Inequity-averse optimization in disaster preparedness and response

by

Mahdi Mostajabdaveh

A Dissertation Submitted to the
Graduate School of Sciences and Engineering
in Partial Fulfillment of the Requirements for
the Degree of

Doctor of Philosophy

in

Industrial Engineering and Operation Management

KOÇ UNIVERSITY

July 10, 2019

Inequity-averse optimization in disaster preparedness and response

Koç University

Graduate School of Sciences and Engineering

This is to certify that I have examined this copy of a doctoral dissertation by

Mahdi Mostajabdaveh

and have found that it is complete and satisfactory in all respects,

and that any and all revisions required by the final

examining committee have been made.

Committee Members:

Prof. Sibel Salman

Assoc. Prof. Deniz Aksen

Asst. Prof. hsan Yankolu

Prof. Sander de Leeuw

Assoc. Prof. Roberto Roberti

Date:

To my beloved wife

ABSTRACT

While efficiency, e.g. minimizing total cost, is the major concern in commercial operations, equity comes forward as an essential requirement in humanitarian operations such as response services. In humanitarian operations, models that address inequity-averseness have received insignificant attention. In this thesis, we aim to incorporate both efficiency and equity objectives into disaster preparedness and response operations. The Gini coefficient has been utilized in the economics literature for assessing income inequity among individuals. In this work, we develop models that incorporate measures of equity based on the Gini coefficient from the economics literature, to balance efficiency and equity.

First, we study the problem of selecting a set of shelter locations in preparation for natural disasters. We incorporate uncertainties concerning demand locations, demand amounts, and road network accessibility into the problem. Our goal is to minimize a measure derived from individual distances that take both efficiency and inequity into account. We use a stochastic model with a set of possible disaster scenarios. Demand, affected population, and traversable road distances between population nodes and facility locations depend on the scenarios. A chance constraint is defined on the total cost of opening the shelters and their capacity expansion. In this stochastic context, a weighted mean of the so-called *ex ante* and *ex post* versions of the inequity-averse objective function under uncertainty are optimized. Since the model can be solved to optimality only for small instances, we developed a tailored genetic algorithm (GA) that utilizes a mixed integer programming subproblem to solve this problem heuristically for larger instances. We compared the performance of the mathematical program and the GA via benchmark instances. It turns out that the GA yields small optimality gaps in a much shorter time for these instances. We run the GA also on

real data of the Kartal district in Istanbul to drive insights to guide decision-makers for preparation. We also implement a bi-objective analysis to approximate the Pareto frontier considering efficiency and equity as separated objectives. Furthermore, we compared ex-ante and ex-post approaches in the objective function.

Second, we study the distribution of relief supplies to the shelters as part of humanitarian relief logistics in response to a natural disaster. We consider the problem of planning vehicle routes between a depot and shelters as well as assigning relief supply delivery amounts to the shelters. We assume the prepositioned relief supply amount will not be sufficient enough to meet all shelters' demands. We define two objectives for the problem: minimization of the total time of relief item distribution and the Gini index related inequity measure on the unsatisfied demand percentages among the shelters. We develop three mathematical formulations for the problem based on the VRP formulation in the literature. We adopt the ϵ-constraint approach to handle our multi-objective problem. We solve the linear relaxation of the problem with column generation, which ensures a good quality lower bound. Moreover, a branch-and-price (B&P) algorithm is proposed to solve the problem efficiently to optimality. Our computational results show the superior performance of the B&P algorithm to solving the alternative models with commercial solvers. Finally, we execute the B&P algorithm on real data of Istanbul and provide some insights to be considered by the decision makers.

ÖZETÇE

Verimlilik, toplam maliyeti minimize etmek yerine, ticari operasyonlarda büyük bir endişe kaynağıdır. Operasyonel yönleri ele alan optimizasyon problemleri göz önünde bulundurulmalıdır. Yöneylem Araştırması literatüründe, kaynak tahsisi, tesis yeri, zamanlama ve telekomünikasyonda bant genişliği tahsisi; işbirlikçi lojistikte maliyet / fayda tahsisi; sağlık hizmetlerinde organ, kan ve ilaç dağılımı; kamu tesisi yeri. Bununla birlikte, insani yardım operasyonlarında, eşitsizlikten kaçınmaya hitap eden modeller nispeten daha az dikkat çekmiştir. Bu çalışmada, afete hazırlık ve müdahale operasyonlarını birleştirmeyi hedefliyoruz.

Bu tezde, önce doğal afetlere hazırlık için bir dizi sığınak yeri seçme problemini inceliyoruz. Talep konumları, talep miktarları ve karayolu ağı erişilebilirliğine ilişkin belirsizlikleri soruna dahil ediyoruz. Amacımız, hem verimliliği hem de eşitsizliği hesaba katan bireysel mesafelerden türetilmiş bir ölçümü en aza indirmektir. Verimlilik, bir kişi tarafından belirlenen barınak yerine ulaşmak için kat edilen ortalama mesafe ile tanımlanır ve eşitsizlik, Gini'nin Ortalama Mutlak Farkı olarak ölçülür. Bir dizi olası felaket senaryosu olan stokastik bir model kullanıyoruz. Talep, etkilenen nüfus ve nüfus düğümleri ile tesis yerleri arasındaki hareketli yol mesafeleri senaryolara dayanmaktadır. Barınakların açılmasının toplam maliyeti ve kapasite artırımı konusunda bir şans kısıtlaması tanımlanmıştır. Bu stokastik bağlamda, belirsizlik altında eşitsizlikten kaçınan amaç fonksiyonunun sözde ex-ante ve ex-post versiyonlarının ağırlıklı bir ortalaması optimize edilmiştir. Model sadece küçük durumlar için iyimserliğe çözülebildiğinden, bu problemi daha büyük durumlar için buluşsal olarak çözmek için karma bir tamsayı programlama alt problemini kullanan özel bir genetik algoritma (GA) geliştirdik. Matematiksel programın ve GA'nın, modelin iyimserliğe veya iyimserliğe yakın bir şekilde çözülebildiği temel örneklerle karşılaştırdık. GA'nın

bu durumlar için çok daha kısa sürede küçük bir iyimserlik boşluğu sağladığı ortaya çıktı. Karar vericilere hazırlık için rehberlik amacıyla iç görüleri sağlamak için, İstanbuldaki Kartal ilçesinin gerçek verileri üzerine de GAyı kullanıyoruz. Ayrıca, nesnel işlevdeki eski ve eski yaklaşımları karşılaştırdık. Ayrı hedefler olarak verimlilik ve eşitliği göz önünde bulundurarak Pareto sınırına yaklaşmak için iki hedefli bir analiz de uyguluyoruz.

İkinci olarak, doğal afetlere yanıt olarak insani yardım lojistiğinin bir parçası olarak yardım malzemelerinin barınaklara dağıtımını inceliyoruz. Depo ve sığınaklar arasında araç rotalarının planlanmasının yanı sıra sığınağa rölyef arz teslimat tutarlarının tahsis edilmesinin problem olduğunu düşünüyoruz. Öngörülen rölyef arz miktarının tüm barınak taleplerini karşılamak için yeterli olmayacağını varsayıyoruz. Problem için iki hedef tanımlarız: sığınaklar arasındaki karşılanmayan talep yüzdeleri üzerindeki toplam serbest bırakma madde dağıtım süresinin ve Gini endeksine bağlı eşitsizlik ölçütünün asgariye indirilmesi. Literatürdeki VRP formülasyonuna dayanarak problem için üç matematiksel formülasyon geliştiriyoruz. Çok amaçlı sorunumuzu ele almak için cons-kısıtlama yaklaşımını benimsiyoruz. Problemin lineer gevşemesini, iyi bir düşük bağlama sağlayan kolon jenerasyonu ile çözeriz. Ayrıca, sorunu iyimserlikle verimli bir şekilde çözmek için branch-and-price (B& P) algoritması önerilmiştir. Hesaplamalı sonuçlarımız, B& P algoritmasının alternatif modelleri ticari çözücülerle çözme konusundaki üstün performansını göstermektedir. Son olarak, B& P algoritmasını İstanbul'un gerçek verileri üzerine uyguluyoruz ve karar vericiler tarafından dikkate alınması gereken bazı bilgileri veriyoruz.

ACKNOWLEDGMENTS

First of all, I would like to express my gratitude to my advisor Prof. Sibel Salman. She has always been a great role model in both research and personal life. I had faced many challenges that seemed impossible to overcome during my Ph.D., but her great ideas and suggestions, calm and supportive attitude, and the enormous believe she had in my research helped me to get through this journey. I appreciate all her contributions of time, ideas, and funding to make my Ph.D. experience productive and stimulating.

Many thanks also to Prof. Walter Guthjar for his thoughtful advises, extended discussions and valuable suggestions through out my Ph.D. research which have contributed greatly to the improvement of the thesis.

I appreciate the time and valuable comments that members of my thesis advisory committee: Prof. Deniz Aksen and Dr. hsan Yankolu genuinely offer to me in the entire process of completing this work. I would also like to thank other thesis committee members: Prof. Sander de Leeuw and Prof. Roberto Roberti for their time and support by reading my thesis, listening to my work, and giving great feedback on it.

I gratefully acknowledge the scholarship and many other side benefits I received from Koc University to undertake my Ph.D. My time at Koc University was made enjoyable in large part due to the many friends that became a part of my life. I am specially grateful for time spent with Meraj Ajam, Milad Mokhtaridoost, Katy Soleimanpour and Sina Khoshsima.

I would also like to say a heartfelt thank you to my Mum, Dad, Mina and Ali for always believing in me and encouraging me to follow my dreams. And Zeynab and Hassan for helping in whatever way they could during this challenging period.

Last but not least, I would like thank my loving, supportive, encouraging, and patient wife Nadia who has been by my side throughout this Ph.D., living every single minute of it, and without whom, I would not have had the courage to embark on this journey in the first place. I should also thank Marron for all of the joy she had brought to our lives these past two years.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURE

CPU: Central Processing Unit

GA: Genetic Algorithm

CG: Column Generation

Gurobi: An optimization solver for various types of optimization problems

CPLEX: An optimization solver for various types of optimization problems

RMP: Restricted Master Problem

MIP: Mixed Integer Programming

B&P: Branch and Price

CL: Candidate Locations

PN: Population Nodes

GMAD:Gini's Mean Absolute Difference

NP-hard: Non-deterministic Polynomial-time hard

SP: Set Partitioning

TSP: Traveling Salesman Problem

ADTS: Average Travelling Distance from the Shelter

GAP : General Assignment Problem

RGA : Randomized Greedy Algorithm

AM: Allocation Model

CVAR: Conditional-Value-At-Risk

VRP: Vehicle Routing Problem

CVRP: Capacitated Vehicle Routing Problem

VNS : Variable Neighborhood Search

Chapter 1

INTRODUCTION

While efficiency, e.g., minimizing total cost, is the major concern in commercial operations, equity comes forward as an essential requirement in humanitarian operations such as response services. Optimization problems addressing operational aspects should aim at providing equitable services to victims in order to make the suggested solutions acceptable. In the Operations Research literature, equity has been addressed in problems such as resource allocation, facility location, scheduling and transportation in various contexts such as bandwidth allocation in telecommunications; cost/benefit allocation in collaborative logistics; organ, blood, and drug allocation in health care; public facility location (Balcik et al. 2010), and hazardous material transportation (Gopalan et al. 1990). However, in humanitarian operations, models that address inequity-averseness have received relatively less attention. In this thesis, we aim to incorporate both efficiency and equity objectives into both disaster preparedness and response operational decisions, namely shelter location and relief item distribution decisions.

1.1 Shelter location problem for disaster preparedness

In general, equity (often synonymously called fairness) aims at equal distribution of benefits or disutilities. For instance, in hazardous material transportion, Gopalan et al. (1990) calculate the risk associated with an accident for each district in a network as a disutility measure. Then for considering equity, they impose a limit on the difference in total risk between every pair of districts. In facility location problems, geographic equity of access to service facilities has been measured in different terms, usually based on the distances between demand points and assigned facilities (see the review articles (Karsu and Morton 2015) and (Ogryczak 2000)). Maximizing the share of population covered within a pre-defined distance or travel time, or minimizing the maximum distance from demand points to facilities (Rawlsian approach) are examples of approaches commonly resorted to in facility location. On the other hand, mean absolute deviation and the Gini coefficient have been utilized in the economics literature for assessing income inequity among individuals. In this study, we develop a model that incorporates measures of equity from the economics literature, to balance efficiency and equity in shelter location decisions under uncertainties concerning demand locations, demand amounts, and road network accessibility.

We study the problem of selecting the locations of shelters to be established as a preparation step for possible natural disasters. Following a disaster, shelters are often provided by organizations or governmental agencies to gather the victims under a safe temporary living space and deliver essential relief services to them, such as food, water, medicine, etc. In case of a disaster, persons from an affected population at different locations go to or are transported to the assigned opened shelters. Informing the people before a disaster about which shelter they are assigned to, as a preparedness measure, saves precious time after a disaster and helps the planning activities. On the side of an individual, the disutility of the travel to the shelter is expressed by the distance to the assigned shelter. Our goal is to minimize a measure derived from individual distances that takes both efficiency and inequity into account. Efficiency is defined by the mean distance travelled by an individual to reach the assigned shelter location, and inequity is measured as Gini's Mean Absolute Difference of the distances.

Since it is not clear in advance where the disaster will strike and which population nodes it will affect to which extent, we use a stochastic model with a set of possible disaster scenarios. Demand, affected population, and traversable road distances between population nodes and facility locations depend on the scenarios. In order to consider also budget limitations, we use a *chance constraint*. This constraint ensures that with a given minimum probability near to unity, the total monetary cost of opening the shelters and expanding their capacities according to the realized demands after the disaster does not exceed the given budget. In the context of the mentioned scenario-based representation of the random demand, the chance constraint will be expressed by its deterministic equivalent. Thus, it requires that the total cost must be within the limit of the given budget in a pre-specified (high) percentage of the scenarios. The chosen percentage represents the service level. To the best of our knowledge, a model with chance constraints has not been studied before in the context of shelter location in disaster logistics.

Multiple organizations across a humanitarian supply chain collaborate with each other for preparedness and response. According to the United Nations International Strategy for Disaster Reduction (UN/ISDR), local governments are responsible for designing and maintaining the preparedness plans such as shelter establishment (Guidance note on recovery shelter, 2010) since they have considerable knowledge of the hazards to which their communities are exposed and the capacity to instruct or engage the local population. Therefore, we assume that the shelter location decision will be taken solely by the local government instead of national governments, regional institutions and Non-Governmental Organisations (NGOs).

To solve our problem we use a modified genetic algorithm and compare its performance against a stochastic program solved by the Cplex solver. Finally, we solve this problem on real data of the Kartal district in Istanbul to yield insights for decision makers.

1.2 Relief item distribution in disaster response

In the aftermath of a disaster, victims need immediate relief supplies such as food, water, blankets, and medicine. These supplies are often provided in a shelter where victims gather after the disaster. To avoid supply shortage, agencies preposition inventory in one or more depots and ship items to shelters after the disaster when the shelters have been established. In this study, we focus on the distribution of relief supplies to the shelters as part of humanitarian relief logistics in disaster response.

To understand the significance of our work, it is beneficial to know how a disaster response operation works. Immediately after a natural disaster has occurred, aid agencies, with the collaboration of local agents, will initiate an assessment of the extent of the disaster. The assessment has to be done continuously in the early stage of a disaster to provide new or updated information on the needs of affected people and the state of damaged infrastructure. This information serves as a basis to approximate the aid items list and their quantities (International Federation of Red Cross and Red Crescent Societies (IFRC), 2008). Based on the disaster preparedness plan and the state of the road network and candidate shelter location, a team of experts informs the local agent where to establish the shelters. Initial relief supply, medical teams, vehicles, and volunteers are gathered at the depots (e.g. relief aid distribution centers). The distribution plan and the relief item allocation to the shelters can be determined after acquiring reliable information on the number of vehicles and volunteers.

The classical Capacitated Vehicle Routing Problem (CVRP) minimizes total transportation costs, however; humanitarian relief organizations are primarily concerned with timely aid delivery. Therefore, in the relief supply distribution, response time is one of the main objectives. Most likely, the prepositioned relief supply amount will not be sufficient enough to meet all the shelters' demands. The decision makers should decide about the amount of supply they are going to deliver to each shelter. Effective relief supply allocation among shelter locations is vital according to the high risk associated with unsatisfied and/or late-satisfied demand. This makes an efficient and equitable supply distribution a critical consideration in humanitarian logistics. We study the problem of distributing the humanitarian aids from a distribution center to established shelter locations through post-disaster transportation network which aims at an efficient and fair distribution of relief items to reduce the human suffering.

The problem addresses two objectives: an inequity measure that ensures the fair distribution of relief supply, an objective that minimizes the total travelling time of all vehicles which encourages a timely distribution. Our problem is concerned with a single item distribution planning since it is common in disaster aid operations that supplies are distributed in the form of standard packages/pallets. Due to the chaotic environment and lack of reliable information after a natural disaster, relief organizations prefer to make daily distribution plans rather than considering multiple days ahead (Huang et al. 2012). Therefore, we consider a problem where decisions have to be made only for a single period and to be revised whenever new real-time information becomes available. To the best of our knowledge, a bi-objective problem that considers both explicit routing decisions and equity in the humanitarian logistics context has not been studied before.

Complementary support to shelters should come from all relevant stakeholders (e.g., local communities, NGOs, international institutions and volunteers). However, immediately after a major disaster only the local government that is in charge of prepositioned inventory at its depot will distribute the relief aids. Therefore, our problem is defined from the viewpoint of such a decision maker. This agency should gather the required information on post-disaster situations such as the shelter locations and their demands, road network status and available vehicles. With this information, the problem presented here will be solved.

Our contributions in this part of the thesis are as follows: We develop two VRP-based MIP formulations for the problem and adapt the ϵ-constraint approach to deal with these two objectives. We also propose a set partitioning formulation and solve its linear relaxation with the column generation approach. A novel heuristic algorithm is proposed to find an initial set of columns that ensure a feasible solution in a reasonable time. We devise a branch-and-price algorithm to solve the problem efficiently. It is not a standard application of the method, since we introduce a novel acceleration method to find feasible solutions more often. We also adapt and execute the initiation heuristic in every node of the B&P tree to find good feasible solutions. Finally, we test our approach on two real data sets from Istanbul's Kartal district and Van province earthquake case study from the literature.

In Chapter 2, we explore the related literature on the equity in disaster preparedness and disaster response. Chapter 3 describes the data sets we adopt from different sources and adapt to our problem. It explains the instance generation procedures for our computational tests. In Chapter 4 we study shelter location and demand allocation problem under uncertainty in the disaster preparedness phase. We provide a stochastic mixed integer programming model for the problem and introduce a metaheuristic approach to solve it efficiently. Chapter 5 investigates relief item routing in the post-disaster environment, where the amount of supply is insufficient and equity over unsatisfied demand rate is explicitly considered as a objective function. Finally Chapter 5.4 concludes the thesis by reviewing contributions, findings and future work.

Chapter 2

LITERATURE REVIEW

At first, we briefly explore the literature of the inequality measures, especially under uncertainty. We divide our literature review into two sections, one on shelter location problems in disaster preparedness and the other one for routing in humanitarian operation which is mostly focused on disaster response. A general survey on inequity-averse optimization has recently been presented by Karsu and Morton (2015). Therein, also some papers with focus on humanitarian logistics applications are reviewed. Nevertheless, these approaches are restricted to a deterministic framework. In a study by Gutjahr and Nolz (2016), several types of objective functions for optimization approaches to humanitarian logistics are discussed, in particular also equity. In this survey, it is observed that in this area, articles dealing with equity are rather scarce and focused on the deterministic case.

In the literature on decision theory, on the other hand, the issue of inequity measurement under uncertainty has found an explicit consideration. For the purpose of our current investigation, four articles are of utmost importance. Ben-Porath et al. (1997) discuss the conceptual difficulty of equity quantification in a context of uncertainty, a difficulty which is mainly caused by the contrast between *ex ante* inequity and *ex post* inequity. In the case of uncertainty, it is not clear whether one should strive for equity from the perspective before a random or uncertain event has happened, or from the perspective after uncertainty has been resolved. As a solution concept, the authors propose the so-called *min-of-means functionals* and study their properties from an axiomatic point of view. Gajdos and Maurin (2004) show that already a very restricted set of reasonable axioms rules out both pure *ex ante* inequity measures and pure *ex post* inequity measures as meaningful quantifications of inequity under uncertainty. Chew and Sagi (2012) propose a one-parameter extension of the Generalized Gini Mean as an inequity measure under uncertainty and investigate it in an axiomatic framework. New theoretical findings are provided in a study by Fleurbaey et al. (2015). All these works, however, aim at economic theory, not optimization. To the best of our knowledge, the current thesis is the first study that uses a decision-theoretic measure for inequity under uncertainty as an objective function of an applied optimization problem and shows how the resulting optimization model can be solved computationally.

In this thesis, humanitarian logistics facility location optimization models and humanitarian relief aid distribution studies are examined in two separate sections. To develop the literature database, first, studies on facility location problems in humanitarian logistics were searched for in journals, books, and conference proceedings and then classified according to their approach towards uncertainty: deterministic and stochastic. As this thesis focuses on a shelter location problem considering the uncertainty in input parameters and the equity, only those papers are included which addresses a shelter location problem or uncertainty in facility location problems or equity. Google Scholar academic search engine was querried using "disaster preparedness", shelter location, equity in humanitarian operations, and fairness in humanitarian logistics as the key search strings. Further, the references in each paper, including books and conference proceedings, were scrutinized to reveal any additional relevant papers.

In the second section of the literature review, the same procedure is implemented to find the relevant studies to relief item distribution in disaster response. Particularly, we selected the papers that address at least one of the following issues: relief item distribution planning, vehicle routing application in humanitarian logistics, and fairness in humanitarian logistics. We found these papers by using humanitarian relief distribution, disaster relief logistics, disaster relief routing and relief routing equity as the search keywords in the Google Scholar search engine.

Most articles identified in the literature search came from a range of journals: European Journal of Operations Research, Operation Research, Socio-Economic Planning Sciences, Computers & Industrial Engineering, Transportation Science, Transportation Research Part B and Part E, OR Spectrum, IISE Transactions, Computer & Operation Research, Expert System with Applications, Int. J. Production Economics, Annals of Operation Research and Applied Mathematical Models among others.

2.1 Facility location problems in humanitarian operations

As defined by the International Federation of Red Cross (IFRC), shelters are safe areas that provide accommodation, climate protection, food and medical care for victims of a disaster. Moreover, they protect the population from possible threats that might arise later on. Selection of suitable sites for the shelters is of critical importance as it affects the performance of an evacuation plan (Kongsomsaksakul et al. (2005)). Determining shelter locations considering evacuees' allocation is now being recognised as a critical decision to successful disaster management and has gained significant attention among researchers in recent years. A notable number of studies have been proposed for shelter and evacuation planning and are discussed in Bayram (2016).

Uster and Dalal (2017) introduced an integrated emergency network design problem in which three levels exist. They decided on the locations of supply facilities, distribution centers and shelters, and allocation of demands and disaster relief goods to those shelters. They adopted two objective functions, one minimizing total cost and the other one minimizing the maximum distance that a disaster victim should travel to reach the assigned shelter. They solve this deterministic multi-objective optimization problem by the ϵ-constraint method and Benders decomposition.

Some researchers took advantage of fuzzy set theory to deal with uncertainty in shelter location problems. Trivedi and Singh (2017) consider multi-objective capacitated shelter location and demand assignment. They represent uncertainty with fuzzy sets. As objective functions, their model minimizes total distance, risk associated with locations, number of sites and uncovered demand, and simultaneously maximizes site suitability score and its degree of public ownership. Another study that adopts multi-objective modeling for evacuation planing is Coutinho-Rodrigues et al. (2012). They consider six objective functions including minimizing total traveled distance, risks related to paths and shelters, number of shelters and total distance to the nearest medical care center. They also define backup paths for demand to shelter allocation in case the main paths fail after the disaster. They test the proposed model at the central area of Coimbra, Portugal. Wang et al. (2016) introduced a bi-objective model that maximizes both road and shelter reliability claiming these two objectives are conflicting. They solve their deterministic model by non-dominating sorting genetic algorithm (NSGA-II). As a case study they applied their model on Wenzhou City in Zhejiang Province, China.

Although the vast majority of the existing evacuation planning models assumes system-optimal (cooperative) behavior, a few researchers focus on selfish (noncooperative) behavior which is more probable during a large evacuation. Ng et al. (2010) introduced a hybrid bilevel model that aims to capture a more realistic victim behavior during the disaster. In designing an evacuation plan, there is a tradeoff between assigning evacuees to the nearest open shelter or using an optimal strategy for the assignments. To address this tradeoff, Bayram et al. (2015) consider that evacuees can only be assigned to an open shelter within a range specified by a tolerance from the distance to the closest shelter. Kılcı et al. (2015) study shelter location considering also equity aspects in their model. Based on Turkish Red Crescent guidelines, they assign priority weights to potential sites and maximize the minimum weight of open shelter areas among selected sites. They include constraints to limit maximum distances, minimum utilization of individual shelter sites and utilization differences between selected sites.

Li et al. (2011) consider two types of shelters, permanent and temporary shelters, and apply two-stage stochastic programming for capturing uncertainty. Permanent shelters' locations and their capacities are treated as first stage decisions, resource allocation and temporary shelter locations are considered as the second stage decisions.

Stochastic Facility Location Problems in Humanitarian Operations

The facility location problem for disaster preparedness has been modeled by stochastic programs, especially by two-stage stochastic programming, in many studies within the past decade. Grass and Fischer (2016) present a comprehensive review on two-stage stochastic programming in disaster management. A recent survey on facility location problems that are related to humanitarian logistics, covering also stochastic models, is provided by Boonmee et al. (2017). Hoyos et al. (2015a) provide a literature review on studies in disaster management that apply Operations Research models with a stochastic component, including those related to facility location.

Rawls and Turnquist (2010) introduce a two-stage stochastic program where the first stage decisions are on opening warehouses and amount of items to be pre-positioned. Uncertainty in demand amounts, arc capacities and survival probabilities of the pre-positioned items are represented by a set of discrete scenarios. Verma and Gaukler (2015) provide a two-stage stochastic programming model of pre-disaster facility location that considers the damage intensity as a random variable. Döyen et al. (2012) propose a two-echelon facility location model where uncertain transportation time and demand values are represented by discrete scenarios. Galindo and Batta (2013a) model the possible destruction of supply points during the disaster event and Paul and MacDonald (2016) model uncertainties such as facility damage and casualty losses as a function of the magnitude of an earthquake while deciding on the location and capacities of distribution centers. Noyan (2012) utilizes Conditional Value-at-Risk (CVaR) as a risk measure in a stochastic model of facility location and material flow. Tricoire et al. (2012) and Rath et al. (2015) propose two-stage programming models for humanitarian location/routing problems under multiple objectives. Song et al. (2019) investigate the shelter location problem considering uncertainty by a qualitative approach. They also try to capture the social and environmental aspect of the problem in the decision making process. They develop a Qualitative Flexible Multiple Criteria method to find the best shelter location as disaster preparedness. They tested their approach on the case of Wenchuan Earthquake in 2008. One of the most destructive earthquakes happened since 1950 in China. They interviewed 4 highly experienced experts to obtain the evaluation criteria, their priority and other necessary information for their approach.

Hu et al. (2016) decide on both reinforcement of road networks and buildings, and facility location with the objectives of minimizing the budget and the risk-induced penalties. Bayram and Yaman (2017) consider the uncertainty in road network, evacuation demand and disruption in shelters by a set of discrete scenarios. They devise a Benders decomposition approach to solve the stochastic shelter location and evacuee allocation problem, with minimizing expected total evacuation time as their objective. In a very recent study, Elçi and Noyan (2018) consider a location-allocation problem for disaster preparedness under demand and network uncertainty. They propose a two-stage stochastic programming model with a chance constraint, which in the first stage decides on the relief facilities' locations and their inventory levels. In the second stage, the allocation of relief supplies are made for each scenario. They also define two types of second stage problems; one problem allows supply shortage and the other one enforces satisfaction of all the demands. To model the decision maker's risk preferences on total random cost, they use conditional value at risk (CVaR) as a risk measure in the objective function. A Benders decomposition based branch and cut algorithm is devised to solve the problem efficiently.

In Renkli and Duran (2015) and Ukkusuri and Yushimito (2008), locations for prepositioning inventory are selected by a path-reliability approach. Ukkusuri and Yushimito (2008) aim at maximizing the probability of sending materials to demand points, while Renkli and Duran (2015) constrain the probability of meeting the demand of an item to be at least a specified amount. Kulshrestha et al. (2011) develop a bilevel model under demand uncertainty and solve it by robust optimization. At the upper level, authorities decide about the number, capacity and locations of shelters and at the lower level, the evacuees choose which shelter to go to.

In most of these studies discussed above, the generation of the scenarios has not been addressed explicitly and the statistical dependency between the scenarios has also not been an explicit topic. Salman and Yücel (2015) suggest a systematic method to generate correlated road network scenarios according to the vulnerability level of the network edges and their spatial closeness while locating emergency response centers. In this work, we generate a sample of scenarios with uncertain demand and travel times resulting from correlated probabilistic failures of road links according to the link failure dependency model in Yücel et al. (2018a), which is explained in Section 3.1.

Very few articles on stochastic facility location for humanitarian purposes address the equity issue. Rezaei-Malek and Tavakkoli-Moghaddam (2014) propose a two-stage stochastic programming model for pre-positioning of relief goods, taking account of equity by means of a fairness level constraint. Noyan et al. (2015) determine locations and capacities of relief distribution points by a two-stage stochastic programming model optimizing the expected total accessibility with a penalty for inequitable solutions. The maximum proportion of unsatisfied demand is used as an equity measure. As far as we know, the combination of a stochastic programming model with the Gini inequity measure, as suggested in the present thesis, is not only new in the humanitarian logistics literature in particular, but in the optimization literature in general.

2.2 Humanitarian relief aid distribution in disaster response

Humanitarian logistics are mostly concerned with distribution relief supplies and transportation of disaster victims from affected areas to medical centers. These operation always involve deciding on the exact vehicle routes. One of the earliest studies on vehicle routing in humanitarian operations addresses the problem of allocating the relief supplies among the demand locations and simultaneously planning the delivery routes within a planning period (Balcik et al. 2008). They propose a two-phase modelling approach which at the first phase determines a delivery schedule for each vehicle and decides on inventory allocation decisions in the second phase. The objective is to minimize the cost of logistics operations plus penalty costs for unsatisfied and late-satisfied demand. They introduce a MIP model to solve this problem and drive some sensitivity analysis on the parameters.

