



VERTICAL AIR –ENTRAINING VORTEX AT AN INTAKE

YASAMEEN MANSOOR A. AL-HAKEEM

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ABSTRACT

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Supervisor: Prof. Dr. Nevzat YILDIRIM

Co-Supervisor: Prof. Dr. Kerem TAŞTAN

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Air-entraining vortices at hydraulic intakes present significant challenges in engineering, leading to efficiency losses, structural vibrations, and potential operational instabilities. While previous research has explored these vortices, most studies have been limited to specific intake geometries and flow conditions, lacking a generalized predictive model. Additionally, due to the complex nature of air-core vortices, deriving a single universal theoretical formula remains elusive. Existing semi-empirical models are typically case-specific, and while numerical methods offer alternative solutions, they often require extensive computational resources and experimental calibration.

By improving an analytical technique to forecast the profile of vertical air-entraining vortices across various intake configurations, this work seeks to overcome these constraints. The method improves on the semi-empirical formula for vertical non-air-entraining vortices (air-core vortex in suspension) that is already available. A trial-and-error solution method was created to predict vortex profiles with more precision since the air-core vortex height in air-entraining vortices cannot be measured directly. The available test data and the results of the present study are compared, and found to be in good agreement.

This research identifies the following advancements:

1. A refined prediction model for the vertical air-entraining vortex profiles .By modifying the available formula for non-air-entraining vortices and incorporating empirical validation, the study establishes a reliable methodology for estimating air-entraining vortex profiles. The validation is based on experimental data from previous available studies, ensuring the proposed model accurately represents real-world vortex behavior.
2. Development of a practical chart and formula for engineers – The study provides a simplified chart that allows engineers to directly determine vortex parameters for downward-flowing intakes without requiring complex numerical simulations.
3. Validation using experimental data sets – The proposed methodology was examined and checked with test data for vertically flowing downward air-entraining vortices ($h/S > 1$) at circular intakes. The results demonstrated a strong correlation between predicted and observed vortex profiles, confirming the accuracy and applicability of the refined model.

The findings of this research offer a significant advancement in vortex prediction methodologies, providing engineers with a practical, validated, and computationally efficient tool for designing hydraulic intakes and mitigating vortex-related inefficiencies. By improving upon existing models and validating them against experimental data, this study enhances the ability to predict and control air-entraining vortices in real-world hydraulic applications

Keywords: Air- core vortex, Intake, Critical submergence, Profile, Air- entrainment

ÖZET

SU ALAMA AĞZINDA DÜŞEY HAVA-GİRİŞLİ ÇEVİRİNTİ

AL-HAKEEM, Yasameen MANSOOR A
İNŞAAT MÜHENDİSLİĞİ YÜKSEK LİSANS TEZİ

Danışman: Prof. Dr. Nevzat YILDIRIM

Ortak Danışman: Prof. Dr. Kerem TAŞTAN

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Hava girişli çevrinti su alma ağız yapılarında ve buna bağlı hidrolik yapılarda verim kayıplarına, yapılarda titreşimlere ve işletimlerinde aksaklıklara sebep olmaktadır. Havalı çevrinti üzerinde geçmişte çok sayıda çalışma yapılmış olmasına rağmen söz konusu çalışmaların her biri (bilhassa deneysel olanları) kendine özgü akım ve geometrik şartlar altında yapılmış olması nedeniyle genelde her çeşit akım ve geometrik şartlarda uygulanabilir değildir. Havalı çevrintinin çok karmaşık bir olay olması nedeniyle havalı çevrinti için geneli kapsayacak tek bir teorik kanun veya formül bulunamamıştır. Mevcut yarı-teorik çalışmalar yapıldıkları özel durumlar ve şartlar için geçerlidir. Diğer taraftan her ne kadar sayısal (nümerik) çalışmalar varsa da bu çalışmaların sağlaması için deneylerin yapılması ve bunlardan elde edilecek verilere ihtiyaç duyulmaktadır.

Bu tez çalışmasının ana hedefi yukarıda belirtilen zorlukların üstesinden gelmek ve su alma ağızında oluşacak hava-girişli düşey çevrintinin şekline ait yarı-teorik (ampirik) genel bir yöntem geliştirmektir. Bu amaç için hava-girişli olmayan düşey çevrintiye (askıda ki hava girişli olmayan düşey çevrintiye) ait mevcut yarı-teorik formülde gerekli düzenlemeler yapılmış olup söz konusu formülün genel olarak tüm hava-girişli düşey çevrintilere uygulanabileceği gösterilmiştir. Hava-girişli çevrintinin tüm düşey yüksekliği (hava-girişli çevrintinin su alma ağız

mansabındaki kısmının hayali olması nedeniyle) ölçülemediğinden ve bilinmeyen sayısının denklem sayısından fazla olması nedeniyle hava-girişli düşey çevrintinin şeklini veren ampirik formülün çözümünde “deneme-yanılma” yöntemi kullanılmıştır. Uygulamaya kolaylık sağlamak için gerekli formül ve grafikler elde edilmiştir. Mevcut deney verileri ile bu tez çalışmasının neticeleri karşılaştırılmış olup aralarındaki uyumun iyi olduğu gösterilmiştir.

