RISK-AVERSE OPTIMIZATION OF WIND-BASED ELECTRICITY GENERATION WITH BATTERY STORAGE

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We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

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As the global installed capacity of wind power increases, various solutions have been developed to accommodate the intermittent nature of wind. Investing in battery storage reduces power fluctuations, improves the reliability of delivering power on demand, and decreases wind curtailment. In the literature, power producers are generally modelled as risk-neutral decision makers, and the focus has been on expected profit maximization. For many privately-held small independent power producers, it is more important to capture their risk-aversion through specialized risk measurements driven by the owners' specific risk preferences, even though the expected value-maximization objective is very desirable for large corporations with diversified investors. We consider a risk-averse, privately-held, small Independent Power Producer interested in investing in a battery storage system and jointly operating the wind farm and storage system with a transmission line connected to the market. We formulate the problem as a Markov decision process (MDP) to find optimal investment, generation, and operational storage decisions. Using dynamic coherent risk measures, we incorporate risk-aversion into our formulation. By choosing the risk measure as first-order mean semi-deviation, we obtain optimal threshold-based policy structure as well as optimal storage investment capacity. We perform a sensitivity analysis on optimal storage capacity with respect to the risk-aversion degree and transmission line limitations.

Keywords: Wind energy, Battery storage, Risk-averse Markov decision process, Dynamic risk measures.

ÖZET

BATARYA DEPOLAMASI İLE RÜZGAR ENERJİSİ BAZLI ELEKTRİK ÜRETİMİNİN RİSKTEN KAÇINAN OPTİMİZASYONU

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Rüzgar enerjisinin küresel kurulu kapasitesi arttıkça, rüzgarın aralıklı doğasına uyum sağlamak için çeşitli çözümler geliştirilmiştir. Batarya depolamasına yatırım yapmak, güç dalgalanmalarını azaltmaya, isteğe bağlı güç sağlamak için güvenilirliği artırmaya ve rüzgar kısıtlamasını azaltmaya yardımcı olur. Literatürde, enerji üreticileri genellikle riske duyarsız karar vericiler olarak modellenir ve odak noktası beklenen kâr maksimizasyonu olmuştur. Beklenen kâr maksimizasyonu hedefi çeşitlendirilmiş yatırımcılara sahip büyük şirketler için çok tercih edilir olsa da, özel sektöre ait küçük bağımsız elektrik üreticisi için, sahibinin belirli risk tercihleri tarafından yönlendirilen özel risk ölçümleri yoluyla riskten kaçınmalarını yakalamak daha önemlidir. Bir batarya depolama sistemine yatırım yapmak ve bir iletim hattı aracılığıyla piyasaya bağlı bir rüzgar çiftliği ve depolama sistemini birlikte işletmekle ilgilenen, riskten kaçınan, özel sektöre ait küçük bir bağımsız elektrik üreticisini göz önüne alıyoruz. Markov karar sürecini (MDP), problemi modellemek ve optimal yatırım, üretim ve operasyonel depolama kararlarını bulmak için kullanıyoruz. Dinamik tutarlı risk önlemleri kullanarak, riskten kaçınmayı modellememize dahil ediyoruz. Risk ölçüsü olarak birinci dereceden ortalama-yarı sapmayı seçerek, optimal depolama yatırım kapasitesinin yanı sıra optimum eşik tabanlı politika yapısını elde ediyoruz. Riskten kaçınma derecesi ve iletim hattı sınırlamalarına göre optimal depolama kapasitesi üzerinde bir hassasiyet analizi yapıyoruz.

Anahtar sözcükler: Rüzgar enerjisi, Batarya depolaması, Riskten kaçınan Markov karar süreci, Dinamik risk ölçüleri.

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Chapter 1

Introduction

During the last 10 years investment in renewable energy has gained significant momentum. In 2021, wind generation has accounted for more than 9 percent of the total electricity generation in the US. The share of wind in the US has even surpassed conventional hydroelectric power for the first time in several years in 2019. The trend is similar in Europe as well, and the share of wind generation in Europe exceeds 20 percent in some countries. The key reasons for this wind momentum in the industry can be attributed to (i) increasing environmental concerns over fossil fuels, (ii) governmental subsidies for wind generation, (iii) fast-improving wind technology and declining costs, and (iv) the development of storage technologies which enables better management of intermittent generation problems.

Over 90 percent of wind generation in the US was supplied by Independent power producers (IPPs) last year. IPPs are private companies that own or operate facilities that generate electricity and sell it to utilities, the central government, and end users. They are part of the unbundled market. IPPs may be big companies, small privately owned businesses, cooperatives, farmers, or even non-energy industrial companies with the ability to pump excess energy into the system.

As argued in the finance literature, large IPPs are mainly interested in maximizing the value of their shareholders. Their managers are incentivized to maximize firm value, which is defined as the total value of expected cash flows appropriately discounted. There are two ways to discount stochastic cash flows: (1) cash flows can be evaluated under the true measure and then discounted using a risk-adjusted discount rate; (2) the risk adjustment can be made to the price process, and one can define a risk-neutral measure and use it to calculate the expected cash flows. The second approach, detailed in Seppi [1], is better under stochastic dynamic programs because under the first approach it is difficult to evaluate the dynamic program due to the stochastic nature of the discount rate, which might not be Markovian in nature. Recently, the risk-neutral measures have been widely adopted in the literature in commodity operations management context (see e.g., [2],[3],[4],[5]).

Although, the value-maximization objective is quite desirable for large corporate firms with many diversified investors; for many privately-held small IPPs it is more relevant to capture their risk-aversion through specific risk measures driven by the owner's specific risk preferences. However, in a multi-period decision model it is not trivial to capture risk-aversion. In this thesis, we present a model to optimize wind generation decisions for a privately-held small IPP whose risk preferences can be captured by a coherent risk measure.

Worldwide installed wind power capacity has risen from 24 GW in 2001 to 837 GW in 2021. Moreover, GWEC Market Intelligence predicts that current policies will result in the addition of 557 GW of additional capacity over the next five years [6]. The increase in the volume of renewable generation resulted in significant amounts of curtailment in generation. Due to the ongoing growth of grid-connected solar and wind capacity, business opportunities for battery storage as a primary resource and as an additional service are opened up. Large-scale battery storage system installation and operating expenses have decreased considerably in recent years. Besides, according to Cole et al. [7], by 2030, costs will be reduced by between 28%-58% relative to 2020. Because of the decreasing prices, each storage facility can accommodate more energy storage capacity. When these two trends are combined investment in battery storage systems proves to be a

key business solution in recent years. In 2020, the total amount invested in battery storage rose by about 40%, reaching USD 5.5 billion [8]. Accordingly, an increasing number of wind farms started to invest in battery systems to integrate them with their existing wind assets. E.g., in the Netherlands, a 10 MW battery energy storage system would enable a 24 MW wind farm to continue producing energy even during periods of surplus entered service in 2019 [9]. For co-location with a wind farm in Finland, a 12MW battery energy storage system is being developed and it will be completed in Spring 2023 [10].

We focus on the impact of risk considerations for a privately-held small IPP. Our modeling of wind generation dynamics is similar to that of Zhou et al. [11], except that we incorporate coherent risk measures into the model and investigate their impact on the investment and operational decisions. We consider a small IPP which owns a wind farm and planning to invest in battery storage. A transmission line with a limited capacity connects the wind farm to the market. The decision maker first determines how much battery storage to incorporate with the generation unit based on the existing capacity of the wind farm. Then, the merchant jointly operates the wind farm and battery storage throughout numerous time periods following the investment. Since we want to explore the impact of risk considerations, we model the problem with a risk-averse approach using a relatively new concept of dynamic risk measures.

Our contributions to the literature can be summarized as follows:

- We provide Markov Decision Process with a finite-horizon model under dynamic risk measures to optimize wind generation decisions,
- We present a threshold-based optimal policy structure,
- We analyze the impacts of transmission line limitation and risk-aversion on the storage investment.

Chapter 2

Literature Review

In this section, we review the studies that are relevant to our problem. The relevant literature is classified into four categories. First, we present storage sizing studies for wind power plants in Section 2.1. Studies considering the management of wind plants equipped with storage are presented in Section 2.2. Researches considering a risk-sensitive approach in aforementioned studies are presented in Section 2.3. Finally, we address the studies considering risk-sensitive MDPs in the general context in Section 2.4. As far as we are aware, no study has considered dynamic risk measures in order to simulate a risk-averse approach for a merchant wind farm with battery storage.

