

ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL

**ADVANCED APPROACHES TO FUZZY COGNITIVE MAPPING
FOR ENHANCING CONVERGENCE, LEARNING, AND PREDICTION**



Ph.D. THESIS

Miraç MURAT

Department of Industrial Engineering

Industrial Engineering Programme

MARCH 2025

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İSTANBUL TEKNİK ÜNİVERSİTESİ ★ LİSANSÜSTÜ EĞİTİM ENSTİTÜSÜ

**YAKINSAMA, ÖĞRENME VE TAHMİN PERFORMANSININ
İYİLEŞTİRİLMESİ İÇİN BULANIK BİLİŞSEL HARİTALARDA İLERİ
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To my family,



FOREWORD

This dissertation focuses on improving the reasoning mechanism of Fuzzy Cognitive Maps (FCMs) to enhance their predictive accuracy. By strengthening the capability of FCMs to model and predict dynamic system behaviors, this thesis aims to contribute to the literature through both theoretical and practical approaches to advancing learning and prediction processes.

I would like to express my sincere gratitude to my advisor, Assoc. Prof. Dr. Umut ASAN, for his valuable guidance and continuous support throughout my doctoral studies. I also extend my thanks to Prof. Dr. Özgür KABAK and Prof. Dr. S. Emre ALPTEKİN for their constructive feedback and developmental contributions during the thesis process. A very special thanks to Assoc. Dr. Ertuğrul AYYILDIZ for his unwavering motivation and encouragement, which were truly invaluable throughout this journey.

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I hope this study will contribute to the FCM literature and inspire further academic and practical research in this field.

December 2024

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ABBREVIATIONS

CM	: Cognitive Mapping
FCM	: Fuzzy Cognitive Map
FF	: Fermatean Fuzzy
F-FCM	: Fermatean Fuzzy Cognitive Map
FFN	: Fermatean Fuzzy Number
FFS	: Fermatean Fuzzy Set
FSC	: Forestry Supply Chain
FF-VIKOR	: Fermatean Fuzzy Vlse KriterijumsaOptimizacija I Kompromisno
FFWA	: Fermatean Fuzzy Weighted Average
HL	: Hebbian Learning
MCDM	: Multi Criteria Decision Making
NA	: Not Applied
pro-FCM	: FCM equipped with the proposed reasoning mechanism
PSO	: Particle Swarm Optimization
sig-FCM	: FCM equipped with sigmoid function
tan-FCM	: FCM equipped with tangent sigmoid function
TFN	: Triangular Fuzzy Numbers
TOPSIS	: Technique for Order Preference by Similarity to Ideal Solutions
VIKOR	: Vlse KriterijumsaOptimizacija I Kompromisno (Multicriteria Optimization and Compromise Solution)



SYMBOLS

G	: Directed graph
W	: Matrix of causal relationships of FCM
C	: The set of concepts
n	: The number of concepts
A	: State vector
N	: The number of state vectors in historical data
t	: Time
a	: Actual activation value
$\hat{a}$	: Predicted activation value
e	: Deviation of actual activation value from estimated activation value
ω	: Normalized form of causal relationships matrix
f	: Activation function
F	: Fitness function
$\mathfrak{U}$	: Solution space for PSO algorithm
S	: Swarm with M particles
t	: Iteration for PSO algorithm
$\mathcal{X}$	: Position of a particle
V	: Velocity of a particle



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ADVANCED APPROACHES TO FUZZY COGNITIVE MAPPING FOR ENHANCING CONVERGENCE, LEARNING, AND PREDICTION

SUMMARY

Fuzzy Cognitive Maps (FCMs) are widely used for modeling complex systems and analyzing their dynamic behaviors. However, traditional FCMs rely on classical reasoning mechanisms, typically based on nonlinear activation functions such as sigmoid or hyperbolic tangent, that often converge to a steady state regardless of initial conditions. While this behavior may be acceptable for certain control-oriented problems, it limits the applicability of FCMs in both prediction and learning-based contexts where modeling non-equilibrium or cyclic behaviors is essential. Additionally, the common reliance on expert-defined relationships introduces subjectivity and inconsistency, which can compromise the reliability of the model, especially when interdependencies are intricate or partially hidden.

This study addresses these limitations by proposing an alternative reasoning mechanism that incorporates indirect relationships through higher-order dependencies. By considering the powers of the relationship matrix, the mechanism captures the deeper structure of causal influence among system concepts. This provides a more comprehensive framework for modeling, predicting and learning complex behaviors.

This thesis is structured around three key parts, each addressing a distinct aspect of the research. The first part introduces the conceptual foundation of the proposed mechanism, focusing on how the inclusion of both direct and indirect relationships enhances the assessment of concept importance in a system. This is demonstrated through an application in a decision-making context under uncertainty, where interrelated criteria are analyzed using fuzzy representations. The results show that accounting for hidden causal relationships improves the model's depth and interpretability.

The second part elaborates the theoretical structure and implementation method. Unlike traditional approaches that rely on nonlinear activation functions, the proposed mechanism uses successive powers of the relationship matrix to represent indirect interrelationships. Each resulting matrix is normalized and aggregated to compute new activation levels. This approach enables dynamic transitions that generate solutions sensitive to initial conditions rather than always converging to a single fixed state. A case study on supply chain vulnerability analysis is presented to demonstrate how the method captures different behavioral patterns under various disruption scenarios.

The third part presents a comprehensive experimental evaluation within the context of FCM learning. Different versions of the reasoning framework, using varied normalization techniques, cumulative and non-cumulative aggregation strategies, and three fitness functions (RMSE, Max-RMSE, and Heaviside), are tested on randomly initialized FCM models. The learning process is supported by the Particle Swarm Optimization (PSO) algorithm, which is used to fine-tune relationship weights based on historical data, reducing reliance on subjective expert inputs. Performance is

compared against classical reasoning mechanisms and a recent exponential normalized reasoning approach. The results show that the proposed mechanism consistently provides higher prediction accuracy. Notably, the Heaviside function yields the best results across most setups.

To ensure an unbiased evaluation, synthetic data were not generated using any of the tested mechanisms. Instead, the experiments utilized a real data about marine ecosystem involving large aquatic species such as dolphins, inspired by early pioneering work in the field. This approach ensured independence of test data and preserved objectivity in performance comparisons. Statistical validation using the Wilcoxon signed-rank test and effect size calculations confirmed the proposed method's superiority in most scenarios.

In conclusion, this thesis presents an effective reasoning framework that extends the learning and prediction capabilities of FCMs. By moving beyond conventional activation-based reasoning and integrating higher-order structural knowledge, the proposed mechanism improves the accuracy of FCMs. It provides valuable tools for modeling complex dynamic systems in domains such as social analysis, ecological management, and strategic planning.

YAKINSAMA, ÖĞRENME VE TAHMİN PERFORMANSININ İYİLEŞTİRİLMESİ İÇİN BULANIK BİLİŞSEL HARİTALARDA İLERİ YAKLAŞIMLAR

ÖZET

Bu tez çalışması, Bulanık Bilişsel Harita (BBH) tabanlı modellemenin etkinliğini artırmak amacıyla BBH'nin çıkarsama mekanizmasında bazı iyileştirmeler yapılmasına odaklanmaktadır. BBH, karmaşık sistemlerin modellenmesinde sıkça tercih edilen, kolay uygulanabilen ve hesaplama açısından pratik bir tekniktir. Bu yöntem, sistemi düğümler ve yönlü oklar yardımıyla harita üzerinde görselleştirir. Sistemin temel unsurları düğümler ile temsil edilirken, bu unsurlar arasındaki ilişkiler ise yönlü oklar aracılığıyla ifade edilir. Bu yönlü oklar, ilişkilerin derecesini gösteren sayısal ağırlıklar taşır. BBH, unsurlar arasındaki ilişkilerin varlığını ve büyüklüğünü kullanarak sistem davranışının analiz edilmesini mümkün kılar.

BBH, karmaşık sistemlerin dinamiklerini modellemede sıklıkla tercih edilmesinin temel sebeplerinden biri kolay yapılandırılabilir olmasıdır. En basit haliyle BBH tekniği, uzman görüşlerine dayanarak sistemin temel unsurlarının ve bunlar arasındaki ilişkilerin belirlenmesine imkân sağlar. Ancak yalnızca uzman görüşüne dayalı bir yaklaşımın bazı sınırlılıkları bulunmaktadır. Uzman görüşlerine başvurmak, genellikle sistemin temel unsurlarının belirlenmesinde sorun yaratmazken, bu unsurlar arasındaki ilişkilerin ve özellikle ilişki güçlerinin belirlenmesinde yetersiz kalabilmektedir. Bu durumun temel nedeni, ele alınan sisteme dair uzmanlık alanlarının farklılığı veya uzmanların sistemi algılama biçimlerinin değişken olmasıdır. Dolayısıyla bu farklılıklar, modelde belirsizliğe ve kesin olmayan sonuçlara yol açabilmektedir. Böyle oluşturulmuş bir BBH modeli ise gerçek sistemin davranışlarını yeterince doğru yansıtamayabilir.

Yukarıda belirtilen sorunlara çözüm arayan araştırmacılar, BBH modellerindeki ilişkilerin güçlerini temsil eden ağırlık değerlerini iyileştirmek amacıyla çeşitli öğrenme algoritmaları geliştirmiştir. Bu algoritmalar, başlangıçta uzman görüşleriyle belirlenen ilişki ağırlıklarını daha hassas biçimde güncellemeyi hedeflemektedir. Böylece uzmanların ortaya koyduğu ilk modele kıyasla, sistemi daha doğru biçimde yansıtan bir model elde edilmesi amaçlanmaktadır. Ağırlıkların güncellenmesi sürecinde, unsurların birlikte nasıl değiştiği analiz edilerek veya sistem davranışına ilişkin mevcut verilerden yararlanılarak modelin doğruluğu artırılmaktadır.

Veri kullanımı için, sistem unsurlarının aktiflik düzeylerini gösteren aktivasyon seviyelerinin, farklı zamanlarda ölçülerek kaydedilmesi gerekmektedir. Başka bir ifadeyle, belirli bir anda sistemdeki her bir unsurun aktivasyon düzeyi, yani durum vektörü, elde edilmelidir. Ardışık anlarda elde edilen bu durum vektörleri, sistemin zamana bağlı davranışının veriye dayalı olarak temsil edilmesini sağlar.

Literatürde araştırmacılar, ilişkilerin ağırlık değerlerini güncellemek amacıyla Genetik Algoritma ve Parçacık Sürüşü Optimizasyonu (PSO) gibi metasezgisel optimizasyon

algoritmalarını sıklıkla kullanmıştır. Burada temel alınan mantık, gözlemlenen verilerde yer alan sistem davranışını mümkün olduğunca doğru biçimde tekrarlayabilecek bir BBH modeli oluşturmaktır. Başka bir ifadeyle, sistemin başlangıç aktivasyon seviyelerinden hareket edilerek, ilerleyen aşamalarda bu aktivasyon seviyelerinin nasıl değiştiğini ve durumlar arası geçişlerin nasıl gerçekleştiğini açıklayan bir davranış modeli geliştirilmesi hedeflenir. Bu süreçte, uzman görüşlerine dayanarak başlangıçta belirlenen ilişki ağırlıkları, gözlenen ardışık durumlar arasındaki geçişleri daha doğru biçimde yansıtacak şekilde güncellenir ve böylece daha hassas bir BBH modeli elde edilir.

Veriye dayanarak ve bazı durumlarda ilişkilerle ilgili belirli kısıtları dikkate alarak, gözlenen sistem davranışını en az hata ile tekrarlayabilen bir model elde etme süreci, literatürde “BBH Öğrenimi” olarak adlandırılmaktadır. BBH Öğrenimi, ilişkilerin yönünü, işaretini ve gücünü (ağırlık değerlerini) belirlemeyi amaçlayan bir yöntemdir.

BBH, karmaşık sistemlerin davranışlarını taklit etmek amacıyla ardışık iki sistem durumu arasındaki geçişi simüle eden bir çıkarsama mekanizması kullanır. Bu mekanizma, mevcut durumdan hareketle unsurlar arasındaki nedensel ilişkileri dikkate alarak sistemi bir sonraki duruma taşır; başka bir ifadeyle, unsurların yeni aktivasyon düzeylerini belirler. Böylece, model belirli bir başlangıç koşulundan başlayarak sistemin gelecekteki durumlarını tahmin eder. Literatürde, durumlar arası geçişleri gerçekleştiren çıkarsama mekanizması için genellikle sigmoid veya tanjant sigmoid gibi doğrusal olmayan aktivasyon fonksiyonları tercih edilmektedir.

Klasik yaklaşımda, önce unsurlar arasındaki etkileşimler, etki yönleri ve şiddetleri tanımlanarak bir BBH modeli oluşturulur. Daha sonra bu model çalıştırılarak, sistemin zaman içindeki davranış örüntüsünü içeren yapay veri setleri elde edilir. Bu veri setleri, BBH öğreniminde başka bir modelin ilişki ağırlıklarını tespit etmek için kullanılır. Başka bir ifadeyle, bilinmeyen parametreleri belirlemek için üretilen yapay verilerden faydalanılır. Fakat böyle bir yöntem, önemli bir yanlışlık riski taşır. Çünkü öğrenme sürecinde kullanılan yapay veriler, oluşturulmak istenen modelle birebir aynı çıkarsama yapısına sahip modellerden elde edilir. Bu durum, BBH modelinin gerçek sistem davranışına uyum sağlama yeteneğini zayıflatabilir ve öğrenilen modelin performansının olduğundan daha iyi görünmesine yol açabilir.

BBH modellerinde klasik çıkarsama mekanizmasının kullanılmasına bağlı daha önemli başka bir sorun daha vardır. Bu sorun, modelin simülasyonu sırasında, farklı başlangıç durumlarından hareket edilmesine rağmen sistemin çoğunlukla aynı veya çok benzer bir son duruma yakınsamasıdır. Genel olarak BBH simülasyonlarında üç temel sistem davranışı ortaya çıkar: sabit duruma yakınsama, durumlar arasında döngüsel hareket ve kaotik davranış. Bunlardan en yaygın karşılaşılanı sabit duruma yakınsamadır. Sistemin zaman içinde belirli bir noktada dengelenmesini ifade eden bu davranış, özellikle fiziksel sistemlerin kontrol edilmesi açısından önemlidir. Örneğin enerji yönetimi, su dağıtımı veya endüstriyel süreçlerin güvenli ve tutarlı biçimde işlemesi için sistemin dengeli bir duruma ulaşması beklenir. Bununla birlikte, özellikle durumlar arasında döngüler veya karmaşık geçişler içeren sosyal sistemlerin modellenmesinde veya başlangıç koşullarına duyarlılık gerektiren tahminlerde ve senaryo analizlerinde klasik çıkarsama mekanizması yeterince etkili değildir. Sosyal sistemler, farklı unsurlar arasında karmaşık, doğrusal olmayan ve beklenmedik dinamik etkileşimlere sahip olduğundan, klasik yöntem bu tür yapıların dinamik değişimlerini doğru yansıtmakta zorlanır. Bu nedenle, sosyal sistemlerin

belirsizliklerini ve deęişkenliklerini daha iyi temsil edebilen, daha esnek ve uyarlanabilir çıkarsama yöntemlerine ihtiyaç duyulmaktadır.

Bu tez çalışması, klasik çıkarsama mekanizmasının özellikle denge durumu hedeflemeyen karmaşık sistemlerdeki yetersizliğini gidermek üzere alternatif bir çıkarsama mekanizması önermektedir. Çalışmanın temel amacı, önerilen mekanizmanın klasik yöntemin eksik kaldığı durumları aşabildiğini ortaya koymaktır.

Bu doğrultuda öncelikle tezin ikinci ve üçüncü bölümlerinde sırasıyla klasik BBH'nin kavramsal, kuramsal altyapısı özetlenmekte ve çıkarsama mekanizmasının sorunları ele alınmaktadır. Sonrasında, tez kapsamında yürütölen üç farklı çalışma üç ayrı bölümde detaylandırılmaktadır. Her bölüm, geliştirilen çıkarsama mekanizmasının farklı yönlerini vurgulamakta, mekanizmanın sağlayacağı potansiyel avantajları farklı uygulama alanları ve yaklaşımlarla ele almaktadır. Böylece, önerilen alternatif yöntemin etkinliği ve esnekliği çeşitli açılardan kapsamlı bir şekilde değerlendirilmektedir.

Birinci kısım (Bölüm 4), tezde önerilen alternatif çıkarsama mekanizmasının kuramsal altyapısını ortaya koymaktadır. Bu mekanizma, doğrudan ilişkilerin yanı sıra dolaylı ilişkileri de modelleme sürecine dahil ederek, sistemin daha derin ve örtük bağlantılarını yakalamayı amaçlamaktadır. Dolaylı ilişkilerin ele alınmasıyla, sistem davranışı daha geniş bir perspektiften incelenebilmekte ve böylece modelin doğruluęu ve temsil yeteneęi artırılmaktadır. Bu bölümde, sistem unsurlarının önem derecelerinin belirlenmesi süreci, unsurların doğrudan ve dolaylı ilişkileri birlikte dikkate alınarak gerçekleştirilmektedir. Buradaki temel varsayıma göre, bir unsurun sistem içindeki önemi, dięer unsurlarla arasındaki bağlantıların sayısına ve bu bağlantıların gücüne baęlı olarak artmaktadır. Bir başka ifadeyle, sistem içinde daha güçlü ve daha fazla ilişkiye sahip unsurlar, model açısından daha kritik ve önemli kabul edilmektedir. Ayrıca, bölümde önerilen bu yeni yaklaşımın uygulanabilirliği, ilişki yapısının belirsizlik içerdiği bir ortamda Çok Kriterli Karar Verme (ÇKKV) bağlamında gösterilmektedir. Bu uygulamada, unsurlar arasındaki doğrudan ve dolaylı nedensel ilişkiler kullanılarak kriterlerin göreceli önemleri değerlendirilmekte ve ilişki ağırlıklarına göre sıralanmaktadır. Böylece, önerilen mekanizmanın belirsiz ve karmaşık sistemlerde kriter önemlerini belirlemedeki esnekliği ve kapsamı, uygulamalı bir örnekle detaylı olarak ele alınmaktadır.

İkinci kısım (Bölüm 5), önerilen alternatif çıkarsama mekanizmasının yöntemsel altyapısını ve uygulama yöntemini ayrıntılı olarak açıklamaktadır. Bu bölümün temel amacı, klasik mekanizmanın kısıtlılıklarını gidererek, karmaşık ve belirsizlik içeren sistemlerin dinamiklerini daha gerçekçi bir şekilde modelleyebilen bir mekanizma ortaya koymaktır. Bu amaç doğrultusunda geliştirilen mekanizma, dolaylı ilişkileri de simölasyon sürecine dahil ederek BBH modellerinin ifade gücünü artırmaktadır. Yöntemin temel farkı, klasik mekanizmada kullanılan ve sıklıkla sabit duruma yakınsamaya neden olan doğrusal olmayan aktivasyon fonksiyonları yerine, ilişki matrislerinin kuvvetlerinin kullanılmasıdır. Bu yaklaşımda, sistemdeki unsurlar arasındaki doğrudan ve dolaylı etkileri içeren ilişki matrisinin kuvvetleri (matrisin belirli bir k 'ıncı kuvvetine kadar) hesaplanmakta ve her bir kuvvet matrisi, içindeki en yüksek deęer baz alınarak normalize edilmektedir. Normalizasyon işlemi sayesinde farklı seviyelerdeki dolaylı ilişkilerin etkileri aynı ölçeęe indirgenerek birleştirilebilmektedir. Ardından normalize edilmiş tüm kuvvet matrislerinin toplamı, başlangıç durum vektörü ile çarpılarak sistemin belirli iterasyonlar sonrasındaki durumu elde edilir. Böylelikle, sistem davranışının başlangıç koşullarına göre

farklılaşması mümkün kılınmakta, tek bir sabit noktaya yakınsama problemi giderilmektedir. Bölümde, geliştirilen mekanizmanın uygulanabilirliği, belirli bir alana yönelik tedarik zincirindeki kırılma faktörlerinin modellenmesi ile gösterilmektedir. Bu örnek uygulamada, farklı senaryolarda faktörlerin nasıl etkileşime geçtiği ve bu etkileşimlerin sistemin genel davranışını nasıl şekillendirdiği analiz edilmektedir. Yapılan analizler, tedarik zincirinde bulunan kritik zayıf noktaların belirlenmesine ve bu zayıflıkların sistem üzerindeki potansiyel etkilerinin daha kapsamlı biçimde anlaşılmasına katkı sağlamaktadır.

Üçüncü kısım (Bölüm 6), önerilen alternatif çıkarsama mekanizmasının kapsamlı ve genelleştirilmiş bir değerlendirmesini içermektedir. Bu bölümde, geliştirilen çıkarsama mekanizmasının performansı farklı senaryolar altında analiz edilerek, mekanizmanın etkinlik derecesi ve klasik çıkarsama mekanizmalarına kıyasla sağladığı avantajları tartışılmaktadır. Ayrıca, önerilen yöntemin sınırlılıkları ve hangi koşullar altında daha verimli çalıştığı incelenerek mekanizmanın genel geçerliliği ve potansiyel kullanım alanları ortaya konmaktadır. Bu bölümde de önceki bölümde olduğu gibi, dolaylı ilişkiler hesaplamaya dahil edilmekte ve doğrusal olmayan aktivasyon fonksiyonlarından kaçınılmaktadır. İlişki matrisinin kuvvetleri alınarak dolaylı etkiler hesaplamaya katılmakta ve elde edilen yeni durum vektörleri normalizasyon işlemine tabi tutulmaktadır. Bu aşamada, farklı normalizasyon tekniklerinin etkinliği analiz edilmekte ve en uygun yöntem belirlenmeye çalışılmaktadır. Böylece dolaylı ilişkilerin sistem modellemesine dahil edilmesinin doğruluğu ve güvenilirliği artırılmaktadır. Ayrıca, dolaylı ilişkilerin ve normalizasyon tekniklerinin farklı kombinasyonları denenerek çeşitli çıkarsama mekanizması tasarımları geliştirilmektedir. Bu analizlerde, hesaplamanın kümülatif (önceki durumların etkisini taşıyan) veya kümülatif olmayan şekilde yapılmasının etkileri de değerlendirilmektedir. Önerilen çıkarsama mekanizmasının, özellikle sabit duruma yakınsama göstermeyen verileri modelleme konusunda klasik yöntemlere göre daha verimli sonuçlar sağlayacağı öne sürülmektedir. Bu doğrultuda oluşturulan farklı çıkarsama mekanizması tasarımları ile yapılandırılan başlangıç BBH modellerinin optimizasyonu PSO algoritması kullanılarak yapılmaktadır. Bu optimizasyon sürecinde Heaviside, RMSE ve Maks-RMSE olmak üzere üç farklı amaç fonksiyonu minimize edilmektedir. Önerilen çıkarsama mekanizması tasarımlarının, söz konusu amaç fonksiyonlarını minimize etme performansları klasik çıkarsama mekanizmaları (sigmoid ve tanjant sigmoid fonksiyonları) ile karşılaştırılarak, yeni yaklaşımın üstünlükleri detaylı biçimde irdelenmektedir.

Aynı bölümde, önerilen çıkarsama mekanizmasının BBH Öğrenimindeki performansını adil biçimde değerlendirmek amacıyla, karşılaştırılan mekanizmalardan herhangi biri kullanılarak veri üretimi yapılmamaktadır. Böylece, tüm çıkarsama mekanizmaları bağımsız veri kümeleri üzerinde değerlendirilmiş ve performans karşılaştırmalarının objektifliği sağlanmaktadır. Veri üretimi için, BBH çalışmalarının öncülerinden olan Dickerson ve Kosko'nun incelediği deniz canlıları ekosistemi modelindeki davranışlar temel alınmaktadır. Bu yaklaşım, ekosistemin karmaşık ve dinamik doğasını gerçekçi senaryolar üzerinden taklit ederek, oluşturulan veri kümelerinin herhangi bir modele bağlı olmamasını garanti altına almaktadır. Bu yöntem sayesinde, önerilen mekanizmanın performans değerlendirmesinde model kaynaklı yanlılıkların önüne geçilmektedir.

Elde edilen bağımsız veri kümeleri üzerinde her bir çıkarsama mekanizması tasarımı, yani farklı normalizasyon yöntemleri, kümülatif hesaplama yaklaşımları ve farklı amaç fonksiyonları, istatistiksel olarak anlamlı sonuçlar elde etmek için 50 kez rastgele

ilişki ağırlıklarına sahip BBH modelleri üzerinde test edilmektedir. Bu tekrarlı değerlendirmelerle, normalizasyon tekniklerinin, hesaplama yaklaşımlarının ve amaç fonksiyonlarının her birinin performansa etkisi derinlemesine analiz edilmektedir. Bu kapsamlı analiz, sonuçların geçerliliğini ve güvenilirliğini artırmaktadır.

Ayrıca, bu tezde önerilen çıkarsama mekanizması, benzer amaçlarla önerilmiş ve üstel normalize edilen çıkarsama yöntemi olarak adlandırılan alternatif bir mekanizmayla da kıyaslanmaktadır. Bu iki mekanizma arasındaki performans karşılaştırması da 30 kez rastgele oluşturulan ilişki ağırlıklarına sahip modeller üzerinden gerçekleştirilmektedir. Yapılan analizlerin sonuçları, önerilen mekanizmanın deneylerin çoğunda daha üstün bir tahmin performansı sergilediğini Wilcoxon işaretli-sıra testi (Wilcoxon signed-rank test) ile istatistiksel olarak göstermektedir. Ayrıca, performans farkının büyüklüğü, etki büyüklüğü (effect size) hesaplarıyla da doğrulanmakta ve farkın pratik olarak önemli olduğu belirlenmektedir. Özellikle, Heaviside amaç fonksiyonunun minimize edildiği senaryolarda önerilen çıkarsama mekanizmasının sürekli olarak daha iyi sonuçlar verdiği görülmektedir. Önerilen mekanizma, üstel normalize edilen çıkarsama yöntemine kıyasla da belirgin biçimde daha iyi bir tahmin performansı sağlamaktadır.

Sonuç olarak, bu tez çalışmasında klasik çıkarsama mekanizmalarının (sigmoid ve tanjant sigmoid gibi aktivasyon fonksiyonlarının) sistem davranışını tek bir sabit noktaya yakınsattığı ve bu nedenle özellikle döngüsel veya karmaşık davranışlar sergileyen tahmin problemlerinde yetersiz kaldığı vurgulanmaktadır. Bu sınırlamayı aşmak amacıyla dolaylı ilişkileri hesaplamaya dahil eden alternatif bir çıkarsama mekanizması geliştirilmekte ve bu mekanizmanın BBH Öğrenimi için daha uygun olduğu gösterilmektedir. Ayrıca, önerilen çıkarsama mekanizmasının probleme özel tasarımlar yoluyla farklı başlangıç durumlarına bağlı olarak farklı nihai durumlara yakınsama ya da döngüsel hareketleri modelleyebileceği kanıtlanmaktadır. Böylelikle önerilen yaklaşımın, yalnızca sabit duruma yakınsama ile sınırlı kalmadığı, karmaşık sistemlerin farklı dinamiklerini de başarılı şekilde modelleyebildiği ortaya konulmaktadır.



1. INTRODUCTION

In complex systems, various elements cooperate in an integrated manner, interplaying dynamically to achieve a common goal or fulfill a particular function. These interplays are not random; instead, they can be systematically modeled to understand the system's behavior and its response under diverse conditions. Such models are powerful tools for analysis, experimentation, and scenario simulation, enabling us to predict, interpret, or even influence system outcomes in a controlled environment. Among the array of modeling techniques available, Fuzzy Cognitive Maps (FCMs) have emerged as a promising approach for representing the interplay among components in complex systems.

Historically, early attempts to comprehend complex systems were grounded in speculative thinking, focusing on isolated observations while overlooking the holistic interconnections among phenomena. Although imaginative, this approach lacked the explanatory depth and predictive power necessary for addressing the complexities of natural and social systems. The emergence of systems theory marked a pivotal shift, emphasizing the importance of viewing systems as interconnected wholes, where each element affects and is affected by others. In such systems, characterized by nonlinear relationships and feedback mechanisms, even minor perturbations can lead to significant and sometimes unpredictable consequences. This realization has reinforced the necessity of understanding relationships within systems for more informed decision-making and to prevent adverse outcomes. Consider, for instance, a forest ecosystem in which trees, animals, microorganisms, and climatic variables exist within an intricate web of interdependencies. A seemingly trivial disruption in one component can cascade through the system, amplifying its effects. Systems thinking, supported by suitable modeling methodologies, such as FCM, thus plays a critical role in managing such complexity, with models acting as simplified representations that offer insights into system structure, function, and dynamics.

FCMs fulfill these requirements and serve as an effective method for modeling complex systems. By blending expert knowledge with quantitative data, FCMs

construct graphical models that capture causal relationships among system elements. This framework allows analysts to comprehend both the static architecture and dynamic evolution of systems. A key strength of FCMs is their capacity to incorporate vague, uncertain, and subjective information, making them particularly suitable for modeling human-centered and decision-oriented systems. Furthermore, FCM simulations facilitate the exploration of system behavior under varying circumstances, supporting the evaluation of possible interventions and forecasting potential developments.

An FCM models a system as a network of interconnected concepts, with each concept representing a particular element, factor, or variable. These elements are linked through directed, weighted relationships that quantify the strength and direction of influence. Although the mathematical formulation of FCMs is relatively simple, their conceptual foundation lies in their ability to express how people understand cause-and-effect relationships in complex systems. The dependence on expert knowledge enables FCMs to reflect nuances that may elude purely data-driven models. This hybrid structure, integrating qualitative insight and quantitative modeling, positions FCMs as a versatile tool across a wide range of domains including the social sciences, engineering, environmental management, and beyond.

At the core of FCMs lies their directed graph structure, consisting of concept nodes and causal connections. Nodes denote the primary variables of interest, while edges represent directed causal relationships between them. Each edge is assigned a weight, indicating the magnitude and polarity of influence exerted by one node on another. These weights can be established either through expert elicitation or inferred from empirical data. The resulting network visualizes complex interrelations and feedback loops, making it easier to grasp interdependence between system elements. Through simulation, the propagation of effects across nodes can be observed, helping identify leverage points or assess the systemic impact of external stimuli. Consequently, FCMs have become an essential tool for evaluating dynamic outcomes in complex systems.

The simulation process in FCMs involves calculating the evolution of activation values for each concept over time. Initially, nodes are assigned activation values based on a defined initial state. These values are then iteratively updated by aggregating the weighted influences from connected nodes, followed by transformation through an activation function. This iterative process continues until the system reaches a steady

state or the desired number of iterations is completed. Simulation outcomes are typically presented as time series graphs or comparative tables, illustrating on how different variables influence one another and how the system responds to external or internal changes. In practice, such simulations are invaluable, for instance, assessing how a species population decline affects an ecosystem or evaluating the impact of policy changes on urban traffic dynamics. By offering a dynamic lens on system evolution, FCM simulations support evidence-based planning and foresight.

Beginning from an initial state, FCM simulations reveal how changes in one part of the system can propagate and influence the overall structure. This dynamic capability is instrumental in scenario analysis, where diverse interventions or developments are assessed for their systemic effects. For example, urban planners may simulate the outcomes of new infrastructure projects to anticipate congestion patterns, while conservationists may explore the ripple effects of climate interventions on ecological stability. Such foresight empowers stakeholders to adopt proactive and preventive measures.

The value of FCM simulations also lies in their ability to support decision-making by visualizing the interplay of different factors and highlighting critical dependencies. FCMs enable decision-makers to test assumptions, evaluate alternative scenarios, and understand trade-offs among competing goals. These features enhance the robustness and transparency of planning processes, particularly in domains where complexity and uncertainty are inherent. The incorporation of simulation outcomes into strategic design ensures that stakeholders can anticipate unintended consequences and adopt measures that balance short-term goals with long-term sustainability.

Beyond simulation, FCMs allow for the estimation of relationship weights through a process known as “learning FCM.” This involves using optimization algorithms, such as evolutionary techniques or swarm intelligence methods, to calibrate the weights in a way that best fits observed data. These algorithms iteratively adjust weights to minimize discrepancies between the model’s predictions and actual system behavior. For example, genetic algorithms may represent weights as chromosomes, evolving them over successive generations, while particle swarm optimization adapts weights through collective learning among candidate solutions. Learning-based FCMs enhance model precision and reliability, bridging expert-driven modeling with data-informed refinement. Incorporating both simulated and real-world behavior into the weight

adjustment process ensures that the models stay grounded in empirical relevance while retaining conceptual clarity.

Despite their advantages, traditional FCMs face certain limitations. A prominent issue arises from the use of fixed nonlinear activation functions in the reasoning mechanism. These functions tend to force the system to converge to a single steady state, regardless of the starting conditions. This behavior undermines the utility of simulations in modeling dynamic, input-sensitive systems where different initial states should lead to distinct trajectories. The problem may not stem from the structure of FCMs itself, but rather from how the reasoning mechanism operates, the mathematical process by which influence propagates and accumulates across the network.

Moreover, reliance on subjective expert input introduces variability and potential bias. While learning algorithms can address this to some extent, they still depend on the reasoning mechanism to simulate system behavior. To improve the robustness and generalizability of FCMs, it is crucial to rethink and enhance the underlying reasoning process. This includes the need to explore alternatives that reduce reliance on fixed functional forms and better capture the diversity of interactions within real-world systems. Without such enhancements, the full potential of FCMs in dynamic systems modeling remains constrained.

This thesis introduces a novel reasoning mechanism aimed at addressing the limitations of traditional FCMs. The proposed mechanism eliminates the reliance on nonlinear activation functions and incorporates both direct and indirect relationships in computing concept activations. By doing so, it allows for more differentiated simulation outcomes and better reflects the interconnected dynamics of real-world systems. The new reasoning mechanism uses matrix power operations to integrate higher-order influence paths, enabling a more nuanced propagation of effects across the concept network.

The key objective of the thesis is to ensure that FCM simulations more faithfully mirror real-world system behavior, particularly under varying initial conditions and when learning from historical data. This is achieved through a framework that applies the proposed reasoning mechanism in different contexts: decision analysis, scenario simulation, and model learning. By testing the mechanism in distinct but interrelated

domains, the thesis explores the generalizability and effectiveness of the approach across multiple modeling challenges.

1.1 Publications and Their Relations

The development of an FCM model still begins with expert knowledge, defining relevant concepts and hypothesized relationships, but the learning phase refines these assumptions using empirical observations. The learning process, when integrated with the proposed reasoning approach, produces models that are both interpretable and data driven. In this way, the thesis bridges the gap between expert insight and computational modeling, while ensuring more realistic representation of dynamic systems.

Learning performance is tightly linked to the quality of the reasoning mechanism. Traditional reasoning processes often fall short in differentiating responses to diverse inputs, especially in systems with feedback loops and indirect dependencies. The proposed mechanism addresses this by capturing cumulative and higher-order influences without resorting to nonlinear transformations. This enables the model to better replicate system dynamics observed in data, and to generate more varied and realistic scenarios. The flexibility introduced through this reasoning framework offers an enhanced capacity to detect subtle transitions in system states, which are often obscured in traditional FCM approaches.

To demonstrate the applicability and advantages of the new reasoning mechanism, this thesis presents three interrelated studies, each tackling a distinct problem context but unified by the underlying methodology. The first study, titled “Electronic Waste Collection Strategy Selection via a Fermatean Fuzzy-Based Decision Support System” (Murat et al., 2025a), applies the mechanism to a multi-criteria decision-making (MCDM) problem, modeling interdependent criteria using an FCM structure. By incorporating indirect relationships into the weight computation, this approach enhances the representational accuracy of the model. The case focuses on selecting optimal electronic waste (e-waste) collection strategies. This domain provides a practical and socially relevant setting in which technical, environmental, and economic criteria intersect.

The second study, titled “Evaluating the Vulnerability of Forestry Supply Chains through Fuzzy Cognitive Map” (Murat et al., 2025b), explores the use of the reasoning mechanism in scenario-based simulations. Here, the goal is to show how different initial conditions can lead to varied system trajectories, thereby enabling more meaningful scenario analysis. This application is demonstrated in the context of forest supply chain (FSC) vulnerabilities, where multiple disruptive factors and their interactions are modeled. The ability to generate multiple plausible future states based on diverse scenarios makes this methodology particularly suitable for strategic planning in sectors exposed to environmental risks and logistical uncertainties.

Finally, the third study with the title of “Learning Fuzzy Cognitive Maps for Prediction Problems Using Swarm Intelligence Based on an Input-Sensitive Reasoning Mechanism” (Murat and Asan, 2025), integrates the proposed mechanism into a learning FCM framework that entirely bypasses the use of nonlinear activation functions, addressing one of the core limitations of traditional learning approaches. Instead, it employs matrix-based computations in combination with normalization techniques. A particular emphasis is placed on evaluating the impact of different normalization strategies as well as cumulative and non-cumulative reasoning paths, on the model’s learning performance. Furthermore, the study explores the use of alternative objective functions in the learning algorithm to optimize model accuracy and generalizability. This extensive evaluation of methodological configurations contributes to a deeper understanding of how the proposed reasoning mechanism influences learning behavior. The approach is validated through a comprehensive application involving a marine ecosystem, where the model is trained on time-series data capturing the dynamics of shoaling marine creature populations and environmental factors.

These three studies are designed to be complementary and sequential. The first study validates the mechanism in a decision context involving indirect influences. The second extends the approach to dynamic simulations under varied initial states, while the third tests its data-driven learning capability. Together, they build a comprehensive case for the effectiveness of the proposed reasoning mechanism. By addressing structural realism, dynamic flexibility, and learning efficiency, the three-part research design positions the thesis at the intersection of theoretical innovation and applied utility.

The structure of the thesis reflects this progression. Following this introduction, the second chapter details the mathematical foundation of FCMs and reviews relevant literature on reasoning mechanisms and learning techniques. The third chapter includes an illustrative example where an FCM is simulated under different initial conditions using traditional reasoning methods. Regardless of the starting values, the system converges to the same equilibrium point, illustrating a key limitation of the conventional reasoning process. This behavior reinforces the motivation to develop an alternative reasoning mechanism capable of generating diverse outcomes based on varying scenarios. The next three chapters present the studies, and their applications described above, each supported by theoretical and empirical analysis. Finally, the conclusion discusses the implications of the findings, highlights the contributions of the thesis, and suggests directions for future research. The comprehensive scope of the study seeks to advance both foundational knowledge and practical capabilities within the field of fuzzy cognitive mapping.

Table 1.1: Key characteristics of the three studies guiding the thesis framework.

Study	Contribution	FCM Model Construction	Application
Electronic Waste Collection Strategy Selection via a Fermatean Fuzzy-Based Decision Support System (Murat et al., 2025a)	Weight calculation based on both direct and indirect relationships	Expert-based	MCDM
Evaluating the Vulnerability of Forestry Supply Chains through Fuzzy Cognitive Map (Murat et al., 2025b)	Future state vectors computations sensitive to initial conditions	Expert-based	Scenario Analysis
Learning Fuzzy Cognitive Maps for Prediction Problems Using Swarm Intelligence Based on an Input-Sensitive Reasoning Mechanism (Murat and Asan, 2025)	Generalization and validation of the proposed reasoning mechanism	Learning FCM	Experimental Studies, a real-life system referenced in nature

Figure 1.1 provides a visual summary of the methodological flow, while Table 1.1 offers a comparative overview of the three studies. Through this work, the thesis aims to expand the capabilities of FCMs as modeling tools and to encourage the development of more responsive and input-sensitive reasoning mechanisms in complex systems modeling. Ultimately, the innovations presented are intended to

stimulate further inquiry into how FCMs can be adapted and improved to better address the intricate, evolving nature of the systems they aim to represent.

