

**BEHAVIOUR AND MODELLING OF REINFORCED CONCRETE
STRUCTURES SUBJECTED TO IMPACT LOADS**

by

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A thesis submitted in conformity with the requirements

for the degree of Doctor of Philosophy

Graduate Department of Civil Engineering

University of Toronto

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ABSTRACT

The analysis and design of reinforced concrete (RC) structures against extreme loads, such as earthquakes, blasts, and impacts, has been an objective of many researchers and designers. As a result of recently elevated terror threat levels in the world, demand for the impact resistant design of buildings has increased. Numerous studies have been conducted to-date toward understanding and developing methodologies predicting the behaviour of RC structures under impact loads. However, the lack of a complete understanding of shear behaviour under high dynamic conditions hindered the efforts for accurate prediction of impact behaviour, since severe shear mechanisms may dominate the behaviour of RC structures when subjected to impact loads. This current study aimed to apply one of the more successful methods of static reinforced concrete shear analysis, the Modified Compression Field Theory (MCFT), to the analysis of dynamic loads, and thus, develop an efficient and reliable tool for impact analysis of RC structures. A two-dimensional nonlinear finite element analysis program for reinforced concrete, VecTor2, developed previously at the University of Toronto for static loads, was modified to include the consideration of dynamic loads, including impacts. VecTor2 uses the MCFT for its computational methodology, along with a wide array of material and behavioural models for reinforced concrete. To verify the performance of VecTor2 and its computational

methodology under impact loads, an experimental program was also undertaken to provide data for corroboration. Eight reinforced concrete beam specimens, four pairs, were tested under free falling drop-weights, impacting the specimens at the mid-span. All specimens had identical longitudinal reinforcement, but varying shear reinforcement ratio, intended to investigate the effects of shear capacity on the impact behaviour. A total of 20 tests were conducted, including multiple tests on each specimen. The test results showed that the shear characteristics of the specimens played an important role in their overall behaviour. All specimens, regardless of their shear capacity, developed severe diagonal shear cracks, forming a shear-plug under the impact point. The VecTor2 analyses of the test specimens were satisfactory in predicting damage levels, and maximum and residual displacements. The methodology employed by VecTor2, based on the MCFT, proved to be successful in predicting the shear-dominant behaviour of the specimens under impact.

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Impact tests conducted as a part of this research were quite a spectacle; they were noisy, dusty, a little dangerous, and therefore, fun! These tests could not be realized without the help and assistance of the University of Toronto Structural Laboratory staff Renzo Basset, John MacDonald, Joel Babbín, Giovanni Buzzeo, and Alan McClenaghan. I thank them all.

Undertaking such a huge task in a foreign country away from my family was sure difficult. On the other hand, it was also a life altering experience made very enjoyable thanks to many good friends I met in Canada, such as Kien Vinh Duong, Serhan Güner, Katrin Habel, David Ho, Karen Liu, Adam Lubell, Nabil Mansour, Phillip Miller, Michael Montgomery, Talayeh Noshiravani, Gülşah Sağbaş, Mohamed Semelawy, Jimmy Susetyo, Liping Xie, Almıla Uzel, and Andrew Voth, just to name a few. Besides my degree, I consider their friendship as the second big prize won in this journey.

To start my studies at the University of Toronto, I arrived in Canada from Turkey on September 11, 2001. Desperately waiting for a phone call to hear that I was safely landed, the horrific events took place on that perhaps one of the most gruesome days in recent history were as if breaking the news to my family that this was not going to be easy. During the course of my studies, despite the thousands of miles between us, my mother, my father, my sister and my grandmother did everything they could to make my life easier and they anxiously waited for me to finish and come back home. I cannot thank them enough for their love, support, and patience. Now that it's over, I going home!



TABLE OF CONTENTS

Abstract	ii
Acknowledgements	iv
Table of Contents	vi
List of Tables	xi
List of Figures	xiii
Notation	xxiii
1 Introduction	1
2 Literature Review	3
2.1 Introduction	3
2.2 Local Response of Reinforced Concrete Structures.....	5
2.3 Global Response of Reinforced Concrete Structures.....	15
2.4 Significance of the Current Study.....	31
3 Finite Element Modelling Of Reinforced Concrete Structures Under Dynamic Loads.....	34
3.1 Introduction	34
3.2 Structural Property Matrices	34
3.2.1 Mass Matrix	36
3.2.2 Damping Matrix.....	38
3.2.3 Stiffness matrix	43

3.2.4	Load Vector.....	50
3.3	Numerical Evaluation of Dynamic Response	51
3.3.1	Newmark's Method of Direct Integration.....	52
3.3.2	Stability and Errors	56
3.4	Dynamic Analysis Algorithms in VecTor2.....	59
3.4.1	Determination of the Modal Periods and the Damping Matrix	59
3.4.2	Direct Integration Method with Secant Stiffness	61
3.4.3	Dynamic Analysis Algorithms	62
3.5	Linear Elastic Verification of VecTor2 Dynamic Analysis.....	64
3.5.1	Static Load.....	65
3.5.2	Free Vibrations.....	66
3.5.3	Impulse Forces	67
3.5.4	Base Accelerations.....	71
4	Experimental Program	75
4.1	Introduction	75
4.2	Test Specimens.....	75
4.3	Test Setup	78
4.4	Material Properties	80
4.5	Instrumentation.....	83
4.5.1	Accelerometers.....	83
4.5.2	LVDT's and Potentiometers	85
4.5.3	Strain Gauges	89
4.5.4	Load Cells	97
4.5.5	Data Acquisition System	98
4.6	Drop-Weights	99
4.7	Test Procedure	102
4.7.1	SS3a-1 (Test Date: July 20, 2005; Drop-weight: 211 kg)	103
4.7.2	SS3a-2 (Test Date: August 8, 2005; Drop-weight: 600 kg)	103
4.7.3	SS3a-3 (Test Date: August 10, 2005; Drop-weight: 600 kg)	104

4.7.4	SS2a-1 (Test Date: August 26, 2005; Drop-weight: 211 kg)	104
4.7.5	SS2a-2 (Test Date: August 31, 2005; Drop-weight: 600 kg)	105
4.7.6	SS2a-3 (Test Date: October 11, 2005; Drop-weight: 600 kg)	106
4.7.7	SS1a-1 (Test Date: November 17, 2005; Drop-weight: 211 kg)	107
4.7.8	SS1a-2 (Test Date: November 23, 2005; Drop-weight: 600 kg)	107
4.7.9	SS1a-3 (Test Date: November 28, 2005; Drop-weight: 600 kg)	107
4.7.10	SS0a-1 (Test Date: January 18, 2006; Drop-weight: 211 kg)	108
4.7.11	SS0a-2 (Test Date: January 23, 2006; Drop-weight: 600 kg)	109
4.7.12	SS3b-1 (Test Date: February 16, 2006; Drop-weight: 600 kg)	110
4.7.13	SS3b-2 (Test Date: February 17, 2006; Drop-weight: 600 kg)	110
4.7.14	SS3b-3 (Test Date: February 21, 2006; Drop-weight: 211 kg)	111
4.7.15	SS2b-1 (Test Date: February 27, 2006; Drop-weight: 600 kg)	111
4.7.16	SS2b-2 (Test Date: March 1, 2006; Drop-weight: 600 kg)	112
4.7.17	SS2b-3 (Test Date: March 3, 2006; Drop-weight: 211 kg)	112
4.7.18	SS1b-1 (Test Date: March 10, 2006; Drop-weight: 600 kg)	113
4.7.19	SS1b-2 (Test Date: March 14, 2006; Drop-weight: 600 kg)	113
4.7.20	SS0b-1 (Test Date: April 7, 2006; Drop-weight: 600 kg)	114
5	Discussion of Test Results	116
5.1	Introduction	116
5.2	Digital Signal Analysis	116
5.2.1	Displacement Data	117
5.2.2	Strain Data	121
5.2.3	Load Cell Data	123
5.2.4	Acceleration Data	126
5.3	Impact Force Measurement	135
5.4	Displaced Shape	139
5.5	Analysis of Crack Patterns	152
5.6	Dynamic Equilibrium	155
5.7	Impact Capacities of Test Specimens	164

5.8	Strain Rates.....	168
5.9	Damping	170
5.10	Conclusion.....	172
6	Nonlinear Finite Element Analyses Of Test Specimens With VecTor2..	174
6.1	Introduction	174
6.2	Finite Element Model.....	174
6.3	Static Analyses of Test Specimens	178
6.4	Impact Analysis of Test Specimens.....	184
6.4.1	Impact Analyses of Undamaged Test Specimens.....	184
6.4.1.1	Mid-span Displacements and Support Reactions	185
6.4.1.2	Reinforcement Strains	190
6.4.1.3	Crack Patterns	198
6.4.2	Impact Analyses of Test Specimens for the Second Impact Tests	207
6.4.2.1	Mid-span Displacements and Support Reactions	208
6.4.2.2	Reinforcement Strains.....	211
6.4.2.3	Crack Patterns.....	217
6.4.3	Effects of Damping Parameters on VecTor2 Impact Analyses.....	223
6.4.4	Effects of Time-Step Size on VecTor2 Impact Analyses	226
6.5	Conclusion.....	229
7	Conclusions.....	230
	References	237
	Appendix A Material Properties of Test Specimens.....	245
A.1	Concrete Properties (December 12, 2005 Cylinder Tests)	246
A.2	Steel Bar Properties	248
A.3	Support Bar Calibration Results	249

Appendix B Technical Data Sheets for the Sensors and the Data Acquisition System....251

Appendix C Photographs and Crack Profiles of Test Specimens 265



LIST OF TABLES

4 EXPERIMENTAL PROGRAM

Table 4.1 Transverse reinforcement ratios and stirrup spacing for the beams	77
Table 4.2 Casting dates of the specimens.....	80
Table 4.3 Cylinder test results.....	81
Table 4.4 Modulus of rupture test results	82
Table 4.5 Steel coupon test results.....	82
Table 4.6 Material Densities	83
Table 4.7 Sensors and connection boards used for data acquisition.....	99

5 DISCUSSION OF TEST RESULTS

Table 5.1 Typical crack widths measured after tests.....	154
Table 5.2 Mass per unit length of specimens	157
Table 5.3 Static capacities of test specimens based on VecTor2 analyses.....	166
Table 5.4 Maximum reaction forces recorded.....	166
Table 5.5 Energy imparted on the specimens.....	167

6 NONLINEAR FINITE ELEMENT ANALYSES OF TEST SPECIMENS WITH VECTOR2

Table 6.1 Material and behavioural models used for concrete	177
Table 6.2 Material and behavioural models used for steel reinforcement.....	178
Table 6.3 Peak values as obtained from the tests and VecTor2 (first impacts).....	187
Table 6.4 Observed and computed peak longitudinal reinforcement strains	196

Table 6.5 Observed and computed peak stirrup strains	197
Table 6.6 Peak values as obtained from the tests and VecTor2 (second impacts).....	210
Table 6.7 Observed and computed peak longitudinal reinforcement strains	216
Table 6.8 Observed and computed peak stirrup strains	216
Table 6.9 Damping properties used in the analyses	224
Table 6.10 Computation times for the analyses	228



LIST OF FIGURES

2 LITERATURE REVIEW

Figure 2.1 Missile impact phenomena (Kennedy 1976).....	4
Figure 2.2 Concrete shell (Rebora et al. 1976)	7
Figure 2.3 Lagrange grid for impact calculation (Attalla and Nowotny 1976).....	8
Figure 2.4 Layout for the two-dimensional computational simulation by Gupta and Sieman (1978).....	9
Figure 2.5 Fragment and target condition 20 μ s after impact (Thoma and Vinckier 1994)	10
Figure 2.6 Penetration of a projectile into concrete (Agardh and Laine 1999).....	11
Figure 2.7 Penetration of a projectile into a reinforced concrete slab (Teng et al. 2004)....	11
Figure 2.8 DEM model (Sawamoto et al. 1998)	12
Figure 2.9 Damage modes of panels (Sawamoto et al. 1998)	13
Figure 2.10 Basic cube model and composition of prisms (Riera and Iturrioz 1998)	13
Figure 2.11 Perforation of a reinforced concrete beam (black lines represent steel bars) ...	14
Figure 2.12 Specimens after the tests (Mylrea 1940)	16
Figure 2.13 Schematic representation of a beam as SDOF system.....	17
Figure 2.14 Spring models for impact (CEB 1988).....	19
Figure 2.15 Typical force-deformation relationship of contact zone, $R_2(\Delta u)$ (CEB 1988).	21
Figure 2.16 Multi-mass model for soft-impact collision (Miyamoto et al. 1994)	23
Figure 2.17 Linking (coupling) procedure for analysis of soft-impact collision	24
Figure 2.18 FEM model (Shirai et al. 1994)	26
Figure 2.19 Test setup and the FEM model of the RC beams (Kishi et al. 2001)	26
Figure 2.20 AUTODYN model of a steel hull structure (Balden et al. 2005)	27
Figure 2.21 Dimensions of reinforced concrete beam (Kishi et al. 2002)	28
Figure 2.22 Crack patterns for beams A36 and B36 (Kishi et al. 2002)	29
Figure 2.23 A simplified model for reaction force versus displacement loop	30

3 FINITE ELEMENT MODELLING OF REINFORCED CONCRETE STRUCTURES UNDER DYNAMIC LOADS

Figure 3.1 Equilibrium of structures.....	35
Figure 3.2 Lumped mass matrix	37
Figure 3.3 Variation of modal damping ratios with natural frequency	40
Figure 3.4 Damping mechanisms (Chopra, 2001)	41
Figure 3.5 Reference systems for reinforced concrete element (Vecchio, 1990)	46
Figure 3.6 Finite element solution procedure.....	48
Figure 3.7 Influence coefficient vector	51
Figure 3.8 Overshooting in numerical direct integration (Chopra, 2001).....	58
Figure 3.9 Flowchart for dynamic analysis with VecTor2	63
Figure 3.10 Finding the coefficients a_0 and a_1 for damping calculations	64
Figure 3.11 Test structure and finite element model	65
Figure 3.12 Static response of the test structure.....	66
Figure 3.13 Comparison between exact and numerical response, free vibration.....	67
Figure 3.14 Impulse forces applied on the test structure	68
Figure 3.15 Notation for the analytical response of applied impulse forces	68
Figure 3.16 Comparison between exact and numerical response, short impulse	70
Figure 3.17 Comparison between exact and numerical response, long impulse	70
Figure 3.18 Imperial Valley Earthquake acceleration record (El Centro–1940)	71
Figure 3.19 Northridge Earthquake acceleration record (Santa Monica–1994)	72
Figure 3.20 Comparison between SDOF and VecTor2 response, Imperial Valley record ..	73
Figure 3.21 Comparison between SDOF and VecTor2 response, Northridge record	74

4 EXPERIMENTAL PROGRAM

Figure 4.1 Specimen dimensions.....	76
Figure 4.2 Specimen cross-section (all dimensions are in millimetres).....	77
Figure 4.3 Naming convention for the beams.....	77

Figure 4.4 Test setup cross section at the supports	79
Figure 4.5 Side view of support (floor beams are not shown).....	79
Figure 4.6 Locations of the accelerometers on the test beams	84
Figure 4.7 Aluminum brackets for mounting the accelerometers on the beam	85
Figure 4.8 Mounting of the accelerometers on the drop-weight	85
Figure 4.9 Displacement sensor locations for SS3a, SS2a, SS1a and SS0a	87
Figure 4.10 Displacement sensor locations for SS3b, SS2b, SS1b and SS0b	88
Figure 4.11 Displacement sensors and their connections to the specimens.....	88
Figure 4.12 Strain gauge glued on a longitudinal bar	89
Figure 4.13 Strain gauge locations for SS3a	90
Figure 4.14 Strain gauge locations for SS3b.....	91
Figure 4.15 Strain gauge locations for SS2a	92
Figure 4.16 Strain gauge locations for SS2b.....	93
Figure 4.17 Strain gauge locations for SS1a	94
Figure 4.18 Strain gauge locations for SS1b.....	95
Figure 4.19 Strain gauge locations for SS0a	96
Figure 4.20 Strain gauge locations for SS0b.....	97
Figure 4.21 Load cell	98
Figure 4.22 Drop-weights	100
Figure 4.23 Drop-weight suspended from the crane with nylon rope.....	100
Figure 4.24 Drop-weight and the columns.....	101
Figure 4.25 Arrangements for the impact point on the beam	102
Figure 4.26 View as seen from the west face, SS3a-2.....	103
Figure 4.27 View as seen from the west face, SS3a-3	104
Figure 4.28 Views as seen from the west face, SS2a-1	105
Figure 4.29 Views as seen from the west face, SS2a-2	105
Figure 4.30 View as seen from the west face, SS2a-3	106
Figure 4.31 Views as seen from the west face, SS1a-2	107
Figure 4.32 Views as seen from the west face, SS1a-3	108
Figure 4.33 View as seen from the west face, SS0a-1	109
Figure 4.34 Views as seen from the west face, SS0a-2	109

Figure 4.35 Views as seen from the west face, SS3b-1	110
Figure 4.36 Views as seen from the west face, SS3b-2	110
Figure 4.37 Views as seen from the west face, SS3b-3	111
Figure 4.38 Views as seen from the west face, SS2b-1	111
Figure 4.39 Views as seen from the west face, SS2b-2	112
Figure 4.40 Views as seen from the west face, SS2b-3	112
Figure 4.41 Views as seen from the west face, SS1b-1	113
Figure 4.42 Views as seen from the west face, SS1b-2	114
Figure 4.43 View as seen from the west face, SS0b-1	115

5 DISCUSSION OF TEST RESULTS

Figure 5.1 Aliasing (squares represent the sampled data from the high-frequency signal)	117
Figure 5.2 Mode shapes and frequencies for test beams	118
Figure 5.3 Mid-span displacement, SS1b-1	120
Figure 5.4 Mid-span displacement, SS2a-1	120
Figure 5.5 Mid-span displacement, SS2a-2	120
Figure 5.6 Strain at Bar #3 Gauge 1, SS1b-1	121
Figure 5.7 Strain at Bar #4 Gauge 1, SS2a-1	122
Figure 5.8 Strain at Bar #4 Gauge 1, SS2a-2	122
Figure 5.9 A closer look at the peak point of strain at Bar #3 Gauge 1, SS1b-1	123
Figure 5.10 Forces as measured by Load Cell A, SS2a-1	124
Figure 5.11 Forces as measured by Load Cell A, SS3b-1	124
Figure 5.12 Forces as measured by Load Cell A, SS1a-2	124
Figure 5.13 Data points at the first peak of the Load Cell A data, SS3b-1	125
Figure 5.14 Accelerations as measured by A1, SS1b-2	126
Figure 5.15 Accelerations as measured by A1, SS0a-1	127
Figure 5.16 Accelerations as measured by A1, SS3a-1	127
Figure 5.17 Data points for the mid-span acceleration measurement of SS1b-2	128

Figure 5.18 Accelerations as measured by A6, SS1b-2	128
Figure 5.19 Accelerations as measured by A6, SS0a-1	129
Figure 5.20 Accelerations as measured by A6, SS3a-1	129
Figure 5.21 Comparison of the A6 data obtained with different sampling rates, SS1a-2	130
Figure 5.22 Power spectrum of the acceleration measured by A6 and recorded by 19.2 kHz sampling rate, SS1a-2.....	131
Figure 5.23 Filtered and unfiltered force-time plots for an impact (Found et al. 1998)	132
Figure 5.24 Comparison of A1 acceleration data and the second-time derivative of the mid- span displacement, SS1b-2.....	133
Figure 5.25 Comparison of A1 acceleration data and the second-time derivative of the mid- span displacement, SS0a-1	133
Figure 5.26 Comparison of A1 acceleration data and the second-time derivative of the mid- span displacement, SS3a-1	134
Figure 5.27 Test setup for drops on a load cell	136
Figure 5.28 Test drop on a load cell from 300 mm – Test 1	137
Figure 5.29 Test drop on a load cell from 300 mm – Test 2	137
Figure 5.30 Test drop on a load cell from 500 mm – Test 1	138
Figure 5.31 Test drop on a load cell from 500 mm – Test 2	138
Figure 5.32 Displacement sensor locations for SS3b, SS2b, SS1b and SS0b	140
Figure 5.33 Displaced shape, SS3b-1	141
Figure 5.34 Displaced shape, SS3b-2.....	141
Figure 5.35 Displaced shape, SS1b-1	142
Figure 5.36 Displaced shape, SS1b-2.....	142
Figure 5.37 Displaced shape, SS0b-1	143
Figure 5.38 Major diagonal crack between P9 and P10, SS3b-2	144
Figure 5.39 South half of SS1b, after SS1b-2	144
Figure 5.40 Unit displaced shape for SS3b, obtained from SS3b-1	145
Figure 5.41 Unit displaced shape for SS1b, obtained from SS1b-1	146
Figure 5.42 Unit displaced shape for SS0b, obtained from SS0b-1	146
Figure 5.43 Displaced shapes as measured and as calculated by Eq. 5.1, SS3b-1	147
Figure 5.44 Displaced shapes as measured and as calculated by Eq. 5.1, SS1b-1	148

Figure 5.45 Displaced shapes as measured and as calculated by Eq. 5.1, SS0b-1	149
Figure 5.46 Comparison of elastic and measured unit displaced shapes	151
Figure 5.47 Comparison of elastic and measured unit displaced shapes, effect of shear-plug	152
Figure 5.48 Typical cracks in a shear-critical specimen (SS1b-1)	153
Figure 5.49 Dynamic free body diagram for the test specimens	155
Figure 5.50 Acceleration distribution along the specimen	156
Figure 5.51 Dynamic equilibrium of forces, SS3a-1	158
Figure 5.52 Dynamic equilibrium of forces, SS2b-1	158
Figure 5.53 Dynamic equilibrium of forces, SS1b-1	159
Figure 5.54 Dynamic equilibrium of forces, SS1b-2	159
Figure 5.55 Dynamic equilibrium of forces, SS0b-1	160
Figure 5.56 Distribution of forces	161
Figure 5.57 Breakdown of resisting forces, SS3a-1	162
Figure 5.58 Vertical cracks due to negative moments	163
Figure 5.59 Inclination of a vertical crack at the overhanging part	164
Figure 5.60 Energy imparted to the specimens	165
Figure 5.61 Calculated strain rates	169
Figure 5.62 Free vibration response of a damped system	170
Figure 5.63 Free vibrations before SS1b-1 test	171
Figure 5.64 Free vibrations after SS1b-1 test	172

6 NONLINEAR FINITE ELEMENT ANALYSES OF TEST SPECIMENS WITH VECTOR2

Figure 6.1 Finite element model	176
Figure 6.2 Static response of SS0	179
Figure 6.3 Static response of SS1	180
Figure 6.4 Static response of SS2	181
Figure 6.5 Static response of SS3	182

Figure 6.6 Static response of test specimens.....	183
Figure 6.7 Comparison of observed and computed responses, SS0a-1.....	185
Figure 6.8 Comparison of observed and computed responses, SS1a-1.....	185
Figure 6.9 Comparison of observed and computed responses, SS2a-1.....	186
Figure 6.10 Comparison of observed and computed responses, SS3a-1	186
Figure 6.11 Comparison of observed and computed responses, SS1b-1	186
Figure 6.12 Comparison of observed and computed responses, SS2b-1	187
Figure 6.13 Comparison of observed and computed responses, SS3b-1	187
Figure 6.14 Observed and computed longitudinal reinforcement strains, SS0a-1.....	191
Figure 6.15 Observed and computed longitudinal reinforcement strains, SS1a-1.....	191
Figure 6.16 Observed and computed longitudinal reinforcement strains, SS2a-1.....	192
Figure 6.17 Observed and computed longitudinal reinforcement strains, SS3a-1.....	192
Figure 6.18 Observed and computed longitudinal reinforcement strains, SS1b-1	193
Figure 6.19 Observed and computed longitudinal reinforcement strains, SS2b-1	193
Figure 6.20 Observed and computed longitudinal reinforcement strains, SS3b-1	194
Figure 6.21 Observed and computed stirrup strains, SS1a-1.....	194
Figure 6.22 Observed and computed stirrup strains, SS2a-1.....	195
Figure 6.23 Observed and computed stirrup strains, SS3a-1.....	195
Figure 6.24 Observed and computed stirrup strains, SS1b-1	195
Figure 6.25 Observed and computed stirrup strains, SS2b-1	196
Figure 6.26 Observed and computed stirrup strains, SS3b-1	196
Figure 6.27 Observed and computed crack profiles, SS0a-1.....	199
Figure 6.28 Observed and computed crack profiles, SS1a-1.....	200
Figure 6.29 Observed and computed crack profiles, SS2a-1.....	201
Figure 6.30 Observed and computed crack profiles, SS3a-1.....	202
Figure 6.31 Observed and computed crack profiles, SS1b-1	203
Figure 6.32 Observed and computed crack profiles, SS2b-1	204
Figure 6.33 Observed and computed crack profiles, SS3b-1	205
Figure 6.34 Comparison of observed and computed responses, SS1a-2	208
Figure 6.35 Comparison of observed and computed responses, SS2a-2	208
Figure 6.36 Comparison of observed and computed responses, SS3a-2	209

Figure 6.37 Comparison of observed and computed responses, SS2b-2	209
Figure 6.38 Comparison of observed and computed responses, SS3b-2	209
Figure 6.39 Observed and computed longitudinal reinforcement strains, SS1a-2.....	211
Figure 6.40 Observed and computed longitudinal reinforcement strains, SS2a-2.....	212
Figure 6.41 Observed and computed longitudinal reinforcement strains, SS3a-2.....	212
Figure 6.42 Observed and computed longitudinal reinforcement strains, SS2b-2	213
Figure 6.43 Observed and computed longitudinal reinforcement strains, SS3b-2	213
Figure 6.44 Observed and computed stirrup strains, SS1a-2.....	214
Figure 6.45 Observed and computed stirrup strains, SS2a-2.....	214
Figure 6.46 Observed and computed stirrup strains, SS3a-2.....	215
Figure 6.47 Observed and computed stirrup strains, SS2b-2	215
Figure 6.48 Observed and computed stirrup strains, SS3b-2	216
Figure 6.49 Observed and computed crack profiles, SS1a-2.....	218
Figure 6.50 Observed and computed crack profiles, SS2a-2.....	219
Figure 6.51 Observed and computed crack profiles, SS3a-2.....	220
Figure 6.52 Observed and computed crack profiles, SS2b-2	221
Figure 6.53 Observed and computed crack profiles, SS3b-2	222
Figure 6.54 Effect of damping on computed response of SS2b-1	225
Figure 6.55 Effect of damping on computed response of SS3b-2.....	226
Figure 6.56 Effect of time-step size on the response, SS3b-1	227

APPENDIX A MATERIAL PROPERTIES OF TEST SPECIMENS

Figure A.1 Concrete stress-strain curves for SS0a and SS0b	246
Figure A.2 Concrete stress-strain curves for SS1a and SS1b	246
Figure A.3 Concrete stress-strain curves for SS2a and SS2b	247
Figure A.4 Concrete stress-strain curves for SS3a and SS3b	247
Figure A.5 Stress-strain curve for No.30 steel bars	248
Figure A.6 Stress-strain curve for D-6 steel bars	248
Figure A.7 Support Bar #1 calibration (south support)	249

Figure A.8 Support Bar #2 calibration (south support)	249
Figure A.9 Support Bar #3 calibration (north support).....	250
Figure A.10 Support Bar #4 calibration (north support).....	250

APPENDIX C PHOTOGRAPHS AND CRACK PROFILES OF TEST SPECIMENS

Figure C.1 SS3a crack profile (West face)	266
Figure C.2 SS3a crack profile (East face)	267
Figure C.3 SS2a crack profile (West face)	268
Figure C.4 SS2a crack profile (East face)	269
Figure C.5 SS1a crack profile (West face)	270
Figure C.6 SS1a crack profile (East face)	271
Figure C.7 SS0a crack profile (West face)	272
Figure C.8 SS0a crack profile (East face)	273
Figure C.9 SS3b crack profile (West face)	274
Figure C.10 SS3b crack profile (East face)	275
Figure C.11 SS2b crack profile (West face)	276
Figure C.12 SS2b crack profile (East face)	277
Figure C.13 SS1b crack profile (West face)	278
Figure C.14 SS1b crack profile (East face)	279
Figure C.15 SS0b crack profile (West face)	280
Figure C.16 SS0b crack profile (East face)	280
Figure C.17 SS3a final photo (West face)	281
Figure C.18 SS3a final photo (East face)	281
Figure C.19 SS2a final photo (West face)	282
Figure C.20 SS2a final photo (East face)	282
Figure C.21 SS1a final photo (West face)	283
Figure C.22 SS1a final photo (East face)	283
Figure C.23 SS0a final photo (West face)	284

Figure C.24 SS0a final photo (East face)	284
Figure C.25 SS3b final photo (West face)	285
Figure C.26 SS3b final photo (East face)	285
Figure C.27 SS2b final photo (West face)	286
Figure C.28 SS2b final photo (East face)	286
Figure C.29 SS1b final photo (West face)	287
Figure C.30 SS1b final photo (East face)	287
Figure C.31 SS0b final photo (West face)	288
Figure C.32 SS0b final photo (East face)	288



NOTATION

a	Acceleration
a	Length of the overhang (Chapter 5)
a_0	Proportionality constant
a_1	Proportionality constant
A	Cross-sectional area
$\mathbf{B}$	Shape function matrix
$\mathbf{c}$	Square damping matrix
d	Effective depth of the beam
$\mathbf{D}$	Material matrix
$\mathbf{D}_c'$	Concrete material matrix expressed with respect to the principal axes 1 and 2
$\mathbf{D}_s'$	Reinforcement material matrix expressed with respect to element's axes
E	Modulus of elasticity
E_a	Absorbed energy
E_{ad}	Design absorbed energy
E_c	Concrete elastic modulus
E^c	Work done by damping forces
$\bar{E}_{c1}$	Secant modulus of concrete for tension
$\bar{E}_{c2}$	Secant modulus of concrete for compression
E^{ext}	Work done by external forces
E^{int}	Work done by internal restoring forces
E_k	Input kinetic energy
E_{kd}	Design input kinetic energy
E^{kin}	Kinetic energy of the system
E_s	Reinforcement modulus of elasticity
$\bar{E}_s$	Elastic modulus for steel reinforcement

f_c'	Concrete peak compressive stress
f_{c1}	Concrete stress in principal direction 1
f_{c2}	Concrete stress in principal direction 2
f_{cd}	Dynamic compressive strength of concrete
f_{cr}	Tensile cracking strength of concrete
f_{cs}	Static compressive strength of concrete
f_D	Damping force
$\mathbf{f}^{\text{ext}}$	External force vector
$\mathbf{f}^{\text{int}}$	Internal restoring force vector
f_{rp}	Modulus of rupture
f_s	Stiffness force
f_s	Stress in the reinforcement
f_t'	Concrete cracking strength
f_{ts}	Dynamic compressive strength of concrete
f_{td}	Dynamic tensile strength of concrete
f_u	Reinforcement ultimate stress
f_{ud}	Dynamic ultimate stress
f_{us}	Static ultimate stress
f_y	Reinforcement yield stress
f_{yd}	Dynamic yield stresses
f_{ys}	Static yield stresses
$\overline{G}_c$	Secant shear modulus
h	Height of the structure
I	Moment of inertia
I	Impact force (Chapter 5)
$\mathbf{k}$	Stiffness matrix for a single element

$\bar{k}$	Secant stiffness
K	Global stiffness matrix
K	Shear deformation correction coefficient
l	Shear span
L	Beam span
L	Total length of the specimen (Chapter 5)
m	Square mass matrix
m	Mass of the structure (Chapter 3)
m	Mass of the steel weight (Kishi et al. 2002)
$\bar{m}$	Mass per unit length
m_1	Mass of the beam
m_2	Impacting mass
M	Bending moment
M_e	Equivalent mass
N	Finite element shape function
N	Total number of modes
p	Applied force
p	Load vector
P	Axial force (Krauthammer et al. 1993)
P	Impulsive force (Feldman and Siess 1956)
P	Point load at the mid-span (Chapter 5)
q	Uniformly distributed dynamic load transverse to beam length
$q(t)$	Displacement history of the single degree of freedom
$q_n(t)$	Response history of mode n

Q	Shear force
Q	Spring force (Feldman and Siess 1956)
$\mathbf{r}$	Influence coefficient vector
R	Total reaction force
R_1	Spring force
R_2	Resistance of the beam
$R_N(t)$	Support reaction force at the north support
$R_S(t)$	Support reaction force at the south support
R_{ud}	Impact shear capacity
t	Time
T	Natural period
$\mathbf{T}_c$	Transformation matrix for concrete elements
$\mathbf{T}_s$	Transformation matrix for reinforcement elements
$\mathbf{u}$	Displacement vector
$u(x, t)$	Displaced shape at time t
$\dot{\mathbf{u}}$	Velocity vector
$\ddot{\mathbf{u}}$	Acceleration vector
$\ddot{u}$	Acceleration
$\ddot{u}(x, t)$	Acceleration of the specimen at time t
u_1	Displacement of the beam
$\ddot{u}_1$	Acceleration of the beam
u_2	Displacement of the impacting mass
$\ddot{u}_2$	Acceleration of the impacting mass
$\ddot{u}_g(t)$	Ground acceleration

V	Element volume (Chapter 3)
V	Impact velocity (Kishi et al. 2002)
V_{us}	Calculated static shear capacity
V_{usd}	Required static shear capacity
w	Transverse displacement of the mid-plane of the beam
α	Ratio of the inertia forces to the total impact force
α	Ratio of static shear capacity to static bending capacity (Kishi et al. 2002)
β	Rotation of the cross section due to bending
β	Direct integration constant (Chapter 3)
δ_{rd}	Design residual displacement
Δt	Time increment
Δu	Differential displacement
$\dot{\epsilon}$	Strain rate
ϵ	Energy balance error tolerance
ϵ_0	Concrete strain corresponding to peak compressive stress
ϵ_{c1}	Concrete strain in principal direction 1
ϵ_{c2}	Concrete strain in principal direction 2
ϵ_s	Strain in the reinforcement
ϵ_y	Reinforcement yield strain
$\phi(x)$	Assumed displaced shape normalized with respect to the mid-span deflection
ϕ_n	Mode shape
γ	Direct integration constant
γ_{xz}	Shear strain

φ	Curvature
ν	Poisson's ratio
ρ	Material density (Chapter 3)
ρ	Displacement amplitude (Chapter 5)
ρ_m	Material density
ρ_i	Reinforcement ratio
ω	Angular frequency
ξ	Damping ratio
$\psi(x)$	Dimensionless unit displaced shape

1 INTRODUCTION

Since the advent of reinforced concrete structures, the analysis and design of these structures against extreme loads, such as earthquakes, blasts, and impacts, has been an objective of many researchers and designers. Amongst several types of extreme loads, impact loads have enjoyed special attention from the military since as early as the beginning of 20th century, for the design of fortification structures against ballistic weapons. Later, the nuclear power industry joined the effort towards better understanding the behaviour of reinforced concrete structures under impact loads, in order to design the containment structures of nuclear power reactors against accidental impact loads, such as plane crashes. Today, the demand for impact resistant design has a wide spectrum, from protective barriers to rock sheds to bridge piers to industrial facilities. Moreover, as a result of recently elevated terror threat levels in the world, impact resistant design of buildings has become the focus of new attention.

Numerous studies have been conducted to-date toward understanding and developing methodologies predicting the behaviour of reinforced concrete structures under impact loads. Along with analytical and experimental methods, recent developments in the area of nonlinear finite element analysis, accompanied by developments in computer technology, provides analysts and designers working in this area with an invaluable tool. Many finite element programs, from simple to sophisticated, have been developed for impact analysis of reinforced concrete. However, during the course of this development, the modelling of reinforced concrete has always proven to be challenging in many aspects. In particular, the shear behaviour of reinforced concrete structures has been a complex issue in modelling. The lack of rational methods for modelling shear behaviour hinders efforts for accurate prediction of impact behaviour, since severe shear mechanisms may dominate the behaviour of reinforced concrete structures when subjected to impact loads. However, recent progress and newly proposed theories for shear modelling have brought new possibilities of improvements in the area. One of the more successful methods proposed for modelling the shear behaviour of reinforced concrete is the Modified Compression Field Theory (MCFT) by Vecchio and Collins (1986), which was later further developed leading

to the Disturbed Stress Field Model (DSFM) (Vecchio 2000). Both methods have been applied to the static analysis of countless reinforced concrete structures with significant success over the years, although their effectiveness in modelling shear behaviour in high dynamic conditions was left unexplored.

This current study aims to apply these successful methods of static reinforced concrete analysis to the analysis of dynamic loads, and thus, develop an efficient and reliable tool for impact analysis of reinforced concrete. A two-dimensional nonlinear finite element reinforced concrete analysis program called VecTor2, developed previously at the University of Toronto for static loads, was modified to include the consideration of dynamic loads, including impacts. VecTor2 uses the Modified Compression Field Theory for its computational methodology, along with a wide array of material and behavioural models for reinforced concrete. To verify the performance of VecTor2 and its computational methodology for impact loads, an experimental program was also undertaken to provide data for corroboration. This test program in itself was another important focus in this study, aimed at providing the literature with comprehensive, well-instrumented test data, which could further be utilized in future studies for the development of new constitutive models and impact analysis methods.

Chapter 2 in this report provides a literature review, presenting the historical progress and the current state of knowledge on the area of impact analysis and design of reinforced concrete structures. Chapter 3 presents the computational methodology of VecTor2, including the details of the newly adapted dynamic analysis algorithms. The experimental program conducted is described in Chapter 4, and test results are discussed in Chapter 5. In Chapter 6, the test specimens are analyzed with VecTor2 and computed results are compared to the test results. Chapter 7 discusses the final conclusions of this study.

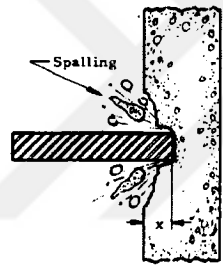
2 LITERATURE REVIEW

2.1 Introduction

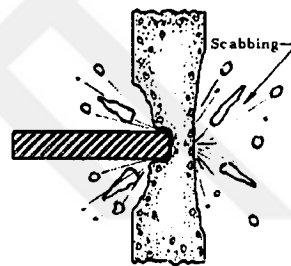
Impact phenomena, the striking of one body - a missile - against another in the broad sense, have long been of interest to many researchers. The earliest studies dealt with the impact problem as a missile striking and penetrating into a semi-infinite medium like earth. These studies, such as by Robins-Euler in 1742, Poncelet in 1830, and Resel in 1895, tried to predict the depth of penetration of the missile as a function of the mass, velocity, and resistance characteristics of the semi-infinite medium (Corbett et al. 1996). However, with the advent of reinforced concrete structures and the emerging need to design such structures against impact loads, this type of approach has proved to be insufficient.

The first contemporary studies on the subject were carried out in the military sector during and after the Second World War. A great amount of experimental research was done for the fortification of reinforced concrete structures against ballistic weapons. However, it was the needs of the nuclear reactor industry which made the research on the impact problem a major and popular subject. Due to the unacceptable consequences of a failure in the containment structures of nuclear reactors, these structures needed to be designed against accidental impact loads, such as aircraft crashes, impacts of tornado generated missiles (automobiles, steel rods, pipes, wooden poles, etc.), and internal accidents resulting from pipe or turbine breaks. As a result, the research in this area not only increased, but also became more systematic. The nuclear power industry was the major driving force of the great majority of research done in respect to the impact problem until the late 1980s. Although the number of new nuclear power plant constructions has declined significantly since then, the methodology developed and the knowledge accumulated have been the foundation of research on the problem that followed. Today, research in the impact response of reinforced concrete elements covers a very wide area of interest: rock shed design for highways, design against accidental loadings in industrial facilities, protective barrier design, and design of sea structures against ice and ship collisions, to name a few.

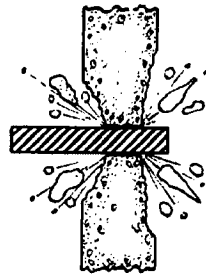
Missile impacts are generally classified as hard and soft impact. Hard impact is the case in which the missile undergoes almost no deformation compared to the impacted structure, whereas in soft impact, the missile itself also deforms significantly. Depending on the nature of the impact, the impacted structure may respond in several ways: 1) it may suffer local damage only, dissipating the majority of the impact energy at or around the impact zone; 2) it may respond to the impact globally through the bending and deformation of the entire reinforced concrete member; 3) it may respond in such a way that it suffers a combination of both local and global damage. Local damage is usually categorized in three levels: 1) penetration of the missile in the front face and scabbing of small pieces of concrete at the back face; 2) significant scabbing of the concrete at both the front and back faces; 3) perforation of the element, with the missile exiting from the back face with a residual velocity (Figure 2.1).



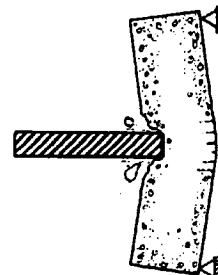
(a) Missile penetration and spalling



(b) Target scabbing



(c) Perforation



(d) Overall target response

Figure 2.1 Missile impact phenomena (Kennedy 1976)

Military studies on the design of fortification structures mainly concentrated on high velocity (150- 1000 m/s) hard impacts that cause extensive local damage without any significant global response. The main objective of these early studies was to develop formulations for determining the required thickness of a reinforced concrete slab, wall or shell to keep the local damage resulting from a design impact at a desired level. These studies were mostly experimental and the formulae developed had very little theoretical basis. However, when the need for civil applications emerged, this approach exhibited a major deficiency since all empirical formulae were limited to the range of available test data which were typically well outside current needs (Kennedy 1976). Therefore, although countless impact tests have been carried out to-date to cover the possible range of impact loads in civil structures, researchers' main objective has been to build a theory for the impact phenomenon to overcome the limitations of the experimental studies. Several theoretical studies were done to-date, but the major breakthrough in impact studies has been the application of numerical methods, such as the finite element method, to analyze the impact loads under various conditions without the need of expensive and time consuming experimental investigations. Numerical methods not only provided valuable tools for the analysis of local damage, but they also enabled the determination of the global response of the structures under impact loads.

In this chapter, an overview of the research on impact of missiles on reinforced concrete structures is provided. The following section briefly presents the history of the research on the local response of reinforced concrete elements. In Section 2.3, the research on the global response of reinforced concrete structures is presented, with particular emphasis on numerical methods used in the literature to-date. The current state of knowledge and the particular contribution of the current study is discussed in Section 2.4. Research on the use of special materials, such as high strength concrete or fibre reinforced concrete for increasing the impact resistance, is omitted in this review for brevity.

2.2 Local Response of Reinforced Concrete Structures

As mentioned earlier, the majority of the research on the local damage of reinforced concrete elements due to impact loads is experimental. Numerous experiments reported in

the literature investigated the effects of various parameters during an impact, such as mass, velocity, shape and deformability of the missile, angle of impact, strength of concrete, and amount of reinforcement, on the local damage level of reinforced concrete slabs, walls and shells. Many studies suggested formulae for the penetration depth of the missile, and minimum thicknesses to prevent scabbing and perforation under impact loading.

Prior to the Second World War, Petry's formulae, first published in 1910 and later modified twice to include effects of concrete strength, were among the most commonly used formulae for the design of reinforced concrete slabs against impact loads. Later, with the contribution of military studies, several other formulae were developed in the area, including the Army Corps of Engineers (ACE) formula, the National Defence Research Committee (NDRC) formula, the Amman & Whitney formula, and the Ballistic Research Laboratory (BRL) formula. A detailed review of these and other formulae and their applications were given by Kennedy (1976) and Corbett et al. (1996). Among these, the NDRC formula was found to be most suitable for nuclear power facility design until the 1970s, since it was based on a physical theory for impact phenomenon and, therefore, could be extrapolated beyond the range of available military test data.

In the 1970s, extensive experimental programs were initiated by the nuclear power industry to determine criteria for dimensioning the containment structures of nuclear power plants against hard impact loads. Several semi-empirical formulae were developed as a result of these studies, among which those of Degen (1980), Chang (1981), Hughes (1984) and Haldar (1982, 1985) have been widely used. There also have been numerous experimental studies to investigate the effects of various parameters, such as reinforcement ratio, use of metallic plates, repeated impacts, soft missiles, fiber-reinforcement, and high strength concrete.

With the development of computers, numerical methods, such as the finite element method (FEM) and the discrete element method (DEM), were also employed by researchers for local damage analysis of reinforced concrete members.

One of the pioneering studies using FEM in this area was done by Rebora et al. (1976). In their study, they proposed a three-dimensional 20-node isoparametric concrete element for modelling concrete, and a one-dimensional 3-node bar element or alternatively a two-dimensional membrane element for modelling steel. For concrete under compression, Saugy's constitutive law (Saugy 1969) was employed, which considers strain rate effects. The failure criteria developed by Zimmerman et al. (1975) was selected for concrete. In their study, they modelled a hypothetical impact of an aircraft on the shield building (outer concrete shell) of a nuclear power plant (Figure 2.2). The impact force was applied as a force-time history on a surface. They obtained predictions of the crack distribution and overall behaviour of the shell, but they also reported some numerical convergence problems due to the extreme local deformations in the vicinity of the load. The study did not make any comparison with test data.

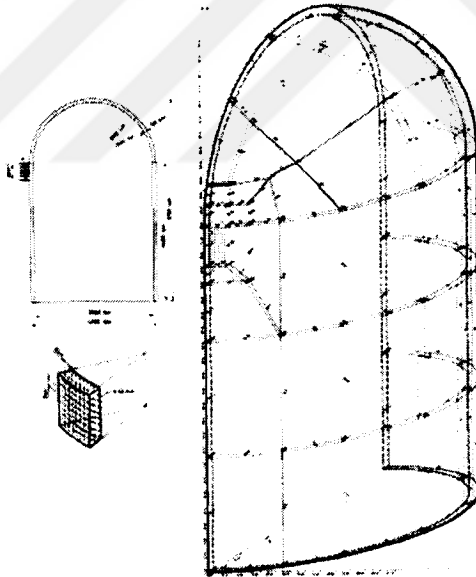


Figure 2.2 Concrete shell (Rebora et al. 1976)

Attalla and Nowotny (1976) used a two-dimensional time-dependent finite differences Lagrange code to model a spherical reinforced concrete cap impacted by a cylindrical missile with a cone head (Figure 2.3). The missile was considered to be rigid compared to the structure (hard impact) and the structure was clamped at the ends, assuming that the effects of the impact would remain local. They proposed a constitutive relation for concrete

in the form of pressure versus density (hydrostat). No test comparisons were provided, but the penetration depth obtained from the analysis was compared with the BRL formula and good agreement was obtained. The analysis also gave the impact force-time history, which the authors suggested be used as a load-time history in a conventional dynamic FEM analysis.

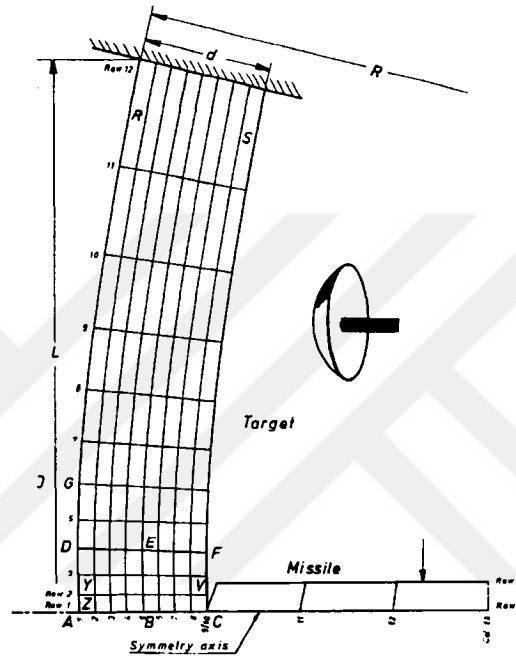


Fig. 1. Lagrange grid for impact calculation.

Figure 2.3 Lagrange grid for impact calculation (Attalla and Nowotny 1976)

Gupta and Seaman (1978) carried out impact experiments on small size reinforced concrete walls and developed a constitutive relationship for concrete under compaction, commonly referred to as the CAP model. They defined two alternative yield surfaces using static and dynamic data and carried out two-dimensional finite difference analyses to model their test specimens (Figure 2.4). The study concluded that the analyses using the yield surface defined by static parameters predicted the damage pattern better compared to the one using dynamic parameters. On the other hand, missile penetration depths obtained using dynamic parameters were found to be more accurate. Later, Adamik and Matejovic (1989) modified the CAP model to include strain rate effects in concrete under tension and obtained better results.

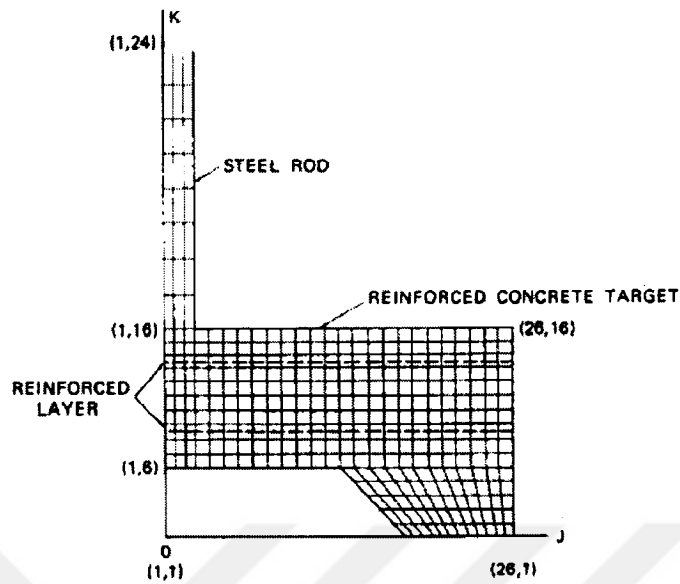


Figure 2.4 Layout for the two-dimensional computational simulation by Gupta and Seaman (1978)

Brown et al. (1979) used a two-dimensional finite difference program to model local failure under impact. They proposed a constitutive model for the dynamic behaviour of concrete in which concrete was assumed to be isotropic and linear elastic up to failure. The authors acknowledged the fact that this was not correct, but they justified the assumption arguing that failure in tension will dominate in many impact problems. However, after comparing the analysis results with impact test data on reinforced concrete panels, they found that the crack pattern predictions were inaccurate. They argued that this discrepancy might be due to lack of plasticity in concrete prior to failure.

Later, with the advancement of computers and numerical methods, much more sophisticated and precise tools were developed to model the penetration of a missile and fracturing in concrete. Thoma and Vinckier (1994), for instance, used a three-dimensional nonlinear finite element code to model the high velocity penetration process with a continuous erosion of the impacting metallic fragment (Figure 2.5). They included a contact algorithm for modelling the contact forces between the missile and the concrete, and proposed a constitutive model for fiber-reinforced concrete under high strain rate loading,

capable of describing the failure and the post-failure behaviour up to complete material crushing.

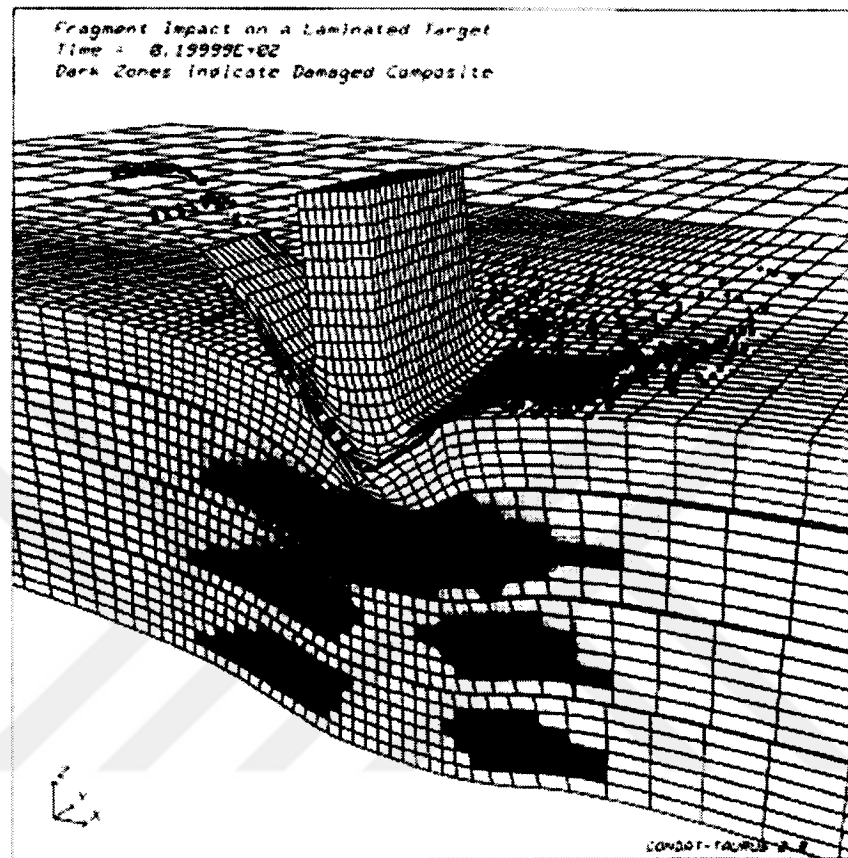


Figure 2.5 Fragment and target condition 20 μ s after impact (Thoma and Vinckier 1994)

Agardh and Laine (1999) used another three-dimensional nonlinear finite element code with a concrete model that accounted for erosion during impact. They modelled the penetration of a projectile into a 60 mm thick reinforced concrete slab (Figure 2.6). Teng et al. (2004) adopted the finite element method by implementing the equivalent inclusion method and considering the reinforced concrete as a homogeneous material, thus simplifying the finite element meshes. They successfully modelled the impact of an ogive-nose projectile on a reinforced concrete slab. They also carried out a numerical parametric analysis of a 50° oblique impact (Figure 2.7).

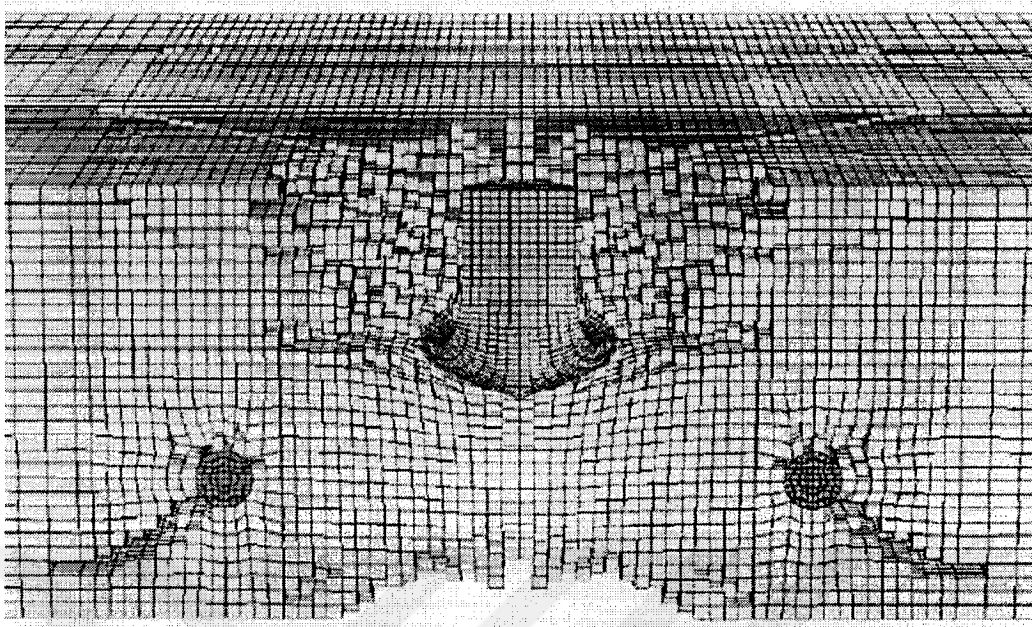


Figure 2.6 Penetration of a projectile into concrete (Agardh and Laine 1999)

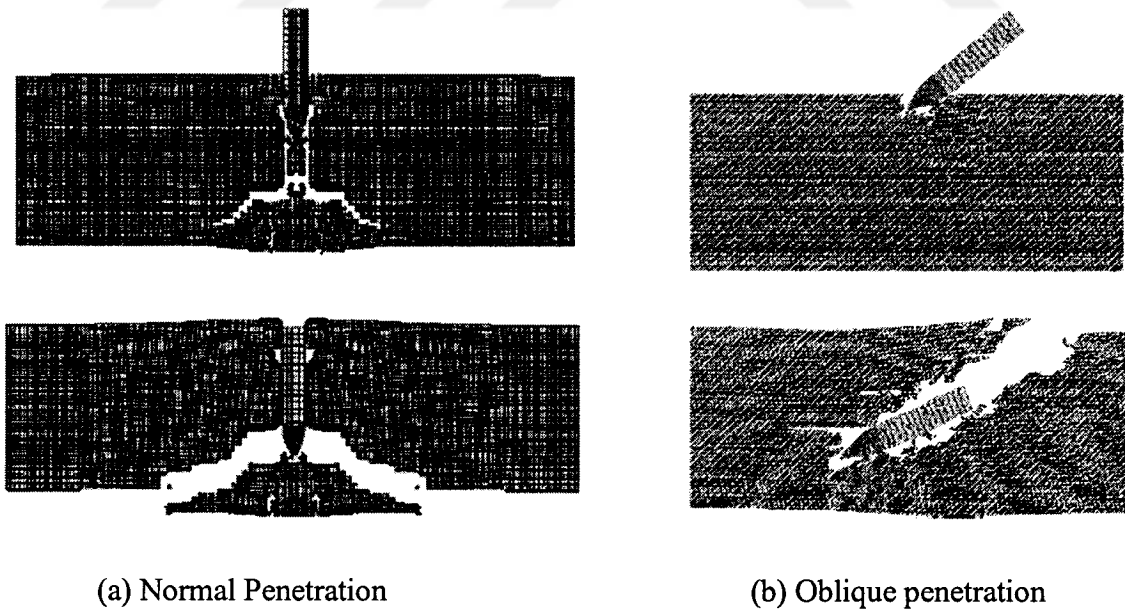


Figure 2.7 Penetration of a projectile into a reinforced concrete slab (Teng et al. 2004)

Use of finite element and finite difference methods in high-deformation and penetration analyses have been criticized by many researchers because of their limitations. These methods involved the solution of partial differential equations of continuum physics and, as a result, the fragmentation that may occur could not be considered. Furthermore, the fracturing process was simplified in these methods. Therefore, alternative methods were developed considering discontinuities in the medium. The discrete element method (DEM) was one of these methods.

For example, Sawamoto et al. (1998) employed the discrete element method to model local damage under impact. In their model, they idealized the reinforced concrete medium as an assemblage of rigid circular elements connected to each other by nonlinear springs and dashpots (Figure 2.8). They proposed constitutive relations and failure criteria representative of concrete for these springs, and modelled a full-scale reinforced concrete panel impact test, successfully predicting the damage (Figure 2.9).

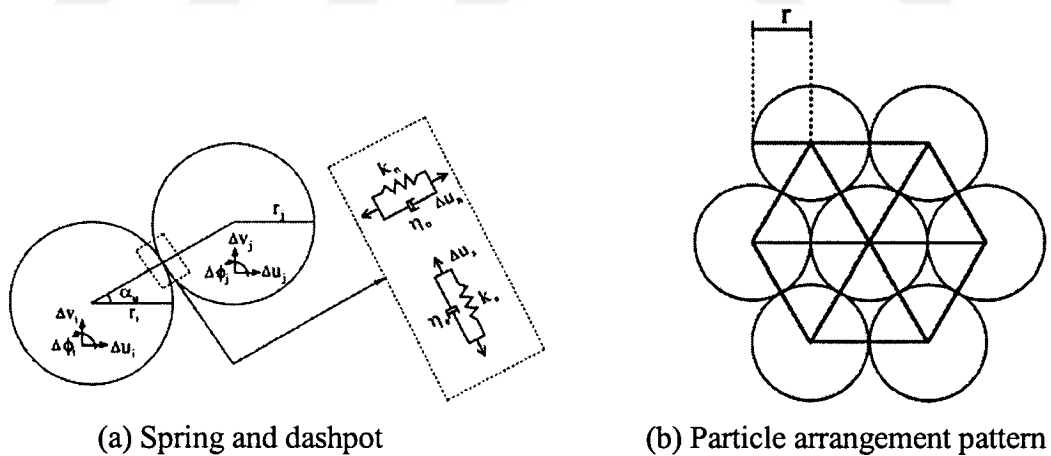


Figure 2.8 DEM model (Sawamoto et al. 1998)

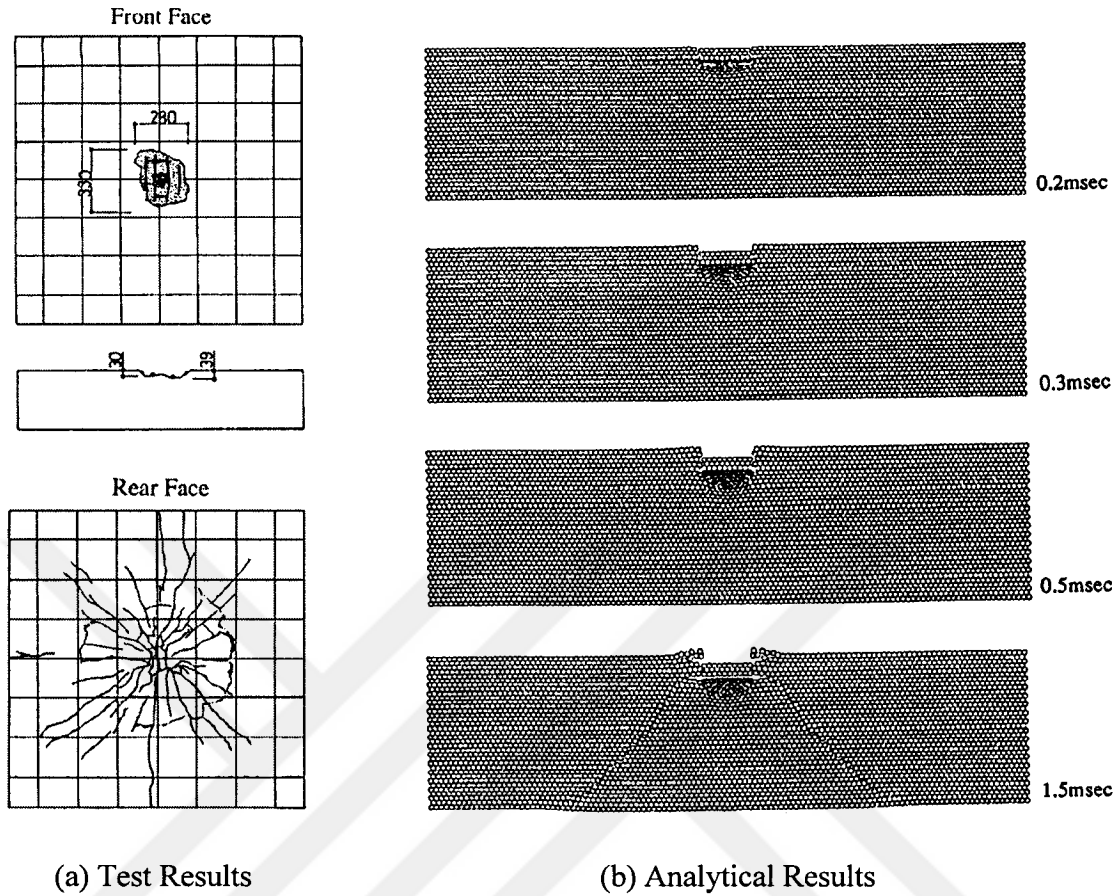


Figure 2.9 Damage modes of panels (Sawamoto et al. 1998)

Riera and Iturrioz (1998) used a model based on the representation of a concrete medium by means of an array of nodal masses interconnected by unidimensional massless elements (Figure 2.10). For these elements, they developed new constitutive relations for concrete to account for fracture energy dissipation as well as residual elastic energy recovery during unloading.

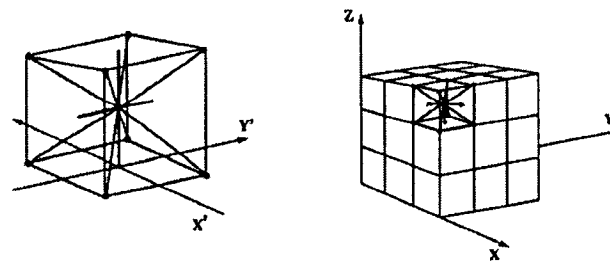


Figure 2.10 Basic cube model and composition of prisms (Riera and Iturrioz 1998)

Magnier and Donzé (1998) used the discrete element method with two-dimensional circular elements to model the impact of rigid spherical-nose missiles on plain and reinforced concrete beams (Figure 2.11). They compared their numerical results with several different empirical prediction formulae and found good agreement.

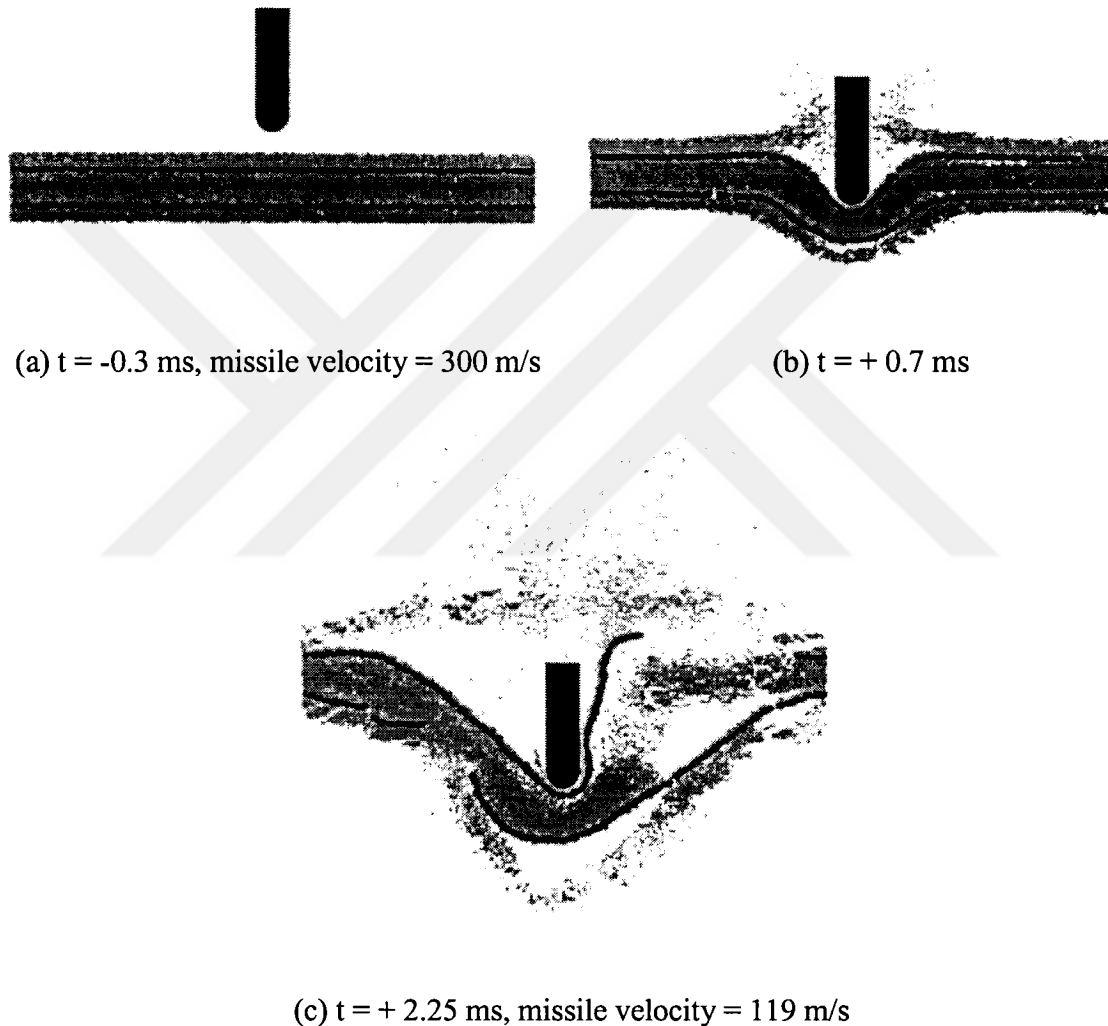


Figure 2.11 Perforation of a reinforced concrete beam (black lines represent steel bars)
(Magnier and Donzé 1998)

Today, although advanced numerical methods have proven to be successful in the analysis of local damage under impact loading, due to their complexity and cost, empirical formulae are still widely preferred over numerical methods. A significant amount of research is still going on to develop analytical formulae for various impact and material conditions.

2.3 Global Response of Reinforced Concrete Structures

Research in the literature on the global response of the concrete structures against impact loads is significantly less in volume compared to the research in local response. This is due to several reasons. Mainly, for a long time, the nuclear reactor industry and the military were the only clients of impact research, and the types of impacts they were interested in were mainly high-velocity hard impacts which caused extensive local damage, leaving very little energy for the structure to dissipate by global deformation. Moreover, impact loads on nuclear reactors or military structures were very low-probability accidental loads; therefore a certain level of damage, such as penetration and back-face scabbing, was anticipated for their design, which again does not leave the structure any significant energy to dissipate. As a result, many researchers believed, and in some cases proved, that the global response of the structure is insignificant and can simply be ignored (Halдар 1985). However, with the growing concern over impulsive blast loads on structures, particularly due to atomic bombs in the pre- and post-World War II era, research on the global response of structures against sudden loads such as impulse and impact loads has increased. This section gives a brief summary of the development of research on the global response of reinforced concrete structures under impact loads. It should be noted that the majority of the research on global response was carried out on reinforced concrete beams, since local damage becomes much more important in the case of thin slabs under impacts. Therefore, studies involving thin slabs are omitted in this summary for brevity.

The earliest studies on global response were mainly qualitative in nature, and were primarily interested in finding the effects of various parameters on impact resistance rather than attempting to explain the overall behaviour. In particular, the effects of the grade of steel bars, regular strength ductile structural steel versus higher strength brittle rail steel, were debated for a long time (Mavis and Richards 1955, Mavis and Greaves 1957). One of

the very first test programs was carried out by Mylrea (1940), in which he tested 10 x 16 inch reinforced concrete beams with 8 feet spans under falling weights. The beams contained different amounts and grades of longitudinal reinforcement and no shear reinforcement. Although the beams were severely damaged with diagonal shear cracks, he concluded that the impact resistance of reinforced concrete beams were quite significant in the presence of even small amounts of reinforcement, since he was not able to rupture any of the longitudinal reinforcement, which defined his failure criteria (Figure 2.12). Anything short of a total collapse was adequate for the industry at the time.

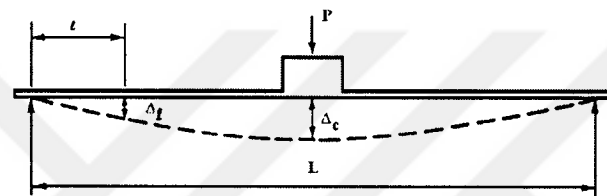


Figure 2.12 Specimens after the tests (Mylrea 1940)

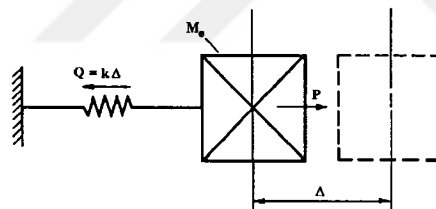
However, as the demand for impact resistant structures increased and the acceptable level of damage in those structures decreased, more comprehensive research emerged in the area. Feldman et al. (1956, 1962) carried out an extensive experimental program to investigate the behaviour of reinforced concrete beams under impulse loading. They tested 43 beams in total; ten were tested under loads applied at the mid-span, and 33 were tested under two-point loading. The beams were 6 x 12 inch in cross section with 9 foot or 12 foot 8 inch spans. The amounts of longitudinal and transverse reinforcement were varied between the test specimens, whereas the concrete strength and yield strength of the reinforcement were

kept constant. Loads were applied to the specimens by a pneumatic loading device which consisted of a piston that loaded the specimen through the sudden release of pressurized gasses in a chamber (explosion or implosion).

For the analytical part of their study, Feldman et al. assumed that the beams vibrated in some definite deflection configuration and only a single coordinate was necessary to define the deflected shape at any given time. This assumption, supported by their experimental observations, reduced the system into a single degree of freedom (SDOF) system as shown in Figure 2.13.



(a) Original Beam



(b) Equivalent SDOF system with no damping

Figure 2.13 Schematic representation of a beam as SDOF system
(Feldman and Siess 1956)

As seen in Figure 2.13, Feldman and Siess surmised that when the impulsive force P was applied to the mass, the motion of the system could be described by Eq. 2.1, ignoring damping:

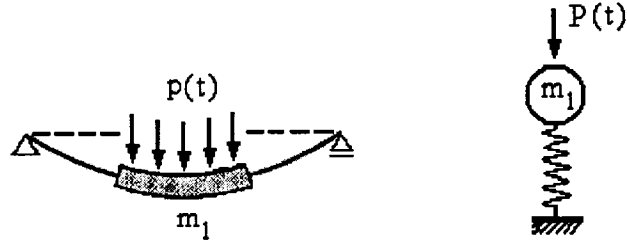
$$M_e \cdot a = P - Q \quad (2.1)$$

where M_e was the equivalent mass that possessed the same kinetic energy as the original system during the vibration, a was the acceleration of the equivalent mass, and Q was the spring force. One of the main objectives of their study was to determine the relationship between the spring force and the deflection in this equivalent single degree of freedom system, which they called the dynamic resistance-deflection relationship.

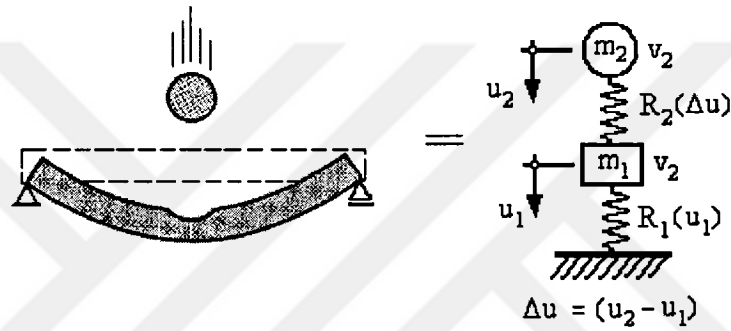
Feldman et al. used two different approaches to achieve this objective. In the first approach, since they could either measure or derive all quantities in Eq. 2.1, they solved the equation for Q and determined the dynamic resistance-deflection characteristics by using the calculated Q and measured deflection. The second approach, on the other hand, involved an iterative procedure. In this approach, they first assumed a bilinear dynamic resistance-deflection relationship and numerically calculated the response according to this assumed relationship. Then, they compared the actual response with the calculated one. If they matched, the assumed relationship was correct. If not, they repeated the analyses with a modified relationship until adequate agreement was achieved. Although it was somewhat impractical, they preferred the second approach over the first to strengthen their belief that such a bilinear function exists for every beam even though its determination may be difficult and time consuming. They proved this point by carrying out numerous analyses for the tested specimens and successfully finding a suitable bilinear function that gave good agreement with the test results.

Although their solution techniques and assumptions were somewhat diversified later, Feldman et al.'s approach of idealizing the beam as a single degree of freedom system was widely accepted for impact analysis and later was recommended by the Comité Euro-International du Béton¹ (CEB) (1988). In their 1988 report, CEB divided the impact problem into soft and hard impact cases and suggested a single spring and a two-spring system for their solution, respectively (Figure 2.14).

¹ This organisation merged with the Fédération Internationale de la Précontrainte (FIP) in 1998 to create a new organisation called the Federation Internationale du Béton (fib).



(a) Single mass model for soft impact



(b) Two mass model for hard impact

Figure 2.14 Spring models for impact (CEB 1988)

Here, mass m_1 represents the mass of the beam, m_2 is the impacting mass, and the spring force R_1 and R_2 represent the stiffness of the beam and the local contact resistance, respectively. In this method, the “participating mass” m_1 of the beam is calculated according to an assumed displaced shape and energy considerations, as mentioned in Feldman and Siess (1956), with the following equation:

$$m_1 = \int_0^L \bar{m} \phi^2(x) dx \quad (2.2)$$

where L is the span, $\bar{m}$ is the mass per unit length and $\phi(x)$ is the assumed displaced shape normalized with respect to the mid-span deflection.

For the soft impact case, the force-time history $p(t)$ is calculated with the dynamics of the crushing body, and the subsequent SDOF analysis is rather simple. For the hard impact case, on the other hand, following equations of equilibrium can be written.

$$m_2 \ddot{u}_2 + R_2 = 0 \quad (2.3)$$

$$m_1 \ddot{u}_1 + R_1 - R_2 = 0 \quad (2.4)$$

where

$$R_1 = R_1(u_1) ; R_2 = R_2(\Delta u_2) ; \Delta u_2 = u_2 - u_1 \quad (2.5)$$

If $R_1(u_1)$ and $R_2(\Delta u_2)$ are known, Eqs. 2.3 and 2.4 can easily be adopted into a time-stepping finite difference scheme and displacement and force time histories can be obtained. However, determining the force-deformation relationships for $R_1(u_1)$ and $R_2(\Delta u_2)$ usually requires a thorough investigation.

Some researchers developed analytical dynamic force-deformation models to determine $R_1(u_1)$ for different types of structural elements under different types of loads. Krauthammer et al. (1990), for example, defined a piecewise multi-linear dynamic resistance function, complete with concrete hysteresis rules, effects of loading rate and shear forces. However, their SDOF model was intended for the case of impulsive forces, where only one mass exists and the load history is calculated or known from the test data, eliminating the need for a second impacting mass. However, for impact loads, either the impact force history needs to be measured or estimated in the manner of an impulse force, or the impacting mass and the spring representing the local contact resistance should be included in the analysis as Eqs. 2.3 and 2.4 suggest. When included, determining the force-deformation relationship for the contact spring (Figure 2.15) is particularly challenging. This relationship would be affected by several factors, such as the surface roughness of both the impacted surface and the impacting mass, friction conditions, shape and incidence angle of the impacting mass, and the material properties. Moreover, the constitutive relations for materials under extremely high hydrostatic compression are difficult to

determine. Nevertheless, there are studies which tested and verified this two-spring approach. Bischoff et al. (1990) conducted simple static penetration tests to determine the force-deformation relationship in the contact zone and then used this information to predict the contact force for drop tests performed on solid polystyrene aggregate concrete blocks. Their model was essentially a one-spring idealization since the structure under consideration was a relatively rigid block, but they were able to predict the contact force with reasonable accuracy for low velocity (< 10 m/s) impacts.

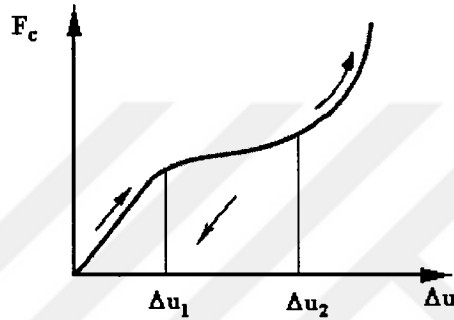


Figure 2.15 Typical force-deformation relationship of contact zone, $R_2(\Delta u)$ (CEB 1988)

Along with simple SDOF approaches, there have also been more comprehensive analytical methods developed. Krauthammer et al. (1993) developed an analytical and numerical approach for the solution of the response of reinforced concrete beams under uniformly distributed impulsive loads. The concept they applied was based on Timoshenko beam theory, which relates the bending moment M and the shear force Q at a given section to the transverse displacement w of the mid-plane of the beam and the rotation β of the cross section due to bending. The governing equations are as follows.

$$\frac{\partial M}{\partial x} - Q = -\rho_m I \frac{\partial^2 \beta}{\partial t^2} \quad (2.6)$$

$$\frac{\partial Q}{\partial x} + q + P \frac{\partial \beta}{\partial x} = \rho_m A \frac{\partial^2 w}{\partial t^2} \quad (2.7)$$

where I is the moment of inertia, ρ_m is the material density, P is the axial force in the member acting at the plastic centroid of the cross section and q is the uniformly distributed dynamic load transverse to beam length. The bending moment M and shear force Q can

further be expressed in terms of the curvature ϕ and shear strain γ_{xz} of the cross section as follows.

$$M = f_2(\phi) \quad (2.8)$$

$$Q = (KA)f_1(\gamma_{xz}) \quad (2.9)$$

where K is the shear deformation correction coefficient and A is the cross-sectional area. In their study, Krauthammer et al. derived analytical relationships for Eqs. 2.8 and 2.9, by using their previous study (Krauthammer et al. 1987) and the compression field theory (Mitchell and Collins 1974), respectively. A set of hysteresis rules similar to the one used in Krauthammer et al. (1990) were also implemented in these equations to take loading-unloading and load reversals into account. They then used the finite difference method for evaluating the time and space derivatives in Eqs. 2.6 and 2.7. In this method, the beam was discretized longitudinally into a finite difference mesh with nodes at equal distances, and time was discretized into equal length time-steps. The entire response-history was obtained numerically through a finite difference time-stepping procedure.

It should be noted that analytical methods similar to the ones mentioned here have some severe limitations in their application. First, as many of them were developed primarily for impulsive loads in which the load-time history was assumed, their implementation to impact loads require an estimation of the time-history of the force exerted on the structure by the impact. Even when the method is developed for impact loads as in CEB (1988), it is still a challenge to model the impact weight and obtain the impact force-time history. Moreover, many of these analytical methods have restrictions on the types of the problems that can be solved. For example, SDOF methods are suitable only for simple structures that have a predominant deflection pattern that can be simplified as an SDOF system, and are not applicable to more complex structures. Loading pattern is also another concern with SDOF methods, as different types of loading patterns, such as an asymmetrical impulsive load, may trigger two or more deflection modes within even a simple structure. Similarly, the methods that are based on continuous governing equations of motion, such as in Krauthammer et al. (1993), are completely restricted to a certain type of loading, and the

whole set of equations have to be re-derived for different types of loading patterns. In order to avoid these restrictions, numerical methods have been preferred over analytical methods, since with methods such as the FEM, it is possible to analyze a much wider range of structures under different types of impact and impulse loads.

Modelling of the impact force still remains a problem even with more general numerical methods. Some researchers have tried to develop relations to estimate the impact forces for a given type of impact. One of the first studies on this issue was carried out by Riera (1968). In his study, he defined the impact force generated by an aircraft collision on a rigid surface, which then could be applied as an impulse load within any analytical or numerical method. Some other studies tried to implement the estimation of the impact force into their numerical analysis. For example, Miyamoto et al. (1991) developed a FEM program for the analysis of reinforced concrete slabs subjected to impulsive loads. They carried out impact tests for the verification of their program, and for the analysis component, the impact load measured during the test was used as an impulsive load input. Later, they adopted an analytical procedure to overcome the need for a known loading function for the analysis of impact loads (Miyamoto et al. 1994). They developed a multi-mass model, which consisted of lumped masses connected with nonlinear axial and rotational springs, to simulate the rigidity and the eccentricity of the impacting body (Figure 2.16). This model was then linked to the finite element program through a procedure (Figure 2.17), which was essentially a back and forth process between the analysis of the impacted structure and the impacting body. This kind of approach is suitable for soft impacts and with impacting bodies that are relatively simple to model with a few lumped masses.

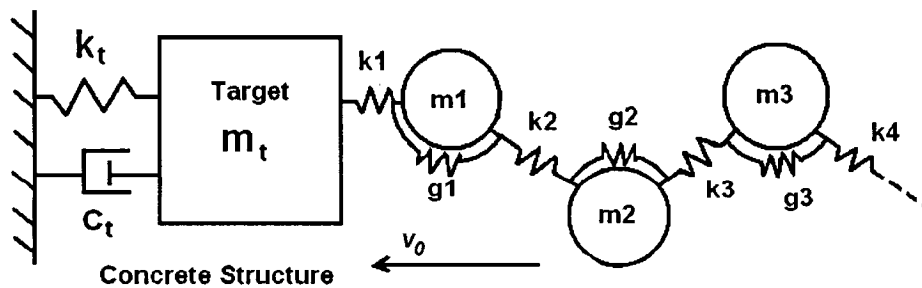
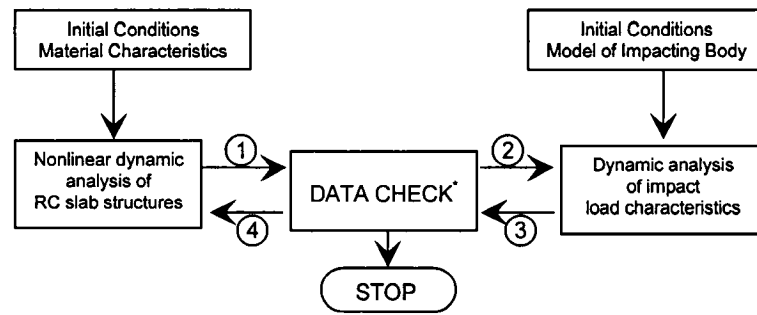


Figure 2.16 Multi-mass model for soft-impact collision (Miyamoto et al. 1994)



* Failure criteria (structural failure) or end of impact collision

- ① Displacement, velocity, acceleration, strain, stress, slab status, effective mass, effective stiffness
- ② Displacement, velocity, acceleration, effective mass, effective stiffness
- ③ Displacement, velocity, acceleration, impact force
- ④ Impact force

Figure 2.17 Linking (coupling) procedure for analysis of soft-impact collision
(Miyamoto et al. 1994)

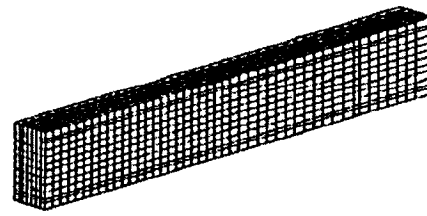
There are few widely used commercial package programs available that are sufficiently comprehensive to model impact problems. These programs mainly differ in their methods for mesh description (Lagrangian, Eulerian, coupled Lagrangian and Euler, arbitrary Lagrangian and Euler, or smooth particle hydrodynamics), time-integration (implicit or explicit), and application area (solids or fluids). A detailed discussion and summary of different computational techniques employed in the area can be found in Hamouda and Hashmi (1996). Time-integration methods will be discussed in detail in Chapter 3.

One of the most popular commercial FEM programs used for high-deformation impact modelling is LS-DYNA, formerly known as DYNA3D. It is a three-dimensional nonlinear FEM program capable of modelling various contact problems including flexible body contact, rigid body contact, edge contact and eroding contact. The program uses various mesh description methods depending on the problem type, and has a comprehensive material library which can be extended by user-defined materials. An explicit time-integration method is implemented into the program. LS-DYNA was initially developed for military purposes; therefore, it has advanced features for modelling missile penetration,

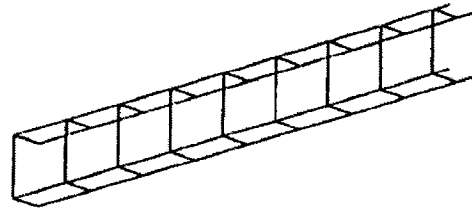
blast response and explosives. As an example of its application to the modelling of missile penetration, the aforementioned study presented in Agardh and Laine (1999) was carried out with LS-DYNA.

Since user-defined material models can easily be introduced into the program, many researchers have used LS-DYNA as a base code to implement their material models. For example, Shirai et al. (1994) implemented a new constitutive relationship and failure criteria for concrete, including strain rate effects, into LS-DYNA and carried out FEM analyses for reinforced concrete beams previously tested. The beams were 2000 mm in span with a square 200 x 200 mm cross section, and they were subjected to the impact of a 70 kg flat-nose hard-steel cylinder with an impact velocity of between 4.6 m/s and 8 m/s. In their FEM model, they modelled only one-quarter of the beam using symmetry. Concrete was modelled using solid elements, whereas reinforcement bars were modelled with beam elements. They idealized the missile as an octagonal column and used slide joints and void elements at the contact zone to simulate the rebound of the missile (Figure 2.18). They compared the strain measurements obtained from the tests to that obtained from the analyses and found reasonable agreement.

A similar study was carried out by Kishi et al. (2001). In their study, using LS-DYNA, they modelled shear-critical beams that they previously tested. In their FEM model, they used simple bi-linear elasto-plastic constitutive relations for concrete and steel, and they modelled the entire test setup including the steel drop-weight and the support system (Figure 2.19). Comparisons of the results obtained from the tests and the FEM models, including displacements, reaction forces and impact forces were reasonably satisfactory.



(a) Concrete

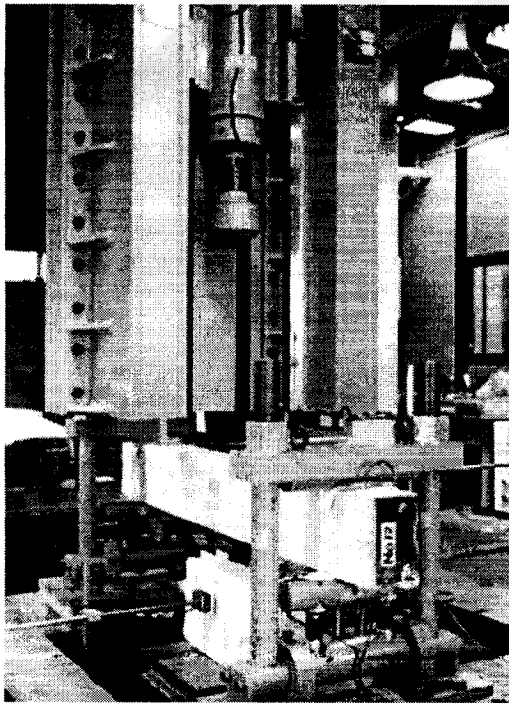


(b) Reinforcement

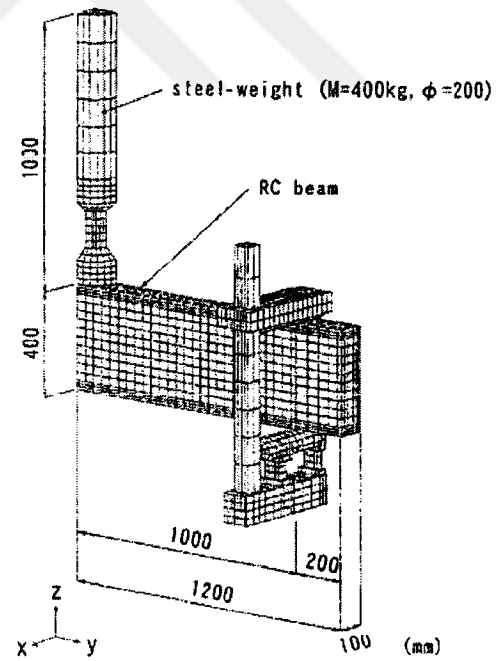


(c) Missile

Figure 2.18 FEM model (Shirai et al. 1994)



(a) Experimental setup



(b) FEM Model

Figure 2.19 Test setup and the FEM model of the RC beams (Kishi et al. 2001)

ABAQUS is another program that is frequently used for impact modelling. The program has several versions available for different type of problems. The version developed for fast dynamic problems is called ABAQUS/Explicit. As the name implies, it uses explicit time-integration procedures for faster solutions, and it supports transient dynamic temperature-displacement, acoustic and coupled acoustic-structural analyses. Similar to LS-DYNA, it is capable of utilizing different types of mesh descriptions according to the problem.

AUTODYN is also a popular package program used for impact analysis. It can use multiple meshing methods at the same time and can switch between the meshes. For instance, a Lagrange-modelled portion of a structure that is starting to severely distort can be remapped into an Euler space, enabling the remainder of the problem to be run using an Euler solution technique. Fluid-structure interactions can be modelled as well. For example, Balden et al. (2005) modelled a steel hull structure subjected to an underwater blast using AUTODYN (Figure 2.20). In their model, water, air and explosive were modelled with Euler meshes, whereas the steel hull structure was modelled with Lagrange shell elements. In their study, they were able to model the load generated by the underwater explosive detonation, and simulate the movement of the structure into the air and the tearing of the steel shell.

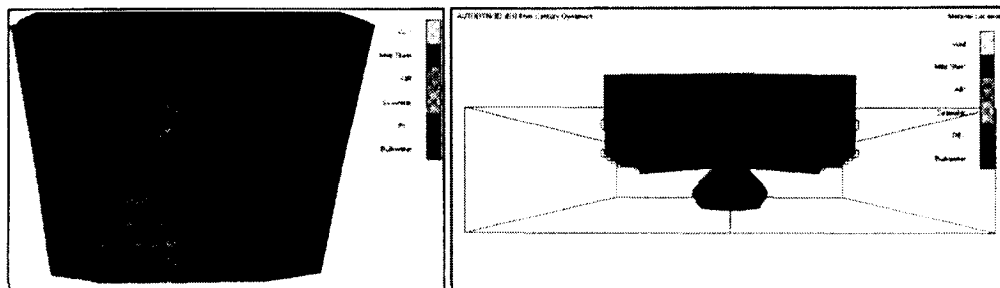


Figure 2.20 AUTODYN model of a steel hull structure (Balden et al. 2005)

It should be noted that, although these package programs offer a wide variety of advanced modelling techniques, they are general purpose analysis programs. In other words, reinforced concrete structures constitute only one of many applications of the technique used. On the other hand, reinforced concrete is a complex material, which may not always be amenable to general modelling techniques. For this reason, numerous theories and methodologies were specifically developed for modelling of reinforced concrete structures

over the years. In particular, modelling the shear behaviour of reinforced concrete structures proved to be challenging, and for a long time heavily depended on purely empirical studies. Only recently have rational theories on the shear behaviour of reinforced concrete been proposed (e.g. Vecchio and Collins 1986, Vecchio 2000). A precise modelling of shear behaviour is particularly important for modelling the impact on reinforced concrete structures, since, evidenced by numerous impact or impulse load tests on reinforced concrete beams and slabs, the high dynamic conditions of an impact may cause shear failure, even if the structure is flexural-critical under static conditions.

Although there has been significant progress in understanding the shear behaviour of reinforced concrete under static loads, limited research has been done on investigating shear mechanisms under high dynamic conditions. Existing research on the subject are mostly empirical studies. In one of the most recent and comprehensive studies in this area, Kishi et al. (2002) carried out a test program to establish a rational impact-resistant design procedure of shear-critical reinforced concrete beams. They tested 27 simply supported rectangular RC beams without shear reinforcement. The cross sectional dimensions of the beams were kept constant at 150 x 250 mm, whereas the ratio of static shear capacity to static bending capacity α , span length L , and main rebar diameter were varied (Figure 2.21). The average concrete compressive strength was 33 MPa, and the reinforcement yield strength was 393 MPa.

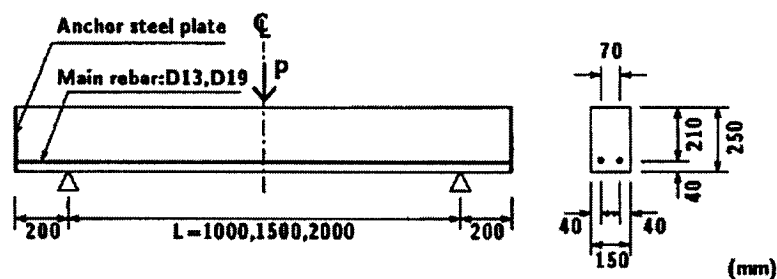


Figure 2.21 Dimensions of reinforced concrete beam (Kishi et al. 2002)

Impact load was applied to the mid-span of the beams by dropping a free-falling 300 kg steel weight. The impact velocity of the weight varied between 1 m/s, 3 m/s, 4 m/s and 5 m/s. During the test, the impact force, the reaction force, and the mid-span displacement

were measured and recorded. Crack patterns of some of the beams after the impact were also sketched and reported (Figure 2.22).

