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**EFFECT OF DELAMINATION ON THE STRENGTH
OF LAMINATED GLASS SHELL STRUCTURE**

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MASTER'S THESIS

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LIST OF SYMBOLS AND ABBREVIATIONS

[A]	:	Matrix containing linear terms of the radial displacement field equation
D₁	:	Flexural Rigidity of the top shell
D₂	:	Flexural Rigidity of the bottom shell
E	:	Glass Young's modulus
e_{1x}, e_{1y}, e_{1xy}	:	Strain components of the top glass shell
e_{2x}, e_{2y}, e_{2xy}	:	Strain components of the bottom
G	:	Glass shear modulus
h₁	:	Thickness of the top glass
h₂	:	Thickness of the bottom glass
h_x	:	Finite difference mesh length in the x direction
h_y	:	Finite difference mesh length in the y direction
n_x, n_y	:	Number of subdivisions in the x and y directions, respectively.
q	:	Uniform applied radial pressure
{RHS}	:	Vector containing the right-hand side of the radial displacement field equation.
t	:	Thickness of the interlayer
u₁	:	In plane displacement for the top plate in the x direction.
u₂	:	In plane displacement for the bottom plate in the x direction.
$\overline{U}_b^{(1)}$	:	Bending strain energy for the top glass
$\overline{U}_b^{(2)}$	:	Bending strain energy for the bottom glass
$\overline{U}_m^{(1)}$	:	Membrane strain energy for the top glass
$\overline{U}_m^{(2)}$	:	Membrane strain energy for the bottom glass

$\overline{U}_{xz}^{(1)} \overline{U}_{yz}^{(1)}$	: Shear strain energy for the interlayer due to γ_{xz} , γ_{yz} , respectively
v_1	: In plane displacement for the top plate in the y direction.
v_2	: In plane displacement for the bottom plate in the y direction
V	: Total potential energy of the system
w	: Radial deflection of the top and bottom glass
$\{w\}$	: Radial displacement vector
$w_{\max}$	: Maximum radial deflection
x, y, z	: Rectangular coordinates
γ_{xz}, γ_{yz}	: Shear strain components of the interlayer
α, β	: Interpolation parameters
μ	: Poisson's ratio for the glass
θ	: Angle undergone by an originally vertical fiber of the interlayer
ϕ	: Angle undergone by an originally horizontal fiber of the interlayer
$\overline{\Omega}$	: Load potential energy function

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ABSTRACT

EFFECT OF DELAMINATION ON THE STRENGTH OF LAMINATED GLASS SHELL STRUCTURES

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Objective: This study primarily focuses on investigating the impact of delamination on the structural integrity of laminated glass shells when subjected to initial delamination. To accomplish this objective, a comprehensive mathematical model is developed specifically for laminated glass shell. The model utilizes the principles of the minimum potential energy principle to derive governing equations involving non-linear terms and boundary constraints. Subsequently, the Finite Difference Method (FDM) is employed to discretize the derived governing equations, resulting in a discrete form model.

Material and Methods: Minimum potential energy theorem, variational method and finite difference method are used to derive the equation for deflection and stress. To obtain convergency the Successive Over Relaxation Method is applied.

Results: Laminated glass shell is a three-dimensional geometrical structure having a thickness much smaller as compared to the other dimensions. A laminated glass shell can exhibit nonlinear behavior and can easily undergo huge displacement under its own weight due to its geometry. As a result of the large displacement, the governing equations are based on large deflection theory. Delamination is also recognized as an undesirable condition that causes a significant deterioration in the strength of laminated glass. The results showed that the effect of delamination on strength is also paramount due to its size and location.

Conclusion: The mathematical model established in this study offers a practical approach to modelling both delaminated and laminated glass shells.

Keywords: Laminated glass shell, Variational method, Nonlinear Behavior, Delamination.

ÖZET

DELAMİNASYONUN LAMİNE CAM KABUK YAPILARININ MUKAVEMETİ ÜZERİNDEKİ ETKİSİ

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Amaç: Bu çalışma ile öncelikle, başlangıçta delaminasyonlu olan lamine cam kabuklarının yapısal bütünlüğü üzerindeki delaminasyonun etkisini araştırmaktır. Bu amaca ulaşmak için, özellikle lamine cam kabuk için kapsamlı bir matematiksel model geliştirilmiştir. Model, türevleri ve sınır koşullarını içeren doğrusal olmayan yönetici denklemleri türetmek için minimum potansiyel enerji ilkeleri kullanılmıştır. Daha sonra, türetilmiş yönetici denklemleri ayırıklaştırmak için Sonlu Farklar Yöntemi (FDM) kullanılmıştır ve bu da ayırık bir form modeliyle sonuçlanmıştır.

Materyal ve Yöntem: Minimum potansiyel enerji teoremi, varyasyonel yöntem ve sonlu elemanlar fark yöntemi, sapma ve gerilme denklemini türetmek için kullanılır. Yakınsama elde etmek için iteratife SOR Yöntemi uygulanır.

Sonuç: Lamine cam kabuk; diğer boyutlara göre çok daha küçük kalınlığa sahip üç boyutlu geometrik bir yapıdır. Lamine cam kabuk, doğrusal olmayan davranış sergileyebilir ve geometrisi nedeniyle kendi ağırlığı altında büyük yer değiştirmelere kolayca maruz kalabilir. Büyük yer değiştirmenin bir sonucu olarak, yönetici denklem büyük sapma teorisine dayanmaktadır. Delaminasyon, lamine camın mukavemetinde önemli bir bozulmaya neden olan istenmeyen bir durum olarak da kabul edilmektedir. Elde ettiğimiz sonuçlara göre, delaminasyonun mukavemet üzerindeki etkisinin boyutu ve konumu nedeniyle önemli olduğunu göstermiştir.

Anahtar kelimeler: Lamina cam kabuk, Varyasyon yöntemi, Doğrusal olmayan davranış, Delaminasyon.

1. INTRODUCTION

1.1 Glass

Glass dates back up to 5000 BCE; it is unknown when and where the manufacturing of glass began. The region of Mesopotamia's Mitannian or Hurrian is where most believe the first glass was invented. Glass objects dating back to 2500 BCE have been discovered in Syria. Glass manufacturing was hard until the discovery of the blowpipe. In 1000 AD, the Egyptian city of Alexandria was the most important center of glass manufacturing. In the 13th and 14th centuries, manufacturing stained glass for churches and cathedral windows throughout Europe reached its peak. Glass use, expansion, and manufacturing of window glass increased quickly after 1890. This became possible after the invention of the sheet glass drawing machine. Glass is therefore one of the most crucial and significant materials in human history. Glassy materials are a key factor in modern human civilization. The invention of glass has sparked a lot of attention and research. The trial-and error-based method of experimentation is an empirical method used in research to understand glass materials. Some of the earliest studies on glass technology were written by the renowned Michael Faraday, who also contributed to early studies on glass. Otto Schott, William E.S. Turner, F. W. Preston, and George W. Morey were among the early researchers in the field of glass science and technology in the late 1800s and early 1900s. Glass is a solid material that is optically transparent, fragile, and amorphous (non-crystalline). Obsidian is a naturally occurring silica-base glass material formed when lava from volcanoes cools rapidly with minimal crystal growth. It is generally produced by melting raw materials at a high temperature and rapidly cooling them to form a solid without a crystalline structure. Glass may be formed into various shapes, including sheets, containers, sculptures, and ornaments, and it is widely used in building, home appliances, optical lenses, electronics, and the arts (Picture 1.1).



Picture 1.1 Ancient glass Ornament

With the development of technology, the usage of glass in the areas of architecture, solar panels, the chemical industry, and the food industry has increased even more. Glass has gained attention as one of the vital components in structural design. Glass has been used in construction for over 2000 years, since the Romans Empire. As of late, endeavors have been made to build structures totally made of glass. Their entire potential has not yet been realized, despite their widespread use. Even though glass has many advantages, some of the following disadvantages hinder its use in the architectural, engineering, and construction fields: brittleness, cost of manufacturing, less impact resistance, maintenance, and unsafe for earthquake-prone areas. The failure of glass is extremely dangerous, as it shows no signs of hardening, strain, or yield. Jelena Savic et al.'s (2013) studies show the right glass choice may enhance a structure's energy efficiency.

1.1.1 Laminated glass

Laminated glass, also known as "Sandwich glass," is a safety type of glass with an interlayer that holds glass pieces together when shattered to minimize the risks. Behind the story

of our modern-day legends of discovery and invention are accidental accounts: Newton's Apple, Watt's kettle, Archimedes bath, and Edouard Benedictus discovery of laminated glass. A well-known French scientist named Edouard Benedictus made a substantial contribution to the study of glass from 1878 to 1930. While carrying out tests in his lab in 1903, Benedictus accidentally made the discovery of laminated glass. He accidentally dropped a glass flask which previously contained cellulose nitrate, a form of plastic substance. The glass pieces did not scatter; they stayed whole even though they were broken. Benedictus understood the unscathed glass had the potential to be a material that would improve safety, and this realization revealed the important consequences of this unexpected event. This coincidental event sparked more study and development in the area of glass technology (Zang and Chen, 2011). Benedictus applied for a patent in 1909 after learning of an automobile accident in which two women suffered serious injuries from glass fragments. To reduce injuries in automobile accidents, he founded the Société du Verre Triplex in 1911. Triplex glass was costly to manufacture and took some time; thus, it was not instantly embraced by the automobile industry. Nevertheless, laminated glass saw extensive use in gas mask eyepieces during World War I. Following approval in 1912, the English Triplex Safety Glass Company took the lead in putting this method into practice. Later, Triplex glass was brought to the US by businesses like Libbey-Owens-Ford and Du Pont, working with Pittsburgh Plate Glass. Two sheets of annealed, heat-strengthened, or tempered glass are combined with an interlayer to create laminated glass. This amalgamation is produced by applying pressure and heat, creating an unbreakable connection. These composite structures are pressure cooked in an autoclave to produce the final bonded product, with the thermoset interlayer fully cross-linked. The interlayer improves the strength, safety, and other features of laminated glass. Many interlayers are employed in the manufacturing of laminated glass; the following are examples of interlayer types: Polyvinyl Butyral (PVB), Ethylene Vinyl Acetate (EVA), Ionoplast Interlayer, Acoustic Interlayer, Ultraviolet (UV) Filtering Interlayer, Fire-Resistant Interlayer, Switchable Interlayer, and Bullet-Resistant Interlayer. The type of interlayer used is determined by the application requirements. This study considers the polyvinyl butyral (PVB) interlayer. Additionally, some vehicle wind shields include higher layers that are PVB-tinted. Moreover, colorful glass panels may be produced by thermosetting interlayer material and laminating colored polycarbonate (PC) sheets. For architectural applications, the patented Dupont Sentry Glass Expressions technique allows for direct printing to the PVB or directly to the glass, followed by laminating. The glass can be exposed without a frame after being appropriately laminated with interlayer. The laminated glass ought not to have any water infiltration, discoloration, or delamination in order to achieve a strong bond. The

thermoplastic family has grown as a result of more recent advancements in glass laminating. PVB's adhesion properties extend to both glass and polymeric interlayers, showcasing its remarkable bonding capabilities. From 2004 forward, the practice of laminating or sandwiching metallized and electroconductive polyethylene terephthalate (PET) interlayers between glass sheets has been adopted to provide a substrate for light-emitting diodes (LEDs). To give a laminated glass panel a permanent translucent hue, colored interlayers can be added. As such, a switchable interlayer can be added to the laminated unit to make a panel that can be clear when a weak electric current is sent through it and opaque when the current is turned off. Properties of laminated glass such as durability, stability, strength, and safety cannot be overlooked. When a car accident occurs, people are protected by laminated glass since the windshield will remain intact. The degree of injury due to breakage of the glass is minimized by preventing flying glass from hitting passengers. Laminated glass provides protection by being hard to break and preventing intruders from entering. Laminated glass can also lessen solar heating, enable building interiors to keep cool, and cut down on energy use. Laminated glass can interfere with sound waves depending on its thickness when used as windows, thereby lowering outside noise pollution. Laminated glass plays a critical role in reducing potential fatalities and injuries during natural disasters like hurricanes and earthquakes by retaining its structural strength and preventing shattering when wind shields or windows have been subjected to strong pressure (Picture 1.2).



Picture 1.2 Laminated glass pane walls and floor slab

1.1.2 Delamination

Laminated composites are distinguished by their high specific strength and stiffness. However, it is important to remember that these materials are very prone to flaws, with delamination being a major worry. The results of delamination can be one or more of the following; imperfection in manufacturing, water immersion, thin PVB, impurities, high humidity, and incorrect temperature parameters.

Delamination is a deterioration or, perhaps, a result of two layers debonding or separating from one another due to a loss in adhesion between glass panes. The start, propagation, and termination of delamination in a pressured laminated glass are significantly affected by the material properties, geometrical factors, and loading parameters. (Chermoshentseva et al., 2016). It is crucial to address this issue for composite materials, as it can cause the material to weaken and reduce its life span. Delamination of the laminated unit can change the way the material resonates, weaken the material, and cause an imbalance in a composite material that is normally symmetric. As previously indicated, weak bonding is the root of delamination. Due to its unfathomable nature,

delamination is frequently unseen until it approaches a critical point or causes a major failure. It is challenging to predict or foretell the recurrence of delamination despite the lack of noticeable external indications. The underlying layers or interfaces experience a decrease in bonding or a weakness, leading to diminished load-bearing capacity and possible performance degradation. Due to the concealed nature of delamination, complex inspection techniques, such as non-destructive testing methods, are required to detect and evaluate its presence. To ensure the reliability and safety of materials and structures susceptible to this type of failure, it is essential to comprehend the factors that contribute to delamination and to develop preventative measures.



Picture 1.3 Delaminated glass window

The sound of a delamination is different as compared to that of an undelamination. The sound of a layered composite is duller than that of a solid composite. However, the transparent nature of laminated glasses makes it easier to spot delamination (Picture 1.3). In general, one or more of the three primary types of interlaminar fracture may result in delamination in laminated composites. These modalities include the peeling or opening mode, the sliding method of shearing, and the twisting or out-of-plane style of shearing.

Many researchers have conducted in-depth research on a variety of delamination characteristics in composite laminates. Studies that dealt with different delamination test methodologies and experimental data reduction techniques were more focused on publishing delamination toughness data for new composite materials. Studies about the effects of materials, the environment, and other factors on the delamination were also conducted. As mentioned above, delamination can be a result of flaws during the production process or an extrinsic influence, like impact force, penetration of water, or oxidation of the interlayer. Other factors like environmental factors like temperature may also accelerate the rate of delamination growth. Unfortunately, not enough research has been done in order to understand how delamination affects the strength of curved laminated glass shells and to gain a basic grasp of delamination fracture mechanisms. The literature's descriptions of various types of composite systems and the results of the current experiment were both in excellent agreement with the theoretical predictions for the mechanisms of delamination. Different approaches have been successfully developed and used in the context of the Finite element method (FEM), such as the J-integral (Drake and Sullivan, 2021). The virtual crack extension technique (Azimi et al., 2017).

Laminated glass can exhibit nonlinear behavior and can easily undergo huge displacement under its own weight due to its geometry. As a result of the large displacement, the governing equation is based on large deflection theory. The complexity of laminated glass is due to the disparity in the materials used: glass and PVB. The elasticity modulus of glass is 10^4 times larger than the elasticity modulus of polyvinyl butyral. Even though PVB is soft for flexural stiffness properties, it produces shear stresses that are crucial for the interaction between glass layers. It is essential to monitor the progressive effect and predict the non-linear behavior of laminated composite glass; in order to do so, more efficient, and accurate methods are required.

Shell is a three-dimensional geometrical structure having a thickness much smaller as compared to other dimensions; it is a light-weight structure, covers a wide area, and plays a significant role in the engineering field. Some examples are pressure tanks, airplane wings, and roofs. As the geometry and thickness of laminated glass play an important role. The use of a shell structure is also applicable to laminated glass. Geometrical complexity results in complex behavior. Due to the complexity of laminated glass behavior, the numerical approach proves to be a useful tool to simulate loading behavior.

Curved laminated glass shells have not been widely used in practical applications due to a lack of understanding of their structural features. When compared to frame buildings,

the inherent curvature of curved structures typically produces peculiar behavioral tendencies. Due to the latter's associated mathematical complexity, most research efforts on curved structures now concentrate on the linear rather than the nonlinear elements of curved structures. Also, when laminated glass shells are put under radial stresses, they often bend a lot in the transverse direction because the glass layers are not very thick compared to the other dimensions. A mathematical model built on the foundation of the nonlinear response principle in engineering has been developed to understand the strength and behavior of shell structures.

This study primarily focuses on investigating the impact of delamination on the structural integrity of laminated glass shells when subjected to initial delamination. To accomplish this objective, a comprehensive mathematical model is developed specifically for laminated glass shells. The model utilizes the principles of the virtual work principle to derive non-linear governing equations involving derivatives and boundary constraints. Subsequently, the Finite Difference Method (FDM) is employed to discretize the derived governing equations, resulting in a discrete-form model. The results obtained from this model are then compared with the results obtained from a laminated glass shell model simulated in the Fortran programming language.

2. PREVIOUS RESEARCH

As the use of laminated glass grows, so does the number of research studies about laminated glass structures. In 1674, George Ravenscroft discovered lead glass as a result of an excess of alkaline salts in the patent he took for crystalline glass. After a laboratory incident, laminated glass was invented by the renowned French chemist Édouard Benedictus, who obtained French and British patents in 1910. Since then, considerable attention has been devoted to the examination and production of the performance of laminated glass (Morgan, 1931). Watkins et al. (1933) illustrated the importance of laminated glass and how it became widespread in production and how legislative action mandated all vehicles be equipped with laminated glass. The literature has presented numerous reports, encompassing both theoretical and experimental investigations, concerning the behavior of laminated glass. Laminated glass was previously used in aircraft and automobile industry. Currently, the utilization of laminated glass as a building material is experiencing a growing trend in popularity (Miller, 1935). However, substantial attention has also been directed towards the examination of the impact of delamination on laminated glass.

Flocker and Dharani (1997) studied the impact of laminated architectural glass from a small, low velocity projectile using finite element analysis. The scenario in which small, hard projectiles strike laminated glass windows in buildings, as is frequently seen during strong winds. They studied several geometric and material factors to see how they affect stress and wave propagation through the three-layer system to the essential interior ply in order to maximize the damage tolerance of laminated glass windows.

By employing the finite difference approach with central differences, the differential equations at discrete locations on the glass plates are formulated, leading to a system of linear algebraic equations. As the algebraic equations exhibit nonlinearity, an iterative strategy is implemented to solve for the displacements. Specifically, a banded matrix solver is utilized for computing lateral displacements, while the strongly implicit method is employed for in-plane displacements. The solution is achieved only when the variable underrelaxation parameter is appropriately applied; otherwise, the solution diverges. The obtained results reveal the formation of complex stress fields, with the maximum stress exhibiting a steady pattern. It initiates at the center, propagates along the x-axis, subsequently traverses the diagonal, and eventually settles in the vicinity of the corner, particularly at a high level of nonlinearity (Aşık, 2003).

In 2009, Elmarakbi et al. presented a novel modified adaptive cohesive element as described in their work. They developed and incorporated these elements into LS-DYNA to enhance the stability of finite element simulations involving the propagation of delamination in composite laminates subjected to transverse loads. They adapted a new way that can examine the initiation and growth of delamination under quasi-static and dynamic mode-I. Although LS-DYNA has huge library, it lacks the continuous cohesive algorithm. The results of the experimental work and the numerical simulations serves as evidence for the new mode's potential.

Sabin (1993) conducted a project is to investigate a composite sample that experiences transverse tensile stress and delamination using an optical fiber strain gage set up as a Michelson interferometer. By straightening out an existing curved piece of composite material, the transverse tensile stress is created. The interferometer's output signal is a representation of the integrated strain along the whole length of the optical fibers. The material's strain can be assessed by embedding the fibers in a composite specimen. This signal is quantitatively examined after being captured on a regular audio tape, transferred, and computer.

Experimental analysis and finite element (FE) modeling are used to explore the growth of delamination cracks in laminated composites by La Saponara et al. (2002). Cross-aligned graphite/epoxy samples were tested under static circumstances. The tested specimens' load-displacement response and crack length are both tracked. A viscous layer is used in the finite element model to simulate delamination and crack development. The experimental load-displacement curves were used to calibrate the cohesiveness parameters. Measurements of strain and crack growth were compared to the FE model. The test results and the predictions of the finite element model match up well. Transverse fracture growth in laminates with notches was also simulated using the same modeling methodology. The evaluation of the fracture toughness of multilayer samples can be made simpler using the proposed FE analysis with viscous layers.

In 2005, Aşık and Tezcan conducted a study that presented an arithmetical method model for the traits of laminated glass, which was subsequently validated through experimental testing. The study also incorporated the use of finite element analysis in the investigation. According to the boundary conditions, it implied that the behavior could either be linear or nonlinear. Because glass beams are so thin, the mathematical model was developed using the minimum total potential energy concept and large deflection assumptions for laminated composite beams.

Foraboschi (2007), studied proposes an analytical model that may be used to estimate stress development and ultimate strength of PVB laminated glass beams during uniaxial

bending. In the architectural glass market, it was used to evaluate the safety and failure strength of laminated glass goods.

The numerical treatment of delamination in laminated composite parts is addressed in a method that is described by Wimmer et al. (2009). The commencement of delamination is predicted using a failure criterion for the first layer, and the spread of delamination is examined using linear elastic fracture mechanics. A cautious assessment of the structure's load-bearing capability is provided by the combination of the initiation and propagation criteria. In addition, the layered growth process's nonlinear structural response, growth stability, and the resultant sensitivity to changes in interfacial characteristics are also identified. Two structures, a curved laminate, and a double-lap shear specimen, were examined to show the effectiveness of the suggested approach.

It is vital to have a solid grasp interfacial adhesion in order to mimic the post-fracture behavior of laminated glass. Butchart and Overend (2012)'s research offers a technique for figuring out interfacial adhesion that is not dependent on test rate. Interfacial adhesion was determined from tear stress experiments that were run at two different loading rates. The measured interfacial adhesion values changed with the test rate, pointing to a potential relationship between the adhesion force and loading rate.

In Naumenko and Eremeyev (2014) studies, a layer-wise theory for the structural characterization of laminates made of glass and solar cells is developed. Deriving partial differential equations involves using governing equations for each layer, kinematic restraints, and interaction forces. To demonstrate the significance of rectification for laminates with soft core layers, closed form analytical solutions are generated.

In Zhang et.al. (2014) study for a better protection of human safety, research on tempered glass fragments that result from air blast pressure was conducted. This study analyzes and presents the fragment characteristics of monolithic tempered glass windows that were seen during blasting testing. It was determined that negative pressure had a substantial impact on both the fragment splash scattering and discharging velocity.

Aşk et al. (2014) conducted research and used numerical solution techniques to build a mathematical model for the nonlinear behavior of a curved laminated glass beam. The resultant partial differential equations were then solved. The results demonstrated that, the shear modulus of the interlayer PVB has an impact on the curved laminated glass, by increasing this interlayer's shear modulus, the curved laminated glass beam can be made stronger than monolithic. The type of support also has a greater effect on displacement than on stresses because the simple supported curved laminated beam has a higher displacement than the fix

support.

Jaśkowiec (2015) modeled the failure of laminated glass due to glass delamination in three dimensions. It was discovered that as the delamination increases, so does the percentage deviation of the system. For lower mode frequencies and shorter delamination lengths, it is also understood that finite element and computational approach are more accurate. Although the percentage deviation for the hinged-hinged beam is lower towards the fixed end and higher in the middle, in the region near the fixed end of the cantilever beam, the observed values are comparatively higher. For the fixed-fixed boundary condition, it has been observed that the percentage deviation from the natural frequency is reduced (Gadelrab, 2015).

The mentioned mathematical model (model proposed by Asik and Tezcan) were further modified and used on delamination analyses of laminated glass beams by (Dural, 2016).

Del Linz et al. (2017) published a study to comprehend the acquired reaction force response of delamination process at actual deformation rates. Prior to this, blast tests were conducted, and the results were utilized to determine the loads placed on the support systems. When laminated glass were subjected to blast loads it was observed that a lot of energy was used during the delamination process.

Vedrtnam (2018) conducted a brief study comparing the delamination strength of laminated glass with various interlayers and nominal thicknesses. The delamination strength of laminated glass is found to be significantly influenced by inter-layer thickness. The statistical study supported the substantial correlation between delamination interlayer type and thickness.

Chen et al. (2021) work, is based on examining energy dissipation rates and delamination strengths. Numerical calculations and experimental investigations are performed on the behavior of PVB laminated glass under quasi-static pressure on delaminated glass with a PVB interlayer.

The current Australian standard, AS1288, enables the use of total glass thickness for short-term load durations but provides relationships for load sharing factors for medium- or long-term load durations. To ascertain whether a method using the laminated glass's actual thickness might be used, a working group was established. The investigation's findings are presented in this document (Kefal and Tessler, 2021).

Although laminated glass is frequently used in construction, delamination is a significant issue. The delamination effect on the strength of laminated glass is examined in this work in relation to constraints and boundary circumstances. There is consistency between the study's outcomes and what the finite element model expected. (Dural and Oyar 2023).

Since its discovery, laminated glass has had broad uses in various fields, including the civil engineering, aerospace design, marine, and automotive industries. Although there has been

an increase in studies assessing laminated glass' mechanical behavior, few have dealt with delamination, and there is less information about the nonlinear behavior of delaminated glass units. The goal of the present research is to explore the effect of delamination on the strength of a laminated glass shell with initial delamination.



3. MATHEMATICAL MODEL FOR LAMINATED GLASS CYLINDRICAL SHELL WITH AN INITIAL DELAMINATION

Nonlinearity in materials can stem from two main factors: material-based nonlinearity and geometric nonlinearity. Material-based nonlinearity arises when the material properties change in response to applied loads, resulting in non-linear force-displacement or stress-strain relationships. On the other hand, geometric nonlinearity arises from kinematic quantities such as strain-displacement relations. Factors like large strains, displacements, and rotations can introduce geometric nonlinearity. To analyze such systems, the minimum potential energy theorem is applied.

In the general case, nonlinearities are present in the description of a solid body's response. When considering the transverse deflection of a shell, it becomes evident that the deflection is not negligible compared to the shell's thickness. As a result, the mid-plane of the shell experiences stresses, leading to the development of nonlinear differential equations.

Laminated glass shell unit shown in Figure 3.1 consist of two thin glass shells and an interlayer placed in-between the glass sheets. The radial displacement in the units is larger than the thickness which makes it highly nonlinearly when subjected to radial pressure and shear. Nonlinear response can also be caused by the difference between modulus of interlayer and glass. Glass has a shear modulus of about 40 GPa, while it is 1000 kPa for interlayer. To analyze behavior of laminated glass the geometric non-linearity of thin glazing under radial loading must be considered. The nonlinear strain-displacement relation must be included in the strain energy expression. To be able to obtain a more accurate and reliable analysis of laminated glass, the large deflection effect should be considered.

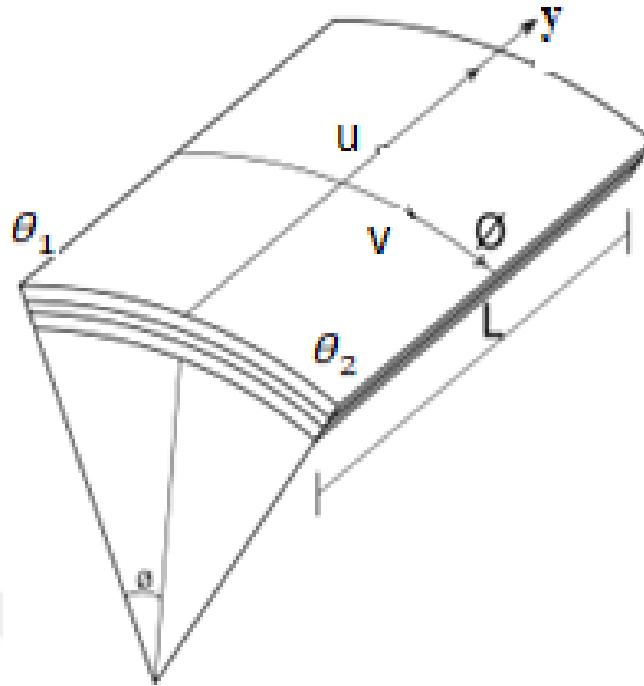


Figure 3.1 Laminated glass shell unit

Therefore, the equilibrium equations that governing the behavior of laminated glass are derived from the principles of large deflection theory. The stress distribution within the cross section of laminated glass shell exhibits a constant coupling stress between the interlayer and two symmetrical triangular stress distributions, each associated with a layered glass ply and centered around its respective neutral axis.

The minimum potential energy theorem is a fundamental concept utilized in physics and engineering. It states that, at low temperatures, a structure or body will deform or displace itself to a position that locally minimizes the total potential energy. The potential energy that is lost during this process is converted into kinetic energy, specifically heat. Consequently, when characterizing the behavior of a solid body, nonlinearities must be considered.

In other to apply minimum potential energy the following assumption are applied for simplification:

Assumptions made to obtain field equation and boundary conditions of glass shells are given as follows:

- shell material is completely homogenous and isotropic
- obeys Hooke's law
- shear deformation is ignored
- normal to mid-plane of the shell before deformation remains normal after deformation.

- for nonlinear behavior, high power in-plane displacement derivatives are ignored
- for nonlinear behavior, membrane stresses are developed in the mid-plane, but are still small compared with the other shell dimensions.

Assumptions made for the interlayer:

- plane section before deformation remains plane after deformation.
- material is homogenous and isotropic
- obeys Hooke's law and interlayer shear modulus is constant.
- linear shear strains are assumed instead of finite strains to introduce a simplification.
- no slip between adjacent faces of plies and interlayer.
- energy stored in the interlayer due to the normal stresses is disregarded as compared to the shear strain energy.

Potential energy, denoted as Π , is the total energy within a system and consists of two components. The first is the elastic strain energy, U , which represents the energy stored in a deformed body and the potential for it to return to its original shape. The second component is the potential energy, V , associated with the applied forces acting on the system. By summing U and V , we obtain the overall potential energy of the system, Π , which encompasses both the stored energy in the deformed body and the energy associated with the applied forces.

$$\Pi = U + V \quad (3.1)$$

Using the assumption stated above and Equation 3.1 in expanded form for glass and interlayer, the total potential energy of the system can be written as:

$$\Pi = U_m^1 + U_m^2 + U_b^1 + U_b^2 + U_{yr}^1 + U_{\theta r}^1 + \Omega \quad (3.2)$$

Where;

U_m^i = membrane strain energy for the shell (i) = 1, 2 for the top and bottom shells, respectively,

U_b^i = bending strain energy for the shell (i),

$U_{yr}^i, U_{\theta r}^i$ = shear strain energy of the interlayer in yr and $r\theta$ planes, respectively,

Ω = potential energy function due to applied loads.

According to Langhaar (1962), the bending strain energy function are given as;

$$U_b^i = \int_{-\theta_1}^{\theta_2} \int_{-a}^a U_b^i d\theta dy$$

$$U_b^i = \frac{E h_i r_i}{24(1-\nu^2)} \int_{-\theta_1}^{\theta_2} \int_{-a_1}^{a_2} ((k_\theta)^2 + (k_y)^2 + 2\nu(k_\theta)(k_y) + (\frac{1-\nu}{2})(k_{\theta y})^2) d\theta dy \quad (3.3)$$

In the equation above k_θ , k_y , $k_{\theta y}$ are the bending strains and they can be expressed as follows

$$k_\theta = \frac{-w_{\theta\theta}}{r_i^2}$$

$$k_y = -w_{yy}$$

$$k_{\theta y} = \frac{-2}{r_i} w_{\theta y}$$

Membrane strain energy function can be expressed as;

$$U_m^i = \int_{-\theta_1}^{\theta_2} \int_{-a}^a U_m^i d\theta dy$$

$$U_m^i = \frac{E h_i r_i}{2(1-\nu^2)} \int_{-\theta_1}^{\theta_2} \int_{-a_1}^{a_2} ((\epsilon_{i\theta})^2 + (\epsilon_{iy})^2 + 2\nu(\epsilon_{i\theta})(\epsilon_{iy}) + (\frac{1-\nu}{2})(\epsilon_{i\theta y})^2) d\theta dy \quad (3.4)$$

In the above equation $\epsilon_{i\theta}$, ϵ_{iy} and $\epsilon_{i\theta y}$ are the nonlinear membrane strains and can be mathematically expressed as follows;

$$\epsilon_{i\theta} = \frac{u_{i\theta} + w}{r_i} + \frac{1}{2} \left(\frac{w_\theta}{r_i} \right)^2$$

$$\epsilon_{iy} = v_{iy} + \frac{1}{2} (w_y)^2$$

$$\epsilon_{i\theta y} = u_{iy} + \frac{v_{i\theta}}{r_i} + \left(\frac{w_\theta w_y}{r_i} \right)$$

The average shear strains, $\gamma_{\theta r}$ and γ_{yr} are written the deformed and undeformed section of the shell unit shown in Figure 3.2.

$$\gamma_{\theta r} = \phi_{\theta} + \theta_{\theta}$$

$$\gamma_{\theta r} = \frac{-\partial w}{\partial \theta} + \frac{\partial u}{\partial r}$$

$$\gamma_{\theta r} = \left(\frac{1}{r_1}\right) \frac{-\partial w}{\partial \theta} + \frac{[u_1 - u_2 - \frac{\partial w}{\partial \theta} (\frac{1}{r_1} \frac{h_1}{2} + \frac{1}{r_2} \frac{h_2}{2})]}{t}$$

$$\gamma_{\theta r} = \frac{(u_1 - u_2 - (\frac{1}{r_1} \frac{h_1}{2} + \frac{1}{r_2} \frac{h_2}{2} + \frac{t}{r_1}) \frac{\partial w}{\partial \theta})}{t}$$

$$\gamma_{yr} = \phi_y + \theta_y$$

$$\gamma_{yr} = \frac{-\partial w}{\partial y} + \frac{\partial v}{\partial r}$$

$$\gamma_{yr} = \frac{-\partial w}{\partial y} + \frac{[v_1 - v_2 - \frac{\partial w}{\partial y} (\frac{h_1}{2} + \frac{h_2}{2})]}{t}$$

$$\gamma_{yr} = \frac{[v_1 - v_2 - (\frac{h_1}{2} + \frac{h_2}{2} + t) \frac{\partial w}{\partial y}]}{t}$$

Substituting shear strain equations, shear strain energy equation becomes,

$$U_{\theta r}^I = \int_V U_{\theta r}^I dV$$

$$U_{\theta r}^I = \int_{-\theta_1}^{\theta_1} \int_{-a}^a \frac{G t r_1}{2} \left(\frac{[u_1 - u_2 - (\frac{1}{r_1} \frac{h_1}{2} + \frac{1}{r_2} \frac{h_2}{2} + \frac{t}{r_1}) \frac{\partial w}{\partial y}]}{t} \right)^2 d\theta dy$$

$$U_{yr}^I = \int_0^t \int_{-\theta_1}^{\theta_1} \int_{-a}^a \frac{1}{2} G \gamma_{yr}^2 dV$$

$$U_{yr}^I = \int_{-\theta_1}^{\theta_1} \int_{-a}^a \frac{Gtr_I}{2} \left(\frac{[v_1 - v_2 - (\frac{h_1}{2} + \frac{h_2}{2} + t) \frac{\partial w}{\partial y}]^2}{t} \right) d\theta dy \quad (3.5)$$

The model is subjected a uniform distributed load and the Load potential equation can written as;

$$\Omega = \int_{-\theta_1}^{\theta_1} \int_{-a}^a \Omega d\theta dy$$

$$\Omega = - \int_{-\theta_1}^{\theta_1} \int_{-a}^a qw d\theta dy \quad (3.6)$$

The presence of a negative sign signifies a decrease in potential energy as the force is displaced in its respective direction.

