ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL OF SCIENCE ENGINEERING AND TECHNOLOGY

UNCAPACITATED MULTIPLE ALLOCATION HUB LOCATION PROBLEM UNDER CONGESTION

Ph.D THESIS

Çağrı ÖZGÜN KİBİROĞLU

Department of Industrial Engineering

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TRAFİK SIKIŞIKLIĞI ALTINDA ÇOK ATAMALI KAPASİTE KISITSIZ ANA DAĞITIM ÜSSÜ YERLEŞİM PROBLEMİ

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To my beloved son,

FOREWORD

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June 2019 Çağrı ÖZGÜN KİBİROĞLU

TABLE OF CONTENTS

Page

ABBREVIATIONS

ADÜ	: Ana Dağıtım Üssü
AP	: Australia Post
B&B	: Branch and Bound
CAB	: Civil Aeronautics Board
CMAHLP	: Capacitated Multiple Allocation Hub Location Problem
CPU	: Central Processing Unit
CSAIHLP	: Capacitated Single Allocation Incomplete Hub Location Problem
CSAHLP	: Capacitated Single Allocation Hub Location Problem
CSAHVP	: Capacitated Single Allocation Hub Covering Problem
CSAHVSP	: Capacitated Single Allocation Hub Set Covering Problem
CSAp-HCP	: Capacitated Single Allocation p-Hub Center Problem
CSAp-HLP	: Capacitated Single Allocation p-Hub Location Problem
CSAp-HMP	: Capacitated Single Allocation p-Hub Median Problem
CSAp-HVP	: Capacitated Single Allocation p-Hub Covering Problem
fix-HLP	: Hub Location Problem with Fixed Cost
GRASP	: Greedy Randomized Adaptive Search Procedure
IHLP	: Incomplete Hub Location Problem
HLP	: Hub Location Problem
HAL	: Hub Arc Location Problem
HVP	: Hub Covering Problem
IP	: Integer Programming
ILP	: Integer Linear Programming
LB	: Lower Bound
LP	: Linear Programming
MATLAB	: MATrix LABoratory
MILP	: Mixed Integer Linear Programming
MINLP	: Mixed Integer Non-Linear Programming
MIP	: Mixed Integer Programming
MIQP	: Mixed Integer Quadratic Programming
NLIP	: Non-Linear Integer Programming

NP : Nondeterministic-Polynomial
O/D : Origin/Destination
PSO : Particle Swarm Optimization
p-HMP : p-Hub Median Problem
p-HCP : p-Hub Center Problem
QIP : Quadratic Integer Programming
r-HLP : r-Allocation Hub Location Problem
TR : Turkish Postal
UB : Upper Bound
UMAHLP : Uncapacitated Multiple Allocation Hub Location Problem
UMAHAL : Uncapacitated Multiple Allocation Hub Arc Location Problem
UMAHVP : Uncapacitated Multiple Allocation Covering Problem
UMAHVMP : Uncapacitated Multiple Allocation Maximal Covering Problem
UMAIHLP : Uncapacitated Multiple Allocation Incomplete Hub Location Problem
UMAIp-HLP : Uncapacitated Multiple Allocation Incomplete p-Hub Location Problem
UMAp-HAL : Uncapacitated Multiple Allocation p-Hub Arc Location Problem
UMAp-HCP : Uncapacitated Multiple Allocation p-Hub Center Problem
UMAp-HLP : Uncapacitated Multiple Allocation p-Hub Location Problem
UMAp-HMP : Uncapacitated Multiple Allocation p-Hub Median Problem
USAHAL : Uncapacitated Single Allocation Hub Arc Location Problem
USAHLP : Uncapacitated Single Allocation Hub Location Problem
USAHVP : Uncapacitated Single Allocation Hub Covering Problem
USAIHLP : Uncapacitated Single Allocation Incomplete Hub Location Problem
USAIHVP : Uncapacitated Single Allocation Incomplete Hub Covering Problem
USAIp-HLP : Uncapacitated Single Allocation Incomplete p-Hub Location Problem
USAIp-HMP : Uncapacitated Single Allocation Incomplete p-Hub Median Problem
USAp-HCP : Uncapacitated Single Allocation p-Hub Center Problem
USAp-HLP : Uncapacitated Single Allocation p-Hub Location Problem
USAp-HMP : Uncapacitated Single Allocation p-Hub Median Problem
USAp-HVP : Uncapacitated Single Allocation p-Hub Covering Problem
USAp-HVSP : Uncapacitated Single Allocation p-Hub Set Covering Problem
USA2HLP : Uncapacitated Single Allocation 2-Hub Location Problem
U2Ap-HMP : Uncapacitated 2-Allocation p-Hub Median Problem

SYMBOLS

G	**:** complete graph
N	**:** set of nodes
P	**:** set of potential hubs
$\boldsymbol{i}$	: origin
$\boldsymbol{j}$	: destination
$\boldsymbol{k}, \boldsymbol{l}$	: potential hubs
$\boldsymbol{n}$	: number of nodes in graph
$\boldsymbol{p}$	: number of potential hubs in graph
$\boldsymbol{\alpha}$	: discount factors of transfer and distribution
$\boldsymbol{\gamma}$	: discount factors of collection
$\boldsymbol{\delta}$	: discount factors of distribution
$\boldsymbol{\tau}$	: represents desired capacity level in terms of hub capacity
$\boldsymbol{W_{ij}}$	: total flow between each pair of origin-destination nodes: i and j
$\boldsymbol{C_{ij}}$	: transportation cost assigned to unit flow from i to j
$\boldsymbol{C_{ijkm}}$	: accumulated transportation cost where the cost between hubs are discounted
$\boldsymbol{F_k}$	: cost of establishing a hub at node
$\boldsymbol{O_k}$	: the capacity of hub k
$\boldsymbol{S_k}$	: represents the quantity calculated by multiplying the desired level capacity and capacities of hubs.
$\boldsymbol{Z_{ik}}$	: flow from node i to hub k
$\boldsymbol{Y_{kl}^i}$	: flow from node i via hubs k and l
$\boldsymbol{X_{lj}^i}$	: flow from node i to node j via hub l
$\boldsymbol{X_{ijkm}}$	: flow departing from node i and ending at node j that is routed through hubs k and m
$\boldsymbol{g_k}$	: total flow through hub k
$\boldsymbol{\Gamma_k}$	: congestion cost function
$\boldsymbol{e}, \boldsymbol{b}$	: positive constants
E	: multiplier of congestion cost

$\boldsymbol{H_k}$: 0-1 integer decision variable of opening k

A : matrix including coefficients of x variables in the constraints of Benders decomposition problem

B : matrix including coefficients of y variables in the constraints of Benders decomposition problem

$\boldsymbol{b}$: vector of right-hand sides parameters of Benders decomposition problem

$\boldsymbol{c^T}$: Objective function coefficient of x variables

$\boldsymbol{f^T}$: Objective function coefficient of y variables

$\boldsymbol{x}, \boldsymbol{y}$: decision variables of Benders decomposition problem

$\widehat{\boldsymbol{y}}$: a solution found by the master problem

$\boldsymbol{u}$: dual variables associated with constraint of Benders decomposition sub-problem

$\widehat{\boldsymbol{u_J}}$ **:** extreme solution of dual problem

$\boldsymbol{\theta}, \boldsymbol{\beta}, \boldsymbol{\varepsilon}, \boldsymbol{\sigma}, \boldsymbol{\varphi}$ **:** dual variables associated with constraint of decomposed sub-problem for proposed model

$\boldsymbol{\theta_{ij}}^{\boldsymbol{h}}, \boldsymbol{\beta_{ijk}}^{\boldsymbol{h}},$

$\boldsymbol{\varepsilon_{ij}}^{\boldsymbol{h}}, \boldsymbol{\sigma_{ij}}^{\boldsymbol{h}}, \boldsymbol{\varphi_{ij}}^{\boldsymbol{h}}$: optimum values derived from sub-problem solutions after h iterations.

H : number of iterations

$\boldsymbol{\eta}$ **:** upper bound for transportation costs

$\boldsymbol{T_k}, \boldsymbol{P_k}$: variables defined to linearization

$\boldsymbol{X_i}$: position vector $(x_{i1}, x_{i2}, \cdots, x_{in})$ of particle swarm optimization

$\boldsymbol{V_i}$: velocity vector $(v_{i1}, v_{i2}, \cdots, v_{in})$ of particle swarm optimization

$\boldsymbol{w}$: inertia factor between $[w_{min}, w_{max}]$

$\boldsymbol{r_1}, \boldsymbol{r_2}$: generated random numbers between [0,1]

$\boldsymbol{c_1}, \boldsymbol{c_2}$: weights associated with the social moves versus cognitive moves

$\boldsymbol{lbest}$: best solution experienced by a particle

$\boldsymbol{gbest}$: best solution experienced by swarm

LIST OF TABLES

Page

LIST OF FIGURES

Page

UNCAPACITATED MULTIPLE ALLOCATION HUB LOCATION PROBLEM UNDER CONGESTION

SUMMARY

Hubs are customized facilities whereby used in transportation, logistics and telecommunication systems. Hubs appear in many various fields such as airline industry, postal or cargo services, supply chain management, computer networks, emergency services. Hubs serve as consolidation, switching and sorting centers, and allow for the replacement of the direct connections between all nodes with fewer indirect connections. Hubs benefit from economies of scales by flow consolidation and dissemination. The flows originated from different nodes are accumulated in a hub and the flows designated to the same node are distributed from this hub rather than mapping directly each of the origin-destination pairs.

Hub location problems could play a decisive role which helps to achieve strategic success with its social, political and economic aspects and these decisions couldn’t modify easily or regularly. Location of hubs is significantly important for the strategical decisions arised from network design such as routing and logistic decisions. Hub location problem deal with the location of hubs and allocation of nodes to hubs. The purpose of the problem is to find the optimum hub location and assign non-hub nodes corresponding hubs at minimum cost or time. Since these two decisions have mutually influenced each other, they should be discussed together. Thus, hub location problem should be considered as a design problem of the networks.

The hub location problem has been an important subject of research since the last quarter of 20th century. Despite of great attention on the hub location problem, there is needed to enlighten researchers about the gaps of existing literature, fields that are rarely investigated and the future directions of the problem. Therefore, this thesis starts with a classification structure covered from basic problem to its existing extensions. A classification structure is proposed by reviewing elaborately selected 400 papers. Then, a detailed literature review is presented, significant studies of each classes are exemplified, and recent trends are examined by grouping topics.

In air transportation, as well as in other transportation networks, it is important to extend the network as large as possible to satisfy the customer needs and to obtain competitive advantage. The hub location problem aims to minimize the transportation and opening costs while taking the advantage of economies of scale where the cost of inter-hub connections is discounted. While the problem focuses on obtaining a minimum set of opening and transportation costs, a relatively small number of hubs are selected. These hubs are likely to be overloaded in comparison to non-hub nodes. Besides, the more nodes are inserted to the network, the more capacity restrictions are imposed to balance the flows between nodes. Essentially, in recent times, the air transportation companies seek to resolve these undesired consequences such as delays or congestion and optimize traffic flow density of their hub airports. Hubs' capacities tend to be sensitive against the increasing volume of traffic flows in time. Hence, considering capacity features of the hub location problem had to be included in the

existing literature. The term “capacity” indicates a limitation on hubs. However, restricting possible hub nodes with capacity levels could not represent a realistic approach to the problem. In recent years, a new term “congestion” has been introduced as an additional flow in excess of the capacity which is imposed on hubs.

In this thesis, the congestion term is investigated to optimize network design and to avoid congestion effect at hubs. A hub network design including congestion aims to prevent the imbalance of flow distribution of selected hubs and the overloads at hubs. Including capacity restrictions to model could design a network without congestion, however this design couldn’t project realistically due to ignoring increases of demands in time, possible changes in capacity resulting from delays or disruptions. So, it is important to introducing congestion in the model independently, but related to capacity, flow consolidation and time parameters.

For practical applications of hub location problem, exceeding capacity in terms of the amount of flow and occurring congestion is unavoidable. Also, once a congestion is occurred or capacity of hub become insufficient, network capabilities as processing time, time dependent services and delays is triggered. Unlike the existing literature, congestion is more involved in capacity level than the amount of flow. Thus, congestion becomes more effective around the maximum capacity level. When a hub faces with congestion situation, all network design is adversely influenced that the associated cost should be charged. Subsequently, more the amount of flow increase, less the congestion affects the network capabilities.

In general hub location problems consider the congestion which is limited under a capacity level or a pre-determined proportion of the capacity. It is significantly crucial to identify reversely due to the fact that the congestion is inevitable. The main contribution of this thesis is to define an innovative congestion cost function which is the proportion of the excess flow on each hub to the total flow through respective hub. This function assists in ensuring the capacity limitation of each hub without an explicit determination. Moreover, both incoming and outgoing flows are summed up to determine total flows through hubs, because airports in passenger transportation may suffer from the intensity of both arriving and departing flights. . In order to observe the effect of the congestion on selection of hubs and allocation decisions, different congestion functions are presented comparatively.

Initially, a mathematical model is proposed which determines the location of hubs and allocation structure of the network and controls the congestion at hubs to balance the flow distribution at network. A mixed integer programming model is formulated as an uncapacitated multiple allocation hub location problem that minimizes the transportation cost, opening cost and congestion cost. Throughout this thesis, proposed model aims to locate hubs whose capacities are larger than a desired level and still congestion on hubs is considered as a penalty cost in the objective function. Therefore, hubs are chosen between nodes which have higher capacities in order to minimize the penalty costs arising from surplus flows on hubs.

While modeling the optimization problems, main concern is the size of the problem meaning the number of decision variables and constraints of the problem. The size of the problem makes the solution procedure more difficult in terms of the complexity and the computational time. Classical solution techniques to achieve the optimal solution remain incapable upon increasing number of decision variables and constraints. To overcome these difficulties, partitioning procedures such as Benders decomposition technique and heuristic algorithms may be applied.

Although mathematical formulations of the model may achieve the optimality, the size of the problem and the computational time make the solution more complicated. Because of this, the proposed non-linear model needs a relatively more effortful solution procedures. Another contribution of this thesis is to develop efficient solution procedures for dealing with complex problems. Consequently, an exact solution procedure, Benders decomposition algorithm, and a heuristic algorithm, particle swarm optimization are developed to solve the proposed model. First, Benders decomposition algorithm is applied to the problem. To apply Benders algorithm, the quadratic congestion cost function is linearized. However, the algorithm become more time-consuming than expected. Secondly, to find near optimal solutions for large-scaled problems, an efficient particle swarm optimization algorithm is proposed. All solution procedures are adapted to proposed model structure. The numerical analyses are performed on Australian Post data set. Both solution procedures are examined comparatively. The various versions of the problem (uncapacitated, capacitated and congested) and problems modeled with three distinct congestion functions are comparatively analyzed. The experiments under different scenarios including the modification of congestion function or the variation in congestion factor, capacity level, economies of scale were made. Besides, the sensitivity analyses to compare the heuristic algorithm's efficiency under different parameters including number of iterations , population size, learning coefficients were examined. Lastly, computational experiments and discussions of all suggested models are presented, and the computational performance of suggested solution procedures are tested.

TRAFİK SIKIŞIKLIĞI ALTINDA ÇOK ATAMALI KAPASİTE KISITSIZ ANA DAĞITIM ÜSSÜ YERLEŞİM PROBLEMİ

ÖZET

Ana dağıtım üsleri, ulaşım, lojistik ve telekomünikasyon sistemlerinde kullanılan özel tesislerdir. Ana dağıtım üsleri, havayolu endüstrisi, posta veya kargo hizmetleri, tedarik zinciri yönetimi, bilgisayar ağları, acil durum hizmetleri gibi birçok farklı alanda ortaya çıkmaktadır. Ana dağıtım üssü, toplama, değiştirme ve dağıtma merkezi olarak hizmet eder ve daha az dolaylı bağlantıya sahip tüm düğümler arasındaki doğrudan bağlantıların değiştirilmesine izin verir. Ana dağıtım üssü, akış konsolidasyonu ve yayılması yoluyla ölçek ekonomisinden yararlanmaktadır. Gidiş-geliş çiftlerinin her birini doğrudan haritalamak yerine, farklı düğümlerden gelen akışlar ana dağıtım üssünde toplanır ve aynı düğüme giden akışlar bu ana dağıtım üssünden dağıtılır.

Ana dağıtım üssü yerleşim problemi, sosyal, politik ve ekonomik yönleriyle stratejik başarıya ulaşmada yardımcı olan belirleyici bir rol oynayabilir ve bu kararlar kolayca veya düzenli olarak değiştirilemez. Ana dağıtım üslerinin konumu, rotalama ve lojistik kararlar gibi ağ tasarımından kaynaklanan stratejik kararlar için özellikle önemlidir. Ana dağıtım üssü yerleşim problemi, merkezlerin konumu ve düğümlerin merkezlere atanması ile ilgilidir. Sorunun amacı, en uygun ana dağıtım üssü konumunu bulmak ve ana dağıtım üslerine karşılık gelen ana dağıtım üssü olmayan diğer düğümleri minimum maliyetle veya zamanla atamaktır. Bu iki karar birbirini karşılıklı olarak etkilediğinden, birlikte tartışılmalıdır. Bu nedenle, ana dağıtım üssü yerleşim problemi, ağ tasarım problemi olarak düşünülmelidir.

Ana dağıtım üssü yerleşim problemi 20. yüzyılın son çeyreğinden bu yana önemli bir araştırma konusu olmuştur. Ana dağıtım üssü yerleşim problemlerine gösterilen büyük öneme rağmen, araştırmacıları mevcut literatürdeki boşluklar, nadiren araştırılan alanlar ve problemin gelecekteki motivasyonları hakkında aydınlatmaya ihtiyaç vardır. Dolayısıyla bu tez, temel problemden mevcut uzantılarına kadar bir sınıflandırma yapısı ile başlamaktadır. Seçilmiş 400 makalenin detaylıca değerlendirilmesi ile bir sınıflandırma yapısı önerilmektedir. Daha sonra ayrıntılı bir literatür taraması sunulmakta, her bir sınıfın önemli çalışmaları örneklenmekte ve son trendler konu başlıkları gruplanarak incelenmektedir.

Hava taşımacılığında ve diğer ulaşım ağlarında, müşteri ihtiyaçlarını karşılamak ve rekabet avantajı sağlamak için ağı mümkün olduğu kadar genişletmek önemlidir. Ana dağıtım üssü yerleşim problemi, merkezler arası bağlantı maliyetinin düşürüldüğü ölçek ekonomilerinden faydalanırken; taşıma ve açılış maliyetlerini en aza indirgemeyi amaçlar. Problem minimum açılış ve ulaşım maliyetlerini belirlemeye odaklanırken, nispeten az sayıda ana dağıtım üssü seçilir. Bu seçilen ana dağıtım üsleri, üs olmayan düğümlere kıyasla aşırı yüklenmiş durumdadırlar. Ayrıca, ağa daha fazla düğüm eklenir, düğümler arasındaki akışları dengelemek için daha fazla kapasite kısıtlaması uygulanır. Temel olarak, son zamanlarda, hava taşımacılığı şirketleri, gecikmeler veya trafik sıkışıklığı gibi istenmeyen sonuçları çözmek ve merkez

havaalanlarının trafik akışı yoğunluğunu optimize etmek istemektedir. Ana dağıtım üslerinin kapasiteleri zaman içindeki artan trafik akışlarına karşı hassas olma eğilimindedir. Bu nedenle, ana dağıtım üssü yerleşim problemleri kapasite özelliklerini düşünerek mevcut literatürde yer almak zorundadır. "Kapasite" terimi ana dağıtım üslerinde bir sınırlamaya işaret eder. Ancak, olası ana dağıtım üslerinin kapasite düzeyleriyle sınırlandırılması, soruna gerçekçi bir yaklaşım getiremez. Son yıllarda, ana dağıtım üslerinde yüklenilen kapasiteyi aşan ilave bir akış olarak yeni bir "sıkışıklık" terimi ortaya çıkmıştır.

Bu tezde, sıkışıklık tabiri üslerdeki sıkışıklık etkisinin önlenmesi ve ağ tasarımının en iyilenmesi için ayrıntılı bir şekilde incelenmektedir. Sıkışıklığı içeren bir ana dağıtım üssü tasarımı, seçilen üslerin akış dağılımındaki dengesizliği ve üslerdeki aşırı yükleri önlemeyi amaçlar. Modele kapasite kısıtlamalarının dahil edilmesi ağ sıkışıklığı olmaksızın bir tasarıma olanak sağlayabilir, ancak bu tasarım zaman içinde gecikmelere veya kesintilere sonuç olacak talep artışlarını, olası kapasite değişikliklerini göz ardı ederek gerçekçi şekilde projelendirilemez. Bu nedenle, modelde sıkışıklığı bağımsız ancak kapasite, akış konsolidasyonu ve zaman parametreleri ile ilgili olarak tanımlamak önemlidir.

Ana dağıtım üssü yerleşim probleminin pratik uygulamalarında, akış miktarı bakımından kapasitenin aşılması ve sıkışıklık meydana gelmesi kaçınılmazdır. Ayrıca, bir sıkışıklık meydana geldiğinde veya ana dağıtım üssünün kapasitesi yetersiz olduğunda, işlem süreleri, zamana bağlı servisler ve gecikmeler gibi ağ yetkinlikleri tetiklenir. Mevcut literatürden farklı olarak, sıkışıklık, akış miktarından ziyade kapasite düzeyi ile ilgilidir. Böylece sıkışıklık, maksimum kapasite seviyelerinde daha etkili hale gelir. Bir ana dağıtım üssü sıkışıklık durumuyla karşılaştığında, bağlantılı maliyetlerin de tahsis edilmesiyle tüm ağ tasarımı olumsuz yönde etkilenir. Ardından, akış miktarı arttıkça, tıkanıklık daha az ağ yeteneklerini etkiler.

Genel olarak ana dağıtım üssü yerleşim problemleri, sıkışıklığı bir kapasite seviyesi veya önceden belirlenmiş bir kapasite oranı altında sınırlı olarak düşünür. Sıkışıklığın kaçınılmaz olması nedeniyle tersini tanımlamak çok önemlidir. Bu tezin ana katkısı, her bir ana dağıtım üssü üzerindeki fazla akışın ilgili ana üsteki toplam akışa oranı olarak yenilikçi bir sıkışıklık maliyet fonksiyonunu tanımlamaktır. Bu işlev, her bir ana dağıtım üssündeki kapasite sınırlandırmasının açık bir tespit yapılmadan sağlanmasına yardımcı olur. Ayrıca, hem gelen hem de giden akışlar, ana dağıtım üslerinden geçen toplam akışları belirlemek için toplanır, çünkü yolcu taşımacılığındaki havaalanları, hem gelen hem de giden uçuşların yoğunluğundan muzdarip olabilir. Sıkışıklığın ana dağıtım üssü yeri seçimi ve tahsis kararları üzerindeki etkisini gözlemlemek için, farklı sıkışıklık fonksiyonları karşılaştırmalı olarak sunulur.

Başlangıç olarak, üslerin konumunu ve ağın tahsis yapısını belirleyen ve ağlardaki akış dağılımını dengelemek için merkezlerde oluşan sıkışıklığı kontrol eden yeni bir matematiksel model önermekteyiz. Ulaştırma, açılış ve sıkışıklık maliyetlerini en küçükleyen kapasite kısıtsız çok atamalı ana dağıtım üssü yerleşim problemi olarak karışık tam sayılı programlama modeli önerilmektedir. Bu tez boyunca, önerilen model kapasiteleri istenilen seviyeden daha yüksek olan ve ana dağıtım üsleri üzerindeki sıkışıklığın gene de amaç fonksiyonunda ceza maliyeti olarak dikkate alındığı ana dağıtım üssü yerlerini bulmayı amaçlamaktadır. Bu nedenle, ana dağıtım üsleri üzerlerindeki fazla akışlardan kaynaklanan ceza maliyetlerini en aza indirmek için, daha yüksek kapasiteye sahip olan düğümler arasından seçim yapılmaktadır.

Optimizasyon problemlerinin modellenmesinde ana endişe, karar değişkenlerinin ve kısıtların sayısı anlamındaki problemin boyutudur. Karmaşıklık ve hesaplama süresi bakımından problemin boyutu çözüm sürecini zorlaştırır. Optimal çözüme ulaşmak için klasik çözüm teknikleri artan karar değişkeni ve kısıt sayısı karşısında yetersiz kalır. Bu zorlukların üstesinden gelmek için, Benders ayrıştırma tekniği gibi ayrıştırma teknikleri ve sezgisel algoritmalar uygulanabilir.

Her ne kadar modelin matematiksel formülasyonları optimal çözümü başarabilse de, problemin boyutu ve hesaplama süresi çözümü daha karmaşık hale getirmektedir. Bu nedenle, önerilen doğrusal olmayan modelin nispeten daha çok çaba gerektiren bir çözüm prosedürüne ihtiyacı vardır. Bu tezin bir başka katkısı, karmaşık problemlerle başa çıkmak için etkili çözüm prosedürleri geliştirmektir. Sonuç olarak, önerilen modeli çözmek için kesin bir çözüm prosedürü olarak Benders ayrıştırma algoritması ve bir sezgisel algoritma olarak parçacık sürüsü optimizasyonu geliştirilmektedir. İlk olarak, Benders ayrıştırma algoritması probleme uygulanmaktadır. Benders algoritmasını uygulamak için karesel sıkışıklık maliyeti fonksiyonu doğrusallaştırılmaktadır. Ancak, algoritma beklenenden daha fazla zaman alıcı hale gelmiştir. İkinci olarak, büyük ölçekli problemler için en yakın optimal çözümleri bulmak için etkili bir parçacık sürüsü optimizasyon algoritması önerilmektedir.

Tüm çözüm prosedürleri önerilen model yapısına uyarlanmıştır. Sayısal analizler AP veri setinde gerçekleştirilmiştir. Her iki çözüm prosedürü de karşılaştırmalı olarak incelenmiştir. Problemin çeşitli versiyonları (kapasite kısıtsız, kapasite kısıtlı ve sıkışık) ve üç ayrı sıkışıklık fonksiyonuyla modellenen problemler karşılaştırmalı olarak analiz edilmiştir. Sıkışıklık fonksiyonunun farklı tanımlanması ya da sıkışıklık faktörü, kapasite seviyesi, ölçek ekonomilerinin değişimi dahil olmak üzere farklı senaryolar altında deneyler yapılmıştır. Ayrıca, iterasyon sayısı, popülasyon büyüklüğü, öğrenme katsayıları gibi farklı parametreler altında sezgisel algoritmanın verimliliğini karşılaştırmak için duyarlılık analizleri incelenmiştir. Son olarak, önerilen tüm modellerin hesaba dayalı deneyleri ve tartışmaları sunulmuştur ve önerilen çözüm prosedürlerinin hesaplama performansları test edilmiştir.

1. INTRODUCTION

Transportation sector provides connections between points, cities and countries around world and make the distances closer. In other words, in an economic sense the need-based displacements of loads and people by benefiting from time and volume is called as transportation. Air transportation is one of the shortest and most reliable way for people and goods such as baggage, cargo and postal. While air transportation benefits the tourism and business sector of countries, it also contributes to socio-cultural development. Air transportation makes it easier to access of the global markets, by the way, the sector plays a crucial role not only in the development of national, but both international and global economy. Nowadays, large number of businesses utilize air transportation to provide qualified time sensitive services and timely production for their customers such as air freight transportation. The transportation opportunities of geographically difficult areas, fast and easy transportation, direct and indirect financial benefits, being an economical catalyst for global business, time effective services and the contribution of socio-cultural developments for regions make the air transportation valuable and unique.

Liberalization, privatization, technological improvements such as non-stop long-range services due to increasing speeds of vehicles, rising affluence of developing countries and globalization lead up the air passenger transportation industry and the industry qualitatively modified according to these trends in the global economy. Thus, air passenger transportation become more competitive, innovative, customer satisfactory oriented with multiple additive services and dominant sector in transportation.

Passenger transportation has been growing at an increasing rate since the 1950s. Annual traffic of global air transportation is 4.1 billion passenger which preferred 41.9 million scheduled flights in 2017. Meanwhile, air passenger transportation has a $704.4 billion of revenue in 2016 and approximately 7,75 trillion passenger kilometers are transported through 45.091 routes around the world. Air passenger transportation industry directly and indirectly generates employment. In 2016 , the industry employed

10.2 million of people, supported 65.5 million jobs and promoted $2.7 trillion of economic activity which is equal to 3.6% of the global gross domestic product. It helps trade enhancement and contributes to the development of industrialized economies.

The International Air Transport Association (IATA) stated that the growth of the air passenger industry will continue, and the passenger statistics will be doubled in 2037. They also claimed that the industry should benefit from the geographical shifting of the air traffic to the East and maximum liberalization policies. Estimated increase of the growth rate for air passenger transportation by IATA is 3.5% per year. It is expected that air passenger traffic over the next 20 years will be doubled by rising to 8.2 billion passenger and the supported economic employment by air passenger transportation may be 100 million jobs globally.

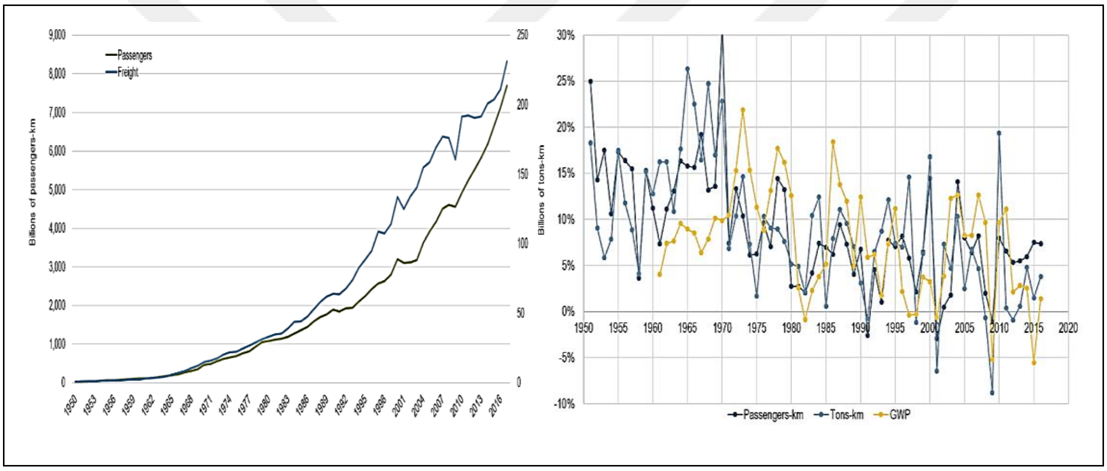


Figure 1.1 : The changes of air passenger/freight carried and air transportation growth from 1950 to 2019.

To be favored in the market and expand the business, all enterprises should take several short- and long-term decisions. These strategic decisions such as resource planning, capacity or investment planning are significant processes spreading over a long time. In air passenger transportation, one of the most important decision is network design. Network design consists of a series of decision as planning routes to be served, service frequency and capacity of transporters. The design plays a critical role to minimize the costs and offer preferable services by customers. The network design aims to determine the locations to be served, select the beginning and ending points and route the flights through network.

After liberalization movement in the 1970's, it is allowed airline actors to establish routings and pricing. These deregulations and privatization modify the system and provoke new generation of industry in terms of cost, quality and security. Significant structural, institutional and legal changes have been made and the world's airline industry has been influenced by liberalization experienced in US. Many countries have liberalized for domestic air system and as well as bilateral agreements were signed. Open Sky projects have been implemented in North America and Europe.

All these overwhelming changes, economic recession periods and competitive environment cause substantial damages and loss in international airline industry. The industry has been forced to reconstruct and modify their network to reduce cost and increase efficiency. With the help of technological improvements which continued since 1930's, airlines have started to construct hub and spoke networks as the most significant outcome of these processes. Hub and spoke networks provide broad distribution especially for international services with a smaller number of intermediate centers. As a result of advantageous implementations of hub networks in aviation, hub networks have been applied successfully in road/rail transportation, telecommunication logistic systems.

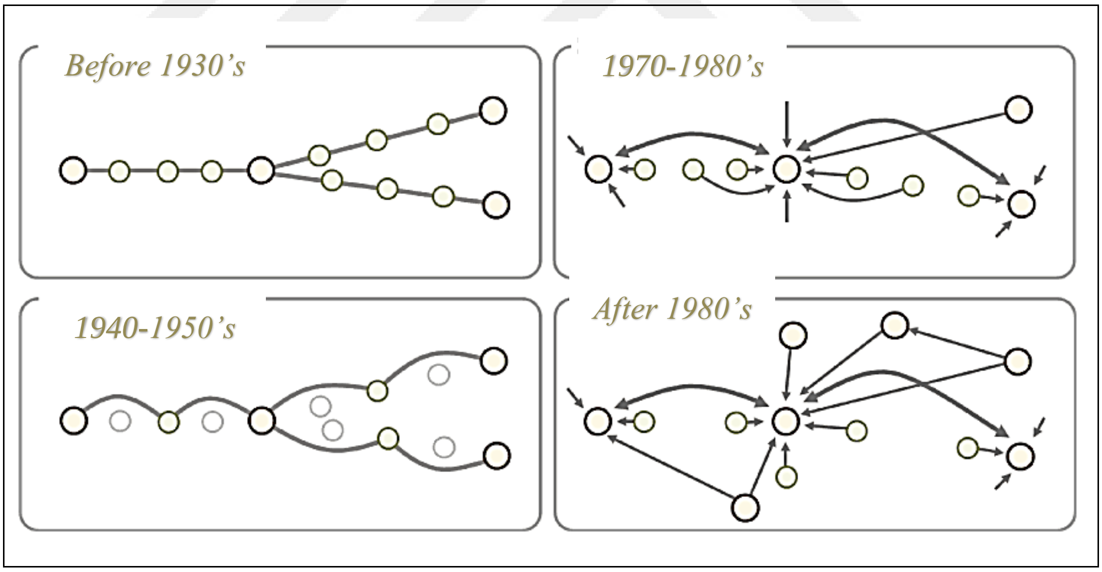


Figure 1.2 : The evolution of networks in air passenger transportation.

From the beginning of aviation industry, there exists intermediate centers namely as major airports. However, before deregulation movements, prohibitions on sectoral policies, market restrictions and technical incapability led companies to provide linear services with multiple stops. After 1940's the distance range of the aircrafts eliminated some points on linear routes. In 1970's the improvements on capacity, size and speed of aircrafts and the specialized needs of the sector have advanced air transportation to service regional, international and intercontinental markets. Thus, the air transportation network has been increasingly complicated. In consequence of the important developments mentioned above, most of the airlines aimed to establish hub and spoke networks. The hub networks provide lower costs, higher frequency services and larger coverage through market. Also, fleet simplicity, fast turnaround times, rapid growth and emphasis on secondary airports are some of the advantages of these networks.

In the hub spoke networks, a centrally located airport serves as a hub. The flows arriving from distinct origins are accumulated in that hub and oriented to distinct destinations or another hub to transfer. This centralization and expansion of the network bring the advantage of scale economies. In addition, hub-spoke networks increase the profit of airlines and provide significant savings in cost for air passenger transportation. It is clear that the routing decisions are influenced by hub location decisions and the inverse of that relationship is also accurate. By the way, the design of hub-spoke networks should consider both the hub location and allocation decisions simultaneously. In this context, hub location problem could be identified as determination of the location of hubs, assignments of flows with appropriate allocation decisions.

Despite of all these improvements and progress in the air passenger transportation, some unfavorable consequences and significant challenges are expected for the sector. These may be occurred due to both hub and spoke systems and rapidly increase of the sector. As mentioned before, global growth of air passenger transportation continues through economies of developing countries. It is equivalent that the sector should serve the increasing number of customers. However, innovative investments combining with the affordable pricing policies cause heavy burden costs. Meanwhile, low cost carriers create greater competition and pressure through entire sector. Thus, the profit in the air passenger transportation decreases.

To overcome severe competition and pricing policies, hub and spoke systems provide better network designs, operational planning and yield management. For instance, generating hub spoke systems improves the global load factors which is the percentage of purchased seats. While, the load factor runs around 65% in 1990’s, it is increased up to 80% globally in 2013. Thus, the utilization level of resources is optimized and their networks are organized effectively such as capacity expansion or reduction in some regions of network. However, high load factors make the networks less flexible against demand fluctuations, delays or disruptions at network.

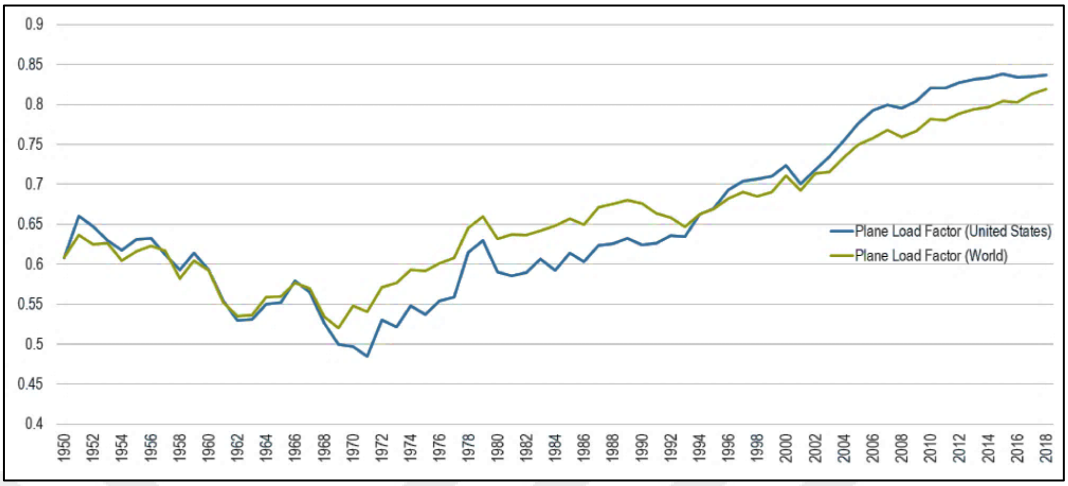


Figure 1.3 : The increase of load factors in years.

Furthermore, airports lead to changes in flight scheduling because of the increase in air traffic. When a flight falls behind its scheduled time in taking off or landing, delay is occurred. According to The Federal Aviation Administration (FAA), the lateness takes longer than 15 minutes, it is a delay. Also, European Union enforces the airlines to bear penalty costs up to €600 per passenger for delays over three hours. As a result of condensing flows at hub spoke systems and increasing demand in sector, the air networks are confronted with pressure on capacities of airports and congestion. To deal with these challenges, airlines prolong the scheduled times for flights. For instance, the flight between New York and Los Angeles is planned to be over 6 hours while same flight was determined as 5 hours in 1960's. Figure 1.4 represents the changes of scheduled times of some flights through years. After implementation of hub spoke systems there is an increase on scheduled times. Designing more reliable, more resistive networks against congestion and delays may optimize the flight scheduling.

Unfortunately, delays are not only concern scheduled times. Nevertheless, the attributes of airports and efficiency of associated ground operations are more relevant. The runways, terminals, capacity, takeoff queues and traffic control infrastructure of an airport may become inadequate and cause congestion. In case of hub spoke networks, the poor conditions are compounded and increase incrementally. The network suffers an intense congestion.

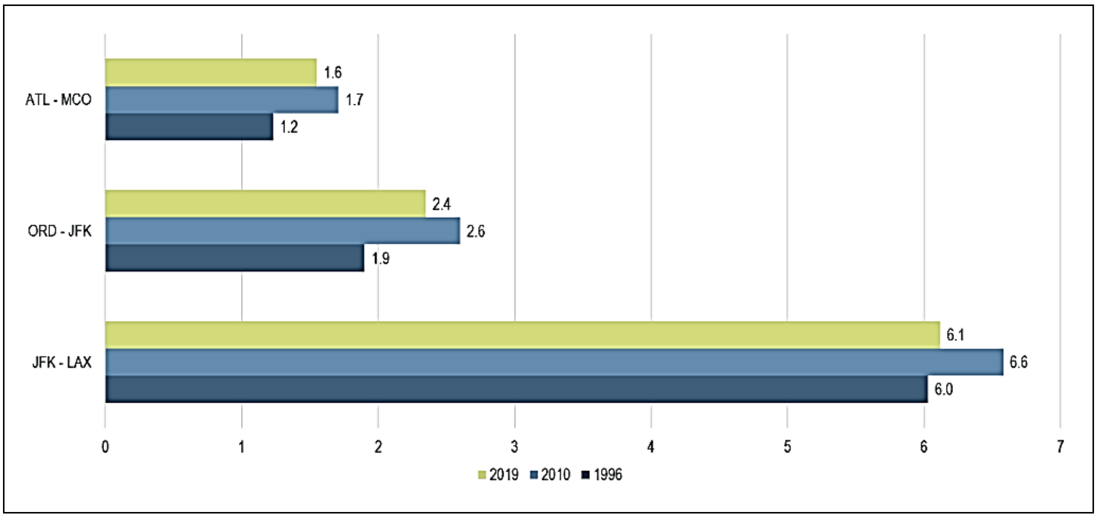


Figure 1.4 : The changes in scheduled times of some flights.

An airline takes advantage of hub networks such as regional superiority of market, higher load factors, large-scale expansion, even sometimes repricing power for airfares. Meanwhile customers benefit from lower costs, flexible services and better connectivity. However, it is not sustainable even in theory. Since the network grow enough, hubbing effects and centralization of the network become more disadvantageous both economically and physically. Capacity features of hubs such as the runways, the number of employees and gates become the vulnerability of the network. Thus, the network is faced with lack of competition, congestion and subsequent delays and loss of traffic.

The hub location problem (HLP) has been an important subject of research since the last quarter of 20th century. In general, hub location problems aim to design networks where the flows of origin-destination pairs are accumulated and distributed through some central nodes named as hubs. The benefits of the economies of scale by flow consolidation between hubs fulfill the expectations of the sector and competitive, progressive motivations of the actors in the network. In other words, in case of non-hub network design, direct connections between nodes cost more and the network becomes more complicated to design.

In air transportation, as well as in other transportation networks, it is important to extend the network as large as possible to satisfy the customer needs and to obtain competitive advantage. The hub location problem aims to minimize the transportation and opening costs while taking the advantage of economies of scale where the cost of interhub connections are discounted. However, more nodes are inserted to the network, the more capacity restrictions are imposed to balance the flows between nodes. Essentially, in recent times, the air transportation companies seek to resolve these undesired consequences and optimize traffic flow density of their hub airports. Hubs' capacities tend to be sensitive against the increasing volume of traffic flows in time. Hence, capacity features of the hub location problem had to be included in the existing literature.

Hub location problems are classified according to capacity features: capacitated and uncapacitated. In the capacitated hub location problems, capacity is considered as a limitation on hubs. However, restricting possible hub nodes with capacity levels could not represent a realistic approach to the problem. Luckily, in recent years, a new term `congestion' has been introduced as an additional flow in excess of the capacity which is imposed on hubs. While the problem focuses on obtaining a minimum set of opening and transportation costs, a relatively small number of hubs are selected. These hubs are likely to be overloaded in comparison to non-hub nodes. In the diverse range of literature congestion effect on the networks is represented implicitly by constraints which are included in the model, similar to capacity constraints. Furthermore, the congestion is represented explicitly in the literature as a delay cost or delay time in the objective function.

In this thesis, it is aimed to introduce a new perspective to hub location problem, within this scope, the features and requirements of air passenger transportation and congestion as a motivation on hub location problem are investigated. Based upon the congestion concept and its apprehension in air passenger transportation, objective function and constraints are formulated and mixed integer programming models are developed as uncapacitated multiple allocation hub location problem. Throughout this thesis, we propose a model locating hubs whose capacities are larger than a desired level and still congestion on hubs is considered as a penalty cost in the objective function. Therefore, hubs are chosen between nodes which have higher capacities in order to minimize the penalty costs arising from surplus flows on hubs.