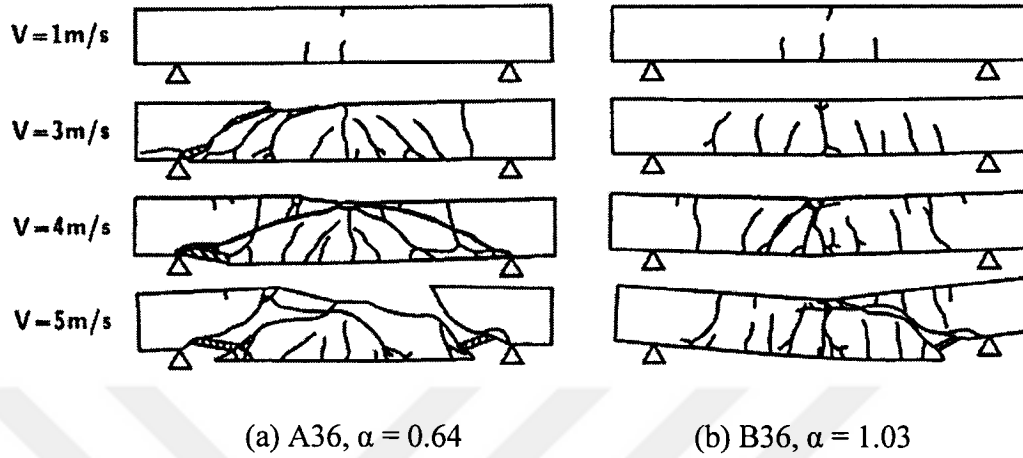


Figure 2.22 Crack patterns for beams A36 and B36 (Kishi et al. 2002)

After examination of the test results, the authors observed that the time-history of the reaction force was similar to the time-history of the mid-span displacement. Therefore, they suggested using the maximum reaction force rather than the maximum impact force as a parameter to estimate the impact load capacity of the beams. They found that, for their test beams, the impact shear capacity R_{ud} was 1.5 times the calculated static shear capacity V_{us} , on average. They also calculated the absorbed energy E_a during the impact by using the loop-area of the reaction force vs. mid-span displacement curve. The input kinetic energy E_k was calculated by the formula $E_k = \frac{1}{2} mV^2$, where m is the mass of the steel weight and V is the impact velocity. For their tests, the authors found that the ratio E_a/E_k ranged between 0.5 and 0.7, with an average value around 0.6. Using these observations and simplifying the reaction force versus mid-span displacement loop as a triangle (Figure 2.23), they derived the required static shear capacity V_{usd} of the beam in terms of design input energy E_{kd} and design residual displacement δ_{rd} as follows:

$$R_{ud} = 1.5V_{usd} \quad (2.10)$$

The absorbed energy E_{ad} , calculated using the simplified reaction force versus mid-span displacement loop (Figure 2.23), can be found as

$$E_{ad} = \frac{R_{ud} \delta_{rd}}{2} \quad (2.11)$$

The relation between design input kinetic energy and the absorbed energy was given as

$$E_{ad} = 0.6 E_{kd} \quad (2.12)$$

Substituting Eqs. 2.10 and 2.11 into 2.12, V_{usd} can be found as

$$V_{usd} = \frac{1.2 E_{kd}}{1.5 \delta_{rd}} = 0.8 \frac{E_{kd}}{\delta_{rd}} \quad (2.13)$$

If the input energy, which is usually the kinetic energy of the missile, is known, then the designer can select an allowable residual displacement and design for sufficient static shear capacity of the member in accordance with Eq. 2.13. This method has the advantage of relating static shear capacity, which can be calculated using code formulations, to the impact shear capacity. This approach significantly simplifies the impact resistant design of shear-critical reinforced concrete members.

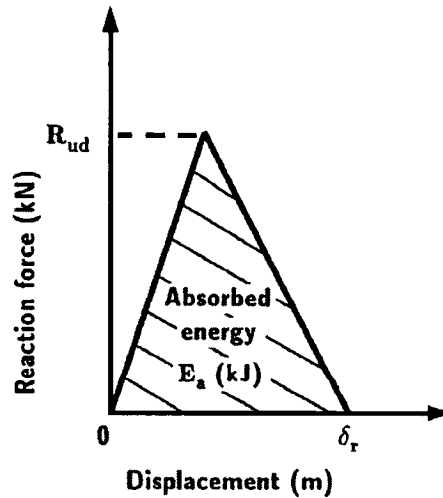


Figure 2.23 A simplified model for reaction force versus displacement loop
(Kishi et al. 2002)

2.4 Significance of the Current Study

As evident in the summary of the literature, the research into the behaviour of reinforced concrete structures under impact loads can roughly be divided into two main areas; research on local response and research on global response.

Research on local response was mainly empirical, since the complicated nature of the problem makes it difficult to rationalize the local behaviour under impact loads. Numerical studies and methods based on FEM or DEM were also carried out, however it has to be noted that modelling such high-deformation response with these methods is significantly time-consuming, requires the use of sophisticated software, and often give results of questionable accuracy.

On the other hand, studies on the global response were more evenly distributed between empirical, numerical and analytical types. There have been experimental studies investigating this issue, usually supported by analytical studies or numerical studies. However, as the physical area of investigation is enlarged from a small local portion of the structure to the structure as a whole, some problems arise in terms of the applicability of the methods developed.

The main problem with the analytical methods for the global response is their general applicability. In the investigation of the local response, the physical area under consideration is either a part of a slab or a shell structure, which can confidently be isolated from its surroundings. In other words, for local response analysis, the overall shape or size of the structure that contains the slab or shell under consideration has little importance since any contribution from the overall structure is deemed negligible. Therefore, any experimental or numerical finding on a small slab can be applicable to a wide variety of situations, regardless of the global structure. However, this is not the case for the global response analysis. When a structure responds to an impact through global deformations, numerous features of the structure comes into effect, including the shape and dimensions of the structure, support conditions, and the amount and layout of the reinforcement. For this reason, most analytical studies are limited only to a certain type of structure under certain conditions, and are not applicable to cases beyond their limitations. As mentioned earlier,

numerical methods were preferred over analytical ones to overcome these limitations. However, where global response analyses are concerned, the FEM or similar methods employed so far are either over-simplified, such as the ones that treat reinforced concrete as a linearly-elastic or elasto-plastic material, or they are over-complicated, such as the ones that employ the sophisticated modelling techniques of a local response for a global structure. Although both approaches can serve well for various specific needs, they have significant drawbacks as well. For instance, simplified methods will give very limited information about the response of the structure and the results obtained are usually less accurate because of the inherent simplifications. On the other hand, sophisticated methods may give more accurate results, but they tend to require tremendously large meshes or very fine time-step increments, which may take several days to solve even with a supercomputer. Moreover, the sophistication in such methods is not always directed towards specific modelling issues relating to reinforced concrete, but rather towards the numerical techniques employed. In other words, a general purpose FEM package that employs high level numerical modelling techniques may not always demonstrate a satisfactory performance for reinforced concrete structures, particularly regarding problematic issues such as modelling the shear behaviour of reinforced concrete which may be critical for successfully modelling the impact behaviour.

This current study is an attempt to fill the gap between highly-simplified and highly-complicated numerical techniques with a technique developed specifically for reinforced concrete and proven to be successful over the years for quasi-static loading. An in-house finite element program called VecTor2, which was originally developed for the static analysis of reinforced concrete structures, was modified to include the consideration of impact loads. VecTor2 is a two-dimensional program, which uses 4-node rectangular or 3-node triangular elements for modelling reinforced concrete. Reinforcement is modelled either as smeared in concrete or with truss bar elements. Along with an extensive library of material and behaviour models for modelling reinforced concrete, VecTor2 employs the Modified Compression Field Theory (Vecchio and Collins 1986) for its computational methodology, which has been one of the most successful rational methods in modelling the shear behaviour of reinforced concrete. The static part of the program has been under

development since mid-1980s, and it has been tested and improved continuously for the past two decades through the course of countless tests carried out at the University of Toronto Laboratories. Successful adaptation of this program for impact loads not only would offer an accurate method for the research in this area, but would also provide a fast tool that could be used for the design of reinforced concrete structures against such loads.

In addition to the code development, the test program carried out in this study supplies the literature with detailed, well-instrumented test data, which can further be utilized for the development and verification of other methods for the analysis of reinforced concrete structures under impact loads.



3 FINITE ELEMENT MODELLING OF REINFORCED CONCRETE STRUCTURES UNDER DYNAMIC LOADS

3.1 Introduction

VecTor2 is a two-dimensional finite element program, used to analyze reinforced concrete structures under various types of loads including static, cyclic and thermal loads. The program is based on Modified Compression Field Theory formulations (Vecchio and Collins, 1986), and has been under continuous development at the University of Toronto since mid the 1980s.

In recent years, studies for the further development of VecTor2 took a direction to include extreme loads; modifications for extreme thermal loads (Zhou, 2004), development of a pre-processor to quantify blast loads on structures (Miller, 2004), and adaptation of new constitutive relations to consider strain rate effects on concrete and steel (Ho, 2004) constituted phases of this development. This current study, along with other aforementioned purposes, includes another phase: to introduce of the capability of handling dynamic loads, such as impact and impulsive loads, as well as loads introduced by base accelerations.

This chapter documents the numerous elements added into the code of VecTor2 to enable dynamic analysis. Section 3.2 gives details of the structural property matrices, such as mass, damping, and stiffness matrices, as well as load vector, which were either added or modified for dynamic analysis. Section 3.3 presents the adapted method for the numerical solution of the dynamic equations of motion, followed by a section that introduces the dynamic algorithms introduced in VecTor2. In the last section of this chapter, the newly adapted dynamic algorithms are tested and verified for a linearly elastic single degree of freedom structure.

3.2 Structural Property Matrices

When a structure is subjected to static loads (i.e. loads that do not change with time), the loads are resisted by the stiffness of the structure alone, as depicted in Figure 3.1. However, if the applied loads change with respect to time, the structure will be set in motion, and the

loads will be resisted by the stiffness and damping of the system, as the structure itself accelerates in the direction of the applied load.

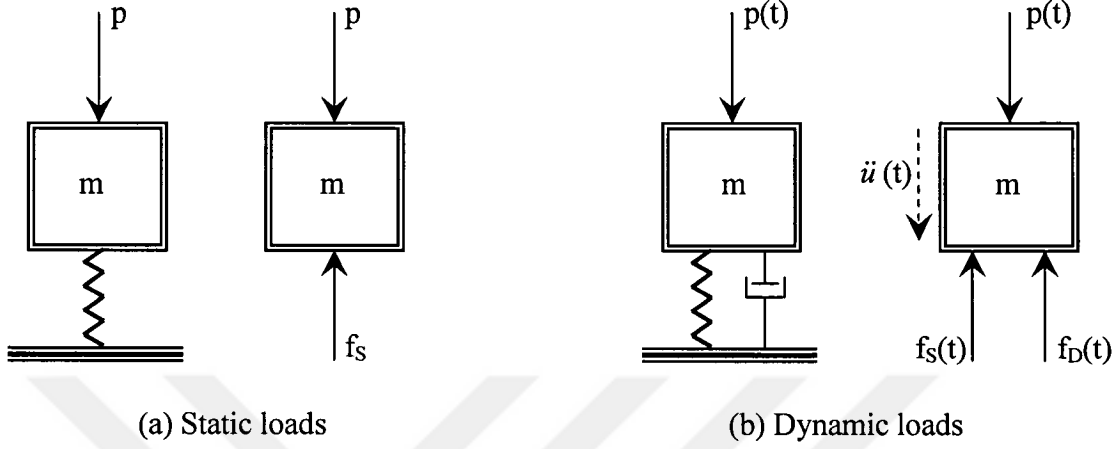


Figure 3.1 Equilibrium of structures

Applying Newton's second law of motion to this dynamic case gives the following relationship, which is well-known as *the equation of motion*:

$$p(t) - f_s(t) - f_D(t) = m\ddot{u}$$

or

$$m\ddot{u} + f_D(t) + f_s(t) = p(t) \quad (3.1)$$

where m is the mass of the structure, $\ddot{u}$ is the acceleration, and f_s , f_D and p are the time dependent stiffness, damping and applied forces, respectively. This relation, derived here for a single degree of freedom (SDOF) system, is also valid for a multi-degree of freedom (MDOF) system and can be represented as follows:

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{k}\mathbf{u} = \mathbf{p}(t) \quad (3.2)$$

where $\mathbf{m}$, $\mathbf{c}$ and $\mathbf{k}$ (with bold characters) are the square mass, damping and stiffness matrices, and $\mathbf{u}$, $\dot{\mathbf{u}}$ and $\ddot{\mathbf{u}}$ are the column vectors of displacement, velocity and accelerations, respectively.

Since Eq. 3.2 is derived from basic principles of mechanics, most dynamic finite element analysis methods involve the solution of this equation in one way or another. This requires establishing the structural property matrices $\mathbf{m}$, $\mathbf{c}$ and $\mathbf{k}$. In this section, formulations for establishing these matrices are examined in detail, since VecTor2 also utilizes Eq. 3.2 to solve for the displacement history under an applied dynamic load.

3.2.1 Mass Matrix

The mass matrix $\mathbf{m}$ is a discrete representation of the mass of the system, mobilized during the dynamic motion. There are two common types of mass matrices: *lumped mass matrices* and *consistent mass matrices*.

A lumped mass matrix is essentially a diagonal matrix in which all the off-diagonal terms are zero. This type of matrix is particularly preferred for modal analysis, since it makes the decoupling of equations of motion easier. The non-zero diagonal elements in a lumped mass matrix are simply calculated by the concentration (lumping) of the total mass of a particular portion of the structure to a single point. The concentrated mass is assigned to the associated degree of freedom, entered as the corresponding diagonal term in the mass matrix. Thus, the sum of the concentrated masses in one direction of motion is equal to the total mass of the structure mobilized in that direction. If rotational degrees of freedom exist, the rotational inertia of the mass is assigned to the corresponding rotational degree of freedom.

Concentrated masses can be calculated and entered as an input, by assigning a mass to a degree of freedom. Alternatively, if the required material densities are known, these masses can be calculated automatically at nodal points. For a four-node two-dimensional plane element, for instance, the total mass of the element can be divided into four and assigned to each node in both directions, as shown in Figure 3.2. For more complex elements, tributary areas can be used to determine the mass corresponding to each node.

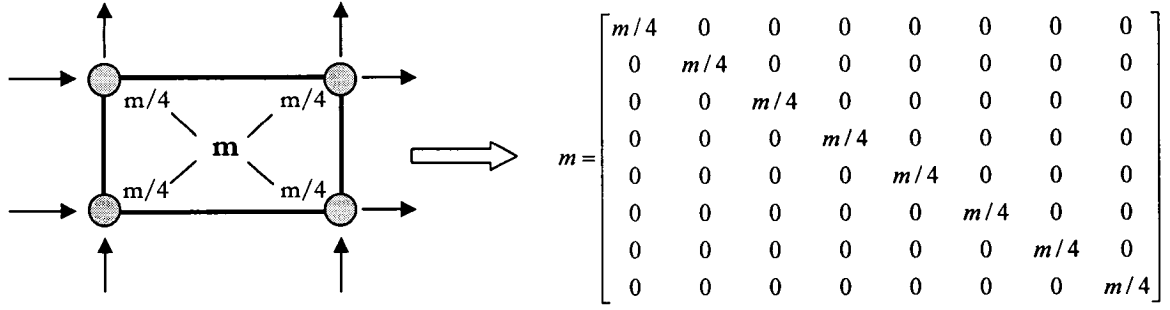


Figure 3.2 Lumped mass matrix

A consistent mass matrix is another type of mass matrix, formed from the finite element shape functions of the element, as presented in the following equation:

$$\mathbf{m} = \int \rho \mathbf{N}^T \mathbf{N} dV \quad (3.3)$$

where $\mathbf{m}$ is the element mass matrix, ρ is the material density, $\mathbf{N}$ is the finite element shape function and V is the element volume. Unlike a lumped mass matrix, a consistent element mass matrix is fully populated. This feature of the consistent mass matrix causes some computational problems, and requires significantly more storage capacity, which makes the lumped mass matrix more appealing. On the other hand, for high-order elements or elements with irregular shape, simply lumping mass to the nodes results in oversimplification of the actual mass distribution. Therefore, some methods were developed to obtain a diagonal mass matrix from a consistent mass matrix, yet avoid the possible discretization errors that may be present in a lumped mass matrix.

One such method is the *HRZ Lumping Scheme* (Hinton et al. 1976, Surana 1978). In this method, only the diagonal terms of the consistent mass matrix are calculated, leaving all off-diagonal terms as zero. The diagonal terms are then scaled such that the sum of the diagonal terms in a direction is equal to the total mass of the structure mobilized in that direction. Diagonal mass matrices obtained by this method are reported to give excellent results for flexural and low-order finite elements. For higher-order plane elements, another method, *Optimal Lumping* (Fix 1972, Fried and Malkus 1975), is preferred.

Optimal Lumping is based on the integration given in Eq. 3.3. When carrying out the integration numerically, if the integration points of the quadrature rule coincide with nodal locations of the element, then no off-diagonal terms are generated and the mass matrix turns out to be diagonal. This method is considered as the optimum solution, since there is no crude simplification or approximation, and the off-diagonal terms are eliminated without compromising the accuracy. However, this method does not produce a diagonal mass matrix if rotational degrees of freedom exist.

The choice of the type of the mass matrix mainly depends on the accuracy needed and the method to be used during the analysis. The lumped mass matrix is a crude approximation, but its computational ease and minimal storage requirements are significant advantages. A consistent mass matrix, on the other hand, is considered more accurate since it is consistent with the finite element discretization of the physical continuum. However, this superior accuracy may sometimes be quite marginal considering the extra storage required and the computational effort spent calculating it. Therefore, for most structural engineering applications, the use of lumped mass matrix is more reasonable².

Since VecTor2's stiffness formulations use low order elements, and significant computational power and storage is already required for determining the stiffness matrix through iterations, a lumped mass matrix is preferred for representing the mass of the structure to facilitate the analyses. The user may enter the lumped masses to individual degrees of freedom, or may have the adapted algorithm calculate the nodal masses from an input material density.

3.2.2 Damping Matrix

Quantifying the damping mechanisms in a structure is rather a complex problem, which has not been well-studied to date. Determining the damping coefficient properties from the physical state of the structure, such as the dimensions, member sizes or materials used, is impractical, and the number of studies in this area in the literature are quite limited. Therefore, damping is usually included in the analyses not as a calculated, well-determined

² A thorough discussion on the different types of mass matrices, and their advantages and disadvantages on particular types of problems can be found in Cook R.D., Malkus, D. S., Plesha, M. E. (1989). *Concepts and Applications of Finite Element Analysis*, John Wiley & Sons, Inc., Chapter 13.

physical property such as the material stiffness, but rather mathematically for its ability to stabilize numerical solutions and take into account energy dissipating mechanisms.

Studies in the literature were mostly focused on obtaining a damping matrix $\mathbf{c}$, which is both compatible with the expected damping ratios of the structural modes, and has practical mathematical properties, such as orthogonality with respect to the modes of vibration. In order to ensure the orthogonality of the damping matrix with respect to modes of vibration, $\mathbf{c}$ can be formed as a linear combination of the mass and stiffness matrices, as shown in the following equation:

$$\mathbf{c} = a_0 \mathbf{m} + a_1 \mathbf{k} \quad (3.4)$$

where a_0 and a_1 are proportionality constants, found as follows.

$$\frac{1}{2} \begin{bmatrix} 1/\omega_i & \omega_i \\ 1/\omega_j & \omega_j \end{bmatrix} \begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix} = \begin{Bmatrix} \xi_i \\ \xi_j \end{Bmatrix} \quad (3.5)$$

Eq. 3.5 can be solved for a_0 and a_1 to yield:

$$a_1 = \frac{2\xi_j\omega_j - 2\xi_i\omega_i}{\omega_j^2 - \omega_i^2}, \quad a_0 = 2\xi_i\omega_i - a_1\omega_i^2 \quad (3.6)$$

where ω_i and ω_j are the angular frequencies, and ξ_i and ξ_j are the damping ratios for modes i and j . In modal analyses, the choice of modes i and j depends on the expected dynamic response of the structure. Generally i is selected as the first mode, whereas j is selected as the largest (or a few smaller than the largest) mode which is expected to have a significant contribution to the overall response. Consequently, the modes between the two selected ones will have slightly smaller damping than the specified ξ_i and ξ_j , whereas the modes below and above will be highly damped, largely eliminating the corresponding modal responses (Figure 3.3). As mentioned previously, there exists no commonly accepted method to determine the damping ratios in advance. However, when the total global response is concerned, there are experimental methods to determine the damping ratios of

the particular modes of vibration in a structure. Accumulated knowledge on the damping characteristics of the analyzed structure type renders the damping ratios predictable to a certain degree. For example, it is commonly accepted that the damping ratios for lightly cracked reinforced concrete structures are approximately 3 to 5% for the fundamental modes, whereas they may approach 10% for heavily damaged reinforced concrete structures.

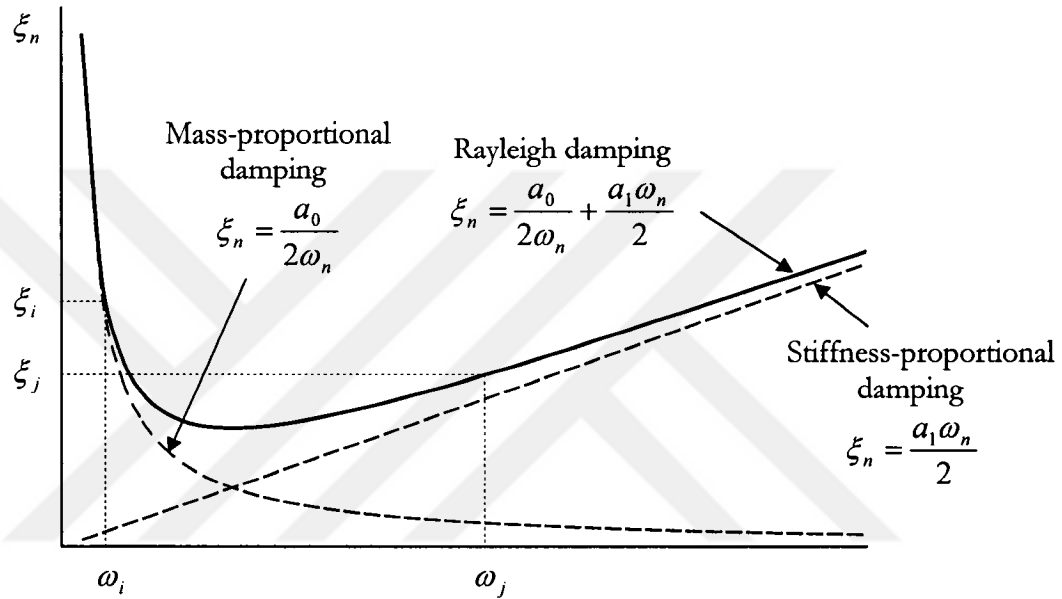
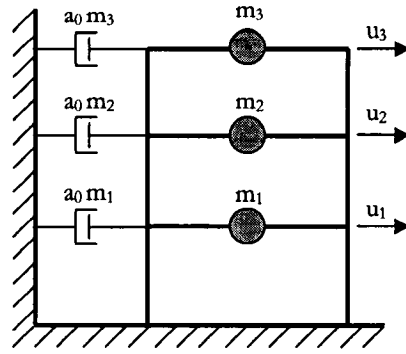


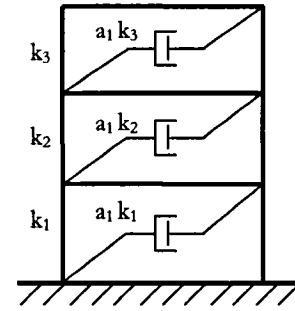
Figure 3.3 Variation of modal damping ratios with natural frequency

This method of determining the damping matrix is called *Rayleigh* or *proportional damping*³. This representation of the structural damping actually combines two types of damping observed in structures: mass-proportional damping and stiffness-proportional damping. Mass-proportional damping can be interpreted as an external resistance to the motion of the structure, whereas stiffness-proportional damping is more similar to the energy dissipation mechanisms developed due to the internal deformations (Figure 3.4). Although stiffness-proportional damping seems to be more logical, it is not effective in modelling the actual damping mechanisms alone. Therefore, a combination of both, as in Rayleigh method, is deemed more appropriate for practical applications.

³ Details of the Rayleigh damping formulation and the equations derived here can be found in Chopra, A.K. (2001). *Dynamics of Structures*, Prentice-Hall, Inc., Chapter 11.



(a) Mass-proportional damping



(b) Stiffness-proportional damping

Figure 3.4 Damping mechanisms (Chopra, 2001)

In the Rayleigh method, damping ratios for two of the modes are specified, and the damping ratios for the remaining modes are calculated accordingly. However, it is also possible to assign the damping ratios for all vibration modes explicitly, by using a method called *Caughey* damping, as shown in the following equation:

$$\mathbf{c} = \mathbf{m} \sum_{i=0}^{N-1} a_i [\mathbf{m}^{-1} \mathbf{k}]^i \quad (3.7)$$

where N is the total number of modes and a_i are constants. If the damping ratios of first j modes are to be specified, then

$$\mathbf{c} = \mathbf{m} \sum_{i=0}^{j-1} a_i [\mathbf{m}^{-1} \mathbf{k}]^i \quad (3.8)$$

and the damping ratio for the vibration mode n is given by

$$\xi_n = \frac{1}{2} \sum_{i=0}^{j-1} a_i \omega_n^{2i-1} \quad (3.9)$$

Coefficients a_i are determined by solving j algebraic equations for the unknowns a_i , $i = 0$ to $j-1$, where ξ_n for each mode are assigned explicitly. It has to be noted that for $j=2$, this method becomes identical with the Rayleigh method. Although it provides more control on

the damping properties, this method is reported to produce numerically ill-conditioned equations. Moreover, it results in fully populated matrices if more than two modal damping ratios are specified, which significantly increases the computational cost and storage requirements. Therefore, in general, Rayleigh damping is preferred over Caughey damping in modal analyses.

Although VecTor2 does not use modal analysis, Rayleigh damping was chosen to introduce damping. As mentioned previously, the need to introduce damping in the analysis mostly arises for numerical considerations, rather than modelling the physical damping present in the system. VecTor2 is already capable of modelling the majority of the energy dissipating mechanisms in a reinforced concrete structure, such as material hysteresis, concrete cracking, and bond slippage. Thus, significant damping would take place in the dynamic response even if no additional viscous damping was introduced. On the other hand, although the viscous response of concrete at the material level was not considered in this study, the presence of viscous damping in the system is still required for the stability of the numerical solution. Details on this issue and the effects of damping on the VecTor2 analyses will be discussed in Chapter 6 through various examples. It should be noted that, despite its known deficiencies, Rayleigh damping was still used in numerous studies for its mathematical convenience, and it is preferred in this study for the same reason. A detailed discussion on the deficiencies of Rayleigh damping and possible remedies can be found in Hall (2005).

In order to implement Rayleigh damping, the vibrational modes of the structure need to be determined. Although the stiffness matrix changes constantly during the nonlinear response, in this study, the initial linear elastic stiffness matrix was used to determine the modes of vibration, which in turn were used to establish the damping matrix as given in Eq. 3.4. In other words, the damping matrix remains constant during the entire analysis, even though the stiffness of the structure changes. This assumption is contrary to the observed damping behaviour in structures, since it is known that damping increases as a structure goes into the nonlinear range and suffers damage. Moreover, the use of a constant damping matrix with Rayleigh damping can be problematic, since it may result in unrealistically high damping forces. In other words, although forces resulting from the resistance of the

structure are limited due to the nonlinear behaviour of the material, damping forces in this method do not have any upper boundary. However, updating the damping matrix during the course of an analysis introduces numerical stability problems, as well as a significant increase in the required computational power; hence, the reasons for selection of a constant damping matrix in this study.

3.2.3 Stiffness matrix

As mentioned previously, VecTor2 has been under development for nearly two decades at the University of Toronto, mainly for the static load analysis of reinforced concrete structures. Therefore, sophisticated modelling techniques for the stiffness formulation of reinforced concrete structures were already in place prior to this study. In this section, a brief review of these formulations is provided. Further details can be found in VecTor2 and Formworks Manual (2003).

VecTor2 uses two-dimensional, 4-node rectangular, 4-node quadrilateral or 3-node triangular elements for modelling reinforced concrete. Reinforcement is modeled either as smeared in concrete or with truss bar elements. The program uses the Modified Compression Field Theory (MCFT) with a rotating smeared-crack approach for modelling reinforced concrete, which is summarized below.

For a single linearly elastic isotropic plane stress element, the material matrix **D**, relating element strains to element stresses, is given as follows for global axes x and y.

$$\sigma = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \mathbf{D} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (3.10)$$

$$\mathbf{D} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{(1-\nu)}{2} \end{bmatrix} \quad (3.11)$$

where E is the modulus of elasticity and ν is the Poisson's ratio of the material. For a cracked concrete element, the material stiffness matrix can be evaluated as

$$\mathbf{D}'_c = \begin{bmatrix} \bar{E}_{c2} & 0 & 0 \\ 0 & \bar{E}_{c1} & 0 \\ 0 & 0 & \bar{G}_c \end{bmatrix} \quad (3.12)$$

where $\bar{G}_c$ is the secant shear modulus, and $\bar{E}_{c2}$ and $\bar{E}_{c1}$ are the secant moduli of concrete, for compression and tension, respectively, defined as follows:

$$\bar{E}_{c1} = \frac{f_{c1}}{\epsilon_{c1}}, \quad \bar{E}_{c2} = \frac{f_{c2}}{\epsilon_{c2}}, \quad \bar{G}_c = \frac{\bar{E}_{c1} \cdot \bar{E}_{c2}}{\bar{E}_{c1} + \bar{E}_{c2}} \quad (3.13)$$

where f_{c1} and f_{c2} are the concrete stresses, and ϵ_{c1} and ϵ_{c2} are the concrete strains in the principal directions. The material matrix $\mathbf{D}'_c$ is expressed with respect to the element's principal axes 1 and 2, corresponding to the directions of the principal average tensile strain and principal average compressive strain (i.e. crack direction). It should be noted that in this formulation, the principal axes of stress are assumed to coincide with the principal axes of strain and are permitted to rotate.

A similar material matrix can also be found for the smeared reinforcement components.

$$\mathbf{D}'_{s_i} = \begin{bmatrix} \rho_i \bar{E}_{s_i} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (3.14)$$

where ρ_i is the reinforcement ratio, and $\bar{E}_{s_i}$ is the elastic modulus for steel reinforcement, defined as:

$$\bar{E}_{s_i} = \frac{f_{s_i}}{\epsilon_{s_i}} \quad (3.15)$$

where f_{si} and ϵ_{si} are the stress and strain in the reinforcement, respectively. A material matrix for each reinforcement component i can be defined as given in Eq. 3.14. After determining the material matrices for concrete and reinforcement components, the total element material matrix $\mathbf{D}$ can be computed as

$$\mathbf{D} = \mathbf{D}_c + \sum_{i=1}^n \mathbf{D}_{s_i} \quad (3.16)$$

where n is the number of reinforcement components and

$$\mathbf{D}_c = \mathbf{T}_c^T \mathbf{D}'_c \mathbf{T}_c, \quad \mathbf{D}_{s_i} = \mathbf{T}_{s_i}^T \mathbf{D}'_{s_i} \mathbf{T}_{s_i} \quad (3.17)$$

In Eq. 3.17, $\mathbf{T}_c$ and $\mathbf{T}_{s_i}$ are transformation matrices, used to rotate the axes of the material matrices to the global x and y axes, as shown below.

$$\mathbf{T} = \begin{bmatrix} \cos^2 \psi & \sin^2 \psi & \cos \psi \sin \psi \\ \sin^2 \psi & \cos^2 \psi & -\cos \psi \sin \psi \\ -2 \cos \psi \sin \psi & 2 \cos \psi \sin \psi & \cos^2 \psi - \sin^2 \psi \end{bmatrix} \quad (3.18)$$

where for the concrete,

$$\psi = \theta + \beta \quad (3.19)$$

and for the reinforcement components

$$\psi = \alpha_i + \beta \quad (3.20)$$

as shown in Figure 3.5.

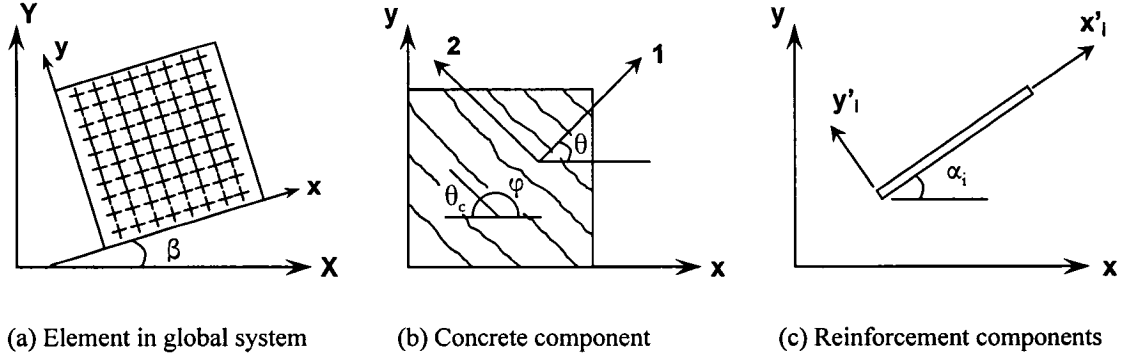


Figure 3.5 Reference systems for reinforced concrete element (Vecchio, 1990)

The constitutive relations for concrete were originally determined from panel tests (Vecchio and Collins, 1986), as follows:

$$f_{c2} = f'_c \frac{2 \left(\frac{\epsilon_{c2}}{\epsilon_o} \right) + \left(\frac{\epsilon_{c2}}{\epsilon_o} \right)^2}{0.8 - 0.34 \left(\frac{\epsilon_{c1}}{\epsilon_o} \right)} \quad \text{for cracked concrete in compression} \quad (3.21)$$

$$f_{c1} = E_c \epsilon_{c1} \quad \text{for concrete in tension, before cracking} \quad (3.22)$$

$$f_{c1} = \frac{f'_t}{1 + \sqrt{500 \epsilon_{c1}}} \quad \text{for concrete in tension, after cracking} \quad (3.23)$$

where E_c is the concrete elastic modulus, f'_t is the cracking strength, f'_c is the peak compressive stress, and ϵ_o is the corresponding compressive strain, as determined from standard cylinder tests. A bilinear constitutive relation can be assumed for steel reinforcement.

$$f_{si} = E_{si} \epsilon_{si} \leq f_{yi} \quad (3.24)$$

It should be noted that the constitutive relations described in Eqs. 3.21 to 3.24 are based on average stresses. However, at a crack, tensile stress in the reinforcement will be higher than average, whereas tensile stress in the concrete will be zero. These local variations may

require the presence of local shear stresses along a crack surface for transmitting the tensile stresses across a crack. On the other hand, the maximum shear stress that can be developed at a crack is limited by several factors such as aggregate size, normal compressive stresses and crack width, which consequently limits the maximum tensile stress that can be transmitted. Therefore, if the shear stress at a crack exceeds the limiting value, the post-cracking tensile stresses in the concrete should be reduced. Detailed discussions regarding this issue can be found in Vecchio and Collins (1986).

It should be noted that these constitutive relationships for concrete have subsequently been updated and improved (Vecchio 2000). After determining the material matrix $\mathbf{D}$, the formulations can be adapted into a finite element form, as follows, to obtain the stiffness matrix $\mathbf{k}$ for a single element.

$$\mathbf{k} = \int \mathbf{B}^T \mathbf{D} \mathbf{B} dV \quad (3.25)$$

where $\mathbf{B}$ is the shape function matrix for the type of the element to be used. Complete stiffness matrices for the 4-node rectangular or 3-node triangular elements that VecTor2 uses can be found in Vecchio (1990). After obtaining the stiffness matrix for a single element, the global stiffness matrix of the structure can be formed by following common finite element procedures. Note that the steel reinforcement also can be modelled discretely as truss bars. In that case, the stiffness matrix of each truss bar is added to the global stiffness matrix.

Since the secant moduli for the elements are not known initially, an iterative solution procedure is required to produce the material matrix $\mathbf{D}$, corresponding to the given element stresses and strains. In the process, secant moduli for concrete are estimated first, followed by the formation and solution of the stiffness matrix for the joint displacements and element strains. Then, secant moduli corresponding to the computed strains are calculated and compared with the initial assumption. Iterations are carried out until the calculated and assumed secant moduli converge (Figure 3.6).

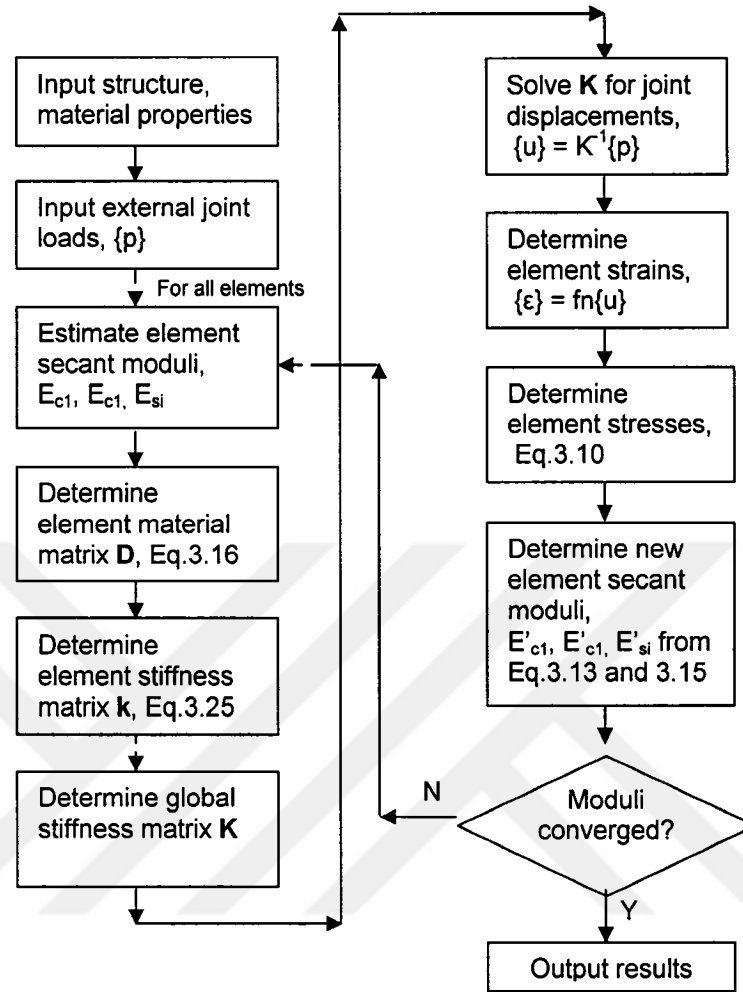


Figure 3.6 Finite element solution procedure

After its initial development, the MCFT was further improved to account for the noncoincident axes of principal strains and principal stresses, and slip along crack surfaces. Constitutive relations of MCFT were updated and improved, and the necessity of checking the shear stresses at cracks was removed. This new method became known as the Disturbed Stress Field Model (DSFM), details of which can be found in Vecchio (2000).

VecTor2 uses the MCFT for its computational methodology along with a wide array of material and behavioural models to account for compression softening, tension stiffening and softening, concrete dilatation, bonding, concrete and reinforcement hysteresis, reinforcement dowel action and reinforcement buckling. Details on these models can be found in the VecTor2 and Formworks Manual (2003). In a recent study by Ho (2004), to

account for the effects of strain rates on material behaviour, the CEB strain rate formulations (1988) were adapted for concrete, and formulations proposed by Malvar (1998) were adapted for reinforcing steel. These formulations can be summarized as follows.

For concrete in compression, as recommended by CEB,

$$\frac{f_{cd}}{f_{cs}} = \left(\frac{\dot{\epsilon}}{\dot{\epsilon}_0} \right)^{1.026\alpha} \quad \text{for } \dot{\epsilon} \leq 30 s^{-1} \quad (3.26)$$

$$\frac{f_{cd}}{f_{cs}} = \gamma \left(\frac{\dot{\epsilon}}{\dot{\epsilon}_0} \right)^{1/3} \quad \text{for } \dot{\epsilon} > 30 s^{-1} \quad (3.27)$$

$$\alpha = \frac{1}{5 + 9 \frac{f_{cs}}{f_{c0}}} \quad (3.28a)$$

$$\gamma = 10^{6.156\alpha - 2.0} \quad (3.28b)$$

where $\dot{\epsilon}_0 = 30 \times 10^{-6}$, $f_{c0} = 10$ MPa, $\dot{\epsilon}$ is the strain rate, and f_{cs} and f_{cd} are the static and dynamic compressive strength of concrete, respectively. For concrete in tension, the following equations govern:

$$\frac{f_{td}}{f_{ts}} = \left(\frac{\dot{\epsilon}}{\dot{\epsilon}_0} \right)^{1.016\delta} \quad \text{for } \dot{\epsilon} \leq 30 s^{-1} \quad (3.29)$$

$$\frac{f_{td}}{f_{ts}} = \beta \left(\frac{\dot{\epsilon}}{\dot{\epsilon}_0} \right)^{1/3} \quad \text{for } \dot{\epsilon} > 30 s^{-1} \quad (3.30)$$

$$\delta = \frac{1}{10 + 6 \frac{f_{cs}}{f_{c0}}} \quad (3.31a)$$

$$\beta = 10^{7.11\delta - 2.33} \quad (3.31b)$$

where $\dot{\varepsilon}_0 = 30 \times 10^{-6}$, $f_{c0} = 10$ MPa, and f_{ts} and f_{td} are the static and dynamic tensile strength of concrete, respectively.

For reinforcing steel, the formulations as proposed by Malvar are as follows:

$$\frac{f_{yd}}{f_{ys}} = \frac{f_{ud}}{f_{us}} = \left(\frac{\dot{\varepsilon}}{10^{-4}} \right)^\alpha \quad (3.32)$$

where f_{yd} and f_{ys} are the static and dynamic yield stresses, and f_{ud} and f_{us} are the static and dynamic ultimate stresses, respectively. The coefficient α is determined differently, depending on whether yield or ultimate stress is calculated.

$$\alpha = \alpha_{fy} = 0.074 - 0.040 \frac{f_y}{414} \quad \text{for yield stress calculations} \quad (3.33)$$

$$\alpha = \alpha_{fu} = 0.019 - 0.009 \frac{f_y}{414} \quad \text{for ultimate stress calculations} \quad (3.34)$$

In these equations, f_y is the static yield stress in MPa, and strain rate $\dot{\varepsilon}$ is in s^{-1} .

3.2.4 Load Vector

As mentioned earlier, VecTor2 uses a secant stiffness, total load approach in its algorithms. Therefore, the load vector $\mathbf{p}(t)$ in Eq.3.2 is the total load acting at the nodes of the structure, including both dynamic time varying loads, as well as constant loads such as self-weight.

Dynamic loads can be introduced in the program in three ways: i) as an initial velocity assigned to lumped masses, ii) as time-varying impulse forces defined by a tri-linear curve, or iii) as base accelerations.

When the dynamic load is introduced as an initial velocity on the lumped masses, the load is introduced in the system through the numerical integration of the dynamic response, rather than a direct calculation of a load vector. Details of the numerical integration scheme

will be explained in the following section. When an impulse force is defined as the dynamic load, the value of the impulsive force is calculated according to the input impulse history, and is then directly added to the total load vector $\mathbf{p}(t)$.

Introducing the dynamic load as base accelerations can be done as follows.

$$\mathbf{p}(t) = -\mathbf{m} \mathbf{r} \ddot{u}_g(t) \quad (3.35)$$

where $\ddot{u}_g(t)$ is the ground acceleration and $\mathbf{r}$ is the influence coefficient vector, which represents the displacements resulting from a unit support displacement in a direction parallel to the base accelerations (Figure 3.7). In VecTor2, the base accelerations are entered as parallel to the global x or y axis. Therefore, for example, for a ground acceleration introduced in the global x direction, all coefficients of $\mathbf{r}$ corresponding to the degrees of freedom in the x direction would be one, whereas the others would be zero.

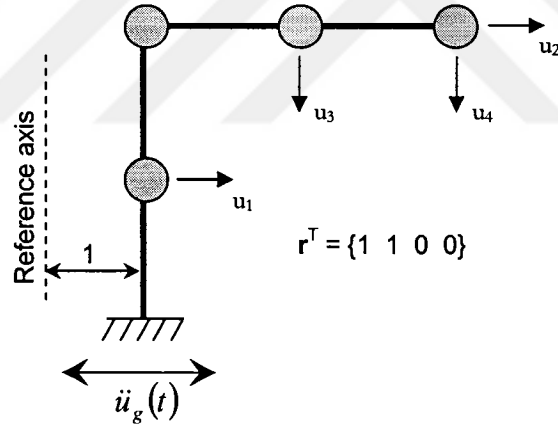


Figure 3.7 Influence coefficient vector

3.3 Numerical Evaluation of Dynamic Response

Numerous methods have been developed to date for the solution of the equation of motion, presented in Eq. 3.2 in matrix form. Most of these methods, such as the mode superposition method, are applicable for linearly elastic structures, for which the structural properties stay constant during the dynamic response. For the case of nonlinear structures, however, step-by-step direct numerical integration of Eq. 3.2 remains as the only generally applicable

method for the solution of the dynamic response. In general, direct numerical integration methods involve satisfying the equation of motion at discrete points in time, starting from time zero and stepping forward with usually constant time intervals. This approach provides a good means for evaluating the inelastic dynamic response, since step-by-step numerical integration treats the inelastic structure similar to an elastic one within a time step.

There exist a large number of methods for the numerical direct integration of the equation of motion. These methods are mainly categorized as *explicit* and *implicit* methods. Explicit methods estimate the response at the end of a time step by using the response values obtained in previous steps, without considering the equilibrium. This results in shorter calculation times, but it also requires a very small time step to ensure the stability and the accuracy of the solution. On the other hand, implicit methods try to satisfy the dynamic equilibrium at the end of the time step, which involves the solution of a system of equations sometimes using iterations. Although implicit methods are computationally more demanding, they are more stable and can handle larger time steps without the loss of accuracy. In this study, implicit methods are used for their advantages in the stability and the accuracy of the solution. In this section, Newmark's method of direct integration is presented in detail, which is adapted in VecTor2 for the evaluation of dynamic response. Issues regarding the stability and accuracy of the method are discussed in Section 3.3.2. Later in the chapter, these formulations are re-derived to handle the problems using secant stiffness, so that they can be applied to the present VecTor2 algorithms.

3.3.1 Newmark's Method of Direct Integration

The method developed by Newmark in his pioneering study published in 1959 is by far the most common method used for evaluating the dynamic response of both elastic and inelastic structures. The method was based on an assumed variation of acceleration in a time step; incremental acceleration, velocity and displacement changes at the end of the time step can be evaluated following this assumption. Here, the formulation will be presented for a SDOF system, and can then be easily adopted for MDOF systems.

At any instant in time t , the dynamic equilibrium of forces acting on a body satisfies the equation of motion, as repeated below for a SDOF system.

$$f_I(t) + f_D(t) + f_S(t) = p(t) \quad (3.36)$$

where $f_I(t)$ is the inertia force. After a time duration of Δt has elapsed, dynamic equilibrium requires

$$f_I(t + \Delta t) + f_D(t + \Delta t) + f_S(t + \Delta t) = p(t + \Delta t) \quad (3.37)$$

as each force acting on the body changes by an increment Δf , which can be presented as follows:

$$f_I(t + \Delta t) = f_I(t) + \Delta f_I(t) \quad (3.38a)$$

$$f_D(t + \Delta t) = f_D(t) + \Delta f_D(t) \quad (3.38b)$$

$$f_S(t + \Delta t) = f_S(t) + \Delta f_S(t) \quad (3.38c)$$

$$p(t + \Delta t) = p(t) + \Delta p(t) \quad (3.38d)$$

Substituting Eq. 3.38 into 3.37 yields the incremental equation of motion:

$$\Delta f_I(t) + \Delta f_D(t) + \Delta f_S(t) = \Delta p(t) \quad (3.39)$$

In other words, to satisfy the dynamic equilibrium, the total change in inertia, damping and stiffness forces within a time increment Δt should be equal to the change in the external load. Assuming that the mass remains constant, Eq. 3.39 can be rewritten as

$$m \cdot \Delta \ddot{u} + c(t) \cdot \Delta \dot{u} + k(t) \cdot \Delta u = \Delta p(t) \quad (3.40)$$

where $\Delta \ddot{u}$, $\Delta \dot{u}$ and Δu are changes in acceleration, velocity and displacement during time interval Δt , respectively. $c(t)$ and $k(t)$ are the values of the tangential damping and stiffness coefficients at time t , corresponding to the beginning of the time interval.

For easier reference, quantities at the beginning of the time interval can be named with subscript i , whereas those at the end of the interval are denoted with $i+1$. Thus, the incremental acceleration, velocity and displacement relations can be expressed as

$$\ddot{u}_{i+1} = \ddot{u}_i + \Delta\ddot{u}_i, \quad \dot{u}_{i+1} = \dot{u}_i + \Delta\dot{u}_i, \quad u_{i+1} = u_i + \Delta u_i \quad (3.41)$$

Similarly, the incremental change in applied force can be expressed as

$$p_{i+1} = p_i + \Delta p_i \quad (3.42)$$

In his study, Newmark proposed a set of equations for the calculation of velocity and displacement at the end of the time step as follows.

$$\dot{u}_{i+1} = \dot{u}_i + (1-\gamma)\Delta t \ddot{u}_i + \gamma \Delta t \ddot{u}_{i+1} \quad (3.43)$$

$$u_{i+1} = u_i + \Delta t \dot{u}_i + \left(\frac{1}{2} - \beta\right) \Delta t^2 \ddot{u}_i + \beta \Delta t^2 \ddot{u}_{i+1} \quad (3.44)$$

where γ and β are constants, determining the variation of the acceleration within the time step. Substituting Eq. 3.41 into Eqs. 3.43 and 3.44 results in following relations.

$$\Delta\dot{u}_i = \Delta t \ddot{u}_i + \gamma \Delta t \Delta\ddot{u}_i \quad (3.45)$$

$$\Delta u_i = \Delta t \dot{u}_i + \frac{\Delta t^2}{2} \ddot{u}_i + \beta \Delta t^2 \Delta\ddot{u}_i \quad (3.46)$$

Eq. 3.46 can be solved for $\Delta\ddot{u}_i$

$$\Delta\ddot{u}_i = \frac{1}{\beta \Delta t^2} \Delta u_i - \frac{1}{\beta \Delta t} \dot{u}_i - \frac{1}{2\beta} \ddot{u}_i \quad (3.47)$$

and substituted into Eq. 3.45, resulting in:

$$\Delta \dot{u}_i = \frac{\gamma}{\beta \Delta t} \Delta u_i - \frac{\gamma}{\beta} \dot{u}_i + \Delta t \left(1 - \frac{\gamma}{2\beta} \right) \ddot{u}_i \quad (3.48)$$

Eqs. 3.47 and 3.48 can be substituted into Eq. 3.40, to obtain the following relation.

$$\left(\frac{1}{\beta \Delta t^2} m + \frac{\gamma}{\beta \Delta t} c_i + k_i \right) \Delta u_i = \Delta p_i + \left(\frac{1}{\beta \Delta t} m + \frac{\gamma}{\beta} c_i \right) \dot{u}_i + \left[\frac{1}{2\beta} m + \Delta t \left(\frac{\gamma}{2\beta} - 1 \right) c_i \right] \ddot{u}_i \quad \dots (3.49)$$

or

$$\hat{k}_i \Delta u_i = \Delta \hat{p}_i \quad (3.50)$$

where

$$\hat{k}_i = \frac{1}{\beta \Delta t^2} m + \frac{\gamma}{\beta \Delta t} c_i + k_i \quad (3.51)$$

and

$$\Delta \hat{p}_i = \Delta p_i + \left(\frac{1}{\beta \Delta t} m + \frac{\gamma}{\beta} c_i \right) \dot{u}_i + \left[\frac{1}{2\beta} m + \Delta t \left(\frac{\gamma}{2\beta} - 1 \right) c_i \right] \ddot{u}_i \quad (3.52)$$

Since $\hat{k}_i$ and $\Delta \hat{p}_i$ are all in terms of known quantities at the beginning of the time interval, they can readily be calculated, and Δu_i can be solved as,

$$\Delta u_i = \frac{\Delta \hat{p}_i}{\hat{k}_i} \quad (3.53)$$

Once Δu_i is obtained, $\Delta \ddot{u}_i$ and $\Delta \dot{u}_i$ can be calculated from Eqs. 3.47 and 3.48, and the acceleration, velocity and displacements at the end of the time step can be calculated using

Eq. 3.41. Similar calculations can be repeated by using the response quantities found for time $i+1$ as the initial values for the next time step, until the required response is obtained.

Newmark argued that any value for γ less than $1/2$ introduces a negative damping which will create a self-excited vibration solely as a result of the numerical procedure. On the other hand, γ greater than $1/2$ will create an artificial numerical damping, in addition to the one already present in the structure. So he concluded that γ should always be taken as $1/2$. The parameter β determines the nature of the variation of acceleration between the beginning and the end of the time step. $\beta = 1/4$ corresponds to constant acceleration; the acceleration is constant during the time step, and its value is the average of the accelerations at the beginning and at the end of the time step. $\beta = 1/6$, on the other hand, corresponds to a linear variation of acceleration during the time interval. These two special cases are referred to as the constant acceleration method and the linear acceleration method, respectively.

3.3.2 Stability and Errors

The selection of time interval Δt is an important element in determining the accuracy and stability of Newmark's direct integration procedure. Lack of stability causes an unbounded oscillation, resulting in limitlessly increasing displacements. Newmark derived a stability limit for Δt as follows.

$$\frac{\Delta t}{T} \leq \frac{1}{\pi} \sqrt{\frac{1}{1-4\beta}} \quad (3.54)$$

where T is the natural period of the system or, for MDOF systems, the period of the highest mode of vibration. Accordingly, for $\beta = 1/4$, $\Delta t / T$ goes to infinity; this implies that for the constant acceleration method, regardless of the choice of the time step, the solution is always unconditionally stable. On the other hand, for the linear acceleration method, $\Delta t / T$ should be less than 0.551 in order to achieve a stable solution.

Another way to ensure a stable solution in time stepping methods is to use the energy balance of the system as an indicator (Belytschko et al. 2000). Any instability in the system

creates spurious generation of energy, violating the conservation of energy. Hence, an energy balance check can detect this instability, even if it is local and not detectable by examining the results. The energy balance check in a MDOF system can be performed as follows⁴:

Using the trapezoidal rule, energy terms can be expressed as

$$E_i^{kin} = \frac{1}{2} \dot{\mathbf{u}}_i^T \mathbf{m} \dot{\mathbf{u}}_i \quad (3.55a)$$

$$E_i^c = E_{i-1}^c + \frac{1}{2} (\dot{\mathbf{u}}_i + \dot{\mathbf{u}}_{i-1})^T \mathbf{c} (\mathbf{u}_i - \mathbf{u}_{i-1}) \quad (3.55b)$$

$$E_i^{int} = E_{i-1}^{int} + \frac{1}{2} (\mathbf{u}_i - \mathbf{u}_{i-1})^T (\mathbf{f}_{i-1}^{int} + \mathbf{f}_i^{int}) \quad (3.55c)$$

$$E_i^{ext} = E_{i-1}^{ext} + \frac{1}{2} (\mathbf{u}_i - \mathbf{u}_{i-1})^T (\mathbf{f}_{i-1}^{ext} + \mathbf{f}_i^{ext}) \quad (3.55d)$$

where E^{kin} is the kinetic energy of the system, E^c is the work done by damping forces, $\mathbf{f}^{int}$ is the internal restoring force vector, E^{int} is the work done by $\mathbf{f}^{int}$, $\mathbf{f}^{ext}$ is the external force vector, and E^{ext} is the work done by $\mathbf{f}^{ext}$. Energy conservation requires that

$$\frac{|E_i^{ext} - E_i^{kin} - E_i^c - E_i^{int}|}{\max(E^{ext}, E^{kin}, E^{int})} \leq \varepsilon \quad (3.56)$$

where ε is the energy balance error tolerance, which is usually selected on the order of 1%. Exceeding an acceptable level of tolerance indicates instability, requiring the reduction of the time step size Δt . An energy balance check procedure was adapted in VecTor2, which calculates and records the energy balance error for every time step. However, VecTor2 does not change the time step itself when the error exceeds the acceptable level; this decision was left to the user.

⁴ Detailed derivations of the energy equations can be found in Christopoulos C. and Filiatrault, A. (2006). *Principles of Supplemental Damping and Seismic Isolation*, IUSS Press, Milan, Italy, pp.502.

The stability condition has little significance in time stepping methods since, generally, much shorter time steps are required in order to obtain an accurate solution, especially for nonlinear structures. Analysis of nonlinear systems with direct integration methods may create some significant sources of error. One of these sources of error is so-called *overshooting*, which is demonstrated in Figure 3.8. In an analysis with constant time intervals, assume point *a* corresponds to displacement u_i on the stiffness load-displacement curve, and at the end of the next step, point *b* is located corresponding to displacement u_{i+1} . However, suppose that the velocity has changed its sign between the time t_i and t_{i+1} , which means that it became zero and changed direction along with the displacement, leading to the unloading portion of the load-displacement curve. In the following time step, the unloading portion of the stiffness curve has to be used, which starts at point *b* and locates point *c* for the displacement u_{i+2} . However, note that the displacement has changed direction not exactly at the end of the time step, but sometime before at point *b'* which would locate point *c'* instead of *c*. These departures from the exact load-displacement curve occur at each reversal of the velocity and they would lead to accumulating errors during the analysis. One remedy for reducing this error is repeating the analysis starting from the beginning of the time interval during which the velocity has changed sign, after subdividing the interval into more time steps; say $\Delta t / 4$. Another solution is through an iterative procedure, changing the time interval after time *i*, to obtain the exact point where the velocity becomes zero.

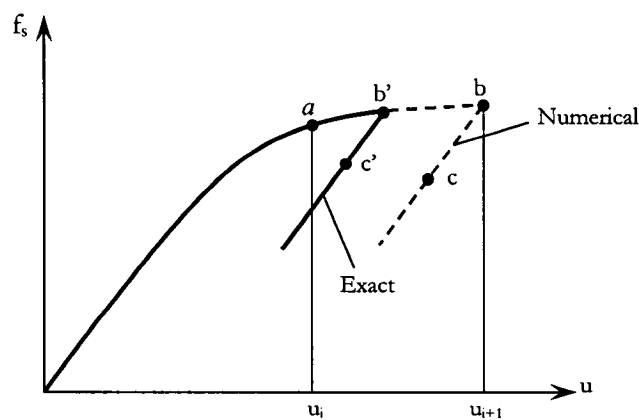


Figure 3.8 Overshooting in numerical direct integration (Chopra, 2001)

Using of tangential properties in Eq. 3.40 is another source of error, since the deviations of the actual stiffness or damping coefficient from the initial tangent value are ignored. Generally, this problem is tackled by iterative procedures such as the Newton-Raphson method. However, using a secant, rather than tangential, formulation of stiffness eliminates this source of error.

It should be noted that since VecTor2 uses secant stiffnesses in its formulations, the direct integration method presented here must be modified before it can be adopted into VecTor2. This modification would not only eliminate the sources of error caused by the use of tangent formulations, but also it would significantly reduce the errors caused by the overshooting phenomenon.

3.4 Dynamic Analysis Algorithms in VecTor2

As mentioned previously, prior to this study, VecTor2 was fully developed for the static analysis of reinforced concrete structures. Therefore, when introducing dynamic analysis capabilities into the program, it was sufficient to add the required coding in the present structure, rather than developing a new program from the outset. This section gives the details of the new codes and algorithms introduced into the present structure of the program, summarized in Figure 3.6.

In this section, first, the method employed for the determination of modal periods is explained, which is required to find the coefficients a_0 and a_1 to establish the damping matrix as previously discussed. Later, the modification of Newmark's direct integration method to utilize the secant stiffness is presented. The details of the new algorithms are given in Section 3.4.3.

3.4.1 Determination of the Modal Periods and the Damping Matrix

In a linearly elastic structure, the dynamic response of a structure can be represented as follows.

$$u(t) = q_n(t)\phi_n \quad (3.57)$$

where ϕ_n is the mode shape and $q_n(t)$ is the response history of mode n , which can be described as a harmonic function.

$$q_n(t) = A_n \cos \omega_n t + B_n \sin \omega_n t \quad (3.58)$$

where A_n and B_n are constants of integration. If Eq. 3.58 is substituted into Eq. 3.57, and damping is ignored, then the equation of motion can be re-written as follows.

$$[-\omega_n^2 \mathbf{m} \phi_n + \mathbf{k} \phi_n] q_n(t) = 0 \quad (3.59)$$

Eq. 3.59 yields the following relation.

$$[\mathbf{k} - \omega_n^2 \mathbf{m}] \phi_n = 0 \quad (3.60)$$

which requires that

$$\det[\mathbf{k} - \omega_n^2 \mathbf{m}] = 0 \quad (3.61)$$

for a nontrivial solution. Since matrices $\mathbf{m}$ and $\mathbf{k}$ are both symmetric and positive-definite, Eq. 3.61 has real and positive roots for ω_n^2 , which determine the modal frequencies of vibration. These roots are also known as eigenvalues, and their calculation presents an eigenvalue problem. It should be noted that a positive-definite mass matrix requires nonzero masses assigned to all degrees of freedom. Therefore, if degrees of freedom without an assigned mass are present in the system, static condensation is required to eliminate them before solving the eigenvalue problem.

There exist several methods for the calculation of the eigenvalues from Eq. 3.61. In a study conducted at the Argonne National Laboratory, a package of FORTRAN subroutines were developed for computing the eigenvalues and eigenvectors of matrices with various properties (Garbow, 1974). This package has been referred as EISPACK, and it was made available to the public for general use. In this study, the subroutines available in this package were used directly in VecTor2 to determine the modal frequencies.

After the modal frequencies are determined, a_0 and a_1 in Eq. 3.4 can be obtained from Eq. 3.6, for specified modes i and j and their corresponding damping ratios. As mentioned earlier, the initial linearly elastic stiffness matrix is used to calculate the damping matrix as in Eq. 3.4, and the damping matrix is kept constant throughout the analysis.

3.4.2 Direct Integration Method with Secant Stiffness

The method presented in Section 3.3.1 can be re-derived to employ secant stiffness properties instead of tangential properties, as follows.

The equation of motion, as in Eq. 3.36 and Eq. 3.37, can be expressed in terms of total loads as follows.

$$m\ddot{u}_i + c\dot{u}_i + \bar{k}_i u_i = p_0 + p_i \quad (3.62)$$

$$m\ddot{u}_{i+1} + c\dot{u}_{i+1} + \bar{k}_{i+1} u_{i+1} = p_0 + p_{i+1} \quad (3.63)$$

where p_0 represents the constant forces, if present, and $\bar{k}$ is the secant stiffness, such that

$$\bar{k}_i = \frac{f_s(t)}{u_i} \text{ and } \bar{k}_{i+1} = \frac{f_s(t + \Delta t)}{u_{i+1}} \quad (3.64)$$

Eq. 3.41 can be substituted into Eq. 3.63 to give

$$m(\ddot{u}_i + \Delta\ddot{u}) + c(\dot{u}_i + \Delta\dot{u}) + \bar{k}_{i+1} u_{i+1} = p_0 + p_{i+1} \quad (3.65)$$

Eq. 3.45 can be substituted into Eq. 3.65; thus

$$m(\ddot{u}_i + \Delta\ddot{u}) + c(\dot{u}_i + \Delta t \ddot{u}_i + \gamma \Delta t \Delta\ddot{u}) + \bar{k}_{i+1} u_{i+1} = p_0 + p_{i+1} \quad (3.66)$$

which can be rearranged as

$$[m + c \gamma \Delta t] \Delta\ddot{u} + m\ddot{u}_i + c\dot{u}_i + c \Delta t \ddot{u}_i + \bar{k}_{i+1} u_{i+1} = p_0 + p_{i+1} \quad (3.67)$$

If Eq. 3.47 is substituted in Eq. 3.67, the following equation can be obtained:

$$\begin{aligned} \frac{[m + c\gamma\Delta t]}{\beta\Delta t^2}u_{i+1} - \frac{[m + c\gamma\Delta t]}{\beta\Delta t^2}u_{i+1} - \frac{[m + c\gamma\Delta t]}{\beta\Delta t}\dot{u}_i - \frac{[m + c\gamma\Delta t]}{2\beta}\ddot{u}_i + \\ m\ddot{u}_i + c\dot{u}_i + c\Delta t\ddot{u}_i + \bar{k}_{i+1}u_{i+1} = p_0 + p_{i+1} \end{aligned} \quad (3.68)$$

Eq. 3.68 can be rearranged as follows.

$$\left[\bar{k}_{i+1} + \frac{m + c\gamma\Delta t}{\beta\Delta t^2} \right] u_{i+1} = p_0 + p_{i+1} + \frac{m + c\gamma\Delta t}{\beta} \left[\frac{u_i}{\Delta t^2} + \frac{\dot{u}_i}{\Delta t} + \frac{\ddot{u}_i}{2} \right] - c[\dot{u}_i + \Delta t\ddot{u}_i] - m\ddot{u}_i \quad \dots (3.69)$$

If the quantities in Eq. 3.69 are replaced with their matrix equivalents, and Eq. 3.4 is substituted, the following equation can be obtained.

$$\begin{aligned} \left[\bar{\mathbf{k}}_{i+1} + \frac{1 + \gamma\Delta t a_0}{\beta\Delta t^2} \mathbf{m} + \frac{\gamma\Delta t a_1}{\beta\Delta t^2} \mathbf{k}_0 \right] \mathbf{u}_{i+1} = \mathbf{p}_0 + \mathbf{p}_{i+1} + \\ \left[\frac{1 + \gamma\Delta t a_0}{\beta} \mathbf{m} + \frac{\gamma\Delta t a_1}{\beta} \mathbf{k}_0 \right] \left\{ \frac{\mathbf{u}_i}{\Delta t^2} + \frac{\dot{\mathbf{u}}_i}{\Delta t} + \frac{\ddot{\mathbf{u}}_i}{2} \right\} - [a_0 \mathbf{m} + a_1 \mathbf{k}_0] \{\dot{\mathbf{u}}_i + \Delta t\ddot{\mathbf{u}}_i\} - \mathbf{m}\ddot{\mathbf{u}}_i \end{aligned} \quad (3.70)$$

where $\mathbf{k}_0$ is initial linearly elastic stiffness matrix used in Eq. 3.4. Note that in this equation, the only unknowns are $\mathbf{k}_{i+1}$ and $\mathbf{u}_{i+1}$; they can be solved through an iterative procedure as explained in the following section.

3.4.3 Dynamic Analysis Algorithms

As mentioned earlier, dynamic analysis algorithms were introduced into the existing structure of VecTor2 as additional subroutines. The flowchart of the dynamic analysis routine implemented is summarized in Figure 3.9. The steps that were modified or added to in the static analysis routine are shown with dotted lines. The flowchart summarizing the calculations for finding the coefficients a_0 and a_1 for damping calculations is presented in Figure 3.10.

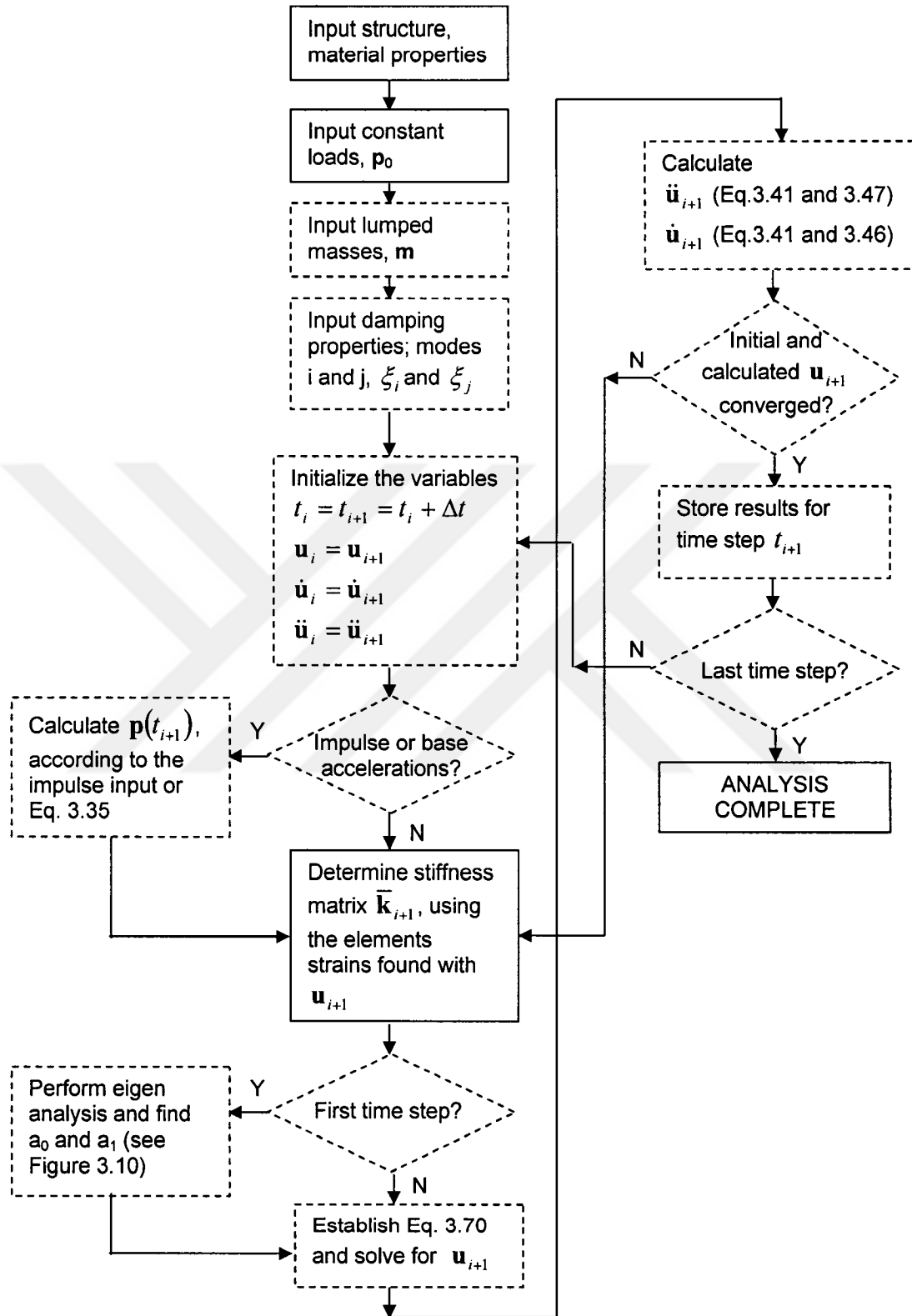


Figure 3.9 Flowchart for dynamic analysis with VecTor2

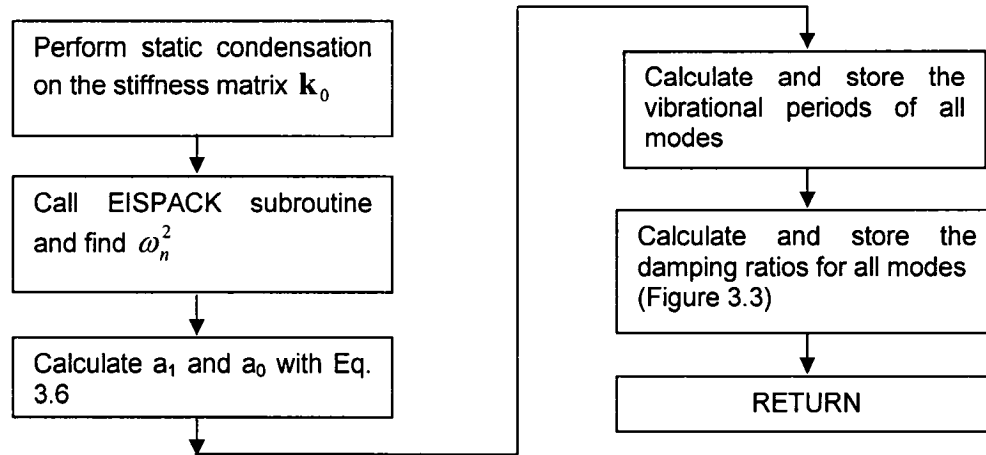


Figure 3.10 Finding the coefficients a_0 and a_1 for damping calculations

3.5 Linear Elastic Verification of VecTor2 Dynamic Analysis

In order to confirm the validity of the new code, a study for verification was carried out on a simple structure. A single-mass fixed-base vertical structure, 100 mm in width and 1000 mm in height, was selected as a test model (see Figure 3.11). The thickness was selected as 50 mm. The structure was modeled in VecTor2 as a planar structure, consisting of 891 nodes and 800 elements of 10 x 12.5 mm in size. The nodes corresponding to the base of the structure were restrained against movements, simulating a fixed-base condition. A 500 kg of lumped mass was assigned to the top middle node (node 886), whereas the rest of the nodes had no mass assigned. Since the selected structure had only one mass, it could be treated as a SDOF system, which would facilitate the derivation of its analytical dynamic response. No damping was assigned since VecTor2 is not capable of incorporating damping for a system with only one mass; a minimum of two masses is required if damping is to be included.

The test structure was subjected to lateral static loads, free vibrations, as well as impulse forces and base accelerations, and it was analyzed as a linearly elastic structure with VecTor2. Results obtained from VecTor2 were compared with analytical solutions. The following sections present details of these comparisons for each type of load considered.

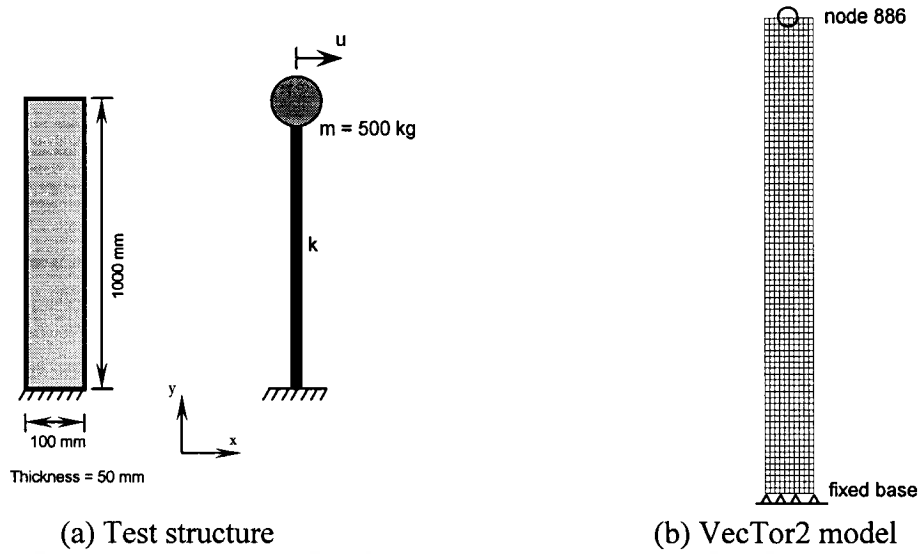


Figure 3.11 Test structure and finite element model

3.5.1 Static Load

The elastic modulus E for the structure was arbitrarily assigned as 40000 MPa. Consequently, the lateral stiffness k of the structure can be calculated by using basic linear elastic relations as follows:

$$k = \frac{3EI}{h^3} = \frac{3(40000) \frac{50(100^3)}{12}}{1000^3} = 500 \text{ N/mm} = 0.5 \text{ kN/mm} \quad (3.71)$$

where I is the moment of inertia and h is the height of the structure. An increasing point load was applied in the x -direction at node 886 to verify the calculated stiffness of the structure. The resulting load-displacement curve is presented in Figure 3.12. The average stiffness of the structure was found as 0.500 kN/mm, which verified the stiffness calculations. Accordingly, the natural period of vibration of the SDOF system was calculated as $T_n = 0.20$ sec. and natural circular frequency was found to be $\omega_n = 31.63$ rad/s.

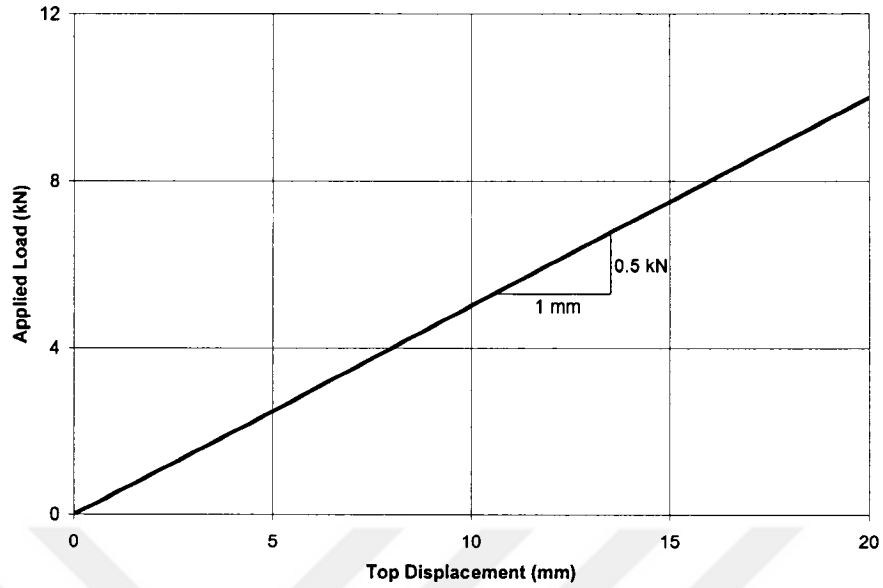


Figure 3.12 Static response of the test structure

3.5.2 Free Vibrations

The test structure was subjected to a free vibration by assigning a 0.5 m/s initial velocity to the mass at node 886. The constant acceleration method was used for the time step procedure. The time step was selected as 0.0001 seconds and results were recorded at every tenth time step, i.e. at 0.001 second intervals. The exact analytical dynamic displacement response can be derived for an undamped SDOF system; it is given by the following equation.

$$u(t) = u(0)\cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n}\sin(\omega_n t) \quad (3.72)$$

The exact analytical response and the response obtained from VecTor2 analysis are compared in Figure 3.13 for the top displacements. As seen in the figure, the results are in almost perfect agreement. Discrepancies between two solutions were found to be less than 1%.

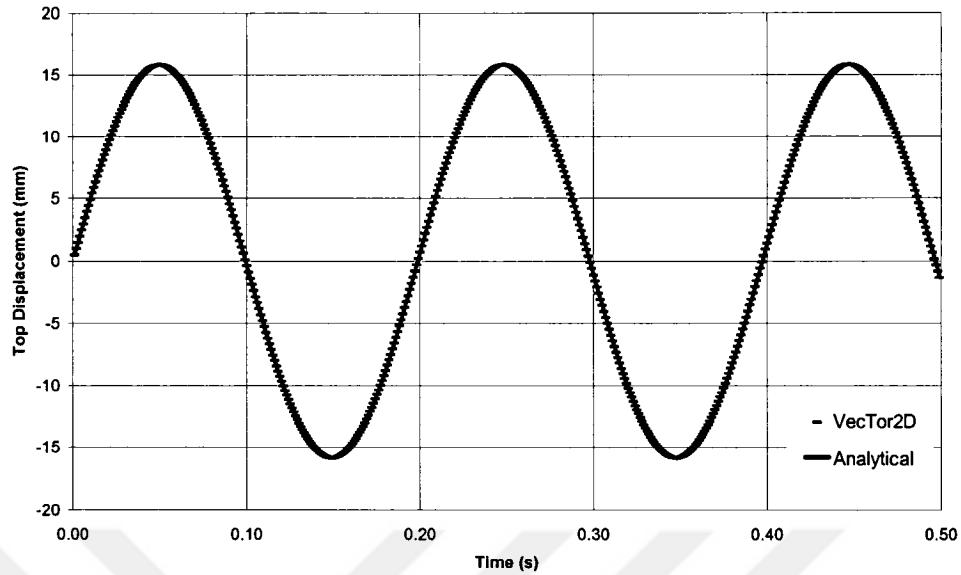


Figure 3.13 Comparison between exact and numerical response, free vibration

3.5.3 Impulse Forces

Two set of impulse forces with different characteristics were applied to the tip of the test structure at node 886. The first set of forces was with a short 0.1 s duration, which basically induced a free vibration, whereas the second set had a 0.8 s duration causing a relatively irregular response. The force-time histories for the applied impulse forces are presented in Figure 3.14. The VecTor2 analysis parameters were identical with the ones used for the free vibration response.

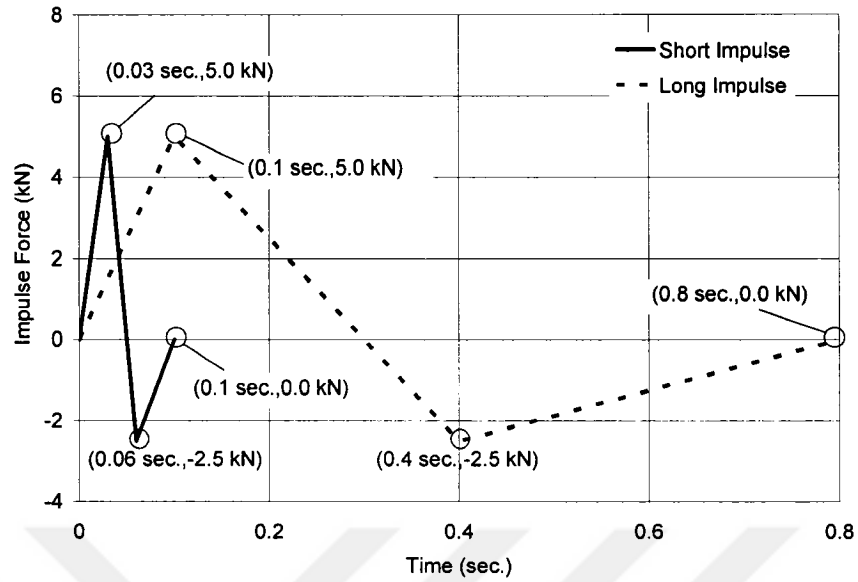


Figure 3.14 Impulse forces applied on the test structure

The exact analytical displacement response of the SDOF structure was calculated by using Duhamel's integral⁵, summarized below. The notations for the variables used in the formulation are defined in Figure 3.15.

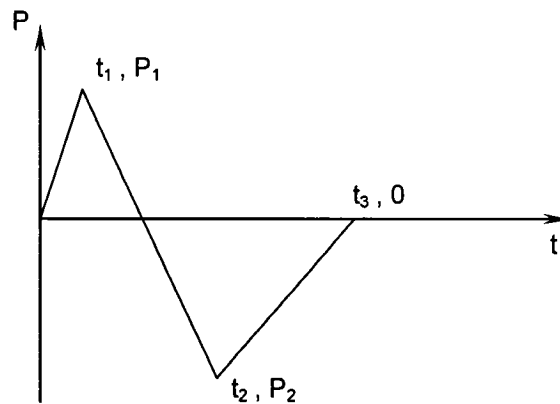


Figure 3.15 Notation for the analytical response of applied impulse forces

⁵ For more information on Duhamel's integral, refer to Chopra, A.K. (2001). *Dynamics of Structures*, Prentice-Hall, Inc., Chapter 4.

For $t \leq t_1$;

$$u(t) = \frac{P_1}{k} \left(\frac{t}{t_1} - \frac{\sin(\omega_n t)}{\omega_n t_1} \right) \quad (3.73)$$

For $t_1 < t \leq t_2$;

$$u(t) = u(t_1) \cos(\omega_n(t - t_1)) + \frac{\dot{u}(t_1)}{\omega_n} \sin(\omega_n(t - t_1)) + \frac{P_1}{k} [1 - \cos(\omega_n(t - t_1))] + \frac{P_2 - P_1}{k} \left(\frac{t - t_1}{t_2 - t_1} - \frac{\sin(\omega_n(t - t_1))}{\omega_n(t_2 - t_1)} \right) \quad (3.74)$$

For $t_2 < t \leq t_3$;

$$u(t) = u(t_2) \cos(\omega_n(t - t_2)) + \frac{\dot{u}(t_2)}{\omega_n} \sin(\omega_n(t - t_2)) + \frac{P_2}{k} [1 - \cos(\omega_n(t - t_2))] - \frac{P_2}{k} \left(\frac{t - t_2}{t_3 - t_2} - \frac{\sin(\omega_n(t - t_2))}{\omega_n(t_3 - t_2)} \right) \quad (3.75)$$

For $t > t_3$;

$$u(t) = u(t_3) \cos(\omega_n(t - t_3)) + \frac{\dot{u}(t_3)}{\omega_n} \sin(\omega_n(t - t_3)) \quad (3.76)$$

where

$$\dot{u}(t_1) = \frac{P_1}{k} \left(\frac{1}{t_1} - \frac{\cos(\omega_n t_1)}{t_1} \right) \quad (3.77)$$

$$\dot{u}(t_2) = -u(t_1) \omega_n \sin(\omega_n(t_2 - t_1)) + \dot{u}(t_1) \cos(\omega_n(t_2 - t_1)) + \frac{P_1}{k} \omega_n \sin(\omega_n(t_2 - t_1)) + \frac{P_2 - P_1}{k} \left(\frac{1}{t_2 - t_1} - \frac{\cos(\omega_n(t_2 - t_1))}{(t_2 - t_1)} \right) \quad (3.78)$$

$$\begin{aligned} \dot{u}(t_3) = & -u(t_2)\omega_n \sin(\omega_n(t_3 - t_2)) + \dot{u}(t_2)\cos(\omega_n(t_3 - t_2)) + \\ & \frac{P_2}{k}\omega_n \sin(\omega_n(t_3 - t_2)) - \frac{P_2}{k}\left(\frac{1}{t_3 - t_2} - \frac{\cos(\omega_n(t_3 - t_2))}{(t_3 - t_2)}\right) \end{aligned} \quad (3.79)$$

Values for $u(t_1)$, $u(t_2)$ and $u(t_3)$ can be obtained from Eqs. 3.73, 3.74 and 3.75, respectively. Comparisons between the analytical response and the response obtained from VecTor2 are presented in Figure 3.16 and Figure 3.17, for the short and long impulse forces, respectively.

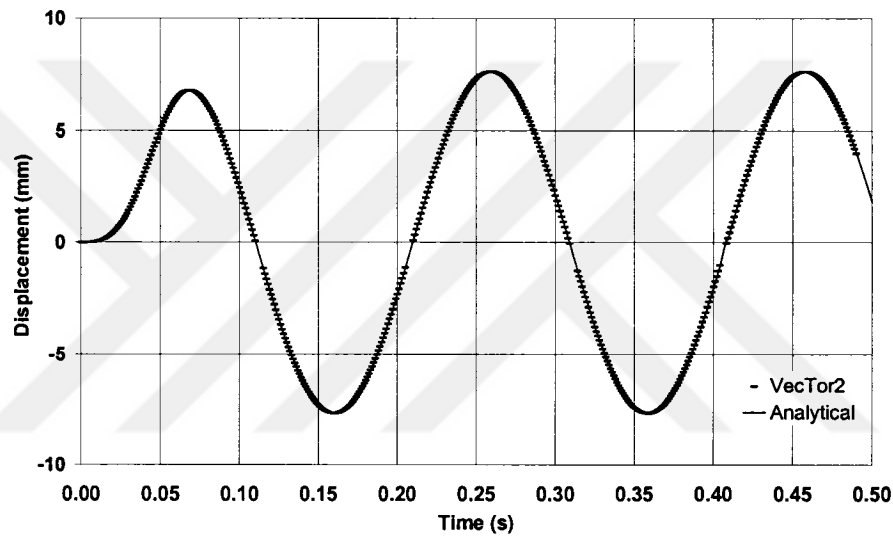


Figure 3.16 Comparison between exact and numerical response, short impulse

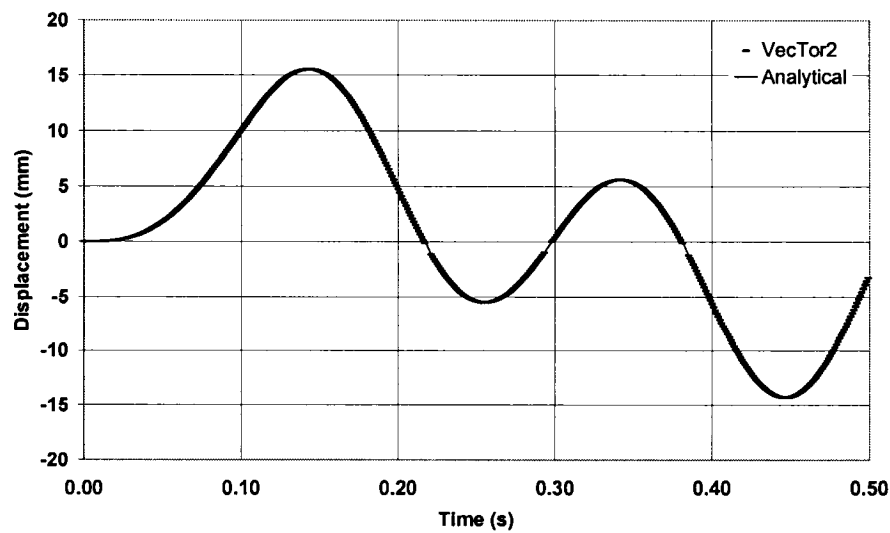


Figure 3.17 Comparison between exact and numerical response, long impulse

As seen in the figures, the agreement between the analytical and VecTor2 responses was almost exact. Discrepancies between two solutions were found to be less than 1%.

3.5.4 Base Accelerations

Two different base acceleration histories were applied to the structure, parallel to the x-direction: the Imperial Valley (El Centro–1940) and Northridge (Santa Monica–1994) earthquake records, both obtained from National Information Service for Earthquake Engineering (NISEE) of University of California, Berkeley. The maximum accelerations observed in the Imperial Valley and Northridge records were 2.1 m/sec^2 and 7.38 m/sec^2 , respectively. The Northridge record was higher in frequency than the Imperial Valley record. The time plots of these records are given in Figure 3.18 and Figure 3.19.

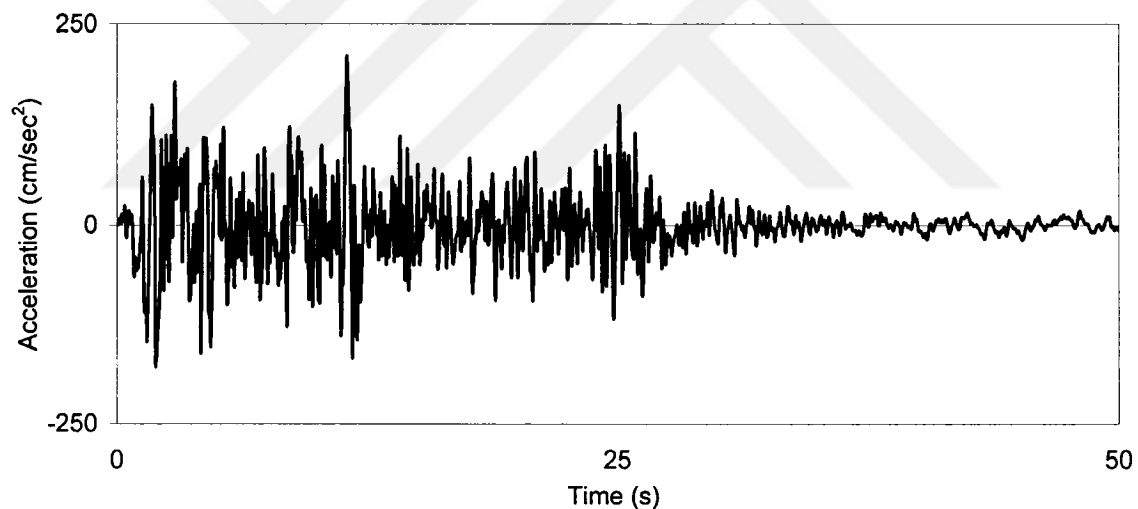


Figure 3.18 Imperial Valley Earthquake acceleration record (El Centro–1940)

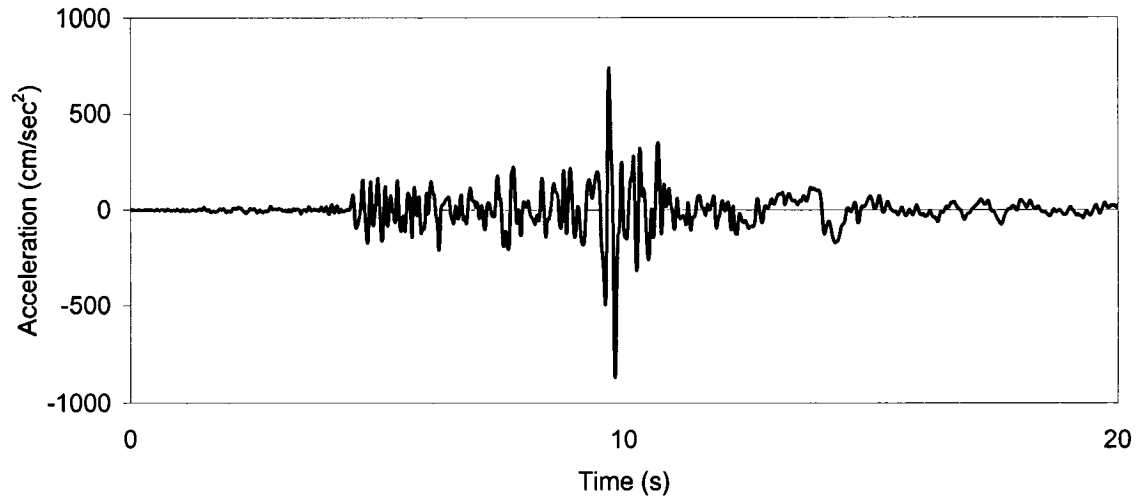
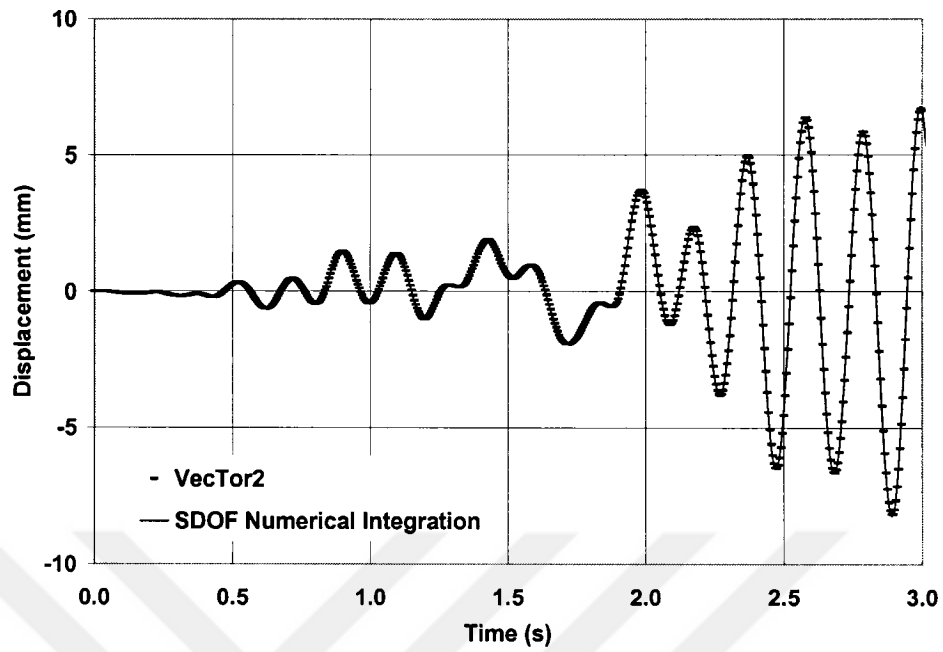


Figure 3.19 Northridge Earthquake acceleration record (Santa Monica–1994)

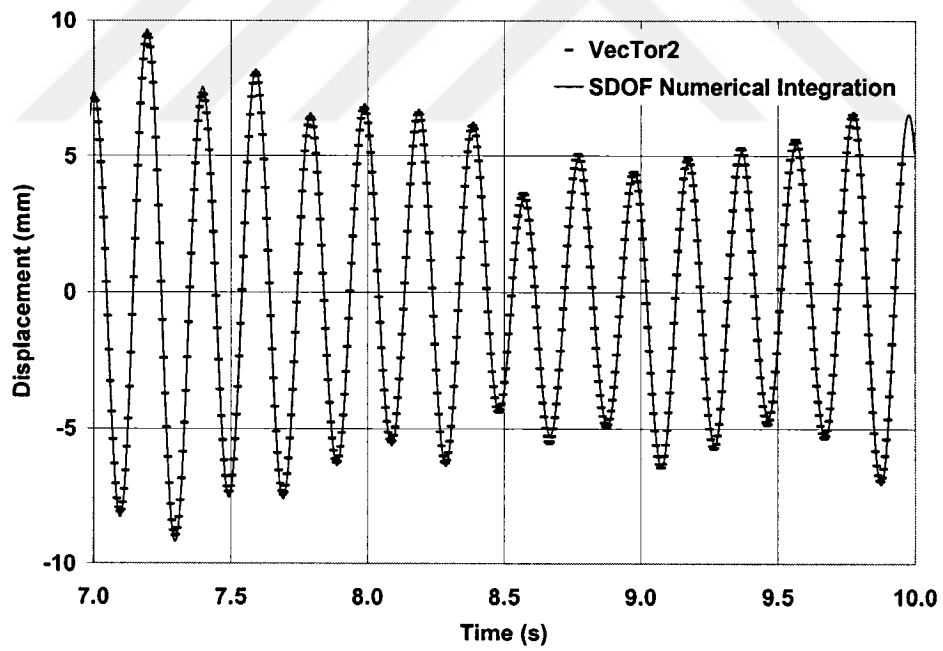
The analyses were carried out with VecTor2 for the first 10 seconds of the response. Analysis parameters were identical with the ones used for the free vibration response.

Since the earthquake records were highly irregular, an analytical closed form determination of the response was not practical. Therefore, the response of the SDOF system was calculated with a spreadsheet by using numerical integration as explained in Section 3.3.1. The results obtained from the spreadsheet and VecTor2 are presented in Figure 3.20 and Figure 3.21, for Imperial Valley and Northridge records, respectively. As seen in the figures, the agreement was almost perfect between two methods. Discrepancies between two solutions were found to be less than 1%.

In conclusion, the analyses carried out for the verification of VecTor2 proved the accuracy of the new dynamic analysis algorithms. Minimal discrepancies occurring between the exact analytical solutions and VecTor2 results can be attributed to the discretization errors inherent with the finite element method formulations.