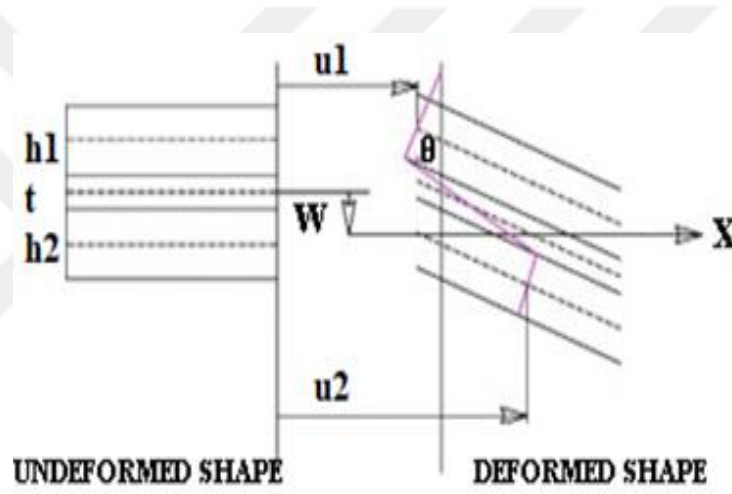


Figure 3. 2 Laminated glass section

Equations above,

$i = 1, 2$ indicate is use to differentiate between top and bottom shells,

E = Young modulus

G = Shear modulus of the interlayer,

h_i = individual glass thickness,

r_i = radius of the shell,

ν = Poisson's ratio,

r_I = radius of the interlayer,

t = interlayer thickness,

w = radial deflection of the laminated glass unit for the top and bottom shell,

u_i, v_i = in the plane displacements of top and bottom glass shells, respectively,

Substituting the above equations, the total potential energy can write as:

$$V = \int_V (U_m^1 + U_m^2 + U_b^1 + U_b^2 + U_{yr}^1 + U_{\theta r}^1 + \Omega) dV \quad (3.7)$$

$$V = \int_{-\theta_1}^{\theta_1} \int_{-a}^a F d\theta dy$$

Here F can be expressed as:

$$\begin{aligned} F = & \frac{Gtr_1}{2} \left[\left(\frac{u_1 - u_2 - \left(\frac{1}{r_1} \frac{h_1}{2} + \frac{1}{r_2} \frac{h_2}{2} + \frac{t}{r_1} \right) \frac{\partial w}{\partial \theta}}{t} \right)^2 + \left(\frac{[v_1 - v_2 - \left(\frac{h_1}{2} + \frac{h_2}{2} + t \right) \frac{\partial w}{\partial y}]}{t} \right)^2 \right] + \frac{Eh_i^3 r_i}{24(1-\nu^2)} \left[\left(-\frac{w_{\theta\theta}^2}{r_i^2} \right)^2 + \right. \\ & (-w_{yy})^2 + 2\nu \left(\frac{w_{\theta\theta}^2}{r_i^2} \right) (-w_{yy}) + \left(\frac{1-\nu}{2} \right) \left(-\frac{2}{r_i} w_{\theta y} \right)^2 \left. \right] + \frac{Eh_i r_i}{2(1-\nu^2)} [e_{iy}^2 + e_{iy}^2 + 2\nu e_{i\theta} e_{iy} + \\ & \left(\frac{1-\nu}{2} \right) e_{i\theta y}^2] - qw \end{aligned} \quad (3.8)$$

Variational method is used to solve this problem. One of the benefits of the variational approach is its ability to cope with fundamental lack of regularity, which is useful for characterizing failure phenomena when Partial differential equation-based formulations are either unavailable or difficult to address using normal approaches (Luca and Ponsiglione, 2021). Variation of the total potential energy of the system with respect to the radial and in plane displacements (u_1 , u_2 , v_1 , v_2 and w) gives the nonlinear governing differential equations.

The governing differential equations of the system are obtained by applying the Euler Equation, which is given by Langhaar (1962), to the total potential energy.

Euler equation in the most general form:

$$\frac{\partial F}{\partial u_i} - \frac{\partial}{\partial \theta} \left(\frac{\partial F}{\partial u_{i,\theta}} \right) - \frac{\partial}{\partial y} \left(\frac{\partial F}{\partial u_{i,y}} \right) + \frac{\partial^2}{\partial \theta^2} \left(\frac{\partial F}{\partial u_{i,\theta\theta}} \right) + \frac{\partial^2}{\partial \theta \partial y} \left(\frac{\partial F}{\partial u_{i,\theta y}} \right) + \frac{\partial^2}{\partial y^2} \left(\frac{\partial F}{\partial u_{i,yy}} \right) = 0 \quad (3.9)$$

u_i = implies u_1 , v_1 , u_2 , v_2 , and w ,

$u_{i,\theta}$ = derivative of u_i with respect to θ ,

$u_{i,y}$ = derivative of u_i with respect to y ,

$u_{i,\theta\theta}$ = second derivate of u_i with respect to θ ,

$u_{i,yy}$ = second derivate of u_i with respect to y ,

$u_{i,\theta y}$ = is the second order cross derivative of u_i

The five nonlinear equations obtained by Vallabhan and Das (1990) for laminated section with respect to radial and in plane displacement, mathematical computations yield the following five nonlinear governing equations of laminated glass shell unit:

$$-\frac{d(N_{1y\theta}r_2)}{dy} - \frac{dN_{1\theta}}{d\theta} - \frac{Gr_1}{t} \left(u_1 - u_2 - \left(\frac{1}{r_1} \frac{h_1}{2} + \frac{1}{r_2} \frac{h_2}{2} + \frac{t}{r_1} \right) \frac{\partial w}{\partial \theta} \right) = 0 \quad (3.10)$$

$$-\frac{d(N_{1y\theta}r_2)}{dy} - \frac{dN_{2\theta}}{d\theta} - \frac{Gr_1}{t} \left(u_1 - u_2 - \left(\frac{1}{r_1} \frac{h_1}{2} + \frac{1}{r_2} \frac{h_2}{2} + \frac{t}{r_1} \right) \frac{\partial w}{\partial \theta} \right) = 0 \quad (3.11)$$

$$-\frac{d(N_{1y\theta})}{d\theta} - \frac{1}{r_1} \frac{dN_{1y}}{dy} - \frac{Gr_1}{t} \left(v_1 - v_2 - \left(\frac{h_1}{2} + \frac{h_2}{2} + t \right) \frac{\partial w}{\partial \theta} \right) = 0 \quad (3.12)$$

$$-\frac{d(N_{2y\theta})}{d\theta} - \frac{1}{r_2} \frac{dN_{2y}}{dy} - \frac{Gr_1}{t} \left(v_1 - v_2 - \left(\frac{h_1}{2} + \frac{h_2}{2} + t \right) \frac{\partial w}{\partial \theta} \right) = 0 \quad (3.13)$$

$$\begin{aligned} & -qr_i + N_{1\theta} + N_{2\theta} - (N_{1y}r_1 + N_{2y}r_2) \left(\frac{\partial^2 w}{\partial y^2} \right) - \left(\frac{N_{1\theta}}{r_1} + \frac{N_{2\theta}}{r_2} \right) \left(\frac{\partial^2 w}{\partial \theta^2} \right) - \\ & (2N_{1\theta y} + 2N_{2\theta y}) \left(\frac{\partial^2 w}{\partial \theta \partial y} \right) + \frac{Gr_1}{t} \left(\frac{h_1}{2} + \frac{h_2}{2} + t \right) \left(\frac{\partial v_1}{\partial y} - \frac{\partial v_2}{\partial y} - \left(\frac{h_1}{2} + \frac{h_2}{2} + t \right) \frac{\partial^2 w}{\partial y^2} \right) + \\ & \frac{Gr_1}{t} \left(\frac{1}{r_1} \frac{h_1}{2} + \frac{1}{r_2} \frac{h_2}{2} + \frac{t}{r_1} \right) \left(\frac{\partial u_1}{\partial \theta} - \frac{\partial u_2}{\partial \theta} - \left(\frac{1}{r_1} \frac{h_1}{2} + \frac{1}{r_2} \frac{h_2}{2} + \frac{t}{r_1} \right) \frac{\partial^2 w}{\partial \theta^2} \right) + \frac{Gr_1}{t} \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \left[u_1 - \right. \\ & \left. u_2 - \left(\frac{1}{r_1} \frac{h_1}{2} + \frac{1}{r_2} \frac{h_2}{2} + \frac{t}{r_1} \right) \frac{\partial w}{\partial \theta} \right] + \frac{Eh_1^3}{12(1-\nu^2)} \left[r_1 \frac{\partial^4 w}{\partial y^4} + \frac{2(1-\nu)}{r_1} \frac{\partial^4 w}{\partial y^2 \partial \theta^2} + \frac{1}{r_1^3} \frac{\partial^4 w}{\partial \theta^4} \right] + \\ & \frac{Eh_2^3}{12(1-\nu^2)} \left[r_2 \frac{\partial^4 w}{\partial y^4} + \frac{2(1-\nu)}{r_2} \frac{\partial^4 w}{\partial y^2 \partial \theta^2} + \frac{1}{r_2^3} \frac{\partial^4 w}{\partial \theta^4} \right] = 0 \end{aligned} \quad (3.14)$$

The extended form of the governing equations can be expressed as follows:

$$\begin{aligned} & \left(\frac{1}{r_1^2} \frac{\partial^2}{\partial \theta^2} + \frac{1-\nu}{2} \frac{\partial^2}{\partial y^2} - \frac{Gr_1(1-\nu^2)}{Eh_1r_1t} \right) u_1 = \frac{\partial w}{\partial \theta} \left(\frac{1}{r_1^2} + \frac{1}{r_1^3} \frac{\partial^2 w}{\partial \theta^2} + \frac{(1-\nu)}{2r_1} \frac{\partial^2 w}{\partial y^2} \right) + \frac{Gr_1(1-\nu^2)}{Eh_1r_1t} \left(-u_2 - \right. \\ & \left. \left(\frac{1}{r_1} \frac{h_1}{2} + \frac{1}{r_2} \frac{h_2}{2} + \frac{t}{r_1} \right) \frac{\partial w}{\partial \theta} \right) - \frac{\partial w}{\partial y} \frac{\partial^2 w}{\partial \theta \partial y} \left(\frac{1+\nu}{2r_1} \right) - \frac{\partial^2 v_1}{\partial \theta \partial y} \left(\frac{1+\nu}{2r_1} \right) \end{aligned} \quad (3.15)$$

$$\left(\frac{1}{r_2^2} \frac{\partial^2}{\partial \theta^2} + \frac{1-v}{2} \frac{\partial^2}{\partial y^2} - \frac{Gr_1(1-v^2)}{Eh_2 r_2 t}\right) u_2 = \frac{Gr_1(1-v^2)}{Eh_2 r_2 t} \left(u_1 - \left(\frac{1}{r_1} \frac{h_1}{2} + \frac{1}{r_2} \frac{h_2}{2} + \frac{t}{r_1}\right) \frac{\partial w}{\partial \theta}\right) - \frac{\partial w}{\partial \theta} \left(\frac{1}{r_2^2} + \frac{1}{r_2^3} \frac{\partial^2 w}{\partial \theta^2} + \frac{(1-v)}{2r_2} \frac{\partial^2 w}{\partial y^2}\right) - \frac{\partial w}{\partial y} \frac{\partial^2 w}{\partial \theta \partial y} \left(\frac{1+v}{2r_2}\right) - \frac{\partial^2 v_2}{\partial \theta \partial y} \left(\frac{1+v}{2r_2}\right) \quad (3.16)$$

$$\left(\frac{\partial^2}{\partial y^2} + \frac{1-v}{2r_1^2} \frac{\partial^2}{\partial \theta^2} - \frac{Gr_1(1-v^2)}{Eh_1 r_1 t}\right) v_1 = \frac{Gr_1(1-v^2)}{Eh_1 r_1 t} \left(-v_2 - \left(\frac{h_1}{2} + \frac{h_2}{2} + t\right) \frac{\partial w}{\partial y}\right) - \frac{\partial w}{\partial y} \left(\frac{v}{r_1} + \frac{\partial^2 w}{\partial y^2} + \frac{(1-v)}{2r_1^2} \frac{\partial^2 w}{\partial \theta^2}\right) - \frac{\partial w}{\partial \theta} \frac{\partial^2 w}{\partial \theta \partial y} \left(\frac{1+v}{2r_1^2}\right) - \frac{\partial^2 u_1}{\partial \theta \partial y} \left(\frac{1+v}{2r_1}\right) \quad (3.17)$$

$$\left(\frac{\partial^2}{\partial y^2} + \frac{1-v}{2r_2^2} \frac{\partial^2}{\partial \theta^2} - \frac{Gr_1(1-v^2)}{Eh_2 r_2 t}\right) v_2 = \frac{Gr_1(1-v^2)}{Eh_1 r_1 t} \left(v_1 - \left(\frac{h_1}{2} + \frac{h_2}{2} + t\right) \frac{\partial w}{\partial y}\right) - \frac{\partial w}{\partial y} \left(\frac{v}{r_2} + \frac{\partial^2 w}{\partial y^2} + \frac{(1-v)}{2r_2^2} \frac{\partial^2 w}{\partial \theta^2}\right) - \frac{\partial w}{\partial \theta} \frac{\partial^2 w}{\partial \theta \partial y} \left(\frac{1+v}{2r_2^2}\right) - \frac{\partial^2 u_2}{\partial \theta \partial y} \left(\frac{1+v}{2r_2}\right) \quad (3.18)$$

$$\begin{aligned} & \frac{E}{12(1-v^2)} \left[\left(\frac{h_1^3}{r_1^3} + \frac{h_2^3}{r_2^3}\right) \frac{\partial^4}{\partial \theta^4} + (r_1 h_1^3 + r_2 h_2^3) \frac{\partial^4}{\partial y^4} + \left(\frac{2h_1^3(1-v)}{r_1} + \frac{2h_2^3(1-v)}{r_2}\right) \frac{\partial^4}{\partial y^2 \partial \theta^2} \right] w + \\ & \frac{E}{(1-v^2)} \left(\frac{h_1}{r_1} + \frac{h_2}{r_2}\right) w - \frac{Gr_1}{t} \left[\left(\frac{1}{r_1} \frac{h_1}{2} + \frac{1}{r_2} \frac{h_2}{2} + \frac{t}{r_1}\right)^2 \frac{\partial^2}{\partial \theta^2} + \left(\frac{h_1}{2} + \frac{h_2}{2} + t\right)^2 \frac{\partial^2}{\partial y^2} - \left(\frac{1}{r_1} \frac{h_1}{2} + \frac{1}{r_2} \frac{h_2}{2} + \frac{t}{r_1}\right) \left(\frac{1}{r_1} - \frac{1}{r_2}\right) \frac{\partial}{\partial \theta}\right] w = qr_1 + \frac{Eh_i}{(1+v)} \left[\left(\frac{\partial u_1}{\partial y} + \frac{1}{r_1} \frac{\partial v_1}{\partial \theta} + \frac{1}{r_1} \frac{\partial w}{\partial \theta} \frac{\partial w}{\partial y}\right)\right] \frac{\partial^2 w}{\partial y \partial \theta} - \frac{Eh_i}{(1-v^2)} \left[\left(\frac{1}{r_1} \frac{\partial u_1}{\partial \theta} + \frac{1}{2r_1^2} \frac{\partial^2 w}{\partial \theta^2}\right) + v \left(\frac{\partial v_1}{\partial y} + \frac{1}{2} \frac{\partial^2 w}{\partial y^2}\right)\right] + \frac{Eh_i}{(1-v^2)r_1} \left[\left(\frac{1}{r_1} \frac{\partial u_1}{\partial \theta} + \frac{w}{r_1} + \frac{1}{2r_1^2} \frac{\partial^2 w}{\partial \theta^2}\right) + v \left(\frac{\partial v_1}{\partial y} + \frac{1}{2} \frac{\partial^2 w}{\partial y^2}\right)\right] \frac{\partial^2 w}{\partial \theta^2} + \frac{Eh_i}{(1-v^2)} \left[\left(\frac{\partial v_1}{\partial y} + \frac{1}{2} \frac{\partial^2 w}{\partial y^2}\right) + v \left(\frac{1}{r_1} \frac{\partial u_1}{\partial \theta} + \frac{w}{r_1} + \frac{1}{2r_1^2} \frac{\partial^2 w}{\partial \theta^2}\right)\right] \frac{\partial^2 w}{\partial y^2} - \frac{Gr_1}{t} \left(\frac{h_1}{2} + \frac{h_2}{2} + t\right) \left(\frac{\partial v_1}{\partial y} - \frac{\partial v_2}{\partial y}\right) - \frac{Gr_1}{t} \left(\frac{1}{r_1} \frac{h_1}{2} + \frac{1}{r_2} \frac{h_2}{2} + \frac{t}{r_1}\right) \left(\frac{\partial u_1}{\partial \theta} - \frac{\partial u_2}{\partial \theta}\right) + \frac{Gr_1}{t} \left(\frac{1}{r_1} - \frac{1}{r_2}\right) (u_1 - u_2) \quad (3.19) \end{aligned}$$

In the above equations, governing equations given in extended form where left side of the equations contains linear terms while non-linear terms are presented on the right side of the equation.

Where;

$$N1_\theta = \frac{Eh_1}{(1-v^2)} (\varepsilon_{1\theta} + v\varepsilon_{1y})$$

$$N2_\theta = \frac{Eh_2}{(1-v^2)} (\varepsilon_{2\theta} + v\varepsilon_{2y})$$

$$N1_y = \frac{Eh_1}{(1 - v^2)}(\varepsilon_{1y} + v\varepsilon_{1\theta})$$

$$N2_y = \frac{Eh_2}{(1 - v^2)}(\varepsilon_{2y} + v\varepsilon_{2\theta})$$

$$N1_{\theta y} = \frac{Eh_1}{2(1 - v)}(\varepsilon_{1\theta y})$$

$$N2_{\theta y} = \frac{Eh_2}{2(1 - v)}(\varepsilon_{2\theta y})$$



4. DERIVATION OF GOVERNING EQUATIONS AND BOUNDARY CONDITIONS AND INCREMENTAL SOLUTION TECHNIQUES.

In the fields of science, engineering, and mathematics, differentiable equations are frequently used to describe and represent a variety of events. It is proving to be one of the most effective tools for modeling, analyzing, and comprehending a wide range of phenomena. When characterizing a solid body's reaction, non-linearities are typically present. Solving these equations can be difficult since they frequently display complex and differing behaviors. As a result, numerical methods and iterative techniques are commonly employed to provide numerical solutions.

Variational method is used to solve the current problem. One of the benefits of the variational approach is its ability to cope with fundamental lack of regularity, which is useful for characterizing failure phenomena when PDE-based formulations are either unavailable or difficult to address using normal approaches (Luca and Ponsiglione, 2021). The total potential energy of the system with respect to the radial and in plane displacements (u_1, u_2, v_1, v_2 and w) gives the nonlinear governing differential equations. The governing differential equations of the system are obtained by applying the Euler Equation, which is given by Langhaar (1962), to the total potential energy.

To simplify the analyze and to reduce the computation time, a quarter of the specimen is analyzed, which simplifies the setup, minimizes its complexity, and eliminates possible sources of error. This can increase the accuracy and dependability of results, thereby raising the overall standard of the study outcome.

The set of equations (3.10–3.14) that describe non-delaminated components and equations (3.15–3.19) that characterize delaminated sections as linear on the left. Using the finite difference approach, these equations are then converted into a system of linear algebraic equations. The equation is structured such that the left side of the equation contains matrix A , while the right side of the equation contains the nonlinear terms and the applied load of the field equation. For this reason, numerical method is used to find approximate solutions to equations. Numerical iteration methods are applied to solve the equations.

For precision in the modelling, it is necessary to carry out the simulation by meshing. The sizes are written as $n_x \times n_y$, where n_x and n_y are the subdivisions in the x and y axes of the radial

displacement, respectively. The obtained equations are applied to the domain by considering the boundary conditions which restrains the behavior at the boundary of unit.

In a study made by Zedan and Schneider (1981) it was found out that “The modified strongly implicit procedure is a significant advance in the state of the art of solving finite-difference equations used to describe field problems. It is derived for a nine-point difference scheme and can be applied to the five-point scheme. It offers superior results when iterations or computational cost are used as the measure of performance, and is less sensitive to control volume aspect ratio, relaxation parameters, and mesh subdivision.”

A set of finite difference equations for the radial deflection field of the equation at a specific location of an undelaminated section of laminated glass is used.

The written forms of the equations are as follows:

$$[A]\{w\}=\{RSH\} \quad (4.1)$$

Where;

$\{w\}$ = the radial displacement vector,

q = The magnitude of the applied pressure,

$i = 3, 4, \dots n_x - 2$;

$j = 3, 4, \dots n_y - 2$.

$$\begin{aligned} RSH_{(i,j)} = & C_{w(i,j)} + B_{w(i+1,j)} + B_{w(i-1,j)} + H_{w(i+2,j)} + H_{w(i-2,j)} + \\ & J_{w(i,j+1)} + J_{w(i,j-1)} + G_{w(i,j+2)} + G_{w(i,j-2)} + F_{w(i+1,j+1)} + F_{w(i+1,j-1)} + \\ & F_{w(i-1,j+1)} + F_{w(i-1,j-1)} \end{aligned} \quad (4.2)$$

Where;

$RSH_{(i,j)}$ is the equation the right.

$(u_1, u_2, v_1, v_2, w,)$, is the nonlinear part of the equation.

$$\{RSH\}_{(i,j)} = \{q + (u_1, u_2, v_1, v_2, w,)\}$$

$$RHS = qr_1 - N_{1\theta} - N_{2\theta} + (N_{1y}r_1 + N_{2y}r_2)wyy + \left(\frac{N_{1\theta}}{r_1} + \frac{N_{2\theta}}{r_2}\right)w\theta\theta + (2N_{1\theta y} + 2N_{2\theta y}) - \frac{Gr_I}{t} \left(\frac{h_1}{2} + \frac{h_2}{2} + t\right) \left(\frac{\partial v_1}{\partial y} - \frac{\partial v_2}{\partial y}\right) - \frac{Gr_I}{t} \left(\frac{1}{r_1} \frac{h_1}{2} + \frac{1}{r_2} \frac{h_2}{2} + \frac{1}{r_I} t\right) \left(\frac{\partial u_1}{\partial \theta} - \frac{\partial u_2}{\partial \theta}\right) - \frac{Gr_I}{t} \left(\frac{1}{r_1} - \frac{1}{r_2}\right) (u_1 - u_2) \quad (4.3)$$

$$C = \left(\frac{6D_1r_1+6D_1r_2}{h_y^4}\right) + \left(\frac{6D_1}{r_1^3h_\theta^4} + \frac{6D_2}{r_2^3h_\theta^4}\right) + \left(\frac{8D_1}{r_1h_\theta^2h_y^2} + \frac{8D_2}{r_1h_\theta^2h_y^2}\right) + \left(\frac{2D_1}{r_1h_1^2} + \frac{2D_2}{r_2h_2^2}\right) + \frac{2Gr_I}{th_y^2} \left(\frac{h_1}{2} + \frac{h_2}{2} + t\right)^2 + \frac{2Gr_I}{th_\theta^2} \left(\frac{h_1}{2} \frac{1}{r_1} + \frac{1}{r_2} \frac{h_2}{2} + \frac{t}{r_I}\right)^2 \quad (4.4)$$

$$B = \left(\frac{-4D_1}{r_1^3h_\theta^4} + \frac{-4D_2}{r_2^3h_\theta^4}\right) + \left(\frac{-4D_1}{r_1h_\theta^2h_y^2} + \frac{-4D_2}{r_1h_\theta^2h_y^2}\right) + \frac{Gr_I}{th_\theta^2} \left(\frac{h_1}{2} \frac{1}{r_1} + \frac{1}{r_2} \frac{h_2}{2} + \frac{t}{r_I}\right)^2 + \frac{Gr_I}{2th_\theta^2} \left(\frac{1}{r_1} - \frac{1}{r_2}\right) \left(\frac{h_1}{2} \frac{1}{r_1} + \frac{1}{r_2} \frac{h_2}{2} + \frac{t}{r_I}\right)^2 \quad (4.5)$$

$$H = \frac{6D_1}{r_1^3h_\theta^4} + \frac{6D_2}{r_2^3h_\theta^4} \quad (4.6)$$

$$F = \left(\frac{2D_1}{r_1h_\theta^2h_y^2} + \frac{2D_2}{r_1h_\theta^2h_y^2}\right) \quad (4.7)$$

$$J = \left(\frac{-4D_1r_1-4D_1r_2}{h_y^4}\right) - \left(\frac{8D_1}{r_1h_\theta^2h_y^2} + \frac{8D_2}{r_1h_\theta^2h_y^2}\right) - \frac{Gr_I}{th_y^2} \left(\frac{h_1}{2} + \frac{h_2}{2} + t\right)^2 \quad (4.8)$$

$$G = \left(\frac{6D_1r_1+6D_1r_2}{h_y^4}\right) \quad (4.9)$$

Equation 3.2 is altered to be able to be used to solve for the delaminated part since it has a different potential energy than the undelaminated part. According to the following, alterations are required for the right-hand side coefficient matrix and vector in the delaminated region:

$$C = \left(\frac{6D_1r_1+6D_1r_2}{h_y^4}\right) + \left(\frac{6D_1}{r_1^3h_\theta^4} + \frac{6D_2}{r_2^3h_\theta^4}\right) + \left(\frac{8D_1}{r_1h_\theta^2h_y^2} + \frac{8D_2}{r_1h_\theta^2h_y^2}\right) + \left(\frac{2D_1}{r_1h_1^2} + \frac{2D_2}{r_2h_2^2}\right) \quad (4.10)$$

$$J = \left(\frac{-4D_1r_1-4D_1r_2}{h_y^4}\right) - \left(\frac{8D_1}{r_1h_\theta^2h_y^2} + \frac{8D_2}{r_1h_\theta^2h_y^2}\right) \quad (4.11)$$

$$B = \left(\frac{6D_1r_1+6D_1r_2}{h_y^4}\right) + \left(\frac{-4D_1}{r_1^3h_\theta^4} + \frac{-4D_2}{r_2^3h_\theta^4}\right) + \left(\frac{-4D_1}{r_1h_\theta^2h_y^2} + \frac{-4D_2}{r_1h_\theta^2h_y^2}\right) \quad (4.12)$$

$$G = \left(\frac{6D_1r_1+6D_1r_2}{h_y^4}\right) \quad (4.13)$$

$$H = \frac{6D_1}{r_1^3 h_\theta^4} + \frac{6D_2}{r_2^3 h_\theta^4} \quad (4.14)$$

$$F = \left(\frac{2D_1}{r_1 h_\theta^2 h_y^2} + \frac{2D_2}{r_1 h_\theta^2 h_y^2} \right) \quad (4.15)$$

$$RHS = qr_1 - N_{1\theta} - N_{2\theta} + (N_{1y}r_1 + N_{2y}r_2)wyy + \left(\frac{N_{1\theta}}{r_1} + \frac{N_{2\theta}}{r_2} \right) w\theta\theta + (2N_{1\theta y} + 2N_{2\theta y}) \quad (4.16)$$

Applying boundary condition to the Equation 3.2 the following equations are obtained:

For i=1; j=1.

$$\frac{1}{4}\{RSH_{(i,j)}\} = \frac{C}{4}W_{(i,j)} + \frac{B}{2}W_{(i+1,j)} + \frac{H}{2}W_{(i+2,j)} + \frac{J}{2}W_{(i,j+1)} + \frac{G}{2}W_{(i,j+2)} + F_{W_{(i-1,j-1)}} \quad (4.17)$$

For i=1; j=2.

$$\begin{aligned} \frac{1}{2}\{RSH_{(i,j)}\} &= \frac{C+G}{2}W_{(i,j)} + B_{W_{(i+1,j)}} + H_{W_{(i+2,j)}} + \frac{J}{2}W_{(i,j+1)} + \frac{J}{2}W_{(i,j-1)} + \\ &\frac{G}{2}W_{(i,j+2)} + F_{W_{(i+1,j+1)}} + F_{W_{(i+1,j-1)}} \end{aligned} \quad (4.18)$$

For i=1; j=3, $n_y - 1$.

$$\begin{aligned} \frac{1}{2}\{RSH_{(i,j)}\} &= \frac{C}{2}W_{(i,j)} + B_{W_{(i+1,j)}} + H_{W_{(i+2,j)}} + \frac{J}{2}W_{(i,j+1)} + \frac{J}{2}W_{(i,j-1)} + \\ &\frac{G}{2}W_{(i,j+2)} + \frac{G}{2}W_{(i,j-2)} + F_{W_{(i+1,j+1)}} + F_{W_{(i+1,j-1)}} \end{aligned} \quad (4.19)$$

For i=1; j=3, n_y

$$\begin{aligned} \frac{1}{2}\{RSH_{(i,j)}\} &= (C - G)W_{(i,j)} + B_{W_{(i+1,j)}} + H_{W_{(i+2,j)}} + \frac{J}{2}W_{(i,j-1)} + G_{W_{(i,j-2)}} + \\ &F_{W_{(i+1,j-1)}} \end{aligned} \quad (4.20)$$

For i=2; j=1.

$$\frac{1}{2}\{RSH_{(i,j)}\} = \frac{C+H}{2}W_{(i,j)} + \frac{B}{2}W_{(i+1,j)} + \frac{B}{2}W_{(i-1,j)} + \frac{H}{2}W_{(i+2,j)} + F_{W_{(i+1,j+1)}} + F_{W_{(i-1,j+1)}} + G_{W_{(i,j+2)}} \quad (4.21)$$

For $i=2; j=2$.

$$\{RSH_{(i,j)}\} = (C + H + G)_{W_{(i,j)}} + B_{W_{(i+1,j)}} + B_{W_{(i-1,j)}} + H_{W_{(i+2,j)}} + J_{W_{(i,j+1)}} + J_{W_{(i,j-1)}} + G_{W_{(i,j+2)}} + F_{W_{(i+1,j+1)}} + F_{W_{(i-1,j+1)}} + F_{W_{(i+1,j-1)}} + F_{W_{(i-1,j-1)}} \quad (4.22)$$

For $i=2; j=3, 4, 5 \dots n_y - 1$

$$\{RSH_{(i,j)}\} = (C + H)_{W_{(i,j)}} + B_{W_{(i+1,j)}} + B_{W_{(i-1,j)}} + H_{W_{(i+2,j)}} + J_{W_{(i,j+1)}} + J_{W_{(i,j-1)}} + G_{W_{(i,j+2)}} + F_{W_{(i+1,j+1)}} + F_{W_{(i-1,j+1)}} + F_{W_{(i+1,j-1)}} + F_{W_{(i-1,j-1)}} \quad (4.23)$$

For $i=2; j=n_y$,

$$\{RSH_{(i,j)}\} = (C - H + G)_{W_{(i,j)}} + B_{W_{(i+1,j)}} + B_{W_{(i-1,j)}} + H_{W_{(i+2,j)}} + J_{W_{(i,j-1)}} - F_{W_{(i-1,j-1)}} + F_{W_{(i+1,j-1)}} + G_{W_{(i,j-2)}} \quad (4.24)$$

For $i=3, 4, 5, \dots n_x - 1; j=n_y$,

$$\frac{1}{2}\{RSH_{(i,j)}\} = \frac{C}{2}W_{(i,j)} + \frac{B}{2}W_{(i+1,j)} + \frac{B}{2}W_{(i-1,j)} + \frac{H}{2}W_{(i-2,j)} + \frac{H}{2}W_{(i+2,j)} + J_{W_{(i-2,j)}} + F_{W_{(i+1,j+1)}} + F_{W_{(i-1,j+1)}} + G_{W_{(i,j+2)}} \quad (4.25)$$

For $i=3, 4, 5, \dots n_x - 1; j=n_y$,

$$\{RSH_{(i,j)}\} = (C - G)_{W_{(i,j)}} + B_{W_{(i+1,j)}} + B_{W_{(i-1,j)}} + H_{W_{(i+2,j)}} + H_{W_{(i-2,j-1)}} + F_{W_{(i-1,j-1)}} + F_{W_{(i+1,j-1)}} + J_{W_{(i,j-1)}} + G_{W_{(i,j-2)}} \quad (4.26)$$

For $i=n_x; j=1$

$$\frac{1}{2}\{RSH_{(i,j)}\} = \frac{C-H}{2}W_{(i,j)} + \frac{B}{2}W_{(i-1,j)} + \frac{H}{2}W_{(i-1,j-1)} + F_{W_{(i-1,j+1)}} + J_{W_{(i,j+1)}} + G_{W_{(i,j+2)}} \quad (4.27)$$

For $i=n_x$; $j=2$.

$$\{RSH_{(i,j)}\} = (C-H+G)_{W_{(i,j)}} + B_{W_{(i-1,j)}} + H_{W_{(i-2,j)}} + F_{W_{(i-1,j-1)}} + F_{W_{(i-1,j+1)}} + J_{W_{(i,j+1)}} + J_{W_{(i,j-1)}} + G_{W_{(i,j+2)}} \quad (4.28)$$

For $i=n_x$; $j=3, 4, 5, \dots, n_y-1$

$$\{RSH_{(i,j)}\} = (C-H)_{W_{(i,j)}} + B_{W_{(i-1,j)}} + H_{W_{(i-2,j)}} + J_{W_{(i,j-1)}} + J_{W_{(i,j+1)}} + F_{W_{(i-1,j-1)}} + F_{W_{(i-1,j+1)}} + G_{W_{(i,j+2)}} + G_{W_{(i,j-2)}} \quad (4.29)$$

For $i=n_x$; $j=n_y$

$$\{RSH_{(i,j)}\} = (C-H-G)_{W_{(i,j)}} + B_{W_{(i-1,j)}} + H_{W_{(i-2,j)}} + J_{W_{(i,j-1)}} + F_{W_{(i-1,j-1)}} + G_{W_{(i,j-2)}} \quad (4.30)$$

The matrix form of circumferential and axial displacements is given below. Full coefficient matrices are obtained for circumferential and axial displacements due to the nature of the problem. Calculation of the deflections u_1 , u_2 , v_1 , and v_2 and the conversion into discrete form are done using the strongly modified implicit approach established by Schneider and Zedan (1981) and Aşık et al. (2005).

$$A_{pv}(i,j) v_2(i,j) = A_{wv}(i,j) v_2(i-1,j) + A_{ev}(i,j) v_2(i+1,j) + A_{sv}(i,j) v_2(i,j-1) + A_{nv}(i,j) v_2(i,j+1) - F_{v2}(i,j) \quad (4.31)$$

$$A_{pu}(i,j) u_2(i,j) = A_{wu}(i,j) u_2(i-1,j) + A_{eu}(i,j) u_2(i+1,j) + A_{su}(i,j) u_2(i,j-1) + A_{nu}(i,j) u_2(i,j+1) - F_{u2}(i,j) \quad (4.32)$$

$$A_{pu}(i,j) u_1(i,j) = A_{wu}(i,j) u_1(i-1,j) + A_{eu}(i,j) u_1(i+1,j) + A_{su}(i,j) u_1(i,j-1)$$

$$+ \text{Anu}(i,j) u1(i,j+1) - \text{Fu1}(i,j) \quad (4.33)$$

$$\begin{aligned} \text{Apv}(i,j) v1(i,j) &= \text{Awv}(i,j) v1(i-1,j) + \text{Aev}(i,j) v1(i+1,j) + \text{Asv}(i,j) v1(i,j-1) \\ &+ \text{Anv}(i,j) v1(i,j+1) - \text{Fv1}(i,j) \end{aligned} \quad (4.34)$$

The coefficient of the laminated glass is expanded as follows:

$$\text{APU}1_{(i,j)} = \frac{(1-\nu)}{h_y^2} + \frac{2}{r_1^2 h_\theta^2} + \frac{G(1-\nu^2)r_I}{Eh_1 tr_1} \quad (4.35)$$

$$\text{AWU}1_{(i,j)} = \text{AEU}1_{(i,j)} = \frac{1}{r_1^2 h_\theta^2} \quad (4.36)$$

$$\text{ANU}1_{(i,j)} = \text{ASU}1_{(i,j)} = \frac{(1-\nu)}{2h_y^2} \quad (4.37)$$

$$\text{APV}1_{(i,j)} = +\frac{(1-\nu)}{h_\theta^2 r_1^2} + \frac{2}{h_y^2} + \frac{G(1-\nu^2)r_I}{Eh_1 tr_1} \quad (4.38)$$

$$\text{AWV}1_{(i,j)} = \text{AEV}1_{(i,j)} = \frac{(1-\nu)}{2r_1^2 h_\theta^2} \quad (4.39)$$

$$\text{ASV}1_{(i,j)} = \text{ANV}1_{(i,j)} = \frac{1}{h_y^2} \quad (4.40)$$

$$\text{APU}2_{(i,j)} = +\frac{(1-\nu)}{h_y^2} + \frac{2}{r_2^2 h_\theta^2} + \frac{G(1-\nu^2)r_I}{Eh_2 tr_2} \quad (4.41)$$

$$\text{AWU}2_{(i,j)} = \text{AEU}2_{(i,j)} = \frac{1}{r_2^2 h_\theta^2} \quad (4.42)$$

$$\text{ANU}2_{(i,j)} = \text{ASU}2_{(i,j)} = \frac{(1-\nu)}{2h_y^2} \quad (4.43)$$

$$\text{APV}2_{(i,j)} = +\frac{(1-\nu)}{h_\theta^2 r_2^2} + \frac{2}{h_y^2} + \frac{G(1-\nu^2)r_I}{Eh_2 tr_2} \quad (4.44)$$

$$\text{AWV}2_{(i,j)} = \text{AEV}2_{(i,j)} = \frac{(1-\nu)}{2r_2^2 h_\theta^2} \quad (4.45)$$

$$\text{ASV}2_{(i,j)} = \text{ANV}2_{(i,j)} = \frac{1}{h_y^2} \quad (4.46)$$

For the delaminated region, the in-plane displacement equations coefficient are arranged as:

$$APU1_{(i,j)} = \frac{(1-\nu)}{h_y^2} + \frac{2}{r_1^2 h_\theta^2} \quad (4.47)$$

$$AWU1_{(i,j)} = AEU1_{(i,j)} = \frac{1}{r_1^2 h_\theta^2} \quad (4.48)$$

$$ANU1_{(i,j)} = ASU1_{(i,j)} = \frac{(1-\nu)}{2h_y^2} \quad (4.49)$$

$$APV1_{(i,j)} = +\frac{(1-\nu)}{h_\theta^2 r_1^2} + \frac{2}{h_y^2} \quad (4.50)$$

$$AWV1_{(i,j)} = AEV1_{(i,j)} = \frac{(1-\nu)}{2r_1^2 h_\theta^2} \quad (4.51)$$

$$ASV1_{(i,j)} = ANV1_{(i,j)} = \frac{1}{h_y^2} \quad (4.52)$$

$$APU2_{(i,j)} = +\frac{(1-\nu)}{h_y^2} + \frac{2}{r_2^2 h_\theta^2} \quad (4.53)$$

$$AWU2_{(i,j)} = AEU2_{(i,j)} = \frac{1}{r_2^2 h_\theta^2} \quad (4.54)$$

$$ANU2_{(i,j)} = ASU2_{(i,j)} = \frac{(1-\nu)}{2h_y^2} \quad (4.55)$$

$$APV2_{(i,j)} = +\frac{(1-\nu)}{h_\theta^2 r_2^2} + \frac{2}{h_y^2} \quad (4.56)$$

$$AWV2_{(i,j)} = AEV2_{(i,j)} = \frac{(1-\nu)}{2r_2^2 h_\theta^2} \quad (4.57)$$

$$ASV2_{(i,j)} = ANV2_{(i,j)} = \frac{1}{h_y^2} \quad (4.58)$$

The following steps are followed in obtaining the iterative solution using the matrix form of equations:

- I. We assumed w ,
- II. The RHS is then calculated from equation (4.1)
- III. $w(i,j)$ is calculated using equation (4.1),
- IV. $w_{(i,j)} = \alpha w(i,j) + (1-\alpha) w_o(i,j)$,

$$V. \text{ The iteration is terminated when, } \frac{\sum_{i,j} |w(i,j) - w_o(i,j)|}{(\text{num} * w_{\max})} \leq 1$$

- VI. Using equation (4.31), $Fu_{1(i,j)}$ is solved and u_1 is obtained.
- VII. From Eqn (4.32), $Fu_{2(i,j)}$ is solved and u_2 is found.
- VIII. From Eqn (4.33), $Fv_{1(i,j)}$ is solved and v_1 is found.
- IX. From Eqn (4.34), $Fv_{2(i,j)}$ is solved and v_2 is found.
- X. Move to step II

α = Relaxation parameter variable

The variable (w) is incorporated using $\left(\frac{\omega_{max}}{h}\right)$, and the in-plane displacement is deduced using the value $\beta = 1.4$.



