PRODUCT DIFFERENTIATION UNDER LOSS AVERSION AND DEMAND UNCERTAINTY

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ABSTRACT

Product Differentiation Under Loss Aversion and Demand Uncertainty

This thesis analyzes, first, the relationship between loss aversion and demand uncertainty for vertical product differentiation, and second, the effect of loss aversion on the degree of horizontal product differentiation. In the vertical product differentiation model, we consider a three-stage duopoly game setting, where we incorporate demand uncertainty. Firms have incomplete information about the buyers' true types and they face demand uncertainty. They announce their qualities before the demand uncertainty is resolved, and they announce their prices after the demand uncertainty is resolved. We find the subgame perfect Nash equilibrium for qualities and prices, and we also calculate the profit of each firm as well as the consumer surplus. We also conduct comparative statics to explain how loss aversion affects the outcomes, including the qualities, prices, profits and consumer surplus. We show that, for a wide variety of utility functions, the high quality firm's equilibrium quality, and the equilibrium prices increase with the degree of loss aversion. Also, both firms benefit from a stronger loss aversion, but consumers are worse off when loss aversion is stronger.

For the horizontal product differentiation model, we also consider a three-stage duopoly game setting, where the firms compete in locations in the first stage, before the buyers determine their reference locations, and compete in prices in the third stage, after the buyers determine their reference locations. Assuming that one firm has a cost advantage, we show that for a range of the degree of the cost advantage, the equilibrium exhibits maximal product differentiation under loss aversion, while we get minimal product differentiation when there is no loss aversion.

ÖZET

Kayıptan Kaçınma ve Talep Belirsizliği Altında Ürün Farklılaştırması

Bu tez, ilk olarak, dikey ürün farklılaştırması için kayıptan kaçınma ve talep belirsizliği arasındaki ilişkiyi ve ikinci olarak, yatay ürün farklılaştırmasının derecesi üzerindeki kayıptan kaçınmanın etkisini analiz eder. Dikey ürün farklılaştırma modelinde, talep belirsizliğini dahil ettiğimiz üç aşamalı bir doupol oyun ortamını ele alıyoruz. Firmalar, alıcıların gerçek tipleri hakkında eksik bilgiye sahiptir ve talep belirsizliği ile karşı karşıyadır. Talep belirsizliği çözülmeden önce kalitelerini, talep belirsizliği ortadan kalktıktan sonra ise fiyatlarını açıklarlar. Kaliteler ve fiyatlar için alt oyun saf Nash dengesini buluyoruz ve ayrıca her bir firmanın kârını ve tüketici fazlasını hesaplıyoruz. Ayrıca, kayıptan kaçınmanın kaliteler, fiyatlar, kârlar ve tüketici fazlası dahil olmak üzere sonuçları nasıl etkilediğini açıklamak için karşılaştırmalı istatistikler üzerinde duruyoruz. Çok çeşitli fayda fonksiyonları için, yüksek kaliteli firmanın denge kalitesinin ve denge fiyatlarının kayıptan kaçınma derecesi ile arttığını gösteriyoruz. Ayrıca, her iki firma da daha güçlü bir kayıptan kaçınmadan fayda sağlar, ancak tüketiciler, kayıptan kaçınma daha güçlü olduğunda daha kötü durumda olurlar.

Yatay ürün farklılaştırma modeli için, ilk aşamada alıcılar referans konumlarını belirlemeden önce firmaların konumlarda rekabet ettiği ve üçüncü aşamada alıcılar referans konumlarını belirledikten sonra fiyatlarda rekabet ettiği üç aşamalı bir duopoli oyun ortamını da ele alıyoruz. Bir firmanın maliyet avantajına sahip olduğunu varsayarak, maliyet avantajının bir derece aralığı için, dengenin kayıptan kaçınma durumunda maksimum ürün farklılaşması gösterdiğini, kayıptan kaçınma olmadığında ise minimum ürün farklılaşması elde ettiğimizi gösteriyoruz.

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CHAPTER 1

INTRODUCTION

In markets with developing technology and an increase in population, buyers face a products with varying quality. In this environment, the buyers try to choose the best product for their needs, while the firms do not have exact information about buyers' valuation for the quality level of a product. In other words, there is uncertainty about the buyers' valuation for quality. For this reason, before the firms announce the products' prices and begin to sell them, they prefer to provide advertisements on television or social media platforms. The firms' approach is to show the quality of the product to buyers to persuade them to purchase the product. The buyers now have some information about the products' quality levels but not the prices. This approach might cause a time lag for the buyers between observing the quality and observing the price of a product. Therefore, within this time lag, the buyers may have an opportunity to form their expectations about which product is best for them, based on the observed quality levels. However, when the prices are announced a buyer may find it optimal to buy a product other than the one she deemed best for herself, just because the price differential may be substantial. In that case, the buyer may realize some difference between the quality she thought she should have gotten and the product quality she actually gets. Also, she may see a difference between what she expected to pay and what she actually paid. More specifically, if two products are announced their qualities first and then their prices, it is natural that the buyers compare the products in the two dimensions regarding qualities and prices according to the behavioral approach. Because of the differentiation between products in terms of qualities and prices, the buyers might feel a gain/loss about what they choose to buy. Such gains and losses may affect their overall utility either as a utility or a disutility. This kind of comparison motivates the loss aversion in our vertical product differentiation model under demand uncertainty.

We analyze the effect of loss aversion on the firms' qualities, prices, profits, and buyers' consumer surplus, in the equilibrium. For this reason, we consider a duopolistic model where the firms compete in a vertical product differentiation environment. The firms have incomplete information in terms of buyers' true valuation for products until they compete in prices. However, they know that the true valuations are drawn from an interval through uniform distribution, but there is some uncertainty regarding the interval itself, that is, demand uncertainty. Thus, the firms compete in qualities considering this demand uncertainty. After the firms learn the exact interval, that is, when the demand uncertainty is resolved, then they compete in prices. The buyers make their purchase decisions after they learn the exact qualities and the prices. Then the profits of firms and the utilities of buyers are realized. We consider loss aversion á la Kahneman and Tversky (1979), and focus on the vertical differentiation. We characterize the equilibrium regarding qualities and prices. The same-sized losses significantly outweigh the value function than same-sized gains, yielding loss-aversion.[1] the way we introduce loss aversion in this setting is as follows. After the qualities are announced, all buyers learn and agree which product is of higher quality and which one is of lower quality. If a buyer chooses to purchase a unit of a product with higher quality and higher price, she compares her purchase decision with the other product, which has a lower quality and a lower price. The buyer has some gain utility due to the higher quality relative to the lower quality product, but she receives a loss disutility due to paying a higher price relative to the lower price of the other product.

We show that the degree of loss aversion affects equilibrium qualities, prices, profits, and consumer surplus. The high quality firm's quality increases with the degree of loss aversion. Both prices in the equilibrium increase with loss aversion as well. Moreover, the firms' profits also increase with the degree of loss aversion,

[1] Also, see Kőszegi and Rabin (2006) for a general reference-dependent preferences model, where the article shows that gain-loss utility appears when there is uncertainty and willingness to pay for a product increases as the expected likelihood of the buy happening increases.

however consumer surplus decreases with loss aversion. We also conduct comparison with our benchmark model, where there is no loss aversion on the buyers' side.

We also analyze a horizontal product differentiation model as in Hotelling's linear city model, and study the effect of loss aversion on the equilibrium degree of product differentiation.[2] In our model, there are two firms, and they first compete in locations and then in prices. The buyers' true locations are drawn from a uniform distribution in an interval $[0, 1]$. Product differentiation comes from the location distance between the buyers' true locations and the locations of the firms. The buyers incur a transportation cost due to every unit of distance between own location and the firm's location, which is captured by, $t > 0$, per unit distance. The location might be explained as a buyer's physical location or characteristic product location. Every buyer has a valuation for a unit product, v. We consider the model in two ways: with and without loss aversion models. In the first stage, the firms choose their locations simultaneously, then the buyers observe the location choice and form their reference points regarding the difference between their locations and firms' locations. In the third stage, the firms compete in prices simultaneously. Once the prices are also observed, the buyers make their purchases and their utilities are realized, as well as the profit of each firm. In this environment, we incorporate loss aversion as follows. When firms announce their locations, each buyer realizes which firm's location is closer to her own location, and sets that firm as her reference point. After the prices are also announced, the buyer makes a purchase decision and when she ends up buying not the reference product, she may experience a loss due to the larger distance she has to incur, but potentially a gain due to a lower she pays. In such an environment, assuming that one firm has a cost advantage, we show that for a range of the degree of the cost advantage, the equilibrium exhibits maximal product differentiation under loss aversion, while we get minimal product differentiation when there is no loss aversion.

[2]See Hotelling (1929) for the canonical location model. See Salop (1979) for the circular city model.

This thesis contributes to the literature by showing the loss aversion affects qualities, prices, profits, and consumer surplus in vertical product differentiation under demand uncertainty. Also, we show that when there is a cost advantage, there exists a threshold for the degree of the cost advantage in the model without loss aversion, which determines whether there is minimal or maximal product differentiation in the equilibrium. Likewise, when there is loss aversion, there is another threshold for the degree of the cost advantage, which determines whether there is minimal or maximal product differentiation in the equilibrium. We show that the latter threshold is bigger and thus, for a range of the degree of the cost advantage, maximal product differentiation emerges under under loss aversion, while minimal product differentiation emerges when there is no loss aversion.

CHAPTER 2

LITERATURE REVIEW

We review the literature in regards to the relevant concepts which are vertical product differentiation, loss aversion, and horizontal product differentiation. We also consider studies that include demand uncertainty and loss aversion concepts in vertical product differentiation. Finally, we focus on the fundamental model and its variations in horizontal differentiation.

We consider the vertical product differentiation in a duopoly under demand uncertainty with a valuation function for buyers where the demand uncertainty is resolved before the price competition of firms. Our model is quite related to a series of papers in the literature. Javier Casado-Izaga (2000) studies the firms' location decisions under uncertainty regarding buyers' tastes. The firms choose their locations regardless of the true taste of the buyers. The paper states that the existence of uncertainty about the buyers' true tastes causes an increase in the degree of product differentiation. Meagher and Zauner (2004) investigates Hotelling's duopoly game under demand uncertainty. They state that increasing uncertainty on demand causes higher differentiation, expected prices, and profits in the equilibrium. The closest study to our model in the literature is, Cheng (2014), who studies a vertical product differentiation in a duopoly setting, under demand uncertainty, where the demand uncertainty is resolved before the price competition. Both fixed and variable cost cases are considered. Increasing uncertainty causes quality differentiation more under variable cost than under fixed cost. Moreover, an increase in demand uncertainty induces higher profits regardless of the cost type. In and Lim (2022) studies a vertical product differentiation in a duopoly game, where there is a demand uncertainty on buyers' tastes. One main difference from our study is that the qualities are exogenously given in the paper. The writer mainly focuses on how uncertainty affects price competition. The study argues that if the quality difference is higher than the cost difference between the two firms, then an increase in the uncertainty causes an increase in high quality firm's market share.

There are various studies that analyze the effect of loss aversion on product differentiation in terms of competition of the firms and profits. These studies investigate the buyers' loss aversion, and they mainly focus on how this concept affects the equilibrium. Our model is quite related to a series of papers in this part of the literature. Hahn, Kim, Kim, and Lee (2018) investigates a price discrimination theory in a monopoly setting, where the buyers are loss averse. When the buyers do not know their willingness to pay for the products, the seller offers a menu, and the buyers form a reference. After the buyers buy a product, they face a gain/loss utility on their references and consumption. The study argues that under the binary types of buyers, the monopolist sees it optimal to leave the screening with an intermediate range of loss aversion if low type, i.e., low willingness to pay, is sufficiently likely. Ahrens, Snower, and Pirschel (2017) analyze an equilibrium theory of price adjustment when the buyers are loss averse. The buyers consider the price changes as endogenous reference points on prices, which are based on the buyers' price expectations in the recent past. The sellers arrange their prices elastically according to the demand curve. The writers found that their results are compatible with the empirical evidence. Also, they show that the firms' reaction to the prices with respect to demand shocks is based significantly on the size, sign, and whether the demand shock is persistent or temporary. The study shows that whereas the prices are more sluggish upwards than downwards concerning transitory demand shocks, the prices are more sluggish downwards than upwards for persistent demand shocks. The degree of the change depends on the magnitude of the demand shock. Heidhues and Kőszegi (2008) study a circular city model, where the buyers are loss averse in terms of prices and match values. The buyers' reference points depend on their expectations. The article presents that although the sellers have various cost distributions, there exists a focal price equilibrium. When the buyers are loss averse, the sellers choose deterministic prices. Heidhues and Kőszegi (2014) conclude that buyers' loss aversion could have an opposite effect, which might induce the seller optimally submit the risk into an otherwise deterministic environment with a low and variable price, but a high regular price. A monopolist can convince the buyers to create a positive probability to buy by choosing a low sale price that is lower than the product value. The article shows that the buyers might purchase at the high regular price because they try to avoid possible loss from not buying. In the study of Rosato (2016), the similar intuition holds. The writer considers optimal pricing and product availability for a seller who sells two substitute goods to loss averse buyers. In the paper, the seller uses one of the goods to form the higher reference points of the buyers with a low bargain and it also charges a high price for the other good that the buyers choose to buy to avoid disappointment when the bargain is not available. In the article Karle and Peitz (2014), there is a circular city model in a duopoly setting. Although some of the buyers are informed about their real tastes, others are not. The paper analyzes the effect of loss aversion on the market outcome regarding both price distributions and match-value dimensions. They present that loss aversion has a pro-competitive impact on the price distribution, but loss aversion has an anti-competitive impact in the match-value dimension. In the symmetric duopoly game, the pro-competitive effect dominates the anti-competitive effect. Karle and Möller (2020) analyze a model with loss averse buyers under the early and late purchase opportunities. The paper focuses on the impact of the advance purchases by comparing the monopoly and duopoly settings, with and without loss aversion. The buyers have a chance to purchase the product early with a lower price, but the late purchase option causes a better match value. Whereas loss aversion has an anti-competitive impact on prices, there is a pro-competitive impact on the discounts. Piccolo and Pignataro (2018) study the impact of loss aversion on product experimentation and tacit collusion. Preventing experimentation is preferable for the firms' joint profit if the buyers are not too loss averse. Conversely, when the buyers are sufficiently loss averse, legalizing experimentation is preferable for firms' joint profit. The writers also find that consumer surplus is maximized by preventing experimentation in a static environment, whereas, it is maximized by allowing experimentation in a dynamic environment. In addition to all, They also argue that vertical differentiation is similar to comparative experimentation that conveys monopolistic power to the firms. Karle and Schumacher (2017) analyze the impact of advertisement on monopolistic profit. When buyers are loss averse, the advertising approach causes a shift in the reference point of the customer. Also, monopolistic sellers choose to supply incomplete information via advertisement, if the monopolistic seller has to. If the monopolistic seller does not have to do, the seller chooses to have no information. Nasiry and Popescu (2012) consider a model that predicted regret on consumer choices and firms' profits and policies in the advance selling context when the buyers are uncertain about their valuations. When the valuation of the buyer is lower than the price, then there exists regret because of the purchase. Moreover, the sellers' profits decrease. The profits might increase if the regret comes from a running discount or the product which is out of stock. Zhou (2011) considers the impact of the reference-dependent preferences in horizontal product differentiation, in a duopolistic price competition model. The buyers do not have any information about the sellers' locations or their locations, and they observe the two products in succession. Moreover, the buyer chooses the first product as her reference point. The article shows that the dominant firm announces its price as randomizing between high and low price levels, whereas, the other firm announces its price as medium price, which is a constant. The article also shows that the existence of loss aversion in the price dimension has a positive impact on the price competition. In contrast, the presence of loss aversion in the product dimension has a negative effect on the price competition. The fundamental variation between the analysis and our study is that we consider the reference-dependent preference as loss aversion on all buyers regarding the location difference between the closest firm and the buyers. Also, we consider endogeneous locations and focus on the effect of loss aversion on the degree of differentiation.