2.1 Storage Sizing in Wind Power Plants

Brunetto and Tina [12] study finding the optimal size of an energy storage system based on hydrogen technology that would be integrated into an existing wind farm. They formulate the problem by both linear programming and dynamic programming. Only the net revenues obtained by using the storage system are considered in the objective function. To find the optimal fuel cell and electrolyzer power and storage capability, the simulations were performed by relaxing one of

the three capacity limitations sequentially. Finally, using recent costs for hydrogen technology and electricity prices from the day-ahead market, multiple simulations are done to perform a cost-effectiveness analysis.

Castronuovo and Lopes [13] consider a merchant wind-hydro system. They work on finding optimal daily and yearly operational strategies for the wind turbines and the hydro generation pumping equipments. The optimal size of the storage is obtained by changing the penalty cost of exceeding the existing storage capacity to zero and observing the maximum capacity demanded for all intervals. By changing the generation capacity of the wind farm, profits are calculated by running a wind-hydro operation and only-wind operation are analyzed.

Brown et al. [14] formulate an LP problem to decide an optimal reservoir capacity as well as the power capacity for a pumped storage station. The annual equivalent of the investment cost and expected operating costs are minimized for several load, wind, and hydropower scenarios. Also, to make sure that the power system can control frequency adequately, a dynamic security constraint is included.

Harsha and Dahleh [15] consider finding the optimal storage size for a renewable energy plant. First, they derive the optimal storage management policy for satisfying all demand while the renewable energy generation amount, demand and prices are all stochastic processes. Then, they obtain an upper bound of the cost of the storage where it is beneficial to make an investment. Sensitivity analysis of the optimal size is also performed and the effects of constant or differential pricing of electricity, presence of ramp, and losses constraint are examined.

Korpaas et al. [16] proposed an optimal operating schedule and sizing of an unspecified storage system given a wind farm connected to the external grid via a transmission line. They assume the decision maker has a demand for electricity, and excess power can be disposed using a dumpload. Unlike our study, electricity prices, wind energy generations and demand load are taken as deterministic. Using dynamic programming, they observe the effect of the storage sizing on the annual revenue by changing the maximum energy storage level parameter.

Choi et al. [17] study the optimal sizing of an energy storage system of a wind farm connected to the market. They formulate the problem as a Markov Decision Process over a finite horizon and obtain an optimal storage management policy. In order to find an optimal storage size, they consider a two-stage optimization where in the first stage the problem is solved for a fixed storage capacity and charge/discharge capacity, and in the second stage, the optimal policy is determined. Annual expected economic value for a specific storage system is obtained by subtracting the case where the storage size is equal to zero and the storage system cost over the time horizon. Then they search for an optimal size by changing the storage size. They find that the storage size and the expected annual economic value of the storage system are positively correlated. They also compare the effect of using the optimal storage management policy they provide and a naive policy on expected annual economic value.

Qi et al. [18] consider the assignment problem of a given set of wind farms to sites and sizing the capacity of an energy storage system and transmission lines for these sites. They assume that the wind producer sells the electricity at a fixed price. They discover that even small-scale energy storage systems can significantly lower the overall estimated cost, but as their capacities increase, their marginal values start to decline more quickly than those of the transmission lines. Energy storage capacity is mostly utilized to manage overflow and does not contribute to reducing energy loss resulting from the charging/discharging efficiencies.

2.2 Management of Wind Farms with Energy Storage

Garcia-Gonzalez [19] study the operation of a wind farm with a pumped-storage plant. They formulate the problem as a two-stage stochastic program where in the first stage bids to the day ahead market are decided and in the second stage, real-time operational decisions related to the pumped-storage plant are decided.

Both cases for the wind farm and the pumped-storage place separate bids and a single bid are considered. They find that when the wind farm and the pumped-storage plant operate coordinately, expected profits will increase as the size of the pumped-storage plant increases. Also, the expected penalty cost will be decreased up to 50%.

Castronuovo and Lopes [20] study optimizing the operation of a pumped storage unit jointly with wind production. As opposed to their previous study mentioned, [13], where they consider a deterministic profile for wind power, they assume that wind power forecasting has some uncertainty. They solve the optimization problem for several scenarios and obtain daily management policies for wind turbines and the pumped storage system.

Kim and Powell [21] address the problem of making energy agreements in advance for wind farms equipped with an unspecified storage system.They assume that forecasting errors for the wind are uniformly distributed, and wind and price processes are stationary. They formulate the problem as an MDP with finite horizon, and they also obtain optimal policy structure for the infinite-horizon case. In order to understand how wind volatility, storage device capacity, and storage losses affect, they derive an formulation for the value of storage.

Jiang and Powell [22] study obtaining the optimal hour-ahead bidding strategy for wind farms with battery storage to maximize revenue. The amount of times a battery can be charged and discharged is limited in order to preserve the battery's lifetime. They formulate the problem as a finite-horizon MDP, where the bidding decisions are made every hour and settlements are done every five minutes. They state that the optimal value function satisfies a monotonicity property, and to solve the problem, they describe an algorithm called Monotone Approximate Dynamic Programming.

Zhou et al. [11] study the management of a wind farm equipped with battery storage and connected to the market via a transmission line with finite capacity. In contrast to the previously mentioned studies, they do not consider bidding. Structural analysis is done for the case where electricity prices are always positive.

They also develop three heuristics and consider the impact of negative prices.

2.3 Risk-Sensitive Energy Storage Management

Morales et al. [23] formulate a stochastic programming problem to obtain the optimal risk-averse bidding policy for a wind power producer trading in the day-ahead, adjustment, and imbalance markets. They do not consider a storage technology, yet they pave the way for detailed research for risk-averse management of energy producers. They choose Conditional Value-at-Risk (CVaR) for incorporating risk and provide a linear programming model to be conveniently solved.

Moazeni et al. [24] study optimal storage management policy under risk consideration. Using conditional value-at-risk as the risk measure, their objective is to minimize the expected cost as well as the risk while satisfying the demand. They do not consider the decision maker as a price taker, and the cost of trading with the grid through an intermediary company is included in the model as transaction costs. Price is modeled as a stochastic process. They solve the problem by using a smoothing technique to approximate non-differentiable CVaR and an augmented Lagrangian method.

There are several researches on risk-averse bidding decisions to be made by wind farm producers in different markets, formulating the risk with CVaR. In Pousinho et al. [25] bidding decisions in the day-ahead market with no storage facility are taken into consideration. Different from, Morales et al. [23], in order to develop wind and price scenarios, they use an artificial intelligence algorithm. Hosseini-Firouz [26] considers a wind farm bidding in day-ahead and adjustment markets. In addition to proposing a new technique to determine imbalance prices, he calculates the optimal amount of reserve needed to support the wind farm in the balancing market. Rodrigues et al. [27] study a wind farm equipped with an energy storage system and compare the coordinated and uncoordinated bidding strategies in day-ahead and spinning reserve markets.

The research on risk-averse operation strategies for virtual power plants is also extensive. A Virtual Power Plant (VPP) is an energy management system that aggregates the capacities of generation units and storage systems, trading on the energy markets. Moghaddam et al. [28] consider a wind farm-cascade hydro system participating in the day-ahead market. In Tajeddini et al. [29], the examined VPP consists of a wind power plant, a photovoltaic, a microturbine, a diesel engine, and a battery bank that participates in the day-ahead and balancing markets. Similarly, Dabbagh and Sheikh-El-Eslami [30], Tan et al. [31], and Gao et al. [32] study optimal risk-averse bidding and operational strategies for VPPs using CVaR to integrate risk.

Yıldıran and Kayahan [33] study optimal risk-averse day-ahead bidding and real-time operation management for a wind power producer with a pumped hydro storage system. Day-ahead bidding problem is modeled as a stochastic programming problem with CVaR chosen as the risk measure. Then, real-time operational decisions are decided through a stochastic model predictive control (SMPC) based algorithm. By comparing several methods and heuristics, they conclude that the SMPC outperforms regardless of the choice of risk-aversion degree.

Pinson et al. [34] study dynamic sizing of an electrical energy storage system. They presume storage as a service in the market, allowing producers to rent the required daily storage capacity. A distribution for the required energy content is obtained by analyzing the joint operation of the wind and storage facility for all possible scenario forecasts. Then, the power producer can adjust their level of risk by selecting a level of storage capacity that falls within a specific quantile of this distribution. They show that a major reduction in storage capacity is achieved via the use of dynamic daily sizing of the required storage without considerably decreasing the producer's profit.

