actions on the sub-regional outcomes i.e how the actions of agents (selling/buying) affect the house price growth rate in a particular region. Another important parameter, $\mu$ provides information about perceptional behavior points which the agents are active buyers and/or sellers in the sub-regions. Therefore, equilibrium behavior of the system could be measured as a combined result of parameters $T$, $\mu$ and $\beta$. Parameter; $\gamma$ represents the shadow price for the mean constraint, which reveals information about the skewness of the marginal distribution . It also helps us to understand the relationship between perceptional behaviors of actors ($\mu$) and actual statistical mean of the data, $\xi$. $\gamma$ becomes zero if $\mu = \xi$, while $\gamma < 0$ represents positive skewness in the marginal distribution.

## 2.4 Model Inference and Results

Figures 2.4 & 2.5 visualize the solution to the maximum entropy program subject to constraints from Appendix A. Parameters are estimated for subsequent time periods; 2000-2003, 2004-2006, 2007/1-2008/6 , 2008/7-2009/12, 2010-2012 and 2013-2015 using the panel data of monthly house price growth rates for the metropolitan areas represented in data sets[11]. The objective is to capture the evolution of housing market behavior both qualitatively and quantitatively throughout the cycle.

Figures 2.4 & 2.5 represent the observed marginal frequencies; $\bar{f}[x]$, fitted (inferred) marginal frequencies; $\hat{f}[x]$ maximum entropy conditional outcome frequencies; $f[x, a]$, $f[x, \bar{a}]$ (left), and estimated conditional action frequencies; $f[a|x]$, $f[\bar{a}|x]$ (right), in which $a$ represents selling action while $\bar{a}$ represents buying action in the market [12]. Model fit results,

[11]Sub-periods are chosen based on distinctive differences in house price growth rate distributions which are represented in Figure 2 by taking information about historical house price movements in the US housing market into account. Then, for each sub-period, monthly house price growth rates of the entire MSAs represented by the data are pooled together for the estimation.

[12]Estimated parameters, related variables, Information distinguishability measure; ID, as a measure of closeness between estimated, observed marginal distributions, and predicted marginal action probabilities "marga=$\{selling, buying\}$"

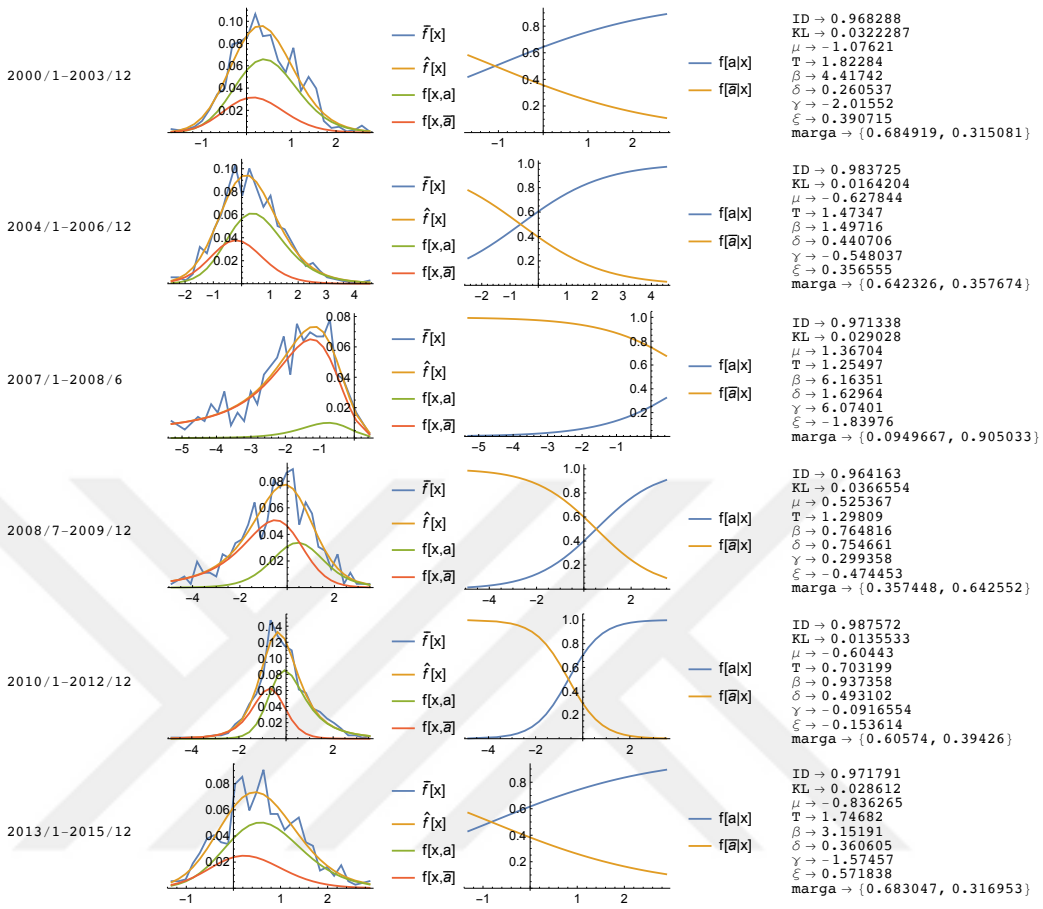


Figure 2.4: **Model Fit Plots (2000-2015) (Based on Inflation Adjusted S&P Case-Shiller Index Monthly House Price Growth Rate for each MSA represented in S&P Case-Shiller Index).** Observed marginal frequencies; $\bar{f}[x]$, fitted (inferred) marginal frequencies; $\hat{f}[x]$, maximum entropy conditional outcome frequencies; $f[x,a]$, $f[x,\bar{A}]$ (left), estimated conditional action frequencies; $f[a|x]$, $f[\bar{a}|x]$ (right), in which $a$ represents selling action while $\bar{a}$ represents buying action in the market.

show that the observed distributions of house price growth rates can be explained as the statistical equilibrium (or dis-equilibrium for some periods) produced by the interactions of buyers and sellers in the sub-housing markets according to a logit quantal response and a given effectiveness of buying/selling in affecting the house price growth rates in these regions.

As can be followed from the Appendix A in more detail, model parameters $T, \mu, \beta$ and $\gamma$, are estimated jointly by minimizing the Kullback-Leibler divergence between the

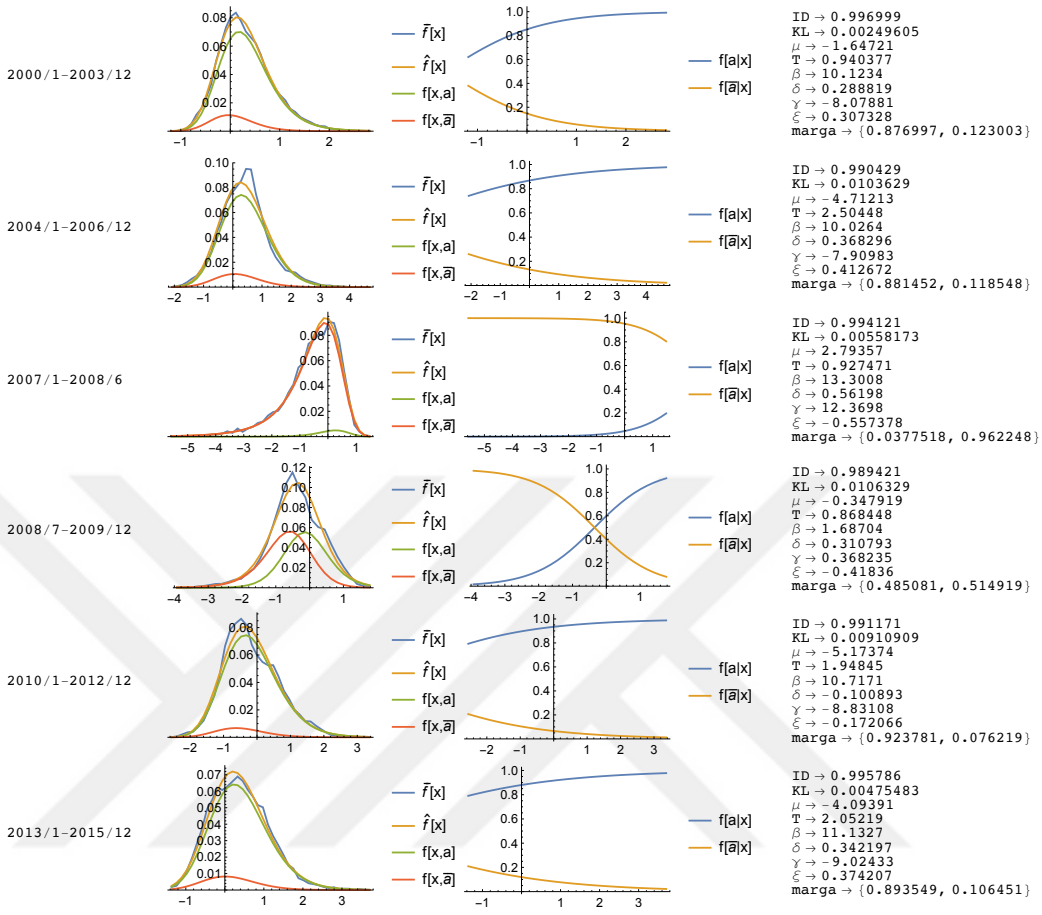


Figure 2.5: **Model Fit Plots (2000-2015) (Based on Inflation Adjusted FMHPI Monthly House Price Growth Rate for each MSA presented in FMHPI index)**Observed marginal frequencies; $\bar{f}[x]$, fitted (inferred) marginal frequencies; $\hat{f}[x]$, maximum entropy conditional outcome frequencies; $f[x,a]$, $f[x,\bar{a}]$ (left), estimated conditional action frequencies; $f[a|x]$, $f[\bar{a}|x]$ (right), in which $a$ represents selling action while $\bar{a}$ represents buying action in the market.

observed marginal frequencies, $\bar{f}[x]$, and inferred marginal frequencies $\hat{f}[x]$, where the inferred frequencies $\hat{f}[x;T,\mu,\beta,\gamma]$ is obtained from the equation 2.7. Results from this process indicate that given the estimated parameters, $T,\mu,\beta,\gamma$, QRSE model generates almost a perfect fit for the observed marginal frequencies in both data sets. Calculated information distinguishabiliy (ID) measures[13], which show the closeness of the model fit to the observed frequencies, indicates that fitted distributions of marginal frequencies, $\hat{f}[x]$

[13]See Appendix for more details on ID measure.

capture approximately 96 to 98% of the information content of the observed frequencies for each sub-period.

### 2.4.1 Interpretation of Estimated Model Parameters

Using the model fit results, Figure 2.6 & 2.7 present the time series of estimated parameters, $T$, $\mu$, $\beta$, $\gamma$, with highest posterior density regions (HPD)[14].

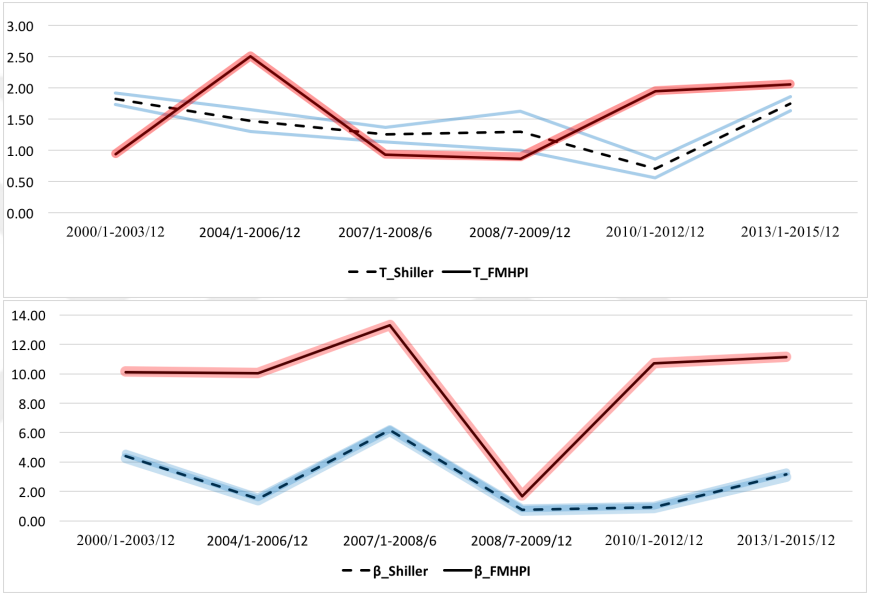


Figure 2.6: **Estimated Parameters with 95% Credible Intervals (HPD)**: $T$ and $\beta$ . Red and Blue lines represent the 95% credible intervals for estimated parameters.

**Interpreting Behavior Temperature, $T$ through the cycle** : As briefly mentioned before, behavioral temperature logit parameter, $T$, determines the endogenous fluctuations of the house price growth rates as an indicator of the responsiveness of the economic agents' actions to the house price changes. The higher the $T$ is the flatter the conditional action frequencies $f[A|x]$ become implying a weaker dependence of selling/buying actions on the house price changes. As a result, fluctuations and individuals' uncertainty of house price change become large. When $T$ gets closer to zero the dependence of actions on house price

[14]See Appendix B for more details on the posterior inferences of parameter estimates.

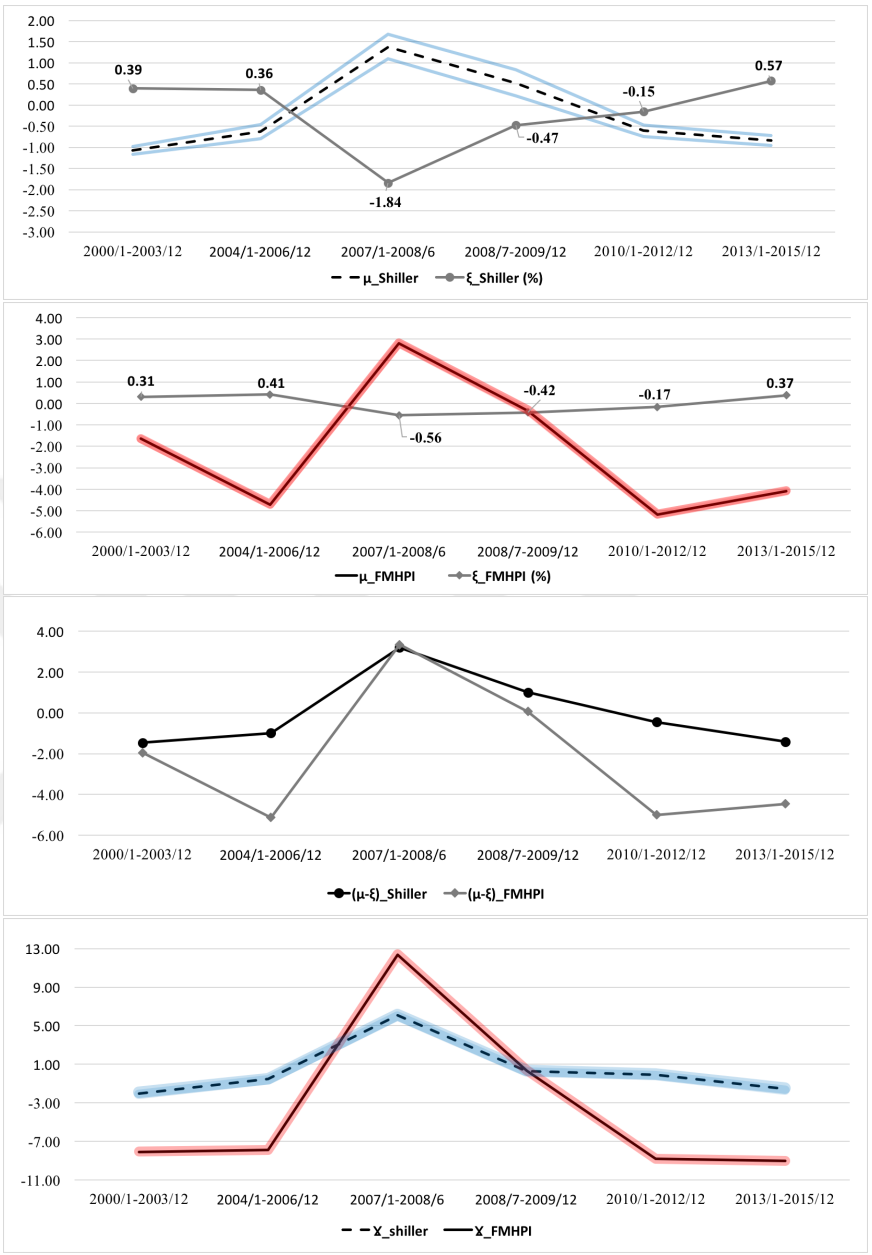


Figure 2.7: **Estimated Parameters with 95% Credible Intervals (HPD)**: $\mu$, actual average house price growth rate $\xi$, $(\mu - \xi)$ and $\gamma$. Red and Blue lines represent the 95% credible intervals for estimated parameters.

changes becomes stronger. This means even a small change in house price growth rate results in an instantaneous response of individuals on choosing their actions.

