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KAHRAMANMARAŞ SÜTÇÜ IMAM UNIVERSITY

GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

INVESTIGATION OF THE SOLUTION BY USING P_N METHOD OF TRANSPORT EQUATION FOR TRIPLET ANISOTROPIC SCATTERING

Pishtiwan Akram HAMA RASHID

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Thesis submitted in candidature for

the degree of Master in

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Prof. Dr. M. Hakkı ALMA …….

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DECLARATION

I hereby declare that all information in the thesis has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Pishtiwan Akram HAMA RASHID

P_N METODU İLE TRİPLET ANİZOTROPİK SAÇILIMLI TRANSPORT EŞİTLİĞİN ÇÖZÜMÜNÜN İNCELENMESİ

(YÜKSEK LİSANS TEZİ)

Pishtiwan Akram HAMA RASHID

ÖZ

Bu tezde, triplet anizotropik saçılımlı transport eşitliğinin çözümü için bir araştırma yapıldı. Saçılım fonksiyonu, Legendre polinomlarının ilk dört terim alınarak ifade edildi. Sonra transport eşitliğinin özdeğerleri, değişik ikincil nötron sayısına ve saçılım katsayılarına göre hesaplandı. Ayrıca, triplet anizotropik saçılma sonuçları; izotropik saçılma, lineer, saf-kuadratik ve saf-triplet anizotropik saçılımın sonuçları ile karşılaştırıldı.

Anahtar Kelimeler: Nötron transport denklemi, P_N metodu, Legendre polinomları, Yüksek derece anizotropik saçılma.

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INVESTIGATION OF THE SOLUTION BY USING P_N METHOD OF TRANSPORT EQUATION FOR TRIPLET ANISOTROPIC SCATTERING

(M.Sc. THESIS)

Pishtiwan Akram HAMA RASHID

ABSTRACT

In this thesis, an investigation is made for the solution of neutron transport equation in the presence of triplet anisotropic scattering. The scattering function has been expressed as series of the first four terms of Legendre polynomials. Then the eigenvalues of transport equation are computed according to the number of secondary neutrons and scattering coefficient for different cases. Also the results of triplet anisotropic scattering are compared with the results of isotropic scattering, linear, pure-quadratic and pure-triplet anisotropic scattering.

Key Words: Neutron transport equation, P_N method, Legendre polynomials, Highly anisotropic scattering.

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P_N METODU İLE TRİPLET ANİZOTROPİK SAÇILIMLI TRANSPORT EŞİTLİĞİN ÇÖZÜMÜNÜN İNCELENMESİ

ÖZET

Bu tezde, homojen, zamandan bağımsız ve tek hızlı, anizotropik saçılımlı nötron transport eşitliğinin çözümü için P_N metot kullanıldı. Legendre polinomlarının ilk dört serisi olarak ifade edilen triplet anizotropik saçılımın varlığında transport eşitliği yeniden düzenildi.

İlk önce, kartezyen ve polar koordinatlarda Maple programı kullanarak saçılım fonksıyonu katsayılarının değişim aralığı hesaplandı. *c* ikincil nötronlara ve saçılım katsayılarına göre transport eşitliğinin özdeğerlerinin sonuçları hesaplandı. Çoğaltkan ($c>1$) ve çoğaltkan olmayan ($c<1$) ortamların durumları araştırıldı. Sonra, sonuçlar tablo ve grafiklerde verildi. Anizotropik saçılımın ilk üç derecelerin arasında karşılaştırmalar yapıldı. Bu tezde, saf-triplet ve triplet anizotropik saçılımın durumları ile yoğunlaşıldı. Sonuç olarak, triplet anizotropik saçılımın özdeğerlerin fiziksel anlamları açıklandı.

İncelemenin sonuçu olarak, triplet anizotropik saçılımın sonuçları saf-triplet anizotropik saçılımın sonuçlarından daha iyi olduğu görüldü.

INVESTIGATION OF THE SOLUTION BY USING P_N METHOD OF TRANSPORT EQUATION FOR TRIPLET ANISOTROPIC SCATTERING

SUMMARY

In this thesis, P_N method is used to solve homogeneous, time independent and one-group neutron transport equation with anisotropic scattering. Transport equation is reconstructed in the presence of triplet anisotropic scattering which is expressed in terms of the first four series of Legendre polynomials.

Firstly, the range of coefficients of scattering function is determined by using maple program in Cartesian and Polar coordinates. And the results of the eigenvalues of transport equation are computed according to the number of secondary neutrons *c* and scattering coefficient. The cases of multiplying ($c>1$) and non-multiplying ($c<1$) media are investigated. Then results are given in the tables and figures. The comparisons among the first three degrees of anisotropic scattering are made. Our specific interest in this thesis is about the case of pure-triplet and triplet anisotropic scattering. The solution of transport equation using P_N method for highly anisotropic scattering is improved. Finally, the physical meaning of eigenvalues and triplet anisotropic scattering are explained.

As a result of the investigation, it is seen that the results of our triplet anisotropic scattering are better than the results of pure-triplet anisotropic scattering.

DEDICATION

This work is dedicated to my mother, Shawbo and my sister, Shawin. Her encouragements and willingnesses to listen to my problems helped to sustain my efforts.

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October-2013

KAHRAMANMARAS

Pishtiwan Akram HAMA RASHID

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LIST OF SYMBOLS AND ABBRIVIATIONS

f_n : Scattering coefficient

$f(\mu, \mu')$: Scattering function for plane geometry

ν : Eigenvalues

$A_n(\nu)$: Eigenfunctions

c : Mean number of secondary neutrons per collision, excluding fission

$\psi(x, \mu)$: Angular neutron flux

$\phi_n(x)$: Neutron flux or moment of neutron flux

$P_n(\mu)$: Legendre polynomial of order n

L : Relaxation length

D : Diffusion length

Ω : Direction of neutron after scattering

Ω' : Direction of neutron before scattering

φ : Azimuthal angle

μ_0 : Scattering cosine

θ : Polar angle

$d\Omega$: Solid angle

∇ : Gradient operator

$\sum_t$: The losses of neutron by all kinds of interactions ($\sum_t$=1)

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1. INTRODUCTION

Technology generally progresses gradually by collecting of basic knowledge and technological developments. In contrast, nuclear engineering was begun with the unexpected discovery of neutrons and nuclear fission, leading to a sudden development of the technology. The first nuclear reactor was made in the middle of Second World War in the military research and development program known as the Manhattan Project.

Table 1.1: The progress of nuclear physics and nuclear engineering by order (Sekimoto, 2007).

Date	Invented by	Subject
1808	Dalton	Atomic theory
1876	Goldstein	Cathode rays
1891	Stoney	Prediction of electrons
1895	Roentgen	X-rays
1896	Becquerel	Radioactivity
1897	Thomson	Cathode rays = electrons
1898	Rutherford	α-rays and β-rays
1900	Planck	Quantum theory
1905	Einstein	Special relativity theory
1911	Rutherford	Atomic model
1912	Thomson	Isotope
1919	Aston	Mass spectrometer
1921	Harkins	Prediction of neutrons
1930	Bothe	Be (α, ?)
1932	Irene and Frederic Joliot-Curie, Chadwick	Be (α, γ), Neutron discovery
1934	Fermi, Szilard	Delayed neutrons, Chain reaction
1939	Hahn, Strassman, Meitner	Discovery of nuclear fission
1942	Fermi	CP–1 made critical
1944	(Hanford, USA)	First plutonium production reactor
1945	(USA)	Test of atomic bomb
1945	(Canada)	Natural uranium heavy water research reactor (ZEEP) made critical
1946	(USA)	Fast reactor (Clementine) made critical
1950		Swimming pool reactor (BSR) made critical
1951	(USA)	Experimental fast breeder reactor (EBR-1) made critical and generates power
1953	(USSR) (United Nations, USA)	Test of hydrogen bomb “Atoms for Peace” Initiative
1954	(USA) (USSR)	Launch of the nuclear submarine “Nautilus” Graphite-moderated water-cooled power reactor (AM-1) generates power

It is a common agreement nowadays that nuclear energy is the only available source able to fulfill current and future energetic demands of mankind without polluting the Earth any further. The environmental impact of nuclear power plants is also substantially smaller than that of conventional power plants.

The nuclear industry has avoided emissions by 38 billion tons of CO_2. That amount is one-third more than CO_2, humans put into the atmosphere for each year from burning fossil fuels. And it is also about one-twelfth of the accumulated CO_2, which human have added to the atmosphere in the period of past 160 years from burning coal, petroleum, and natural gas. This is a rough estimation, yet it shows that nuclear reactors are a great factor in lowering CO_2 emissions.

Nuclear power plants produce none of these air pollutants, and emit only trace amounts of radioactive gases. Here the fact that nuclear plants do not consume oxygen like conventional plants has been the overriding factor in their adoption for the undersea fleet. Furthermore, ships powered by nuclear plants need to be refueled only after long periods of operation. For this reason, many of the newer surface vessels of the world's navies are nuclear powered.

Preceding its useful utilization, such as generation of electricity, material irradiation or for medical purposes, its extraction within a nuclear reactor must occur. Nuclear reactors are sophisticated devices composed of diverse materials like fuel, coolant, regulation and structural materials. All these constituents are arranged in a highly heterogeneous manner due to various safeties, technological and economic considerations. Depending on the time spent within the reactor, both their physical and mechanical properties change and some of them (like fuel elements of control rods) are also subject to repeated rearrangement.

Design of such reactors and analysis of their various operational modes is therefore, a complicated task that encompasses several areas of science and engineering. At its start, however, determination of neutronic conditions within the reactor core plays a crucial role and received a substantial attention in the field of reactor physics in past decades.

Neutron scattering is now used by more than 5000 researchers World-wide, and the ambit of the method is continuously becomes wider. 45 years ago, neutron scattering was an exotic tool for solid state physicists and crystallographers, but today it serves communities as wide as Biology, Planetary Science, Earth Sciences, Engineering,

Nanoscience, and Cultural Heritage. In brief, neutrons are used in all scientific fields that deal with condensed matter (Lefmann, 2007).

There are many areas of nuclear reactor studies where, the calculation performance is highly important. In the fuel loading optimization process, for example, the steady state equation has to be evaluated many times in order to find the optimal condition. Another area, where the need for a fast core neutronic solver is even aggravated because a quasi-static approach cannot be used, is the analysis of the transient states of the reactor, such as the startup, shutdown or abrupt control rod movement during an accident.

Several forms of the neutron transport equation exist. The integro-differential formulation, the most popular form in neutron transport equation, is presented in section 2.2. However, there are other forms of the transport equation and are being used for example, integral and adjoint forms.

The optimal operation is achieved by automating the regulation devices to balance neutron production from fission chain reaction and their loss due to capture and out of core leakage. Such automation foremost requires knowledge of a long-term behaviour of any tested core configuration, which may be obtained by performing a steady-state analysis of the reactor core. Methods used for these analyses should accurately calculate the neutron multiplication factor (characterizing the departure of the core from the desired equilibrium state) and spatial distribution of neutron flux (from which subsequently, the important reaction rates, power and heat distribution can be obtained for further steps of reactor design or assessment). Since, there is usually a multitude of possible core configurations; all calculations should proceed swiftly to be practically applicable.

In this thesis, the neutron transport equation is solved. And highly anisotropic scattering is analyzed for the core and moderator. Then the computation of eigenvalues has been done which is an important parameter in the reactors.

This thesis is organized as follow. In section 1.2., the applications of neutron transport theory will be described. And in section 2, the solution methodology will be given for neutron transport equation. Then the obtained results from the solution of neutron transport equation will be discussed in section 3.

1.1. Literature Review

Basically the field of neutron transport related to the migration of neutrons through media. And an example of neutron transport theory is the designing of nuclear reactors. To obtain a complete description of the neutron population, theoretically the distribution of neutrons should specify in space, velocity, and time. The function of distribution satisfies the linearized Boltzmann transport equation. This equation has been known for almost a century and later has been studied in connection with radiative transfer.

The earliest known approximated solution to the one-speed neutron transport equation is mostly attributed to Davison (1958), followed by Case and Zweifel (1967). Their studies were the first attempts to convert the one-speed neutron transport equation to an algebraic set of eigenvalue equations within a spectral domain.

During the past few decades, polynomial expansion analysis has played an important role in the methods for solving the neutron transport equation. The earliest known form of this equation has been presented by Davison and treated by Case's singular eigenfunction expansion method (Case, 1967).

Polynomial approximation methods for the solution of transport problems have been applied, firstly, by Conkie (1959) for the solution in slab geometry. And Ganapol (1986), who proposed the use of perturbation to develop a numerical method for solving the one-dimensional transport problem by polynomial reconstruction. In some recent studies (Anli et al., 2006; Yaşa et al., 2006), polynomial expansions have been successfully applied in eigenvalue spectrum calculations and criticality of bare/reflected slab geometries for isotropic scattering.

Critical slab was calculated by Kerner (1967) and Kschwendt (1971) for the high state values of *c*. İnönü (1973), because of difficult solution of highly anisotropic problems for different geometries, developed a model for anisotropic scattering. This model includes forward and backward scattering (Yaşa, 2002; Yaşa and Anli, 2009).

Many researchers studied linear anisotropic scattering (Eynde, 2005). But for quadratic case, very little work has been done. That is studied by Lathrop and Leonard (1965). Dahl et al. (2003) by using the numerical computation of eigenvalues in neutron transport equation have obtained numerical results. The most detailed calculations on the linear anisotropic scattering made by Eynde(2005) and Sjöstrand (1976).

Basic studies on linear and quadratic anisotropic scattering mostly are made by Sjöstrand and Protopopescu (1981). Riyait and Ackroyd (1987), by testing one-dimensional neutron transport of multigroup problems for highly anisotropic scattering obtained numerical results.

Atalay (1995) examined the case of anisotropic scattering by the method of eigenfunction. The basic work which is related to Linear anisotropic scattering behavior of the critical eigenvalues of the one-dimensional transport operator was uncovered by Sahni (1995, 1997).

Using P_N approximation to analysis neutron transport theory has become almost a traditional method. One-dimensional slab geometry for the case of anisotropic scatter by using Marshak boundary conditions, Yıldız (2001) have been determined the critical thickness of the system using the P_N method.

In generally two things are important about neutron transport equation. First, finding out what are the significant parameters of the problem, what factors influence the output, and how. Second, being able to compute numerical results.

Sallah and Margeanu (2007) make a study about the randomness effect in the medium properties in the presence of the pure-triplet anisotropic scattering ($f_1=f_2=0$, $f_3\neq0$) on the ensemble-average partial neutron fluxes. Partial neutron fluxes are obtained. And deterministic solution is obtained by using Pomraning-Eddington approximation. For the average partial neutron fluxes, numerical results are calculated for different values of the single scattering albedo and changing the parameters characterizing the random medium.

In this thesis, we made investigation for the solution of neutron transport equation in the presence of triplet ($f_1\neq0$, $f_2\neq0$, $f_3\neq0$) anisotropic scattering, which is different with pure-triplet anisotropic scattering. Our scattering function has been expressed as the first four series of Legendre polynomials. In this investigation, we obtained different eigenvalues for triplet anisotropic scattering.

1.2. Neutron Transport Theory

As it is known, the study of a transport equation is extremely important and interesting in transport theory. Different methods have been developed to investigate, the task of searching methods that generate accurate results to transport problems. The term "Transport Theory" is used to describe the transport of particles, whether they are photons of electromagnetic radiation, including light photons, x-rays or gamma rays, neutrons, or charged particles such as electrons and protons, through a host medium under rarefied conditions. Some examples of transport processes are shown in Figure 1.1.