In the article d'Aspremont, Gabszewicz, and Thisse (1979), the writers analyze the Hotelling model, which is introduced by Hotelling (1929). They show that the sellers have a tendency to maximize their product differentiation; on the contrary, Böckem (1994) shows that the maximum differentiation is not robust when the buyers have any outside options to spend their money. These articles are close to our study in some parts because we also analyze the product differentiation behavior of the firms under some circumstances.

CHAPTER 3

MODEL: VERTICAL PRODUCT DIFFERENTIATION

We consider a vertical product differentiation model, where two firms, firm 1 and firm 2 are engaged in a three-stage duopoly game. Firms compete both in qualities and in prices, under demand uncertainty. In the first stage, the firms simultaneously announce their qualities, q_1 and q_2, which are commonly observed. In the second stage, the buyers' demand uncertainty is resolved and becomes common knowledge. Finally, in the third and final stage, the firms, after observing the demand, announce their prices, p_1 and p_2, simultaneously. Once the prices are announced, the demand each firms gets is realized, as well as the profit level of each firm.

Each buyer has a type, in terms of their marginal valuation for a product's quality. A type is represented by ϑ, and it is uniformly distributed in the interval $[\underline{\vartheta}, \underline{\vartheta} + 1]$. Additionally, there is demand uncertainty, modeled by an uncertainty regarding $\underline{\vartheta}$. In other words, the interval where the types are drawn is uncertain, during the first stage where firms pick their qualities. In the second stage, $\underline{\vartheta}$ is drawn from a uniform distribution with an interval [0, 1/2], and then each buyer draws her own type from $[\underline{\vartheta}, \underline{\vartheta} + 1]$. Thus, each buyer learns her realized ϑ and what the real $\underline{\vartheta}$ is in the second stage, before the price competition stage (third stage). However, the firms in the first stage only know that the distribution that $\underline{\vartheta}$ is drawn from, but they learn the realization of $\underline{\vartheta}$ right before the price competition. Thus, the firms are uncertain about demand of the buyers when they announce their qualities, but they know the real demand when they announce their prices. When $\underline{\vartheta}$ turns out to be high, then the valuations for the quality of the product are overall high.

Each buyer has a unit demand, that is, each buyer buys exactly one unit of the product, either from firm 1 with the quality q_1 at price p_1, or from firm 2 with the quality q_2 at price p_2. When a buyer with a type ϑ purchases a unit of product from firm i, then she gets the following intrinsic utility

$$U(\vartheta, q_i, p_i) = \vartheta v(q_i) - p_i$$

where $v(q_i)$ is a strictly increasing and strictly concave function of quality, q_i is the quality of firm i's product, and p_i is firm i's price, for $i = 1, 2$.

When the quality and price competition stages are over, both q_i and p_i for $i = 1, 2$ will be common knowledge. For any given (q_1, q_2) and (p_1, p_2), there is a marginal type of buyer, whose type is denoted by $\hat{\vartheta}(q_1, q_2, p_1, p_2)$, and she is indifferent between purchasing 1 unit from firm 1 and purchasing 1 unit from firm 2. Then, $\hat{\vartheta}$ is given by

$$\hat{\vartheta}v(q_1) - p_1 = \hat{\vartheta}v(q_2) - p_2$$

Without loss of generality, suppose $q_2 > q_1$ and $p_2 > p_1$. Then all buyers with a type less than $\hat{\vartheta}$ buy from firm 1 and all buyers with a type more than $\hat{\vartheta}$ buy from firm 2. Then, a buyer with a type ϑ has the following intrinsic utility.

$$U(\vartheta, q_1, q_2, p_1, p_2) = \begin{cases} \vartheta v(q_1) - p_1 & \text{if } \vartheta < \hat{\vartheta} \\ \vartheta v(q_2) - p_2 & \text{if } \vartheta > \hat{\vartheta} \end{cases}$$

We assume that qualities and prices are non-negative: $q_i \geq 0$ and $p_i \geq 0$, for $i \in 1, 2$. Quality q_i may be zero (thus a corner solution), but cannot be negative. We also assume, without loss of generalization, $q_2 > q_1$ which means that quality of firm 2 is always higher than quality of the firm 1. Given $q_2 > q_1$, if $p_1 \geq p_2$ then no buyer would buy from firm 1, that is, it would be $\hat{\vartheta} = 0$. But this cannot be an equilibrium since firm 1 can decrease it's price in order to get some demand. Thus, when we have $q_2 > q_1$, it will be accompanied with the assumption that $p_2 > p_1$. Now, given $q_2 > q_1$ and $p_2 > p_1$, therefore given , the demand each firm gets will be given as follows

$$D_1(q_1, q_2, p_1, p_2) = \hat{\vartheta}(q_1, q_2, p_1, p_2) - \underline{\vartheta}$$

and

$$D_2(q_1, q_2, p_1, p_2) = 1 + \underline{\vartheta} - \hat{\vartheta}(q_1, q_2, p_1, p_2)$$

The firms have a symmetric quality cost function, given by $c_1(q) = c_2(q) = c(q)$, which is strictly increasing and strictly convex, that is, $c' > 0$ and $c'' > 0$. If the cost of quality is *fixed* (incurred before the price competition) then the profit expression of firm i will be

$$\pi_i(q_1, q_2, p_1, p_2) = p_i D_i(q_1, q_2, p_1, p_2) - c(q_i),$$

and if the cost of quality is *variable* (incurred during the price competition) then the profit expression of firm i will be

$$\pi_i(q_1, q_2, p_1, p_2) = [p_i - c(q_i)] D_i(q_1, q_2, p_1, p_2)$$

where $D_i(q_1, q_2, p_1, p_2)$ is firm i's demand under the quality and price levels (q_1, q_2, p_1, p_2).

The firms announce their qualities, q_i, simultaneously. The quality pair, (q_1, q_2), are observed publicly by the buyers. However, at this point, the firms are unaware of the total demand because of the uncertainty over the buyers' type, ϑ, but the buyers know the real $\underline{\vartheta}$. Then, the realized $\underline{\vartheta}$ is observed by the firms. Then, they simultaneously compete in prices, (p_1, p_2), and announce them publicly. Finally, each buyer makes a choice about which firm to purchase from, based on the qualities of the products q_i, own type ϑ and the prices of the products p_i, and the profit of each firm and the utility of each buyer are realized. The chain of the events is summarized as follows:

Stage 1: Firms simultaneously announce their qualities, q_1, and q_2, and the qualities become common knowledge.

Stage 2: The uncertainty on ϑ is resolved and the realized $\underline{\vartheta}$, therefore the distribution of the buyers' types, becomes common knowledge.

Stage 3: Firms simultaneously announce their prices, p_1, and p_2, based on total demand and then the prices become common knowledge for the buyers.

Stage 3.1: Each buyer chooses which firm to buy from after observing the qualities and the prices, (q_1, q_2), own type, ϑ, and the prices, (p_1, p_2).

Stage 3.2: The profits of the firms and the utilities of the buyers are realized.

Now we introduce loss aversion of the buyers into our model and describe it in detail.

3.1 Loss Aversion

The total utility of a loss-averse buyer includes two different components. The first is the consumption/intrinsic utility that a buyer obtains from purchasing one unit of a product net of paying the price for a product. The consumption/intrinsic utility is equal to $\vartheta v(q_i) - p_i$ where $v(q_i)$ is a strictly increasing and strictly concave function of quality, for $i = 1, 2$. The second component is a possible loss/gain utility based on the product's quality and price, q_i and p_i that the buyer decides to buy from and the other product's quality and price, q_i and p_i.

The loss/gain utility component for a buyer comes from the ex-post comparison between the two products in terms of qualities and prices. The gain/loss utility is additively separable in terms of two dimensions, which are quality and monetary terms, i.e price. When a buyer purchases a product from firm i with a quality q_i and a price p_i, her reference point is set to be the purchase a unit of product from the other firm, firm i_{-1} with a quality q_{i-1} and a price p_{i-1}. In other words, if the buyer's choice is to buy from firm i and $q_i > q_{i-1}$ and $p_i > p_{i-1}$, then the buyer feels a gain because of obtaining a relatively high quality, but feel a loss because of paying a relatively high price. We assume that the high-quality product belongs to the firm 2, i.e., $q_2 > q_1$, and the lower price belongs to the firm 1, i.e., $p_2 > p_1$. Suppose a buyer with a type ϑ decides to buy from firm 2. Then, the buyer experiences a gain/loss from the difference between the qualities and the prices as follows. It will induce a gain due to higher quality, $q_2 - q1$, but at the same time, she will face a loss due to paying a higher price, $\lambda(p_2 - p_1)$ where $\lambda > 1$ and it reflects the degree of loss aversion.

Assuming that the types $\vartheta < \hat{\vartheta}$ buy from firm 1, and the types $\vartheta > \hat{\vartheta}$ buy from firm 2, the utility function of a loss-averse buyer with a realized type ϑ is as follows.

$$U(\vartheta, q_1, q_2, p_1, p_2, \lambda) = \begin{cases} \vartheta v(q_1) - p_1 + (p_2 - p_1) - \lambda(q_2 - q_1) & \text{if } \vartheta < \hat{\vartheta} \\ \vartheta v(q_2) - p_2 - \lambda(p_2 - p_1) + (q_2 - q_1) & \text{if } \vartheta > \hat{\vartheta} \end{cases}$$

A buyer with type $\vartheta < \hat{\vartheta}$, without loss of generality, decides to buy from firm 1, and she feels a gain due to paying less compared to buying from firm 2, but she feels a loss due to obtaining a lower quality product compared to buying from firm 2. On the other hand, a buyer with type $\vartheta > \hat{\vartheta}$, without loss of generality, decides to buy from firm 2, and she feels a loss due to paying more compared to buying from firm 1, but she feels a gain due to obtaining a higher quality product compared to buying from firm 1.

CHAPTER 4

ANALYSIS: FIXED COST

We first focus on a benchmark model. We keep the same structure as our model except for loss aversion because we mainly focus on the loss-aversion effect on equilibrium qualities and prices. In other words, there is a demand uncertainty in our benchmark model, but there is not gain/loss utility component in the buyers' utility. Moreover, our benchmark model has the same chain of events. The demand uncertainty comes from $\underline{\vartheta}$. [1]

4.1 Model Analysis without Loss Aversion

As explained earlier in the model section, each buyer has a type as ϑ that is drawn from a uniform distribution over $[\underline{\vartheta}, \underline{\vartheta} + 1]$ and also there is a demand uncertainty which stems from $\underline{\vartheta}$ being drawn from a uniform distribution over [0, 1/2]. Each buyer knows her realized type ϑ and the realized $\underline{\vartheta}$. However, the firms do not know the realized $\underline{\vartheta}$ and ϑ of any buyer at the first stage, where they compete in qualities.

$$U(\vartheta, q_1, q_2, p_1, p_2) = \begin{cases} \vartheta v(q_1) - p_1 & \text{if } \vartheta < \hat{\vartheta} \\ \vartheta v(q_2) - p_2 & \text{if } \vartheta > \hat{\vartheta} \end{cases}$$

After the qualities and prices are announced by the firms, each buyer has two options, either purchase from firm 1 or purchase from firm 2. However, there is a buyer who is indifferent between buying from firm 1 or firm 2. For this buyer, the utility of buying from firm 1 is the as the utility of buying from firm 2. We first find this indifferent buyer's type through $U(\vartheta, 1) = U(\vartheta, 2)$, where we abuse notation with denoting the net utility of a buyer with ϑ who buys from firm i as $U(\vartheta, i)$. Denoting

[1] We consider $\underline{\vartheta} = 0$ and with loss/gain utility as an alternative benchmark model, but in this case we realized that we lost the demand uncertainty with taking only one possibility of the uncertainty in terms of $\underline{\vartheta}$ in uniform distribution over [0, 1/2]. For this reason, we only focus on the case with demand uncertainty and without loss/gain utility as a benchmark model.

the type of this indifferent buyer with $\hat{\vartheta}$, we have

$$\hat{\vartheta}(q_1, q_2, p_1, p_2) = \frac{p_2 - p_1}{v(q_2) - v(q_1)}$$

Note that $v(q_i)$ is a strictly increasing and strictly concave function of quality, with respect to q_i where $i \in 1, 2$. If $\hat{\vartheta} > \vartheta$, then the buyer chooses to purchase the product from firm 1 with quality q_1 and price p_1, otherwise she decides to buy from firm 2 with quality q_2 and price p_2. Thus, the demand each firm gets will be given as follows

$$D_1(q_1, q_2, p_1, p_2) = \hat{\vartheta}(q_1, q_2, p_1, p_2) - \underline{\vartheta}$$

$$D_2(q_1, q_2, p_1, p_2) = 1 + \underline{\vartheta} - \hat{\vartheta}(q_1, q_2, p_1, p_2)$$

The profits of the firm 1 and firm 2 as follows

$$\pi_1(q_1, q_2, p_1, p_2) = p_1 D_1(q_1, q_2, p_1, p_2) - c(q_1) = p_1(\hat{\vartheta} - \underline{\vartheta}) - c(q_1)$$

$$\pi_2(q_1, q_2, p_1, p_2) = p_2 D_2(q_1, q_2, p_1, p_2) - c(q_2) = p_2(1 + \underline{\vartheta} - \hat{\vartheta}) - c(q_2)$$

Note that the firms have a symmetric quality cost function, given by $c_1(q) = c_2(q) = c(q)$, which is strictly increasing and strictly convex, that is, $c' > 0$ and $c'' > 0$.