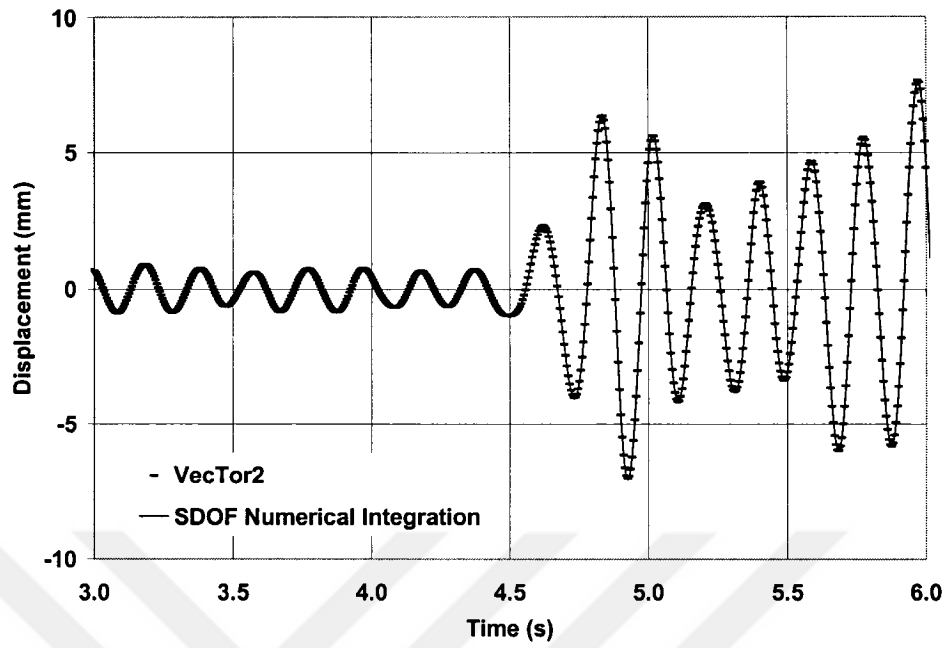


(a) 0 - 3.0 second response

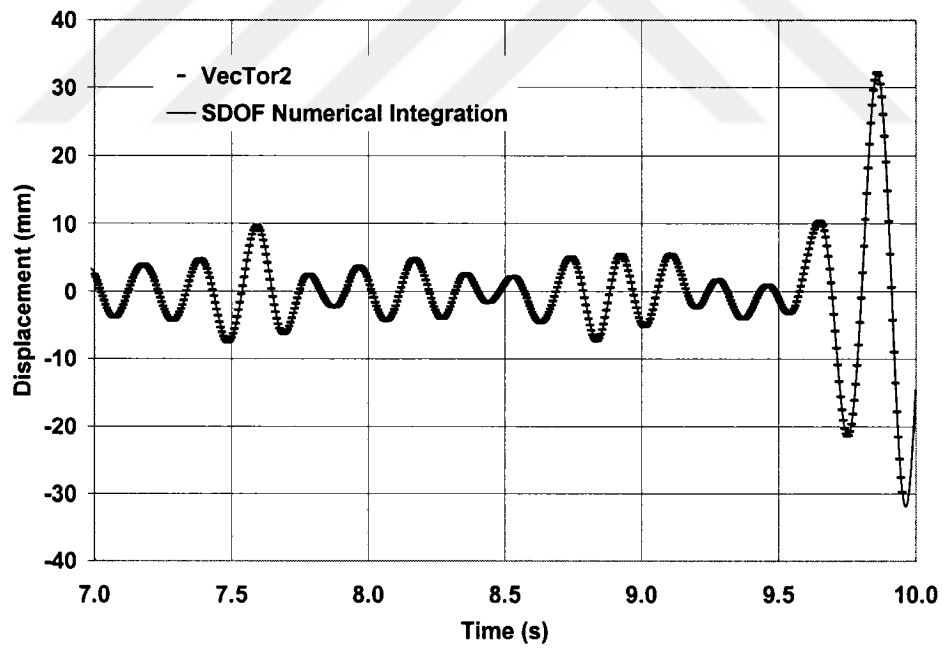


(b) 7.0 – 10.0 second response

Figure 3.20 Comparison between SDOF and VecTor2 response, Imperial Valley record



(a) 3.0 – 6.0 second response



(b) 7.0 – 10.0 second response

Figure 3.21 Comparison between SDOF and VecTor2 response, Northridge record

4 EXPERIMENTAL PROGRAM

4.1 Introduction

As discussed in the previous chapter, VecTor2 is a sophisticated nonlinear finite element analysis (NLFEA) program, which gives substantial detailed information regarding the analysed structure. On the other hand, most of the impact experiments reported in the literature were either qualitative in nature, or, because of the complexities involved in the instrumentation of an impact test, collected and reported very little information during the experiment. Therefore, a well-instrumented test program was needed in order to verify the new algorithms introduced into VecTor2 for impact analysis. For this purpose, eight beams (four pairs) with varying shear reinforcement ratio were tested under falling impact weights at the University of Toronto Structural Laboratories. This chapter documents the details of the test program, including specimen properties, test setup, instrumentation and test procedures. Test results and discussions are given in the following chapters.

4.2 Test Specimens

Eight reinforced concrete beams were constructed as specimens for this test program. The dimensions of the test specimens were determined using experience gained from a previous pilot test program conducted by Ho (2004). In that program, the dimensions of the specimens were 560 mm in height and 300 mm in width, and it was observed that it was not possible to induce a significant level of damage in them with the drop-weights available. Therefore, to investigate damage levels of a wider range, the width and the height of the specimens were reduced. The final dimensions of 250 mm width, 410 mm height and 4880 mm length were selected. The specimens were tested under simply supported conditions with a shear span of 1500 mm, leaving a 940 mm overhang at each end (see Figure 4.1).

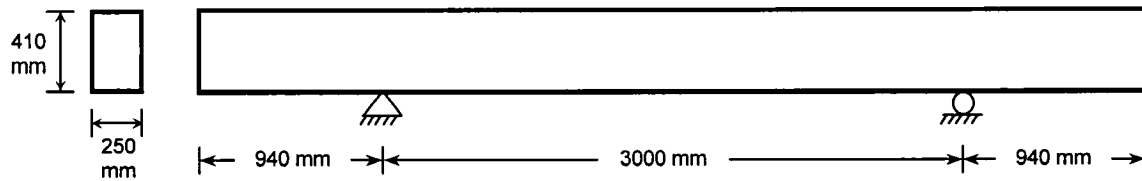


Figure 4.1 Specimen dimensions

For this test program, it was decided to have a duplicate of each specimen in order to investigate the effects of different impact drop-weights on undamaged specimens. An identical twin of a specimen would also be valuable in case of an unexpected result in the first test, which might warrant a repeat of the test with either the same or different type of load. Therefore, the specimens were cast as four pairs; that is, with identical geometry and reinforcement. The pairs were cast at the same time, using the same concrete batch, to ensure having the same concrete material properties for both.

All specimens had the same amount and configuration of longitudinal reinforcement. They were doubly-reinforced with the longitudinal reinforcement bars placed symmetrically along the height of the specimens, with the intention of having the same member properties in both positive and negative flexure. CSA Standard G30.18 No.30 bars, with a 700 mm^2 cross-sectional area and a 29.9 mm nominal diameter, were used as longitudinal reinforcement. They were placed as two bars at the bottom and two at the top, spanning the entire length of the specimen. A 38 mm (1.5 inches) clear cover was provided between the top and the bottom beam surfaces and the bars. The transverse reinforcement ratios for the specimens were varied as 0.0%, 0.1%, 0.2% and 0.4%. Closed stirrups were used, which were made from U.S. Standard D-6 deformed wire with a 38.71 mm^2 cross-sectional area and a 7.01 mm diameter (see Figure 4.2).

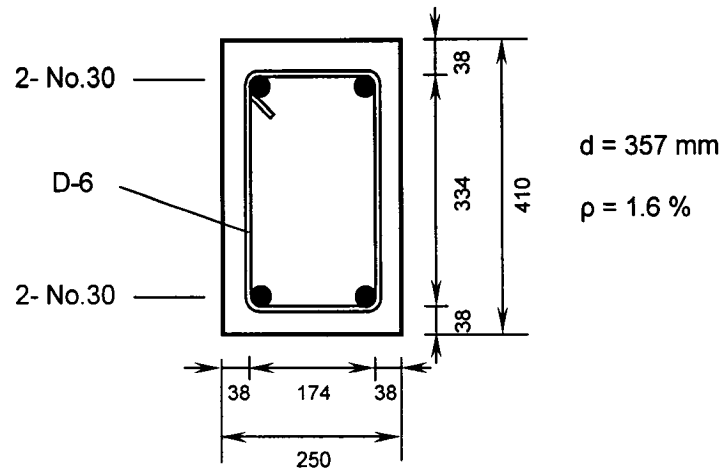


Figure 4.2 Specimen cross-section (all dimensions are in millimetres)

For easier reference, the specimens were named according to their transverse reinforcement ratio, the only property intentionally varied between the pairs. Figure 4.3 explains the convention adopted for naming the specimens, used throughout this report. The transverse reinforcement ratios and stirrup spacings for the specimens are given in Table 4.1.

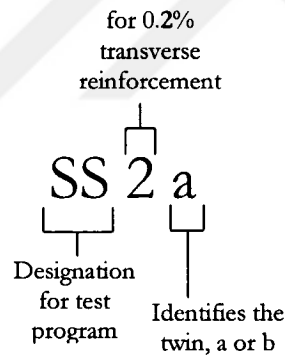


Figure 4.3 Naming convention for the beams

Table 4.1 Transverse reinforcement ratios and stirrup spacing for the beams

Specimen	Transverse Reinforcement Ratio	Stirrup Spacing (mm)
SS0a, SS0b	0.0%	-
SS1a, SS1b	0.1%	300
SS2a, SS2b	0.2%	200
SS3a, SS3b	0.3%	100

In this report, locations on the specimen will be referenced according to their orientation in the laboratory. For example, the beams were longitudinally oriented in the north-south direction, so the ends of the beam will be referred to as the north end or the south end, whereas the faces of the specimens will be referred to as the east face or the west face.

4.3 Test Setup

All of the specimens were tested under the same simply supported conditions. From similar tests done in the literature, as well as the observations during pilot test program, it was expected that the beams would experience uplift from the supports during the impact. This behaviour, if not prevented, would render the data collected after the first bounce-back of the impact-induced vibration useless, thus severely limiting the amount of useful data. Therefore, in this test program, it was decided to employ an arrangement to prevent uplift of the specimen without creating any moments at the supports during vibrations.

Uplift of specimens at the supports was prevented by holding down the beam with No.30 reinforcement bars henceforth referred to as support bars. The bars were threaded at both ends and bolted at the top end to a hollow structural steel (HSS) section, which crossed the specimen in width (see Figure 4.4). At the bottom end, in order to enable free-rotation, the bars were fitted to spherical bearings which were supported by steel floor beams bolted to the strong floor. Between the specimen and the HSS section at the top, a hinge was placed. With this setup, the specimens were able to rotate freely with minimal secondary moments at the supports (see Figure 4.5).

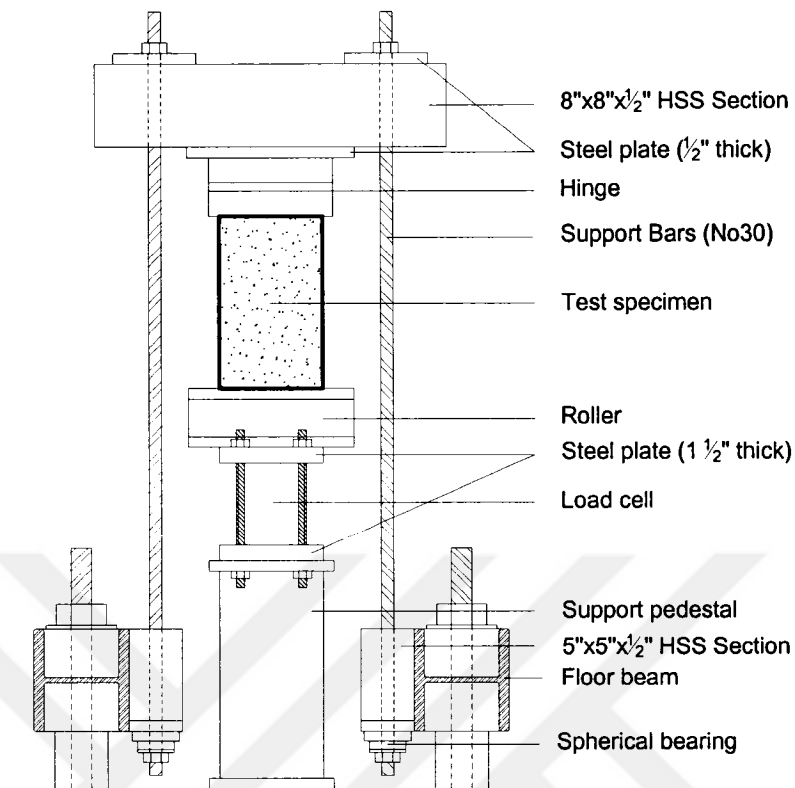


Figure 4.4 Test setup cross section at the supports

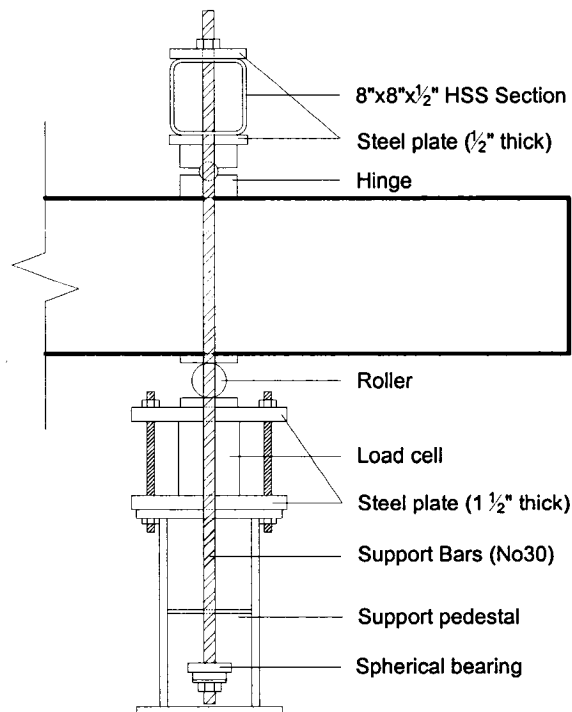


Figure 4.5 Side view of support (floor beams are not shown)

The supports bars were identified with numbers from 1 to 4. They were fitted with strain gauges and calibrated under tension, so that by measuring their strains it would be possible to calculate the force in the bars (calibration results are given in Appendix A). To prevent any possible uplifting, the bars were pre-tensioned with about 15 kN force by tightening the bolts at the threaded ends and monitoring the strain readings from the gauges.

The specimens sat on bottom roller supports at both ends, whereas hinges were used at the top (see Figure 4.5). Although these support conditions rendered the system longitudinally unrestrained, it was chosen to prevent any possible axial load developing during the testing which would be difficult to measure and would further complicate analyses. As a result of this instability in the test setup, it was observed by both video records and a displacement sensor that the specimens typically vibrated slightly in the longitudinal direction during the tests. The effects of this movement on displacement measurements were found to be insignificant.

4.4 Material Properties

The concrete used for the specimens was obtained from a local ready-mix concrete company, Dufferin Custom Concrete Group. Delivery orders specified 25 MPa minimum concrete strength, with 10 mm maximum aggregate size and 100 mm slump. No chemical additives were required. As mentioned previously, the specimens were cast two at a time and both specimens were cast from a single batch. Table 4.2 gives the casting dates.

Table 4.2 Casting dates of the specimens

Specimen	Casting Date
SS3a, SS3b	June 21, 2004
SS2a, SS2b	August 31, 2004
SS1a, SS1b	September 23, 2004
SS0a, SS0b	October 25, 2004

During the casting, several standard cylinder samples (150 mm in diameter and 300 mm in height) were taken to determine the compressive strength of the concrete. Three cylinders from each batch were kept in a curing room and tested according to the ASTM C39 Standard at the 28th day after the casting. Some cylinders were kept in the same room with the beam specimens, and they were tested on December 21, 2005 after the first four specimens were tested, which was roughly mid-way through the test program. Testing dates and the results of the cylinder tests are summarized in Table 4.3. The values shown in Table 4.3 are the averages of the results obtained from three cylinders from the same batch. Stress-strain curves obtained from the cylinders tested on December 21, 2005 are presented in Appendix A. Note that the typical age of the beam specimens, at the time of impact testing, was more than one year. Hence, any strength variation arising from cylinder testing before and after the impact testing was minimal.

Table 4.3 Cylinder test results

	28 th day		December 21, 2005	
	Peak compressive stress, f'_c (MPa)	Strain at peak stress (ϵ_0)	Peak compressive stress, f'_c (MPa)	Strain at peak stress (ϵ_0)
SS0a, SS0b	39.9	Not measured	50.1	2.32×10^{-3}
SS1a, SS1b	34.9	1.83×10^{-3}	44.7	2.36×10^{-3}
SS2a, SS2b	39.4	1.65×10^{-3}	47.0	2.42×10^{-3}
SS3a, SS3b	37.6	1.70×10^{-3}	46.7	2.51×10^{-3}

In addition to the cylinder tests, standard modulus of rupture samples (152 x 152 x 508 mm prisms) were also taken and tested, in order to determine the tensile strength of the concrete. One set of the samples was tested at the 28th day of casting, whereas another set was tested at the end of the test program on April 6, 2006. The modulus of rupture tests were carried out according to the ASTM C78-94 Standard, and the dates and summary of the results are given in Table 4.4. Similar to the cylinder tests, the values in Table 4.4 are the averages of the results obtained from two samples from the same batch. The direct tensile strength of concrete was estimated as 0.55 times the modulus of rupture. Note that the samples tested on the 28th day after casting had higher tensile strengths since they were kept in a curing

room, whereas the other samples were kept in the same room with the beam specimens; hence, they were subjected to drying shrinkage reducing their tensile strength.

Table 4.4 Modulus of rupture test results

	28 th day		April 6, 2006	
	Modulus of rupture, f_{rp} (MPa)	Tensile Strength*, f_{cr} (MPa)	Modulus of rupture, f_{rp} (MPa)	Tensile Strength*, f_{cr} (MPa)
SS0a, SS0b	6.6	3.6	5.9	3.2
SS1a, SS1b	6.0	3.3	5.6	3.1
SS2a, SS2b	6.1	3.4	6.2	3.4
SS3a, SS3b	Not tested	Not tested	5.8	3.2

* Estimated as $f_{cr} = 0.55 f_{rp}$

Standard tensile coupon tests were carried out to determine the properties of the steel reinforcement. The results of these tests, for the two types of bars that were used, are summarized in Table 4.5, whereas the complete stress-strain curves are presented in Appendix A. Note that the D-6 bars were not heat-treated, so they did not have a flat-yield plateau; their yield stresses were calculated using a 0.2% offset to the initial portion of the response curve.

Table 4.5 Steel coupon test results

	Area (mm ²)	Nominal Diameter (mm)	Yield Strain, ϵ_y ($\times 10^{-3}$)	Yield Stress, f_y (MPa)	Ultimate strength, f_u (MPa)	Modulus of Elasticity, E_s (MPa)
No.30	700	29.9	2.38	464	630	195 000
D-6	38.71	7.0	3.18	605	652	190 250

Since they would be required in the analysis phase of the study, the densities of concrete and steel bars were also measured. The density of concrete was determined by weighting the cylinder samples, whereas the steel density was determined by weighting a certain length of the bar. Average results are summarized in Table 4.6.

Table 4.6 Material Densities

Material	Density
Concrete (SS3a, SS3b)	2425 kg/m ³
Concrete (SS2a, SS2b)	2420 kg/m ³
Concrete (SS1a, SS1b)	2473 kg/m ³
Concrete (SS0a, SS0b)	2437 kg/m ³
Steel (No.30 bar)	5.3 kg/m
Steel (D-6 stirrup)	0.48 kg/stirrup

4.5 Instrumentation

As mentioned previously, one of the main reasons in carrying out such a test program was to address the lack of sufficient experimental data in the literature. Therefore, the specimens in this test program were instrumented to capture the displacements, accelerations, steel bar strains and support forces. The following sections give the details and specifications of the sensors connected to the specimens, as well as of the data acquisition system. It should be noted that as the test program progressed, some modifications were made to the instrumentation to improve the data collected. Specific details of the instrumentation for the individual specimens are given in relevant sections.

4.5.1 Accelerometers

Seven accelerometers were used for each test. Five were mounted on the specimen to measure the accelerations during the impact-induced vibrations, and two were mounted on the drop-weight to measure the impact force. The accelerometers were EGA-F type miniature accelerometers produced by Entran Sensors & Electronics, and they had ± 1000 g range with ± 3000 g over-range limit and 4000 Hz natural frequency. The manufacturer's technical data sheet is given in the Appendix B.

Five accelerometers were mounted on the west face of the specimen along the south half; four of them were mounted equally spaced between the mid-span and the support, while the fifth was mounted on the middle point of the overhanging portion. The five accelerometers

on the specimen were denoted as A1 to A5, and those on the drop-weights were labelled as A6 and A7. Their locations on the specimen are given in Figure 4.6.

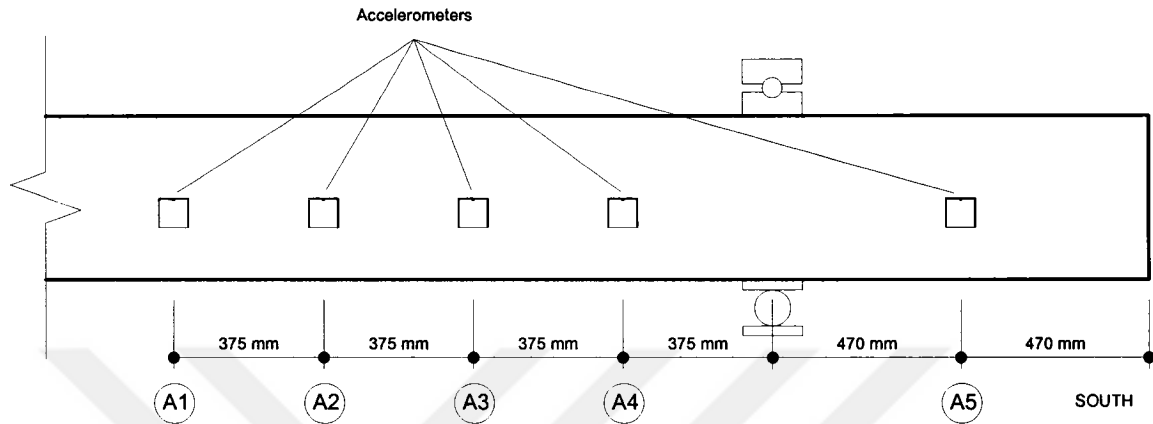
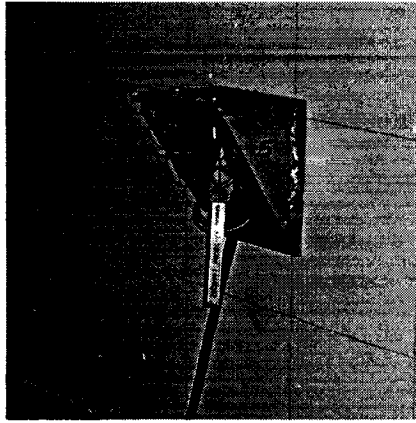
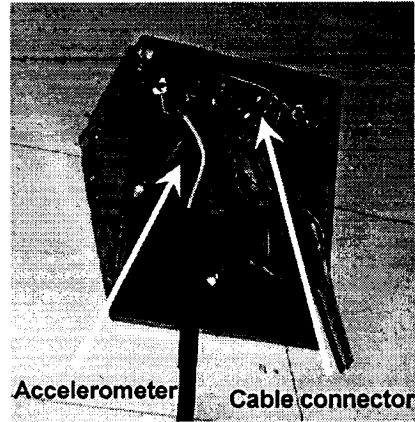


Figure 4.6 Locations of the accelerometers on the test beams

Since the accelerometers had very small dimensions, special brackets were manufactured to mount them onto the specimen (see Figure 4.7). The brackets were made of 50 x 75 mm aluminum angles, reinforced with triangular pieces welded on the sides. The accelerometer was bolted close to the edge to minimize the effects of the vibration of the bracket itself, and a cable connector was mounted to the edge for connection of the cables. The bracket was then fixed at the mid-height of the specimen with three screws. Two accelerometers on the drop-weights were mounted using the same brackets, but with a different electrical connection (see Figure 4.8). Two brackets were screwed to the opposite sides of the weight to obtain an average reading between the two of them, thus reducing the possible measurement errors due to rocking of the weight during impact.



(a) Side view



(b) Bottom view

Figure 4.7 Aluminum brackets for mounting the accelerometers on the beam

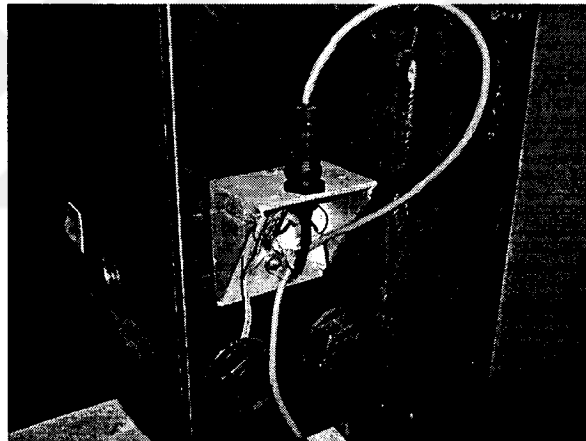


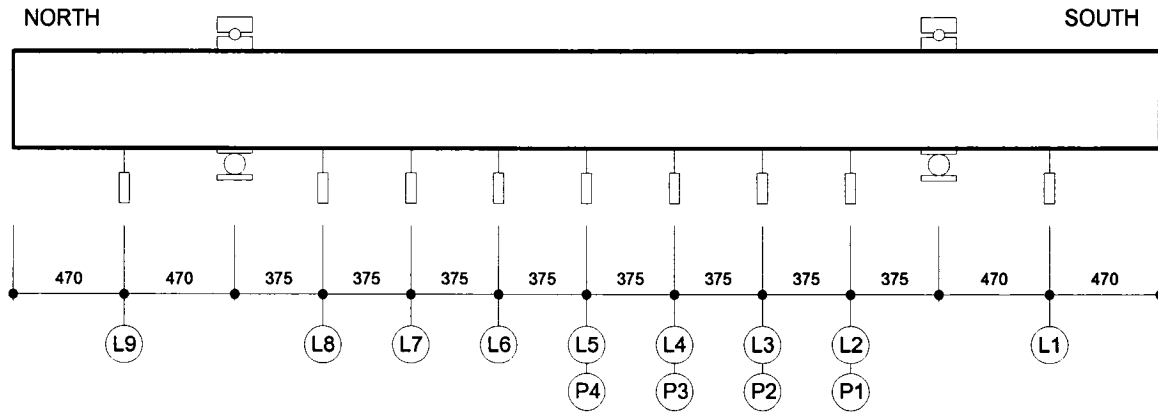
Figure 4.8 Mounting of the accelerometers on the drop-weight

4.5.2 LVDT's and Potentiometers

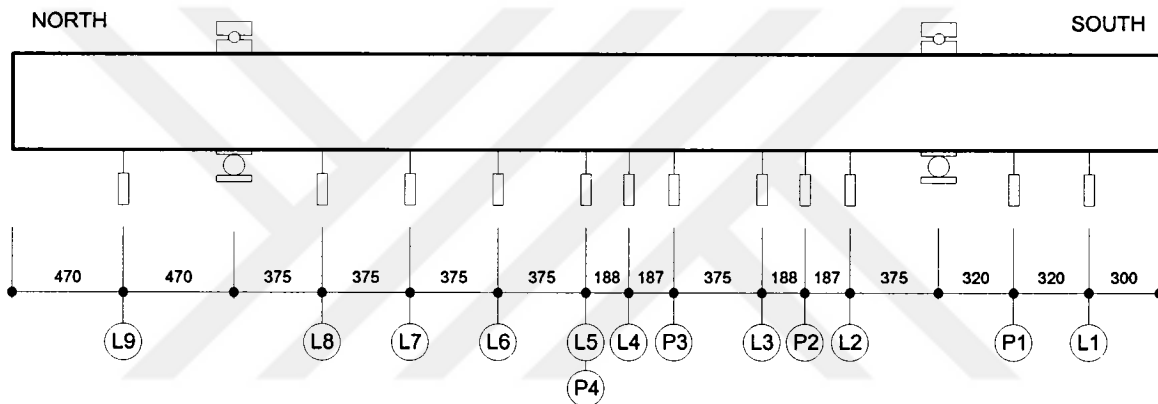
Linear variable differential transformers (LVDT) and linear motion position transducers (potentiometers) were used to measure the displacements along the specimens during impact. The LVDTs used were Schlumberger DC25 type with ± 50 mm or ± 75 mm ranges, and the potentiometers were Duncan Electronics Models 602 and 604 with ± 50 mm or ± 100 mm ranges, respectively. The manufacturer's technical data sheets for these sensors are given in the Appendix B.

For the entire test program, nine LVDTs and 16 potentiometers were used; the LVDTs were denoted as L1 to L9, and the potentiometers were denoted as P1 to P16. The numbers and locations of the displacement sensors were varied between the specimens as the tests progressed. Initially, nine LVDTs and four potentiometers were used. For the first specimen, SS3a, the potentiometers were placed at the same locations as the LVDTs to compare the readings obtained from two different types of sensors. Since the comparisons were satisfactory for the first specimen, three of the potentiometers were later moved to other locations in order to obtain more displacement data, and four more potentiometers were added since it was observed that the number of different locations of displacement measurements were not sufficient to obtain an accurate displaced shape (see Figure 4.9). At this stage, one of the new potentiometers was placed at the north end of the specimen to measure the longitudinal movement during the test. Later, in light of the experience gained from the tests of the first four specimens, all LVDTs were replaced with potentiometers for the second half of the program, since the potentiometers were found to generate significantly less noisy signals compared to LVDTs (see Figure 4.10). The potentiometer previously used to measure the longitudinal movement (P8) was moved under the specimen and its function was replaced with one of the LVDTs (L5). In all specimens, two sensors were attached at the mid-span (100 mm apart), not only to detect any possible rocking movement, but also to have a back-up reading for this important measurement.

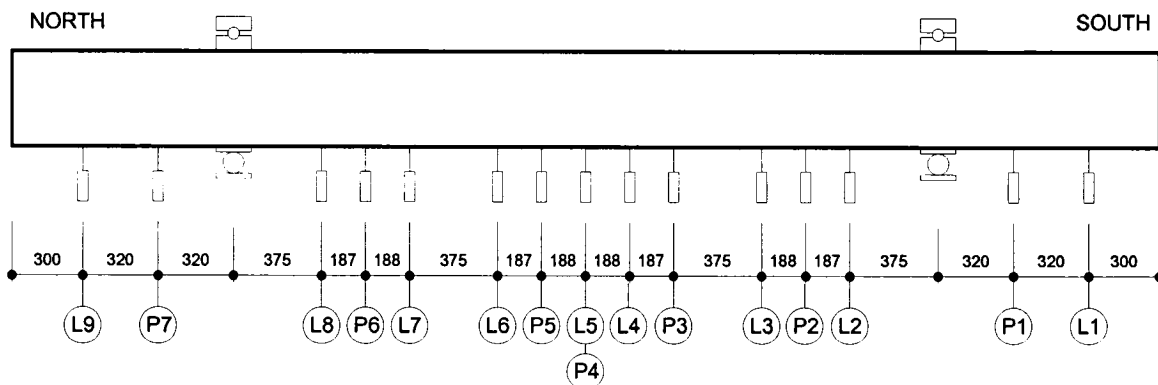
Displacement sensors were located at the bottom of the specimens, fixed to magnetic bases or aluminum brackets. They were protected from possible spalling of concrete by wood blocks and plywood, supported by two steel pedestals under the specimens, symmetrically placed at approximately 550 mm away from the mid-span. The core connecting rods of the sensors were extended using aluminum rods, which were connected to the specimen at the mid-width. To connect the aluminum extensions to the specimens, holes were drilled into the bottom of the specimen and 50 mm long threaded steel rods were glued into the holes with epoxy. Aluminum rods were connected to these threaded rods with plastic shrink tubes (see Figure 4.11). Before each test, all LVDTs and potentiometers were adjusted with a bubble level to make sure they were perfectly vertical.



(a) SS3a



(b) SS2a



(c) SS1a and SS0a (P8 for longitudinal movement is not shown)

Figure 4.9 Displacement sensor locations for SS3a, SS2a, SS1a and SS0a
(All dimensions are in millimetres)

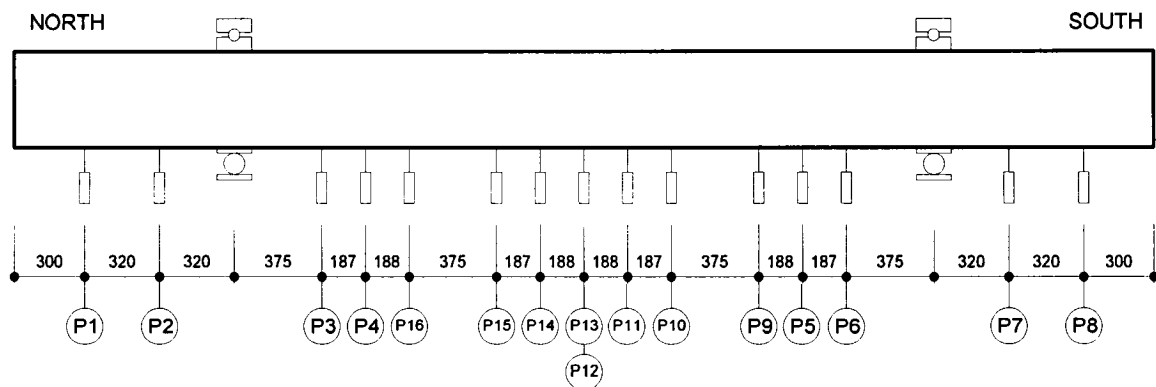
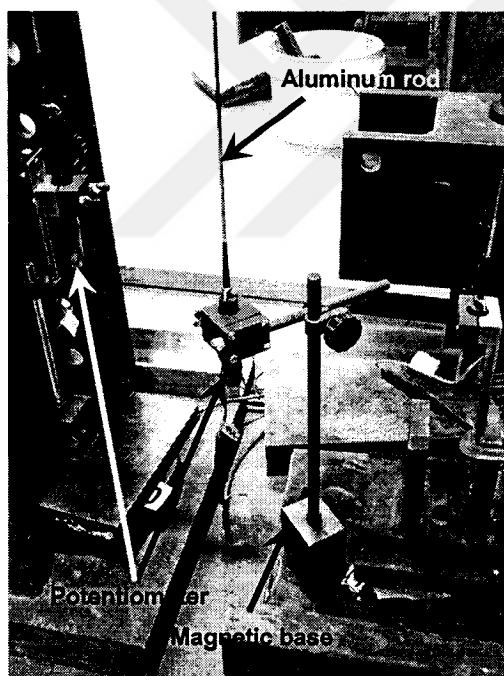
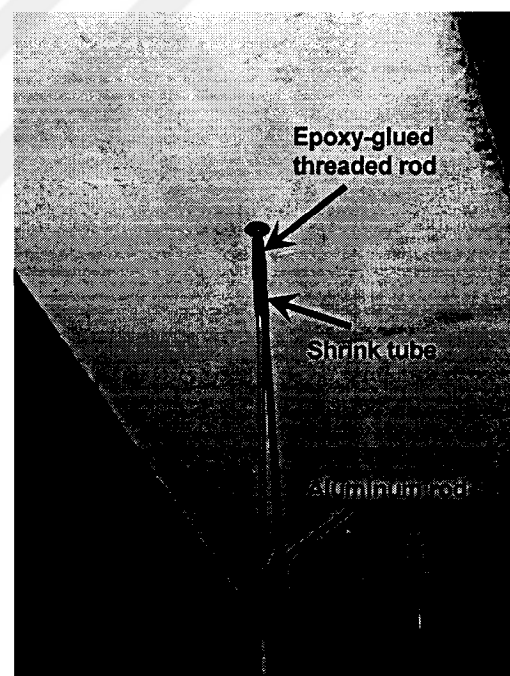


Figure 4.10 Displacement sensor locations for SS3b, SS2b, SS1b and SS0b
(L5 for longitudinal movement is not shown, all dimensions are in millimetres)



(a) Potentiometers



(b) Connection to the specimen

Figure 4.11 Displacement sensors and their connections to the specimens

4.5.3 Strain Gauges

All longitudinal bars were fitted with strain gauges. The gauges were TML type FLA-5-11 with a 5 mm gauge length, manufactured by Tokyo Sokki Kenkyujo Co. Ltd. The bar surfaces at the locations of the gauges were lightly grinded and cleaned using alkaline and acidic chemicals, and the gauges were glued longitudinally using the glue provided by the manufacturer. Figure 4.12 shows a photo of a glued strain gauge, before covering it with paraffin wax and aluminum tape to protect against water in the concrete mix during the casting.

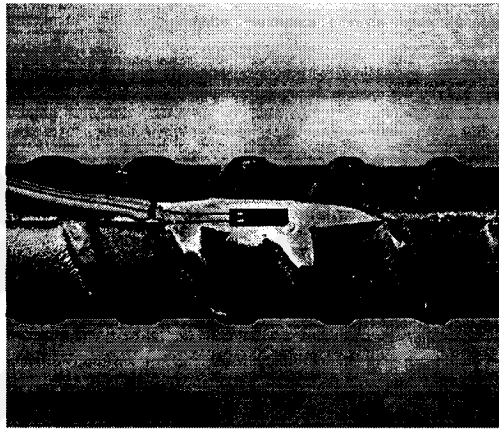
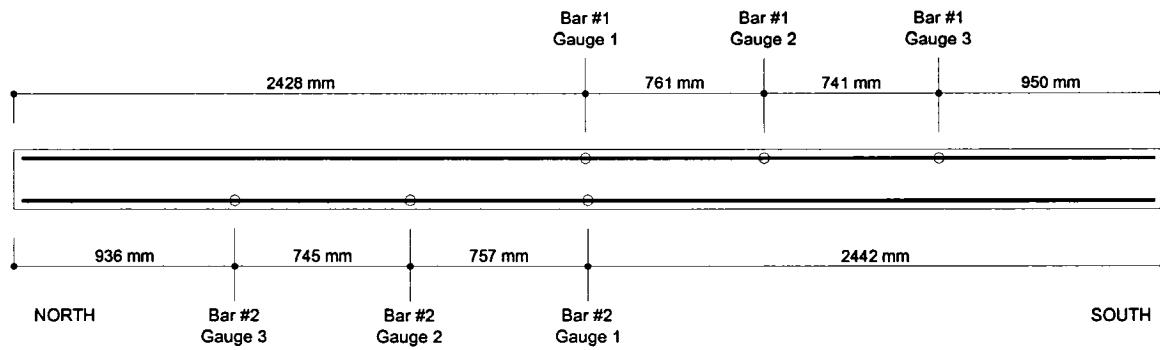
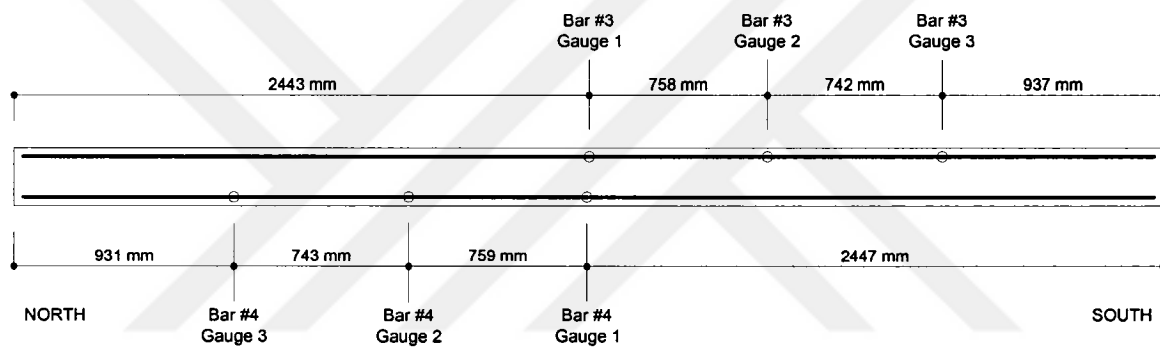


Figure 4.12 Strain gauge glued on a longitudinal bar

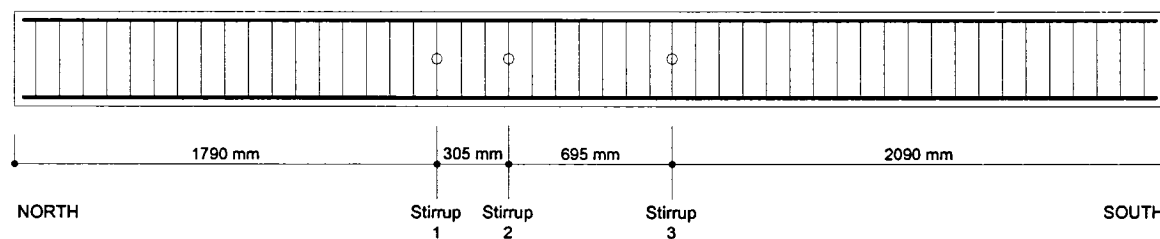
The strain gauges were glued at three locations on all 4880 mm long longitudinal bars: one at the middle and two at the third-points between the support point and mid-span, giving a 750 mm spacing. Three of the stirrups were also each fitted with a strain gauge at the mid-height: two at one side of the mid-span and one on the other side, roughly 300 mm apart. However, exact locations varied. The exact locations of the gauges for each specimen, as measured before casting, are given from Figure 4.13 to Figure 4.20. For reference to gauges, the longitudinal bars were denoted with numbers from 1 to 4 for each specimen and the strain gauges were denoted with numbers along with the bar number, such as “Bar #1 Gauge 2”. Stirrups were denoted with numbers as well, such as “Stirrup 3”. Support bars were also equipped with two strain gauges on the opposite sides, and they were calibrated to measure the axial load on the bars. They were referenced with letters a or b, and named with the bar name, such as “Support Bar #1a”.



(a) Top layer longitudinal bars (top view)

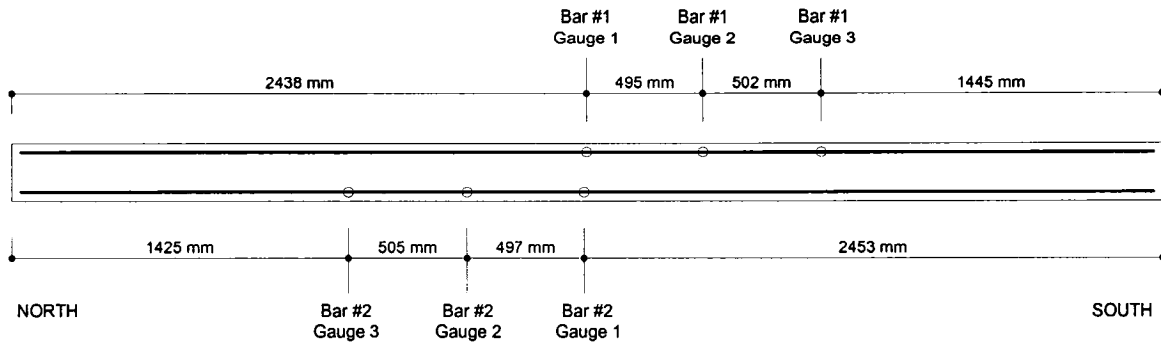


(b) Bottom layer longitudinal bars (top view)

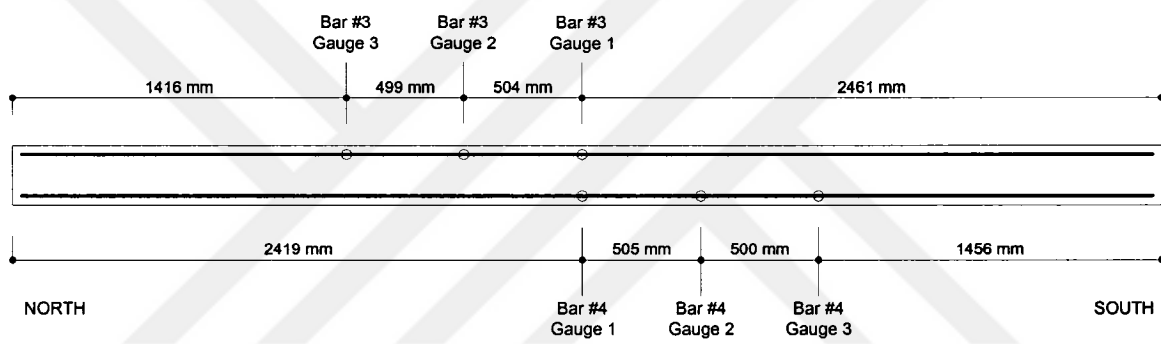


(c) Stirrups (as seen from the west face)

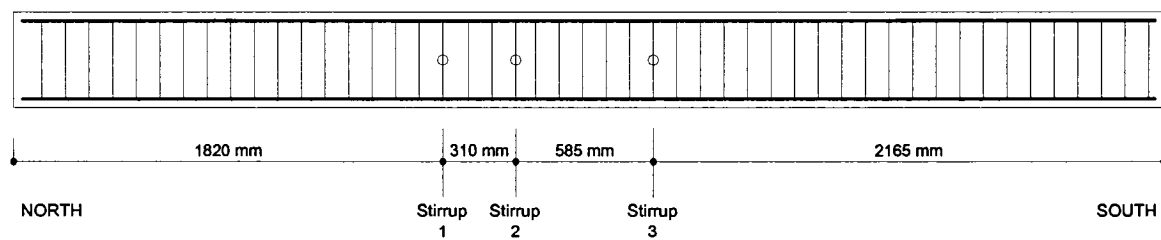
Figure 4.13 Strain gauge locations for SS3a



(a) Top layer longitudinal bars (top view)

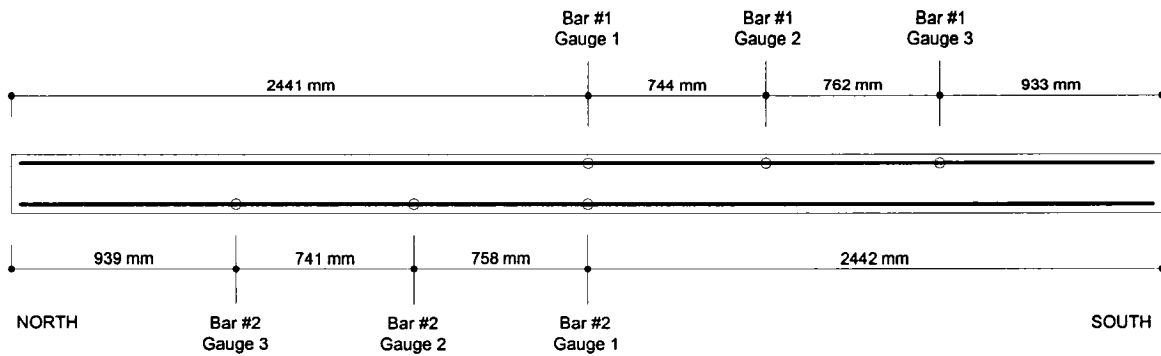


(b) Bottom layer longitudinal bars (top view)

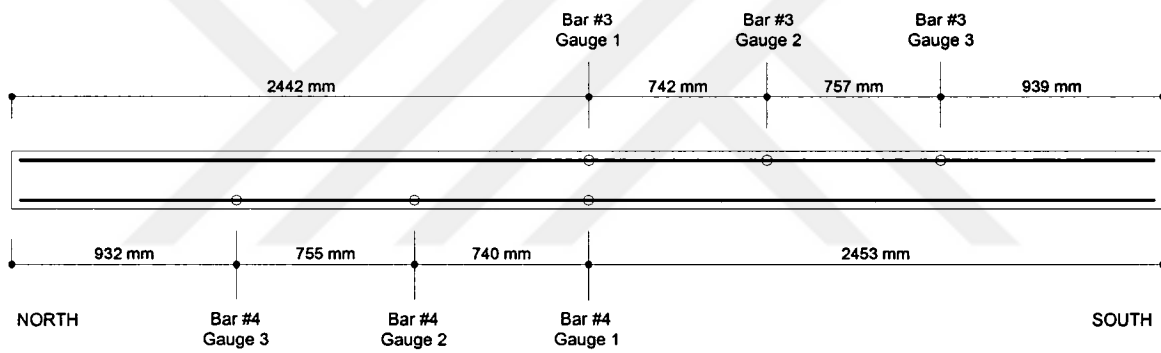


(c) Stirrups (as seen from the west face)

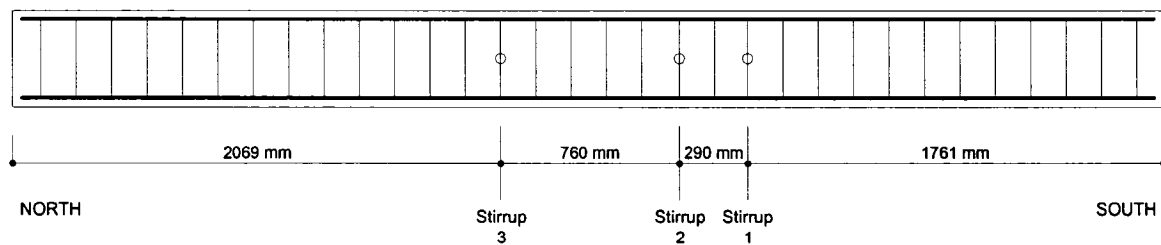
Figure 4.14 Strain gauge locations for SS3b



(a) Top layer longitudinal bars (top view)

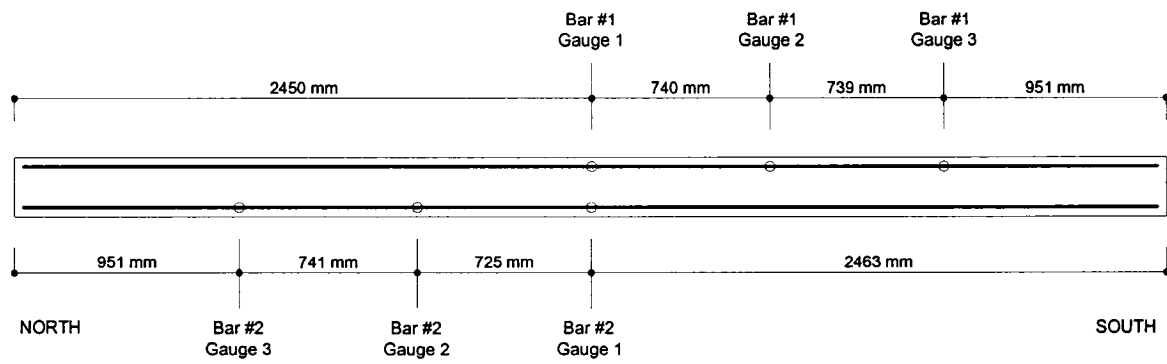


(b) Bottom layer longitudinal bars (top view)

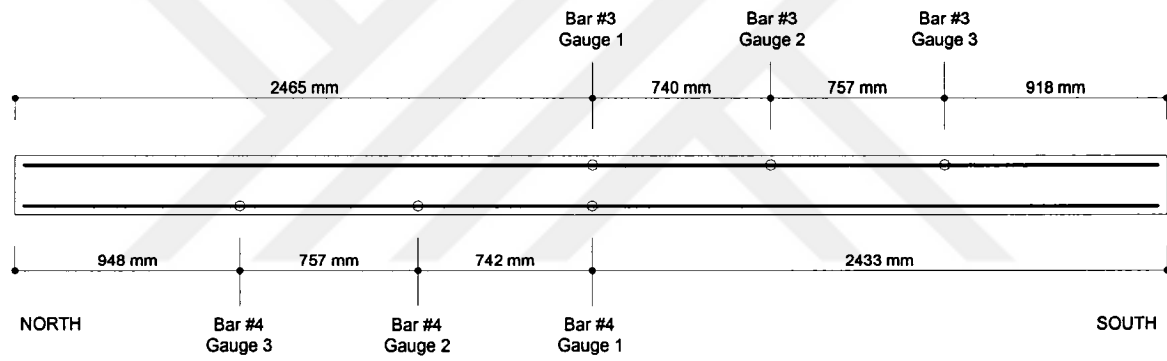


(c) Stirrups (as seen from the west face)

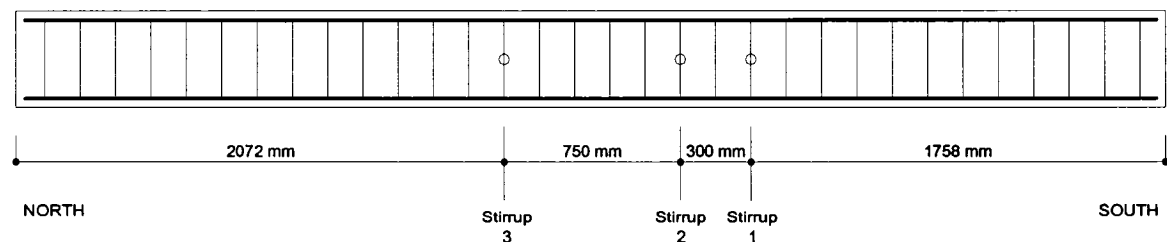
Figure 4.15 Strain gauge locations for SS2a



(a) Top layer longitudinal bars (top view)

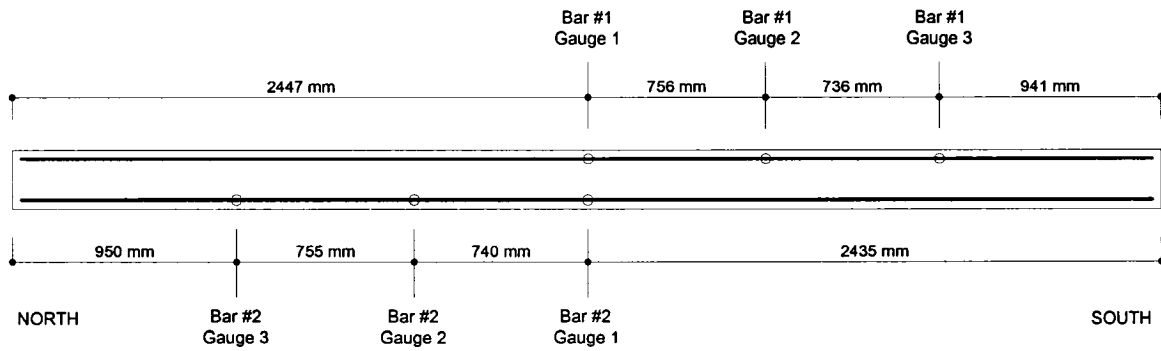


(b) Bottom layer longitudinal bars (top view)

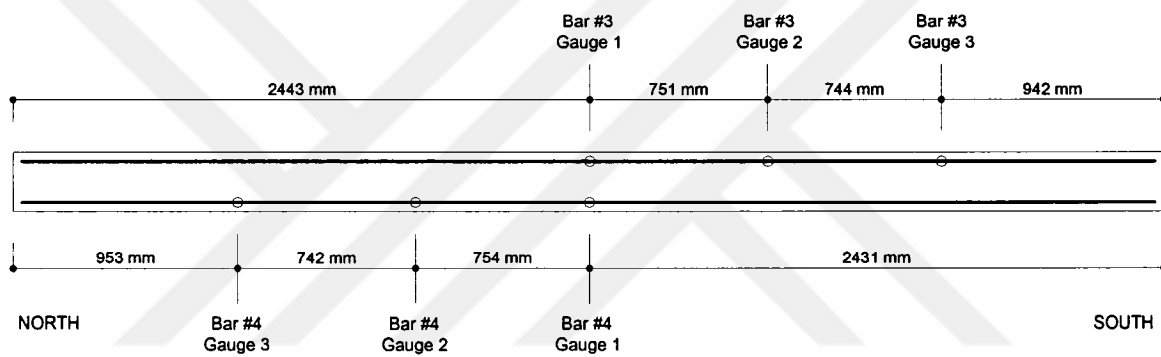


(c) Stirrups (as seen from the west face)

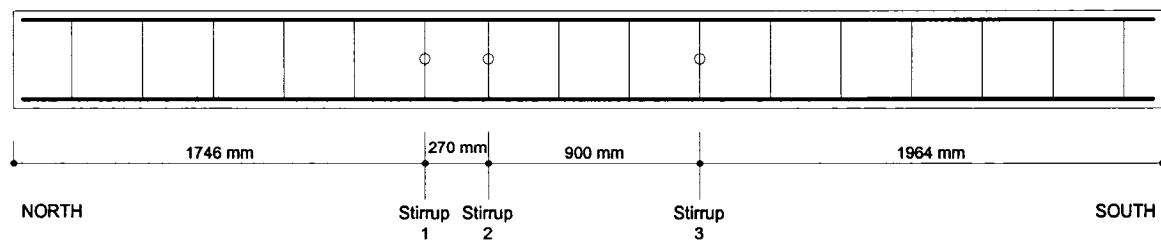
Figure 4.16 Strain gauge locations for SS2b



(a) Top layer longitudinal bars (top view)

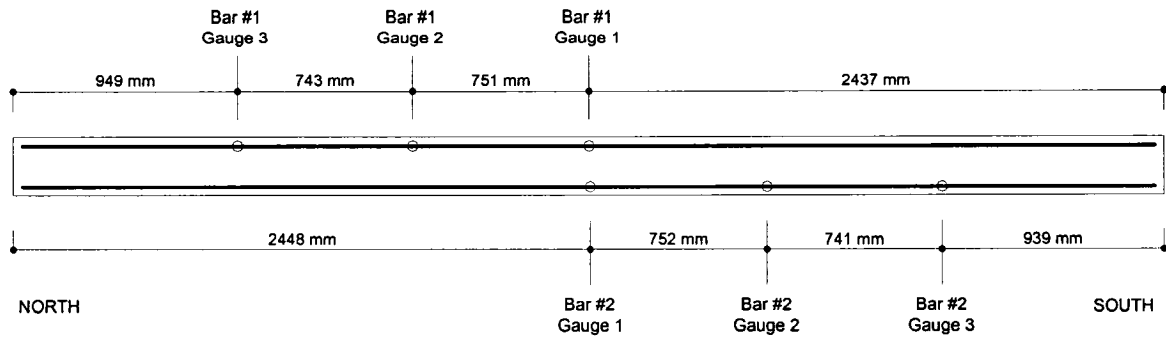


(b) Bottom layer longitudinal bars (top view)

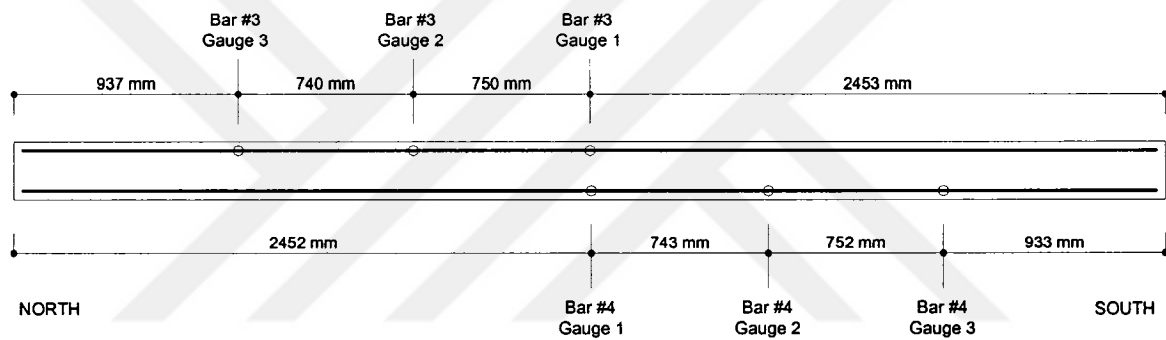


(c) Stirrups (as seen from the west face)

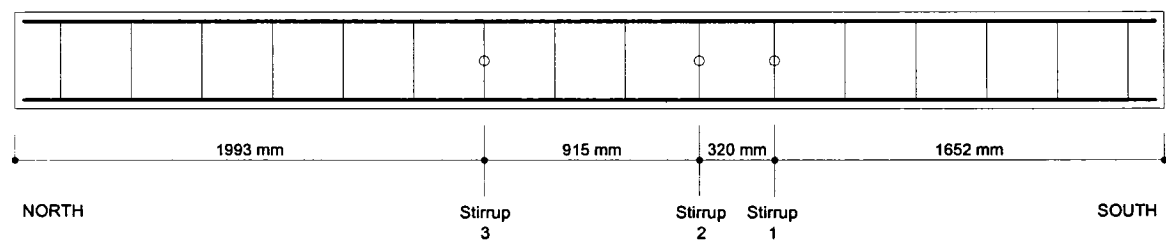
Figure 4.17 Strain gauge locations for SS1a



(a) Top layer longitudinal bars (top view)

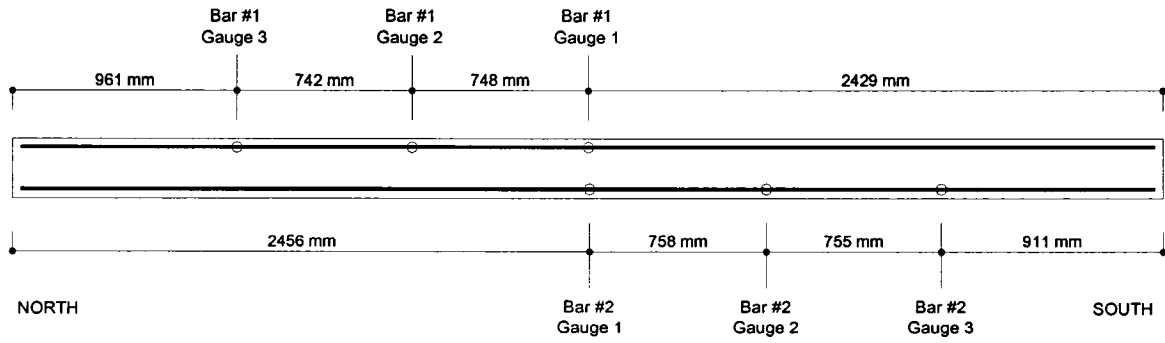


(b) Bottom layer longitudinal bars (top view)

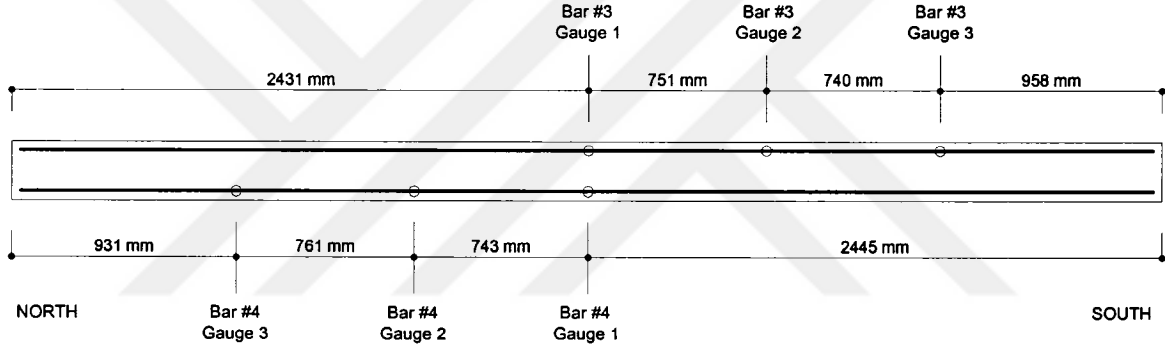


(c) Stirrups (as seen from the west face)

Figure 4.18 Strain gauge locations for SS1b

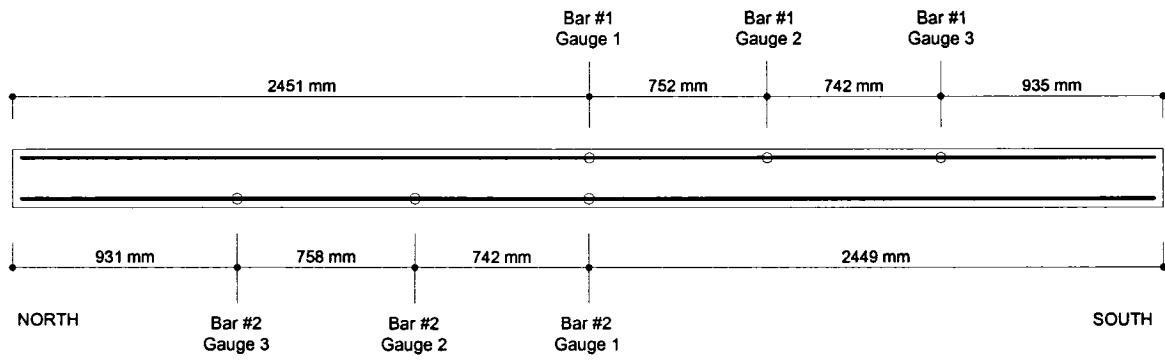


(a) Top layer longitudinal bars (top view)

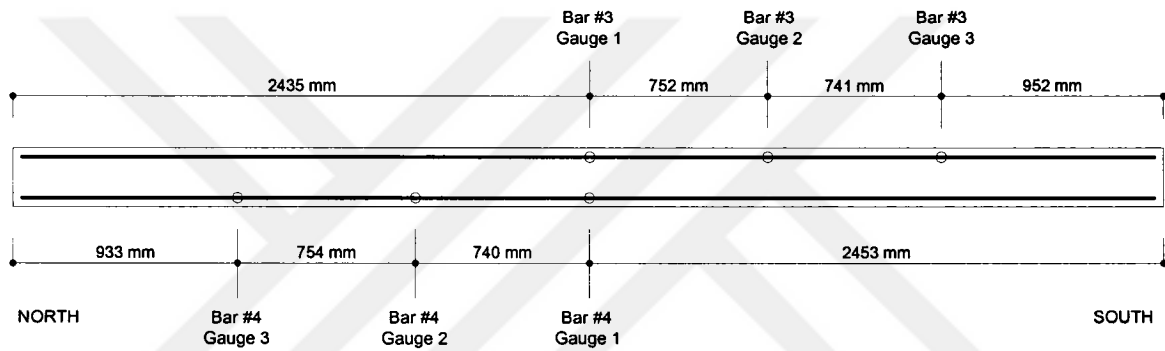


(b) Bottom layer longitudinal bars (top view)

Figure 4.19 Strain gauge locations for SS0a



(a) Top layer longitudinal bars (top view)



(b) Bottom layer longitudinal bars (top view)

Figure 4.20 Strain gauge locations for SS0b

4.5.4 Load Cells

Two load cells were used at the supports to measure the reaction forces. They were manufactured at the University of Toronto Structural Laboratories and each had a 3000 kN maximum load capacity. The load cells were calibrated, before the tests, under static load up to 800 kN. They were placed between two 25 mm thick steel plates to ensure an even contact (see Figure 4.21). The load cell at the south support was denoted as Load Cell A, whereas the one at the north support was denoted as Load Cell C.

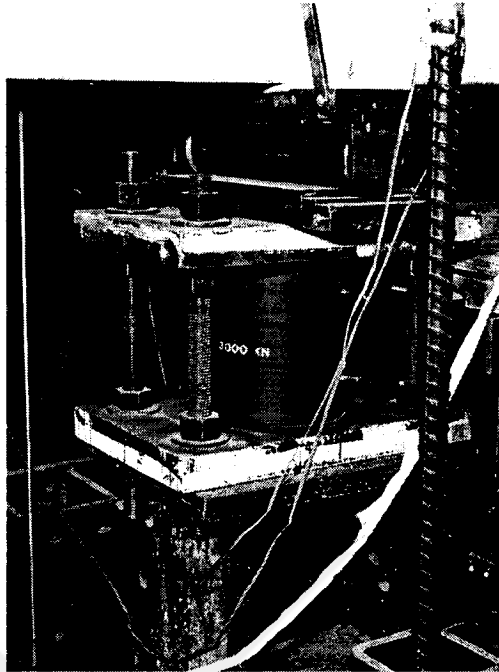


Figure 4.21 Load cell

4.5.5 Data Acquisition System

The digital data acquisition system used, model MGCplus, was manufactured by Hottinger Baldwin Messtechnik GmbH (HBM). The system had a modular system for the connection boards. The connection boards and types used for the test program are summarized in Table 4.7. Technical information regarding the data acquisition system and the connection boards can be found in Appendix B and on the manufacturer's website (<http://www.hbm.com>).

The data collected were processed real-time with a software called catman[®], which was also developed by HBM. In addition to data acquisition and recording capabilities, catman[®] also has built-in post-processing features, including digital signal processing and filtering.

Table 4.7 Sensors and connection boards used for data acquisition

Sensors	Type of Connection	No. of Available	No. of Boards	Sampling
	Board	Channels	Used	Rate
Potentiometers	AP836	8	2	2.4 kHz
LVDT's	AP801	8	3	2.4 kHz
Accelerometers	AP810	8	1	2.4 kHz
Load Cells	AP801	8	3	2.4 kHz
Strain Gauges	AP801	8	3	2.4 kHz
Accelerometers (in some tests)	AP11	1	1	19.2 kHz

4.6 Drop-Weights

Two different drop-weights were used for the testing; a lighter weight of 211 kg and a heavier weight of 600 kg (see Figure 4.22). They were manufactured by filling a 300 mm square HSS section with concrete. The heavier drop-weight had additional thick steel plates welded on both sides of the HSS section for extra mass. Both drop-weights were 304 mm (2 feet) in length and had a hook at the top. They were fitted with two accelerometers, as discussed in Section 4.5.1.

Impact loads were applied via the free-fall of the drop-weights onto the specimens at the mid-span. The weights were hung on the hook of a crane by a nylon rope (see Figure 4.23). After raising the weight up to the required height, the nylon rope was cut with a saw, resulting in the fall of the weight on the specimen. The drop-weights fell clear between two I-shaped steel columns; the columns were utilized both to ensure hitting the right point on the specimen, as well as to avoid any possible accidents by keeping the weight on the specimen after the fall. Wooden blocks were mounted on both sides of the weight to act as a guide between the columns (see Figure 4.24). The blocks had about 30 mm clearance between the web and the flanges of the column, so that it would be an ideal free-fall and there would not be any friction between the columns and the drop-weight during the fall.



(a) Light drop-weight (211 kg)

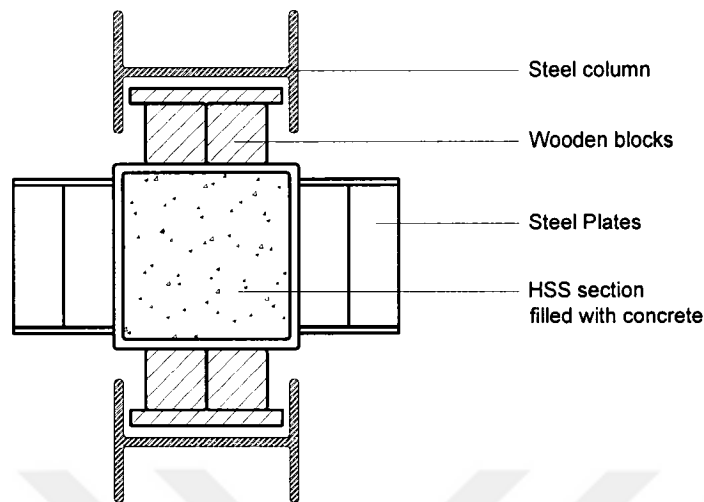


(b) Heavy drop-weight (600 kg)

Figure 4.22 Drop-weights



Figure 4.23 Drop-weight suspended from the crane with nylon rope



(a) Fitting of the 600 kg weight between the columns
(top view)



(b) Steel columns

Figure 4.24 Drop-weight and the columns

In order to obtain a well-distributed impact force, special assemblies were placed on the specimen at the impact point. Three different arrangements were used during the course of the test program (see Figure 4.25). In the first test, a ball-socket assembly was bolted onto a 50 mm thick, 300 mm square steel plate. The assembly was placed on top of the specimen at the mid-span, so that the drop-weight would hit the ball-socket first, and the impact load would be transferred to the bottom plate at the centre. Using this assembly, the intention was to have a perfectly centered impact on the specimen, even if the drop-weight hit it off-centre. Z-shaped brackets were used on both sides of the assembly to fix it to the specimen and prevent rebound after the impact.

However, in the first test, this arrangement caused a significant longitudinal movement on the specimen, since the slightly off-centre hit on the ball-socket caused horizontal forces and the specimen was not supported for longitudinal movements. Therefore, for the subsequent test, the ball-socket was removed; however, the bottom 125 mm square plate of the ball-socket assembly was left on the larger 300 mm square steel plate. Similar to the ball-socket assembly, the aim was to obtain a centralized hit on the specimen. However, this arrangement caused a longitudinal movement as well.

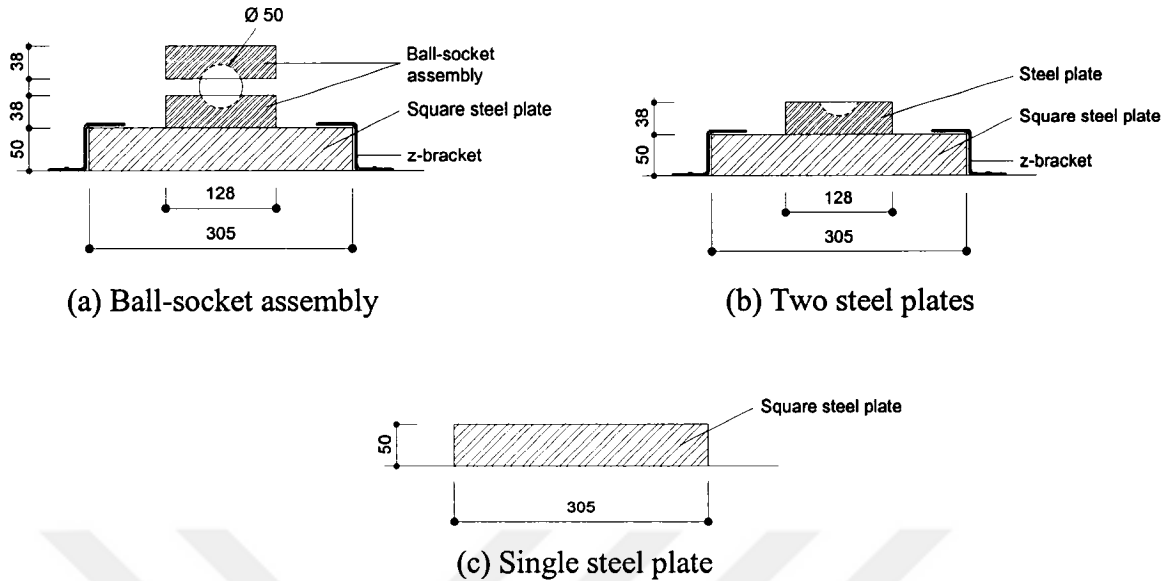


Figure 4.25 Arrangements for the impact point on the beam
(All dimensions are in millimetres)

For the following test, the whole assembly was removed and the drop-weight was allowed to hit the beam directly. This caused extensive local crushing of the concrete at the impact point, possibly caused by an uneven contact of the drop-weight with the beam surface. Therefore, for the remainder of the test program, only a 50 mm thick 300 mm square steel plate was employed on the beam. Using this setup, a seemingly uniform contact was obtained, without any significant horizontal forces induced. The Z-brackets were also removed later, since they were not strong enough to prevent the jump of the steel plate.

4.7 Test Procedure

As mentioned previously, all specimens were tested under impact loads induced by free-falling drop-weights. For all tests, the weights were dropped from a clear height of 3.26 m above the specimen, resulting in an 8.0 m/s calculated impact velocity. All specimens, except SS0b, were tested more than once. With the a-series specimens, the beams were tested once with the 211 kg drop-weight first, then twice with the 600 kg weight. For b-series specimens, the order was reversed; they were tested twice with the 600 kg drop-weight first, and then once with the 211 kg weight. Using this procedure, the total impact energy imparted to the specimens was kept constant between twin specimens.

In addition to the data collected by the sensors, after each test, the cracks developed were marked, and their widths were measured. The test data obtained from the sensors are available in digital format at University of Toronto - VecTor Analysis Group's website (<http://www.civ.utoronto.ca/vector>), whereas the crack maps sketched after the tests are presented in Appendix C. The following sections summarize the main observations made during the tests. The number following the specimen name indicates the sequence of the particular test performed on that specimen. In the photos presented, labels attached next to the cracks indicate the width of the crack in millimetres.

4.7.1 SS3a-1 (Test Date: July 20, 2005; Drop-weight: 211 kg)

The specimen cracked at various locations. A 0.15 mm wide diagonal crack, with approximately 45 degrees inclination, opened on the north half of the specimen. The crack profile was not symmetric; the cracks on the south half were less inclined.

The ball-socket assembly was employed at the impact point. The specimen shifted towards the north by a considerable amount. A residual longitudinal displacement of 30 mm was measured.

4.7.2 SS3a-2 (Test Date: August 8, 2005; Drop-weight: 600 kg)

New diagonal cracks developed on both halves of the beam and the crack profile became more symmetric. Diagonal crack widths were as large as 1.1 mm (see Figure 4.26). A vertical 0.8 mm crack opened around the mid-span.

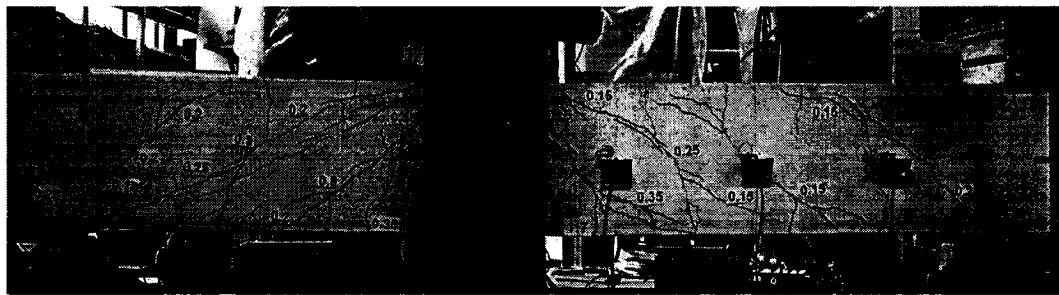


Figure 4.26 View as seen from the west face, SS3a-2

The ball-socket assembly was removed for this test. Instead, two steel plates were left as shown in Figure 4.25(b). A longitudinal movement was still observed; however, it was not as significant as in SS3a-1.

The connection of displacement sensor L6 was broken. Therefore, the readings obtained from this sensor had to be discarded.

4.7.3 SS3a-3 (Test Date: August 10, 2005; Drop-weight: 600 kg)

Minor new cracks developed. Diagonal cracks forming a shear-plug under the impact point widened significantly. However, no significant changes in the widths of the cracks away from the shear-plug were observed (see Figure 4.27).

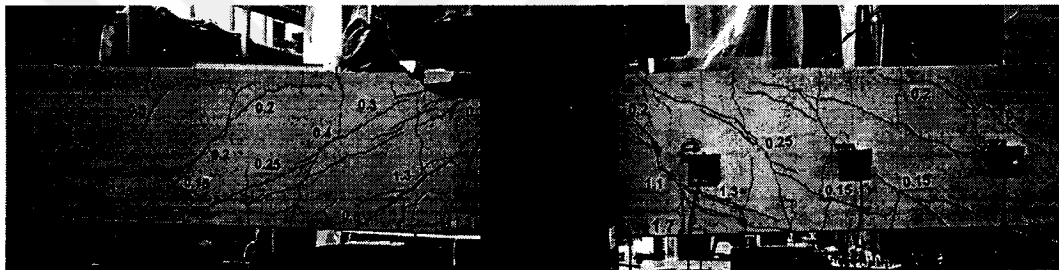


Figure 4.27 View as seen from the west face, SS3a-3

The steel plates at the impact point were removed and the drop-weight was allowed to hit the specimen directly. However, extensive spalling on the east face and crushing of concrete around the mid-span was observed. It was suspected that an uneven contact of the drop-weight caused this. Therefore, it was decided to keep the 300 mm square steel plate on the impact point for the rest of the test program.

4.7.4 SS2a-1 (Test Date: August 26, 2005; Drop-weight: 211 kg)

Several minor cracks developed in the beam. A 0.2 mm wide diagonal crack opened on the north half close to the mid-span. The crack profile was not symmetric; cracks on the south half were less inclined (see Figure 4.28).

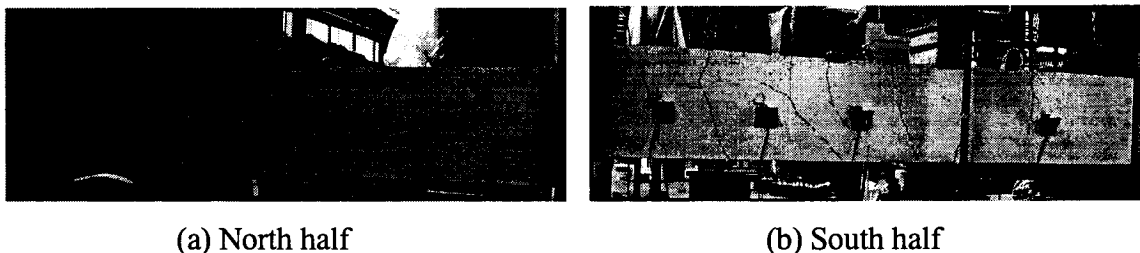


Figure 4.28 Views as seen from the west face, SS2a-1

The cable connecting accelerometer A6 on the drop-weight to the data acquisition system was crushed under the drop-weight. Therefore, the readings obtained from this sensor had to be discarded.

4.7.5 SS2a-2 (Test Date: August 31, 2005; Drop-weight: 600 kg)

Several new cracks developed. On the north half, the diagonal crack originally formed in SS2a-1 widened, reaching 3 mm in width. No significant new diagonal cracks formed except one close to the mid-span. On the south half, however, several new diagonal cracks developed with narrow widths. In other words, one side had a single, wide diagonal crack, whereas the other side had several narrow diagonal cracks (see Figure 4.29). The vertical crack at the mid-span was 1.8 mm wide, extending from the bottom to the top of the beam.

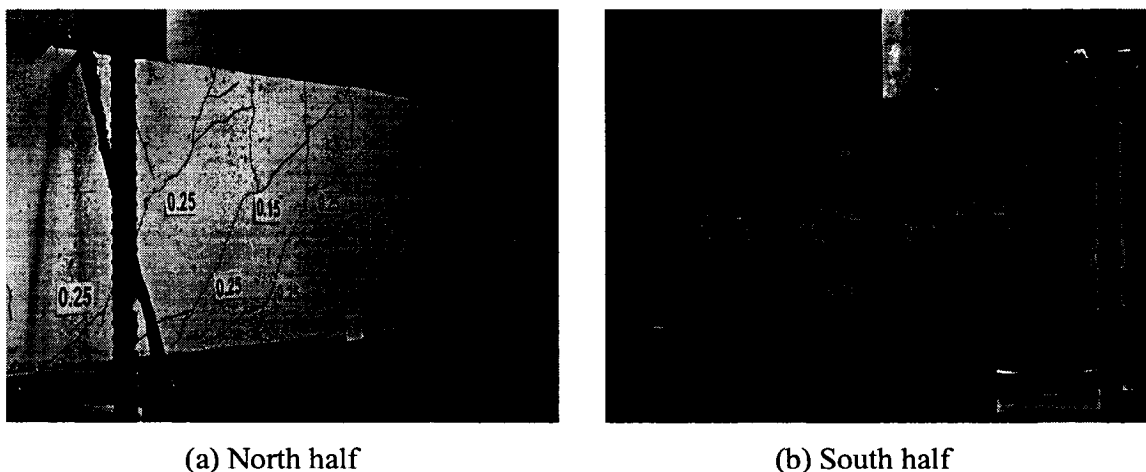


Figure 4.29 Views as seen from the west face, SS2a-2

The connection of displacement sensor P3 was destroyed. Connecting cables for accelerometer A1 were also broken, and both accelerometers on the drop-weight, A6 and

A7, were saturated. The strain gauge on Stirrup 3, which was passing through the wide diagonal crack on north side, also stopped functioning, possibly due to excessive deformation. Therefore, readings obtained from these sensors had to be discarded.

4.7.6 SS2a-3 (Test Date: October 11, 2005; Drop-weight: 600 kg)

No new significant cracks developed. However, the pre-existing diagonal cracks forming the shear-plug widened to 5 mm (see Figure 4.30). Concrete under the impact point crushed and some spalling of the clear cover occurred under the specimen around the north part of the shear plug. The width of the vertical crack at the mid-span remained unchanged; however, some horizontal cracks developed under the impact point on the east side.

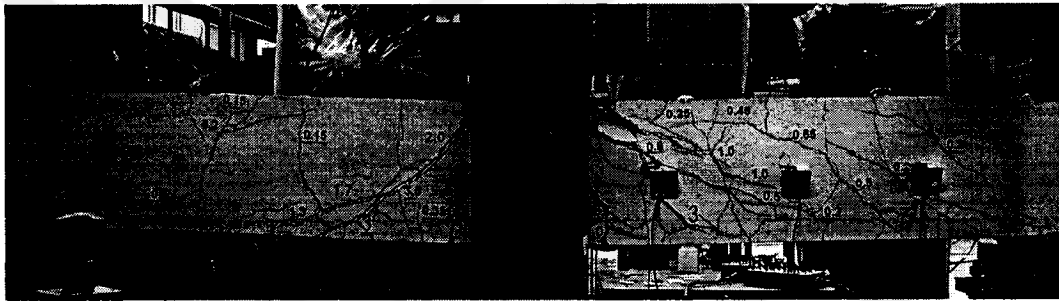


Figure 4.30 View as seen from the west face, SS2a-3

For this test, one of the accelerometers on the drop-weight, A6, was connected to a new connection board, AP11, with a 19.2 kHz sampling rate. With a higher sampling rate, the intent was to capture the high-frequency components of the acceleration data obtained from the drop-weight. Additionally, it was suspected that the peak acceleration of the drop-weight may have been missed with the low sampling rate, especially since the accelerometers were saturated in the previous test, although the peak value recorded was well within range. With a higher sampling rate, the intention was to capture the true peak value. During the test, A6 recorded a much higher peak than A7, which was also connected to the drop-weight. However, judging from the signal recorded, the sampling rate was still found to be insufficiently low. Detailed discussions on this issue are given in the following chapter.

4.7.7 SS1a-1 (Test Date: November 17, 2005; Drop-weight: 211 kg)

Diagonal cracks developed on both sides. The cracks forming a shear-plug under the impact point reached up to 0.25 mm in width.

The connecting cables for accelerometer A7 on the drop-weight were destroyed. Therefore, readings obtained from this sensor had to be discarded.

4.7.8 SS1a-2 (Test Date: November 23, 2005; Drop-weight: 600 kg)

Several new minor cracks developed (see Figure 4.31). The diagonal cracks forming the shear-plug widened significantly, reaching up to 3.5 mm in width. Another diagonal crack, extending from the south support and propagating horizontally at the top, attained a width of up to 4 mm. On the south half, the top cover concrete was loosened near the impact point. Significant spalling of the bottom cover concrete occurred close to the south support.



Figure 4.31 Views as seen from the west face, SS1a-2

The connection of displacement sensor L2 was broken because of the concrete spalling. The strain gauge on Stirrup 1 stopped functioning, possibly due to excessive deformation. Accelerometer A6 on the drop-weight was connected in parallel to both the 2.4 kHz and 19.2 kHz connection boards. However, because of some complications, the AP11 19.2 kHz connection board was removed after this test, and A6 was connected to an AP801 2.4 kHz connection board similar to the other sensors.

4.7.9 SS1a-3 (Test Date: November 28, 2005; Drop-weight: 600 kg)

The specimen suffered extensive damage (see Figure 4.32). The impact force punched through the shear-plug, and significant spalling of clear cover occurred at both sides. In this test, the south half completely lost its strength as a result of the widened diagonal crack

extending from the support. The longitudinal bars were mostly exposed between the supports and the mid-span, and they were severely bent. Two of the stirrups, 1350 and 1650 mm away from the south end, were ruptured.

The connections of displacement sensors P2, L3, P3, L4, L5, P4, P5, L7 and P6 were destroyed because of extensive concrete spalling. Strain gauges Bar#1 Gauge 1, Bar#1 Gauge 2, Bar#4 Gauge 2, Bar#3 Gauge 1, Bar#3 Gauge 2, Stirrup 2 and Stirrup 3 stopped functioning, either because of excessive deformation or broken cables inside the specimen. Therefore, readings obtained from these sensors had to be discarded.

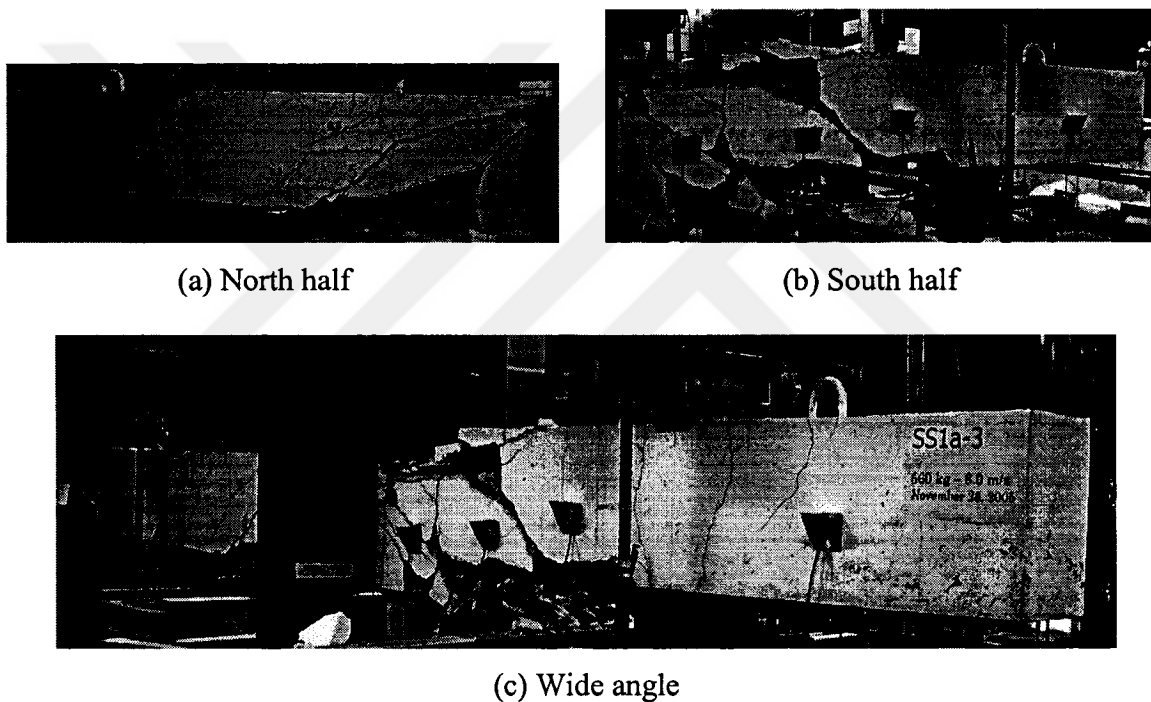


Figure 4.32 Views as seen from the west face, SS1a-3

4.7.10 SS0a-1 (Test Date: January 18, 2006; Drop-weight: 211 kg)

Several diagonal cracks developed on both halves; however, the cracks on the south half were slightly wider, reaching 0.5 mm in width. The main diagonal cracks were horizontal at the top, travelling towards the impact point (see Figure 4.33).

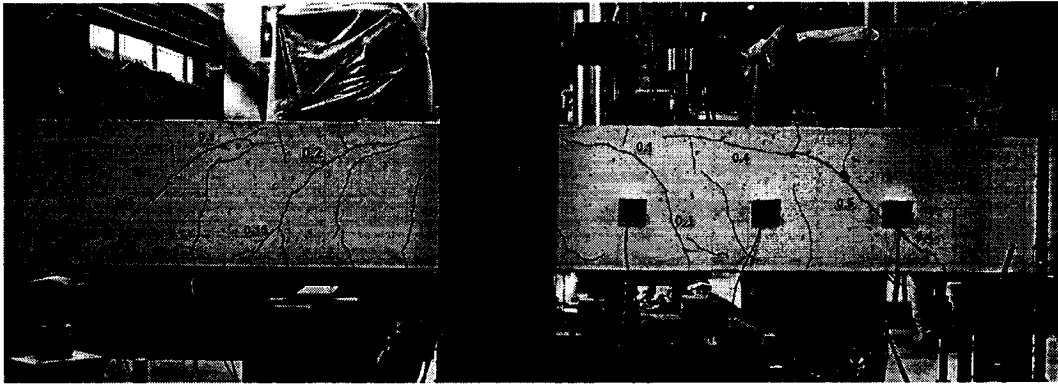


Figure 4.33 View as seen from the west face, SS0a-1

4.7.11 SS0a-2 (Test Date: January 23, 2006; Drop-weight: 600 kg)

The specimen suffered extensive damage, with a crack pattern similar to that in SS1a-3 (see Figure 4.34). Significant spalling of concrete occurred along both at the top and the bottom of the specimen. The longitudinal bars at the bottom were exposed along a one metre distance from both supports. The top longitudinal bars were also exposed along the middle two metres of length. The longitudinal bars were severely bent, and it was clear that the specimen remained standing only because of the bending capacity of these bars.



(a) North half



(b) South half



(c) Wide angle

Figure 4.34 Views as seen from the west face, SS0a-2

The connections of all displacement sensors, except for L1, P1, P7 and L9, were destroyed due to extensive spalling of concrete.

4.7.12 SS3b-1 (Test Date: February 16, 2006; Drop-weight: 600 kg)

Several diagonal cracks developed on both halves of the beam (see Figure 4.35). The widths of the diagonal cracks were 0.4 mm maximum; however, the vertical cracks that developed at the mid-span reached 1 mm in width.

The strain gauge on Stirrup 3 stopped functioning. Therefore, readings obtained from this sensor had to be discarded.

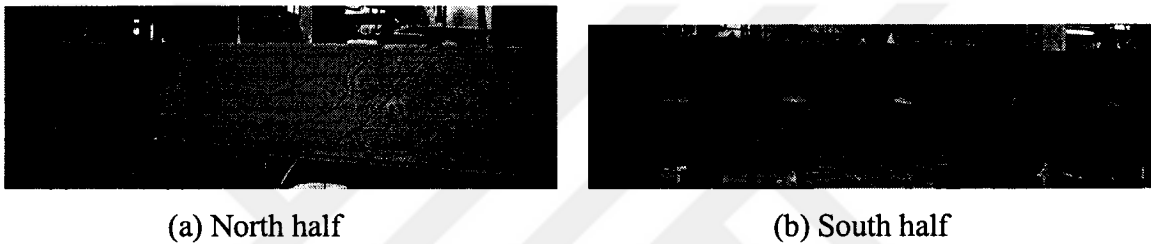


Figure 4.35 Views as seen from the west face, SS3b-1

4.7.13 SS3b-2 (Test Date: February 17, 2006; Drop-weight: 600 kg)

The diagonal cracks forming the shear-plug widened significantly, reaching up to 2 mm in width on the south half (see Figure 4.36). Few new cracks developed, and the widths of the cracks away from the shear-plug remained virtually unchanged. The vertical cracks at the mid-span widened somewhat, reaching 2 mm in width. A minor amount of concrete spalling was observed at the bottom of the specimen.

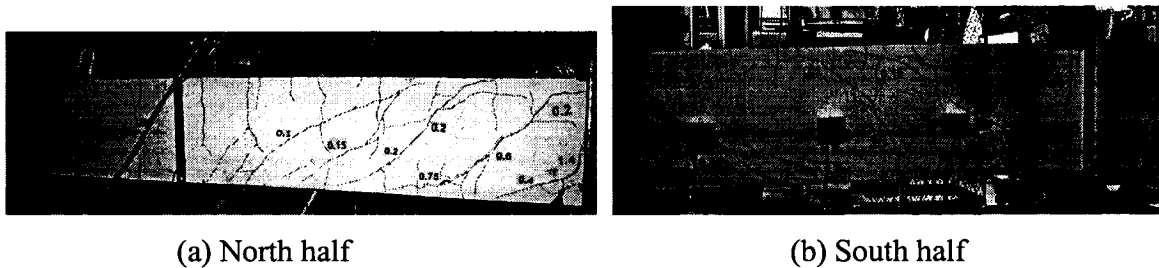


Figure 4.36 Views as seen from the west face, SS3b-2

The strain gauge Bar#3 Gauge 2 stopped functioning, and the connecting cables for accelerometer A1 were broken. Therefore, readings obtained from these sensors had to be discarded.

4.7.14 SS3b-3 (Test Date: February 21, 2006; Drop-weight: 211 kg)

In general, the widths of the cracks formed in the previous tests changed little, and no significant new cracks developed (see Figure 4.37). Some spalling of concrete occurred along the bottom of the specimen.

The connections of displacement sensors P12, P13 and P14 were broken because of concrete spalling. Therefore, readings obtained from these sensors had to be discarded.