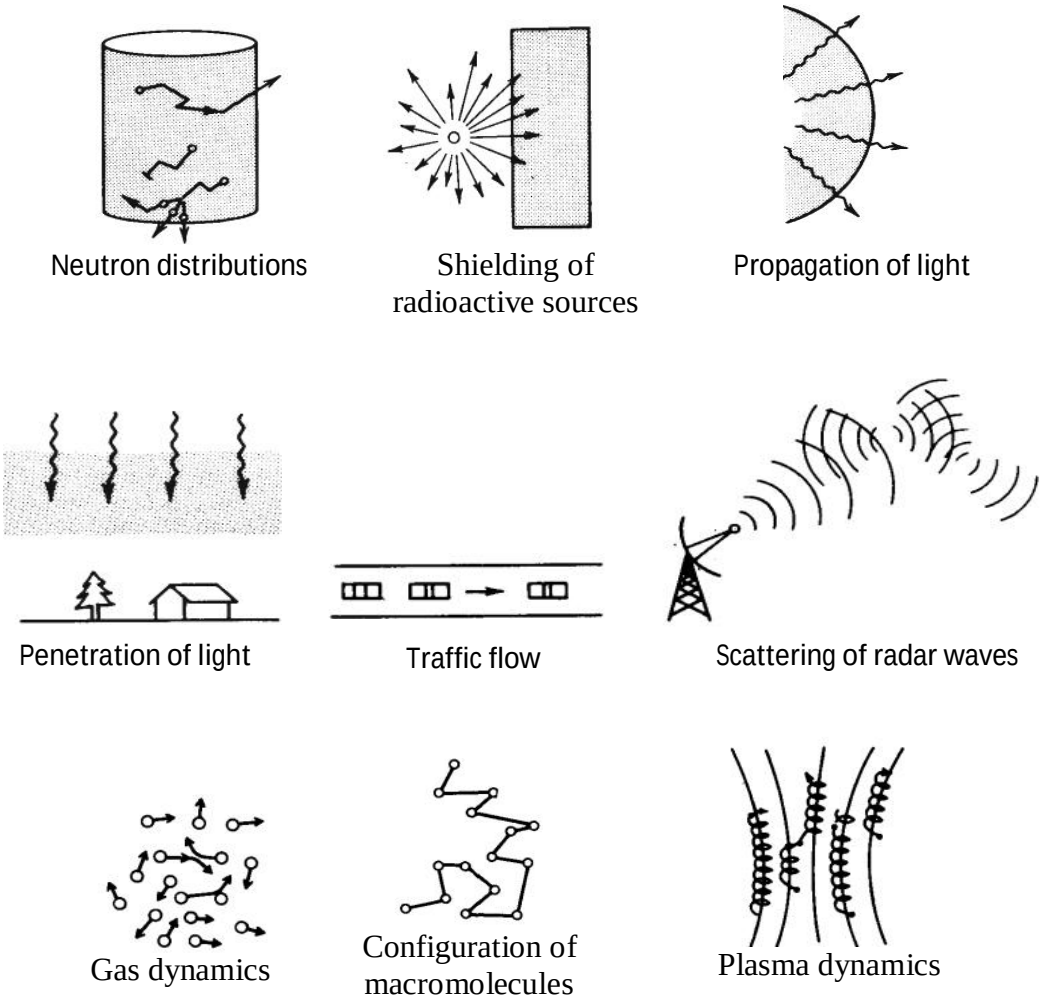


Figure 1.1: Examples of transport processes (Duderstadt, 1978)

Some examples of transport processes are:

1) Neutron distribution in nuclear reactors.
2) Shielding of radioactive sources.
3) Propagation of light through stellar matter,
4) Penetration of light through the planetary atmosphere,
5) Traffic flow.
6) Gas dynamics.
7) Scattering of radar waves from the atmospheric turbulence.
8) Configuration of macromolecules.
9) Plasma dynamics.

The neutron transport theory is considered as the foundation of reactor modeling and design. The neutron transport equation is considered to be an expression governing the neutron distribution in a reactor core. However, the generality and inherent complexity of this integro-differential equation make it exceedingly difficult to solve. Typically, simplifying approximations are made to reduce its complexity, and in cases such as diffusion theory, lead to a system of Partial Differential Equations (PDEs). Hence we present a concise introduction to transport theory, which serves to highlight the natural hierarchy of the approximations that are inherent in any reactor model.

An important implication of this hierarchy is that the terminology employed in virtually all approximate reactor models has its ancestry in transport theory. Thus, our overview of transport theory permits the introduction of this terminology in its natural environment.

2. MATERIAL AND SOLUTION METHODOLOGY

2.1. The Legendre Polynomials

The Legendre polynomials, sometimes called Legendre functions of the first kind, are solutions of the second order differential equation known as the Legendre differential equation:

$$(1-\mu^2)y''(\mu)-2\mu y'(\mu)+n(n+1)y(\mu)=0. \tag{2.1}$$

If n is an integer, the functions are polynomials. The Legendre polynomials, named after Adrien-Marie Legendre, $P_n(\mu)$, n = 0, 1, 2, …, with $\mu \in [-1,1]$ are plotted in the Figure 2.1 up to n = 4.

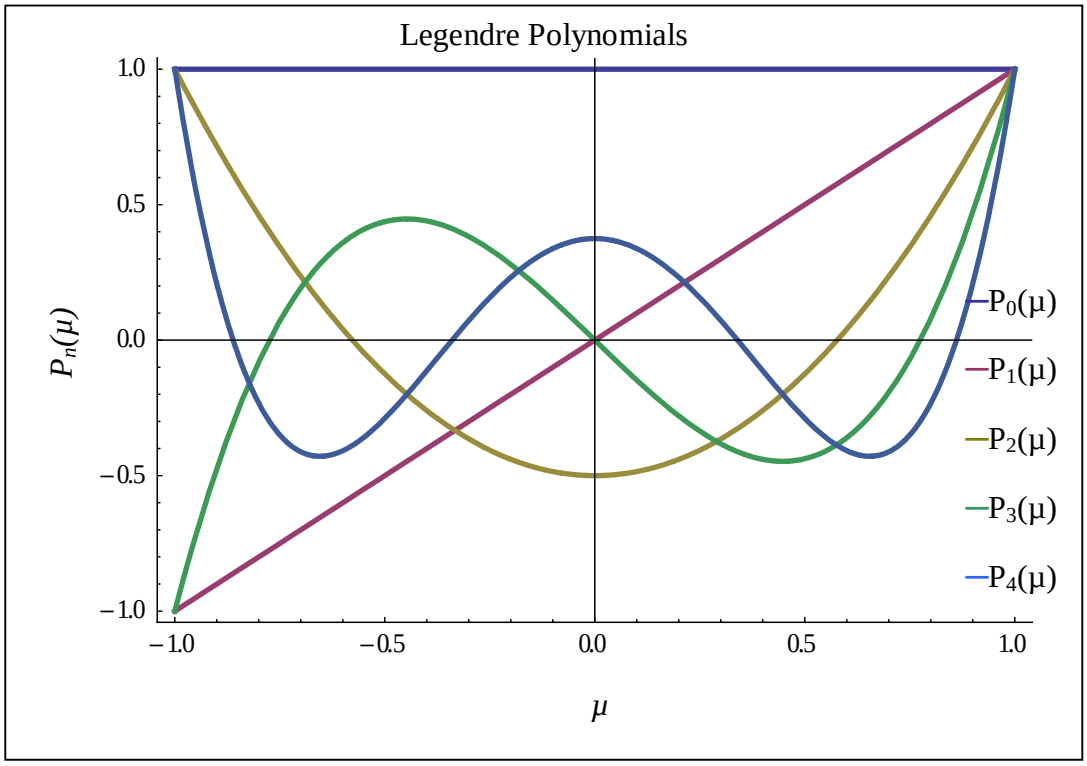


Figure 2.1: The graphs of Legendre polynomials (n =0, 1,.., 4).

The Legendre ordinary differential equation occurs when solving Laplace's equation in spherical coordinates. It may be solved using the standard power series method, in which case the solution is finite (i.e. the series converges) provided $|\mu|<1$ is satisfied. Furthermore, it is finite at μ = ±1 provided n is a non-negative integer. In this case, the solutions form a polynomial sequence of orthogonal polynomials.

The plot of the Legendre polynomials help in understanding the reasons for their orthogonality and the fact that odd or even numbered polynomials are odd or even functions, having symmetric positive and negative roots.

Each Legendre polynomial $P_n(\mu)$ is an n^{th}-degree polynomial, that may be expressed using (Ivory–Jacobi) Rodrigues' formula as,

$$P_n(\mu) = \left(\frac{1}{2^n n!}\right)\frac{d^n}{d\mu^n}[(\mu^2 - 1)^n]. \tag{2.2}$$

An important property of the Legendre polynomials is that they are orthogonal with respect to the L^2 inner product on the interval $-1 \le \mu \le 1$, i.e. they satisfy,

$$\int_{-1}^{1} P_n(\mu) P_m(\mu) d\mu = \frac{2}{2n+1}\delta_{mn}. \tag{2.3}$$

Where, δ_{mn} denotes the Kronecker delta, equal to 1 if $m = n$, and otherwise, it is zero. Some examples of Legendre polynomial can be given by using Rodrigues' formula in equation (2.2) as,

$$n = 0, \quad P_0(\mu) = 1 \tag{2.4}$$

$$n = 1, \quad P_1(\mu) = \left(\frac{1}{2^1 !}\right)\frac{d}{d\mu}[(\mu^2 - 1)^1] = \mu \tag{2.5}$$

$$n = 2, \quad P_2(\mu) = \left(\frac{1}{2^2 2!}\right)\frac{d^2}{d\mu^2}[(\mu^2 - 1)^2] = \frac{1}{2}(3\mu^2 - 1) \tag{2.6}$$

$$n = 3, \quad P_3(\mu) = \left(\frac{1}{2^3 3!}\right)\frac{d^3}{d\mu^3}[(\mu^2 - 1)^3] = \frac{1}{2}(5\mu^3 - 3\mu) \tag{2.7}$$

The other Legendre polynomials are given in Table 2.1.

Table 2.1: Obtains some examples of Legendre polynomials.

n	$P_n(\mu)$
0	1
1	μ
2	$\frac{1}{2}(3\mu^2-1)$
3	$\frac{1}{2}(5\mu^3-3\mu)$
4	$\frac{1}{8}(35\mu^4-30\mu^2+3)$
5	$\frac{1}{8}(63\mu^5-70\mu^3+15\mu)$
6	$\frac{1}{16}(231\mu^6-315\mu^4+105\mu^2-5)$
7	$\frac{1}{16}(429\mu^7-693\mu^5+315\mu^3-35\mu)$
8	$\frac{1}{128}(6435\mu^8-12012\mu^6+6930\mu^4-1260\mu^2+35)$
9	$\frac{1}{128}(12155\mu^9-25740\mu^7+18018\mu^5-4620\mu^3+315\mu)$
10	$\frac{1}{256}(46189\mu^{10}-109395\mu^8+90090\mu^6-30030\mu^4+3465\mu^2-63)$

Some useful properties of these polynomials are given further,

$$P_n(-\mu)=\begin{cases} P_n(\mu) & n \quad even, \\ -P_n(\mu) & n \quad odd. \end{cases} \tag{2.8}$$

The polynomials satisfy the following recurrence relations which also are known as Bonnet's recursion formula:

$$\mu P_n(\mu)=\frac{1}{2n+1}[(n+1)P_{n+1}(\mu)+nP_{n-1}(\mu)]. \tag{2.9}$$

2.2. The Associated Legendre Function

The associated Legendre function, $P_n^m(\mu)$ is defined for integral values of m=0, 1, 2, …, n by the formula,

$$P_n^m(\mu)=(-1)^m(1-\mu^2)^{m/2}\frac{d^m P_n(\mu)}{d\mu^m}, \tag{2.10}$$

from this equation it is known that $P_n^0(\mu) = P_n(\mu)$. If equation (2.2) is used for $P_n(\mu)$, then it follows that,

$$P_n^m(\mu) = \frac{(-1)^m}{2^n n!}(1-\mu^2)^{m/2}\frac{d^{n+m}}{d\mu^{n+m}}(\mu^2-1)^n. \tag{2.11}$$

This equation can be used to extend the definition of $P_n^m(\mu)$ to negative integral values of m such that $|m| \le n$. It can then be shown that,

$$P_n^{-m}(\mu) = (-1)^m \frac{(n-m)!}{(n+m)!}P_n^m(\mu). \tag{2.12}$$

The associated Legendre functions satisfy the orthogonality relation,

$$\int_{-1}^{1} P_n^m P_n^m(\mu)\, d\mu = (-1)^m \frac{2}{2n+1}\frac{(n+m)!}{(n-m)!}\delta_{nn}, \tag{2.13}$$

and the recurrence relations,

$$\mu P_n^m(\mu) = \frac{1}{2n+1}[(n-m+1)P_{n+1}^m(\mu) + (n+m)P_{n-1}^m(\mu)], \tag{2.14}$$

$$(\mu^2-1)\frac{dP_n^m(\mu)}{d\mu} = n\mu P_n^m(\mu) - (n+m)P_{n-1}^m(\mu). \tag{2.15}$$

Here are some examples of associated Legendre functions,

$\mu = cos\theta$

$$\begin{aligned}
&P_1^1(\mu) = -(1-\mu^2)^{1/2}, && P_1^1(\cos\theta) = -\sin\theta,\\
&P_2^1(\mu) = -3(1-\mu^2)^{1/2}\mu, && P_2^1(\cos\theta) = -3\sin\theta\cos\theta,\\
&P_2^2(\mu) = -3(1-\mu^2)^{1/2}, && P_2^2(\cos\theta) = 3\sin^2\theta.
\end{aligned} \tag{2.16}$$

Hence equation (2.11) can be expressed as,

$$P_n^m(\mu) = (-1)^m \sin^m\theta \frac{d^m P_n(\mu)}{d\mu^m};\quad |m| \le n. \tag{2.17}$$

In this representation θ_0 is the scattering angles of neutron and cosine of the scattering angles is given, as shown in Figure 2.5.

$$\mu_0 = \cos\theta_0 = \Omega.\Omega' = P_1(\mu_0)$$
$$\mu_0 = (\sin\theta\cos\varphi\,\mathbf{i} + \sin\theta\sin\varphi\,\mathbf{j} + \cos\theta\,\mathbf{k}).(\sin\theta'\cos\varphi'\,\mathbf{i} + \sin\theta'\sin\varphi'\,\mathbf{j} + \cos\theta'\,\mathbf{k})$$

$$
\begin{aligned}
&= \sin\theta \sin\theta' \cos\varphi \cos\varphi' + \sin\theta \sin\theta' \sin\varphi \sin\varphi' + \cos\theta \cos\theta' \\
&= \sin\theta \sin\theta' \ (\cos\varphi \cos\varphi' + \sin\varphi \sin\varphi') + \cos\theta \cos\theta' \\
&= \sin\theta \sin\theta' \ \cos(\varphi - \varphi') + \cos\theta \cos\theta' \\
&= (-P_1^1(\mu) \,.\, -P_1^1(\mu')) \ \cos(\varphi - \varphi') + P_1(\mu) P_1(\mu') \\
&= P_1(\mu) P_1(\mu') + P_1^1(\mu) P_1^1(\mu') \cos(\varphi - \varphi').
\end{aligned}
\tag{2.18}
$$

Then a Legendre polynomial can be conveniently transcribed according to addition theorem:

$$
P_n(\mu_0) = P_n(\mu) P_n(\mu') + 2 \sum_{m=1}^{n} P_n^m(\mu) P_n^m(\mu') \cos(\varphi - \varphi'). \tag{2.19}
$$

If we assume rotational symmetry in the φ' direction, then we can take the integration of equation (2.19) over this angle for 2π. The second term of the equation becomes zero because $\sin\varphi'|_0^{2\pi} = 0$, the equation remains as,

$$
P_n(\mu_0) = P_n(\mu) P_n(\mu'). \tag{2.20}
$$

2.3. Neutron Transport Equation

One of the fundamental aims of nuclear reactor design is to have a suitable prediction of neutron production and loss rates. Predictions come from the solution of the neutron conservation equation, and consequently the importance of the neutron Boltzmann equation. This equation can be used to determine the distribution of neutrons or photons in a reactor as a function of position, energy, time and direction. The basic form of the neutron transport equation in its integro-differential form, in terms of angular flux expressed as (Attia, T.A., 1976),

$$
\begin{aligned}
&\frac{1}{v} \frac{\partial \psi(r, \Omega, E; t)}{\partial t} + \Omega . \nabla \psi(r, \Omega, E; t) + \Sigma(r, E) \psi(r, \Omega, E; t) = \\
&\int_{E'} \int_{\Omega'} (\Sigma(r, E') \ f(r, \Omega', E' \to \Omega, E) \psi(r, \Omega', E'; t) + S(r, \Omega, E; t)) d\Omega' dE',
\end{aligned}
\tag{2.21}
$$

where, we use the following notations:

r: Position vector of a neutron.

v : Neutron speed.

$\psi(r, \Omega, E; t)$: Neutron angular flux, i.e. expected number of neutrons at (r, Ω,E,t).

$\Sigma(r,E)$: Total macroscopic cross section of neutrons.

$f(r,\Omega',E' \to \Omega,E)$: Scattering function.

$S(r,\Omega,E;t)$: All the external neutron sources.

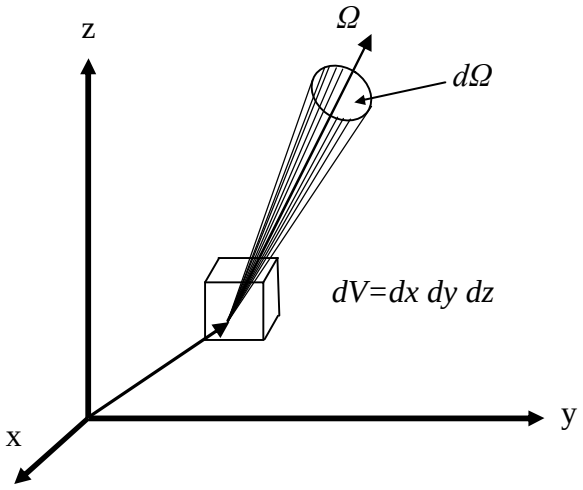


Figure 2.2: The volume element *dV* and the directional element *dΩ*.

