82427

ANTENNA DIVERSITY AND BEAMFORMING FOR CDMA MOBILE SYSTEMS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
THE MIDDLE EAST TECHNICAL UNIVERSITY

BY

ALİME YANARTAŞ ÖZYILDIRIM

82427

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE

DEGREE OF

DOCTOR OF PHILOSOPHY

IN

THE DEPARTMENT OF ELECTRICAL AND ELECTRONICS

ENGINEERING

T.C. YÜKSEKÖĞRETİM KURULU
DOKÜMANTASYON MERKEZİ

SEPTEMBER 1999

Approval of the Graduate School of Natural and Applied Sciences.

Prof. Dr. Tayfur Öztürk
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Doctor of Philosophy.

Prof. Dr. Fatih Canatan
Head of Department

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Doctor of Philosophy.

Prof. Dr. Yalçın Tanık
Supervisor

Examining Committee Members

Prof. Dr. Mete Severcan

Prof. Dr. Yalçın Tanık

Prof. Dr. Mehmet Şafak

Assoc. Prof. Dr. Sencer Koç

Assist. Prof. Dr. Halit Oğuztüzün

ABSTRACT

ANTENNA DIVERSITY AND BEAMFORMING FOR CDMA MOBILE SYSTEMS

Özyıldırım, Alime Yanartaş

Ph.D., Department of Electrical and Electronics Engineering

Supervisor: Prof. Dr. Yalçın Tanık

September 1999, 137 pages

Antenna arrays with beamforming and space diversity improve the performance of mobile communication systems by suppressing cochannel interference and mitigating multipath fading. However, this improvement is reduced by correlation between the antennas. Uncorrelated fading can be achieved if the spacing between the antennas is much greater than half wavelength. The result of this placement is undesired grating lobes in the beam patterns of uniform linear arrays. Therefore, the uniform arrays can not be suitable for this aim. In this thesis, the use of the nonuniform linear array is proposed as a promising solution.

The spacings of nonuniform arrays are generated randomly according to the predetermined criterion. The performance of the nonuniform arrays is investigated for near (LOS) and far users (i.e., multipath exists) in a CDMA system. A Rayleigh correlated channel model is derived for the case of nonuniform array. In order to expose the impacts of correlation on the system performance, a detailed investigation is carried out. Simulation results show that the nonuniform arrays provide better overall system performance than that of the uniform arrays with the same number of antennas, when the average spacing is greater than half wavelength.

Since simulations are computationally very intensive, analytical studies are performed in multivariate framework. Exact expressions for the conditional probability density function (pdf) of SIR and outage probability are accomplished by computing a hypergeometric function with matrix arguments. By exploiting a simplifying assump-

tion, the unconditional pdf and outage probability are derived. The analytical results are confirmed by simulations.

Keywords: Antenna Arrays, Beamforming, Space Diversity, Interference Suppression, Multipath Fading, CDMA

ÖZ

CDMA GEZGİN SİSTEMLERİ İÇİN ANTEN ÇEŞİTLEMESİ VE IŞIN DEMETİ OLUŞTURULMASI

Özyıldırım, Alime Yanartaş
Doktora, Elektrik Elektronik Mühendisliği Bölümü
Tez Yöneticisi: Prof. Dr. Yalçın Tanık

Eylül 1999, 137 sayfa

Işın demeti oluşturulması ve uzam çeşitlemesi ile anten dizilimleri, aynı kanalı kullanan girişimleri bastırırak ve çok yollu sönümü azaltarak gezgin haberleşme sistemlerinin başarımlarını arttırır. Ama bu başarım, antenler arasındaki ilinti nedeniyle azalır. Eğer, antenler arasındaki uzaklık dalga boyunun yarısından oldukça büyükse, ilintisiz sönüme ulaşılır. Bu yerleşimin sonucunda, düzgün doğrusal dizilimlerin anten ışıma örüntüsünde istenmeyen loblar oluşur. Bu nedenle, düzgün doğrusal dizilimler bu amaca uygun değildir. Bu tezde, düzgün yerleşimli olmayan doğrusal dizilimler umut verici bir çözüm olarak önerilmiştir.

Düzgün yerleşimli olmayan dizilimlerin anten uzaklıkları önceden belirlenen bir kritere göre rastgele olarak üretilmiştir. CDMA sisteminde, düzgün yerleşimli olmayan anten dizilimlerinin başarımları, hem yakın (LOS) hem de uzak kullanıcılar (çok yollu sönümün var olduğu durumlar) için araştırılmıştır. Düzgün yerleşimli olmayan dizilimler için Rayleigh ilintili bir kanal modeli türetilmiştir. İlintilerin sistem başarımı üzerindeki etkilerini ortaya çıkarmak için ayrıntılı bir araştırma yapılmıştır. Benzetim sonuçları, ortalama anten uzaklığı yarım dalga boyundan büyük olduğunda, düzgün yerleşimli olmayan dizilimlerin, aynı anten sayısına sahip olan düzgün dizilimlere göre daha iyi sistem başarımı sağladığını göstermiştir.

Benzetim çözümlemeleri hesaplama açısından oldukça yoğun olduklarından, anali-

tik çalışmalar çoklu-olasılıksal değişken ortamında yapılmıştır. Matris değişkenli hipergeometrik fonksiyonun hesaplanması ile SIR'ın koşullu olasılık yoğunluk işlevi (pdf) ve eşik olasılığı için tam ifadeler elde edilmiştir. Analizleri basitleştiren bir yaklaşım kullanılarak, koşulsuz pdf ve eşik olasılığı türetilmiştir. Analitik sonuçlar benzetimler aracılığı ile doğrulanmıştır.

Anahtar Kelimeler: Anten Dizilimleri, Işın Demeti Oluşturulması, Uzam Çeşitlemesi, Girişim Bastırma, Çok Yollu Sönüm, CDMA

ACKNOWLEDGMENTS

I wish to express my deep gratitude to Prof. Dr. Yalçın Tanık for his knowledgeable supervision and constructive criticism in the development of this thesis.
Special thanks to ASELSAN Inc. particularly Communications Division for financial support for attendance technical meeting. I would like to thank to my sincere friends for being such a good listener when I felt hopeless during my graduate studies.
I am grateful to my parents for their moral and financial support throughout my life, and for encouraging me to pursue a higher education.
I am indebted to my husband for his help in writing this manuscript, and his patience and encouragement throughout this study.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

The past decade has shown that the demand for wireless communications is huge. In general, wireless systems have been categorized in two groups, depending on their range of operation. Wide area wireless systems involve highly mobile users, whose primary concern is to provide high speed and high quality communication at any time at any location in the world. For local area wireless systems, on the other hand, convenience is the primary factor since wireless is basically substitution of a local wireline.

The wireless users have generally common concern about cost, size, weight, talk and standby time, voice quality, channel availability, ease of the use and privacy. However, the user requirements that drive design tradeoffs and technologies for local area and wide area wireless systems can be diametrically opposed in the application.

The local area application has been started with the invention of cordless telephones to provide tetherless voice communications in the home. Analog technologies with proprietary standards were employed in almost all cordless applications. Although voice communication demands dominate big market segment in the local area wireless industry, data communications have been growing in importance. Especially, the popularity of Internet has triggered to explode the data service demands. In general, voice and data communications have the same design constraints such as size, weight, operation time of the user unit, however, data communication needs primarily higher data rate and higher quality. Thus, digital technology has penetrated in the local area wireless communication. Although cordless systems have been basically the alternative for a wireline system, digital technologies have been extending cordless communications to cover low mobility, pedestrian application in environments such as offices, buildings,

shopping malls, airport, etc.

The immense demand for wireless mobile communications results in the need for efficient use of available frequency spectrum. In the early 1980's cellular era started to attain primarily a higher user capacity with a higher quality as compared to conventional telephone systems. The main idea is to reuse the available frequency spectrum in different geographical locations. The key requirements driving cellular mobile systems can be stated as follows: high mobility, capability of handoff from one cell to the next, spectral efficiency, coverage, and user capacity.

The cellular scheme has been rapidly adopted throughout the world. There have been a considerable progress and a considerable amount of work going on in the area of cellular mobile communications. The evolution of the cellular systems has resulted in the availability of the wireless mobile systems in the commercial market.

The great increases on the number of cellular subscribers beyond expectations have introduced the demand for more efficient technology. Thus, digital technology has also penetrated in cellular mobile systems as well as cordless telephone systems. Since digital signals are quite resistant to noise and interference, they allow much larger number of mobile users to be supported within the allocated frequency band, provide better voice quality, and permit a range of value-added services such as data communications. Digital signals can offer not only improved capacity but also possibilities of secure voice and data transmissions [1]. Moreover, with appropriate source coding and channel coding, digital techniques offer significant reduction either in the required transmitter power or the transmission bandwidth.

In digital cellular systems, different forms are used. With multiple access techniques, numerous mobile users share simultaneously a specified radio spectrum. The three major access schemes are frequency division multiple access (FDMA), time division multiple access (TDMA), and code division multiple access (CDMA). In FDMA, the frequency spectrum is divided into disjoint bands that are assigned to different users. A TDMA scheme allocates different time slots to different mobile units using the same carrier frequency. A CDMA scheme, on the other hand, is a spread spectrum technique that uses a unique code for each user. In addition to FDMA, TDMA and CDMA, space division multiple access (SDMA) techniques are widely employed in wireless communications. With SDMA, user accesses can be apportioned to space or angle domain as well as frequency, time and code domains in FDMA, TDMA and

CDMA respectively. Since a single access scheme does not appear to be the best for all situations in mobile communication systems, hybrid schemes are widely employed in practical applications [2].

The first-generation cellular systems were introduced in analog form to provide local mobile speech services, and afterwards they were extended to countrywide coverage. Various different standard systems have been developed and adopted: AMPS (Advanced Mobile Phone Service) in United States, NTT (Nippon Telephone and Telegraph) systems in Japan, TACS (Total Access Communication Systems) in United Kingdom, NMT (Nordic Mobile Telephones) in European countries, and so on [3]. The first-generation systems were based on FDMA scheme. Capacity and quality were the major problems in these systems as well as incompatible systems.

The developments in the digital technology gave birth to the second-generation systems providing digital voice services and short message services. The pan European GSM (Global System for Mobile Communications) in Europe, PDC (Personal Digital Cellular) in Japan, IS-54/136 (Interim Standard) in North America employ a hybrid FDMA/TDMA scheme. In addition to IS-54/136, IS-95, which is based on FDMA/CDMA scheme has been developed in North America. The advancements of new digital cordless telephone technologies brought about the supplementary second-generation systems such as PHS (Personal Handy Phone System) in Japan, DECT (Digital Enhanced Cordless Telephone) in Europe, and PACS (Personal Access Communication Services) in North America. Although the second-generation systems and their supplements cover local, national, and international services, they will still have one major drawback in terms of a universal service facility that will enable users to roam freely between different types of networks anywhere in the world, and allow them communicate with anyone at any time at any place [3].

As moving into the next millennium, wireless systems seem to play a continuing significant role to meet communication needs of the twenty-first century. The total number of wireless subscribes is approximately 100 million now, however it is expected to be in excess of 500 million in the year 2001 [1, 4]. This tremendous growth does not reflect only demand for voice communication but also for other communication services involving data, video, and image. The major trend in wireless communication is towards mobile multimedia services involving nearly all communication services. Hence, in order to meet future needs, and handle a wide range of customer demands, third generation

wireless systems have been brought about.

The third-generation system specifications and standard, a worldwide standard, named as FPLMTS (Future Public Land Mobile Telephone System) or IMT-2000 (International Mobile Telecommunications) are being formulated by ITU (International Telecommunications Union). In Europe, this is supported by the UMTS (Universal Mobile Telecommunications System) programs within the European community. UMTS is intended to provide multi-function, compatible digital mobile radio system that would integrate paging, cordless, and cellular systems, as well as low-earth orbit (LEO) satellites into one universal mobile system [5]. Thus, a pocket telephone would be served in almost any location on earth. As a result of the ongoing research throughout the world, it seems that either CDMA or one of the hybrid multiple access schemes based on CDMA in conjunction with TDMA or FDMA will be employed as the future multiple access scheme [6].

Spread spectrum techniques are known to be quite resistant to fading and multipath effects because of implicit diversity capability, which becomes more pronounced as the bandwidth increases well above the *coherence bandwidth* of the channel. Of many potential uses for spread spectrum communication in commercial applications, direct sequence (DS) code division multiple access (CDMA) appears to be the most popular. In 1989, Qualcomm Inc. proposed the use of DS-CDMA scheme to overcome the capability limits of second-generation systems. Qualcomm has further developed the idea of a DS-CDMA digital cellular system into actual implementation that has been adopted as an Interim Standard (IS-95). It is claimed that the capacity improvement of CDMA over TDMA or FDMA is in the order of 4 to 6 [7]. The fundamental reason for this increase is due to fact that DS-CDMA can reuse the same frequency for all cells. In addition to frequency reuse that results in high spectrum efficiency, CDMA also offers some attractive features such as inherent diversity, voice activity gain and so on. Due to these favorable features, CDMA has been accepted as a promising technique for future cellular communication systems.

A key element in DS-CDMA is DS spreading that can be performed by using low-correlation spreading codes. Since all users share the same frequency spectrum in DS-CDMA, correlation values among codes determine multiple access interference (MAI) in the system. On the other hand, there is a major disadvantage associated with CDMA, which is known as the near-far problem: The signals closer to the base station of interest

arrive with a higher power than signals far away from the base station, which increases MAI. The most popular solution is the use of power control mechanism within the cell. Since every user is a source of interference to every other user, it makes power control the most important aspect of a CDMA system. However, power control cannot ensure acceptable interference level from adjacent cells: The signals from these cells may arrive with higher power levels due to independent fading.

From all above, DS-CDMA tend to be interference limited while FDMA and TDMA are bandwidth limited. This implies that MAI places a limit on the number of simultaneously active users in CDMA systems, and thereby any reduction in MAI will result in a linear increase in the capacity of CDMA systems. In cellular communications systems, the capacity is limited due to cochannel interference regardless of cell sizes, whereas the system performance is limited due to the mobile channel. Thus, the understanding of propagation characteristics play a crucial role in the efficient use of a transmission medium.

The existence of reflectors and scatterers in a mobile environment changes amplitude, phase, time and angle of arrival of the transmitted wave. Hence, the received signal from a mobile is considered as a combination of multiple versions of the transmitted signal, which is referred to as *multipath propagation.* When all multipaths stem from the vicinity of the mobile, the received signal varies quickly as the mobile changes its location. This is often referred to as *fast fading.* However, *slow fading* occurs as the mobile travels over a much larger distance, and the scattering environment changes significantly.

Due to multipath nature of propagation, the multiple and delayed copies of the transmitted signal result in spreading the received signal in time. The inverse of this delay spread is known as *coherence bandwidth* of the channel. If the bandwidth of a transmitted signal is above the coherence bandwidth of the channel, the different frequency components of the transmitted signal arrive at a receiver, at different times, and such a channel is known as *frequency selective* (i.e. time dispersive). On the other hand if the fading is independent of frequency, it is referred to as *flat fading*, which is the most difficult channel in spread spectrum applications. All of above channel impairments degrade the performance of cellular mobile systems significantly. In order to overcome the impairments caused by fading various types of techniques such as diversity, forward error correction (FEC), equalization, etc. need to be employed

appropriately.

Diversity techniques can be employed in time domain with the use of FEC, in the frequency domain through the transmission of a wideband signal, in space domain with the use of two or more antennas, which are spaced sufficiently apart. Space division multiple access (SDMA) or *space diversity* (also known as *antenna diversity*) has received the most attention in mitigating multipath fading distortion. With the use of sufficiently separated antennas, the fading envelopes of the received signals become uncorrelated, and then all received signals are combined to collect the energy from different paths.

Due to practical difficulties in implementing multiple antennas for a mobile, multiple antennas are generally located at the base station. However, Vaughan [8] presents a discussion of multiple antennas at the mobile unit. Very early systems employed the base station antenna diversity to improve the quality of the communication link. The newer systems have used multiple antennas to intensify the average received signal power of the intended user while suppressing the average interference power. The concept of *optimum combining* or *beamforming* has been developed as a natural extension of antenna diversity scheme. The process of beamforming implies weighting the received signals at antennas, and thereby adjusting their amplitudes and phases such that when added together they form the desired beam. With optimum combining, capacity enhancement is also achieved due to interference suppression. Recently, real time *adaptive antenna arrays* have been employed, which are intended to maximize the instantaneous signal-to-interference-plus-noise ratio (SINR) at the output of the array.

There has been a considerable amount of recent interest in developing adaptive array techniques for mobile communication systems [9, 10, 11, 12, 13, 14, 15]. It has been concluded that adaptive arrays with beamforming and space diversity improve the system performance by increasing the channel capacity and spectrum efficiency, extending range of coverage, suppressing cochannel interference, and mitigating multipath fading and time dispersion. Further performance improvement can be attained when the antenna arrays are used in conjunction with other techniques such as equalization, interference cancellation, FEC, etc. When an antenna array is employed with coding schemes together, a better BER performance is achieved than no-coding schemes [16]. An antenna array operating in a combined mode of space diversity and equalization using a tapped delay line filter for the individual antenna provides the potential bene-

fits in mobile communication systems. Space diversity is useful to overcome flat fading, whereas equalizers are normally used to reduce intersymbol interference (ISI) caused by frequency selective fading. Hence, combined scheme, which is referred to as *spatial-temporal filtering*, offers an effective means to combat the effects of time dispersion as well as flat fading [17, 18, 19]. Moreover, antenna arrays with other diversity combining schemes such as MMSE (Minimum Mean Square Error), MLSE (Maximum Likelihood Sequence Estimation) receivers are also used to combat ISI and multipath fading in wideband mobile communications [20, 21, 22, 23].

In general, with optimum combining, an M element antenna array can null up to $M-1$ interfering signals, and thus permit M simultaneous signals in the same spectrum. A combination of interference canceling techniques and an antenna array provides a means to reduce the effect of interferers, and thereby improves the performance of the system as well as capacity [11, 24, 25, 26, 27].

Basically an adaptive antenna array system is composed of spatially distributed antennas, receiver for each antenna, and signal processor that generates a weight vector for combining antenna outputs. In order to combine antenna outputs properly, the weight vector based on the statistics of the received signals should be determined. Fundamentally, the objective is stated such that the beamformer response is optimized with respect to a predetermined criterion, therefore the output of the beamformer consists of minimal contribution from noise and interference. There are a number of criteria to determine the optimum weight: The most common ones are *minimum mean square error* (MMSE), *maximum signal-to-interference+noise ratio*, and *maximum likelihood* (ML) criteria [28]. Several algorithms have been proposed to find the optimum weights based on these criteria. Some of the algorithms require either the knowledge of a reference signal, a training signal or the direction of the intended signal to accomplish the prescribed objectives. Some other algorithms have also been proposed that exploit properties of signals to avoid the requirement of training signals or reference signals [12, 29, 30]. These techniques are known as *blind beamforming techniques.*

Due to multipath fading and mobility, the mobile radio channel varies with time, and it must be estimated and tracked during communication. There has been a wide range of research to update the beamformer weights. Most popular ones are *least mean square* (LMS), *direct matrix inversion* (DMI), and *recursive least squares* (RLS) algorithms. The performances of these algorithms have been investigated in different

applications of mobile radio systems to track the desired and interfering signals [11, 31].

Considering the future needs of wireless communications, it is apparent that CDMA schemes with adaptive arrays have a potential to offer significant performance and capacity improvement. There has also been a wide range of research covering different aspects of CDMA systems with adaptive antenna arrays [12, 32, 33, 34, 35, 36].

In the antenna array concept, multiple antennas are arranged in various forms depending on the application and the physical environment. The most common form of an antenna array is the *linear array*, which consists of a finite number of antennas arranged along a straight line. The linear array has two well-known structures: *uniformly spaced array* and *nonuniformly spaced array*. The former has multiple antennas with equal inter-element spacing, whereas the latter has antennas with unequal inter-element spacing.

In the beamforming concept, the antenna array beampattern is of paramount importance. The array beampattern is described by the width of the main lobe and the level of sidelobes. The former impacts the resolution of the array (i.e., accuracy of angle of arrival estimation), and the latter determines the ability of interference rejection. Previous studies on optimum combining assumed independent fading of the received signals at different antennas. The signals from the receiving antennas are weighted proportionally to their individual SINR, and then summed. This is referred to as *maximal ratio combining* (MRC) that provides the optimum solution in narrowband communications. However, in mobile communications since the signals arrive mainly from a certain direction, and the same interfering signal can be present at each antenna, MRC cannot provide the optimum solution. The essential condition for space diversity is that sufficient decorrelation of fading signals be attained. When the correlation between antennas is high, the signals tend to fade simultaneously, and thereby the diversity benefit of an array is significantly reduced.

Depending on the correlation characteristics, uncorrelated fading can be achieved if the spacing between antennas is much greater than half wavelength. As the spacing exceeds half wavelength significantly, undesired grating lobes that have the same gain as the main lobe occur in the array beam pattern of the uniformly spaced array. This problem is most severe for line of sight (LOS) users in which case multipath effects are negligible. Since antenna arrays are also intended to suppress MAI in addition to combating multipath signals, uniform arrays do not seem to be suitable. Therefore, we

propose the use of nonuniformly spaced arrays to achieve the compromises. There has been a widespread interest in the subject of nonuniformly spaced array design. Most of methods are based on the uniformly spaced array with a spacing of half wavelength. Then, the spacings are perturbed slightly to accomplish the predetermined objectives such as sidelobe reduction, desired beam pattern, etc. Since available methods in the literature are not suitable to our problem, we have tried to constitute a nonuniform array by maximizing SINR at the output of the beamformer. However, finding optimum spacings is an extremely difficult problem, and conventional optimization techniques cannot be applied directly. Therefore, we have formed a nonuniform linear array by generating spacings randomly according to predetermined statistics. The length of array (i.e. aperture size) and the number of antennas are selected as the same for both uniform and nonuniform arrays so that the average spacings of both arrays are equal.

In this thesis, we have explored the performance of nonuniformly spaced arrays with joint fading reduction and interference suppression for a CDMA mobile system in a correlated Rayleigh fading environment. We developed a model of DS-CDMA single cell system with an antenna array. The main features of the model are briefly stated as follows: We consider a linear array located at the base station, which uses the array to receive and transmit to the mobile user. Since the mobile to the base station (reverse link) presents the most difficult part of the problem in CDMA cellular systems, we particularly consider the reverse link of a CDMA system. We make the assumption of perfect power control, i.e. the average power received by an antenna for each user is kept at a predetermined level. The CDMA system of concern is assumed to have a sufficiently small cell so that the users' signals arrive at the base station approximately at the same time.

In order to understand the array performance with joint fading reduction and interference suppression, the effects of correlation between antennas are investigated. The fading correlation determines the spacing between antennas to attain sufficient space diversity. We use a local scattering channel model where multipath signals arrive at the antenna array within a range of angles due to scatterers in the vicinity of a mobile. Multipath components are described with different arriving angles, strengths and phases. This implies that the multipath signals can be resolved in space but not in time. The channel vectors are modeled as mutually independently distributed complex Gaussian random vectors, with zero mean and different covariance matrices. This de-

scribes a Rayleigh flat fading channel vector. The closed form expressions are used to determine the correlation as a function of angle of arrival, angle spread, and antenna spacing [37].

We have first carried out analyses and investigations for LOS users. Results have revealed that the proposed nonuniform array provides better performance than the uniform array having the same average spacing. The performance measures are selected as *outage probability* (i.e., a probability that SINR will be below a certain threshold) and *bit error rate* (BER).

In order to expose the impacts of correlation, we have dealt with the performance of far users. The proposed nonuniformly spaced array has been only included in the analysis since the average spacing plays a crucial role in determining space diversity. The degree of space diversity gain depends on the antenna spacing and angle spread of arriving signals. A detailed investigation for the effects of correlation on the overall system performance has been carried out through computer simulations. The investigations have concluded some important remarks: As the angle spread decreases the antenna spacing should be increased in order to obtain space diversity gain since a small spread causes a high correlation between antennas. The performance becomes better even with large antenna spacing. For a moderate angle spread, a moderate spacing provides sufficient performance improvement.

In the earlier phases of our work, the performance of CDMA systems with optimum combining was studied in a correlated Rayleigh fading channel by using Monte Carlo simulations. However, the simulations are computationally very intensive since a wide range of parameters are required to determine the system performance successfully. This problem brings about the necessity of analytical investigation of systems. However, an exact analysis of such a system is extremely complicated due to fading and the large number of interferers compared to the number of antennas. In order to simplify the analysis, we have first neglected the thermal noise. This assumption is generally valid since CDMA systems are primarily interference limited.

Other contributions of this thesis are the derivations of the *probability density function* (pdf) of SIR, and the corresponding outage probability in a correlated Rayleigh fading environment. When the number of interferers exceeds the number of antennas, the array is unable to suppress every interfering signal, and thus a moderate increase in SIR can result in a significant increase in the system capacity. This case is of particular

interest.

The maximum SIR at the output of the optimum combiner involves random variables in quadratic form containing an inverse matrix. The random vector denoted as the channel vector of the desired user is composed of correlated complex Gaussian random variables. The matrix called as total interference covariance matrix consists of mutually independently complex Gaussian distributed column vectors. In fact, each column vector having correlated Gaussian random variables represents one of the channel vectors of interferers. The quadratic form in Gaussian random variables have been studied in multivariate framework. Khatri [38] discussed the pdf of a family of quadratic forms approximately 30 years ago. In all his studies, the matrix is comprised of independently and identically distributed (i.i.d.) random vectors. This corresponds to i.i.d. interferers having complex Gaussian distributions with zero mean and the same covariance matrix. However, in our case, all channel vectors are independently distributed with zero mean but they have different covariance matrices. Therefore, Khatri's result cannot be applied to our problem directly.

Recently the closed form expressions for the pdf of SIR have been accomplished in [39, 40] by using Khatri's results directly. The pdf of SIR is obtained for equal power interferers in [39]. It is shown that the pdf of the SIR at the output of the optimum combiner has a Hotelling T^2 distribution. Similarly, in [40] an exact closed form expression is derived with multiple interferers, each having arbitrary power. Both of the studies assume that all channel vectors including the desired one are i.i.d., and thus the pdf of the SIR does not depend on the covariance matrices or correlation between antennas.

When the interfering signals have non-identical distribution, an exact analysis of the pdf is extremely difficult. However, we have proposed an approach, which makes Khatri's result approximately applicable to our problem. For large number of interferers, the distribution of the total interference covariance matrix is expected to remain almost the same when we assume that all channel vectors are i.i.d with a suitable covariance matrix, while actually they are not. In fact, intensive simulations have verified that our approach is quite valid, which will be presented in Chapter 6.

The pdf of the SIR conditioned on covariance matrices of the desired user and interferers is given in terms of *hypergeometric functions* with two matrix arguments. Hypergeometric functions have been mostly used as analytical representation in multivariate

statistics theory, and there have been a few works about their numerical computations in the literature. Fortunately, Gao and Smith [41] managed to compute hypergeometric functions of matrix arguments based on [141]. We have employed their approach to compute the hypergeometric function of interest.

Although the computation of the hypergeometric function is very lengthy and cumbersome, the resulting pdf is remarkably simple, and this leads to the evaluation of outage probability, which is actually the cumulative distribution of the SIR.

The resulting pdf and outage probability are derived for given covariance matrices of the desired user and interfering users. These conditional probabilities are used to obtain unconditional probabilities. At this point, a key question is which interferer covariance matrix should provide the best result? Firstly, we have concentrated on the total interference covariance matrix conditioned on the channel vectors. Then, the average interferer covariance matrix is defined, and employed to determine the unconditional probabilities. Finally, the unconditional pdf and outage probability are approximated by considering all possible values of the desired user covariance matrix. Throughout the thesis, the angle spreads were selected as the same for all mobile users so that the covariance matrices depend only on the angles of arrivals.

The results obtained from formulas have been compared to those from Monte Carlo simulations. It has been observed that there is a good agreement between the theory and simulations.

The main contributions brought by this thesis can be summarized as follows:

1. The use of nonuniformly spaced array is proposed for the application of CDMA systems in a correlated Rayleigh fading environment.

2. The nonuniformly spaced array performance is investigated for LOS and far users. A mobile channel model is derived for the case of nonuniformly spaced array. A detailed investigation of the impacts of correlation between antennas is carried out to emphasize the gain by means of space diversity in addition to beamforming gain.

3. An exact derivation of the conditional pdf of the SIR is performed with the assumption of i.i.d. interferers. The derivation is accomplished in multivariate framework by computing the hypergeometric function of matrix arguments. The pdf result is employed to derive the conditional outage probability.

4. The unconditional pdf and outage probability are then approximated based on item (3). The average interferer covariance matrix is defined, and the unconditional probabilities are evaluated by considering all possible values of the desired user in the angle ranges of interest.

The outline of the thesis is summarized as follows:

Chapter 2 presents a brief overview of cellular mobile communications, covering mainly basic concepts and features of cellular radio, propagation characteristics of mobile radio channels, and multiple access schemes.

Chapter 3 is an overview of spread spectrum systems. A detailed discussion about DS-CDMA systems are outlined by defining DS-CDMA systems, and describing advantages and disadvantages of CDMA systems.

Chapter 4 covers a detailed discussion about adaptive antenna arrays, and their application to mobile communication systems. In this chapter, fundamental antenna array terminology is presented in order to follow the rest of the thesis. This chapter also contains discussions on previous studies as well as on progress and trends of adaptive beamforming techniques.

Chapter 5 presents the performance analyses of antenna arrays with joint fading reduction and interference reduction for a CDMA mobile system. In this chapter, the core subject of the thesis is given, which is the use of nonuniformly spaced array. The previous studies on the design of nonuniform arrays are reviewed, and the proposed design is then presented. The performance analysis includes description of a CDMA system, characterization of a mobile radio channel, derivation of optimum beamforming weights and SINR, and the definition of performance criteria. The results of uniform and nonuniform arrays for LOS users are presented in this chapter. Additionally, the performance of nonuniformly spaced arrays is investigated for far users in a correlated Rayleigh fading environment. The impact of fading correlation on the performance is studied as a function of angle of arrival, angle spread, and antenna spacing.

Chapter 6 analyzes the statistical behavior of maximum SIR at the output of the beamformer in a correlated Rayleigh fading environment when the number of DS-CDMA users is larger than the number of antennas. The pdf analysis of the SIR is carried out in multivariate theory framework. Based on i.i.d. interfering signals, the expression for the conditional pdf is given through a hypergeometric function with

matrix arguments. This brings about the need for computation of hypergeometric functions. In order to facilitate the presentation of the derivation, the basic features of hypergeometric functions with matrix arguments are overviewed. Exact conditional pdfs are obtained without using hypergeometric functions for two special cases: (i) the number of antennas is equal to 1, (ii) all mobile users including the desired one are i.i.d. For the general case, the computation approaches of hypergeometric functions with matrix arguments are touched on. Then, the evaluation steps of hypergeometric functions are presented. After a lengthy derivation, the closed form expression for the conditional pdf is accomplished. The very simple nature of this pdf leads to evaluation of the conditional outage probability as the cumulative distribution of the SIR. In order to achieve unconditional probabilities, the average interferer covariance matrix is defined. Consequently, approximate unconditional probabilities are determined by considering all possible values of the desired user's covariance matrix. This chapter also includes the results to verify our theoretical results by means of computer simulations.

Finally, chapter 7 focuses on the conclusions and future directions.

CHAPTER 2

CELLULAR MOBILE COMMUNICATION SYSTEMS

2.1 Introduction

Recently, the demand for wireless mobile communications has been growing at an explosive rate. The realization of wireless requires high speed and high quality information exchange between subscribers located anywhere in the world and high user capacity without increase in RF spectrum. In order to achieve this goal and to overcome the operational limitations of conventional mobile telephone systems, cellular mobile communication systems have been developed.

A conventional mobile telephone system was planned by selecting one or more channels from a specific frequency allocation for use in autonomous geographic areas. Since the coverage area was planned to be as large as possible, high power transmitters were needed. Moreover, the mobile user calling in one region had to reinitiate the call while moving into a new region since the previous call would be dropped.

Since each frequency channel served one user at a time in a whole area, the conventional telephone systems could not use the frequency spectrum efficiently. Actually, the main problem in mobile communication systems is the limitation of available frequency bands and an explosive demand for mobile radios. Therefore, instead of the simple mobile telephone system described above, a new system that would operate within a limited assigned frequency band and would serve an almost unlimited number of users in unlimited areas was needed. Cellular systems have been developed to make efficient use of frequency spectrum by dividing the service area in to smaller areas known as *cells*, each containing a subset of mobile subscribers of the cellular system. The available frequency bands can be reused in different cells.

In this chapter, a brief overview of cellular mobile communications will be introduced to understand its principle of working, and the terminology required to understand the rest of the thesis will be presented. The characterization of mobile radio systems comprises basic concepts of cellular systems, operation of cellular systems, existing cellular systems, mobile radio channel and multiple access schemes.

2.2 Basic Cellular System

The communication within a cell is provided by a *base station*, which is responsible for serving calls to or from the mobile users located in its cell. Cellular radio systems involve two radio links: the radio link from the mobile to base station is called the *reverse link* (*uplink*), whereas the link from the base station to the mobile is termed as the *forward link* (*downlink*).

Each cell in a mobile communication system is assigned to its own base station with the allocated frequency for communication. There are basically three different types of cells based on their size as follows [42]:

1. *Macrocells* : A cellular system with large cells which are typically up to 30 km in radius is referred as a macro cellular system. A mobile with high mobility is assigned to a macro cellular system. For outdoor communications, macro cellular systems are employed. The signals arrive at the base station in the order of several microseconds.

2. *Microcells* : This type of cells correspond to small cells with high user density. Small sized cells with the radii in the order of few hundred meters are denoted as microcells. Small cell systems provide the efficient use of frequency, allow large frequency reuse, and thereby result in an increased user capacity. However, it needs frequent handoff processes due to the small cell size. The delay spread in a microcellular system is in the order of hundreds of nanoseconds.

3. *Picocells* : When the cell size is about tens of meters, covering areas such as inside buildings, underground stations, shopping centers, such systems are termed as picocellular systems. The delay spread is in the order of a hundred of a nanosecond.

The cell shape is ideally considered as hexagonal to simplify the planning and design of a cellular system since it approaches a circular shape that is the ideal power coverage area.

A basic cellular system is composed of three major parts: *mobile units*, a *base station*, and a *mobile telephone switching office* (*MTSO*).

Mobile unit provides the interface between the mobile user and the base station. Each mobile unit contains its own radio equipment such as a control unit, a transceiver, an antenna etc. Every cell contains a base station, which is connected to a switching center via radio links, communicates to mobiles, and connects them to the MTSO. The base stations are usually located at the center of the cell or at the alternate corners. The centrally located cell concept is generally applied in small cities which need fewer cell sites. In this case, omnidirectional antennas are employed to communicate the surrounding mobiles. However, in big cities, the base stations are located at the corners and directional antennas are used instead of omnidirectional antennas [43].

The MTSO is the most important part of a cellular system. The base stations are connected to MTSO that serves as a controller to base stations. The MTSO interfaces the mobile units and public switching telephone network (PSTN). The major task of MTSO can be counted as paging, locating and handoff.

Frequency Reuse : Frequency reuse is the main concept of the cellular mobile radio system. Each cell has a fixed number of radio channels. When the cells are suitably separated geographically, the mobile users in different cells can simultaneously use the same frequency channel. This is called frequency reuse that can drastically increase the capacity of the spectrum efficiency since the transmitters are distributed with moderate power throughout the coverage area instead of covering a complete area from one transmitter site with high power. However, if the system is not properly designed, severe interference may occur. Interference due to use of the same channel is called *cochannel interference.* In order to combat this effect and to evaluate the system performance, *frequency reuse distance* D is described. It is the minimum distance that allows the same frequency to be reused.

Cell Splitting : Cell splitting is one of the essential features of the cellular concept to improve spectrum efficiency. Each cell has a fixed number of channels whereas when the user density starts to increase and the available channels in a cell cannot provide sufficient mobile calls, the cell can be divided into smaller cells. This process is called

cell splitting.

Cell Sectorization : Sectorization refers to the case where a cell is divided into several sectors, each of which is served by the same base station. This can be achieved by employing directional antennas such that the energy is directed in each sector by separate antennas. Sectorization results in increased channel capacity similar to cell splitting. However, all sectors use the same base station, the establishment of a new base station is not required. With sectorization, the cochannel interference is reduced as the energy is directed in the central direction of a sector. The most commonly used form of sectorization is achieved by dividing the cell into three sectors with 120^o beamwidth directional antennas.

2.3 Operation of Cellular Systems

The basic operation of cellular systems can be divided into four sections [42, 44]:

1. *Mobile Originated Call* : When a mobile unit is switched on, it scans the control or set-up channels. Then, it is tuned to the channel with the strongest signal, usually arriving from the nearest base station. The set-up channels are used for initiating or setting up calls. They can be classified by usage in two types: access channel and paging channel. The former is utilized for mobile initiating calls whereas the latter for the base station originating calls. Whenever the mobile unit is tuned to the base station, it identifies itself and establishes authorization to use the network. The base station sends this request to the MTSO, which then assigns a radio traffic channel to the mobile unit. The control channels are used by all mobiles in the service area and cannot be used for data traffic. This traffic channel assignment is relayed to the unit via respective base station. The mobile unit tunes itself to this channel and the MTSO completes the rest of the call.

2. *Network Originated Call* : When someone calls a mobile unit, the MTSO sends a paging message to certain base stations based on the mobile unit number and a search algorithm. Each base station transmits the paging message through its control channels. The mobile unit recognizes its own identification on the strongest control channel. The mobile unit responds to the base station, which then informs the MTSO about the location of the mobile unit. The MTSO assigns a channel, and the call is established.

3. *Call Termination* : When the mobile user or the other party wants to terminate a call, a particular signal is transmitted to the base station, and both sides set the channel free.

4. *Handoff* : The subscriber movement may cause it to be far away from the respective base station and the signals from the base station to the mobile may become weak. This distance is detected by MTSO, which monitors the signal strength. The MTSO reassigns a new traffic channel via base station close to the mobile user. This process is called as handoff (handover) and is generally transparent to the subscriber. During this process, the call continues without interrupting the call or alerting the user.