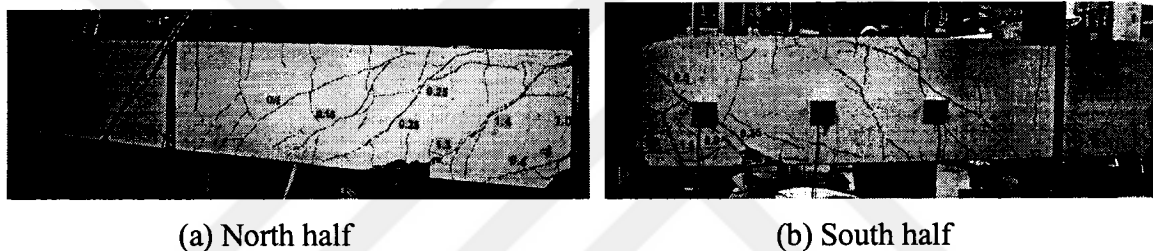


Figure 4.37 Views as seen from the west face, SS3b-3

4.7.15 SS2b-1 (Test Date: February 27, 2006; Drop-weight: 600 kg)

Several diagonal cracks developed (see Figure 4.38). One diagonal crack on the north half reached 1.8 mm in width. However, the inclinations of the cracks were slightly different between the east and west faces; the diagonal cracks on the west side were steeper. There were also some diagonal cracks at the top and the bottom of the specimen, indicating the presence of torsion. The vertical crack at the mid-span was 1.3 mm wide.

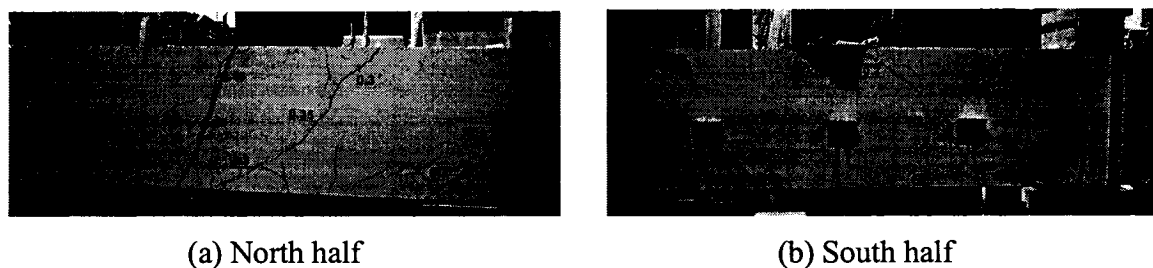


Figure 4.38 Views as seen from the west face, SS2b-1

4.7.16 SS2b-2 (Test Date: March 1, 2006; Drop-weight: 600 kg)

A new diagonal crack formed on north half, and some cracks propagated in length (see Figure 4.39). The width of the cracks forming the shear-plug increased, reaching 2 mm. Significant concrete spalling occurred on the west face close to the mid-span, and at the bottom of the specimen along both the north and south halves.

Accelerometer A2 came loose from the specimen, due to concrete spalling. The strain gauge on Stirrup 2 stopped functioning, possibly due to excessive deformation. Therefore, the readings obtained from these sensors had to be discarded.

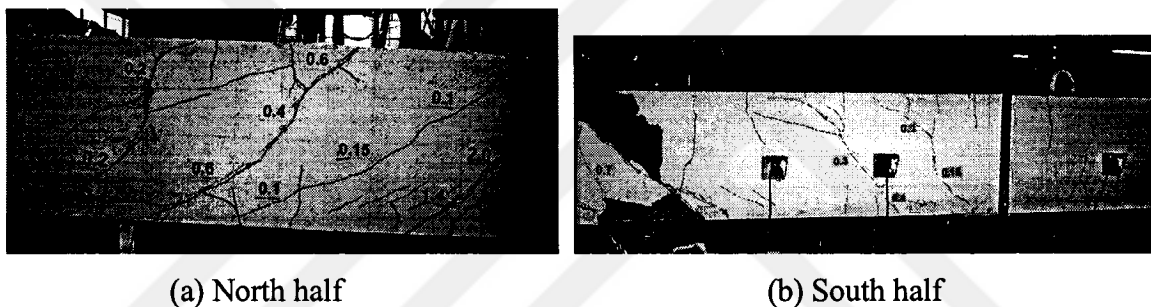


Figure 4.39 Views as seen from the west face, SS2b-2

4.7.17 SS2b-3 (Test Date: March 3, 2006; Drop-weight: 211 kg)

Few changes were observed in either the crack profile or in the crack widths (see Figure 4.40). Some concrete spalling occurred along the bottom of the specimen, and along the top on the north half near the impact point.

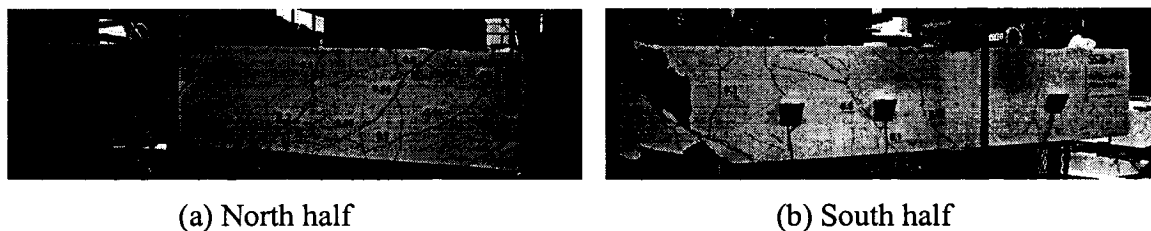


Figure 4.40 Views as seen from the west face, SS2b-3

4.7.18 SS1b-1 (Test Date: March 10, 2006; Drop-weight: 600 kg)

Diagonal cracks of up to 5 mm wide developed (see Figure 4.41). The main diagonal crack on the north half was 3 mm wide. A 5 mm wide diagonal crack starting from the south support became horizontal, reaching the impact point at the mid-span. A 0.9 mm wide vertical crack developed at the mid-span.

Strain gauges Bar#4 Gauge 1, Bar#4 Gauge 2, Bar#4 Gauge 3 and Stirrup 3 stopped functioning. Therefore, the readings obtained from these sensors had to be discarded.

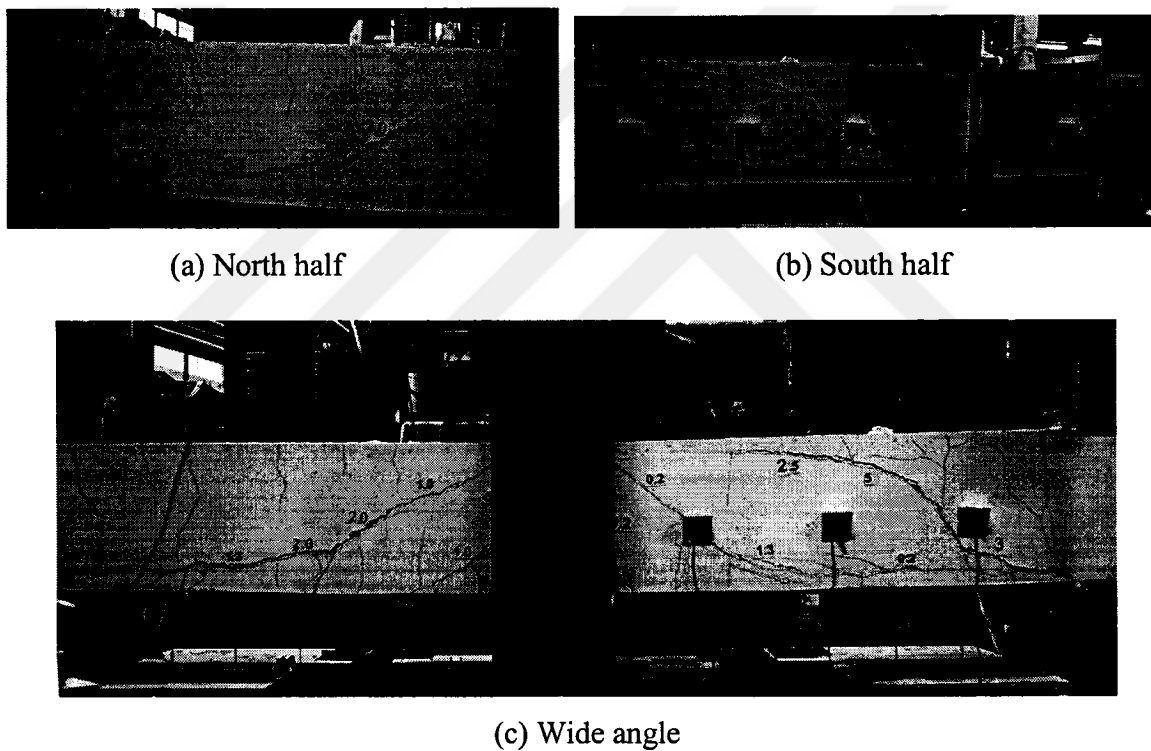


Figure 4.41 Views as seen from the west face, SS1b-1

4.7.19 SS1b-2 (Test Date: March 14, 2006; Drop-weight: 600 kg)

The specimen suffered extensive damage similar to that in SS1a-3 (see Figure 4.42). The diagonal crack extending from the south support reached 30 mm in width. Significant spalling occurred along both the top and the bottom of the specimen, exposing the longitudinal bars. The exposed longitudinal bars were bent, and a stirrup crossing the wide

diagonal crack was fractured at 1400 mm from the south end. Another stirrup at 1700 mm from the south end experienced opening of the top hook. There were no significant changes in the cracks beside the ones forming the shear-plug, including the vertical cracks at the mid-span. Horizontal cracks developed above the south support at the level of the bottom longitudinal reinforcement, indicating a bond failure of the bars.

Strain gauges Bar#2 Gauge 1, Bar#2 Gauge 2, Bar#4 Gauge 3, Stirrup 1 and Stirrup 2 stopped functioning, possibly due to excessive deformations and broken cables inside the specimen. The connections of displacement sensors P5, P6 and P9 were broken because of concrete spalling. Therefore, the readings obtained from these sensors had to be discarded.

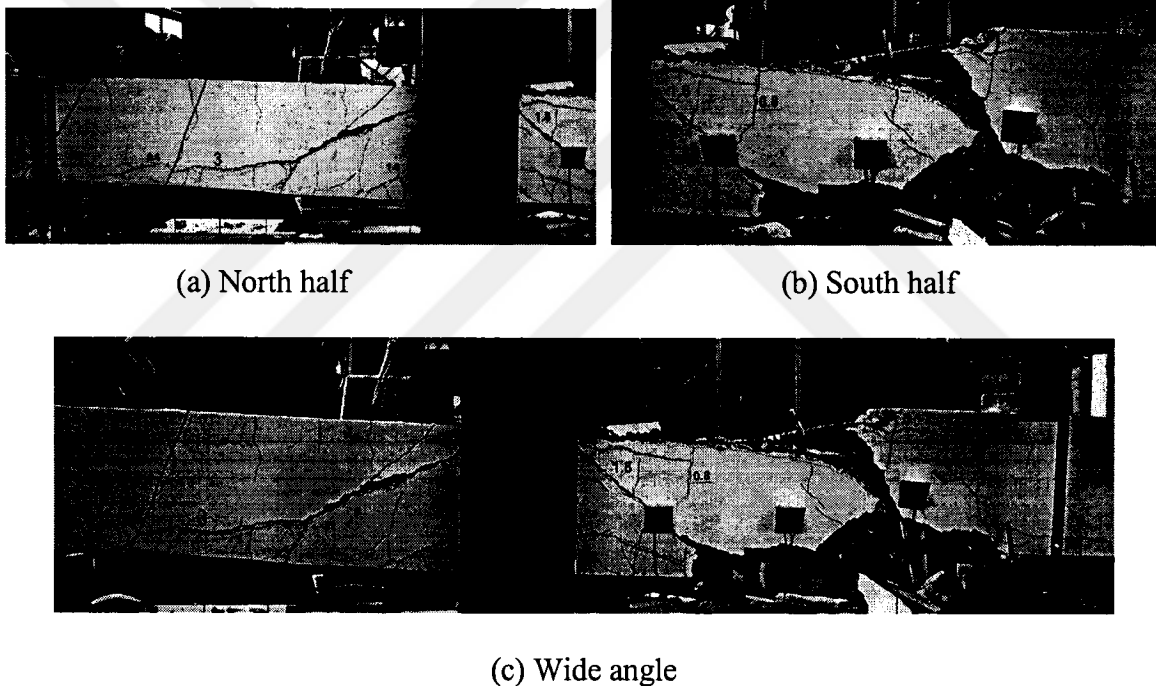


Figure 4.42 Views as seen from the west face, SS1b-2

4.7.20 SS0b-1 (Test Date: April 7, 2006; Drop-weight: 600 kg)

The specimen suffered extensive damage (see Figure 4.43). The drop-weight punched through, causing massive concrete spalling both at the top and the bottom of the specimen. The longitudinal reinforcing bars were exposed and severely bent. There were also signs of bond failure of the reinforcement bars at the supports.

All strain gauges, except Bar#1 Gauge 2, Bar#1 Gauge 3, Bar#2 Gauge 2 and Bar#2 Gauge 3, were damaged due to broken cables inside the specimen. Similarly, the connections of all displacement sensors, except P1, P2, P7 and P8, were damaged.

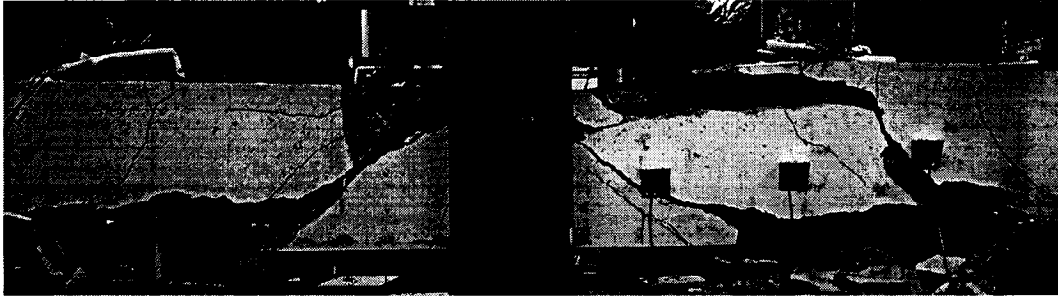


Figure 4.43 View as seen from the west face, SS0b-1

5 DISCUSSION OF TEST RESULTS

5.1 Introduction

Due to the typically very high rates of loading and highly dynamic behaviour of the specimens, impact testing has many complications (Bischoff 1998). Especially in cases where the experimental results are used for the validation of an analysis method, they should be examined carefully, and possible experimental errors should be identified and corrected or filtered out if possible.

Therefore, this chapter is mostly devoted to a critical discussion of the experimental results. Measurements taken during the tests are examined and analyzed. Particular issues, such as the digital signal analyses of the measurements, displaced shapes, force equilibrium, and strain rates are explained with sample representative readings. The complete test data obtained from the sensors are available in digital format at the University of Toronto - VecTor Analysis Group's website (<http://www.civ.utoronto.ca/vector>).

5.2 Digital Signal Analysis

As mentioned in Chapter 4, data from the sensors were collected with a digital data acquisition system at a 2.4 kHz sampling rate. In other words, continuous analog signals generated by the sensors were read and recorded on computer at a rate of 2400 times per second. This process is referred to as digitisation. Digitised data is a discrete representation of the original continuous analog signal; therefore, it is only an approximation, from which the original signal can later be "recovered". One of the main factors affecting the quality of this approximation, and hence the recovered signal, is the sampling rate.

The suitability of the sampling rate mainly depends on the highest frequency present in the response. The *Nyquist's sampling theorem* states that, for an accurate digitisation, the sampling rate should be at least twice the maximum frequency response present in the original analog signal (Stearns and Hush 1990). In other words, the highest frequency that can accurately be represented with a set of digitised data is less than one-half of the sampling rate. As seen in Figure 5.1, a signal that is digitised at a rate less than twice its

highest frequency causes an incorrect recovery of the continuous signal and, consequently, loss of data. This phenomenon is called *aliasing*. In order to ensure the validity and quality of the collected digitised data, a careful examination is required to determine if aliasing is present.

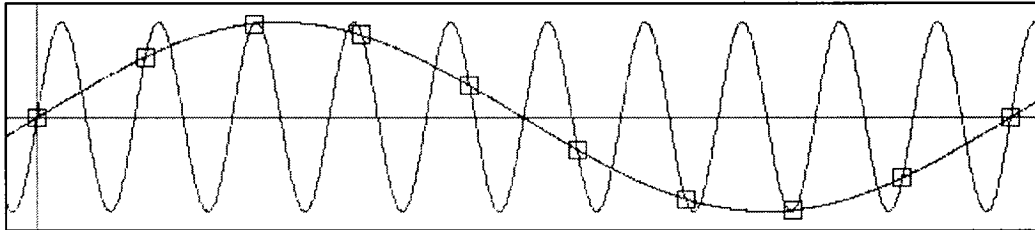


Figure 5.1 Aliasing (squares represent the sampled data from the high-frequency signal)

Although, for this experimental program, it is not possible to verify either the presence or level of aliasing with certainty, an analysis of the data, accompanied by some analytical or numerical estimates of the physical response, can reveal if it should be a concern. The following sections examine the data collected by different sensors and evaluate their validity.

5.2.1 Displacement Data

A linear-elastic vibration analysis would be useful in determining the free-vibration frequency content of the test specimens and seeing if the sampling rate is sufficiently high to capture the displacements accurately. To calculate the vibration modes and the frequencies of the specimens, the finite element package SAP2000 was used. The material properties of SS3a were used, as a representative case, to calculate the mass and the elastic stiffness of the specimen. The results of the analysis for the first ten mode shapes are summarized in Figure 5.2. The mode shapes related to the axial deformations are ignored. It should be noted that as the specimen cracks and suffers damage, its vibration periods will increase and thus the frequencies of the modes will decrease. In other words, the elastic frequencies calculated are the upper bound estimates of the actual inelastic response.

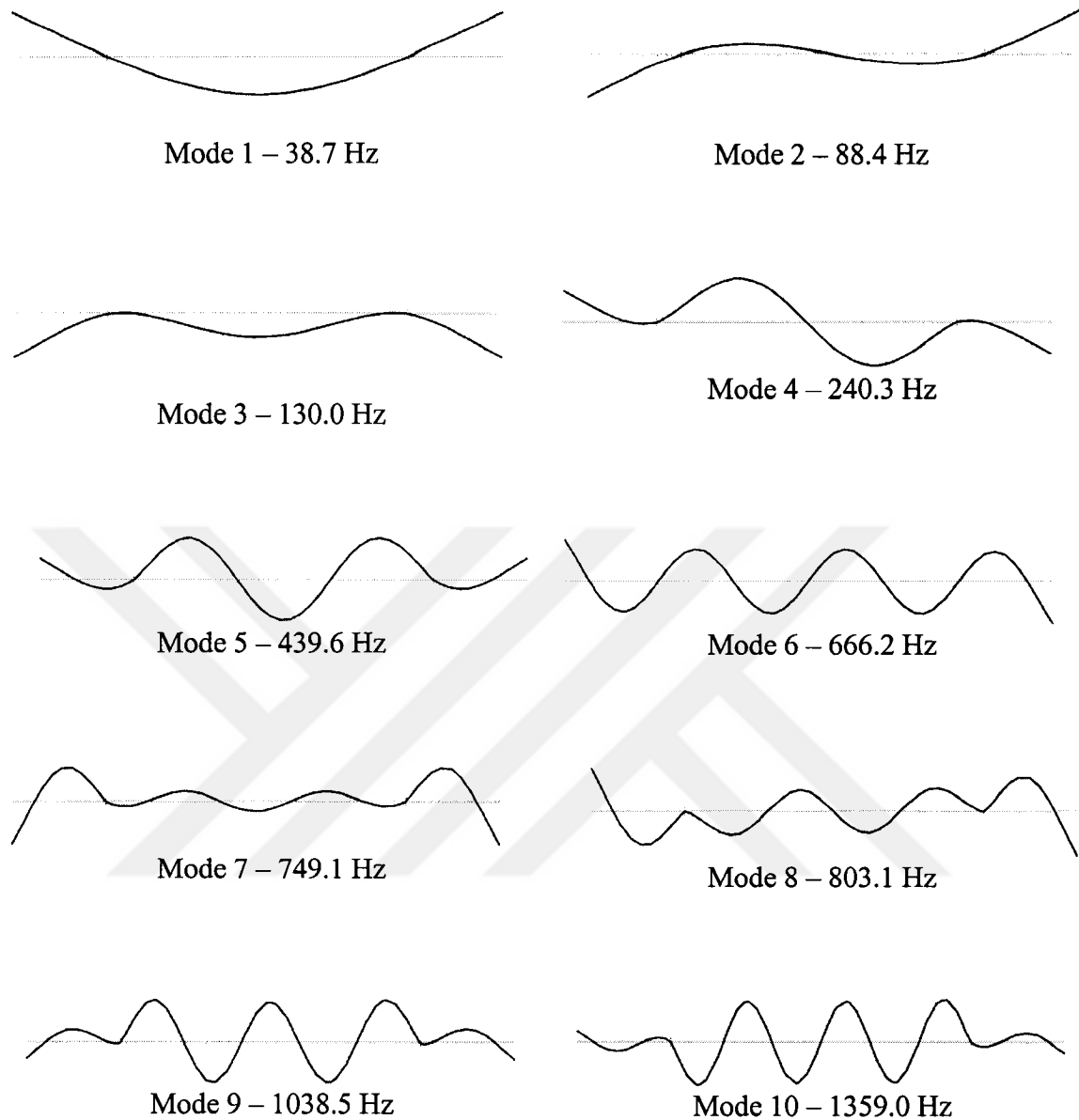


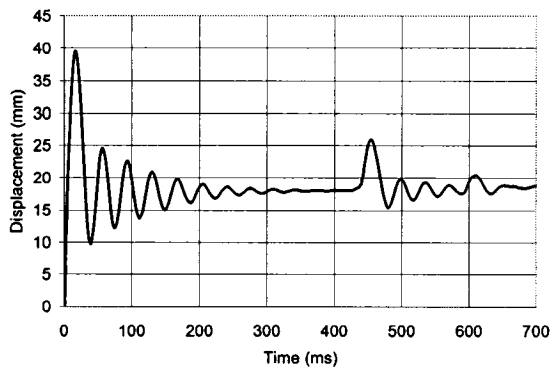
Figure 5.2 Mode shapes and frequencies for test beams

As can be seen from Figure 5.2, the expected frequency content for the first nine modes of vibration are within the range of the maximum frequency that can be captured with a 2400 Hz sampling rate, which is 1200 Hz as the sampling theorem states. The cumulative elastic modal mass participation factor for the first nine modes is 89%, which means the higher modes of vibration have only a small effect on the overall response.

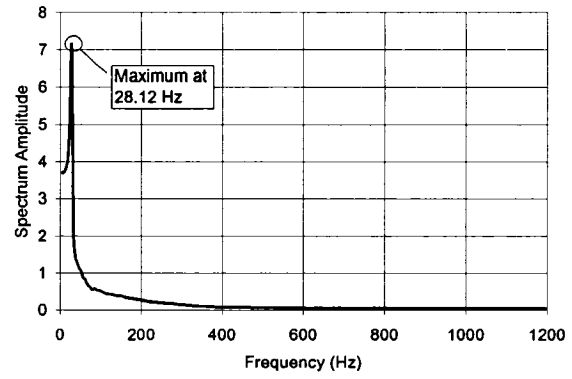
Similar to the elastic response, the actual inelastic response, as measured during the test, can also be examined for its vibration frequencies. A common method used to determine the actual frequency content of a digitised signal is to calculate its *power spectrum*. Basically, any harmonic wave can be represented as the composition of a series of sinusoidal waves, commonly known as *Fourier series*, with different amplitude and frequencies. The power spectrum of a signal gives information on the composition of these waves, in terms of their frequencies and corresponding magnitudes. The higher the magnitude of a composing sinusoidal wave, the higher will be its effect on the overall shape of the response. The methods and procedures needed to derive the power spectrum of a signal is a broad issue and its details can be found elsewhere (Stearns and Hush 1990). In this study, software program catman[®], which was also used for data acquisition as mentioned in Chapter 4, was used to obtain the power spectrum of the measured signals.

Since the specimens were loaded at the mid-span, and the predominant first mode of vibration has the shape of a beam statically loaded at the mid-span, the displacements measured at the mid-span can be taken as a representative measurement to determine the frequency content of the overall displacement response. For the following discussions, three different tests were considered: SS1b-1 for high levels of damage (Figure 5.3), SS2a-1 for low levels of damage (Figure 5.4), and SS2a-2 for intermediate levels of damage (Figure 5.5). Note that downward displacements are assumed as being in the positive direction in the figures.

Power spectrums from these three displacement measurements were derived with catman[®]; they also are presented in Figures 5.3 to 5.5. Note that the second peak in the response corresponds to the second hit of the drop-weight after it bounces off the specimen, and it was excluded from the power spectrum. In other words, only the first 400 ms of the measurements were considered for power spectrum calculations.

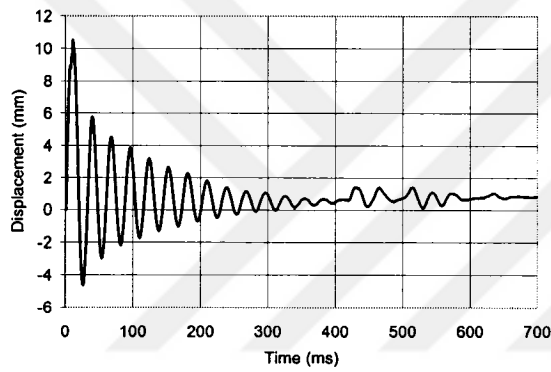


(a) Displacement

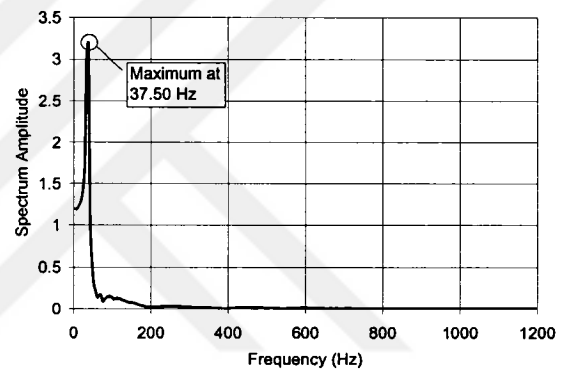


(b) Power spectrum

Figure 5.3 Mid-span displacement, SS1b-1

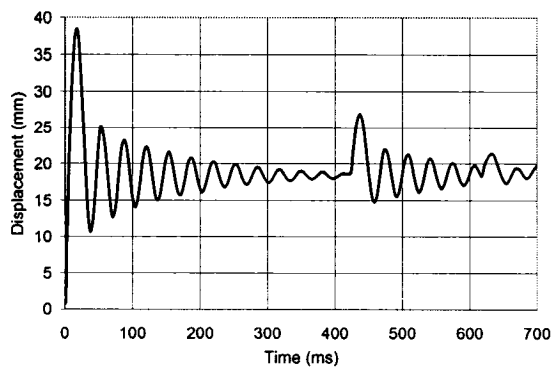


(a) Displacement

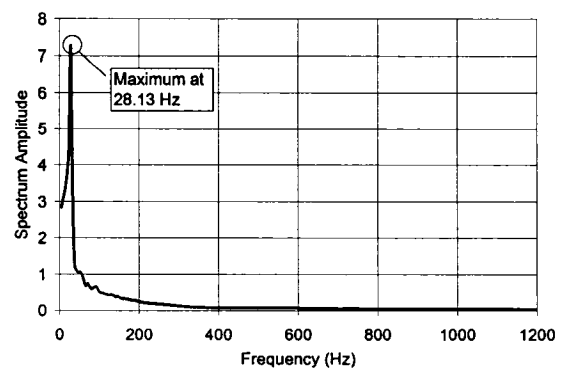


(b) Power spectrum

Figure 5.4 Mid-span displacement, SS2a-1



(a) Displacement



(b) Power spectrum

Figure 5.5 Mid-span displacement, SS2a-2

As seen in the preceding figures, all three measurements have a distinctive predominant frequency at around 30 Hz. As expected, the low-damage level response, such as the one observed in SS2a-1, has a higher predominant frequency, close to the elastic first mode frequency. When a specimen suffers more damage, its predominant frequency drops, reflecting a reduced stiffness. For all three cases, there is a sharp drop in the amplitudes of the frequencies after the peak, and the responses do not contain any significant component above 400 Hz. Although theoretically there is still a possibility of the existence of higher amplitudes in the higher frequency range, considering the elastic mode shapes and frequencies, it is highly unlikely that a frequency component higher than 1200 Hz will have a significant effect on the response. In other words, the 1200 Hz maximum frequency that can be captured with the available sampling rate seems to be sufficient for an accurate measurement of the displacements.

5.2.2 Strain Data

Similar analyses can be performed for the other sensors as well. To investigate the response of the strain gauges, in a manner similar to the investigation of the displacement responses, SS1b-1, SS2a-1 and SS2a-2 were selected. The strain gauges at the mid-span of the respective specimens were chosen for the frequency analysis; Bar #3 Gauge 1 for SS1b-1 (Figure 5.6), and Bar #4 Gauge 1 for SS2a-1 (Figure 5.7) and SS2a-2 (Figure 5.8). Their power spectra were derived using catman[®], and are represented in the respective figures.

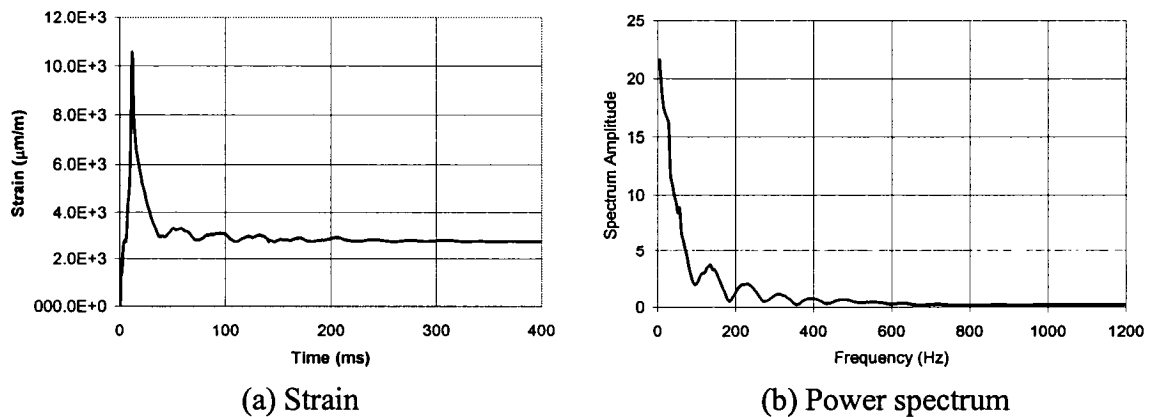


Figure 5.6 Strain at Bar #3 Gauge 1, SS1b-1

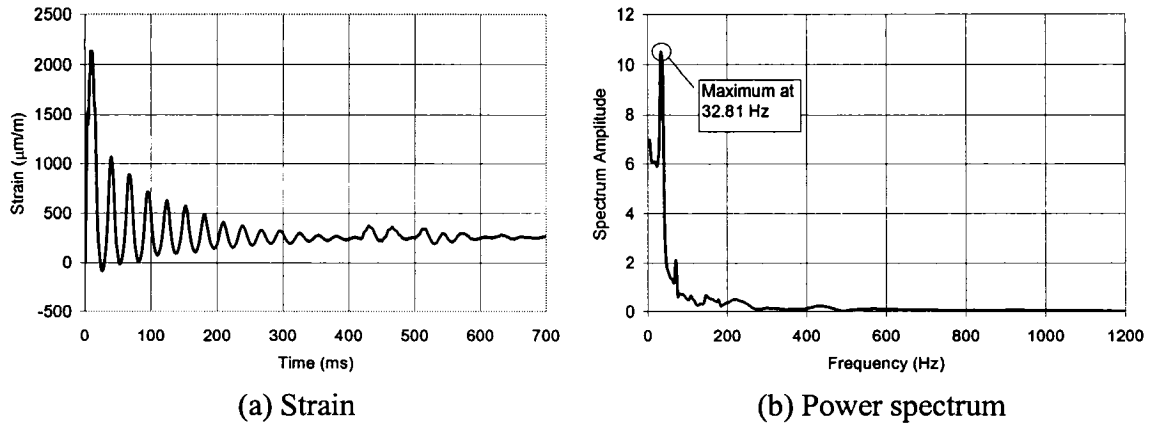


Figure 5.7 Strain at Bar #4 Gauge 1, SS2a-1

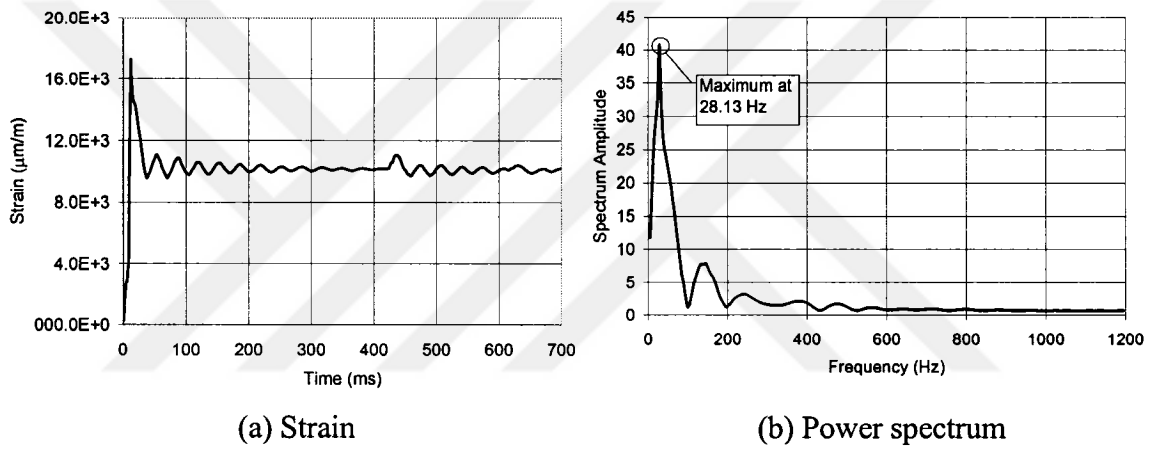


Figure 5.8 Strain at Bar #4 Gauge 1, SS2a-2

As seen in Figure 5.6, Bar #3 Gauge 1 shows significant yielding and deformation in SS1b-1. Therefore, the corresponding power spectrum is rich in low frequencies, due to the irregular nature of the vibrations after the impact. However, unlike with displacements, there are still frequency components in the 400-500 Hz range. SS2a-2 is very similar to SS1b-1, with the exception of a distinctive peak frequency and relatively regular vibrations after the impact. With SS2a-1, on the other hand, the specimen suffered much less damage, and the reinforcement bar barely yielded at the mid-span. The resulting power spectrum has a distinctive peak, close to the one observed in the displacement measurements, and the frequency components become insignificant after 300 Hz. In all three cases, similar to the displacement measurements, 1200 Hz seems to be a sufficiently high frequency to accurately capture the response of the strain gauges. However, unlike the displacement

measurements, some significant higher frequency components may still exist due to the high deformation rates of the yielded bars, which may result in misleading peak strains. For instance, a close look at the peak point of the Bar #3 Gauge 1 of SS1b-1 shows that there may be a higher peak between the two sampling points at the peak, as shown in Figure 5.9 with dotted lines. On the other hand, for most of the strain gauge measurements in this experimental program, this error was found to be insignificant.

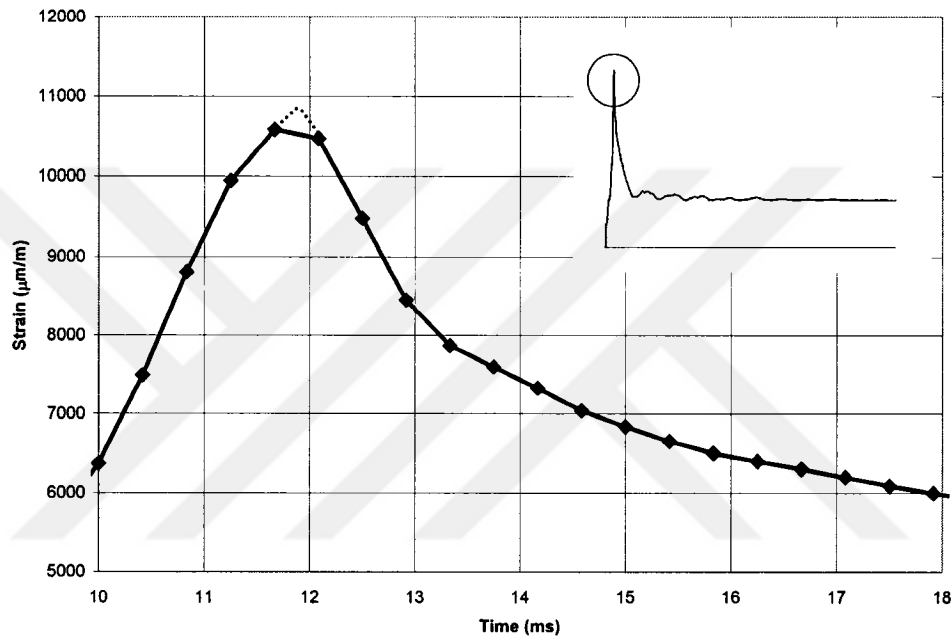
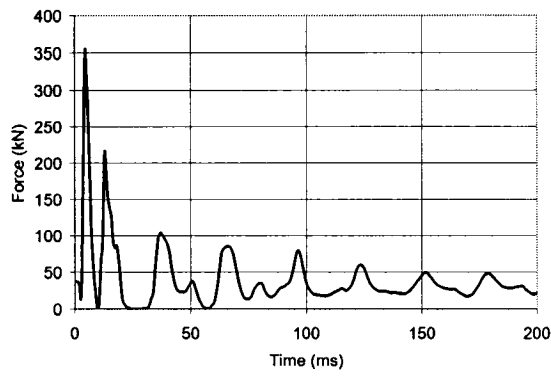


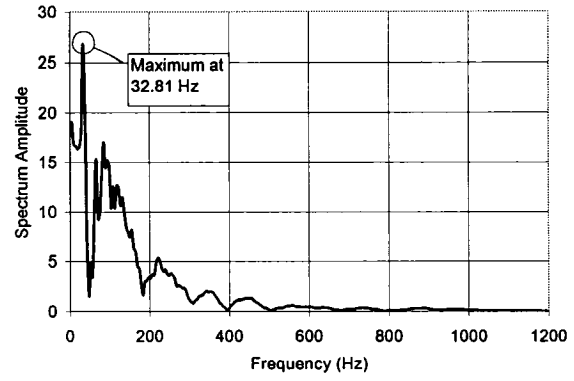
Figure 5.9 A closer look at the peak point of strain at Bar #3 Gauge 1, SS1b-1

5.2.3 Load Cell Data

To investigate the data obtained from the load cells, load cell readings from three different tests were considered: SS2a-1 for a low peak force (Figure 5.10), SS3b-1 for a high peak force (Figure 5.11), and SS1a-2 for an intermediate peak force (Figure 5.12). Readings from the load cell at the south support, Load Cell A, were used for the analyses. Power spectrums of the data are also presented in the respective figures. It should be noted that there was an initial load on the load cells due to the prestressing of the support bars. Therefore, forces obtained directly from the load cells are not the actual support forces.

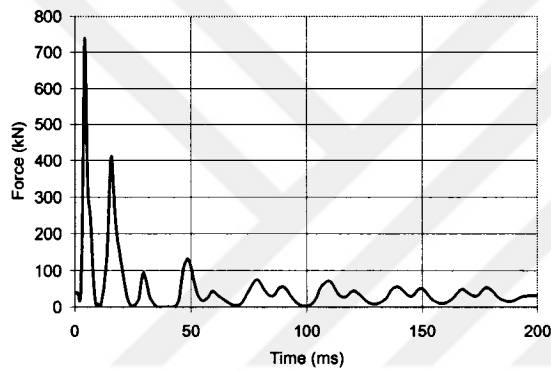


(a) Force

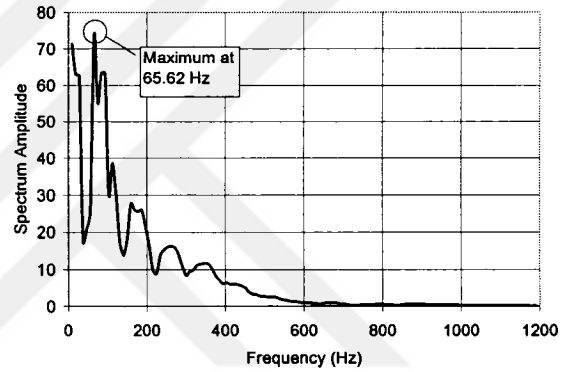


(b) Power spectrum

Figure 5.10 Forces as measured by Load Cell A, SS2a-1

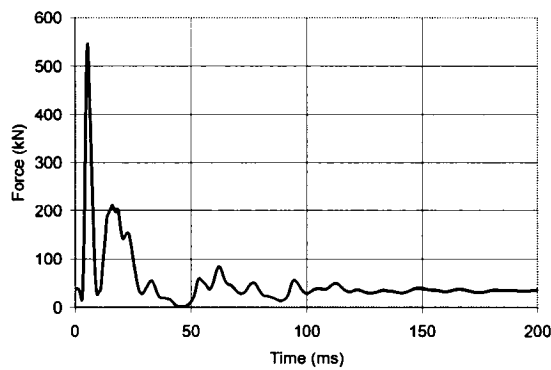


(a) Force

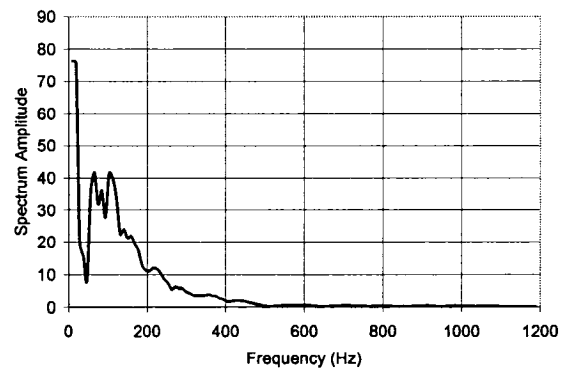


(b) Power spectrum

Figure 5.11 Forces as measured by Load Cell A, SS3b-1



(a) Force



(b) Power spectrum

Figure 5.12 Forces as measured by Load Cell A, SS1a-2

As can be seen in Figures 5.10 to 5.12, the load cell data have considerable frequency components up to 600 Hz. However, the amplitudes for the higher frequency components are mostly insignificant. The oscillations in force are mainly irregular in nature; hence, there is no distinctive single dominant frequency component. This behaviour can partially be attributed to the change in force transfer mechanisms due to asymmetric damage and cracking of the specimen, as well as the change in the prestressing forces at the support bars due to the movement and rotation of the specimen at the support points. A close inspection of the data points at the first peak of the SS3b-1 measurement reveals that there may be higher frequency components contributing to the response and, as a result, the peak force may be underestimated. However, the error caused by this is not significant. Although caution is still required with regards to the peak values, the 2.4 kHz sampling rate appears to be sufficient to capture the load cell response with reasonable accuracy.

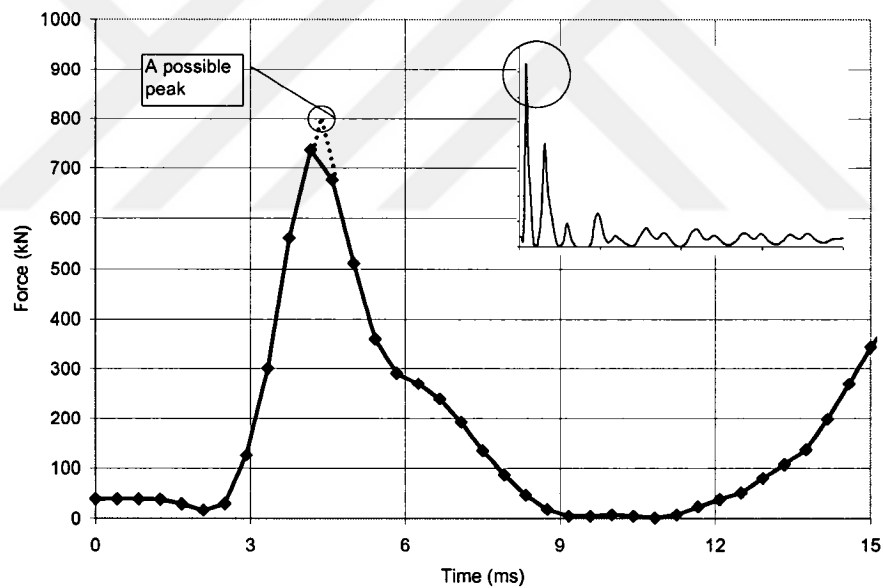


Figure 5.13 Data points at the first peak of the Load Cell A data, SS3b-1

5.2.4 Acceleration Data

Similar analyses were also carried out for the accelerometer measurements. Accelerometers were mounted on both the drop-weight and the specimen, and their response characteristics were different to some extent. Typically, the accelerometers mounted on the drop-weight registered significantly higher peak accelerations compared to those obtained from accelerometers mounted on the specimen. Therefore, the accelerometer measurements were examined in two categories for the following discussions: accelerometers mounted on the specimen, and accelerometers mounted on the drop-weight.

For the accelerometers on the specimen, measurements taken by the accelerometer mounted at the mid-span, A1, were used for sample readings. Three tests were chosen as representative cases: SS1b-2 for high peak acceleration (Figure 5.14), SS0a-1 for low peak acceleration (Figure 5.15), and SS3a-1 for intermediate peak acceleration (Figure 5.16). Their power spectrums are also presented in the respective figures. Note that accelerations in the direction of gravity are assumed as negative in the figures. It should be noted that, since the accelerometers were not sufficiently sensitive to record the subsequent low level vibrations after the first impact, their responses die much quicker than the displacement responses. Therefore, only the initial portions (approximately the first 100 ms) of the accelerometer responses are presented here.

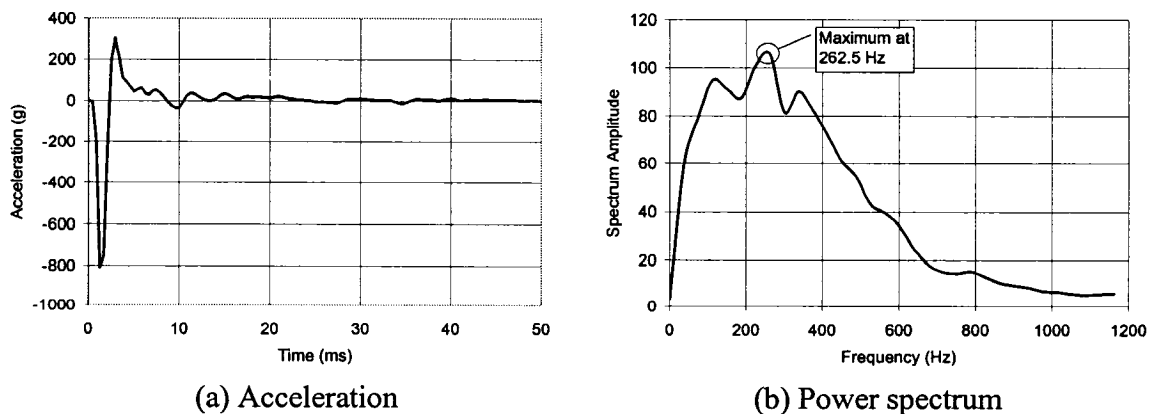


Figure 5.14 Accelerations as measured by A1, SS1b-2

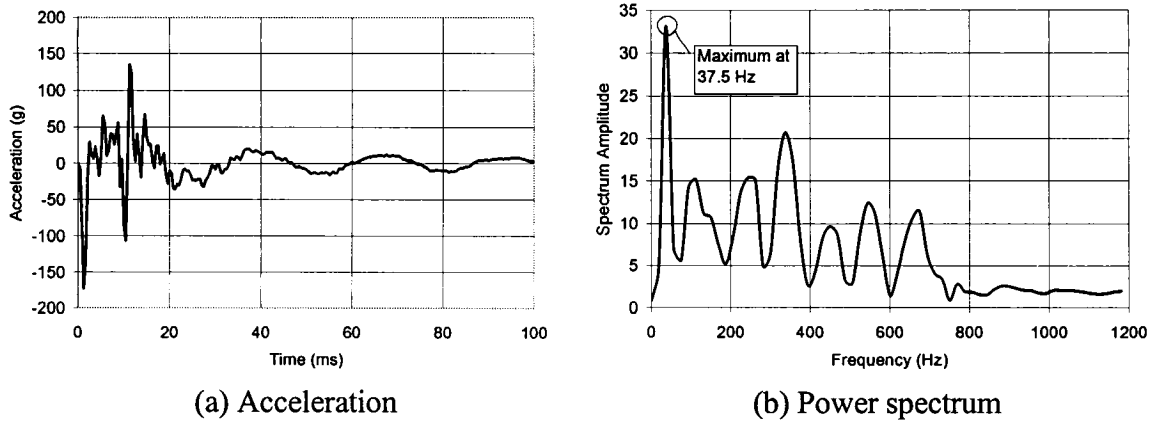


Figure 5.15 Accelerations as measured by A1, SS0a-1

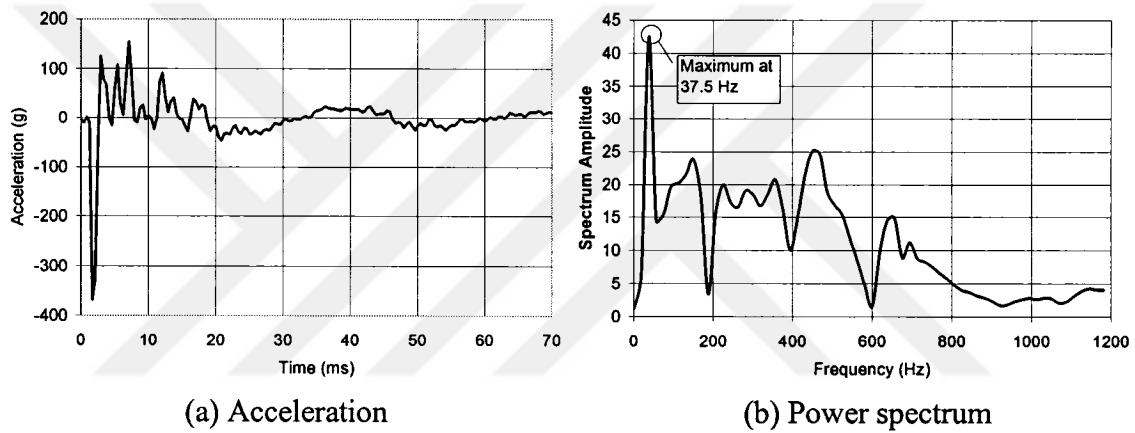


Figure 5.16 Accelerations as measured by A1, SS3a-1

As seen in Figures 5.14 to 5.16, the acceleration responses were significantly richer in high frequencies compared to displacement and strain responses, and it is likely that the exclusion of frequencies higher than 1200 Hz affects the accuracy of the measurement. For instance, a close inspection of the data points for the mid-span acceleration measurement of SS1b-2 reveals that the actual peak acceleration may have been missed due to the low sampling rate, since the acceleration increased very rapidly, and few data points could be taken during the rise (Figure 5.17). Unlike with the strain gauge measurements, this error seems to be much more significant in some of the acceleration measurements.

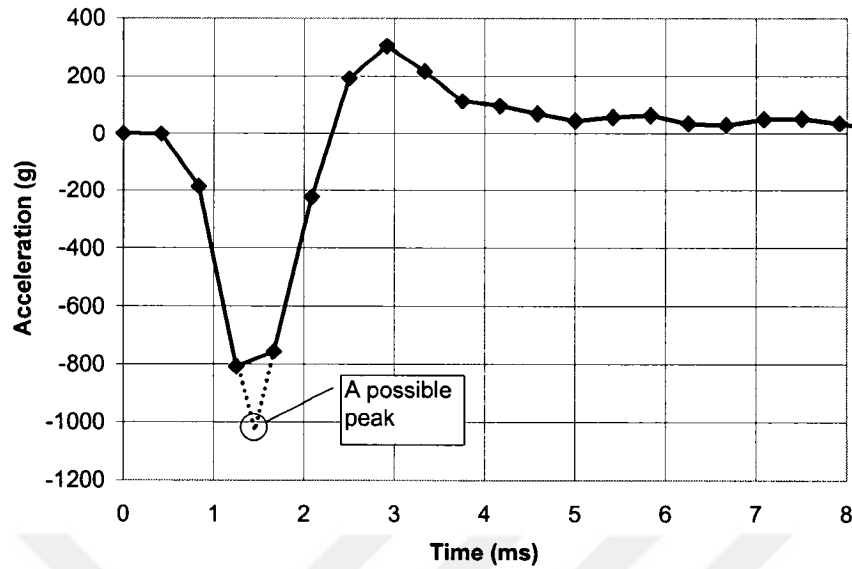


Figure 5.17 Data points for the mid-span acceleration measurement of SS1b-2

A similar situation is also observed in the accelerations measured by the accelerometers mounted to the drop-weight. Figures 5.18, 5.19, and 5.20 present the accelerations as measured by A6, along with their power spectrums, for SS1b-2, SS0a-1, and SS3a-1, respectively. As before, the accelerations in the direction of gravity are assumed as negative in the figures.

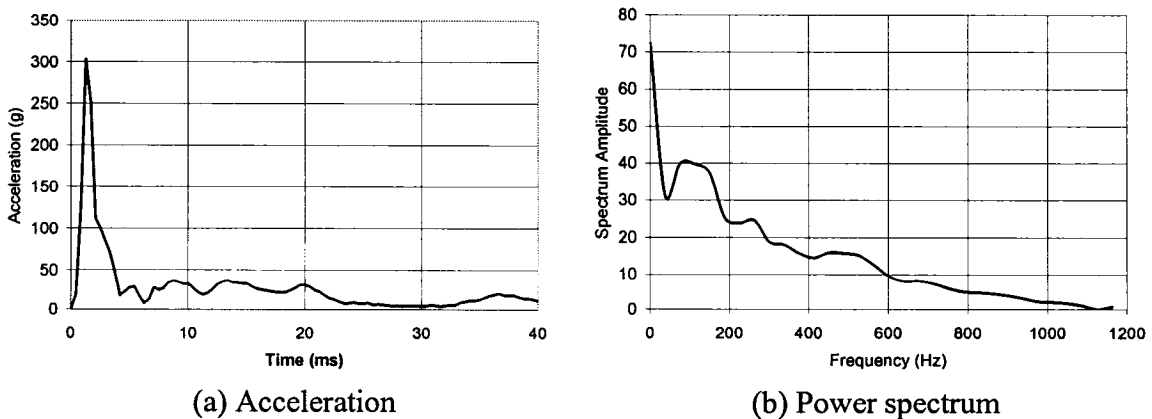


Figure 5.18 Accelerations as measured by A6, SS1b-2

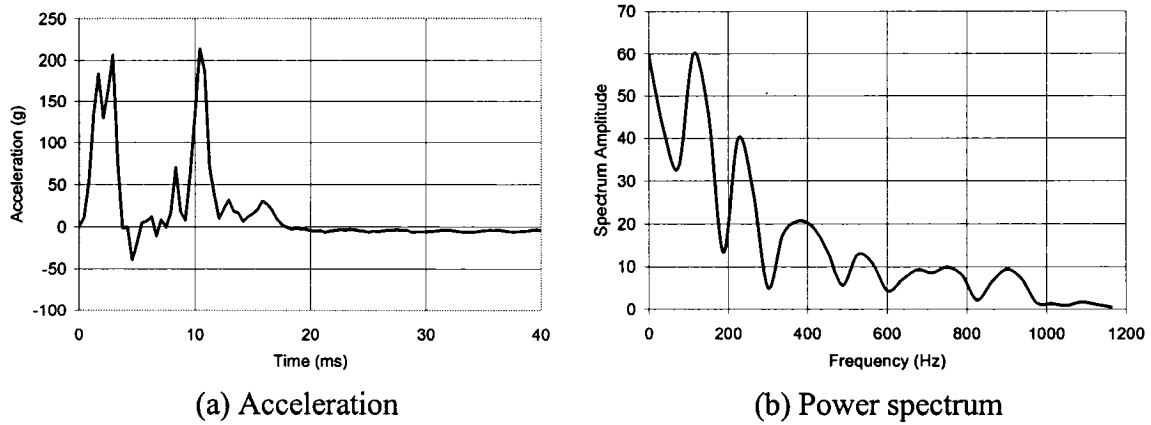


Figure 5.19 Accelerations as measured by A6, SS0a-1

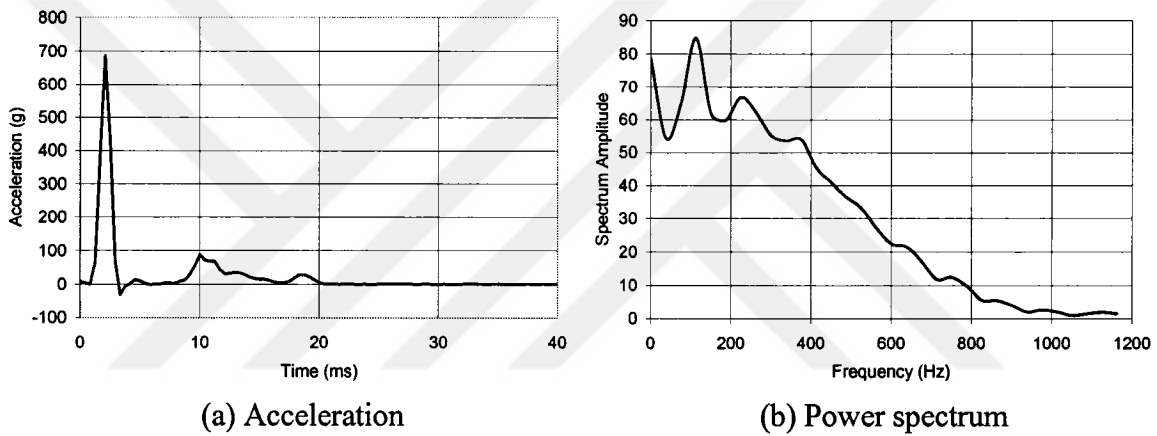


Figure 5.20 Accelerations as measured by A6, SS3a-1

Similar to the accelerometers mounted on the specimen, the responses measured by the accelerometers on the drop-weight contained significant high-frequency components, and the information missed due to the low sampling rate may be significant. As mentioned in the previous chapter, because of this suspicion, A6 was connected parallel to both a 2.4 kHz and a 19.2 kHz sampling rate connection board in SS1a-2. A comparison of the data, obtained from the same sensor but recorded by different connection boards, shows that the suspicions are correct; some high-frequency response were missed due to the low sampling rate (Figure 5.21).

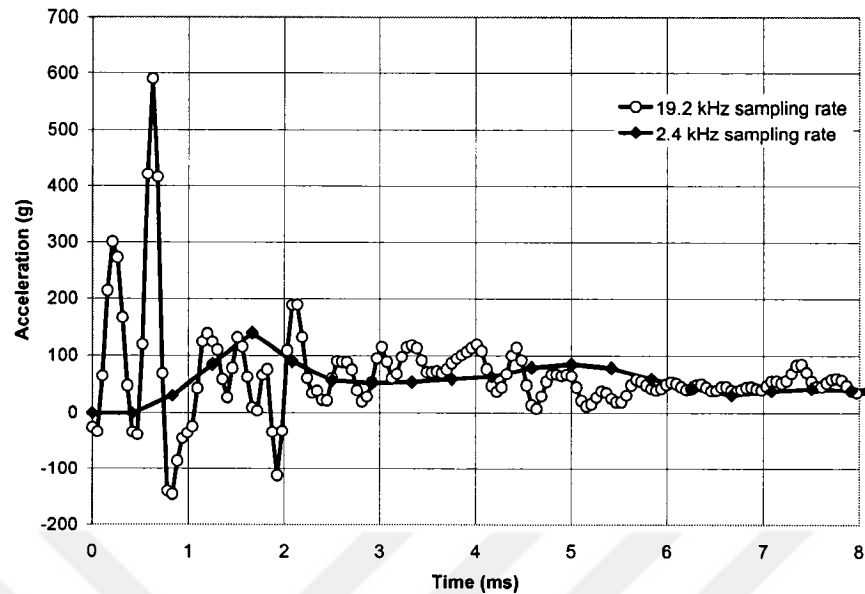


Figure 5.21 Comparison of the A6 data obtained with different sampling rates, SS1a-2

As seen in Figure 5.21, the accelerometer actually experienced a much higher acceleration at a much earlier stage, but it was missed with the 2.4 kHz sampling rate. Not only the peak acceleration, but also the high frequency oscillations were not recorded with the lower sampling rate connection board. The power spectrum of the acceleration measured by A6 and recorded with 19.2 kHz sampling rate shows that there exist significant high-frequency components in the response (Figure 5.22). Note that in Figure 5.22, the frequency components higher than 1200 Hz are omitted with the 2.4 kHz sampling rate.

As demonstrated with the sample measurements, the acceleration measurements recorded in most of the tests were questionable to some extent. The acceleration responses were much richer in frequency content than the displacement and strain responses, and the available 2.4 kHz sampling rate was not entirely sufficient to capture the actual acceleration responses accurately.

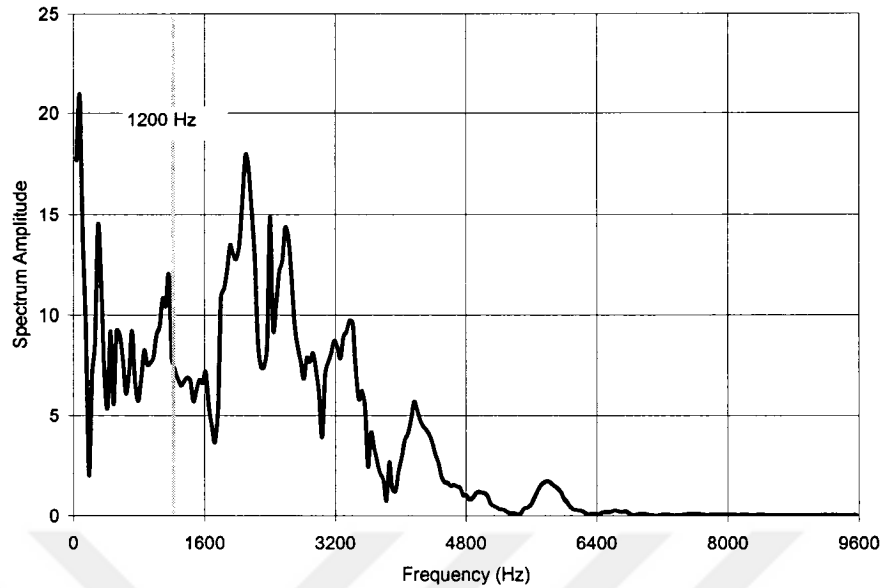


Figure 5.22 Power spectrum of the acceleration measured by A6 and recorded by 19.2 kHz sampling rate, SS1a-2

On the other hand, it is not correct to dismiss the acceleration data completely. For the accelerometers on the drop-weight, it should be noted that the experiments were essentially a hard impact test; impact force was induced by the hit of a relatively rigid steel drop-weight on a thick steel plate placed on the specimen. The steel-on-steel contact would induce significant high-frequency vibrations in the drop-weight itself, which in turn would be picked up by the accelerometers mounted on it. Therefore, it is safe to say that the high frequency response captured, or failed to be captured, by these accelerometers was not entirely a result of the actual accelerations experienced by the total mass of the drop-weight, but rather is partially a result of the vibrations caused by the stress-waves traveling through the drop-weight at the moment of impact. This problem, distinguishing the high-frequency vibrations from the actual response, was experienced in many other similar experiments and has been a concern of many researchers (Bischoff 1998, Found et al. 1998). One solution proposed was to capture the data with the highest possible sampling rate, and then apply a low-pass digital filter on the captured data to eliminate the frequency components higher than the expected response frequency, as determined from a preliminary response analysis (Figure 5.23). For example, in a similar beam impact test, May et al. (2005) captured the acceleration data with a sampling rate as high as 50 MHz, and applied a low-pass digital

filter at 800 Hz to eliminate the unwanted high-frequency vibrations. As can be seen in Figure 5.23, filtering of the data significantly eliminates the high peaks, and has an averaging effect on the remainder of the response. Since the sampling rate is already too low to capture the high-frequency vibrations entirely, it is not possible to make an accurate assessment of the situation in the current study. However, it is possible to say that the low sampling rate acts as an unintentional low-pass filter, which eliminates the effects of the high frequency vibrations on the drop-weight from the collected data.

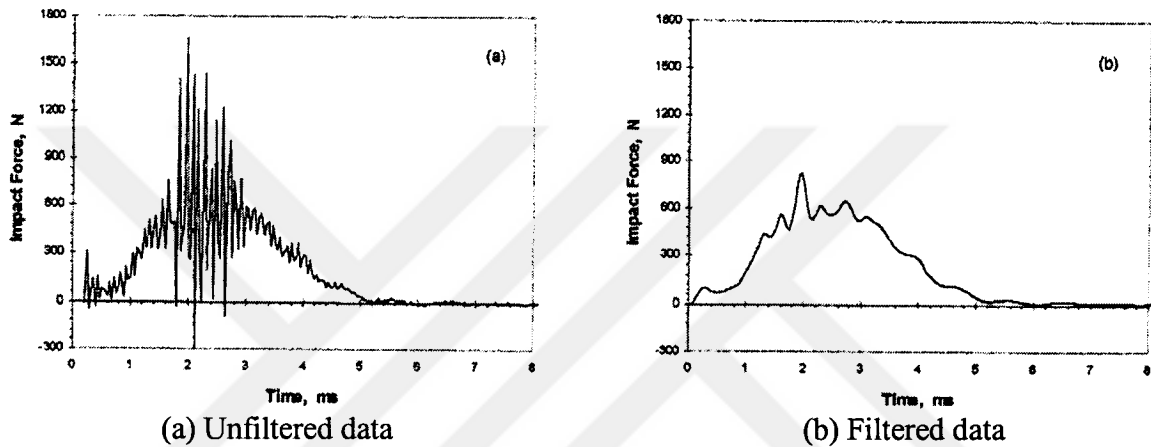


Figure 5.23 Filtered and unfiltered force-time plots for an impact (Found et al. 1998)

The accelerometers mounted on the specimen were different from those on the drop-weight, in the sense that they measured the movement of the specimen and did not have complications due to high-frequency vibrations. The only concern was with regard to the vibrations of the aluminum brackets on which they were mounted; therefore, the brackets were strengthened with triangular pieces and the accelerometer itself was bolted close to the inner edge, next to the beam's surface. Since it was shown previously that the displacement data were reliable, their second time-derivative would provide an indication of the accelerations that the beam specimen actually experienced, and hence, if the acceleration data are reliable. Figures 5.24, 5.25, and 5.26 provide this comparison for the accelerations measured by A1 at the mid-span, and the second-derivatives of the displacements measured at the same location, for SS1b-2, SS0a-1, and SS3a-1, respectively. The second time-derivatives of the displacements were calculated with catman[®].

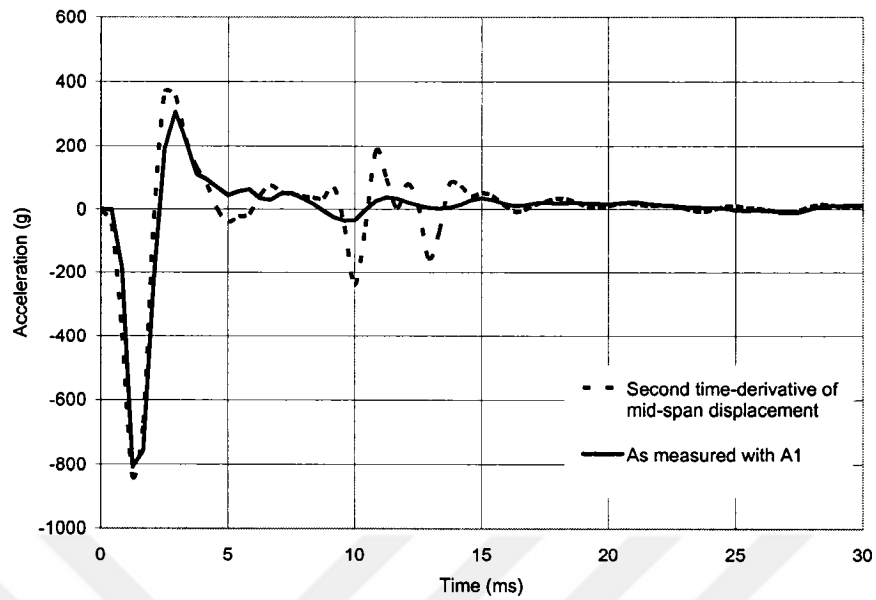


Figure 5.24 Comparison of A1 acceleration data and the second-time derivative of the mid-span displacement, SS1b-2

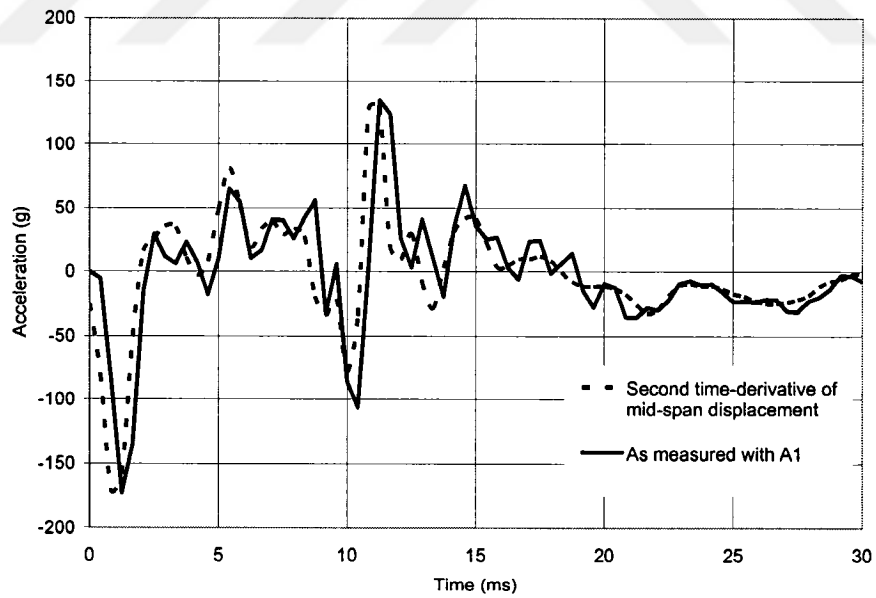


Figure 5.25 Comparison of A1 acceleration data and the second-time derivative of the mid-span displacement, SS0a-1

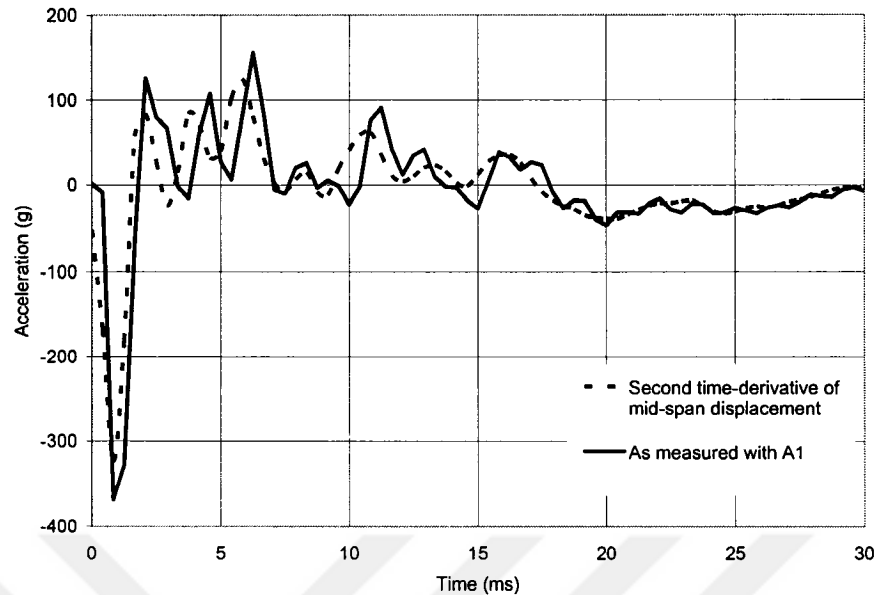


Figure 5.26 Comparison of A1 acceleration data and the second-time derivative of the mid-span displacement, SS3a-1

Figures 5.24 to 5.26 show that there is good agreement between the accelerations measured by A1 and the second time-derivatives of the displacement measured at the same location. Some discrepancies exist, but the peak accelerations agree especially well, despite the possible errors in capturing the peak acceleration as mentioned before. It should be noted that still it would not be possible to capture a high frequency response component by taking the second time-derivative of the displacements, since it is also derived from discrete data sampled at 2.4 kHz rate. But it is an important observation that the data obtained by two different sensors agrees well, which provides some validation of the reliability of the acceleration data recorded by the accelerometers mounted on the specimen.

Although caution is still required, it can be concluded that the data obtained by the accelerometers mounted on the specimen have reasonable accuracy. However, the same cannot be said for the accelerometers mounted on the drop-weight, since it is difficult to assess the actual response with the available sampling rate. As mentioned previously, the 2.4 kHz sampling rate may act as a filter on the high frequency vibrations. But the information at hand is not adequate to use these acceleration data confidently. Therefore, data obtained from drop-weight mounted accelerometers should not be used as a primary parameter.

For the forthcoming discussions, the acceleration data, whether measured on the specimen or on the drop-weight, are used as if they are correct. This assumption may affect the accuracy of some calculations, but the methodology presented can still be considered valid.

5.3 Impact Force Measurement

Measuring the impact force in an impact experiment presents some unique challenges. One particular solution is to place a force measuring device, such as a load cell, between the drop-weight and the structure. This approach does have some drawbacks, since the presence and the arrangement of a heavy load cell may itself affect the contact and the measurements (Bischoff 1998). If it is a hard impact, measuring the acceleration of the drop-weight is another solution to this problem; the impact force can simply be calculated as the acceleration multiplied by the mass of the drop-weight. This approach also requires caution, however, due to the stress-waves propagating through the drop-weight, as previously discussed. A third approach to calculating the impact force is to take indirect measurements, such as the displacements, reaction forces and accelerations on the impacted body, and perform an inverse analysis to determine the impact force that caused them (Inoue 2001). However, such an analysis is rather complex and mostly applicable to linear elastic structures, for which the overall structural behaviour can be modelled accurately.

In this experimental program, the second approach was preferred over the others. Both drop-weights were weighed to determine their masses, and two accelerometers were mounted on them to measure their accelerations during impact. To determine the impact force history, the mass of the drop-weight was multiplied by the average of the accelerations measured by two accelerometers, A6 and A7. It should be noted that in some tests, one or both accelerometers were saturated, or sustained broken cables. Therefore, in those tests, only the available acceleration data were used to determine the impact force history.

In order to verify the reliability of this approach, simple tests were conducted by dropping the drop-weights onto a load cell. The 211 kg drop-weight was dropped on a 3000 kN capacity load cell, which was sandwiched between two steel plates. A spherical bearing was placed at the contact point, to minimize the effects of an eccentric hit on the load cell.



Figure 5.27 Test setup for drops on a load cell

Four drop tests were conducted; two from a 300 mm height, and two from a 500 mm height. The drop heights were kept small to avoid damage to the load cell. Load cell readings, along with the individual and average acceleration readings multiplied by the mass, are presented in Figures 5.28 and 5.29 for the 300 mm drops, and in Figures 5.30 and 5.31 for the 500 mm drops. Note that only the first hit of the drop-weight is presented, since the measurements taken for the second hit after the drop-weight bounced were unreliable due to the movements of the steel plates.

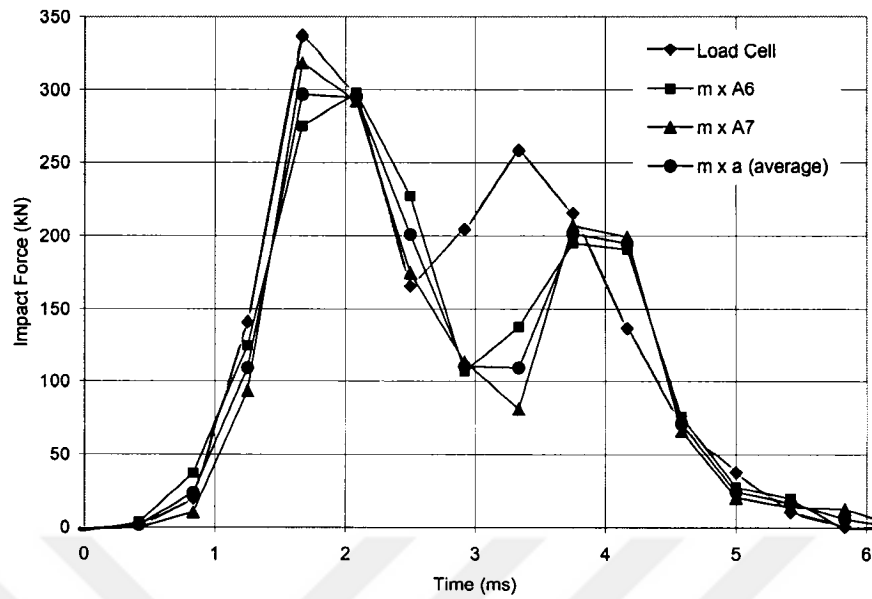


Figure 5.28 Test drop on a load cell from 300 mm – Test 1

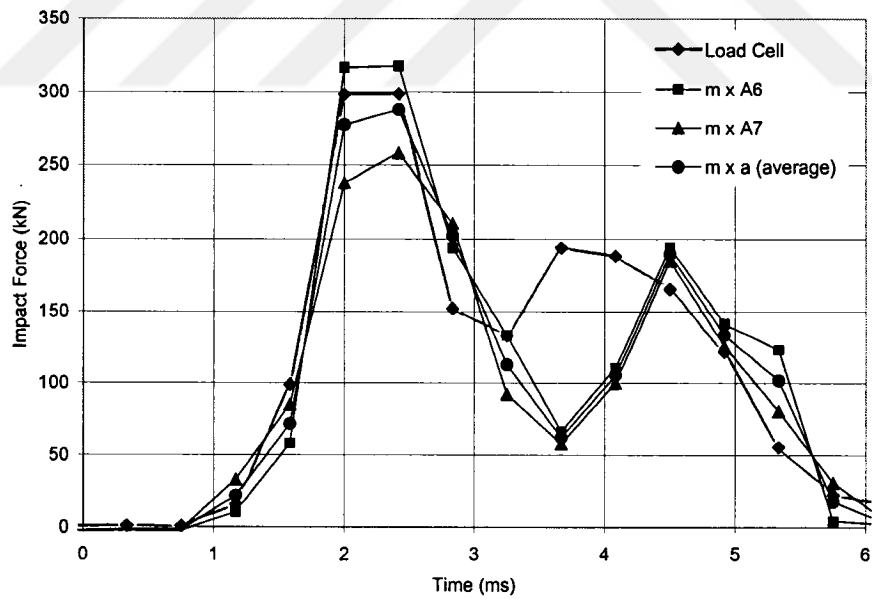


Figure 5.29 Test drop on a load cell from 300 mm – Test 2

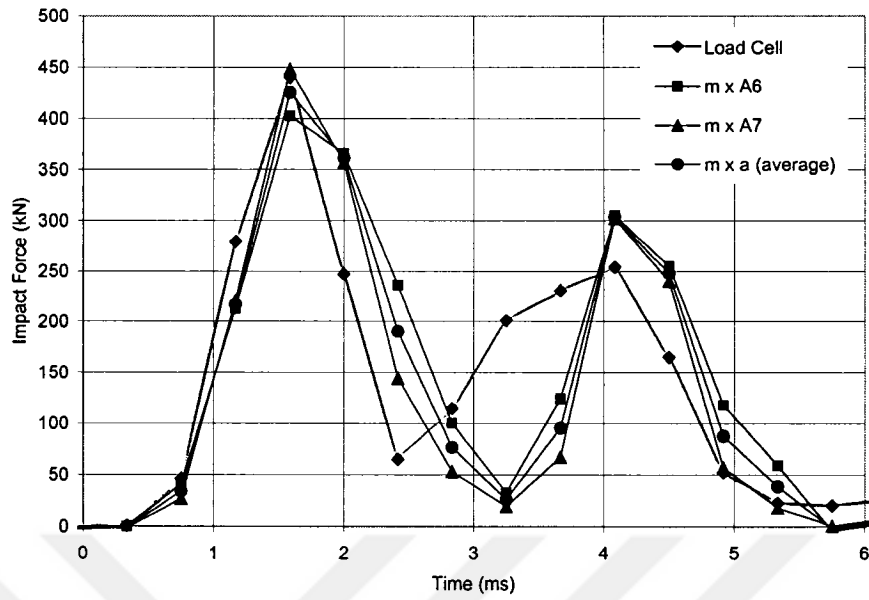


Figure 5.30 Test drop on a load cell from 500 mm – Test 1

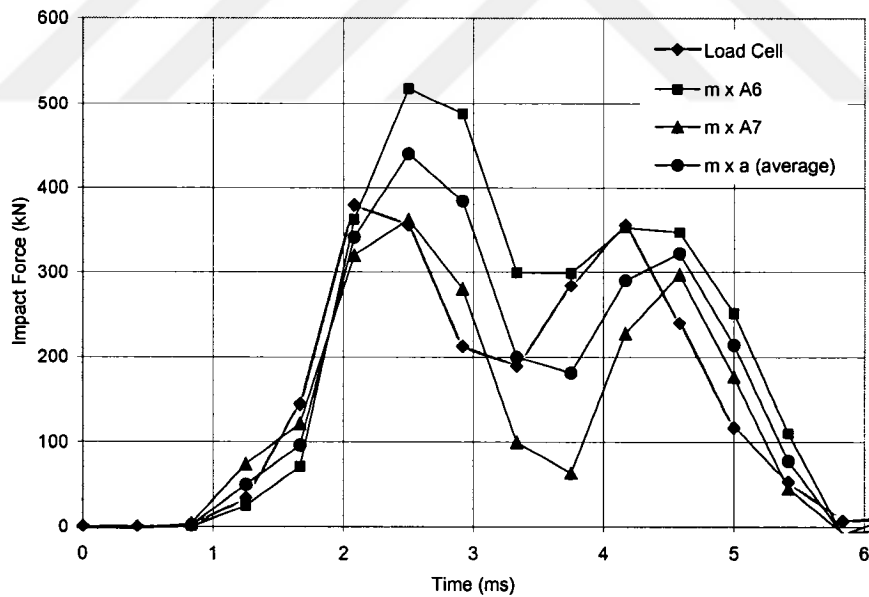


Figure 5.31 Test drop on a load cell from 500 mm – Test 2

As seen in the figures, there are some discrepancies between the load cell readings and the forces calculated by using the acceleration data. For instance, in all four tests, there exists a second peak immediately following the first one, which shows high inconsistency between the load cell and the acceleration data. This second peak may have been caused by the movement of the plates or the rotation at the spherical bearing, and can be discarded for the comparison. When only first peak is considered, it can be seen that the agreement is strong in general. However, the average of the two accelerometers did not give a better result for all cases; for example, in the first 500 mm drop, one accelerometer registers almost exactly the same force as the load cell, whereas the average of the two accelerometers is a relatively poor estimation. But in the second 300 mm drop, the average acceleration reading gives better results. In the second 500 mm drop, on the other hand, there is a high discrepancy between the acceleration data and the load cell data as far as the peak load is concerned.

It should be noted that this test setup was devised for quick verification and it was not a proper, well-designed one. Some deficiencies existed, such as the free-movement of the plates, and the eccentric drop and consequently rocking of the drop-weight. Therefore, the complete force history is not dependable. However, the generally good first peak agreement between the load cell and the acceleration readings suggest that the correlation is valid and that accelerations can be used to determine the impact force.

5.4 Displaced Shape

Displacement data play an important role in the analysis of the specimens, and are likely the most useful type of data collected for several reasons. First, displacement data are usually the easiest and the most direct type to acquire and compare with the results of any numerical or analytical method, and they can provide important information about the validity of the methodology. Moreover, some analytical methods start with an assumed displaced shape, such as those employing single-degree-of-freedom (SDOF) analysis, to derive the inertia and other forces acting on a specimen; a well-predicted displaced shape is critical to the accuracy of such methods. For these reasons, special attention was paid to the displacement measurements made during the test program, and the specimens were equipped with as many displacement sensors as was possible to capture the displaced shape

during the tests. In this section, the displaced shapes of the specimens are examined, to discern the general behaviour as well as test the validity of the SDOF assumption that many analytical methods use. During the test program, it was observed that the initial response time of the LVDTs and potentiometers were different, and hence, the displaced shape obtained by a mix of these two types of displacement sensors contained errors. Therefore, in the second half of the test program, all LVDTs were replaced with potentiometers, which had all consistent and faster response rates. For that reason, only those specimens, namely the b-series specimens, are used for sample demonstrations in the following discussions.

As mentioned before, 16 displacement sensors were connected to the specimen along the bottom face, measuring the displacements at 15 different locations (see Figure 5.32). Assuming zero displacement at the supports, and extrapolating the end displacements by using the slope of the overhanging parts, the displaced shape of the specimens can be obtained for each sampling time. These shapes, obtained up to the first peak of the vibrations, are presented in Figures 5.33 to 5.36, for SS3b-1, SS3b-2, SS1b-1 and SS1b-2, respectively. For SS0b-1, since most of the displacement sensors were disconnected from the specimen due to excessive concrete spalling, only about the first 20 ms of displacement response were captured successfully, as presented in Figure 5.37. Note that, in all figures, displaced shapes are given in 1.25 ms intervals as the specimens deflected downward, and they are shown as seen from the west face of the specimens. Recorded data points were connected by smoothened lines.

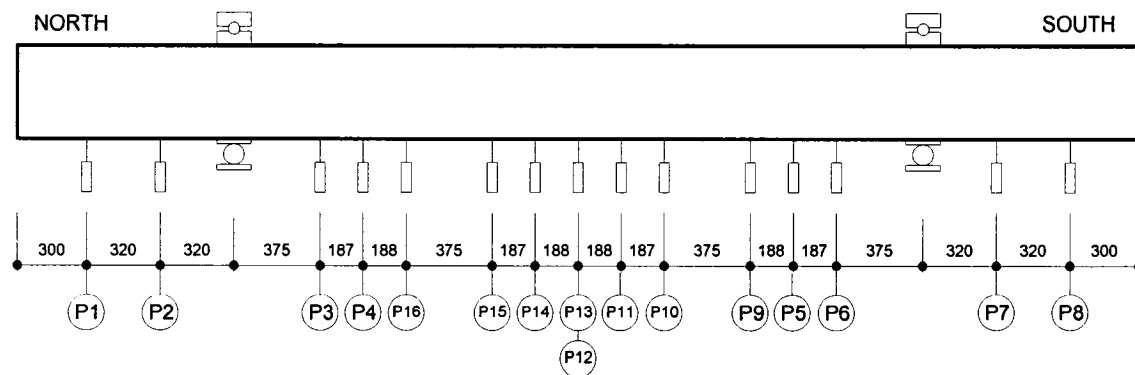


Figure 5.32 Displacement sensor locations for SS3b, SS2b, SS1b and SS0b
(L5 for lateral movement is not shown, all dimensions are in millimetres)

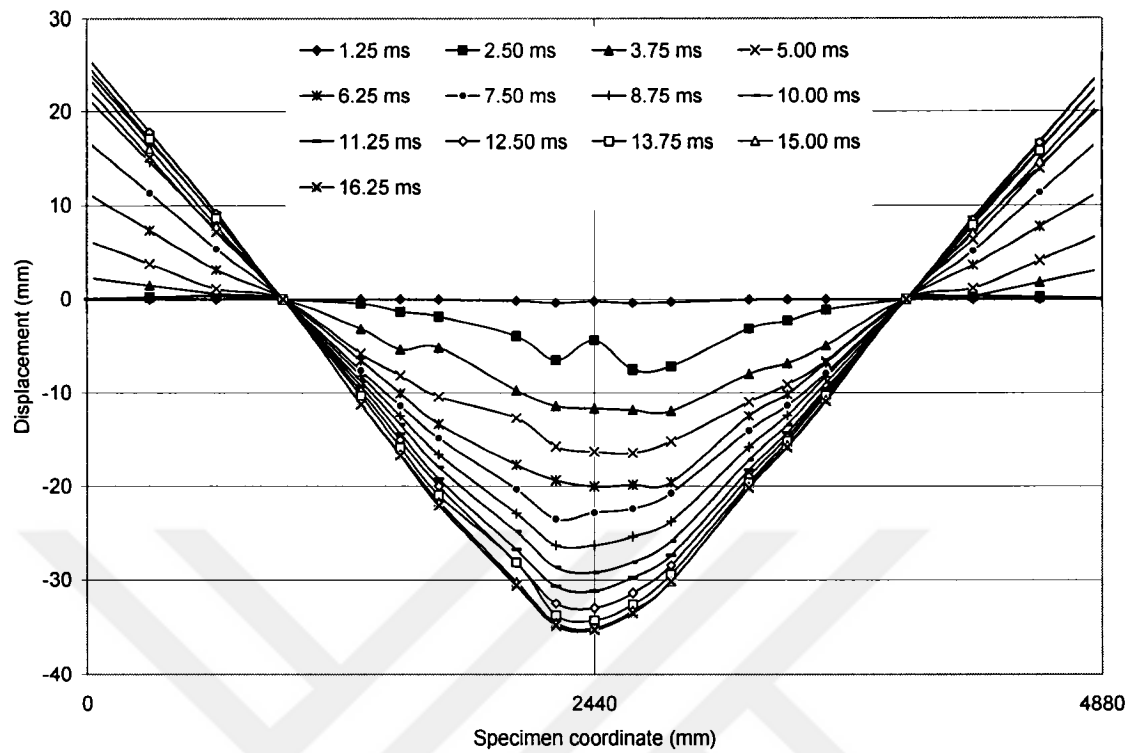


Figure 5.33 Displaced shape, SS3b-1

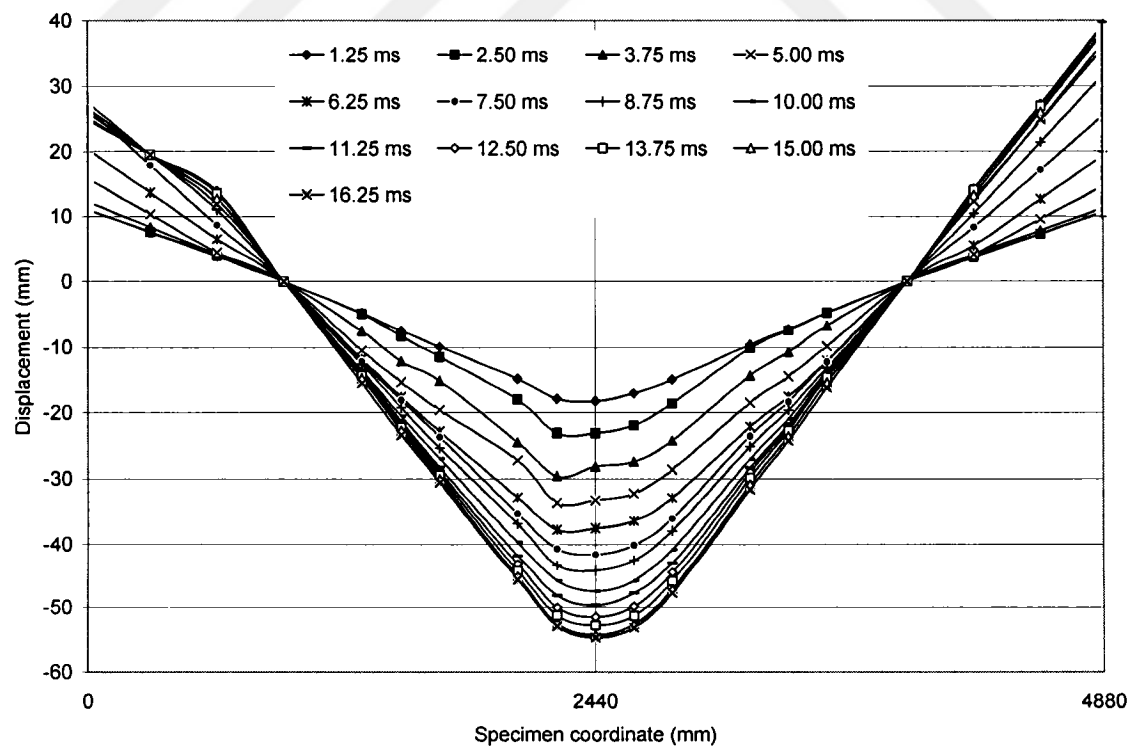


Figure 5.34 Displaced shape, SS3b-2

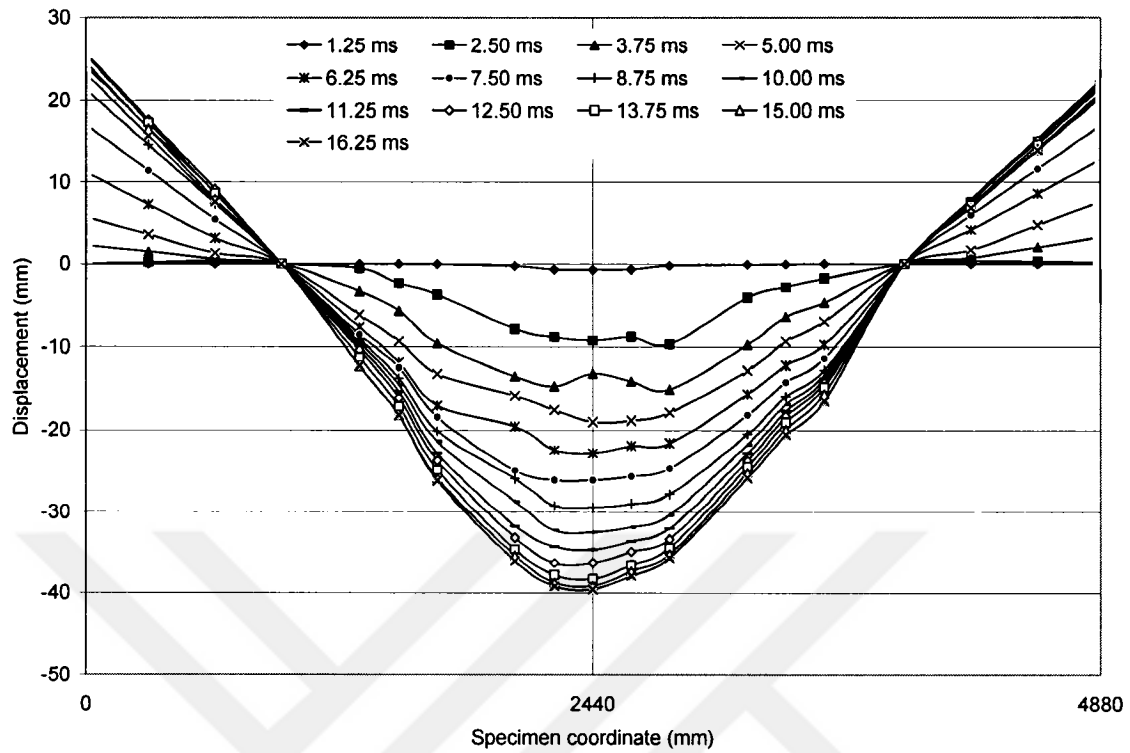


Figure 5.35 Displaced shape, SS1b-1

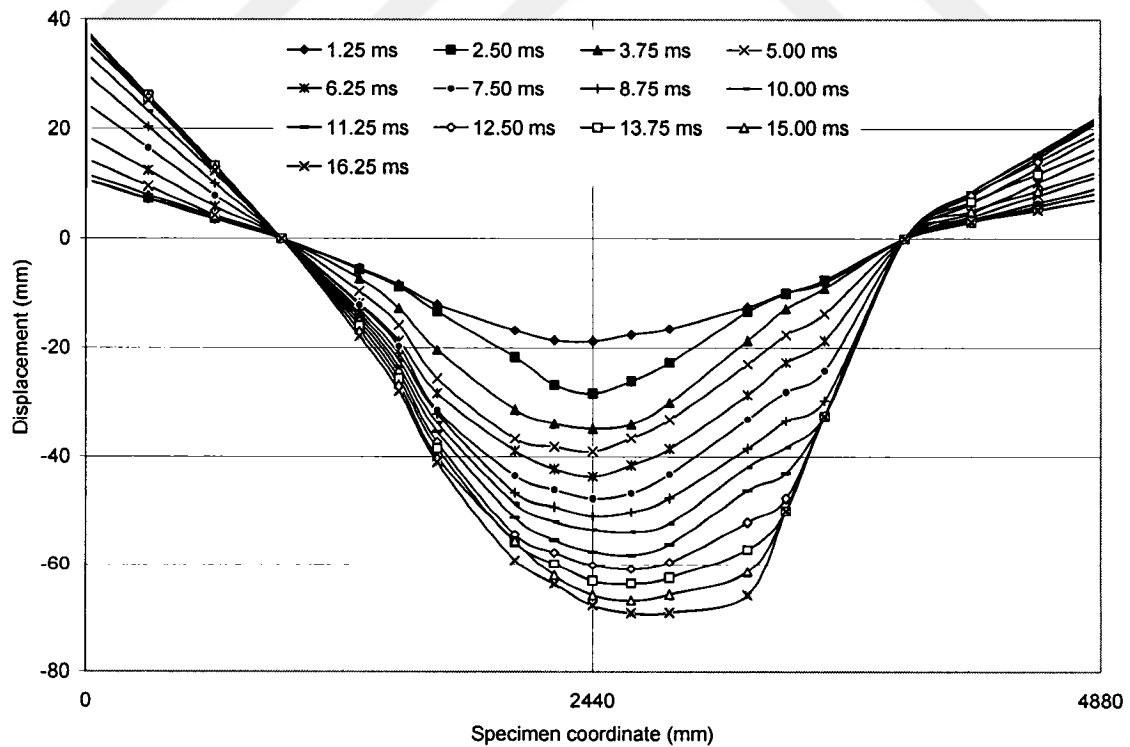


Figure 5.36 Displaced shape, SS1b-2

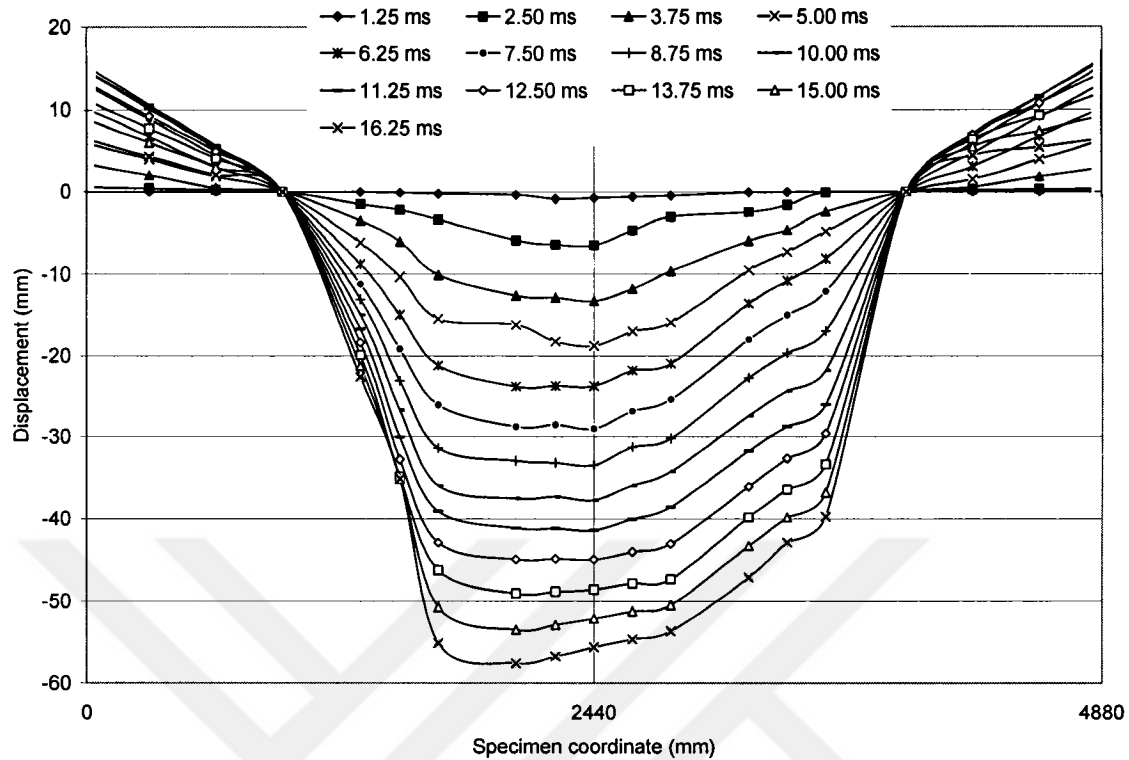


Figure 5.37 Displaced shape, SS0b-1

One distinctive feature of the displaced shapes is the formation of the shear-plug. As mentioned previously, nearly all specimens developed major diagonal cracks, starting at the top near the impact point and traveling down at about a 45-degree angle into both halves. In other words, instead of bending with the shape of a beam loaded at the mid-span, the drop-weight punched out the middle segment of the specimen, commonly referred to as forming a shear-plug. This behaviour can be observed in the displaced shapes to some extent; but it should be noted that, although the data points are connected with a continuous line in the figures, in reality there exists a discontinuity roughly between the locations of P9 and P10, and between P15 and P16, due to the propagation of the major diagonal cracks (Figure 5.38). Displacement sensors were not placed close enough to capture the precise location of this discontinuity; therefore, the exact profile of the shear-plug could not be captured. Nevertheless, it can be seen that the middle three displacements are very close in all tests, indicating the motion of the shear-plug. This behaviour is less apparent in the specimen with the highest stirrup ratio, SS3b, since the stirrups significantly reinforced the specimen against shear, allowing the bending of the middle segment to a certain degree. In SS1b, on

the other hand, there was notably less bending in the middle segment, evidenced by the comparable displacements recorded by the middle five sensors. In SS1b-2, the diagonal crack at the south half opened wide, separating the plug from the rest of the beam on one side (Figure 5.39), which is observed in the displaced shape as a sharp angle. SS0b had no stirrups, hence the middle segment punched through with almost no visible bending deformations; the shear-plug formation can be observed clearly in the displaced shape.

