

T.R.
GEBZE TECHNICAL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

**LIGHTNING ANALYSIS FOR DISPERSIVE GROUND USING
FINITE DIFFERENCE TIME DOMAIN METHOD**

OSMAN KURNAZ
A THESIS SUBMITTED FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY
ELECTRONICS ENGINEERING DEPARTMENT

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T.C.
GEBZE TEKNİK ÜNİVERSİTESİ
FEN BİLİMLERİ ENSTİTÜSÜ

DİSPERSİF TOPRAK İÇİN
ZAMAN UZAYI SONLU FARKLAR
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SUMMARY

Modeling of lightning problems using the Finite Difference Time Domain (FDTD) method for dispersive ground has been popular in recent years. However, it is seen that the research on the topic is not in sufficient level and most of the lightning problems remain unsolved with the FDTD. In this work, the lightning electromagnetic pulses are computed for the dispersive ground using the FDTD in two-dimensional cylindrical coordinates. Although FDTD solutions to general dispersive media have been proposed decades ago, they have been partially applied to the lightning problem or have been applied with some approximations. The novelty of this work is that a Recursive Convolution (RC) FDTD approach is proposed to represent the Smith-Longmire soil model as the multi-pole Debye medium without using any approximation. Not only the Smith-Longmire but also the soil models of Scott, Messier, Visacro-Portela, Portela, Visacro-Alipio, Alipio-Visacro and CIGRE that have fractional frequency-dependence are investigated by proposing the representation of the ground with Cole-Cole medium, which is realized for the first time in lightning problems. The maximum of computational times of the proposed approaches do not exceed 1.8 times that of the classical FDTD solution. According to the obtained results, using dispersive soil model makes a difference of more than 100% in electromagnetic field values for some models compared to using lossy ground with constant parameters. Additionally, the difference decreases as the water content of the soil increases. Thus, using dispersive soil model in the simulation is more significant for the soils of dry type.

Key Words: Lightning, Finite Difference Time Domain (FDTD), Recursive Convolution (RC), Dispersive Soil Models, Debye Model, Cole-Cole Model.

ÖZET

Zaman Uzayı Sonlu Farklar (ZUSF) yöntemi ile dispersif toprak için yıldırım problemini çözmek son yıllarda yoğun rağbet görmektedir. Ancak bu konu hakkındaki çalışmalar yeterli düzeyde olmayıp, birçok yıldırım problemi ZUSF ile halen çözülmemiştir. Bu çalışmada, yıldırım kaynaklı elektromanyetik alanlar dispersif toprak için iki boyutlu silindirik koordinatlarda ZUSF kullanılarak hesaplanmaktadır. Genel dispersif ortamlar için ZUSF çözümlerinin on yıllar önce önerilmesine karşın bu çözümler henüz yıldırım problemine kısmen veya bazı yaklaşımlar kullanılarak uygulanmıştır. Bu tezin yenilikçi tarafı herhangi bir yaklaşım kullanmadan Smith-Longmire toprak modelinin çok kutuplu Debye ortamı şeklinde modellenmesi için Yinelemeli Konvolüsyon (YK) ZUSF tekniğinin önerilmesidir. Buna ek olarak, yıldırım problemlerinde ilk kez toprağın Cole-Cole ortamı olarak modellenmesi gerçekleştirilmiş ve bu sayede kesirli frekans bağımlılığına sahip olan Scott, Messier, Visacro-Portela, Portela, Visacro-Alipio, Alipio-Visacro ve CIGRE toprak modelleri de araştırılmıştır. Önerilen yaklaşımların maksimum benzetim süresinin klasik ZUSF yöntemi ile elde edilen sürenin 1.8 katını aşmadığı görülmüştür. Elde edilen sonuçlara göre, dispersif toprak modeli kullanmak bazı modellerde sabit parametrelili kayıplı toprak modeli kullanmaya göre elektromanyetik alan değerlerinde % 100'ü geçen farklar oluşturmaktadır. Ek olarak, topraktaki su miktarı arttıkça fark azalmaktadır. Bu sebeple, kuru tipteki topraklar için benzetimde dispersif toprak modeli kullanmak daha önemli hale gelmektedir.

Anahtar Kelimeler: Yıldırım, Zaman Uzayı Sonlu Farklar (ZUSF), Yinelemeli Konvolüsyon (YK), Dispersif Toprak Modelleri, Debye Modeli, Cole-Cole Modeli.

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LIST OF ABBREVIATIONS AND ACRONYMS

<u>Abbreviations</u>	<u>Explanations</u>
<u>and Acronyms</u>	
$\nabla \times$	: Curl Operator
$\vec{E}$	: Electric Field Vector
$\vec{H}$	: Magnetic Field Vector
$\vec{J}$	: Current Density Vector
$\vec{D}$	: Electric Displacement Vector
ϵ	: Electric Permittivity
ϵ_0	: Electric Permittivity in Free Space
ϵ_r	: Relative Electric Permittivity
ϵ_s	: Static Dielectric Constant
ϵ_∞	: Infinite Frequency Dielectric Constant
μ	: Magnetic Permeability
σ	: Conductivity
σ_0	: Low Frequency Conductivity
ρ	: Resistivity
A	: Ampere
A/m	: Ampere per meter
ABC	: Absorbing Boundary Condition
ADE	: Auxiliary Differential Equation
dB	: Decibel
DC	: Direct Current

FDTD	: Finite Difference Time Domain
F/m	: Farad per meter
Hz	: Hertz
IFT	: Inverse Fourier Transform
kA	: Kiloampere
kHz	: Kilohertz
k Ω .m	: Kiloohm-meter
LEMP	: Lightning Electromagnetic Pulse
LFC	: Low Frequency Conductivity
max	: Maximum
MHz	: Megahertz
μ s	: Microsecond
ms	: Millisecond
mS/m	: Millisiemens per meter
Ω .m	: Ohm-meter
RC	: Recursive Convolution
s	: Second
S/m	: Siemens per meter

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1. INTRODUCTION

1.1. Importance of the Subject

Lightning has been a curiosity attracting phenomenon since the beginning of the mankind. The first experimental works on the lightning are claimed to have been realized in the 18th century with a metallic kite by Benjamin Franklin [Jernegan, 1928] who is a famous politician and scientist, and also the inventor of the lightning rod. On the other hand, theoretical infrastructure of the lightning problem was constituted in the 20th century by Sommerfeld who developed the theory about the evaluation of electromagnetic field radiated by a dipole antenna over finitely conducting ground [Sommerfeld, 1949]. Since there is no analytical solution to the so-called Sommerfeld integral, it can be solved numerically in frequency domain, but the integral is improper integral with a highly oscillatory and slow convergent integrand [Liu et al., 2017]. For this reason, an approximation to the Sommerfeld integrals in frequency domain, named as the Cooray-Rubinstein (CR) approximation [Cooray, 1992], [Rubinstein, 1996], has been proposed for the low-cost computation and has become very popular. Later, the counterpart of CR formula in time domain has been proposed [Caligaris et al., 2008]. The time-domain CR formula has still been improved with new approaches by the researchers as in [Liu et al., 2019]. Although the CR formula presents a simple solution for finitely conducting ground in the frequency domain, its time-domain version is quite complex and brings some limitations [Andreotti et al., 2018]. Moreover, this complexity is even for the lossy ground with constant parameters and a CR solution for frequency-dependent (dispersive) ground is not currently available.

The Finite Difference Time Domain (FDTD) method that provides a full-wave solution to Maxwell equations was first applied to the field of lightning electromagnetic pulse (LEMP) computation at the beginning of this millennium by modeling the ground using a perfect electric conductor (PEC) [Sartori and Cardoso, 2000]. Then, the ground was modeled as lossy dielectric with constant parameters using the FDTD for the protection of grounding system from lightning effects [Tanabe, 2001]. Later, more researchers were interested in the application of the FDTD to the lightning problem with an accelerating increase in recent years because it is a direct

time-domain method, it is very efficient in the analysis of transient signals and can easily deal with nonlinear components.

The FDTD researchers solved the lightning problem using a finitely conducting ground for long years and this is still a popular research topic. Nevertheless, many frequency-dependent soil models are available in the literature that are already used in the lightning analysis with frequency domain methods. For example, Method of Moments (MoM) [Nazari et al., 2021], Finite Element Method (FEM) [Salarieh et al., 2020], Transmission Line Matrix (TLM) method [Diniz et al., 2021], Hybrid Electromagnetic (HEM) method [Vasconcellos et al., 2021] and analytical methods [Li et al., 2020] can be addressed as the methods recently used in the lightning problem in the frequency domain. Furthermore, huge errors exceeding 100% are reported to introduce if lossy ground with constant parameters are used instead of the dispersive ground [Visacro and Alipio, 2012].

The problem with the frequency domain methods is that, whether it is a numerical method or an analytical one, and whether it is applied to the ground with constant parameters or dispersive ground, general approach is computation of the electromagnetic fields in frequency domain first, and then, transforming the frequency domain results to the time domain by inverse transform like Laplace or Fourier. However, such an indirect method may introduce spurious information due to bandwidth or time step restrictions [Barbosa et al., 2013]. In addition, these techniques require computations at several frequencies to obtain a time domain waveform resulting in a need of more computational resources [Delfino et al., 2009], [Barbosa and Paulino, 2019]. Furthermore, choosing a greater number of frequencies increases the computational needs whereas choosing a smaller number of frequencies results in low accuracy. Also, the effect of the chosen frequency interval on the solution accuracy is ambiguous. Researchers seek direct time-domain methods for a while not only for these reasons, but also for the fact that lightning has effects mostly on transient responses of the systems [Alipio and Visacro, 2017]. On the other hand, considering the research including the dispersive ground analysis, most of these studies are based on finding equivalent circuit parameters especially for grounding analyses or using some approximate techniques like vector fitting (VF) which is based on the rational functions [Salvador et al., 2020].

There exist studies in the literature dealing with the dispersive ground using the FDTD especially in recent years. However, available research did not reach a sufficient

level yet. In the next section, a summary of the existing studies on the topic can be found.

1.2. Literature Review

In this section, the techniques used to obtain the electromagnetic fields in general dispersive media by means of the FDTD method are summarized first. Then, the studies on solving the lightning problem for the dispersive ground using the FDTD are reviewed.

1.2.1. FDTD Method for General Dispersive Media

The FDTD method has emerged based on the utilization of constant constitutive parameters, namely, electric permittivity (ϵ), magnetic permeability (μ), and conductivity (σ). Nonetheless, not all the media have constant parameters, rather there are several dispersive media types whose parameters are varying with the frequency. Debye medium, Plasma medium, Lorentz medium, and Cole-Cole medium can be listed as examples of such types. One can find more media types in literature; however, they are generally derived from the mentioned ones. For instance, Cole-Davidson, Havriliak-Negami, and Raicu dispersion relations are all different versions of Cole-Cole equations and include fractional powers of frequency.

Due to the classical FDTD method's being not sufficient to solve Maxwell equations in dispersive media, additional techniques were proposed to get the FDTD solutions in such kind of media. In literature, mainly two techniques draw attention: Recursive Convolution (RC) and Auxiliary Differential Equation (ADE) [Aksoy, 2023].

RC technique is based on the computation of discretized convolution integral belonging to dielectric susceptibility function of the medium in time domain. The convolution integral is the sum of all values at previous time steps, which results in a huge storage requirement in the code. For this reason, the FDTD update equation in the RC technique includes a term that represents the convolution integral and must be defined as an iterative structure. This requires the condition that the susceptibility function of the medium must be in the form of exponential function in time domain [Aksoy, 2023]. Since the Debye medium obeys this rule, the FDTD solutions to

Maxwell equations in Debye medium can be obtained with the RC technique. The FDTD utilization in Debye medium was first proposed in 1990 [Luebbers et al., 1990]. The researchers computed the reflection coefficient in the air-water interface over a wide frequency range using the FDTD in one-dimension (1D).

In the ADE technique, the FDTD update equations are constituted by using the polarization currents starting from the Ampere's law that relates the current density with the polarization. Contrary to the RC technique which expresses the relationship between the displacement and the electric field with a convolution integral, the ADE technique is based on the approach that expresses this relationship with a differential equation [Inan and Marshall, 2011, ch. 10]. This concept is realized by taking the inverse Fourier Transform (IFT) of the relationship between displacement and the electric field instead of taking the IFT of the susceptibility function of the medium. The first solution of the FDTD problem with the ADE was published by Joseph et al. who computed reflection coefficients over a wide frequency range for a Gaussian pulse incident upon a half-space Lorentz medium [Joseph et al., 1991].

The FDTD solution of Maxwell equations in a Debye medium can be achieved by using either RC or ADE technique. On the other hand, solution for the Cole-Cole medium is not that simple because the frequency term in the Cole-Cole equation has a fractional power.

After taking the IFT of the equation with the fractional power, fractional differential equations arise in the time domain which makes difficult to reach a straightforward solution. Some approaches have been proposed in the literature for the solution to this problem such as convergence with the sum of exponential functions, convergence with rational functions, and Z transformation. All these approaches require curve fitting with complex values or solving the polynomial roots [Aksoy, 2023].

In this thesis, both Debye and Cole-Cole models are used to represent the frequency-dependent ground. For the Debye model, the research by [Luebbers et al., 1990] who used the RC technique is made basis for our work. For the Cole-Cole model, we started with the research by [Torres et al., 1996] who used the sum of exponential functions in addition to the ADE technique (which may be called as fractional-FDTD) and sought out the applicability of their study to the lightning problem.

1.2.2. FDTD Method for Lightning over Dispersive Ground

Zhang et al. analyzed lightning induced voltages using subsequent return stroke striking to tall objects. As the dispersive soil models, Scott, Smith-Longmire, Visacro-Alipio and Portela models are used. They found electromagnetic fields over a PEC ground by using the 2D FDTD first, and then computed its Fourier Transform (FT). Later, they used the Wait's approximation for vertical field and CR formulation for horizontal field in frequency domain, and finally they applied IFT on the field components along with the Agrawal coupling model to calculate induced voltages. Even though the authors used the FDTD method in their research including the dispersive ground, they used the FDTD just for PEC ground and continued with frequency domain approaches [Zhang et al, 2015].

Kuklin presented modifications of the model of thin wire buried in Debye media and carried out computations of ground potential rise for the validation. They used the 3D FDTD and the ADE formulation. Visacro-Alipio soil model was used as the dispersive ground model and defined as the multi-pole Debye model. The soil model was fitted to the Debye model by using a hybrid approach of particle swarm/least squares optimization. In their research, good approximation is possible if resistivity is close to certain value [Kuklin, 2016].

Alipio and Visacro conducted research on a hemispherical electrode buried in dispersive soil with Alipio-Visacro soil model and computed the induced voltages and currents. They proposed VF technique which is based on the approach of sum of rational functions to represent the convolution of the susceptibility term with the electric field in time domain, and also modeled the electric permittivity of the ground with sum of multi-pole Debye functions by fitting the parameters. Analytical solutions are compared with the numerical ones and the experimental data. Additionally, they computed the results in frequency domain and used the IFT. Although the authors did not use the FDTD method, they used RC technique in time domain and they claimed that their proposed method would be useful in time domain solutions of the lightning problem especially for the ones with the FDTD method [Alipio and Visacro, 2017].

Oliveira et al. computed lightning-induced voltages and currents for the grounding electrode under dispersive ground by adopting Alipio-Visacro soil model and approximated the soil parameters with Pade coefficients. They used the FDTD method along with the ADE approach in 3D cartesian coordinates. They compared the

lightning induced voltages with the results for lossy ground with constant parameters and also with their own measurements. Their own FDTD-based codes modified to include dispersive case is used in the computation and CPML is adopted as ABC. In their algorithm, the use of double-precision is not enough, and use of high-precision variables is necessary to avoid numerical divergence. Moreover, complex-numbered electric fields due to the complex Pade coefficients must be stored and updated. However, all these factors increase the needed computer RAM and have negative effect on accuracy [Oliveira et al., 2017].

Sun et al. computed underground LEMP for dispersive flat ground using the FDTD method in 2D cylindrical coordinates. They adopted a semi-analytical RC (SARC) algorithm to take soil dispersion into account and used Smith-Longmire soil model. They used VF scheme to approximate the frequency-dependent soil parameters and compared their horizontal electric field results with the CR formula and also with the constant-parameter results. In addition, different water contents of the soil are included, and the performances of first order Mur ABC and CPML ABC are also compared using a Gaussian point source [Sun et al., 2018].

Pal and Kumar investigated the effect of soil dispersion and soil ionization on the current evolution of the return stroke and on the underground fields in close range by adopting the FDTD method in 2D coordinates. The model of Smith-Longmire is adopted as the dispersive soil model and wide range of soil conductivities are considered. Although the authors did not share their FDTD equations in explicit way, they pronounced that the soil dispersion is taken into account by the VF method [Pal and Kumar, 2021].

Rizk et al. computed lightning-induced voltages on multi-conductor overhead lines due to nearby first and subsequent return strokes using the method of FDTD in 2D coordinates. In their research, different parameters affecting induced voltage levels such as line height, distance between line and the channel, stroke channel velocity, and soil conductivity are investigated. Frequency-dependence of the ground is included by adopting Alipio-Visacro soil model. The electrical parameters are modeled as the sum of Debye functions by utilization of VF and RC techniques [Rizk et al., 2022].

Carvalho and Silveira adopted Alipio-Visacro soil model and used an FDTD-based computation model to investigate the performance of grounding electrodes. They followed and extended the approach used in [Oliveira et al., 2017] to represent the ground with the Debye medium. They adopted the ADE formulation along with

the approximation with Pade coefficients. The thin wire approximation for Debye medium proposed before in [Kuklin, 2016] was also used in the research. Also, the effects of different Absorbing Boundary Conditions such as second order Liao, third order Liao, and the CPML on the results and computation time were investigated in the study. The results were compared with the HEM model and an agreement between the results were found provided that the frequency-dependence of the ground is included in the simulation. They found that the CPML and the third order Liao gives the most accurate results, and also that adoption of a 6-pole model is sufficient to represent the dispersion of the ground without extending the computation time unnecessarily [Carvalho and Silveira, 2022].

Kuklin followed a calculation procedure for the grounding potential rise both in homogeneous and inhomogeneous soils and found that using apparent properties of soil results in important errors for electrical grounding. In their study, which is the extension of their research in [Kuklin, 2016], Alipio-Visacro soil model is modeled as multi-pole Debye medium by hybrid particle swarm/least squares optimization and the resistivity and permittivity parameters found by the ADE formulation using the 3D FDTD are compared with the initial values of the parameters. The soil models of Portela and Messier are additionally used in the study [Kuklin, 2022].

1.3. Motivation

Global climate change is a problem known for long years. This results in sudden weather changes and increased lightning storm activity [Price and Rind, 1994]. More countries and more people will probably be affected by this problem in the coming years. Thus, it seems that lightning will be on the front burner for the world in the future more than it was in the past.

Researchers have followed a couple of methods and procedures for the last few decades for the development of accurate lightning protection and detection systems modeling. However, it is understood that these works are not complete yet and there is indeed a demand for developing new time domain methods.

There are techniques well-known by the FDTD society that propose solutions to the Maxwell equations in dispersive media. However, these techniques seem not to be sufficiently investigated by the lightning society even though there are few research most of which use approximation techniques like VF. Therefore, the aim of this thesis

is to apply these techniques, namely the RC-FDTD and fractional-FDTD, to the lightning problem to obtain novel FDTD applications for the dispersive ground models existing in the literature.

1.4. Contributions

First of all, the RC-FDTD method is applied to dispersive Smith-Longmire soil model without using any approximation and a multi-pole Debye equation whose parameters are the same with those of Smith-Longmire is proposed, which is a procedure realized for the first time. The RC technique in a modified form was applied to Smith-Longmire model by other researchers before, but they used the VF approximation.

Secondly, for the other dispersive soil models (seven models with fractional powers), fractional-FDTD is applied to LEMP problem by modeling the ground as Cole-Cole medium for the first time. In this case, a curve fitting approximation is adopted. Regarding the grounding problem, Pade approximation and multi-pole Debye approach were applied to fractional Alipio-Visacro soil model before, but no researcher has adopted Cole-Cole model yet.

Another contribution is that the FDTD solution without using Absorbing Boundary Condition is proposed by determining the geometrical limits of the solution domain with simple formulas.

Finally, a graphical user interface that includes the FDTD solution to eight dispersive soil models has been coded in Matlab for easy computations and future applications.

1.5. Thesis Content

In section 1, a general information about the thesis topic and review of the literature are given. General information regarding the formation of lightning, modeling of lightning current, and frequency-dependent soil models are presented in section 2. In section 3, the FDTD for the case of lossy ground with constant parameters is reviewed first, and then the formulations for the frequency-dependent soil models are derived. In section 4, results are presented. Finally, concluding ideas are given in section 5.

2. LIGHTNING

Comprehensive explanations and models about the mechanisms related to the lightning such as charging process in clouds, lightning flash development, and lightning discharge stages are available in the literature. In this thesis, only basic terminology and rudiments of lightning electromagnetic especially related to the application of FDTD are mentioned.

2.1. Basic Knowledge about Lightning Electromagnetics

Lightning flash is the term used for the lightning discharge between two clouds or between the cloud and the ground. The latter is generally named as *cloud-to-ground* discharge. As shown in Figure 2.1, there are four types of cloud-to-ground lightning: downward negative lightning, upward negative lightning, downward positive lightning, and upward positive lightning.

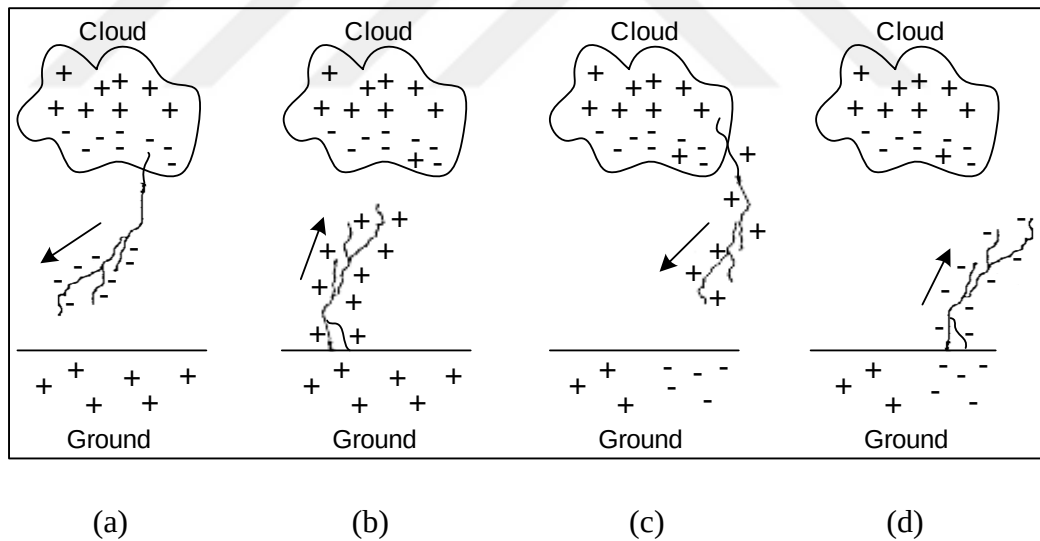


Figure 2.1: Types of lightning for cloud-to-ground discharge: (a) Downward negative lightning (b) Upward negative lightning (c) Downward positive lightning (d) Upward positive lightning. The initial leader is illustrated for each type.

All four types are called as cloud-to-ground discharges independent of the direction. Approximately 90 percent of global cloud-to-ground lightning activity includes downward negative lightning and nearly 10 percent of it includes downward positive lightning. The other two types, namely upward negative and upward positive

lightning, occur if tall objects higher than 100 m or objects on mountain tops exist. *Lightning strike* is the term used for the lightning discharge containing an object on the ground or in the atmosphere, and the term *stroke* is used for the cloud-to-ground discharges. Each stroke contains a *leader* whose direction is downward and a *return stroke* whose direction is upward. *Continuing current* may be involved in the stroke just after the return stroke. *M-components* is the name given to the perturbations in the steady continuing current in the lightning channel. *Stepped* leaders initiate *first* strokes and *dart* leaders initiate *subsequent* strokes that flows in the previously formed channel [Rakov and Uman, 2003].

The most significant stages in the formation of a typical downward negative lightning strike are illustrated in Figure 2.2. After the initiation of the stepped leader whose each step is around 50 m, it goes down towards the ground together with its several branches. After the attachment of the leader with the ground, a current named as the first return stroke flows in the conducting path formed by the leader in upward direction and neutralizes the charge transfer made by the leader [Baba and Rakov, 2016]. Then, the dart leader whose each step is around 10 m flows towards the ground and another return stroke current flows in upward direction after that. There may be a few return strokes and continuing currents in a strike; however, according to Rakov and Uman, nearly 20 percent of the lightning flashes are composed of a single stroke and about 30 to 50 percent of flashes contain continuing currents [Rakov and Uman, 2003]. All the lightning activity occurs within 200 to 300 ms.

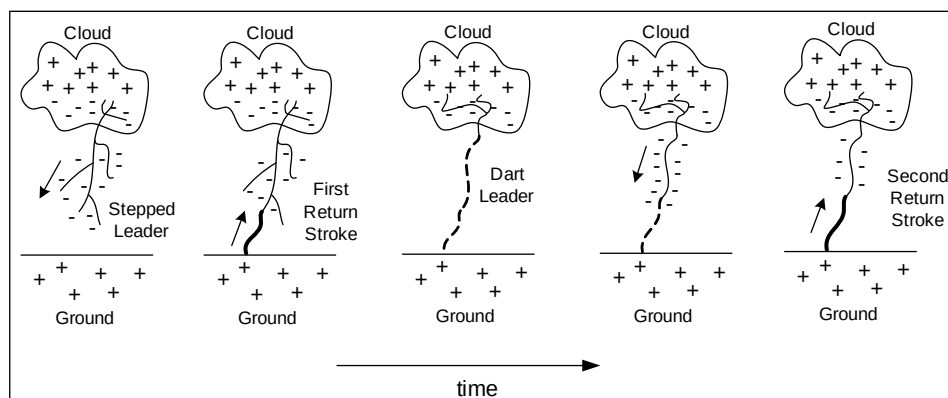


Figure 2.2: Formation of a typical lightning strike with two return strokes.

The return strokes occurring after the first one are named as *subsequent* return strokes. Figure 2.3 shows time-resolved drawing of a typical downward negative flash

and Uman, 2003]. Thus, as compatible with the literature, only the return strokes are used as the lightning sources in simulations presented in this thesis.

2.2. Return Stroke Models

2.2.1. Model Types

The existing lightning return stroke models can be distinguished into four categories according to the governing equations [Rakov and Uman, 2003]. These are *gas dynamic* models, *electromagnetic* models, *distributed circuit* models, and *engineering* models.

In gas dynamic models, a short segment of a cylindrical plasma column that is assumed to be straight and cylindrically symmetric is considered to be driven by resistive heating due to the time-varying current. These models relate the radius of lightning channel with the varying current and resistance (R) per unit length is calculated using this radius. The main disadvantage is that the spatial distribution of the current along the channel is not considered by the gas dynamic models [Rakov and Uman, 1998].

In electromagnetic models, Maxwell's equations are solved to yield the current distribution along the lightning channel using numerical techniques in time or frequency domains and representation of the channel as a vertical wire with finite radius is involved [Cooray, 2012, ch. 8]. Since the propagation speed of the wave along the conductor in air is same with the speed of the light, the medium including the return stroke is modified in a few ways to decrease the propagation speed to conform to the realistic return stroke speeds. According to Cooray, reduction of the speed with such artificial methods makes the electromagnetic models doubtful about their capability of pure derivation of the return stroke properties due to the lack of physical representation [Cooray, 2014].

Distributed circuit models consider the lightning channel as an RLC (per-unit-length resistance, inductance, and capacitance) transmission line. By assuming the electromagnetic waves guided by the transmission line have a transverse electromagnetic (TEM) field structure, telegrapher's equations that can be derived from Maxwell's equations are solved to find the voltage and current. However, the TEM assumption is not valid for a vertical conductor above ground since any current

attenuates as it propagates upward along a vertical conductor. This incorrect assumption in the vicinity of lightning channel leads to incorrect current distribution along the channel [Baba and Rakov, 2007].

Engineering models give the spatial and temporal current distribution along the lightning channel by relating the channel current at any height and any time to the current at the base of the channel. In this model type, propagating speed of the return stroke and the current attenuation with height can easily be controlled with a simple mathematical expression [Baba and Rakov, 2016]. These models do not pay much attention to the physics of the return stroke; rather, achieving an agreement between the predicted and experimentally measured electromagnetic fields is preferred [Rakov and Uman, 1998]. The engineering models are the most widely accepted type in the literature especially in “field to line coupling” problems, thanks to the balance between computational efforts and their ability to represent the electromagnetic fields sufficiently [Brignone et al., 2019]. As a result, using an engineering return stroke model is preferred in this thesis because of its both wide usage and relatively simple mathematical structure that can be easily applied to the method of FDTD.

2.2.2. Engineering Models

Several engineering models can be expressed by using a single equation:

$$I(z', t) = u\left(t - \frac{z'}{v_{rs}}\right) P(z') I\left(0, t - \frac{z'}{v_r}\right) \quad (2.1)$$

where $I(z', t)$ is the current at the height z' and at time t , u is the Heaviside function, $P(z')$ is the current attenuation factor, v_{rs} is the speed of return stroke, $I(0, t)$ is the current at the base of the channel, and v_r is the propagation speed of current wave. The engineering models widely used in the literature are the TL (transmission line) model [Uman and McLain, 1969], the MTLL (modified transmission line model with linear current decay with height) model [Rakov and Dulzon, 1987], the MTLE (modified transmission line model with exponential current decay with height) model [Nucci et al., 1988], the BG model [Bruce and Golde, 1941], and the TCS (traveling current source) model [Heidler, 1985]. The value of v_r and $P(z')$ in addition to the expression of $I(z', t)$ for each model are shown in Table 2.1. In the table, H represents the height

of the lightning channel, λ is defined as the current decay height constant, and c is the speed of the light in free space.

Table 2.1: $P(z')$, v_r , and $I(z', t)$ for the five engineering models.

Model	$P(z')$	v_r	$I(z', t)$ for $t \geq z'/v_{rs}$
TL	1	v_{rs}	$I\left(0, t - \frac{z'}{v_{rs}}\right)$
MTLL	$1 - \frac{z'}{H}$	v_{rs}	$\left(1 - \frac{z'}{H}\right) I\left(0, t - \frac{z'}{v_{rs}}\right)$
MTLE	$e^{-z'/\lambda}$	v_{rs}	$(e^{-z'/\lambda}) I\left(0, t - \frac{z'}{v_{rs}}\right)$
BG	1	∞	$I(0, t)$
TCS	1	$-c$	$I\left(0, t + \frac{z'}{c}\right)$

According to observations by optical techniques, typical values of the return stroke speed v_{rs} are found in the range from $c/3$ to $c/2$ [Rakov, 2007].

2.2.3. Base Current

In literature, there are a couple of ways to define the base current of the channel $I(0, t)$. The first proposed function to represent the lightning channel was *double exponential* function [Bruce and Golde, 1941], which is as follows:

$$I(0, t) = I_0(e^{-\alpha t} - e^{-\beta t}) \quad (2.2)$$

where α and β are constants, and I_0 is the normalization factor to adjust the peak of the current. The function value is zero for $t < 0$. This function has been used in return stroke representation for long years and it is still used by some researchers. Nonetheless, an unrealistic convex wavefront is constituted by this model [Rakov and Uman, 1998]. Also, the first derivative of the function has a discontinuity at the onset time $t = 0$.

Later, another function known as the Heidler function was proposed [Heidler, 1985], which is defined as

$$I(0, t) = \frac{I_0}{\eta} \frac{\left(\frac{t}{\tau_1}\right)^n}{1 + \left(\frac{t}{\tau_1}\right)^n} e^{-\frac{t}{\tau_2}} \quad (2.3.a)$$

$$\eta = e^{-\frac{\tau_1}{\tau_2} \left(n \frac{\tau_2}{\tau_1}\right)^{\frac{1}{n}}} \quad (2.3.b)$$

where I_0 is a factor to adjust the current peak, η is the amplitude correction factor, n is an exponent having a value between 2 and 10, τ_1 and τ_2 are the front and decay time constants, respectively.

Apart from these two functions, many models have been proposed to represent the return stroke base current in an accurate way such as the *current impulse model* [Nucci et al., 1990] that is a combination of the double exponential and Heidler functions and *pulse function model* [Chen et al., 2002] that is a modified version of the Heidler function. Several models having different modifications made on these functions have been presented in the literature so far. A new approach based on the calculation of Prony series and time-domain VF was also proposed [Delfino et al., 2012]. However, these models are not commonly used. The most widely used base current model in the literature is the Heidler function in equation (2.3) due to its fulfilling the basic needs for the return stroke representation like concave-shaped wavefront and non-zero first derivative. A sum of at least two Heidler functions with different parameters or sum of Heidler and double exponential functions are often used especially for representation of the subsequent return stroke. Therefore, the Heidler function is used in this thesis to represent the base current in the simulations.

2.2.4. Comparison of Engineering Models

In this section, comparison of the engineering models is shown to see the difference between the predicted longitudinal current distributions along the channel. The mentioned five engineering models; namely TL, MTLL, MTLE, BG, and TCS models are compared using a sample base current. The base current in equation (2.3) is defined using the parameters of the model in [Li et al., 2020] because it is a widely used base current model in the literature. These parameters are given in Table 2.2 for both first and subsequent return stroke currents. The latter is constituted by the sum of two Heidler functions that can be seen in the table. The channel height H and the

current decay height constant λ are assumed to be 7.5 km and 2 km, respectively. The return stroke speed v_{rs} is assumed to be half of the speed of the light ($c/2$).

Table 2.2: Base current parameters for first and subsequent return strokes.

Stroke Type	I_{01} (kA)	τ_{11} (s)	τ_{12} (s)	n_1	I_{02} (kA)	τ_{21} (s)	τ_{22} (s)	n_2
First Stroke	28	1.8	95	2	-	-	-	-
Subsequent Stroke	10.7	0.25	2.5	2	6.5	2	230	2

Figure 2.4 illustrates the first and subsequent return stroke currents obtained at the base of the channel. The current at the channel base is the same for all model types and the results deviate from each other as the height increases along the channel. These deviations are depicted in Figure 2.5 and Figure 2.6 for the first and the subsequent return strokes, respectively.

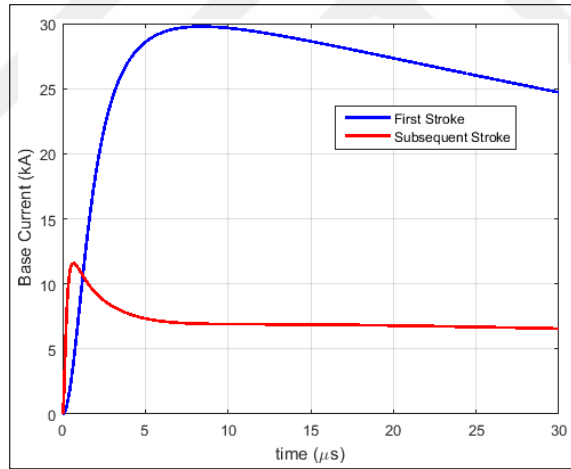


Figure 2.4: First and subsequent stroke currents at the base of the channel.

For the first return stroke, the current variations with respect to time at the heights of 500 m and 1 km are shown in Figure 2.5. It is observed that the models of BG and TCS are close to each other whereas the other models greatly differ from these two. The transmission line models, namely TL, MTLL, and MTLE differ from each other in a great manner, too. It is seen that the differences between the models increase as the height increases. The same observations can be made for the subsequent return stroke results that are shown in Figure 2.6. It is obviously seen in the results that all

the model values are non-zero for $t \geq z'/v_{rs}$ corresponding to $3.33 \mu\text{s}$ and $6.66 \mu\text{s}$ for the heights of 500 m and 1 km, respectively. A gradual increase is observed in the rising portion of the TL, MTLL, and MTLE models whereas an abrupt increase in the BG and TCS models draws attention.

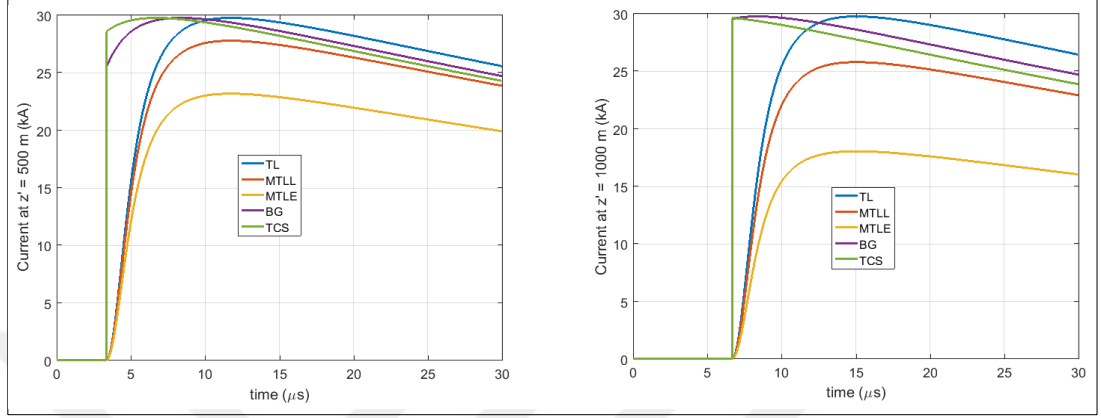


Figure 2.5: First stroke current for different engineering models at $z' = 500 \text{ m}$ (left) and $z' = 1 \text{ km}$ (right).

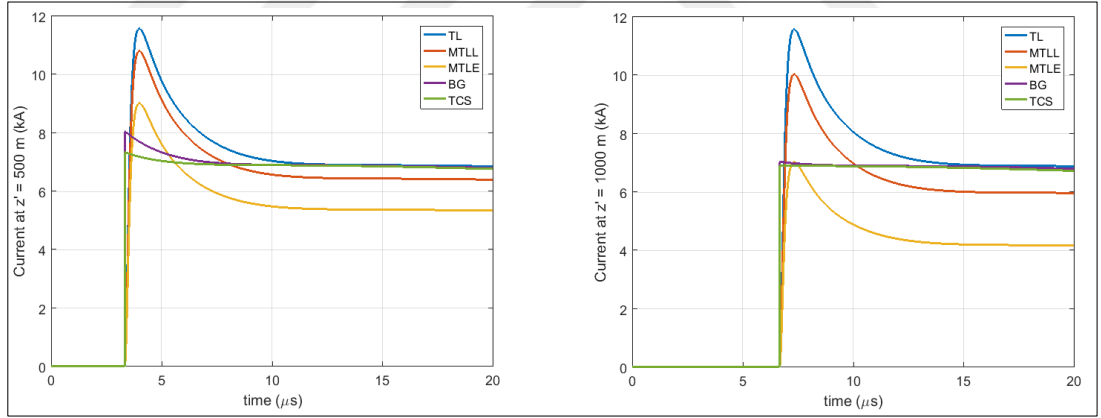


Figure 2.6: Subsequent stroke current for different engineering models at $z' = 500 \text{ m}$ (left) and $z' = 1 \text{ km}$ (right).

In fact, usage of the BG and TCS models are not very common in the literature especially in recent years. The claim by the BG model that no propagation exists does not have any physical validity [Brignone, 2019]. Thus, it can be accepted as a special case of the TL model with infinity v_{rs} [Rakov and Uman, 2003]. On the other hand, if the TCS model is adopted in an FDTD simulation, then numerical oscillations resulting in instability would arise due to the discontinuity at the current propagation [Baba and Rakov, 2016]. Also, the TL model is mathematically too simple and may not represent

the return stroke in an accurate way, however, it can be used as a practical model to predict the initial field peak from the current peak or vice versa [Rakov and Uman, 2003]. Therefore, it can be concluded that the most adopted engineering models in lightning simulations in recent years are the MTLL and MTLE models.

2.3. Frequency-Dependent Soil Models

The phenomenon of lightning is closely related to the electrical structure of the ground because it often gives damage to structures under the ground or close to the ground like the overhead transmission lines. For this reason, frequency-dependent (dispersive) soil models attract the attention of the researchers in the lightning society. Several dispersive soil models were defined in the last few decades. The most common dispersive models one may encounter in the literature can be listed in chronological order as in the following:

- Scott model [Scott, 1966],
- Smith-Longmire model [Smith and Longmire, 1975],
- Messier model [Messier, 1985],
- Visacro-Portela model [Visacro and Portela, 1985],
- Portela model [Portela, 1999],
- Visacro-Alipio model [Visacro and Alipio, 2012],
- Alipio-Visacro model [Alipio and Visacro, 2014],
- CIGRE model [CIGRE Working Group C4.33, 2019].

These eight dispersive models mentioned above are reviewed and compared with each other in the next sections. Also, solutions to lightning problem for these models using the FDTD method are investigated within the context of the thesis.

2.3.1. Scott Model

In Scott model, the relative electric permittivity and conductivity of the soil with respect to the frequency are defined according to following expressions:

$$\varepsilon_r(f) = 10^D \quad (2.4.a)$$

$$D = 5.491 + 0.946 \log_{10}(\sigma_0) - 1.097 \log_{10}(f) + 0.069 \log_{10}^2(\sigma_0) - 0.114 \log_{10}(f) \log_{10}(\sigma_0) + 0.067 \log_{10}^2(f) \quad (2.4.b)$$

$$\sigma(f) = 10^K \quad (2.4.c)$$

$$K = 0.028 + 1.098 \log_{10}(\sigma_0) - 0.068 \log_{10}(f) + 0.036 \log_{10}^2(\sigma_0) - 0.046 \log_{10}(f) \log_{10}(\sigma_0) + 0.018 \log_{10}^2(f) \quad (2.4.d)$$

where the frequency f is between 100 Hz and 1 MHz, and σ_0 is the low frequency conductivity (LFC) measured at 100 Hz in mS/m. $\sigma(f)$ is also given in mS/m. We can rearrange these expressions so that they are rewritten as follows:

$$\varepsilon_r(f) = 10^{5.491} (10^3 \sigma_0)^{0.946} [(10^3 \sigma_0)^{0.069}]^{(3 + \log_{10} \sigma_0)} \times f^{-1.097} (f^{-0.114})^{(3 + \log_{10} \sigma_0)} (f^{0.067})^{\log_{10}(f)} \quad (2.5.a)$$

$$\sigma(f) = 10^{-3} 10^{0.028} (10^3 \sigma_0)^{1.098} [(10^3 \sigma_0)^{0.036}]^{(3 + \log_{10} \sigma_0)} \times f^{-0.068} (f^{-0.046})^{(3 + \log_{10} \sigma_0)} (f^{0.018})^{\log_{10}(f)} \quad (2.5.b)$$

where both σ_0 and $\sigma(f)$ are in S/m.

2.3.2. Smith-Longmire Model

According to the Smith-Longmire soil model that is based on the Scott's data, relative permittivity (ε_r) and conductivity (σ) of the soil are expressed as follows:

$$\varepsilon_r(f) = \varepsilon_\infty + \sum_{q=1}^{13} \frac{a_q}{1 + \left(\frac{f}{F_q}\right)^2} \quad (2.6.a)$$

$$\sigma(f) = \sigma_0 + 2\pi\varepsilon_0 \sum_{q=1}^{13} a_q F_q \frac{\left(\frac{f}{F_q}\right)^2}{1 + \left(\frac{f}{F_q}\right)^2} \quad (2.6.b)$$

$$\varepsilon_{\infty} = 5 \quad (2.6.c)$$

$$\sigma_0 = 8 \times 10^{-3} \times \left(\frac{p}{10}\right)^{1.54} \quad (2.6.d)$$

$$F_q = \left(\frac{p}{10}\right)^{1.28} \times 10^{q-1} \quad (2.6.e)$$

where σ_0 is the LFC at 100 Hz in S/m and $\sigma(f)$ is given in S/m, ε_0 is the electric permittivity of free space, f is the frequency ranging from DC to 5 MHz, p is the water percentage of soil, and a_q is a coefficient presented in Table 2.3 for different q values.

Table 2.3: a_q coefficients.

q	a_q
1	3.4×10^6
2	2.74×10^5
3	2.58×10^4
4	3.38×10^3
5	5.26×10^2
6	1.33×10^2
7	2.72×10^1
8	1.25×10^1
9	4.8×10^0
10	2.17×10^0
11	9.8×10^{-1}
12	3.92×10^{-1}
13	1.73×10^{-1}

2.3.3. Messier Model

Messier model is also based on Scott's data and defines the relative electric permittivity and conductivity of the soil as

$$\varepsilon_r(f) = \frac{\varepsilon_{\infty}}{\varepsilon_0} \left(1 + \sqrt{\frac{\sigma_0}{\pi f \varepsilon_{\infty}}} \right) \quad (2.7.a)$$

$$\sigma(f) = \sigma_0 \left(1 + \sqrt{\frac{4\pi f \varepsilon_\infty}{\sigma_0}} \right) \quad (2.7.b)$$

$$\varepsilon_\infty = 8\varepsilon_0 \quad (2.7.c)$$

where σ_0 is the LFC in S/m and the frequency f is between 100 Hz and 1 MHz. $\sigma(f)$ is given in S/m and ε_∞ is in F/m.

2.3.4. Visacro-Portela Model

In Visacro-Portela model,

$$\varepsilon_r(f) = 2.34 \times 10^6 \left(\frac{1}{\sigma_0} \right)^{-0.535} f^{-0.597} \quad (2.8.a)$$

$$\sigma(f) = \sigma_0 \left(\frac{f}{100} \right)^{0.072} \quad (2.8.b)$$

where σ_0 is the LFC measured at 100 Hz in S/m. The expressions are based on the laboratory measurements in the frequency range between 100 Hz and 2 MHz.

2.3.5. Portela Model

In Portela model,

$$\varepsilon_r(f) = \frac{\Delta_i}{\varepsilon_0} \left(\frac{1}{2\pi 10^6} \right)^\xi (2\pi f)^{\xi-1} \quad (2.9.a)$$

$$\sigma(f) = \sigma_0 + \Delta_i \cot\left(\frac{\pi}{2} \xi\right) \left(\frac{f}{10^6} \right)^\xi \quad (2.9.b)$$

where σ_0 is the LFC measured at 100 Hz in mS/m, ξ is an adjustable parameter, and Δ_i is the value of $\omega\varepsilon$ at 1 MHz depending on the soil model, and frequency f is between 100 Hz and 2 MHz. In literature, ξ and Δ_i are generally accepted as 0.706 and 11.71 mS/m, respectively. $\sigma(f)$ is given in mS/m.