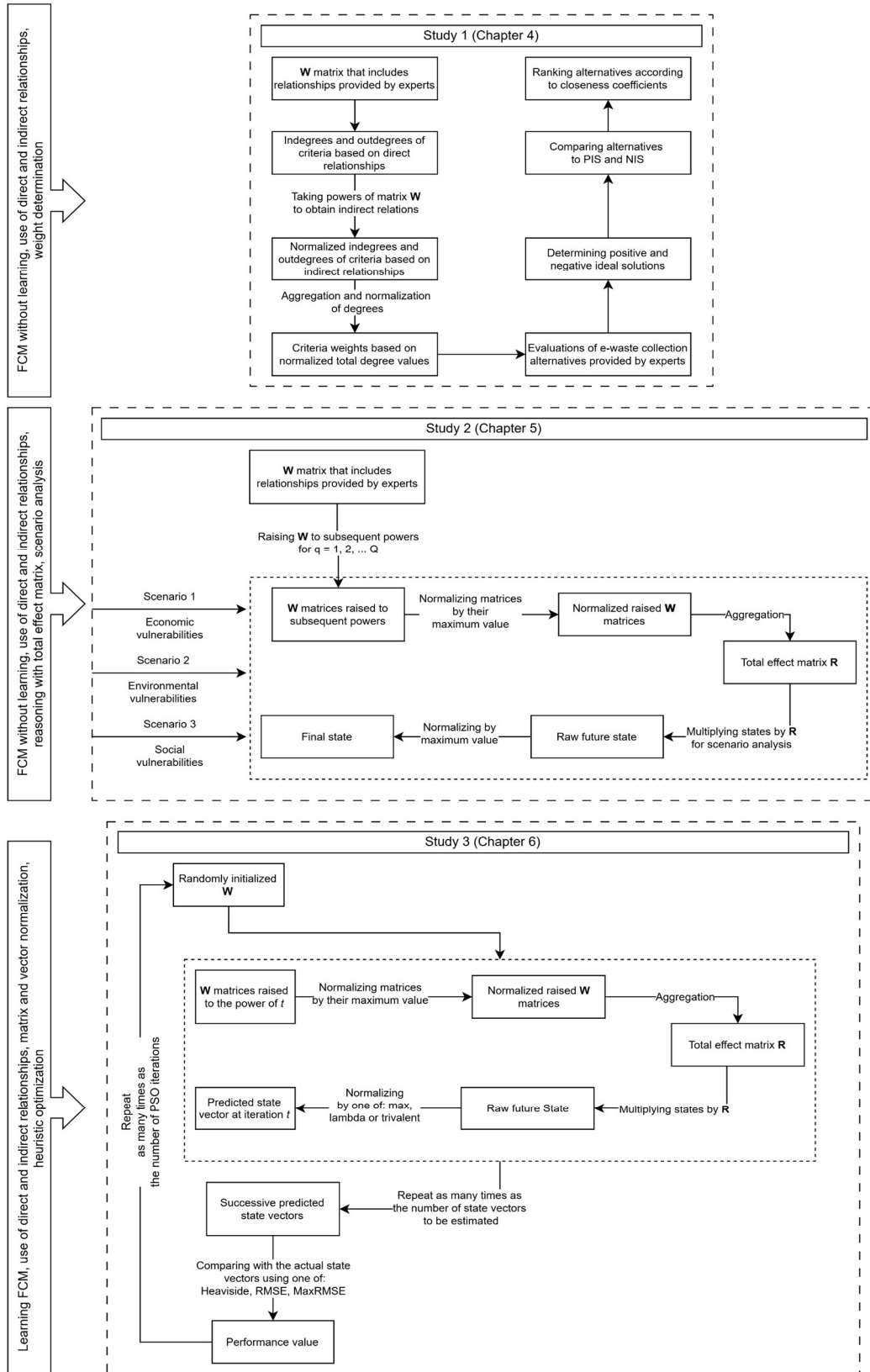


Figure 1.1: Methodological flow and linkages between the three studies.

2. FUZZY COGNITIVE MAPS

Complex systems span a wide range of disciplines, including social, economic, environmental, and biological frameworks. These systems are inherently dynamic and nonlinear, with components that are interconnected in ways that challenge traditional analysis and modeling methods. To address these challenges, fuzzy cognitive mapping offers an effective approach that captures and represents the interdependencies within such systems. This approach involves multiple processes, including determining components, identifying their relationships, quantifying the magnitudes of these relationships, and synthesizing them into a map called fuzzy cognitive map (FCM). Static and dynamic analyses are then performed on the resulting map, often using mathematical tools such as relationship matrices to evaluate system behavior and dynamics.

FCMs are powerful soft computing tools by enabling modeling, analysis, and simulation of systems with interdependent variables. They build upon Axelrod's (1976) Cognitive Maps (CMs) and extend their capabilities through introducing fuzzy logic by Kosko (1986). As a qualitative model, FCM explains the dynamic behavior of a complex system by considering its defined variables and causal interdependencies (Gray et al., 2014). This allows for a more nuanced representation of relationships within a system, making FCMs particularly suited for analyzing complex behaviors and uncertainties.

Cognitive Maps (CMs), as originally proposed by Axelrod, provide a visual framework for representing how people conceptualize relationships within a system. However, Kosko (1986) highlighted a fundamental limitation of CMs: their reliance on binary logic (i.e., Aristotelian logic, see Kosko (1993)), which evaluates concepts and their relationships strictly as either "existent" or "non-existent." This binary framework restricts their ability to represent real-world complexities, which often involve varying degrees of influence between system components. To address this limitation, Kosko (1986) introduced fuzzy logic into CMs, allowing causal connections to be assigned weights. These weights quantify both the direction,

indicating positive or negative influences, and the strength of relationships within the system (Özesmi and Özesmi, 2004), making abstract concepts tangible and enabling precise simulations of system behavior. The combination of these weighted relationships offers a detailed view of how changes in one component propagate through and influence the entire system.

Introducing fuzzy logic into the mapping process reflects the real-world need to manage uncertainty and ambiguity. Various systems, whether natural or human-made, involve relationships that are neither fully deterministic nor binary. FCMs enable the representation of these “fuzzy” relationships, acknowledging the nonlinear and dynamic nature of most decision-making environments. This makes FCMs a valuable tool for not only visualizing interconnections but also simulating potential outcomes of changes within a system.

FCMs also serve as system modeling tools that identify the core elements or concepts and the relationships among them, which together contribute to achieving the objectives of the system. By visually representing these concepts and their causal relationships, as illustrated in Figure 2.1, FCMs construct a network map that embodies the structure of the system. For example, in modeling a lake ecosystem, key concepts might include “fish population,” “algae concentration,” “nutrient levels,” “water temperature,” and “presence of invasive species.” The causal relationships between these elements are assigned weights that reflect both their strength and direction, integrating the dynamics of the lake ecosystem. This visual representation allows decision-makers to intuitively grasp how individual components interact and influence system behavior. Moreover, the visual framework provides the foundation for further analysis, making it possible to explore not only the system’s structure but also its evolution. Such an approach aids in identifying leverage points within the system, where targeted interventions could yield the most significant outcomes.

The construction of FCMs begins with identifying the key concepts and causal relationships within a system. This process typically involves stakeholder engagement, where domain experts collaborate to determine the most relevant elements and their interconnections (Groumpos, 2010). These elements are represented as nodes, while the relationships between them are expressed as weighted edges. Weights can be assigned based on expert knowledge or derived through data-driven methods. In cases where sufficient data is available, machine learning algorithms can also aid in

determining these weights, ensuring a robust model that reflects real-world dynamics. Once the FCM is constructed, simulations can be performed to explore how changes in one concept propagate through the network and influence other concepts. These simulations rely on mathematical calculations to analyze the cascading effects of interventions, enabling decision-makers to explore “what-if” scenarios and predict potential system behaviors. For instance, in the lake ecosystem model, reducing nutrient levels can be simulated to observe its effects on algae concentration, fish populations, and overall ecological health.

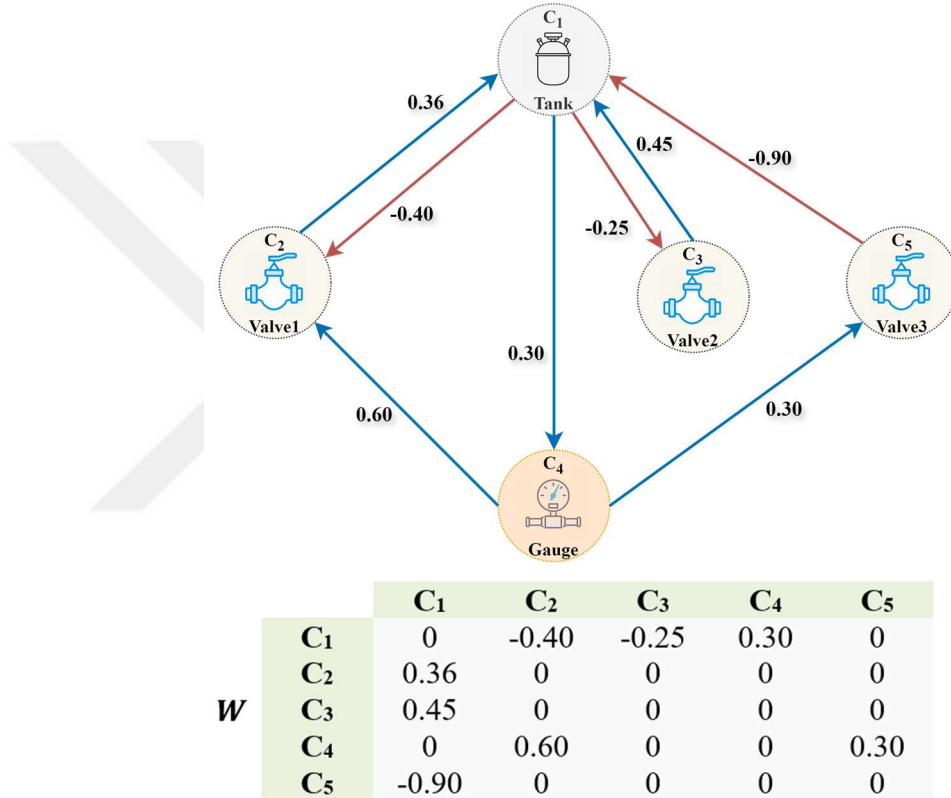


Figure 2.1: An FCM graph showing the directed causal relationships among five concepts (c_i) and their weights w_{ij} in a relationship matrix W for a control process (Parsopoulos et al., 2003).

FCMs facilitate both static and dynamic analyzes of complex systems. Static analysis focuses on the structural properties of the network, such as identifying central nodes, examining connectivity patterns, and understanding the relative importance of different components. In contrast, dynamic analysis involves simulating the evolution of a system over time to examine how changes in specific concepts influence the overall behavior of the system. The visual representation offered by FCMs directly supports these analyzes, enabling researchers to move seamlessly from understanding

the static structure of a system to exploring its dynamic interactions. By combining these analyzes, FCMs can reveal the intricate cause-effect relationships that govern the system's dynamics and allow decision-makers to expect the long-term impacts of interventions.

One of the key advantages of FCMs lies in their interpretability and flexibility. Unlike many artificial intelligence methods that act as black boxes, FCMs allow users to understand the relationships and reasoning processes that drive system behavior. This transparency, combined with the ability to handle uncertainties and nonlinear dynamics, has led to the widespread application of FCMs across various fields. In engineering, FCMs have been used to assess risks and optimize processes (Bonvicini et al., 2015), while in economics, they help analyze market dynamics and policy impacts (Gupta and Gupta, 2017). In environmental sciences, FCMs are employed to evaluate the effects of interventions on ecosystems (Olazabal et al., 2018), while in human management, they facilitate the modeling of decision-making processes (Mago et al., 2013). Applications in politics show the utility of FCMs for understanding and predicting policy outcomes (Löhe et al., 2015). Despite these advantages, one limitation of FCMs is that the quality of the model depends heavily on the quality of the data or expert knowledge used to construct it. Subjective biases in weight assignment or insufficient data can lead to models that do not fully capture the dynamics of the system being studied.

By integrating concepts and weighted causal relationships, FCMs enable the exploration of both current system behavior and future scenarios. Their capacity to perform simulations under varying conditions allows decision-makers to acquire valuable insights into how interventions or changes in key variables might impact system outcomes. The combination of qualitative and quantitative analysis makes FCMs an effective tool for addressing real-world problems that involve uncertainty, complexity, and interconnected dynamics.

2.1 Fundamentals of Fuzzy Cognitive Maps

Integrating fuzzy logic provides FCMs with a powerful mathematical foundation, allowing for a more flexible and realistic modeling of dynamic systems. This integration supports better visualization, manipulation, and quantification of system

dynamics, ultimately making it possible to acquire richer insights into the behavior of systems (Gray et al., 2014).

An FCM of a system is illustrated as a directed graph, denoted as $\mathbf{G} = (\mathbf{C}, \mathbf{W})$. Here:

- $\mathbf{C} = \{C_1, C_2, \dots, C_n\}$ represents the set of nodes in the FCM, which are known as “concepts” in FCM terminology. Here, n represents the total number of concepts included in the system. Concepts are the fundamental elements or decision variables that are affected by each other. For example, in a lake ecosystem, a concept might represent the fish population, while another concept could represent algae concentration, and so on.

Concepts serve as building blocks for modeling the complex interdependencies within the system. Each concept is associated with an activation level, which reflects its current state or importance within the system. The activation level of concept C_i , denoted by A_i where $i = 1, \dots, n$, typically ranges between 0 and 1, where 0 indicates an inactive state and 1 represents full activation, while intermediate values represent partial activation. These activation levels provide a snapshot of the system at a time and are essential for analyzing the behaviors of systems. This representation makes FCMs an intuitive tool for representing complex interdependencies within systems (Papageorgiou and Salmeron, 2013).

- The matrix of $\mathbf{W} = [w_{ij}]_{n \times n}$ forms the backbone of FCM. It encapsulates the causal relationships between concepts. Each element w_{ij} represents the weight of the causal influence from concept C_i to concept C_j . These weights take values within the range of $[-1, 1]$ and indicate the direction and strength of causal relationships. The absolute magnitude of a weight reflects the strength of the relationship, with higher values representing stronger causal effects. Depending on its sign, the specific nature of a relationship is classified into three categories as outlined below (Papageorgiou and Salmeron, 2013):
- Positive causal influence: A positive weight ($w_{ij} > 0$) indicates a causal increase, which means an increase in the activation level of C_i will lead to an increase in the activation level C_j , and vice versa. For example, in a lake ecosystem, an increase in algae concentration might positively influence the fish population growth if the algae serve as a food source.

- **Negative causal influence:** A negative weight ($w_{ij} < 0$) signifies a causal decrease, that is, an increase in the activation level C_i results in a decrease in the activation level of C_j , and vice versa. For example, an increase in invasive species might have a negative impact on the fish population, with a weight of -0.7 indicating a strong detrimental effect.
- **No Influence:** A weight of zero ($w_{ij} = 0$), as the name suggests, indicates the absence of a direct causal relationship between C_i and C_j . Additionally, $w_{ii} = 0$ as concepts do not directly influence their own activation level in this framework.

By quantifying the impact of different concepts on one another, $\mathbf{W}$ provides a crucial foundation for understanding and analyzing the overall behavior of the system. This importance stems from its role as the mathematical representation of the graphical structure of the FCM, as illustrated in Figure 2.1, which also serves as a basis for computational analysis.

The conceptual foundation of FCMs allows for analyzing systems in various ways, offering different perspectives that deepen our understanding of modeled systems. FCMs can be subjected to both static and dynamic analyzes (Christoforou and Andreou, 2017), each offering unique insights into system behavior. Static analysis provides a snapshot of the overall structure and significance of different concepts, while dynamic analysis simulates system evolution.

2.1.1 Static analysis of FCMs

Static analysis of FCMs helps to understand the overall structure of a system. By focusing on the structural attributes of an FCM, such as connectivity, centrality, input/output concepts analysis, and density, static analysis helps reveal the inherent structure and expected behavior of the system (Soner et al., 2015; Tsadiras et al., 2001). These tools rely on graph-theoretic computations and relationship matrices, enabling the evaluation of system complexity and the identification of key concepts within the interconnected network. For instance, calculating model density helps assess overall connectivity and complexity, while the significance of individual concepts can be determined by summing the absolute values of their associated

weights. The concept with the highest cumulative absolute weight is deemed the most influential and/or dependent, highlighting its critical role in the system.

Connectivity refers to the pattern of links between concepts. A high level of connectivity can indicate the complexity of the system. For example, in a lake ecosystem, there may be high connectivity among concepts such as “trophic levels,” “algae concentration,” “water temperature,” and “fish population,” reflecting close interrelations within the food chain.

Centrality measures identify the most critical or central concepts within the network. Concepts with high centrality play a critical role in the system’s dynamics. For instance, “water temperature” can be considered a central concept in a lake ecosystem, as it affects algae growth, fish metabolism, and the solubility of oxygen in the water.

Input/Output Analysis involves identifying concepts that influence the system externally (input) and those that are influenced by the system (output). In the lake ecosystem example, “nutrient flow” from surrounding agricultural land may be an important input, whereas the “health of the fish population” could be seen as a key output reflecting the overall health of the ecosystem.

2.1.2 Dynamic analysis of FCMs

Dynamic analysis of FCMs extends the scope of understanding beyond what static analysis can provide, allowing for a deeper examination of how systems evolve over time. By analyzing relationships among concepts, dynamic analysis provides a comprehensive understanding of the interdependencies within the system and their capacity to influence. This enables a detailed description of system behavior over time, uncovering how changes in one concept propagate through the network and influence the activity of other concepts. Understanding the role of indirect relationships is particularly valuable for determining the importance levels of concepts, as their cumulative effects can significantly impact the system’s overall state.

Through dynamic analysis, users can simulate what-if scenarios under various conditions, which is crucial for understanding system dynamics, predicting future states (Khan et al., 2004; Słoń, 2014), and aiding in decision-making (Abbaspour Onari and Jahangoshai Rezaee, 2022; Baykasoğlu and Gölcük, 2015; Papageorgiou, 2011; Yu and Tzeng, 2006). In this context, a system’s state refers to the combination of activation levels of all concepts that define the system at a specific moment. By

modeling transitions between successive states, dynamic analysis helps track how systems evolve over time. At any time step t , these activation levels collectively form the state vector $\mathbf{A}^t = [A_1^t, A_2^t, \dots, A_n^t]$, which represents the overall system state.

The activation levels of the concepts in a system are calculated iteratively, with each concept's activation level influenced by the activation levels of other connected concepts. Each iteration generates a new state of the system, which reflects the evolving dynamics of the entire system when considering all iterations. This iterative process is heavily based on a reasoning mechanism. The reasoning mechanism is central to how the system evolves, reflecting changes in activation levels. It involves calculations that update the activation levels of each concept in the FCM.

The process of determining successive state vectors, starting from an initial state vector $\mathbf{A}^0$ that encapsulates the initial activation levels of all concepts (Doukas and Nikas, 2020), is referred to as simulating the FCM model of the system or, more simply, conducting an FCM simulation. Simulating the FCM model involves an iterative process where the activation levels of the criteria are calculated using functions defined by the reasoning mechanism, based on the relationships specified in $\mathbf{W}$. Different reasoning mechanisms are available, determined by the specific reasoning rules applied that may account for the prior activation level of a concept and its self-connection. The general mathematical expression of the reasoning mechanism used to calculate activation levels during successive iterations in a simulation is as follows (Koutsellis et al., 2022; A. Sharma et al., 2022; Stylios and Groumpos, 1999):

$$A_i^{t+1} = f \left(\theta_1 A_i^t + \theta_2 \sum_{j=1, j \neq i}^n w_{ji} A_j^t \right) \quad (2.1)$$

In equation 2.1, n represents the number of concepts, and A_i^t represents the activation level of concept C_i at time step t . The parameter θ_1 reflects the influence of the previous activation level of C_i and provides one-step memory function to the FCM by assigning a non-zero value, while θ_2 represents the interconnected influence of other concepts on the new activation value of C_i . In this thesis, both parameters were set to 1. Finally, f is the activation function that squashes activation values into a particular range to enable comparison across concepts. This iterative calculation enables the

modeling of dynamic interactions among concepts, simulating the causal dependencies that evolve within the system.

The activation function f , often chosen to be a sigmoid, ramp, or linear function, squashes the resulting activation values to fit within a range of $[0, 1]$ or $[-1, 1]$ (Knight et al., 2014). Sigmoid and hyperbolic tangent functions are commonly used to map these activation levels in practical implementations (Bueno and Salmeron, 2009). In simulations, the activation level of a concept is iteratively influenced by others according to the activation function used in equation 2.1 until a termination condition is met. Depending on the particularities of the FCM structure (i.e., relationships and their strengths), the activation function, and initial states, the system's activation levels may converge to a fixed state (fixed-point attractor), cycle between states (limit-cycle), or even display chaotic behavior (Tsadiras, 2008). A system is deemed to have converged or reached a fixed equilibrium point if the activation levels stabilize over iterations.

Dynamic analysis of FCMs also allows for evaluating a system's response under different initial states. This iterative updating process (i.e., the reasoning mechanism) is repeatedly applied to simulate the system's trajectory. It can reveal whether the system converges to a stable state, oscillates between different states, or exhibits chaotic and unpredictable patterns. For instance, simulating the effect of introducing an invasive species into a lake ecosystem can be achieved by setting the initial activation level of the corresponding concept to a high value and observing how this change impacts other concepts over time. Through these simulations, the spread and long-term influence of such interventions on the ecosystem can be better understood.

Dynamic analysis of FCMs offers detailed insights into the system by evaluating aspects such as the final activation levels of concepts, duration or activation levels within cycles, and trends in activation levels during simulation for key concepts.

2.2 Related Works and Extensions of FCMs

This thesis focuses primarily on the FCM reasoning mechanism and learning FCM. The literature was reviewed in this context, concentrating on these fundamental aspects.

FCMs have garnered significant interest from researchers as a powerful tool for modeling and analysis. A search conducted in the Scopus database using the keyword “fuzzy cognitive map*” reveals the extent of this interest. Specifically, 3397 studies were identified with no additional filters applied. Notably, 2365 of these studies, approximately 70%, were published in the last decade (between 2014-2024). This growing interest highlights the applicability and effectiveness of FCMs in addressing complex problems across various domains.

This thesis conducts analysis on the reasoning mechanism of FCMs and tests a novel reasoning mechanism using the FCM learning approach, which frames the scope of the literature review. A comprehensive search was conducted in the Scopus database, focusing on “Article title, Abstract, Keywords” fields, using selected keywords or keyword groups relevant to the study. Table 2.1 presents the number of studies identified through this literature search.

The initial search retrieved abstracts for 1,388 scientific studies containing keywords related to FCM reasoning mechanisms and FCM learning, with no time restrictions. After merging the results from different keyword queries and eliminating duplicate records, the dataset was refined to 1,263 unique studies. During this process, priority was given to publications written in English to ensure consistency and accessibility for further analysis.

In order to focus on the most recent developments, the literature review prioritized studies published within the last ten years. This timeframe ensures a relevant and up-to-date background for the present study while maintaining a comprehensive perspective. As a result, 827 articles were identified. Although many of these studies contained relevant keywords related to reasoning mechanisms in FCMs, a significant portion did not directly aim to improve or modify the reasoning process itself. Therefore, a manual filtering step was carried out to assess whether each study genuinely focused on enhancing the reasoning mechanism. This in-depth screening led to a subset of approximately 70 studies, which explicitly address the development, modification, or improvement of FCM reasoning mechanisms. Notably, several of these studies approach reasoning enhancement through the integration of learning-based techniques, such as weight adaptation or the calibration of causal relationships.

Table 2.1: Number of scientific publications on FCMs retrieved from the Scopus database based on keywords.

Search Query Used in the Literature Review	Number of Studies	
	All Time	2014-2024
“fuzzy cognitive map*” OR “fuzzy causal map”	3397	2365
“fuzzy causal map*”	5	1
“fuzzy cognitive map*” AND “reasoning*”	305	160
“fuzzy cognitive map*” AND “inference*”	383	218
“fuzzy cognitive map*” AND “reasoning mechanism*”	14	9
“fuzzy cognitive map*” AND “inference mechanism*”	25	13
“fuzzy cognitive map*” AND “reasoning engine*”	7	2
“fuzzy cognitive map*” AND “inference engine*”	45	13
“fuzzy cognitive map*” AND (“reasoning*” OR “inference*”) AND (“mechanism*” OR “engine*”)	127	64
“fuzzy cognitive map*” AND “converge*”	118	88
“fuzzy cognitive map*” AND “convergence*”	98	71
“fuzzy cognitive map*” AND “activation*”	78	52
“fuzzy cognitive map*” AND “activation function*”	27	19
“fuzzy cognitive map*” AND “activation rule*”	2	2
“fuzzy cognitive map*” AND “transfer function*”	22	13
“fuzzy cognitive map*” AND “transformation function*”	12	9
“fuzzy cognitive map*” AND “learning*”	804	568
“learning fuzzy cognitive map*” OR “fuzzy cognitive map* learning*” OR “learning FCM*” OR “FCM learning*”	123	83
“learn* fuzzy cognitive map*” OR “fuzzy cognitive map* learn*” OR “learn* FCM*” OR “FCM learn*”	143	95

FCMs have been widely used as a versatile modeling tool for representing and simulating complex systems. At their core, FCMs rely on activation functions to determine the activation levels of interconnected concepts based on causal relationships. Classical FCM models typically use nonlinear activation functions, such as sigmoid and hyperbolic tangent sigmoid, to compute these activation values. However, such functions present certain drawbacks, including the risk of convergence to identical points regardless of initial states and difficulties in adequately capturing cyclic behaviors (Hatwágner and Kóczy, 2021). To address these limitations, recent research has focused on improving the reasoning mechanisms within FCMs and integrating them with advanced learning methodologies to enhance their capabilities

(Ghazanfari et al., 2007; Nápoles et al., 2019; Orang et al., 2022; Tsadiras, 2008). The following review categorizes these studies into three major groups: improvements to existing activation functions, exploration of new reasoning mechanisms, and integrating learning methodologies to optimize FCM structures.

Traditional nonlinear functions, such as the sigmoid and hyperbolic tangent, have been modified to improve FCM performance. For example, researchers have focused on optimizing the parameters of these functions, such as altering the slope or threshold of the sigmoid function, to enhance system sensitivity and efficacy (Hatwágner et al., 2017; Nápoles, et al., 2018a). Similarly, modifications to the hyperbolic tangent activation function have been examined to establish improvements in convergence points, uniqueness conditions, and stability (Harmati and Koczy, 2019). Despite these modifications, the fundamental reasoning mechanism remains unchanged, and improvements are often limited in their ability to completely overcome the original functions' limitations.

Some of the earliest modifications aimed at enhancing reasoning capabilities involved using activation functions based on certainty factors (Tsadiras and Margaritis, 1997). More recent studies have incorporated innovative approaches, such as integrating wavelet activation functions (K. Wu et al., 2017), allowing for more nuanced concept activation and limiting excessive expression levels. These explorations have shown promising advances, yet the improvements have often been incremental rather than transformational (P. K. Singh and Chudasama, 2017a).

To address the limitations of traditional activation functions, various studies have proposed alternative approaches to the reasoning mechanisms of FCMs. These alternative approaches often aim to enhance flexibility, adaptivity, and overall reasoning capabilities. For example, Nápoles et al. (2022) introduced a normalization-like activation function into FCMs that showed improved convergence properties and helped prevent concept saturation. Later, Nápoles et al. (2024) proposed a neural cognitive mapping approach using an “exponentially normalized activator,” focusing on relative changes rather than absolute concept values.

In addition, normalization techniques have been applied to scale concept values and balance the influence of weights in FCMs. Various researchers have also explored different mechanisms to strengthen inference, such as integrating fuzzy if-then rules

to increase model adaptability and enhance classification capabilities (Guo, 2022; Song et al., 2011). Other examples include the use of rough set theory (X. Li and Luo, 2020) to extend the applicability of FCMs for both continuous and discrete data, as well as approaches involving rule-based FCMs incorporating fuzzy set semantics (Zdanowicz and Petrovic, 2018).

Another notable innovation was presented by Frias et al. (2018), who leveraged a computational paradigm based on linguistic terms rather than numerical concepts and relationships, using triangular fuzzy numbers for reasoning. Falcon et al. (2019) further integrated knowledge granules, such as intervals, fuzzy sets, and rough sets, demonstrating their applicability in time-series analysis and pattern classification (Peng et al., 2024). However, these approaches were often constrained to specific applications, limiting their generalizability across diverse domains.

Beyond the development of new activation functions and reasoning mechanisms, integrating learning methodologies has played a crucial role in advancing the performance and accuracy of FCMs. Learning methods in FCM are broadly categorized into three types: Hebbian-based methods, error-driven methods, and hybrid approaches. These learning approaches aim to determine the optimal weights of interrelationships between concepts through a structured algorithm, incorporating both historical data and expert insights (Alizadeh et al., 2007; Alizadeh and Ghazanfari, 2009).

Hebbian-based learning methodologies build upon unsupervised mechanisms to adjust weights in response to changes in concept values. Differential Hebbian Learning (Dickerson and Kosko, 1994), Balanced Differential HL (Hueriga, 2002), and Active HL (Papageorgiou et al., 2004) are among the variations of Hebbian-based learning FCMs. These approaches rely heavily on expert intervention and parameter tuning, making the process labor-intensive and dependent on domain expertise (S. J. Li and Shen, 2004; Stach et al., 2008).

Error-driven learning algorithms aim to optimize the weight matrix by minimizing the deviation between predicted and expected outcomes, introducing a novel approach to enhancing learning within FCMs. Such methods have led to significant advances, including the use of metaheuristic optimization tools such as Particle Swarm Optimization (PSO) (Salmeron et al., 2017) and its variants, like memetic PSO (Petalas

et al., 2007, 2009) and multi-leader PSO (P. Liu and Liu, 2017). These tools have been instrumental in constructing FCMs that are less reliant on expert-based weight assignment and more capable of learning from historical data (P. K. Singh and Chudasama, 2017b).

Supervised learning methods for FCMs, such as PSO and its derivatives, provide an efficient way to optimize the weight matrices, often using historical datasets (Hengjie et al., 2007; León, et al., 2011a; León, et al., 2011b; Pedrycz, 2010). Other swarm intelligence techniques, such as Ant Colony Optimization (Chen et al., 2012) and Artificial Bee Colony (Yesil et al., 2013), have also showed success in training FCMs by optimizing the weight structure.

In the realm of evolutionary algorithms, studies have explored the use of Genetic Algorithms (Stach et al., 2005), Imperialist Competitive Algorithms (Ahmadi et al., 2015), and Dynamic Multi-agent Genetic Algorithms (J. Liu et al., 2016) to enhance the learning process of FCMs. Evolution-based metaheuristics, such as Evolution Strategy (Koulouriotis et al., 2001), have also been used for optimizing weight configurations (Chi and Liu, 2016; Shen et al., 2020; C. Wang et al., 2021). Besides these, hybrid learning methodologies combining Hebbian and error-driven methods have emerged as promising alternatives (Papageorgiou et al., 2005; Ren, 2012; Yazdi and Lucas, 2008; Zhu and Zhang, 2008). By leveraging the strengths of both techniques, these hybrid approaches have been shown to enhance FCM training for complex systems, offering a balanced solution that reduces the limitations inherent in individual methodologies.

Other hybrid approaches have employed reinforcement learning paradigms to model reasoning in dynamic systems (Tlili et al., 2021). By integrating fuzzy rule bases and transition maps, these methods facilitate effective representation of system states, thereby supporting enhanced decision-making capabilities. Population-based metaheuristics such as Asexual Reproduction Optimization (Salmeron et al., 2019), Immune Algorithm (Chunmei, 2009), Big Bang-Big Crunch (Urbas, 2010), and Extended Great Deluge Algorithm (Baykasoglu et al., 2011), and methods by (Motlagh et al., 2015) have further contributed to the expansion of the learning toolbox for FCMs.

The literature on reasoning approaches and learning methodologies for FCM demonstrates significant progress in enhancing the flexibility, adaptability, and reliability of FCMs. Studies on modifying activation functions, developing novel reasoning mechanisms, and integrating advanced learning methodologies, such as error-driven, Hebbian-based, and hybrid approaches, all contribute to overcoming traditional limitations and broadening the applicability of FCMs in real-world scenarios. Nonetheless, many proposed methods have remained limited in addressing the sensitivity to initial conditions and examining its impact on learning, and there remains a need for generalizable, widely applicable solutions that can comprehensively address the diverse requirements of dynamic and complex systems.





3. CONVERGENCE CHALLENGES IN FUZZY COGNITIVE MAPS: THEORY AND SIMULATION EVIDENCE

As discussed before, FCMs are widely used to model the dynamic behavior of complex systems by capturing causal relationships among system components. Through iterative simulations, FCMs can analyze system responses to various scenarios, enabling predictive reasoning and decision support. However, the convergence behavior of FCMs, i.e., whether the system stabilizes and how it does so, presents both critical opportunities and limitations depending on the application domain.

3.1 Understanding Convergence in FCMs

Convergence in FCMs refers to the stabilization of concept activation levels through iteration, typically to a fixed point. This behavior is fundamental to many FCM applications, particularly in domains where system stability, predictability, or equilibrium is essential (Kosko, 1986; Papageorgiou and Groumpos, 2005). For instance, in control systems or engineering applications, convergence ensures that a modeled system reaches a functional and repeatable state that supports automation and operational consistency.

In decision-making applications, such as multi-criteria decision making (MCDM), convergence can assist in generating consistent weightings (Hajek and Prochazka, 2018) or averaged assessments based on expert input. In these cases, convergence enhances the reliability and transparency of the decision-making process.

Nevertheless, in complex and adaptive systems, such as those involving human behavior, socio-economic dynamics, or environmental variability, convergence may become a liability. Models that always converge to the same fixed point, regardless of the initial state, fail to capture the full spectrum of possible system trajectories (Carvalho, 2013; Khan and Quaddus, 2004). This limitation is especially problematic in domains requiring flexibility and scenario sensitivity.

3.2 When Convergence Becomes a Limitation

In systems driven by human decisions or behavioral variability, such as organizational behavior or socio-political modeling, convergence to a single outcome undermines the system's capacity to reflect alternative futures. It restricts the model's adaptability, preventing it from generating diverse responses to varied initial inputs (Papageorgiou et al., 2004; Stylios et al., 1997).

While convergence simplifies the interpretation and implementation of FCM-based models, it also poses a fundamental question: Is stability always desirable? In many real-world applications, the answer is no. Excessive convergence masks the complexity of systems, limiting their explanatory and exploratory power. Furthermore, repetitive convergence may lead to oversimplified conclusions, resulting in decisions that ignore the system's inherent uncertainty and multifaceted dynamics.

3.3 Theoretical and Computational Perspectives

From a computational perspective, convergence is influenced by several factors, including the structure of the weight matrix, the choice of transfer functions (e.g., sigmoid, hyperbolic tangent), and the iteration method used. For example, certain configurations of weights and thresholds can produce cyclic or even chaotic behavior instead of convergence to a single fixed point (Dickerson and Kosko, 1994; Lee and Kwon, 2010).

Some studies have addressed the theoretical underpinnings of convergence through analytical approaches. Boutalis et al. (2009) introduced sufficient conditions for the existence and uniqueness of fixed points based on the steepness of sigmoid functions. These results were later expanded to address global exponential stability using Lyapunov-based methods (Lee and Kwon, 2010).

Despite these efforts, the dynamic behavior of FCMs is still typically explored through empirical simulation. This involves extensive scenario testing, which is computationally expensive and may not guarantee coverage of all stable states (Dickerson and Kosko, 1994). Thus, there remains a need for methods that enhance convergence without sacrificing the model's sensitivity to input variations.

3.4 Demonstrating the Convergence Problem

To illustrate these challenges, this chapter presents practical simulation-based examples using FCMs with different transfer functions, specifically the sigmoid and the hyperbolic tangent sigmoid. In the examples, FCM models, one that is adopted from Harmati et al. (2023) and one that is designed for human reliability in a manufacturing process, are simulated using varying initial activation states. In all tested configurations, the model consistently converges to the same final state, regardless of the initial inputs.

This outcome demonstrates a key limitation of conventional FCMs: their inability to reflect variability and complexity in human-centered or uncertain systems. Such limitations raise concerns about the model's capacity to support decision-making in domains where multiple valid outcomes exist, and initial conditions should matter.

As the first illustrative example, the study by Harmati et al. (2023) presents a case in which, under certain conditions of the relationship matrix, the FCM iteration rule (i.e., reasoning mechanism) produces the same equilibrium point for every initial activation vector. The relationship matrix used in their work is given in equation 3.1:

$$W_1 = \begin{bmatrix} 0 & -1 & 0.5 & 0 & 0 \\ 0 & 0 & -0.5 & 0.5 & -0.5 \\ -1 & 1 & 0 & -0.5 & 0 \\ 0 & 1 & -1 & 0 & -0.5 \\ -1 & 0 & 1 & -0.5 & 0 \end{bmatrix} \quad (3.1)$$

Simulations based on the relationship matrix in equation 3.1 and the FCM rule from equation 2.1, starting from the initial states shown in Figure 3.1, yield the results also presented in the same figure.

In the simulations, the non-cumulative computations are carried out with $\theta_1 = 0$ and $\theta_2 = 1$, while the cumulative computation uses $\theta_1 = \theta_2 = 1$. The activation function f is defined as the sigmoid function shown in equation 3.2:

$$f_{sigmoid}(x) = \frac{1}{1 + e^{-\lambda x}} \quad (3.2)$$

where λ is a parameter that controls the steepness of the sigmoid curve. In this simulation, λ was set to 1.

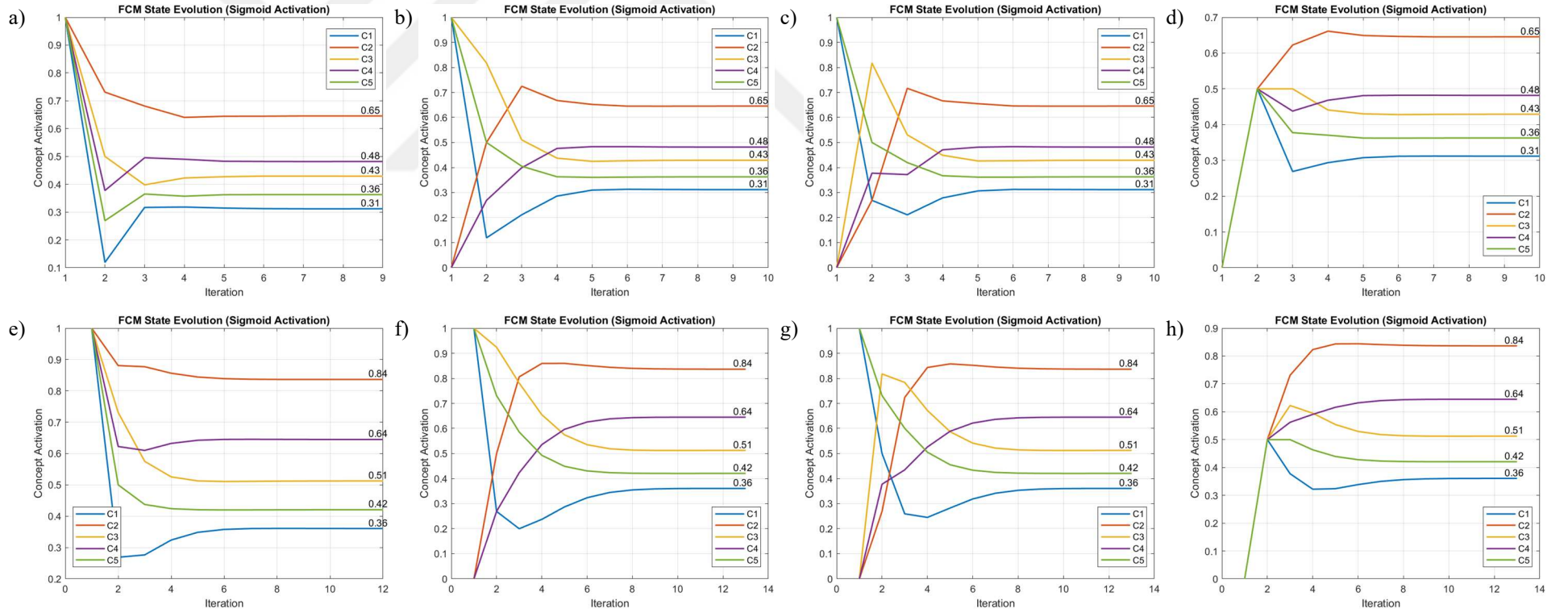


Figure 3.1: FCM simulations with non-cumulative computation starting from the initial state vectors a. $[1 \ 1 \ 1 \ 1 \ 1]$, b. $[1 \ 0 \ 1 \ 0 \ 1]$, c. $[1 \ 0 \ 0 \ 0 \ 1]$, and d. $[0 \ 0 \ 0 \ 0 \ 0]$; and with cumulative computation starting from the state vectors e. $[1 \ 1 \ 1 \ 1 \ 1]$, f. $[1 \ 0 \ 1 \ 0 \ 1]$, g. $[1 \ 0 \ 0 \ 0 \ 1]$, and h. $[0 \ 0 \ 0 \ 0 \ 0]$.