2.4 Risk-Sensitive MDPs

Risk sensitivity in MDPs was initiated by Howard and Matheson [35]. Introducing an exponential utility function and a constant risk-aversion coefficient, they design an MDP to maximize the certain equivalent of the reward. Later, a discounted infinite horizon version of this formulation is extended by Porteus [36]. Jaquette [37] formulates the problem by using the utility function at each stage. Several studies consider the average cost criterion on the discrete-time risk-sensitive MDPs. MDPs with countable state spaces were studied by Hernández-Hernández and Marcus [38], Cavazos-Cadena and Fernández-Gaucherand [39], and Borkar and Meyn [40]; whereas MDPs with finite state and action spaces are studied by Cavazos-Cadena and Hernández-Hernández [41]. Bäuerle and Rieder [42] consider minimizing the certainty equivalent of the total and discounted cost over a finite and an infinite horizon and analyze both risk-averse and risk-seeking cases. Chung and Sobel [43] consider risk-sensitive discrete-time MDPs with discounting in the exponential utility function. Coraluppi and Marcus [44] investigate discrete-time, finite-state MDPs both for the finite horizon and the discounted infinite horizon using risk-sensitive and minimax criteria. Patek [45] examines MDPs with an absorbing state and compact constraint sets using an exponential risk-averse objective function and total cost criterion. Applications of risk-sensitive MDPs formulated by exponential utility functions can be found in the literature. Liu et al. [46] study the problem of optimal bidding problem for risk-averse auction agents by maximizing the expected utility of the profit for concave exponential utility functions. Barz and Waldmann [47] extend a risk-neutral single-leg revenue management problem to a risk-averse problem using an exponential utility function.

Another way to incorporate risk into MDPs is by formulating the objective function to minimize probability risk. In other words, the total discounted rewards not exceeding a specified value target is ensured. This concept is introduced by Wu and Lin [48], and discounted and undiscounted cases are studied by Ohtsubo and Toyonaga [49] and Ohtsubo [50], respectively. Ohtsubo [51] considers minimizing risk model in a stochastic shortest path problem. Boda et al. [52]

consider optimal management of a retirement savings account. They study the problem for a finite time horizon with a target hitting time criteria.

Filar et al. [53] consider risk-aversion in MDPs with both average reward criterion and total discounted reward criterion by using variance-based penalties in the objective function. In Bäuerle and Ott [54], the problem of minimizing the AVaR of the discounted cost is considered. They study both the finite and infinite horizon problems. They propose dynamic programming algorithm to solve the problem. In Carpin et al. [55], the undiscounted total cost criterion is used together with the AVaR metric. Borkar and Jain [56] propose a finite-horizon constrained MDP model to maximize expected total reward with a CVaR constraint imposed on the terminal cost. They also develop offline and online stochastic approximation algorithms. Haskell and Jain [57] use a convex analytic approach to solve risk-aware MDPs. They use both the expected utility approach and CVaR to incorporate risk. Also, they consider expressing risk in the constraints based on stochastic dominance and chance constraints. In most cases, they obtain a linear programming problem or an approximation by an LP sequence to formulate the problem. Chow et al. [58] show that for a given error budget, a CVaR objective can also be interpreted as the expected cost under the worst-case (robust) modeling errors, in addition to reflecting risk sensitivity. They also propose an approximate value-iteration algorithm for CVaR MDPs and analyze its convergence rate. Xia [59] study maximization of mean minus variance metrics in MDPs. The theory of sensitivity-based optimization is utilized to examine this problem from a novel angle. The performance difference formula is developed to directly quantify the difference between any two policies' mean-variance combination metrics and the necessary condition for the optimal policy is derived. They propose a policy iteration-type algorithm to find the optimal policy and prove its convergence. They also conduct a numerical experiment on reducing the power fluctuation in a wind power equipped with a battery storage system to show the effectiveness of their approach.

Delage and Mannor [60] propose a chance-constrained model for infinite-horizon MDPs with finite state and action spaces. They consider reward and transition probability uncertainty separately. Their formulation corresponds to

the Value-at-Risk (VaR) risk measure. Meraklı and Küçükkyavuz [61] suppose that both action costs and transition probabilities are subject to uncertainty and have a joint distribution. They apply their proposed models and solution methodology to an inventory management problem for slow-onset disaster relief. Xu et al. [62] use VaR and CVaR to formulate a safety constraint for a maintenance model. They obtain risk-aware maintenance policies.

Riedel [63] introduces dynamic risk measures and the axiom of dynamic (time) consistency. Later, Ruszczyński [64] extends this theory into sequential optimization problems and proves that time-consistent dynamic risk measures can be written as compositions of one-step conditional risk measures. He formulates risk-averse MDPs for both finite horizon and discounted infinite horizon settings by introducing Markov risk measures. He develops a risk-averse value iteration algorithm for both problems and a risk-averse policy iteration algorithm for the discounted infinite horizon problem and proves its convergence. Çavuş and Ruszczyński [65] extend the formulation for undiscounted transient Markov decision process with infinite state space and compact action sets. They demonstrate their theory on an asset selling problem and an organ transplantation problem. Çavuş and Ruszczyński [66] develop computational methods to solve risk-averse MDPs with one-step conditional risk measures. They develop value and policy iteration algorithms and prove their convergence. Jiang and Powell [67] consider the problem of minimizing a quantile-based risk measure of the sequence of future costs at each stage for a finite-horizon MDP. They propose approximate dynamic programming (ADP) algorithm to solve the problem. They illustrate their algorithm on a risk-averse bidding problem with an energy storage system. Bäuerle and Glauner [68] consider MDPs with both finite horizon and infinite horizon discounted cost criteria using dynamic risk measures. They use Borel state and action spaces and unbounded cost functions. They identify the required properties of risk measures to prove the validity of the Bellman equation.

Our research considers both the capacity sizing problem and the problem of finding the optimal coordinated management policy. We consider an established wind farm to be equipped with battery storage. We model the wind and price as stochastic processes. The price process is modeled using the method in Jiang and

Powell [22] consisting of normal and spike regimes. We assume that the prices are always positive. Several researches make use of battery storage in order to find the optimal bidding strategy. However, similar to Zhou et al. [11], we do not consider bidding in our problem. We formulate the model as a finite-horizon MDP. In order to incorporate risk into our model, we use dynamic risk measures. We discretize the problem and solve the exact model with finite state and action spaces by backward dynamic programming algorithm.

Table 2.1: Comparison of selected studies

	Optimization			Formulation		
	Management Policy	Storage Sizing	Storage Type	Deterministic	Stochastic	Risk-Aversion
Brunetto and Tina [12]		✓	hydrogen-based	✓	✓	
Castronuovo and Lopes [13]	✓	✓	pumped-hydro	✓		
Harsha and Dahleh [15]	✓	✓	unspecified		✓	
Korpaas et al. [16]	✓	✓	unspecified	✓		
Choi et al. [17]	✓	✓	battery		✓	
Garcia-Gonzalez et al. [19]	✓		pumped-hydro		✓	
Castronuovo and Lopes [20]	✓		pumped-hydro		✓	
Kim and Powell [21]	✓		unspecified	✓		
Jiang and Powell [22]	✓		battery	✓		
Zhou et al. [11]	✓		battery		✓	
Moazeni et al. [24]	✓		unspecified		✓	✓
Rodrigues et al. [27]	✓		unspecified		✓	✓
Moghaddam et al. [28]	✓		cascade-hydro		✓	✓
Tajeddini et al. [29]	✓		battery		✓	✓
Yıldıran and Kayahan [33]	✓		pumped-hydro		✓	✓
Pinson et al. [34]		✓	electrical ESS		✓	✓
Our Study	✓	✓	battery		✓	✓

Chapter 3

Preliminaries

In the present section, we will provide the dynamic risk measure concept proposed in [64]. For a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, let $J : \Omega \to \mathbb{R}$ be the random cost and $\mathcal{J} = \mathcal{L}_p(\Omega, \mathcal{F}, \mathbb{P})$ be the space of possible outcomes for $p \in [1, \infty]$. Remember that, as stated in [69], a risk measure $\rho : \mathcal{J} \to \mathbb{R}$ is a coherent risk measure on $\mathcal{J}$ if it satisfies the following four axioms:

$$\rho(\alpha X + (1-\alpha)Y) \leq \alpha\rho(X) + (1-\alpha)\rho(Y),\ \forall X, Y \in \mathcal{J} \text{ and } \forall \alpha \in [0,1] \quad (3.1)$$

$$\text{If} X \preceq Y,\ \text{then } \rho(X) \leq \rho Y,\ \forall X, Y \in \mathcal{J} \quad (3.2)$$

$$\rho(a + X) = a + \rho(X),\ \forall X \in \mathcal{J} \text{ and } a \in \mathbb{R} \quad (3.3)$$

$$\rho(\beta X) = \beta\rho(X),\ \forall X \in \mathcal{J} \text{ and } \forall \beta \geq 0 \quad (3.4)$$