By performing a neutron balance within a small volume element *dV* as shown in Figure 2.2 and representing the rate of change of the number of neutrons as the difference of their productions and losses, the neutron transport equation (2.21) is obtained. And this equation has been found to be entirely satisfactory for solving most problems in reactor physics.

In the equation (2.21), there are different terms with having different physical meanings; we are starting by introducing each term from the left to the right side:

First term: represents the time rate of change of the neutron angular density.
Second term: represents the net loss of neutrons of interest due to leakage as well as streaming through its surface.
Third term: represents the loss of neutrons due to collisions with nuclei; which includes both absorption and scattering collision.

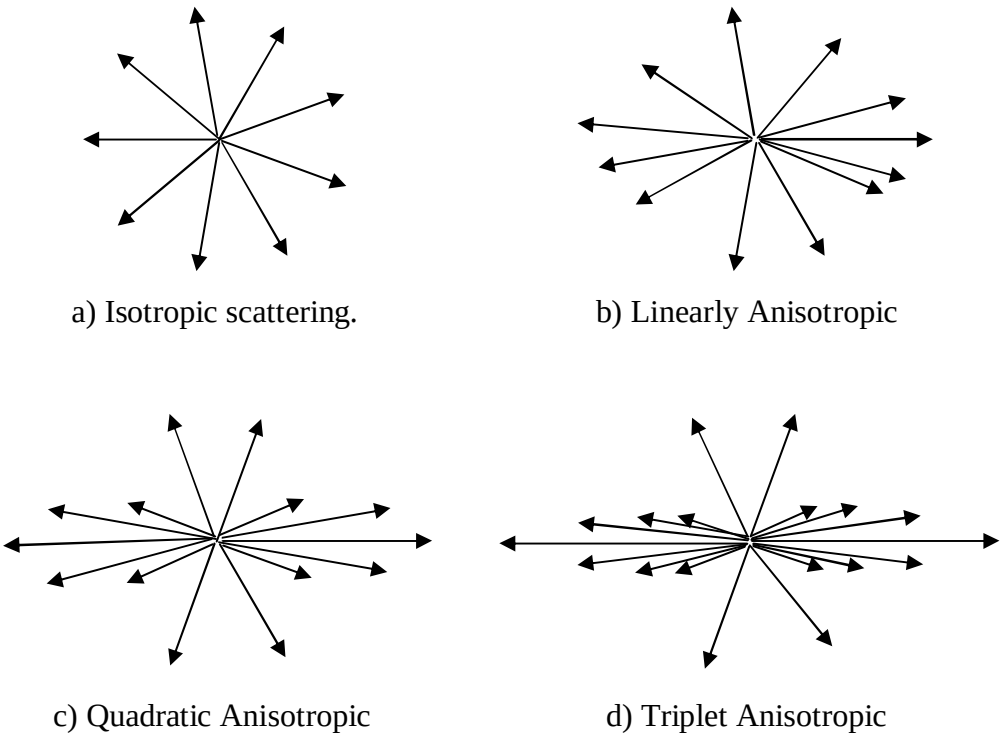
And from right side, we have:
First term: gives all neutrons, in the unit volume under consideration, of different directions *Ω'* and energy *E'*.
Second term: represents all the external neutron sources.

Some cases for neutron transport equation:

1) Steady state (Time independent): The steady state of a reactor is characterized by a non-changing neutron population.
2) One speed (one group) = the equation is independent of energy.
3) Homogeneous (source free): neutron cross section is independent of spatial position.

Main types of scattering:

1) Isotropic scattering: Scattering function = $1/4\pi$ (constant).
2) Anisotropic scattering: Scattering function is not constant, depends on the angle of scattering. These are shown in Figure 2.3.



a) Isotropic scattering. b) Linearly Anisotropic

c) Quadratic Anisotropic d) Triplet Anisotropic

Figure 2.3: Isotropic and anisotropic (linearly, quadratic and triplet) scattering.

2.4. Neutron Transport Equation for Plane Geometry

In order to solve equation (2.21) analytically, it should be simplified by integration over one or more of the independent variables. For example, homogeneous time independent one-group neutron transport equation for anisotropic scattering can be expressed as,

$$\mu \frac{\partial \psi(x,\mu)}{\partial x} + \psi(x,\mu) = c \int_{\Omega'} f(\Omega' . \Omega)\, \psi(x,\mu') d\Omega', \tag{2.22}$$

where:

$\psi(x,\mu)$: Neutron flux.

$f(\Omega' . \Omega)$: Scattering kernel (scattering function).

x : The spatial coordinate.

μ : The angular coordinate ($\Omega . \hat{x}$).

c : The number of secondary neutrons per collision.

Ω: Direction of neutron after scattering.

Ω' : Direction of neutron before scattering.

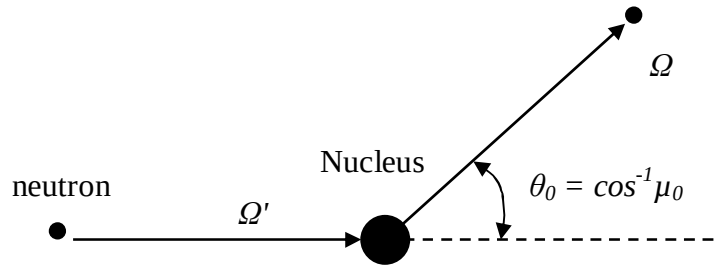


Figure 2.4: Direction of neutron before and after scattering (μ_0 is the cosine of the scattering angle of neutron).

It is important to notice that the scattering of a neutron on any nucleus in nuclear reactor applications will not basically depend on the incoming neutron direction. Therefore, the differential scattering cross section will not depend on the incoming neutron direction of motion, but it will depend on the scattering direction (or the change of the direction). This dependence is described as a function of scattering angle (θ_0) as shown in Figure 2.4 or the cosine of the scattering angle ($\mu_0 = cos\theta_0$).

The scattering law for interaction between radiation and particle may be written as $f(\Omega, \Omega')d\Omega$ and quantitatively is the probability that the particle in the direction denoted by the unit vector Ω'. It will be scattered into the solid angle $d\Omega$ about the unit vector Ω with dΩ=dμ'dφ. For much media of interest, we can write $f(\Omega, \Omega') = f(\cos\theta_0)$. Where, $\cos\theta_0 = \Omega.\Omega'$.

Now the equation (2.22) can be expressed as following,

$$\mu \frac{\partial \psi(x,\mu)}{\partial x} + \psi(x,\mu) = c\int_0^{2\pi}\int_{-1}^{1} f(\mu_0)\,\psi(x,\mu')\,d\mu' d\varphi, \tag{2.23}$$

suppose that neutron scattering from collisions have an isotropic distribution, then scattering function becomes,

$$f(\mu_0) = \frac{1}{4\pi}. \tag{2.24}$$

For plane geometry, ψ is a function of x and μ only, which is cosine of the angle between the positive x-axis. And the angular dependence of ψ, may be expanded in a series of Legendre polynomials with coefficients,

$$\psi(x,\mu) = \sum_{n=0}^{N} \frac{2n+1}{2}\phi_n(x)P_n(\mu). \tag{2.25}$$

Where, the $P_n(\mu)$ are the Legendre polynomials, and the $\phi_n(x)$ are the Legendre moment, $N \to \infty$. From the orthogonality properties it becomes,

$$\phi_n(x) = \int_{-1}^{1} \psi(x,\mu)\,P_n(\mu)d\mu. \tag{2.26}$$

One advantage of the Legendre expansion for the neutron flux $\psi(x,\mu)$ is that the first two terms, have a simple physical meaning as shown in equations (2.4-5),

$$\phi_0(x) = \int_{-1}^{1} \psi(x,\mu)d\mu = \text{Flux Density} \tag{2.26.a}$$

$$\phi_1(x) = \int_{-1}^{1} \mu\,\psi(x,\mu)d\mu = \text{Directional Flux Density} \tag{2.26.b}$$

$$\phi_2(x) = \frac{1}{2}\int_{-1}^{1} (3\mu^2 - 1)\,\psi(x,\mu)d\mu \tag{2.26.c}$$

$$\phi_3(x) = \frac{1}{2}\int_{-1}^{1} (5\mu^3 - 3\mu)\,\psi(x,\mu)d\mu. \tag{2.26.d}$$

2.5. Anisotropic Scattering Phase Function

Anisotropic scattering phase functions which can be expanded in terms of Legendre polynomials, for various cases are given as followings:

1) Linear anisotropic scattering

$$f(\mu,\mu')=\frac{1}{2}[1+3f_1P_1(\mu)P_1(\mu')]. \tag{2.27}$$

2) Quadratic anisotropic scattering

$$f(\mu,\mu')=\frac{1}{2}[1+3f_1P_1(\mu)P_1(\mu')+5f_2P_2(\mu)P_2(\mu')]. \tag{2.28}$$

3) Pure-quadratic anisotropic scattering

$$f(\mu,\mu')=\frac{1}{2}[1+5f_2P_2(\mu)P_2(\mu')]. \tag{2.29}$$

4) Triplet anisotropic scattering (Our consideration)

$$f(\mu,\mu')=\frac{1}{2}[1+3f_1P_1(\mu)P_1(\mu')+5f_2P_2(\mu)P_2(\mu')+7f_3P_3(\mu)P_3(\mu')]. \tag{2.30}$$

5) Pure-triplet anisotropic scattering (It exists in literature)

$$f(\mu,\mu')=\frac{1}{2}[1+7f_3P_3(\mu)P_3(\mu')]. \tag{2.31}$$

2.6. Neutron Transport Equation with Triplet Anisotropic Scattering

There is an increasing need to take into account anisotropic scattering in many branches of physics. The accurate evaluation of certain parameters needs to take into account the anisotropy of neutron scattering. These parameters are important for nuclear reactor systems and other physical sciences. A nuclear reactor is a complex system consisting of fuel, moderators, coolant, control rods, and support structure. The homogeneous one-speed Boltzmann neutron transport equation with highly anisotropic scattering for plane geometry is given by equation (2.23), and our considered formula of triplet anisotropic scattering is given in equation (2.30). By the end of this section, neutron transport equation is represented with our consideration of triplet anisotropic scattering for plane geometry.

If the scattering function is an anisotropy, except in special cases, can be developed into a Legendre polynomial series of the argument $\mu_0 = \Omega \cdot \Omega'$. Where, Ω and Ω' are the directions of neutrons before and after collision, μ_0 is the cosine of the difference of scattering angles as shown in Figure 2.5. θ and φ are the axial and azimuthal angles, respectively, of Ω.

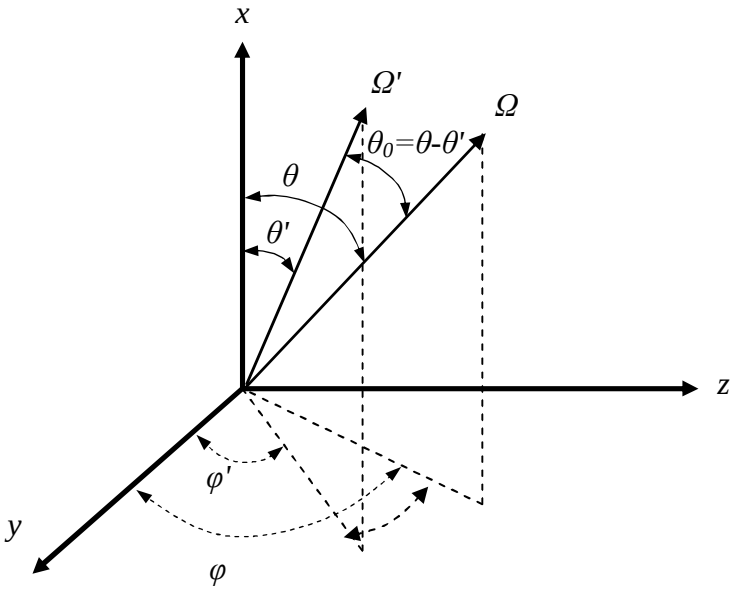


Figure 2.5: Direction of neutron before and after collisions.

Now we assume that we can express the scattering function as an infinite series of Legendre polynomials ($N \rightarrow \infty$) and in terms of the full-range Legendre polynomials,

$$f(\mu_0) = \sum_{n=0}^{N} \frac{2n+1}{4\pi} f_n P_n(\mu_0), \tag{2.32}$$

where, $f(\mu_0)$ is the scattering phase function. It can be expanded in terms of Legendre polynomials for triplet scattering as,

$$f(\mu_0) = \frac{1}{4\pi}[f_0 P_0(\mu_0) + 3f_1 P_1(\mu_0) + 5f_2 P_2(\mu_0) + 7f_3 P_3(\mu_0)], \tag{2.33}$$

here, f_n is called the scattering coefficient or expansion coefficient with $f_0 = 1$ in equation (2.33). The first term of the expansion corresponds to the isotropic scattering. The second term of this expansion represents linearly anisotropic scattering. The third term of the expansion represents the quadratic anisotropic scattering. The fourth term of the expansion represents the triplet anisotropic scattering.

The neutron transport equation for anisotropic scattering can be expressed as,

$$\mu \frac{\partial \psi(x,\mu)}{\partial x} + \psi(x,\mu) = c \int_0^{2\pi} \int_{-1}^{1} f(\mu_0)\, \psi(x,\mu')\, d\,\mu' d\,\varphi. \tag{2.34}$$

Then equation (2.33) can be substituted into the neutron transport equation,

$$\mu \frac{\partial \psi(x,\mu)}{\partial x} + \psi(x,\mu) =$$
$$\frac{c}{4\pi} \int_0^{2\pi} \int_{-1}^{1} [f_0 P_0(\mu_0) + 3 f_1 P_1(\mu_0) + 5 f_2 P_2(\mu_0) + 7 f_3 P_3(\mu_0)]\, \psi(x,\mu')\, d\,\mu' d\,\varphi. \tag{2.35}$$

By using equation (2.20), the right side of the transport equation can be rewritten as,

$$= \frac{c}{2} [P_0(\mu) \int_{-1}^{1} f_0 P_0(\mu') \psi(x,\mu') d\,\mu' + 3 P_1(\mu) \int_{-1}^{1} f_1 P_1(\mu') \psi(x,\mu') d\,\mu' +$$
$$5 P_2(\mu) \int_{-1}^{1} f_2 P_2(\mu') \psi(x,\mu') d\,\mu' + 7 P_3(\mu) \int_{-1}^{1} f_3 P_3(\mu')\, \psi(x,\mu') d\,\mu']. \tag{2.36}$$

According to the equations of (2.25) and (2.26) n^{th} Legendre moment of the neutron flux can be given as following,

$$\phi_0(x) = \int_{-1}^{1} P_0(\mu') \psi(x,\mu') d\,\mu'$$
$$\phi_1(x) = \int_{-1}^{1} P_1(\mu') \psi(x,\mu') d\,\mu'$$
$$\phi_2(x) = \int_{-1}^{1} P_2(\mu') \psi(x,\mu') d\,\mu'$$
$$\phi_3(x) = \int_{-1}^{1} P_3(\mu') \psi(x,\mu') d\,\mu'. \tag{2.37}$$

After the substitutions the equations (2.37) in to the equation (2.36), it becomes,

$$= \frac{c}{2} [P_0(\mu) f_0 \phi_0(x) + 3 P_1(\mu) f_1 \phi_1(x) + 5 P_2(\mu) f_2 \phi_2(x) + 7 P_3(\mu) f_3 \phi_3(x)], \tag{2.38}$$

then we replace the Legendre polynomial according to the Table 2.1, then the neutron transport equation for triplet scattering is obtained as,

$$\mu \frac{\partial \psi(x,\mu)}{\partial x} + \psi(x,\mu) = \frac{c}{2} [f_0 \phi_0(x) + 3\mu f_1 \phi_1(x) + \frac{5}{2}(3\mu^2 - 1) f_2 \phi_2(x)$$
$$+ \frac{7}{2}(5\mu^3 - 3\mu) f_3 \phi_3(x)]. \tag{2.39}$$

In this thesis, as shown in equation (2.39), we increased the degree of anisotropy of the scattering function.

2.7. Solution of P_N Method

Time independent one-group neutron transport equation for plane geometry is given in equation (2.23), the angular flux is given in equation (2.25). And the scattering function as an infinite series of Legendre polynomials is given in equation (2.32).