As shown in Figure 3.1, regardless of the initial state vector used, both non-cumulative and cumulative computations lead the system to converge to the same final state. In such a case, determining the current state of the system provides little to no insight into its future behavior, reducing the practical value of the simulation for predictive analysis. Furthermore, this outcome highlights the issue raised by Nápoles et al. (2024) even when the initial activation values are set to zero for all concepts, as seen in Figure 3.1(d) and Figure 3.1(h), some concepts still become activated, which is inconsistent with real-world expectations.

Another example demonstrates the convergence behavior when the tangent sigmoid function, defined in equation 3.3, is used. As in the previous case, the reasoning mechanism described in equation 2.1 is applied.

$$f_{tanh}(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \quad (3.3)$$

The relationship matrix used in this example is provided in equation 3.4.

$$W = \begin{bmatrix} 0.000 & 0.000 & 0.000 & 0.323 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.430 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.622 & 0.576 & 0.000 & -0.259 & 0.180 & 0.259 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & -0.516 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.180 \\ 0.000 & 0.000 & -0.516 & 0.381 & 0.000 & 0.000 & 0.000 & 0.473 & 0.510 \\ -0.254 & -0.254 & -0.473 & 0.434 & 0.572 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & -0.180 & 0.000 & 0.515 & 0.000 & 0.259 & 0.000 & 0.189 \\ 0.000 & 0.000 & 0.000 & 0.212 & 0.513 & 0.000 & 0.000 & 0.000 & 0.000 \end{bmatrix} \quad (3.4)$$

The simulation results corresponding to this example are illustrated in the graphs presented in Figure 3.2. As in the previous case, all four simulations, each initiated from a different initial state vector, converge to the same final set of concept activation values. However, in this instance, it is notable that the system required more iterations to reach convergence. A potential explanation for this behavior lies in the use of the tangent sigmoid function, which maps activation values to the relatively wide interval of $[-1,1]$. Although this function offers mathematical smoothness and symmetry, the broader output range may introduce delays in convergence and complicate interpretation in practical contexts. In particular, this scale may not be intuitive or meaningful in many real-world decision-making problems, where activation values are often expected to fall within the normalized interval $[0,1]$, representing levels of agreement, importance, or likelihood. As such, while the tangent sigmoid function may

offer some advantages in terms of dynamics, its interpretability and practical applicability remain limited in many use cases.

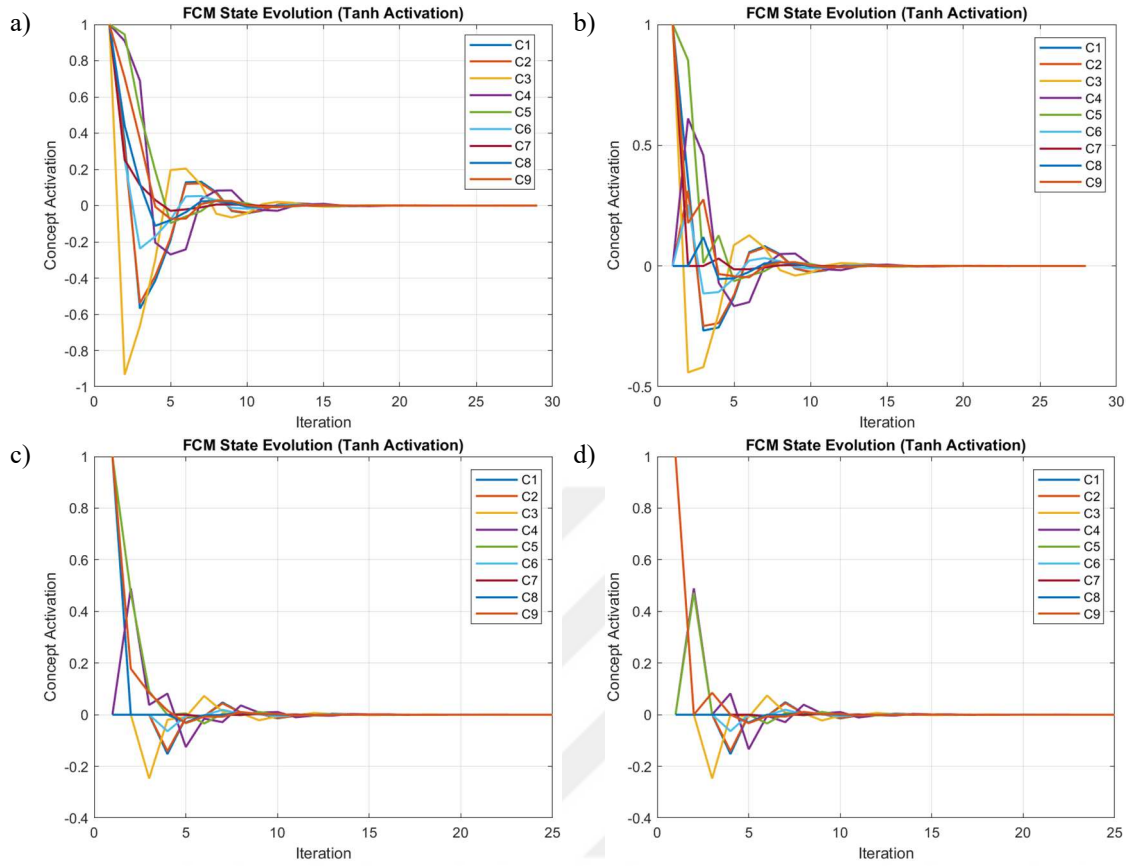


Figure 3.2: FCM simulations with non-cumulative computation starting from the state vectors a. $[1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1]$, b. $[1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1]$, c. $[1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1]$, and d. $[1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1]$.

4. THE ROLE OF INDIRECT RELATIONSHIPS IN FCM: APPLICATION OF FERMATEAN FCM TO E-WASTE COLLECTION STRATEGY SELECTION

In this chapter, our exploration of FCMs is extended by focusing on indirect relationships between concepts. The goal is to highlight the role of indirect relationships in providing a more complete understanding of system dynamics and their influence on determining the importance levels of concepts. We also introduce an approach that incorporates both direct and indirect effects in the calculation of weights, which will serve as a foundation for following chapters.

In traditional dynamic analyses with classical FCMs, simulations are based solely on direct relationships. However, focusing exclusively on direct relationships has its limitations, particularly with regard to capturing the nuanced dynamics of complex systems.

To address this limitation, it is critical to consider indirect relationships, which account for the propagated effects that concepts exert on each other beyond immediate, one-step interrelations. These indirect relationships can be determined by taking powers of the relationship matrix, effectively incorporating multiple layers of influence into the model. This approach allows us to build a total effect matrix that includes both direct and indirect relationships, offering a more holistic representation of the system. The importance of including indirect relationships is tied to overcoming the convergence problem caused by linear activation functions, which was elaborated on in Chapter 3. By independently calculating effects beyond direct relationships, we can achieve a more nuanced understanding of how the concepts interact over time.

Indirect relationships bring significant additional information and value to FCM analysis. While direct relationships describe the immediate influence between concepts, indirect relationships reveal how concepts impact each other through intermediary pathways. This extended view provides a richer understanding of the system's structure and behavior, which is particularly useful for determining the relative importance of each concept within the system. By calculating and combining

the influence that each concept receives (indegree) and exerts (outdegree) through both direct and indirect pathways, a single total degree value is derived for each concept. This comprehensive approach allows for a more informed assessment of the weights assigned to the concepts.

This chapter also introduces the application carried out in the study titled “Electronic Waste Collection Strategy Selection via a Fermatean Fuzzy-Based Decision Support System” (Murat et al., 2025a). In this study, we demonstrate the practical implementation of calculating weights using both direct and indirect relationships in FCMs. Specifically, we take the powers of the relationship matrix to derive the total effect matrix, allowing us to evaluate the overall influence of each concept. This approach was used to determine the weights of selection criteria for identifying the most suitable e-waste collection method.

The focus on indirect relationships is not only central to this chapter but will also be revisited in Chapters 5 and 6, where further developments and extensions are presented. By analyzing both direct and indirect relationships, it is aimed at enhancing the reliability of FCMs by modeling complex decision-making processes. The incorporation of indirect influences enables a more comprehensive evaluation of system dynamics, ultimately supporting more effective decision-making strategies.

4.1 An MCDM Framework Using Direct and Indirect Relationships in Fermatean FCMs

In the context of selecting e-waste collection strategies, fuzzy logic offers a promising approach for effectively managing inherent uncertainty and subjectivity in decision-making processes. Unlike traditional methods that rely solely on precise mathematical models and objective data, fuzzy logic allows the incorporation of partial truth values and subjective inputs, thereby enabling more flexible and realistic evaluation of various criteria and their trade-offs (Ayyildiz et al., 2020). Often, strategy selection problems encompass variables that cannot be measured numerically, such as security, management, and environmental sensitivity, which may be conflicting in nature. Fuzzy logic provides a versatile methodology for evaluating these criteria, transcending rigid mathematical models and embracing the uncertainties and ambiguities commonly encountered in real-life scenarios. Consequently, to address the complex nature of e-waste collection strategy selection, this study proposes a novel integration of FCM and

Technique for Order Preference by Similarity to Ideal Solutions (TOPSIS) methodologies in an Fermatean Fuzzy (FF) environment.

In this study, Fermatean Fuzzy Sets (FFSs) and FCM are employed to enhance the decision-making process. FFSs provide an extended framework for better managing uncertainty in expert evaluations. This enables experts to express their opinions with greater flexibility by allowing higher values for the degrees of membership and non-membership while maintaining the condition that their sum, along with the degree of hesitancy, remains within a broader allowable range compared to other fuzzy set types (Senapati and Yager, 2020). Particularly, FFSs excel at dealing with uncertain information in complex problems, making it highly suitable for evaluating e-waste collection strategies by allowing more nuanced analyses of criteria. On the other hand, FCM enables examination of how variations in one criterion affect others by modeling causal interrelationships between them. This approach identifies the criteria that exert a greater influence and are more susceptible to change. As a result, the significance of the selection criteria for e-waste collection strategies can be systematically determined. Thus, integrating FCM and TOPSIS methodologies in an FF environment provides a robust methodology for identifying the most appropriate e-waste collection strategy based on expert opinion.

The details of the proposed methodology are presented in this section. Initially, the FCM method proposed by Kosko (1986) is extended to FFSs to determine the criteria weights. Subsequently, the TOPSIS method introduced by Hwang and Yoon (1981) and extended to FFSs by Senapati and Yager (2020) is employed to identify the best strategy based on expert opinions. By integrating FCM with TOPSIS in an FF environment, this study offers a comprehensive and innovative approach to address the challenges of e-waste collection strategy selection, effectively addressing system complexity while ensuring a more flexible representation of real-world uncertainties. The flow of the proposed integrated fuzzy methodology is illustrated in Figure 4.1.

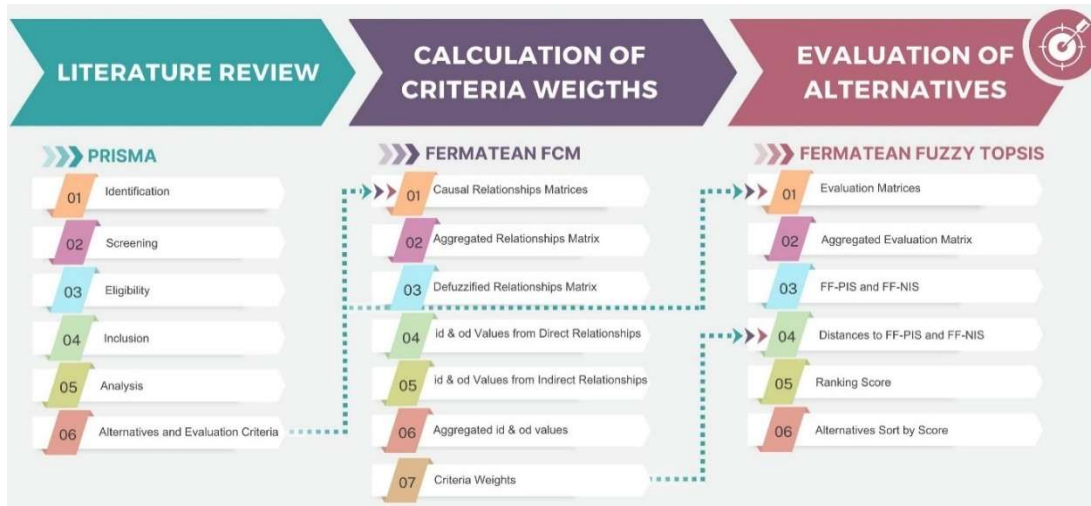


Figure 4.1: Flow of the proposed integrated decision-making methodology.

4.1.1 Fermatean Fuzzy Sets

Fuzzy logic is a robust mathematical framework that addresses uncertainty in decision-making. Unlike binary logic, it accounts for degrees of truth, which is essential in complex systems (Kahraman, 2008). It is ideal for controlling nonlinear processes with uncertain information (Erdogan and Ayyildiz, 2022). Instead of exact values, qualitative assessments can be conducted that incorporate subjective judgment (Ayyildiz, 2022). Traditional fuzzy sets use membership functions, whereas intuitionistic fuzzy set theory, an extension of classical fuzzy sets, adds non-membership and hesitancy functions, offering a more nuanced approach (Atanassov, 1999).

This study employs a more advanced fuzzy logic approach that goes beyond traditional methods to better handle uncertainty and ambiguity in decision-making processes. Fermatean fuzzy sets (FFSs) extend intuitionistic and Pythagorean fuzzy sets by providing a more flexible framework for managing indeterminate and uncertain information (Senapati and Yager, 2020). This enhanced capability allows FFSs to address the limitations often found in previous fuzzy models (Senapati and Yager, 2019). By offering a refined method for translating subjective, linguistic assessments into precise mathematical terms, FF logic proves to be one of the most effective tools for dealing with imprecise data (Mishra and Rani, 2021)

FFSs are particularly well-suited to tackling complex decision-making problems where ambiguity is present because they allow for an independent definition of the degree of indeterminacy, something that most other fuzzy systems lack (D. Liu et al.,

2019). This affords decision-makers greater flexibility in assigning membership and non-membership values, leading to more accurate reflections of their judgments (Gül, 2021). Therefore, FFSs provides a pioneering tool for dealing with unreliable, ambiguous, and imprecise information within fuzzy environments (D. Liu et al., 2019), making them an essential resource for tackling real-world strategy selection challenges in waste management. By providing a more adaptable structure, FF logic is ideal for evaluating criteria in strategy selection, particularly in cases like sustainable waste management where uncertainty is high.

4.1.2 Preliminaries of Fermatean Fuzzy Sets

Senapati and Yager (2020) introduced FFSs as an extension of the Intuitionistic and Pythagorean fuzzy sets. FFSs offer a novel approach to representing unreliable and imprecise information in fuzzy systems (Garg et al., 2020; Simic et al., 2021).

Definition 4.1: An FFS on a fixed universal non-empty set $\mathbf{U}$ is mathematically defined as the following equation (Senapati and Yager, 2019):

$$\tilde{F} = \{ \langle u, \tilde{F}(\mu_{\tilde{F}}(u), \nu_{\tilde{F}}(u)) \rangle | u \in \mathbf{U} \} \quad (4.1)$$

where $\mu_{\tilde{F}}(u): \mathbf{U} \mapsto [0,1]$ and $\nu_{\tilde{F}}(u): \mathbf{U} \mapsto [0,1]$ are the membership and non-membership degrees of u to $\tilde{F}$, and for every $u \in \mathbf{U}$, they satisfy the following equation:

$$0 \leq (\mu_{\tilde{F}}(u))^3 + (\nu_{\tilde{F}}(u))^3 \leq 1 \quad (4.2)$$

The hesitancy degree of the element $u \in \mathbf{U}$ is calculated as follows:

$$\pi_{\tilde{F}}(u) = \sqrt[3]{1 - (\mu_{\tilde{F}}(u))^3 - (\nu_{\tilde{F}}(u))^3} \quad (4.3)$$

For the sake of convenience, aligning with the representation introduced by Senapati and Yager, (2019, 2020), $\tilde{\eta} = \tilde{F}(\mu_{\tilde{\eta}}, \nu_{\tilde{\eta}})$ is defined as a Fermatean fuzzy number (FFN).

Let $\tilde{A} = \tilde{F}(\mu_{\tilde{A}}, \nu_{\tilde{A}})$ and $\tilde{B} = \tilde{F}(\mu_{\tilde{B}}, \nu_{\tilde{B}})$ are two FFNs.

Definition 4.2: $\tilde{A}$ multiplied by positive scalar λ is given as follows (Simic et al., 2021):

$$\lambda \tilde{A} = \left(\sqrt[3]{1 - (1 - \mu_{\tilde{A}}^3)^\lambda}, v_{\tilde{A}}^\lambda \right) \quad (4.4)$$

Definition 4.3: The positive power λ of $\tilde{A}$ is given as follows (Simic et al., 2021):

$$\tilde{A}^\lambda = \left(\mu_{\tilde{A}}^\lambda, \sqrt[3]{1 - (1 - v_{\tilde{A}}^3)^\lambda} \right) \quad (4.5)$$

Definition 4.4: The score value of $\tilde{A}$ is obtained by the following equation (Gonzales et al., 2022):

$$S_F(\tilde{A}) = \frac{1}{2} [(\mu_{\tilde{A}}^3 - v_{\tilde{A}}^3 - \ln(1 + \pi_{\tilde{A}}^3)) + 1] \quad (4.6)$$

Definition 4.5: The Manhattan distance between $\tilde{A}$ and $\tilde{B}$ can be calculated as follows:

$$D(\tilde{A}, \tilde{B}) = \frac{1}{2} [|\mu_{\tilde{A}}^3 - \mu_{\tilde{B}}^3| + |v_{\tilde{A}}^3 - v_{\tilde{B}}^3| + |\pi_{\tilde{A}}^3 - \pi_{\tilde{B}}^3|] \quad (4.7)$$

Definition 4.6: The FF Weighted Average (FFWA) and FF Weighted Geometric (FFWG) operators are defined as the following equations (Senapati and Yager, 2019):

$$FFWA(\tilde{\mathcal{F}}_1, \tilde{\mathcal{F}}_2, \dots, \tilde{\mathcal{F}}_n) = \tilde{F} \left(\sum_{i=1}^n w_i \mu_{\tilde{\mathcal{F}}_i}, \sum_{i=1}^n w_i v_{\tilde{\mathcal{F}}_i} \right) \quad (4.8)$$

where $\tilde{\mathcal{F}}_i = \tilde{F}(\mu_{\tilde{\mathcal{F}}_i}, v_{\tilde{\mathcal{F}}_i})$ and $w = (w_1, w_2, \dots, w_n)^T$ are the set of FFNs and weight vectors, respectively.

4.1.3 Fermatean Fuzzy Cognitive Map

Section 2.1 reveals that the attributes of concepts within an FCM can be unveiled through static analysis. Building upon this principle, the FCM can be crafted to analyze concepts and assign their respective weights.

The creation of an FCM is deeply rooted in the insights of domain experts, particularly in cases where data is scarce or the concepts themselves are difficult to quantify precisely (Poczeta et al., 2015). Expert knowledge guides every step of the process, from structuring the map to defining relationship strengths and selecting meaningful

input data (Mkhitarian et al., 2022). When variables are vague or imprecise, as in this section, the expertise of specialists becomes indispensable.

Yet, decision-making processes present a challenge, experts often struggle to provide precise values for w_{ij} , as the variables at hand are too elusive to quantify. To address this, predefined sets of linguistic variables are introduced for w_{ij} , enabling a more accurate reflection of expert insights. This method empowers experts to estimate weight ranges even when precise values cannot be given. By weaving linguistic variables into the framework, this approach provides a strong bridge between qualitative judgments and the quantitative rigor of fuzzy numbers.

In this section, linguistic expressions are employed to advance the clarity and precision of expert input within the FCM framework. Notably, fuzzy numbers are seamlessly integrated into the construction process, enabling experts to articulate their perspectives more accurately. The methodology for determining the weights of the selection criteria, which incorporates both direct and indirect relationships within the F-FCM, unfolds in the following steps:

Step 1. Experts are first briefed on the selection criteria and then asked about the relationships among the criteria and their strengths. Experts use the terms of the linguistic variable in Table 4.1 to describe the strength of relationships. This process generates individual F-FCMs that illustrate criterion interdependence according to each expert's perspective.

Table 4.1: Linguistic variable.

Linguistic Term	μ	ν
No Impact (NI)	0	1
Low Impact (LI)	0.4	0.5
Medium Impact (MI)	0.7	0.2
High Impact (HI)	0.9	0.1

Step 2. Individual F-FCMs are aggregated using equation 4.8 to obtain the aggregated F-FCM. The relationships in this F-FCM are mathematically encapsulated by the following matrix $\widetilde{W}$.

$$\widetilde{\mathbf{W}} = [\widetilde{w}_{ij}]_{n \times n} \quad (4.9)$$

where $\widetilde{w}_{ij}$ denotes the magnitude of the causal impact from C_i to C_j expressed with FFNs. All $\widetilde{w}_{ii}$ values are represented as (0, 1).

Step 3. The $\widetilde{\mathbf{W}}$ is defuzzified according to equation 4.6 to acquire crisp relationship matrix $\mathbf{W} = [w_{ij}]$, i.e, crisp FCM, where w_{ij} represents the crisp value of the relationship between C_i and C_j .

Step 4. For each node (i.e., criterion), metrics called indegree (id) and outdegree (od) are calculated by referring to the direct relationships in $\mathbf{W}$. id is the cumulative strength of incoming relationships for a node and od is the cumulative strength of outgoing relationships. They are calculated using the following equations (Özesmi and Özesmi, 2004):

$$id_i = \sum_{j=1}^n |w_{ji}| \quad (4.10)$$

$$od_i = \sum_{j=1}^n |w_{ij}| \quad (4.11)$$

where n is the number of criteria.

Step 5. To fully determine the significance of the criteria, the strength of relationships in causal chains of full-length is considered. Indirect relationships and their potential effects are included in the calculation (Soner et al., 2015).

Step 5.1. The relationship matrix $\mathbf{W}$ is raised to increasing powers $q = 2, 3, \dots, n$. This exponentiation terminates when q reaches a certain value Q where the orders of the concepts in terms of both row and column sums remain unchanged between two consecutive iterations, indicating a state of stability (Soner et al., 2015).

Step 5.2. The normalized id and od measures of the criteria for each matrix raised to the q^{th} power are calculated according to the following equations:

$$Nid_i^q = \frac{\left(\max_{i=1,\dots,n} id_i\right) (id_i^q)}{\left(\max_{i=1,\dots,n} id_i^q\right)} \quad (4.12)$$

$$Nod_i^q = \frac{\left(\max_{i=1,\dots,n} od_i\right) (od_i^q)}{\left(\max_{i=1,\dots,n} od_i^q\right)} \quad (4.13)$$

where id_i^q and od_i^q represent the indegree and outdegree values of criterion i , respectively, calculated from the matrix obtained by raising matrix $\mathbf{W}$ to the power of q .

Step 5.3. For each criterion, aggregated normalized id value id_i^{agg} and aggregated normalized od value od_i^{agg} are calculated as in equations 4.14 and 4.15, respectively.

$$id_i^{agg} = (id_i + Nid_i^2 + \dots + Nid_i^Q)/Q \quad (4.14)$$

$$od_i^{agg} = (od_i + Nod_i^2 + \dots + Nod_i^Q)/Q \quad (4.15)$$

Step 6: id_i^{agg} and od_i^{agg} measures are added for each criterion as follows (Özesmi and Özesmi, 2004):

$$td_i = id_i^{agg} + od_i^{agg} \quad (4.16)$$

where td_i represents the total degree value of criterion i .

Step 7. Each td_i value is normalized by dividing it by the sum of all total degree values as in equation 4.17, to obtain each criterion importance ψ_i .

$$\psi_i = td_i / \sum_{i=1}^n td_i \quad (4.17)$$

The algorithmic procedure for calculating weights using the F-FCM methodology is presented in Algorithm 1.

4.1.4 Fermatean Fuzzy TOPSIS

TOPSIS, proposed by Hwang and Yoon (1981), identifies the best alternative by maximizing its distance from the negative ideal solution and minimizing its distance

Algorithm 1: F-FCM-based weight determination algorithm

Input:

n : the number of evaluation criteria

E : the number of experts

$\tilde{W}^k = [\tilde{w}_{ij}^k]$, $1 < i < n, 1 < j < n, 1 < k < E$: FF relationships matrix obtained from expert k // see Table 4.1

Output:

ψ : the vector of criteria weights

Begin

1. $\tilde{W} \leftarrow$ aggregation of $\tilde{W}^k$ matrices
 - 1.1. check the number of relationships for criteria pairs
 - 1.2. if there are 4 or more relationships, replace them with their FFN equivalents; otherwise, assign no relationship
 - 1.3. aggregate the equivalent relationships in the matrices using the FFWA operator // see equation 4.8
2. $W \leftarrow$ defuzzify aggregated relationship matrix $\tilde{W}$ // see equation 4.6
3. $id_i \leftarrow$ sum the absolute values of the incoming relationships for each node i // see equation 4.10
4. $od_i \leftarrow$ sum the absolute values of the outgoing relationships for each node i // see equation 4.11
5. Calculation of normalized indegrees and outdegrees by raising W to the q^{th} power
 - 5.1. $q \leftarrow 2$
While $q \leq n$
 - 5.2. $\omega^q \leftarrow \text{power}(W, q)$
 - 5.3. $Nid_i^q \leftarrow$ calculate the normalized sum of absolute indegrees for ω^q // see equation 4.12
 - 5.4. $Nod_i^q \leftarrow$ calculate the normalized sum of absolute outdegrees for ω^q // see equation 4.13
 - 5.5. if q equals Q // see Step 5.1 of F-FCM
set $q = n$
 - 5.6. $id_i^{agg} \leftarrow$ arithmetic mean of all indegrees for node i // see equation 4.14
 - 5.7. $od_i^{agg} \leftarrow$ arithmetic mean of all outdegrees for node i // see equation 4.15
6. $td_i \leftarrow$ add id_i^{agg} to od_i^{agg} for node i // see equation 4.16
7. $\psi = [\psi_i] \leftarrow$ normalize td_i with the sum of all td_i // see equation 4.17

End

from the positive ideal solution, which emphasizes maximizing beneficial criteria and minimizing costs (Hezer et al., 2021; James et al., 2021). While effective in handling multiple alternatives and criteria (Jati, 2012), it struggles with imprecision and uncertainty (Erdogan and Ayyildiz, 2022). To address this, Fermatean Fuzzy TOPSIS (FF-TOPSIS) is applied in this study to evaluate e-waste collection strategies for Türkiye, with its steps summarized below (Erdogan and Ayyildiz, 2022):

Step 1: Experts evaluate each of the m strategies with respect to n criteria using the linguistic terms provided in Table 4.2. These evaluations result in a matrix $\tilde{X}^k = [\tilde{x}_{ij}^k]_{n \times m}$ for each expert k , where $\tilde{x}_{ij}^k$ corresponds a FFN $\tilde{F}(\mu_{\tilde{x}_{ij}^k}, v_{\tilde{x}_{ij}^k})$.

Table 4.2: Linguistic terms for FF-TOPSIS.

Linguistic Term	μ	v
Extreme High (EH)	0.9	0.1
Very High (VH)	0.8	0.1
High (H)	0.7	0.2
Moderate High (MH)	0.6	0.3
Moderate (M)	0.5	0.4
Moderate Low (ML)	0.4	0.5
Low (L)	0.25	0.6
Very Low (VL)	0.1	0.65
Extreme Low (EL)	0.1	0.9

Step 2: Expert opinions are aggregated using the FFWA operator, as defined in equation 4.8, to create the aggregated FF decision matrix $\tilde{X}$, shown in equation 4.18.

$$\tilde{X} = [\tilde{x}_{ij} = \tilde{F}(\mu_{\tilde{x}_{ij}}, v_{\tilde{x}_{ij}})]_{n \times m} \quad (4.18)$$

Step 3: FF Positive Ideal Solution (FF-PIS) and FF Negative Ideal Solution (FF-NIS) are determined using the score function given by equation 4.6.

$$\begin{aligned}
S^+ &= \begin{cases} \max_i \langle S_F(\tilde{x}_{ij}) \rangle \mid j = 1, 2, \dots, m ; \text{for beneficial criterion} \\ \min_i \langle S_F(\tilde{x}_{ij}) \rangle \mid j = 1, 2, \dots, m ; \text{for cost criterion} \end{cases} \\
&= \{(\mu_1^+, v_1^+), (\mu_2^+, v_2^+), \dots, (\mu_n^+, v_n^+)\}
\end{aligned} \quad (4.19)$$

$$\begin{aligned}
S^- &= \begin{cases} \min_i \langle S_F(\tilde{x}_{ij}) \rangle \mid j = 1, 2, \dots, m ; \text{for beneficial criterion} \\ \max_i \langle S_F(\tilde{x}_{ij}) \rangle \mid j = 1, 2, \dots, m ; \text{for cost criterion} \end{cases} \\
&= \{(\mu_1^-, v_1^-), (\mu_2^-, v_2^-), \dots, (\mu_n^-, v_n^-)\}
\end{aligned} \quad (4.20)$$

Step 4: Weighted Manhattan distances from the FF-PIS and FF-NIS are calculated for each strategy ζ_j using equations 4.21 and 4.22, respectively.

$$D^+(\varsigma_j, S^+) = \sum_{i=1}^n \psi_i \left(\frac{1}{2} \left[|(\mu_{ij})^3 - (\mu_i^+)^3| + |(v_{ij})^3 - (v_i^+)^3| + |(\pi_{ij})^3 - (\pi_i^+)^3| \right] \right) \quad (4.21)$$

$$D^-(\varsigma_j, S^-) = \sum_{i=1}^n \psi_i \left(\frac{1}{2} \left[|(\mu_{ij})^3 - (\mu_i^-)^3| + |(v_{ij})^3 - (v_i^-)^3| + |(\pi_{ij})^3 - (\pi_i^-)^3| \right] \right) \quad (4.22)$$

Step 5: Closeness coefficients are calculated.

$$\phi_j = \frac{D^-(\varsigma_j, S^-)}{D^-(\varsigma_j, S^-) + D^+(\varsigma_j, S^+)} \quad (4.23)$$

Step 6: Strategies are ranked based on their closeness coefficients; the highest coefficient indicates the best option.

Algorithm 2 summarizes the key steps of the FF-TOPSIS method.

Algorithm 2: Sequential algorithm of FF-TOPSIS method.

Input:

n : the number of evaluation criteria and m : the number of strategies

E : the number of experts

$\tilde{X}^k = [\tilde{x}_{ij} = \tilde{F}(\mu_{\tilde{x}_{ij}}^k, v_{\tilde{x}_{ij}}^k)], 1 < i < n, 1 < j < m, 1 < k < E$: FF
evaluation matrix obtained from experts

ψ : the vector of criteria weights

Output:

L : List of strategies sorted in terms of preference

Begin

1. $\tilde{X} \leftarrow \text{aggregation}(\tilde{X}^t)$ // see equation 4.8
2. $S^+ \leftarrow \text{positive_ideal_solution}(\tilde{X})$ // see equation 4.19
3. $S^- \leftarrow \text{negative_ideal_solution}(\tilde{X})$ // see equation 4.20
4. $D^+ \leftarrow \text{weighted_distance}(\tilde{X}, S^+, \psi)$ // see equation 4.21
5. $D^- \leftarrow \text{weighted_distance}(\tilde{X}, S^-, \psi)$ // see equation 4.22
6. $\phi \leftarrow \text{closeness_coefficient}(\tilde{X}, D^+, D^-)$ // see equation 4.23
7. $L \leftarrow \text{ranking_strategies}(\phi)$

End

4.2 Applying the Proposed Framework to a Real-World Case

As a leading emerging market in its region and with its increasing urbanization, Türkiye has witnessed a significant increase in the use of electronic devices such as smartphones, laptops, and household appliances. The increase in disposal income in Turkey has paralleled rapid technological advancements globally, resulting in a shorter lifespan for these products and a shorter time for their classification as waste. Consequently, the management of e-waste for Türkiye has become a vital issue, necessitating effective strategies for its collection, recycling, and disposal. The widespread use of electronic devices has aided in technical development and economic expansion; however, it has also raised concerns regarding public health and the environment related to the improper disposal of e-waste. If not adequately controlled, the presence of hazardous compounds such as lead, mercury, and cadmium in electronic equipment poses substantial threats to human health and the environment.

In this context, the development and implementation of efficient waste collection strategies are crucial for mitigating the adverse impacts of e-waste on the environment and public health. Efficient waste collection strategies support resource conservation and the circular economy, in addition to facilitating appropriate disposal and recycling of e-waste. Türkiye can minimize the negative effects of e-waste buildup while maximizing the economic and environmental benefits associated with sustainable waste practices by using structured methods for e-waste management. This section presents a real-world case study evaluating alternative e-waste collection strategies. This case study highlights the need for proactive measures to improve e-waste management in Türkiye.

4.2.1 Criteria Set

Thirteen evaluation criteria were identified, as seen in Figure 4.2, based on a literature review and discussion with experts.

Accessibility (C1) evaluates the ease of access to waste collection services for various demographics and geographic areas. It considers factors such as proximity to collection points, availability of transportation options, and inclusivity for individuals with disabilities or limited mobility.



Figure 4.2: Criteria for selecting e-waste collection strategies based on literature review.

Energy consumption (C3) evaluates the consumed energy throughout the collection process. This includes the energy expended in the transportation, processing, and disposal stages. Strategies aimed at lower energy consumption prioritize fuel-efficient vehicles, optimized routes to minimize travel distances, and use energy-efficient processing technologies. This criterion is crucial for gauging both the environmental sustainability and economic feasibility of waste collection strategies (Mousavi et al., 2022; W. Wu et al., 2023).

Governmental support (C4) gages the alignment of the strategy with national or local policies and laws. This reflects the extent to which relevant administrative bodies endorse and promote the adoption of the collection strategy (T. Adar and Delice, 2019; Luo et al., 2021).

Initial costs (C5) encompass all expenses associated with acquiring the resources, equipment, labor, information, and land necessary to implement the strategy. It provides a comprehensive view of the financial investment required for the implementation of the chosen waste collection technology, facilitating cost-effectiveness analysis and financial planning (Chakraborty and Saha, 2022; Kumar et al., 2022).

Operability (C6) assesses a strategy's user-friendly features, installation simplicity, infrastructure demands, and requisite expertise for personnel. This criterion provides a holistic understanding of how easily the chosen strategy can be integrated into existing systems, emphasizing its practicality and ease of use in the context of waste collection, transportation, and disposal (Govind Kharat et al., 2019; Jiang et al., 2009).

Operation costs (C7) encompass all expenditures incurred during the ongoing operation of the strategy. This includes energy, transportation, maintenance, preliminary preparation, and disposal costs (Kumar et al., 2022; Shanta et al., 2024).

Pollution potential (C8) refers to the strategy's impact on environmental components, including water, air, and soil. This criterion assesses the emissions contributing to global warming while also considering the technology's capacity to create ergonomic discomforts such as noise and odor (E. Adar et al., 2020; Mousavi et al., 2022; Shanta et al., 2024; Zou and Zhang, 2022).

Public acceptance (C9) assesses the level of societal approval for the chosen e-waste collection strategy. It gauges the strategy's compatibility with socio-cultural structures, its potential to create employment, and public awareness of its positive and negative aspects. This criterion delves into a strategy's integration with community values, ensuring a holistic understanding of its societal impact and fostering positive engagement (T. Adar and Delice, 2019; Govind Kharat et al., 2019).

Scalability (C10) assesses the potential of each e-waste collection strategy to adapt and expand over time to accommodate changes in waste generation rates, technological advancements, and population growth.

Staff availability (C11) refers to the evaluation of human resource requirements for operating and maintaining the chosen strategy. This criterion includes the availability of skilled personnel essential for effective utilization (Arıkan et al., 2017; Chakraborty and Saha, 2022).

Timing (C12) encompasses the duration from waste generation to arrival at the processing plant, including the collection, waiting, and transportation periods. Timely waste collection yields benefits, such as reduced stockpile volumes, streamlined processing, and optimized resource allocation within the selected strategy. Streamlining collection processes by minimizing delays enhances operational efficiency throughout the waste management chain (Kumar et al., 2022).

Work safety (C13) ensures the well-being and protection of workers involved in waste collection activities. This includes measures to minimize occupational hazards and mitigate the risks associated with handling e-waste. Strategies prioritizing work safety implement proper training protocols, provide personal protective equipment, and adhere to safety regulations and guidelines (Kheybari et al., 2019; Sagnak et al., 2021).

Five alternative e-waste collection strategies were defined based on a literature review and expert opinion.

Curbside collection (S1) refers to the practice of placing waste, including e-waste, at curbside or designated collection points outside residences or businesses for scheduled pickup by waste management services (de Souza et al., 2016; Nowakowski, 2017).

Door-to-door collection (S2) involves waste management personnel or contractors visiting individual residences or businesses to collect e-waste directly from the premises (S. Singh et al., 2022).

Collection Centers (S3) are designated facilities where individuals can dispose of their e-waste for proper recycling or disposal (Bolat et al., 2021).

Mobile Collection Units (S4) are vehicles equipped with collection bins and processing equipment that travel to different locations, such as community events, schools, and businesses, to collect e-waste directly from the public (Nowakowski et al., 2020).

Retailer Take-Back (S5) involves retailers accepting old or unwanted electronic devices from customers for recycling or disposal when purchasing new devices (de Souza et al., 2016; Dwivedy et al., 2015).

4.2.2 Weight Determination

FCM plays a crucial role in selecting an e-waste collection strategy by modeling the complex relationships between various influencing factors. e-waste collection involves multiple criteria, all interconnected. FCM helps decision makers understand how changes in one criterion affect other aspects of the system. This causal mapping is essential for navigating trade-offs between factors and ensuring comprehensive analysis. FCM helps identify the factors that have the most significant impact on the overall system. In waste management, prioritization is essential for understanding the criteria that should be emphasized when selecting a strategy. Moreover, FCM allows the incorporation of subjective expert opinions and uncertain data by applying fuzzy logic, making it particularly effective for handling the ambiguity often found in waste management problems. FCM also supports consensus-building among stakeholders by integrating diverse perspectives into a shared understanding of the system, further enhancing the decision-making process.

The evaluation criteria discussed in the previous section are considered to mutually influence each other. Expert input was gathered to capture these interrelations, which were incorporated into the process of determining the criteria weights used to evaluate the alternatives. To achieve this, the FCM method was applied in an FF environment. The relationship weights between the criteria, which are key parameters in the F-FCM model, were established through a combination of expert opinions and iterative testing. The weights were further refined through a consensus-building process to ensure accuracy and reliability.

In order to ensure a comprehensive understanding of the complexities involved in e-waste collection strategy selection, a diverse group of experts was selected. The experts were chosen based on their relevant experience and expertise in fields related to waste management, environmental science, decision-making processes, and related technologies. Specifically, the selection criteria included a minimum of five years of professional experience in waste management or a closely related discipline, a demonstrated history of work or academic publications in the field, and a proven track record of engaging in MCDM methodologies. This diverse expertise enabled a more comprehensive view during the evaluation process, ensuring that various perspectives were represented. Nine experts participated in the study and were asked to identify the relationships between the above-described criteria and evaluate the alternatives based on predetermined criteria. The expert profiles are presented in Table 4.3.

Table 4.3: Expert profiles.

Expert	Profile
E1	Environmental Scientist with expertise in waste management policies and sustainable practices.
E2	Researcher in Environmental Science focusing on sustainable waste management practices and their impact on urban ecosystems.
E3	Environmental Engineer with extensive research experience in e-waste management and recycling technologies.
E4	Economist focusing on the economic aspects of waste management and resource allocation.
E5	Experienced in urban planning within a municipality, providing contributions to the decision-making process regarding waste collection methods in sustainable cities.
E6	Sustainability Consultant advising businesses on eco-friendly practices and waste reduction strategies.
E7	Policy Analyst with expertise in environmental policies and their implications for waste management systems.
E8	An academic specialized in decision-making and marketing, with expertise in various quality management projects and hands-on experience in implementing strategies.
E9	Urban Planner with expertise in urban infrastructure and waste collection strategies

The experts were first asked to explain how the criteria for evaluating the e-waste collection strategies affected each other. The experts explained how a change (either an increase or decrease) in one criterion's characteristic affects another criterion using the linguistic terms provided in Table 4.1 and indicated whether this effect was positive or negative. The opinions of the experts are presented in the appendix with Table A.1.