(3.1)-(3.4) correspond to convexity, monotonicity, translation invariance, and positive homogeneity axioms in order. Consider filtration $\mathcal{F}_1 \subset \mathcal{F}_2 \subset ... \subset \mathcal{F}_T \subset \mathcal{F}$. Let J_t be the random cost for $t \in 1, ..., T$. Assume that, $\mathcal{F}_1 = \{\emptyset, \Omega\}$, which is the trivial σ-algebra. Then, introduce the spaces of possible outcomes as $\mathcal{J}_t = \mathcal{L}_p(\Omega, \mathcal{F}, \mathbb{P})$ for $p \in [1, \infty]$ and $t \in 1, ..., T$. Let $\mathcal{J}_{t,T} = \mathcal{J}_t \times ... \times \mathcal{J}_T$. A mapping $\rho_{t,T} : \mathcal{J}_{t,T} \to \mathcal{J}_t$ for $1 \leq t \leq T$ satisfying the monotonicity axiom is described as a conditional risk measure. A sequence of conditional risk measures for $t = 1, ..., T$ is defined as dynamic risk measure. One-step conditional risk measure $\rho_t : \mathcal{J}_{t+1} \to \mathcal{J}_t$, $t = 1, ..., T-1$ is defined as $\rho_t(J_{t+1}) = \rho_{t,t+1}(0, J_{t+1})$. Then, by

following the recursive equation, a dynamic risk measure can be represented as below.

$$\rho_{t,T}(J_t, ..., J_T) = J_t + \rho_t(J_{t+1} + \rho_{t+1}(J_{t+2} + ... + \rho_{T-1}(J_T)...))) \tag{3.5}$$

Mean-semi deviation risk measure is an important example of one-step conditional risk measures. Mean-semi deviation of order p for $\kappa = [0, 1]$ can be represented as follows:

$$\rho(J_{t+1}) = \mathbb{E}[J_{t+1}|\mathcal{F}_t] + \kappa\mathbb{E}[((J_{t+1} - \mathbb{E}[J_{t+1}|\mathcal{F}_t])_+)^p|\mathcal{F}_t]^{\frac{1}{p}} \tag{3.6}$$

Note that, $\kappa = 0$ represents the risk-neutral case, while $\kappa = 1$ represents the most risk-averse case. Risk-aversion degree increases as order p or κ increases. We incorporate risk into our problem by using first-order mean semi-deviation risk measure.

Consider a Markov decision process, defined by a state space $x_t \in \mathcal{X}$, an action space $a_t \in \mathcal{A}$, and cost functions r_t, for $t = 1, 2, ..., T + 1$. Remember that a sequence of decisions π_t is called as a policy, i.e. $\Pi = \{\pi_t\}_{t=1}^T$. Stage-wise costs are denoted by J_t. Each policy Π corresponds to a cost sequence $J_t = r_t(x_t, a_t)$, $t = 1, 2, ..., T$, and $J_{T+1} = r_{T+1}(x_{T+1})$. Using one-step conditional risk measures $\rho_t : \mathcal{J}_{t+1} \to \mathcal{J}_t$, a dynamic measure of risk for a policy Π is defined as below.

$$\begin{aligned} D(\Pi, x_1) = r_1(x_1, a_1) + \rho_1(r_2(x_2, a_2) + \rho_2(r_3(x_3, a_3) + ... \\ +\rho_{T-1}(r_T(x_T, a_T) + \rho_T(x_{T+1})))...)) \end{aligned} \tag{3.7}$$

Then, it is shown that the optimal value functions defined for a fixed $x_t = x$ can be formulated as below.

$$
\begin{aligned}
v_t(x) = \min_{\pi_t,\ldots,\pi_T} = \{ & r_t(x, a_t) + \rho_t(r_2(x_{t+1}, a_{t+1}) + \ldots \\
& + \rho_{T-1}(r_{T-1}(x_T, a_T) + \rho_T(r_{T+1}(x_{T+1})))\ldots)\}.
\end{aligned}
\tag{3.8}
$$

Chapter 4

Problem Definition and Formulation

We model a risk-averse wind farm interested in investing in battery storage connected to the market via a transmission line with a finite capacity. We assume that the wind farm investment has been completed. At time $t = 0$, given the existing capacity of the wind farm, the investor decides how much battery storage to integrate with the wind farm. After the investment, the firm jointly operates the wind farm and battery storage over multiple periods to optimize its risk-averse utility function. Using dynamic risk measures, we formulate the model as a Markov decision process with a finite horizon. Following the battery investment, at each time $t \in \mathcal{T} := \{1, ..., T\}$, the merchant will make a decision on how much energy to generate from the wind turbine, purchase or sell electricity from the market, and charge or discharge the battery. We assume that the wind farm is a small player, so its decisions do not affect the market prices. A graphical representation of the wind-storage system is depicted in Figure 4.1. Note that dashed lines represent the energy flow.

Decisions are made at the beginning of each period t. The timeline of the decision-making procedure can be seen in Figure 4.2. Buying and selling electricity in a single period is not allowed. In addition, we suppose that the efficiency of

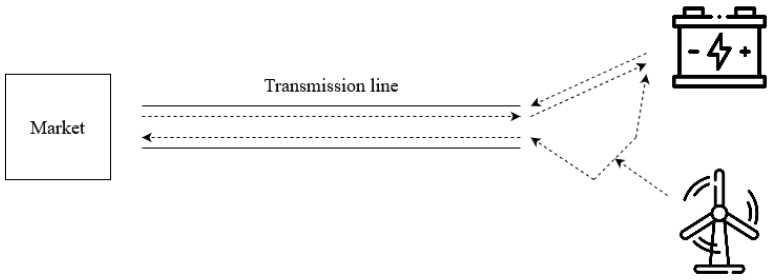


Figure 4.1: Visual illustration of the wind-storage system

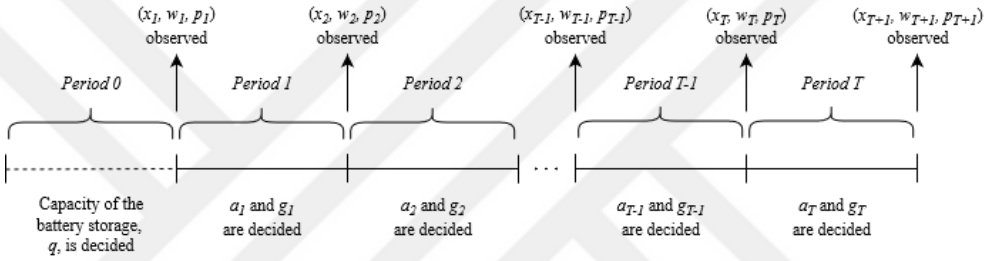


Figure 4.2: Graphical description of the decision-making procedure

charging and discharging will remain constant during the entire problem horizon.

A summary of the notation used is presented in Table 4.1.

4.1 Parameters

The unit cost of the battery storage per MW capacity will be denoted as c. This cost will be normalized according to the problem horizon and battery life. The battery can be charged and discharged only to some extent in a single period. We will define the maximum charge and discharge amounts in a period by θ_c, θ_d, respectively.

Wind turbines have a finite capacity that limits the amount of energy they can generate. We will denote the generation capacity in a single period as θ_g, assuming $\theta_g \geq 0$. θ_{tr} indicates the maximum amount of energy that can flow in the transmission line, assuming $\theta_{tr} > 0$. Also, we assume that $\theta_g + \theta_d \geq \theta_{tr}$. Otherwise, the transmission capacity will not be constraining. Charging and

Table 4.1: Notation

Decision Variables	
q	Energy capacity of the battery storage (MWh)
a_t	Inventory change decision in period t (MWh)
g_t	Generation decision in period t (MWh)
State Variables	
x_t	Electricity inventory of the battery storage at the beginning of period t (MWh)
w_t	Available wind energy in period t (MWh)
$\vec{p}_t$	Price-component pair in period t
Parameters	
c	Unit cost of the battery storage (\$/MWh)
θ_c	Charging capacity of the battery storage (MW)
θ_d	Discharging capacity of the battery storage (MW)
θ_g	Generation capacity of the wind farm (MWh)
θ_{tr}	Capacity of the transmission line (MW)
α	Charging efficiency of the battery storage
β	Discharging efficiency of the battery storage
τ	Transmission efficiency of the transmission line

discharging the battery are subject to some energy loss. Let α, β denote the efficiencies of the storage facility for charging and discharging, respectively. Note that $\alpha, \beta \in (0, 1]$. Finally, the transmission efficiency of the transmission line is denoted by τ, where $\tau \in (0, 1]$.