5. NUMERICAL SOLUTION AND RESULTS

In this study, mathematical model developed by Dural and Oyar (2023), to analyze laminated glass plate is modified for laminated glass shell subjected to initial delamination. Under pressure, laminated glass shell units exhibit extremely complicated and nonlinear behavior. The analysis process of the laminated glass unit involves two key steps. First, the total potential energy of the system is expressed in terms of displacements, capturing the energy stored within the unit. Then, nonlinear governing differential equations are derived using a variational approach and the minimum potential energy theorem, which minimize the total potential energy of the system and describe its nonlinear behavior. These steps are essential for understanding and characterizing the complex behavior of laminated glass units. The obtained equations are solved numerically. The model can describe the behavior of various glass types. In order to solve the nonlinear governing equations of differentiation, the finite difference approach is used once the equations have been obtained. This discretization involves approximating the partial differential equations with a system of algebraic equations. Subsequently, the resultant matrix systems are solved by using the programming language software Fortran. This software employs numerical techniques to efficiently and accurately solve the matrix equations, providing solutions for the behavior of the glass shell reaction. The Excel program is used to draw the graphs for the stress, radial displacements, and axial and circumferential displacements.

In this study, three laminated glass shell units are analyzed, each having a distinct ratio of initial delamination. These specimens are 1.24 m in curved direction and 1.44 m in the longitudinal directions. In the laminated glass shell unit, each glass sheet has the same thickness as $h_1 = h_2 = 5$ mm and the thickness of the interlayer is 0.38 mm. The unit has a total thickness of 10.38 mm. The glass Poisson's ratio is taken as 0.25 and Young's modulus as 72 GPa. The interlayer shear modulus and Poisson's ratio as 1000 kPa and 0.29 respectively. Physical characteristics of shell unit are given in Table 5.1.

Three different types of laminated shell glass with 15 %, 30 % and 45% of initial delamination named as Dlm 15%, Dlm 30%, and Dlm 45% respectively. layered glass (two glass layer without bond) and monolithic glass (single glass sheet) are used in this study. Figure 5.1 shows representation of laminated glass, layered glass, and monolithic glass components, whereas Figure 5.2 shows the region of delamination in delaminated specimens.

	Poisson's Ratio	Modulus		Dimensions(mm)		
		E	G	Thickness	Length	Curvature
GLASS 1	0.25	72 GPa	28.8 GPa	5	1440	1240
PVB	0.29	2900 kPa	1000 Kpa	0.38	1440	1240
GLASS 2	0.25	72 GPa	28.8 GPa	5	1440	1240

Table 5.1 Laminated glass shell physical characteristics

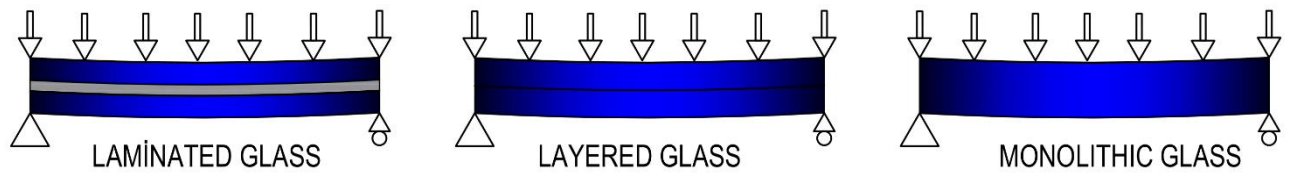


Figure 5.1 Section of Laminated glass, Layered glass, and Monolithic glass

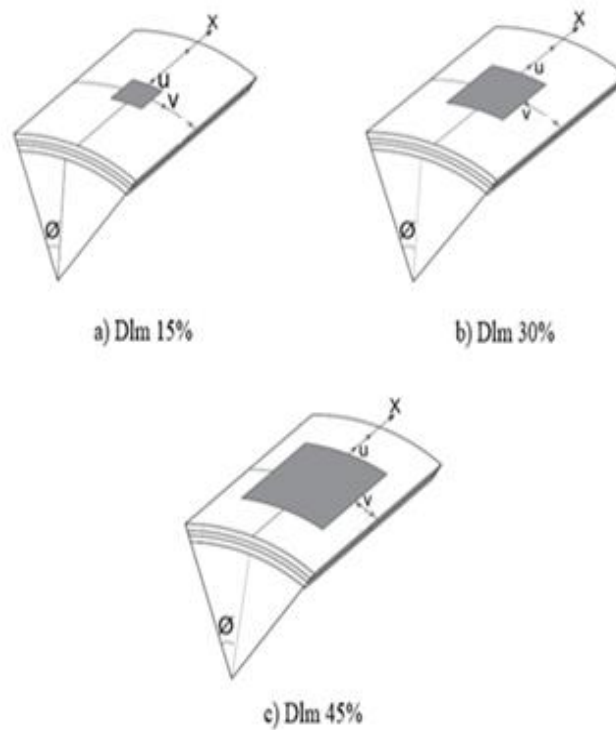


Figure 5.2 Representation of delaminated specimens

5.1. Numerical Results of a Fixed-Supported Laminated Glass Shell Under Uniformly Distributed Load.

In the first step of the study, laminated glass shell units subjected to fix supported boundary conditions are analyzed for layered, monolithic, laminated, and delaminated units. Figures 5.3 and 5.4 illustrate the difference between the linear and nonlinear methods for the fix supported laminated glass shell's behavior. The quotient obtained as a result of dividing the thickness of each glass by the maximum value of glass deflection is the nonlinearity level. The vertical axes show the normalized radial deflection, and the horizontal axes represent the load value. Overall, the deflection between the linear and nonlinear graphs increases while the load increases, while they are very close to each other for small load values. It is observed that the normalized radial deflections from zero to 10 kPa are essentially identical to one another, after which the gap in between starts to widen.

Figure 5.3 illustrates that the normalized radial deflection values of the linear and non-linear models are around 0.033 and 0.031, respectively, at 100 kPa. The linear and non-linear stress values are plotted in Figure 5.4. Likewise, in the radial deflection, the relationship between the linear and non-linear models is becoming more pronounced as the load increases. The values of stress obtained from the linear model are found to be lower than those obtained from the non-linear model. From these figures, we can conclude that as the load increases, the importance of nonlinear analysis increases

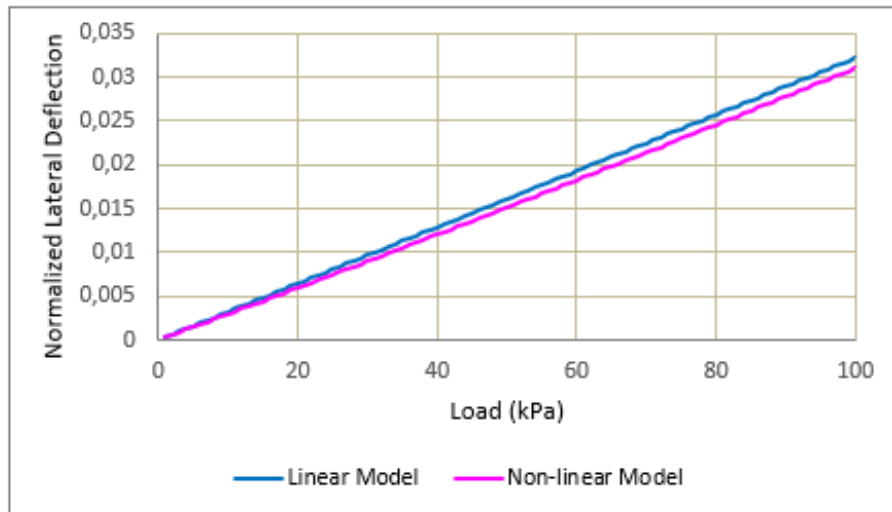


Figure 5.3 The load-normalized radial deflection ($\frac{w_{max}}{h}$) relationship for a fixed-supported laminated glass shell.

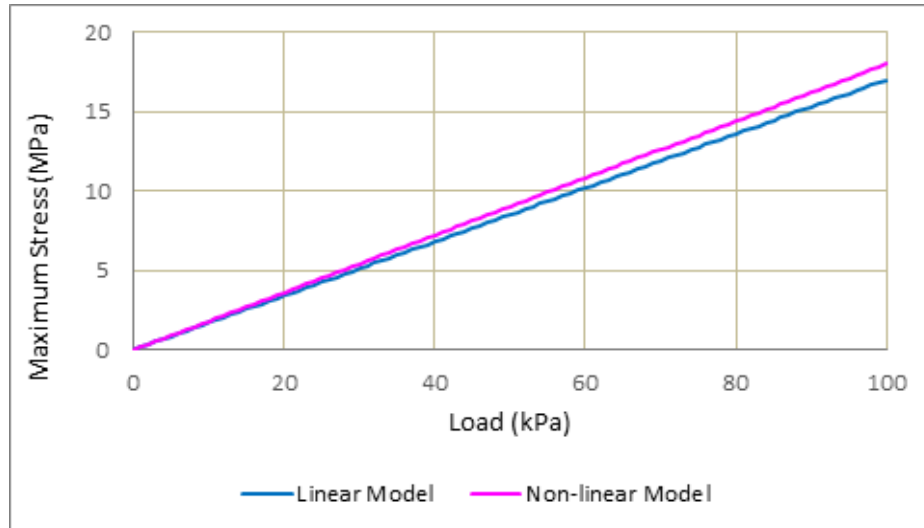


Figure 5.4 The stress-load relationship for a laminated glass shell with a fixed support.

The comparison of linear and non-linear model stress and normalized deflection values for Dlm 30%, are shown in Figures 5.5 and 5.6. From Figure 5.5, the normalized deflection of the non-linear model is around 0.025 at $P = 100$ kPa and 0.0275 for the linear model. The maximum stress level is around 16 MPa and 18 MPa for the linear and non-linear models, respectively, as seen in Figure 5.6. It is observed that Dlm 30% glass shell deflects less than laminated glass shell, which implies that the interlayer is still in place and successfully keeps the glass layers together, ensuring the structural integrity of the laminated glass shell. The laminated glass shell distributes the applied pressure more evenly as a result of this structural continuity, which increases its deflection capacity.

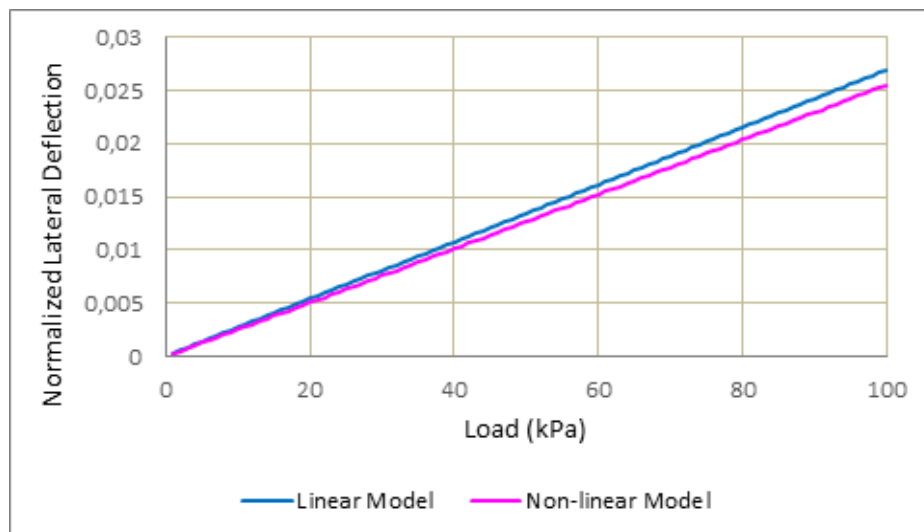


Figure 5.5 The load-normalized radial deflection ($\frac{w_{max}}{h}$) relationship for fixed-supported delaminated glass shell. (Dlm 30%)

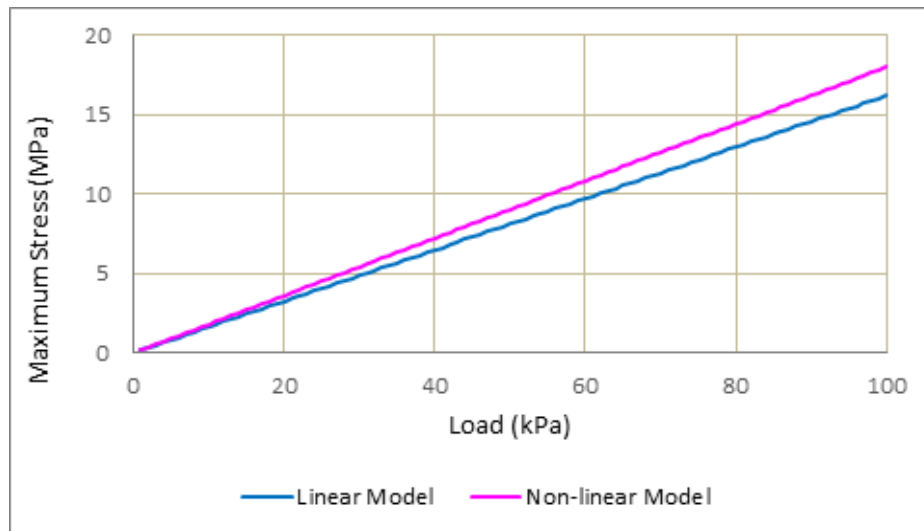


Figure 5.6 The stress-load relationship for a delaminated glass shell for fixed supported delaminated glass shell (Dlm 30%)

The revised model was equally effective at predicting the reaction of laminated, delaminated, layered, and monolithic glass shells in the current study. Figures 5.7 and 5.8 show the comparison of behaviors among the mentioned units. The graphs are plotted to indicate the displacement and maximum stress values versus load. There are important differences between these units. From Figure 5.7, it is observed that the displacement of Dlm 45%, is less as compared to Dlm 30%, and Dlm 15%. The effect of delamination in terms of displacements increases as the percentage of delamination increases. From Figure 5.8, the stress lines of Dlm 45% and Dlm 30% almost overlap. The material's stress and displacement values for varying load values identify the mechanical behavior of the unit. It is apparent from the figures that different percentage values of delamination cause varying mechanical behavior for laminated glass shell units.

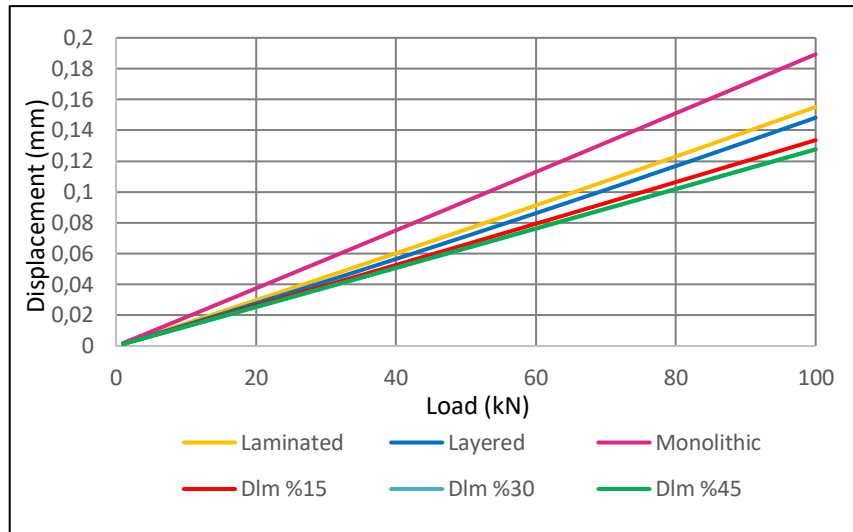


Figure 5.7 Maximum displacement versus load

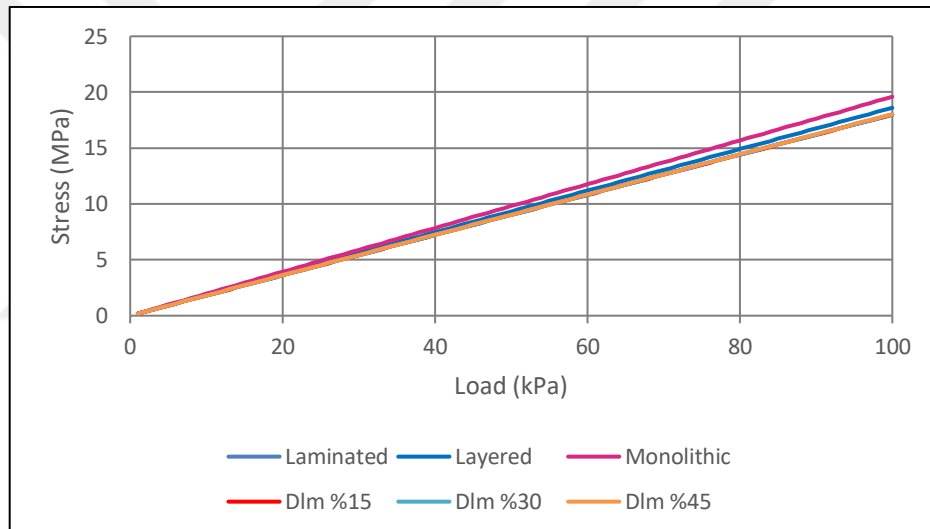


Figure 5.8 Stress-load relationship

The relationship between the in-plane displacements (mm) (u_1 and u_2), which represent the top and bottom glass sheets, respectively, is shown in Figures 5.9 and 5.10. The top glass's in-plane displacement line is shown in Figure 5.9, while the bottom glass's line is shown in Figure 5.10. As shown in the figures, the axial displacement values switch signs along the diagonal of the unit; the top glass displaces less as compared to the bottom glass. For delaminated glass units, the top glass layer displaces positively before displacing negatively. The bottom glass sheet displacement lines of delaminated glass units take both positive and negative values, in contrast with those of laminated layered and monolithic glass units.

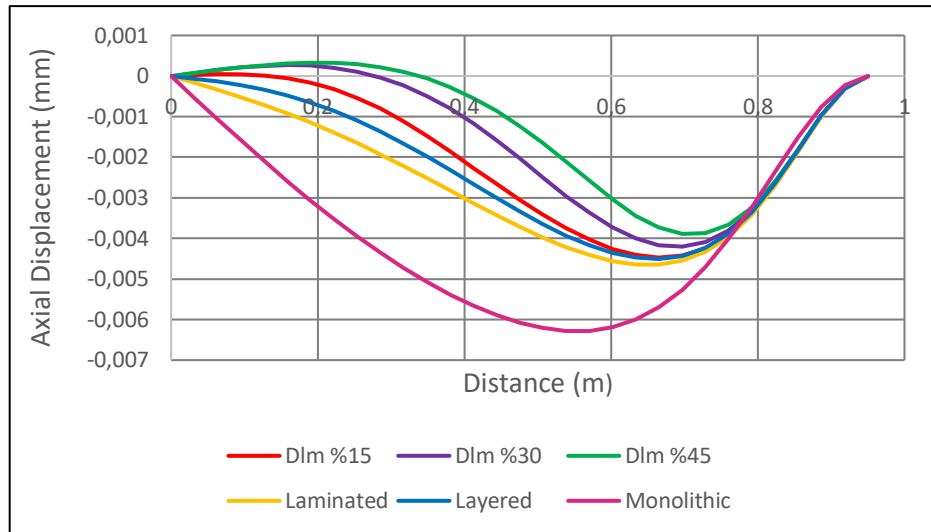


Figure 5.9 The in-plane displacement of the top glass unit along diagonal

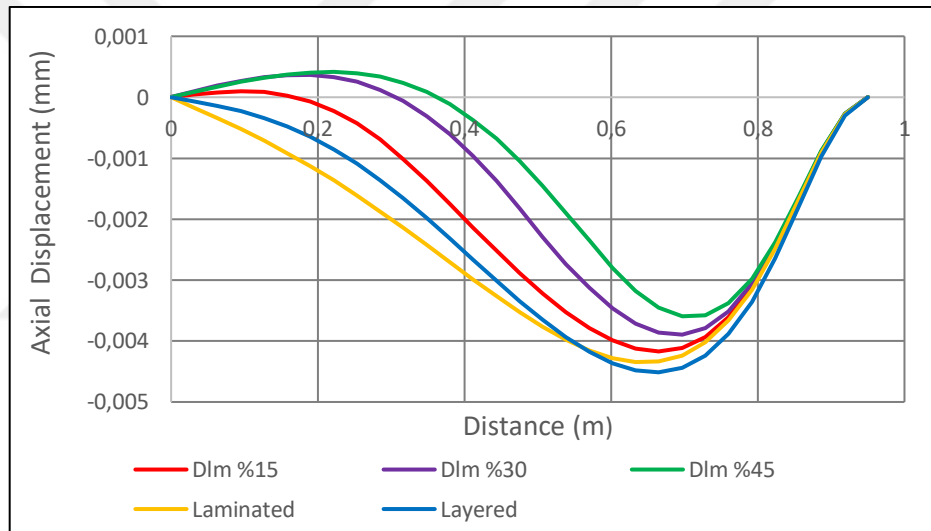


Figure 5.10 The in-plane displacement of the bottom glass unit along diagonal

Plots of the circumferential displacements in the θ -direction (v_1 and v_2) are shown in Figures 5.11 and 5.12. These plots represent the top glass and bottom glass units, respectively. There is no displacement at the boundary of the unit on the top and bottom glass shells. The maximum displacement is observed towards the corner of the unit. Examine the nature of changes in the signs of the displacement curves in the units between the distances (0.0–0.5) and (0.5–0.9). From Figure 5.11, It can be observed that, in decreasing order of displacement, monolithic, layered, and laminated glass increase positively at the start of the range of distance mentioned before decreasing between the distances of (0.5–0.9), whereas Dlm 15%, Dlm 30%, and Dlm 45% have negative displacement between the ranges. From Figure 5.12, which shows

the diagonal displacement of the bottom surface, laminated glass has the highest positive displacement at the start, followed by layered, Dlm 15%, Dlm 30%, and Dlm 45%. Even though monolithic glass can withstand a little bit of the force applied on the top surface, on the bottom surface it is the weakest. Undelaminated glass, having displaced little on the top and bottom surfaces and showing a significant strength to deflection among the units due to the perfect bond between glass surfaces.

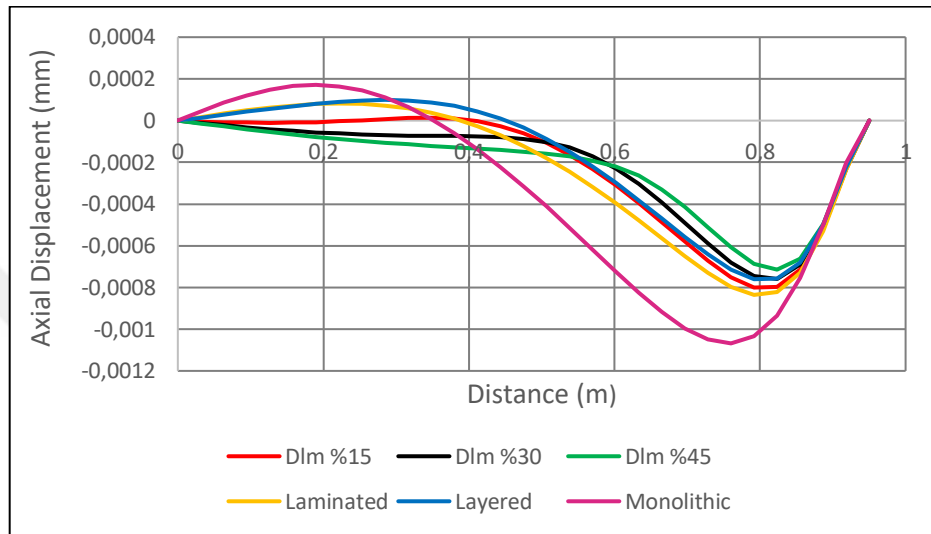


Figure 5.11 The y-axis displacement of the top glass unit diagonally

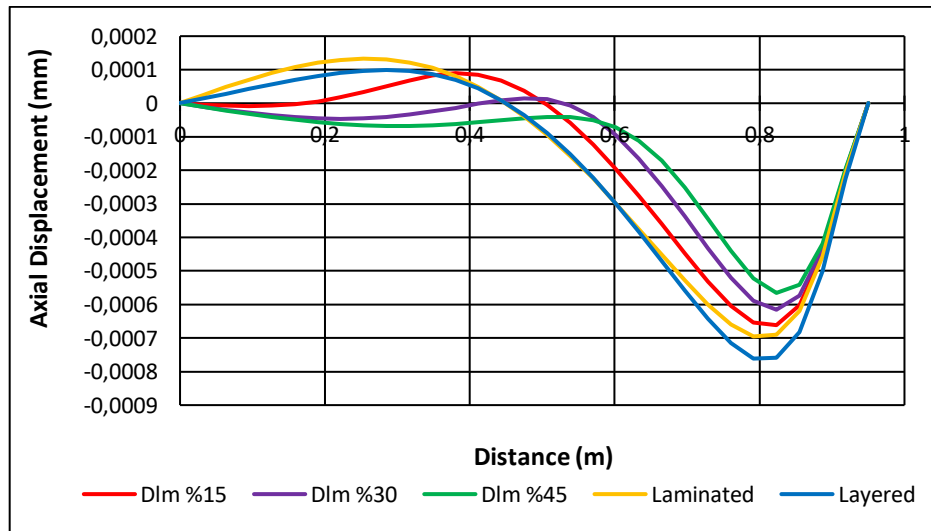


Figure 5.12 The y-axis displacement of the bottom glass unit diagonally.

A representation of the diagonally radial displacements, radial displacements in the θ -direction, and y-direction of the glass units is shown in Figure 5.13, 5.14, and 5.15, respectively.

The highest value of displacement is observed at the center, while it is zero at the corner of the unit. From the graph, monolithic glass is seen to have a higher radial displacement as compared to other glass units. It can be observed from the graphs that, even though there are rapid displacements of monolithic glass at the start in all considered directions, its mechanical properties to withstand stress are less compared to other units as the distance increases. Laminated glass, layered glass, Dlm 15%, Dlm 30%, and Dlm 45%, on the other hand, show an increase in displacement as the distance increases before decreasing. The mechanical properties due to the interlayer between laminated glass (Dlm 15%, Dlm 30%, and Dlm 45%) present the ability to withstand stress at a given distance more than others. The effect of delamination on the decreasing the strength of to withstand the applied load is also observed.

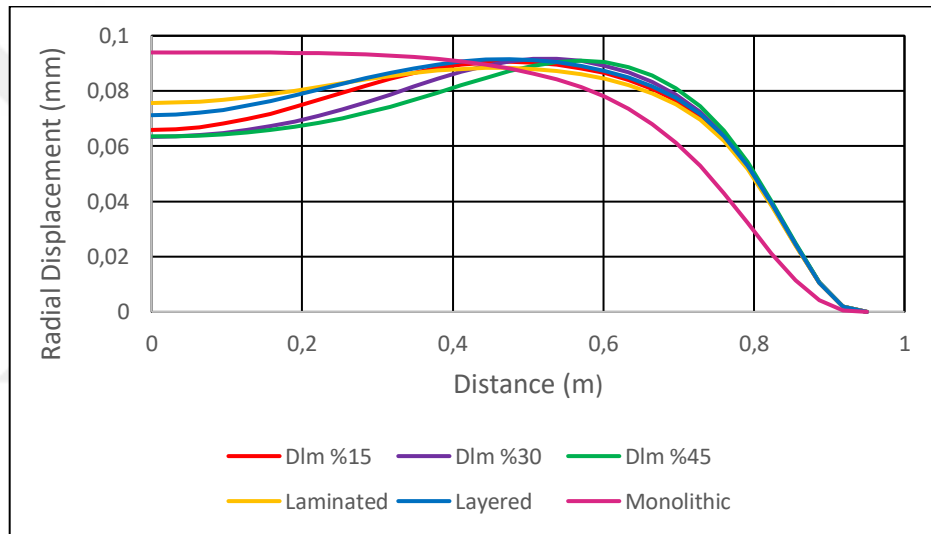


Figure 5.13 Diagonal radial displacement

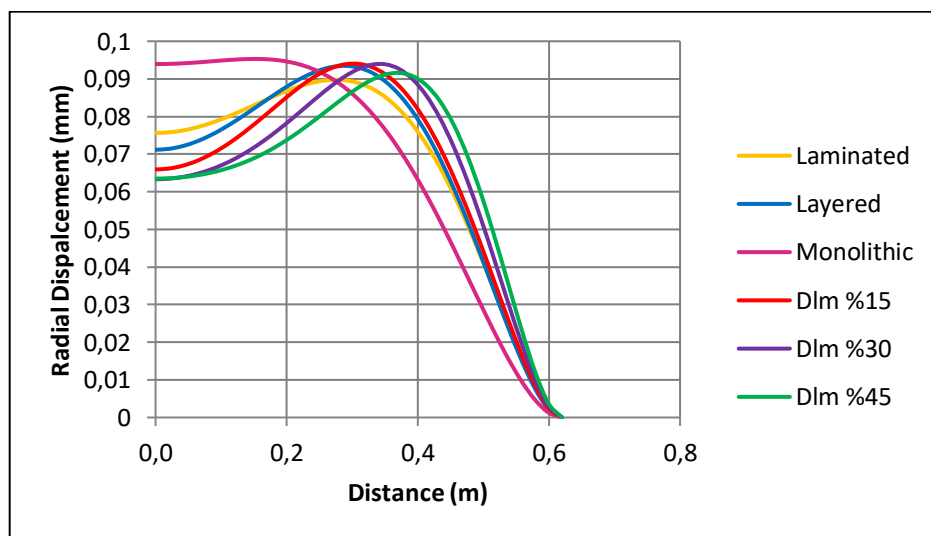


Figure 5.14 Radial displacement in the θ direction

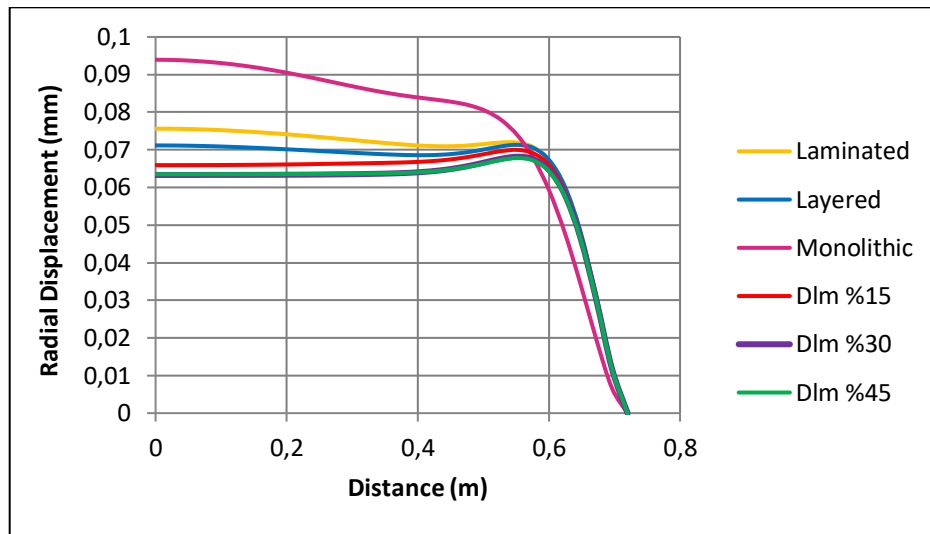


Figure 5.15 Radial displacement in y direction

According to the maximum principal stress failure predictor, fracture will happen whenever a component made of brittle material is subjected to a multiaxial stress where the maximum principal stress anywhere exceeds its local strength. Figures 5.16, 5.17, 5.18, and 5.19 show the maximum and minimum principal stresses of the bottom and top glass surfaces along the center, where $y = 0$. In Figure 5.16, maximum principal stress values are observed between the distances of 0.4 and 0.5 m (max. stress region). The graphs gradually increase before decreasing. The maximum principal stresses (S_1) on the bottom glass surface take the least value at the boundary of the unit. In the max. stress region, stresses of Dlm 45% are higher than those of other units, while they are smaller than those of other units at the edge of the unit. Figure 17 shows the value of minimum principal stresses (S_2) through the centerline. From Figure 5.17, it is observable that the location of minimum stress S_2 is between 0.3 and 0.45 for delaminated units. While max value of S_2 develops at the center of shell for monolithic glass it develops at a distance 0.19m for laminated glass and 0.25 m for layered glass unit. The minimum principal stress values of delaminated units are almost the same at the center of unit. Dlm 45% has a maximum value between the distances of 0.4 and 0.5 m, and monolithic glass has the minimum values at the same range of distances Figures 5.18 and 5.19 represent S_1 and S_2 on the top surface of laminated glass through the center of the shell unit at $y = 0$. S_1 on the top glass surface are maximum at the edge of unit. S_1 stresses are minimum between 0.37 and 0.46 meters. S_2 on top surface take their maximum value at the edge of the shell unit. Due to the direct contact of force on the glass surface, while the maximum principal stress is approximately

5 MPa, the minimum principal stress is approximately 1 MPa. There is compression at the center of the unit and maximum values at the boundaries.

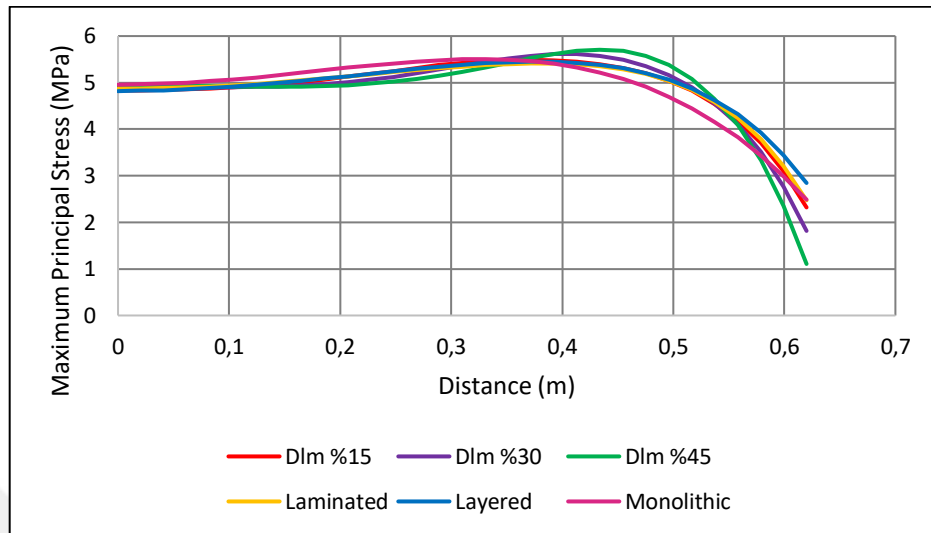


Figure 5. 16 Central maximum principal stresses (S1) through the centerline at y=0 (Bottom glass surface)

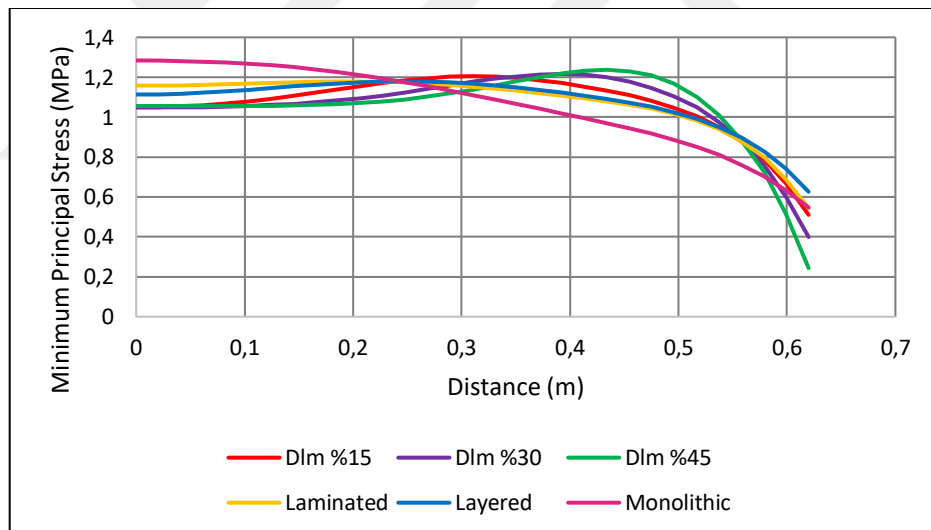


Figure 5. 17 Central maximum principal stresses (S2) through the centerline at y = 0 (Bottom glass surface)

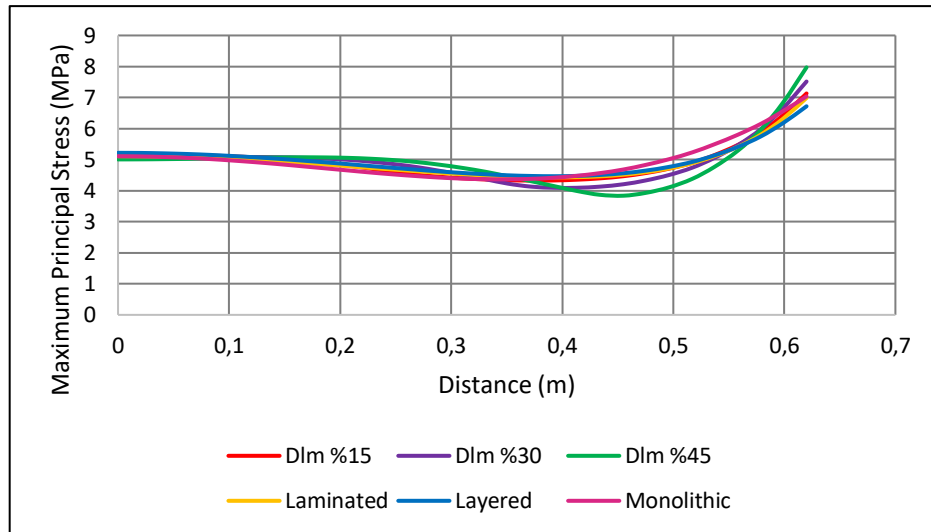


Figure 5.18 Central maximum principal stresses (S1) through the centerline at $y = 0$ (Top glass surface).