We use Subgame Perfect Nash equilibrium as our solution concept. Thus, the way we proceed is in the spirit of backward induction. First given a pair of qualities, (q_1, q_2), we solve the Nash equilibrium in the price competition in the subgame following this quality pair. Then, we will use the price equilibrium in the quality competition.

Now, in order to solve for the price equilibrium in the subgame with a quality pair (q_1, q_2), we first solve for the best response functions of each firm. To do this we maximize the profit of each firm with respect to own price. The best response functions are then given by

$$p_1(q_1, q_2, p_2, \underline{\vartheta}) = \frac{p_2 - \underline{\vartheta}(v(q_2) - v(q_1)}{2}$$

$$p_2(q_1, q_2, p_1, \underline{\vartheta}) = \frac{p_1 + (1 + \underline{\vartheta})(v(q_2) - v(q_1)}{2}$$

Solving these best response functions, we get the equilibrium prices in the subgame as follows.

$$p_1(q_1, q_2, \underline{\vartheta}) = \frac{(1 - \underline{\vartheta})(v(q_2) - v(q_1))}{3} \tag{4.1}$$

$$p_2(q_1, q_2, \underline{\vartheta}) = \frac{(2 + \underline{\vartheta})(v(q_2) - v(q_1))}{3} \tag{4.2}$$

Now we plug these prices into $\underline{\vartheta}$ to obtain the indifferent type in the equilibrium in this subgame. The indifferent type's location is moving in the same direction with $\underline{\vartheta}$. The indifferent type is then given as follows

$$\hat{\vartheta}(\underline{\vartheta}) = \frac{1 + 2\underline{\vartheta}}{3} \tag{4.3}$$

Note that the firms make their quality decisions under the demand uncertainty. In other words, the firms do not know the exact demand of the buyers before they announce their quality decisions, q_1 and q_2. When we substitute (4.1), (4.2), and (4.3) into the profits, then we obtain the following reduced profit expressions for a given $\underline{\vartheta}$.

$$\pi_1(q_1, q_2, \underline{\vartheta}) = \frac{(1 - \underline{\vartheta})^2(v(q_2) - v(q_1))}{9} - c(q_1)$$

$$\pi_2(q_1, q_2, \underline{\vartheta}) = \frac{(2 + \underline{\vartheta})^2(v(q_2) - v(q_1))}{9} - c(q_2)$$

Since the firms do not know the realization of $\underline{\vartheta}$, they need to take the expectation over $\underline{\vartheta}$ and find own expected profit. Thus, the two firms make their quality decision by maximizing their expected profits. The expected profits are given by

$$E\pi_1(q_1, q_2) = \int_0^{1/2} \frac{(1-\underline{\vartheta})^2(v(q_2)-v(q_1))}{9} - c(q_1)f(\underline{\vartheta})d\underline{\vartheta}$$

$$E\pi_2(q_1, q_2) = \int_0^{1/2} \frac{(2+\underline{\vartheta})^2(v(q_2)-v(q_1))}{9} - c(q_2)f(\underline{\vartheta})d\underline{\vartheta}$$

The firms know that the $\underline{\vartheta}$ is drawn from a uniform distribution over [0, 1/2]. Then, with using a uniform distribution properties, the firms get $E(\underline{\vartheta}) = 1/4$ and $E(\underline{\vartheta}^2) = \text{Variance}(\underline{\vartheta}) + E^2(\underline{\vartheta}) = 1/12$. Thus the expected profit functions of the firm 1 and the firm 2 are given by

$$E\pi_1(q_1, q_2) = \frac{7(v(q_2)-v(q_1))}{108} - c(q_1)$$

$$E\pi_2(q_1, q_2) = \frac{61(v(q_2)-v(q_1))}{108} - c(q_2)$$

The firms choose their own quality levels simultaneously to maximize their own expected profits. The first order conditions give the following:

$$-\frac{7v'(q_1)}{108} - c'(q_1) = 0 \tag{4.4}$$

$$\frac{61v'(q_2)}{108} - c'(q_2) = 0 \tag{4.5}$$

Note that the result in (4.4) shows $q_1 = 0$ which means firm 1 has a corner solution in this benchmark model. Also, $q_1 = 0$ means $v(q_1) = 0$. However, the result in the (4.5) shows that $q_2 > 0$ which means firm 2 has an interior solution. Thus, we can calculate the reduced profits of the firms as

$$\pi_1(q_1, q_2) = \frac{7v(q_2)}{108}$$

$$\pi_2(q_1, q_2) = \frac{61v(q_2)}{108} - c(q_2)$$

4.2 Closed Form Example: without Loss Aversion

In this subsection, we provide a specific example, specifying functional forms for $v(\cdot)$ and $c(\cdot)$. We let

$$v(q_i) = \frac{1-(1-q_i)^2}{2}$$

and

$$c(q_i) = \frac{q_i^2}{2}$$

where $i \in 1, 2$. As we assumed in the model, $v(\cdot)$ is a strictly increasing and strictly concave function. Also, $c(\cdot)$ is a strictly increasing and strictly convex function. When we use these $v(\cdot)$ and $c(\cdot)$ functions, we get the following equations as a result of the first order conditions with respect to q_1 and q_2, respectively.

$$-\frac{7(1-q_1)}{108} - q_1 = 0 \tag{4.6}$$

$$\frac{61(1-q_2)}{108} - q_2 = 0 \tag{4.7}$$

Note that (4.6) and (4.7) are the first order conditions for q_1 and q_2, respectively. Now, we find q_1^* and q_2^*. Note that since we assumed $q_1 \geq 0$ and $q_2 \geq 0$, q_1 cannot be negative. Since the right hand side of (4.6) is always negative, an interior q_1 is not possible and the profit of firm 1 is decreasing with q_1. Thus, it must be that $q_1 = 0$. We also solve for q_2 and find the following

$$q_1^* = 0$$

$$q_2^* = \frac{61}{169}$$

The results show that q_1 is a corner solution, but q_2 is an interior solution. The equilibrium prices are as follows:

$$p_1(q_1, q_2, \underline{\vartheta}) = \frac{(1-\underline{\vartheta})(2-q_1-q_2)(q_2-q_1)}{6}$$

$$p_2(q_1, q_2, \underline{\vartheta}) = \frac{(2+\underline{\vartheta})(2-q_1-q_2)(q_2-q_1)}{6}$$

Plugging q_1^* and q_2^* into the above price expressions, we get the prices in the equilibrium outcome.

$$p_1(\underline{\vartheta}) = (1-\underline{\vartheta})(0.099)$$

$$p_2(\underline{\vartheta}) = (2+\underline{\vartheta})(0.099)$$

These prices show that p_1 is decreasing with the demand, but p_2 is increasing with the demand, as larger $\underline{\vartheta}$ indicates a higher demand. Given the equilibrium qualities, now we can find the equilibrium profits as follows.

$$\pi_1 = \frac{7v(q_2)}{108} - c(q_1) = 0.0192$$

$$\pi_2 = \frac{61v(q_2)}{108} - c(q_2) = 0.1019$$

Note that profit of the firm 2 is higher than profit of firm 1.

4.3 Model Analysis with Loss Aversion

In this section we incorporate loss aversion in our model. Each buyer's type ϑ is drawn from a uniform distribution over $[\underline{\vartheta}, \underline{\vartheta} + 1]$, where $\underline{\vartheta}$ is drawn from a uniform distribution over [0, 1/2]. Each buyer knows her realized type ϑ and the realized $\underline{\vartheta}$. However, the firms do not know the realized $\underline{\vartheta}$ and ϑ at the first stage. In addition to all, the utility function of a loss-averse buyer now has a gain/loss utility component. The loss aversion parameter is $\lambda > 1$, and it is the weight for the losses, while the gains have a weight 1. Moreover, we assume that all buyers are loss-averse with the same λ. A type ϑ buyer who is loss-averse has the following utility function

$$U(\vartheta, q_1, q_2, p_1, p_2, \lambda) = \begin{cases} \vartheta v(q_1) - p_1 + (p_2 - p_1) - \lambda(q_2 - q_1) & \text{if } \vartheta < \hat{\vartheta} \\ \vartheta v(q_2) - p_2 - \lambda(p_2 - p_1) + (q_2 - q_1) & \text{if } \vartheta > \hat{\vartheta} \end{cases}$$

Solving $U(\vartheta, 1) = U(\vartheta, 2)$ (abusing notation), we get the indifferent buyer's type, $\hat{\vartheta}$ as follows

$$\hat{\vartheta}(q_1, q_2, p_1, p_2, \lambda) = \frac{(2+\lambda)(p_2 - p_1)}{(1+\lambda)(q_2 - q_1) + v(q_2) - v(q_1)}$$

Note that $v(q_i)$ is is a strictly increasing and strictly concave function of quality, with respect to q_i where $i \in 1, 2$. Note that if $\hat{\vartheta} > \vartheta$, then the buyer chooses to purchase product from firm 1 with quality q_1 and price p_1, otherwise she decides to buy from firm 2 with quality q_2 and price p_2. If the buyer buys from firm 1, she receives a gain in terms of lower price paid, $+(p_2 - p_1)$, and a loss in terms of lower quality difference, $-\lambda(q_2 - q_1)$. If the buyer buys from firm 2, then losses and gains are flipped.

Similar to the benchmark model, the demand functions for firm 1 and firm 2 are as follows

$$D_1(q_1, q_2, p_1, p_2, \lambda) = \hat{\vartheta}(q_1, q_2, p_1, p_2) - \underline{\vartheta}$$

$$D_2(q_1, q_2, p_1, p_2, \lambda) = 1 + \underline{\vartheta} - \hat{\vartheta}(q_1, q_2, p_1, p_2)$$

The profits of the firm 1 and firm 2 as follows

$$\pi_1(q_1, q_2, p_1, p_2, \lambda) = p_1 D_1(q_1, q_2, p_1, p_2) - c(q_1) = p_1(\hat{\vartheta} - \underline{\vartheta}) - c(q_1)$$

$$\pi_2(q_1, q_2, p_1, p_2, \lambda) = p_2 D_2(q_1, q_2, p_1, p_2) - c(q_2) = p_2(1 + \underline{\vartheta} - \hat{\vartheta}) - c(q_2)$$

The best response price functions are as follows.

$$p_1(q_1, q_2, p_2, \underline{\vartheta}, \lambda) = \frac{p_2}{2} - \frac{\underline{\vartheta}((1+\lambda)(q_2 - q_1) + v(q_2) - v(q_1))}{2(2+\lambda)} \tag{4.8}$$

$$p_2(q_1, q_2, p_1, \underline{\vartheta}, \lambda) = \frac{p_1}{2} + \frac{(1+\underline{\vartheta})((1+\lambda)(q_2 - q_1) + v(q_2) - v(q_1))}{2(2+\lambda)} \tag{4.9}$$

Now solving (4.8) and (4.9) together, we get the equilibrium prices in terms of qualities and parameters, in the subgame following the quality pair (q_1, q_2).

$$p_1(q_1, q_2, \underline{\vartheta}, \lambda) = \frac{(1-\underline{\vartheta})((1+\lambda)(q_2 - q_1) + v(q_2) - v(q_1))}{3(2+\lambda)} \tag{4.10}$$

$$p_2(q_1, q_2, \underline{\vartheta}, \lambda) = \frac{(2+\underline{\vartheta})((1+\lambda)(q_2-q_1)+v(q_2)-v(q_1))}{3(2+\lambda)} \tag{4.11}$$

Note that the prices are directly affected by the value of the λ. Now, we can deduce the indifferent buyer's type in the equilibrium, by plugging the prices we found into $\hat{\vartheta}$.

$$\hat{\vartheta} = \frac{1+2\underline{\vartheta}}{3} \tag{4.12}$$

Note that the firms make their decisions under the demand uncertainty and know that the buyers are loss averse. In other words, the firms do not know the exact demand of the buyers for the qualities before the firms announce the quality decisions, q_1 and q_2. When we substitute (4.10), (4.11), and (4.12) into the profits, we obtain the the profit expressions, for a given $\underline{\vartheta}$, in terms of qualities as follows

$$\pi_1(q_1, q_2, \underline{\vartheta}, \lambda) = \frac{(1-\underline{\vartheta})^2((1+\lambda)(q_2-q_1)+v(q_2)-v(q_1))}{9(2+\lambda)} - c(q_1)$$

$$\pi_2(q_1, q_2, \underline{\vartheta}, \lambda) = \frac{(2+\underline{\vartheta})^2((1+\lambda)(q_2-q_1)+v(q_2)-v(q_1))}{9(2+\lambda)} - c(q_2)$$

The expected profits are then as follows.

$$E\pi_1(q_1, q_2, \lambda) = \int_0^{1/2} \frac{(1-\underline{\vartheta})^2((1+\lambda)(q_2-q_1)+v(q_2)-v(q_1))}{9(2+\lambda)} - c(q_1)f(\underline{\vartheta})d\underline{\vartheta}$$

$$E\pi_2(q_1, q_2, \lambda) = \int_0^{1/2} \frac{(2+\underline{\vartheta})^2((1+\lambda)(q_2-q_1)+v(q_2)-v(q_1))}{9(2+\lambda)} - c(q_2)f(\underline{\vartheta})d\underline{\vartheta}$$

Simplifying these expressions, we get the expected profit functions of each firms as follows.

$$E\pi_1(q_1, q_2, \lambda) = \frac{7((1+\lambda)(q_2-q_1)+v(q_2)-v(q_1))}{108(2+\lambda)} - c(q_1)$$

$$E\pi_2(q_1, q_2, \lambda) = \frac{61((1+\lambda)(q_2-q_1)+v(q_2)-v(q_1))}{108(2+\lambda)} - c(q_2)$$

The firms choose the qualities to maximize their own expected profits simultaneously. Then, the first order conditions with respect to q_1 and q_2 give us the following:

$$-\frac{7((1+\lambda)+v'(q_1))}{108(2+\lambda)} - c'(q_1) = 0 \tag{4.13}$$

$$\frac{61((1+\lambda)+v'(q_2))}{108(2+\lambda)} - c'(q_2) = 0 \tag{4.14}$$

Similar to the benchmark model, the equation in (4.13) shows that $q_1 = 0$, since the left hand side is negative. This means firm 1 has a corner solution, and $v(q_1) = 0$. However, the equation (4.14) shows that $q_2 > 0$ which means firm 2 has an interior solution. Thus, in the equilibrium we have $q_1^* = 0$ and q_2^* is characterized by the equation (4.14).