Berkoune et al. (2012) focused on the allocation of relief items to (e.g., water, food, medical goods and survival equipment) to demand nodes with a heterogeneous fleet of vehicles. They introduced a mathematical model that minimizes the total duration of all vehicles trips. The proposed model does not reflect the vehicle routing aspect of the problem, instead it only focuses on supply allocation. They proposed a construction heuristic to generate a set of feasible solutions and a genetic algorithm to improve the quality of these solutions. Nolz et al. (2011) studied the problem of drinking water distribution into a subgroup of population nodes in disaster response. Each vehicle tour visits a subset of all population nodes and install a water tank in the visited nodes. The nearby nodes can be served by the ones with installed water tank. They considered three objectives; minimization of various risks associated with relief supplies, minimization of a measure based on the inhabitants travel times, and minimization of vehicle tour length. They developed a Memetic Algorithm to find Pareto-optimal solutions for the problem.

Najafi et al. (2013) investigated the distribution of relief commodities and injured people in the earthquake response. They proposed a multi-objective, multi-item, and multi-period stochastic model to this end. To handle the uncertainty in the model, they used a robust optimization approach. Three minimization objectives of the model are; total waiting time of unserved victims, total commodity distribution time, and total number of vehicles. To solve the deterministic robust counterpart of the model they arranged the objective functions hierarchically and solve each phase by CPLEX 12.1. Abounacer et al. (2014) developed a model that decides on the number and location of humanitarian aid distribution centers as well as distribution of humanitarian aid from these centers to demand points. Three minimization objectives are considered; the total transportation time, the number of persons needed to open and operate the all distribution centers and the total unsatisfied demand. They utilized an epsilon-constraint method combined with an approximate method to generate a good approximation of the Pareto front of the model. Their problem did not include the routing decisions for each vehicle.

Chang et al. (2014) studied a vehicle routing-scheduling problem that distributes relief items from several supply points to demand points in a fair and efficient manner with a fixed number of vehicles. The objectives were to minimize unsatisfied demand, total distribution time and total transportation cost. Multiple depots were taken into account and each demand point has its own deterministic arrival time window and demand. They developed a greedy-search-based multi-objective genetic algorithm to approximate the Pareto front curve efficiently. The algorithm efficacy was verified by a simulation model based on the ChiChi earthquake in Taiwan. Huang et al. (2015) determined the relief item distribution planning and allocation simultaneously. A multi-objective model is developed to formulate the problem with three objectives related to lifesaving utility, delay cost, and fairness. The proposed model is a multi-period multi-depot model that allows arrivals of supplies and demands at different times. They transformed the multi-objective model to a single objective model by weighted sum approach and changed the weights to approximate the Pareto curve. They design a Variational Inequality algorithm to solve the problem. Ahmadi et al. (2015) determined the location of distribution centers and vehicle routes by developing a multi-objective multi-depot location-routing model which minimizes the total distribution time, penalty cost of unsatisfied demand and fixed costs of opening distribution centers. They considered a partial road network failure in the aftermath of an earthquake and the cost of unsatisfied demand. They also extended this model to a two-stage stochastic program to accommodate possible earthquake scenarios in the preparation phase. To solve the large instances for the deterministic model they devised a *Variable Neighborhood Search* (VNS) algorithm. They tested the proposed VNS on real-world instances from the San Francisco district.

Moreno et al. (2018) optimized location, transportation, and fleet sizing decisions. They allow multiple trips for each vehicle. Considering uncertainty regarding demand, supply and route availability, they developed a two-stage stochastic programming model. They mainly focused on application of the model in recurrent weather-related disasters such as floods and landslides. In the first stage which correspond to pre-disaster phase, the problem decided on location of distribution centers and vehicle fleet size and type. After the disaster strikes, in the second stage, they decided on commodities flow and vehicles types to carry these flows. However, they assumed that each vehicle only visits one distribution center in its route, the simplified assumption that have been used in many relief distribution problems. The model minimizes both expected logistic cost and human suffering (monetary cost of depriving victims of emergency commodities over time). This multi-objective model solved by bi-level hierarchical optimization and two heuristic approaches. Liu et al. (2018) investigated the problem of post-disaster relief goods distribution and injured people transportation. They proposed a multi-commodity multi-period mathematical model that accounts for supply and demand uncertainty with objective of minimization of the unsatisfied demand. However, their proposed model does not decide on exact vehicle routes. A robust optimization approach is adopted to cope with these uncertainties. This model is tested on samples based on an earthquake occurred in Sichuan Province, China in 2008. Penna et al. (2018) investigated a last mile delivery problem which is a VRP with multiple depots, multi-trips, heterogeneous fleet, and site-dependent constraints. Although they emphasize on the problem application in humanitarian logistics, the objective of the problem is to satisfy all demands, while minimizing travel costs and fixed costs. To solve the model efficiently, they proposed a hybrid meta-heuristic that utilizes both a set partitioning formulation and a multi-start local search. In this research project, they closely collaborated with crisis management organisations.

An interesting vehicle routing application in disaster response operations was investigated by Duque et al. (2016). They considered the problem of scheduling and routing the emergency repair crew to visit the damaged facilities. According to their definition, a node is considered accessible if there is a path from the depot to that node which contains only undamaged nodes. The aim of the problem is to minimize the weighed sum of all accessibility times. They developed a dynamic programming model to formulate the problem. They also proposed a greedy-randomized constructive approach to find near optimal solutions and compared its performance with dynamic programming approach. Pérez-Rodríguez and Holguín-Veras (2015) integrated relief supply distribution, vehicle fleet sizing and inventory allocation decisions. They defined social costs as a sum of logistics and deprivation costs and developed a mixed-integer non-linear programming models based on their previous work(Holguín-Veras et al. 2013). Their model explicitly considers routing decisions into account. However, the proposed model became complex to solve. They decomposed the problem into an inventory allocation and a routing problem and finally resorted to a point-to-point delivery instead of the routing problem. Heuristics are devised to solve real size instances.

Holguín-Veras et al. (2012) first discussed the difference between commercial and humanitarian logistic. Then they identified seven key elements of humanitarian logistics models. They emphasize on the difference between problems which address the disaster response in long term and immediate disaster response phase. Finally the article identified the existing gaps in the literature. Anaya-Arenas et al. (2014) presented a general review over Relief distribution network literature. The readers are encouraged to refer to (Habib et al. 2016) and (Özdamar and Ertem 2015) for a review on mathematical models in humanitarian supply chain, including the routing models. Finally, Ertem et al. (2017) explored the studies that address multi-modal transportation in humanitarian logistics. A list of all humantarian operations management review papers and their focused area can be found in Kovacs and Moshtari (2018).

2.2.1 Fairness in humanitarian logistics

A growing body of humanitarian logistics literature addresses fairness in aspect of humanitarian relief aid operations. The common fairness attributes among these studies are demand satisfaction, emergency response time, and deprivation time. In this section, we provide a literature review on the studies that use optimization models to determine fair resource allocation in humanitarian logistics. Campbell et al. (2008) was among the first studies that incorporated inequity concept to the disaster relief supply distribution. They introduced two inequity objectives; minimization of maximum arrival time to demand points and minimization of average arrival time to Travelling Salesman Problem (TSP) and Vehicle Routing Problem (VRP) in the disaster response context. To solve these models, they present local search heuristics which could prove the significant changes in optimal solution comparing to conventional objectives for VRP and TSP. An interesting application of humanitarian logistic combined with inequity is introduced in the study by Cao et al. (2016). They investigated the breast-milk distribution and network expansion problem under supply and demand uncertainty which decides on new distribution enters a location, routing and deliverers amount. To adopt equity to the model, they use a maximin approach on the percentage of uncovered demand in each region. They solved the problem with an integer rounding-based heuristic. They tested this heuristic on data from the national breast-milk supply chain in South Africa.

Lin et al. (2011) investigated the problem of delivering prioritized items to demand nodes via the transportation network over a planning horizon. The study assumed supplies are unlimited and the demand is deterministic. They introduced a tour-based model that has three minimization objectives; unsatisfied demand and late demand cost, total traveling cost and the maximum difference of satisfaction between two demand points. They devise a GA-based heuristic and a decomposition assignment heuristic approach to solve the problem. Finally, they compared the performance of these two approaches. Vitoriano et al. (2011) proposed two network flow model to capture the material and patient logistics in disaster response. They introduced several measures as objectives of these models; namely, total cost, distribution time, equity of the delivery amounts, demand node priority, reliability of network edges, and security of designated routes. They adopt a goal programming approach to handle these multi-objective models. Finally, they applied their model to the Haiti earthquake case.

Equitable aid distribution among disaster victims where equity is defined in term of differences between individual service-level is studied by Huang et al. (2012). They defined the service-level for each aid recipient by delivery speed and amount. A set partitioning approach is introduced to formulate the problem. Three objectives are introduced which measure the followings; transportation costs, quick response, and fairness of service-level. They drive some observations about the impact of these objectives separately on the solution structure. Furthermore, a greedy random adaptive search procedure (GRASP) is introduced to solve the problem for large instances. Finally, they concluded that multiple deliveries to demand nodes with small vehicles leads to a more equitable solution. However, this will add to the difficulty of logistics planning in reality.

An integrated supply chain operation in disaster response is studied by Afshar and Haghani (2012). They proposed a model that decides on vehicle routing, pick up or delivery, optimal locations for temporary facilities. Equity among aid recipient ensured by enforcing a constraint on the minimum service level to each victim. The proposed formulation is evaluated by some instances to determine the properties of the optimization problem. However, the model's size increases rapidly when the extension of the operations increases and could not be solved by commercial solvers in a reasonable time.

Gralla et al. (2014) suggested that prioritization of commodities and delivery locations can serve as a mean to promote equity, and evaluated this objective in relation to other measures by interviewing field experts in humanitarian logistics. Noyan et al. (2015) developed two-stage stochastic programming for last mile relief network design problem, which determines the locations and capacities of relief aid distribution points (PODs) in the first stage. In the second stage, the model decides on the allocation of demand locations to the PODs and distributing available supplies among the PODs. They considered the uncertainty as a set of possible scenarios for post-disaster relief demand and transportation network conditions. They characterize equity into two ways; equitable accessibility to PODs from demand points and equitable supply allocation to PODs. Their objective function maximizes the expected total accessibility of PODs/relief supplies by individuals in demand points. They developed a Benders decomposition-based branch-and-cut algorithm to solve the model. Their proposed approach was tested on real-world data from the 2011 Van earthquake.

Ransikarbum and Mason (2016) presented a multi-objective optimization model which decides on the supply distribution and network restoration during post-disaster operations. The equity objective function maximizes the minimum percentage of satisfied demand. Two other objectives are the minimization of unsatisfied demand and total network cost. They applied the model to a regional case study from Hazus earthquake and provided decision-makers with equitable restoration and distribution plans. They did not provide any solution method to solve a model for large instances in a reasonable time. Ferrer et al. (2016) developed a last-mile aid distribution planning problem which decides on each vehicle route and amount of delivery to each demand node. They considered road reliability and security issues when planning a shipment. To evaluate the feasible solutions, they defined six measures; namely, operation cost, operation time, equity according to unsatisfied demand, service priority, reliability and security of the distribution plan. The objective function is defined as a weighted sum over all these measures. They introduced a GRASP heuristic to find good solutions in a reasonable time for the problem.

Holguín-Veras et al. (2013) investigated the use of social costs objectives into humanitarian logistics problems. They argue that the best way to capture a relief aid distribution impacts on the disaster victims is through using a deprivation cost function. Deprivation cost is defined as the monetary cost of human suffering which depends on the amount of time that an individual is without service. They introduced a nonlinear function to calculate the deprivation cost. However, they did not specify how the function parameters estimation can be done. Following this study, some researchers utilized derivation cost or its linear approximation as the equity measure in humanitarian logistics problems such as Loree and Aros-Vera (2018), Ni et al.

(2018).

Noyan and Kahvecioğlu (2018) studied the last mile relief network design problem with resource reallocation between local distribution centers (LCD). To address accessibility and equity, they developed the concept of coverage sets which uses the accessibility scores and enforce a supply allocation policy. They proposed a two-stage stochastic programming model which decides on the location of LDCs and distribution points in the first stage and on reallocation amounts on the second stage. They did not consider details regarding the delivery to demand points and assumed that the individuals come to the closest distribution point to collect relief aids. They developed an exact delayed cut generation-based solution algorithm to solve the model effectively. Gutjahr and Fischer (2018) showed that the selection of the inequity measure have a great impact on the equity in the final solution and sometimes minimization of deprivation cost can lead to a very high degree of inequity. Instead, they suggested to minimize a measure proportional to Ginis means absolute difference of the undesirable attribute. Huang and Rafiei (2019) introduced a vehicle routing based multi-period relief distribution problem with a time window and split delivery. They compared equity measures regarding delivery quantities, arrival times and deprivation times under different situations such as ample and insufficient supply and ample and insufficient vehicle capacity. They solved their model for small instances and claim that equity measures show different behaviors depending on the ratios of supply, vehicle capacity, and demand.

Yu et al. (2018) approached the multi-period resource allocation problem in the disaster response phase with a dynamic programming method. They considered three optimization measures simultaneously; namely, accessibility cost, deprivation cost, and end of period penalty cost. Their proposed model was a non-linear mixed integer programming model that can be solved for small instances with dynamic programming. However, to solve it for large-size instances, they used the piecewise linear method to linearise the non-linear terms. This study did not consider the vehicle routing decision and focused on timely resource allocation to demand points.

Sabouhi et al. (2018) introduced an integrated model that addresses transferring injured people from affected areas to hospitals, transportation of evacuees from affected areas to shelters as well as the distribution of relief items shelters. They addressed the demand uncertainty by a robust optimization approach. They adopted a minimax approach towards equity of transportation times. The proposed multi-objective model is a location-allocation model with a weighted sum of all objectives. They interpreted the results yielded by the model in a real case study.

Fianu and Davis (2018) investigated the allocation of donated supplies to food banks under supply uncertainty in an equitable manner. They used a discrete-time, discrete state Markov decision process to model the problem. The equity is defined by the amount of delivery to each individual. Their computational results on a real case study showed that maximum equity attains by allocating supply to each demand point in proportion to its population. Vahdani et al. (2018) studied a relief distribution network design considering the short-term repair of infrastructures. They introduced a location-routing model that decides on the location of distribution centers, exact vehicle routes and delivery amounts from a depot to the opened centers. The first and the second objectives minimize the maximum vehicle route traveling time and the total cost, respectively. The third objective maximizes the minimum reliability of routes. To solve this multi-objective model, they developed two metaheuristic algorithms; a non-dominated sorting genetic algorithm and a multi-objective particle swarm optimization. Finally, they presented a comparison between these two methods and a sensitivity analysis on the model parameters.

Eisenhandler and Tzur (2018) introduced humanitarian pickup and distribution problem which arises in donated food distribution. The problem decides on which supplier sites and demand points should be visited as well as the amount of food should be picked up and delivered to them using a single vehicle. The problem simultaneously incorporates vehicle routing decisions. The problem seeks an allocation strategy combined with routing decisions under vehicle capacity and driver's working hour limitations which respects the equity among the population. The objective function of the model is the combination of the total number of delivered units and the Gini coefficient of the deliverers. To solve the problem for realistic-size instances, they provided a heuristic solution method and tested its performance on two real datasets; an Israeli food bank and Houston food bank. Following this study, Eisenhandler and Tzur (2019) presented a new mathematical formulation for the same problem based on a segment-based approach. They introduced a new heuristic algorithm to solve this model effectively and extended the solution approach to solve problems with a more complex structure(e.g. multi-vehicle, time window). Their computational results showed the superiority of this new heuristic approach over the one presented in Eisenhandler and Tzur (2018).

Our relief item distribution problem can be considered as a variation of the problem proposed by Eisenhandler and Tzur (2018) and Eisenhandler and Tzur (2019) but with different applications and considerations. We aim at the relief item distribution as an immediate response to a natural disaster where the supply is pre-allocated in the depot prior to the disaster. In contrast to their model, our model formulates a multi-vehicle problem with a fixed number of vehicles. We have to visit all demand points/shelters to deliver relief aids. Furthermore, our problem is unique since it incorporates equity measure related to delivery amounts and total traveling time as its two objectives.

Interested readers can refer to Matl et al. (2017) for a comprehensive review on equity in vehicle routing problem in general context. Table (2.1) summarizes the characteristics of related studies in equitable relief aid distribution in comparison with the model proposed in this study. We only explored the studies that explicitly considered equity in a mathematical model.

Table 2.1: Equity in relief aid distribution studies

Article Name	Equity attributes	Equity approach	M\ S	St\ D	Problem domain	Routing aspect
Tzeng et al. (2007)	DSR	Maximin	M	D	RAD	AL
Campbell et al. (2008)	AT	Minmax	S	D	RAD	VRP \TSP
Lin et al. (2011)	DSR	EM	M	D	RAD with priority	VRP \ route based
Vitoriano et al. (2011)	DA	EM	M	D	RAD and patient logistic	NF
Huang et al. (2012)	DA\AT	EM	S	D	RAD	VRP \ route based
Afshar and Haghani (2012)	Service level	Constraint	S	D	LMRND	NF
Bozorgi-Amiri et al. (2013)	DSR	Maximin	M	ST	RAD	NF
Renkli and Duran (2015)	Travel distance	Constraint	S	ST	RIP	NF
Noyan et al. (2015)	Accessibility\DA	EM	S	ST	LMRND	AL
Cao et al. (2016)	Unsatisfied demand rate	Minmax	S	ST	distribution network expansion	NF
Tofighi et al. (2016)	Travel distance	Minmax	M	ST	LMRND	NF
Ransikarbum and Mason (2016)	DSR	Maximin	M	D	RAD and network restoration	NF
Pradhananga et al. (2016)	Deprivation cost	EM	S	ST	RAD and resource allocation	NF
Ferrer et al. (2016)	Unsatisfied demand	EM	M	D	LMRND	VRP
Mohammadi et al. (2016)	DSR difference	Minmax	M	ST	RIP	NF
Üster and Dalal (2017)	Travel distance	Minmax	M	D	Preparedness network design	NF
Sabouhi et al. (2018)	AT	Minmax	M	ST	Victim and patient transportation	NF
Fianu and Davis (2018)	DA	EM	S	ST	FRD	NF
Vahdani et al. (2018)	AT	Minmax	M	D	RAD network design	VRP
Noyan and Kahvecioğlu (2018)	Accessibility\DA	Equity measure	S	ST	LMRND	NF
Eisenhandler and Tzur (2018)	DA	EM (Gini)	S	D	FRD	VRP
Eisenhandler and Tzur (2019)	DA	EM (Gini)	S	D	FRD	VRP
Huang and Rafiei (2019)	DA \AT \deprivation times	EM	S	D	RAD	VRP
Our second problem	Unsatisfied demand rate	EM(Gini related)	M	D	RAD	VRP

DSR:Demand satisfaction rate, **DA**: Delivery amount, **AT**: Arrival Time, **EM**: Equity measure, **S**: single objective, **M**: multi-objective, D: deterministic, **ST**: stochastic, **RAD**: Relief aid distribution, **LMRND**: Last mile relief network design, **RIP**: Relief item prepositioning, **FRD**: Food rescue and delivery, **VRP**: vehicle routing problem, **NF**: Network flow, **AL**: Allocation .

From the literature review we can see that there are two primary approaches applied toward relief aid distribution planning problem. Most studies investigate this problem on macro-scale which means they decide on allocation of the supply to demand nodes. Generally this type of study combines distribution decisions with other decision such as locating the distribution centers and deciding on inventory levels. On the other hand, a number of studies look at the problem from a micro-scale. They are mostly concerned with vehicle routing problem that considers vehicles number, type and capacity. We identify our problem as this type.

In summary, the literature on relief aid distribution that incorporates explicit vehicle routing decisions with an adequate equity measure is still quite limited. Therefore in Chapter 5, a distribution plan that is deciding on the vehicle routes and delivery amount to shelter locations to minimize two objectives; inequity regarding the relief supply distribution and the total traveling time, is presented for the first time. None of the previously mentioned studies introduced an exact solution method that provides a solution for problem sizes arise in real situation in short period of time on the problems similar to ours. In disaster response problems, applying the optimal solution as soon as possible is essential. Therefore, a branch-and-price method for equitable relief item routing is developed to achieve this goal which is our unique contribution.

Chapter 3

DATA GENERATION AND CASE STUDIES

This section will describe how data generated to be used for computational tests. We used three data sets. The first one is adopted from OR-library benchmark data for capacitated warehouse location problem. The second one is based on the Kartal district in Istanbul and the third one is based on a Van province in Turkey 2011 earthquake real data.

3.1 Benchmark data from OR-library

The propose of generating these benchmark data is mainly to test the performance of the GA that devised for our stochastic shelter location problem. These instances are obtained from the OR-library and are originally suggested for the capacitated warehouse location problem by Beasley (1988). We need to generate some parameter values in order to adapt the mentioned benchmark data set to our shelter location problem. These parameters are budget, expansion costs, service level, scenario dependent demands and distances. In this section we explain how we generate these parameters in detail. The expansion cost of CL j (η_j) is generated by selecting a random value between $0.5 \cdot \frac{f_j}{c_j}$ and $2 \cdot \frac{f_j}{c_j}$, where c_j is the pre-defined capacity and f_j is the cost of shelter j establishment. The mean budget, when we need to open all facilities at candidate sites is estimated by the formula,

$$C_{mm} = \max\left[\left(\frac{\sum_{s\in S}\sum_{k\in K} b_k^s}{|S|} - \sum_{j\in J} c_j\right), 0\right] \cdot \frac{\sum_{j\in J}\eta_j}{|J|} + \sum_{j\in J} f_j.$$

In real world situations we would not have the budget to open all the facilities and/or to pay all the associated expansion costs. Hence, we set the budget as a proportion of the above approximation by multiplying it with multiplier ξ. The selected value for ξ in all instances is 0.7. We set the service level parameter α equal to 0.95 for all instances.

Scenarios are generated to represent the uncertainty as close as to real world situations as possible. Scenarios consist of demands and distances between each pair of PNs and CLs. Demands are proportional to the population size and are mainly influenced by two factors, distance from the epicenter and the severity of the earthquake. We consider three possible cases for the disaster severity. It can have low, moderate or high impact on the population, with equal probabilities. The severity coefficient is β^1 which is equal to 0.2, 0.5 and 0.8 for low, moderate and high disaster severity, respectively. In order to see the affect of these parameter on the problem we generate demand with three other set of values for β^1 in section 4.3.4. Similarly, we define a coefficient to indicate how distance from the epicenter affects demand. Here, with d_{ek} denoting the Euclidean distance between PN k and the epicenter e, the distance coefficient is defined as $\beta^2 = 1 - d_{ek}/d_{max}$, where $d_{max} = \max_{k \in K} d_{ek}$, so that the demand decreases with increasing distance. Finally we select a random value between 0 and $2\beta^1\beta^2 \cdot w_k$ as the demand of PN k.

The road network is represented by $G = (V, E)$, an undirected connected graph where V is the set of nodes (consisting of candidate facility locations, population nodes, road junction points) and E is the set of edges. To generate distances in scenarios, we consider the network after the disaster, which may have some roads blocked by debris or impassable for other reasons. We denote this (possibly partially damaged) post-disaster road network in scenario s by G^s. In a realistic approach, how the disaster affects a certain edge should not be independent from other edges; naturally, edges with a small distance to each other have a high probability of being jointly affected. In order to address this important issue, we use the link failure dependency model presented in Yücel et al. (2018a) to generate G^s as follows. Each edge $e \in E$ in the network has a survival probability p_e. The edges are divided into mutually exclusive sets according to Euclidean distances of their endpoints from the epicenter. We divide E into three subsets, E_n as the set of nodes that are near to the epicenter, E_m as the set of nodes moderately close to the epicenter and E_f as the set of farthest nodes. The E_n members have the lowest survival probabilities and E_f edges have higher survival probabilities. Within these sets, the survival probability of the edges are dependent on each other. Between different sets, edge failures are assumed as independent. We generate the initial p_e for edges in E_n, E_m and E_f by randomly selecting a value within $[0.7, 0.8]$, $[0.8, 0.9]$ and $[0.9, 1]$ intervals, respectively.These intervals are selected similar to the ones are calculated in Yücel et al. (2018b) for prior link survival probability in Istanbul road network. If e^1 and e^2 are two edges in the same dependency set and $p_{e^1} \geq p_{e^2}$, the failure of the stronger edge (e^1) would affect the survival probability of the weaker edge such that, p_{e^2} is reduced to $(1-\nu)p_{e^2}$, where $0 \leq \nu \leq 1$. The pseudocode of the sampling algorithm is given in Algorithm 1. In this algorithm the parameter ν determines the degree of dependency between edges

Algorithm 1 Network Sampling Algorithm

Partition E in three subsets according to distance from epicenter $\{E_n, E_m, E_f\}$
for $\bar{E} \in \{E_n, E_m, E_f\}$ **do**
  $E' \leftarrow \bar{E}$
  while $E' \neq \emptyset$ **do**
    Find the edge with maximum p_e in E', denote it by $\hat{e}$
    Generate a random number r between 0 and 1
    if $r \geq p_{\hat{e}}$ **then**
      Block the edge $\hat{e}$ in G_s
      Remove $\hat{e}$ from E'
      for $e \in E'$ **do**
        $p_e = (1-\nu) \cdot p_e$
      end for
    end if
  end while
end for

in the same dependency set. A high value of ν increases the chance of generating a

Table 3.1: Instance specifications

Instance name	Benchmark data name	# PN	# CL	# S	Instance name	Benchmark data name	# PN	# CL	# S
cap61_50	cap61	50	16	50	cap114_50	cap114	50	50	50
cap61_100	cap61	50	16	100	cap114_100	cap114	50	50	100
cap61_200	cap61	50	16	200	cap114_200	cap114	50	50	200
cap61_500	cap61	50	16	500	cap114_500	cap114	50	50	500
cap63_50	cap63	50	16	50	cap123_50	cap123	50	50	50
cap63_100	cap63	50	16	100	cap123_100	cap123	50	50	100
cap63_200	cap63	50	16	200	cap123_200	cap123	50	50	200
cap63_500	cap63	50	16	500	cap123_500	cap123	50	50	500
cap74_50	cap74	50	16	50	cap124_50	cap124	50	50	50
cap74_100	cap74	50	16	100	cap124_100	cap124	50	50	100
cap74_200	cap74	50	16	200	cap124_200	cap124	50	50	200
cap74_500	cap74	50	16	500	cap124_500	cap124	50	50	500
cap83_50	cap83	50	25	50	cap131_50	cap131	50	50	50
cap83_100	cap83	50	25	100	cap131_100	cap131	50	50	100
cap83_200	cap83	50	25	200	cap131_200	cap131	50	50	200
cap83_500	cap83	50	25	500	cap131_500	cap131	50	50	500
cap92_50	cap92	50	25	50	cap132_50	cap132	50	50	50
cap92_100	cap92	50	25	100	cap132_100	cap132	50	50	100
cap92_200	cap92	50	25	200	cap132_200	cap132	50	50	200
cap92_500	cap92	50	25	500	cap132_500	cap132	50	50	500
cap101_50	cap101	50	25	50	cap133_50	cap133	50	50	50
cap101_100	cap101	50	25	100	cap133_100	cap133	50	50	100
cap101_200	cap101	50	25	200	cap133_205	cap133	50	50	200
cap101_500	cap101	50	25	500	cap133_500	cap133	50	50	500
cap112_50	cap112	50	50	50	cap134_50	cap134	50	50	50
cap112_100	cap112	50	50	100	cap134_100	cap134	50	50	100
cap112_200	cap112	50	50	200	cap134_200	cap134	50	50	200
cap112_500	cap112	50	50	500	cap134_500	cap134	50	50	500

disconnected network. In our instances, we set ν equal to 0.1, hence maintaining a reasonable level of dependency.Our experience with bigger values resulted in frequent disconnected networks. We have to maintain the network connectivity in all scenarios, therefore if the resulting network is disconnected, we discard it and generate another network. After obtaining the G^s, we calculate the distances d_{kjs} between CLs and PNs in scenario $s \in S$ by applying a shortest path algorithm for each pair.

We select 14 instances from the OR-library benchmark data with different sizes and structures, and generate 4 different scenario sizes, 50, 100, 200 and 500 for each of them. Thus, we have 56 benchmark instances in total. Table 3.1 shows all 56 instances' names and specifications. The first column is the name we assign to each instance. The second column states the related instance from the OR-library. Columns $\#PN$, $\#CL$ and $\#S$ refer to the number of population nodes, number of candidate locations and number of scenarios, respectively.

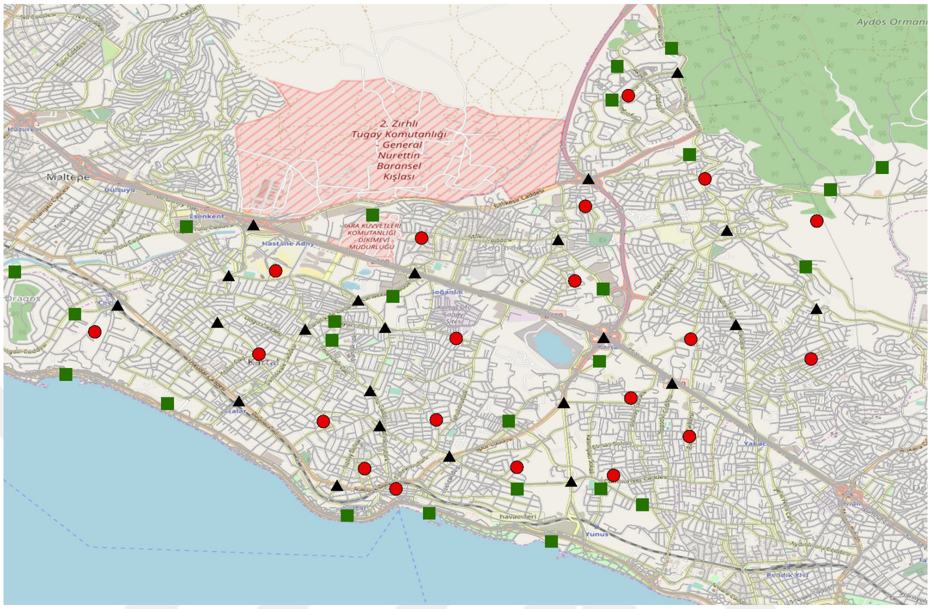

Figure 3.1: Kartal district map and the node locations

3.2 Istanbul-Kartal real data

We adapt the real data of the Kartal district of Istanbul provided by Kılcı et al. (2015). This dataset specifies the candidate shelter locations, their capacities, population nodes, locations and their number of inhabitants. It contains 20 PNs that represent 20 sub-districts in Kartal, as well as 25 CLs selected according to various criteria. The PNs are considered as the centers of each sub-district. We illustrate the Kartal district map, PNs and CLs in Figure 3.1. The population nodes are represented by red dots and candidate locations are represented by green squares. The black triangles indicate the junction points that we selected. We were able to generate a larger number of scenarios (2000), as the GA can handle this problem size.