Bu araştırma aşağıdaki ilerlemeleri ortaya koymaktadır.

- 1- Hava-girişli düşey çevrintinin geometrik şeklini veren yarı-teorik (ampirik) bir yöntem geliştirilmiştir. Hava-girişli olmayan düşey çevrinti için geliştirilmiş olan mevcut yarı-teorik formül hava-girişli düşey çevrinti için de kullanılabilir duruma getirilmiştir. Bulunan yarı-teorik formülün ve yöntemin mevcut deney verileriyle karşılaştırılması yapılmış olup söz konusu yöntemin gerçek hayatta uygulanabilirliği ispatlanmıştır.
- 2- Gerçek hayatta uygulamaya yönelik olarak mühendisler için kolaylık sağlayan formül ve grafikler geliştirilmiştir.
- 3- Deney verileriyle geçerliliğin ispatı – Bu tez çalışmasında bulunan yöntemin doğruluğunu göstermek ve kontrol etmek için bulunan formül ve yöntem daire kesitli su alma ağız yapılarında ki hava-girişli düşey çevrintiyle ilgili deney verileriyle karşılaştırılmış ve aralarındaki uyumun iyi olduğu görülmüştür.

Bu tez çalışmasındaki bulgular uygulamada ki mühendislere su alma ağız yapılarında oluşan hava-girişli düşey çevrintiyle ilgili önemli bir katkı sağlamaktadır. Bu çalışmada geliştirilen yöntem gerçek hayatta hava-girişli düşey çevrintilerle ilgili sorunların çözümüne çok yardımcı olacaktır.

Anahtar Kelimeler: Havalı çevrinti, su alma yapısı, kritik batıklık, Profil ,hava girişi

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LIST OF SYMBOLS

α_{LBP}	:Magnitude of α at $z/h = 1$.
α_s	:Magnitude of α at $z/h = S/h$.
b_1	:Horizontal Distance of the Center Point of the Intake to the Right Side Wall of the Canal.
b_2	:Horizontal Distance of the Center Point of the Intake to Left Side Wall of the Canal.
C	:Vertical Distance (Clearance) of the Intake Level to the Bottom of the Canal .
CSSS	:Critical Spherical Sink Surface.
D	:Internal Diameter of the Intake.
g	:Acceleration Due to Gravity.
h	:Total Depth (Vertical Height) of the Profile of the Air-core Vortex.
H_o	:The Elevation of the Lower Closed End tip Point of the Vertical Air Core.
H_r	:The Vertical Distance of the Profile of the Air-core Vortex at r .
ℓ	:Distance of the End (Rear) Wall to the Intake Center.
ρ	:Fluid Density (Water Density).
PIV	Particle Image Velocimetry.
Q	:Intake Discharge.
r	:Radial Distance from the Center of the Air -core Vortex.
r_m	:Radius at the Maximum Tangential Velocity that Divides the Inner Viscous-core and the Outer Inviscid-zone or Vortex Core Radius (Outer limit Radius of the Viscous Region).
r_{mid}	:Radius at the ,Mid-depth of the Air-core Vortex Profile.
$\bar{R}$	: r/r_{mid} = Dimensionless Radius.
RSM	:Reynold's stress Model.

S	:Submergence.
S_c	:Critical Submerge Depth.
SPH	:Smoothed Particle Hydrodynamics.
U_∞	:Average Approach Flow Velocity.
V	:Average Intake Velocity.
V_r	:The Radial Velocity.
V_θ	:The Tangential Velocity.
z	:Vertically Downward Axis.
$(z/h)_{BP}$	: z/h for its Corresponding Break Point (BP).
$(z/h)_{so}$	: z/h at the Intersection Point of the Linear Line Section of Variation of α with the Horizontal axis of z/h (or line of $\alpha = 0$).
Γ	:Imposed Circulation.
σ	:Surface Tension.
ν	:Kinematic Viscosity.