In addition to define congestion function of hub location problem properly, a mixed integer non-linear programming is proposed. In order to observe the effect of the congestion on selection of hubs and allocation decisions, 3 different congestion function are presented comparatively. Furthermore, this thesis explores the congestion effects by presenting novel function differed from convex cost function defined by

Elhedhli and Hu (2005) and Kleinrock function defined by Camargo et al. (2011). We propose a new congestion function whose scalar multiplier generated by the average transportation costs of the network. Our proposed congestion function is calculated by the amount of the surplus capacity per total flow accumulated at hubs. In the studies up to date, the amount of the congestion occurred at a hub has been limited with the capacity of that hub. Whereas, in this thesis, the congestion is defined as an excess of the amount of the total flows accumulated at hub that is more than that hub's capacity. Similarly, most of the studies in hub location literature assume that the amount of total flows through hub is calculated as the sum of incoming flows. However, in this thesis, both incoming and outgoing flows are summed up to determine total flows through hubs, because airports in passenger transportation may suffer from the intensity of both arriving and departing flights.

Since the hub location problems are NP-hard, developing solution procedures for these problems is a major concern. Determining both location and allocation decisions make the solution techniques complicated for HLPs. Since the middle of 1990s, hub location problem literature has shown significant advance in developing algorithms to obtain high quality solutions in a reasonable time. Specific real-life requirements make the problem more complicated to model and solve. Thus, a novel approach is needed to develop better performing solution procedures. Exact, heuristic, meta-heuristic and combinatorial algorithms are employed to solve HLPs. Besides, exact solution techniques are common in the literature; many heuristic algorithms attempt to either solve them more efficiently or to obtain the best solution, not both at the same time. Tabu search, greedy randomized adaptive search procedure, genetic algorithms, variable neighborhood search and simulated annealing are some of the heuristic methodologies used frequently to solve these problems. However, congestion is one of the recent research subjects to develop more realistic and insightful hub location models. There is a very small number of studies focusing on congestion in the hub location literature where only exact solution procedures are proposed.

Currently, developing heuristic algorithms are competing for large scale instances, short computational time and minimum gap to optimum. Accordingly, this thesis proposes an efficient particle swarm optimization algorithm to deal with the congested uncapacitated hub location problem.

In Chapter 2, an extensive literature review on hub location problem is presented. This section includes future direction of the literature classified according to the core of the studies. The mathematical models of uncapacitated multiple allocation hub location problem and the proposed model for UMAHLP under congestion are presented in Chapter 3. Also, the congestion formulation is expressed elaborately. Chapter 4 describes the general framework of Benders decomposition algorithm and Particle Swarm Optimization algorithm and highlights the adaptation of these algorithm to proposed problem. The numerical experiments performed on AP data sets and computational experiments of proposed problem will be presented in Chapter 5. The thesis concludes with general discussions and future directions in Chapter 6.

2. HUB LOCATION PROBLEM

Hubs are customized facilities whereby used in transportation, logistics and telecommunication systems. Hubs appear in many various fields such as airline industry, postal or cargo services, supply chain management, computer networks, emergency services. Hubs serve as consolidation, switching and sorting centers, and allow for the replacement of the direct connections between all nodes with fewer indirect connections. Hubs benefit from economies of scales by flow consolidation and dissemination. The flows originated from different nodes are accumulated in a hub and the flows designated to the same node are distributed from this hub rather than mapping directly each of the origin-destination pairs.

Hub location problems could play a decisive role which helps to achieve strategic success with its social, political and economic aspects and these decisions couldn't modify easily or regularly. Location of hubs is significantly important for the strategical decisions arised from network design such as routing and logistic decisions. Hub location problem deal with the location of hubs and allocation of nodes to hubs. The purpose of the problem is to find the optimum hub location and assign non-hub nodes corresponding hubs at minimum cost or time. Since these two decisions have mutually influenced each other, they should be discussed together. As a result, HLP is considered as a design problem of the networks.

Hub location problems consider determining possible hub locations while origin and destination pairs are given as a set of n nodes. Some of the common parameters of classical hub location problem such as cost, time, distance related to network, flows or demands between nodes and discount factor are deterministic.

Non-linearity in the objective function of HLP differed from earlier facility location problem. A naïve assumption for HLP may be allocation of non-hub nodes to hubs based on the shortest distance. It may be a solution for the facility location problems. However, this naïve assumption may not guarantee the optimality for HLP due to quadratic objective function. Thus, the nodes should be assigned to hubs in an optimal way. The figure 2.1 illustrates the different network structures related to different problematic approaches.

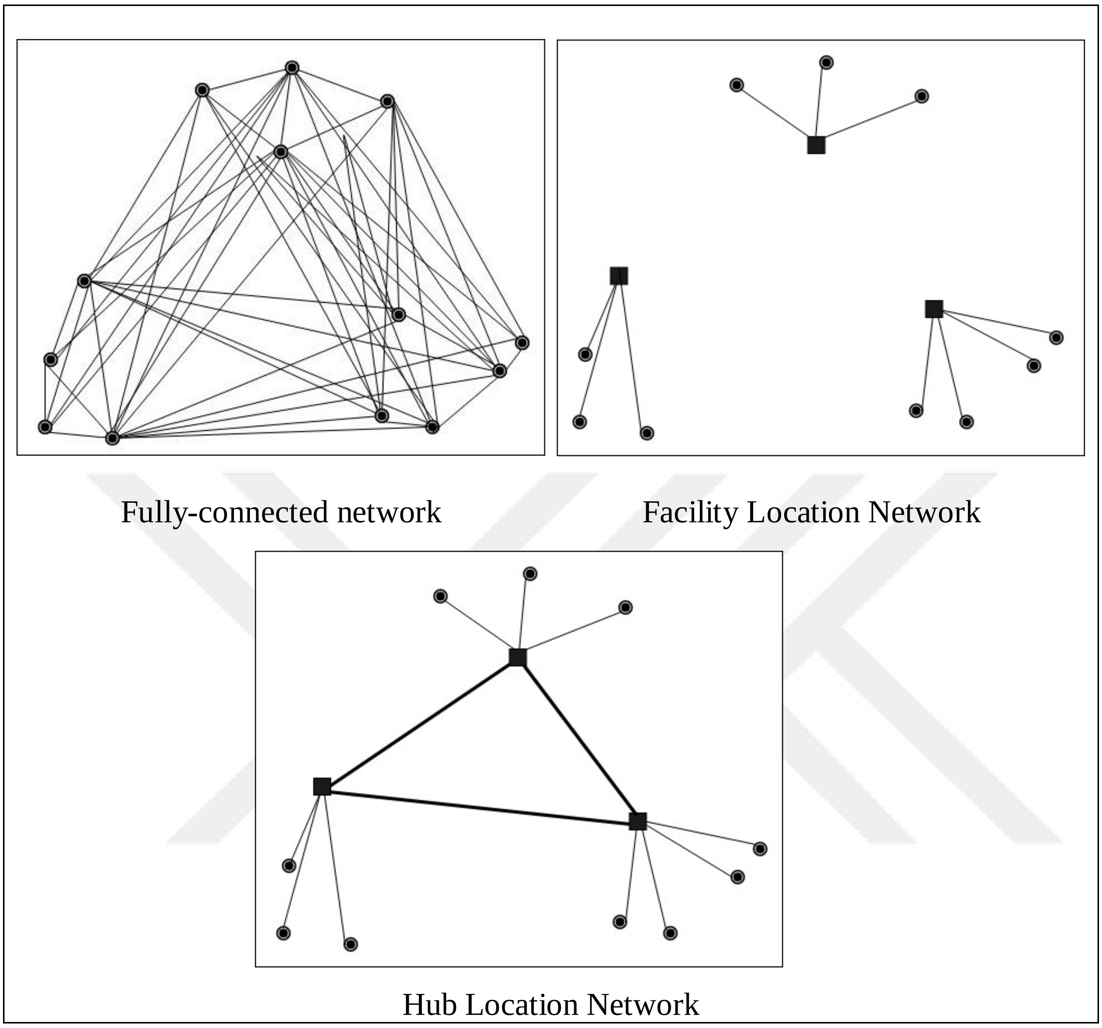


Figure 2.1 : Distinct network configurations.

Under the term hub location problem (HLP) various optimization models and solution procedures have been presented and discussed in the literature. Since the first publications, academic work focused on the core of the problem which is to design the optimum network. Subsequent works more and more attempted to extend the problem by integrating various constrains or objectives, like minimax, uncertainty, inter-hub connectivity alternatives. The increasing number of the distinct and complex modeling on the subject oriented the researchers to develop sophisticated solution procedures.

In this thesis, we have reviewed 400 publications dealing with or related to the network hub location problem. To build an empirical literature review, databases of Web of Science, Scopus and university libraries are utilized as research areas. Figure 2.2

shows the number of publication types in the reviewed hub literature and the distribution of journals published related works about hub location problem.

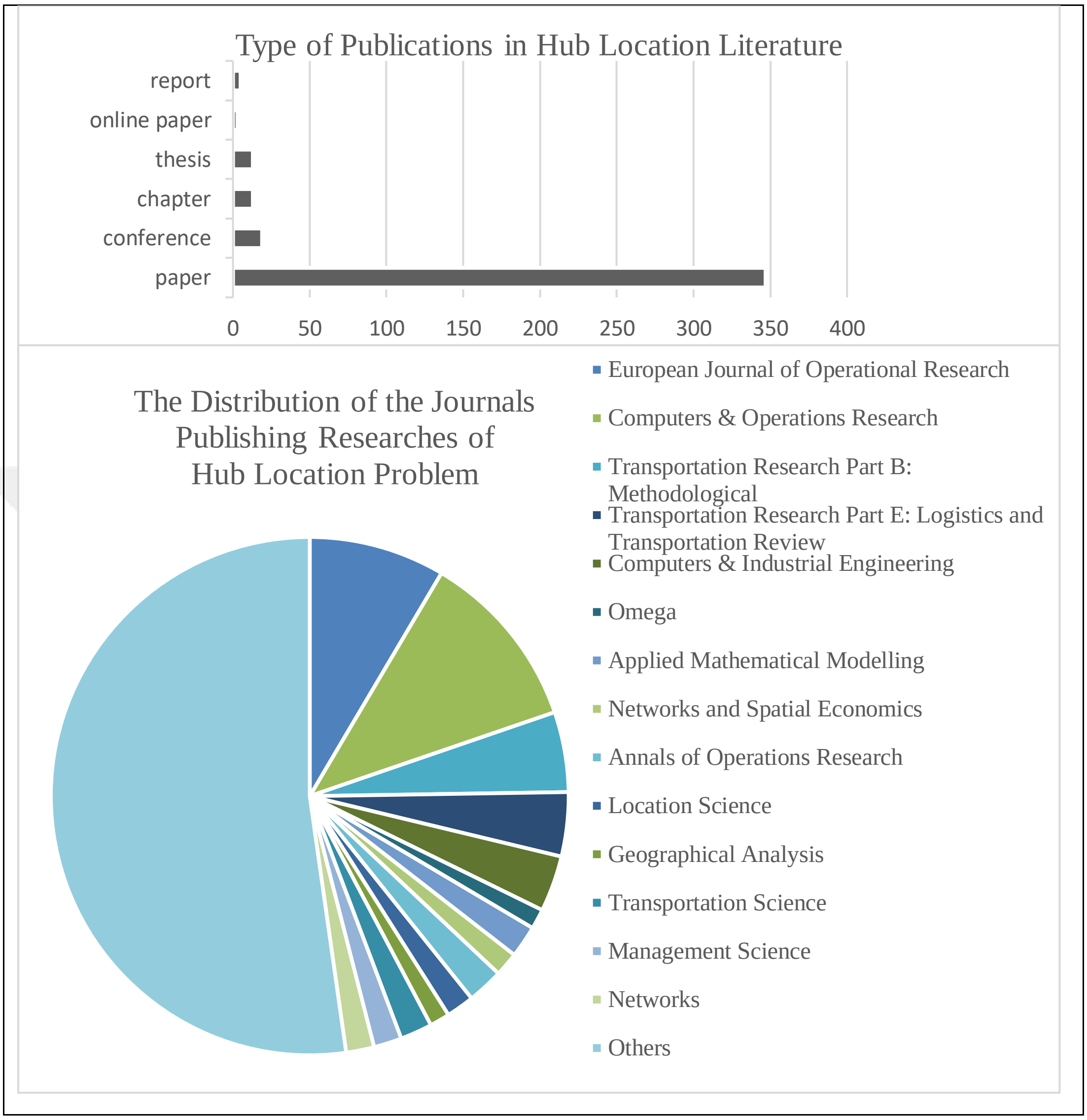


Figure 2.2 : The overview of the publications reviewed.

Many papers have been published on hub location problem so far. Figure 2.3 shows that the number of papers has increased in recent years. The emphasis of papers was on modeling of hub problems in the early years, on optimization and completing models in the following years, on solution methods from the 2000's, and finally on defining new versions of the problem and exact solution procedures in recent years. Considering the large variety of extensions on the subject, it is astonishing that there remains a very considerable gap between academic discussions and practical applications, up to now.

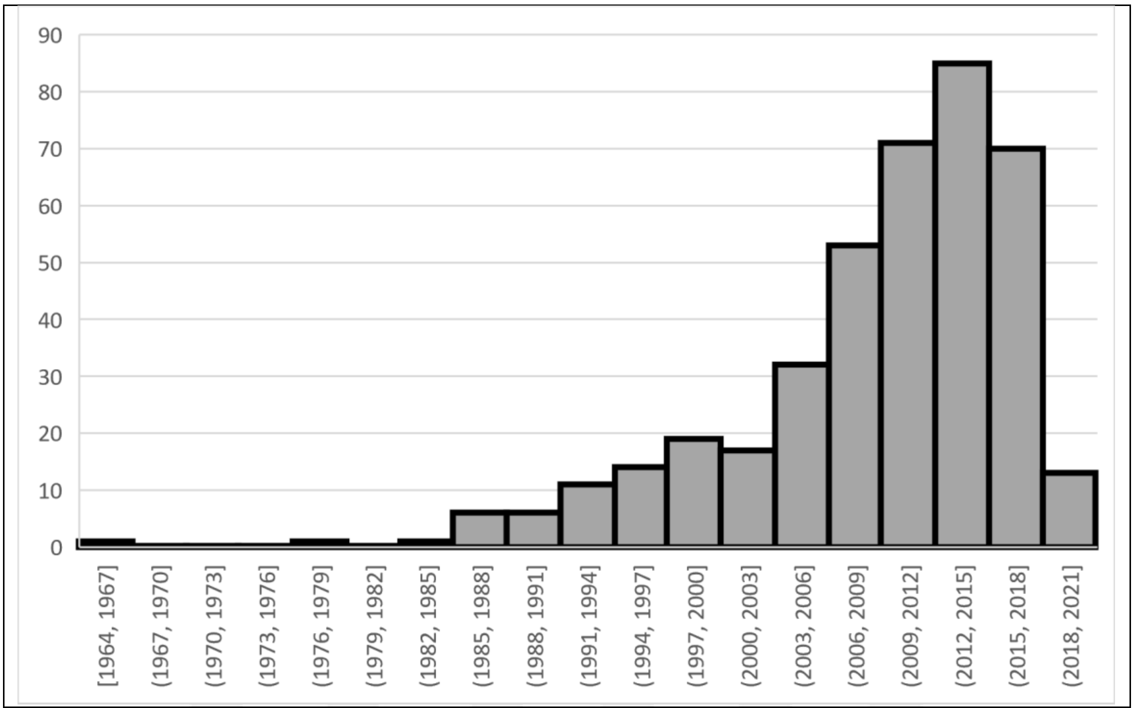


Figure 2.3 : The number of publications according to years.

As mentioned before, despite of great attention on the HLP, there is needed to enlighten researchers about the gaps of existing literature, fields that are rarely investigated and the future motivations of the problem. A reliable, consistent classification of the problem may help to achieve this aim.

A classification enables researchers to recognize and compare their own work within the literature, to possess precisely their research area and to find out the appropriate solution methods. By determining various possible pattern of the problem as different objectives, constraint or solution procedures which exists as similar as real life aspects, HLP become more complicated but factual. Furthermore, classifying and structuring existing literature enlighten the direction and challenges of future researches and specify the motivation of future progress in the literature.

For that purposes, a classification structure is proposed by reviewing elaborately selected 400 papers in the literature. Figure 2.4 represents proposed classification structure. This section summarizes HLP research by presenting the classification structure covered from basic problem to its existing extensions. Then, by using this classification structure, the sub-classes of the literature are explained in detail and significant studies of that classes are exemplified. Finally, recent trends in the literature are classified and examined by topic.

2.1 A Proposed Structure of the Classification for HLP Extensions

Hub location problems are based on a set of continuing patterns which could be defined as assumptions or logic of the problem. Identifying each pattern will ease the problem complexity to understand. Also, it will create a guide to characterize the problems and put them into subgroups. The real-life applications are renewing with different and additional requirements such as strategical and organizational decisions. Consequently, the real-life applications mutually interacting with theoretical framework of the problem. Hence this thesis focuses on categorization of the elements that specify the structure of the problem.

Classical hub location problems are constructed based on three basic assumption:

- The network becomes "complete" by direct connections among each hub pair.
- Inter-hub connections benefit from the economies of scale (α).
- It is not permitted the non-hub nodes are connected directly.

O'Kelly and Miller (1994) and Campbell (1994a) provide the earliest reviews about the structure of HLP. Although O'Kelly and Miller (1994) proposed eight assumptions of HLP corresponding allocation choices, hub interconnection, etc., Campbell's four classes of HLP gained more currency. Another well-known review, Klincewicz (1998) considered application field-oriented hub location such as telecommunication or transportation. O'Kelly (1998) investigated hub networks in perspective of air passenger and air express freight applications. Another review analyzing the effects of the economies of scale for passenger airlines and package delivery systems was presented by Bryan and O'Kelly (1999). Also, Hekmatfar and Pishvaee (2009) reviewed the problem by mentioning its application areas and solution procedures. Kara and Taner (2011) provided a five-fold taxonomy to construct a generalized framework of HLP. Most beneficial reviews on HLP to give brilliant evaluative comprehension are a book chapter by Campbell et al. (2002) and a survey by Alumur and Kara (2008a).

For recent and up-to-date reviews and classifications of the problem, see Hamacher and Meyer (2009), Contreras and Fernandez (2012), Campbell and O'Kelly (2013), Campbell (2013a), Farahani et al. (2013), O'Kelly (2015) and Contreras (2015).

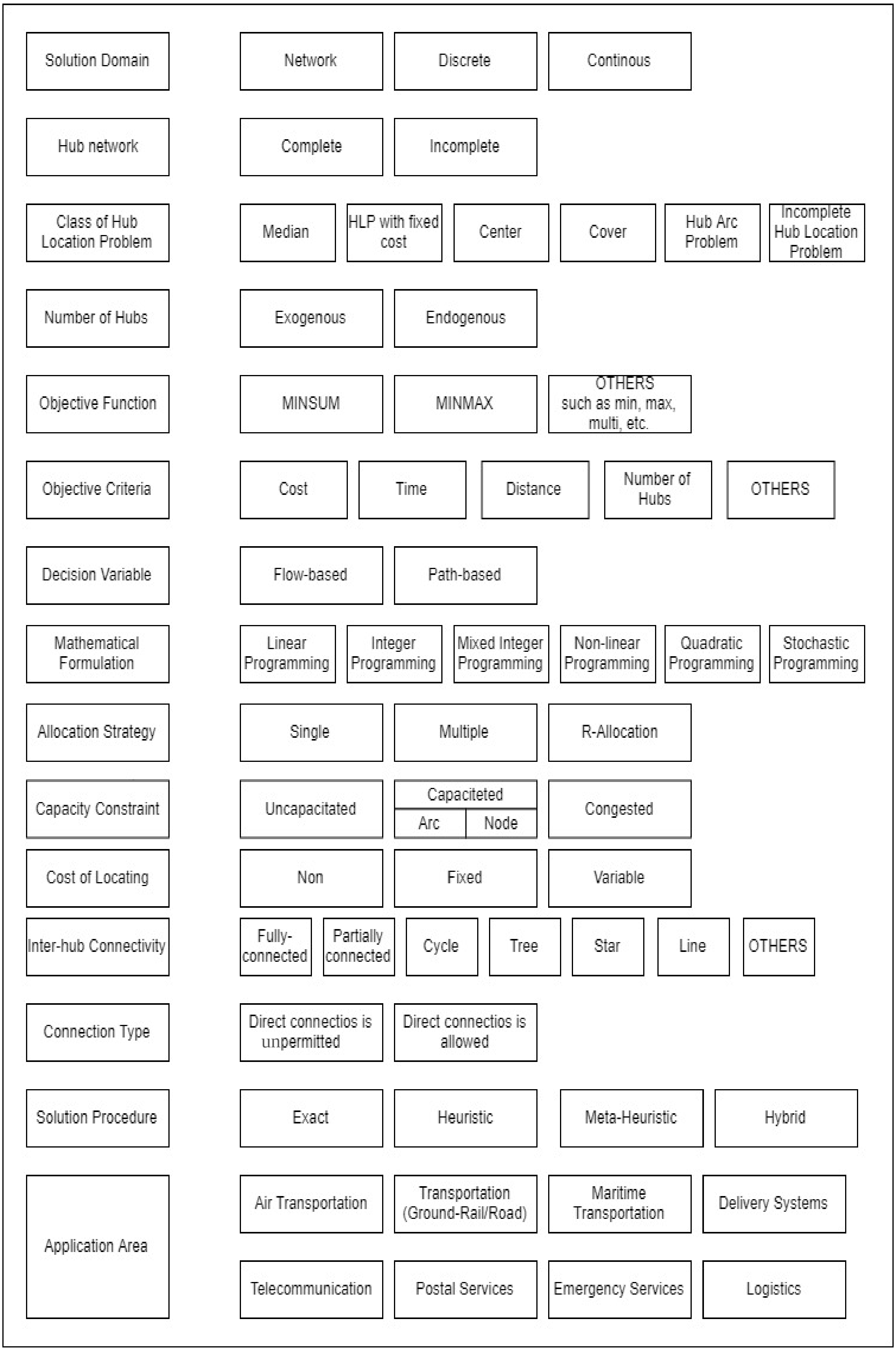


Figure 2.4 : The classification scheme of HLP.

There are conflicting abbreviations related to different types of HLP in the literature. Therefore, a common taxonomy could provide more comprehensive literature to researchers. By the way, HLP extensions could be identified in terms of mutual factors or application areas. Moreover, a deeper investigation on the existing literature could determine whether proposed solution techniques are capable of dealing with various modeling of the problem or any future progress is required. Figure 2.4 classifies the problem into 16 subgroups.

2.2 HLP in Dependency of Objective Function

The main goal of hub location problem is locating hubs and allocating non-hub nodes so that the network is optimally designed. Distinct network features could become optimality criterion as objective. Thereby, objective functions of the problem varied as shown in Figure 2.5. Minsum, minmax, min and max are frequently used in modeling. Most common and earliest objective function is minimizing sum of the costs at network including cost of the collection, transfer and distribution, cost of opening hub, cost of establishing links or arcs and other specified costs. Other common function is minimizing maximum time, distance or cost between node pairs.

The objective functions differ by general classes of the problem such as median, center or covering. P-median hub location problem and hub location problem with fixed cost minimize the sum of the costs occurring related to network design. Cost-oriented models can deal with congestion or delay cost, routing costs, costs associated to mode of transportation or vehicle and costs for reliable networks design.

On the other hand, center and covering problems consider objectives such as time or distance. These models aim to minimize number of hubs, length and time related to delay or flow processing and minimizing maximum length or time between pairs. Although cost oriented models are strongly practical motivation to design real-world networks, service-oriented models yield promising results for delivery systems, emergency systems and telecommunication.

The maximization objective is not a familiar function in the literature. However, maximizing covered flow between nodes is used to model some covering problems. Profit maximization is recent objective functions used in modeling. Designing a network with reliability or competition contribute to introduce novel objective functions.

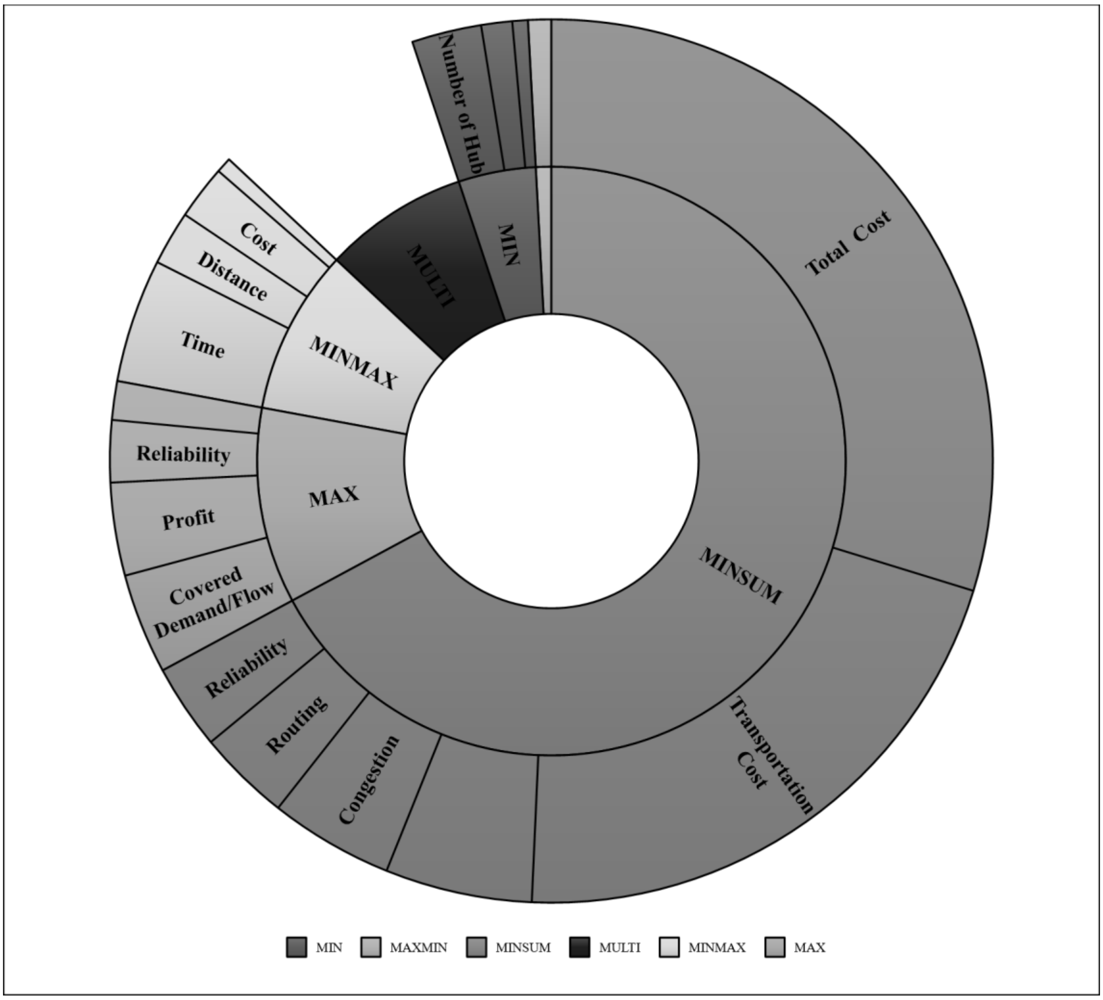


Figure 2.5 : The objective functions of the models presented in the literature.

Although most of papers deals with only one objective function, remaining attributes can be projected by secondary formulations as boundary constraints. Incorporating multiple objectives such as cost oriented models with time restrictions or service-oriented models with bounded cost functions provides more complicated problems to solve and more realistic approach to model the problem.

Correia et al. (2010a) formulated a minisum model as capacitated single allocation HLP where a value is considered for the maximum and minimum number of nodes allocating to hubs. Hwang and Lee (2012) evaluated covering problem while travelling time is limited to a pre-determined deadline. Minimizing the number of hubs under a maximum travel time limits a reasonable objective for the problem, opening and operating hubs require high set-up and operating costs.

In general, most of the hub location problems deal with single objective function, multi-objective hub location problem is recent trend in the literature. Costa et al. (2008) first multi-objective modeling, discussed both minimization of total cost and maximum process times at hubs. Barutçuoğlu (2009) proposed an interactive procedure to solve hub location problem with time and cost related objectives. Köksalan and Soylu (2010) proposed p-median formulations where the objectives of first formulation is minimizing total cost and minimizing transportation cost of network. Their other formulation simultaneously reduces total and congestion cost due to delays at hubs. Also, Mirzaei and Bashiri (2010) studied multi objective HLP including both minisum of total cost and minimax of time between pairs.

Çetiner, Sepil, & Süral (2010) introduced hub location routing problem minimizing the total transportation cost and the number of vehicles used in network. Yaman and Elloumi (2012) analyzed star/star network using the formulations of p-center and p-median. Their first model minimizes the length of the longest path in the star network while the second model minimizes the total routing cost ensuring upper bound constraints of path lengths obtained by first one. Lu & Xie (2014) incorporated hub covering problem with cost objective. Soylu and Katip (2019) proposed a network design of air transportation to minimize 2-stop journeys in order to improve customer satisfaction and traditional total cost in the same time.

2.3 . HLP in Dependency of Classes

Through early hub location researches, problems are classified analogous with facility location versions. Four fundamental classes of the problem are first defined in the study of Campbell (1994a). P-hub median minimizes the total transportation costs. Hub location problem with fixed cost utilizes same objective attaching the opening cost of hubs. By this way, it is not required to pre-determine the number of hubs and it becomes a decision variable of the model.

Earliest p-hub center problems minimize the maximum cost of transportation between nodes. Later, objective criterion changes into maximum time or distance between node pairs. Hub covering decides the locations of hubs to cover the network while keeping the attributes of pairs below a predetermined value. Alternative objectives are minimizing the total cost of opening hubs or the number of hubs. Figure 2.6 visualizes the distribution of distinct classes of the problem through the literature.

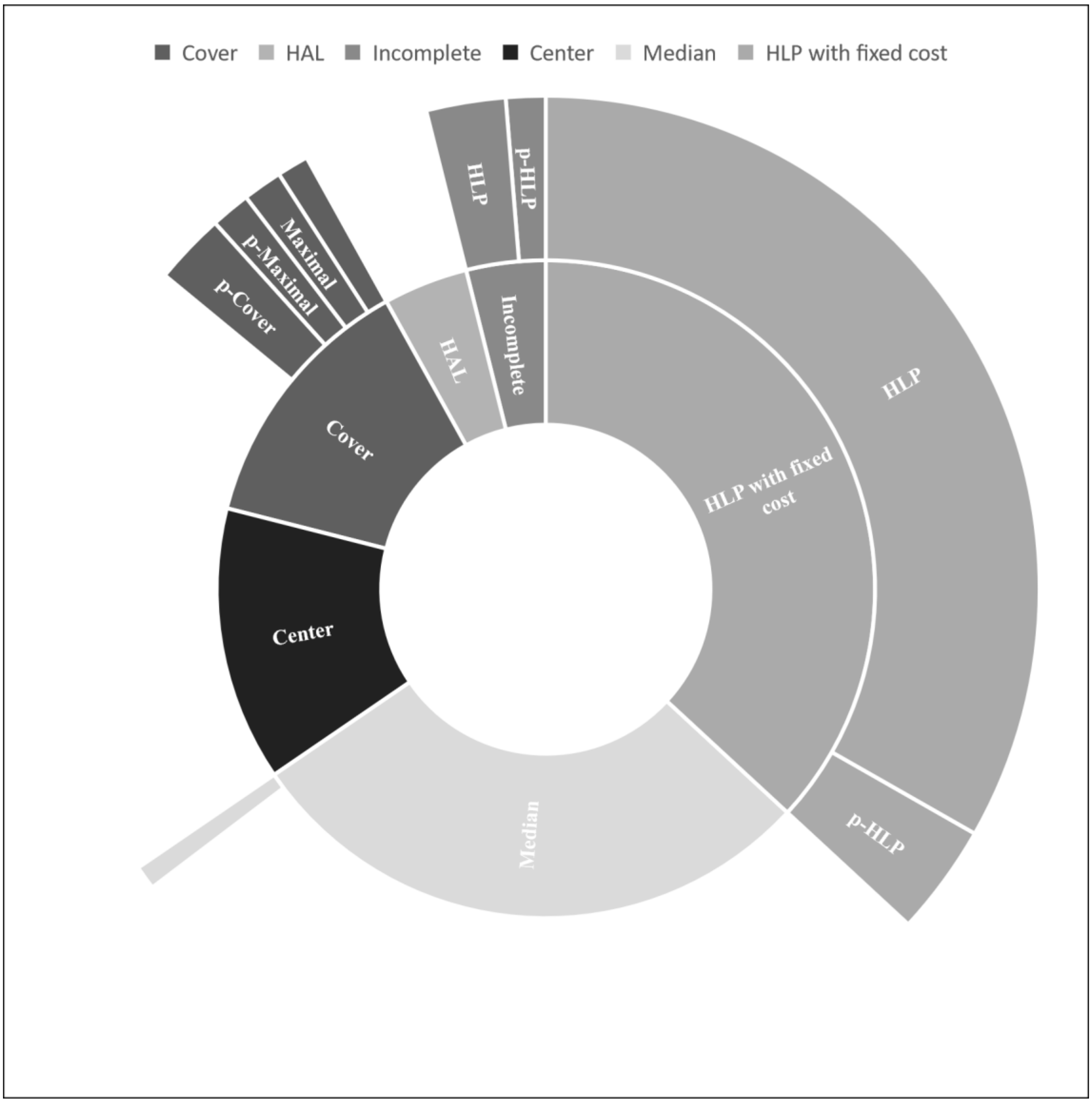


Figure 2.6 : The general classes of HLP.

2.3.1 P-hub median problem (p-HMP)

Given a network with n nodes and pre-determined number (p) of hubs and a set of flows between nodes., p- hub median problems minimize the total transportation cost (time, distance, etc.) of the network. O'Kelly (1986a) was the first to introduce hub location problem which consists of one or two hubs. The first mathematical formulation show that origin-destination pairs would decrease from $\binom{n}{2}$ to $(n-1)$ if hub exists.

Subsequently, the quadratic cost function is structured by O'Kelly (1986b) while studying the cost terms in a network. O'Kelly (1987) introduces the integer binary modeling with a quadratic objective function which is the sum of the flow costs. Since it is assumed that the connections between hubs and non-hub nodes cost more than the

Table 2.1 : Milestones of general classes of hub location literature.

	Authors	Milestones
single p-median	O'Kelly (1986a), (1986b), (1987)	first mathematical formulation as quadratic integer programming and representing CAB data
	Aykin (1990)	allocation decisions
	Klincewicz (1991), (1992)	heuristic based solution procedure
	Campbell (1994b)	first linear integer programming formulation with flow threshold
	Ernst and Krishnamoorthy (1996)	defining "flow variables"
	O'Kelly et al. (1996)	improving the linearization scheme for the models and representing symmetric flow data AP
	Campbell (1996a)	proposing new heuristics
	Sohn and Park (1998)	studying model with symmetric flow proportional to distance
	Ebery (2001)	revising formulation with fewer variables and constraint
	O'Kelly (2009)	extending their previous work (1986) by locating hubs in a rectilinear system.
	Yaman (2011)	introducing r-allocation where the number of nodes allocated to hubs is bounded by a pre-determined value.
	Puerto et al. (2011)	introducing ordered median where the hubs to select are ordered related to their flow concentration
	Campbell (2013a)	modeling continuous approximation for time definite hub location problem
	Peker et al. (2015)	identifying key characteristic of optimal hubs by means of their ability for preference
	Meier et al. (2016)	proposing compact formulations with fewer variables by linearization from the Euclidean distance and cost
multiple p-median	Campbell (1992)	first formulation of multiple allocation
	Campbell (1994b)	first linear integer programming formulation with flow threshold
	Skorin-Kapov et al. (1996)	presenting tighter and stronger formulations
	Sasaki et al. (1999)	special case of the problem with one stop path
	Boland et al. (2004)	improving lower bound by preprocessing and proposing flow cover constraint for capacitated version
	Kimms (2006)	proposing a piecewise linear cost function using flow dependent discounts and a fixed cost for connections.
	Çetiner et al. (2010)	introducing routing policies while designing network with vehicle assignments under desired service level
	Campbell J. F., 2013b	comparing and classifying hub location problem according to formulation of economies of scale
	Corberán et al. (2019)	Improving polyhedral description of model to develop exact procedures

Table 2.1. (continued): Milestones of general classes of hub location literature.

	Authors	Milestones
single allocation hub location	O'Kelly (1992a)	introducing "fixed cost"
	Aykin (1994)	introducing "capacity" with direct connections allowed
	Campbell (1994b)	first formulation of integer linear programming with use of minimum flow threshold
	Ernst and Krishnamoorthy (1999),	formulation for capacitated version as mixed integer programming
	Labbe and Yaman (2004)	defining valid and facet inequalities
	Costa et al. (2008)	multi objective formulation combining total cost and service time objectives
	Correia et al. (2010a)	introducing a set of capacity level where capacities of hubs become a decision variable
multiple allocation hub location problem with fixed	Campbell (1994b)	first formulation of integer linear programming for capacitated and uncapacitated versions
	Klincewicz (1996)	developing B&B procedure
	Ebery et al. (2000)	developing a B&B method for capacitated problem
	Nickel et al. (2001)	introducing fixed cost for both hub and spoke edges
	Hamacher et al. (2004)	improving polyhedral study of model
	Yaman and Carello (2005)	formulating the cost of using an edge as stepwise cost function
	Yaman (2005)	introducing "modular capacity" term
	Marin at al. (2005)	reducing size of variables for capacitated version of the problem where the flows defined as splittable
	Labbe et al. (2005)	presenting relaxations of both capacitated and uncapacitated versions
	O'Kelly (2012)	incorporating environmental concerns with hub location problem
	Taherkhani and Alumur (2019)	applying r-allocation strategy with profit maximization objective under location routing network
p-hub center	Campbell (1994b)	first formulation of integer linear programming
	Ernst and Krishnamoorthy (2000)	revise formulation based on their previous work (1996)
	Kara and Tansel (2000)	presenting a linearization of the formulation with minmax cost between hubs
	Yaman et al. (2007)	introducing multiple stopovers allocated to hubs through paths where arrival times measured as transient times
	Campbell et al. (2007)	investigating allocation strategies on the model
	Ernst et al. (2009)	comparing distinct index formulations by means of computational performance
	Gavriliouk, 2009	presenting mathematical simplification of the problem
	Yaman and Elloumi (2012)	designing first hierarchical star-star network
	Liang, 2013	improving the hardness and approximation by two-level allocation or rounding of linear relaxations

Table 2.1. (continued): Milestones of general classes of hub location literature.

	Authors	Milestones
p-hub covering	Campbell (1994b),	first formulation of integer linear programming
	Kara and Tansel (2003),	presenting a linearization of the formulation
	Wagner (2004b),	proposing preprocessing techniques
	Kara and Tansel (2001)	introducing set covering modeling
	Wagner (2008)	comparing formulations of covering model with quantity independent and dependent discount factor
	Alumur and Kara (2009)	modeling by bounding the number of stopovers concept through paths with time restrictions
	Hwang and Lee (2012)	introducing maximal covering models
	Lowe and Sim (2013)	combining median and covering problem approaches
	Peker & Kara (2015)	reformulating of maximal covering model with decrement of the number of variables and constraints
	Ernst et al. (2018)	comparing reformulations such as sizing of model, tightness or lifting of constraints by means of computational performance
	Dukkancı et al. (2019)	modeling with environmental objectives
arc	Campbell et al. (2003),	introducing arc location problem
	Campbell et al. (2005a),	proposing solution procedures appropriate to hub arc location problem
	Campbell et al. (2005b),	presenting median, center, fixed cost and cover classes of hub arc location model
	Campbell (2009)	presenting time definite modeling blending cost and service-oriented approaches under arc location problem
	Contreras and Fernandez (2014)	revised arc location formulation with a supermodular function.
	Gelareh et al. (2015)	incorporating setup costs for both hub nodes and arcs
incomplete	Klincewicz (1998)	examining network structure of general classes of hub location problem including incomplete one
	Rodriguez and Gonzalez (2008)	modeling arc-capacitated incomplete network
	Alumur et al. (2009)	first modeling of incomplete network
	Gelareh and Nickel (2011)	proposing 4-index formulation for incomplete modeling
	O'Kelly et al. (2015a)	new 3-index formulation while modeling fixed and variable arc costs for incomplete hub backbone system
	Mahmutoğulları and Kara (2015)	designing general classes of the problem by allowing direct links and routing between non-hub nodes with bounded constraints or penalized objective
	Campbell et al. (2015)	proposing fixed and variable arc cost with restrictions on inter-hub connections related to their establishing cost.

connections between hubs, O'Kelly introduced a discount factor (α) namely economies of scale. In his formulation, the number of hubs is known, and the opening costs and capacity restrictions of the hubs are not included to the model.

Aykin (1990) investigated the allocation decisions on hub location problem. Campbell (1992) linearized the formulation of the multiple allocation p-hub median problem. Campbell (1994b) provides linearized formulations for the single allocation. Campbell (1994b) also proposed another formulation with flow thresholds for p-median problem where a restriction on an amount of flow through a path to utilize the corresponding connection. Campbell's studies (1994, 1996) majored on extending hub location problems and linearizations of all formulations associated with classes of the problem. Ernst and Krishnamoorthy (1996) reformulated the problem by reducing the size of the model with flow variables introduced. Ebery et al. (2001) also revised the formulations by reducing size of variables and constraints of the models.Klincewicz (1998) examined eight different structure of network including allowing direct connections by relaxation of general assumption of the problem. Elhedhli and Hu (2005) included congestion cost in the objective of p-median problem. Yaman (2009) evaluated p-hub median problem with variable capacities of arcs. Çetiner et al. (2010) incorporated routing decisions with p-median hub location problem. This iterative two-stage solution procedure included multiple p-median problem and multiple vehicle routing problem with tour lengths. P-hub median is investigated according to novel allocation strategy: r-allocation by Yaman (2011). The study limits the number of non-hub nodes allocated to each hub. Garcia et al. (2012) reformulated the model with $O(n^2)$ variables for the first time in the literature. Thus, their branch and cut algorithm becomes beneficial to solve larger instances. Lin et al. (2012) considered capacitated version of the problem with integral constraints on the paths that formed by connections between nodes and hubs. Meier et al. (2016) proposed compact formulations with fewer variables by linearization from the Euclidean structure of distance and cost. Corbean et al. (2019) improved polyhedral description of model to develop exact procedures

2.3.2 Hub location problem with fixed cost (fix-HLP)

P-hub median problems ignores the opening cost of hubs. O'Kelly (1992a) first introduced the single allocation hub location problem with fixed costs. Since the opening cost of hubs are included to the model, minimization of the total costs helps to determine the number of hubs to open. Also, the capacity parameters for hubs may be included to the models. Aykin (1994) introduced capacitated versions of the problem allowing direct connections where capacities limited the flow through hubs. Campbell (1994b) linearized the formulations for all possible versions of the problem. Nickel et al. (2001) modeled fixed cost associated with both hub edges and spoke edges. Yaman (2005) introduced modular capacity and examined polyhedral results of the problem. Yaman and Carello (2005) propose a stepwise cost function to model fixed cost of using an edge.

As earlier solution procedures of HLP with fixed cost is represented by Helm and Venkataramanan (1998). They presented Branch and Bound (B&B) method and genetic algorithm. Following this paper, Ernst and Krishnamoorthy (1999) studied on simulated annealing to obtain upper bound and random descent heuristic. Ebery et al. (2000) provided shortest path heuristic algorithms. Mayer and Wagner (2002) developed a B&B method to obtain good lower bounds to tighten their formulation and improve the computational performance of their algorithm. Topcuoglu et al. (2005) also proposed genetic algorithm whereas Chen (2007) presented a hybrid heuristic including the simulated annealing, tabu list and improvement procedures. Chen (2007)'s heuristic showed better performance in term of quality and time of solution than the heuristic of Topcuoglu et al. (2005).

Labbe and Yaman (2004) and b) presented some facet-defining valid inequalities on hub location problem with fixed costs. Marin et al. (2006) extended their earlier formulation relaxing the assumption of satisfying triangle inequality of costs. Labbe et al. (2005) introduced capacity of hubs in terms of the traffic that passes through it and developed a branch-and-cut algorithm. Martin and Gonzalez (2008) proved that Benders decomposition becomes incapable relatively to Double Benders decomposition or their branch and cut algorithms based on decomposition techniques. One of the results of this paper is their proposed heuristic is capable of solving capacitated HLP instances that cannot be managed by MIP solvers. Contreras et al. (2009a) formulated the model with path variables and achieved tight lower bounds with a Lagrangean relaxation. Later, Contreras et al. (2010) proposed a branch-and-price algorithm combining Lagrangean relaxation and column generation techniques which was capable to solve instances up to 200 nodes.