To make this data set consistent with our stochastic shelter location problem, we have to generate scenarios as well as a budget, the establishment and expansion costs, and the service level. We take the service level α equal to 0.95. We consider the case that all establishment costs are the same and equal to 1000. The budget and expansion costs are randomly generated by the same procedure that we used for the benchmark instances in Section 3.1.

Before generating the scenarios, first we need to build the road network. In order to do that we identify 23 main road junction points and add them to the node set. Then we calculate the shortest path between each pair of nodes using ArcGIS Pro 2.0.0 with real road network data. We construct a network with 68 nodes and the edges between each pair of them considering the shortest path value as the length of the edge. To have an approximation of the real network, we omit the edges which have length more than 2 km. As a result, our network has 1206 edges and 68 nodes. Scenarios are generated in the same way as we generated scenarios for the benchmark instances using the Network Sampling Algorithm presented in Section 3.1. We test six different instances with different number of scenarios: 50, 100, 200, 500, 1000 and 2000. In these six instances the scenario sets are generated independently. For example, the 50 scenarios do not necessarily appear among the 100 scenarios.

We also use the Kartal case study for our second problem where the distribution of the relief item among established shelter locations is concerned. We adopt the solution of an instance with 2000 scenarios to find the location of the established shelter. Twelve location are selected for shelter establishment. The depot location is selected based on the real location of a relief supply distribution center in the region. We generate 30 instances with different parameter. For each instance we randomly select one scenario to determine the post-disaster demands and distances. We assume that all individuals assigned to a shelter will reside at that shelter after the disaster. Therefore, shelters' population are equal to the summation of its assigned PNs' demand. Without loss of generality we consider each individual is in need for one unit of the relief supply which means the shelter demand is equal to the population who gathered there after the disaster. We suppose that 3 identical vehicles are available to distribute the relief supply among the shelter locations. The maximum tour length Ψ is obtained by dividing the number of nodes by m and multiply it to the average travelling time between two nodes.

$$\Psi = \zeta^1 \lceil \frac{n}{m} \rceil \times \bar{t} \tag{3.1}$$

where $\bar{t} = \sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n}(t_{i,j})/(n-1)^2$ and ζ^1 is a coefficient that controls maximum tour length. For instances where we aim at generating an abundant maximum tour length we fix $\zeta^1 = 2$ and for instances with tight maximum tour length we set $\zeta^1 = 1$. We assume that the total available supply at the depot is equal to $\zeta^2 = 70\%$ percentage of the total demand that means $C = \zeta^2 \sum_i d_i$. The capacity of a vehicle is calculated as $Q = \zeta^3 \frac{C}{m}$ where $'zeta^3$ is a control parameter which is between $[0.2, 1]$. For a regular capacity we select $\zeta^3 = 0.5$ and for a very tight capacity we set $\zeta^3 = 0.2$. The reason to pick such a tight capacity is to see the performance of the algorithm under extreme conditions. Finally, the ϵ value is fixed to $0.85m \times \Psi$.

3.3 Van earthquake case study

Turkey has been subjected to several severe earthquakes during the past decades. One of those most destructive earthquakes hit eastern Turkey in Van province on October 23, 2011. The earthquake death toll rised upto 600 people, and 2,500 injured persons. More than 600,000 people were affected by the 7.2 magnitude earthquake and its' after-shocks. As our next case study, we adopt the realistic data from the 2011 Van earthquake. We especially use this data for our second problem, the relief item distribution problem in disaster response operations. The data has been made made available to us by Noyan et al. (2015). There are 94 settlements that have been effected by the earthquake and one local distribution center which had the responsibility to distribute the relief items immediately after the disaster. Noyan et al. (2015) studied a problem that decided on locating the relief aid distribution points (PODs) in a limited number of districts, their capacity, allocating supply to these distribution centers from the LDC and assignment of demand points to PODs as preparedness for a similar earthquake. They developed a two-stage stochastic programming model to formulate the problem. To capture the uncertainty in demand amounts and the road network damage situation, they construct a set of discrete scenarios. The demand scenarios for each district were generated according to real-world data including population, earthquake strength, and distance to the epicenter. Interested readers are referred to

(Noyan et al. 2015) Section 6.1 for a more detailed discussion on how the scenarios are generated. They estimated a base demand for each of the 94 districts and for a single scenario a value around these base demand are selected. Especially, they divided all the districts into three categories; *destructive*, *damaging*, and *very strong or strong* according to the intensity of the earthquake. To obtain the demand for a scenario, they multiplied its base value by a deviation factor, where these factors are selected uniformly from the intervals $[0.75, 1.30]$, $[0.75, 1.20]$, and $[0.75, 1.10]$, depending on the intensity category of the demand nodes.

In our study, we focus on relief aid distribution in the post-disaster phase where the demands and the road network damaged state are known. Therefore, instead of constructing different scenarios, we generate instances with different characteristics and solve our relief routing problem for each instance seperately. To generate demands, we exactly follow the above described approch, which was used in Noyan et al. (2015). However, to generate distances, we utilize the network sampling algorithm (Algorithm 1) that was described in Section 3.1. The network has 94 demand points which also can act as intermediate points and one LDC which is the depot that stores all the prepositioned supply and the vehicles are positioned at the depot. To have a better estimation of the road network, we added 14 road junction points to 94 demand points and build a complete graph $G = (V, E)$, where V is the set of all nodes (junction, demand points and LDC) and E is the set of edges that connect nodes. We assume that there is a direct edge between every two nodes that have a real distance less than 3 kilometres. The travelling time on the graph edges have been calculated by the ArcGIS Pro 2.3.3 software. Graph G connects all the nodes together with the real street distances as the weight of an edge between every pair of nodes. Fig 3.2 presents the Van province map and the location of demand and junction points, and the depot. The demand nodes are represented by red dots and the roads junction points are shown by green triangles. The blue star indicates the depot location where the available supplies are stored.

To consider post disaster road damage when generating distances, we use the link

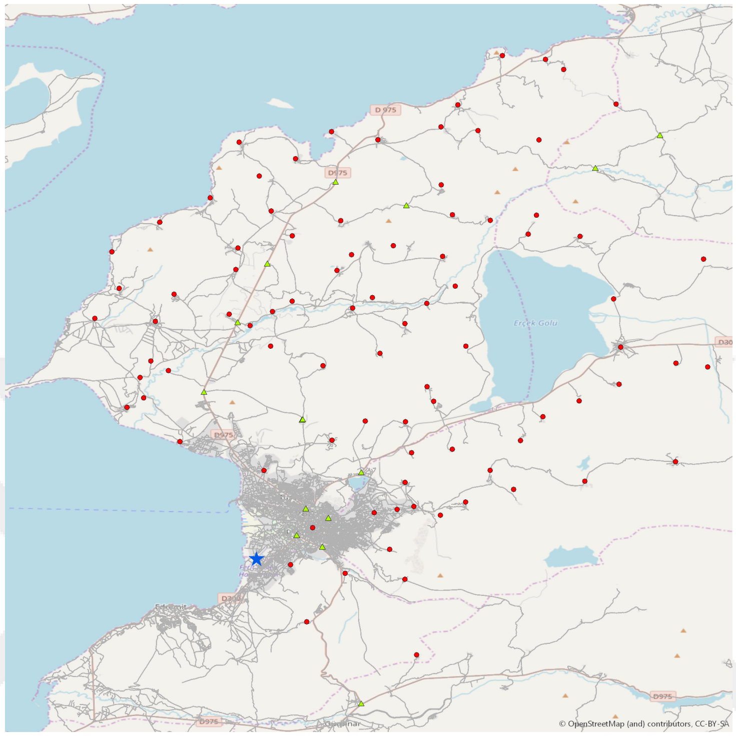

Figure 3.2: Van province map and the network node locations

failure dependency model presented in Yücel et al. (2018b). We divide the edges set in to three sets; E_d, E_a, E_s according to damage intensity category of their endpoints. The edges that have one or two endpoints with destructive damage intensity are placed in set E_d. From the remaining edges, the ones having one or two endpoints with damaging intensity are assigned to set E_a. Set E_s contains the edges that do not belongs to two other sets. Each edge $e \in E$ in G has a survival probability p_e, which is the probability that the edges remain intact after the disaster. For each of our sample instances, we assign the initial p_e values to the edges in the sets E_d, E_a and E_s by selecting a value within the $[0.7, 0.8]$, $[0.8, 0.9]$ and $[0.9, 1]$ intervals, respectively. The Algorithm 1 determines which edges are going to fail and which will remain functional in the post-disaster network. We use this information to calculate the shortest path between the demand nodes themselves and the depot.

In order to reduce the size of the problem, Noyan et al. (2015) clustered the demand nodes and created new networks with 15, 30, 60 nodes. The demand related to each clustered nodes are aggregated and assigned to the cluster centroid, which represents the cluster. Relief distribution operation in Noyan et al. (2015) was only considered as distributing the supply from the depot to selected PODs. However, here we consider the case that we are going to deliver the supply directly from the depot to the demand points. It is reasonable to only solve the problem on 15, 30 and 60 node-networks, instead of the original 94 node-network since the people can reach to cluster centroids easily. We set the number of vehicles equal to 3, 5 and 9 for instances with 15, 30 and 60 shelters, respectively. Totally, we generate 45 random instances, 15 for each of the problem sizes. We divide the number of nodes by the number of vehicles and multiply it by the average travelling time between any two nodes to obtain the maximum tour length Ψ. We calculate Ψ by an approach similar to the one in Section 3.2. The total available supply at the depot is equal to $C = \zeta^2 \sum_i d_i$, where $\zeta^2 = 0.7$. The vehicle capacity and the ϵ value are calculated by the same procedure we explained in Section 3.2 for Kartal instances. Finally, the value for ϵ is fixed to $0.85m \times \Psi$.

Chapter 4

INEQUITY-AVERSE SHELTER LOCATION FOR DISASTER PREPAREDNESS

The chapter is organized as follows. The first section describe the problem and introduce the mixed integer programming (MIP) model for the problem. The genetic algorithm is explained in Section 4.2. In Section 4.3 we report our experimental results on OR-lib benchmark instances and the Istanbul data. Finally, we present concluding remarks in Section 4.4.

4.1 Problem definition

Let J be a set of candidate locations (CLs) for the facilities, which we call "shelters" although the model admits a more general interpretation, e.g., one where the facilities are local distribution centers, medical units, etc. Let $|J| = m$. A set K of population nodes (PNs) is given, where $|K| = n$. Population node $k \in K$ has w_k inhabitants. To capture the uncertainty, we define a finite set S of scenarios, where each scenario s has a certain occurrence probability, p_s. Depending on where the disaster strikes and how severe it is, a demand of b_k^s $(0 \leq b_k^s \leq w_k)$ occurs in PN k. Here we consider demand as a portion of the population. Thus, b_k^s can be interpreted as the number of people from PN k who have to be transported to a shelter under scenario s, but the model is general enough also to allow other interpretations. In scenario s, the demand vector is $(b_1^s, \ldots, b_n^s)$. Furthermore, the distance matrix is $D = (d_{kjs})$, where d_{kjs} is the (network) distance between PN k and CL j in scenario s. Note that in view of possible road failures, the distance between two points depends on the scenario.

Each CL j has a pre-defined capacity c_j. Since assignments are independent of scenarios, the pre-decided demand allocations to CL j can exceed the capacity of the facility. Whenever this happens, a recourse action takes place where the facility is expanded at the price of a unit expansion (penalty) cost η_j, in order to be able to accommodate all assigned victims. The decision on expansion differs for each scenario since demands are dependent on the scenarios. Opening a shelter in CL j incurs an establishment cost f_j, independent from the scenarios. A budget C limits the total cost, i.e. the sum of all establishment costs and expansion costs. If the budget is exceeded, then there must be unassigned individuals. To maintain a certain service level α, the total cost must not exceed the budget with probability of at least α.

The location decisions are represented by a binary vector $x = (x_j)$, where $x_j = 1$ if a shelter is opened at CL j and $x_j = 0$ otherwise, for all $j \in J$. The assignment of the affected individuals in PN k to the open shelter at location j is shown by variable y_{kj}, where y_{kj} takes the value 1 if we assign PN k to the open shelter j, otherwise it takes the value 0, for all $k \in K$, $j \in J$.

Uncertainty in the input data is one of the main issues when dealing with disaster preparedness optimization models. In case of large-scale disaster, data may not be available or easily accessible for everyone. This situation introduces uncertainty into parameters such as demands, costs, and travel times.

Using stochastic programming with probability distributions is meaningful only when a certain action can be repeated several times. There is no repetition in the occurrence of disasters. As such, it is hard or even impossible/meaningless to estimate probabilistic distributions for uncertain parameters in this context. On the other hand, robust optimization focuses on the worst case and is better able to control risk while stochastic programming emphasizes on expected values. However, the stochastic programming formulation can easily accommodate risk considerations. One way to do that is through a chance constraint. Moreover, the results of both methods depend strongly on the model for the uncertain parameters either the uncertainty set or the probabilistic scenarios employed in the optimization.

The other option that is adopted in this thesis, is to use stochastic programming with discrete scenarios for the disaster and incorporating a chance constraint to manage risk in the problem. According to Galindo and Batta (2013b) modelling disaster related uncertainties by use of scenarios might be the best option.

To feel safe after a disaster, a person would want to be with his/her acquaintances. Being close to acquaintances is very important to maintain the social structure of society. We assumed that a person has a degree of familiarity with the people that live in the same district as him/her. Thus, we assumed that all people that live in the same district are assigned to one single shelter.

Note that we consider the assignment of PNs to CLs as a decision that is made in a pre-disaster stage, contrary to some other stochastic models for disaster response which assume that the assignment of individuals to shelters is done only after the disaster, i.e., after the uncertainty has been resolved and all demands are known. In many cases, the latter is a reasonable assumption with the potential to increase the efficiency of the solution by letting the assignment decision depend on the actual random scenario. However, there are other situations where a pre-assignment of people to shelters is advantageous: Knowing the assignments already in the preparedness stage allows the planner to tell (and even train) the individuals which shelter they should go to in case of a disaster, and by which evacuation path. This may be especially important for children and old people. It will facilitate a faster evacuation after the disaster and avoid chaos. In addition, it is often the case that telecommunication infrastructure is disrupted by the disaster, which will delay the disaster relief operations and may lead to very undesirable consequences if an assignment to shelters depends on information that is only available after the disaster has stricken. Therefore, we assume a pre-disaster allocation of people to shelters in the current work.

Generally, the assignment of PNs to CLs can be done by three approaches; user equilibrium (UE), nearest assignment (NA), and system optimal (SO) assignment. User equilibrium models could take more involved considerations into account such as a preference for open shelters which are expected not yet to be congested (cf. Gutjahr and Dzubur (2016)). However, the UE approach is not realistic for assigning the people to shelters during a disaster for the following reasons. In the UE model, it is assumed that the evacuees have full information on travel times on every possible route and they can find the optimal routes. Disasters and evacuations are rare events with unusual traffic demand resulting in different traffic conditions. As a result, victims do not have the opportunity to learn from past experience which routes minimize their evacuation time Pel et al. (2012).

Victim choice could be modeled as simple as seeking the nearest opened shelter (nearest assignment, NA). However, modeling this assumption in realistic settings in our problem is not straightforward. The nearest shelter can differ from the viewpoint of the victims in the same district as a result of their limited knowledge of the post-disaster road network situation. We consider disruptions in the network without the information being passed onto the victims. It is extremely hard/impossible to model realistic NA since we have to account for the subjective opinion of the people on the nearest (easiest to access) shelter. Moreover, this assumption will conflict with two other assumptions of the problem; single-source assumption, and scenario independent PNs-CLs assignments. Another downside of the NA approach may be higher costs for capacity expansion.

For these reasons, we do not consider the NA and UE types of models here and instead use the system optimal (SO) approach. We argue that this approach gives us the advantage of informing the people which shelter they should go to in case of a disaster and by which evacuation path.

In the objective function of the problem, we want to account for both efficiency and equity. Different quantifications of equity have been proposed in the literature (Karsu and Morton 2015, Marsh and Schilling 1994). These approaches include, among others, the Rawlsian (i.e., minimum or maximum) measure, the lexicographic extension of the Rawlsian measure, the mean deviation, the variance, the range, the Gini index (also called Gini coefficient), or Hoover's concentration index. In the present work, we have decided to adopt the Gini index for two reasons. First, this index has been extensively used in economics to represent inequality and will, therefore, be easily understood by managers with economic education. Secondly, the index has favorable theoretical properties, as scale invariance with respect to multiplication by positive constants, symmetry (switching two individuals will not change the index), and validity of the Pigou-Dalton Principle of Transfers which basically says that if a small amount of a good is transferred from an individual owing more to an individual owing less, then the inequality index will improve or at least not become worse. This is a crucial property for an efficient inequity measure which has been mostly overlooked by operations research scholars when addressing equity. According to Gutjahr and Nolz (2016) the most used approach to address inequity in humanitarian operations studies is to minimize the worst value of a disutility measure (Rawlsian approach). However, this approach suffers from lacking the Pigou-Dalton Principle of Transfers which leads to unfair distribution of the services. In recent years, some researchers show interest in using the Gini index as an inequity measure in humanitarian operation studies. Gutjahr and Fischer (2018) discussed using Ginis mean absolute difference of the deprivation costs is a proper metric for considering inequity in humanitarian logistics. Eisenhandler and Tzur (2018) and Eisenhandler and Tzur (2019) adopted a Gini index related measure in relief items delivery and pick up problem.

The Gini index is typically formulated for benefits as wealth or income, we need it for disadvantages or costs. The Gini index for costs (in our case: distances) $d^{(i)}$ ($i = 1, \ldots, N$) of N individuals can be defined analogously as the Gini index for benefits as

$$G = \frac{1}{2N} \frac{\sum_{i=1}^{N} \sum_{j=1}^{N} |d^{(i)} - d^{(j)}|}{\sum_{i=1}^{N} d^{(i)}} \tag{4.1}$$

(see Drezner et al. (2009), Yitzhaki and Schechtman (2013)). The minimum value of G is 0 (perfect equity), the maximum value is 1 (extreme inequity). However, we do not use the Gini index itself as an objective function since minimization of the Gini index alone would result in equitable, but very inefficient solutions, as argued in Drezner et al. (2009). Therefore, what we need as an objective function is an *inequity-averse aggregation function* (taking also efficiency into account) rather than an inequity measure alone. For this reason, we replace the Gini index G for distances by a linear combination $\mathcal{I} = \mu + \lambda\Delta$. Therein, $\mu = \frac{1}{N} \sum_{i=1}^{N} d^{(i)}$ is the *mean distance*, and $\Delta = 2\mu G$ is *Gini's Mean Absolute Difference* of the distances.

The choice of a linear combination of the efficiency measure μ and the equity measure Δ as the objective is intuitively plausible, and the decision to take rather $\Delta = 2\mu G$ than G itself as the equity measure is easily explained by the requirement that both measures have to be expressed in the same units if they are to be combined to a weighted sum. (Note that G is dimensionless.) However, there are deeper reasons justifying the choice of $\mathcal{I} = \mu + \lambda\Delta$, which can be elucidated by the theory of *symmetric comonotonically linear functionals* from the inequity measurement literature, see Ben-Porath et al. (1997). In order to apply the theory of inequality under uncertainty developed in Ben-Porath et al. (1997), we have to verify that the framework of *symmetric comonotonically linear functionals* used there includes our proposed inequity-averse aggregation function $\mathcal{I} = \mu + \lambda\Delta$ (as it is the case for the Rawlsian inequity measure, see (Ben-Porath et al. 1997)). We shall show that for $0 \leq \lambda \leq 1/2$, the measure $\mathcal{I} = \mu + \lambda\Delta$ is indeed a symmetric comonotonically linear functional, defined by a probability vector on the sorted distances. A related result has already been shown in Porath and Gilboa (1994) for the case of (income) maximization instead of (distance) minimization. Note that the switch from maximization to minimization is not completely trivial (as it has also been observed in Drezner et al. (2009)): the idea of simply subtracting distances from an upper bound to get a maximization problem has the drawback that the Gini index is not translation-invariant, which would make the value of the index dependent on the chosen bound. Therefore, we give an explicit proof for the situation treated in the present thesis.

Following Ben-Porath et al. (1997), we characterize a symmetric comonotonically linear functional on a set $\{d_1, \ldots, d_N\}$ of variables by a vector of "rank-dependent" probabilities q_i ($i = 1, \ldots, N$) to be assigned as weights to the *sorted* variable values. Let $d^{(1)} \leq d^{(2)} \leq \ldots \leq d^{(N)}$ be the sorted variable values. While in Ben-Porath et al. (1997), the variables correspond to payoffs, they are distances (i.e., costs) in our case, so d_i stands now for the distance to be traversed by the individual who is the i-th *best*-off (instead of worst-off as in Ben-Porath et al. (1997)). Consequently, we have to reverse the assumption $q_1 \geq q_2 \geq \ldots \geq q_N$ made in Ben-Porath et al. (1997) to the assumption $q_1 \leq q_2 \leq \ldots \leq q_N$. The functional is given by

$$\mathcal{I}(d) = \sum_{i=1}^{N} q_i \, d^{(i)}. \tag{4.2}$$

The special case $q_i = 1/N$ for all i corresponds to the utilitarian measure, and the special case $q_N = 1$ and $q_i = 0$ $(i < N)$ corresponds to the Rawlsian measure. As mentioned in Ben-Porath et al. (1997), a functional of the form (4.2) can be viewed as a so-called "min-of-means" functional

$$\mathcal{I}(d) = \min_{q \in C} q \cdot d \tag{4.3}$$

by choosing C as the convex hull of all permutations of $(q_i)_{i=1}^N$.

An alternative representation of the Gini index derives from the well-known *Lorenz curve* and is given by

$$G = \frac{1}{N} \cdot \left(N + 1 - 2\, \frac{\sum_{i=1}^{N} (N+1-i)\, d^{(i)}}{\sum_{i=1}^{N} d^{(i)}} \right) \tag{4.4}$$

where it is now essential that the $d^{(i)}$ are sorted in ascending order as assumed above. Elementary calculations show that the two expressions (4.1) and (4.4) are equivalent. In addition to G, we consider the mean distance $\mu = \frac{1}{N} \sum_{i=1}^{N} d^{(i)}$.

Proposition. For $0 \leq \lambda \leq 1/2$, the weighted sum $\mathcal{I}(d) = \mu + \lambda\,\Delta$ with $\Delta = 2\mu G$ is a symmetric comonotonically linear functional of the form (4.2) with a vector q of probabilities $q_i \geq 0$ satisfying $\sum_{i=1}^{N} q_i = 1$ and $q_1 \leq \ldots \leq q_N$.

Proof. By insertion of (4.4) for G we get

$$\Delta = 2\mu G = \frac{2}{N^2} \sum_{i=1}^{N} [(N+1) - 2(N+1-i)]\, d^{(i)} = \frac{2}{N^2} \sum_{i=1}^{N} (2i - N - 1)\, d^{(i)}. \tag{4.5}$$

Using this expression, we further get

$$\mathcal{I}(d) = \frac{1}{N} \sum_{i=1}^{N} d^{(i)} + \lambda\, \frac{2}{N^2} \sum_{i=1}^{N} (2i - N - 1)\, d^{(i)} = \frac{2}{N^2} \sum_{i=1}^{N} \left\{ \frac{N}{2} + \lambda (2i - N - 1) \right\} d^{(i)}.$$

Let us denote the coefficients of the variables $d^{(i)}$ by

$$q_i = \frac{2}{N^2}\left\{\frac{N}{2} + \lambda(2i - N - 1)\right\} \quad (i = 1, \ldots, N).$$

Obviously, $q_1 \leq \ldots \leq q_N$. Moreover, since $q_1 = 1/N - 2\lambda(N-1)/N^2 \geq 1/N - 1/N = 0$ because of $\lambda \leq 1/2$, we have $q_i \geq 0$ for all i. Finally, $\sum_{i=1}^{N}(2i - N - 1) = 0$, and therefore $\sum_{i=1}^{N} q_i = \frac{2}{N^2} \cdot \frac{N}{2} \cdot N = 1$.

Note that some of the coefficients of the variables $d^{(i)}$ on the r.h.s. of (4.5) are negative, so neither Gini's Mean Absolute Difference Δ nor the Gini index G itself would allow a representation as required in the Proposition.

To represent *uncertainty* in our problem, we follow the approach in Ben-Porath et al. (1997). The key difficulty treated therein, the contrast between *ex ante* inequity and *ex post* inequity, can be stated in formal terms as follows: Suppose we have an inequity measure $\mathcal{I}(f)$ assigning to each vector $(f(i))_{i\in I}$ of costs of all individuals $i \in I$ a real number, and let $f(s,i)$ denote the cost of individual i under random scenario s. Then it is not clear whether $\mathcal{I}(E\,[f(s,i)])$ or $E\,([\mathcal{I}(f(s,i))]$ should be optimized, where $\mathcal{I}$, acting on i, is the given inequity measure, and E, acting on s, is the mathematical expectation. Should we try to minimize the inequity of the expected outcome (*ex ante* consideration), or rather the expected inequity of the outcome (*ex post* consideration)?

To see the difference between the two criteria, consider the following example from Ben-Porath et al. (1997). Let us compare the three "social policies" f, g and h below, where a social policy f is a function $f(s,i)$ assigning costs to scenarios and individuals as above. In the example, two scenarios s and t with equal probabilities 0.5 and two individuals a and b are considered.

f	a	b
s	0	0
t	1	1

g	a	b
s	0	1
t	1	0

h	a	b
s	0	1
t	0	1

The *ex ante* inequity perspective is indifferent with respect to the social policies f and g since they entail the same expected cost of 0.5 for each of the two individuals, whereas it prefers f and g over the social policy h which entails unequal expected costs of 0 and 1 for individual a and b, respectively. The *ex post* inequity perspective, on the contrary, prefers f over both g and h, since f leads with probability one to a final overall state of equity, whereas g and h, between which the ex-post perspective is indifferent, result with probability one in a final overall state of inequity. Roughly summarizing we can say that *ex ante* optimizes equity of chances, whereas *ex post* optimizes the chance of equity. It is not obvious which criterion is more appropriate in humanitarian logistics. On one hand, humanitarian organizations would certainly not like to see big differences in the well-being and safety of victims after a disaster (*ex post* consideration). On the other hand, using exclusively the *ex post* criterion may discourage private or communal initiatives to protect against the consequences of disasters in advance by own efforts.

Ben-Porath et al. (1997) define a broad class of "reasonable" inequity measures taking both *ex ante* inequity and *ex post* inequity into account. A special case are weighted means of $\mathcal{I}(E\,[f(s,i)])$ and $E\,[\mathcal{I}(f(s,i))]$. This is the approach we follow here. As shown in Gajdos and Maurin (2004) (see Theorem 9), it is arguable from theoretical reasons that the two weights are chosen as equal. Therefore, in this study, we shall mainly choose

$$\frac{1}{2}\,\mathcal{I}(E\,[f(s,i)]) + \frac{1}{2}\,E\,[\mathcal{I}(f(s,i))]. \tag{4.6}$$

as the objective. Based on the inequity-averse aggregation function $\mathcal{I}(d) = \mu(d) + \lambda\Delta(d)$, where d is the vector of all distances from individuals to assigned opened shelters, we specify now the objective functions for the *ex ante* and for the *ex post* case, respectively. The objective function for the combined case results as the weighted mean (4.6).

***Ex ante* problem:** First of all, we compute the expected distance to be traversed by an inhabitant of PN k. With the defined notation, in scenario s, an inhabitant i of PN k has a probability of b_k^s/w_k to be affected by the disaster. If (s)he is affected by the disaster, (s)he faces a distance of $d_{ks} = \sum_{j\in J} y_{kj} d_{kjs}$ to the assigned shelter.