Now we substitute both equations of (2.25) and (2.32) in to equation (2.23),

$$\mu\sum_{n=0}^{N}\frac{2n+1}{2}P_n(\mu)\frac{\partial\phi_n(x)}{\partial x}+\sum_{n=0}^{N}\frac{2n+1}{2}P_n(\mu)\phi_n(x)$$
$$=c\,2\pi\sum_{n=0}^{N}\frac{2n+1}{4\pi}f_n P_n(\mu)\sum_{n=0}^{N}\frac{2n+1}{2}\phi_n(x)\int_{-1}^{1}P_n(\mu)P_n(\mu')d\mu'. \tag{2.40}$$

And Legendre polynomials satisfy the orthogonality relation over the interval $-1\le\mu\le 1$ and the recurrence relations,

$$\int_{-1}^{1}P_m(\mu)P_n(\mu)d\mu=\frac{2}{2n+1}\delta_{mn},$$
$$\mu P_n(\mu)=\frac{1}{2n+1}[(n+1)P_{n+1}(\mu)+nP_{n-1}(\mu)], \tag{2.41}$$

then after some rearrangement we get the general form,

$$(n+1)\frac{d\phi_{n+1}(x)}{dx}+n\frac{d\phi_{n-1}(x)}{dx}+(2n+1)(1-cf_n\delta_{n0}+cf_n\delta_{n1}+cf_n\delta_{n2}$$
$$+cf_n\delta_{n3})\phi_n(x)=0,\quad n=0,1,2,.....,N. \tag{2.42}$$

Where, δ_{mn} denotes the Kronecker delta,

$$\delta_{mn}=\begin{cases}1, & n=m\\ 0, & n\neq m\end{cases} \tag{2.43}$$

with the requirement that $\phi_{-1}(x)$ is zero and considering the first *N*+1 of these equations and setting $d\phi_{N+1}(x)/dx=0$. The One may employ solution procedure of the homogeneous equation (2.42) to introduce the *Ansatz*:

$$\phi_n(x)=A_n(\nu)e^{\frac{-x}{\nu}}. \tag{2.44}$$

Where, $A_n(\nu)$ is the eigenfunctions and ν corresponding the eigenvalues.

Eigenvalue spectrum of the transport operator consists of two parts. They are the discrete spectrum (ν>1) and the continuous spectrum (-1<ν<1). We represent a method and corresponding computer code to get all eigenvalues for triplet anisotropic scattering.

1) For n=0, P_0 approximation

$$\frac{d\phi_1(x)}{dx} + (1 - cf_0)\phi_0(x) = 0 \tag{2.45}$$

By substituting the equation (2.44) in to the equation (2.45), we can get $A_1(\nu)$ as shown in equation (2.46),

$$\begin{aligned}
&-\frac{1}{\nu}(A_1(\nu)e^{\frac{-x}{\nu}}) + (1 - cf_0)A_0(\nu)e^{\frac{-x}{\nu}} = 0 \\
&-A_1(\nu) + \nu A_0(\nu)(1 - cf_0) = 0 \\
&A_1(\nu) = \nu A_0(\nu)(1 - cf_0), \quad A_0(\nu) = 1 \\
&A_1(\nu) = \nu(1 - cf_0), \quad f_0 = 1 \\
&A_1(\nu) = \nu(1 - c).
\end{aligned} \tag{2.46}$$

2) For n=1, P_1 approximation

$$\frac{2}{3}\frac{d\phi_2(x)}{dx} + \frac{1}{3}\frac{d\phi_0(x)}{dx} + (1 - cf_1)\phi_1(x) = 0 \tag{2.47}$$

$$\frac{2}{3}\frac{d}{dx}(A_2(\nu)e^{\frac{-x}{\nu}}) + \frac{1}{3}\frac{d}{dx}(A_0(\nu)e^{\frac{-x}{\nu}}) + (1 - cf_1)A_1(\nu)e^{\frac{-x}{\nu}} = 0$$

$$\frac{2}{3}(-\frac{1}{\nu})(A_2(\nu)e^{\frac{-x}{\nu}}) + \frac{1}{3}(-\frac{1}{\nu})(A_0(\nu)e^{\frac{-x}{\nu}}) + (1 - cf_1)A_1(\nu)e^{\frac{-x}{\nu}} = 0$$

$$A_2(\nu) = \frac{3\nu A_1(\nu)(1 - cf_1) - 1}{2}. \tag{2.48}$$

3) For n=2, P_2 approximation

$$\frac{3}{5}\frac{d\phi_3(x)}{dx} + \frac{2}{5}\frac{d\phi_1(x)}{dx} + (1 - cf_2)\phi_2(x) = 0 \tag{2.49}$$

$$\frac{3}{5}\frac{d}{dx}(A_3(\nu)e^{\frac{-x}{\nu}}) + \frac{2}{5}\frac{d}{dx}(A_1(\nu)e^{\frac{-x}{\nu}}) + (1 - cf_2)A_2(\nu)e^{\frac{-x}{\nu}} = 0$$

$$\frac{3}{5}(-\frac{1}{\nu})(A_3(\nu)e^{\frac{-x}{\nu}}) + \frac{2}{5}(-\frac{1}{\nu})(A_1(\nu)e^{\frac{-x}{\nu}}) + (1 - cf_2)A_2(\nu)e^{\frac{-x}{\nu}} = 0$$

$$A_3(\nu) = \frac{5\nu A_2(\nu)(1 - cf_2) - 2A_1(\nu)}{3}. \tag{2.50}$$

4) For n=3, P_3 approximation

$$\frac{4}{7}\frac{d\phi_4(x)}{dx}+\frac{3}{7}\frac{d\phi_2(x)}{dx}+(1-cf_3)\phi_3(x)=0 \tag{2.51}$$

$$\frac{4}{7}\frac{d}{dx}(A_4(\nu)e^{\frac{-x}{\nu}})+\frac{3}{7}\frac{d}{dx}(A_2(\nu)e^{\frac{-x}{\nu}})+(1-cf_3)A_3(\nu)e^{\frac{-x}{\nu}}=0$$

$$\frac{4}{7}(-\frac{1}{\nu})(A_4(\nu)e^{\frac{-x}{\nu}})+\frac{3}{7}(-\frac{1}{\nu})(A_2(\nu)e^{\frac{-x}{\nu}})+(1-cf_3)A_3(\nu)e^{\frac{-x}{\nu}}=0$$

$$A_4(\nu)=\frac{7\nu A_3(\nu)(1-cf_3)-3A_2(\nu)}{4}. \tag{2.52}$$

5) For n=4, P_4 approximation

$$\frac{5}{9}\frac{d\phi_5(x)}{dx}+\frac{4}{9}\frac{d\phi_3(x)}{dx}+\phi_4(x)=0 \tag{2.53}$$

$$\frac{5}{9}\frac{d}{dx}(A_5(\nu)e^{\frac{-x}{\nu}})+\frac{4}{9}\frac{d}{dx}(A_3(\nu)e^{\frac{-x}{\nu}})+A_4(\nu)e^{\frac{-x}{\nu}}=0$$

$$\frac{5}{9}(-\frac{1}{\nu})(A_5(\nu)e^{\frac{-x}{\nu}})+\frac{4}{9}(-\frac{1}{\nu})(A_3(\nu)e^{\frac{-x}{\nu}})+A_4(\nu)e^{\frac{-x}{\nu}}=0$$

$$A_5(\nu)=\frac{9\nu A_4(\nu)-4A_3(\nu)}{5}. \tag{2.54}$$

From the equations (2.42) and (2.44) we can drive general form for $A_n(\nu)$,

$$(n+1)\frac{d}{dx}(A_{n+1}(\nu)e^{-\frac{x}{\nu}})+n\frac{d}{dx}(A_{n-1}(\nu)e^{-\frac{x}{\nu}})$$

$$+(2n+1)(1-cf_n\delta_{n0}+cf_n\delta_{n1}+cf_n\delta_{n2}+cf_n\delta_{n3})A_n(\nu)e^{-\frac{x}{\nu}}=0$$

$$(n+1)(-\frac{1}{\nu})(A_{n+1}(\nu)e^{-\frac{x}{\nu}})+n(-\frac{1}{\nu})(A_{n-1}(\nu)e^{-\frac{x}{\nu}})$$

$$+(2n+1)(1-cf_n\delta_{n0}+cf_n\delta_{n1}+cf_n\delta_{n2}+cf_n\delta_{n3})A_n(\nu)e^{-\frac{x}{\nu}}=0$$

$$(n+1)A_{n+1}(\nu)+nA_{n-1}(\nu)+(2n+1)(1-cf_n\delta_{n0}+cf_n\delta_{n1}+cf_n\delta_{n2}$$
$$+cf_n\delta_{n3})\nu A_n(\nu)=0,\quad n=0,1,2,...,N. \tag{2.55}$$

P_1 Approximation:

We assume the angular flux is linearly anisotropic, for this approximation it requires equation (2.42) with n=0 and n=1 then by setting $d\phi_2(x)/dx = 0$. We get equations (2.56) and (2.57) respectively.

$$\frac{d\phi_1(x)}{dx} + (1 - cf_0)\phi_0(x) = 0, \tag{2.56}$$

$$\frac{d\phi_0(x)}{dx} + 3(1 - cf_1)\phi_1(x) = 0, \tag{2.57}$$

$$\phi_1(x) = -\frac{1}{3(1 - cf_1)}\frac{d\phi_0(x)}{dx}. \tag{2.57.a}$$

And from the definition of the Fick's law the equation can be given as,

$$J(r) = -D\nabla\phi(r), \tag{2.58}$$

equation (2.58) becomes equivalent to equation (2.57.a). Then the diffusion coefficient can be obtained as,

$$D = \frac{1}{3(1 - cf_1)}. \tag{2.59}$$

If we substitute equation (2.57.a) in equation (2.56), hence neutron diffusion equation for linearly anisotropic scattering can be written as,

$$\frac{d^2\phi_0(x)}{dx^2} - 3(1 - cf_0)(1 - cf_1)\phi_0(x) = 0. \tag{2.60}$$

And the diffusion length is given as,

$$L = \frac{1}{\sqrt{3(1 - c)(1 - cf_1)}}. \tag{2.61}$$

In order to obtain eigenvalues, we used equation (2.48) by setting $A_2(\nu)$=0,

$$3\nu A_1(\nu)(1 - cf_1) - 1 = 0, \tag{2.62}$$

then we substitute $A_1(\nu)$ from equation (2.46), the equation of eigenvalues will be,

$$\nu = \pm\frac{1}{\sqrt{3(1 - cf_0)(1 - cf_1)}}. \tag{2.63}$$

Also by using matrix, the eigenvalues can be obtained for low order approximation as shown in equation (2.65). Where, the determinant of the matrix should be equal to zero.

$$(1-c f_0)A_0(\nu)-\frac{1}{\nu}A_1(\nu)=0, \tag{2.64}$$
$$\frac{1}{3\nu}A_0(\nu)+(1-c f_1)A_1(\nu)=0,$$

$$M=\begin{bmatrix} 1/3\nu & (1-c f_1) \\ (1-c f_0) & -1/\nu \end{bmatrix}. \tag{2.65}$$

Eigenvalues spectrum of the transport operator consists of two parts. They are the discrete spectrum (ν>1) and the continuous spectrum (-1<ν<1). The discrete eigenvalues ν is known as relaxation length L and characterizes the diffusive properties of the system. And the diffusion coefficient can be calculated by the below equation,

$$D=\nu^2(1-c f_0). \tag{2.66}$$

Some notes about discrete eigenvalues for values of c are given as following:

1) c = 0 (only absorbing case), there are no discrete eigenvalues.
2) c = 1(the critical case), there is a double root at ∞ and, in the case of anisotropic scattering, possibly others.
3) c < 1 (the subcritical case), non-multiplying medium, discrete eigenvalues are real.
4) c > 1 (the supercritical case), multiplying medium, discrete eigenvalues are real or purely imaginary.
5) General properties of discrete eigenvalues are,
   a) simple.
   b) occur in plus-minus pairs.
   c) $|\nu| \geq 1$.
   d) are finite in number.

In this thesis, the discrete eigenvalues with the performance of these properties are given in the tables in section 3.

3. DISCUSSION AND RESULTS

In this research, we investigate the effect of triplet anisotropic scattering for the solution of neutron transport equation. And the scattering function has been expressed as the first four terms of Legendre polynomials in equation (2.33).

Since it is almost impossible to obtain the analytical solution for triplet anisotropic scattering, the investigation of triplet anisotropic scattering and the effect of parameter f_3 ($f_1 \neq 0$, $f_2 \neq 0$) on eigenvalues of transport equation can be obtained in an easy way with maple.

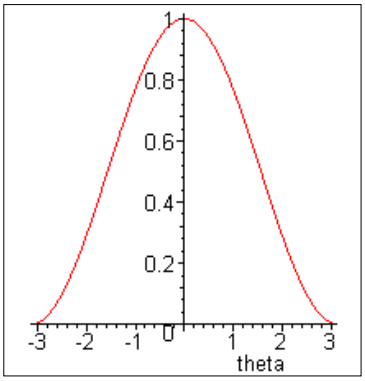
In the beginning, we determined the range of scattering coefficients from plotting Cartesian and polar coordinates of the scattering phase function. And considerations are done for various scattering approximations such as linear, pure-quadratic ($f_1=0$, $f_2 \neq 0$), pure-triplet ($f_1= 0$, $f_2= 0$, $f_3 \neq 0$) and triplet ($f_1 \neq 0$, $f_2 \neq 0$, $f_3 \neq 0$) anisotropic scattering in equation (2.33). Maple program is used for these processes. Then we derived a formula for the scattering coefficients as shown in equation (3.1). Later eigenvalues are calculated for isotropic scattering, linear anisotropic; pure-quadratic, pure-triplet and triplet anisotropic scattering. All calculations are done according to the number of secondary neutrons and scattering coefficient for different orders. And also eigenvalues are calculated for each case of multiplying and non-multiplying (Sallah, 2007).

We determined the range of scattering coefficients from the figures with attending to two points. The intensity of phase function have to be positive values (i.e. the probability must not be negative values), and the other, value of scattering coefficients have to be chosen correctly (the values must be converged). While the effect of highly anisotropic scattering on eigenvalues are investigated, a formula for expansion coefficients is derived as,

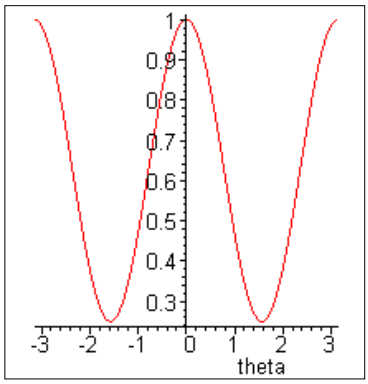
$$f_n = \frac{1}{\alpha^n}, \quad \text{while} \quad \alpha = 5 \quad \text{and} \quad n = 0,1,2,3. \tag{3.1}$$

This formula gives correct value for scattering coefficients, because these coefficients give positive and converging values in the solution. The scattering coefficients are given by using this formula for the case of triplet anisotropic scattering in Figure 3.2. To compare the result of the scattering coefficients by equation (3.1), we plot more figures for various

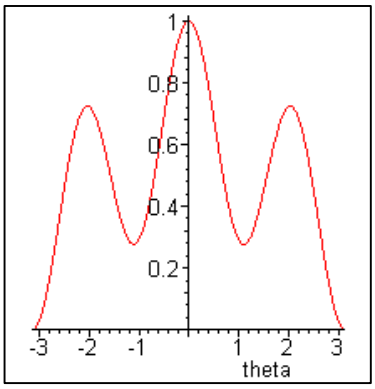
cases as shown in Figure 3.1. There are different formulae about the scattering coefficient in the literature that have been studied by some other researchers, as Garcia and Siewert (1988).



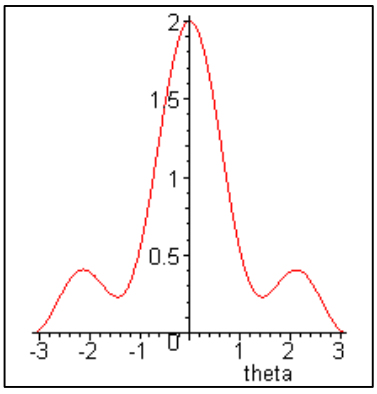
a) Linear anisotropic scattering (f_1 = 0.33).



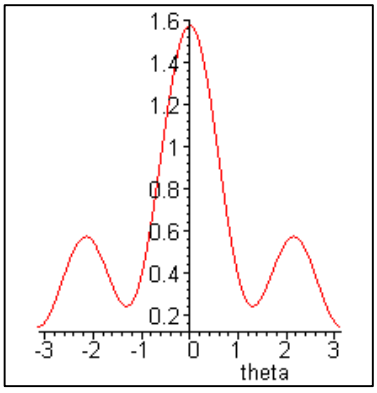
b) Pure-quadratic anisotropic scattering (f_1 = 0 and f_2 = 0.2).