Correia et al. (2010a) introduced a set of capacity levels to determine optimal capacity level for each located hub. Their model determined the appropriate size of the hubs. They suggested that the capacitated single allocation HLP in the literature has some gaps in terms of incomplete formulation leading to optimal solutions that are not feasible. They reformulated the problem leading to the inclusion of an additional set of constraints. Contreras et al. (2012) have considered multiple capacity levels as a decision variable where their model determines the capacity level associated to hubs. A new definition on demands as splittable and non-splittable commodities are represented. O'Kelly (2012) investigated environmental concerns on hub location problem where fuel burns and aircraft cost are presented as objective. Takerhani and Alumur (2019) proposed r-allocation modeling where the objective is profit maximization including classical cost with routing costs.

2.3.3 P- hub center problem (p-HCP)

Analogously to p-center problem, P-hub center problems aim to minimize the maximum attributes (cost, time, etc.) of node pairs. The study of Campbell (1994b) once again the first formulation of the p-hub center problem in the literature. Three types of minimization are represented in his study:

- Cost/time of each connection between nodes.
- Cost/time of any single connection between nodes
- Cost/time of each inter-hub connections

Campbell (1994b) stated that p-hub center problems are beneficial to design perishable or time sensitive networks. He presented formulations for all versions of p-hub center problem. Kara and Tansel (2000) compared their combinatorial formulation which has (n^2) binary variables with three linearizations of the formulation in the Campbell (1994b)'s study. Pamuk and Sepil (2001) proposed first heuristic combining single-relocation heuristic and tabu search to solve p-hub center problem in a reasonable time without trapping by local optima. Ernst et al. (2002a) also studied the multiple allocation formulations and proposed a shortest path-based B&B method.

Hamacher and Meyer (2006) proposed a binary search algorithm for both center and covering problems. In this paper for covering problems, the feasibility polyhedron is analyzed and several classes of facet defining valid inequalities are identified.

Campbell et al. (2007) presented various complexity results and provided integer programming formulations for both uncapacitated and capacitated cases. Yaman, Kara and Tansel (2007) introduced multiple stopovers allocated to hubs through paths where arrival times measured as transient times. Mahdi and Masoud (2008) is analyzed p-center problem in term of qualitative and quantitative variables in which the fuzzy logic has to apply. Gavrilioluk (2009) is studied on the large size of networks exceeding 200 nodes. The paper preferred to use aggregation which is able to bound errors to search the resulting errors and decrease them. The result of the paper is that adapting meta-heuristics for p-hub center problem become complex but still considerable for the future.

Ernst et al. (2009) have proved that single/multiple allocation p-hub center problems are NP-hard even if the economic discount factor is equal to zero. Their proposed formulation performed computationally better than the best-known formulation from Kara and Tansel (2000). Meyer et al. (2009) developed a hybrid heuristic combining ant colony optimization to get better upper bounds and B&B method to achieve optimality. They showed that their algorithm performed better than traditional MIP-solver. Yaman & Elloumi (2012) incorporated star network topology with center hub location problem. Liang (2013) improved the hardness and approximation of hub center model by two-level allocation or rounding of linear relaxations.

2.3.4 Hub covering problem (HVP)

Hub covering problems aim to maximize the coverage of the network in terms of the number of allocated node or the amount of the flows routed where hubs should locate in a pre-determined distance range to self-allocated nodes. Campbell (1994b)'s first formulation of covering models classifies three coverage criteria for hubs:

- the cost/time of any route (i-j-k-m) restricted by a pre-determined value,
- the cost/time of each route restricted by a pre-determined value,
- the cost/time of each routes between hubs and origins or destinations restricted by a pre-determined value,

Hub covering models are separated into two sub groups according to their objectives. Hub set-covering problem minimizes the opening cost of hubs to cover all flows at network; whereas maximal hub-covering problem maximizes the amount of flows covered with a given time, cost or distance restriction of routes or pre-determined number of hubs to open.

Kara and Tansel (2001) introduced first set covering modeling by defining minimization of number of hubs. Kara and Tansel (2003) compared their combinatorial formulation with the linearized formulations of Campbell (1994b) and proved that the single allocation set covering problem is NP hard. Wagner (2004b) proposed preprocessing techniques which eliminate some hubs to open. By the way, their formulations with reducing variables and constraints perform better than Kara and Tansel (2003)'s formulation and developed a further procedure for aggregating some constraints. Tan and Kara (2007) proposed a cargo delivery model with hub covering formulation. To increase the applicability, two different variations are suggested. The first one is related to weights of each alternative hub location which are analyzed by decision makers using multi attribute utility theory. The second one is planned to create more realistic model by considering different service schedules such that subsequent deadlines are in 24 hours.

Wagner (2008) compared formulations of covering model with quantity independent and dependent discount factor. Alumur and Kara (2009) improved stopover concept through paths by bounding the number of stops with time restriction. An imperialist competitive algorithm is represented by Ghodsi et al. (2010) to deal with the characteristics of cover problem such as tight radius and capacity bounds. The formulations with different coverage constraints are first investigated by Karimi and Bashiri (2011). The significant index in this paper is that the coverage is considered between two nodes not only an origin and destination pair which could be beneficial to model different strategies of real-life problems. Zarandi et al. (2012) introduced new approach to covering problem which nodes at network should cover for q time. Hwang and Lee (2012) presented maximal covering hub location models by introducing maximization of covered demand as an objective.

Peker and Kara (2015) reformulated of maximal covering model with decrement of the number of variables and constraints. Ernst et al. (2018) also studied on hub set-covering problem similar to their previous study on p-hub center (2002a) and compared reformulations such as sizing of model, tightness or lifting of constraints by means of computational performance. They developed an implicit enumerative method.

2.3.5 Hub arc location problem (HAL)

Most of the literature of HLP assume that the hub network connecting hub nodes is complete. For real-world applications, this assumption isn’t accurate. Some studies concentrate on relaxing the complete hub network assumption and proposing more flexible network designs.

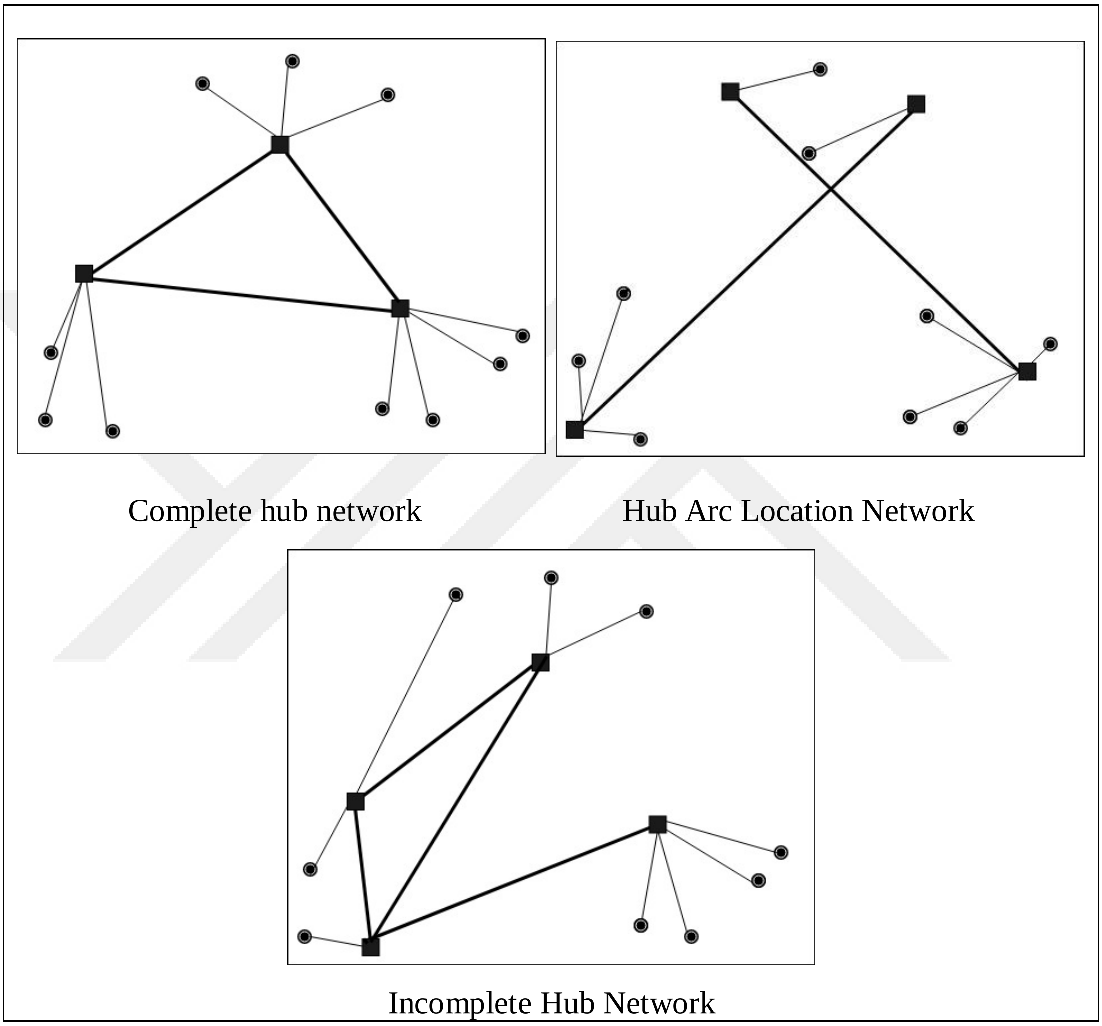


Figure 2.7 : Illustrations of network structure.

The research on hub arc location problems are pioneering studies which relax this assumption. Campbell et al. (2003) introduced first hub arc location modeling which aim to locate hub pairs to form an arc rather than individual hubs. In this study, they mainly attended to develop an enumerative algorithm to solve large scaled hub location problems. Campbell et al. (2005a) introduced general framework of hub arc location problem to improve designs for hub network. They both relaxed the assumption of fully connected network and considered the discounted cost on arcs rather than every hub to hub connection. They presented four special cases related to network structure in detail and analyzed the results comparatively. Campbell et al. (2005b) improved their previous study providing integer formulations of HAL and computational methods such as preprocessing methods and enumeration-based algorithm.

Following studies considered incorporating hub arc location problems with different aspects of the problem such as time definite modeling (Campbell, 2009), competition (Sasaki et al., 2009), modular transportation (Alumur et al., 2012) and dynamic modeling (Gelareh et al.,2015). Contreras and Fernandez (2014) proposed a new formulation for HAL as the minimization of a supermodular function where the model incorporates separate setup costs for the hub nodes and arcs. Alibeyg et al (2016) introduced a profit maximization for hub arc location problem. Alibeyg et al (2017) developed an exact solution procedure to solve their formulation.

2.3.6 Incomplete hub location problem

Early hub location studies adopted some assumption about fundamentals of hub network design as mentioned above. Campbell (1994b) classified hub location problems corresponding to eight main assumption such as economic benefit of routing flows through hub, visiting at most two hubs and multiple or single allocation. Moreover, constructing fully interconnected hubs and prohibition of direct connections between non-hub nodes are the main assumptions to design the structure of the network.

However, it is sometimes possible to design more cost, time and service effective incomplete hub network than a complete one. Thus, the total cost in designing complete networks due to unnecessarily increase of costs such as opening a hub or constructing multiple links may be reducible. Although these assumptions make the problem generic or idealized and simple to model and solve, they also move the studies far from the application specific models with real-life constraints and objectives. To widen the problem’s scope, more interesting and flexible network structures have been investigated by recent literature leading to incomplete hub networks.

Incomplete hub location problem insists on only ensuring all connectivity of the flows at network and doesn’t adhere to any network structure. This means that demand associated with flows should be routed between node pairs.

O’Kelly and Miller (1994) first analyzed incomplete networks throughout eight topological protocols for hub networks. In their study, one of the protocols of hub network design connects directly non-hub nodes to each other. Klincewicz (1998) examined distinct topology alternatives including relaxing fully connected assumption and introduced fixed costs to establish hub arc. Nickel et al. (2001) presented multiple allocation hub location problem while minimizing total cost including a new term "fixed costs of constructing hub connections".

After these pioneering studies, Alumur et al. (2009) introduced first incomplete hub location problem. They addressed four classes of HLP (median, HLP with fixed cost, center, and covering) with incomplete design, but connected hub level and proposed efficient mathematical formulations. Alumur et al. (2009) proved that even some hub links of complete network carry low amounts of flows, it is still employed the discount factor as economies of scale. Thus, the usage of these low amounted hubs and associated connections increase total cost of network. To consider time efficiency of the network, they proved that constructing a complete hub networks may not be necessity to guarantee the service quality of network.

Campbell (2010) introduced isolated hubs which are not connected to any hub arc. Gelareh and Nickel (2011) presented a tighter formulation of incomplete hub location problem with four-index variables. Subsequent studies incorporated different aspects of the problem such as competition (Lüer-Villagra and Marianov, 2013), routing policies (Karimi and Setak, 2014) and uncertainty (Martins de Sá et al., 2018a; 2018b). Some of other studies considered distinct cost structures to establish the components of network such as variable arc costs (Campbell et al, 2015; Akgün and Tansel, 2018, O'Kelly et al., 2015a). Camargo et al. (2017) proposed a restriction of the number of stops through routing of every node pairs to handle service levels. Dai et al. (2019) presented a heuristic algorithm which measures the quality of pairs to construct hub connections.

2.4 HLP in Dependency of Allocation Structure

In classical hub location problem, two different allocation strategy (single and multiple) is investigated. While single allocation forces each non-hub node to utilize only one hub to transmit the flows to associated destination, multiple allocation allows a non-hub node to connect all possible p hubs. Figure 2.9 shows the number of the studies applying these strategies in their model in the literature.

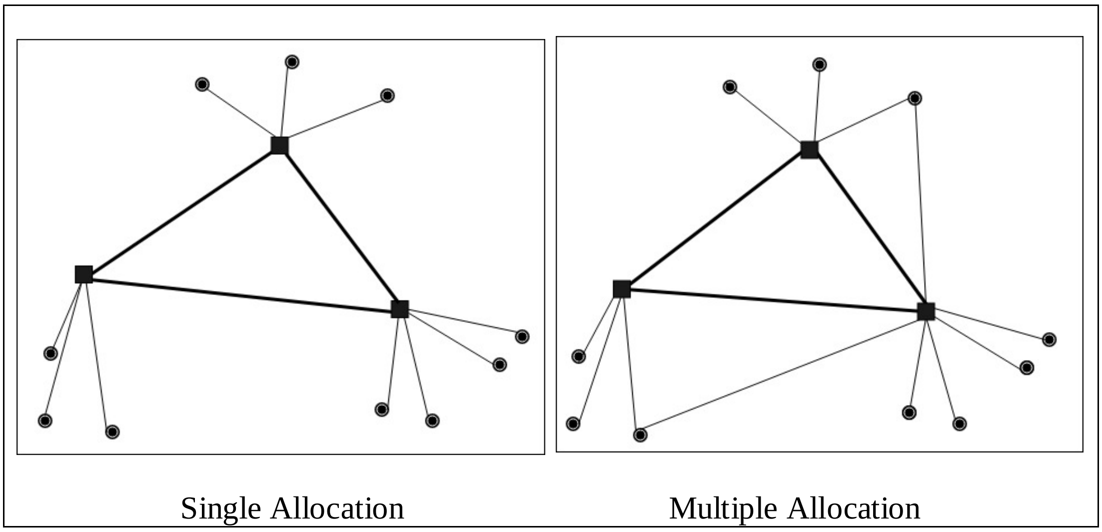


Figure 2.8 : Illustrations of the allocation strategies.

Both versions of the allocation strategy might be inconvenient for real-life applications of the problem. A network whose non-hub node allocated to all possible hubs become economically unreasonable. Whereas, single allocation undesirably increases the routing cost. Through these observations, Yaman (2011) proposed a new perspective for allocation structure: "r-allocation". His model specified the number of non-hub nodes allocated to hubs to predetermined value (r). More recently, some studies considered to develop solution methodologies to solve r-allocation modeling including heuristics based on threshold accepting method (Chen, 2013), scatter search (Marti et al., 2015), general variable neighborhood search (Todosijević et al., 2017; Jankovic and Stanimirovic, 2017) and Benders decomposition algorithm (Mokhtar et al., 2019a).

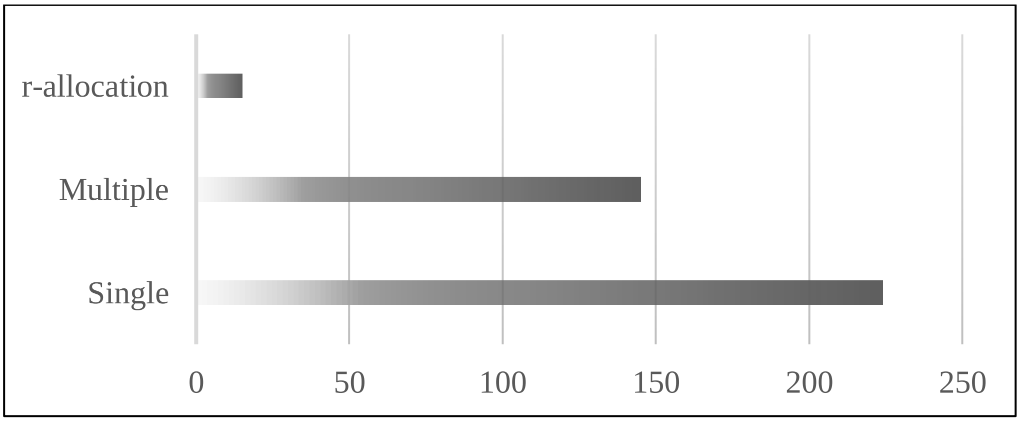


Figure 2.9 : The distribution of the allocation strategies in the literature.

2.5 HLP in Dependency of Mathematical Formulation

Hub location research provides wide range of mathematical formulations including quadratic, integer, integer linear, mixed integer, non-linear and stochastic programming. Figure 2.10 shows the distribution of these formulation for our reviewed literature.

In the mixed-integer programming (MIP) formulations, the decision variables which represent the routes of flows between nodes of mixed integer programming (MIP) formulations may be represented in two class: path-based formulation and flow-based formulation. Figure 2.11 shows the number of the studies using these formulations in the literature.

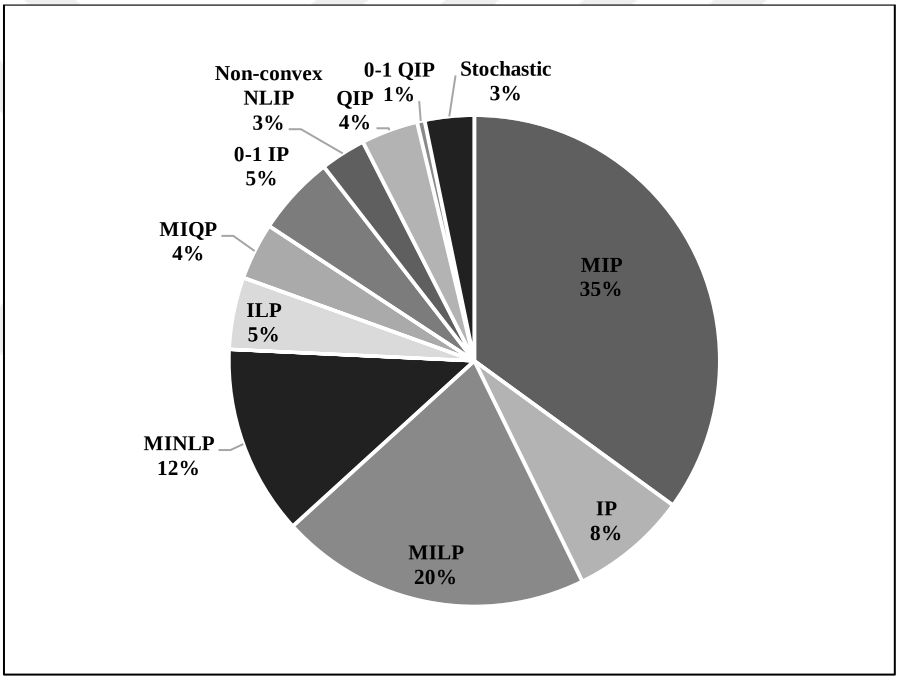


Figure 2.10 : The distribution of mathematical formulations in the literature.

Path-based formulations utilizes decision variables to define their routes from origin to destination visiting associated hub/hubs. This formulation presents the decision variable X_{ijkm} as the fraction the flow from origin (i) to destination (j) through edge of hubs (k, m). Skorin-Kapov et al. (1996) and Campbell (1994b) studied hub location problems with path-based formulations like many other studies. Although path-based formulations are requiring $O(n^4)$ variables and $O(n^3)$ constraints, they provide tight lineer programming relaxation bounds.

Flow-based formulation utilizes decision variables to define their route originated from a node to associated hub/hubs. This formulation presents the decision variables Y_{ikm} as as the fraction the flow started from origin (i) and routed through edge of hubs (k, m).

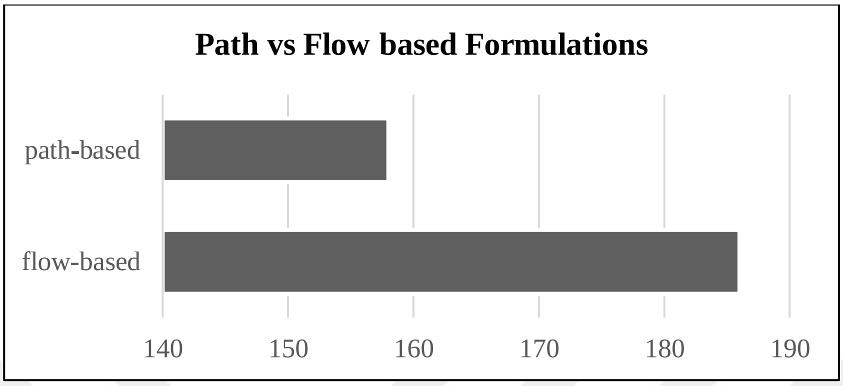


Figure 2.11 : The number of studies utilized distinct formulations in the literature.

Ernst et al. (1998, 1999) and Boland (2004) studied hub location problems with flow-based formulations as in most studies in literature. Since, flow-based formulations are requiring $O(n^3)$ variables and $O(n^2)$ constraints, the formulations have fwer variables and constraints compared to the path-based formulations. To model multiple allocation hub location problems, additional flow variables need to be defined as x_{iim} which represent the fraction of the amount of the flow originated from (i) to destinated to (j) through hub (m).

2.6 HLP in Dependency of Capacity Constraint

Hub location researches considered capacity as a natural and important extension of the problem. Thus, two classes of the hub location problem as capacitated or uncapacitated considered whether or not the amount of the flows through the network (at hub, on link or arcs) are restricted to a pre-determined value. This means that the quantity of flow that can be redirected or sorted by a hub will be restricted to some maximum volume, known as its capacity. Capacity limitations may be applied on a hub/ hub-arc which represent the amount of the flow accumulated at that hub/hub-arc or on a link/vehicle which represent the amount of the flow transmitted through that link/vehicle. Hub capacity may be regarded as the maximum number of passengers a node can deal with when it is selected as a hub. For example, the number of runways and the number of slots assigned to the airline company may constitute the hub capacity. On the other hand, arc capacity may represent the number of available aircrafts for the airline company on that arc. In postal delivery, the volume of mail that hubs can sort is limited by time constraints. This, in turn, limits the volume of mail that can be sorted at mail sorting centres. Hence, that volume or the limited time can be defined as capacity restrictions on the hub nodes and these restrictions are only applied to the traffic arriving at the hub directly from non-hub nodes as this is where the sorting takes place. Also, different functions may represent the capacity of a network element such as node, hub, arc, link or vehicle. Splitting/Non-splitting commodities over node pairs, incoming/outcoming flows at hubs and time needed to process the flows at nodes are utilized to formulate capacity. Figure 2.12 illustrates the proportion of the capacity considerations in the literature.

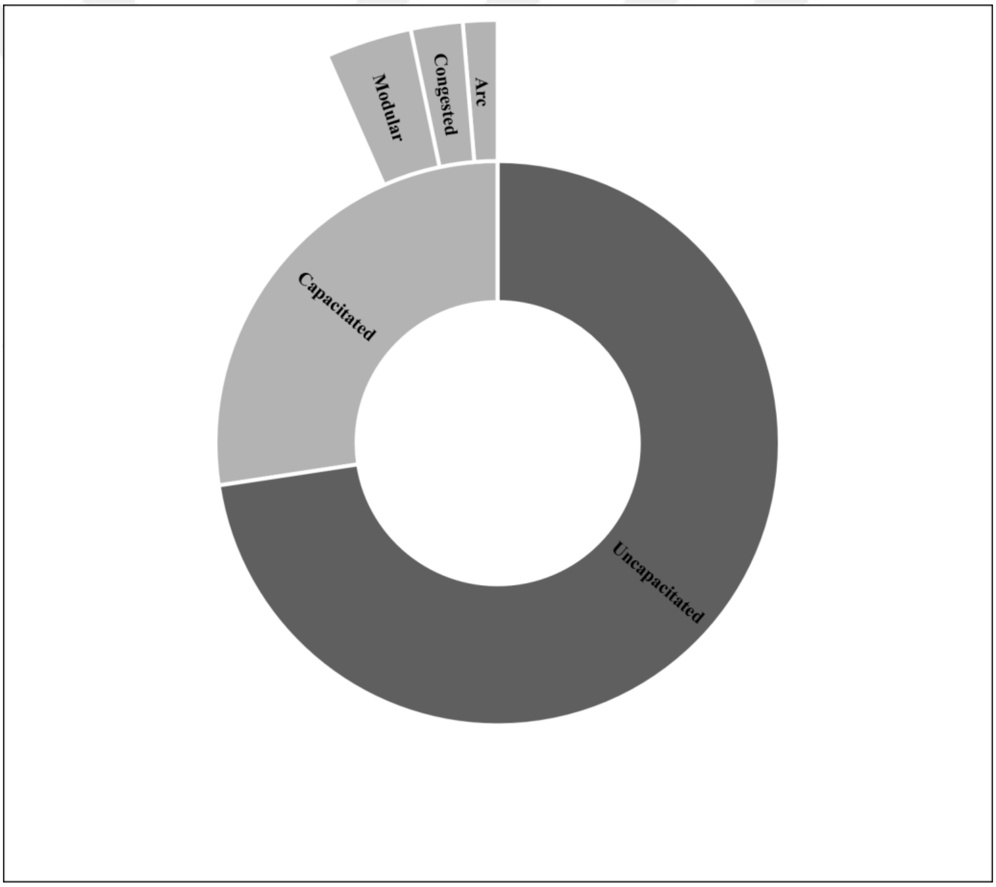


Figure 2.12 : The distribution of the studies modeled as uncapacitated or capacitated in the literature.

Campbell (1994b) presented the prior capacitated hub location problem. The capacity restrictions impose on both incoming and outcoming flows through hubs. Aykin (1994, 1995) defined capacity as only incoming flows at hubs and nodes if they are directly connected to each other. Bryan (1998) determined capacity as a restriction on hub arcs.

Marin (2005b), Rodriquez-Martin and Salazar Gonzalez (2008) are the examples of the studies examined splitting cases of commodities. Inherently, splittable commodities are natural and desirable concern to model multiple allocation problems. Yaman and Carello (2005) and Yaman (2008) introduced modular link capacities with quadratic constraints and defined the capacity as all flows passing through a hub. They also, imposed the capacity restriction on the links to establishing hubs.

Recently, there are some studies incorporated capacity restrictions with congestion. Costa et al. (2008) determined congestion as waiting times at hubs according to its capacity. To avoid congestion at hubs and balance the flow distribution of the network, Marianov and Serra (2003) proposed capacitated hub location problems as a queueing model. Elhedhli and Hu (2005), Rodriquez at al. (2007), Elhedhli and Wu (2010), Ishfaq and Sox (2012), Rahimi et al. (2016) continued to refer capacity consideration as a queuing model. Correia et al. (2010a) investigated multiple capacity levels where the capacities of each hub are a part of decision. Subsequent studies such as Correia et al. (2011), Sender and Clausen (2011), Contreras et al. (2012) and Chen (2013) studied on locating potential hubs and evaluating their capacity levels.

2.7 HLP in Dependency of Application Areas

Hub location problems become promoting research field over the past three decades due to its various and extensive application areas. Telecommunication and transportation have been main application areas to determine the fundamentals of the problem. Almost half of the studies encountered in the literature are idealized generic models that are not intended to specific application which we labelled this type of studies as hub network in Figure 2.13. Whereas remaining studies are employed to reflect targeted application's features with associated constraints and objectives.

Hub location problems may apply to freight and passenger transportation where three basic modes of transportation exist: land (road, rail, pipeline, etc.), water (marine, liner shipping) and air. Moreover, delivery networks, logistic and supply chain networks and trucking are suitable to apply hub location problem with their specific features and requirements.

Among the works that presented models and formulations associated with air transportation, relationship between cost and flow distributions, direct flights or flights with single or multiple stops and topological innovations are mainly examined. Bryan and O'Kelly (1999) studied on the economies of scale for passenger transportation to model flow dependent cost. Jaillet et al. (1996), Watanabe et al. (2008) and Campbell (2013a) focused on cost modeling depend on economies of scale, as well. Also, most of the studies investigated several network structures designed for air transportation such as Sasaki et al. (1999) for one stop flights, Campbell et al. (2003) for isolated hubs, Campbell et al. (2005a, 2005b) for incomplete network. Lastly, Karow (2003) and Teymourian et al. (2011) studied on the reliability of airports.

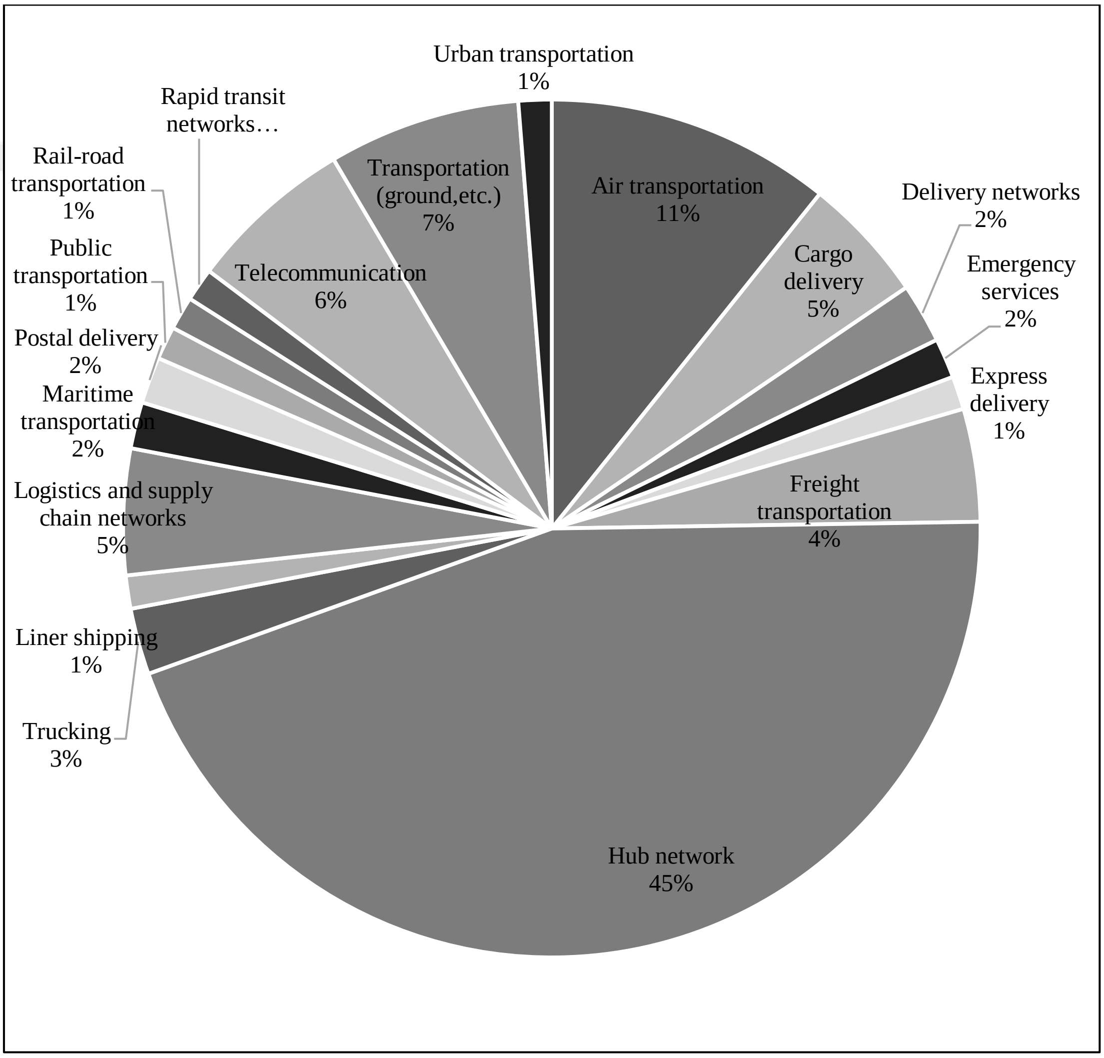


Figure 2.13 : Application areas encountered in the literature.

Different authors have proposed specified models and formulation to represent airline transportation features for various HLPs. Among them we can mention Yang and Chiu (2016), Miranda Jr. et al. (2011) for uncertain network environment, Özgün-Kibiroğlu et al. (2019), Kawasaki (2012) Mayer and Sinai (2002), Kostiuk et al. (1998) and

Grove and O'Kelly (1986) for congestion effect, Sasaki et al. (2014), Redondi et al. (2011), Adler and Smilowitz (2007), Martin and Roman (2004, 2003), Marianov et al. (1999) and Skorin-Kapov (1998) for competition and Soylu and Katip (2019) and Çitftçi and Şevkli (2015) for multi-objective modeling.

Service oriented networks involve postal delivery, cargo delivery, express delivery and emergency services. O'Kelly and Lao (1991) presented hub location problem with modular transportation for express delivery. Aykin and Brown (1992) incorporated hub location with postal delivery perspective. Ernst and Krishnamoorthy (1996, 1999) studied on the problems of cargo delivery. These application areas essentially need to concentrate on service quality of network.

Among them we can mention Kara and Tansel (2001) and Tan and Kara (2007) for time definite modeling, Vasconcelos et al. (2011), Alumur et al. (2012a) and Serper and Alumur (2016) for modular transportation, Lin and Chen (2004), Yaman et al. (2007), Alumur and Kara (2009) and Karimi (2018) for network topology such as hierarchical and incomplete, Çetiner et al. (2010), Rieck et al. (2013), Rodriquez-Martin et al. (2014), Bostel et al (2016) and Kartal et al. (2018) for routing policies and Masaeli et al. (2018) for scheduling policies.

Rapid transit networks and urban/public transportation are similar applications which doesn't require designing fully inter-connected hubs at network. Thus, Maheo et al. (2017), Martins de Sá et al. (2015a, 2015b), Owsiński et al. (2015), Gelareh and Nickel (2011), Gelareh (2008) and Nickel et al. (2001) considered incomplete hub networks coherent with associated applications. Some of the studies such as Meier and Clausen (2015), Catanzaro et al. (2011), Lin (2010), Campbell (2009), Lin and Lee (2010) and Cunha and Silva (2007) worked on trucking networks.

Recently, logistics and supply chain networks become novel application areas for hub location problems where we can mention Groothedde et al. (2005), Ishfaq and Sox (2012), Puerto et al. (2011, 2013) Lin et al. (2013), Hörhammer (2014), Campbell et al. (2015) and Mokhtar et al. (2019b).

In telecommunication applications, studies focus on the cost of the opening hubs or hub arcs. This significant distinction leads the earlier researches to develop novel modeling of the problem and associated solution procedures where we can mention Klincewicz (1998), Lee et al. (1993, 1996), Carello et al. (2004), Yaman and Carello

(2005), O'Kelly et al. (2006), Thomadsen and Larsen (2007), Labbe and Yaman (2008), Yaman and Elloumi (2012) and Sen and Krishnamoorthy (2017). On the other hand, telecommunication networks should maintain networks capabilities continuously. Some of the studies, Grover and Tipper (2005), Kim (2008), Kim and O'Kelly (2008), Kim (2012), Yıldız and Karaşan (2015) and Rostami et al. (2018) worked on reliability concept of the problem.

2.8 HLP in Dependency of Solution Methodologies

Various solution approaches have been presented through the development of the literature. Prior hub location models were simplified by some fundamentals as mentioned before. Thus, both exact and heuristic algorithms proposed to solve the problem. Most of the studies have practiced on CAB and AP data sets to evaluate their results. Figure 2.14 illustrates the distribution of the data sets in the literature.

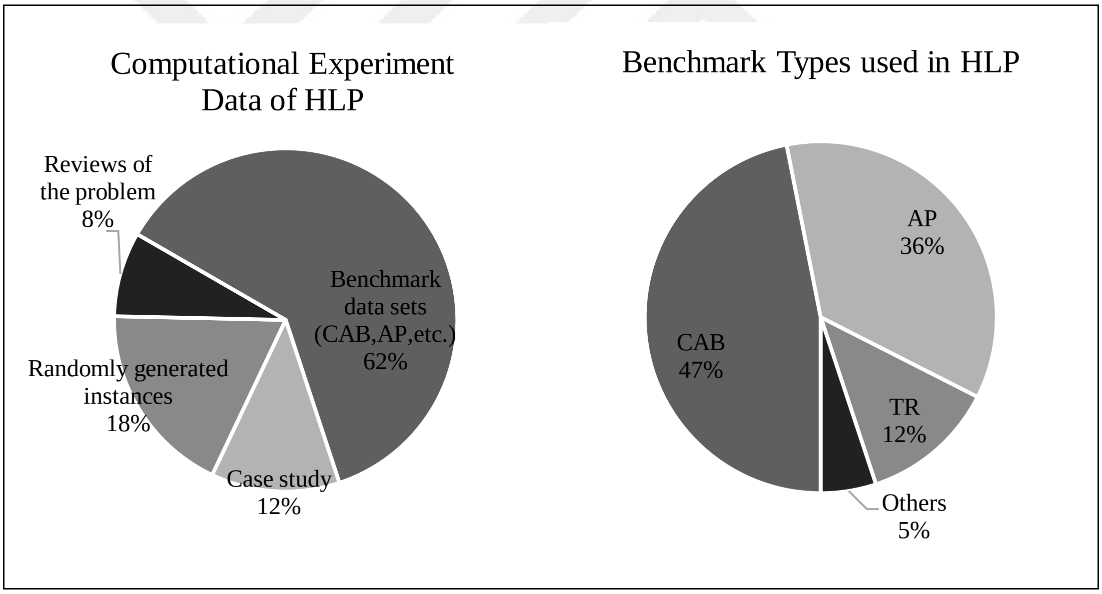


Figure 2.14 : Data profile used in HLP applications.

Since the design of the networks and modeling of the problem became more complicated with new aspects such as time definite modeling, cost functions formulated as piece-wise linear function and large-scaled networks, the hub location researches have tended to improve solution techniques in order to achieve optimality or high-quality solution and provide better upper/lower bounds.

After succeeding progress in developing solution procedures, the literature concentrated on larger scaled problems and computational time effectiveness. Figure

2.15 shows the variety of the solution approaches proposed in the literature and the frequency of their usage.

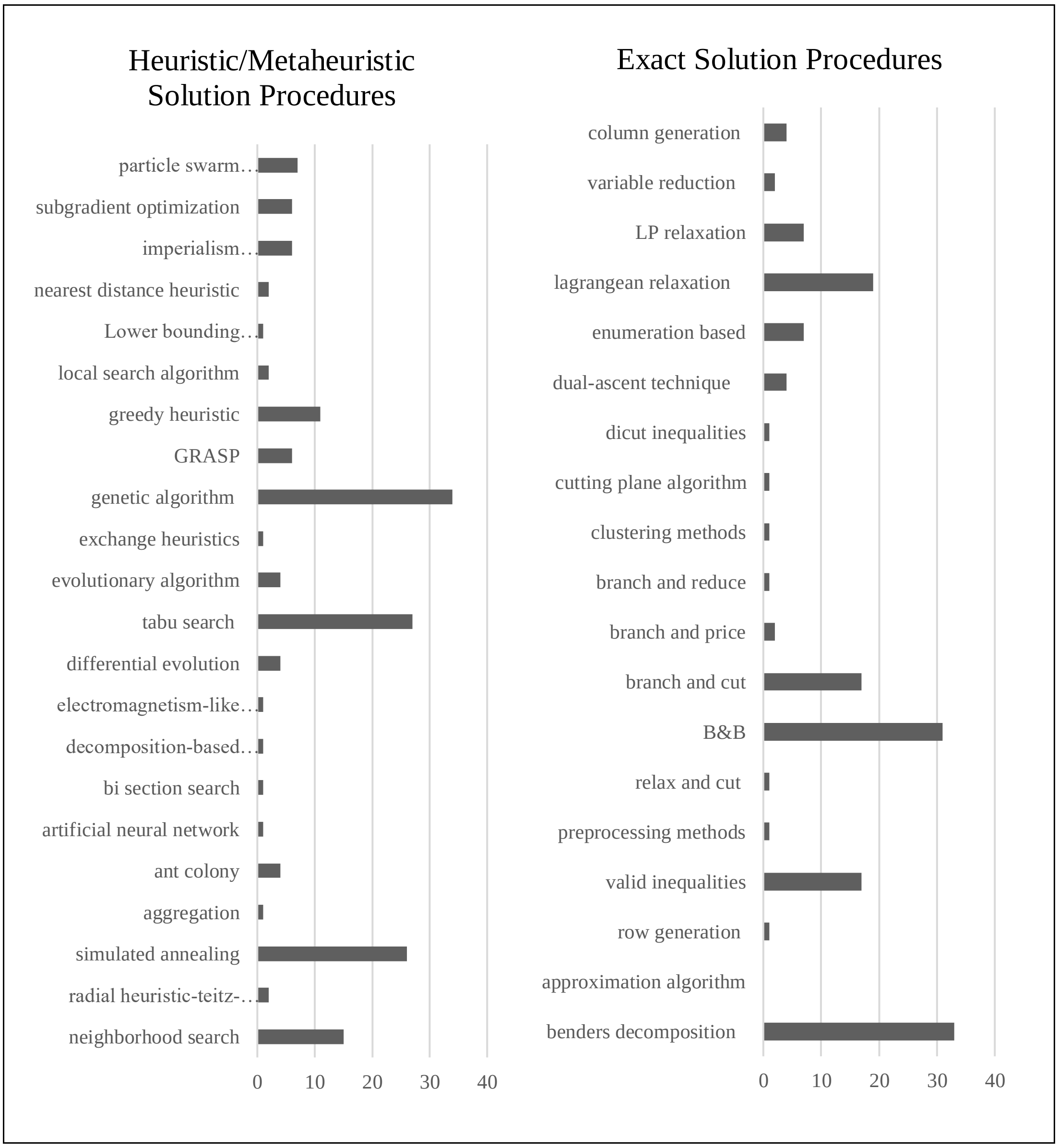


Figure 2.15 : Solution procedures encountered in the literature.

Hub location problem is classified as NP hard problems in optimization. Although such a great variety of solution procedures are introduced and extended in the literature, there exist few studies on complexity results of HLP. Ernst and Krishnamoorthy (1998b), Sohn and Park (2000), Kara and Tansel (2000,2003), Campbell et al. (2007), Gavriliouk (2009), Ernst et al. (2009), Contreras et al., 2010), Liang (2013), Campbell (2013a), Contreras and Fernandez (2014), Bordini and Vignatti (2017), Chen et al. (2018) are some examples of these studies.

To achieve optimality, exact solution procedures are preferable. However, exact solution procedures require more dedicated effort and costly computational burden. Preprocessing techniques benefit from reducing size of the problem rather than working on the complexity of the problem. Boland et al. (2000, 2004), Skorin-Kapov et al. (1996) reformulated tighter models by pre-processing.