This is correct because we make sure that every population node is assigned only to one shelter. Therefore, the expected distance for an arbitrary person from PN k is $\bar{d}_k = \sum_{s \in S} p_s \cdot (b_k^s / w_k) \cdot d_{ks}$. There are $N = \sum_{k \in K} w_k$ individuals in total. The Gini index of the values $\bar{d}_k$ (note that each value $\bar{d}_k$ has to be taken with multiplicity w_k for the w_k inhabitants of PN k) can easily be computed, using representation (4.1):

$$G = \frac{1}{2N} \cdot \frac{\sum_{k \in K} \sum_{k' \in K} w_k \, w_{k'} \, |\bar{d}_k - \bar{d}_{k'}|}{\sum_{k \in K} w_k \, \bar{d}_k}$$

$$= \frac{1}{2N} \cdot \frac{\sum_{k \in K} \sum_{k' \in K} \left| \sum_{s \in S} p_s \left(w_{k'} \, b_k^s \, \sum_{j \in J} y_{kj} d_{kjs} - w_k \, b_{k'}^s \, \sum_{j \in J} y_{k'j} d_{k'js} \right) \right|}{\sum_{k \in K} \sum_{s \in S} \sum_{j \in J} p_s b_k^s y_{kj} d_{kjs}}.$$

Since the mean μ of the values $\bar{d}_k$ (taken with their multiplicities w_k) is

$$\frac{1}{N} \sum_{k \in K} \sum_{s \in S} \sum_{j \in J} p_s \, b_k^s \, y_{kj} d_{kjs},$$

we get the following formula for the objective function (to be minimized) of the *ex ante* problem:

$$F_{ante} = \mu + 2\lambda\mu G = \frac{1}{N} \sum_{k \in K} \sum_{s \in S} \sum_{j \in J} p_s \, b_k^s \, y_{kj} d_{kjs} + \frac{\lambda}{N^2} \sum_{k \in K} \sum_{k' \in K} \left| \sum_{s \in S} p_s \left(w_{k'} \, b_k^s \sum_{j \in J} y_{kj} d_{kjs} - w_k \, b_{k'}^s \sum_{j \in J} y_{k'j} d_{k'js} \right) \right|. \quad (4.7)$$

***Ex post* problem:** Now we have to start with the computation of the Gini index (and of the inequity-averse aggregation function $\mathcal{I}(d)$) for a fixed scenario s. For a given scenario s, we reduce each PN k to those b_k^s inhabitants that are affected by the disaster, and disregard the remaining $w_k - b_k^s$ inhabitants. We do not take these inhabitants into account since we want to judge equity only among those that are candidates for actions, i.e., assignments to shelters. This is possible in the *ex post* framework, since after scenario realization, it is already known who has been affected by the disaster and who not. Let $N'_s = \sum_{k \in K} b_k^s$ be the total number of affected people in scenario s. Then, with d_{ks} as in the *ex ante* case, and again considering multiplicities (this time by the number of affected individuals instead of the number of all individuals), we get:

$$G^s = \frac{1}{2N'_s} \cdot \frac{\sum_{k\in K}\sum_{k'\in K} b_k^s\, b_{k'}^s\, |d_{ks} - d_{k's}|}{\sum_{k\in K} b_k^s\, d_{ks}}, \quad s \in S.$$

Since the mean μ of the distances (considering their multiplicities) is $\mu^s = \frac{1}{N'_s}\sum_{k\in K} b_k^s\, d_{ks}$ ($s \in S$), the objective function of the *ex post* problem results by taking the expected value over the random scenarios as

$$\begin{aligned} F_{post} = E[\mu^s + 2\lambda\mu^s G^s] &= \sum_{s\in S} p_s \frac{1}{N'_s} \sum_{k\in K}\sum_{j\in J} b_k^s\, y_{kj} d_{kjs} \\ &+ \lambda \sum_{s\in S} p_s \frac{1}{N'^2_s} \sum_{k\in K}\sum_{k'\in K} b_k^s\, b_{k'}^s\, |\sum_{j\in J} y_{kj} d_{kjs} - \sum_{j\in J} y_{k'j} d_{k'js}|. \end{aligned} \tag{4.8}$$

Both the *ex ante* and the *ex post* problem can be cast into MIP formulations, as it will be shown in the next subsection. For example, the *ex ante* problem can be linearized by defining

$$\tau_{k,k'} = \sum_{s\in S} p_s \left(w_{k'}\, b_k^s \sum_{j\in J} y_{kj} d_{kjs} - w_k\, b_{k'}^s \sum_{j\in J} y_{k'j} d_{k'js} \right)$$

and introducing the auxiliary variables $\tau^+_{k,k'} = \max(\tau_{k,k'}, 0)$ and $\tau^-_{k,k'} = \max(-\tau_{k,k'}, 0)$.

4.1.1 Formulation of the model

We are now in the position to present the suggested MIP models. All that is still missing is the chance constraint, expressing the condition that with at least a certain pre-specified probability α, the solution remains feasible even after shelter expansions, i.e., $Pr(\text{total cost} \le C) \ge \alpha$. Since randomness is represented in our model by the possible scenarios $s \in S$ with assigned probabilities p_s, the chance constraint takes the form $\sum_{s\in S} p_s z_s \ge \alpha$, where z_s is the indicator variable for the event that the total cost in scenario s is smaller or equal to the budget C. The formulations below give the deterministic equivalents of our chance-constrained stochastic programs. Table 4.1 lists the decision variables of the models.

***Ex ante* problem:**

$$\min\ \mu + 2\lambda\mu G = \frac{1}{N} \sum_{k\in K}\sum_{s\in S}\sum_{j\in J} p_s\, b_k^s\, y_{kj} d_{kjs} + \frac{\lambda}{N^2} \sum_{k\in K}\sum_{k'\in K} (\tau^+_{k,k'} + \tau^-_{k,k'}) \quad \text{s.t.} \tag{4.9}$$

Table 4.1: Decision Variables

Notation	Description	Variable type
x_j	1 if CL j is opened, 0 otherwise	Binary
y_{kj}	1 if all individuals in PN k are assigned to CL j, 0 otherwise	Binary
z_s	1 if total cost meets the budget in scenario s, 0 otherwise	Binary
t_{js}	Amount of exceeded demand assigned to CL j in scenario s	Continuous

$$\tau^+_{k,k'} - \tau^-_{k,k'} = \sum_{s\in S} p_s \left(w_{k'}\, b^s_k \sum_{j\in J} y_{kj} d_{kjs} - w_k\, b^s_{k'} \sum_{j\in J} y_{k'j} d_{k'js} \right) \qquad \forall k,\, k' \in K \tag{4.10}$$

$$y_{kj} \leq x_j \qquad \forall k \in K,\; j \in J \tag{4.11}$$

$$\sum_{j\in J} y_{kj} = 1 \qquad \forall k \in K \tag{4.12}$$

$$\sum_{k\in K} b^s_k\, y_{kj} \leq t_{js} + c_j \qquad \forall j \in J,\; s \in S \tag{4.13}$$

$$\sum_{j\in J} f_j x_j + \eta_j\, t_{js} \leq C + M_0(1 - z_s) \qquad \forall s \in S \tag{4.14}$$

$$\sum_{s\in S} p_s z_s \geq \alpha \tag{4.15}$$

$$x_j \in \{0,1\} \quad \forall j \in J, \quad z_s \in \{0,1\} \quad \forall s \in S$$

$$y_{kj} \in \{0,1\} \qquad \forall k \in K,\; j \in J$$

$$t_{js} \geq 0 \qquad \forall j \in J, s \in S \tag{4.16}$$

$$\tau^+_{k,k'}, \tau^-_{k,k'} \geq 0 \qquad \forall k,\, k' \in K \tag{4.17}$$

To reformulate the objective function in linear terms, we introduced auxiliary variables $\tau^+_{k,k'}$ and $\tau^-_{k,k'}$ as discussed above and added constraints (4.10). Constraint (4.11) ensures that a PN can only be served by an opened shelter. Constraint (4.12) guarantees that each PN is assigned to exactly one shelter. Constraint (4.13) calculates the exceeded capacity for each CL in each scenario. Constraint (4.14) defines the indicator variable z_s for the satisfaction of the budget constraint in scenario s. If we meet the budget in scenario s, then z_s has no reason to become zero; otherwise, z_s will be zero. Our chance constraint (4.15) imposes that the budget should be met with probability at least α. Finally, the large number M_0 can be replaced by $\sum_{j \in J} (f_j + \eta_j (N - c_j))$.

***Ex post* problem:**

$$\min \sum_{s \in S} \sum_{k \in K} \sum_{j \in J} \frac{1}{N'_s} p_s b_k^s \, y_{kj} d_{kjs} + \lambda \sum_{s \in S} p_s \frac{1}{N'^2_s} \sum_{k \in K} \sum_{k' \in K} b_k^s \, b_{k'}^s \, (\rho^+_{kk's} + \rho^-_{kk's}) \quad \text{s.t.} \tag{4.18}$$

$$\rho^+_{kk's} - \rho^-_{kk's} = \sum_{j \in J} y_{kj} d_{kjs} - \sum_{j \in J} y_{k'j} d_{k'js} \quad \forall s \in S, \; k, k' \in K \tag{4.19}$$

$$\rho^+_{kk's} \, \rho^-_{kk's} \geq 0 \quad \forall k, \, k' \in K, \, s \in S \tag{4.20}$$

constraints (4.11) – (4.16)

where $\rho^+_{kk's}$ and $\rho^-_{kk's}$ are three-indexed nonnegative auxiliary variables that help us to make the objective function linear.

Combined problem:

$$\begin{aligned} \min \; \gamma & \left(\frac{1}{N} \sum_{k \in K} \sum_{s \in S} \sum_{j \in J} p_s \, b_k^s \, y_{kj} d_{kjs} + \frac{\lambda}{N^2} \sum_{k \in K} \sum_{k' \in K} (\tau^+_{k,k'} + \tau^-_{k,k'}) \right) \\ & + (1 - \gamma) \left(\sum_{s \in S} \sum_{k \in K} \sum_{j \in J} \frac{1}{N'_s} p_s b_k^s \, y_{kj} d_{kjs} + \lambda \sum_{s \in S} p_s \frac{1}{N'^2_s} \sum_{k \in K} \sum_{k' \in K} b_k^s \, b_{k'}^s \, (\rho^+_{kk's} + \rho^-_{kk's}) \right) \end{aligned} \tag{4.21}$$

s.t. constraints (4.10) – (4.17) and (4.19) – (4.20).

In the objective function, γ is the weight of the *ex ante* measure which is a constant parameter. The special cases $\gamma = 1$ and $\gamma = 0$ yield the *ex ante* and the *ex post* problem, respectively. The special case $\gamma = 1/2$ yields the problem corresponding to (4.6).

For the evaluation of our experimental results in Section 4.3, we will calculate the average distance that each individual has to travel to reach the assigned shelter (ADTS), given by the first them of (4.7) for *ex ante* approach and by the first term of (4.8) for *ex post* approach. We also define a *combined* ADTS by

$$\frac{1}{2} \cdot \left(\frac{1}{N} \sum_{k \in K} \sum_{s \in S} \sum_{j \in J} p_s \, b_k^s \, y_{kj} d_{kjs} \right) + \frac{1}{2} \cdot \left(\sum_{s \in S} \sum_{k \in K} \sum_{j \in J} \frac{1}{N_s'} \, p_s b_k^s \, y_{kj} d_{kjs} \right).$$

On the other hand, inequity will be judged by Gini's mean absolute difference (GMAD). For the *ex ante* and the *ex post* version of GMAD, we use the second term of (4.7) and (4.8), respectively:

$$GMAD_{exante} = \frac{1}{N^2} \sum_{k \in K} \sum_{k' \in K} \left| \sum_{s \in S} p_s \left(w_{k'} \, b_k^s \sum_{j \in J} y_{kj} d_{kjs} - w_k \, b_{k'}^s \sum_{j \in J} y_{k'j} d_{k'js} \right) \right|$$

$$GMAD_{expost} = \sum_{s \in S} p_s \frac{1}{N_s'^2} \sum_{k \in K} \sum_{k' \in K} b_k^s \, b_{k'}^s \left| \sum_{j \in J} y_{kj} d_{kjs} - \sum_{j \in J} y_{k'j} d_{k'js} \right|.$$

Analogously to the combined ADTS, a combined GMAD is calculated as the weighted mean of $GMAD_{exante}$ and $GMAD_{expost}$ with equal weights. Finally, *ex ante*, *ex post* and combined versions of the Gini index result as the quotients GMAD/$(2 \cdot$ ADTS) for the respective versions of GMAD and ADTS.

4.2 Solution algorithm

In principle, the model from Section 4.1.1 can be solved by a MIP solver. We used Cplex 12 for this purpose. However, as it shall be detailed in Section 4.3, it turned out that the computational limits of this solution approach exclude larger test instances, as they may well occur in practice: The model cannot be solved to optimality for instances with more than about 50 PNs, 50 CLs and 500 scenarios. Therefore, in the present section, we develop a genetic algorithm (GA) to solve the presented problem approximately for realistic sizes of the problem. GAs are widely used to solve combinatorial optimization problems and work well on location-allocation problems (Jaramillo et al. 2002), (Alp et al. 2003). The performance of GAs is highly dependent on the search space size. Hence, a GA may fail to converge to a global optimum in a large search space and may take a long execution time (Derbel et al. 2012). In order to restrict the solution space, we follow a commonly used approach: Any location-allocation problem can be decomposed into a location problem ("master problem") and a repeatedly solved allocation problem ("subproblem"). Consequently, we can search for location decisions on the level of the master problem by the GA, and for each such decision, knowing which candidate locations are chosen, we can find the optimal allocation along with the objective function value. However, this may still take extensive runtime. Therefore, to solve the allocation problem quickly, we utilize a greedy algorithm on the level of the subproblem. A complicating factor here is the chance constraint. We propose a method to tackle this complication and discuss it in the remainder of this section.

4.2.1 Overview of the genetic algorithm

A GA is a population-based metaheuristic and works with a population of solutions at each iteration to obtain a near-optimal solution to an optimization problem. GAs were originally introduced by (Holland 1992), based on the natural selection concept that drives biological evolution. Solutions are represented by chromosomes, and a population of chromosomes defines a generation. The GA starts with a fixed number of initial solutions and maintains this number as the population size through the generations. In each iteration, new chromosomes are generated as offspring of two chromosomes or as the result of mutating one chromosome. The next generation is built by selecting "good" solutions from the current population and recently generated offspring, and eliminating less good solutions. The algorithm continues to evolve populations from one generation to another until a stopping criterion is met. Let Pop_t, PA_t and C_t respectively represent the population, parents and offspring sets in the current generation t. The general implementation structure of the GA is described in Algorithm 2, and followed in this section. The related section to each component of the algorithm is specified by the section reference number.

Algorithm 2 General structure of the genetic algorithm

Input: problem data, GA parameters

Output: the best solution

t ← 1

Generate the initial population Pop_1 as described in section 4.2.2;

Evaluate all members of Pop_1 using the section 4.2.3;

while the **termination criteria** does not hold **do**

  Create solution set PA_t as the parents set from members of Pop_t respecting the **parents selection** (section 4.2.4);

  Select two chromosome from PA_t and create two children by the **crossover** (section 4.2.4);

  If the children are distinct add them to C_t;

  Select one chromosome form PA_t and create one child by the **mutation** (section 4.2.4);

  If the child is unique add it to C_t;

  Evaluate all members of C_t by the **fitness evaluation** (section 4.2.3);

  Select Pop_{t+1} from PA_t and C_t by the **survivors selection** (section 4.2.4);

  t ← t+1;

end while

Return the best solution in the final population

4.2.2 Solution representation and initial solution

In our algorithm we allow the GA to iterate over location decisions. The allocation decisions are made within the fitness function evaluation procedure while we know the values of $(x_j)_{j\in J}$ representing the current location decision. The chromosomes are the backbone of the GA. They represent the solutions by a one-to-one relationship. As the location decisions are modeled by binary variables, the binary string encoding is a natural choice. This means each of our GA chromosomes is a one-dimensional array vector $x = (x_1, \ldots, x_m)$ of 0-1 values x_j with length equal to the number m of candidate locations.

To obtain an initial solution, we randomly assign 0 or 1 to each gene j of each chromosome. However, to help the algorithm to avoid generating infeasible solutions as much as possible, we calculate an upper bound for the number of open facilities and respect this bound during the assignment of 1-bits to the initial solution. At first, we sort the CLs according to their establishment costs in non-decreasing order. Let us change the indices accordingly, such that $f_1 \leq f_2 \leq ... \leq f_m$ is the resulting arrangement of establishment costs. Now, we define $m_{max} = \max\{j : \sum_{i=1}^{j} f_i \leq C\}$. We make sure that the number of open facilities in the initial solution and in any other solution generated during the algorithm will not exceed m_{max}. This produces a good chance to obtain solutions that respect the chance constraint.

4.2.3 Fitness evaluation

To evaluate the quality of the chromosomes, we need to define a fitness function which should relate to the objective function and the feasibility of the solutions. Therefore, for given location solution (chromosome) x, we want to obtain an optimal allocation $y = (y_{kj})$ and the corresponding objective function value assigned to (x, y). Knowing the opened facilities, we can determine the best allocation by solving a MIP model which we call the allocation model. Given solution x, let $J' = \{j \in J : \ x_j = 1\}$ be the set of open locations. The **allocation model (AM)** is as follows:

$$\min\ \gamma\left(\frac{1}{N}\sum_{k\in K}\sum_{s\in S}\sum_{j\in J'} p_s\, b_k^s\, y_{kj} d_{kjs} + \frac{\lambda}{N^2}\sum_{k\in K}\sum_{k'\in K}(\tau_{k,k'}^{+}+\tau_{k,k'}^{-})\right) + (1-\gamma)\left(\sum_{s\in S}\sum_{k\in K}\sum_{j\in J'}\frac{1}{N'_s}\, p_s b_k^s\, y_{kj} d_{kjs} + \lambda\sum_{s\in S} p_s\,\frac{1}{N'^2_s}\sum_{k\in K}\sum_{k'\in K} b_k^s\, b_{k'}^s\,(\rho_{kk's}^{+}+\rho_{kk's}^{-})\right) \tag{4.22}$$

$$\text{s.t. } \tau_{k,k'}^{+}-\tau_{k,k'}^{-} = \sum_{s\in S} p_s\left(w_{k'}\, b_k^s \sum_{j\in J'} y_{kj} d_{kjs} - w_k\, b_{k'}^s \sum_{j\in J'} y_{k'j} d_{k'js}\right) \qquad \forall k,\, k'\in K \tag{4.23}$$

$$\rho_{kk's}^{+}-\rho_{kk's}^{-} = \sum_{j\in J'} y_{kj} d_{kjs} - \sum_{j\in J'} y_{k'j} d_{k'js} \qquad \forall s\in S,\ k,k'\in K \tag{4.24}$$

$$\sum_{j\in J'} y_{kj} = 1 \qquad \forall k\in K \tag{4.25}$$

$$\sum_{k\in K} b_k^s\, y_{kj} \le t_{js} + c_j \qquad \forall j\in J',\ s\in S \tag{4.26}$$

$$\sum_{j\in J'} f_j x_j + \eta_j\, t_{js} \le C + M_0(1-z_s) \qquad \forall s\in S \tag{4.27}$$

$$\sum_{s\in S} p_s z_s \ge \alpha \tag{4.28}$$

$$y_{kj}\in\{0,1\} \qquad \forall k\in K,\ j\in J'$$

$$t_{js}\ge 0 \quad \forall j\in J', s\in S,$$

$$\tau_{k,k'}^{+},\tau_{k,k'}^{-}\ge 0 \quad \forall k,\, k'\in K, \quad \rho_{kk's}^{+},\, \rho_{kk's}^{-}\ge 0 \quad \forall k,\, k'\in K,\, s\in S \tag{4.29}$$

Solving this model for every chromosome x considered by the GA is time consuming. We require an efficient heuristic to find near-optimal allocations. By relaxing constraints (4.27) and (4.28) and ignoring the Gini related part of the objective function and the constraints (4.23) and (4.24), as well as removing the t_{js} variables, we obtain a *stochastic generalized assignment problem* (SGAP). Note that since we omit constraints (4.27) and (4.28), the SGAP solutions may not be feasible. However, we consider a penalty for violating these constraints in the fitness function of the GA in

Section 4.2.3. There are some heuristic approaches in the literature to solve the deterministic GAP efficiently, one of which is the greedy approach proposed by (Romeijn and Morales 2000). Before adapting their method, we need to deal with the uncertain demand which appears in constraint (4.26). A reasonable approach is to substitute the demands b_k^s with the expected demands $\bar{b}_k = \sum_{s \in S} p_s b_k^s$, keeping in mind that this will underestimate the demand in some scenarios (feasibility will finally be checked by the GA). With all these modifications, we obtain the deterministic allocation model named GAP presented below:

$$\min \quad f_{GAP} = \sum_{k \in K} \sum_{j \in J'} \left(\gamma \cdot \frac{\sum_{s \in S} p_s b_k^s d_{kjs}}{\sum_{k \in K} w_k} + (1-\gamma) \cdot \sum_{s \in S} \frac{p_s b_k^s d_{kjs}}{\sum_{k \in K} b_k^s} \right) \cdot y_{k,j} \tag{4.30}$$

$$\sum_{j \in J'} y_{kj} = 1 \qquad \forall k \in K$$

$$\sum_{k \in K} \bar{b}_k y_{kj} \leq c_j \qquad \forall j \in J'$$

$$y_{kj} \in \{0,1\} \qquad \forall k \in K,\ j \in J'.$$

Since we do not consider capacity expansion in the GAP problem and use expected demand instead of actual demand, some scenarios may confront us with expansion costs caused by the allocations that result as the solutions of GAP. Therefore, it is possible that the GAP solution violates the chance constraint (constraint (4.28)). If the greedy algorithm yields a feasible solution to the main problem, we consider it as an acceptable (approximate) solution; otherwise, we try to find a feasible allocation. We propose two approaches which can be combined. First, the algorithm from Romeijn and Morales (2000) can be changed to a randomized greedy algorithm. Shortly speaking, this means that we allow selection from the top β percentage of the allocation choices randomly instead of choosing the best one. We repeat the randomized greedy approach until it gives us a feasible allocation or until a certain number of repetitions is reached. In this manner, the chance of finding a feasible allocation is increased. Details will be presented in Section 4.2.3. Secondly, in the case of promising solutions, we solve the AM to optimality by the MIP solver, as explained later in

Section 4.2.5.

Randomized Greedy Algorithm

Suppose there are M open facilities in the current chromosome x and n population nodes as before. As before, J' denotes the set of open facilities. Furthermore, let K' be the set of nodes that are initially unassigned. In order to apply the idea of the algorithm presented in Romeijn and Morales (2000), we have to define a weight function that scores all possible assignments $k \mapsto j$ of PNs to CLs. So that the algorithm will assign PNs to open facilities according to this function. The most common choice for the weight function is to use the assignment cost which is the coefficient of y_{kj} in the objective function of GAP. Therefore, focusing on the special case where the coefficient γ is chosen as $1/2$, we define the weight function as

$$\varphi(k,j) = \frac{\sum_{s \in S} p_s b_k^s d_{kjs}}{\sum_{k \in K} w_k} + \sum_{s \in S} \frac{p_s b_k^s d_{kjs}}{\sum_{k \in K} b_k^s}$$

and extend it later by an additional term to a function $g(k,j)$. Below we present the pseudocode of the randomized greedy algorithm.

As the proposed greedy approach does not guarantee that the allocations necessarily satisfy the omitted constraints,we use in the GA a penalty function approach by defining a suitable feasibility score. The fitness function is obtained then as a combination of an optimality score and the feasibility score.

Fitness Function

Let us denote the set of solutions in the current population as V, and let v be an element of this set. Infeasibility is defined as the violation of the chance constraint. To obtain this violation, first we need to calculate the total cost incur by current location (x^v) and allocation (y^v) solutions. The total establishment cost can be calculated as $\sum_{j \in J'} f_j x_j^v$. However, to calculate the total expansion cost we need to know the t_{js} variables' values. Since we do not want to pay more than necessary for capacity expansion in an optimal solution, the t_{js} variables can obtained from constraint (4.26)

Algorithm 3 Randomized Greedy Algorithm (RGA) to solve GAP

set $K' = K, \quad c'_j = c_j \quad \forall j \in J'$

set $\kappa_j = H/c_j \quad \forall j \in J'$, where H is a constant

while $K' \neq \emptyset$ **do**

  for $k \in K'$ **do**

    $g(k,j) = \varphi(k,j) + \kappa_j \, \bar{b}_k \quad \forall j \in J'$

    $j_k = \arg\min_{j \in J'} \{g(k,j)\}$

    $\Psi_k = \min_{j' \in J', j' \neq j_k} (g(k,j') - g(k,j_k))$

  end for

  sort $(\Psi_k)_{k \in K}$ in non-increasing order

  define Ω as the set of indices of the top β percentage of sorted Ψ_k values

  let $\hat{k}$ be a randomly selected element of Ω

  assign PN $\hat{k}$ to facility $j_{\hat{k}}$

  update the remaining capacity: $c'_{j_{\hat{k}}} = c'_{j_{\hat{k}}} - \bar{b}_{\hat{k}}$

  set $K' = K' \setminus \{\hat{k}\}$

end while

such as: $t^v_{js} = max(c_j - \sum\limits_{k \in K} b^s_k \, y^v_{kj}, 0)$. Thus the total cost will be $\sum_{j \in J'} f_j x^v_j + \eta_j \, t^v_{js}$. According to constraint (4.27), if the total cost meets the budget in scenario s, then $z^v_s = 1$; otherwise $z^v_s = 0$. Now, let $\rho(v) = \max\{\alpha - \sum_{s \in S} p_s z^v_s, \, 0\}$ for each solution $v \in V$. The feasibility score for v is $\delta(v) = 1 - \rho(v)/\rho_{max}$, where $\rho_{max} = \max_{v \in V}\{\rho(v)\}$. Note that any feasible solution will get the score 1 and the infeasible solution with the maximum degree of chance constraint violation will get a score 0. Let $F(v)$ denote the objective function value of a given solution v. The optimality score can be written as $\theta(v) = F_{min}/F(v)$, where $F_{min} = \min_{v \in V}\{F(v)\}$. Finally, the fitness function is computed as:

$$\phi(v) = \theta(v)^{\alpha_t} \cdot \rho(v)^{1-\alpha_t}\,, \tag{4.31}$$

where value α_t is a number between two pre-defined constants α_{min} and α_{max}, and t is the current iteration. This parameter helps us to control the weight of optimality and feasibility scores over the iterations. We update α_t by $\alpha_{t+1} = \alpha_t - (\alpha_{max} - \alpha_{min})/T$, where T is the total number of iterations, and start with $\alpha_1 = \alpha_{max}$.

4.2.4 Genetic operators and termination criterion

Selection: Solutions or individuals in the current population that contribute to the generation of the next generation individuals are commonly called parents. Selection of parents can effect the quality and diversity of the next generation. We assign a selection probability P_v to every individual $v \in V$ and select individuals for the next generation according to this probability. Denote the rank of individual v in the population based on its fitness value $\phi(v)$ by r, and the population size by $nPop$. Then the selection probability of individual v is chosen as

$$P_v = \frac{2-\chi}{nPop} + \frac{2r(\chi-1)}{nPop(nPop-1)}, \tag{4.32}$$

where χ is a parameter in the interval $[1,2]$ that shapes the probability distribution. When the GA wants to go from one generation to the next one, some individuals will survive and the others will be omitted from the population. There are several methods to perform the survivor selection. Among them we adopt the $(\mu+\lambda)$ selection method as described in Eiben et al. (2003). It means that we select the top $nPop$ of solutions from the parent and offspring pool of chromosomes to be retained.

Crossover: Crossover is the move that makes two new chromosomes from two parents selected randomly. We adopt the three common operators for binary chromosomes suggested in Eiben et al. (2003); namely, one-point crossover, two-point crossover and uniform crossover. In general, we can not state which of these operators performs best; therefore, we select one of them randomly, with equal probabilities, each time we need to apply a crossover.

Mutation: Mutation is designed to create a new individual from a randomly selected parent by some random and unbiased change in a certain number of its genes. Mutation has to be an explorative operator in the GA (Eiben and Schippers 1998). Therefore, it needs to cover a wide neighborhood space. To achieve this, we use the following three moves and randomly choose one of them with equal probability until the number of changed genes reaches a pre-specified value. (i) *Open a facility:* We select a closed facility and open it. (ii) *Close a facility:* We select an opened facility and close it. (iii) *Two-point swap:* An open facility and an unopened facility are selected randomly, and their indicator variables x_j are swapped.

Termination Criterion: The execution of the GA is continued until a maximum number of iterations is reached, or a certain number of successive generations provides no improvement. We refer to the latter as *idle populations*.

4.2.5 Enhancing the GA

As mentioned before, to increase the chance of finding an optimal solution, we solve the AM to optimality for some selected chromosomes. In order to have a systematic approach to select these chromosomes, we use the information that the randomized greedy algorithm (RGA) provides about the chromosomes. Note that the GAP is a relaxation of the AM. Thus, if the optimal solution of GAP is a feasible solution to AM, it is already optimal. Hence, the difficulty arises from solutions that are infeasible for AM. To reduce the number of chromosomes for which we solve AM to optimality, the selected chromosome should satisfy the following conditions: (i) The allocations produced by RGA should be infeasible to AM. (ii) The objective function value of the allocations produced by RGA should be less than or equal to $(1+\tau)\cdot F_{min}$, where F_{min} is the best objective value from the previous iteration, with τ denoting a parameter that is tuned experimentally.

4.3 Computational results and case study

To test the performance of the proposed GA, we generate a number of instances based on the benchmark data set provided by Beasley (1988) from the OR-library for the capacitated warehouse location problem. We compare the runtime and the quality of the solutions achieved by our algorithm to the solutions obtained by solving the MIP model given in Section 4.1.1 by Cplex 12.5.1 in instances that are solvable by Cplex within two hours. We also test our algorithm on real data from a district of Istanbul by augmenting the data of Kılcı et al. (2015). In all of the instances, we consider the case of an earthquake as the natural disaster when generating the scenarios. We first present the GA parameter settings. Next we describe how the benchmark instances are generated and then present the computational results for these instances. After that we explain how we prepare the Istanbul data and present the corresponding results. We also perform several analyses using the Istanbul data.

Computational results on OR-library benchmark data

In this section we present the computational results on the 56 benchmark instances generated based on ORlib benchmak data that we describe in Section 3.1. All computations are conducted on a computer with two 28-core Intel Xeon E5-2690 v4 processors, running at 2.6 GHz with 35 MB cache, 128 GB RAM and with windows 10. To solve the MIP model we use Gams 24.1.2 and the Cplex 12.5.1 solver. We implemented the GA using Python 2.7 and the allocation model (AM) inside the GA is solved by Gurobi 7.5.

To have reliable results we execute 5 replications when using the GA and report the best and average objective values found in Table 4.2 alongside with the optimal solution of MIP. In Table 4.2 the first column shows the instance names, the next three columns are outputs of the MIP model solved by Cplex. *Opt. Objval* shows the optimal value of the objective function obtained by Cplex. When Cplex was unable to solve the problem optimally, we report the best feasible solution found within the 2-hour time limit. *Runtime* is the time that Cplex spends to reach the final solution, and *LB* is the reported lower-bound by Cplex. Furthermore, the '-' sign means that we could not calculate any value because Cplex did not provide enough information. The next three columns are related to the results of the GA runs. *Best objval*, *Avg. objval* and *Avg. time* indicate the best objective value, average objective value and average runtime of 5 GA replications in seconds, respectively. ODP(%) shows the difference between the objective function value obtained by Cplex (Z_C) and the best objective function value obtained by the GA (Z_{GA}) in terms of percentages of Z_C. Similarly, the GAP(%) column which shows the optimality gap is derived as GAP(%)$= 100 \cdot (Z_{GA} - LB)/LB$.