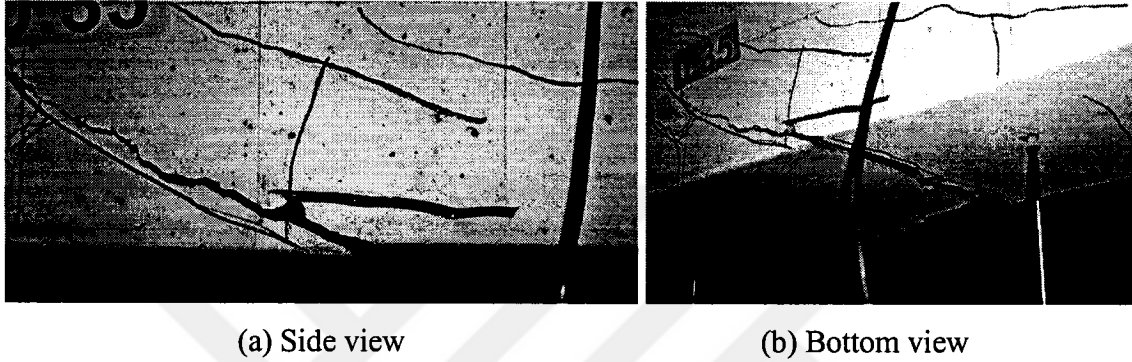


Figure 5.38 Major diagonal crack between P9 and P10, SS3b-2

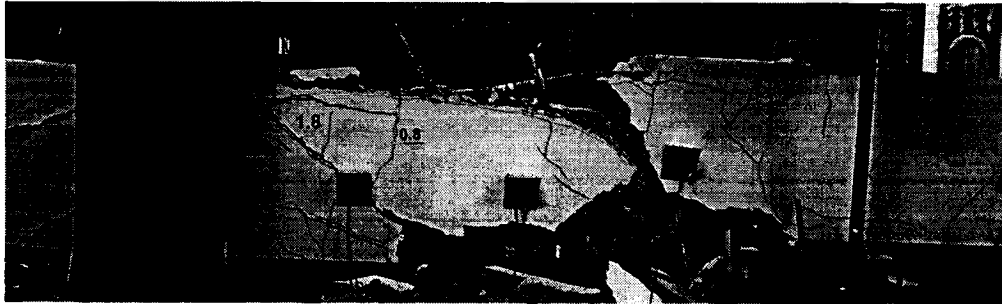


Figure 5.39 South half of SS1b, after SS1b-2

Determining whether the specimen deflected as a SDOF system is another problem which requires further investigation. The number of independent displacements required to define the displaced shape of a structure is termed the number of *degrees of freedom*. Therefore, in a single degree of freedom system, one single displacement should be sufficient to define the displaced shape. Consequently, the history of that displacement would give the whole displacement history of the structure, which can be expressed as,

$$u(x,t) = q(t)\psi(x) \quad (5.1)$$

where $q(t)$ is the displacement history of the degree of freedom, $\psi(x)$ is the dimensionless unit displaced shape, and $u(x,t)$ is the displaced shape at time t . $\psi(x)$ can be a continuous function, but it can be a discrete representation of the displaced shape as well.

For these specimens, if they are assumed to be SDOF systems, the obvious choice of the single degree of freedom would be the mid-span displacement, since they were loaded at the mid-span. The unit displaced shape $\psi(x)$ can be taken as a discrete function, defined at the locations of the displacement measurements. If a function $\psi(x)$ exists, such that the displaced shape is a multiple of it at any given time, then it can be proven that the system is a SDOF system.

If the system is a SDOF system, $\psi(x)$ can be chosen as the displaced shape of the specimen at any given time, normalized with respect to the mid-span displacement. For the following discussions, the displaced shapes of the specimens at the time of peak mid-span displacement were chosen as prospective functions $\psi(x)$, which are presented in Figures 5.40 to 5.42, for SS3b, SS1b and SS0b, respectively. Note that x is the coordinate from the north (left) end of the specimen. Displacements at the supports were assumed to be zero.

x (mm)	$\psi(x)$	x (mm)	$\psi(x)$
0	0.64	2625	-0.95
300	0.43	2810	-0.86
620	0.20	3185	-0.57
940	0.00	3370	-0.45
1315	-0.32	3555	-0.31
1505	-0.47	3940	0.00
1690	-0.62	4260	0.18
2065	-0.87	4580	0.40
2255	-0.99	4880	0.60
2440	-1.00		

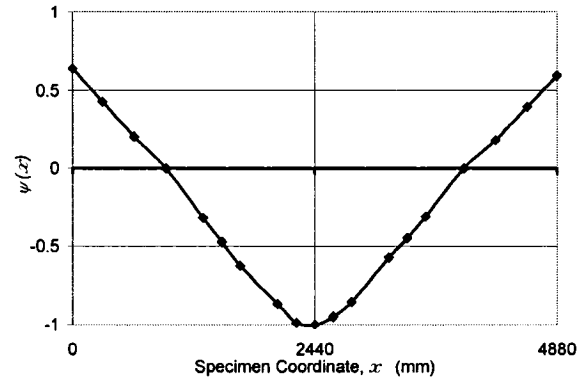


Figure 5.40 Unit displaced shape for SS3b, obtained from SS3b-1

x (mm)	$\psi(x)$	x (mm)	$\psi(x)$
0	0.59	2625	-0.96
300	0.40	2810	-0.90
620	0.19	3185	-0.65
940	0.00	3370	-0.52
1315	-0.31	3555	-0.42
1505	-0.46	3940	0.00
1690	-0.66	4260	0.17
2065	-0.91	4580	0.35
2255	-0.99	4880	0.52
2440	-1.00		

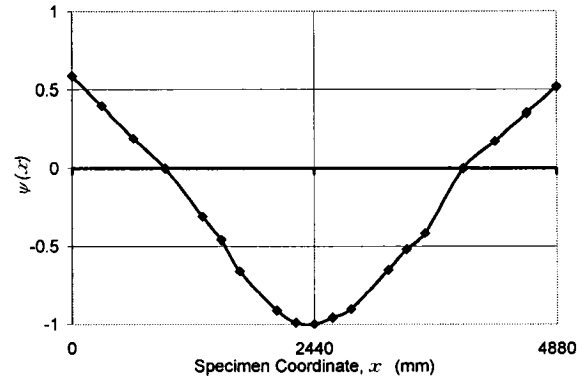


Figure 5.41 Unit displaced shape for SS1b, obtained from SS1b-1

x (mm)	$\psi(x)$	x (mm)	$\psi(x)$
0	0.12	2625	-0.98
300	0.08	2810	-0.96
620	0.04	3185	-0.85
940	0.00	3370	-0.77
1315	-0.41	3555	-0.72
1505	-0.63	3940	0.00
1690	-0.99	4260	0.08
2065	-1.04	4580	0.10
2255	-1.02	4880	0.12
2440	-1.00		

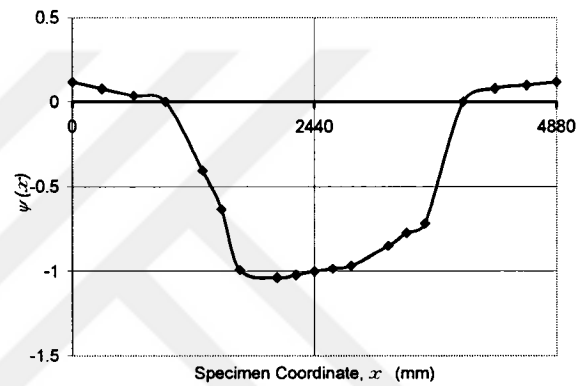
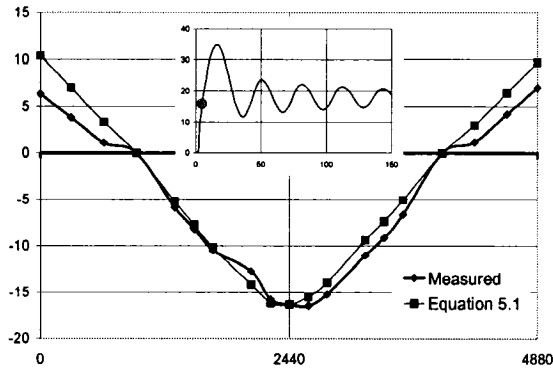
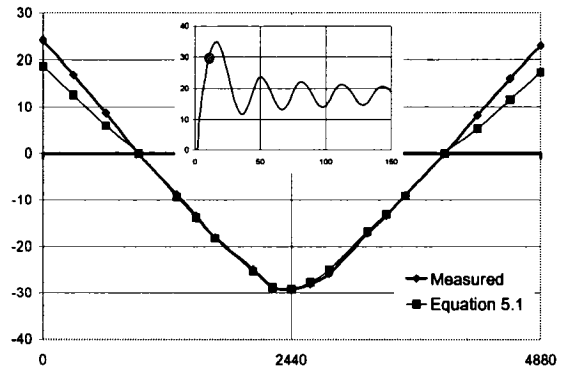


Figure 5.42 Unit displaced shape for SS0b, obtained from SS0b-1

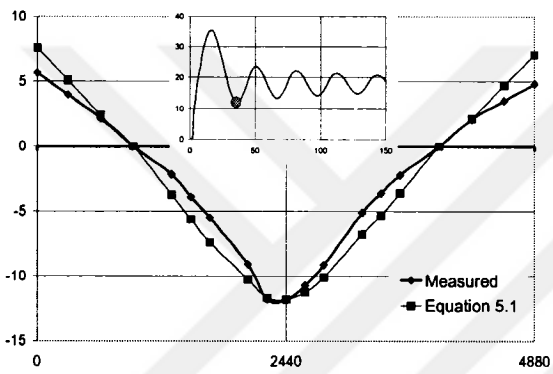
Once the unit displaced shapes are determined, the measured displaced shapes of the specimens can be compared with the ones found from Eq. 5.1. This comparison is made for several different stages of the responses, and presented in Figures 5.43 to 5.45, for SS3b-1, SS1b-1, and SS0b-1, respectively.



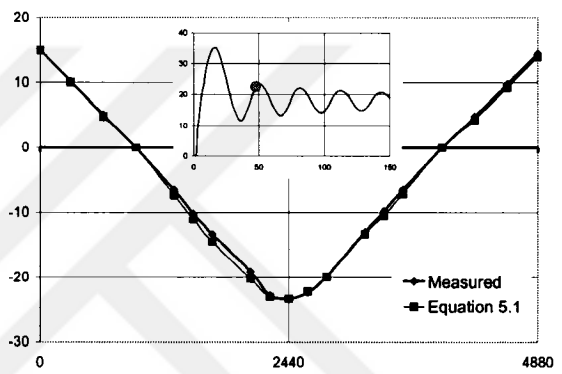
(a) 5 ms



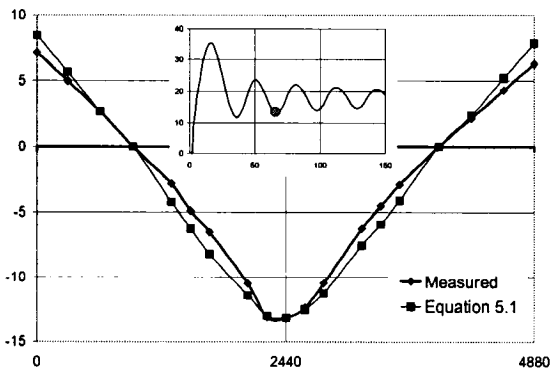
(b) 10 ms



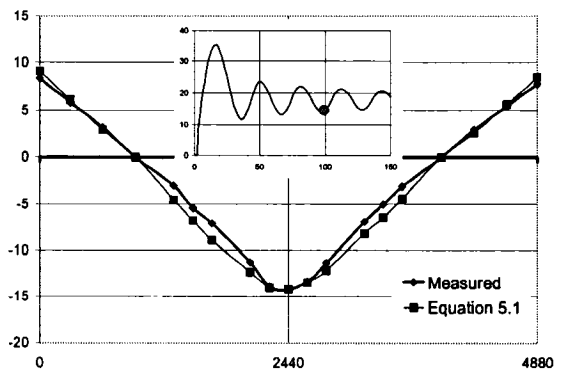
(c) 35 ms



(d) 48.75 ms

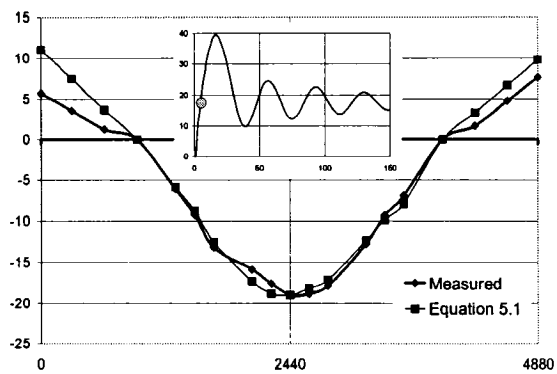


(d) 66.25 ms

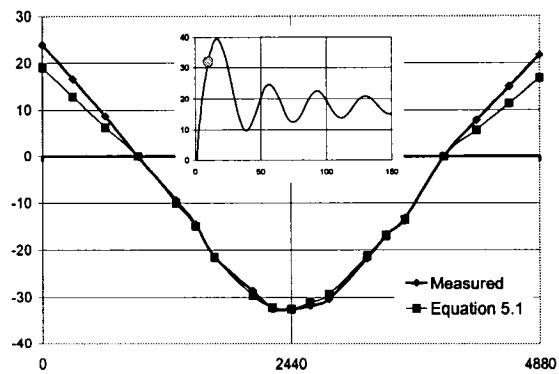


(e) 98.75 ms

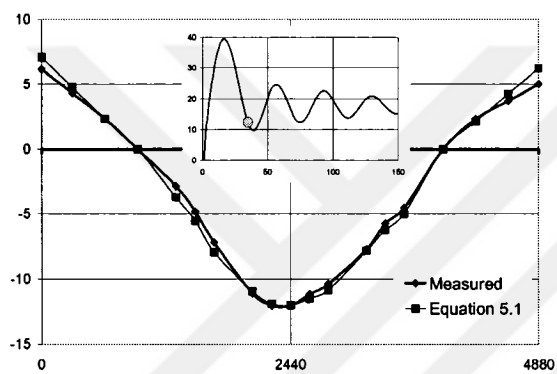
Figure 5.43 Displaced shapes as measured and as calculated by Eq. 5.1, SS3b-1



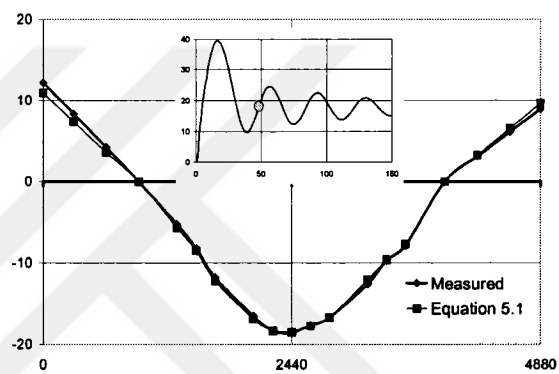
(a) 5 ms



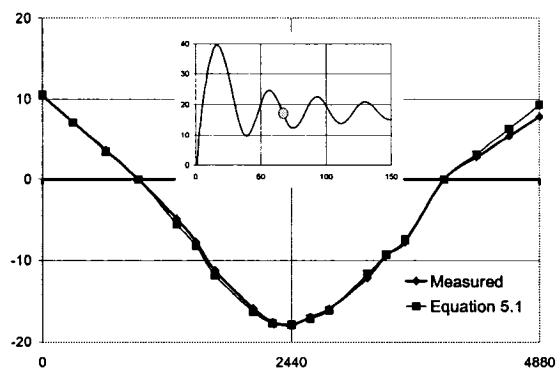
(b) 10 ms



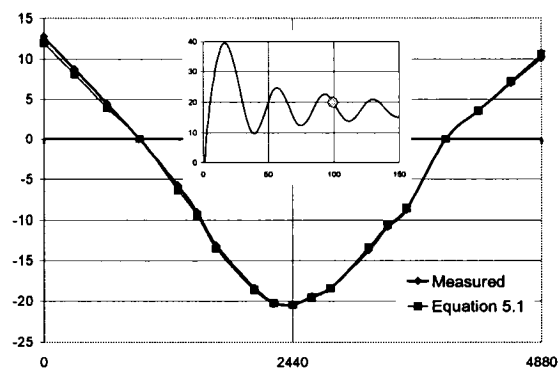
(c) 35 ms



(d) 48.75 ms



(d) 66.25 ms



(e) 98.75 ms

Figure 5.44 Displaced shapes as measured and as calculated by Eq. 5.1, SS1b-1

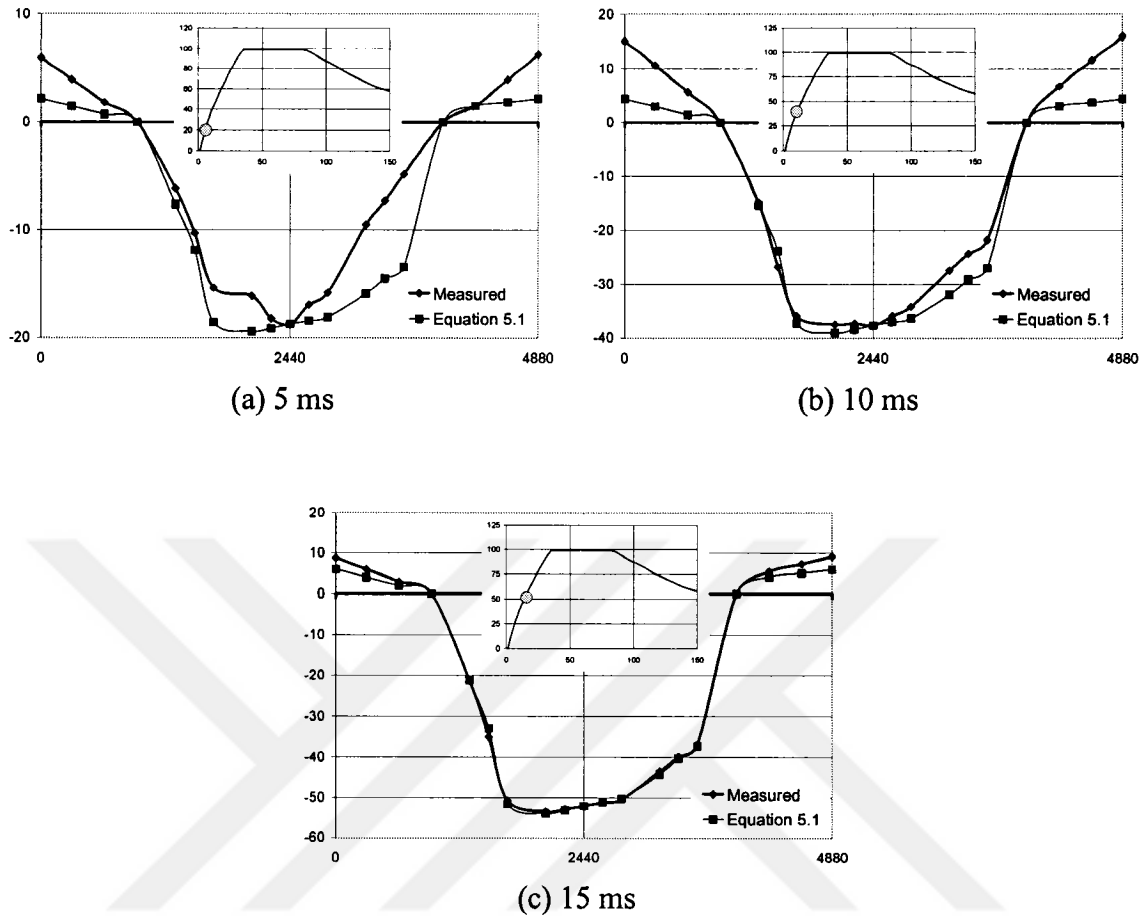


Figure 5.45 Displaced shapes as measured and as calculated by Eq. 5.1, SS0b-1

As seen in the preceding figures, in general, the specimen has a dominant deflected shape that mostly fits into a SDOF system. However, some major discrepancies also exist.

As mentioned previously, the unit displaced shapes were derived from the displaced shape of the specimens at the time of the peak mid-span displacement. This displaced shape corresponds to the state of the specimen when it has just suffered the most amount of damage in terms of crack formation. Therefore, it is safe to say that the specimen had fewer cracks before that point, and the number and width of the cracks do not change significantly afterwards. Consequently, the displaced shape close to and after the peak fits better into the SDOF unit displaced shape. This behaviour is most visible in SS1b and SS0b, which suffered more damage since they contained minimal or no transverse reinforcement. However, for SS3b, which was strongly reinforced in the transverse direction, there exist

larger discrepancies, since the specimen suffered less damage and it was still able to vibrate in multiple frequencies which did not fit into the SDOF unit displaced shape.

In all cases, on the other hand, the overhanging parts at the ends of the specimens do not fit into the SDOF behaviour. In both SS3b-1 and SS1b-1, these overhanging parts first lag behind the SDOF shape, and later surpass it, exhibiting behaviour similar to a whip effect, which continues during the whole response history. This is actually an expected behaviour; this structure may be represented as a SDOF system when it is loaded statically, but dynamically, it is a multi-degree-of-freedom system because of its distributed mass and inertia. In SS0b, this whip effect is less visible, since the specimen suffered extensive damage and the overhanging parts quickly lost their integrity with the rest of the specimen.

Formation of the shear-plug is another factor that complicates the treatment of the specimen as a SDOF system. The methods that employ the SDOF system assumption usually choose the static elastic displaced shape of the specimen as the unit displaced shape. In this case, the displaced shape of an elastic beam with a point load at the mid-span would be the logical choice for the unit displaced shape, as given in Eqs. 5.2 and 5.3.

$$u(x) = -\frac{9P}{16EI}x + \frac{8.46P}{16EI} \quad \text{for } x < 0.94 \text{ m} \quad (5.2)$$

$$u(x) = \frac{P}{12EI}(x - 0.94)^3 - \frac{9P}{16EI}(x - 0.94) \quad \text{for } 0.94 \text{ m} < x < 2.44 \text{ m} \quad (5.3)$$

where $u(x)$ is the displacement, P is the point load at the mid-span, E is the elastic modulus, I is the moment of inertia, and x is the coordinate measured in metres from the left end of the beam. Note that the displaced shape is symmetrical around the mid-span (i.e. $x = 2.44$ m). The elastic displaced shape can be normalized with respect to the mid-span displacement and compared to the unit displaced shapes presented in Figures 5.40 to 5.42. This comparison is given in Figure 5.46.

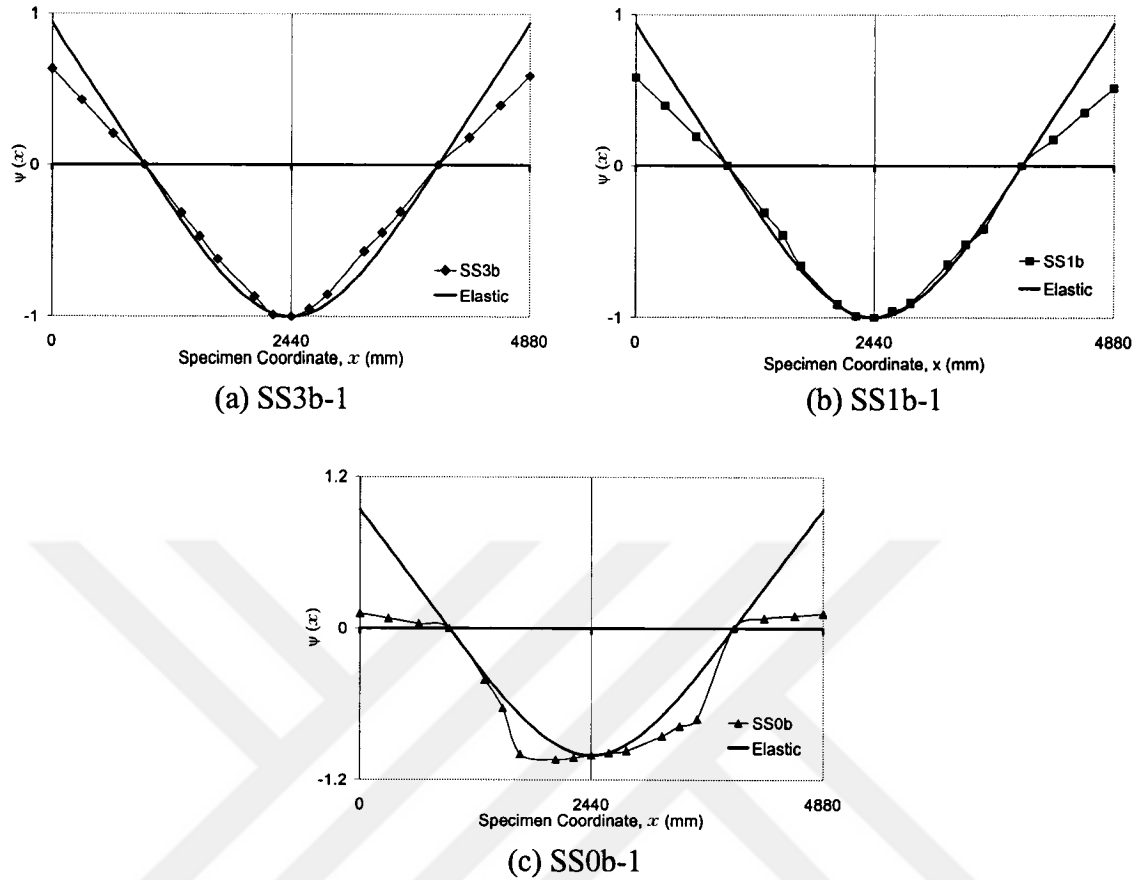


Figure 5.46 Comparison of elastic and measured unit displaced shapes

Although it might appear that the segment between the supports actually does correspond well to the elastic displaced shape for SS3b-1 and SS1b-1, some discrepancies exist between the elastic and measured shapes due to the shear-plug formation and the resulting discontinuities in the displaced profile. These discrepancies would become more visible if the elastic shape was fit into the slope of the specimen at the supports instead of the mid-span displacement, as shown in Figure 5.47. In this figure, it can be seen that the measured displaced shape around the mid-span actually deviates considerably from the elastic shape. This deviation is more obvious in SS1b-1, for which the shear-plug was more prominent. For SS0b-1, on the other hand, the elastic displaced shape is quite different from the actual one, since the drop-weight essentially punched through the specimen, without inducing noticeable bending. In all specimens, the elastic shape fails to capture the deflections of the overhanging parts; as discussed before, this is mostly a result of the multi-degree of freedom behaviour of the specimen.

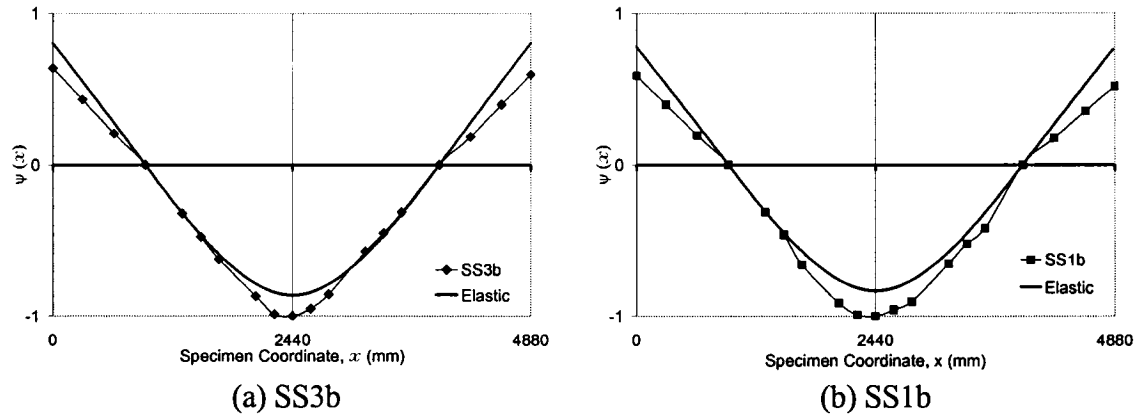


Figure 5.47 Comparison of elastic and measured unit displaced shapes, effect of shear-plug

In conclusion, it can be said that the specimens did not behave in the manner of a SDOF system; the beam overhangs and the shear-plug formation introduced complexities that cannot be modelled with a SDOF model. Although the segments of the specimens between the supports mostly followed a definite displacement pattern as seen in Figures 5.43 and 5.44, determination of this pattern would be a difficult task, which defeats the purpose of a simple SDOF analysis. Using the elastic displaced shape also contains errors, since it fails to consider the shear-plug formation, which can be very significant in structures with little or no reinforcement against shear.

5.5 Analysis of Crack Patterns

All beams were analyzed with VecTor2 under static load conditions (the details of these static analyses will be presented in Chapter 6). These analyses suggested that SS3 and SS2 would experience a ductile flexure-critical behaviour, whereas SS1 and SS0 would be shear-critical, under static loading. However, during the impact testing, regardless of their static behaviour, all specimens developed severe diagonal cracks, starting from the impact point and propagating downwards with an angle of approximately 45 degrees, forming shear-plugs. In addition, several diagonal cracks parallel to the major shear-plug cracks also developed, along with some flexural cracks at the mid-span and at the supports. These flexural cracks usually propagated vertically through the height of the beams, although the vertical cracks at the mid-span started from the bottom surface and the cracks at the supports started from the top surface. Several other short vertical cracks, starting from the

top of the specimens, were also observed in most of the specimens due to negative bending moments at the initial stages of the impact. Mechanisms involved in the formation of these vertical cracks are discussed in Section 5.6. Complete crack maps of the specimens after each test are presented in Appendix C.

The crack patterns and failure modes varied between specimens depending on their static capacities. For example, in the flexure-critical specimens, SS2 and SS3, flexural cracks at the mid-span were more visible and wider, whereas they were shorter and narrower in SS1 and SS0. In SS2 and SS3, subsequent impacts on the damaged specimens did not change the widths of the cracks located beyond the major diagonal cracks forming the shear-plug; increasing deformations were mostly accommodated by the widening of shear cracks forming the shear-plug. However, in SS1 and SS0, another diagonal crack developed alongside the shear-plug, starting from the supports, propagating at an angle of approximately 45 degrees upwards, and becoming horizontal close to the top before reaching to the impact zone (Figure 5.48). These cracks were quite similar in nature to the typical shear cracks that result in failure of shear-critical beams under static loads, and they became wider with subsequent impacts, causing the failure of the specimens in SS1a-3, SS0a-2, SS1b-2 and SS0b-1.

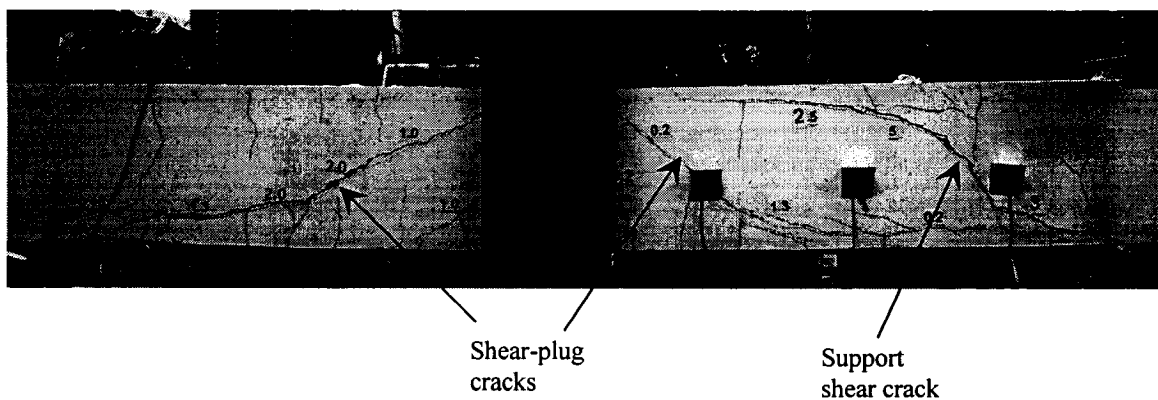


Figure 5.48 Typical cracks in a shear-critical specimen (SS1b-1)

As an indication of damage level, crack widths were measured and test specimens were photographed after each test. Typical crack widths measured after each test are summarized

in Table 5.1. Note that the damage was not always symmetric; therefore, the crack widths in Table 5.1 are the maximum values; see Appendix C for details. Some cracks were either not measured or it was not possible to measure them due to extensive spalling; such cases are denoted by a question mark.

Table 5.1 Typical crack widths measured after tests

Test	Diagonal crack width (shear-plug) mm	Flexural crack width (mid-span) mm	Support shear crack width mm	Test	Diagonal crack width (shear-plug) mm	Flexural crack width (mid-span) mm	Support shear crack width mm
SS3a-1	0.15	?	-	SS3b-1	0.50	1.00	0.10
SS3a-2	1.10	1.30	0.25	SS3b-2	2.00	2.00	0.20
SS3a-3	1.80	?	0.40	SS3b-3	?	2.00	0.20
SS2a-1	0.20	< 0.10	-	SS2b-1	1.80	1.10	0.20
SS2a-2	3.00	1.80	0.20	SS2b-2	2.00	1.80	0.20
SS2a-3	5.00	1.80	0.20	SS2b-3	> 10.00	1.80	0.20
SS1a-1	0.30	-	0.20	SS1b-1	2.00	0.90	5.00
SS1a-2	3.50	0.60	4.00	SS1b-2	5.00	0.90	> 10.00
SS1a-3	?	0.60	> 10.00	SS1b-3	Not conducted		
SS0a-1	0.20	-	0.50	SS0b-1	> 10.00	< 0.10	> 10.00
SS0a-2	> 10.00	-	>10.00	SS0b-2	Not conducted		
SS0a-3	Not conducted			SS0b-3	Not conducted		

Observations made from the development of crack profiles and crack widths revealed that the failure modes were determined mainly by the static behaviour of the specimens. In the flexure-critical specimens, shear-plugs developed faster than did the support shear cracks. Their capacities were higher, comparing the damage they suffered to the damage shear-critical specimens suffered, after three impacts. Although they developed pronounced shear-plugs as well, failure of the shear-critical specimens was not entirely a result of the shear-plugs, but rather the shear failure at the supports. In other words, the shear strengths of flexure-critical specimens were sufficient to carry the shear forces beyond the shear-plug to the supports; subsequent impacts pushed the shear-plug down further, without creating significant damage in the rest of the specimen. However, this was not the case for shear-critical members. These specimens did not have sufficient strength to transmit the shear forces to the supports. Consequently, the shear forces beyond the shear-plug caused extensive damage between the shear-plug and the supports, ultimately failing the beam in this segment. In any case, the flexural behaviour of all specimens was minimal. The flexure-critical beams developed some wide vertical cracks, but their widths were always

less than the width of the diagonal shear cracks. Since the deformations caused by the shear-plugs dominated the displacement profile, the shear-critical specimens developed much narrower flexural cracks.

5.6 Dynamic Equilibrium

When a drop-weight impacts a specimen, the resulting impact force is resisted by the stiffness of the specimen, while the specimen accelerates in the direction of the impact force. The acceleration of the specimen creates inertia forces, equal in magnitude to the mass times acceleration. If the direction of these inertia forces is taken as opposite to the direction of the accelerations, a state of force equilibrium can be established for that point in time. This is referred to as *D'Alembert's principle of dynamic equilibrium*. Ignoring the damping forces, a free body diagram for this state of dynamic equilibrium can be derived as shown in Figure 5.49 for the test specimens.

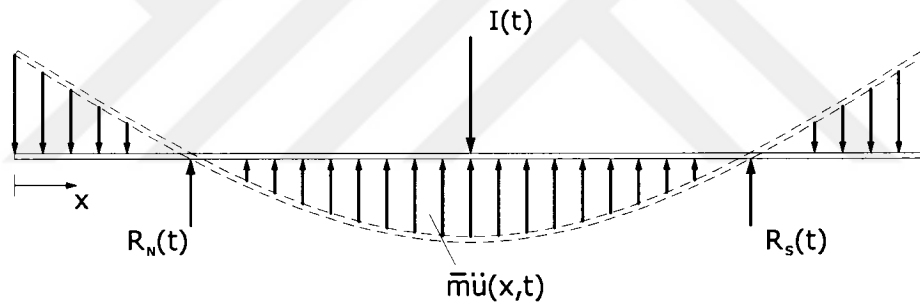


Figure 5.49 Dynamic free body diagram for the test specimens

In Figure 5.49, $I(t)$ is the impact force, $\bar{m}$ is the mass per unit length, $\ddot{u}(x,t)$ is the acceleration of the specimen, and $R_N(t)$ and $R_S(t)$ are the support reaction forces at the north and south supports, respectively. According to this free body diagram, the vertical force equilibrium of the specimen at any time t can be expressed as follows:

$$\int_0^L \bar{m} \ddot{u}(x,t) dx + R_N(t) + R_S(t) - I(t) = 0 \quad (5.4)$$

where L is the total length of the specimen. In this test program, all quantities in Eq. 5.4 were measured during the tests, providing an opportunity to verify the test data for dynamic equilibrium and examine the corresponding behaviour of the specimens.

As mentioned in the previous chapter, accelerations on the specimen were measured by accelerometers attached to the south half of the specimen at five different locations. Assuming a symmetric acceleration distribution around the mid-span, and a linear variation of the accelerations between two adjacent locations (Figure 5.50), the integral in Eq. 5.4 can be calculated in segments as follows:

$$\int_0^L \bar{m} \ddot{u}(x, t) dx = \bar{m} \left[0.375 \left(\frac{A1(t) + A2(t)}{2} \right) + 0.375 \left(\frac{A2(t) + A3(t)}{2} \right) + 0.375 \left(\frac{A3(t) + A4(t)}{2} \right) + \frac{0.375}{2} A4(t) + 0.940 A5(t) \right] \times 2 \quad (5.5)$$

In Eq. 5.5, $A1(t)$ to $A5(t)$ refer to the acceleration measurements by the accelerometers in m/s^2 , and $\bar{m}$ is in tonnes/m, which gives the result of the equation in kilonewtons (kN). The mass per unit length of each specimen, calculated using the material densities given in the previous chapter, are presented in Table 5.2.

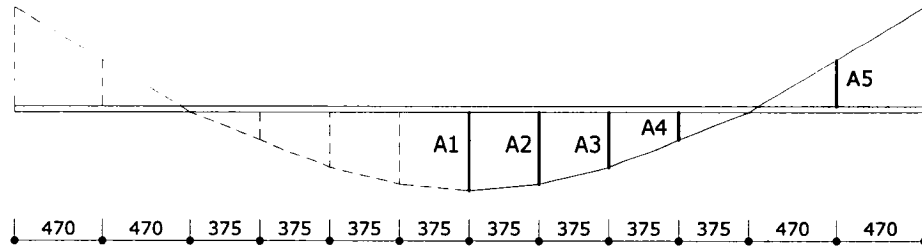


Figure 5.50 Acceleration distribution along the specimen
(All dimensions are in millimetres)

Table 5.2 Mass per unit length of specimens

Specimen	Density
SS3a, SS3b	267 kg/m
SS2a, SS2b	265 kg/m
SS1a, SS1b	268 kg/m
SS0a, SS0b	264 kg/m

The support reactions $R_N(t)$ and $R_S(t)$ were obtained from the load cell readings, after taking the prestressing forces in the support bars into consideration. The impact forces $I(t)$ were calculated as the product of the accelerations of the drop-weight multiplied by its mass. Possible errors related to the acceleration measurements of the drop-weight were discussed in Section 5.2.4 and 5.3. In this section, the acceleration data obtained from the two accelerometers mounted on the drop-weight were assumed as correct and used directly for the impact force calculations. However, in some tests, only one accelerometer data was used instead of the average of the two, since not both gave reliable data at all times.

After obtaining all the required terms, the dynamic equilibrium in Eq. 5.4 was checked for several test specimens. Five tests are presented here as examples: SS3a-1 (Figure 5.51) for a test with the light weight, SS2b-1 (Figure 5.52) and SS1b-1 (Figure 5.53) for a test with the heavy weight, and SS1b-2 (Figure 5.54) and SS0b-1 (Figure 5.55) for a heavily damaged specimen. For comparison, the summation of inertia and reaction forces are shown against the impact force, $I(t)$; note that, ideally, they should be equal. Only the initial portions of the responses are shown for easier comparison.

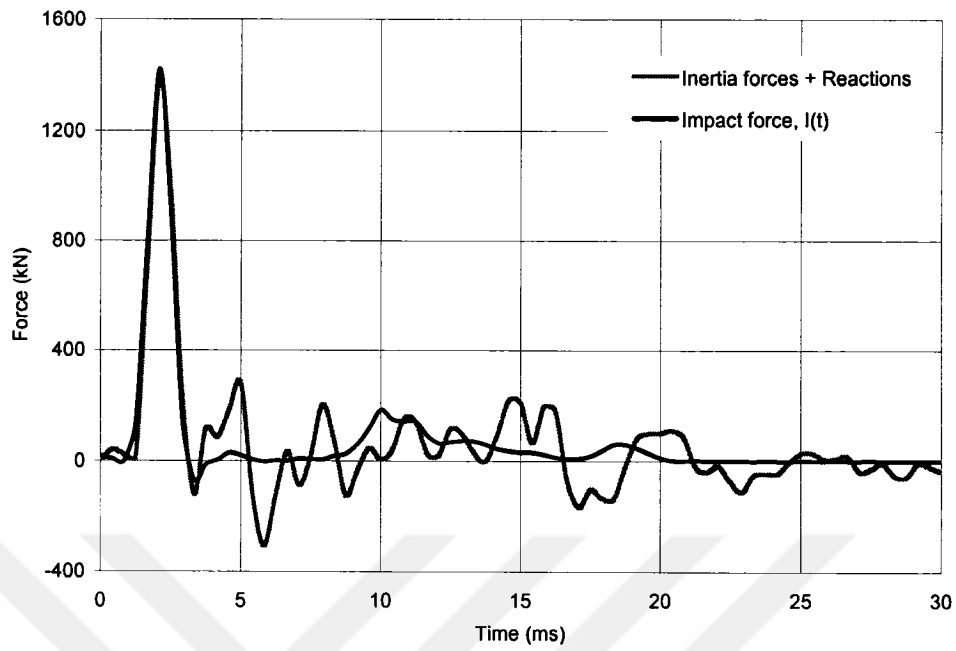


Figure 5.51 Dynamic equilibrium of forces, SS3a-1

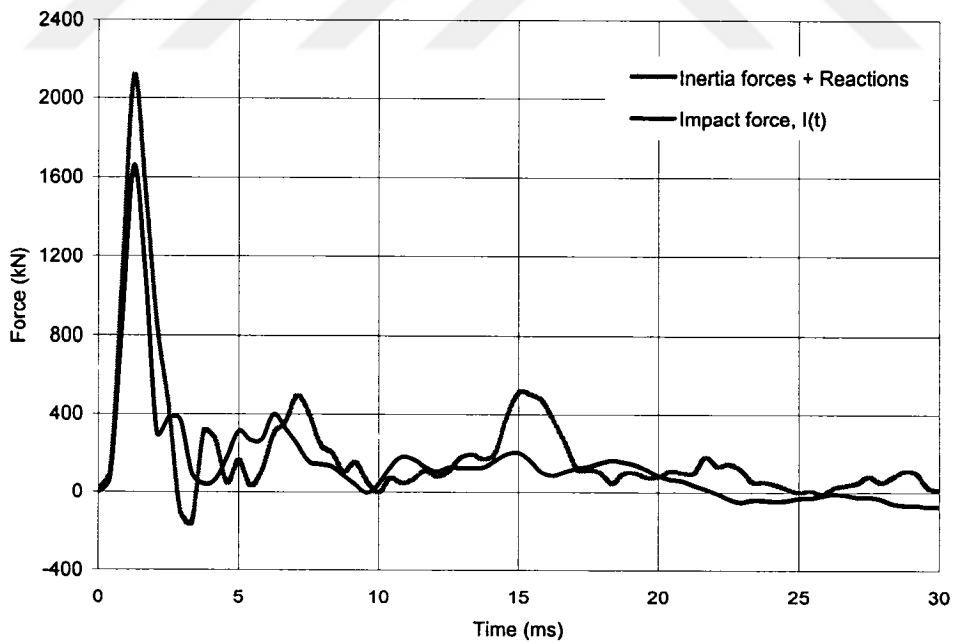


Figure 5.52 Dynamic equilibrium of forces, SS2b-1

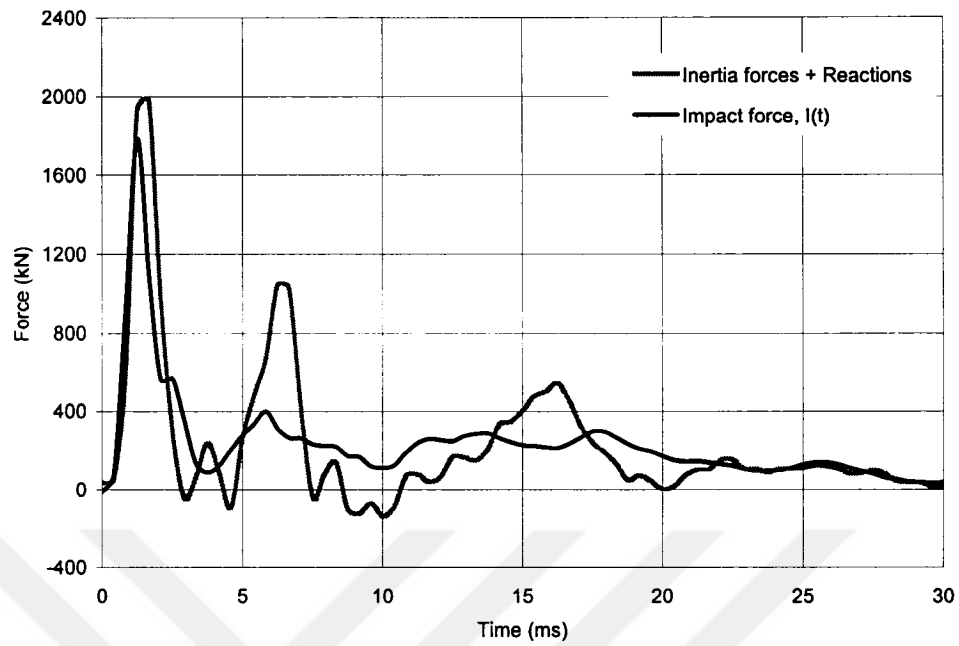


Figure 5.53 Dynamic equilibrium of forces, SS1b-1

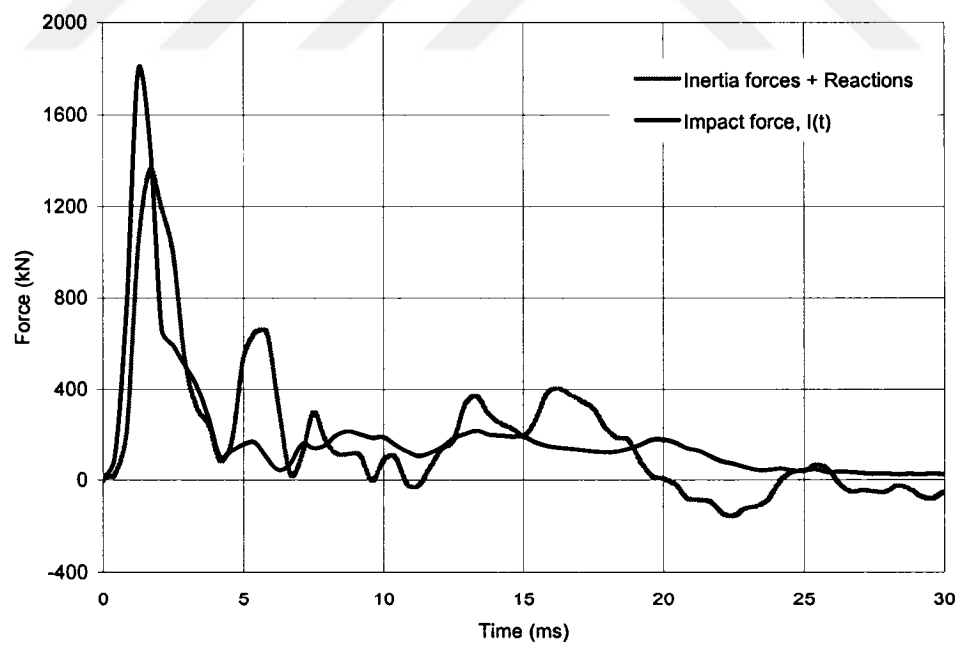


Figure 5.54 Dynamic equilibrium of forces, SS1b-2

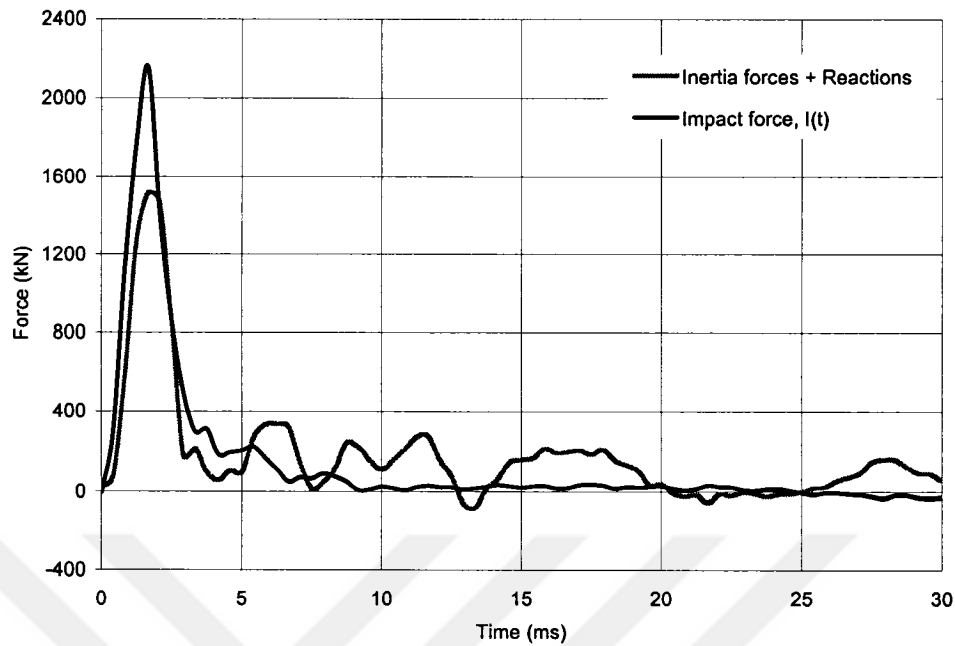


Figure 5.55 Dynamic equilibrium of forces, SS0b-1

It should be noted that the problems related to the sampling rate and the capturing of the high-frequency response components highly affects the accuracy of the dynamic force calculations. In particular, the comparison of peak forces contains inaccuracies since the forces had fast-rising peaks in the initial portions of the responses, which were defined by only a few points given the available sampling rate. Moreover, the scarcity of the locations of acceleration measurement also introduced errors. Nevertheless, the general comparison between the applied impact force and the resisting reactions and inertia were satisfactory to a certain degree. For example, the peak impact and resisting forces exhibited an almost perfect match for SS3a-1, as seen in Figure 5.51. For SS2b-1 and SS1b-1, on the other hand, the resisting forces had a 10 to 20% higher peak than the impact force. This trend was opposite for the tests where the specimens suffered extensive damage; in SS1b-2 and SS0b-1, the peak impact forces were much higher than the peak resisting forces. For these specimens, the high discrepancy can be attributed to the extensive spalling of the concrete cover of the specimen. In other words, a certain portion of the impact energy was spent on projecting large pieces of concrete off the specimen. After the peak, in all tests, the resisting forces exhibited some fluctuations, whereas the drop-weight bounced back and lost contact with the specimen. From an equilibrium point of view, although the specimens continued to

vibrate, such fluctuations in the sum of forces should not have existed since the inertia forces should have cancelled out with the reaction forces, as shown in Eq. 5.4, for the case where $I(t)$ is equal to zero. These fluctuations were thought to be mostly a result of measurement errors, since noise in the system, both electrical and physical, such as that caused by cracking and spalling of concrete, were registered in the accelerometers and load cells as unrealistic readings. The effects of these factors diminished in later stages and the sum of resisting forces dropped to zero as expected.

A further investigation, similar to the one described by CEB (1998), can reveal more about the distribution of forces along the specimen. As stated in Eq. 5.4, the impact force is resisted by a combination of inertia and reaction forces. The distribution of these forces, and the resulting shear and moment distributions, can be determined by using the basic principles of static equilibrium, as shown in Figure 5.56.

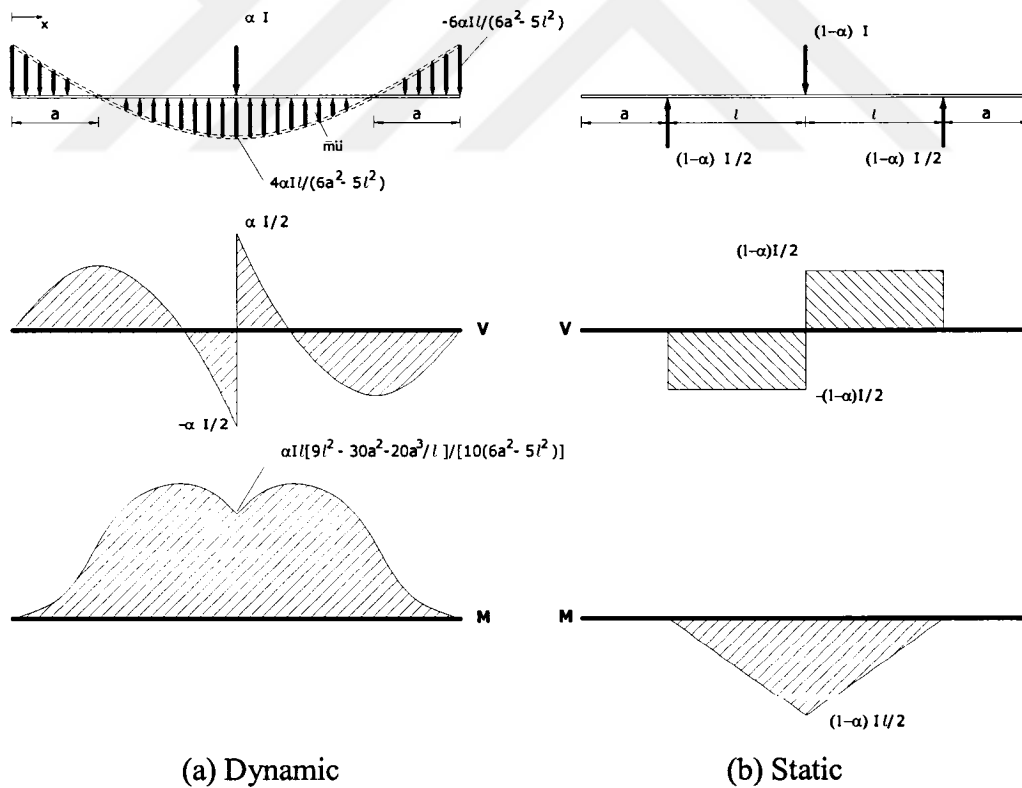


Figure 5.56 Distribution of forces

In Figure 5.56, a is the length of the overhang, l is the shear span, I is the impact force, and α is the ratio of the inertia forces to the total impact force. The inertia force distribution is assumed proportional to the elastic displaced shape given in Eqs. 5.2 and 5.3. The total reaction force at the supports can be found as follows:

$$R = (1 - \alpha) \frac{I}{2} \quad (5.6)$$

which can be checked against the test results, to find the value of α . In SS3a-1, for example, where the peak resisting forces and the impact force were in almost perfect agreement, the peak impact force was 1421 kN and the average peak support reaction at the same point in time was 13.1 kN, giving $\alpha = 0.98$. The same calculations done for other tests also gave α values close to 1.0 for the first peak, meaning that almost all the impact force was resisted by the inertia forces at the first instant, before it was transmitted to the supports. This can be seen more clearly when the resisting forces are broken down to their components, as done in Figure 5.57 for SS3a-1.

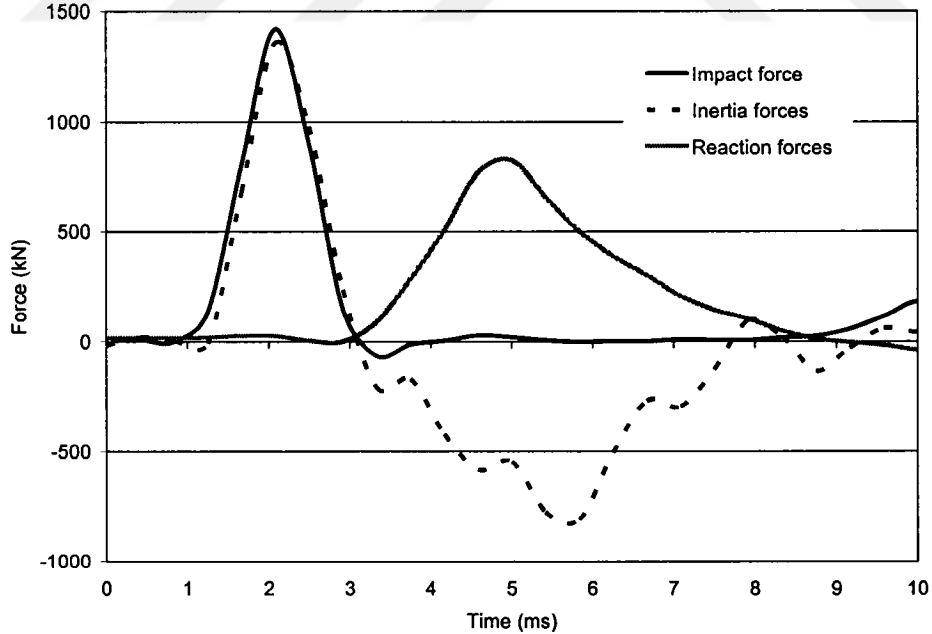


Figure 5.57 Breakdown of resisting forces, SS3a-1

This phenomenon also explains the formation of the shear-plug. As seen in the moment diagrams in Figure 5.56, the mid-span moment for the case of $\alpha = 1.0$ is much smaller than it is for the static case of $\alpha = 0.0$. If values of $a=0.94$ m and $l = 1.5$ m are substituted, it can be found that the maximum moment for $\alpha = 1.0$ is $0.437 I$, whereas for $\alpha = 0.0$, it is $0.75 I$. However, the mid-span shear force is independent of α , always equal to $I/2$. Therefore, in the initial stages of the response, the specimen experiences the same shear force but significantly smaller moments than it would under static loading, thus becoming shear-critical, forming the diagonal shear cracks and the subsequent shear-plug.

Another consequence of α being close to 1.0 in the initial stages of the response is the vertical cracks observed along the specimen. As seen in the moment diagram presented in Figure 5.56, $\alpha = 1.0$ creates negative moments along the entire length of the specimen. These negative moments resulted in vertical cracks starting from the top and propagating downwards (see Figure 5.58). Typically, these cracks were spaced closer on the specimens with higher stirrup ratio compared to the specimens with larger stirrup spacing. They were characterized by narrow crack widths (usually around 0.10 mm), since they formed in the very early stages of the response and closed quickly after the negative moments diminished, i.e. as the α -value approached zero. Vertical cracks developed at the overhanging parts as well; however, they had an inclination towards the supports as they propagate downwards, due to the high shear forces at the supports along with negative moments when $\alpha = 1.0$ (see Figure 5.59).

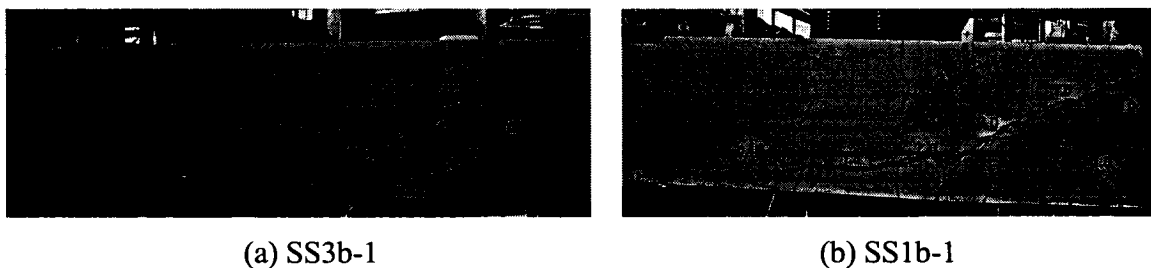


Figure 5.58 Vertical cracks due to negative moments

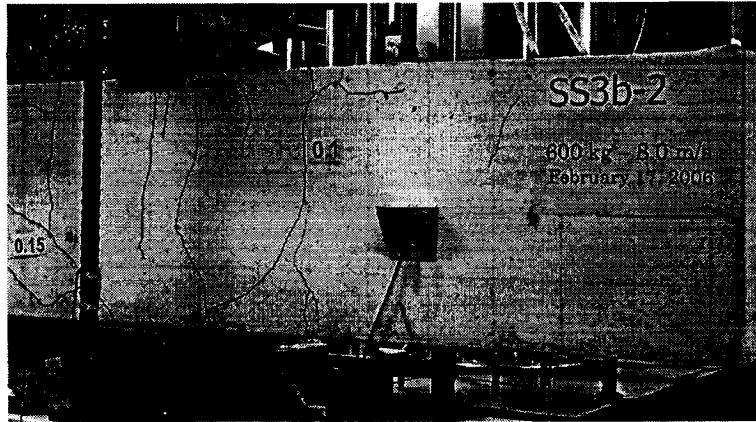


Figure 5.59 Inclination of a vertical crack at the overhanging part

5.7 Impact Capacities of Test Specimens

The test program carried out in this study was not directed towards the determination of the impact capacities of the specimens; a study with that purpose would require a significantly larger number of specimens to observe and quantify the behavioural patterns. However, some observations can still be made regarding the impact capacities based on the energy imparted to the specimens and the reaction forces recorded.

As discussed in Chapter 4, all specimens with the exception of SS0b were subjected to multiple impacts. With the a-series specimens, the beams were first tested once with the light drop-weight, followed by two tests with the heavy drop-weight. With the b-series specimens, the order was reversed; they were tested twice with the heavy weight and one last time with the light weight. This arrangement of delivering the impact energy was intended to observe the effect of previous damage on the resistance characteristics of the specimens. Note that the total impact energy imparted to the specimens was constant between the specimens, except for those that failed prior to the second or third impact. Calculating the kinetic energy of the drop-weights with the commonly used formula ($E = \frac{1}{2} mv^2$), the impact energy imparted to the specimens during each test can be summarized as in Figure 5.60.

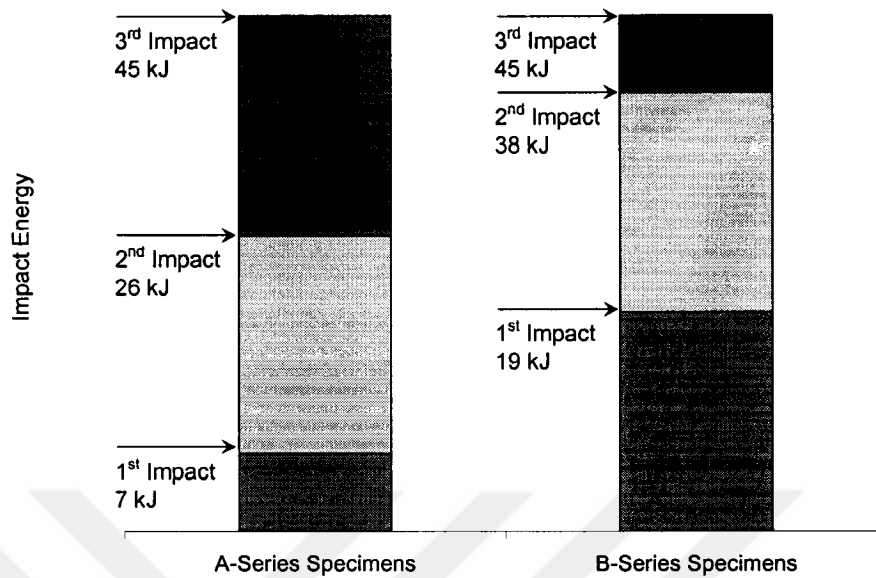


Figure 5.60 Energy imparted to the specimens

The efforts made to determine the effect of delivering the impact energy in varying order were mostly inconclusive. Clearly, the impact capacities of some specimens were exceeded before the third impacts. For example, SS1a experienced extensive damage after the third impact, whereas SS1b suffered approximately identical levels of damage after the second impact. Therefore, in energy terms as depicted in Figure 5.60, it is safe to say that the impact energy capacity of SS1 lay somewhere between 26 and 38 kJ. A similar conclusion can be drawn for SS0; its impact capacity was between 7 and 19 kJ. However, no significant difference was observed between the a-series specimens and the b-series specimens for SS3 and SS2, when their crack profiles were compared after the third impact tests. On the other hand, their crack profiles were not identical either. Since only two specimens of the same type were available for comparison, it was not possible to determine if these differences in the crack profile were insignificant, or if they were a result of a consistent behavioural difference caused by the impact chronology.

A comparison between the dynamic and static properties can be made in order to observe the behavioural difference between two cases. As mentioned before, all beams were analyzed with VecTor2 under static load conditions (the details of these static analyses will be presented in Chapter 6). Table 5.3 presents the maximum reaction forces calculated by

VecTor2 for each specimen. Static energy dissipation capacities, based on the area under static mid-span displacement-support reaction curve, are also calculated and presented in Table 5.3. Maximum reaction forces recorded during the impact tests are summarized in Table 5.4. Table 5.5 compares the energy imparted to the specimens with the static energy dissipation capacities of the specimens.

Table 5.3 Static capacities of test specimens based on VecTor2 analyses

Specimen	Maximum Reaction Force (kN)	Energy Dissipation Capacity (kJ)	Failure Mode
SS3a, SS3b	184	26.9	Flexural
SS2a, SS2b	178	18.5	Flexural
SS1a, SS1b	159	4.6	Shear
SS0a, SS0b	120	2.4	Shear

Table 5.4 Maximum reaction forces recorded

Test	Maximum Reaction Force Recorded (kN)	Max. Reaction Force Recorded / Calculated Capacity	Test	Maximum Reaction Force Recorded (kN)	Max. Reaction Force Recorded / Calculated Capacity
SS3a-1	431	2.34	SS3b-1	682	3.71
SS3a-2	822	4.47	SS3b-2	725	3.94
SS3a-3	783	4.26	SS3b-3	613	3.33
SS2a-1	327	1.84	SS2b-1	592	3.33
SS2a-2	651	3.66	SS2b-2	633	3.56
SS2a-3	779	4.38	SS2b-3	589	3.31
SS1a-1	356	2.24	SS1b-1	625	3.93
SS1a-2	517	3.25	SS1b-2	571	3.59
SS1a-3	314	1.97	SS0b-1	399	3.33
SS0a-1	305	2.54			
SS0a-2	190	1.58			

Table 5.5 Energy imparted on the specimens

Test	Total Energy Imparted (kJ)	Total Energy Imparted / Static Energy Dissipation Capacity	Test	Total Energy Imparted (kJ)	Total Energy Imparted / Static Energy Dissipation Capacity
SS3a-1	7	0.26	SS3b-1	19	0.71
SS3a-2	26	0.97	SS3b-2	38	1.41
SS3a-3	45	1.67	SS3b-3	45	1.67
SS2a-1	7	0.38	SS2b-1	19	1.02
SS2a-2	26	1.41	SS2b-2	38	2.05
SS2a-3	45	2.43	SS2b-3	45	2.43
SS1a-1	7	1.52	SS1b-1	19	4.13
SS1a-2	26	5.65	SS1b-2	38	8.26
SS1a-3	45	9.78	SS0b-1	19	7.92
SS0a-1	7	2.92			
SS0a-2	26	10.83			

It has to be noted that, in an impact test, a part of the energy imparted to the specimen is dissipated through the damage, cracking and permanent deformations of the specimen, whereas the remainder is absorbed and released by other means, such as kinetic energy during the vibrations. In other words, total energy dissipated by the specimen at each impact is always somewhat less than the input impact energy. Therefore, effect of a single impact with a 45 kJ energy would not be identical to three impacts with a total energy of 45 kJ. As a result, comparing the total energy of three impacts directly to the static energy dissipation capacity of the specimens can be misleading. On the other hand, such a comparison can still provide a qualitative assessment regarding the impact capacities of the specimens. For example, it can be seen that the dynamic energy dissipation capacities of the specimens are higher than the static capacities in general. The first impacts on b-series specimens imparted 19 kJ to the specimens, exceeding the static energy dissipation capacity of all beams except SS3. However, except for SS0b, none of the specimens suffered significant damage in the first impact. Similarly, although the total energy of 45 kJ imparted to the specimens is twice larger than the static energy capacity of SS3 and SS2, these specimens did not exhibit a total failure at the end of three impacts. In other words, due to the energy dissipated through numerous other mechanisms (such as damping and material

hysteresis), the dynamic energy dissipation capacity of the specimens were significantly higher than their static capacity.

A similar observation can be made for support reactions as well. As can be seen from the comparisons between the static capacities of the specimens and the maximum reactions recorded, the static capacity of the specimens were severely exceeded during the tests. However, in most cases, the specimens exhibited sufficient resistance against these high reaction forces. This behaviour was observed in other similar studies as well (e.g. Kishi et al. 2002). This sharp increase in the capacity can be attributed to the enhancing effects of the strain rate on the strength of the materials, as well as the highly dynamic conditions and the short duration of the peak forces affecting the failure mechanisms. It should be noted that, since this current experimental study was not aimed at a thorough investigation of this phenomenon, no major effort was spent towards explaining the actual mechanisms involved in determining the impact capacities of the specimens.

5.8 Strain Rates

The stress-strain behaviour for most structural materials is dependent on the rate of loading. Static loading conditions are generally assumed for loading rates that correspond to 10^{-6} to 10^{-4} s^{-1} . Impact loads introduce considerably higher strain rates in structures, resulting in different behaviour in both concrete and steel relative to the static case. The effects of strain rate on material behaviour are a major topic of concern, studied by many researchers (CEB 1988, Bischoff and Perry 1991, Malvar 1998). Most of the formulations developed define the material behaviour for certain strain rate intervals. Therefore, it is essential to determine the range of strain rates and their variation in the structure for a better analysis.

In this test program, no measurements were taken to measure the strains in concrete. However, as mentioned previously, all longitudinal steel bars and some of the stirrups were fitted with strain gauges. In this section, the strain rates as measured by some of the strain gauges are presented. Sample calculated strain rates from SS3a-1, SS3a-3, SS3b-1 and SS1b-1 are shown in Figure 5.61.

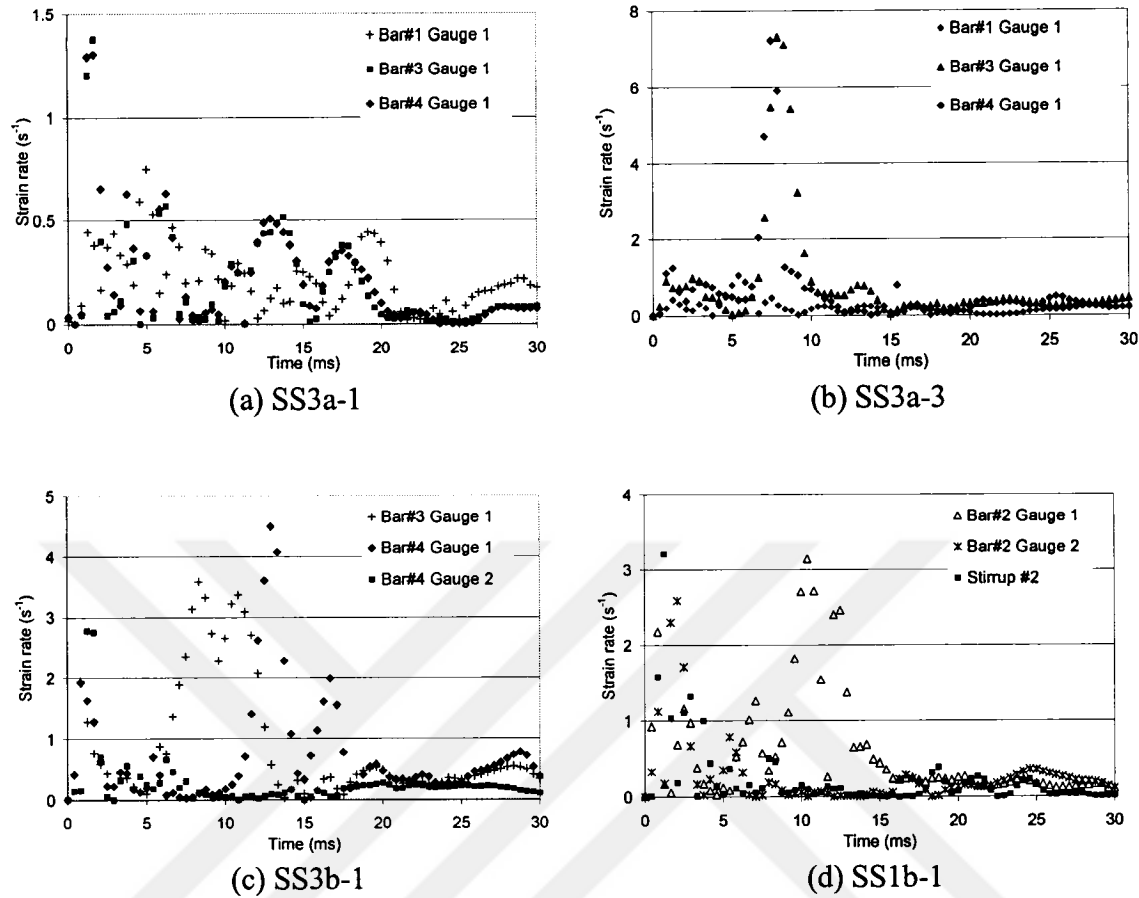


Figure 5.61 Calculated strain rates

As seen in Figure 5.61, the strain rates varied significantly between the specimens. In SS3a-1, for example, the specimen suffered minimal damage and none of the bars yielded. The maximum rate observed in this test was 1.4 s^{-1} at the mid-span, and the strain rates mostly remained under 0.5 s^{-1} for the duration of the response. However, some bars yielded during subsequent drops, and strain rates attained values up to 7.3 s^{-1} . This same trend was observed in SS3b-1 and SS1b-1. In SS3b-1, the twin specimen of SS3a was impacted with the heavier weight first, and consequently some bars yielded at the mid-span. The strain rate at these locations reached 4.5 s^{-1} , which was also the case for SS1b-1. As would be expected, since they lose most of their stiffness after the yield point, the yielded steel bars showed less resistance against strain and registered higher strain rates than bars that did not yield. In general, however, after the peak deformation, the strain rate in the steel bars

fluctuated between 0.1 s^{-1} and 0.5 s^{-1} during the later stages of vibration, before it reduced to minimal levels as the response died out.

5.9 Damping

Damping mechanisms in a structure are very complex, and currently there exists no practical way to calculate the damping analytically. However, some techniques do exist to measure the apparent damping in a structure. One of these techniques is to determine the damping by measuring the free vibration response of the structure. A damped free vibration response decays in time as shown in Figure 5.62, with an envelope curve given by Eq. 5.7:

$$y(t) = \rho e^{-\xi \omega_n t} \quad (5.7)$$

where ρ is the displacement amplitude, ξ is the damping ratio, ω_n is the natural angular frequency, and t is the elapsed time. Free vibration can be induced with the hit of a hammer, and the resulting response can be fitted into the exponential envelope to determine its decay rate and apparent damping ratio.

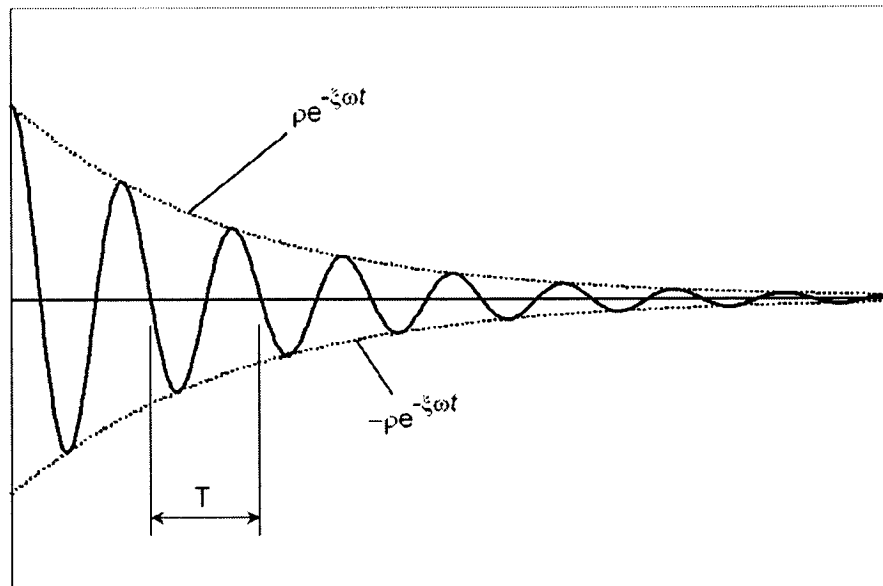


Figure 5.62 Free vibration response of a damped system

During the test program, several attempts were made to induce free vibrations in a beam specimen by hitting it with a rubber hammer at the mid-span. However, because of the significant size and stiffness of the specimens, it was not possible to induce sufficiently high displacements that could reliably be measured with the available displacement sensors. Yet, during the testing of SS1b, attempts to get a free vibration response were successful. Although still very small, some measurable free vibration displacements were obtained before and after test SS1b-1, which are presented in Figures 5.63 and 5.64, respectively. The peak points of the responses were fitted with exponential curves, also shown in the respective figures. Equations for these envelope curves are as follows:

$$y(t) = 0.0705 e^{-5.65t} \quad \text{prior to SS1b-1} \quad (5.8)$$

$$y(t) = 0.0584 e^{-4.26t} \quad \text{after SS1b-1} \quad (5.9)$$

The apparent period of vibration, T , was calculated as 0.0155 s prior to the test, and 0.0337 s after the test, corresponding to angular natural frequencies of 406 rad/s and 186 rad/s, respectively.

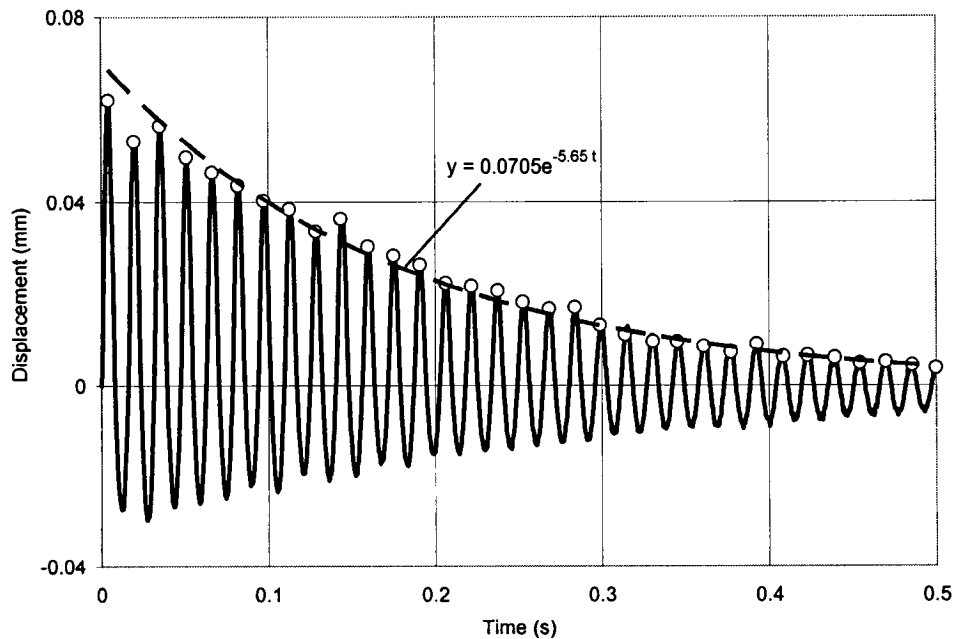


Figure 5.63 Free vibrations before SS1b-1 test

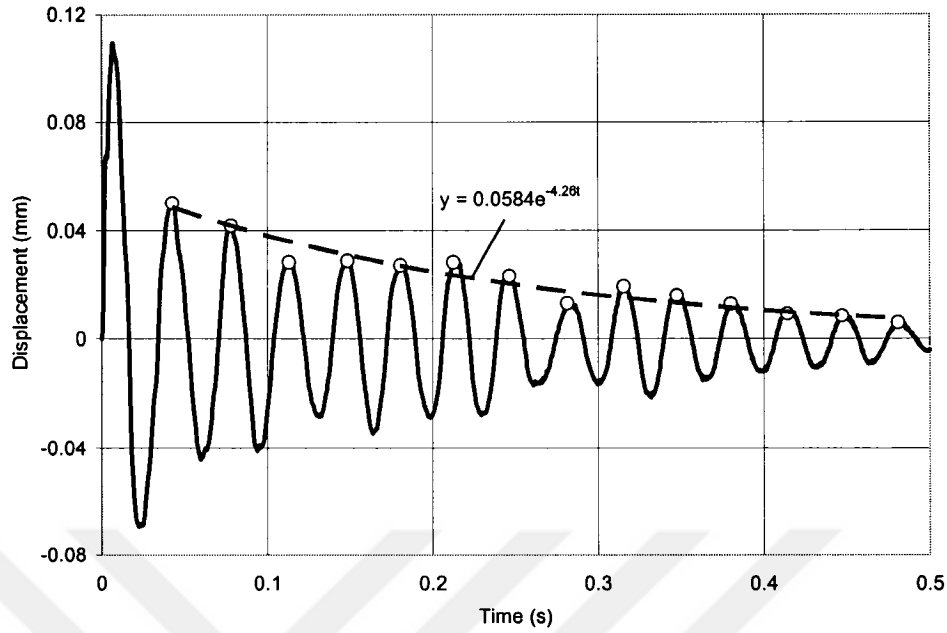


Figure 5.64 Free vibrations after SS1b-1 test

By using the Eqs. 5.8 and 5.9, the apparent damping ratios before and after the test were calculated as 1.4% and 2.3%, respectively. As expected, damping increased as the specimen experienced some damage.

It should be noted that the level of displacements for these hammer tests were still low for a reliable measurement. Moreover, high frequency response components may have higher influence in such low amplitude responses, probably causing the irregularities at the peak points as can be seen in figures. Nevertheless, the damping values obtained are in the expected range for a reinforced concrete structure, and they can be used as a rough approximation to the actual damping present in the specimen.

5.10 Conclusion

To sum up, the experimental program was successful in providing a substantial amount of high quality impact test data for further research. Close examinations of test data, such as in comparing the displaced shape with the elastic shape, measuring the impact force, and establishing the dynamic force equilibrium, showed that the test data were of reasonable accuracy, and thus are suitable as a verification database for calibrating analytical modelling. Digital signal analysis of certain data, such as acceleration and load cell

measurements, revealed some possible measurement errors and inaccuracies due to the somewhat low sampling rate used; nevertheless, these errors were not found to be significant to the extent where they would alter the validity of the test results.



6 NONLINEAR FINITE ELEMENT ANALYSES OF TEST SPECIMENS WITH VECTOR2

6.1 Introduction

As mentioned previously, one of the main objectives of the experimental program was to obtain reliable test data to verify the dynamic analysis algorithms newly implemented in VecTor2. This chapter presents the VecTor2 analyses of the test specimens for the impact loads applied in the experimental program.

In the following section, the VecTor2 finite element model of the test specimens is described, presenting details of the mesh, material models and other analysis parameters. Section 6.3 describes the static analyses of the test specimens. Section 6.4 presents the impact analyses simulating the experimental program and discusses the effects of damping and time-step size on the analyses. Section 6.5 includes the concluding remarks regarding the analyses presented in this chapter.

6.2 Finite Element Model

A total of 1008 rectangular elements were used to represent the concrete, the drop-weight and the support plates, and 124 truss bar elements were used to model the longitudinal steel reinforcement (Figure 6.1). The mesh included 1098 nodes. Taking advantage of the symmetric load and support conditions, only one-half of a beam was modelled. All nodes at the centreline of the beam were restrained against translations in the x-direction.

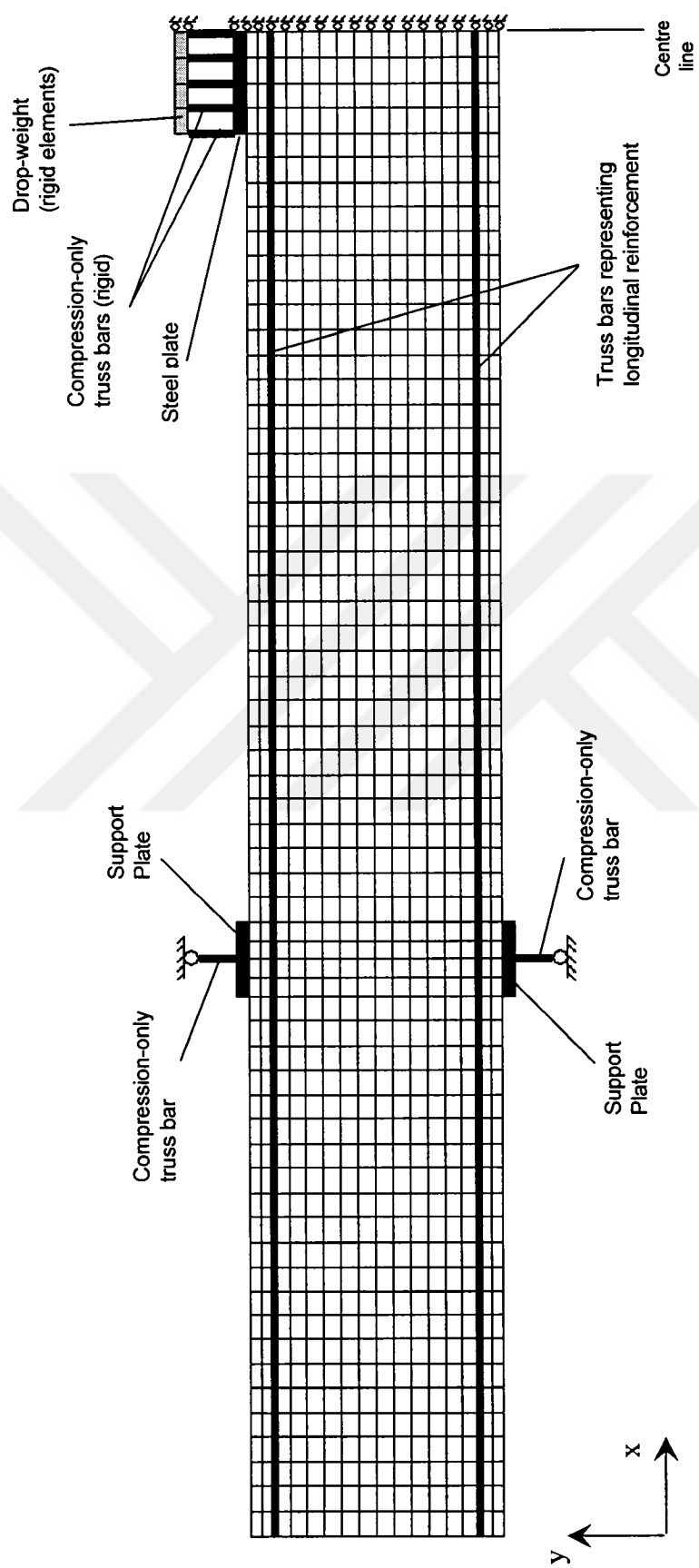


Figure 6.1 Finite element model

The 150 mm wide steel bearing plates at the supports were modelled with four elements placed at the top and bottom support points, with higher stiffness compared to the concrete elements. As mentioned in Chapter 4, the specimens were restrained against upward movements at the supports by support bars, which held the specimen down from the top. To simulate these conditions, supports were introduced in the model with compression-only truss bars connected to the elements representing the support plates at one end, and fixed against translations at the other end. With this arrangement, uplift forces were supported by the top truss bar, avoiding any artificial tension stresses in the specimen that would occur if the bottom support was restrained against translations in y-direction. These truss bars were assigned calculated stiffness properties to simulate the actual stiffness of the bottom support and the support truss bars used in the tests. The top support bar was assigned the stiffness of two No.30 steel bars of 1665 mm length. The stiffness of the 500 mm high bottom pedestal, manufactured by welding steel plates with 14420 mm² total cross-sectional area, was calculated and assigned to the bottom truss bar. In the stiffness calculations, the elastic modulus of steel was taken as 200000 MPa. The load cells, steel plates and rollers at the bottom supports were assumed rigid.

The drop-weight was modelled using four rigid rectangular elements, each 40 x 20 mm in size. The stiffness properties of these elements were assigned very high values to ensure a rigid behaviour. The elements modelling the drop-weight were connected to the specimen by five compression-only truss bars, so that when the drop-weight bounced back, it would not pull up on the specimen. These truss bars were also assigned very high stiffness properties, and they were connected to the elements representing the steel plate placed at the point of impact.

A total of 992 rectangular elements were used to model the concrete. The dimensions of the elements were varied in the model to accommodate the nodal locations in accordance with the locations of the truss bars representing the longitudinal reinforcement and the elements representing the support plates. Elements of 18 x 40 mm dimension were used in the clear cover area, and of 30 x 28 mm dimension were used at the support location. The remainder of the elements were 40 x 28 mm. The Popovics (1973) formulation for normal strength concrete was selected as the compression base curve, since it was found to best fit the

stress-strain curve obtained from the tests of the cylinder specimens. The concrete tensile strength was calculated with the formula $f_t' = 0.33\sqrt{f_c'}$ as recommended by CSA Standard A23.3-04, since tensile strengths found from the modulus of rupture tests are known to over-estimate the actual tensile strength of concrete when used with VecTor2. The model proposed by Palermo and Vecchio (2002) was used to model the hysteretic response of concrete. All other material and behavioural models used for concrete were the default models of VecTor2, which are summarized in Table 6.1. Details on the formulations of these models can be found in the VecTor2 and Formworks Manual (2002).

Table 6.1 Material and behavioural models used for concrete

Material Property	Material Model
Concrete Compression Base Curve	Popovics (NSC)
Concrete Compression Post-Peak	Modified Park-Kent
Concrete Compression Softening	Vecchio 1992-A
Concrete Tension Stiffening	Modified Bentz
Concrete Tension Softening	Linear
Concrete Tension Splitting	Not Considered
Concrete Confinement Strength	Kupfer / Richart
Concrete Dilatation	Variable - Kupfer
Concrete Cracking Criterion	Mohr-Coulomb (Stress)
Concrete Crack Width Check	Crack Limit (Agg/5)
Concrete Hysteresis	NL w/ Decay (Palermo)
Slip Distortion	Vecchio-Lai

The transverse steel reinforcement was modelled as smeared within the concrete, since the stirrups provided were regularly distributed along the specimen. The longitudinal steel reinforcement was modelled with 124 truss bars. Each truss bar element was directly connected to concrete elements; that is, perfect bond was assumed. All the material and behavioural models used for steel reinforcement were the default models of VecTor2, as summarized in Table 6.2. Details on the formulations of these models can be found in the VecTor2 and Formworks Manual (2002).