Now, using $q_1^* = 0$, we can simplify the profit of each firm, in the equilibrium, as functions of q_2^* and λ only, as follows:

$$E\pi_1(q_1^* = 0, q_2^*, \lambda) = \frac{7((1+\lambda)q_2^* + v(q_2^*))}{108(2+\lambda)} \tag{4.15}$$

$$E\pi_2(q_1^* = 0, q_2^*, \lambda) = \frac{61((1+\lambda)q_2^* + v(q_2^*))}{108(2+\lambda)} - c(q_2^*) \tag{4.16}$$

4.4 Closed Form Example: with Loss Aversion

Now, we provide a specific example, specifying functional forms for $v(\cdot)$ and $c(\cdot)$. We let

$$v(q_i) = \frac{1-(1-q_i)^2}{2}$$

and

$$c(q_i) = \frac{q_i^2}{2}$$

where $i \in 1, 2$. As we assumed in the model, $v(\cdot)$ is a strictly increasing and strictly concave function. Also, $c(\cdot)$ is a strictly increasing and strictly convex function. When we use these $v(\cdot)$ and $c(\cdot)$ functions, we get the following equations as a result of the first order conditions with respect to q_1 and q_2, respectively.

$$-\frac{7((1+\lambda)+(1-q_1))}{108(2+\lambda)} - q_1 = 0 \tag{4.17}$$

$$\frac{61((1+\lambda)+(1-q_2))}{108(2+\lambda)} - q_2 = 0 \tag{4.18}$$

Note that (4.17) and (4.18) are the first order conditions for q_1 and q_2, respectively. Now, again $q_1^* = 0$ and q_2^* solves equation (4.18). When solving (4.18) for q_2 we get the following

$$q_1^* = 0$$

$$q_2^* = \frac{61(2+\lambda)}{108(2+\lambda)+61}$$

Now plugging q_1^* and q_2^*, into the price expressions we found earlier, we get the equilibrium prices as follows.

$$p_1(\underline{\vartheta},\lambda) = (\frac{1-\underline{\vartheta}}{3(2+\lambda)})(\frac{61(1+\lambda)(2+\lambda)}{108(2+\lambda)+61} + \frac{1-(\frac{(47(2+\lambda)+61)^2}{108(2+\lambda)+61})}{2})$$

$$p_2(\underline{\vartheta},\lambda) = (\frac{2+\underline{\vartheta}}{3(2+\lambda)})(\frac{61(1+\lambda)(2+\lambda)}{108(2+\lambda)+61} + \frac{1-(\frac{(47(2+\lambda)+61)^2}{108(2+\lambda)+61})}{2})$$

Note that equilibrium prices depend on the observed (before the price competition) demand level. Also, these equilibrium prices show that p_1 is decreasing with the demand, but p_2 is increasing with the demand,as larger $\underline{\vartheta}$ indicates a higher demand. Given the equilibrium qualities, now we can find the equilibrium expected profits as follows.

$$E\pi_1(\lambda) = (\frac{7}{108(2+\lambda)})(\frac{61(1+\lambda)(2+\lambda)}{108(2+\lambda)+61} + \frac{1-(\frac{(47(2+\lambda)+61)^2}{108(2+\lambda)+61})}{2})$$

$$E\pi_2(\lambda) = (\frac{61}{108(2+\lambda)})(\frac{61(1+\lambda)(2+\lambda)}{108(2+\lambda)+61} + \frac{1-(\frac{(47(2+\lambda)+61)^2}{108(2+\lambda)+61})}{2}) - \frac{(\frac{61(2+\lambda)}{108(2+\lambda)+61})^2}{2}$$

How these equilibrium qualities, equilibrium prices and expected equilibrium profit levels are affected by the degree of loss aversion, λ, is analyzed in the next section.

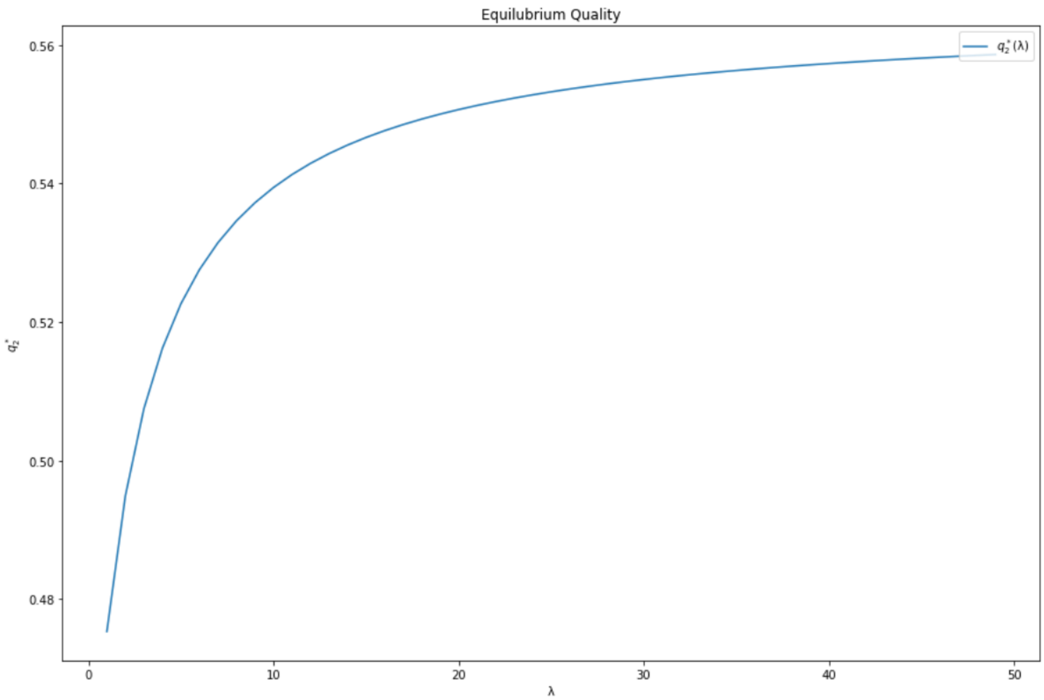

Figure 1. Quality of Firm 2 in the equilibrium

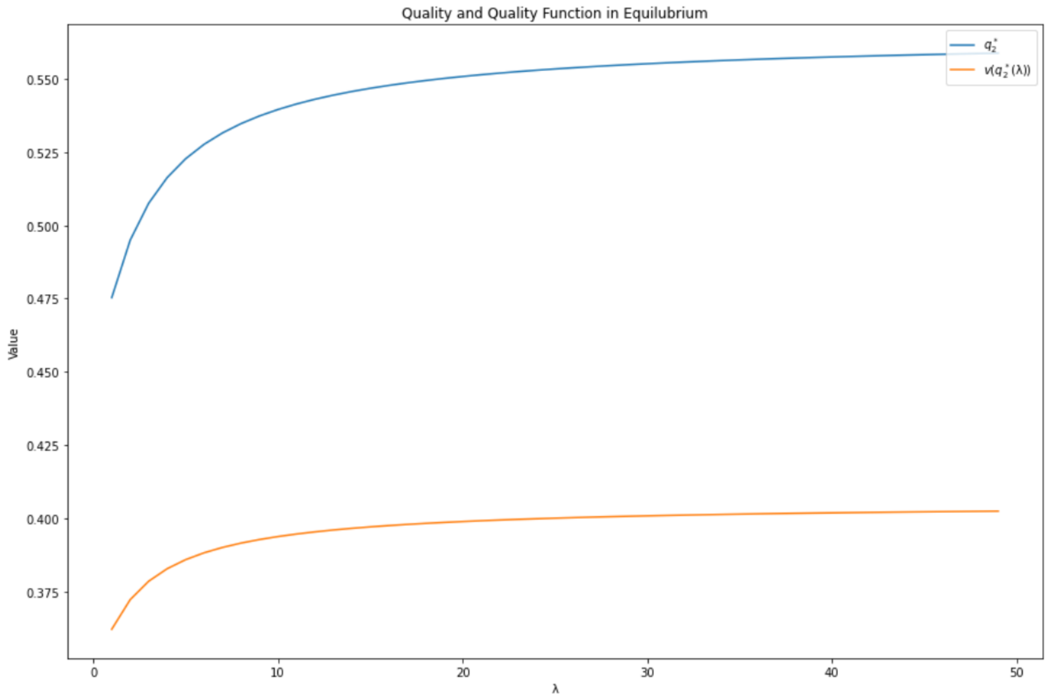

Figure 2. Quality and quality function of Firm 2 in the equilibrium

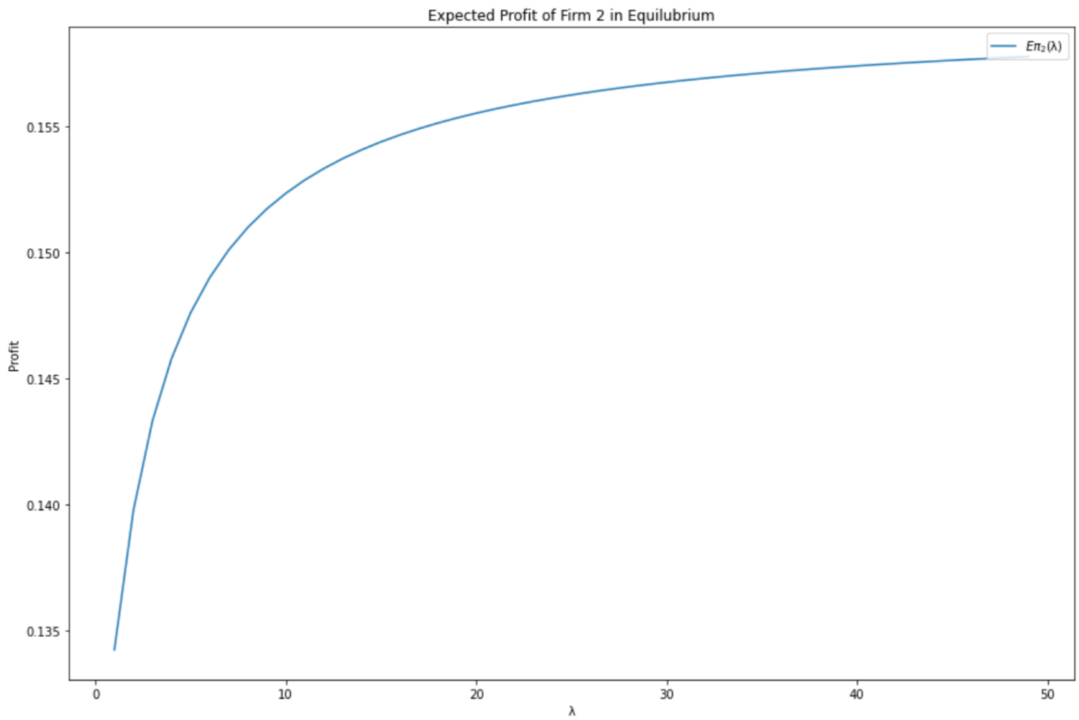

Figure 3. Expected profit of Firm 2 in the equilibrium

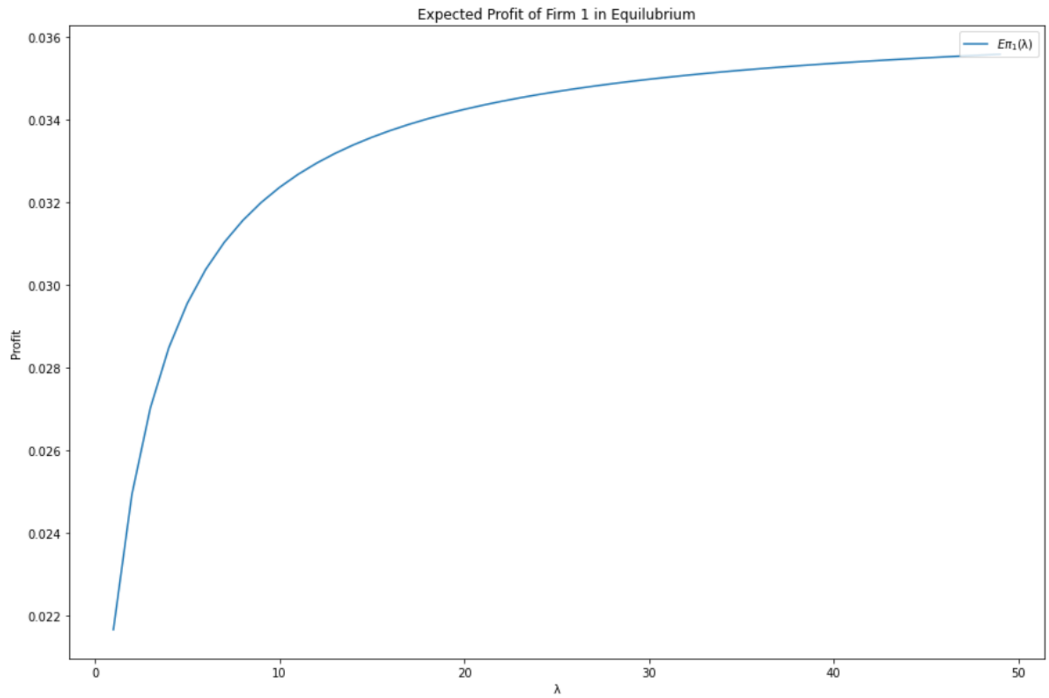

Figure 4. Expected profit of Firm 1 in the equilibrium

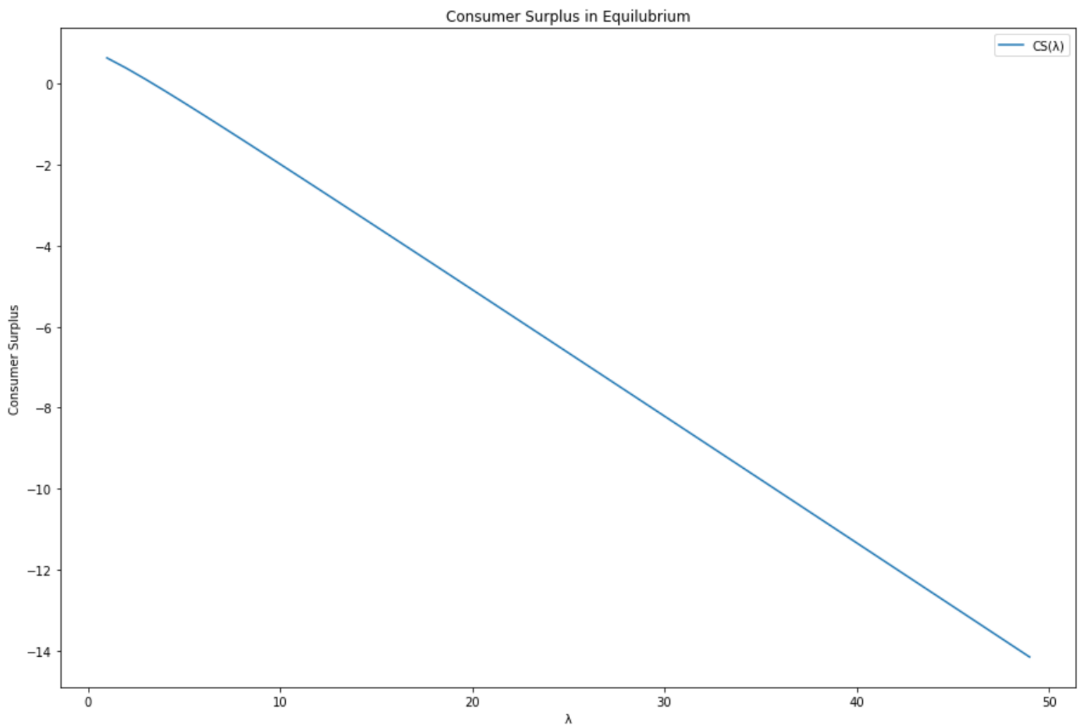

Figure 5. Consumer surplus in the equilibrium

4.5 Main Result

In this section, we investigate the effect of loss aversion on the equilibrium prices, qualities, profits, and also on consumer surplus. Also, we compare the profits under no loss aversion, to those with loss aversion with both $\lambda > 1$ and also to those with loss aversion with $\lambda = 1$. Now, we state these comparative statics' result formally.