The negative values in the ODP(%) column show that the GA was sometimes able to find better solutions compared to Cplex. This is possible because in those cases Cplex reached its time limit and could not find the optimal solution. The instances where Cplex reached the time limit are indicated by displaying the values in the runtime column in bold. As it can be seen in Table 4.2, out of 56 instances, the GA obtained the optimal solution in 27 of them. In addition, the optimality gap for instances where Cplex could find the optimal solution is 0.42 percentage on the average. By comparing *Best objval* and *Avg. objval* columns, we can see that in 23 instances out of 56, the value of these two columns are equal. This means that the GA found the same solution in all 5 replications. In other 23 cases the difference between these two values is insignificant, indicating the robustness of the algorithm.

Let us analyze how the problem size affects the runtime of Cplex and of the GA. The average of the runtime values over instances of the same size are seen from Table 4.2. As we go from smaller instances towards bigger ones, the runtime increases gradually. It can be observed that the changes in the average runtime of Cplex are much larger than those in the average runtime of the GA. Thus, contrary to Cplex, the GA performs in a fairly robust fashion.

4.3.1 Case study

As mentioned before, we also executed the GA on the data of the Kartal district of Istanbul explained in Section 3.2 We present the results for these six problems in Table 4.3. The column labels are the same as the ones in Tables 3.1 and 4.2. We observe that the runtime increases almost linearly with the number of scenarios. We also observe that the best objective value starts to converge as the number of scenarios reaches 2000. Therefore, for the remaining analyses we fixed the number of scenarios to 2000.

We show the solutions of Istanbul_50 and Istanbul_2000 instances on the Kartal

Table 4.2: Computational results of benchmark instances

Instance name	Cplex			GA			ODP(%)	GAP(%)
	Opt. Objval	runtime	LB	Best objval	Avg. objval	Avg. time		
cap61_50	16531.92	102.30	16531.92	16533.06	16533.06	55.76	0.01	0.01
cap61_100	17467.68	499.88	17467.68	17467.87	17467.87	112.43	0.00	0.00
cap61_200	16379.25	1782.18	16379.25	16379.27	16379.27	170.13	0.00	0.00
cap61_500	14922.02	3912.79	14922.02	14922.02	14922.02	515.76	0.00	0.00
cap63_50	19157.14	113.53	19157.14	19160.62	19160.62	55.71	0.02	0.02
cap63_100	16476.17	397.65	16476.17	16477.25	16477.25	97.04	0.01	0.01
cap63_200	16271.46	1996.16	16271.46	16272.83	16273.74	196.43	0.01	0.01
cap63_500	18426.67	4867.42	18426.67	18431.65	18431.65	423.81	0.03	0.03
cap74_50	16632.20	128.44	16632.20	16635.10	16635.10	59.19	0.02	0.02
cap74_100	18511.62	423.16	18511.62	18512.90	18512.90	123.62	0.01	0.01
cap74_200	15400.32	1245.10	15400.32	15401.09	15401.09	177.79	0.01	0.01
cap74_500	15591.65	3694.83	15591.65	15592.11	15592.11	396.55	0.00	0.00
cap83_50	17754.41	172.48	17754.41	17755.31	17755.31	111.88	0.01	0.01
cap83_100	16271.02	509.00	16271.02	16271.34	16271.34	195.03	0.00	0.00
cap83_200	14054.35	1669.56	14054.35	14054.43	14055.24	316.03	0.00	0.00
cap83_500	13917.38	**7314.22**	13531.13	13888.25	13888.25	944.73	-0.21	2.64
cap92_50	15805.19	115.07	15805.19	15806.03	15806.03	152.31	0.01	0.01
cap92_100	14590.47	425.44	14590.47	14591.90	14591.90	216.98	0.01	0.01
cap92_200	15108.19	1663.68	15108.19	15108.72	15108.72	369.66	0.00	0.00
cap92_500	13320.44	6712.17	13320.44	13661.09	13663.81	823.43	2.56	2.56
cap101_50	14984.40	111.90	14984.40	14984.51	14988.31	104.73	0.00	0.00
cap101_100	15221.47	505.99	15221.47	15221.52	15222.41	172.27	0.00	0.00
cap101_200	14527.47	1062.70	14527.47	14527.51	14527.51	293.16	0.00	0.00
cap101_500	13412.97	5956.88	13412.97	13777.95	13777.95	696.12	2.72	2.72
cap112_50	216.97	323.14	216.97	228.31	235.04	428.46	5.23	5.23
cap112_100	202.37	2518.78	202.37	202.37	207.67	620.99	0.00	0.00
cap112_200	172.19	**7275.09**	144.70	172.19	173.96	3479.98	0.00	19.00
cap112_500	-	**7200.00**	-	104.22	105.24	2447.80	-	-
cap114_50	209.77	310.47	209.77	222.06	224.41	403.37	5.86	5.86
cap114_100	197.70	2249.10	197.70	199.08	201.00	631.58	0.70	0.70
cap114_200	137.70	**7278.26**	121.30	137.70	138.28	1060.87	0.00	13.52
cap114_500	-	**7200.00**	-	121.74	122.84	5261.95	-	-
cap123_50	216.97	341.99	216.97	216.97	218.97	400.01	0.00	0.00
cap123_100	177.74	1636.07	177.74	177.80	181.89	668.38	0.04	0.04
cap123_200	148.82	6350.66	148.82	148.82	151.31	1222.15	0.00	0.00
cap123_500	-	**7200.00**	-	115.07	115.07	2410.27	-	-
cap124_50	207.06	253.54	207.06	208.45	217.52	390.11	0.67	0.67
cap124_100	184.01	1786.61	184.01	184.01	184.66	682.19	0.00	0.00
cap124_200	141.89	5813.65	141.89	141.89	149.30	1044.83	0.00	0.00
cap124_500	-	**7200.00**	-	108.14	108.14	2937.39	-	-
cap131_50	201.01	262.90	201.01	202.87	206.38	413.08	0.92	0.92
cap131_100	174.64	2018.68	174.64	174.64	174.64	649.20	0.00	0.00
cap131_200	144.17	6841.22	144.17	144.17	146.79	1089.66	0.00	0.00
cap131_500	1675.20	**7382.61**	-	109.13	109.76	2592.21	-93.49	-
cap132_50	219.64	284.70	219.64	219.64	224.90	412.00	0.00	0.00
cap132_100	188.19	2158.22	188.19	188.19	188.19	722.54	0.00	0.00
cap132_200	139.89	5238.09	139.89	139.89	142.51	1118.06	0.00	0.00
cap132_500	1662.07	**7395.70**	-	104.78	106.54	2246.33	-93.70	-
cap133_50	249.10	317.60	249.10	249.10	249.10	371.40	0.00	0.00
cap133_100	197.34	1900.03	197.34	197.53	198.83	632.65	0.10	0.10
cap133_205	136.31	5558.90	136.31	136.31	137.87	1039.95	0.00	0.00
cap133_500	1615.03	**7201.36**	-	107.58	107.95	2695.12	-93.34	-
cap134_50	225.98	302.87	225.98	225.98	234.70	378.46	0.00	0.00
cap134_100	191.78	1428.87	191.78	191.78	191.78	646.91	0.00	0.00
cap134_200	143.44	5591.36	143.44	143.44	144.16	1070.47	0.00	0.00
cap134_500	1634.33	**7393.58**	-	108.19	108.53	2341.24	-93.38	-

Table 4.3: Results of the Kartal case

Instance	# PN	# CL	# S	Best obj	Avg. obj	Avg. time
Istanbul_50	20	25	50	1661.54	1662.73	29.16
Istanbul_100	20	25	100	1924.25	1925.01	94.42
Istanbul_200	20	25	200	1365.15	1365.15	122.90
Istanbul_500	20	25	500	1380.99	1380.99	179.90
Istanbul_1000	20	25	1000	1403.97	1403.98	334.34
Istanbul_2000	20	25	2000	1410.87	1410.87	600.05

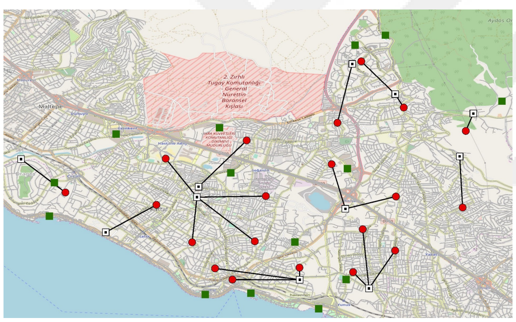

Figure 4.1: Solution of Istanbul_50 instance

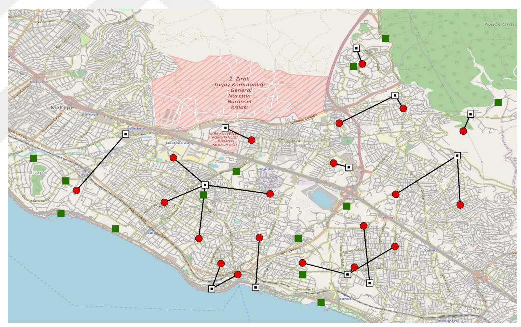

Figure 4.2: Solution of Istanbul_2000 instance

map to see the differences entailed by the number of scenarios. Figures 4.1 and 4.2 demonstrate the locations where we decide to open facilities and how we allocate the population nodes to those open facilities. Open facilities are identified with white squares and allocations are shown by black lines between PNs and assigned CLs. We see that the open facility locations change significantly between the two solutions.

4.3.2 Comparison between ex ante *and* ex post *measures*

As explained in Section 4.1.1, *ex ante* and *ex post* inequity measurements are two different approaches towards inequity quantification under uncertainty. We suggest to use a combination of these two concepts, but it may be instructive to see their differences if applied in pure form. Thus, in this subsection, we compare the solutions

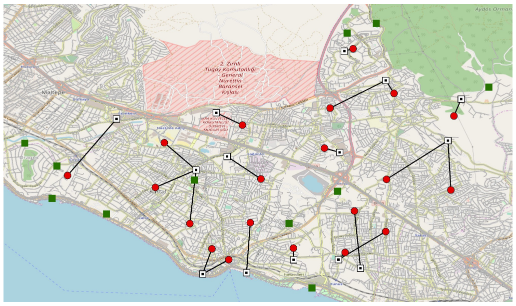

Figure 4.3: *Ex ante* problem solution

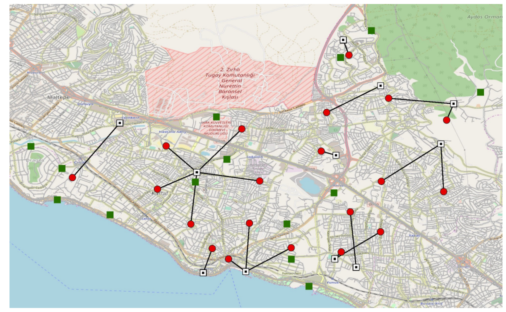

Figure 4.4: *Ex post* problem solution

Table 4.4: Evaluation of the solutions of the problem versions

Problem	*Ex ante*	*Ex post*	Combined
ATDS	1135.16	1147.35	1137.61
GMAD	610.24	580.64	599.85

found by solving the *ex ante* and the *ex post* versions of the problem, defined in Section 4.1.1, separately from each other. This is achieved by setting the parameter γ in (4.21) to the values 1 and 0, respectively. We select the instance Istanbul_2000 as our test case. The solutions of the two problem versions are presented in Figures 4.3 and 4.4. The solution to the original problem where $\gamma = 0.5$ is given in Figure 4.2.

Table 4.4 presents the values of the two objective function components to compare the solutions of the *ex ante*, the *ex post*, and the combined problem. The second line of Table 4.4 shows the mean distance measure ADTS, calculated by the combined approach. Similarly, the third line shows Gini's mean absolute difference GMAD, again computed by the combined approach. There is no solution dominating another one. We cannot conclude that one solution is superior to the other; as discussed in Section 4.1, the choice depends on whether one rather strives for "equal chances" or for "chances of equity". However, it can be observed that eight facility locations are common in the solutions.

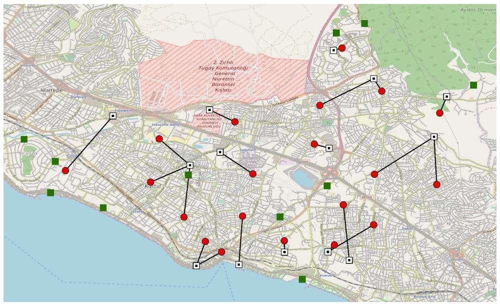

Figure 4.5: Optimizing the combined ADTS for Istanbul_2000

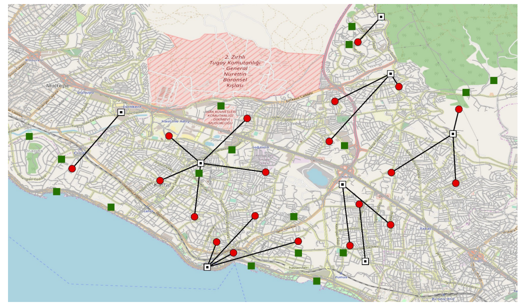

Figure 4.6: Optimizing the combined GMAD for Istanbul_2000

4.3.3 Tradeoff between efficiency and equity

A major contribution of this work is the introduction of a Gini-related component in the objective function while modeling a stochastic shelter location problem. Therefore, it is worthy to see how this component affects the fairness of the best solution provided by the GA. We first solve the Istanbul_2000 instance without the Gini part in the objective function by setting λ equal to zero, meaning that we neglect equity. Then, we determine the solution obtained for the other extreme case where the objective function is replaced by GMAD of distances (this corresponds to the limiting case for $\lambda \to \infty$). The two solutions are shown in Figures 4.5 and 4.6. We can see that the distances traveled are generally longer but more equitable in Figure 4.6, compared to Figure 4.5. It is rather clear that the solution in Figure 4.6 pays a too high price for equity. For example, it assigns the upmost PN a more remote shelter than necessary, which is certainly undesirable. So we are interested in compromise solutions obtained by taking weighted means with varying weights λ. The theory of symmetric comonotonically linear functionals guarantees that the above-mentioned undesirable behavior cannot occur as long as $\lambda \leq 0.5$.

4.3.4 Parameters sensitivity analysis

First, we employ a sensitivity analysis on the disaster severity coefficient β^1 which effects the PNs' demand in each scenario. Throughout our computational tests, we consider β^1 is equal to 0.2, 0.5 and 0.8 for low, moderate and high disaster severity, respectively. However, we test three other cases where β^1 is equal to [0.4, 0.7, 1], [0.3,0.6,0.9] and [0.1,0.4,0.7]. We call these instances S1, S2 and S4, respectively and name the regular one S3. We generate 500 scenarios for the Kartal case study and solve the problem with the proposed GA. Table 4.5 reports the objective value (Obj), ADTS, GMAD and Gini index of these instances. Note that column *Demand* presents the average-sum of all PNs' demand over the scenarios.

Table 4.5: Sensitivity analysis on the disaster severity

Instance	Demand	Obj	ADTS	GMAD	Gini
S1	72175.42	1358.35	1055.63	605.44	0.287
S2	62836.95	1330.29	1031.52	597.53	0.290
S3	52668.51	1409.68	1100.09	619.18	0.281
S4	43147.76	1249.03	965.33	567.40	0.294

As the parameter for disaster severity increases from S4 to S1, we can see the total demand is increasing. However, we do not observe a clear pattern in the objective value, average distance traveled and the Gini coefficient as the demand increases.

Until now we consider the service level and the budget as the same in all our computational tests. In the remainder of this section, we define 4 different cases based on the Istanbul_2000 instance and report various measures. Furthermore, to observe the behavior of the solutions under different values of λ, we solve the problem for each of the two extreme λ values ($\lambda = 0$ and $\lambda \to \infty$, respectively), as well as for the compromise approaches where λ is equal to 0.25, 0.5 and 0.75. For the indicated theoretical reasons, λ should be at most 0.5. However, it is interesting also to see how the solution changes when a higher weight is assigned to the equity measure.

The obtained values for ADTS, for GMAD, and for the Gini index (both in an *ex ante* and an *ex post* version) are presented in Tables 4.6, 4.7, 4.8 and 4.9. These tables present the results of four cases with varying service level and budget. In the

Table 4.6: Evaluation measures for Case I ($\alpha = 0.95$, $\xi = 0.7$) under different values of λ

Problem	ADTS	GMAD	EAGI	EPGI	MaxED	MinED	RangeED
$\lambda = 0$	1135.15	610.24	0.2107	0.2760	2269.23	659.42	1609.81
$\lambda = 0.25$	1135.87	605.36	0.2111	0.2734	2312.65	659.42	1653.23
$\lambda = 0.5$	1137.61	599.85	0.2106	0.2702	2312.65	755.62	1557.03
$\lambda = 0.75$	1148.42	582.28	0.2065	0.2593	2312.65	931.91	1380.74
$\lambda \to \infty$	1191.97	565.06	0.2094	0.2404	2696.41	931.91	1764.50

original setting we set the budget multiplier ξ equal to 0.7 and service level α equal to 0.95 which defines the Case I. Other cases values for ξ and α are introduced in the tables' captions.

Since the columns in the four tables show the same measures, we just explain those in Table 4.6. The first column indicates the weight λ. The second column reports the combined measure for the mean distance that an individual has to traverse in order to reach the assigned shelter (ADTS), cf. end of Section 3. The third column shows Gini's mean absolute difference of distances (GMAD) for each problem, again computed by the combined approach. The two next columns, EAGI and EPGI, present the Gini index itself, calculated in a pure *ex ante* and a pure *ex post* consideration, respectively. The columns MaxED and MinED indicate the maximum and the minimum expected distance of a population node from the assigned shelter. The last column shows the range of these expected distances, i.e., the difference between MaxED and MinED.

As the weight of the GMAD increases in the objective function, the general trend is an improvement in GMAD values and increase in ADTS values. However, in Table 4.9 problem with $\lambda = 0.5$ (also in Table 4.7 problem with $\lambda \to \infty$) behaves on the contrary in terms of GMAD. This might be attributed to having non-optimal solutions.

Figure 4.7 illustrates how the Gini index – we take here the combined version –

Table 4.7: Evaluation measures for Case II ($\alpha = 0.95$, $\xi = 0.3$) under different values of λ

Problem	ADTS	GMAD	EAGI	EPGI	MaxED	MinED	RangeED
$\lambda = 0$	1188.69	583.10	0.2030	0.2505	2460.87	659.42	1801.45
$\lambda = 0.25$	1189.48	581.65	0.2024	0.2497	2460.87	755.62	1705.25
$\lambda = 0.5$	1192.59	574.38	0.2128	0.2443	2696.41	755.62	1940.79
$\lambda = 0.75$	1205.12	563.06	0.2061	0.2370	2696.41	931.91	1764.5
$\lambda \to \infty$	1212.72	572.96	0.2126	0.2391	2696.41	931.91	1764.5

Table 4.8: Evaluation measures for Case III ($\alpha = 0.98$, $\xi = 0.7$) under different values of λ

Problem	ADTS	GMAD	EAGI	EPGI	MaxED	MinED	RangeED
$\lambda = 0$	1135.15	610.24	0.2107	0.2760	2269.23	659.42	1609.81
$\lambda = 0.25$	1141.15	605.16	0.2131	0.2716	2312.65	659.42	1653.23
$\lambda = 0.5$	1140.63	592.10	0.2102	0.2657	2312.65	755.62	1557.03
$\lambda = 0.75$	1151.44	573.43	0.2060	0.2543	2312.65	931.91	1380.74
$\lambda \to \infty$	1191.97	565.06	0.2094	0.2404	2696.41	931.91	1764.5

Table 4.9: Evaluation measures for Case IV ($\alpha = 0.98$, $\xi = 0.3$) under different values of λ

Objective	ADTS	GMAD	EAGI	EPGI	MaxED	MinED	RangeED
$\lambda = 0$	1217.44	598.42	0.2078	0.2505	3295.25	659.42	2635.83
$\lambda = 0.25$	1218.74	591.39	0.2072	0.2470	3295.25	755.62	2539.63
$\lambda = 0.5$	1221.34	589.86	0.2160	0.2446	3295.25	755.62	2539.63
$\lambda = 0.75$	1252.62	595.18	0.2180	0.2400	3185.71	931.91	2253.8
$\lambda \to \infty$	1307.67	580.61	0.1677	0.2287	3186.29	931.91	2254.38

and the mean distance change with the "weight of equity" λ. It can be seen that by increasing λ from 0 to 0.25, the Gini index is decreased, but the mean distance stays at almost the same level. Also if we further increase λ from 0.25 to 0.5, the mean distance increases only marginally, compared to a much stronger decrease in the Gini index. Therefore, selecting λ equal to 0.5, which is still in line with theoretical recommendations, may be a reasonable choice: it already provides a perceptible reduction of the Gini index (and of the range between maximal and minimal distances) at negligible costs in terms of the average distance. When judging the magnitude of the Gini index reduction, one should be aware that for location decisions, a comparably high "base level" of inequity is unavoidable, since wherever a facility is established, it obviously favours the inhabitants in its immediate neighborhood.

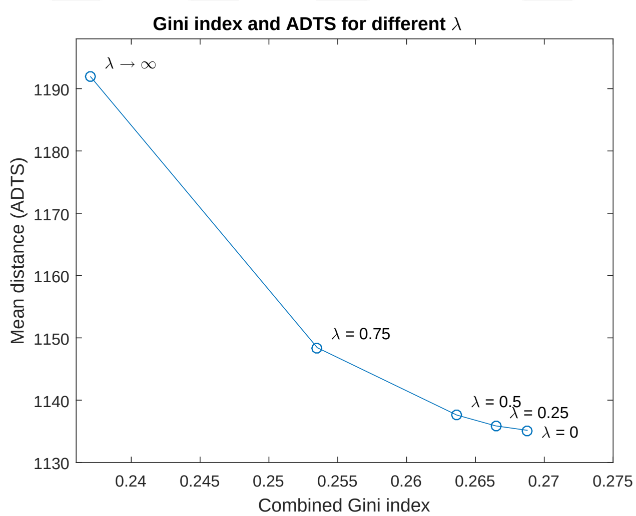

Figure 4.7: Mean distance and Gini index in dependence of λ for Istanbul_2000

4.3.5 Bi-objective analysis

We also carried out a (rough) *bi-objective* analysis by determining an approximation to the Pareto frontier of the problem with objectives "(expected) combined Gini index" (f_{CGI}) and "(expected) mean distance" (f_{ADTS}). At first we minimized each of these two objectives separately. Let us call the minimum value found for mean distance

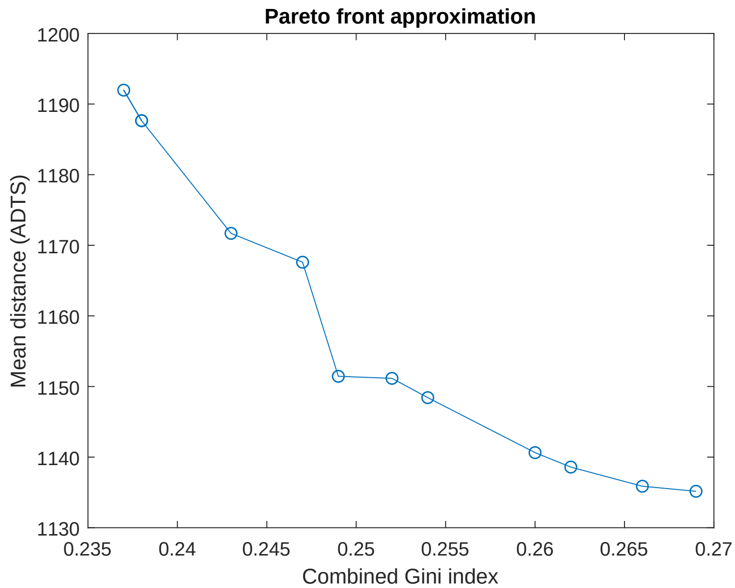

Figure 4.8: Approximation of the full range of the Pareto frontier for Istanbul case.

and combined Gini index, f_{ADTS}^{min} and f_{CGI}^{min}, respectively. Then we construct a new objective by taking a linear combination of normalization of these two objectives.

$$f = \omega \times \frac{f_{CGI}}{f_{CGI}^{min}} + (1-\omega) \times \frac{f_{ADTS}}{f_{ADTS}^{min}}$$

In order to explore the full range of the Pareto frontier, the single-objective problems were solved by our GA for the 21 different values $0, 0.05, 0.10, ..., 0.95, 1$ of the weight ω. The following results refer to the Istanbul_2000 case as described before. Dominated solutions with respect to the two indicated objective functions were removed. Figure 4.8 shows the Pareto frontier approximation. Note that in the bi-objective analysis, the second term of the weighted sum is the (normalized) Gini index and not Gini's mean absolute difference, as in the results on which Figure 4.7 is based. Obviously, also so-called unsupported solutions (Pareto-optimal solutions that are not optimizers of the single-objective weighted-sum problem for some weight ω) have a nonzero chance of being found in this way, since the GA solves the problem only heuristically.

4.4 Conclusion

In this chapter we introduced a stochastic shelter location and demand allocation problem for natural disaster preparation that aims to maximize an objective combining efficiency and equity. To the best of our knowledge, this work is the first that includes a measure derived from the Gini index within a stochastic optimization framework to account for fairness. We represented uncertainty in demand and transportation network damage by means of a set of scenarios and suggested to obtain the scenarios by generating the network damage randomly with respect to link-level correlation. We considered uncertainty in the objective by a combination of *ex ante* and *ex post* inequity concepts from the literature on decision theory.

First we formulated the problem as a MIP model and showed that it cannot be solved efficiently by a commercial MIP solver on benchmark instances. We tested problem with sizes that would arise in real applications. We next developed a GA for our problem and introduced specialized enhancements in the algorithm by utilizing a relaxed allocation model in order to improve the performance. We used the benchmark instances to compare the exact solutions with the GA solutions. Our results showed that the GA outperforms Cplex in terms of runtime in almost all instances. Furthermore, the GA obtains either the optimal solutions or solutions that are very close to optimal ones, with 0.42 percentage optimality gap on the average over different problem sizes.

We also executed the GA on real data from the Kartal district of Istanbul provided by (Kılcı et al. 2015) expanded with additional parameters. The GA solved this instance with 20 PNs, 25 CLs, and 2000 scenarios in around 10 minutes in our computational platform. We analyzed the tradeoff between efficiency (mean distance) and equity (Gini's mean absolute difference of distances), and we also compared the solutions obtained from pure *ex ante* and *ex post* approaches.

A future research direction concerns the replacement of the allocation subproblem with a representation of the behavior of disaster victims when seeking for shelter. In addition, different alternative approaches for incorporating inequity aversion into optimization might also deserve investigation. Moreover, our model assumes a risk-neutral decision maker. An interesting topic of future research is the inclusion of measures for risk averseness, which is especially appropriate for disaster relief applications. A suitable way to do that would be to add penalties for the Conditional-Value-at-Risk (CVaR), or to impose constraints on the CVaR. For an example of the application of CVaR penalties or CVaR constraints in humanitarian logistics, we refer the reader to Elçi and Noyan (2018) and Noyan et al. (2017), respectively. We carried out preliminary experiments using CVaR constraints on total excess demand on the one hand and total capacity expansion on the other hand. Although in these experiments, the additional constraints did not change the properties of the solutions to a large extent, the topic deserves a more comprehensive investigation in the future. In particular, efficient algorithms (both exact and heuristic) should be developed for this extension.

Distributionally-robust optimization is becoming popular as the information required is in between robust optimization and stochastic programming. This approach considers an "ambiguity set" of all of the possible distributions of the input data and optimize the objective hedging against the worst-case distribution (Delage and Ye 2010). This approach has very promising applications in disaster management problems where the distribution of the input data can be guessed. Our problem also provides the opportunity to expand based on this approach, especially by the ambiguous chance constraint concept introduced by Erdoğan and Iyengar (2006).

Chapter 5

INEQUITY-AVERSE RELIEF ITEM ROUTING IN DISASTER RESPONSE

This chapter considers the problem of planning vehicle routes between a depot and shelter locations and assigning relief supply delivery amounts to the shelters simultaneously. As the problem concerns the disaster response phase, we assume that the demand d_i for relief supply is known at each shelter i since the number of victims in each shelter will be known. We are concerned with a single commodity or a package combining several commodities. We consider the situation that the prepositioned supply C in the depot is less than the total demand in shelters. A fleet of homogeneous vehicles with size of m, each with a capacity of Q is based at the depot. Each shelter has to be visited exactly once. No vehicle visits a set of shelters whose total delivery amount exceeds Q. We assume that the travel times between the depot and the shelters and among the shelters are known. Routes start from the depot and collectively visit all the shelters on the post-disaster network. Each vehicle is assigned to one route. Two objectives are defined for the problem. We minimize the total travelling time of the vehicles, which is the summation of all routes travelling time. As an inequity measure, we minimize a measure defined based on the Gini's absolute differences of the unsatisfied demand rate among the shelters which addresses both equity and efficacy. This problem is *NP*-hard as it is a natural generalization of the TSP. In the next section, we introduce the mathematical formulations for the problem.

5.1 Problem definition and mathematical modeling

Table 5.1: Parameters

Parameter	Definition
Q	Vehicles capacity
d_i	Demand for relief supplies at shelter node i
t_{ij}	Time it takes to travel between nodes i and j
C	Total amount of available relief supply
Ψ	Maximum delivery time (time when the last supply is delivered)

The problem can be defined on a directed graph $G = (V, A)$. The node set $V = \{0, ..., n, n+1\}$ contains the depot that is represented by both node 0 and node $n+1$ and n shelters. All shelters require a delivery of relief supply, which are shown by subset $V_c = V \backslash \{0, n+1\}$. The arcs set A is equal to $\{(i, j)|\forall i, j \in V,\ i \neq n+1,\ j \neq 0\}$ with travel times $t_{i,j}$ for each arc which are calculated according to the shortest path between i and j in the post disaster road network. The travel time symmetry is not ensured which means $t_{i,j}$ is not necessarily equal to $t_{j,i}$. For $i = 1, ..., n$, shelter i has a known integer demand d_i with $0 < d_i \leq Q$. The amount of prepositioned relief supply at the depot is shown by C. There are m available vehicles at the depot. Table 5.1 shows the notation for the input parameters of the model. In this section, we present the formulations of the problem based on the mathematical model for the capacitated vehicle routing problem (CVRP).