4.2 State Variables

The merchant will observe the electricity inventory of the battery storage at the beginning of period t, denoted by x_t. The domain of x_t is $\mathcal{X} := [0, q]$, where q is the capacity of the storage facility decided on Period 0. Wind energy availability, or the amount of electricity that can be generated by wind turbines in period t, is denoted by w_t. Note that the domain of w_t is $\mathcal{W} := [0, \theta_g]$, since generation capacity will limit the generation amount. $\vec{p}_t$ denotes the relevant price information. It consists of two elements, namely, regime type and error level. The price of electricity in period t is a function of time t and the combination of the seasonality factor and the $\vec{p}_t$ explained in Section 6.1, $\vec{p}_t \in \mathcal{P} \subseteq \mathbb{R}^2$. We assume

that electricity prices are always positive. State at time t consists of these three components and can be denoted by $S_t = (x_t, w_t, \vec{p}_t)$. Note that the state space, $\mathcal{S} := \mathcal{X} \times \mathcal{W} \times \mathcal{P}$.

4.3 Decision Variables

In period 0, the energy capacity of the storage facility, q will be decided where $q > 0$. After making a decision about q, starting from period 1 in each period t, the decision maker will evaluate the system state S_t, and then make the decisions regarding storage and generation, denoted by a_t and g_t. a_t denotes the inventory change action, $a_t \in \mathbb{R}$. Charging the storage corresponds to a_t having positive values, whereas discharging storage corresponds to a_t having negative values. Generation amount in period t is denoted by $g_t \in \mathbb{R}^+$. A positive generation amount in a period can be used to charge the battery and/or sold through the transmission line.

4.4 Transition Functions

From period t to $t+1$, storage inventory level variable, x_t, changes by the amount of inventory change action a_t as $x_{t+1} = x_t + a_t$. w_t and $\vec{p}_t$, changes according to Markovian stochastic processes which are explained in detail in Section 6.1 and 6.2.

4.5 Constraints

We defined the immediate revenue function as below. $P_t(\vec{p}_t)$ denotes the price considering the seasonality factor, given that the price state is $\vec{p}_t$, which consists of a regime component and an error component.

$$R(a_t, g_t, \vec{p}_t) := \begin{cases} -P_t(\vec{p}_t)(a_t/\alpha - g_t)/\tau & \text{if } a_t > \alpha g_t, \\ P_t(\vec{p}_t)(g_t - a_t/\alpha)\tau & \text{if } 0 \leq a_t \leq \alpha g_t, \\ P_t(\vec{p}_t)(g_t - \beta a_t)\tau & \text{if } a_t < 0. \end{cases} \tag{4.1}$$

The first case in (4.1) represents purchasing cost since changing inventory level being greater than the generated energy requires purchasing energy from the market. The second case represents selling revenue, when the inventory change level is positive and less than generated energy, excess energy is sold. The third case represents selling revenue from selling discharged energy.

$$[(a_t/\alpha - g_t)/\tau - \theta_{tr}]\mathbb{1}(a_t > \alpha g_t) \leq 0 \tag{4.2}$$

$$(g_t - a_t/\alpha - \theta_{tr})\mathbb{1}(0 \leq a_t \leq \alpha g_t) \leq 0 \tag{4.3}$$

$$(g_t - \beta a_t - \theta_{tr})\mathbb{1}(a_t \leq 0) \leq 0 \tag{4.4}$$

Constraints (4.2)-(4.4) ensure that the transmission capacity is not exceeded.

$$g_t \leq w_t \tag{4.5}$$

Constraint (4.5) represents that the generation level cannot exceed available wind energy.

$$-x_t \leq a_t \leq q - x_t \tag{4.6}$$

$$-\theta_d \leq a_t \leq \theta_c \tag{4.7}$$

Constraints (4.6) and (4.7) specify the limits of the inventory change action. The inventory change level cannot be less than the negative of the energy that

is available in the storage. It also cannot be greater than the remaining storage energy capacity. Also, the inventory change level will be restricted between the charging and the discharging capacities of the storage facility.

4.6 Objective Function

Using the cost functions $R(a_t, g_t, \vec{p}_t)$, and one-step conditional risk measure ρ_t, the dynamic measure of risk of our problem is constructed as our objective function. It is similar to the formulation given in (3.7).

$$\min cq + \rho_0(-R(a_1, g_1, \vec{p}_1) + \rho_1(-R(a_2, g_2, \vec{p}_2) + \rho_2(-R(a_3, g_3, \vec{p}_3) + ... + \rho_{T-1}(-R(a_T, g_T, \vec{p}_T))...))) \quad (4.8)$$

Then, using finite horizon formulation in [64], demonstrated in (3.8), we construct the optimal value function as follows:

$$V_0(w_0, \vec{p}_0) = \min_q \{cq + \rho_0(V_1(0, w_1, \vec{p}_1))\}, \quad (4.9)$$

$$V_t(x_t, w_t, \vec{p}_t) = \min_{(a_t, g_t) \in \Psi(x_t, w_t, q)} \{-R(a_t, g_t, \vec{p}_t) + \rho_t(V_{t+1}(x_t + a_t, w_{t+1}, \vec{p}_{t+1}))\}, \quad \forall t \in \mathcal{T} \quad (4.10)$$

where $\Psi(x_t, w_t, q)$ denotes action pairs set (a_t, g_t) in $\mathbb{R} \times \mathbb{R}^+$ satisfying the constraints (4.2)-(4.7) according to the decision q made at stage 0. Also, $V_{T+1}(S_{T+1}) = 0$ for each $S_{T+1} \in \mathcal{S}$.

When the first-order mean-semi deviation is used as our dynamic risk measure, the optimal value function will take the form below:

$$V_t(x_t, w_t, \vec{p}_t) = \min_{(a_t, g_t) \in \Psi(x_t, w_t, q)} \{-R(a_t, g_t, \vec{p}_t) + \mu + \kappa \mathbb{E}[[V_{t+1}(x_t + a_t, w_{t+1}, \vec{p}_{t+1}) - \mu]_+ | S_t]\}, \quad \forall t \in \mathcal{T} \quad (4.11)$$

where $\mu = \mathbb{E}[V_{t+1}(x_t + a_t, w_{t+1}, \vec{p}_{t+1})|S_t], \quad \kappa \in [0, 1]$.

Chapter 5

Structural Analysis

In this chapter, the structural properties of the optimal policy will be examined. We will begin with an assumption.

Assumption 5.1 *We have* $\mathbb{E}[|P_v(\vec{(p)}_v)||\vec{p}_t] < \infty$ *for each* $t, v \in \mathcal{T}$, $v \geq t$, *and* $\vec{p}_t \in \mathcal{P}$.

We state in Proposition 5.1 that the optimal value function in the inventory level is convex, given all the other state variables.

Proposition 5.1 *Suppose* $P_t(\vec{p}_t) > 0$ *for each* $t \in \mathcal{T}$ *and* $\vec{p}_t \in \mathcal{P}$. *For each* $t \in \mathcal{T}$, $V_t^*(x_t, w_t, \vec{p}_t)$ *is convex in* x_t *for each particular* $(w_t, \vec{p}_t) \in \mathcal{W} \times \mathcal{P}$.