2.3.6. Visacro-Alipio Model

Visacro and Alipio defined a new model based on the field measurements in different areas, which is different from the previous ones based on using the soil samples. In their model, relative electric permittivity and conductivity are defined as

$$\varepsilon_r(f) = \begin{cases} 1.3 + 7.6 \times 10^3 f^{-0.4} & f \geq 10 \text{ kHz} \\ 192.2 & f < 10 \text{ kHz} \end{cases} \quad (2.10.a)$$

$$\sigma(f) = \sigma_0 \left\{ 1 + \left[1.2 \times 10^{-6} \left(\frac{1}{\sigma_0} \right)^{0.73} \right] (f - 100)^{0.65} \right\} \quad (2.10.b)$$

where σ_0 is the LFC in S/m, and frequency f is between 100 Hz and 4 MHz. $\sigma(f)$ is given in S/m.

2.3.7. Alipio-Visacro Model

Alipio and Visacro proposed one more model using the same data set of the Visacro-Alipio model. In Alipio-Visacro model,

$$\varepsilon_r(f) = \frac{\varepsilon'_\infty}{\varepsilon_0} + \frac{10^{-3} \sigma_0 h(\sigma_0) \tan \left(\frac{\pi}{2} \xi \right)}{2\pi \varepsilon_0 (1 \text{ MHz})^\xi} f^{\xi-1} \quad (2.11.a)$$

$$\sigma(f) = \sigma_0 + \sigma_0 h(\sigma_0) \left(\frac{f}{1 \text{ MHz}} \right)^\xi \quad (2.11.b)$$

$$h(\sigma_0) = 1.26(\sigma_0)^{-0.73} \quad (2.11.c)$$

$$\xi = 0.54 \quad (2.11.d)$$

$$\varepsilon'_\infty = 12\varepsilon_0 \quad (2.11.e)$$

where σ_0 is the LFC in mS/m, and frequency f is between 100 Hz and 4 MHz. $\sigma(f)$ and ε'_∞ are given in mS/m and F/m, respectively.

2.3.8. CIGRE Model

CIGRE model that is based on the same data set of Alipio and Visacro is recommended in the technical brochure of CIGRE (International Council on Large Electric Systems). According to the CIGRE model,

$$\varepsilon_r(f) = 12 + 9.5 \times 10^4 \sigma_0^{0.27} f^{-0.46} \quad (2.12.a)$$

$$\sigma(f) = \sigma_0 + 4.7 \times 10^{-6} \sigma_0^{0.27} f^{0.54} \quad (2.12.b)$$

where σ_0 is the LFC in S/m, and frequency f is between 100 Hz and 4 MHz. $\sigma(f)$ is given in S/m.

2.3.9. Comparison of the Models

As can be seen in equation (2.4.a) through (2.12.b), the dispersive soil models are highly dependent on the LFC (σ_0) of the soil in addition to the frequency f . For this reason, the value of σ_0 has a great effect on the relative electric permittivity $\varepsilon_r(f)$ and conductivity $\sigma(f)$ so the soil models should be examined for different σ_0 values. In the literature, typical σ_0 values are in the range between 0.1 mS/m and 10 mS/m [Cooray, 2012, ch. 7]. Thus, in this section, the models of Scott, Smith-Longmire, Messier, Visacro-Portela, Portela, Visacro-Alipio, Alipio-Visacro and CIGRE are compared using three different σ_0 values: 0.1 mS/m, 1 mS/m, and 10 mS/m. According to equation (2.6.d), these values correspond to the water percentages (p) of 0.58%, 2.59%, and 11.56%, respectively. Although some models are defined for frequencies up to 4 MHz, the frequency range between 100 Hz and 1 MHz has been used in the comparisons because it is the common frequency region for all models. The soil resistivity (ρ) is also included in comparisons since it is commonly used in the literature depending on the application. It is simply equal to the reciprocal of the conductivity and defined as

$$\rho(f) = \frac{1}{\sigma(f)}. \quad (2.13)$$

In Figure 2.7, conductivity (σ) in mS/m and relative electric permittivity (ϵ_r) of the models are compared for the σ_0 value of 0.1 mS/m. The comparisons for the σ_0 values of 1 mS/m and 10 mS/m are also depicted in Figure 2.8 and Figure 2.9, respectively.

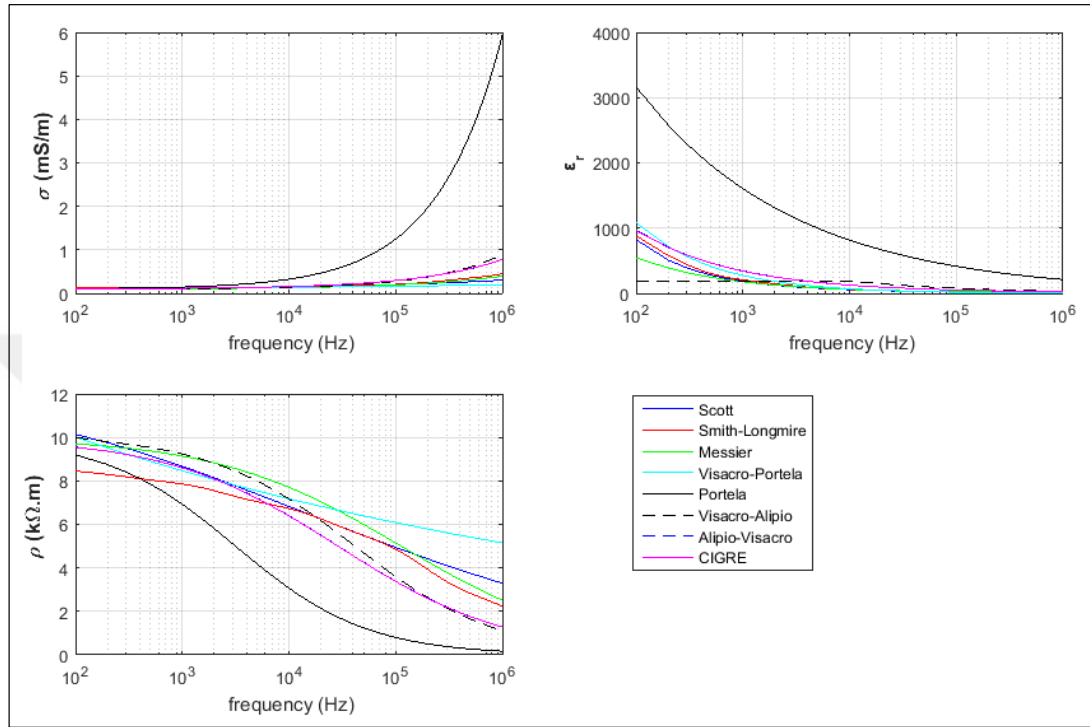


Figure 2.7: Conductivity (top left), electric permittivity (top right) and resistivity (bottom left) comparisons of the dispersive soil models for σ_0 of 0.1 mS/m.

As can be observed in Figure 2.7, the prediction made by the Portela model shows a great deviation from those of the other models for σ_0 value of 0.1 mS/m. For 1 mS/m, the prediction of Portela deviates again from those of the others especially considering the conductivity, as seen in Figure 2.8. On the other hand, the fact that Portela model is observed to be compatible with the others is observed in Figure 2.9 but the model of Visacro-Portela differs from the others in a huge manner for the case of 10 mS/m especially for the conductivity. For the permittivity, all the models seem to make similar predictions for 1 mS/m and 10 mS/m except for the Visacro-Alipio model in low frequencies, though.

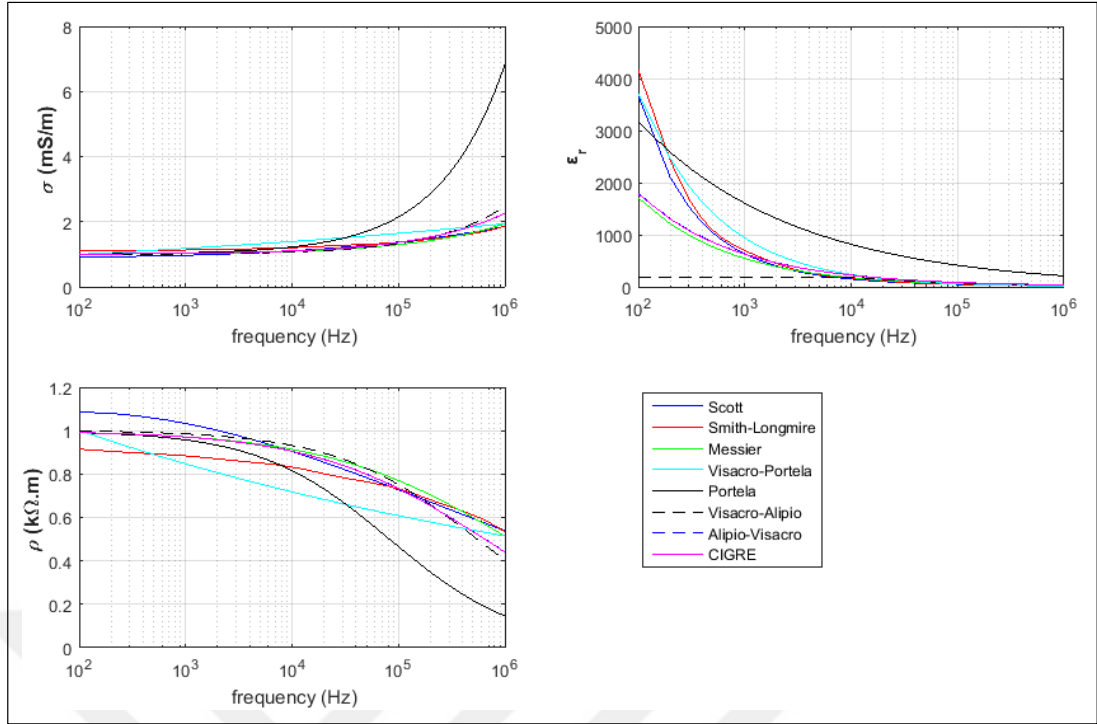


Figure 2.8: Conductivity (top left), electric permittivity (top right) and resistivity (bottom left) comparisons of the dispersive soil models for σ_0 of 1 mS/m.

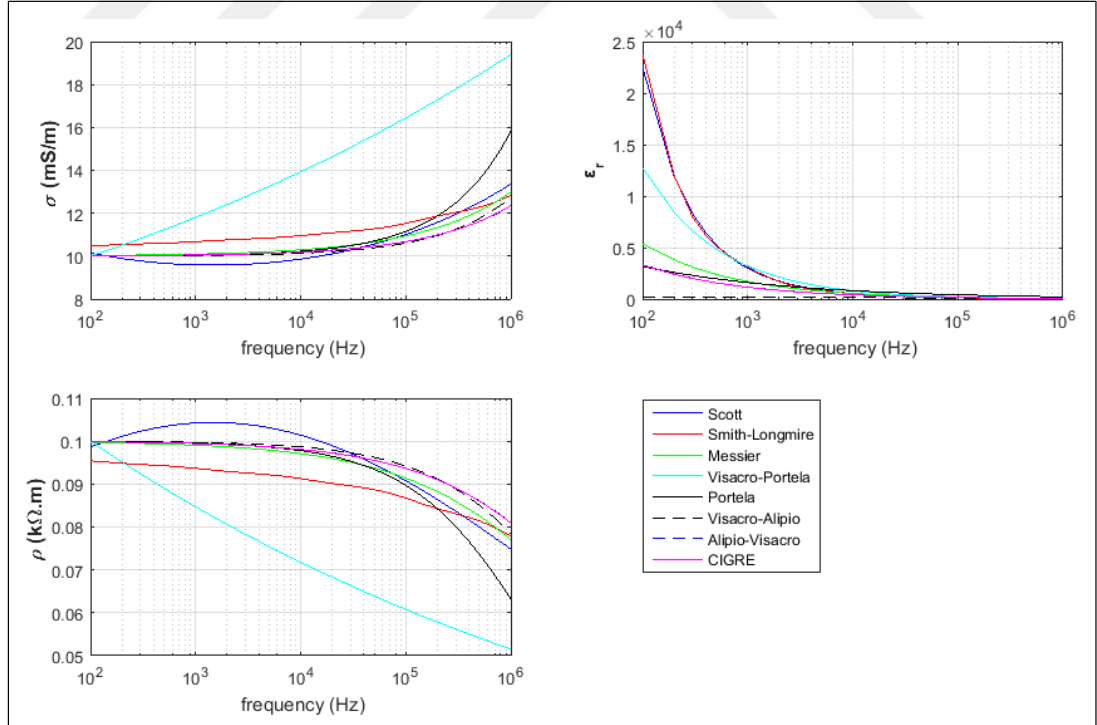


Figure 2.9: Conductivity (top left), electric permittivity (top right) and resistivity (bottom left) comparisons of the dispersive soil models for σ_0 of 10 mS/m.

3. FDTD FORMULATIONS FOR LIGHTNING

The method of FDTD provides a full-wave solution to compute the electric and magnetic fields by solving the Maxwell equations iteratively. According to Baba and Rakov, main advantages of the FDTD method are its being based on a simple procedure in electric and magnetic fields, its having a relatively easy programming, its capability of solving complex geometries and including nonlinear effects, and its providing solution for broadband applications with single run [Baba and Rakov, 2016].

There is an increasing interest in the FDTD method for solving the lightning problem in the last two decades. In the beginning of 2000s, the researchers modeled the ground as a perfect electric conductor, which is far from a realistic modeling. Later, the ground was modeled as a dielectric material with constant electrical parameters, namely electrical permittivity, and conductivity. The modeling of the ground with constant parameters is still preferred by most of the lightning society. On the other hand, in the last few years, some researchers put solutions by adopting the FDTD method using the dispersive soil models to model the ground in a more realistic way by including the variation of the electrical parameters with respect to the frequency. Although their studies present valuable results, they usually include some approximations that decreases the accuracy and increase the intermediate steps before reaching the solution. Nevertheless, there are techniques to put a solution for dispersive media using the FDTD, which are well-known by the FDTD society and not worked out sufficiently by the lightning society. In spite of the fact that there are some studies which are reviewed in section 1.2 attempting to use these techniques, they still used approximations to reach the solution.

In this thesis, these FDTD techniques are applied to the lightning problem without using any approximation by adopting the dispersive soil models reviewed in section 2.3. One can find many applications related to the lightning such as computation of electromagnetic fields, surge computation on overhead transmission lines, surges in power stations, wind turbines, and buildings, surges on grounding electrodes and underground cables, coupling in airborne vehicles and electric vehicles [Baba and Rakov, 2014]. All these applications except for the electromagnetic field computation are beyond the scope of this thesis and may be future research topics, though. The focus of this thesis is the computation of electromagnetic fields at both

close and far distances using the general lightning problem. First, the lightning problem is solved using the ground with constant electrical parameters to understand the simple case. Then, solution for the dispersive soil models is introduced in the following parts.

In the literature, the FDTD solution of the lightning problem is obtained in either three dimensional (3D) cartesian coordinate system, or two dimensional (2D) cylindrical coordinate system thanks to the axisymmetric nature of the problem. The researchers often prefer the latter because the required computer resources and the computational time are lower for the 2D system. The 2D cylindrical coordinate system is preferred in this thesis for the same purpose.

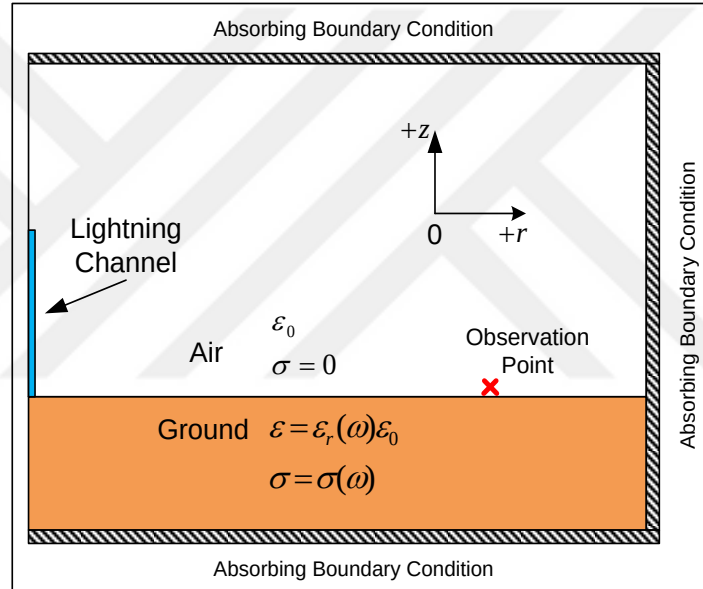


Figure 3.1: General lightning problem in 2D cylindrical coordinates.

The scenario for the general lightning strike problem in 2D is illustrated in Figure 3.1. The horizontal and vertical axes of the coordinate system are represented by r and z , respectively. The lightning channel is located above the ground at the origin $r = 0$. The top, right and bottom walls of the solution domain are terminated by the Absorbing Boundary Conditions which are required in the FDTD method to truncate the solution domain and reviewed in the next sections. ϵ_0 is the electrical permittivity of the free space and ϵ_r is the relative electrical permittivity of the ground. ϵ_r is constant for soils with constant parameters and frequency-dependent for dispersive soils. The conductivity of the ground σ is also constant for soils with constant parameter models and frequency-dependent for the dispersive soils.

3.1. FDTD Method for Soils with Constant Parameters

Maxwell equations in simple homogenous media are given as

$$\nabla \times \vec{H}(\vec{r}, t) = \varepsilon \frac{\partial \vec{E}(\vec{r}, t)}{\partial t} + \vec{J}(\vec{r}, t) \quad (3.1.a)$$

$$\nabla \times \vec{E}(\vec{r}, t) = -\mu \frac{\partial \vec{H}(\vec{r}, t)}{\partial t} \quad (3.1.b)$$

where $\nabla \times$ is the curl operator, $\vec{r}$ is position vector, t is the time, ε is the electrical permittivity of the medium, μ is the magnetic permeability of the medium, and $\vec{E}(\vec{r}, t)$, $\vec{H}(\vec{r}, t)$, and $\vec{J}(\vec{r}, t)$ are electric field, magnetic field, and current density vectors, respectively. $\vec{J}(\vec{r}, t)$ includes both conduction current which equals $\sigma \vec{E}(\vec{r}, t)$ (σ is the medium conductivity) and the source current [Balanis, 2012, ch. 1]. In 3D cylindrical coordinates, the general system of equations can be expressed as [Inan and Marshall, 2011, ch. 4]

$$\frac{\partial E_r}{\partial t} = \frac{1}{\varepsilon} \left[\frac{1}{r} \frac{\partial H_z}{\partial \varphi} - \frac{\partial H_\varphi}{\partial z} \right] - \frac{\sigma}{\varepsilon} E_r \quad (3.2.a)$$

$$\frac{\partial E_\varphi}{\partial t} = \frac{1}{\varepsilon} \left[\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} \right] - \frac{\sigma}{\varepsilon} E_\varphi \quad (3.2.b)$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon} \frac{1}{r} \left[\frac{\partial(r H_\varphi)}{\partial r} - \frac{\partial H_r}{\partial \varphi} \right] - \frac{\sigma}{\varepsilon} E_z - \frac{1}{\varepsilon} J_s \quad (3.2.c)$$

$$\frac{\partial H_r}{\partial t} = \frac{1}{\mu} \left[\frac{\partial E_\varphi}{\partial z} - \frac{1}{r} \frac{\partial E_z}{\partial \varphi} \right] \quad (3.2.d)$$

$$\frac{\partial H_\varphi}{\partial t} = \frac{1}{\mu} \left[\frac{\partial E_z}{\partial r} - \frac{\partial E_r}{\partial z} \right] \quad (3.2.e)$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu} \frac{1}{r} \left[\frac{\partial E_r}{\partial \varphi} - \frac{\partial(r E_\varphi)}{\partial r} \right] \quad (3.2.f)$$

after omitting the vector signs where $\frac{\partial}{\partial(\cdot)}$ is the partial derivative operator and r , φ , and z represent the horizontal, azimuthal, and vertical dimensions, respectively. It is worth noting that J_s is the only source current density in the system resulting from the return stroke current defined at $r = 0$.

For reduction to 2D coordinates, since the lightning problem is axisymmetric in the azimuthal direction, we can equate the derivative of components in the azimuthal direction to zero, namely $\partial(\cdot)/\partial\varphi = 0$. The lightning current is in the vertical direction so vertical component of the magnetic field H_z is also zero. Then, in TM_z (transverse magnetic) mode, equation (3.2) reduces to

$$\frac{\partial E_r}{\partial t} = -\frac{\sigma}{\varepsilon} E_r - \frac{1}{\varepsilon} \frac{\partial H_\varphi}{\partial z} \quad (3.3.a)$$

$$\frac{\partial E_z}{\partial t} = -\frac{\sigma}{\varepsilon} E_z + \frac{1}{r\varepsilon} \frac{\partial(rH_\varphi)}{\partial r} - \frac{1}{\varepsilon} J_s \quad (3.3.b)$$

$$\frac{\partial H_\varphi}{\partial t} = \frac{1}{\mu} \left[\frac{\partial E_z}{\partial r} - \frac{\partial E_r}{\partial z} \right] \quad (3.3.c)$$

in 2D cylindrical coordinates. Equation (3.3) shows the system of equations governing the relation between the electromagnetic fields in the lightning problem with constant parameters.

3.1.1. FDTD Update Equations

The FDTD grid in 2D cylindrical coordinates is shown in Figure 3.2. The problem domain given in Figure 3.1 are divided into $M \times N$ rectangle cells whose horizontal and vertical dimensions are Δr and Δz , respectively. Each rectangle in the grid represents the unit cell of the FDTD lattice formed by the scheme of Yee [Taflove and Hagness, 2005]. The horizontal component of the electric field E_r is placed at the edge of each unit cell in the horizontal dimension and the vertical component of the electric field E_z is placed at the edge of each unit cell in the vertical dimension. The azimuthal magnetic field H_φ is located at the center of each unit cell. The dimensions of the unit cell (Δr , Δz) depend on the studied frequency range and the objects existing in the problem domain.

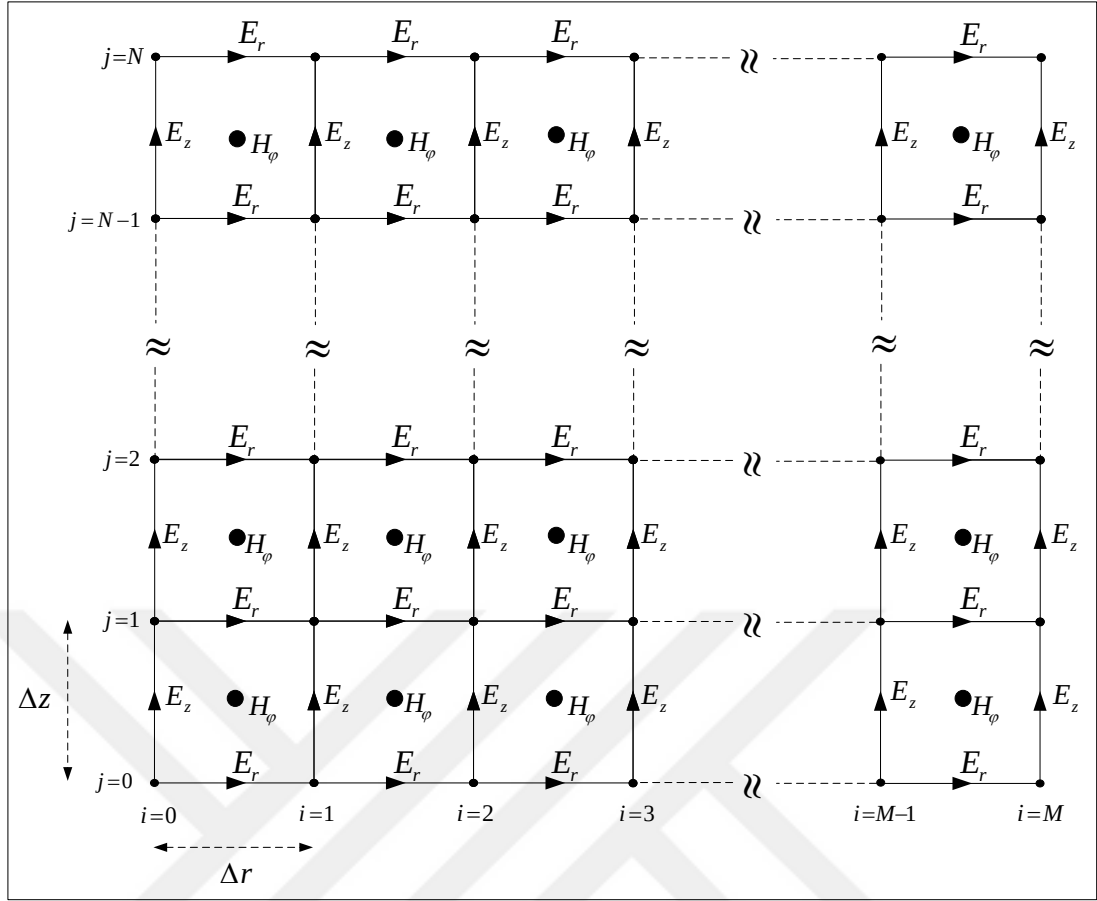


Figure 3.2: FDTD grid in 2D cylindrical coordinates.

If equation (3.3.a) is discretized by using the central difference scheme [Taflove and Hagness, 2005], we obtain

$$\frac{E_r|_{i+\frac{1}{2},j}^{n+1} - E_r|_{i+\frac{1}{2},j}^n}{\Delta t} = -\frac{\sigma}{\varepsilon} E_r|_{i+\frac{1}{2},j}^{n+\frac{1}{2}} - \frac{1}{\varepsilon} \frac{1}{\Delta z} \left[H_\phi|_{i+\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} - H_\phi|_{i+\frac{1}{2},j-\frac{1}{2}}^{n+\frac{1}{2}} \right]. \quad (3.4)$$

In equation (3.4), the first term $E_r|_{i+\frac{1}{2},j}^{n+1}$ represents the E_r at the position $\left(\left[i + \frac{1}{2}\right] \Delta r, j \Delta z\right)$ and at the time $(n+1)\Delta t$. The other terms $E_r|_{i+\frac{1}{2},j}^n$, $H_\phi|_{i+\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}}$, and $H_\phi|_{i+\frac{1}{2},j-\frac{1}{2}}^{n+\frac{1}{2}}$ can be considered accordingly. The electric field term at the half time step $E_r|_{i+\frac{1}{2},j}^{n+\frac{1}{2}}$ resulting from the conduction current density can be replaced with the average of the E_r terms at half step left and right indices over the time interval between $n\Delta t$ and $(n+1)\Delta t$. In this case, it is written that

$$\frac{E_r|_{i+\frac{1}{2},j}^{n+1} - E_r|_{i+\frac{1}{2},j}^n}{\Delta t} = -\frac{\sigma}{\varepsilon} \frac{E_r|_{i+\frac{1}{2},j}^{n+1} + E_r|_{i+\frac{1}{2},j}^n}{2} - \frac{1}{\varepsilon} \frac{1}{\Delta z} \left[H_\varphi|_{i+\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} - H_\varphi|_{i+\frac{1}{2},j-\frac{1}{2}}^{n+\frac{1}{2}} \right]. \quad (3.5)$$

Equation (3.5) can be rearranged as

$$E_r|_{i+\frac{1}{2},j}^{n+1} = E_r|_{i+\frac{1}{2},j}^n - \frac{\sigma \Delta t}{\varepsilon} \frac{E_r|_{i+\frac{1}{2},j}^{n+1} + E_r|_{i+\frac{1}{2},j}^n}{2} - \frac{1}{\varepsilon} \frac{\Delta t}{\Delta z} \left[H_\varphi|_{i+\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} - H_\varphi|_{i+\frac{1}{2},j-\frac{1}{2}}^{n+\frac{1}{2}} \right] \quad (3.6.a)$$

$$\left(1 + \frac{\sigma \Delta t}{2\varepsilon}\right) E_r|_{i+\frac{1}{2},j}^{n+1} = \left(1 - \frac{\sigma \Delta t}{2\varepsilon}\right) E_r|_{i+\frac{1}{2},j}^n - \frac{1}{\varepsilon} \frac{\Delta t}{\Delta z} \left[H_\varphi|_{i+\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} - H_\varphi|_{i+\frac{1}{2},j-\frac{1}{2}}^{n+\frac{1}{2}} \right]. \quad (3.6.b)$$

Now, the update equation of E_r can be found by writing the term at the most forward time:

$$E_r|_{i+\frac{1}{2},j}^{n+1} = \frac{2\varepsilon - \sigma \Delta t}{2\varepsilon + \sigma \Delta t} E_r|_{i+\frac{1}{2},j}^n - \frac{2\Delta t}{(2\varepsilon + \sigma \Delta t)\Delta z} \left[H_\varphi|_{i+\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} - H_\varphi|_{i+\frac{1}{2},j-\frac{1}{2}}^{n+\frac{1}{2}} \right]. \quad (3.7)$$

If equation (3.3.b) is discretized in a similar way, the update equation of E_z can easily be found as

$$E_z|_{i,j+\frac{1}{2}}^{n+1} = \frac{2\varepsilon - \sigma \Delta t}{2\varepsilon + \sigma \Delta t} E_z|_{i,j+\frac{1}{2}}^n + \frac{2\Delta t}{(2\varepsilon + \sigma \Delta t)r_i \Delta r} \left[r_{i+\frac{1}{2}} H_\varphi|_{i+\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} - r_{i-\frac{1}{2}} H_\varphi|_{i-\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} \right]. \quad (3.8)$$

The source current in (3.3.b) is not included in the update equation in (3.8) because it only exists at $r = 0$ and it is not existent elsewhere. The situation at $r = 0$ is handled in the next section.

The update equation of H_ϕ is also obtained as

$$H_\phi|_{i+\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} = H_\phi|_{i+\frac{1}{2},j+\frac{1}{2}}^{n-\frac{1}{2}} + \frac{\Delta t}{\mu\Delta r} \left[E_z|_{i+1,j+\frac{1}{2}}^n - E_z|_{i,j+\frac{1}{2}}^n \right] - \frac{\Delta t}{\mu\Delta z} \left[E_r|_{i+\frac{1}{2},j+1}^n - E_r|_{i+\frac{1}{2},j}^n \right] \quad (3.9)$$

after discretizing equation (3.3.c) in a similar way.

In the update equations, Δr and Δz are spatial steps in horizontal and vertical dimensions, respectively, Δt is the unit time step and should be compatible with the Courant stability condition [Taflove and Hagness, 2005], and ϵ, σ and μ all have the respective values for each i and j in the grid with $i = 0, 1, \dots, M$ and $j = 0, 1, \dots, N$.

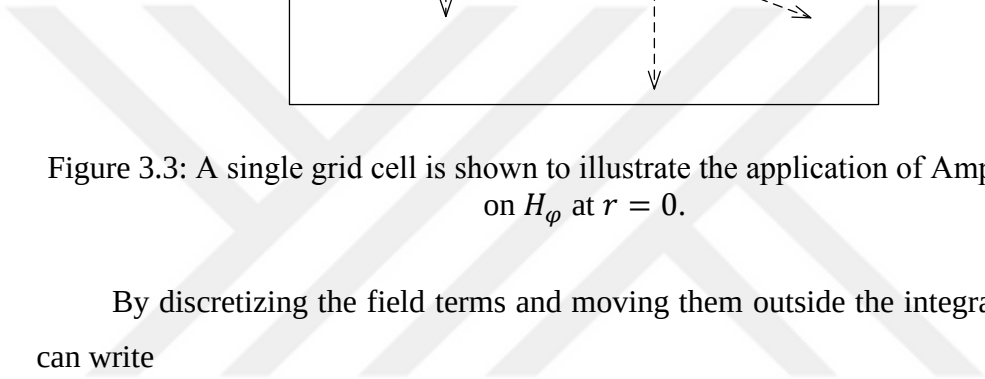
3.1.2. Singularity Treatment

It should be noted that the update equation of E_z in (3.8) contains terms multiplied or divided by the horizontal distance r . If the problem is not carefully handled, this leads to singularity at $r = 0$. Adapting from [Balanis, 2012, ch. 1], the Maxwell's equation derived from Ampere's law can be expressed in integral form as

$$\oint_c \vec{H}(\vec{r}, t) \cdot d\vec{l} = \iint_S \left(\epsilon \frac{\partial \vec{E}(\vec{r}, t)}{\partial t} + \sigma \vec{E}(\vec{r}, t) + \vec{J}_s(\vec{r}, t) \right) \cdot d\vec{S} \quad (3.10)$$

where c represents the circular contour enclosing the vertical electric field at $r = 0$ and having a radius of $\Delta r/2$, S is the surface surrounded by the contour c , $\vec{J}_s(\vec{r}, t)$ is the source electric current density resulting from the return stroke current at $r = 0$, and $\sigma \vec{E}(\vec{r}, t)$ is the conduction electric current density. The graphical interpretation of the situation in equation (3.10) is illustrated in Figure 3.3.

Assuming that H_ϕ at $\Delta r/2$ radial distance is constant along the contour c and that E_z is constant across the surface S , the equation (3.10) can be solved for the update equation of E_z at $r = 0$.



By discretizing the field terms and moving them outside the integral signs, we can write

can write

$$H_\phi|_{\frac{1}{2}, j+\frac{1}{2}}^{n+\frac{1}{2}} \oint_c dl = \left(\varepsilon \frac{\partial E_z}{\partial t} \Big|_{0, j+\frac{1}{2}}^{n+\frac{1}{2}} + \sigma E_z|_{0, j+\frac{1}{2}}^{n+\frac{1}{2}} + J_s \right) \iint_S dS. \quad (3.11)$$

After defining $E_z^{n+\frac{1}{2}}$ as the average of E_z^{n+1} and E_z^n

$$E_z|_{0,j+\frac{1}{2}}^{n+\frac{1}{2}} = \frac{E_z|_{0,j+\frac{1}{2}}^{n+1} + E_z|_{0,j+\frac{1}{2}}^n}{2}, \quad (3.12)$$

it can be written as follows:

$$H_\varphi|_{\frac{1}{2}j+\frac{1}{2}}^{n+\frac{1}{2}} \cdot \left(2\pi \frac{\Delta r}{2}\right) = \left[\frac{\varepsilon}{\Delta t} \left(E_z|_{0,j+\frac{1}{2}}^{n+1} - E_z|_{0,j+\frac{1}{2}}^n\right) + \frac{\sigma}{2} \left(E_z|_{0,j+\frac{1}{2}}^{n+1} + E_z|_{0,j+\frac{1}{2}}^n\right) + J_s\right] \cdot \left[\pi \left(\frac{\Delta r}{2}\right)^2\right] \quad (3.13.a)$$

$$\left(\frac{4}{\Delta r}\right) H_{\varphi}|_{\frac{1}{2}, j+\frac{1}{2}}^{n+\frac{1}{2}} = \left(\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}\right) E_z|_{0, j+\frac{1}{2}}^{n+1} + \left(\frac{\sigma}{2} - \frac{\varepsilon}{\Delta t}\right) E_z|_{0, j+\frac{1}{2}}^n + J_s \quad (3.13.b)$$

$$\frac{2\varepsilon + \sigma\Delta t}{2\Delta t} E_z|_{0, j+\frac{1}{2}}^{n+1} = \frac{2\varepsilon - \sigma\Delta t}{2\Delta t} E_z|_{0, j+\frac{1}{2}}^n + \frac{4}{\Delta r} H_{\varphi}|_{\frac{1}{2}, j+\frac{1}{2}}^{n+\frac{1}{2}} - J_s. \quad (3.13.c)$$

The source current density J_s can be defined as

$$J_s = \frac{I_s}{\pi \left(\frac{\Delta r}{2}\right)^2} \quad (3.14)$$

where I_s is the return stroke source current that can be defined by using one of the return stroke models reviewed in section 2.2. Then, the update equation of E_z can be written as follows:

$$E_z|_{0, j+\frac{1}{2}}^{n+1} = \frac{2\varepsilon - \sigma\Delta t}{2\varepsilon + \sigma\Delta t} E_z|_{0, j+\frac{1}{2}}^n + \frac{8\Delta t}{(2\varepsilon + \sigma\Delta t)\Delta r} H_{\varphi}|_{\frac{1}{2}, j+\frac{1}{2}}^{n+\frac{1}{2}} - \frac{8\Delta t}{(2\varepsilon + \sigma\Delta t)\pi(\Delta r)^2} I_s|_{0, j+\frac{1}{2}}^{n+\frac{1}{2}} \quad (3.15)$$

where I_s is equal to zero for the region under the ground.

3.2. FDTD Method for Dispersive Soils

Dispersive soil models are reviewed in section 2.3. All the soil models except for the Smith-Longmire can be named as the *fractional soil models* because the corresponding equations of each include frequency terms with fractional power. On the other hand, the Smith-Longmire model has frequency terms with integer power and needs to be treated differently from the fractional ones.

This section can be separated into two main parts. First, after finding electric susceptibility of the ground using the complex ground permittivity, solution to the lightning problem for the Smith-Longmire model is obtained by adopting the approach proposed by [Luebbers et al., 1990] that introduced the RC-FDTD (Recursive Convolution FDTD) method. Then, the solutions for the fractional soil models are

obtained after fitting the parameters of the electrical susceptibility by adopting the fractional-FDTD method introduced by [Torres et al., 1996].

3.2.1. Derivation of Complex Permittivity

The frequency-dependent complex electric permittivity of the ground can be derived using the Ampere-Maxwell equation:

$$\nabla \times \vec{H}(\vec{r}, t) = \frac{\partial \vec{D}(\vec{r}, t)}{\partial t} + \vec{J}(\vec{r}, t) \quad (3.16)$$

where $\vec{D}(\vec{r}, t)$ is the electric displacement vector and $\vec{J}(\vec{r}, t)$ is the conduction current density vector in a source-free medium. In frequency domain, equation (3.16) can be written as

$$\nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \sigma(\omega) \vec{E}(\vec{r}, \omega) \quad (3.17.a)$$

$$\nabla \times \vec{H}(\vec{r}, \omega) = j\omega \varepsilon_0 \varepsilon_r(\omega) \vec{E}(\vec{r}, \omega) + \sigma(\omega) \vec{E}(\vec{r}, \omega) \quad (3.17.b)$$

$$\nabla \times \vec{H}(\vec{r}, \omega) = \overbrace{[j\omega \varepsilon_0 \varepsilon_r(\omega) + \sigma(\omega)]}^{j\omega \varepsilon(\omega)} \vec{E}(\vec{r}, \omega). \quad (3.17.c)$$

As shown in equation (3.17.c), *complex permittivity* (ε) including the conductivity, and then, a related *effective relative complex permittivity* (ε_{eff}) [Cavka et al., 2014] can be defined as in the following:

$$\varepsilon(\omega) = \frac{j\omega \varepsilon_0 \varepsilon_r(\omega) + \sigma(\omega)}{j\omega} = \varepsilon_0 \varepsilon_r(\omega) + \frac{\sigma(\omega)}{j\omega} \quad (3.18.a)$$

$$\varepsilon_{eff}(\omega) = \frac{\varepsilon(\omega)}{\varepsilon_0} = \varepsilon_r(\omega) - j \frac{\sigma(\omega)}{\omega \varepsilon_0} \quad (3.18.b)$$

where $\omega = 2\pi f$ and f is frequency.

3.2.2. FDTD Method for Smith-Longmire Model

3.2.2.1. Derivation of Electric Susceptibility

Using the notation ω instead of f , $\varepsilon_r(f)$ in equation (2.6.a) and $\sigma(f)$ in equation (2.6.b) can be rewritten as

$$\varepsilon_r(\omega) = \varepsilon_\infty + \frac{a_1}{1 + \left(\frac{\omega}{2\pi F_1}\right)^2} + \dots + \frac{a_{13}}{1 + \left(\frac{\omega}{2\pi F_{13}}\right)^2} \quad (3.19.a)$$

$$\sigma(\omega) = \sigma_0 + 2\pi\varepsilon_0 \left(\frac{a_1 F_1 \left(\frac{\omega}{2\pi F_1}\right)^2}{1 + \left(\frac{\omega}{2\pi F_1}\right)^2} + \dots + \frac{a_{13} F_{13} \left(\frac{\omega}{2\pi F_{13}}\right)^2}{1 + \left(\frac{\omega}{2\pi F_{13}}\right)^2} \right). \quad (3.19.b)$$

After substituting (3.19.a) and (3.19.b) into (3.18.b), we can write $\varepsilon_{eff}(\omega)$ as

$$\begin{aligned} \varepsilon_{eff}(\omega) = & \varepsilon_\infty + \frac{a_1}{1 + \left(\frac{\omega}{2\pi F_1}\right)^2} + \dots + \frac{a_{13}}{1 + \left(\frac{\omega}{2\pi F_{13}}\right)^2} \\ & - \frac{j}{\omega\varepsilon_0} \left[\sigma_0 + 2\pi\varepsilon_0 \left(\frac{a_1 F_1 \left(\frac{\omega}{2\pi F_1}\right)^2}{1 + \left(\frac{\omega}{2\pi F_1}\right)^2} + \dots + \frac{a_{13} F_{13} \left(\frac{\omega}{2\pi F_{13}}\right)^2}{1 + \left(\frac{\omega}{2\pi F_{13}}\right)^2} \right) \right] \end{aligned} \quad (3.20.a)$$

$$\begin{aligned} \varepsilon_{eff}(\omega) = & \varepsilon_\infty - j \frac{\sigma_0}{\omega\varepsilon_0} + \left(\frac{a_1}{1 + \left(\frac{\omega}{2\pi F_1}\right)^2} - j \frac{\left(\frac{a_1}{2\pi F_1}\right)\omega}{1 + \left(\frac{\omega}{2\pi F_1}\right)^2} \right) + \dots \\ & + \left(\frac{a_{13}}{1 + \left(\frac{\omega}{2\pi F_{13}}\right)^2} - j \frac{\left(\frac{a_{13}}{2\pi F_{13}}\right)\omega}{1 + \left(\frac{\omega}{2\pi F_{13}}\right)^2} \right). \end{aligned} \quad (3.20.b)$$

At this step, we suppose a complex rational function in the form of $\frac{K}{1+j\omega\tau}$ where K and τ are constants, and then find its real and imaginary parts:

$$\frac{K}{1+j\omega\tau} = \text{Re} \left\{ \frac{K}{1+j\omega\tau} \right\} - j \text{Im} \left\{ \frac{K}{1+j\omega\tau} \right\} \quad (3.21.a)$$

$$\frac{K}{1+j\omega\tau} = \frac{K(1-j\omega\tau)}{(1+j\omega\tau)(1-j\omega\tau)} = \underbrace{\frac{K}{1+(\omega\tau)^2}}_{\text{Real part}} - j \underbrace{\frac{K\tau\omega}{1+(\omega\tau)^2}}_{\text{Imaginary part}}. \quad (3.21.b)$$

If we let $K = a_1, a_2, \dots, a_{13}$ and $\tau = \frac{1}{2\pi F_1}, \frac{1}{2\pi F_2}, \dots, \frac{1}{2\pi F_{13}}$, we can easily see that the structure of each term inside the brackets in equation (3.20.b) is the same with that of equation (3.21.b). Then, equation (3.20.b) can be rearranged as follows:

$$\varepsilon_{eff}(\omega) = \varepsilon_\infty + \frac{\sigma_{DC}}{j\omega\varepsilon_0} + \left(\frac{a_1}{1+j\frac{\omega}{2\pi F_1}} \right) + \dots + \left(\frac{a_{13}}{1+j\frac{\omega}{2\pi F_{13}}} \right) \quad (3.22.a)$$

$$\varepsilon_{eff}(\omega) = \varepsilon_\infty + \frac{\sigma_0}{j\omega\varepsilon_0} + \sum_{q=1}^{13} \frac{a_q}{1+j\frac{\omega}{2\pi F_q}}. \quad (3.22.b)$$

In fact, the rational function in (3.21) has the form of Debye equation [Debye, 1929] which describes the behavior of polar dielectrics at various frequencies; that is,

$$\varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1+j\omega\tau} = \varepsilon' - j\varepsilon'' = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1+j\omega^2\tau^2} - j \frac{(\varepsilon_s - \varepsilon_\infty)\omega\tau}{1+j\omega^2\tau^2} \quad (3.23)$$

where ε' and ε'' represent the real and imaginary parts of the relative complex permittivity, respectively, ε_s is the dielectric constant under DC and ε_∞ is the dielectric constant at infinite frequency, and τ is the relaxation time. Now, electric susceptibility $\chi(\omega)$ can be defined as a multi-pole Debye model with modified parameters and we can write

$$\chi_D(\omega) = \sum_{q=1}^{13} \frac{a_q}{1+j\frac{\omega}{2\pi F_q}} \quad (3.24.a)$$

$$\varepsilon_{eff}(\omega) = \varepsilon_\infty + \frac{\sigma_0}{j\omega\varepsilon_0} + \chi_D(\omega) \quad (3.24.b)$$

where the subscript D in χ_D denotes “Debye”. In fact, the first two terms in the right-hand side of equation (3.24.b) already exist in the case of non-dispersive lossy ground with constant electric parameters, which can easily be seen if the frequency-dependent terms in equation (3.19.a) and equation (3.19.b) are equated to zero. $\chi_D(\omega)$, on the other side, represents the frequency-dependent part of these equations.

Since the Smith-Longmire model is fitted to the Debye model with analytical approach, it is trivial to calculate the error between the two models. As an example, Figure 3.4 shows the errors between the Smith-Longmire model and its corresponding Debye fit for the LFC (σ_0) of 1 mS/m. Similar results are obtained for other σ_0 values. The errors between the models include only the numerical errors resulting from the computer capability so they are zero most of the time and maximum of the errors does not exceed 10^{-15} . For this reason, in Figure 3.4, the error plots look not smooth due to infinite values in dB.