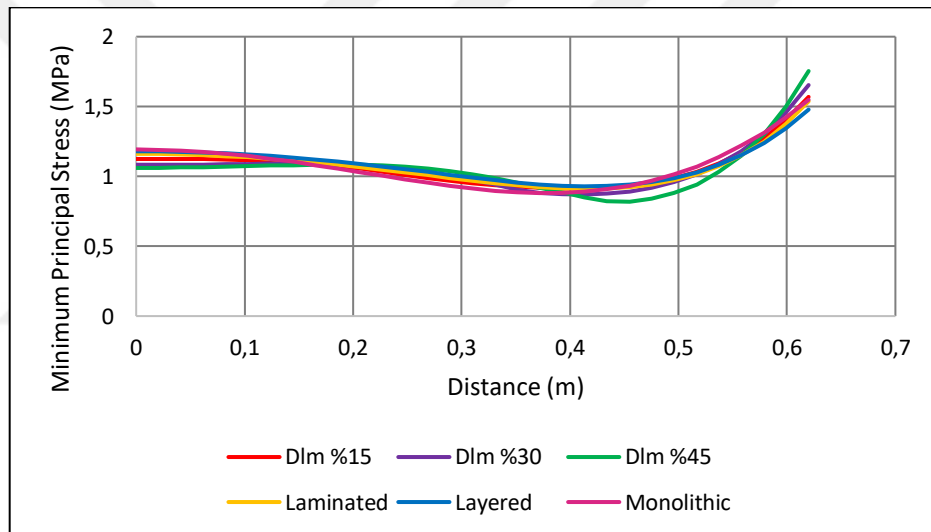


Figure 5.19 Central minimum principal stresses (S2) through the centerline at $y = 0$ (Top glass surface)

The maximum and minimum principal stresses of top glass at $\theta = 0$ are shown in figures 5.20 and 5.21. The maximum principal stress of all specimens on top surface tends to be uniform at 5 MPa at 0.5 distance, after which the stress decreases. S1 distribution of monolithic glass slightly increases from other unit at a distance 0.4 meters. The monolithic glass stress rapidly decreases more after 0.5 m, which implies the failure of the monolithic top glass surface. Figure 5.21 shows the distribution of S2 stress on the top glass surface of unit. While the minimum stress distribution of laminated and delaminated glass unit seems to absorb the energy a little bit more than the layered glass due to the interlayer, the monolithic eventually fails earlier.

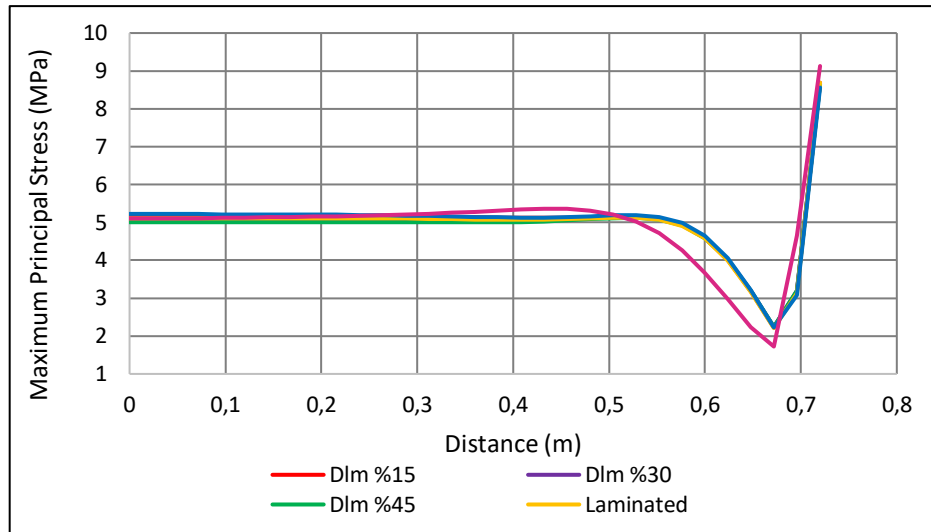


Figure 5.20 Central maximum principal stresses (S1) through the centerline at $\theta = 0$ (Top glass surface)

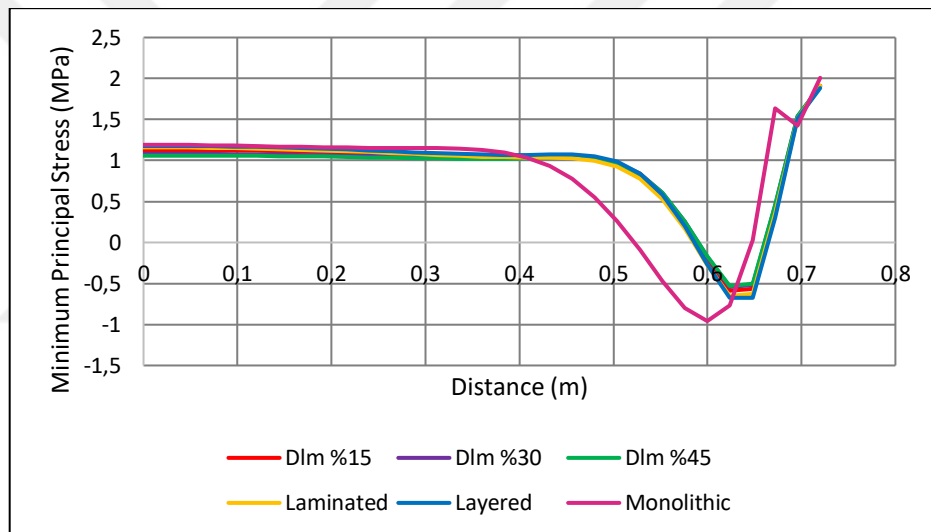


Figure 5. 21 Central minimum principal stresses (S2) through the centerline at $\theta = 0$ (Top glass surface)

The principal stresses S1 and S2 of the bottom surface of the shell at $\theta = 0$ line are shown in Figures 5.22 and 5.23. Maximum and minimum principal stresses are observed as tensile on the bottom surface. While S2 stresses of units have a minimal stress of 1.75 MPa, the corresponding S1 value is approximately 5 MPa for other units. The minimum principal stress (S2) of the top glass at $\theta = 0$ is greater as compared to the stress on the bottom glass.

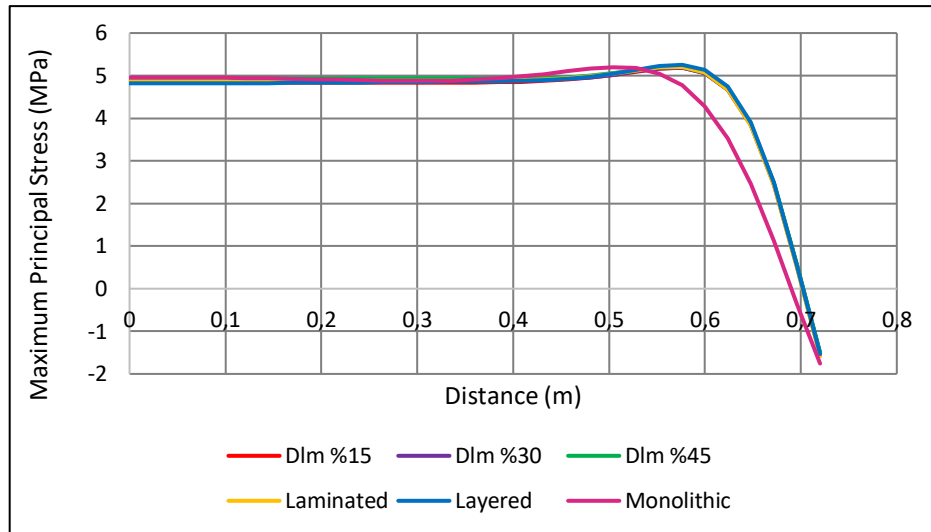


Figure 5.22 Central maximum principal stresses S1 at $\theta = 0$ (Bottom glass surface)

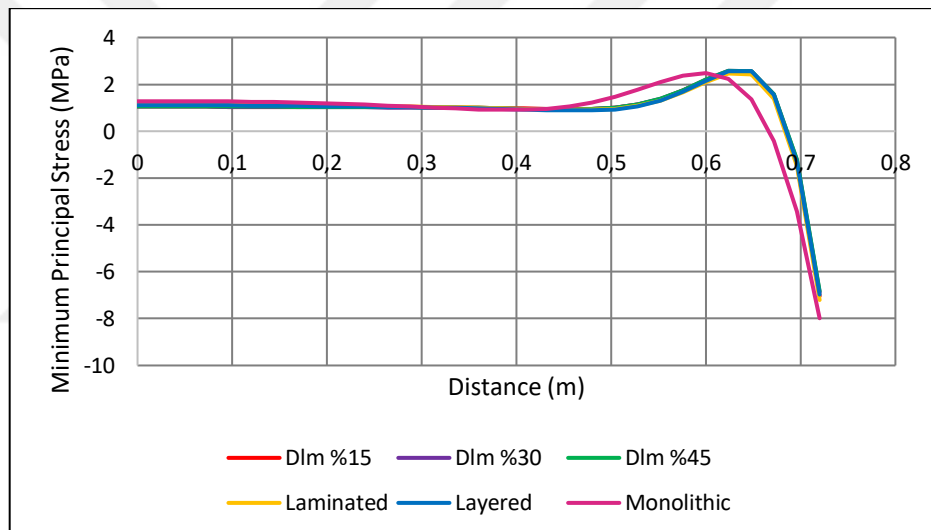


Figure 5.23 Central minimum principal stresses S2 at $\theta = 0$ (Bottom glass surface)

The maximum and minimum principal stresses of the top and bottom glass shells along the diagonal are shown in figures 5.24 to 5.27, respectively for the applied 50 kPa load value. The maximum stress of the glass shows a gradual decrease in stress from 5 MPa as the distance increases on the top surface in Figure 5.24; the maximum stress at the bottom, on the other hand, shows an incremental increase in stress as the distance increases between 0.0 and 0.7 in Figure 5.26. The stresses on the bottom surface in θ -direction and y-direction maintain a constant value for a longer distance, while the diagonal direction shows a rapid stress increase which implies the stress on the bottom glass is higher than the top glass along the diagonal in Figure 5.26. The minimum principal stress (S2) on the top and bottom glass surfaces along the

diagonal is shown in figures 5.25 and 5.27. In figure 5.25, the minimum stress on the top glass exhibits both tensile (0-0.65 m) and compressive behavior (0.65-0.85 m), respectively.

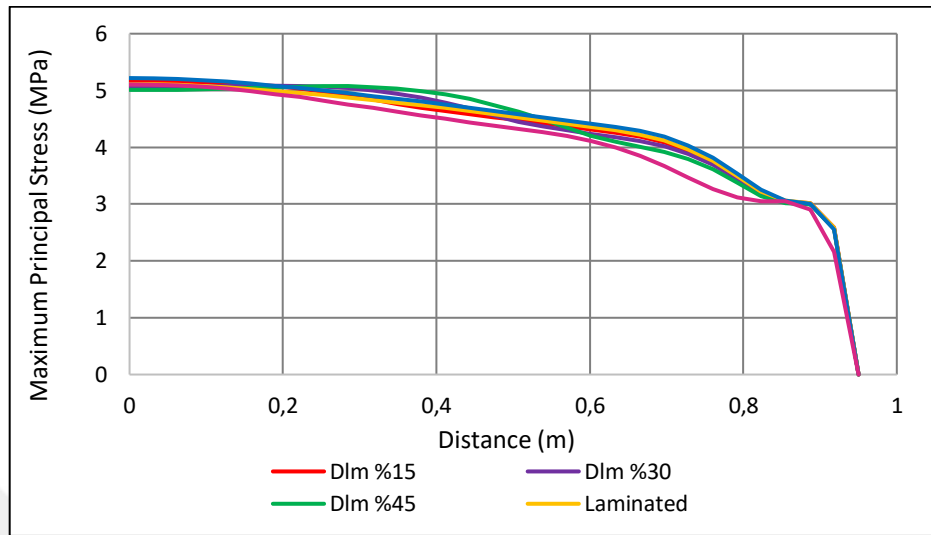


Figure 5.24 Top glass surfaces maximum principal stress (S1) through the diagonal

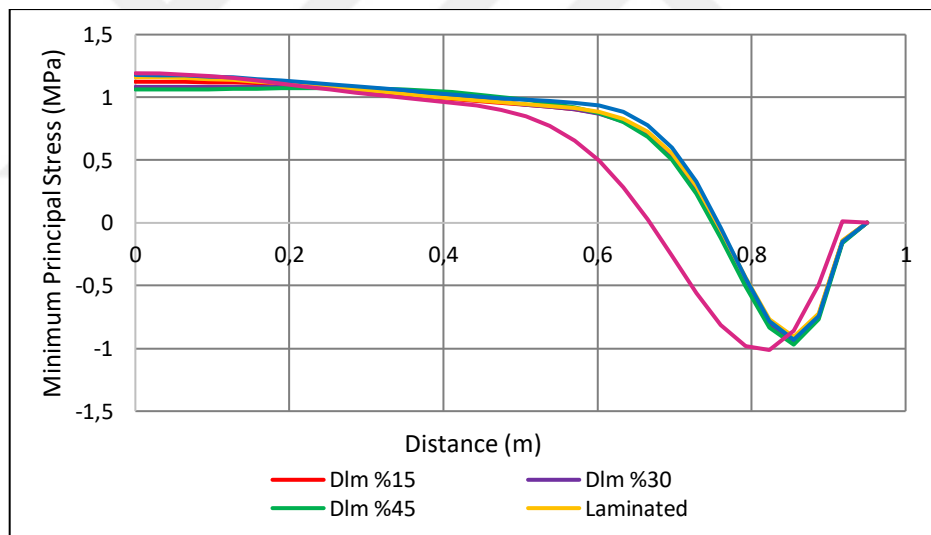


Figure 5.25 Top glass surfaces minimum principal stress (S2) through the diagonal

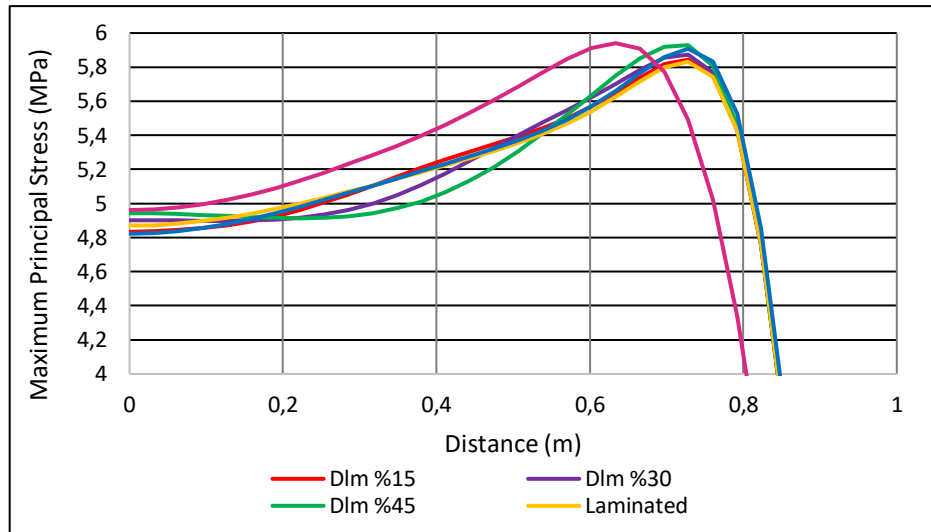


Figure 5. 26 Bottom glass surfaces maximum principal stress (S1) through the diagonal

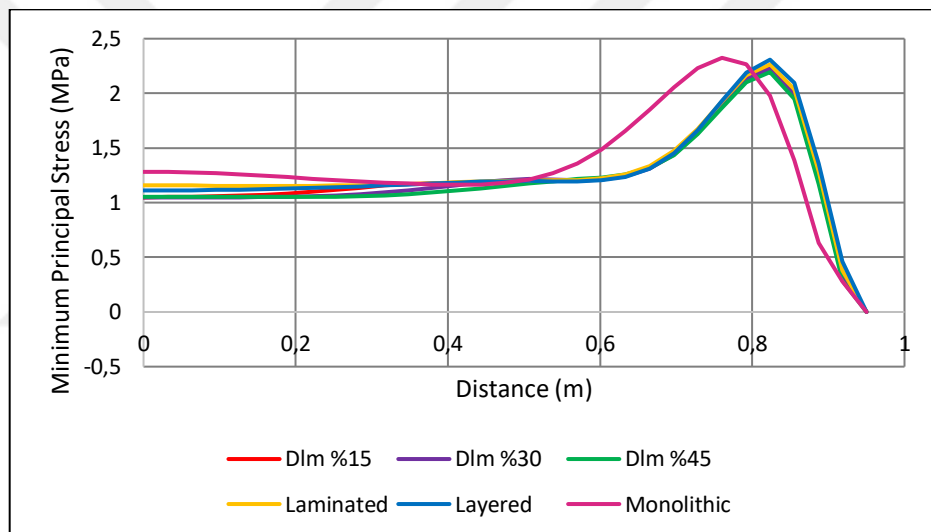


Figure 5. 27 Bottom glass surfaces minimum principal stress (S2) through the diagonal

The principal stress versus load on the top and bottom surface of laminated glass and Dlm 45% glass are plotted in Figures 5.30 to 5.33. Both the top and bottom surfaces of laminated glass and Dlm 45% have positive tensile principal stresses, which is a significant stress that might affect the material's properties of failure or deformation. The top glass surface principal stress of laminated glass (S1) is higher as compared to that of Dlm 45% glass at 100 kPa pressure as shown in Figures 5.28 and 5.29. In Figures 5.30 and 5.31, it can also be seen that even though the bottom surfaces are seen to have the same stress values, there is a slight difference. Looking at figure 5.30, at 40 kPa, the laminated glass has a stress value (S1) less

than 4 MPa, while in figure 5.33 of Dlm 45%, the principal stress value (S1) is approximately 4 MPa.

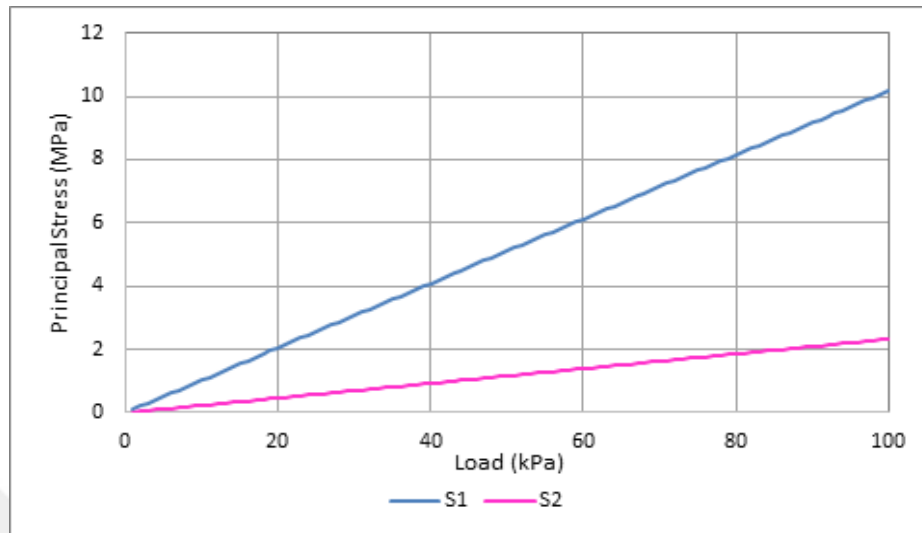


Figure 5.28 Principal stress on top surface of laminated unit

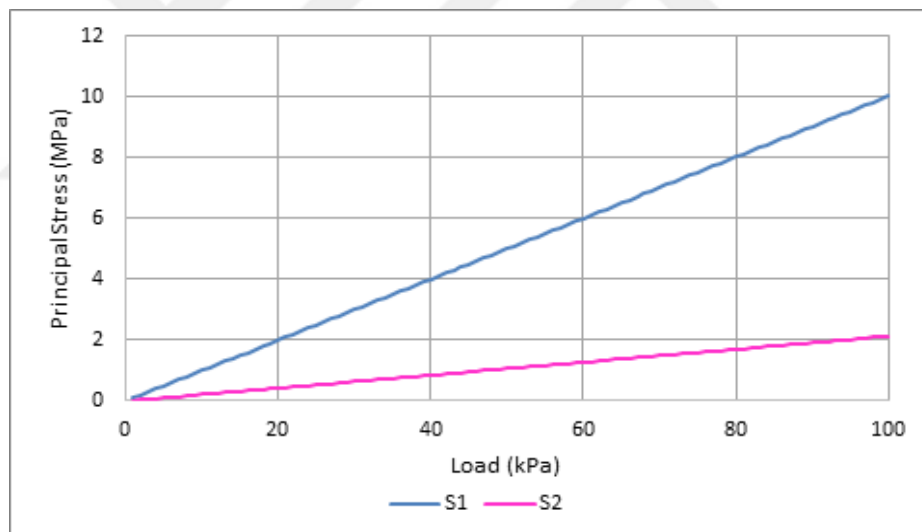


Figure 5.29 Principal stress on top surface of Dlm 45% unit

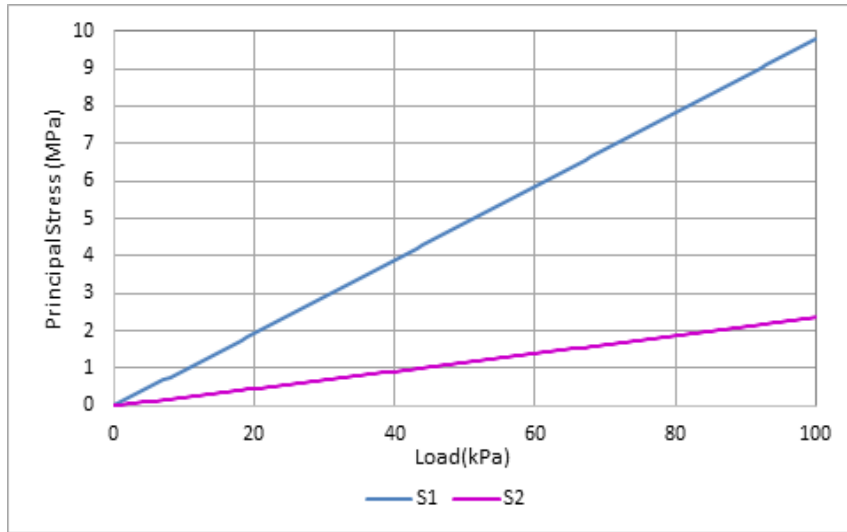


Figure 5. 30 Principal stress on bottom surface of laminated unit

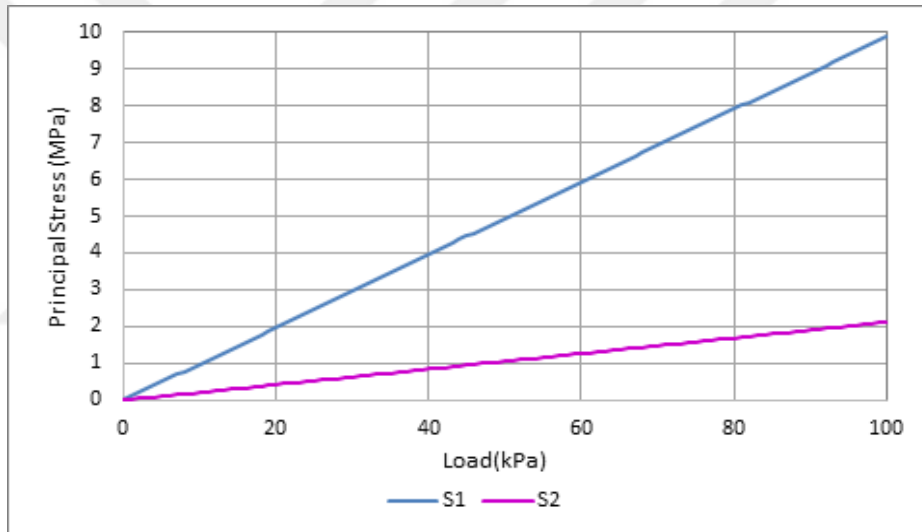


Figure 5. 31 Principal stress on bottom surface of Dlm 45% unit

The mechanical behavior of the material is also influenced by the strength factor, a parameter used to quantify its ability to resist pressure or load. A strength factor of 1 implies the material is in critical condition and has reached its limit; any additional stress would cause permanent deformation or damage. The strength factor is calculated as a ratio between the maximum principal stress of the laminated glass and the maximum principal stress of the monolithic glass unit. The strength factor of glasses is shown in Figure 5.34.

$$\text{Strength Factor} = \frac{\text{Monolithic glass maximum principal stress}}{\text{Laminated Glass maximum principal stress}}$$

From Figure 5.32, it can be seen that laminated and delaminated glass have a strength

factor of more than 1.0; a safety factor of more than 1 implies that the imposed load or stress is less than the maximum stress that the materials can withstand. The range of the strength factor is 1.2 to 1.5. With an increase in the load, as shown in Figure 5.34, laminated glass has the least strength factor, followed by Dlm 15%, Dlm 45%, and Dlm 30%, with the highest value of safety factor.

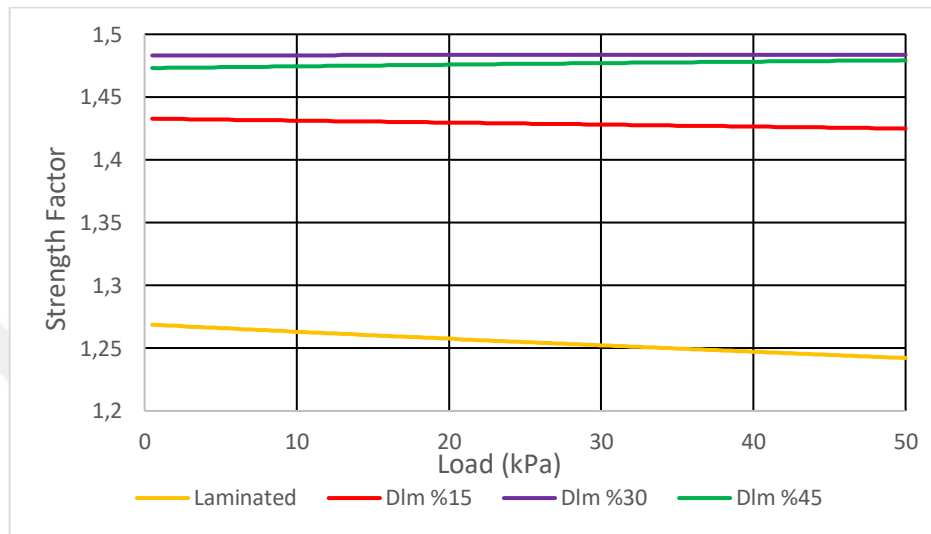


Figure 5. 32 Strength factor of fixed supported units.

Figure 5.33 shows the contour map of the maximum principal stress contours for laminated glass shells (Top Surface), and figure 5.34 shows those of the minimum stresses under 50 kPa pressure. These graphs show the regions of highest principal stress and stress magnitude over the surface. The top surface of the laminated glass shell is subjected to high principal stress around the edges. These stresses are tensile and cause the material to pull or stretch. These stresses have an effect on the shells and overall structural integrity. The distribution of stress at the top surfaces is quite similar, with much higher concentrations occurring at the support. S2 stresses can be both tension and compression through the surface of the shell. There are visible signs of compression stress on the minimum principal stress contours close to the edges.

Figures 5.35 and 5.36 illustrate the S1 and S2 stress contours on the bottom surfaces of the laminated glass shell. On figure 5.35, the stress of the bottom surface is tension, and it is maximum at a point close to the edge of the unit; on the other hand, S2 stresses are both positive

and negative, as seen in Figure 5.36. They are compressive around the 0.7 m distance, and they are at their maximum in a region close to the center of the quarter shell.

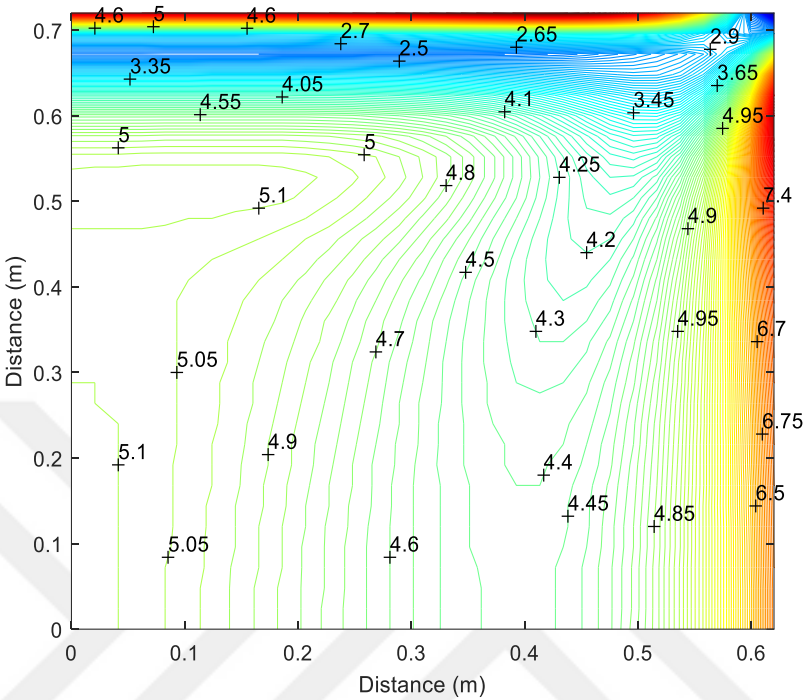


Figure 5. 33 Maximum principal stress contours for laminated glass shells (Top Surface)

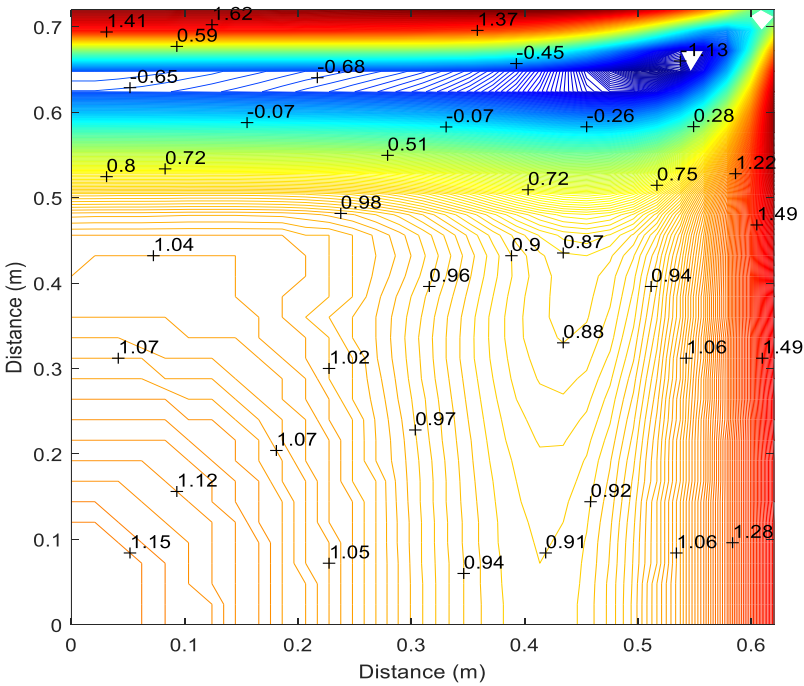


Figure 5.34 Minimum principal stress contours for laminated glass shells (Top Surface)

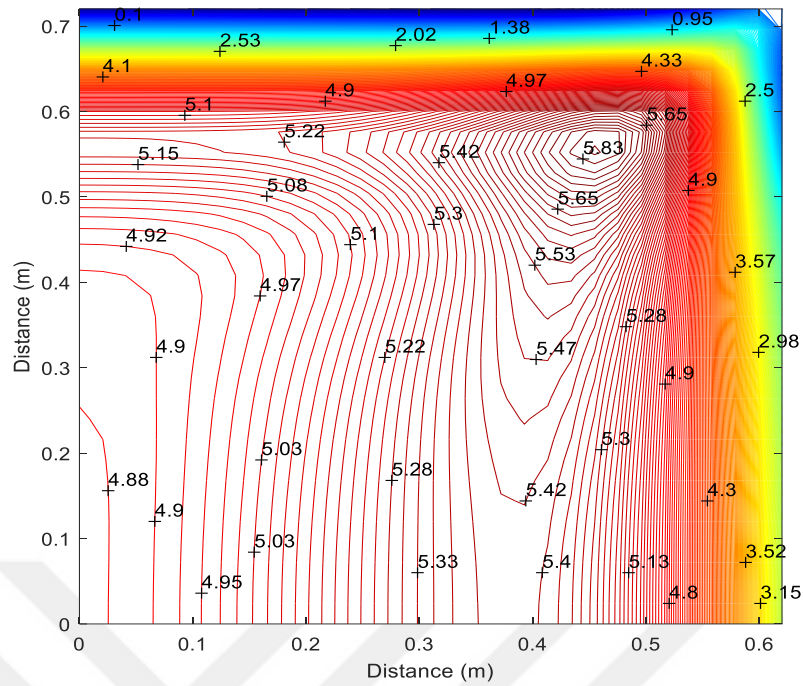


Figure 5.35 Maximum principal stress contours for laminated glass shells (Bottom Surface)

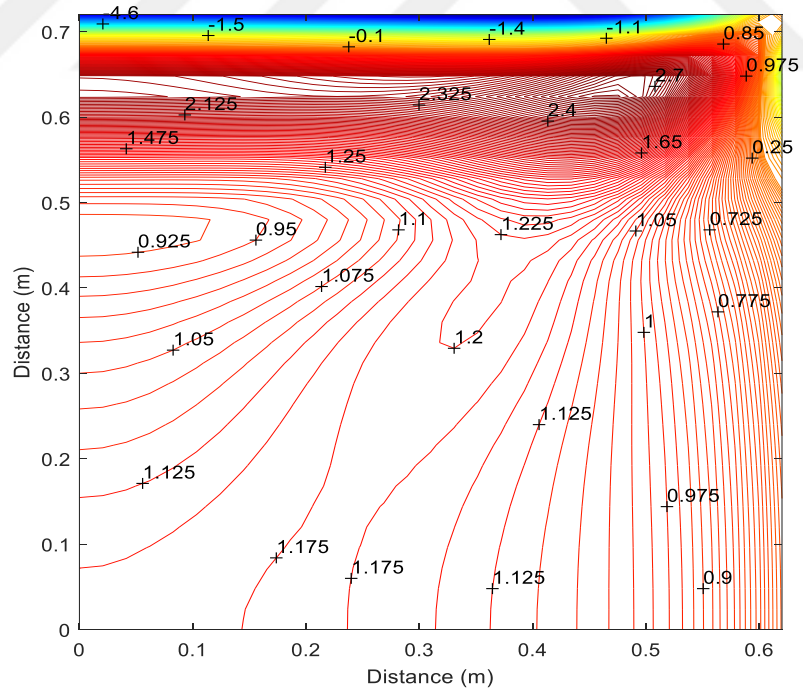


Figure 5.36 Minimum principal stress contours for laminated glass shells (Bottom Surface)

Figures 5.37 and 5.39 illustrate the contours of the maximum principal stress contours on top and bottom surfaces for layered glass, and figures 5.38 and 5.40 illustrate the minimum

principal stress contours of layered glass specimens. Figure 5.37 illustrates that the maximum principal stress is positive through the surface, and its maximum value is located on the edges of the glass. When Figure 5.37 is compared with Figure 5.33, it can be seen that the stress distributions of laminated and layered units are quite similar. Top surface minimum principal stresses (S2) are tension; compressive stresses can also be observed on the small part of the top edge in Figure 5.38. Figures 5.39 and 5.40 illustrate the maximum (S1) and minimum (S2) principal stresses on the bottom surface of layered glass. The maximum principal stress on the edge is lower and higher between the distances of 0.5 m and 0.6 m. In a nutshell, the bottom glass exhibits less compressive minimum principal stress, and a tensile behavior can also be observed on the top glass surface.