Proposition 1: When buyers are loss averse with $\lambda > 1$,

(i) $q_1^* = 0$,

(ii) $\frac{dq_2}{d\lambda} > 0$ if and only if $1 > v'(q_2^*)$.

We have already established that $q_1^* = 0$, through observing equation (4.13). Also, q_2^* is characterized by equation (4.14), which is given below

$$\frac{61(1 + \lambda + v'(q_2))}{108(2 + \lambda)} - c'(q_2) = 0$$

We analyze how the quality of firm 2 changes with λ, that is we aim to find the sign of $\frac{dq_2}{d\lambda}$. To do this, we employ the Implicit Function Theorem (IFT), using $F(q_2; \lambda) = 0$, where

$$F(q_2; \lambda) = \frac{61(1 + \lambda + v'(q_2))}{108(2 + \lambda)} - c'(q_2) = 0$$

Now, using IFT gives us the following:

$$\frac{dq_2}{d\lambda} = -\frac{\frac{\partial F(q_2;\lambda)}{\partial \lambda}}{\frac{\partial F(q_2,\lambda)}{\partial q_2}}$$

The partial derivatives of $F(q_2; \lambda)$ with respect to λ and q_2 are sufficient to analyze $\frac{dq_2}{d\lambda}$. The partial derivative of $F(q_2; \lambda)$ with respect to λ is as follows:

$$\frac{\partial F(q_2; \lambda)}{\partial \lambda} = \frac{183(1 - v'(q_2))}{4(9(2 + \lambda))^2}$$

The partial derivative of $F(q_2; \lambda)$ with respect to q_2 is as follows:

$$\frac{\partial F(q_2; \lambda)}{\partial q_2} = \frac{61v''(q_2)}{108(2 + \lambda)} - c''(q_2)$$

Then, we get the $\frac{dq_2}{d\lambda}$ as follows:

$$\frac{dq_2}{d\lambda} = -\frac{183(1 - v'(q_2))}{3(2 + \lambda)(61v''(q_2) - 108(2 + \lambda)c''(q_2))}$$

Thus, since we need to consider the sign of the $\frac{dq_2}{d\lambda}$, we analyze the sign of the numerator and the denominator part by part. We start analyzing with the denominator.

$$3(2 + \lambda)61v''(q_2) < 0 \tag{4.19}$$

$$108(2 + \lambda)c''(q_2) > 0 \tag{4.20}$$

(4.19) is negative because $\lambda > 1$ and $v(\cdot)$ is a strictly concave function, which means the second derivation of $v''(q_2)$ is negative. (4.20) is positive because both $\lambda > 1$ and $c''(q_2) > 0$ because $c(\cdot)$ is a strictly convex function. Now, putting these together, we find that the denominator is negative.

$$3(2 + \lambda)(61v''(q_2) - 108(2 + \lambda)c''(q_2)) < 0$$

Now, we look at the sign of the numerator, $183(1 - v'(q_2))$, which is determined by the sign of $1 - v'(q_2)$. Therefore the sign of $\frac{dq_2}{d\lambda}$ is the same as the sign of $1 - v'(q_2)$. We observe that whenever $1 > v'(q_2^*)$, we have $\frac{dq_2}{d\lambda} > 0$ and whenever $1 \leq v'(q_2^*)$, we have $\frac{dq_2}{d\lambda} \leq 0$. Thus, we get

$$\frac{dq_2}{d\lambda} > 0 \Longleftrightarrow 1 > v'(q_2^*)$$

We also check some commonly used concave function examples, to see whether we have $1 > v'(q_2^*)$. For instance, consider the following functions.

$$v(q) = \ln(q + 1) \tag{4.21}$$

$$v(q) = \frac{1 - (1 - q)^2}{2} \tag{4.22}$$

$$v(q) = \sqrt{q} \tag{4.23}$$

For all these functions, we have $1 > v'(q_2^*)$. Thus, assuming $1 > v'(q_2^*)$, we obtain that if buyers are getting more loss averse, the quality of firm 2 is in the equilibrium is higher. In other words, there is a positive relationship between q_2^* and λ.

Proposition 2: When buyers are loss averse with $\lambda > 1$, if $1 > v'(q_2^*)$ and $q_2^* > v(q_2^*)$, then $\frac{dp_2}{d\lambda} > 0$ and $\frac{dp_1}{d\lambda} > 0$.

Recall the equilibrium prices of each firm in the subgame that follows a given quality pair, (q_1, q_2).

$$p_1(q_1, q_2, \underline{\vartheta}) = \frac{(1-\underline{\vartheta})((1+\lambda)(q_2-q_1)+v(q_2)-v(q_1))}{3(2+\lambda)}$$

$$p_2(q_1, q_2, \underline{\vartheta}) = \frac{(2+\underline{\vartheta})((1+\lambda)(q_2-q_1)+v(q_2)-v(q_1))}{3(2+\lambda)}$$

In order to analyze how these prices change with respect to λ, we look for the signs of the derivatives $\frac{dp_1}{d\lambda}$ and $\frac{dp_2}{d\lambda}$, which are as follows:

$$\frac{dp_1}{d\lambda} = \frac{\partial p_1}{\partial \lambda} + \frac{\partial p_1}{\partial q_1}\frac{dq_1}{d\lambda} + \frac{\partial p_1}{\partial q_2}\frac{dq_2}{d\lambda}$$

$$\frac{dp_2}{d\lambda} = \frac{\partial p_2}{\partial \lambda} + \frac{\partial p_2}{\partial q_1}\frac{dq_1}{d\lambda} + \frac{\partial p_2}{\partial q_2}\frac{dq_2}{d\lambda}$$

Since we have, $q_1^* = 0$ and $\frac{dq_1}{d\lambda} = 0$, we get $\frac{\partial p_1}{\partial q_1}\frac{dq_1}{d\lambda} = 0$ and $\frac{\partial p_2}{\partial q_1}\frac{dq_1}{d\lambda} = 0$. Now, we start with $\frac{dp_1}{d\lambda}$. We first consider $\frac{\partial p_1}{\partial \lambda}$.

$$\frac{\partial p_1}{\partial \lambda} = \frac{3(1-\underline{\vartheta})(q_2-v(q_2))}{(3(2+\lambda))^2}$$

When $q_2 > v(q_2)$, we have $\frac{\partial p_1}{\partial \lambda} > 0$. Now consider $\frac{\partial p_1}{\partial q_2}$.

$$\frac{\partial p_1}{\partial q_2} = \frac{(1-\underline{\vartheta})(1+\lambda+v'(q_2))}{3(2+\lambda)}$$

The above expression is always positive since $1 > \underline{\vartheta}$ and $v'(q_2) > 0$. Therefore, $\frac{\partial p_1}{\partial q_2} > 0$. When $1 > v'(q_2)$, we have $\frac{dq_2}{d\lambda} > 0$ byProposition 1. Thus we get

$$\frac{dp_1}{d\lambda} > 0$$

Now consider $\frac{dp_2}{d\lambda}$. For $\frac{\partial p_2}{\partial \lambda}$ we have

$$\frac{\partial p_2}{\partial \lambda} = \frac{3(2+\underline{\vartheta})(q_2 - v(q_2))}{(3(2+\lambda))^2}$$

When $q_2^* > v(q_2^*)$, we have $\frac{\partial p_2}{\partial \lambda} > 0$. Also, we have,

$$\frac{\partial p_2}{\partial q_2} = \frac{(2+\underline{\vartheta})(1+\lambda+v'(q_2))}{3(2+\lambda)}$$

This is clearly positive. We also have $\frac{dq_2}{d\lambda} > 0$ when $1 > v'(q_2)$ by Proposition 1. Thus, we get

$$\frac{dp_2}{d\lambda} > 0$$

Note that our assumption $q_2^* > v(q_2^*)$ holds for a variety of concave functions. For instance for those given in (4.22) and (4.23) it is always the case that $q_2 > v(q_2)$, therefore $q_2^* > v(q_2^*)$ must hold as well.

Proposition 3: When buyers are loss averse with $\lambda > 1$, if $1 > v'(q_2^*)$ and $q_2^* > v(q_2^*)$, then $\frac{d\pi_2}{d\lambda} > 0$ and $\frac{d\pi_1}{d\lambda} > 0$.

We have the equilibrium profit levels given by (4.15) and (4.16).

$$E\pi_1(q_1^* = 0, q_2^*, \lambda) = \frac{7((1+\lambda)q_2^* + v(q_2^*))}{108(2+\lambda)}$$

$$E\pi_2(q_1^* = 0, q_2^*, \lambda) = \frac{61((1+\lambda)q_2^* + v(q_2^*))}{108(2+\lambda)} - c(q_2^*)$$

To show the effect of λ, on these equilibrium profit levels, let's calcualte $\frac{d\pi_1}{d\lambda}$ and $\frac{d\pi_2}{d\lambda}$.

$$\frac{d\pi_1}{d\lambda} = \frac{\partial \pi_1}{\partial \lambda} + \frac{\partial \pi_1}{\partial q_1}\frac{dq_1}{d\lambda} + \frac{\partial \pi_1}{\partial q_2}\frac{dq_2}{d\lambda}$$

$$\frac{d\pi_2}{d\lambda} = \frac{\partial\pi_2}{\partial\lambda} + \frac{\partial\pi_2}{\partial q_1}\frac{dq_1}{d\lambda} + \frac{\partial\pi_2}{\partial q_2}\frac{dq_2}{d\lambda}$$

Again, using $\frac{dq_1}{d\lambda} = 0$, we have

$$\frac{d\pi_1}{d\lambda} = \frac{\partial\pi_1}{\partial\lambda} + \frac{\partial\pi_1}{\partial q_2}\frac{dq_2}{d\lambda}$$

$$\frac{d\pi_2}{d\lambda} = \frac{\partial\pi_2}{\partial\lambda} + \frac{\partial\pi_2}{\partial q_2}\frac{dq_2}{d\lambda}$$

Consider $\frac{d\pi_1}{d\lambda}$. For $\frac{\partial\pi_1}{\partial\lambda}$ we have

$$\frac{\partial\pi_1}{\partial\lambda} = \frac{7(q_2 - v(q_2))}{108(2+\lambda)^2}$$

When $q_2 > v(q_2)$, we have $\frac{\partial\pi_1}{\partial\lambda} > 0$. For $\frac{\partial\pi_1}{\partial q_2}$ we have

$$\frac{\partial\pi_1}{\partial q_2} = \frac{7(1+\lambda+v'(q_2))}{108(2+\lambda)}$$

which is positive. We also have $\frac{dq_2}{d\lambda} > 0$ when $1 > v'(q_2^*)$, by Proposition 1. Therefore, we get

$$\frac{d\pi_1}{d\lambda} > 0$$

Now consider $\frac{d\pi_2}{d\lambda}$. For $\frac{\partial\pi_2}{\partial\lambda}$ we have

$$\frac{\partial\pi_2}{\partial\lambda} = \frac{61(q_2 - v(q_2))}{108(2+\lambda)^2}$$

When $q_2 > v(q_2)$, we have $\frac{\partial\pi_2}{\partial\lambda} > 0$. For $\frac{\partial\pi_2}{\partial q_2}$ we have

$$\frac{\partial\pi_2}{\partial q_2} = \frac{61(1+\lambda+v'(q_2))}{108(2+\lambda)} - c'(q_2)$$

Note that since this is the first order condition of firm 2 with respect to q_2, it must be zero at q_2^*. Thus, we must have $\frac{\partial\pi_2}{\partial q_2} = 0$. Then, $\frac{\partial\pi_2}{\partial q_2}\frac{dq_2}{d\lambda} = 0$, implying $\frac{d\pi_2}{d\lambda} = \frac{\partial\pi_2}{\partial\lambda}$. As the right hand side is positive, we get

$$\frac{d\pi_2}{d\lambda} > 0$$

Now we analyze the consumer surplus in the equilibrium. Using $q_1^* = 0$, the consumer surplus for a given $\underline{\vartheta}$ is as follows:

$$CS(\underline{\vartheta}) = \int_{\hat{\vartheta}}^{\underline{\vartheta}+1} \vartheta v(q_2) - p_2 + \vartheta q_2 - \lambda(p_2 - p_1) d\vartheta + \int_{\underline{\vartheta}}^{\hat{\vartheta}} -p_1 - \lambda\vartheta q_2 + (p_2 - p_1) d\vartheta$$

where $\hat{\vartheta}(\underline{\vartheta}) = \frac{1}{3}(1 + 2\underline{\vartheta})$. Then the expected consumer surplus, before the demand uncertainty is resolved, is as follows

$$CS = \int_0^{1/2} \left(\int_{\hat{\vartheta}}^{\underline{\vartheta}+1} \vartheta v(q_2) - p_2 + \vartheta q_2 - \lambda(p_2 - p_1) d\vartheta + \int_{\underline{\vartheta}}^{\hat{\vartheta}} -p_1 - \lambda\vartheta q_2 + (p_2 - p_1) d\vartheta \right) f(\underline{\vartheta}) d\underline{\vartheta}$$

We plug $\hat{\vartheta}$, p_1, and p_2, then we get:

$$CS = \int_0^{1/2} \frac{(5\underline{\vartheta}^2+14\underline{\vartheta}+8)(v(q_2)+q_2) - q_2\lambda(1+4\underline{\vartheta}-5\underline{\vartheta}^2)}{9} - \frac{((1+\lambda)q_2+v(q_2))((4+\underline{\vartheta}+4\underline{\vartheta}^2)+\lambda(2+5\underline{\vartheta}+2\underline{\vartheta}^2))}{9(2+\lambda)} f(\underline{\vartheta}) d\underline{\vartheta}$$

Solving these integral expressions we get CS as follows

$$CS = \frac{143(q_2 + v(q_2)) - 19q_2\lambda}{108} - \frac{(55 + 41\lambda)((1 + \lambda)q_2 + v(q_2))}{108(2 + \lambda)}$$

Proposition 4: When buyers are loss averse with $\lambda > 1$, if $1 > v'(q_2)$, then $\frac{dCS}{d\lambda} < 0$ if and only if λ is large enough.