The model presented here is adapted from the two-index formulation for CVRP. This formulation is easy to understand, implement and adapt to extra assumptions. We refer to this formulation as the *2I* model later in this work. The decision variables are as defined in Table 5.2.

$$\min \sum_{(i,j)\in A} t_{i,j} x_{i,j} \tag{5.1}$$

$$\min D \cdot \mathcal{I} \tag{5.2}$$

Table 5.2: Decision Variables

Notation	Description	Variable type
$x_{i,j}$	1, if the vehicle directly travels from node i to node j	Binary
v_i	The delivery amount of relief supply to node i	Non-negative
w_i	The amount of vehicle load when reaching node i	Non-negative
u_i	The time that the vehicle visits node i	Non-negative

s.t.

$$\sum_{j=1,j\neq i}^{n+1} x_{i,j} = 1, \quad \forall i \in V_c, \tag{5.3}$$

$$\sum_{j=0}^{n} x_{j,i} = \sum_{j=1}^{n+1} x_{i,j}, \qquad i \in V_c \tag{5.4}$$

$$\sum_{j=1}^{n} x_{0,j} = m, \tag{5.5}$$

$$\sum_{i=1}^{n} x_{i,n+1} = m, \tag{5.6}$$

$$v_i \leq d_i, \qquad \forall i \in V_c \tag{5.7}$$

$$\sum_{i=1}^{n} v_i \leq C, \qquad \forall i \in V_c \tag{5.8}$$

$$u_j \geq u_i + t_{i,j} + (1 - x_{i,j})M_1, \qquad \forall i, j \in A \tag{5.9}$$

$$u_j \leq \Psi, \qquad \forall j \in V_c \tag{5.10}$$

$$w_j \geq w_i + v_j + (1 - x_{i,j})M_2, \qquad \forall i, j \in A \tag{5.11}$$

$$w_{n+1} \leq Q, \tag{5.12}$$

$$x_{i,j} \in \{0, 1\}, \quad \forall (i, j) \in A,$$

$$v_i, u_i, w_i \geq 0 \qquad \forall i \in V_c \tag{5.13}$$

The first objective function minimizes the total travelling time of all vehicles. The second objective function is an inequity measure which will be described in detail in

Section 5.1.1. Constraint (5.3) enforces each shelter location to be visited only once. Constraint (5.4) keeps the balance between incoming and outgoing flow to each node. Each vehicle assigned to one route therefore in total we have m route. A route must start from the depot (node 0) and end at the depot (node n+1), this is ensured by Constraints (5.5) and (5.6). Delivery amounts to each shelter should be less than the shelter's demand, which is implied by Constraint (5.7). The total delivered amount of shelters is restricted by total available supply C in the Constraint (5.8). Constraint (5.9) calculates the arrival time to shelter locations and act as subtour elimination constraint as well. M_1 is a big number equal to the total demand. Constraint (5.10) limits the length of a route by Ψ. Note that we do not consider the time of the returning trip to the depot in route travelling time calculation therefore, we do not restrict u_{n+1} by Ψ. Variable w_i and constraint (5.11) are defined to show the amount of vehicle load when visiting node i, where M_2 is equal to $N \times \max\limits_{i,j \in A} (t_{i,j})$. Constraint (5.12) makes sure that the delivery amounts to shelters in one route respect the vehicle capacity. We only need to bound w_{n+1} by Q, since it shows the maximum vehicle loads among all vehicles according to constraint (5.11).

5.1.1 Equity objective function

Unsatisfied demand is an inevitable part of our problem resulted from the insufficient supply assumption. Deciding on the amount of relief supply delivered to shelter locations plays a crucial role to achieve an equitable humanitarian aid distribution. Therefore, We dedicate the first objective function of the problem to minimizing an equity measure on unsatisfied demand rate. We choose Gini index to build our inequity measure. Minimizing the Gini index alone will lead to an equitable solution, but most probably very inefficient solution. We use inequity-averse aggregation function ($\mathcal{I}$) rather than an inequity measure alone as it is described in Gutjahr and Fischer (2018) and Mostajabdaveh et al. (2019). We define $\mathcal{I} = \mu + \lambda \cdot \Delta$ where μ is the utilitarian measure $\mu = \frac{1}{N}\sum\limits_{i=1}^{N} c^{(k)}$ and Δ is Gini's mean absolute difference $\Delta = \frac{1}{N^2}\sum_{k=1}^{N}\sum_{l=1}^{N}|c^{(k)} - c^{(l)}|$. Therein c^k denotes the cost of individual k. For $0 \leq \lambda \leq 1/2$, the measure above satisfies the Axioms of an equitable aggregation function, in particular, it is strictly monotonous (cf. (Karsu and Morton 2015)). In our study, we assume that the demand of each beneficiary is one unit, and define the unsatisfied demand for all individual k in shelter i as $c^{(k)} = 1 - v_i/d_i$. Note that this is always non-negative since the delivery amount to shelter i v_i is less than the demand d_i. We calculate the μ and Δ specific to our problem as such.

$$\mu = \frac{1}{D}\sum_i (d_i - v_i) = 1 - \frac{1}{D}\sum_i v_i$$

$$\Delta = \frac{1}{D^2}\sum_i \sum_j |d_i v_j - d_j v_i|,$$

where D is the total demand and is equal to $\sum_i d_i$. We multiply $\mathcal{I}$ by D and adopt the resulting expression as the equity objective function in our problem.

$$D \cdot \mathcal{I} = \sum_{i \in V_c}(d_i - v_i) + \frac{\lambda}{D}\sum_{i \in V_c}\sum_{j \in V_c}|d_i v_j - d_j v_i| \tag{5.14}$$

The non-linear term of the objective function can be linearize by a similar approach we used the end of Section 4.1. Let us define variables $\iota^+_{i,j} = max\{0, \iota_{i,j}\}$ and $\iota^-_{i,j} = \{0, -\iota_{i,j}\}$ where

$$\iota_{i,j} = d_i v_j - d_j v_i, \quad \forall i, j \in V_c.$$

Using these variables, the second term of the objective function (5.14) can be rewritten as

$$\sum_{i \in V_c}(d_i - v_i) + \frac{\lambda}{D}\sum_{i \in V_c}\sum_{j \in V_c}\iota^+_{i,j} + \iota^-_{i,j},$$

providing the following constraints are added to the model:

$$\begin{aligned} \iota^+_{i,j} - \iota^-_{i,j} = d_i v_j - d_j v_i, \quad & \forall i, j \in V_c, \\ \iota^+_{i,j}, \iota^-_{i,j} \geq 0 \quad & \forall i, j \in V_c. \end{aligned} \tag{5.15}$$

5.2 *Exact solution method*

Generally, the models introduced in Section 5.1 can be solved by a MIP solver. We used Gurobi 8.1 for this purpose. However, it turned out that this solution approach cannot solve the medium and large instances which normally occur in practice. Therefore, we present an exact solution method in this section to solve medium and large sized instances in a reasonable time.

First, we explain the transformation of our bi-objective model to a single objective problem by the ϵ constraint method in Section 5.2.1. Section 5.2.2 presents a Set Partitioned (SP) formulation for the problem. In Section 5.2.3 we develop a column generation approach which solves the linear relaxation of the SP formulation and provides a lower bound on the problem. Section 5.2.4 introduce the initial heuristic approach which generates some feasible solutions to initiate the column generation. Finally, in Section 5.2.5 we present our Branch and Price algorithm to solve the problem to the optimality.

5.2.1 Multi-objective approach

To solve our two-objective problem we utilize the ϵ-constraint method which transforms the problem to a single objective optimization problem with an additional constraint. The objective with higher priority is considered as the main objective function and the other objective will be bounded by ϵ. Let us assume $f_1(x)$ and $f_2(x)$ are the two objectives to minimize and F be the feasible space of the problem.

$$\begin{aligned} &\min f_1(x) \\ &\quad f_2(x) \leq \epsilon \\ &\quad x \in F \end{aligned} \tag{5.16}$$

The most common approach to calculate the range of ϵ is to individually optimizing each objective function. Lets assume f_1^{min} and f_2^{min} are the optimal values of solving the problem with individual objective functions $f_1(x)$ and $f_2(x)$ where X_1^* and X_2^* are the corresponding optimal solutions. We define $f_2^{max} = f_2(X_1^*)$, the value of $f_2(x)$ in the optimal solution of the $f_1(x)$.

The drawback of the conventional ϵ-constraint method is the lack of efficiency in its Pareto solutions. An inefficient solution is a solution that one of objective functions can be improved without degenerating the other objective functions. There is no guarantee for the solutions of the conventional ϵ-constraint method to be efficient. Mavrotas (2009) designed augmented ϵ-constraint method to overcome this drawback. First, the inequality constraint (5.17) is transformed into an equality by introducing slack variable $s \geq 0$. Then, the main objective function is augmented by adding the normalized slack variable. The augmented ϵ-constraint method transforms the problem to

$$\begin{aligned} &\min f_1(x) + \gamma \times \frac{s}{\theta} \\ &\quad f_2(x) + s = \epsilon \\ &\quad x \in F \end{aligned} \tag{5.17}$$

where γ is a coefficient between $[10^{-6}, 10^{-3}]$, and θ is the range of $f_2(x)$ values which is calculated as $\theta = f_2^{max} - f_2^{min}$.

To adopt this approach we choose the inequity measure as the main objective function and convert the maximum tour length minimization into a ϵ constraint. By increasing ϵ value from f_2^{min} to f_2^{max} we can find the Pareto-front. However, in the remaining of the section, we will focus on solving the problem with a fix ϵ.

5.2.2 Set partitioning formulation

A *route-delivery* is defined by a sequence of nodes visited by a vehicle as well as their delivery quantities which take shelter demands and vehicle capacity into account. In addition, the length of every route-delivery tour should respect the maximum tour length impose by Ψ. Let R be the set of all possible route-deliveries starting from the depot. For a given route-delivery $r \in R$, binary variable y_r is 1 if and only if route-delivery r belongs to the optimal solution, q_i^r is the delivery amount to node i, indicator $a_{i,r}$ takes 1 if node i is visited within route-delivery r. The total tour length of route-delivery r is shown by t^r. We define N_r to be the set of nodes that are visited by route-delivery r. Now we can present the SP formulation of the problem.

$$\min \sum_{i \in V_c}(d_i - \sum_{r \in R} q_i^r y_r) + \frac{\lambda}{D}\sum_{i \in V_c}\sum_{j \in V_c} \iota_{ij}^+ + \iota_{ij}^- + \gamma \frac{\epsilon - \sum_{r \in R} t^r y_r}{\theta} \tag{5.18}$$

s.t.

$$\iota_{ij}^+ - \iota_{ij}^- = (d_i \times \sum_{r \in R} q_j^r y_r) - (d_j \times \sum_{r \in R} q_i^r y_r) \qquad \forall i, j \in V_c \tag{5.19}$$

$$\sum_{r \in R} t^r y_r \leq \epsilon \tag{5.20}$$

$$\sum_{r \in R} a_{ir} y_r = 1, \quad \forall i \in V_c, \tag{5.21}$$

$$\sum_{r \in R} y_r \leq m \tag{5.22}$$

$$\sum_{r \in R}\sum_{i \in N_r} q_i^r y_r \leq C \tag{5.23}$$

$$\iota_{ij}^+,\, \iota_{ij}^- \geq 0, \quad \forall i, j \in V_c \tag{5.24}$$

$$y_r \in \{0, 1\}, \quad \forall r \in R, \tag{5.25}$$

The objective function (5.18) is composed of two parts, the first part is the equity measure of the delivery amount and the second part is related to the total travelling time objective that is adopted from the augmented ϵ-constraint method. Constraint (5.19) is designed to linearize the absolute value term in the objective function. Total travelling time of vehicles is restricted by ϵ in the constraint (5.20). Each node should be visited only once, which is insured by Equation (5.21). Constraints (5.22) and (5.23) limit the number of vehicles and relief item supply, respectively.

In Section (5.2.3) we describe the column generation approach to solve the linear relaxation of this formulation and then in Section (5.2.5) we introduce the branch and price algorithm which solves the presented model to the optimality.

5.2.3 Column generation

In order to solve the integer relaxation of the formulation presented in the previous section, we adopt a Column Generation (CG) approach. Column generation is an iterative procedure that instead of solving the problem with a complete set of columns (e.g. route-deliveries) starts with a small set of columns and creates columns dynamically until the optimality condition meet. At each iteration, CG solves a master problem that optimizes the objective function under a restricted set of route-deliveries $\bar{R}$. The master problem is referred to as Restricted Master Problem or RMP. RMP solution provides dual variables to be transferred to the objective function of a subproblem. The subproblem (pricing problem) generates new route-delivery, which will be added to $\bar{R}$. Master problem is re-optimized according to updated set $\bar{R}$. The process continues until subproblem cannot find a route-delivery that improves the RMP objective value.

The RMP corresponds to the integer relaxation of the SP formulation introduced in Section 5.2.2. Thus RMP is defined by (5.18)-(5.24), where set R is replaced by $\bar{R}$. Note that the upper bound imposed by the integer relaxation of variable y_r, $0 \leq y_r \leq 1$, is redundant, since (5.21) impose all y_r variables to be one at most.

To start CG we need at least one feasible solution for the problem. To this end, we devise a problem-specific heuristic to construct a set of route-deliveries that constitute a feasible solution. Once the RMP is solved by a solver with the initial set of columns, its dual solution is used to define the objective function of the subproblem. The subproblem aims to find a route with the most negative reduced cost for variable y_r. Later, this route is added to RMP as a new column. We again solve the RMP to start the next iteration. The algorithm continues until subproblem cannot find a route-delivery with negative reduced cost. The last solution of RMP will be the optimal solution. We first introduce the subproblem in Section 5.2.3. Next, in Section 5.2.4, we describe the constructive heuristic that is used to generate the initial solution for RMP.

Subproblem

The subproblem decides on a single vehicle route that starts from the depot and collectively visits some nodes considering maximum tour length. Subproblem also determines the amount of deliveries to the visited nodes by respecting the vehicle capacity. The objective function of the subproblem minimizes the reduced cost of y_r variables. Therefore, we calculate the dual of RMP to obtain the reduced cost. Let us define the dual variables , $\pi^1_{i,j}$, π^2, π^3_i, π^4 and π^5 correspond to constraints (5.19), (5.20), (5.21), (5.22) and (5.23), respectively. The dual of RMP is as follows:

$$\max \epsilon\pi^2 + \sum_i \pi^3_i + m\pi^4 + C\pi^5$$

s.t.

$$\sum_i \sum_j (d_j q^r_i - d_i q^r_j)\pi^1_{i,j} + t^r\pi^2 + \sum_{i \in N_r} a_{ir}\pi^3_i$$
$$+ \pi^4 + \sum_{i\in N_r} \pi^5 q^r_i \leq - \sum_{i \in N_r} q^r_i - \frac{\gamma}{\theta} t^r \qquad \forall r \in \bar{R} \tag{5.26}$$
$$\frac{-\lambda}{D} \leq \pi^1_{ij} \leq \frac{\lambda}{D} \tag{5.27}$$
$$\pi^2, \pi^4, \pi^5 \geq 0 \tag{5.28}$$

The reduced cost for variable y_r can be calculated from constraint (5.26) which is

$$c_r = - \sum_{i\in V_c} \sum_{j \in V_c} (d_j q^r_i - d_i q^r_j)\pi^1_{i,j} - t^r\pi^2 - \sum_{i\in V_c} a_{ir}\pi^3_i - \pi^4 - \sum_{i \in V} (\pi^5 + 1) q^r_i - \frac{\gamma}{\theta} t^r.$$

Now we define the variables needed to formulate the subproblem. Binary variable $x_{i,j}$ takes one if and only if edge $i-j$ is used by the vehicle. q_i is the amount of delivery to the shelter location i. a_i is the binary variable that shows if demand point i is visited by the vehicle or not. Parameter notations are the same as the ones introduced in Section 5.1. The mathematical formulation of the subproblem is presented below.

$$\min \; - \sum_{i\in V_c} \sum_{j\in V_c} \bar{\pi}^1_{i,j}(d_j q_i - d_i q_j) - \sum_{(i,j)\in E} (\bar{\pi}^2 + \frac{\gamma}{\theta}) t_{i,j} x_{i,j} - \sum_{i \in V_c} \bar{\pi}^3_i a_i - \bar{\pi}^4 - \sum_{i\in V_c} (\bar{\pi}^5 + 1) q_i \tag{5.29}$$

s.t.

$$\sum_{(i,j)\in E} t_{i,j} x_{i,j} \leq \Psi \tag{5.30}$$

$$\sum_{i \in V_c} x_{0,i} = 1 \tag{5.31}$$

$$\sum_{i \in V_c} x_{i,n+1} = 1 \tag{5.32}$$

$$\sum_{j \in V_c} x_{i,j} - \sum_{j \in V_c} x_{j,i} = 0, \qquad \forall i \in V_c \tag{5.33}$$

$$\sum_{j \in V_c} x_{i,j} = a_i \qquad \forall i \in V_c \tag{5.34}$$

$$\sum_{(i,j), i \in S j \in S} x_{i,j} \leq \sum_{i \in S, i \neq k} a_i, \qquad \forall S \subset V_c, \forall k \in S \tag{5.35}$$

$$q_i \leq d_i a_i, \qquad \forall i \in V_c \tag{5.36}$$

$$\sum_{i \in V_c} q_i \leq Q \tag{5.37}$$

$$x_{i,j} \in \{0,1\}, \quad \forall i, j \in V$$

$$a_i \in \{0,1\}, \quad \forall i \in V$$

$$q_i \geq 0, \quad \forall i \in V_c \tag{5.38}$$

Objective function (5.29) is the reduced cost of a route-delivery (column) in RMP. Note that $\bar{\pi}^1_{i,j}$, $\bar{\pi}^2$, $\bar{\pi}^3_i$, $\bar{\pi}^4$ and $\bar{\pi}^5$ are constant values. Constraint (5.30) ensures that the route length does not exceed the specified maximum tour length. Constraints (5.32) and (5.31) make sure that the tour starts at the depot and end at the depot. The balance between incoming and outgoing flow for each shelter is kept by Equality (5.33). Constraint (5.34) specifies the relation between a_i and $x_{i,j}$ variables such that if node i is not visited in the route then all variables $x_{i,j}$, $\forall j \in V$ should be zero. Subtour elimination is enforced by inequality (5.35) which is adopted from Feillet et al. (2005) for TSP with profit. The delivery amount of node i should be limited by its demand d_i which imposed by constraint (5.36). Constraint (5.37) restricts the total delivery amounts in the route by the vehicle capacity.

To solve the subproblem, we use Gurobi 8.0.1 solver implemented in Python 2.7.10. Our computational test shows that the solver is able to tackle the model for fairly large instances in a reasonable amount of time.

5.2.4 Constructive heuristic

The *Initiation Heuristic* (IH) algorithm aims to provide a number of feasible solutions for RMP. At first, we fix the delivery amounts, then we employ a Sweep algorithm to construct m routes that visit all of the shelters with respect to the vehicle capacity. Sweep heuristic is introduced by (Gillett and Miller 1974) for CVRP. Since these routes might not yield a feasible solution according to maximum tour length Ψ and total traveling time ϵ, we employ a Tabu Search heuristic to find a feasible solution for RMP. The Algorithm 4 demonstrate the overall structure of the IH.

The Sweep algorithm sets the depot location at the center of a polar system and calculates the polar-coordinate angle for each shelter location. The algorithm starts with a given starting angle and increases this value until it hits the first node. The first cluster is built containing only this node. The current angle increased again until a radius from the depot hit the next shelter location. We check if by adding the current node to the current cluster the total delivery amount of the cluster exceeds the vehicle capacity Q. If it is possible, we add the current node to the cluster, if not we create the next cluster. The maximum number of clusters should not exceed m. The algorithm continues until all nodes assigned to a cluster. If it is not possible to maintain the total delivery amount for all clusters less than Q with m clusters, we ignore the vehicle capacity limit for the last cluster. The Algorithm 5 describe the steps of the Sweep algorithm in details. Note that $v_{(1)}$ is the first element in shelters set V_c.

Sweep specifies the shelter that should be visited by one vehicle, however, it would not determine the sequence of their visits. The vehicle tour length for one cluster should be less than Ψ and the sum of all routes traveling time should be less than ϵ. Our best chance to meet these conditions is to find a sequence of nodes that minimize the route tour length. Therefore, we formulate a Travelling Salesman Problem (TSP) for each cluster and solve it to find the optimal solution. Assume that the set of shelter in one cluster is defined as N_c which also includes the depot node 0 as the starting point of the tour. The binary variable $x_{i,j}, \forall i, j \in N_c, i \neq j$ indicates if the

Algorithm 4 Initiation Heuristic

Input: G, Ψ, depot location, number of initial solution ρ

Output: A set of routes-deliveries $\bar{R}$.

Fix the delivery amount q'_i for each node i such as $q'_i = \frac{\min\{C, mQ\}}{\sum_i d_i} \times d_i$.

$k, k' \leftarrow 0$

while $k < \rho$ **do**

  Randomly select a starting angle between 0 and 360.

  Generate m clusters with **Sweep algorithm** and the given starting angle.

  Optimize the node sequence for each cluster by solving related **ITSP** model.

  Assign $q_i^r = q'_i \; \forall \, i \in V_c$ and create m new route-deliveries.

  Create the set of generated route-deliveries RD.

  if $t^r \leq \Psi, \forall r \in RD$ and $\sum_{r \in RD} t^r \leq \epsilon$ **then**

    Append RD to $\bar{R}$.

    $k = k + 1$

  else

    Use the **Tabu search algorithm** to make infeasible route-deliveries feasible and create RD'.

    if $t^r \leq \Psi, \forall r \in RD'$ and $\sum_{r \in RD'} t^r \leq \epsilon$ **then**

      Append RD' to $\bar{R}$.

      $k = k + 1$

    else

      $k' = k' + 1$

    end if

  end if

  if $k' >= 2$ **then**

    $q'_i = 0.85 \times q'_i, \; \forall i \in V_c$

    $k' = 0$

  end if

end while

Return $\bar{R}$

Algorithm 5 Sweep Algorithm

Input: V_c, q'_i $\forall i \in V_c$, Q, m and the starting angle α.

Output: Clusters set.

Calculate the polar coordinate angle for every node i in V_c.

Shift nodes' angle by α.

Sort V_c according to the new angles value.

$k \leftarrow 1$

Create the empty cluster C_k and define $R_k = Q$ as the remaining capacity of the cluster k.

while V_c is not $\emptyset$ **do**

  Remove node $v_{(1)}$ from V_c.

  if $q'_{v_{(1)}} \leq R_k$ **then**

    Add $v_{(1)}$ to C_k.

    Update the remaining capacity of the current cluster $R_k = R_k - q'_{v_{(1)}}$

  else if $k = m$ **then**

    Add $v_{(1)}$ to C_k.

  else

    $k = k + 1$

    Add $v_{(1)}$ to C_k.

    $R_k = Q - q'_{v_{(1)}}$.

  end if

  Return $\{C_1, C_2, ..., C_m\}$

end while

vehicle goes directly from node i to j in optimal solution or not. We formulate the mathematical model of this TSP in the following and refer to it as ITSP later on.

ITSP mathematical model

$$\min \sum_{i\in N_c} \sum_{j\in N_c, j\neq 0} t_{i,j}x_{i,j} \tag{5.39}$$

s.t.

$$\sum_{i\in N_c} x_{0,i} = 1 \tag{5.40}$$

$$\sum_{j\in N_c} x_{j,i} = 1, \qquad \forall i \in N_c \tag{5.41}$$

$$\sum_{j\in N_c} x_{i,j} = 1 \qquad \forall i \in N_c \tag{5.42}$$

$$\sum_{i,j\in S} x_{i,j} \leq |S| - 1, \qquad \forall S \subset N_c \setminus \{0\}, \tag{5.43}$$

$$x_{i,j} \in \{0, 1\}, \quad \forall i, j \in V$$

The objective function and constraints are the common ones for every TSP problem. We implement this mathematical model by python 2.7.10 and solve it with IP Gurobi 8.0.1 solver. To decrease the execution time of the IP solver, we add the subtour elimination constraints (5.43) dynamically when they are needed using the lazy callback option of Gurobi.

Tabu search algorithm We devise a Tabu search heuristic based on TABUROUTE algorithm introduced in Gendreau et al. (1994) to generate a feasible solution for the problem given an infeasible solution. Tabu search is an iterative algorithm which starts with a given solution s and tries to find a better solution by moving to a neighborhood solution in each iteration. To implement the algorithm we first need to define a move that can create a new solution based on the current solution. We define this move as selecting one node to remove from its original route and to reinsert it into another route. This move can explore a large neighborhood of solutions. In every iteration, we explore several candidate moves and accept a move that can make the current solution feasible or improve its objective value. The accepted move is

Table 5.3: Tabu search parameters

Parameter	Description	Value
η	Number of nodes that are selected for reinsertion in each iteration	$\lceil \frac{n}{5} \rceil$
$[\omega_{min}, \omega_{max}]$	Bounds on the number of iterations for which a move is considered tabu	$[2, 6]$
κ_{max}	Maximum number consecutive iteration with no improvement in objective value	20
g	scaling factor used in infeasible solution objective value	0.1
p_1	Number of nearest nodes to node v	$\lceil \frac{n}{3} \rceil$

considered tabu for a certain number of iterations, which means the algorithm will avoid repeating it in these iterations. The proposed algorithm is described in Algorithm 6. First, we define the notation used in the algorithm's pseudocode. Table (5.3) demonstrates the algorithm parameters description, their notation, and assigned values.

To understand the pseudocode in Algorithm 6, we need to define its components first. We refer to the solution that has attained the best objective value at every point in time during the Tabu search algorithm execution as BS. Similarly BFS is the solution with the best objective value among feasible solutions. The primal aim of the Tabu search algorithm is to find a set of solutions that are feasible due to the maximum tour length and maximum total travelling time. To facilitate achieving this goal, we evaluate the solutions by their total travelling time. A solution s consist of at most m route-deliveries. Let r be a route-delivery in set RD_s which presents all route-deliveries belongs to solution s. The objective value for solution s is calculated as the following.

$$F(s) = \sum_{r \in RD_s} t^r + \nu \left(\sum_{r} \max(0, \sum_{i \in V_c} q_i^r - Q) \right)$$

$$+\xi\left(\sum_{r\in RD_s}\max(0,t^r-\Psi)+\max(0,\sum_{r\in RD_s}t^r-\epsilon)\right),$$

where ν and ξ are the coefficient of the violated vehicle capacity and the traveling time. According to our primarily tests we set both of these parameters equal to 1000. We define CS to indicate the current solution and CA as the set of candidate solutions for CS of the next iteration. CA is populated with potential solutions such as PS that is obtained by a single move (v_{ps}, r_{ps}) from the current solution. Move (v_{PS}, r_{PS}) presents removing node v_{PS} from its original route in CS and inserting it to the best position in route r_{PS}. To obtain the best position for the inserted node we solve the ITPS model by Gurobi 8.0.1. This is also the case for the route that we remove v_{PS} from. $Z(v)$ is the number of times that node v is reinserted during the algorithm execution.

5.2.5 Branch and Price

The column generation solves the linear relaxation of the SP formulation, however, we are interested to find an integer feasible solution of the model. We propose a branch-and-price (B&P) algorithm which is, a branch-and-bound algorithm that in every node the lower bound is computed by solving the linear relaxation of the SP formulation with CG. If CG provides a fractional solution, a branch is made by dividing the feasible region of integer solutions in a way that the two resulting subspaces exclude the fractional solution. Then again each of these new problems will be solved by CG. Under three conditions we do not need to explore a node further, (I) if the CG's solution is integer which will provide an upper bound on the problem,(II) the related problem is infeasible, (III) The lower bound provided by CG is greater than or equal to the best upper bound. The algorithm continues to branch until the lower bound and the upper bound converge to the same number.

The most frequent branching rule used in the literature of vehicle routing problems is branching on arcs variable in the compact formulation. However, this approach is not efficient enough to obtain integer solutions rapidly Dayarian et al. (2015).

Algorithm 6 Tabu search algorithm

Input: Algorithm parameters, initial solution s, Q, Ψ, ϵ.
Output: One or more feasible solution.
$BS \leftarrow s$
$\kappa \leftarrow 0$
$Z(v) = 0, \forall v \in V_c$
while $\kappa \leq \kappa_{max}$ **do**
Create set S which contains η nodes randomly selected from V_c.
for v in S **do**
Find p_1 nearest nodes to node v
Find the set of routes RI that does not contain v but atleast contain one of its p_1 nearest nodes.
for route r in RI **do**
Insert node v in the best position in route r obtained by solving ITSP and build the potential solution PS.
Calculate $F(PS)$.
if move (v,r) is in tabu list **then**
if PS is feasible and F(PS) ¡ F(CS) or infeasible and F(PS) ¡ F(BS) **then**
Add PS to CA set.
end if
else
$F(PS) = F(PS) + m^2 g \times Z(v_{ps})$
Add PS to CA set.
end if
end for
end for
Find the solution with minimum objective value in CA and call it PS'.
if $F(PS') \leq F(CS)$ **then**
$CS \leftarrow PS'$
if CS is feasible and $F(CS) < F(BFS)$ **then**
$BFS \leftarrow CS$
end if
if CS is not feasible and $F(CS) < F(BS)$ **then**
$BS \leftarrow CS$
end if
Randomly generate h between ω_{min} and ω_{max}.
Keep move $(v_{PS'}, r_{PS'})$ in tabu list for h iterations.
$Z(v_{PS'}) = Z(v_{PS'}) + 1$
end if
end while

Therefore, here we choose to branch on the edges instead of arcs as suggested by Desaulniers et al. (2015). One of the advantages of this branching scheme is that integrating the new constraint is rather simple. We calculate $x_{i,j}$ values in the optimal solution of the RMP such that

$$x_{i,j} = \sum_{r} y_r b^r_{i,j}$$

where $b^r_{i,j}$ indicates if route-delivery r use arc (i, j) or not. Selection of branching variable can affect $B\&P$ conversion speed. Some researchers suggest using an evaluation procedure to select the branching variable. However, given the size of possibilities for branching using an evaluation technique before branching can increase the run time of the algorithm drastically. Therefore, we use a simple heuristic rule by branching on the variable that has the closest value to 0.5. Our computational result shows the effectiveness of this approach.

Branching creates two new nodes; one with the implicit constraints $x_{i,j} = 0$ and $x_{j,i} = 0$ which we call left node and one with implicit constraint $x_{i,j} = 1$ or $x_{j,i} = 1$ which we refer to as the right node. In the left child node we eliminate the columns that their corresponding route traverse arc (i, j) or (j, i) from $\bar{R}$. Both (i, j) and (j, i) are removed from graph G and suproblem is updated accordingly.