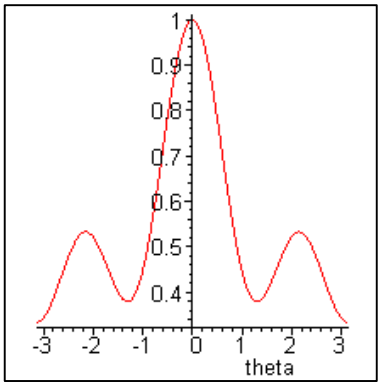
c) Pure-triplet anisotropic scattering (f_1 = 0, f_2 = 0 and f_3 = 0.142).



d) Triplet anisotropic scattering (f_1 = 0.33, f_2 = 0.2 and f_3 = 0.142).



e) Triplet anisotropic scattering (f_n = 0.142).

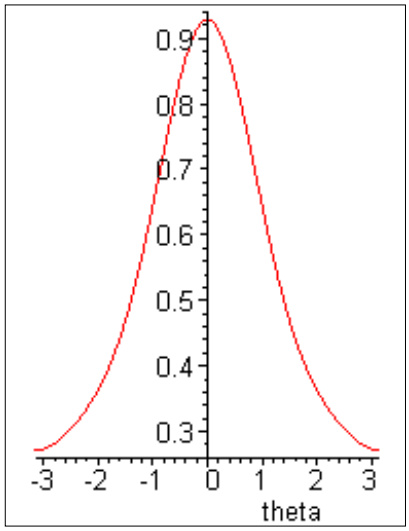


f) Triplet anisotropic scattering (f_n = 0.0666).

Figure 3.1: The ranges of scattering coefficient in the scattering phase function for linear, pure-quadratic, pure-triplet and triplet anisotropic scattering.

In Figure 3.1 the ranges of scattering coefficients are denoted by using maple program with choosing the true graphs for linear, pure-quadratic and pure-triplet anisotropic scattering as $f_1 \leq 0.33$, $f_2 \leq 0.2$ and $f_3 \leq 0.142$ respectively (Türeci, 2005). In Figure 3.1.d, we used the scattering coefficients of pure cases for triplet anisotropic scattering. In Figure 3.1.e, the minimum value of scattering coefficients of pure cases is chosen for triplet anisotropic scattering as $f_n = 0.142$.

The value of scattering coefficient is chosen for correct figure as $f_n = 1/15$ by trial and error in Figure 3.1.f. But the problem in here is that, the values of scattering coefficients are not converging. As a result, we decide to formulate the scattering coefficients which allow converging with increasing the orders.



($f_1 = 0.2$, $f_2 = 0.04$ and $f_3 = 0.008$).

Figure 3.2: The range of scattering coefficients with respect to equation (3.1) for triplet anisotropic scattering.

Physical meaning for isotropic scattering and linear, quadratic and triplet anisotropic scattering:

The ranges of scattering coefficient are determined from the polar coordinates of scattering phase function. From changing the value of scattering coefficients, we can mention the physical meanings. In Figure 3.3, there is symmetry property for isotropic scattering. By another meaning, the intensity of phase function is the same in all direction.

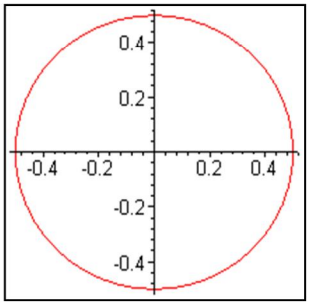
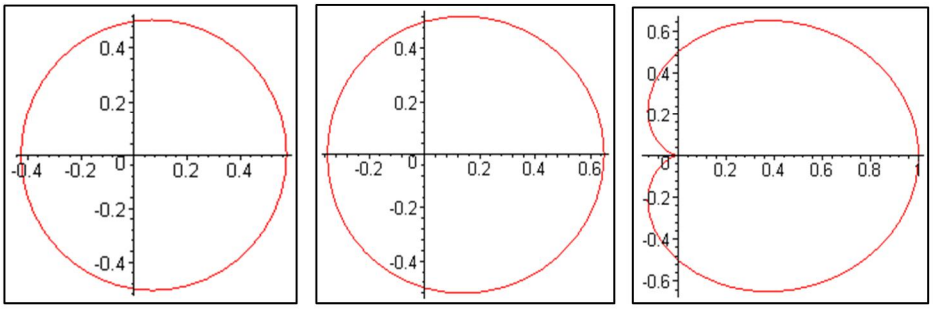
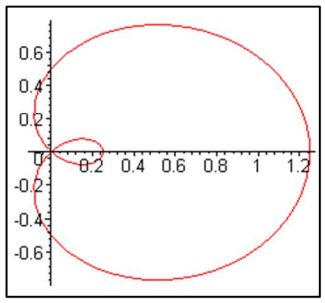


Figure 3.3: Isotropic scattering drawn from polar coordinate in equation (2.24).



a) f_1=0.05 b) f_1=0.1 c) f_1=0.33

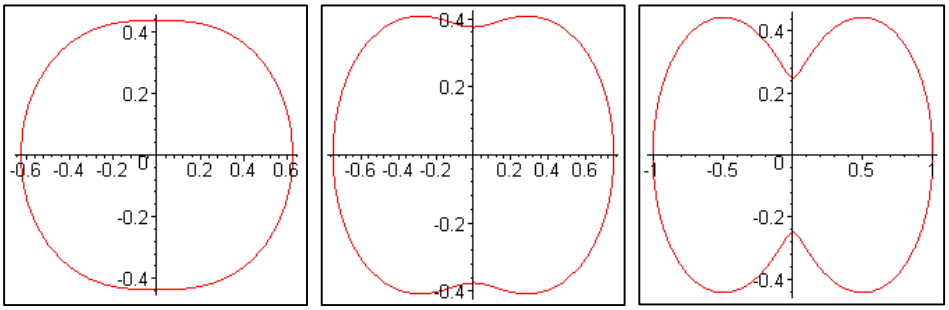
Figure 3.4: Linear anisotropic scattering with changing the scattering coefficient.



f_1=0.5

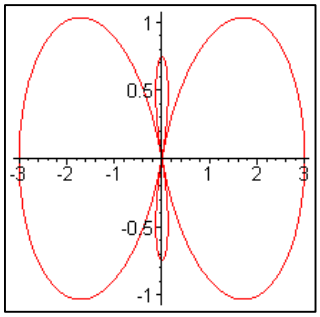
Figure 3.5: Linear anisotropic scattering that the scattering coefficient is out of its range.

In Figure 3.4, there is asymmetry property for linear anisotropic scattering that is why in the literature called as asymmetry parameter. In Figure 3.5 the scattering coefficient is out of its range and shows the errors.



a) f_2=0.05 b) f_2=0.1 c) f_2=0.2

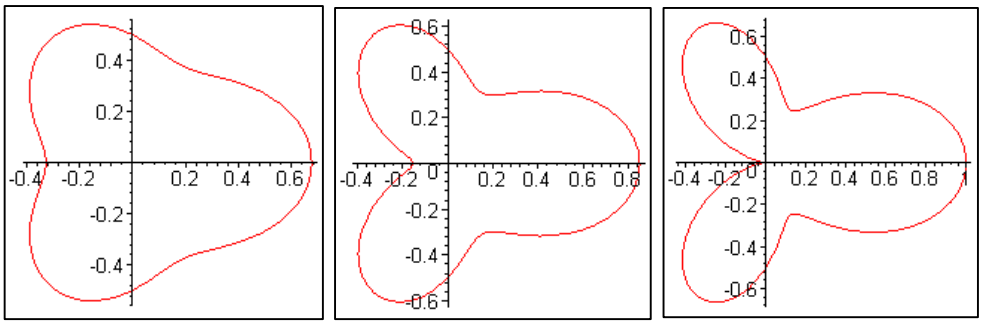
Figure 3.6: Pure-quadratic anisotropic scattering with changing the scattering coefficient.



f_2=1

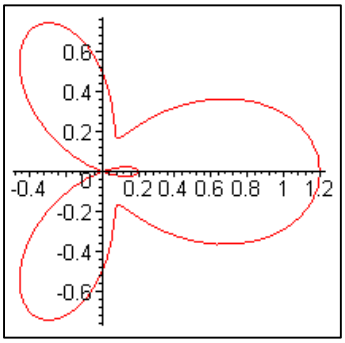
Figure 3.7: Pure-quadratic anisotropic scattering that the scattering coefficient is out of its range.

In Figure 3.6, there is forward property for pure-quadratic anisotropic scattering that is why in the literature called as forward parameter. In Figure 3.7 the scattering coefficient is out of its range.



a) f_3=0.05 b) f_3=0.1 c) f_3=0.142

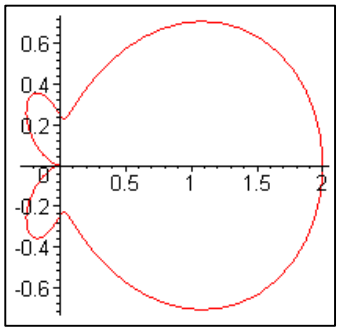
Figure 3.8: Pure-triplet anisotropic scattering with changing the scattering coefficient.



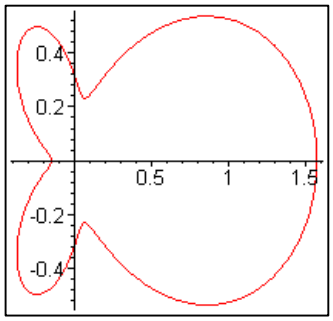
f_3=0.2

Figure 3.9: Pure-triplet anisotropic scattering that the scattering coefficient is out of its range.

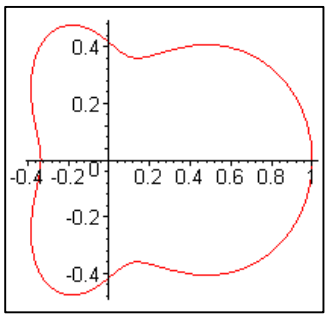
In Figure 3.8, there is high asymmetry property for pure-triplet anisotropic scattering. Because pure-triplet anisotropic scattering is related to the angle of higher power as the scattering phase function includes $cos\theta$ *and* $cos^3\theta$. Figure 3.9 shows errors for the range of f_3 which must be $f_3 \leq 0.142$.



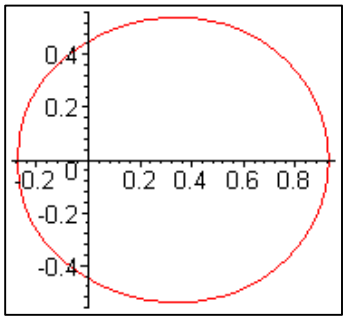
a) f_1 =0.33, f_2=0.2 and f_3=0.142



b) f_n=0.142



c) f_n=0.0666



d) f_1 =0.2, f_2=0.04 and f_3=0.008

Figure 3.10: Triplet anisotropic scattering with the different maximum values of scattering coefficients.

In Figure 3.10, large ranges of scattering coefficients are represented for triplet anisotropic scattering. Figure 3.10.d shows the polar coordinate of triplet scattering phase function by involving the values of equation (3.1). Later the eigenvalues are calculated by involving the values of scattering coefficient in equation (3.1) that considered by us.

Table 3.1: Eigenvalues of pure-triplet anisotropic scattering for $f_3 = 0.14$ and $c<1$.

	$N = 2$	$N = 4$	$N = 8$	$N = 9$
$c = 0.0$	0.5773502692	0.8611363115	0.9602898565	0.9681602395
		0.3399810436	0.7966664774	0.8360311073
			0.5255324099	0.6133714327
			0.1834346425	0.3242534234
				0.0
$c = 0.5$	0.816496580	1.015406544	1.051828606	1.052848889
		0.4228245261	0.8646485709	0.8950640818
			0.5859347148	0.6727652308
			0.2029548503	0.3566617050
				0.0
$c = 0.8$	1.290994449	1.406783622	1.410646979	1.410649967
		0.4938303994	0.9128256055	0.9335443867
			0.6280657117	0.7149386434
			0.2163849365	0.3779803451
				0.0
$c = 0.9$	1.825741858	1.903566456	1.904205918	1.904205943
		0.5202389273	0.9245671229	0.9426420100
			0.6408065399	0.7269826154
			0.2211183262	0.3850367677
				0.0
$c = 0.95$	2.581988897	2.635380251	2.635488336	2.635488336
		0.5335667611	0.9297239310	0.9466720795
			0.6467274030	0.7324332544
			0.2235264003	0.3885110886
				0.0
$c = 0.98$	4.082482905	4.115593454	4.115604064	4.115604064
		0.5415319332	0.9326308270	0.9489592333
			0.6501237380	0.7355190439
			0.2249833105	0.3905728176
				0.0
$c = 0.99$	5.773502692	5.796756874	5.796758729	5.796758729
		0.5441756263	0.9335718526	0.9497023953
			0.6512293082	0.7365173248
			0.2254708497	0.3912558023
				0.0

Table 3.2: Eigenvalues of pure-triplet anisotropic scattering for $f_3 = 0.14$ and $c>1$.

	$N = 2$	$N = 4$	$N = 8$	$N = 9$
$c = 1.01$	5.773502692 I	5.750570412 I	5.750568597 I	5.750568597 I
		0.5494399546	0.9354150508	0.9511622570
			0.6534003974	0.7384691243
			0.2264486669	0.3926149523
				0.0
$c = 1.02$	4.082482905 I	4.050282550 I	4.050272402 I	4.050272402 I
		0.5520586324	0.9363181945	0.9518797012
			0.6544658894	0.7394229251
			0.2269389055	0.3932909778
				0.0
$c = 1.05$	2.581988897 I	2.532187648 I	2.532090919 I	2.532090919 I
		0.5598493797	0.9389593360	0.9539862829
			0.6575822854	0.7421978503
			0.2284146964	0.3953040408
				0.0
$c = 1.1$	1.825741858 I	1.757994495 I	1.757481729 I	1.757481715 I
		0.5725638347	0.9431589477	0.9573637302
			0.6625120225	0.7465454152
			0.2308899839	0.3986052452
				0.0
$c = 1.2$	1.290994449 I	1.202886039 I	1.200377328 I	1.200376483 I
		0.5966579889	0.9509615116	0.9637374027
			0.6714210107	0.7542927119
			0.2358888612	0.4049797672
				0.0
$c = 1.3$	1.054092553 I	0.9553835557 I	0.9494921128 I	0.9494838818 I
		0.6186023480	0.9582026187	0.9697733207
			0.6791733772	0.7609533632
			0.2409312305	0.4110122135
				0.0
$c = 1.4$	0.9128709292 I	0.8088425008 I	0.7986224409 I	0.7985849707 I
		0.6382702787	0.9650870937	0.9756198540
			0.6859248743	0.7667256970
			0.2459898326	0.4166715489
				0.0
$c = 1.5$	0.8164965809 I	0.7103302615 I	0.6952923540 I	0.6951792783 I
		0.6557944554	0.9717585430	0.9813813390
			0.6918305164	0.7717778548
			0.2510358017	0.4219429782
				0.0
$c = 1.6$	0.7453559925 I	0.6390204240 I	0.6190328017 I	0.6187684432 I
		0.6714371710	0.9783200594	0.9871334511
			0.6970294776	0.7762455489
			0.2560398093	0.4268257043
				0.0
$c = 1.7$	0.6900655593 I	0.5847831957 I	0.5599555223 I	0.5594353142 I
		0.6854964581	0.9848478891	0.9929330249
			0.7016402177	0.7802359062

			0.2609732991	0.4313298477
				0.0
c = 1.8	0.6454972244 I	0.5420212971 I	0.5126094428 I	0.5117043274 I
		0.6982556549	0.9914003057	0.9988243381
			0.7057607744	0.7838328259
			0.2658096763	0.4354732293
				0.0
c = 1.9	0.6085806194 I	0.5073662017 I	0.4737040085 I	0.4722669913 I
		0.7099631166	0.9980234050	1.004843191
			0.7094711659	0.7871020232
			0.2705253242	0.4392784816
				0.0
c = 2.00	0.5773502692 I	0.4786616564 I	0.4411155528 I	0.4389888204 I
		0.7208281152	1.004754950	1.011019616
			0.7128363430	0.7900952077
			0.2751003481	0.4427707156
				0.0

In Table 3.1 for c<1, the eigenvalues of pure-triplet anisotropic scattering are calculated. The diffusion length or eigenvalue is increasing with increasing the number of secondary neutrons. And also the diffusion length is increasing with increasing the orders.

In Table 3.2, the diffusion length or eigenvalue of pure-triplet anisotropic scattering is decreasing with increasing the number of secondary neutrons for c>1. And also the diffusion length is decreasing with increasing the orders. For c=0.99 and c=1.01, the values of diffusion length in high orders (N>8) nearly become the same. The obtained results are the same with Salah's results (Sallah, 2007).