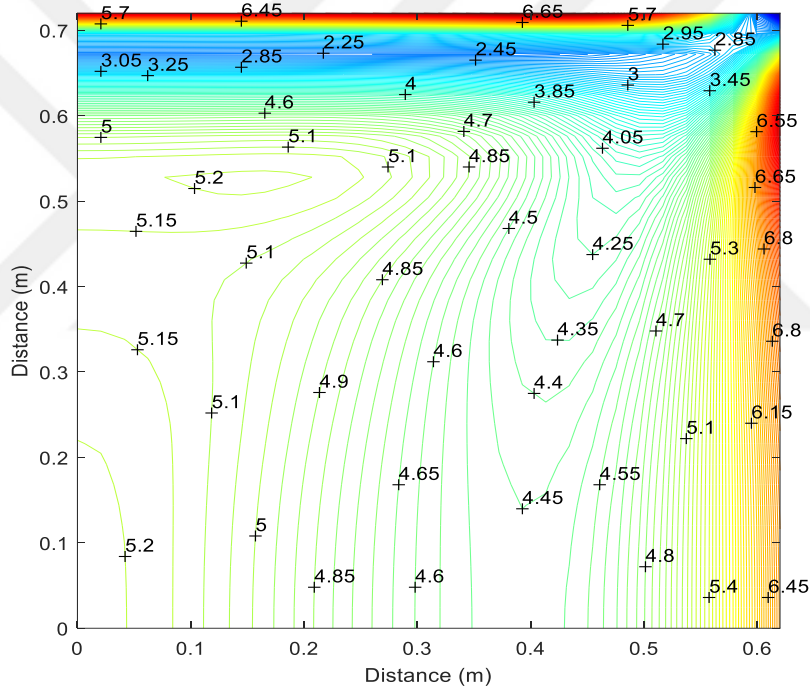
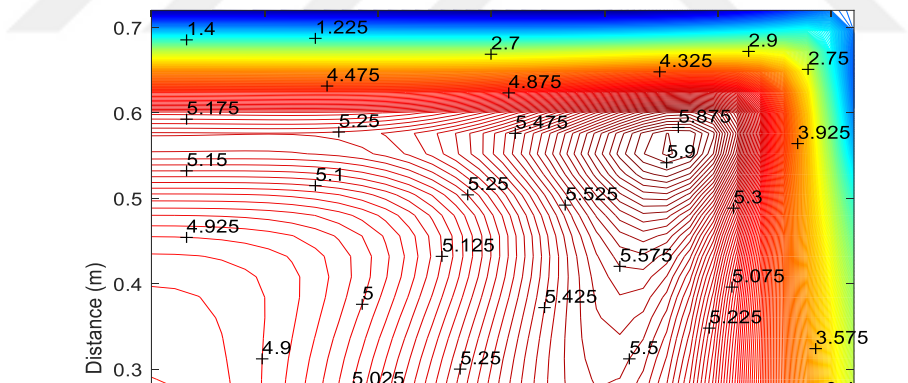
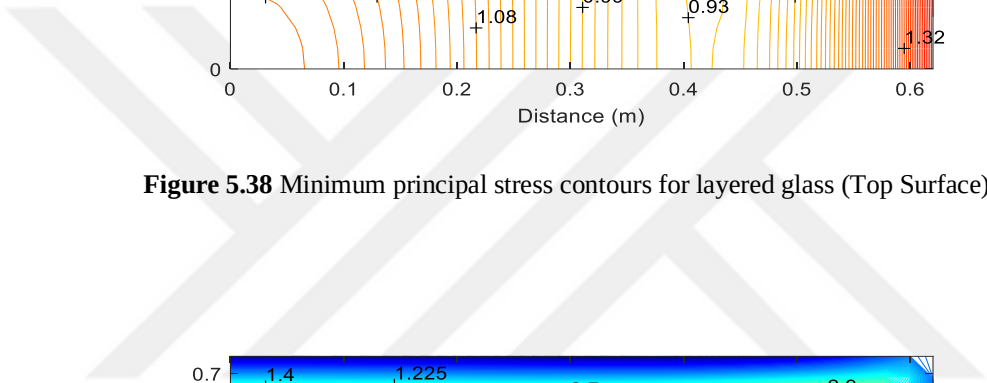


Figure 5. 37 Maximum principal stress contours for layered glass (Top Surface)



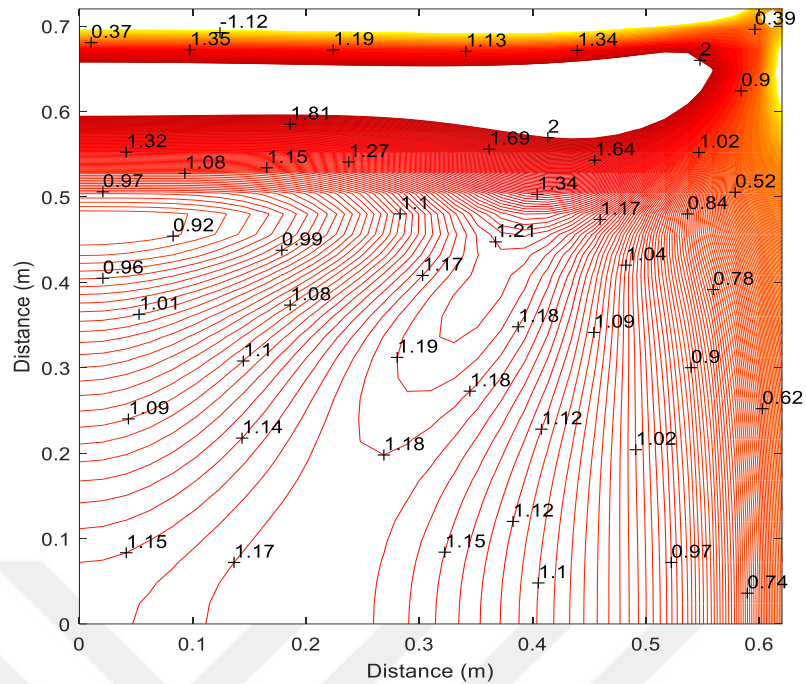


Figure 5.40 Minimum principal stress contours for layered glass (Bottom Surface)

Figures 5.41 to 5.44 present maximum principal stress contours and minimum principal stress distributions on the top and bottom parts of the monolithic glass unit. In Figure 5.41, the top surface of the monolithic glass exhibits a tensile behavior, with higher stresses observed along the edges. These stresses indicate a tendency for the material to expand. Figure 5.41 illustrates the maximum principal stress distribution on the top surface; the bottom surface also has positive stress distributions. However, the stresses along the edges are comparatively lower than in other areas of the surface. In summary, the analysis of maximum and minimum principal stresses reveals both the stress values of the top and bottom surfaces of the monolithic glass. The top surface exhibits higher stresses along the edges, while the bottom surface shows lower

stress levels in those regions. These findings provide valuable insights into the stress distribution and behavior of the monolithic glass unit.

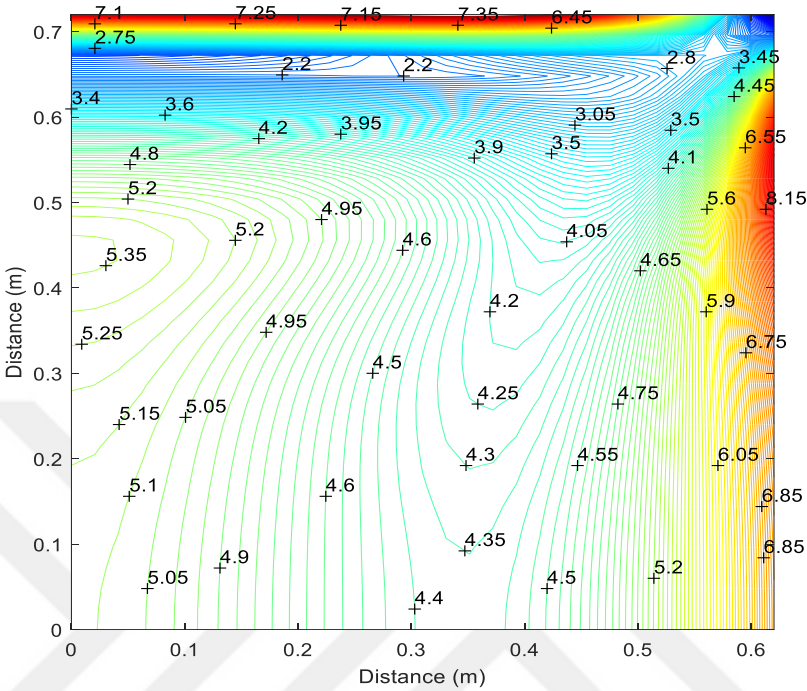


Figure 5.41 Maximum principal stress contours for monolithic glass (Top Surface)

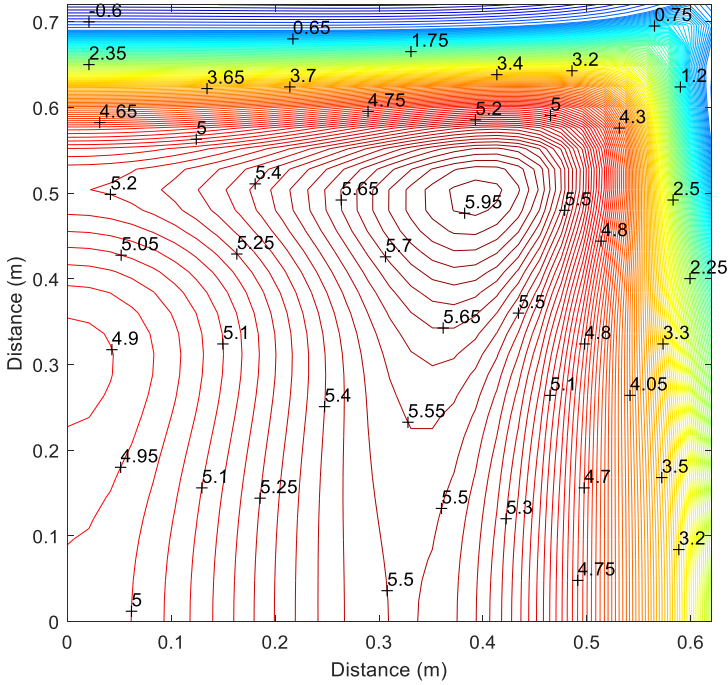


Figure 5.42 Maximum principal stress contours for monolithic glass (Bottom Surface)

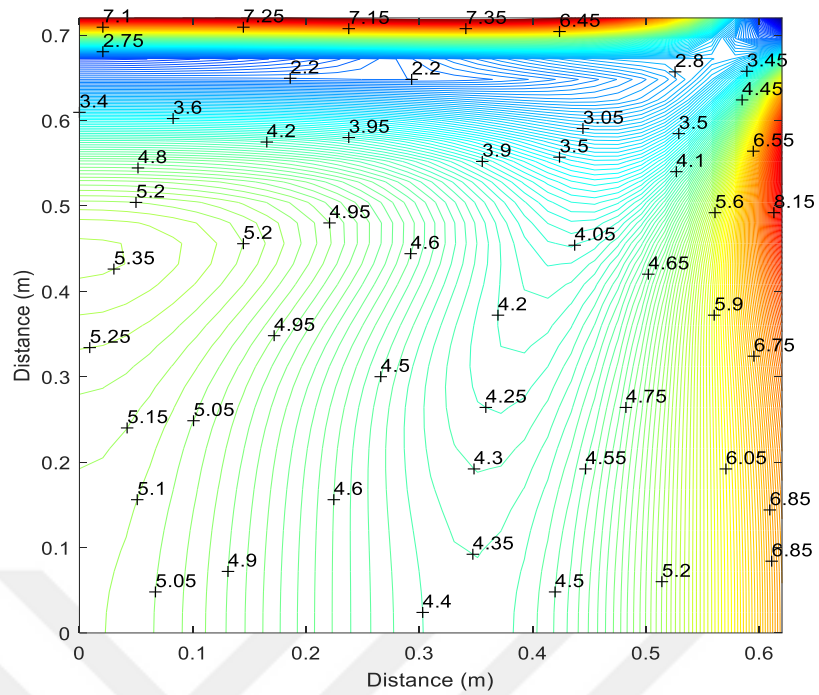


Figure 5.43 Minimum principal stress contours for monolithic glass (Top Surface)

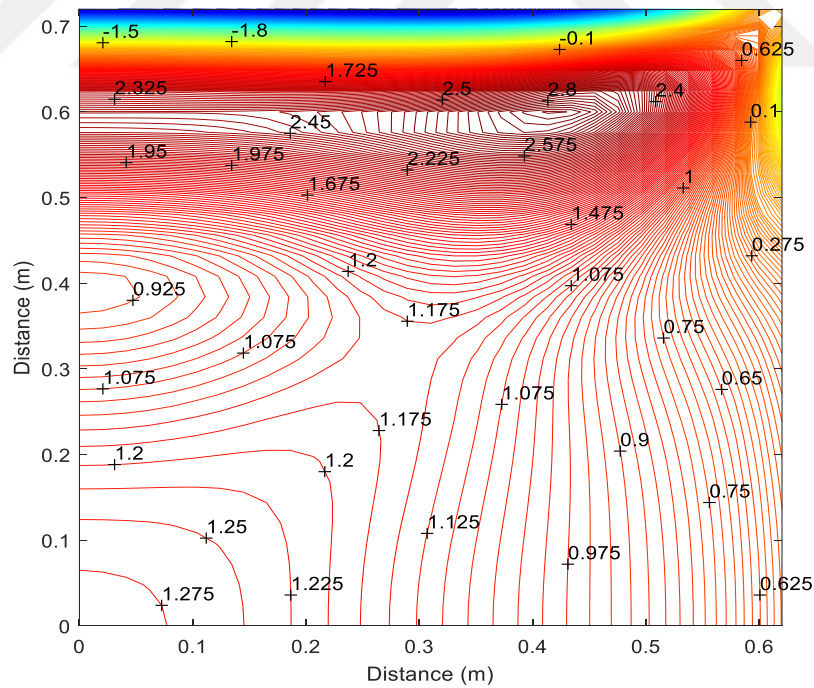


Figure 5.44 Minimum principal stress contours for monolithic glass (Bottom Surface)

Figures 5.45 and 5.46 illustrate the maximum principal stress contours on the top and bottom parts of the of Dlm 15%. Figures 5.47 and 5.48 illustrate the minimum principal stresses.

The maximum principal stresses are in tension for both the top and bottom surfaces of the unit, with stress high on the edges for the top surface and less stress observed on the bottom surface.

The minimum principal stress contours for the top of the delaminated unit (Dlm 15%) can be seen in Figure 5.47, and the bottom of the unit can be seen in Figure 5.48. Stresses are high on the edges and evenly distributed in the middle. Even though a small portion of figure 5.47 exhibits compressive stress near the top edge, a wide area of the top part exhibits tensile stress. On the surface below, high tensile stresses are observed almost in the same region where compressive stresses are observed for the top surface, and compression is observed on the top edge.

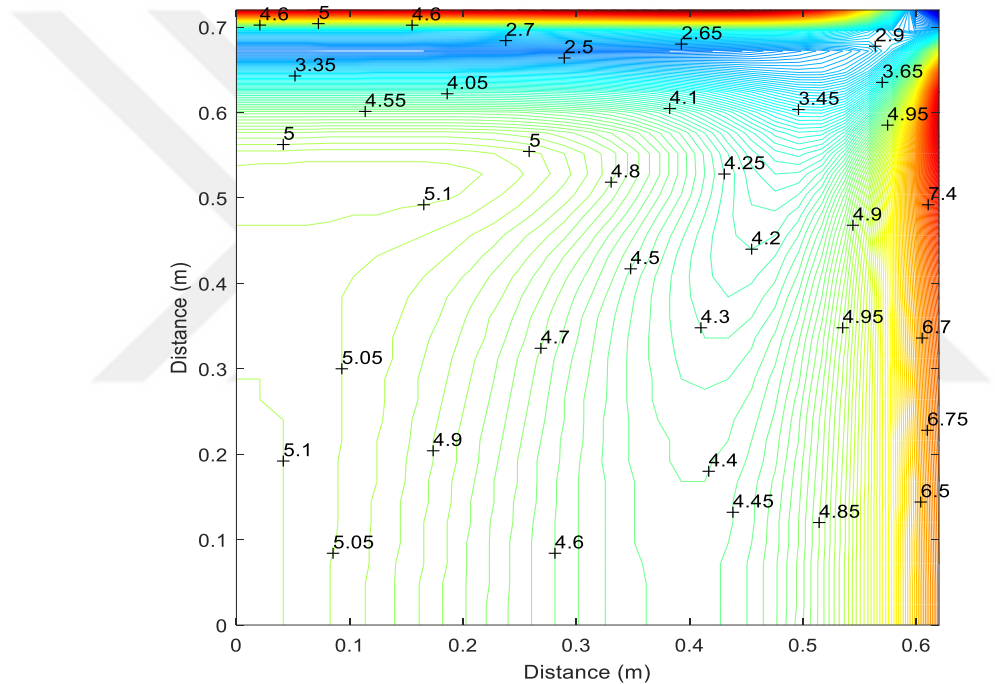


Figure 5.45 Maximum principal stress contours on the top surface for Dlm 15%

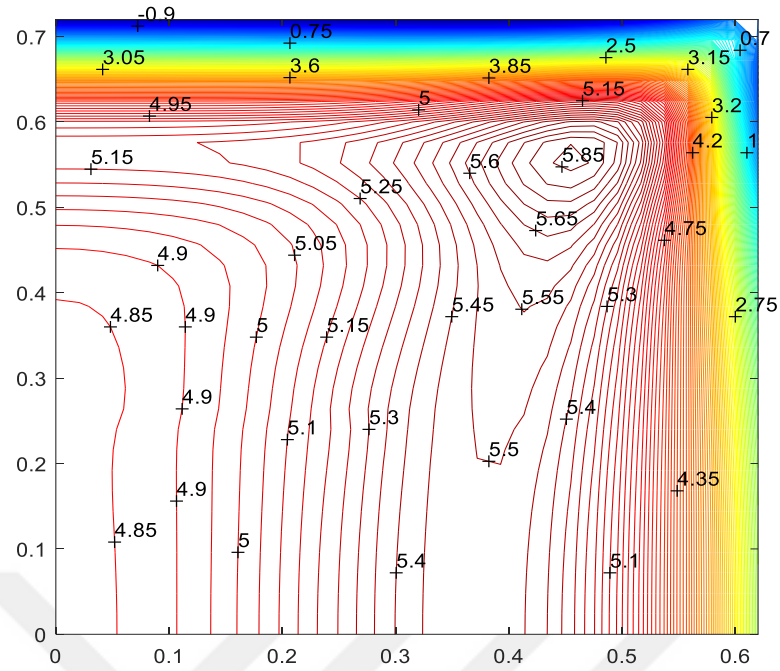


Figure 5.46 Maximum principal stress contours on the bottom surface for Dlm 15%

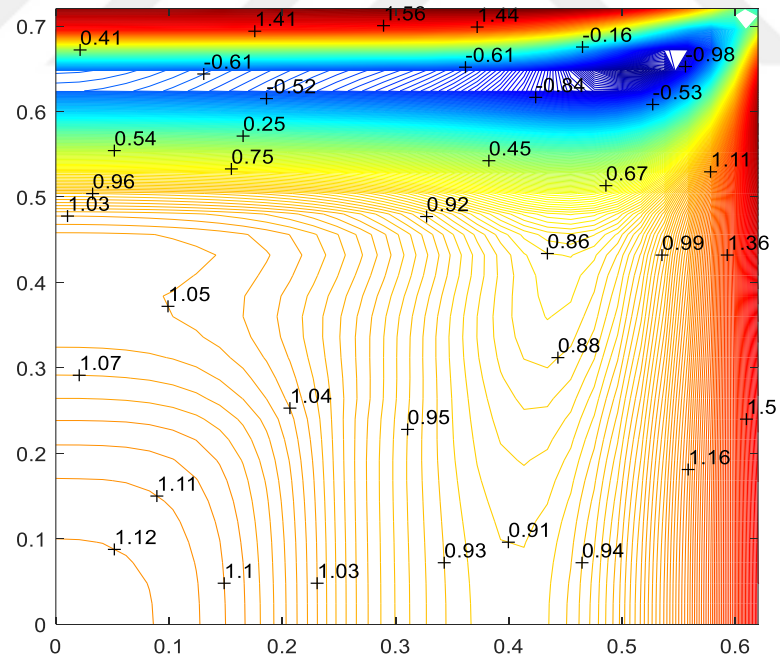


Figure 5.47 Minimum principal stress contours on the top surface for Dlm 15%

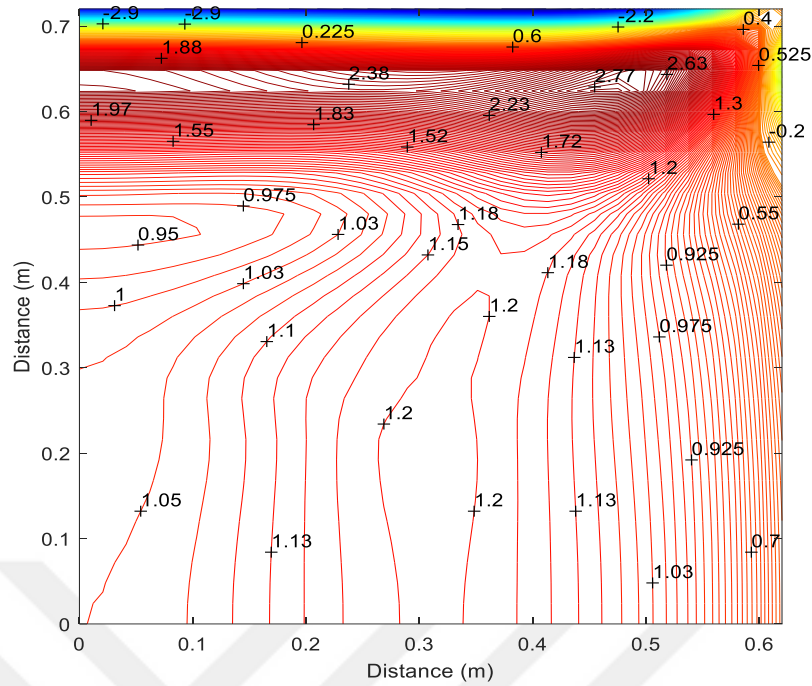


Figure 5.48 Minimum principal stress contours on the bottom surface for Dlm 15%

Figures 5.49 and 5.50 present the maximum principal stress (S_1) contours on the top and bottom surfaces of the delaminated glass unit (Dlm 30%), respectively. On the top surface of the unit, maximum tensile stress is observed along the edges of the quarter plate. In contrast, when comparing the top and bottom surfaces, the maximum principal stress on the bottom surface is lower at support and shows some traits of compression on the top edge. Figures 5.51 and 5.52 show the minimum principal stress contours on the top and bottom surfaces of Dlm 30% glass. The top surface has a region of compression near the top edges and high compression on the edge; stress is evenly distributed in the middle of the top glass. On the bottom surface, on the other hand, the concentration of tension is higher as compared to the top glass, even though compression is visible at the top edge.

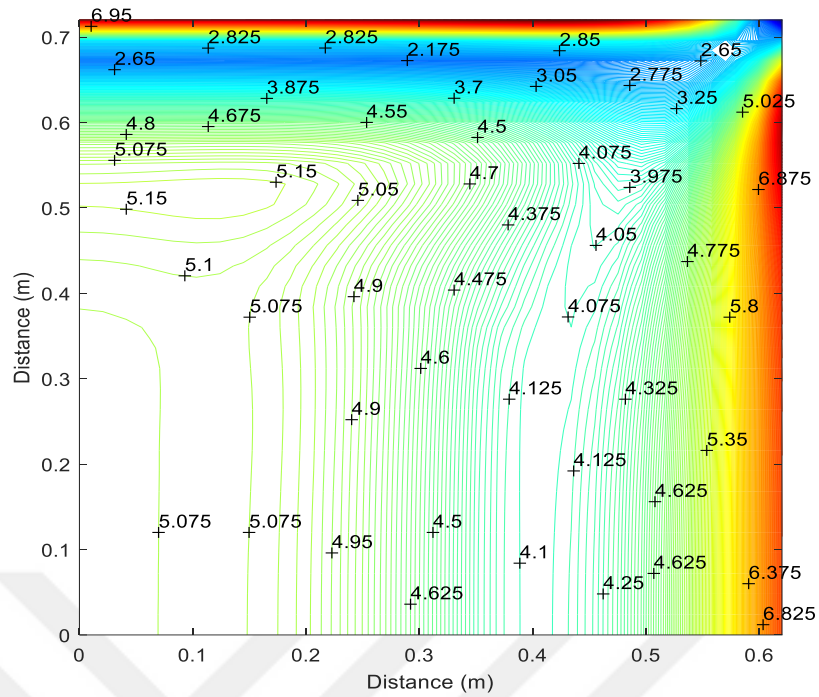


Figure 5.49 Maximum principal stress contours on the top surface for Dlm 30%

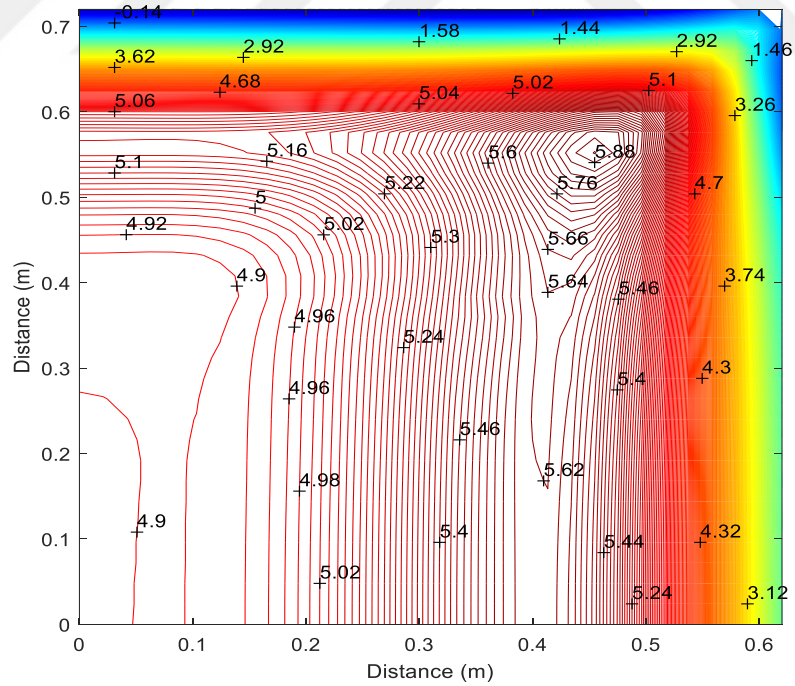


Figure 5.50 Maximum principal stress contours on the bottom surface for Dlm 30%

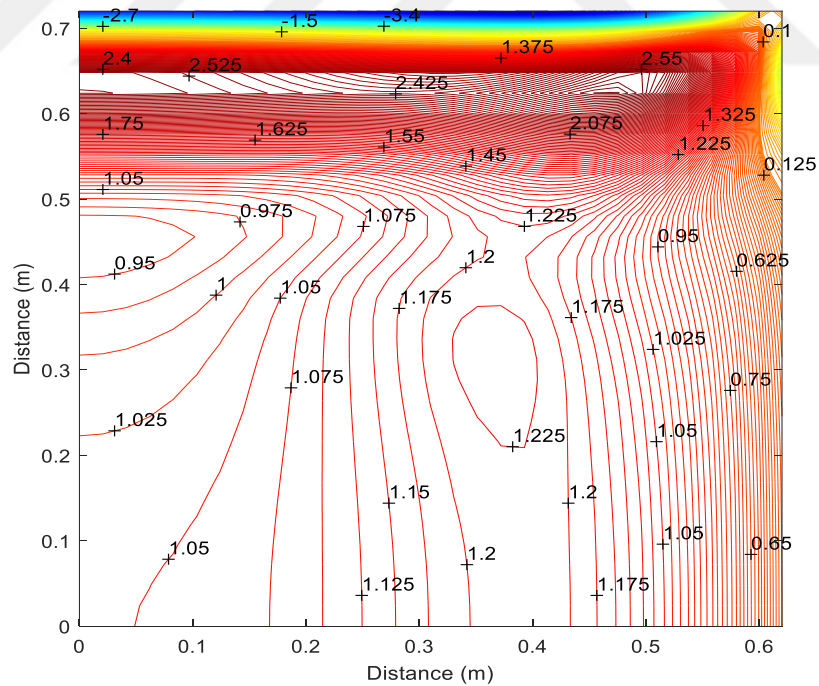
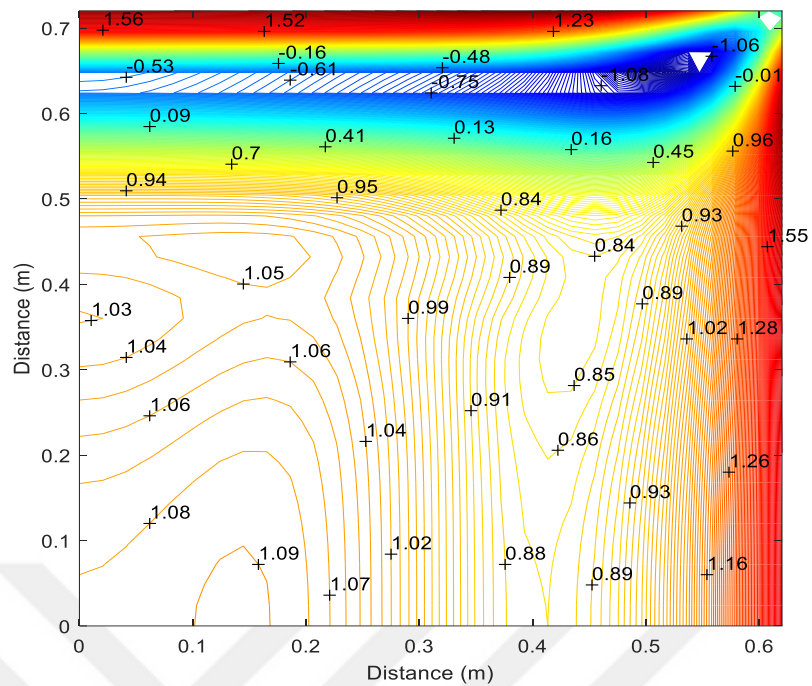


Figure 5.52 Minimum principal stress contours on the bottom surface for DIm 30%

Figures 5.53 and 5.54 illustrate the contour of maximum principal stress for the surfaces on the top and bottom parts of Dlm 45% glass. Tensile stresses are observed on the top surface,

with high stress concentrations occurring at the edges of the quarter plate. In contrast, the maximum principal stress concentration at the bottom surface in the middle region is higher than at the edges. An opposite stress concentration is observed on the top and bottom glass after the 0.5 m and 0.6 m distance regions.

Figures 5.55 and 5.56 illustrate the minimum principal stress contours for Dlm 45%'s top and bottom surfaces. A significant portion of the top surface is under tensile stress. The stress on the edges is higher than in the middle portion. A compression stress is observed in a small part of the top region.

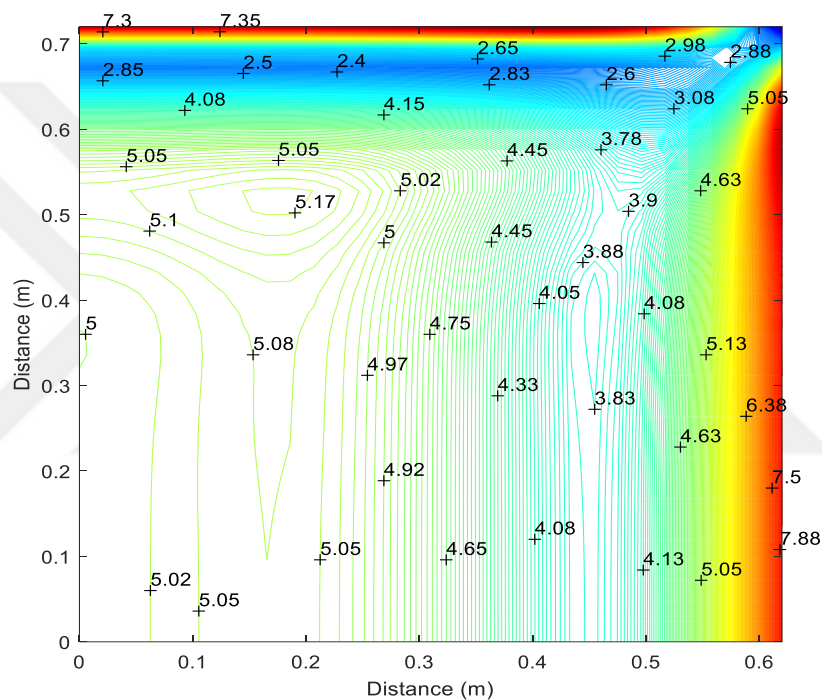


Figure 5.53 Maximum principal stress contours on the top surface for Dlm 45%

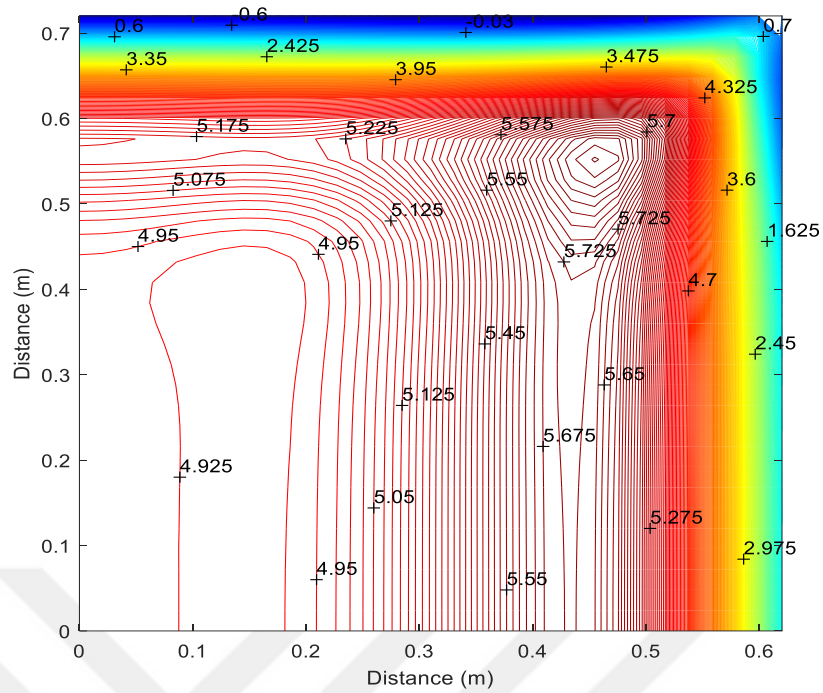


Figure 5.54 Maximum principal stress contours on the bottom surface for Dlm 45%

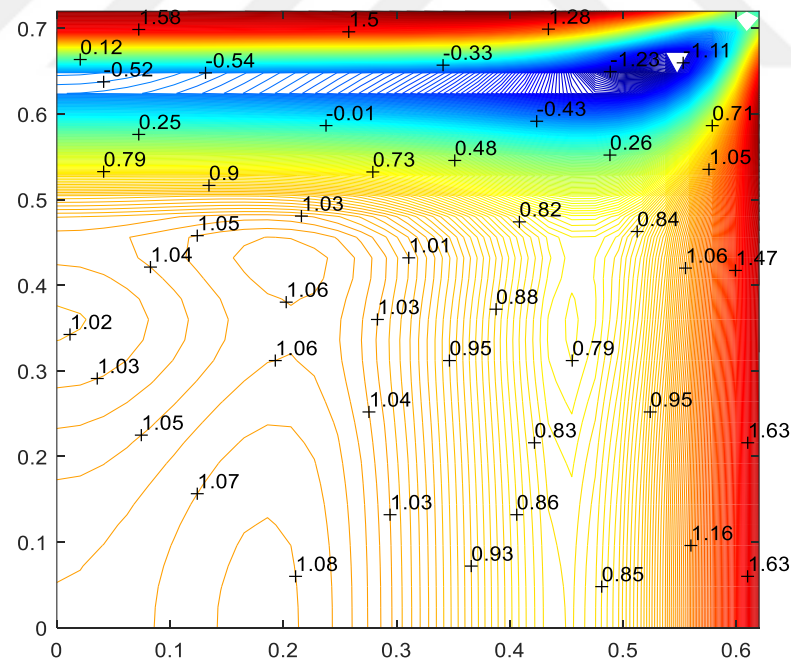


Figure 5.55 Minimum principal stress contours on the top surface for Dlm 45%

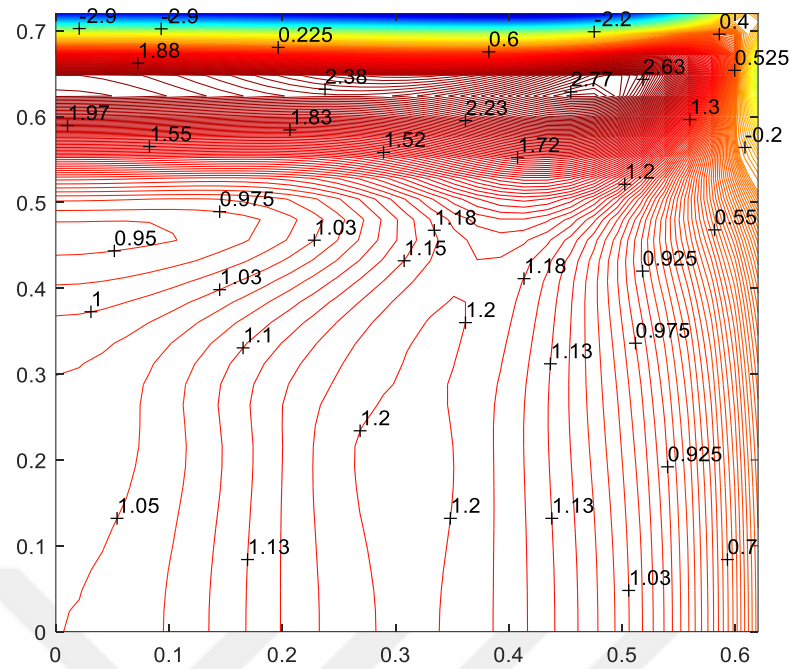


Figure 5.56 Minimum principal stress contours for Dlm 45% bottom Surface

In a nutshell, the following radial displacement contours are generated to visually show how the specimen's deformation or displacement patterns change and where stress is concentrated. Which also helps us determine which specimen displaces more and in which region it displaces more. This helps evaluate structural integrity, validate analytical models, and improve conceptual efficiency.