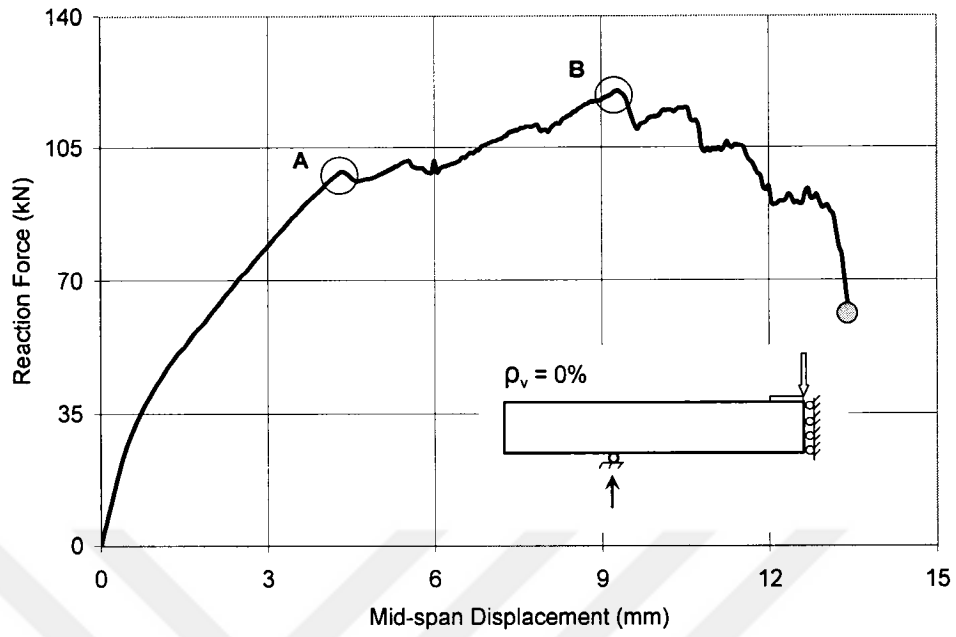
Table 6.2 Material and behavioural models used for steel reinforcement

Material Property	Material Model
Steel Hysteresis	Seckin Model
Rebar Dowel Action	Tassios (Crack Slip)
Rebar Buckling	Asatsu Model

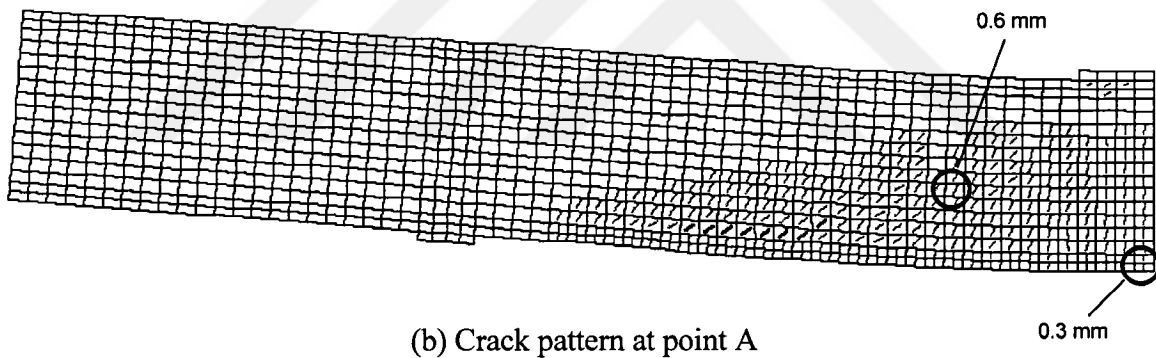
Mass data, as presented in Chapter 4, was used to calculate the lumped masses at the nodes. The masses of the rectangular concrete elements were equally divided into four and assigned to their nodes. Similarly, the masses of the truss bars were divided into two and added to the corresponding nodal masses. The mass of the drop-weight was also distributed to the nodes of the elements used in its modelling. It should be noted that, since only half of the test setup was modelled, only one half of the drop-weight mass was included in the model. The lumped masses of the drop-weight were activated only in the y-direction for the dynamic analyses. All other lumped masses on the specimen were activated in both the x- and y-directions.

6.3 Static Analyses of Test Specimens

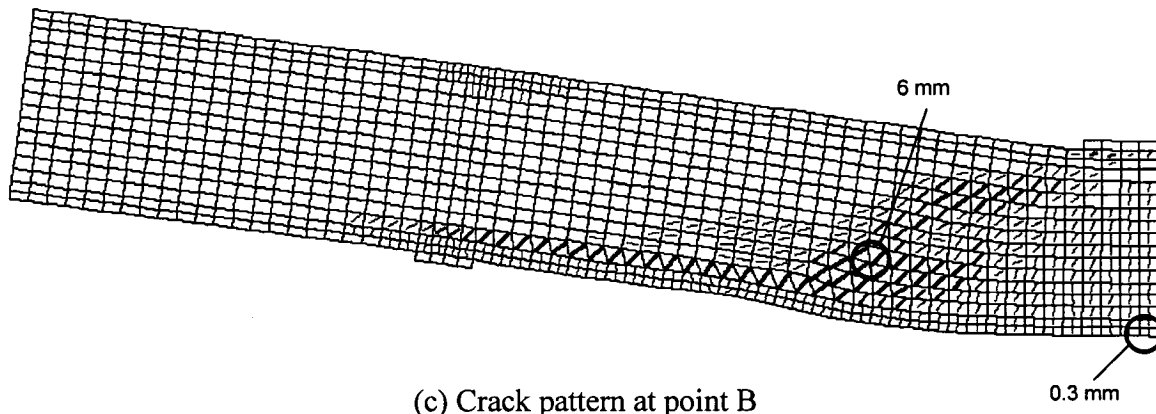
Although no static tests were conducted on the test specimens, information regarding their static behaviour may help in understanding their dynamic behaviour as well. For this reason, static load analyses of the specimens were carried out using VecTor2. The static load on the specimens was introduced by applying increasing displacements at the node corresponding to the centre of the steel plate at the impact point. The displacement load was applied in the negative y-direction at 0.05 mm increments. All analysis parameters were set as indicated in Tables 6.1 and 6.2. The support reaction force versus mid-span displacement curves, obtained from VecTor2 analyses of the four different types of specimens, are presented in Figures 6.2 to 6.5. Crack patterns at two load stages, and the corresponding crack widths at critical locations, are also presented in the same figures. The crack patterns were obtained with Augustus (Copyright © Evan Bentz), the post-processor of VecTor2. Figure 6.6 presents the static response of all specimens in a single graph for easier comparison.



(a) Reaction force – mid-span displacement curve

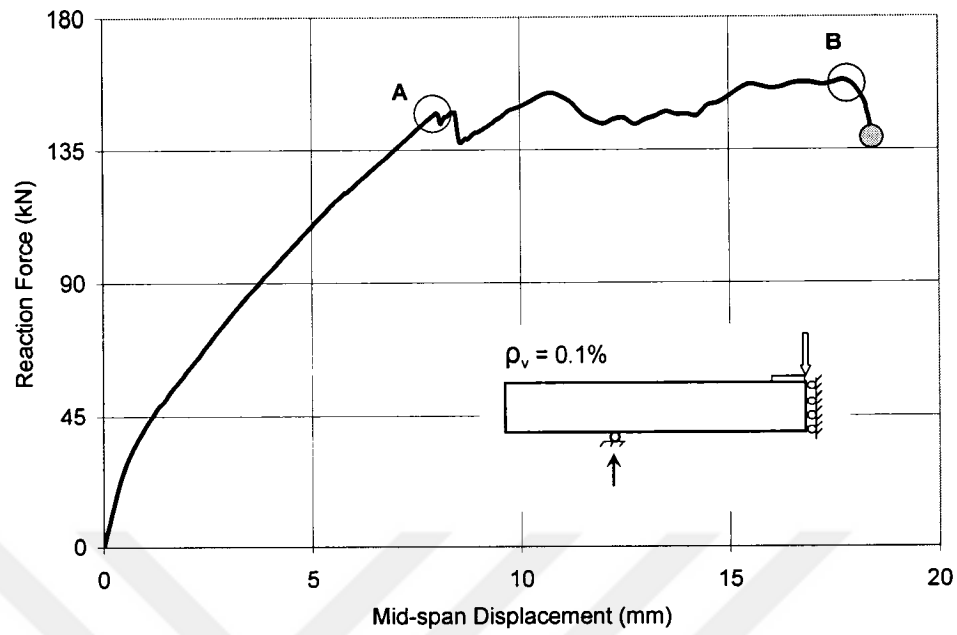


(b) Crack pattern at point A

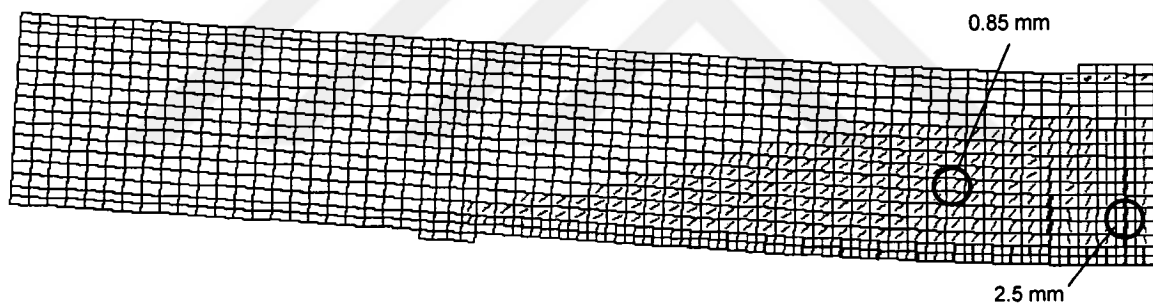


(c) Crack pattern at point B

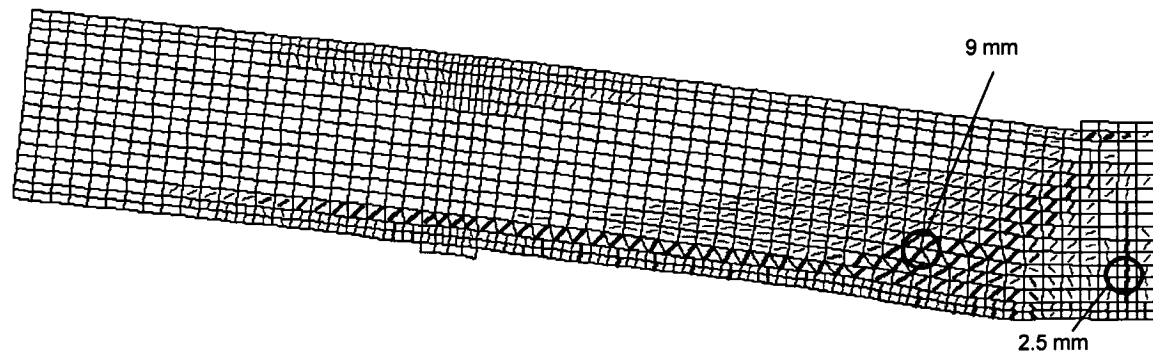
Figure 6.2 Static response of SS0



(a) Reaction force – mid-span displacement curve

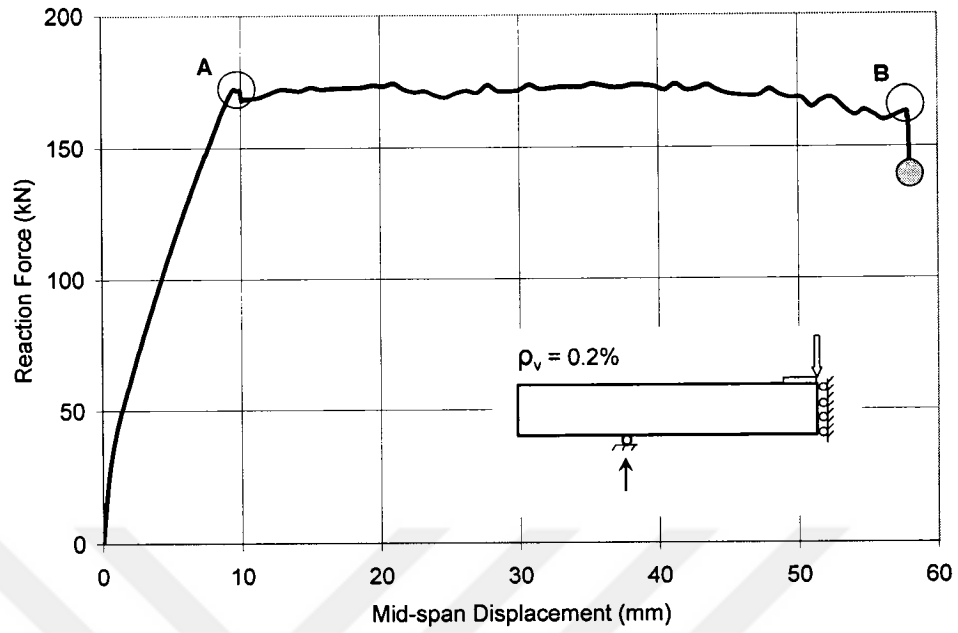


(b) Crack pattern at point A

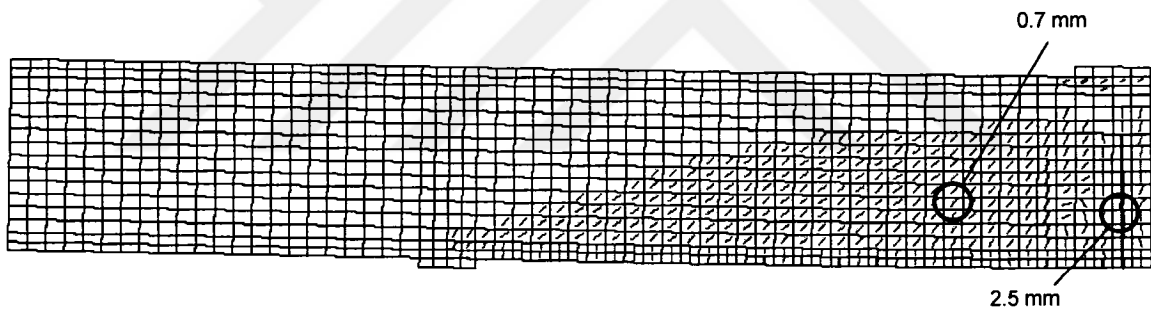


(c) Crack pattern at point B

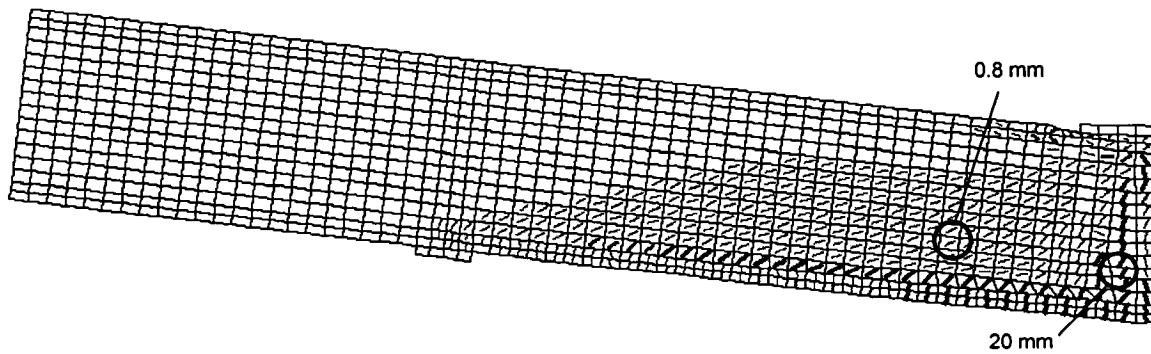
Figure 6.3 Static response of SS1



(a) Reaction force – mid-span displacement curve

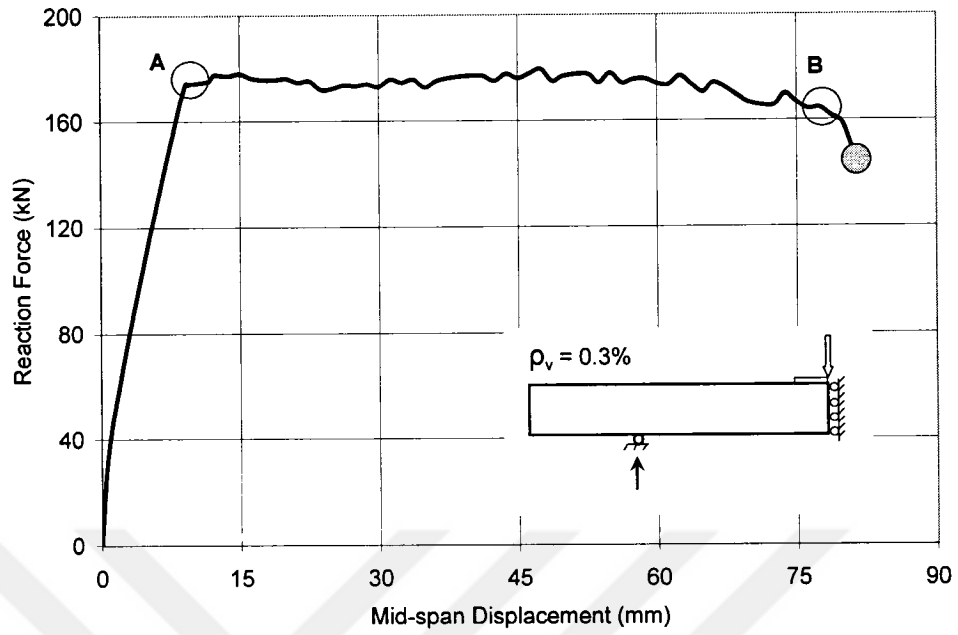


(b) Crack pattern at point A

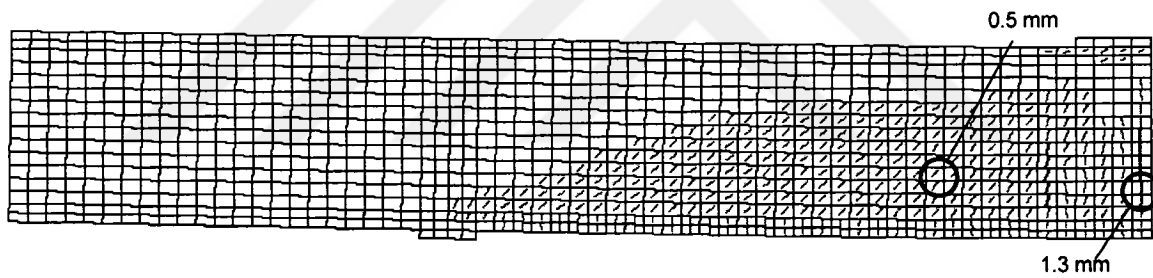


(c) Crack pattern at point B

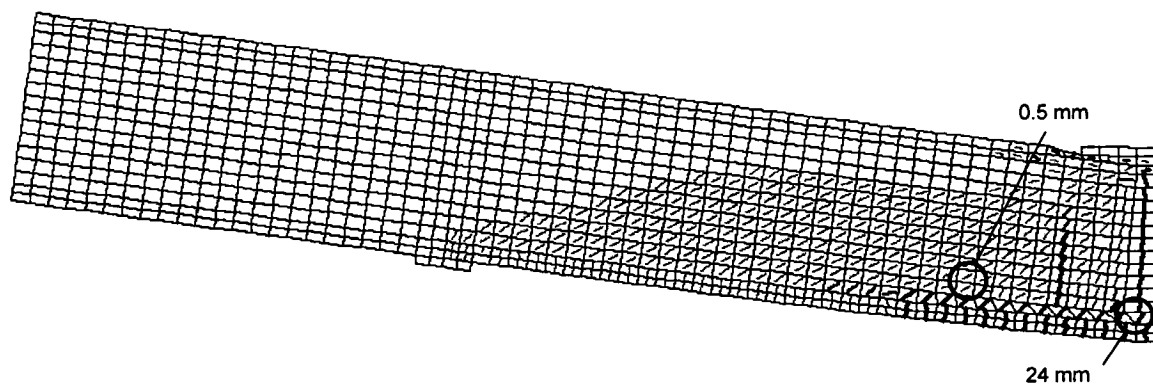
Figure 6.4 Static response of SS2



(a) Reaction force – mid-span displacement curve



(b) Crack pattern at point A



(c) Crack pattern at point B

Figure 6.5 Static response of SS3

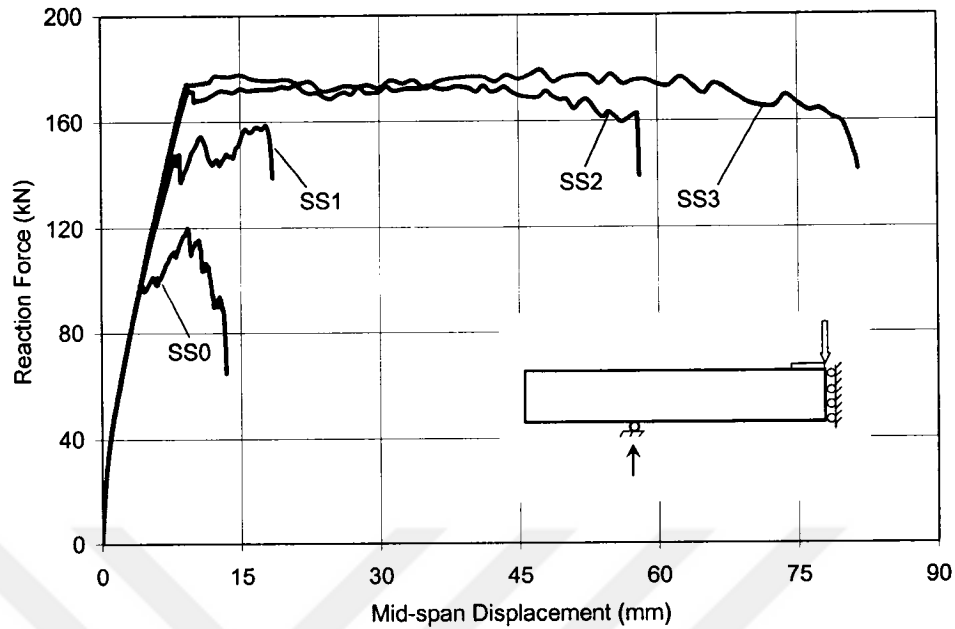


Figure 6.6 Static response of test specimens

As seen in the figures, SS2 and SS3 are predicted to have flexural ductile responses with long yield plateaus. SS0 and SS1, on the other hand, show brittle responses. They both fail in shear, evidenced by the wide diagonal cracks. SS0 has significantly lower capacity than SS1, since it does not contain transverse reinforcement. Both SS0 and SS1 develop vertical flexural cracks at the mid-span. However, it should be noted that these cracks do not widen significantly with increasing deformations. In other words, increasing deformations are mostly accommodated by widening of the diagonal cracks for both specimens.

The static analyses show that the test specimens cover a range of member behaviours, from brittle to ductile. This affords an opportunity to investigate the correspondence between static behaviour and behaviour under impact load conditions.

6.4 Impact Analysis of Test Specimens

The impact loads on the test specimens were simulated by assigning the impact velocity of the drop-weight to the nodal masses representing the drop-weight. Analyses were first carried out on undamaged specimens. VecTor2 records the final state of the specimen at the end of an analysis, in terms of residual displacements, element stiffnesses and other structural properties, which can later be used to initiate subsequent load analysis on the same structure. Using this feature, second and third impact tests were modelled by starting from the previous damaged state of the specimen.

In the following section, the analyses carried out on undamaged specimens are presented. Mid-span displacements, support reactions, rebar strains, and crack patterns as measured during the tests and calculated from VecTor2 analyses are compared. Section 6.4.2 presents similar analyses and comparisons for the second impact tests. The effects of time-step length and damping parameters on the analysis results are discussed in Sections 6.4.3 and 6.4.4, respectively.

6.4.1 Impact Analyses of Undamaged Test Specimens

In all analyses, the constant acceleration method was utilized. The time-step length selected for the numerical integration was 0.0001 seconds. Damping ratios for the first and second modes of vibration were selected by a parametric study, determining the minimum viscous damping required to stabilize the solutions. For SS1a-1, SS2a-1 and SS3a-1, the first mode of vibration was assigned 0.01% of critical damping, whereas the second mode was assigned 1% of critical damping. For SS0a-1, the first and second modes were assigned 1% and 5% of critical damping, respectively. For the b-series specimens, the damping ratio for the first mode was 0.01% of critical damping, and for the second mode it was reduced to 0.5% of critical damping. Issues regarding the selection of these analysis parameters are discussed in Sections 6.4.3 and 6.4.4. The following sections present the comparisons of mid-span displacements, support reactions, rebar strains, and crack patterns as measured during the tests and calculated from VecTor2 analyses. The VecTor2 analysis of SS0b-1 is not included, since, during the test, the specimen suffered extensive damage beyond the analysis capabilities of VecTor2.

6.4.1.1 Mid-span Displacements and Support Reactions

The following figures present the comparisons of mid-span displacements and support reaction forces, as observed and computed with VecTor2. Peak displacements and support reactions are summarized in Table 6.3.

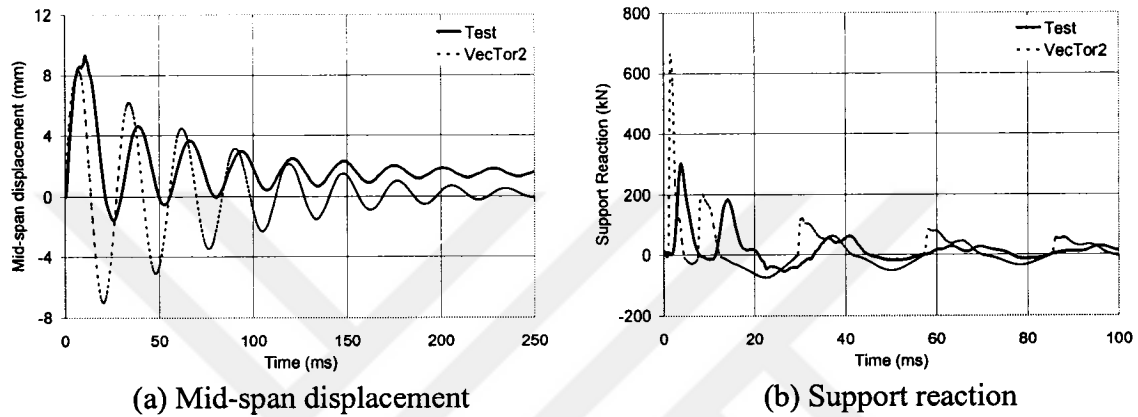


Figure 6.7 Comparison of observed and computed responses, SS0a-1

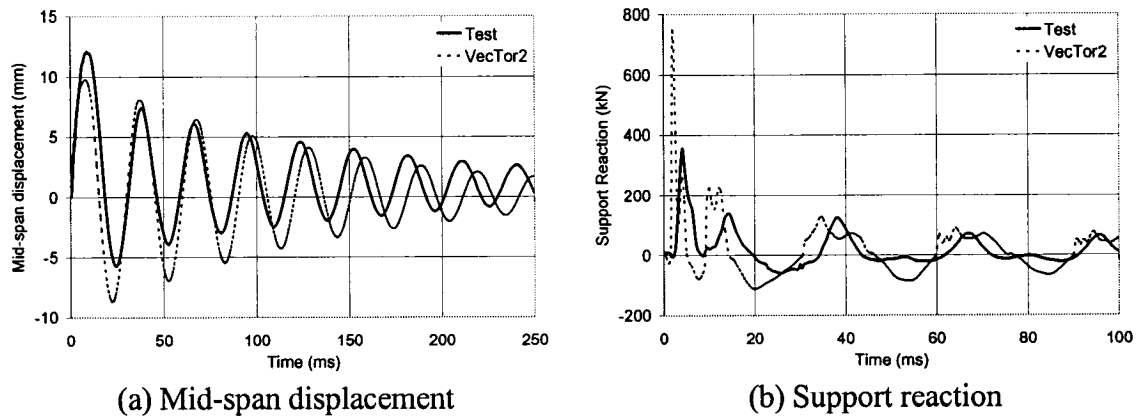
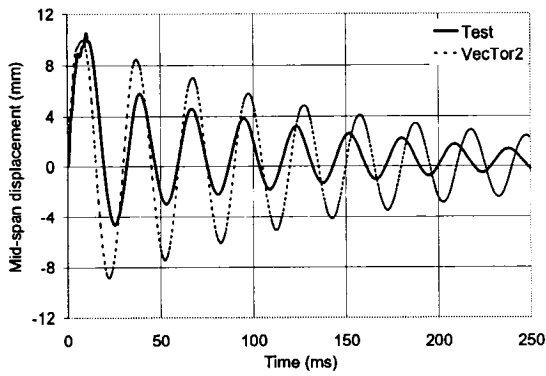
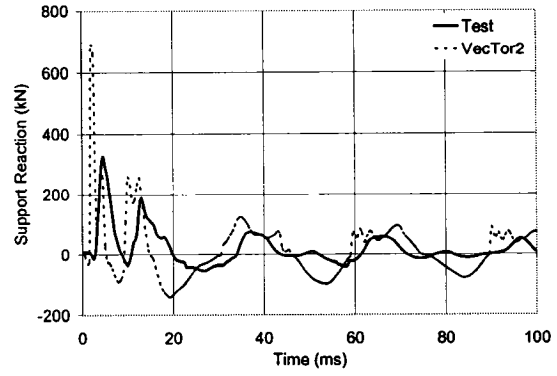


Figure 6.8 Comparison of observed and computed responses, SS1a-1

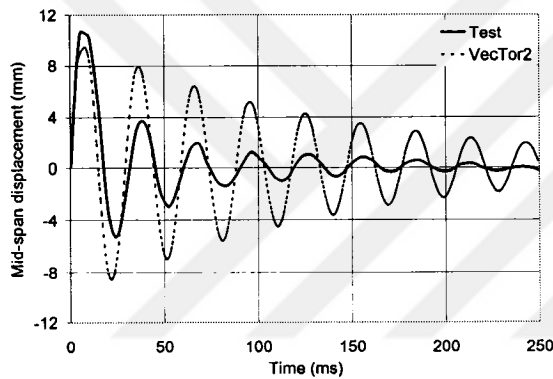


(a) Mid-span displacement

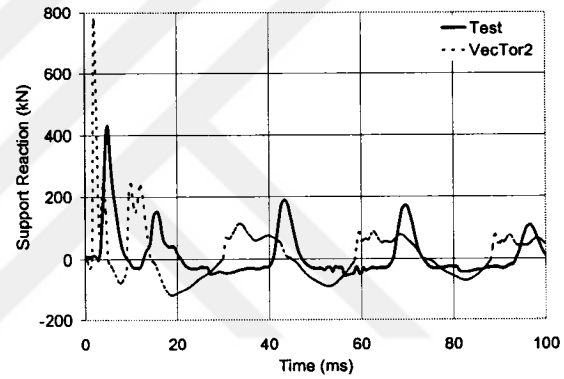


(b) Support reaction

Figure 6.9 Comparison of observed and computed responses, SS2a-1

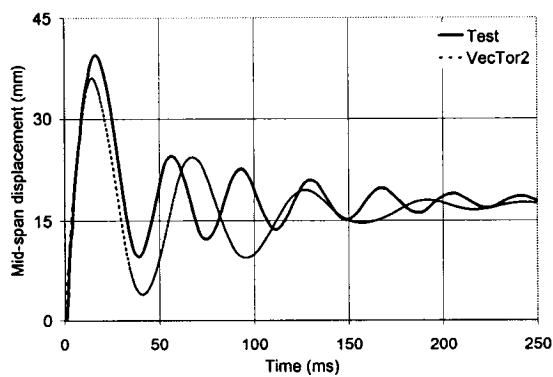


(a) Mid-span displacement

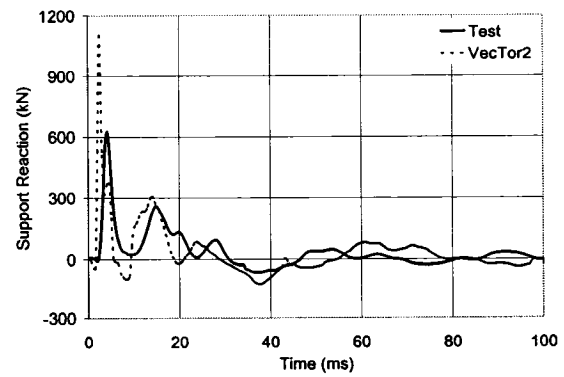


(b) Support reaction

Figure 6.10 Comparison of observed and computed responses, SS3a-1

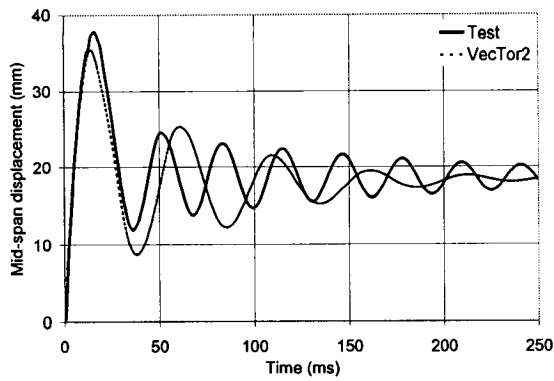


(a) Mid-span displacement

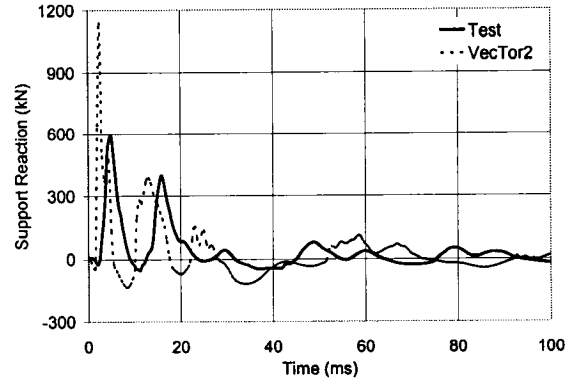


(b) Support reaction

Figure 6.11 Comparison of observed and computed responses, SS1b-1

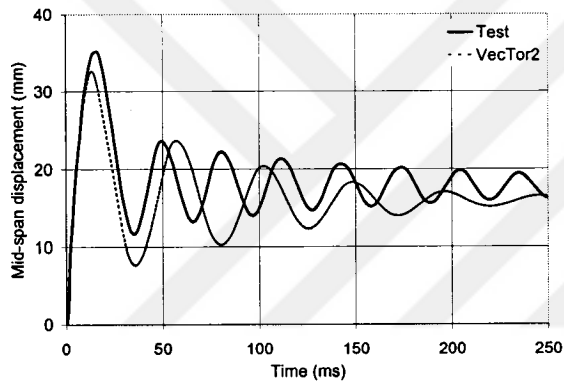


(a) Mid-span displacement

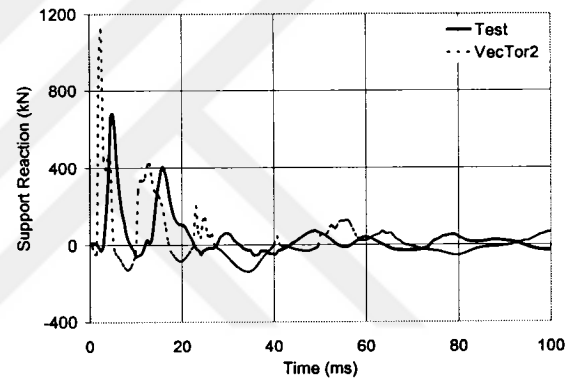


(b) Support reaction

Figure 6.12 Comparison of observed and computed responses, SS2b-1



(a) Mid-span displacement



(b) Support reaction

Figure 6.13 Comparison of observed and computed responses, SS3b-1

Table 6.3 Peak values as obtained from the tests and VecTor2 (first impacts)

Test	Peak Displacements (mm)			Residual Displacements (mm)			Peak Support Reactions (kN)		
	Test (mm)	VecTor2 (mm)	Error (%)	Test (mm)	VecTor2 (mm)	Error (%)	Test (kN)	VecTor2 (mm)	Error (%)
SS0a-1	9.3	8.3	10.1	1.6	0.2	87.5	305	658	115.7
SS1a-1	12.1	9.8	18.8	0.9	0.3	66.7	356	740	107.5
SS2a-1	10.0	8.9	4.7	0.5	0.3	40	327	691	111.4
SS3a-1	10.7	9.5	11.4	0.0	0.1	-	431	770	78.8
SS1b-1	39.5	36.1	8.8	17.7	17.1	3.4	625	1110	77.6
SS2b-1	37.9	35.4	6.4	18.5	18.5	0.0	592	1145	93.3
SS3b-1	35.3	32.7	7.4	17.7	16.1	9.0	682	1121	64.5

As seen in the preceding figures, some discrepancies exist between the test results and the VecTor2 analyses. Some typical characteristics were observed in the comparisons among a-series and b-series specimens. Typically, the peak displacements were underestimated in all tests by VecTor2. However, in the b-series specimens, the maximum discrepancy was 8.8%, whereas in the a-series specimens, the discrepancies were higher, reaching up to 18.8%. Post-peak vibrations showed discrepancies as well. In the a-series specimens, the periods of vibration (i.e., the time elapsed between two consecutive peaks) found from VecTor2 analyses were slightly shorter than the periods measured during testing. However, in the b-series specimens, the opposite was observed; VecTor2 estimated longer periods of vibration. Displacement responses for the b-series specimens damped out more rapidly in the VecTor2 analyses compared to the actual measured response, whereas the post-peak damping characteristics were captured somewhat better in a-series specimens. Support reactions, on the other hand, were poorly estimated for all specimens. VecTor2 greatly overestimated the peak reaction forces, although the post-peak fluctuations were captured better to a certain extent.

Discrepancies in displacement response can be attributed to several factors. First, it should be noted that the finite element model is an ideal representation of the actual test setup. In other words, certain deficiencies and irregularities in the test setup, such as the flexibility of the test apparatus, uneven contact of the support rollers with the specimens, uneven contact of the drop-weight with specimens, and vibrations in the longitudinal direction, are inherently omitted in the computer modelling of the setup, since it is difficult, if not impossible, to detect and measure these deficiencies due to the very rapid nature of an impact test. From this point of view, for example, it can be seen that the discrepancies observed in the peak and residual displacements for the a-series specimens are actually quite negligible, although they appear to be high on a percentage basis. The highest discrepancy in these specimens was around 2 mm for peak displacements, and a mere fraction of a millimetre for residual displacements, and thus within an acceptable range that can be attributed to the deficiencies in the test setup. The b-series specimens, on the other hand, suffered more damage since they were impacted with a heavier weight, and consequently the measured displacements were higher compared to the a-series specimens.

Deficiencies in the test setup played a relatively smaller role in their response; thus, errors in the peak and residual displacements were smaller in these tests, percentage wise, and within an acceptable range.

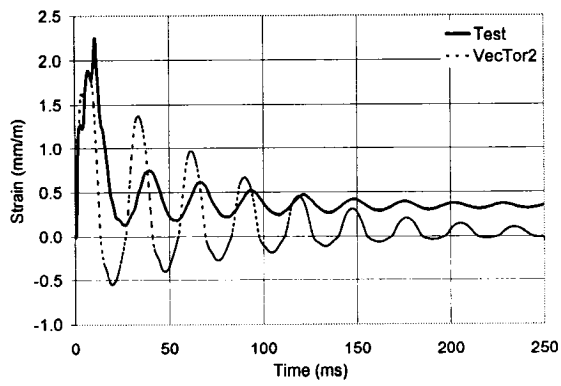
Almost without exception, VecTor2 captured the initial stiffnesses accurately for all specimens. However, rapid damping and long vibrational periods for the b-series specimens showed that the VecTor2 simulations overestimated the damage levels in these specimens. On the other hand, for the a-series specimens, the VecTor2 simulations estimated somewhat stiffer responses than the actual measured responses, with shorter periods of vibration and less damping. These discrepancies in the post-peak vibrational displacement response point to possible deficiencies in the material and behavioural models within VecTor2, such as in the concrete and steel hysteresis models, and in the strain rate and cracking formulations. For example, the concrete hysteresis models are critical in modelling the vibrational characteristics; however, most models describing the hysteresis rules for concrete, including the model used in this study, were developed under static cyclic loading conditions. Therefore, such sets of hysteresis rules are expected to create some errors when used in high strain rate situations, such as with impact loads. Moreover, strain rate formulations for concrete and steel are mostly developed for constant strain rates, whereas strain rates change rapidly during an impact-induced vibration as discussed in Section 4.6. Similarly, crack formulations, such as those that estimate crack slip deformations and crack shear stresses arising from aggregate interlock, are mainly developed for static load cases. Intuitively, it would be expected that under high dynamic conditions, these material behaviours would deviate somewhat from the static load conditions. Shear crack formulations play an especially important role for modelling the shear plug, which dominates the overall behaviours of the specimens as discussed in Chapter 5.

Large discrepancies in the reaction forces are more likely to be a numerical problem, rather than a material behaviour modelling problem. Truss bars representing the supports were assigned the estimated stiffness of the actual supports, which were significantly stiffer than the rest of the structure. The high stiffnesses assigned to these truss bars create numerical convergence problems, since small changes in the strains of these truss bars correspond to a substantial change in the force, making it difficult to satisfy overall member force

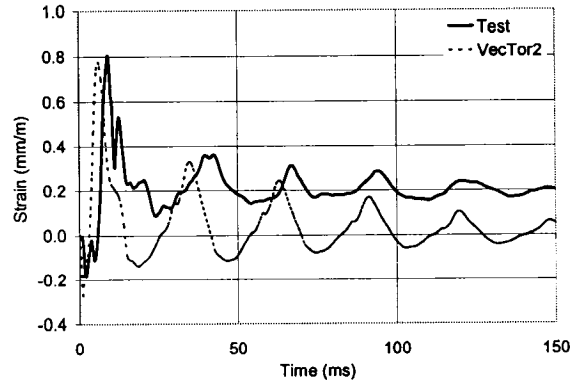
equilibrium between iterations. Moreover, as discussed in Chapter 3, some errors due to the low sampling rate, particularly regarding the peak forces, were suspected in the load cell measurements during the tests. In other words, there is a possibility that the actual peak support reactions were higher than those measured, which may contribute to the observed large discrepancy in peak forces, especially considering that the lower level post-peak reaction forces were estimated with greater accuracy.

6.4.1.2 Reinforcement Strains

The reinforcement strains, as measured during the tests and calculated from VecTor2 analyses, were also compared. For longitudinal reinforcement, strains at the mid-span and support (in the bottom layer of bars) were selected for comparison. Truss bar strains in the VecTor2 model corresponding to these gauge locations were compared to the strain gauge readings from the tests. Note that, for the mid-span strains, averages of the readings obtained from two strain gauges were used. Transverse reinforcement was modelled as smeared reinforcement in VecTor2. Therefore, average reinforcement strains calculated for the elements at the location of the stirrup strain gauges were used to compare with the test measurements. Strain comparisons for longitudinal reinforcement are presented in Figures 6.14 to 6.20, whereas the comparisons for stirrups are presented in Figures 6.21 to 6.26. The maximum values for longitudinal reinforcement and stirrups are summarized in Tables 6.4 and 6.5, respectively. Exact locations of the strain gauges can be found in Chapter 4.

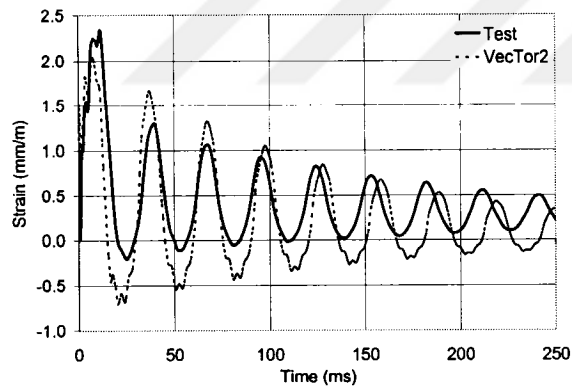


(a) Bar #3 Gauge 1 and Bar #4 Gauge 1 (mid-span)

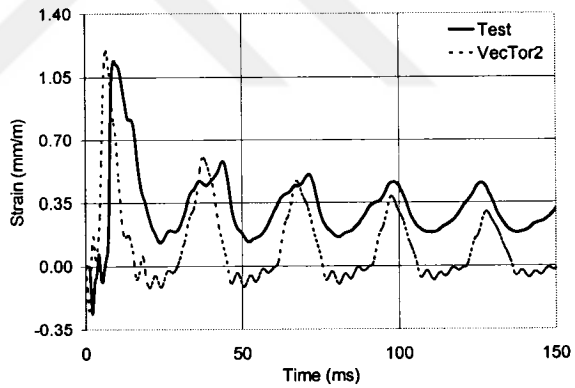


(b) Bar #4 Gauge 3 (support)

Figure 6.14 Observed and computed longitudinal reinforcement strains, SS0a-1

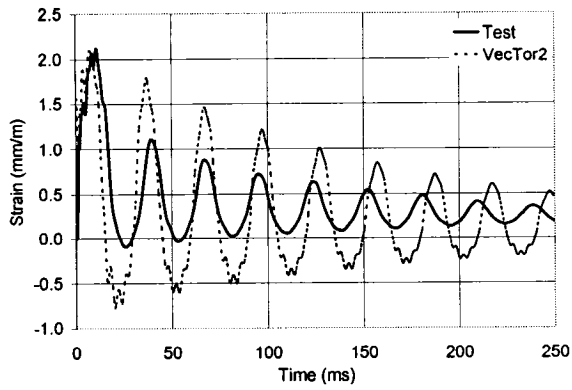


(a) Bar #3 Gauge 1 and Bar #4 Gauge 1 (mid-span)

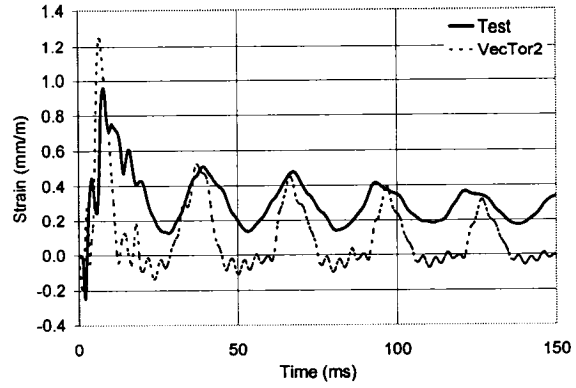


(b) Bar #4 Gauge 3 (support)

Figure 6.15 Observed and computed longitudinal reinforcement strains, SS1a-1

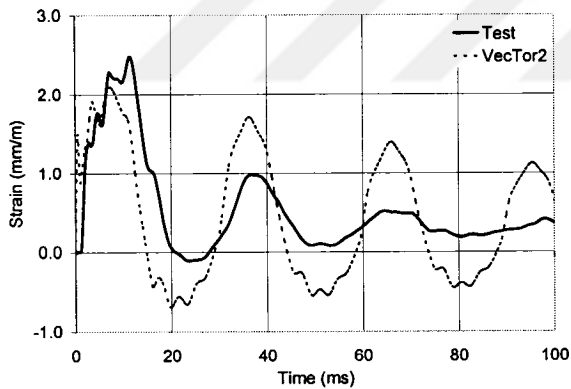


(a) Bar #3 Gauge 1 and Bar #4 Gauge 1
(mid-span)

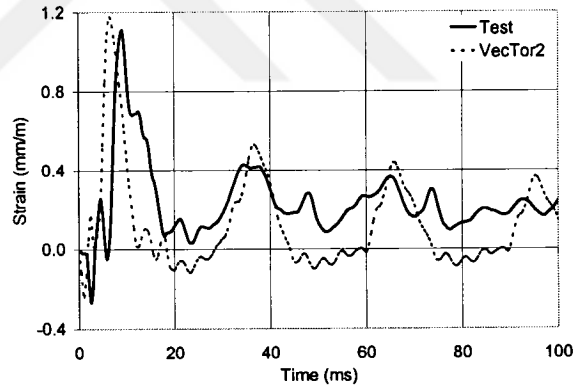


(b) Bar #4 Gauge 3
(support)

Figure 6.16 Observed and computed longitudinal reinforcement strains, SS2a-1

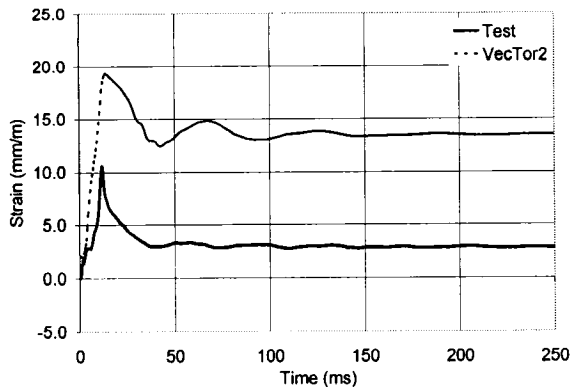


(a) Bar #3 Gauge 1 and Bar #4 Gauge 1
(mid-span)

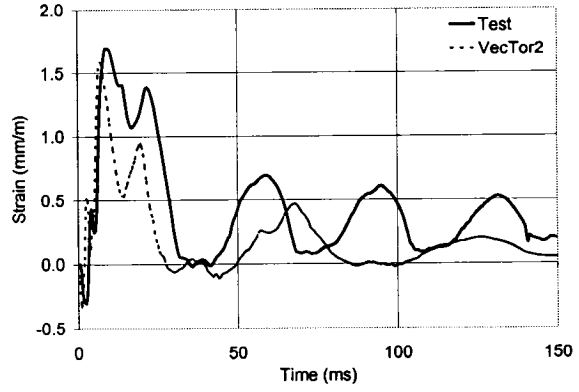


(b) Bar #4 Gauge 3
(support)

Figure 6.17 Observed and computed longitudinal reinforcement strains, SS3a-1

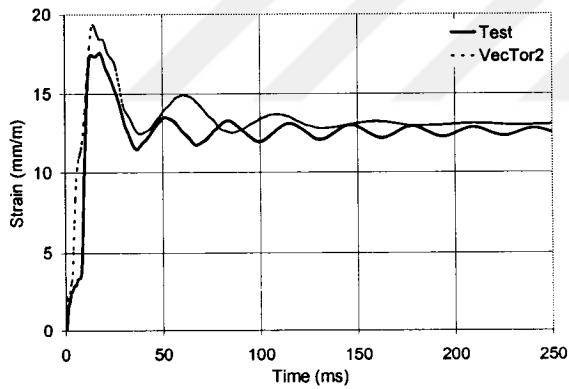


(a) Bar #3 Gauge 1
(mid-span)

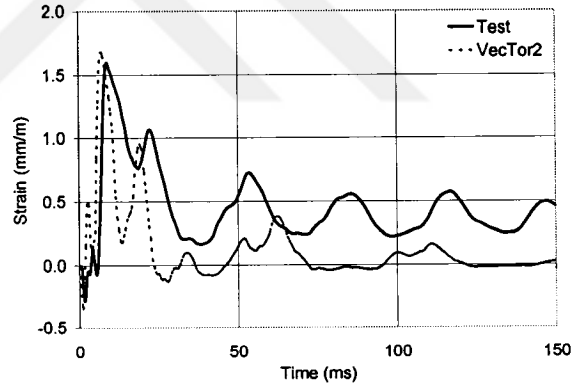


(b) Bar #3 Gauge 3
(support)

Figure 6.18 Observed and computed longitudinal reinforcement strains, SS1b-1

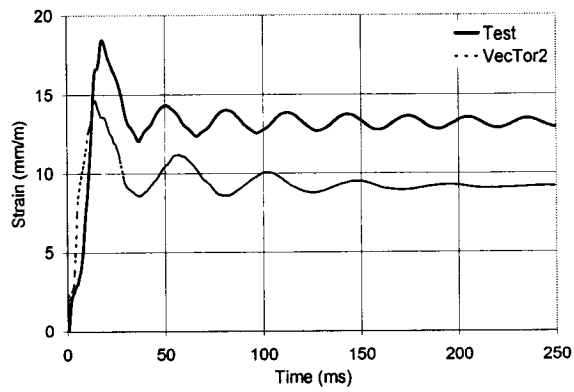


(a) Bar #3 Gauge 1 and Bar #4 Gauge 1
(mid-span)

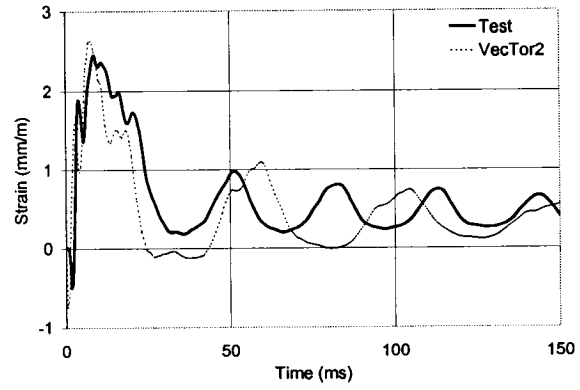


(b) Bar #4 Gauge 3
(support)

Figure 6.19 Observed and computed longitudinal reinforcement strains, SS2b-1

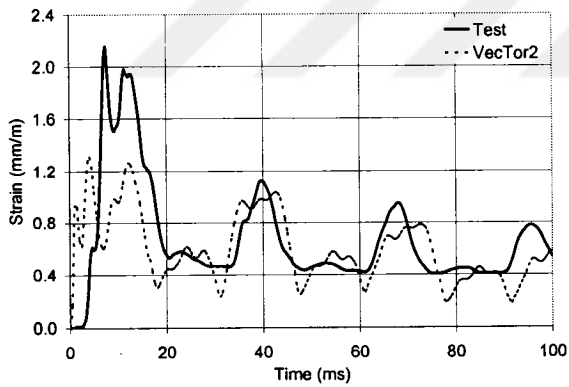


(a) Bar #3 Gauge 1 and Bar #4 Gauge 1 (mid-span)

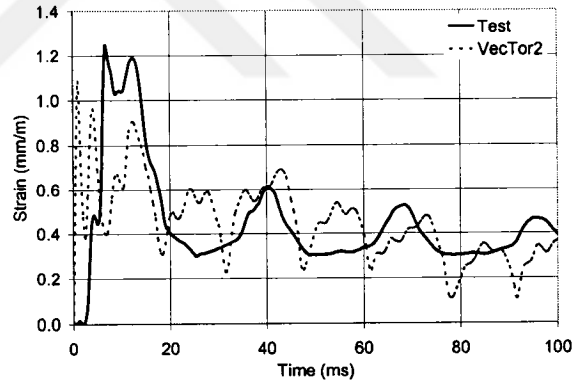


(b) Bar #4 Gauge 3 (support)

Figure 6.20 Observed and computed longitudinal reinforcement strains, SS3b-1

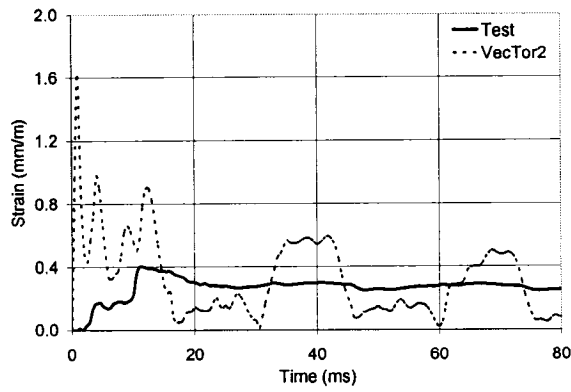


(a) Stirrup 1

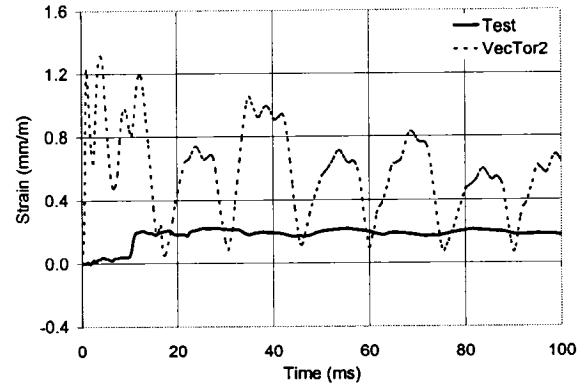


(b) Stirrup 2

Figure 6.21 Observed and computed stirrup strains, SS1a-1

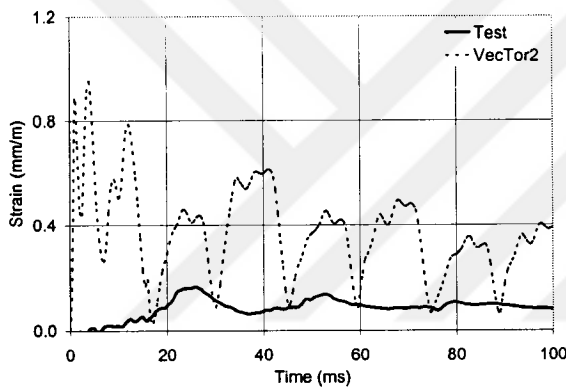


(a) Stirrup 1

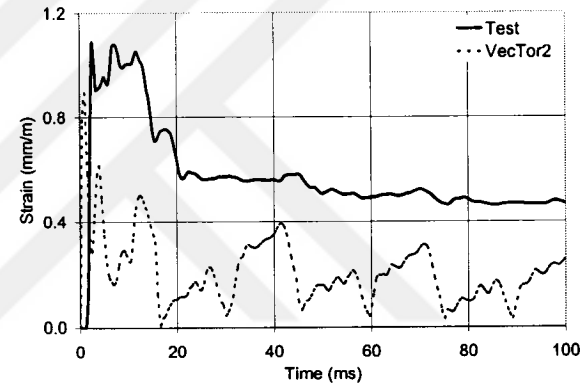


(b) Stirrup 2

Figure 6.22 Observed and computed stirrup strains, SS2a-1

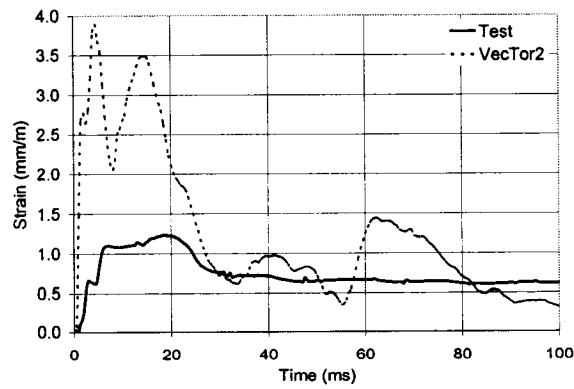


(a) Stirrup 1

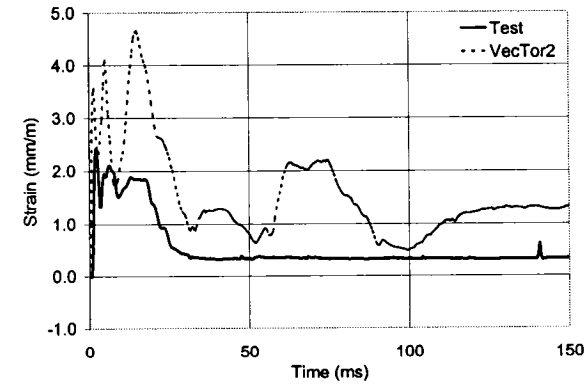


(b) Stirrup 2

Figure 6.23 Observed and computed stirrup strains, SS3a-1



(a) Stirrup 1



(b) Stirrup 2

Figure 6.24 Observed and computed stirrup strains, SS1b-1

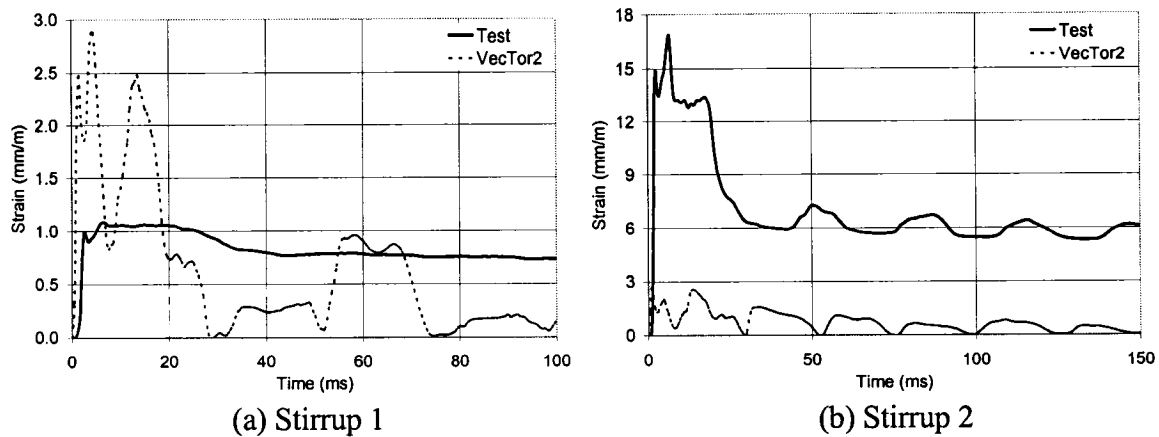


Figure 6.25 Observed and computed stirrup strains, SS2b-1

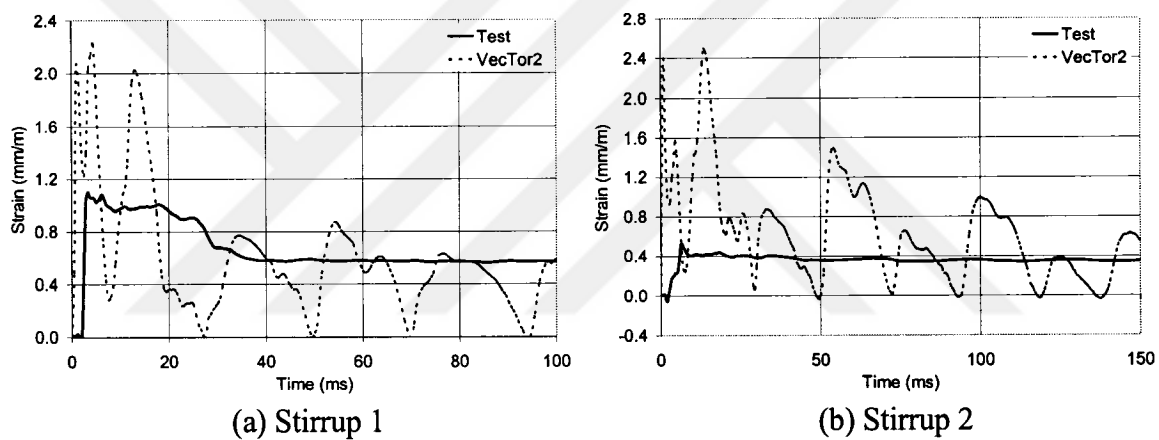


Figure 6.26 Observed and computed stirrup strains, SS3b-1

Table 6.4 Observed and computed peak longitudinal reinforcement strains

Test	Peak Mid-span Strains (mm/m)			Peak Support Strains (mm/m)		
	Test	VecTor2	Error (%)	Test	VecTor2	Error (%)
SS0a-1	2.30	1.90	16.1	0.80	0.78	3.4
SS1a-1	2.30	2.05	12.5	1.10	1.20	4.9
SS2a-1	2.13	2.11	0.7	0.96	1.25	30.4
SS3a-1	2.47	2.09	15.2	1.11	1.18	6.3
SS1b-1	10.59	19.34	82.7	1.70	1.59	6.1
SS2b-1	17.62	19.36	9.9	1.60	1.69	5.5
SS3b-1	18.50	14.64	20.9	2.45	2.64	7.8

Table 6.5 Observed and computed peak stirrup strains

Test	Peak Stirrup 1 Strains (mm/m)			Peak Stirrup 2 Strains (mm/m)		
	Test	VecTor2	Error (%)	Test	VecTor2	Error (%)
SS0a-1	-	-	-	-	-	-
SS1a-1	2.15	1.32	38.9	1.25	1.09	12.7
SS2a-1	0.40	1.62	300.5	0.22	1.32	485.8
SS3a-1	0.16	0.95	479.5	1.08	0.90	16.9
SS1b-1	1.24	3.91	215.5	2.43	4.66	91.6
SS2b-1	1.09	2.89	165.5	16.87	2.76	83.7
SS3b-1	1.10	2.23	102.5	0.53	2.50	373.1

As seen in the figures, VecTor2 predicted the longitudinal reinforcement strains with reasonable accuracy. Peak strains were generally predicted with very good accuracy. Post-peak responses were also predicted well. On the other hand, strains in the stirrups were predicted poorly.

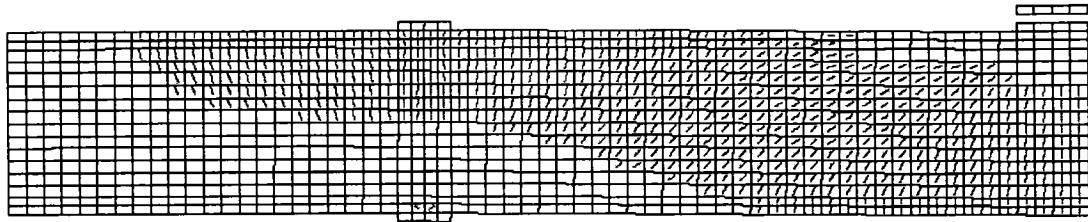
The discrepancies in the comparisons can be attributed to several factors. First, it should be noted that measuring strains in a reinforcing bar is a delicate issue. As mentioned in Chapter 4, strain gauges are glued on the bar surface, and embedded in concrete with the bar. Preparing the bar surface before gluing the gauge requires special attention to ensure that the gauge is in full and secure contact, so that it can deform parallel to the bar. However, even minor scratches or barely noticeable irregularities on the bar surface may compromise the accuracy of the strain readings. Moreover, especially in high deformation states, the gauge may partially or completely lose its contact with the bar, or it may suffer significant damage, as occurred during several tests in this test program. Note that, in SS1b-1 and SS3b-1, predictions for the mid-span strains were very poor, although the strains at the support were predicted well. In these tests, the strains at the mid-span were exceptionally high, possibly causing a loss in their accuracy and resulting in the high discrepancies observed between the measured and computed responses.

Another reason for the poor prediction of the stirrup strains is the method of modelling the stirrups in VecTor2 analysis. As mentioned before, stirrups were included in the model as

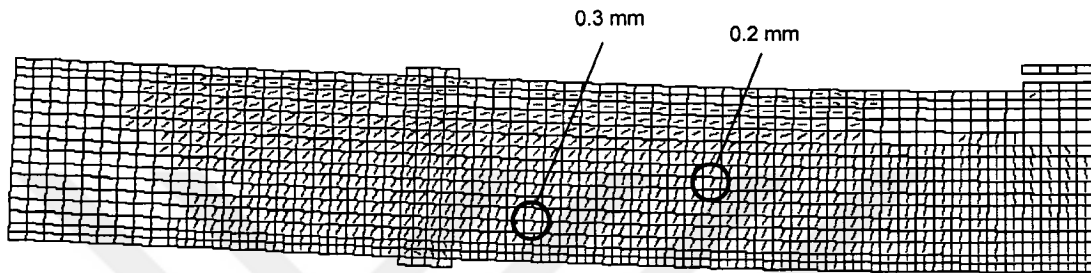
smeared reinforcement in concrete. Calculated strains for smeared reinforcement are average values, making the prediction of the strains at a particular location difficult especially if the amount of reinforcement is small. Moreover, as seen in almost all tests, wide diagonal cracks passing through the stirrups cause very high strain concentrations on the bars at the crack locations. As a result, strains measured by a strain gauge significantly depend on the gauge's proximity to a crack. Therefore, calculated average strains can be drastically different than the measured, as seen in these analyses.

6.4.1.3 Crack Patterns

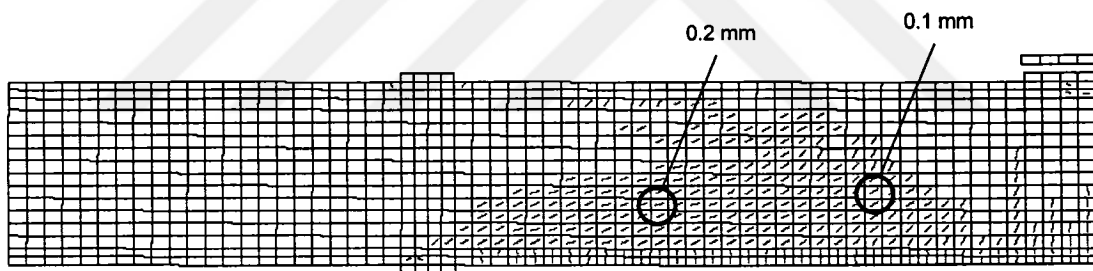
Crack patterns as calculated by VecTor2 were compared with the patterns obtained from the tests (see Figures 6.27 to 6.33). It should be noted that, as a result of VecTor2's rotating crack approach, crack direction at a load stage is determined by the principal axis of stress calculated for that load stage. In other words, the direction of cracks constantly changes, and VecTor2 does not report the crack directions of the preceding load stages in the output. Therefore, a crack profile as drawn by Augustus does not reflect the cracking history of the structure, but rather belongs only to that particular load stage. For this reason, several load stages need to be examined for a complete analysis of the estimated crack pattern. In this section, the crack patterns estimated at the initial stages of the response (negative moment phase as mentioned in Chapter 5), at the time the peak mid-span displacement occurred, and at the final resting stage of the specimen are presented and compared with the crack maps obtained from the test results. Note that the crack widths measured after the tests relate to the final resting stage of the specimen. Crack widths calculated by VecTor2 for the time when the peak mid-span displacement occurred are also presented to give an idea about the levels of predicted maximum crack widths.



(a) Calculated crack profile at 1 ms



(b) Calculated crack profile at 7 ms (peak displacement)

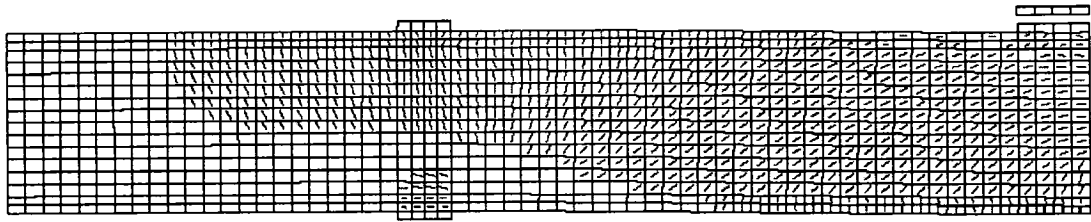


(c) Crack profile at the final resting stage

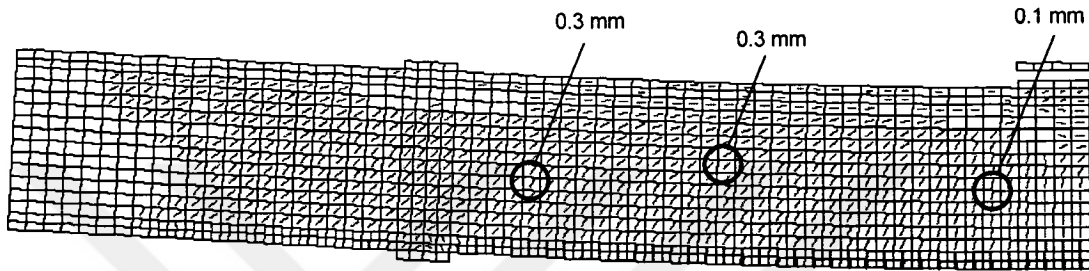


(d) Observed crack profile (west face)

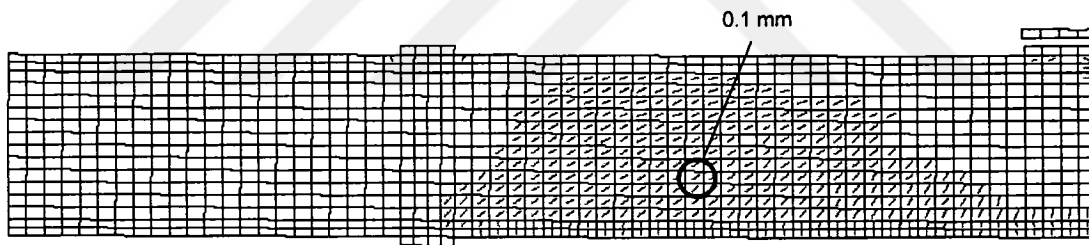
Figure 6.27 Observed and computed crack profiles, SS0a-1



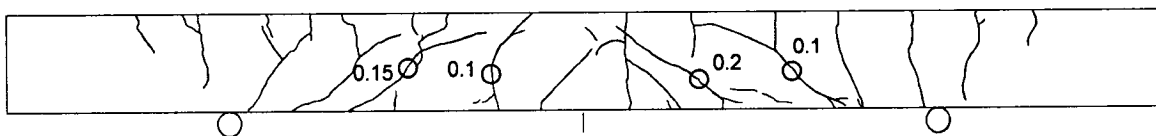
(a) Calculated crack profile at 1 ms



(b) Calculated crack profile at 9 ms (peak displacement)

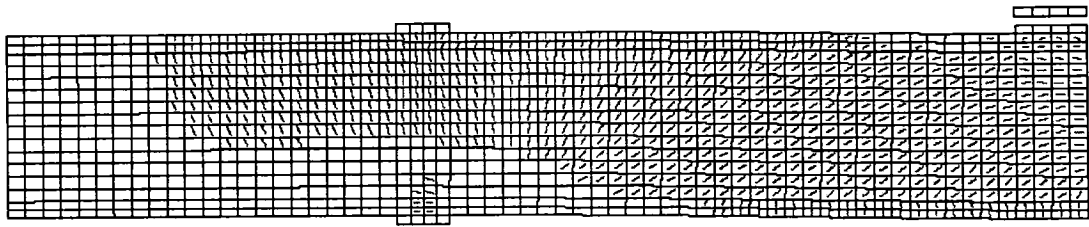


(c) Crack profile at the final resting stage

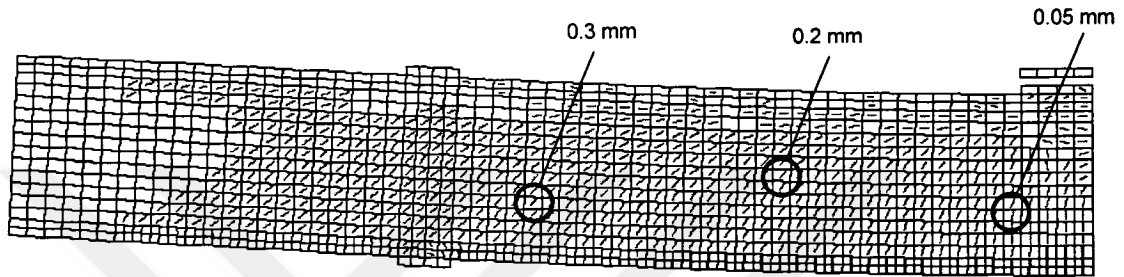


(d) Observed crack profile (west face)

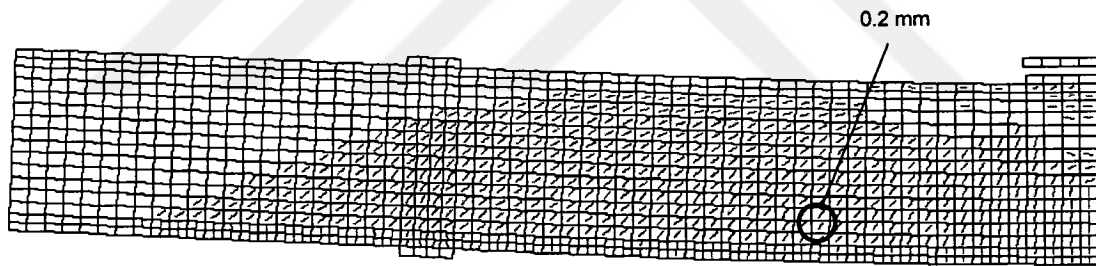
Figure 6.28 Observed and computed crack profiles, SS1a-1



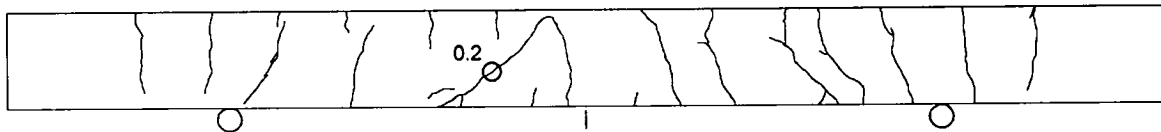
(a) Calculated crack profile at 1 ms



(b) Calculated crack profile at 9 ms (peak displacement)

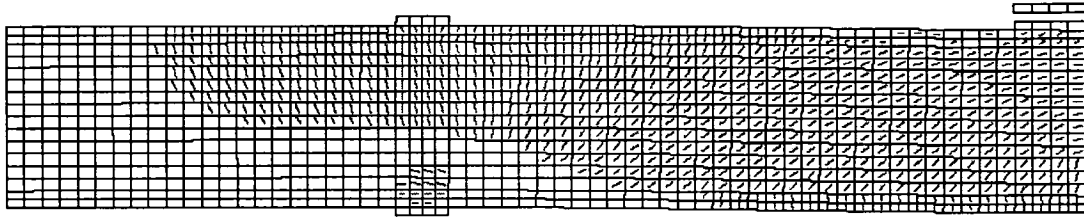


(c) Crack profile at the final resting stage

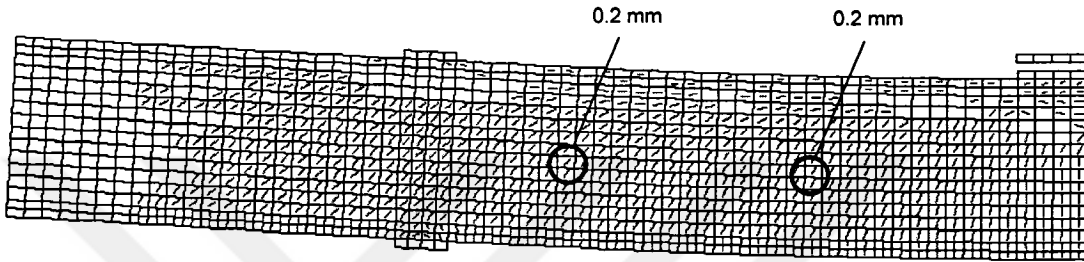


(d) Observed crack profile (west face)

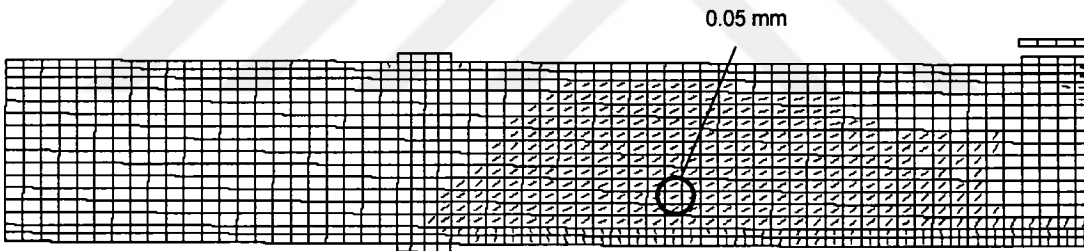
Figure 6.29 Observed and computed crack profiles, SS2a-1



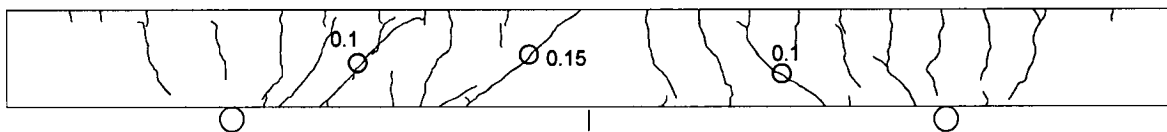
(a) Calculated crack profile at 1 ms



(b) Calculated crack profile at 8 ms (peak displacement)

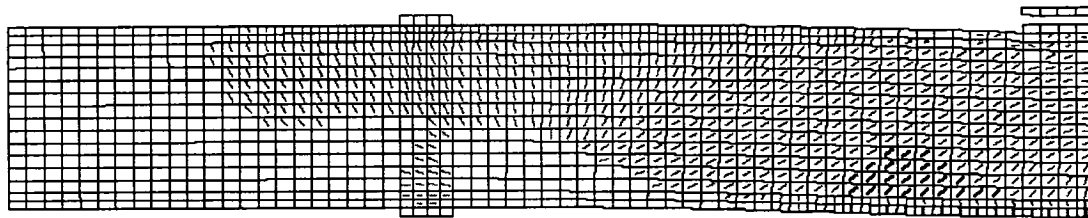


(c) Crack profile at the final resting stage

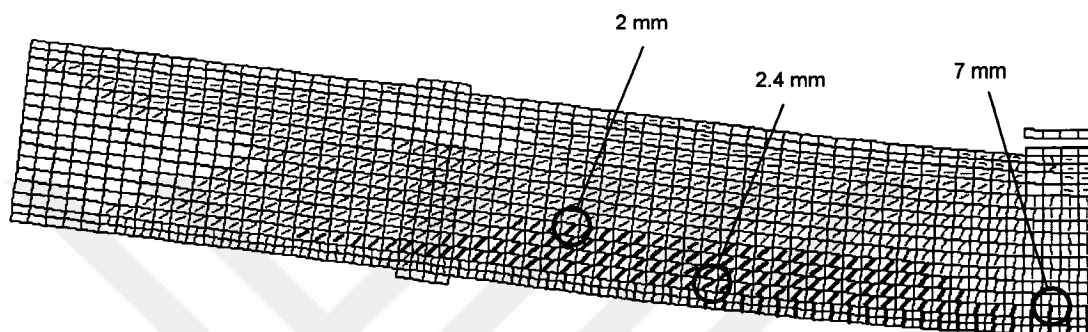


(d) Observed crack profile (west face)

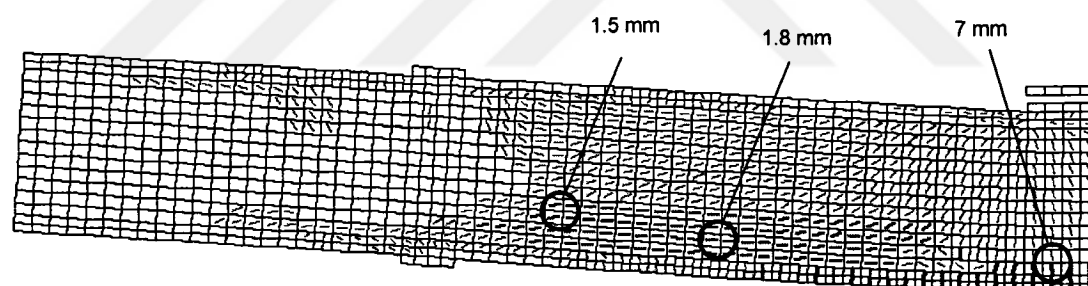
Figure 6.30 Observed and computed crack profiles, SS3a-1



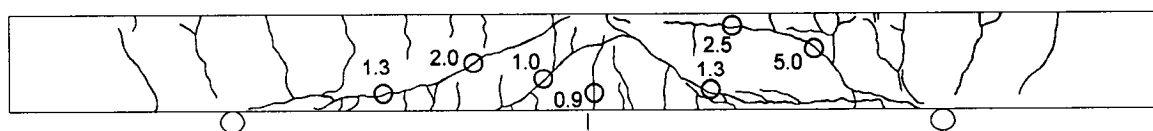
(a) Calculated crack profile at 1 ms



(b) Calculated crack profile at 15 ms (peak displacement)

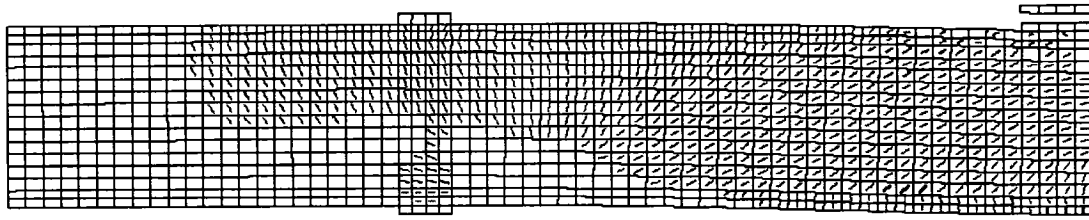


(c) Crack profile at the final resting stage

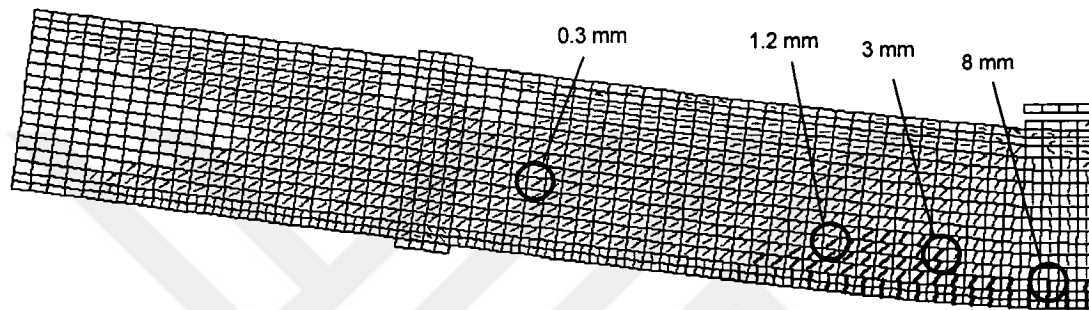


(d) Observed crack profile (west face)

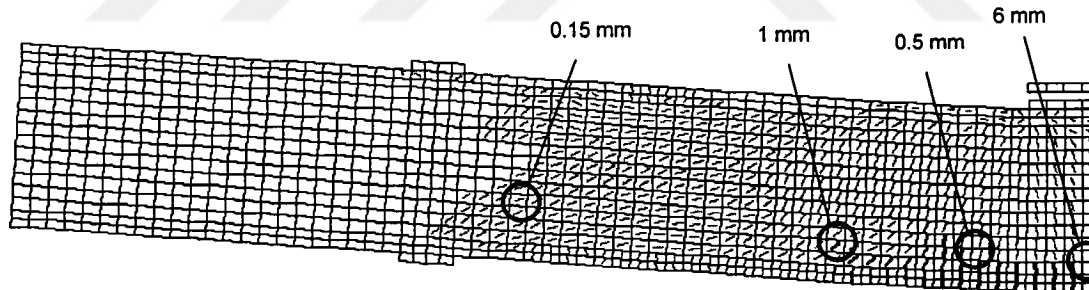
Figure 6.31 Observed and computed crack profiles, SS1b-1



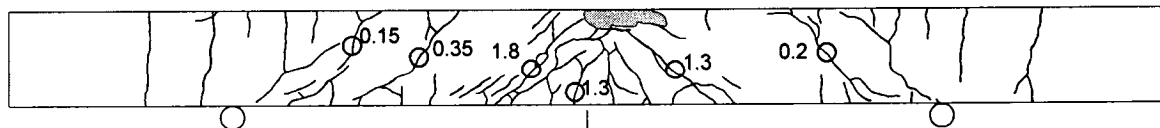
(a) Calculated crack profile at 1 ms



(b) Calculated crack profile at 15 ms (peak displacement)

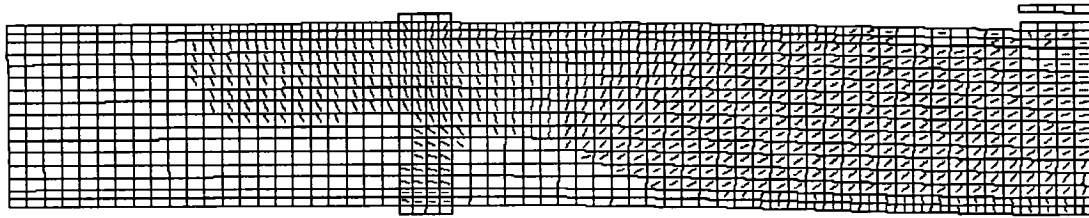


(c) Crack profile at the final resting stage

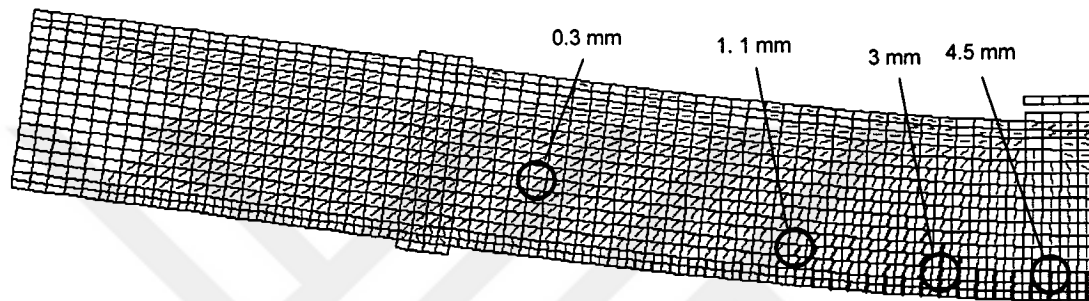


(d) Observed crack profile (west face)

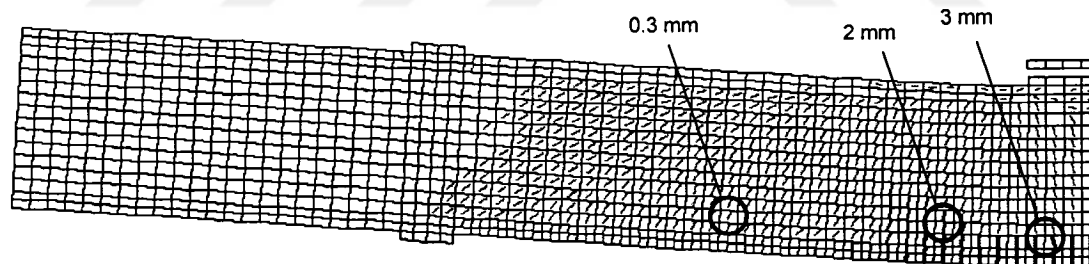
Figure 6.32 Observed and computed crack profiles, SS2b-1



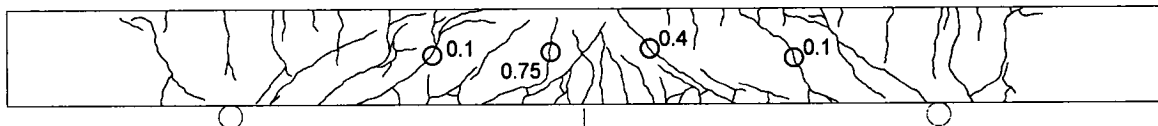
(a) Calculated crack profile at 1 ms



(b) Calculated crack profile at 14 ms (peak displacement)



(c) Crack profile at the final resting stage



(d) Observed crack profile (west face)

Figure 6.33 Observed and computed crack profiles, SS3b-1

As seen in the figures, VecTor2 generally predicted the crack profile well. Vertical cracks starting from the top face were predicted correctly at the early stages of the response. Vertical cracks at the overhanging part and their slight inclination towards the supports were also predicted accurately. VecTor2 calculations support the idea that these cracks occur at the very early stages of the response, as argued in Chapter 5. For the later stages of the response, VecTor2 accurately predicted the formation of the diagonal cracks. On the other hand, there were some discrepancies as well. For example, VecTor2 predicted wide vertical cracks close to the mid-span in all tests. However, vertical cracks measured after the tests at the mid-span were not as wide, or, as observed in some tests, there were no vertical cracks at the mid-span. Note that, especially in b-series specimens, these unrealistically wide vertical cracks at the mid-span are another indication to the over-estimated levels of damage by VecTor2 in these specimens, as argued in Section 6.4.1.1.

It should be noted that, in a vibrational response, cracks may close due to reversed deformations. VecTor2 can predict the crack closing, however, Augustus draws only the cracks exceeding a certain crack width. Therefore, elements shown without a crack in the predicted crack profiles may actually have been calculated as cracked by VecTor2, but not drawn by Augustus since they were closed and consequently had a very narrow width. On the other hand, closing of the cracks can make their detection difficult after the test, especially if they had a narrow width when they initially formed. In other words, some cracks predicted by VecTor2 but not observed after the test may have actually developed during the test as well, but they may have gone undetected since they were completely closed later.

6.4.2 Impact Analyses of Test Specimens for the Second Impact Tests

Similar analyses were carried out for the second impact tests on the same specimens. VecTor2 has the capability to start from the damaged condition of a previous analysis and apply a new load on the structure. By using this feature, the second impacts on the same specimens were analyzed, starting from the results of the analyses carried out on the undamaged specimens. It should be noted that this approach causes accumulating errors, since the first analyses were not error free, and hence, these subsequent analyses starts from a state that contains errors. On the other hand, the errors carried forward were likely not significant, evidenced by the good accuracy in residual displacements obtained from the first analyses, allowing for stable solutions with reasonable accuracies. However, attempts to analyze the same specimens for the third impact tests failed, since the accumulated errors from the first and second impact analyses, coupled with the numerical problems related to the significantly reduced stiffnesses of the highly-damaged specimens after the first two impact tests, made it impossible to obtain a numerically stable solution.

The analysis parameters for the second impact tests were kept identical with those of the first analyses, except for the damping coefficients. Damping ratios for the first and second modes of vibration were selected by a parametric study, determining the minimum viscous damping required to stabilize the solutions. For the a-series specimens, the damping ratio for the first mode was assigned as 0.1% of critical damping, whereas the second mode was assigned 1% of critical damping. For SS3b-2, 0.1% and 3% of critical damping were assigned to first and second modes, respectively. For SS2b-2, the damping ratio for the first mode was assigned a value of 0.1% of critical damping, whereas the second mode was assigned 5% of critical damping. Issues regarding the selection of these analysis parameters are discussed in Sections 6.4.3 and 6.4.4. The following sections present the comparisons of mid-span displacements, support reactions, rebar strains, and crack patterns as measured during the tests and calculated from VecTor2 analyses.

6.4.2.1 Mid-span Displacements and Support Reactions

Figures 6.34 to 6.38 present the comparisons of mid-span displacements and support reaction forces, as measured during the tests and found from VecTor2 analyses. Peak displacements and support reactions are summarized in Table 6.6. It should be noted that VecTor2 analyses for SS1b-2 and SS0a-2 are not included, since, during testing, these specimens suffered extensive damage beyond the analysis capabilities of VecTor2.