Table 3.3: Eigenvalues of triplet anisotropic scattering for $f_n = 1/15$ and $c<1$.

	$N = 2$	$N = 4$	$N = 8$	$N = 9$
$c = 0.0$	0.5773502692	0.8611363115	0.9602898565	0.9681602395
		0.3399810436	0.7966664774	0.8360311073
			0.5255324099	0.6133714327
			0.1834346425	0.3242534234
				0.0
$c = 0.5$	0.8304547985	1.036055171	1.066742457	1.067436560
		0.4204783174	0.8658211185	0.8970231299
			0.5824962135	0.6695371769
			0.2039758577	0.3565427355
				0.0
$c = 0.8$	1.326862231	1.450890439	1.454109156	1.454110935
		0.4898705830	0.9050857245	0.9277010611
			0.6225094201	0.7080581662
			0.2185327188	0.3787535991
				0.0
$c = 0.9$	1.883108943	1.967846099	1.968391692	1.968391709
		0.5162305410	0.9145909878	0.9348129787
			0.6352433578	0.7196523927
			0.2237635752	0.3863759743
				0.0
$c = 0.95$	2.667852642	2.726510726	2.726604276	2.726604276
		0.5297327149	0.9188156325	0.9379842973
			0.6413285925	0.7250657668
			0.2264467317	0.3901898710
				0.0
$c = 0.98$	4.222756096	4.259337154	4.259346421	4.259346421
		0.5378788364	0.9212099902	0.9397890101
			0.6448749704	0.7281843129
			0.2280775259	0.3924736250
				0.0
$c = 0.99$	5.974009853	5.999750257	5.999751883	5.999751883
		0.5405963608	0.9219869115	0.9403760436
			0.6460387390	0.7292020811
			0.2286245296	0.3932336538
				0.0

Table 3.4: Eigenvalues of triplet anisotropic scattering for $f_n = 1/15$ and $c>1$.

	$N = 2$	$N = 4$	$N = 8$	$N = 9$
$c = 1.01$	5.978278528 I	5.952796833 I	5.952795234 I	5.952795234 I
		0.5460291916	0.9235110277	0.9415299551
			0.6483380295	0.7312050226
			0.2297235595	0.3947514197
				0.0
$c = 1.02$	4.228792920 I	4.192943406 I	4.192934432 I	4.192934432 I
		0.5487426778	0.9242588825	0.9420973518
			0.6494733533	0.7321902407
			0.2302755555	0.3955090054
				0.0
$c = 1.05$	2.677397763 I	2.621625891 I	2.621539534 I	2.621539534 I
		0.5568605638	0.9264494048	0.9437642326
			0.6528210711	0.7350813467
			0.2319412748	0.3977756705
				0.0
$c = 1.1$	1.896608105 I	1.819979793 I	1.819514722 I	1.819514713 I
		0.5702623865	0.9299408795	0.9464379610
			0.6582032478	0.7396883308
			0.2347488646	0.4015287049
				0.0
$c = 1.2$	1.345954755 I	1.244273696 I	1.241928005 I	1.241927361 I
		0.5962301674	0.9364390495	0.9514783176
			0.6682224840	0.7481488863
			0.2404728151	0.4089102647
				0.0
$c = 1.3$	1.102970993 I	0.9867805735 I	0.9811195050 I	0.9811128539 I
		0.6205852151	0.9424580794	0.9562315370
			0.6772788862	0.7556987279
			0.2463222646	0.4160728309
				0.0
$c = 1.4$	0.9587062361 I	0.8338903684 I	0.8238392974 I	0.8238074031 I
		0.6430077062	0.9481485253	0.9608060257
			0.6854480201	0.7624602731
			0.2522695733	0.4229639541
				0.0
$c = 1.5$	0.8606629658 I	0.7309139078 I	0.7158401267 I	0.7157394199 I
		0.6634552165	0.9536188036	0.9652792628
			0.6928263659	0.7685504966
			0.2582829442	0.4295457790
				0.0
$c = 1.6$	0.7886003723 I	0.6563045881 I	0.6359603952 I	0.6357155661 I
		0.6820641619	0.9589480126	0.9697078059
			0.6995137347	0.7740723999
			0.2643275511	0.4357953072
				0.0
$c = 1.7$	0.7328416801 I	0.5995505542 I	0.5739720965 I	0.5734738566 I
		0.6990547700	0.9641948683	0.9741338750
			0.7056036500	0.7791128852

			0.2703670357	0.4417030489
				0.0
c = 1.8	0.6881023532 I	0.5548219540 I	0.5242321166 I	0.5233399092 I
		0.7146673900	0.9694037875	0.9785896885
			0.7111791587	0.7837436647
			0.2763652203	0.4472706834
				0.0
c = 1.9	0.6512205543 I	0.5185981519 I	0.4833300014 I	0.4818781906 I
		0.7291289308	0.9746090164	0.9831003465
			0.7163117975	0.7880233126
			0.2822878295	0.4525083477
				0.0
c = 2.00	0.6201736729 I	0.4886192838 I	0.4490612329 I	0.4468669957 I
		0.7426391877	0.9798374597	0.9876857839
			0.7210621544	0.7919995267
			0.2881040090	0.4574320203
				0.0

In the same manner of pure-triplet; in Table 3.3 for c<1, the diffusion length of triplet anisotropic scattering is increasing with increasing the number of secondary neutrons. And also the diffusion length is increasing with increasing the orders.

In Table 3.4, the diffusion length of triplet anisotropic scattering is decreasing with increasing the number of secondary neutrons for c>1. And also the diffusion length is decreasing with increasing the orders. For c=0.99 and c=1.01, the values of diffusion length in high orders (N>8) nearly become the same.

Table 3.5: Eigenvalues of triplet anisotropic scattering for $f_1 = 0.2$, $f_2 = 0.04$, $f_3 = 0.008$ and $c<1$.

	$N = 2$	$N = 4$	$N = 8$	$N = 9$
$c = 0.0$	0.5773502692	0.8611363115	0.9602898565	0.9681602395
		0.3399810436	0.7966664774	0.8360311073
			0.5255324099	0.6133714327
			0.1834346425	0.3242534234
				0.0
$c = 0.5$	0.8606629658	1.065139891	1.088552982	1.088950227
		0.4147352900	0.8690289187	0.9004078768
			0.5793901021	0.6676560723
			0.2030279877	0.3545996764
				0.0
$c = 0.8$	1.408590425	1.534307516	1.536465892	1.536466564
		0.4746965571	0.8982304253	0.9227737366
			0.6132340135	0.6990905321
			0.2167827468	0.3749773324
				0.0
$c = 0.9$	2.016194596	2.103378248	2.103739689	2.103739695
		0.4968587008	0.9045932495	0.9273623640
			0.6239087539	0.7083524359
			0.2217261913	0.3820076234
				0.0
$c = 0.95$	2.868876553	2.929805730	2.929867667	2.929867667
		0.5081937923	0.9073808309	0.9293623448
			0.6290719797	0.7127299224
			0.2242659678	0.3855500407
				0.0
$c = 0.98$	4.552986373	4.591227156	4.591233300	4.591233300
		0.5150476954	0.9089523432	0.9304890643
			0.6321100757	0.7152773693
			0.2258115657	0.3876817656
				0.0
$c = 0.99$	6.446918626	6.473887216	6.473888295	6.473888295
		0.5173385826	0.9094611359	0.9308538712
			0.6331125836	0.7161136380
			0.2263303773	0.3883931781
				0.0

Table 3.6: Eigenvalues of triplet anisotropic scattering for $f_1 = 0.2$, $f_2 = 0.04$, $f_3 = 0.008$ and $c>1$.

	$N = 2$	$N = 4$	$N = 8$	$N = 9$
$c = 1.01$	6.463056119 I	6.436234392 I	6.436233329 I	6.436233329 I
		0.5219271210	0.9104577949	0.9315686348
			0.6351021642	0.7177672039
			0.2273734056	0.3898170501
				0.0
$c = 1.02$	4.575808502 I	4.537983027 I	4.537977056 I	4.537977056 I
		0.5242238934	0.9109461775	0.9319189966
			0.6360892011	0.7185846273
			0.2278976170	0.3905294397
				0.0
$c = 1.05$	2.904964469 I	2.845674658 I	2.845616960 I	2.845616960 I
		0.5311189505	0.9123744614	0.9329442772
			0.6390191640	0.7210002683
			0.2294810041	0.3926680042
				0.0
$c = 1.1$	2.067245576 I	1.984675835 I	1.984362207 I	1.984362204 I
		0.5426004246	0.9146454147	0.9345773497
			0.6437985204	0.7249086129
			0.2321556029	0.3962348448
				0.0
$c = 1.2$	1.480872194 I	1.367953580 I	1.366329472 I	1.366329232 I
		0.5653399299	0.9188626952	0.9376248784
			0.6529735073	0.7323194645
			0.2376361703	0.4033627745
				0.0
$c = 1.3$	1.225357703 I	1.091980659 I	1.087925927 I	1.087923348 I
		0.5874909214	0.9227753638	0.9404770295
			0.6616622292	0.7392585560
			0.2432862620	0.4104558557
				0.0
$c = 1.4$	1.075828707 I	0.9274876763 I	0.9200097858 I	0.9199968806 I
		0.6088102947	0.9265024726	0.9432225307
			0.6699097503	0.7458081202
			0.2490966853	0.4174843896
				0.0
$c = 1.5$	0.9759000729 I	0.8161538347 I	0.8044891440 I	0.8044464820 I
		0.6291847074	0.9301355276	0.9459300547
			0.6777699997	0.7520443402
			0.2550554233	0.4244230613
				0.0
$c = 1.6$	0.9038769076 I	0.7351414280 I	0.7187748337 I	0.7186661049 I
		0.6486122225	0.9337481561	0.9486560043
			0.6852993194	0.7580348177
			0.2611477250	0.4312516807
				0.0
$c = 1.7$	0.8494119857 I	0.6733407724 I	0.6519809987 I	0.6517489937 I
		0.6671695500	0.9374027508	0.9514497225

			0.6925523016	0.7638379497
			0.2673563672	0.4379554397
				0.0
c = 1.8	0.8068715305 I	0.6245820565 I	0.5981189358 I	0.5976835801 I
		0.6849798338	0.9411551444	0.9543571538
			0.6995793909	0.7695032463
			0.2736620973	0.4445247387
				0.0
c = 1.9	0.7728981596 I	0.5851213802 I	0.5535808405 I	0.5528393284 I
		0.7021882092	0.9450580374	0.9574235920
			0.7064256806	0.7750720053
			0.2800442321	0.4509546856
				0.0
c = 2.00	0.7453559925 I	0.5525392130 I	0.5160433698 I	0.5148720675 I
		0.7189462327	0.9491636584	0.9606959117
			0.7131304310	0.7805780247
			0.2864813702	0.4572443863
				0.0

In Table 3.5, the eigenvalues are calculated by involving the value of scattering coefficients in equation (3.1). The diffusion length of triplet anisotropic scattering is increasing with increasing the number of secondary neutrons for c<1. And also the diffusion length is increasing with increasing the orders.

In Table 3.6, the diffusion length of triplet anisotropic scattering is decreasing with increasing the number of secondary neutrons for c>1. And also the diffusion length is decreasing with increasing the orders. For c=0.99 and c=1.01, the values of diffusion length in high orders (N>8) nearly become the same.

Table 3.7: Eigenvalues of isotropic scattering and linear, pure-quadratic, pure-triplet and triplet anisotropic scattering for $N = 4$, $f_n = 0.14$, $f_0 = 1$, $c<1$ and $n = 1, 2, 3$.

	Isotropic Scattering	Linear Anisotropic Scattering	Pure-quadratic Anisotropic Scattering	Pure-triplet Anisotropic Scattering	Triplet Anisotropic Scattering
$c = 0.0$	0.8611363115 0.3399810436	0.8611363115 0.3399810436	0.8611363115 0.3399810436	0.8611363115 0.3399810436	0.8611363115 0.3399810436
$c = 0.5$	1.011376584 0.4093819671	1.044945632 0.4108718939	1.027237302 0.4179548288	1.015406544 0.4228245261	1.065195461 0.4333989681
$c = 0.8$	1.404738391 0.4660324475	1.488820461 0.4666194052	1.420316455 0.4891251635	1.406783622 0.4938303994	1.507226897 0.5190551724
$c = 0.9$	1.902731844 0.4865741343	2.034503203 0.4867573420	1.914438416 0.5172845274	1.903566456 0.5202389273	2.047791256 0.5533166922
$c = 0.95$	2.635068997 0.4968778209	2.829687173 0.4969281828	2.643458682 0.5319361769	2.635380251 0.5335667611	2.838984216 0.5712812612
$c = 0.98$	4.115512636 0.5030227972	4.430589824 0.5030312821	4.120832346 0.5408434734	4.115593454 0.5415319332	4.436391634 0.5822594222
$c = 0.99$	5.796728080 0.5050608168	6.245664345 0.5050629729	5.800489474 0.5438254506	5.796756874 0.5441756263	6.249743727 0.5859453265

Table 3.8: Eigenvalues of isotropic scattering and linear, pure-quadratic, pure-triplet and triplet anisotropic scattering for $N = 4$, $f_n = 0.14$, $f_0 = 1$, $c>1$ and $n = 1, 2, 3$.

	Isotropic Scattering	Linear Anisotropic Scattering	Pure-quadratic Anisotropic Scattering	Pure-triplet Anisotropic Scattering	Triplet Anisotropic Scattering
$c = 1.01$	5.750541215 I 0.5091173350	6.205994419 I 0.5091195597	5.746785311 I 0.5498018414	5.750570412 I 0.5494399546	6.201968141 I 0.5933497285
$c = 1.02$	4.050199455 I 0.5111345012	4.374492281 I 0.5111435343	4.044895227 I 0.5527939093	4.050282550 I 0.5520586324	4.368840561 I 0.5970657624
$c = 1.05$	2.531853973 I 0.5171338297	2.741035624 I 0.5171927252	2.523524239 I 0.5617713760	2.532187648 I 0.5598493797	2.732328833 I 0.6082541647
$c = 1.1$	1.757034591 I 0.5269219538	1.909363643 I 0.5271725337	1.745485102 I 0.5766672362	1.757994495 I 0.5725638347	1.897714888 I 0.6269603783
$c = 1.2$	1.200164425 I 0.5454699847	1.313121207 I 0.5465691671	1.184896796 I 0.6057165209	1.202886039 I 0.5966579889	1.299029046 I 0.6640606651
$c = 1.3$	0.9505386853 I 0.5623363805	1.046113909 I 0.5649504375	0.9335570474 I 0.6330652343	0.9553835557 I 0.6186023480	1.032200844 I 0.6999577274
$c = 1.4$	0.8017837257 I 0.5773502691	0.8868643424 I 0.5821184862	0.7842949686 I 0.6582474058	0.8088425008 I 0.6382702787	0.8746844475 I 0.7341099861
$c = 1.5$	0.7011305395 I 0.5905310241	0.7789724674 I 0.5980065618	0.8291793423 0.6612010014 I	0.7103302615 I 0.6557944554	0.7693844944 I 0.7664036006
$c = 1.6$	0.6278301077 I 0.6020171195	0.7003393467 I 0.6126488077	0.6111227833 I 0.7020881524	0.6390204240 I 0.6714371710	0.6937414030 I 0.7970045190
$c = 1.7$	0.5717750112 I 0.6120013978	0.6402165530 I 0.6261425257	0.5558181332 I 0.7212193801	0.5847831957 I 0.6854964581	0.6367298726 I 0.8262089641
$c = 1.8$	0.5273616207 I 0.6206876316	0.5926400917 I 0.6386159847	0.5122079000 I 0.7388980836	0.5420212971 I 0.6982556549	0.5922369143 I 0.8543459033
$c = 1.9$	0.4912030186 I 0.6282671079	0.5539954570 I 0.6502062161	0.4768418890 I 0.7554103325	0.5073662017 I 0.7099631166	0.5565784280 I 0.8817285247
$c = 2.00$	0.4611214048 I 0.6349087654	0.5219497392 I 0.6610459853	0.4475113402 I 0.7710034331	0.4786616564 I 0.7208281152	0.5273971722 I 0.9086362598

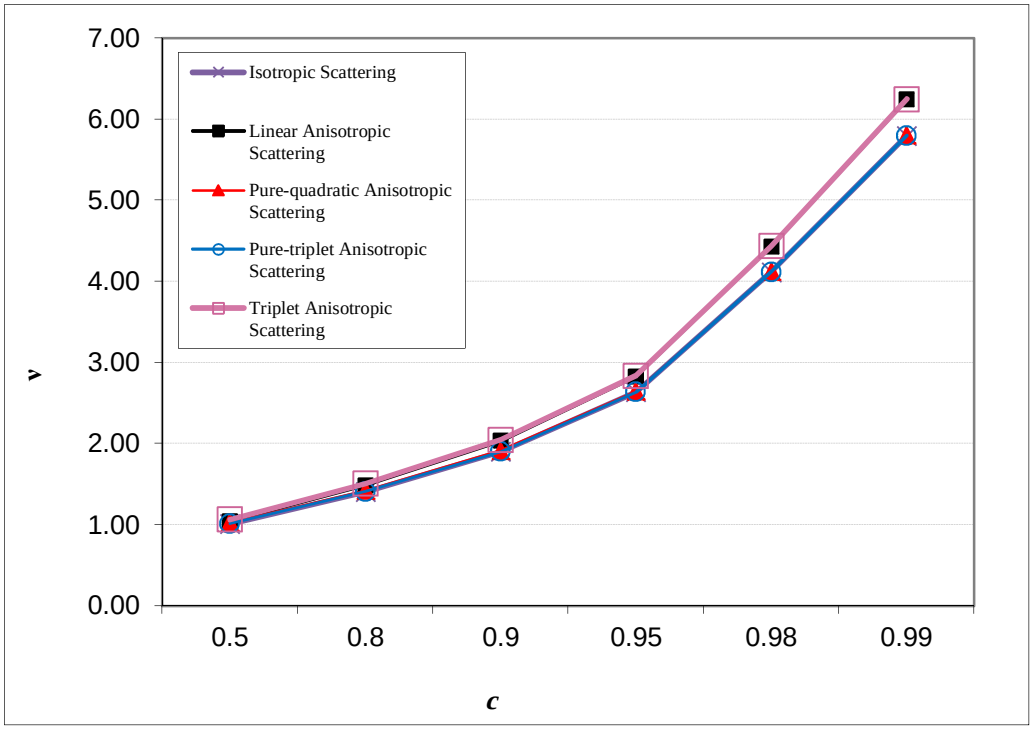


Figure 3.11: For N = 4, f_n =0.14 and c<1, the diffusion lengths or eigenvalues for different values of c, by comparing isotropic scattering and the first three degrees of anisotropic scattering.