Temperature behavior, thus provides us with two quantitative measures to interpret the behavior of the individuals in a certain time period. First, it measures how responsive the

action frequency of economic agents (buying/ selling) to changes in house price growth rates at the average house price growth rate, because derivative of the conditional action function at $x = \mu$ is $\frac{1}{4T}$. To illustrate $T = 1.82$ for the early pre-crisis period (2000/1-2003/12) based on Case-Shiller index, indicates that a 1% increase in the house price growth rate near the average growth rate would raise the frequency of selling action by 0.13%. Based of FMHPI index results, $T = 0.94$ for the same period indicates that the same increase in house price growth rate near the average would raise the frequency of selling action by 0.26%. The difference between the two results tells us that any increase in house price growth rate would be more effective to attract sellers to take action in the market with a smaller $T$, who are more responsive to the changes in house price growth rates.

Second, $T$ defines the region where the action is sensitive to house price changes: the value of $x$ for which the conditional action frequency is $p$, with $0 < p < 1$ is $x[p, T] = TLog[p/(1-p)]$. For example, based on Case-Shiller Index data, $T = 1.82$ implies that the region of house price growth rate deviations where the frequency of selling lies between 0.05 and 0.95 is $\{-5.36\%, 5.36\%\}$ while it is $\{-2.76\%, 2.76\%\}$ based on FMHPI data for the period of 2000-2003/12. Accordingly, this tells us about how tight the quantal response link functions are[15].

The time series of $T$, estimated from the Case-Shiller Index data in Figure 2.5 demonstrates a stable pattern with a smooth decline between 2000 and 2009 from $T = 1.82$ to $T = 1.29$. In the early post-crisis period (2010-2012), it drops sharply to its minimum level, $T = 0.7$, and starts to increase dramatically after 2012 nearly reaching to its early pre-crisis period level, $T = 1.74$ in the end of the cycle.

From the FMHPI index, $T$ seems to follow a more volatile pattern as compared to results from the Case-Shiller Index. Starting from, $T = 0.94$, in early pre-crisis period, it reaches its maximum level of $T = 2.5$, in late pre-crisis period (2004-2006). A sharp decline in $T$ occurs during the crisis (2007-2009) reaching approximately the same level as in the case

[15]Please see Figure (3,4) for the quantal response link function plots, which represent conditional action distributions $f[a|x]$ and $f[\bar{a}|x]$

of Case-Shiller Index data, $T = 0.86$. After the crash, it starts increasing steadily above its late pre-crisis period level of $T = 2.05$ towards the end of the cycle between 2013 and 2015.

Patterns in the behavior of the temperature parameter, $T$, based on Case-Shiller Index data, which represent the largest metropolitan cities in the US, imply that typical sellers/ buyers were less responsive to the changes in the house price growth rates at the beginning of the cycle, but they started to eliminate the uncertainties about the house price changes, that in turn, strengthened the dependency between actions and house price changes. In general, one would expect increasing uncertainties of individuals' actions of buying and selling to changes in house prices when the market gets closer to collapse. Yet, here, individuals seems to build stronger ties to the house price movements before the crisis, which might be due to increasing optimism about the future of the housing market, such that every increase in growth rate of house prices encouraged house sellers to be more responsive to house price increases in the market.

On the other hand, in case of FMHPI index data, which represents a larger number of metropolitan regions, $T$ follows a sharp increase during the late pre-crisis period (2004-2006) signaling the worsening responsiveness of individuals' actions to house price changes. When the crash occurs responsiveness to the market signals seems to be regained but starts to deteriorate again right after the crash.

**Interpreting $\mu$, $\gamma$ and their relationship with actual average house price growth rate $\xi$ through the cycle** : Results from the model fit for both data sets from Figure 2.4 & 2.5 show that marginal probability of selling action, represented in "marga" [16], was higher than the marginal probability of buying action for the sub-periods of pre and post-crisis, while it is the opposite during the crash. Domination of marginal probability of selling action before and after the crisis coincide with increasing average house price growth rate $\xi$. Here, parameters $\mu$, $\gamma$ and the difference between $\mu$ and the average actual house price growth

[16]The first probability in "marga" represents marginal probability of selling action while the second probability is the marginal probability of buying action.

rate, $\xi$ could shed some light on the system behavior to distinguish different characteristics of different sub-periods throughout the cycle.

$\mu$ provides useful information about the perceptions of economic agents, i.e. gaining interest in taking action in the market. For example, $\mu < \xi$ means that sellers decide to sell, although mean house price growth rate of the region is less than the average house price growth rate of the market. Except during the crisis period (2007-2009), $\mu$ stays below the actual average house price growth rate for both data sets pointing out that the market participants were more willing to sell before and after the crash, although regional house price growth rate was well below the market average. This is because opportunity cost of keeping an empty house during the boom periods is higher than it is during the bust since there is a chance that house values might decline in the next period (Krainer, 2001); hence, the payoff of selling becomes higher during the boom, while it is the opposite for the bust and crash periods. During the crisis, $\mu$ exceeds $\xi$ signaling a deterioration in individuals willingness to sell. After the crisis periods, their perceptions about the market seem to return to pre-crisis periods where $\mu < \xi$.

In addition to the relationship between $\mu$ and $\xi$, another important piece of information that $\mu$ provides us with is that housing market was more inclined to be “frothy" and speculative before and after the crash meaning that the market was more vulnerable to create a housing market bubble. This can be observed from the perception parameter $\mu$ being negative for pre and post-crisis periods for both data sets (except during the crash it becomes positive).

Another way of presenting the individuals’ perceptions and willingness of selling/buying can be followed by the measure - $(\mu - \xi)$, from Figure 2.7. By reaching its peak during the crash, $(\mu - \xi)$ renders how overcautious the market participants become when the system gets closer to collapse. As can be seen from the same figure, individuals start to become more careful about their actions beginning in the late pre-crisis period, where $(\mu - \xi)$ starts to increase. When it peaks during the crash, it means individuals prefer to wait until regional

house price growth rate is well above the market average.

The estimation of $\gamma$ parameter informs us about the skewness of the marginal distributions, which in turn, helps us to interpret the relationship between $\mu$ and the actual mean $\xi$. When $\mu$ is equal to $\xi$, $\gamma = 0$ indicates perfect symmetry in the marginal distribution, $\bar{f}[x]$. On the other hand, $\mu < \xi$ implies positive skewness, meaning that some of the regions are experiencing house price increases well above the market average. This might mean that the market is actually in the boom cycle and building upon the housing bubble. As can be seen from Figure 2.7, $\gamma$ was negative before and after the crash demonstrating a positive skewness in the marginal distributions, which also coincides with the boom periods in the housing markets while house price growth rates were increasing. During the crisis, $\gamma$ shifts from negative to positive values indicating an increase in skewness to the left caused by negative growth rate values in most of the regions due to the market crash. Based on these results, one can argue that estimated $\gamma$ parameter is a useful indicator for differentiating between boom and bust periods in the market.

Overall, after the early pre-crisis sub-period (2000-2003), sellers start to become more cautious about their actions which can be observed from the relationship between $\mu$, $\xi$ and $\gamma$. Although marginal probability of selling (which is represented in “marga" from Figures 2.4 & 2.5) was higher than the buying action during the periods before the crash, it starts to decline after 2003, and buying becomes more attractive than before, and the system becomes less positively skewed to the right before the crash. Periods of Crisis , in which sharp declines in the house price growth rates occurred in most of the regions, could be represented as vanishing willingness to sell in the market (increasing $\mu - \xi$), and increasing negative skewness of marginal distribution of the house price growth rates (positive values of $\gamma$) .

After the crash, both $\mu$ and $\gamma$ decline and get closer to their pre-crisis levels emphasizing the similarities in the behavior of the system between pre-crisis and post-crisis periods.

**Interpreting Negative Feedback Mechanism, $\beta$, through the cycle** : As mentioned before, $\beta$ is the Lagrangian multiplier for the impact constraint $\delta$ that limits the impact of selling on the change of house price growth rates in sub-regions. Therefore, it represents the impact of actions on the house price changes providing information about how effective is the market. The lower the $\beta$ is, the lower the market effectiveness. This in turn means that the negative feedback mechanism from actions on house price changes, was deteriorated. For example, in a well functioning market an increase in house price growth rate in a particular region would attract sellers to sell. Once the actions are taken, if the market has an highly effective negative feedback mechanism, it would cause a decline in house price growth rate of this particular region. On the other hand, if the mechanism does not work properly, house price growth rate in the region would keep increasing.

Time series of estimated $\beta$ for both data sets from Figure 2.6 show that impact of selling/buying actions on forcing house price growth rates in the regions towards the average growth rate of the market was much weaker between 2004-2006 than 2000-2003 indicating a deterioration in the negative feedback mechanism right before the crash.This behavior of the market creates a plausible environment for unsustainable housing price bubble before the crash.

During the first half of the crisis, $\beta$ increases to keep the system in equilibrium but since the willingness of selling action drops dramatically, strong negative feedback mechanism (higher $\beta$ ) would not prevent market from collapsing. As a result, after the first half of the crisis, $\beta$ reaches its lowest level indicating a dysfunctional negative feedback from the actions . To illustrate, if deterioration in the negative feedback mechanism would not occur between 2004-2006, strong willingness of selling action of individuals, represented by the relationship between $\mu$ and $\xi$, would cause house price growth rates to decline preventing prices to grow further. Since this was not the case between 2004 and 2006, price growth rates keep appreciating. When the appreciation reaches to unsustainable levels, market collapses in 2007 leading further decline in the effectiveness of negative feedback

mechanism between 2008/7 and 2009/12. After the collapse, $\beta$ starts to rise again signaling the recovery of the market forces, which brings the system back to statistical equilibrium.

These results allow us to argue that a deteriorating negative feedback mechanism with worsening willingness of individuals to sell during the crisis, is one of the main characteristics of market behavior and observed divergence from the statistical equilibrium during the collapse as presented in Figure 2.2. Therefore, the trends in $\beta$ and $\mu$ provide a strong evidence that the cause of the bubble cannot be explained only by the increasing demand for the housing in the market because implicit mechanism behind the boom and the housing market bubble was the relationship between the actions and the outcome price changes. Conventional explanations for the housing market behavior such as a positive feedback mechanism from the buyers, or speculations in the market (Case and Shiller, 2003; Shiller, 2008b) could only explain some part of housing bubble creation. If the negative feedback mechanism between the actions and house price changes were working effectively, supply of housing by sellers would be able to prevent the bubble from happening, and the system would not be able to diverge from the strong statistical equilibrium of the early 2000's.

## 2.5 Conclusion

In this paper, it has been shown that employing the maximum entropy method with plausible theoretical frameworks as the constraints, we are able to overcome the incomplete information problem resulted from unavailability of data for individuals' actions. As a result, we were able to fully determine the statistical model explained as an equilibrium joint distribution of observed house price growth rate data and unobservable buy/sell actions of individuals given the predicted model parameters, $T, \mu, \beta$ and $\gamma$. The recovered joint distribution of house price growth rates $x$ and individual actions $\{a, \bar{a}\}$, which implies the marginal distribution of house price growth rates - $\hat{f}[x]$ is then compared to the observed distribution of $x$ - $\bar{f}[x]$, using the Kullback-Leibler (KL) divergence. Since KL Divergence is a good approximation to log posterior probability for the multinomial model for any

parameters, we were able to estimate the conditional posterior distributions of each model parameter holding others at their maximum posterior estimates.

Estimations from the both data sets used in the analysis (S&P Case Shiller Index and FMHPI index) follow strikingly similar patterns throughout the cycle proving the strength of the QRSE model's explanatory power of defining the behavior and dynamics of social systems.

Predicted parameters of the model $T$, $\mu$, $\beta$ and $\gamma$ revealed significant characteristics of the US housing market dynamics for pre-crisis, crisis and post-crisis periods during the most recent housing market cycle in the US. Model fit estimations show strong statistical equilibrium before the crisis. Deviations from the equilibrium start to arise between 2004 and 2006, and the first half of the crisis period (2007-2008/6) represents the disequilibrium in the housing market [17]. The system seems to start recovering during the second half of the collapse (2008/7 -2009/12). After the crisis, the market returns back to statistical equilibrium between 2010 and 2015. [18]

Based on our analysis, we argue that increasing average house price growth rates before the housing market collapse created an optimistic environment in a way that weakening responsiveness of individuals' action on house price growth rate movements (higher $T$ compared to crash period) before the collapse is one of the reasons that caused system to build up a period of housing boom/ bubble. This may be because glamorous expectations for the future of housing market deteriorated individuals' capability of processing the market signals. Individuals' responsiveness reached its maximum during the crisis when the collapse showed itself as a fact rather than just being a possibility.

In addition to weak responsiveness of individuals to market outcomes (higher $T$) between

[17]It is worth mentioning that QRSE model is a statistical equilibrium model, and it might not be able to produce powerful results for the out-of-equilibrium cases such as housing market collapse because forces that keep the system in the statistical equilibrium might not work as well during the crash. In our case, self-organizing dynamics of the system seem to be weakened during the crisis; however, they were still strong enough to be measured in this study.

[18]Positive average house price growth rates above early pre-crisis period between 2013 and 2015 could be signaling a new housing boom cycle which might necessitate further analysis but is outside of the scope of this study.

2004-2006, strong willingness of sellers to act in the same period seems to represent the speculative market behaviors before the crisis. Along with a weakened negative feedback mechanism between actions and house price changes (lower $\beta$), strong willingness to sell seems to aggravate further increases in house price growth rates triggering the housing bubble during the pre-crisis periods.

Between 2007-2009, even though the feedback mechanism recovered during the first half of the crisis (2007-2008/6) (higher $\beta$), the market could not escape from the collapse due to the fact that high responsiveness and deteriorated willingness of individuals' actions to declining price growth rates (lower $T$ and broadening gap between $\mu$ and $\xi$) prevented selling actions to take place. As a result, the forces keeping the system in statistical equilibrium became unsustainable, and the system-dynamics collapsed leading to dis-equilibrium in the market between 2007 and 2008. Finally, after the crash, average actual growth rate of the market and estimated parameters start to return to their pre-crisis values revealing a recovery of statistical equilibrium after the collapse.