CHAPTER I

INTRODUCTION

1.1 IMPORTANCE OF PREDICTION OF THE PARAMETERS RELATING TO THE AIR- ENTRAINING VORTEX AT A CIRCULAR INTAKE

Water intake structures are essential to hydraulic systems, such as those found in cooling water intakes for power generation facilities, hydroelectric power plants, and agricultural irrigation systems. These intakes have been meticulously designed to reliably and easily drain large amounts of water. However, air-entraining vortices may form at the intake under some operating conditions. Air entrainment from these vortices causes significant mechanical damage and reduces the system operation efficiency [1-4]. Predicting that the general circumstances that produce these vortices will enhance the system design and save costly operational downtime requires as much experience as feasible. Predicting what causes vortices to be aspirated is extremely useful for a wide range of applications.

1.1.1 Operational Efficiency

Vortex-induced air entrainment to the intake significantly reduces the water flow efficiency [5] and [6]. Because the air entrainment lowers water density, it affects the efficiency of energy generation in hydropower plants .

1.1.2 Cavitation and Mechanical Damage

Entrained air in the flow can initiate cavitation by creating vapor bubbles that collapse at higher pressure. When these bubbles violently collapse, they can cause significant mechanical damage to pumps, turbines, and intake structures [7] and [8]. Systems operating at high-pressure or high-velocity flows cannot avoid this situation. Additionally, cavitation creates vibrations, which can slowly damage the components over a long time period .

1.1.3 Design Optimization and Safety

Prediction of the properties of a vortex will allow engineers to design intake systems that are not only difficult but also better performance [9] and [10]. However, engineers can automatically prevent the development of a more or less dense vortex by changing the geometry, including the submergence depth of intake structures. To subdue the rotational vortex patterns, designers might also incorporate non-air-entrainment solutions into the mixing tank design, such as spoil of plates and veins. Moreover, this strategic method helps improving the operational reliability and, most importantly, decrease the maintenance costs.

1.2 BACKGROUND

Water from oceans, lakes, rivers, or reservoirs has been utilized by humans for agriculture, power production, household consumption, and industrial processes via intakes. However, a major wastewater issue has emerged in the contemporary age due to the loss of water supplies. underscoring the importance of managing water resources effectively and carefully. To minimize the cost and operational issues, We need to use more effective design standards while creating water intake structures. Two fundamental issues arise are the sedimentation of water in the intake and the swirling flow problems that occur at the entrance or within the pump intake. Pump operational difficulties can arise from sedimentation problems, as they either convey sediment with the diverted flow or deposit it in the intake, disrupting the flow.

Meanwhile, swirling flow problems commonly related to vortex formation can lead to swirls and air entrainment at the pump suction bell [11]. The rotational motion of fluid around an axis perpendicular to the free surface forms a vortex, a common phenomenon in nature [12]. Vortices are made up of different approaches to the intake, shear layers of high-velocity gradients, and rotational waves caused by obstacles in the incoming flow [13].

1.3 CLASSIFICATION OF INTAKES

Types of Intakes: To create a straightforward classification, two different factors can be used (Figure 1) as mentioned in the studies [11], [14], and [15].

- The first one is the intake direction.

- The second one pertains to structural differences, specifically the placement of the intake in relation to the walls and floor of the reservoir .