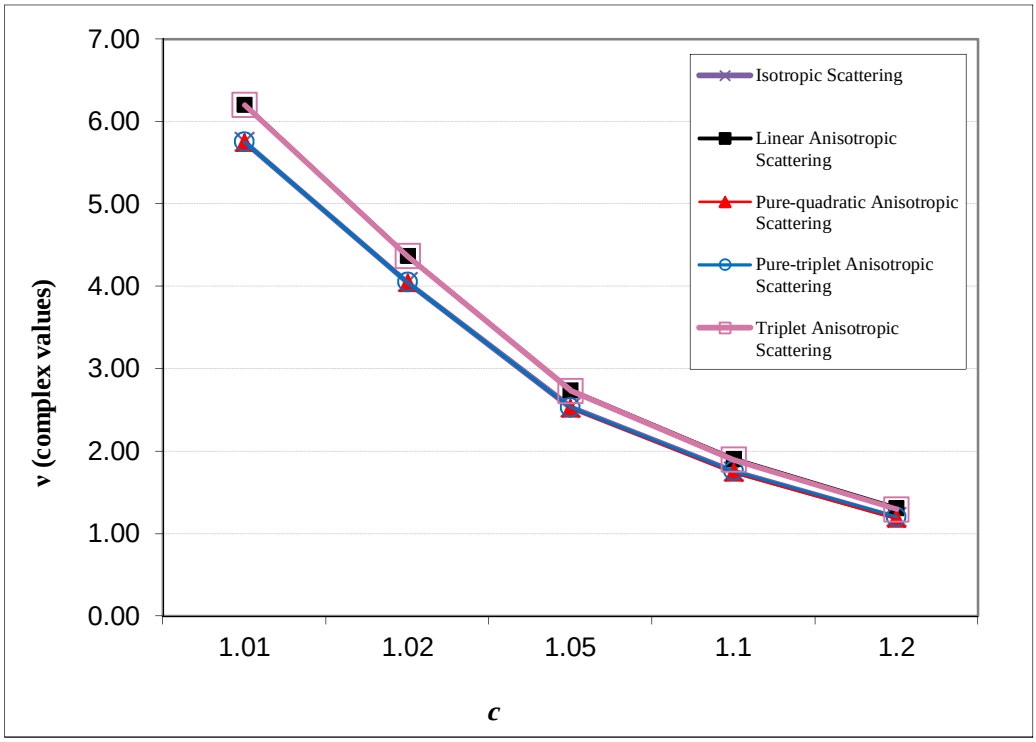


Figure 3.12: For N = 4, f_n =0.14 and c>1, the diffusion length for different values of c, by comparing isotropic scattering and the first three degrees of anisotropic scattering.

Table 3.9: Eigenvalues of isotropic scattering and linear, pure-quadratic, pure-triplet and triplet anisotropic scattering for $N = 4$, $f_n = 1/15$, $f_0 = 1$, $c<1$ and $n = 1, 2, 3$.

	Isotropic Scattering	Linear Anisotropic Scattering	Pure-quadratic Anisotropic Scattering	Pure-triplet Anisotropic Scattering	Triplet Anisotropic Scattering
$c = 0.0$	0.8611363115 0.3399810436	0.8611363115 0.3399810436	0.8611363115 0.3399810436	0.8611363115 0.3399810436	0.8611363115 0.3399810436
$c = 0.5$	1.011376584 0.4093819671	1.026880221 0.4100940257	1.018641239 0.4134109516	1.013211702 0.4156263126	1.036055171 0.4204783174
$c = 0.8$	1.404738391 0.4660324475	1.442895684 0.4663136519	1.411669922 0.4766283857	1.405644601 0.4786714619	1.450890439 0.4898705830
$c = 0.9$	1.902731844 0.4865741343	1.962164731 0.4866617496	1.907897580 0.5005040791	1.903099347 0.5017659863	1.967846099 0.5162305410
$c = 0.95$	2.635068997 0.4968778209	2.722566184 0.4969018622	2.638759095 0.5126834843	2.635205760 0.5133747912	2.726510726 0.5297327149
$c = 0.98$	4.115512636 0.5030227972	4.256886492 0.5030268419	4.117848799 0.5200113634	4.115548114 0.5203020616	4.259337154 0.5378788364
$c = 0.99$	5.796728080 0.5050608168	5.998029631 0.5050618441	5.798379165 0.5224521923	5.796740716 0.5225998634	5.999750257 0.5405963608

Table 3.10: Eigenvalues of isotropic scattering and linear, pure-quadratic, pure-triplet and triplet anisotropic scattering for $N = 4$, $f_n = 1/15$, $f_0 = 1$, $c>1$ and $n = 1, 2, 3$.

	Isotropic Scattering	Linear Anisotropic Scattering	Pure-quadratic Anisotropic Scattering	Pure-triplet Anisotropic Scattering	Triplet Anisotropic Scattering
$c = 1.01$	5.750541215 I 0.5091173350	5.954490265 I 0.5091183938	5.748893830 I 0.5273258838	5.750554024 I 0.5271736440	5.952796833 I 0.5460291916
$c = 1.02$	4.050199455 I 0.5111345012	4.195317215 I 0.5111387976	4.047873732 I 0.5297569883	4.050235901 I 0.5294480245	4.192943406 I 0.5487426778
$c = 1.05$	2.531853973 I 0.5171338297	2.625268346 I 0.5171617861	2.528204452 I 0.5370168800	2.532000279 I 0.5362118154	2.621625891 I 0.5568605638
$c = 1.1$	1.757034591 I 0.5269219538	1.824823631 I 0.5270404378	1.751976743 I 0.5489546875	1.757455456 I 0.5472433693	1.819979793 I 0.5702623865
$c = 1.2$	1.200164425 I 0.5454699847	1.250078238 I 0.5459847327	1.193465403 I 0.5718838864	1.201359316 I 0.5681261413	1.244273696 I 0.5962301674
$c = 1.3$	0.9505386853 I 0.5623363805	0.9924796843 I 0.5635464237	0.9430511415 I 0.5930838235	0.9526708745 I 0.5870950731	0.9867805735 I 0.6205852151
$c = 1.4$	0.8017837257 I 0.5773502691	0.8388751193 I 0.5795293977	0.7940224511 I 0.6122658018	0.8048980109 I 0.6039930351	0.8338903684 I 0.6430077062
$c = 1.5$	0.7011305395 I 0.5905310241	0.7348614404 I 0.5939021377	0.6934058167 I 0.6294088829	0.7051975747 I 0.6188844036	0.7309139078 I 0.6634552165
$c = 1.6$	0.6278301077 I 0.6020171195	0.6590771493 I 0.6067474443	0.6203148956 I 0.6446618948	0.6327834694 I 0.6319592646	0.6563045881 I 0.6820641619
$c = 1.7$	0.5717750112 I 0.6120013978	0.6011203543 I 0.6182099070	0.5645546642 I 0.6582507983	0.5775359487 I 0.6434552847	0.5995505542 I 0.6990547700
$c = 1.8$	0.5273616207 I 0.6206876316	0.5552199700 I 0.6284564636	0.5204691274 I 0.6704174379	0.5338526317 I 0.6536103002	0.5548219540 I 0.7146673900
$c = 1.9$	0.4912030186 I 0.6282671079	0.5178826244 I 0.6376523902	0.4846419273 I 0.6813877931	0.4983539431 I 0.6626396720	0.5185981519 I 0.7291289308
$c = 2.00$	0.4611214048 I 0.6349087654	0.4868578706 I 0.6459491979	0.4548797979 I 0.6913594589	0.4688713356 I 0.6707286778	0.4886192838 I 0.7426391877

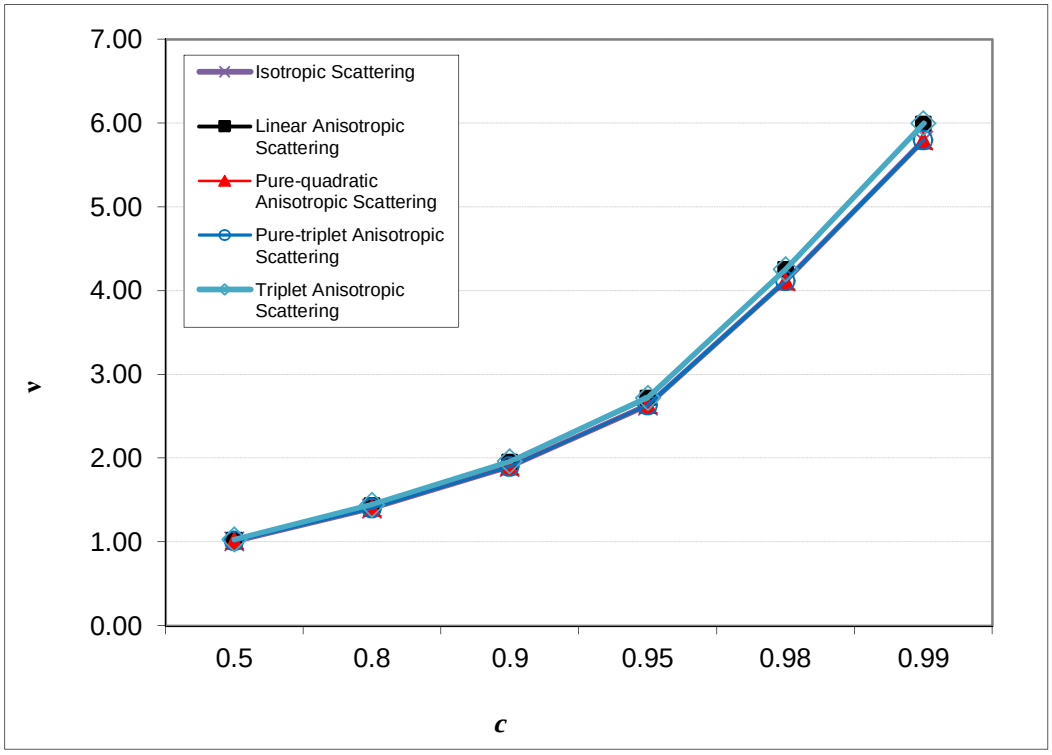


Figure 3.13: For $N = 4$, $f_n = 0.0666$ and $c<1$, the diffusion length for different values of c, by comparing isotropic scattering and the first three degrees of anisotropic scattering.

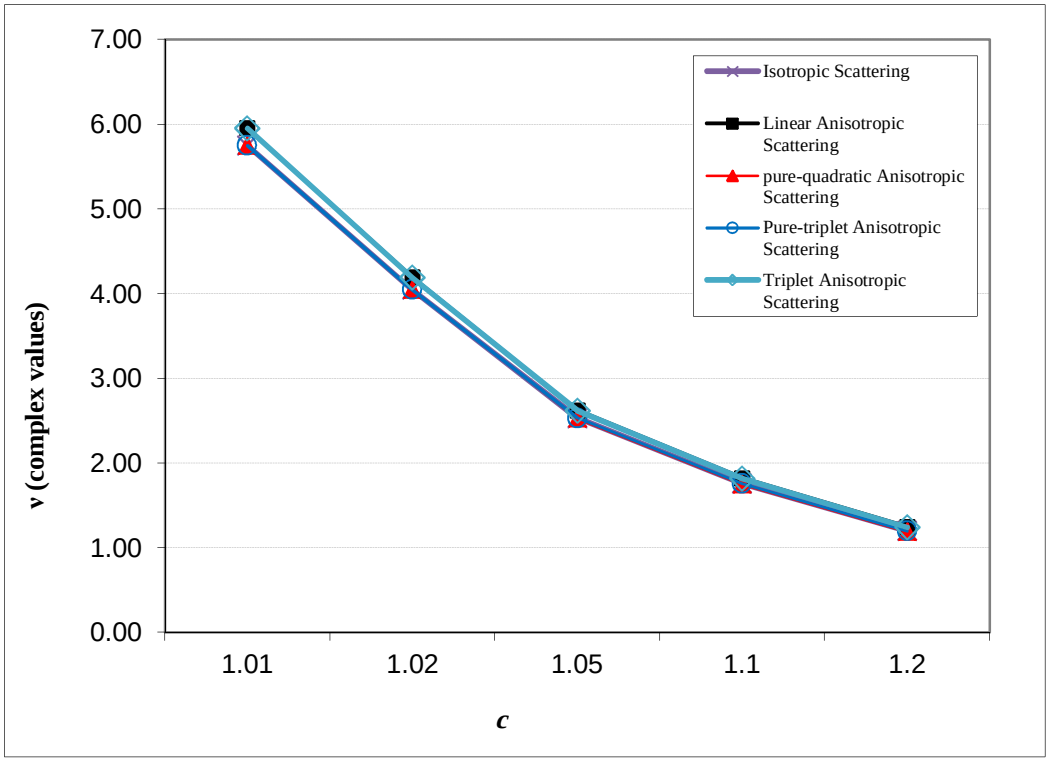


Figure 3.14: For $N = 4$, $f_n = 0.0666$ and $c>1$, the diffusion length for different values of c, by comparing isotropic scattering and the first three degrees of anisotropic scattering.

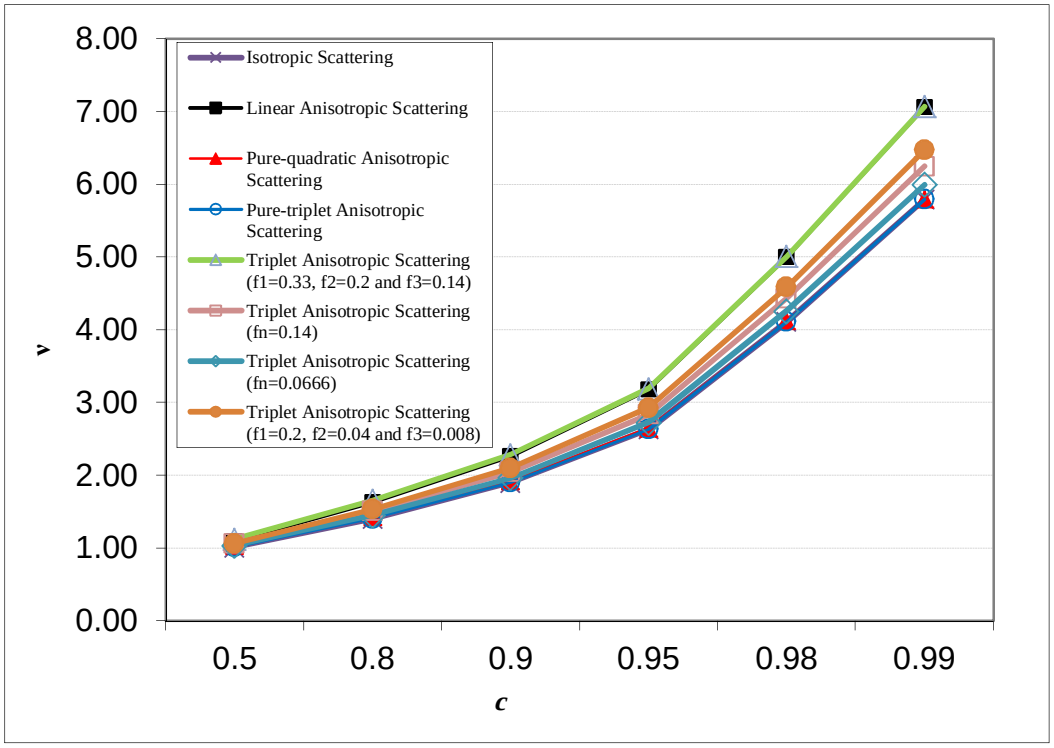


Figure 3.15: For $N = 4$, f = max. values and $c<1$, the diffusion length for different values of c, by comparing isotropic scattering and the first three degrees of anisotropic scattering.