The next step involved aggregating expert opinions to derive the final relationship matrix. Each expert opinion matrix was adjusted to include their FFN equivalents, and these matrices were then combined using equation 4.8. During this process, weights for the relationships between criteria were aggregated under the condition that at least four experts expressed an opinion other than No Impact (N) for the specific relationship. Any relationships with three or fewer opinions were assigned a weight of 0. The resulting aggregated relationship matrix is presented in Table 4.4.

Applying Definition 4.4, the crisp relationships of the FCM shown in Figure 4.3 were then derived from the aggregated FF relationship matrix.

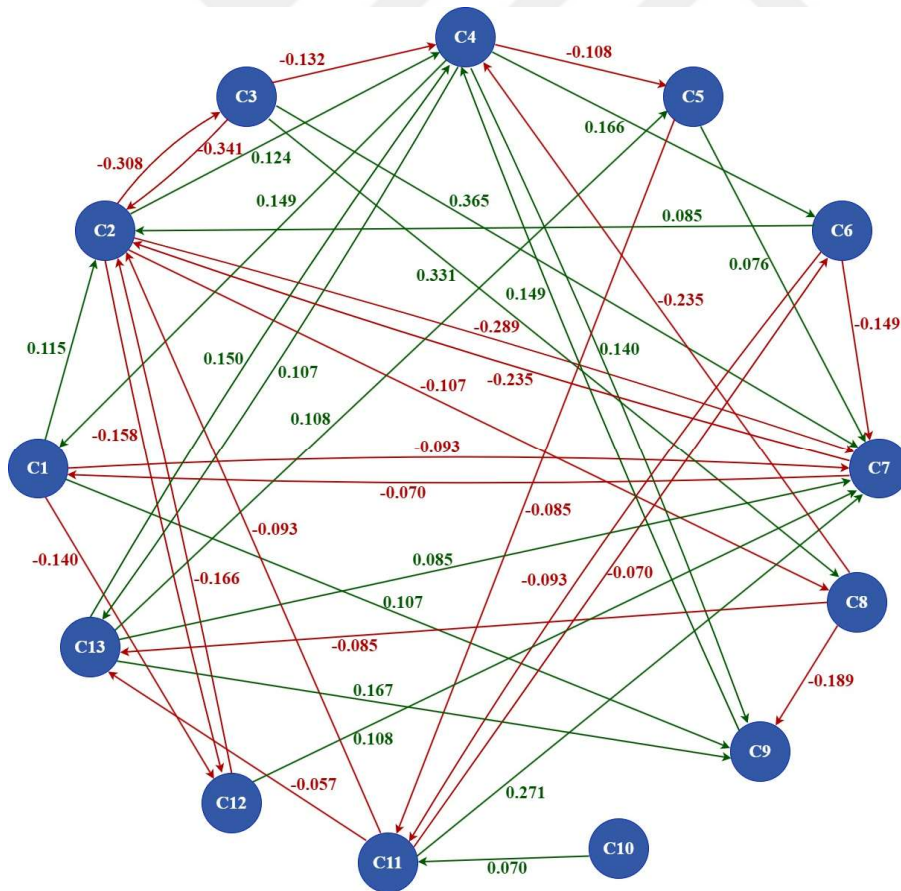


Figure 4.3: FCM illustrating interrelationships between selection criteria with crisp aggregated relationships.

Table 4.4: Aggregated FF relationship matrix.

	C1		C2		C3		C4		C5		C6		C7		C8		C9		C10		C11		C12		C13	
	μ	ν	μ	ν	μ	ν	μ	ν	μ	ν	μ	ν	μ	ν	μ	ν	μ	ν	μ	ν	μ	ν	μ	ν	μ	ν
C1	0.000	1.000	0.311	0.644	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.256	0.689	0.000	1.000	0.300	0.667	0.000	1.000	0.000	1.000	0.356	0.589	0.000	1.000
C2	0.000	1.000	0.000	1.000	0.600	0.367	0.322	0.622	0.000	1.000	0.000	1.000	0.578	0.378	0.300	0.667	0.000	1.000	0.000	1.000	0.000	1.000	0.389	0.556	0.000	1.000
C3	0.000	1.000	0.633	0.333	0.000	1.000	0.333	0.600	0.000	1.000	0.000	1.000	0.656	0.289	0.622	0.322	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000
C4	0.367	0.567	0.000	1.000	0.000	1.000	0.000	1.000	0.289	0.656	0.400	0.533	0.000	1.000	0.000	1.000	0.356	0.589	0.000	1.000	0.000	1.000	0.000	1.000	0.300	0.667
C5	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.233	0.733	0.000	1.000	0.000	1.000	0.000	1.000	0.244	0.711	0.000	1.000	0.000	1.000
C6	0.000	1.000	0.244	0.711	0.000	1.000	0.000	1.000	0.167	0.800	0.000	1.000	0.367	0.567	0.000	1.000	0.000	1.000	0.000	1.000	0.256	0.689	0.000	1.000	0.000	1.000
C7	0.211	0.744	0.511	0.444	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000
C8	0.000	1.000	0.000	1.000	0.000	1.000	0.511	0.444	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.444	0.511	0.000	1.000	0.000	1.000	0.000	1.000	0.244	0.711
C9	0.000	1.000	0.000	1.000	0.000	1.000	0.367	0.567	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000
C10	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.211	0.744	0.000	1.000	0.000	1.000
C11	0.000	1.000	0.256	0.689	0.000	1.000	0.000	1.000	0.000	1.000	0.211	0.744	0.556	0.389	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.178	0.778
C12	0.000	1.000	0.400	0.533	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.289	0.656	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000
C13	0.000	1.000	0.000	1.000	0.000	1.000	0.378	0.578	0.289	0.656	0.000	1.000	0.244	0.711	0.000	1.000	0.400	0.533	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000

The importance of the criteria was then initially analyzed by considering their direct relationships. The indegree (*id*) and outdegree (*od*) values for each criterion were calculated using equations 4.10 and 4.11, and the results are shown in Table 4.5.

Table 4.5: Degree of importance based on direct relationships.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
<i>id</i>	0.2189	1.0345	0.3079	0.7883	0.2152	0.2370	1.4345	0.4379	0.6031	0.0000	0.2474	0.2986	0.2484
<i>od</i>	0.4551	0.9854	1.1683	0.6700	0.1609	0.3256	0.3053	0.5086	0.1485	0.0703	0.4907	0.2742	0.5085

When considering the indicators of direct interaction among the criteria, it is observed that the cumulative effect is most prominent toward the “Operation costs (C7)” criterion, with a *id* value of 1.4345. This shows that even slight changes in other criteria can significantly alter “Operation costs.” On the other hand, in terms of directly influencing other criteria, the “Energy consumption (C3)” criterion is evaluated as effective with a *od* value of 1.1683, followed by the “Efficiency (C2)” criterion with a *od* value of 0.9854. In addition to these direct effects, secondary or higher-order interaction forces should also be analyzed.

Further calculations were performed to determine the overall importance of the criteria by considering indirect relationships. The relationship matrix was raised to different powers, and *id* and *od* values were calculated for each resulting matrix. The criteria in each matrix were then ranked based on their *id* and *od* values. For powers five and higher, the rankings remained unchanged according to both *id* and *od* values. Therefore, the assessment of criterion importance considered indirect relationships up to power five. Accordingly, the normalized *id* and *od* values are presented in Table 4.6.

Table 4.6: Normalized degrees based on indirect relationships.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
<i>Nid</i>_i²	0.5184	1.3312	0.7575	0.9462	0.2653	0.3538	1.4345	0.5054	0.6141	0.0000	0.0954	0.4624	0.3220
<i>Nid</i>_i³	0.3915	1.2494	0.6647	0.8468	0.2213	0.2666	1.4345	0.6373	0.5472	0.0000	0.0894	0.4596	0.2421
<i>Nid</i>_i⁴	0.4114	1.2992	0.6982	0.9238	0.2126	0.2675	1.4345	0.6417	0.5837	0.0000	0.0787	0.4586	0.2713
<i>Nid</i>_i⁵	0.4176	1.2778	0.7017	0.9303	0.2256	0.2797	1.4345	0.6490	0.5968	0.0000	0.0750	0.4619	0.2762
<i>Nod</i>_i²	0.3258	1.0435	1.1683	0.3560	0.1076	0.2890	0.4376	0.3794	0.1653	0.0573	0.3749	0.3272	0.2793
<i>Nod</i>_i³	0.3812	1.0575	1.1683	0.2739	0.1106	0.3192	0.4555	0.2353	0.0898	0.0448	0.4271	0.3754	0.2199
<i>Nod</i>_i⁴	0.4129	1.1122	1.1683	0.2879	0.1293	0.3586	0.5021	0.1823	0.0742	0.0548	0.4675	0.4109	0.1941
<i>Nod</i>_i⁵	0.4251	1.1054	1.1683	0.2940	0.1377	0.3748	0.5136	0.1735	0.0757	0.0582	0.4870	0.4235	0.1979

The degree values for each criterion were aggregated using equations 4.14, 4.15, and 4.16, as shown in Table 4.7.

Table 4.7: Final degrees based on direct and indirect relationships.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
td_i	0.3044	0.8843	0.6901	0.4859	0.1374	0.2363	0.7220	0.3346	0.2691	0.0220	0.2179	0.3040	0.2123

These values were then normalized based on their sums, leading to the determination of the criteria weights, as seen in Figure 4.4.

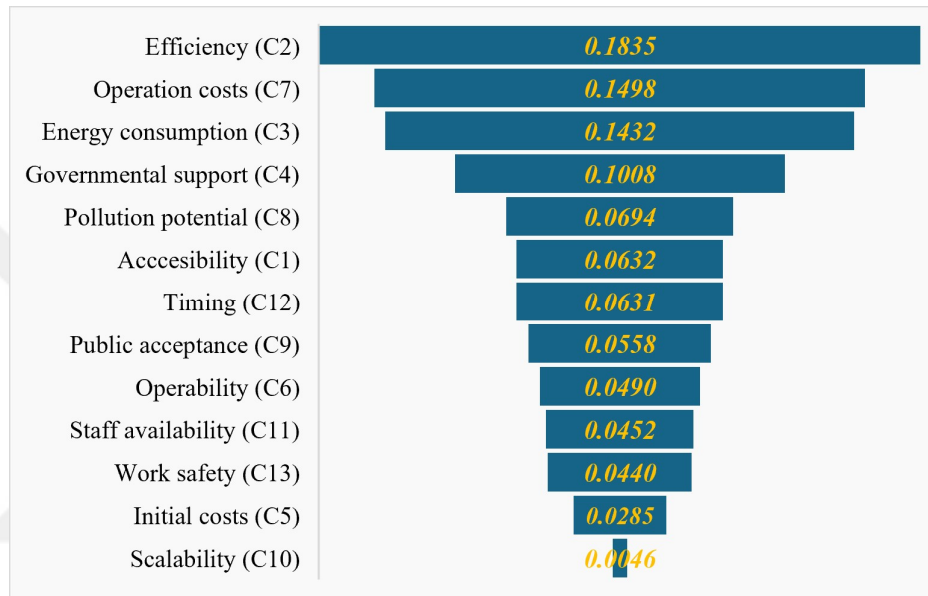


Figure 4.4: Weights of evaluation criteria for selecting e-waste collection alternatives.

The weighting of the criteria plays a crucial role in evaluating the factors that influence strategy selection. The analysis revealed that “Efficiency” holds the highest importance, with a weight of 0.184, emphasizing its role in increasing productivity and operational efficiency in e-waste collection efforts (Petrović et al., 2016). Efficiency of time and money spent is an important factor in evaluating waste collection systems (Aliahmadi et al., 2020, 2021; Topaloglu et al., 2018). Following closely is “Operation costs” with a weight of 0.150, indicating the significance of financial considerations for sustainability. The collection and transportation of waste form a large part of the reverse supply chain in each waste system. The costs of running these systems must be effectively reduced, and sustainability must be ensured (Jovanovic et al., 2016). These costs also include the maintenance costs of the system (Arıkan et al., 2017). “Energy consumption” ranked third at 0.143, underscoring the critical role of energy-efficient practices in minimizing environmental impact and optimizing resource use.

Waste collection systems often seek to minimize environmental harm while consuming the least amount of energy possible to benefit society as much as possible (Seker, 2022). Although “Scalability” and “Initial costs” received lower weights (0.005 and 0.029), they still require attention due to their long-term viability and scalability potential contribution to the decision-making process. Factors such as “Accessibility”, “Governmental support”, “Public acceptance”, as well as “Timing”, “Staff availability”, and “Work safety” are relatively less influential but offer valuable insights into the broader context of e-waste collection strategies, in line with the multifaceted nature of decision-making processes and the need for a comprehensive consideration of different criteria.

The final criterion rankings above are similar to the results of the importance analysis based on direct relationships. When considering indirect relationships that account for the long-term consequences of criterion interrelationships, the criterion rankings remain the same although the weights differ. This weight adjustment is made possible by accounting for indirect relationships.

4.2.3 Alternative Evaluation

In this section, we present an evaluation of alternative e-waste collection strategies. The evaluated strategies include Curbside Collection (S1), Door-to-door Collection (S2), Collection Centers (S3), Mobile Collection Units (S4), and Retailer Take-Back (S5). Each strategy represents a distinct approach to e-waste collection, ranging from scheduled pickups to drop-off facilities and mobile collection units. To assess the effectiveness of these strategies, a comprehensive analysis based on FF-TOPSIS was conducted by considering 13 criteria relevant to e-waste management. The evaluation involved input from nine experts who used the linguistic terms provided in Table 4.2 to evaluate the alternatives. Table 4.8 displays alternative-criteria- evaluations of the experts. It is important to note that the row headings in Table 4.8 represent the alternatives, while the column headings correspond to the criteria.

The parameters of the FF-TOPSIS method, boundary values of linguistic terms in Table 4.2, were set based on established guidelines in the literature to ensure accurate decision-making. Specifically, we considered the membership and non-membership degrees for each criterion, allowing for greater flexibility and precision in capturing

uncertainty and vagueness. To enhance performance, these parameters were carefully calibrated using expert opinions and linguistic terms.

Table 4.8: Alternative evaluation matrices for experts.

		C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
Expert-1	S1	M	VH	L	MH	MH	H	M	ML	MH	H	M	ML	H
	S2	VH	MH	ML	MH	M	H	MH	M	H	MH	VH	M	ML
	S3	L	H	L	H	H	H	M	L	M	H	L	L	H
	S4	H	MH	H	M	ML	MH	MH	M	H	M	MH	MH	ML
	S5	VH	M	VH	M	ML	M	H	MH	VH	L	H	VH	L
Expert-2	S1	L	L	MH	M	VL	M	ML	H	L	L	L	H	ML
	S2	EH	MH	H	M	H	MH	MH	VH	H	VH	VL	MH	L
	S3	MH	VH	ML	H	VH	EH	L	ML	MH	MH	H	M	MH
	S4	VH	H	VH	MH	EH	H	VH	VH	H	H	VH	ML	M
	S5	EH	ML	VH	ML	M	ML	VH	VH	EH	ML	VH	L	L
Expert-3	S1	ML	VH	VL	M	L	H	L	L	EH	EH	L	VL	H
	S2	VH	H	VH	M	VH	VH	VH	M	VH	VH	VH	H	M
	S3	VL	MH	L	M	MH	ML	L	VL	ML	ML	L	EL	H
	S4	H	H	H	M	MH	H	MH	M	H	H	H	M	M
	S5	EH	L	EH	ML	L	H	EH	VH	EH	L	EH	VH	L
Expert-4	S1	M	H	ML	M	H	MH	M	M	MH	H	M	ML	ML
	S2	H	M	H	ML	ML	MH	MH	H	M	H	H	MH	ML
	S3	MH	H	MH	M	MH	M	L	MH	MH	M	H	MH	MH
	S4	MH	M	H	M	ML	M	M	ML	H	M	M	MH	H
	S5	M	ML	H	M	ML	MH	H	M	H	M	MH	M	M
Expert-5	S1	H	H	MH	H	H	H	H	L	H	H	VH	M	M
	S2	EH	VH	EL	H	L	H	MH	L	MH	H	VH	H	L
	S3	H	H	ML	H	MH	H	H	L	MH	MH	MH	VH	ML
	S4	MH	MH	H	H	H	H	H	L	MH	M	MH	VH	MH
	S5	H	EH	EH	H	VH	VH	EH	L	MH	H	EH	H	H
Expert-6	S1	H	H	H	L	H	MH	H	MH	MH	H	H	VH	VH
	S2	VH	H	VH	ML	M	H	M	ML	VH	M	VH	VH	H
	S3	ML	L	L	H	L	H	M	H	ML	ML	L	L	H
	S4	EH	H	VL	ML	ML	MH	ML	ML	VH	VH	L	VL	ML
	S5	EH	VH	VL	L	VL	ML	EL	H	VH	H	VL	L	VH
Expert-7	S1	M	VH	M	H	L	ML	VL	L	H	H	L	H	ML
	S2	VH	ML	H	MH	H	H	MH	H	MH	L	L	ML	H
	S3	ML	H	ML	H	H	VH	ML	M	ML	MH	M	VH	MH
	S4	L	H	MH	M	M	MH	L	M	M	H	MH	H	MH
	S5	H	ML	H	M	H	M	MH	H	MH	L	L	ML	H
Expert-8	S1	M	H	L	H	VL	H	L	VL	M	L	L	M	H
	S2	L	L	M	M	L	H	M	L	L	H	L	H	M
	S3	EL	VL	L	M	H	L	H	L	L	L	M	H	M
	S4	H	M	VH	H	L	L	VH	M	VH	M	H	M	M
	S5	EH	M	EH	M	M	M	EH	MH	EH	L	VH	M	M
Expert-9	S1	H	VH	L	H	H	VH	VH	ML	MH	H	EH	ML	VH
	S2	EH	H	VL	M	VL	H	M	H	VH	H	EH	VH	VL
	S3	ML	H	H	VH	VH	VH	VH	MH	M	MH	L	H	H
	S4	ML	MH	VH	H	H	M	H	MH	M	ML	ML	MH	H
	S5	L	MH	VH	H	MH	MH	H	M	M	H	MH	EH	H

Next, the assessment matrices were converted into FFNs using the scale presented in Table 4.4. Subsequently, the optimal solutions for each criterion were identified, as shown in Table 4.9, which presents the FF-PIS and FF-NIS. Criteria C3, C5, C7, C8, C11, and C12 were categorized as cost criteria, whereas the remaining criteria were categorized as benefit criteria.

Table 4.9: Positive and negative ideal solutions for each criterion.

		C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
FF-PIS	μ	0.761	0.694	0.389	0.644	0.456	0.689	0.467	0.383	0.744	0.639	0.444	0.500	0.611
	ν	0.167	0.200	0.489	0.256	0.422	0.211	0.417	0.494	0.189	0.256	0.433	0.394	0.289
FF-NIS	μ	0.394	0.528	0.733	0.494	0.639	0.556	0.700	0.606	0.472	0.444	0.628	0.644	0.422
	ν	0.506	0.378	0.194	0.400	0.256	0.344	0.244	0.289	0.422	0.433	0.283	0.256	0.461

In the context of selecting an e-waste collection strategy, the distances between each alternative and ideal solutions were computed using the criteria weights derived from the F-FCM. The positive and negative distances between each alternative and ideal solutions are presented in Table 4.10.

Table 4.10: Distances from ideal solutions.

Alternative	$D(S_j, S^+)$	$D(S_j, S^-)$
S1: Curbside Collection	0.044	0.802
S2: Door-to-door Collection	0.103	0.816
S3: Collection Centers	0.079	0.804
S4: Mobile Collection units	0.115	0.776
S5: Retailer Take-back	0.172	0.500

The scores of the alternatives were determined using equation 4.23, as depicted in Figure 4.5. Subsequently, the outcomes of the final score computation are used to arrange the alternative strategies in order of preference, ultimately identifying Curbside Collection (S1) as the optimal e-waste collection strategy. Thus, drawing from the specified criteria and expert insights, it is inferred that Curbside Collection (S1) presents the most advantageous circumstances for the advancement of e-waste collection efforts.

Curbside Collection emerged as the preferred strategy among experts, garnering recognition as the optimal alternative for five of the 13 criteria evaluated. Notably, experts highlighted Curbside Collection as particularly effective in terms of efficiency (C2), emphasizing its ability to streamline waste collection processes and maximize resource utilization. Furthermore, its favorable ranking in initial cost (C5) highlights

its cost-effectiveness and feasibility in implementation, which indicates its potential economic advantages.

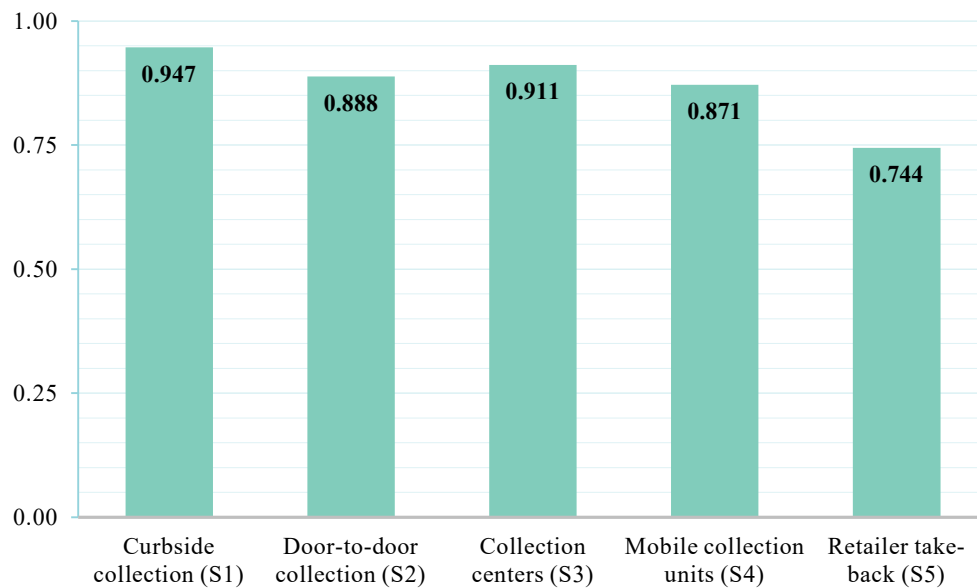


Figure 4.5: Final scores of the e-waste collection strategies.

In addition, Curbside Collection's commendable performance in operation cost (C7) underscores its potential for long-term sustainability and financial efficiency in waste management operations. Furthermore, experts have suggested that Curbside Collection holds promise in reducing the environmental impacts associated with waste disposal practices, noting its low pollution potential (C8). Finally, Curbside Collection's recognized effectiveness in reducing time (C12) requirements underlines its capacity to speed up waste collection processes and increase overall operational efficiency within the chosen strategy.

4.2.4 Sensitivity Analysis

In this section, the sensitivity of the proposed weighting approach is examined under various weight calculation schemes. The methodology is based on the assumption that criteria are interrelated rather than independent, thus incorporating both direct and indirect causal relationships. To evaluate the impact of this modeling assumption, multiple weight sets were generated by systematically varying the order of indirect relationships considered in the computation.

Table 4.11 presents the resulting weights for each criterion under five different configurations, each reflecting a specific combination of direct and indirect relationships.

Table 4.11: Sensitivity of criteria weights to the order of indirect relationships included in the computation.

Relationship Type	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
Only direct	0.0555	0.1663	0.1216	0.1201	0.0310	0.0463	0.1433	0.0779	0.0619	0.0058	0.0608	0.0472	0.0623
Direct and 2nd-Order Indirect	0.0606	0.1754	0.1358	0.1102	0.0299	0.0481	0.1441	0.0731	0.0611	0.0051	0.0482	0.0544	0.0542
Direct and 2nd & 3rd-Order Indirect	0.0615	0.1798	0.1405	0.1041	0.0290	0.0481	0.1476	0.0726	0.0582	0.0046	0.0463	0.0590	0.0488
Direct and 2nd to 4th-Order Indirect	0.0624	0.1826	0.1423	0.1021	0.0285	0.0484	0.1490	0.0707	0.0566	0.0046	0.0455	0.0615	0.0458
Direct and 2nd to 5th-Order Indirect*	0.0632	0.1835	0.1432	0.1008	0.0285	0.049	0.1498	0.0694	0.0558	0.0046	0.0452	0.0631	0.0440

* indicates the reference scheme used in the proposed methodology

The ranking results based on these weight sets reveal that the relative importance of several criteria, particularly C1, C6, C9, C11, C12, and C13, varied depending on the order of indirect relationships included. In contrast, the remaining criteria maintained stable rankings across all configurations. Among all scenarios, Efficiency (C2) consistently appeared as the most influential criterion, underlining its importance in e-waste collection strategy selection. Conversely, Scalability (C10) was consistently identified as the least influential criterion.

To assess how these changes in weights affected final decision outcomes, the FF-TOPSIS method was applied to each configuration. The resulting scores of the alternatives are presented in Table 4.12.

Table 4.12: Alternative scores based on different weighting schemes.

Relationship Type	S1	S2	S3	S4	S5
Only direct	0.9467	0.8858	0.9143	0.8743	0.7498
Direct and 2nd-Order Indirect	0.9463	0.8869	0.9113	0.8718	0.7471
Direct and 2nd- & 3rd-Order Indirect	0.9473	0.8873	0.9114	0.8712	0.7452
Direct and 2nd- to 4th-Order Indirect	0.9475	0.8875	0.9112	0.8709	0.7444
Direct and 2nd- to 5th-Order Indirect*	0.9475	0.8877	0.9110	0.871	0.7441

* indicates the reference scheme used in the proposed methodology

While the computed weights noticeably influenced the importance ranking of some criteria, no change in the final ranking of alternatives was observed across all schemes. This outcome suggests that, in this specific case, the ranking stability is more closely associated with the expert evaluations used in FF-TOPSIS rather than with variations

in criteria weights. To further investigate whether this stability was coincidental, a second sensitivity analysis was conducted using five randomly generated weight sets, as shown in Table 4.13.

Table 4.13: Randomly generated weight sets for sensitivity analysis.

Weight set	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
Set 1	0.0408	0.0919	0.0720	0.0444	0.0841	0.0998	0.1112	0.1086	0.0960	0.1013	0.0604	0.1117	0.0973
Set 2	0.0706	0.0822	0.0477	0.0429	0.0793	0.0885	0.0414	0.0634	0.1048	0.0563	0.0720	0.0704	0.0676
Set 3	0.0391	0.0989	0.0870	0.0804	0.0720	0.1044	0.1127	0.0883	0.0850	0.0573	0.0519	0.0884	0.0856
Set 4	0.0517	0.1107	0.1083	0.0530	0.1042	0.0555	0.0598	0.1041	0.0426	0.0683	0.0780	0.0901	0.0394
Set 5	0.1048	0.0494	0.0743	0.0595	0.0452	0.0677	0.0858	0.0670	0.0767	0.1115	0.1094	0.1033	0.0977

FF-TOPSIS scores were recalculated using each of these weight sets. The resulting values are visualized in Figure 4.6.

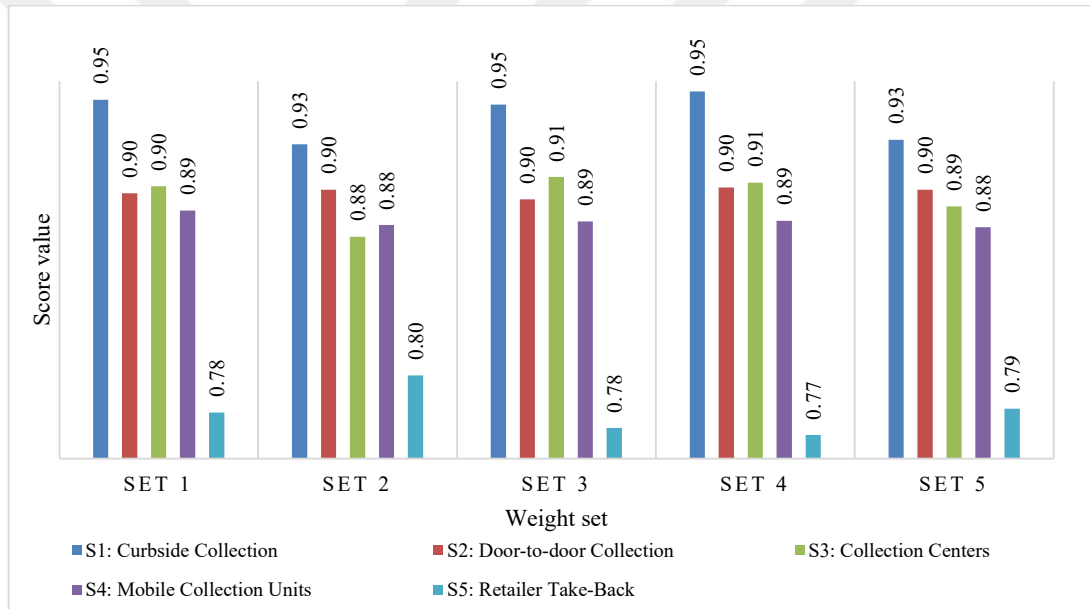


Figure 4.6: Results of the sensitivity analysis.

As shown in Figure 4.6, the positions of alternatives remained largely consistent across different random weight sets. Only the rankings of the strategies Door-to-Door Collection (S2) and Collection Centers (S3) vary relative to each other, and for the weight Set 2 and Set 5, Door-to-Door Collection emerges as the more preferable strategy. The findings for this specific decision problem confirm that the expert-based performance evaluations in FF-TOPSIS had a stronger influence on the final rankings than the relative variation in criteria weights. The following section examines the robustness and consistency of the proposed approach by conducting comparative

evaluations with Fermatean Fuzzy Vlse Kriterijumsa Optimizacija I Kompromisno Resenje (FF-VIKOR) across multiple alternative scenarios.

4.2.5 Comparative Analysis

To gain deeper insights into the robustness and consistency of the evaluation, the e-waste collection strategies were ranked using both FF-TOPSIS and FF-VIKOR (see (Gül, 2021) for details). Although both approaches employ FFSs to handle uncertainty, they differ in their ranking philosophy: FF-TOPSIS emphasizes proximity to the ideal solution, while FF-VIKOR seeks a compromise solution by considering the balance between the closest and farthest alternatives.

Given the strategic importance of cost-effectiveness in e-waste management, particularly in systems that integrate urban mining principles, which involve high treatment costs and require consistent input flows, this analysis compares three distinct weighting scenarios. The baseline scenario reflects the original weight set derived from expert evaluation and the established relationship matrix. The cost-oriented scenario acknowledges the economic pressures in the sector by assigning cost-related criteria twice the importance of benefit-related ones, with equal weights within each group. Conversely, the benefit-oriented scenario prioritizes benefit-related criteria, assigning them double the weight of cost-related ones while maintaining internal balance. The weight distributions for each scenario are presented in Table 4.14. This scenario-based approach underlines the importance of the collection method, which plays a key role in ensuring system efficiency and supporting sustainability across the e-waste management lifecycle.

Table 4.14: Criteria weights according to scenarios.

Weight Set	C1	C2	C3*	C4	C5*	C6	C7*	C8*	C9	C10	C11*	C12*	C13
Baseline	0.0632	0.1835	<i>0.1432</i>	0.1008	<i>0.0285</i>	0.049	<i>0.1498</i>	<i>0.0694</i>	0.0558	0.0046	<i>0.0452</i>	<i>0.0631</i>	0.0440
Cost-oriented	0.0526	0.0526	<i>0.1053</i>	0.0526	<i>0.1053</i>	0.0526	<i>0.1053</i>	<i>0.1053</i>	0.0526	0.0526	<i>0.1053</i>	<i>0.1053</i>	0.0526
Benefit-oriented	0.1000	0.1000	<i>0.0500</i>	0.1000	<i>0.0500</i>	0.1000	<i>0.0500</i>	<i>0.0500</i>	0.1000	0.1000	<i>0.0500</i>	<i>0.0500</i>	0.1000

* criteria in italics show cost criteria, others benefit

A summary of the comparative results is presented in Table 4.15.

Both FF-TOPSIS and FF-VIKOR identified Curbside Collection (S1) as the best-performing strategy under baseline and cost-oriented scenarios, confirming its strength in terms of cost-effectiveness across different weighting schemes. However, in the

benefit-oriented scenario, FF-VIKOR shifted its top ranking to Door-to-Door Collection (S2), reflecting its sensitivity to benefit-related criteria such as accessibility and operability. Nevertheless, S1 remained highly competitive, ranking second in this case. In contrast, the FF-TOPSIS rankings demonstrated greater stability, with Curbside Collection consistently emerging as the highest-ranked strategy in every scenario.

Table 4.15: Results of comparative analysis.

Scenario Strategy	FF-TOPSIS						FF-VIKOR					
	Baseline		Cost-oriented		Benefit-Oriented		Baseline		Cost-oriented		Benefit-Oriented	
	Score	Rank	Score	Rank	Score	Rank	Score	Rank	Score	Rank	Score	Rank
S1: Curbside Collection	0.948	1	0.953	1	0.918	1	0.000	1	0.000	1	0.006	2
S2: Door-to-door Collection	0.888	3	0.901	3	0.904	2	0.765	4	0.134	2	0.002	1
S3: Collection Centers	0.911	2	0.915	2	0.876	4	0.681	3	0.663	4	0.669	4
S4: Mobile Collection Units	0.871	4	0.886	4	0.881	3	0.548	2	0.336	3	0.291	3
S5: Retailer Take-Back	0.744	5	0.776	5	0.794	5	1.000	5	1.000	5	1.000	5

Some divergence is also observed in the mid-tier rankings. While Door-to-door Collection (S2), Collection Centers (S3), and Mobile Collection Units (S4) maintained consistent rankings in FF-TOPSIS under the baseline and cost-oriented scenarios, FF-VIKOR yield different rankings. This reflects FF-VIKOR's tendency to favor compromise solutions, possibly valuing operational flexibility more heavily.

This stability makes FF-TOPSIS particularly suitable for decision-making contexts where consistency and cost-effectiveness are key priorities, and where gradual shifts in evaluation criteria are expected. Additionally, both methods consistently ranked Retailer Take-Back (S5) as the least favorable option, across all scenarios, reinforcing confidence in FF-TOPSIS's ability to reliably exclude underperforming strategies.

FF-TOPSIS is validated by its alignment with FF-VIKOR in critical outcomes but stands out for its resilience across scenarios. It is preferable in contexts where greater robustness to changes in weighting schemes and cost-effectiveness are valued, and the objective is to identify the best-performing option rather than compromise solutions.

4.3 Results and Discussion

As e-waste generation continues to grow at an alarming rate, identifying effective collection strategies is essential for mitigating environmental impact and promoting sustainability. This study evaluated multiple e-waste collection strategies based on 13 critical criteria, which were weighted using F-FCM to capture the complexity and uncertainty inherent in the decision-making process. By applying FF-TOPSIS, the study ranked the strategies, providing practical insights into their relative effectiveness. This section discusses the key findings from the rankings, compares them with existing literature, and highlights their significance for stakeholders involved in e-waste management.

The results demonstrate that efficiency, with a weight of 0.1835, is the most influential factor, which highlights its critical role in ensuring that e-waste collection strategies are optimized for performance. Operational costs and energy consumption closely follow, with weights of 0.1498 and 0.1432, respectively, underlining the importance of both financial sustainability and minimizing environmental impact for effective waste management. Governmental support also emerged as an essential criterion, with a weight of 0.1008, underscoring the necessity of regulatory frameworks and incentives in promoting effective e-waste management practices. In contrast, criteria such as scalability and initial costs have notably lower weights, at 0.0046 and 0.0285, implying that although these aspects are not irrelevant, they are not the primary drivers of the decision-making process.

The ranking derived from FF-TOPSIS offers further insights into the strengths and weaknesses of each strategy. Curbside collection emerged as the top-ranked strategy, with a score of 0.9463, demonstrating significant efficiency, operational costs, and public acceptance. This approach benefits from established infrastructure, streamlining logistics and encouraging high participation rates. However, it may require strong government support and public awareness, particularly in areas with low recycling participation. Following closely, collection centers ranked second with a score of 0.9108, indicating their strong performance in energy consumption and operational costs, providing a dedicated drop-off point that enhances convenience. Nevertheless, accessibility remains a challenge, as not all residents may find these centers easily reachable. Door-to-door collection, while convenient and supportive of

governmental initiatives, ranked third with a score of 0.8895, but it faces concerns regarding higher operational costs and energy consumption. Mobile collection units ranked fourth at 0.8709, effectively addressing accessibility issues in hard-to-reach areas, yet they entail significant resource demands for maintenance and scheduling. Lastly, retailer take-back programs, with the lowest score of 0.7837, can promote responsible consumer behavior but struggle with limited scalability and varying levels of public acceptance. Overall, curbside collection and collection centers appear to be the most promising options; however, the strengths and weaknesses of each strategy emphasize the need for a tailored approach to e-waste collection that considers community-specific needs and resources.

Policymakers and stakeholders can leverage these insights to enhance existing e-waste management frameworks, promote public engagement, and improve infrastructure. Additionally, the study highlights the necessity for further research into innovative approaches and technologies that can optimize e-waste collection, ultimately contributing to more sustainable practices in the management of e-waste.

4.4 Implications and Concluding Remarks

Efficiency plays a central role in determining the most suitable strategy for e-waste collection (Nowakowski et al., 2020). An efficient system maximizes the amount of e-waste collected, keeping harmful materials out of landfills and minimizing improper disposal practices. This, in turn, reduces environmental damage caused by toxic substances leaching into soil and water (Ikhlayel, 2017). Moreover, proper collection enables the recovery and reuse of valuable resources, such as precious metals and rare earth elements, decreasing the need for raw material extraction and mitigating the environmental degradation associated with mining activities (Z. Li et al., 2019).

From a policy standpoint, emphasizing efficiency can shape regulations that support environmentally responsible practices, such as Extended Producer Responsibility (EPR) programs (Atasu and Subramanian, 2012) and improved labeling on electronic devices to inform consumers about appropriate disposal methods. At the management level, operational efficiency translates to cost savings and better performance for waste handlers and recyclers. Lean management principles, innovative technologies, and data-driven route optimization can reduce waste, energy use, and costs (Goyal et al., 2019). Efficient logistics and transportation systems, including electric or hybrid

vehicles, further reduce emissions and reliance on fossil fuels. Additionally, establishing nearby recycling facilities and strategically locating drop-off centers can significantly minimize transportation distances and energy consumption(Yong et al., 2019).

Public engagement and awareness are equally critical. Informing consumers about the environmental and health risks associated with e-waste can promote responsible recycling. Through partnerships among government agencies, manufacturers, retailers, and recyclers, a comprehensive, cost-effective, and scalable e-waste management system can be developed.

Among the evaluated alternatives, Curbside Collection emerges as the most appropriate strategy. Its accessibility eliminates common participation barriers by integrating directly into residents' daily routines, encouraging widespread compliance and increasing the volume of properly disposed e-waste. By leveraging existing municipal waste infrastructure, curbside collection optimizes route planning and reduces operational costs and environmental impact. From a policy perspective, it aligns with sustainability goals and offers a practical solution for urban implementation.

The main objective of the study in this chapter was to identify effective e-waste collection strategies by evaluating alternatives through an FF-based decision support system. The proposed approach combines F-FCM for determining interdependent criterion weights and FF-TOPSIS for ranking alternatives. Criteria such as accessibility, efficiency, operating cost, energy consumption, and pollution potential were considered, and expert opinion was used to construct the FCM and evaluate the alternatives. This methodology provides a holistic view of the decision problem, reflecting real-world interdependencies among criteria.

When conducting such an evaluation, determining criterion weights in a way that reflects not only their individual importance but also their interdependence is essential for producing a realistic assessment. This was achieved using FCMs that enable the modeling of both direct and indirect relationships among criteria. The ability of FCMs to represent negative relationships is particularly valuable in this context, as it allows for a more nuanced and accurate representation of real-world dynamics. While other methods such as DEMATEL and Analytic Network Process (ANP) also aim to capture

interdependencies, often by raising the direct-relation matrix to a power to account for indirect effects, these approaches are not well-suited to handle negative causal relationships (Y. Sun et al., 2021; Tzeng and Huang, 2011). Most rely on non-negative matrices and are designed under the assumption that all influences are either neutral or positive. In contrast, FCMs inherently support negative causal values, making them more suitable for evaluating complex systems like e-waste management, where criteria may have both reinforcing and inhibiting effects on one another.