To see how consumer surplus changes with λ, let's analyze $\frac{dCS}{d\lambda}$.

$$\frac{dCS}{d\lambda} = \frac{\partial CS}{\partial \lambda} + \frac{\partial CS}{\partial q_1}\frac{dq_1}{d\lambda} + \frac{\partial CS}{\partial q_2}\frac{dq_2}{d\lambda}$$

Note that since $\frac{dq_1}{d\lambda} = 0$, we have

$$\frac{dCS}{d\lambda} = \frac{\partial CS}{\partial \lambda} + \frac{\partial CS}{\partial q_2}\frac{dq_2}{d\lambda}$$

By Proposition 1, we have $\frac{dq_2}{d\lambda} > 0$. Now, for $\frac{\partial CS}{\partial \lambda}$ we have

$$\frac{\partial CS}{\partial \lambda} = -\frac{27v(q_2) + q_2(41\lambda^2 + 164\lambda + 137)}{108(2+\lambda)^2} - \frac{19q_2}{108}$$

which is negative, $\frac{\partial CS}{\partial \lambda} < 0$. We continue with $\frac{\partial CS}{\partial q_2}$ as follows:

$$\frac{\partial CS}{\partial q_2} = \frac{143(v'(q_2)+1) - 19\lambda}{108} - \frac{(55+41\lambda)(1+\lambda+v'(q_2))}{108(2+\lambda)}$$

Note that $\frac{\partial CS}{\partial q_2} < 0$ for large enough λ. To see this we show:

(i) $\frac{143(v'(q_2)+1)-19\lambda}{108} - \frac{(55+41\lambda)(1+\lambda+v'(q_2))}{108(2+\lambda)} = 0$ for some $\hat{\lambda}$

(ii) $\frac{143(v'(q_2)+1)-19\lambda}{108} - \frac{(55+41\lambda)(1+\lambda+v'(q_2))}{108(2+\lambda)}$ is decreasing in λ

Let start with (i).

$$\frac{143(v'(q_2)+1) - 19\lambda}{108} - \frac{(55+41\lambda)(1+\lambda+v'(q_2))}{108(2+\lambda)} = 0$$

$$(1 + v'(q_2)(231 + 102\lambda) - \lambda(93 + 60\lambda) = 0$$

Note that the solution of the equation (4.104) gives the $\hat{\lambda}$. There are two solution for the equation.

$$\hat{\lambda_1} = \frac{1}{40}(-\sqrt{1156v'(q_2)^2 + 6364v'(q_2) + 6169} + 34v'(q_2) + 3) \tag{4.24}$$

$$\hat{\lambda_2} = \frac{1}{40}(\sqrt{1156v'(q_2)^2 + 6364v'(q_2) + 6169} + 34v'(q_2) + 3) \tag{4.25}$$

Since any λ cannot be negative, then the threshold $\hat{\lambda}$ which makes $\frac{\partial CS}{\partial q_2} = 0$ is (4.25). Let continue with (ii).

We need to show that $\frac{\partial CS}{\partial q_2 \partial \lambda} < 0$:

$$\frac{\partial CS}{\partial q_2 \partial \lambda} = -\frac{19}{108} - \frac{(27v'(q_2) + 41\lambda^2 + 164\lambda + 137)}{108(2+\lambda)^2}$$

which confirms $\frac{\partial CS}{\partial q_2 \partial \lambda} < 0$, which shows (ii)

Thus we get

$$\frac{\partial CS}{\partial q_2} < 0$$

for all $\lambda > \hat{\lambda}$.

We also have that $\frac{dq_2}{d\lambda} > 0$ by Proposition 1 when $1 > v'(q_2^*)$. As a result, $\frac{\partial CS}{\partial q_2}\frac{dq_2}{d\lambda} < 0$ for large enough λ. Thus, for $\lambda > \hat{\lambda}$, we get

$$\frac{dCS}{d\lambda} < 0$$

Proposition 5: When buyers are loss averse with $\lambda > 1$, if $q_2^* > v(q_2^*)$, then the profits of the firms satisfy,

$$\pi_i(\lambda) \geq \pi_i(\lambda = 1) > \pi_i(NoLA)$$

for $i = 1, 2$.

Using $q_1^* = 0$, we have the profits of the firms as follows.

$$\pi_1(\lambda) = \frac{7((1+\lambda)q_2 + v(q_2))}{108(2+\lambda)}$$

$$\pi_2(\lambda) = \frac{61((1+\lambda)q_2 + v(q_2))}{108(2+\lambda)} - c(q_2)$$

$$\pi_1(\lambda = 1) = \frac{7(2q_2 + v(q_2))}{324}$$

$$\pi_2(\lambda = 1) = \frac{61(2q_2 + v(q_2))}{324} - c(q_2)$$

$$\pi_1(NoLA) = \frac{7v(q_2)}{108}$$

$$\pi_2(NoLA) = \frac{61v(q_2)}{108} - c(q_2)$$

First, to see $\pi_1(\lambda) \geq \pi_1(\lambda = 1)$, check

$$\frac{7((1+\lambda)q_2 + v(q_2))}{108(2+\lambda)} \geq \frac{7(2q_2 + v(q_2))}{324}$$

This is equivalent to

$$(\lambda - 1)(q_2 - v(q_2)) \geq 0$$

which holds under our assumptions. Thus, we get $\pi_1(\lambda) \geq \pi_1(\lambda = 1)$. Now, to see $\pi_1(\lambda = 1) > \pi_1(\text{NoLA})$, note that it is equivalent to

$$\frac{7(2q_2 + v(q_2))}{324} > \frac{7v(q_2)}{108}$$

which is equivalent to

$$q_2 > v(q_2)$$

Thus, $\pi_1(\lambda = 1) > \pi_1(\text{NoLA})$ holds. Thus, we get

$$\pi_1(\lambda) \geq \pi_1(\lambda = 1) > \pi_1(\text{NoLA})$$

Note that even if we take $\lambda = 1$, the profit of firm 1 under loss aversion is higher than the profit of firm 1 when there is no loss aversion. We continue with firm 2 and want to show $\pi_2(\lambda) \geq \pi_2(\lambda = 1)$:

$$\frac{61((1+\lambda)q_2 + v(q_2))}{108(2+\lambda)} - c(q_2) \geq \frac{7(2q_2 + v(q_2))}{324} - c(q_2)$$

This is again equivalent to

$$(\lambda - 1)(q_2 - v(q_2)) \geq 0$$

which holds under our assumption. Thus, $\pi_2(\lambda) \geq \pi_2(\lambda = 1)$. To see $\pi_1(\lambda = 1) > \pi_1(\text{NoLA})$, check

$$\frac{61(2q_2 + v(q_2))}{324} - c(q_2) > \frac{61v(q_2)}{108} - c(q_2)$$

which also holds since $q_2 > v(q_2)$. Thus, we get

$$\pi_2(\lambda = 1) > \pi_2(NoLA)$$

We get

$$\pi_2(\lambda) \geq \pi_2(\lambda = 1) > \pi_2(NoLA)$$

CHAPTER 5

MODEL:HORIZONTAL PRODUCT DIFFERENTIATION

In this chapter, we consider a horizontal product differentiation model where two firms, firm A and firm B are engaged in a three-stage duopoly game. Buyers draw their locations which are distributed uniformly over the interval [0, 1], in the first stage. Firms only know the distribution but does not know the location of any given buyer. In the second stage, the firms choose their locations, z_A and z_B, simultaneously. Here, without loss of generality, z_A is the distance of firm A from point 0, and z_B is the distance of firm B from point 1. Moreover, the locations become common knowledge for the buyers. Lastly, in the third and final stage, the firms, announce their prices p_A and p_B, simultaneously. Once the prices are announced, buyers choose which firm to buy from, and then the demand each firms gets is realized, as well as the profit of each firm.

Each buyer has a unit demand, that is each buyer chooses to buy exactly one unit of product, either from firm A with the firm's location z_A at the price p_A, or from firm B with the location z_B at price p_B. The buyers have a per distance transportation cost given by t where $t > 0$. Thus, a buyer who is located at x faces with the transportation cost as $t(x - z_A)^2$ if the buyer chooses to buy from firm A, and cost as $t(1 - z_B - x)^2$ if the buyer chooses to buy from firm B, under the quadratic transportation cost and where $1 - z_B \geq z_A$. When a buyer with a location x purchases a unit of product from firm i, then she gets the following intrinsic utility

$$U(x, i) = \begin{cases} v - p_A - t(x - z_A)^2 & \text{if she buys from firm A} \\ v - p_B - t(1 - z_B - x)^2 & \text{if she buys from firm B} \end{cases}$$

When the location choice and price competition stages are over, both z_i and p_i for $i = A, B$ will be common knowledge. For any given (z_A, z_B) and (p_A, p_B), there is a marginal location of buyer, whose location is denoted by $\hat{x}(z_A, z_B, p_A, p_B)$, and she is indifferent between purchasing 1 unit from firm A and purchasing 1 unit from firm B.

Then, $\hat{x}$ is given by the x that solves

$$v - p_A - t(x - z_A)^2 = v - p_B - t(1 - z_B - x)^2$$

Without loss of generality, suppose $1 - z_B \geq z_A$. Then all buyers with a type less than $\hat{x}$ buy from firm A and all buyers with a location more than $\hat{x}$ buy from firm 2. Then, a buyer with a location x has the following intrinsic utility.

$$U(x, z_A, z_B, p_A, p_B) = \begin{cases} v - p_A - t(x - z_A)^2 & \text{if } x < \hat{x} \\ v - p_B - t(1 - z_B - x)^2 & \text{if } x > \hat{x} \end{cases}$$

We assume that location and prices are non-negative: $z_i \geq 0$ and $p_i \geq 0$, for $i \in A, B$. Location z_i may be zero (thus a maximal differentiation if it is zero for both $i = A, B$), but cannot be negative. We also assume, $1 - z_B \geq z_A$ which means that firm B is always to the right hand side of firm A or they located at the same location (thus a minimal differentiation). Now, given $1 - z_B \geq z_A$, the demand each firm gets will be given as follows

$$D_A(z_A, z_B, p_A, p_B) = \hat{x} = \frac{1 - z_B + z_A}{2} + \frac{p_B - p_A}{2t(1 - z_B - z_A)}$$

and

$$D_B(z_A, z_B, p_A, p_B) = 1 - \hat{x} = \frac{1 + z_B - z_A}{2} - \frac{p_B - p_A}{2t(1 - z_B - z_A)}$$

The firms have unit cost of a product, given by c_A and c_B. Thus, the profit expression of firm i will be

$$\pi_i(z_A, z_B, p_A, p_A) = [p_i - c_i]D_i(z_A, z_B, p_A, p_B)$$

where $D_i(z_A, z_B, p_A, p_B)$ is firm i's demand under the location choice and price levels (z_A, z_B, p_A, p_B).

The firms announce their locations, z_i, simultaneously. The location pair, (z_A, z_B), are observed publicly by the buyers. Then, they simultaneously compete in prices, (p_A, p_B), and announce them publicly. Finally, each buyer makes a choice about which firm to purchase from, based on the locations of the products z_i, own location x and the prices of the products p_i, and the profit of each firm and the utility of each buyer are realized. The chain of the events is summarized as follows:

Stage 1: Each buyer learns her own true location, x, distributed uniformly over the interval $[0, 1]$.

Stage 2: Firms announce their locations simultaneously.

Stage 3: Firms simultaneously announce their prices, p_A, and p_B.

Stage 3.1: Each buyer chooses which firm to buy from after observing the locations, (z_A, z_B), and the prices, (p_A, p_B), and own location, x.

Stage 3.2: The profits of the firms and the utilities of the buyers are realized.

Now we introduce loss aversion of the buyers into our model and describe it in detail.

5.1 Loss Aversion

The total utility of a loss-averse buyer includes two different components. The first is the consumption/intrinsic utility that a buyer obtains from purchasing one unit of a product net of paying the price for a product. The consumption/intrinsic utility is equal to $v - p_B - t(1 - z_B - x)^2$ (if she buys from firm B) or $v - p_A - t(x - z_A)^2$ (if she buys from firm A), where v is the valuation of the buyer for a product. As it is standard in the literature, we assume that this value, v is the same for each buyer and it is large enough, so that each buyer buys exactly one unit. Thus, the market is covered. The second component is a possible loss/gain utility which is based on the reference point of the buyer and the buyer's actual purchase decision. The reference point may be a firm's location z_i, but the buyer may make the actual purchase from the other firm with location z_{-i}, maybe because the price p_i turned out to be much higher than the price p_{-i}, $i = A, B$. In this case, the buyer experiences a loss/gain utility due to the difference in the distance to the reference point and the distance she incurs in the actual purchase. That is, the loss/gain utility component for a buyer comes from the ex-post comparison between the two products (reference and purchased one) in terms of transportation cost. We assume that, when the firms announce their locations, z_A and z_B, the buyers observe the locations and form their reference product as follows:

$$R(x) = \begin{cases} A & \text{if } x - z_A < 1 - z_B - x \\ B & \text{if } x - z_A > 1 - z_B - x \end{cases}$$

Thus, each buyer, looking at the firms' locations, sets the reference location as the closer firm's location. After the reference locations are set, the price competition takes place and prices are announced. Observing the the prices, a buyer may not purchase from the firm in her reference location. Then, a loss disutility emerges in terms of larger distance the buyer has to travel. The loss/gain utility component includes only loss part in terms of transportation cost.