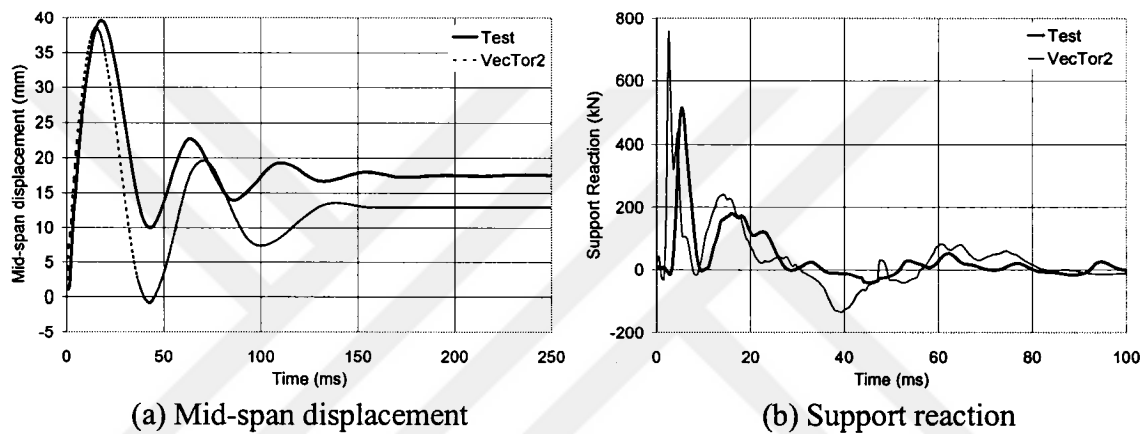


Figure 6.34 Comparison of observed and computed responses, SS1a-2

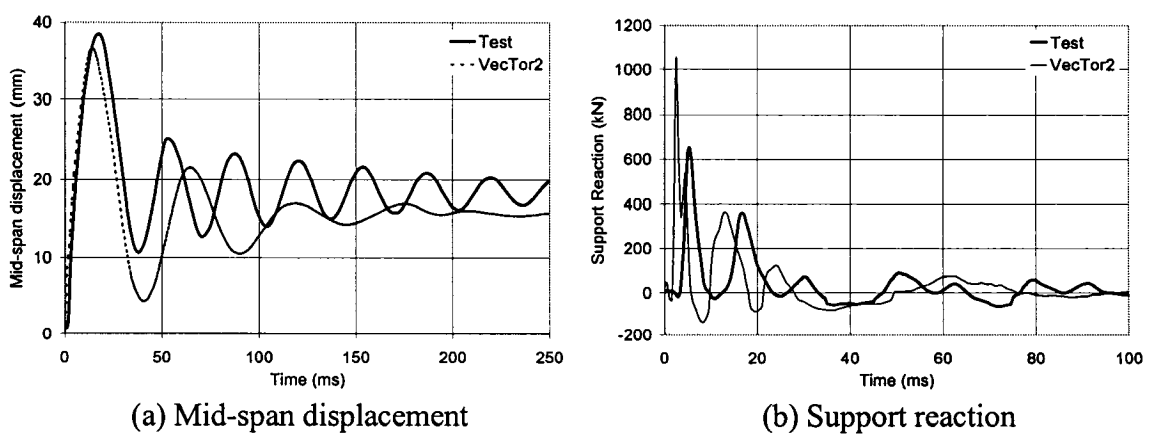
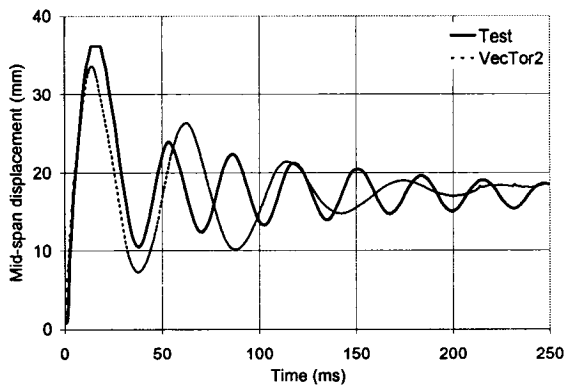
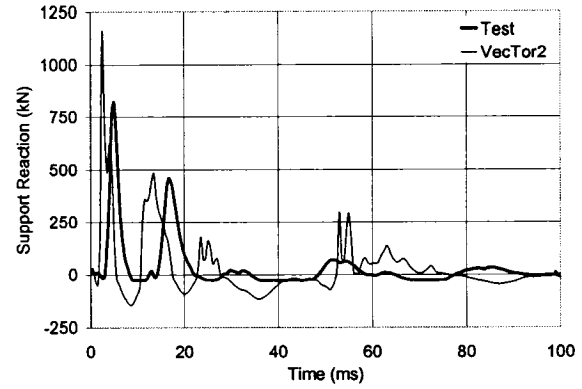


Figure 6.35 Comparison of observed and computed responses, SS2a-2

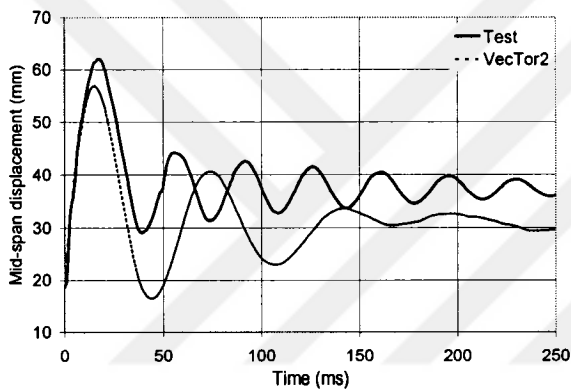


(a) Mid-span displacement

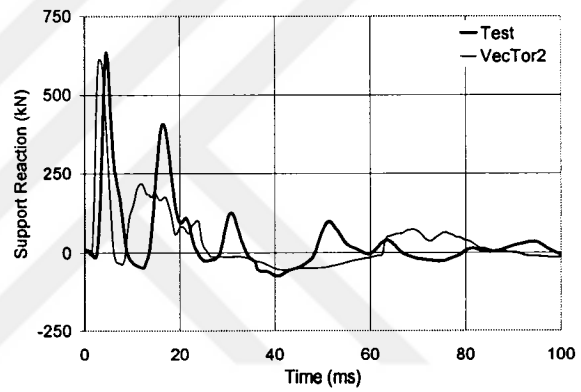


(b) Support reaction

Figure 6.36 Comparison of observed and computed responses, SS3a-2

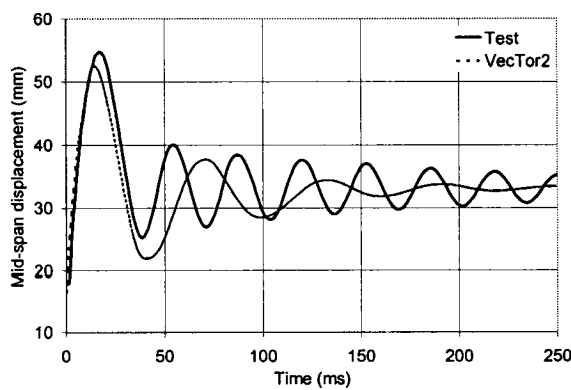


(a) Mid-span displacement

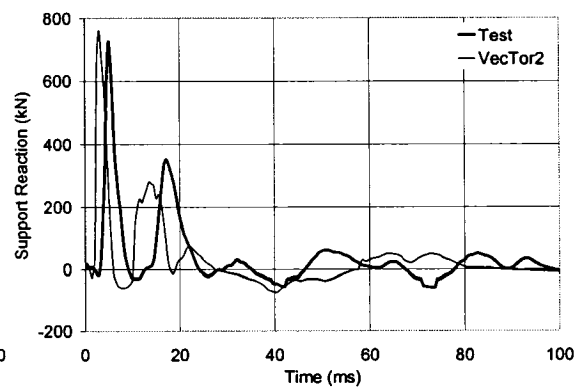


(b) Support reaction

Figure 6.37 Comparison of observed and computed responses, SS2b-2



(a) Mid-span displacement



(b) Support reaction

Figure 6.38 Comparison of observed and computed responses, SS3b-2

Table 6.6 Peak values as obtained from the tests and VecTor2 (second impacts)

Test	Peak Displacements (mm)			Residual Displacements (mm)			Peak Support Reactions (kN)		
	Test (mm)	VecTor2 (mm)	Error (%)	Test (mm)	VecTor2 (mm)	Error (%)	Test (kN)	VecTor2 (mm)	Error (%)
SS1a-2	39.6	38.7	2.4	17.5	12.9	26.3	517	747	44.6
SS2a-2	38.5	36.7	4.6	18.6	15.8	15.1	651	1050	61.3
SS3a-2	36.3	33.7	7.1	17.0	18.4	8.2	822	1152	40.2
SS2b-2	62.1	56.9	8.3	37.5	42.9	14.4	633	612	3.3
SS3b-2	54.8	52.6	4.1	33.3	33.0	0.9	725	762	5.2

As seen in the preceding figures, VecTor2 generally predicted the displacement responses well. Peak displacements were predicted with less than 10% error. Residual displacements were also predicted well. However, discrepancies exist for the post-peak vibrations; the computed displacement responses damped out quicker for all beams than did the responses measured during the tests. The support reactions were estimated better than in the first impact analyses. For a-series specimens, although the calculated peak support reactions were considerably higher than the measured reactions, the errors were significantly lower than with the first impact analyses. On the other hand, the peak reactions were estimated with great accuracy for b-series specimens.

It seems that the factors that caused the discrepancies in the first impact test analyses persisted for the second impact analyses as well. Nevertheless, considering the high levels of damage the specimens suffered, it is remarkable that VecTor2 estimated the peak displacements and some peak support reactions with very good levels of accuracy. Residual displacements were estimated with reasonable accuracy as well. Levels of error in the peak support reactions were also reduced, although, for a-series beams, they were still beyond an acceptable range. It should be noted that the peak support reactions were predicted with very good accuracy in highly damaged specimens. This suggests that the calculated extremely high peaks in the first impact analyses of undamaged specimens or the second impact analyses of relatively strong specimens may be related to the unrealistic contribution of high frequency response components to the numerical solution. Note that SS3b and SS2b were damaged significantly during the second impacts; thus, their fundamental vibrational

mode became more dominant and the contributions of higher modes were significantly reduced. Therefore, the effects of high frequency components on the support reactions were less in these tests. However, further investigation is still required to determine the exact mechanisms involved in the over-prediction of the peak support reactions.

6.4.2.2 Reinforcement Strains

Strain comparisons for longitudinal reinforcement are presented in Figures 6.39 to 6.43, whereas the comparisons for stirrups are presented in Figures 6.44 to 6.48. The maximum values for longitudinal reinforcement and stirrups are summarized in Tables 6.7 and 6.8, respectively. The exact locations of the strain gauges can be found in Chapter 4.

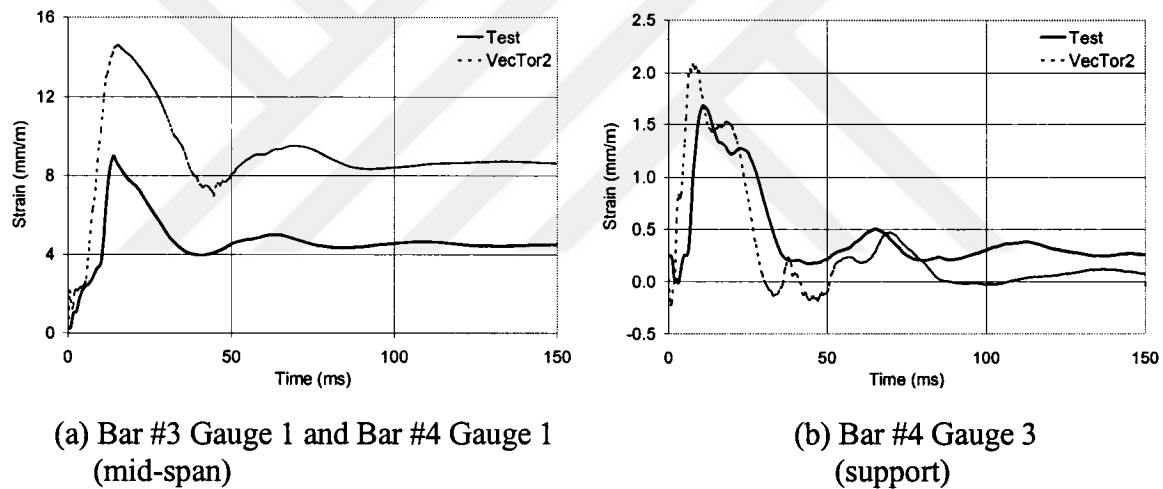
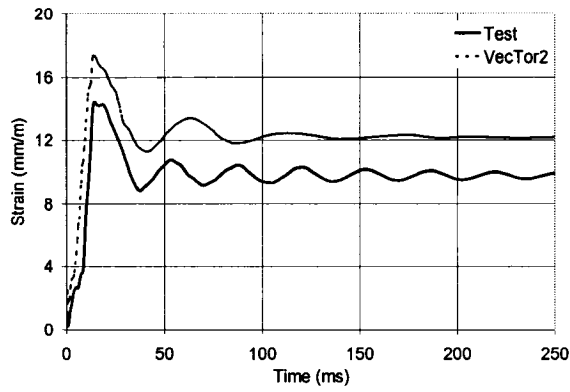
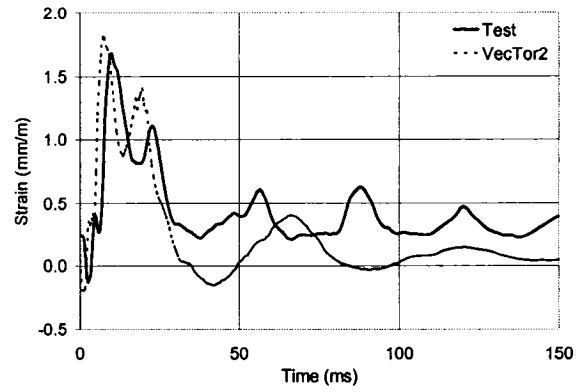


Figure 6.39 Observed and computed longitudinal reinforcement strains, SS1a-2

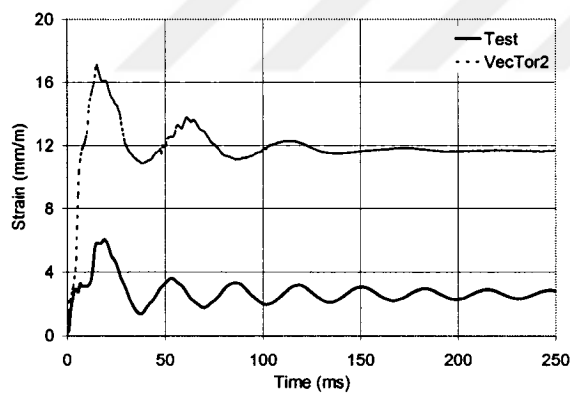


(a) Bar #3 Gauge 1 and Bar #4 Gauge 1
(mid-span)

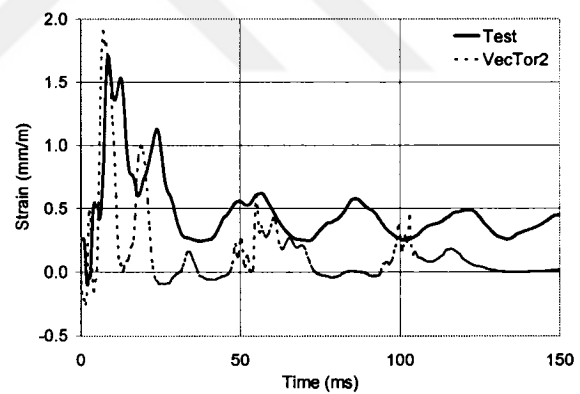


(b) Bar #4 Gauge 3
(support)

Figure 6.40 Observed and computed longitudinal reinforcement strains, SS2a-2

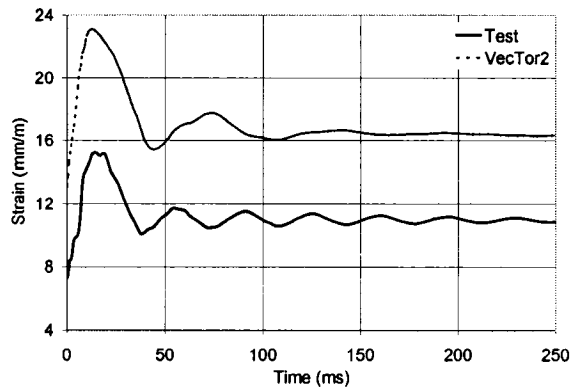


(a) Bar #3 Gauge 1 and Bar #4 Gauge 1
(mid-span)

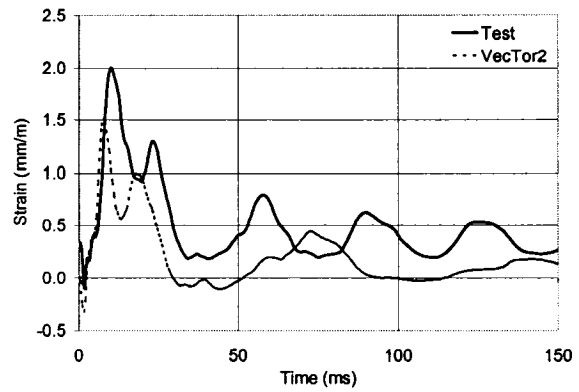


(b) Bar #4 Gauge 3
(support)

Figure 6.41 Observed and computed longitudinal reinforcement strains, SS3a-2

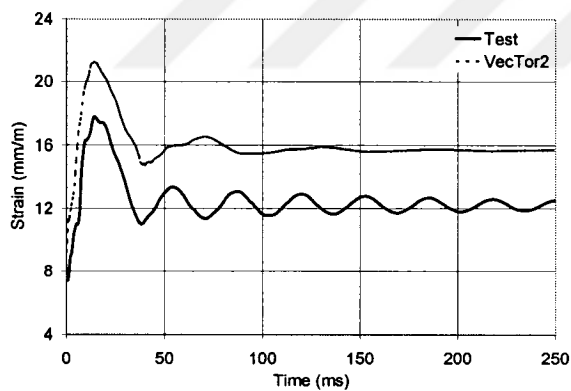


(a) Bar #3 Gauge 1 and Bar #4 Gauge 1
(mid-span)

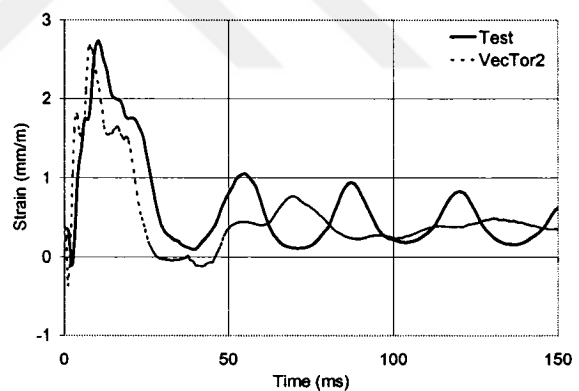


(b) Bar #4 Gauge 3
(support)

Figure 6.42 Observed and computed longitudinal reinforcement strains, SS2b-2

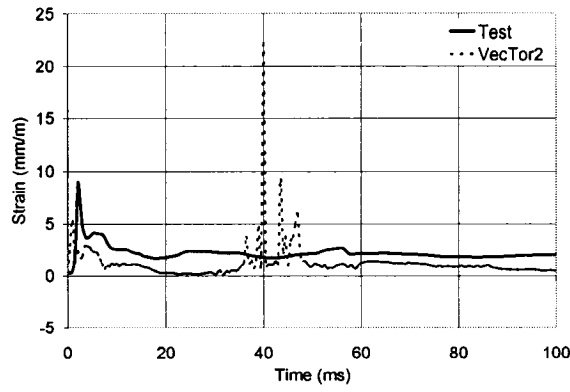


(a) Bar #3 Gauge 1 and Bar #4 Gauge 1
(mid-span)

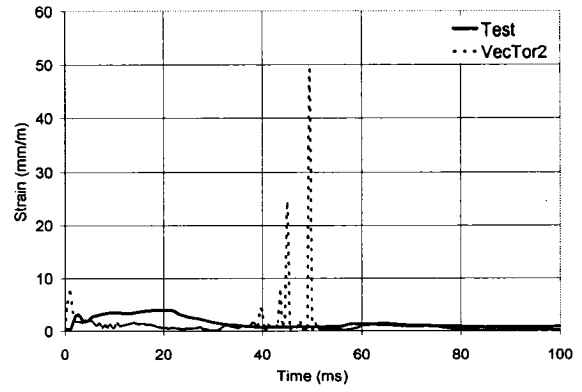


(b) Bar #4 Gauge 3
(support)

Figure 6.43 Observed and computed longitudinal reinforcement strains, SS3b-2

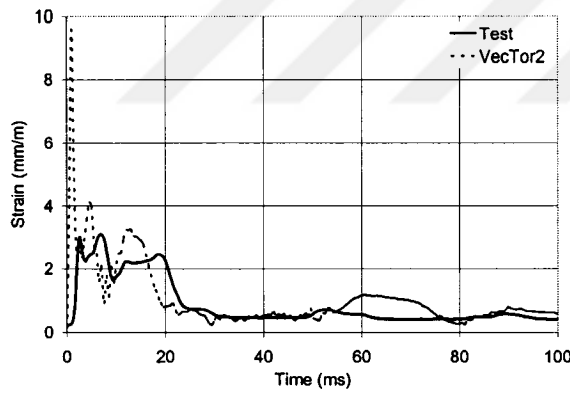


(a) Stirrup 1

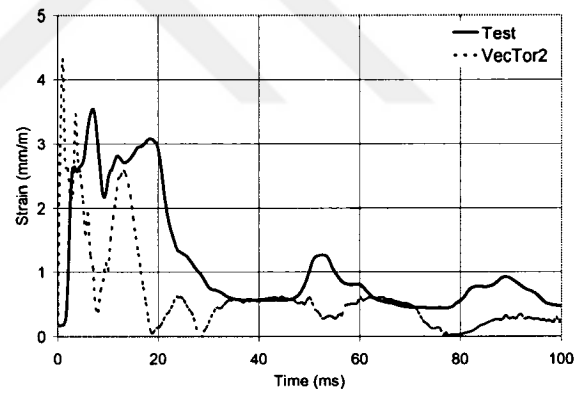


(b) Stirrup 2

Figure 6.44 Observed and computed stirrup strains, SS1a-2

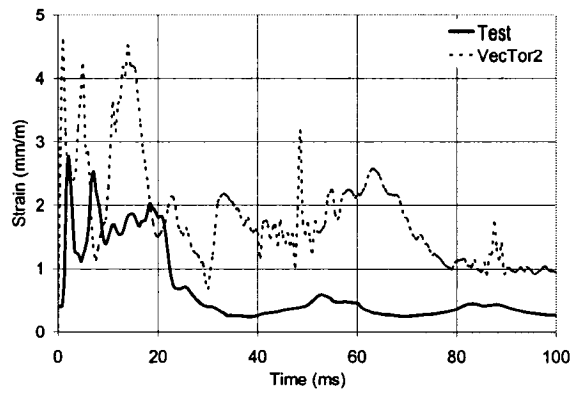


(a) Stirrup 1

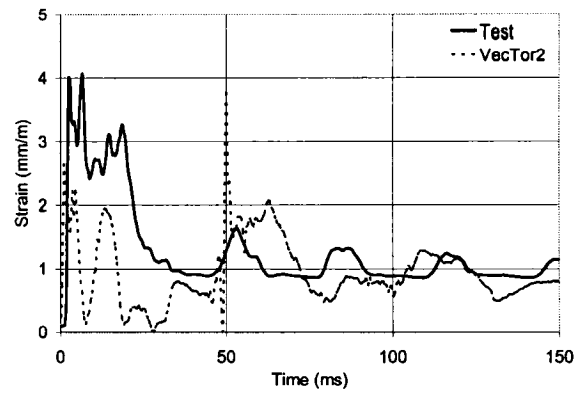


(b) Stirrup 2

Figure 6.45 Observed and computed stirrup strains, SS2a-2

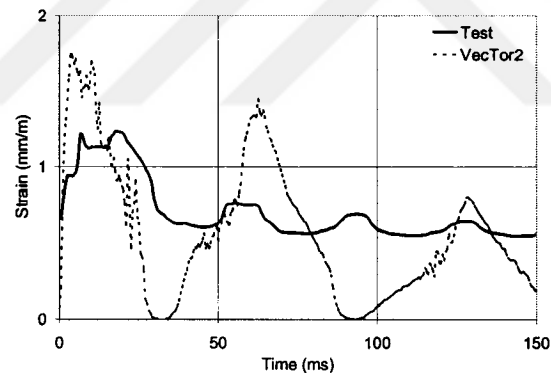


(a) Stirrup 1



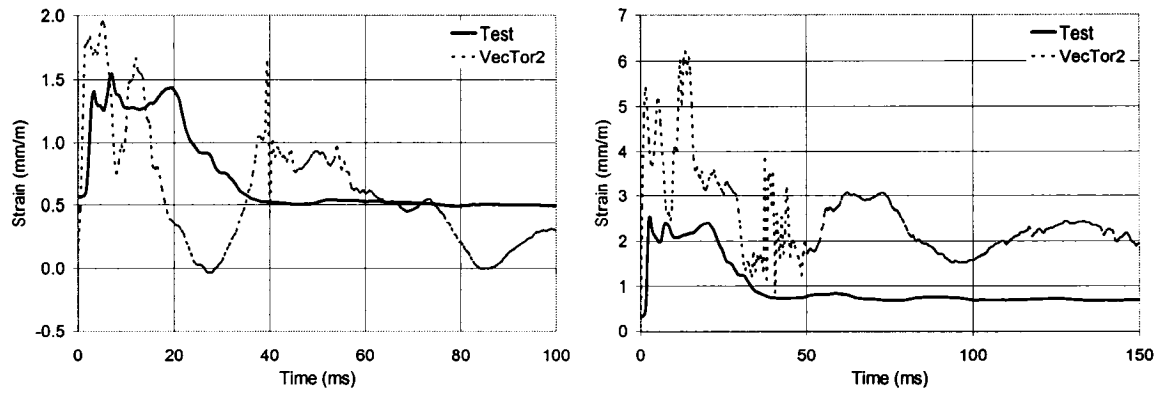
(b) Stirrup 2

Figure 6.46 Observed and computed stirrup strains, SS3a-2



(a) Stirrup 1

Figure 6.47 Observed and computed stirrup strains, SS2b-2



(a) Stirrup 1

(b) Stirrup 2

Figure 6.48 Observed and computed stirrup strains, SS3b-2

Table 6.7 Observed and computed peak longitudinal reinforcement strains

Test	Peak Mid-span Strains (mm/m)			Peak Support Strains (mm/m)		
	Test	VecTor2	Error (%)	Test	VecTor2	Error (%)
SS1a-2	8.99	14.59	62.3	1.69	2.09	23.7
SS2a-2	14.45	17.41	20.5	1.68	1.82	8.2
SS3a-2	6.07	17.16	182.9	1.71	1.91	12.0
SS2b-2	15.21	23.14	52.1	2.01	1.51	24.6
SS3b-2	17.81	21.25	19.4	2.73	2.67	2.2

Table 6.8 Observed and computed peak stirrup strains

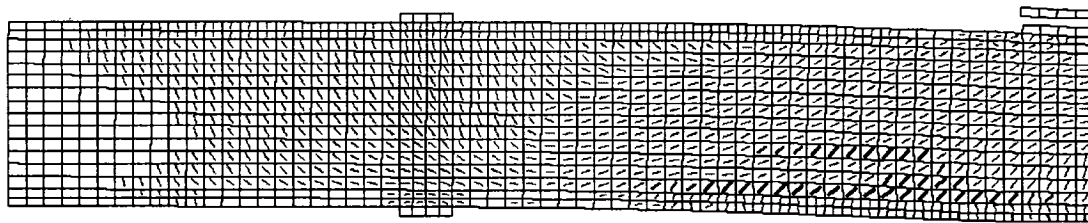
Test	Peak Stirrup 1 Strains (mm/m)			Peak Stirrup 2 Strains (mm/m)		
	Test	VecTor2	Error (%)	Test	VecTor2	Error (%)
SS1a-2	8.83	22.4	153.4	4.13	49.25	1092.3
SS2a-2	3.11	9.63	209.6	3.54	4.33	22.4
SS3a-2	2.78	4.64	67.3	4.06	3.78	6.7
SS2b-2	1.24	1.75	41.5	-	-	-
SS3b-2	1.55	1.96	26.5	2.56	6.24	144.3

As seen in the figures, VecTor2 predicted longitudinal reinforcement strains with reasonable accuracy. Strains at the mid-span were generally over-estimated. However, the general shapes of these strain responses were very similar; VecTor2 predictions were mainly parallel to the responses measured in the tests. On the other hand, VecTor2 calculations for the support strains were notably more accurate. As previously observed with the impact analyses on undamaged specimens, stirrup strains were predicted rather poorly. Unusual spikes were observed in the responses of some stirrups in SS1a-2 and SS3a-2, possibly due to local extensive deformations calculated for the reinforced concrete elements used to extract stirrup strains. Other possible causes of the discrepancies in strain responses were discussed in Section 6.4.1.2.

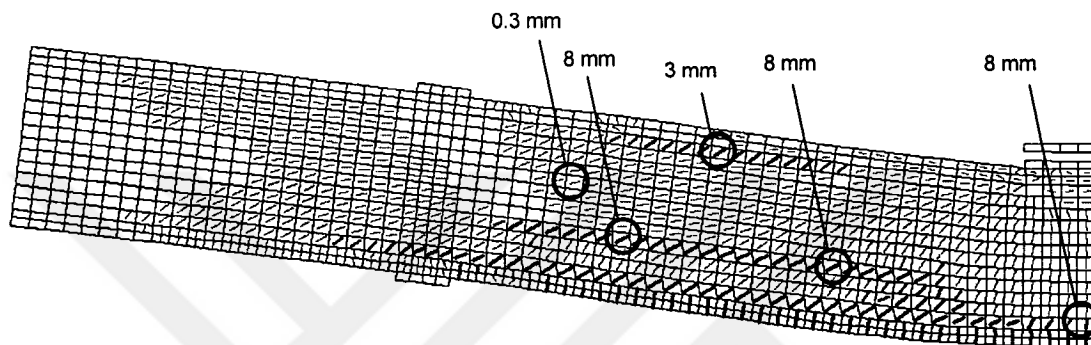
6.4.2.3 Crack Patterns

The crack patterns calculated by VecTor2 are compared in Figures 6.49 to 6.53 with the patterns obtained from the tests. Similar to the comparisons made for the impact analyses on undamaged specimens, the crack patterns calculated at the initial stages of the response, at the time the peak mid-span displacement occurred, and at the final resting stage of the specimen are presented for comparison. Typical calculated crack widths are also presented in the respective figures.

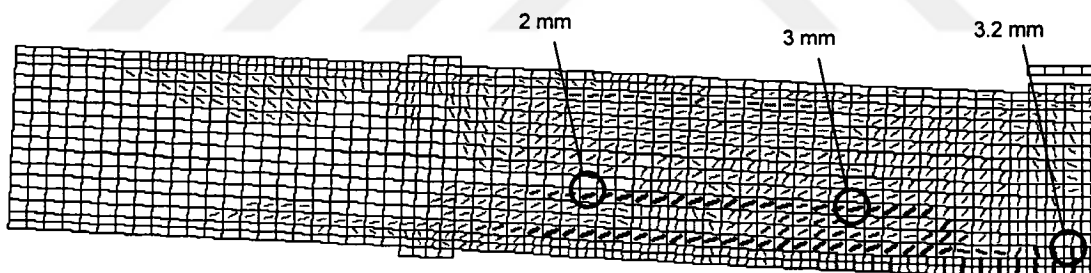
As seen in the figures, VecTor2 predicted general crack patterns well. Extensive cracking at the level of bottom reinforcement were predicted accurately; note that most specimens either developed horizontal cracks at this level, or they suffered spalling of the bottom cover concrete. Wide diagonal cracks forming the shear-plug were also predicted successfully. In the tests, it was observed that the diagonal cracks forming the shear-plug were wider close to the bottom; this also was observed in VecTor2 calculations. However, some discrepancies exist in the crack width predictions. For example, VecTor2 over-estimated the widths of the vertical cracks at the mid-span. Probable reasons of the discrepancies in crack width calculations were discussed in Section 6.4.1.3.



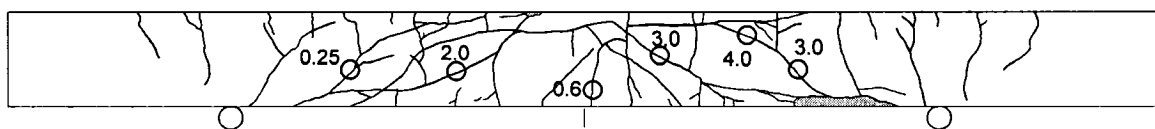
(a) Calculated crack profile at 1 ms



(b) Calculated crack profile at 16 ms (peak displacement)

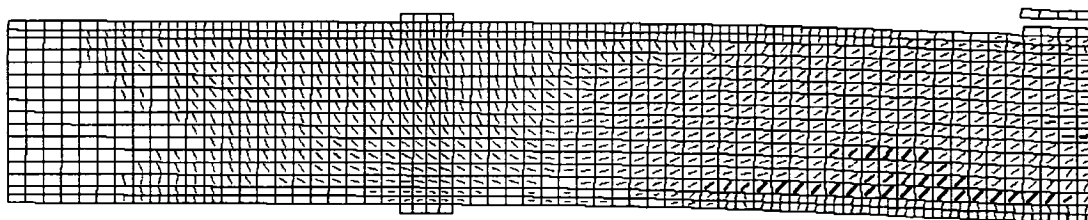


(c) Crack profile at the final resting stage

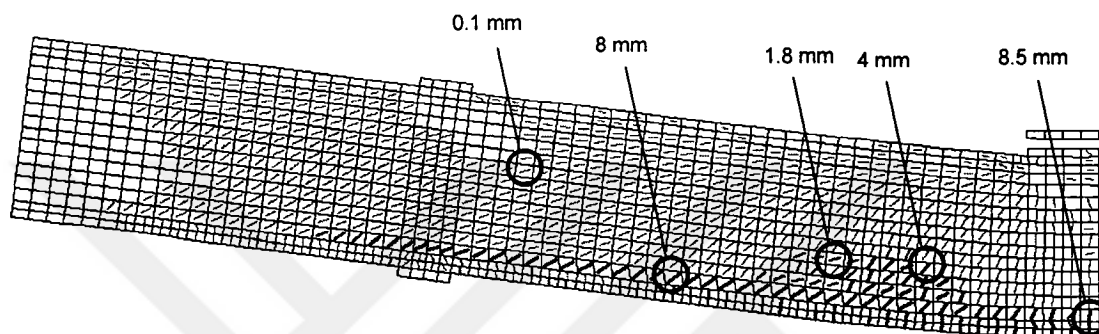


(d) Observed crack profile (west face)

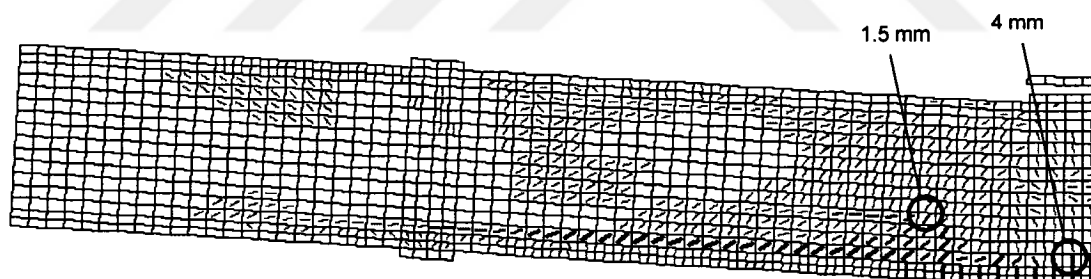
Figure 6.49 Observed and computed crack profiles, SS1a-2



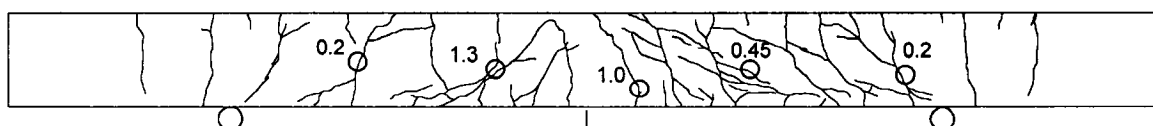
(a) Calculated crack profile at 1 ms



(b) Calculated crack profile at 14 ms (peak displacement)

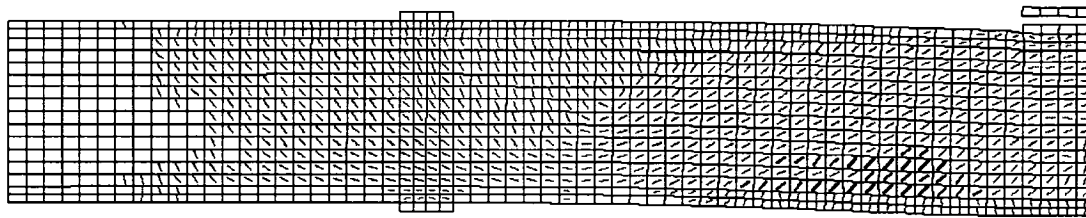


(c) Crack profile at the final resting stage

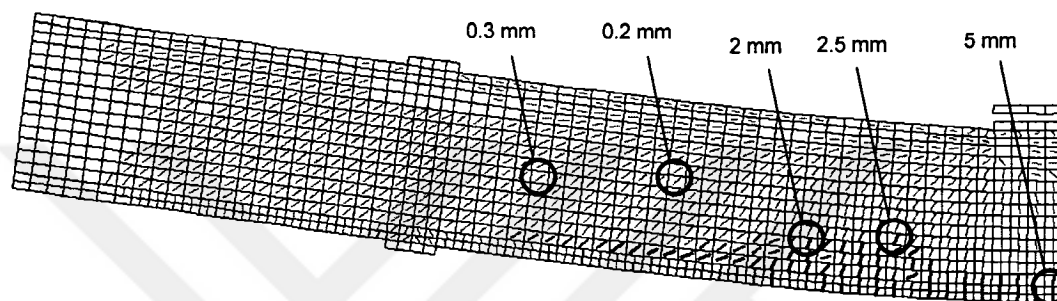


(d) Observed crack profile (west face)

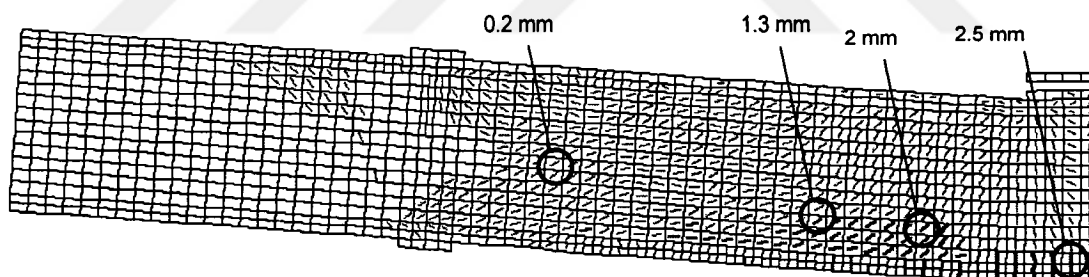
Figure 6.50 Observed and computed crack profiles, SS2a-2



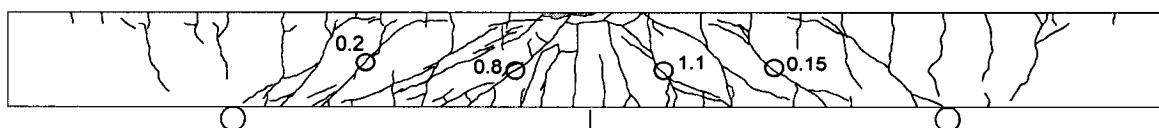
(a) Calculated crack profile at 1 ms



(b) Calculated crack profile at 14 ms (peak displacement)

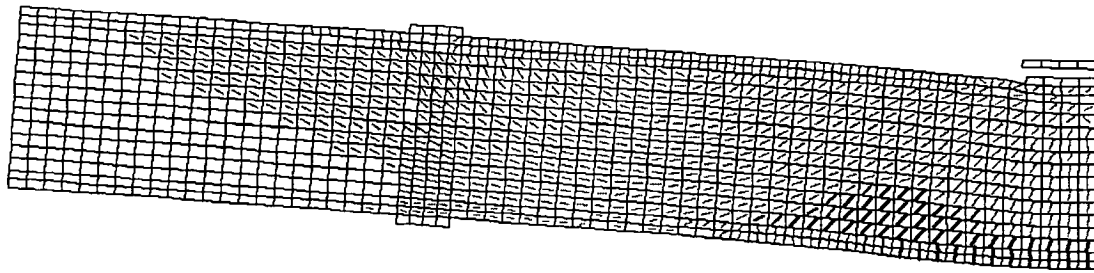


(c) Crack profile at the final resting stage

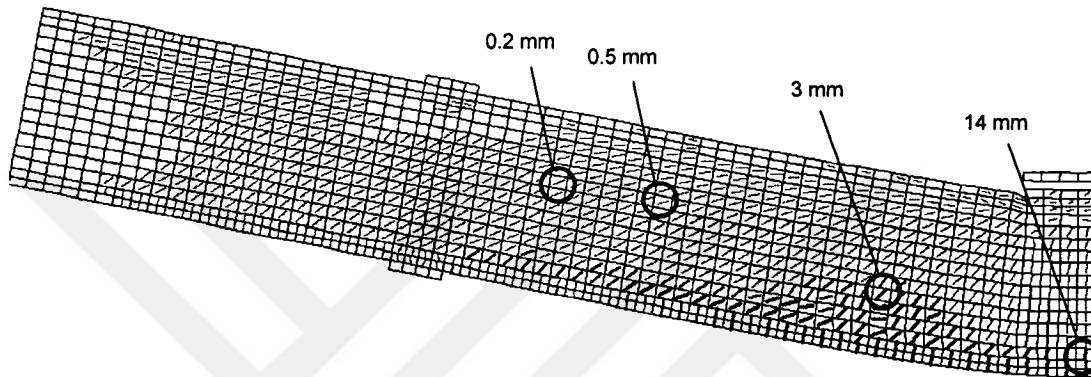


(d) Observed crack profile (west face)

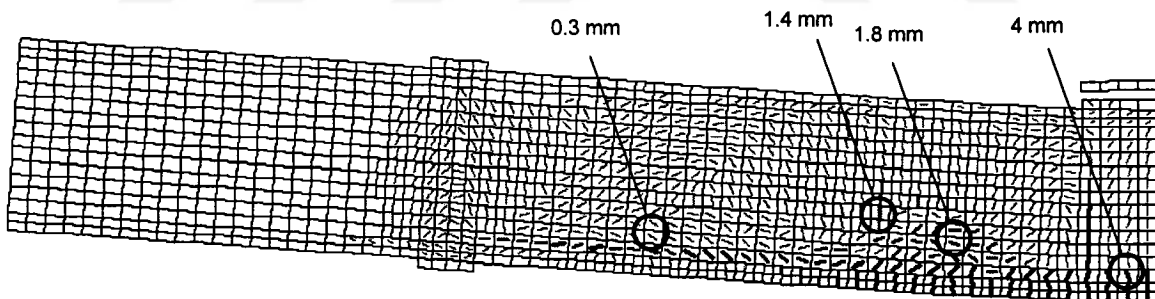
Figure 6.51 Observed and computed crack profiles, SS3a-2



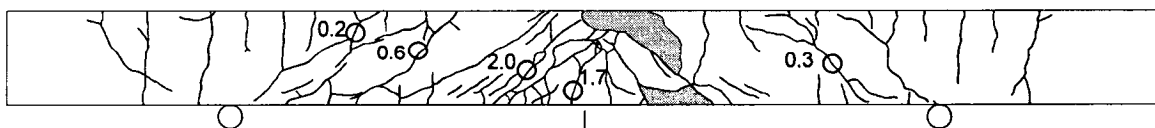
(a) Calculated crack profile at 1 ms



(b) Calculated crack profile at 15 ms (peak displacement)

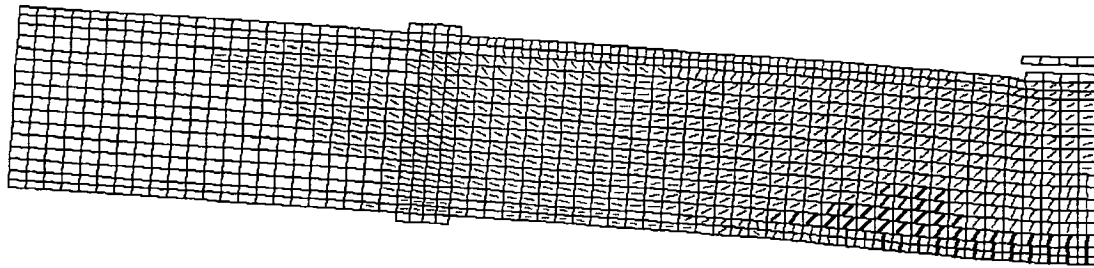


(c) Crack profile at the final resting stage

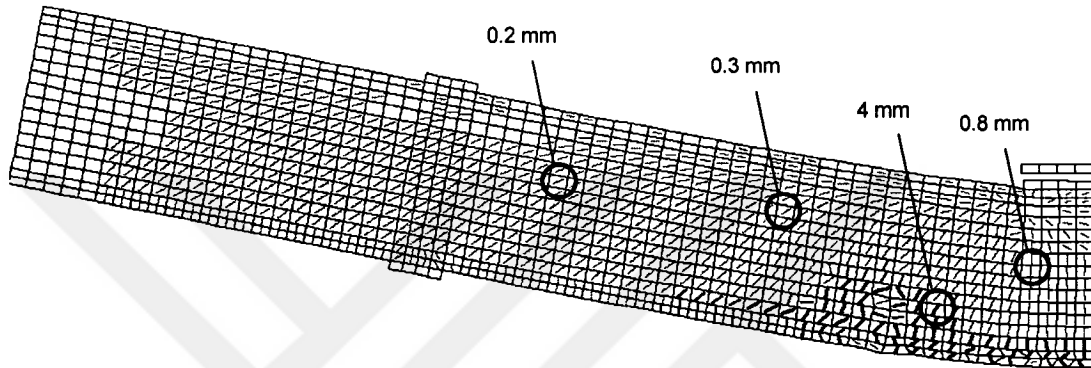


(d) Observed crack profile (west face)

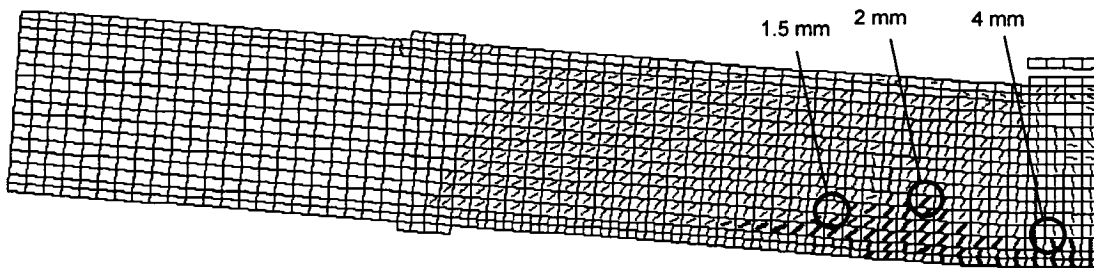
Figure 6.52 Observed and computed crack profiles, SS2b-2



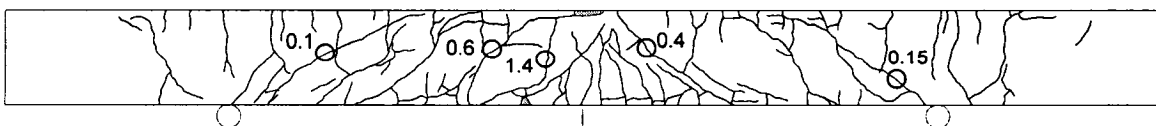
(a) Calculated crack profile at 1 ms



(b) Calculated crack profile at 15 ms (peak displacement)



(c) Crack profile at the final resting stage



(d) Observed crack profile (west face)

Figure 6.53 Observed and computed crack profiles, SS3b-2

6.4.3 Effects of Damping Parameters on VecTor2 Impact Analyses

As discussed earlier in Chapter 3, it is difficult to calculate the damping properties of a structure from its physical state, and the number of studies in this area is quite limited. This lack of productivity in investigating the detailed damping characteristics of materials has several reasons. First, for most structures that are analyzed and designed for dynamic loads, such as earthquakes, there exist numerous non-structural factors that contribute significantly to the damping mechanisms, which are highly impractical to consider in an analysis that is carried out for design purposes. In other words, even if methods for accurately determining the damping properties of a bare structure existed, it still would not be possible to model the true damping characteristics of a finished structure with non-structural components in place. Moreover, from a designer's point of view, damping is not a major factor affecting the design parameters, such as the forces in members. Considering the great levels of uncertainty in more important parameters, such as expected earthquake loads, devoting much effort on the precise calculations of a parameter that does not have a significant influence on the final design is not practical. Therefore, in practice, rough estimates of damping ratios, usually recommended by design codes, are used for the structural analyses carried out for design purposes. However, when the complete vibration history of a structure under a dynamic load is sought, damping has an undeniable effect on most analytical methods, since it may influence the displacement amplitudes significantly. Therefore, sophisticated computational methods developed to simulate complete vibrational histories require educated estimates and detailed modelling of damping characteristics, rather than vague approximations. Researchers, who utilize such computational methods, have followed several strategies for obtaining an accurate modelling of damping in concrete structures. One approach to solving the problem is to measure the damping characteristics of the structure using techniques similar to the one described in Section 5.9, and incorporate the measured damping into the model by using one of the methods summarized in Section 3.2.2. This approach is, of course, only applicable when the analysis of an existing structure is required. As an example, Inoue et al. (1997) followed this approach to simulate the vibrations of a shear wall subjected to base accelerations. Another method incorporating damping is to include energy dissipating mechanisms, such as material yielding and hysteresis, in the structural model. Although this approach may not completely eliminate

the need to include viscous damping in the structural model, it reduces the sensitivity of the analysis results to the assigned viscous damping ratios, since the majority of the damping mechanisms are partially modelled through more accurate material modelling. Filippou et al. (1992), for example, developed a finite element model for the analysis of reinforced concrete subassemblages using this method. In their model, they included plasticity and hysteresis models for concrete and steel, and introduced additional viscous damping by using Rayleigh method, as described in Section 3.2.2.

In this current study, the second approach was preferred, as discussed in Section 3.2.2. Since VecTor2 was capable of modelling the majority of energy dissipating mechanisms, only a minimal amount of viscous damping was needed to stabilize the numerical solutions. Before finalizing the analyses, a parametric study was carried out to determine the damping ratios to be assigned to the vibrational modes. The damping ratios for the first and second modes of vibration were continuously reduced, until the solutions lost their stability. The smallest amount of viscous damping that resulted in a stable solution was selected for the final analyses of the test specimens. These values are summarized in Table 6.1. Note that the viscous damping ratios assigned to the first and second modes had to be increased for the second impact tests, possibly as a result of the damage suffered in the first impact test. Similarly, specimens with lower strength, such as SS0a and SS2b in the second impact test, had to be assigned damping values higher than those used for the other specimens in the same group, in order to ensure a stable solution.

Table 6.9 Damping properties used in the analyses

Specimen	First Impact Test		Second Impact Test	
	1 st Mode Damping (% of critical)	2 nd Mode Damping (% of critical)	1 st Mode Damping (% of critical)	2 nd Mode Damping (% of critical)
SS0a	1.00	5.00	-	-
SS1a	0.01	1.00	0.10	1.00
SS2a	0.01	1.00	0.10	1.00
SS3a	0.01	1.00	0.10	1.00
SS1b	0.01	0.50	-	-
SS2b	0.01	0.50	0.10	5.00
SS3b	0.01	0.50	0.10	3.00

The effects of damping parameters on the solutions are presented in Figures 6.54 and 6.55, for SS2b-1 and SS3b-1, respectively. Note that the legends in these figures show the damping ratios assigned to the first and second modes. As can be seen in the figures, low levels of damping rendered the solutions unstable, causing an unbounded increase of displacements; a stable solution could only be achieved by increasing the viscous damping. Increasing the damping further beyond this stability limit simply decreased the displacement response. This method of introducing damping was found to be quite effective and accurate, especially considering that the analyses carried out with these limiting damping ratios also resulted in the best estimates of the actual response.

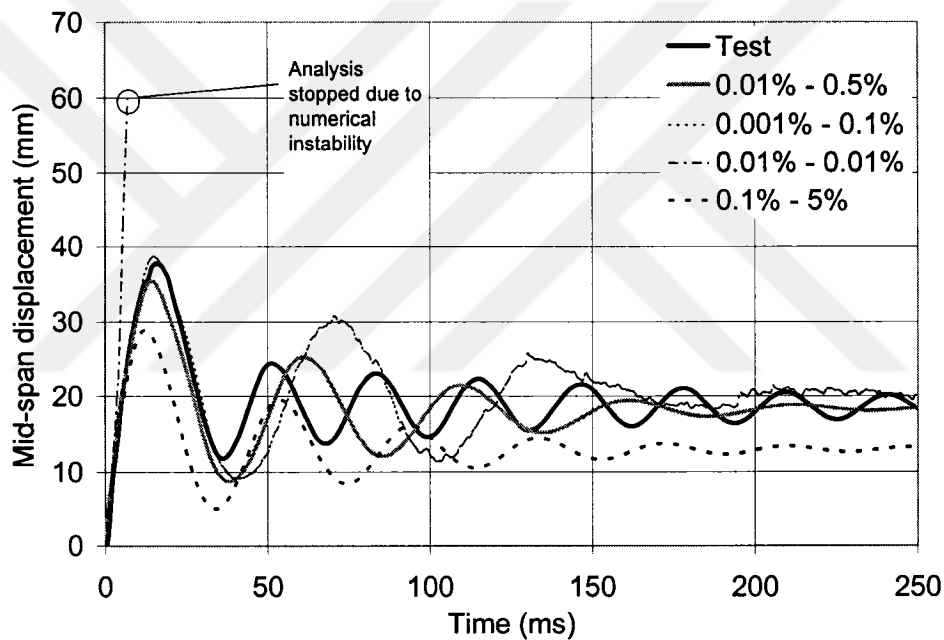


Figure 6.54 Effect of damping on computed response of SS2b-1

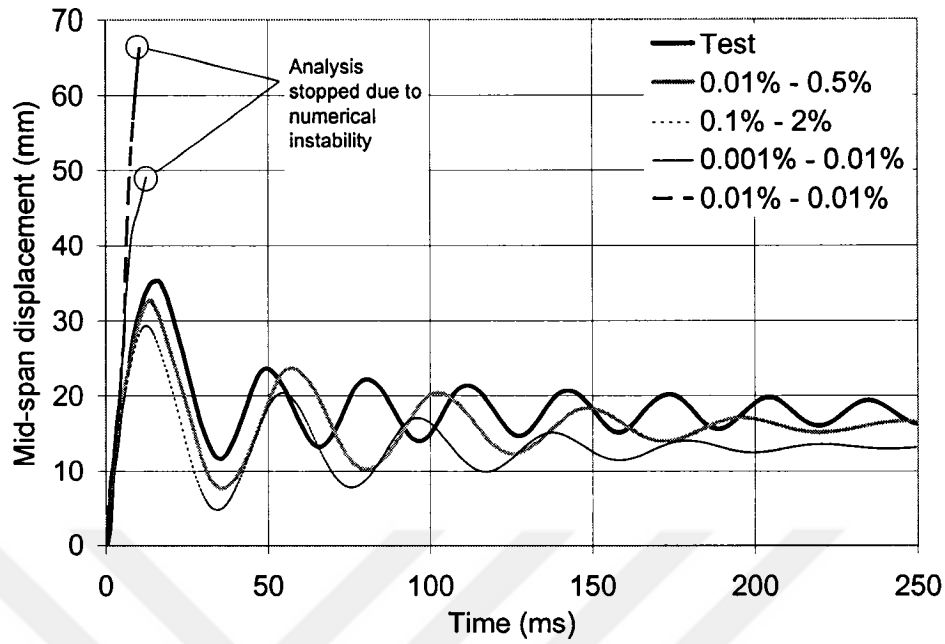


Figure 6.55 Effect of damping on computed response of SS3b-2

6.4.4 Effects of Time-Step Size on VecTor2 Impact Analyses

As mentioned previously, the time-step size for numerical integration was selected as 0.0001 seconds for all analyses. The effects of the time-step size on the accuracy of the analyses were investigated by varying the time-step size for some of the analyses. Results obtained by using different time-step sizes for the analysis of SS3b-1 are presented in Figure 6.56.

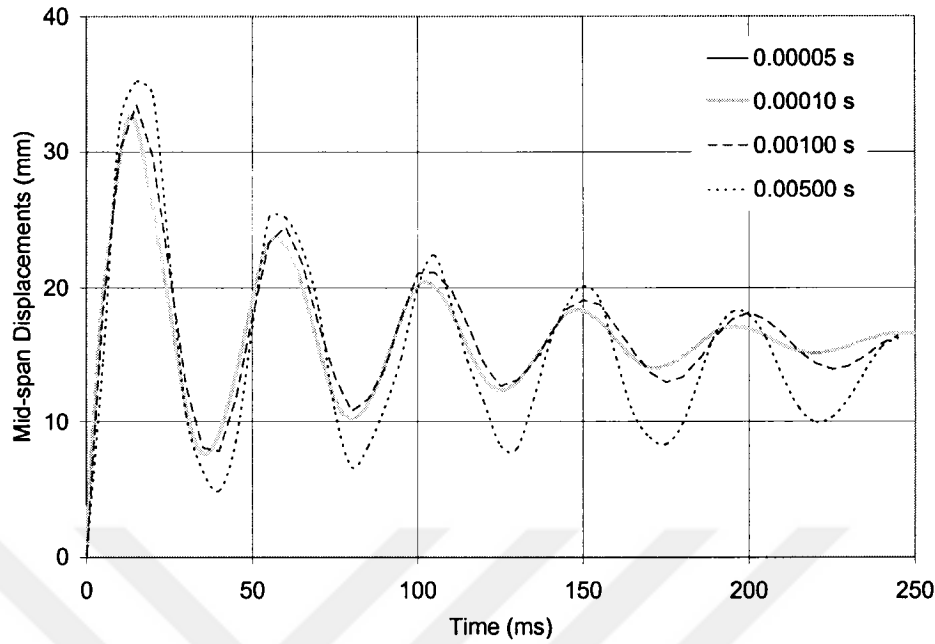


Figure 6.56 Effect of time-step size on the response, SS3b-1

As can be seen from the figure, the analyses carried out with a time-step half the size of the original did not result in any significant change in the response, an indication that the selected time-step size was sufficiently short for an accurate solution. Larger time-step sizes, on the other hand, significantly changed the results, causing increased displacement amplitudes and accumulating errors. Analyses presented in Figure 6.56 were carried out with an Intel® Pentium IV® computer with a 2.26 GHz CPU speed and 512 MB of RAM. Elapsed times required to complete 250 ms long analyses were recorded to investigate the effect of time-step size on the computation time; these are summarized in Table 6.10. As can be seen from this table, although shorter time-steps required a larger number of steps to complete an analysis for a given time duration, the total time elapsed did not increase by the same ratio, since the iterations converged faster with a shorter time-step.

Table 6.10 Computation times for the analyses

Time-step Size	Number of Time Steps	Time Elapsed
0.00005 s	5000	7 hours 10 minutes
0.00010 s	2500	5 hours 48 minutes
0.00100 s	250	43 minutes
0.00500 s	50	17 minutes

It should be noted that, as mentioned in Chapter 3, an energy balance check performed at every time-step is a faster and more accurate method to determine if the selected time-step is sufficiently short for an accurate analysis. VecTor2 was adapted with the energy balance calculation algorithms as described in Section 3.3.2. However, the presence of very stiff or rigid elements, such as the rigid rectangular elements used to model the drop-weight or the rigid truss-bar elements connecting the drop-weight to the specimen, resulted in extremely high energy terms in the overall energy balance check, since very small changes in the strains calculated for these elements created high energy values. A close examination of the forces and deformations of these elements revealed that these high energy terms were created since, although they were assigned with a very high stiffness compared to the structure, these very stiff elements were not perfectly rigid. It was found that, for an accurate calculation of the energies dissipated by the structure and the damping forces, these rigid elements should be excluded from the structure stiffness matrix and damping matrix when performing the energy balance check. However, such additional steps needed extra storage, user input, and computation time; hence, the energy balance check was not performed for the analyses presented in this chapter. Note that assigning very high stiffness values to rigid elements yielded results of sufficiently good accuracy when the overall impact response of the structure was concerned. On the other hand, further studies are required to implement alternative ways of introducing elements intended-to-be-rigid into VecTor2 models to facilitate the energy balance calculations.

6.5 Conclusion

As discussed in Chapter 3, one of the main goals of this study was to develop a numerical tool for the impact analysis of reinforced concrete structures, by modifying a nonlinear finite element analysis program originally developed for the static analysis of reinforced concrete structures. VecTor2 has shown considerable success over the years in modelling the static behaviour of reinforced concrete, especially in problematic cases such as with shear-critical structures. A successful adaptation of this program for dynamic loads would provide an accurate and reliable analysis method for reinforced concrete structures under impact loads.

Although some deficiencies were discovered, comparisons between the VecTor2 simulations and the measurements obtained from the test program showed that this goal was largely achieved. The especially good performance of VecTor2 in modelling the impact response of damaged specimens, such as in the case of second impact tests, was remarkable, considering that very few methods in the literature are suitable for such an analysis. Modelling of energy dissipating mechanisms in VecTor2 were also satisfactory to a certain degree, since the addition of only a small amount of viscous damping was sufficient to stabilize the numerical solutions and obtain accurate responses. It should be noted that similar finite element methods may require much higher viscous damping, sometimes up to 5%, for stable and accurate solutions even though they include other energy dissipating mechanisms such as concrete hysteresis (Filippou et al. 1992). On the other hand, these analyses also revealed that certain behavioural models regarding the material hysteresis should be improved for high strain rate loads, since the post-peak vibrations obtained from the analyses showed relatively poor correlations with the test results. Problems regarding the modelling of supports and calculation of reaction forces should also be addressed, although they are more likely to be numerical problems rather than deficiencies in the methodology.

7 CONCLUSIONS

The study presented in this thesis had several focus points. The main objective was to develop a computational tool for the analysis of reinforced concrete structures under impact loads. For this purpose, a sophisticated two-dimensional nonlinear finite element program developed earlier at the University of Toronto, VecTor2, was modified to include the consideration of dynamic loads. Another focus of this study was to devise a comprehensive test program, not only to verify the newly adapted dynamic analysis routines of VecTor2, but also to supply the literature with detailed, well-instrumented test data that would contribute to our understanding on the behaviour of reinforced concrete structures under impact loads.

The test program carried out in this study was successful in providing a substantial amount of high quality impact test data. Eight reinforced concrete beam specimens, four pairs, were tested under free falling drop-weights, impacting the specimens at the mid-span. All specimens had identical longitudinal reinforcement, but varying shear reinforcement ratios, intended to investigate the effects of shear capacity on the impact behaviour. The specimens were well-instrumented; displacements, strains on reinforcing bars, support reactions and accelerations along the beams were recorded with 49 sensors. A total of 20 tests were conducted, including multiple tests on each specimen. Observations made during the tests, and examination of the data collected, led to the following conclusions:

- (i) The shear characteristics of the specimens played an important role in their overall behaviour. All specimens, regardless of their shear capacity, developed severe diagonal shear cracks, forming a shear-plug under the impact point. Specimens with higher shear capacity were able to sustain more impacts and absorb more energy, whereas the ones with lower shear capacity suffered extensive damage under the same or smaller impact loads. Thus, methods developed to predict the response of reinforced concrete under impact loads must consider shear mechanisms for accurate modelling. In other words, even if the member is flexure-critical under static load conditions, ignoring shear

mechanisms with methods that assume flexural behaviour can lead to significant errors in the computed dynamic responses.

- (ii) As a result of the presence of overhanging parts, and the formation of shear-plugs, the displaced shapes of the test specimens did not fit into single degree of freedom behaviour. Comparisons of the measured displaced shapes with the elastic shape revealed severe discrepancies as well. Therefore, simplified single degree of freedom methods, developed for impact analysis and employing elastic displaced shapes to derive the deformation characteristics of a structure, should be used with caution, especially for the structures with little or no shear reinforcement.
- (iii) Because of the highly dynamic nature of the responses, impact forces at the initial stages of response were mainly resisted by the inertia of the specimens, before the forces reached the supports. Therefore, the mass and geometric properties of a structure, such as the span length of a beam, are important factors in resisting the impact forces. In other words, it is not possible to estimate the impact capacity of a beam accurately based only on its cross-sectional properties.
- (iv) Strain rates observed in the tests, as measured by strain gauges fitted to the longitudinal reinforcing bars, attained values of up to 7.3 s^{-1} . Levels of strain rates varied according to the level of deformations in the bars; that is, yielded bars recorded higher strain rates. Moreover, strain rates varied extensively during the response. Therefore, formulations developed to predict the response of steel or concrete under high strain rate conditions may not accurately represent the structural behaviour under impact loads, since they are mostly derived for constant strain rate conditions.
- (v) The apparent damping ratios, measured on one of the test specimens prior to and after an impact test, were calculated as 1.4% and 2.3%, respectively. This was

consistent with the common knowledge that damping increases when a structure suffers damage.

The test specimens were analyzed with VecTor2 for the applied impact loads. A finite element mesh, consisting of 1098 nodes, 1008 rectangular concrete elements, and 124 truss bar elements was utilized for the analyses. The drop-weight, modelled with four rigid rectangular elements, was connected to the elements representing the specimen with five compression-only rigid truss bars, so that it would not apply any pulling forces on the specimens when it bounced back. The impact load was introduced to the model by assigning the impact velocity of the drop-weight to the nodal masses representing the drop-weight. The following conclusions were reached after comparing the analyses results with the test results:

- (i) VecTor2 performed well in predicting damage levels, maximum displacements, and residual displacements. The methodology employed by VecTor2, based on the Modified Compression Field Theory and the Disturbed Stress Field Model, proved to be successful in predicting the shear-dominant behaviour of the specimens under impact.
- (ii) The dynamic analysis algorithm implemented into VecTor2 performed well. The modification of Newmark's direct integration method, to employ secant stiffness properties instead of tangential properties, was successful.
- (iii) Post-peak vibration characteristics, compared between the test results and the VecTor2 analyses results, showed some discrepancies. These discrepancies were mostly attributed to the deficiencies regarding the modelling of hysteresis behaviour of concrete and steel under high strain rate conditions.
- (iv) The support reactions were poorly predicted by VecTor2. This was most likely a result of numerical problems stemming from inadequate modelling of the support conditions.

- (v) VecTor2 was successful in predicting the response of the damaged specimens for the second impact tests on the same specimens. Agreement between the test and analysis results for the peak and residual displacements was strong. This indicates that VecTor2 is capable of accurately modelling the reduced stiffness of a reinforced concrete structure after it suffers considerable levels of damage.
- (vi) Attempts to model the test specimens for the third impact tests on the same specimens failed because of numerical convergence problems with VecTor2. Considering the very high levels of damage that the specimens typically suffered during the third impact, VecTor2 was found to have deficiencies regarding modelling a state of damage beyond a certain degree.
- (vii) The simplified approach followed for modelling the drop-weight and impact load proved to be successful. This approach completely eliminated the need for an impact force history. Predetermination of impact force history in an impact event generally requires extremely sophisticated methods; however, the only required parameters for the impact analysis with VecTor2 were the drop-weight mass and its impact velocity, which are usually known parameters for design problems.
- (viii) VecTor2 successfully modelled the energy dissipating mechanisms. Only a slight amount of viscous damping, incorporated in the Rayleigh method used, was needed to numerically stabilize the solutions. The majority of the damping was supplied by other behavioural mechanisms such as material hysteresis and concrete cracking. The fact that externally assigned viscous damping had little role in the calculated response is important, since it is difficult to estimate the damping characteristics of a structure. Successful modelling of energy dissipating mechanisms by VecTor2 eliminated an uncertain parameter from the computer model.
- (ix) VecTor2 was computationally efficient. The time required to complete an analysis was typically significantly shorter than with some other well-known

sophisticated finite element programs. Using short time steps for improved accuracy did not have a severe adverse effect on computational times, since iterations converged faster with shorter time steps.

Overall, VecTor2 was found to be an efficient and accurate tool for the analysis of reinforced concrete structures under impact loads. Nevertheless, the following recommendations for future research are made with the aim of further improvement:

- (i) The hysteresis models for concrete and steel in VecTor2 should be improved for the consideration of strain rate effects. Experimental work would be necessary for developing improved constitutive relations in this area.
- (ii) The inclusion of high strain rate effects within shear crack formulations, such as those used to estimate crack slip deformations and crack shear stresses arising from aggregate interlock, should be investigated. Better predictions of these behaviours under high strain rates would contribute to improved modelling of the shear-plug, and enhance the overall prediction of the response under impact loads.
- (iii) Numerical problems regarding the support modelling should be addressed, and alternative ways to model the support conditions should be investigated.
- (iv) The constitutive relations can be improved with respect to modelling high damage levels. Alternatively, some advanced numerical methods, such as the ones that employ advanced techniques of damage modelling, can be investigated for their adaptability into VecTor2.
- (v) Methods for modelling of soft and oblique impacts should be explored for a wider coverage of impact load types.
- (vi) Adaptation of dynamic analysis capabilities into the three-dimensional counterparts of VecTor2, namely VecTor3 and VecTor4, would expand the

applicability of the methods employed by VecTor2 to the impact analysis of a wider array of structures.

It should be noted that the focus in this study was not towards developing a design procedure for the reinforced concrete structures against impact loads; hence, little effort was applied in that direction. However, the experimental program undertaken revealed some useful information, and raised some interesting questions, that could provide a good basis for future efforts to develop impact design procedures. In light of the experience gained in this test program, the following recommendations can be made for future experimental studies to be carried out for this purpose:

- (i) A test program aimed at determining the impact capacity of reinforced concrete structures should include a significantly higher number of specimens, to cover a wider range of parameters that could affect the overall behaviour.
- (ii) The effects of various impact parameters, such as mass and velocity of the missile, should be investigated by using drop-weights with different masses and a wider range of impact velocities.
- (iii) The effects of the structural mass and geometry should be investigated by testing beam specimens with different spans. Accelerometers should be closely spaced in order to accurately determine the inertia forces acting on the specimen.
- (iv) Measurements made on such impacts test should be recorded with a higher sampling rate, preferably in the several hundred kilohertz range.
- (v) Displacement measurements should be made with more advanced methods, such as with optical techniques, to prevent the loss of important data due to concrete spalling, observed in majority of the tests in this test program.

- (vi) More advanced instrumentation, such as a high capacity load cell attached on the drop-weight, should be utilized for more reliable measurements of the impact forces.
- (vii) High-speed cameras should be utilized for a better understanding of the crack formation and propagation mechanisms.



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APPENDIX A

MATERIAL PROPERTIES OF TEST SPECIMENS



A.1 Concrete Properties (December 12, 2005 Cylinder Tests)

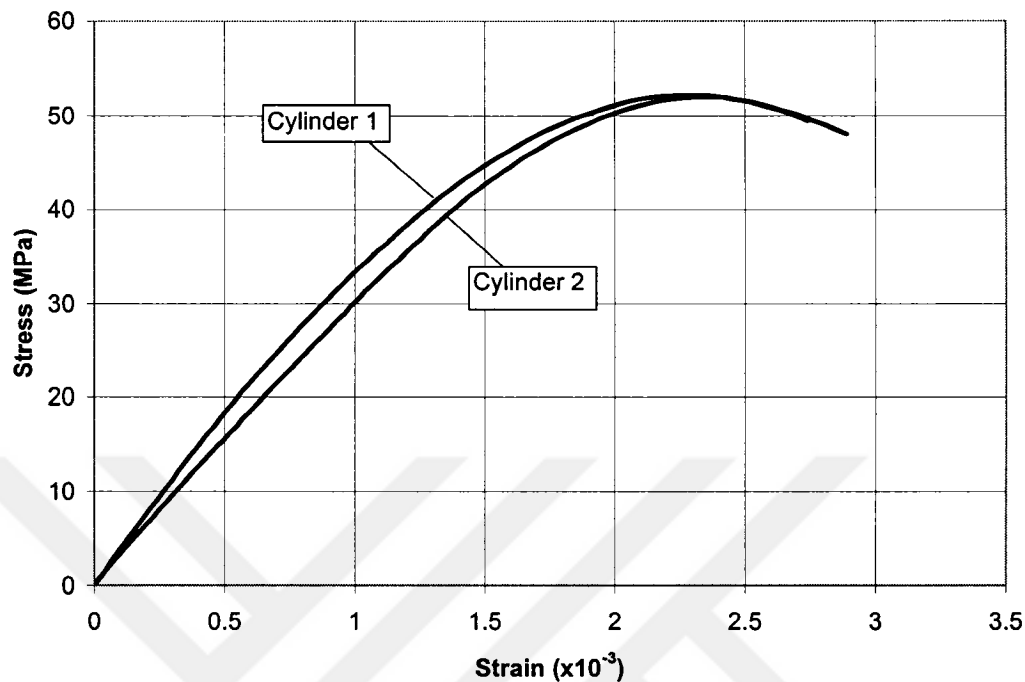


Figure A.1 Concrete stress-strain curves for SS0a and SS0b

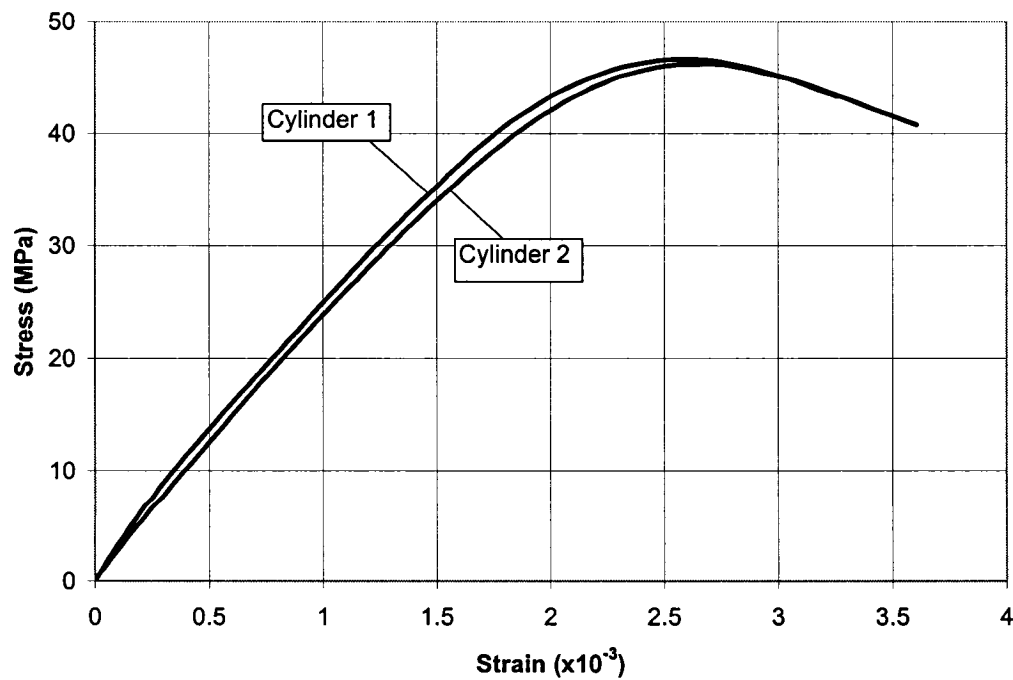


Figure A.2 Concrete stress-strain curves for SS1a and SS1b

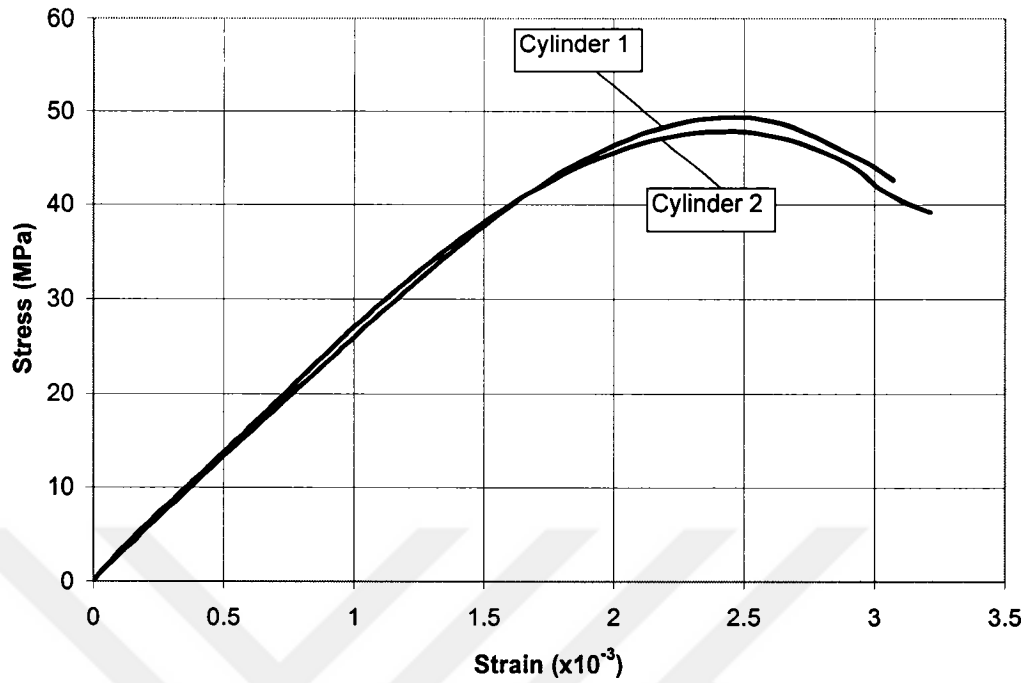


Figure A.3 Concrete stress-strain curves for SS2a and SS2b

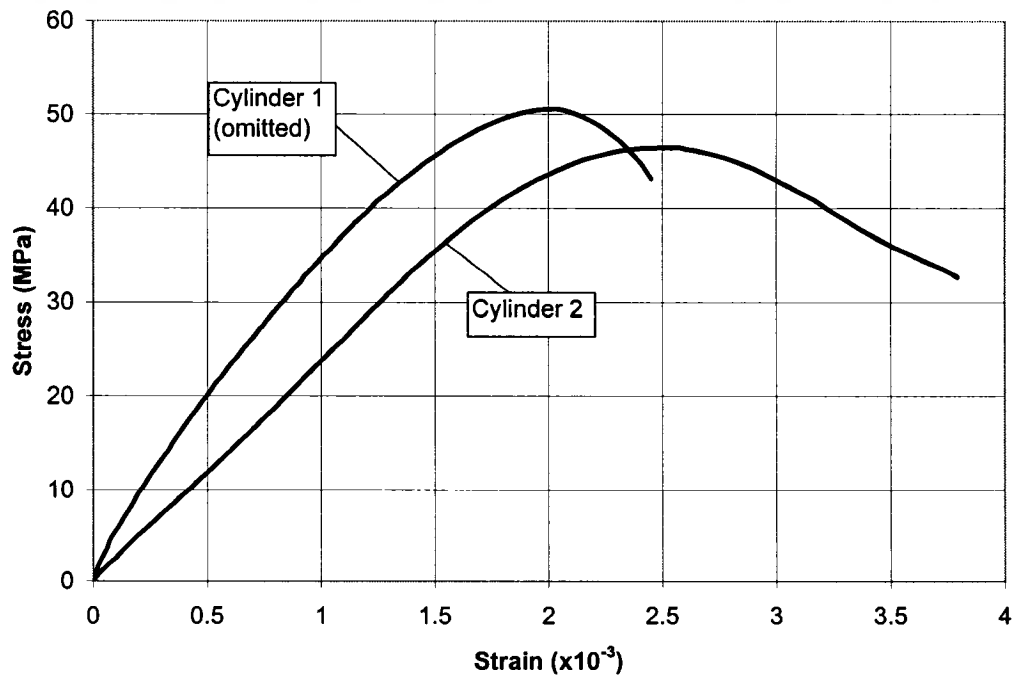


Figure A.4 Concrete stress-strain curves for SS3a and SS3b

A.2 Steel Bar Properties

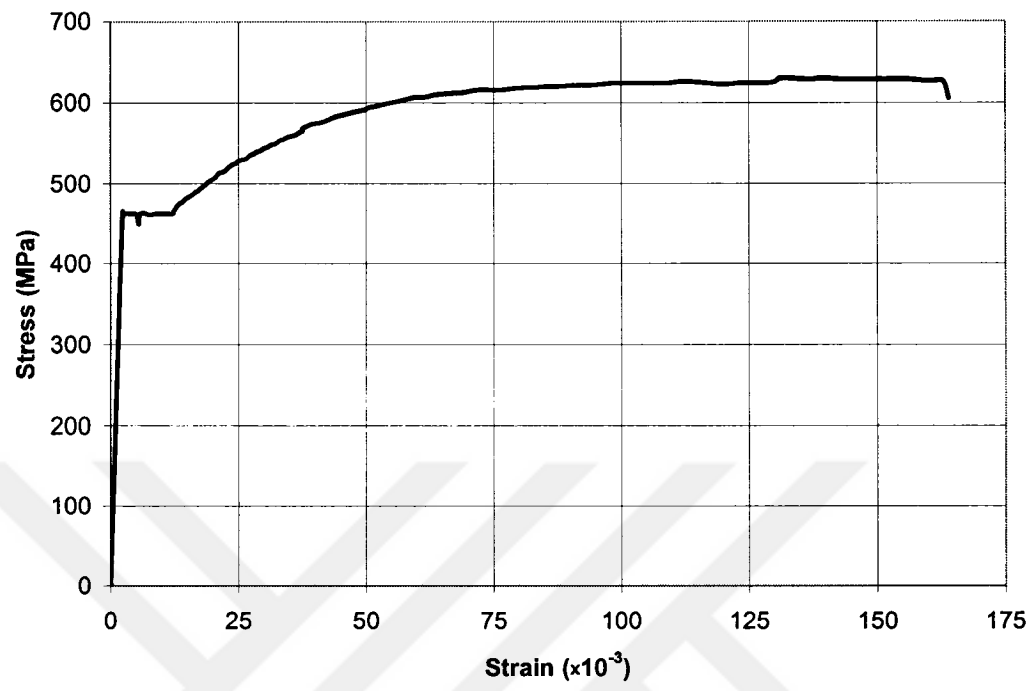


Figure A.5 Stress-strain curve for No.30 steel bars

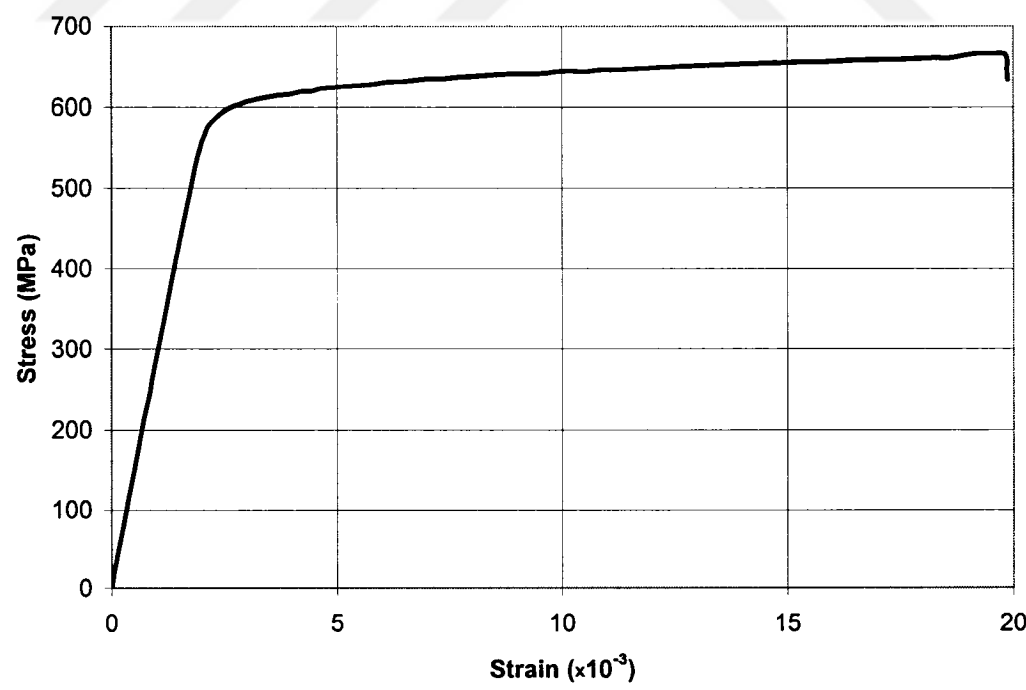


Figure A.6 Stress-strain curve for D-6 steel bars

A.3 Support Bar Calibration Results

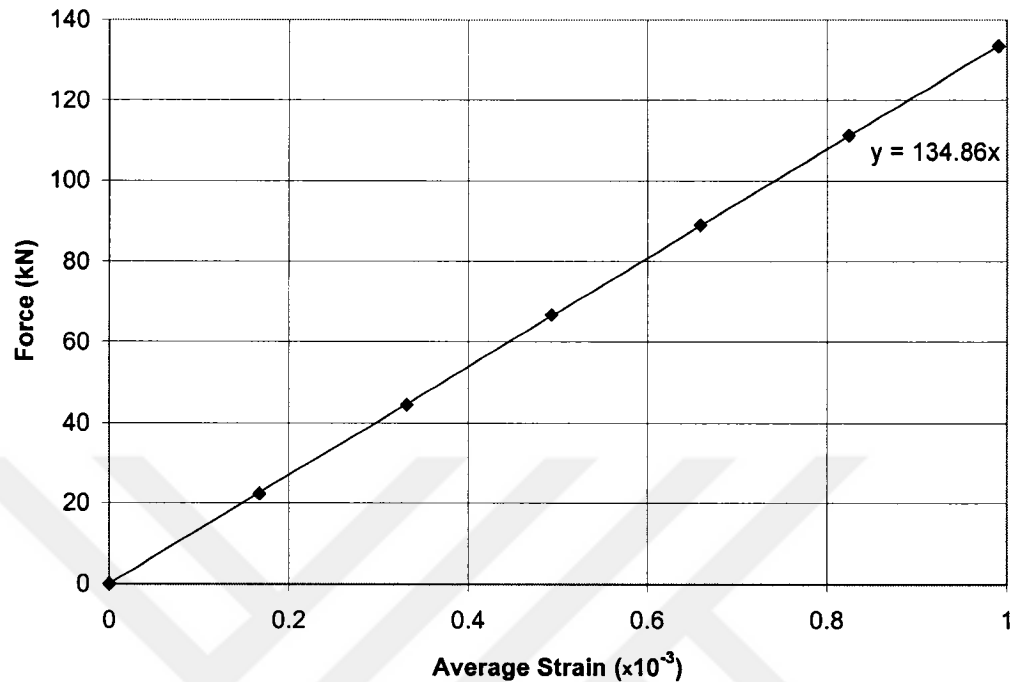


Figure A.7 Support Bar #1 calibration (south support)

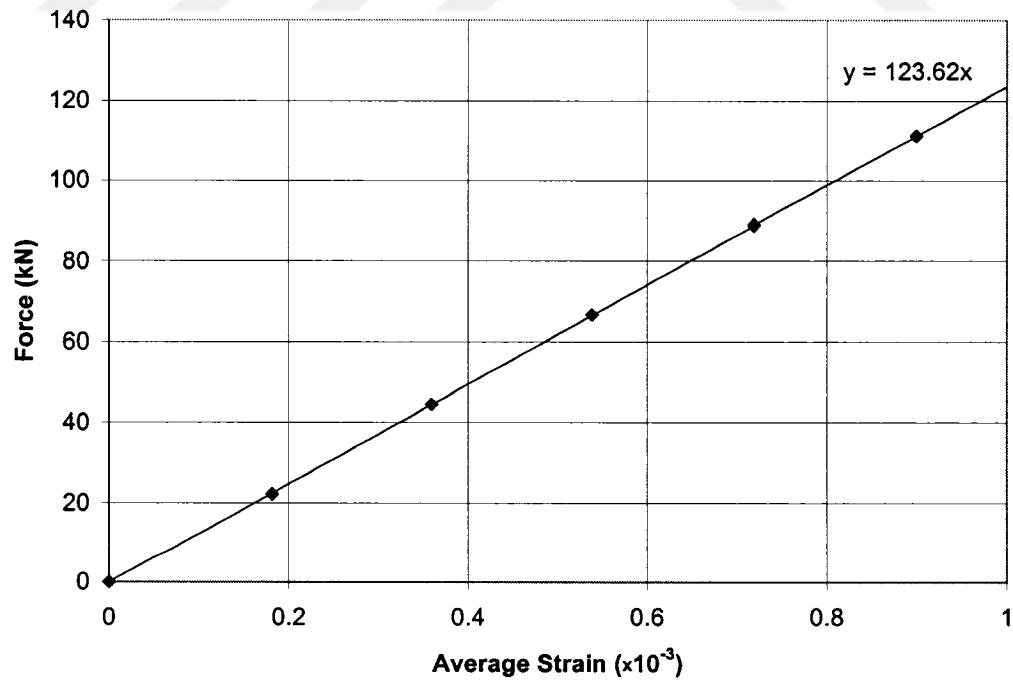


Figure A.8 Support Bar #2 calibration (south support)

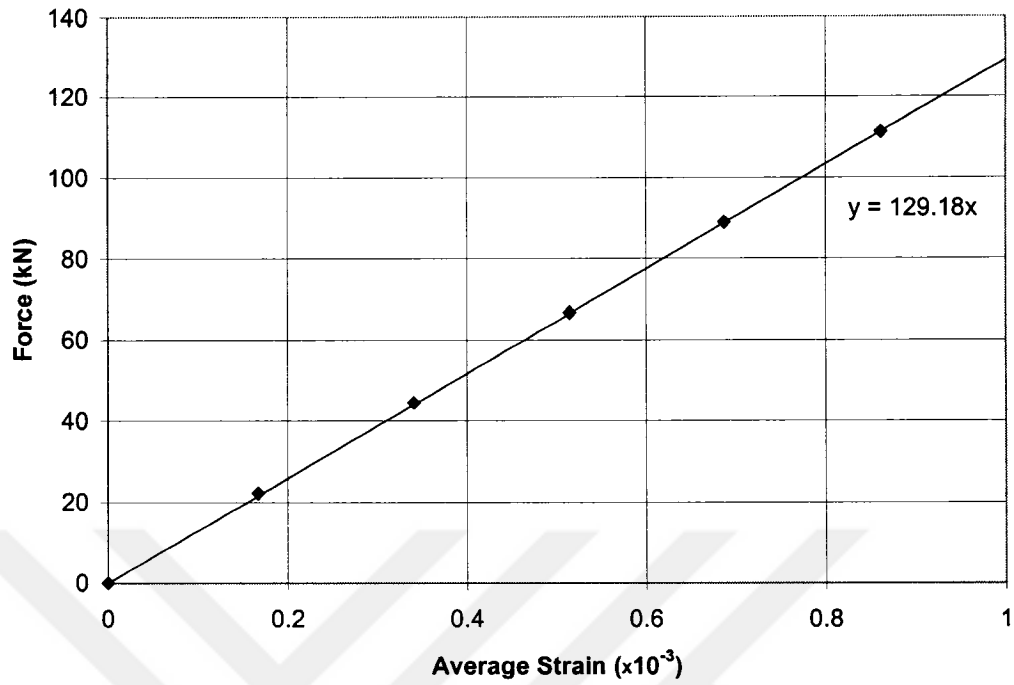


Figure A.9 Support Bar #3 calibration (north support)

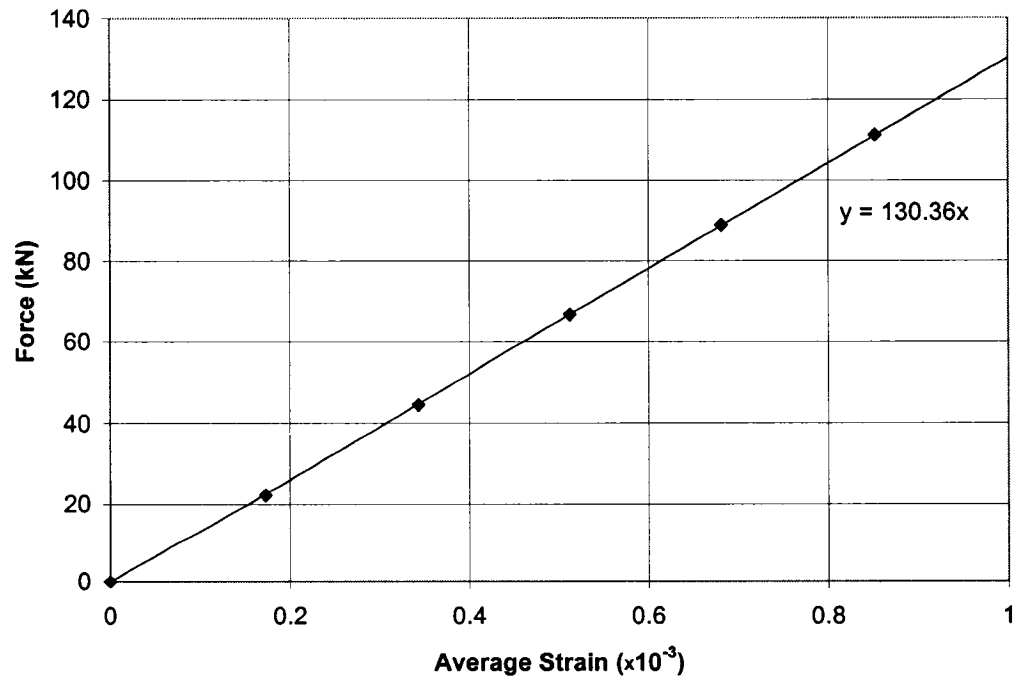


Figure A.10 Support Bar #4 calibration (north support)

APPENDIX B

TECHNICAL DATA SHEETS FOR THE SENSORS AND THE DATA ACQUISITION SYSTEM



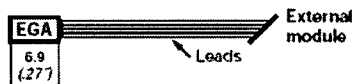
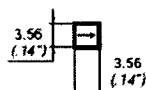
EGA Series Accelerometers

Miniature - Smallest Size & Weight

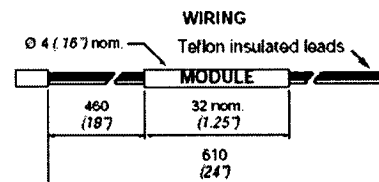


EGA, EGA-F & EGA-R

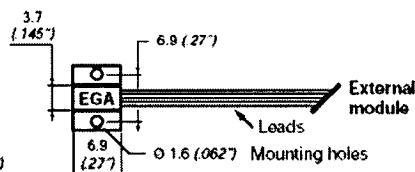
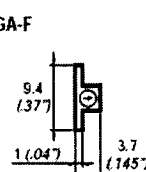
EGA



Weight without leads: 1/2 gram nom.

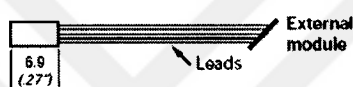


EGA-F

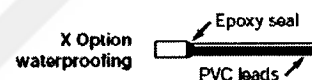


Weight without leads: 1/2 gram nom.

EGA-R



Weight without leads: 1/2 gram nom.



Dim: mm (inches)

EGA Series

g RANGES "FS"	g OVERRANGE LIMIT	FREQUENCY RESPONSE $\pm 1/2$ dB nom./min.	NATURAL FREQUENCY nom.	SENSITIVITY mV/g nom.	OUTPUT "FSO" mV nom.
± 5	± 25	0 to 150/70 Hz	300 Hz	15	± 75
± 10	± 50	0 to 250/120 Hz	500 Hz	12.5	± 125
± 25	± 125	0 to 500/240 Hz	1000 Hz	5	± 125
± 50	± 250	0 to 600/290 Hz	1200 Hz	4	± 200
± 100	± 500	0 to 750/360 Hz	1500 Hz	2.5	± 250
± 250	± 1250	0 to 1000/500 Hz	2000 Hz	1	± 250
± 500	± 2500	0 to 1500/750 Hz	3000 Hz	0.5	± 250
± 1000	± 3000	0 to 2000/1000 Hz	4000 Hz	0.25	± 250
± 2500	± 5000	0 to 3000/1400 Hz	6000 Hz	0.1	± 250
± 6000	± 10000	0 to 3500/2000 Hz	8000 Hz	0.05	± 250

EXCITATION: 15VDC
 IMPEDANCE IN: 1000 Ω nom. typ.
 IMPEDANCE OUT: 450 Ω nom.
 NON-LINEARITY: $\pm 1\%$
 TRANSVERSE SENSITIVITY: 3% max.
 DAMPING RATIO AT 20°C (70°F): 0.7 nom. (0.3 to 1)
 THERMAL ZERO SHIFT: $\pm 2.5\text{mV}/50^\circ\text{C}$ ($\pm 2.5\text{mV}/100^\circ\text{F}$)
 THERMAL SENSITIVITY SHIFT (TSS): $\pm 2.5\%/50^\circ\text{C}$ ($\pm 2.5\%/100^\circ\text{F}$)
 OPERATING TEMPERATURE: -40°C to 120°C (-40°F to 250°F)
 COMPENSATED TEMPERATURE: 20°C to 80°C (70°F to 170°F)
 ZERO OFFSET AT 20°C (70°F): $\pm 15\text{mV}$ typ.

"Off-the-Shelf" Stocking Program

Entran®  www.entran.com	EGA ACCELEROMETERS Miniature Smallest Size & Weight	Entran Sensors & Electronics USA: Fairfield, NJ UK: Garston, Watford, Herts, England Europe: Les Clayes-sous-Bois, France		
		SPECIFICATION	ISSUE	PAGE
		EGAS0001U	PA0	1 of 2

5-28

EGA specifications continued ...

Options and Accessories:

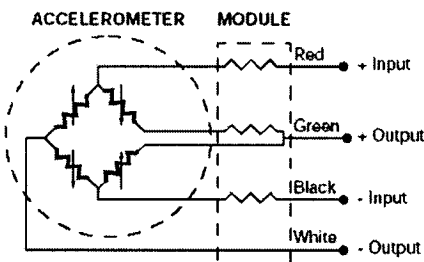
COMPENSATED TEMPERATURE RANGES:	STANDARD	= 20°C to 80°C (70°F to 170°F)
	Z1	= -20°C to 40°C (0°F to 100°F)
	Z2	= 0°C to 60°C (32°F to 140°F)
	Z4	= 40°C to 90°C (100°F to 200°F)
	Z*	= Non-standard, contact Entran
5 WIRE BRIDGE WIRING FOR ADJUSTABLE ZERO OFFSET:	5	= 5 Wire
EXCITATION VOLTAGE:	STANDARD	= 16VDC
	V00	= Replace "00" with Excitation between 1 and 15. If less than 15, Sensitivity (FSO) will decrease accordingly.
	V*	= Non-standard Excitation with standard FSO and non-standard TSS, contact Entran.
SPECIAL LEAD LENGTH:	L00F	= Replace "00" with total length in feet.
	L00M	= Replace "00" with total length in meters.
SPECIAL MODULE LOCATION:	M00F	= Replace "00" with distance between sensor and module in feet.
	M00M	= Replace "00" with distance between sensor and module in meters.
WATERPROOFING LEAD EXIT:	X	= Short term waterproofing. Limited to 105°C (220°F).
CONNECTOR WIRED TO LEADS:	C	= Microtech type male or equivalent (w/o mate)
	R	= RJ Telephone type male (w/o mate)
MATING CONNECTORS FOR CONNECTOR OPTIONS:	See Cable and Connector Bulletin	
TRIAXIAL MOUNTING BLOCKS:	MTG-A1 & A1M for mounting 3 EGA (see EG Accessories)	

Model Number construction:

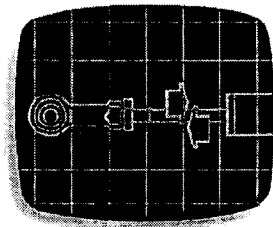
EGA Series	-F Housings	- 100 Range	- /Z1/L2M/R Options
	* * -F -R		C or R L00F or L00M M00F or M00M V1 thru V15 or V* X Z1,Z2,Z4, or Z* 5

"Off-the-Shelf" Stocking Program

Wiring:



Entran®	EGA ACCELEROMETERS	SPECIFICATION	ISSUE	PAGE
		EGAS0001U	PA0	2 of 2



MODEL 602

The Duncan 600 Series provides an extremely accurate method of position sensing in applications that require a rugged instrument operating in a tight area. Its long life, infinite resolution and very smooth output deliver high reliability in critical measurements.

Seven models provide a choice of electrical travel from 1" (25mm) to 12" (305mm), and include a floating shaft design to accommodate shaft/interface connecting misalignments. Accessories for the 600 Series include a rod end bearing and mounting bracket.

ELECTRICAL SPECIFICATIONS

Electrical Travel:
1.00" (25mm) to 12.00" (305mm) (See table)

Resistance Range:
1.0K to 12.0K Ω standard (See table)

Resistance Tolerance:
 $\pm 20\%$

Independent Linearity:
 ± 0.70 to 0.08% (See table)

Power Rating at 70°C:
0.25 watts
Per inch of electrical travel derated
to 0 watts at 125°C

Output Smoothness:
0.1%

Resolution:
Infinite

Insulation Resistance at 500 VDC:
1000 megohms

Dielectric Strength:
500 VRMS

Temperature Range:
-55 to +125°C

MECHANICAL SPECIFICATIONS

Mechanical Travel:
Electrical travel ± 0.1 inch min.