Enumerative algorithms are beneficial to provide optimal solution of the problem by listing all possible choices that should be determined such as allocation of non-hub nodes or the amount of the flows transmitted between pairs. Aykin, (1995a), Ernst and Krishnamurthy (1998a, 1998b), Abdinnour-Helm and Venkataramanan (1998b), Klincewicz (2002), Campbell et al. (2003), Lin and Chen (2004), Campbell et al. (2005b), Sasaki et al (2009), Lin (2010), Sasaki et al. (2014), Mahmutoğulları and Kara (2016) developed enumerative algorithms.

The vast majority of the literature have used linear programming-based approaches such as dual ascent techniques, lagrangean relaxation, Benders decomposition, branching techniques and column-row generation. Dual ascent technique as the name implies takes the dual formulation of the problem and tightens that formulation. Lee et al. (1996), Klincewicz (1996), Sung and Jin (2001), Mayer and Wagner (2002), Canovas et al. (2007), Meyer et al. (2009) presented some of examples of that technique.

Branching techniques, like branch and bound, branch and cut, branch and price and etc., is most common technique used in optimization. Branch and bound explores the feasible region of the problem by dividing original problem and capturing smaller parts of the problem. As shown in Figure 2.15, researchers preferred at most this technique. Moreover, branch and bound may incorporated with other solution approaches as LP relaxation. Aykin (1994, 1995a), Ernst and Krishnamoorthy (1996, 1998, 1999), Klincewicz (1998), Ebery et al. (2000), Nickel et al. (2001), Ebery (2001), Labbe et al. (2005), Ernst et al. (2009), Berman and Wang (2010), Ishfaq and Sox (2012), Puerto et al. (2013, 2016), An et al. (2015), Alumur et al. (2015), Alibeyg et al. (2016, 2017), Tanash et al. (2017), Ernst et al. (2018) applied branch and bound to solve the problem.

Branch and cut improves branch and bound algorithm with additional cutting planes to tighten LP relaxations. This technique increases the size of the problem optimally solved. Some of the examples of this technique are Yaman and Carello (2005), Marin

(2005b), Yaman et al. (2007), Rodriguez-Martin and Salazar-Gonzalez (2008), Labbé and Yaman (2008), Contreras et al. (2010b, 2013, 2015, 2016), Catanzaro et al. (2011), García et al. (2012), Rodríguez-Martín et al. (2014), Yıldız and Karaşan (2015), Zetina et al. (2016), Rothenbächer et al. (2016), Boccia et al. (2018), Corbean et al. (2109).

Lagrangean relaxation is a partitioning technique which mostly provide tighter bounds to achieve optimality in less time. Lagrange Relaxation takes out some constraints which make the problem difficult to solve from the model and replace them into the objective function by weighting them with their dual variables. Aykin and Brown (1992), Lee et al. (1996), Pirkul and Schilling (1998), Elhedhli and Hu (2005), Yaman (2008), Contreras et al. (2009a, 2009b, 2009c), Gelareh et al. (2010), Elhedhli and Wu (2010), Ishfaq and Sox (2011), Lin et al. (2012), Karimi and Setak (2014), He et al. (2015), Rostami et al. (2016), Neamatian Monemi et al. (2017), Dukkanci and Kara (2017), Alkaabneh et al. (2019) investigated Lagrangean Relaxation on solving HLP.

Benders decomposition algorithm is another partitioning technique which applied the solution procedure of the problems extracted from the original one iteratively. The master problem and sub-problems derived from the original problem are solved iteratively and obtained solutions are attached to respective problems until achieve the optimal solution. Since one of the contributions of this thesis is solving proposed model by Benders decomposition, the issue will be examined in Chapter 4.

Some authors such as Camargo et al. (2008, 2009a, 2009b, 2011, 2013, 2017), Martins de Sá et al. (2013, 2015a, 2015b, 2018a, 2018b), Contreras et al. (2011b, 2012) and Meraklı and Yaman (2016, 2017) incorporated Benders decomposition on distinct models of the problem. Gelareh (2008), Gelareh and Pisinger (2011), Miranda Junior et al. (2011), Camargo and Miranda (2012) utilized Benders decomposition algorithm for HLP with classical feasibility and optimality Benders cuts, multiple cuts or pareto optimal cuts. Recent studies on Benders decompositions are Hult et al. (2014), Campbell et al. (2015), Gelareh et al. (2015), O'Kelly et al. (2015b), Bayraktar (2016), Rostami et al. (2018), Real et al. (2018) and Mokhtar et al. (2019b).

Heuristic algorithms aim to find best solution between all feasible solutions for optimization problems. It's questionable that heuristic algorithms achieve the optimality. Their results may be approximate or inaccurate. To solve the optimization problems heuristic algorithms is applied due to preferring an effective and fast solution rather than optimality and accuracy. Various heuristic methods are encountered in the hub location literature. Earlier problems, especially while the size of the network is larger, or the nature of the modeling is complicated, tend to apply heuristic algorithms to obtain possible best solution. Moreover, easy implementation of a heuristic algorithm allows them appearing frequently in the literature.

Greedy algorithms run iteratively while scanning local optima of the problem to get the global optimal. Among a set of candidates, the algorithm selects the best ones for feasibility and optimality evaluations. O'Kelly (1987), Klincewicz (1992), Campbell (1996a) are earlier examples of greedy algorithms. Some recent studies such as Zhang et al. (2017), Owsinski et al. (2015), Contreras and Fernandez (2014), Peiro et al. (2014), Lin et al. (2013) implemented greedy heuristic to solve HLP. Also, Contreras et al. (2013, 2016), Martins de Sá et al. (2015b), Alkaabneh et al. (2019) proposed hybrid algorithm which integrated GRASP with some exact procedures.

Neighborhood algorithms start from one feasible solution and proceed iteratively to a neighbor solution if there exist an improvement in terms of objective function among the neighbors. Soylu and Katip (2019), Dai et al. (2019), Brimberg et al. (2017), Chaharsooghi et al. (2017), De Carvalho et al. (2017), Hoff et al. (2017), Jankovic and Stanimirovic (2017), Neamatian Monemi et al. (2017), Talbi and Todosijević (2017), Todosijević et al. (2017), Serper and Alumur (2016), Zhai et al. (2016), Gelareh et al. (2015), Gelareh and Nickel (2011), Rodriguez-Martin and Salazar-Gonzalez (2008) developed some hybrid and heuristic algorithm based on local search.

Genetic algorithms are most common evolutionary solution techniques used in the optimization problems. Genetic algorithm generates a population with feasible solutions as candidates. Later, algorithm improves the quality of population's solutions with some specific operators such as mutation, crossover. The very first studies applying genetic algorithms to HLP are Abdinnour-Helm and Venkataramanan (1998) and Abdinnour-Helm (1998).

Abbasi and Niknam (2017), Zhai et al. (2016), Pasandideh et al. (2015), Ghodratnama et al. (2013b) and Wagner (2007) compared and improved genetic algorithm to the other heuristics (simulated annealing, tabu search, particle swarm optimization, etc.). Some recent studies on genetic algorithms are Lüer-Villagra et al. (2019), Damgacıoğlu et al. (2015), Lüer-Villagra and Marianov (2013), Rieck et al. (2013), Meng and Wang (2011) and Demirci (2010).

Tabu search is another evolutionary technique whose process resembles to local search process. However, tabu search allows the moves may return to previous moves which are accepted as non-improved to prevent trapping in local optima. Also, tabu search is one of the first proposed heuristic algorithms in the hub location literature. Klincewicz (1992, 2002), Skorin-Kapov and Skorin Kapov (1994, 1995), Abdinnour- Helm (1998) are studied tabu search to solve their models. Recent studies incorporated with tabu search are Yaman and Carello (2005), Yaman et al. (2007), Wagner (2007), Kim (2008), Chen and Wu (2008), Çalık et al. (2009), Ishfaq and Sox (2010,2011), Lu and Xie (2014), Silva and Cunha (2017), Karimi (2018).

Simulated annealing explores the global optimal among the all local optima. Aykin (1995a) proposed the first simulated annealing algorithm. Subsequently, Ernst and Krishnamoorthy (1996,1999), Chen (2006), Sim (2007), Rodriguez et al. (2007), Chen (2007,2008), Teymourian et al. (2011), Geramianfar et al. (2013), Parvaresh et al. (2014), Meier and Clausen (2015), Dabidian et al. (2015), Ghaffarinasab et al. (2018a) presented some simulated algorithm procedures.

To solve hub location problems, subgradient optimization mostly is associated with lagrangean relaxation as a hybrid solution procedure. Subgradient optimization runs iteratively on convex functions to minimize. Pirkul and Schilling (1998), Contreras et al. (2009b, 2009c), Gelareh et al. (2010), Elhedhli and Wu (2010), Dukkanci and Kara (2017) are successful implementations of subgradient optimization for HLP.

Particle Swarm optimization is very recent method encountered in the literature as another evolutionary algorithm. Yang et al. (2013b) first studied particle swarm optimization improved with parametric decomposition. Only a few studies exist in the literature such Bailey et al. (2013), Wu and Wang (2014), Sen and Krishnamoorthy (2017) and Özgün-Kibiroğlu et al., 2019). A detailed explanation and survey of this algorithm will be included in the Chapter 4.

For all these exact and heuristic algorithms, various programming language and software packages are utilized. Figure 2.16 shows the distribution of programming techniques in the literature.

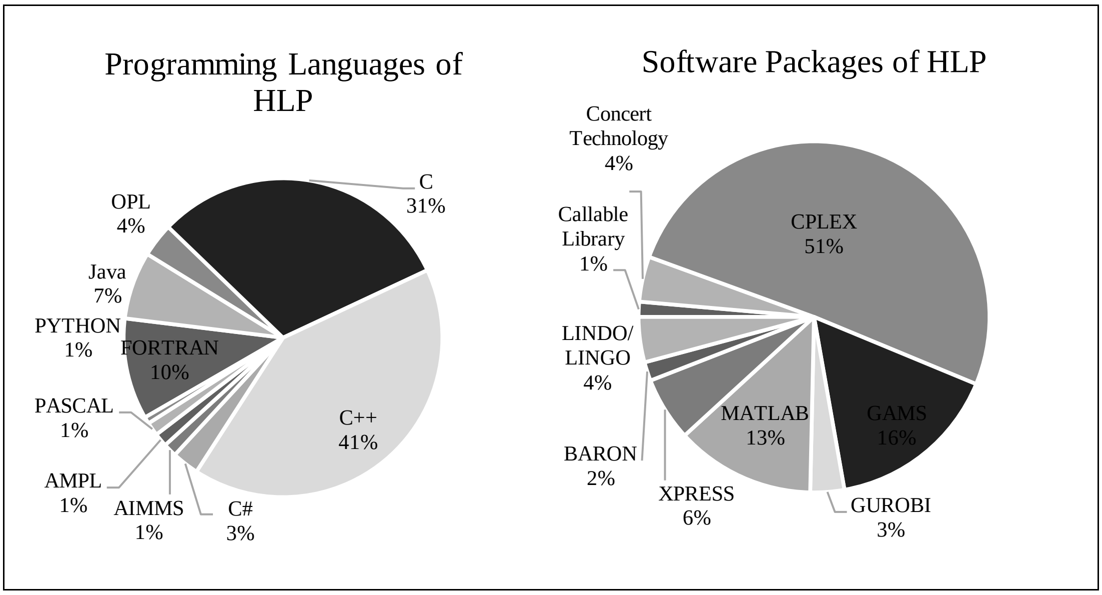


Figure 2.16 : The computational tools used in the hub location problems.

2.9 Future Research Trends

Hub location researches focus on defining the basics of the problem, formulating new problems and developing solution algorithms until 2000's. The researches mainly investigated p-hub median, HLP with fixed cost and small-scale instances. Earlier studies explored deeply the problem in terms of various allocation types, capacity constraints, more realistic modeling and solution approaches to handle large-scale problem sizes and different type of application areas. However, by the findings and classifications of novel extensions of the problem and network structure, especially Campbell's work (2005a, 2005b), the attention on HLP moves forward to improving models, formulations to reflect real-life requirements properly and extensively. Figure 2.17 shows the main issues that the existing literature is interested in.

After 2000's the studies focused on p-center and hub covering problems. The researches attended to develop exact and heuristic solution procedures. Through accelerating of the progress on hub location studies and expanding context of the problem with the knowledge and observations gathered by previous studies, future motivations include network structure where mostly fully interconnected hubs assumption is relaxed, problems with novel covering, routing, allocation strategies, time-based formulations such as time definite, delay constraints, congestion on hubs, maximization objectives to measure user/owner preference or competition, stochastic and fuzzy approach. All these efforts make HLP more attractive in literature and realistic for applications. The classification of future trends of HLP could be useful to understand the context of problem and to determine the motivation needed to improve the literature in this section.

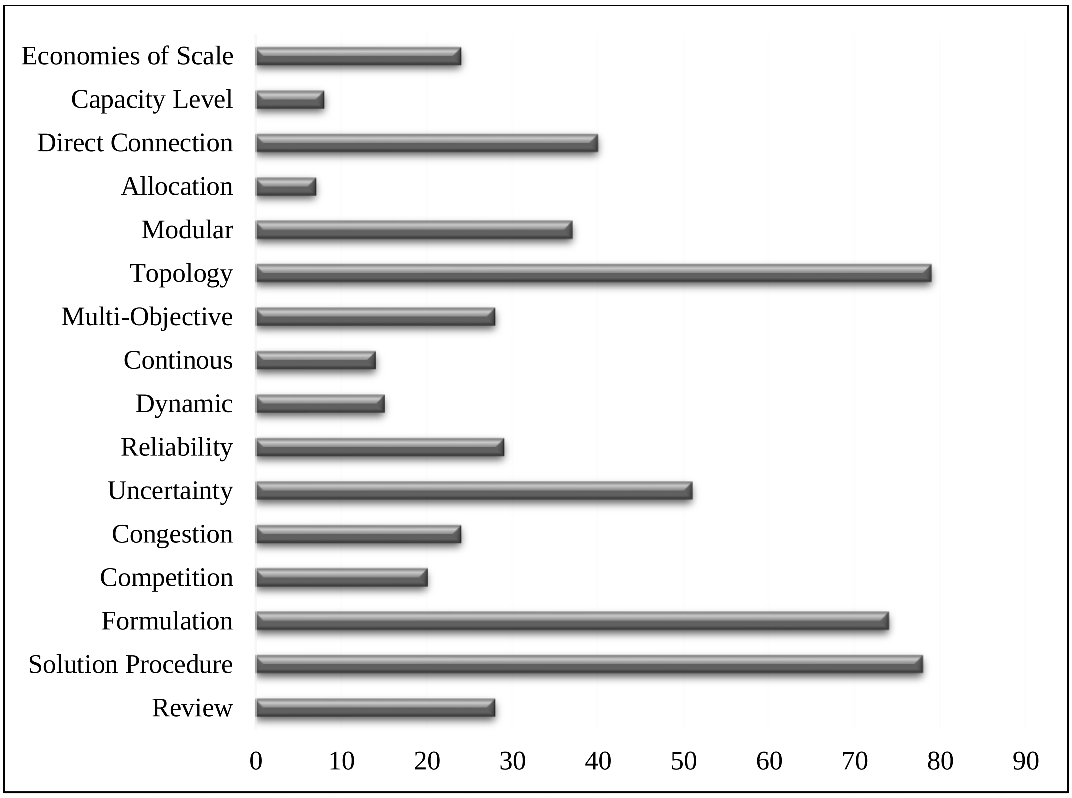


Figure 2.17 : Main issues that the hub location literature is interested in.

2.9.1 HLP in uncertain environment

Classical hub location problems assume that the inputs are precisely known and deterministic. Ignoring uncertainty of the data used in the models makes the findings and solutions highly questionable. Moreover, some of the parameters, especially demand, vary in time. These changes cause the problem to solve misguidedly. Also, hub location problems are solved as a part of designing hub networks which are time-consuming strategic decisions. Most of these decisions include uncertainty. Uncertainty is one of the factors that effects most of these decisions. Also, all these decisions are applied within a considerable time. Beside most of the application areas of HLP are time sensitive systems. On-time delivery is notable factor to design the networks.

A further aspect of the uncertainty is punctuality of the transportation time. Therefore, input could not be precise exactly. Defining probabilistic or fuzzy variables for demands, time or etc. and modelling stochastic formulations or adding stochastic constraints provide accurately results and realistic approach to HLP. While dealing with uncertainty, it is possible to define an interval associated to a set or a probability distribution to describe the behavior.

HLP under uncertain environment in the literature are rarely seen. Most of the studies incorporated uncertainty through demand and cost parameters. Marianov and Serra (2003) considered M/D/c queue theory to represent stochasticity at the hubs and evaluated capacity as the number of airplanes at a node. Sim (2007) presented the problem with uncertain travel times. Sim et al. (2009) considered minimum service level of hub-center problem with a chance-constrained formulation. Yang (2009) developed hub location and routing model to represent seasonal changes of the demand with a two-stage stochastic programming.

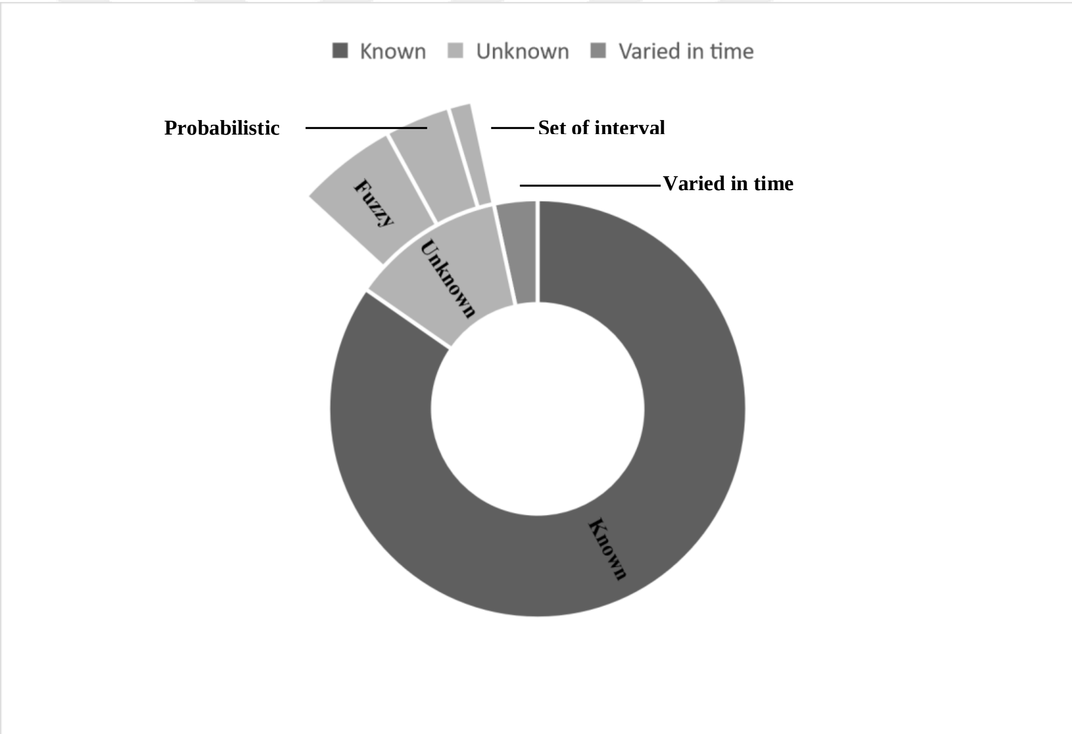


Figure 2.18 : Parameters used in hub literature.

Mirakhorli (2010) presented fuzzy radius cover and travel times defined as trapezoidal fuzzy number. Contreras et al. (2011a) presented 3 different stochastic model with uncertainty on demands and transportation as a two-stage integer stochastic programming. Zhai et al. (2012) modeled uncertain demands as random vectors with two stage stochastic programming.

Table 2.2 : Milestones of hub location literature in terms of uncertainty.

Authors	Model	Milestones	Solution Procedure
Correire et al. (2018)	CMAHLP	casting stochasticity under multi-period hub location problem where demand is modeled with scenario-based uncertainty	valid inequalities
Martins de Sá et al. (2018a)	UMAIHLP	traversing times on hub arcs, and on spokes are modeled within uncertainty intervals and problem is handled with robust optimization approach	Benders decomposition
Martins de Sá et al. (2018b)	UMAIHLP	Demand and fixed setup cost are modeled with uncertainty	Benders decomposition-neighborhood search
Meraklı and Yaman (2017)	CMAHLP	incorporating both demand uncertainty defined as hose model and capacity constraints	Benders decomposition
Parizi et al. (2017)	UMAp-HLP	modeling non-linear robust hub location problem	Benders decomposition
Qin and Gao (2017)	USAp-HLP	modeling flows as independent zigzag uncertain variables with regular uncertainty distributions	genetic algorithm
Talbi and Todosijević (2017)	UMAp-HLP	introducing the quantification of the robustness of a solution in the presence of uncertainties	neighborhood search
Zhalechian et al. (2017a)	CSAp-HLP	propose fuzzy programming to solve multi objective problem with uncertain parameters including capacity level, demands, transportation costs and times	evolutionary algorithm
Meraklı and Yaman (2016)	UMAp-HMP	Demand is represented with a special polyhedral uncertainty model known as the hose model	Benders decomposition
Zetina et al. (2016)	UMAHLP	proposing a robust model with uncertain demand and transportation cost and comparing the solutions of deterministic, stochastic and robust models	cutting plane algorithm
Zhai et al. (2016)	UMAHLP	modeling hybrid uncertain environments	genetic algorithm-variable neighborhood search
Dabidian et al. (2015)	CMAHLP	extending the problem with stochastic vehicle and hub capacities	simulated annealing
Yang and Liu (2015)	USAp-HCP	travel times are characterized by fuzzy random variables.	sample average approximation-tabu search with parametric decomposition
Hult et al. (2014)	USAp-HCP	modeling stochastic travel times on links and developing exact solution procedure	Benders decomposition

Table 2.2 (continued): Milestones of hub location literature in terms of uncertainty.

Authors	Model	Milestones	Solution Procedure
Shahabi and Unnikrishnan (2014)	UMA/SAHLP	demand uncertainty by means of an ellipsoidal uncertainty set	simulated annealing
Bashiri et al. (2013)	CSAp-HCP	incorporating capacitated models while qualitative parameters defined as fuzzy	genetic algorithm
Davari et al. (2013)	USAIHVP	incorporating incomplete network design with the fuzziness of exact location of demands	neighborhood search
Ghodratnama et al. (2013a)	CSAHVP	production facilities, time horizons and transporter vehicles are defined in fuzzy environment to solve multi-objective hub covering problem	
Yang et al. (2013b)	USAp-HCP	improved their study with a risk aversion formulation	particle swarm optimization
Yang et al. (2013a)	USAp-HCP	travel times are characterized by normal fuzzy vectors with jointed possibility distribution	genetic algorithm
Alumur et al. (2012c)	USA/MAHLP	modeling setup cost without a known probability and demands associated with stochasticity and proposing minimax regret model and two-stage stochastic program	
Zhai et al. (2012)	USAHLP	proving that two-stage stochastic programming model is equivalent to a single-stage minimum risk p-model	
Contreras et al. (2011a)	UMAHLP	proved that stochastic problems have equivalent deterministic versions while demand and transportation costs are defined as stochastically	Benders decomposition
Yang et al. (2011)	USAp-HCP	extended the literature by assuming discrete random travel time	B&B
Demirci (2010)	USAp-HCP	proposing meta-heuristic to solve the problem with probabilistic travel times	genetic algorithm
Mirakhorli (2010)	CSAHVSP	modeling fuzzy radius cover and travel times defined as trapezoidal fuzzy numbers	genetic algorithm
Chou (2010)		proposed a fuzzy multiple criteria decision-making model	
Sim et al. (2009)	USAp-HVP	modeling travel times associated with stochasticity as chance constraint	radial heuristic-teitz-bart heuristic
Yang (2009)	USAHLP	modeling demand associated with stochasticity and established a stochastic programming model	
Mahdi and Masoud (2008)	CSAp-HCP	studying in fuzzy environment for demand zone traffic and other parameters	
Sim (2007)	USAp-HCP	modeling uncertain travel times	simulated annealing-teitz-bart heuristic

An extended study about the uncertainty context of HLP is represented by Alumur et al. (2012c). The paper explicated uncertainty in the setup costs and stochastic demands under the different type of allocation strategies individually and mutually. Yang et al. (2013a) improved their study with a risk aversion formulation. Shahabi and Unnikrishnan (2014) modeled an ellipsoidal uncertainty set to represent demands. Dabidian et al. (2015) extended the problem with stochastic vehicle and hub capacities. Zetina et al. (2016) proposed a robust model with uncertain demand and transportation cost and compared the solutions of deterministic, stochastic and robust models.

Meraklı and Yaman (2016) proposed a special polyhedral uncertainty model to define demand parameters. Talbi and Todosijević (2017) introduced the quantification of the robustness of a solution in the presence of uncertainties. Martins de Sá et al. (2018a) modeled the traversing times on hub arcs, and on spokes. Correire et al. (2018) incorporated stochasticity with multi-period hub location problem where demand is modeled with scenario-based uncertainty.

2.9.2 Reliable HLP

In location science, reliability is defined as the ability of a network to perform successfully. Considering reliability under hub location modeling is investigated very recently. Hub location problem is closely associated with reliability due to consolidation of all flow at network through hubs. Any failure, malfunction or loss of hub in hub network effects unfavorably entire network's ability. Consistency of a hub or hub arc is crucial to maintain the operations in the network. It is important to state that since occurrence of any failure could not exactly identify as deterministic, uncertainty and reliability issues are engaged.

Karow (2003) introduced virtual hub concept where potential locations are determined to direct the flows in case of disruption. Grover & Tipper (2005) examined reliability function related to failures and other significant patterns of reliability. O'Kelly et al. (2006) designed probabilistic successful communication through a network to avoid congestion or loss of traffic between nodes. Kim H. (2008) introduced reliable hubs while designing hub network to assure equitable flow distribution and prevent congestion and degradation. Kim H. (2012) extended their work where alternative locations are utilized as backup hubs.

Table 2.3 : Milestones of hub location literature in terms of reliability.

Authors	Model	Milestones	Solution Procedure
Mokhtar et al. (2019b)	U2Ap-HMP	modeling survivable hub network	Benders decomposition
Rostami et al. (2018)	USAHLP	modeling failure of hubs with uncertainty	Benders decomposition
Chaharsooghi et al. (2017)	UMAHLP	modeling hub operations probabilistically failed under scenario	neighborhood search
Kim and Ryerson (2017)	USA/MAHLP	generic model of the ad-hoc hub location problems	
Musavi and Bozorgi-Amiri (2017)	USAp-HLP	optimizing network in three dimensions of costs, responsiveness and environmental	non-dominated sorting genetic algorithm
Azizi et al. (2016)	USAp-HMP	introducing backup hubs and alternate routes in case of disruption	genetic algorithm
An et al. (2015)	USAp-HMP	modeling robust hub failure rates while backup hubs and alternate routes maintain service	B&B
Bashiri and Rezanezhad (2015)	CSAp-HVP	capabilities inside of hub facilities	non-dominated sorting genetic algorithm
O'Kelly (2015)		introducing resilience and vulnerability concept in survivable hub network	
Sadeghi et al. (2015)	USAp-HVP	modeling disruption with link capacities satisfied in a Truncated Erlang distribution function	differential evolution
Yıldız and Karaşan (2015)	UMAHVP	introducing revised regenerator location problem as hub covering problem	branch and cut
Eghbali et al. (2014)	USAp-HVP	hub covering formulation with reliability	non-dominated sorting genetic algorithm

Table 2.3 (continued): Milestones of hub location literature in terms of reliability.

Authors	Model	Milestones	Solution Procedure
Hamidi et al. (2014)	USAHLP	modeling preventive reliability by operating real and fake hubs	
Parvaresh et al. (2014)	USAp-HLP	incorporating multi objective modeling with reliability	branch&reduce and simulated annealing
Lei (2013)	UMAp-HMP	designing network in case of breakdown by identifying which hubs lead most severe degradation in the network	simulated annealing-tabu search
Kim (2012)	UMAp-HMP	designing network where backup hubs are distinct from the original hub and only used in case an original hub fails.	
Davari et al. (2010)	USAp-HMP	designing reliable hub networks using fuzzy goal programming	simulated annealing
Kim and O'Kelly (2009)	USA/MAp-HLP	new formulations for reliability factor which the probability is associated with inter-hub flows transmitted in a given time without failure	simulated annealing
Kim (2008)	USA/MAp-HLP	defining reliable hubs while designing hub network to assure equitable flow distribution and prevent congestion and degradation	
O'Kelly et al. (2006)		modeling probabilistic successful communication to deliver traffic without congestion or loss between O/D pairs	exhaustive search-tabu search
Grover and Tipper (2005)		examining reliability function related to failures and various patterns in the reliability	
Karow (2003)	CSAHLP	introducing virtual hub concept	

Hamidi et al. (2014) investigated preventive reliability by operating real and fake hubs. Yıldız & Karaşan (2015) modeled a revised regenerator location problem as hub covering problem. O'Kelly M. E. (2015) introduced resilience and vulnerability concept in survivable hub network. An et al. (2015) presented robust hub failure rates while backup hubs and alternate routes maintain service quality. Mokhtar et al. (2019b) presented improved models to design survivable hub network.

2.9.3 Topological network structure of HLP

In classical hub location problem, the models assume that every hub are fully interconnected. However, configuration of network structure may become more complicated than that simplified assumption, especially in some application such as telecommunication or public transportation. There exist few studies (O'Kelly and Miller, 1994; Klincewicz, 1998; Nickel et al., 2001) considering how a hub network should be designed until 2005. Campbell et al. (2005a, b) introduced hub arc location problem which allow flexible structure of backbone network. After then, rapidly expanding studies have introduced new models that provide alternatives for network topology. Incomplete, Hierarchical, Tree, Star, Line and Cycle hub location problems are some of the examples. Hub location problems take advantage of these structure related to associated purpose of design. Rapid transit systems have a unique pattern that the path consists of multiple stops through a route. Thus, hubs may be connected by means of a line to design such a structure. To deal with the routing decisions in logistic, the design of hub network structure may resemble a cycle. Star hub problems arises in designing service quality considerations where hubs are assigned to central hub and other nodes connected to hubs.

Hub arc location researches incorporated setup costs for the hub nodes and hubs arcs (Contreras and Fernández, 2014; Gelareh et al., 2015). Moreover, some studies presented hub arc location problems that impose topological structures, such as tree–star (Contreras et al., 2010), star–star (Labbé and Yaman, 2008), ring–star (Contreras et al., 2016), and hub lines (Martins de Sá et al., 2015a; 2015b).

Campbell (2009) introduced isolated hubs for time definite models. This study showed that adding hubs and hub arcs could increase the service level. Also, isolated hubs that are not adjacent to a hub arc could improve the service level with more economical costs than adding associated hubs.

Table 2.4 : Milestones of hub location literature in terms of topology.

	Authors	Model	Solution Procedure	Milestones	Direct
ARC	Alibeyg et al. (2017)	UMAHVMP	B&B	developing exact solution procedure to solve their previous formulation	Non
	Alibeyg et al. (2016)	UMAHVMP	B&B	introducing profit oriented objective function for hub arc location problem	Non
	Contreras and Fernández (2014)	USAp-Hq-AL	Branch and cut - greedy heuristic	proposing mew formulation for hub arc location problem as the minimization of a supermodular function.	Non
	Alumur et al. (2012b)	USAp-HLP	Valid inequalities	incorporating multi modal network design with arc location problems	Non
	Campbell (2009)	UMAp-HAL		elimination of potential path due to service level time restriction	Non
	Sasaki et al. (2009)	UMAHAL	Enumeration based	incorporating competition with arc location problems	Non
	Campbell et al. (2005a)	UMAHAL		presenting four special cases of the general multiple allocation hub arc location model	Yes
	Campbell et al. (2005b)	USAHAL	Enumeration based	proposing hub arc location problem where a fixed number of hub arcs is located while minimizing the total transportation costs	Yes
	Campbell et al. (2003)	UMAHAL	Enumeration based	introducing hub arcs which seeks to locate a specified number of discounted arcs for complete networks	Non
CLUSTER	Mirabi and Seddighi (2017)	CMAHLP	Ant colony-simulated annealing	modeling network with predetermined cluster areas to select hubs	Yes
	Wagner (2007)	USAp-HMP	genetic algorithm-tabu search	network design by clustering of nodes and selecting of hubs within sub-groups	Yes
	Sung and Jin (2001)	USAHLP	Dual-based	network design by clustering of nodes and selecting of hubs within cluster	Yes
CYCLE	Contreras et al. (2016)	USAp-HLP	Branch and cut-GRASP	proposing exact and heuristic approaches	Non
	Contreras et al. (2015)	USAp-HLP	Branch and cut	proposing exact solution procedure	Non
	Contreras et al. (2013)	USAHLP	Branch and cut-GRASP	designing network while hubs are connected by means of cycle	Non

Table 2.4 (continued): Milestones of hub location literature in terms of topology.

	Authors	Model	Solution Procedure	Milestones	Direct
HIERARCHICAL	Real et al. (2018)	USAIHLP	Benders decomposition	proposing three-layer network design for different geographic regions of air transportation while routing flows within the network at minimum cost	Non
	Dukkanci and Kara (2017)	USAp-HVP	lagrangean relaxation-subgradient optimization	designing three-layer network as ring-star-star network with multimodal (trucks and airplane) options with time restrictions	Non
	Rabbani et al. (2017a)	USAHLP	differential evolution-genetic algorithm	modeling three-layer network as ring-star-star network while speed, bandwidth etc. are considered in modelling	Non
	Alumur et al. (2012a)	USAp-HVP	valid inequalities	designing two-level network as star-incomplete-star network with multimodal options and time-definite deliveries	Non
	Fontes and Goncalves (2015)	UMAIHLP		modeling hierarchical network where alternative paths for flows	Non
	Yaman and Elloumi (2012)	USAp-HCP/HMP	valid inequalities	presenting two formulation for median and center problem in two-level network as star network with time bounds	Non
	Davari and Zarandi (2012)	USAp-HMP	local search algorithm	incorporating fuzziness of flow with hierarchical hub location	Non
	Lin (2010)	UMAHLP	enumeration based	designing 3-layer hub network design with intermodal transportation and time restrictions which provide a secondary route for node-hub connections.	Non
	Yaman (2009)	USAp-HMP	valid inequalities	designing three-level network as star-complete-star network with time-definite restriction	Non
	Thomadsen and Larsen (2007)	UMAHLP	branch and price	designing two-layered network which consists of clusters of nodes defining an access network and a backbone network.	Non
	Lin and Chen (2004)	UMAHLP	enumeration based	designing clustered layers for network to determine the fleet sizes and schedules while ensuring a given service level	Non
LINE	Martins de Sá et al. (2015a)	UMAIp-HLP	Benders decomposition	minimizing total weighted travel time	Yes
	Martins de Sá et al. (2015b)	UMAIp-HLP	Benders decomposition-GRASP	extending previous work multiple lines in network where the number of the hubs in each line is a decision variable	Non

Table 2.4 (continued): Milestones of hub location literature in terms of topology.

	Authors	Model	Solution Procedure	Milestones	Direct
INCOMPLETE	Dai et al. (2019)	USAIp-HLP	neighborhood search	proposing a heuristic based on estimation of node pair quality for establishing hub links	Yes
	Martins de Sá et al. (2018a)	UMAIHLP	Benders decomposition	modeling robust incomplete hub location problem with uncertain service times	Non
	Martins de Sá et al. (2018b)	UMAIHLP	Benders decomposition-neighborhood search	modeling problem with uncertainties on travel demands and fixed setup cost, but without direct/access link cost	Yes
	Akgün and Tansel (2018)	UMAp-HMP	decomposition-based algorithms	incomplete network design with flow-independent costs on the hub and access arcs and different discount rates for each type of movement	Yes
	Camargo et al. (2017)	UMAIHLP	Benders decomposition	incomplete network design with the restriction of the number of hops (i.e. stops) in a path to handle service levels	Yes
	Campbell et al. (2015)	UMAIHLP	Benders decomposition	proposing fixed and variable arc cost where hub links can also be constrained by the fixed cost for establishing hub links.	Yes
	O'Kelly et al. (2015a)	UMAp-HLP	LP relaxation	new 3-index formulation while modeling fixed and variable arc costs for incomplete hub backbone system	Yes
	Karimi and Setak (2014)	UMAIHLP	lagrangean relaxation	incorporating routing policies with incomplete network design	Non
	Davari et al. (2013)	USAIHVP	neighborhood search	incorporating fuzziness with incomplete network design	Non
	Lüer-Villagra and Marianov (2013)	UMAHLP	genetic algorithm	incorporating competition with incomplete network design	Non
	Gelareh and Nickel (2011)		Benders decomposition-neighborhood search	proposing 4-index formulation for incomplete modeling	Yes
	Campbell (2010)	UMAHAL		introducing isolated hubs with time definite restrictions	Non
	Alumur and Kara (2009)	USAHVP		allowing at most three hub stops on a route from any origin to destination	Non
	Alumur et al. (2009)	USAIpHMP/HLP/HVP/HCP	valid inequalities	proposing formulations for general classes of single allocation hub location problems with incomplete hub networks where q-hub links connection is established	Non
	Calık et al. (2009)	USAIHVP	tabu search	incomplete network with service time bound constraint	Non

Table 2.4 (continued): Milestones of hub location literature in terms of topology.

	Authors	Model	Solution Procedure	Milestones	Direct
INCOMPLETE	Gelareh (2008)	UMAHLP	Benders decomposition-greedy heuristic	incorporating multi-period modeling with incomplete hub network	Non
	Rodriguez-Martin and Salazar-Gonzalez (2008)	CMAHLP	branch and cut-neighborhood search	incomplete network design where hub to hub arcs are not assumed as completely connected	Non
	Yaman et al. (2007)	USAp-HCP	Branch and cut-tabu search	incomplete network design where multiple nodes may allocate by hubs by some stopover	Yes
	Nickel et al. (2001)	UMAHLP	B&B	introducing fixed cost for locating hub arcs while hub network is completely interconnected.	Yes
	Klincewicz (1998)			examining distinct topology alternatives including network relaxing fully connected assumption and fixed cost for hub arcs	
	O'Kelly and Miller (1994)			presenting eight topological protocols for hub networks including incomplete one	Yes
STAR	Chen et al. (2018)	USAp-HCP	approximation algorithm	developing approximation algorithm on star network	Non
	Labbé and Yaman (2008)	USAHLP	branch and cut	modeling problem as star-star network design	Non
	Yaman (2008)	CSAp-HMP	lagrangean relaxation	strengthening star network formulation and proposing heuristic algorithms	Non
TREE	Blanco and Marín (2019)	UMAp-HLP	valid inequalities	introducing tree hub location problem with upgrading where a pre-specified number of hubs are upgraded with a reduction on the cost	Yes
	Moghaddam and Sedehzadeh (2014)	CSAIHLP	imperialism competitive algorithm	incorporating multimodal concept with tree hub location problem	Non
	Martins de Sá et al. (2013)	USAHLP	Benders decomposition	designing a branch-and-Benders-cut strategy to solve	Non
	Contreras et al. (2010b)	USAp-HMP	branch and cut	improving their model by reducing variables and constraints of previous models.	Non
	Contreras et al. (2009b)	USAp-HLP	lagrangean relaxation-subgradient optimization	modeling four index p-median formulation by tightening models with tree network	Non

Table 2.4 (continued): Milestones of hub location literature in terms of topology.

	Authors	Model	Solution Procedure	Milestones	Direct
ROUTING	Dukkancı et al. (2019)	USAp-HVP		incorporating multi-objective approach while routing the deliveries within a predetermined service time limit	Non
	Taherkhani and Alumur (2019)	USA/MA/rHLP		considering distinct allocation strategies on hub location routing problem while maximizing total profit	Yes
	Boccia et al. (2018)	CSAp-HLP	branch and cut	extending the problem to the multi commodity flow case	Non
	Karimi (2018)	CSAHVP	valid inequalities-tabu-search	considering the service time constraint for any path in the location and routing decisions	Non
	Kartal et al. (2017)	USAp-HMP	ant colony-simulated annealing	incorporating hub location routing with p-median formulation	Yes
	Bayraktar (2016)	USAIp-HLP	Benders decomposition-Iterative Clustering-Routing Heuristic	introducing stronger formulation while incorporating the cost of the routing in the local tours as a function of both distances traversed and flow carried.	Yes
	Bostel et al. (2016)	CSAIHLP	memetic algorithm	extending the problem by the limitation of the capacity of hubs and the number of nodes consisting local tours	Non
	Zameni and Razmi (2015)	USAp-HMP	genetic algorithm	incorporating multimodal with hub location routing problem where transportation cost was formulated by a vehicle routing problem between nodes of each hub	Non
	Rodríguez-Martín et al. (2014)	USAp-HMP	branch and cut	modeling hub location routing problem with an assignment of single vehicle to route allocation of non-hub nodes	Non
	Camargo et al. (2013)	USAHLP	Benders decomposition	revised formulation for hub location routing problem with fixed cost of hub establishing, vehicle assignment and non-hub allocations to develop exact solution procedure	Non
	Rieck et al. (2013)	CSAHLP	genetic algorithm	modeling hub location routing problem in a 3-layered network	Non
	Catanzaro et al. (2011)	UMAHLP	branch and cut	improving the problem incorporating graph partitioning and routing features where network is partitioned into sub-networks, locating at least one hub in each sub-network	Yes
	Çetiner et al. (2010)	UMAp-HMP	iterative heuristic	introducing hub location routing problem determining how to construct a routing system through the allocations to hub while satisfying the desired service level.	Non
	Aykin (1995b)	UMAp-HMP	enumeration based heuristic	introducing hub location routing problems	Yes

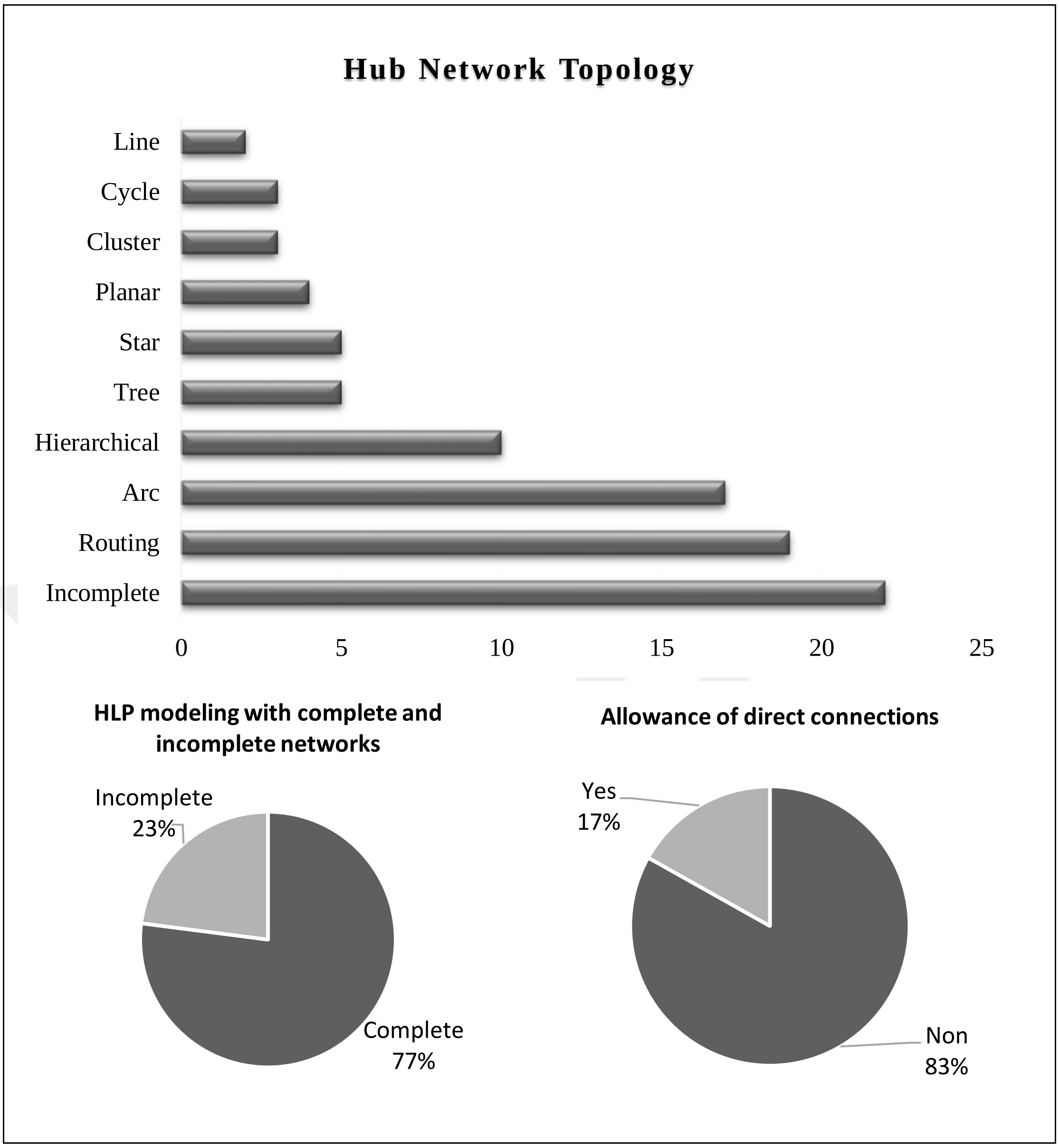


Figure 2.19 : Graphics of network topology in the literature.