Figures 5.57 to 5.60 show the deformation of laminated glass, layered glass, monolithic glass, and Dlm 45% glass, respectively. The center region of the units undergoes greater radial displacement as compared to the edges due to boundary conditions. Laminated glass, when compared to the other specimens, shows comparatively smaller displacement concentrations. As a result of the displacement being more uniformly distributed, the deformation pattern is also more consistent. Figure 5.58 shows that layered glass, in contrast with laminated glass, displays relatively higher displacement concentrations. In Figure 5.61, monolithic glass, shows a substantial concentration of displacement between the 0 and 0.3 horizontal distances. Which indicate a greater deflection concentration compared to laminated and layered glass. Figure 5.60 displays that Dlm 45%, has the largest concentration of displacement among the studied specimens. The delaminated region shows a significantly higher displacement. It is also observed from the figures that the displacement contours of delaminated units differ from those of laminated, layered, and monolithic units.

Overall, the monolithic glass and layered glass specimens exhibit the least substantial displacement concentrations, followed by Dlm 45%. Comparatively speaking to the other specimens, the laminated glass specimen shows smaller displacement concentrations.

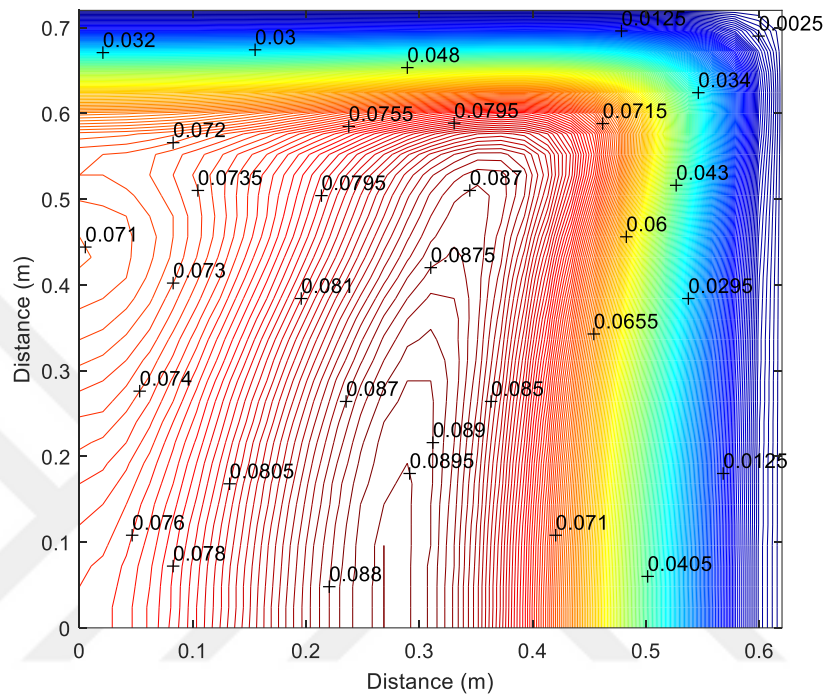


Figure 5.57 Laminated glass shell contours for radial displacement

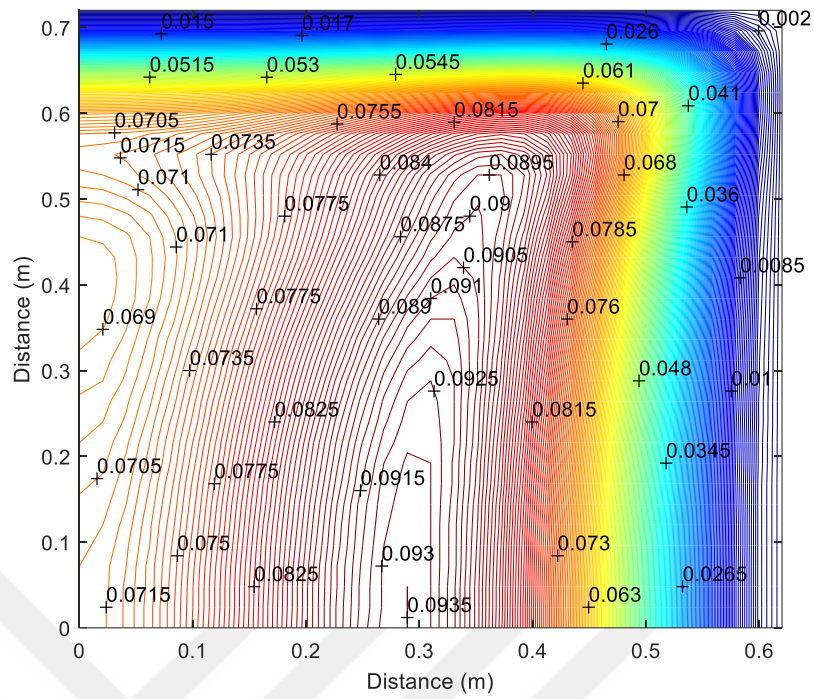


Figure 5.58 Layered glass contours for radial displacement.

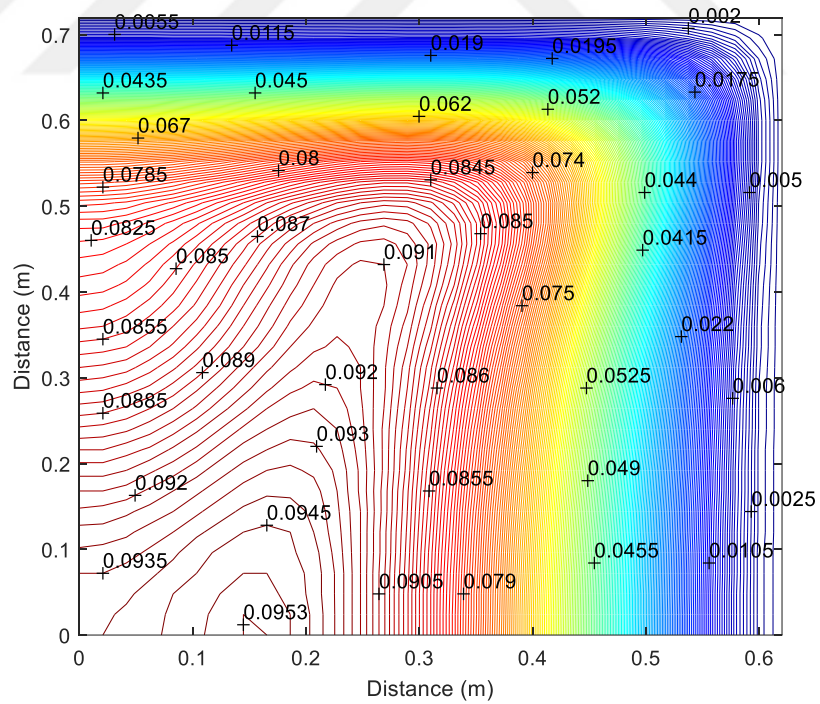


Figure 5.59 Monolithic glass contours for radial displacement.

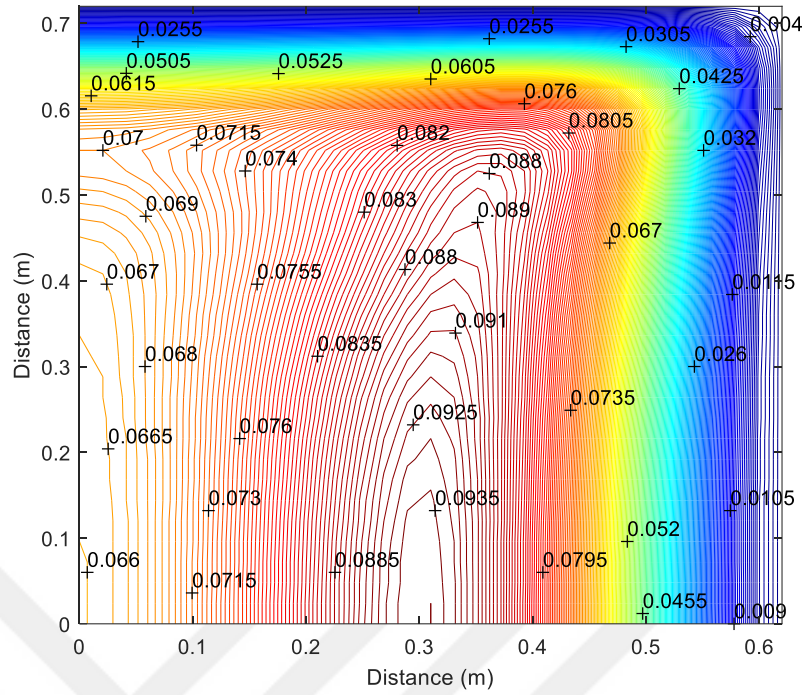


Figure 5.60 D1m 45%glass shell contours for radial displacement.

5.2. The Uniformly Distributed Load Behavior of a Simply-Supported Laminated Glass Shell Using Numerical Modeling.

The current model is modified for simply supported boundary conditions to explore how the constraints affect the performance of delaminated glass shells. A beam that has simply supported ends that are supported but have no momentary resistance and are free to spin. These specimens are 1.24 m in the curved direction and 1.44 m in the longitudinal direction. Simply supported laminated shell glass is put to the test; each glass sheet has the same thickness ($h_1 = h_2 = 5$ mm), and the thickness of the interlayer is 0.38 mm. The unit has a total thickness of 10.38 mm. The shell-glass Poisson's ratio is taken as 0.25 and Young's modulus as 72 GPa. The interlayer shear modulus and Poisson's ratio are 0.29 and 1000 kPa, respectively.

For the solution of a simply supported unit, the resulting governing equations can be used. The boundary conditions are as follows for a simply supported glass under uniform pressure:

When $\theta = 0$:

- $\frac{\partial w}{\partial x} = 0$

- $\frac{\partial v_1}{\partial \theta} = 0$
- $\frac{\partial v_2}{\partial \theta} = 0$
- $u_1 = 0,$
- $u_2 = 0,$

At $\theta = \theta_1$:

- $w = 0,$
- $\frac{\partial w}{\partial x} = 0$
- $u_1 = 0,$
- $u_2 = 0,$
- $v_1 = 0,$
- $v_2 = 0,$

At $y = 0$:

- $v_1 = 0,$
- $v_2 = 0,$
- $\frac{\partial u_1}{\partial y} = 0$
- $\frac{\partial u_2}{\partial y} = 0$
- $\frac{\partial w}{\partial \theta} = 0$

At $y = y_1$:

- $\omega = 0,$
- $\frac{\partial w}{\partial \theta} = 0$
- $u_1 = 0,$
- $u_2 = 0,$
- $v_1 = 0,$
- $v_2 = 0,$

Figures 5.63 to 5.66 show the linear and nonlinear responses of units as a function of normalized maximum deflection against load. The graph is used to determine the deflection

response of the specimens. The linear portion denotes the portion of the correlated connection between load and deflection, while the nonlinear component denotes the deviation from linearity as a result of geometric changes or material behavior. The vertical axes have normalized radial deflection, and the load is on the horizontal axes.

Figure 5.61 is plotted for laminated glass. The linear and nonlinear models are readily differentiated until the load is 100 kPa, after which the nonlinearity model radial deflection increases more than the linear model. For both fixed-supported and simply supported units, until the load is 20 kPa, the two models are rarely distinguished; the deflection of the nonlinear response is higher than the linear deflection of the unit under the same loading. The nonlinear model deflects above the linear model. The nonlinearity level of laminated units is observed at 0.06. Figures 5.62–5.64 indicate the graphs for Dlm 15%, Dlm 30%, and Dlm 45%. The nonlinearity levels of delaminated units are 0.54, 0.522, and 0.521, respectively. The deflection value obtained using the linear solution is almost 1.05 times the deflection obtained using the nonlinear solution for applied 50 kPa pressure, while the same ratios are 1.08 and 1.09 for delaminated units, which have 30 and 45 percent delamination, respectively. It is seen that as the delamination percentage increases, the difference between the nonlinear and linear responses increases, and the importance of nonlinear modelling in the solution becomes more important. This incrementation between the linear and nonlinear graphs also indicates the reduction in stiffness of the units; as the delamination percentage increases, the stiffness reduces. It was also observed that the constraint types also play a significant role and can cause linearity to diverge.

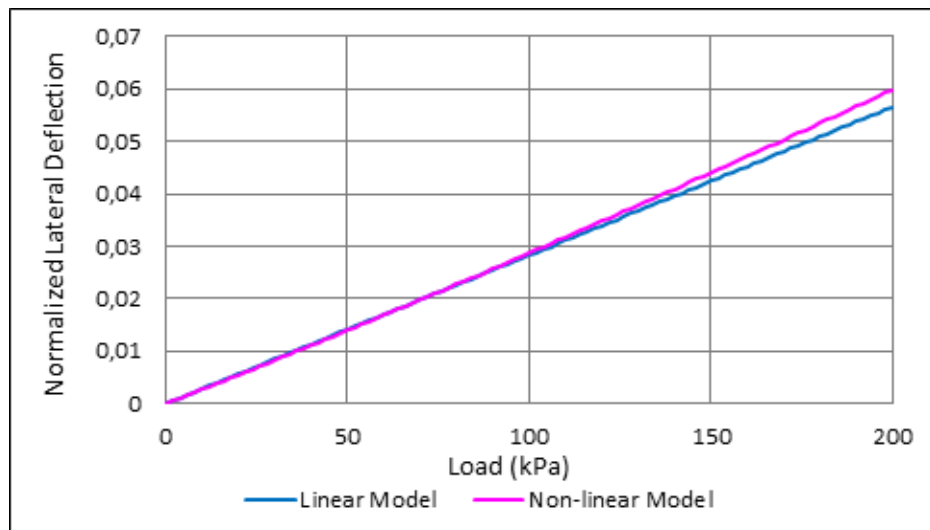


Figure 5.61 Normalized maximum deflection ($\frac{w_{max}}{h}$) against the load for simply supported laminated glass shell.

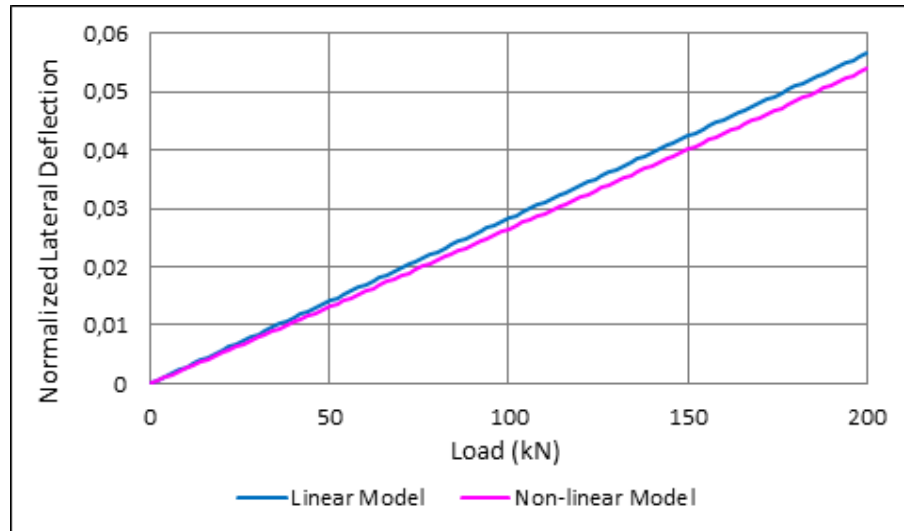


Figure 5.62 Normalized maximum deflection ($\frac{w_{max}}{h}$) against the load for simply supported delaminated glass shell (Dlm 15%)

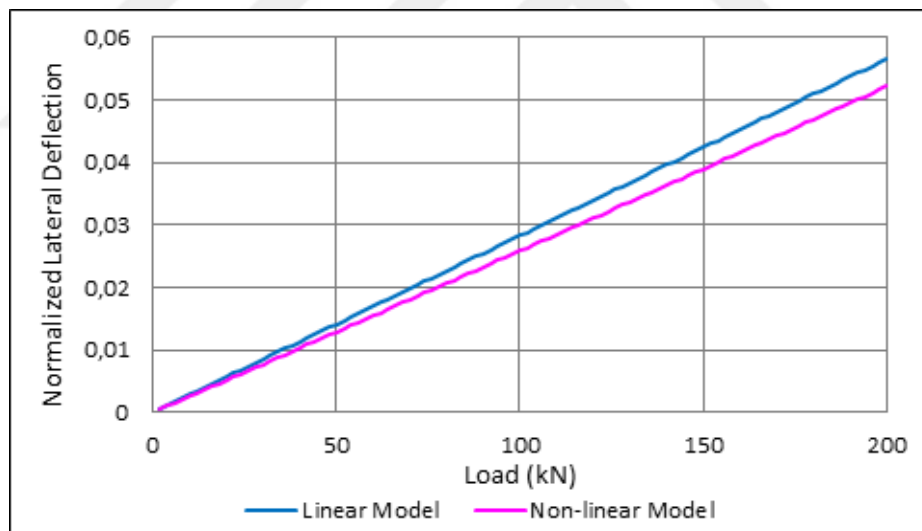


Figure 5.63 Normalized maximum deflection ($\frac{w_{max}}{h}$) against the load for simply supported delaminated glass shell (Dlm 30%)

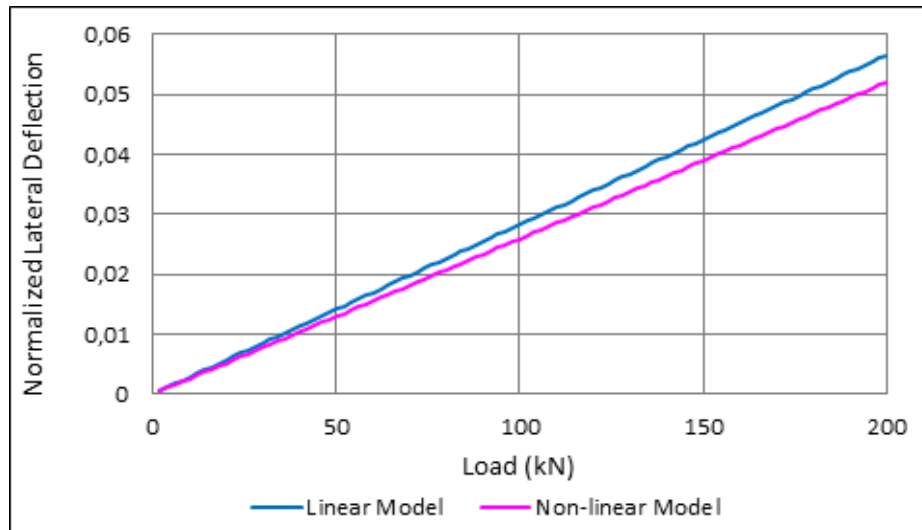


Figure 5. 64 Normalized maximum deflection ($\frac{w_{max}}{h}$) against the load for simply supported delaminated glass shell (Dlm 45%)

Figures 5.65 and 5.66 show the comparison of maximum displacements and stresses of laminated glass, layered glass, monolithic glass, and delaminated glasses (Dlm 15%, Dlm 30%, and Dlm 45%, respectively). Monolithic glass has the largest deflection, whereas Dlm 45% exhibits the lowest deflection. Laminated glass deflects more than layered glass because of its flexibility and damping, allowing great deflection under load. In Figure 5.68, the stress is compared; the maximum and minimum stress are observed in monolithic and Dlm 45% units, respectively.

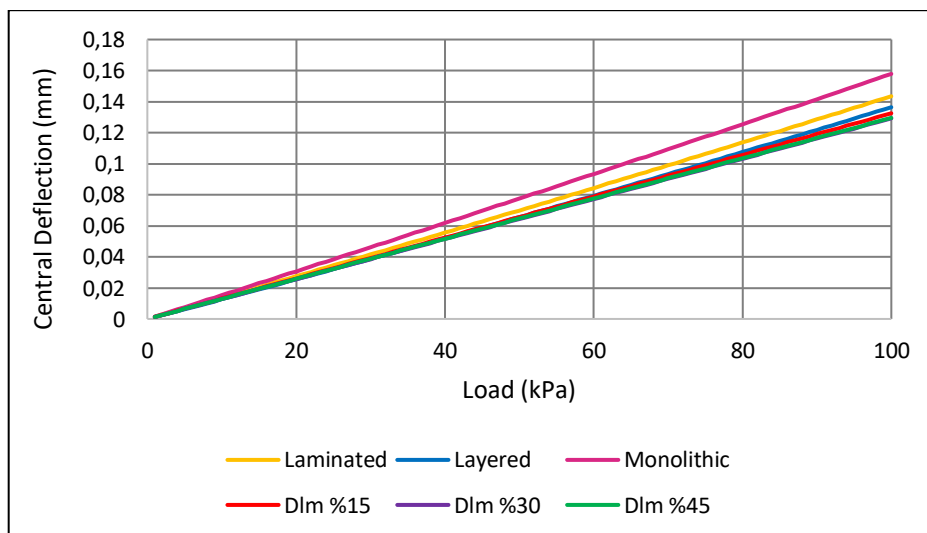


Figure 5.65 Comparison of maximum central deflection

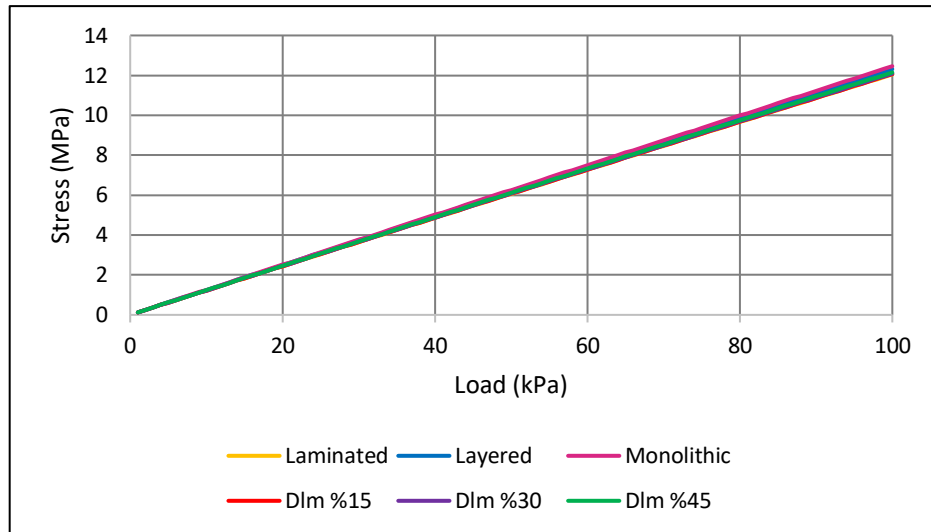


Figure 5.66 Comparison of maximum stresses

Figures 5.67 and 5.68 show the top and bottom glass units' respective circumferential displacements (u_1 and u_2) along the diagonal. The circumferential displacements are zero at the starting point of the quarter plate, as we anticipated. The circumferential displacements start to increase to their maximum at a certain point on the first quartal, after which they displace into negative values. On the other hand, the monolithic unit exhibits the least displacement.

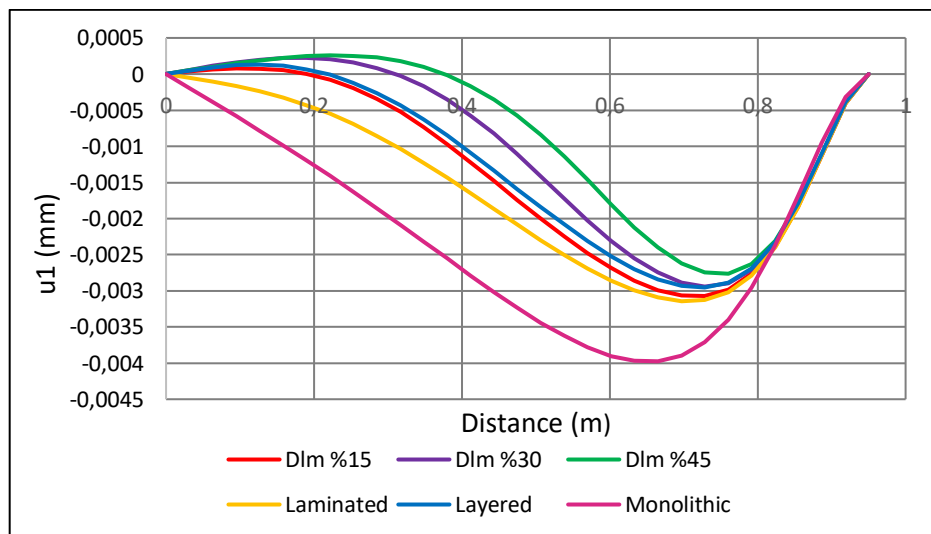


Figure 5.67 Circumferential displacement through the diagonal (Top glass surface)

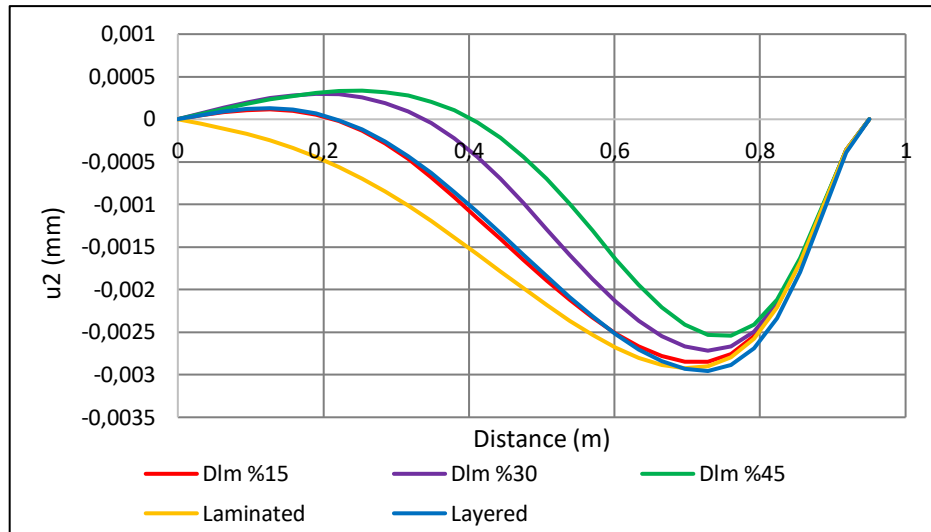


Figure 5.68 Circumferential displacement through the diagonal Bottom glass surface)

Figures 5.69 and 5.70 show the top and bottom glass units' respective axial displacements (v_1 and v_2) along the diagonal. The axial displacements are zero at the starting point, as we anticipated. An increment in axial displacement is observed from zero to 0.6 m for monolithic and layered glass, after which it displaces negatively. Minimum displacements observed for a delaminated unit that has 15% delamination (Dlm 15%)

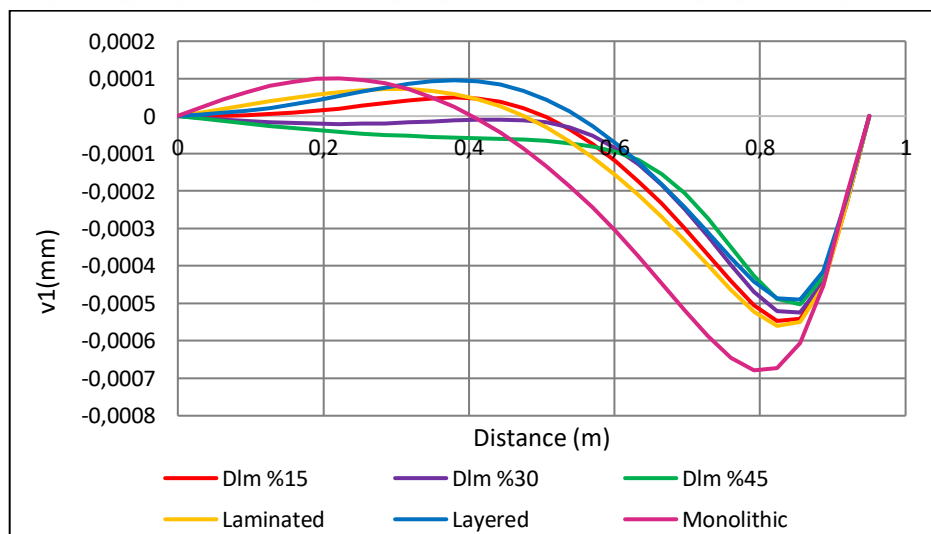


Figure 5.69 Axial displacement through the diagonal (Top glass surface)

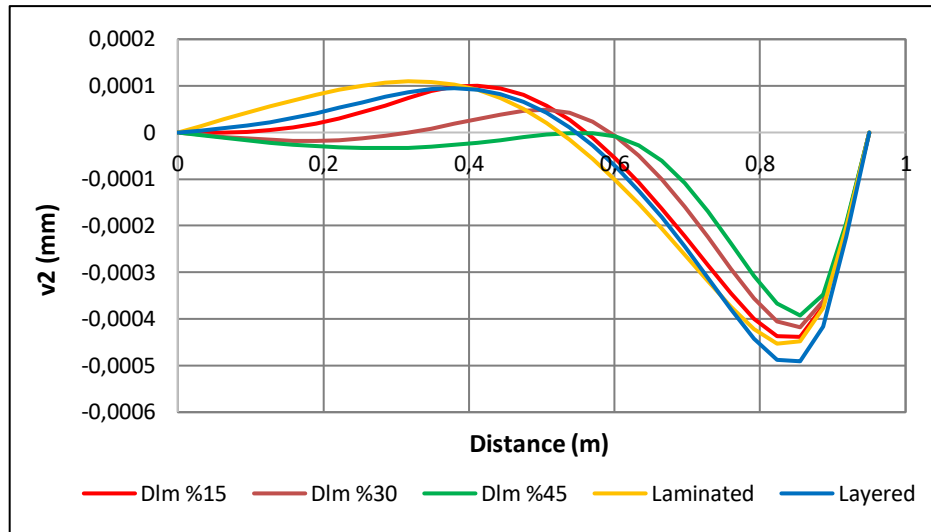


Figure 5.70 Axial displacement through the diagonal (Bottom glass surface)

The radial deflections at the center of the unit in θ and y directions are plotted in Figures 5.71 and 5.72, while the corresponding displacements in the diagonal of the unit are illustrated in Figures 5.73. The radial deflections are zero at the edges and at the corners of the unit. Radial deflections reach their maximum value between 0.28 and 0.4 m in the θ direction. Double curvature, which is a result of nonlinear response, can be observed in the figures. Despite being zero at the edges, radial displacements are at their maximum in the middle. The effect of delamination is clearly visible for the fixed-supported units compared to the simple-supported units.

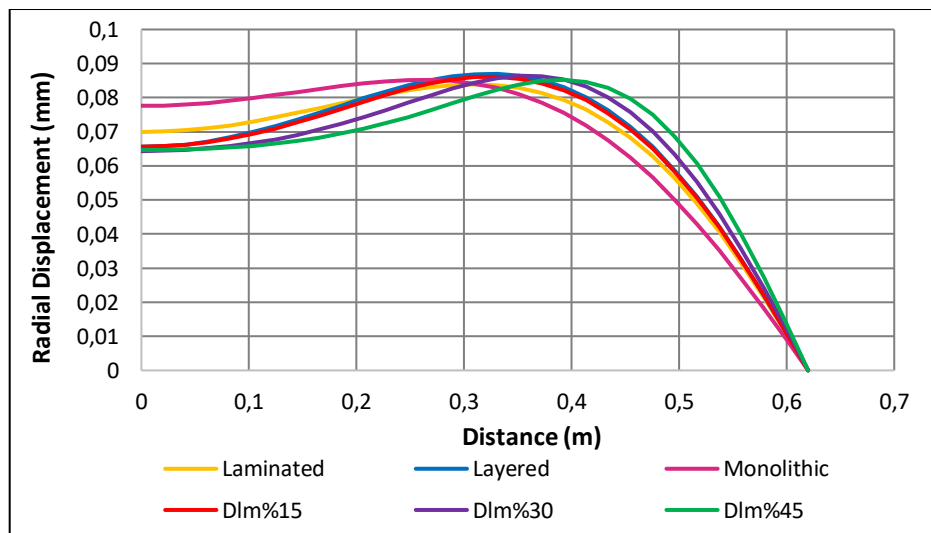


Figure 5.71 Central radial displacement at $y = 0$

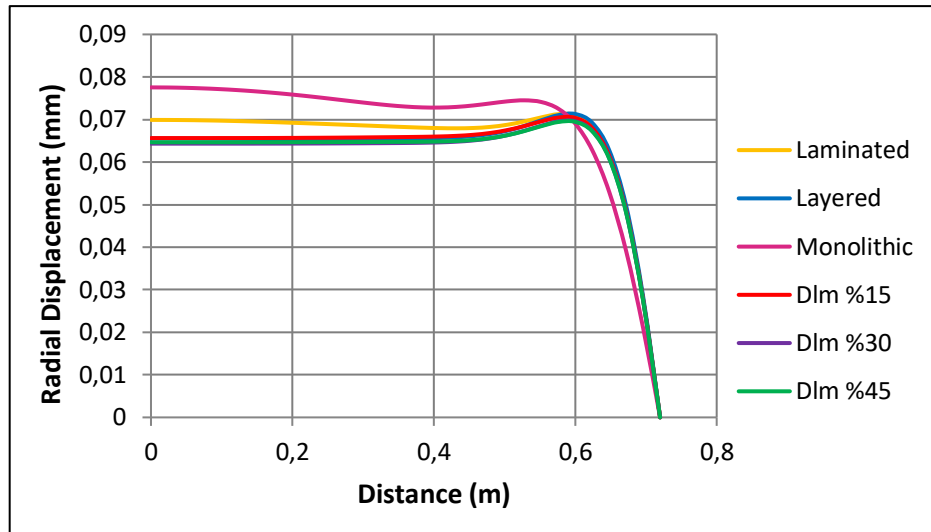


Figure 5.72 Central radial displacement at $\theta=0$

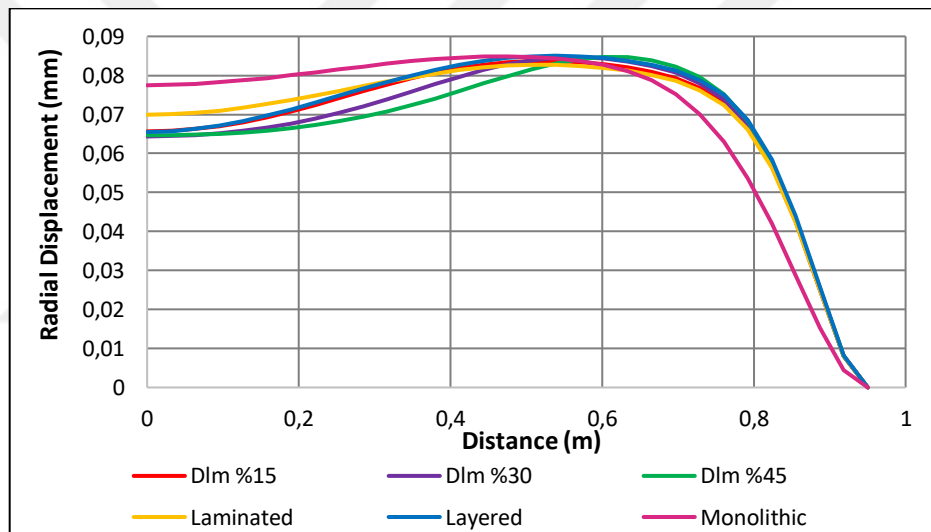


Figure 5.73 radial displacement through the diagonal

Figures 5.74 and 5.75 show the maximum stress distributions (S1) on the surfaces of the unit along the centerline at $y=0$ with applied $P = 50$ kPa pressure. As can be observed from the figures, at the applied pressure value of $P = 50$ MPa, the surface's maximum stress has positive values for both top and bottom glass along the q direction of the shell. The surfaces have stresses that reach their maximum between 0.4 and 0.45 meters from the plate's center for both surfaces.