2.4 Existing Cellular Systems

Looking at today's cellular products, we can find many different cellular systems employed in different parts of the world. The basic features of foremost digital cellular systems will be given below.

European Digital Global System for Mobile (GSM) : In 1987, European countries agreed to set up a Pan-European digital cellular standard called GSM (Special Mobile Group) specification. The basic objectives of this contract are to provide pan European roaming, which extend compatibility through all the European countries, and intercommunication with the integrated service digital network (ISDN). CEPT (Conference Europeenne des Postes at Telecommunications) reserved two frequency bands to be used by the GSM system [45], namely 890-915 MHz for the direction mobile unit to base station (uplink) and 935-960 MHz for the direction base station to mobile unit (downlink).

The band in both directions is divided into 124 channels. GSM uses the FDMA/TDMA techniques. It is a combination of frequency division with carriers spaced at 200 kHz intervals and time division with 8 logical channels per carrier. The data rate is 270.883 kbits/s carried over the radio interface using GMSK (Gaussian Minimum Shift Keying) modulation. Since such a high data rate is used in GSM, adaptive equalization techniques are implemented to overcome the multipath whose delay spread is assumed to extend up to 16 μs.

North American TDMA Digital Cellular System (IS-54) : While European and

Japanese standards will be applied to completely new system operating in dedicated frequency bands, the present North American standard specifications are enhanced, rather than replaced [5]. To meet the increasing demand for cellular phones, the digital cellular subcommittee developed the North American Digital Cellular standard, IS-54, as a replacement for the existing analog AMPS system. A digital voice capability was added to new user equipment firstly. In order to provide the conversion of individual analog channels to digital operation, the carrier spacing and carrier frequency of IS-54 are selected as 30 kHz and 800 MHz respectively like AMPS. As GSM, IS-54 uses FDMA/TDMA techniques, with user traffic and control channels built upon the logical channels provided by TDMA time slots. IS-54 adopts $\pi/4$ DQPSK (Differential Quaternary Phase Shift Keying) modulation. The data transmission rate is 48.6 kbits/s.

North American CDMA Digital Cellular System (IS-95) : QUALCOMM proposed a new digital cellular system based on spread spectrum CDMA as an alternative to IS-54's TDMA standard [46, 47]. It is optimized for service (voice quality, voice activity, handoff) and environment (time varying fading, radial attenuation, etc.). It has high capacity, which is achieved by instantaneous optimal utilization of all constituents: Frequency, time, space and voice activity. The spread spectrum bandwidth is 1.25 MHz in 800 MHz and transmission rate is 9.6 kbits/s. QPSK is used as the modulation technique.

This system offers higher capacity than traditional TDMA and FDMA systems due to the potential advantages of CDMA. According to statistics of telephone conversation, the duty cycle of each voice is less than 35%. The vocoder can detect the pauses in speech signal and this signal is used to gate transmitter on and off. In TDMA and FDMA systems, it seems to be hard to implement voice activity because of time delays associated with reassigning channel resources during speech pauses. However, CDMA systems can exploit the voice activity so the mutual interference between CDMA users can be significantly reduced, and thus the spectral efficiency or network capacity increases.

In CDMA systems, since the different spreading code is assigned for every user on the same frequency spectrum, there is no need to employ actually the frequency reuse concept.

IS-95 uses time and path diversity to overcome the effects of frequency selective

multipath fading. Time diversity is obtained by the use of forward error correction (FEC) and interleaving. Path diversity is inherent from CDMA approach due to using of wide band spreading signals. The wideband signals allow different propagation paths to be separated and thus provide the receiver with several independently fading signal paths. The path diversity is accomplished by the use of RAKE receiver, which resolve and combine the multipath signals.

In IS-95, a base station uses three sectored antennas to reduce the multiple access interference and thereby increase system capacity. Since every user is a source of interference to other users, transmitter power control becomes a significant problem as in all CDMA systems. IS-95 uses open loop and closed loop power control algorithms to attain an acceptable system performance.

2.5 Mobile Radio Channel

In mobile communications, the channel seems to be the greatest challenge as a communication medium for reliable communications. The propagation conditions and channel characteristics are important for the efficient use of a transmission medium. Many measurements have been performed to reveal the mobile channel characteristics, and the channel models have been developed based on these empirical data [48, 49].

Due to random position of the mobile and obstacles, the mobile channel cannot be characterized by a deterministic model. In order to analyze the performance of mobile communication system a statistical channel model should be used. The received signal may consist of a line-of-sight (LOS) signal and other non-LOS signals, which are scattered and reflected by surroundings in the vicinity of the mobile. In most circumstances, due to location of structures between the mobile and the base station, a LOS path may not exist. The received signal may comprise a large number of signals having randomly distributed amplitudes, phases, and time delays. The mobile radio propagation is usually characterized by three factors as *path loss*, *short term fading* (*fast fading*) and *long term fading* (*shadowing*).

Path loss is mainly a function of the distance between the transmitter and the receiver and is generally measured in dB. It is defined as the difference (in dB) between the received power and the transmitter power. In free space, the path loss is proportional to the second power of the distance between the transmitter and the receiver.

Experimental results show that the received power falls off inversely with approximately the fourth power of the distance in urban areas and with about the sixth power of the distance in indoor areas [42].

The transmitted signal is strongly affected by its surroundings such as buildings, hills, streets, moving vehicles. The amplitude of the signal fluctuates over the travel path. This type of fluctuation is named as *fading*. When a signal is transmitted, more than one replica of the signal may be received due to diffraction, scattering and reflections, and this situation is called as *multipath propagation*. The rapid change in signal amplitude over a short period of time or travel distance proportional to wavelength is referred as fast fading. In addition to rapid amplitude changes, multipath causes random frequency modulation due to varying Doppler shifts, time dispersion due to multipath propagation delays. The short term fading statistics is mainly characterized by two distributions: Rayleigh and Rician. The envelope of the received signal resulting from multipath propagation is Rayleigh distributed. When there is a LOS path between the transmitter and the receiver in addition to multipaths, the envelope of the received signal is considered as Rician distributed.

In addition to multipath fading, both LOS and other paths may cause attenuation due to large obstructions such as large buildings, hills, which is referred to as shadowing or long term fading and changes with the density and height of the obstacles, the width of the streets, etc. This type of propagation characterizes the signal strength over large travel distance (several hundreds or thousand of meters). The mean of the signal strength in a mobile channel is described by slow fading models, which is usually modeled by Lognormal distribution.

Factors Effecting Multipath Fading [5]

1. *Motions of Vehicle and Surroundings* : The movement in a mobile causes the received frequency to differ from the transmitted frequency due to *Doppler shifts* resulting from its relative motion. Since the received signal consists of many paths, the relative velocity of the mobile differs from one multipath to another, it causes different Doppler shifts. This can be viewed as a spreading of the transmitted frequency and is termed as *Doppler spread*, which determines the rate of fluctuations in the received signal. Clearly, time invariant or slowly changing channels have a small Doppler spread.

2. *The Transmission Bandwidth of the Signal* : Multipath causes time dispersion on the received signal. Time dispersion of the channel and the bandwidth of the transmitted signal define the type of fading, which the transmitted signal suffers. If the transmission bandwidth is greater than the coherence bandwidth of the multipath channel, the received signal will be distorted. The maximum delay of multipath component, which is above a particular threshold in power delay profile, is named as the *delay spread* of the channel. The coherence bandwidth is inversely proportional to the delay spread. Similar to the delay spread in time domain, coherence bandwidth characterizes the channel in frequency domain. The coherence bandwidth is a measure of the maximum frequency difference for which signals are strongly correlated in amplitudes. If the bandwidth of signal is narrower than coherence bandwidth, the amplitude of the received signal will change rapidly but the signal is not distorted in time. This type of fading is called as *flat fading.* Otherwise, the received signal will undergo *frequency selective fading.* Under this condition, the received signal consists of multiple version of the transmitted signal, which are faded and delayed in time, and hence the received signal is distorted.

2.6 Multiple Access Techniques

The basic aim of present day mobile communication systems is to maximize the capacity in terms of the number of users. This has resulted in a concentrated effort to develop more spectrally efficient modulation methods. An important aspect of radio communication by many users in a common medium is the sharing of common resources which is referred to as *multiple access.*

Multiple access refers to the simultaneous transmission by numerous users to or through a common receiving point. The overall channel is divided into sub-channels and assigned to mobile radios. There are three fundamental approaches to access and share a common channel resource:

1. Frequency Division Multiple Access (FDMA)
2. Time Division Multiple Access (TDMA)
3. Code Division Multiple Access (CDMA)

Recently, Space Division Multiple Access (SDMA) scheme has also been considered as a new form of multiple access.

Since each scheme has its own advantages and disadvantages, it does not appear that a single scheme fits best for all situations in mobile communication systems. In fact, most practical systems use hybrid access schemes such as a hybrid FDMA/TDMA structure (eg. GSM), a hybrid FDMA/CDMA structure (eg. IS-95), and so on.

2.6.1 Frequency Division Multiple Access (FDMA)

The simplest and oldest way is to divide up the total radio channel capacity in frequency domain. FDMA splits the frequency band into disjoint sub-bands and multiple radios can communicate using a sub-band or frequency. There would be no interference between radio signals whose spectra occupy disjoint parts of the total available frequency band. The channels are assigned to users who request services. FDMA can be applied in both analog and digital cellular mobile systems. The main features of FDMA are as follows [5]:

- The FDMA channel carries only one connection at a time.
- If a FDMA channel is not in active use, then it sits idle and it cannot be used by other users to increase or share capacity.
- FDMA is usually implemented in narrowband systems since the symbol time is large compared to the average delay spread. This implies that the intersymbol interference can be negligible and thus, little or no equalization is required in FDMA narrowband systems.
- Due to continuous transmission scheme in FDMA systems, fewer bits are required for overhead purposes such as synchronization, framing, etc. as compared to TDMA.
- The cost of FDMA systems is higher than that of TDMA systems. A separate transceiver is needed to use at the base station for each mobile in its cell. Moreover, since the transmitter and receiver operate at the same time, the mobile unit uses duplexers. This results in an increase in the cost of FDMA system.

2.6.2 Time Division Multiple Access (TDMA)

TDMA is another basic way of multiple access. Here, time axis is divided into disjoint slots where any two mobile radios communicate using the assigned time slots. The main features of TDMA are given as follows [5]:

- In principle, TDMA and FDMA are equal in spectral efficiency. However, in practice TDMA is more flexible than FDMA because time slots can be easily changed without requiring hardware changes in the system.
- TDMA uses different time slots for transmission and reception, thus duplexers are not required.
- Since the transmission rate is naturally high as compared to FDMA systems, adaptive equalization is usually necessary in TDMA systems because of larger bandwidth.
- Due to discontinuous transmissions, handoff process is much simpler for a mobile unit, since it is able to listen for other base stations during idle time slots.
- Because of bursty transmission, synchronization is of vital importance in TDMA systems. TDMA transmissions occur in burst, and this requires the receivers to be synchronized precisely for each data burst.
- TDMA has the advantage that it is possible to allocate different numbers of time slots per frame to different users to enhance the data rate.

2.6.3 Code Division Multiple Access (CDMA)

A CDMA scheme (direct sequence) is a spread spectrum technique that uses a separate code for each user. A narrowband modulated signal is multiplied by a very wide bandwidth signal called the *spreading signal* or *code*. All mobile units in a CDMA system use the same carrier frequency and may transmit at the same time. The users are not separated by different time slots or frequency bands but by different pseudorandom (PN) codes. These codes are sufficiently long sequences with low cross correlation among themselves. The receiver performs a time correlation operation to detect only the intended spreading code. Other CDMA signals, which occupy the same spectrum,

appear as random noise. In order to obtain the desired signal, the receiver needs to know the spreading code used by the mobile unit, for despreading.

Since all CDMA signals within a cell use the same bandwidth, the power of multiple users at a receiver determines the noise floor. If the power of each user is not controlled, the strongest mobile signal will capture the demodulator at a base station. This is known as the *near-far problem* on the reverse link. In order to combat near-far problem, *power control* is used in most CDMA systems. Power control on the reverse link attempts to ensure that all signals from mobile units within a coverage area arrive at the base station at the same power. The main features of CDMA are given as [5]:

- Many users share the same frequency. Either Time Division Duplexing (TDD) or Frequency Division Duplexing (FDD) may be used.

- Unlike TDMA or FDMA, CDMA has a soft capacity limit. In theory, there is no strict limit on the number of users. Increasing the number of users results in a graceful degradation of performance. This is known as *soft degradation.*

- Since a CDMA scheme is a spread spectrum method, there is inherent protection against jamming and message privacy.

- Multipath fading effects may be considerably reduced because the transmitted signal is spread over a large spectrum. A RAKE receiver can be used to collect the time delayed versions of the transmitted signal.

- The capacity of a CDMA system can be increased by using the voice activity that can be described as stopping transmission when voice or data activity is absent and thereby considerably reducing multiple access interference (MAI) to other users.

- Since the same frequency bands are used in all cells of a CDMA system, soft handoff process can be provided. MTSO monitors a particular user's signal from two or more base stations. The MTSO selects the best version of the signal at any time without switching the frequency.

Since this thesis is focused on CDMA systems, a detailed discussion on CDMA will be given in Chapter 3.

2.6.4 Space Division Multiple Access (SDMA)

Another form of multiple access is known as space division multiple access [50]. In cellular systems, a large geographical region is divided into a large number of cells. This allows the same carrier frequency to be used in different cells. In fact, this is a primitive form of SDMA since the signals that are transmitted at the same carrier frequency are separated by a spatial distance [5]. In order to further increase the capacity, for example, 120^o sectorial beams at different carrier frequencies can be used within a cell. These frequencies are reused in other sectors of other cells to reduce interference. Another form of SDMA is to use independently steered high gain narrow radiation beams at the same carrier frequency to provide service to each user within a cell. The latest SDMA form uses *adaptive antenna arrays* at the base station. Basically, an antenna array with *beamforming* or *combining* points to the intended mobile user and nulls out the interfering ones. This provides increased coverage and reduced cochannel interference, leading to reduced reuse distance.

CHAPTER 3

SPREAD SPECTRUM AND CDMA SYSTEMS

3.1 Introduction

Spread spectrum radio communications that have been primarily used in military communication since it resists jamming and provides a low probability of detection (or intercept), is now on the margin of explosive commercial applications. There has also been a growing interest in the use of this technique for mobile communications systems, timing and positioning systems, satellite systems, etc. [2, 47, 51].

Over the past several years, multiple access techniques in wireless communications applications have become a very active area of research and development. The primary multiple access techniques are known as TDMA, FDMA and CDMA. Although there does not appear to be a single multiple access scheme that is superior to others in all circumstances, due to spread spectrum characteristics CDMA offers certain advantages over other schemes, especially, due to a potential capacity.

Spread Spectrum is a means of transmission that has a bandwidth much greater than the minimum bandwidth required to transmit the digital information. The spreading of the spectrum is accomplished by using any one of the spreading modulation methods. Despreading is achieved by correlating the received spread spectrum signal with a similar local reference signal.

There are several basic techniques to spread a signal. One method of spreading the spectrum of a data signal is to modulate the signal using a very wideband signal, and hence, the transmission is dominated by the spreading signal. Bandwidth spreading by a wideband signal is called *Direct Sequence Spread Spectrum* (DSSS).

A second method for widening the spectrum of a data modulated carrier is to change

the carrier frequency in a pseudorandom (PN) manner. Since a data modulated carrier hops from one frequency to the next, this type of spread spectrum is called *Frequency Hopping* (FH) *Spread Spectrum.* In the receiver, there is an identical PN generator and frequency hopping is removed by mixing with a local oscillator signal that is hopping synchronously with received signal.

A third method for spectrum spreading is to employ both direct sequence and frequency hopping spreading techniques in a hybrid form (FH/DS), which offers certain advantages over the direct sequence and frequency hopping techniques. Hybrid forms are widely used in military spread spectrum systems and are currently the only practical way of achieving extremely wide spectrum spreading.

There are two basic elements in the design of spread spectrum systems. The bandwidth expansion is much greater than unity, i.e.; the transmitted signal must occupy a bandwidth, which is much larger than the information bit rate. Spreading can mitigate the severe effects of interference, such as multipath reception in mobile radio communication.

A second element is the spreading sequence, which is pseudorandom (PN), i.e.; the signal is similar to random noise and difficult to demodulate by receivers other than the desired one. The same spreading signal is used in the receiver operating in synchronism with the transmitter to despread the received signal so that the original data can be recovered. Spreading sequences have mainly the following properties: easy to generate, long periods, and difficult to reconstruct from a short segment [52]. Linear feedback shift register (LFSR) sequences have these specific properties (except the last one), and they have been widely employed in spread spectrum systems.

There are numerous reasons for spreading the spectrum, and if it is performed properly, certain distinct benefits can be achieved. Some of them are given as follows [52, 53]:

- Antijamming,
- Antiinterference,
- Low probability of intercept, i.e., hiding a signal that is transmitted,
- Achieving message privacy,
- Accurate universal timing,

- High resolution ranging,

- Code division multiplexing.

In this chapter, a brief description of DSSS is presented. A detailed information about CDMA spread spectrum techniques is outlined by defining CDMA systems, advantages of CDMA spread spectrum systems, and power control for CDMA applications.

3.2 Direct Sequence Spread Spectrum (DSSS)

Direct sequence systems are the best known and most widely used spread spectrum systems because of their relative simplicity from the standpoint that they do not require a high speed, fast settling frequency synthesizers [54]. Direct sequence transmission spreads the spectrum not periodically by changing the frequency but by modulating the information bearing signal with a very wideband digital signal. This wideband character is used to achieve enhanced performance in the presence of interference and multipath propagation. Moreover, the spreading signal has some properties that facilitate the demodulation of the transmitted signal by the intended receiver, and that render demodulation as difficult as possible to unintended receiver. The most common forms of DSSS systems used in practice are known as Binary Phase Shift Keying (BPSK) Direct Sequence Spreading and Quadrature Phase Shift Keying (QPSK) Direct Sequence Spreading.

Assuming BPSK signaling, the spreading can be accomplished by multiplying the information bearing signal at a low-rate R_b by a baseband code sequence at a high-rate R_c. Then, the product is phase modulated to obtain a wideband signal with a rate R_c. The duration corresponding to the spreading sequence called *chip* is denoted as the chip duration T_c, which is reciprocal of R_c. Similarly, the reciprocal of R_b denoted as T_b defines the transmission time of an information signal, which is called as bit duration. The ratio of the spread bandwidth to the unspread bandwidth, $R_c/R_b = T_b/T_c$ is referred to as the *processing gain* (L), which also gives the number of chips per information-bearing signal.

3.3 Code Division Multiple Access (CDMA)

Spread spectrum techniques, which have been primarily established for antijam and multipath rejection applications as well as for accurate ranging and tracking, have also been proposed for CDMA to provide simultaneous communication in the same radio channel [7].

With DSSS-CDMA each user is assigned a different spreading code, which has a low cross correlation with the codes of other users. Many CDMA users occupy the same bandwidth and appear as random noise to each other. Hence, the primary advantage of CDMA system is its ability to tolerate a fair amount of interference compared to other conventional systems that must carefully assign frequency or time slots. Moreover, CDMA system uses noise like signals, where each user has a unique code, and thereby privacy is inherent.

CDMA exhibits the greatest advantage over TDMA and FDMA systems in cellular applications since CDMA can reuse the same spectrum for all cells. In FDMA and TDMA the frequency management is always a critical task to reduce the real time interference. With CDMA the capacity is increased by a large percentage of frequency factor [7].

Although CDMA offers numerous benefits compared to other schemes, there are mainly two disadvantages associated with CDMA. These problems are related with 'self jamming' and 'near-far' effect. The former arises from the fact that the spreading sequences of the different users may have large cross correlations, hence the design of the spreading sequence is the primary challenge of CDMA systems. In order to ensure the minimal interference between any pair of users of the same frequency spectrum, the different code pairs should have low cross correlations. The most commonly employed sequences are the binary maximal length linear feedback shift register sequences, which are named as *m-sequences*. In a typical CDMA system, the assignment of m-sequences to mobile users is achieved as follows: phases (i.e., one chip delay) of a m-sequence are assigned to users of a cell, and different m-sequences are assigned to different cells. For a single cell, the correlation between different phases of the m-sequence, referred to as autocorrelation, determines the co-cell (intra-cell) interference. On the other hand, correlation between different phases of different m-sequences specifies the amount of interference (named as inter-cell interference) in a multiple cell environment.

T.C. YÜKSEKÖĞRETİM KURULU
DOKÜMANTASYON MERKEZİ

CDMA systems are implemented as either synchronous CDMA (S-CDMA) or asynchronous CDMA [47]. In a S-CDMA network, all base stations are synchronized to each other, and each user's transmission is aligned in time at the base station. For such a system, where the delay spread is in the order of chip duration (negligible), codes with low autocorrelation at phases except zero, and low cross correlation are sufficient for a proper spreading. However, in asynchronous systems, each user's signal can arrive at any time within the chip duration so that the partial autocorrelation of a sequence is more pronounced. Therefore, the key parameters in a CDMA system appear to be partial autocorrelation and cross correlation of spreading sequences. There exists a vast amount of research on the design and optimization of spreading codes with good correlation properties in the literature [55, 56, 57, 58, 59]. Other than m-sequences, Gold codes [60] and Kasami sequences [61], which are generated from m-sequences, have received paramount attentions for spreading purposes. In contrast to binary sequences, complex sequences have also been proposed.

In mobile communication, random geographic distribution of transmitters, multipath and fading causes the reception of mobile signals to be at different levels. The signals closer to the base station of interest are received with a high power than signals far away from the base station. This strongest received signal will capture the demodulator at a receiver and may result in a dropping call for far user. This problem is known as near-far problem for which power control appears to be the most popular method to combat. In a CDMA system every user is a source of interference to every other user, which makes power control a key element in its current implementation.

The capacity of CDMA is interference limited, while it is bandwidth limited in FDMA and TDMA. Therefore, any reduction in the interference will result in a linear increase in the capacity of CDMA. By maintaining constant power level throughout a cell, interference can be kept at a nominal level so that capacity is increased. Power control does not only mitigate the near-far problem and minimize interference, but also provides reduction on the power consumption of mobile units.

Since CDMA offers attractive features such as the high spectrum efficiency, soft capacity, inherent diversity against multipath fading, simplified frequency planning and easy system deployment, CDMA seems to be a promising technique for radio access in near future cellular mobile and personal communications. In the following subsection, the main benefits of CDMA over TDMA and FDMA techniques are set to forth in

detail. Then, the power control mechanism is presented.

3.3.1 CDMA Benefits

3.3.1.1 Spectrum Efficiency and Channel Capacity

Spectrum efficiency is the amount of traffic for a given system with certain spectrum allocation. An increase in the number of users of mobile communication systems, without a loss of performance, results in the increase of the spectrum efficiency. Channel capacity refers to the maximum data rate, which system could provide. An improved channel capacity leads to more users at a specified data rate, implying a better spectrum efficiency. In that sense, these two terms are used interchangeably in the mobile communication systems [42].

Let γ_b be the required E_b/N_o to achieve a specified bit error probability. Let N be the number of users in a cell of interest. All signals are assumed to be transmitted at a controlled power level to arrive at the base station with the same power, P. With the transmission bandwidth W, and the additive Gaussian noise power spectral density N_o', the power spectral density of the noise-plus-interference N_o is given by

$$N_o = (P/W)(N-1) + N_o'. \tag{3.1}$$

Since CDMA systems are primarily interference limited and $N \gg 1$, γ_b is approximately equal to

$$\gamma_b = \frac{P T_b}{(P/W) N} = \frac{W T_b}{N} \tag{3.2}$$

where T_b is the bit duration. Since $T_b = 1/R_b$ and $W/R_b = L$ as defined in Section 3.2, the number of users is easily obtained as

$$N = \frac{L}{E_b/N_o}. \tag{3.3}$$

The multicell capacity is found based on the capacity of a single cell. CDMA systems tend to be self-interference limited so that the multiple access interference sets the limit on the number of simultaneously active users. According to the statistics, in a typical full duplex two-way telephone conversation, the duty cycle of each voice is less than 35%. With CDMA, it is possible to reduce the transmission rate when the silence occurs, and thereby the mutual interference between CDMA users can be drastically reduced. However, in either FDMA or TDMA, it may be hard to exploit the

voice activity because of time delays associated with reassigning channel during speech pauses.

Another essential feature of CDMA is that special frequency reuse are not required because of use of different spreading codes for users. In contrast to FDMA and TDMA, CDMA system performance is primarily limited by average interference (from all users within the range) instead of adjacent cell interference.

The concept of cell sectorization is an other fundamental feature of the cellular mobile systems. The base station uses sectored antennas to reduce the multiple access interference. When the directional three antennas are used (covering 120^o), the interference is simply divided by three since on the average each antenna receives only in the direction of one-third of the mobiles, and thereby increases system capacity by three [46].

The above attributes determines multicell capacity in terms of processing gain L, E_b/N_o, voice duty cycle d, frequency reuse f_u, and the number of sectors S in the cell site antenna. It is given mathematically [46]

$$N = \frac{L}{E_b/N_o} \frac{1}{d} f_u S. \tag{3.4}$$

In contrast to FDMA and TDMA, there is no exact limit on the number of active users in CDMA system. With TDMA and FDMA, when the number of users reaches the maximum value, no additional user can be handled anymore. There is a soft relationship between the number of users and the grade of service in CDMA systems. The system can decide to allow a small degradation in the bit error rate and increase the number of available channels. This feature is referred as *soft degradation.*

3.3.1.2 Diversity Techniques

Digital signal transmission through mobile radio environment is severely degraded by fading. Diversity techniques are widely used to overcome fading induced impairments. There is a wide range of diversity implementation with a relatively low cost in practice. These techniques are virtually implemented at the receivers, and are based on the notion that impairments due to fading occur in reception when the channel is in a deep fade. With diversity techniques, the replicas of the transmitted signal over independently fading channels are provided so that the probability that all signal components will fade simultaneously is reduced drastically [53].

Diversity approaches can be realized in the frequency domain through the transmission of wideband signals, in the time domain through the use of error-correcting codes or in the space domain through the use of multiple antennas that are sufficiently spaced far apart. Diversity gain in different domains is multiplicative. Hence, a large gain may be achieved by using more than one form of diversity. In order to make use of any of diversity techniques, combining methods that offer significant performance and capacity improvement must be used in the combination of diversity approaches. In this section, some major diversity techniques, which are inherent from CDMA and simple to be applied, are presented.

Frequency Diversity : Wideband CDMA offers frequency diversity, by spreading the signal energy over a wide bandwidth. The channel type is frequency selective since the transmission bandwidth is much greater than the coherence bandwidth of the channel. With FDMA, each user has a limited frequency band, thus FDMA has no diversity by distributing the symbol energy along the frequency axis. However, both TDMA and CDMA have frequency diversity, because they spread the symbol energy over the entire channel.

Path Diversity : The path diversity is another capability of CDMA spread spectrum systems. Most pseudorandom codes have essentially zero autocorrelation value for time offsets greater than one chip time. Thus, when the difference in path delays for the various paths exceeds the PN chip duration, the wide bandwidth PN allows different propagation paths to be separated. A transmitted signal having bandwidth much greater than the coherence bandwidth of the channel is utilized to provide the receiver with several independently fading signal paths. RAKE receiver, invented by Price and Green (1958), resolves these signals, and hence, profits from the diversity. In other words, RAKE receiver collects the energy from the various transmission paths. Then, resolved signals are used in a diversity combining scheme in order to maximize the total desired signal-to-noise energy at the input of the demodulator. Optimum diversity combining can be achieved if a Maximal Ratio Combining (MRC) technique is implemented in the form of a RAKE receiver. In the MRC process, signals (paths) are weighted proportionally to their individual signal-to-noise power ratios and then summed. By using a RAKE receiver with MRC scheme, an important loss in performance occurs only if all paths fade at the same time, as when a path suffers from a fast fading which cannot be tracked, there are other paths still to be processed by the

RAKE receiver.

Time Diversity : Time diversity can best be obtained by the use of the interleaving and forward error correction codes. Even though spread spectrum techniques are used to overcome multipath fading and thus provide a reliable and high quality channel, the spectral efficiency of the system can be poor. In order to combat the effects of fading, Forward Error Correction (FEC) coding can be employed with a relatively small increase in data rate overhead, and thus, the spectral efficiency of the system can be enhanced. FEC coding reduces the BER of the uncoded channel by adding redundancy to the coded data signal. This can be implemented using different codes such as block, convolutional and trellis codes. In CDMA spread spectrum systems optimum performance is obtained if the channel errors seen by the FEC scheme are independent (uncorrelated) of each other. Since errors often occur in bursts, it is necessary to interleave the FEC data to get uncorrelated errors in consecutive data bits. Hence, time diversity is achieved. Time diversity can be provided in all digital systems that can tolerate a higher transmitted symbol rate, which is required for error correction process. Due to deep fading in FDMA, burst errors occur more frequently. TDMA, because of burst transmission, is vulnerable to short time impulsive noise. Since continuous transmission is used in CDMA system, FEC and interleaving are not a problem as in FDMA and TDMA.

Space (Antenna) Diversity : Space diversity, also known as antenna diversity, is widely used in wireless mobile communication systems. Multiple antennas, which are spaced sufficiently far apart, are employed at the receiver (and sometimes at the transmitter). In a Rayleigh fading environment, the multipath components of a signal are received by antennas at different propagation delays so that the received signal will undergo independent fading.

Space diversity forms are classified into three categories :

1. Selection diversity,
2. Maximal ratio combining (MRC),
3. Equal gain combining (EGC).

Selection diversity is the simplest method, in which the signal levels on antennas are monitored simultaneously, and the best is selected at any time. This is a sub-

optimum technique because of using the information from only one antenna. In MRC method, the signals from the receiving antennas are weighted proportionally to their individual signal-to-noise ratios (SNR), and then summed. The MRC scheme provides an output SNR equal to the sum of individual SNRs, and thus it has the advantage of producing an acceptable output even if none of the individual signals are themselves acceptable. Therefore, it is known as the optimum solution. However, this method requires continuous estimation of amplitude and phase at the outputs of every diversity receiver. In certain cases, it can not be convenient to estimate SNRs properly. For such cases, the antenna weights are all set to unity but each branch is co-phased to provide equal gain combining (EGC). The performance of EGC is inferior to MRC, but is superior to selection diversity [5].

Space diversity can easily be provided in FDMA and TDMA systems. However, in CDMA due to spread spectrum features multipath signals arriving with different propagation delays can be received separately, and can easily be combined.

3.3.2 Soft Handoff and Cell Diversity

In the CDMA cellular system, the same frequency is used in all cells. When a mobile unit enters a new cell's coverage area, the mobile unit detects the pilot channel of the new cell and reports it to the original cell. The new cell assigns a modulator-demodulator (modem) unit to the call while the old cell continues to handle the call. Thus, handoff process is initiated. The base station modem searches for and finds the mobile's signal. It also begins transmitting the outbound signal to the mobile, for which the mobile unit searches. When the mobile unit finds the new base station signal, it starts to listen to it. The mobile sends a control message to indicate that the handoff process is complete [46].

Additionally, the soft handoff process provides cell diversity. In this diversity mode, the process is initiated as explained before. The mobile unit determines that a neighboring base station has a signal strength high enough to allow acceptable quality in detection. Then, it transmits a control message to the base station and requests the cell diversity mode. The system controller responds by connecting the call to the new base station modem. It also performs diversity combining of the signals received by the two base stations. Likewise, the mobile unit combines the transmitted signals from the

base stations. The cell diversity mode is terminated when the mobile unit determines that only one base station is providing adequate signals for quality reception.

3.3.3 Power Control

The primary use of power control is to maximize the capacity of a CDMA mobile system in terms of the number of simultaneous mobile users that can be served in a given system spectrum. The system capacity is maximized if the transmit power of each user is controlled so that its signal arrives at the base station with the minimum required SNR. With power control, the same power level is received at the base station for each user in the same cell regardless of his position or propagation loss.

In the case of no power control, when a mobile user signal arrives at the base station with too low received power, a high level interference is experienced by this mobile, and thereby results in a high bit error rate. If the received power is too high, the performance of this mobile will be very good, but it will augment interference to all other mobile users in the cell. Power control scheme is investigated separately for reverse and forward links, which are described as mobile to base, and base to mobile respectively. A useful discussion on the power control mechanism for CDMA system can be found in [62, 63].

Reverse Link Power Control : The reverse link of a CDMA system is typically designed to be asynchronous, where near-far problem degrades drastically the system performance. In an asynchronous CDMA system, the spreading codes of the different users are not orthogonal, hence interference due to the nonzero cross correlation of the spreading codes is summed on a power basis. Power control is a crucial solution of the near-far problem, which attempts to ensure that all signals from mobiles in a given cell arrive at the base station with equal power.

Two different power control mechanisms are employed for the reverse link : *open loop* and *closed loop* power control. In open loop power control, mobile units estimate the path loss from the base station. Reception of a strong signal shows that the mobile is either close to the base station or has an unusually good path to the base station. The main goal of the open loop power control is to rapidly adjust transmit power of mobiles according to changes in received power from the base station. Each mobile unit adjusts each transmitter power by itself. However, open loop power control may be

insufficient since the forward and reverse link propagation characteristics are usually not identical due to widely separated center frequencies in reverse link and forward link directions. In addition to open loop power control mechanism, closed loop power control can be employed on the reverse link. Closed loop power control is also fast but not as fast as open loop power control. The base station takes active role in the closed loop power control. The base station monitors the received signal power from each user and compares it to a preset threshold. Then, the base station warns the mobile unit whether to decrease or to increase its transmitted power on the forward link.

A combination of open and close loop power control on the reverse link is used in IS-95 CDMA system [46].

Forward Link Power Control : All mobiles in a given cell receive the base station signal of their own cell at the same power level so that the forward link does not suffer from the near-far problem. Moreover, in a single cell environment, no power control is required for forward link since for a mobile any interference from other mobiles remains at the same level relative to the desired signal. However, in the case of multiple cells, the power control is essential for mobiles that suffer from the large inter-cell interference. This case may occur when a mobile reaches the cell boundary [2]. Interference from neighboring base stations fade independently so that performance degrades for any level of interference.

Prior to transmission, each mobile measures the quality of the received signal from the base station, and periodically sends the results to the base station. If the signal quality is below the nominal value, the measurement results are immediately transmitted to the base station. Then, the base station evaluates the power needs of individual mobile in its cell. The base station can either increase its total power that causes an increase on the interference level of other mobiles in the neighboring cell or can redistribute power from mobile units with good quality to the far mobiles.

CHAPTER 4

ADAPTIVE ANTENNA ARRAYS

4.1 Introduction

The immense demand for mobile radio communication without increases in RF spectrum motivates the need for more efficient techniques to improve spectrum utilization. Adaptive antenna arrays have been proposed as a promising solution to this challenge. It has been shown that adaptive arrays can improve the system performance by increasing the channel capacity and spectrum efficiency, extending range of coverage, steering multiple beams to track any mobiles. Furthermore, adaptive arrays help in improving the system performance by mitigating the effects of multipath fading and time dispersion. It also reduces cochannel interference, system complexity and cost, BER and outage probability.

In traditional systems, coverage problem has been solved by reducing cell sizes and using conventional antenna systems at the base station. However, though coverage problem seems to be overcomed, it has some significant drawbacks. As the number of cells increases, the number of base stations also increases at the same quantity, and thus causes increased cost and complexity of the system. Fortunately, an adaptive antenna array increases the cell coverage substantially through antenna gain and interference rejection, and thus results in a cost-effective system.

Mobile communication suffers from fading due to multipath propagation. Traditionally, antenna arrays have been used to overcome the problem of fading in mobile radio channels. Two or more antennas at the receiver (or transmitter) are spaced sufficiently far so that their fading envelopes become uncorrrelated. This is referred as *antenna* or *space diversity*, which is mostly employed at the receiver. With space diversity, the

received signals from antennas are combined to mitigate the multipath fading of the intended signal and reduce the power of the interfering signals.

Another main problem of mobile radio systems is the presence of interfering signals. Antenna arrays provide the capability to suppress the interferences. By optimally combining signals received by different antennas, the contribution of the undesired interferences is reduced while the power of a desired signal is enhanced. This process is exactly the process of *adaptive beamforming*, which has been primarily developed in radar [64] and sonar systems [65]. On the other hand, even though *optimum combining* concept has been developed for communications systems, optimum combining and adaptive beamforming are basically the same. In the literature, they are utilized interchangeably for mobile communication systems.

Adaptive beamforming provides a means for separating a desired signal from interfering signals. An adaptive beamformer is able to automatically optimize array pattern by adjusting the elemental control weights until a predetermined objective function is satisfied. Adaptive beamforming technology is also referred to as *smart antenna* technology in some literature. The use of the term 'smart' reflects the ability of an antenna to adapt to the environment in which it operates [50]. Smart antennas and adaptive beamforming are also used interchangeably.

In this chapter, the basic principles of antenna arrays and adaptive beamforming processes are presented by covering main topics. The paramount advantages of arrays in mobile radio systems are also discussed. It is shown how an antenna array can improve the performance of a mobile communication system.

4.2 Antenna Arrays

4.2.1 Basic Antenna Array Parameters

We present some of the terms and definitions commonly used in the concept of antenna arrays, which are particularly relevant to issues addressed throughout the thesis [50, 28].
Far Field or Plane Wave Assumption : In far field (or plane wave) assumption, the sources and receivers are assumed to be located sufficiently far from each other such that the signals from these sources arrive at the array in a planar fashion.
Isotropic Antennas : The isotropic antennas can radiate or receive energy uniformly in all directions. In fact, an isotropic antenna is a sole hypothetical definition in theory,

it does not exist in the real world.

Array Pattern : The array pattern is the far field radiation pattern of an array of isotropically radiating elements. It is generally denoted by $F(\theta, \phi)$, where θ is the azimuth angle and ϕ is the elevation angle.

Main Lobe : The main lobe of an antenna denotes the lobe that contains the direction of maximum radiation power.

Beamwidth : The angular width of the main lobe (as measured with a certain criterion) in the far-field radiation is called as beamwidth of an antenna.