# CHAPTER III

# Wealth Concentration, Income Distribution, and Alternatives for the USA

## 3.1 Introduction

In the "long run," or at least in two-century simulations of the demand-driven model of economic growth described herein, the share of the top one percent of US households in the distribution of wealth could rise from around 40% to 60%. Ever-increasing concentration of wealth along the lines suggested by Thomas Piketty (2014) is not likely to occur.

On the other hand, it would be very difficult to reduce wealth inequality significantly just by using policy measures such as taxes on capital gains or net worth. Andrew Mellon, Secretary of the Treasury under the string of Republican presidents in the 1920s, said that "During a Depression, assets return to their rightful owners." In fact, he was wrong. Under almost all circumstances, assets move toward "rightful" households who already hold a preponderant share. To get that share back below 40% and keep it there, the market needs an actor with countervailing power against the accumulation of the very rich. Creation of an institution such as a public wealth fund to out-save the savers would be required to mitigate their control of national net worth.

Wealth of households comprises residential capital plus a stock of claims on business firms which own productive capital and on financial liabilities issued by government and

the rest of the world. Income is a flow of payments per unit time – along with capital gains saving from payments flows cumulates into the stocks. In the short run income inequality can be reduced by one-shot measures such as boosting low wages or raising taxes at the top. But even if applied aggressively these moves will not erase income differentials between the rich and the poor or middle class unless new resources can be directed toward lower income households. Higher labor productivity (output per person-hour) can provide such a flow of resources. After 1980, however, US productivity gains favored the top one percent because wage growth did not keep up. Economic surplus moved from almost all other households toward the very rich.

Without the potential benefits of productivity growth, the economy resembles a zero-sum game in which the saving required to finance new assets created by investment or a fiscal deficit comes largely from households with high incomes. If output is fixed, then the lower their saving rate, the higher their income (mostly from profits) must be. John Maynard Keynes (1930) called this theorem of accounting a "widow's cruse."[1] In fact, shifts in output do affect savings flows as emphasized a few years later by Keynes (1936), and go together with income redistribution to balance demand injections and saving leakages in the simulations reported below.

## 3.2 Outline

The road to these conclusions takes interesting turns. We proceed from the abstract to the concrete, beginning with a review of distribution and growth theories proposed by Cambridge University economists half a century ago. They concentrated on social relations and class conflict, with lessons soon forgotten by the mainstream but brought back into relevance by recent debate. Cambridge theory emphasizes how different income flows and wealth stocks affect the economic prospects of the social classes. The widow's cruse is

[1]In King James Bible used by Keynes, 1 Kings 17: 9-16 describes the prophet Elijah's flight from evil King Ahab. God provided Elijah and the widow with whom he was boarding oil in a cruse and flour. The more oil they consumed, the more they received.

one example. A more realistic narrative is that in a demand-driven growth model there can be a medium-term inverse relationship between the profit share and the level of economic activity, consistent with the data. Under standard assumptions, both mainstream growth theory and the widow's cruse impose the wrong sign on this linkage[2].

Another complication that almost all growth theory lacks is an adequate description of how households get access to income from profits. Their capital gains on assets such as equity are an important vehicle. We take up their significance in the income and wealth distributions in the USA. A key empirical point for the business sector is that financial transfers (dividends and interest) to households plus capital losses due to rising equity prices may outstrip net profits. Business suffers paper losses which are transformed into apparent household increases in wealth. Capital gains at the top together with erosion of the labor share created a strong shift in the income distribution toward the richest one percent of US households. Their income share rose by ten percent after the mid-1980s – an enormous change by historical standards (Taylor et al., 2014). Our simulation model is constructed on the basis of the theory and data. We end the paper with its results and alternative scenarios of change. The model incorporates the usual convention of setting up a medium term macro equilibrium consistent with the national income and financial accounts, and then tracing its dynamics toward a "long run" steady state with all relevant variables growing at the same rate.

## 3.3 Growth and the distribution of income and wealth

Initial stylized analysis can be constructed around the functional (or profits vs. wages) income distribution, assuming that physical capital is the sole component of wealth. A mapping toward the size distribution is discussed below. For the moment, all profits are supposed to accrue directly to households, leaving out business saving, financial transfers,

[2]In mainstream models there can be a negative relationship if a parameter called the elasticity of substitution exceeds one – a value usually rejected by standard econometrics applied to the data.

and capital gains. Households typically own only residential capital (subject to mortgage obligations) along with debt and equity claims on firms which hold productive plant and equipment. Financial linkages in the form of payments flows as well as claims on assets are all treated as veils covering but not affecting ultimate ownership of capital. A useful additional simplification is to follow Luigi Pasinetti (1962, 1974) in assuming that there are two classes of households – "capitalists" who receive profits on the capital they own, and "workers" who get the rest of income.

## 3.4 Demand and income distribution over the cycle

We construct a medium run around the business cycle. Karl Marx observed a century and a half ago that an upswing in economic activity increases labor's bargaining power so that the share of profits in output (say $\pi$) tends to fall, i.e. there is a high employment "profit squeeze."[3] In the same time frame, activity itself (conveniently measured by the output/capital ratio $u = X/K$ with $X$ as output and $K$ as the capital stock) is stimulated by increased investment due to a higher profit rate $r = \pi u$. Aggregate demand appears to be "profit-led."

There is controversy in the literature as to whether aggregate demand is profit- or wage-led. When one takes the size distribution of income into account, however, this distinction blurs. As emphasized by Piketty and illustrated below, wage dispersion has vastly increased over recent decades; moreover poor people have low positive or negative saving rates. Households near the bottom of the distribution receive a combination of wages and income transfers, and increasing their pay has a minimal impact on retained earnings. One can safely say that demand can be led by increases both in profits and low (partly wage) incomes.

The combination of a profit-squeeze distribution and profit-led output generates a predator-prey cycle along lines first formalized by Richard Goodwin (1967) Contemporary versions of Goodwin such as Barbosa-Filho and Taylor (2006) and Kiefer and Rada

[3] As noted above, mainstream models postulate the opposite sign – $\pi$ rises when output swings up.

(2014) treat the wage share $\psi = 1 - \pi$ as predator and capacity utilization as prey. Both papers extend Goodwin in setting up dynamics of capacity utilization and the wage share. In continuous time (with $\dot{u} = du/dt$ etc.) we have

$$\dot{u} = h(u, \psi) \tag{3.1}$$

as a representation of the Keynes (output adjustment) side of the model and

$$\dot{\psi} = j(u, \psi) \tag{3.2}$$

describing Marx-Goodwin shifts in distribution.

These differential equations will be locally stable if $dh/du < 0$ and $dj/d\psi < 0$. In the medium run, effective demand will be profit-led if $dh/d\psi < 0$. There will be a profit squeeze if $dj/du > 0$. The opposite signs of the latter two "cross partial" derivatives suggest that a cycle is likely.

Figure 3.1 illustrates the dynamics. The $\dot{u} = 0$ and $\dot{\psi} = 0$ schedules are "nullclines" showing combinations of $u$ and $\psi$ that respectively hold the time-derivatives $\dot{u}$ and $\dot{\psi}$ to zero. The small arrows show the directions in which $u$ and $\psi$ move when they are away from their nullclines. The negative slope of the $\dot{u} = 0$ schedule can be interpreted to mean that aggregate demand is profit-led in the medium run; the positive slope of the $\dot{\psi} = 0$ curve indicates that there is a profit squeeze à la Marx when output and employment go up.

Suppose that there is an initially low level of $u$ as at point A. Capital utilization will begin to rise toward its nullcline, and the wage share will fall. Later in the cycle $\psi$ will begin to increase. Eventually the rising labor share forces the trajectory to cross the $\dot{u} = 0$ nullcline, and output declines from its cyclical peak. In a bit more detail, if $\omega$ is the real wage and $\xi$ is labor productivity then $\psi = \omega/\xi$. Productivity typically goes up as an economy emerges from a slump so $\psi$ falls. After a time a tighter labor market means that $\omega$ begins to rise, and $\psi$ increases after the trajectory crosses the $\dot{\psi} = 0$ nullcline. Over recent decades in the

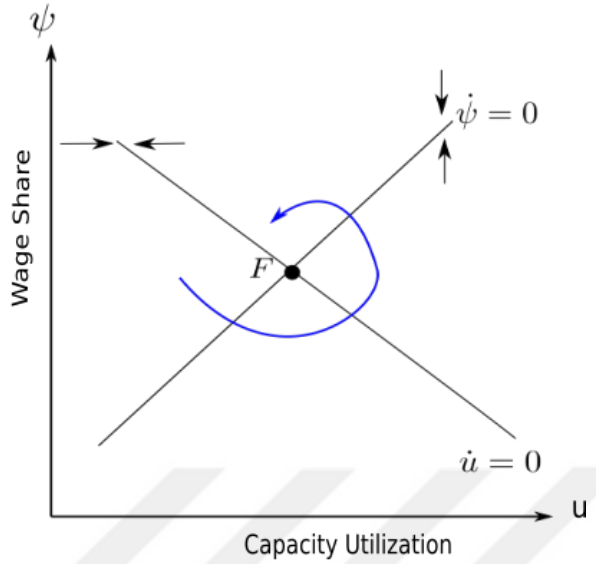


Figure 3.1: Cyclical dynamics between the wage share $\psi$ and capital utilization $u$.

USA, cyclicality has persisted but the whole relationship appears to have shifted downward (see below), increasing the profit share across cycles and contributing to growing income concentration at the top of the distribution.

Using long-term quarterly data for 13 wealthy OECD economies, Kiefer and Rada fit a discrete-time cross-country econometric model like the one illustrated in Figure 3.1, obtaining nullclines with relatively steep slopes in the $(u, \psi)$ plane, i.e. the economies are weakly profit-led but demonstrate a robust profit squeeze when economic activity rises.[4] Typical fluctuations of the variables over the cycle are in the range of five to ten percentage points.

In the figure the intersection of the nullclines establishes a focal point $F$ around which the variables cycle. For our purposes, Kiefer and Rada's most significant result is that $F$ has moved southwest over time. Over four decades the wage share has dropped by around five percent and capacity utilization by two percent (before the Great Recession). Because the fall in $u$ is proportionately less than the decrease in $\psi$, the profit rate $r = (1 - \psi)u$ has gone up.

[4]Alternatively, the profit share rises sharply when capital utilization declines – a point of relevance below.

## 3.5 Growth with capitalists and workers

Now we can take up the dynamics of wealth concentration. To keep the presentation as simple as possible, business cycle fluctuations are suppressed by replacing the Kiefer-Rada nullclines with "level" relationships.

As assumed above, capital $K$ is the sole component of wealth. It is not a "factor of production" subject to diminishing returns as in neoclassical models but rather serves to scale the system. Its associated profit flow $rK = \pi X$ is somehow made directly available to households, presumably via financial payments and capital gains. They do not engage in transactions involving debt.

The rich class of hereditary capitalists or rentiers hold capital $K_c$ to generate their income $rK_c$. Workers hold the rest of capital to get profits $r(K - K_c)$ and receive labor income as well. Shares of income saved by capitalists and workers are $s_c$ and $s_w$ respectively, with $s_c > s_w$. Their saving means that accumulation $\dot{K}^S$ can be expressed as

$$\dot{K}^S = s_c r K_c + s_w[X - rK_c] - \delta K \tag{3.3}$$

with $\delta$ as the rate of depreciation.

Dividing through by $K$, recalling that $r = \pi u$, and letting $Z = K_c/K$ gives

$$\hat{K}^S = [(s_c - s_w)\pi Z + s_w]u - \delta \tag{3.4}$$

In this equation $\hat{K}^S$ is the capital stock growth rate ($\hat{K} = \dot{K}/K$) permitted by available saving at economic activity level $u$ which depends on $\pi$ and the capital control ratio $Z$. A "profit effect" on saving and growth operates through changes in the income distribution ($\pi$) and eventually wealth ($Z$). Mainstream growth theory determines $\hat{K}^S$ in Eq. 3.4 from full employment, a neoclassical production function, etc., while fixing $u$ and $\pi$ at any time. With $Z$ set by "history" and pre-determined saving and depreciation rates the capital stock

growth rate follows directly.[5]

To clarify assumptions built into Eq. 3.4, it helps to think in terms of a $19^{th}$ century distinction between not consuming a share of income (or "abstinence" on the right-hand side) and transforming the resulting saving into capital ("accumulation" on the left). Growth theory equates the two flows. Even assuming that in practice households can manage this transformation there is no reason to expect that higher fiscal or foreign "saving" (a government surplus or external deficit) will automatically feed into higher investment. See the simulations below.

Keynesian models bridge the abstinence/accumulation gap by bringing in an independent investment function and thereby effective demand. One way to do so is to follow Kalecki (1971) and introduce profit-driven investment,

$$\dot{K}^I = g_0 K + \alpha \pi X - \delta K$$

or

$$\hat{K}^I = g_0 + \alpha r - \delta = g_0 + \alpha \pi u - \delta. \tag{3.5}$$

In these equations $g_0$ represents Keynesian animal spirits and capital stock growth responds to the profit rate (parameter $\alpha > 0$).[6]

By setting excess effective demand to zero, $\dot{K}^I - \dot{K}^S = 0$, and solving we get an expression for $u$,

$$[[(s_c - s_w)Z - \alpha]\pi + s_w]u = g_0 \tag{3.6}$$

A higher $Z$ raises saving and thereby reduces capital utilization in a demand driven model (the paradox of thrift applies). If aggregate demand is profit-led, i.e.$(s_c - s_w)Z - \alpha < 0$, $u$ will respond positively to $\pi$.

[5]The growth Eqns. 3.3 or 3.4 and 3.8a below will tend toward steady states on their own dynamics. Mainstream models always impose an additional restriction saying that steady state capital stock and population growth rates are equal, but in general there is no reason for this assumption to apply.

[6]Investment could also depend on u directly in a version of the accelerator, but we leave that out for simplicity. It makes profit-led demand less likely.

In effect Eq. 3.6 replaces the $\dot{u} = 0$ nullcline in Figure 3.1 for determination of $u$. Similarly, the relationship

$$\pi = \theta(u) \tag{3.7}$$

with $d\theta/du < 0$ replaces the $\dot{\psi} = 0$ nullcline to set $\pi$ (and $\psi$) as a function of $u$.

See Figure 3.2. Following Kiefer and Rada the $\pi(u)$ schedule is steep, signaling a strong profit squeeze. A contractionary demand shock such as an increase in $Z$ or a reduction in government spending would shift the u($\pi$,$Z$) schedule to the left. The result would be an increase in $\pi$ and a drop in $u$.[7] The implication is that $r = \pi u$ could move either way. In the simulations presented below a fall in capital utilization tends to be associated with a small increase in the profit rate and rate of growth. In other words, higher concentration of wealth can lead to low employment without a big growth pay-off in the long run.[8]

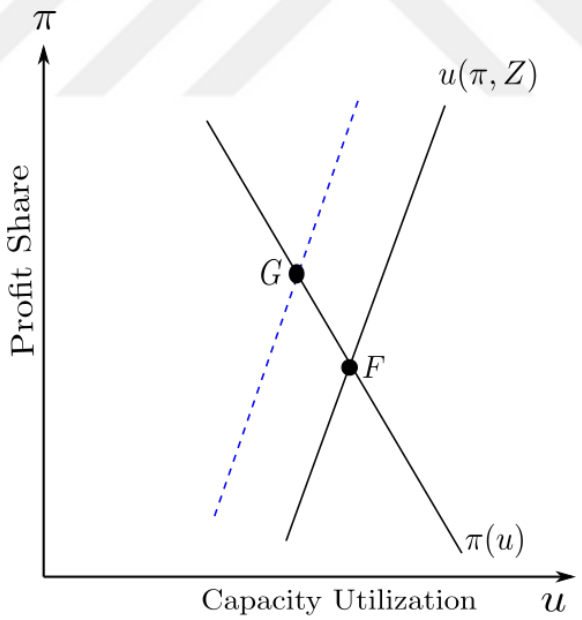


Figure 3.2: : Determination of the profit share $\pi$ and capital utilization $u$. The dashed line shows the effect of a higher value of the capital control ratio $Z$.