In the right node where at least one vehicle should traverse arc (i, j) or (j, i), we eliminate all route-delivers from $\bar{R}$ that visit node i (node j) but do not directly go to the node j (node i). To avoid generating routes that do not visit i and j consecutively during CG, we add the following constraints to the subproblem. Let define $\bar{E}$ as the set of edges should be visited.

$$x_{0,j} = a_j, \quad \forall i, j \in \bar{E},\, i = 0, \tag{5.44}$$

$$x_{i,j} + x_{j,i} = a_i, \quad \forall i, j \in \bar{E},\, i, j \neq 0, \tag{5.45}$$

$$a_i = a_j \quad \forall i, j \in \bar{E},\, i, j \neq 0. \tag{5.46}$$

As mentioned before branching eliminates some columns from $\bar{R}$ and makes it quite possible that the resulting set cannot provide a feasible solution to the RMP.

Therefore, we run the IH to find some feasible solutions prior to solving RMP at each node of the B&P tree. We regard a node in $B\&P$ tree as infeasible when its corresponding graph becomes disconnected as a result of omitting some edges, or the subproblem cannot find a feasible route-delivery. Infeasibility also arises when edges in $\bar{E}$ make a cycle that does not contain the depot.

Acceleration routine

To accelerate the branch-and-price algorithm, we develop a method that helps to update the upper bound more frequently. At each node of the $B\&P$ tree when the CG provides a lower bound with a specific column set R^{CG}, we build and solve a MIP model that provides an integer feasible solution to the problem. The model will choose a subset of route-deliveries from R^{CG}. It ignores the delivery amounts associated with route-deliveries and instead decides on the amount of deliveries to each shelter point with respects to the vehicle capacity and total available supply. We define variable $\varphi_{i,r}$ as the delivery amount to node i in route-delivery r, which is equal to $q_i y_r$. The model is presented below.

$$\min \sum_{i=1}^{n}(d_i - \sum_{r\in R^{CG}} \varphi_{i,r}) + \frac{\lambda}{D}\sum_{i=1}^{n}\sum_{j=1}^{n} \iota_{ij}^{+} + \iota_{ij}^{-} + \frac{\gamma}{\theta}\left(\epsilon - \sum_{r\in R^{CG}} t^r y_r\right) \tag{5.47}$$

s.t.

(5.20) - (5.22), (5.24), (5.25)

$$\iota_{ij}^{+} - \iota_{ij}^{-} = (d_i \times \sum_{r\in R^{CG}} \varphi_{i,r}) - (d_j \times \sum_{r\in R^{CG}} \varphi_{i,r}) \qquad \forall i,j \in V_c \tag{5.48}$$

$$\varphi_{i,r} \leq q_i, \qquad \forall i \in N_r, \forall r \in R^{CG} \tag{5.49}$$

$$\varphi_{i,r} \leq d_i y_r, \qquad \forall i \in N_r, \forall r \in R^{CG} \tag{5.50}$$

$$\varphi_{i,r} \geq q_i - d_i(1 - y_r), \qquad \forall i \in N_r, \forall r \in R^{CG} \tag{5.51}$$

$$\sum_{i\in N_r} \varphi_{i,r} \leq Q, \qquad \forall r \in R^{CG} \tag{5.52}$$

$$q_i \leq d_i, \qquad \forall i \in V_c \tag{5.53}$$

$$\sum_{i\in V_c} q_i \leq C \tag{5.54}$$

$$\varphi_{i,r} \geq 0 \quad \forall i \in V_c, \forall r \in R^{CG}$$
$$q_i \geq 0, \quad \forall i \in V_c \tag{5.55}$$

The objective function and the constraint (5.48) are exactly defined to serve the same purpose as the ones in set partitioning formulation in Section 5.2.2. Inequalities (5.49)-(5.49) are establishing a linear relation between variable $\varphi_{i,r}$ and two variables q_i and y_r. Constraint (5.52) ensures that the delivery amounts respect the vehicle capacity. Constraint (5.53) implies that supply delivery quantities should not exceed the demand of each shelter. Finally, Constraint (5.54) limits the total delivery amounts by the available supply at the depot. We solve this model with Gurobi 8.0.1 in python 2.7.10. This model can be solved in a reasonable time for the instance sizes we used in our computational results. The size of columns set $\bar{R}$ is relatively remain unchanged during the algorithm execution, which means the model size also remains the same. We use Gurobi *cutoff* option to stop the solver earlier when it determines that the problem's optimal value will be less than the cutoff parameter. In these cases, this model cannot provide a better upper bound than the current one. This option will terminate the run when it determines the optimal objective values will be worse than the specified value. The cutoff parameter value is fixed to the current upper bound.

5.3 Computational results

In this section, we use two data sets to examine the performance of the proposed exact solution method. First, both the B&P algorithm and the 2I model are utilized to solve 45 instances based on the real data of the Van province earthquake with 3 different sizes. We note that the data preparation process was described in Section 3.3. Second, we execute the B&P algorithm on 30 instances from the Karatl case study with different characteristics that are described in Section 3.2. We also report the results of solving the 2I formulation (refer to the subsection **??**) on the same instances.

We conduct the computational experiments by Python 2.7.10 on a workstation with two 28-core Intel Xeon E5-2690 v4 processors, running at 2.6 GHz with 35 MB cache, 128 GB RAM running under the Windows 10 operating system. All the mathematical models (e.g. the subproblem, ITSP, and 2I) are solved with the Gurobi 8.0.1 solver. In principle, our problem arises after a disaster when time plays a crucial role. Solution methods should provide a solution as fast as possible. Therefore, to have a reasonable run time, we limit the execution time of the B&P algorithm and the model. Model runtime is restricted by 2 hours for all instances, since the optimality gap does not improve significantly after 2 hours. On the other hand, we set the B&P algorithm runtime restriction according to the size of the problem, but always let it be less than or equal to 2 hours. We define the relative gap between the best-known solution (UB) and the best lower bound (LB) as

$$Gap = \frac{UB - LB}{UB} \times 100.$$

When solving the problem with both the 2I model and B&P, we set the minimum relative gap equal to 0.01 as another termination criterion. To write the RMP objective function, we need to find the θ value for each instance. From Section 5.2.1 recall that the θ value is calculated such that, we should solve the problem with the two objectives separately. This way we can calculate the maximum and the minimum total traveling time of the vehicles. However, in the current context that would be inapplicable due to long runtimes. Therefore, we resort to solving the linear relaxation of these problems by the CG approach. Table 5.4 gives the θ vales for the Van instances and Table 5.5 present these values for the Kartal instances.

5.3.1 Computational results on Van earthquake data

This section presents the computational results on 45 instances derived from the Van province data set. This data set is originally prepared and made available to us by Noyan et al. (2015) and adapted to our problem in Section 3.3. The instances are constructed in three categories with three different number of shelters, namely 15, 30 and 60. We also derive instances with tight (T), very tight (VT) vehicle capacity and restricted tour length (VTL) to see the performance of the proposed

Table 5.4: The θ values for the Van instances

Instance name	θ value	Instance name	θ value	Instance name	θ value
Van_15_1	16297.96	Van_30_1	44001.16	Van_60_1	112873.36
Van_15_2	19225.65	Van_30_2	48370.43	Van_60_2	115516.79
Van_15_3	16751.35	Van_30_3	48613.01	Van_60_3	115233.86
Van_15_4	15768.82	Van_30_4	47592.91	Van_60_4	117027.96
Van_15_5	14030.97	Van_30_5	50639.34	Van_60_5	116225.51
Van_15_6	18707.94	Van_30_6	48138.27	Van_60_6	95417.78
Van_15_7	16426.92	Van_30_7	54011.49	Van_60_7	132076.21
Van_15_8	18318.05	Van_30_8	40663.17	Van_60_8	118666.37
Van_15_9	18377.21	Van_30_9	38255.19	Van_60_9	97276.54
Van_15_10	15197.33	Van_30_10	55978.17	Van_60_10	137120.47
Van_15_11	12574.63	Van_30_11	37224.98	Van_60_11	112299.75
Van_15_12	11486.86	Van_30_12	32508.78	Van_60_12	93084.31
Van_15_13	13057.99	Van_30_13	39019.28	Van_60_13	125643.96
Van_15_14	11614.27	Van_30_14	32438.29	Van_60_14	111880.08
Van_15_15	11428.87	Van_30_15	38920.49	Van_60_15	121827.74

Table 5.5: The θ values for the Kartal instances

Instance name	θ value	Instance name	θ value	Instance name	θ value
Kartal_T_1	15917.71	Kartal_VT_1	15576.81	Kartal_VTL_1	10477.95
Kartal_T_2	16620.40	Kartal_VT_2	16470.81	Kartal_VTL_2	12934.80
Kartal_T_3	22297.95	Kartal_VT_3	15904.01	Kartal_VTL_3	15956.31
Kartal_T_4	21138.68	Kartal_VT_4	16292.41	Kartal_VTL_4	14358.75
Kartal_T_5	17732.94	Kartal_VT_5	18026.58	Kartal_VTL_5	9136.17
Kartal_T_6	17641.63	Kartal_VT_6	14795.14	Kartal_VTL_6	12105.22
Kartal_T_7	15482.51	Kartal_VT_7	18484.56	Kartal_VTL_7	9917.60
Kartal_T_8	17851.65	Kartal_VT_8	16004.99	Kartal_VTL_8	9920.64
Kartal_T_9	17590.31	Kartal_VT_9	18923.35	Kartal_VTL_9	13141.27
Kartal_T_10	21663.87	Kartal_VT_10	19111.35	Kartal_VTL_10	12917.26

method in extreme situations. We explain the parameter values to generate these instances later in this section. Tables 5.6, 5.7 and 5.8 report the computational results obtained by implementing both the B&P algorithm and solving the 2I model on these instances. The naming of the columns is the same for all these tables. The first column shows the instance name. The first number in the name of an instance indicates the number of shelters that should be visited by the vehicles. The second number demonstrates the instance identification number. Note that the number of available vehicles for instances with 15, 30 and 60 nodes are 3, 5 and 9, respectively. Column *Type* indicates the vehicle capacity status where T and VT refer to tight and very tight vehicle capacity, respectively. Instance type VTL indicates that both the vehicle capacity and the maximum tour length are very tight. To have a tight vehicle capacity, we set ζ^3 (see Section 3.3) equal to 0.5. However, for a very tight vehicle capacity ζ^3 is set to 0.2. In instances of type VTL, the maximum tour length is calculated by (3.1) with $\zeta^1 = 1$. Note that in other instance types ζ^1 is equal to 2. Next four columns in Table 5.6 present the results obtained by solving the 2I model with the MIP solver. The first two columns, *UB* and *LB*, report the best feasible

Table 5.6: Van earthquake case study results on the 15 node-network with 3 vehicles

Instance name	Type	Model				B&P				UB-Gap%	LB-Gap%
		UB	LB	Time	Gap%	UB	LB	Time	Gap%		
Van_15_1	T	142638.74	142631.29	1.66	0.01	142632.70	142632.70	19.243	0.00	0.00	0.00
Van_15_2	T	143815.65	143809.49	0.28	0.00	143810.56	143810.20	14.973	0.00	0.00	0.00
Van_15_3	T	139494.32	139487.35	0.27	0.00	139488.81	139488.71	18.565	0.00	0.00	0.00
Van_15_4	T	140654.74	140646.66	0.25	0.01	140648.11	140648.11	23.901	0.00	0.00	0.00
Van_15_5	T	140200.69	140192.37	0.24	0.01	140193.99	140193.99	26.007	0.00	0.00	0.00
Van_15_6	VT	157263.57	152716.96	7200.09	2.98	157293.51	157259.93	1809.057	0.02	-0.02	2.89
Van_15_7	VT	169210.63	165388.89	7200.09	2.31	169682.67	169206.40	1802.637	0.28	-0.28	2.26
Van_15_8	VT	155805.12	151188.59	7200.11	3.05	156273.73	155802.65	1800.114	0.30	-0.30	2.96
Van_15_9	VT	159072.60	155027.04	7200.78	2.61	159299.95	159070.10	1803.977	0.14	-0.14	2.54
Van_15_10	VT	163207.43	159160.88	7200.76	2.54	163742.29	163203.51	1801.161	0.33	-0.33	2.48
Van_15_11	VTL	152773.18	148468.38	7200.07	2.90	152863.29	152766.60	1804.354	0.06	-0.06	2.81
Van_15_12	VTL	168460.02	164469.59	7200.12	2.43	168508.41	168450.87	1805.941	0.03	-0.03	2.36
Van_15_13	VTL	159873.28	155736.21	7200.06	2.66	160063.76	159865.54	1801.799	0.12	-0.12	2.58
Van_15_14	VTL	162461.31	158327.78	7200.12	2.61	162528.01	162453.60	1805.648	0.05	-0.04	2.54
Van_15_15	VTL	160383.73	155922.33	7200.10	2.86	160624.27	160373.05	1807.716	0.16	-0.15	2.78
Average					1.80				0.10	-0.10	1.75

solution and the lower bound found until the termination. The Column *Time* shows the execution time of the model. The relative gap between the lower bound and the upper bound (optimality gap) as defined in the previous section is reported by the Column *Gap*. The next four columns under *B&P* exactly have the same names and definitions as the ones under *Model*. Finally, the two last column, *UB-Gap* and *LU-Gap*, represent the gap between upper bound (lower bound) reported by the Model and the B&P algorithm in percentage.

We use "-" in the columns UB and Gap, where the model can not provide any value for the upper bound within the 2 hours time limit. Two termination criteria are applied to the *B&P* algorithm: (i) maximum execution time of 1800, 3600 and 7200 seconds for instances with size 15, 30 and 60, respectively. (ii) minimum relative gap of 0.01%. The algorithm run time exceeds the defined limit for some instances, especially for the ones with 60 shelters. This is because we checked the elapsed time after completing the CG every two nodes. Solving such big instances is a very challenging task. The B&P algorithm found a very high-quality solution for the 60-node instances with an average gap of 0.96 %. On the other hand, the model only

Table 5.7: Van earthquake case study results on the 30 node-network with 5 vehicles

Instance name	Type	Model				B&P				UB-Gap%	LB-Gap%
		UB	LB	Time	Gap%	UB	LB	Time	Gap%		
Van_30_1	T	145082.99	145076.98	2.13	0.00	145079.25	145079.24	2085.37	0.00	0.00	0.00
Van_30_2	T	156988.08	156979.94	2.65	0.01	157179.18	156982.67	3602.19	0.13	-0.12	0.00
Van_30_3	T	155593.49	155592.01	7.58	0.00	156806.36	155592.41	3612.97	0.77	-0.77	0.00
Van_30_4	T	148049.88	148043.38	2.30	0.00	148045.14	148045.05	2016.34	0.00	0.00	0.00
Van_30_5	T	159615.61	159609.39	2.18	0.00	160374.52	159610.33	3632.67	0.48	-0.47	0.00
Van_30_6	VT	170444.35	166837.14	7201.09	2.16	172718.22	170431.67	3732.44	1.32	-1.32	2.11
Van_30_7	VT	166735.06	163136.96	7200.96	2.21	170684.30	166701.70	3772.04	2.33	-2.31	2.14
Van_30_8	VT	157852.62	153696.61	7201.25	2.70	160487.86	157771.55	5126.93	1.69	-1.64	2.58
Van_30_9	VT	165529.62	161425.11	7201.01	2.54	167775.77	165519.72	4789.15	1.34	-1.34	2.47
Van_30_10	VT	165752.51	161929.71	7200.85	2.36	167981.54	165671.24	3666.55	1.38	-1.33	2.26
Van_30_11	VTL	160276.36	156403.36	7200.43	2.48	162086.85	160249.05	3660.47	1.13	-1.12	2.40
Van_30_12	VTL	158186.68	154002.82	7200.95	2.72	158834.33	158177.51	3618.58	0.41	-0.41	2.64
Van_30_13	VTL	169682.38	166147.20	7201.35	2.13	170237.53	169664.16	3619.17	0.34	-0.33	2.07
Van_30_14	VTL	165362.97	161384.36	7200.95	2.47	165820.03	165332.90	3695.43	0.29	-0.28	2.39
Van_30_15	VTL	168168.59	164576.73	7201.15	2.18	168927.38	168058.47	3633.47	0.51	-0.45	2.07
Average					1.60				0.81	-0.79	1.54

Table 5.8: Van earthquake case study results on the 60 node-network with 9 vehicles

Instance name	Type	Model				B&P				UB-Gap%	LB-Gap%
		UB	LB	Time	Gap%	UB	LB	Time	Gap%		
Van_60_1	T	161777.18	161775.41	496.07	0.00	163114.32	161776.61	7334.60	0.82	-0.82	0.00
Van_60_2	T	163284.86	163215.99	7201.17	0.04	164373.52	163216.44	7394.76	0.70	-0.66	0.00
Van_60_3	T	153801.84	153800.90	780.00	0.00	155084.59	153801.40	7230.94	0.83	-0.83	0.00
Van_60_4	T	150207.41	150143.13	7201.58	0.04	151535.42	150143.61	7233.29	0.92	-0.88	0.00
Van_60_5	T	163182.09	163180.30	2779.49	0.00	165267.08	163181.82	7250.13	1.26	-1.26	0.00
Van_60_6	VT	166162.40	163186.39	7201.64	1.82	167447.60	166012.44	7485.71	0.86	-0.77	1.70
Van_60_7	VT	173435.75	170810.29	7202.32	1.54	174645.82	173356.20	8051.47	0.74	-0.69	1.47
Van_60_8	VT	171395.84	168632.25	7202.60	1.64	172300.76	171231.03	8217.65	0.62	-0.53	1.52
Van_60_9	VT	164363.24	161348.31	7202.31	1.87	165628.38	164188.92	7538.25	0.87	-0.76	1.73
Van_60_10	VT	169090.94	166109.32	7201.60	1.79	170670.75	168873.95	7355.45	1.05	-0.93	1.64
Van_60_11	VTL	171590.45	164652.04	7201.21	4.21	172768.72	171288.48	7530.53	0.86	-0.68	3.87
Van_60_12	VTL	176006.91	168925.13	7202.73	4.19	177432.82	175742.65	7642.61	0.95	-0.80	3.88
Van_60_13	VTL	173486.65	166606.66	7201.81	4.13	174187.92	173235.27	7618.23	0.55	-0.40	3.83
Van_60_14	VTL	164269.06	157480.16	7201.41	4.31	166405.79	163583.81	7711.60	1.70	-1.28	3.73
Van_60_15	VTL	161385.47	154271.83	7201.08	4.61	163760.60	161061.29	7231.69	1.65	-1.45	4.22
Average					2.01				0.96	-0.85	1.84

was able to obtain the average gap 2.01 %. For all of the instances, our proposed algorithm could either verify the optimality of a solution or find a very high-quality solution with an average gap of 0.62 % within the specified time limit. On the other hand, the average relative gap of 1.80 % reported by the model indicates the poorer performance of this solution method.

The negative vales in Column *UB-Gap* indicates that the model could find a better upper bound than the B&P algorithm. On average the model could provide a 0.1%, 0.79% and 0.85% better feasible solutions for instances with 15, 30 and 60 nodes, respectively. On the other hand, the B&P algorithm could find 1.75%, 1.54% and 1.84% better lower bounds on the average for 15, 30 and 60-nodes instances. Although it is possible that in some instance the model provide slightly better upper bounds, we cannot solely rely on this approach because of its lower bound poor quality. Fig. 5.1 demonstrates the reported gap by both the B&P algorithm and the model for instances of types VT and VTL. The horizontal axis shows instance names and the vertical axis indicates the optimality gap reported by the two solution methods. The blue bars display the model results and the red bars show the B&P algorithm optimality gap. From Figure 5.1 it can be seen that the optimality gap reported by the model is almost always higher than the one reported by the B&P algorithm. Especially for the instance type VTL where the model optimality gap has a meaningful increase. However, the optimality gap for the B&P algorithm has a very insignificant changes for these instances. It is remarkable that in spite of the shorter run time of the B&P, the relative gap is always much smaller than the one reported by the model, especially for large-sized resource-restricted instances. On average, the gap between the upper bound and the lower bound for the B&P algorithm is 2.9 times smaller than the model on the instances where the model could provide an upper bound.

In summary, our computational results on the Van data set suggest that the B&P algorithm is more successful than the 2I model solution approach. It can provide the optimal solution or a very high-quality solution in a reasonable time. Therefore, the proposed B&P algorithm is reliable to be utilized for decision making in post-disaster

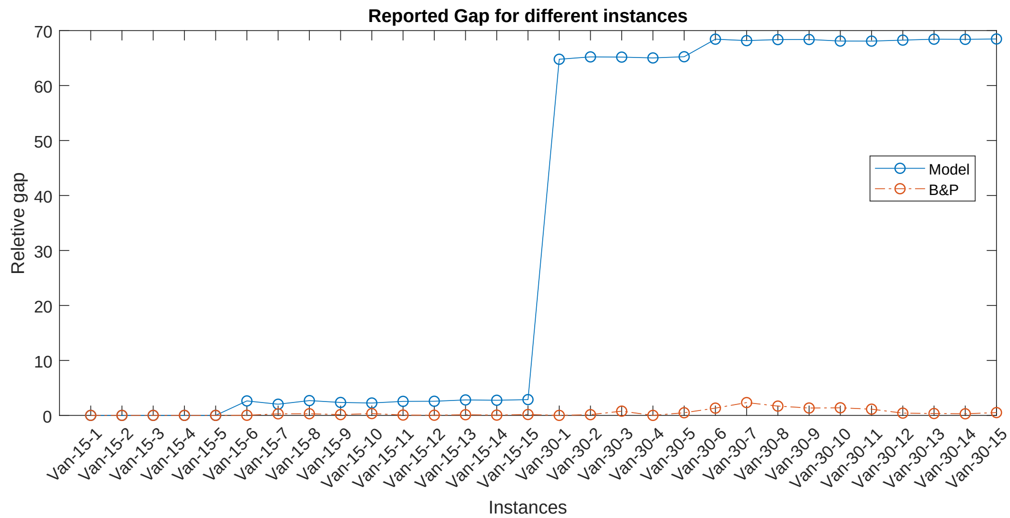

Figure 5.1: Optimality gap of the model and the B&P algorithm

situations when both time and solution quality are of the utmost importance.

5.3.2 Computational results on Kartal case study

In this section, we executed the B&P algorithm on the data of the Kartal district of Istanbul provided by Kılcı et al. (2015). We explained the instance preparation in Section 3.2. Table 5.9 reports the computational results provided by both B&P and 2I model on these instances. The first column indicates the name of the instances. The second part of the instance's name demonstrates the instance type which is differentiated by the vehicle capacity and maximum tour length, namely T, VT and VTL. The data generation parameters for each instance types are exactly the same as the ones described in Section 5.3.1. Columns UB, LB, Time, Gap, UB-Gap and LB-Gap are similar to the ones described for Table 5.6. The maximum execution time of the model and the B&P algorithm is set to 7200 and 1800 seconds, respectively.

In these instances the average optimality gap obtained by the B&P algorithm is 44 times smaller than the average gap reported by the 2I model. In spite the 4 times longer runtime of the model. We illustrate the relative gap reported by the model and the B&P algorithm in Fig. 5.2. Similar to Fig. 5.1, the blue bars shows the

Table 5.9: Kartal case study results with 13 nodes and 3 vehicles

Instance name	Model				B&P				UB-Gap%	LB-Gap%
	UB	LB	Time	Gap%	UB	LB	Time	Gap%		
Kartal_T_1	47263.16	28923.87	7200.13	38.80	47676.85	47218.14	1815.45	0.96	-0.87	38.74
Kartal_T_2	38755.28	33049.74	7200.22	14.72	38755.28	38743.50	1809.54	0.03	0.00	14.70
Kartal_T_3	24411.78	19794.44	7200.09	18.91	24418.24	24395.14	1815.43	0.09	-0.03	18.86
Kartal_T_4	16333.38	13151.06	7200.09	19.48	16347.33	16328.53	1806.58	0.12	-0.09	19.46
Kartal_T_5	18006.27	12459.87	7200.12	30.80	18147.25	17988.61	1803.07	0.87	-0.78	30.73
Kartal_T_6	39470.85	21575.86	7200.13	45.34	39768.61	39424.99	1800.87	0.86	-0.75	45.27
Kartal_T_7	43238.45	38832.33	7200.07	10.19	43241.28	43218.48	1801.09	0.05	-0.01	10.15
Kartal_T_8	35765.48	26690.86	7200.13	25.37	35977.10	35692.22	1802.74	0.79	-0.59	25.22
Kartal_T_9	46356.92	32503.77	7200.11	29.88	46372.43	46290.82	1812.86	0.18	-0.03	29.78
Kartal_T_10	42632.65	35555.06	7200.21	16.60	42665.05	42598.92	1802.68	0.15	-0.08	16.54
Kartal_VT_1	49932.64	44179.46	7200.17	11.52	50032.87	49875.94	1811.09	0.31	-0.20	11.42
Kartal_VT_2	58818.11	55067.25	7200.08	6.38	58818.11	58795.20	1802.08	0.04	0.00	6.34
Kartal_VT_3	54608.44	48597.29	7200.29	11.01	54608.44	54583.14	1803.34	0.05	0.00	10.97
Kartal_VT_4	43067.43	41425.55	7200.22	3.81	43068.35	43058.45	1802.20	0.02	0.00	3.79
Kartal_VT_5	34623.40	32455.17	7200.06	6.26	34768.27	34566.95	1808.05	0.58	-0.42	6.11
Kartal_VT_6	36282.64	36032.32	7200.06	0.69	36291.28	36240.94	1801.42	0.14	-0.02	0.58
Kartal_VT_7	41236.08	36078.52	7200.22	12.51	41267.51	41196.94	1802.58	0.17	-0.08	12.42
Kartal_VT_8	53961.38	51371.48	7200.13	4.80	53969.35	53931.81	1804.06	0.07	-0.01	4.75
Kartal_VT_9	37229.77	35395.95	7200.06	4.93	37244.21	37205.31	1800.19	0.10	-0.04	4.86
Kartal_VT_10	41891.54	37809.15	7200.19	9.75	41892.24	41872.22	1804.26	0.05	0.00	9.70
Kartal_VTL_1	41351.54	37843.04	7200.17	8.48	41450.94	41312.63	1815.37	0.33	-0.24	8.40
Kartal_VTL_2	55877.04	54883.33	7200.13	1.78	55981.68	55777.18	1810.41	0.37	-0.19	1.60
Kartal_VTL_3	55095.25	48208.71	7200.26	12.50	55308.73	55065.80	1807.23	0.44	-0.39	12.45
Kartal_VTL_4	38853.81	34446.95	7200.19	11.34	38933.67	38839.20	1800.71	0.24	-0.21	11.31
Kartal_VTL_5	27585.52	26935.03	7200.05	2.36	27637.76	27452.47	1814.13	0.67	-0.19	1.88
Kartal_VTL_6	28991.14	27678.14	7200.16	4.53	29047.65	28957.89	1810.10	0.31	-0.19	4.42
Kartal_VTL_7	46552.10	43759.39	7200.23	6.00	46568.65	46503.41	1800.47	0.14	-0.04	5.90
Kartal_VTL_8	53442.08	49016.82	7200.13	8.28	53467.11	53387.69	1802.20	0.15	-0.05	8.19
Kartal_VTL_9	44161.17	41935.45	7200.09	5.04	44229.58	44125.09	1815.56	0.24	-0.15	4.96
Kartal_VTL_10	34859.02	31594.93	7200.08	9.36	34963.44	34849.80	1809.70	0.33	-0.30	9.34
Average				13.05				0.30	-0.20	12.96

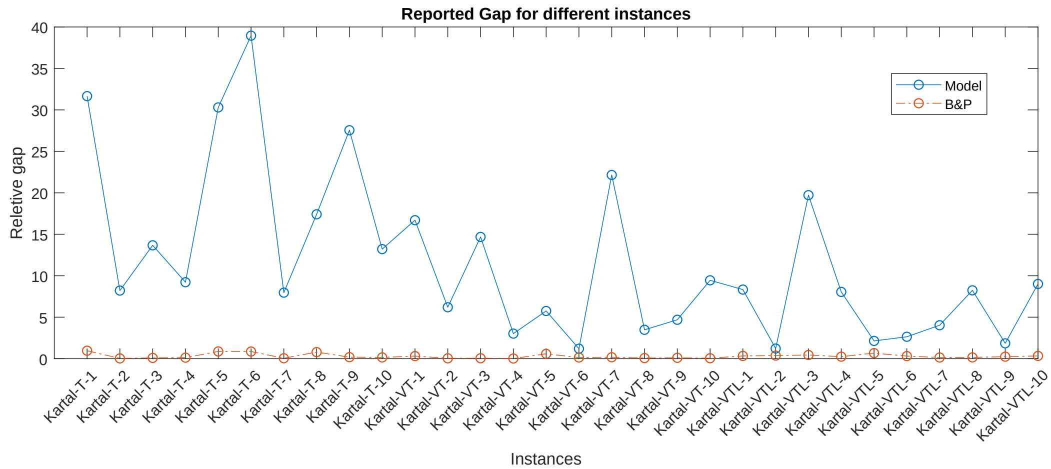

Figure 5.2: Optimality gap of the model and the B&P algorithm

gaps obtained from the model and the red bars shows the ones yielded from the B&P algorithm.

As can be seen in Fig. 5.2, the B&P algorithm can finds solutions with very small gaps for almost all of Kartal instances. However, the model's gap acts unpredictably due to the vehicle capacity and maximum tour length tightness. Again this computational results show the superiority of the B&P algorithm over the 2I model solution method regarding the solution quality and the execution time.

5.4 Conclusion

In this chapter, we studied a problem of distributing relief supplies to a set of shelters for a depot as part of humanitarian relief logistics in disaster response. As the problem concerns the disaster response phase, we assume that the demand for relief supply is known at each shelter since the number of individuals in each shelter will be known. We are concerned with a single commodity or a package combining several commodities. We consider the situation that the prepositioned supply in the depot is less than the total demand in shelters in the immediate response to the disaster. In our setting, a fleet of homogeneous vehicles with specific size each with the same capacity is based at the depot. Our goal is to minimize both the total traveling time of the vehicle and an inequity measure regarding the unsatisfied demand. As the inequity measure, we introduced an inequity-averse aggregation function based on the Gini's absolute differences of unsatisfied demand rate among the shelters.