Sidelobes : Sidelobes represent lobes in any directions other than that of main lobe.

Grating Lobes : In antenna array, if the inter-element spacing is too large compared to half wavelength ($\lambda/2$), several main lobes will occur in the antenna array pattern. These lobes are referred to as grating lobes.

Aperture Size : The geometrical length of an array is denoted as the aperture size of the array.

4.2.2 Array Configurations

Multiple antennas can be arranged geometrically in a various number of ways to provide an adequate coverage over a certain desired spatial area. The physical configurations of antennas affect various aspects of adaptive array systems such as resolution, the type of processing for estimating angle of arrivals, etc.

In order to select array configurations, it is necessary to consider the following critical factors [28]:

1. Resolution
2. Angular Coverage
3. Number of antennas
4. Sidelobe level

In general, the performance of an array enhances linearly as the number of antennas increases. However, as the number of array elements increases, the cost and complexity of the array also increases, therefore there is a basic trade-off in the design of array systems.

In general, an antenna array consists of identical elements, although it is not necessary. However, non-identical elements result in a complicated array. In the array concept, multiple antennas are arranged in a variety of ways to constitute an array depending on the application and the physical limitations. The most common form of an antenna array is the *linear array*, in which antennas are placed along a line.

The linear array is most often used to meet the requirement of a large coverage in one dimension. By using linear arrays, one-dimensional information of signal can be solely obtained. In the case of applications where both azimuth and elevation angles are required, a two-dimensional array is utilized.

Most common forms of two-dimensional arrays are *planar arrays* and *circular arrays*. Multiple antennas are placed on a plane to form a planar array instead of locating antennas in a straight line in the case of linear arrays. In fact, a circular array is a special configuration of planar arrays, where antennas are placed on a circle. Planar arrays provide additional variables that can be used to control and shape the array beam pattern. The main beam of the array can be steered towards any point in its half plane. The planar array is most frequently used in radar applications because it is the most flexible of all antenna types. With circular arrays, beams can be steered through 360^o and thus complete coverage can be obtained.

There exist many discussions on the physical configuration of arrays in the literature [28]. The detailed analysis of planar and circular arrays is beyond the scope of this thesis, hence we only deal with linear arrays on the application of our interest.

4.3 Linear Antenna Arrays

The linear array, which comprises antennas located along a straight line, is one of most commonly used array structures in many different applications due to its simplicity and its high-resolution property. Using linear arrays has long been an attractive solution to severe problems in communication areas. Most studies have concerned with combining antenna outputs to accomplish a predetermined array beam pattern. Thus, a beam is formed in a preferred direction while rejecting the interferences from other directions. This combining is referred to as *beamforming* in the context of antenna arrays. The process of beamforming implies weighting the antenna outputs, thereby adjusting the amplitudes and phases of antenna outputs such that when they are summed together

they form the desired beam.

Multiple antennas can be arranged along a line with equal inter-element spacing, which is referred to as *uniformly spaced array*. This structure is intensively used in both theory and practice because of its simplicity on computations and the ease of obtaining a desired radiation pattern.

In selecting the linear array configuration, it is essential to bear in mind the important parameters, which are the number of antennas, aperture size of the array, and sidelobe level. From beamforming point of view, the beamwidth and the level of sidelobes are critical factors to achieve a preferred beam pattern. The former impacts the accuracy of angle of arrival estimations or the resolution of the array. The sidelobe level control is substantial in the context of interference rejection.

One way of increasing resolution is to increase the number of array elements, however, the cost and complexity of the array also increases. The other way is to increase the length of the array for a given number for antennas, which results in an increase on the inter-element spacing. Although inter-element spacing of much greater than half wavelength ($\lambda/2$) improves the spatial resolution of a uniformly spaced linear array, the undesired grating lobes can also appear in the beam pattern. Placing antennas with unequal spacing overwhelm the problem of grating lobes. This type of a linear array is called *nonuniformly spaced linear array*, which is also referred to as *sparse*, *thinned*, *aperiodic*, and *space-tapered array* in the literature. A major advantage of the use of nonuniform linear arrays lies in the fact that the nonuniform array can provide a better radiation pattern than that of the uniform linear array with the same number of antennas, when the average spacing is greater than $\lambda/2$. A detailed summary of nonuniform array is provided in [67].

The problem of designing antenna arrays in which the elements are not equally spaced has been studied for approximately 40 years. A fundamental motivation for these works is to achieve high resolution, reduction on the sidelobe levels compared to an equispaced array with the same number of elements. In some radar applications such as a microwave imaging system, a very large aperture size is required, however it is impractical to locate antennas equally spaced so that unequal spacing is inevitable [68]. Furthermore, the use of nonuniformly spaced arrays has become popular for the process of null steering, which is performed by controlling element positions [69], or by adjusting the amplitudes and phases in a linear combiner.

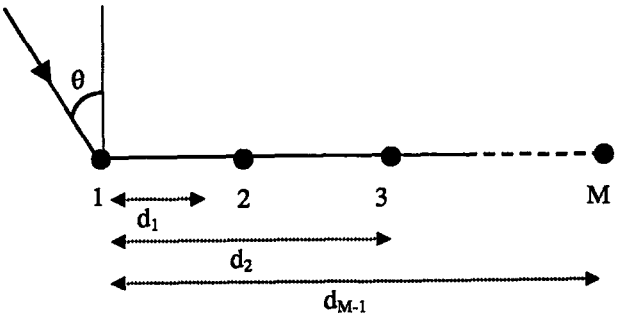

Figure 4.1: An illustration of a linear array

In order to point out the effects of spacing and number of antennas on the beam pattern of a linear array, the directional beam pattern is introduced for uniformly spaced arrays. Consider an array of M identical elements receiving a signal from a single point source in the far field, and the antenna array and the source are in the same plane. A typical linear array is illustrated in Figure 4.1. Let the first antenna be taken as the reference, d_{m-1} represents the distance between *mth* array element and the first element in terms of the wavelength. When a plane wave impinges upon the array at an angle θ with respect to the array normal, the signal arrives at element *(m+1)* sooner than at element-m, since the differential distance along two ray paths is $(d_{m+1} - d_m)\sin\theta$. Thus, the time taken by a plane wave arriving from the source in the direction θ and measured from the *mth* element is given by

$$\tau_m = \frac{d_{m-1}}{c}\sin\theta \tag{4.1}$$

where c is the velocity of light. For a linear array of equispaced antennas with inter-element spacing of d, the delay expression becomes

$$\tau_m = (m-1)\frac{d}{c}\sin\theta. \tag{4.2}$$

By setting the phase of the signal at the reference antenna arbitrarily to zero, the corresponding phase shift at *mth* antenna can be expressed as

$$\psi_m = \frac{2\pi d_{m-1}}{\lambda}\sin\theta \tag{4.3}$$

where λ is the wavelength corresponding to carrier frequency.

Let $\mathbf{x}(t)$ be an induced narrowband signal on the uniformly spaced array. Narrowband assumption implies that the bandwidth of the signal is sufficiently small and the

aperture size of the array is not so large that the signal stays almost constant during propagation through the array. Hence, every element receives a copy of the signal that is time delayed with respect to the reference antenna. The array response can be evaluated by considering the phasor sum of signal contributions from each elements as follows:

$$\mathbf{y}(t) = \sum_{m=1}^{M} \mathbf{x}(t) e^{j\psi_m}. \tag{4.4}$$

The directional beam pattern of the array is defined as [28]

$$F(\theta) = \sum_{m=1}^{M} e^{j\psi_m} \tag{4.5}$$

which can be expressed in terms of vector inner product

$$F(\theta) = \mathbf{V^T v} \tag{4.6}$$

where $\mathbf{V} = [\ 1 \quad 1 \quad \cdots \quad 1\ ]^T$, $\mathbf{v}$ is the antenna array response vector and is given by $\mathbf{v} = \left[\ 1 \quad e^{j2\pi \frac{d}{\lambda} \sin\theta} \quad \cdots \quad e^{j2\pi(M-1)\frac{d}{\lambda}\sin\theta}\ \right]^T$, and $(.)^T$ denotes transpose.

The normalized array pattern in decibels (dB) is given by

$$G(\theta) = 10 \log_{10} \left\{ \frac{|F(\theta)|^2}{M} \right\}. \tag{4.7}$$

The maximum response of $F(\theta)$ is obtained for the angles of $\theta = 2\pi k$. Maintaining the inter-element spacing $d/\lambda = 0.5$, the normalized array patterns are given in Figure 4.2 and 4.3 for 3 and 7 elements. The antenna beam is steered towards the antenna boresight (i.e., $\theta = 0^o$). From these figures it is concluded that as the number of antennas increases the main lobe beamwidth decreases, whereas the number of sidelobes and pattern nulls increase.

The spacing effects are indicated in Figure 4.4, 4.3 and 4.5 for a 7 element array with d/λ taking the values of 0.1, 0.5 and 1 respectively. The normalized array pattern does not have nulls if the spacing is too small. Moreover, in practice array elements cannot be located much closer than $\lambda/2$ due to mutual coupling effect. As the spacing increases, nulls and sidelobes begin to appear in the array pattern. Spacing of greater than $\lambda/2$ improves spatial resolution of the array, however, the grating lobes also occur since the signals align exactly and add coherently at undesired angles.

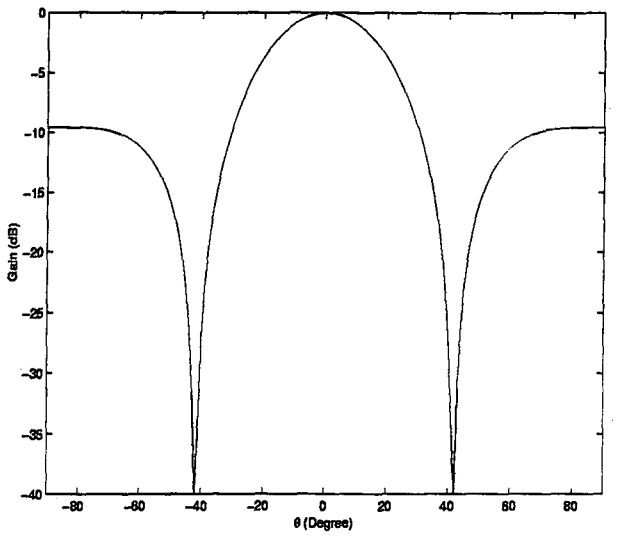

Figure 4.2: The normalized array beam pattern for $M = 3$ and $d/\lambda = 0.5$

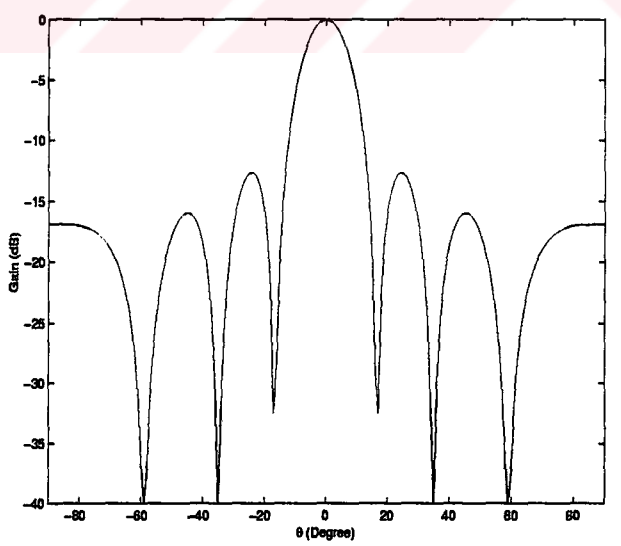

Figure 4.3: The normalized array beam pattern for $M = 7$ and $d/\lambda = 0.5$

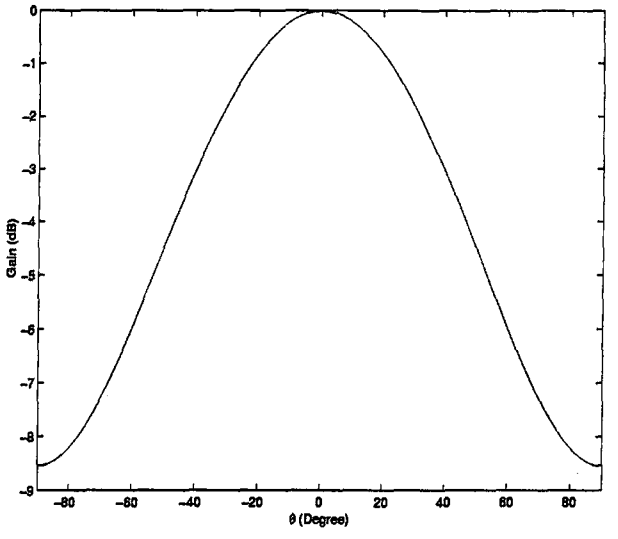

Figure 4.4: The normalized array beam pattern for $M = 7$ and $d/\lambda = 0.1$

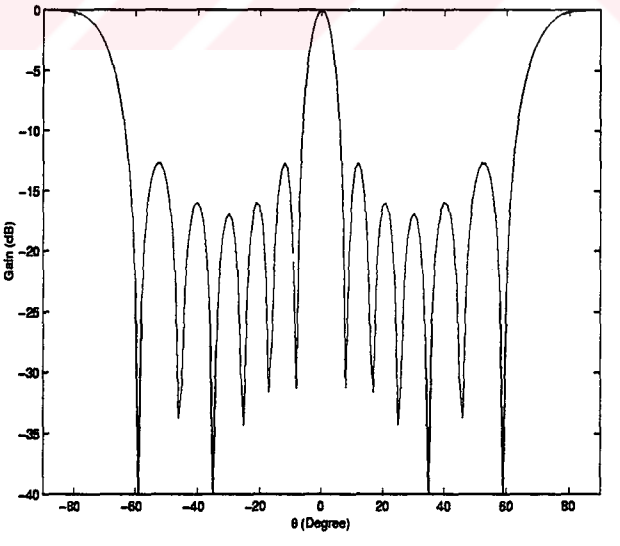

Figure 4.5: The normalized array beam pattern for $M = 7$ and $d/\lambda = 1$

4.4 Adaptive Beamforming

With beamforming, energy radiated by an antenna array is focused along a specific direction to either receive a signal from that direction or transmit a signal to that direction. The practical implementation of beamforming was pioneered by Howells in the late 1950's [70]. He invented an intermediate frequency sidelobe canceller (SLC) for radar applications. The notion for a fully adaptive beamforming was developed in 1965 by Applebaum [71]. The SLC was also included as a special case in his work. He proposed an algorithm based on the maximization of SNR at the array output.

Another major contribution to adaptive antennas was made in 1967 by Widrow and colleagues [72]. They independently developed another approach for controlling an adaptive array using least mean squares minimization method, which is now known as LMS algorithm. Although Applebaum's maximum SNR algorithm and Widrow's LMS algorithm were developed independently and based on the different criteria, they are basically very similar. For stationary signals, both of them converge to the optimum Wiener solution [50].

Capon [73] proposed a different method to solve the adaptive beamforming problem in 1969. He developed a new beamformer in which the beamformer weight vector is chosen so as to minimize the variance (i.e., average power) of the beamformer output. In order to ensure that the desired signal is obtained with a certain gain and phase, a constraint is used so that the inner product of the beamformer weight and array response vector is kept unity. The output of the beamformer with this constrained minimization is referred to as *minimum variance distortionless response* (MVDR) beamformer, which is attributed to Capon.

Following the pioneering studies of Howells, Applebaum, Widrow, and Capon there has been widespread interest and a vast amount of researches in the field of adaptive beamforming. In addition, numerous research papers, tutorial papers, and textbooks devoted to the adaptive arrays can be found in the literature [50, 75, 42, 74].

4.4.1 Criteria for Adaptive Beamforming

Consider a uniform linear array with M identical elements, which operates in a signal environment where a desired signal $s_0(t)$, thermal noise, and interfering signals

$\{s_i(t)\}_{i=1}^{N-1}$ exist. The received signal at the antenna array is represented by

$$\mathbf{x}(t) = \mathbf{s}(t) + \mathbf{i}(t) + \mathbf{n}(t). \tag{4.8}$$

Here, $\mathbf{s}(t)$ denotes the desired signal vector given by $s_0(t)\mathbf{v}_0$ where

$$\mathbf{v}_0 = \begin{bmatrix} 1 & e^{j2\pi\frac{d}{\lambda}\sin\theta_0} & \dots & e^{j2\pi\frac{d}{\lambda}(M-1)\sin\theta_0} \end{bmatrix}^T, \tag{4.9}$$

and $\mathbf{i}(t)$ represents the composite of $N-1$ cochannel interferers equal to $\sum_{i=1}^{N-1} s_i(t)\mathbf{v}_i$ where $\mathbf{v}_i$ denotes the array response vector for the *ith* interferer and is given by

$$\mathbf{v}_i = \begin{bmatrix} 1 & e^{j2\pi\frac{d}{\lambda}\sin\theta_i} & \dots & e^{j2\pi\frac{d}{\lambda}(M-1)\sin\theta_i} \end{bmatrix}^T. \tag{4.10}$$

Finally, $\mathbf{n}(t)$ is the additive noise vector, which is modeled as a complex Gaussian random vector with zero mean and covariance $E\{\mathbf{n}(t_1)\mathbf{n}^H(t_2)\} = \sigma^2\mathbf{I}\delta(t_1 - t_2)$ where $\mathbf{I}$ is MxM identity matrix, σ^2 is the antenna noise variance, δ represents Dirac delta function, and $(.)^H$ denotes complex conjugate transpose. This assumption for noise implies that the noise is both *spatially* and *temporarily* white. Let us further define $\mathbf{u}(t)$ as the total interference-plus-noise vector to simplify notations ($\mathbf{u}(t) = \mathbf{i}(t) + \mathbf{n}(t)$).

Let $\mathbf{w}$ be weight vector corresponding to the desired user to combine antenna outputs linearly. The output of beamformer $y(t)$ is equal to

$$y(t) = \mathbf{w}^H\mathbf{x}(t). \tag{4.11}$$

Now, the criteria for choosing the optimum weights will be given as follows.

Minimum Mean Square Error (MMSE) : If the desired signal $s_0(t)$ is known, the error between the beamformer output and the desired signal can be minimized. In fact, the knowledge of $s_0(t)$ removes the necessity of beamforming. However, in many applications, some sort of reference (or training) signal, which means a priori and explicit knowledge about signals of interest, may be available to generate a 'desired' signal $d(t)$. The square of error between the beamformer output and reference signal is given by

$$\varepsilon^2 = \left(d(t) - \mathbf{w}^H\mathbf{x}(t)\right)^2. \tag{4.12}$$

Taking the expectation and carrying out some basic manipulation, the mean square error is

$$E\{\varepsilon^2\} = E\{d^2(t)\} - 2\mathbf{w}^H\mathbf{r}_{xd} + \mathbf{w}^H\mathbf{R}_{xx}\mathbf{w} \tag{4.13}$$

where $\mathbf{r}_{xd} = E\{d(t)\mathbf{x}(t)\}$ and $\mathbf{R}_{xx} = E\{\mathbf{x}(t)\mathbf{x}^H(t)\}$. Finding gradient vector of (4.13) with respect to $\mathbf{w}$ and setting the result to zero yields

$$\mathbf{R}_{xx}\mathbf{w} = \mathbf{r}_{xd}. \tag{4.14}$$

Finally, the optimum weight is

$$\mathbf{w}_{MMSE} = \mathbf{R}_{xx}^{-1}\mathbf{r}_{xd} \tag{4.15}$$

which is known as the Wiener-Hopf equation in matrix form and its solution is generally referred to as the *Wiener* solution.

If $s_0(t) = d(t)$, then $\mathbf{r}_{xd} = E\{d^2(t)\mathbf{v}_0\}$ and $\mathbf{R}_{xx} = E\{d^2(t)\}\mathbf{v}_0\mathbf{v}_0^H + \mathbf{R}_{uu}$ where $\mathbf{R}_{uu} = E\{\mathbf{u}(t)\mathbf{u}^H(t)\}$. Using the matrix inversion lemma, the optimum MMSE weight can be expressed as

$$\mathbf{w}_{MMSE} = \beta\mathbf{R}_{uu}^{-1}\mathbf{v}_0 \tag{4.16}$$

where β is a scalar quantity and it equals to

$$\beta = \frac{E\{d^2(t)\}}{1 + E\{d^2(t)\mathbf{v}_0^H\mathbf{R}_{uu}^H\mathbf{v}_0\}}. \tag{4.17}$$

Maximum Signal-to-Interference-Plus-Noise Ratio : The weights can be obtained by maximizing the SINR at the output of the beamformer. The array output is rewritten as

$$\mathbf{x}(t) = \mathbf{s}(t) + \mathbf{u}(t). \tag{4.18}$$

The total signal power at the beamformer output can be expressed as $E\{|\mathbf{w}^H\mathbf{s}(t)|^2\}$, and the output undesired signal power is $E\{|\mathbf{w}^H\mathbf{u}(t)|^2\}$. Assuming that $\mathbf{R}_{ss} = E\{\mathbf{s}(t)\mathbf{s}^H(t)\}$ and $\mathbf{R}_{uu} = E\{\mathbf{u}(t)\mathbf{u}^H(t)\}$ are known, the SINR is given as

$$\text{SINR} = \frac{\mathbf{w}^H\mathbf{R}_{ss}\mathbf{w}}{\mathbf{w}^H\mathbf{R}_{uu}\mathbf{w}}. \tag{4.19}$$

Taking the derivative of (4.19) with respect to $\mathbf{w}$ and setting the result to zero yields

$$\mathbf{R}_{ss}\mathbf{w} = \lambda\mathbf{R}_{uu}\mathbf{w} \tag{4.20}$$

which is a generalized eigenvalue problem [76]. The value of $\text{SINR} = \frac{\mathbf{w}^H\mathbf{R}_{ss}\mathbf{w}}{\mathbf{w}^H\mathbf{R}_{uu}\mathbf{w}}$ is bounded by the minimum and maximum eigenvalues of the symmetric matrix $\mathbf{R}_{uu}^{-1}\mathbf{R}_{ss}$. The maximum eigenvalue is the optimum value of SINR and the corresponding unique eigenvector denotes the optimum weight. Therefore, we have

$$\mathbf{R}_{ss}\mathbf{w} = \text{SINR}\,\mathbf{R}_{uu}\mathbf{w}. \tag{4.21}$$

If $\mathbf{s}(t) = s_0(t)\mathbf{v}_0$, $\mathbf{R}_{ss} = E\left\{s_0^2(t)\right\}\mathbf{v}_0\mathbf{v}_0^H$, and the optimum weight is

$$\mathbf{w}_{SINR} = \beta\mathbf{R}_{uu}^{-1}\mathbf{v}_0 \tag{4.22}$$

where $\beta = \frac{E\{s_0^2(t)\}}{\text{SINR}}\mathbf{v}_0^H\mathbf{w}_{SINR}$.

Thus, the optimum weights for maximization of SINR can also be expressed in terms of the Wiener solution.

Maximum Likelihood (ML) : If the desired signal and its characteristics are both known, the maximum likelihood beamformer is used to ensure a good signal reception. Choosing the weights with respect to the maximum likelihood criterion requires the assumption that interference-plus-noise signals have a multivariate Gaussian distribution. Recalling that the array output

$$\mathbf{x}(t) = s_0(t)\mathbf{v}_0 + \mathbf{u}(t), \tag{4.23}$$

then the likelihood function is defined as

$$\Lambda = \frac{1}{\pi^M\left|\mathbf{R}_{uu}\right|}e^{-[\mathbf{x}(t)-\hat{s}_0(t)\mathbf{v}_0]^H\mathbf{R}_{uu}^{-1}[\mathbf{x}(t)-\hat{s}_0(t)\mathbf{v}_0]}. \tag{4.24}$$

The weight based on ML criterion is achieved by estimating the desired signal $\hat{s}_0(t)$ that maximizes (4.24). The solution can be obtained by taking the derivative of $\ln(\Lambda)$ with respect to $\hat{s}_0(t)$ and setting result to zero as

$$0 = \frac{\partial ln(\Lambda)}{\partial\hat{s}_0(t)} = -2\mathbf{v}_0^H\mathbf{R}_{uu}^{-1}\mathbf{x}(t) + 2\hat{s}_0(t)\mathbf{v}_0^H\mathbf{R}_{uu}^{-1}\mathbf{v}_0. \tag{4.25}$$

It follows that the estimation of $\hat{s}_0(t)$ is

$$\hat{s}_0(t) = \frac{\left(\mathbf{R}_{uu}^{-1}\mathbf{v}_0\right)^H\mathbf{x}(t)}{\mathbf{v}_0^H\mathbf{R}_{uu}^{-1}\mathbf{v}_0} = \mathbf{w}_{ML}^H\mathbf{x}(t). \tag{4.26}$$

Hence, the optimum weight vector has the form as

$$\mathbf{w}_{ML} = \beta\mathbf{R}_{uu}^{-1}\mathbf{v}_0 \tag{4.27}$$

where β is a complex coefficient and equal to $\frac{1}{\mathbf{v}_0^H\mathbf{R}_{uu}^{-1}\mathbf{v}_0}$. Similar to SINR maximization criterion, the ML optimum weight is also given by the Wiener solution. Note that in MMSE and maximum SINR criteria the probability density functions (pdf) of signals were not of interest, whereas for ML the pdfs of them must be available. Also, since for the Gaussian case all the results are the same and ML solution is considered as optimum, MMSE and maximum SINR criteria are both optimum for Gaussian signals.

4.4.2 Adaptive Algorithms

In the previous section, it has been shown that the optimum weights using different criteria are all given by the Wiener solutions weights. In spite of the different coefficients (β), all criteria yield the same SINR, which is given by

$$\text{SINR} = E\{s_0^2(t)\}\mathbf{v}_0^H \mathbf{R}_{uu}^{-1}\mathbf{v}_0. \tag{4.28}$$

Hence, one of these criteria can be used to optimize SINR. However, the choice of adaptive algorithms for deriving the optimum weights is extremely important since it directly impacts the implementation characteristics of the algorithms such as the speed of convergence, hardware complexity. In this section, a number of common adaptive algorithms will be touched on.

Least Mean Squares (LMS) : The LMS algorithm is most commonly employed to estimate the optimal weight continually. This algorithm is based on a well-known optimization method referred to as the steepest descent algorithm, which recursively computes and updates the beamformer weights. The LMS algorithm may be implemented in two ways: constrained LMS and unconstrained LMS. According to the steepest descent method, for the unconstrained LMS algorithm, the weight update equation at time *n+1* is given by

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu\mathbf{x}^H(n)\epsilon(n) \tag{4.29}$$

where μ is a gain constant controlling the convergence characteristics of the weight vector $\mathbf{w}(n)$, $\mathbf{x}(n)$ is the received and sampled signal vector or generally referred to as the *array snapshot* at the *nth* instant, and ϵ is the error given by

$$\epsilon(n) = d(n) - \mathbf{w}^H\mathbf{x}(n). \tag{4.30}$$

The LMS algorithm is the least complex weight adaptation algorithm. However, the convergence rate depends heavily on the eigenstructure of $\mathbf{R}_{xx}$ matrix, and it may not be as good as some other algorithms. When the eigenvalues of $\mathbf{R}_{xx}$ are widely spread, convergence can be slow and this may be insufficient in fast changing environment.

Direct Matrix Inversion (DMI) : One way to increase the convergence and adaptation rate is to use direct inversion of the covariance matrix $\mathbf{R}_{xx}$ in (4.15). If the desired signal and interfering signals are known a priori, the covariance matrix can be calculated and the optimum weight vector can be evaluated by using the sample direct

matrix inversion (DMI) [28]. However, in practice, the signals are not known and environment can change fast. Therefore, the weight vector must be updated continuously to adapt to varying conditions.

This algorithm replaces $\mathbf{R}_{xx}$ with its estimate $\hat{\mathbf{R}}_{xx}$, which is generally obtained by using a simple averaging over N samples

$$\hat{\mathbf{R}}_{xx} = \frac{1}{N}\sum_{n=1}^{N}\mathbf{x}(n)\mathbf{x}^H(n) \tag{4.31}$$

where $\hat{\mathbf{R}}_{xx}$ denotes the estimate of the covariance matrix and $\mathbf{x}(n)$ represents the array signal sample at the *nth* instant. In order to find the optimum weight vector, $\mathbf{r}_{xd}$ is also similarly estimated, then the optimum beamformer weight is given by

$$\mathbf{w} = \hat{\mathbf{R}}_{xx}^{-1}\hat{\mathbf{r}}_{xd}. \tag{4.32}$$

In contrast to the LMS algorithm, the DMI approach is the most computationally complex algorithm because of the requirement of matrix inversion. However, DMI provides the fastest convergence rate that is independent of the eigenstructure of $\mathbf{R}_{xx}$.

Recursive Least Square Algorithm (RLS) : One important concern with DMI algorithm is its modification for time varying environment. On the other hand, the primary drawback of LMS algorithm is the occurrence of the slow convergence. In order to prohibit problems induced by both algorithms, RLS approach is employed. Instead of sliding window, an exponential forgetting function is used to estimate both $\mathbf{R}_{xx}$ and $\mathbf{r}_{xd}$, namely,

$$\hat{\mathbf{R}}_{xx}(n+1) = \gamma\hat{\mathbf{R}}_{xx}(n) + \mathbf{x}(n)\mathbf{x}^H(n) \tag{4.33}$$

$$\hat{\mathbf{r}}_{xd}(n+1) = \gamma\hat{\mathbf{r}}_{xd}(n) + d^*(n)\mathbf{x}(n) \tag{4.34}$$

where γ is the forgetting factor, $0 < \gamma \leq 1$. It is used to deemphasize the old samples on the update equations, providing that the statistical variations can be followed. The expression of the optimum weights requires the matrix inversion of $\hat{\mathbf{R}}_{xx}$, and thus the process of updating and inversing can be combined to update the inverse of $\hat{\mathbf{R}}_{xx}$ recursively by using Matrix Inversion Lemma. The Kalman algorithm is used to update the beamforming weights iteratively.

There are a number of forms for the implementation of RLS algorithm, where it is indicated that the RLS algorithm is the most fastest algorithm, and is typically an order of magnitude faster than the LMS algorithm [77, 78].

4.4.3 Adaptive Beamforming Techniques

Array signal processor involves the manipulation of signals induced on the antennas. In order to combine the array outputs the weight vector is generally evaluated based on the criteria and algorithms given in the preceding section. All criteria and algorithms work properly if some sort of reference signal is present. Reference signals imply that a priori and explicit information or knowledge about the signals of interests [50]. Explicit reference can be categorized as *spatial reference* and *temporal reference*. A spatial reference mainly means the angle of arrival (AOA) of a desired signal. A temporal reference can be a training sequence or a pilot signal that correlates with the desired signal to a certain extent.

The conventional spatial reference algorithms are related to direction finding (DF) techniques. There are a vast number of direction of arrival (DOA) estimation methods that can be classified as wavenumber estimation techniques and parametric estimation techniques [50]. Wavenumber estimation techniques are only based on the decomposition of a covariance matrix whose elements comprise estimates of the correlation between the signals at the antennas of an array. The parametric estimation techniques mainly involve a variety of maximum likelihood estimation methods in which generally a specific likelihood function is formed for the given signals. This technique searchs directions that maximizes the likelihood function. Some of common spatial reference techniques are spectral estimation methods, MVDR (Minimum Variance Distortionless Response) estimator, maximum entropy method (MEM), eigenstructure methods, MUSIC (Multiple Signal Classification), ESPRIT (Estimation of Signal Parameters via Rotational Invariance Technique), CLOSEST, WSF (Weighted Subspace Fitting) [74]. All these DOA techniques require the knowledge of the *array manifold* to estimate DOA and the corresponding array response vector. Array manifold is defined as the collection of all array response vectors over the parameter range of interest Θ and is mathematically denoted as

$$\mathcal{A} = \{v(\theta_i) \mid \theta_i \in \Theta\}. \tag{4.35}$$

Temporal reference techniques use a reference or training signal to distinguish the desired signal from the interfering signals in such a way that this signal is correlated with the desired signal and uncorrelated with any interference. The major virtue of the temporal reference technique is no requirement for the knowledge of the DOA, hence

a system employing this technique has less complexity, and inexpensive cost compared to that DOA reference based systems.

In the case where the explicit reference signal does not exist, spatial-temporal reference techniques can not be employed. For this application, a blind adaptive beamforming scheme have been proposed and developed. Blind techniques use the same criteria and adaptation methods as well as other traditional adaptive beamforming techniques. A blind adaptive beamforming system forms its own reference signal based on the implicit properties of the desired signal instead of requiring an explicit reference signal. This technique exploits hidden signal characteristics such as constant modulus (CM), cyclostationarity, decision-directed, and other similar signal features [79, 80, 81, 82]. These signal-structure based beamforming techniques are very vigorous against different propagation conditions.

4.5 Use of Antenna Arrays in Mobile Communication Systems

The investigation of antenna arrays in communications has begun approximately for two decades ago. The main aim was to improve the performance of receivers in the presence of strong jamming, especially in military communications. Adaptive antennas have been used for TDMA satellite communications and spread spectrum communications. The mobile systems include outdoor-mobile, indoor-mobile, satellite-mobile and satellite-satellite communication systems. Although the implementation and application of adaptive arrays differ from one mobile system to other, the use of antenna array has mainly the same goal that is to improve the system performance. Since we concentrate on outdoor mobile communications, the use of antenna arrays in outdoor mobile systems are primarily discussed.

The use of antenna arrays in mobile radio communications to combat cochannel interference was pioneered by Yeh and Redulink [83]. They indicated that with a large number of antennas, the frequency reuse was probable to accomplish very high spectrum efficiency. They also showed that with a moderate number of space diversity branches, much higher spectrum efficiency was achievable compared to a non-diversity system. By using 24 branches for a base station and 20 branches for a mobile, 100% relative spectrum efficiency can be attainable. Bogacehv and Kiselev [84] dealt with a similar concept approximately in the same time. The capability of adaptive arrays

and their potential to increase the spectrum efficiency in mobile radio systems has been gradually recognized by the industry [50].

Recently, antenna arrays have been used in various configurations for mobile communications systems to primarily combat cochannel interference, time dispersion, and multipath fading. The following section will present fundamental benefits of using antenna arrays in mobile communications, and the main improvements that can be achieved by using an antenna array.

4.5.1 Advantages of Using Antenna Arrays

In mobile communication systems, adaptive arrays with multiple antennas offer paramount benefits over a single antenna. The improvement obtained by using an array can be expressed in a number of ways. It is able to reduce cochannel interference and multipath fading, and thus providing better quality of services such as reduced BER and outage probability. Multiple beam capability can be exploited to serve many users, and thus it results in increased spectral efficiency. It is able to adapt its beam as the mobile location changes, resulting in reduction on handoff rate.

In mobile communications, signals suffer from time dispersion (or delay spread) due to multipath propagation where received signals consist of a large number of delayed paths from different directions. An array provides the capability to form beams in certain direction and nulls in other directions to discard some of the delayed paths in two ways. In transmit mode, an array focuses its energy in the required direction, which provides to reduce reflections and thus results in a reduction in the delay spread. In receive mode, multipath fading can be combatted by space diversity combining, by adding the signals belonging to different clusters after compensating for delays, and by canceling delayed signals emanating from different directions than that of the intended signal [42].

A distinguished ability of an antenna array is to reduce cochannel interferences. This capability is exploited in transmit mode as well as in receiver mode. In transmitting mode, a beam is directed to the site where the intended receiver is likely to be. Hence, by focusing the energy in a certain direction, interference to other directions is reduced. Cochannel interference is further reduced in transmit mode by forming specialized beams with nulls in the directions of other receivers. The fundamental advantages of

using antenna arrays are presented in the following paragraphs:

Coverage : Antenna arrays with adaptive beamforming can increase the coverage significantly through antenna gain and interference rejection. In a noise limited environment, the coverage can be improved by a factor of $M^{1/n}$, where M represents the number of antennas and n denotes the propagation loss exponent. For example, if $n = 4$ and $M = 16$, the coverage can be doubled compared to the omnidirectional antenna case [50]. Moreover, increased coverage area results in the ease of deployment and reduction on the deployment costs.

Capacity : Depending upon the propagation conditions, adaptive antenna arrays improve the capacity or the spectrum efficiency substantially with respect to the single antenna case. Adaptive beamforming can increase the number of available communication channels via directional communication links. The improvement depends on the number of antennas, and the amount of dynamic channel assignment (DCA). Adaptive arrays with DCA allow the use of the same frequency in all cells, and provide that multiple mobiles operating on the same RF channel are separated spatially.

The reduction on the interferences and multipath fading due to adaptive beamforming enhances the SIR or SINR. Thus, the use of an antenna array increases the transmission data rate. In noise limited systems, the SNR can be improved by a factor of $10 \log M$ on the average depending on the modulation scheme. In interference limited environment, an additional improvement may be attainable due to interference rejection capability against directional interferers.

Handoff : In general, if the number of users in a cell exceeds its capacity, the cell is split into new cells, each of which has its own base station and new frequency. Thus, this reduced cell size results in an increase on the handoff rate. Using adaptive arrays with independent beams, the capacity is enhanced instead of cell splitting, and thus handoff rate is reduced.

In many cases, adaptive array technology provides locations of mobiles that can be used in handoff process, which is generally transparent to mobile users. Handoff process is required to maintain a sustaining call when a mobile is either far away from its base station or at the cell boundary. By accurately estimating mobiles' positions with adaptive arrays, handoff process is rather simplified.

Base station and Mobile Transmit Powers : For transmitting antennas, the antenna gain and the amount of power supplied to the antenna determine *effective*

isotropic radiation power (EIRP), which is the product of them. Actually, EIRP denotes the power radiated by an isotropic antenna, which radiates uniformly in all directions with the same main beam power density as the antenna of concern. Using adaptive beamforming with M antennas, the maximum EIRP required per user on a specific is reduced by $10 \log M$ dB compared to the case of without adaptive antennas, hence the base station transmitter power is reduced.

Similarly, since adaptive antennas provide additional gain of $10 \log M$ dB at base station receivers, the transmission power levels for mobile units can be lessened on average by at least $10 \log M$ dB. Consequently, this gain relaxes the requirements on batteries for mobile units.