Proof of Proposition 5.1 can be seen in Section A.1. Let y_t denote the inventory level of its subsequent period. Meaning that, $y_t := x_t + a_t$. Optimal continuation function in period t is defined as $O_t(\cdot, w_t, \vec{p}_t) := \rho_t[V_{t+1}(\cdot, w_{t+1}, \vec{p}_{t+1})]$.

$$X_t^{(1)}(w_t, \vec{p}_t) := \arg\min_{y_t \in \mathcal{X}} \{O_t(y_t, w_t, \vec{p}_t) + P_t(\vec{p}_t) y_t / (\alpha\tau)\} \tag{5.1}$$

$$X_t^{(2)}(w_t, \vec{p}_t) := \arg\min_{y_t \in \mathcal{X}} \{O_t(y_t, w_t, \vec{p}_t) + P_t(\vec{p}_t)\tau y_t/\alpha\} \tag{5.2}$$

$$X_t^{(3)}(w_t, \vec{p}_t) := \arg\min_{y_t \in \mathcal{X}} \{O_t(y_t, w_t, \vec{p}_t) + P_t(\vec{p}_t)\beta\tau y_t\} \tag{5.3}$$

Fixing $\vec{p}_t \in \mathcal{P}$, we will find $a_t^*(S_t)$ for each region Γ^0, Γ^1 and Γ^2, which are defined as below:

$$\Gamma^0 := \{(x_t, w_t) \in \mathcal{X} \times \mathcal{W} : w_t \geq \theta_{tr} + \min\{q - x_t, \theta_c\}/\alpha\} \tag{5.4}$$

$$\Gamma^1 := \{(x_t, w_t) \in \mathcal{X} \times \mathcal{W} : \theta_{tr} \leq w_t < \theta_{tr} + \min\{q - x_t, \theta_c\}/\alpha\} \tag{5.5}$$

$$\Gamma^1 := \{(x_t, w_t) \in \mathcal{X} \times \mathcal{W} : 0 \leq w_t < \theta_{tr}\} \tag{5.6}$$

Note that, these regions partition $\mathcal{X} \times \mathcal{W}$. Then, the optimal policy structure is explained in Proposition 5.2.

Proposition 5.2 *For each $t \in \mathcal{T}$, suppose that $P_t(\vec{p}_t) > 0$. Then, for each $t \in \mathcal{T}$, it holds that (i) $X_t^{(1)} \leq X_t^{(2)} \leq X_t^{(3)}$, (ii) $g_t^*(S_t) = \min\{w_t, \theta_{tr} + \min\{q - x_t, \theta_c\}/\alpha\}$, (iii) $a_t^*(S_t) = \min\{q - x_t, \theta_c\}$ if $(x_t, w_t) \in \Gamma^0$;*

$$a_t^*(S_t) = \begin{cases} \min\{X_t^{(1)} - x_t, \alpha(\tau\theta_{tr} + w_t), \theta_c\} & \text{if } x_t \in [0, X_t^{(1)} - \alpha w_t], \\ \min\{X_t^{(2)} - x_t, \alpha w_t\} & \text{if } x_t \in (X_t^{(1)} - \alpha w_t, X_t^{(2)} - \alpha(w_t - \theta_{tr})], \\ \alpha(w_t - \theta_{tr}) & \text{if } x_t \in (X_t^{(2)} - \alpha(w_t - \theta_{tr}), q], \end{cases}$$

if $(x_t, w_t) \in \Gamma^1$, and $\alpha w_t < \min\{X_t^{(1)}, \theta_c\}$;

$$a_t^*(S_t) = \begin{cases} \min\{X_t^{(2)} - x_t, \alpha w_t\} & \text{if } x_t \in [0, X_t^{(2)} - \alpha(w_t - \theta_{tr})], \\ \alpha(w_t - \theta_{tr}) & \text{if } x_t \in (X_t^{(2)} - \alpha(w_t - \theta_{tr}), q], \end{cases}$$

if $(x_t, w_t) \in \Gamma^1$, $\alpha(w_t - \theta_{tr}) < X_t^{(2)}$, *and either* $X_t^{(1)} \leq \alpha w_t < \theta_c$ *or* $\alpha w_t \geq \theta_c$;

$$a_t^*(S_t) = \alpha(w_t - \theta_{tr}),$$

if $(x_t, w_t) \in \Gamma^1$, $\alpha(w_t - \theta_{tr}) \geq X_t^{(2)}$, *and either* $X_t^{(1)} \leq \alpha w_t < \theta_c$ *or* $\alpha w_t \geq \theta_c$;

$$a_t^*(S_t) = \begin{cases} \min\{X_t^{(1)} - x_t, \alpha(\tau\theta_{tr} + w_t), \theta_c\} & \text{if } x_t \in [0, X_t^{(1)} - \alpha w_t], \\ \min\{X_t^{(2)} - x_t, \alpha w_t\} & \text{if } x_t \in (X_t^{(1)} - \alpha w_t, X_t^{(2)}], \\ 0 & \text{if } x_t \in (X_t^{(2)}, X_t^{(3)}], \\ \max\{X_t^{(3)} - x_t, (w_t - \theta_{tr})/\beta, -\theta_d\} & \text{if } x_t \in (X_t^{(3)}, q], \end{cases}$$

if $(x_t, w_t) \in \Gamma^2$, *and* $\alpha w_t < \min\{X_t^{(1)}, \theta_c\}$;

$$a_t^*(S_t) = \begin{cases} \min\{X_t^{(2)} - x_t, \alpha w_t\} & \text{if } x_t \in [0, X_t^{(2)}], \\ 0 & \text{if } x_t \in (X_t^{(2)}, X_t^{(3)}], \\ \max\{X_t^{(3)} - x_t, (w_t - \theta_{tr})/\beta, -\theta_d\} & \text{if } x_t \in (X_t^{(3)}, q], \end{cases}$$

if $(x_t, w_t) \in \Gamma^2$, *and either* $X_t^{(1)} \leq \alpha w_t < \theta_c$ *or* $\alpha w_t \geq \theta_c$.

The optimal policy structure has a similar pattern to that of Zhou et al. [11]. However, in our study, the optimal continuation function (O_t) formulation is not the same since we use a risk measure. Then, calculations of $X_t^{(1)}, X_t^{(2)}, X_t^{(3)}$, and hence the threshold values differ from [11].

Chapter 6

Computational Study

In this section, we will describe the settings of the computational study we conduct and present the results with their interpretations. In Section 6.1 and 6.2, we explain the characteristics of the price process and wind process, respectively. In Section 6.3, we provide the data used in this study and numerical results with our managerial insights will be presented in Section 6.4.

6.1 Price Process

We have implemented second variation of the price process considered in Jiang and Powell [22]. For each hour of the day $t \in \{1, ..., 24\}$, let Reg_t denote the price regime as below.

$$Reg_t = \begin{cases} 0, & \text{normal regime} \\ 1, & \text{spike regime} \end{cases} \tag{6.1}$$

Let $Se(t)$ be the deterministic seasonal component, ϵ_t, ϵ_t^s denote discrete i.i.d. random variables which represent the error in normal regime and spike regime respectively. Price process P_t is represented as follows:

$$P_t = Se(t) + \mathbb{1}_{\{Reg_t=0\}}\epsilon_t + \mathbb{1}_{\{Reg_t=1\}}\epsilon_t^s \tag{6.2}$$

Deterministic seasonality component is expressed as a sinusoidal function.

$$Se(t) = 15\sin(2\pi t/12) + 20 \tag{6.3}$$

Transition probabilities of the Markov chain Reg_t are defined as follows:

$$p_{ij}(t) = \mathbb{P}(Reg_{t+1} = j | Reg_t = i) \tag{6.4}$$

Let p_t denotes the probability of transitioning from the normal regime to the spike regime, $p_t = p_{0,1}(t)$, and q_t from the spike regime to the normal regime, $q_t = p_{1,0}(t)$. p_t is defined for a constant chosen $\lambda_p \leq 1$ as below whereas q_t is taken as a constant λ_q.

$$p_t = \lambda_p[\sin(2\pi t/12) + 1]/2, \quad p_t \in [0, \lambda_p] \tag{6.5}$$

$$q_t = \lambda_q \tag{6.6}$$

Error terms ϵ_t and ϵ_t^s are assumed to have the same finite support with 6 elements and have discrete pseudo-normal distribution. Price process parameters for the low, average, and high volatility cases can be further examined in Table 6.1.