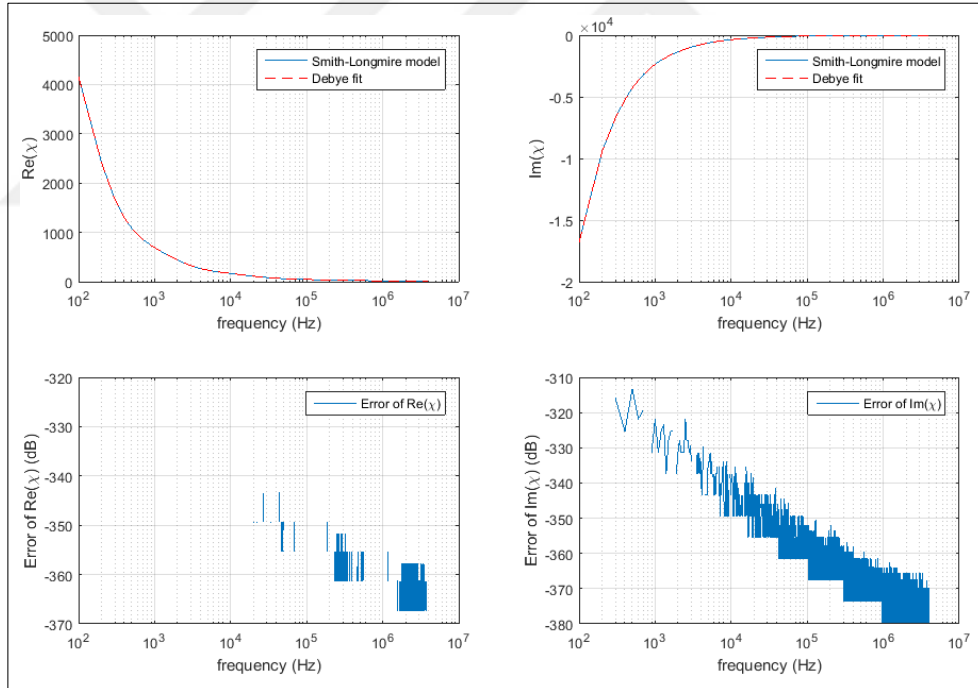


Figure 3.4: Comparison of real parts (top left) and imaginary parts (top right) of the Smith-Longmire model and its Debye fit for σ_0 of 1 mS/m. Corresponding errors are shown at the bottom of each one.

3.2.2.2. FDTD Update Equations

The relation with the displacement vector $\vec{D}(\vec{r}, \omega)$ and the electric field vector $\vec{E}(\vec{r}, \omega)$ can be written as

$$\vec{D}(\vec{r}, \omega) = \varepsilon_0[\varepsilon_\infty + \chi_D(\omega)]\vec{E}(\vec{r}, \omega) \quad (3.25)$$

in frequency domain, and as

$$\vec{D}(\vec{r}, t) = \varepsilon_0\varepsilon_\infty\vec{E}(\vec{r}, t) + \varepsilon_0 \int_0^t \vec{E}(\vec{r}, t - \xi) \chi_D(\xi) d\xi \quad (3.26)$$

in time domain, where ξ is a dummy variable of the integration.

In 2D cylindrical coordinates and assuming TM_z mode, Maxwell's equations in linear isotropic dispersive media are:

$$\frac{\partial D_r}{\partial t} = -\sigma_0 E_r - \frac{\partial H_\phi}{\partial z} \quad (3.27.a)$$

$$\frac{\partial D_z}{\partial t} = -\sigma_0 E_z + \frac{1}{r} \frac{\partial(r H_\phi)}{\partial r} \quad (3.27.b)$$

$$\frac{\partial H_\phi}{\partial t} = \frac{1}{\mu} \left[\frac{\partial E_z}{\partial r} - \frac{\partial E_r}{\partial z} \right]. \quad (3.27.c)$$

Equation (3.27.a) can be discretized as

$$\frac{D_r|_{i+\frac{1}{2},j}^{n+1} - D_r|_{i+\frac{1}{2},j}^n}{\Delta t} = -\sigma_0 E_r|_{i+\frac{1}{2},j}^{n+1} - \frac{H_\phi|_{i+\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} - H_\phi|_{i+\frac{1}{2},j-\frac{1}{2}}^{n+\frac{1}{2}}}{\Delta z}. \quad (3.28)$$

At this stage, we will pursue the procedure introduced by [Luebbers et al., 1990] who formulated update equations for dispersive media having the Debye model relation. The difference is that they used single-pole Debye model whereas we will derive update equations for dispersive soil with the Smith-Longmire model using the susceptibility equation in (3.24.a) represented with the multi-pole Debye equation.

If equation (3.26) is discretized using $t = n\Delta t$, displacement vector D_r^n can be expressed as

$$D_r(t) \approx D_r(n\Delta t) = D_r^n \quad (3.29.a)$$

$$D_r^n = \varepsilon_0 \varepsilon_\infty E_r^n + \varepsilon_0 \int_0^{n\Delta t} E_r(n\Delta t - \xi) \chi_D(\xi) d\xi. \quad (3.29.b)$$

After adding spatial indices and taking E_r outside the integral, we can write

$$D_r|_{i+\frac{1}{2},j}^n = \varepsilon_0 \varepsilon_\infty E_r|_{i+\frac{1}{2},j}^n + \varepsilon_0 \sum_{m=0}^{n-1} E_r|_{i+\frac{1}{2},j}^{n-m} \int_{m\Delta t}^{(m+1)\Delta t} \chi_D(\xi) d\xi \quad (3.30.a)$$

$$D_r|_{i+\frac{1}{2},j}^{n+1} = \varepsilon_0 \varepsilon_\infty E_r|_{i+\frac{1}{2},j}^{n+1} + \varepsilon_0 \sum_{m=0}^n E_r|_{i+\frac{1}{2},j}^{n+1-m} \int_{m\Delta t}^{(m+1)\Delta t} \chi_D(\xi) d\xi. \quad (3.30.b)$$

Then, subtracting D_r^n from D_r^{n+1} , we get

$$\begin{aligned} D_r|_{i+\frac{1}{2},j}^{n+1} - D_r|_{i+\frac{1}{2},j}^n &= \varepsilon_0 \varepsilon_\infty [E_r|_{i+\frac{1}{2},j}^{n+1} - E_r|_{i+\frac{1}{2},j}^n] + \varepsilon_0 E_r|_{i+\frac{1}{2},j}^{n+1} \int_0^{\Delta t} \chi_D(\xi) d\xi \\ &+ \varepsilon_0 \sum_{m=0}^{n-1} E_r|_{i+\frac{1}{2},j}^{n-m} \left[\int_{(m+1)\Delta t}^{(m+2)\Delta t} \chi_D(\xi) d\xi - \int_{(m)\Delta t}^{(m+1)\Delta t} \chi_D(\xi) d\xi \right]. \end{aligned} \quad (3.31)$$

For simplification, we let

$$\chi_m = \int_{m\Delta t}^{(m+1)\Delta t} \chi_D(\xi) d\xi \quad (3.32.a)$$

$$\Delta \chi_m = \chi_m - \chi_{m+1}. \quad (3.32.b)$$

Then, it can be written

$$\begin{aligned} D_r|_{i+\frac{1}{2},j}^{n+1} - D_r|_{i+\frac{1}{2},j}^n &= \\ &\varepsilon_0 (\varepsilon_\infty + \chi_0) E_r|_{i+\frac{1}{2},j}^{n+1} - \varepsilon_0 \varepsilon_\infty E_r|_{i+\frac{1}{2},j}^n - \varepsilon_0 \sum_{m=0}^{n-1} E_r|_{i+\frac{1}{2},j}^{n-m} \Delta \chi_m. \end{aligned} \quad (3.33)$$

Combining equation (3.33) with (3.28), E_r^{n+1} can be obtained as

$$E_r|_{i+\frac{1}{2},j}^{n+1} = \frac{1}{\varepsilon_\infty + \chi_0 + \frac{\sigma_0 \Delta t}{\varepsilon_0}} \times \left[\varepsilon_\infty E_r|_{i+\frac{1}{2},j}^n + \sum_{m=0}^{n-1} E_r|_{i+\frac{1}{2},j}^{n-m} \Delta \chi_m - \frac{\Delta t}{\varepsilon_0 \Delta z} \left(H_\varphi|_{i+\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} - H_\varphi|_{i+\frac{1}{2},j-\frac{1}{2}}^{n+\frac{1}{2}} \right) \right]. \quad (3.34)$$

In the equation above, the terms other than the convolution (summation) term are fine. Nevertheless, the convolution term requires storing all the past time values at each time step, which means a great increase in the computation time. This problem can be overcome using the technique called Recursive Convolution (RC) since the susceptibility term χ has an exponential form in time domain. Different from the study by [Luebbers et al., 1990] who found a recursive relation for a single term having an exponential form, we will constitute a recursive relation for the sum of exponential terms.

Using the inverse Fourier Transform, it is given that

$$\frac{1}{\beta + j\omega} \xrightarrow{iFT} e^{-\beta t} u(t) \quad (3.35.a)$$

$$\frac{1}{1 + j\frac{\omega}{\beta}} \xrightarrow{iFT} \beta e^{-\beta t} u(t) \quad (3.35.b)$$

where $u(t)$ is the unit step function. Then, the frequency domain susceptibility term in equation (3.24.a) can be expressed as

$$\chi_D(t) = \sum_{q=1}^{13} [a_q (2\pi F_q) e^{-(2\pi F_q)t} u(t)] \quad (3.36)$$

in the time domain. Now, χ_0 in equation (3.34) can be calculated as follows:

$$\chi_0 = \int_0^{\Delta t} \chi_D(\xi) d\xi = \int_0^{\Delta t} \left\{ \sum_{q=1}^{13} [a_q (2\pi F_q) e^{-(2\pi F_q)\xi} u(\xi)] \right\} d\xi \quad (3.37.a)$$

$$\chi_0 = \sum_{q=1}^{13} \left[a_q (2\pi F_q) \int_0^{\Delta t} e^{-(2\pi F_q)\xi} u(\xi) d\xi \right] \quad (3.37.b)$$

$$\chi_0 = \sum_{q=1}^{13} \left[a_q (2\pi F_q) \left(-\frac{1}{2\pi F_q} \right) e^{-(2\pi F_q)\xi} \Big|_{\xi=0}^{\xi=\Delta t} \right] \quad (3.37.c)$$

$$\chi_0 = \sum_{q=1}^{13} a_q [1 - e^{-(2\pi F_q)\Delta t}], \quad (3.37.d)$$

and $\Delta\chi_m$ can be similarly calculated as

$$\Delta\chi_m = \int_{m\Delta t}^{(m+1)\Delta t} \chi_D(\xi) d\xi - \int_{(m+1)\Delta t}^{(m+2)\Delta t} \chi_D(\xi) d\xi \quad (3.38.a)$$

$$\Delta\chi_m = \sum_{q=1}^{13} a_q \left[-e^{-(2\pi F_q)\xi} \Big|_{m\Delta t}^{(m+1)\Delta t} + e^{-(2\pi F_q)\xi} \Big|_{(m+1)\Delta t}^{(m+2)\Delta t} \right] \quad (3.38.b)$$

$$\Delta\chi_m = \sum_{q=1}^{13} a_q \left[e^{-(2\pi F_q)\xi} \Big|_{(m+1)\Delta t}^{m\Delta t} + e^{-(2\pi F_q)\xi} \Big|_{(m+1)\Delta t}^{(m+2)\Delta t} \right] \quad (3.38.c)$$

$$\begin{aligned} \Delta\chi_m = \sum_{q=1}^{13} a_q & \left[e^{-(2\pi F_q)m\Delta t} (1 - e^{-(2\pi F_q)\Delta t}) \right. \\ & \left. + e^{-(2\pi F_q)m\Delta t} (e^{-2(2\pi F_q)\Delta t} - e^{-(2\pi F_q)\Delta t}) \right] \end{aligned} \quad (3.38.d)$$

$$\Delta\chi_m = \sum_{q=1}^{13} a_q [e^{-(2\pi F_q)m\Delta t} (1 - 2e^{-(2\pi F_q)\Delta t} + e^{-2(2\pi F_q)\Delta t})] \quad (3.38.e)$$

$$\Delta\chi_m = \sum_{q=1}^{13} a_q [e^{-(2\pi F_q)m\Delta t} (1 - e^{-(2\pi F_q)\Delta t})^2]. \quad (3.38.f)$$

Now, it is aimed to find a recursive relation between the electric field and the convolution term in equation (3.34). For simplicity, we define $A_q = a_q(1 - e^{-b_q\Delta t})^2$ and $b_q = 2\pi F_q$. In a simplified form, χ_0 and $\Delta\chi_m$ can be rewritten as

$$\chi_0 = \sum_{q=1}^{13} a_q [1 - e^{-b_q\Delta t}] \quad (3.39.a)$$

$$\Delta\chi_m = \sum_{q=1}^{13} A_q e^{-b_q m\Delta t}. \quad (3.39.b)$$

To simplify the convolution term, we introduce ψ_r that is expressed as

$$\psi_r|_{i+\frac{1}{2},j}^n = \sum_{m=0}^{n-1} E_r|_{i+\frac{1}{2},j}^{n-m} \Delta\chi_m. \quad (3.40)$$

If ψ_r terms are written recursively in an open form, it is found that

$$\psi_r|_{i+\frac{1}{2},j}^1 = E_r|_{i+\frac{1}{2},j}^1 \Delta\chi_0 \quad (3.41.a)$$

$$\psi_r|_{i+\frac{1}{2},j}^2 = E_r|_{i+\frac{1}{2},j}^2 \Delta\chi_0 + E_r|_{i+\frac{1}{2},j}^1 \Delta\chi_1 \quad (3.41.b)$$

$$\psi_r|_{i+\frac{1}{2},j}^3 = E_r|_{i+\frac{1}{2},j}^3 \Delta\chi_0 + E_r|_{i+\frac{1}{2},j}^2 \Delta\chi_1 + E_r|_{i+\frac{1}{2},j}^1 \Delta\chi_2 \quad (3.41.c)$$

$\vdots$

$$\psi_r|_{i+\frac{1}{2},j}^{n-1} = E_r|_{i+\frac{1}{2},j}^{n-1} \Delta\chi_0 + E_r|_{i+\frac{1}{2},j}^{n-2} \Delta\chi_1 + \cdots + E_r|_{i+\frac{1}{2},j}^1 \Delta\chi_{n-2} \quad (3.41.d)$$

$$\psi_r|_{i+\frac{1}{2},j}^n = E_r|_{i+\frac{1}{2},j}^n \Delta\chi_0 + E_r|_{i+\frac{1}{2},j}^{n-1} \Delta\chi_1 + \cdots + E_r|_{i+\frac{1}{2},j}^1 \Delta\chi_{n-1} \quad (3.41.e)$$

where

$$\Delta\chi_0 = A_1 + A_2 + \cdots + A_{13} \quad (3.42.a)$$

$$\Delta\chi_1 = A_1 e^{-b_1 \Delta t} + A_2 e^{-b_2 \Delta t} + \dots + A_{13} e^{-b_{13} \Delta t} \quad (3.42.b)$$

$$\Delta\chi_2 = A_1 (e^{-b_1 \Delta t})^2 + A_2 (e^{-b_2 \Delta t})^2 + \dots + A_{13} (e^{-b_{13} \Delta t})^2 \quad (3.42.c)$$

$\vdots$

$$\Delta\chi_{n-2} = A_1 (e^{-b_1 \Delta t})^{n-2} + A_2 (e^{-b_2 \Delta t})^{n-2} + \dots + A_{13} (e^{-b_{13} \Delta t})^{n-2}. \quad (3.42.d)$$

$$\Delta\chi_{n-1} = A_1 (e^{-b_1 \Delta t})^{n-1} + A_2 (e^{-b_2 \Delta t})^{n-1} + \dots + A_{13} (e^{-b_{13} \Delta t})^{n-1}. \quad (3.42.e)$$

If the expressions of $\Delta\chi_m$ in equation (3.42.a) through equation (3.42.e) are substituted into $\psi_r|_{i+\frac{1}{2},j}^n$ in equation (3.41.e) to find a recursive formula, the following expressions

$$\begin{aligned} \psi_r|_{i+\frac{1}{2},j}^n &= E_r|_{i+\frac{1}{2},j}^n (A_1 + \dots + A_{13}) \\ &\quad + E_r|_{i+\frac{1}{2},j}^{n-1} (A_1 e^{-b_1 \Delta t} + \dots + A_{13} e^{-b_{13} \Delta t}) + \dots \\ &\quad \dots + E_r|_{i+\frac{1}{2},j}^1 [A_1 (e^{-b_1 \Delta t})^{n-1} + \dots + A_{13} (e^{-b_{13} \Delta t})^{n-1}] \end{aligned} \quad (3.43.a)$$

$$\begin{aligned} \psi_r|_{i+\frac{1}{2},j}^n &= \\ &\left(E_r|_{i+\frac{1}{2},j}^n A_1 + E_r|_{i+\frac{1}{2},j}^{n-1} A_1 e^{-b_1 \Delta t} + \dots + E_r|_{i+\frac{1}{2},j}^1 A_1 (e^{-b_1 \Delta t})^{n-1} \right) + \dots \\ &+ \left(E_r|_{i+\frac{1}{2},j}^n A_{13} + E_r|_{i+\frac{1}{2},j}^{n-1} A_{13} e^{-b_{13} \Delta t} + \dots + E_r|_{i+\frac{1}{2},j}^1 A_{13} (e^{-b_{13} \Delta t})^{n-1} \right) \end{aligned} \quad (3.43.b)$$

can be written. In equation (3.43.b), the field ψ_r^n is observed to be related with the same field at the previous time step ψ_r^{n-1} . We can express this recursive relation as

$$\begin{pmatrix} \psi_{r,1}|_{i+\frac{1}{2},j}^n \\ \psi_{r,2}|_{i+\frac{1}{2},j}^n \\ \vdots \\ \psi_{r,13}|_{i+\frac{1}{2},j}^n \end{pmatrix} = E_r|_{i+\frac{1}{2},j}^n \cdot \begin{pmatrix} A_1 \\ A_2 \\ \vdots \\ A_{13} \end{pmatrix} + \begin{pmatrix} e^{-b_1 \Delta t} \\ e^{-b_2 \Delta t} \\ \vdots \\ e^{-b_{13} \Delta t} \end{pmatrix} \cdot \begin{pmatrix} \psi_{r,1}|_{i+\frac{1}{2},j}^{n-1} \\ \psi_{r,2}|_{i+\frac{1}{2},j}^{n-1} \\ \vdots \\ \psi_{r,13}|_{i+\frac{1}{2},j}^{n-1} \end{pmatrix} \quad (3.44.a)$$

$$\psi_r|_{i+\frac{1}{2},j}^n = \psi_{r,1}|_{i+\frac{1}{2},j}^n + \psi_{r,2}|_{i+\frac{1}{2},j}^n + \cdots + \psi_{r,13}|_{i+\frac{1}{2},j}^n. \quad (3.44.b)$$

After substituting $a_q(1 - e^{-b_q\Delta t})^2$ into A_q and $2\pi F_q$ into b_q , it is found that

$$\begin{aligned} \psi_r|_{i+\frac{1}{2},j}^n &= \begin{pmatrix} \psi_{r,1}|_{i+\frac{1}{2},j}^n \\ \psi_{r,2}|_{i+\frac{1}{2},j}^n \\ \vdots \\ \psi_{r,13}|_{i+\frac{1}{2},j}^n \end{pmatrix} = E_r|_{i+\frac{1}{2},j}^n \cdot \begin{pmatrix} a_1(1 - e^{-(2\pi F_1)\Delta t})^2 \\ a_2(1 - e^{-(2\pi F_2)\Delta t})^2 \\ \vdots \\ a_{13}(1 - e^{-(2\pi F_{13})\Delta t})^2 \end{pmatrix} \\ &\quad + \begin{pmatrix} e^{-(2\pi F_1)\Delta t} \\ e^{-(2\pi F_2)\Delta t} \\ \vdots \\ e^{-(2\pi F_{13})\Delta t} \end{pmatrix} \cdot \begin{pmatrix} \psi_{r,1}|_{i+\frac{1}{2},j}^{n-1} \\ \psi_{r,2}|_{i+\frac{1}{2},j}^{n-1} \\ \vdots \\ \psi_{r,13}|_{i+\frac{1}{2},j}^{n-1} \end{pmatrix}. \end{aligned} \quad (3.45)$$

Equation (3.45) can also be written in closed form as follows:

$$\psi_r|_{i+\frac{1}{2},j}^n = E_r|_{i+\frac{1}{2},j}^n \cdot \Delta\chi_0 + \sum_{q=1}^{13} \left[(e^{-(2\pi F_q)\Delta t}) \psi_{r,q}|_{i+\frac{1}{2},j}^{n-1} \right]. \quad (3.46)$$

After substituting ψ_r^n into equation (3.34), we can finally obtain the update equation of the electric field E_r

$$E_r|_{i+\frac{1}{2},j}^{n+1} = \frac{1}{\varepsilon_\infty + \chi_0 + \frac{\sigma_0\Delta t}{\varepsilon_0}} \times \quad (3.47)$$

$$\left[\varepsilon_\infty E_r|_{i+\frac{1}{2},j}^n + \psi_r|_{i+\frac{1}{2},j}^n - \frac{\Delta t}{\varepsilon_0\Delta Z} \left(H_\varphi|_{i+\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} - H_\varphi|_{i+\frac{1}{2},j-\frac{1}{2}}^{n+\frac{1}{2}} \right) \right]$$

where χ_0 is found as

$$\chi_0 = \sum_{q=1}^{13} a_q [1 - e^{-(2\pi F_q)\Delta t}]. \quad (3.48)$$

To obtain the update equation for E_z , a similar procedure can easily be followed by discretizing equation (3.27.b), then we can end up with the following equation:

$$E_z|_{i,j+\frac{1}{2}}^{n+1} = \frac{1}{\varepsilon_\infty + \chi_0 + \frac{\sigma_0 \Delta t}{\varepsilon_0}} \times \left[\varepsilon_\infty E_z|_{i,j+\frac{1}{2}}^n + \psi_z|_{i,j+\frac{1}{2}}^n + \frac{\Delta t}{\varepsilon_0 r_i \Delta r} \left(r_{i+\frac{1}{2}} H_\varphi|_{i+\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} - r_{i-\frac{1}{2}} H_\varphi|_{i-\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} \right) \right] \quad (3.49)$$

where ψ_z can be written as

$$\psi_z|_{i,j+\frac{1}{2}}^n = E_z|_{i,j+\frac{1}{2}}^n \cdot \Delta \chi_0 + \sum_{q=1}^{13} \left[(e^{-(2\pi F_q) \Delta t}) \psi_{z,q}|_{i,j+\frac{1}{2}}^{n-1} \right]. \quad (3.50)$$

Update equation for H_φ is the same with that in equation (3.9) everywhere in the solution domain. In air part of the lightning problem, the update equations in (3.7) and (3.8) can also be used for E_r and E_z , respectively. However, in the soil part, the update equations in (3.47) and (3.49) should be used where ψ_r and ψ_z are dependent on the convolution of respective electric fields with susceptibility and updated recursively.

3.2.2.3. Singularity Treatment

As in the case with constant parameters, the update equation of E_z contains singularity due to the terms in denominator in the last term of equation (3.49). To overcome the singularity problem at $r = 0$, we can follow the same procedure in section 3.1.2 and solve for the update equation of E_z as

$$H_\varphi|_{\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} \oint_c dl = \left(\frac{\partial D_z}{\partial t} \Big|_{0,j+\frac{1}{2}}^{n+\frac{1}{2}} + \sigma_0 E_z|_{0,j+\frac{1}{2}}^{n+\frac{1}{2}} + J_s \right) \iint_S dS \quad (3.51.a)$$

$$H_\varphi|_{\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} \cdot \left(2\pi \frac{\Delta r}{2} \right) = \left[\frac{1}{\Delta t} \left(D_z|_{0,j+\frac{1}{2}}^{n+1} - D_z|_{0,j+\frac{1}{2}}^n \right) + \sigma_0 \left(E_z|_{0,j+\frac{1}{2}}^{n+1} \right) + J_s \right] \cdot \left[\pi \left(\frac{\Delta r}{2} \right)^2 \right]. \quad (3.51.b)$$

Using equations (3.33) and (3.40), it can be written that

$$\begin{aligned} \left(\frac{4\Delta t}{\varepsilon_0 \Delta r}\right) H_\varphi|_{\frac{1}{2}j+\frac{1}{2}}^{n+\frac{1}{2}} &= (\varepsilon_\infty + \chi_0) E_z|_{0,j+\frac{1}{2}}^{n+1} - \varepsilon_\infty E_z|_{0,j+\frac{1}{2}}^n \\ &\quad - \psi_z|_{0,j+\frac{1}{2}}^n + \frac{\sigma_0 \Delta t}{\varepsilon_0} \left(E_z|_{0,j+\frac{1}{2}}^{n+1}\right) + \frac{\Delta t}{\varepsilon_0} J_s \end{aligned} \quad (3.52.a)$$

$$\begin{aligned} \left(\varepsilon_\infty + \chi_0 + \frac{\sigma_0 \Delta t}{\varepsilon_0}\right) E_z|_{0,j+\frac{1}{2}}^{n+1} &= \varepsilon_\infty E_z|_{0,j+\frac{1}{2}}^n \\ &\quad + \psi_z|_{0,j+\frac{1}{2}}^n + \frac{4\Delta t}{\varepsilon_0 \Delta r} H_\varphi|_{\frac{1}{2}j+\frac{1}{2}}^{n+\frac{1}{2}} - \frac{\Delta t}{\varepsilon_0} J_s. \end{aligned} \quad (3.52.b)$$

After substituting equation (3.14) into J_s , we obtain the following update equation at $r = 0$ including the current source:

$$\begin{aligned} E_z|_{0,j+\frac{1}{2}}^{n+1} &= \frac{1}{\varepsilon_\infty + \chi_0 + \frac{\sigma_0 \Delta t}{\varepsilon_0}} \times \\ &\quad \left[\varepsilon_\infty E_z|_{0,j+\frac{1}{2}}^n + \psi_z|_{0,j+\frac{1}{2}}^n + \frac{4\Delta t}{\varepsilon_0 \Delta r} H_\varphi|_{\frac{1}{2}j+\frac{1}{2}}^{n+\frac{1}{2}} - \frac{4\Delta t}{\varepsilon_0 \pi (\Delta r)^2} I_s \right]. \end{aligned} \quad (3.53)$$

3.2.3. FDTD Method for Fractional Soil Models

Many researchers tried to put a fractional derivative definition since the 17th century when the concept of fractional derivative emerged for the first time [Khalil et al., 2014]. According to the Riemann–Liouville definition, which is one of the most popular definitions, fractional derivative of $f(t)$ that is a continuous function in the interval $[a, T]$, $T > a$, is given by [Scherer et al., 2011]

$$D^\alpha f(t) = \frac{\partial^\alpha f(t)}{\partial t^\alpha} = \frac{1}{\Gamma(m - \alpha)} \frac{d^m}{dt^m} \int_a^t \frac{f(u)}{(t - u)^{\alpha+1-m}} du \quad (3.54)$$

where m is a positive integer, α is the differentiation order and is a positive real number in the interval $m - 1 \leq \alpha < m$, and Γ is the gamma function given by [Abramowitz and Stegun, 1964]

$$\Gamma(z) = \int_0^{\infty} t^{z-1} e^{-t} dt, \quad \text{Re}(z) > 0 \quad (3.55.a)$$

$$\Gamma(z + 1) = z\Gamma(z). \quad (3.55.b)$$

Assuming $0 \leq \alpha < 1$ and the function $f(t)$ is limited in the interval $(0, t)$, $t \leq T$ for the causality reason, we can write

$$D^\alpha f(t) = \frac{1}{\Gamma(1 - \alpha)} \frac{d}{dt} \int_0^t (t - u)^{-\alpha} f(u) du \quad (3.56.a)$$

$$= \frac{1}{\Gamma(1 - \alpha)} \frac{d}{dt} \int_0^t u^{-\alpha} f(t - u) du. \quad (3.56.b)$$

The constitution of the FDTD update equations for the fractional derivative definition in equation (3.56) was achieved by [Torres et al., 1996]. First, we will address their method for the general media in section 3.2.3.1, and then adapt their technique to the lightning problem by using the fractional soil models mentioned in section 2.3.

3.2.3.1. FDTD Method for General Cole-Cole Media

In the frequency domain, the relation between the polarization of the medium $\vec{P}(\vec{r}, \omega)$ and the electric field $\vec{E}(\vec{r}, \omega)$ is expressed as

$$\vec{P}(\vec{r}, \omega) = \varepsilon_0 \chi(\omega) \vec{E}(\vec{r}, \omega) \quad (3.57)$$

where $\chi(\omega)$ is the electric susceptibility and can be defined as a Cole-Cole model [Cole and Cole, 1941]:

$$\chi_{cc}(\omega) = \frac{\varepsilon_s - \varepsilon_\infty}{1 + (j\omega\tau)^\alpha} \quad (3.58)$$

where cc denotes the abbreviation for Cole-Cole, ε_s and ε_∞ are the static and infinite frequency dielectric constants, ω is the angular frequency, τ is the relaxation time, and α is a rational number between 0 and 1. Here, it can easily be seen that equation (3.58) turns to the Debye model in equation (3.23) for $\alpha = 1$.

If we define $\Delta\varepsilon = \varepsilon_s - \varepsilon_\infty$, then we can write

$$\vec{P}(\vec{r}, \omega) = \varepsilon_0 \frac{\Delta\varepsilon}{1 + (j\omega\tau)^\alpha} \vec{E}(\vec{r}, \omega) \quad (3.59.a)$$

$$\tau^\alpha (j\omega)^\alpha \vec{P}(\vec{r}, \omega) + \vec{P}(\vec{r}, \omega) = \varepsilon_0 \Delta\varepsilon \vec{E}(\vec{r}, \omega). \quad (3.59.b)$$

Fourier Transform of the fractional derivative of the function $f(t)$ is given by [Tseng et al., 2000]

$$FT\left\{\frac{\partial^\alpha}{\partial t^\alpha} f(t)\right\} = (j\omega)^\alpha FT\{f(t)\}. \quad (3.60)$$

Then, the expression in equation (3.59.b) can be written as

$$\tau^\alpha \frac{\partial^\alpha}{\partial t^\alpha} \vec{P}(\vec{r}, t) + \vec{P}(\vec{r}, t) = \varepsilon_0 \Delta\varepsilon \vec{E}(\vec{r}, t) \quad (3.61)$$

in time domain. Using only the time indices and omitting the spatial indices for the purpose of simplification, the discretized version of equation (3.61) at the time step $(n + \frac{1}{2})\Delta t$ can be written as

$$\tau^\alpha \frac{\partial^\alpha}{\partial t^\alpha} P(t) \Big|_{(n+\frac{1}{2})\Delta t} + P^{n+\frac{1}{2}} = \varepsilon_0 \Delta\varepsilon E^{n+\frac{1}{2}}. \quad (3.62)$$

If the integration term in equation (3.56.b) is defined as $I(t)$, then the fractional derivative of the polarization can be written as follows:

$$\left. \frac{\partial^\alpha}{\partial t^\alpha} P(t) \right|_{(n+\frac{1}{2})\Delta t} = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} I(t) \Big|_{(n+\frac{1}{2})\Delta t} \quad (3.63)$$

where

$$I(t) = I^n = \int_0^{n\Delta t} u^{-\alpha} P(n\Delta t - u) du. \quad (3.64)$$

By discretizing $\frac{d}{dt} I(t)$ according to the classical FDTD principle, equation (3.63) can be rewritten as

$$\left. \frac{\partial^\alpha}{\partial t^\alpha} P(t) \right|_{(n+\frac{1}{2})\Delta t} = \frac{1}{\Gamma(1-\alpha)} \frac{I^{n+1} - I^n}{\Delta t}. \quad (3.65)$$

We can write I^{n+1} as

$$I^{n+1} = \int_0^{(n+1)\Delta t} u^{-\alpha} P((n+1)\Delta t - u) du \quad (3.66.a)$$

$$\begin{aligned} I^{n+1} = & \int_0^{\Delta t} u^{-\alpha} P((n+1)\Delta t - u) du + \int_{\Delta t}^{2\Delta t} u^{-\alpha} P((n+1)\Delta t - u) du + \\ & \dots + \int_{n\Delta t}^{(n+1)\Delta t} u^{-\alpha} P((n+1)\Delta t - u) du. \end{aligned} \quad (3.66.b)$$

According to the basic FDTD principle, $P(t)$ over each Δt interval is assumed to the average of $P(n\Delta t)$ and $P((n+1)\Delta t)$. Also, the integration in equation (3.66) is, in fact, the convolution of $t^{-\alpha}$ with $P((n+1)\Delta t)$. Thus, equation (3.66.b) can be interpreted as the multiplication of $\int u^{-\alpha}$ with the flipped version of the sequence $P^{\frac{1}{2}}, P^{\frac{3}{2}}, \dots, P^{n-\frac{1}{2}}, P^{n+\frac{1}{2}}$ term by term; that is,

$$I^{n+1} = P^{n+\frac{1}{2}} \int_0^{\Delta t} u^{-\alpha} du + \dots + P^{\frac{1}{2}} \int_{n\Delta t}^{(n+1)\Delta t} u^{-\alpha} du. \quad (3.67)$$

I^n can be written in a similar way as

$$I^n = P^{n-\frac{1}{2}} \int_0^{\Delta t} u^{-\alpha} du + \dots + P^{\frac{1}{2}} \int_{(n-1)\Delta t}^{n\Delta t} u^{-\alpha} du. \quad (3.68)$$

The evaluation of the integration terms in equations (3.67) and (3.68) are as follows:

$$\int_{m\Delta t}^{(m+1)\Delta t} u^{-\alpha} du = \frac{u^{1-\alpha}}{1-\alpha} \Big|_{u=m\Delta t}^{u=(m+1)\Delta t} = \frac{\Delta t^{1-\alpha}}{1-\alpha} [(m+1)^{1-\alpha} - m^{1-\alpha}] \quad (3.69)$$

Then, after combining equations (3.65), (3.67), (3.68), and (3.69), fractional derivative of the polarization is expressed as

$$\begin{aligned} \frac{\partial^\alpha}{\partial t^\alpha} P(t) \Big|_{(n+\frac{1}{2})\Delta t} &= \frac{1}{\Gamma(1-\alpha)} \frac{\Delta t^{1-\alpha}}{1-\alpha} \times \\ &\frac{1}{\Delta t} \sum_{m=0}^n \left(P^{n+\frac{1}{2}-m} - P^{n-\frac{1}{2}-m} \right) [(m+1)^{1-\alpha} - m^{1-\alpha}]. \end{aligned} \quad (3.70)$$

The summation term in equation (3.70) is somehow similar to that in equation (3.34) because it requires to store all the past time values of $P(t)$. This issue can be overcome by finding a recursive relation for the summation term. In fact, the term $[(m+1)^{1-\alpha} - m^{1-\alpha}]$ is a decreasing function of m and can be written as the sum of N decaying exponentials as in the following

$$[(m+1)^{1-\alpha} - m^{1-\alpha}] = \sum_{k=1}^N a_k e^{-b_k m} \quad (3.71)$$

where N is an integer number depending on the value of α . Now, it is just a matter of finding a and b coefficients, which can easily be found through the process of curve fitting on a computer program. After including the relation in equation (3.55.b), the fractional derivative of the polarization can be expressed as

$$\left. \frac{\partial^\alpha}{\partial t^\alpha} P(t) \right|_{(n+\frac{1}{2})\Delta t} = \frac{\Delta t^{-\alpha}}{\Gamma(2-\alpha)} \sum_{k=1}^N \sum_{m=0}^n a_k \left(P^{n+\frac{1}{2}-m} - P^{n-\frac{1}{2}-m} \right) e^{-b_k m}. \quad (3.72)$$

If we define $\Psi_k^{n+\frac{1}{2}}$ as

$$\Psi_k^{n+\frac{1}{2}} = \sum_{m=0}^n a_k \left(P^{n+\frac{1}{2}-m} - P^{n-\frac{1}{2}-m} \right) e^{-b_k m}, \quad (3.73)$$

then equation (3.72) is written as

$$\left. \frac{\partial^\alpha}{\partial t^\alpha} P(t) \right|_{(n+\frac{1}{2})\Delta t} = \frac{\Delta t^{-\alpha}}{\Gamma(2-\alpha)} \sum_{k=1}^N \Psi_k^{n+\frac{1}{2}}. \quad (3.74)$$

Since each $\Psi_k^{n+\frac{1}{2}}$ term is a sum of exponential functions, then a recursive relation between $\Psi_k^{n+\frac{1}{2}}$ and its previous value $\Psi_k^{n-\frac{1}{2}}$ is observed to exist, which is written as

$$\Psi_k^{n+\frac{1}{2}} = a_k \left(P^{n+\frac{1}{2}} - P^{n-\frac{1}{2}} \right) + e^{-b_k} \Psi_k^{n-\frac{1}{2}}. \quad (3.75)$$

Then, it can be written that

$$\left. \frac{\partial^\alpha}{\partial t^\alpha} P(t) \right|_{(n+\frac{1}{2})\Delta t} = \frac{\Delta t^{-\alpha}}{\Gamma(2-\alpha)} \sum_{k=1}^N a_k \left(P^{n+\frac{1}{2}} - P^{n-\frac{1}{2}} \right) + e^{-b_k} \Psi_k^{n-\frac{1}{2}}. \quad (3.76)$$

After substituting the fractional derivative of the polarization in equation (3.76) into equation (3.62) and rearrangement of the expression, the polarization at the most forward time step is obtained as

$$P^{n+\frac{1}{2}} = B \left[\left(A \sum_{k=1}^N a_k \right) P^{n-\frac{1}{2}} - A \sum_{k=1}^N e^{-b_k} \Psi_k^{n-\frac{1}{2}} + \varepsilon_0 \Delta \varepsilon E^{n+\frac{1}{2}} \right] \quad (3.77)$$

where

$$A = \frac{\tau^\alpha \Delta t^{-\alpha}}{\Gamma(2 - \alpha)} \quad (3.78.a)$$

$$B = \left(1 + A \sum_{k=1}^N a_k \right)^{-1}. \quad (3.78.b)$$

The displacement vector including the convolution of the electric field and the susceptibility in equation (3.26) can be rewritten as

$$\vec{D}(\vec{r}, t) = \varepsilon_0 \varepsilon_\infty \vec{E}(\vec{r}, t) + \varepsilon_0 \int_0^t \vec{E}(\vec{r}, t - \xi) \chi_{cc}(\xi) d\xi \quad (3.79.a)$$

$$\vec{D}(\vec{r}, t) = \varepsilon_0 \varepsilon_\infty \vec{E}(\vec{r}, t) + \vec{P}(\vec{r}, t). \quad (3.79.b)$$

Then, using the Maxwell's equation for magnetic field, it is given for a source-free medium that

$$\nabla \times \vec{H}(\vec{r}, t) = \varepsilon_0 \varepsilon_\infty \frac{\partial}{\partial t} \vec{E}(\vec{r}, t) + \frac{\partial}{\partial t} \vec{P}(\vec{r}, t). \quad (3.80)$$

After discretizing equation (3.80) for the horizontal component of the electric field E_r , it can be written in 2D cylindrical coordinates that

$$-\frac{\partial H_\phi}{\partial z} = \varepsilon_0 \varepsilon_\infty \frac{E_r^{n+\frac{1}{2}} - E_r^{n-\frac{1}{2}}}{\Delta t} + \frac{P_r^{n+\frac{1}{2}} - P_r^{n-\frac{1}{2}}}{\Delta t}. \quad (3.81)$$

by using the time discretization only. The electric field at the most forward time step is found as

$$E_r^{n+\frac{1}{2}} = E_r^{n-\frac{1}{2}} + \frac{1}{\varepsilon_0 \varepsilon_\infty} \left[P_r^{n-\frac{1}{2}} - P_r^{n+\frac{1}{2}} - \Delta t \frac{\partial H_\phi}{\partial z} \right]. \quad (3.82)$$

After substituting equation (3.77) into equation (3.82), the expression of the electric field can be obtained as

$$E_r^{n+\frac{1}{2}} = E_r^{n-\frac{1}{2}} - \frac{\Delta t}{\varepsilon_0 \varepsilon_\infty} \left(\frac{\partial H_\varphi}{\partial z} \right) + \frac{1}{\varepsilon_0 \varepsilon_\infty} \times \left\{ P_r^{n-\frac{1}{2}} - B \left[\left(A \sum_{k=1}^N a_k \right) P_r^{n-\frac{1}{2}} - A \sum_{k=1}^N e^{-b_k \Psi_{r,k}} P_r^{n-\frac{1}{2}} + \varepsilon_0 \Delta \varepsilon E_r^{n+\frac{1}{2}} \right] \right\} \quad (3.83.a)$$

$$E_r^{n+\frac{1}{2}} = E_r^{n-\frac{1}{2}} - \frac{\Delta t}{\varepsilon_0 \varepsilon_\infty} \left(\frac{\partial H_\varphi}{\partial z} \right) + \frac{1}{\varepsilon_0 \varepsilon_\infty} \times \left[\left(1 - AB \sum_{k=1}^N a_k \right) P_r^{n-\frac{1}{2}} + AB \sum_{k=1}^N e^{-b_k \Psi_{r,k}} P_r^{n-\frac{1}{2}} - B \varepsilon_0 \Delta \varepsilon E_r^{n+\frac{1}{2}} \right] \quad (3.83.b)$$

$$\left(1 + \frac{B \Delta \varepsilon}{\varepsilon_\infty} \right) E_r^{n+\frac{1}{2}} = E_r^{n-\frac{1}{2}} + \frac{1}{\varepsilon_0 \varepsilon_\infty} \times \left[\left(1 - AB \sum_{k=1}^N a_k \right) P_r^{n+\frac{1}{2}} + AB \sum_{k=1}^N e^{-b_k \Psi_{r,k}} P_r^{n+\frac{1}{2}} + \Delta t \frac{\partial H_\varphi}{\partial z} \right]. \quad (3.83.c)$$

Finally, we obtain the update equation of electric field for Cole-Cole medium as

$$E_r^{n+\frac{1}{2}} = \frac{\varepsilon_\infty}{\varepsilon_\infty + B \Delta \varepsilon} E_r^{n-\frac{1}{2}} + \frac{1}{\varepsilon_\infty + B \Delta \varepsilon} \times \left[\left(1 - AB \sum_{k=1}^N a_k \right) \frac{P_r^{n+\frac{1}{2}}}{\varepsilon_0} + \frac{AB}{\varepsilon_0} \sum_{k=1}^N e^{-b_k \Psi_{r,k}} P_r^{n-\frac{1}{2}} + \frac{\Delta t}{\varepsilon_0} \frac{\partial H_\varphi}{\partial z} \right] \quad (3.84)$$

where A and B are the same with those in equation (3.78).

3.2.3.2. FDTD Update Equations

The update equation for the general Cole-Cole medium is available in the previous section. Now, the update equations customized for the lightning problem are presented.

In the previous section 3.2.3.1, half steps are used for the time indices of polarization and electric fields in order to write the expressions compatible with the

technique in [Torres et al., 1996]. In this section, half time steps are used for the magnetic field, and integer time steps are used for the polarization and electric fields to conform to the general approach in sections 3.1 and 3.2.2.

Including the conduction electric current density as in equation (3.27.a), the formulation in equation (3.80) can be rewritten as

$$\varepsilon_0 \varepsilon_\infty \frac{\partial}{\partial t} E_r + \frac{\partial}{\partial t} P_r = -\sigma_0 E_r - \frac{\partial H_\varphi}{\partial z} \quad (3.85)$$

using the horizontal electric field component E_r . If the steps in equations (3.81) through (3.84) are followed, the update equation for E_r can easily be constituted as

$$E_r|_{i+\frac{1}{2},j}^{n+1} = \frac{1}{\varepsilon_\infty + B\Delta\varepsilon + \frac{\sigma_0\Delta t}{\varepsilon_0}} \times \left[\varepsilon_\infty E_r|_{i+\frac{1}{2},j}^n + \left(1 - AB \sum_{k=1}^N a_k\right) \frac{P_r|_{i+\frac{1}{2},j}^n}{\varepsilon_0} + \frac{AB}{\varepsilon_0} \sum_{k=1}^N e^{-b_k} \Psi_{r,k}|_{i+\frac{1}{2},j}^n \right] - \frac{\Delta t}{\varepsilon_\infty + B\Delta\varepsilon + \frac{\sigma_0\Delta t}{\varepsilon_0}} \frac{H_\varphi|_{i+\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} - H_\varphi|_{i+\frac{1}{2},j-\frac{1}{2}}^{n+\frac{1}{2}}}{\varepsilon_0 \Delta z} \quad (3.86)$$

by writing all spatial and time indices. In a similar way, update equation for the E_z is

$$E_z|_{i,j+\frac{1}{2}}^{n+1} = \frac{1}{\varepsilon_\infty + B\Delta\varepsilon + \frac{\sigma_0\Delta t}{\varepsilon_0}} \times \left[\varepsilon_\infty E_z|_{i,j+\frac{1}{2}}^n + \left(1 - AB \sum_{k=1}^N a_k\right) \frac{P_z|_{i,j+\frac{1}{2}}^n}{\varepsilon_0} + \frac{AB}{\varepsilon_0} \sum_{k=1}^N e^{-b_k} \Psi_{z,k}|_{i,j+\frac{1}{2}}^n \right] + \frac{\Delta t}{\varepsilon_\infty + B\Delta\varepsilon + \frac{\sigma_0\Delta t}{\varepsilon_0}} \frac{r_{i+\frac{1}{2}} H_\varphi|_{i+\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} - r_{i-\frac{1}{2}} H_\varphi|_{i-\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}}}{\varepsilon_0 r_i \Delta r}. \quad (3.87)$$

The update equations for the polarization can be expressed as

$$P_r|_{i+\frac{1}{2},j}^{n+1} = \left(AB \sum_{k=1}^N a_k \right) P_r|_{i+\frac{1}{2},j}^n - AB \sum_{k=1}^N e^{-b_k} \Psi_{r,k}|_{i+\frac{1}{2},j}^n + B \varepsilon_0 \Delta \varepsilon E_r|_{i+\frac{1}{2},j}^{n+1} \quad (3.88.a)$$

$$P_z|_{i,j+\frac{1}{2}}^{n+1} = \left(AB \sum_{k=1}^N a_k \right) P_z|_{i,j+\frac{1}{2}}^n - AB \sum_{k=1}^N e^{-b_k} \Psi_{z,k}|_{i,j+\frac{1}{2}}^n + B \varepsilon_0 \Delta \varepsilon E_z|_{i,j+\frac{1}{2}}^{n+1} \quad (3.88.b)$$

where the auxiliary variables are updated using

$$\Psi_{r,k}|_{i+\frac{1}{2},j}^{n+1} = a_k \left(P_r|_{i+\frac{1}{2},j}^{n+1} - P_r|_{i+\frac{1}{2},j}^n \right) + e^{-b_k} \Psi_{r,k}|_{i+\frac{1}{2},j}^n \quad (3.89.a)$$

$$\Psi_{z,k}|_{i,j+\frac{1}{2}}^{n+1} = a_k \left(P_z|_{i,j+\frac{1}{2}}^{n+1} - P_z|_{i,j+\frac{1}{2}}^n \right) + e^{-b_k} \Psi_{z,k}|_{i,j+\frac{1}{2}}^n. \quad (3.89.b)$$

It is worth noting again that the update equation for H_ϕ is the same with that in equation (3.9) everywhere in the solution domain. In the air part of the lightning problem, the update equations in (3.7) and (3.8) can also be used for E_r and E_z , respectively. On the other hand, the update equations in (3.86) and (3.87) should be used along with the equations in (3.88) and (3.89) in the soil part.