Yaman (2007) introduced incomplete hub networks where access network imposes no condition on allocation pattern of nodes to hubs. Some earlier studies addressed incomplete networks by allowing direct connection between non-hub nodes (Aykin, 1994, 1995; Sung and Jin, 2001).

Hierarchical network design, nodes and their associated hubs are categorized by the level of the services with layers or clusters where the difference between service level may arise from flow patterns, service availability, spatial configuration and objectives. Cargo delivery is common application to design hierarchical hub networks. Kim and Tcha (1992) presented two-layered tree-star hub network structure. Lin and Chen (2004) designed clustered layers for network to determine the fleet sizes and schedules while ensuring a given service level. Thomadsen and Larsen (2007) examined two-layered network whereas Yaman (2009) studied three-layered star network to minimize the link installation cost.

Tree hub location is one of the examples where p hubs should be located on a network and connected by the means of a tree. Star hub location consists of a central hub connecting remaining nodes directly to themselves. Cycle hub location provides network designs where all hubs are connected by means of a cycle and all the flow between nodes should be circulated with these connections.

Hub location routing problems aim to locate hubs and determine the service types and the routes between nodes simultaneously. Network design of routing problems show similarity with cycle ones in terms of topology. However, employment of cost parameters is more complicated in routing problems.

2.9.4 HLP with mode of transportation

While modeling the mode of transportation, an additive decision should be considered in addition to classical HLP decisions: Which transporters could utilize to ensure the connection through network and how the network should be designed with different possible transportation modes?

The aim of HLP improved with the perspective of transportation modes to decide on the location of hubs, allocation of the non-hub nodes to hubs and which type of transportation to utilize on node/hub pairs. The choice of the transportation modes is named as hub location decisions in inter or multi modal network. If multiple transportation modes are assigned through a path/route, that is referred as multi-modal transportation. If only single transportation mode is assigned to the routes of network, then it is referred as intermodal transportation.

O'Kelly and Lao (1991) introduces intermodal transportation where hub location problem utilizes air and ground transportation on the connections of network. Racunica and Wynter (2005) investigated increasing share of rail in intermodal transport using hub networks for freight rail. Ishfaq and Sox (2011) reflected distinct features of each mode of transportation in terms of costs. The transportation costs, modal connectivity costs and fixed costs are presented under service time requirements.

Table 2.5 : Milestones of hub location literature in terms of modular network.

Authors	Model	Milestones	Solution Procedure
Mokhtar et al. (2019a)	UMAp-HMP	extending the problem by incorporating three transfer modes and two kinds of hubs	
Rothenbächer et al. (2016)	UMAHAL	considering multiple transshipments of requests at hubs, transport time limits for requests, request splitting, and outsourcing possibilities.	
Masaeli et al. (2018)	USAHLP	network design depending on whether holding costs are incorporated and whether the vehicles are of different types	
Dukkanci and Kara (2017)	USAp-HVP	multimodal network with trucks and airplane options with time parameters	lagrangean relaxation-subgradient optimization
Zhalechian et al. (2017b)	CSAp-HLP	incorporating possibilistic-stochastic uncertainty with intermodal network	differential evolution-hybrid imperialist competitive algorithm
Rahimi et al. (2016)	CN(C)SAp-HMP	modeling different transportation modes utilized at each hub-hub link	differential evolution-simulated annealing
Serper and Alumur (2016)	CSAHLP	alternative transportation modes and different types of vehicles where economies of scale are modeled to reflect the trade-off between the capacities of different types of vehicles, and the purchasing, operational and transportation costs	neighborhood search
Aygün et al. (2015)		different transportation cost associated to transportation mode (air and ground)	
He et al. (2015)	CSAHLP	developing heuristic combining branch&bound, lagrangian relaxation and linear programming relaxation	lagrangean relaxation
Maheo et al. (2017)	UMAHAL	different transportation costs for alternative transportation modes for public transportation	Benders decomposition
Sedehzadeh et al. (2015)	CSAp-HVP	modeling transportation time which depends on transportation mode	simulated annealing
Zameni and Razmi (2015)	USAp-HMP	incorporating routing policies with multi-modal modeling	genetic algorithm
Moghaddam and Sedehzadeh (2014)	CSAIHLP	modeling all parameters varied according to transportation modes	imperialism competitive algorithm

Table 2.5 (continued): Milestones of hub location literature in terms of modular network.

Authors	Model	Milestones	Solution Procedure
Mohammadi et al. (2013)	CSAp-HVP	designing network with the limitation of the number of the transporters depending on transportation mode	imperialism competitive algorithm
Oktal and Özger (2013)	UMAHLP	modeling transportation mode as aircraft range and associated trip cost with runway availability and cargo traffic continuity	
Özger and Oktal (2013)	USAp-HMP	distance and capacity restrictions for vehicle selection at network	
Alumur et al. (2012b)	USAp-HLP	designing the graph model partitioned into two subgraphs, each for one mode of transfer, and possibly a set of arcs to connect the two subgraphs	valid inequalities
Alumur et al. (2012a)	USAp-HVP	relaxing full interconnection assumption by modeling multi-modal hub arc network using different transportation mode with associated costs for two level networks under time definite restrictions	valid inequalities
Ishfaq and Sox (2012)	UMAp-HMP	incorporating delay at hubs with intermodal models	B&B-tabu search
Gelareh and Pisinger (2011)	UMAHLP	modeling fleet deployment between paths with associated capacities and arrival frequencies	Benders decomposition
Ishfaq and Sox (2011)	UMAp-HMP	applying different costs of carriers for different transportation mode choice	lagrangean relaxation-tabu search
Meng and Wang (2011)	CSAHLP	incorporating multi-type containers with multiple stakeholders for intermodal transportation	genetic algorithm
Ishfaq and Sox (2010)	UMAp-HLP	modeling rail/road intermodal network under service level constraints	tabu search
Groothedde et al. (2005)		developing a multi-modal network using a combination of transportation choices	
Racunica and Wynter (2005)	UMAHLP	increasing utilization of rail system in intermodal transportation	variable reduction
Arnold et al. (2004)	USAHLP	locating rail/road terminals for freight transportation	
O'Kelly and Lao (1991)	USA2-HLP	allocation strategies using air and ground transportation	

Another study by Ishfaq and Sox (2012) is investigated the utilization of separate cost structures for related transportation modes and service performance influenced by hub delays. Oktay and Özgel (2013) considered distinct range of aircraft as mode of transportation where allocation decisions are affected from runway availability and traffic continuity.

Gelareh and Pisinger modeled the fleet deployment between paths with associated capacities and arrival frequencies. Alumur et al. (2012a) presented a review on mode of transportation They investigated incomplete hub location problem. Their model aims to select appropriate transportation modes on hub links. They proposed distinct transportation costs and service times associated with different modes of transportation. Alumur et al. (2012b) is also studied hierarchical multimodal HLP with time definite deliveries.

Mohammadi et al. (2013) designing network with the limitation of the number of the transporters depending on transportation mode. Maheo et al. (2015) and Aygün et al. (2015) applied different transportation costs for different mode of transportations. Serper and Alumur (2016) proposed alternative transportation modes and different types of vehicles where economies of scale are modeled to reflect the trade-off between the capacities of different types of vehicles, and the purchasing, operational and transportation costs. Zhalechian et al. (2017b) incorporated possibilistic-stochastic uncertainty with intermodal network. Dükkancı and Kara (2017) designed multimodal network with trucks and airplane options with time parameters. Mokhtar et al. (2019a) extended the multimodal hub location problem by incorporating three transfer modes and two kinds of hubs

2.9.5 Dynamic HLP

In classical hub location problems, parameters of network are assumed as constant. However, through a time horizon these parameters such as demand, cost, capacity or processing time are varied. In most of the application area, demand fluctuations are obviously realized. According to competition, economic and governmental enforcements cause variations of cost. Thus, a model ignoring these changes may misguide the solution of the problem. Representing the dynamic nature of the problem may become essential to enrich the problem literature. Gelareh (2008) introduced multi-period hub location problem for designing public transportation.

Table 2.6 : Milestones of hub location literature in terms of dynamic modeling.

Authors	Model	Milestones	Solution Procedure
Correire et al. (2018)	CMAHLP	incorporating stochasticity on demands with dynamic modeling	valid inequalities
Masaeli et al. (2018)	USAHLP	modeling shipment scheduling where delivered shipments and economies of scale decisions are planned with network design during time horizon	
Bashiri et al. (2017)	USAHLP	modeling mobile facilities relocatable during time	genetic algorithm
Alumur et al. (2015)	CSA/MAHLP	introducing allocation of non-hub nodes, modular capacities, additive hubs or gradually expanding capacities and associated cost varied in time	B&B
Ebrahimi-zadea et al. (2015)	USAp-HVSP	modeling covering radius varied in time	genetic algorithm
Gelareh et al. (2015)	UMAp-HLP	incorporates several features of maritime and land transport practices such as budget constraints and periodical fluctuations in demand and costs	Benders decomposition-neighborhood search
Hörhammer (2014)	CSAHLP	modeling operating, opening, closing, and up/downsizing of hubs while demands and fixed costs are varied during the planning horizon	
Ghodratnama et al. (2013a)	CSAHVP	incorporating fuzziness and multi objective modeling	
Taghipourian et al. (2012)	CSAHLP	incorporating fuzziness on demands with dynamic modeling	
Teymourian et al. (2011)	CMAHLP	modeling capacity decisions with opening virtual hub due to weather conditions during time horizon	simulated annealing
Campbell (2010)	UMAHAL	introducing isolated hub to meet network requirements changed in time such as demand or geographical expansion	
Contreras et al. (2009c)	UMAHLP	modeling network with the decisions of opening/closing hubs while demands are varied in different time periods	lagrange relaxation
Gelareh (2008)	UMAIHLP	modeling network by opening/closing hubs or hub arcs where associated costs are varied in time	Benders decomposition-greedy heuristic
Campbell (1990)	USA/MAHLP	O/D points are scattered randomly over the service region	

Table 2.7 : Milestones of hub location literature in terms of competition.

Authors	Model	Milestones	Solution Procedure
Ghaffarinasab et al. (2018a)	USA/MAp-HMP/HCP	modeling allocation strategies in a competitive network	simulated annealing
Abbasi and Niknam (2017)	UMAHLP	modeling customer preference based on only price with logit function	genetic algorithm-simulated annealing
Neamatian Monemi et al. (2017)	CMAHLP	modeling cooperation and competition for a duopoly market with evaluation of travel time and fuel consumption and travel fare	lagrangean relaxation-local search algorithm
Mahmutoğulları and Kara (2016)	UMAp-HMP/HCP	new formulations such as medainoid and centeroid to model each competitor in a duopoly market	enumeration based
Sasaki et al. (2014)	UMAHAL	new formulation of competition with two firm under hub arc problem	enumeration based
Asgari et al. (2013)	CMAHLP	modeling both competition and cooperation	B&B
Lüer-Villagra and Marianov (2013)	UMAHLP	modeling new-entrant competition with a logit function to define customer preferences	genetic algorithm
Redondi et al. (2011)		measuring the competition between airport hubs based on an analysis of travel times	
Gelareh et al. (2010)	UMAp-HLP	modeling stepwise attraction function	lagrangean relaxation-subgradient optimization
Eiselt and Marianov (2009)	UMAHLP	extending their model with customer preference based on time and cost defined by gravity like utility function	
Lin and Lee (2010)	CSAp-HMP	modeling integral constrained non-cooperative game theoretic model for time-definite network in an oligopolistic market	lagrangean relaxation
Sasaki et al. (2009)	UMAHAL	designing hub network with the relaxation of fully interconnection assumption	enumeration based
Adler and Smilowitz (2007)	USAp-HMP	developing a game theoretic approach to design network with potential mergers and alliances	
Sasaki (2005)	UMAp-HMP	Introducing a restriction where captured market share should exceed a threshold limit	greedy heuristic
Martin and Román (2004)		extended their previous model to highlight the dynamical nature of the problem	
Martin and Román (2003)		analyzing problem through a spatial competition game	
Sasaki and Fukushima (2001)	UMAHLP	modeling competition between leader and medium firms and formulating non-linear logit function to determine captured demand	
Marianov et al. (1999)	UMAp-HLP	introducing competition with distinct objectives of leader and follower	tabu search-teitz-bart heuristic
Skorin-Kapov (1998)	USAp-HLP	developing a cooperative game theory approach to exploit economies of scale	

Campbell (2010) proposed implementation of isolated hubs in network to meet demands and locational expansion varied in time. Hörhammer (2014) presented the model altering network design with new hubs or closed hubs and their varied capacities in time. Alumur et al. (2015) extended the problem similarly where the capacities of hubs are gradually changed. Correire et al. (2018) incorporated dynamic modeling with demand stochasticity.

2.9.6 HLP with competition

In general, hub location problem assumed that only single possessor should manage all flows and all possible nodes to design network. However, most of the hub network application such as cargo delivery, passenger transportation comprises highly intense competition environment. Rivals compete to each other to design time sensitive, profit-driven hub, centralized or expanded hub networks. Competition issues have been investigated antecedently.

Distinct cases reflecting different competition or cooperation conditions such as duopoly, leader-follower companies, etc. modeled as hub location problem. Skorin-Kapov (1998) introduced hub location problem with a cooperative game theory where the users provide each other economic advantage by their economies of scale factor associated with flows. Marianov et al. (1999) introduced competition on hub location problem where competitors try to maximize their own demand covered.

Recent studies proposed different formulation on competition: non-linear logit utility function (Sasaki and Fukushima, 2001), logit utility function (Lüer-Villagra and Marianov, 2013; Abbasi and Niknam, 2017), stepwise attraction function (Gelareh et al., 2010), gravity like utility function (Eiselt and Marianov, 2009). Sasaki et al. (2009) presented Stackelberg hub location problem where the rivals design the network subsequently to each other. Their model also relaxed the fully interconnected hub assumption. Sasaki et al. (2014) revised their study with modeling hub arc location problem. Mahmutoğulları and Kara (2016) presented novel approach to competitive hub location problem in a duopoly where competitors prefer different modeling of network as medianoid and centeroid.

2.9.7 HLP with economies of scale

Another much debated issue of hub location problem is the assumption about discount factor on inter-hub links. This flow-independent discount integrated with the fully interconnected hub network design is projected to concentrate the flows on hubs. By the way, this consolidation provides cost advantage on increasing amount of flows. On the other hand, ignoring amount of flows between hub links may simplify the modeling of the problem. However, solutions discussed in the literature state that this flow independent discount factor may be applied to hubs accounting small amounts of flows whereas there exist other nodes, especially access arcs, with much larger amount of flows. Also, this assumption leads the problem to locate hubs incorrectly and decide consequential misallocation of non-hub nodes. All time or cost oriented objectives may be affected by this inaccurate network design.

From the earlier hub location researches, the economies of scale issue have been considered with distinct approaches. O'Kelly et al. (1996) presented a prior study by analyzing the sensitivity of the solutions related to economies of scale. Jaillet et al. (1996) proposed vehicle-based cost function to determine the cost of flows between hubs. O'Kelly and Bryan (1998) introduced first flow-dependent cost modeling. Bryan (1998) formulated flow-dependent discount as a piece-wise linear function where the cost function decreases nonlinearly and inversely as the amount of the flow increases. Horner and O'Kelly (2001) formulated a non-linear concave cost function to describe flow-dependent cost for entire connections in the network. Racunica and Wynter (2005) modeled same cost function only for spoke and inter-hub connections. Kimms (2006) proposed a combination of a fixed charge cost for hub arcs and variable flow-dependent cost for links.

While Campbell et al. (2005) relaxed the fully interconnected hub network assumption, they also provided a novel formulation of discount factor of flow-cost as fixed hub arc cost to establish. Alumur et al. (2009) described the problem with fixed cost for both hubs and hub arcs, except spoke arcs.

Table 2.8 : Milestones of hub location literature in terms of economies of scale.

Authors	Model	Milestones for Economies of Scale	Solution Procedure
Fard et al. (2019)	USAp-HMP	transportation cost is a stepwise function of the number of vehicles.	Probabilistic analysis
Lüer-Villagra et al. (2019)	USAp-HLP	cost of the flow on an arc modeled as piecewise-linear function	Genetic algorithms
Alkaabneh et al. (2019)	USAHLP	modeling the economies-of-scale as a concave piece-wise linear function	Lagrangean relaxation-GRASP
Akgün and Tansel (2018)	UMAp-HMP	flow- independent costs on the hub and access arcs	Decomposition-based algorithms
Camargo et al. (2017)	IHLP	modeling distinct fixed and variable cost for arcs or spokes for incomplete hub location problem	Benders decomposition
Rabbani et al. (2017b)	USAp-HCP	flow dependent costs	Genetic algorithm
Taner &Kara (2016)	USAp-HMP	revising cost function according to forecast of flow intensities considering hubbing effects	
Meier et al. (2016)	USAp-HMP	economies of scale with Euclidian distance and quadratic cost function	Row generation
O'Kelly et al. (2015a)	UMAp-HLP	modeling distinct fixed cost independent of the flows and variable cost dependent of the flows for arcs	LP relaxation
O'Kelly et al. (2015b)	UMAHLP	investigating price sensitive flow demands	Benders decomposition
Campbell J. F. (2013b)		comparing models with flow-dependent discounts, vehicle-based models, and modular cost models	
Mirzaghafour (2013)	CSA/MAHLP	step-wise cost function to model the flow dependency of transportation costs at the links	
Gelareh S. (2010)	UMAp-HMP	variable discount factor for economies of scale	
Ishfaq and Sox (2010)	UMAp-HLP	Introducing piece-wise linear cost function	Tabu search
Camargo et al. (2009b)	UMAHLP	new and tighter formulation of O'Kelly and Bryan (1998)	Benders decomposition

Table 2.8 (continued): Milestones of hub location literature in terms of economies of scale.

Authors	Model	Milestones for Economies of Scale	Solution Procedure
Alumur et al. (2009)	USAIHLP	modeling economies of scale with fixed costs for hubs and for hub arcs, but not for the spokes	valid inequalities
Watanabe et al. (2008)		economies of scale using non-linear cost function of distances and demands	
Cunha & Silva (2007)	USAHLP	variable discount factors of inter-hub links which varies according to the amount of flow	Genetic algorithm
Kimms (2006)	U SA/MA HLP/p-HMP	comparing models with fixed link usage cost and flow dependent discounts	
Campbell et al. (2005a) Campbell et al. (2005b)	USAHAL	locating the discounted hub arcs	enumeration based heuristic
Racunica & Wynter (2005)	UMAHLP	non-linear concave cost function for spokes and inter-hub links	Variable reduction
Klincewicz J. G. (2002)	UMAp-HMP	effective solution for models with flow dependent discounts	Tabu search-GRASP
Horner & O'Kelly (2001)	UMAHLP	discount factors of inter-hub links and other links depending on the amount of flow using a non-linear concave cost function	
Bryan D. (1998)	UMAp-HMP	modeling flow-dependent discounts for inter-hub links as linear piece-wise function to mimic nonlinear decreasing function with increasing flows for spokes and inter-hub links	
O'Kelly (1998)		investigating air passenger transportation under flow dependent discounts	
O'Kelly & Bryan (1998)		modeling flow-dependent discounts for inter-hub links	
Jaillet et al. (1996)	CSA/MAHLP	vehicle based cost function	valid inequalities
O’Kelly et al. (1996)	USA/MAp-HMP	prior study of the concept: inter-hub discount factor	

Subsequent works extended the economies of scale issue with different cost formulations as piece-wise function (Ishfaq and Sox, 2010; Alkaabneh et al, 2019), stepwise function (Mirzaghafour, 2013; Fard et al., 2019), variable cost function (Gelareh, 2010; O’Kelly et al., 2015a; Camargo et al., 2017) and vehicle-based functions. (Campbell, 2013b).

O’Kelly et al. (2015b) investigated price sensitive flow demands. Meier et al. (2016) formulated economies of scale as quadratic cost function of Euclidean distance. Taner and Kara (2016) revised cost function according to flow intensities rather than the amount to consider hubbing effects. Akgün and Tansel (2018) introduced classical economies of scale discount factor on access arcs.

2.9.8 HLP with congestion

Classification of facilities as capacitated or uncapacitated is applied to hub location problem likewise other location problems. Although considering networks without capacity restriction as uncapacitated hubs/hub arcs, makes the problem less complicated to solve, it is not accepted as appropriate and realistic approach. However, especially for deterministic hub location network, limiting capacities of the network to a specific and pre-determined value leads the solution of the problem to a false prediction of network design. Thus, it could be identified as a prejudge of the network capabilities.

Since both classes of uncapacitated and capacitated hub location problem may become insufficient to design hub network, it is required to develop another way of thinking. First, a network ignoring capacity restrictions cause the problem to be solved based on distances between pairs or density of flows. Capacity restrictions could not project the changes in demands, the variability of time parameters, potential bottleneck, peak times or failures over network and geographical expansion of the network. Finally, as mentioned before, dynamic nature of hub network and pre-determined capacity restrictions are controversial. Thus, controlling the capacity of the network or balancing the distribution of flows according to capacities may become more appropriate rather than limiting the capacity to design optimal network.

Table 2.9 : Milestones of hub location literature in terms of congestion.

Authors	Model	Milestones for Congestion	Solution Procedure
Özgün-Kibiroğlu et al. (2019)	UMAHLP	Formulating congestion cost which is not limited to capacity level of hub	Particle swarm optimization
Lüer-Villagra et al. (2019)	USAp-HLP	Modeling the cost of the flow on any arc as a piecewise-linear function	Genetic algorithms
Alkaabneh et al. (2019)	USAHLP	Modeling the economies-of-scale as a concave piece-wise linear function and congestion as a convex function	Lagrangean relaxation-GRASP
Alumur et al. (2018)	CMA/SAHLP	Modeling congestion using piecewise-linear function with varied capacity and service levels	
Kian and Kargar (2016)	USAp-HLP	Modeling nonlinear congestion cost.	
Rahimi, et al. (2016)	CSAp-HMP	Modeling congestion via a M/M/c-queuing system for hub covering problem	Differential evolution-simulated annealing
Yang and Chiu (2016)	CMAHLP	Investigating congestion under stochastic demand	
Wu and Wang (2014)	UMApHMP	Proposing heuristic to solve convex congestion cost function	Particle swarm optimization
Camargo and Miranda (2012)	USAHLP	Investigating congestion which is defined as power law function under competition	Benders decomposition
Ishfaq and Sox (2012)	UMAp-HMP	Defining congestion as a G/G/1-queuing system where hub delays modeled as time restriction	B&B-tabu search
Kawasaki (2012)		Social preferences of hub network versus scheduling advantages	
Camargo et al. (2011)	USAHLP	Comparing two congestion cost function (power-law/ Kleinrock)	Benders decomposition-outer

Table 2.9 (continued): Milestones of hub location literature in terms of congestion.

Authors	Model	Milestones for Congestion	Solution Procedure
Miranda Junior et al. (2011)	UMAHLP	Modeling congestion as a power-law function of total flow through hub under stochastic demand	Benders decomposition
Mohammadi et al. (2011)	CSAHVP	Modeling as M/M/c queuing system	Imperialism competitive algorithm-genetic algorithm
Elhedhli and Wu (2010)	CSAp-HLP	Kleinrock function: modeling as M/M/c queuing system where congestion is limited with capacity level	Lagrangean relaxation-subgradient optimization
Köksalan and Soylu (2010)	UMAp-HMP	Measuring congestion as demand/capability ratio	Evolutionary algorithm
Camargo et al. (2009a)	UMAHLP	Proposing Benders decomposition to handle power-law congestion cost function for multiple allocation	Benders decomposition
Rodriguez et al. (2007)	CSAp-HMP	Modeling as M/M/1 queuing system	Simulated annealing
Elhedhli and Hu (2005)	USAp-HLP	Modeling congestion as a convex non-linear cost function to balance distribution of flows where economies of scale captured by concave cost function	Lagrangean relaxation
Marianov and Serra (2003)	UMAHLP	Modeling as M/D/c queuing system	Tabu search
Mayer and Sinai (2002)		Detailed review of congestion and hubbing for air transportation	
Kostiuk et al. (1998)		Possible economic impacts of increased air traffic congestion on the air transportation industry	
Grover and O'Kelly (1986)	CSAp-HMP	Prior definition of congestion where total delay is calculated for each node of network	

Congestion could be described as an undesirable ineffectiveness of facility to process the flow traversing over itself. Congestion which is named differently as network congestion or traffic congestion is discussed in both transportation and telecommunication fields. Queuing, synchronization of flows in terms of speed, time or amount, bottlenecks, variations in time or expansion of the network could be the reasons of the congestion. Also, hubbing effect over network is considered as a trigger of congestion. The earlier academic efforts on congestion investigated deeply in terms of formulating, measuring, pricing, determining its possible causes and results. Studies by Vickrey (1969) Palma and Marchal (1998), Kostiuk et al. (1998), Gillen and Levinson (1999) and Mayer and Sinai (2002) are the examples of the theorical works on congestion.

Initially, delays occurred by processing of flows at hub have seen as a major cause of congestion. Most of the studies considered the hub network as a queuing model and proposed capacitated hub location problem which the capacity constraint is formulated as arrival rates, processing or delay times. Marianov and Serra (2003), Rodriquez et al. (2007), Mohammadi et al. (2011), Ishfaq and Sox (2012), Rahimi et al. (2016) studied hub location problems incorporated with queuing theory.

Later, recent studies defined congestion as a part of decision and an element of objective function. The modeling congestion in the objective function in a cost or time related manner provide more homogeneous network and effective allocation of non-hub nodes. Distinct congestion cost function proposed in the literature: convex nonlinear function (Elhedhli and Hu, 2005; Camargo et al., 2009a; Miranda Jr. et al., 2011; Camargo and Miranda, 2012; Kim and Kargar, 2016; Alkaabneh et al, 2019), Kleinrock function (Elhedhli and Wu, 2010; Camargo et al., 2012) and piece-wise functions (Luer-Villagra et al., 2019).

In hub location literature, there are different capacity definitions such as sum of incoming flows or all flows traversing through a hub. Similar to the definitions of the capacity, congestion definitions differentiate much. In general hub location problems consider the congestion which is limited under a capacity level or a pre-determined proportion of the capacity. It is significantly crucial to identify reversely due to the fact that the congestion is inevitable. Our main contribution in this thesis is to define an innovative congestion cost function which will be explain in the next chapter in detail.

Furthermore, the hub location literature related to congestion will be deeply presented in Chapter 3.

2.9.9 Other promising research areas

Until now, we explored most of the hub location problems in the literature and the motivations carrying the hub literature to the future. Some issues such as continuous hub location problem, survivable networks or planar topology of the network are exceptional studies in the literature and still rare. Also, uninvestigated complex formulations to identify an element of the network or entire network, more complicated network designs involving novel constraints or objective functions, combining different aspects of application fields, multi-objective nature of the problem and hybrid solution procedures to handle large-scaled problem may become alternative directions of the future efforts.

To explore the gaps in the literature, theoretical progress of HLP and real-life applications, this thesis focused on recently studied papers and the future trends that the topic is moving to. Even though previous studies and newly interested research areas are scattered in the literature, more realistic models and more computational effective solution procedures should be examined. The hub location problem literature still stands open to new ideas by the previous discussions and adaptations of realistic need of application fields. Thus, it is natural to propose more complex modeling, present better formulations of the problem and develop more sophisticated solution procedure to deal with them.

3. MODEL FORMULATION

In this chapter, we introduce uncapacitated multiple allocation problem with congestion. Section 3.1 provides the motivation and definition of the problem. Section 3.2 presents the congestion cost function used in the proposed model. Section 3.3 summarizes the previous mathematical models and final section explains the proposed mathematical model. The set of indices, parameters, decision variables and scalars which are used in the mathematical models are listed in the following tables.

Table 3.1 : The list of notation used in the modeling.

Set of Indices	
N	set of nodes
P	set of potential hubs
Parameters used in modeling	
W_{ij}	traffic flows between each pair of origin-destination nodes i and j
C_{ij}	transportation cost assigned to unit flow from i to j
C_{ijkm}	accumulated transportation cost where the cost between hubs are discounted
F_k	cost of establishing a hub at node k
γ, α, δ	discount factors of collection, transfer and distribution costs respectively
O_k	capacity of hub k
τ	desired capacity level in terms of hub capacity $(70\%, 85\%, 100\%)$
S_k	quantity of desired capacity of hub k $(S_k = \tau . O_k)$
E	multiplier of the proposed congestion function including a scalar 'e' and the average transportation costs of the network '$(\frac{\sum_i C_{ij}}{n})$' $(E = e.\frac{\sum_i C_{ij}}{n})$
Decision variables used in modeling	
Z_{ik}	flow from node i to hub k
Y_{kl}^i	flow from node i via hubs k and l
X_{lj}^i	flow from node i to node j via hub l
X_{ijkm}	flow departing from node i and ending at node j that is routed through hubs k and
g_k	total flow through hub k
Γ_k	congestion cost function
H_k	binary variable representing the decision of installing hub k

Table 3.2 : The list of scalars used in the modeling.

Scalars	
a, e, b	positive constants used in congestion cost functions

Table 3.3 shows the congestion cost functions used in the mathematical models.

Table 3.3 : The list of congestion cost functions.

Congestion cost functions	
$\Gamma_k = e. {g_k}^b$	Power-law congestion cost
$\Gamma_k(g_k) = a. (\frac{g_k}{O_k - g_k})$	Kleinrock congestion cost
$\Gamma_k(g_k) = e. (\frac{\sum_i C_{ij}}{n}) (\frac{g_k - S_k}{g_k})$	Proposed congestion cost

3.1 Motivation and Problem Definition

To model practical and profound hub location problems encountered in the air passenger transportation, we aim to provide a mathematical approach designing balanced hub networks. Owing to detailed literature research, it is observed that hub location problems fulfil the need of airline sector. In this section, we present our main discussions and evaluations from hub location literature and sector related approaches. Then, we will propose our mathematical model starting from these evaluations.

Air transportation sector includes specific and distinctive factors which are correspondingly related with each other such as airports, air passengers, direct routes or routes with stopovers, airlines and ground handling services. Transportation from origin and destination is handled by airports. Routes are planned either directly or multiple stops through a path. Airlines take advantage of central airports to organize their flights such as national or international. Planning routes of flights is a long-term decision consisting customer preferences, choice of central airports, distance or duration of flights and etc. to achieve competitive or cost minimization advantage.

Competition in air transportation, airlines try to offer price and time sensitive flights and more options for origin-destination pairs. By the way, their network design becomes more complicated and deals with larger flows. Employment of central airports brings flexibility in terms of alternative routes, ground operations, flow consolidation and cost-effective network where economies of scale are generated similar to hub location problem literature. Although the routing through hubs provides discounted travel cost by economies of scale, this may increase the amount of flow at hubs. While hub network design tends to increase amounts of flows at hub, it is crucial to focus on congestion occurred at hubs.

Including capacity restrictions to model could design a network without congestion, however this design couldn't project realistically due to ignoring increases of demands in time, possible changes in capacity resulting from delays or disruptions. Thus, it is important to introducing congestion in the model independently, but related to capacity, flow consolidation and time parameters.

The congestion cost function is aimed at balancing flow between hubs, avoiding the case where some interhub links are over-utilized compared to others. This fact is recognized by Ebery (2001) who imposes capacities on incoming flows.

Congestion has been mentioned for the first time in the paper of Grove and O'Kelly (1986). They have demonstrated the effect of the accumulated flows on a single hub triggered by scheduled delays on a network. Yet the first consideration of the congestion within a hub-spoke network was that of Marianov and Serra (2003). They have studied on a M/D/c queuing theory to design a network. They proposed a probabilistic model as referring to capacity constraints due to the waiting customers in the network and subsequently linearized the model. A tabu search algorithm has been used to handle the computational complexity of the model. Another solution procedure, simulated annealing, has been used to solve a similar problem by Rodriguez, Alvarez and Barcos (2007). Similarly, most of the earlier studies have considered congestion as a constraint while modeling capacitated versions of the problem.

On the other hand, there exists some distinct approaches to deal with congestion in a hub network. Costa, Captivo and Climaco (2008) have suggested a multi-criteria model which aimed to minimize both transportation cost and maximum waiting time at hubs. Thereby, congestion has been utilized as a parameter which determines waiting times at hubs according to its level related to capacity. Some studies in the literature, such as hub arc location problem have achieved the purpose of avoiding congestion and excessive delays at hubs (Elhedhli & Hu, 2005). Hub arc location problems try to assign hubs while balancing their flows. Thereby, they simplified the congestion problem. Rodriguez-Martin and Salazar-Gonzalez (2008) have developed a Benders decomposition algorithm to tackle a capacitated hub location problem based on a multi-commodity formulation where the network is assumed to be incomplete graph. Their model has been similar to the one presented by Sohn and Park (2000) for the fixed-charge capacitated network design problem.

Camargo, Miranda, and Ferreira (2009) have stated that modeling congestion through capacity constraints on the flows does not mimic the exponential nature of congestion effect. Congestion introduced in the model as a capacity constraint causes the model to disregard the realistic structure of the network. Limiting flows according to capacities avoids achieving credible and sustainable allocation of hubs and network designs.

The paper of Elhedhli and Hu (2005) has been the first work where congestion is introduced in the objective function. They have modeled a single allocation hub location problem with a convex cost function in which the delay cost is assumed to be a power-law function of capacity utilization. They linearized the non-linear model by using a piecewise linear function and then solved it by Lagrangean relaxation. Consecutively, the multiple allocation version of this problem was studied by Camargo et al. (2009a). They have introduced a Benders decomposition algorithm to deal with the non-linear part of the problem.

Both studies have proposed congestion as a function of total consolidated flows at selected hubs. Afterward, Elhedhli and Wu (2010) have modified the congestion as the ratio of total flow to the surplus capacity by modeling the network as M/M/1 queues. They proposed a Lagrangean heuristic algorithm where subproblems are solved by the cutting plane method and subgradient optimization. Camargo, Miranda, and Ferreira (2011) considered a different cost function called Kleinrock average delay function and compared this function with power-law function in the modeling single allocation hub location problem.

Camargo and Miranda (2012) presented newer congestion perspectives according to user or owner networks and evaluated their results and network designs. Since their problem possesses a highly combinatorial nature with the non-linearity associated to congestion, they have worked on a generalized Benders decomposition algorithm. Also, their next study aimed to solve large scale problems by that algorithm. Ishfaq and Sox (2012) proposed a row generation procedure to solve large scale problems.

Rahimi, Tavakkoli-Moghaddam, Mohammadi, and Sadeghi (2016) have presented a multi-objective model for a multi-modal hub location problem under uncertainty considering congestion at hubs. Alumur, Nickel, Rohrbeck, and Saldanha-da gama (2018) have proposed a model which is determining location and capacities of hubs, allocation of non-hub nodes and routes of flow through the network while providing the delivery between all of the pairs within the given service time. Limiting the service time on the network helps to balance the flows and optimize the congestion effects.

There is a very small number of studies focusing on hub location problem with congestion in the literature where mainly exact solution procedures are proposed. Table 2.9 summarizes the studies mentioned above and their solution techniques used to solve the problem.

In this study, we are constructing our mathematical model to focus on congestion which is also one of the main challenges in air passenger transportation. With the observations above, we propose a new mathematical model which determines the location of hubs and allocation structure of the network and controls the congestion occurred at hubs to balance the flow distribution at network. We formulate a multiple allocation hub location problem that minimizes the transportation cost, opening cost and congestion cost. We also consider different congestion cost functions to represent occupation of hubs while modeling the problem.

3.2 Congestion Function

Congestion refers to a network state where a node or link transmits larger flows that it may weaken network service quality, resulting in queuing delay or loss of demand. In a congested network, response time slows with reduced network throughput. Congestion occurs when there is insufficient capabilities of hub or network and network flows exceeds capacity. As mentioned before, in air transportation one of the recent challenges is to prevent or at least reduce the congestion in hub airports.

When the total flow through a hub or an airport exceed the available capacity, the congestion is occurred and causes delay or loss of traffic. The capacity term here may refer the number of flights occupied runways, the number of gateways allocated to flights, the operation capability of ground handling services or customer services or the number of the people serviced in the airport. Thus, congestion may be occurred due to insufficiently capacitated facilities, slow operation speed at the departments of airport/airline, malfunction or failure on operations such as a disaster, bad conditions on airside, heavy ground handling services, poor scheduling, malpractice of staff or heterogenous traffic of air network. Beside the operational issues, airport and routing related problems become key focus to decrease congestion and ensure a safety level for a reliable network.

While a possible increase in demand of an airport, peak times or seasons of the airport and varied customer-sector preferences for a specific route already lead to congested networks, hub spoke systems, flying between busy airports and recent open skies projects even worsen capacity related congestion.

Avoiding congestion completely is practically not reasonable. Because of sectoral structure and theoretical framework of hub network systems, occurrence of congestion in air transportation is inevitable. Congestion related delays and traffic losses at airports may require additional investments in the infrastructure of air systems. However, optimizing the use of existing system elements in terms of capacity and applying this strategy of efficiency over entire network may become beneficial and advantageous rather than costly reorganizations. Hub spoke systems should be designed by focusing on sufficient capacity levels to select hub airports. Moreover, efficient utilization of available capacities of each hub should be optimized by evenly allocating air traffic to hubs. Allocation decisions are important to ensure homogeneous traffic distribution as possible over network.

In the hub location literature, congestion, firstly represented as capacity restriction. Congestion is modelled as the amount of flows accumulated at hubs, delay occurred by processing the flows at hubs or total time to process the flows through hubs. In general, queuing theory is utilized to formulate congestion function. However, these formulations are more related to capacity features.

In order to formulate congestion cost, congestion effect should be measured by proportional to amount of flow or time needed to process that flow. In the literature, there is two distinct formulation to represent congestion. First one is a convex cost function that increases exponentially as more flows are routed through that hub by Elhedhli and Hu (2005). They justified their formulation with the discussions of the studies of Palma and Marchal (1998) and Gillen and Levinson (1999). Gillen and Levinson (1999) stated that the delay cost, defined as congestion, is assumed as a power-law function of airport utilization related to capacity by US-Federal Aviation Administration.

Second cost function of congestion is represented as the Kleinrock average delay function where hubs are assumed as a M/M/1 queue under steady-state conditions. This formulation is also supported by the study of Kleinrock (1964). The congestion effect is calculated by total flow per residual of capacity at hubs. Both of congestion functions assume that the total amount of flow is limited of the capacity of hubs. Although the studies on congestion refer as uncapacitated, the models are restricted congestion function with capacity parameter.

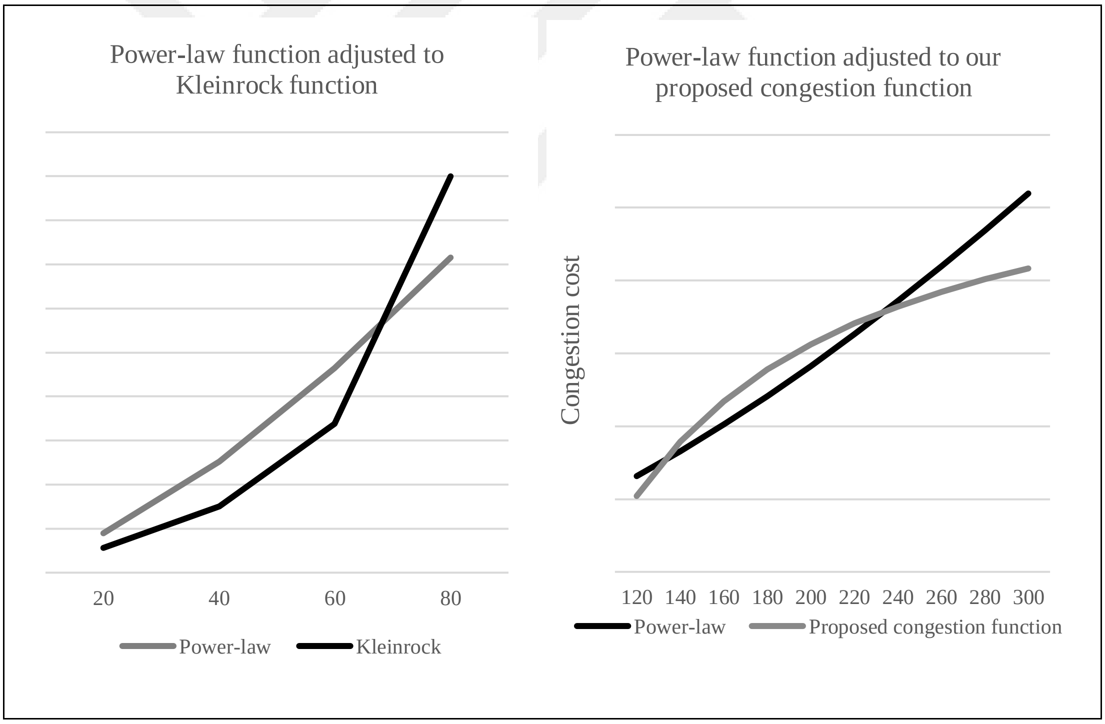


Figure 3.1 : Graphical representations of distinct formulations of congestion.

The distinct congestion functions are decipted in Figure 3.1, comparatively. Kleinrock function and power-law function are increasing on [0,+∞), proper convex and smooth, and it is normally used to estimate delay costs in airport applications. It is seen from left side of Figure 3.1 that Kleinrock function costs more then power-law fuction, when total flows accumulated at hub are minimum than desired capacity level. On the contrary, when the total flows accumulated at hubs are larger than desired level, power-law function results higher costs. The power law function increases rapidly as more traffic goes through hubs.

Kleinrock function limits the total flows accumulated at hubs under capacity of respective hub and the power-law function doesn't include the capacity of respective hub as a parameter. However, regardless of which desired capacity level is pre-determined or even minimum deviations are foreseen, in airport applications, the unkown nature of the operations and the system may cause congestion. This congestion leads delays, disruption and loss of traffic in the network. Finally, the network should bear the cost occurred due to congestion. Thus, the congestion can not be considered that it does not exceed the capacity. The occasions where congestion exceeds capacity should be appropriately reflected to the mathematical model. In this study, we relax this assumption by proposing a novel function of congestion which could exceed the capacity at hub. Since the definition of congestion implies that the capacity is exceeded at hubs. The proposed congestion function is formulated as equation below:

$$\Gamma_k(g_k) = e.(\frac{\sum_i C_{ij}}{n}).(\frac{g_k - \tau.O_k}{g_k}) \tag{3.1}$$

Our proposed congestion function is calculated by the amount of the surplus capacity $(g_k - \tau.O_k)$ per total flow accumulated at hub (g_k). This is reasonable approach to represent congestion effect. As shown in Figure 3.2, the proposed congestion function are increasing on $[0, e.(\frac{\sum_i C_{ij}}{n})]$, while g_k are in range between $[\tau.O_k, +\infty]$. It is a concave function which is tremendously increasing while total flows accumulated at hubs are around the capacity level of respective hub.

In Figure 3.1, the power-law and proposed congestion functions are adjusted to show the difference between them. While the values of g_k is converging to the capacity level of respective hub, the proposed function costs more than power-law function as seen in Figure 3.1.