Table 6.1: Price process parameters

Parameters	Low	Average	High
Support	$\{13.5, 14, ..., 16\}$	$\{12, 14, ..., 22\}$	$\{6, 14, ..., 46\}$
$(\mu_\epsilon, \sigma_\epsilon)$	$(14, 0.5)$	$(14, 2)$	$(14, 8)$
$(\mu_{\epsilon^s}, \sigma_{\epsilon^s})$	$(14.75, 1)$	$(17, 4)$	$(26, 16)$
λ_p	0.8	0.8	0.8
λ_q	0.7	0.5	0.3

6.2 Wind Process

We formulated the wind process as a discrete pseudo-normal distribution with parameters $(\mu_w, \sigma_w) = (0.75, 0.2)$, $(\mu_w, \sigma_w) = (0.75, 0.5)$ and $(\mu_w, \sigma_w) = (0.75, 0.8)$ for low, average and high wind volatility respectively. We assumed that the w_t has support as follows:

$$\mathcal{W} = \{0.125, 0.25, 0.375, 0.5, 0.625, 0.75, 0.875, 1, 1.125, 1.25, 1.375\} \tag{6.7}$$

6.3 Experiment Settings

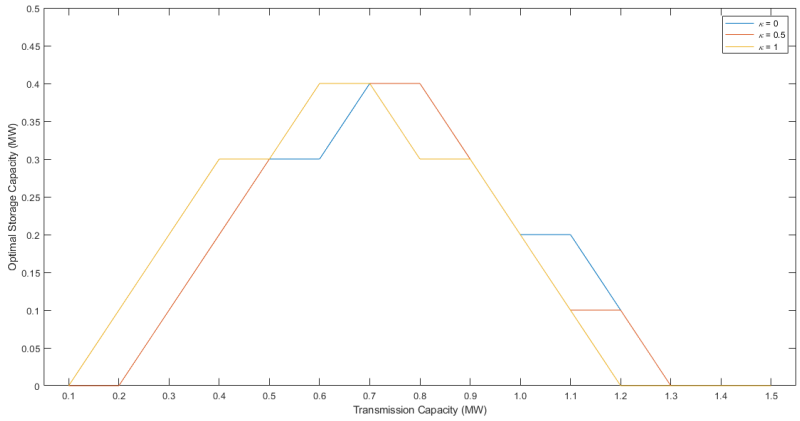
We set the horizon as $T = 6$, each period representing 1 hour. We discretize the inventory state space between 0 and q by 0.1 MW increments, i.e. $\mathcal{X} = \{0, 0.1, ..., q\}$ where $q \in \{0, 0.1, 0.2, ..., 1\}$ MW. We consider a battery that can be fully charged or discharged in 1 hour. Available wind energy state space is taken as $\mathcal{W} = \{0.125, 0.25, 0.375, 0.5, 0.625, 0.75, 0.875, 1., 1.125, 1.25, 1.375\}$. We also consider the transmission line capacity between 0.1 and 1.5 MW with 0.1 MW increments, i.e. $\theta_{tr} \in \{0.1, 0.2, ..., 1.5\}$ MW. Following the same approach in Zhou et al. [11], to find the optimal policy, we consider discretized action pairs as well as extreme points of the feasible region. For the case when the action pair is not from the discretized action space, to determine the value of the optimal continuation function O_t, we use linear interpolation between the two discretized inventory levels that are adjacent to the resulting non-discrete next-stage inventory level. When alternative optimal action pairs are available, we choose the pair that leads to the highest inventory level in the subsequent stage. We examine both the risk-neutral and risk-averse problems by selecting $\kappa \in \{0, 0.5, 1\}$. The lifetime of the battery is assumed to be 10 years, and unit cost of the investment is rationed according to the problem horizon. Values of the parameters are presented in Table 6.2.

Table 6.2: Values of the parameters

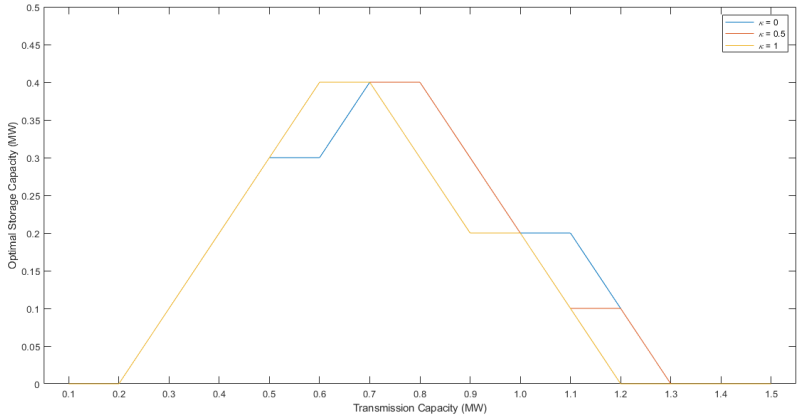
Parameters	Values
c	12.9315 \$/MW
θ_c	q
θ_d	q
θ_g	1.5 MW
α	0.85
β	1
τ	0.97

6.4 Numerical Results

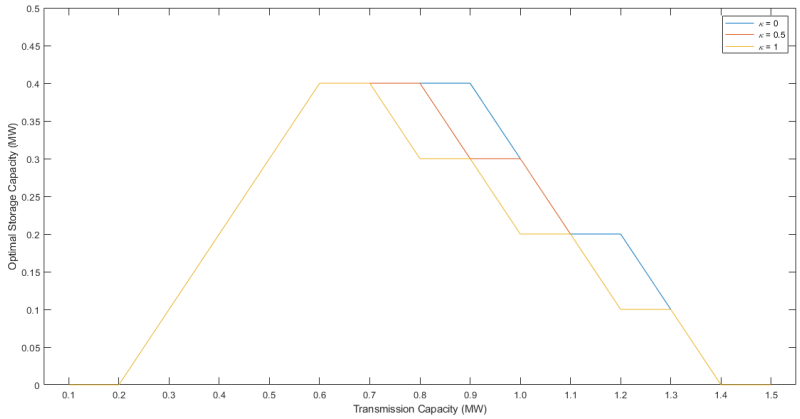
In this section, numerical results and managerial insights are delivered. We solve the problem by backward dynamic programming algorithm coded in Python 3.8. In order to find all of the extreme points, the Double Description Method described in Motzkin et al. [70] is used. Optimal battery size q for each instance is found by linearly searching among the values $q \in \{0, 0.1, 0.2, ..., 1\}$. For each transmission line capacity and κ value, optimal battery capacity q can be seen in Figures 6.1 and 6.2. In all of the graphs, we can observe that the optimal storage capacity follows a version of a bell curve. Since we employ a discretized version of our problem, it cannot follow the bell shape perfectly, but we can see that optimal storage capacity increases, remains constant, and then decreases as transmission line capacity increases. When transmission line capacity is less than or equal to 0.7 MW, the optimal battery capacity for the risk-averse problem is consistently higher than or equal to the other two cases. When transmission line capacity exceeds 0.7 MW, however, this pattern is reversed, and optimal battery capacity lowers as the degree of risk-aversion increases. We can deduce that, for low transmission line capacities, risk-averse decision maker seeks to invest in additional capacity to avoid the transmission line's limitation. Whereas, when there is sufficient transmission line capacity, a risk-averse decision maker will invest in less capacity since he is more concerned with the investment cost. In Figure 6.1, the effect of the price volatility on the optimal battery capacity can be observed. When we compare Figures 6.1b and 6.1c, we can see that as price volatility increases, the optimal battery size rises. In Figure 6.2, we can observe the wind volatility on the optimal battery capacity. As the volatility of the available wind energy increases, optimal battery capacity increases generally.