3.2.3.3. Singularity Treatment

Following the same procedure in section 3.2.2.3, the update equation of E_z given in equation (3.87) at the singularity point $r = 0$ can simply be written as in the following:

$$\begin{aligned}
E_z|_{0,j+\frac{1}{2}}^{n+1} &= \frac{1}{\varepsilon_\infty + B\Delta\varepsilon + \frac{\sigma_0\Delta t}{\varepsilon_0}} \times \\
&\left[\varepsilon_\infty E_z|_{0,j+\frac{1}{2}}^n + \left\{ 1 - AB \sum_{k=1}^N a_k \right\} \frac{P_z|_{0,j+\frac{1}{2}}^n}{\varepsilon_0} + \frac{AB}{\varepsilon_0} \sum_{k=1}^N e^{-b_k} \Psi_{z,k}|_{0,j+\frac{1}{2}}^n \right] \\
&+ \frac{4\Delta t}{\varepsilon_\infty + B\Delta\varepsilon + \frac{\sigma_0\Delta t}{\varepsilon_0}} \left[\frac{H_\varphi|_{\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}}}{\varepsilon_0\Delta r} - \frac{I_s}{\varepsilon_0\pi(\Delta r)^2} \right] \quad (3.90)
\end{aligned}$$

where the return stroke current source I_s is zero in the air region.

3.2.3.4. Cole-Cole Parameters for Fractional Soil Models

The fractional soil models are reviewed in section 2.3. The powers of the frequency terms in fractional soil models are all fractional, which is also the case for the Cole-Cole model. If we fit the parameters of the soil models to the parameters of Cole-Cole model, then FDTD solutions for all these soil models can easily be obtained.

Here, the formulations of these soil models are fitted to the electric susceptibility formula that represents the Cole-Cole model given in equation (3.58). The Cole-Cole model is a complex function with real input parameters, namely the static dielectric constant ε_s , the infinite frequency dielectric constant ε_∞ , the angular frequency ω , the relaxation time τ , and the constant α that is between 0 and 1. The frequency is already defined in the system up to a few MHz depending on the soil model so ω is not required to be fitted. Also, we can define the difference of static and infinite frequency dielectric constants as a single term and write:

$$\Delta\varepsilon = \varepsilon_s - \varepsilon_\infty. \quad (3.91)$$

Then, the three terms, $\Delta\varepsilon$, τ , and α are sufficient to be fitted to the soil models.

Based on the original paper by [Cole and Cole, 1941], real and imaginary parts of the Cole-Cole model can be written as follows:

$$\chi_{cc}(\omega) = \frac{\Delta\varepsilon}{1 + (j\omega\tau)^\alpha} = \text{Re}\{\chi_{cc}(\omega)\} - j\text{Im}\{\chi_{cc}(\omega)\} \quad (3.92.a)$$

$$\text{Re}\{\chi_{cc}(\omega)\} = \frac{\Delta\varepsilon \left[1 + (\omega\tau)^\alpha \cos\left(\frac{\pi}{2}\alpha\right) \right]}{1 + 2(\omega\tau)^\alpha \cos\left(\frac{\pi}{2}\alpha\right) + (\omega\tau)^{2\alpha}} \quad (3.92.b)$$

$$\text{Im}\{\chi_{cc}(\omega)\} = \frac{\Delta\varepsilon(\omega\tau)^\alpha \sin\left(\frac{\pi}{2}\alpha\right)}{1 + 2(\omega\tau)^\alpha \cos\left(\frac{\pi}{2}\alpha\right) + (\omega\tau)^{2\alpha}}. \quad (3.92.c)$$

Since the formulas in equation (3.92) are complicated, following an analytical fitting procedure similar to that carried out in section 3.2.2.1 is not a simple task. For this reason, fitting has been realized numerically by using the built-in function “*lsqcurvefit*” in MATLAB [Mathworks, 2023]. First, the fractional soil models reviewed in section 2.3 have been rearranged using the formula in equation (3.18.b) before the fitting procedure. Secondly, the obtained expression has been separated into real and imaginary parts, and then real input parameters of the Cole-Cole model have been obtained after imaginary parts of the input parameters had been set to zero. Since the formulas of soil models depend on the σ_0 , three different σ_0 values have been considered in model fitting: 0.1 mS/m, 1 mS/m, and 10 mS/m, as in section 2.3.9.

On the other hand, the update equations in both section 3.2.2 and section 3.2.3 are constituted based on the expression in equation (3.24.b) where constant terms like ε_∞ and σ_0 exist in addition to the frequency dependent susceptibility term. Nevertheless, such summand terms do not exist in all fractional soil models such as the Scott and Visacro-Portela models where all terms are in the multiplier form. For this reason, these models can be considered to be incompatible for the application of the FDTD method at a first glance; however, a fitting procedure may still be applied to compare the obtained performance with those of the other models after assuming that the summand terms are zero.

The effective relative complex permittivity can be generalized for all fractional soil models and written as follows [Cavka et al., 2014]:

$$\varepsilon_{eff}(\omega) = \varepsilon_r(\omega) - j \frac{\sigma(\omega)}{\omega\varepsilon_0} = \varepsilon'_\infty - j \frac{\sigma'_0}{\omega\varepsilon_0} + \chi(\omega). \quad (3.93)$$

The constant terms, ε'_{∞} and σ'_0 in equation (3.93), used in each model are summarized in Table 3.1. It should be noted that $\chi(\omega)$ is dependent on σ_0 . This is why different σ_0 values are used in the fitting of $\chi(\omega)$. However, σ'_0 is different from σ_0 and equals zero for the models of Scott and Visacro-Portela.

Table 3.1: Constant terms used in the soil models.

Model Term	Scott	Messier	Visacro- Portela	Portela	Visacro- Alipio	Alipio- Visacro	CIGRE
ε'_{∞}	0	8	0	0	1.3	12	12
σ'_0	0	σ_0	0	σ_0	σ_0	σ_0	σ_0

The susceptibility part $\chi(\omega)$ is fitted using the Cole-Cole expression for each soil model in the next sections. The errors between the soil model and the corresponding fitted model are calculated according to the following formula:

$$Error(dB) = 20 \log_{10} \frac{|V_{fitted} - V_{model}|}{\max(|V_{model}|)}. \quad (3.94)$$

where V_{model} is the data set of the values of the soil model and V_{fitted} is the data set of the values of the fitted Cole-Cole equivalent obtained at the frequency range.

Obtained Cole-Cole parameters; $\Delta\varepsilon$, τ , and α for all models are given in Table 3.2 for each σ_0 value.

In Figure 3.5 through Figure 3.7, the susceptibility $\chi(\omega)$ of the models are shown for different σ_0 values. Both real and imaginary parts of $\chi(\omega)$ are shown in the upper part of each figure. In the bottom of each figure, corresponding errors between the soil models and the fitted models that are calculated using equation (3.94) are shown.

As can be seen in Figure 3.5 showing the fitting performance for σ_0 of 0.1 mS/m, the fittings for the models of Messier, Portela, Alipio-Visacro and CIGRE seem to be fairly good for both real and imaginary parts. There exist almost perfect matches for these models since their error values are below -40 dB within the defined frequency range. The fittings for Scott and Visacro-Portela models are good for the imaginary part whereas their performances are worse for the real part. The worst fitting performance has been obtained for the Visacro-Alipio model especially at low

frequencies if both parts are considered. In fact, obtaining good performances for both parts is a problem also confronted by other researchers [Kuklin, 2016].

Table 3.2: Obtained Cole-Cole parameters for the models.

Model	σ_0 (mS/m)	$\Delta\epsilon$	τ (s)	α
Scott	0.1	8792860.3930	1.1055	0.9466
	1	8792860.3930	1.6382	0.9792
	10	19.3881	0.9192	0.1139
Messier	0.1	100104.3711	27.1732	0.5006
	1	238928.0695	15.3897	0.5008
	10	1961118.8974	105.5255	0.5002
Visacro-Portela	0.1	283161531.3745	46.0983	0.9396
	1	84772550.2507	0.9074	0.9661
	10	19.3316	0.8899	0.1141
Portela	0.1	119630.8502	229.9104	0.2954
	1	119629.7565	229.9031	0.2954
	10	119607.4567	229.7533	0.2954
Visacro-Alipio	0.1	4265.3447	0.4501	0.3617
	1	4265.3447	0.0147	0.4103
	10	5352.9890	0.0057	0.4504
Alipio-Visacro	0.1	121428.6944	35.6519	0.4606
	1	121428.6944	7.9347	0.4612
	10	386145.8002	25.8169	0.4607
CIGRE	0.1	134614.6990	38.5409	0.4611
	1	134614.6990	9.8128	0.4616
	10	394307.3830	26.6136	0.4612

The case of 1 mS/m LFC gives a similar result as seen in Figure 3.6. Very similar comments can be made about the fitting performances for the LFC of 1 mS/m whereas it can easily be seen that fitting for the Visacro-Alipio model greatly improves compared to the case of 0.1 mS/m. Fitting performances of the models for 10 mS/m are illustrated in Figure 3.7. Awesome results are again obtained for the Messier, Alipio-Visacro and CIGRE models. The model of Portela is also fairly good. The fitting results for the model of Visacro-Alipio is not good, but it is acceptable with high errors. However, the Scott and Visacro-Portela models give awful fitting results since the values of the corresponding fitted models are at least hundred times lower than those of the soil models at low frequencies. Thus, it is meaningless to run simulations using the Scott and Visacro-Portela models for 10 mS/m.

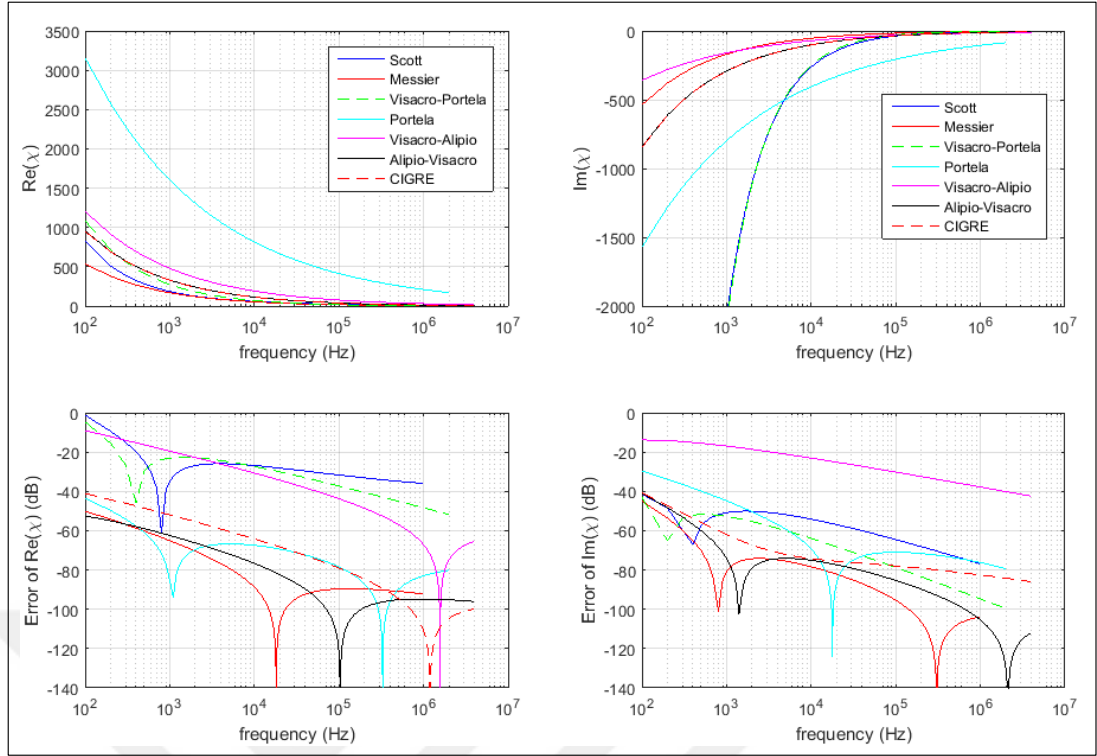


Figure 3.5: Comparison of real parts (top left) and imaginary parts (top right) of the models and the corresponding fitting errors of real parts (bottom left) and imaginary parts (bottom right) for σ_0 of 0.1 mS/m.

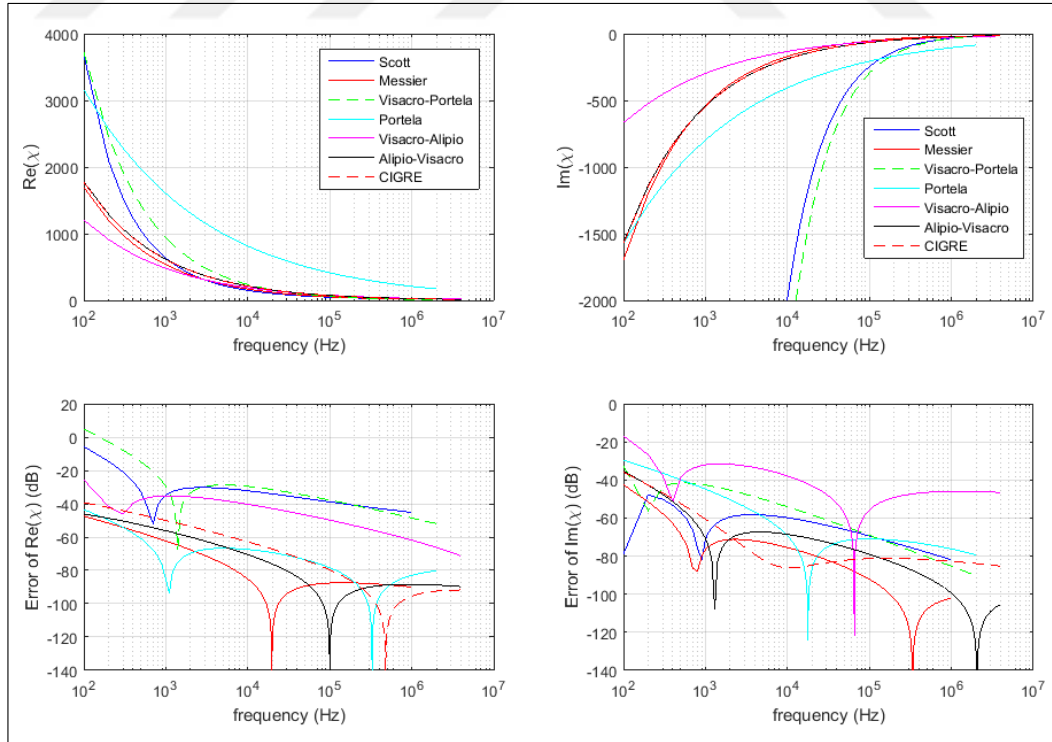


Figure 3.6: Comparison of real parts (top left) and imaginary parts (top right) of the models and the corresponding fitting errors of real parts (bottom left) and imaginary parts (bottom right) for σ_0 of 1 mS/m.

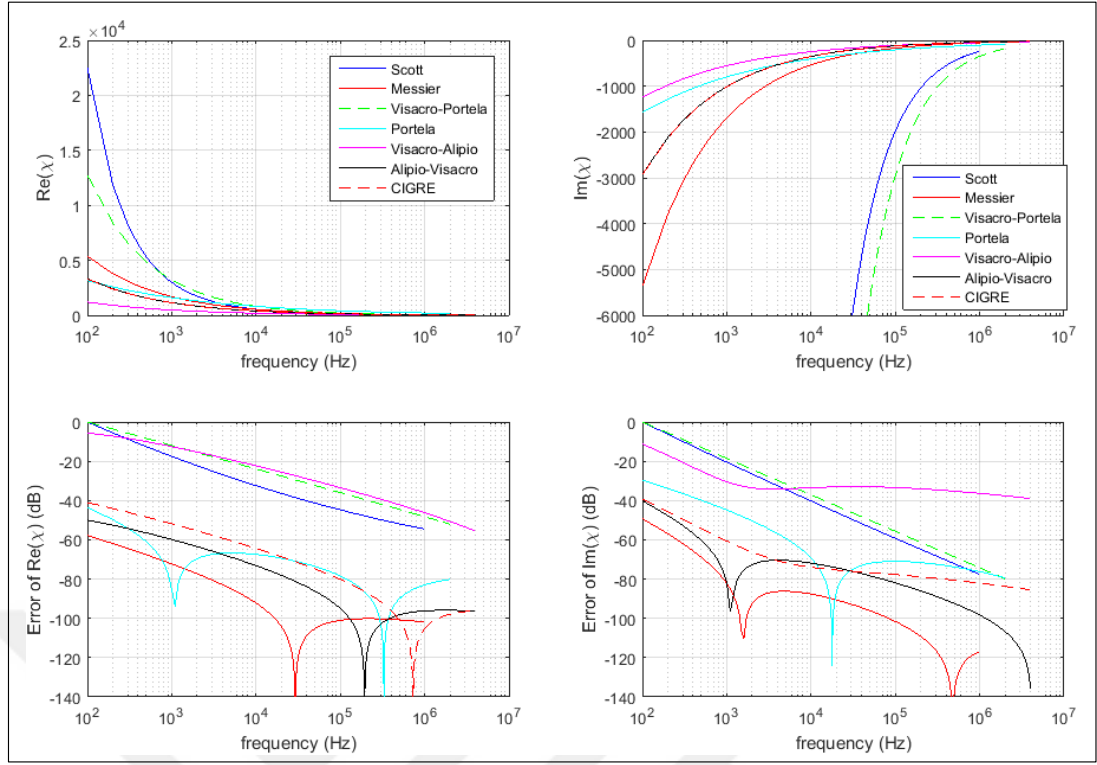


Figure 3.7: Comparison of real parts (top left) and imaginary parts (top right) of the models and the corresponding fitting errors of real parts (bottom left) and imaginary parts (bottom right) for σ_0 of 10 mS/m.

3.2.3.5. Parameters of Exponential Sums

According to equation (3.71), the decreasing function of m can be written as the sum of N decaying exponentials that is written as:

$$f(m) = [(m+1)^{1-\alpha} - m^{1-\alpha}] = \sum_{k=1}^N a_k e^{-b_k m}. \quad (3.95)$$

As the maximum m value increases, the difficulty of finding a_k and b_k coefficients of $f(m)$ increases and the fitting performance decreases. On the other hand, the decaying speed of $f(m)$ increases so the importance of large m values decrease as α in the Cole-Cole model increases. The change of $f(m)$ with respect to m for different α values is given in Figure 3.8. As seen in the figure, the function $f(m)$ decays very rapidly for large α values.

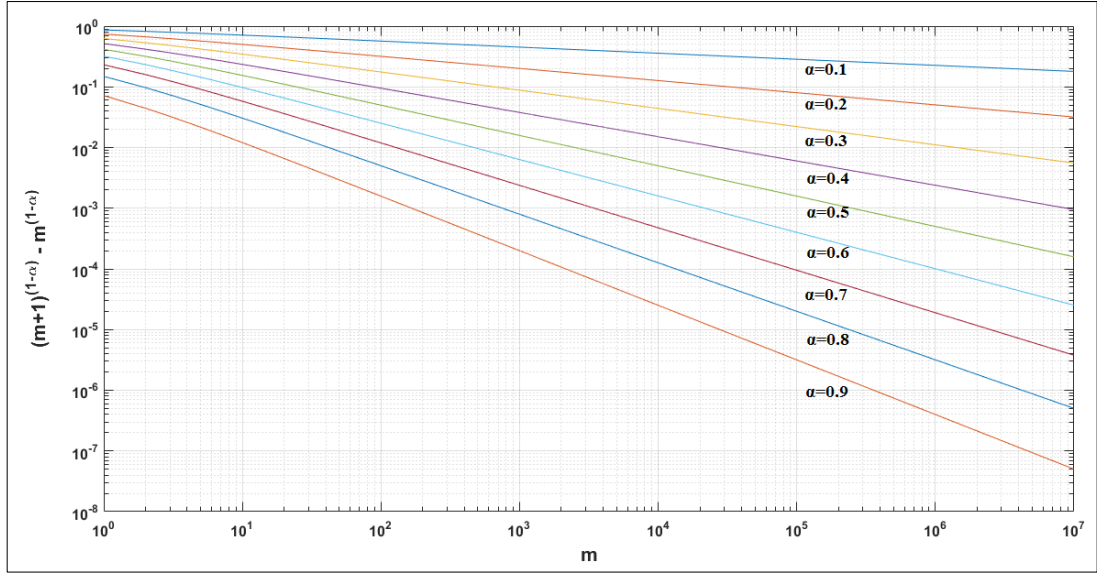


Figure 3.8: $f(m) = [(m + 1)^{1-\alpha} - m^{1-\alpha}]$ with respect to m for different α values. Both axes are shown in logarithmic scale for a better readability.

As the maximum m value increases, it is hard to find the coefficients regarding the computational resources. The fitting performance is also worse for large m values. In fact, the value of m is related to the n^{th} time step in the simulation and the maximum m value is generally in the order of a few hundreds to a few thousands in a typical lightning simulation if the horizontal distance is not very far away from the channel. In our fitting, maximum m values were taken as values between 3000 and 5000 depending on the value of α .

The α values desired as input for the fitting procedure are given in section 3.2.3.4 and summarized in Table 3.3 for different soil models and σ_0 values. The Curve Fitting Toolbox in MATLAB has been used for fitting [Mathworks, 2023]. The tool uses the *method of least squares* and present two different algorithms: *Trust-Region* and *Levenberg-Marquardt*. The choice of the algorithm gives different results so either of them should be selected before the fitting. In this work, both algorithms were tried and the one giving the better result was selected. The maximum value of m , the algorithm used in the fitting process corresponding to each σ_0 and α , and the number of k obtained in the fitting process are given in Table 3.3. No fitting is required for the models of Scott and Visacro-Portela for σ_0 of 10 mS/m because the fitting performances of the corresponding Cole-Cole models are already awful as shown in section 3.2.3.4 so fitting of $f(m)$ for 10 mS/m will obviously give wrong

results. Thus, the Scott and Visacro-Portela models will be fitted for only 0.1 mS/m and 1 mS/m.

Table 3.3: Maximum m value, the algorithm used, and the number of k in the fitting of each model.

Soil Model	σ_0 (mS/m)	α	Maximum m	Algorithm	Number of k
Scott	0.1	0.9466	5000	Levenberg-Marquardt	6
	1	0.9792	5000	Levenberg-Marquardt	5
	10	0.1139	-	-	-
Messier	0.1	0.5006	5000	Levenberg-Marquardt	7
	1	0.5008	5000	Levenberg-Marquardt	11
	10	0.5002	4000	Trust-Region	12
Visacro-Portela	0.1	0.9396	5000	Levenberg-Marquardt	4
	1	0.9661	4000	Levenberg-Marquardt	6
	10	0.1141	-	-	-
Portela	0.1	0.2954	5000	Trust-Region	9
	1				
	10				
Visacro-Alipio	0.1	0.3617	3000	Trust-Region	14
	1	0.4103	3000	Levenberg-Marquardt	15
	10	0.4504	3000	Trust-Region	11
Alipio-Visacro	0.1	0.4606	5000	Levenberg-Marquardt	15
	1	0.4612	4000	Levenberg-Marquardt	6
	10	0.4607	5000	Levenberg-Marquardt	19
CIGRE	0.1	0.4611	4000	Levenberg-Marquardt	6
	1	0.4616	4000	Levenberg-Marquardt	9
	10	0.4612	4000	Levenberg-Marquardt	6

The fitting performances of each model are shown in Figure 3.9 through Figure 3.11. The errors belonging to each model for the same LFC are plotted on the same figure. The plots show the errors between the function $f(m)$ and its fitted model that is equal to the sum of exponentials with a_k and b_k coefficients. The errors have been calculated using the formula in equation (3.94). Although maximum value of m has been taken as values between 3000 and 5000 in the fitting process due to the restriction of computational resources, the error calculations have been extended up to $m = 10^6$ to see the errors of the fitted models in a wider range.

Figure 3.9 displays the results obtained for σ_0 of 0.1 mS/m. It is observed that errors of all fittings are below -40 dB excluding the region of very large m values. Except for the Portela and Visacro-Alipio models, the errors are very low even in that region where the value of m is far larger than the maximum m value needed in a typical lightning simulation. The fittings of the Scott and Visacro-Portela models seem to be

the best among the model fittings owing to their having large α values because large α results in a rapid decay of the function $f(m)$ as shown in Figure 3.8.

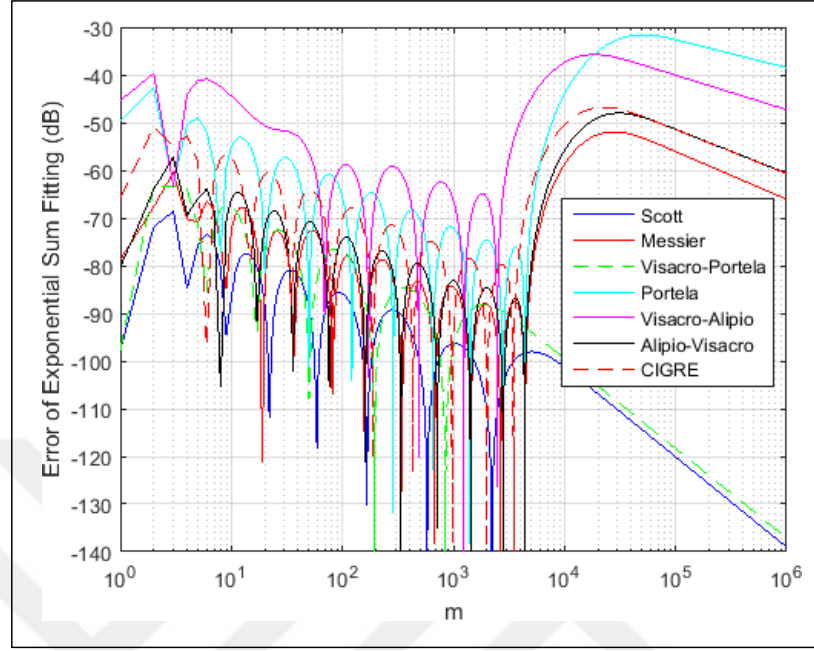


Figure 3.9: Errors of exponential sum fittings for the models for σ_0 of 0.1 mS/m.

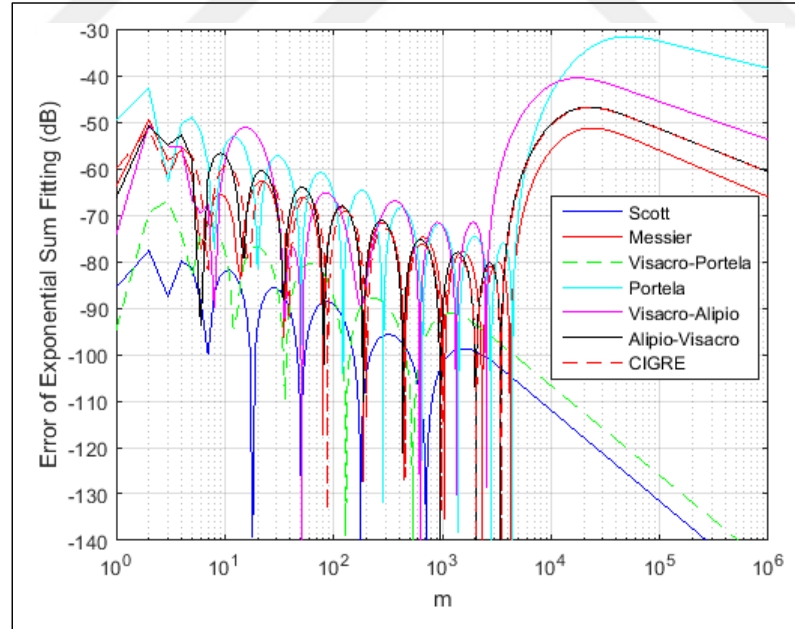


Figure 3.10: Errors of exponential sum fittings for the models for σ_0 of 1 mS/m.

The errors of the fittings for σ_0 of 1 mS/m and 10 mS/m are shown in Figure 3.10 and Figure 3.11, respectively. It is seen that similar results with the case of 0.1 mS/m have been obtained for 1 mS/m and 10 mS/m. Except for the Portela model, the

errors even in the region of large m are lower than -40 dB and they are even lower than -100 dB in some regions. Therefore, the fitting process can be told to be realized with a perfect match by looking at the overall performances of the models. Generally speaking, the five models, namely Scott, Messier, Visacro-Portela, Alipio-Visacro, and CIGRE have good results. Also, it is worth noting that the errors of Scott and Visacro-Portela models are not shown in 10 mS/m results because they are not fitted for that LFC.

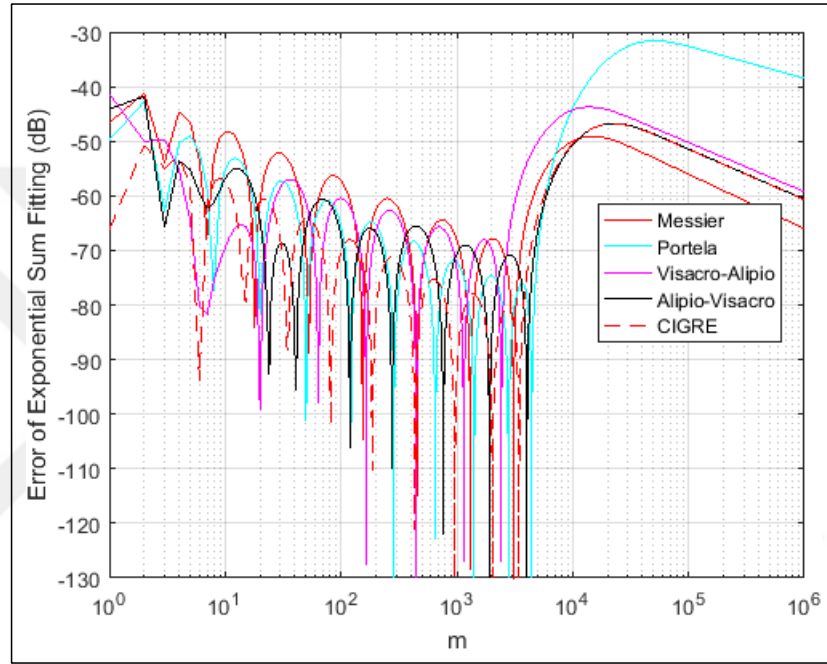


Figure 3.11: Errors of exponential sum fittings for the models for σ_0 of 10 mS/m.

3.3. Absorbing Boundary Condition (ABC)

In the FDTD method, the boundaries of the simulation domain should be truncated properly so that unwanted reflections from the boundaries are absorbed to assume the infinite space conditions in the simulation. Absorbing Boundary Condition (ABC) is the name given to the condition applied to the fields at the boundary. Although there are several ABC types, they can be divided into two main groups: differential type ABCs and material type ABCs. The first is applied at only the edges of the boundary whereas the latter is applied on a layer having a thickness of several grid cells. The most used ABCs in the lightning society are Mur type ABCs [Mur,

1981], 2nd order Liao ABC [Liao et al., 1984], and Convolution Perfectly Matching Layers (PML) [Roden and Gedney, 2000].

Formulations of 1st order and 2nd order Mur ABCs in 2D cylindrical coordinates can be found in [Mimouni et al., 2007] and [Xu et al., 1999], respectively. 2nd order Liao formulations are given in [Baba and Rakov, 2016] for cartesian coordinates. The same formulations at the top and bottom edges of the solution domain can be used in 2D cylindrical coordinates. At the right edge of the domain, the implementation of [Yu et al., 2007] can be applied. Finally, the CPML formulation is given in [Sun et al., 2018] in 2D cylindrical coordinates. In addition to these ABCs, an ABC-free solution is proposed in this thesis for the solution of the LEMP problem since the result is the same up to the defined end time of the simulation.

3.3.1. Solution without ABC

Among the ABCs mentioned, Mur type ABCs and Liao type ABC require the phase velocity in the medium. In literature, this desired phase velocity is generally calculated assuming that the ground is a good dielectric [Mimouni et al., 2007], or assuming a single source frequency for phase velocity calculation [Yu and Simpson, 2011]. Nonetheless, both approaches possess low accuracy due to the frequency-dependent nature of dispersive soils and structure of the lightning stroke having no single frequency. Also, their reflection coefficients are functions of incident angle so that they are successful only under certain conditions [Inan and Marshall, 2011, ch. 8].

The CPML is accepted as appropriate for dispersive mediums since it can be implemented on all medium types and was shown to introduce low error at the boundaries [Roden and Gedney, 2000]. For this reason, it has been implemented in lightning problems in recent years. On the other hand, the CPML is applied on a layer with several cells and pre-simulations on test grid are needed to define the parameters desired in the PML region. In the case of lightning problem, the lightning channel is a few kilometers long so the test grid should be at least a few kilometers high, which elongates the computation time of the testing process. Besides, its formulation is more complicated than that of Mur and Liao so implementation of the CPML increases the volume of the written code. Moreover, it may greatly increase the simulation time if the fields are computed at a distant point using a computer with restricted resources.

Even for a small problem space, a time increase of 25% due to the CPML utilization is reported [Carvalho and Silveira, 2022].

In fact, although using an ABC may be mandatory for other FDTD applications depending on the problem, there is no need to use ABC for the solution of lightning problem if the dimensions of the solution domain are properly selected. As an example, if the fields at a point 25 km away from the channel are desired to be computed, then it is enough to choose the horizontal dimension of the solution domain as around 30 km for a typical lightning problem unless an ABC is needed. Since the minimum horizontal dimension should be greater than 25 km, putting an ABC, for instance at 28 km may not have much meaning because it does not provide a significant gain in simulation time. In addition, ABCs even the CPML cannot absorb the waves totally and introduce error up to some extent. Furthermore, it is observed in the literature that the researchers use ABC in their lightning simulations whereas they select the solution domains suitable so that the waves reflecting from the domain edges do not reach the observation point at all, which makes using an ABC in their code unnecessary. Indeed, the researchers put ABC anyway, but they already apply ABC-free solution without telling it. In this thesis, some simple formulas are proposed to calculate the required simulation dimensions without the need to use any ABC.

For the general return stroke model, the lightning channel is biased from its base and the waves propagate upward with a certain speed. On the other hand, lightning strikes flat ground as well as towers and mountain tops. The proposed model is shown in Figure 3.12. Even though only the flat ground case is within the context of this thesis, we put the channel base at a point with certain height above the ground level in order to cover the general case in the formulation.

We can suppose that the first waves coming to observation point after being reflected from right and top walls follow *path1* and *path2*, respectively. h_1 and h_2 are the height of the channel base and height of the observation point from the surface of the ground, respectively. d is the horizontal distance to the observation point from the channel. L_r is the horizontal dimension of the simulation domain, L_z is the vertical dimension above ground and L_g represents the ground thickness.

To make analysis without observing the effects of the waves reflected from the walls, we can write the following relations:

$$|path1| \geq R + c_0 T_s \quad (3.96.a)$$

$$|path2| \geq R + c_0 T_s \quad (3.96.b)$$

where c_0 is the speed of the light in free space, R is the direct distance between the channel base and the observation point, T_s is the source duration desired to be observed at the output, $|path1|$ and $|path2|$ are the total lengths of *path1* and *path2*, respectively. The physical meaning of equation (3.96) is that the waves following the direct way R arrive at the observation point and are observed throughout the T_s time after the waves' arrival. During this time, the waves reflected from the walls should not arrive at the observation point for the realization of ABC-free solution.

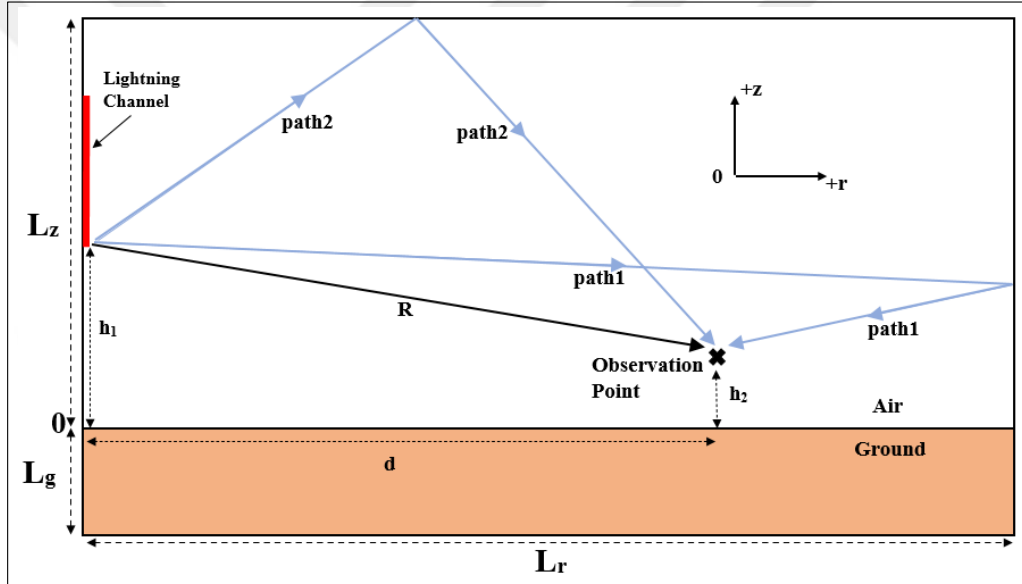


Figure 3.12: The model used to obtain dimensions for ABC-free solution.

We can write that

$$R = \sqrt{d^2 + (h_1 - h_2)^2} \quad (3.97)$$

using Pythagorean theorem, and that

$$|path1| = \sqrt{(2L_r - d)^2 + (h_1 - h_2)^2} \quad (3.98.a)$$

$$|path2| = \sqrt{[2L_z - (h_1 + h_2)]^2 + d^2} \quad (3.98.b)$$

using the *method of images* as in [Rappaport, 1996].

3.3.1.1. Selection of Horizontal Dimension (L_r)

By putting equation (3.97) and equation (3.98.a) into equation (3.96.a), it can be written that

$$\sqrt{(2L_r - d)^2 + (h_1 - h_2)^2} \geq \sqrt{d^2 + (h_1 - h_2)^2} + c_0 T_s. \quad (3.99)$$

If the equation above is rearranged by leaving L_r alone in the left side, we get

$$L_r \geq \frac{d}{2} + \frac{1}{2} \sqrt{d^2 + (c_0 T_s)^2 + 2c_0 d T_s \sqrt{1 + \frac{(h_1 - h_2)^2}{d^2}}} \quad (3.100)$$

where the Taylor expansion approximation

$$\sqrt{1 + \frac{(h_1 - h_2)^2}{d^2}} \approx 1 + \frac{(h_1 - h_2)^2}{2d^2} \quad (3.101)$$

can be used when

$$d \gg |h_1 - h_2|. \quad (3.102)$$

Then, using equation (3.101), we get

$$L_r \geq \frac{d}{2} + \frac{1}{2} \sqrt{d^2 + (c_0 T_s)^2 + 2c_0 d T_s \left[1 + \frac{(h_1 - h_2)^2}{2d^2} \right]} \quad (3.103.a)$$

$$L_r \geq \frac{d}{2} + \frac{1}{2} (c_0 T_s + d) \sqrt{1 + \frac{c_0 T_s (h_1 - h_2)^2}{d (c_0 T_s + d)^2}}. \quad (3.103.b)$$

If equation (3.102) is satisfied, then the equation below

$$d(c_0T_s + d)^2 \gg c_0T_s(h_1 - h_2)^2 \quad (3.104)$$

is already satisfied. To show equation (3.104) is satisfied, we can write it by opening the parenthesis in the left side and dividing both sides by c_0T_s :

$$\left[c_0T_sd + \frac{d^3}{c_0T_s} + d^2 \right] + d^2 \gg (h_1 - h_2)^2 \quad (3.105)$$

Since the term in parenthesis in the left side of equation (3.105) is greater than zero, then equations (3.104) and (3.105) are already satisfied because d^2 is already much greater than $(h_1 - h_2)^2$. So, we can apply Taylor expansion approximation again and get the following:

$$L_r \geq \frac{d}{2} + \frac{1}{2}(c_0T_s + d) \left(1 + \frac{c_0T_s(h_1 - h_2)^2}{2d(c_0T_s + d)^2} \right) \quad (3.106.a)$$

$$L_r \geq \frac{d}{2} + \frac{1}{2}(c_0T_s + d) + \frac{1}{2} \frac{c_0T_s(h_1 - h_2)^2}{2d(c_0T_s + d)} \quad (3.106.b)$$

$$L_r \geq d + \frac{c_0T_s}{2} \left(1 + \frac{(h_1 - h_2)^2}{2d(c_0T_s + d)} \right). \quad (3.106.c)$$

It is easy to see that the formula in equation (3.106.c) reduces to the basic form:

$$L_r \geq d + \frac{c_0T_s}{2} \quad (3.107)$$

if h_1 and h_2 have the same values.

3.3.1.2. Selection of Vertical Dimension (L_z)

According to equation (3.96.b), it can be written that

$$\sqrt{[2L_z - (h_1 + h_2)]^2 + d^2} \geq \sqrt{d^2 + (h_1 - h_2)^2} + c_0T_s. \quad (3.108)$$

Using a similar approach as in section 3.3.1.1, L_z can be written as

$$L_z \geq \frac{h_1 + h_2}{2} + \frac{1}{2} \sqrt{(h_1 - h_2)^2 + (c_0 T_s)^2 + 2c_0 d T_s \sqrt{1 + \frac{(h_1 - h_2)^2}{d^2}}}. \quad (3.109)$$

If $d \gg |h_1 - h_2|$, L_z becomes

$$L_z \geq \frac{h_1 + h_2}{2} + \frac{1}{2} \sqrt{(h_1 - h_2)^2 + (c_0 T_s)^2 + 2c_0 d T_s \left[1 + \frac{(h_1 - h_2)^2}{2d^2} \right]} \quad (3.110.a)$$

$$L_z \geq \frac{h_1 + h_2}{2} + \frac{1}{2} \sqrt{(h_1 - h_2)^2 + c_0 T_s \left(c_0 T_s + 2d + \frac{(h_1 - h_2)^2}{d} \right)}. \quad (3.110.b)$$

Indeed, in most of the lightning problems in the literature, h_1 is zero, and h_2 is zero or has a small value. If both are assumed to be zero, then it simplifies to

$$L_z \geq \frac{1}{2} \sqrt{c_0 T_s (c_0 T_s + 2d)}. \quad (3.111)$$

3.3.1.3. Determination of Simulation Time (T_{max})

If the end time of simulation is defined as T_{max} , then it can be found by dividing equation (3.96) by c_0 . Then, T_{max} can be written as follows:

$$T_{max} \leq T_s + \frac{R}{c_0} \quad (3.112)$$

If equation (3.97) is put into equation (3.112), it can easily be found as

$$T_{max} \leq T_s + \frac{\sqrt{d^2 + (h_1 - h_2)^2}}{c_0}. \quad (3.113)$$

3.3.1.4. Selection of Ground Thickness (L_g)

Analyses of reflections from the perfect conductor walls at the right and top are straightforward, but the ground should be handled in a special manner. Some waves are transmitted through the soil and some others are reflected from the interface between the air and ground. In this case, making an exact analysis of the waves may be hard. However, we can obtain a proper ground thickness for the simulation after making some assumptions. The exact expression of the phase velocity is given by

$$v = \frac{\omega}{\beta} \quad (3.114.a)$$

$$\beta = \omega\sqrt{\mu\epsilon} \sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2}} \quad (3.114.b)$$

where β is the phase constant of the wave in the lossy ground [Balanis, 2012, ch. 4]. The velocity formula can be rearranged as follows:

$$v = \frac{1}{\sqrt{\mu\epsilon_r\epsilon_0}} V(\omega) = \frac{c_0}{\sqrt{\epsilon_r}} V(\omega) \quad (3.115.a)$$

$$V(\omega) = \frac{1}{\sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon_r\epsilon_0}\right)^2}}} \quad (3.115.b)$$

where c_0 is the speed of the wave in free space and ϵ_r is the dielectric constant of the ground. Since $\sigma/(\omega\epsilon_r\epsilon_0)$ in equation (3.115.b) is always greater than zero, $V(\omega)$ is always lower than unity. This is shown graphically in Figure 3.13 for constant ϵ_r and σ using different σ values. ϵ_r of the ground was taken as 10 in the computation. As seen in Figure 3.13, the value of $V(\omega)$ converges to 1 as the frequency increases. Changing the dielectric constant of the ground results in a shift in the horizontal axis but does not affect the convergence of $V(\omega)$ to unity.