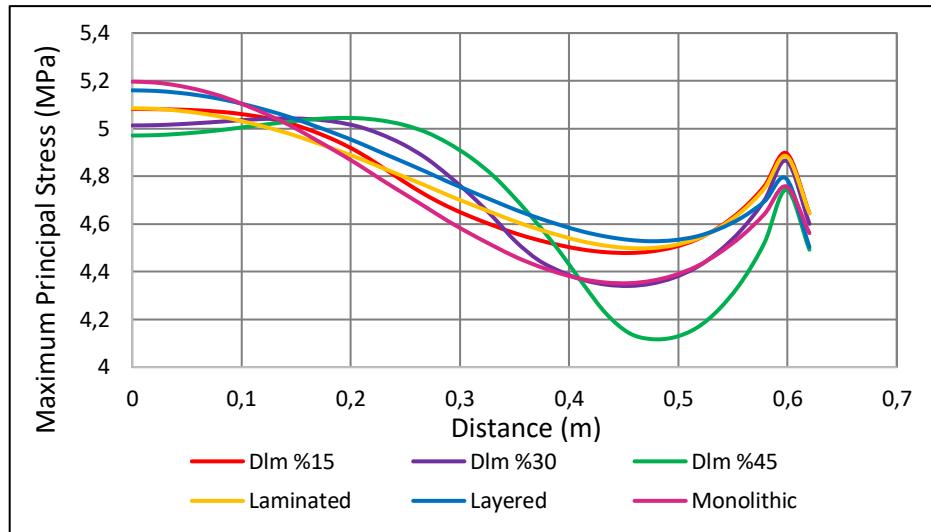


Figure 5.74 Central maximum stress at $y = 0$ (Top glass surface)

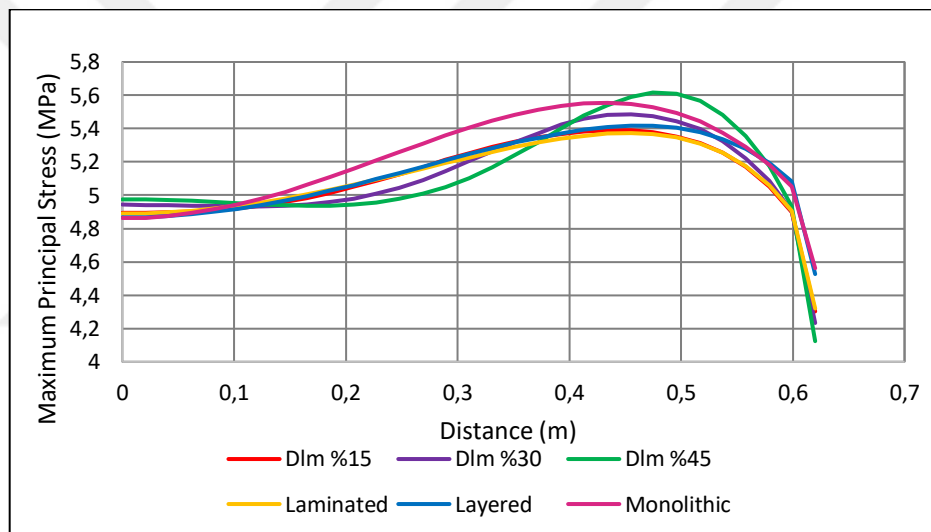


Figure 5.75 Central maximum stress at $y = 0$ (Bottom glass surface)

Figures 5.76 and 5.77 show the minimum stress distributions on the surfaces of the top and bottom units along the centerline at $y = 0$. The surface's minimum stress has positive values for both top and bottom glass in the θ direction. As can be observed from Figure 5.76, the Dlm 45% glass unit has the minimum stress on the top surface at a distance of 0.5 m and around 0.45 m for the other specimens. The surfaces have stresses that reach their maximum between 0.4 and 0.5 meters from the plate's center for the bottom surfaces. Dlm 45%'s minimum stress is higher than that of the bottom surface.

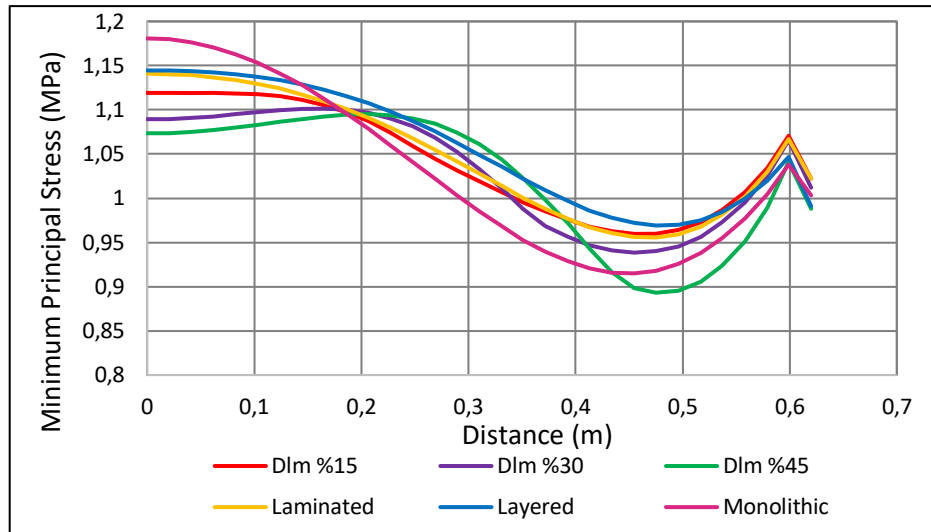


Figure 5.76 Central minimum stress at $\gamma=0$ (Top glass surface)

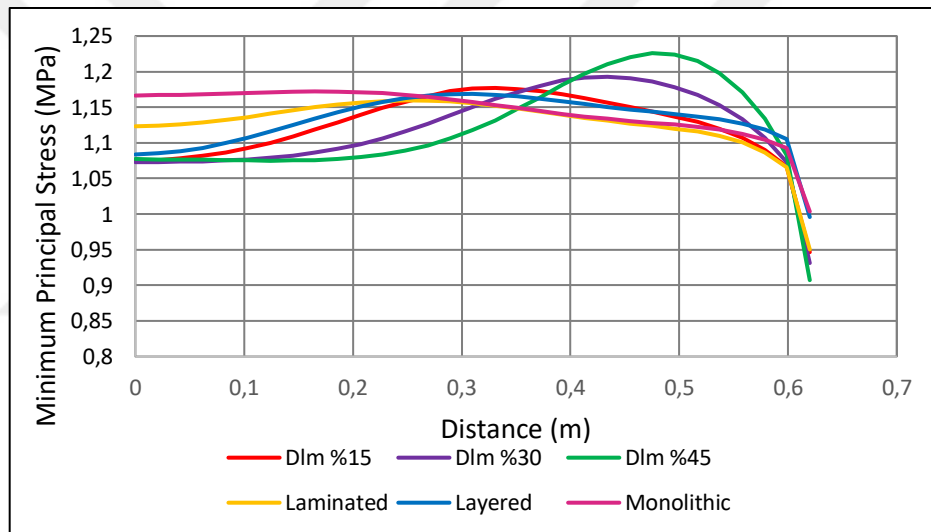


Figure 5.77 Central minimum stress at $\gamma=0$ (Top glass surface)

The maximum stresses along the centerline of the plate unit at $\theta = 0$ for applied $P = 50$ kPa pressure are plotted in Figures 5.78 and 5.79. The top and bottom surfaces of a simply supported unit are tensioned through the centerline, unlike the maximum stress on the bottom surfaces of a fixed unit, where compression is observed.

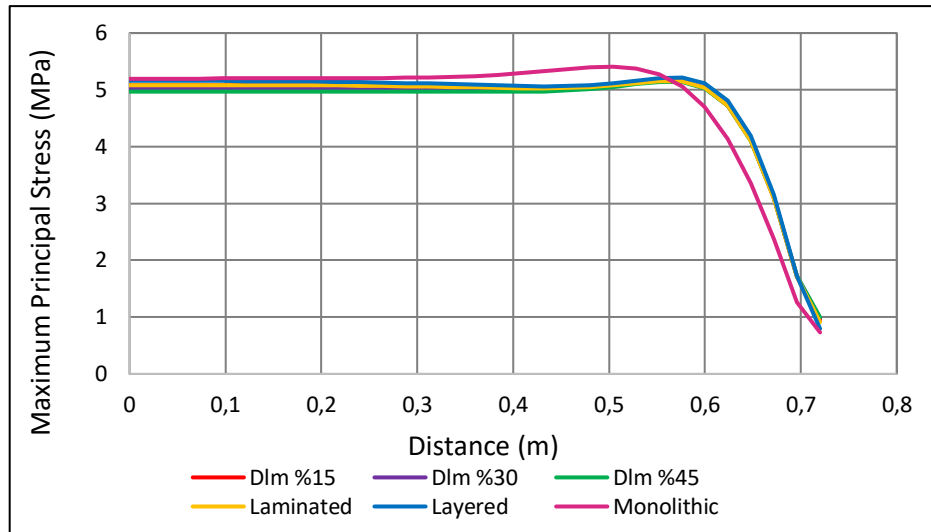


Figure 5.78 Central maximum stress at $\theta=0$ (Top glass surface)

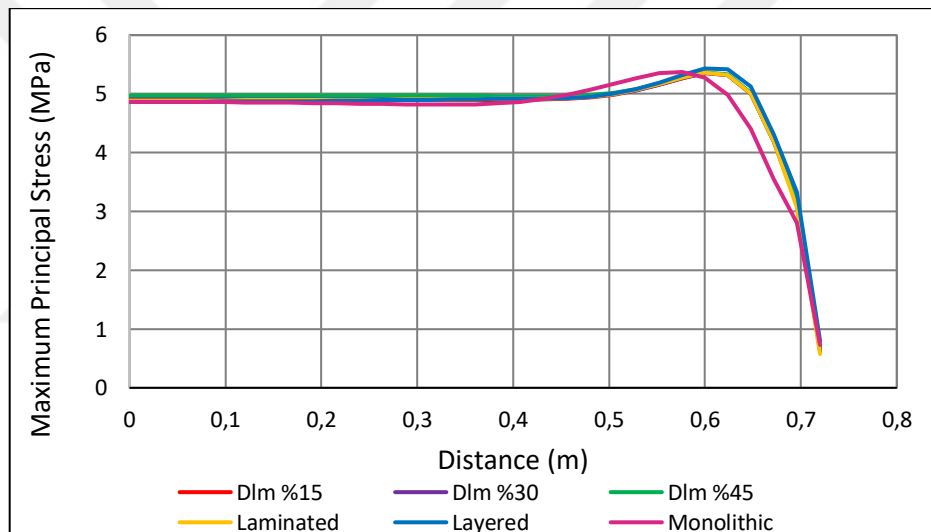


Figure 5.79 Central maximum stress at $\theta=0$ (Bottom glass surface)

The minimum stresses along the centerline of the plate unit at $x = 0$ for applied $P = 5$ kPa pressure are plotted in Figures 5.82 and 5.83. The unit's top surface has a minimum stress line that starts as tension and is roughly constant up to 0.4 m distance from the glass's center, at which point a compressive behavior can be observed. The minimum stress lines on the unit's bottom surface reach tension at a distance of 0.5 m, where they reach their maximum value.

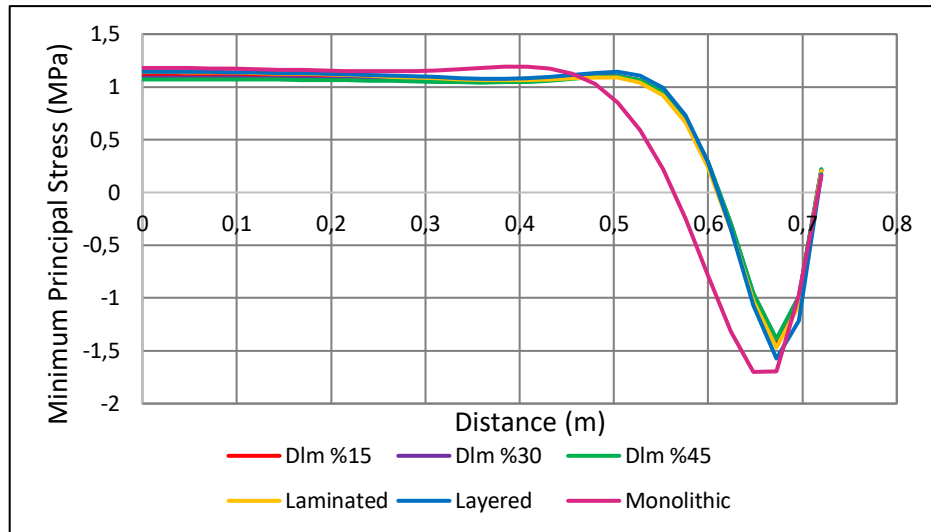


Figure 5.80 Central minimum stress at $\theta= 0$ (Top glass surface)

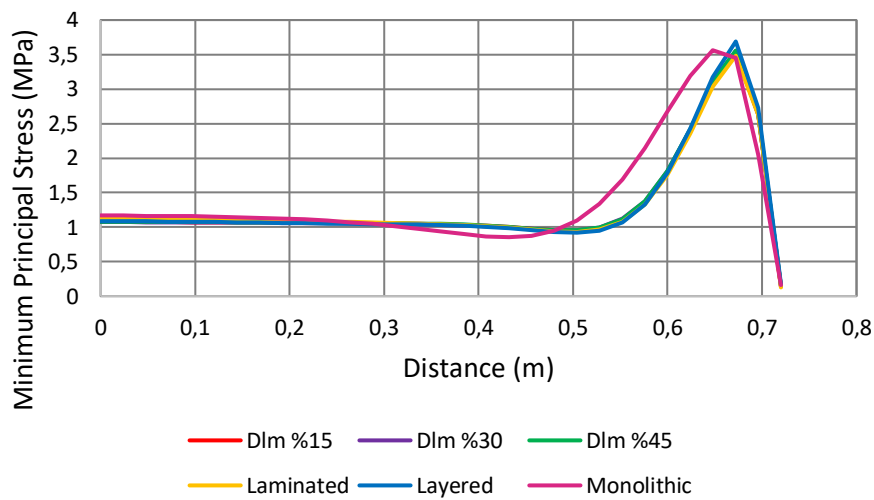


Figure 5.81 Central minimum stress at $\theta= 0$ (Bottom glass surface)

Data on the principal stresses (S1 and S2) versus load is shown in Figures 5.82 and 5.83. The behavior of the top glass of a laminated and delaminated (Dlm 45%) glass shell is depicted in a graph. The stresses on the top surfaces of a laminated glass unit are shown in Figure 5.82. At a load of 50 kPa, the laminated glass has a maximum principal stress of 5.1 MPa at 50 kPa and a minimum principal stress of 1.2 MPa at 50 kPa. The principal stresses (S1 and S2) are tensile stresses. Figure 5.83 shows the maximum and minimum principal stresses against load for delaminated glass (Dlm 45%), which is similar to laminated glass. However, stress values for the linear and nonlinear graph lines are marginally lower than for laminated glass at the same load rate (50 MPa).

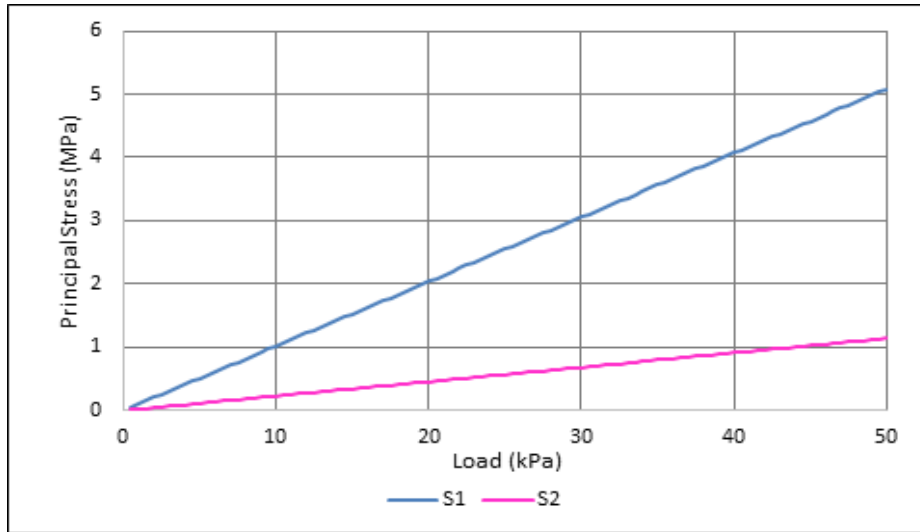


Figure 5.82 Principal stresses of laminated glass (Top surface)

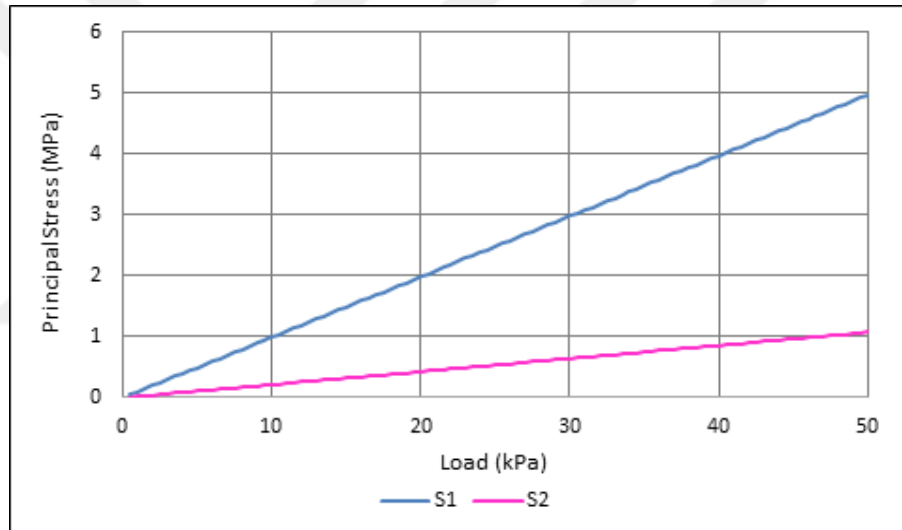


Figure 5.83 Principal stresses of Dlm 45% (Top surface)

Principal stresses for the bottom surfaces of the laminated and delaminated (Dlm 45%) glass shell are illustrated in Figures 5.84 and 5.85 against various load values. The stress indicators do not alter. Both the maximum and minimum graph lines exhibit smaller stress results than the top glass surface. Maximum and minimum principal stresses on the bottom surface of Dlm 45% are approximately the same as those in laminated glass. In comparison to fixed support, stresses are less for simply supported.

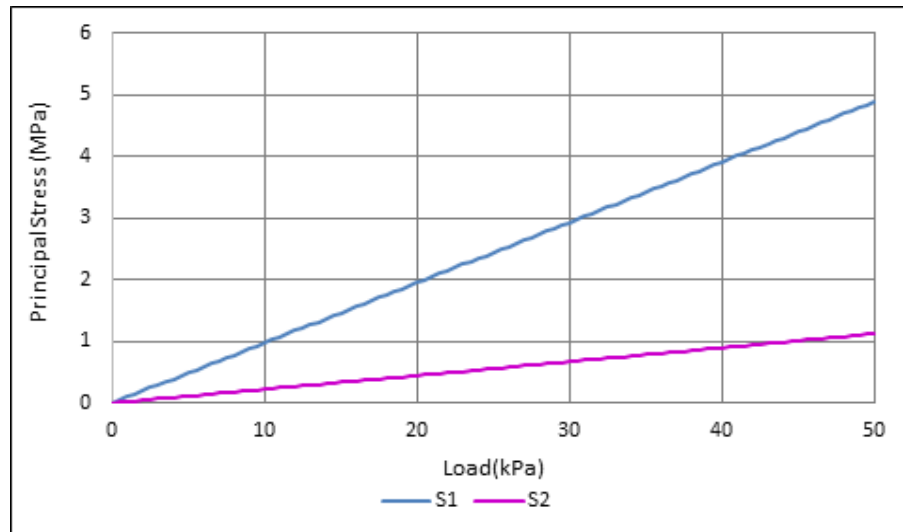


Figure 5.84 Principal stresses of laminated glass (Bottom surface)

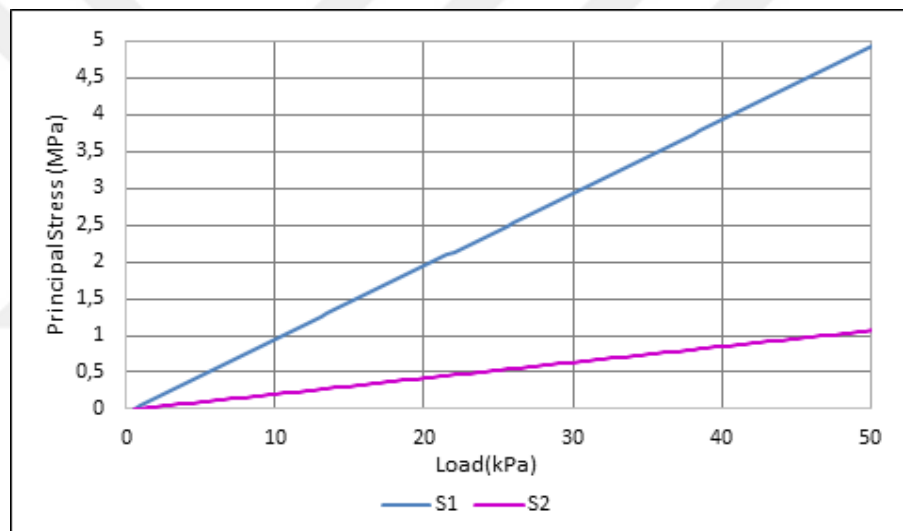


Figure 5.85 Principal stresses of Dlm 45% (Bottom surface)

Figures 5.86 to 5.89 depict the maximum and minimum principal stress values on the top and bottom surfaces of Dlm 15% glass, which is simply supported at both ends. Specifically, Figures 5.86 and 5.87 display the contours of the maximum principal stress on the top and bottom surfaces of Dlm 15% glass, respectively. On the top surface, the maximum principal stress is in tension and is highest at a distance 0.53 meters from the horizontal centerline, gradually decreasing towards the top edge. The bottom surface also exhibits maximum stress close to the corner located at coordinates 0.6 m vertically and 0.5 m horizontally.

Additionally, Figures 5.88 and 5.89 present the contours of the minimum principal stress on the top and bottom surfaces of Dlm 15% glass, respectively. Towards the center of the top surface, high tensional values of minimum principal stress and compression stress can be

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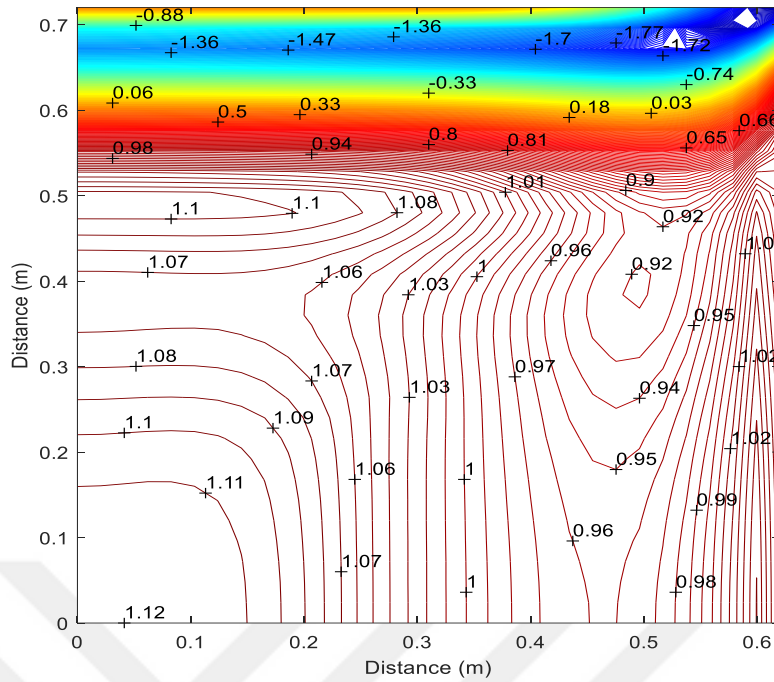


Figure 5.88 Minimum principal stress contours (S2) for Dlm 15% (Top surface)

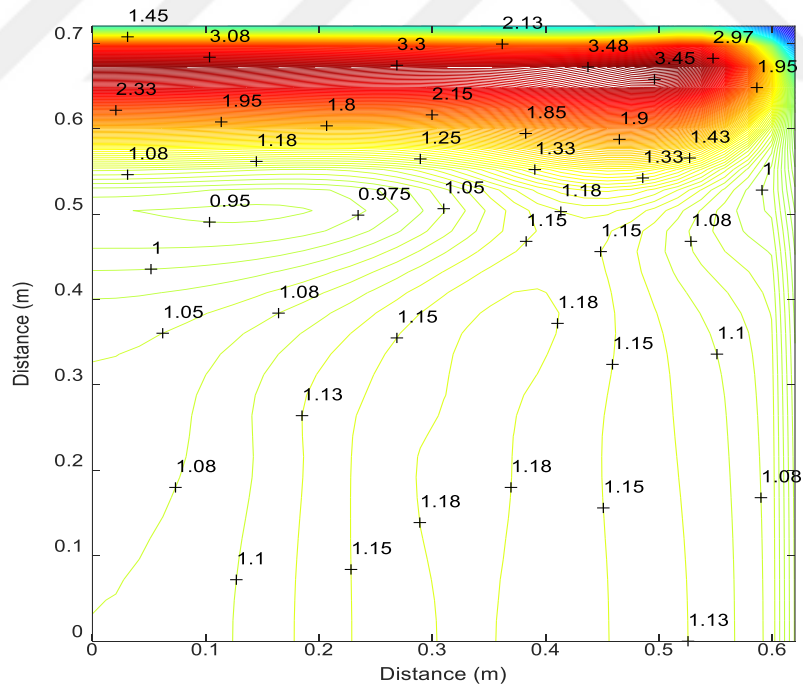


Figure 5.89 Minimum principal stress (S2) contours for Dlm 15% (Bottom surface)

Figure 5.90 represents the maximum stress contours that are given for the top surface of the delaminated unit, Dlm 30%, whereas Figure 5.91 represents the contours for the bottom

surface of the corresponding unit. The stress at the top surface is high near the top edge at around 0.55 m vertical distance and around 0.2 m horizontal distance on the quarter unit. Stress values at the edge are much smaller compared to the middle region. On the bottom surface, the stresses are higher than those on the top surface. Figures 5.92 and 5.93 illustrate the minimum principal stress (S2) on the top and bottom surfaces of Dlm 30% glass. The minimum stress on the top surface is tension near the center and compression at the top edge. The bottom surface is all tension, with high stress values near the top edge.

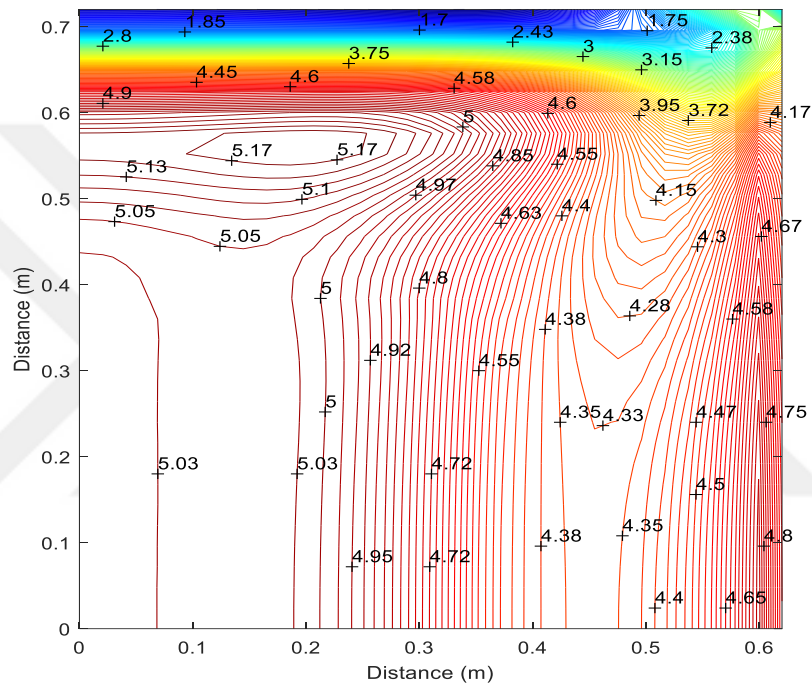


Figure 5.90 Maximum principal stress contours (S1) for Dlm 30% (Top surface)

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The maximum principal stress contours for the top laminated glass unit are shown in Figure 5.98, and the bottom unit is represented in Figure 5.99. At a distance of about 0.1 m and 0.15 m in both the horizontal and vertical directions from the quarter unit's center, the top surface has its maximum value. The value of maximum stress on the top surface is slightly smaller than that on the bottom surface, and it moves to the corner of the unit. Figures 5.100 and 5.101 show the S2 stress distribution of the top and bottom glass surfaces. The top surfaces are in compression on the upper edge of the quarter plate and tension towards the center. Everywhere on the bottom surface is in tension, while the top edge has high stresses.

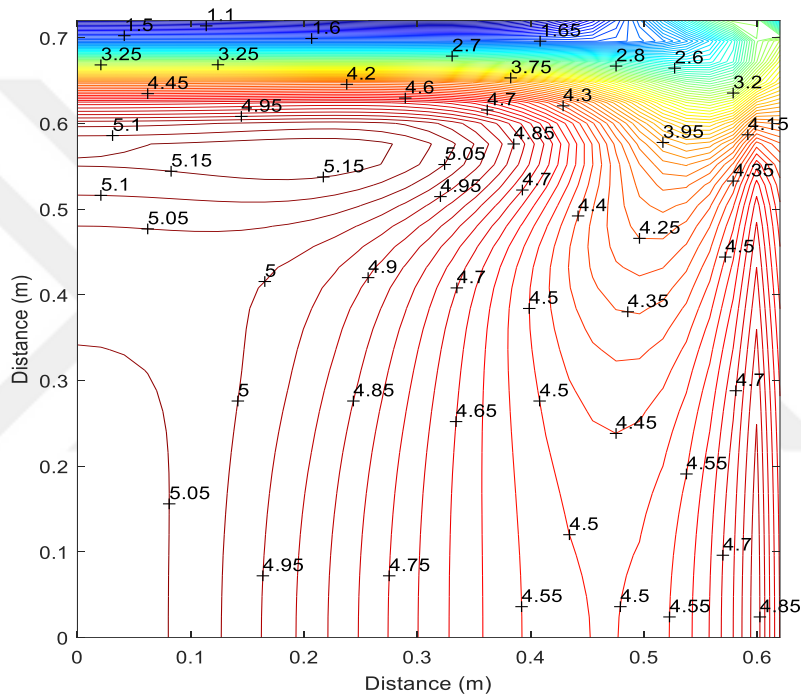


Figure 5.98 Maximum principal stress contours (S1) for laminated glass shell (Top surfaces)

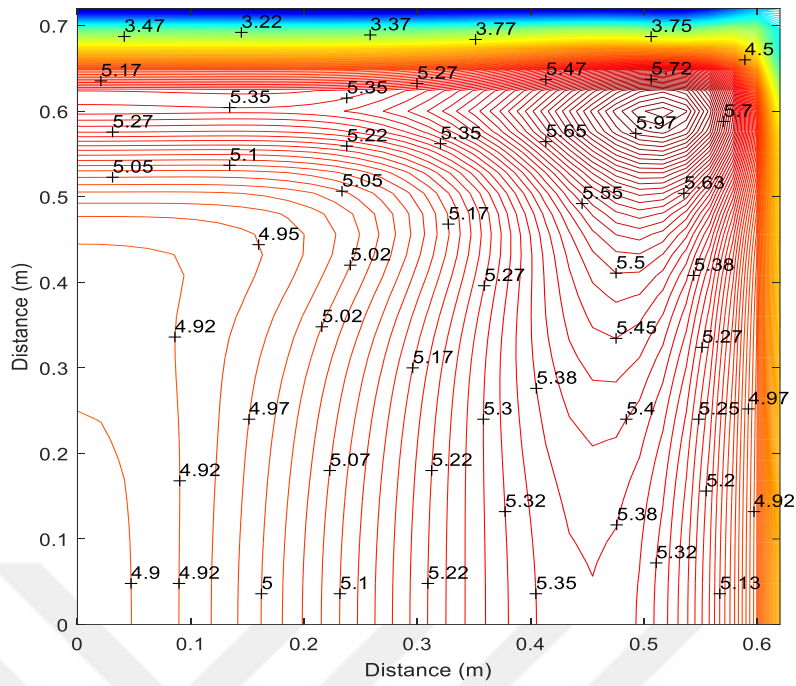


Figure 5.99 Maximum principal stress contours (S1) for laminated glass shell (Bottom surface)

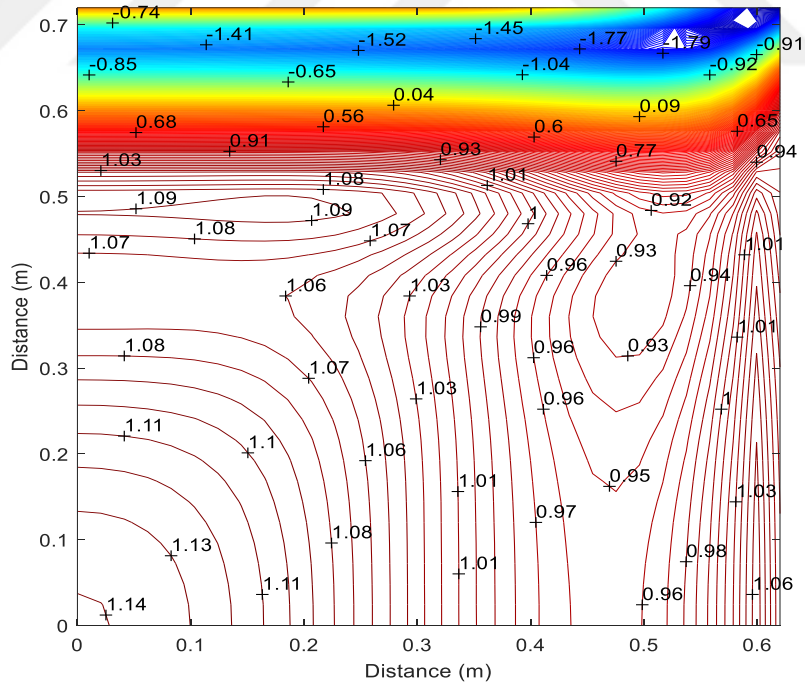


Figure 5.100 Minimum principal stress (S2) contours for laminated glass shell (Top surface)

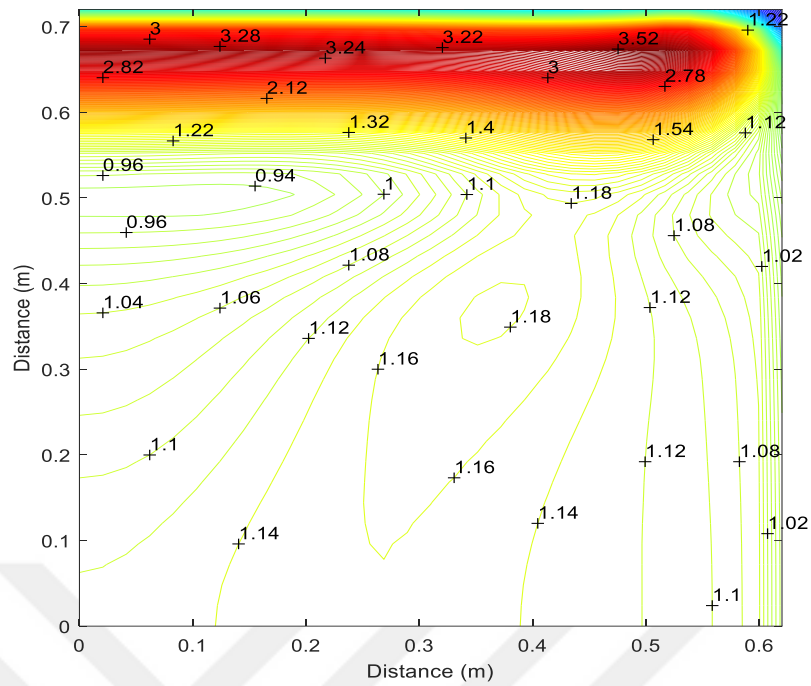


Figure 5.101 Minimum principal stress (S2) contours for laminated glass shell (Bottom surface)

The maximum principal stress contours for the top of the layered glass unit are represented by Figure 5.102, and those for the bottom unit are represented by Figure 5.103. The top surface is in tension, with its maximum principal stress at a distance of 0.2 and 0.6 meters in horizontal and vertical directions, respectively. The bottom surface's maximum value is observed in the upper right corner of the layered unit. When compared to the top surface, high tension stress is seen on the bottom surface. The top surface S2 stresses of the layered units are shown in Figure 5.104, while the ones on the bottom surface are shown in Figure 5.105. On the top surface, the high-tension minimum principal stress values can be seen moving towards the

center, and compressive stresses are seen at the edge. Tension stress can be found all over the bottom surface.