The utility of a buyer at location x, whose reference product is firm j, but buys from firm i, is denoted as $U(x, i; j)$. Then, we have the following possible utility expressions, which are $U(x, A; A)$, $U(x, A; B)$, $U(x, B; B)$, and $U(x, B; A)$. Without loss of generality, we assume $1 - z_B \geq z_A$. As before, denoting $\lambda > 1$ as the weight for losses, these utilities are as follow:

$$U(x, A; A) = v - p_A - t(x - z_A)^2$$

$$U(x, B; B) = v - p_B - t(1 - z_B - x)^2$$

$$U(x, B; A) = v - p_B - t(1 - z_B - x)^2 - \lambda t((1 - z_B - x)^2 - (x - z_A)^2)$$

$$U(x, A; B) = v - p_A - t(x - z_A)^2 - \lambda t((x - z_A)^2 - (1 - z_B - x)^2)$$

For instance, a buyer with a location x, chooses her reference product as firm B if $x - z_A > 1 - z_B - x$ holds. However, if the buyer decides to purchase from firm

A, then she faces a loss due to the longer distance she actually travels relative to the distance in her reference point, in which case the loss is given by $\lambda t((x - z_A)^2 - (1 - z_B - x)^2)$. Also, note that the loss/gain utility component is seen only in $U(x, A; B)$ and $U(x, B; A)$.

5.2 Model Analysis without Loss Aversion

We used standard horizontal product differentiation with variable cost as a benchmark model. As explained earlier in the model section, each buyer has a location as x that is drawn from a uniform distribution over [0, 1]. Each buyer knows her own true type x. However, the firms do not know the realized x of any buyer at the first stage.

$$U(x, z_A, z_B, p_A, p_B) = \begin{cases} v - p_A - t(x - z_A)^2 & \text{if } p_A + t(x - z_A)^2 < p_B + t(1 - z_B - x)^2 \\ v - p_B - t(1 - z_B - x)^2 & \text{if } p_A + t(x - z_A)^2 > p_B + t(1 - z_B - x)^2 \end{cases}$$

After the firms' locations and products' prices are announced by the firms, each buyer has two options, either purchase from firm A or purchase from firm B. However, there is a buyer who is indifferent between buying from firm A or firm B. For this buyer, the utility of buying from firm A is the same as the utility of buying from firm B. We first find this indifferent buyer's location through $U(x, B) = U(x, A)$, where we abuse notation with denoting the net utility of a buyer with at location at x who buys from firm i as $U(x, i)$. Denoting the type of this indifferent buyer with $\hat{x}$, we have

$$\hat{x}(z_A, z_B, p_A, p_B) = \frac{p_B - p_A}{2t(1 - z_B - z_A)} + \frac{1 - z_B + z_A}{2}$$

where we assumed $1 - z_B \geq z_A$.

If $\hat{x} > x$, then the buyer chooses to purchase the product from firm A with price p_A, otherwise she decides to buy from firm B with price p_B. Thus, the demand each firm gets will be given as follows

$$D_A(z_A, z_B, p_A, p_B) = \hat{x} = \frac{p_B - p_A}{2t(1 - z_B - z_A)} + \frac{1 - z_B + z_A}{2}$$

$$D_B(z_A, z_B, p_A, p_B) = 1 - \hat{x} = \frac{1 + z_B - z_A}{2} - \frac{p_B - p_A}{2t(1 - z_B - z_A)}$$

The profits of the firm A and firm B as follows

$$\pi_A(z_A, z_B, p_A, p_B) = (p_A - c_A)D_A(z_A, z_B, p_A, p_B) = (p_A - c_A)\hat{x}$$

$$\pi_B(z_A, z_B, p_A, p_B) = (p_B - c_B)D_B(z_A, z_B, p_A, p_B) = (p_B - c_B)(1 - \hat{x})$$

Note that the firms have a unit cost of product, given by c_A and c_B. Now, in order to solve for the price equilibrium with a location pair (z_A, z_B), we first solve for the best response functions of each firm. To do this we we maximize the profit of each firm with respect to own price. The best response functions are then given by

$$p_A(z_A, z_B, p_B, c_A) = \frac{p_B + c_A}{2} + \frac{2t(1 - z_B - z_A)(1 - z_B + z_A)}{4}$$

$$p_B(z_A, z_B, p_A, c_B) = \frac{p_A + c_B}{2} + \frac{2t(1 - z_B - z_A)(1 + z_B - z_A)}{4}$$

Solving these best response functions, we get the equilibrium prices as follows.

$$p_A(z_A, z_B, c_A, c_B) = \frac{2c_A + c_B}{3} + \frac{t(1 - z_B - z_A)(3 - z_B + z_A)}{3}$$

$$p_B(z_A, z_B, c_A, c_B) = \frac{c_A + 2c_B}{3} + \frac{t(1 - z_B - z_A)(3 + z_B - z_A)}{3}$$

Now, we focus on location choice of the firms. In the location competition, we consider the profit maximization for both firms.

$$\frac{d\pi_A}{dz_A} = (p_A - c_A)\left(\frac{\partial D_A}{\partial z_A} + \frac{\partial D_A}{\partial p_B}\frac{dp_B}{dz_A}\right)$$

$$\frac{d\pi_B}{dz_B} = (p_B - c_B)\left(\frac{\partial D_B}{\partial z_B} + \frac{\partial D_B}{\partial p_A}\frac{dp_A}{dz_B}\right)$$

We have $\frac{\partial\pi_A}{\partial p_A}\frac{\partial p_A}{\partial z_A} = 0$ due to the Envelope theorem, $\frac{\partial\pi_A}{\partial p_A} = 0$ because firm A maximizes with respect to price in the third stage. Also, we have $\frac{\partial\pi_B}{\partial p_B}\frac{\partial p_B}{\partial z_B} = 0$ with the same reason for firm B, due to the Envelope Theorem.

Thus, we focus on direct effect of z_A and z_B on profit which is called demand effect and indirect effect through the change of firm B's product price for firm A and also the change of firm A's product price for firm B, respectively. The summation of the two effects is the total effect, which gives the direction of the location choices of the firms. In other words, using the fact that for firm i, $(p_i - c_i) > 0$, if the total effect is less than zero, $\frac{d\pi_B}{dz_B} < 0$, then the firm i always wants to move to the corner of her side. For instance, if $i = A$, then the firm A always want to move left in these conditions.

Let's start with location choice of firm A. We need to find the demand effect first.

$$\frac{\partial D_A}{\partial z_A} = \frac{1}{2} + \frac{c_B - c_A}{6t(1 - z_B - z_A)^2} + \frac{z_B - z_A}{3(1 - z_B - z_A)}$$

Let's continue with the strategic effect of firm A.

$$\frac{\partial D_A}{\partial p_B}\frac{dp_B}{dz_A} = \frac{z_A - 2}{3(1 - z_B - z_A)}$$

Now, we can calculate the total effect of firm A as demand effect together with the strategic effect as follows:

$$\frac{1}{2} + \frac{z_B - 2}{3(1 - z_B - z_A)} + \frac{c_B - c_A}{6t(1 - z_B - z_A)^2}$$

Consider the location choice of firm B. We need to find the demand effect first.

$$\frac{\partial D_B}{\partial z_B} = \frac{1}{2} - \frac{c_B - c_A}{6t(1 - z_B - z_A)^2} - \frac{z_B - z_A}{3(1 - z_B - z_A)}$$

The strategic effect of firm B is given by

$$\frac{\partial D_B}{\partial p_A}\frac{dp_A}{dz_B} = \frac{z_B - 2}{3(1 - z_B - z_A)}$$

The total effect of firm B is then

$$\frac{1}{2} + \frac{z_A - 2}{3(1 - z_B - z_A)} - \frac{c_B - c_A}{6t(1 - z_B - z_A)^2}$$

5.3 Model Analysis with Loss Aversion

After the firms compete for location choice, but before the prices are revealed, the buyers form their reference products based on the observed firm locations, as follows:

$$R(x, z_A, z_B) = \begin{cases} A & \text{if } x < \frac{1}{2}(1 + z_A - z_B) \\ B & \text{if } x > \frac{1}{2}(1 + z_A - z_B) \end{cases}$$

After the firms' locations and products' prices are announced by the firms, each buyer has two options, either purchase from firm A or purchase from firm B. However, there is a buyer who is indifferent between buying from firm A or firm B. For this buyer, the utility of buying from firm A is the same as the utility of buying from firm B. We first find this indifferent buyer's location, denoted $\hat{x}$. Assuming $1 - z_B \geq z_A$, recall that for a buyer with location x, we have four possible utility levels, which depend on her reference point and her actual purchase. Denoting the net utility of a buyer with at location x, whose reference product is j and she buys from firm i as $U(x, i; j)$, we have

$$U(x, A; A) = v - p_A - t(x - z_A)^2$$

$$U(x, B; B) = v - p_B - t(1 - z_B - x)^2$$

$$U(x, B; A) = v - p_B - t(1 - z_B - x)^2 - \lambda t((1 - z_B - x)^2 - (x - z_A)^2)$$

$$U(x, A; B) = v - p_A - t(x - z_A)^2 - \lambda t((x - z_A)^2 - (1 - z_B - x)^2)$$

We consider the two possible case: $U(x, B; B) = U(x, A; B)$ and $U(x, A; A) = U(x, B; A)$ in order to get $\hat{x}$.

(i) For $U(x, B; B) = U(x, A; B)$ we get

$$\hat{x}(z_A, z_B, p_A, p_B, \lambda) = \frac{p_B - p_A}{2t(1+\lambda)(1 - z_B - z_A)} + \frac{1 - z_B + z_A}{2}$$

(ii) For $U(x, A; A) = U(x, B; A)$ we get

$$\hat{x}(z_A, z_B, p_A, p_B, \lambda) = \frac{p_A - p_B}{2t(1+\lambda)(1 - z_B - z_A)} + \frac{1 - z_B + z_A}{2}$$

Thus, we get

$$\hat{x}(z_A, z_B, p_A, p_B, \lambda) = \frac{|p_B - p_A|}{2t(1+\lambda)(1 - z_B - z_A)} + \frac{1 - z_B + z_A}{2}$$

At this point, WLOG we assume that $p_B \geq p_A$. Thus the relevant comparison is $U(x, B; B) = U(x, A; B)$. Denoting the type of this indifferent buyer with $\hat{x}$, we have

$$\hat{x}(z_A, z_B, p_A, p_B) = \frac{p_B - p_A}{2t(1+\lambda)(1 - z_B - z_A)} + \frac{1 - z_B + z_A}{2}$$

where $\lambda > 1$, $p_B \geq p_A$ and $1 - z_B \geq z_A$

If $\hat{x} > x$, then the buyer chooses to purchase the product from firm A with price p_A, otherwise she decides to buy from firm B with price p_B. Thus, the demand each firm gets will be given as follows

$$D_A(z_A, z_B, p_A, p_B) = \hat{x} = \frac{p_B - p_A}{2t(1+\lambda)(1 - z_B - z_A)} + \frac{1 - z_B + z_A}{2}$$

$$D_B(z_A, z_B, p_A, p_B) = 1 - \hat{x} = \frac{1 + z_B - z_A}{2} - \frac{p_B - p_A}{2t(1+\lambda)(1 - z_B - z_A)}$$

The profits of the firm A and firm B are as follows

$$\pi_A(z_A, z_B, p_A, p_B) = (p_A - c_A)D_A(z_A, z_B, p_A, p_B) = (p_A - c_A)\hat{x}$$

$$\pi_B(z_A, z_B, p_A, p_B) = (p_B - c_B)D_B(z_A, z_B, p_A, p_B) = (p_B - c_B)(1 - \hat{x})$$

Note that the firms have a unit cost of product, given by c_A and c_B. Now, in order to solve for the price equilibrium with a location pair (z_A, z_B), we first solve for the best response functions of each firm. To do this we we maximize the profit of each firm with respect to own price. The best response functions are then given by

$$p_A(z_A, z_B, p_B, c_A, \lambda) = \frac{p_B + c_A}{2} + \frac{(1+\lambda)t(1 - z_B - z_A)(1 - z_B + z_A)}{2}$$

$$p_B(z_A, z_B, p_A, c_B, \lambda) = \frac{p_A + c_B}{2} + \frac{(1+\lambda)t(1 - z_B - z_A)(1 + z_B - z_A)}{2}$$

Solving these best response functions, we get the equilibrium prices in the price competition subgame, in terms of locations, as follows.

$$p_A(z_A, z_B, c_A, c_B) = \frac{2c_A + c_B}{3} + \frac{(1+\lambda)t(1 - z_B - z_A)(3 - z_B + z_A)}{3}$$

$$p_B(z_A, z_B, c_A, c_B) = \frac{c_A + 2c_B}{3} + \frac{(1+\lambda)t(1 - z_B - z_A)(3 + z_B - z_A)}{3}$$

Now, we focus on location choice of the firms. In the location competition, we consider the profit maximization for both firms.

$$\frac{d\pi_A}{dz_A} = (p_A - c_A)\left(\frac{\partial D_A}{\partial z_A} + \frac{\partial D_A}{\partial p_B}\frac{dp_B}{dz_A}\right)$$

$$\frac{d\pi_B}{dz_B} = (p_B - c_B)\left(\frac{\partial D_B}{\partial z_B} + \frac{\partial D_B}{\partial p_A}\frac{dp_A}{dz_B}\right)$$

We have $\frac{\partial \pi_A}{\partial p_A}\frac{\partial p_A}{\partial z_A} = 0$ due to the Envelope theorem, since $\frac{\partial \pi_A}{\partial p_A} = 0$ because firm A maximizes with respect to price in the third stage. Also, we have $\frac{\partial \pi_B}{\partial p_B}\frac{\partial p_B}{\partial z_B} = 0$ with the same reason for firm B, due to the Envelope Theorem.

Thus, we focus on direct effect of z_A and z_B on profit which is the combination of demand effect and the indirect effect through the change of firm B's product price for firm A and also the change of firm A's product price for firm B, respectively. Using the fact that for firm i, $(p_i - c_i) > 0$, if the total effect is less than zero, $\frac{d\pi_B}{dz_B} < 0$, then the firm i always wants to move to the corner of her side. For instance, if i = A, then the firm A always want to move left in that case.