The findings highlight the importance of prioritizing both operational efficiency and environmental sustainability. Efficient systems ensure timely collection, minimize resource waste, and enhance processing facility performance. Meanwhile, energy consumption and pollution potential are critical for evaluating the ecological impact of each method. Curbside Collection outperformed other strategies across multiple dimensions, particularly in terms of efficiency, cost-effectiveness, and environmental performance, as validated by expert assessments.

However, successful implementation requires overcoming barriers such as infrastructure costs, which may be mitigated through public funding or subsidies. Public-private partnerships can improve coordination, optimize resources, and foster shared responsibility. Public engagement campaigns, educational initiatives, and transparency in waste practices are also essential to sustaining participation and support for recycling programs.

Despite its contributions, this study has certain limitations. The use of expert evaluations, while valuable, introduces an element of subjectivity. Broader expert participation in future studies could help reduce this bias. Moreover, due to data availability constraints, some relevant criteria may have been excluded from the model, potentially affecting its comprehensiveness. Addressing these limitations in future research can further enhance the robustness of e-waste management assessments.

Future studies could also explore additional fuzzy set extensions, such as hesitant, picture, or circular intuitionistic fuzzy sets, or incorporate alternative uncertainty modeling techniques like evidence theory or rough set theory. These approaches may offer improved representation of ambiguity in complex decision-making. Furthermore,

integrating hybrid methods, such as group decision-making or machine learning-enhanced fuzzy systems, could further enrich the proposed methodology.

By providing a flexible, context-sensitive decision-making framework, this study offers valuable guidance to policymakers, practitioners, and stakeholders aiming to develop more sustainable, efficient, and adaptive e-waste collection systems.



5. A NEW REASONING MECHANISM: APPLICATION OF FCM TO FORESTRY SUPPLY CHAIN VULNERABILITY ASSESSMENT

This chapter presents the foundations of a novel reasoning mechanism for FCMs, expanding on the work introduced in the previous chapter. It aims to demonstrate the advantages of this new approach over classical reasoning methods, particularly in overcoming key limitations associated with traditional FCM reasoning mechanism. Emphasis is placed on how the proposed mechanism enhances the adaptability and effectiveness of FCMs in complex decision-making contexts, with a particular focus on dynamic analysis and decision making-oriented applications.

The classical FCM reasoning mechanism often relies on nonlinear activation functions, such as the sigmoid and hyperbolic tangent function, which are used to simulate the interdependence between concepts over time. While these functions can be effective in capturing certain dynamics, they also exhibit significant limitations. One major issue is insensitivity to initial states, meaning that, regardless of different starting conditions, the model often converges to the same state. Additionally, threshold function sensitivity poses a problem, as different threshold functions produce varying state values, complicating the consistency of predictions.

The reasoning mechanism introduced in this chapter proposes an alternative that aims to overcome these challenges. Instead of relying on nonlinear activation functions, the approach is based on taking the power of the relationship matrix, effectively generating a reachability matrix that captures both direct and indirect relationships among concepts. This new method allows for the creation of a total effect matrix that better represents the influence of dynamics within the system. By multiplying the initial vector with this total effect matrix and applying normalization, it becomes possible to simulate different system behaviors from a variety of starting points, thereby offering a more flexible and dynamic approach to prediction.

The advantages of the new mechanism are numerous. First, it provides greater sensitivity to initial states, allowing the model to yield different outcomes depending on the starting state, which is particularly beneficial in prediction problems. Second,

by incorporating both forward and backward analysis, the new method enables decision-makers to not only predict future states but also determine the necessary initial states to reach a desired target state. This capability significantly enhances strategic planning opportunities, as it provides a way to model how interventions can influence future outcomes in a more precise manner.

The new reasoning mechanism is particularly useful in MCDM contexts, where interdependence between different elements needs to be considered to prioritize vulnerabilities or assess potential strategies. For instance, in the study titled “Evaluating the Vulnerability of Forestry Supply Chains through Fuzzy Cognitive Map,” (Murat et al., 2025b), the proposed reasoning mechanism is used to assess vulnerabilities in forest supply chains. This study emphasizes the interdependency between vulnerabilities and utilizes the new reasoning model to prioritize these vulnerabilities under different scenarios. Unlike previous approaches that primarily used the sigmoid function, this study showcases the potential of the new mechanism to adapt to different initial states, although it does not make a direct comparison with the sigmoid-based model.

The new mechanism builds upon previous research that explored alternatives involving the reachability matrix. The method presented here allows for different results to emerge in different scenarios, offering a more adaptable and nuanced approach to modeling complex systems. By capturing the cumulative effects of both direct and indirect relationships, the new mechanism enables the modeling of diverse behaviors and responses, which is critical for systems characterized by multiple, interrelating factors.

This chapter also explores how this new mechanism can be applied to simulate vulnerabilities under various strategies, providing decision-makers with insights into how different interventions can lead to different outcomes. By utilizing the proposed approach, it becomes possible to model and predict the behavior of complex systems, providing an effective tool for addressing the uncertainties inherent in multi-faceted decision environments.

Overall, the introduction of this new reasoning mechanism represents a significant step forward in enhancing the capabilities of FCMs. By moving away from traditional nonlinear activation functions and incorporating a more comprehensive approach to

system influence dynamics, this chapter aims to provide a deeper understanding of how FCMs can be effectively used for strategic planning, prediction, and decision-making in complex systems.

5.1 Introducing the Core Structure of the Proposed Reasoning Mechanism through Scenario-Based MCDM

The study in this chapter uses a decision-making approach to reveal the factors associated with FSC vulnerability using FCM based on expert opinions enhanced with triangular fuzzy numbers (TFNs). The approach follows the steps described below and depicted in Figure 5.1:



Figure 5.1: An overview of the steps involved in the proposed approach.

Step 1. Engage experts to determine and evaluate factors.

Step 2. Determine the factors causing FSC vulnerability.

Step 3. Request experts to evaluate factors. Experts are encouraged to use the linguistic terms in Table 5.1 to assess relationship magnitude and specify direction as “positive” or “negative.” The linguistic scale in the table is adapted and simplified from the work of C. C. Sun (2010).

Table 5.1: Linguistic terms to evaluate relationships.

Linguistic term	Triangular fuzzy number (l,m,u)
High (H)	(0.7, 0.8, 0.9)
Medium (M)	(0.4, 0.5, 0.6)
Low (L)	(0.1, 0.2, 0.3)

Step 4. Aggregate the individual evaluations to acquire a single fuzzy evaluation matrix.

Step 5. Defuzzify the evaluation matrix to transform the fuzzy evaluation matrix into a crisp evaluation matrix.

Step 6. Perform FCM simulations to analyze vulnerability-driving situations across various scenarios.

5.1.1 Preliminaries of Triangular Fuzzy Numbers

This section briefly reviews some basic definitions of fuzzy sets and numbers, which are used throughout this paper unless otherwise noted. A fuzzy set $\tilde{A}$ is defined by a membership function $\mu_{\tilde{A}}(u)$, which assigns a real number within the range of $[0, 1]$ to each element u in the universe of discourse $\mathbf{U}$. $\tilde{A}$ is convex if and only if inequality 5.1 holds for all u_1 and u_2 in $\mathbf{U}$, and $\lambda \in [0, 1]$ (Yesil et al., 2014).

$$\mu_{\tilde{A}}(\lambda u_1 + (1 - \lambda)u_2) \geq \min(\mu_{\tilde{A}}(u_1), \mu_{\tilde{A}}(u_2)) \quad (5.1)$$

A fuzzy number is a fuzzy subset of $\mathbf{U}$ that is characterized by being both convex and normal. According to fuzzy set theory, classical sets are encompassed within a broader category of fuzzy sets; thus, crisp numbers are a specific instance of fuzzy numbers because they share all the same properties. A crisp number $\mathbb{x}$ can be represented by a fuzzy number $\tilde{\mathcal{T}}$ defined by the membership function given by equation 5.2. Crisp numbers that are interpreted as fuzzy numbers are commonly referred to as fuzzy singletons.

$$\mu_{\tilde{\mathcal{T}}}(u) = \begin{cases} 0, & u < \mathbb{x} \\ 1, & u = \mathbb{x} \\ 0, & u > \mathbb{x} \end{cases} \quad (5.2)$$

A triangular fuzzy number (TFN) $\tilde{\mathcal{T}}$ can be described by a triplet (a^L, a^M, a^U) . The membership function $\mu_{\tilde{\mathcal{T}}}(u)$ is defined as given in equation 5.3 (van Laarhoven and Pedrycz, 1983):

$$\mu_{\tilde{\mathcal{T}}}(u) = \begin{cases} 0, & u < a^L \\ \frac{u - a^L}{a^M - a^L}, & a^L \leq u \leq a^M \\ \frac{u - a^U}{a^M - a^U}, & a^M \leq u \leq a^U \\ 0, & u \geq a^U \end{cases} \quad (5.3)$$

where $a^L \leq a^M \leq a^U$, with a^L and a^U representing the lower and upper bounds of the support of $\tilde{a}$, respectively, and a^M denoting the modal value.

The basic mathematical operations of summation, multiplication, and scalar multiplication related to positive TFNs, $\tilde{a} = (a^L, a^M, a^U)$ and $\tilde{b} = (b^L, b^M, b^U)$, are given by equations 5.4, 5.5, and 5.6, respectively.

$$\tilde{a} \oplus \tilde{b} = (a^L + b^L, a^M + b^M, a^U + b^U) \quad (5.4)$$

$$\tilde{a} \otimes \tilde{b} = (a^L b^L, a^M b^M, a^U b^U) \quad (5.5)$$

$$\lambda \otimes \tilde{a} = (\lambda a^L, \lambda a^M, \lambda a^U), \quad \lambda > 0 \quad (5.6)$$

Equation 5.7 can be used to aggregate n TFNs (Frias et al., 2018).

$$\tilde{A} = \sum_{i=1}^n w_i \tilde{a}_i \quad (5.7)$$

where w_i is the weight of TFN $\tilde{a}_i$ and $\sum_{i=1}^n w_i = 1$.

For defuzzification of $\tilde{a}$, equation 5.8 can be used (Irvanizam et al., 2018).

$$\mathbb{D}(\tilde{a}) = a^L + \frac{(a^U - a^M)}{4} \quad (5.8)$$

5.1.2 Fundamental Design of the Proposed FCM Reasoning Approach

FCMs are a well-established soft computing tool that enable the modeling of dynamic interrelationships among system concepts and their effects on the system as a whole. By representing concepts as nodes and their relationships as weighted and directed edges, FCM provides a framework for visualizing intuitive relationships among system concepts (Podvesovskii and Isaev, 2018). These relationships are further analyzed through matrix-based mathematical modeling to capture system dynamics.

However, FCM has been criticized for producing the same final vector regardless of changes in the initial state vectors. Asan and Kadaifçi (2020) propose an approach for obtaining the final vector through simulations that account for both direct and indirect relationships, in contrast to the classical FCM method, which only considers direct relationships at each simulation step. This study extends the reachability matrix

calculation method introduced by Asan and Kadaifçi (2020). The reachability matrix calculation is further developed as shown in equation 5.9. The proposed approach normalizes the matrices, which have been raised to their respective powers, according to the maximum weight w_{max}^k W^k and then sums the normalized matrices.

$$\mathfrak{R} = \sum_{k=1}^n \omega^k \quad (5.9)$$

where ω^k is the normalized form of W^k , as specified in equation 5.10:

$$\omega^k = \frac{W^k}{w_{max}^k} \quad (5.10)$$

It continues by simulating the system starting from an initial state, by multiplying the initial state A_0 and $\mathfrak{R}$. Thus, the final state of the system is acquired as $A_0 \times \mathfrak{R}$. To interpret this final state vector, a final step that involves normalizing this vector by its maximum value is performed. Equation 5.11 gives the normalized final state vector.

$$A_f = \frac{A_0 \times \mathfrak{R}}{\max(A_0 \times \mathfrak{R})} \quad (5.11)$$

This revised calculation provides the final state based on a one-step simulation to draw inferences on the system's behavior originating from a specific initial state.

5.2 Application of the Basic Form of the Proposed Reasoning Mechanism

The application section of this study focuses on the complexities of FSC by identifying critical vulnerability factors through a rigorous literature review. Recognizing the interconnected nature of these factors and their potential for diverse impacts, this study employs FCM as a robust analytical tool. FCM is instrumental in visualizing and quantifying the intricate relationships among these vulnerabilities and their subsequent effects on the FSC system.

5.2.1 Determining Criteria Set

The literature review identified the factors in Table 5.2 as the causal areas of vulnerability. It has been recognized that FSC operates in a complex and dynamic

environment, influenced by several interconnected factors. These factors can be categorized into environmental, economic, social, and operational dimensions.

Table 5.2: Factors causing vulnerability in FSC.

Factor Category	Factors	Source
Environmental	Natural disturbance (C1)	(Roos, 2023; J. Yang and Liu, 2018)
	Climate change (C2)	(Roos, 2023)
Economic	Market demand (C3)	(J. Yang and Liu, 2018)
	Economic downturn (C4)	(Q. Liu and Ning, 2023)
Social	Workforce availability (C5)	(Hoffmann et al., 2018)
	Community acceptance (C6)	(Barnett, 2018)
Operational	Technological advancement (C7)	(Pandirwar et al., 2023)
	Environmental regulations (C8)	(Garrett et al., 2021)
	Trade policies (C9)	(Zhang et al., 2021)
	Resource availability (C10)	(M. Wang et al., 2023)
	Contingency planning awareness (C11)	Expert view
	Government support (C12)	(Vermeulen and Kok, 2012)
	Collaboration (C13)	(S. K. Sharma et al., 2023)
	Supply chain structure convenience (C14)	(Greenslade et al., 2021)

Environmental factors include natural disturbances (C1) and climate change (C2). Natural disturbances refer to events such as wildfires, insect infestations, and extreme weather events, which may significantly disrupt forest ecosystems, leading to timber shortages, infrastructure damage, and increased harvesting costs. For instance, large-scale wildfires can destroy vast areas of forests, reducing timber availability for years and disrupting transportation routes. Climate change encompasses factors like rising temperatures, changing rainfall patterns, and increased frequency of extreme weather events, posing long-term threats to forest health and productivity. Such conditions can lead to reduced timber growth, increased pest and disease outbreaks, and altered forest composition, affecting overall supply chain stability.

Economic factors are market demand (C3) and economic downturn (C4). Market demand reflects fluctuations in the demand for forest products, driven by factors such as economic cycles, construction activity, and consumer preferences. These shifts can trigger price volatility, overstocking, and shortages. Sudden increases in demand may strain resources and cause bottlenecks, whereas declines can lead to reduced revenues and potential closures. Economic downturn refers to recessions and financial crises that severely affect the forestry industry. Reduced consumer spending during such periods diminishes demand for forest products, depresses timber prices, and results in job losses across the supply chain. Moreover, limited financial resources may hinder investments in forest management and infrastructure.

Social factors include workforce availability (C5) and community acceptance (C6). Workforce availability refers to the presence of skilled labor, particularly in specialized areas like forestry and logging. A shortage of skilled labor can constrain operations, drive up costs, and reduce the overall efficiency of the supply chain. This challenge is often exacerbated by aging labor pools and competition from other industries. Community acceptance captures the degree to which local populations support forest operations. Social conflicts, environmental concerns, and public perceptions of industry effects can pose significant obstacles. Maintaining a positive social license to operate is critical for ensuring continued access to forest resources and minimizing potential disruptions.

Operational factors consist of technological advancements (C7), environmental regulations (C8), trade policies (C9), resource availability (C10), contingency planning awareness (C11), government support (C12), collaboration (C13), and supply chain structure convenience (C14). Technological advancements contribute to improved efficiency, cost reduction, and enhanced sustainability. However, these advancements also demand substantial investments and workforce training. The rapid pace of innovation may pose difficulties for smaller firms with limited resources. Environmental regulations must be adhered to in order to ensure sustainable practices, yet they can also increase operational costs and complexity. Strict regulatory frameworks may limit access to forest resources and necessitate investments in compliance technologies such as pollution control and monitoring systems. Trade policies shape market access, pricing, and competitive dynamics through mechanisms such as tariffs, quotas, and trade agreements. Shifts in trade policy can introduce uncertainty and require supply chains to be restructured or adapted. Resource availability is often constrained by land-use changes, resource competition, or depletion, which can hamper production and drive up costs. In addition, variability in timber quality and quantity may also affect product consistency and processing efficiency. Contingency planning awareness highlights the importance of preparing for potential disruptions, including natural disasters, economic shocks, or supply chain failures, to ensure business continuity and enhance resilience. Government support plays a crucial role in shaping industry performance through regulations, policy direction, and financial incentives. Supportive governmental measures can stimulate investment, drive innovation, and foster a favorable business climate. Collaboration

among stakeholders, such as landowners, loggers, processors, and other partners—can streamline operations, reduce costs, and enhance supply chain robustness. Shared planning, information exchange, and coordinated risk management are key to minimizing disruptions. Supply chain structure convenience refers to how the configuration of the supply chain, its level of complexity, number of intermediaries, transportation demands, and inventory practices, affects its resilience. More complex and interconnected supply chains tend to be more vulnerable to disruptions than simpler, integrated ones.

Understanding the interplay among these factors is crucial for developing resilient FSC. In addition, by proactively identifying and addressing potential challenges, stakeholders within the industry can enhance long-term sustainability and profitability.

5.2.2 Building FCM

After determining the factors associated with FSC vulnerability, four domain experts were engaged in constructing the FCM model by evaluating the relationships between them. All experts are knowledgeable about forestry and the supply chain. Employing the structured linguistic scale in Table 5.1, the experts qualitatively assessed the strength and direction of these relationships.

Table 5.3: Relationships between the FSC vulnerability factors.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14
C1		0,094		0,138						-0,469	0,138			
C2	0,275									-0,319				
C3		0,063		0,106		0,063				-0,244		0,213		
C4			-0,650		-0,031		-0,138	0,138	-0,138	0,213				-0,213
C5										0,213			0,319	
C6			0,213		0,319							0,138	0,169	
C7					0,256				0,063					0,425
C8	-0,169	-0,244							0,138	0,063	0,138			0,138
C9			0,319	-0,213						0,138			0,138	
C10	0,213													
C11						0,063		0,213						
C12			0,244		0,213		0,288	0,394	0,319	0,138	0,244			
C13						0,094					0,063			0,575
C14							0,275		0,288				0,394	

The qualitative expert judgments were then transformed into quantifiable data using TFNs, enabling the subsequent aggregation of multiple expert opinions using equation 5.1 into a consolidated relationship matrix. Relationships with two or more opinions expressed by experts were directly incorporated into the matrix, whereas solitary opinions were considered negligible. The aggregated matrix was subsequently

defuzzified using equation 5.8 and transformed into the crisp relationship matrix shown in Table 5.3, thereby facilitating a clearer representation of factor interactions. The derived relationship matrix in Table 5.3 is the foundation for constructing the map shown in Figure 5.2.

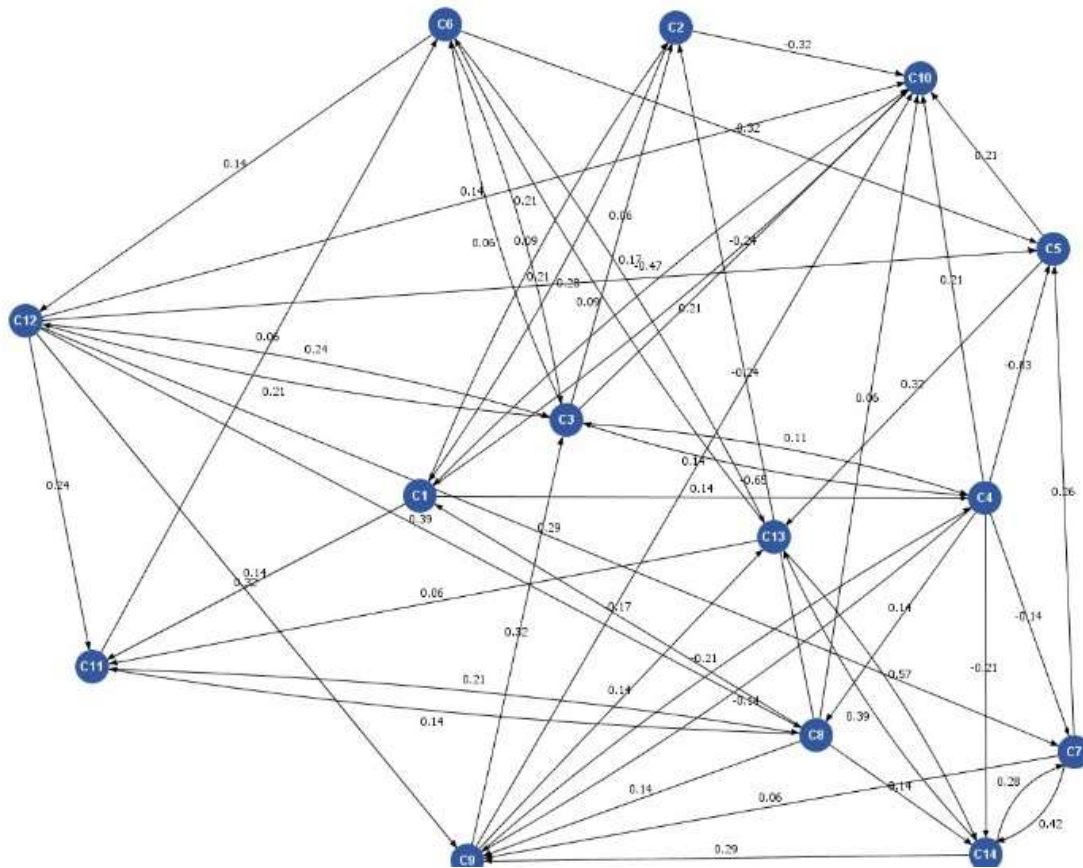


Figure 5.2: FCM model for FSC vulnerability.

5.3 Results and Discussion

The map given in Figure 5.2 graphically depicts the intricate connections among the vulnerability factors within the FSC. According to the map, the most central factor, with a centrality value of 2.306, indicating the sum of the absolute magnitudes of its direct interactions, is the convenience of the Supply Chain Structure (C14). This finding aligns with previous studies that emphasize the importance of a well-structured supply chain in mitigating vulnerabilities (He and Turner, 2021; Ivanov, 2017). Government support (C12), market demand (C3), and source availability (C10) are, respectively, the next most central factors. These are significantly influenced by or have a substantial impact on others through their associated interactions. Dubey et al. (2023) highlighted how government interventions can shape market dynamics and

supply chain effectiveness. The factor most influenced and altered by other factors is Source Availability (C10), with an in-degree value of 1.794, indicating the sum of the absolute magnitudes of the relationships affecting it. This means that any change in FSC could significantly impact the raw material sources that sustain the system. A concern has been expressed by Raulier et al. (2014) regarding the impact of natural disruptions on timber supply. This reinforces the need for effective risk management strategies that address the vulnerabilities of resource availability in FSCs (Jüttner et al., 2003). The other two factors that also have high centrality values are market demand (C3) and Supply Chain Structure (C14), respectively. Finally, Government support (C12) and Economic downturns (C4), with out-degree values of 1.837 and 1.519 respectively, which is the opposite of in-degree, are the factors with the highest potential to cause changes in the FSC system dynamics. This observation is consistent with findings from Pettit et al. (2019), who argued that government policies and economic conditions are pivotal in shaping the resilience and adaptability of supply chains.

To understand the dynamic behavior of the FSC under varying conditions, analyses were conducted based on the simulation of cause–effect relationships in the map. A sustainability-centric approach was adopted, with three distinct scenarios formulated, each targeting a specific sustainability dimension. Each scenario is characterized by unique initial state vector encompassing actions relevant to its respective sustainability objectives.

The first scenario, which considers environmental impacts, was initialized with an initial state vector of $[1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$. This scenario assumes that initially only the factors of natural disturbance (C1) and climate change (C2) are active, meaning that FSC is vulnerable to only these two factors. Therefore, by simulating the system under the influence of these two factors, changes in the FSC dynamics, including environmental factors and others, were observed. Accordingly, FSC dynamics with direct and indirect relationships directed the system to a state of $[0.792, 0.860, 0.316, 1.000, 0.571, 0.700, 0.426, 0.793, 0.376, 0.000, 0.850, 0.638, 0.225, 0.082]$. The system behavior in terms of the initial and final states is illustrated in Figure 5.3.

The initial state vector for the second scenario, focusing on economic factors, was set to $[0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0]$ to determine how economic-oriented vulnerabilities

impact other vulnerability-causing factors in FSC. Such an initial state triggers most sources in the FSC, as indicated by the final state vector $[0.921, 0.881, 0.333, 1.000, 0.646, 0.777, 0.461, 0.872, 0.398, 0.954, 0.806, 0.765, 0.186, 0.000]$, except for a few. For instance, while changes in demand that support the flow within the supply chain can encourage the enrichment of resources for the forestry industry, economic slowdowns may cause forest areas to shrink as investors redirect their resources to more profitable ventures. It is also clear from Figure 5.4 that Demand-driven vulnerabilities have a weaker impact when other vulnerabilities come into play.

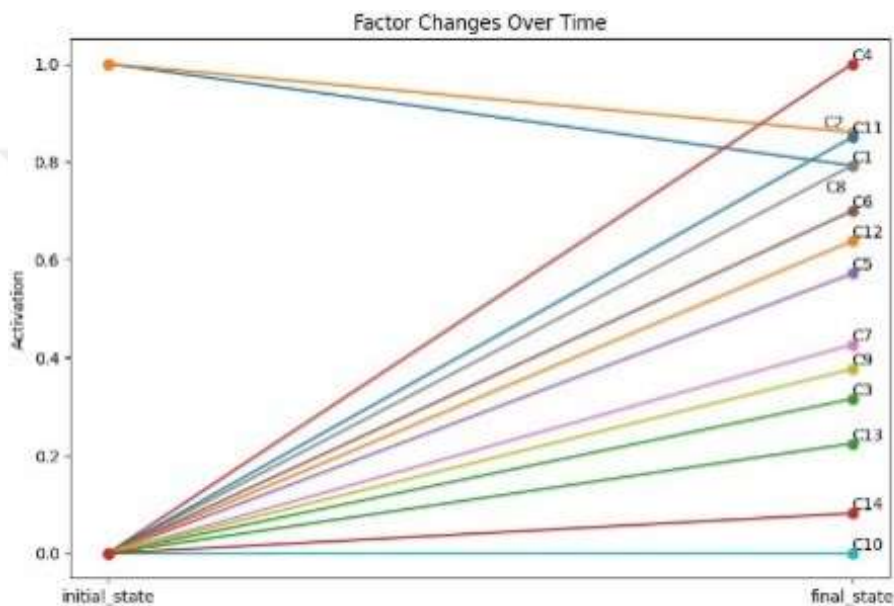


Figure 5.3: Emerging of vulnerability causing factors in terms of scenarios.

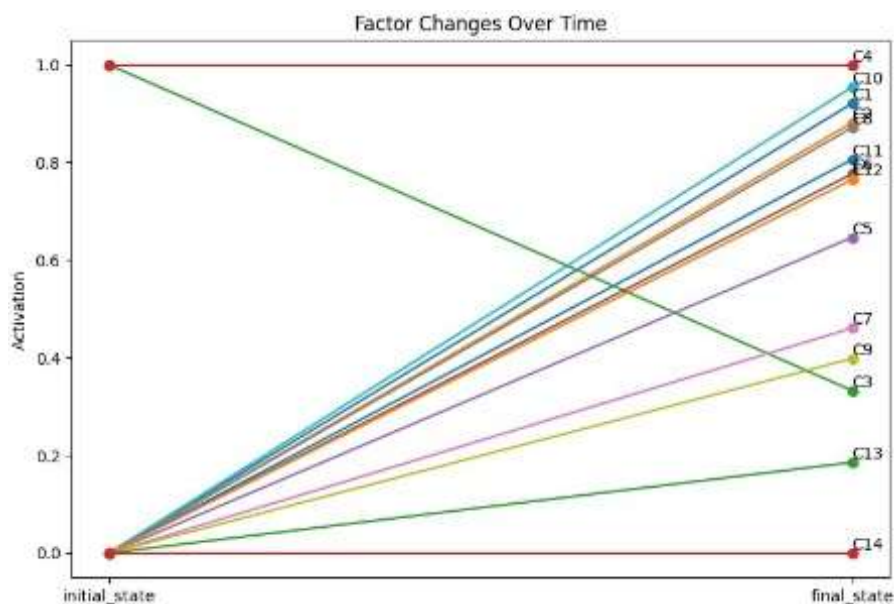


Figure 5.4: Emerging of vulnerability causing factors in terms of scenarios.

Finally, the third scenario addresses vulnerabilities related to social factors. Here, the initial state vector was set to $[0,0,0,0,1,1,0,0,0,0,0,0,0,0]$. The result of this scenario, $[0.082, 0.075, 0.487, 0.000, 0.383, 0.232, 0.502, 0.199, 0.582, 0.175, 0.229, 0.245, 0.9, 1.000]$, social vulnerabilities, except for two factors, do not show high potential for causing vulnerabilities in other factors as seen in Figure 5.5. Due to the direct correlation of workforce availability (C5) and community acceptance (C6) with the adoption of FSC activities, their vulnerabilities can only trigger vulnerabilities within the supply chain structure convenience (C14) and collaboration (C13).

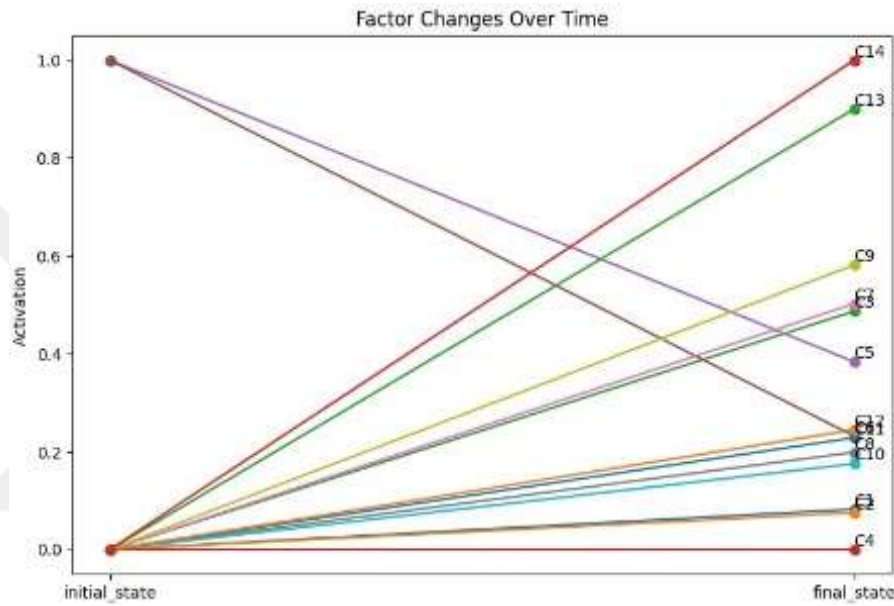


Figure 5.5: Emerging of vulnerability causing factors in terms of scenarios.

By simulating each scenario using its corresponding initial state vector, this study explored the potential trajectories of the FSC system under different conditions. The resulting system states provide valuable insights into the vulnerabilities that may emerge and their potential impacts on the FSC's overall performance and resilience.

5.4 Concluding Remarks

FSC plays a significant role in sustainable forest management and global economic stability. However, its complex and interconnected nature exposes it to vulnerabilities, ranging from environmental disruptions to economic fluctuations and social challenges. This study employed FCM as a novel approach to comprehensively assess these vulnerabilities.

Through detailed review and analysis, key factors affecting FSC vulnerability were identified across environmental, economic, social, and operational dimensions. Factors such as climate change, market demand volatility, workforce shortages, and regulatory complexities emerged as critical. FCM provided a structured framework to model the intricate relationships and feedback loops among these factors, offering insights into their combined impacts on FSC resilience.

The application of FCM highlighted the centrality of supply chain structure, government support, and resource availability in shaping FSC dynamics. By visualizing these interdependencies, administrators can better understand and prioritize strategies to mitigate risks and enhance sustainability. Simulation scenarios underscored the sensitivity of FSCs to environmental stressors and economic shocks, reinforcing the need for adaptive strategies and contingency planning.

Additionally, this study identified gaps in current research, particularly the limited integration of diverse vulnerability factors and the underutilization of analytical tools like FCM, despite the ease of expert evaluation enabled by TFNs.

A key limitation is the absence of a comprehensive sensitivity analysis that considers how different expert backgrounds might influence factor assessment. The study does not account for perspectives from groups such as environmental activists, industry professionals, or policymakers, nor does it compare the influence of different forest management systems, such as state-managed versus privately managed, on factor weightings.

Future research should address these gaps by conducting sensitivity analyses to evaluate how diverse expert opinions may affect assessments, and by comparing management systems to understand their impact on vulnerability. Such efforts would improve the model's robustness and broaden its relevance.

In conclusion, understanding FSC vulnerability requires a holistic approach that encompasses environmental, economic, and social dimensions. By applying methodologies like FCM, stakeholders can navigate uncertainties, foster sustainable practices, and support the long-term viability of forest resources and dependent communities.

6. AN IMPROVED LEARNING FCM APPROACH TO PREDICTION PROBLEMS USING AN INPUT-SENSITIVE REASONING MECHANISM

This chapter delves into the integration of learning approaches into FCMs, emphasizing how learning mechanisms can enhance FCM's ability to model and predict system behavior effectively. FCMs are constructed using causal relationships between concepts, with nodes representing different elements of a system. This construction often relies on expert knowledge, making the process inherently complex, especially for systems with numerous interrelated components. Expert disagreements arising from differing perspectives, skills, and backgrounds further complicate the task. To address these challenges, learning methods inspired by artificial neural networks have been developed to automate and enhance the efficiency of determining and revising these relationships.

Learning methods in FCM are broadly categorized into three types: Hebbian-based methods, error-driven methods, and hybrid approaches. Hebbian-based learning works by strengthening links between co-activated concepts and weakening those between rarely activated concepts. Error-driven learning, on the other hand, aims to minimize the error between the desired and actual outcomes of FCMs by adjusting the causal relation weights using a gradient descent algorithm. Hybrid learning approaches combine aspects of both Hebbian and error-driven rules, providing a balanced mechanism for optimizing the weights in the FCM model.

The primary goal of these learning approaches is to iteratively improve the weights of causal relations to ensure that the FCM accurately reflects the behavior of the system being modeled. In this context, the reasoning mechanism plays a critical role in determining the activation levels of concepts at specific time points, simulating the system's behavior in a virtual environment. Most traditional FCMs use a sigmoid activation function in this reasoning mechanism. However, the choice of activation function can significantly impact the accuracy and adaptability of the model. This chapter addresses the limitations posed by conventional activation functions, such as

gradient saturation, which hampers the model's ability to capture varied behaviors in complex systems.

To overcome these limitations, this chapter elaborates the novel reasoning mechanism, introduced in the previous chapters, tailored specifically for error-driven learning FCMs. This mechanism moves away from relying on standard activation functions and instead employs exponentiation and normalization techniques that offer enhanced responsiveness to initial states. Such sensitivity is essential for prediction and scenario-based analysis, as it allows for more nuanced simulations that can differentiate outcomes based on initial states. This enhancement is crucial for improving both the learning process and the FCM's ability to model and predict a system's long-term behavior, especially when loops and indirect relationships are involved.

The effectiveness of the proposed reasoning mechanism is demonstrated through a series of experimental studies designed to evaluate its performance under various conditions. These experiments employ a dataset that is independent of both the proposed and traditional reasoning mechanisms, providing a robust basis for comparative analysis. This dataset, developed from real-life scenarios, allows us to assess the impact of different reasoning mechanisms in a realistic context. One key aspect of these experiments is the evaluation of normalization techniques, as well as the comparison between cumulative and non-cumulative calculations. Fitness functions, which measure the error between predicted and actual outputs, are utilized to assess the performance of the models. Additionally, out-of-sample error is used as a metric to evaluate the generalization capability of the proposed mechanism.

In this chapter, we also discuss our main study titled “Learning Fuzzy Cognitive Maps for Prediction Problems Using Swarm Intelligence Based on an Input-Sensitive Reasoning Mechanism” (Murat and Asan, 2025). This study provides a detailed explanation of the proposed reasoning mechanism and its integration with the learning FCM. It also discusses experimental designs and the use of real-world data to validate the new mechanism, which stands in contrast to the typical use of artificially generated data in similar studies.

This chapter presents several key contributions to the field of learning FCMs. Firstly, it proposes a new reasoning mechanism that extends beyond traditional optimization methods, making it effective for both control and prediction problems. By improving

sensitivity to initial states, the reasoning mechanism enhances the accuracy of scenario planning and prediction, making it possible to model cycles, which traditional methods often struggle to represent. This versatile approach proves suitable for control systems and is particularly effective for prediction problems, broadening the application range of FCMs. In addition, it provides a comprehensive analysis of various activation and normalization techniques, comparing sigmoid, tangent sigmoid, and new normalization techniques to understand their impact. Unlike many studies that use synthetic data, we employ real-world scenarios, adding practical relevance and reliability to our results. Moreover, the methodology is designed to function autonomously without the need for expert intervention, though it maintains the flexibility to include constraints if necessary.

By addressing the challenges associated with traditional learning FCM, this chapter aims to advance the capabilities of FCMs in accurately modeling complex systems. The novel reasoning mechanism not only improves the learning accuracy of FCMs but also ensures that the models are adaptable to varying initial states and capable of providing reliable predictions for scenario-based analyses. These advancements position FCMs as more robust tools for understanding and predicting the behaviors of systems characterized by uncertainty and complexity.

6.1 An Improved Learning FCM Approach to Prediction Problems Using an Input-Sensitive Reasoning Mechanism

6.1.1 FCM using the proposed reasoning mechanism

FCM offers significant advantages for modeling complex system relationships and predicting dynamic, time-evolving behavior. Nonetheless, some researchers remain cautious about its ability to simulate long-term behavior under diverse conditions due to various limitations (Asan and Kadaifci, 2020; Karagiannis and Groumpos, 2013; Koutsellis et al., 2022; Mpelogianni and Groumpos, 2016; Nápoles et al., 2014; Vergini and Groumpos, 2016). One of the notable drawbacks in simulating FCM is the insensitivity of steady-state values to initial states. This means that if the reasoning mechanism of FCM reaches a stable state, known as a unique fixed point, it will always produce the same output state vector regardless of the initial input or starting conditions (see (Nápoles et al., 2016, 2018a)). As a result, the system will be unable to differentiate between different patterns or scenarios that have different meanings or

contexts, since it will always converge to the same end state. Although the standard reasoning mechanism may work effectively in control scenarios where the objective is to produce a consistent response regardless of the initial state vector, it is not suitable for adjusting FCM models used in prediction scenarios because of its limited ability to generalize (Nápoles et al., 2020). In line with these arguments, Asan and Kadaifci (2020) reported two drawbacks of the classical reasoning mechanism. They explored alternative FCM settings using the well-known traffic example (Kosko, 1993) to determine whether the reasoning mechanism yields the same output under varying initial states (i.e., scenarios). Their findings indicate that simulations performed with different initial state vectors consistently produce the same steady state values across all scenarios. They also found that the reasoning mechanism produces for the same data completely different steady state values depending on the choice of the threshold function. Their research provides clear evidence of a restricted reliability of the classical reasoning mechanism. To address this issue, they proposed a reasoning mechanism that considers all possible indirect relationships (Asan and Kadaifci, 2020). The mechanism involves a simulation with a total effect matrix created by consecutively powering and summing the relationship matrix.

This study proposes a reasoning mechanism that not only exploits indirect relations but also offers flexibility on the requirement to use traditional activation functions that can cause issues related to convergence. This study focuses on prediction systems, in contrast to those modeling control systems that typically converge to an equilibrium point. Hence, it introduces a novel prediction model capable of replicating data representing the consecutive states of a system, effectively characterizing its behavior over time.