In terms of Figure 3.2, both neoclassical and widow's cruse models share a similar point

[7]With $\pi$ on the vertical axis, the shift northwest from $F$ to $G$ is consistent with a southwest move in Figure 3.1.

[8]As discussed below, households in the top one percent provide around 60% of private saving in the USA. A shift in the income distribution in their favor can significantly reduce the multiplier.

of view. Their $\pi(u)$ schedule has a shallow positive slope, cutting the effective demand $u(\pi, Z)$ curve from the left. An increase in aggregate demand shifts $u(\pi, Z)$ up (to the right), so $u$, $\pi$, and $r = \pi u$ all increase. As described above, such a profit boost or wage squeeze when aggregate demand rises is not observed in the data. We can do better by treating $u$, $r$, and $\hat{K}$ as being determined by $\pi$ and $Z$. To do so, we have to bring in dynamics of $Z$.

## 3.6 Steady states

Following Pasinetti, the growth rate of capitalists' capital is

$$\hat{K_c} = s_c \pi u - \delta. \tag{3.8a}$$

Pasinetti concentrated on properties of the steady state, but the focus here is on dynamics of growth. The concentration ratio $Z$ may converge to a steady state at which $\hat{K_c} = \hat{K} = \bar{g}$. The growth rate $\bar{g}$ is endogenous in the present model, while both Piketty and Pasinetti treat it as exogenous, perhaps determined from the supply side. Either way, we can restate Eq. 3.8a as an extension of a famous formula due to Pasinetti,[9]

$$s_c r - \delta = \bar{g}. \tag{3.8b}$$

For a given $\bar{g}$, the profit rate will be higher, the lower is the capitalist saving rate $s_c$ – a manifestation of the widow's cruse.

To analyze macro dynamics, with $\hat{Z} = \hat{K_c} - \hat{K}$ we can use Eqns. 3.4 & 3.8a to get a differential equation for $Z$,

$$\dot{Z} = [[s_c(1 - Z) + s_w Z]\pi - s_w]Zu \tag{3.9}$$

[9]Piketty emphasizes a neoclassical explanation for wealth concentration relying on the inequality $r > g$. With $s_c < 1$ and $\delta > 0$, Eq. 3.8b already shows that this condition holds at steady state, regardless of neoclassical theory.

subject to the restrictions Eqns. 3.6 & 3.7. The question at hand is whether $Z$ will converge toward a steady state with $\dot{Z} = 0$ and $\hat{K}_c = \hat{K}$.

More compact equations are easier to work with. Let

$$f(Z) = [s_c(1 - Z) + s_w Z]\pi - s_w$$

$$= (s_c\pi - s_w) - (s_c - s_w)\pi Z \tag{3.10}$$

For future reference the derivative of $f$ is

$$df/dZ = f' = [-(s_c - s_w)\pi + [s_c(1 - Z) + s_w Z]\pi'] \tag{3.11}$$

in which $\pi' = d\pi/dZ > 0$ from Figure 3.2. From the same source we also have $u' < 0$. Note that $f'$ can take either sign. The second derivative is

$$f'' = [-2(s_c - s_w)\pi' + [s_c(1 - Z) + s_w Z]\pi'']$$

which we will assume to be negative (that is, $\pi'' = \frac{d\pi^2}{dZ^2}$ is not strongly positive).

With this notation we have

$$\dot{Z} = fZu \tag{3.12}$$

and

$$d\dot{Z}/dZ = Z[f'u + fu'] + fu. \tag{3.13}$$

Eqns. 3.9 or 3.12 permits several steady state solutions with $\dot{Z} = 0$ to exist. One can describe them in terms of the profit share response parameter $\pi'$ and the capitalist saving rate $s_c$. Pasinetti concentrated on a steady state with $0 < Z < 1$, illustrated in Figure 3.3. There can be a stable Pasinetti equilibrium at point $P$, with $f(Z) = 0$ and $d\dot{Z}/dZ = f'Zu$ from Eq. 3.13. With a low relatively low value of $\pi'$, $f' < 0$ from Eq. 3.11 so the equilibrium is stable. Also, $dr/dZ < 0$ and the capital stock growth rate $\hat{K}$ is a decreasing function of $Z$

from Eq. 3.5.[10]

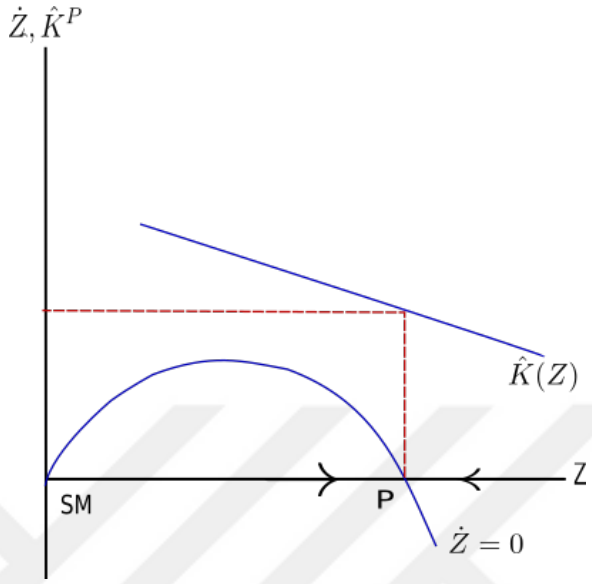


Figure 3.3: Stable Pasinetti equilibrium at point $P$. The growth rate $\hat{K}$ is a declining function of $Z$ with steady state level $\hat{K}^P$

## 3.7 Capital and income in the long run

What happens to the concentration ratio $Z$ in steady state? To a large extent, steady growth is "supply constrained" insofar as all stocks and flows must increase at the same rate. In the medium run, $\pi$ and $u$ determine investment and the rate of growth $g$. Because $g$ is constant at a steady state, it sets the levels of these and other variables.

At the Pasinetti solution (ignoring depreciation for simplicity), "flow-stock" ratios must equalize across comparable variables. Say's Law applies and the paradox of thrift is nullified in the "long run" (if we ever get there).

Specifically, if $S_w$ and $S_c$ are savings flows of workers and capitalists and $S$ is total saving then it must be true that $S_w/K_w = S_c/K_c = S/K = \hat{K} = g$

[10]There can also be a "dual" equilibrium at $Z = 0$, labeled $SM$ after a paper by Paul Samuelson and Franco Modigliani (1966). It resembles Piketty's set-up when $s_c = s_w = s$ and there is no class distinction in saving rates. The shift of equilibrium from $SM$ to $P$ is an example of a saddle-node or transcritical bifurcation of the differential Eq. 3.9 or 3.12 – there is a regime change parameterized by the rentier saving rate $s_c$.

We can parameterize these relationships using lower case $s_w$ and $s_c$ for saving rates, letting $W = (1 - \pi)X$ be the wage bill, and noting that $K_w = K - K_c$. Then these equations can be restated as

$$s_w[r(K - K_c) + W]/(K - K_c) = s_c r K_c/K_c = g. \tag{3.14}$$

A version of Pasinetti's formula:

$$g = s_c r$$

follows immediately. The profit rate adjusts to changes in $s_c$ and $g$.

More interesting is that fact that the capital concentration ratio adjusts as well. One can use Eq. 3.14 to derive

$$s_w r + s_w W/(K - K_c) = s_c r.$$

Since $K - K_c = K(1 - Z)$ we get

$$1 - Z = [s_w/(s_c - s_w)](W/rK) = [s_w/(s_c - s_w)][(1 - \pi)/\pi]. \tag{3.15}$$

Shares of wealth in the long run are intimately related to the distribution of income between profits and wages, and thereby social change and redistributive policy. The workers' share of capital, $1 - Z$, increases with $s_w$ and decreases with $s_c$ and $\pi$. The former effect reflects the observation of James Meade (1964) in that wages are a potential source of savings unavailable to rentiers, setting an upper limit to $Z$.[11] A corollary is that because of their different sources of income both classes will co-exist in steady state, contrary to neoclassical models such as one by Joseph Stiglitz (2014) in which income sources are blended and the highest saving group takes over all wealth.

Although it only applies at a steady state, Eq. 3.15 points to another aspect of the

[11]Meade's argument shows why $Z$ will not rise to a value of one. Even so, the upper bound may be high. Plausible numbers are $s_c = 0.5$, $s_w = 0.1$, and $\pi = 0.4$. These numbers generate a steady state value for $Z$ of 0.625, in the range reported by the simulation model below.

accumulation process. At any time, there is a given stock of capital which has to be held by either capitalists or workers. Because $s_c > s_w$ the former have the upper hand in accumulating wealth. This observation is consistent with Mellon's views about the "rightful owners" of assets – they have a lot of money and save a lot.

The power of owners historically has been constrained by public or societal interventions such as direct taxation, imposed income redistribution, levies on capital, and confiscation or morally sanctioned renunciation of wealth as under Islam. Charles Lindblom (1977) famously observed that the state in principle, although constrained in practice, can always use its powers of coercion to seize capital and save or invest the proceeds for its own purposes, as will be illustrated later. But it would need to have an ownership and investment vehicle on hand.

## 3.8 Income, wealth, and capital gains

As noted above, Pasinetti (1962, 1974); Piketty (2014) presume that households get direct access to all profits $rK = \pi X$. Available data do not fit the hypothesis. In the national accounts households receive financial transfers $F$ (interest and dividends ) while the corporate business sector has (usually positive) saving $S_b$. In the data $\pi X \approx F + S_b$ (minor discrepancies arise due to transfers outside the financial sector). How can households get their hands on $S_b$? The answer is capital gains.

To pursue the details, simple macroeconomic accounting is helpful. We begin with the income and product accounts and proceed to flows of funds and balance sheets. Figure 3.4 lays out a social accounting matrix for an economy without government, foreign trade, and (for the moment) depreciation.

The key accounting convention is that sums of corresponding row and columns should be equal. There are still two classes (the number will grow to three in the simulations presented later). All households receive income from various sources, so we replace the worker and capitalist labels with "middle income" and "rich." There is a goods-producing

| | COSTS | | | | | | |
|---|---|---|---|---|---|---|---|
| | | MIDDLE | RICH | BUSINESS | INT.& DIV. | INVESTMENT | TOTALS |
| OUTLAYS | | $C_m$ | $C_r$ | | | $I$ | $X$ |
| | | | | | | | |
| MIDDLE | | | | | | | |
| Wages | $\Psi_m X$ | | | | | | $Y_m$ |
| Financial | | | | | $F_m$ | | |
| | | | | | | | |
| RICH | | | | | | | $Y_r$ |
| Wages | $\Psi_r X$ | | | | | | |
| Financial | | | | | $F_r$ | | |
| | | | | | | | |
| BUSINESS | | | | | | | |
| Profits | $\pi X$ | | | | | | $Y_b$ |
| | | | | | | | |
| INT.&DIV. | | | | $F_b$ | | | $Y_f$ |
| | | | | | | | |
| Macro Balance | | $S_m$ | $S_r$ | $S_b$ | 0 | $-I$ | 0 |
| | | | | | | | |
| TOTAL | $X$ | $Y_m$ | $Y_r$ | $Y_b$ | $Y_f$ | 0 | |

Figure 3.4: Simplified social accounting matrix for economy with "middle" and "rich" classes

business sector, and "finance" which receives interest and dividend payments from business and distributes them (without incurring costs) to households.[12]

The first row gives the decomposition of output $X$ into consumption and gross investment; the first column is a breakdown of costs. Middle-income households receive labor payments $\psi_m X$ from business and transfers $F_m$ from finance to give income $Y_m = \psi_m X + F_m$. In the corresponding column they consume $C_m$ and save $S_m$ in the row for "macro balance." The accounting for rich households with income $Y_r$ is similar. After paying labor, business has gross profits $\pi X$ . A portion $F_b = F_m + F_r$ is transferred to finance for distribution to households; the rest is saved as $S_b$. The macro balance row states that total saving equals investment. Substitution among the entries shows that

$$\pi X = F_b + S_b \tag{3.16}$$

exactly. Profits are exhausted by financial transfers and business saving.[13] The transfers all flow to households.

## 3.9 Rents and Capital Gains

Piketty and Stiglitz emphasize that increases in asset valuations have contributed to the concentration of wealth. One mechanism can be capitalization of "rent," a tricky concept. Tracing back beyond Adam Smith, rent can be viewed as payment for the use of some asset; in other words it is a monopoly price.[14] Along Lindblom's lines the threat of coercion (legally sanctioned or otherwise) underlies a monopoly position. Rents and "rent-seeking" (Krueger, 1974) are a vehicle for mainstream economists to bring class and power into discussion.

Rent-seeking interprets some rents as claims on income associated with social relation-

[12]This treatment of finance approximates the set-up of the US national accounts.

[13]In SAMs based on US data, the equality of profits to financial transfers plus business saving holds to about three significant digits.

[14]Duncan Foley (2006) provides a lucid analysis of the economics of rent over the centuries since Smith.

ships which do not themselves create income. For example a firm bribes politicians to get legislation increasing its profits.[15] This version was popularized by Gordon Tullock (1967). Even he, however, emphasized difficulties in quantifying rent-creating claims. Krueger ran the research department of the World Bank in the 1980s. It devoted a lot of effort toward putting numbers on rents in developing countries, with shaky results.

Stiglitz points to rents, legitimate or not, as sources of wealth inequality if they are "capitalized." That is if some rental flow $\rho$ is associated with the stock of an asset and there is a perfect capital market for the asset in question, then its value will be $\rho/i$ with $i$ as the relevant interest rate. Capitalization of (say) urban rents or subsidies to Texas cotton growers can give high valuations to the associated buildings or land. Like Tullock and Krueger he provides little guidance about the magnitude of these asset prices. Nor is it clear how the capital gains involved make their way from business accounts to households.

A more direct route is to ask how households can access the income embodied in gross business saving, which in the USA fluctuates in the range of $1.5 trillion (roughly ten percent of GDP) per year. Realized capital gains on equity and the stock of residential housing are observable and can function as the transmission mechanism by increasing asset values.[16] We can quickly demonstrate the accounting in connection with simplified (omitting money and bonds) balance sheets in Figure 3.5.

The term $E$ stands for equity outstanding (treated as a liability of business) at price $P_e$. Besides equity, households hold capital $K_h$ (residential housing). Their saving is $S_h$, wealth is $\Omega_h$, and debt is $D_h$. Let a "dot" over a variable represent its change over time and a "hat"

[15]Which relationships generate income is another tricky question. Import quotas are legally sanctioned public-private relationships often used to exemplify targets of rent-seeking. But they helped stimulate industrialization in Korea, so were they unproductive?

[16]This point has been recognized for a long time, and is built into a perfect foresight equation for capital gains proposed by Myron Gordon (1959). Using notation introduced immediately below, let the dividend/capital stock ratio be $\Delta$ and $q = (P_e E)/K$. Then the Gordon equation (ignoring returns to bonds, taxation, and other complications) becomes $r = \hat{P}_e + \Delta/q$, or profits are exhausted by dividends and capital gains on equity. The formula is not rejected by the data.

| HOUSEHOLDS | | | | BUSINESS | |
|---|---|---|---|---|---|
| | | | | | |
| $P_eE$ | $D_h$ | | | $K_b$ | $P_eE$ |
| | | | | | |
| $K_h$ | $\Omega_h$ | | | | $D_b$ |
| | | | | | $\Omega_b$ |

Figure 3.5: Balance sheets for all households and business

its growth rate. The household flow of funds can be written as

$$S_h + \dot{D}_h = I_h + P_e\dot{E},$$

i.e. saving plus new debt finance residential investment plus purchases of newly issued equity at the going price. A negative value of $\dot{D}_h$ would represent new net lending by households, for example to the banking system in the form of higher deposits. Share buybacks imply that $\dot{E} < 0$, presumably accompanied by an rise in $P_e$.