Let's start with location choice of firm A. We need to find the demand effect first.

$$\frac{\partial D_A}{\partial z_A} = \frac{1}{2} + \frac{c_B - c_A}{6t(1+\lambda)(1-z_B-z_A)^2} + \frac{z_B - z_A}{3(1-z_B-z_A)}$$

The strategic effect of firm A is

$$\frac{\partial D_A}{\partial p_B}\frac{dp_B}{dz_A} = \frac{z_A - 2}{3(1-z_B-z_A)}$$

Now, we can calculate the total effect of firm A as the sum of demand effect and strategic effect.

$$\frac{1}{2} + \frac{z_B - 2}{3(1-z_B-z_A)} + \frac{c_B - c_A}{6t(1+\lambda)(1-z_B-z_A)^2}$$

Consider the location choice of firm B. We need to find the demand effect first.

$$\frac{\partial D_B}{\partial z_B} = \frac{1}{2} - \frac{c_B - c_A}{6t(1+\lambda)(1-z_B-z_A)^2} - \frac{z_B - z_A}{3(1-z_B-z_A)}$$

The strategic effect of firm B is

$$\frac{\partial D_B}{\partial p_A}\frac{dp_A}{dz_B} = \frac{z_B - 2}{3(1-z_B-z_A)}$$

The total effect of firm B is then

$$\frac{1}{2} + \frac{z_A - 2}{3(1-z_B-z_A)} - \frac{c_B - c_A}{6t(1+\lambda)(1-z_B-z_A)^2}$$

5.4 Main Result

In this subsection, we investigate the effect of loss aversion on the maximal product differentiation result. We assume that firm A has some cost advantage, that is, $c_A < c_B$. To make the analysis easier we assume the following: $c_A = 0$ and $c_B = c > 0$.

In the benchmark model, without loss aversion and with no cost advantage ($c_B = c_A = c$), the total effect of a location change (towards the other firm) is negative for both firms. Thus, firms in the location equilibrium move away from each other as much as possible, locating themselves at the two different end points. Thus, there is maximal product differentiation.

In the case, when firm A has a cost advantage and there is no loss aversion, there is a threshold value, $\hat{c}$, such that for any $c > \hat{c}$, the total effect for firm A is positive, that is, firm A would like to move towards the other firm. Thus, the maximal product differentiation result is no longer valid for $c > \hat{c}$.

However, when there is loss aversion, the threshold value, $\tilde{c}$, is such that for $c < \tilde{c}$ the total effect is negative, and for $c > \tilde{c}$ the total effect is positive. Thus, for $c < \tilde{c}$ we have the maximal product differentiation result. Moreover, we find that $\tilde{c} > \hat{c}$, which means that for all $c \in (\hat{c}, \tilde{c})$, we get maximal product differentiation result only if buyers are loss averse. Now, we show this result in two steps.

Proposition 6: When there is no loss aversion, but there is a cost advantage for firm A, there exists a $\hat{c}$ such that for all $c > \hat{c}$, it is optimal for firm A to move towards the other firm. Thus, for $c > \hat{c}$ maximal product differentiation result does not hold.

We find the total effect of firm A as

$$\frac{1}{2} + \frac{z_B - 2}{3(1 - z_B - z_A)} + \frac{c_B - c_A}{6t(1 - z_B - z_A)^2}$$

When we take $c_B = c_A = c$, then we find that the total effect is negative. However, when we take $c_B = c > 0$ and $c_A = 0$, then we get

$$\frac{1}{2} + \frac{z_B - 2}{3(1 - z_B - z_A)} + \frac{c}{6t(1 - z_B - z_A)^2}$$

To see that there is a threshold $\hat{c}$, such that the total effect is positive for all $c > \hat{c}$, check

$$\frac{1}{2} + \frac{z_B - 2}{3(1 - z_B - z_A)} + \frac{c}{6t(1 - z_B - z_A)^2} > 0$$

Thus, the total effect of a location increase for firm A is positive whenever

$$c > t(1 + z_B + 3z_A)(1 - z_A - z_B)$$

Denoting $\hat{c} = t(1 + z_B + 3z_A)(1 - z_A - z_B)$, we get that for all $c > \hat{c}$, the total effect of firm A is positive and firm A does not move to extreme anymore.

Proposition 7: When there is loss aversion, and there is a cost advantage for firm A, there exists a $\tilde{c}$ such that for all $c < \tilde{c}$, it is optimal for firm A to move away from the other firm. Firm B also moves away from firm A. Thus, for $c < \tilde{c}$ maximal product differentiation result holds.

Under loss aversion, we find the total effect for firm A as follows

$$\frac{1}{2} + \frac{z_B - 2}{3(1 - z_B - z_A)} + \frac{c_B - c_A}{6t(1 + \lambda)(1 - z_B - z_A)^2}$$

When we take $c_B = c_A = c$, then we find that the total effect is negative. However, when we take $c_B = c > 0$ and $c_A = 0$, then we get

$$\frac{1}{2} + \frac{z_B - 2}{3(1 - z_B - z_A)} + \frac{c}{6t(1 + \lambda)(1 - z_B - z_A)^2}$$

To see that there is a threshold $\tilde{c}$, such that the total effect is positive for all $c > \tilde{c}$, check

$$\frac{1}{2} + \frac{z_B - 2}{3(1 - z_B - z_A)} + \frac{c}{6t(1 - z_B - z_A)^2} > 0$$

Thus, the total effect of a location increase for firm A is positive whenever

$$c > (1 + \lambda)t(1 + z_B + 3z_A)(1 - z_A - z_B)$$

Denoting $\tilde{c} = (1+\lambda)t(1+z_B+3z_A)(1-z_A-z_B)$, we get that for all $c > \tilde{c}$, the total effect of firm A is positive, and for all $c < \tilde{c}$, the total effect of firm A is negative. When $c < \tilde{c}$ we get maximal product differentiation since firm B also wants to move away from firm A.

Proposition 8: When there is cost advantage, $c_B = c < 0$ and $c_A = 0$, then for any given location pair (z_A, z_B), there exist two thresholds, $\tilde{c}$ and $\hat{c}$ such that $\hat{c} < \tilde{c}$ and for all $\hat{c} < c < \tilde{c}$, there is maximal product differentiation only if buyers are loss averse. Moreover, as λ becomes larger, the range for c values for which the maximal product differentiation occurs only if buyers are loss averse, expands.

For any given location pair (z_A, z_B), we get these two thresholds from the previous two propositions.

$$\hat{c} = t(1+z_B+3z_A)(1-z_A-z_B)$$

$$\tilde{c} = (1+\lambda)t(1+z_B+3z_A)(1-z_A-z_B)$$

Since $\lambda > 1$ we have $\hat{c} < \tilde{c}$ and the result follows from the two propositions above. Also, we find that when λ increases, $\tilde{c}$ increases but $\hat{c}$ stays the same, proving the last argument in the proposition.

CHAPTER 6
DISCUSSION

When we depict the model to study the relationship between loss aversion and the vertical product differentiation model under demand uncertainty, we considered analyzing both fixed and variable cost (of quality) cases. We first focused on the fixed cost case and characterized the subgame perfect Nash equilibrium of the game with and without loss aversion. Also, we conducted comparative statics for qualities, prices, expected profits, and consumer surplus. We also studied a closed form example to check the degree of loss aversion effect on the equilibrium values. However, when we focus on the variable cost case, the general results are considerably complicated to carry out the comparative statics. Then, we focus on the closed-form solution to get less complicated results. We use the function $v(q_i) = \frac{1-(1-q_i)^2}{2}$ and $v(q_i) = \ln(q_i + 1)$ as a quality value function while keeping the cost function as $\frac{q_i^2}{2}, i \in 1, 2$. Unfortunately, even with the specific functional forms, we cannot obtain closed form solutions, especially on the quality competition side. It is an essential point that the complication appears in both with and without loss aversion models. We also tried to use MATLAB software to solve the equations, but the solution could not be reached due to the complexity of the equations. Our actual aim is to add the variable cost case analyses for both with and without loss aversion, but we cannot progress to solve the complexity. Still, below we show the best response functions for both with and without loss aversion models under variable cost.

We start with the best response functions of prices of the without loss aversion model under variable cost. The best response functions are as follows.

$$p_1(q_1, q_2, p_2, \underline{\vartheta}) = \frac{p_2 + c_1 - \underline{\vartheta}(v(q_2) - v(q_1))}{2}$$

$$p_2(q_1, q_2, p_1, \underline{\vartheta}) = \frac{p_1 + c_2 + (1 + \underline{\vartheta})(v(q_2) - v(q_1))}{2}$$

When we plug the $p_1(q_1, q_2, p_2, \underline{\vartheta})$ and $p_2(q_1, q_2, p_1, \underline{\vartheta})$ each other, we get the prices as:

$$p_1(q_1, q_2, \underline{\vartheta}) = \frac{2c(q_1) + c(q_2) + (1 - \underline{\vartheta})(v(q_2) - v(q_1))}{3}$$

$$p_2(q_1, q_2, \underline{\vartheta}) = \frac{2c(q_2) + c(q_1) + (2 + \underline{\vartheta})(v(q_2) - v(q_1))}{3}$$

The first order condition result for the qualities of firms, q_1 and q_2, respectively. First order condition with respect to q_1

$$-\frac{2c'(q_1)(c(q_2) - c(q_1))}{9} + \frac{c'(q_1)}{12} - \frac{1}{4}(c'(q_1)(v(q_2) - v(q_1)) + v'(q_1)(c(q_2) - c(q_1)))$$
$$-\frac{2v'(q_1)(v(q_2) - v(q_1))}{9} + \frac{v'(q_1)}{18} = 0$$

First order condition with respect to q_2

$$-\frac{45c'(q_2)}{108} - \frac{1}{12}(c'(q_2)(v(q_2) - v(q_1)) + v'(q_2)(c(q_2) - c(q_1))) - \frac{41}{54}v'(q_2)(v(q_2) - v(q_1))$$
$$+\frac{2}{9}c'(q_2)(c(q_2) - c(q_1)) + \frac{102v'(q_2)}{108} = 0$$

We continue with the best response functions of prices of the with loss aversion model under variable cost. The best response functions are as follow.

$$p_1(q_1, q_2, p_2, \underline{\vartheta}, \lambda) = \frac{p_2 + c(q_1)}{2} - \frac{\underline{\vartheta}((1 + \lambda)(q_2 - q_1) + v(q_2) - v(q_1))}{2(2 + \lambda)}$$

$$p_2(q_1, q_2, p_1, \underline{\vartheta}, \lambda) = \frac{p_1 + c(q_2)}{2} + \frac{(1 + \underline{\vartheta})((1 + \lambda)(q_2 - q_1) + v(q_2) - v(q_1))}{2(2 + \lambda)}$$

When we plug the $p_1(q_1, q_2, p_2, \underline{\vartheta}, \lambda)$ and $p_2(q_1, q_2, p_1, \underline{\vartheta}, \lambda)$ each other, we get the prices as:

$$p_1(q_1, q_2, \underline{\vartheta}, \lambda) = \frac{(1 - \underline{\vartheta})((1 + \lambda)(q_2 - q_1) + v(q_2) - v(q_1))}{3(2 + \lambda)} + \frac{2c(q_1) + c(q_2)}{3}$$

$$p_2(q_1, q_2, \underline{\vartheta}, \lambda) = \frac{(2 + \underline{\vartheta})((1 + \lambda)(q_2 - q_1) + v(q_2) - v(q_1))}{3(2 + \lambda)} + \frac{2c(q_2) + c(q_1)}{3}$$

The first order condition result for the qualities of firms, q_1 and q_2, respectively. First order condition with respect to q_1

$$\frac{(2+\lambda)(c(q_2)-c(q_1))}{9((1+\lambda)(q_2-q_1)+v(q_2)-v(q_1))}\left(\frac{(c(q_2)-c(q_1)(1+\lambda+v'(q_1))}{(1+\lambda)(q_2-q_1)+v(q_2)-v(q_1)}-2c'(q_1)\right)$$
$$-\frac{7(1+\lambda+v'(q_1))}{108(2+\lambda)}-\frac{c'(q_1)}{6}=0$$

First order condition with respect to q_2

$$\frac{(2+\lambda)(c(q_2)-c(q_1))}{9((1+\lambda)(q_2-q_1)+v(q_2)-v(q_1))}\left(2c'(q_2)-\frac{(c(q_2)-c(q_1)(1+\lambda+v'(q_2))}{(1+\lambda)(q_2-q_1)+v(q_2)-v(q_1)}\right)$$
$$+\frac{61(1+\lambda+v'(q_2))}{108(2+\lambda)}-\frac{c'(q_2)}{2}=0$$

As it can be seen from the best response functions in the quality competition, the analysis is extremely involved and complicated.

CHAPTER 7

CONCLUSION

Many buyers pay attention to the product's quality and price in real life. The approach is quite a normal situation because when a buyer purchases a product, she wants to enjoy the quality, she wants to get the product as cheaply as possible. Indeed, some buyers mainly pay attention to the product's quality that they might purchase, whereas, others primarily focus on the product's price. Suppose there are two options for buyers, and a buyer purchases a high-quality product with paying high pricing. At that moment, the buyer knows that she can enjoy the high-quality product more than the low-quality one. However, she also knows that she buys more to obtain a high-quality product than the low one. The buyer's decision depends on the valuation of the product her own. However, the buyer might expect another scenario, what if she decides to buy the other option, which is low quality and price. In this case, it is natural to consider the loss aversion concept in the buyers' preferences. Another point is that the firms generally announce their product's qualities before the prices are announced. At the same time, the firms have foresight about the buyers' product valuation, but they might only be sure about the buyers' purchasing decisions once the purchase is realized. It is very natural to consider demand uncertainty for firms. This thesis study investigates the firms' decisions regarding qualities and prices within demand uncertainty and loss aversion concept under a duopolistic competition setting for vertical product differentiation. Also, the thesis analyzes the firms' decisions in terms of locations and prices within demand uncertainty and loss aversion concept under a duopolistic competition setting for horizontal product differentiation.

We consider a duopolistic vertical product differentiation model where the firms compete in quality and prices. The firms do not know the real types of buyers when they compete in quality. After the qualities are announced, the firms learn the buyers' real types of valuations. We consider a reference-dependent setting to build buyers' preferences and involve loss aversion in the utility function. After the demand uncertainty is resolved and the firms compete in prices, each buyer purchases a unit of the product. Therefore, the buyers can compare the realized and reference product in terms of quality and price.

We also consider Hotelling's type of horizontal product differentiation model using quadratic transportation cost and production cost. The firms decide to move extremes in the fundamental or without loss aversion model. We find in the without-loss aversion model that if a firm has a cost advantage, then there is a cost threshold that turns the firm's decision from moving extreme. When we investigate the same issue in the loss aversion case, we find that the threshold cost value exists and is higher than the threshold cost value derived from the without-loss aversion model.

The thesis study highlights the relationship between the loss aversion concept and product differentiation models. This study aids in filling a theoretical gap in the behavioral approach of industrial organization.

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