The FCM equipped with the proposed reasoning mechanism (here after pro-FCM) incorporates direct and indirect relationships between concepts in order to simulate the long-term behavior of the system. Indirect relationships are included in the calculations by raising the weight matrix W to its successive powers. The new state of the system at any given time t (for cumulative calculation) is determined using equation 6.1:

$$\mathbf{A}^t = \mathbf{A}_{initial} + \mathbf{A}_{initial} \sum_{\tau=1}^t \boldsymbol{\omega}^{\tau} \quad (6.1)$$

where $\mathbf{A}^t$ is the vector showing the state of the system after t iterations from the initial state $\mathbf{A}_{initial}$. $\sum_{\tau=1}^t \boldsymbol{\omega}^{\tau}$ denotes the total effect matrix at iteration t , where $\boldsymbol{\omega}^{\tau}$ is the normalized form of the τ^{th} power of the $\mathbf{W}$ matrix, as detailed in equation 6.2. Directly using the power of $\mathbf{W}$ means weights have increasing influence on the calculation of state vectors compared to activation levels as values in the τ^{th} power of $\mathbf{W}$ grows in parallel with the increasing number of iterations.

$$\boldsymbol{\omega}^{\tau} = \text{norm}(\mathbf{W}^{\tau}) \quad (6.2)$$

In each iteration, similar to classical FCM reasoning, the difference between two consecutive state vectors is examined to determine convergence or repetition in system states. Convergence is assumed when the maximum change in the activation levels of concepts at an iteration falls below a set threshold. $\Delta \mathbf{A}^t$, a vector representing this change in the activation levels, is calculated as shown equation 6.3. If this threshold is not met but $\Delta \mathbf{A}^t$ equals any previous one, a repeated pattern of system behavior is identified.

$$\Delta \mathbf{A}^t = \mathbf{A}^t - \mathbf{A}^{t-1} = \mathbf{A}_{initial} \boldsymbol{\omega}^t \quad (6.3)$$

Considering the new mechanism, it is important to recognize two critical requirements that have emerged:

- i. The first requirement is obtaining the normalized total effect matrix, $\sum_{i=1}^t \boldsymbol{\omega}^i$, to ensure that it has the properties of a weight matrix. This normalization ensures a balance between the influences of the relationship matrix and the current activation levels in the computation of new state vectors. Therefore, $\mathbf{W}^i$ is normalized to obtain $\boldsymbol{\omega}^i$.
- ii. The second requirement is to normalize the new state vector $\mathbf{A}^t$ to a convenient interval. Unlike traditional FCM, which uses activation functions like log-sigmoid or hyperbolic tangent, pro-FCM calculates concept activation levels in the new state vector, as shown in equation 6.1, without such limiting functions. This can lead to activation levels in an unbounded range. Consequently, normalizing each new state

vector (the normalization methods are detailed in Section 4.2), for example, to the $[0, 1]$ range, is necessary to ensure comparability between consecutive state vectors.

6.1.2 Learning FCM based on the proposed reasoning mechanism

Learning FCM involves discovering relationships between concepts from a multivariate time series, which is longitudinal historical data. Each time series variable corresponds to a concept, with individual data points reflecting the system status in relation to that concept. Using automated algorithms such as HL or heuristic-based methods is crucial for exploring the extensive solution space when constructing FCMs from this data (Stach et al., 2010).

This study employs heuristic-based learning methods for FCMs, tailored to work with the proposed reasoning mechanism. The objective is to demonstrate that using this mechanism in data-driven FCMs improves learning performance by reducing prediction error. While the basic philosophy of the proposed reasoning mechanism has been previously established as reasonable in expert knowledge-based FCMs (Asan and Kadaifci, 2020), this study addresses areas that require further development. Specifically, Asan and Kadaifci focused primarily on the modeling capabilities of the mechanism but overlooked aspects such as the activation functions and potential convergence concerns. Additionally, they did not specify criteria for terminating simulations, an issue that this study also seeks to address.

The ability to model system behavior for FCMs using the proposed reasoning mechanism is investigated through a learning process in this study. The reasoning mechanism is enhanced using total effect matrix and state vector normalization techniques, which move away from traditional sigmoid and tangent sigmoid functions. In addition, the learning process involves examining various fitness functions to align with the logic of historical data prediction.

6.1.2.1 Learning FCM based on meta-heuristic algorithm

Determining the causality of FCMs typically relies on either expert knowledge or data analysis. In cases where expert input is lacking or insufficient, methods like Hebbian and heuristic learning are employed to learn causal relations from historical data (Alizadeh et al., 2007). This study employs the first and a widely favored heuristic method PSO algorithm (Abdelbar et al., 2005; Parsopoulos et al., 2003, 2004) for

training FCMs using the proposed reasoning mechanism for constructing FCMs and modeling data. It is used because of its simplicity and minimal parameter requirements (L. Li and Liu, 2011). It's important to note that this study does not involve improving any heuristic algorithm, instead the primary concern is to improve the reasoning mechanism and, thereby, the learning performance of FCMs.

Particle swarm optimization algorithm

PSO is an evolutionary computation technique and a part of the swarm intelligence algorithms known for heuristic solutions (Eberhart and Shi, 2001; Mavrovouniotis et al., 2017). Similar to other algorithms in this class, PSO operates with a population (i.e., a swarm) of individuals, termed particles, each representing a potential solution. Particles systematically search for the solution space to arrive at a near-optimal solution by drawing on the collective intelligence of the swarm.

A particle, in the PSO framework, is characterized by its position and velocity, possessing a memory that encapsulates a portion of the collective experience of the swarm. The position, indicative of a potential solution, is updated over time according to both individual and collective memories. Each particle records its best position ever attained, $pbest$, in its memory, while the swarm collectively tracks the best position across the particles, $gbest$, in the collective memory (D. Wang et al., 2018). Particle positions are dynamically adjusted on the basis of their velocity and are influenced by both individual and collective swarm memories. Consequently, the updated particles form a new generation of the swarm that is more likely to converge to the most reasonable solution.

Suppose $\mathbf{U} \subset \mathbb{R}^D$ is a D-dimensional solution space, and $\mathbf{S}$ is a swarm that consists of M particles from $\mathbf{U}$. The position of particle i at the iteration t and its velocity are represented by the D-dimensional vectors $\mathbf{x}_i^t = (x_{i,1}^t, x_{i,2}^t, \dots, x_{i,D}^t)$ and $\mathbf{v}_i^t = (v_{i,1}^t, v_{i,2}^t, \dots, v_{i,D}^t)$, respectively. Where $x_{i,d}^t$ and $v_{i,d}^t$ are the values at the iteration t for a corresponding variable d ($1 \leq d \leq D$) of the problem considered, here each d corresponds to a relationship between any two concepts. Both the position and velocity can be restricted to the limits of $[\mathbf{x}_{min}^t, \mathbf{x}_{max}^t]$ and $[\mathbf{v}_{min}^t, \mathbf{v}_{max}^t]$, respectively.

The new swarm at iteration $t + 1$ is generated by updating the velocities of the particles with equation 6.4 and the positions with equation 6.5 (D. Wang et al., 2018).

$$\mathbf{V}_i^{t+1} = \chi[\omega \mathbf{V}_i^t + c_1 r_1 (\mathbf{P}_i^t - \mathbf{X}_i^t) + c_2 r_2 (\mathbf{P}_g^t - \mathbf{X}_i^t)], \quad \forall i \in \{1, 2, \dots, M\} \quad (6.4)$$

$$\mathbf{X}_i^{t+1} = \mathbf{X}_i^t + \mathbf{V}_i^{t+1}, \quad \forall i \in \{1, 2, \dots, M\} \quad (6.5)$$

where $\mathbf{P}_i^t$ is the *pbest* of particle i , $\mathbf{P}_g^t$ is the *gbest* at iteration t . χ is the *constriction factor* and ω is *inertia weight*, and they are parameters to control the magnitude of the velocity to avoid falling into the trap of local optima. c_1 (*cognitive*) and c_2 (*social*) acceleration constants, and r_1, r_2 are uniform random vectors on the interval $[0, 1]$, which control the convergence speed.

The particles are evaluated through a designated fitness function F , which gauges the quality of the proposed solution offered by each particle. F additionally leads to the determination of $\mathbf{P}_i^{t+1}$ and $\mathbf{P}_g^{t+1}$ by equations 6.6 and 6.7, respectively, until the terminal iteration.

$$\mathbf{P}_i^{t+1} = \begin{cases} \mathbf{P}_i^t & : F(\mathbf{X}_i^{t+1}) \geq F(\mathbf{P}_i^t) \\ \mathbf{X}_i^{t+1} & : F(\mathbf{X}_i^{t+1}) < F(\mathbf{P}_i^t) \end{cases}, \quad \forall i \in \{1, 2, \dots, M\} \quad (6.6)$$

$$\underset{\mathbf{P}_g^{t+1} \in \mathbf{S}}{\operatorname{argmin}} F(\mathbf{P}_g^{t+1}) = \{\mathbf{P}_g^{t+1} \mid F(\mathbf{P}_g^{t+1}) \leq F(\mathbf{P}_i^{t+1}), \quad \forall \mathbf{P}_i^{t+1} \in \mathbf{S}\} \quad (6.7)$$

where *argmin* is a function that yields the points in $\mathbf{S}$ where F is minimized. $\mathbf{P}_i^0$, the best position of the i^{th} particle in the initial iteration, corresponds to the position of the particle.

In this study, F is structured as a procedure involving the development of an FCM prediction model for each particle, followed by an evaluation of the predictive performance of the model. The procedure initiates with the creation of an FCM, proceeds with FCM simulations to make predictions, and ends with measuring the deviation between the predictions and historical data. The measured deviation of a particle serves as an indicator of the prediction accuracy of the candidate solution.

Each particle generated by the PSO algorithm represents a potential solution for developing a predictive FCM corresponding to the n -square relationship matrix $\mathbf{W}$. Particles are D -dimensional vectors where $D = n * n$ and $w_{ij} = 0 \mid i = j$. In the proposed training process, a candidate FCM is derived by converting a vector-form

particle into a $\mathbf{W}$. The candidate FCM is simulated for the number of iterations matching the count of consecutive state vectors in the historical data, beginning with the initial state vector. The FCM predicts a series of contiguous state vectors, each aligning with a state vector from historical data. The deviation between the historical and predicted state vectors serves as a performance measure for a particle and indicates how accurately the candidate relationship matrix $\mathbf{W}$ represents the system.

During each iteration of FCM training, where the PSO algorithm updates the particles, the prediction performance of candidate FCMs is evaluated using three different fitness functions. These functions, which are central to performance evaluation, focus on minimizing error: i. Heaviside, ii. Root Mean Square Error (RMSE) and iii. Maximum of Root Mean Square Errors (max-RMSE). The Heaviside function evaluates the accuracy of predictions by counting the number of incorrectly predicted concepts across the dataset. RMSE measures the deviations in the activation levels of concepts, providing an error calculation for the entire dataset. In contrast, max-RMSE focuses on the largest discrepancies in the state vectors rather than individual concept variations.

In optimizing the Heaviside fitness function, the differences between the corresponding values in historical and predicted data are converted into binary format. Deviations of 0.1 or more (10% deviation) from the actual data are marked as incorrect predictions and coded as 1; deviations less than this threshold are deemed correct and coded as 0. This threshold is lower than the optimal error rate of 15% as suggested in the literature (Wilson et al., 2019) for modeling binary data but can be adjusted based on the specific problem requirements. The Heaviside value, as outlined in equation 6.8, represents the average number of incorrect predictions.

$$error\ rate = \frac{1}{Nn} \sum_t^N \sum_j^n e_j^t, \quad e_j^t = 1 \text{ if } A_j^t \not\cong \hat{A}_j^t \text{ and } e_j^t = 0 \text{ if } A_j^t \cong \hat{A}_j^t \quad (6.8)$$

where N is the number of state vectors in the historical data (i.e. the count of simulation steps), and n is the number of concepts of the simulated system. e_j^t is the error of C_j of the historical state vector at time t . A_j^t shows the actual activation value of C_j and $\hat{A}_j^t$ shows the post-simulation output of the candidate relationship matrix at time t .

Here, $\not\approx$ indicates that $\hat{A}_j^t$ differs from the actual activation value of A_j^t by more than a predetermined threshold ratio.

When using the RMSE fitness function, the goal is to minimize the discrepancies between the predicted activation values of concepts and those of the corresponding concepts in the historical data. The output of the RMSE is the root mean of all deviations in the activation values, calculated as shown in equation 6.9. It measures the prediction error in the same units as the historical data.

$$RMSE = \sqrt{\frac{1}{Nn} \sum_t^N \sum_j^n (A_j^t - \hat{A}_j^t)^2} \quad (6.9)$$

The final fitness function, max-RMSE, focuses on the largest root-mean-square deviation between the corresponding state vectors in the historical and predicted data. The calculation method for the max-RMSE is outlined in equation 6.10.

$$e^t = \sqrt{\frac{1}{n} \sum_j^n (A_j^t - \hat{A}_j^t)^2}, \quad t = 1, 2, \dots, N \quad (6.10)$$

$$maxRMSE = \max(e^t), \quad t = 1, 2, \dots, N$$

where e^t is the prediction error corresponding to the state at the time t .

6.1.2.2 Proposed meta-heuristic optimization-based learning FCM

The steps of the metaheuristic optimization-based learning FCM described above are summarized as follows and are illustrated in Figure 6.1.

Step 1. The key concepts defining the system in question are identified (Tsadiras et al., 2001). The activation levels of these concepts are observed at regular intervals throughout a cycle period or until the system converges. This results in a collection of state vectors representing the system behavior over successive time points. The sequence of these state vectors constitutes the historical data of the system.

Step 2. The size of the position of a particle is set based on the number of concepts. The number of particles (i.e., the swarm size), the number of iterations required to run the PSO algorithm, and the relevant hyperparameters are defined. Next, the positions and velocities of all particles are initialized.

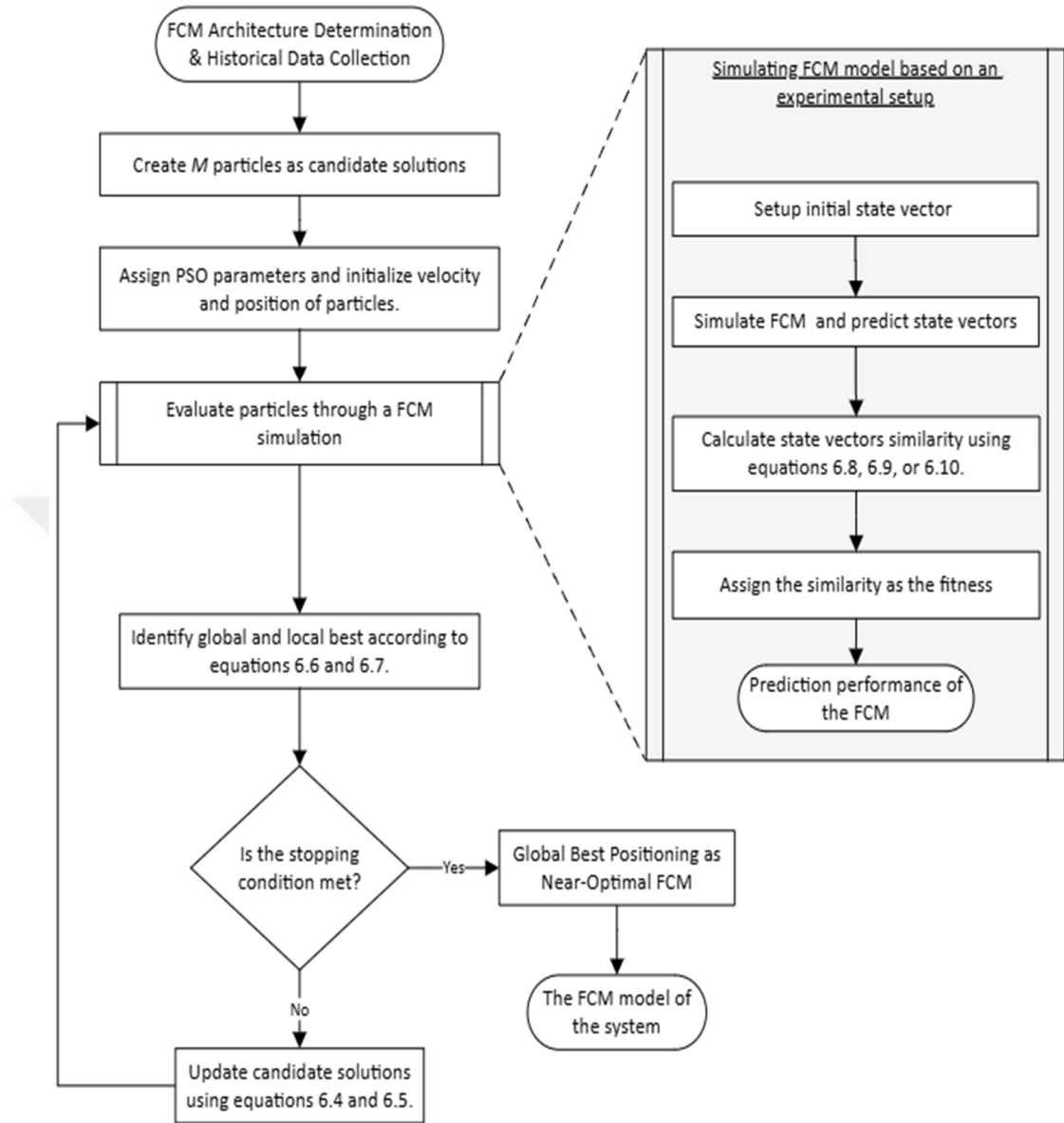


Figure 6.1: Flowchart of the metaheuristic optimization based learning FCM.

Step 3. The suitability of each particle is evaluated using fitness functions. For this purpose, the particles are converted to a square matrix, with each matrix representing a candidate FCM.

Step 3.1. Beginning with the first vector in the historical data, which is the initial state of the system, candidate FCMs are simulated a number of times equal to the count of successive vectors in the historical data.

Step 3.2. The deviation between the predicted consecutive state vectors and those in the historical data is determined using equations 6.8, 6.9, or 6.10. The deviation reflects the prediction quality of a candidate FCM.

Step 4. The fitness value of each particle in the current iteration is determined using equation 6.6, whereas the global best fitness value among all particles up to the current iteration is determined using equation 6.7.

Step 5. It is checked if the stopping criterion is met. If not, particle velocities and positions are updated according to equations 6.4 and 6.5, creating new candidate solutions. Step 3, in turn, is revisited to evaluate these new candidates. On the other hand, if the stopping criterion is met, proceed to step 6.

Step 6. The position of the particle that has yielded the global best so far is determined as the near-optimal FCM for the system.

6.2 Experimental Approach

This study builds on the advantages of the proposed method outlined in Section 6.1.1 introducing a noteworthy advancement in constructing FCMs. This new approach relies solely on historical data, eliminating the need for expert opinions. It adopts a training process similar to supervised learning, where FCMs learn to replicate the behavior of a complex system based on specific inputs using historical system data.

6.2.1 Dataset

Two critical aspects related to dataset selection are crucial for this approach. First, the data must demonstrate clear causal relationships between concepts. Second, the data samples (i.e., state vectors) should be collected in sequence. To our knowledge, datasets produced and used in this manner in FCM literature are rare. As addressed, most FCM research traditionally leans on an expert-based approach instead of a data-driven approach. Where data-driven models are employed, they often rely on simulated data from classical FCMs with randomly assigned weights rather than real-world data.

The use of datasets from simulations of randomly constructed FCMs unfairly benefits optimization methods in training FCMs adopted with the same reasoning mechanism. It would be unreasonable to compare algorithm performance using such data. This

study avoids this problematic approach in generating data by adopting the method of Dickerson and Kosko (1994), allowing the gathering of historical data that is more representative and is relatively free from any prevailing FCM algorithm.

To implement pro-FCM, we used datasets derived from the study of Dickerson and Kosko (1994) examining dolphin herd behavior. They developed an FCM model reflecting the goals and actions in the life of a dolphin, featuring 10 bivalent concepts to virtualize dolphin herd life. In this model, the activation levels of concepts take values from $\{0, 1\}$, where 1 indicates an active concept. The causal relationships between concepts are expressed with trivalent weights of $\{-1, 0, 1\}$. Table 6.1 illustrates these causal relationships and Table 6.2, Table 6.3 and Table 6.4 show how the activation levels vary across different scenarios.

Table 6.1: Concepts and causal relationships in the dolphin herd case (Dickerson and Kosko, 1994).

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈	C ₉	C ₁₀
Hunger (C₁)	0	-1	-1	0	0	1	0	0	0	0
Companionship (C₂)	0	0	0	0	1	0	0	0	0	0
Fatigue (C₃)	0	0	0	1	1	-1	-1	0	0	-1
Rest (C₄)	1	0	-1	0	0	-1	-1	0	0	-1
Herd clustering (C₅)	1	0	1	0	0	0	0	0	-1	0
Food search (C₆)	0	0	0	0	-1	0	1	0	0	0
Chase food (C₇)	0	0	0	0	0	0	0	1	0	0
Catch and eat food (C₈)	-1	1	-1	0	1	0	-1	0	0	0
Survival threat (C₉)	0	0	0	0	1	-1	-1	-1	0	1
Run away (C₁₀)	-1	-1	1	1	-1	-1	-1	-1	-1	0

Dickerson and Kosko examined the natural behaviors of organisms to generate sequential datasets, a method that also inspired the generation of datasets used in this study. They monitored the instantaneous states (snapshots) of a herd's cyclic behavior over time through a straightforward FCM simulation. This simulation revealed the transitions between these states. Notably, Dickerson and Kosko's simulation approach differs from prevailing FCM simulations, which typically use sigmoidal or related activation functions. Instead, they used a binary function that maps the activation of concepts to 0 or 1 based on a threshold value of 0.5. Accordingly, they simulated their approach starting from a scenario where only a life-threatening by a shark to the dolphin herd exists (that is, $C_9 = 1$). The resulting state vectors are detailed in Table 6.2.

Table 6.2 shows that the herd in state t_0 enters a cyclical behavior pattern after a while, unless there is an exogenous effect on any concept. The herd, in turn, reaches the first state of its circular behavior at time t_3 and returns to the same state after seven iterations.

Table 6.2: Snapshots of the system behavior after only C_9 is active.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
t_0	0	0	0	0	0	0	0	0	1	0
t_1	0	0	0	0	1	0	0	0	0	1
t_2	0	0	1	1	0	0	0	0	0	0
t_3	1	0	0	1	1	0	0	0	0	0
t_4	1	0	0	0	0	0	0	0	0	0
t_5	0	0	0	0	0	1	0	0	0	0
t_6	0	0	0	0	0	0	1	0	0	0
t_7	0	0	0	0	0	0	0	1	0	0
t_8	0	1	0	0	1	0	0	0	0	0
t_9	1	0	1	0	1	0	0	0	0	0
t_{10}	1	0	0	1	1	0	0	0	0	0

We generated an additional dataset by simulating the herd's behavior, starting the initial state vector from the second case identified by Dickerson and Kosko. This vector signifies a scenario in which the life-threatening situation is no longer present. By applying the same binary reasoning mechanism, we obtained the behavior pattern outlined in Table 6.3. This dataset simplifies the training of FCMs by eliminating the need to predict the preliminary states leading up to the herd's cyclical behavior pattern, a notable challenge in the earlier dataset.

Table 6.3: Snapshots of the system behavior after C_3 , C_4 , and C_5 are active.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
t_0	0	0	1	1	1	0	0	0	0	0
t_1	1	0	0	1	1	0	0	0	0	0
t_2	1	0	0	0	0	0	0	0	0	0
t_3	0	0	0	0	0	1	0	0	0	0
t_4	0	0	0	0	0	0	1	0	0	0
t_5	0	0	0	0	0	0	0	1	0	0
t_6	0	1	0	0	1	0	0	0	0	0
t_7	1	0	1	0	1	0	0	0	0	0
t_8	1	0	0	1	1	0	0	0	0	0

In Table 6.3, the first state (t_0) represents the herd at rest having overcome a threat and exhausted from their efforts to resist becoming prey. The table reveals that after only a single iteration, the herd enters a seven-step cyclical behavior, beginning

immediately after the initial state. In a nutshell, the herd has only a circular pattern of behavior.

It is evident that both datasets in Table 6.2 and Table 6.3 consist only of binary activation values. To demonstrate the applicability of the proposed method with continuous-valued activation levels, an alternative mapping method to 0.5 threshold mapping is explored. In this method, the new state vector is calculated by multiplying the state vector and the relationship matrix. However, rather than mapping these results to 0 or 1, we normalize them based on the highest value in the vector. Any activation level with a negative value in the normalized vector is replaced with 0. Following this, as shown in Table 6.4, the behavior pattern of the dolphin herd across consecutive times displays a cyclical pattern that stabilizes over a longer period compared to the binary datasets.

Table 6.4: Snapshots of the system (continuous-valued) behavior after C_3 , C_4 , and C_5 are active.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
t_0	0	0	1	1	1	0	0	0	0	0
t_1	1	0	0	0.5	0.5	0	0	0	0	0
t_2	1	0	0	0	0	0.5	0	0	0	0
t_3	0	0	0	0	0	1	0.5	0	0	0
t_4	0	0	0	0	0	0	1	0.5	0	0
t_5	0	0.5	0	0	0.5	0	0	1	0	0
t_6	0	0.67	0	0	1	0	0	0	0	0
t_7	1	0	1	0	0.67	0	0	0	0	0
t_8	0.67	0	0	1	1	0	0	0	0	0
t_9	1	0	0	0	0	0	0	0	0	0
t_{10}	0	0	0	0	0	1	0	0	0	0
t_{11}	0	0	0	0	0	0	1	0	0	0
t_{12}	0	0	0	0	0	0	0	1	0	0
t_{13}	0	1	0	0	1	0	0	0	0	0
t_{14}	1	0	1	0	1	0	0	0	0	0
t_{15}	1	0	0	1	1	0	0	0	0	0
t_{16}	1	0	0	0	0	0	0	0	0	0

6.2.2 Experimental setups

The evaluation of pro-FCM was conducted through a series of experimental studies. In these, we developed FCMs with the ability to learn the relationships between concepts, using historical data introduced in Section 6.2.1 as a guide. To determine the effectiveness of the proposed reasoning method, we compared these FCMs, which

incorporated our proposed approach, with those using traditional reasoning mechanisms, ensuring identical experimental conditions.

The evaluation methodology included testing various techniques at different stages of the learning FCM process. These techniques encompassed diverse normalization methods for the total effect matrix and state vectors, various performance measures, and both cumulative and non-cumulative calculation approaches. Notably, Dickerson and Kosko (1994) solely employed a non-cumulative method for calculating state vectors.

Each experimental setup is defined using a distinct combination of techniques. We thoroughly tested these setups, totaling 18 different setups for each of the three datasets (i.e., scenarios). The experimental studies were repeated 50 times for each of the proposed and two classical FCM methods. For each of the 18 different setups in each scenario, these repetitions began with a learning procedure using the same randomly generated initial relationship matrix. To measure the performance of the proposed FCM reasoning mechanism in these varied setups, we compared it with two classical FCM reasoning methods: sig-FCM and tan-FCM, which use the standard reasoning mechanism including log-sigmoid and hyperbolic tangent activation functions, respectively.

Figure 6.2 displays the techniques used and their role in forming an experimental setup. In this study, an experimental setup is denoted, for example, as 'lambda~heaviside~NA'. Here, 'lambda' refers to the vector normalization technique, 'heaviside' to the fitness function, and 'NA' (Not Applied) signifies that the cumulative calculation was not performed. It is important to note that the matrix normalization technique 'maximum normalization' is excluded from the notation, as no alternative techniques are considered.

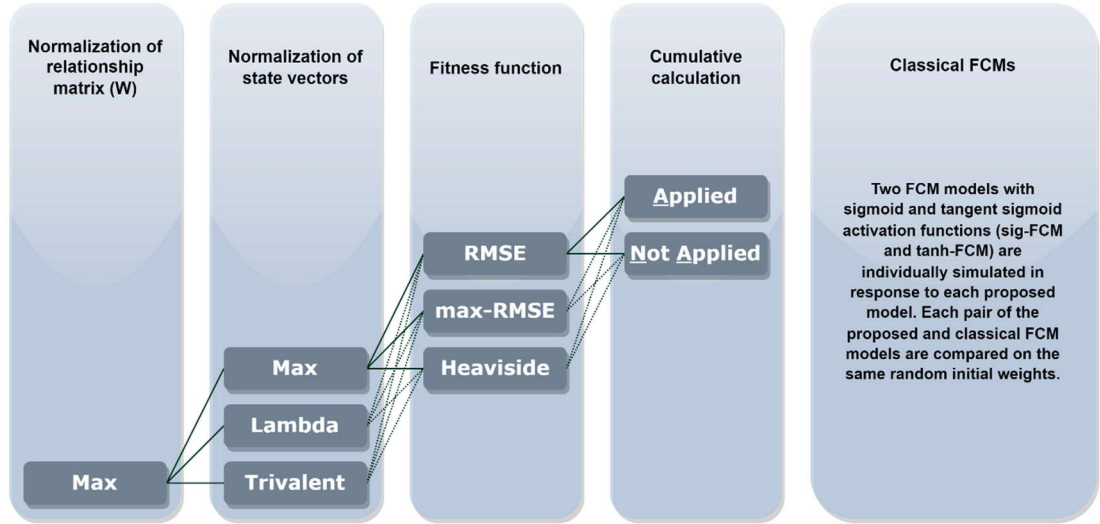


Figure 6.2: Design of experimental setups.

The maximum normalization for the weight matrix raised to its powers is adopted based on the findings of Murat and Asan (2021), which demonstrated its effectiveness. Equation 6.11 yields the normalized matrix ω^i , where $\max_{j,k} |w_{jk}^i|$ is the largest absolute value in W^i .

$$\omega^i = \frac{W^i}{\max_{j,k} |w_{jk}^i|} \quad (6.11)$$

In each simulation iteration, we tested three methods for normalizing the state vectors to ensure comparability. The first method is the maximum normalization f_{max} . In this approach, a state vector is normalized based on its largest absolute element. In addition, following this normalization, any element with a negative value is set to zero, as shown in equation 6.12.

$$f_{\max(A_i)} = \begin{cases} 0, & \frac{A_i}{\max_i A_i} < 0 \\ \frac{A_i}{\max_i A_i}, & \frac{A_i}{\max_i A_i} \geq 0 \end{cases} \quad (6.12)$$

where A_i is the activation value of C_i ($1 \leq i \leq n$) in the state vector to be normalized.

The second method of normalization, referred to as Lambda, adjusts the activation values to be near either 0 or 1. This method, as described in equation 6.13, employs a sigmoid function with a λ value of 8. In contrast to the sigmoid function with $\lambda = 1$,

which is commonly used in the literature, f_{λ} operates more like a step function, similar to the reasoning approach used by Dickerson and Kosko (1994).

$$f_{\lambda}(A_i) = \frac{1}{1 + e^{-8(A_i)}} \quad (6.13)$$

The third method, trivalent normalization $f_{trivalent}$, is described in equation 6.14. This method adjusts activation values that fall outside the $[0, 1]$ interval to match the interval endpoints, offering more flexibility for FCM modeling than f_{λ} .

$$f_{trivalent}(A_i) = \begin{cases} 0, & A_i < 0 \\ A_i, & 0 \leq A_i \leq 1 \\ 1, & A_i > 1 \end{cases} \quad (6.14)$$

The experimental study also compares alternative performance measurement approaches. Heaviside is based on binary-valued activation level comparisons, whereas RMSE and max-RMSE compare continuous predicted activation levels against actual ones. The activation levels in the predicted state vectors are ideally binary to match the above historical data. However, normalizing activation levels using the max, lambda, or trivalent methods results in continuous values within the range of $[0, 1]$. In such instances, prediction accuracy may hinge on the degree to which these continuous-valued activation levels align with the actual binary-valued levels.

The Heaviside fitness function evaluates this alignment, allowing a 10% deviation from the actual activation level. Activation levels falling within this margin are deemed accurate predictions. Therefore, the success of a model is measured by the proportion of accurately predicted activation levels relative to the total number of actual levels.

For more precise predictions, directly comparing the continuous values of the activation levels in the predicted state vectors can be more effective. In this approach, the focus is on the magnitude of deviation from the actual activation level, which is interpreted as a measure of closeness. RMSE and max-RMSE consider the average magnitude of deviations. RMSE provides a comprehensive measure, generating a single metric for all historical data. Max-RMSE, however, specifically evaluates the deviation of each state vector from its historical counterpart on an individual activation

level basis. It highlights the pair of predicted and actual state vectors that exhibit the maximum deviation.

6.2.3 Comparison with an alternative reasoning mechanism

In addition to comparing the proposed reasoning method with classical approaches, we offer a thorough evaluation by testing it against exponential normalized reasoning, a reasoning mechanism recently introduced by Nápoles et al. (2024) that, similar to the proposed one, operates without relying on traditional activation functions like sigmoid or hyperbolic tangent. This comparison allows us to evaluate the proposed method's reasoning capabilities against a similarly unconventional reasoning mechanism. It demonstrates the effectiveness of the proposed method in a broader context of alternative approaches.

Where conventional FCM frameworks rely on nonlinear functions to drive concept activation, exponential normalized reasoning takes a different route. It leverages the normalization of changes in concept inputs, followed by an exponentiation phase, to shape the activation dynamics of concepts. The core of this method is the modulation of concept activation levels through exponential scaling. While this process offers a solution to the saturation pitfalls of classic activation functions and brings an added layer of flexibility in managing nonlinearity, our interest in it is purely for comparative purposes. Since the pro-FCM method distinguishes itself from such function-based reasoning, evaluating its predictions against exponential normalized reasoning provides an opportunity to examine how these distinct approaches compare. As Nápoles et al. (2024) underscore, exponential normalized reasoning facilitates a direct and adaptable modeling of reasoning behaviors, especially in the face of shifting initial conditions.

To break down the mechanics: let $\Delta A^{(t)} = [\Delta A_1^{(t)}, \dots, \Delta A_i^{(t)}, \dots, \Delta A_n^{(t)}]$ denote the set of changes of concepts' activation levels. The activation of a given concept C_i at iteration $(t + 1)$ is calculated as follows:

$$A_i^{t+1} = A_i^t + \Delta A_i^{t+1} \theta_i^{t+1} \quad (6.15)$$

where θ_i^{t+1} represents the maximum amount of resources and

$$\Delta A_i^{t+1} = S\left(\sum_{j=1}^N \Delta A_j^t w_{ji}\right) \left(\left|\sum_{j=1}^N \Delta A_j^t w_{ji}\right| / \sum_{j=1}^N |w_{ji}|\right)^{\beta_i} \quad (6.16)$$

In this formulation, $S(\cdot)$ is the sign function, and β_i is a parameter that adjusts the degree of nonlinearity in concept activation. The term $\sum_{j=1}^N |w_{ji}|$ serves as an upper bound for the concept's capacity, ensuring a realistic normalization process where the activation values lie within the $[0,1]$ interval. As shown in Equation 6.16, the changes in the activation levels of concepts at an iteration rely on the raw changes in the activation levels that are exponential normalized.

Nápoles et al. (2024) states that this normalization operation mitigates the saturation issues inherent in traditional activation functions, where extreme input values lead to minimal changes in output. Following normalization, the exponentiation step, regulated by the parameter β_i modulates the concept's response, allowing for both contraction and expansion of activation levels. When $\beta_i = 1$, the reasoning behaves linearly; values of compress input variations, suitable for densely connected networks, while $0 < \beta_i < 1$ dilates input changes, ideal for sparsely connected systems.

For optimizing weights, the PSO algorithm will be applied along with the exponential normalized reasoning mechanism. Similar to the prior experiments, PSO iteratively adjusts these weights to optimize accuracy by refining fitness functions. This ensures the framework's capacity to accurately recreate the dataset and thus provides a foundation for reliable matrix comparisons.

The next section presents the results of the comparative analyses of the pro-FCM method against sig-FCM and tan-FCM, along with a comparison to the more recent exponential normalization method, providing insights into performance consistency of the proposed method.

6.3 Findings

Subsection 6.3.1 provides the results from comparisons of pro-FCM models against two classical methods: sig-FCM, which uses the log-sigmoid activation function, and tan-FCM, which employs the hyperbolic tangent activation function. These comparisons were conducted through simulations based on learning FCM, focusing on

predicting datasets for the scenarios outlined in Section 6.2.1 .2.1. The results of the comparison between the pro-FCM method and the exponential normalized reasoning method are presented in Subsection 6.3.2.

6.3.1 Results of the comparison with classical methods

For each experimental setup illustrated in Figure 6.2, separate simulations were conducted for pro-FCM, sig-FCM, and tan-FCM, all starting from the same random initial weight matrix. The 18 alternative configurations of pro-FCM were generated by considering all possible combinations of the factors: normalization of state vectors, fitness function and cumulative calculation status. To allow for statistical comparisons, each learning FCM-based simulation was repeated 50 times using different random initial weight matrices for each setup. The resulting prediction errors, calculated according to the defined fitness functions, were then analyzed to compare the reasoning mechanisms across the data scenarios.

The violin plots in Figures 6.3–6.11 illustrate the distribution of prediction errors across 50 replicates for the pro-FCM (green), sig-FCM (dark pink), and tan-FCM (cream) reasoning methods. Each violin's width represents the density of data points at different error values, providing insight into the variability and concentration of errors for each method.

Figures 6.3, 6.4, and 6.5 showcase the prediction errors of pro-FCM, sig-FCM and tan-FCM, calculated using the Heaviside fitness function for data from Scenario-1, Scenario-2, and Scenario-3. As a rule, lower prediction errors correspond to better prediction performance, providing a quantitative evaluation of the methods. Pro-FCM's violin plots are narrower and more concentrated around lower error values, reflecting a higher density of smaller errors. In contrast, sig-FCM shows a broader and more uniform distribution, indicating a wider error spread without a single central tendency. On the other hand, tan-FCM demonstrates a skewed distribution toward higher error values, with its density extending into larger errors, emphasizing its tendency for greater prediction inaccuracies compared to pro-FCM.

These figures show that the pro-FCM consistently outperforms the others in terms of prediction accuracy (i.e., lower prediction error) across all experimental setups when evaluated using the Heaviside fitness function. For example, in Scenario-1 (Figure 6.3), the proposed method successfully predicts approximately 83% of the concepts,

corresponding to an error rate of 0.1725, within a predefined acceptable margin of closeness (here, 10%) to the actual values, as seen in the experimental setup max~heaviside~NA. Conversely, in the setup showcasing the best performance of the sig-FCM method, the success rate is 51%, while the highest performance of tan-FCM reaches only 50% for the data from Scenario-1.



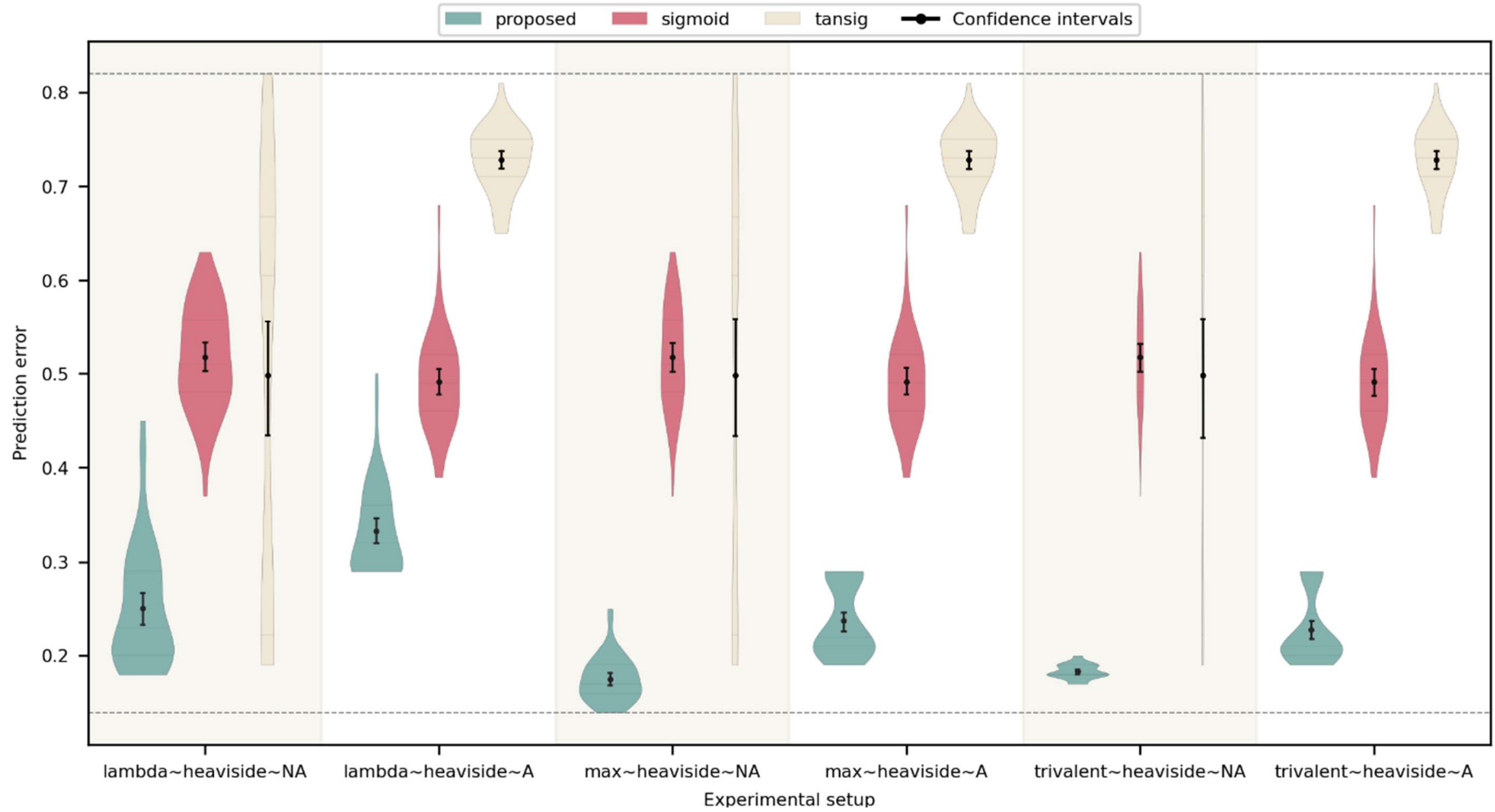


Figure 6.3: Prediction performance of reasoning methods for Scenario-1 according to the Heaviside fitness function.