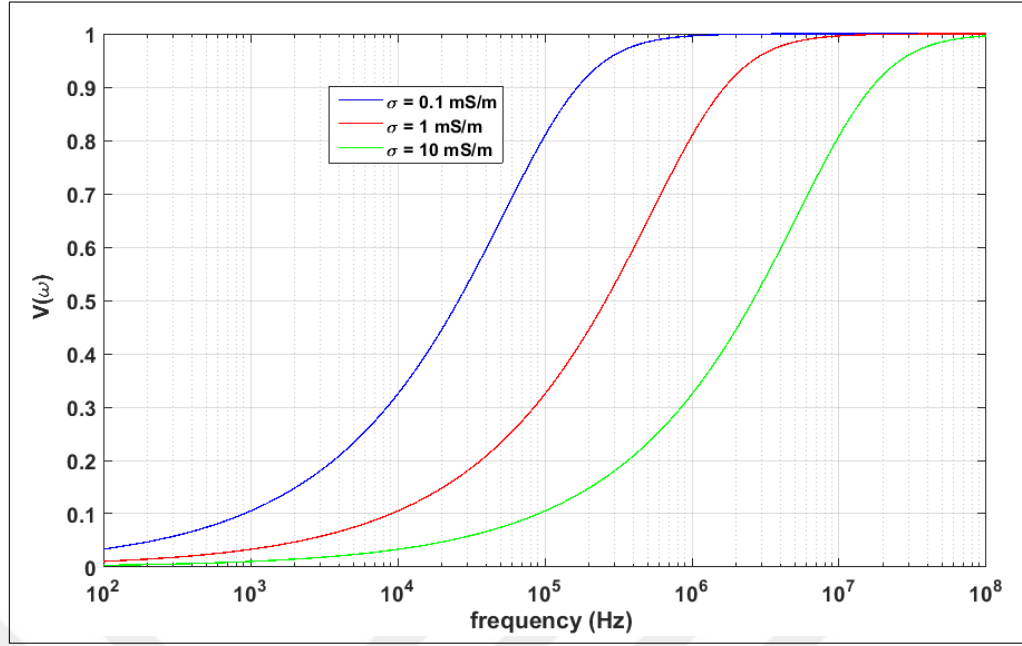


Figure 3.13: The value of $V(\omega)$ in equation (3.115.b) for different σ values.

If the ground is considered as a dispersive soil model so ε_r and σ in equation (3.115.b) are taken as frequency-dependent, the result is shown in Figure 3.14 for the common frequency range of the models. All the dispersive models are included and σ_0 is taken as 1 mS/m in computations. The same result is observed in Figure 3.14; that is, the value of $V(\omega)$ for all dispersive soil models is lower than 1.

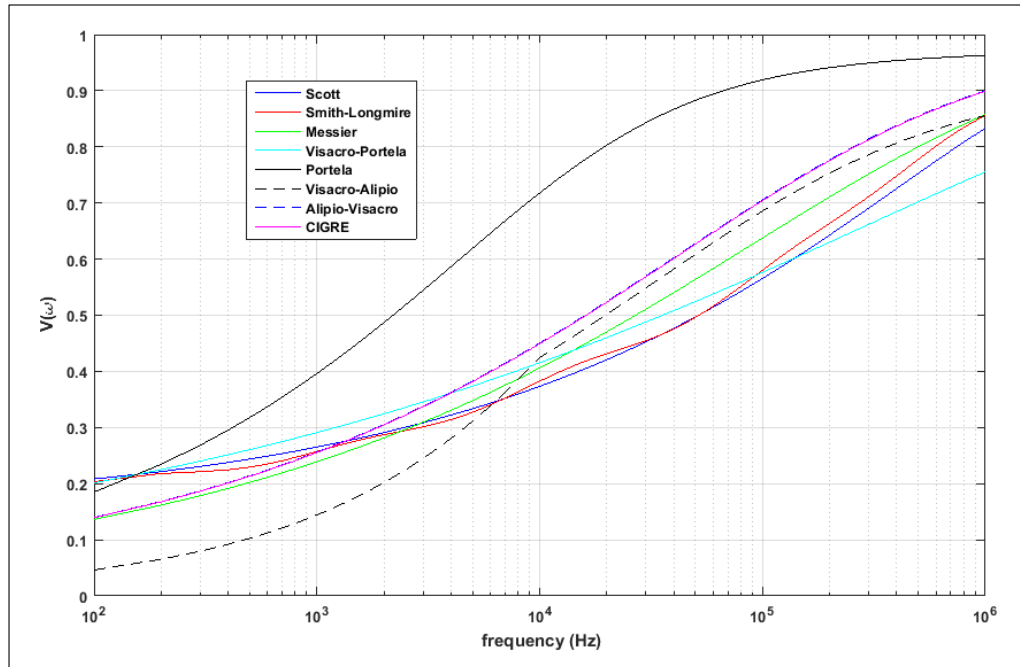


Figure 3.14: The value of $V(\omega)$ for dispersive soil models (σ_0 is 1 mS/m).

Thus, the velocity of the wave in the ground can be written as

$$v \leq \frac{c_0}{\sqrt{\epsilon_r}}. \quad (3.116)$$

If v is taken equal to $c_0/\sqrt{\epsilon_r}$ and the ground thickness value L_g is found accordingly, this may not be the optimum thickness, but it will guarantee the ABC-free solution.

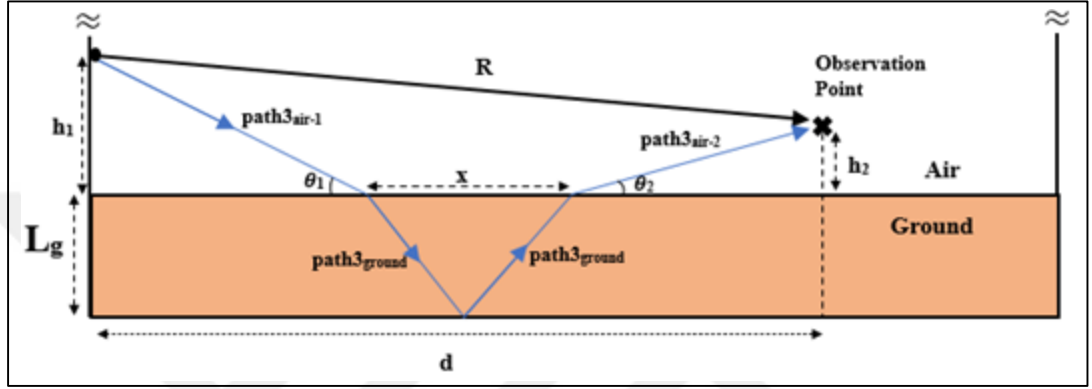


Figure 3.15: Scenario of the lightning waves reflecting from the bottom boundary.

Let us assume a wave path as in Figure 3.15. We want the arrival time of the wave traveling through the $path3$ is greater than that of the wave following R in addition to the source duration T_s . In this case, we can write that

$$\frac{|path3_{air-1}|}{c_0} + \frac{|path3_{ground}|}{v} + \frac{|path3_{air-2}|}{c_0} \geq \frac{R}{c_0} + T_s \quad (3.117.a)$$

$$\frac{|path3_{air-1}|}{c_0} + \frac{|path3_{ground}|}{\frac{c_0}{\sqrt{\epsilon_r}}} + \frac{|path3_{air-2}|}{c_0} \geq \frac{R}{c_0} + T_s \quad (3.117.b)$$

$$\sqrt{\epsilon_r}|path3_{ground}| + \{|path3_{air-1}| + |path3_{air-2}|\} \geq R + c_0 T_s \quad (3.117.c)$$

It can be written that

$$|path3_{ground}| = \sqrt{(2L_g)^2 + x^2} \quad (3.118)$$

using the method of images. Assuming $\theta_1 = \theta_2$ and defining x as the horizontal distance through which the wave travels inside the ground, we can write

$$|path_{3_{air-1}}| + |path_{3_{air-2}}| = \sqrt{(d-x)^2 + (h_1 + h_2)^2} \quad (3.119)$$

using the triangle similarity. Then, equation (3.117.c) can be rewritten as

$$\begin{aligned} \sqrt{\epsilon_r} \sqrt{(2L_g)^2 + x^2} + \sqrt{(d-x)^2 + (h_1 + h_2)^2} \geq \\ \sqrt{d^2 + (h_1 - h_2)^2} + c_0 T_s \end{aligned} \quad (3.120)$$

and we can find the ground thickness value as

$$\begin{aligned} L_g \geq \frac{1}{2\sqrt{\epsilon_r}} \times \\ \sqrt{\left[\sqrt{d^2 + (h_1 - h_2)^2} + c_0 T_s - \sqrt{(d-x)^2 + (h_1 + h_2)^2} \right]^2 - \epsilon_r x^2}. \end{aligned} \quad (3.121)$$

The formula above contains an unknown term x . Even though direct calculation of L_g is not possible due to this unknown term, the value of L_g can be found by sweeping x from 0 to d and taking the maximum of obtained $L_g(x)$ values, which is not a hard task using a computer.

If $h_1 = h_2 = 0$ and $\epsilon_r > 1 + c_0 T_s / d$, equation (3.121) is reduced to a simpler form and we can find global maximum of $L_g(x)$ analytically by equating the derivative of $L_g(x)$ to zero. In this case,

$$L_g(x) \geq \frac{1}{2\sqrt{\epsilon_r}} \sqrt{[d + c_0 T_s - (d-x)]^2 - \epsilon_r x^2} \quad (3.122.a)$$

$$L_g(x) \geq \frac{1}{2\sqrt{\epsilon_r}} \sqrt{x^2(1 - \epsilon_r) + (c_0 T_s)^2 + 2xc_0 T_s}. \quad (3.122.b)$$

We can write the derivative of $L_g(x)$ as

$$\frac{d}{dx}L_g(x) = \frac{1}{2\sqrt{\varepsilon_r}} \frac{2(1 - \varepsilon_r)x + 2c_0T_s}{2\sqrt{x^2(1 - \varepsilon_r) + (c_0T_s)^2 + 2xc_0T_s}} \quad (3.123)$$

After equating the expression above to zero, we get

$$(1 - \varepsilon_r)x + c_0T_s = 0 \quad (3.124.a)$$

$$x = \frac{c_0T_s}{\varepsilon_r - 1}. \quad (3.124.b)$$

By putting the obtained value of x into equation (3.122.b), we can easily reach the desired value as follows:

$$L_g \geq L_g(x)|_{x=\frac{c_0T_s}{\varepsilon_r-1}} \quad (3.125.a)$$

$$L_g \geq \frac{1}{2\sqrt{\varepsilon_r}} \sqrt{\left(\frac{c_0T_s}{\varepsilon_r - 1}\right)^2 (1 - \varepsilon_r) + (c_0T_s)^2 + 2\left(\frac{c_0T_s}{\varepsilon_r - 1}\right)c_0T_s} \quad (3.125.b)$$

$$L_g \geq \frac{1}{2\sqrt{\varepsilon_r}} \sqrt{(c_0T_s)^2 \left(1 - \frac{1}{\varepsilon_r - 1} + \frac{2}{\varepsilon_r - 1}\right)} \quad (3.125.c)$$

$$L_g \geq \frac{c_0T_s}{2\sqrt{\varepsilon_r}} \sqrt{\frac{\varepsilon_r}{\varepsilon_r - 1}} \quad (3.125.d)$$

$$L_g \geq \frac{c_0T_s}{2\sqrt{\varepsilon_r - 1}}. \quad (3.125.e)$$

The L_g value found in this manner may be higher than an optimal ground thickness, but it guarantees the ABC-free solution because it is the maximum of all possible ground thickness values obtained by equation (3.121).

3.3.1.5. Underground Observation Point

Suppose that an underground observation point close to surface with a depth of g and horizontal distance d is of interest as in Figure 3.16, which is the general case

in lightning problems. In this case, the values calculated from equation (3.106.c) and equation (3.110.b) obtained by equating h_2 to zero can be used for L_r and L_z , respectively. These values will automatically satisfy the needed top and right dimensions within the time T_{max} in equation (3.113) because the paths of the reflected waves are longer than those in the case of above-ground and the speed of the waves are slower after being transmitted to the ground. Also, the L_g value obtained by equation (3.121) or equation (3.125.e) will be valid in most of the cases since the depth g is generally low (not more than a few tens of meters) in lightning problems. Nonetheless, a higher L_g value may be needed depending on the observation depth and the horizontal distance. In this case, a higher ground thickness can be selected by supposing that the ground thickness is increased by ΔL_g , as shown in Figure 3.16. Since both *path3* and *path4* are in the same medium, we can assume

$$|path4| - |path3| \geq g. \quad (3.126)$$

Using the method of images, it can be written that

$$\sqrt{(2(L_g + \Delta L_g) - g)^2 + d^2} - \sqrt{(2L_g)^2 + d^2} \geq g \quad (3.127)$$

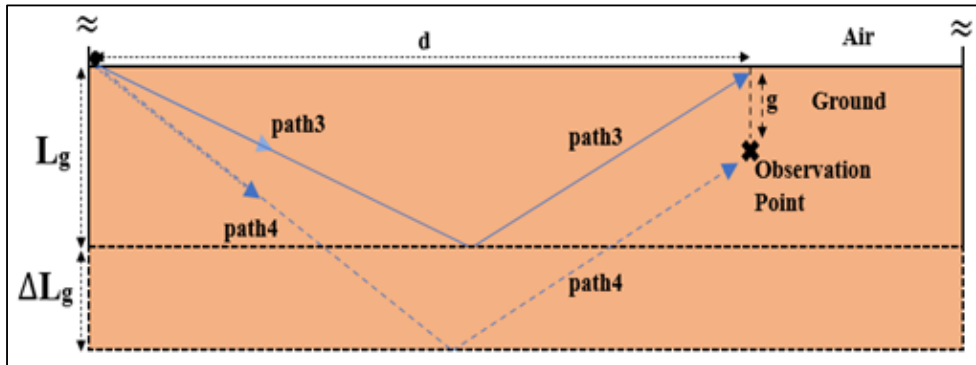


Figure 3.16: Scenario of lightning waves reflecting from the bottom of thicker ground.

After rearranging, we obtain

$$L_g + \Delta L_g \geq \frac{g}{2} + \frac{1}{2} \sqrt{g^2 + (2L_g)^2 + 2g \sqrt{(2L_g)^2 + d^2}}. \quad (3.128)$$

Also, the end time of simulation can be expanded by the following approximation:

$$T_{max,g} \cong T_{max} + \frac{g}{(c_0/\sqrt{\epsilon_r})} \quad (3.129)$$

where the last term represents the time travelled by the wave in the ground.

3.3.2. Performance of ABC-free solution

A sample simulation has been run to show the performance of the proposed ABC-free solution along with the other ABCs. Overground LEMP at the observation point of 5 km have been computed. The spatial grid of 5 m $\times$ 5 m is used. The same channel base current with the first stroke defined in Table 2.2 (section 2.2.4) is adopted. Smith-Longmire model is used as the dispersive soil model. Taking d as 5 km, h as zero and T_s as 50 μ s, we calculated L_r as 12.5 km, L_z as 9682.5 m (which is rounded to 9.685 km for the purpose of proper gridding), T_{max} as 66.7 μ s, and L_g as 3.75 km using equation (3.107), equation (3.111), equation (3.113), and equation (3.125.e), respectively.

For the ABCs, the research given at the beginning of section 3.3 have been used. For the Mur and Liao type ABCs, the phase velocity in the soil has been calculated considering the source center frequency, as suggested in [Yu and Simpson, 2011]. The phase velocity is calculated as in [Balanis, 2012, ch. 4].

The performances of the ABCs together with the proposed ABC-free solution are shown in Figure 3.17 where all the three field components are illustrated. The “reference” plot seen in the figure represents the computation for a solution domain whose dimensions are large enough so that the reflected waves never come to the observation point within the simulation time.

As can be seen in Figure 3.17, the results are the same and the FDTD simulation boundaries do not need any ABC up to the time T_{max} if the dimensions proposed are used. After this time, the reflected waves arrive at the observation point and the result deviates from the reference solution, which is also true for cases of Mur, Liao and the CPML with smaller error margins. Therefore, the computation of LEMP can be realized without using ABC if the proposed dimensions and the simulation time are adopted.

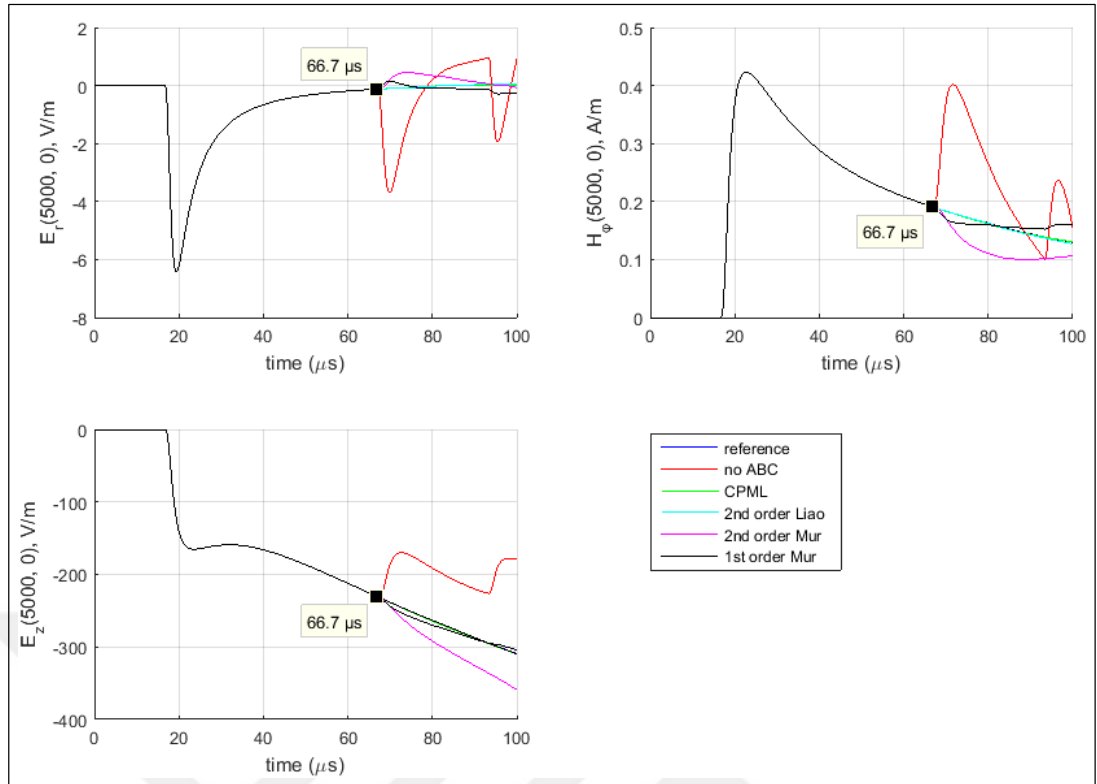


Figure 3.17: Comparison of the electromagnetic fields with ABCs and the proposed ABC-free solution.

4. RESULTS

In this section, the obtained results are presented. First, development of graphical user interface and the used parameters in the simulation are explained shortly. Then, the numerical results obtained in the simulations are shown for different observation point distances, heights, stroke types and ground conductivities. Discussions on the results are also presented.

4.1. Development of Graphical User Interface

A Graphical User Interface (GUI) has been developed to obtain the simulation results faster since running the codes without a GUI for dozens of configurations such as different distances, heights, stroke types, and soil models occupies more time and sometimes may lead to mistakes. Also, a GUI has the potential to facilitate making applications in the future.

First, the update equations derived in section 3 along with the return stroke parameters have been coded separately for each dispersive soil model reviewed in section 2. Then, a GUI has been developed by combining all the codes belonging to the soil models. The developed GUI was coded as simple as possible so that it can be used by a user who does not know the technical details about the soil models. The general view of the GUI is shown in Figure 4.1.

The input parameters required for the simulation are requested at the top of GUI panel. After the input parameters are entered, the simulation can be run, and the obtained results can be combined and saved by the control buttons in the middle part. The obtained results belonging to three field components; E_r , E_z , and H_ϕ are located at the bottom part of the panel and they are plotted with respect to time. Finally, the used return stroke currents at the desired altitude can be viewed at top right part of the panel.

The input parameters part of the GUI can be seen in Figure 4.2. The input parameters asked in the GUI are separated into four main categories: the return stroke current parameters, gridding and the location of observation point, electrical parameters of the ground, and the dimensions of the problem space. If any of the parameters in this part are not filled or unsuitable entrance is made, the user is warned

before running the simulation. Also, multiple input entrance is possible in some fields like horizontal distance and vertical height of the observation point.

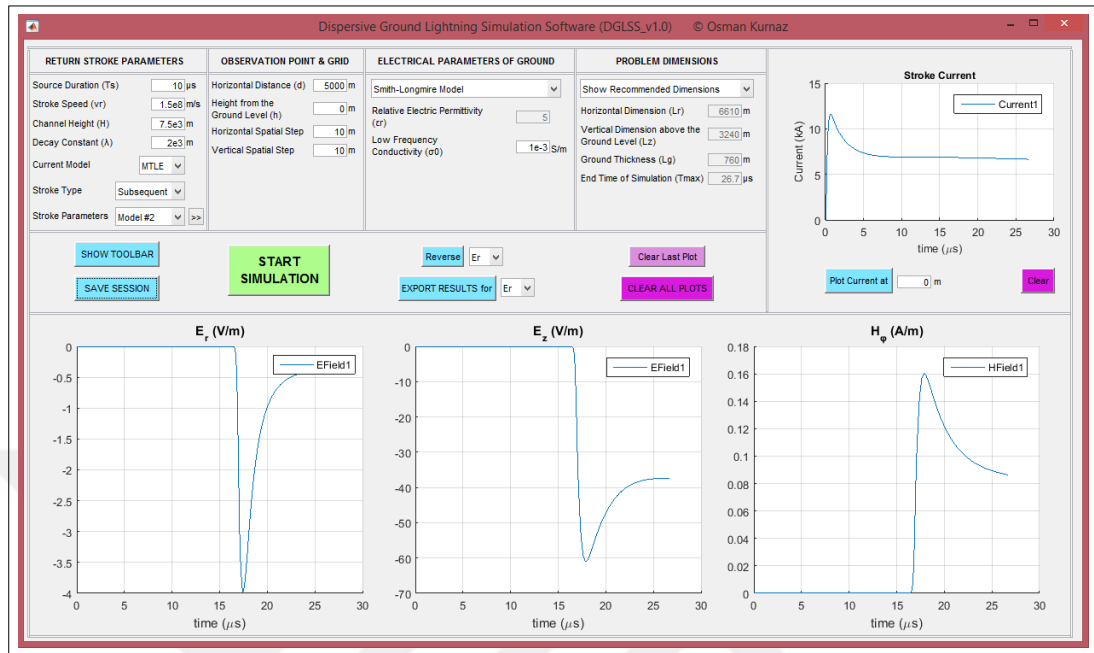


Figure 4.1: General view of the developed GUI for a sample simulation.

RETURN STROKE PARAMETERS	OBSERVATION POINT & GRID	ELECTRICAL PARAMETERS OF GROUND	PROBLEM DIMENSIONS
Source Duration (Ts) 30 μs	Horizontal Distance (d) 500 m	Alipio-Visacro Model	Enter Manually
Stroke Speed (vr) 1.5e8 m/s	Height from the Ground Level (h) 0 m	Relative Electric Permittivity (εr) 12	Horizontal Dimension (Lr) 5055 m
Channel Height (H) 7.5e3 m	Horizontal Spatial Step 5 m	Low Frequency Conductivity (σ0) 0.001 S/m	Vertical Dimension above the Ground Level (Lz) 4800 m
Decay Constant (λ) 2e3 m	Vertical Spatial Step 5 m		Ground Thickness (Lg) 1360 m
Current Model MTLE			End Time of Simulation (Tmax) 31.7 μs
Stroke Type First			
Stroke Parameters Model #2			

Figure 4.2: Input parameters section of the GUI.

4.2. Simulation Parameters

4.2.1. Problem Domain Parameters

The problem space is selected as 32.5 km × 19.5 km in horizontal and vertical dimensions, respectively. The ground thickness is set to 3.75 km. Both Δr and Δz are chosen as 5 m so that the simulation covers the defined frequency range of all dispersive soil models. The unit FDTD time step Δt is selected as compatible with the

Courant stability condition. The simulations were repeated for the LFC (σ_0) of 0.1 mS/m, 1 mS/m, and 10 mS/m.

4.2.2. Return Stroke Parameters

Both first and subsequent return strokes are considered in the simulation. The channel height H and the decay constant λ are set to 7.5 km and 2 km, respectively. The propagation speed of the return stroke v_{rs} is set to 1.5×10^8 m/s corresponding to half of the speed of light and assumed constant along the channel. The same channel base current parameters, which are given in Table 2.2, and the same type (MTLE) of the return stroke in [Li, 2020] are adopted. The used return stroke currents at the base of the lightning channel in time domain and their corresponding Fast Fourier Transform (FFT) results in frequency domain are shown in Figure 4.3.

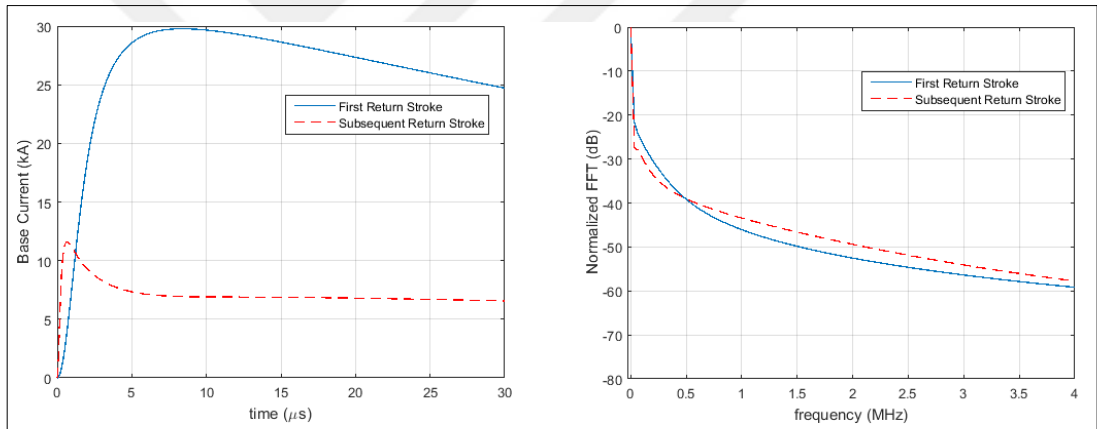


Figure 4.3: Return stroke currents at the channel base (left) and their normalized FFT results in dB (right).

4.3. Numerical Results

Configurations including two return stroke types (first and subsequent), three different σ_0 values (0.1 mS/m, 1 mS/m, 10 mS/m), four observation point distances (50 m, 500 m, 5 km, 25 km), and three vertical observation point levels (10 m under the ground, just over the ground, 10 m above the ground) have been considered in the simulation campaign. All computations for these configurations were repeated for each dispersive soil model. The point 10 m above the ground is considered for the

transmission lines which are often affected by the lightning strikes. The point 10 m below the ground is considered for the grounded cables and grounding electrodes.

In simulations, it was observed that the models of Scott and Visacro-Portela have stability problem and the iterations diverge. For this reason, they are excluded from the results and the remaining models have been plotted together on the same graph. All the field components E_r , E_z and H_ϕ are included in the results.

To compare the models better, the difference between the results of each model and that of the lossy ground with constant parameters are calculated using the following formula:

$$RD = 100 \times \left(\frac{\max |D_i - L_i|}{\max |L_i|} \right), \quad i = 1, 2, \dots, N. \quad (4.1)$$

RD in equation (4.1) is the abbreviation for *relative difference*. N is the number of data points for the results, which is also equal to the number of time steps in the simulation. D_i and L_i are the values of dispersive and lossy ground results at each i^{th} data point, respectively. Thus, RD represents the maximum difference between each dispersive model and the constant lossy model in percent. The results for each configuration and the corresponding RD values are given in the following sections.

4.3.1. Results for LFC (σ_0) of 1 mS/m

4.3.1.1. Overground Results

The RD values obtained for the models over the ground for the first and subsequent return strokes are given in Table 4.1 and Table 4.2, respectively. The graphical illustrations regarding different distances for both first and subsequent strokes are shown in Figure 4.4 through Figure 4.11.

According to the results obtained over the ground, the first remarkable consequence is that using dispersive soil models lead to significant changes only on the E_r and has almost no effect on E_z and H_ϕ especially at close distances and has little effects at far ranges. Secondly, rise times of the results are shorter if dispersive model is used, which is more evident with the usage of subsequent stroke at far range. Regarding the dispersive models, it is apparent that the model of Portela gives more

distinct results compared to others. Its RD value is more than twice the nearest RD value in most configurations. In fact, this is not an unforeseen outcome because its predictions of relative permittivity and conductivity are also distinct from those of other models, which can be seen in section 2.3.9. On the other hand, the models of Smith-Longmire and Messier generally give similar results whereas Visacro-Alipio, Alipio-Visacro and CIGRE give results close to each other. Moreover, the results of Alipio-Visacro and CIGRE are almost the same for all configurations.

Table 4.1: RD values obtained over the ground with σ_0 of 1 mS/m using first stroke.

Model Distance		Smith-Longmire	Messier	Portela	Visacro-Alipio	Alipio-Visacro	CIGRE
50 m	E_r	21.6358	19.6851	51.1373	22.8172	24.0849	24.0672
	E_z	0.0382	0.0387	0.1523	0.0483	0.0506	0.0505
	H_φ	0.1309	0.1396	0.3395	0.1678	0.1724	0.1722
500 m	E_r	17.1135	17.0979	50.5056	20.6038	21.5149	21.4948
	E_z	0.1027	0.1102	0.2825	0.1338	0.1378	0.1376
	H_φ	0.6184	0.6595	1.7450	0.8025	0.8276	0.8267
5 km	E_r	17.1135	17.6272	50.2126	21.0269	21.9595	21.9407
	E_z	2.6316	2.7383	7.4842	3.3064	3.4243	3.4207
	H_φ	3.3626	3.4961	9.6009	4.2228	4.3745	4.3698
25 km	E_r	16.9941	16.4400	45.8918	19.3448	20.2801	20.2651
	E_z	6.6205	6.6233	18.9933	7.9206	8.2645	8.2571
	H_φ	6.6602	6.6618	19.1328	7.9673	8.3138	8.3064

Table 4.2: RD obtained over the ground with σ_0 of 1 mS/m using subsequent stroke.

Model Distance		Smith-Longmire	Messier	Portela	Visacro-Alipio	Alipio-Visacro	CIGRE
50 m	E_r	30.0270	33.1116	78.7276	40.5017	41.5916	41.5502
	E_z	0.2889	0.3265	0.8529	0.4090	0.4209	0.4205
	H_φ	1.8984	2.1152	4.3992	2.5643	2.6435	2.6409
500 m	E_r	28.4843	30.9705	72.8419	37.2779	38.1546	38.1244
	E_z	1.4748	1.6302	3.8473	1.9963	2.0396	2.0373
	H_φ	7.6443	8.4472	20.1512	10.3584	10.6011	10.5890
5 km	E_r	24.5772	26.2530	60.6940	31.2300	32.0078	31.9780
	E_z	15.2949	16.6657	46.8158	20.2726	20.7206	20.7050
	H_φ	15.4582	16.8437	47.4144	20.5005	20.9562	20.9407
25 km	E_r	22.3503	23.2191	54.9935	27.4000	28.2440	28.2169
	E_z	17.4539	18.5718	57.0859	22.6815	23.3817	23.3550
	H_φ	17.5147	18.6373	57.3574	22.7649	23.4682	23.4415

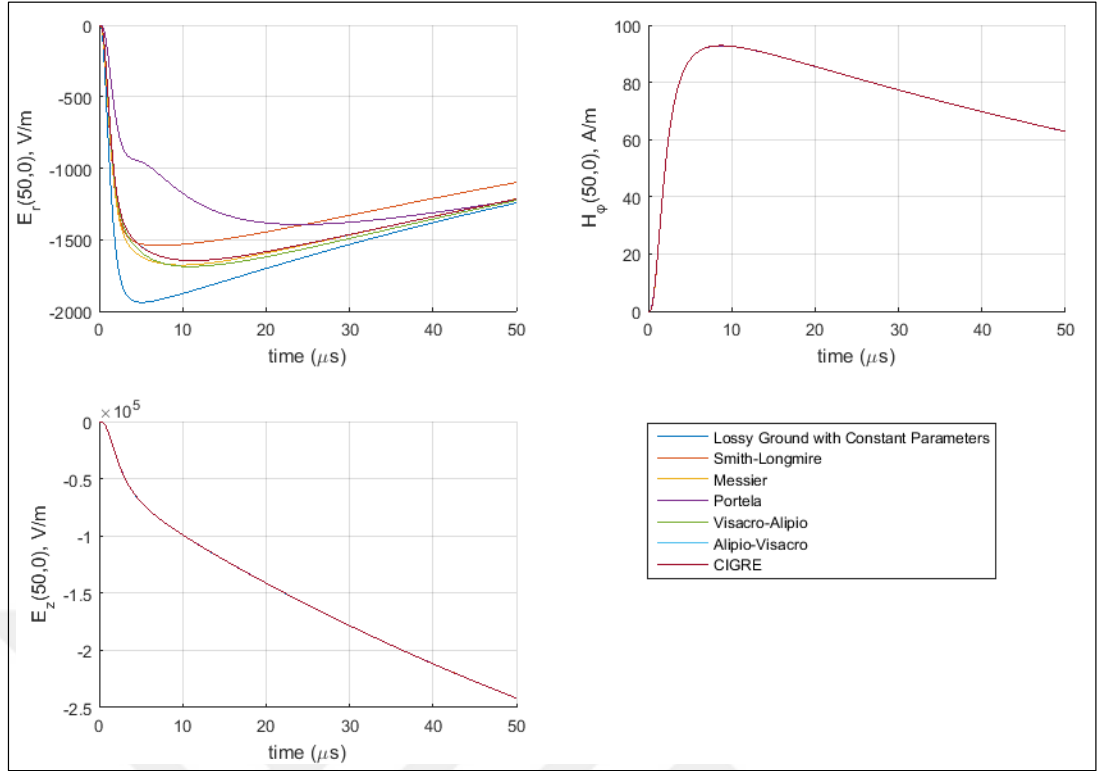


Figure 4.4: E_r (top left), H_ϕ (top right), and E_z (bottom left) observed at 50 m away using first stroke.

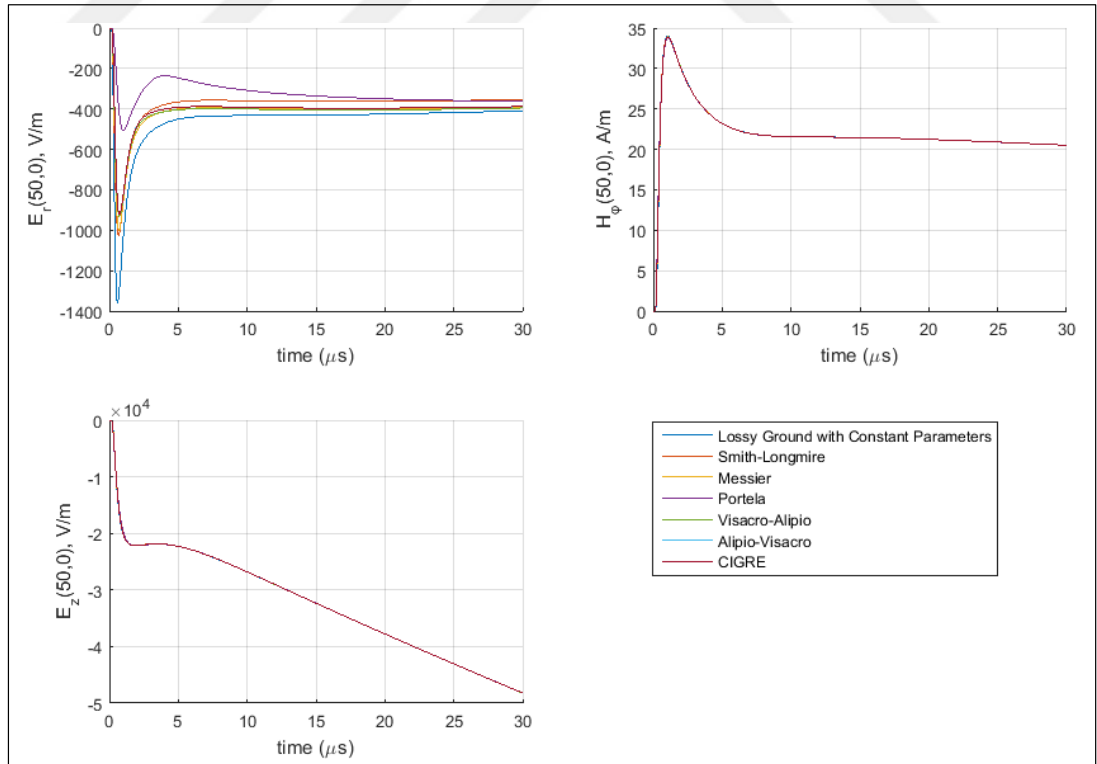


Figure 4.5: E_r (top left), H_ϕ (top right), and E_z (bottom left) observed at 50 m away using subsequent stroke.

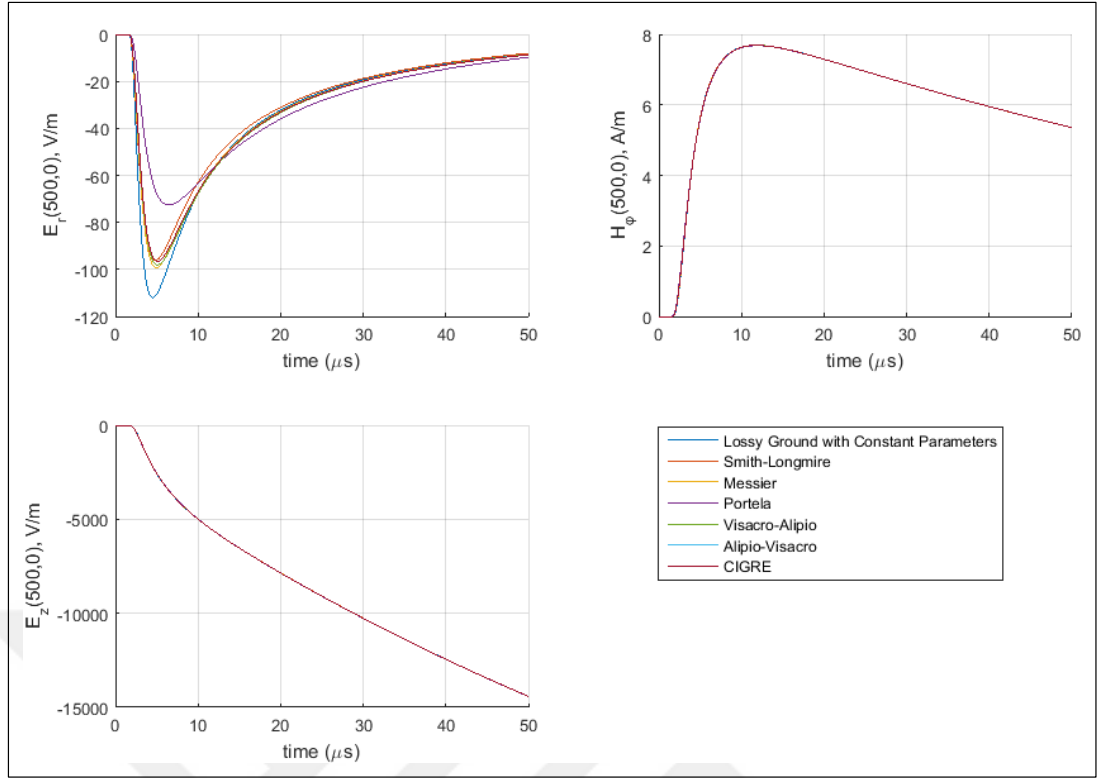


Figure 4.6: E_r (top left), H_ϕ (top right), and E_z (bottom left) observed at 500 m away using first stroke.

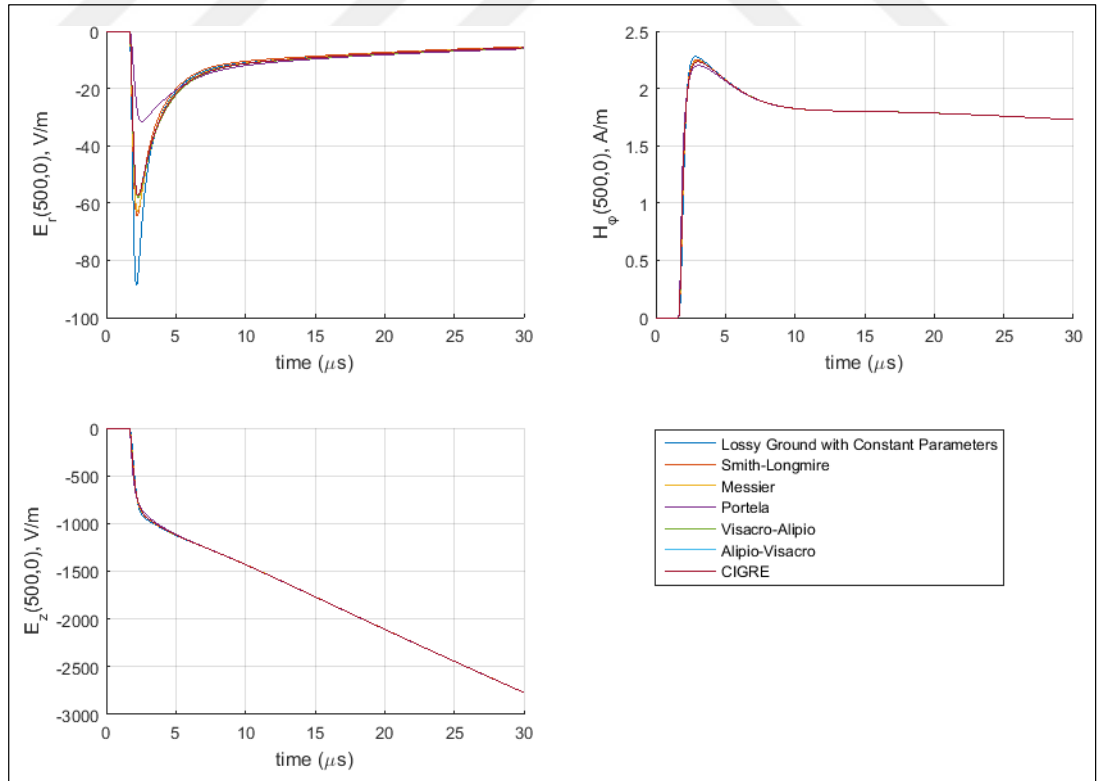


Figure 4.7: E_r (top left), H_ϕ (top right), and E_z (bottom left) observed at 500 m away using subsequent stroke.

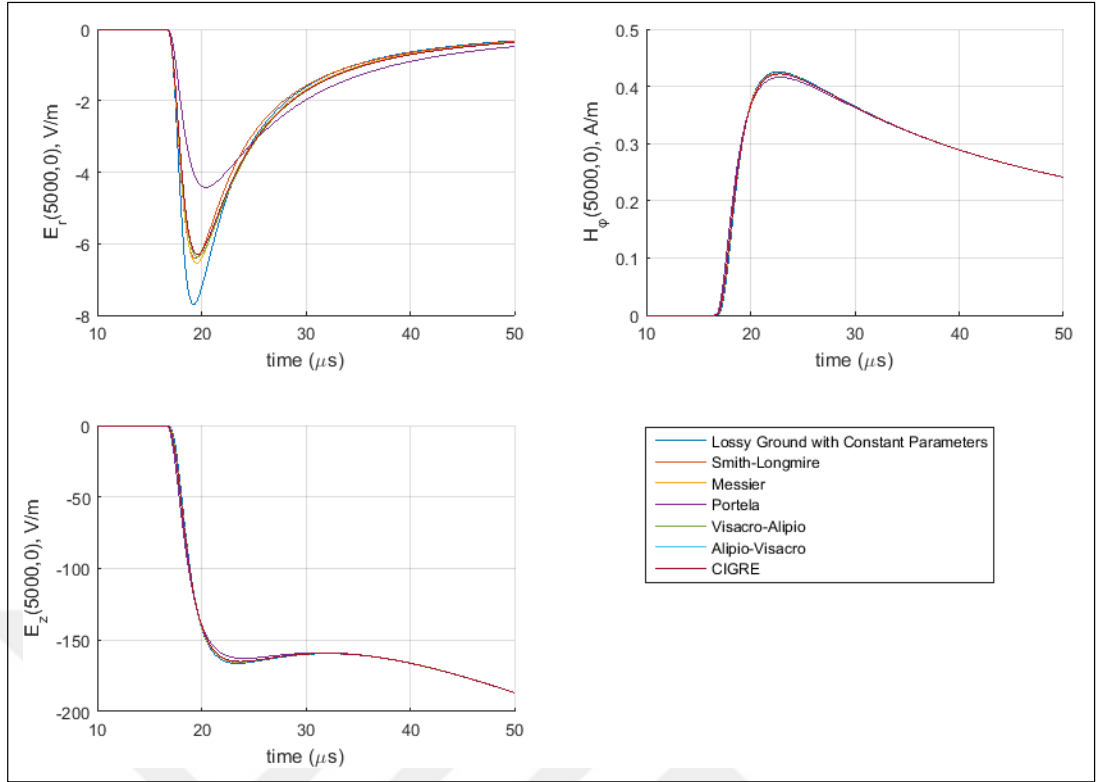


Figure 4.8: E_r (top left), H_ϕ (top right), and E_z (bottom left) observed at 5 km away using first stroke.