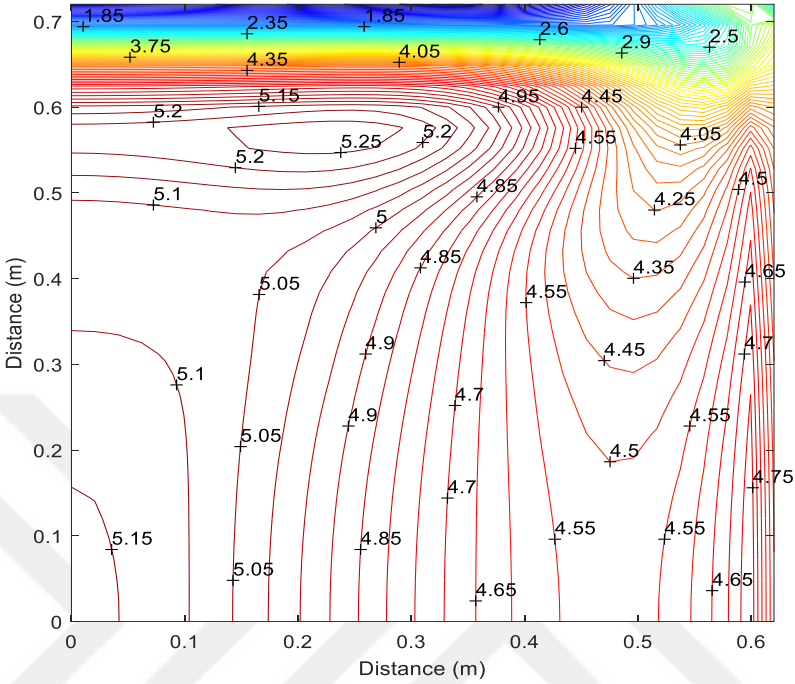


Figure 5.102 Maximum principal stress contours (S1) for layered glass shell (Top surface)

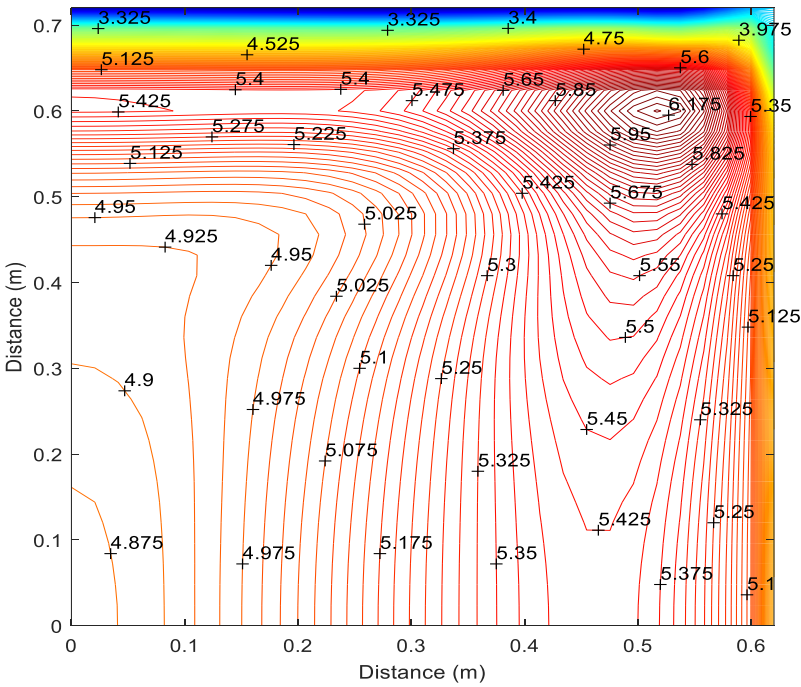


Figure 5. 103 Maximum principal stress contours (S1) for layered glass shell (Bottom surface)

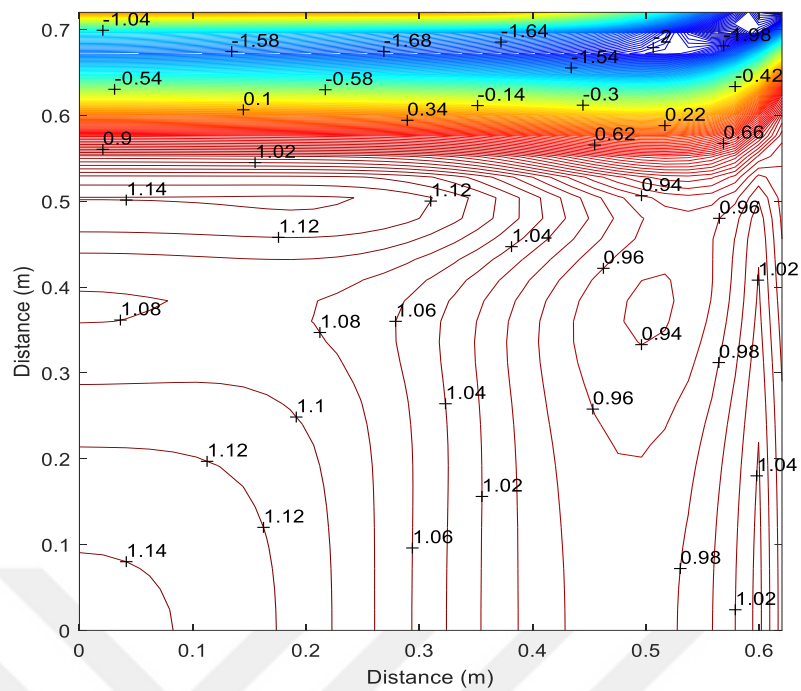


Figure 5.104 Minimum principal stress (S2) contours for layered glass shell (Top surface)

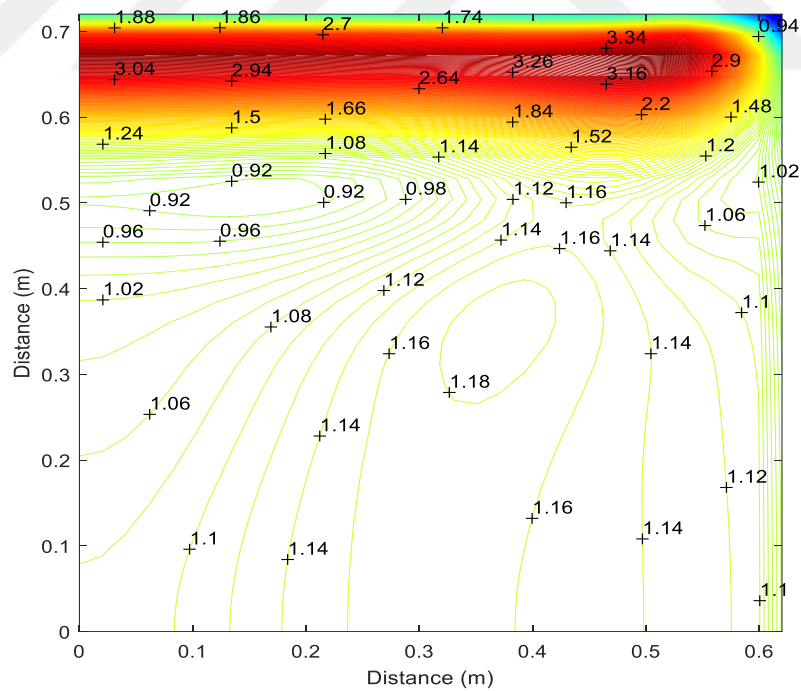


Figure 5.105 Minimum principal stress (S2) contours for layered glass shell (Bottom surface)

The maximum principal stress contours for the top monolithic glass unit are represented in Figure 5.106, and those for the bottom unit are represented in Figure 5.107. The maximum

principal stress (S_1) on the bottom surface of the monolithic unit is also tension, and its maximum value is located near the right top corner, whereas the maximum principal stress on the top surface is all tension, and its maximum is located towards the top left edge. Figures 5.108 and 5.109 show the minimum principal stress (S_2) contours for the top and bottom of the monolithic unit. The top surface units are compressed towards the top edge of the unit, whereas the bottom surface units have a minimum principal stress that is entirely tensioned and high towards the top edge.

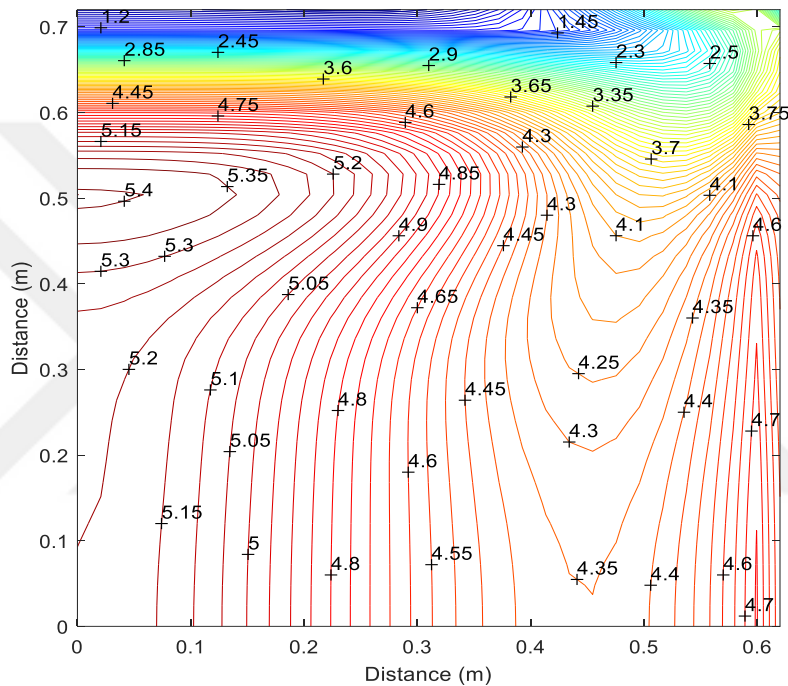


Figure 5.106 Maximum principal stress (S_1) contours for monolithic glass shell (Top surface)

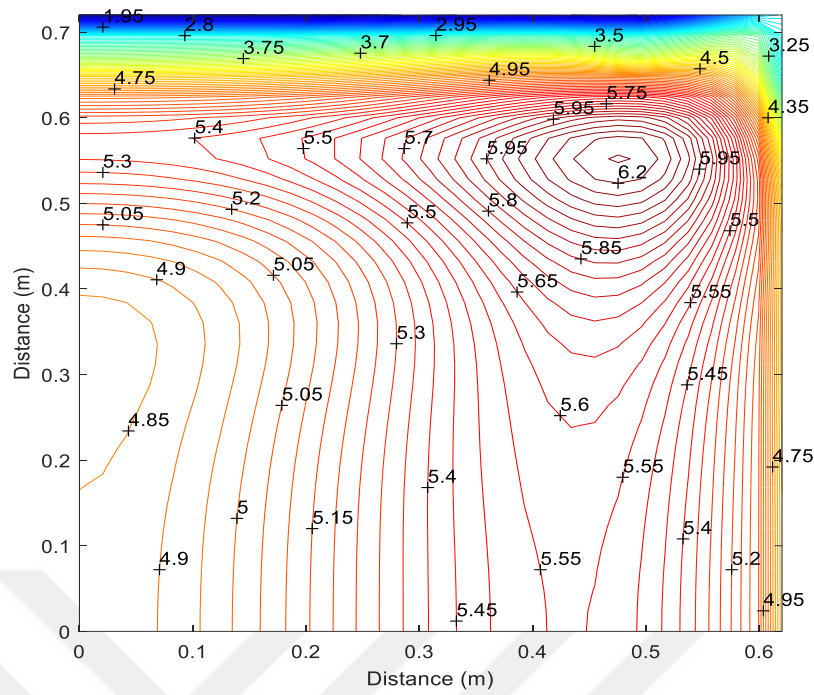


Figure 5.107 Maximum principal stress (S1) contours for monolithic glass shell (Bottom surface)

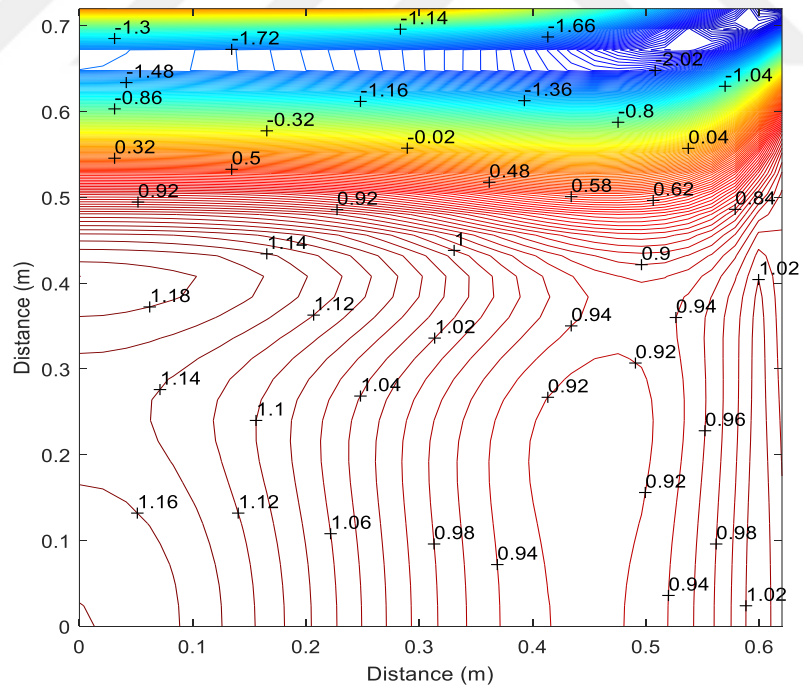


Figure 5.108 Minimum principal stress (S2) contours for monolithic glass shell (Top surface)

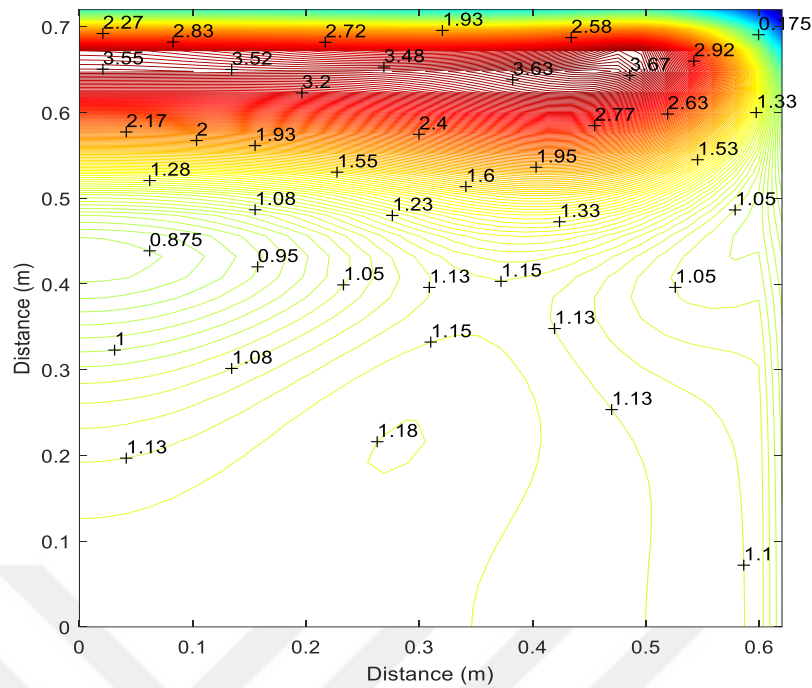


Figure 5.109 Minimum principal stress (S2) contours for monolithic glass shell (Bottom surface)

The radial displacement contours for Dlm 15%, Dlm 30%, and Dlm 45% laminated, layered, and monolithic units are given in figures 5.110 to 5.115. The radial displacement contours are plotted in order to analyze the deflections in detail. Radial displacements take their maximum value at a point on the centerline close to the vertical edge of the unit. When compared to a fixed support, the displacement of a simple supported unit is smaller. The contour

maps below demonstrate that displacements are largest for Dlm 45%, while they are smallest for laminated glass specimens.

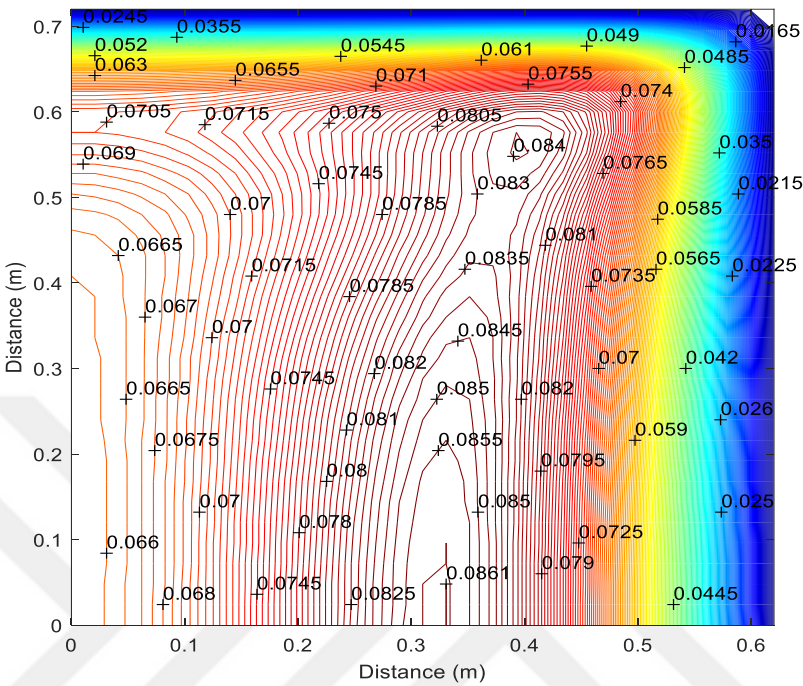


Figure 5.110 Contours for radial displacement of Dlm 15% glass

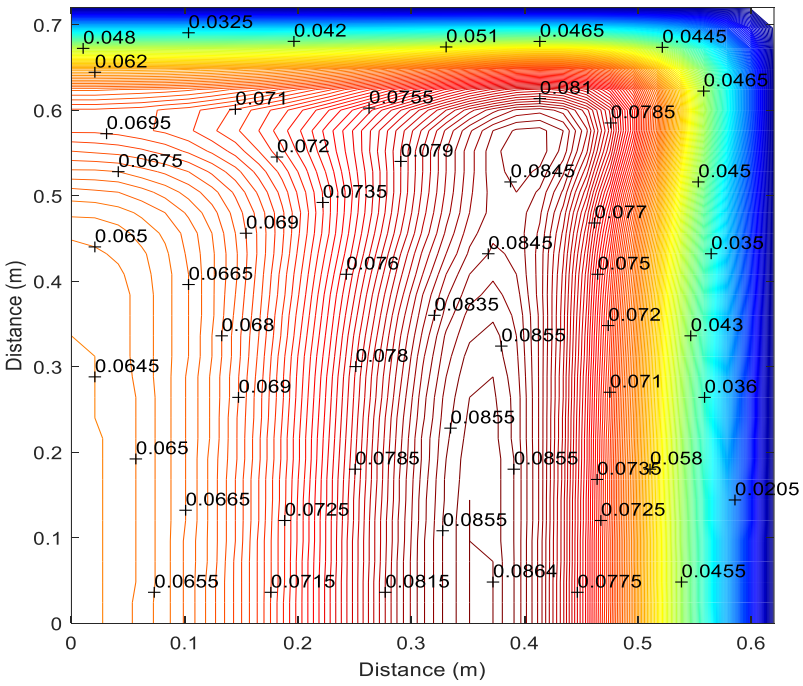


Figure 5.111 Contours for radial displacements of Dlm 30% glass

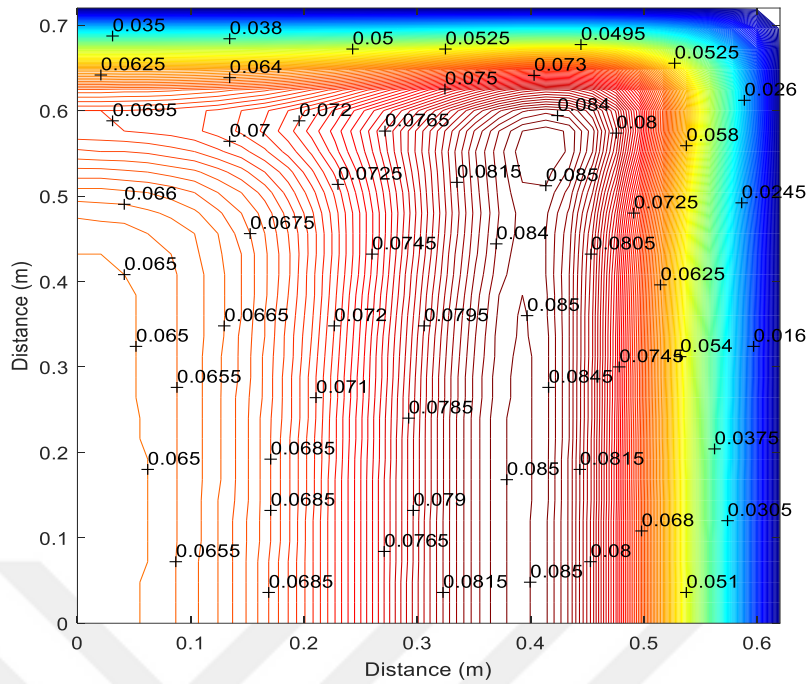


Figure 5.112 Contours for radial displacements Dlm 45% glass shell

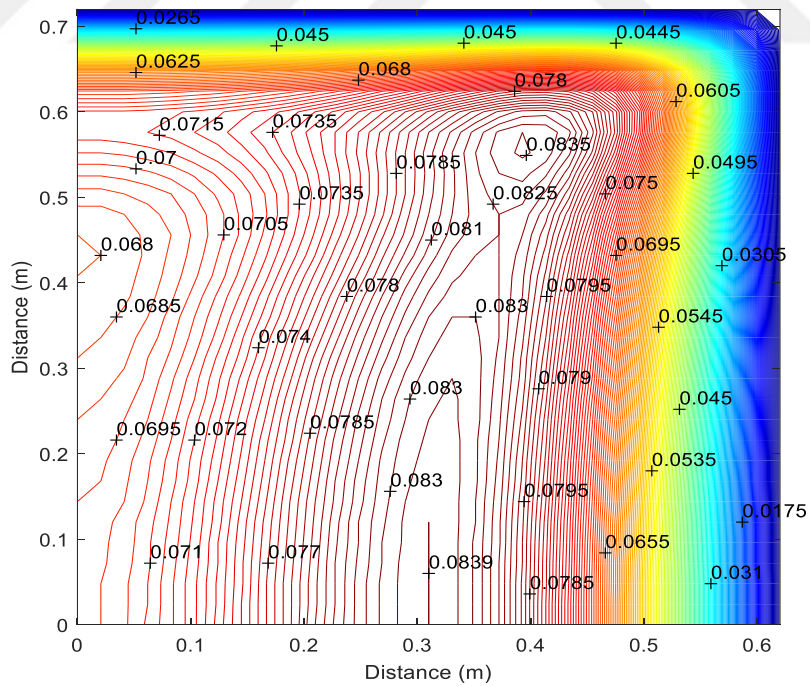


Figure 5.113 Contours for radial displacements of laminated glass shell

6. CONCLUSION AND SUGGESTIONS

To ascertain the impact of delamination on the strength of laminated glass shells under distributed loads, this research uses the modified equation of Dural's (2011) nonlinear bending theory. The corresponding geometric and structural characteristics distinguish a shell from a plate. The emphasis is on the delaminated portion of the glass shell, and the boundary conditions and field equations are rearranged for both linear and nonlinear equations. Using interpolation, convergent solutions are attained. Based on the obtained field equation and using the iterative solution method, a model for the analyze of laminated glass shells with initial delamination is generated.

The mentioned field equations and boundary conditions of laminated glass units are rearranged in the delaminated region of the glass shell. Obtained field equations are transformed into nonlinear algebraic equation systems for delaminated and undelaminated regions of the glass unit. This means the mathematical model established in this study offers a practical approach to modelling both delaminated and laminated glass shells.

The minimum potential energy theorem and calculus of variation are used to derive five nonlinear partial differential equations with simple and fixed supported boundary conditions. The effect of membrane stresses on fixed-supported glass shells makes the same set of equations give nonlinear results for both laminated and delaminated glass shells. A nonlinear strain tensor is taken into consideration in the analysis of laminated glass shells since it is found to be a result of geometric nonlinearity.

In the derivation of the model, as the first step, the total potential energy of the laminated glass shell is written by summing membrane, bending, shear, and load strain energies. Using a minimum potential energy theorem and variational approach, differential equations entailing nonlinear variables are derived with fixed and simply supported constraints for undelaminated and delaminated sections of the glass shell. Considering whether there is delamination or not, two different equation systems are used in the solution. In the delaminated sections, shear stress is ignored as a result of the separation between the layers. The obtained equations are solved by modifying the FORTRAN computer code developed by Dural (2011).

The model is suitable for the solution of monolithic and layered glass shells. This allows us to determine the advantage of using a polyvinyl butyral interlayer. In the current model, it is possible to investigate the linear and nonlinear responses of units. This provides us with information about the load value at which nonlinear response should be considered to obtain

the true behavior of the unit. Differentiating between linear and nonlinear deflection graphs starts at different load levels for fixed and simple supported units. This means that constraints and the type of support cannot be ignored in the analysis process. Various relations between deflection, stress, and load are evaluated. The comparison of the responses of laminated and delaminated glass specimens is evaluated. It is also observed that the nonlinearity level of delaminated units is lower than that of laminated units. The conclusion is that this model can provide accurate predictions about the linear or nonlinear response of a delaminated glass shell subjected to a variety of load levels and boundary conditions. The response of laminated glass shell units with initial delamination under various loadings, boundary conditions, and different delamination ratios is provided and put into the service of engineers, designers, and manufacturers. Strength values of shells subjected to fixed and simply supported boundary conditions are attained and evaluated for comparison. It has been figured out that the ratio of delamination has an effect on the response of laminated glass shells.

The concentration is highest at the edges and decreases as it moves towards the center. The displacement distribution matches the curvature of the shell, with the most displacement at the center and lessening as it moves outward. It is also observed that, for initially delaminated glass shells, as the percentage of delamination increases, the stress concentration towards the edges on both tops and bottoms increases. A fixed-supported laminated glass shell expresses a different behavior than a simple-supported shell. The region of high stress is much closer to the support. Stress is more uniformly distributed throughout the shell. It can also be established that the location of the delamination also affects the stress and displacement of the laminated glass shell. Despite the fact that the behavior of monolithic and layered glass beams acts as a restriction for the behavior of laminated glass shells, delaminated glass units violate these constraints.

6.1. Suggestions for Future Studies

Further investigation can be done for different laminated glass shells with initial delamination under uniformly distributed loads at different temperatures and load rates. Finite element analysis program and software can be used, and experiments can be conducted for the verification of the model. In the current study, delamination is considered at the center of different ratios. Further investigations can be conducted to analyze the effect of delamination location by considering the same amount of delamination in different regions of the unit.

REFERENCE

- Abarca, A. M. (2003). Tests performed on laminated glass bonded with a new interlayer composition (*Doctoral dissertation, Texas Tech University*)
- Aşık, M. Z. (2003). Laminated glass plates: revealing of nonlinear behavior. *Computers and Structures*, 81(28-29), 2659-2671.
- Aşık, M. Z., and Dural, E. (2014). Effect of delamination on the strength of laminated glass plate structures. In *Applied Mechanics and Materials* (Vol. 487, pp. 181-184). Trans Tech Publications Ltd.
- Aşık, M. Z., and Tezcan, S. (2005). A mathematical model for the behavior of laminated glass beams. *Computers and Structures*, 83(21-22), 1742-1753.
- Aşık, M. Z., Dural, E., Yetmez, M., and Uzhan, T. (2014). A mathematical model for the behavior of laminated uniformly curved glass beams. *Composites Part B: Engineering*, 58, 593-604.
- Azimi, M., Mirjavadi, S. S., Asli, S. A., and Hamouda, A. M. S. (2017). Fracture analysis of a special cracked lap shear (CLS) specimen with utilization of virtual crack closure technique (VCCT) by finite element methods. *Journal of Failure Analysis and Prevention*, 17(2), 304-314.
- Behr, R. A., Minor, J. E., and Norville, H. S. (1993). Structural behavior of architectural laminated glass. *Journal of Structural Engineering*, 119(1), 202-222.
- Butchart, C., and Overend, M. (2012). Delamination in fractured laminated glass. In *International Conference at glasstec, Düsseldorf, Germany*.
- Chermoshentseva, A. S., Pokrovskiy, A. M., and Bokhoeva, L. A. (2016). The behavior of delaminations in composite materials-experimental results. In *IOP Conference*

Series: *Materials Science and Engineering* (Vol. 116, No. 1, p. 012005). IOP Publishing.

- Chen, X., Rosendahl, P. L., Chen, S., and Schneider, J. (2021). On the delamination of polyvinyl butyral laminated glass: Identification of fracture properties from numerical modelling. *Construction and Building Materials*, 306, 124827.
- Cerracchio, P., Gherlone, M., Di Sciuva, M., and Tessler, A. (2015). A novel approach for displacement and stress monitoring of sandwich structures based on the inverse Finite Element Method. *Composite Structures*, 127, 69-76.
- Davies, P., and Cadwallader, R. (1984). Delamination Issues with Laminated Glass—Causes and Prevention. In *Glass Processing Days* (pp. 427-430).
- Del Linz, P., Hooper, P. A., Arora, H., Wang, Y., Smith, D., Blackman, B. R., and Dear, J. P. (2017). Delamination properties of laminated glass windows subject to blast loading. *International journal of impact engineering*, 105, 39-53.
- De Luca, L., and Ponsiglione, M. (2021). Variational models in elasticity [J]. *Mathematics in Engineering*, 3(2), 1-4.
- Ding, W. (2000). Delamination analysis of composite laminates. University of Toronto.
- Drake, D. A., and Sullivan, R. W. (2021). Interfacial fracture energy of a single cantilevered beam specimen using the J-integral method. *International Journal of Fracture*, 229(2), 185-194.
- Dural, E., and Oyar, F. (2023, January). Effect of delamination size, location and boundary conditions on the behavior of a laminated glass plate. In *Structures* (Vol. 47, pp. 121-133).
- Zang, M. Y., and Chen, S. H. (2011). Laminated glass. *Wiley Encyclopedia of Composites*, 1-7.
- Elmarakbi, A. M., Hu, N., and Fukunaga, H. (2009). Finite element simulation of delamination growth in composite materials using *LS-DYNA*. *Composites Science*

and Technology, 69(14), 2383-2391.

Foraboschi, P. (2007). Behavior and failure strength of laminated glass beams. *Journal of Engineering Mechanics*, 133(12), 1290-1301.

Faraday, M. (1833, December). The Bakerian lecture. On the manufacture of glass for optical purposes. In Abstracts of the Papers Printed in the Philosophical Transactions of the Royal Society of London (No. 2, pp. 383-385). London: *The Royal Society*.

Flocker, F. W., and Dharani, L. R. (1997). Stresses in laminated glass subject to low velocity impact. *Engineering Structures*, 19(10), 851-856.

Gadelrab, R. M. (1996). The effect of delamination on the natural frequencies of a laminated composite beam. *Journal of Sound and Vibration*, 197(3), 283-292. (A)
CPNI, E. (2013). Influence of delamination of laminated glass on its blast performance'.

Gadelrab, R. M. (1996). The effect of delamination on the natural frequencies of a laminated composite beam. *Journal of Sound and Vibration*, 197(3), 283-292.

Galuppi, L., and Royer-Carfagni, G. F. (2012). Effective thickness of laminated glass beams: new expression via a variational approach. *Engineering Structures*, 38, 53-67.

Garg, N., Prusty, B. G., Ooi, E. T., Song, C., Pearce, G., and Phillips, A. W. (2020). Application of scaled boundary finite element method for delamination analysis of composite laminates using cohesive zone modelling. *Composite Structures*, 253, 112773.

Hellen, T. K. (1975). On the method of virtual crack extensions. *International Journal for numerical methods in engineering*, 9(1), 187-207.

Jaśkowiec, J. (2015). Numerical modeling mechanical delamination in laminated glass by

XFEM. *Procedia Engineering*, 108, 293-300.

Kangkang, W. A. N. G., Libin, Z. H. A. O., Haiming, H. O. N. G., Zhang, J., And Yu, G. O. N.

G. (2021). Parameter studies and evaluation principles of delamination damage in laminated composites. *Chinese Journal of Aeronautics*, 34(7), 62-72.

Kefal, A., and Tessler, A. (2021). Delamination damage identification in composite shell structures based on Inverse Finite Element Method and Refined Zigzag Theory. In *Developments in the Analysis and Design of Marine Structures* (pp. 354-363). CRC Press.

Krawczyk, P., Frey, F., and Zieliński, A. P. (2007). Large deflections of laminated beams with interlayer slips: Part 1: model development. *Engineering Computations*, 24(1), 17-32.

La Saponara, V., Muliana, H., Haj-Ali, R., and Kardomateas, G. A. (2002). Experimental and numerical analysis of delamination growth in double cantilever laminated beams. *Engineering Fracture Mechanics*, 69(6), 687-699.

Langhaar, H., and Boresi, A. (1962). Buckling and Post-Buckling of Elastic Shells. In *Collected Papers on Stability of Shell Structures-1962. National Aeronautics and Space Administration*.

MacLeod, C. (1987). Accident or design? George Ravenscroft's patent and the invention of lead-crystal glass. *Technology and Culture*, 28(4), 776-803.

Mauro, J. C., and Zanotto, E. D. (2014). Two centuries of glass research: historical trends, current status, and grand challenges for the future. *International Journal of Applied Glass Science*, 5(3), 313-327.

Mauro, J. C., and Zanotto, E. D. (2014). Two centuries of glass research: historical trends, current status, and grand challenges for the future. *International Journal of Applied Glass Science*, 5(3), 313-327.

- Miller, R. A. (1935). Symposium Materials of Construction in the Building Industry. Glass in Building. *Industrial and Engineering Chemistry*, 27(11), 1291-1293.
- Morey, G. W. (1934). The constitution of glass. *Journal of the American Ceramic Society*, 17(1-12), 315-328.
- Morgan, W. L. (1931). Manufacture and characteristics of laminated glass. *Industrial and Engineering Chemistry*, 23(5), 505-508.
- Naumenko, K., and Eremeyev, V. A. (2014). A layer-wise theory for laminated glass and photovoltaic panels. *Composite Structures*, 112, 283-291.
- Preston, F. (1922). Structure of abraded surface glasses. *Trans. Opt. Soc*, 23(3), 141-162.
- Sabin, G. E. (1993). Delamination in composite materials as observed using an optical fiber strain gage. NAVAL ACADEMY ANNAPOLIS MD.
- Savić, J., Đurić-Mijović, D., and Bogdanović, V. (2013). Architectural glass: Types, performance and legislation. *Facta universitatis-series: Architecture and Civil Engineering*, 11(1), 35-45.
- Schneider, G. E., and Zedan, M. (1981). A modified strongly implicit procedure for the numerical solution of field problems. *Numerical Heat Transfer*, 4(1), 1-19.
- Teotia, M., and Soni, R. K. (2018). Applications of finite element modelling in failure analysis of laminated glass composites: A review. *Engineering Failure Analysis*, 94, 412-437.
- Torabi, Keivan and Shariati-Nia, Mostafa and Heidari-Rarani, Mohammad. (2015). Effect of delamination size and location on natural frequencies of laminated composite beams.
- Vallabhan, C. G., Das, Y. C., Magdi, M., Asik, M., & Bailey, J. R. (1993). Analysis of laminated glass units. *Journal of Structural Engineering*, 119(5), 1572-1585.
- Vedrtnam, A. (2018). Experimental and simulation studies on delamination strength of laminated glass composites having polyvinyl butyral and ethyl vinyl acetate inter-

- layers of different critical thicknesses. *Defence Technology*, 14(4), 313-317.
- Wang, X. E., Yang, J., Wang, F., Liu, Q. F., and Xu, H. (2018). Simulating the impact damage of laminated glass considering mixed mode delamination using FEM/DEM. *Composite Structures*, 202, 1239-1252. (5)
- Watkins, G. B., and Harkins, W. (1933). Laminated Safety Glass. *Industrial and Engineering Chemistry*, 25(11), 1187-1192.
- Wimmer, G., Schuecker, C., and Pettermann, H. E. (2009). Numerical simulation of delamination in laminated composite components—A combination of a strength criterion and fracture mechanics. *Composites Part B: Engineering*, 40(2), 158-165.
- Wimmer, G., Schuecker, C., and Pettermann, H. E. (2009). Numerical simulation of delamination in laminated composite components—A combination of a strength criterion and fracture mechanics. *Composites Part B: Engineering*, 40(2), 158-165.
- Zhang, X., Hao, H., and Wang, Z. (2014). Experimental investigation of monolithic tempered glass fragment characteristics subjected to blast loads. *Engineering Structures*, 75, 259-275.
- Zou, Z., Reid, S. R., and Li, S. (2003). A continuum damage model for delamination in laminated composites. *Journal of the Mechanics and Physics of Solids*, 51(2), 333-356.

T.C.

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SCIENTIFIC ETHICAL STATEMENT

I hereby declare that I composed all the information in my master's thesis entitled
EFFECT OF DELAMINATION ON THE STRENGTH OF LAMINATED GLASS
SHELL STRUCTURES within the framework of ethical behavior and academic rules,
and that due references were provided and for all kinds of statements and information
that do not belong to me in this study in accordance with the guide for writing the thesis.
I declare that I accept all kinds of legal consequences when the opposite of what I have
stated is revealed.

Muaz IBRAHİM

07/ 08 /2023

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