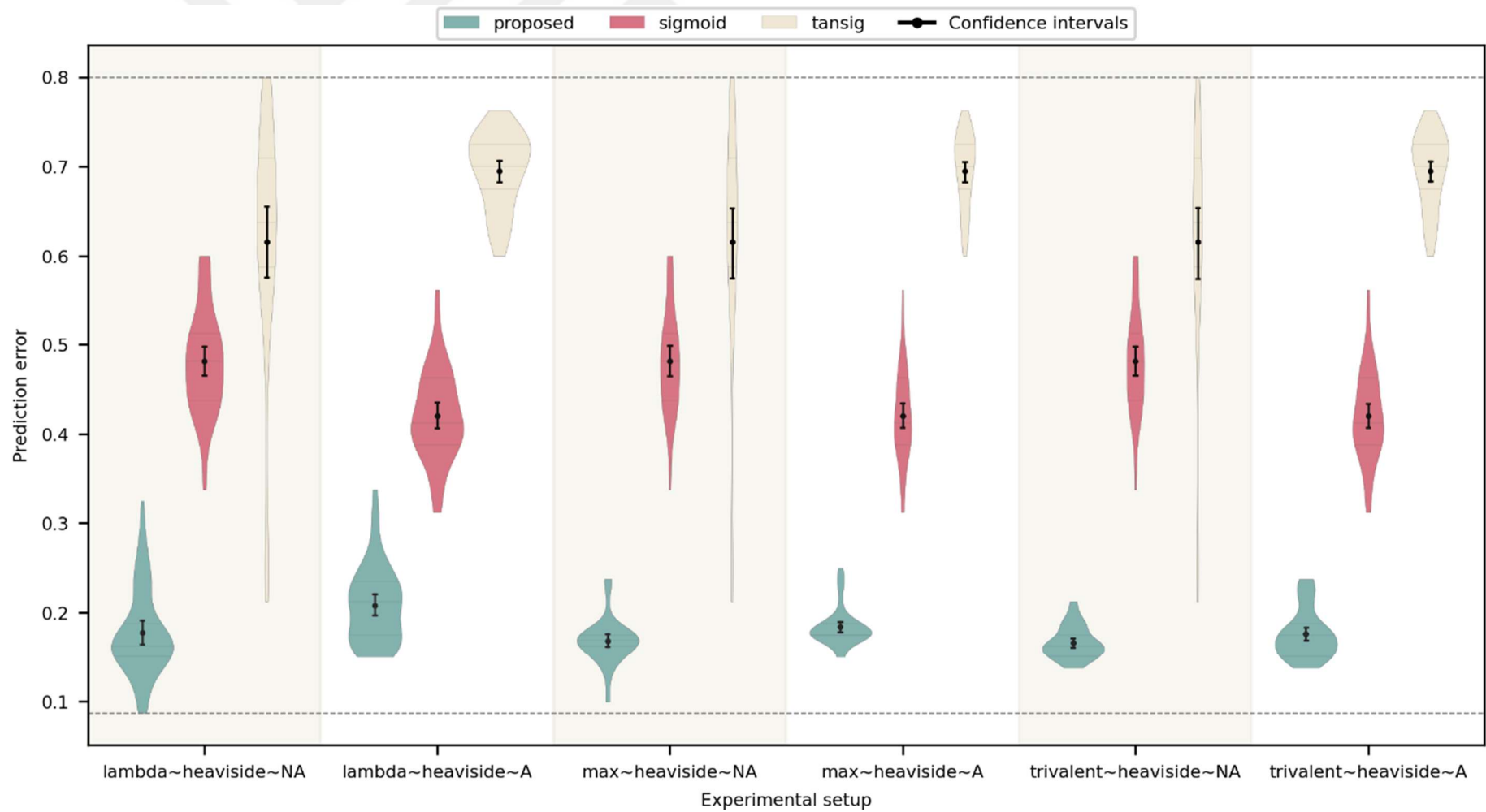


Figure 6.4: Prediction performance of reasoning methods for Scenario-2 according to the Heaviside fitness function.

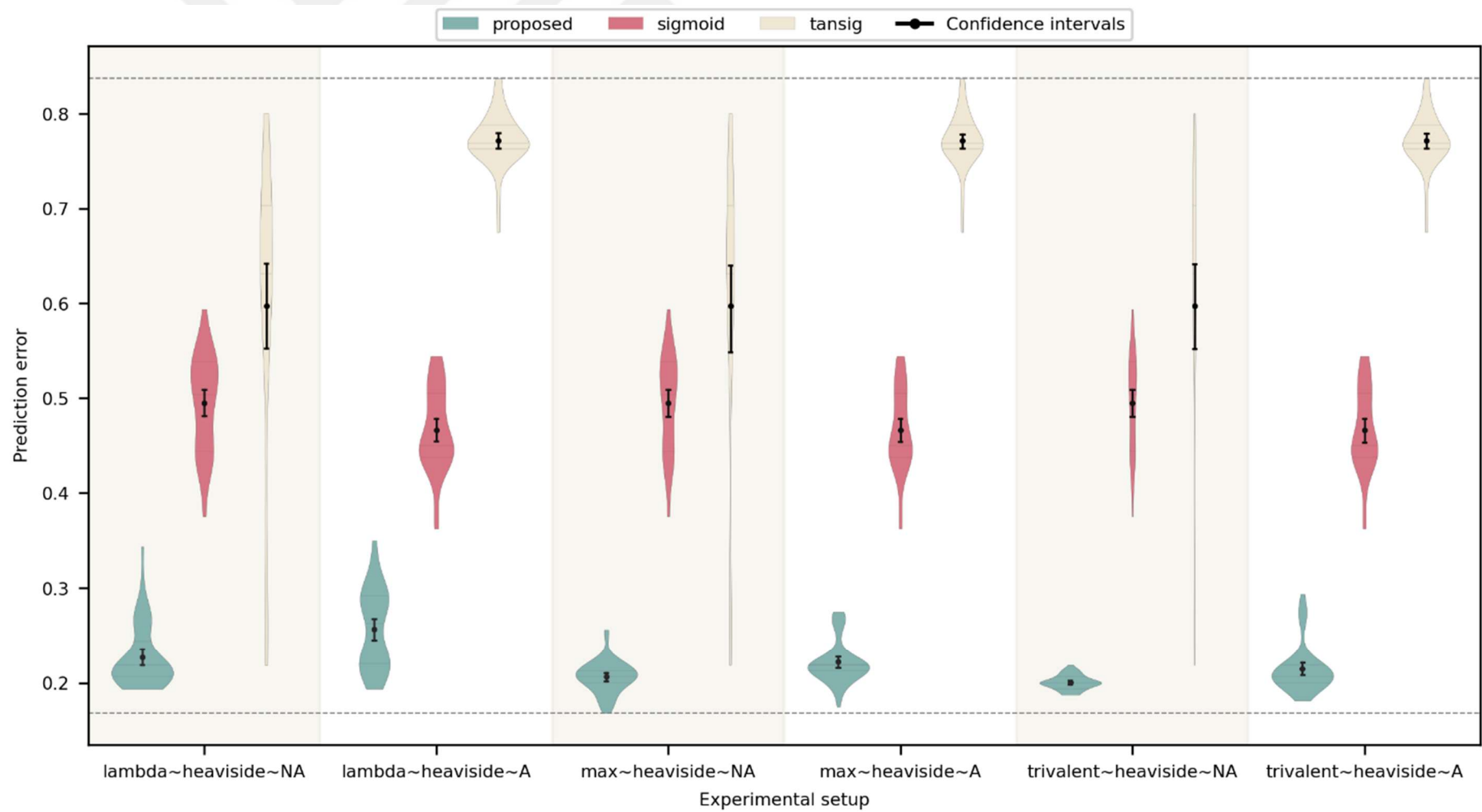


Figure 6.5: Prediction performance of reasoning methods for Scenario-3 according to the Heaviside fitness function.

Figures 6.6, 6.7, and 6.8 present the results of the experimental setups using the max-RMSE fitness function. The violin plots reveal notable differences in error distributions: pro-FCM and tan-FCM exhibit narrower shapes with greater distance between the lowest and highest prediction errors, indicating less concentration around a specific error value. The large distance between the lowest and highest prediction errors in pro-FCM suggests that it represents models with the best individual performances, even if not on average. In contrast, sig-FCM shows a more compact error range with short, wide violins, reflecting tightly clustered predictions. This aligns with the 95% confidence interval analysis, where sig-FCM achieves smaller error intervals, highlighting its stability in predictions.

The results from Figures 6.6, 6.7 and 6.8 clearly indicate that the proposed method achieves superior prediction performance. However, it is important to note that in some setups, the prediction performance of the proposed method is relatively similar to, and occasionally slightly less effective than, other reasoning mechanisms. Consequently, the results indicate that the proposed method achieves superior prediction performance in most setups, while in some setups its performance is relatively comparable to or occasionally slightly less effective than other reasoning mechanisms.

The results from Figures 6.9, 6.10, and 6.11, which pertain to experimental setups using the RMSE fitness function, demonstrate again that the pro-FCM models achieve the superior prediction performance compared to other reasoning mechanisms. The proposed method exhibits a wide range between the lowest and highest prediction error values, with the lowest prediction errors consistently observed in this method. Sig-FCMs, on the other hand, generate prediction errors resembling those of the worst-performing tan-FCMs, following a distribution akin to normal. Notably, except for the setups involving lambda vector normalization, the pro-FCM models also excel in average prediction performance compared to other reasoning mechanisms. In scenarios without lambda vector normalization, the superiority of the pro-FCM models is clear, while in the lambda normalization setups, their performance is still competitive, though not always the best.

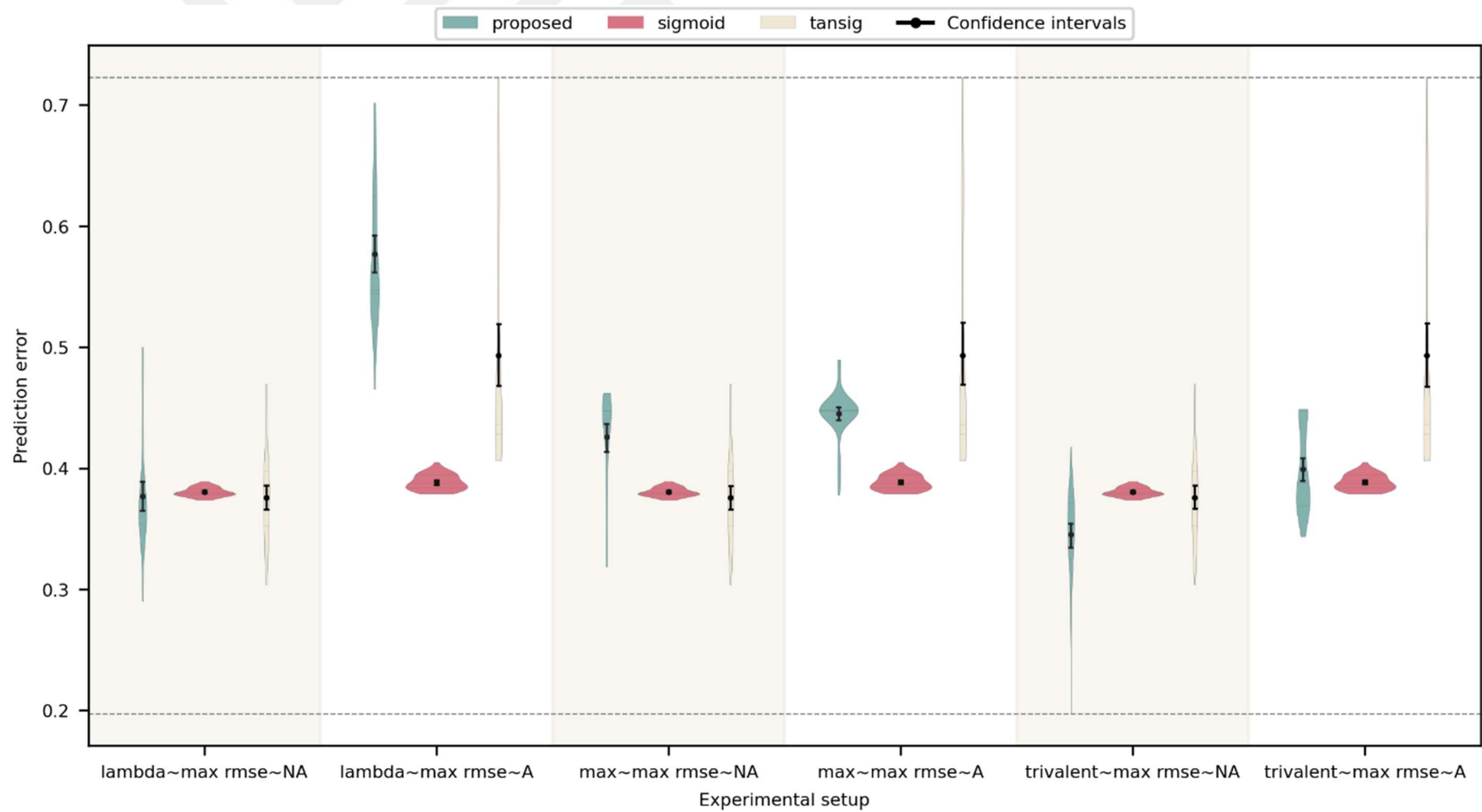


Figure 6.6: Prediction performance of reasoning methods for Scenario-1 according to the max-RMSE fitness function.

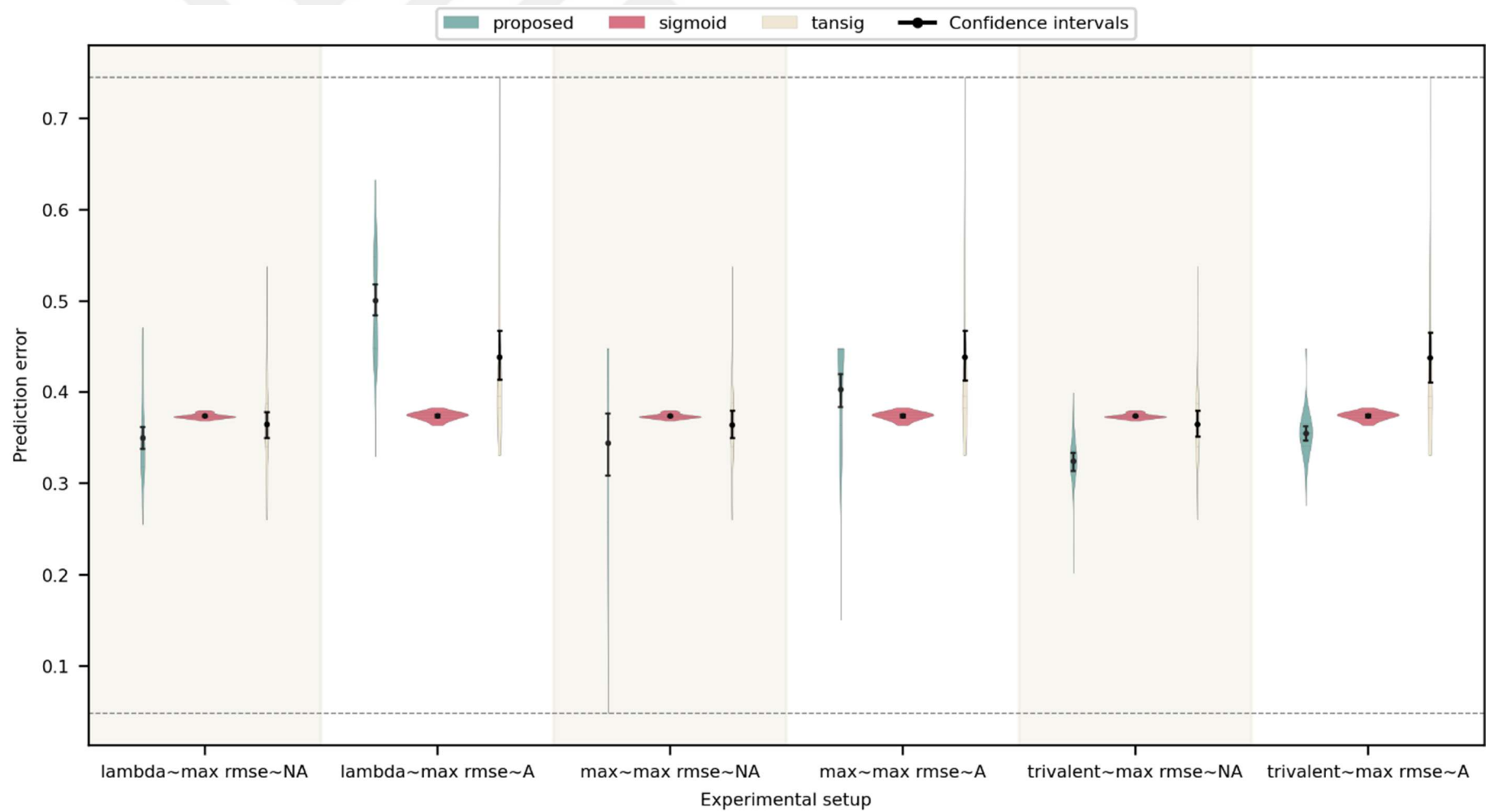


Figure 6.7: Prediction performance of reasoning methods for Scenario-2 according to the max-RMSE fitness function.

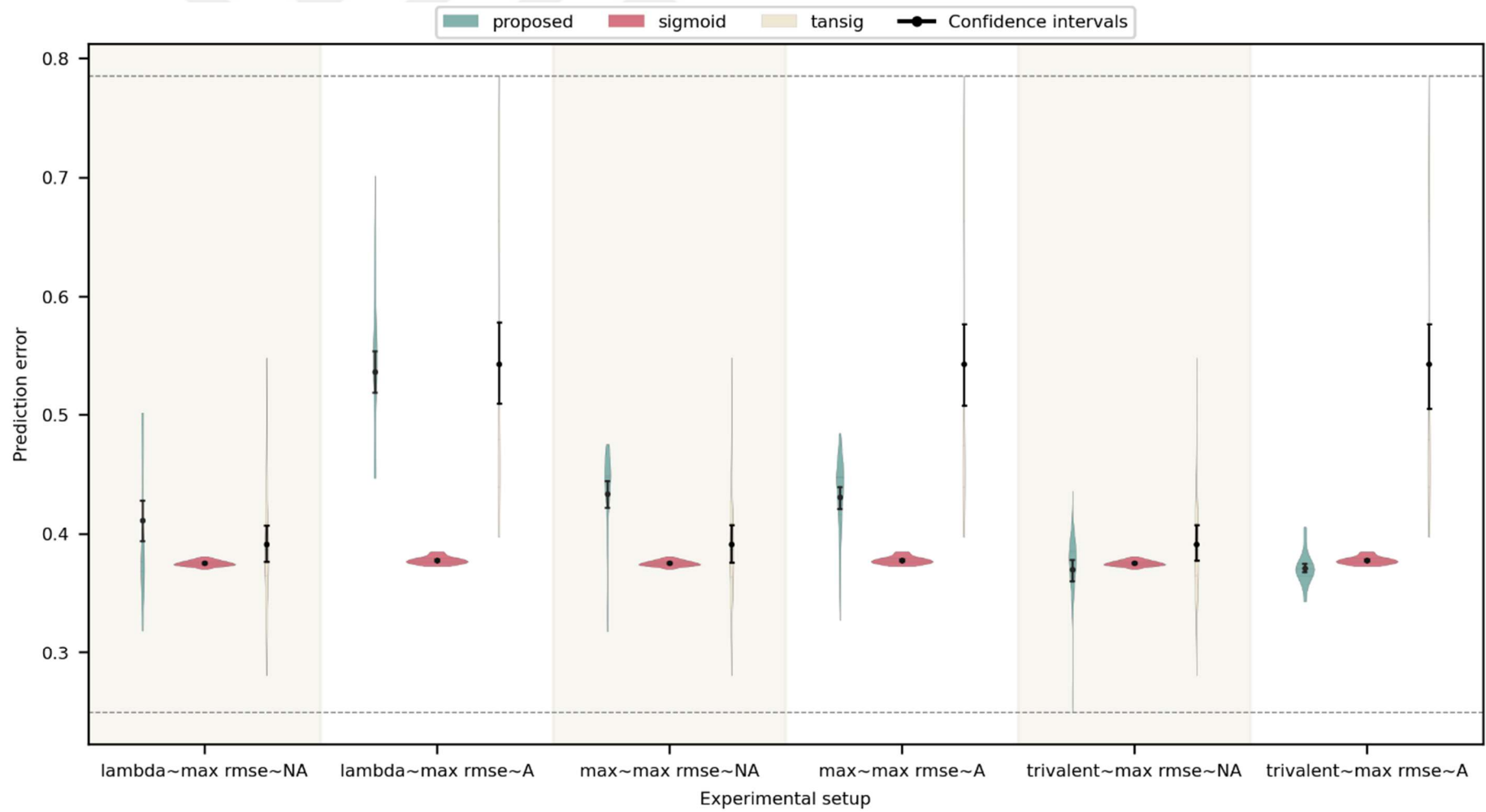


Figure 6.8: Prediction performance of reasoning methods for Scenario-3 according to the max-RMSE fitness function.

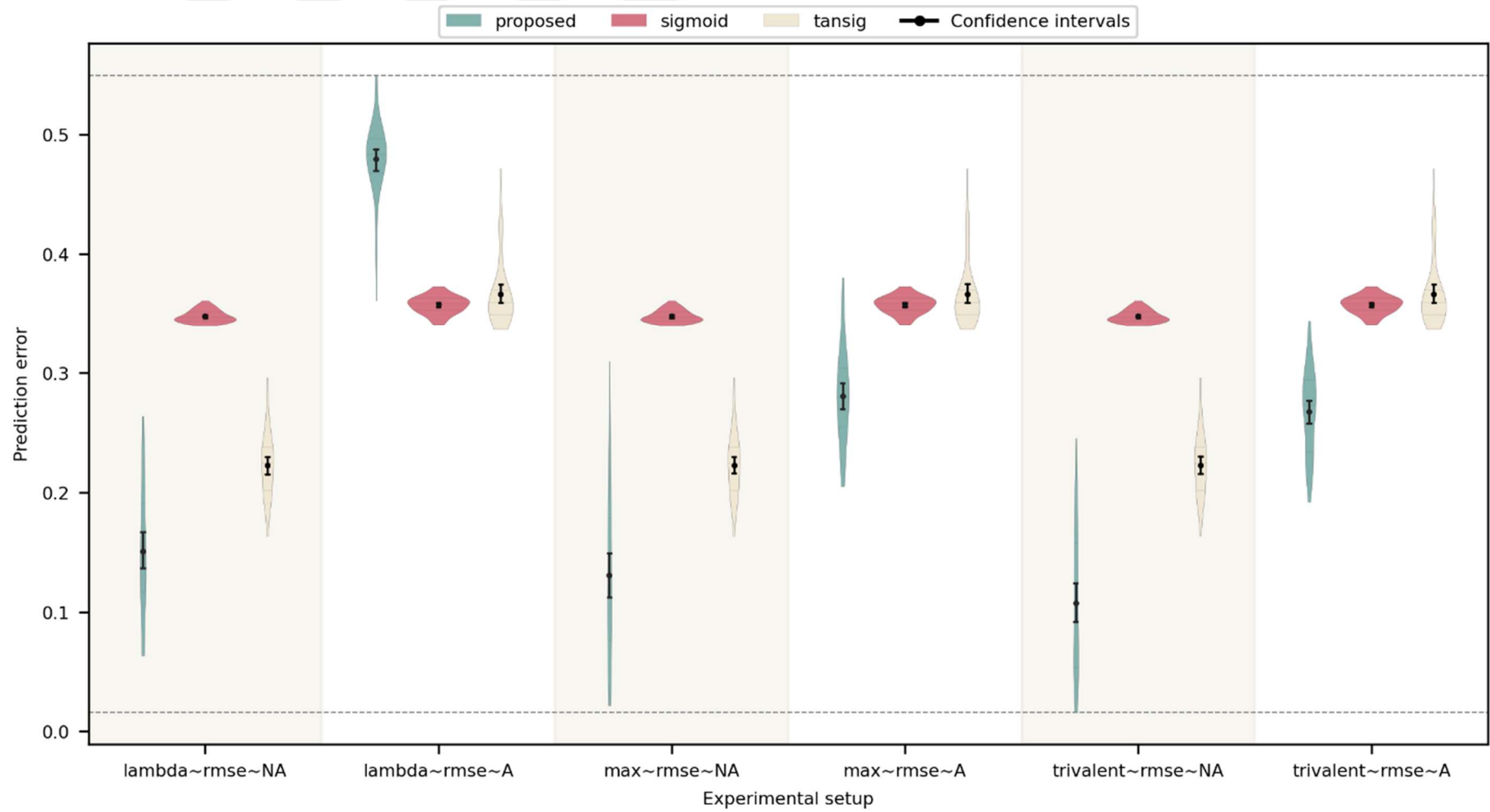


Figure 6.9: Prediction performance of reasoning methods for Scenario-1 according to the RMSE fitness function.

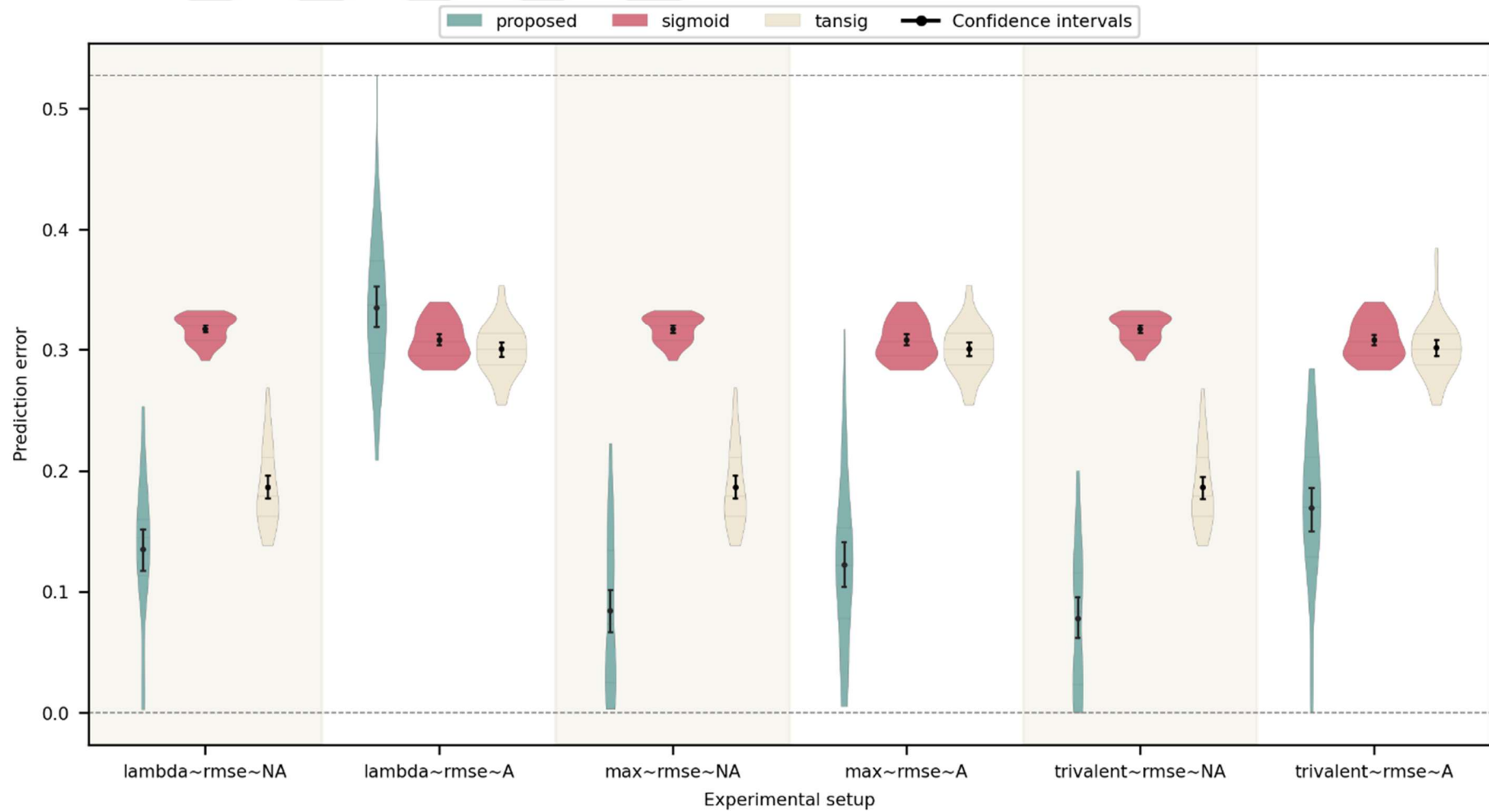


Figure 6.10: Prediction performance of reasoning methods for Scenario-2 according to the RMSE fitness function.

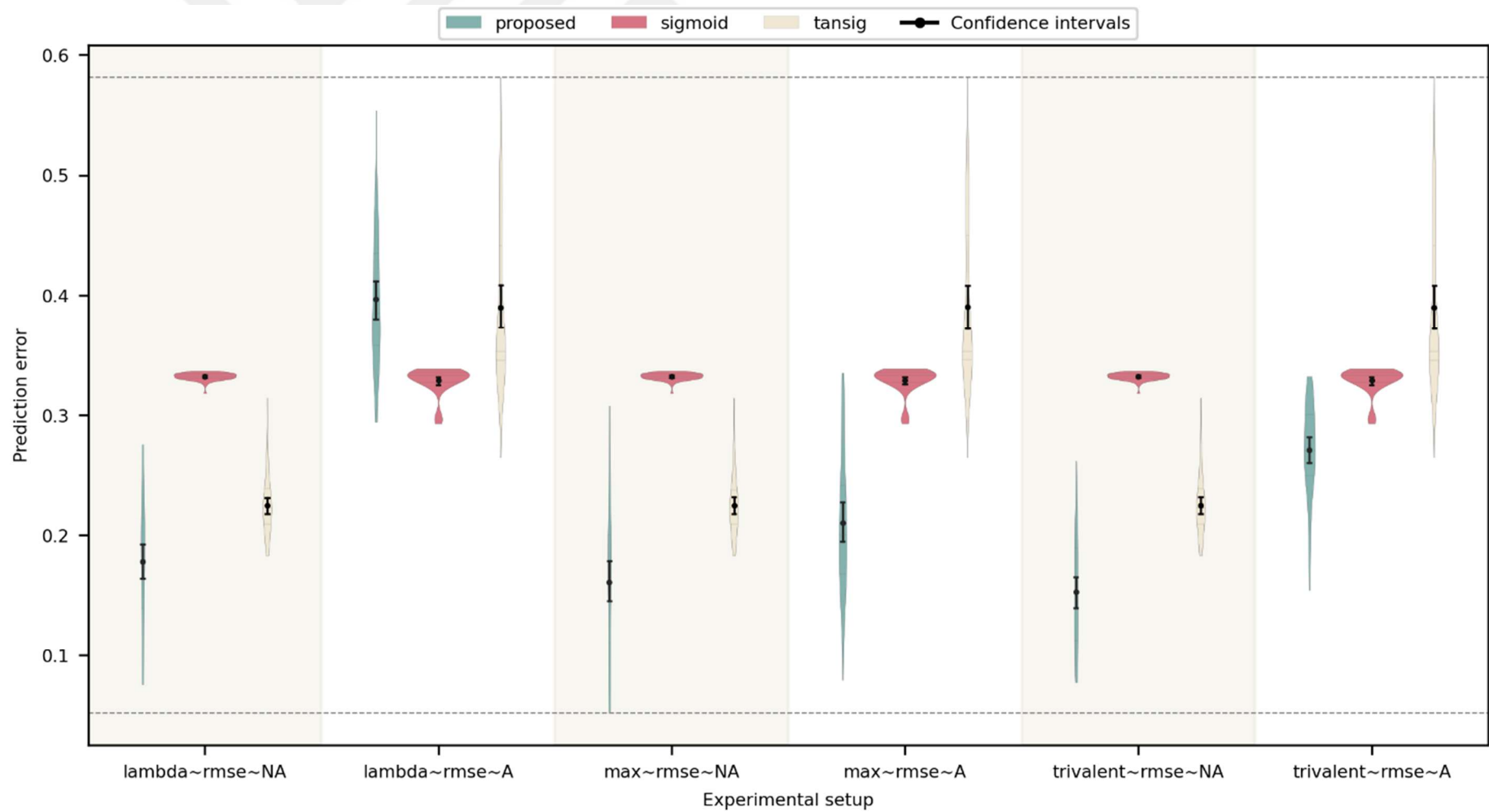


Figure 6.11: Prediction performance of reasoning methods for Scenario-3 according to the RMSE fitness function.

Statistical testing was applied to assess whether the differences in prediction performance among reasoning mechanisms across experimental setups were statistically significant or merely coincidental. In each setup, the mechanisms were compared pairwise. Given that most data, each with 50 error rates, did not meet the normality assumption, the Wilcoxon signed-rank test was used.

The Wilcoxon signed-rank test, a non-parametric statistical method, is used to infer conclusions about the median value of a population without making specific assumptions about data distribution. It is particularly effective for paired-sample designs, which are prevalent in studies in which the same subject is measured under varying conditions or treatments. The aim is to compare the differences between these conditions or treatments for each subject (Taheri and Hesamian, 2013). In this study, the test is employed to conduct pair-wise comparisons of prediction performances derived from experimental outcomes using the same initial relationship weight.

Table 6.5 displays the average prediction performances from the experiments for the Scenario-1 dataset, along with the results of the statistical test on these averages. The pro-FCM outperforms the other mechanisms, particularly in experimental setups not including lambda vector normalization or max-RMSE fitness function, where it achieves the best results. Statistically significant improvements in performance by the proposed method are evident in most experimental setups, with the exceptions of 'lambda~rmse~A', 'lambda~max.rmse~A', 'max~max.rmse~NA', and 'max~max.rmse~A', where the classical reasoning mechanism with sigmoid function surpasses. In other setups 'lambda~max.rmse~NA' and 'trivalent~max.rmse~A', where pro-FCM could not show superiority, other methods did not provide a statistically significant superiority.

The superior performance of Pro-FCM in experimental setups is also supported by the effect size statistics r with a value of approximately 0.60. The r statistic, a measure of Cohen's effect size, is interpreted as a large effect in treatments when it exceeds 0.5 (Coolican, 2013).

Table 6.5: Summary of the average performances for 50 repetitions and statistical tests, Scenario-1.

Setting	mean (std)*			Wilcoxon signed-rank test stat. (effect size)**		
	pro-FCM	sig-FCM	tan-FCM	pro vs. sig	pro vs tan	sig vs. tan
lambda~heaviside~NA	0.251 (0.064)	0.517 (0.056)	0.498 (0.234)	0.001 (0.616)	0.001 (0.485)	0.406 (0.083)
lambda~heaviside~A	0.333 (0.048)	0.491 (0.052)	0.728 (0.035)	0.001 (0.612)	0.001 (0.616)	0.001 (0.616)
max~heaviside~NA	0.175 (0.024)	0.517 (0.056)	0.498 (0.234)	0.001 (0.616)	0.001 (0.614)	0.406 (0.083)
max~heaviside~A	0.238 (0.038)	0.491 (0.052)	0.728 (0.035)	0.001 (0.610)	0.001 (0.610)	0.001 (0.610)
trivalent~heaviside~NA	0.183 (0.008)	0.517 (0.056)	0.498 (0.234)	0.001 (0.616)	0.001 (0.603)	0.406 (0.083)
trivalent~heaviside~A	0.228 (0.037)	0.491 (0.052)	0.728 (0.035)	0.001 (0.616)	0.001 (0.616)	0.001 (0.616)
lambda~rmse~NA	0.151 (0.055)	0.348 (0.005)	0.222 (0.026)	0.001 (0.615)	0.001 (0.539)	0.001 (0.615)
lambda~rmse~A	0.479 (0.032)	0.358 (0.008)	0.367 (0.028)	0.001 (0.615)	0.001 (0.615)	0.213 (0.125)
max~rmse~NA	0.131 (0.072)	0.348 (0.005)	0.222 (0.026)	0.001 (0.615)	0.001 (0.543)	0.001 (0.615)
max~rmse~A	0.280 (0.040)	0.358 (0.008)	0.367 (0.028)	0.001 (0.608)	0.001 (0.602)	0.213 (0.125)
trivalent~rmse~NA	0.108 (0.061)	0.348 (0.005)	0.222 (0.026)	0.001 (0.615)	0.001 (0.598)	0.001 (0.615)
trivalent~rmse~A	0.268 (0.036)	0.358 (0.008)	0.367 (0.028)	0.001 (0.615)	0.001 (0.615)	0.213 (0.125)
lambda~max.rmse~NA	0.377 (0.045)	0.381 (0.004)	0.376 (0.037)	0.010 (0.256)	0.847 (0.019)	0.247 (0.116)
lambda~max.rmse~A	0.577 (0.058)	0.389 (0.006)	0.493 (0.096)	0.001 (0.615)	0.001 (0.438)	0.001 (0.615)
max~max.rmse~NA	0.426 (0.04)	0.381 (0.004)	0.376 (0.037)	0.001 (0.545)	0.001 (0.484)	0.247 (0.116)
max~max.rmse~A	0.445 (0.020)	0.389 (0.006)	0.493 (0.096)	0.001 (0.609)	0.149 (0.144)	0.001 (0.615)
trivalent~max.rmse~NA	0.345 (0.038)	0.381 (0.004)	0.376 (0.037)	0.001 (0.550)	0.001 (0.339)	0.247 (0.116)
trivalent~max.rmse~A	0.399 (0.035)	0.389 (0.006)	0.493 (0.096)	0.141 (0.147)	0.001 (0.549)	0.001 (0.615)

* Values in brackets indicate standard deviation; ** Values in brackets indicate effect size.

Table 6.6 displays the results for the Scenario-2 dataset. The pro-FCM outperforms the other mechanisms, especially in experimental setups involving trivalent normalization. Statistically significant improvements in performance by the proposed method are evident across all experimental setups, with the exceptions of ‘lambda~max rmse~A’, ‘lambda~rmse~A’, and ‘max~max rmse~A’. Note that in the setup ‘max~max rmse~NA’, the differences in prediction performances between any pair of reasoning mechanisms are not statistically significant. In the rest of the experimental setups, the pro-FCM consistently exhibited better performance, as supported by r with a value of approximately 0.6.

Finally, Table 6.7 provides the results for the Scenario-3 dataset. The pro-FCM again outperforms the other mechanisms in most setups. In some setups where vector normalization is denoted by lambda, the sig-FCM shows a performance advantage over pro-FCM. Among the setups in which pro-FCM performs better, it does not statistically significantly outperform sig-FCM only in ‘trivalent~max.rmse~A’. The

performance of pro-FCM is further supported by effect size statistics r with values of approximately 0.6.

Table 6.6: Summary of the average performances for 50 repetitions and statistical tests, Scenario-2.