The increase in capital is

$$\dot{K}_h = I_h - \delta_h K_h \tag{3.17}$$

with $\delta_h$ as a rate of depreciation.

"Differentiating the balance sheet" in Figure 3.5 gives

$$\dot{\Omega}_h = \dot{P}_eE + P_e\dot{E} + I_h - \delta_h K_h - \dot{D}_h.$$

Substitution from the flow of funds then shows that

$$\dot{\Omega}_h = S_h - \delta_h K_h + \dot{P}_eE \tag{3.18}$$

An increase in household wealth is generated by net saving and capital gains.

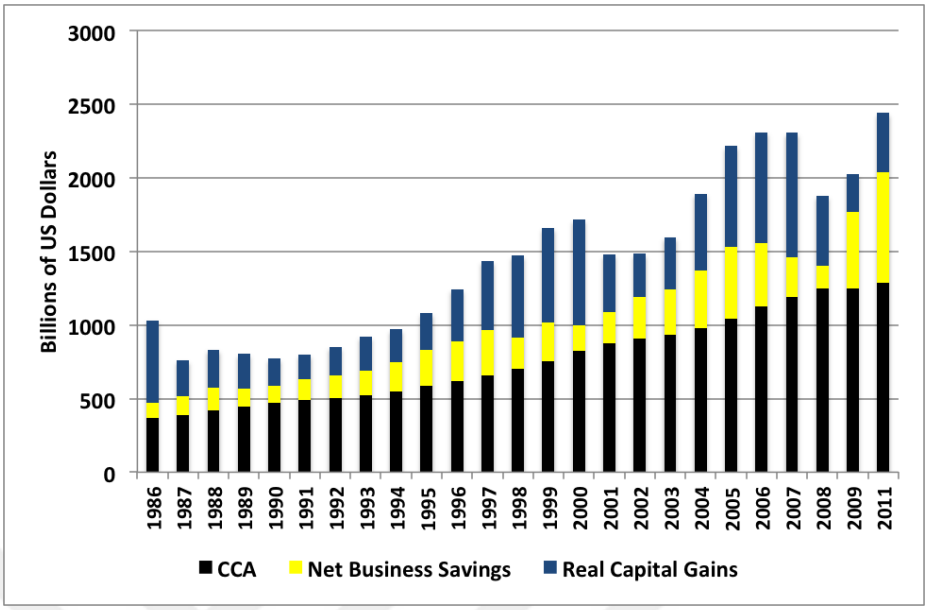


Figure 3.6: Net business saving, business depreciation, and household capital gains

Households' increases in $P_e$ are losses for business so the corresponding expression is

$$\dot{\Omega}_b = S_b - \delta_b K_b - \dot{P}_e E \tag{3.19}$$

with $S_b$ as profits minus interest, dividends, direct taxes, etc.

The empirical question at hand from Eq. 3.19 is how net business saving $S_b - \delta_b K_b$ compares to capital losses $-\dot{P}_e E$ due to rising prices of equity. Figure 3.6 presents data on all realized capital gains (including residential housing) since the mid-1980s. On the whole, yearly gains by households are greater than net business saving so $\dot{\Omega}_b < 0$. Sums exceeding profits are transferred to households via financial flows and asset price changes, running down firms' net worth.

What forces cause capital gains is a vexed and open issue. Capitalization may enter the process by driving up property valuations which get channeled to owner households. Low interest rates beginning in the mid-2000s contributed. A more fundamental factor is the corporate governance philosophy of maximizing shareholder value (and one might add the strategy of executives paid in the form of stock options to prop their payments up). One

manifestation is the movement toward share buybacks, which may reach a trillion dollars in 2015.

For future reference, finally note that income as defined in terms of the Figure 3.4 SAM, does not include capital gains (the "Haig-Simons" definition of income does not apply). Rather, they enter into changes in wealth as defined in Eqns. 3.18) & 3.19. As illustrated below, a tax on capital gains thereby has different macroeconomic impacts than, say, a tax on interest or dividends.

## 3.10 Incomes and wealth

Now it is time to look at the numbers, to see if theory can be adjusted to data so that it can be used to say something about US distributive prospects.

### 3.10.1 Income flows to the classes

The Congressional Budget Office (CBO, 2012) presents estimates of income from different sources for groups of households all across the distribution, from 1986 through 2011. We rescaled the CBO numbers (including capital gains) to be consistent with the national income and product accounts, to give insight into their macroeconomic significance.[17] Here we work with a three-class breakdown, treating the top one percent as rich, the group between the $60^{th}$ and $99^{th}$ percentiles as middle income, and the rest as lower income.

Look first at households in the top one percent of the income size distribution. These people generate most household saving and hold substantial wealth, including equity which produces capital gains. Figure 3.7 summarizes their mean income levels per household over time – late in the decade it was more than $2 million per year. The green segments toward the bottom of the bars show that the well-off did receive growing pre-tax income from labor compensation, around seven percent of the total economy-wide. Bigger chunks come from

[17]For details, see Taylor et al. (2014).

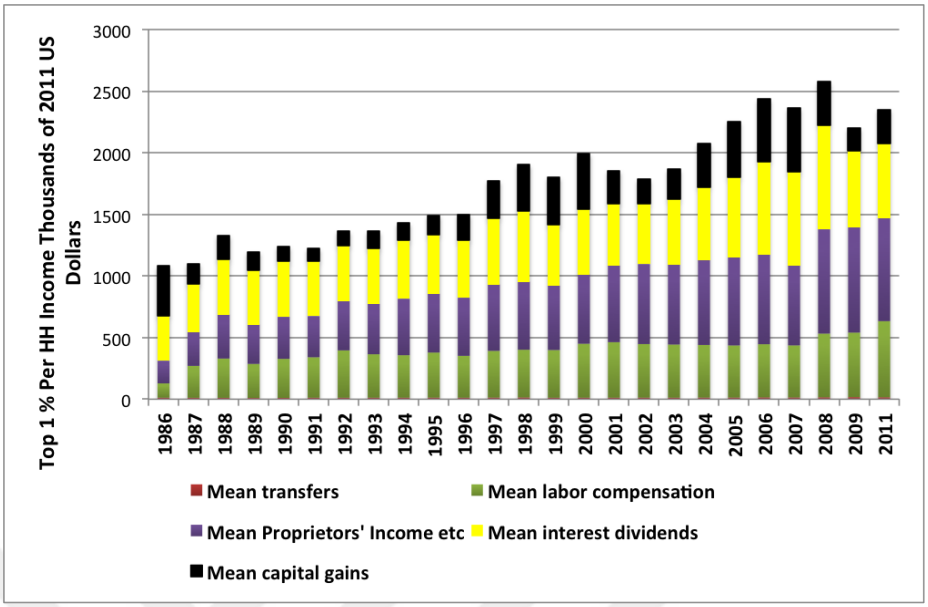


Figure 3.7: Real per household incomes top 1% of households

interest and dividends along with rents and proprietors' incomes like lawyers' fees. Capital gains were substantial in many years.

The chart indicates that rich households steadily gained income from all sources. Over two decades their share of the household total rose by around 10% – a very large change for such an indicator. After Federal taxes, income disparities remain large. The top one percent pay somewhat higher overall rates (23% of pre-tax income as opposed to 18% for all households). Their direct taxes are more progressive but they are scarcely touched by regressive FICA employment taxes.[18]

In 2011 there were 1.1 million households in the top one percent with a mean pre-tax income of $2.073 million, not including capital gains, for a total of $2.28 trillion. GDP was $15.52 trillion so the top group absorbed 14.7% of total output. Including capital gains, in 2014 they probably got close to three trillion. Fifteen percent of GDP represents enormous economic power.

Figure 3.8 shows middle class income sources which sum to 55% of the total. Note the

[18]One might add that Federal income tax rates decline at the highest income levels, e.g.17.6 percent for the richest 0.001 percent of households with average "adjusted gross income" from their tax forms exceeding $60 million per year.

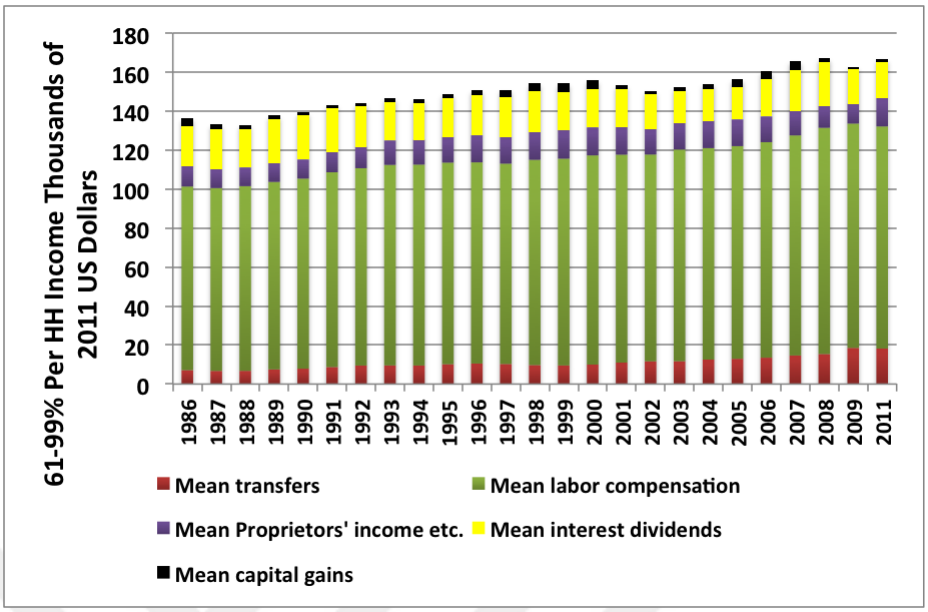


Figure 3.8: Real per household incomes 61-99% of households

difference in the scales between Figures 3.7 and 3.8. The top one percent's income is factor of ten higher than the middle class's. It simply does not fit with flows to the other 99% of households.

Middle class mean income is around $160,000. These households have positive saving rates and visible net worth, largely concentrated in housing. Labor compensation makes up almost 70% of their income and 65% of the total across all three classes. The pre-tax level increased from around $95,000 per household in 1986 to $110,000 in 2011 – a modest growth rate of half a percent per year. In the USA, a visible share of labor pay is absorbed by FICA, explaining the 21% tax rate that these households pay (their direct tax rate is 10.5%).

Other significant income sources are interest and dividends, proprietors' incomes and government transfers such as Social Security, Medicare, unemployment insurance, and (at lower income levels) food stamps and Medicaid.

Figure 3.9 shows that households in the bottom 60% are highly dependent on labor income for their mean level of $55,000. In 2011, transfers were around two-thirds of wages. In reported consumer expenditure data, this group has a negative savings rate, meaning that

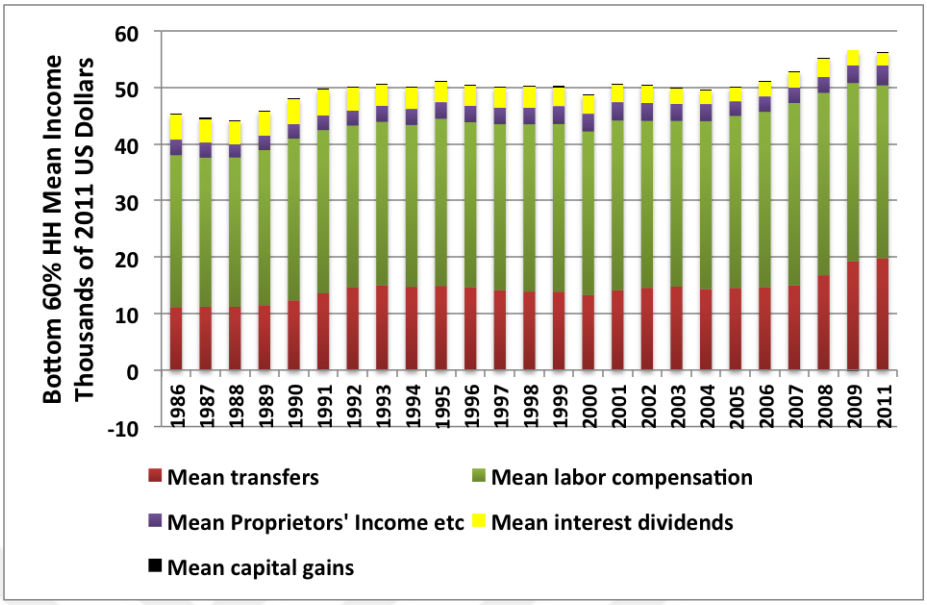


Figure 3.9: Real per household incomes bottom 60% of households

people spend more than they receive. Their average wealth is close to zero, so that financial incomes are very small.

### 3.10.2 Distribution of wealth

Figure 3.10 presents a snapshot of the US distribution of wealth in 2012, based on the Federal Reserve's Financial Accounts and Edward Wolff's breakdown of the size distribution of holdings which comprise physical capital, the value of equity, and other financial instruments ("bonds"). Assets held by each group of actors have a negative sign; liabilities and net worth are positive. The numbers don't add up –reported holdings of bonds by households and finance exceed the supply from firms, government, and the rest of the world.

In any case, consistent with the data on flows in Figure 3.6, the business sector has negative net worth. As noted above, the value of equity outstanding ($29,483 trillion) substantially exceeds firms' capital stock ($19,903 trillion).

In contrast to growth models, total wealth of the private sector – households (and nonprofit institutions) and business combined – includes financial claims besides the value of fixed assets or "capital" $K$. Government debt and net foreign assets both contribute to

| | CAPITAL | BONDS | EQUITY | NET WORTH |
|---|---|---|---|---|
| ALL HOUSEHOLDS | -16312.3 | -23769.1 | -28960.5 | 69041.9 |
| Lower 80% | -3154.7 | -3176.8 | -1332.2 | 7663.7 |
| Upper 20% | -13157.6 | -20592.3 | -27628.3 | 61378.2 |
| Lower 99% | -11923.1 | -15909.8 | -16768.1 | 44601.1 |
| Upper 1% | -4389.2 | -7859.3 | -12192.4 | 24440.8 |
| FIRMS | -19903.3 | 1774.7 | 29482.6 | -11354.0 |
| GOVERNMENT | -12508.0 | 16862.8 | -279.9 | -4074.9 |
| FINANCE | | -5131.6 | -1047.2 | 6178.8 |
| R.O.W | | 3175.3 | 805.1 | -3980.4 |
| Col.Sum | -48723.6 | -7087.9 | 0.1 | 55811.4 |

Figure 3.10: : US assets (-) vs. liabilities (+) divided into capital stock, bonds and equity, and net worth by sector and household income groups, 2012. Levels in US$ billions.
Sources: Assets, liabilities, net worth from Financial Accounts (Federal Reserve, 2014); capital stock from Survey of Current Business (October, 2013), household groupings computed based on Wolff (2012).

private net worth. Capital is the dominant entry.

Households in the table are split into the top 1% and the bottom 99% of holders of wealth (the 20%-80% split is also included for reference). The top 1% directly own 35.4% of household net worth, consistent with other estimates. They hold only 26.9% of household capital stock, basically residential housing (the top 20% hold 80.7%!). Households overall hold roughly one-third of capital, and business holds around one-half. The rest, infrastructure basically, is owned by the government.

## 3.11 SAM representation

The next step is to set up a simulation model to track dynamics of wealth accumulation over time. We begin with a quick description of the data base.