BER and Outage Probability Improvement : BER provides a good indication of the performance of a particular modulation scheme. In mobile environments, signals suffer from deep fades, which can lead to outage or a complete loss of the signal. Outage probability is another performance measure for a mobile radio system, which is the probability of failing to obtain satisfactory reception in the presence of noise and interference.

Adaptive antenna arrays provide the capability to reduce cochannel interferences and multipath fading, and thereby result in reduced BER and outage probability. There are intensive research studies quantifying the performance improvement by the use of adaptive antennas in terms of the BER and the outage probability. Winters [85] presented a detailed analysis and simulation results for BER in a PSK system with flat fading and interference. The results showed that BER improvement increases with the number of antennas, and there is a large BER reduction compared to the case of a single antenna. He also indicated that adaptive beamforming systems require less SIR than that of maximal ratio combining. The experimental and analytical results for BER were reported in [11] to show that with optimum combining a reduction in BER is provided for frequency selective fading channels as well as flat fading channels. Winters continued his studies about optimum combining for indoor environment in [86] to demonstrate its impact in reducing BER for a PSK system.

The analytical and simulation results of BER are presented for CDMA systems with adaptive beamforming in [87], where RLS algorithm is used to adjust the weights of an array. Results on the improvement in the outage probability by means of an adaptive array for a CDMA system is presented in [88]. It is shown that for a given outage

probability, adaptive antenna arrays allow an increase in the number of users in a cell, resulting in higher system capacity.

The performance enhancement brought about by the use of adaptive antennas is discussed in [89, 10, 90, 91]. The study in [89] shows that how much cochannel interference reduction can be achieved by the use of adaptive antennas. The fading and shadowing effects have been considered in the analysis. It is observed that the outage probability decreases as the number of beams used by a base station. The detailed analysis and simulation results given in [90, 91] conclude that with optimum combining significant reduction is achieved in the outage probability. A comparison of the outage probability performance using selection diversity and optimum combining shows that the latter provides a reduced threshold for a given outage probability [10].

4.5.2 Applied Adaptive Beamforming Techniques

An important concern for adaptive beamfoming techniques is how well the numerous criteria and algorithms operate in practice. There has been a considerable amount of interest on the application and implementation of these algorithms for mobile radio systems. Most of these studies have been devoted to the reverse link part of the communication since the spatial information is readily available on the reverse link. However, in recent years some research effort has also been devoted to the forward link case [92, 93].

Application of LMS, DMI and RLS algorithms to estimate the beamforming weights to operate in mobile radio systems has been studied in many papers. Although the LMS algorithm has the least complexity, Winters [85] showed that the LMS algorithm has lower dynamic range; for example, it has 20 dB for a specific scenario. In mobile communications, signal levels can vary more than 20 dB; hence power control is necessary if the LMS is employed. In order to overcome the dynamic range limitation, the use of normalized LMS algorithm is proposed [77]. The convergence rate is an important parameter for operation of algorithms in mobile communications because of fading. Thus, the beamforming weights are updated continuously to adapt the signal environment. In [31], the performance of DMI and LMS algorithms are compared for the North American digital radio system IS-54. It is indicated that LMS does not perform as well as DMI in a rapidly fading environment. However, DMI's complexity increases much

faster than that of LMS. There are a number of considerations for the application of DMI algorithms in different communication areas such as GSM mobile systems, indoor radio systems, mobile satellite communication systems [94, 95, 96]. The main drawback of DMI algorithms is the requirement of matrix inversion. This shortcoming can be avoided by the use of RLS algorithm that replaces the inversion of the covariance matrix by a simple scalar division at each step. A computer-based simulation of the RLS and LMS algorithm indicates that the former provides better performance than the latter in flat-fading channels [97]. Adaptive arrays in combination with RLS algorithm is considered for the reverse link of a cellular system employing CDMA in [98] to highlight capacity improvement by an array.

In addition to these algorithms, other adaptive techniques have also been proposed and studied in mobile radio systems to either overcome imperfections or improve the performance of the LMS, DMI and RLS algorithms. Some of these techniques are the conjugate gradient, the eigenanalysis algorithm, the linear least squares error (LLSE) algorithm, and the neural networks [50].

Direction finding methods have been studied to indicate their potentials in mobile communications systems. Since MUSIC algorithm is a relatively simple and efficient eigenstructure method of DOA estimations, it is the most widely studied one. There are a number of variations of MUSIC algorithm such as Root-MUSIC, Constrained MUSIC, and Beam-Space MUSIC algorithm. ESPRIT is known as a computationally efficient and robust algorithm for DOA estimation, and there have been a vast number of research work about its application in mobile communications. More detailed information on various issues of DOA estimation may be found in the tutorial paper [74].

In IS-54 systems, each slot includes a known 28-bit synchronization sequence and 12-bit user identification sequence. In [11], the use of this synchronization sequence is proposed to generate the temporal reference signal. Results show that the system capacity is improved by reducing the frequency-reuse factor and increasing SINR with the use of adaptive beamforming. Similar to IS-54, GSM system consists of a training sequence in each data burst. It has been proposed to use this sequence to create a reference signal for beamforming [94]. A temporal reference scheme has also been proposed for a digital cordless telephone DECT system [99].

In mobile communications the presence of multipaths may make the knowledge of

the array manifold unreliable, which results in DOA techniques to be almost useless. Most of the DOA algorithms are based on eigen-decomposition concept, which may be difficult for implementation. In addition, all DOA techniques assume that the number of cochannel interfering signals is less than the number of antennas of an array. In contrast to DOA techniques, temporal reference or training sequence based beamforming algorithms do not require the information on the DOA. These techniques operate well under multipath fading environment since there is no restriction about the multipath angle spread. However, these techniques need the use of training sequences that reduce the throughput of the system. Moreover, these techniques require robust carrier and symbol recovery for training signals to operate properly, but this can be difficult in the existence of interference. Hence, one innovative trend in mobile communications systems is the use of blind adaptive beamforming techniques. The CMA (Constant Modulus Algorithm) has been applied to advanced mobile phone service (AMPS) and IS-54 signals [100], GMSK signals [101] to investigate how well it operates in the presence of fading and cochannel interference. In [81] a decision directed algorithm was investigated, and it was compared to CMA as well as to SCORE algorithm. It is shown that the convergence rate of the decision directed approach is much faster than other algorithms, and it reaches its theoretical performance limit after 50 symbols.

For CDMA systems, the use of a joint spatial-temporal process have been proposed in the presence of both multipath fading and interference. This process is implemented by using an adaptive beamformer with a RAKE receiver [12]. Before beamforming, code-filtering is employed at each antenna for individual user in the system. The eigenstructure of the pre and post-correlation array covariance matrices is exploited to estimate the channel vector and the corresponding beamforming weight. This approach is also a blind technique in the sense that a training sequence is not required although the perfect knowledge of the spreading sequence is assumed for each user.

CHAPTER 5

ANTENNA ARRAYS FOR CDMA MOBILE SYSTEMS

5.1 Introduction

Since CDMA offers better performance in terms of user capacity and spectrum efficiency compared to TDMA and FDMA, there has been intensive research work for its use in digital cellular systems. In CDMA systems, all mobiles utilize the same radio frequency for transmitting to the base station. Although each user is identified by its own spreading sequence, it can interfere with other users due to cross correlation between spreading sequences. Hence, the presence of multiple access interference (MAI) place limits on the system capacity. Moreover, the performance of the CDMA system is limited by the multipath fading.

Antenna arrays with optimum combining have been shown to be an attractive solution to combat multipath fading of the desired signal, to suppress the MAI, and thus they improve the performance of mobile radio communication systems. With optimum combining or beamforming, the signals received by antennas are combined to intensify the intended signal power while reducing the interfering signal powers.

The reverse link from the mobile unit to the base station presents the most difficult problem in CDMA cellular systems for several reasons [87]. First of all, the base station has complete power control of all transmitted signals on the forward link, however due to different propagation paths between each user and the base station, the transmitted power from each mobile must be dynamically controlled to prevent near-far problem. Second, transmit power is limited by battery consumption at the mobile unit so that the mobile unit power may not be increased to a desired value at all times. Thus, the use of adaptive arrays at the base station has received considerable attraction to

enhance the performance of CDMA mobile systems.

On the other hand, adaptive arrays have also been investigated for both the reverse link and the forward link to increase the capacity of multicellular CDMA mobile systems [102]. The increase in capacity stems from reducing the amount of cochannel interference from other users inside both the corresponding cell and the neighboring cells. The well-known IS-95 CDMA standard already uses sectorized antennas, each of which covers 120^o beamwidth in both transmission and reception [46].

A major advantage of spread spectrum communications is their potential to exploit the multipath structure of the received signal. The multipaths are characterized by their time delays, amplitudes and angle of arrivals. The well-known RAKE receiver estimates the path delays, amplitudes, and combines them optimally however the spatial characteristics of the received signal cannot be exploited by using a single antenna. The antenna arrays are required to attain an efficient path combination scheme. In order to identify the spatial structure, angle of arrival (AOA) estimation techniques should be incorporated into the system. Actually, because of the number of multipaths exceeding the number of antennas in mobile environment, the traditional AOA techniques (e.g. MUSIC and ESPRIT) cannot be easily applied. However, since in CDMA system each user is identified by its spreading sequence, this ability makes it possible to identify both the temporal and spatial structure of the multipaths, and this scheme has been addressed in [103].

Although path diversity in conjunction with adaptive beamformer mitigates some problems of CDMA system, sufficient fading reduction may not be achieved in practical applications. In traditional array processing techniques, antennas are placed at a half wavelength ($\lambda/2$) or smaller intervals such that a high correlation exists between signals across the array. However, most of the combining schemes including maximal ratio combiners assume independent fading of the desired signal and interfering signals at the antennas so that these approaches cannot operate properly in a correlated fading environment. When the correlation between antennas is high, the signals at the antennas tend to fade at the same time, and thereby the ability of antennas against fading is drastically reduced. This reduction can be avoided by providing means for independent fading at the antennas (space diversity). It is possible to place antennas sufficiently apart so that the fading signals received from different antennas become uncorrelated.

The combination of space diversity and path diversity has already been studied in [104]. This study involves a quantative evaluation of fading reduction effects of the hybrid diversity, combining them with those of space diversity alone and path diversity alone. The equispaced array with a spacing of 2λ is used in the analysis. It is shown that the hybrid diversity scheme is promosing in environments where the delay spread of a transmission is $1\mu s$ or less.

The multipath fading reduction effect due to space diversity depends on the number of antennas M and the correlations among the antennas. Salz and Winters [37] showed that independent fading at each antenna can be obtained if the spacing between antennas is much greater than half wavelength. In conventional systems, uniform linear arrays are the most commonly used among antenna arrays. However, as the spacing exceeds $\lambda/2$ significantly, undesired grating lobes that have the same gain as the main lobe occur in the array beam pattern of the uniformly spaced array. This problem is the most severe for line of sight (LOS) users in which case multipath effects are negligible. Since antenna arrays are also intended to suppress the interfering signals in addition to combating multipath signals, uniform arrays do not seem to be suitable. Therefore, the use of the nonuniform linear array can be investigated as a promising method.

In the previous chapter, we qualitatively discussed the impacts of antenna arrays on mobile radio communication systems. In this chapter, we quantify the performance improvement brought about by the use of an antenna array in a CDMA system, covering the relevant topics: We give a description of the mobile channel characteristics in a Rayleigh flat fading environment with multiple antennas. We present the design of a nonuniformly spaced array by discussing the previous studies. We describe the reverse link propagation medium for both LOS users and far users (i.e. multipath exists) to make a quantitative analysis, which includes the derivation of MAI and signal-to-interference-plus-noise ratio (SINR), and beamformer weights. Finally, we discuss how much the performance of LOS users differs in the case of uniform and nonuniform arrays. For far users, we investigate how much the performance improvement can be achieved by the use of a nonuniform array. The performance improvement is quantified by comparing the outage probabilities and bit error probabilities.

5.2 Mobile Radio Channel with Multiple Antennas

The mobile channel can place paramount limitations on the performance of mobile communications systems. In order to overwhelm channel impairments it is required to use a channel model, which is representative of the real environment in the development stages of a mobile system.

The mobile radio channel model is primarily employed to achieve realistic and reliable results while predicting and comparing the performance of different systems. There have been a large number of channel models based on both measurements and theoretical works for single antenna systems [105, 48, 49, 106, 107, 108]. These models describe path loss, power delay profiles and Doppler spectrum. Recently, there has been a growing interest on the application of adaptive antenna arrays in mobile systems, and there are also a number of studies on the modeling of mobile radio channels with multiple antenna case [37, 110, 111]. For multiple antenna systems, the direction of arrival (DOA) is also included to constitute a valid channel model. The most severe fading in mobile communications occurs when there is no line of sight (LOS) between the mobile and the base station. This section describes a Rayleigh fading channel model with the local scattering in a multiple case system, which has been proposed by Salz and Winters [37].

The receiver motion or motion of the objects in the environment causes time variation of the channel. If the relative motion between the transmitter and receiver is not too much, the channel characteristic can be assumed to be at least wide sense stationary over a short period of time or travel distance.

The multiple antenna channel model is generally referred to as vector channel model. Throughout the analysis, we assume that the signal envelope remains constant while the signal propagates across the antenna array. This implies that the frequency responses of the antennas (and the whole array) are supposed to be approximately flat over the signal bandwidth. This assumption can be valid for both narrowband and wideband signals if the aperture size of the array is small relative to the inverse bandwidth of the signal. Thereby, time delays due to propagation across the antenna array are modeled as phase shifts [12].

A linear array is employed at the receiver, which consists of M identical elements, each giving the same response for any given direction. A simple plane wave model

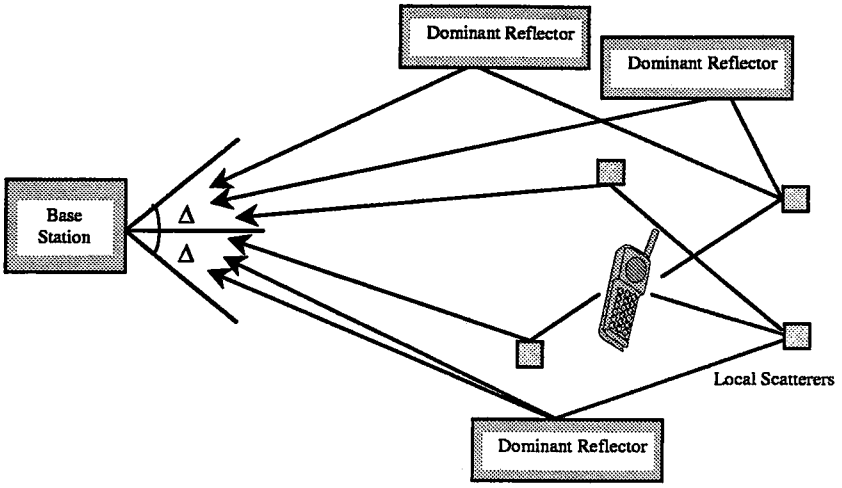

Figure 5.1: An illustration of a typical mobile radio propagation environment

assumption is used, and the whole structure has only two dimensions in order not to complicate the analysis. Let *1st* antenna be the reference on the array, and the array response vector be given by

$$\mathbf{v}(\theta) = \begin{bmatrix} 1 & e^{j2\pi d_1 \sin\theta} & e^{j2\pi d_2 \sin\theta} & \dots & e^{j2\pi d_{M-1} \sin\theta} \end{bmatrix}^T \tag{5.1}$$

where θ is the angle between the arriving signal and the normal to the array, and d_{m-1} denotes the distance between *mth* and *1st* antennas in terms of the wavelength λ. In mobile communications, each propagation path arriving to base station has also its own angle of arrival as well as time delay, amplitude and phase. However, single antenna cannot exploit these angles of arrivals. Note that (5.1) implies that the array elements are isotropic.

The illustration of a typical mobile radio propagation situation is depicted in Figure 5.1. The location of reflectors and scatterers determine the angle of arrival of each path to the base station antenna array. The signals often arrive at the base station antennas mainly from a given direction. For example, in rural or suburban areas, antenna arrays are located on a very high position at the base station. Hence, the antenna array may have a LOS path to the mobile with local scattering signals that arrive mainly within a given range of angles or beamwidth [37]. It is assumed that signals arrive from mobile to base station within an interval $[-\Delta, +\Delta]$ referred to as angle spread, centered at the angle of θ. Figure 5.2 shows a typical scenario.

The probability density function (pdf) for angle of arrivals depends on the spatial

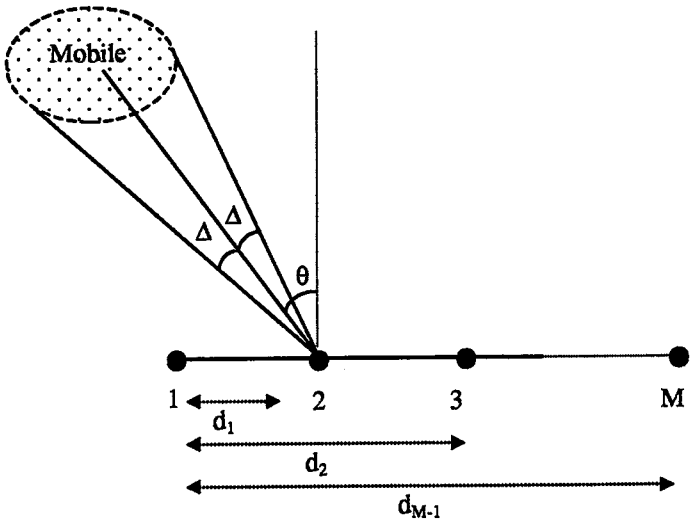

Figure 5.2: Wireless environment where all signals from mobile arrive at base station within $\mp\Delta$ at angle θ

distribution of scatterers. The pdf of angle of arrival is usually described by certain distributions such as cosine-shape, Gaussian, or uniform. Although, the uniform distribution appears difficult to be justified physically, it is widely employed due to its simplicity. In this study also, the probability density function of angle of arrival is assumed to be uniform:

$$f_\theta(\theta_i) = \begin{cases} \frac{1}{2\Delta} & -\Delta + \theta \leq \theta_i < \Delta + \theta \\ 0 & \text{elsewhere} \end{cases} \tag{5.2}$$

where Δ is the angle spread of the mean angle of arrival of θ_i.

Let $s(t)$ be the baseband equivalent of the transmitted signal. If the inverse bandwidth of the transmitted signal is much higher than the multipath delay spread of the channel ($W^{-1} \gg \Gamma$), i.e. $s(t)$ is a narrowband signal, then $s(t - t_1) \approx s(t - t_0)$ where $t_0 \in [\min_l t_l, \max_l t_l]$ [12]. Then, the baseband equivalent of the received signal at the antenna array is (if the additive white Gaussian noise is ignored)

$$\mathbf{y}(t) \approx s(t - t_0) \sum_{l=1}^{L} \xi_l \, e^{j\phi_l} \mathbf{v}(\theta_l) \tag{5.3}$$

where ξ_l denotes the amplitude of *lth* path (sometimes referred to as reflector coefficient) and ϕ_l represents the phase. For the time varying case, these parameters will vary with the time. In (5.3), the combination of all terms excluding the transmitted signal is

defined as the *channel vector*

$$\mathbf{a} = \sum_{l=1}^{L} \xi_l \, e^{j\phi_l} \mathbf{v}(\theta_l) \tag{5.4}$$

and the impulse response of channel is described by **a**:

$$\mathbf{h}(t) = \delta(t - t_0)\mathbf{a}. \tag{5.5}$$

If ϕ_ls are i.i.d. random variables over $[0, 2\pi]$, and ξ_ls are also i.i.d. random variables, the resulting **a** is a complex random vector. Furthermore, if the sum includes a large number of terms, then the complex channel vector will approach a zero mean complex Gaussian random vector [37, 12]. Hence, the complex channel vector can be characterized by its mean (zero mean) and covariance matrix, i.e.,

$$\mathbf{\Sigma}_a = E\left\{\mathbf{a}\mathbf{a}^H\right\}. \tag{5.6}$$

This covariance matrix has a paramount importance to evaluate correlation between array elements, which affects the whole system performance. $\mathbf{\Sigma}_a$ can be written as

$$\mathbf{\Sigma}_a = E\left\{\sum_{l=1}^{L}\sum_{k=1}^{L} \xi_l \xi_k \, e^{j\phi_l} e^{-j\phi_k} \mathbf{v}(\theta_l)\mathbf{v}^H(\theta_k)\right\}. \tag{5.7}$$

As aforementioned assumptions based on the ϕ_ls are used, (5.7) is reduced to

$$\mathbf{\Sigma}_\mathbf{a} = \sum_{l=1}^{L} E\left\{\xi_l^2\right\} \mathbf{v}(\theta_l)\mathbf{v}^H(\theta_l). \tag{5.8}$$

According to the assumptions that there are a large number of terms in the sum, i.e. $L \to \infty$ and θ_ls are uniformly distributed over $[-\Delta + \theta, \Delta + \theta]$, the sum can be expressed as an integral:

$$\mathbf{\Sigma}_v = \frac{1}{2\Delta} \int_{-\Delta+\theta}^{\Delta+\theta} \sigma_\xi^2 \, \mathbf{v}(\rho)\mathbf{v}^H(\rho) \, d\rho \tag{5.9}$$

where $\sigma_\xi^2 = \sum_{l=1}^{L} E\{\xi_l^2\}$. Then, the normalized channel vector can be characterized by $\mathbf{\Sigma}_a/\sigma_\xi^2$ for convenience; in fact this ratio is the correlation matrix of $\mathbf{v}(\theta)$, which is defined as $\mathbf{\Sigma}_\mathbf{v} = E\{\mathbf{v}(\theta)\mathbf{v}^H(\theta)\}$.

Remembering the array response vector definition in (5.1), *(m, n)th* element of $\mathbf{\Sigma}_v$ is given by

$$\mathbf{\Sigma}_v(m,n) = \frac{1}{2\Delta} \int_{-\Delta+\theta}^{\Delta+\theta} e^{j2\pi(d_m - d_n)\sin\rho} \, d\rho. \tag{5.10}$$

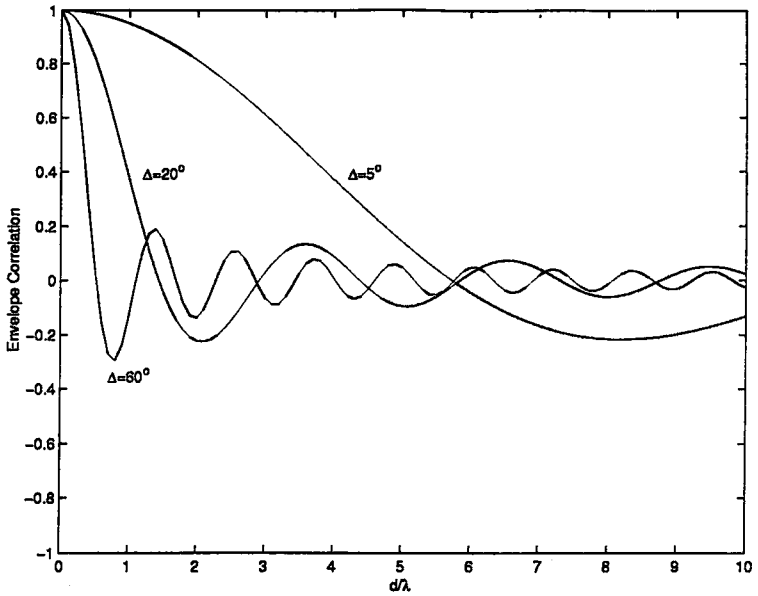

Figure 5.3: Correlation versus antenna spacing for different angle spreads when $\theta = 0^o$

Using the well-known series representations [112], i.e.,

$$\begin{aligned} \cos(z\sin\rho) &= J_0(z) + 2\sum_{i=1}^{\infty} J_{2i}(z)\cos(2i\rho) \\ \sin(z\sin\rho) &= 2\sum_{i=0}^{\infty} J_{2i+1}(z)\sin((2i+1)\rho), \end{aligned} \tag{5.11}$$

we have

$$Re\{\mathbf{\Sigma}_v(m,n)\} = J_0(z_{m,n}) + 2\sum_{i=1}^{\infty} J_{2i}(z_{m,n})\cos(2i\theta)\frac{\sin(2i\Delta)}{2i\Delta} \tag{5.12}$$

$$Im\{\mathbf{\Sigma}_v(m,n)\} = 2\sum_{i=0}^{\infty} J_{2i+1}(z_{m,n})\sin((2i+1)\theta)\frac{\sin((2i+1)\Delta)}{(2i+1)\Delta} \tag{5.13}$$

where J_0 is Bessel function of integer order and

$$z_{m,n} = 2\pi(d_m - d_n). \tag{5.14}$$

The envelope correlation $\eta_v(m,n)$ is defined as

$$\eta_v(m,n) = \sqrt{(Re\{\mathbf{\Sigma}_v(m,n)\})^2 + (Im\{\mathbf{\Sigma}_v(m,n)\})^2}. \tag{5.15}$$

From (5.12) and (5.13) it is seen that when $\Delta = 0^o$, $Re\{\mathbf{\Sigma}_v(m,n)\} = J_0(z_{m,n})$ and $Im\{\mathbf{\Sigma}_v(m,n)\} = 0$. When the signal arrives from broadside (i.e., $\theta = 0^o$) the imaginary part is zero and the envelope correlation reduces to the real part as in the case of

$\Delta = 0^o$. This case is depicted in Figure 5.3 for $\Delta = 5^o$, $\Delta = 20^o$ and $\Delta = 60^o$ for the uniformly spaced array. This figure indicates that the correlation decreases with increasing angle spread. In addition, as the angle spread decreases, the first zero in the correlation occurs at a larger antenna spacing. Specifically, the first zero crossing occurs at $d/\lambda \approx 30/\Delta$ [37]. The effect of angle of arrival on the correlation is depicted in Figure 5.4 for $\Delta = 20^o$. When the signal arrives at an angle other than broadside, the required antenna spacing for low correlation increases, and the envelope correlation is never zero for all values of $\theta \neq 0^o$ and $\Delta < 180^o$ (i.e., η_v is zero if $Re\{\mathbf{\Sigma}_v\}$ and $Im\{\mathbf{\Sigma}_v\}$ have zero crossings at exactly the same spacing). The impacts of angle spread on the correlation can be summarized as follows:

1. If the angle spread is small, the outputs of an array are highly correlated, and all paths indicate approximately the same direction. Hence, the complex channel vector exhibits single direction, and it varies only in amplitude and phase. This type of fading is called as *nonspace selective fading*. The conventional direction finding (DF) techniques (e.g. MUSIC or ESPRIT) can be used to determine direction of arrival. However, when the signal from one of the antennas is in deep fade, all others will be in the same fading so that space diversity cannot be achieved.

2. If the angle spread is large, the signals from different antennas have low correla-

tions. Therefore, the complex channel vector has changes in both amplitude and directions and it does not point to any certain direction. This type of fading is referred to as *space selective*. In this case, the conventional DF methods cannot be employed. On the other hand, because of independent fading, the array provides space diversity gain.

If the bandwidth of the incoming signal increases, the narrowband assumption will no longer be valid, and hence the received signal consists of multiple delayed version of the transmitted signal. Based on Turin's work [113] multipath components with arrival times smaller than the inverse bandwidth of the transmitted signal cannot be resolved at the receiver, and they are combined into a single path. Hence, each resolvable signal is, in fact, comprised of numerous unresolvable paths. The baseband equivalent of the time invariant multipath fading channel is described as

$$\mathbf{h}(t) = \sum_{p=1}^{P} \delta(t - \tau_p)\mathbf{a}_p \tag{5.16}$$

where P is the number of resolvable paths. The complex channel vector includes a linear combination of a large number of paths. Each complex channel vector $\mathbf{a}_p$ is characterized by its mean of arrival of θ_p and angle spread of Δ_p. The elements of $\mathbf{a}_p$ are zero mean complex Gaussian correlated random variables. With the assumption of independence of different resolvable paths, the covariance matrix of the complex channel vector is given

$$\begin{cases} \sigma_l^2 \mathbf{\Sigma}_v(\theta_l, \Delta_l) & \text{if} \quad l = k \end{cases}$$

synthesis, the main concern is to produce a beampattern that suitably approximates a desired one. In most studies, the desired beampattern has been achieved by adjusting the inter-element spacings according to certain optimization objectives.

Many early attempts for designing nonuniform arrays were based on series expansions of the array beampattern. Equating the truncated series with the desired, these equations are solved for antenna gain [114] and for location [115]. The same problem has been addressed in [116, 117], and using spatial tapering of the array elements sidelobes are reduced. An alternative approach based on random element positions has been proposed by Lo [118]. The array patterns are selected as samples from a probability distribution that exhibit optimum statistical properties. In 1968, Moffet [117] has proposed a class of nonuniform arrays referred to as minimum redundancy arrays, which are linear arrays for a given number of array elements with maximum resolution (i.e., minimum beamwidth) at the expense of relatively high peak sidelobe level.

The studies given in [120, 121, 122] are based on the solution of a series of equations. In [120, 121] a set of linear equations are iteratively solved to attain a desired beam pattern with low sidelobe levels. Each equation is written for a specified point on the antenna beam pattern. The iteration is started with a uniform array having a spacing of $\lambda/2$, and then the beam pattern is rewritten by assuming that every antenna is shifted with a small distance with respect to its uniform location. The perturbation methods differ in two methods. At the end of each iteration, new array positions are obtained and they are used to generate the antenna beam pattern to initiate the next iteration. However in [122] a single equation is solved iteratively to obtain one of the inter-element spacings. The others are generated according to a simple relation.

The dynamic programming has been used to search array element positions of a symmetric linear array with the objective of minimizing peak sidelobe levels [123]. Bratkovic [124] extended this work with the constraint of monotonically increasing element spacing away from the array center to achieve the minimax solution with guaranteed optimality. An alternative design for a nonuniform array based on the constrained optimization is presented in [125] where the nonlinear cost function is chosen as to select the array with the minimum number of antennas. The response of the array is controlled by a set of linear inequality response constraints.

The problem of designing nonuniformly spaced array has also been considered in conjunction with null steering methods, which are used for placing nulls in arbitrary

directions in the antenna beam pattern in order to suppress directional interferers. The process of null steering based on Taylor series expansion has been carried out by controlling the element positions [126, 69]. The former minimizes the mean square value of the array pattern in certain directional regions while imposing one or several directional nulls constraints with respect to an error element position vector. However, the latter involves null control by using an array beam pattern perturbation technique in which either minimization of the sum of the square of the total element position perturbations or the sum of the squares of the relative element perturbations is used as a constraint. The experimental results for this approach are presented in [127] to prove the validity of null steering by controlling the element positions.

Large arrays are rather difficult to optimize with respect to certain criteria in order to obtain desired beam pattern. Traditional optimization methods such as simplex, Powell's method, conjugate gradient are not well for optimizing a large number of parameters. Instead 'thinning' approach has been used in the application of large arrays. Thinning an array means turning off some elements in a uniformly spaced array [128]. This approach is much simpler than the more general problem of spacing the elements nonuniformly. Simulated annealing and genetic algorithms [129], well-known optimization techniques, have been proposed to handle large antenna arrays because they have no limit on the number of optimization variables. Ruf [130] used simulated annealing to optimize low-redundancy linear arrays. Haupt [128] employed genetic algorithms to produce the lowest maximum relative sidelobe level for both linear and planar arrays. The global nature of these algorithms and the lack of derivative information result in them to converge very slowly compared to other methods. Therefore, they may not be suitable for real-time pattern control, however they are quite useful for optimizing large arrays.

5.3.2 The Proposed Array

As mentioned earlier, if the correlation between antennas is high, the space diversity is not achievable or significantly reduced. Therefore, we first concentrate on the correlation characteristics. For example, as seen from Figure 5.3, a spacing of 2λ yields sufficiently small correlation for a moderate angle spread, e.g. $\Delta = 20^o$. However, this spacing causes grating lobes on the array beam pattern of uniform arrays, and thereby

T.C. YÜKSEKÖĞRETİM KURULU
DOKÜMANTASYON MERKEZİ

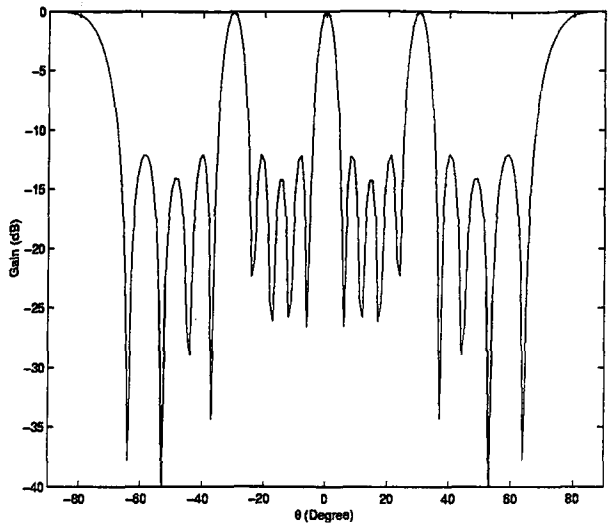

Figure 5.5: The normalized array beam pattern for uniformly spaced array with $M = 5$ and $d/\lambda = 2$

this kind of array cannot achieve the compromises. Therefore, we propose the use of nonuniformly spaced array.

At very early beginning of this study, our aim was to obtain the optimum inter-element spacing in order to maximize SINR. However, it was seen that finding optimum spacings is an extremely difficult problem, and the conventional optimization techniques cannot be directly employed. We then focused on the previous studies on the design of nonuniformly spaced arrays. Most of the approaches discussed in the preceding section begin with a uniform array having inter-element spacing of $\lambda/2$. Then the spacings are perturbed slightly to achieve the predetermined objective such as sidelobe reduction, desired beam pattern, etc. It was seen that these schemes are not suitable for our problem, and hence we propose an approach to find the spacings, in which the spacings are generated randomly according to a predetermined statistics as follows:

We have selected the average spacing as 2λ. Since in practice array elements cannot be much closer than $\lambda/2$ due to mutual coupling, the minimum spacing is chosen as $\lambda/2$. The number of antennas and the aperture size of the array are selected as the same as that of the uniform array having a spacing of 2λ. The nonuniformly spaced array arrangement is accomplished iteratively by generating random points on a line. The first element on the array is taken as the reference. The other spacings are checked to see whether they satisfy the minimum spacing of $\lambda/2$. If one does not satisfy this condition, it is regenerated and the iteration procedure is repeated. The generation

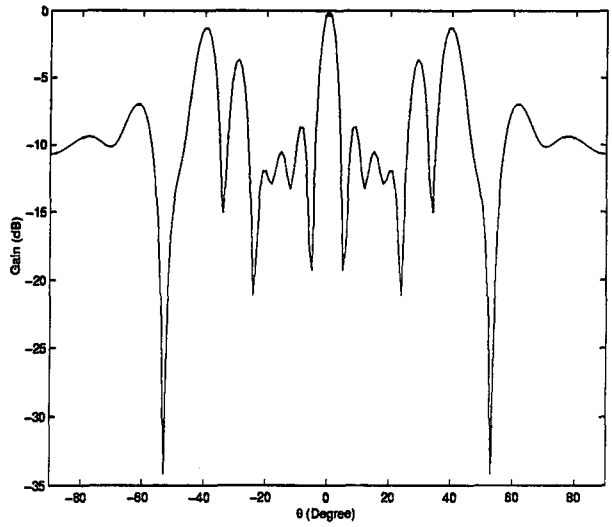

Figure 5.6: The normalized array beam pattern for nonuniformly spaced array with $M = 5$ and $d/\lambda = 2$

process continues until all spacings meet the minimum spacing criterion. All spacings are normalized according to the last spacing value.

Table 5.1: Antenna spacings for M=5

	d_0	d_1	d_2	d_3	d_4
Uniform	0	2	4	6	8
Nonuniform	0	1.94	3.4	6.4	8

Table 5.2: Antenna spacings for M=7

	d_0	d_1	d_2	d_3	d_4	d_5	d_6
Uniform	0	2	4	6	8	10	12
Nonuniform	0	1.1	3.5	4.5	6.2	7.3	12

Table 5.1 and 5.2 presents the examples of the obtained spacings for the number of antennas $M = 5$ and $M = 7$ via uniform spacings respectively. These spacings have been selected among the generated ones, which provide the best SINR values. The average spacing was chosen as 2λ. The corresponding aperture sizes are 8λ and 12λ.

The normalized array beam pattern plots are depicted in Figure 5.5 and 5.6 for the uniform and nonuniform arrays with 5 antennas, respectively. Similarly, Figure 5.7 and 5.8 show the normalized array beam patterns for $M = 7$ antennas. As can be

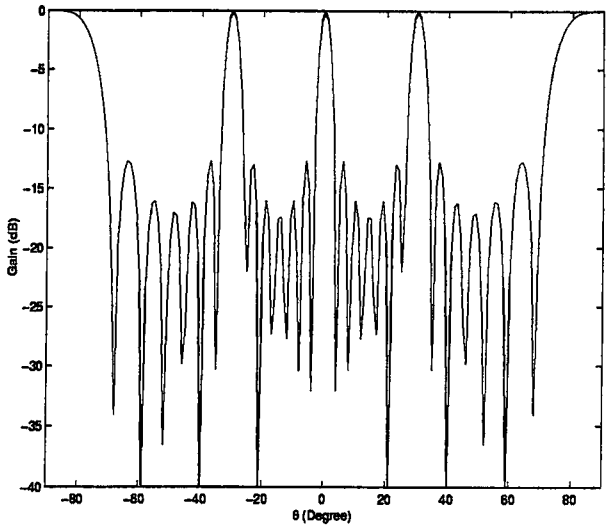

Figure 5.7: The normalized array beam pattern for uniformly spaced array with $M = 7$ and $d/\lambda = 2$

seen, grating lobes appear in the uniform array patterns (Figure 5.5 and Figure 5.7) in contrast to the patterns of the nonuniform arrays (Figure 5.6 and Figure 5.8) .