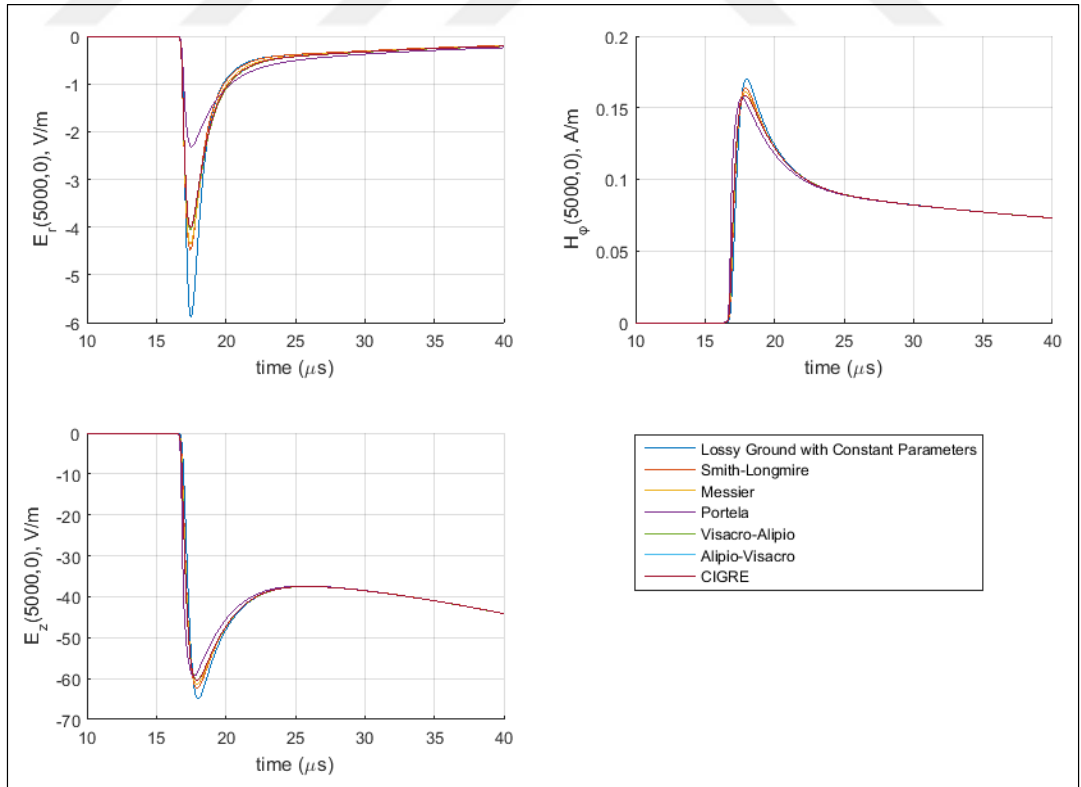


Figure 4.9: E_r (top left), H_ϕ (top right), and E_z (bottom left) observed at 5 km away using subsequent stroke.

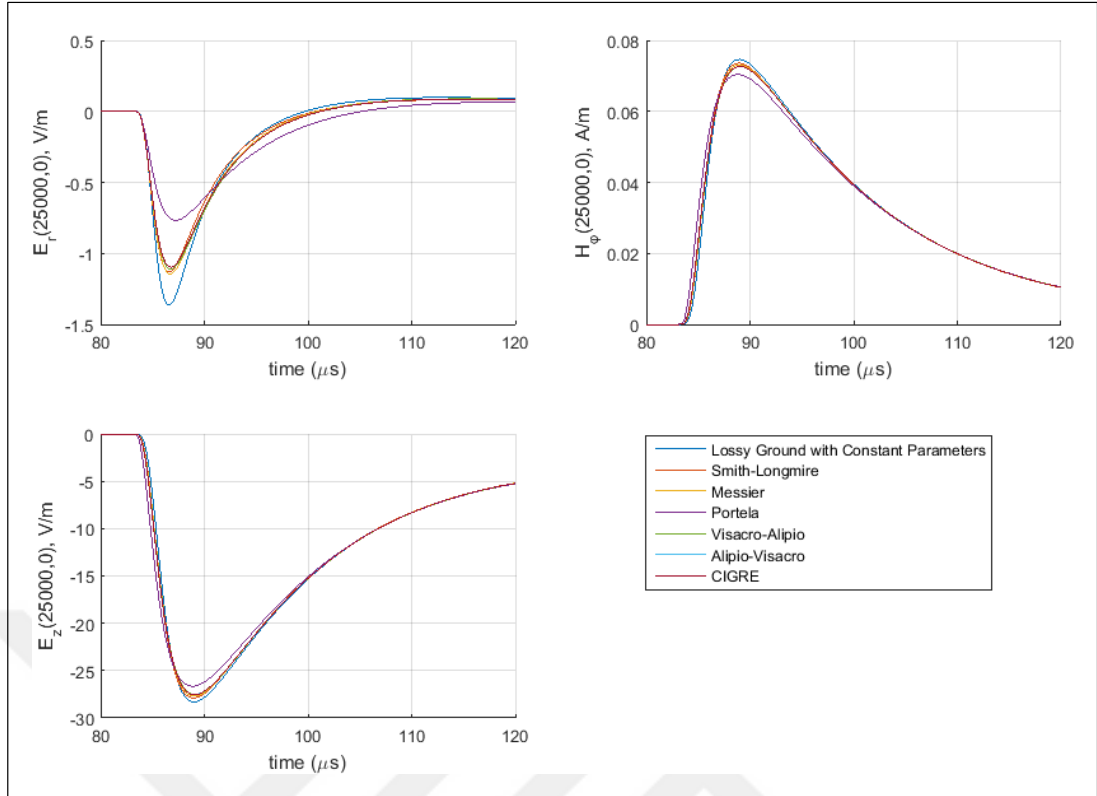


Figure 4.10: E_r (top left), H_ϕ (top right), and E_z (bottom left) observed at 25 km away using first stroke.

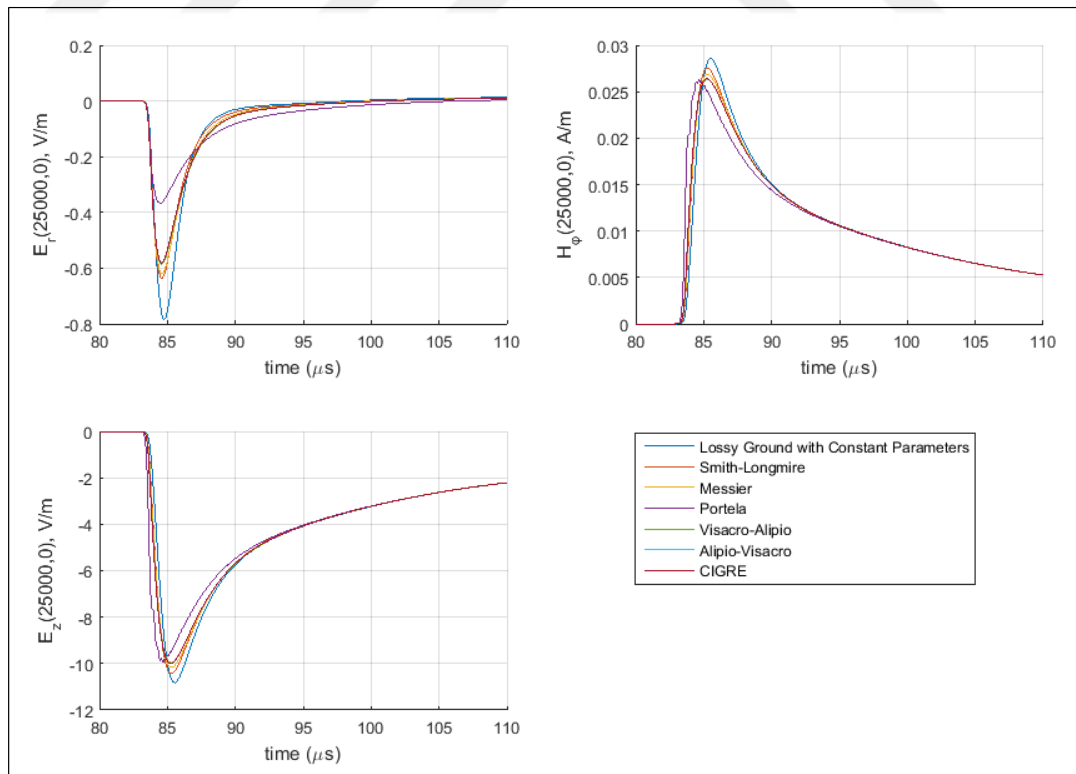


Figure 4.11: E_r (top left), H_ϕ (top right), and E_z (bottom left) observed at 25 km away using subsequent stroke.

4.3.1.2. Above-Ground Results

The RD values obtained above the ground for the first and subsequent return strokes are given in Table 4.3 and Table 4.4, respectively. The graphical illustrations regarding different distances for both first and subsequent strokes are shown in Figure 4.12 through Figure 4.19.

According to the results obtained above the ground, the deviation of the Portela model is apparent again. Besides, using dispersive models has important effects on E_r at far ranges as in the overground case whereas these effects are very little especially at 50 m. Independent of the used model, it is observed that the intensity of E_r is quite stronger especially at close distances compared to the overground case and that the polarity of the field is also reversed at 50 m and 500 m. Similar to the overground results, the models of Smith-Longmire and Messier gives similar results and the results of Visacro-Alipio, Alipio-Visacro and CIGRE models are close to each other for both stroke types. Furthermore, the RD values for E_z and H_ϕ increase as the distance increases, so it can be said that they are directly proportional to the horizontal distance, which is also valid for the overground results.

Table 4.3: RD obtained at 10 m above the ground with σ_0 of 1 mS/m using first stroke.

Model Distance		Smith- Longmire	Messier	Portela	Visacro- Alipio	Alipio- Visacro	CIGRE
50 m	E_r	0.6879	0.6316	1.6602	0.7357	0.7750	0.7745
	E_z	0.0632	0.0619	0.1975	0.0744	0.0779	0.0778
	H_ϕ	0.1059	0.1129	0.2728	0.1356	0.1394	0.1393
500 m	E_r	4.2624	4.2577	12.5836	5.1310	5.3580	5.3530
	E_z	0.0984	0.1056	0.2711	0.1283	0.1321	0.1319
	H_ϕ	0.5985	0.6380	1.6913	0.7764	0.8008	0.7999
5 km	E_r	18.3892	18.2681	52.1060	21.7917	22.7608	22.7414
	E_z	2.6107	2.7161	7.4313	3.2800	3.3972	3.3936
	H_ϕ	3.3374	3.4692	9.5371	4.1909	4.3416	4.3370
25 km	E_r	17.1113	16.5471	46.2285	19.4700	20.4134	20.3983
	E_z	6.6006	6.6028	18.9460	7.8965	8.2396	8.2322
	H_ϕ	6.6403	6.6413	19.0856	7.9433	8.2890	8.2816

Table 4.4: RD obtained at 10 m above the ground with σ_0 of 1 mS/m using subsequent stroke.

Model Distance		Smith-Longmire	Messier	Portela	Visacro-Alipio	Alipio-Visacro	CIGRE
50 m	E_r	2.2116	2.4260	5.6584	2.9458	3.0205	3.0176
	E_z	0.3020	0.3362	0.8615	0.4159	0.4269	0.4266
	H_ϕ	1.4460	1.6083	3.3428	1.9537	2.0087	2.0067
500 m	E_r	19.7877	21.5243	51.0950	25.9441	26.5575	26.5364
	E_z	1.3983	1.5465	3.6597	1.8963	1.9378	1.9356
	H_ϕ	7.2838	8.0541	19.2673	9.8832	10.1192	10.1076
5 km	E_r	25.0261	26.7172	62.0061	31.7899	32.5885	32.5580
	E_z	15.1322	16.4872	46.5351	20.0614	20.5060	20.4907
	H_ϕ	15.2946	16.6670	47.1407	20.2894	20.7421	20.7267
25 km	E_r	22.4221	23.2830	55.2222	27.4774	28.3271	28.2996
	E_z	17.3878	18.5025	57.0632	22.6005	23.2996	23.2729
	H_ϕ	17.4495	18.5687	57.3356	22.6833	23.3865	23.3599

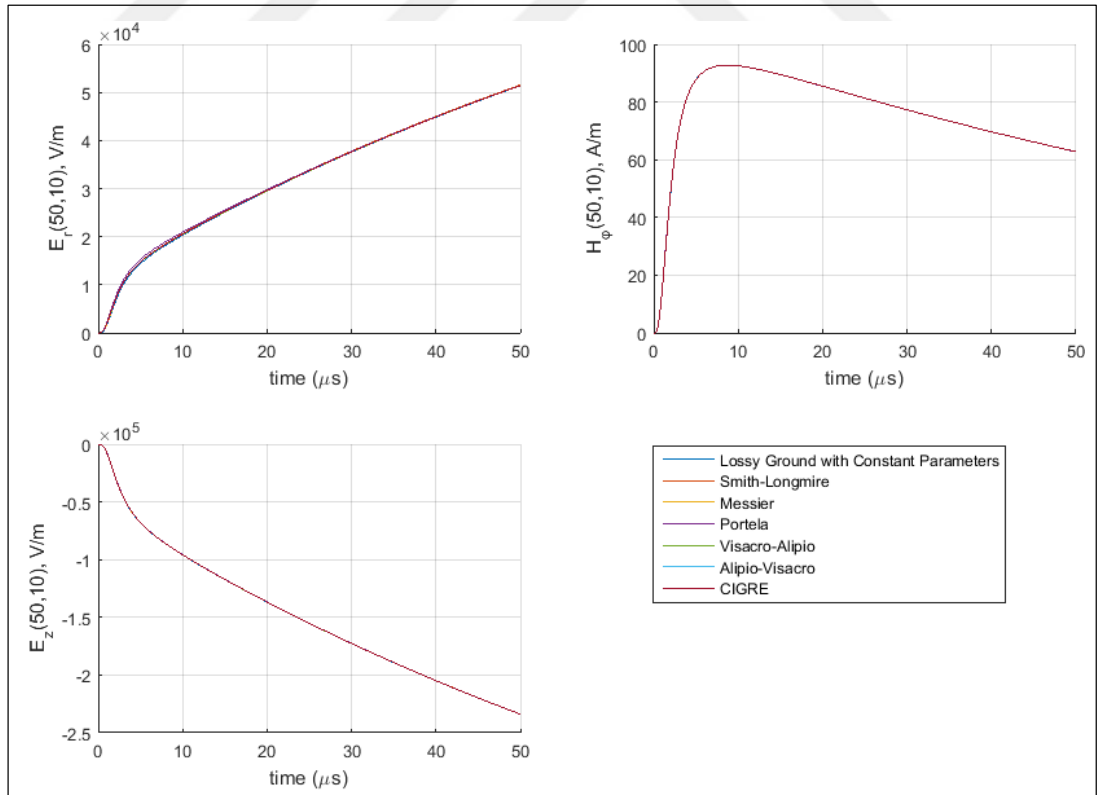


Figure 4.12: E_r (top left), H_ϕ (top right), and E_z (bottom left) observed at 50 m away using first stroke.

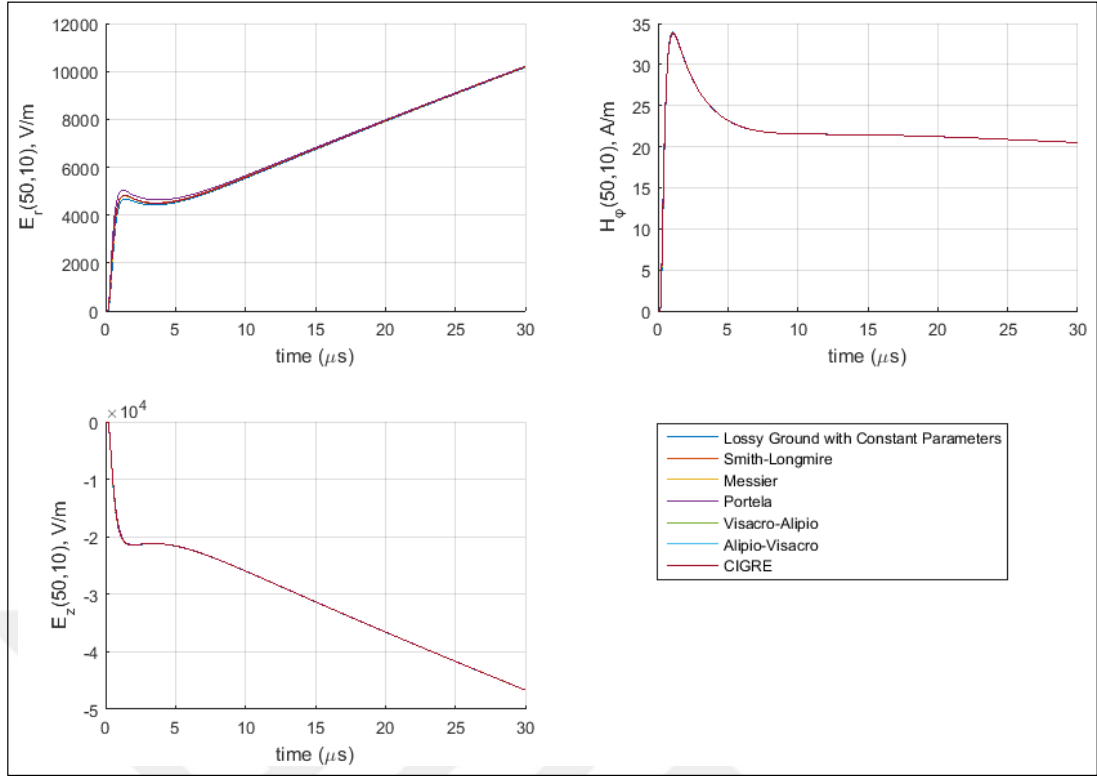


Figure 4.13: E_r (top left), H_ϕ (top right), and E_z (bottom left) observed at 50 m away using subsequent stroke.

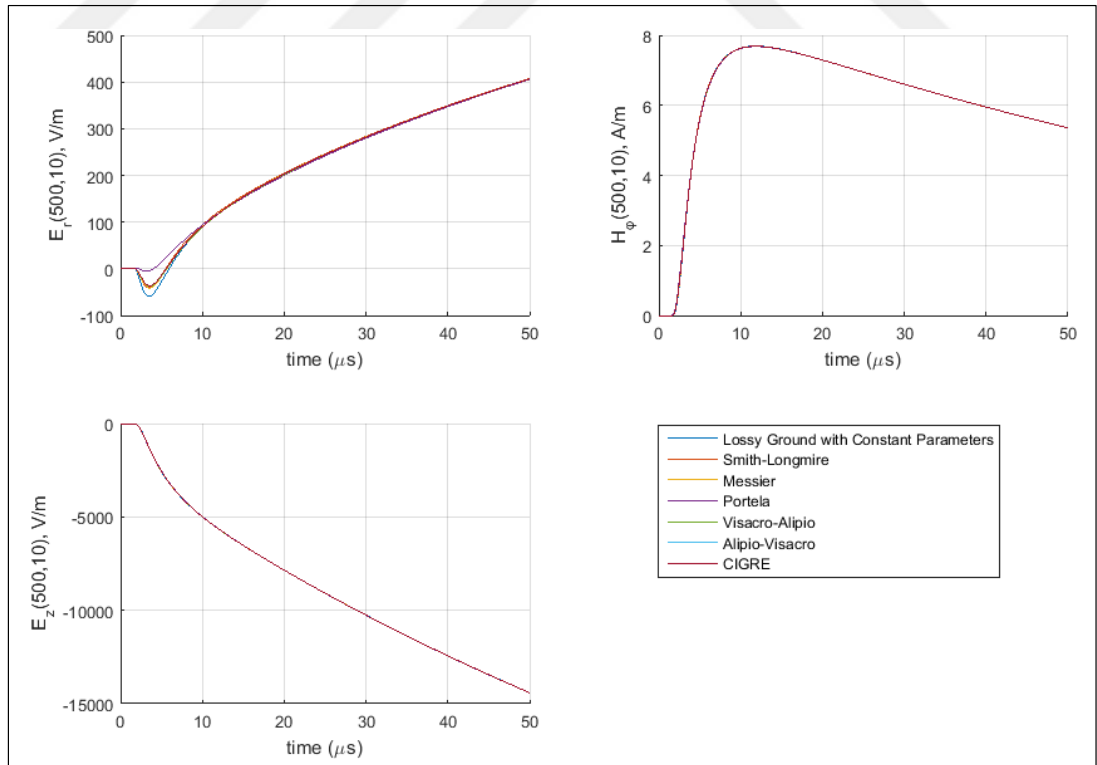


Figure 4.14: E_r (top left), H_ϕ (top right), and E_z (bottom left) observed at 500 m away using first stroke.

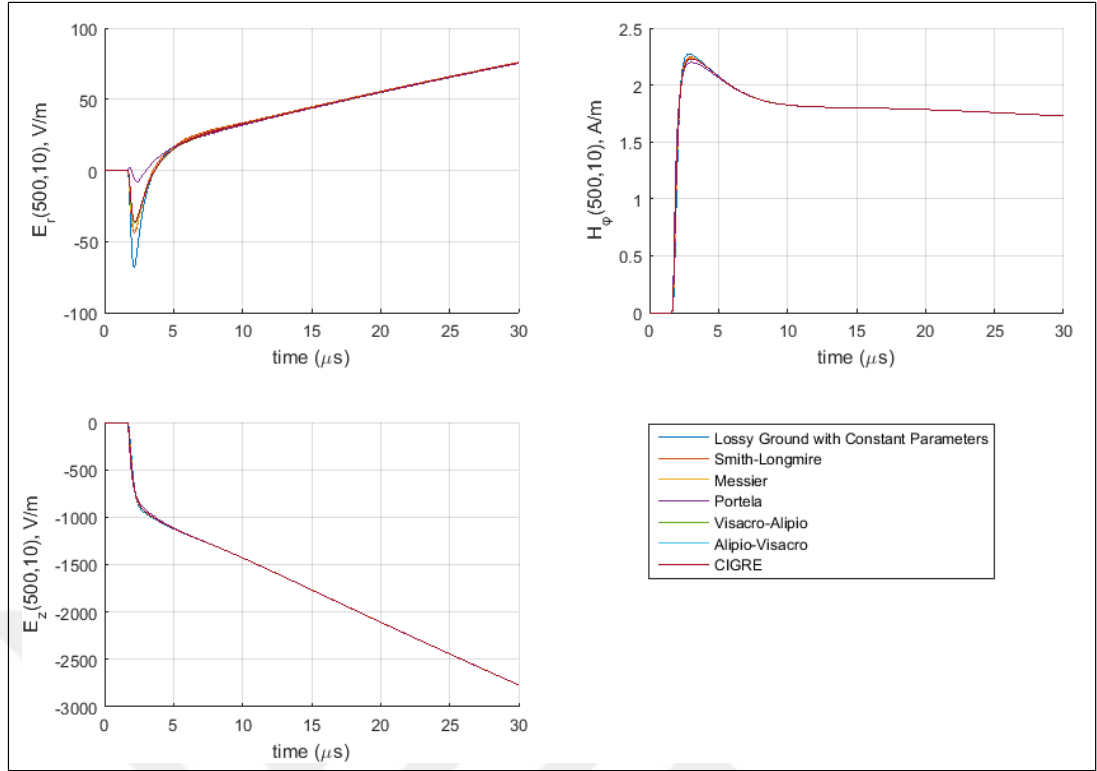


Figure 4.15: E_r (top left), H_ϕ (top right), and E_z (bottom left) observed at 500 m away using subsequent stroke.

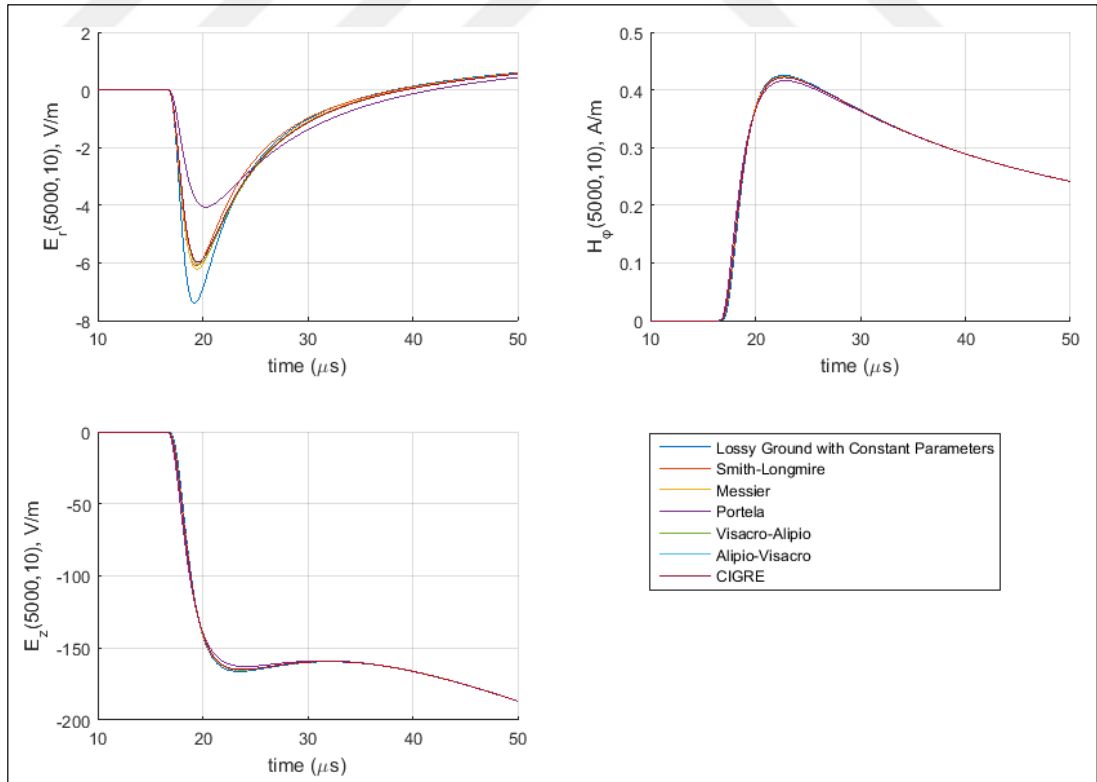


Figure 4.16: E_r (top left), H_ϕ (top right), and E_z (bottom left) observed at 5 km away using first stroke.

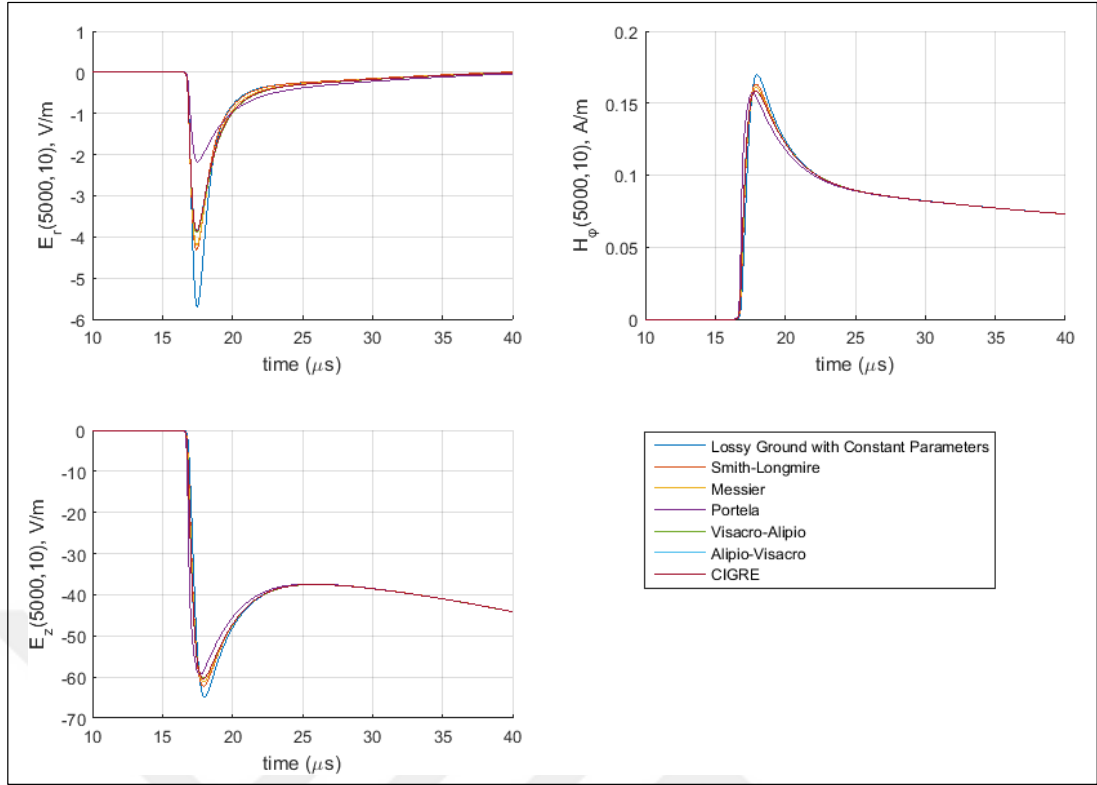


Figure 4.17: E_r (top left), H_ϕ (top right), and E_z (bottom left) observed at 5 km away using subsequent stroke.

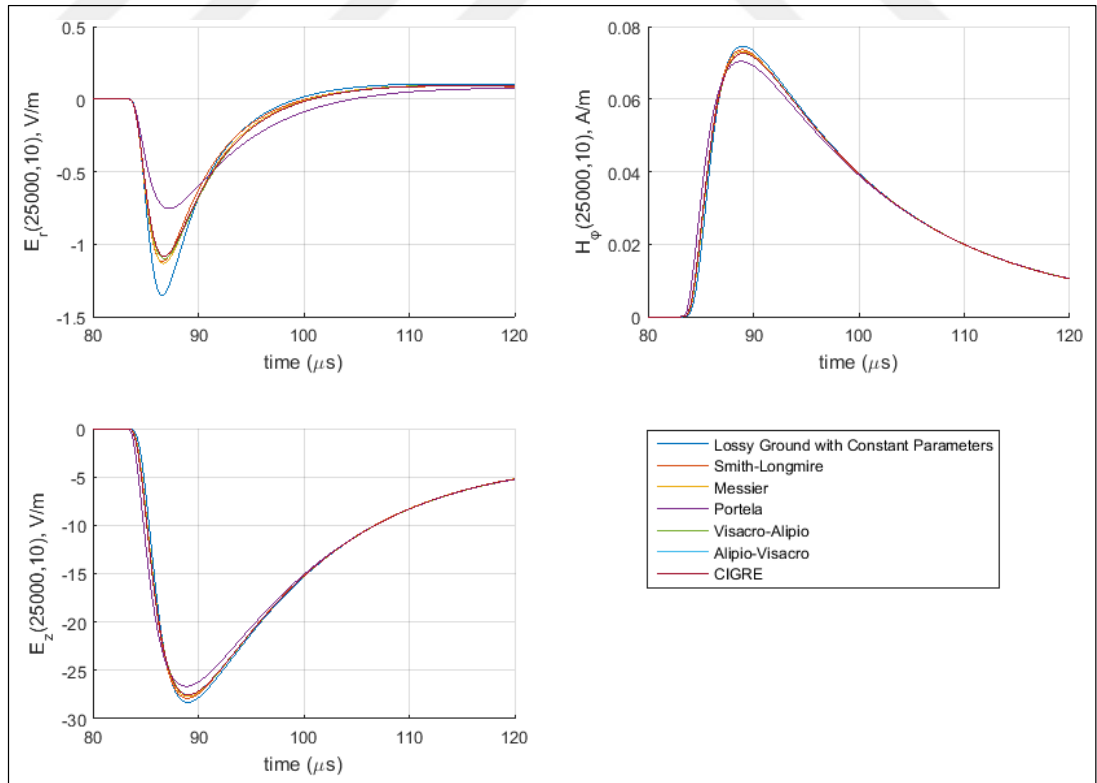


Figure 4.18: E_r (top left), H_ϕ (top right), and E_z (bottom left) observed at 25 km away using first stroke.

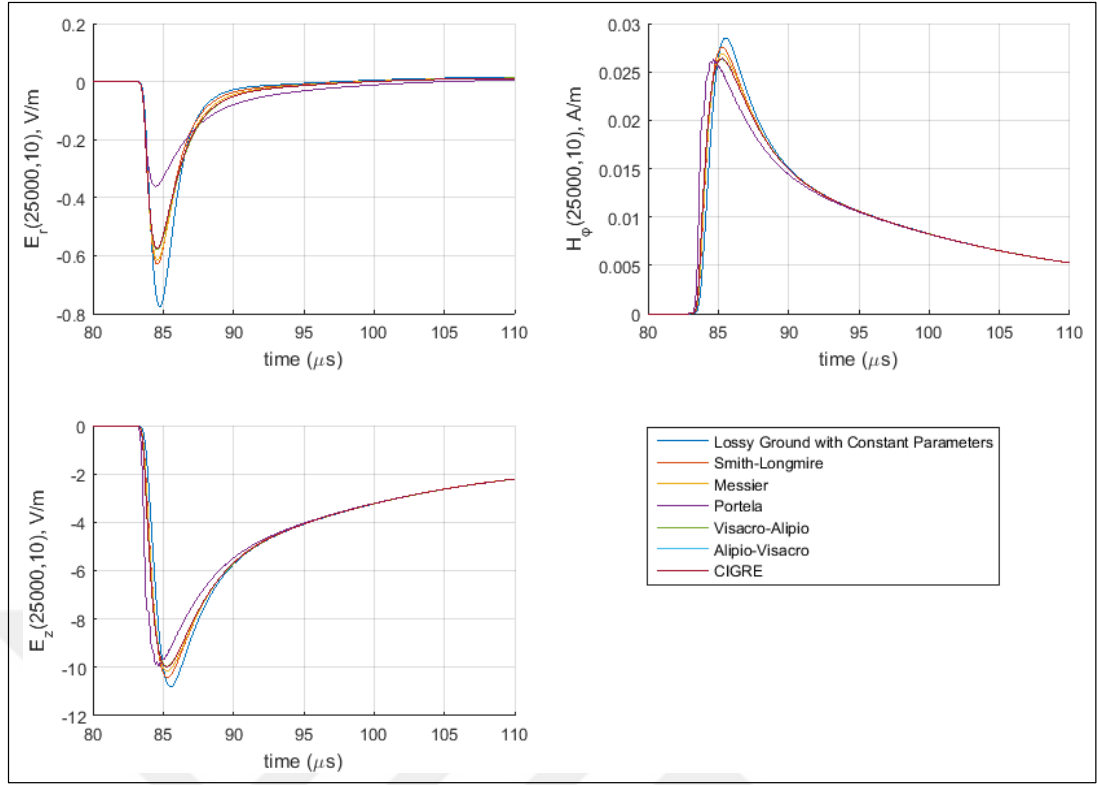


Figure 4.19: E_r (top left), H_ϕ (top right), and E_z (bottom left) observed at 25 km away using subsequent stroke.

4.3.1.3. Underground Results

The RD values obtained above the ground for the first and subsequent return strokes are given in Table 4.5 and Table 4.6, respectively. The graphical illustrations regarding different distances for both first and subsequent strokes are shown in Figure 4.20 through Figure 4.27.

According to the underground results, both E_r and E_z are greatly affected by the usage of dispersive model. Moreover, the E_z is more affected than the E_r for all configurations. This is not just related with the absolute peak values but also related with the shorter rise time in dispersive model results. Considering the RD values, the H_ϕ field is quite affected at close distances for the subsequent stroke, which is contrary to the previous results where it is affected at the far range for the overground and above-ground cases. Regarding the similarities of dispersive models, similar comments in the previous sections can be made; that is, Portela is the most distinct one and Visacro-Alipio, Alipio-Visacro and CIGRE are very close to each other.

Table 4.5: RD obtained at 10 m under the ground with σ_0 of 1 mS/m using first stroke.

Model Distance		Smith-Longmire	Messier	Portela	Visacro-Alipio	Alipio-Visacro	CIGRE
50 m	E_r	21.9684	20.2712	55.0268	23.6678	24.8751	24.8585
	E_z	24.9508	24.2962	72.4088	29.4400	30.5554	30.5357
	H_ϕ	2.7693	2.9217	16.7258	3.8447	4.0454	4.0388
500 m	E_r	19.6083	19.5742	59.7608	23.6735	24.7313	24.7074
	E_z	38.0450	39.5526	86.8727	46.9121	48.3638	48.3189
	H_ϕ	2.3978	2.2976	11.4728	2.8446	3.0186	3.0155
5 km	E_r	19.8919	19.6050	59.0300	23.4442	24.5397	24.5186
	E_z	37.1423	38.2354	85.5636	45.1924	46.6681	46.6265
	H_ϕ	2.1818	2.0023	9.0984	2.3318	2.4882	2.4862
25 km	E_r	17.9569	16.9901	50.0979	19.9041	20.9850	20.9688
	E_z	32.6320	32.3442	78.0245	37.9762	39.5168	39.4864
	H_ϕ	2.7952	2.8320	4.4200	3.2365	3.3179	3.3157

Table 4.6: RD obtained at 10 m under the ground with σ_0 of 1 mS/m using subsequent stroke.

Model Distance		Smith-Longmire	Messier	Portela	Visacro-Alipio	Alipio-Visacro	CIGRE
50 m	E_r	45.0476	49.8401	97.1417	60.3890	62.3920	62.3320
	E_z	68.3806	73.2854	99.6815	82.4037	84.1177	84.0723
	H_ϕ	16.1322	18.4393	67.2265	24.3044	25.4395	25.4018
500 m	E_r	37.8165	41.7540	95.1380	51.6313	53.2203	53.1640
	E_z	59.5240	64.3541	99.2328	74.4419	76.0951	76.0411
	H_ϕ	8.2207	9.4003	51.1507	13.1147	13.7232	13.6993
5 km	E_r	25.7370	27.2253	82.3841	33.0293	33.9983	33.9605
	E_z	43.7373	47.1661	95.3374	56.1565	57.3945	57.3504
	H_ϕ	4.8276	4.9000	28.0041	5.4664	5.6618	5.6616
25 km	E_r	21.0443	21.0052	55.8473	24.5212	25.5082	25.4908
	E_z	34.0216	35.1984	83.6874	41.7400	43.0005	42.9561
	H_ϕ	8.4645	8.9845	8.7024	10.5361	10.7531	10.7454

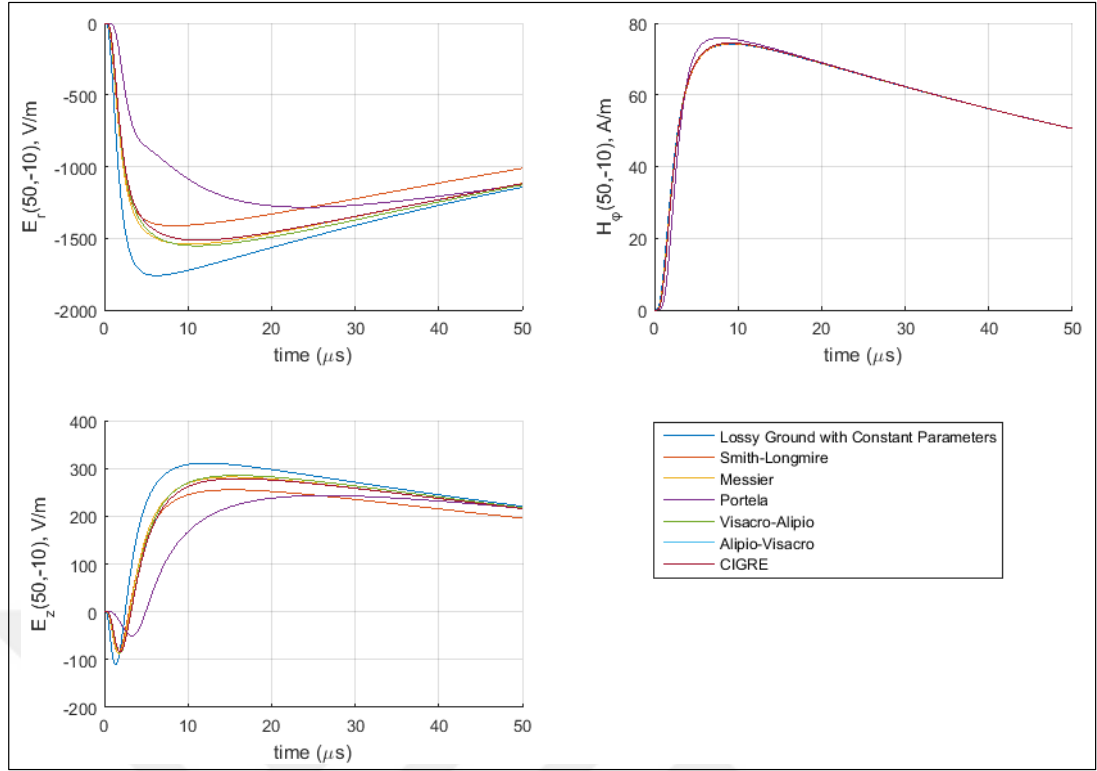


Figure 4.20: E_r (top left), H_ϕ (top right), and E_z (bottom left) observed at 50 m away using first stroke.

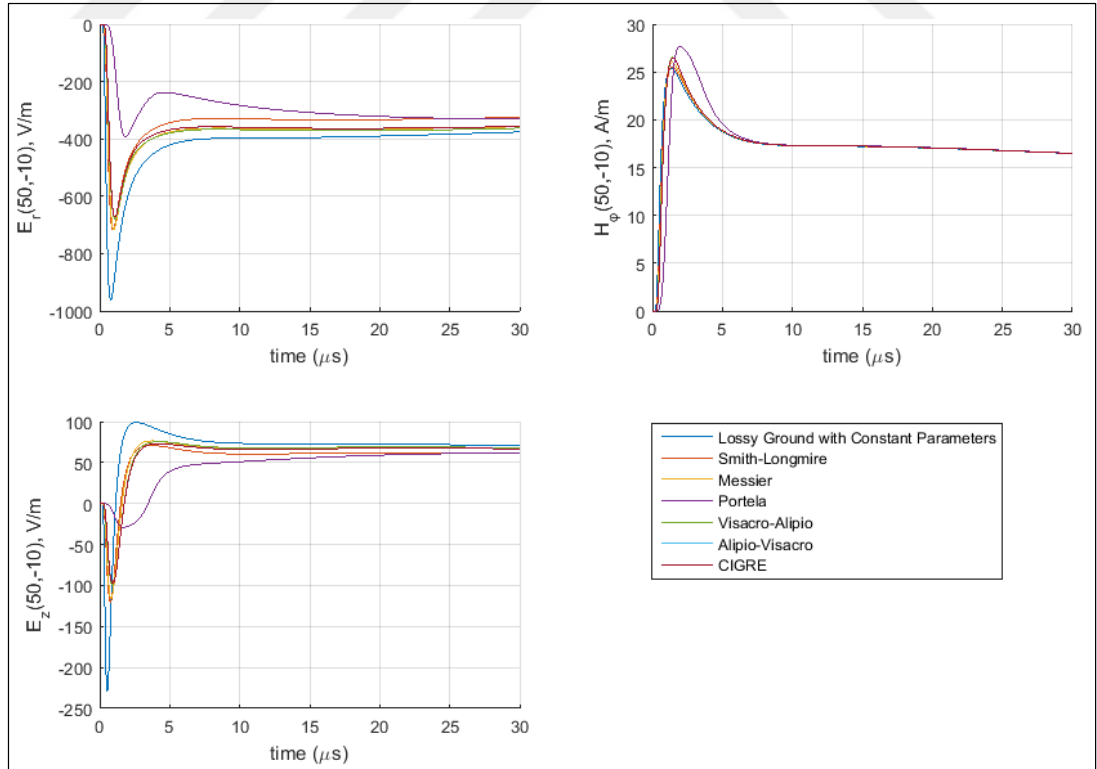


Figure 4.21: E_r (top left), H_ϕ (top right), and E_z (bottom left) observed at 50 m away using subsequent stroke.

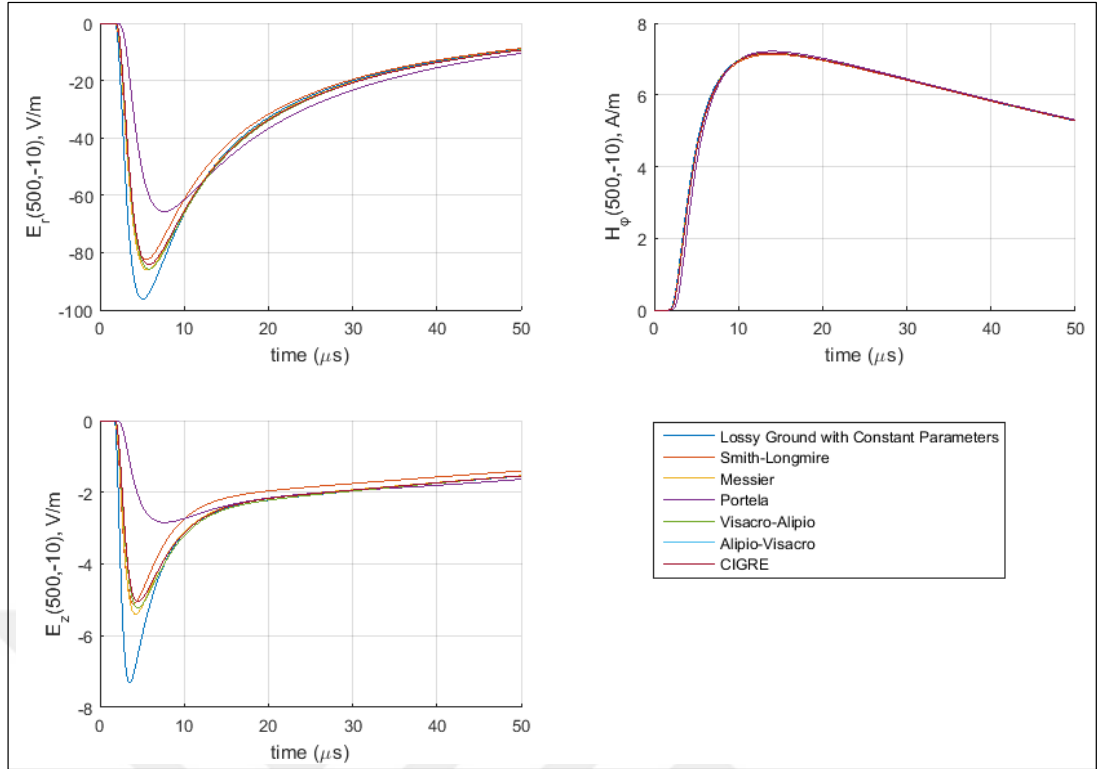


Figure 4.22: E_r (top left), H_ϕ (top right), and E_z (bottom left) observed at 500 m away using first stroke.