Figure 3.11 is a social accounting matrix illustrating how the three income classes fit into the national economy. The accounting principles are the same as in Figure 3.4. As noted above, the household size distribution has been rescaled to the national income and product accounts (NIPA). The data refer to 2008, a relatively "normal" year for the US economy as it started to slip into recession.

Several points about the SAM's numbers are worth noting. Total supply in the matrix is $16848.1 billion, comprising imports (2556.5) plus GDP (14291.6), as shown in the first column. The first row illustrates how current and capital spending flows break down. Both government and the rest of the world figure in the accounting. Total income flows for the three household groups are 3815.69 (lower), 7504.73 (middle), and 2416.58 (rich).

In the relevant column, transfers (interest and dividends) from the finance sector to households total 2165.4, and in the corresponding row they pay 257.5 to finance. The net transfer is 1907.7. Gross business saving is 1410.1 (in the business expenditure column toward the bottom). In line with Eq. 3.16, the financial transfer and saving sum to 3317.8, close to profits of 3318.3 in the first column.

The top two household groups get the lion's share of financial transfers. The flows are

| 2008 | | | CURRENT EXPENDITURE | | | | | | | CAPITAL EXPENDITURE | | | TOTAL |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | HOU | | | BUS | GOV | ROW | INT & DIV | HOU | BUS | GOV | |
| | | | *Bottom 60th* | *Mid 61-99%* | *Top 1%* | | | | | | | | |
| | **Uses of Total Supply** | | 5038.03 | 4696.82 | 300.65 | | 2381.00 | 1846.80 | | 529.20 | 1558.40 | 497.20 | **16848.10** |
| HOU | ***Bottom 60th*** | **2436.08** | | | | **22.78** | **1140.27** | | **216.56** | 157.33 | | | **3815.69** |
| | Labor Income | 2236.26 | | | | | | | | | | | |
| | Non Labor Income | 199.82 | | | | 22.78 | 1140.27 | | 216.56 | | | | |
| | **Mid 61-99th** | **5775.39** | | | | **13.71** | **686.16** | | **1029.47** | 306.29 | | | **7504.73** |
| | Labor Income | 5269.78 | | | | | | | | | | | |
| | Non Labor Income | 505.61 | | | | 13.71 | 686.16 | | 1029.47 | | | | |
| | **Top 1%** | **1480.93** | | | | **0.32** | **15.97** | | **919.37** | 65.58 | | | **2416.58** |
| | Labor Income | **562.27** | | | | | | | | | | | |
| | Non Labor Income | 918.67 | | | | 0.32 | 15.97 | | 919.37 | | | | |
| | **BUS** | **3318.30** | | | | | | **270.70** | **2746.90** | | | | **6335.90** |
| | Profits | 3318.30 | | | | | | | | | | | |
| | Transfers | | | | | | | 270.70 | | | | | |
| | Int Div | | | | | | | | 2746.90 | | | | |
| | **GOV** | **1280.90** | **326.19** | **1555.91** | **630.60** | **382.00** | | **18.80** | **115.50** | | | | **4309.90** |
| | Indirect Taxes, CCA and OS | 1280.90 | | | | | | | | | | | |
| | Direct Taxes | | (4.03) | 853.75 | 585.98 | 382.00 | | 18.80 | | | | | |
| | Contributions Received | | 330.22 | 702.16 | 44.62 | | | | | | | | |
| | Int Div | | | | | | | | 115.50 | | | | |
| | **ROW** | **2556.50** | **20.30** | **43.16** | **2.74** | **90.50** | **60.80** | | **631.60** | | | | **3405.60** |
| | Imports | 2556.5 | | | | | | | | | | | |
| | Transfers | | 20.30 | 43.16 | 2.74 | 90.50 | 60.80 | | | | | | |
| | Int Div | | | | | | | | 631.60 | | | | |
| | **INT DIV Disbursed** | | 78.57 | 148.90 | 30.03 | 4416.50 | 400.30 | 585.50 | | | | | **5659.80** |
| | **Total Expenditures** | | **5463.09** | **6444.79** | **964.03** | **4925.80** | **4684.50** | **2721.80** | **5659.40** | **529.20** | **1558.40** | **497.20** | **33448.20** |
| | | | | | | | | **Net Lending** | | | | | |
| HOU | ***Bottom 60th*** | | **(1647.40)** | | | | | | | | | | |
| | ***Mid 61-99th*** | | | 1059.94 | | | | | | (529.20) | | | **335.90** |
| | ***Top 1%*** | | | | 1452.56 | | | | | | | | |
| | **BUS** | | | | | 1410.10 | | | | | (1558.40) | | **(148.30)** |
| | **GOV** | | | | | | (374.60) | | | | | (497.20) | **(871.80)** |
| | **ROW** | | | | | | | 683.80 | | | | | **683.80** |
| | **INT DIV** | | | | | | | | 0.40 | | | | **0.40** |
| | | | | | | | | | | **BALANCE** | | | |

Figure 3.11: Three-class US social accounting matrix for 2008

919.37 to the rich (38% of their income) and 1029.47 (13.7%) to the middle class. Note that there is also a very large transfer within the business sector, which includes financial firms. That is, business pays out a total of 4416.5 in interest and dividends, and receives 2746.9 back.

To borrow a European phrase, the US economy is a fiscal and financial "transfer union." Net fiscal flows benefit the bottom 60% of households. They receive 1140.27 in government transfers (almost 30% of their income) and pay slightly negative direct taxes. On the other hand they send 330.22 to the government sector as "social contributions" or FICA and other employment taxes (8.6% of their income).

At the other end of the distribution, the top one percent get minimal government transfers (15.97) but pay 630.6 (26% of income) in direct taxes. Their social contributions are minimal at 44.62.

The middle class pays government more than it receives. Outgoing flows are direct taxes of 853.75 (11.6% of income) and contributions of 702.16 (9.4%). Incoming transfers are 686.16 (9.1%).

On balance, the bottom 60% have net inflows from the government sector of 814.1. The middle class have an outflow of 869.75 and the top one percent lay out 614.6. Total transfers to households are 1842.4, or 12.9% of GDP. In Europe, such flows are on the order of 20% of GDP.

The bottom rows of the table show saving and investment. The sign convention is that saving is positive and investment negative. The external deficit or "foreign saving" is 683.8 or 4.8% of GDP. Government saving is -374.6. Public investment represents another outlay of -497.2 so that the overall financial outflow is -871.8 or 6.1% of GDP. The increase in foreign claims on the US economy is less than new government borrowing so that private sector wealth increases by 871.8-683.8=188. Private saving from households and business must therefore finance private investment plus the fiscal deficit net of capital inflows or 2087.6+188=2275.6.

Low income households have negative saving of -1647.4, or a reported 43.2% of income. Below-zero saving at the bottom of the size distribution shows up consistently in the Bureau of Economic Analysis consumer expenditure data that we used.[19] The saving shortfall from low incomes is offset by the middle class (1059.94 or 14.1% of income) and the rich (1452.56 or 60.1%). Such large saving differentials suggest that model simulations will in part be driven by redistribution across the classes.

## 3.12 Model specification

In the data, middle and high income households somewhat resemble Pasinetti's abstract workers and capitalists. Minor transfers aside, the main difference is that the rich receive wages and proprietors' incomes along with their interest, dividends, and (in the wealth account) capital gains. Low income households hold only a few percent of total wealth. With their negative saving, they certainly boost effective demand. So in the simulations we track incomes and expenditures for all three classes as well as for business, government, and the rest of the world, but keep track of wealth accumulation only for the upper two classes.

The ratio of wealth held by the rich to the household total (assuming away the modest net worth of the bottom group) becomes a key distributive indicator. To measure income inequality we use "Palma ratios" of disposable incomes per household of the top group to the lower two. Proposed by José Gabriel Palma (2011), these ratios focus attention on income disparities between the classes (as opposed to taking an average across incomes as in a Gini coefficient).

At a given time, macro equilibrium is determined in a simple demand-driven specification. Details of a very similar model are presented in Taylor et al. (2014). Briefly, effective demand is set by household and business investment, government spending, and exports as "injections." The main "leakages" are for taxes net of transfers, imports, and

[19] One can speculate about what consistently reported negative saving "really" represents. Movements of households' income positions, understated and/or undeclared incomes, and unpaid informal debt are the most obvious explanations.

saving. Financial transfers representing interest and dividends shift a portion of total profits to households as in Eq. 3.16. The parameter specification makes demand profit-led. The price level $P$ is set as a mark-up on wage and import costs. In equation form we have

$$P = (\Lambda + eaP^*)/(1 - \tau - \Xi - \Pi)$$

with $\Lambda$ as total labor cost per unit output, $eaP^*$ import cost (driven by the exchange rate $e$, import/output coefficient $a$, and world price level $P^*$), $\tau$ indirect tax rate, $\Xi$ proprietors' income rate (across household groups), and $\Pi$ the profit mark-up rate. A higher level of economic activity reduces $\Pi$ as discussed above. It is straightforward to solve for output and price variables, with the other entries in the SAM following from accounting relationships. GDP at current prices is $PX - eaP^*X$.

Over time, levels of injections, transfers, etc. are scaled to capital stocks or wealth so that the system can grow. Leakages such as taxes and imports are scaled to the level of output. Capital stock growth is governed by equations like (3.17) for the household classes, business, and government. Real wealth accumulation comes from Eq. 3.18 for households and Eq. 3.19 for business which must be modified to take a variable price level into account, i.e.

$$\dot{\Omega}_h = (S_h/P) - \delta_h K_h + \dot{P}_e E$$

and

$$\dot{\Omega}_b = (S_b/P) - \delta_b K_b - \dot{P}_e E$$

with $S_h$ and $S_b$ as nominal saving flows.

In the simulations reported here, we set $\dot{P}_e$ in each year to drive $\dot{\Omega}_b$ to zero, so that all business saving is transferred to households via "capital gains." Profits are 100% directed to households as in the theoretical literature discussed above. Demand-driven macroeconomic adjustment applies in the model's medium run so that, for example, higher government spending will increase output and the fiscal deficit with multipliers of around 1.7 and 0.7.

Over time, solutions tend toward a steady state with all variables growing at the same rate. Away from the steady state, distributive changes mix with output adjustment. In the accounting of Figure 3.11, private saving must finance investment plus the fiscal deficit minus the external deficit on current account. In line with the "Keynesian stability condition" built into model, investment responds less strongly than saving to perturbations so we focus on shifts in the latter.

## 3.13 Simulation runs

The simulations aim to examine factors influencing the distributions of income and wealth among the household groups. We begin with an illustrative base run, and go on to variant solutions.

### 3.13.1 Base run and variants

Figure 3.12 shows results from a base run of the model (blue curves) and a couple of variants over time. We ran the simulations out to 250 years to illustrate convergence properties of the system (computer time is cheap). Of course only the first few decades are policy-relevant.[20] The purpose of the base run is to provide a stable benchmark against which distributive variations can be tested. It is not a forecast or projection, and in some ways is unrealistic.[21] For the moment, potential increases in labor productivity and wage levels are ignored – the economy is zero-sum.

Trajectories of the variables converge toward steady state levels over a century or so, typical of dynamics built around accumulation equations such as (3.17). Over 50 years, capital utilization $u = X/K$ goes up by about a percentage point, accompanied by decreases in the price level and profit share. Contrary to recent experience the wealth share $Z$ of

[20]The model was solved over time using Mathematica.

[21]Notably, steady state wealth/output ratios are "too high." The reason is that a high initial level of government net borrowing (6.1% of GDP) propagates over time, adding to household net worth. We chose not to repair this problem by gradually reducing the deficit/GDP ratio because so doing creates strong fluctuations in solution trajectories for the first few decades.

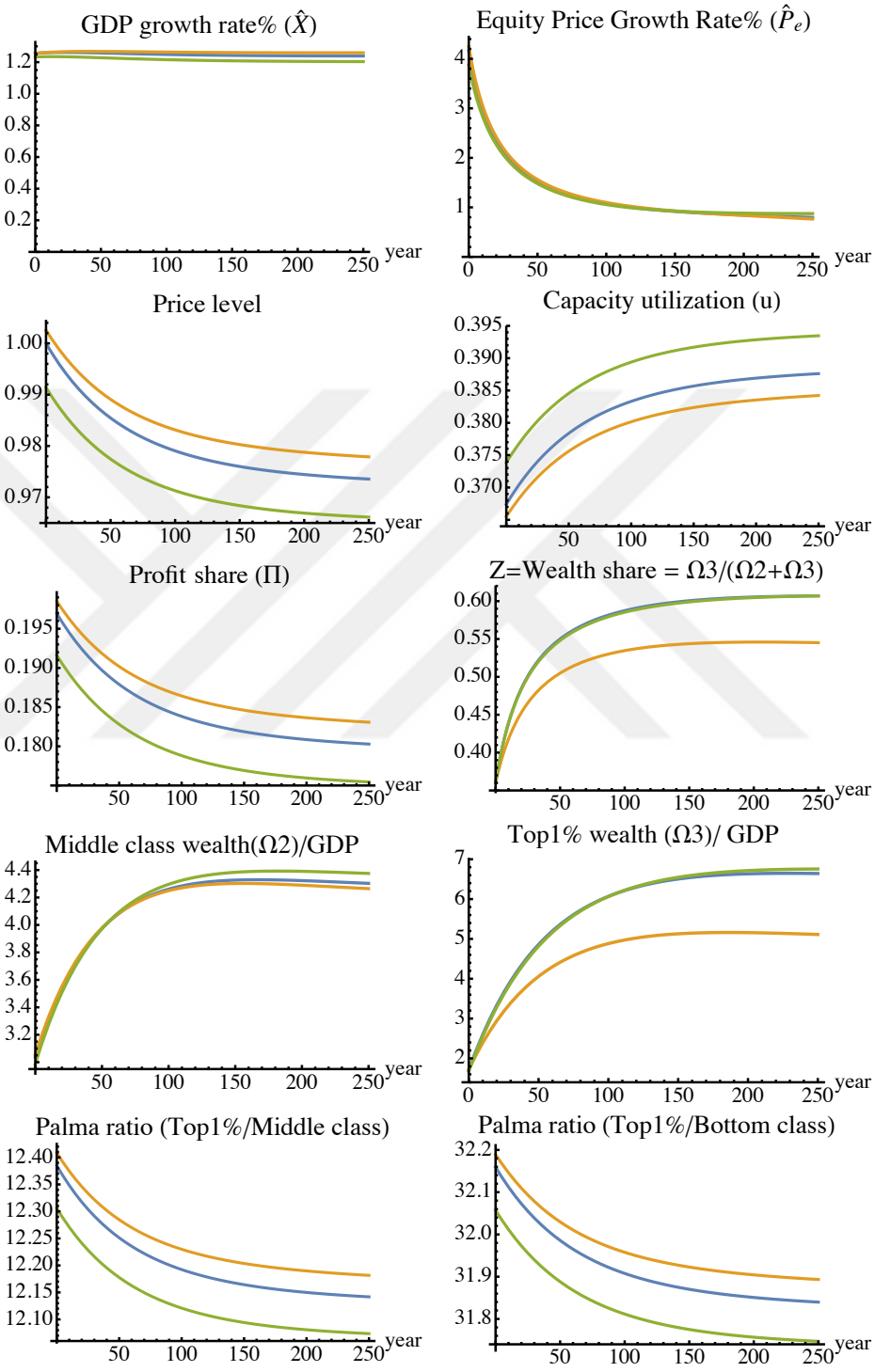


Figure 3.12: 70% increase in tax rate on high income households ($\tau$) is represented by ORANGE, and 10% increase in government spending (G) represented by GREEN, while BLUE represents the ORIGINAL baseline

rich households rises sharply and then settles down at a level of around sixty percent. The modest reductions in $P$ and $\Pi$ are accompanied by slight decreases in the Palma ratios, calculated with respect to disposable income. In short, wealth inequality rises and income inequality stays put.