Throughout the thesis, we employ this approach to form nonuniformly spaced linear arrays. We first generate the spacings for the nonuniform array having the average spacing of λ. Then, the obtained spacings are scaled to attain another nonuniform array with a larger average spacing, i.e,

$$\mathbf{d}_{new} = \alpha \mathbf{d} \tag{5.18}$$

where α is the scaling factor, $\mathbf{d}$ is the spacing vector having the previously generated spacings with the average spacing of λ, and $\mathbf{d}_{new}$ denotes the new spacing vector that will be generated.

5.4 LOS Users

5.4.1 System Model

In this section, we develop a model of a CDMA system with an antenna array to determine the SINRs of LOS users at the output of the array. Then, this model is employed to show the amount of improvement using the nonuniform array.

We consider a linear array located at the base station, which uses the array to receive and transmit signals from and to the mobile user. Since the reverse link presents the most challenging part in a CDMA mobile system, this link is particular of interest.

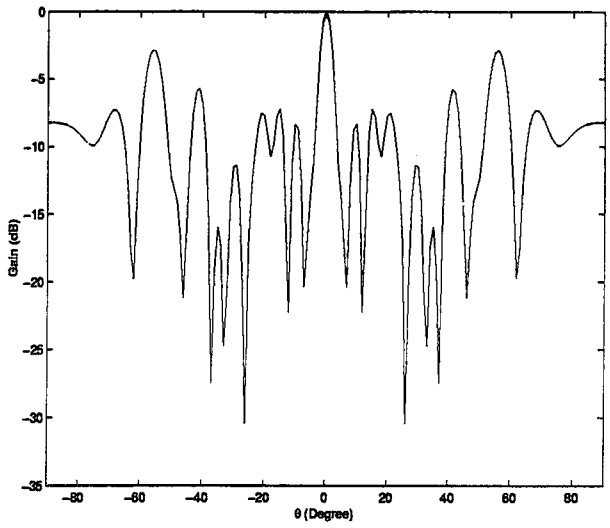

Figure 5.8: The normalized array beam pattern for nonuniformly spaced array with $M = 7$ and $d/\lambda = 2$

In order to simplify the analysis, we consider only a single cell environment, however this analysis can be extended to multicell environment by considering the intra-cell interference. The CDMA system of concern is assumed to have a small cell, in which cell size is sufficiently small so that the users' signals arrive at the base station approximately at the same time. A typical illustration of base station antenna array with M identical elements is depicted in Figure 4.1. A simple plane wave model is used, and the whole structure has only two dimensions in order to not complicate the analysis.

Let N be the number of active users randomly distributed within the cell. The number of users is assumed to be much larger than the number of antennas. The DSSS-CDMA system employing BPSK is considered in the analysis. The transmitted signal from *ith* user is given by

$$s(t) = Re\{u_i(t)e^{j2\pi f_c t}\} \tag{5.19}$$

where f_c represents the carrier frequency, and the lowpass equivalent signal $u_i(t)$ is given by

$$u_i(t) = \sqrt{P_i}b_i(t)c_i(t). \tag{5.20}$$

Here, $b_i(t)$ is a data sequence taking the values of ∓ 1 and having duration of T_b seconds, $c_i(t)$ is the spreading sequence belonging to *ith* mobile, and P_i is given by

$$P_i = \frac{1}{T_b}\int_0^{T_b} u_i^2(t)\,dt. \tag{5.21}$$

The spreading code is defined as

$$c_i(t) = \sum_{l=0}^{L-1} c_{i,l}\, p(t - lT_c) \tag{5.22}$$

where $c_{i,l}$ is the *lth* chip of the *ith* spreading sequence, each of duration of T_c, and $p(t)$ is the chip pulse shape. In our analysis, $c_{i,l}$s are assumed to be independently and identically distributed (i.i.d.) random variables taking values of ∓ 1 with equal probability, and $p(t)$ is assumed to be a Nyquist pulse. The processing gain is defined as $L = T_b/T_c$.

As far as the reverse link is concerned, the perfect power control is assumed within the cell; the average power received by an antenna for each user is kept at the predetermined level, P. In addition, the perfect carrier and bit synchronization are accomplished at the receiver.

Considering LOS users, the complex baseband output of the array is given by

$$\mathbf{x}(t) = \sum_{i=1}^{N} \sqrt{P} b_i(t) c_i(t) \mathbf{a}_i + \mathbf{n}(t) \tag{5.23}$$

where $\mathbf{a}_i$ is the Mx1 complex channel vector or antenna array response vector for arriving signal from *ith* user, and $\mathbf{n}(t)$ represents the Mx1 additive noise vector with zero mean and covariance

$$E\left\{\mathbf{n}(t_1)\mathbf{n}^H(t_2)\right\} = \sigma_n^2 \mathbf{I}\delta(t_1 - t_2) \tag{5.24}$$

where σ_n^2 is the noise variance per antenna, $\mathbf{I}$ is MxM identity matrix and δ denotes the Dirac delta function. Equation (5.24) implies that the noise is spatially and temporarily white.

The array response vector of *ith* user is given by

$$\mathbf{a}_i = \begin{bmatrix} 1 & e^{j2\pi d_1 \sin\theta_i} & e^{j2\pi d_2 \sin\theta_i} & \dots & e^{j2\pi d_{M-1} \sin\theta_i} \end{bmatrix}^T \tag{5.25}$$

where d_{m-1} represents the distance between *mth* array element and the reference (i.e., $d_0 = 0$) in terms of λ that is the wavelength corresponding to the carrier frequency f_c, and θ_i denotes the angle of arrival (AOA) of the *ith* user, which is the angle between the arriving signal and the array normal.

The received signals at the output of the array are combined to maximize SINR. The first procedure in reception is to despread the combiner output by the desired user's spreading code, which is in syhronism with the transmitted spreading code.

Let k be the intended user. Recalling the array output given in (5.23), the undesired component due to MAI and noise is denoted by $\mathbf{i}(t)$ and is given by

$$\mathbf{i}(t) = \sum_{\substack{i=1 \\ i \neq k}}^{N} \sqrt{P} b_i(t) c_i(t) \mathbf{a}_i + \mathbf{n}(t). \tag{5.26}$$

The output of the array is correlated with the spreading code $c_k(t)$ to recover the *kth* user's signal. The correlation signal vector for the *lth* transmitted bit is given by

$$\mathbf{y}_k(l) = \int_{(l-1)T_b}^{lT_b} c_k(t) \mathbf{x}(t)\, dt. \tag{5.27}$$

The signals at the despreader output are combined by the weight vector $\mathbf{w}_k$. The output of the beamformer is

$$z_k(t) = \mathbf{w}_k^H \mathbf{y}_k(t). \tag{5.28}$$

In order to combine the despread signals optimally and to recover the *kth* signal, the receiver should choose the weight vector $\mathbf{w}_k$. In other words, the array can be steered in the desired user's direction such that the user of concern is always illuminated by the main beam. The optimization criterion is chosen as maximizing SINR of the *kth* user.

The data bits $\{b_i\}_{i=1}^{N}$ are assumed to be i.i.d. with zero mean. Taking the expectation with respect to the data bits and the additive noise, the average desired signal and the average interference-plus-noise powers can be found as

$$\overline{S} = P r_{k,k}^2 \mathbf{w}_k^H \mathbf{R}_{aa} \mathbf{w}_k \tag{5.29}$$

$$\overline{I} = \mathbf{w}_k^H \mathbf{R}_{nn} \mathbf{w}_k \tag{5.30}$$

where

$$\mathbf{R}_{aa} = \mathbf{a}_k \mathbf{a}_k^H \tag{5.31}$$

$$\mathbf{R}_{nn} = P \sum_{\substack{i=1 \\ i \neq k}}^{N} r_{i,k}^2 \mathbf{a}_i \mathbf{a}_i^H + \sigma_n^2 r_{k,k} \mathbf{I}. \tag{5.32}$$

$r_{i,k}$ denotes the cross correlation value between the *ith* and *kth* spreading sequences with $i \neq k$, and is defined as

$$r_{i,k} = \int_{(l-1)T_b}^{lT_b} c_i(t) c_k(t)\, dt. \tag{5.33}$$

Note that $r_{k,k}$ represents the energy of the kth user's spreading code. The corresponding average SINR is given by

$$\text{SINR} = \frac{\overline{S}}{\overline{I}} = P\, r_{k,k}^2 \frac{\mathbf{w}_k^H \mathbf{R}_{aa} \mathbf{w}_k}{\mathbf{w}_k^H \mathbf{R}_{nn} \mathbf{w}_k}. \tag{5.34}$$

This is a standard quadratic form, and the maximization of $\frac{\mathbf{w}_k^H \mathbf{R}_{aa} \mathbf{w}_k}{\mathbf{w}_k^H \mathbf{R}_{nn} \mathbf{w}_k}$ is a well-known optimization problem. This problem has already been solved in Section 4.4.1. This ratio is bounded by the minimum and maximum eigenvalues of the symmetric matrix $\mathbf{R}_{nn}^{-1}\mathbf{R}_{aa}$. The maximum eigenvalue λ_{max} satisfying

$$\mathbf{R}_{nn}^{-1}\mathbf{R}_{aa}\mathbf{w}_k = \lambda_{max}\mathbf{w}_k \tag{5.35}$$

is the optimum value of SINR (i.e., $\text{SINR} = \lambda_{max}$). Corresponding to this value, there is a unique eigenvector $\mathbf{w}_k$ that represents the optimum beamforming weight. Hence, we have

$$\mathbf{R}_{aa}\mathbf{w}_k = \text{SINR}\,\mathbf{R}_{nn}\mathbf{w}_k. \tag{5.36}$$

This is a generalized eigenvalue problem, and the $\mathbf{w}_k$ that maximize the SINR can be found directly without estimating the array response vector $\mathbf{a}_k$.

There have been a vast number of methods to select the weight vector. Most of them focus on direction of arrival (DOA) estimation techniques discussed in the previous chapter. In general, the DOA algorithms operate properly if the number of signals is less than the number of antennas. However, in CDMA systems both the number of users and multipaths far exceed the number of antennas. Therefore, the conventional DOA estimation techniques may not be applicable, and the choice of an algorithm is of paramount importance for CDMA applications.

A number of algorithms to obtain beamforming weights have been investigated for the reverse link of a cellular CDMA system in [35]. The purpose of that study was to compare algorithms in terms of the performance and complexity in a Rayleigh flat fading environment. The algorithms of concern are eigenfilter method [77], Stanford method (a) [131], Stanford method (b) [132], and optimum combining method [11]. Comparing all algorithms, it was concluded that the best strategy for the receiver seems simply to estimate the array response vector $\mathbf{a}_k$ for the desired user.

On the other hand, the estimation techniques of both beamforming weights and antenna array response vectors are beyond the scope of the thesis, we assume that the

array response vector $\mathbf{a}_k$ is perfectly estimated. Since $\mathbf{R}_{aa} = \mathbf{a}_k\mathbf{a}_k^H$, then we obtain $\mathbf{w}_k$ as Wiener solution

$$\mathbf{w}_k = \beta\mathbf{R}_{nn}^{-1}\mathbf{a}_k \tag{5.37}$$

where β is a complex quantity and equal to

$$\beta = \frac{1}{\text{SINR}}\mathbf{a}_k^H\mathbf{w}_k. \tag{5.38}$$

The optimum value for the average SINR is given by

$$\text{SINR} = P\,r_{k,k}^2\,\mathbf{a}_k^H\mathbf{R}_{nn}^{-1}\mathbf{a}_k. \tag{5.39}$$

The formula (5.39) is rewritten by the use of assumption of random binary spreading sequences. Despreading with respect to the *kth* user leads to a processing gain of L; The MAI and noise power at each antenna is suppressed by a factor of L ($\sqrt{L}$ in amplitude) on the average compared to the desired user signal power. Then, we obtain

$$\text{SINR} = PL\mathbf{a}_k^H\mathbf{R}^{-1}\mathbf{a}_k \tag{5.40}$$

where $\mathbf{R}$ is called the *total interference-plus-noise covariance matrix*, and is defined as

$$\mathbf{R} = P\sum_{\substack{i=1\\ i\neq k}}^{N}\mathbf{a}_i\mathbf{a}_i^H + \sigma_n^2\mathbf{I}. \tag{5.41}$$

5.4.2 Performance Criteria

The performance of a CDMA mobile communication system can be characterized by several different measures. In the mobile environment, all users usually change their geographical positions arbitrarily, the resulting SINR is a statistical quantity. The oldest and widely used criterion is the average SINR. However, the average SINR cannot always lead informative and realistic results because of mobile radio channel, which is characterized by various impairments such as fading, multipath, and Doppler spread.

The performance of a mobile system is also characterized by its average bit error rate (BER) and outage probability. BER provides a good indication of a particular modulation scheme, and gives the average probability of errors occurred on the transmitted symbols. However, the drawback of BER is the lack of information about the type of errors, for example bursty errors.

In a fading mobile environment, it is probable that a transmitted signal suffers deep fades, which cause outage or a complete loss of the signal. Outage probability is another

important statistical measure in mobile systems, which operate in a fading environment with multiple interferers. There are two main definitions of outage probability: 1) the probability of the BER exceeding a certain threshold that is required for satisfactory reception, 2) the probability that SINR will be below a certain threshold for acceptable performance (i.e., $P_o = P(SINR \leq \gamma)$ where γ denotes the predetermined threshold). In some cases, these definitions can be used interchangeably. Throughout the thesis, we employ the second definition as the performance measure.

Although there are some simple analytical techniques to compute BER in a slow, flat fading environment, BER in a selective channel, and the computation of outage probability are often made through computer simulation. In such difficult cases, it is sometimes possible to derive upper and lower bounds for BER [133]. In antenna array concept, it is obvious that an exact analysis of BER and outage probability is rather complicated due to existence of MAI and fading.

In order to evaluate the probability of error in a slow, flat fading environment, one must average the probability of error of the particular modulation over the possible ranges of SINR. We assume that the MAI is modeled as additive white Gaussian noise. From [5] the validity of this assumption can be seen with 31 users and code length of 31. The BER as a function of the SINR, (γ_b per bit) for a BPSK modulation, is given by

$$P_b(\gamma_b) = \frac{1}{2} erfc(\sqrt{\gamma_b}). \tag{5.42}$$

In fact, this probability of error is a conditional error probability, where the condition is that γ_b is fixed. In order to evaluate the average BER, we need to average (5.42) over the pdf of γ_b, namely,

$$P_e = \int_0^\infty P_b(\gamma) f_{\gamma_b}(\gamma)\, d\gamma. \tag{5.43}$$

There are also other performance measures as well as BER and outage probability for a CDMA mobile system. The most common measures are known as the blocking probability, system capacity in terms of the number of users, and the coverage area [134].

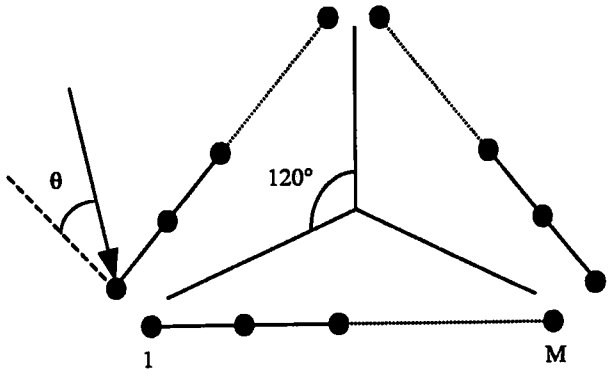

Figure 5.9: A simulation scenario

5.4.3 Simulation and Results

This section presents the simulation results for the reverse link performance of a CDMA cellular system employing both uniformly spaced and nonuniformly spaced arrays.

To begin with, we assume that the base station has 3 sectors, each having an antenna array with M identical elements, as shown in Figure 5.9. The nonuniformly spaced arrays are generated as described in Section 5.3.2. The spacings of nonuniform arrays are given in Table 5.1 and Table 5.2. We also assume that the mobiles are independently and randomly distributed within the cell. The mobile user location is regarded as the angle of arrival in the simulation that is assumed to be uniformly distributed over $[-60^o, 60^o]$.

CDMA is primarily interference limited, so that the noise variance is selected as 15 dB below the average user received signal power with negligible effect on the performance. The spreading sequences with length of 31 are assumed to be truly random sequences. Moreover, it is assumed that these sequences are randomly assigned to mobile users. We also make the assumption of perfect power control.

The results of 20000 simulation runs are used to obtain statistical behavior of SINRs, outage probability, and average BER. Table 5.3 lists the statistical information of the average SINR for both uniformly spaced and nonuniformly spaced arrays with 5 antennas, the average spacing of 2λ. The results for $M = 7$ are given in Table 5.4.

We know that the minimum improvement that can be attainable is $10 \log M$ dB in noise limited environment. The impact of the number of antennas on the output SINR can be shown by the use of two tables. Taking the mean values from Table 5.3 and Table 5.4 for one kind of arrays, it is seen that there is an approximately 2 dB

Table 5.3: Statistics of SINR for M=5

	Uniform Array	Nonuniform Array
max (SINR)	23 dB	12.7 dB
min (SINR)	2.9 dB	4.4 dB
mean (SINR)	7.8 dB	7.7 dB
median (SINR)	7.3 dB	7.6 dB
std (SINR)	3.43	1.48

Table 5.4: Statistics of SINR for M=7

	Uniform Array	Nonuniform Array
max (SINR)	34.5 dB	17 dB
min (SINR)	3.4 dB	5.7 dB
mean (SINR)	10 dB	9.7 dB
median (SINR)	8.9 dB	9.4 dB
std (SINR)	24.16	2.77

difference, whereas the increase in the number of antennas provides 1.4 dB (i.e., $10\log 7 - 10\log 5$). The additional improvement is due to the interference suppression capability of arrays.

As can be also seen from both Table 5.3 and Table 5.4, although the average SINRs belonging to uniform and nonuniform arrays are approximately the same, there are differences in other statistical values. Therefore, the average SINR alone cannot provide adequate information about the performance. Hence, the average BER and the outage probability seems to be more meaningful for almost all communication services.

The resulting SINR distribution plots are depicted in Figure 5.10 and 5.11 for uniform and nonuniform arrays with 7 antennas. From the tables and the figures, it is seen that the SINR distribution for the uniform array has tails longer than that of the nonuniform array; there exist extreme SINR values for the uniform arrays. Furthermore, the SINR distributions for the nonuniform arrays are condensed around their means with noticeable small standard deviations.

Figure 5.12 shows the resulting outage probabilities for both uniform and nonuniform arrays with 7 antennas. Figure 5.13 also depicts the same plots, which are zoomed in. Similar to previous results, it is obtained that extreme SINR values occur for the case of the uniform array. It is obvious that the performance of the nonuniform array

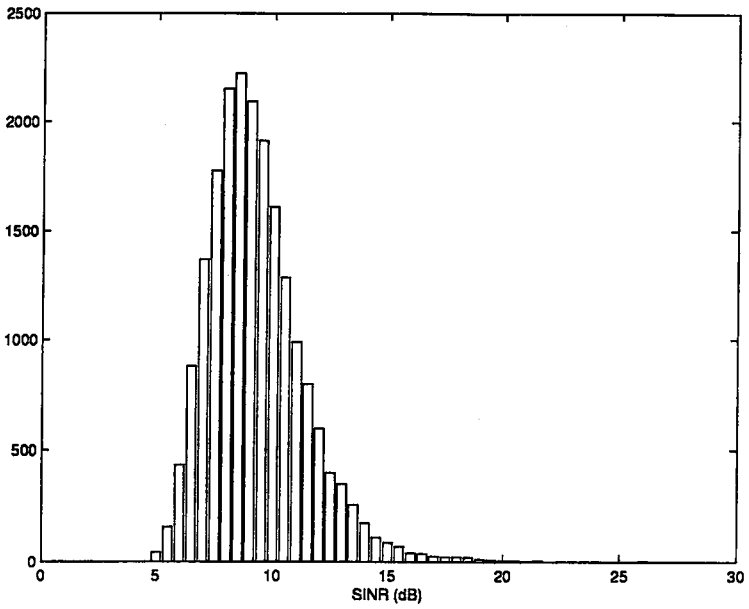

Figure 5.10: Histogram of SINR for uniform array having spacing $d/\lambda = 2$ with M=7

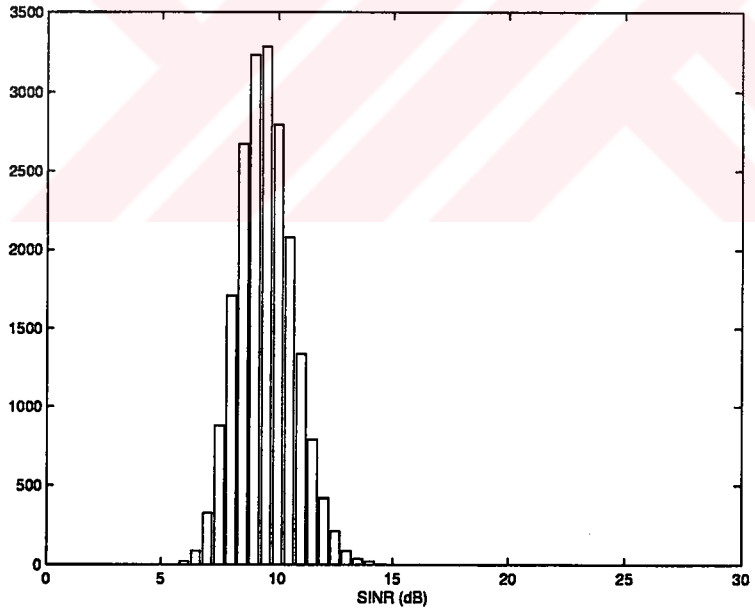

Figure 5.11: Histogram of SINR for nonuniform array having average spacing $d/\lambda = 2$ with M=7

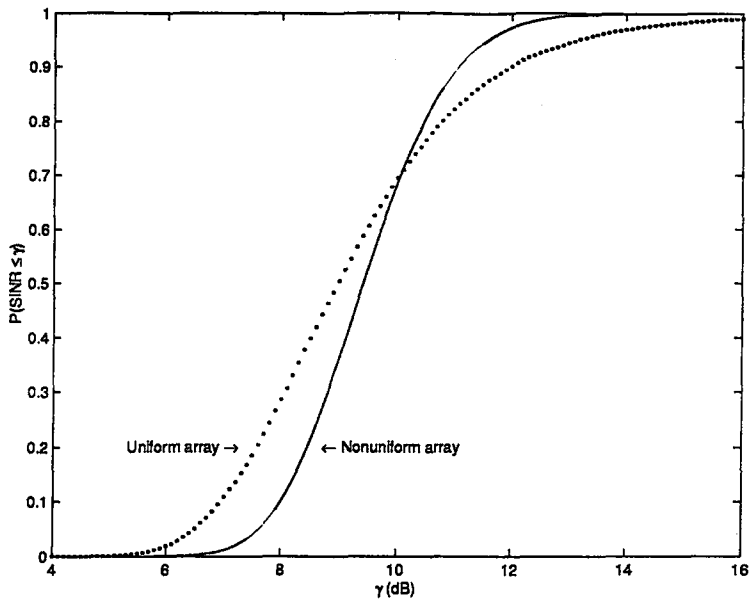

Figure 5.12: Outage probabilities for uniform and nonuniform arrays with $M = 7$

is better than that of the uniform array especially at low SINR values providing a gain of 1.5-2 dB.

The average BER is evaluated for both uniform and nonuniform arrays as

$$P_e = 2.8810^{-4} \quad \text{for uniform array}$$

$$P_e = 7.5810^{-5} \quad \text{for nonuniform array.}$$

Thus, the average BER performance of the nonuniform array is better than that of uniform array as expected, since the lower SINR values occur for the uniform array in contrast to the nonuniform array.

The results reveal that the nonuniform array can provide better performance for LOS users when large antenna spacing is required [135, 136].

5.5 Far Users

5.5.1 System Model

In the previous section, we proposed the use of the nonuniformly spaced array to attain space diversity in addition to suppressing MAI. In this section, we investigate the performance of antenna arrays in a Rayleigh flat fading environment considering correlation between antennas. The results are solely presented for the nonuniformly

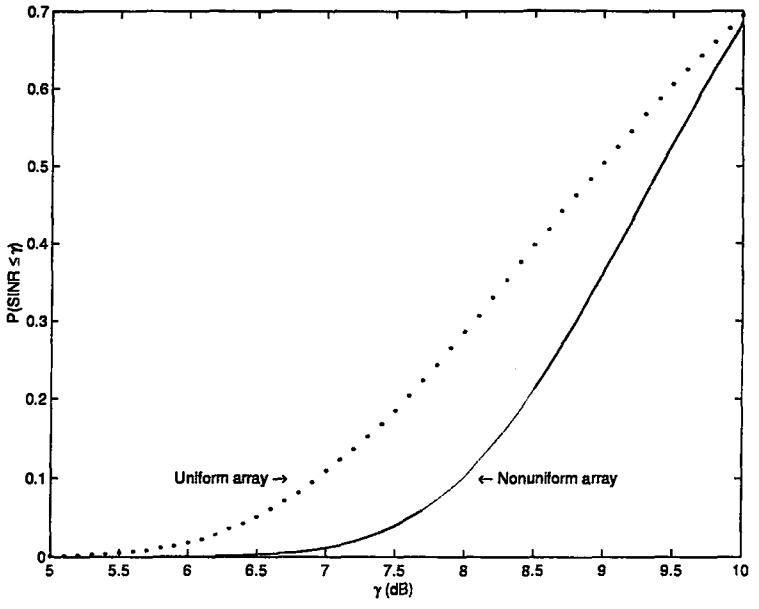

Figure 5.13: Outage probabilities for uniform and nonuniform arrays with $M = 7$ (zoomed in)

spaced array since the number of antennas and the aperture sizes are selected as the same for both arrays so that the average spacings for both arrays are equal.

Since the far users are of interest, we have to include the statistical mobile radio channel characteristics in both analysis and simulation. As is well-known, mobile radio receivers involve many multipaths from each transmitter. The signals received by the base station from a mobile are considered as resulting mainly from reflectors and scatterers in the vicinity of the mobile. Thus, we employ the local scattering channel model described in Section 5.2, where multipath components are modeled with different arriving angles, strengths and phases. This implies that multipath signals can be resolved in space but not in time.

Actually, the system model of concern has the same features as in the case of LOS users except the mobile radio channel. For convenience, the assumptions made in this section is briefly stated as follows:

The reverse link is considered in a single cell environment. The base station has a nonuniformly spaced array with M identical elements. The system employs BPSK modulation with DSSS-CDMA scheme. The corresponding spreading codes are selected as random codes with length of L. There is a relatively large number of users, N compared to the number of antennas. Perfect power control is employed, i.e., the

average power received by an antenna for each user is kept at the determined level, P. The maximum delay experienced by the signals is assumed to remain less than a chip period; this implies flat fading environment so that intersymbol interference (ISI) does not occur.

Consider a scenario where there are N users randomly distributed in a cell. Neglecting the propagation delay, the complex baseband output of the array is

$$\mathbf{x}(t) = \sum_{i=1}^{N} \sqrt{P} b_i(t) c_i(t) \mathbf{a}_i + \mathbf{n}(t) \tag{5.44}$$

where $b_i(t)$ is the data sequence (∓ 1), $c_i(t)$ is the spreading code, and $\mathbf{a}_i$ is the Mx1 complex channel vector for arriving signal from *ith* mobile user, and $\mathbf{n}(t)$ represents spatially and temporarily white complex Mx1 noise vector.

Considering the flat fading environment, the received signal is approximated by a sum of large number of unresolvable paths that arrive at the base station within the angle spread $\mp\Delta$ at angle θ with respect to the array normal. Based on Section 5.2, the channel vector $\mathbf{a}_i$ is modeled as independently distributed complex Gaussian random vectors with zero mean and covariance matrix

$$\mathbf{\Sigma}_i = E\left\{\mathbf{a}_i \mathbf{a}_i^H\right\}. \tag{5.45}$$

Assuming the model of uniform angle distribution of arrival over Δ, $\mathbf{\Sigma}_i$ has the correlation properties given in (5.12) and (5.13). Moreover, we assume that the fading rate is much less than the symbol rate so that the channel vectors are constant over the symbol duration.

Following the same steps as LOS users in the previous section, the beamformer weight that maximize SINR for *kth* user is obtained as

$$\mathbf{w}_k = \beta \mathbf{R}^{-1} \mathbf{a}_k \tag{5.46}$$

where β is a complex quantity and $\mathbf{R}$ is given by

$$\mathbf{R} = P \sum_{\substack{i=1 \\ i \neq k}}^{N} \mathbf{a}_i \mathbf{a}_i^H + \sigma_n^2 \mathbf{I}. \tag{5.47}$$

5.5.2 Simulation and Results

This subsection presents the performance of linear antenna arrays in the correlated Rayleigh flat fading environment. The performance measures are selected as the outage

probability and the average BER as described in Section 5.4.2. In order to bring about the performance improvement due to beamforming and space diversity, a series Monte Carlo simulations are carried out.

Our simulations are mainly based on the following assumptions:

- There are 31 users randomly distributed around the base station of a single cell.
- The base station has a nonuniformly spaced array with M identical elements.
- The angle of arrival for each user has the same uniform distribution over $[-60^o, 60^o]$.
- The angle spread is identical for all mobiles in the service zone.
- As far as the reverse link is concerned, perfect power control is employed within the cell.
- The spreading codes are random codes with length of 31.
- Noise power is 15 dB below the user received power.

Before giving the simulation results, we present how the channel vectors are generated. For given θ_i, Δ and the antenna spacings, the correlation matrix $\mathbf{\Sigma}_i$ is constructed by using the expressions given (5.12) and (5.13). Then, a $2M$ vector ζ_i is generated as having independent, zero mean Gaussian random variables with a variance of $1/2$. The channel vector for each user is obtained as

$$\mathbf{a}_i = \mathbf{\Sigma}_i^{1/2} \zeta_i. \tag{5.48}$$

The results of 20000 simulation trials are used to attain the outage probability and the average BER. For each trial, the angles of arrivals of mobiles are generated independently. The correlation matrix for each mobile is computed as a function of the antenna spacing, the angle spread and the angle of arrival. Then, the channel vector is generated randomly with respect to (5.48).

We first investigate beamforming gain in a Rayleigh fading channel with MAI. Figure 5.14 indicates the outage probability for $\Delta = 20^o$ and the average antenna spacing of $d = 2\lambda$ for $M = 3$, $M = 5$ and $M = 7$. It is obvious that the performance improvement is directly proportional to the number of antennas due to beamforming gain.

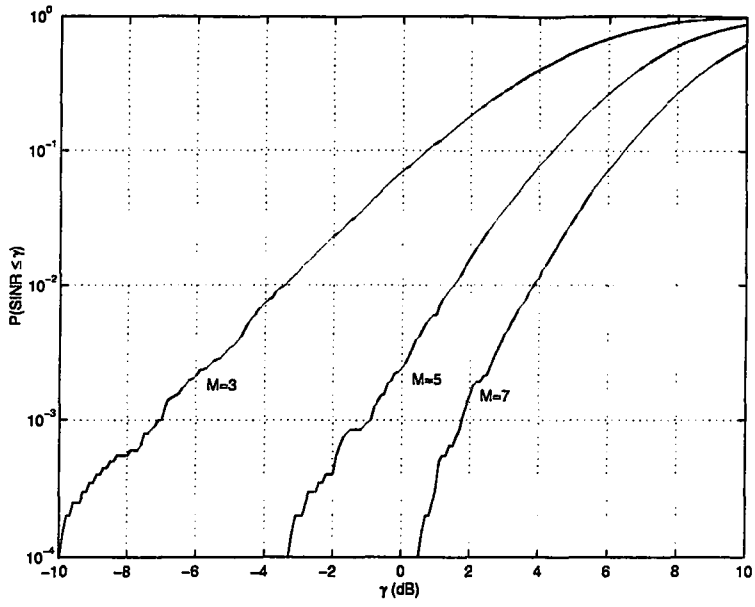

Figure 5.14: Outage probabilities for $\Delta = 20^o$ and $d/\lambda = 2$

Next, we study the impact of the angle spread on the performance. For fixed $M = 7$ and $d = 2\lambda$, the outage probability results are depicted in Figure 5.15 for different angle spreads $\Delta = 5^o$, $\Delta = 20^o$ and $\Delta = 60^o$. As can be seen from this figure, with a small angle spread lower SINR values occur not in the case of a large angle spread. This is due to the fact that at small angle spread, the received multipath signals for each user experience the same fading at all antennas, and thus the array cannot provide any space diversity. However, for large angle spread, the received signals at different antennas become uncorrelated, which leads to a space diversity gain in addition to beamforming gain.

Figure 5.16 shows the resulting average BERs for different values of Δ via the average antenna spacing. At a fixed average spacing, for example $d = \lambda$, when we compare the results for different angle spreads, we can see that large angle spread improves the BER performance as well the outage probability. Furthermore, the BER performance improves with increasing antenna spacing, as expected: For a fixed angle spread, as the antenna spacing increases, the signal fading at antennas become independent so that a gain of space diversity is achieved in addition to beamforming gain. For a small angle spread, for example $\Delta = 5^o$, this additional gain is more pronounced with increasing antenna spacing. An interesting point is that the BER performance remains almost constant as long as Δ greater than 20^o, i.e., the correlation between antennas is at a

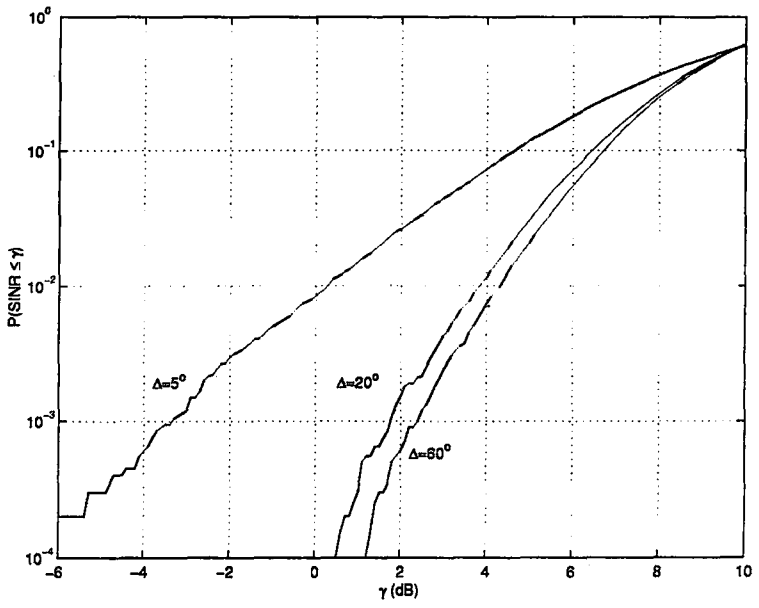

Figure 5.15: Outage probabilities for different angle spreads for M=7 and $d/\lambda = 2$

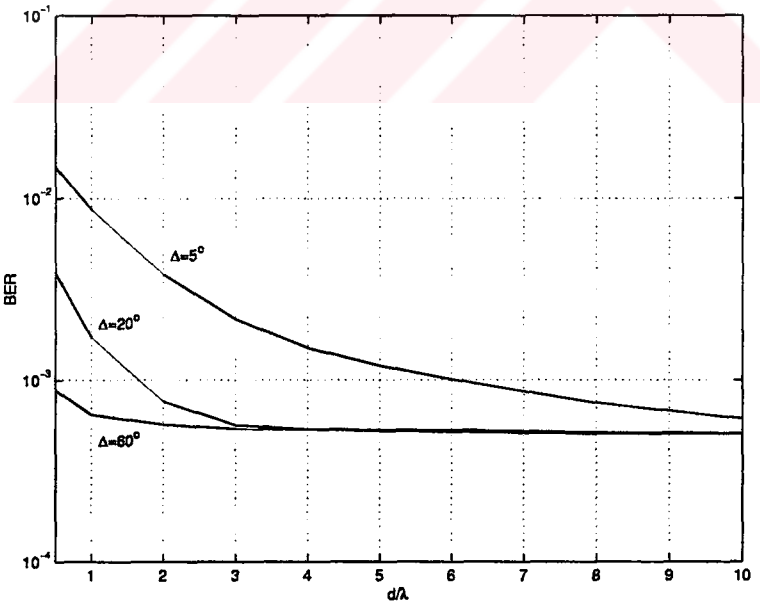

Figure 5.16: BER versus antenna spacing for $M = 7$

tolerable level.

From Figure 5.15 and 5.16, it is concluded that with the average spacing of 2λ there is a little performance degradation as long as the angle spread of the received signals is greater than 20^o. However, this tolerable angle spread can be reduced even further by increasing the average antenna spacing.

In conclusion, since the signals can stem from any direction in mobile communication systems, larger antenna spacing is required to attain a better performance of antenna array that combats fading and suppress interference. Due to grating lobes in the antenna beam pattern of a uniformly spaced array, the use of a nonuniformly spaced array can be seen as a promising method [137].

CHAPTER 6

THE STATISTICAL ANALYSIS OF SIR

6.1 Pdf Analysis of SIR

Most of the performance measures are related to signal-to-interference ratio (SIR), and are generally expressed in terms of SIR. Since in a particular time SIR of a mobile depends on the spatial positions of all mobiles, SIR is a random variable. In antenna array concept, the distribution of the SIR is related to multivariate analysis. In this section we will obtain an expression for the probability density function of SIR through a hypergeometric function ${}_1F_0$ with matrix arguments.

In order to facilitate the probability density function (pdf) analysis of SIR, we recall the SIR definition, which was given by a detailed description in the preceding chapter. We assume that thermal noise is neglected since CDMA system is primarily interference limited. The maximum SIR, v, is given by

$$v = L\mathbf{a}_k^H \mathbf{R}^{-1} \mathbf{a}_k \tag{6.1}$$

where L is processing gain, $\mathbf{a}_k$ denotes the channel vector for the desired user, *kth* user. The total interference covariance matrix $\mathbf{R}$ conditioned on channel vectors $\mathbf{a}_i$s is given by

$$\mathbf{R} = \sum_{\substack{i=1 \\ i \neq k}}^{N} \mathbf{a}_i^H \mathbf{a}_i. \tag{6.2}$$

It is assumed that all channel vectors $\{\mathbf{a}_j\}_{j=1}^N$ are modeled as independently distributed Mx1 complex Gaussian random vectors with zero mean and covariance matrix $\mathbf{\Sigma}_j$, which is defined as

$$\mathbf{\Sigma}_j = E\left\{\mathbf{a}_j \mathbf{a}_j^H\right\}. \tag{6.3}$$

By definition $\mathbf{\Sigma}_j$ is a positive definite and Hermitian matrix, and thus its inverse exists. When $\mathbf{\Sigma}_j > 0$, i.e., positive definite and $N-1 \geq M$, the matrix $\mathbf{R}$ is positive definite with probability one [39]. If independent fading at each antenna is available, $\mathbf{\Sigma}_j = \mathbf{I}$ where $\mathbf{I}$ denotes an identity matrix with dimension of M.