Actuation Force:
2 oz. max.

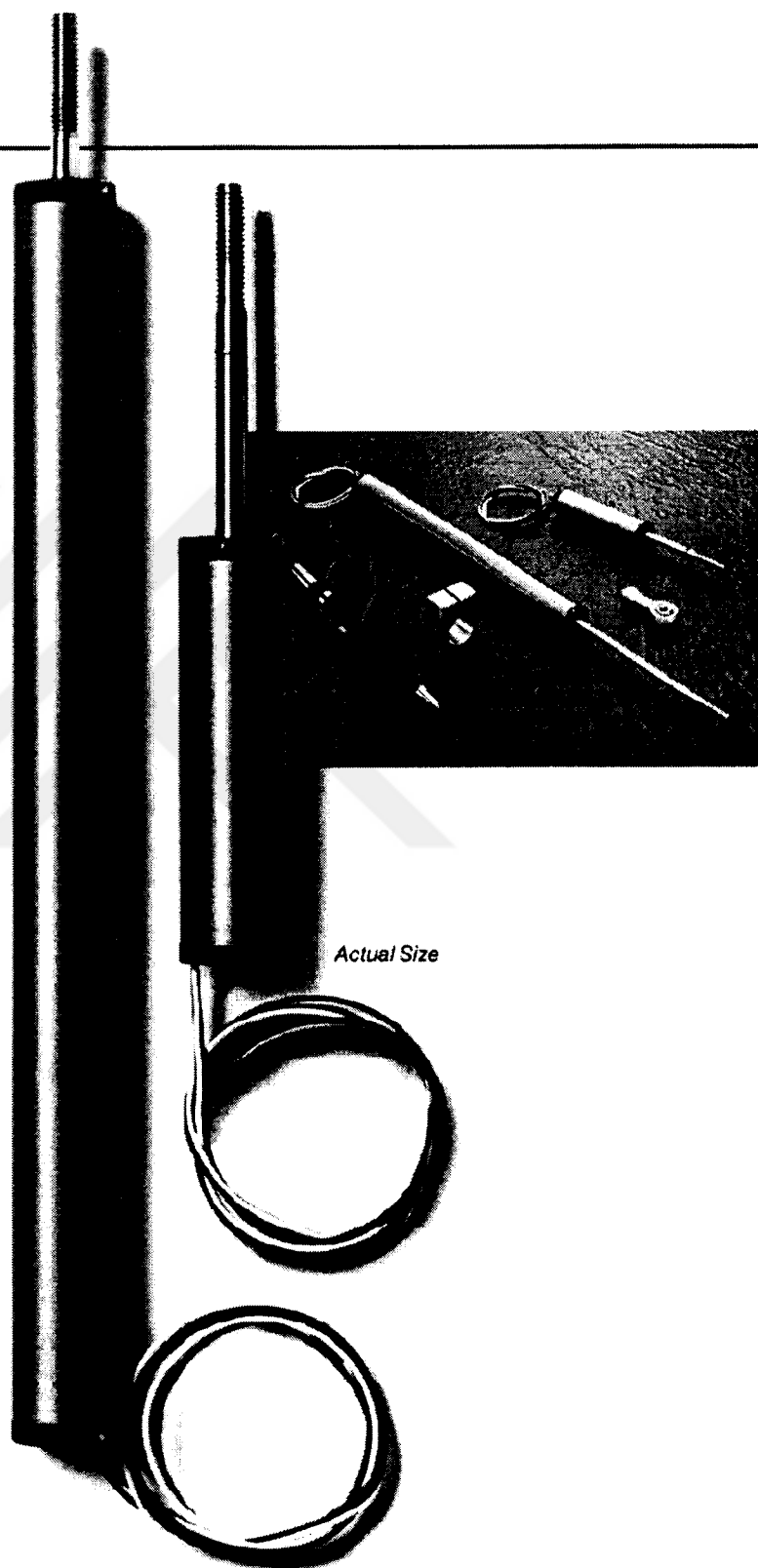
Repeatability:
Within .0005 in.

Life:
10 x 10⁶ Cycles

Shaft Alignment:
Floating shaft design allows for shaft/interface
misalignment up to 0.010" (.25).
Shaft rotates freely

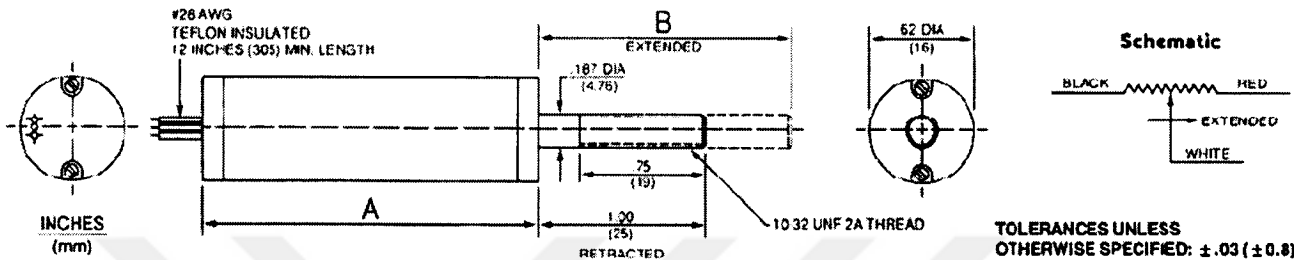
NOTES:

- 1 Available in dual configuration.
- 2 Most specifications may be altered to meet specific requirements.
- 3 Other travels available.



LINEAR MOTION POSITION SENSOR INFINITE RESOLUTION

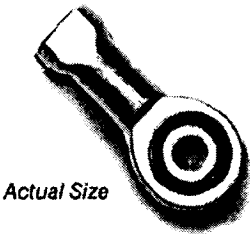
OUTLINE SPECIFICATIONS



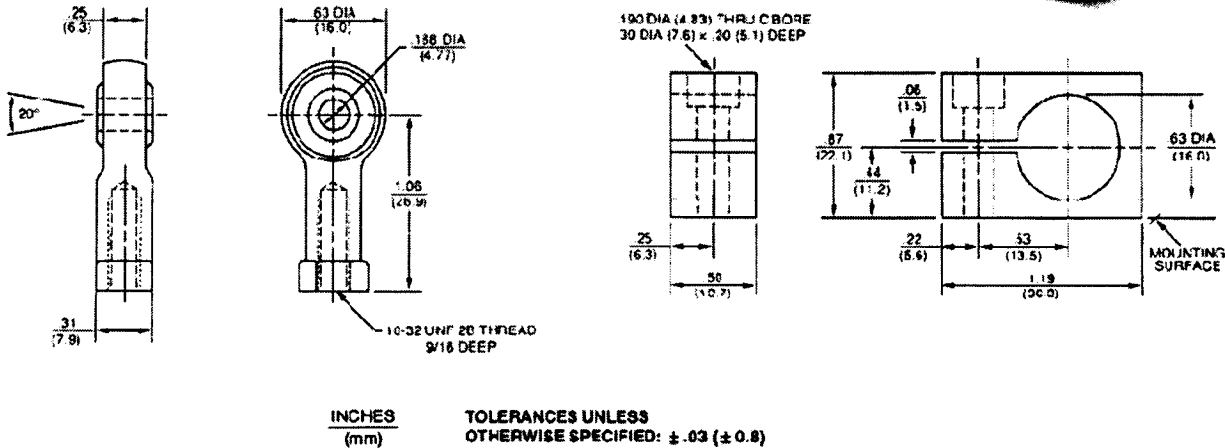
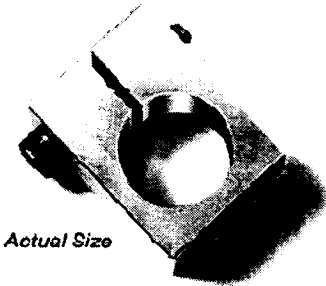
MODEL	602
Resistance K Ω	2.0
Linearity $\pm$ %	0.35
Electrical Travel inches (mm)	2.00 (51)
Case Dimensions	
A inches (mm)	3.50 (89)
B inches (mm)	3.10 (79)

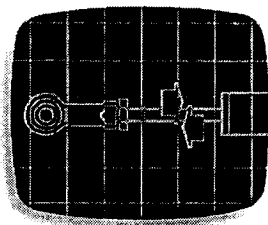
ACCESSORIES

ROD END BEARING
600-52



MOUNTING BRACKET
600-51





MODEL 604

The Duncan 600 Series provides an extremely accurate method of position sensing in applications that require a rugged instrument operating in a tight area. Its long life, infinite resolution and very smooth output deliver high reliability in critical measurements.

Seven models provide a choice of electrical travel from 1" (25mm) to 12" (305mm), and include a floating shaft design to accommodate shaft/interface connecting misalignments. Accessories for the 600 Series include a rod end bearing and mounting bracket.

ELECTRICAL SPECIFICATIONS

Electrical Travel:
1.00" (25mm) to 12.00" (305mm) (See table)

Resistance Range:
1.0K to 12.0K Ω standard (See table)

Resistance Tolerance:
 $\pm 20\%$

Independent Linearity:
 ± 0.70 to 0.08% (See table)

Power Rating at 70°C:
0.25 watts
Per inch of electrical travel derated
to 0 watts at 125°C

Output Smoothness:
0.1%

Resolution:
Infinite

Insulation Resistance at 500 VDC:
1000 megohms

Dielectric Strength:
500 Vrms

Temperature Range:
-55 to +125°C

MECHANICAL SPECIFICATIONS

Mechanical Travel:
Electrical travel ± 0.1 inch min.

Actuation Force:
2 oz. max.

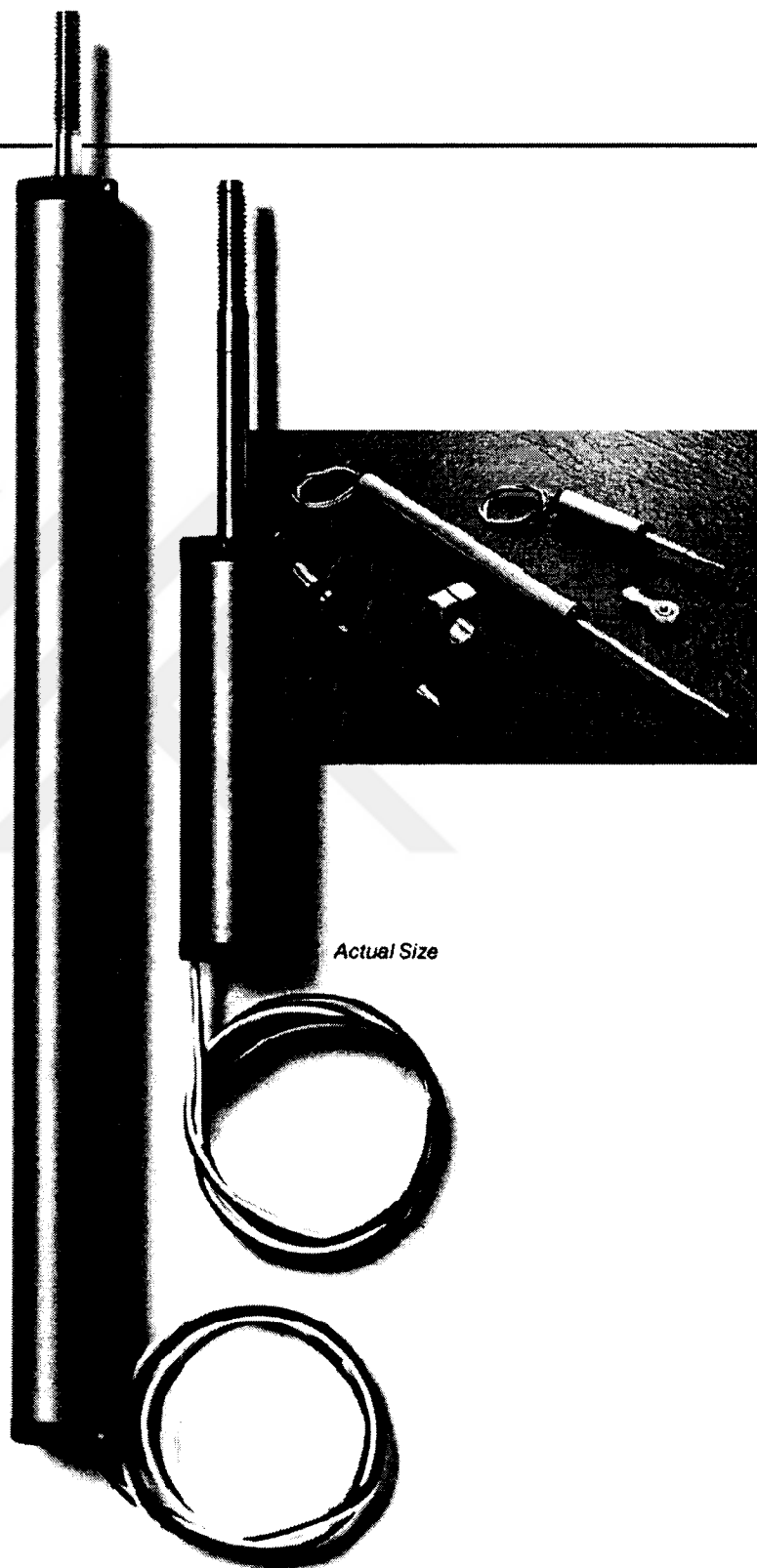
Repeatability:
Within .0005 in.

Life:
10 x 10⁶ Cycles

Shaft Alignment:
Floating shaft design allows for shaft/interface
misalignment up to 0.010" (.25).
Shaft rotates freely

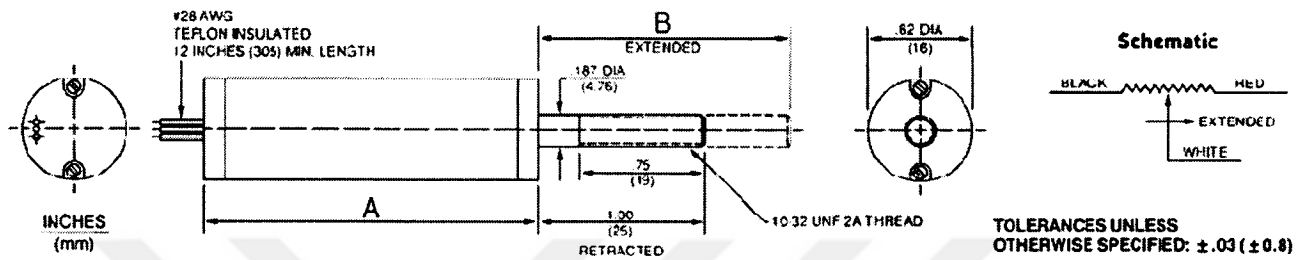
NOTES:

- 1 Available in dual configuration.
- 2 Most specifications may be altered to meet specific requirements.
- 3 Other travels available.



LINEAR MOTION POSITION SENSOR INFINITE RESOLUTION

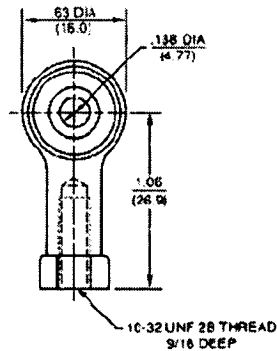
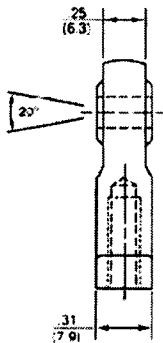
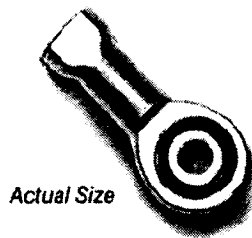
OUTLINE SPECIFICATIONS



MODEL	604
Resistance K Ω	4.0
Linearity $\pm$ %	0.15
Electrical Travel inches (mm)	4.00 (102)
Case Dimensions	
A inches (mm)	5.50 (140)
B inches (mm)	5.10 (129)

ACCESSORIES

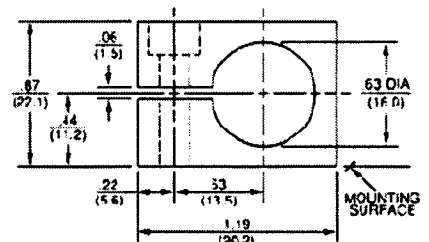
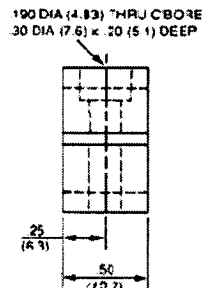
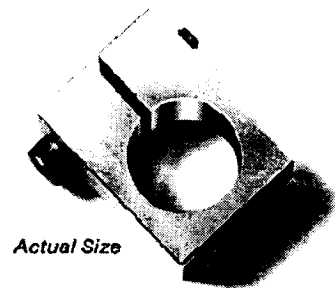
ROD END BEARING
600-52



INCHES
 (mm)

TOLERANCES UNLESS
 OTHERWISE SPECIFIED: $\pm .03$ (± 0.8)

MOUNTING BRACKET
600-51



TML STRAIN GAUGE TEST DATA

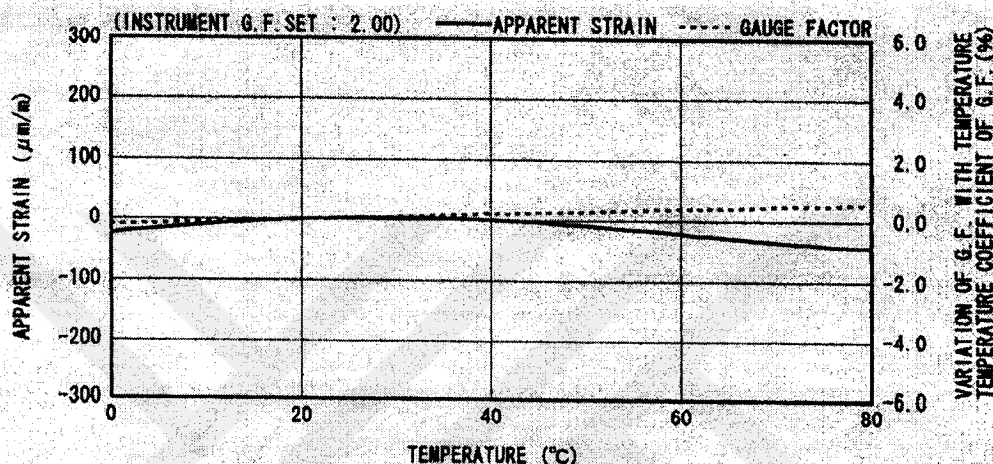
GAUGE TYPE : FLA-5-11
 LOT NO. : A513611
 GAUGE FACTOR : 2.12 $\pm 1\%$
 ADHESIVE : P-2

TESTED ON : SS 400
 COEFFICIENT OF THERMAL EXPANSION : 11.8 $\times 10^{-6}/^{\circ}\text{C}$
 TEMPERATURE COEFFICIENT OF G.F. : $+0.1 \pm 0.05 \%$ $/^{\circ}\text{C}$
 DATA NO. : A0479

THERMAL OUTPUT (ϵ_{app} : APPARENT STRAIN)

$$\epsilon_{app} = -2.33 \times 10^{-1} + 1.57 \times T - 1.21 \times 10^{-2} \times T^2 - 4.97 \times 10^{-4} \times T^3 + 4.55 \times 10^{-5} \times T^4 \quad (\mu\text{m/m})$$

TOLERANCE : $\pm 0.85 [(\mu\text{m/m})/^{\circ}\text{C}]$, T : TEMPERATURE



ひずみゲージ取扱いの注意事項

- 上記の特性データは、リード線の取付けによる影響を含んでおりません。裏面記載のリード線の測定値への影響に従って補正してください。
- ゲージの使用温度は、接着剤の耐熱温度などにより変わります。
- 絶縁抵抗などの点検は、印加電圧を50V以下にしてください。
- ゲージリード線に無理な力を加えないでください。
- ゲージ裏面に接着剤を塗布して接着してください。
- ひずみゲージの裏面は脱脂洗浄してありますので、汚さないように取扱いしてください。
- ゲージの包装を開封後は、乾燥した場所で保管してください。
- ご使用に際してご不明な点などがございましたら、当社までお問い合わせください。

CAUTIONS ON HANDLING STRAIN GAUGES

- The above characteristic data do not include influence due to lead wires. Correct the data in accordance with the influence of lead wires on measured values described overleaf.
- The service temperature of strain gauge depends on the operating temperature of adhesive, etc.
- Check of insulation resistance, etc. should be made at a voltage of less than 50V.
- Do not apply an excessive force to the gauge leads.
- Apply an adhesive to the back of a strain gauge and stick the gauge to a specimen.
- As the back of strain gauge has been degreased and washed, do not contaminate it.
- After unpacking, store strain gauges in a dry place.
- If you have any questions on strain gauges or installation, contact TML or your local agent.

Made in Japan



株式会社 東京測器研究所

〒140-8560 東京都品川区南大井 6-8-2
 TEL 03-3763-5611
 FAX 03-3763-6128

Tokyo Sokki Kenkyujo Co., Ltd.

8-2, Minami-Ohi 6-Chome
 Shinagawa-ku, Tokyo 140-8560

TML ひずみゲージの取扱い方法

基本的な接着手順

1. **接着剤の選択** 使用する条件に合った、ひずみゲージ用の接着剤を選びます。
2. **表面処理** 被接着面のさび・塗料などを除去し、120～180番(アルミ用:240～320番)のサンドペーパーで軽く磨きアセトンなどで清浄して、ゲージ接着位置をけがしてください。
3. **ゲージ接着** 接着剤の取扱説明書を参照してください。
4. **キュアリング** 接着剤の取扱説明書を参照してください。
5. **点検** ひずみゲージのゲージ抵抗値・絶縁抵抗値の点検を行ってください。
6. **結線** リード線はゲージ端子を使用して、はんだ付けで接続してください。
必要に応じて防湿処理を施してください。

リード線の測定値への影響

- リード線の温度変化による影響。
(3線式結線法では、温度影響はありません。)

$$\varepsilon l = \frac{r \cdot L \cdot \alpha \cdot \Delta T}{K(R + r \cdot L)} \quad \text{<式1>}$$

εl = リード線の熱出力
 r = リード線 1m 当たりの往復の抵抗値 (Ω/m)
 L = リード線の長さ (m)
 α = リード線の抵抗温度係数
 (銅線 = $3.9 \times 10^{-3} / ^\circ\text{C}$)
 ΔT = 温度変化量
 K = ゲージ率
 R = ゲージ抵抗

- リード線の結線によるゲージ率の補正。
・ 2線式の場合

$$K_o = \frac{R}{R + r \cdot L} \cdot K \quad \text{<式2>}$$

- ・ 3線式の場合

$$K_o = \frac{R}{R + \frac{r \cdot L}{2}} \cdot K \quad \text{<式3>}$$

K_o = 補正したゲージ率

HANDLING METHOD OF TML STRAIN GAUGES

Basic Bonding Procedures

1. **Select adhesive**
Select an adhesive most suitable for test conditions.
2. **Surface treatment**
Remove grease, rust, paint, etc. from the bonding surface of a specimen, lightly polish with an abrasive paper of #120 ~ 180 (#240 ~ 320 for aluminium), wipe with acetone, etc. and mark gauge installation position.
3. **Gauge Installation**
Refer to the operation manual of adhesive.
4. **Adhesive curing**
Refer to the operation manual of adhesive.
5. **Gauge installation check**
Check gauge resistance and insulation resistance.
6. **Lead wire attachment**
Solder lead wires to the strain gauges through connecting terminals.
If necessary, apply waterproof coating.

Influence of Lead Wires on Measured Values

- Influence of temperature variation of lead wires
(3-wire system is independent of temperature.)

$$\varepsilon l = \frac{r \cdot L \cdot \alpha \cdot \Delta T}{K(R + r \cdot L)} \quad \text{<Equation 1>}$$

where εl = thermal output of lead wires
 r = total resistance per meter of lead wires (Ω/m)
 L = length of lead wires (m)
 α = temperature coefficient of resistance of lead wires
 (copper wire = $3.9 \times 10^{-3} / ^\circ\text{C}$)
 ΔT = temperature variation
 K = gauge factor
 R = gauge resistance

- Gauge Factor Correction due to Lead Wire Attachment
・ In case of 2-wire system

$$K_o = \frac{R}{R + r \cdot L} \cdot K \quad \text{<Equation 2>}$$

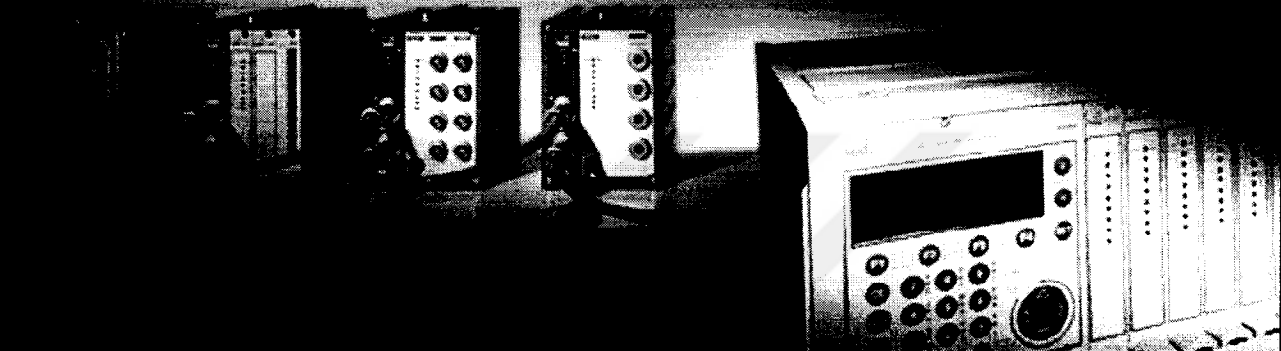
- ・ In case of 3-wire system

$$K_o = \frac{R}{R + \frac{r \cdot L}{2}} \cdot K \quad \text{<Equation 3>}$$

where K_o = corrected gauge factor

リード線 1m 当たりの往復の抵抗値 Total Resistance per Meter of Lead Wires

構成 (心数/直径) Lead wires (number of cores / diameter)	ポリイミド線 polyimide	ポリイミド線 polyimide	7/0.12	10/0.12	12/0.18	20/0.18
リード線の直径または断面積 Diameter or cross sectional area of lead wires	ϕ 0.14mm	ϕ 0.18mm	0.08 mm ²	0.11 mm ²	0.3 mm ²	0.5 mm ²
1m 当たりの往復の抵抗値 Total resistance per meter	2.5 Ω/m	1.5 Ω/m	0.44 Ω/m	0.32 Ω/m	0.12 Ω/m	0.07 Ω/m



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... the Ultimate Modular, Scalable
and Configurable System



CONNECTION BOARDS

Number of inputs

AP01



1

AP03



1

AP07/1



1

AP08



1

AP09



1

AP11



1

AP13



1

AP14



1

AP17¹⁾



1

AP18



1

AMPLIFIER MODULES

SINGLE CHANNEL AMPLIFIERS

Single
Channel
Amplifiers

Excitation

Accuracy
class⁵⁾

Multi
Channel
Amplifiers

Accuracy
class⁵⁾

ML01B

0.03



ML 801

0.05

(0.1 mit AP 401)
(0.1 mit AP 409)

ML10B

DC³⁾

0.03



ML 460

0.03

ML30B

TF⁴⁾

600 Hz

0.03

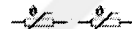


aML35B

TF⁴⁾

75 Hz

0.03



ML38

TF⁴⁾

225 Hz

0.0025



ML50B

TF⁴⁾

600 Hz

0.03



ML55B

TF⁴⁾

4,800 Hz

0.03



ML55BS6

TF⁴⁾

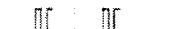
9,600 Hz

0.03



ML60B

0.01



T10

1) max. 4 AP in use depending on supply T10F

2) ML70 can be used without connection board

3) DC - Direct Current

4) TF - Carrier frequency

5) Accuracy class - the biggest individual error of the specification

AP401	AP409	AP460	AP801	AP809	AP810	AP814i	AP815i	AP835	AP836			AP71	AP72	AP75	AP77	AP78
4 electr. isolated	4 electr. isolated	4 electr. isolated	8	8	8	8	8	8	8			2 x CAN	2 serial	8 dig. in 8 dig. out	ProfibusDP	10 analog out
MULTICHANNEL AMPLIFIERS										SPECIAL FUNCTION MODULES						

Special Function Modules		Description	

EXPLANATION OF SYMBOLS

	Strain gage (resistive) full bridge
	Strain gage (resistive) half bridge
	Strain gage (resistive) quarter bridge
	Inductive full bridge
	Inductive half bridge
	Piezo-resistive transducer
	Thermo resistors
	Thermocouples
	Voltage
	Current
	Passive piezoelectric transducer
	Current fed piezoelectric transducer
	Torque/rotary speed T32, T34, T10 opt. KF1

	Rotary speed T4WA
	Impulse counter/frequency
	Digital output
	Digital input
	Analog output
	ProfibusDP
	CANBus
	RS232C, RS422, RS485 I/O
	Potentiometer 200Ω - 5000Ω
	T10 torque transducer family, except option KF1

Software for your MGCplus



catman® Enterprise

catman® Professional

catman® Express

catman® Express

- Ready-made modules for plug-and-play, measurement and visualization
- Supports export formats such as Excel, ASCII, DIAdem®, nSoft, and RPC.



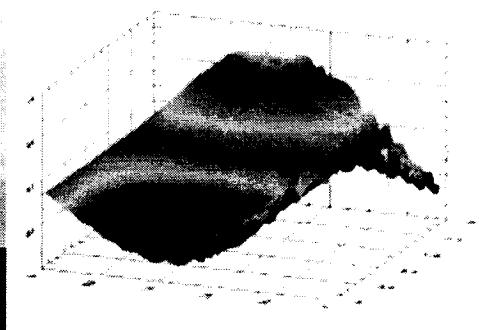
catman® Enterprise

- Easily manages up to 10,000 channels
- Several client PCs are connectable with a measurement server
- Distributes data in local and global networks.



catman® Professional

- Powerful tool for setup, measurement, visualization, and analysis
- Report generation, including all setup parameters for traceability
- More than 400 mathematical functions including modules for stress analysis, statistics and signal analysis.

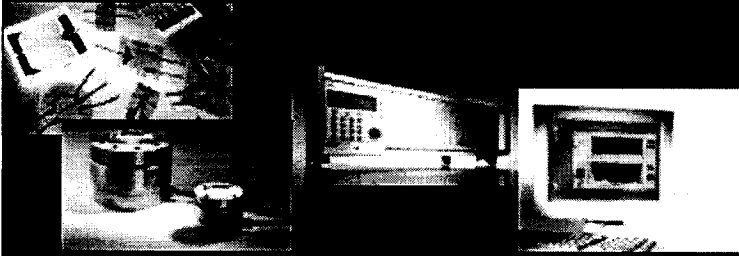


Other Software for MGCplus

- DIAdem® (National Instruments)
- BEAM (AMS)
- LabView®, DasyLab® (National Instruments) via ActiveX®

MGCplus Assistant

- Easy setup of the MGCplus unit
- Several user profiles
- Diagnosis and service functions.



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Fax +49 6151 803 9100

E-mail: info@hbm.com
Internet: www.hbm.com

APPENDIX C

PHOTOGRAPHS AND CRACK PROFILES OF TEST SPECIMENS



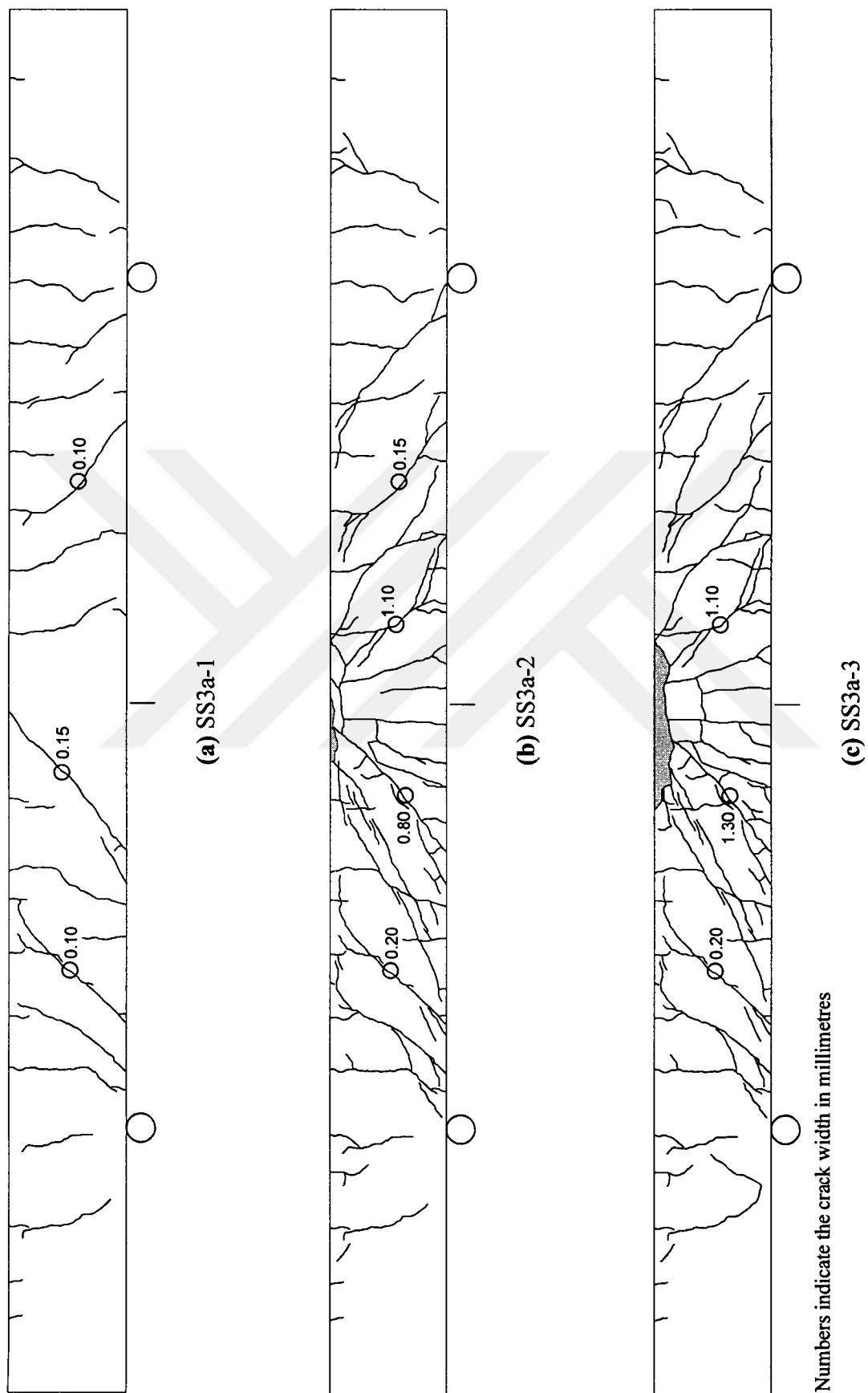


Figure C.1 SS3a crack profile (west face)

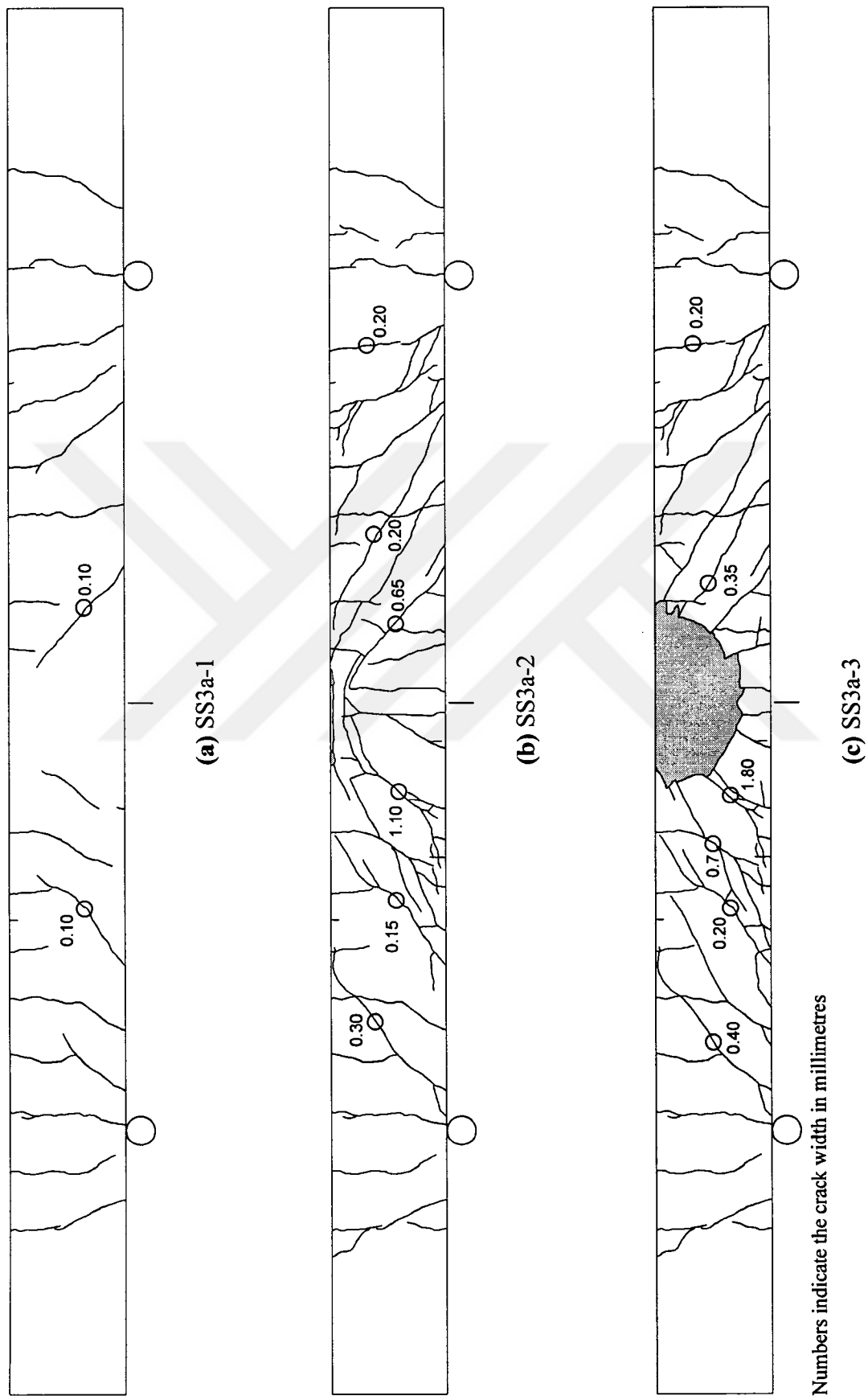


Figure C.2 SS3a crack profile (east face)

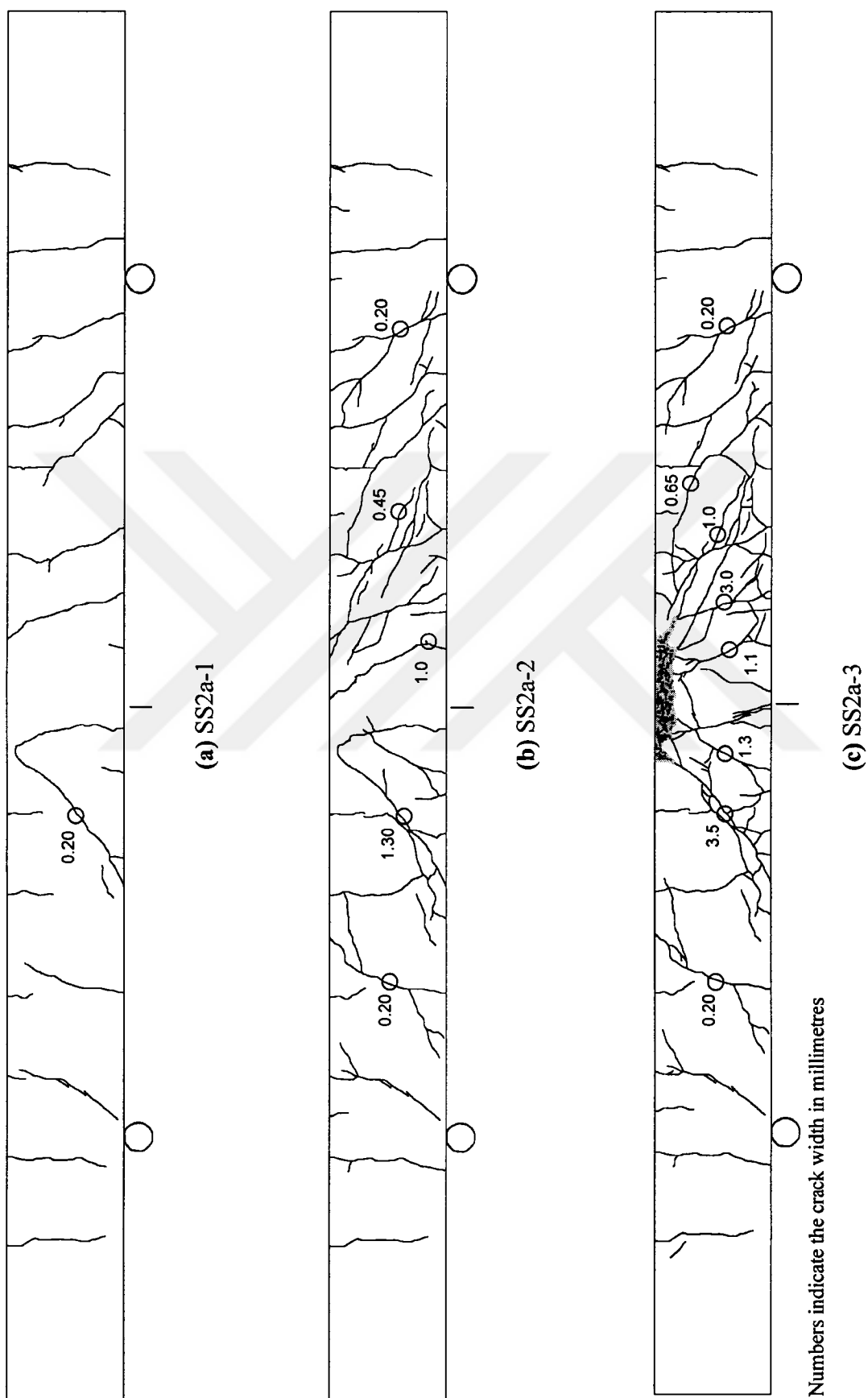


Figure C.3 SS2a crack profile (west face)

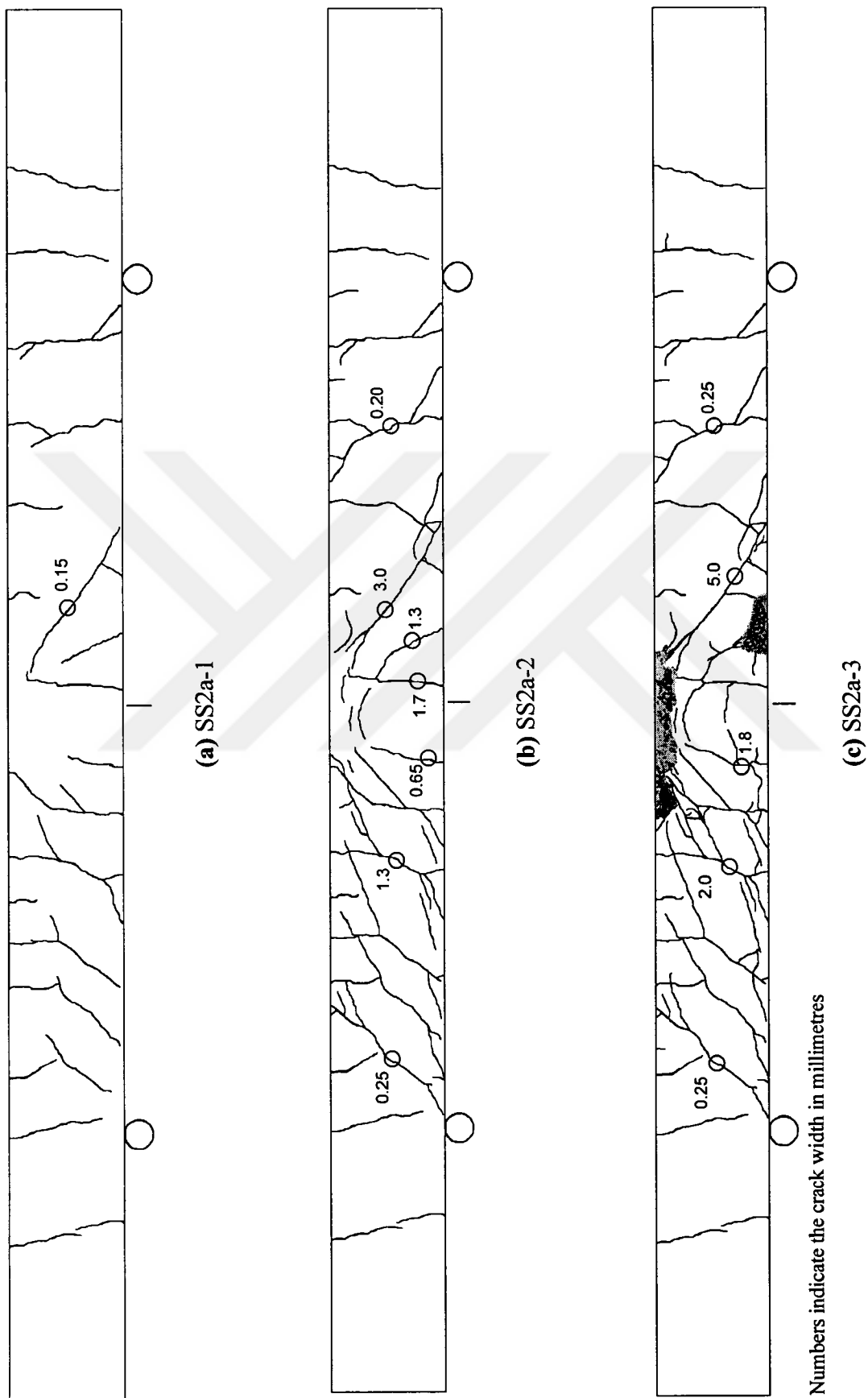


Figure C.4 SS2a crack profile (east face)

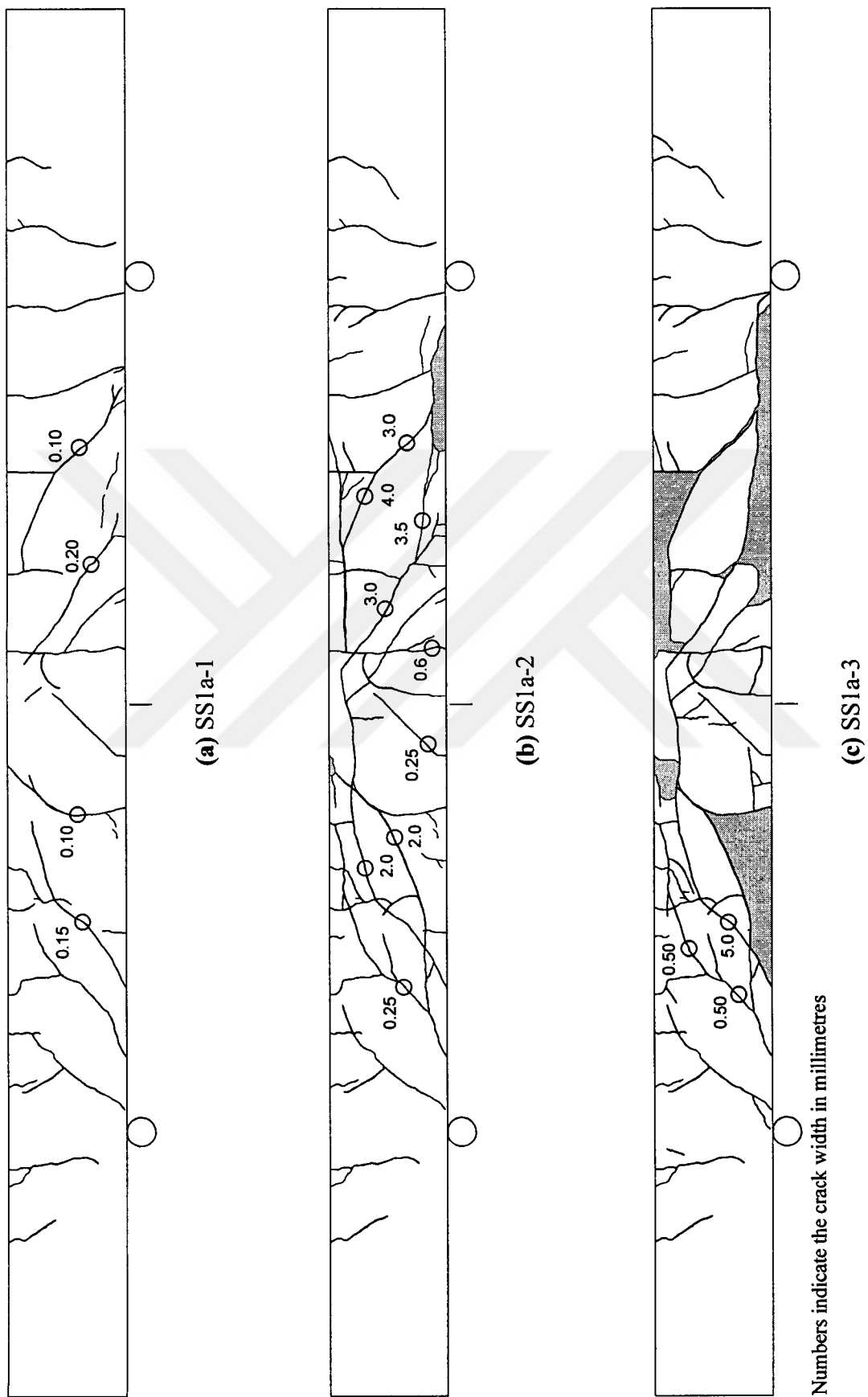


Figure C.5 SS1a crack profile (west face)

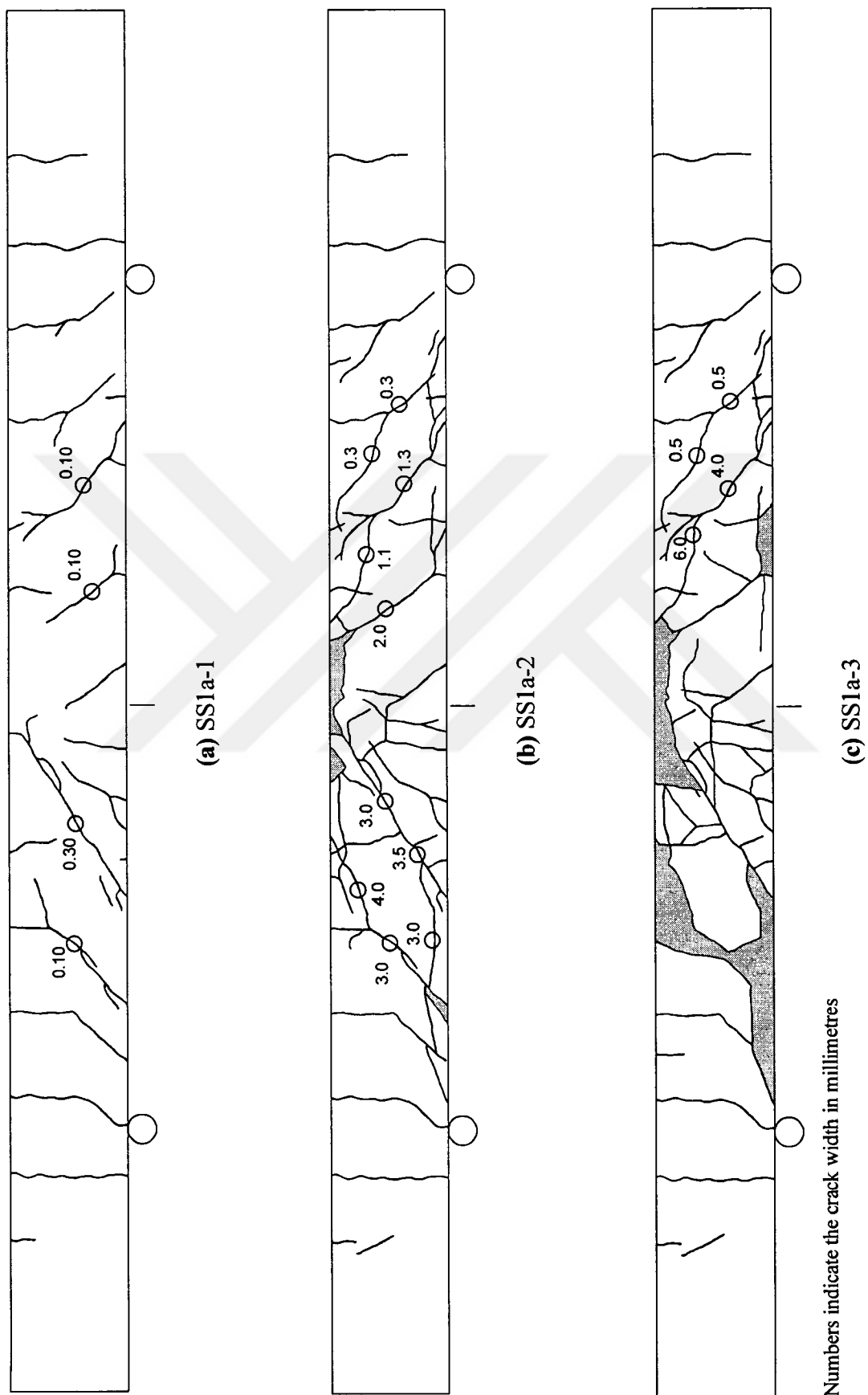


Figure C.6 SS1a crack profile (east face)

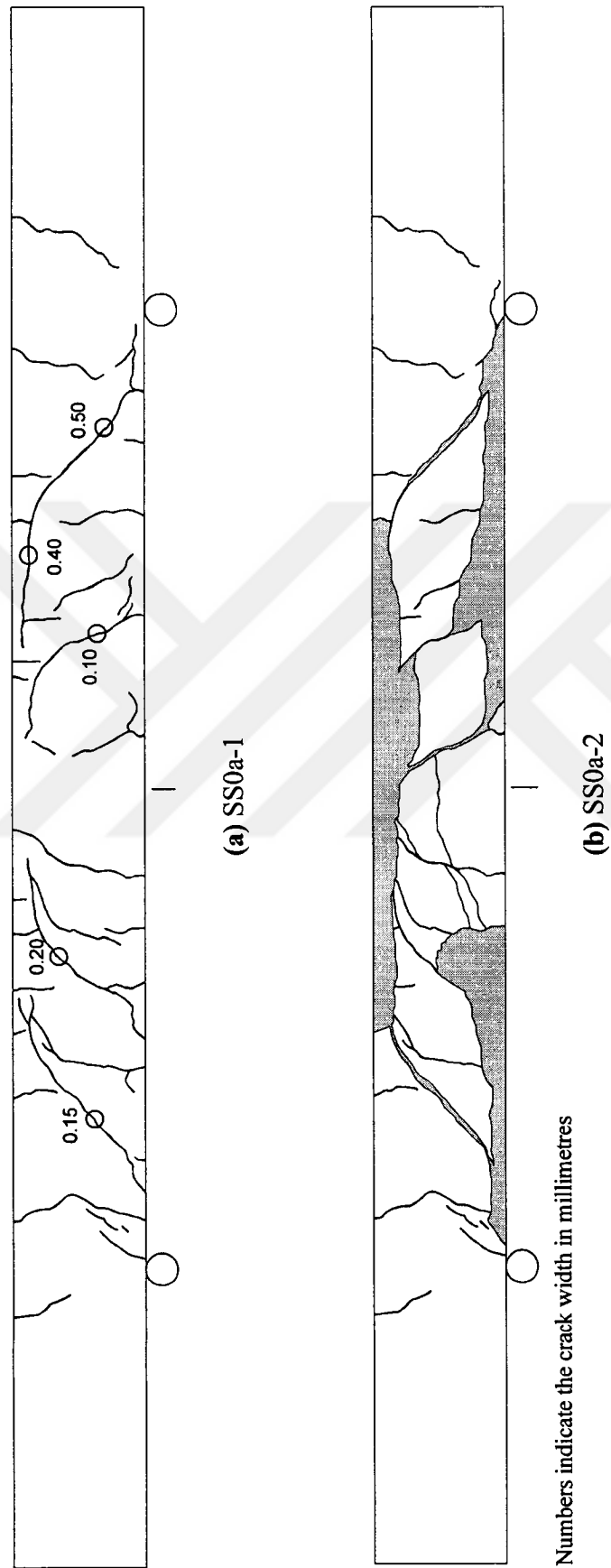


Figure C.7 SS0a crack profile (west face)

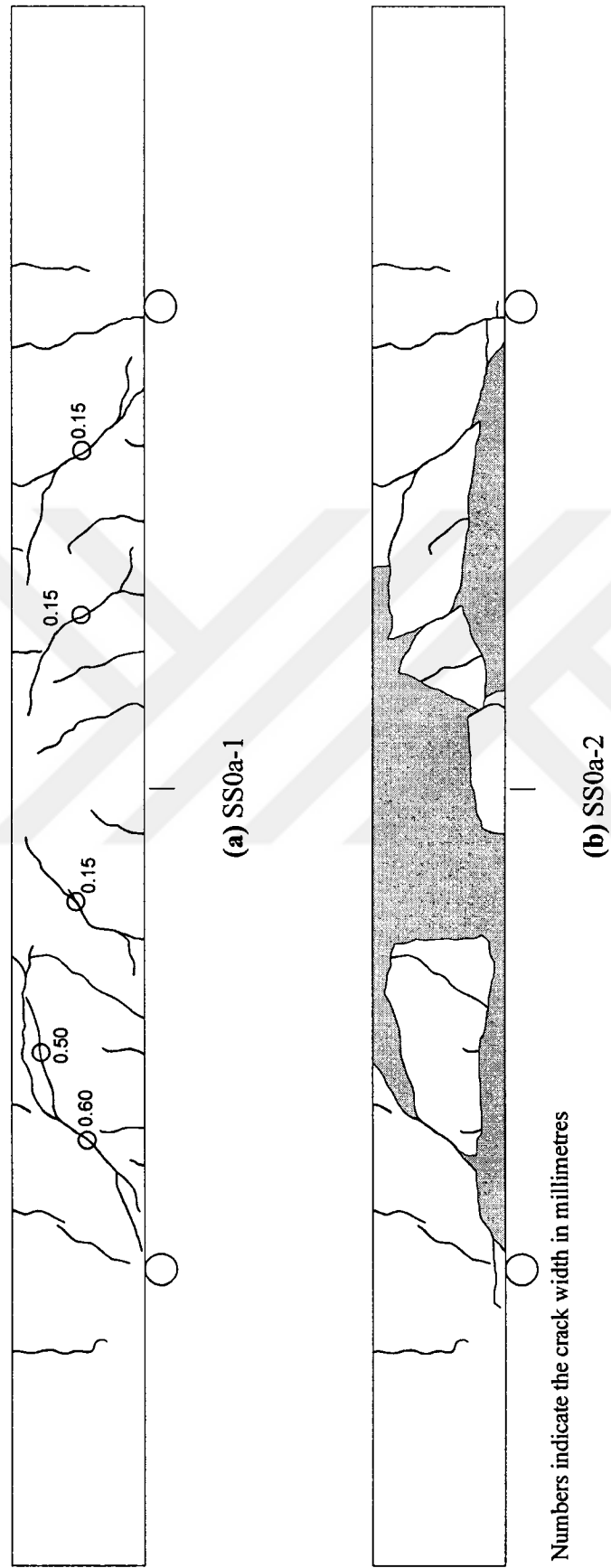


Figure C.8 SS0a crack profile (east face)

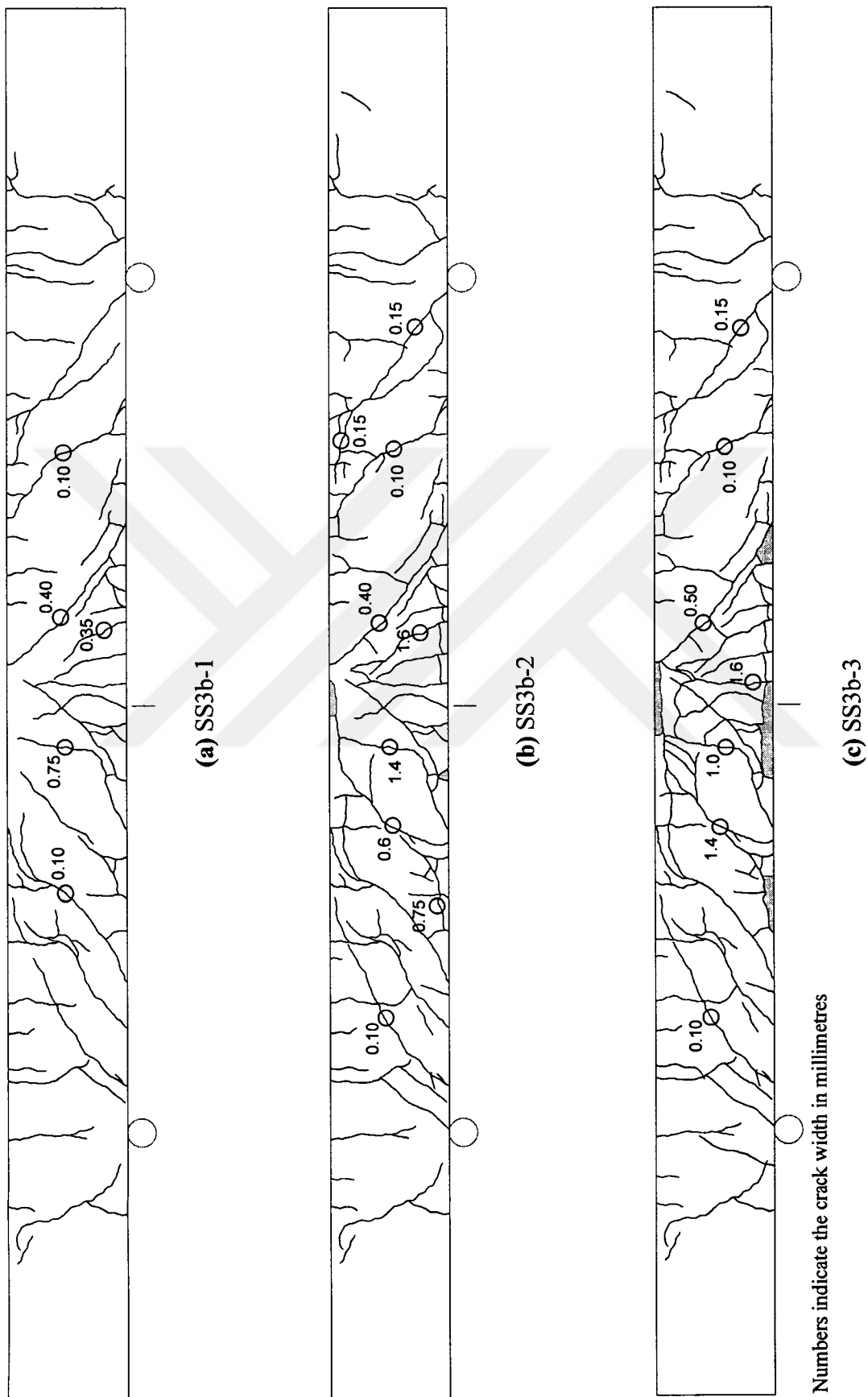


Figure C.9 SS3b crack profile (west face)

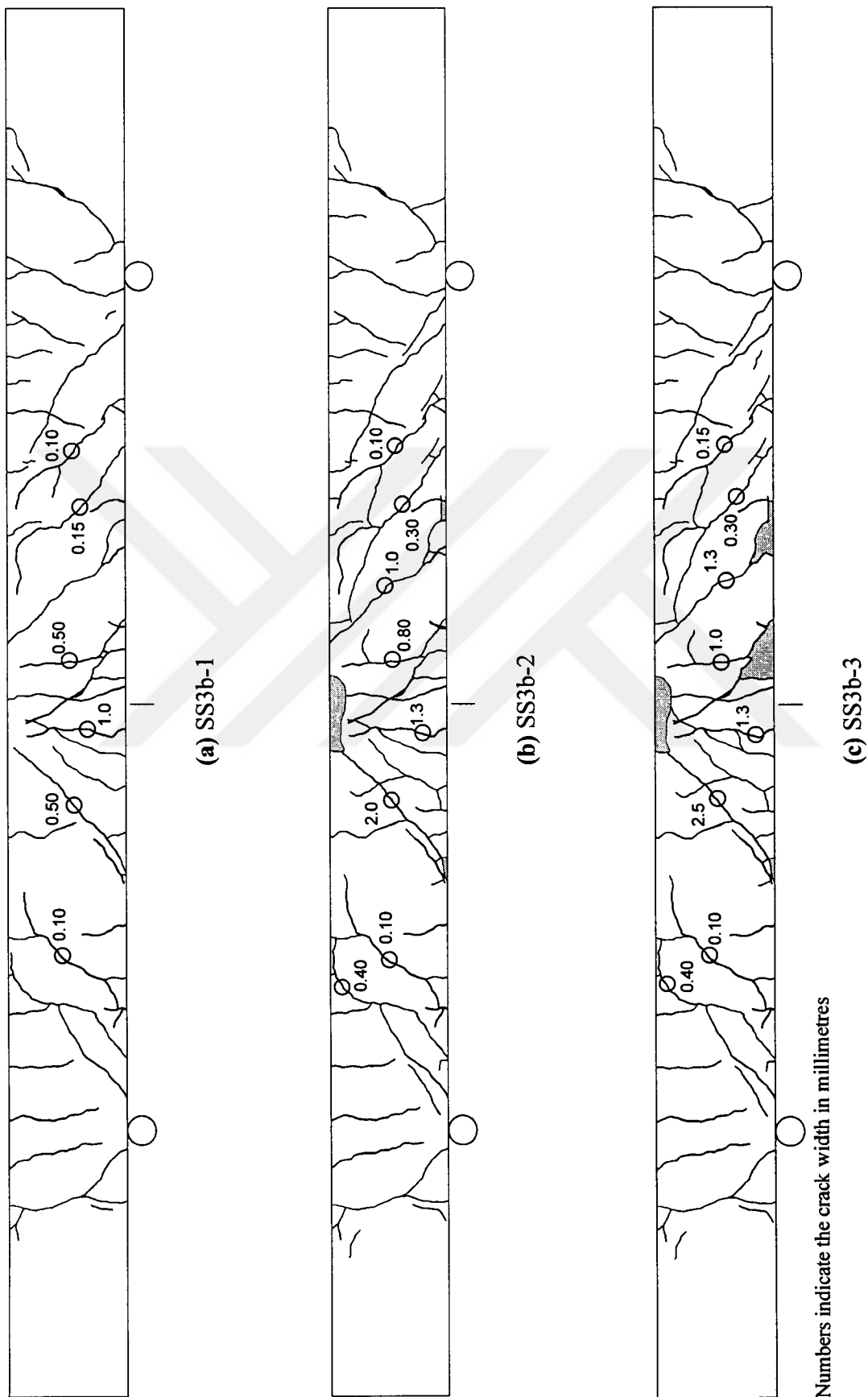


Figure C.10 SS3b crack profile (east face)

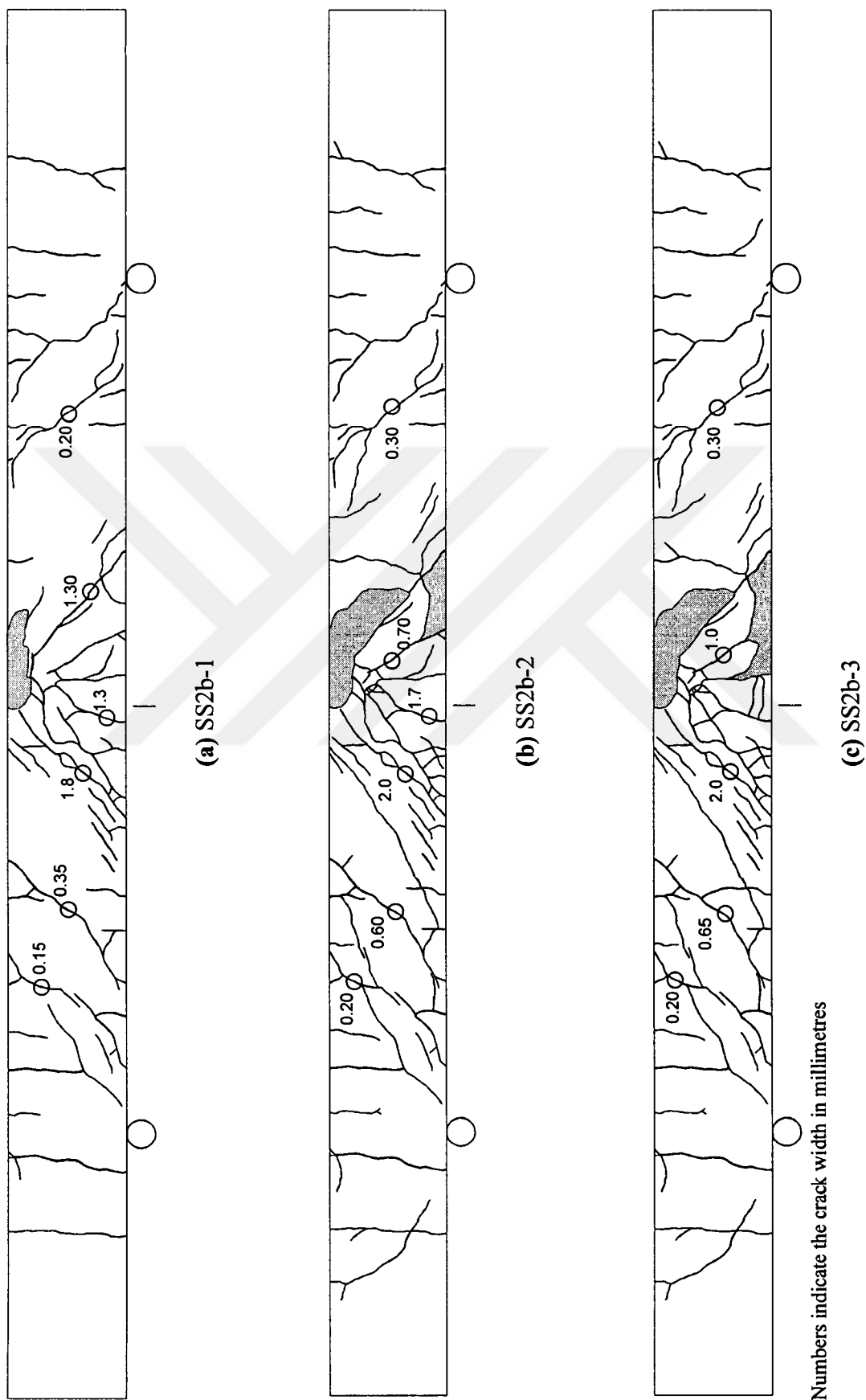


Figure C.11 SS2b crack profile (west face)

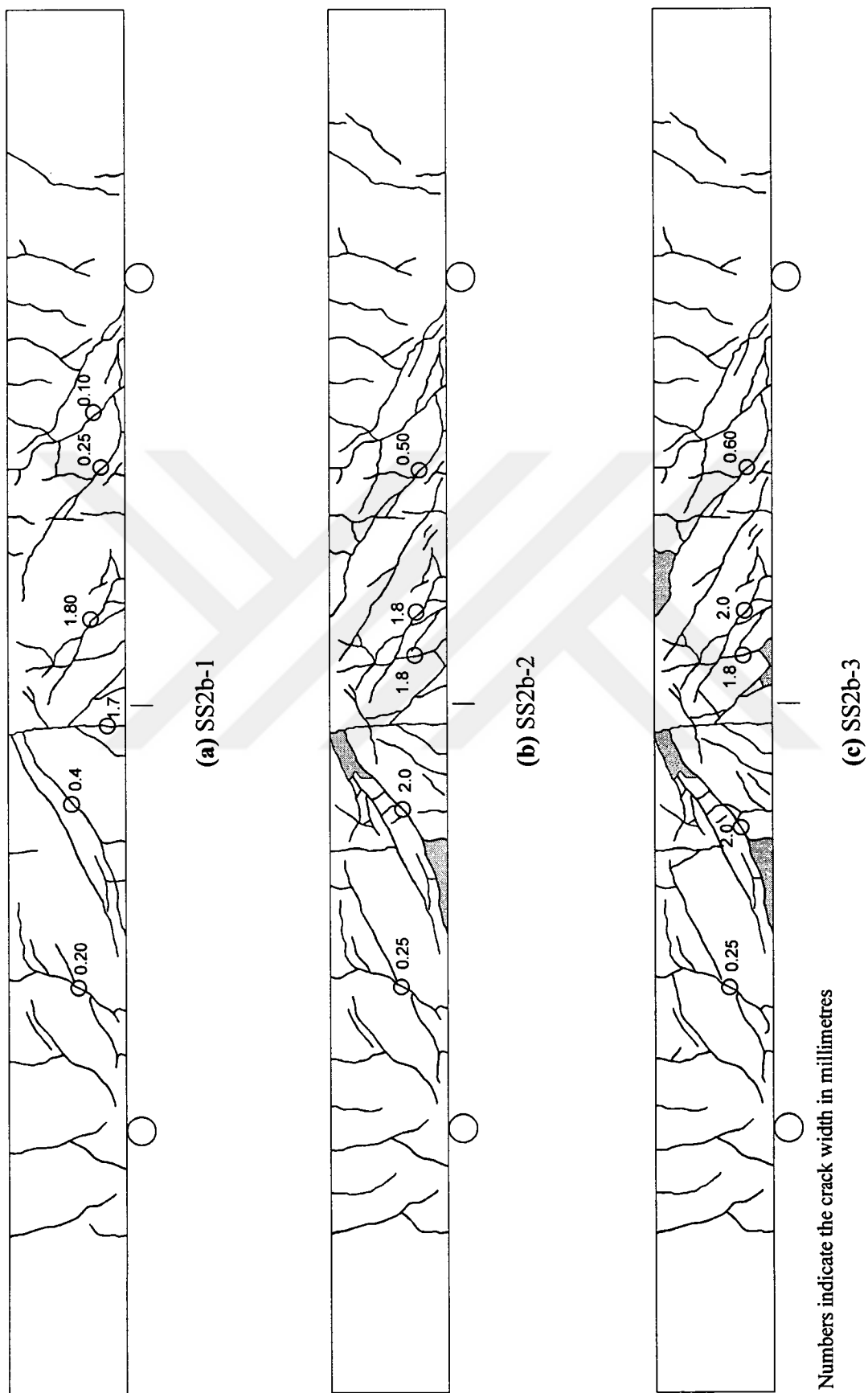


Figure C.12 SS2b crack profile (east face)

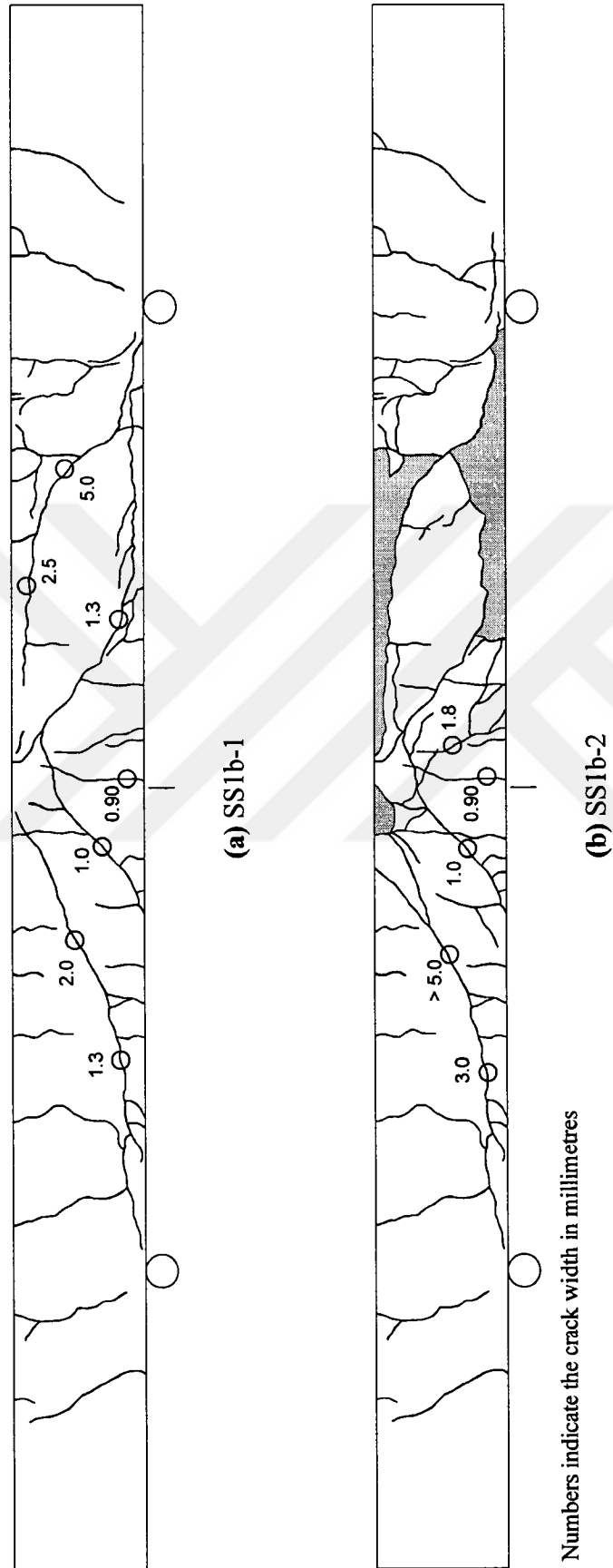


Figure C.13 SS1b crack profile (west face)

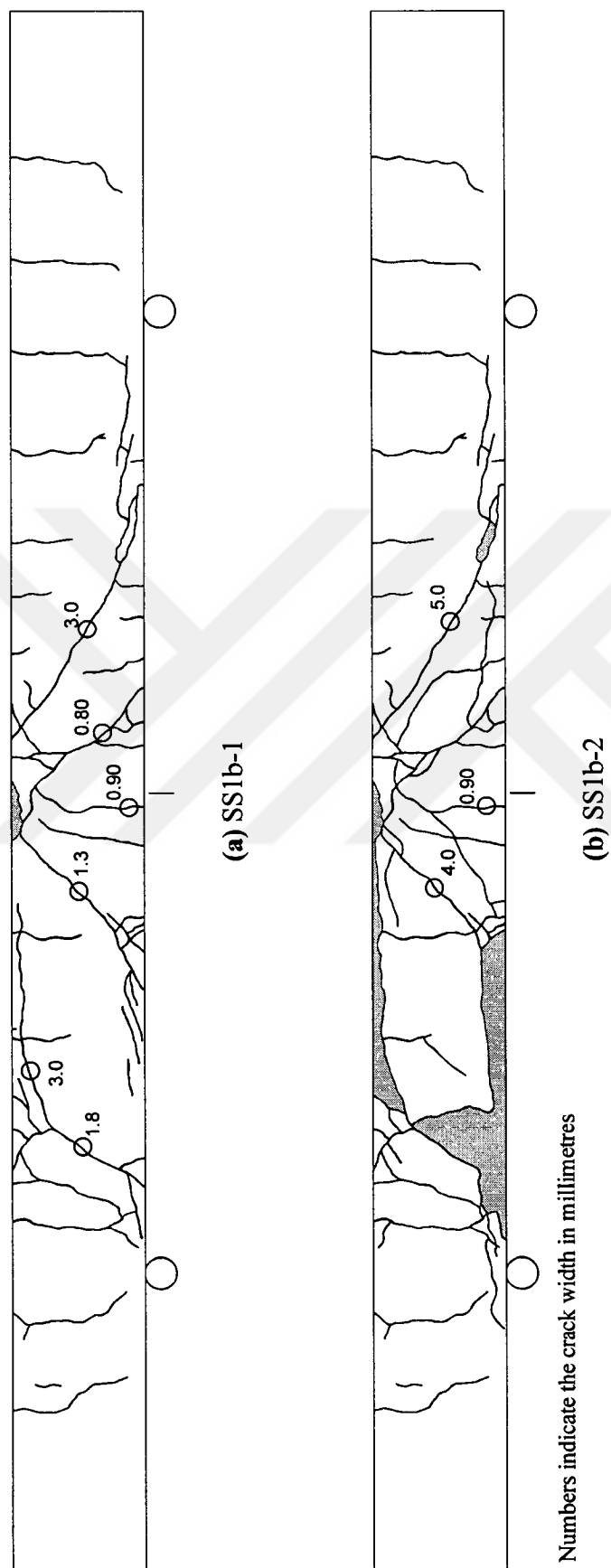


Figure C.14 SS1b crack profile (east face)

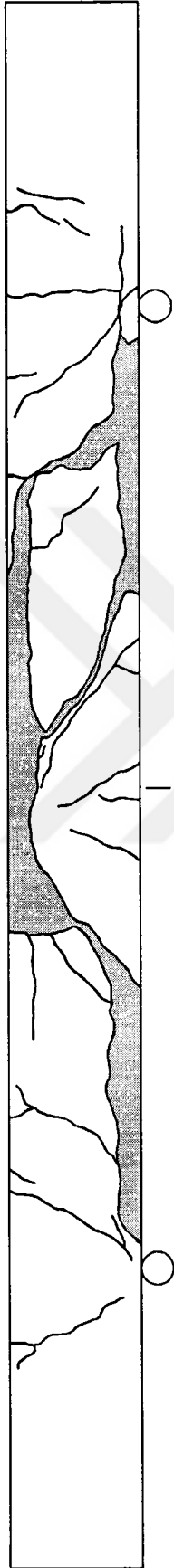


Figure C.15 SS0b crack profile (west face)



Figure C.16 SS0b crack profile (east face)

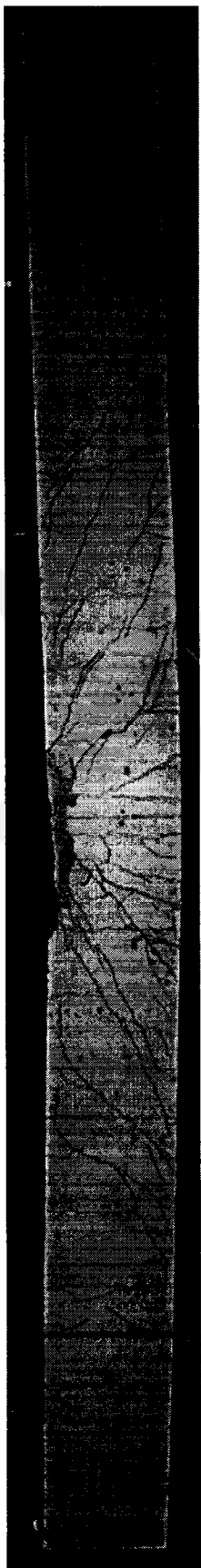


Figure C.17 SS3a final photo (west face)

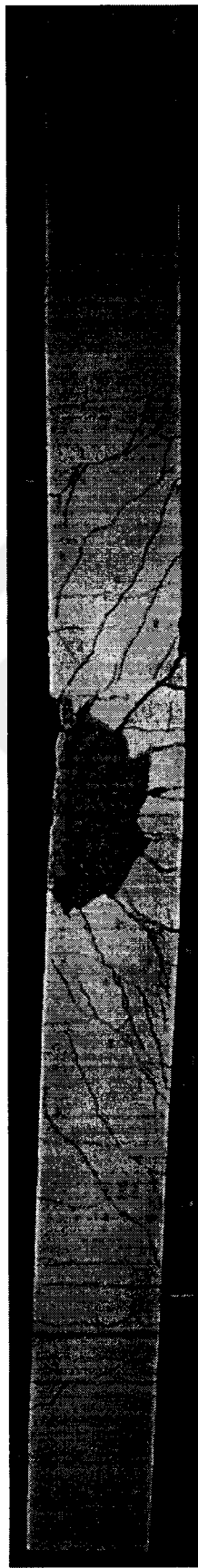


Figure C.18 SS3a final photo (east face)

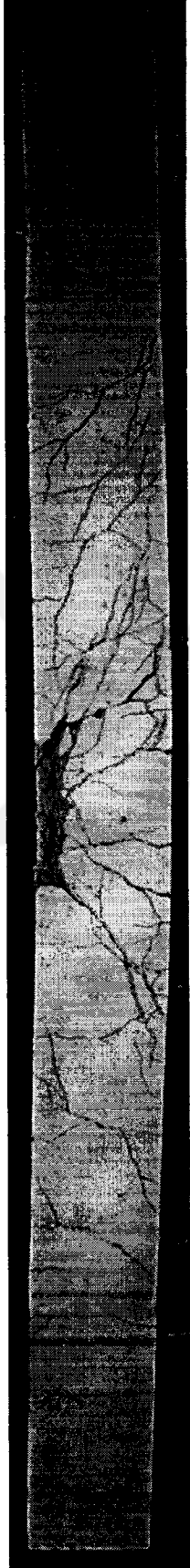


Figure C.19 SS2a final photo (west face)

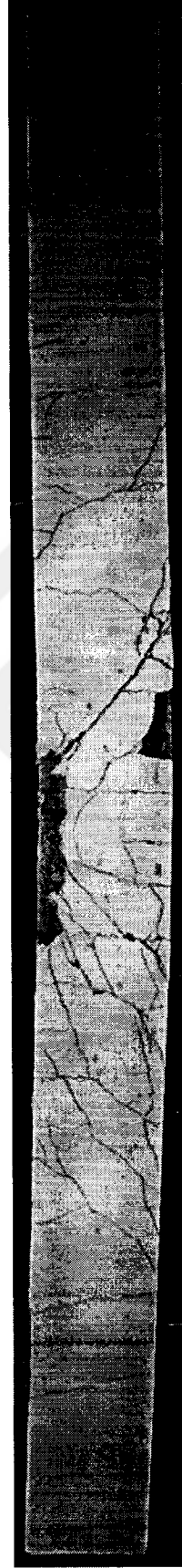


Figure C.20 SS2a final photo (east face)

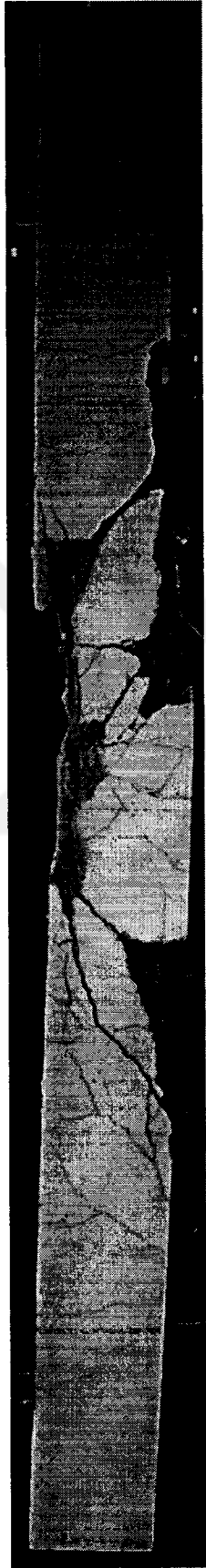


Figure C.21 SS1a final photo (west face)

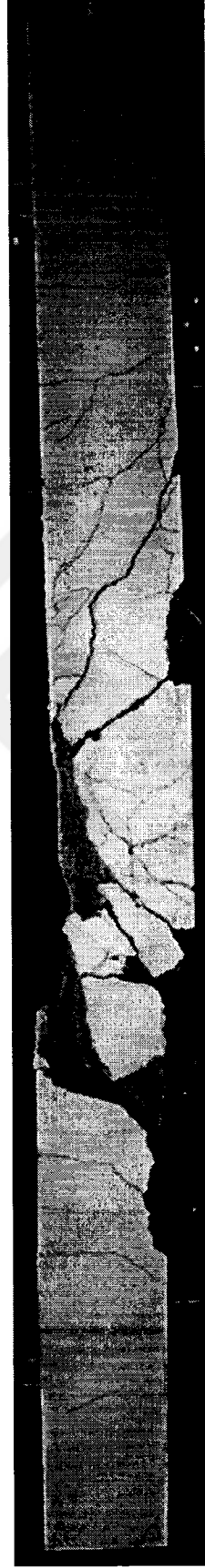


Figure C.22 SS1a final photo (east face)



Figure C.23 SS0a final photo (west face)



Figure C.24 SS0a final photo (east face)

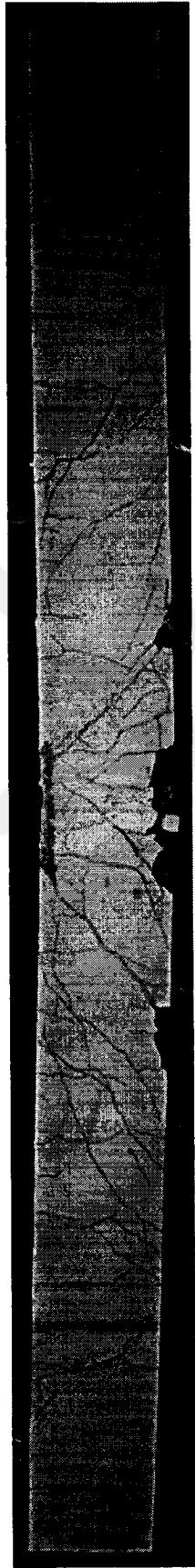


Figure C.25 SS3b final photo (west face)

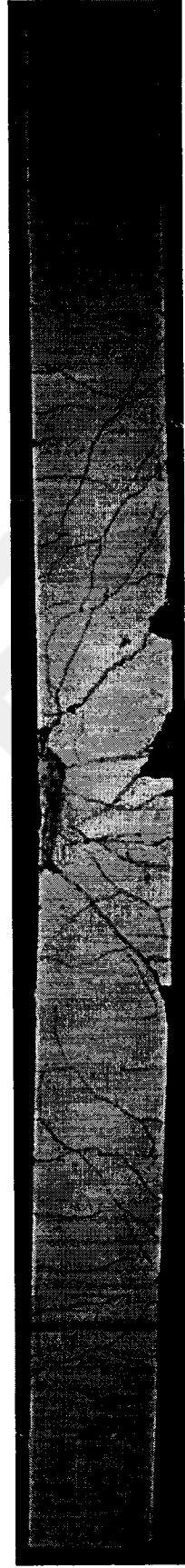


Figure C.26 SS3b final photo (east face)

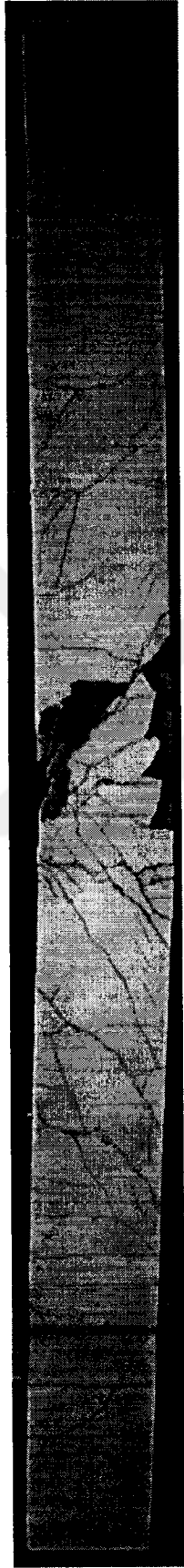


Figure C.27 SS2b final photo (west face)

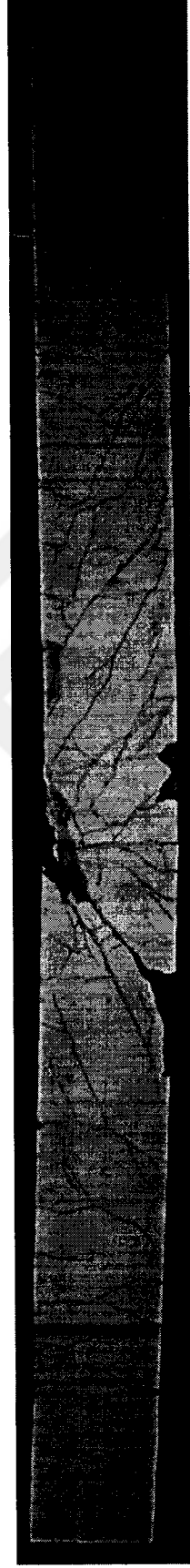


Figure C.28 SS2b final photo (east face)

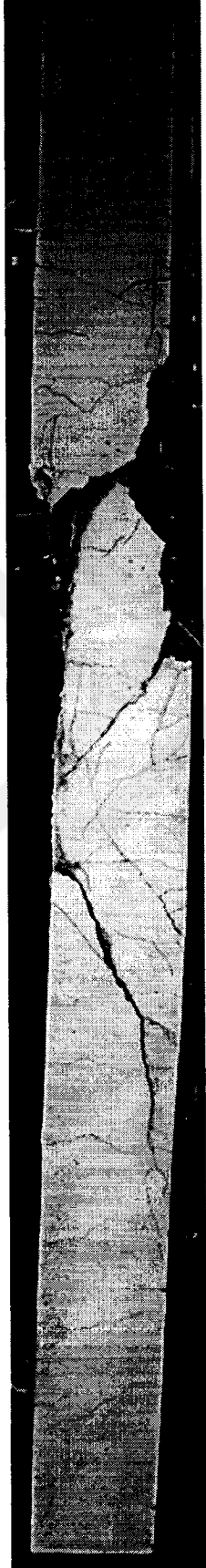


Figure C.29 SS1b final photo (west face)

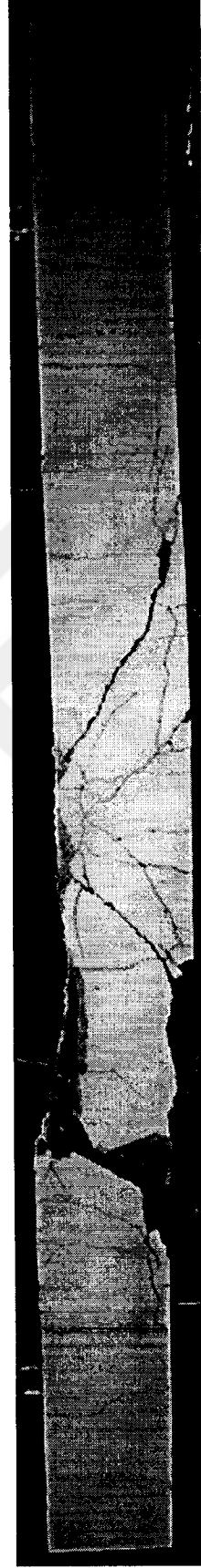


Figure C.30 SS1b final photo (east face)

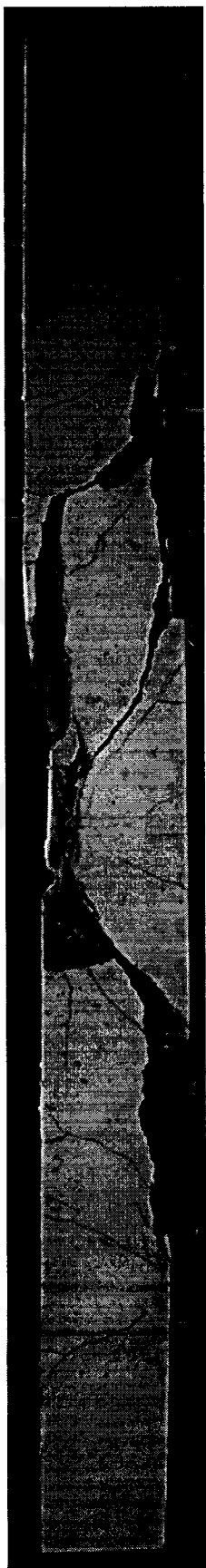


Figure C.31 SS0b final photo (west face)

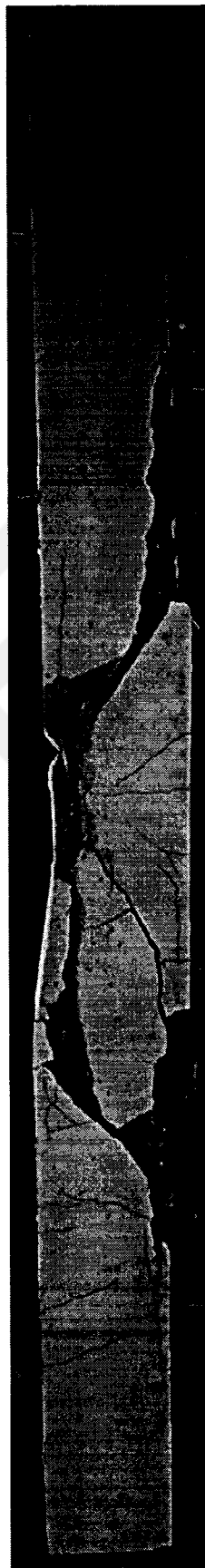


Figure C.32 SS0b final photo (east face)