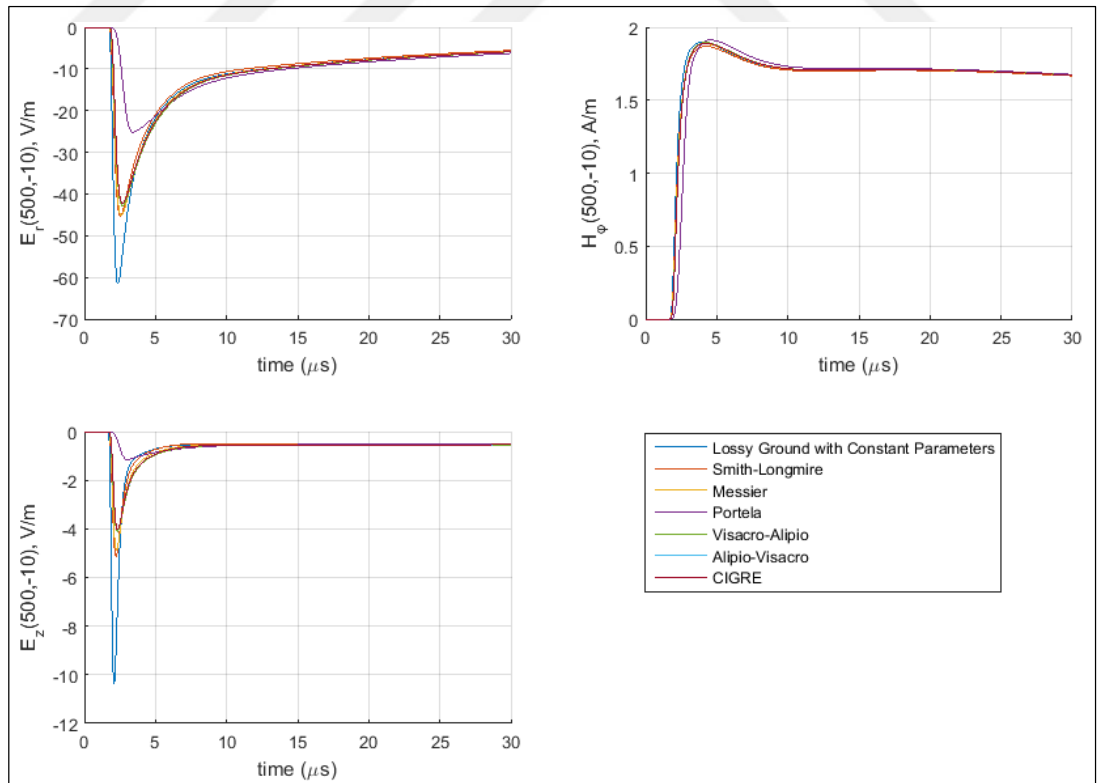


Figure 4.23: E_r (top left), H_ϕ (top right), and E_z (bottom left) observed at 500 m away using subsequent stroke.

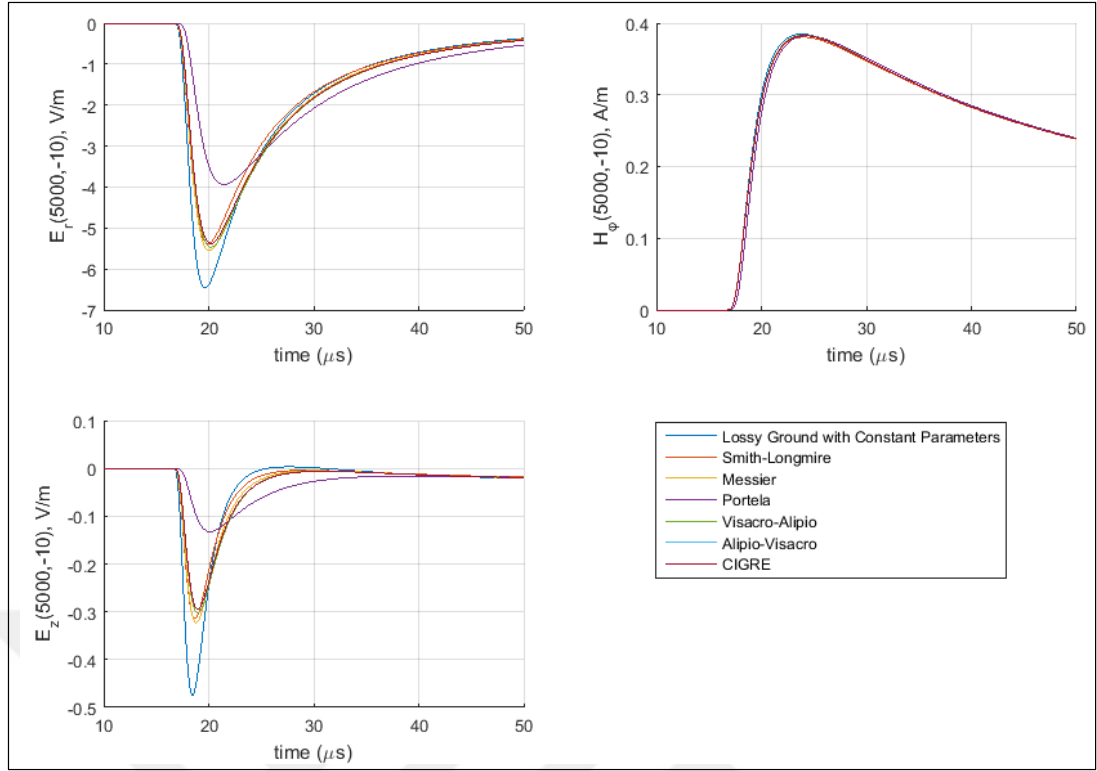


Figure 4.24: E_r (top left), H_ϕ (top right), and E_z (bottom left) observed at 5 km away using first stroke.

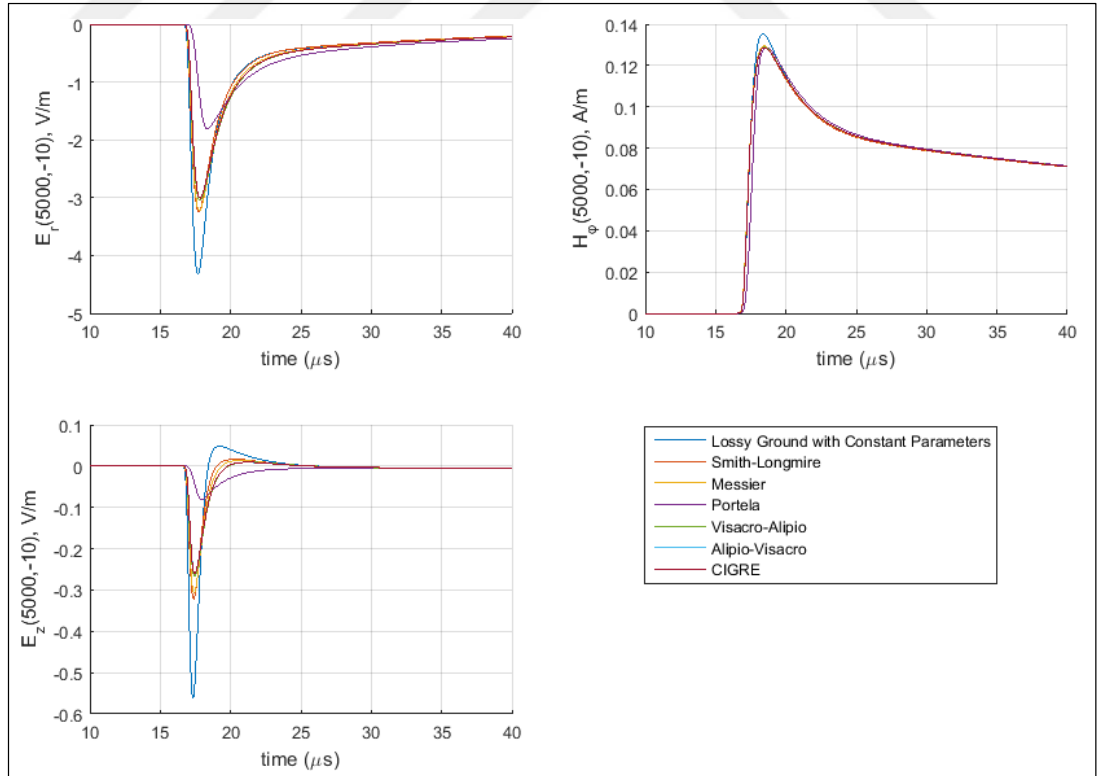


Figure 4.25: E_r (top left), H_ϕ (top right), and E_z (bottom left) observed at 5 km away using subsequent stroke.

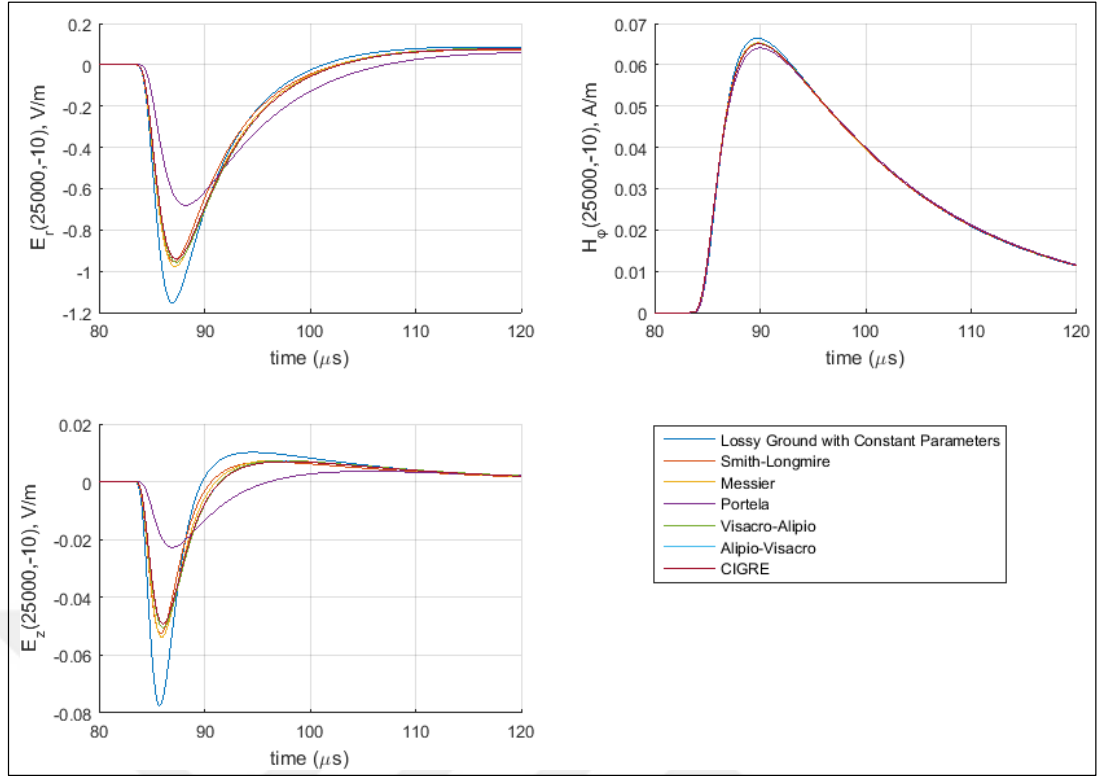


Figure 4.26: E_r (top left), H_ϕ (top right), and E_z (bottom left) observed at 25 km away using first stroke.

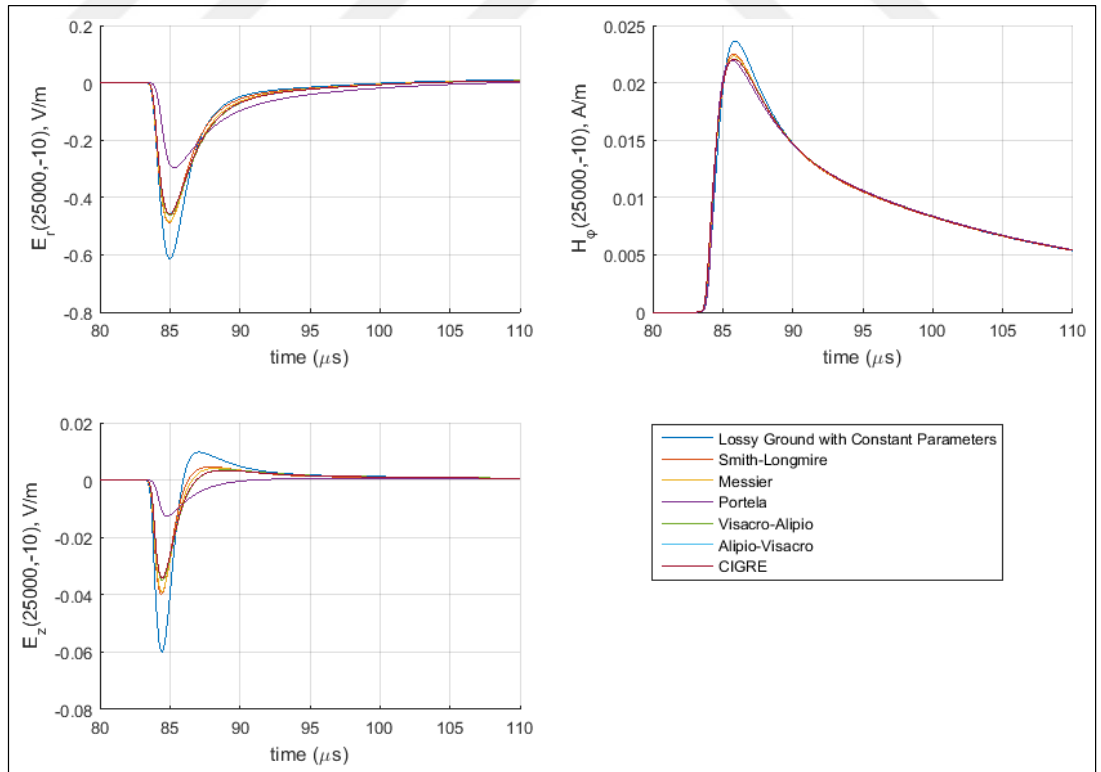


Figure 4.27: E_r (top left), H_ϕ (top right), and E_z (bottom left) observed at 25 km away using subsequent stroke.

4.3.2. Results for Other σ_0 Values

In this section, graphical illustrations are not included to avoid non-essential bulk of figures because the waveforms of the results are very similar to those for σ_0 of 1 mS/m except for the amplitudes of the obtained results. Since the differences of the models are interpreted better by reading the tables, the RD values are given in tables as in the previous section.

4.3.2.1. Results for 0.1 mS/m

The RD values obtained over the ground, above the ground, and under the ground for both first and subsequent return strokes are given in Table 4.7 through Table 4.12. According to the results, using σ_0 of 0.1 mS/m increases the absolute amplitudes of the obtained waveforms for all the configurations. Not only the amplitudes but also the obtained RD values increase for both stroke types as the σ_0 decreases. Even the fields E_z and H_ϕ which are affected little by the model change in overground and above-ground results for 1 mS/m are observed to be much more affected by the dispersive models for 0.1 mS/m especially at the far range. Thus, it can be said that all values including the field intensities and the differences between the dispersive and constant lossy results increase with decreasing σ_0 .

Table 4.7: RD obtained over the ground with σ_0 of 0.1 mS/m using first stroke.

Model Distance		Smith- Longmire	Messier	Portela	Visacro- Alipio	Alipio- Visacro	CIGRE
50 m	E_r	46.3340	43.6640	90.8294	54.2348	60.2842	61.4799
	E_z	0.3624	0.3463	0.6585	0.4237	0.4640	0.4721
	H_ϕ	1.5745	1.5298	2.4624	1.8369	1.9458	1.9690
500 m	E_r	31.4705	29.8204	77.7294	39.1896	43.8412	44.8456
	E_z	0.4586	0.4421	0.9789	0.5687	0.6171	0.6285
	H_ϕ	2.8349	2.7230	6.2765	3.5235	3.8448	3.9201
5 km	E_r	31.9568	30.0567	75.7986	38.3683	43.2500	44.2566
	E_z	9.6658	9.1656	23.0227	11.9705	13.2873	13.5816
	H_ϕ	12.0764	11.4464	29.0915	14.9785	16.6486	17.0211
25 km	E_r	30.5598	28.3274	72.6255	35.5524	40.8110	41.8461
	E_z	19.4784	18.1523	51.9442	23.9150	27.3604	28.0848
	H_ϕ	19.7816	18.4316	53.0163	24.3036	27.8192	28.5589

Table 4.8: RD obtained over the ground with σ_0 of 0.1 mS/m using subsequent stroke.

Model Distance		Smith-Longmire	Messier	Portela	Visacro-Alipio	Alipio-Visacro	CIGRE
50 m	E_r	60.5943	58.9254	92.8912	70.8700	75.1265	76.0103
	E_z	0.9406	0.9386	0.9577	0.9055	0.8721	0.8611
	H_φ	5.2389	5.1551	8.6152	5.8612	6.0377	6.0835
500 m	E_r	37.0376	35.7427	82.1557	46.6959	50.7697	51.7599
	E_z	2.4936	2.4627	6.3207	3.3224	3.5774	3.6425
	H_φ	12.8933	12.7243	33.0472	17.1890	18.5492	18.8913
5 km	E_r	36.4437	34.5958	78.8806	44.0883	48.4968	49.4748
	E_z	26.1354	25.0487	69.8544	34.2015	37.6798	38.5798
	H_φ	28.9308	27.7249	77.9469	37.9231	41.8086	42.8114
25 km	E_r	35.0077	32.9682	76.7947	41.4247	46.5484	47.5919
	E_z	33.2788	31.1394	103.2461	43.0395	48.8819	50.2917
	H_φ	33.7096	31.5409	105.0250	43.6353	49.5734	51.0080

Table 4.9: RD obtained at 10 m above the ground with σ_0 of 0.1 mS/m using first stroke.

Model Distance		Smith-Longmire	Messier	Portela	Visacro-Alipio	Alipio-Visacro	CIGRE
50 m	E_r	16.9521	15.9760	33.2177	19.8421	22.0545	22.4918
	E_z	0.9119	0.8588	1.7576	1.0607	1.1796	1.2028
	H_φ	1.2349	1.1995	1.9488	1.4414	1.5275	1.5460
500 m	E_r	29.7377	28.1724	73.2919	36.9852	41.3778	42.3250
	E_z	0.4406	0.4248	0.9394	0.5462	0.5925	0.6035
	H_φ	2.7580	2.6489	6.1037	3.4267	3.7392	3.8124
5 km	E_r	32.4606	30.5252	77.0902	38.9754	43.9458	44.9706
	E_z	9.6026	9.1056	22.8920	11.8943	13.2039	13.4966
	H_φ	12.0077	11.3811	28.9507	14.8958	16.5582	16.9290
25 km	E_r	30.6431	28.4003	72.8756	35.6467	40.9263	41.9653
	E_z	19.4414	18.1175	51.8750	23.8718	27.3123	28.0358
	H_φ	19.7447	18.3969	52.9474	24.2605	27.7713	28.5100

Table 4.10: RD obtained at 10 m above the ground with σ_0 of 0.1 mS/m using subsequent stroke.

Model Distance		Smith-Longmire	Messier	Portela	Visacro-Alipio	Alipio-Visacro	CIGRE
50 m	E_r	26.1594	25.4277	40.5137	30.5595	32.3968	32.7814
	E_z	1.3763	1.3385	2.1112	1.5825	1.6745	1.6926
	H_φ	4.0662	4.0032	6.9408	4.6665	4.8420	4.8886
500 m	E_r	42.5183	41.0195	94.5533	53.6192	58.3166	59.4580
	E_z	2.3853	2.3558	6.0636	3.1790	3.4236	3.4860
	H_φ	12.4391	12.2749	31.9197	16.5762	17.8962	18.2269
5 km	E_r	36.8116	34.9399	79.8641	44.5435	49.0165	50.0073
	E_z	25.9136	24.8394	69.4630	33.9240	37.3824	38.2753
	H_φ	28.7420	27.5454	77.6593	37.6873	41.5564	42.5557
25 km	E_r	35.0630	33.0156	76.9862	41.4881	46.6288	47.6754
	E_z	33.2068	31.0723	103.1310	42.9541	48.7896	50.1977
	H_φ	33.6378	31.4740	104.9093	43.5503	49.4812	50.9130

Table 4.11: RD obtained at 10 m under the ground with σ_0 of 0.1 mS/m using first stroke.

Model Distance		Smith-Longmire	Messier	Portela	Visacro-Alipio	Alipio-Visacro	CIGRE
50 m	E_r	46.3237	43.6512	90.7619	54.2141	60.2653	61.4608
	E_z	46.6300	44.2090	91.6742	54.9383	60.8138	62.0105
	H_φ	1.0402	1.0331	19.3545	2.0454	2.5842	2.7181
500 m	E_r	32.7967	31.0680	81.8367	40.8922	45.7730	46.8274
	E_z	61.3938	59.4254	97.4157	71.5662	76.1855	77.1470
	H_φ	1.5728	1.4281	13.0954	1.9512	2.4343	2.5414
5 km	E_r	32.0852	30.0971	77.2410	38.4233	43.4698	44.5042
	E_z	55.6306	53.2864	95.7047	65.3201	70.6032	71.6841
	H_φ	9.0743	8.6830	8.6207	10.8657	11.6998	11.8713
25 km	E_r	30.1250	27.7893	71.9318	34.7954	40.0928	41.1185
	E_z	48.4978	45.5334	92.9791	56.6762	63.1334	64.3925
	H_φ	17.3619	16.2542	37.1262	21.1717	23.9114	24.4775

Table 4.12: RD obtained at 10 m under the ground with σ_0 of 0.1 mS/m using subsequent stroke.

Model Distance		Smith-Longmire	Messier	Portela	Visacro-Alipio	Alipio-Visacro	CIGRE
50 m	E_r	59.8944	58.2247	91.7280	69.9261	74.1391	75.0015
	E_z	66.6778	67.4386	99.8926	81.7940	86.1136	86.8288
	H_ϕ	15.9686	16.4051	77.9191	25.2687	29.5942	30.3566
500 m	E_r	37.9428	36.5389	93.0010	48.0728	52.4300	53.4992
	E_z	66.1487	65.1297	99.0594	77.4330	80.9728	81.7616
	H_ϕ	3.4318	3.2466	45.6868	3.9202	5.3744	5.6524
5 km	E_r	35.1333	33.2924	77.1807	42.2629	46.7247	47.6817
	E_z	56.0123	53.9951	95.8751	66.9403	71.5556	72.6227
	H_ϕ	22.2768	21.5653	28.9374	28.0638	30.1296	30.5955
25 km	E_r	33.7897	31.6730	73.6110	39.5793	44.6469	45.6606
	E_z	47.7647	45.0758	92.8473	57.1741	63.1715	64.4009
	H_ϕ	29.7264	27.9604	73.3960	38.0854	42.7359	43.8355

4.3.2.2. Results for 10 mS/m

The RD values obtained over the ground, above the ground, and under the ground for both first and subsequent return strokes are given in Table 4.13 through Table 4.18. According to the results, the same conclusion with the previous section can be inferred from the reversal side; that is, both field intensities and the RD values decrease as the σ_0 increase from 1 mS/m to 10 mS/m for both stroke types.

Table 4.13: RD obtained over the ground with σ_0 of 10 mS/m using first stroke.

Model Distance		Smith-Longmire	Messier	Portela	Visacro-Alipio	Alipio-Visacro	CIGRE
50 m	E_r	8.5632	7.6490	16.2653	4.4714	6.3485	6.3613
	E_z	0.0110	0.0097	0.0207	0.0056	0.0080	0.0080
	H_ϕ	0.0150	0.0166	0.0378	0.0104	0.0146	0.0146
500 m	E_r	8.0932	6.9857	14.5543	4.0547	5.7521	5.7660
	E_z	0.0136	0.0182	0.0333	0.0181	0.0182	0.0182
	H_ϕ	0.0845	0.0882	0.1993	0.0881	0.0881	0.0881
5 km	E_r	8.6988	7.7547	16.0968	4.5491	6.4358	6.4494
	E_z	0.3987	0.4053	0.8966	0.2491	0.3510	0.3508
	H_ϕ	0.5180	0.5246	1.1604	0.3222	0.4542	0.4539
25 km	E_r	8.5113	7.4489	14.9800	4.3587	6.1473	6.1612
	E_z	1.2356	1.2088	2.5940	0.7352	1.0348	1.0351
	H_ϕ	1.2409	1.2130	2.6031	0.7377	1.0384	1.0386

Table 4.14: RD obtained over the ground with σ_0 of 10 mS/m using subsequent stroke.

Model Distance		Smith-Longmire	Messier	Portela	Visacro-Alipio	Alipio-Visacro	CIGRE
50 m	E_r	17.4727	21.4481	46.3219	14.3242	19.8899	19.8791
	E_z	0.0567	0.0733	0.1622	0.0496	0.0688	0.0689
	H_φ	0.3111	0.3925	0.7508	0.2786	0.3790	0.3782
500 m	E_r	17.4387	21.4401	46.1866	14.3651	19.8740	19.8618
	E_z	0.3146	0.3960	0.7658	0.2813	0.3812	0.3805
	H_φ	1.6754	2.1015	4.1085	1.4932	2.0313	2.0275
5 km	E_r	15.1226	17.9924	38.9385	11.9741	16.3989	16.3965
	E_z	5.2530	6.5078	14.0974	4.3667	6.0480	6.0434
	H_φ	5.2652	6.5142	14.1046	4.3709	6.0548	6.0498
25 km	E_r	13.3240	14.9602	29.5959	9.8963	13.4115	13.4089
	E_z	7.6765	8.9952	20.3434	5.8026	8.0835	8.0814
	H_φ	7.6740	8.9974	20.3311	5.8016	8.0789	8.0765

Table 4.15: RD obtained at 10 m above the ground with σ_0 of 10 mS/m using first stroke.

Model Distance		Smith-Longmire	Messier	Portela	Visacro-Alipio	Alipio-Visacro	CIGRE
50 m	E_r	0.0546	0.0484	0.1022	0.0283	0.0401	0.0402
	E_z	0.0112	0.0098	0.0206	0.0057	0.0081	0.0081
	H_φ	0.0118	0.0130	0.0295	0.0081	0.0114	0.0114
500 m	E_r	0.5757	0.4973	1.0366	0.2887	0.4096	0.4106
	E_z	0.0131	0.0182	0.0320	0.0181	0.0182	0.0182
	H_φ	0.0819	0.0881	0.1929	0.0881	0.0881	0.0881
5 km	E_r	9.9063	8.8283	18.3258	5.1782	7.3260	7.3416
	E_z	0.3958	0.4021	0.8896	0.2471	0.3482	0.3480
	H_φ	0.5147	0.5206	1.1516	0.3197	0.4506	0.4504
25 km	E_r	8.7178	7.6271	15.3380	4.4624	6.2938	6.3079
	E_z	1.2322	1.2049	2.5860	0.7328	1.0314	1.0317
	H_φ	1.2374	1.2091	2.5951	0.7353	1.0350	1.0352

Table 4.16: RD obtained at 10 m above the ground with σ_0 of 10 mS/m using subsequent stroke.

Model Distance		Smith-Longmire	Messier	Portela	Visacro-Alipio	Alipio-Visacro	CIGRE
50 m	E_r	0.3099	0.3825	0.8235	0.2539	0.3537	0.3535
	E_z	0.0536	0.0674	0.1489	0.0451	0.0629	0.0630
	H_ϕ	0.2321	0.2925	0.5664	0.2083	0.2824	0.2818
500 m	E_r	3.0938	3.8090	8.2208	2.5490	3.5318	3.5298
	E_z	0.2966	0.3736	0.7240	0.2642	0.3602	0.3595
	H_ϕ	1.5817	1.9915	3.8969	1.4103	1.9170	1.9135
5 km	E_r	16.2332	19.3078	41.9802	12.8298	17.5893	17.5874
	E_z	5.1886	6.4297	13.9392	4.3136	5.9768	5.9722
	H_ϕ	5.2020	6.4380	13.9458	4.3175	5.9830	5.9780
25 km	E_r	13.4868	15.1210	29.9715	9.9945	13.5471	13.5446
	E_z	7.6430	8.9573	20.2727	5.7773	8.0497	8.0476
	H_ϕ	7.6411	8.9602	20.2607	5.7759	8.0458	8.0434

Table 4.17: RD obtained at 10 m under the ground with σ_0 of 10 mS/m using first stroke.

Model Distance		Smith-Longmire	Messier	Portela	Visacro-Alipio	Alipio-Visacro	CIGRE
50 m	E_r	11.9839	10.2908	22.0343	5.9128	8.4282	8.4478
	E_z	12.7573	8.7316	17.2374	4.8180	6.9460	6.9160
	H_ϕ	3.0644	2.4783	5.3791	1.3770	1.9836	1.9883
500 m	E_r	10.9422	9.0421	18.9378	5.1517	7.3395	7.3571
	E_z	19.0846	17.6244	36.9367	10.5497	14.8697	14.8904
	H_ϕ	2.4959	1.9050	3.9848	1.0481	1.5085	1.5114
5 km	E_r	12.3089	10.5910	22.4096	6.1006	8.6791	8.6992
	E_z	21.5173	20.6935	43.2833	12.5592	17.6439	17.6592
	H_ϕ	3.2270	2.5826	5.5148	1.4351	2.0635	2.0683
25 km	E_r	12.1296	10.2749	21.2428	5.9000	8.3798	8.3986
	E_z	20.7233	19.4927	40.0636	11.7571	16.4911	16.5139
	H_ϕ	3.2960	2.5855	5.3519	1.4310	2.0552	2.0590

Table 4.18: RD obtained at 10 m under the ground with σ_0 of 10 mS/m using subsequent stroke.

Model Distance		Smith- Longmire	Messier	Portela	Visacro- Alipio	Alipio- Visacro	CIGRE
50 m	E_r	23.8636	27.1717	58.6242	17.3651	24.4023	24.3932
	E_z	35.8190	41.9871	77.9156	28.5551	38.8253	38.8132
	H_ϕ	8.7953	9.7405	24.4425	5.9151	8.5293	8.5255
500 m	E_r	22.7856	25.9567	55.9897	16.6346	23.3293	23.3212
	E_z	39.9206	47.4023	83.3237	33.0944	44.4399	44.4179
	H_ϕ	6.3769	7.0339	17.7032	4.2730	6.1553	6.1526
5 km	E_r	22.9488	25.8026	56.2476	16.4302	23.0443	23.0358
	E_z	37.1869	43.8317	80.0506	30.2436	40.7968	40.7773
	H_ϕ	8.2735	8.9838	22.7584	5.4229	7.8056	7.8020
25 km	E_r	19.7757	21.2774	47.7384	13.2870	18.7032	18.6972
	E_z	32.3979	37.0539	72.2377	24.5828	33.6687	33.6556
	H_ϕ	6.7184	6.8110	17.2586	4.0262	5.7894	5.7885

4.4. Interpretation of the Results

In the previous section, both numerical results are provided, and short interpretations of the dispersive models are given. In this section, the RD values of the models are plotted together with respect to the distance and the LFC (σ_0) to observe the model behaviors better from a different point of view. First, the RD values obtained for each dispersive model are plotted with respect to the horizontal distance. Then, they are plotted with respect to the LFC values. Both comparisons are made for both first and subsequent strokes. In comparisons, the field components that are the most affected by the dispersive nature of the soil are used: the E_r is used for overground and above-ground results, and both E_r and E_z are used for underground results.

The RD values for E_r with respect to the distance are plotted in Figure 4.28 and Figure 4.29 under first and subsequent strokes, respectively. As stated in section 4.3, the deviation of the Portela model from the others is evident. For both first and subsequent stroke results, the models of Smith-Longmire and Messier give similar results. Also, Visacro-Alipio, Alipio-Visacro and CIGRE models compute the fields close to each other. The models of Alipio-Visacro and CIGRE are even almost the same. These deductions are observed to be valid at all heights of the observation, namely over the ground, above the ground, and under the ground.

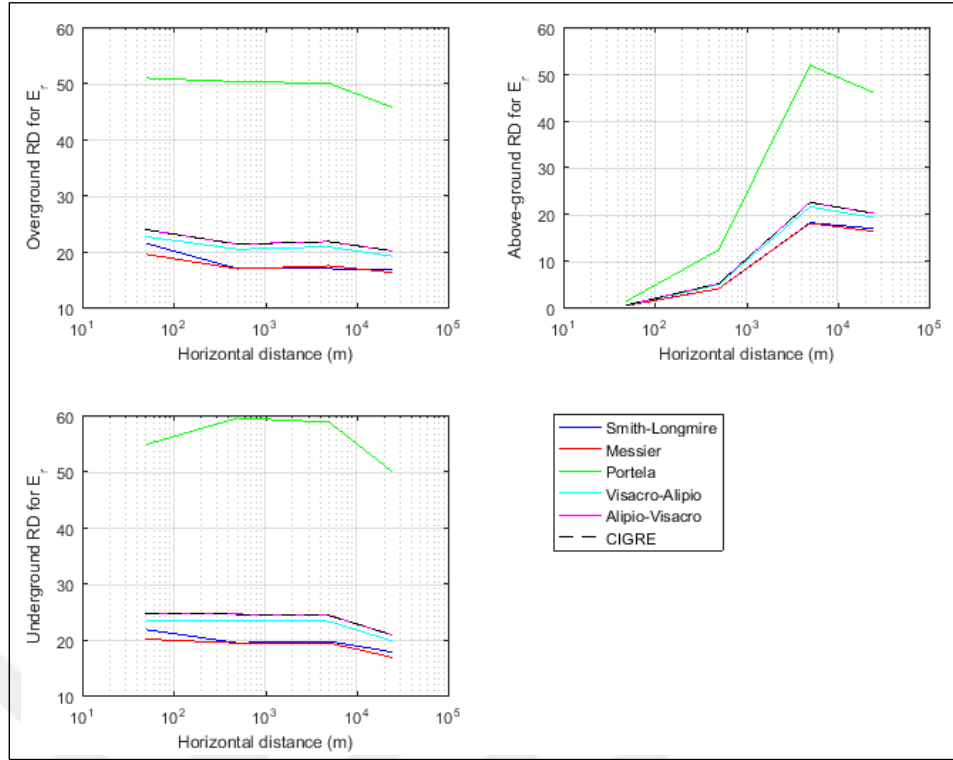


Figure 4.28: Overground (top left), above-ground (top right), and underground (bottom left) RD values for E_r obtained under first stroke with respect to the horizontal distance.

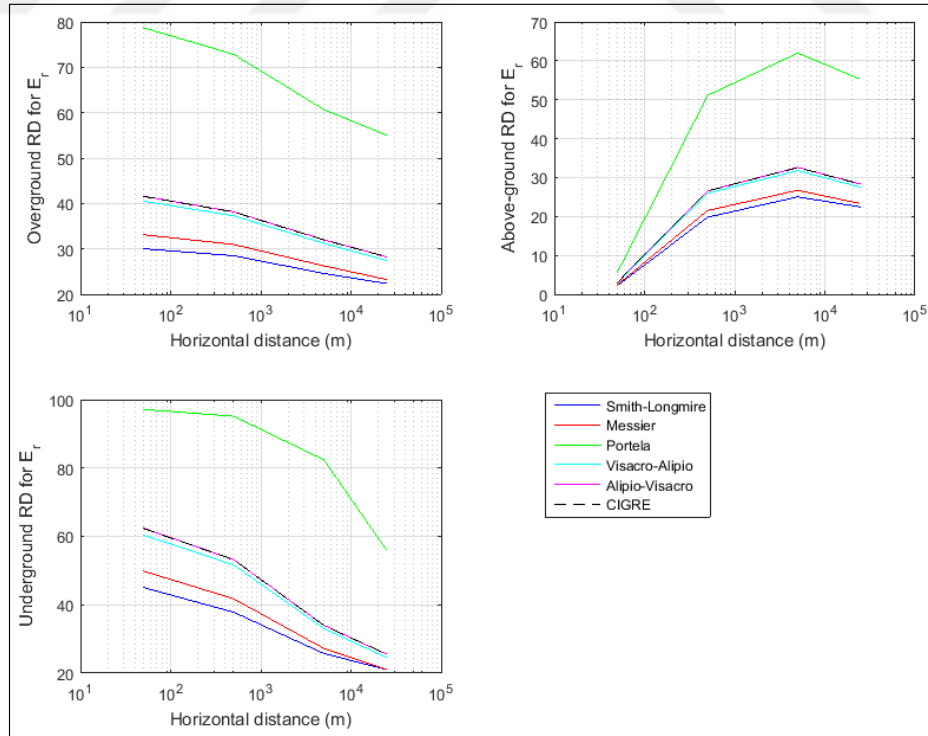


Figure 4.29: Overground (top left), above-ground (top right), and underground (bottom left) RD values for E_r obtained under subsequent stroke with respect to the horizontal distance.

The same comments can be made about the E_z results obtained under the ground, which are given in Figure 4.30 for both first and subsequent strokes. Both E_r and E_z results show that the obtained RD values under subsequent stroke are higher than those obtained under the first stroke. On the other hand, a descending structure is observed for the overground and underground RD values of the E_r whereas those for the E_z are in descending order just for the subsequent stroke.

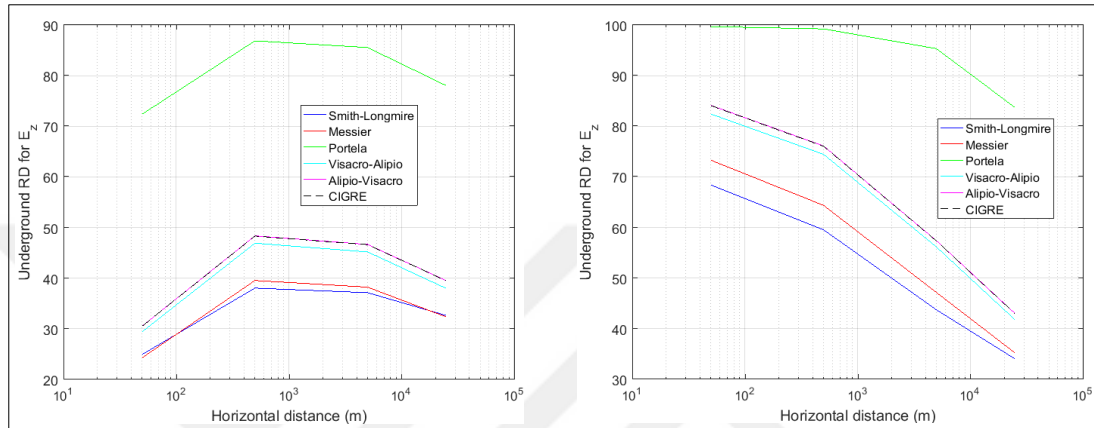


Figure 4.30: Underground RD values for E_z obtained under first stroke (left) and subsequent stroke (right) with respect to the horizontal distance.

The RD values obtained for each model are also plotted with respect to the LFC values. The results for all configurations including the overground, above-ground and underground results under both first and subsequent strokes can be seen in Figure 4.31 through Figure 4.38. The RD values at 50 m, 500 m, 5 km, and 25 km are separately illustrated in the figures.

The first eye catching conclusion that can be deduced from the results is that RD values for all the configurations decrease as the LFC increases if the exceptions in some models of the underground results under the subsequent stroke (Figure 4.37 and Figure 4.38) are ignored. Secondly, the fact that the Portela model computes distinct results compared to the other models is again observed apparently in these results. Furthermore, it is seen in general that the differences between the models, even the difference of Portela model, decrease as the LFC increases. Even though these differences decrease for 1 mS/m and increase again for 10 mS/m in some configurations, one can easily infer that higher LFC decreases the significance of using dispersive soil model by looking at the general tendency of the results.

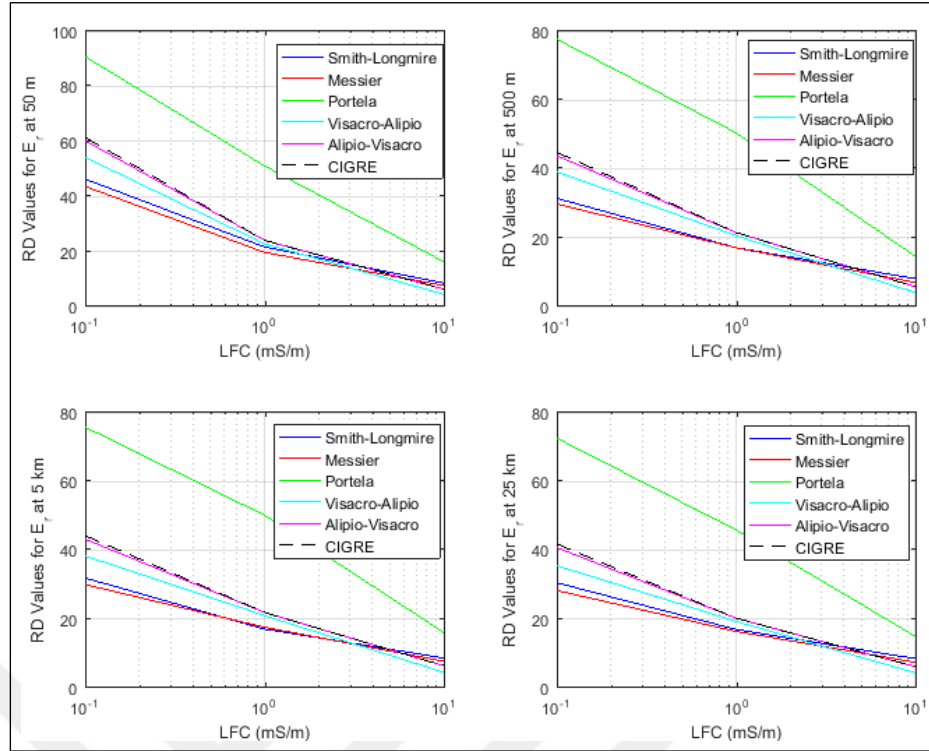


Figure 4.31: Overground RD values for E_r obtained under first stroke at 50 m (top left), 500 m (top right), 5 km (bottom left), and 25 km (bottom right) with respect to the LFC.

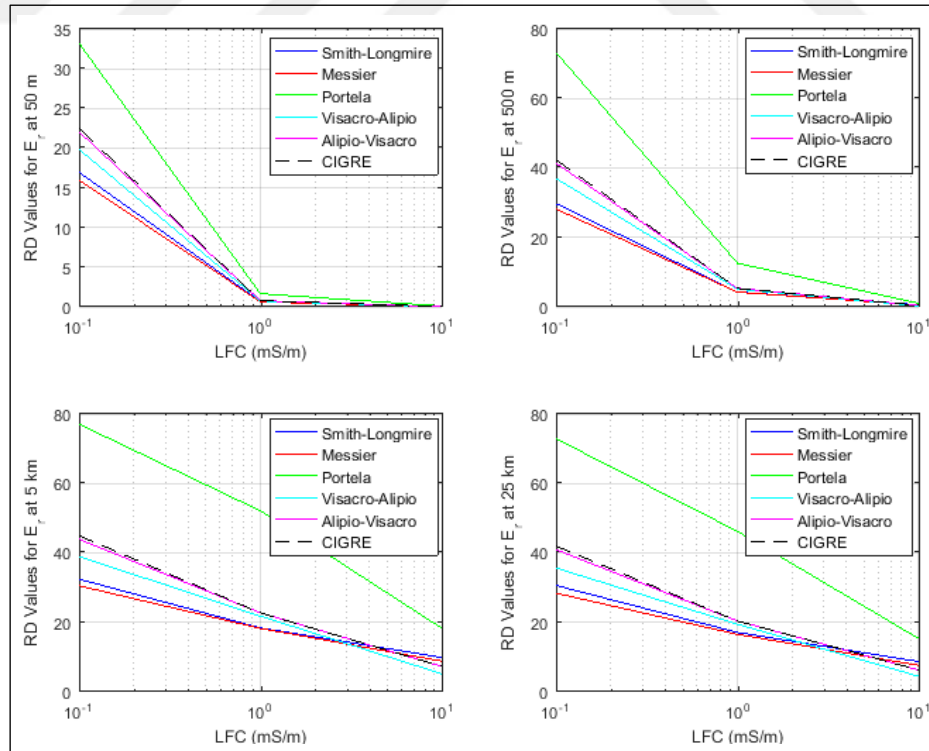


Figure 4.32: Above-ground RD values for E_r obtained under first stroke at 50 m (top left), 500 m (top right), 5 km (bottom left), and 25 km (bottom right) with respect to the LFC.

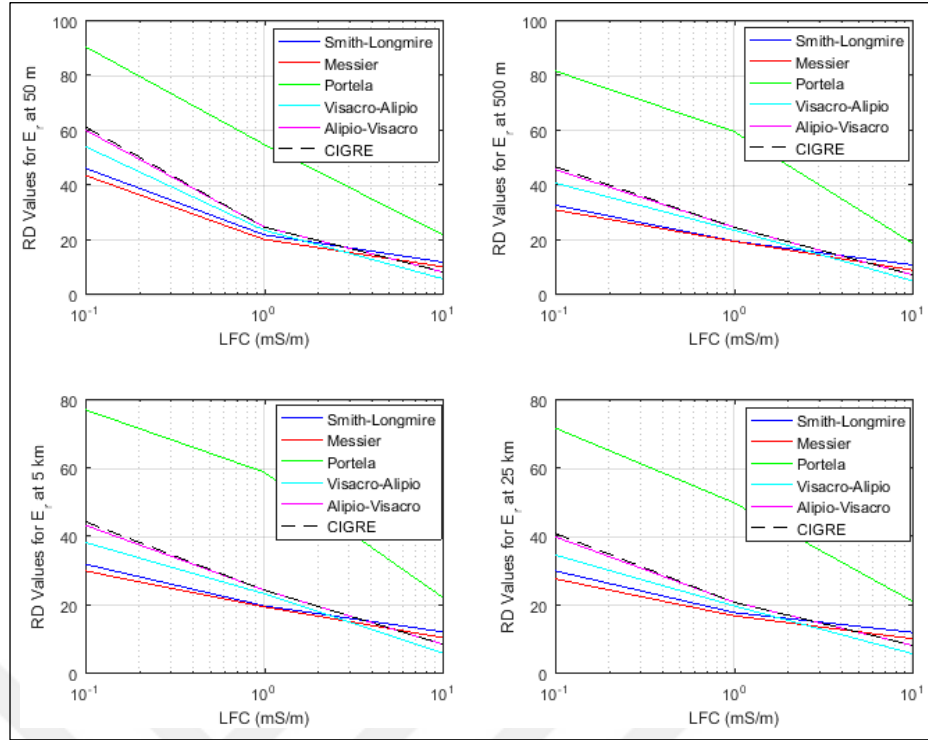


Figure 4.33: Underground RD values for E_r obtained under first stroke at 50 m (top left), 500 m (top right), 5 km (bottom left), and 25 km (bottom right) with respect to the LFC.