Government spending over time is proportional to the capital stock. The green curves show what happens when the factor of proportionality is increased by a substantial ten percent. As is to be expected, $u$ increases, leading to profit squeeze reductions in $\Pi$ and $P$. Wealth accumulation is unaffected, with the blue and green curves overlapping for wealth levels $\Omega_2$ and $\Omega_3$ and the concentration ratio $Z$. The Palma ratios slide down slightly because of the lower mark-up rate.

The orange curves illustrate responses to an increase in the tax rate on high income households by 70%, from about 24% to 41%. (The level was chosen to have roughly the opposite effect to higher spending.) Overall saving increases (the government's marginal saving or abstinence rate is 100%) so capital utilization goes down in line with the paradox of thrift. Despite the upper income tax increase, Palma ratios for disposable income increase slightly because reduced output cuts into income flows below the top. Over 50 years, wealth concentration goes down by a few points. The implication of both variants is that relatively large one-shot fiscal adjustments will not have much impact on inequality over time.

### 3.13.2 Capital Gains

As mentioned above capital gains measured by the growth rate of the price of equity $\hat{P}_e$ are supposed to mediate transfers of net business saving to households. Eq. 3.19 describes the mechanism. It is tempting to tax capital gains. Compared to the base run, the effects of a 70% profits/capital gains tax are shown in Figure 3.13.

Because the Haig-Simons definition of incomes is not built into the model (see above) the effects of the tax on effective demand and income inequality are minimal. As with the tax on high income households in Figure 3.12, there are small adjustments in wealth

accumulation.

### 3.13.3 Wages and productivity

For short, we call mean labor income per household in the three groups "wages." An immediate question is how changes in such payments will affect the macro-economy. Figure 3.14 shows what happens when the wage of the poor household group is increased by 40%, the middle group gets an increase of 20%, and labor payments per household for the top group are cut by 40%. These changes lie at the extremes of proposals being advanced in 2015.

Compared to the base run (blue vs. orange), these shifts kick off a big increase in capital utilization via effective demand. The resulting profit squeeze slows investment and the rate of growth. The distribution of wealth shifts against the upper income group but even so the concentration ratio $Z$ tends toward a long-run ratio exceeding one-half. More interesting is that in a growing economy a large once-off shift in labor payments does not strongly affect income inequality – the Palma ratios drop by only a few percent. At least in the present model, static wage adjustments cannot redress unequalizing income trends over the past three decades.

Figure 3.15 tells a different story. The underlying assumptions are that labor productivity increases over time and that nominal wage levels grow faster. In a bit more detail productivity $\xi$ is assumed to grow at 1.5% per year initially, with the growth rate tending toward zero as $\xi$ converges toward twice its level in the simulation's base year. Wages are assumed to grow two percent faster than productivity.

There is a pay-off in terms of greater income equality. The Palma ratios decline visibly. Despite favorable wage trends, however, the simulated reductions in the ratios are less than the increases from 11.8 to 33 (lower income) and from 4.3 to 12.7 (middle income) between 1986 and 2008 in our data set. Decreases in wealth concentration are small.

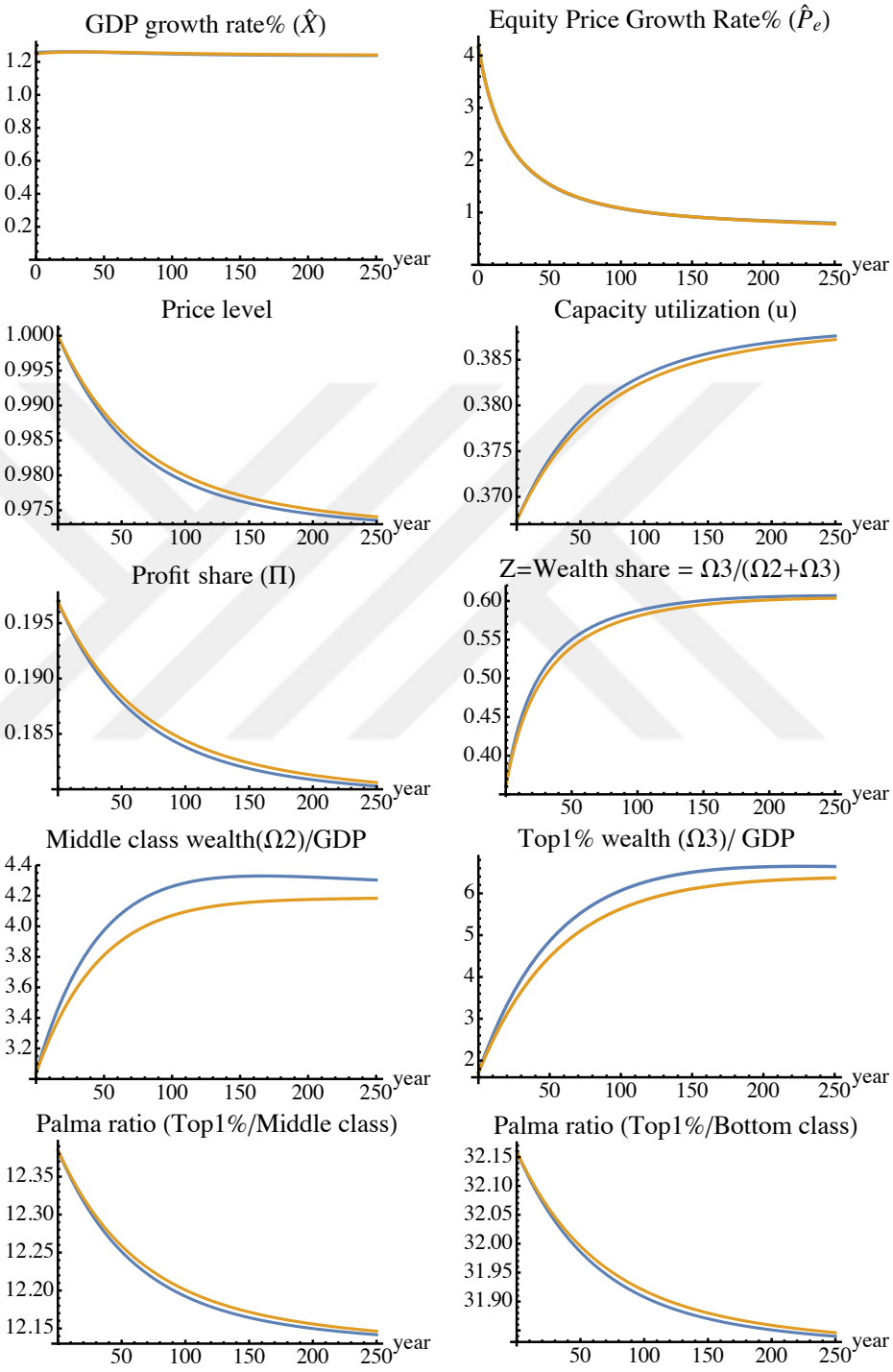


Figure 3.13: 70% tax on capital gains for middle and highest income groups, both transferred to government income. Tax on capital gains is represented by ORANGE line while BLUE line represents the original baseline.

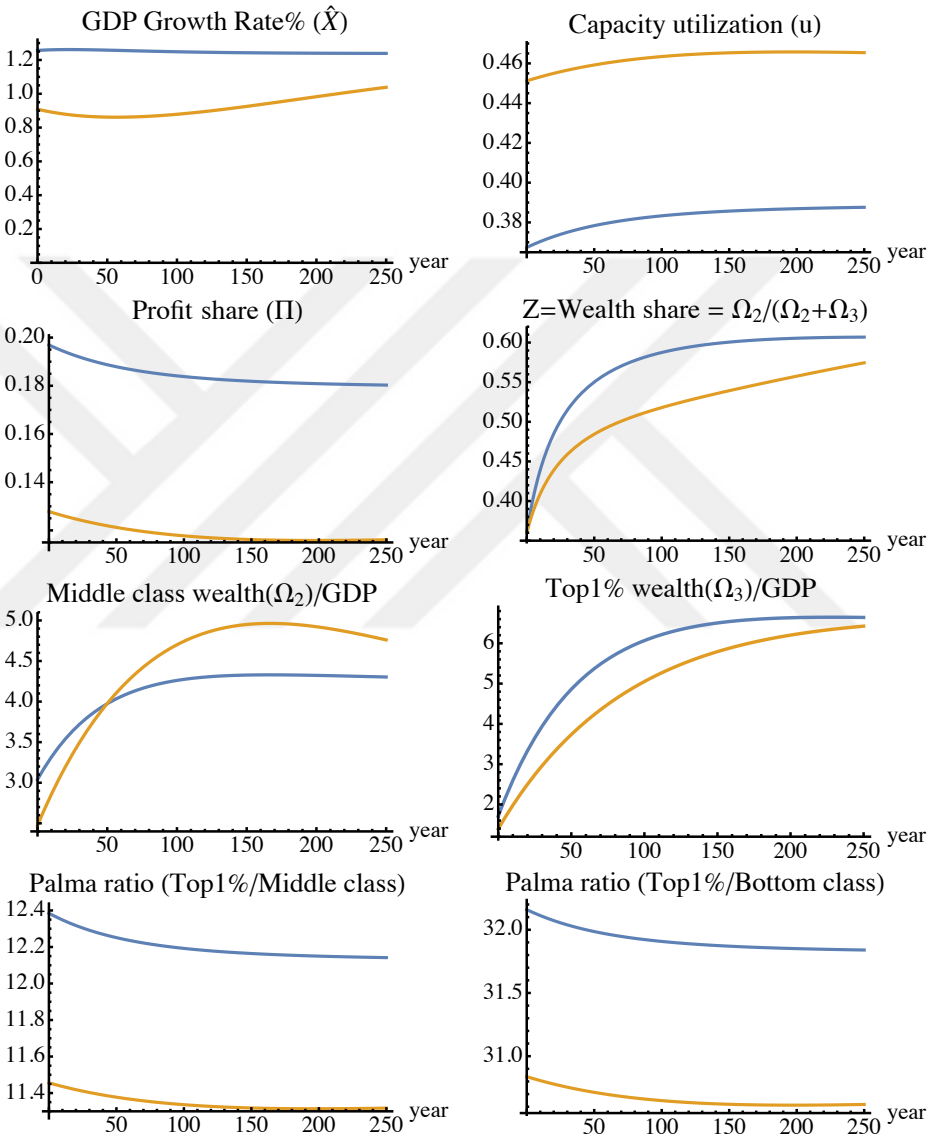


Figure 3.14: ORANGE line represents wage changes for different income groups. (40% increase for bottom group, 20% increase for middle group and 40% decline for top 1% of households) while baseline is represented by BLUE line.

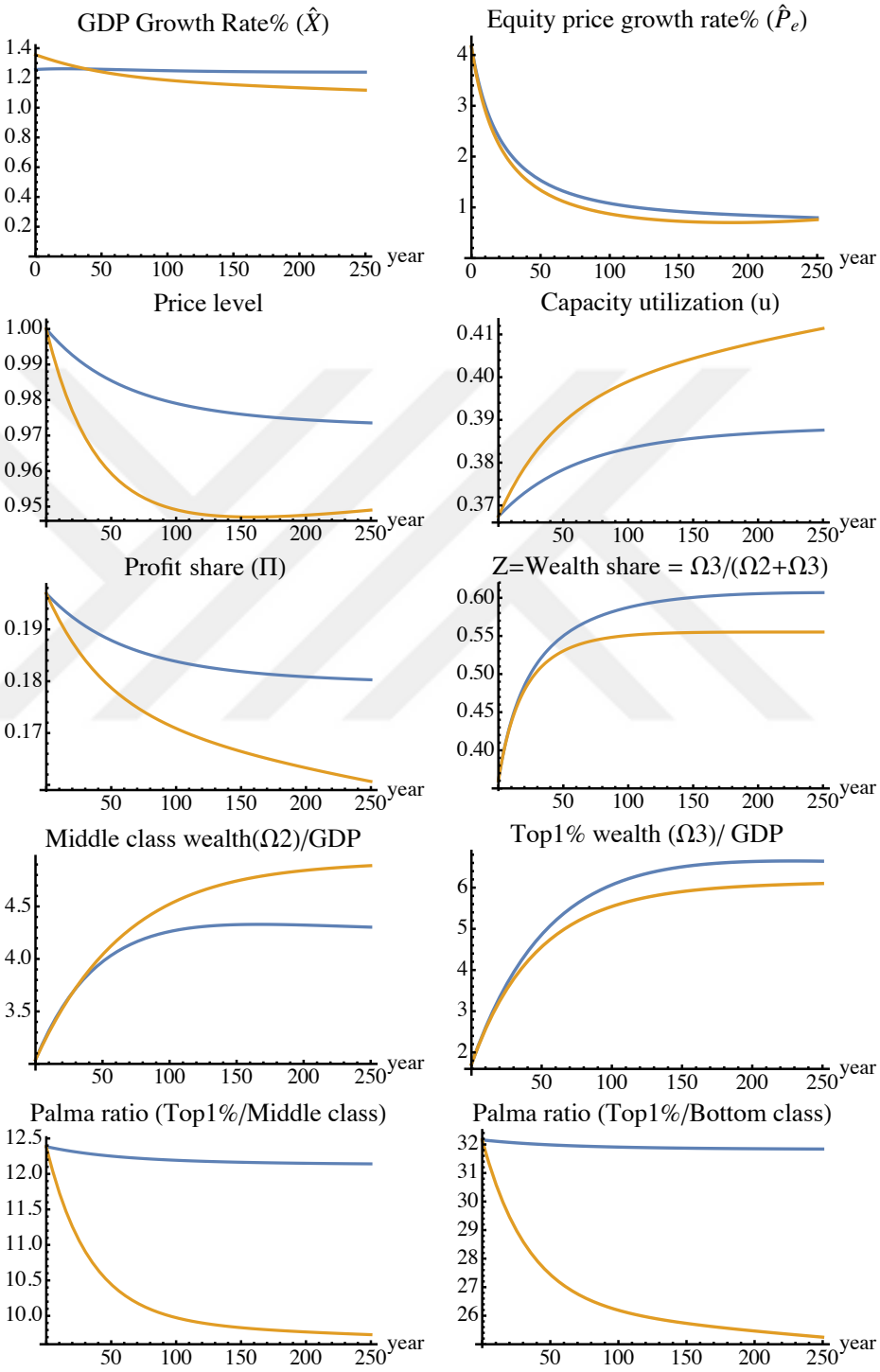


Figure 3.15: Original baseline (BLUE line) vs labor Baseline with an increase in labor productivity and wage growth (ORANGE line)

### 3.13.4 Wealth fund

As we (and Andrew Mellon) noted at the outset, owners of wealth tend to maintain their positions because their large stocks of assets generate high incomes from which their saving rate is high. The macro economy channels their abstinence into ongoing accumulation.

One way to overcome this advantage is through a public wealth fund which can become an alternative vehicle for accumulation. Perhaps the best-known proposal is still the one put forward 65 years ago by the Swedish trade union economists Gösta Rehn and Rudolf Meidner. Part of the plan involved a fund to be built up from new special shares to be issued by firms which would support workers' pensions. The fund's accumulation would provide a counterweight to private sector saving.[22] In the recent period, the "oil fund" in Norway ($900 billion in assets) and the California pension fund CalPERS ($300 billion) provide practical examples of well-known publicly controlled institutions.

We can borrow this idea by imposing a tax on capital gains (by assumption equal to net business profits). But rather than just passing the revenues to the government to be saved uselessly as in Figure 3.13, they could be transferred to a wealth fund for accumulation. Growing from a zero base, the fund would also receive a share proportional to its size from interest and dividend payments on its investments.