Moreover, the proposed function continues its increasing trend even the capacity level is exceeded. After the total flow accumulated at hub are equal to doubled capacity level of respective hub, the increasing trend of the proposed function is slowed down. By the way, the outcome of the power-law congestion function become higher than the proposed one. This slow convergence is reasonable that the network couldn't become more vulnerable after that amount of congestion occurred.

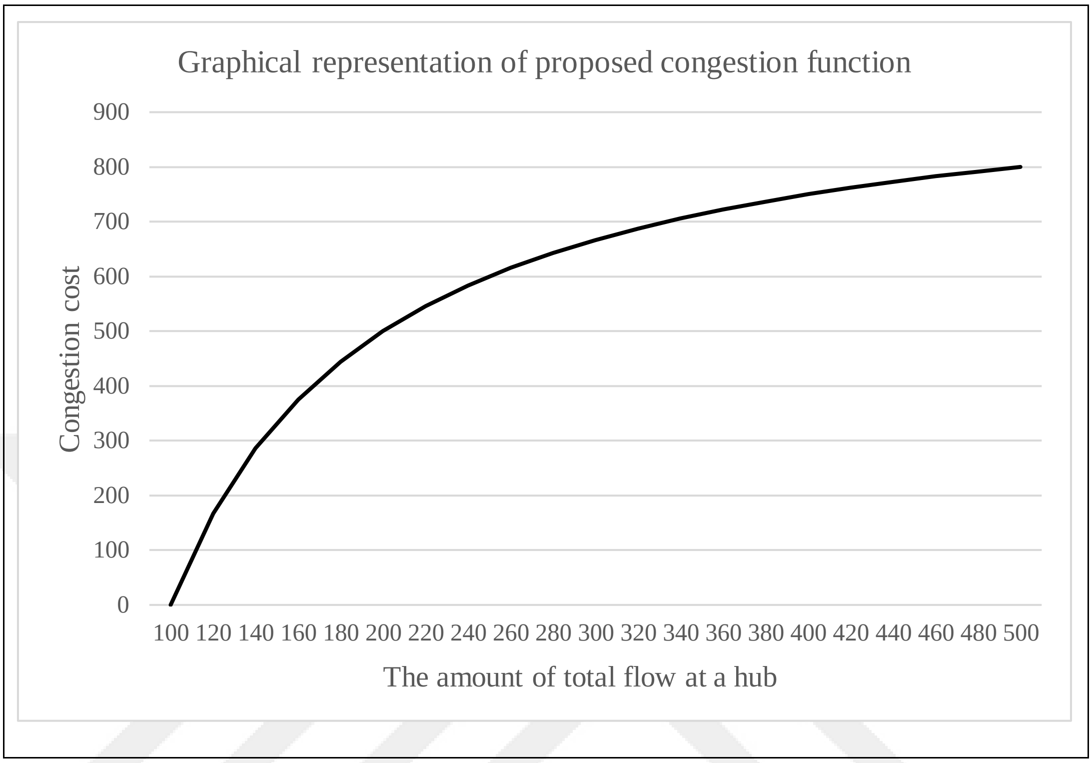


Figure 3.2 : Graphical representation of the proposed formulation of the congestion.

Furthermore, the power-law congestion cost function helps only to avoid the congestion with a dominating formula, but it doesn't aim to balance homogeneity of the flow distribution among selected hubs. This is because the function leaves the capacity parameter of respective hubs out of the formula. In this respect, our proposed function is beneficial to define congestion properly.

For practical applications of hub location problem, exceeding capacity in terms of the amount of flow and occurring congestion is unavoidable. In which situation the congestion is occurred ?

- When the total amount of the flows accumulated at hub converges to the maximum capacity level of respective hub
- When the total amount of the flows accumulated at hub surpasses to the desired level of the capacity at respective hub

- When the total amount of the flows accumulated at hub exceeds the desired level of the capacity at respective hub

Also, once a congestion is occurred or capacity of hub become insufficient, network capabilities as processing, time dependent services and delays is triggered. Unlike the existing literature, congestion is more involved in capacity level than the amount of flow. Thus, congestion becomes more effective around the maximum capacity level. When a hub faces with congestion situation, all network design is adversely influenced that the associated cost should be charged. The delays occuring due to the congestion, the re-scheduling of operations, flights or employees, the reorientation of the customers such as repayment, cost of retards or ticketing and the revision of the network design are some of the problems encountered in air passenger transportation.

Early on, a network is suffered from these losses, the difference between the costs before and after increases sharply. However, as the congestion continues, the increase in the costs incurred will become stagnant and stationary. Subsequently, the more the amount of flow increase, the less the congestion affects the network capabilities.

According to this reasoning, an optimal network design should locate hubs which have appropriate capacities to prevent congestion and allocate non-hub nodes where the flow distribution is balanced over network.

3.3 Congested Uncapacitated Multiple Allocation Hub Location Problem

In this section, we first present previous mathematical models in literature which provide basis to model our problem. We define the decision variables and mixed integer programming models step by step. Then we present the proposed model with quadratic cost function to our problem.

We consider the uncapacitated multiple allocation hub location problem, where each node can be assigned to more than one hub. Given a complete graph $G = (N, P)$, where $N = \{1,2,\cdots,n\}$ is the set of nodes, $P = \{1,2,\cdots,p\}$ is the set of potential hubs, and $P \subseteq N$. There is a flow traffic, W_{ij} between each pair of origin-destination nodes: i and j $(i, j \in N, i \neq j)$. Also, a transportation cost, C_{ij}, is assigned to unit flow from i to j. In general, transportation cost is calculated proportional to distance between origin-destination pairs and therefore Euclidian.

Thus, by referring a hub-spoke network consisting of interacting points and three types of links for collection, transfer, and distribution; the following assumptions are required for our model:

- Hubs are fully connected, and all flow between pairs of spokes go through hubs, i.e. direct shipment between spokes is not allowed.
- Hubs have finite capacity, but could have higher capacity from desired level
- Each spoke could be assigned to more than one hub.

While constructing our mathematical models, we are enlightened by the studies of Ernst and Krishnamoorthy (1998a): uncapacitated multiple allocation p-hub location problem, Ebery et al. (2000): multiple allocation version of the capacitated p-hub median location problem and Camargo et al. (2009a): uncapacitated multiple allocation hub location problem with congestion function.

We next present the first formulation for the uncapacitated multiple allocation p-median hub location problem UMApHMP published by Ernst and Krishnamoorthy (1998a). The parameters of their model are given as flows, with minor notational changes: are defined as follows:

- F_k: the cost of establishing a hub at node k
- γ, α, δ: the collection, transfer and distribution coefficients respectively
- O_k: the capacity of hub k

The variables in the formulations are defined as:

Z_{ik}: the flow from node i to hub k

- Y_{kl}^{i}: the flow from node i via hubs k and l
- X_{lj}^{i}: the flow from node i to node j via hub l
- $H_k = \begin{cases} 1, & if\ node\ k\ is\ a\ hub \\ 0, & otherwise \end{cases}$

With these definitions, Ernst and Krishnamoorthy (1998a) stated UMApHLP as follows:

$$Min \sum_i \left(\gamma \sum_k C_{ik} Z_{ik} + \alpha \sum_k \sum_l C_{kl} Y_{kl}^i + \delta \sum_k \sum_l C_{lj} X_{lj}^i \right) \tag{3.2}$$

s.t.

$$\sum_k H_k = p \, , \tag{3.3}$$

$$\sum_k Z_{ik} = \sum_j W_{ij} \, , \qquad \forall i \tag{3.4}$$

$$\sum_l X_{lj}^i = W_{ij} \, , \qquad \forall i,j \tag{3.5}$$

$$\sum_l Y_{kl}^i + \sum_j X_{kj}^i - \sum_l Y_{lk}^i - Z_{ik} = 0 \, , \quad \forall i,k \tag{3.6}$$

$$Z_{ik} \leq \sum_j W_{ij} \, H_k \, , \qquad \forall i,k \tag{3.7}$$

$$X_{lj}^i \leq \sum_i W_{ij} H_l \qquad \forall i, \mathrm{j}, l \tag{3.8}$$

$$X_{lj}^i \, , Y_{kl}^i \, , Z_{ik} \ \geq 0 \, , \quad H_k \in \{0,1\} \, , \qquad \forall \, i,j,k,l \in N \tag{3.9}$$

The objective function defined as equation 3.2 is a minimization of the sum of transportation costs. Equation 3.3 limits the number of hubs opened. Equation 3.4 and 3.5 represent the flow equations for the network. Equation 3.6 controls the flows equivalence at hubs. Equation 3.7 ensures that the total flow originated from node i should be directed by a hub k. Equation 3.8 ensures that the total flow between every i and j should be destinated by a hub l. Equation 3.9 is the non-negativity constraints.

As a second formulation, we summarize the formulation suggested by Ebery et al. (2001). They proposed a formulation of capacitated multiple allocation version which does not only decides on the optimal location of hubs and allocation of non-hub nodes to hubs, but also the number of hubs that are required. Thus, there is no need to determine the number of hubs. They considered the fixed cost for establishing a hub in the objective function. Capacity constraint is added to limit the flows at selected hub.

$$Min \sum_i \left(\gamma \sum_k C_{ik} Z_{ik} + \alpha \sum_k \sum_l C_{kl} Y^i_{kl} + \delta \sum_k \sum_l C_{lj} X^i_{lj} \right) + \sum_k F_k H_k \tag{3.10}$$

s.t.

$$\sum_k Z_{ik} = \sum_j W_{ij} \;, \qquad \forall i \tag{3.11}$$

$$\sum_l X^i_{lj} = W_{ij} \;, \qquad \forall i,j \tag{3.12}$$

$$\sum_i Z_{ik} \leq O_k H_k \;, \qquad \forall k \tag{3.13}$$

$$\sum_l Y^i_{kl} + \sum_j X^i_{kj} - \sum_l Y^i_{lk} - Z_{ik} = 0 \;, \quad \forall i,k \tag{3.14}$$

$$Z_{ik} \leq \sum_j W_{ij} H_k \;, \qquad \forall i,k \tag{3.15}$$

$$X^i_{lj} \leq W_{ij} H_l \;, \qquad \forall i,j,l \tag{3.16}$$

$$X^i_{lj}, Y^i_{kl}, Z_{ik} \geq 0 \;, \; H_k \in \{0,1\} \;, \quad \forall\, i,j,k,l \in N \tag{3.17}$$

Unlike previous one, objective function shown in equation 3.10 represents the sum of transportation and opening costs. Also, equation 3.13 restricts the amount of the flow through a hub by its respective capacity.

In order to represent congestion effect on multiple allocation hub location problem, a formulation with usage of four-index variables is suggested by Elhedhli and Hu (2005). They revised the objective function which is the sum of the congestion cost and total transportation cost, they considered uncapacitated single allocation p-hub location problem. They introduced a convex cost function which represents the convergence of the flows accumulated through hubs at peak hours. Their congestion function is a power-law function of total flows at hubs. Although 3-index variable usage has substantially fewer variables than this formulation, the model is reputed to have a tight linear programming bound. So, it probably becomes a stronger formulation than the others in solving large scale hub location problems.

Camargo et al. (2009a) proposed a multiple allocation version of the formulation of Elhedhli an Hu (2005) with some revisions. Camargo et al. (2009a) evolved out the formulation of Hamacher et al. (2004) due to their formulation's theoretical properties and better structure. Since, their model doesn't restrict the number of hubs opened, Camargo et al. (2009a) revised the objective function including transportation cost, opening cost and congestion cost.

Their formulation uses four index variables and parameters which are defined below:

- F_k: the cost of establishing a hub at node k
- γ, α, δ: discount factors of collection, transfer and distribution costs respectively
- C_{ijkm}: the accumulated transportation cost where the cost between hubs are discounted.
- X_{ijkm}: the flow departing from node i and ending at node j that is routed through hubs k and m
- g_k: the total flow through hub k
- Γ_k $(= e.{g_k}^b)$: the power-law congestion cost function where e and b are positive constants.
- $H_k = \begin{cases} 1, & if\ node\ k\ is\ a\ hub \\ 0, & otherwise \end{cases}$

$$Min \sum_i \sum_j \sum_k \sum_m C_{ijkm}\ X_{ijkm} + \sum_k (F_k H_k + \Gamma_k(g_k)) \tag{3.18}$$

s.t.

$$\sum_i \sum_j \sum_m X_{ijkm} + \sum_i \sum_j \sum_{m \neq k} X_{ijkm} = g_k \ , \quad \forall k \tag{3.19}$$

$$\sum_k \sum_m X_{ijkm} = W_{ij} \ , \quad \forall i, j \tag{3.20}$$

$$\sum_m X_{ijkm} + \sum_{m \neq k} X_{ijkm} \leq W_{ij} H_k \quad \forall \ i, j, k \tag{3.21}$$

$$X_{ijij} \geq W_{ij}(H_i + H_j - 1) \quad \forall \ i, j \tag{3.22}$$

$$\sum_{k \neq j} X_{ijik} \geq W_{ij}(H_i - H_j) \quad \forall i, j \tag{3.23}$$

$$\sum_{k \neq i} X_{ijkj} \geq W_{ij}(H_j - H_i) \quad \forall i, j \tag{3.24}$$

$$X_{ljkm} \geq 0 \ , \ H_k \in \{0,1\} \ , \ g_k \geq 0 \ , \quad \forall \ i, j, k, l \in N \tag{3.25}$$

$$C_{ijkm} = \gamma C_{ik} + \alpha C_{km} + \delta C_{mj} \tag{3.26}$$

$$\Gamma_k(g_k) = e. {g_k}^b \tag{3.27}$$

The congestion cost function in the formulation is calculated by the flow through each hub. This function homogenizes the flow load along the selected hubs, thus decreases overall congestion (Camargo et al., 2009).

The objective function shown in equation 3.18 is the sum of total transportation cost, opening costs and congestion cost. Equation 3.19 represents the total amount of the flows at hub k. Equation 3.20 assures that the flow between i and j should be routed through hub k and m. Equation 3.21 prevents routing the flows of every origin-destination pairs without using any hubs. Equation 3.22 represents the special case whether i and j are located as hubs. In case of either i or j are located as hubs, equation 3.23 and 3.24 ensure that the routes between origin-destination includes two hubs. In the paper of Camargo et al. (2009a) the congestion function has been calculated proportional of all the flow through the hub which is considered increasing properly convex and smooth on [0, +∞). There are other different congestion functions in the literature as mentioned in the previous subsection.

3.4 The Proposed Uncapacitated Multiple Allocation Hub Location Problem under Congestion

Several mixed integer linear programming models have been proposed for uncapacitated multiple allocation hub location problem (UMAHLP). Our model is an extension of the models summarized above. This extension essentially includes additional constraints and a reconstruction of the objective function.

The objective function is revised in such a way that a congestion cost is added as a penalty cost in the proposed model. This congestion cost is determined by the excess of accumulated flows at hubs from capacities of hubs. This surplus of the flow at a given hub is multiplied by the average transportation cost of corresponding hub and a positive scalar e. The capacity constraint of the model by Ebery et al. (2000) is removed from proposed model. As for notations used in our model, the following parameters are defined:

- F_k: the cost of establishing a hub at node k
- γ, α, δ: represents the discount factors of collection, transfer and distribution costs respectively.
- O_k: represents the capacity of hub k.
- τ: represents desired capacity level in terms of hub capacity $(70\%, 85\%, 100\%)$.
- S_k: represents the quantity calculated by multiplying the desired level capacity and capacities of hubs.
- C_{ijkm}: the transportation cost where the cost between hubs are discounted.
- g_k: represents the total accumulated flow through hub k.
- Γ_k $\left(= e.\left(\frac{\sum_i C_{ij}}{n}\right)\left(\frac{g_k - S_k}{g_k}\right)\right)$: represents the appropriate congestion cost function for surplus of the capacity with positive scalar e.

The variables in the formulations are defined as:

- X_{ijkm}: the flow departing from node i and ending at node j that is routed through hubs k and m.
- $H_k = \begin{cases} 1, & if\ node\ k\ is\ a\ hub \\ 0, & otherwise \end{cases}$

Using these notations above, the model for uncapacitated multiple allocation hub location problem (UMAHLP) with congestion can be stated as follows:

$$Min \sum_{i}\sum_{j}\sum_{k}\sum_{m} C_{ijkm} X_{ijkm} + \sum_{k}(F_k H_k + \Gamma_k(g_k)) \tag{3.28}$$

s.t.

$$\sum_{i}\sum_{j}\sum_{m} X_{ijkm} + \sum_{i}\sum_{j}\sum_{m\neq k} X_{ijkm} = g_k \ , \qquad \forall k \tag{3.29}$$

$$\sum_{k}\sum_{m} X_{ijkm} = W_{ij} \qquad \forall i,j \tag{3.30}$$

$$\sum_{m} X_{ijkm} + \sum_{m\neq k} X_{ijkm} \leq W_{ij}.H_k \qquad \forall\, i,j,k \tag{3.31}$$

$$X_{ijij} \geq W_{ij}(H_i + H_j - 1) \qquad \forall\, i,j \tag{3.32}$$

$$\sum_{k\neq j} X_{ijik} \geq W_{ij}(H_i - H_j) \qquad \forall i,j \tag{3.33}$$

$$\sum_{k\neq i} X_{ijkj} \geq W_{ij}(H_j - H_i) \qquad \forall i,j \tag{3.34}$$

$$X_{ijkm} \geq 0 \ , \ H_k \in \{0,1\} \ , \ g_k \geq 0 \ , \qquad \forall\, i,j,k,l \in N \tag{3.35}$$

$$C_{ijkm} = \gamma C_{ik} + \alpha C_{km} + \delta C_{mj} \tag{3.36}$$

$$\Gamma_k(g_k) = e.\left(\frac{\sum_i C_{ij}}{n}\right)\left(\frac{g_k - S_k}{g_k}\right) \tag{3.37}$$

$$S_k = \tau O_k \tag{3.38}$$

The equation 3.28 minimizes the total cost comprising the transportation cost, opening and congestion costs. Constraint defined in equation 3.29 determines the total amount of the flows through hub k. Equation 3.30 assures that every route from i to j should be transferred through hubs. Equation 3.31 prevents the flow consolidation on a node unless it is selected as hub. Equation 3.32 represents the special case whether i and j are located as hubs. In case of either i or j are located as hubs, equation 3.33 and 3.34 ensure that the routes between origin-destination includes two hubs. Equation 3.35 is to identify the non-negativity of variables and binary variables. Equation 3.36-3.38 represents the equations of the parameters utilized in the model such as total transportation cost, congestion function and pre-determined service levels of hubs.

4. SOLUTION METHODOLOGIES

In Chapter 4, we describe general framework of Benders decomposition algorithm with some basic concepts and definitions. Firstly, we explain how to Benders decomposition algorithm apply to mixed integer programming models. Master problem, subproblems and generation of Benders cuts are constructed through algorithm. Subsequently, we modify the Benders decomposition algorithm to apply our problem. To cope with the non-linear part of the proposed models, we present a linearization of the model. Secondly, we describe the general framework of the particle swarm optimization algorithm. Then, we develop particle swarm optimization algorithm for proposed model by demonstrating each step of the algorithm.

4.1 Benders Decomposition Algorithm

While modeling the optimization problems like our problem, main concern is the size of the problem meaning the number of decision variables and constraints of the problem. The size of the problem makes the solution procedure more difficult in terms of the complexity and the computational time. Classical solution techniques to achieve the optimal solution remain incapable upon increasing number of decision variables and constraints. To overcome these difficulties, partitioning procedures such as Benders decomposition technique may be applied.

Unlike classical solution procedures, partitioning procedure divide the process of solution into parts. Benders decomposition algorithm is a popular solution technique to solve NP hard problems such as integer, mixed integer, non-linear, quadratic and stochastic programming. Since Benders (1962) invented the algorithm for mixed integer problems, most of the studies including combinatorial optimization and stochastic programming problems apply frequently. Main principle of the algorithm is splitting the problem into smaller part where one includes integer variables as master problem and other includes continuous variables or other complicating variables as sub-problem. Master problem become a kind of relaxation of the original problem due to extracting some variables from the problem. The main advantage of the algorithm is discarding some complicating variables. Thus, the algorithm makes the problem significantly easier to solve. Moreover, the algorithm improves the overall computational performance by structuring the problem.

The basic principles of the algorithm are to reformulate the problem and to reduce the complexity and size of the problem. The algorithm aims to make the problem resolvable by extracting some set of variables from the problem. Benders decomposition algorithm decomposes the decision variables into the specific subsets.

Firstly, the algorithm tries to solve master problem with one of the subsets of decision variables. By the way, solution procedure obtains the relaxation of the original problem and defines it as master problem. Then, sub-problems are constructed with the remaining decision variables to be solved. If the solutions of these sub-problems show that the master problem are not feasible, they are utilized to generate cuts. These Benders cuts are added iteratively to the master problem and become good upper bounds to achieve optimality. At each iteration, master problem is reformulated by adding a new constraint generated from Benders cut. The solution of the dual formulation of the sub-problem is utilized to construct this new constraint. The algorithm is repeated step by step until the optimal solutions of the master and sub-problems become equal or the difference between the lower and upper bounds is smaller than a pre-specified value.

Finally, these procedures are iteratively repeated until the optimal solution is attained. Thus, the complexity of the problem is handled by solving a serial of simpler problem rather than a large scaled problem.

4.2 Benders Decomposition Methodology

Benders decomposition algorithm is based on the weak duality of linear programming. A mixed integer problem is represented (Taşkın, 2010) with the notations as follows:

$$
\begin{aligned}
&MIN\ c^T x + f^T y \\
&Ax + By \leq b \\
&y \in Y \subseteq \mathbb{R}^q \\
&x \geq 0
\end{aligned}
\tag{4.1}
$$

By fixing the value of y variables, we can reformulate the problem as follows:

$$f^T y + \begin{array}{c} MIN\ c^T x \\ Ax \leq b - B\hat{y} \\ x \geq 0 \end{array} \tag{4.2}$$

In the formulation 4.2, remaining equations from $f(\hat{y})$ can be separated from the original problem to get an easier solution. The following equations represent the general form of the Benders sub-problem.

$$\begin{array}{c} MIN\ c^T x \\ Ax \leq b - B\hat{y} \\ x \geq 0 \end{array} \tag{4.3}$$

The strong duality state that the optimal solution of the original problem should be equal to the dual formulation of the equations 4.3. The dual formulation can be written as below by associating dual variables $\boldsymbol{u}$ with constraints:

$$\begin{array}{c} MAX\ (b - B\hat{y})\ u^T \\ A^T u \leq c \\ u\ \leq 0 \end{array} \tag{4.4}$$

Let u_j be an extreme point that maximizes the problem defined by equations 4.4. z is an auxiliary variable complementing the objective function of the master problem. The dual of the sub-problem is revised as follows:

$$\begin{array}{c} MIN\ z \\ (b - B\hat{y})\ {u_j}^T \leq z \\ z\ unrestricted \end{array} \tag{4.5}$$

If we replace the z and y variables into $c^T x$ in the original problem, master problem of the algorithm is revised as follows.

$$
\begin{aligned}
&MIN\ f^T y + z \\
&(b - By)\ u_j^T \leq z \\
&y \in Y \\
&z\ unrestricted
\end{aligned}
\qquad (4.6)
$$

Finally, Benders decomposition algorithms enables us to obtain the following two problems to solve (Taşkın, 2010):

$$
\begin{aligned}
(MP) \qquad & MIN\ f^T y + z \\
& s.t \\
& (b - By)\ u_j^T \leq z \\
& (b - By)\ u_j^T \leq 0 \\
& y \in Y \\
(DSP) \qquad & MAX\ (b - By)^T u \\
& s.t \\
& A^T u \leq c \\
& u \geq 0
\end{aligned}
\qquad (4.7)
$$

If the sub-problem in equation 4.3 is feasible by fixing y variable, the dual sub-problem is bounded. So, an optimality Benders cut is obtained. Conversely, infeasible sub-problem leads to generating a feasibility cut achieved from dual sub problem. First constraint of the master problem in equation 4.7 is identified as optimality benders cut and the second constraint of the master problem is feasibility cut.

The optimality cuts provide strong lower bound achieved by the master problem, whereas feasibility cuts guarantee the validity of lower bound to exercise in the original problem. It is mainly accepted that the convergence of Benders decomposition algorithm strengthening with the optimality cuts is faster than one with the feasibility cuts. Benders decomposition algorithm achieve the optimality iteratively. Both master and sub-problem are solved through each iteration and the sub-problem solutions are added to the master problem to narrow the feasible region of the problem. Adding the violated Benders cuts to master problem continues until all constraint generated form cuts are satisfied at a relaxed master problem solution. Since the master problem is a relaxation, its optimal value provides a lower bound on the optimal value and an upper bound is given by the expectation of the optimal values of the sub-problems. This row generation approach is the opposite of column generation used in Dantzig-Wolfe decomposition. Consequently, these basic steps of the Benders decomposition should be adapted to our congested hub location problem.

4.2.1 Adapting Benders decomposition algorithm to our problem

In this section, we will focus on how we adapt Benders decomposition to our specific problem as shown in the following equations.

$$Min \sum_{i}\sum_{j}\sum_{k}\sum_{m} C_{ijkm} X_{ijkm} + \sum_{k}(F_k H_k + \Gamma_k(g_k)) \tag{4.8}$$

s.t.

$$\sum_{i}\sum_{j}\sum_{m} X_{ijkm} + \sum_{i}\sum_{j}\sum_{m\neq k} X_{ijkm} = g_k\ , \qquad \forall k \tag{4.9}$$

$$\sum_{k}\sum_{m} X_{ijkm} = W_{ij} \qquad \forall i,j \tag{4.10}$$

$$\sum_{m} X_{ijkm} + \sum_{m\neq k} X_{ijkm} \leq W_{ij} H_{k_k} \qquad \forall\ i,j,k \tag{4.11}$$

$$X_{ijij} \geq W_{ij}(H_i + H_j - 1) \qquad \forall\ i,j \tag{4.12}$$

$$\sum_{k\ \neq j} X_{ijik} \geq W_{ij}(H_i - H_j) \qquad \forall i,j \tag{4.13}$$

$$\sum_{k\neq i} X_{ijkj} \geq W_{ij}(H_j - H_i) \qquad \forall i,j \tag{4.39}$$

$$X_{ijkm} \geq 0\ ,\ H_k \in \{0,1\}\ ,\ g_k \geq 0\ , \qquad \forall\ i,j,k,l \in N \tag{4.40}$$

$$C_{ijkm} = \gamma C_{ik} + \alpha C_{km} + \delta C_{mj} \tag{4.16}$$

$$\Gamma_k(g_k) = e(\frac{\sum_i C_{ij}}{n})\,(\frac{g_k - S_k}{g_k}) \quad (4.41)$$

$$S_k = \tau.\ O_k \quad (4.42)$$

By separating 0-1 integer variable H_k, the master problem of Benders decomposition algorithm can be written as follows:

$$Min \sum_k F_k . H_k + SP(y) \quad (4.19)$$

s.t.

$$H_k \in \{0,1\} \quad (4.2043)$$

Hence, the sub-problem for fixed variable $H_k = \widehat{H_k}$ decomposed from the original problem is written as the following model:

$$SP(y) = Min \sum_i \sum_j \sum_k \sum_m C_{ijkm}\ X_{ijkm} + \sum_k \Gamma_k(g_k) \quad (4.21)$$

s.t.

$$\sum_k \sum_m X_{ijkm} = W_{ij} \qquad \forall i,j \quad (4.22)$$

$$\sum_m X_{ijkm} + \sum_{m \neq k} X_{ijkm} \leq W_{ij}\widehat{H_k} \qquad \forall\, i,j,k \quad (4.23)$$

$$X_{ijij} \geq W_{ij}(\widehat{H_i} + \widehat{H_j} - 1) \qquad \forall\, i,j \quad (4.24)$$

$$\sum_{k\ \neq j} X_{ijik} \geq W_{ij}(\widehat{H_i} - \widehat{H_j}) \qquad \forall i,j \quad (4.25)$$

$$\sum_{k \neq i} X_{ijkj} \geq W_{ij}(\widehat{H_j} - \widehat{H_i}) \qquad \forall i,j \quad (4.26)$$

In the sub problem above the equations 4.9 and 4.15 from original problem are also satisfied , since there are feasible flows satisfying that equations through master problem. The following model is formulated as the dual of the model above with the dual variables $\theta \geq 0$, $\beta \geq 0, \varepsilon \geq 0, \sigma \geq 0, \varphi \geq 0$ associated with the constraints:

$$Max \quad \theta_{ij}W_{ij} - \sum_{k} \beta_{ijk}W_{ij}\widehat{H}_k + \varepsilon_{ij}W_{ij}(\widehat{H}_i + \widehat{H}_j - 1) + \sigma_{ij}W_{ij}(\widehat{H}_i - \widehat{H_j}) + \varphi_{ij}W_{ij}(\widehat{H}_j - \widehat{H_i}) \tag{4.28}$$

s.t.

$$\theta_{ij} - \beta_{ijk} \leq C_{ijkk} + \widehat{\tau_k} \qquad \forall\, k \neq i, k \neq j \tag{4.29}$$

$$\theta_{ij} - \beta_{ijk} - \beta_{ijm} \leq C_{ijkm} + \widehat{\tau_k} + \widehat{\tau_m} \qquad \forall\, k \neq i, k \neq m, m \neq j \tag{4.30}$$

$$\theta_{ij} - \beta_{iji} + \sigma_{ij} \leq C_{ijii} + \widehat{\tau_i} \tag{4.31}$$

$$\theta_{ij} - \beta_{iji} + \varphi_{ij} \leq C_{ijjj} + \widehat{\tau_j} \tag{4.32}$$

$$\theta_{ij} - \beta_{iji} - \beta_{ijj} + \varepsilon_{ij} \leq C_{ijij} + \widehat{\tau_i} + \widehat{\tau_j} \tag{4.33}$$

$$\theta_{ij} - \beta_{iji} - \beta_{ijk} + \sigma_{ij} \leq C_{ijik} + \widehat{\tau_i} + \widehat{\tau_k} \qquad \forall\, k \neq i, k \neq j \tag{4.34}$$

$$\theta_{ij} - \beta_{ijj} - \beta_{ijk} + \varphi_{ij} \leq C_{ijkj} + \widehat{\tau_k} + \widehat{\tau_j} \qquad \forall\, k \neq i, k \neq j \tag{4.35}$$

$$\theta_{ij}, \beta_{ijk} \in \mathbb{R}, \qquad \varepsilon_{ij}, \sigma_{ij}, \varphi_{ij} \geq 0 \qquad \forall\, i, j, k, l \in N \tag{4.36}$$

Since the constraints of dual subproblem are enough to ensure feasibility, it is known that SP is bounded. Due to the structure of equation 4.21, the subproblem has convex objective function with linear constraints. So, Karush–Kuhn–Tucker conditions are necessary and sufficient for optimality. By the way, the generated Benders cuts for master problem become the following inequality:

$$\begin{aligned} \eta \geq\ & \theta_{ij}W_{ij} - \sum_{k} \beta_{ijk}W_{ij}\widehat{H}_k + \varepsilon_{ij}W_{ij}(\widehat{H}_i + \widehat{H}_j - 1) \\ & + \sigma_{ij}W_{ij}(\widehat{H}_i - \widehat{H_j}) + \varphi_{ij}W_{ij}(\widehat{H}_j - \widehat{H_i}) \\ & \theta_{ij}W_{ij} - \sum_{k} \beta_{ijk}W_{ij}\widehat{H}_k + \varepsilon_{ij}W_{ij}(\widehat{H}_i + \widehat{H}_j - 1) + \sigma_{ij}W_{ij}(\widehat{H}_i - \widehat{H_j}) \\ & + \varphi_{ij}W_{ij}(\widehat{H}_j - \widehat{H_i}) \geq 0 \end{aligned} \tag{4.37}$$

Finally, mathematical model of master problem is transformed into the below equations:

$$Min \sum_{k} F_k . H_k + \eta$$

s.t.

$$\eta \geq \theta_{ij} W_{ij} - \sum_{k} \beta_{ijk} W_{ij} \widehat{H}_k + \varepsilon_{ij} W_{ij} (\widehat{H}_i + \widehat{H}_j - 1) + \sigma_{ij} W_{ij} (\widehat{H}_i - \widehat{H_j}) + \varphi_{ij} W_{ij} (\widehat{H}_j - \widehat{H_i}) \tag{4.38}$$

$$\eta, \theta, \beta, \varepsilon, \sigma, \varphi \geq 0$$

Every generation of the constraints presented in equation 4.37 are added iteratively. After h iterations, we obtain the optimal value of master problem. f^h determined by vector H represents the opening cost of hub. $\widehat{\tau_k}^h$ determined by vector H represents the congestion cost of the network. By the help of this dual formulation, $\theta_{ij}{}^h$, $\beta_{ijk}{}^h$, $\varepsilon_{ij}{}^h$, $\sigma_{ij}{}^h$, $\varphi_{ij}{}^h$ are the optimum values derived from sub-problem solutions after h iterations. η , which is an auxiliary variable complementing the objective function of the master problem, becomes an upper bound for transportation costs. By the way, master program revised as below:

$$Min\, \eta + f^h$$

s.t.

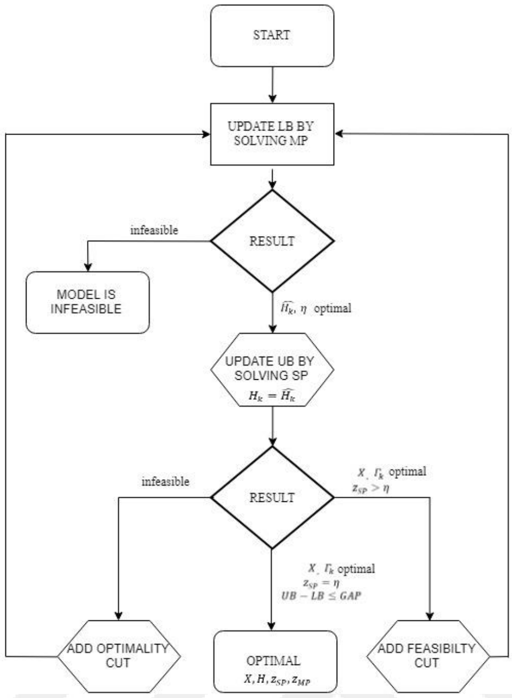
$$\eta + \theta_{ij}{}^h . W_{ij} + \varepsilon_{ij}{}^h . W_{ij} . (H_i + H_j - 1) + \sigma_{ij}{}^h . W_{ij} . (H_i - H_j) + \varphi_{ij}{}^h . W_{ij} . (H_i - H_j) \geq \sum_{i} \beta_{ijk}{}^h . W_{ij} . H_k \tag{4.39}$$

$$\eta \geq 0, \; H_k \in \{0,1\} , \qquad \forall\, k$$

An outline of Benders decomposition method for HLP model is illustrated in Figure 4.1 and the pseudocode of the algorithm is shown in Figure 4.1.

In Figure 4.1, Z_{MP} and Z_{SP} den.otes the actual optimal value of master and subproblem, respectively where UB and LB are upper and lower bounds of original problem. GAPis the value of the pre-specified tolerance for the difference between UB and LB.

Moreover, $\widehat{H_k}$, η, ${\theta_{ij}}^h$, ${\beta_{ijk}}^h$, ${\varepsilon_{ij}}^h$, ${\sigma_{ij}}^h$, ${\varphi_{ij}}^h$, f^h, $\widehat{\tau_k}^h$ are the values from the solutions from the MP and DSP at iterations h.



Pseudocode of the algorithm of Benders decomposition

Initialization

$UB = +\infty, LB = -\infty, h = 1$

$\textbf{While } \frac{UB - LB}{UB} < GAP, \textbf{Do}$

Begin

Solve the MP in the equation 4.39

$LB \leftarrow z_{MP}$

Do $LB = z_{MP}$ *and Update the values of* $\widehat{H_k}$ *in the SP*

${\widehat{H_k}}^h \leftarrow H^*$ *and* $\eta^k \leftarrow \eta^*$

Calculate $\widehat{\tau_k}^h$

For** routing variables **Do

Begin

Solve the SP in the equations 4.28 − 4.36

using the optimal values of ${\theta_{ij}}^h, {\beta_{ijk}}^h, {\varepsilon_{ij}}^h, {\sigma_{ij}}^h, {\varphi_{ij}}^h$

$\textbf{If } \eta^k < z_{SP}$ *to generate cuts,* ***Then***

Begin

Calculate $\sum_k F_k . H_k + \eta$

Add the Benders cut in the equation 4.38 *to the MP*

End

End

$\textbf{If } z_{SP} < UB$, *Update* $UB = z_{SP}$, ***Then***

$h = h + 1$

End

Figure 4.1 : Algorithmic Schemes of Benders decomposition.

To apply Benders algorithm, we should consider the quadratic nature of the congestion cost function. This may lead us not to determine the optimal solutions of sub problems.

Thus, in the next chapter we will linearize the quadratic formulation of our primal model, then we will apply Benders decomposition.

4.2.2 Linearization of the proposed model

As mentioned before, our proposed model has quadratic cost function to determine congestion.

$$Min \sum_i \sum_j \sum_k \sum_m C_{ijkm}.\ X_{ijkm} + \sum_k (F_k Y_k + \Gamma_k(g_k))$$

s.t.

$$\sum_i \sum_j \sum_m X_{ijkm} + \sum_i \sum_j \sum_{m \neq k} X_{ijkm} = g_k, \qquad \forall k \tag{4.40}$$

$$\Gamma_k(g_k) = e\left(\frac{\sum_i C_{ij}}{n}\right)\left(\frac{g_k - S_k}{g_k}\right)$$

To linearize the model, a new variable is defined as below:

$$P_k = \left(\frac{g_k - S_k}{g_k}\right) \tag{4.41}$$

$$P_k = \left(\frac{\sum_i \sum_{i<j} W_{ij} H_k - \sum_k O_k H_k}{\sum_i \sum_{i<j} W_{ij} H_k}\right) \tag{4.42}$$

$$\sum_i \sum_{i<j} W_{ij} H_k = \left(\frac{1}{1-P_k}\right) \sum_k O_k H_k = \sum_k O_k \left(\frac{1}{1-P_k} * H_k\right) \tag{4.43}$$

All equations are re-written by defining another new variable T_k. Also, the selected hubs are equal to p; $.\sum_k H_k = p$. The equations become as follow:

$$T_k = \left(\frac{1}{1-P_k} H_k\right)$$

$$\sum_k T_k = p\left(\frac{1}{1-P_k}\right) \tag{4.44}$$

Providing this inequalities between variables; $0 \leq T_k \leq H_k$ and the condition $H_k = 0$; T_k is force to be equal to 0 ($T_k = 0$). Thus; we obtain the concave function $(\frac{1}{1-P_k})$. This function can be converged using tangential cutting planes.

$$\left(\frac{1}{1-P_k}\right) = min\left[\frac{1}{\left(1-P_k^h\right)^2} + \left(\frac{1}{1-P_k^h}\right)^2\right] \tag{4.45}$$

$$\left(\frac{1}{1-P_k}\right) \leq \frac{1}{\left(1-P_k^h\right)^2} + \left(\frac{1}{1-P_k^h}\right)^2, \forall\, h\; \in H \tag{4.46}$$

$$\sum_k T_k \leq \frac{1}{\left(1-P_k^h\right)^2} + \left(\frac{1}{1-P_k^h}\right)^2, \forall\, h\; \in H \tag{4.47}$$

Finally, our proposed model is represented as:

$$Min \sum_i \sum_j \sum_k \sum_m C_{ijkm}\; X_{ijkm} \quad + \sum_k (F_k\, Y_k + \Gamma_k(g_k)) \tag{4.48}$$

s.t.

$$\sum_i \sum_j \sum_m X_{ijkm} + \sum_i \sum_j \sum_{m \neq k} X_{ijkm} = g_k\,, \qquad \forall k \tag{4.49}$$

$$\sum_k \sum_m X_{ijkm} = W_{ij} \qquad \forall i,j \tag{4.50}$$

$$\sum_m X_{ijkm} + \sum_{m \neq k} X_{ijkm} \leq W_{ij}.H_k \qquad \forall\, i,j,k \tag{4.51}$$

$$X_{ijij} \geq W_{ij}(H_i + H_j - 1) \qquad \forall\, i,j \tag{4.52}$$

$$\sum_{k\; \neq j} X_{ijik} \geq W_{ij}(H_i + H_j - 1) \qquad \forall i,j \tag{4.53}$$

$$\sum_{k \neq i} X_{ijkj} \geq W_{ij}(H_j - H_i) \qquad \forall i,j \tag{4.54}$$

$$\Gamma_k(g_k) = e\left(\frac{\sum_i \sum_{i<j} C_{ij}}{N}\right) P_k \tag{4.55}$$

$$\sum_k T_k \leq \frac{1}{\left(1-P_k^h\right)^2} + \left(\frac{1}{1-P_k^h}\right)^2, \forall\, h\; \in H \tag{4.56}$$

$$0 \leq T_k \leq H_k, \qquad \forall\, i,k,l \tag{4.57}$$

$$X_{ijkm} \geq 0\ ,\ H_k \in \{0,1\}\ ,\ g_k \geq 0\ , \qquad \forall\, i,j,k,l \in N \tag{4.58}$$

$$C_{ijkm} = \gamma C_{ik} + \alpha C_{km} + \delta C_{mj} \tag{4.59}$$

$$\Gamma_k(g_k) = e\left(\frac{\sum_i C_{ij}}{n}\right)\left(\frac{g_k - S_k}{g_k}\right) \tag{4.60}$$

Benders algorithm doesn't especially need that linearization operations to determine lower bounds. The algorithm may utilize pareto optimal cuts taking the advantages of the duality to achieve optimality. However, these revisions could speed up the computational performances of the algorithm.

4.3 Particle Swarm Optimization Algorithm

Particle swarm optimization (PSO) is one of the evolutionary algorithms used in the optimization literature. It is a population based heuristic algorithm that mimics the movements of bird flocks and fish schools in the nature. Particle swarm optimization which has been first proposed by Kennedy and Eberhart (1995) is one of the heuristic algorithms recently utilized in most of the optimization problems. Then, Shi and Eberhart (1999) have developed this evolutionary technique for engineering and computer science. Since this technique performs better with low computational cost and easy implementation, researchers focus on more this technique than existing evolutionary techniques such as genetic algorithm. The algorithm is developed based on three simple rules:

- The information of located food obtained by a bird immediately is transmitted to the swarm.
- The swarm is oriented to the target or food related to each particle's states, but not directly.
- Every member of the swarm has their own independent instinctual behavior and memory of past experiences. This is applied to the algorithm as a component of the structure.

Particle Swarm Optimization (PSO) is a bio-inspired algorithm that mimics the intelligent swarming behavior (information exchange and sharing) that flock of birds deploy in order to swarm in the direction of a good outcome or target (Sen and Krishnamoorthy, 2018). In the context of an optimization problem, a PSO algorithm starts with different points in the solution space to attain global optimal solution and shares the acquired information to determine good search spaces. Each particle is an initial feasible solution. Many such particles are randomly generated, and these moves around in the solution space under some well-defined guidelines (Sen and Krishnamoorthy, 2018).

To apply our hub location model, each particle is a feasible solution which is represented by two different arrays: selected hubs and allocation array. Installing hubs are randomly selected and all nodes are randomly allocated to hubs. Thus, the particles are generated, and each particle has the information of selected hubs, assignments of non-hub nodes and the objective function value of that network design.

PSO is initialized with random solutions assigned to the population, similarly to genetic algorithm (GA) (Jia, Zheng, Qu, and Khan, 2011). In contrast to GA, some features such as a randomized velocity is associated with each potential solution Then, these potential solutions, namely particles, are distributed over the problem space. Each particle improves its earlier best solution in the problem space which is achieved so far through previous experiences to stochastically converge towards the best solution in the swarm. The notation $lbest$ represents best solution of each particles. Another indicator of the progress of the swarm in terms of overall best fitness value and its location decision is the best solution of any particle in the swarm. The notation $lbest$ represents best solution of the swarm. The algorithm runs iteratively while the velocity of each particle is varied according to its lbest and gbest locations.

Every particle improves its earlier position through previous experience to stochastically converge towards the best position in the swarm. Whereas the position of the particle holding the best fitness value of the swarm is entitled as global best; the position of the particle holding best fitness value of itself until the current state is defined as local best. These values are kept in the memory of algorithm. Consequently, PSO algorithm is based on a convergence where the positions of the particles in swarm are getting closer to the best position of the swarm.