We developed two new mathematical formulations for the problem. However, these mathematical models cannot solve the problem efficiently. The augmented ϵ-constraint method is adapted to handle these bi-objective models. We show that the selected model can only obtain the optimal solution for small and less challenging instances. In addition, a Set Partitioning (SP) formulation is also developed for the problem. We utilized a column generation approach to solve the linear relaxation of the SP model to provide a tight lower bound on the problem. A branch-and-price algorithm is also devised to solve the problem with sizes that would arise in real situations. We compare our exact solution approach with the model on the Van province earthquake in Turkey and Kartal district of Istanbul real data sets, amounting to a total of 75 instances. Our computational results on the Van earthquake data showed that the B&P algorithm can provide a relative gap equal to or better than the model in all of the instances. Moreover, in medium and large sized instances, the algorithm can find very good bounds in spite of the fact that the model fails to obtain an optimality gap less than 2% within 2 hours. We executed the B&P algorithm along with the model again on real data that derived from Kartal district of Istanbul. This data was genearted using the results of Mostajabdaveh et al. (2019) on the stochastic shelter location in disaster preparedness on the same data set. Our computational results on the Kartal data again verify the better performance of the B&P algorithm.

The proposed B&P algorithm can be improved in a few ways. Solving the subproblem by a heuristic approach can accelerate the column generation which boosts the computational power of the algorithm. It is possible to adopt our proposed constructive heuristic in Section 5.2.4 to generate columns with negative reduced cost. However, this change can also prolong CG convergence and requires further investigation. Moreover, the impact of different branching scheme on algorithm performance can be the subject of another study.

The work presented in this chapter can be extended to the cases with multiple depots and a heterogeneous fleet of vehicles for supply distribution. In addition, different alternative approaches for incorporating inequity aversion into optimization might also deserve investigation (see Erbeyoğlu and Bilge (2019)).

Chapter 6

CONCLUSION

In this thesis, we studied two problems related to optimization of disaster logistics operations. We provided efficient solution methodologies specific to the two problems under consideration.

As our first problem, which is related to disaster preparedness, we introduced a stochastic shelter location and demand allocation problem for natural disaster preparation that aims to maximize an objective combining efficiency and equity. To the best of our knowledge, this study is the first that includes a measure derived from the Gini index within a stochastic optimization framework to account for fairness (Chapter 4). We represented uncertainty in demand and transportation network damage by means of a set of scenarios and suggested to obtain the scenarios by generating the network damage randomly with respect to link-level correlation. We considered uncertainty in the objective by a combination of *ex ante* and *ex post* inequity concepts from the literature on decision theory.

In the first study, we formulated the problem as a MIP model and showed that it cannot be solved efficiently by a commercial MIP solver on benchmark instances. We tested problem with sizes that would arise in real applications. We next developed a GA for our problem and introduced specialized enhancements in the algorithm by utilizing a relaxed allocation model in order to improve its performance. We used the benchmark instances to compare the exact solutions with the GA solutions. Our results showed that the GA outperforms Cplex in terms of runtime in almost all instances. Furthermore, the GA obtains either the optimal solutions or solutions that are very close to optimal ones, with 0.42 percentage optimality gap on the average over different problem sizes.

We also executed the GA on real data from the Kartal district of Istanbul provided by (Kılcı et al. 2015) expanded with additional parameters. The GA solved this instance with 20 PNs, 25 CLs, and 2000 scenarios in around 10 minutes in our computational platform. We analyzed the tradeoff between efficiency (mean distance) and equity (Gini's mean absolute difference of distances), and we also compared the solutions obtained from pure *ex ante* and *ex post* approaches.

In the second problem of the thesis, we focus on the fair distribution of relief supplies to the shelters as part of humanitarian relief logistics in disaster response. This problem is a bi-criteria multi-vehicle routing problem in which we decide on delivery amounts as well as vehicle routes. We devised a Gini index based measure of the unsatisfied demand (assuming that shortage will occur) as the main objective function. As the second objective, we minimize the total traveling time of all vehicles. Even the special cases of this problem where just a single objective is assumed are difficult to solve.

We develop a compact MIP formulations of the problem. However, the medium and large sized instances cannot be solved with the models. Therefore, a set partitioning formulation was developed for the problem and its linear relaxation was solved with a column generation procedure to provide a tight lower bound. Furthermore, we developed a branch-and-price algorithm to solve the problem into optimality in a reasonable runtime. This algorithm took advantage of problem-specific constructive heuristics and an acceleration routine. Finally, we tested our approach on two real-world data sets and compared its performance with the selected model. We used our results from the first study on the shelter establishment problem on the Kartal data to generate a case study for this problem. Moreover, we executed the B&P algorithm on the Van earthquake data and compared the results with the model regarding the runtime and solution quality. Both Kartal and Van earthquake data are obtained from the literature and adapted to our problem. The computational results shows the absolute superiority of the presented approach to the model. On average the optimality gap reported by the B&P algorithm is 44 times smaller than the one reported by the MIP model on Kartal data.

Disaster relief operations should provide efficient and equitable services to victim populations. This thesis offers decision support tools to the humanitarian aid organizations by considering both service equity and efficiency. The proposed decision-making framework and an analysis of the mathematical models and the solution methods can guide the decision-makers on disaster preparedness and disaster response phases. Especially, they provide a systematic and more effective approach to shelter location and relief aid distribution decisions and hence, contribute positively to the relief of disaster victims. Our first problem helps the decision makers to decide on the shelter location as a disaster preparedness plan. Our results promise fair and efficient access to the shelter for every individual in the population. By using the proposed method, aid organizations can inform the population of their assigned shelter location in advance of the disaster. This can help to mitigate the possible confusion for disaster victims in chaotic post-disaster situations.

The second study of the thesis focuses on the equitable distribution of the relief supplies among the demand nodes. This problem arises immediately in the aftermath of a disaster when the aid organizations should deliver the aid supplies to a population distributed over a geographical region. In the post-disaster context, a fast, efficient and equitable solution with enough level of detail is needed. Our solution to this problem not only offers a fast delivery, but also ensure an equitable distribution of the relief supply amount to the disaster victims. The solution approach is designed to deliver a high-quality solution as fast as possible. Solution quality and solver runtime are the two most important aspects of the decision making in the immediate disaster response phase.

On the methodological side, this thesis contributes to the literature by developing a powerful genetic algorithm for location-allocation problems and an innovative branch-and-price algorithm to solve routing-delivery problems. The Genetic algorithm can be utilized for any two-stage model, (e.g. Location-allocation models). The proposed column generation approach and the branch-and-price algorithm can be used directly to solve single-period inventory-routing problems. These methods are particularly designed to decide on routing and delivery quantity decisions simultaneously. However, with some adaptations, they can solve general inventory-routing problems and vehicle routing with delivery and pick up problems.

As uncertainty is the indispensable component of the disaster relief operations, it is worthwhile to investigate how to address uncertainty with alternative methods other than the one presented in this thesis. One powerful tool to model uncertainty is Robust Optimization and particularly because of the disaster nature Adjustable Robust Optimization (ARO) can be utilized to cope with the uncertainty. In a disaster preparedness problem such as the one we address in chapter 4, the decision we took before the disaster should be revised after some part of the uncertainty reveals itself. For example, the PNs-CLs allocation can be changed during the recourse action when the locations of road blockages become available. Hoyos et al. (2015b) survey the studies with an uncertain component in disaster relief operations. They cannot find any study that takes advantage of the adaptive methods in disaster management operations. On the other hand, recently, Yankoglu et al. (2019) explore the ARO and its applications. They suggest using this approach to deal with the uncertainty in humanitarian logistics problem. Studies that incorporate adaptive approaches such as ARO are very scarce and to the best of our knowledge are limited to Ben-Tal et al. (2011). Using such approaches can make the problem more realistic and should be followed as a future research direction.

Humanitarian operations, especially in disaster response operations, are involved with risks. This thesis can be extended by incorporating risk-averseness considerations into the defined problems. Additionally, the inclusion of the deprivation cost to our inequality-averse models can be studied in future works. Equity is one of the most important aspects of social sustainability, however, other aspects of sustainability are equally important. Recently, some studies focused on the sustainability aspect of humanitarian operations (see Dubey and Gunasekaran (2016) and Kunz and Gold (2017)). One interesting extension of this thesis is to include long-term sustainability concerns into the first problem.

BIBLIOGRAPHY

Abounacer, R., Rekik, M., and Renaud, J. (2014). An exact solution approach for multi-objective location–transportation problem for disaster response. *Computers & Operations Research*, 41:83–93.

Afshar, A. and Haghani, A. (2012). Modeling integrated supply chain logistics in real-time large-scale disaster relief operations. *Socio-Economic Planning Sciences*, 46(4):327–338.

Ahmadi, M., Seifi, A., and Tootooni, B. (2015). A humanitarian logistics model for disaster relief operation considering network failure and standard relief time: A case study on san francisco district. *Transportation Research Part E: Logistics and Transportation Review*, 75:145–163.

Alp, O., Erkut, E., and Drezner, Z. (2003). An efficient genetic algorithm for the p-median problem. *Annals of Operations Research*, 122(1):21–42.

Anaya-Arenas, A. M., Renaud, J., and Ruiz, A. (2014). Relief distribution networks: a systematic review. *Annals of Operations Research*, 223(1):53–79.

Balcik, B., Beamon, B. M., and Smilowitz, K. (2008). Last mile distribution in humanitarian relief. *Journal of Intelligent Transportation Systems*, 12(2):51–63.

Balcik, B., Iravani, S. M., and Smilowitz, K. (2010). A review of equity in nonprofit and public sector: a vehicle routing perspective. *Wiley Encyclopedia of Operations Research and Management Science*.

Bayram, V. (2016). Optimization models for large scale network evacuation planning and management: A literature review. *Surveys in Operations Research and Management Science*, 21(2):63 – 84.

Bayram, V., Tansel, B. C., and Yaman, H. (2015). Compromising system and user interests in shelter location and evacuation planning. *Transportation Research Part B: Methodological*, 72:146 – 163.

Bayram, V. and Yaman, H. (2017). Shelter location and evacuation route assignment under uncertainty: A benders decomposition approach. *Transportation Science*. Articles in Advance.

Beasley, J. E. (1988). An algorithm for solving large capacitated warehouse location problems. *European Journal of Operational Research*, 33(3):314–325.

Ben-Porath, E., Gilboa, I., and Schmeidler, D. (1997). On the measurement of inequality under uncertainty. *Journal of Economic Theory*, 75(1):194–204.

Ben-Tal, A., Do Chung, B., Mandala, S. R., and Yao, T. (2011). Robust optimization for emergency logistics planning: Risk mitigation in humanitarian relief supply chains. *Transportation research part B: methodological*, 45(8):1177–1189.

Berkoune, D., Renaud, J., Rekik, M., and Ruiz, A. (2012). Transportation in disaster response operations. *Socio-Economic Planning Sciences*, 46(1):23–32.

Boonmee, C., Arimura, M., and Asada, T. (2017). Facility location optimization model for emergency humanitarian logistics. *International Journal of Disaster Risk Reduction*, 24(Supplement C):485 – 498.

Bozorgi-Amiri, A., Jabalameli, M., and Al-e Hashem, S. M. (2013). A multi-objective robust stochastic programming model for disaster relief logistics under uncertainty. *OR Spectrum*, 35(4):905–933.

Campbell, A. M., Vandenbussche, D., and Hermann, W. (2008). Routing for relief efforts. *Transportation Science*, 42(2):127–145.

Cao, W., Çelik, M., Ergun, Ö., Swann, J., and Viljoen, N. (2016). Challenges in service network expansion: An application in donated breastmilk banking in south africa. *Socio-Economic Planning Sciences*, 53:33–48.

Chang, F.-S., Wu, J.-S., Lee, C.-N., and Shen, H.-C. (2014). Greedy-search-based multi-objective genetic algorithm for emergency logistics scheduling. *Expert Systems with Applications*, 41(6):2947–2956.

Chew, S. H. and Sagi, J. S. (2012). An inequality measure for stochastic allocations. *Journal of Economic Theory*, 147(4):1517–1544.

Coutinho-Rodrigues, J., Tralho, L., and Alada-Almeida, L. (2012). Solving a location-routing problem with a multiobjective approach: the design of urban evacuation plans. *Journal of Transport Geography*, 22:206 – 218.

Dayarian, I., Crainic, T. G., Gendreau, M., and Rei, W. (2015). A column generation approach for a multi-attribute vehicle routing problem. *European Journal of Operational Research*, 241(3):888–906.

Delage, E. and Ye, Y. (2010). Distributionally robust optimization under moment uncertainty with application to data-driven problems. *Operations research*, 58(3):595–612.

Derbel, H., Jarboui, B., Hanafi, S., and Chabchoub, H. (2012). Genetic algorithm with iterated local search for solving a location-routing problem. *Expert Systems with Applications*, 39(3):2865–2871.

Desaulniers, G., Rakke, J. G., and Coelho, L. C. (2015). A branch-price-and-cut algorithm for the inventory-routing problem. *Transportation Science*, 50(3):1060–1076.

Döyen, A., Aras, N., and Barbarosoğlu, G. (2012). A two-echelon stochastic facility location model for humanitarian relief logistics. *Optimization Letters*, 6(6):1123–1145.

Drezner, T., Drezner, Z., and Guyse, J. (2009). Equitable service by a facility: Minimizing the gini coefficient. *Computers & Operations Research*, 36(12):3240–3246.

Dubey, R. and Gunasekaran, A. (2016). The sustainable humanitarian supply chain design: agility, adaptability and alignment. *International Journal of Logistics Research and Applications*, 19(1):62–82.

Duque, P. A. M., Dolinskaya, I. S., and Sörensen, K. (2016). Network repair crew scheduling and routing for emergency relief distribution problem. *European Journal of Operational Research*, 248(1):272–285.

Eiben, A. E. and Schippers, C. A. (1998). On evolutionary exploration and exploitation. *Fundamenta Informaticae*, 35(1-4):35–50.

Eiben, A. E., Smith, J. E., et al. (2003). *Introduction to Evolutionary Computing*, volume 53. Springer.

Eisenhandler, O. and Tzur, M. (2018). The humanitarian pickup and distribution problem. *Operations Research*, 67(1):10–32.

Eisenhandler, O. and Tzur, M. (2019). A segment-based formulation and a matheuristic for the humanitarian pickup and distribution problem. *Transportation Science*, just accepted.

Elçi, Ö. and Noyan, N. (2018). A chance-constrained two-stage stochastic programming model for humanitarian relief network design. *Transportation Research Part B: Methodological*, 108:55–83.

Erbeyoğlu, G. and Bilge, U. (2019). A rich-vrp model for assessing the performance of alternative linear objective functions in effectiveness-fairness trade-off. Technical Report FBE/IE-03/2019-3, Department of Industrial Engineering, Boğaziçi University, Istanbul.

Erdoğan, E. and Iyengar, G. (2006). Ambiguous chance constrained problems and robust optimization. *Mathematical Programming*, 107(1-2):37–61.

Ertem, M. A., İşbilir, M., and Arslan, A. Ş. (2017). Review of intermodal freight transportation in humanitarian logistics. *European Transport Research Review*, 9(1):10.

Feillet, D., Dejax, P., and Gendreau, M. (2005). Traveling salesman problems with profits. *Transportation science*, 39(2):188–205.

Ferrer, J. M., Ortuño, M. T., and Tirado, G. (2016). A grasp metaheuristic for humanitarian aid distribution. *Journal of Heuristics*, 22(1):55–87.

Fianu, S. and Davis, L. B. (2018). A markov decision process model for equitable distribution of supplies under uncertainty. *European Journal of Operational Research*, 264(3):1101–1115.

Fleurbaey, M., Gajdos, T., and Zuber, S. (2015). Social rationality, separability, and equity under uncertainty. *Mathematical Social Sciences*, 73:13–22.

Gajdos, T. and Maurin, E. (2004). Unequal uncertainties and uncertain inequalities: an axiomatic approach. *Journal of Economic Theory*, 116(1):93–118.

Galindo, G. and Batta, R. (2013a). Prepositioning of supplies in preparation for a hurricane under potential destruction of prepositioned supplies. *Socio-Economic Planning Sciences*, 47(1):20–37.

Galindo, G. and Batta, R. (2013b). Review of recent developments in or/ms research in disaster operations management. *European Journal of Operational Research*, 230(2):201–211.

Gendreau, M., Hertz, A., and Laporte, G. (1994). A tabu search heuristic for the vehicle routing problem. *Management science*, 40(10):1276–1290.

Gillett, B. E. and Miller, L. R. (1974). A heuristic algorithm for the vehicle-dispatch problem. *Operations research*, 22(2):340–349.

Gopalan, R., Kolluri, K. S., Batta, R., and Karwan, M. H. (1990). Modeling equity of risk in the transportation of hazardous materials. *Operations Research*, 38(6):961–973.

Gralla, E., Goentzel, J., and Fine, C. (2014). Assessing trade-offs among multiple objectives for humanitarian aid delivery using expert preferences. *Production and Operations Management*, 23(6):978–989.

Grass, E. and Fischer, K. (2016). Two-stage stochastic programming in disaster management: A literature survey. *Surveys in Operations Research and Management Science*, 21(2):85 – 100.

Gutjahr, W. J. and Dzubur, N. (2016). Bi-objective bilevel optimization of distribution center locations considering user equilibria. *Transportation Research Part E: Logistics and Transportation Review*, 85:1–22.

Gutjahr, W. J. and Fischer, S. (2018). Equity and deprivation costs in humanitarian logistics. *European Journal of Operational Research*, 270(1):185–197.

Gutjahr, W. J. and Nolz, P. C. (2016). Multicriteria optimization in humanitarian aid. *European Journal of Operational Research*, 252(2):351–366.

Habib, M. S., Lee, Y. H., and Memon, M. S. (2016). Mathematical models in humanitarian supply chain management: A systematic literature review. *Mathematical Problems in Engineering*, 2016.

Holguín-Veras, J., Jaller, M., Van Wassenhove, L. N., Pérez, N., and Wachtendorf, T. (2012). On the unique features of post-disaster humanitarian logistics. *Journal of Operations Management*, 30(7-8):494–506.

Holguín-Veras, J., Pérez, N., Jaller, M., Van Wassenhove, L. N., and Aros-Vera, F. (2013). On the appropriate objective function for post-disaster humanitarian logistics models. *Journal of Operations Management*, 31(5):262–280.

Holland, J. H. (1992). Genetic algorithms. *Scientific American*, 267(1):66–73.

Hoyos, M. C., Morales, R. S., and Akhavan-Tabatabaei, R. (2015a). Or models with stochastic components in disaster operations management: A literature survey. *Computers & Industrial Engineering*, 82(Supplement C):183 – 197.

Hoyos, M. C., Morales, R. S., and Akhavan-Tabatabaei, R. (2015b). Or models with stochastic components in disaster operations management: A literature survey. *Computers & Industrial Engineering*, 82:183–197.

Hu, S.-L., Han, C.-F., and Meng, L.-P. (2016). Stochastic optimization for investment in facilities in emergency prevention. *Transportation Research Part E: Logistics and Transportation Review*, 89:14–31.

Huang, K., Jiang, Y., Yuan, Y., and Zhao, L. (2015). Modeling multiple humanitarian objectives in emergency response to large-scale disasters. *Transportation Research Part E: Logistics and Transportation Review*, 75:1–17.

Huang, K. and Rafiei, R. (2019). Equitable last mile distribution in emergency response. *Computers & Industrial Engineering*, 127:887–900.

Huang, M., Smilowitz, K., and Balcik, B. (2012). Models for relief routing: Equity, efficiency and efficacy. *Transportation Research Part E: Logistics and Transportation Review*, 48(1):2–18.

Jaramillo, J. H., Bhadury, J., and Batta, R. (2002). On the use of genetic algorithms to solve location problems. *Computers & Operations Research*, 29(6):761–779.

Karsu, Ö. and Morton, A. (2015). Inequity averse optimization in operational research. *European Journal of Operational Research*, 245(2):343–359.

Kılcı, F., Kara, B. Y., and Bozkaya, B. (2015). Locating temporary shelter areas after an earthquake: A case for turkey. *European Journal of Operational Research*, 243(1):323–332.

Kongsomsaksakul, S., Yang, C., and Chen, A. (2005). Shelter location-allocation model for flood evacuation planning. *Journal of the Eastern Asia Society for Transportation Studies*, 6:4237–4252.

Kovacs, G. and Moshtari, M. (2018). A roadmap for higher research quality in humanitarian operations: A methodological perspective. *European Journal of Operational Research*.

Kulshrestha, A., Wu, D., Lou, Y., and Yin, Y. (2011). Robust shelter locations for evacuation planning with demand uncertainty. *Journal of Transportation Safety & Security*, 3(4):272–288.

Kunz, N. and Gold, S. (2017). Sustainable humanitarian supply chain management–

exploring new theory. *International Journal of Logistics Research and Applications*, 20(2):85–104.

Li, L., Jin, M., and Zhang, L. (2011). Sheltering network planning and management with a case in the gulf coast region. *International Journal of Production Economics*, 131(2):431 – 440.

Lin, Y.-H., Batta, R., Rogerson, P. A., Blatt, A., and Flanigan, M. (2011). A logistics model for emergency supply of critical items in the aftermath of a disaster. *Socio-Economic Planning Sciences*, 45(4):132–145.

Liu, Y., Lei, H., Zhang, D., and Wu, Z. (2018). Robust optimization for relief logistics planning under uncertainties in demand and transportation time. *Applied Mathematical Modelling*, 55:262–280.

Loree, N. and Aros-Vera, F. (2018). Points of distribution location and inventory management model for post-disaster humanitarian logistics. *Transportation Research Part E: Logistics and Transportation Review*, 116:1–24.

Marsh, M. T. and Schilling, D. A. (1994). Equity measurement in facility location analysis: A review and framework. *European Journal of Operational Research*, 74(1):1–17.

Matl, P., Hartl, R. F., and Vidal, T. (2017). Workload equity in vehicle routing problems: A survey and analysis. *Transportation Science*, 52(2):239–260.

Mavrotas, G. (2009). Effective implementation of the ε-constraint method in multi-objective mathematical programming problems. *Applied mathematics and computation*, 213(2):455–465.

Mohammadi, R., Ghomi, S. F., and Jolai, F. (2016). Prepositioning emergency earthquake response supplies: A new multi-objective particle swarm optimization algorithm. *Applied Mathematical Modelling*, 40(9-10):5183–5199.

Moreno, A., Alem, D., Ferreira, D., and Clark, A. (2018). An effective two-stage stochastic multi-trip location-transportation model with social concerns in relief supply chains. *European Journal of Operational Research*, 269(3):1050–1071.

Mostajabdaveh, M., Gutjahr, W. J., and Salman, F. S. (2019). Inequity-averse shelter location for disaster preparedness. *IISE Transactions*, 51(8):809–829.

Najafi, M., Eshghi, K., and Dullaert, W. (2013). A multi-objective robust optimization model for logistics planning in the earthquake response phase. *Transportation Research Part E: Logistics and Transportation Review*, 49(1):217–249.

Ng, M., Park, J., and Waller, S. T. (2010). A hybrid bilevel model for the optimal shelter assignment in emergency evacuations. *Computer-Aided Civil and Infrastructure Engineering*, 25(8):547–556.

Ni, W., Shu, J., and Song, M. (2018). Location and emergency inventory pre-positioning for disaster response operations: Min-max robust model and a case study of yushu earthquake. *Production and Operations Management*, 27(1):160–183.

Nolz, P. C., Semet, F., and Doerner, K. F. (2011). Risk approaches for delivering disaster relief supplies. *OR spectrum*, 33(3):543–569.

Noyan, N. (2012). Risk-averse two-stage stochastic programming with an application to disaster management. *Computers & Operations Research*, 39(3):541–559.

Noyan, N., Balcik, B., and Atakan, S. (2015). A stochastic optimization model for designing last mile relief networks. *Transportation Science*, 50(3):1092–1113.

Noyan, N. and Kahvecioğlu, G. (2018). Stochastic last mile relief network design with resource reallocation. *OR Spectrum*, 40(1):187–231.

Noyan, N., Merakli, M., and Kucukyavuz, S. (2017). Two-stage Stochastic Programming under Multivariate Risk Constraints with an Application to Humanitarian Relief Network Design. *ArXiv e-prints*.

Ogryczak, W. (2000). Inequality measures and equitable approaches to location problems. *European Journal of Operational Research*, 122(2):374–391.

Özdamar, L. and Ertem, M. A. (2015). Models, solutions and enabling technologies in humanitarian logistics. *European Journal of Operational Research*, 244(1):55–65.

Paul, J. A. and MacDonald, L. (2016). Location and capacity allocations decisions to mitigate the impacts of unexpected disasters. *European Journal of Operational Research*, 251(1):252–263.

Pel, A. J., Bliemer, M. C., and Hoogendoorn, S. P. (2012). A review on travel behaviour modelling in dynamic traffic simulation models for evacuations. *Transportation*, 39(1):97–123.

Penna, P. H. V., Santos, A. C., and Prins, C. (2018). Vehicle routing problems for last mile distribution after major disaster. *Journal of the Operational Research Society*, 69(8):1254–1268.

Pérez-Rodríguez, N. and Holguín-Veras, J. (2015). Inventory-allocation distribution models for postdisaster humanitarian logistics with explicit consideration of deprivation costs. *Transportation Science*, 50(4):1261–1285.

Porath, E. B. and Gilboa, I. (1994). Linear measures, the gini index, and the income-equality trade-off. *Journal of Economic Theory*, 64(2):443–467.

Pradhananga, R., Mutlu, F., Pokharel, S., Holguín-Veras, J., and Seth, D. (2016). An integrated resource allocation and distribution model for pre-disaster planning. *Computers & Industrial Engineering*, 91:229–238.

Ransikarbum, K. and Mason, S. J. (2016). Multiple-objective analysis of integrated relief supply and network restoration in humanitarian logistics operations. *International Journal of Production Research*, 54(1):49–68.

Rath, S., Gendreau, M., and Gutjahr, W. J. (2015). Bi-objective stochastic programming models for determining depot locations in disaster relief operations. *International Transactions in Operational Research*, 23(6):997–1023.

Rawls, C. G. and Turnquist, M. A. (2010). Pre-positioning of emergency supplies for disaster response. *Transportation Research Part B: Methodological*, 44(4):521–534.

Renkli, Ç. and Duran, S. (2015). Pre-positioning disaster response facilities and relief items. *Human and Ecological Risk Assessment: An International Journal*, 21(5):1169–1185.

Rezaei-Malek, M. and Tavakkoli-Moghaddam, R. (2014). Robust humanitarian relief logistics network planning. *Uncertain Supply Chain Management*, 2(2):73–96.

Romeijn, H. E. and Morales, D. R. (2000). A class of greedy algorithms for the generalized assignment problem. *Discrete Applied Mathematics*, 103(1):209–235.

Sabouhi, F., Tavakoli, Z. S., Bozorgi-Amiri, A., and Sheu, J.-B. (2018). A robust possibilistic programming multi-objective model for locating transfer points and shelters in disaster relief. *Transportmetrica A: Transport Science*, pages 1–28.

Salman, F. S. and Yücel, E. (2015). Emergency facility location under random network damage: Insights from the istanbul case. *Computers & Operations Research*, 62:266–281.

Song, S., Zhou, H., and Song, W. (2019). Sustainable shelter-site selection under uncertainty: A rough qualiflex method. *Computers & Industrial Engineering*, 128:371–386.

Tofighi, S., Torabi, S. A., and Mansouri, S. A. (2016). Humanitarian logistics network design under mixed uncertainty. *European Journal of Operational Research*, 250(1):239–250.

Tricoire, F., Graf, A., and Gutjahr, W. J. (2012). The bi-objective stochastic covering tour problem. *Computers & operations research*, 39(7):1582–1592.

Trivedi, A. and Singh, A. (2017). A hybrid multi-objective decision model for emergency shelter location-relocation projects using fuzzy analytic hierarchy process and goal programming approach. *International Journal of Project Management*, 35(5):827 – 840.

Ukkusuri, S. V. and Yushimito, W. F. (2008). Location routing approach for the humanitarian prepositioning problem. *Transportation research record*, 2089(1):18–25.

Uster, H. and Dalal, J. (2017). Strategic emergency preparedness network design integrating supply and demand sides in a multi-objective approach. *IISE Transactions*, 49(4):395–413.

Üster, H. and Dalal, J. (2017). Strategic emergency preparedness network design integrating supply and demand sides in a multi-objective approach. *IISE Transactions*, 49(4):395–413.

Vahdani, B., Veysmoradi, D., Shekari, N., and Mousavi, S. M. (2018). Multi-objective, multi-period location-routing model to distribute relief after earthquake by considering emergency roadway repair. *Neural Computing and Applications*, 30(3):835–854.

Verma, A. and Gaukler, G. M. (2015). Pre-positioning disaster response facilities at safe locations: An evaluation of deterministic and stochastic modeling approaches. *Computers & Operations Research*, 62:197–209.

Vitoriano, B., Ortuño, M. T., Tirado, G., and Montero, J. (2011). A multi-criteria optimization model for humanitarian aid distribution. *Journal of Global optimization*, 51(2):189–208.

Wang, W., Yang, S., Hu, F., He, S., Shi, X., Meng, Y., and Shi, M. (2016). Integrated optimization model for shelter allocation and evacuation routing with consideration of reliability. *Transportation research record*, 2599(1):33–42.

Yankoglu, I., Gorissen, B. L., and den Hertog, D. (2019). A survey of adjustable robust optimization. *European Journal of Operational Research*, 277(3):799 – 813.

Yitzhaki, S. and Schechtman, E. (2013). More than a dozen alternative ways of spelling Gini. In *The Gini Methodology*, pages 11–31. Springer.

Yu, L., Zhang, C., Yang, H., and Miao, L. (2018). Novel methods for resource allocation in humanitarian logistics considering human suffering. *Computers & Industrial Engineering*, 119:1–20.

Yücel, E., Salman, F. S., and Arsik, I. (2018a). Improving post-disaster road network accessibility by strengthening links against failures. *European Journal of Operational Research*, 269(2):406 – 422.

Yücel, E., Salman, F. S., and Arsik, I. (2018b). Improving post-disaster road network accessibility by strengthening links against failures. *European Journal of Operational Research*, 269(2):406–422.