For convenience, we let $\mu = v/L$, SIR can be written as a function of μ

$$\mu = \mathbf{a}_k^H \mathbf{R}^{-1} \mathbf{a}_k. \tag{6.4}$$

In what follows, we will obtain the pdf of μ.

The random variable μ is a quadratic form in complex Gaussian random variables. The distributions of such random variables have been studied in multivariate framework for many years [38, 109, 138]. Khatri [38] discussed the distributions of a family of quadratic forms approximately 30 years ago. In all his studies, the matrix $\mathbf{R}$ is comprised of independently and identically distributed (i.i.d.) random variables, which corresponds to i.i.d. interferers having a complex Gaussian distribution with zero mean and the same covariance matrix. However, in our case, all channel vectors are independently distributed with zero mean but with different covariance matrices. Therefore, Khatri's result cannot be applied to our problem directly.

Recently, the closed form expressions of the pdf of the SIR have been obtained with random channel vectors, which are i.i.d., by using Khatri's results directly. Shah and Haimovich [39] carried out distribution analysis in a multivariate framework. It is shown that the pdf of the SIR in the case of optimum combining has a Hotelling T^2 distribution for equal power interferers. Similarly, in [40], an exact closed form expression was derived with multiple interferers, each having arbitrary power. Moreover, that work was extended to the general case including additive Gaussian noise. Since in both studies all users including the desired one have the same covariance matrix, the resulting densities do not depend on the covariance matrix or the correlation between antennas. This implies that the same performance is achieved regardless of whether the fading at array elements is independent or not.

Recalling our problem, one would expect that an exact analysis of the pdf of such random variables to be rather complicated when the interfering signals have non-identical distribution. However, based on the previous simulation results we propose an approach, which makes Khatri's result approximately applicable to our problem. For large number $(N-1)$ of interferers, the distribution of $\mathbf{R}$ is expected to remain almost

the same when we assume that $\mathbf{a}_i$s are i.i.d. with a suitable covariance matrix, while actually they are not. In fact, intensive simulations have verified that our approach is quite valid, which will be presented in the following sections.

Khatri [38] appears to be the first person to derive the expression for pdf of a quadratic form as $\mathbf{Z} = \mathbf{X}^H \left(\mathbf{Y}\mathbf{Y}^H\right)^{-1} \mathbf{X}$ where $\mathbf{X}$:pxn and $\mathbf{Y}$:pxm are complex Gaussian random matrices and $n \leq p \leq m$.

Explicitly, $\mathbf{X}$:pxn is a matrix of random complex variates such that the column vectors are i.i.d. as multivariate normals with zero mean vectors. The probability density function is given by

$$\pi^{-pn} |\mathbf{\Sigma}|^{-n} |\mathbf{B}|^{-p} e^{-tr\mathbf{\Sigma}^{-1}\mathbf{X}\mathbf{B}^{-1}\mathbf{X}^H} \quad . \tag{6.5}$$

where $tr(\cdot)$ and $|\cdot|$ stand for trace and determinant of a matrix, respectively, $\mathbf{\Sigma}$ and $\mathbf{B}$ are Hermitian positive definite matrices. The random matrix $\mathbf{Y}$:pxm is independently distributed of $\mathbf{X}$ and its columns are i.i.d. as multivariate normals with zero mean vectors. The pdf is given by

$$\pi^{-pm} |\mathbf{\Sigma}_1|^{-m} e^{-tr\mathbf{\Sigma}_1^{-1}\mathbf{Y}\mathbf{Y}^H} \tag{6.6}$$

where $\mathbf{\Sigma}_1$ is Hermitian positive definite matrix.

Then, for $n \leq p \leq m$, the probability density function of $\mathbf{Z}$ is given in terms of a hypergeometric function as follows

$$\tilde{\Gamma}_p(m+n) \left\{\tilde{\Gamma}_p(m)\tilde{\Gamma}_n(p)\right\}^{-1} |\mathbf{B}|^{-p} |\mathbf{\Omega}|^{-n} |\mathbf{Z}|^{p-n} |\mathbf{I}_n + (q\mathbf{B})^{-1}\mathbf{Z}|^{-m-n} \tag{6.7}$$

$$\cdot {}_1F_0\left(m+n; \mathbf{\Omega}^*, \mathbf{Z}\left(\mathbf{Z}+\mathbf{B}q\right)^{-1}\right)$$

where $q > 0$ is an arbitrary constant, $\mathbf{\Omega}^* = \mathbf{I}_p - q\mathbf{\Omega}^{-1}$ and $\mathbf{\Omega} = \mathbf{\Sigma}^{1/2}\mathbf{\Sigma}_1^{-1}\mathbf{\Sigma}^{1/2}$,

$$\tilde{\Gamma}_m(a) = \pi^{\frac{1}{2}m(m-1)} \prod_{i=1}^{m} \Gamma(a-i+1), \tag{6.8}$$

$$\Gamma(a) = (a-1)!, \tag{6.9}$$

and $\Gamma(\cdot)$ is the gamma function.

The last term in (6.7) denotes a hypergeometric function of matrix arguments. The standard expression of a hypergeometric function with matrix arguments is given as an infinite sum of zonal polynomials involving an inner summation over partitions [139]. Constantine [140] and James [109] pioneered the use of these functions in the

representation of the distribution of random matrices. The general hypergeometric function with Hermitian matrices $\mathbf{X}$ and $\mathbf{Y}$ is given by

$$ {}_pF_q\left(a_1,\ldots,a_p;b_1\ldots b_q;\mathbf{X},\mathbf{Y}\right)=\sum_{k=0}^{\infty}\sum\nolimits_{\kappa}\frac{[a_1]_\kappa\cdots[a_p]_\kappa}{[b_1]_\kappa\cdots[b_q]_\kappa}\frac{C_\kappa(\mathbf{X})\,C_\kappa(\mathbf{Y})}{k!\,C_\kappa\left(\mathbf{I}_m\right)} \tag{6.10} $$

where $\sum_\kappa$ denotes the summation over all partitions $\kappa=(k_1,\cdots,k_m)$, $k_1\geq k_2\geq\cdots k_m\geq 0$, $k_1+k_2+\cdots+k_m=k$, $C_\kappa(\mathbf{X})$ is a symmetric homogeneous polynomial of degree k in the *latent roots* (characteristics roots or eigenvalues) of $\mathbf{X}$ and is referred to as zonal polynomial, and $a_1,\ldots a_p$ and $b_1,\ldots b_q$ are real or complex constants. No denominator parameter b_j is allowed to be equal to zero, or an integer, or half integer $\leq\frac{1}{2}(m-1)$, otherwise some of the denominators in the series will vanish [139]. The multivariate hypergeometric coefficient $[a]_\kappa$ is given by

$$ [a]_\kappa = \prod_{i=1}^{m}(a-i+1)_{k_i}\,, \tag{6.11} $$

$$ (a)_k = a\,(a+1)\cdots(a+k-1)\,, \tag{6.12} $$

$$ (a)_0 = 1, $$

and $(a)_k$ denotes the usual *Pochhammer* symbol.

In (6.10), $\mathbf{X}$ and $\mathbf{Y}$ are complex symmetric matrices, and it is assumed that $p\leq q+1$. The convergence properties of hypergeometric series in (6.10) is summarized in the theorem by Gross and Richards [141] as follows:

1. If $p\leq q$, then the series converges for all $\mathbf{X}$ and $\mathbf{Y}$.

2. If $p=q+1$, then the series converges for $\|\,\mathbf{X}\,\|\|\,\mathbf{Y}\,\|<1$, where $\|\,\mathbf{X}\,\|$ represents the maximum of the absolute value of the characteristics roots of $\mathbf{X}$.

3. If $p>q+1$, then the series diverges for all $\mathbf{X}\neq 0$ and $\mathbf{Y}\neq 0$.

It is obvious from (6.10), the series provides the same result regardless of the order matrices $\mathbf{X}$ and $\mathbf{Y}$ in the expression, i.e.,

$$ {}_pF_q\left(a_1,\ldots,a_p;b_1,\ldots,b_q;\mathbf{X},\mathbf{Y}\right)={}_pF_q\left(a_1,\ldots,a_p;b_1,\ldots,b_q;\mathbf{Y},\mathbf{X}\right). \tag{6.13} $$

Also, when one of the matrices is identity matrix, this function reduces to the one-matrix function of (6.10), namely,

$$ {}_pF_q\left(a_1,\ldots,a_p;b_1,\ldots,b_q;\mathbf{X},\mathbf{I}_m\right)={}_pF_q\left(a_1,\ldots,a_p;b_1,\ldots,b_q;\mathbf{X}\right). \tag{6.14} $$

Finally, if $m = 1$ (i.e., one dimensional matrix), the function (6.14) reduces to the classical hypergeometric function with a scalar argument; that is

$$ {}_pF_q\left(a_1,\ldots,a_p;b_1,\ldots,b_q;x\right) = \sum_{k=0}^{\infty} \frac{(a_1)_k \cdots (a_p)_k}{(b_1)_k \cdots (b_q)_k} \frac{x^k}{k!}. \tag{6.15} $$

It is clear from (6.15), the hypergeometric function of a single scalar argument is defined as an infinite power series.

Although, the theory about hypergeometric function with matrix arguments has been discussed in detail by many researchers, there appears to be no much-detailed information about their computation in the literature. There have been some attempts to compute hypergeometric functions in terms of zonal polynomials, however, they do not provide satisfactory results for the numerical computation, and thus we do not employ zonal polynomials. In order to avoid a sophisticated study, we will present a brief summary of zonal polynomials in Appendix A. Another representation about hypergeometric function is given as solutions to certain differential equations [139], however, this approach also appears to be improper for numerical computation.

After stating some important properties of hypergeometric function briefly, we continue the discussion of the pdf analysis. Our problem is a special case of (6.7) where $n = 1$, $p = M$, $m = N - 1$ and $\mathbf{B} = \mathbf{I}_M$. The random variable μ in (6.4) is equivalent to $\mathbf{Z}$ in (6.7), in which case $\mathbf{a}_k = \mathbf{X}$ and $\mathbf{R} = \mathbf{Y}\mathbf{Y}^H$. Assuming the desired user channel vector $\mathbf{a}_k$ is complex Gaussian distributed with zero mean and covariance matrix $\mathbf{\Sigma}_k$, and all interfering signals' channel vectors are i.i.d complex Gaussian distributed with zero mean and $\mathbf{\Sigma}_I$, the pdf of μ is given by [38]

$$ f_\mu(\mu) = \tilde{\Gamma}_M(N)\left\{\tilde{\Gamma}_M(N-1)\tilde{\Gamma}_1(M)\right\}^{-1} |\mathbf{\Omega}|^{-1}\mu^{M-1} \tag{6.16} $$

$$ \cdot (1+\mu/q)^{-N}\, {}_1F_0\left(N;\mathbf{\Omega}^*,\mu/(\mu+q)\right) $$

where $\mathbf{\Omega}^* = \mathbf{I_M} - q\mathbf{\Omega}^{-1}$ and $\mathbf{\Omega} = \mathbf{\Sigma}_k^{1/2}\mathbf{\Sigma}_I^{-1}\mathbf{\Sigma}_k^{1/2}$. It is noted that ${}_1F_0$ is a function of a multivariate coefficient of N, a matrix argument of $\mathbf{\Omega}^*$ and a scalar argument of $\mu/(\mu+q)$. By performing some algebraic manipulation, (6.16) is simplified to

$$ f_\mu(\mu) = \frac{\Gamma(N)}{\Gamma(N-M)\Gamma(M)} \frac{q^N}{(q+\mu)^N} \mu^{M-1} |\mathbf{\Omega}|^{-1}\, {}_1F_0\left(N;\mathbf{\Omega}^*,\mu/(\mu+q)\right). \tag{6.17} $$

6.1.1 Exact Pdf of SIR for $M = 1$

For $M = 1$, all channel vectors reduce to complex values; that is $\mathbf{a}_j = a_{j1}$ for all $j = 1, 2, \ldots, N$ where a_{j1} is a complex Gaussian random variable with zero mean and a variance $E\{|a_{j1}|^2\} = 1$. Then, we have

$$\mu = \mathbf{a}_k^H \mathbf{R}^{-1} \mathbf{a}_k = \frac{|a_{k1}|^2}{\sum\limits_{\substack{i=1 \\ i \neq k}}^{N} |a_{i1}|^2}. \tag{6.18}$$

Let y_1 be $|a_{k1}|^2$ and y_2 be $\sum_{\substack{i=1 \\ i \neq k}}^{N} |a_{i1}|^2$. In this case, y_1 has a χ_2^2 (chi-square) distribution with 2 degrees of freedom; that is

$$f_{y_1}(y_1) = e^{-y_1} \qquad y_1 \geq 0. \tag{6.19}$$

Since y_2 is a sum of $N-1$ i.i.d. complex Gaussian random variables, each having a χ_2^2 distribution, it also has a $\chi_{2(N-1)}^2$ distribution namely,

$$f_{y_2}(y_2) = \frac{1}{\Gamma(N-1)} y_2^{N-2} e^{-y_2} \qquad y_2 \geq 0. \tag{6.20}$$

From the result in [142], the pdf of $\mu = y_1/y_2$ is given by

$$f_\mu(\mu) = \int_0^\infty y_2 f_{y_1}(\mu y_2) f_{y_2}(y_2)\, dy_2. \tag{6.21}$$

Expanding the integral by $(N-1)$ times, we have

$$f_\mu(\mu) = \frac{N-1}{(1+\mu)^N}. \tag{6.22}$$

Finally, the pdf of SIR or $v = L\mu$ is found as

$$f_v(v) = \frac{N-1}{L} \frac{1}{(1+v/L)^N}. \tag{6.23}$$

6.1.2 Exact Pdf of SIR for i.i.d. Users and $N > M$

In this case, all channel vectors including the desired user's one have i.i.d. distribution, i.e., $\mathbf{\Sigma}_k = \mathbf{\Sigma}_I$, and thus $\mathbf{\Omega}$ reduces to an identity matrix and the hypergeometric function in (6.10) reduces to

$${}_1F_0(N; (1-q)\mathbf{I}_M, \mu/(\mu+q)) = \sum_{k=0}^{\infty} \sum_{\kappa} \frac{[N]_\kappa C_\kappa((1-q)\mathbf{I}_M)\, C_\kappa(\mu/(\mu+q))}{k!\, C_\kappa(\mathbf{I}_M)}. \tag{6.24}$$

From the consequence **C.2** of zonal polynomials given in Appendix A, $C_\kappa((1-q)\mathbf{I}_M) = (1-q)^k C_\kappa(\mathbf{I}_M)$. Then, we have

$${}_1F_0(N; (1-q)\mathbf{I}_M, \mu/(\mu+q)) = \sum_{k=0}^{\infty} \sum_{\kappa} \frac{[N]_\kappa (1-q)^k C_\kappa(\mu/(\mu+q))}{k!}. \tag{6.25}$$

Since $\mu/(\mu+q)$ is just a scalar argument, from first consequence **C.1** in Appendix A, (6.25) reduces to the classical hypergeometric series with a scalar argument given in (6.15), namely,

$$\begin{aligned} {}_1F_0\left(N;(1-q)\mathbf{I}_M,\mu/(\mu+q)\right) &= \sum_{k=0}^{\infty}\frac{(N)_k}{k!}(1-q)^k\left(\frac{\mu}{\mu+q}\right)^k \\ &= {}_1F_0\left(N;(1-q)\left(\mu/(\mu+q)\right)\right). \end{aligned} \tag{6.26}$$

The hypergeometric function ${}_1F_0\left(N;(1-q)\left(\mu/(\mu+q)\right)\right)$ is referred to as a *binomial hypergeometric series*, and is given by [109]

$${}_1F_0\left(N;(1-q)\left(\mu/\mu+q\right)\right) = \left(1-(1-q)\frac{\mu}{\mu+q}\right)^{-N}. \tag{6.27}$$

Substituting this result into (6.17) and canceling the common terms, the pdf of μ is obtained

$$f_\mu(\mu) = \frac{\Gamma(N)}{\Gamma(N-M)\Gamma(M)}\frac{\mu^{M-1}}{(1+\mu)^N}. \tag{6.28}$$

This distribution of μ is known as the *Null distribution of the Hotelling's* T^2 statistics [143]. Using the transformation $v = L\mu$ leads to

$$f_v(v) = \frac{\Gamma(N)}{\Gamma(N-M)\Gamma(M)}L^{N-M}\frac{v^{M-1}}{(L+v)^N}. \tag{6.29}$$

6.1.3 Pdf Analysis for General Case and $N > M$

Although the expression (6.17) seems to be rather simple, the numerical computation of the hypergeometric function is rather difficult. These types of functions have been mostly used as analytical representation in multivariate statistics theory. Furthermore, there have been a few works about their numerical computation in the literature. Calculating the hypergeometric function via zonal polynomials results in a daunting computational task, and may result in some accuracy problem [144]. Gao [144] attempted to compute ${}_1F_0(\cdot)$ with matrix arguments in terms of zonal polynomials but he could not reach a viable numerical result. He also dealt with as asymptotic expression for the hypergeometric function, which is proposed by Krishnaiah [145], however this method also did not lead to a satisfactory approximation.

Gross and Richards [141] presented an expression for the hypergeometric function of matrix arguments in terms of the classical hypergeometric function of a scalar argument.

Although that work is completely theoretical, Gao and Smith [41] managed to compute the hypergeometric function based on [141]. In our particular case of interest, we will employ their approach to compute the hypergeometric with matrix arguments.

From [141] the hypergeometric function with two matrix arguments is written in terms of the classical hypergeometric function with certain matrices $\mathbf{\Omega}^*$ and $\mathbf{U}^*$ of dimension MxM

$$
\begin{aligned}
&{}_pF_q\left(a_1,\cdots,a_p;b_1,\cdots,b_q;\mathbf{\Omega}^*,\mathbf{U}^*\right)=c_{p,q} \qquad (6.30)\\
&\cdot\frac{\det\left({}_pF_q\left(a_1-M+1,\cdots,a_p-M+1;b_1-M+1,\cdots,b_q-M+1;u_i^*w_j^*\right)\right)}{V\left(\mathbf{\Omega}^*\right)V\left(\mathbf{U}^*\right)}
\end{aligned}
$$

where

$$
\begin{aligned}
c_{p,q} &= \beta_M\frac{\prod_{j=1}^{M}\prod_{i=1}^{q}\left(b_i-M+1\right)_{M-j}}{\prod_{j=1}^{M}\prod_{i=1}^{p}\left(a_i-M+1\right)_{M-j}},\\
\beta_M &= \prod_{j=1}^{M}\Gamma(j),
\end{aligned}
$$

$w_1^*<w_2^*<\cdots<w_M^*$ and $u_1^*<u_2^*<\cdots<u_M^*$ are the distinct eigenvalues of $\mathbf{\Omega}^*$ and $\mathbf{U}^*$ respectively, and $V(.)$ denotes a Vandermonde determinant that is given by

$$
V\left(\mathbf{\Omega}^*\right)=\prod_{1\le i<j\le M}\left(w_i^*-w_j^*\right).
$$

From (6.30), ${}_1F_0\left(N;\mathbf{\Omega}^*,\mathbf{U}^*\right)$ is expressed as

$$
{}_1F_0\left(N;\mathbf{\Omega}^*,\mathbf{U}^*\right)=c_{1,0}\frac{\det\left({}_1F_0\left(N-M+1;u_i^*w_j^*\right)\right)}{V\left(\mathbf{\Omega}^*\right)V\left(\mathbf{U}^*\right)} \qquad (6.31)
$$

where

$$
\begin{aligned}
c_{1,0} &= \prod_{j=1}^{M}\Gamma(j)\frac{1}{\prod_{j=1}^{M}\left(N-M+1\right)_{M-j}},\\
\left(N-M+1\right)_{M-j} &= \left(N-M+1\right)\left(N-M+2\right)\cdots\left(N-j\right),\\
{}_1F_0\left(N-M+1;u_i^*w_j^*\right) &= \left(1-u_i^*w_j^*\right)^{-(N-M+1)}.
\end{aligned}
$$

Here, we let $w_j^*=1-qw_j$ where w_j is an eigenvalue of $\mathbf{\Omega}^{-1}=\mathbf{\Sigma}_k^{-1/2}\mathbf{\Sigma}_I\mathbf{\Sigma}_k^{-1/2}$, and $w_1>w_2>\cdots>w_M$. From (6.17), ${}_1F_0$ actually has one matrix argument $\mathbf{\Omega}^*$ and a scalar argument $\mu/(\mu+q)$, and thereby we cannot use (6.31) directly. Therefore, we set $\mathbf{U}^*=diag\left(u_1^*,u_2^*,\cdots,u_M^*\right)=diag\left(0,0,\cdots,\mu/(\mu+q)\right)$ to construct a MxM matrix.

Case Study for $M=2$

For $M=2$, $\mathbf{U}^*=diag\left(u_1^*,u_2^*\right)=diag\left(0,\mu/(\mu+q)\right)$, $\mathbf{\Omega}^*=\mathbf{I}_2-q\mathbf{\Omega}^{-1}$ and the eigenvalues of $\mathbf{\Omega}^*$ are denoted as $w_1^*<w_2^*$, which are $1-qw_1$ and $1-qw_2$ respectively.

In order to compute the pdf of μ, we first compute the following determinant

$$\det\left({}_1F_0\left(N-1;u_i^*w_j^*\right)\right) \stackrel{\text{def}}{=} \begin{vmatrix} {}_1F_0\left(N-1;u_1^*w_1^*\right) & {}_1F_0\left(N-1;u_2^*w_1^*\right) \\ {}_1F_0\left(N-1;u_1^*w_2^*\right) & {}_1F_0\left(N-1;u_2^*w_2^*\right) \end{vmatrix}$$

$$= \begin{vmatrix} 1 & \left(\frac{q}{\mu+q}\right)^{-(N-1)}\left(1+\mu w_1\right)^{-(N-1)} \\ 1 & \left(\frac{q}{\mu+q}\right)^{-(N-1)}\left(1+\mu w_2\right)^{-(N-1)} \end{vmatrix}. \tag{6.32}$$

This determinant reduces to

$$\det\left({}_1F_0\left(N-1;u_i^*w_j^*\right)\right) = \frac{(\mu+q)^{N-1}}{q^{N-1}}\left[\left(1+\mu w_2\right)^{-(N-1)} - \left(1+\mu w_1\right)^{-(N-1)}\right]. \tag{6.33}$$

The Vandermonde determinants and the constant $c_{1,0}$ are evaluated as

$$V\left(\mathbf{U}^*\right) = \prod_{1\le i<j\le 2}\left(u_i^*-u_j^*\right) = -\frac{\mu}{\mu+q}, \tag{6.34}$$

$$V\left(\mathbf{\Omega}^*\right) = \prod_{1\le i<j\le 2}\left(w_i^*-w_j^*\right) = q\left(w_2-w_1\right), \tag{6.35}$$

$$c_{1,0} = \prod_{j=1}^{2}\Gamma(j)\frac{1}{\prod_1^2(N-1)_{2-j}} = \frac{\Gamma(1)\,\Gamma(2)}{N-1}. \tag{6.36}$$

Combining all evaluated values, the pdf of μ in (6.16) is

$$f_\mu(\mu) = \frac{\Gamma(N)}{\Gamma(N-2)\Gamma(2)}\frac{q^N}{(q+\mu)^N}\,\mu\,(w_1w_2) \tag{6.37}$$

$$\cdot\left[\frac{\Gamma(2)}{N-1}\frac{\frac{(\mu+q)^{N-1}}{q^{N-1}}\left[\left(1+\mu w_2\right)^{-(N-1)}-\left(1+\mu w_1\right)^{-(N-1)}\right]}{-\frac{\mu}{\mu+q}\,q\left(w_2-w_1\right)}\right].$$

By simplifying the expression, we get

$$f_\mu(\mu) = (N-2)\,\frac{w_1w_2}{w_1-w_2}\left[\frac{1}{\left(1+\mu w_2\right)^{N-1}} - \frac{1}{\left(1+\mu w_1\right)^{N-1}}\right]. \tag{6.38}$$

In our general case, we cannot apply (6.31) directly since $\mathbf{U}^* = diag\,(0,0,\ldots,\mu/(\mu+q))$ has more than one zero eigenvalues. Hence, the necessity of distinct eigenvalues condition is invalidated, and Vandermonde determinant results in zero. However, Gao and

Smith [41] proposed an approach, in which case the equal eigenvalues are perturbed to different eigenvalues, and then permitted to converge to the same value, namely,

$$ {}_1F_0\left(N;\mathbf{\Omega}^*,\mathbf{U}^*\right)=\lim_{\varepsilon_i\to 0} c_{1,0}\frac{\det {}_1F_0\left(N-M+1;u_iw_j^*\right)}{V\left(\mathbf{\Omega}^*\right)V(\mathbf{U})} \tag{6.39} $$

where $\mathbf{U}=diag\left(0,\varepsilon_1,\varepsilon_2\ldots,\mu/(\mu+q)\right)$ and $0<\varepsilon_1<\varepsilon_2<\cdots<\varepsilon_{M-2}<\mu/(\mu+q)$. A particular $q>0$ and a set of $\{\varepsilon_j\}_{j=1}^{M-2}$ are selected to satisfy $\parallel \mathbf{\Omega}^* \parallel\parallel \mathbf{U} \parallel< 1$ so that the hypergeometric function converges absolutely.

Similar to case study for $M=2$, the determinant is defined as

$$ det\left({}_1F_0\left(N-M+1;u_iw_j^*\right)\right)\stackrel{\text{def}}{=} \tag{6.40} $$

$$ \begin{vmatrix} {}_1F_0\left(N-M+1;u_1w_1^*\right) & {}_1F_0\left(N-M+1;u_2w_1^*\right) & \cdots & {}_1F_0\left(N-M+1;u_Mw_1^*\right)\\ {}_1F_0\left(N-M+1;u_1w_2^*\right) & {}_1F_0\left(N-M+1;u_2w_2^*\right) & \cdots & {}_1F_0\left(N-M+1;u_Mw_2^*\right)\\ \cdots & \cdots & \cdots & \cdots\\ {}_1F_0\left(N-M+1;u_1w_M^*\right) & {}_1F_0\left(N-M+1;u_2w_M^*\right) & \cdots & {}_1F_0\left(N-M+1;u_Mw_M^*\right) \end{vmatrix}. $$

Using the properties of $det(A)=det(A^T)$, we have

$$ det\left({}_1F_0\left(N-M+1;u_iw_j^*\right)\right)= \tag{6.41} $$

$$ \begin{vmatrix} 1 & \cdots & 1\\ \left(1-\varepsilon_1(1-qw_1)\right)^{-(N-M+1)} & \cdots & \left(1-\varepsilon_1(1-qw_M)\right)^{-(N-M+1)}\\ \vdots & & \vdots\\ \left(1-\varepsilon_{M-2}(1-qw_1)\right)^{-(N-M+1)} & \cdots & \left(1-\varepsilon_{M-2}(1-qw_M)\right)^{-(N-M+1)}\\ \left(1-\frac{\mu}{\mu+q}(1-qw_1)\right)^{-(N-M+1)} & \cdots & \left(1-\frac{\mu}{\mu+q}(1-qw_M)\right)^{-(N-M+1)} \end{vmatrix}. $$

Before going further, we define some functions to simplify the notations:

$$ f_j(x)=\left(1-x(1-qw_j)\right)^{-(N-M+1)}, \tag{6.42} $$

$$ \mathbf{f}(x)=\left[\ f_1(x)\quad f_2(x)\quad \ldots\quad f_M(x)\ \right]. \tag{6.43} $$

Then, the cumbersome determinant expression is written in terms of $\mathbf{f}(x)$ as

$$\det\left({}_1F_0\left(N-M+1\right);u_iw_j\right)\right) = \det\left(\mathbf{f}(0),\mathbf{f}(\varepsilon_1),\ldots,\mathbf{f}(\varepsilon_{M-2}),\mathbf{f}(\mu/(\mu+q))\right). \quad (6.44)$$

Let us now calculate $V(\mathbf{\Omega}^*)$, and $c_{1,0}$

$$\begin{aligned} V\left(\mathbf{\Omega}^*\right) &= \prod_{1\le i<j\le M}\left(w_i^*-w_j^*\right) \\ &= q^{\frac{M(M-1)}{2}}\prod_{1\le i<j\le M}\left(w_j-w_i\right), \quad (6.45) \\ c_{1,0} &= \prod_{j=1}^{M}\Gamma(j)\,\frac{1}{\prod_{j=1}^{M}(N-M+1)_{M-j}}. \quad (6.46) \end{aligned}$$

Similarly, $V(\mathbf{U}) = \prod_{1\le i<j\le M}(u_i-u_j) = (-1)^{\frac{M(M-1)}{2}}\prod_{1\le i<j\le M}(u_j-u_i)$. The Vandermonde determinant of $\mathbf{U}$ is written explicitly as

$$V(\mathbf{U}) = (-1)^{\frac{M(M-1)}{2}}\begin{vmatrix} 1 & 1 & \ldots & 1 \\ u_1 & u_2 & \ldots & u_M \\ u_1^2 & u_2^2 & \ldots & u_M^2 \\ \vdots & \vdots & & \vdots \\ u_1^{M-1} & u_2^{M-1} & \ldots & u_M^{M-1} \end{vmatrix}. \quad (6.47)$$

By inserting the eigenvalues into the determinant, we have

$$V(\mathbf{U}) = (-1)^{\frac{M(M-1)}{2}}\begin{vmatrix} 1 & 1 & \ldots & 1 & 1 \\ 0 & \varepsilon_1 & \ldots & \varepsilon_{M-2} & \frac{\mu}{\mu+q} \\ 0 & \varepsilon_1^2 & \ldots & \varepsilon_{M-2}^2 & \left(\frac{\mu}{\mu+q}\right)^2 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & \varepsilon_1^{M-1} & & \varepsilon_{M-2}^{M-1} & \left(\frac{\mu}{\mu+q}\right)^{M-1} \end{vmatrix}. \quad (6.48)$$

Let us now define some other functions

$$g_j(x) = x^{j-1}, \quad (6.49)$$

$$\mathbf{g}(x) = [\ g_1(x)\ \ g_2(x)\ \ \ldots\ \ g_M(x)\ ]^T. \quad (6.50)$$

Hence,

$$V(\mathbf{U}) = (-1)^{\frac{M(M-1)}{2}}\det\left(\mathbf{g}(0),\mathbf{g}(\varepsilon_1),\ldots,\mathbf{g}(\varepsilon_{M-2}),\mathbf{g}(\mu/(\mu+q))\right). \quad (6.51)$$

In order not to further complicate the analysis, we first deal with

$$\frac{\det {}_1F_0\left(N-M+;u_iw^{*j}\right)}{V(\mathbf{U})}=(-1)^{-\frac{M(M-1)}{2}}\frac{\det\left(\mathbf{f}(0),\mathbf{f}(\varepsilon_1),\ldots,\mathbf{f}(\varepsilon_{M-2}),\mathbf{f}(\mu/(\mu+q))\right)}{\det\left(\mathbf{g}(0),\mathbf{g}(\varepsilon_1),\ldots,\mathbf{g}(\varepsilon_{M-2}),\mathbf{g}(\mu/(\mu+q))\right)}. \tag{6.52}$$

From Cauchy's mean value theorem, the right hand side of (6.52) can be written as

$$\begin{aligned}
&\frac{\det\left(\mathbf{f}(0),\mathbf{f}(\varepsilon_1),\mathbf{f}(\varepsilon_2),\ldots,\mathbf{f}(\varepsilon_{M-2}),\mathbf{f}(\mu/(\mu+q))\right)}{\det\left(\mathbf{g}(0),\mathbf{g}(\varepsilon_1),\mathbf{g}(\varepsilon_2),\ldots,\mathbf{g}(\varepsilon_{M-2}),\mathbf{g}(\mu/(\mu+q))\right)}= \\
&\quad=\frac{\det\left(\mathbf{f}(0),\mathbf{f}^{(1)}(\xi_1),\mathbf{f}(\varepsilon_2),\ldots,\mathbf{f}(\varepsilon_{M-2}),\mathbf{f}(\mu/(\mu+q))\right)}{\det\left(\mathbf{g}(0),\mathbf{g}^{(1)}(\xi_1),\mathbf{g}(\varepsilon_2),\ldots,\mathbf{g}(\varepsilon_{M-2}),\mathbf{g}(\mu/(\mu+q))\right)} \\
&\qquad\vdots \\
&\quad=\frac{\det\left(\mathbf{f}(0),\mathbf{f}^{(1)}(\xi_1),\mathbf{f}^{(1)}(\xi_2)\ldots,\mathbf{f}^{(1)}(\xi_{M-2}),\mathbf{f}(\mu/(\mu+q))\right)}{\det\left(\mathbf{g}(0),\mathbf{g}^{(1)}(\xi_1),\mathbf{g}^{(1)}(\xi_2),\ldots,\mathbf{g}^{(1)}(\xi_{M-2}),\mathbf{g}(\mu/(\mu+q))\right)} \\
&\quad=\frac{\det\left(\mathbf{f}(0),\mathbf{f}^{(1)}(\xi_1),\mathbf{f}^{(2)}(\xi_2),\ldots,\mathbf{f}^{(1)}(\xi_{M-2}),\mathbf{f}(\mu/(\mu+q))\right)}{\det\left(\mathbf{g}(0),\mathbf{g}^{(1)}(\xi_1),\mathbf{g}^{(2)}(\xi_2),\ldots,\mathbf{g}^{(1)}(\xi_{M-2}),\mathbf{g}(\mu/(\mu+q))\right)} \\
&\qquad\vdots \\
&\quad=\frac{\det\left(\mathbf{f}(0),\mathbf{f}^{(1)}(\xi_1),\mathbf{f}^{(2)}(\xi_2),\ldots,\mathbf{f}^{(M-2)}(\xi_{M-2}),\mathbf{f}(\mu/(\mu+q))\right)}{\det\left(\mathbf{g}(0),\mathbf{g}^{(1)}(\xi_1),\mathbf{g}^{(2)}(\xi_2),\ldots,\mathbf{g}^{(M-2)}(\xi_{M-2}),\mathbf{g}(\mu/(\mu+q))\right)}
\end{aligned} \tag{6.53}$$

where $\mathbf{f}^j$ denotes *jth* derivative of $\mathbf{f}(\cdot)$, and $0\le\xi_i\le\varepsilon_i$ for $i=1,2,\ldots,M-2$. Since $\mathbf{f}^j(x)$ and $\mathbf{g}^j(x)$ are continuous at $x=0$, we are ready to evaluate the limiting case

$$\lim_{\varepsilon_i\to 0}\frac{\det\left({}_1F_0\left(N-M+1;u_iw_j^*\right)\right)}{V(\mathbf{U})}=(-1)^{-\frac{M(M-1)}{2}} \tag{6.54}$$

$$\cdot\frac{\det\left(\mathbf{f}(0),\mathbf{f}^{(1)}(0),\mathbf{f}^{(2)}(0),\ldots,\mathbf{f}^{(M-2)}(0),\mathbf{f}(\mu/(\mu+q))\right)}{\det\left(\mathbf{g}(0),\mathbf{g}^{(1)}(0),\mathbf{g}^{(2)}(0),\ldots,\mathbf{g}^{(M-2)}(0),\mathbf{g}(\mu/(\mu+q))\right)}.$$

The determinants can be evaluated separately through the use of Vandermonde determinants and Laplace's expansion. However, although the computation is straightforward, the evaluation is very lengthy and cumbersome so that they will be presented in Appendix B.

The resulting determinants are given by

$$\det\left(\mathbf{f}(0),\mathbf{f}^{(1)}(0),\ldots,\mathbf{f}^{(M-2)}(0),\mathbf{f}(\mu/(\mu+q))\right) = (-1)^{\frac{(M-1)(M-2)}{2}} q^{\frac{(M-1)(M-2)}{2}} \tag{6.55}$$

$$\cdot\left(\frac{q}{\mu+q}\right)^{-(N-M+1)} \prod_{i=1}^{M-2} (N-M+1)_i$$

$$\cdot \sum_{k=1}^{M} (-1)^{M+k} (1+\mu w_k)^{-(N-M+1)} \prod_{\substack{1\le i<j\le M \\ i,j\neq k}} (w_j - w_i)\,.$$

and

$$\det\left(\mathbf{g}(0),\mathbf{g}^{(1)}(0),\ldots,\mathbf{g}^{(M-2)}(0),\mathbf{g}(\mu/(\mu+q))\right) = \left(\frac{\mu}{\mu+q}\right)^{M-1} \prod_{j=1}^{M-2} j!\,. \tag{6.56}$$

From (6.56), (6.51) reduces to

$$V(\mathbf{U}) = (-1)^{\frac{M(M-1)}{2}} \left(\frac{\mu}{\mu+q}\right)^{M-1} \prod_{j=1}^{M-2} j!\,. \tag{6.57}$$

Substituting all the calculated values of (6.45), (6.46), (6.55) and (6.57) into (6.39), and performing simplification, the resulting hypergeometric function becomes

$${}_1F_0\left(N;\mathbf{\Omega}^*,\mathbf{U}^*\right) = (-1)^{-(M-1)} \frac{(M-1)!}{(N-M+1)_{M-1}} \mu^{-(M-1)} \frac{q^{-N}}{(\mu+q)^{-N}} \tag{6.58}$$

$$\cdot \frac{\sum_{k=1}^{M} (-1)^{M+k} (1+\mu w_k)^{-(N-M+1)} \prod_{\substack{1\le i<j\le M \\ i,j\neq k}} (w_j - w_i)}{\prod_{1\le i<j\le M} (w_j - w_i)}.$$

This result is inserted into (6.17), and the pdf of μ is evaluated as

$$f_\mu(\mu) = (-1)^{-(M-1)} (N-M) \prod_{j=1}^{M} w_j \tag{6.59}$$

$$\cdot \frac{\sum_{k=1}^{M} (-1)^{M+k} (1+\mu w_k)^{-(N-M+1)} \prod_{\substack{1\le i<j\le M \\ i,j\neq k}} (w_j - w_i)}{\prod_{1\le i<j\le M} (w_j - w_i)}.$$

This density function is further simplified through the following reduction

$$\frac{\prod_{\substack{1\le i<j\le M \\ i,j\neq k}} (w_j - w_i)}{\prod_{1\le i<j\le M} (w_j - w_i)} = \frac{1}{\prod_{j\neq k}(-1)^{k+1}(w_j - w_k)} \tag{6.60}$$

and the resulting pdf of μ is

$$f_\mu(\mu) = (N-M)\prod_{j=1}^{M} w_j \sum_{k=1}^{M} \frac{(1+\mu w_k)^{-(N-M+1)}}{\prod_{j\neq k}(w_j - w_k)}. \tag{6.61}$$

Finally, the pdf of the SIR, $v = L\mu$ is obtained as

$$f_v(v) = \frac{(N-M)}{L}\prod_{j=1}^{M} w_j \sum_{k=1}^{M} \frac{(1+vw_k/L)^{-(N-M+1)}}{\prod_{j\neq k}(w_j - w_k)}. \tag{6.62}$$

It should be emphasized that this expression is only true for $N > M$.