For Meade's reasons the fund would not take over the economy – rich households have income sources besides finance. Nevertheless, the orange curves in Figure 3.16 shows that with the fund supported by a 70% tax on capital gains, the share of rich households in total wealth grows for only a few decades and then starts to taper off. The fund's share steadily increases.[23]

An obvious question regards the uses of the fund's assets. The Norwegian oil fund, for

[22]Blackburn (2005) provides a concise review of the plan. Incidentally, Gösta Rehn was not related to the Finn Swede Olli Rehn, the EU commissioner notorious for his austerian views during the post-2008 financial crisis.

[23]In a Pasinetti world, if there is a tax at rate t on gross profits $rK$ used to finance a wealth fund one can use the logic underlying Eq. 3.15 to show that that the fund's steady state share in total wealth would be $s_f t/s_c(1-t)$, with $s_f$ as its saving rate (presumably close to one). If $s_f = 1$ and $s_c = 0.5$ then even a 20% tax on would give the fund ownership of one-half of capital in the long run.

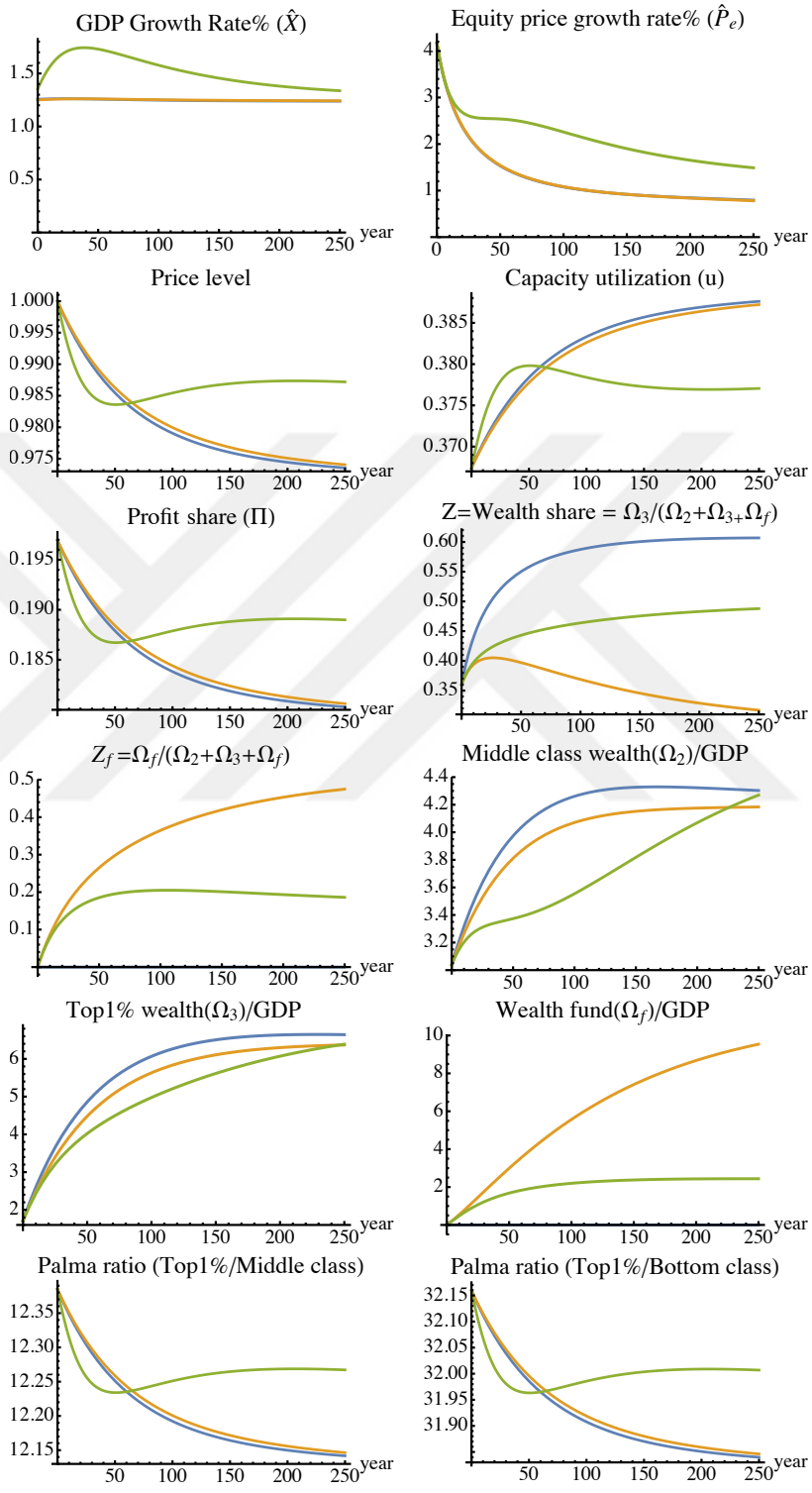


Figure 3.16: Original baseline (BLUE line) vs. wealth fund. ORANGE line represents wealth fund without transfer to government saving, and GREEN line represents wealth fund with 2% transfer to government saving

example, has a target of shifting four percent annually of its assets to the government. In the model here, at least, such a transfer is not feasible in the long run. The wealth fund's share of the total gets up to around ten percent and then steadily declines. The green curves in Figure 3.16 shows what happens if two percent of fund assets are transferred to government saving and capital formation. The fund's wealth share goes up and then stabilizes while the rich households' share continues to rise. An aggressive public fund appears to be needed to make a real dent in the concentration of wealth.

## 3.14 Bottom Line

Wage and productivity dynamics on one hand and a wealth fund on the other involve dynamic processes which are largely independent of each other. Figure 3.17 shows what happens when the two are combined. Income inequality and wealth concentration both go down.

The simulations imply that continuing trends toward equalization over decades would be required if income and wealth inequality is to decline. This result is robust insofar as it mainly is built upon the accounting rules underlying Figures 3.10 and 3.11. Policy is relevant only if it can support sustained growth in wages and productivity along with a vehicle to divert abstinence and accumulation toward public ends.

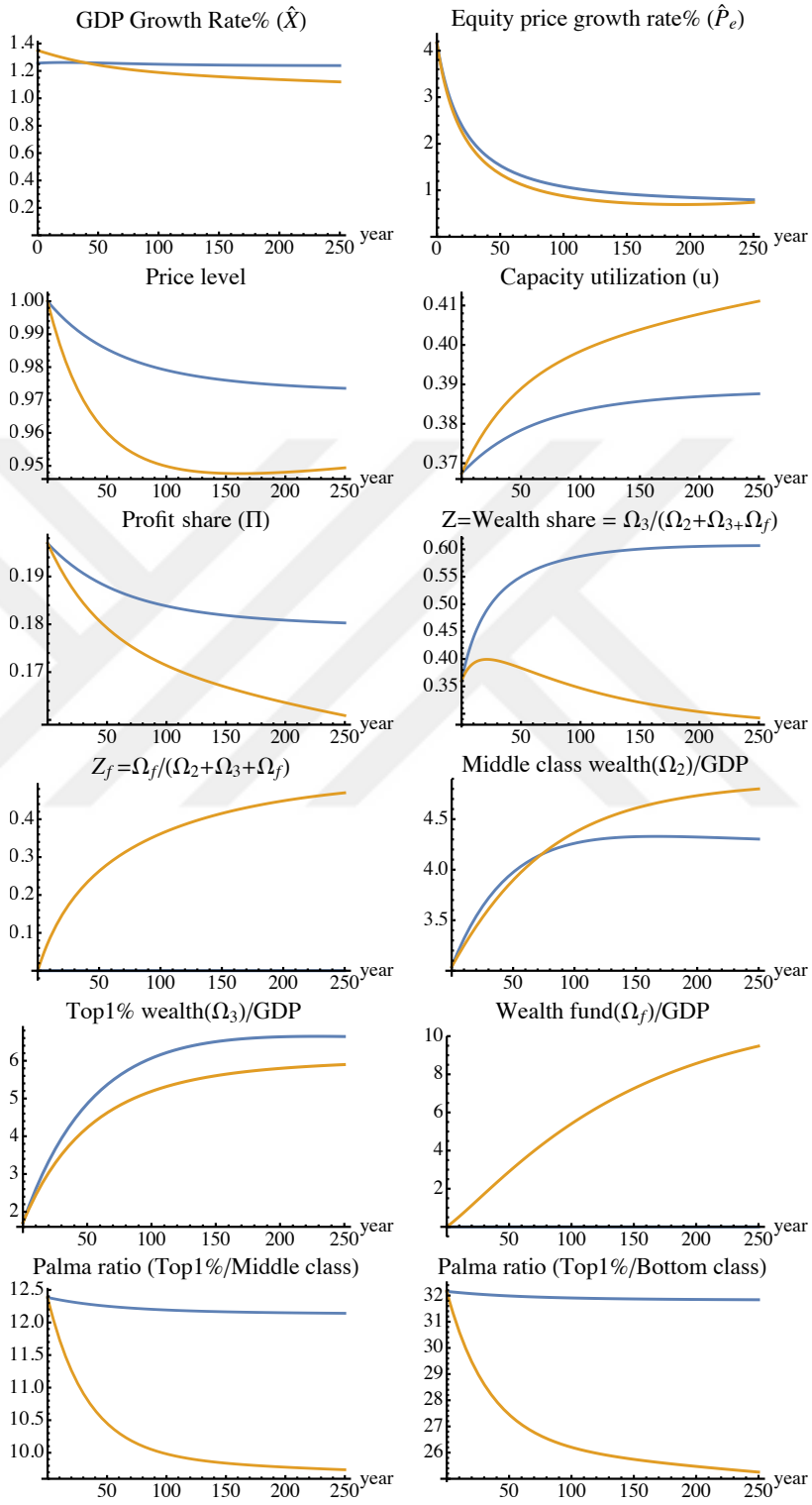


Figure 3.17: Original baseline (BLUE line) vs. combination of wealth fund (no transfer to government saving) with labor productivity growth and wage growth (ORANGE line)

# APPENDICES

# APPENDIX A

# Maximum Entropy Program of the QRSE Model for the Housing Market

This section presents the solution to the maximum entropy program, which is used to predict the marginal frequencies, $f[x]$, subject to three constraints; the constraint on the difference between the weighted conditional expected outcomes, Eq (6), a constraint on the mean value of the outcome, $\int f[x]xdx = \xi$, and the assumption that the conditional action is a logit function with parameters $\mu \quad and \quad T$ .

Incorporating the assumption of quantal response, constraint on the difference between the weighted conditional expected outcomes, Eq (6) with the conditional action constraint takes a more compact form which is a hyperbolic tangent function that is:

$$\begin{aligned} f[sell]E[x|sell] - f[buy]E[x|buy] &= \int (f[sell|x]f[x] - f[buy|x])f[x])xdx \\ &= \int \left(\frac{1}{1+e^{-\frac{(x-\mu)}{T}}} - \frac{e^{-\frac{(x-\mu)}{T}}}{1+e^{-\frac{(x-\mu)}{T}}}\right) f[x]xdx \\ &= \int Tanh\left[\frac{(x-\mu)}{2T}\right] f[x]xdx \leq \delta \end{aligned}$$

Since the data on actions are not available, the entropy of the joint distribution can be

written in terms of the marginal distribution $f[x]$, as:

$$H = -\int f[x]Log[f[x]]dx + \int H_{\mu,T}[x]dx$$

where $H_{\mu,T}$ is the binary entropy function:

$$H_{\mu,T}[x] = -\left(\int f[x]Log\left[\frac{1}{1+e^{\frac{-(x-\mu)}{T}}}\right] + \int f[x]Log\left[\frac{e^{\frac{-(x-\mu)}{T}}}{1+e^{\frac{-(x-\mu)}{T}}}\right]\right)$$

Therefore, maximum entropy program that determines the marginal distribution $f[x]$ given the parameters $T, \mu, \delta, \xi$ can be expressed as:

$$\begin{aligned} \underset{f[x]\geq 0}{\text{Max}} \quad & -\int [f[x]Log[f[x]]dx + \int f[x]H_{\mu,T}[x]dx \\ \text{subject to} \quad & \int f[x]dx = 1 \\ & \int f[x]xdx = \xi \\ & \int Tanh\left[\frac{(x-\mu)}{2T}\right]f[x]xdx \leq \delta \end{aligned}$$

The Lagrangian to this maxent program, hence is :

$$\begin{aligned} \mathcal{L}[f[x], \lambda, \beta] = & -\int [f[x]Log[f[x]]dx + \int f[x]H_{\mu,T}[x]dx - \lambda\left(\int f[x]dx - 1\right) \\ & - \gamma\left(\int f[x]xdx - \xi\right) - \beta\left(\int Tanh\left[\frac{(x-\mu)}{2T}\right]f[x]xdx - \delta\right) \end{aligned}$$

The associated first order conditions for maximizing entropy of the joint frequencies and conditional frequency require:

$$\frac{\partial \mathcal{L}}{\partial f[x]} = -Log[f[x]] - 1 - \lambda + H_{\mu,T}[x] - \gamma\mu - \beta Tanh\left[\frac{(x-\mu)}{2T}\right]x = 0$$

Solving for $f[x]$ gives the maximum entropy (recovered) marginal distribution $\hat{f}[x]$ with normalization factor:

$$\hat{f}[x] = \frac{e^{H_{\mu,T}[x]} e^{-\gamma x} e^{\beta Tanh[\frac{x-\mu}{2T}]x}}{\int e^{H_{\mu,T}[x]} e^{-\gamma x} e^{\beta Tanh[\frac{x-\mu}{2T}]x} dx}$$

# A.1

## Posterior Probability Estimates

As mentioned above, predicted marginal distribution $\hat{f}[x]$ from equation (7) is a Kernel to the maximum entropy program, and together with the parameters $T, \mu, \beta, \gamma$, provides a multinomial distribution for the model. In a Bayesian sense, observed marginal distribution, $\bar{f}[x]$ is just a sample distribution from the multinomial distribution. We use the Kullback-Leibler divergence as an approximation to the log posterior probability for the multinomial model since it allows us to make posterior inferences about the parameter estimations[1].

Kullback-Leibler divergence measures the discrepancy between the observed marginal frequencies, $\bar{f}[x]$ and predicted marginal frequencies $\hat{f}[x; T, \mu, \beta, \gamma]$ inferred from the maximum entropy Kernel as:

$$D_{KL}[\hat{f}[x], \bar{f[x]}] = \sum \hat{f}[x] log[\frac{\hat{f}[x]}{\bar{f}[x]} \tag{A.1}$$

.

As a result, KL divergence provides us with a tool to compare the observed marginal frequency distribution with the predicted marginal distribution. If $\hat{f}[x] = \bar{f}[x]$, $D_{KL}$ becomes zero indicating that two distribution are the same. Therefore, smaller the KL divergence is closer the observed distribution to the predicted one and better the fit.

Model parameters $T, \mu, \beta, \gamma$ are estimated jointly by minimizing the $D_{KL}$ function between the observed and inferred marginal frequencies. In order to measure the closeness of the model fit, we used information distinguishability criteria (ID) introduced by Soofi and Retzer (2002). ID measure shows approximately how much of the informational content of

[1]Please see Scharfenaker and Foley (2017a) where Kullback-Leibler divergence is proved to be a good approximation to the log posterior probability for the multinomial model.

the observed frequencies to be captured from the results of the maximum entropy program.

Finally, to make inferences about the posterior parameters and their credible intervals, conditional distributions of each parameter, holding all others at their maximum posterior probability estimates $\bar{\Omega}$, (where the model parameters are $\Omega = \{T, \mu, \beta, \gamma\}$), are calculated.

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