For an optimization problem, the decision variable $X = (x_1, x_2, \cdots, x_n)$ is n-dimensional variable, and the variables are called the particles. Assuming the solution space is n-dimensional solution space, each particle has two state descriptions: position $X_i = (x_{i1}, x_{i2}, \cdots, x_{in})$ and velocity $V_i = (v_{i1}, v_{i2}, \cdots, v_{in})$. The position X_i represents a feasible solution of the problem. Particles move in a multidimensional solution space and replace iteratively. Thus, their positions or states which are used to identify them like velocity change at each iteration. The target of each particle is the best solution achieved in the swarm. Thereby, particles are oriented to the optimum solution respectively to their current positions and states and previous experiences and the experiences of their neighbors (Wu and Wang, 2014). A velocity vector is utilized to describe the direction and the magnitude of the movement of each particle. At each iteration this velocity function updates the particles to direct them to be desired search direction. Updating the particles includes two sub-processes. One is an update for velocity vector and the second is a position change of the particle related to its movement. The velocity update function is given in the following equation:

$$v_{ik+1} = w_k v_{ik} + c_1 r_1 (lbest_{ik} - x_{ik}) + c_2 r_2 (gbest_{ik} - x_{ik}) \tag{4.44}$$

In the equation above, $w \in [w_{min}, w_{max}]$ is the inertia factor. r_1, r_2 are generated randomly between [0,1]. c_1, c_2 are the weights associated with the social moves versus cognitive moves. c1 represents the social learning coefficient of the particles and is useful to calculate the local optimum while updating the velocity vector. In other words, c_1 helps each particle to move based on its own previous experiences. In the same manner, c_2 represents the cognitive learning coefficient which identifies the contribution of the global optimum in recalculating velocity vector. Kennedy and Eberhart (1995) and Shi and Eberhart (1999) have shown that when $c_1$and c_2 parameters take the common value of 2, the algorithm performs better. The position update function is given in the following equation. Finally, the next position of the particle is calculated as follow:

$$x_{ik+1} = x_{ik} + v_{ik+1} \tag{4.45}$$

In order to prevent a particle from leaving far away out of the search space, the elements of the velocity vector are restricted in a predetermined minimum and maximum values. If the velocity of a particle would reach at w_{max}, then the velocity is left at w_{max} and could not exceed w_{max}. Figure 4.1 represents a pseudocode of the algorithm.

```
Pseudocode of the algorithm of PSO

Identify all parameters neede and assign values
Initalize all particles;
while termination criteria (optimum or number of iteration)is not met
do
{
for each particle do
Calculate fitness value;
If fitness value < particle'sbest − known value (local optimum);
end if
}
end for
Choose the particle with the best fitness value of all paritcles
as the global best position (global optimum);
for each particle do
{
Calculate particle velocity;
Update particle position;
Calculate fitness value;
Update local optimum;
Update global optimum;
}
end for
```

Figure 4.2 : Pseudocode of PSO.

4.4 Adaptation of PSO to Solve Congested UMAHLP

The heuristic algorithm, PSO, is a real-coded algorithm and is generally exercised to solve continuous optimization problems. For discrete optimization problems, the general structure of the algorithm would remain same, but it could still be adapted with the help of some hybrid approaches such as genetic operations or list-based representation. The hub network is represented as an $n \times n$ matrix to express the flows or costs between nodes. Similarly, the selected nodes to become hubs can be expressed in a binary $n \times 1$ matrix where a '1' means an allocation and a '0', otherwise. So, these representations can make our discrete hub location problem solvable by PSO's designed to solve the continuous optimization problems.

Next, we design a modified PSO algorithm, the representations used in the procedure, the feasibility conditions defined in the model. The algorithm is improved by genetic operators. Also, the particle positions are improved and updated by a local search strategy. Following improvements have been applied to enhance the performance of the new algorithm, including among others:

- Feasibility of particle positions is kept by the help of representation. Note that this representation does not consist of scalars such as distance of a particle to the best position, but two n dimensional arrays: $LocatingArray$ and $AllocatingArray$;
- Most PSO algorithms employ two values, $lbest$ and $gbest$ which are determined using respective update formulas ($lbest$ and $gbest$ denote the objective function values at local and global particle positions, respectively).
- To get a new feasible solution, the particle positions should be updated. In the proposed algorithm, we use the genetic operator 'mutation' in the update formula to update the particle positions;

A local search strategy is applied to the initial population and the update process. This strategy decreases the probability of a particle's getting trapped in a local optimum to further enhance the search ability of the algorithm. The proposed algorithm will be detailed in the following subsections.

4.4.1 Particle representation

This algorithm utilizes two n-dimensional arrays (vectors): $LocatingArray$ ($\boldsymbol{x}$) and $AllocatingArray$ ($\boldsymbol{h}$) to represent a feasible solution (i.e., a particle position). The total number of nodes, n, at network becomes the dimension of these arrays. $LocatingArray$ consists of 1's and 0's, whether a node is assigned as a hub or not. $AllocatingArray$ represents the allocation decisions of no-hub nodes where the nodes take the number of its corresponding hub as a value. Figure 4.2 illustrates an example of $AllocatingArray$ (i.e., [2, 2, 3, 2, 7, 3, 7, 3, 3, 7]) for a network consisting of 10 nodes. Moreover, each hub is evaluated as their associated flows go through themselves in the $AllocatingArray$. The location of hubs and allocation of non-hubs are described simply and accurately at once by these representations.

On the other hand, the algorithm utilizes a cost matrix derived from unit cost matrix of nodes (C_{ij}) through location and allocation assignments of particle by evaluating alternative paths for (i, j) pairs according to minimum cost. A direct link between (i, j) is not permitted, but the algorithm tries to obtain minimum cost for each (i, j) pair by selecting different hubs to assign them.

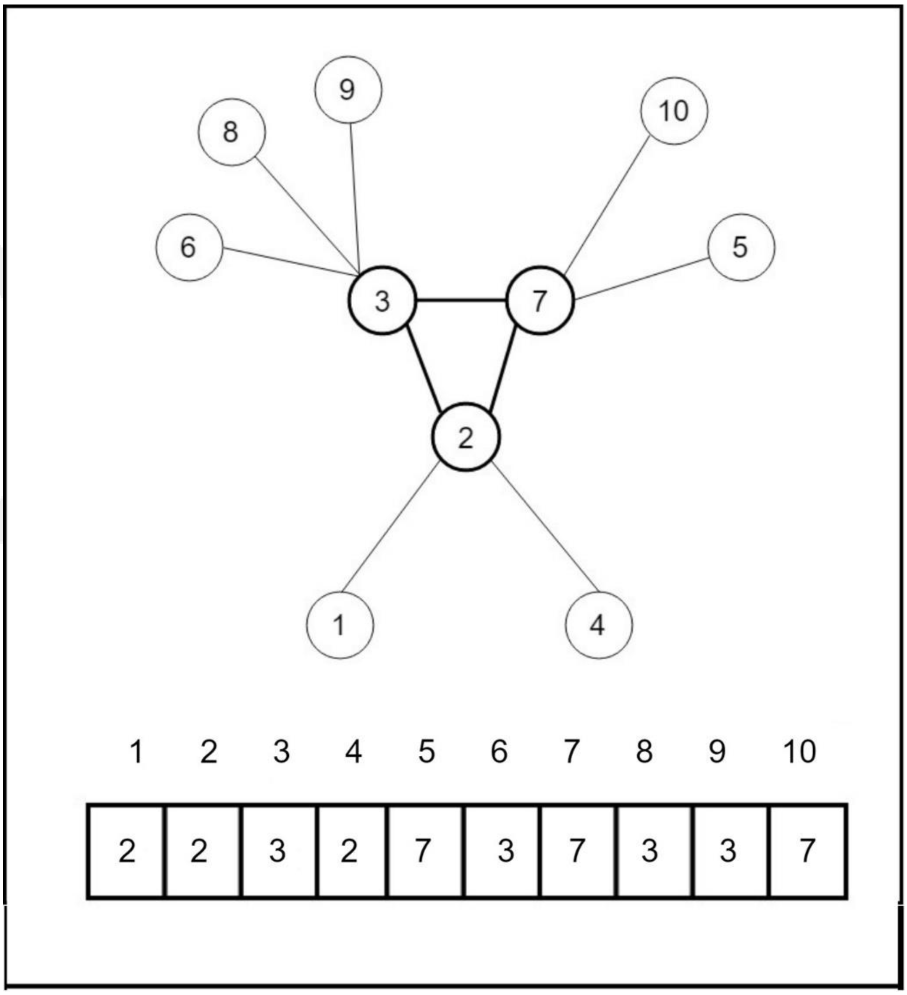


Figure 4.3 : A list-based network encoding.

4.4.2 Particle evaluation

The generation of initial particle positions starts with creating two random arrays: $LocatingArray$ and $AllocatingArray$. An $n \times n$ matrix $(\hat{X})$ is generated randomly. After its diagonal elements are sorted sequentially, appropriate number of hubs is selected to form $LocatingArray$. To establish $LocatingArray$, the number of hubs is not predetermined just limited to total number of nodes in network. Differently from earlier uncapacitated models; a predetermined number of hubs (p) to be selected does not exist. So, the number of hubs to select is also randomly determined. Generating of particles sequentially up to population size, all feasible particles, namely swarm, are produced. Total cost of the network which includes transportation, opening and congestion costs is utilized as fitness function to evaluate the particles. As a result, particles of smaller objective values are assigned larger fitness values.

4.4.3 Particle update rule

Initially, particle swarm optimization was designed to solve continuous optimization problems. Thus, some improvements are required to adapt the procedure to discrete optimization (Bailey, Ombuki-Berman, & Asobiela, 2013). In order to overcome the difficulties of this discrete structure, the update rules of the heuristic algorithm are modified. The algorithm takes the magnitude of the change during particle update to be the difference between a particle as current position and the best-known position in the swarm and neighborhood. So, the PSO algorithm even in discrete space aims essentially to design an update procedure considering both effect of the local position and global position in some manner.

Sen and Krishnamoorthy (2018) have stated that this process of information exchange is done through two mechanisms in the literature such as defining an import mechanism to copy certain elements of the best-known solution and defining some genetic operators (crossover, mutation, etc.).

In our algorithm, new feasible solutions are obtained in update procedure by using genetic operator ‘mutation’. This mutation operator applies to $LocatingArray$ of a particle and corresponding $AllocatingArray$ is modified. Genetic operator ‘mutation’ randomly modifies the particle positions by converting first generated $n \times n$ matrix called ‘$\hat{X}$’. Swapping rows, columns, elements and diagonals of this matrix or exchanging selected hubs to non-hub nodes are the functions of mutation process. After that, the corresponding $AllocatingArray$ is also modified. Figure 4.3 shows some examples of these functions.

Considering premature convergence is still the main deficiency of many PSO algorithm (Hu, Shi, Eberhart, 2004), it is possible for the proposed algorithm to possibly get trapped in a state where all the particles will become unable to discover better solutions. Thus, the local search strategy is applied to improve particle positions.

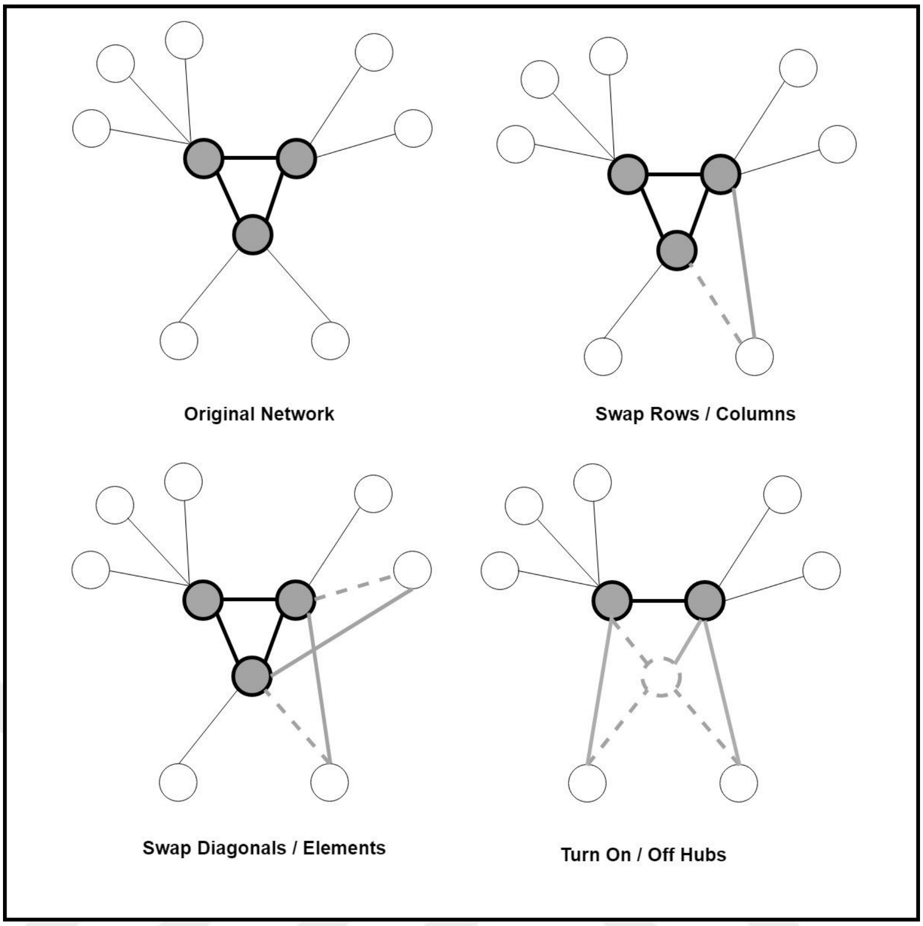


Figure 4.4 : Swap and change hub functions.

4.4.4 Exploitative processes

In the literature, one of the reasons that PSO is attractive is that the algorithm converges very fast comparatively to other heuristic algorithms. This speed of the convergence becomes advantageous in the optimization; however, it causes a stagnation after finding a local minimum. A local search procedure as an optimization method can become beneficial to master this difficulty and reach at better solutions. The local search procedure developed by Aarts and Lenstra (2003) consists of generating a local optimal solution by exploring the neighborhood of a given initial solution. However, a local optimal solution in a neighborhood structure is not necessarily a local optimal in another neighborhood structure (Yang, Liu, & Zhang, 2011). For this reason, the use of several neighborhood structures helps guiding the local search to converge less rapidly to a local optimum (Yang et al., 2011).

Our local search procedure attends generating neighborhood solutions by modifying the $AllocatingArray$ matrix. It does not affect the location of hubs but rearranges the allocation decisions. Shifting the allocated non-hub node to another hub or swapping two allocated non-hub nodes to each other are the operations done by local search. These reassignments are applied until $AllocatingArray$ cannot show more improvements.

5. COMPUTATIONAL RESULTS

In this thesis, a design of hub network where congestion may be occurred due to overflows at hubs are constructed. Although mathematical formulations of the model may achieve the optimality, the size of the problem and the computational time make the problem more complicated. Therefore, different solution procedures are needed to solve the problem.

First, we propose Benders decomposition algorithm. However, the algorithm become more time-consuming that expected. To find near optimal solutions for large-scaled problems, Particle Swarm Optimization algorithms are proposed throughout this thesis. We present the results of two proposed solution procedure in this section.

The results of distinct models presented in Chapter 3 are evaluated deeply. We also discuss the effectiveness of the two functions of congestion: power-law and delay congestion. We present the performance evaluation of proposed algorithm in terms of both solution quality and computation time. We evaluate the proposed algorithms using AP data sets. We also compare the results of the various data instances in terms of algorithm performance such as CPU time (seconds) or gap (the relative difference between obtained results (i.e. total costs) and the best result (i.e. “min. total cost”) of the heuristic algorithm.

In this chapter computational studies to test both the mathematical formulations and solution methodologies are constructed. We code Benders decomposition algorithm in C++ and run it on a laptop with Intel Core i7 CPU, 3.5 GHz processor and 16 GB RAM. We use *“Concert Technology"* package of IBM CPLEX 12.7 (CPLEX) to apply Benders decomposition. The particle swarm optimization algorithm is coded in MATLAB Version R2012a. Runs were made on a laptop with Intel Core i7 CPU, 3.5 GHz processor and 16 GB RAM. Table 5.1 indicates empirically established the parameters of our algorithm. Each experiment is organized with 50 runs of the algorithm and were terminated after 250 iterations. The learning factors are assigned as the best convergence of the proposed heuristic algorithm.

Table 5.1 : Particle Swarm Optimization Parameters.

Parameter,	Value
Iterations	250
Swarm size	150
Inertia Factor	0,9
Social Learning Factor	1.5
Cognitive Learning Factor	2

AP data sets are utilized for our computational experiments. Ernst and Krishnamoorthy (1996) have presented the AP data set derived from case of Australia Post. All data sets contain distance, cost and flow parameters for associated network sizes which can be acquired from OR-Library (Beasley, 1990a).

On the other hand, the AP data set includes the fixed costs and capacities for the nodes which consists of 200 nodes. Here, it is important to recall the definitions of capacity and fixed cost terms. The term "capacity" represents the quantity of flow that can be redirected or sorted by a hub. The term "fixed cost" represents the cost associated with the establishment of a node as a hub. The AP sets possess the inequalities such as $\gamma \neq \delta$; $W_{ij} \neq W_{ji}; W_{ij} \neq 0$. Thus, the discount factors for collection, transfer and distribution costs γ, α, δ are valued as (3, 0.75, 2).

The AP data sets are classified related to two types of fixed costs and capacities: loose (L) and tight (T). Ernst and Krishnamoorthy organized the data set where tight fixed cost is assigned to the nodes with high volume of flows. Similarly, the nodes with high volume of flows is associated with tight capacities. Thus, there exists four cases of the data instances: LL, LT, TL and TT.

Before pointing out the computational experiments conducted, it is beneficial to illustrate one of the optimal solutions, for the AP10LL instance with $\alpha = 0{,}75$ is provided, to depict the possible, solution to our problem. The optimal solutions are achieved for both algorithms, Benders decomposition and particle swarm optimization. The objective function is determined as 225.563,184 and the hub nodes 4, 5 are selected. The non-hub nodes 2,6 are assigned directly to hub 4 and remaining nodes (1,3,7,8,9,10) to hub 5.

However, for AP50LT instance with $\alpha = 0{,}75$, the optimal solutions of proposed algorithms are differed from each other. In the heuristic solution, the number of selected hubs is 5 whereas, the network consists of 4 hubs in Benders decomposition solution. Hubs 14,19, 39,46,48 are selected in heuristic solution whereas 46 is extracted from hub set in exact solution.

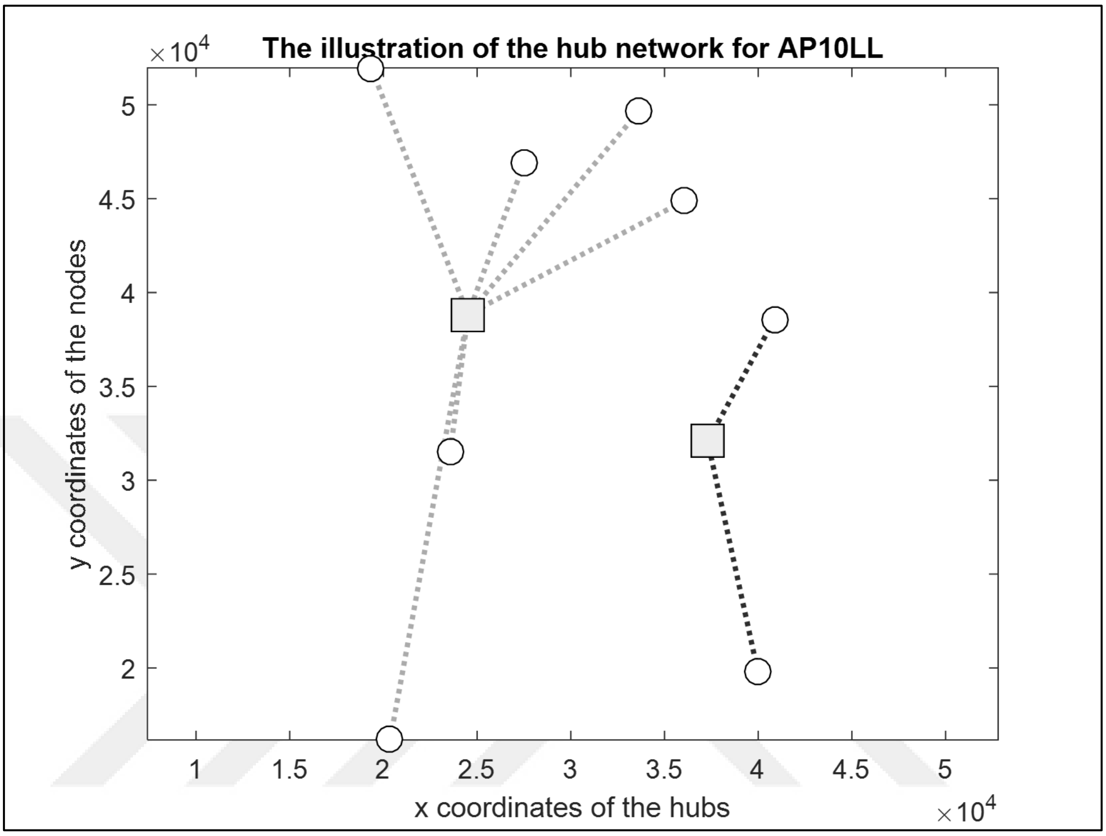


Figure 5.1 : Optimal Solution for AP10LL with $\alpha = 0{,}75$.

According to optimal values of the solutions, heuristic algorithm obtains better values in terms of minimization. This difference is the consequence of the Benders decomposition computational time restrictions. The figure 5.2 show the optimal solutions of two proposed algorithm.

Moreover, the assignments of non-hub nodes are different in each solution. For instance, the non-hub node 32 is assigned to hub 46 in heuristic solution where there is closer solution, hub 32 which we could see in figure of Benders decomposition. Here, it is important to recall that the heuristic procedure improves its assignments during update process iteratively and may become incapable of achieve all improvements due to iteration restrictions. This situation reveals the importance of developing efficient algorithms to solve hub location problems.

As mentioned above, particle swarm optimization algorithm is implemented using MATLAB. Before investigating the evaluation of the proposed congested model, the computational results of the problem by distinct solution procedures should be compared. For now, on, the heuristic algorithm (MATLAB) is compared with both *"Concert Technology"* package of IBM CPLEX 12.7 (CPLEX) and its implementation as a default Benders decomposition. CPLEX 12.7 helps to solve HLP instances of sizes less than 40.

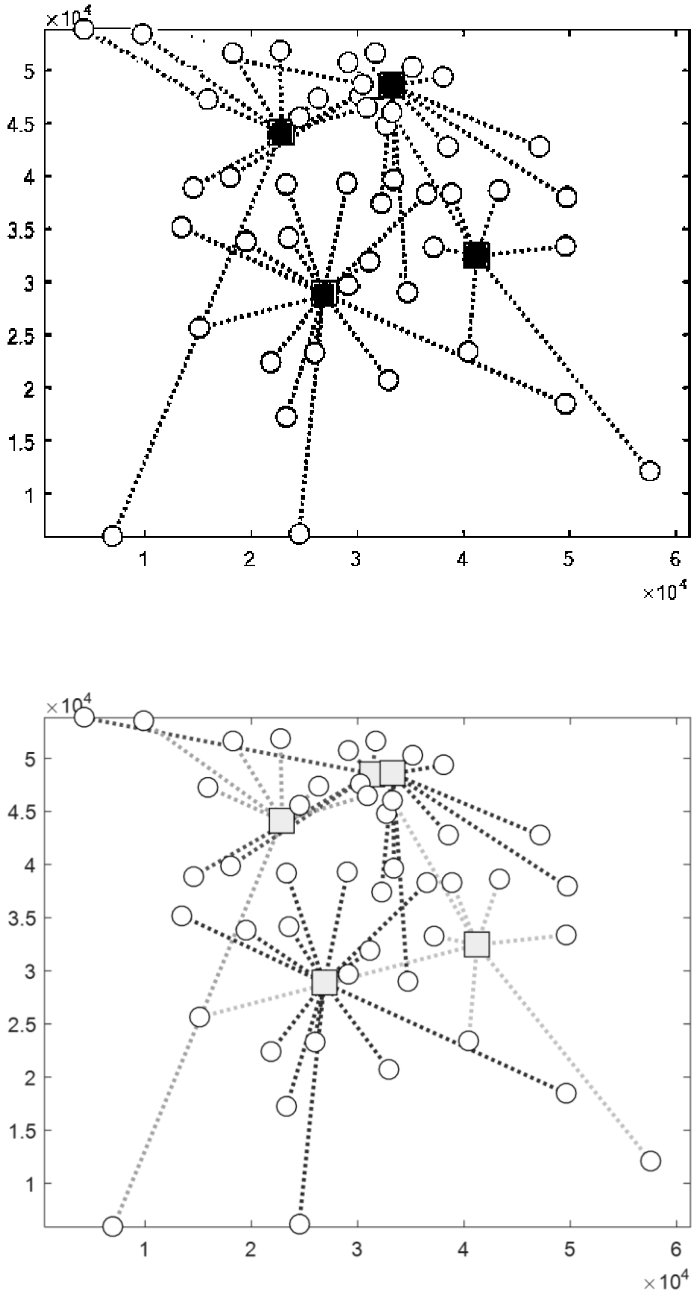


Figure 5.2 : Optimal Solutions for AP50LT (obtained by Benders decomposition top and obtained by PSO bottom).

In larger instances CPLEX is incapable of solving the problem mainly due to the insufficient memory capacities. Benders decomposition as a feature of CPLEX enhances the capability to solve larger instances under time constraints. Table 5.2 visualizes the performance comparison of the three-solution procedure.

Table 5.2 : Performance comparison of PSO and Benders decomposition.

		PSO		CPLEX	CPLEX featured by Benders decomposition	
	Optimal Value	Avg. no. of Iterations	CPU Times (sec.)	CPU Times (sec.)	Avg. no. of Iterations	CPU Times (sec.)
10LL	222158	127	49,85	0,49	6	0,13
10TT	277407	138	51,77	0,74	6	0,15
20LL	231533	138	55,37	8,74	6	0,42
20TT	292165	138	58,14	9,15	7	0,61
25LL	233905	140	69,47	50,75	7	3,07
25TT	312877	153	71,89	58,02	9	3,96

In Table 5.2, the instances are denoted by the notation nFC. This notation represents the network size (n), the types of fixed costs (F) and capacities (C). For example, 25LT means that we are presenting the solution of the respective problem in a network consisting of 25 nodes with loose fixed costs and tight capacity levels. Optimal value in Table 5.2 represents the optimal solution of the congested hub location problem.

When comparing three solution procedure, the number of the nodes become the most effective factor to determine the computational times. As seen from Table 5.2, both CPLEX solvers have smaller computational times while the instances are up to 40 nodes. However, with large scale data and tight constraints, MATLAB achieves less computational performance. Table 5.3 shows the computational efforts of three solution procedure for the instances from 40 to 200 nodes. In Table 5.3, asterix means that due to insufficent memory capacity CPLEX can not generate the solution of the problem.

As Camargo et al. (2011) stated that the larger instances have almost 6GB of data on the full diagonals of the constraint matrix making the problem unmanageable on regular computers without aid of decomposition techniques, commercial solver procedures cannot find the optimal solution for such instances in reasonable CPU times. Previously, Camargo et al. (2008) remarked that instances from sizes 40 to 200 have not yet solved to optimality for uncapacitated multiple allocation hub location problem in the literature.

Table 5.3 : Performance comparison of PSO and Benders decomposition for size more than 40 nodes.

	PSO		CPLEX	CPLEX featured by Benders decomposition	
	Avg. no. of Iterations	Times (sec)	Times (sec)	Avg. no. of Iterations	Times (sec)
40LL	162	69.85	*	10	109.38
40TT	228	71.77	*	12	147.22
50LL	208	89.47	*	11	431.70
50TT	227	91.89	*	15	494.55
100LL	241	245.29	*	14	2036.15
100TT	243	243.25	*	15	3146.05
200LL	245	748.92	*	17	79348.05
200TT	248	768.60	*	18	102249.10

Since modeling congestion with UMAHLP makes the problem obviously non-linear and harder to handle the complexity of the problem than UMAHLP, the proposed algorithm outperforms the others. Also, Alumur et al. (2018) tested their models on 100-node AP instances and claimed that larger instances are not solvable within an acceptable optimality gap in a reasonable time. They suggested that there is an obvious need to develop tailored exact or heuristic solution methodologies for these problems in the future.

5.1 Comparison of Solutions of Uncapacitated, Capacitated and Congested Hub Location Models

In Table 5.4, we summarize the best solutions of the heuristic algorithm for uncapacitated, capacitated and congested UMAHLP with different instances of AP data set.

Table 5.4 : Solutions of different versions of HLP for AP data sets for Γ= 0.8, $\alpha = 0.75$, E = 500.

	UNCAPACITATED						CAPACITATED			CONGESTED		
	Optimal Set of Hubs			Min. Total Cost			Optimal Set of Hubs	# of Hubs	Min Total Cost	Optimal Set of Hubs	# of Hubs	Min Total Cost
Instance	$p = 2$	$p = 3$	$p = 4$	$p = 2$	$p = 3$	$p = 4$						
40LL	14-28	11-12-28	6-11-22-28	229978,7	233979,4	246153,9	14-29	2	230642,3	14-29	2	230642,3
40TL	14-19	1-14-19	1-14-19-22	284900,2	302541,1	321123	14-19	2	284900,2	14-19	2	287223,4
40LT	14-28	11-22-28	6-11-22-28	229978,7	233980,7	247345,4	14-26-30	3	258871,5	14-19-26-30	4	283111,9
40TT	14-19	14-19-22	1-14-19-22	284900,2	303414,1	321252,6	14-25-40	3	344864,3	14-19-25-38	4	369192,3
50LL	15-36	3-27-35	6-22-27-35	228173,6	234528,3	247248,2	15-35	2	229126,9	15-35	2	229305,1
50TL	17-48	17-41-48	3-27-41-48	288757,3	308189,7	329536,4	3--24	2	304902,4	3--24	2	310497,3
50LT	15-36	3-27-35	3-22-27-35	228173,6	234528,3	245984	14-26-35-48	4	272709,5	14-25-32-46	4	291300,3
50TT	17-48	24-27-48	3-27-41-48	288757,3	289273,7	329536,4	12-26-41-48	4	389715,7	6-25-26-41	4	454483,1
100LL	29-73	29-52-70	29-39-46-73	231194,7	240180,2	255681,1	30-73	2	239957,0	30-75	2	245827,1
100TL	5-52	5-19-52	5-19-41-52	298916,3	320409,9	348255,8	44-52	2	343075,4	44-52	2	345837,3
100LT	29-73	29-52-70	29-39-46-73	231194,7	240180,2	255681,1	29-72-76	3	265035,0	26-29-68-69	4	289697,0
100TT	5-52	5-19-52	5-19-41-52	298916,3	318659,2	343441,8	5-34-41-76	4	482479,7	5-18-34-41-95	5	547864,0
	$p = 5$	5-12-19-41-52		376449,5								
200LL	43-148	40-195-148	30-61-120-148	228221,2	238115,4	260167,6	43-139	2	239092,7	43-159	2	249480,6
200TL	50-120	50-90-186	51-54-122-168	265837,4	277764,9	286923,6	95-186	2	281684,0	73-133-141-158	4	329721,0
200LT	43-148	40-195-148	30-61-120-148	228221,2	238115,4	260167,6	80-133-170	3	288161,9	96-111-133-171	4	326262,8
200TT	50-120	50-90-186	51-54-122-168	265837,4	277764,9	286923,6	52-53-168-186	4	328402,6	47-54-57-96-186	5	395954,7
	$p = 5$	25-52-57-113-186		313676,6								

For the congested model, the capacity level is determined as 80%. So, the problem congestion function is calculated for the excess flow at hubs which is larger than 80% of the hub capacity. The congestion function multiplies the excess flow at hubs by parameters: a scalar 'e' and the average of the transportation costs of the network '$(\frac{\sum_i c_{ij}}{n})$'.These parameters '$E = (\frac{e\sum_i c_{ij}}{n})$' are settled as 500.

The first analysis has been made to show the variation of the total cost and selected hubs comparatively for the uncapacitated, capacitated and congested versions of the problem. The obtained results are shown in Table 5.4. The congested version of the problem calculates the congestion cost proportional between the excess flow and total flow through each hub. This set of experiments are made to sophisticate the problem to learn the different behaviors of the model such as selected hubs or optimum cost range.

As shown in Table 5.4, if number of preferred hubs (p) to locate increases, total cost also increases. Although the uncapacitated model behaves independant from the capacities of hubs and prefers to locate hubs with lower opening costs, the capacitated and congested models tend to select nodes as hub with especially higher capacities and lower opening costs. The difference of the total costs between the models comes from locating greater number of hubs to optimize the flow distribution and hub loads.

Furthermore, in Table 5.4, the capacity levels turn out to be quite different in various models. The capacitated model utilizes 100% of the capacity whereas the congested one utilizes 80%. The congested hub location problem resembles the capacitated version of the problem in selecting hubs, especially for the instances with loose capacity. However, in the instances of tight capacity level, the congested HLP locates more hubs to balance the flow distribution and hub loads resulting in an increase in the total cost.

5.2 Comparison of Solutions of Hub Location Models with Different Congestion Cost Functions

In this section, we analyze the results of the models with different congestion functions: power-law function of total flow at hubs, power-law function of residual capacity of hubs and the proposed congestion cost function.

Table 5.5 : Solutions of different congested HLP for AP data sets for Γ=0.8, α=0.75, $\left(\frac{e\sum_i C_{ij}}{n}\right)$=500.

	CONGESTED BY $e.g_k{}^b$			CONGESTED BY $e.(g_k - S_k)^b$			CONGESTED BY $e.(\frac{\sum_i c_{ij}}{n}).(\frac{g_k - S_k}{g_k})$		
	Optimal Set of Hubs	# of Hubs	Min. Total Cost	Optimal Set of Hubs	# of Hubs	Min. Total Cost	Optimal Set of Hubs	# of Hubs	Min. Total Cost
40LL	11-22-28	3	292673,49	14-29	2	230642,33	14-29	2	230642,33
40TL	14-19-22	3	360427,31	14-19	2	286714,54	14-19	2	287223,43
40LT	11-22-28	3	292476,24	14-26-30	3	272063,51	14-19-26-30	4	283111,95
40TT	14-19-22	3	360749,43	6-14-19-38	4	351377,04	14-19-25-38	4	369192,38
50LL	15-27-35	3	295656,57	15-35	2	229214,48	15-35	2	229305,15
50TL	17-48	2	370061,13	3-24	2	309337,97	3-24	2	310497,33
50LT	2-27-35	3	295637,30	6-26-32-35	4	269247,98	14-25-32-46	4	291300,37
50TT	17-48	2	370061,13	6-26-41-48	4	388721,77	6-25-26-41	4	454483,11
100LL	29-52-71	3	306500,69	30-75	2	245700,63	30-75	2	245827,12
100TL	5--52	2	395220,73	44-52	2	344548,32	44-52	2	345837,31
100LT	26-52-70	3	307500,13	29-72-76	3	270174,31	26-29-68-69	4	289697,03
100TT	5-52	2	395302,57	5-41-52-95	4	450980,55	5-18-34-41-95	5	547864,09
200LL	57-95-148	3	316219,25	43-159	2	244263,93	43-159	2	249480,68
200TL	50-120-126	3	348833,17	95-186	2	291320,52	73-133-141-158	4	329721,01
200LT	50-120-126	3	312385,22	48-133-171	3	300085,79	96-111-133-171	4	326262,80
200TT	50-120-126	3	349662,55	57-113-168-186	4	329655,07	47-54-57-96-186	5	395954,70

Table 5.6 : Comparison of flow distribution ratios for UMAHLP with non-congested case and three different congestion cases.

	Flow Ratios of the Hubs													
	NON-CONGESTED		CONGESTED BY e. ${g_k}^b$			CONGESTED BY e. $(g_k - S_k)^b$				CONGESTED BY $e.(\frac{\sum_i C_{ij}}{n}).(\frac{g_k - S_k}{g_k})$				
	Hub no.		Hub no.			Hub no.				Hub no.				
	1	2	1	2	3	1	2	3	4	1	2	3	4	5
40LL	0,38	0,62	0,30	0,28	0,41	0,36	0,64	-	-	0,36	0,64	-	-	-
40TL	1	-	0,30	0,40	0,29	0,41	0,59	-	-	0,43	0,57	-	-	-
40LT	0,38	0,62	0,29	0,30	0,41	0,32	0,34	0,34	-	0,22	0,22	0,29	0,27	-
40TT	1	-	0,30	0,40	0,29	0,20	0,27	0,25	0,28	0,26	0,21	0,29	0,24	-
50LL	0,32	0,68	0,27	0,33	0,41	0,37	0,63	-	-	0,38	0,62	-	-	-
50TL	1	-	0,43	0,57	-	0,34	0,66	-	-	0,36	0,64	-	-	-
50LT	0,32	0,68	0,26	0,31	0,43	0,22	0,26	0,28	0,24	0,18	0,27	0,31	0,24	-
50TT	1	-	0,43	0,57	-	0,23	0,34	0,18	0,25	0,22	0,30	0,29	0,19	-
100LL	0,35	0,65	0,27	0,29	0,44	0,36	0,64	-	-	0,36	0,64	-	-	-
100TL	1	-	0,21	0,79	-	0,57	0,43	-	-	0,58	0,42	-	-	-
100LT	0,35	0,65	0,31	0,31	0,38	0,33	0,34	0,33	-	0,23	0,26	0,25	0,26	-
100TT	1	-	0,38	0,62	-	0,20	0,30	0,17	0,33	0,15	0,15	0,22	0,24	0,25
200LL	0,35	0,65	0,31	0,28	0,41	0,36	0,64	-	-	0,60	0,40	-	-	-
200TL	1	-	0,29	0,44	0,27	0,54	0,46	-	-	0,19	0,28	0,26	0,26	-
200LT	0,35	0,65	0,28	0,34	0,38	0,33	0,35	0,32	-	0,22	0,25	0,28	0,25	-
200TT	1	-	0,24	0,34	0,42	0,26	0,22	0,24	0,28	0,17	0,16	0,18	0,24	0,25

The results from Table 5.5 proves that the selected hubs are evolving according to the network size, congestion models and parameters. In particular; if one assigned more weight to congestion, the allocation of flows varies accordingly. The proposed congested model costs more than the other versions, because the scalar of the congestion function 'E' suppresses the model in such a way that the congestion cost gets smaller, partly balancing of the total cost. The model works naturally in such a way that hub loads stay strictly in their desired range. As an example, if this scalar 'E' is equal to 50, the total costs of the instances converge to other versions values and the selected hubs show more similarity.

The second analysis has been designed to analyze the distribution of hub flows affected by congestion and the variation of the cost (transportation/opening/congestion). The instances have been solved without congestion and using distinct functions of the congestion cost. The analysis assesses how the flows through hubs concentrate to the installed hubs. The results show that the number of installed hubs is highly affected from the contribution of congestion cost. Table 5.6 shows that the number of selected hubs is changing according to the congestion function, even if the congestion does not exist.

First power-law function which estimates all flows through hubs endeavors to balance the distribution of flows at hubs. It is not interested in preventing the overload of hubs. That is demonstrated by the transportation cost ratios seen in Table 5.7 which are less than the cost ratio of other functions. Second power-law function which deals only the excess flow at hubs considers both the avoidance of capacity surpluses and the homogeneity of the flow distribution. Still, there have been some occurrence of overloads of hubs which cause some minor congestion cost in the larger instances.

The proposed congestion cost strictly achieves the avoidance and the homogeneity by increasing the number of selected hubs. Thereby, it has higher opening cost percentage than other functions. The proposed function provides more reliable network in terms of the capacity usage. It is clear from Table 5.6 that the congestion whichever function is defined imposes the selected hubs on sharing even sum of flows through themselves. If the network has tight capacity, the ratio of congestion cost per total cost tends to be in the range. Compared to a non-congested network, the flow ratios of hubs become well balanced.

Table 5.7 : Comparison of cost ratios for UMAHLP with different congestion functions.

	NON-CONGESTED				CONGESTED BY $e.g_k{}^b$				CONGESTED BY $e.(g_k - S_k)^b$				CONGESTED BY $e.(\frac{\sum_i C_{ij}}{n}).(\frac{(g_k - S_k)}{g_k})$			
	Hub #	Percentage of Costs			Hub #	Percentage of Costs			Hub #	Percentage of Costs			Hub #	Percentage of Costs		
		TC	FC	CC		TC	FC	CC		TC	FC	CC		TC	FC	CC
40LL	2	0,76	0,24	0	3	0,54	0,27	0,19	2	0,77	0,23	0	2	0,77	0,23	0
40TL	1	0,86	0,14	0	3	0,59	0,26	0,15	2	0,78	0,22	0	2	0,78	0,22	0
40LT	2	0,76	0,24	0	3	0,54	0,27	0,19	3	0,63	0,32	0,05	4	0,61	0,38	0
40TT	1	0,86	0,14	0	3	0,59	0,26	0,26	4	0,55	0,44	0,01	4	0,55	0,45	0
50LL	2	0,75	0,25	0	3	0,54	0,27	0,27	2	0,74	0,26	0	2	0,74	0,26	0
50TL	1	0,82	0,18	0	2	0,52	0,26	0,22	2	0,70	0,30	0	2	0,70	0,30	0
50LT	2	0,75	0,25	0	3	0,55	0,26	0,19	4	0,56	0,43	0,01	4	0,56	0,44	0
50TT	1	0,82	0,18	0	2	0,52	0,26	0,22	4	0,49	0,47	0,04	4	0,51	0,47	0,02
100LL	2	0,77	0,23	0	3	0,56	0,26	0,18	2	0,75	0,25	0	2	0,75	0,25	0
100TL	1	0,89	0,11	0	2	0,63	0,15	0,22	2	0,68	0,32	0	2	0,68	0,32	0
100LT	2	0,77	0,23	0	3	0,57	0,26	0,17	3	0,68	0,29	0,02	4	0,64	0,36	0
100TT	1	0,89	0,11	0	2	0,63	0,15	0,21	4	0,54	0,40	0,06	5	0,53	0,47	0
200LL	2	0,80	0,20	0	3	0,60	0,23	0,17	2	0,76	0,24	0	2	0,76	0,24	0
200TL	1	0,86	0,14	0	3	0,61	0,23	0,16	2	0,82	0,17	0	4	0,64	0,36	0
200LT	2	0,80	0,20	0	3	0,59	0,23	0,17	3	0,64	0,30	0,06	4	0,66	0,34	0
200TT	1	0,86	0,14	0	3	0,62	0,22	0,16	4	0,66	0,32	0,02	5	0,63	0,37	0,00

Table 5.8 : Comparison of hub loads for UMAHLP with different congestion functions.

	NON-CONGESTED				CONGESTED BY e. $(g_k - S_k)^b$				CONGESTED BY $e.(\frac{\sum_i c_{ij}}{n}).(\frac{g_k - S_k}{g_k})$			
	Optimal Set of Hubs	Hub Loads			Optimal Set of Hubs	Hub Loads			Optimal Set of Hubs	Hub Loads		
		MIN	AVG	MAX		MIN	AVG	MAX		MIN	AVG	MAX
40LL	14-28	0,63	3,33	6,03	14-29	0,60	0,67	0,74	14-29	0,60	0,67	0,74
40TL	19	1,38	1,38	1,38	14-19	0,69	0,75	0,81	14-19	0,71	0,75	0,79
40LT	14-28	1,12	2,35	3,57	14-26-30	0,93	0,96	0,98	14-19-26-30	0,66	0,76	0,80
40TT	19	3,70	3,70	3,70	6-14-19-38	0,73	0,85	0,95	14-19-25-38	0,76	0,78	0,80
50LL	15-36	0,65	4,03	7,41	15-35	0,76	0,78	0,80	15-35	0,77	0,78	0,79
50TL	24	1,25	1,25	1,25	3--24	0,45	0,64	0,82	3--24	0,48	0,64	0,80
50LT	15-36	19,68	31,36	43,03	6-26-32-35	0,70	0,80	0,96	14-25-32-46	0,56	0,71	0,78
50TT	24	1,88	1,88	1,88	6-26-41-48	0,75	0,93	1,16	6-25-26-41	0,80	0,80	0,81
100LL	30-73	0,47	0,83	1,19	30-75	0,48	0,63	0,78	30-75	0,48	0,63	0,78
100TL	52	1,88	1,88	1,88	44-52	0,73	0,77	0,81	44-52	0,74	0,77	0,79
100LT	30-73	3,85	4,65	5,45	29-72-76	0,88	0,90	0,92	26-29-68-69	0,69	0,76	0,79
100TT	52	9,45	9,45	9,45	5-41-52-95	0,87	1,15	1,58	5-18-34-41-95	0,43	0,70	0,80
200LL	90-148	1,59	1,63	1,67	43-159	0,58	0,70	0,82	43-159	0,64	0,71	0,77
200TL	122	1,84	1,84	1,84	95-186	0,79	0,80	0,82	73-133-141-158	0,79	0,79	0,80
200LT	90-148	2,44	2,56	2,67	48-133-171	0,97	0,98	0,98	96-111-133-171	0,66	0,74	0,78
200TT	122	4,68	4,68	4,68	57-113-168-186	0,87	0,93	1,01	47-54-57-96-186	0,56	0,69	0,78

The third analysis assesses how the selected hubs load their associated flows related to their capacity. A hub load is the ratio between the total flow through a hub and its capacity level.