6.2 Exact Result for Outage Probability

The simplicity of the formula (6.62) leads to the evaluation outage probability quickly. The outage probability is defined as the probability that SIR will be below a certain threshold:

$$P_o = P(v \le v_{op}) = \int_0^{v_{op}} f_v(v)\, dv. \tag{6.63}$$

It is noted that the outage probability denotes the *cumulative distribution function* of the SIR.

The result of the integration is given by

$$P_o = 1 - \prod_{j=1}^{M} w_j \sum_{k=1}^{M} \frac{(1+v_{op}w_k/L)^{-(N-M)}}{w_k \prod_{j\neq k}(w_j - w_k)}. \tag{6.64}$$

6.3 Unconditional Pdf and Outage Probability Analysis

In the preceding section, the pdf and the outage probability have already been derived for given $\mathbf{\Sigma}_k$ and $\mathbf{\Sigma}_I$. In fact the formulas (6.62) and (6.64) reflect the conditional probabilities, namely,

$$f_v(v|\mathbf{\Sigma}_k, \mathbf{\Sigma}_I) = \frac{(N-M)}{L}\prod_{j=1}^{M} w_j \sum_{k=1}^{M} \frac{(1+vw_k/L)^{-(N-M+1)}}{\prod_{j\neq k}(w_j - w_k)} \tag{6.65}$$

$$P_o(v \le v_{op}|\mathbf{\Sigma}_k, \mathbf{\Sigma}_I) = 1 - \prod_{j=1}^{M} w_j \sum_{k=1}^{M} \frac{(1+v_{op}w_k/L)^{-(N-M)}}{w_k \prod_{j\neq k}(w_j - w_k)}. \tag{6.66}$$

In order to simulate the mobile radio system performance, a number of assumptions have been made in the previous chapter: Every mobile changes its spatial position regardless of previous location. The angle spread and antenna spacings are fixed for all mobiles. From these assumptions, it is obvious that the covariance matrices depend only on the spatial position of mobiles or the angle of arrivals. Hence, the pdf of v in (6.65) can be conditioned on the angle of arrivals θ_k and θ_I instead of $\mathbf{\Sigma}_k$ and $\mathbf{\Sigma}_I$ respectively.

The unconditional pdf of the SIR can be determined exactly by considering all possible values of angle of arrivals such that

$$f_v(v) = \int_{\mathbf{\Theta}} f_v(v|\mathbf{\Theta})\, f(\mathbf{\Theta})\, d\mathbf{\Theta} \tag{6.67}$$

where $\mathbf{\Theta} = \{\theta_1, \theta_2, \ldots, \theta_k, \ldots, \theta_N\}$ and $f(\mathbf{\Theta})$ is the joint pdf.
Unfortunately, the exact analysis of this function is extremely difficult. Instead, the pdf can be approximated, depending on $\mathbf{\Sigma}_k$ and $\mathbf{\Sigma}_I$ or θ_k and θ_I. A key question is: *Which covariance matrix $\mathbf{\Sigma}_I$ provides the best result?* Firstly, we focus on the total interference covariance matrix $\mathbf{R}$ conditioned on the channel vectors $\mathbf{a}_i$s. The average of $\mathbf{R}$ can be obtained by

$$E\{\mathbf{R}\} = E_{\mathbf{\Theta}_I}\left\{E\{\mathbf{R}|\,\mathbf{\Theta}_I\}\right\} = \int_{\mathbf{\Theta}_I} E\{\mathbf{R}|\,\mathbf{\Theta}_I\}\, f(\mathbf{\Theta}_I)\, d\mathbf{\Theta}_I \tag{6.68}$$

where $\mathbf{\Theta}_I = \{\theta_1, \ldots, \theta_{k-1}, \theta_{k+1}, \ldots, \theta_N\}$.

Due to the assumption of independent interferers, and substitution of (6.2) into (6.68), we have

$$E\{\mathbf{R}\} = \sum_{\substack{i=1 \\ i\neq k}}^{N} \int_{\theta_i} E\left\{\mathbf{a}_i\mathbf{a}_i^H|\,\theta_i\right\} f(\theta_i)\, d\theta_i. \tag{6.69}$$

Considering uniformly distributed interferers throughout the angle range of $[-\phi, \phi]$, and $E\{\mathbf{a}_i\mathbf{a}_i^H|\,\theta_i\} = \mathbf{\Sigma}_{i|\theta_i}$ is the same for all interfering signals, then, the average interferer covariance matrix is defined as

$$\overline{\mathbf{\Sigma}}_I \overset{\text{def}}{=} \int_{-\phi}^{\phi} \mathbf{\Sigma}_{I|\theta_i}\, f(\theta_i)\, d\theta_i. \tag{6.70}$$

Finally, the approximate unconditional pdf of the SIR, which depends only θ_k can be evaluated from

$$f_v(v) \approx \int_{-\phi}^{\phi} f\left(v|\theta_k, \overline{\theta}_I\right) f(\theta_k)\, d\theta_k. \tag{6.71}$$

Similarly, the approximate unconditional outage probability can be calculated by considering all possible values of angle of arrivals

$$P_o = \int_{\Theta} P_o(v \leq v_{op}|\Theta)\, f(\Theta)\, d\Theta. \tag{6.72}$$

Following the same steps as the unconditional pdf analysis, the overall outage probability is evaluated from

$$P_o \approx \int_{-\phi}^{\phi} P\left(v \leq v_{op}|\, \theta_k, \overline{\theta}_I\right) f(\theta_k)\, d\theta_k \tag{6.73}$$

where $P\left(v \leq v_{op}|\, \theta_k, \overline{\theta}_I\right)$ is given in (6.64).

6.4 Simulation and Results

In this section, we present the results to show the validity of our theoretical results by means of Monte Carlo simulations. Our simulations are based on the following assumptions: In a CDMA system, a large number of users N randomly distributed around the base station of a single cell communicate simultaneously with the base station. The nonuniformly spaced linear array with identical M elements is located at the base station. Perfect power control is employed. The length of spreading sequence is selected as 31. The angle spread is identical for all mobiles in the service zone.

To begin with, we assume that there are 10 users uniformly distributed over $[-60^o, 60^o]$. The array has 5 antennas, which are assumed to be nonuniformly spaced with average spacing of 2λ. The angle spread is selected as 20^o for each user. Moreover, it is assumed that the desired user signal impinges on the array from 0^o, and interferers impinge on the array from 20^o. The number of samples used in the Monte Carlo simulations was 100000. The covariance matrices are calculated only once at the beginning of the simulation, and then the channel vectors are generated randomly according to the complex Gaussian distribution with zero mean and these covariance matrices at each trial. The conditional pdfs and outage probabilities of SIR obtained from (6.65) and (6.66), and by the simulation are resulted in Figure 6.1 and 6.2 respectively [146].

The results of 20000 simulation runs were used to estimate the unconditional pdf and unconditional outage probability of SIR. For each trial, the arriving angles are generated randomly regardless of the previous values. The covariance matrix for each mobile is computed as a function of the antenna spacing, the angle spread and the angle

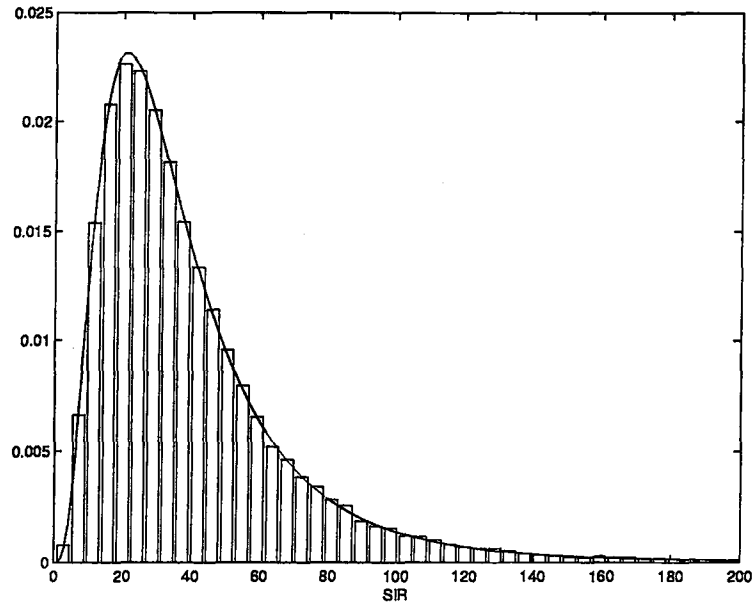

Figure 6.1: Comparison of the pdf of SIR using both theory and simulation for $N = 10$, $M = 5$, $\Delta = 20^o$, $d/\lambda = 2$, angle range $[-60^o, 60^o]$ and '$-$' denotes theory

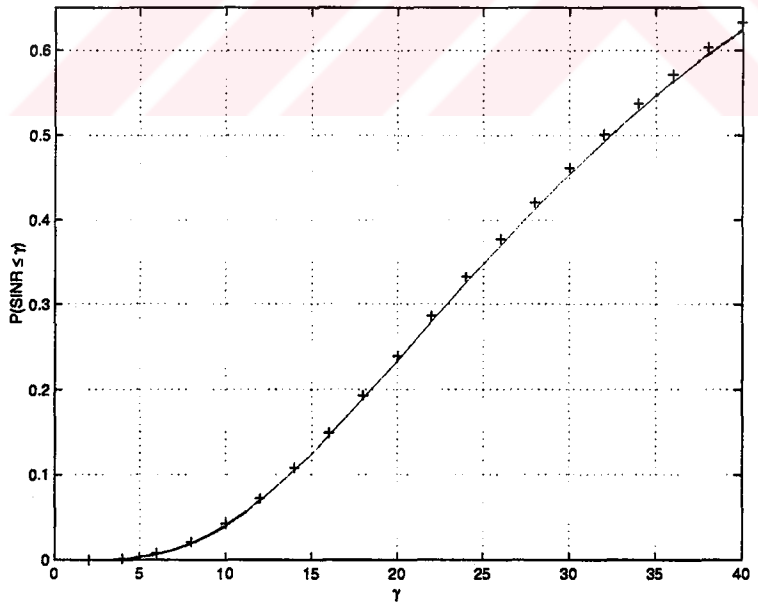

Figure 6.2: Comparison of the outage probability using both theory and simulation for $N = 10$, $M = 5$, $\Delta = 20^o$, $d/\lambda = 2$, angle range $[-60^o, 60^o]$ and '$+$' denotes theory

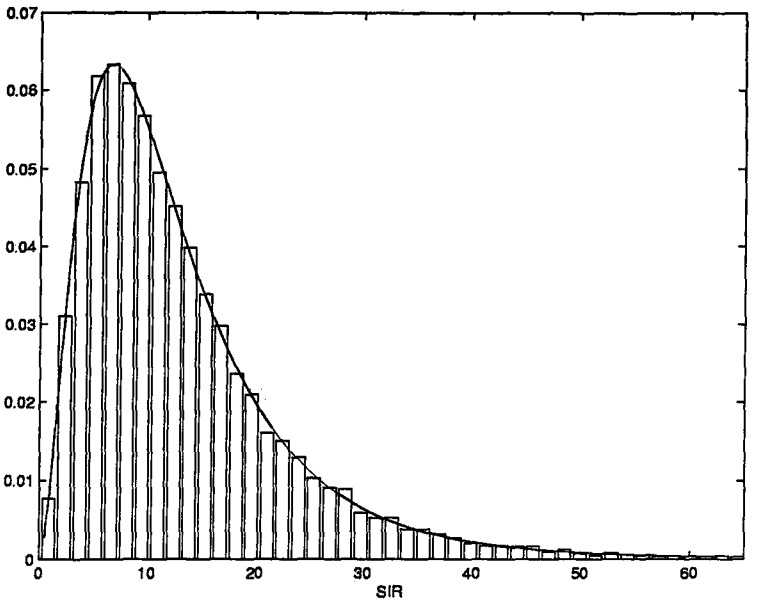

Figure 6.3: Comparison of the pdf of SIR using both theory and simulation for $N = 11$, $M = 3$, $\Delta = 20^o$, $d/\lambda = 2$, angle range $[-20^o, 20^o]$ and '$-$' denotes theory

of arrival. Then, the corresponding channel vector is generated randomly according to the complex Gaussian distribution with zero mean and previously calculated covariance matrix at each run. Finally, the SIRs as obtained by the simulation are presented as a histogram. The outage probability is obtained by counting the SIRs below a predetermined threshold.

For fixed antenna spacing and fixed angle spread for both the desired user and the interfering users, the unconditional pdf and the unconditional outage probability are calculated by using the formulas (6.71) and (6.73). Figure 6.3 depicts the pdf obtained from both the formula and simulation. The number of users $N = 11$, the number of antennas $M = 3$, the angle spread $\Delta = 20^o$, and the antenna spacing $d/\lambda = 2$ are used in the simulation. The users are assumed to be randomly located over $[-20^o, 20^o]$.

Figure 6.4 shows the resulting pdfs from both theory and simulation for $N = 31$, $M = 7$, $\Delta = 5^o$, $d/\lambda = 5$, and the range of the arriving angles $[-60^o, 60^o]$.

Another pdf result is presented in Figure 6.5, in which case $N = 11$, $M = 3$, $\Delta = 30^o$, $d/\lambda = 3$, and the angle range $[-50^o, 50^o]$. The corresponding outage probabilities are depicted in Figure 6.6.

In Figure 6.7, the outage probabilities are given for $N = 11$, $M = 5$, $\Delta = 20^o$, $d/\lambda = 1$, and the angle range $[-20^o, 20^o]$.

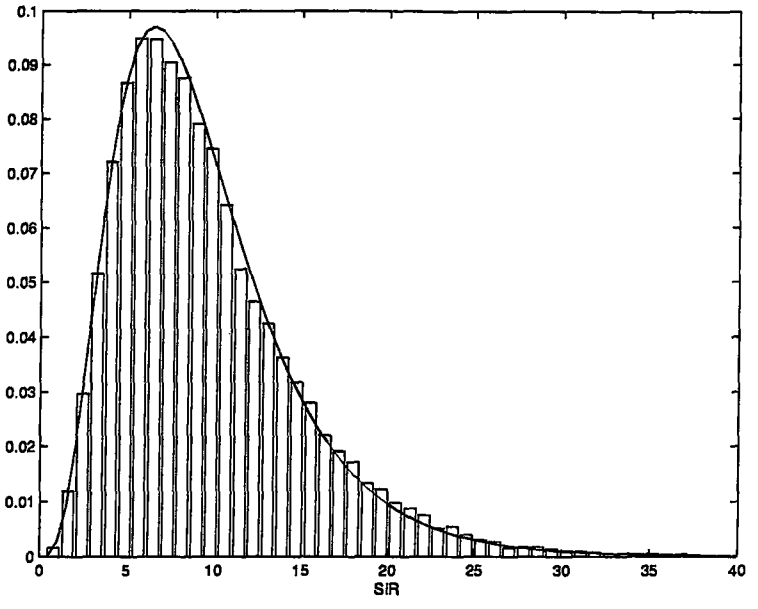

Figure 6.4: Comparison of the pdf of SIR using both theory and simulation for $N = 31$, $M = 7$, $\Delta = 5^o$, $d/\lambda = 5$, angle range $[-60^o, 60^o]$ and '$-$' denotes theory

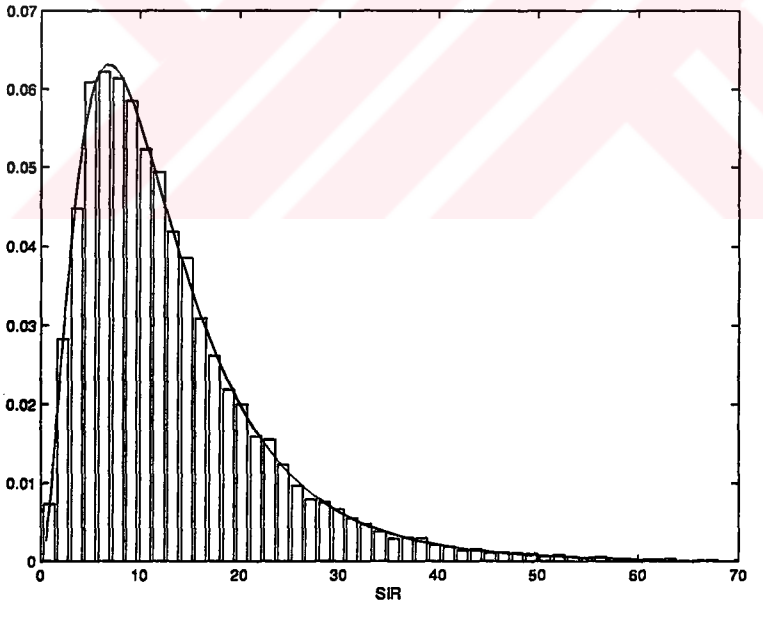

Figure 6.5: Comparison of the pdf of SIR using both theory and simulation for $N = 11$, $M = 3$, $\Delta = 30^o$, $d/\lambda = 3$, angle range $[-50^o, 50^o]$ and '$-$' denotes theory

T.C. YÜKSEKÖĞRETİM KURULU
DOKÜMANTASYON MERKEZİ

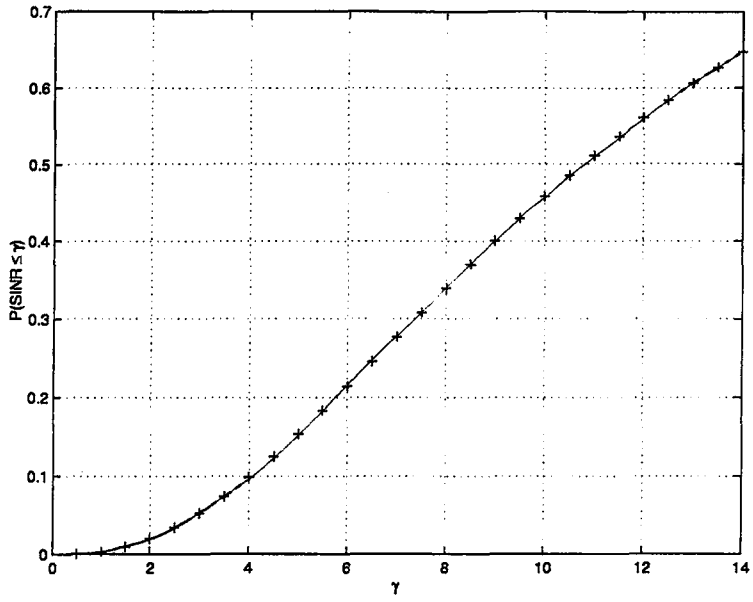

Figure 6.6: Comparison of the outage probability using both theory and simulation for $N = 11$, $M = 3$, $\Delta = 30^o$, $d/\lambda = 3$, angle range $[-50^o, 50^o]$ and '+' denotes theory

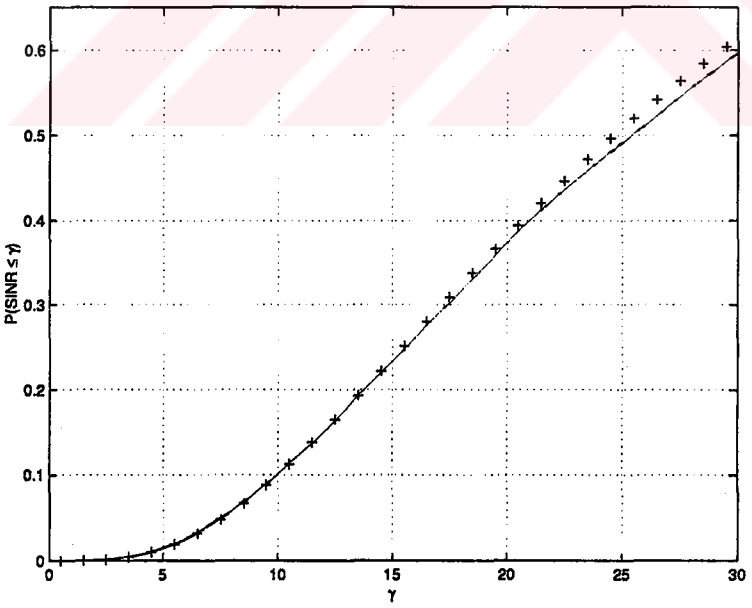

Figure 6.7: Comparison of the outage probability using both theory and simulation for $N = 11$, $M = 5$, $\Delta = 20^o$, $d/\lambda = 1$, angle range $[-20^o, 20^o]$ and '+' denotes theory

From these figures, one can see that there is an excellent match between the probability functions as obtained from the formulas and by the simulations. Hence, these analytical expressions can constitute a simple, quick, and robust tool to evaluate both the pdf and the outage probability so that they remove intensive computational necessities of simulations [147, 148].

CHAPTER 7

CONCLUSIONS

In this thesis, briefly, the performance of antenna arrays with joint fading reduction and interference reduction has been investigated in detail for a CDMA cellular system in a correlated Rayleigh fading environment. The use of nonuniformly spaced array has been proposed to overcome the problem induced by uniformly spaced array with a spacing of much larger than half wavelength. The statistical analysis of maximum SIR at the output of beamformer has been carried out.

In order to meet tremendous demand for wireless communication, CDMA schemes in conjunction with adaptive arrays have recently attracted significant interest. Adaptive arrays with optimum combining (beamforming) intensify the intended signal power while suppressing the interference power. Antenna arrays have also been employed to mitigate multipath fading effects by means of space diversity, which is achieved by using multiple antennas spaced sufficiently apart. Consequently, the application of antenna arrays in CDMA mobile system is of particular interest.

We have developed a DS-CDMA single cell model that has been used throughout the thesis. For simplification, we have assumed that the system employs a BPSK modulation. However, all analyses are valid and easily extended for a more complex modulation scheme, for example, QPSK. We have focused on the reverse link, and assumed that linear antenna arrays are located at the base station. Since CDMA systems may become useless without power control, we have assumed that perfect power control is employed within the cell. The spreading codes have been selected as random codes instead of specific spreading codes such as m-sequences, Gold codes, Kasami sequences, etc. Actually, we do not deal with correlation properties of particular

codes specifically, and the most important theme is the contribution of cross correlation values into MAI.

We have employed a model of local scattering where multipaths arrive at the base station within a range of angles. Based on this, a closed form expression for correlation between antennas has been derived as a function of the angle of arrival, angle spread, and antenna spacing. The fading correlation has been utilized to determine the antenna spacing in order to get sufficient space diversity gain in addition to beamforming gain. The results have indicated that antenna spacings that are much larger than half wavelength is required to achieve a tolerable correlation value.

As the spacing exceeds half wavelength significantly, undesired grating lobes having the same gain as main lobe occur in the beam pattern of uniformly spaced array. Hence, uniformly spaced arrays cannot achieve the compromises. We have proposed the use of nonuniformly spaced array to combat multipath fading and suppress interference. For the purpose of constituting a nonuniform array with an average spacing of much larger than half wavelength, a detailed literature survey has been done; however, it did not provide appropriate results. Traditional design methods are generally based on uniform arrays with a spacing of half wavelength, and they permit perturbation of spacings slightly to achieve the predetermined objectives such as sidelobe reduction, desired beam pattern, etc. Therefore, we first have attempted to find optimum spacings by maximizing SINR at the output of the beamformer. However, due to complex structure of SINR, we have elected to generate spacings randomly according to a predetermined statistics.

The results of uniform and nonuniform linear arrays for LOS users have been obtained in terms of average SINR, and they have been compared to each other in terms of outage probability and BER. Results have revealed that nonuniformly spaced array provide a better performance than uniformly spaced arrays.

In order to investigate the impact of correlation on the system performance, studies have been carried out to simulate a scenario where far users exist. For far users, multipath components with different arriving angles, strengths and phases have been considered. The channel vectors are modeled as mutually independent distributed complex Gaussian random vectors with zero mean and different covariance matrices. The correlation between antennas has been considered as a function of angle of arrival, angle spread and antenna spacing. The results have been obtained solely for the case of

nonuniformly spaced arrays since the number of antennas and the length of arrays have been selected as the same so that the average spacings of both arrays are equal. Through the use of numerous computer simulations, thousands of SINRs have been obtained from which outage probability and BER have been calculated for a given average spacing and an angle spread. A major observation is that as the angle spread decreases antenna spacing should be increased to accomplish space diversity since a small spread causes a high correlation between the antennas. The performance becomes better with a large spacing even with a small angle spread. In mobile communications, since signals can arrive at a base station from any direction, large antenna spacing is required to avoid high correlation between fading signals received by different antennas. This result also supports our claim about the use of nonuniformly spaced array to combat multipath fading in addition to reducing interference.

It is apparent that assessment of performance of such a system through Monte Carlo simulation is computationally intensive and time consuming. Analytical expressions always provide more useful insights, and avoid the necessity for time consuming simulation. This has triggered us to attempt analysis of pdf of the SINR at the beamformer output. In order to simplify the analysis we have neglected thermal noise. This assumption is particularly valid for CDMA systems, which are primarily interference limited.

Due to the random spatial positions of mobile users and random channel, SIR is a random variable having quadratic forms in Gaussian variables. Khatri [38] discussed such random variables in the case of i.i.d. interferes in multivariate environment. One would expect that an exact analysis of pdf of such a random variable is quite complicated when the interfering signals have non-identical distribution. However, we have proposed an approach, which makes Khatri's result approximately applicable to our problem. For large number of interferers, the distribution of the total interference covariance matrix is expected to remain almost the same when the channel vectors are assumed to be i.i.d. with a suitable covariance matrix, while in fact they are not. This approach has been verified by intensive simulations. Based on this assumption, the analytical formula for the pdf of the SIR has been obtained, including a hypergeometric function with matrix arguments.

Although the pdf expression seems to be trivial, the numerical computation has been rather difficult due to absence of detailed work about hypergeometric functions.

Gao and Smith [41] recently developed an approach for evaluating the hypergeometric function. Adapting this approach to our problem, the pdf of the SIR has been determined. Although the evaluation of the hypergeometric function of interest is very lengthy and cumbersome, the resulting pdf is quite simple, and this leads to derive the outage probability, which is actually equivalent to the cumulative distribution of the SIR.

The derived expressions in fact reflect the conditional pdf and the conditional outage probability since they have been evaluated for given covariance matrices. In order to attain unconditional pdf and unconditional outage probability, we have first focused on the assumptions made in Chapter 5 to model channel vectors. Since angle spread and antenna spacings were fixed for all mobiles, the resulting covariance matrices for the desired user and intereferers only depend on spatial positions or angle of arrivals of mobiles. Thus, unconditional probabilities can be determined by considering all possible values of angle of arrivals. However, at this point there is a problem of finding the optimum interferer covariance matrix due to the assumption of i.i.d. interferers. This problem has been solved by describing the average interferer covariance matrix, which almost provides an exact result.

Finally, the approximate unconditional pdf and outage probability have been accomplished. Our theoretical results have been verified by means of Monte Carlo simulations. Results have revealed that there is a very good match between the theory and the simulations. The obtained analytical expressions constitute a simple, quick and robust tool to attain the statistical characteristics of SIR so that they remove intensive computational necessities of simulations. The simplicity of these formulas also facilitates the investigation of the effects of correlations on the performance easily and robustly.

Although this thesis has answered several questions about application of antenna arrays for a CDMA cellular system, there are several other questions that remain as challenging topics as follows:

1. Throughout the thesis we analyzed the situation for a single cell environment. This work can be extended to a multicellular environment to simulate a real cellular environment.

2. Since the flat fading channel is the most difficult channel in mobile communica-

tions, we considered the performance of antenna arrays for correlated flat fading channels. However, signals also arrive at the base station with different time delays as well as different angles, strengths and phases. These multipaths with different time delays can be exploited by temporal receivers such as well-known RAKE receivers. Therefore, the performance of nonuniformly spaced array can be investigated for a CDMA system in conjunction with a temporal receiver.

3. These analytical results are planned to be used for the nonuniformly spaced array system design, which is employed in cellular mobile radio systems. The analytical expressions provide performance studies to be carried out over a much wider range of parameters such as the number of users, the number of antennas, different processing gain, etc.

4. The analytical formulas have been obtained for a special aim, however, actually the SIR has a general quadratic form with Gaussian vectors. Therefore, the derived analytical expressions can be utilized in different field of theoretical applications such as adaptive filters, economics, statistics, and so on.

REFERENCES

[1] J. Uddenfeldt, "Digital Cellular–Its Roots and Its Future", *Proc. IEEE*, vol. 86, no. 7, pp. 1319–1324, July 1998.

[2] R. Kohno, R. Meidan and L.B. Milstein, "Spread Spectrum Access Methods for Wireless Communications," *IEEE Commun. Mag.*, pp. 58–67, Jan. 1995.

[3] R. Prasad, "Overview of Wireless Personal Communications: Microwave Perspectives," *IEEE Commun. Mag.*, pp. 104–108, April 1997.

[4] M. Shafi, A. Hashimoto, M. Umehira, S. Ogose, and T. Murase, "Wireless Communications in the Twenty-First Century: A Prespective," *Proc. IEEE*, vol. 85, no. 10, pp. 1622–1637, Oct. 1997.

[5] T.S. Rappaport, *Wireless Communications*. Prentice Hall, 1996.

[6] P.W. Baier, P. Jung, and A. Klein, "Taking the Challenge of Multiple Access for Third Generation Cellular Mobile Radio Systems–A European View," *IEEE Commun. Mag.*, vol. 34, pp. 82–89, Feb. 1996.

[7] K.S. Gilhousen, I.M. Jacobs, R. Padovani, A.J. Viterbi., L.A. Weaver. and C.E. Wheatly, "On the Capacity of a Cellular CDMA System," *IEEE Trans. Veh. Technol.*, vol. VT-40, no. 2, pp. 303–311, May 1991.

[8] R.G. Vaughan, "On Optimum Combining at Mobile," *IEEE Trans. Veh. Technol.*, vol. 37, pp. 181–188, Nov. 1988.

[9] S. Anderson, M. Milnert, M. Viberg, and B. Wahlberg, "An Adaptive Array for Mobile Communication Systems," *IEEE Trans. Veh. Technol.*, vol. 40, no. 1, pp. 230–236, Feb. 1991.

[10] M. Barrett and R. Arnott, "Adaptive Antennas for Mobile Communications, *Electron. Commun. Eng. J.*, vol. 6, pp. 203–214, Aug. 1994.

[11] J.H. Winters, J. Salz and R.D. Gitlin, "The Impact of Antenna Diversity on the Capacity of Wireless Communication Systems," *IEEE Trans. Commun.*, vol. COM-41, no. 4, pp. 1740–1751, April 1994.

[12] A.F. Naguib, *Adaptive Antennas for CDMA Wireless Networks*, Ph.D. Thesis, Stanford University, San Francisco (CA), 1996.

[13] K.J. Molnar and G.E. Bottomley, "Adaptive Array Processing MLSE Receivers for TDMA Digital Cellular/PCS Communications," *IEEE J. Select. Areas Commun.*, vol. 16, no. 8, pp. 1340–1350, Oct. 1998.

[14] X. Wang and H.V. Poor, "Robust Adaptive Array for Wireless Communications," *IEEE J. Select. Areas Commun.*, vol. 16, no. 8, pp. 1352–1366, Oct. 1998.

[15] V.K. Garg, "Application of Adaptive Array Antenna to a TDMA Cellular/PCS Systems," *IEEE Commun. Mag.*, pp. 148–152, Oct. 1997.

[16] C.L.B. Despins, D.D. Falconer, and S.A. Mahmoud, "Compound Strategies of Coding, Equalization and Space Diversity for Wideband TDMA Indoor Wireless Channels," *IEEE Trans. Veh. Technol.*, vol. 41, pp. 369–379, 1992.

[17] R. Kohno, "Spatial and Temporal Precoding and Equalization Using Adaptive Array Antenna for Mobile Radio Communications," 32^{rd} *Annual Alerton Conf. on Commun., Control and Computing*, Oct. 1995.

[18] I. Yoshii and R. Kohno, "Adaptive Spatial and Temporal Optimum Receivers," *Proc. ICT*, vol. 3, pp. 341–345, Porto Carras (Greece), June 1998.

[19] N. Ishii and R. Kohno, "Spatially and Temporally Joint Transmitter-Receiver Using an Adaptive Array Antenna," *IEICE Trans. Commun.*, vol. E 79-B, no. 13, pp. 361–367, March 1996.

[20] W.H. Sheen and G.L. Stuber, "MLSE Equalization and Decoding for Multipath-Fading Channels," *IEEE Trans. Commun.*, vol. 39, pp. 1455–1464, Oct. 1991.

[21] R. Krenz and K. Wesalowski, "Comparative Study of Space Diversity Techniques for MLSE Receivers in Mobile Radio," *IEEE Trans. Veh. Technol.*, vol. 46, pp. 634–641, Aug. 1997.

[22] Jr. Forney, "Maximum Likelihood Sequence Estimation of Digital Sequences in the Presence of Intersymbol Interference," *IEEE Trans. Inform. Theory*, vol. IT-18, pp. 363–378, May 1972.

[23] M.V. Clark, L.J. Greenstein, W.K. Kennedy, and M. Shafi, "MMSE Diversity Combining for Wide-Band Digital Cellular Radio," *IEEE Trans. Commun.*, vol. 40, no. 6, pp. 1128–1135, June 1992.

[24] R. Kohno, H. Imai, M. Hatari, and S. Pasupathy, "Combination of an Adaptive Antenna Array and a Canceler of Interference for Direct-Sequence Spread Spectrum Multiple Access System," *IEEE J. Select. Areas Commun.*, vol. 8, no. 4, pp. 675–682, May 1990.

[25] I.J. Gupta, "Adaptive Arrays for Multiple Simultaneous Desired Signals," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-19, no. 5, pp. 761–767, Sept. 1983.

[26] F. Haber, V. Bar-Ness, C.C. Yeh, "An Adaptive Interference Canceling Array Utilizing Hybrid Techniques," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-19, no. 5, pp. 795–803, Sept. 1983.

[27] T.S. Lee and T.T. Lin, "Coherent Interference Suppression with Complementally Transformed Adaptive Beamformer," *IEEE Trans. Antennas Propagat.*, vol. 46, no. 5, pp. 609–617, May 1998.

[28] R.A. Monzingo and T.W. Miller, *Introduction to Adaptive Arrays.* Wiley, 1980.

[29] M.D. Zoltowski and A. Kannan, "Blind Separation of Co-Channel Digital Signals Under Imperfect Carrier Recovery," *Proc. IEEE MILCOM*, Monterey (CA), 1997.

[30] K. Yao, R.E. Hudson, C.W. Reed, D. Chen, and F. Lorenzelli, "Bilnd Beamforming on a Randomly Distributed Sensor Array System," *IEEE J. Select. Areas Commun.*, vol. 16, no. 8, Oct. 1998.

[31] J.H. Winters, "Signal Acquisition and Tracking with Adaptive Arrays in Digital Mobile Radio System IS-54 with Flat Fading," *IEEE Trans. Veh. Technol.*, vol. VT-42, no. 4, pp. 377-384, July 1993.

[32] S. Choi and D. Yun, "Design of an Adaptive Antenna Array for Tracking the Source of Maximum Power and Its Application to CDMA Mobile Communications," *IEEE Trans. Antennas Propagat.*, vol. 45, no. 9, pp. 1393–1404, Sept. 1997.

[33] S. Bulroje, J.F. Diourois, J. Zeidler, and L. Milstein, "Performance for DS-CDMA Receivers Using Space-Path Diversity," *Proc. IEEE MILCOM*, Monterey (CA), 1997.

[34] G.V. Tsoulos and M.A. Beach, "DS-CDMA Microcellular Networks with Adaptive Antennas," *Proc. IEEE GLOBECOM*, pp. 581–585, London, 1996.

[35] J.S. Thompson, P.H. Grant and B. Mulgrew, "Performance of Antenna Array Receiver Algorithms for CDMA," *Proc. IEEE GLOBECOM*, pp. 570–574, London, Nov. 1996.

[36] J. E. Miller and S.L. Miller, "Performance of DS-SS-CDMA Systems Using Partial Power Control and a Base Station Antenna Array," *Proc. IEEE MILCOM*, pp. 491–495, 1995.

[37] J. Salz and J.H. Winters, "Effect of Fading Correlation on Adaptive Arrays in Digital Mobile Radio," *IEEE Trans. Veh. Technol.*, vol. VT-43, no. 4, pp. 1049–1057, Nov. 1994.

[38] C.G. Khatri,"On Certain Distribution Problems Based on Positive Definite Quadratic Function in Normal Vectors," *Ann. Math. Stat.*, vol. 37, pp. 468–479, 1966.