(a) Low price volatility

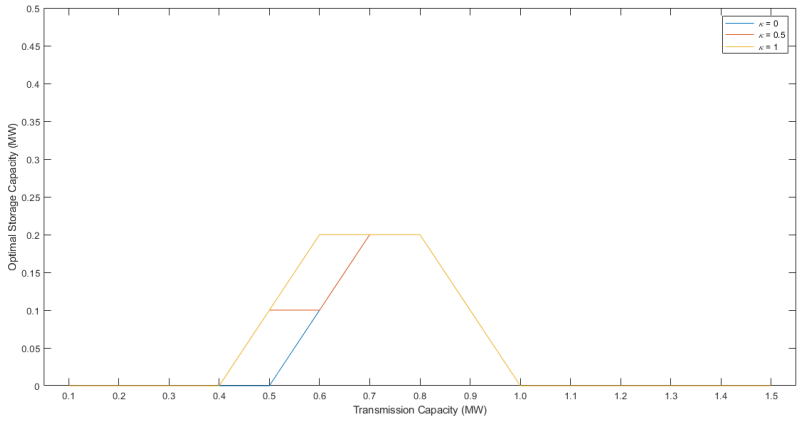


(b) Average price volatility

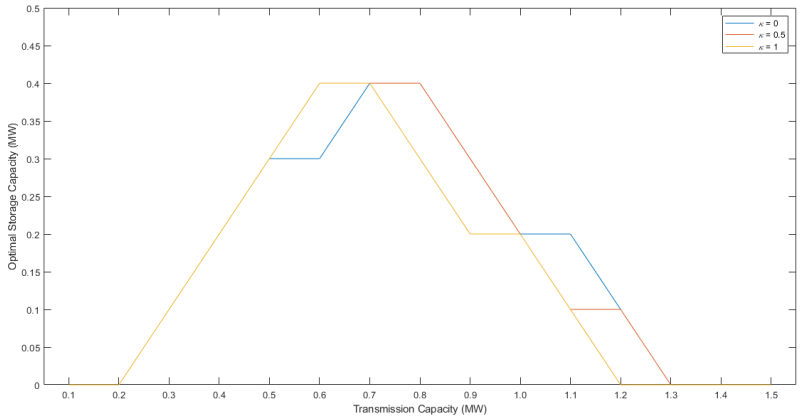


(c) High price volatility

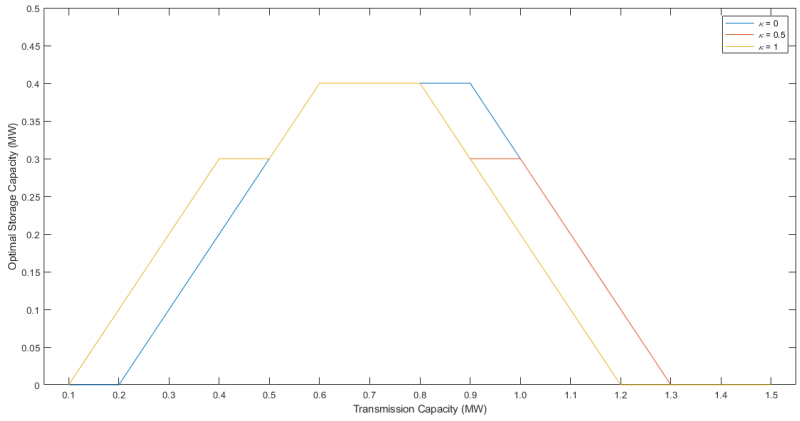
Figure 6.1: Optimal battery sizes under varying price parameters



(a) Low wind volatility



(b) Average wind volatility



(c) High wind volatility

Figure 6.2: Optimal battery sizes under varying wind parameters

Chapter 7

Conclusion

In this thesis, we study the optimal sizing of the battery storage and the coordinated management of the wind plant with battery storage. Our system is subject to wind power and electricity price uncertainty. We formulate the problem as a Markov decision process and model the risk-averse behavior by using dynamic risk measures.

We show that the model is convex and that optimal policy has a threshold-based structure. We model the price uncertainty using a two-regime system in [22]. We solve the problem by applying backward dynamic programming algorithm and find the optimal battery size to invest in. Also, the effects of risk-aversion and the transmission line capacity on the optimal battery size are analyzed. We show that the optimal battery size for varying transmission line capacities follows a bell-shaped pattern. We conclude that risk aversion and optimal investment capacity are positively correlated for low transmission line capacities but negatively correlated for high transmission line capacities. Also, optimal battery capacity generally increases as the variability of available wind energy increases.

For future research, we can consider risk-averse bidding problem in our setting using dynamic risk measures. Also, alternative renewable energy systems such

as solar and hydropower can be integrated into our model. More complex wind and price scenario generation techniques can be used to represent the uncertainty more accurately. Since the problem size is substantial for an exact solution, different heuristics can be developed and their performances can be analyzed.

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Appendix A

Proofs of Structural Results

A.1 Proof of Proposition 5.1

Using finite horizon formulation in Ruszczyński [64], we are going to construct an equivalent model to (4.11):

$$V_t(x_t, w_t, \vec{p}_t) = \min_{(a_t^{IG}, a_t^{IB}, a_t^W, g_t^I, g_t^S) \in \Psi^1(x_t, w_t)} \{-P_t(\vec{p}_t)(g_t^S + \beta a_t^W)\tau + P_t(\vec{p}_t)\frac{a_t^{IB}}{\alpha\tau} + \rho_t(V_{t+1}(x_t + a_t^{IG} + a_t^{IB} - a_t^W, w_{t+1}, \vec{p}_{t+1}))\} \qquad \text{for } t = 1, ..., T$$

$$V_{T+1}(x_{T+1}, w_{T+1}, \vec{p}_{T+1}) = 0$$

where a_t^{IG} denotes the amount of inventory injection from energy generated, a_t^{IB} is the amount of inventory injection from energy bought, a_t^W is the amount of inventory withdrawal. Also, g_t^I denotes the amount of energy generated and used for charging the storage, g_t^S denotes the amount of energy generated for sale.

Suppose $P_t(\vec{p}_t) > 0$ for each $t \in \mathcal{T}$ and $\vec{p}_t \in \mathcal{P}$. We need to show that for each $t \in \mathcal{T}$, $V_t(x_t, w_t, \vec{p}_t)$ is convex in x_t for each given $(w_t, \vec{p}_t) \in \mathcal{W} \times \mathcal{P}$. The

argument holds trivially at $t = T + 1$ since $V_{T+1}(x_{T+1}, w_{T+1}, \vec{p}_{T+1}) = 0$. Using induction, assume that the argument holds at each stage from $k+1$ to T. Consider stage k, fix $(w_k, \vec{p}_k) \in \mathcal{W} \times \mathcal{P}$. The set $\mathcal{C} := \{(x_k, a_k^{IG}, a_k^{IB}, a_k^W, g_k^I, g_k^S) | x_k \in \mathcal{X}, (a_k^{IG}, a_k^{IB}, a_k^W, g_k^I, g_k^S) \in \Psi^1(x_k, w_k)\}$ is convex because $\mathcal{X} \equiv [0, q]$ is convex and $\Psi^1(x_k, w_k)$ is polyhedral, and hence, convex. The first two terms of the objective function of $V_k(x_k, w_k, \vec{p}_k)$ are linear in (g_k^S, a_k^W) and a_k^{IB}, respectively, and they do not depend on x_k, a_k^{IG} and g_k^I. The induction hypothesis implies that $V_{k+1}(x_k + a_k^{IG} + a_k^{IB} - a_k^W, w_{k+1}, \vec{p}_{k+1})$ is jointly convex in $(x_k, a_k^{IG}, a_k^{IB}, a_k^W)$ for each given w_{k+1} and $\vec{p}_{k+1}$. From $(A1)$ and $(A2)$ in Ruszczyński [64] we understand that ρ_t is a non-decreasing and convex function. Using the composition rule in Boyd [71], we are able to conclude that $\rho_k(V_{k+1}(x_k + a_k^{IG} + a_k^{IB} - a_k^W, w_{k+1}, \vec{p}_{k+1}))$ is also jointly convex in $(x_k, a_k^{IG}, a_k^{IB}, a_k^W)$. Also, this function is independent of g_k^I and g_k^S. Hence, the objective function is jointly convex on $\mathcal{C}$.

Also, we need to show that for each $t \in \mathcal{T}$ and $S_t \in \mathcal{S}$ it holds that $V_t(S_t) > -\infty$. When $t = T+1$ the claimed property is true as $V_{T+1}^*(x_{T+1}, w_{T+1}, \vec{p}_{T+1}) = 0$. For each stage $t \in \mathcal{T}$ the quantity sold or bought is limited by θ_{tr}/τ. Then, $V_t(x_t, w_t, \vec{p}_t) \geq -P_t(\vec{p}_t)\theta_{tr}/\tau + \rho_t(V_{t+1}(x_t + a_t, w_{t+1}, \vec{p}_{t+1}))$. Assuming $P_t(\vec{p}_t) < \infty$, we can conclude that the cost functions are bounded. Then using Theorem 4 in [64], we can conclude that $\rho_t(V_{t+1}(x_t + a_t, w_{t+1}, \vec{p}_{t+1})$ will also be bounded. Hence, $V_t(S_t) > -\infty$.

The set $\Psi^1(x_k, w_k)$ is nonempty for every $x_k \in \mathcal{X}$, the objective function is jointly convex on $\mathcal{C}$ and $V_k(x_k, w_k, \vec{p}_k) > -\infty$. Then, using Theorem A.4 in Porteus [72], we can conclude that $V_k(x_k, w_k, \vec{p}_k)$ is convex on $\mathcal{X}$. Then by induction, the argument holds at each stage from 0 to $T + 1$.

A.2 Proof of Proposition 5.2

Since convexity of the value functions hold, proof of proposition 2 will be followed from [11].