It is clearly stated in Table 5.8 that a hub in the non-congested networks could be loaded with an overhead flow, but in a congested network, congestion functions improve the hub loads. Anyway, the existence of some hub load ratios whose values are greater than 1 verifies that the congestion function aims to minimize the average congestion and does not imitate the capacity constraints. However, the proposed congestion function limits the hub load to a prespecified capacity usage percentage, too.

As shown in Table 5.8, average hub loads are less than 80%; because the proposed model tries to suppress the hub load less than the capacity usage used in the congestion function to set the minimum congestion cost. However, the model congested by power-law function does not force the model to utilize hubs below the prespecified level. Thereby, the model provides lower total cost as mentioned before. As can be seen from Table 5.8, for 50TT, 100TT and 200TT instances, congested model with power-law causes some overloads at hubs superior than desired level such as 1,16 1.58 and 1.01.

5.3 Sensitivity Analyses of Parameters Used in the Proposed Congested HLP

The remaining analyses investigate the variations of the model proposed according to the parameters such as capacity level (70%, 85%, 100%), economies of scale (0.2, 0.4, 0.6, 0.8), congestion function scalar 'E' (0, 25, 50, 100, 125, 250, 500, 1000, 5000). These experiments are made with the AP50LT instances.

In Table 5.9, 5.10, 5.11 and Figure 5.3, the sensitivity of the model versus different parameters are summarized.

Figure 3 shows that after the scalar 'E' takes a value larger than 100, the congestion cost becomes zero. Thus, the capacity limitation through hubs is ensured. While the transportation cost stays in the range from 150000 to 200000, the opening costs may vary too much, which may correspondingly increase the total cost.

Furthermore, Table 5.9 and Table 5.10 represent the behavior of the model with respect to balance of the hub flows and hub loads.

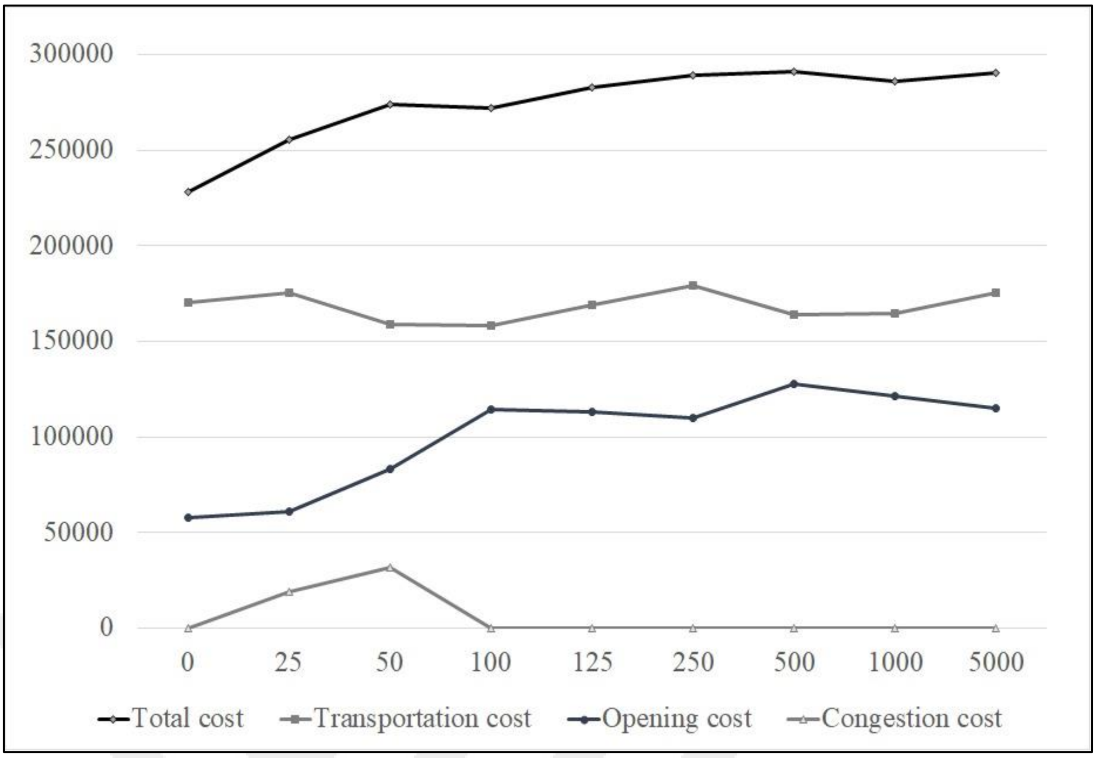


Figure 5.3 : Variation of cost components versus congestion function scalar 'E'.

The imbalance ratio of the hub flows decreases inversely proportional to the scalar 'E'. Likewise, an increase of the value of the scalar 'E' pushes the hub loads under the prespecified capacity usage percentage.

Table 5.9 : Network statistics of UMAHLP versus congestion function as scalar 'E'.

$E = e.(\frac{\sum_i C_{ij}}{n})$	Total Hub Flow (%)				Imbalance Ratio	Hub Loads				Average Hub Load
0	0,32	0,68			2,16	19,69	43,04			31,36
25	0,21	0,79			3,74	0,78	3,20			1,99
50	0,19	0,28	0,52		2,69	0,72	0,79	2,12		1,21
100	0,17	0,27	0,32	0,24	1,88	0,63	0,75	0,79	0,80	0,74
125	0,18	0,29	0,31	0,23	1,78	0,66	0,80	0,79	0,70	0,73
250	0,20	0,30	0,27	0,23	1,74	0,74	0,79	0,76	0,76	0,76
500	0,18	0,27	0,31	0,24	1,69	0,56	0,72	0,77	0,78	0,71
1000	0,26	0,28	0,28	0,19	1,49	0,79	0,78	0,69	0,61	0,72
5000	0,18	0,29	0,32	0,22	1,48	0,66	0,80	0,77	0,79	0,76

Table 5.10 shows the network statistics such as selected hubs, flow distribution, hub loads and cost ratios, while the capacity usage level is varied. Under different capacity usage levels, the model tends to select more hubs.

The total cost decreases proportionally to the opening cost ratio. This decrease indicates that the model restricts the selection of hubs with lower opening cost to extend permitted by the capacity usage level. The hubs’ loads do not exceed their capacity level, yet the more capacity level increases, the higher average hub load become.

Table 5.10 : Network statistics of UMAHLP versus distinct capacity usages.

Capacity Usage (%)	Hubs	Total Cost	Cost Ratio	Flow Distribution Ratio	Hub Loads	Average Hub Load
75	6-25-26-32-48	303776,82	0,56 0,44 0	0,19 0,22 0,22 0,23 0,14	0,69 0,60 0,62 0,57 0,64	0,63
80	14-25-32-46	291300,37	0,56 0,44 0	0,18 0,27 0,31 0,24	0,56 0,72 0,77 0,78	0,71
85	14-26-32-38	284024,13	0,57 0,43 0	0,27 0,16 0,33 0,24	0,84 0,44 0,83 0,84	0,73
90	6-26-32-38	275493,14	0,58 0,42 0	0,19 0,22 0,35 0,24	0,71 0,62 0,87 0,84	0,76
100	6-26-32-38	266053,31	0,61 0,39 0	0,21 0,35 0,23 0,21	0,78 0,98 0,58 0,98	0,83

In the existing literature, the values of economies of scale are equal to (3,0,75,2). To analyze the sensitivity of discount factor on models, the AP data set is modified as symmetrical. Therefore, the values of discount factors for collection, transfer and distribution costs (γ, α, δ) are settled as (1, 0.75, 1).

In addition, the tests are made under different values of economies of scale factor such as (0.2, 0.4, 0.6, 0.8) on AP50LT instance. Table 5.11 demonstrates that the model tends to utilize the hubs less than the other values for the greater economies of scale factor.

Table 5.11 : Detailed model results versus economies of scale factor for AP50LT instance.

Economies of Scale Factor (α)	Hubs	Total Cost	Cost Ratio			Flow Distribution Ratio			Hub Loads			Average Hub Load
0.2	6-26-32	155157,2	0,48	0,52	0	0,26	0,35	0,39	0,97	0,96	0,96	0,97
0.4	6-26-32	159388,2	0,49	0,51	0	0,27	0,36	0,37	0,88	0,99	0,93	0,93
0.6	12-26-32	163372,4	0,51	0,49	0	0,25	0,35	0,40	0,82	0,99	0,98	0,93
0.8	12-26-32	167556,2	0,52	0,48	0	0,26	0,35	0,39	0,84	0,99	0,96	0,93

5.4 Performance Evaluations of Particle Swarm Optimization Algorithm

The final analysis aims to express the performance of the algorithm relatively to the different versions of congestion (non-congested, power-law and proposed congested) and to evaluate how the particle swarm optimization parameters influence the solution procedure. The performance of the algorithm is examined with the factors such as computational times (CPU), number of iterations and gap which is the percentage of the relative deviation of each solution of heuristic run from the best solution. It is expected that the computational times increases proportional to the size of network. However, the computational times vary also with the allocation patterns, types of costs or capacities and the congestion factor as shown in Table 5.12.

Generally, the problems with tight fixed costs, capacities and with a high congestion factor require higher computational time. Furthermore, the power-law congestion model becomes more time consuming than the proposed model. The computational times increase much more if the power-law function is applied to all flow through hubs.

In Table 5.12 and Figure 5.4, the outputs of the algorithm for different spectrum of the problems (number of nodes, congestion factor, different set of cost and capacities) can be visualized.

Table 5.12 : The heuristics' efficiency for different congestion models.

	CAPACITATED			CONGESTED BY $e.\,{g_k}^b$			CONGESTED BY $e.\,(g_k - S_k)^b$			CONGESTED BY $e.\,(\frac{\sum_i c_{ij}}{n}).\,(\frac{g_k - S_k}{g_k})$		
	Avg. no. of Iterations	CPU Times (Sec)	Gap (%)	Avg. no. of Iterations	CPU Times (Sec)	Gap (%)	Avg. no. of Iterations	CPU Times (Sec)	Gap (%)	Avg. no. of Iterations	CPU Times (Sec)	Gap (%)
40LL	187,35	70,33	99,20%	214,60	77,06	97,25%	189,70	77,94	95,29%	162	69,85	94,90%
40TL	173,65	71,47	99,63%	178,25	71,86	98,63%	166	78,22	93,49%	194,40	70,01	94,67%
40LT	223,40	72,86	93,83%	209,35	73,25	97,83%	225,65	79,10	92,60%	224	72,02	88,61%
40TT	227,24	71,92	94,35%	179,45	71,91	98,24%	202,65	78,02	93,76%	228,30	71,71	87,76%
50LL	201,85	92,84	99,16%	226,25	93,69	97,87%	195,10	100,12	92,42%	208,90	89,47	91,38%
50TL	178,95	93,11	96,67%	198,40	92,98	97,65%	185,55	100,29	94,66%	199,75	89,82	94,22%
50LT	224,40	93,56	95,49%	218,60	93,77	98,05%	231,70	102,12	90,44%	231,75	92,06	83,32%
50TT	221,38	93,58	92,21%	200,65	93,35	98,35%	234,55	102,05	90,45%	227,70	91,89	81,56%
100LL	230,50	248,05	96,06%	242,55	253,56	97,55%	236,15	276,31	92,08%	241,95	245,29	93,85%
100TL	235,45	238,53	97,40%	236,55	250,86	95,80%	237,45	292,76	89,48%	240	244,70	84,10%
100LT	246,75	242,28	91,88%	233,70	261,58	97,59%	244,75	295,64	87,03%	244,40	249,69	80,18%
100TT	244,15	243,87	89,51%	234,35	257,54	94,63%	246,05	295,93	86,73%	243,25	249,98	78,12%
200LL	248,80	751,22	94,27%	247,10	816,66	97,20%	248,05	892,70	84,95%	248,20	748,92	85,65%
200TL	249,40	735,89	95,88%	248,90	802,38	96,91%	247,55	898,88	93,77%	244,55	764,53	76,52%
200LT	248,50	743,80	92,69%	247,14	900,09	95,87%	248,85	908,88	93,57%	246,80	780,48	79,95%
200TT	249	750,26	81,86%	247,78	825,84	97,48%	249,10	942,69	83,20%	245,40	768,60	75,84%

In Table 5.12, the iteration columns represent the average number of iterations for each heuristic run where the best solution of the algorithm is fixed and achieved. While the number of iterations is closer to 250, the heuristic algorithm might need more iterations to optimize the allocation decisions and the flow distributions for large data instances with tight factors. The instances with loose capacities or costs can be optimized elaborately, thus the gap results lie between 90% and 95%. Consequently, the earlier formulations are not as effective as the proposed formulation in consideration of the results.

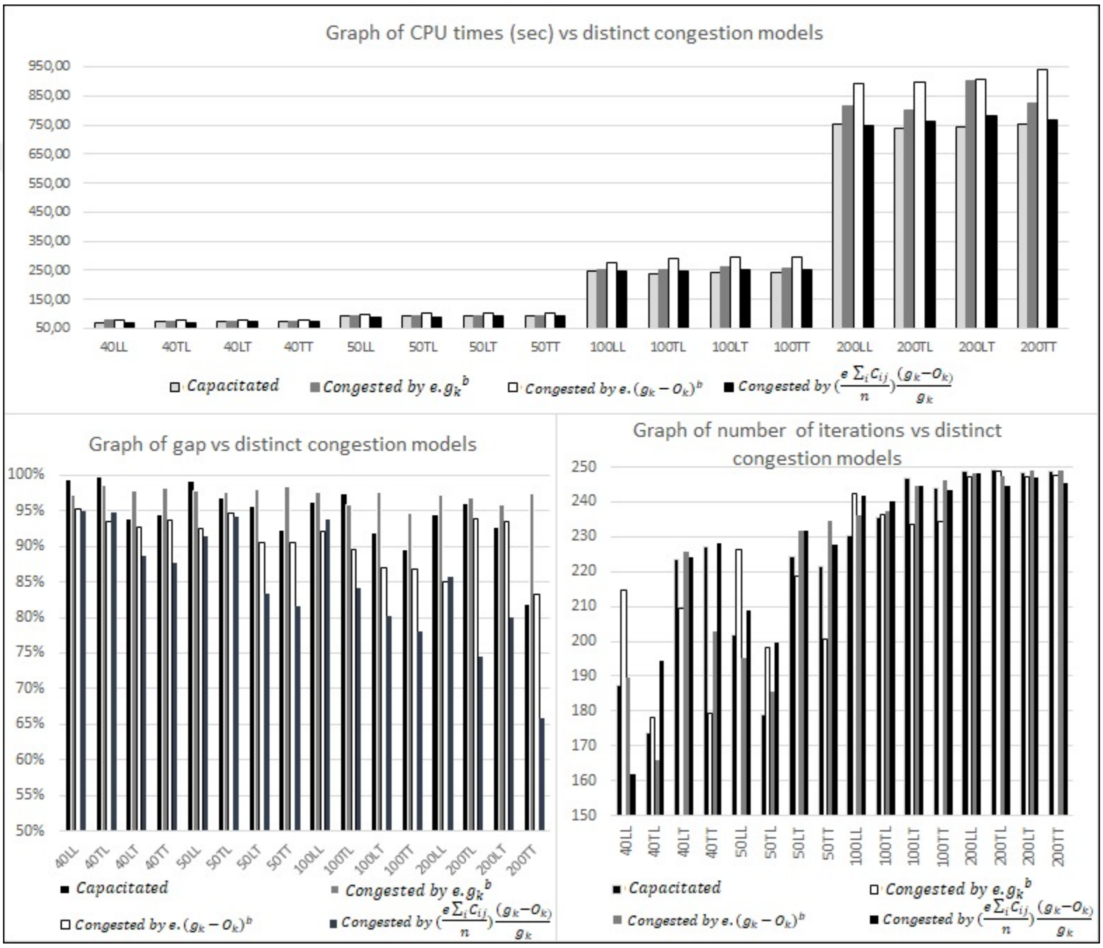


Figure 5.4 : The computational performance of the algorithm for distinct congested models.

Our proposed algorithm can achieve the optimality or find the best possible solution for the instances. There are some irregularities that the heuristic algorithm turned out to get worse results.

Generally, increasing number of nodes may cause ineffectiveness for the heuristic algorithms. In case most of these results could be classified as more complicated problems with tight capacities and fixed costs.

One of the significant cases is AP data instance 200TT with a gap of 75%. These can be explained as our heuristic solution is caught at a locally optimal neighborhood. The enrichment of the particle update and the local search procedure with more genetic operators can improve the results. Although the heuristic algorithm could take more time to compute, increasing the number of iterations in each run may become another improvement. The average computation times are 70, 90, 245, 765 seconds with gaps of 91%, 88%, 84%, 80% for networks with 40, 50, 100, 200 nodes, respectively.

Additionally, the maximum iteration number, swarm size, learning coefficients are varied to observe the heuristics' performance. The experiments show that the more iteration, the less gap achieved. The swarm size is an efficient factor to achieve the optimum, but it has a sharp negative effect on computational time. The learning coefficients may cause the problem to trap in a local optimum.

Figure 5.5 shows the sensitivity analysis of the heuristic algorithm for the chosen values of the selected parameters. Iteration numbers larger than 200 give better result for all cases, but the computational times show an excessive increase.

According to the gap results, it is advantageous to determine the iteration number as 250. The population size behaviors can be explained similarly. The best results are obtained in the range of 150-250 of population.

Incorporating of the learning coefficients for the particles can improve the effectiveness of the solution. The computation time is not affected by the change of the learning coefficients. In order to achieve better gap which are closer to optimum, the social learning coefficients should be set in the range from 1 to 2 and the cognitive learning coefficients should be equal to 2.

As it can be observed in Figure 5.5, higher ranges of the learning coefficients produce worse results in terms of gap which are more distant from optimum, because the heuristic algorithm converges fast and the particles are caught by their own local optimum.

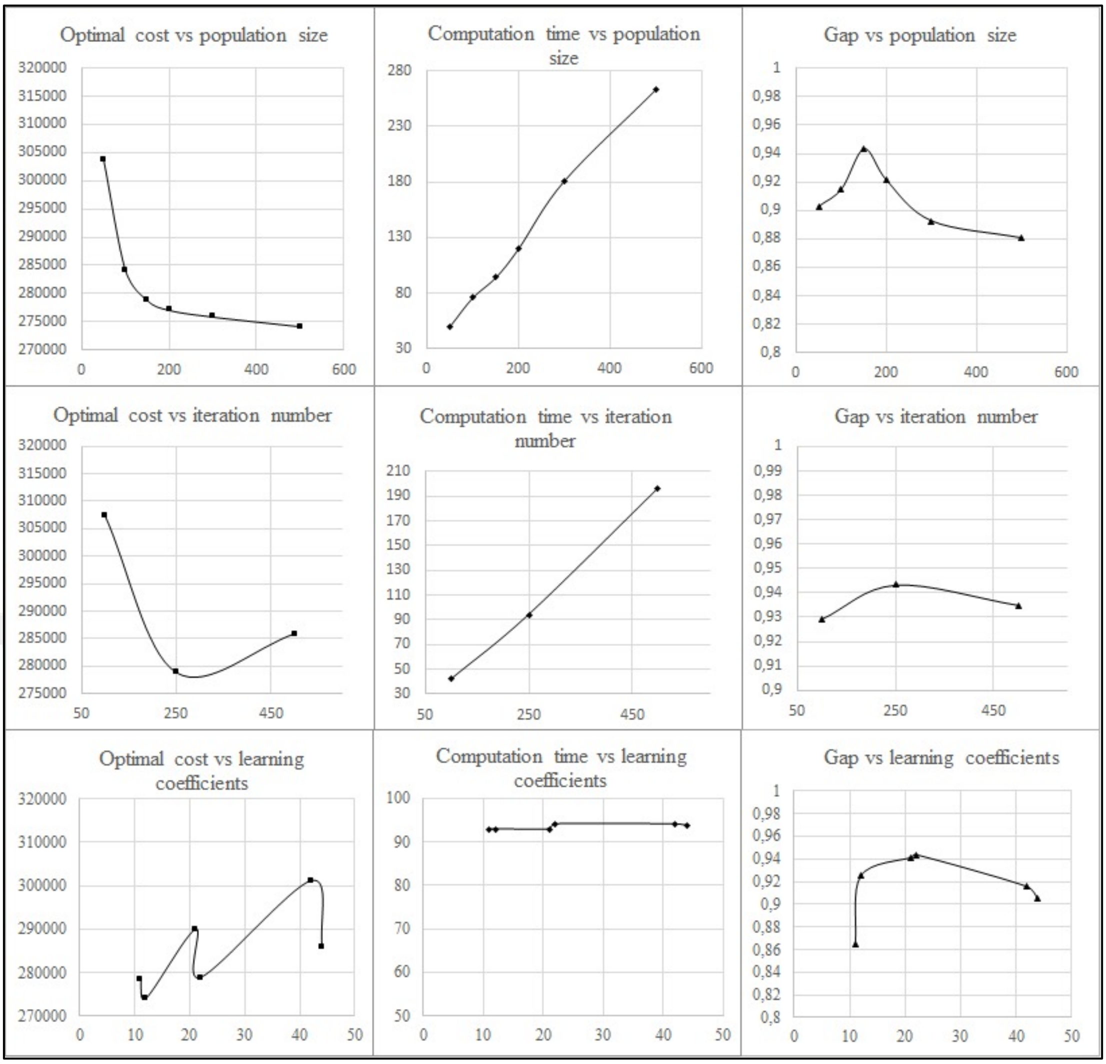
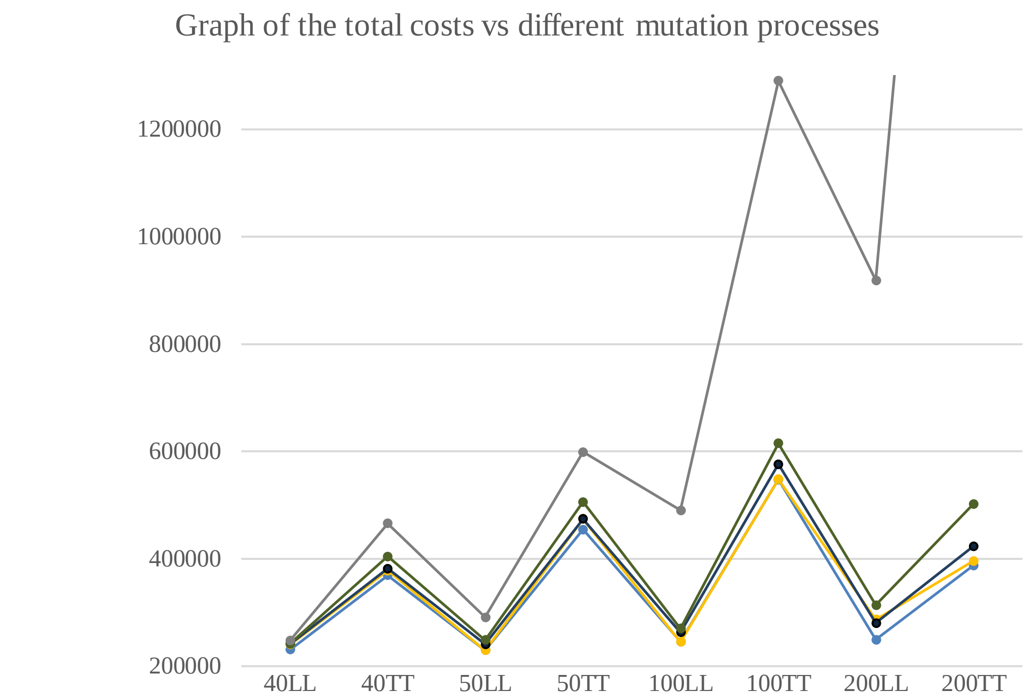


Figure 5.5 : Sensitivity of the heuristics' performance.

Moreover, in order to show the contribution of the genetic operator, mutation used in the update process and in the local search procedure, the particle swarm optimization is applied without mutation and with some of the operations of mutation separately. As mentioned before, swapping rows, columns, elements and diagonals of allocation matrix or exchanging opening hubs selection with non-hub ones are some of the functions of mutation process. These functions are applied to the update process one by one in sequence (i.e. not randomly) to see their corresponding improvement effects.

Figure 5.4 shows the changes in the solutions of the cases which are update process with swapping rows/columns, swapping elements/diagonals of the allocation matrix, turning on/off hubs and without mutation operator.

The results show that smaller total costs are achieved when the mutation operator is applied. The most useful operation of the mutation is swapping rows and columns, since this operation improves simultaneously both hub locations and allocation of non-hubs.



	40LL	40TT	50LL	50TT	100LL	100TT	200LL	200TT
ORIGINAL	230642	369192	229305	454483	245827	547864	249481	387516
SWAP R./C.	240532	378074	229264	474200	245240	548030	286692	395955
TURN ON/OFF	241498	381428	240995	474200	263158	576332	280430	423749
SWAP E./D.	242137	404612	248522	505584	270070	614764	313828	502255
NO MUTATION	247966	465692	291181	598720	490222	1290653	918912	2917541

Figure 5.6 : Comparison of the different mutation processes for pso algorithm.

On the other hand, the worst performing operation is swapping elements and diagonals of the allocation matrix. This is because this operation focuses only on the allocation decision of the network. In addition, it is important to remark that the computational times decrease by approximately 10%, while the mutation is not utilized in the update process and in the local search.

To sum up, the obtaining results from the analyses are listed as below:

- For the instances with loose parameters, congested hub location problems show similarity with the capacitated version in terms of selected hubs
- For the instances with tight parameters, the congested hub location problmes tend to select more hubs than uncapacitated and capacitated versions of the problem. The model aims to minimize congestion cost by selecting additional hubs to the network. So, the numbers of seleceted hub is more than the other

versions. Correspondingly, the total cost of congested network becomes higher.

- The results revealed that the congested hub location problems design more reliable and flexible networks.
- The congested models with distinct functions show some significant differences in terms of the flow ratios, the cost ratio and the hub loads.
- For the instances with tight parameters, the proposed model selects more hub than distinctly congested models. However, the difference between totat cost of the models changes slightly.
- The results of the proposed model show that the flows between non-hub and hub nodes distributed evenly to avoid an overhead loading at hubs.
- A minor congestion is occurred in the models with power-law congestion function, except the proposed model.
- Similarly, instead of the presence of some hub loads exceed its own capacity at the power-law congestion model, the proposed model provides strictly the hub loads less than a prespecified capacity level.
- The sensitivity analyses of the parameters in the proposed model state that the opening costs vary according to the pressure of the congestion function. This is consistent with the previous results.
- The imbalance ratio of the hub flows decreases inversely proportional to the multiplier of the congestion function. It is important to highlight whether the congestion multiplier affects excessively, the imbalance ratio between flows on hubs approaches to value of 1.
- As the capacity usage level increases, the model could focus on minimizing total cost by selecting hubs with lower opening costs. Otherwise, the proposed model prefers hubs with appropriate capacity levels regardless of their opening costs.
- Generally, the problems with tight fixed costs, capacities and with a high congestion factor require higher computational time. Furthermore, the power-law congestion model becomes more time consuming than the proposed model.

The computational times increase much more if the power-law function is applied to all flow through hubs.

- The optimal values of the parameters used in particle swarm optimization and the most benefical genetic operators are determined to improve the algorithm.
- Computational analyses of the performance of the heuristic algorithm show that the algorithm makes the solution procedure easier, especially in dealing with the non-linearity of the model.

6. CONCLUSIONS AND RECOMMENDATIONS

This thesis has considered congested multiple allocation hub location problem with a recent congestion definition. This definition aim to prevent the imbalance of flow distribution of selected hubs and the overloads at hubs, not to control strictly network with the capacity restrictions. A congestion cost function has been introduced as a convex function. By the way the problem becomes a nonlinear mixed integer programming problem. This revised congestion function is the proportion of the excess flow on each hub to the total flow through respective hub. This function assists in ensuring the capacity limitation of each hub without an explicit determination. Because of this, the proposed non-linear model needs a relatively more effortful solution procedure.

A heuristic algorithm, PSO, has been developed to handle the problem effectively, especially for the large instances. As far as known, this is the first study developing a heuristic solution procedure on multiple allocation HLP with congestion in the literature. The algorithm makes the solution procedure easier, especially in dealing with the non-linearity of the model. The algorithm has shown great effectiveness being able to solve large instances under reasonable computational times. However, the exact solution methodologies do not show comparable success to solve the NP-hard problems with some of the instances examined in this thesis.

AP data set is utilized to evaluate the models and the proposed algorithms. 16 distinct data instances were generated by classifying in number of nodes, tight-loose capacity and fixed costs. The analyses under different scenarios including the modification of congestion function or the variation in congestion factor, capacity level, economies of scale were made. Besides the sensitivity analyses to compare the heuristic efficiency under different parameters including number of iterations, population size, learning coefficients were examined.

The results revealed that a congested network causes higher total cost than a capacitated one for tight capacities or fixed cost. The number of selected hubs becomes the basic factor of that increase. To obtain a more homogeneous network, the congested problem tends to select more hubs and tolerate higher opening costs. Among the different models of congestion, total costs share similarity except from the congestion model which is a convex function of all flow through a hub and ends up with an exorbitant cost. The total cost and selected hubs differ slightly. Only, the proposed model provides more strict capacity usage causing slight increase of the total cost. However, the congested models show some significant differences in terms of the flow ratios, the cost ratio and the hub loads. Firstly, the transportation cost of a non-congested network is higher than a congested network. All congested models do not imitate the capacity constraint as mentioned before, so a minor congestion cost ratio exists in the solution except the proposed model. Subsequently, the flows between non-hub and hub nodes distributed evenly to avoid an overhead loading at hubs. Though the effect of the proposed model on the flow ratio of each hubs is more uniform than the power-law congestion model. Eventually, the hub loads at a non-congested network turn out to be more severe overload such as 7 times more than the capacities of hubs. Whereas the congested networks try to fix up the loading of flows to hubs according to their capacities. Instead of the presence of some hub loads exceed its own capacity at the power-law congestion model, the proposed model provides the hub loads less than a prespecified capacity level.

The analyses of the proposed model under different parameters such as congestion scalar, capacity level and discount factor presented the behaviors of the network under different conditions. It is important to highlight whether the congestion scalar affects excessively, the imbalance ratio between flows on hubs approaches to value of 1. Furthermore, the capacity level becomes dominant on the number of hubs selected. As high as the capacity is utilized, the number of hubs at a network is decreases. Finally, while the discount factor is greater, the hubs selected are utilized under their capacity level. Apart from that, both version of the discount factor tends to design network similarly.

In conclusion all of the previous results show that the congested networks behave unlike from the previous versions of the problem and become a distinctive aspect of the problem which will be investigated deeply as a further study. Congestion could not be thought only a control factor on the capacity of selected hubs, it is an additional cost, which affect the network design to ensure sustainability, reliability and foresight of the network’s requirement. Thus, considering both time, capacity and cost components of the congestion becomes a visionary approach. Besides, congestion could be defined in different mathematical and logical formulations. The flow lower than capacity and the flow larger than capacity could be identified as distinct congestion formulations such as cost or delay. For future research, presenting newer functions to characterize the congestion such as linear, exponentially incremental, piecewise linear function opens new prospects to the problem. Thereby, the structure of both network and mathematical model is evolving.

This thesis has presented a particle swarm optimization heuristic algorithm which is relatively a new solution procedure used in hub location problem. The analyses have demonstrated that the proposed heuristic algorithm may become an advantageous engineering tool to solve large instances with time effectiveness. Furthermore, our algorithm can be improved to get good upper bounds and get lower gap results for hub location problems. Future research may try to attach more improvement procedures and some genetic operators to update and exploitative processes. Comparing particle swarm optimization with distinct heuristic algorithms in terms of computational performance may be another future direction to investigate. Moreover, Benders decomposition algorithm may be enriched with branch and bound techniques to solve large scale problems. Consequently, future researches can be done to improve the model and proposed algorithm to reflect the real-life requirements of an HLP network. Future works may study other appropriate mathematical models and relevant solution procedures on HLP problems to achieve better computational performances and solutions.

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APPENDICES

APPENDIX A: Part of Particle Swarm Optimization MATLAB codes
APPENDIX B: Part of Benders Decomposition Algorithm CPLEX C++codes

APPENDIX A : Part of Particle Swarm Optimization MATLAB codes

```
%% Problem Definition

%model=SelectModel();                    % Select Model

CostFunction=@(xhat) MyCost(xhat,model);    % Cost Function
VarSize=[model.N model.N];      % Decision Variables Matrix Size
nVar=prod(VarSize);            % Number of Decision Variables

VarMin=0;          % Lower Bound of Decision Variables
VarMax=1;          % Upper Bound of Decision Variables

%% PSO Parameters

MaxIt=250;      % Maximum Number of Iterations

nPop=150;        % Population Size (Swarm Size)

w=1;               % Inertia Weight
wdamp=0.99;         % Inertia Weight Damping Ratio
c1=2.0;             % Personal Learning Coefficient
c2=2.0;             % Global Learning Coefficient

% Constriction Coefficients
% phi1=2.05;
% phi2=2.05;
% phi=phi1+phi2;
% chi=2/(phi-2+sqrt(phi^2-4*phi));
% w=chi;          % Inertia Weight
% wdamp=1;        % Inertia Weight Damping Ratio
% c1=chi*phi1;    % Personal Learning Coefficient
% c2=chi*phi2;    % Global Learning Coefficient
% Velocity Limits
VelMax=0.1*(VarMax-VarMin);
VelMin=-VelMax;

%% Initialization

empty_particle.Position=[];
empty_particle.Cost=[];
empty_particle.Sol=[];
empty_particle.Velocity=[];
empty_particle.Best.Position=[];
empty_particle.Best.Cost=[];
empty_particle.Best.Sol=[];

particle=repmat(empty_particle,nPop,1);
```

```
BestSol.Cost=inf;

for i=1:nPop

    % Initialize Position
    particle(i).Position=unifrnd(VarMin,VarMax,VarSize);

    % Initialize Velocity
    particle(i).Velocity=zeros(VarSize);

    % Evaluation
    [particle(i).Cost, particle(i).Sol]=CostFunction(particle(i).Position);

    % Update Personal Best
    particle(i).Best.Position=particle(i).Position;
    particle(i).Best.Cost=particle(i).Cost;
    particle(i).Best.Sol=particle(i).Sol;

    % Update Global Best
    if particle(i).Best.Cost<BestSol.Cost

        BestSol=particle(i).Best;

    end

end

BestCost=zeros(MaxIt,1);

%% PSO Main Loop

for it=1:MaxIt

    for i=1:nPop

        % Update Velocity
        particle(i).Velocity = w*particle(i).Velocity ...
            +c1*rand(VarSize).*(particle(i).Best.Position-particle(i).Position) ...
            +c2*rand(VarSize).*(BestSol.Position-particle(i).Position);

        % Apply Velocity Limits
        particle(i).Velocity = max(particle(i).Velocity,VelMin);
        particle(i).Velocity = min(particle(i).Velocity,VelMax);

        % Update Position
        particle(i).Position = particle(i).Position + particle(i).Velocity;

        % Velocity Mirror Effect
        IsOutside=(particle(i).Position<VarMin | particle(i).Position>VarMax);
        particle(i).Velocity(IsOutside)=-particle(i).Velocity(IsOutside);
```

```
    % Apply Position Limits
    particle(i).Position = max(particle(i).Position,VarMin);
    particle(i).Position = min(particle(i).Position,VarMax);

    % Evaluation
    [particle(i).Cost, particle(i).Sol] = CostFunction(particle(i).Position);

    % Mutation
    NewParticle=particle(i);
    NewParticle.Position=Mutate(particle(i).Position, model);
    [NewParticle.Cost, NewParticle.Sol]=CostFunction(NewParticle.Position);
    if NewParticle.Cost<=particle(i).Cost || rand < 0.1
       particle(i)=NewParticle;
    end

    % Update Personal Best
    if particle(i).Cost<particle(i).Best.Cost

       particle(i).Best.Position=particle(i).Position;
       particle(i).Best.Cost=particle(i).Cost;
       particle(i).Best.Sol=particle(i).Sol;

       % Update Global Best
       if particle(i).Best.Cost<BestSol.Cost

          BestSol=particle(i).Best;

       end

    end

end

% Local Search based on Mutation
for k=1:3
    NewParticle=BestSol;
    NewParticle.Position=Mutate(BestSol.Position, model);
    [NewParticle.Cost, NewParticle.Sol]=CostFunction(NewParticle.Position);
    if NewParticle.Cost<=BestSol.Cost
       BestSol=NewParticle;
    end
end

BestCost(it)=BestSol.Cost;

disp(['Iteration ' num2str(it) ': Best Cost = ' num2str(BestCost(it))]);

w=w*wdamp;
```

```
    %Plot Best Solution
    %figure(1);
    PlotSolution(BestSol.Sol,model);
    pause(0.01);

end
%% Results

figure;
plot(BestCost,'LineWidth',2);
xlabel('Iteration');
ylabel('Best Cost');
```

APPENDIX B : Part of Benders Decomposition Algorithm C++ codes

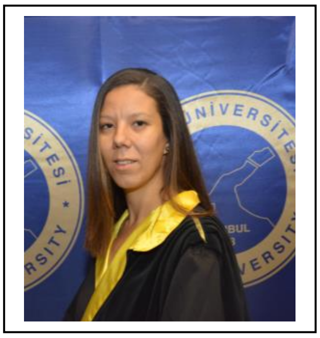
```
#include <ilcplex/ilocplex.h>

ILOSTLBEGIN

typedef IloArray<IloNumArray> TwoDMatrix;
typedef IloArray<IloNumVarArray> IloNumMatrix;
typedef IloArray<IloNumMatrix> IloNumMatrix3;
typedef IloArray<IloNumMatrix3> IloNumMatrix4;
typedef IloArray<IloNumVarArray> NumVarMatrix;
typedef IloArray<NumVarMatrix>   NumVar3Matrix;
typedef IloArray<NumVar3Matrix>   NumVar4Matrix;
typedef IloArray<NumVar4Matrix>   NumVar5Matrix;
int
main(int argc, char **argv) {

        IloEnv env;

        try {

                IloInt i, j, m, t, e = 500, a = 0;

                IloInt teta = 1;

                IloInt HUB, NODE;

                IloNumArray capacity(env);

                IloNumArray fix_hubcost(env);
                TwoDMatrix flow(env), link_cost(env);

                const char* filename = "tt.txt";

                if (argc > 1)

                        filename = argv[1];
                env.out() << filename << endl;

                ifstream file(filename);

                if (!file) {

                        cerr << "ERROR: could not open file '" << filename

                                << "' for reading" << endl;

                        cerr << "usage:   " << argv[0] << " <file>" << endl;

                        throw(-1);
```

```
}

file >> capacity >> fix_hubcost >> link_cost >> flow;
HUB = capacity.getSize();
NODE = fix_hubcost.getSize();
env.out() << HUB << endl;
env.out() << NODE << endl;
env.out() << link_cost << endl;
env.out() << fix_hubcost << endl;
env.out() << flow[0][1] << endl;
env.out() << capacity[1] << endl;


IloModel model(env);
IloNumVarArray open(env, NODE, 0, 1, ILOINT);
env.out() << open << endl;

IloNumVarArray congestion(env, NODE, 0, IloInfinity, ILOFLOAT);
env.out() << congestion << endl;


IloNumMatrix4 qq(env, NODE);
for (i = 0; i < NODE; i++) {
	qq[i] = IloNumMatrix3(env, NODE);
	for (j = 0; j < NODE; j++) {
		qq[i][j] = IloNumMatrix(env, NODE);
		for (t = 0; t < NODE; t++) {
			qq[i][j][t] = IloNumVarArray(env, NODE, 0,
IloInfinity, ILOFLOAT);
		}
	}
}
env.out() << qq[1][1][1][1] << endl;

/////////////////// //1
IloExpr phub(env);
for (t = 0; t < NODE; t++) {
	phub += open[t];
}
model.add(phub == 3);
phub.end();

//2
for (i = 0; i < NODE; i++) {
	for (j = 0; j < NODE; j++) {
		IloExpr d(env);
		for (t = 0; t < NODE; t++) {
			for (m = 0; m < NODE; m++) {
				d += qq[i][j][t][m];
```

```
                        }
                    }
                    model.add(d == flow[i][j]);
                    d.end();
                }
            }
            //3
            for (i = 0; i < NODE; i++) {
                for (j = 0; j < NODE; j++) {
                    for (t = 0; t < NODE; t++) {
                        IloExpr v(env);
                        for (m = 0; m < NODE; m++) {
                            v += qq[i][j][t][m];
                        }
                        for (m = 0; m != t && m < NODE; m++) {
                            v += qq[i][j][t][m];
                        }
                        model.add(v <= flow[i][j] * open[t]);
                        v.end();
                                                        }
                }
            }



            IloExpr obj(env);
            for (t = 0; t < NODE; t++) {
                obj += ((fix_hubcost[t] * open[t]) + 500*(1-
teta*capacity[t]/congestion[t])*open[t]);
            }
            IloExpr objo(env);
            for (i = 0; i < NODE; i++) {
                for (j = 0; j < NODE; j++) {
                    for (t = 0; t < NODE; t++) {
                        for (m = 0; m < NODE; m++) {
                            obj += (3*link_cost[i][t] +
0.75*link_cost[t][m] + 2*link_cost[m][j])*qq[i][j][t][m] ;
                        }
                    }
                }
            }

            model.add(IloMinimize(env, obj));
            obj.end();

            IloCplex cplex(env);
            cplex.extract(model);
            IloCplex::LongAnnotation bendersdec =
cplex.newLongAnnotation(IloCplex::BendersAnnotation, 1);
```

```
			for (t = 0; t < NODE; t++) {
				cplex.setAnnotation(bendersdec, open[t], 0);
			}

			cplex.solve();

			cplex.out() << "Solution status: " << cplex.getStatus() << endl;
			IloNum tolerance =
cplex.getParam(IloCplex::Param::MIP::Tolerances::Integrality);
			cplex.out() << "Optimal value: " << cplex.getObjValue() << endl;

			for (t = 0; t < NODE; t++) {
				if (cplex.getValue(open[t]) >= 1 - tolerance) {
					cplex.out() << "HUB " << t << " is open";
					cplex.out() << endl;
				}
				else {

					cplex.out() << "noHUB " << endl;
				}

				}
			}

			catch (IloException& e) {
			cerr << " ERROR: " << e << endl;
		}
		catch (...) {
			cerr << " ERROR" << endl;
		}
		system("pause");
		env.end();
		system("pause");
		return 0;
}
```

CURRICULUM VITAE



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- **Özgün-Kibiroğlu, Ç.**, Serarslan, M. N. and Topcu, Y. İ. (2019). Particle swarm optimization for uncapacitated multiple allocation hub location problem under congestion. *Expert Systems with Applications, 119,* 1-19.

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- **Özgün-Kibiroğlu, Ç.** 2009: The Selection of a Model of E-Waste Management by Fuzzy AHP. International Symposium on Intelligent Manufacturing Systems, June 16, 2009 Sakarya Üniversitesi, İstanbul.
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