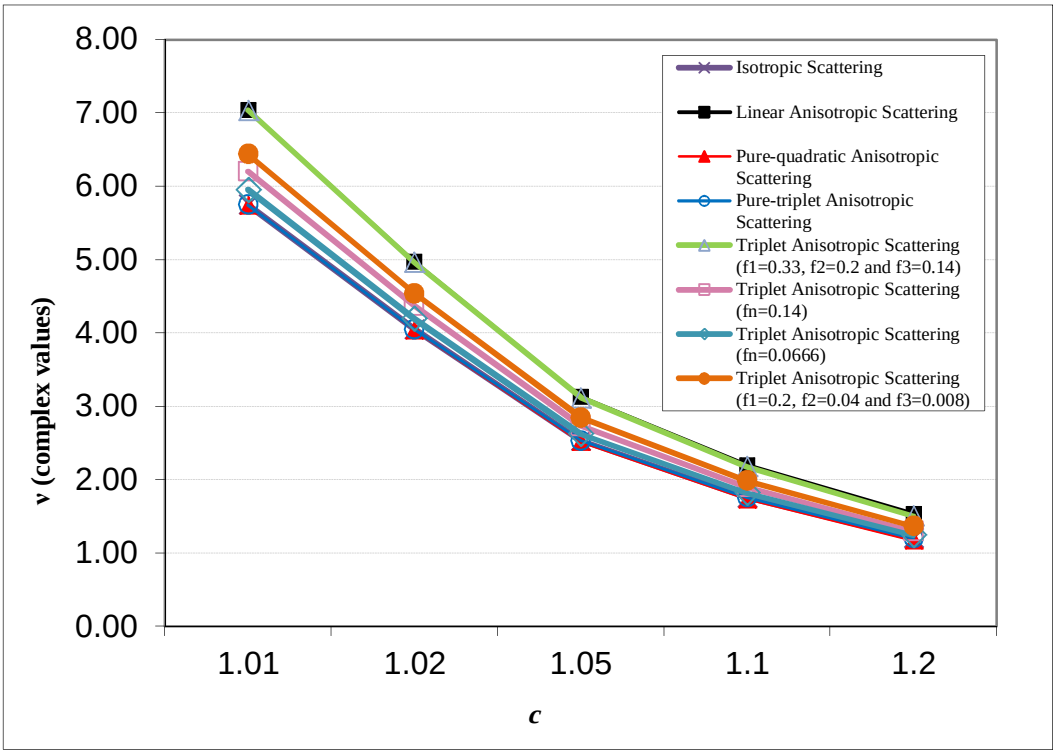


Figure 3.16: For $N = 4$, f = max. values and $c>1$, the diffusion length for different values of c, by comparing isotropic scattering and the first three degrees of anisotropic scattering.

Table 3.11: Eigenvalues of linear, pure-quadratic and pure-triplet anisotropic scattering for $N = 4$, $c = 0.99$ and $n = 1, 2, 3$.

	Linear Anisotropic Scattering	Pure-quadratic Anisotropic Scattering (f_1=0)	Pure-triplet Anisotropic Scattering (f_1=0, f_2=0)
f_n = max.	7.064383333 0.5050658919	5.802502240 0.5634094204	5.796756874 0.5441756263
f_n = 0.14	6.245664345 0.5050629729	5.800489474 0.5438254506	5.796756874 0.5441756263
f_n = 0.0666	5.998029631 0.5050618441	5.798379165 0.5224521923	5.796740716 0.5225998634
$f_n = \frac{1}{a^n}, a = 5$	6.472813543 0.5050638956	5.797691341 0.5152825542	5.796729507 0.5070726920

Table 3.12: Eigenvalues of linear, pure-quadratic and pure-triplet anisotropic scattering for $N = 4$, $c = 1.01$ and $n = 1, 2, 3$.

	Linear Anisotropic Scattering	Pure-quadratic Anisotropic Scattering (f_1=0)	Pure-triplet Anisotropic Scattering (f_1=0, f_2=0)
f_n = max.	7.042697145 I 0.5091225869	5.744771174 I 0.5704957652	5.750570412 I 0.5494399546
f_n = 0.14	6.205994419 I 0.5091195597	5.746785311 I 0.5498018414	5.750570412 I 0.5494399546
f_n = 0.0666	5.954490265 I 0.5091183938	5.748893830 I 0.5273258838	5.750554024 I 0.5271736440
$f_n = \frac{1}{a^n}, a = 5$	6.437312055 I 0.5091205147	5.749580357 I 0.5198108466	5.750542661 I 0.5111865895

Table 3.13: Eigenvalues of linear, quadratic and triplet anisotropic scattering for $N = 4$, $c = 0.99$ and $n = 1, 2, 3$.

	Linear Anisotropic Scattering	Triplet Anisotropic Scattering ($f_1 \neq 0$, $f_2 \neq 0$)
f_n = max.	7.064383333 0.5050658919	7.071417416 0.6070524360
$f_n = 0.14$	6.245664345 0.5050629729	6.249743727 0.5859453265
$f_n = 0.0666$	5.998029631 0.5050618441	5.999750257 0.5405963608
$f_n = \frac{1}{a^n}, a = 5$	6.472813543 0.5050638956	6.473887216 0.5173385826

Table 3.14: Eigenvalues of linear, quadratic and triplet anisotropic scattering for $N = 4$, $c = 1.01$ and $n = 1, 2, 3$.

	Linear Anisotropic Scattering	Triplet Anisotropic Scattering ($f_1 \neq 0$, $f_2 \neq 0$)
f_n = max.	7.042697145 I 0.5091225869	7.035626745 I 0.6156894313
$f_n = 0.14$	6.205994419 I 0.5091195597	6.201968141 I 0.5933497285
$f_n = 0.0666$	5.954490265 I 0.5091183938	5.952796833 I 0.5460291916
$f_n = \frac{1}{a^n}, a = 5$	6.437312055 I 0.5091205147	6.436234392 I 0.5219271210

In Table 3.11, the eigenvalue of pure anisotropic scattering is decreasing with increasing the degree of anisotropy for $c<1$. But in Table 3.13, the eigenvalue of non-pure anisotropic scatterings is increasing with increasing the degree of anisotropy for $c<1$.

It is well known that anisotropy increases the diffusion length. In this thesis, it is seen that the obtained diffusion lengths by using our scattering function are greater than the diffusion lengths obtained by pure-triplet anisotropic.

We calculate the eigenvalues for pure-triplet anisotropic scattering according c, by comparing the different orders as shown in Tables 3.1-2. The obtained results are the same with Salah's results (Sallah, 2007). The diffusion length is increasing with increasing the number of secondary neutrons for $c<1$ as shown in Figure 3.11. But the diffusion length is decreasing with increasing the number of secondary neutrons for $c>1$ as shown in Figure 3.12. These cases are true for all scatterings (see Tables 3.1-14 and Figures 3.11-16).

The eigenvalues or diffusion lengths are increasing with increasing the orders for the case of $c<1$, but the diffusion lengths is decreasing with increasing the orders for the case of $c>1$ in all the scatterings. And for $c=0.99$ and $c=1.01$, the values of diffusion length in high orders ($N>8$) nearly become the same. These results are given in Tables 3.1-6.

The diffusion length is increasing with increasing the scattering coefficient for the case of $c<1$ and $c>1$, for all scatterings except $c=0$. The diffusion lengths are the same for all scatterings and there are no discrete eigenvalues for the case of $c=0$ (see appendix Table 1 and Table 2).

In order to see the different between the degrees of anisotropy, we fixed the different scattering coefficients for linear, pure-quadratic and pure-triplet anisotropic scattering. In Table 3.11, the eigenvalues are decreasing with increasing the degree of anisotropy for $c<1$.

But in Table 3.13, we also fixed the different scattering coefficients for linear and triplet anisotropic scattering. The eigenvalues are increasing with increasing the degree of anisotropy for $c<1$. And in both cases of pure and non-pure, the eigenvalues are changing in a very small amount for different degrees of anisotropy.

As results of this thesis, we can draw the following conclusions:

1. We improve the pure-triplet anisotropic scattering by adding new terms to the phase function.
2. The scattering coefficients of triplet anisotropic scattering are predicted by a formula.
3. For the comparison of eigenvalues of isotropic scattering, pure-quadratic and pure-triplet anisotropic scattering are calculated with different situations.
4. The results of triplet anisotropic scattering are more suitable than pure-triplet anisotropic scattering.

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APPENDIX Table 1: Eigenvalues of isotropic scattering and linear, pure-quadratic, pure-triplet and triplet anisotropic scattering for $N = 4$, f = maximum values, and $c<1$.

	Isotropic Scattering f_0=1	Linear Anisotropic Scattering f_1=0.33	Pure-quadratic Anisotropic Scattering f_1=0, f_2=0.2	Pure-triplet Anisotropic Scattering f_1=0, f_2=0, f_3=0.14	Triplet Anisotropic Scattering f_1=0.33, f_2=0.2, f_3=0.14	Triplet Anisotropic Scattering f_n = 0.14	Triplet Anisotropic Scattering f_n = 0.0666	Triplet Anisotropic Scattering f_1= 0.2, f_2= 0.04, f_3= 0.008
c = 0.0	0.8611363115	0.8611363115	0.8611363115	0.8611363115	0.8611363115	0.8611363115	0.8611363115	0.8611363115
	0.3399810436	0.3399810436	0.3399810436	0.3399810436	0.3399810436	0.3399810436	0.3399810436	0.3399810436
c = 0.5	1.011376584	1.097475982	1.034791834	1.015406544	1.125544609	1.065195461	1.036055171	1.065139891
	0.4093819671	0.4128605062	0.4217619101	0.4228245261	0.4400205141	0.4333989681	0.4204783174	0.4147352900
c = 0.8	1.404738391	1.632634930	1.428345289	1.406783622	1.660558317	1.507226897	1.450890439	1.534307516
	0.4660324475	0.4673945201	0.5000791613	0.4938303994	0.5320744956	0.5190551724	0.4898705830	0.4746965571
c = 0.9	1.902731844	2.267351542	1.920612262	1.903566456	2.289176787	2.047791256	1.967846099	2.103378248
	0.4865741343	0.4870012567	0.5323288230	0.5202389273	0.7830872859	0.5533166922	0.5162305410	0.4968587008
c = 0.95	2.635068997	3.179571792	2.647923285	2.635380251	3.195073710	2.838984216	2.726510726	2.929805730
	0.4968778209	0.4969957750	0.5494063755	0.5335667611	0.5901853058	0.5712812612	0.5297327149	0.5081937923
c = 0.98	4.115512636	5.003117740	4.123675478	4.115593454	5.013034627	4.436391634	4.259337154	4.591227156
	0.5030227972	0.5030427419	0.5598853487	0.5415319332	0.6027841353	0.5822594222	0.5378788364	0.5150476954
c = 0.99	5.796728080	7.064383333	5.802502240	5.796756874	7.071417416	6.249743727	5.999750257	6.473887216
	0.5050608168	0.5050658919	0.5634094204	0.5441756263	0.6070524360	0.5859453265	0.5405963608	0.5173385826

APPENDIX Table 2: Eigenvalues of isotropic scattering and linear, pure-quadratic, pure-triplet and triplet anisotropic scattering for $N = 4$, f = maximum values, and $c>1$.

	Isotropic Scattering f_0=1	Linear Anisotropic Scattering f_1=0.33	Pure-quadratic Anisotropic Scattering f_1=0, f_2=0.2	Pure-triplet Anisotropic Scattering f_1=0, f_2=0, f_3=0.14	Triplet Anisotropic Scattering f_1=0.33, f_2=0.2, f_3=0.14	Triplet Anisotropic Scattering f_n = 0.14	Triplet Anisotropic Scattering f_n = 0.0666	Triplet Anisotropic Scattering f_1= 0.2, f_2= 0.04, f_3=0.008
c = 1.01	5.750541215 I 0.5091173350	7.042697145 I 0.5091225869	5.744771174 I 0.5704957652	5.750570412 I 0.5494399546	7.035626745 I 0.6156894313	6.201968141 I 0.5933497285	5.952796833 I 0.5460291916	6.436234392 I 0.5219271210
c = 1.02	4.050199455 I 0.5111345012	4.972451207 I 0.5111558593	4.042048214 I 0.5740551056	4.050282550 I 0.5520586324	4.962431640 I 0.6200571524	4.368840561 I 0.5970657624	4.192943406 I 0.5487426778	4.537983027 I 0.5242238934
c = 1.05	2.531853973 I 0.5171338297	3.131109929 I 0.5172738117	2.519044242 I 0.5847796651	2.532187648 I 0.5598493797	3.115203164 I 0.6333508687	2.732328833 I 0.6082541647	2.621625891 I 0.5568605638	2.845674658 I 0.5311189505
c = 1.1	1.757034591 I 0.5269219538	2.198947875 I 0.5275236738	1.739267022 I 0.6027164451	1.757994495 I 0.5725638347	2.176454456 I 0.6561072286	1.897714888 I 0.6269603783	1.819979793 I 0.5702623865	1.984675835 I 0.5426004246
c = 1.2	1.200164425 I 0.5454699847	1.536632420 I 0.5481799491	1.176728461 I 0.6381584929	1.202886039 I 0.5966579889	1.505597483 I 0.7035837472	1.299029046 I 0.6640606651	1.244273696 I 0.5962301674	1.367953580 I 0.5653399299
c = 1.3	0.9505386853 I 0.5623363805	1.243183665 I 0.5690004744	0.9246054154 I 0.6720377750	0.9553835557 I 0.6186023480	1.207165517 I 0.7531623311	1.032200844 I 0.6999577274	0.9867805735 I 0.6205852151	1.091980659 I 0.5874909214
c = 1.4	0.8017837257 I 0.5773502691	1.069688939 I 0.5899945048	0.7752512710 I 0.7037005239	0.8088425008 I 0.6382702787	1.031197924 I 0.8043946416	0.8746844475 I 0.7341099861	0.8338903684 I 0.6430077062	0.9274876763 I 0.6088102947
c = 1.5	0.7011305395 I 0.5905310241	0.9531853710 I 0.6112495403	0.6750820075 I 0.7330541383	0.7103302615 I 0.6557944554	0.9140545619 I 0.8571586807	0.7693844944 I 0.7664036006	0.7309139078 I 0.6634552165	0.8161538347 I 0.6291847074
c = 1.6	0.6278301077 I 0.6020171195	0.8692037860 I 0.6329339206	0.6028120429 I 0.7603518442	0.6390204240 I 0.6714371710	0.8307010916 I 0.9116956499	0.6937414030 I 0.7970045190	0.6563045881 I 0.6820641619	0.7351414280 I 0.6486122225
c = 1.7	0.5717750112 I 0.6120013978	0.8059459679 I 0.6552988384	0.5480115892 I 0.7859879257	0.5847831957 I 0.6854964581	0.7688874963 I 0.9685745940	0.6367298726 I 0.8262089641	0.5995505542 I 0.6990547700	0.6733407724 I 0.6671695500
c = 1.8	0.5273616207 I 0.6206876316	0.7569205438 I 0.6786850804	0.5048996067 I 0.8103760404	0.5420212971 I 0.6982556549	0.7217877184 I 1.028651635	0.5922369143 I 0.8543459033	0.5548219540 I 0.7146673900	0.6245820565 I 0.6849798338
c = 1.9	0.4912030186 I 0.6282671079	0.7182279263 I 0.7035397613	0.4699995026 I 0.8338964163	0.5073662017 I 0.7099631166	0.6852678454 I 1.093066806	0.5565784280 I 0.8817285247	0.5185981519 I 0.7291289308	0.5851213802 I 0.7021882092
c = 2.00	0.4611214048 I 0.6349087654	0.6873803165 I 0.7304492625	0.4410931168 I 0.8568813672	0.4786616564 I 0.7208281152	0.6566802325 I 1.163298934	0.5273971722 I 0.9086362598	0.4886192838 I 0.7426391877	0.5525392130 I 0.7189462327

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