[39] A. Shah and A.M. Haimovich, "Performance Analysis of Optimum Combining in Wireless Communications with Rayleigh Fading and Cochannel Interference," *IEEE Trans. Commun.*, vol. 46, no. 4, pp. 473–479, Apr. 1998.

[40] H. Gao, P.J. Smith and M.V. Clark,"Theoretical Reliability of MMSE Linear Diversity Combining in Rayleigh-Fading Additive Interference Channels," *IEEE Trans. Commun.*, vol. 46, no. 5, pp. 666–672, May 1998.

[41] H. Gao and P. Smith, "Exact SINR Calculations for Optimum Linear Combining in Wireless Systems," to appear in *IEEE Trans. Commun.*

[42] L. C. Godara,"Applications of Antenna Arrays to Mobile Communications, Part I: Performance Improvement, Feasibility, and System Considerations," *Proc. IEEE*, vol. 85, no. 7, pp. 1031–1060, July 1997.

[43] S.C. Gupta, R. Viswanathan and R. Muammar R., "Land Mobile Radio Systems," *IEEE Commun. Mag.*, vol. 23, no. 6, pp. 34–45, June 1985.

[44] W.C.Y Lee, *Mobile Cellular Telecommunication Systems*. Mc-Graw-Hill, 1989.

[45] M. Rahnema," Overview of the GSM System and Protocol Architecture," *IEEE Commun. Mag.*, vol. 31, no. 4, pp. 92–100, April 1993.

[46] Qualcomm Inc., *An Overview of the Application of Code Division Multiple Access (CDMA) to Digital Cellular Systems and Personal Cellular Networks*, Tech. Rep. EX60-10010, May 1992.

[47] Cylink Corporation, *Final Report–A Spread Spectrum S-CDMA Wireless Telephone System*, May 1992.

[48] A. Suzuki, "A statistical Model for Urban Radio Propagation Channel," *IEEE Trans. Commun.*, vol. COM-25, no. 7, pp. 673–680, July 1977.

[49] H. Hashemi, "Simulation of the Urban Radio Propagation Channel," *IEEE Trans. Veh. Technol.*, vol. VT-28, no. 3, pp. 213–224, Aug. 1979.

[50] J. Litva, T. Kwok and Y. Lo, *Digital Beamforming In Wireless Communications*. Artech House, 1996.

[51] D.L. Schilling, R.L. Pickholtz and L.B. Milstein, "Spread Spectrum Goes Commercial," *IEEE Spectrum*, pp. 40–45, Aug. 1990.

[52] R.L. Pickholtz, L.B. Milstein and D.L. Schilling, "Theory of Spread Spectrum Communications–A Tutorial," *IEEE Trans. Commun.*, vol. COM-30, no.5, pp. 855–884, May 1992.

[53] J.G. Proakis, *Digital Communications*. 2^{nd} edition, McGraw-Hill, 1989.

[54] R. C. Dixon, *Spread Spectrum Systems with Commercial Applications*. 3^{rd} edition, John Wiley & Sons, 1994.

[55] D.A. Shedd and D.V. Sarwate, "Construction of Sequence with Good Correlation Properties," *IEEE Trans. Inform. Theory*, vol. IT-25, no. 1, pp. 94–97, Jan. 1979.

[56] D.V. Sarwate and M.B. Pursley, "Cross Correlation Properties of Pseudorandom and Related Sequences," *Proc. IEEE*, vol. 68, no. 5, pp. 593–619, May 1980.

[57] U.C. Fiebig and M. Schell, "Correlation Properties of Extended M-Sequences," *Electron. Lett.*, vol. 29, no. 20, pp. 1753–1755, Sept. 1993.

[58] H. Fukusama, R. Kohno and H. Imai, "Design of Pseudonoise Sequences with Good Odd and Even Correlation Properties for DS/CDMA," *IEEE J. Select. Areas Commun.*, vol. JSAC-12, no. 5, pp. 828–836, June 1994.

[59] F.M. Özlütürk, S. Tantarana and A.W. Lam, "Performance of DS/SS Communications with MPSK Signalling and Complex Signature Sequences," *IEEE Trans. Commun.*, vol. COM-43, no. 2/3/4, pp. 1127–1133, Feb./March/April 1995.

[60] R. Gold, "Optimal Binary Sequences for Spread Spectrum Multiplexing," *IEEE Trans. Inform. Theory*, vol. IT-13, no. 5, pp. 619–621, Oct. 1967.

[61] T. Kasami, "Weight Distribution Formula for Some Class of Cyclic Codes," Coordinated Science Laboratory, University of Illinois, Urbana, Tech. Rep. R-285 (AD632574), April 1966.

[62] S. Ariyavisitakul and L.F. Chang, "Signal and Interference Statistics of a CDMA System with Feedback Power Control," *IEEE Trans. Commun.*, vol. COM-41, no. 11, pp. 1626–1634, Nov. 1993.

[63] A. Jalali and P. Mermelstein, "Effect of Diversity, Power Control, and Bandwidth on the Capacity of Microcellular CDMA Systems," *IEEE J. Select. Areas Commun.*, vol. JSAC-12, no. 5, pp. 952–961, June 1994.

[64] T.E. Curtis, "Digital Beamforming for Sonar System," *IEE Proc. Pt.F.*, vol. 127, pp. 257–265, Aug. 1980.

[65] P. Barton, "Digital Beamforming for Radar," *IEE Proc. Pt.F.*, vol. 127, pp. 266–277, Aug. 1980.

[66] J.H. Winters, "On the Capacity of Radio Communication Systems with Diversity in a Rayleigh Fading Environment," *IEEE J. Select. Areas Commun.*, vol. SAC-5, no. 5, pp. 871–878, June 1987.

[67] M.I. Skolnik, "Nonuniform Antenna Arrays," in *Antenna Theory, Part I*, edited by R.E. Collin and F.J. Zucker, McGraw Hill, 1969.

[68] H.J. Li, G.T. Huana and S.L. Yen, "Nonuniformly Spaced Array Imaging," *IEEE Trans. Antennas Propagat.*, vol. 41, no. 3, March 1993.

[69] T.H. Ismail and M.M. Dawoud, "Null Steering in Phased Arrays by the Element Positions," *IEEE Trans. Antennas Propagat.*, vol. 39, no. 11, pp. 1561–1566, Nov. 1991.

[70] P.W. Howells, "Intermediate Frequency Sidelobe Canceler," *Technical Report*, US Patent 3202990, May 1959.

[71] S. Applebaum, "Adaptive Arrays," *Technical Report* SPL TR-66-001, Syracuse Univ., Res. Corp. Report, 1965.

[72] B. Widrow, P.E. Montey, L.J. Griffiths and B.B. Goode, "Adaptive Antenna Systems," *Proc. IEEE*, vol. 55, pp. 2143–2159, Dec. 1967.

[73] J. Capon, "High Resolution Frequency Wave Number Spectrum Analysis," *Proc. IEEE*, vol. 57, pp. 1408–1418, 1969.

[74] L.C. Godara, "Application of Antenna Arrays to Mobile Communications, Part II," *Proc. IEEE*, vol. 85, no. 8, pp. 1195–1215, Aug. 1997.

[75] R.T. Compton, *Adaptive Antennas: Concepts and Applications.* Prentice Hall, 1988.

[76] G.H. Golub and C.F. Loan, *Matrix Computations.* The John Hopkins University Press, 2^{nd} edition, 1989.

[77] S. Haykin, *Adaptive Filter Theory.* 3^{rd} edition, Prentice Hall, 1996.

[78] P. Fabre and C. Gueguen, "Improvement of the Fast Recursive Least-Squares Algorithms via Normalization: A comparative study," *IEEE Trans. Acoust. Speech Signal Proc.*, vol. ASSP-34, pp. 296-308, 1986.

[79] J.R. Treichler and B. Agee, "A New Approach to Multipath Correlation of Constant Modulus," *IEEE Trans. Acoust., Speech, Signal Proc.*, vol. ASSP-31, pp. 459–472, April 1983.

[80] B.G. Agee, S.V. Scheel and W.A. Gardner, "Spectral-Self Coherence Restoral: A New Approach to Blind Adaptive Signal Extraction Using Antenna Arrays," *Proc. IEEE*, vol. 78, pp. 753–767, April 1990.

[81] A. Swindlehurst, S. Daas and J. Yang, "Analysis of a Decision Directed Beamformer," *IEEE Trans. Signal Proc.*, vol. 43, pp. 2920–2927, Dec. 1995.

[82] L. Tong, Y. Inouye and R. Liu, "Waveform-Preserving Blind Estimation of Multiple Independent Services," *IEEE Trans. Acoust., Speech, Signal Proc.*, vol. ASSP-41, no. 7, pp. 2461–2470, July 1993.

[83] Y.S. Yeh and D.O. Reudink, "Efficient Spectrum Utilization for Mobile Radio Systems Using Space Diversity," *IEEE Trans. Commun.*, vol. COM-30, pp. 447–455, March 1982.

[84] V.M. Bogachev and I.G. Kiselev, "Optimum Combining of Signals in Space-Diversity Reception," *Telecomm., Radio Eng.*, vol. 34/35, pp. 83–85, Oct. 1980.

[85] J.H. Winters, "Optimum Combining in Digital Mobile Radio with Co-Channel Interference," *IEEE J. Select. Areas Commun.*, vol. JSAC-2, no. 4, pp. 528–539, July 1984.

[86] J.H. Winters, "Optimum Combining for Indoor Radio Systems with Multiple Users," *IEEE Trans. Commun.*, vol. COM-35, no. 11, pp. 1222-1230, Nov. 1987.

[87] J.C. Liberti and T.S. Rappaport, "Analytical Results for Capacity Improvements in CDMA," *IEEE Trans. Veh. Technol.*, vol. 43, no. 13, pp. 680–690, Aug. 1994.

[88] A.F. Naguib and A. Paulraj, "Performance of CDMA Cellular Networks with Base Station Antenna Array," *Proc. Int. Zurich Seminar on Digital Commun.*, Zurich, pp. 87–100, March 1994.

[89] S.C. Swales, M.A. Beach and J.P. McGeehn, "The Performance Enhancement of Multibeam Adaptive Antennas for Cellular Land Mobile Radio Systems," *IEEE Trans. Veh. Technol.*, vol. VT-39, no. 1, pp. 56–67, Feb. 1990.

[90] P. Balaban and J. Salz, "Optimum Diversity Combining and Equalization in Data Transmission with Application to Cellular Mobile Radio-Part I: Theoretical Considerations," *IEEE Trans. Commun.*, vol. COM-40, no. 5, pp. 885-894, May 1992.

[91] P. Balaban and J. Salz, "Optimum Diversity Combining and Equalization in Data Transmission with Application to Cellular Mobile Radio-Part II: Numerical Results," *IEEE Trans. Commun.*, vol. COM-40, no. 5, pp. 895-907, May 1992.

[92] A.F. Naguib and A. Paulraj, "Performance of CDMA Cellular Networks with Base Station Antenna Arrays: The Downlink," *Proc. ICC*, New Orleans, vol. 2, pp. 795–799, May 1994.

[93] D. Gerlach and A. Paulraj, "Adaptive Transmitting Antenna Arrays with Feedback," *IEEE Signal Proc. Lett.*, vol. 1, pp. 150–152, Oct. 1994.

[94] E. Lindskog, "Making SMI Beamforming Insensitive to the Sampling Timing for GSM Signals," *Proc. 6th IEEE Int. Symp., Personal, Indoor and Mobile Radio Commun.*, Toronto, pp. 664–668, Sept. 1995.

[95] C. Passerini, M. Missiroli, G. Riva and M. Frullone, "Adaptive Antenna Arrays for Reducing the Delay Spread in Indoor Radio Channels," *Electron. Lett.*, vol. 32, pp. 280–281, 1996.

[96] T. Gebaur and H.G. Gockler, "Channel-Individual Adaptive Beamforming for Mobile-Satellite Communications," *IEEE J. Select. Areas Commun.*, vol. JSAC-13, pp. 429–448, May 1995.

[97] J. Fernandez, I.R. Corden and M. Barrett, "Adaptive Algorithms for Optimal Combining in Digital Mobile Communication Systems," *Proc. Inst. Elect. Eng. 8th Int. Conf. Antennas and Propagation*, Edinburg, pp. 983–986, 1993.

[98] Y. Wang and J.R. Cruz, "Adaptive Antenna Arrays for the Reverse Link of CDMA Cellular Communication Systems," *Electron. Lett.*, vol. 30, pp. 1017–1018, 1994.

[99] J. Wigard, P.E. Mogenson, F. Frederikson and O. Norklit, "Evaluation of Optimum Diversity Combining in DECT," *Proc. 6th IEEE Int. Symp. Personal, Indoor, and Mobile Radio Commun.*, Toronto, pp. 653–657, Sept. 1995.

[100] P. Petrus and J.H. Reed, "Least Squares CM Adaptive Arrays for Cochannel Interference Rejection for AMPS and IS-54 Signals," *in Wireless'95*, Calgary, July 1995.

[101] T. Ohgane, T. Shimura, N. Matsuzawa and H. Sasaoka, "An Implementation of a CMA Adaptive Array for High GMSK Transmission in Mobile Communications," *IEEE Trans. Veh. Technol.*, vol. 42, no. 3, pp. 282–288, Aug. 1993.

[102] A.F. Naguib, A. Paulraj and T. Kailath, "Capacity Improvement with Base-Station Antenna Arrays in Cellular CDMA," *IEEE Trans. Veh. Technol.*, vol. VT-43, no. 3, pp. 691–698, Aug. 1994.

[103] B. Khalaj, A. Paulraj and T. Kailath, "Antenna Arrays for CDMA Systems with Multipath," *Proc. IEEE MILCOM*, pp. 624–628, Boston (MA), 1993.

[104] H. Iwai, T. Shiokawa anf Y. Karasawa, "An Investigation of a Space-Path Hybrid Diversity Scheme for Base Station Reception in CDMA Mobile Radio," *IEEE J. Select. Areas Commun.*, vol. 12, no. 5, pp. 962–969, June 1994.

[105] G.L. Turin, "A Statistical Model of Urban Multipath Propagation," *IEEE Trans. Veh. Technol.*, vol. VT-21, pp. 673–680, July 1977.

[106] A.A. Saleh and R. Valenzula, " A Statistical Model for Indoor Multipath Propagation," *IEEE J. Select. Areas Commun.*, vol. JSAC-5, no. 2, pp. 128–137, Feb. 1987.

[107] W.R. Braun and U. Dersch, "A Physical Mobile Radio Channel Model," *IEEE Trans. Veh. Technol.*, vol. VT-40, no. 2, pp. 472–482, May 1991.

[108] COST 207 *Digital land Mobile Radio Communications*, Final Report, Commission of the European Communities, 1989.

[109] A.T. James, "Distributions of Matrix Variates and Latent Roots Derived from Normal Samples," *Ann. Math. Stat.*, vol. 35, pp. 475–501, 1964.

[110] J. Fuhl, A.F. Molisch and E. Bonek, "Unified Channel Model for Mobile Radio Systems with Smart Antennas," *IEE Proc.-Radar, Sonar Navig.*, vol. 145, no. 1, pp. 32–41, Feb. 1998.

[111] S.C. Kim, T. Chang, S. Park, L. Song, M.S. Lee and H.M. Kim, "A Vector Channel Modeling and Adaptive Beamforming for Multipath Propagation Using Base-Station Antenna Arrays," *Proc. IEEE MILCOM*, Monterey, CA, 1997.

[112] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables.* Dover Inc., 1965.

[113] G.L. Turin, "Introduction to Spread Spectrum Antimultipath Techniques and Their Application to Urban Digital Radio," *Proc. IEEE*, vol. 68, pp. 328–353, March 1980.

[114] H. Unz, "Linear Arrays with Arbitrarily Distributed Elements," *IRE Trans. Antennas Propagat.*, vol. AP-8, pp. 222–223, 1960.

[115] A. Ishimaru and Y.S. Chen, "Thinning and Broadbanding Antenna Arraysby Unequal Spacings," *IEEE Trans. Antennas Propagat.*, vol. AP-13, pp. 34–42, 1965.

[116] D. King, R. Packhard, and R. Thomas, "Unequally-Spaced, Broad-Band Antenna Arrays," *IRE Trans. Antennas Propagat.*, vol. AP-8, pp. 380-384, 1960.

[117] A.T. Moffet, "Array Factors with Nonuniform Spacing Arrays," *IRE Trans. Antennas Propagat.*, vol. AP-20, pp. 131–136, 1962.

[118] Y. Lo, "A mathematical Theory of Antenna Arrays with Randomly Spaced Elements," *IEEE Trans. Antennas Propagat.*, vol. AP-12, pp. 257–268, 1964.

[119] A.T. Moffet, "Minimum-Redundancy Linear Arrays," *IEEE Trans. Antennas Propagat.*, vol. 16, no. 2, pp. 172–175, 1968.

[120] R.F. Harrington, "Sidelobe Reduction by Nonuniform Element Spacing," *IRE Trans. Antennas Propagat.*, vol. 3, pp. 187–192, 1961.

[121] F. Hodjat and S.A. Hovanessian, "Nonuniformly Spaced Linear and Planar Array Antennas for Sidelobe Reduction," *IEEE Trans. Antennas Propagat.*, vol. AP-26, no. 2, pp. 198–204, March 1978.

[122] M.A. Hassan, "Simple-Iterative Method for Pattern Synthesis Using Nonuniformly Spaced Arrays," *Electron. Lett.*, vol. 26, no. 13, pp. 891–892, June 1990.

[123] M.I. Skolnik, G. Nemhauser and J. Sherman, "Dynamic Programming Applied to Unequally Spaced Elements," *IEEE Trans. Antennas Propagat.*, vol. AP-12, pp. 35–43, 1964.

[124] A. Bratkavic, "Computer Determination of Spaces for Broad-Band Nonuniform Arrays," *IEEE Trans. Antennas Propagat.*, vol. AP-21, pp. 211–213, 1973.

[125] R.M. Leahy and B.D. Jeffs, "On the Design of Maximally Sparse Beamforming Arrays," *IEEE Trans. Antennas Propagat.*, vol. 39, no. 8, Aug. 1991.

[126] B.P. Ng, M.H. Er, C. Kot, "Linear Array Geometry Synthesis with Minimum Sidelobe Level and Null Control," *IEE Proc.-Microw. Antennas Propag.*, vol. 141, no. 3, pp. 162–166, June 1994.

[127] M.M. Dawoud and T.H. Ismail, "Experimental Verification of Null Steering by Element Position Perturbations," *IEEE Trans. Antennas Propagat.*, vol. 40, no. 11, pp. 1431–1434, Nov. 1992.

[128] R. Haupt, "Thinned Arrays Using Genetic Algorithms," *IEEE Trans. Antennas Propagat.*, vol. 42, no. 7, pp. 993–999, July 1994.

[129] L. Davis, *Genetic Algorithms and Simulated Annealing.*, Morgan-Kaufman, 1987.

[130] C.S. Ruf, "Numerical Annealing of Low-Redundancy Linear Arrays," *IEEE Trans. Antennas Propagat.*, vol. 41, no. 1, pp.85–90, Jan. 1993.

[131] B. Suard, A.F. Ayman, G. Xu and A. Paulraj, "Performance of CDMA Mobile Communication System Using Antenna Array," *Proc. IEEE ICASSP*, pp. 153–156, Minneapolis (MN), April 1993.

[132] B.H. Khalaj, A. Paulraj and T. Kailath, "Spatio-Temporal Channel Estimation Techniques for Multiple Access Spread Spectrum System with Antenna Arrays," *Proc. IEEE ICC*, Seattle (WA), pp. 1520–1524, June 1995.

[133] S. Haykin, *Communication Systems.* 2^{nd} edition, Wiley, 1978.

[134] A.F. Naguib and A. Paulraj, "Effects of Multipath and Base Station Antenna Arrays on Uplink Capacity of Cellular CDMA," *Proc. IEEE GLOBECOM*, pp. 395–399, San Francisco (CA), 1994.

[135] A.Y. Özyıldırım and Y. Tanık, "Gezgin İletisim Sistemlerinde Düzgün Olmayan Anten Dizilerinin Tasarımı," *Proc. SIU*, vol. 1, pp. 34–39, Kuşadası, Aydın, May 1997.

[136] A.Y. Özyıldırım and Y. Tanık, "Antenna Diversity and Adaptive Beamforming for CDMA Mobile Systems," *Proc. ICT*, vol. 4, pp. 91–95, Porto Carras (Greece), June 1998.

[137] A.Y. Özyıldırım and Y. Tanık, "Performance of Antenna Arrays with Joint Fading Reduction and Interference Suppression for CDMA Mobile Systems," *Proc. IEEE Veh. Technol. Conf.*, Houston (TX), May 1999.

[138] K.H. Biyari and W.C. Lindsey, "Statistical Distributions of Hermitian Quadratic Forms in Complex Gaussian Variables," *IEEE Trans. Inform. Theory*, vol. 39, no. 3, pp. 1076–1082, May 1993.

[139] R. J. Muirhead, *Aspects of Multivariate Statistical Theory*, Wiley, 1982.

[140] A.G. Constantine, "Some Noncentral Distribution Problems in Multivariate Analysis," *Ann. Math. Stat.*, vol. 34, pp. 1270–1285, 1963.

[141] K.I. Gross and D.ST.P. Richards, "Total Positivity, Spherical Series, and Hypergeometric Functions of Matrix Argument," *J. Approx. Theory*, vol. 59, pp. 224–246, 1989.

[142] A. Papoulis, *Probability, Random Variables, and Stochastic Processes*, 2^{nd} edition, McGraw-Hill, 1984.

[143] A.M. Khirsagar, *Multivariate Analysis*, Marcel Dekker, 1972.

[144] H. Gao, "The Design and Analysis of Some Computationally Intensive Communication System," *Ph.D. Thesis*, Victoria Univ., Wellington, New Zelland, April 1996.

[145] P.R. Krishnaish, *Multivariate Analysis V*, North-Holland Pub. Co., 1980.

[146] A.Y. Özyıldırım and Y. Tanık, "SIR Statistics in Antenna Arrays in the Presence of Correlated Rayleigh Fading," *Proc. IEEE Veh. Technol. Conf. Fall*, Amsterdam, Sept. 1999 (accepted for presentation).

[147] A.Y. Özyıldırım and Y. Tanık, "Distribution Analysis of SIR in Antenna Arrays under Rayleigh Correlated Fading," *Proc. IEEE GLOBECOM*, Rio de Janeiro, Dec. 1999 (accepted for presentation).

[148] A.Y. Özyıldırım and Y. Tanık, "Outage Probability Analysis of a CDMA System with Antenna Arrays in a Correlated Rayleigh Environment," *Proc. IEEE MILCOM*, Atlantic City (NJ), Oct. 1999 (accepted for presentation).

[149] A.M. Parkhurst and A.T. James, "Zonal Polynomials of Order 1 through 12," In *Select. Tables in Math. Stat.*, pp. 199–388, 1974, American Mathematical Society, Providence, R.I.

[150] M.L. Mc Laren, "Coefficients of the Zonal Polynomials," *Appl. Stat.*, vol. 25, pp. 82–87, 1976.

[151] L.K. Kates, *Zonal Polynomials*, Ph.D. Thesis, Princeton University, 1980.

[152] C.R. Wylie and L.C. Parker, *Advanced Engineering Mathematics.* 5^{th} edition, McGraw Hill, 1985.

Appendix A

ZONAL POLYNOMIALS

The *zonal polynomials* are defined in terms of partitions of positive integers [139]. Let k be a positive integer; a partition κ of k is written as $\kappa = (k_1, \ldots, k_m)$ where $k_1 + k_2 + \cdots + k_m = k$ and $k_1 \geq k_2 \geq \cdots \geq k_m$. if $\kappa = (k_1, \ldots, k_m)$ and $\lambda = (l_1, \ldots, l_m)$ are two partitions of k, then $\kappa > \lambda$ if $k_i > l_i$ for the first index i for which the parts are unequal. For example, if $k = 5$,

$$\kappa = (2, 2, 1) > \lambda = (2, 1, 1). \tag{A.1}$$

Supposing that κ and λ are two partitions of k with $\kappa > \lambda$ and $y_1, \ldots, y_m$ are m variables, the *monomial* $y_1^{k_1}, \ldots, y_m^{k_m}$ is said to be of higher weight than the monomial $y_1^{l_1}, \ldots, y_m^{l_m}$.

Definition of Zonal Polynomials :

Let $\mathbf{Y}$ be a mxm symmetric matrix with latent roots (characteristics roots or eigenvalues) $y_1, \ldots, y_m$, and let $\kappa = (k_1, \ldots, k_m)$ be a partition of k into more than m parts. The *zonal polynomial* of $\mathbf{Y}$ corresponding to κ, denoted by $C_\kappa(\mathbf{Y})$ that is a symmetric, homogeneous polynomial of degree k in the latent roots $y_1, \ldots, y_m$ satisfies the following three conditions [139]

(i) The term of highest weight in $C_\kappa(\mathbf{Y})$ is $y_1^{k_1}, \ldots, y_m^{k_m}$, i.e.,

$$C_\kappa(\mathbf{Y}) = d_\kappa\, y_1^{k_1} \ldots y_m^{k_m} + \text{ terms of lower weight} \tag{A.2}$$

where d_κ is a constant.

(ii) $C_\kappa(\mathbf{Y})$ is an eigenfunction of the differential operator $\Delta_\mathbf{Y}$ given by

$$\Delta_\mathbf{Y} = \sum_{i=1}^{m} y_i^2 \frac{\partial}{\partial y_i^2} + \sum_{i=1}^{m} \sum_{\substack{j=1 \\ j \neq i}} \frac{y_i^2}{y_i - y_j} \frac{\partial}{\partial y_i}. \tag{A.3}$$

(iii) As κ varies all partitions of k the zonal polynomials have unit coefficients in the expansion of $(tr\mathbf{Y})^k$; that is

$$(tr\mathbf{Y})^k = (y_1 + \cdots + y_m)^k = \sum_{\kappa}^{m} C_\kappa(\mathbf{Y}). \tag{A.4}$$

There are two important **consequences** of this definition as follows:

(C.1) If $m = 1$, condition (iii) becomes $y^k = C_k(\mathbf{Y})$ so that zonal polynomials of a matrix variable are analogous to powers of a single variable.

(C.2) If β is a constant then the fact that $C_\kappa(\mathbf{Y})$ is homogeneous of degree k implies that $C_\kappa(\beta\mathbf{Y}) = \beta^k C_\kappa(\mathbf{Y})$.

Calculation of Zonal Polynomials :

In order to construct an algorithm for the calculation of zonal polynomials, monomial symmetric functions are defined. If $\kappa = (k_1, \ldots, k_m)$, the monomial function of $y_1, \ldots, y_m$ is expressed as

$$M_\kappa(\mathbf{Y}) = \sum \cdots \sum y_{i_1}^{k_1} \cdots y_{i_p}^{k_p} \tag{A.5}$$

where p is the number of nonzero parts in the partition κ, and the summation is over the distinct permutations $(i_1, \ldots, i_p)$ of p different integers from $1, \ldots, m$. Hence,

$$M_\kappa(\mathbf{Y}) = y_1^{k_1} \ldots y_m^{k_m} + \text{symmetric terms.} \tag{A.6}$$

From condition (i) and the fact that $C_\kappa(\mathbf{Y})$ is symmetric and homogeneous of degree k, $C_\kappa(\mathbf{Y})$ can be expressed in terms of the monomial symmetric functions as

$$C_\kappa(\mathbf{Y}) = \sum_{\lambda \leq \kappa} c_{\kappa,\lambda} M_\lambda(\mathbf{Y}) \tag{A.7}$$

where $c_{\kappa,\lambda}$s are constants, and the summation is over all partitions λ of k with $\lambda \leq \kappa$. The recurrence relation for the coefficient $c_{\kappa,\lambda}$ is given by

$$c_{\kappa,\lambda} = \sum_{\lambda < \mu \leq \kappa} \frac{[(l_i + t) - (l_j - t)]}{\rho_\kappa - \rho_\lambda} c_{\kappa,\mu} \tag{A.8}$$

where

$$\rho_\kappa = \sum_{i=1}^{m} k_i(k_i - i), \tag{A.9}$$

$\lambda = (l_1, \ldots, l_m)$ and $\mu = (l_1, \ldots, l_{i+t}, \ldots, l_{j-t}, \ldots, l_m)$ for $t = 1, \ldots, l_j$ such that when the parts of the partition μ are arranged in descending order, μ is below λ and below

or equal to κ. The summation in (A.8) is over all such μ, including possibly non descending ones, and an empty sum is taken to be zero.

This recurrence relation determines $C_\kappa(\mathbf{Y})$ uniquely once the coefficient of the term of the highest weight is given. From condition (iii) it follows that $\kappa = (k)$, the coefficient of the term of highest weight in $C_{(k)}(\mathbf{Y})$ is unity, i.e; $c_{(k),(k)} = 1$. This determines all other coefficients $c_{k,\lambda}$ in the expansion (A.6) of $C_{(k)}(\mathbf{Y})$ in terms of monomial symmetric functions. These determine, in turn, the coefficient of the term of highest weight in $C_{(k-1,1)}(\mathbf{Y})$, and once this is known, the recurrence relation gives all the other coefficients, and so on.

The coefficients $c_{\kappa,\lambda}$ of the monomial symmetric functions $M_\lambda(\mathbf{Y})$ in $C_\kappa(\mathbf{Y})$ are depicted in Table A.1 for $k = 2$ to 4. Parkhurst and James [149] tabulated these values up to $k = 12$. For larger values of k, tabulation of zonal polynomials appears in prohibitive in terms of space; indeed, for $k = 12$, there exist already 77 zonal polynomials corresponding to the 77 partitions of 12. On the other hand, Mc Laren [150] employed the recurrence relation (A.8) as the basis of his subroutine. In 1980, Kates [151] offered an alternative method of calculation of zonal polynomials by computing sums of products of moments of independent normal variables.

Table A.1: Coefficients of monomial symmetric functions $M_\lambda(\mathbf{Y})$ in the zonal polynomial $C_\kappa(\mathbf{Y})$.

$k = 2$:

$\kappa \backslash \lambda$	(2)	(1,1)
(2)	1	2/3
(1,1)	0	4/3

$k = 3$:

$\kappa \backslash \lambda$	(3)	(2,1)	(1,1,1)
(3)	1	3/5	2/5
(2,1)	0	12/5	18/5
(1,1,1)	0	0	2

$k = 4$:

$\kappa \backslash \lambda$	(4)	(3,1)	(2,2)	(2,1,1)	(1,1,1)
(4)	1	4/7	18/35	12/35	8/35
(3,1)	0	24/7	16/7	88/21	32/7
(2,2)	0	0	16/5	32/15	16/5
(2,1,1)	0	0	0	16/3	64/5
(1,1,1,1)	0	0	0	0	16/5

Appendix B

CALCULATION OF DETERMINANTS

We will calculate the determinants in (6.54) separately through the use of Vandermonde determinants and Laplace's expansion. Let us recall the following functions

$$\begin{aligned} f_j(x) &= (1-x(1-qw_j))^{-(N-M+1)} \\ \mathbf{f}(x) &= [\ f_1(x) \quad f_2(x) \quad \dots \quad f_M(x)\ ] \qquad \text{(B.1)} \\ g_j(x) &= x^{j-1} \\ \mathbf{g}(x) &= [\ g_1(x) \quad g_2(x) \quad \dots \quad g_M(x)\ ]^T. \qquad \text{(B.2)} \end{aligned}$$

Evaluating the derivatives, we have

$$\det\left(\mathbf{f}(0), \mathbf{f}^{(1)}(0), \mathbf{f}^{(2)}(0), \dots, \mathbf{f}^{(M-2)}(0), \mathbf{f}(\mu/(\mu+q))\right) = \qquad \text{(B.3)}$$

$$\begin{vmatrix} 1 & \dots & 1 \\ (N-M+1)_1(1-qw_1) & \dots & (N-M+1)_1(1-qw_M) \\ \vdots & & \vdots \\ (N-M+1)_{M-2}(1-qw_1)^{M-2} & \dots & (N-M+1)_{M-2}(1-qw_M)^{M-2} \\ \left(1-\frac{\mu}{\mu+q}(1-qw_1)\right)^{-(N-M+1)} & \dots & \left(1-\frac{\mu}{\mu+q}(1-qw_M)\right)^{-(N-M+1)} \end{vmatrix}.$$

Using matrix algebra, the common multiplicative terms are taken out such that

$$\det\left(\mathbf{f}(0), \mathbf{f}^{(1)}(0), \mathbf{f}^{(2)}(0), \dots, \mathbf{f}^{(M-2)}(0), \mathbf{f}(\mu/(\mu+q))\right) = \left(\frac{q}{\mu+q}\right)^{-(N-M+1)} \qquad \text{(B.4)}$$

$$\cdot \prod_{i=1}^{M-2} (N-M+1)_i \begin{vmatrix} 1 & \dots & 1 \\ (1-qw_1) & \dots & (1-qw_M) \\ \vdots & \vdots & \vdots \\ (1-qw_1)^{M-2} & \dots & (1-qw_M)^{M-2} \\ (1+\mu w_1)^{-(N-M+1)} & \dots & (1+\mu w_M)^{-(N-M+1)} \end{vmatrix}.$$

From the matrix theory, the determinant of a square matrix is equal to the sum of the products of the elements of its any row (or any column) and their respective cofactors. This is referred to as Laplace's expansion [152]. Through the use of Laplace's expansion at *Mth* row and *kth* column, we have

$$\det\left(\mathbf{f}(0), \mathbf{f}^{(1)}(0), \mathbf{f}^{(2)}(0), \dots, \mathbf{f}^{(M-2)}(0), \mathbf{f}(\mu/(\mu+q))\right) = \left(\frac{q}{\mu+q}\right)^{-(N-M+1)} \tag{B.5}$$

$$\cdot \prod_{i=1}^{M-2} (N-M+1)_i \sum_{k=1}^{M} (-1)^{M+k} (1+\mu w_k)^{(N-M+1)}$$

$$\cdot \begin{vmatrix} 1 & \dots & 1 & 1 & \dots & 1 \\ (1-qw_1) & \dots & (1-qw_{k-1}) & (1-qw_{k+1}) & \dots & (1-qw_M) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ (1-qw_1)^{M-2} & \dots & (1-qw_{k-1})^{M-2} & (1-qw_{k+1})^{M-2} & \dots & (1-qw_M)^{M-2} \end{vmatrix}.$$

Let $\mathcal{L}$ be determinant in (B.5), which is readily expressed in terms of determinant of a Vandermonde determinant, namely,

$$\begin{aligned} \mathcal{L} &= (-1)^{\frac{(M-1)(M-2)}{2}} \\ &\quad \cdot \det(\text{Vandermonde}[\ 1-qw_1 \ \dots \ 1-qw_{k-1} \ \ 1-qw_{k+1} \ \dots \ 1-qw_M\ ]) \\ &= (-1)^{\frac{(M-1)(M-2)}{2}} \prod_{\substack{1\le i<j\le M \\ i,j\neq k}} ((1-qw_i)-(1-qw_j)) \\ &= (-1)^{\frac{(M-1)(M-2)}{2}} q^{\frac{(M-1)(M-2)}{2}} \prod_{\substack{1\le i<j\le M \\ i,j\neq k}} (w_j - w_i). \end{aligned} \tag{B.6}$$

Finally, (B.5) reduced to

$$\det\left(\mathbf{f}(0),\mathbf{f}^{(1)}(0),\ldots,\mathbf{f}^{(M-2)}(0),\mathbf{f}(\mu/(\mu+q))\right)=(-1)^{\frac{(M-1)(M-2)}{2}}q^{\frac{(M-1)(M-2)}{2}} \tag{B.7}$$

$$\cdot\left(\frac{q}{\mu+q}\right)^{-(N-M+1)}\prod_{i=1}^{M-2}(N-M+1)_i$$

$$\cdot\sum_{k=1}^{M}(-1)^{M+k}(1+\mu w_k)^{-(N-M+1)}\prod_{\substack{1\le i<j\le M\\ i,j\neq k}}(w_j-w_i).$$

The second determinant is evaluated by first taking corresponding derivatives. Then, we get

$$\det\left(\mathbf{g}(0),\mathbf{g}^{(1)}(0),\mathbf{g}^{(2)}(0),\ldots,\mathbf{g}^{(M-2)}(0),\mathbf{g}(\mu/(\mu+q))\right)= \tag{B.8}$$

$$\begin{vmatrix} 1 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 1! & 0 & \ldots & 0 & \frac{\mu}{\mu+q} \\ 0 & 0 & 2! & \ldots & 0 & \left(\frac{\mu}{\mu+q}\right)^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & (M-2)! & \left(\frac{\mu}{\mu+q}\right)^{M-2} \\ 0 & 0 & 0 & \ldots & 0 & \left(\frac{\mu}{\mu+q}\right)^{M-1} \end{vmatrix}.$$

Hence, the determinant is obtained as

$$\det\left(\mathbf{g}(0),\mathbf{g}^{(1)}(0),\mathbf{g}^{(2)}(0),\ldots,\mathbf{g}^{(M-2)}(0),\mathbf{g}(\mu/(\mu+q))\right)=\left(\frac{\mu}{\mu+q}\right)^{M-1}\prod_{j=1}^{M-2}j!. \tag{B.9}$$

VITA

Alime Yanartaş Özyıldırım was born in Konya, on August 1967. She received her B.Sc. and M.Sc. degrees from Electrical and Electronics Engineering Department of the Middle East Technical University, in July 1989 and December 1991, respectively. She studied the performance assessment of a CDMA system using an urban mobile channel model in her M.Sc. thesis.

During the period October 1989-June 1994, she worked as a research assistant at the Electrical and Electronics Engineering Department of the Middle East Technical University. At the same duration, she also worked in the EURO-COST 231 Mobile Communication Project. Since June 1994, she has been working as a research engineer at Communications Division in ASELSAN Inc. She has been a partner of the team developing a mobile network based on the spread spectrum. Her current research interests lie in the areas of adaptive antenna arrays, signal statistics, digital mobile communications, spread spectrum systems, and their applications to various kinds of practical communication systems.

T.C. YÜKSEKÖĞRETİM KURULU
DOKÜMANTASYON MERKEZİ