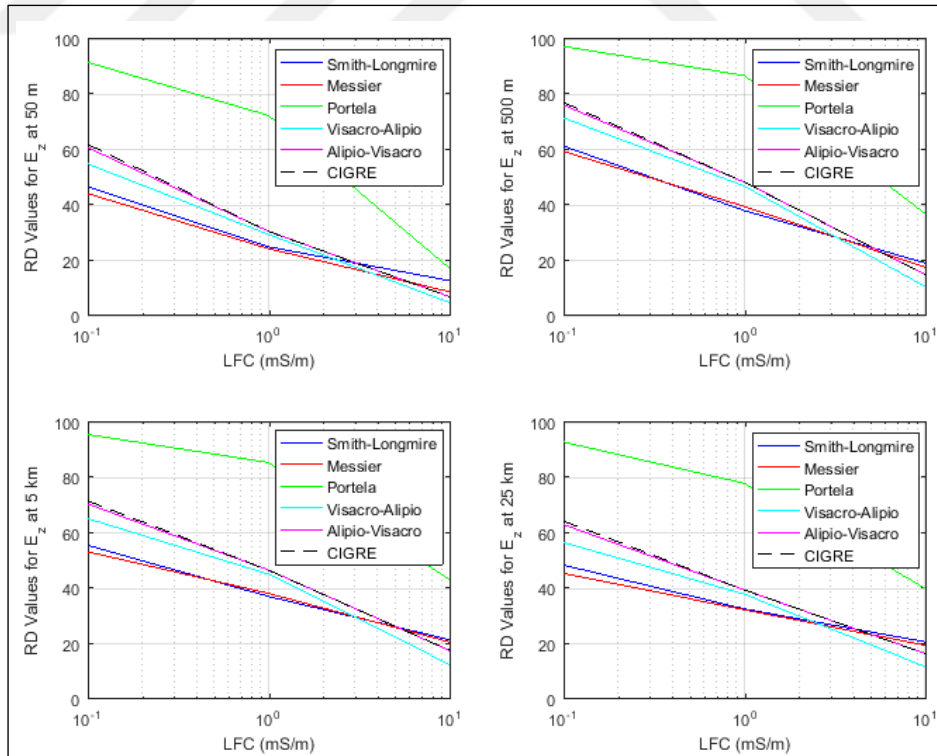


Figure 4.34: Underground RD values for E_z obtained under first stroke at 50 m (top left), 500 m (top right), 5 km (bottom left), and 25 km (bottom right) with respect to the LFC.

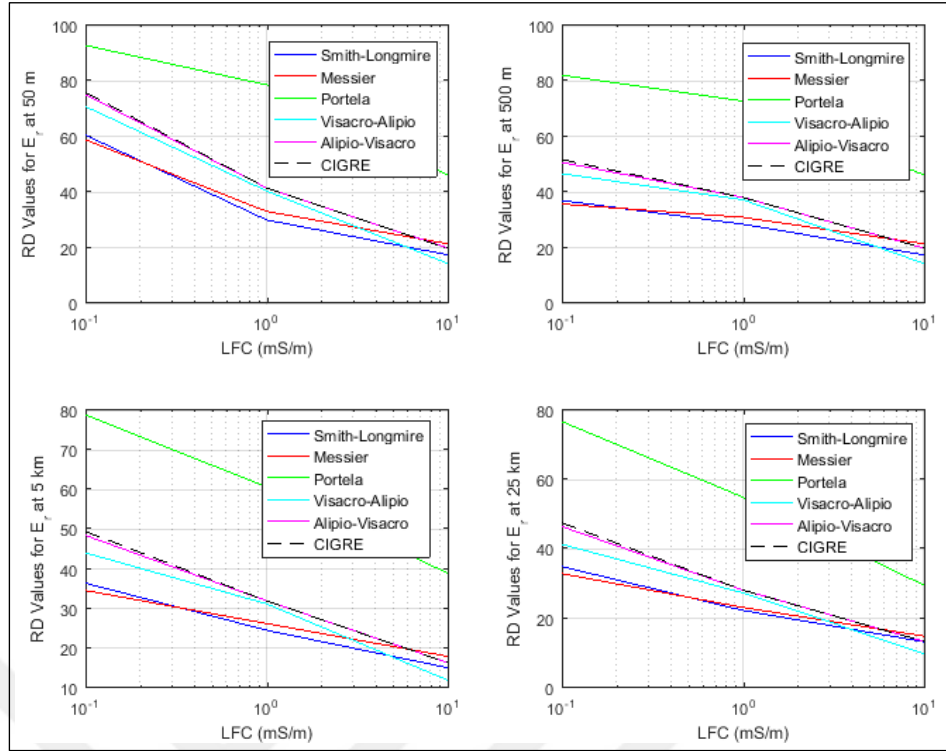


Figure 4.35: Overground RD values for E_r obtained under subsequent stroke at 50 m (top left), 500 m (top right), 5 km (bottom left), and 25 km (bottom right) with respect to the LFC.

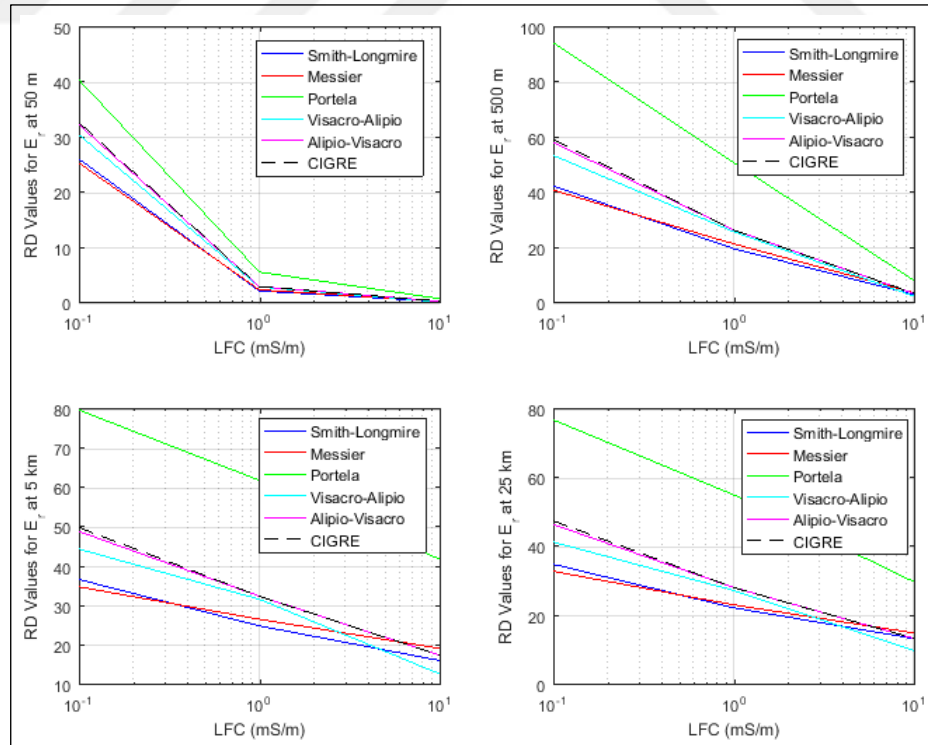


Figure 4.36: Above-ground RD values for E_r obtained under subsequent stroke at 50 m (top left), 500 m (top right), 5 km (bottom left), and 25 km (bottom right) with respect to the LFC.

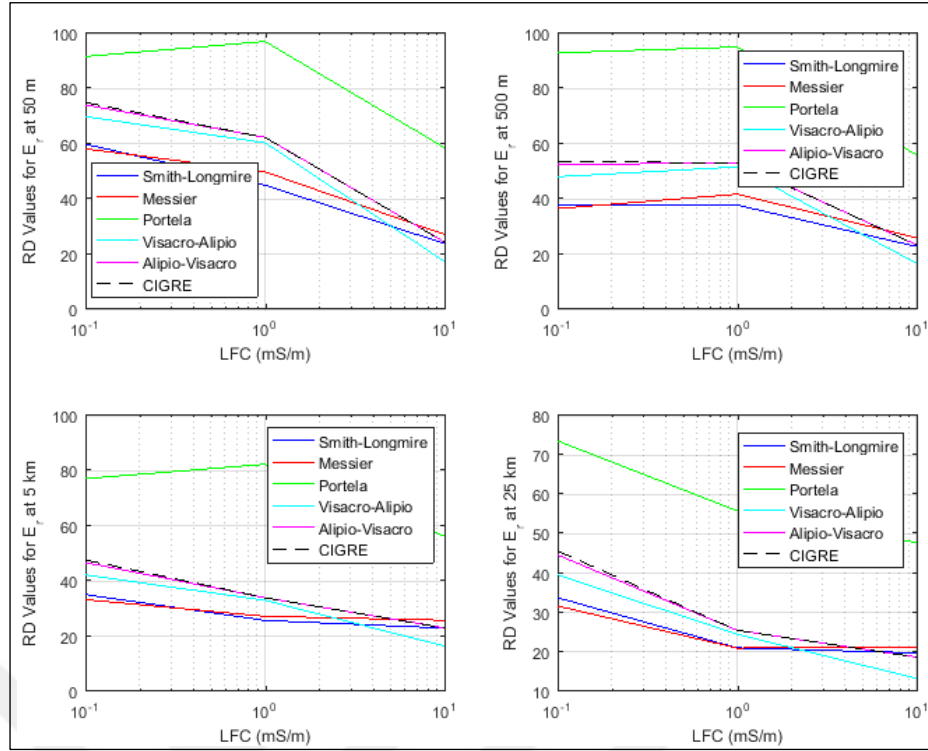


Figure 4.37: Underground RD values for E_r obtained under subsequent stroke at 50 m (top left), 500 m (top right), 5 km (bottom left), and 25 km (bottom right) with respect to the LFC.

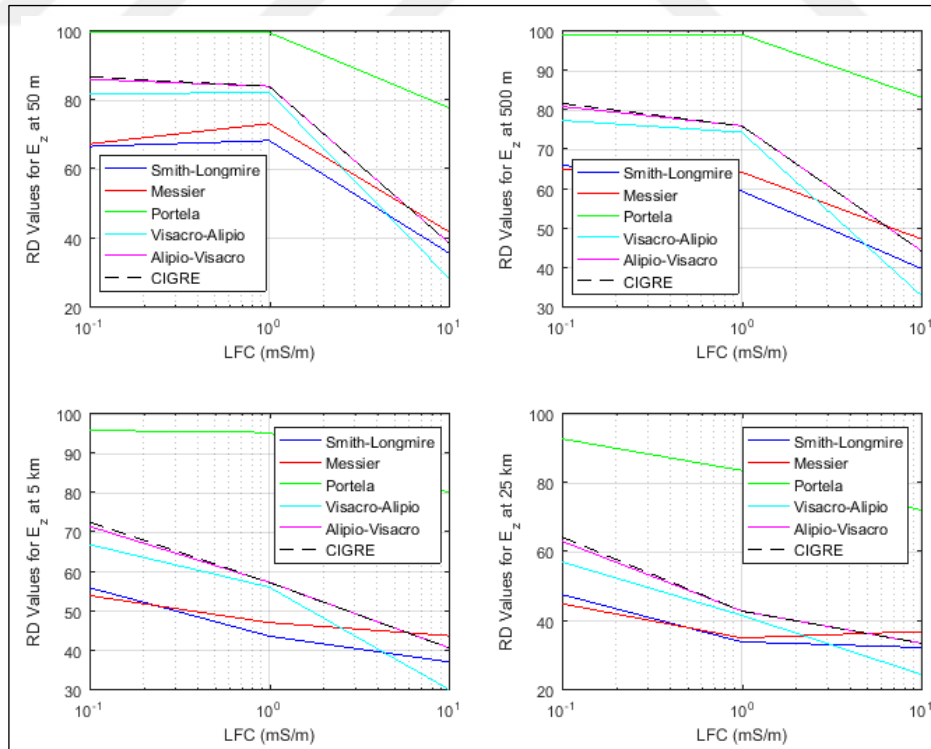


Figure 4.38: Underground RD values for E_z obtained under subsequent stroke at 50 m (top left), 500 m (top right), 5 km (bottom left), and 25 km (bottom right) with respect to the LFC.

To sum up the comments made about the results throughout section 4, it can be said that the LEMP are greatly affected by the frequency-dependence of the ground. The differences more than 100 percent are obtained in the results for some field components. However, the difference between the results of dispersive model and lossy ground model with constant parameters depends on the LFC of the soil, which is related to the soil moisture content. As the LFC increases, the importance of using dispersive model decreases. This means that using dispersive models in the simulation is less important for the wet soils and more important for the dry soils.

Different dispersive models give different results. The question of which model gives the most accurate results may not have a precise answer because the fitted data of most of the investigated dispersive models were measured using different soil types. On the other hand, the Portela model gives very distinct results. This distinction is also observed on its prediction of the electrical parameters of the soil in section 2.3.9. This may be related to the lack of reliable measured data of the Portela model, which is also stated in [Pascoalato et al., 2022].

The other models give more close results. Generally speaking, the results of the Alipio-Visacro and the CIGRE models are almost the same. The model of Visacro-Alipo also gives close results to those of Alipio-Visacro and CIGRE. The models of Smith-Longmire and Messier give close results to each other, but their results are distinct from those of the other three models. Although the differences between the models decrease and the results get closer as the LFC of the ground increases, the model differences for the same LFC are generally preserved at both close and far horizontal distances. It can be said that vertical position of the observation point has no prominent effect on the results of the models, and it leads to different behaviors depending on the field components and the distance. In general, the overground and underground E_r fields tend to decrease as the horizontal distance increases whereas the E_r field above the ground increases as the distance increases except for the results at 25 km.

4.5. Verification of the Results

First, the developed codes for lossy ground with constant parameters are verified by comparing the results with the literature. Later, the results for dispersive ground are verified.

4.5.1. Verification for Lossy Ground with Constant Parameters

The results using the update equations in section 3.1 are compared with the results of Aoki et al. who computed lightning pulses propagating over lossy ground [Aoki et al., 2015]. In the comparison, all parameters are adopted as the same with those in that research, namely the same channel base current, same return stroke type, and same spatial and time steps. The comparison results at 5 km that belong to the E_r and E_z are illustrated in Figure 4.39 and Figure 4.40, respectively.

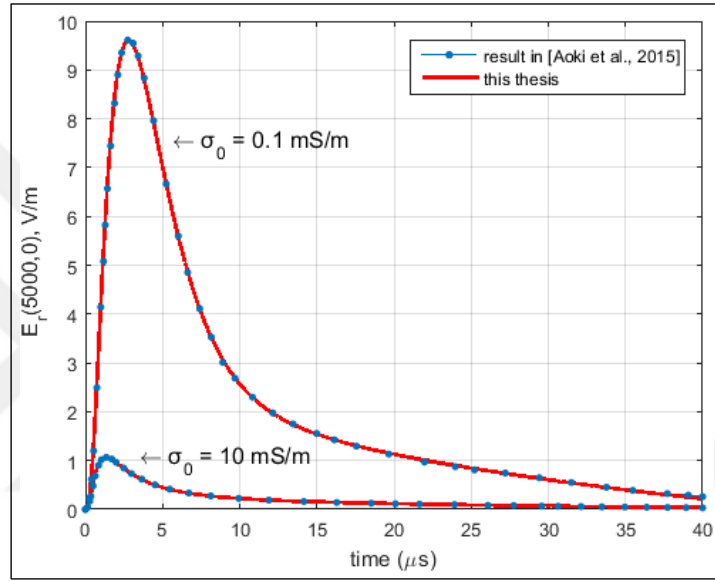


Figure 4.39: The comparison of E_r at 5 km for σ_0 of 0.1 mS/m and 10 mS/m.

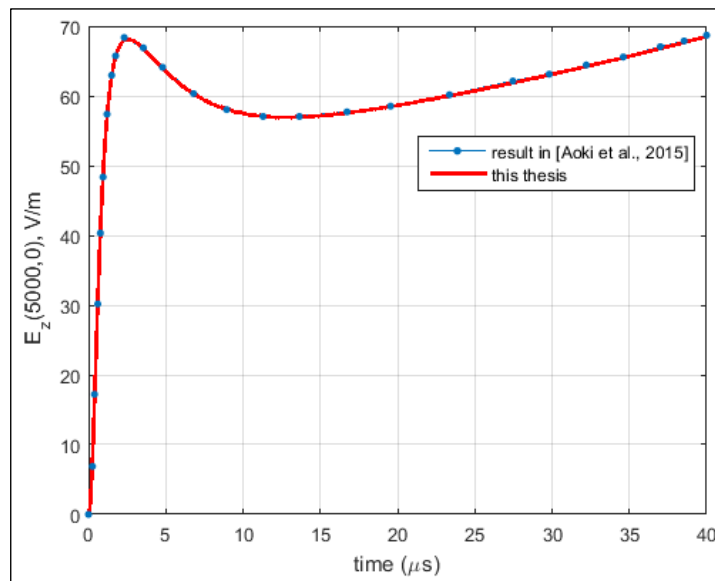


Figure 4.40: The comparison of E_z at 5 km for σ_0 of 10 mS/m.

The E_r field is compared for both 0.1 mS/m and 10 mS/m LFC, and the E_z field is compared for 10 mS/m. As observed in the figures, it is obvious that the developed code and the results in [Aoki et al., 2015] perfectly match, which means a very successful verification (The polarities of the fields are reversed because it is published in the corresponding study in this manner).

4.5.2. Verification for Dispersive Ground

The obtained results for the dispersive ground are verified with other studies in literature computing LEMP. In verification, our results are compared with measurement, FDTD method, and frequency domain methods, respectively.

4.5.2.1. Comparison with Measurement

Since it requires special measurement tools, there are limited numbers of institutions in the world conducting measurements by producing triggered lightning strikes. In an alternative way, the effects of lightning are simulated with tools that generates transient signals like the surge generator and they are commonly used for research on the grounding problem. In addition, there are some measurement results in the literature showing the measured voltage levels of the VLF (very low frequency) components radiated from triggered lightning.

We compared our E_z result with the measurement in [Li et al., 2020] that was measured at 85 km away from the triggered lightning strike. The measured return stroke of the triggered lightning has also been reproduced from the measurement of return stroke, and the corresponding Heidler function parameters are found as in Table 4.19. The measured return stroke and the reproduced counterpart are shown in Figure 4.41.

For the given reproduced return stroke model, the obtained overground E_z field at 85 km away from the channel is shown in Figure 4.42 for the Smith-Longmire model representing the Debye solution and the Messier model representing the Cole-Cole solution. Spatial grid of $10\text{ m} \times 10\text{ m}$ and σ_0 of 10 mS/m are adopted in the simulation. The model results are compared with the measurement in [Li et al., 2020]. The peak value of the measurement is 5 V/m whereas that of Smith-Longmire model is obtained as 4.86 V/m and that of Messier model is obtained as 4.76 V/m. Even though the

obtained general waveform is a little different from that of the measurement, the peak values rather agree with each other.

Table 4.19: Heidler function parameters of the reproduced return stroke.

Stroke Type	I_{01} (kA)	τ_{11} (s)	τ_{12} (s)	n_1	I_{02} (kA)	τ_{21} (s)	τ_{22} (s)	n_2
Reproduced Stroke	16	0.5	16	8	6	11	25	2.8

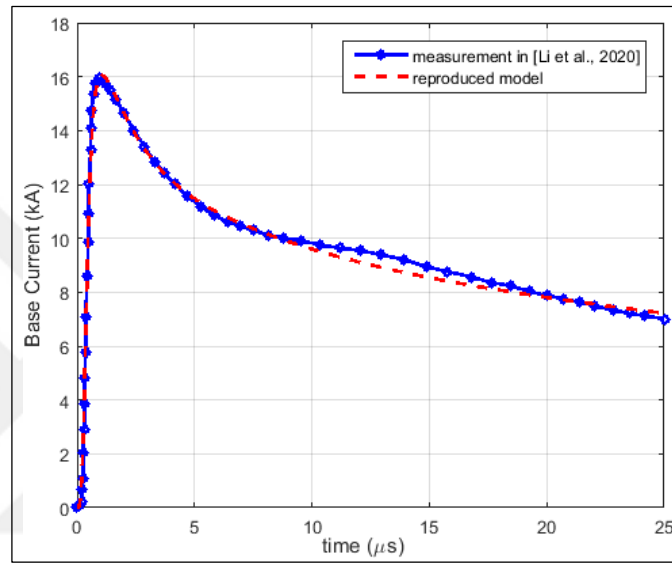


Figure 4.41: The reproduced return stroke model compatible with the measurement.

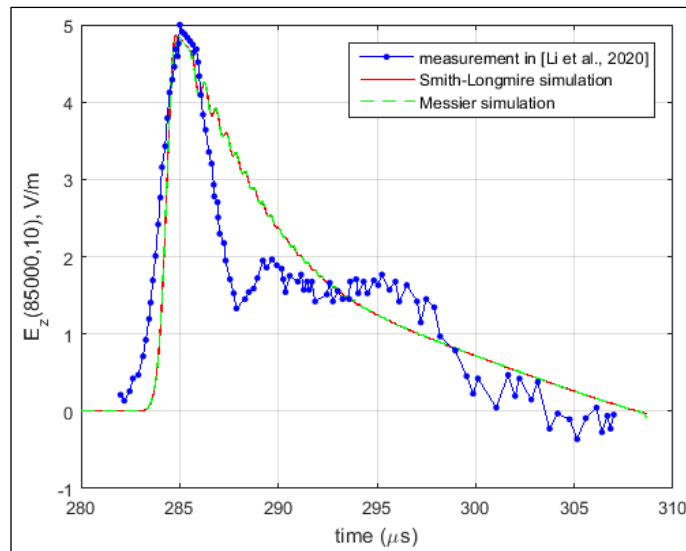


Figure 4.42: Comparison of Smith-Longmire and Messier models with the measurement in literature at 85 km away (The polarity of the field is reversed to compare with the result in the aforementioned study).

It is worth mentioning that the conductivity value of the ground belonging to the measurement site is ambiguous. For this reason, it may be possible to get the simulation results overlap (at least peak values) to the measurement result by adjusting the LFC to the desired value. Thus, from the standpoint of the writer of this thesis, although getting the simulation results compatible with the measurement is significant, comparing the simulation results with such measurements is not a strong idea especially for testing the accuracy of the written codes where little details can change the accuracy.

4.5.2.2. Comparison with FDTD Method

The results for Smith-Longmire model in this thesis are compared with the results of Sun et al. who are the only authors in the literature publishing dispersive FDTD computations of the LEMP [Sun et al., 2018]. They adopted the Smith-Longmire model and semi analytical RC with the VF approach to compute underground E_r field. The same parameters with those in the corresponding paper regarding the return stroke current and a spatial grid of $1 \text{ m} \times 1 \text{ m}$ are adopted. Our calculations are successfully verified at underground observation points as depicted in Figure 4.43 and Figure 4.44.

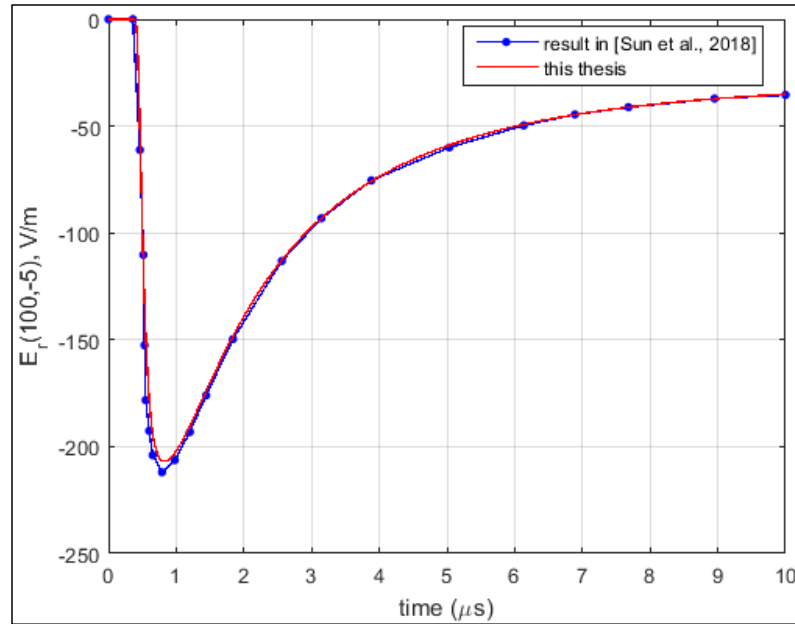


Figure 4.43: Comparison of Smith-Longmire result for observation point 100 m away from the channel and 5 m under the ground for σ_0 of 3 mS/m.

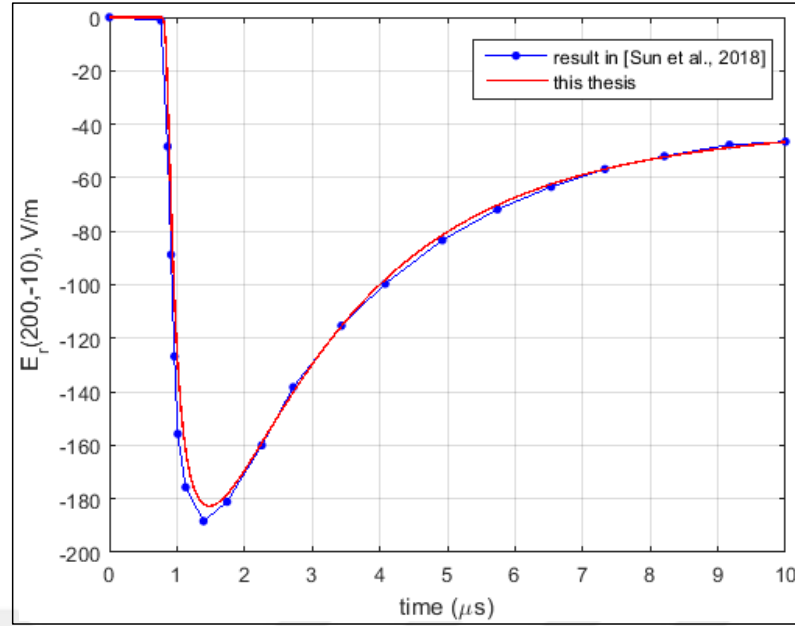


Figure 4.44: Comparison of Smith-Longmire result for observation point 200 m away from the channel and 10 m under the ground for σ_0 of 0.5 mS/m.

In Figure 4.43, the E_r result at 100 m away from the channel and 5 m under the ground is compared with that in [Sun et al., 2018]. It is observed that the two results are almost perfectly matched. The E_r result at 200 m away from the channel and 10 m under the ground is also compared in Figure 4.44. It is seen that a perfect match also exists between these two results except for a small region around the absolute peak point, which may be due to the approximation adopted in the aforementioned research.

4.5.2.3. Comparison with Frequency Domain Methods

In this section, the results for both Smith-Longmire model and Messier model are compared. The result for the Smith-Longmire model is compared with the result in [Delfino et al., 2009] and that for the Messier model is compared with the results in [Li et al, 2020].

Delfino et al. adopted the Smith-Longmire model for the dispersive ground and followed a procedure to compute LEMP starting from analytical equations in frequency domain [Delfino et al., 2009]. The same return stroke parameters used in the research are adopted in the comparison and a spatial grid of 0.5 m $\times$ 0.5 m is used. Our computations are successfully verified for the E_r at above-ground observation point at 800 m away and 10 m above the ground. The comparison of the results is depicted in Figure 4.45. The results are almost perfectly matching up to the absolute

peak point and there is a slight difference between the two results after that point. This difference may result from the numerical IFT they applied for converting the results from frequency domain to time domain, which is a problem mentioned in [Barbosa et al., 2013] before.

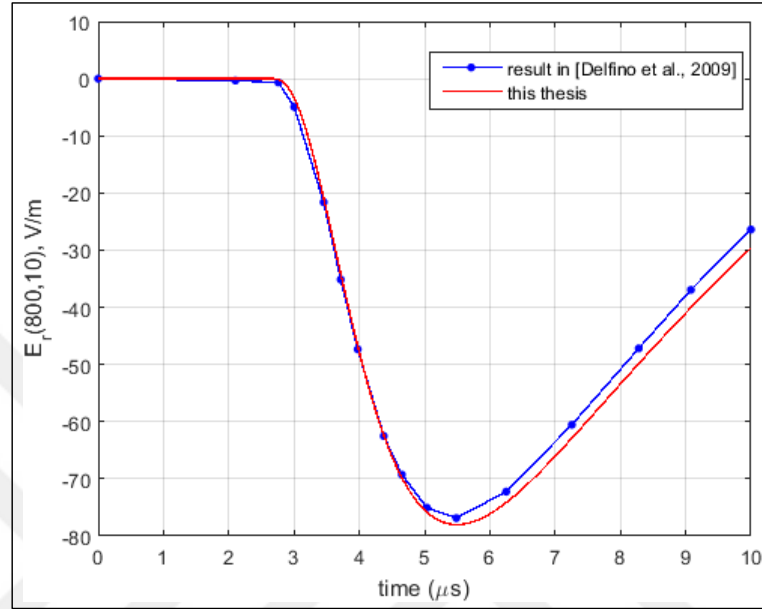


Figure 4.45: Comparison of Smith-Longmire result at observation point 800 m away from channel and 10 m above the ground for σ_0 of 0.23 mS/m.

Li et al. adopted the Messier model for the dispersive ground and computed LEMP using analytical equations in frequency domain where the effects of both dispersion and soil stratification are investigated [Li et al., 2020]. The same return stroke parameters used in this research are adopted in the comparison and a spatial grid of $2 \text{ m} \times 2 \text{ m}$ is used. Our computations are successfully verified with their E_r result obtained for homogenous soil at observation point of 5 km away and 10 m above the ground. The comparison of the results is shown in Figure 4.46. There is a little difference between the results around and after the absolute peak point, which is very similar to the case of previous comparison with the results in [Delfino et al., 2009].

To seek the reason of the difference between our computations and the frequency domain methods, it might be useful to compare the constant parameter lossy ground result with that of the aforementioned paper because the lossy ground results in this thesis exhibit a perfect match with the FDTD results in the literature (section 4.5.1). In this case, the comparison is given in Figure 4.47 where the results with constant parameters at the same observation point are shown. It is observed in the lossy ground

comparison that very similar difference with that of the dispersive ground has been obtained. Therefore, it is apparent that the difference between the two results is related to the difference between the procedure of time domain and frequency domain methods and it is not related to the techniques applied in this thesis.

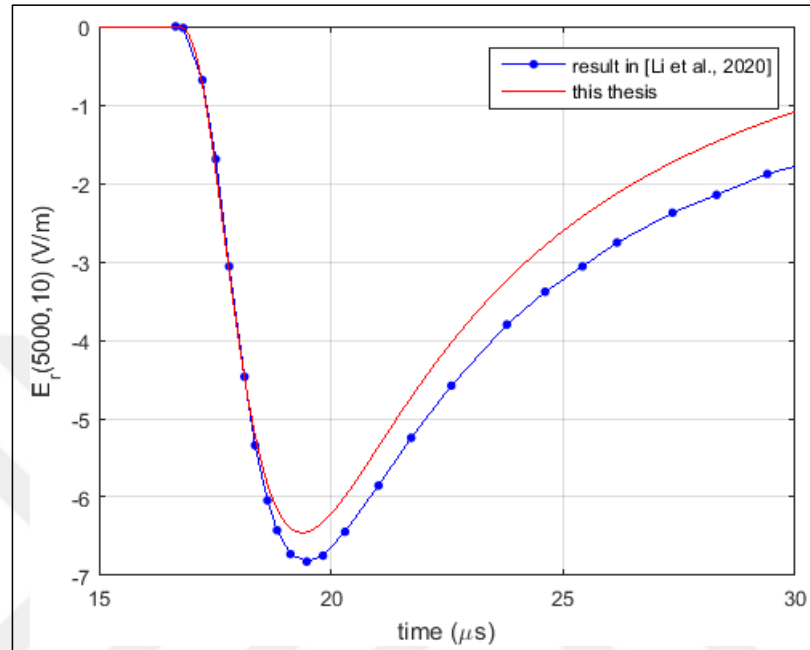


Figure 4.46: Comparison of Messier result at a point 5 km away from the channel and 10 m above the ground for σ_0 of 1 mS/m.

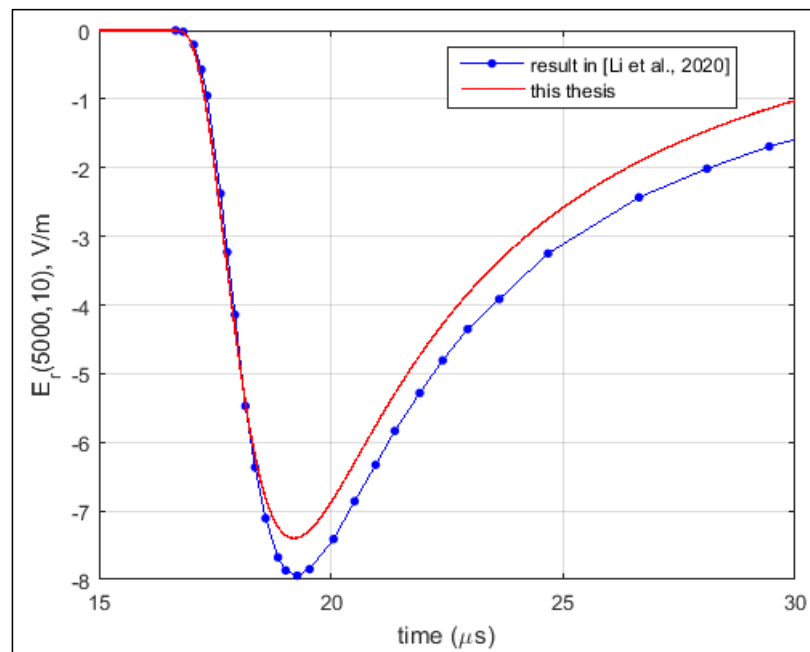


Figure 4.47: Comparison of result for lossy ground with constant parameter at the same observation point of dispersive case for σ_0 of 1 mS/m.

4.6. On the Stability of Scott and Visacro-Portela Models

In section 3.2.3, we followed a fitting procedure anyhow despite saying that the models of Scott and Visacro-Portela seem to be incompatible for the FDTD application. A relatively good fitting has been obtained at least for the imaginary part of the susceptibility equation for σ_0 of 0.1 mS/m and 1 mS/m. Then, we have seen that the simulations for these models give unstable results and were excluded from the comparisons (section 4.3), though. However, it has been observed in the simulation campaign that Visacro-Portela model has convergent results if larger spatial steps (Δr , Δz) are selected. For example, stable results are obtained if spatial steps are larger than 6 m for 1 mS/m results and if they are larger than 45 m for 0.1 mS/m results. The comparisons of the overground results at 500 m away if the spatial steps were both 10 m for 1 mS/m are shown in Figure 4.48.

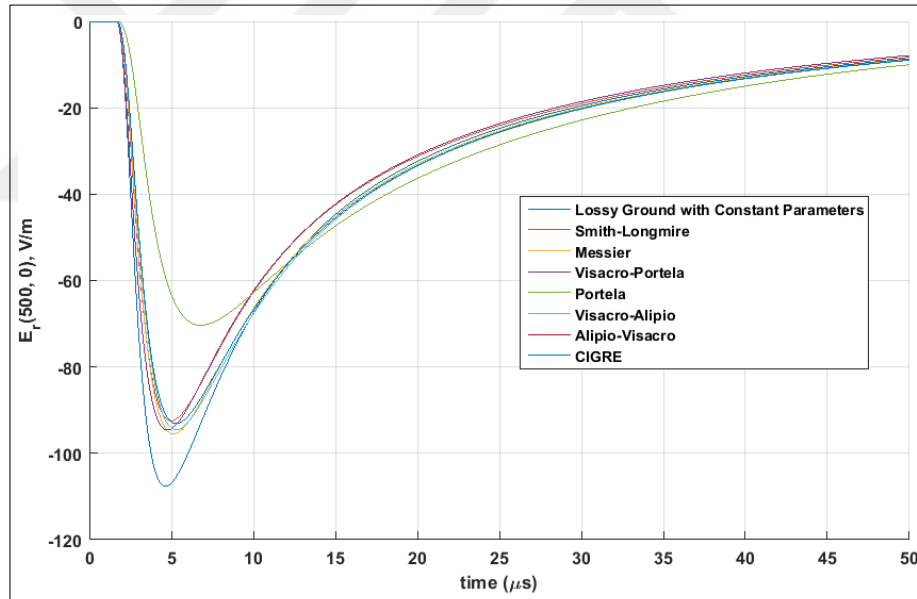


Figure 4.48: The overground results including the Visacro-Portela model observed at 500 m away using first stroke for 1 mS/m for a grid of 10 m $\times$ 10 m.

As observed in Figure 4.48, the Visacro-Portela model gives a result quite close to that of other models. Moreover, Visacro-Portela gives rather stable results for other type of sources like different type of engineering return stroke model, stroke type (first or subsequent), and different base currents if large spatial steps are adopted. In addition, it is seen that the absolute values of the results decreased after adopting larger grid steps, which is an expected result since it is a well-known problem that larger

steps decrease accuracy in the FDTD. On the other hand, the step size intensities for the Scott model to give stable results are found to be larger than 120 m for σ_0 of 1 mS/m, which is a huge number decreasing the accuracy too much.

Independent of the model, the fact that Scott and Visacro-Portela models have stability issues is observed for step sizes smaller than certain values. Smaller step size means smaller wavelength in the FDTD simulations that results in increasing the covered frequency range. For this reason, the problem of stability may be related to the high-frequency effects mentioned in [Kuklin, 2022]. Since the stability analysis is beyond the scope of this thesis, it may be a future research topic or new approaches may be developed to represent these soil models using the Cole-Cole model.

4.7. Computation Time

Computation times of the codes for each dispersive ground model and also for the lossy ground case are measured in Matlab. All the computations for the models have been realized with a standard user laptop with 2.4 GHz Intel Core i7-5500U CPU, Nvidia GeForce 940M GPU, 8 GB of RAM, and 480 GB of SSD Hard Disk.

If ABC-free solution proposed in section 3.3.1 is adopted and the first return stroke with the period of 50 μ s is used, obtained computation times for the configurations in section 4.3 are shown in Table 4.20 for different distances.

Table 4.20: Computation times (in hours) for the models.

		Constant Lossy	Smith- Longmire	Scott	Messier	Visacro- Portela	Portela	Visacro- Alipio	Alipio- Visacro	CIGRE
50 m	0.1 mS/m	0.49	0.82	0.57	0.60	0.52	0.67	0.81	0.82	0.60
	1 mS/m	0.51	0.85	0.57	0.73	0.59	0.67	0.83	0.58	0.65
	10 mS/m	0.52	0.85	-	0.76	-	0.68	0.73	0.93	0.61
500 m	0.1 mS/m	0.55	0.91	0.66	0.70	0.61	0.73	0.89	0.91	0.66
	1 mS/m	0.56	0.93	0.63	0.80	0.67	0.73	0.88	0.64	0.72
	10 mS/m	0.57	0.93	-	0.82	-	0.71	0.77	0.99	0.64
5 km	0.1 mS/m	1.27	1.98	1.39	1.46	1.28	1.56	1.82	1.88	1.39
	1 mS/m	1.26	1.97	1.34	1.68	1.38	1.56	1.88	1.39	1.56
	10 mS/m	1.26	1.98	-	1.73	-	1.57	1.69	2.10	1.39
25 km	0.1 mS/m	6.66	9.49	6.37	6.70	6.13	7.26	8.13	8.24	6.66
	1 mS/m	6.68	9.32	6.21	7.55	6.51	7.21	8.37	6.46	7.25
	10 mS/m	6.69	9.32	-	7.76	-	7.21	7.54	9.07	6.62

If the time increases of the models compared to the constant lossy case are calculated, almost a linear relation is observed between the computation times and the number of coefficients used in the summation terms of the update equations for the corresponding soil models. The number of coefficients for the Smith-Longmire is 13 as can be seen in section 3.2.2.2 and the number of coefficients for the fractional models are given in Table 3.3 in section 3.2.3.5 for each model. The time increases (in percent) of the models with respect to the number of coefficients are depicted in Figure 4.49 for different distances. As can be seen, amount of increase in the computation time is almost linearly proportional to the number of coefficients. Also, it is evident that the increase in computation time drops with the increase in horizontal distance.

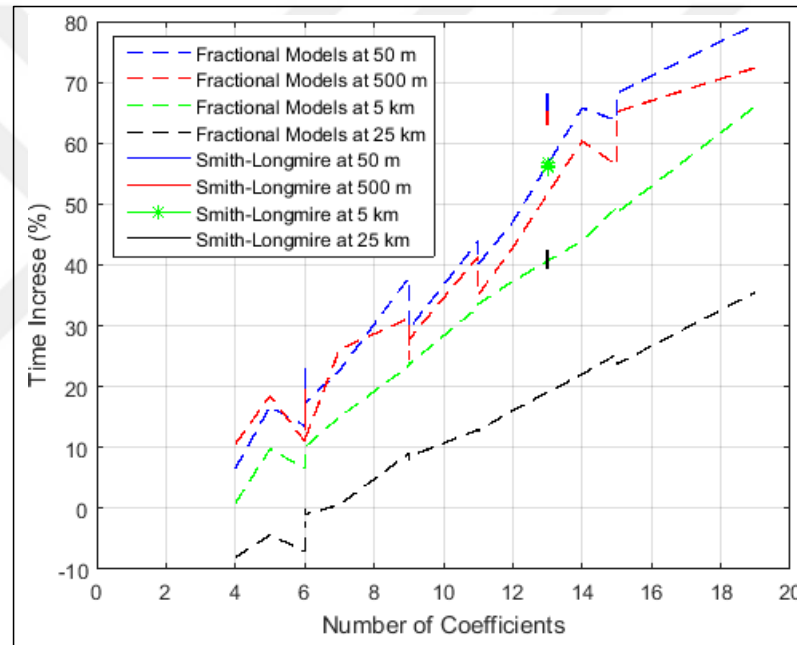


Figure 4.49: The computation time increase (in percent) of dispersive models with respect to the lossy case for different distances.

5. CONCLUSION

In this thesis, computation of the LEMP for the dispersive ground are realized using the FDTD method in 2D cylindrical coordinates. In the computations, eight different soil models are adopted to investigate the applicability of the method of FDTD to the lightning problems with the dispersive ground. The ground is represented using both Debye and Cole-Cole models. The parameters of Smith-Longmire model are used to express the ground as a multi-pole Debye model and the RC-FDTD is applied without any approximation. The remaining seven dispersive models are represented by the Cole-Cole function since the powers of the frequency terms in these models are fractional. The fractional-FDTD that is based on the ADE formulation is applied to these models by fitting the soil model parameters to Cole-Cole function parameters. The obtained results are successfully verified with the literature.

In simulations, computations for each dispersive model were repeated for the configurations of four horizontal distances, three vertical positions, two stroke types, and three soil LFC values. The results for the models of Scott and Visacro-Portela are excluded from the results due to their suffering from the stability problem and divergence of their results. According to the comparison results of the remaining models, using dispersive soil models instead of the finitely conducting ground becomes more important for the low soil LFC and more than 100% differences are obtained for some field components in the computations. The results of Portela model are distinct from those of the others. Smith-Longmire and Messier give similar results in most of the configurations whereas the models of Visacro-Alipio, Alipio-Visacro and CIGRE give close results to each other.

Considering the approach of ground representation, the model of Smith-Longmire is fitted to the Debye model very successfully because no approximation has been assumed in model fitting, which is superior to approaches of other studies in the literature. For the fractional soil models, a fitting procedure involving numerical errors up to some extent has been followed to fit the soil model parameters to the model of Cole-Cole. Among the fractional models, the best fitting performances have been obtained for Messier, Alipio-Visacro, and CIGRE. Thus, these models can be viewed to have more accuracy than the models of Visacro-Alipio and Portela regarding the numerical fitting. However, this does not mean that they give the best results in the

real conditions because it is worth remembering that these soil models were constituted by the respective owners using different soil types.

The application of the FDTD to the LEMP problem for the dispersive ground has been popular in recent years. Therefore, the researchers can benefit from the approaches proposed in this thesis. On the other hand, this thesis computes only the LEMP so the other lightning related problems such as lightning effects on transmission lines, power networks, grounding electrodes and buried cables can be solved using the proposed approaches as future works. Moreover, the optimization approaches used in the fitting procedure of fractional soil models can be developed more in the future. Also, the Scott and Visacro-Portela models can be investigated more, and new techniques can be developed to solve the stability problem of these models arising in case of using smaller spatial steps.

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BIOGRAPHY

Osman Kurnaz received his Bachelor of Science in Electrical and Electronics Engineering from Middle East Technical University in 2008 and his Master of Science in Electrical and Electronics Engineering from Akdeniz University in 2012. During his Master, he worked with the School of Electrical and Electronics Engineering at Akdeniz University as a research assistant. Later, he worked at TUBITAK-UME as a researcher and TUBITAK-BILGEM as an expert researcher. Now, he is working towards the degree of Doctor of Philosophy at the Department of Electronics Engineering in Gebze Technical University.



APPENDICES

Appendix A: List of Publications

Kurnaz O., Aksoy S., (2022), “Debye representation of frequency dependent ground for LEMP analysis with RC-FDTD”, IEEE Transactions on Electromagnetic Compatibility, 64 (6), 2149–2156.

Kurnaz O., Aksoy S., (2021), “Electromagnetic fields radiated by lightning return stroke over lossy ground with rock formation”, IEEE Transactions on Electromagnetic Compatibility, 63 (5), 1444–1451.

Appendix B: Note

This thesis was supported by the Scientific and Technological Research Council of Turkey through the short-term 1002 project under grant number 122E327. The GUI has been developed and the research by [Kurnaz and Aksoy, 2022] (in Appendix A) has been published within the context of the project.