Setting	Mean (std)*			Wilcoxon signed-rank test stat. (effect size)**		
	pro-FCM	sig-FCM	tan-FCM	pro vs. sig	pro vs tan	sig vs. tan
lambda~heaviside~NA	0.178 (0.049)	0.482 (0.059)	0.616 (0.147)	0.001 (0.616)	0.001 (0.615)	0.001 (0.424)
lambda~heaviside~A	0.208 (0.043)	0.420 (0.051)	0.695 (0.042)	0.001 (0.616)	0.001 (0.616)	0.001 (0.616)
max~heaviside~NA	0.168 (0.027)	0.482 (0.059)	0.616 (0.147)	0.001 (0.616)	0.001 (0.616)	0.001 (0.424)
max~heaviside~A	0.184 (0.021)	0.420 (0.051)	0.695 (0.042)	0.001 (0.616)	0.001 (0.616)	0.001 (0.616)
trivalent~heaviside~NA	0.166 (0.018)	0.482 (0.059)	0.616 (0.147)	0.001 (0.616)	0.001 (0.616)	0.001 (0.424)
trivalent~heaviside~A	0.176 (0.027)	0.420 (0.051)	0.695 (0.042)	0.001 (0.616)	0.001 (0.616)	0.001 (0.616)
lambda~rmse~NA	0.135 (0.061)	0.318 (0.011)	0.186 (0.033)	0.001 (0.615)	0.001 (0.439)	0.001 (0.615)
lambda~rmse~A	0.336 (0.061)	0.308 (0.016)	0.301 (0.022)	0.004 (0.291)	0.001 (0.359)	0.043 (0.203)
max~rmse~NA	0.084 (0.065)	0.318 (0.011)	0.186 (0.033)	0.001 (0.615)	0.001 (0.591)	0.001 (0.615)
max~rmse~A	0.122 (0.070)	0.308 (0.016)	0.301 (0.022)	0.001 (0.614)	0.001 (0.615)	0.043 (0.203)
trivalent~rmse~NA	0.078 (0.062)	0.318 (0.011)	0.186 (0.033)	0.001 (0.615)	0.001 (0.598)	0.001 (0.615)
trivalent~rmse~A	0.169 (0.066)	0.308 (0.016)	0.302 (0.024)	0.001 (0.615)	0.001 (0.615)	0.068 (0.183)
lambda~max.rmse~NA	0.350 (0.045)	0.373 (0.003)	0.364 (0.052)	0.001 (0.323)	0.076 (0.177)	0.056 (0.191)
lambda~max.rmse~A	0.500 (0.065)	0.374 (0.005)	0.438 (0.099)	0.001 (0.614)	0.002 (0.317)	0.001 (0.496)
max~max.rmse~NA	0.344 (0.115)	0.373 (0.003)	0.364 (0.053)	0.347 (0.094)	0.524 (0.064)	0.054 (0.193)
max~max.rmse~A	0.403 (0.063)	0.374 (0.005)	0.438 (0.099)	0.001 (0.366)	0.466 (0.073)	0.001 (0.496)
trivalent~max.rmse~NA	0.324 (0.033)	0.373 (0.003)	0.364 (0.052)	0.001 (0.603)	0.001 (0.411)	0.056 (0.191)
trivalent~max.rmse~A	0.355 (0.028)	0.374 (0.005)	0.437 (0.099)	0.001 (0.471)	0.001 (0.512)	0.001 (0.496)

* Values in brackets indicate standard deviation; ** Values in brackets indicate effect size.

Table 6.7: Summary of the average performances for 50 repetitions and statistical tests, Scenario-3.

Setting	mean (std)*			Wilcoxon signed-rank test stat. (effect size)**		
	pro-FCM	sig-FCM	tan-FCM	pro vs. sig	pro vs tan	sig vs. tan
lambda~heaviside~NA	0.227 (0.033)	0.495 (0.051)	0.598 (0.165)	0.001 (0.616)	0.001 (0.612)	0.001 (0.340)
lambda~heaviside~A	0.257 (0.042)	0.466 (0.044)	0.772 (0.028)	0.001 (0.616)	0.001 (0.616)	0.001 (0.616)
max~heaviside~NA	0.206 (0.017)	0.495 (0.051)	0.598 (0.165)	0.001 (0.616)	0.001 (0.616)	0.001 (0.340)
max~heaviside~A	0.222 (0.022)	0.466 (0.044)	0.772 (0.028)	0.001 (0.616)	0.001 (0.616)	0.001 (0.616)
trivalent~heaviside~NA	0.201 (0.007)	0.495 (0.051)	0.598 (0.165)	0.001 (0.616)	0.001 (0.616)	0.001 (0.340)
trivalent~heaviside~A	0.215 (0.025)	0.466 (0.044)	0.772 (0.028)	0.001 (0.616)	0.001 (0.616)	0.001 (0.616)
lambda~rmse~NA	0.178 (0.055)	0.332 (0.003)	0.225 (0.025)	0.001 (0.615)	0.001 (0.435)	0.001 (0.615)
lambda~rmse~A	0.397 (0.057)	0.329 (0.012)	0.390 (0.068)	0.001 (0.582)	0.454 (0.075)	0.001 (0.835)
max~rmse~NA	0.161 (0.062)	0.332 (0.003)	0.225 (0.025)	0.001 (0.615)	0.001 (0.504)	0.001 (0.615)
max~rmse~A	0.210 (0.058)	0.329 (0.012)	0.390 (0.067)	0.001 (0.615)	0.001 (0.610)	0.001 (0.584)
trivalent~rmse~NA	0.153 (0.048)	0.332 (0.003)	0.225 (0.025)	0.001 (0.615)	0.001 (0.589)	0.001 (0.615)
trivalent~rmse~A	0.271 (0.040)	0.329 (0.012)	0.390 (0.068)	0.001 (0.614)	0.001 (0.607)	0.001 (0.584)
lambda~max.rmse~NA	0.411 (0.060)	0.375 (0.002)	0.391 (0.054)	0.022 (0.230)	0.086 (0.172)	0.056 (0.191)
lambda~max.rmse~A	0.536 (0.063)	0.378 (0.003)	0.543 (0.124)	0.001 (0.615)	0.791 (0.027)	0.001 (0.615)
max~max.rmse~NA	0.433 (0.040)	0.375 (0.002)	0.391 (0.054)	0.001 (0.576)	0.001 (0.394)	0.055 (0.192)
max~max.rmse~A	0.431 (0.034)	0.378 (0.003)	0.543 (0.125)	0.001 (0.577)	0.001 (0.469)	0.001 (0.615)
trivalent~max.rmse~NA	0.370 (0.032)	0.375 (0.002)	0.391 (0.054)	0.757 (0.031)	0.018 (0.237)	0.056 (0.191)
trivalent~max.rmse~A	0.371 (0.013)	0.378 (0.003)	0.543 (0.124)	0.001 (0.330)	0.001 (0.615)	0.001 (0.615)

* Values in brackets indicate standard deviation; ** Values in brackets indicate effect size.

To evaluate the effectiveness and generalization capabilities of the pro-FCM in prediction, a model trained on a dataset with $N - 1$ consecutive state vectors can be used to predict the final (N^{th}) state vector. First, the last state vector in each dataset is set aside as the test data. Subsequently, as described above, the FCM for each experimental setup is trained using the previous $N - 1$ state vectors. After training, the trained FCM is resimulated from the initial state vector of the training dataset to predict the test state vector. The prediction performance of the FCM is then evaluated by comparing the actual test state vector with the predicted state vector.

Table 6.8 summarizes the prediction performance of the trained FCMs across datasets including the first $N - 1$ state vectors. The results in the table are related to the experimental setups including Heaviside fitness function which shows dominant high performance in out-of-sample error. It is important to note that the values in the table are the average error rates derived from 50 trials. Therefore, the values after the decimal point should be interpreted as the proportion of incorrectly predicted concepts relative to the total number tested, rather than as an exact rate for any individual simulation.

Table 6.8: Prediction performance of trained FCMs across all training datasets.

Setting	Scenario 1			Scenario 2			Scenario 3		
	pro-FCM	sig-FCM	tan-FCM	pro-FCM	sig-FCM	tan-FCM	pro-FCM	sig-FCM	tan-FCM
lambda~heaviside~NA	0.17	0.48	0.66	0.23	0.50	0.61	0.25	0.53	0.35
lambda~heaviside~A	0.21	0.45	0.71	0.26	0.47	0.77	0.31	0.52	0.73
max~heaviside~NA	0.16	0.48	0.66	0.21	0.50	0.62	0.17	0.53	0.35
max~heaviside~A	0.18	0.45	0.71	0.22	0.47	0.77	0.25	0.52	0.73
trivalent~heaviside~NA	0.14	0.48	0.66	0.21	0.50	0.61	0.17	0.53	0.35
trivalent~heaviside~A	0.16	0.45	0.71	0.21	0.47	0.78	0.22	0.52	0.73

The average errors for 50 trained FCMs for each setup with the Heaviside fitness function in the prediction of the test state vector are provided in Table 6.9. A key observation is that pro-FCM's predictions fall within the $\pm 10\%$ tolerance range defined by the Heaviside function more frequently than those of the other mechanisms, resulting in a lower overall prediction error. Specifically, in Scenario-2 and under the 'trivalent~heaviside~NA' setup, the proposed method stands out by accurately predicting 9 of 10 concepts. Conversely, FCMs using sigmoid or hyperbolic tangent functions managed acceptable predictions for at most five concepts.

Further analysis of Table 6.9 reveals subtle performance differences. Although the proposed method shows comparable results to the sig-FCM method in Scenario-3,

where activation levels are continuous, it excels in scenarios with binary activation levels. In both Scenario-1 and Scenario-2, the proposed approach significantly outperforms other methods in terms of accurate predictions. These findings illustrate the robustness of the proposed learning approach under various conditions. It successfully predicts consecutive state vectors, representing system behavior over N periods, and effectively determines the state vector at any specific time t , based on the previous $t - 1$ state vectors.

Table 6.9: Prediction performance of trained FCMs across the test dataset.

Setting	Scenario 1			Scenario 2			Scenario 3		
	pro-FCM	sig-FCM	tan-FCM	pro-FCM	sig-FCM	tan-FCM	pro-FCM	sig-FCM	tan-FCM
lambda~heaviside~NA	0.29	0.47	0.74	0.18	0.49	0.55	0.32	0.40	0.45
lambda~heaviside~A	0.21	0.37	0.71	0.23	0.45	0.82	0.36	0.36	0.74
max~heaviside~NA	0.28	0.47	0.74	0.17	0.49	0.57	0.30	0.40	0.45
max~heaviside~A	0.25	0.37	0.71	0.20	0.45	0.82	0.32	0.36	0.74
trivalent~heaviside~NA	0.30	0.47	0.74	0.11	0.49	0.55	0.30	0.40	0.45
trivalent~heaviside~A	0.23	0.37	0.71	0.19	0.45	0.82	0.32	0.36	0.73

As discussed previously, this study does not include an analysis of model-error fitness functions in addition to the data-error fitness functions used to evaluate the approaches for learning FCM. Model-error, which compares the weights of the learned FCM with those of the true model, is applicable when true weights are known or specific targets for the weights are set (Chen et al., 2012, 2015). As discussed previously, this study does not include an analysis of model-error fitness functions in addition to the data-error fitness functions used to evaluate the approaches for learning FCM. Model-error, which compares the weights of the learned FCM with those of the true model, is applicable when true weights are known or specific targets for the weights are set (Chen et al., 2012, 2015). However, in real-world scenarios where actual weights are typically unavailable, the use of these functions is often limited. Therefore, many studies prioritize developing models that accurately predict system behavior, even when the true relationships remain latent. Niskanen pointed out the neural network-driven nature of FCMs and highlighted that in practical applications, the preference lies in the modeling performance of an FCM rather than a comprehensive interpretation of its weights (Niskanen, 2019).

Moreover, based on experiments reported in the literature (Chen et al., 2012, 2015; Stach et al., 2005; Z. Yang and Liu, 2020), algorithms that overlook weights in their optimization process because of the reasons explained above can result in solutions

with low data-error but relatively high model-error. Essentially, a reduction in data-error does not necessarily lead to a decrease in model-error, as highlighted by Stach et al. (Stach et al., 2010). This discrepancy often arises from the potential to reach suboptimal solutions.

The primary goal of this study is to create FCM models capable of reproducing historical data, rather than focusing on minimizing the discrepancy between the weights in the constructed FCMs and those in the relationship matrix shown in Table 6.1. It is presumed that an actual FCM model, which is responsible for generating system data, does not exist. To ensure fairness in the evaluation, no FCM simulation technique using the proposed reasoning mechanism or threshold functions like sigmoid and hyperbolic tangent is employed for data reproduction, as this could give an unfair advantage. For instance, if sig-FCM were used to generate historical data, a common practice in literature, it would unfairly benefit over other FCM methods in data prediction.

Instead, data generation in this study was confined to simple mathematical calculations that replicate natural observations on paper. This allowed unbiased comparison by avoiding the use of FCM models as the source of data in the experimental study. Consequently, the study does not need to pursue models that can produce data aligned with a real-world model. Furthermore, the study explores a different reasoning mechanism, acknowledging that the predicted relationship matrix might not always be optimal due to the possibility of suboptimal conditions.

6.3.2 Results of the comparison with exponential normalized reasoning

In this section, the performance of pro-FCM within the Learning FCM framework is compared to that of the exponential normalized reasoning mechanism, which shares a similar objective. Although Nápoles et al. (2024) suggests that the β parameter can be determined based on expert opinions or extracted from data, they employ the values 0.8, 1.0, and 1.2 for experimental purposes. In this study, the exponential normalized reasoning integrated into Learning FCM was examined separately for these three β values. For each selected value, all concepts were assigned the same β , and the methods are labeled as “exp. norm. with $\beta=0.8$ ”, “exp. norm. with $\beta=1.0$ ”, and “exp. norm. with $\beta=1.2$.” For comparisons with each pro-FCM setup, 30 simulations were conducted. To ensure paired comparisons, the simulations for pro-FCM and

exponential normalized reasoning methods were performed using the same randomly initialized weights.

As explained in equations 6.15 and 6.16, to calculate the activation levels of concepts, it is necessary to identify the maximum increase in resources for each iteration. As did Nápoles et al. (2024), the maximum increment θ , equal to the initial activation levels, is set to 1 for all concepts in this study.

After predicting state vectors using the exponential normalized reasoning method in learning FCM, each state vector in the predicted dataset was normalized to the $[0,1]$ range through maximum normalization. Nápoles et al. (2024) do not apply this step due to their resource-amount based approach.

Figures 6.12, 6.13, and 6.14 present the comparisons of the prediction errors between pro-FCM vs. exponential normalized reasoning methods across three different scenarios

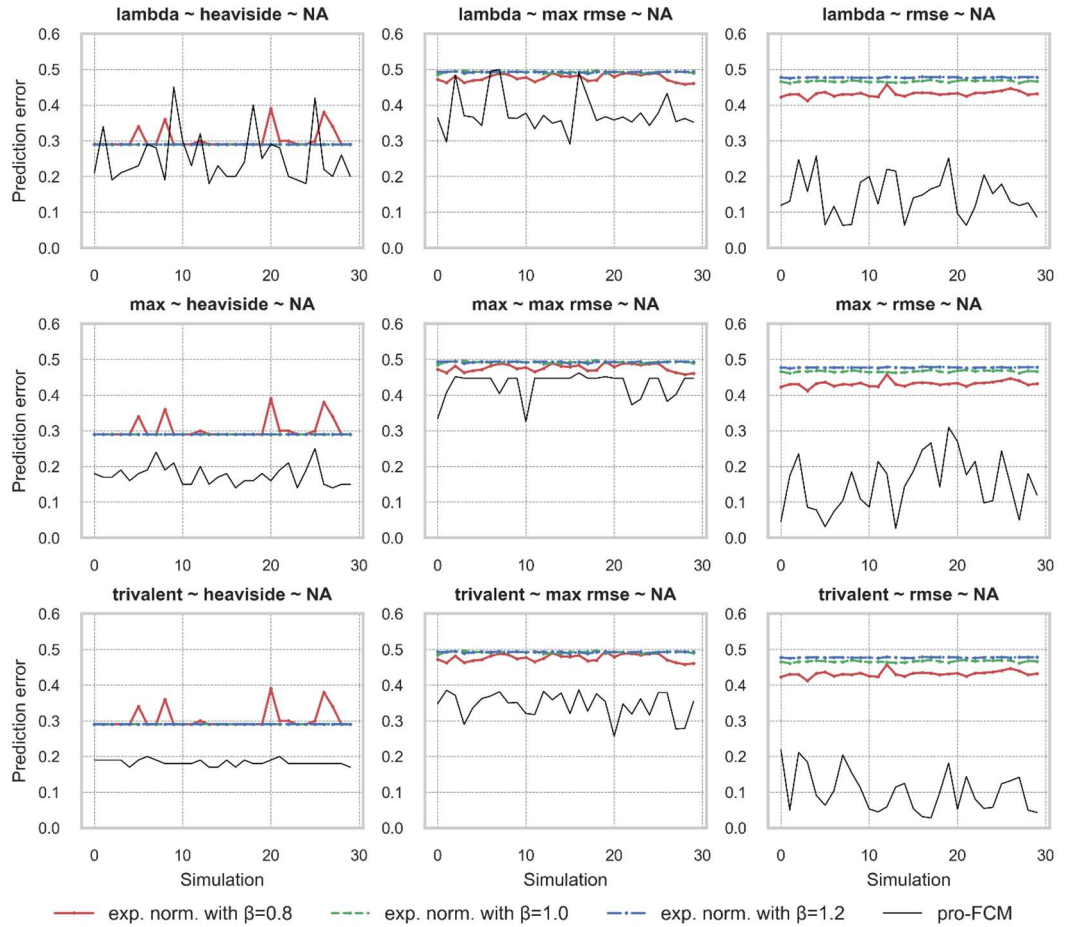


Figure 6.12: Comparison of the learning FCM performance of the exponential normalized reasoning mechanism and pro-FCM across the setups for Scenario-1.

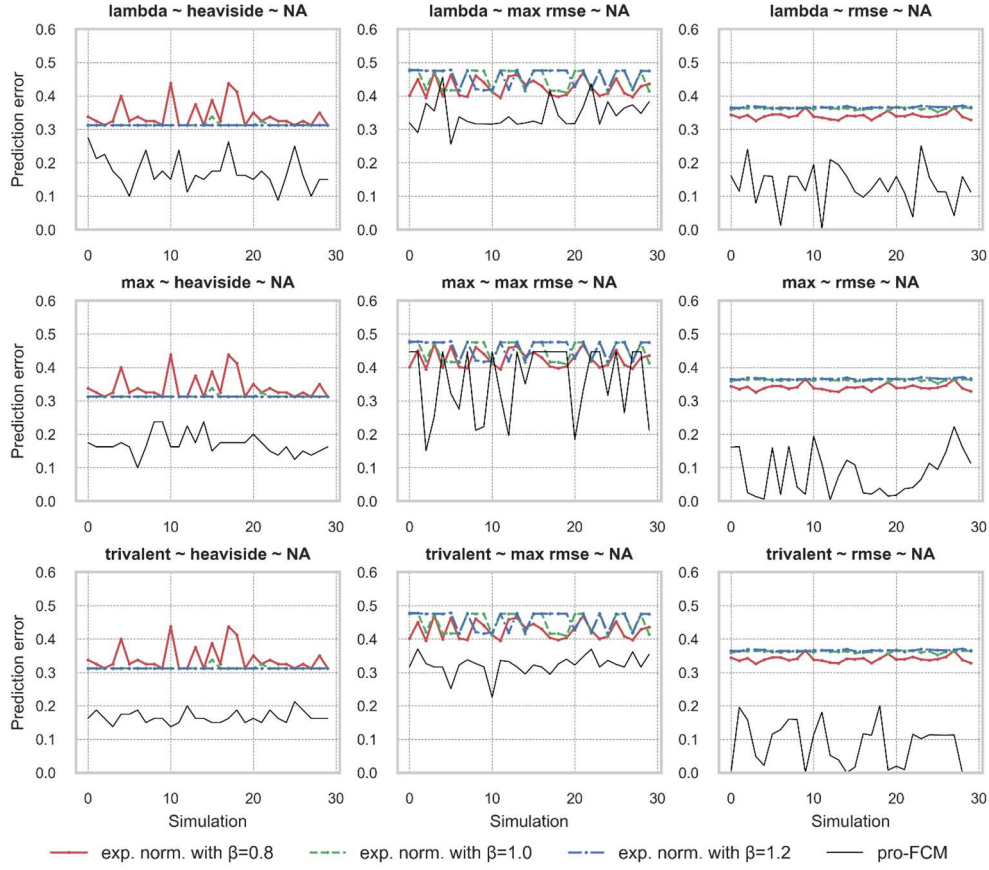


Figure 6.13: Comparison of the learning FCM performance of the exponential normalized reasoning mechanism and pro-FCM across the setups for Scenario-2.

As shown in Figures 6.12, 6.13, and 6.14, pro-FCM achieves lower prediction errors in multiple setups, demonstrating its effectiveness as a predictive model. Despite the larger fluctuations in pro-FCM's prediction errors compared to sig-FCM and tan-FCM, the presence of lower errors within these fluctuations indicates pro-FCM's potential to identify the optimal model. In contrast, exponential normalized reasoning requires careful selection of the β parameter to achieve low errors, making its optimization process more demanding.

Statistical analyses presented in Table 6.10 provide further evidence supporting the success of pro-FCM. In Scenario 3, for the max~max.rmse~NA setup, pro-FCM does not show a significant difference compared to exponential normalized reasoning with $\beta = 0.8$ ($p = 0.681$) and $\beta = 1.0$ ($p = 0.109$), while a significant but low-effect difference appears for $\beta = 1.2$ ($p = 0.022$, $r = 0.295$). However, in all other setups, pro-FCM achieves statistically significant differences with notable effect sizes, confirming its capability as a strong predictive model. For example, in the lambda~heaviside~NA

setup, pro-FCM shows significant differences in Scenario 1 with p -values as low as 0.005 and 0.013, indicating meaningful improvements.

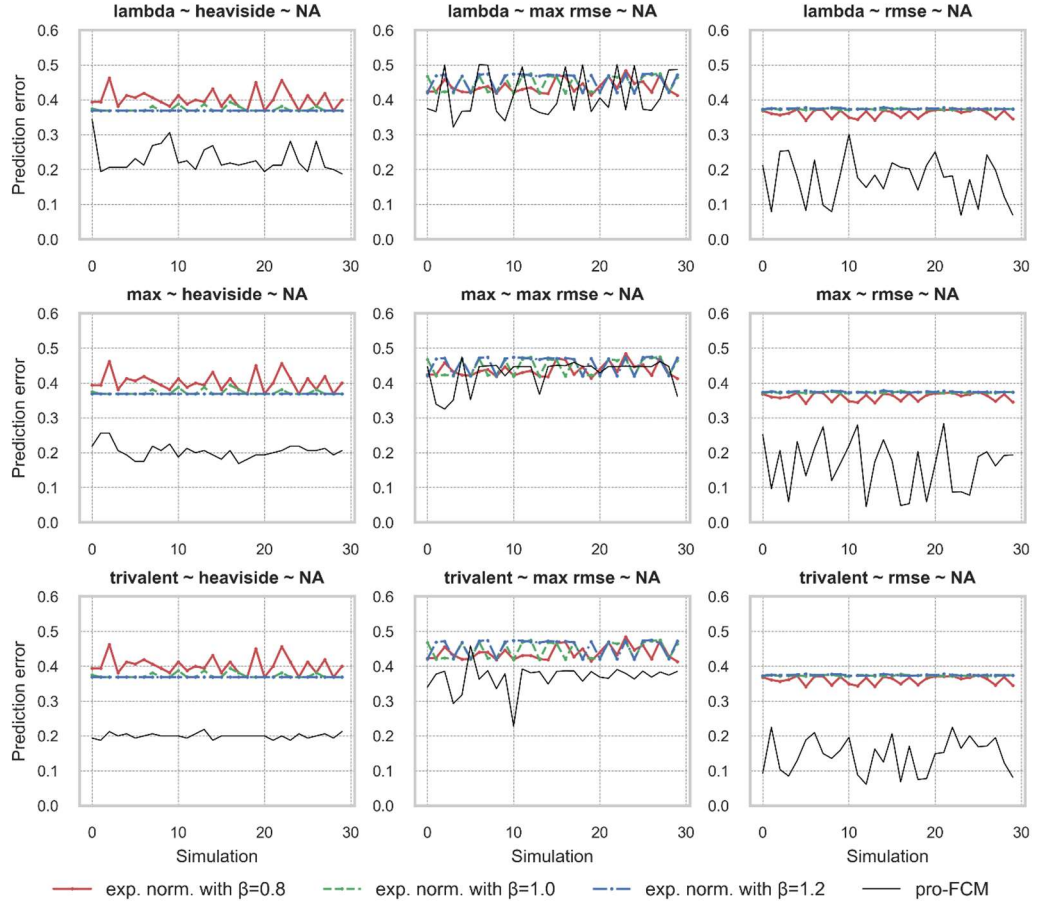


Figure 6.14: Comparison of the learning FCM performance of the exponential normalized reasoning mechanism and pro-FCM across the setups for Scenario-3.

Overall, the results confirm that pro-FCM not only enables lower prediction errors but also generally outperforms exponential normalized reasoning in a statistically meaningful way. While exponential normalized reasoning remains stable, its dependence on β selection limits its flexibility. These findings suggest that pro-FCM is a reliable choice for reasoning in FCMs, with room for further refinement to ensure consistently robust advantages.

Table 6.10: Statistical comparison for prediction errors of pro-FCM vs exponentially normalized reasoning.

Wilcoxon signed-rank test p -value (effect size)**									
Setup	Scenario-1			Scenario-2			Scenario-3		
	pro vs 0.8	pro vs 1.0	pro vs 1.2	pro vs 0.8	pro vs 1.0	pro vs 1.2	pro vs 0.8	pro vs 1.0	pro vs 1.2
lambda~heaviside~NA	0.005 (0.363)	0.013 (0.321)	0.013 (0.321)	0.000 (0.617)	0.000 (0.619)	0.000 (0.618)	0.000 (0.616)	0.000 (0.617)	0.000 (0.617)
max~heaviside~NA	0.000 (0.617)	0.000 (0.617)	0.000 (0.617)	0.000 (0.617)	0.000 (0.620)	0.000 (0.620)	0.000 (0.617)	0.000 (0.617)	0.000 (0.618)
trivalent~heaviside~NA	0.000 (0.620)	0.000 (0.625)	0.000 (0.625)	0.000 (0.618)	0.000 (0.622)	0.000 (0.622)	0.000 (0.617)	0.000 (0.620)	0.000 (0.623)
lambda~rmse~NA	0.000 (0.616)	0.000 (0.616)	0.000 (0.616)	0.000 (0.616)	0.000 (0.616)	0.000 (0.616)	0.000 (0.616)	0.000 (0.616)	0.000 (0.616)
max~rmse~NA	0.000 (0.616)	0.000 (0.616)	0.000 (0.616)	0.000 (0.616)	0.000 (0.616)	0.000 (0.616)	0.000 (0.616)	0.000 (0.616)	0.000 (0.616)
trivalent~rmse~NA	0.000 (0.616)	0.000 (0.616)	0.000 (0.616)	0.000 (0.616)	0.000 (0.616)	0.000 (0.616)	0.000 (0.616)	0.000 (0.616)	0.000 (0.616)
lambda~max.rmse~NA	0.000 (0.589)	0.000 (0.600)	0.000 (0.605)	0.000 (0.584)	0.000 (0.597)	0.000 (0.613)	0.096 (0.215)	0.015 (0.313)	0.002 (0.396)
max~max.rmse~NA	0.000 (0.616)	0.000 (0.616)	0.000 (0.616)	0.031 (0.279)	0.003 (0.382)	0.000 (0.592)	0.681 (0.053)	0.109 (0.207)	0.022 (0.295)
trivalent~max.rmse~NA	0.000 (0.616)	0.000 (0.616)	0.000 (0.616)	0.000 (0.616)	0.000 (0.616)	0.000 (0.616)	0.000 (0.605)	0.000 (0.605)	0.000 (0.611)

** Values in brackets indicate effect size.

6.4 Discussion

As highlighted earlier, the majority of studies in the literature concentrate on heuristics and learning algorithms, while only few explore reasoning mechanisms. Thus, research in this field rely on a few well-established reasoning approaches. Accordingly, the following discussion of the results was structured around alternative reasoning approaches rather than specific studies.

The experimental comparisons demonstrate the enhanced performance of the proposed pro-FCM reasoning mechanism over widely-used classical methods and the more recently introduced exponential normalized reasoning across multiple scenarios. The findings highlight the robustness, accuracy, and generalization capabilities of pro-FCM, particularly in handling diverse initial conditions and complex system behaviors. Below, we discuss the strengths and limitations of the proposed method.

The pro-FCM consistently outperformed sig-FCM and tan-FCM in prediction accuracy across all scenarios, as evidenced by the narrower and more concentrated error distributions in the violin plots. For instance, in Scenario-1, pro-FCM achieved a much higher prediction accuracy, significantly outperforming sig-FCM and tan-FCM. This improvement is attributed to the novel reasoning mechanism's ability to incorporate indirect relationships and respond effectively to varying initial conditions, which classical methods struggle to address. The Heaviside fitness function emerged

as the most effective, consistently yielding the lowest prediction errors, further supporting the proposed method’s efficacy.

Statistical analysis using the Wilcoxon signed-rank test confirmed the superiority of pro-FCM, with effect sizes ($r \approx 0.6$) indicating a large and meaningful improvement in prediction performance. While pro-FCM occasionally performed similarly to or slightly worse than classical methods in specific setups (e.g., $\lambda \sim \text{rmse} \sim A$ and $\max \sim \max.\text{rmse} \sim NA$), these cases were exceptions rather than the norm. These findings underscore pro-FCM’s robustness and reliability across diverse experimental conditions.

The generalization capability of pro-FCM was evaluated by predicting the final (N^{th}) state vector using models trained on $N-1$ consecutive state vectors. The results revealed that pro-FCM consistently underpredicted or overpredicted activation levels less frequently than sig-FCM and tan-FCM, with a notable example being the accurate prediction of 9 out of 10 concepts in Scenario-2. This performance highlights pro-FCM’s ability to generalize well to new data, a critical requirement for real-world applications.

The comparison with exponential normalized reasoning further validated pro-FCM’s effectiveness. While exponential normalization demonstrated stability (see also (Karagiannis and Groumpos, 2013; Mpelogianni and Groumpos, 2018; Nápoles et al., 2024)), its performance heavily depended on the careful selection of the β parameter, limiting its flexibility. In contrast, pro-FCM achieved lower prediction errors across multiple scenarios without such dependencies. Statistical analysis confirmed that pro-FCM performed successfully in most setups, with significant p -values and notable effect sizes. All these findings suggest that pro-FCM is a reliable and effective method for reasoning in learning FCMs.

Despite its strengths, pro-FCM also has certain limitations, as highlighted in the experimental comparisons. For instance, its performance in setups involving lambda vector normalization was occasionally comparable to or slightly worse than classical methods, indicating room for improvement in handling specific normalization techniques. Additionally, the study focused on learning FCM models that effectively reproduce historical data, rather than prioritizing the alignment of the learned FCM weights with those in the true relationship matrix (i.e., model-error fitness). Model-

error is applicable only when true weights are known or specific targets for the weights are set (Chen et al., 2012, 2015). However, in real-world scenarios where actual weights are typically unavailable, the use of these fitness functions is often limited. Therefore, many studies prioritize developing models that accurately predict system behavior, even when the true relationships remain latent. Niskanen (2019) pointed out the neural network-driven nature of FCMs and highlighted that in practical applications, the preference lies in the modeling performance of an FCM rather than a comprehensive interpretation of its weights.

Moreover, based on experiments reported in the literature (Chen et al., 2012, 2015; Stach et al., 2005; Z. Yang and Liu, 2020), algorithms that overlook weights in their optimization process because of the reasons explained above can result in solutions with low data-error but relatively high model-error. Essentially, a reduction in data-error does not necessarily lead to a decrease in model-error, as highlighted by Stach et al. (2010). This discrepancy often arises when the optimization process settles on suboptimal solutions.

Therefore, the primary objective of this thesis is to develop FCM models that effectively minimize data error, ensuring accurate representation and reproduction of historical data patterns. It is presumed that an actual relationship matrix, which is responsible for generating system data, does not exist. To ensure a fair evaluation, no FCM simulation technique using the proposed reasoning mechanism or threshold functions like sigmoid and hyperbolic tangent is employed for data reproduction, as their inclusion could introduce an unfair advantage. For instance, if sig-FCM were used to generate historical data, a common practice in literature, it would unfairly benefit over other FCM methods in data prediction.

Instead, data generation in this study was confined to simple mathematical calculations that replicate natural observations on paper. This allowed unbiased comparison by avoiding the use of common FCM methods as the source of data in the experimental study. Consequently, the study does not need to pursue models that can produce data aligned with a real-world model. Furthermore, the study explores a different reasoning mechanism, acknowledging that the predicted relationship matrix might not always be optimal due to the possibility of suboptimal conditions.

7. CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH

This thesis focuses on improving the modeling accuracy and reliability of Fuzzy Cognitive Maps (FCMs), particularly in their application to prediction problems. FCMs are widely recognized for their ability to model complex systems by capturing causal relationships among their components, thus providing insights into both the elements and the behaviors that emerge from their interactions. While FCMs have shown great promise in modeling control systems that operate under balanced and ideal conditions, they face significant challenges when applied to prediction problems, especially those that attempt to replicate the dynamics of complex systems.

The primary challenge addressed in this thesis is the convergence issue that classical FCMs exhibit when modeled with different initial conditions. Traditional FCMs, which utilize nonlinear activation functions such as the sigmoid function, tend to converge to a single state regardless of the starting point. This behavior limits the ability of FCMs to perform scenario analyses, particularly in complex, real-world environments where multiple possible outcomes must be modeled based on different initial conditions.

To overcome this limitation, this thesis proposes a novel reasoning mechanism that does not rely on nonlinear activation functions. Instead, the new mechanism integrates both direct and indirect relationships by taking the power of the relationship matrix, which provides a more comprehensive representation of the system's dynamics. This approach results in the creation of a total effect matrix, allowing the model to reach different final states from different initial conditions, thus making scenario analysis possible. This enhanced mechanism addresses the shortcomings of traditional FCM models and provides a more nuanced and flexible framework for modeling systems with diverse potential behaviors.

The contributions of this thesis were developed in several key phases. First, we demonstrated that classical FCMs consistently converge to the same state, even when different initial conditions are applied. This convergence issue hinders the ability of FCMs to effectively represent diverse scenarios, particularly in predictive scenarios.

We showed that when using the traditional FCM reasoning mechanism, scenario analysis is not feasible due to the fixed outcomes produced.

Next, we explored the potential of indirect relationships within FCMs by using them to calculate the weight of concepts in an MCDM application. This approach allowed us to demonstrate the value of indirect relationships, emphasizing that both direct and indirect influences play critical roles in accurately modeling systems.

Subsequently, to model the dynamics of complex systems we introduced a new reasoning mechanism that incorporates indirect relationships. By taking the power of the relationship matrix, we created a total effect matrix that enables the model to reach varying states from different initial conditions. This improvement provides the necessary foundation for conducting effective scenario analyses, allowing for the modeling of systems with multiple potential final outcomes.

Finally, the integration of the proposed reasoning mechanism into the learning process of FCMs also offers a systematic generalization and validation. Different normalization techniques, as well as cumulative and non-cumulative activation level calculations, were tested to evaluate the effectiveness of the proposed method. The results of these experiments, conducted on datasets derived independently of any specific reasoning mechanism, showed that the new mechanism significantly enhances the learning capabilities of FCMs, enabling them to better reflect the behaviors observed in real-world systems. The statistical analysis of these results confirmed that the new reasoning mechanism provides a robust framework for modeling systems that successfully predict different behaviors depending on their initial conditions.

In conclusion, this thesis has demonstrated that the newly proposed reasoning mechanism can significantly improve the modeling capabilities of FCMs, particularly in the context of prediction problems. By addressing the convergence issues associated with traditional FCMs and incorporating a more comprehensive approach to influence dynamics, the new mechanism allows for more flexible and accurate modeling of complex systems. This advancement makes it possible to create FCM models that can effectively simulate diverse scenarios, offering valuable tools for decision-making in environments characterized by uncertainty and complexity. The proposed approach represents a substantial step forward in the development of FCMs, enhancing their

applicability to a broader range of problems and enabling more effective modeling of systems that require dynamic adaptability.





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APPENDICES

APPENDIX A: Evaluation matrices



APPENDIX A: Evaluation matrices

Table A.1: Evaluation of Experts.

Expert 1													
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
C1				+L			-L		+M	+M			
C2			-H					-H					
C3		-H		-L			+M	+H					
C4	+M								+M				
C5				-L									
C6							-L			+M	-L		
C7	-L									-L			
C8		-H	+M	-M					-M				-M
C9				+L									
C10	+M			+L			-L						
C11							+M		+M				
C12		-M	+M				+L						
C13				+L			+L		+M		+L		
Expert 2													
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
C1							-L					-M	
C2			-H	+M			-M		+L			-L	
C3		-H		-M	-L	-L	+M	+L	-L	-L			
C4	+L				-L		-L		+L				
C5				-L									
C6		+L	-L				-L				-M		
C7		-H		-M		-M			-L				
C8				-M					-L				-L
C9				+M								-L	
C10			-L	+L	+L	+L					+L		
C11				+M		-L	+L		+L				-L
C12		-L							-L				
C13				+L		+L			+M				
Expert 3													
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
C1						+M			+H				
C2			-H	+M			-M						
C3		-M		-L			+M	+H					
C4					-M	+L	-L		+M				
C5		-L				+L	-L				-L		+L
C6		+L		+M	+L		-L				-L		
C7		-M	+L					-L			+L	-L	
C8				-H			-L		-H				-M
C9		+H		+L		+M							
C10					+M								
C11		-L				-L	+M		+L				-L
C12		-M	+M										
C13		+L		+H	+L	+M	+L		+M				

Table A.1 (continued): Evaluation of Experts.

Expert 4													
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
C1		+L									+L		
C2				+L			-L		+L			-L	
C3				-L									
C4	+L				-L	+L							+L
C5			+M				-L				-L		
C6	+L				+M								
C7	-L		+M		+L						+L		
C8	-L		+L										
C9				+L				-L					
C10													
C11	+L	-L											-L
C12													+L
C13		+L		+L	+L				+L		+L		
Expert 5													
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
C1		+H	-M	+M								-M	
C2	+M		-M				-M	-M				-M	
C3	+M	-M					+M	+M					
C4	+M				-M	+M							
C5													
C6				+M			-M						
C7	-M	-M			+M	-M				-M		-M	-M
C8	-M	-M									+M		
C9	+M			+M				-H					
C10													
C11	+M						+M					-M	
C12	-M	-M				-M	+M	+M			+M		
C13					+M		+M						
Expert 6													
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
C1			-L		-M		-M						
C2	+M			+M								-M	
C3				-L	-L		+L	+M		+M	+L	+M	
C4	+L				-L	+M							
C5	+M								-M		-M	+M	
C6									+M	+M			
C7			+L										
C8						-L	-M		-M	+M		+L	
C9			-M		+M					+L			-H
C10			-M				-M	+M	+L		+M		
C11					+M					+M			
C12		-M						+L	+M				+M
C13				+L	+M			-M				+M	

Table A.1 (continued): Evaluation of Experts.

Expert 7													
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
C1		+M			-L		-L		+M		+L	-M	
C2			-L	+L			-H	-M					
C3		-M		-M			+H	+M	-M				
C4						+M			+M				+M
C5	+M	-L		-M					-M				
C6		+M					-M		+L	+M	-L		
C7		-M							-L				-M
C8				-M		-M			-L				
C9	+L					+M	-L			+L		-M	
C10	+M										+L		
C11		-L				-M	+M					-L	-L
C12			+L				+L						
C13							+M		+L				
Expert 8													
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
C1		+L	-L				-L		+L			-L	
C2			-M				-H	-L				-L	
C3		-H					+H	+L					
C4									+M				+H
C5						+L	-L	-L		+M	-M		+L
C6					+L						-L		
C7		-M											
C8		-L		-H					-H				-L
C9													
C10							-L				+L		
C11		-L			+L		+H						
C12		-L					+M						
C13				+H	+L			-L	+M				
Expert 9													
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
C1		+L										-M	
C2	+M		-H			+M	-H					-H	
C3		-H					+H	+H					
C4	+M					+M							+M
C5		-H				+M	-H						+L
C6		+M					-M					-M	+L
C7	-L	-H											
C8			+M	-M				-H		+L	+M		
C9			-M	+M				-H		+L	-M		
C10	+M		-L										
C11		-M			+H	-L	+H					-M	
C12	-L						+L						
C13						+L				+L			

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