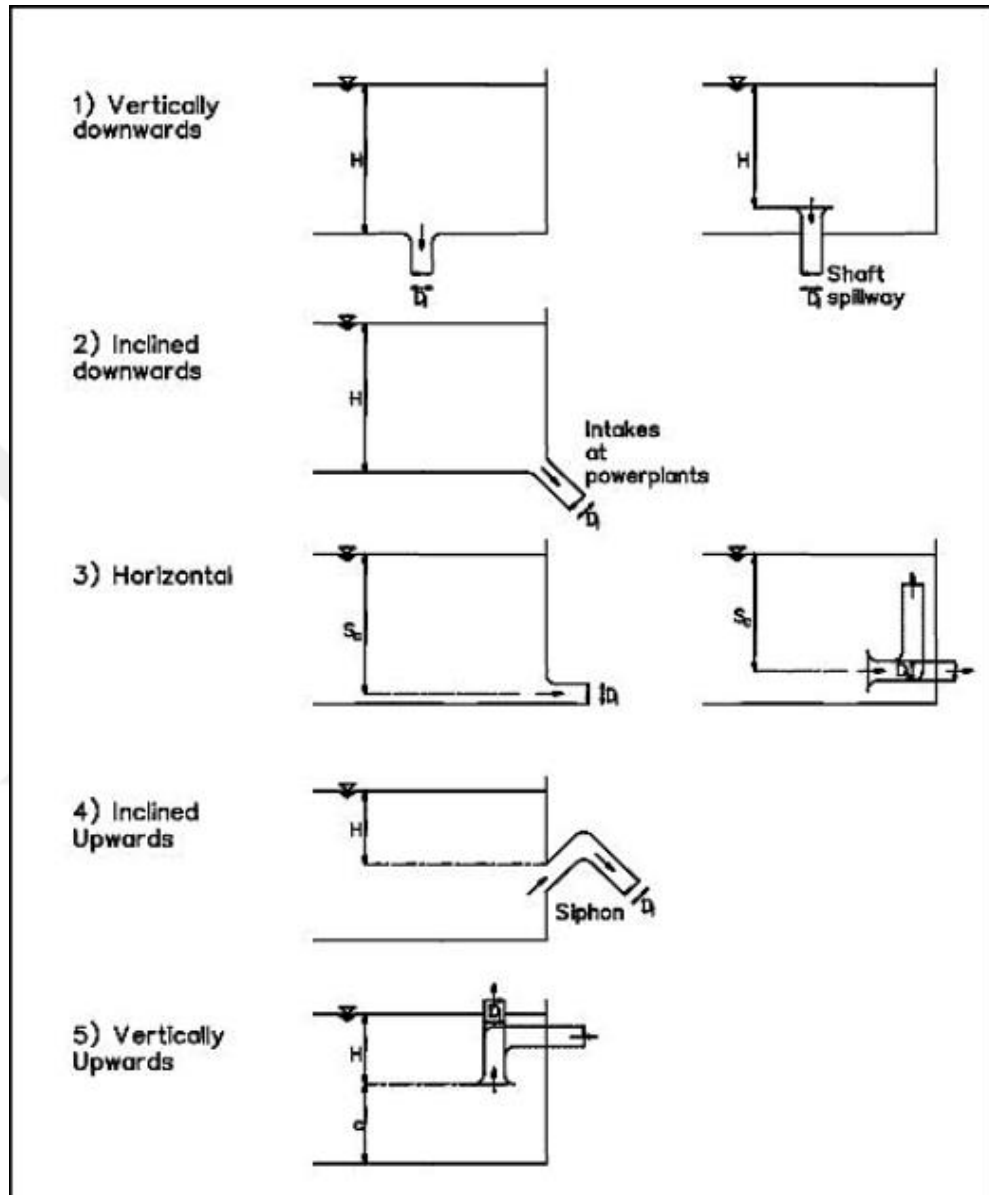


Figure 1 : Classification of Intakes [11]

1.4 THE SOURCES OF VORTICITY

Numerous vorticity sources for various geometrical flow conditions have been defined by researchers in the literature. The eccentricity of the approach flow in relation to the intake structure and the asymmetric approach flow have been shown to be the two most effective elements in creating vortices. Additionally, as shown in Figure 2, [11], [15], and [16] established three basic vorticity sources. They claimed that the primary causes of vortices are impediments that create rotating wakes, shear layers with strong velocity gradients, and variable intake orientations in comparison to a symmetric approach flow.

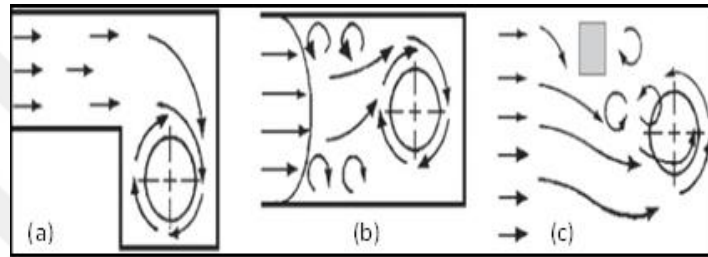


Figure 2 : Vorticity Sources (a) Offset (b) Velocity Gradients (c) Obstruction [11], [15], and [16]

1.5 THE PROBLEM OF VORTICES

As mentioned by researchers in [12], [15] and [17], the vortex problem has been divided into the following categories:

- a- Free surface vortices.
- b- Sub-surface vortices.

Originating from the free water surface, surface vortices can range in power and appearance from a minor dimple to a robust circulation. This strong vortex might draw air or floating debris into the pump column.

On the other hand, subsurface vortices are further divided into sidewall-attached, back wall, and floor vortices, each of which refers to how connected the vortices at one end are to the corresponding boundary walls. When the pressure is low, a powerful subsurface vortex with a reasonably strong core can cause dissolved water and air to separate, creating bubbles in the pump intake flow.

1.6 CLASSIFICATION OF FREE SURFACE VORTICES AT INTAKES

To accurately classify, a few visual measurement methods are used. Visual notions like swirl, eddy, dimple, and vortex tail should be introduced before describing these kinds. As seen in Figure 3, the terms swirl, eddy, dimple, and vortex tail are used to describe how the water surface appears depending on the kind of vortex. Swirl motion, which can only be seen by reflecting light, initiates vortices. When it becomes stronger, it becomes a dimple, and when it enters the intake, it adopts the shape of a vortex tail.

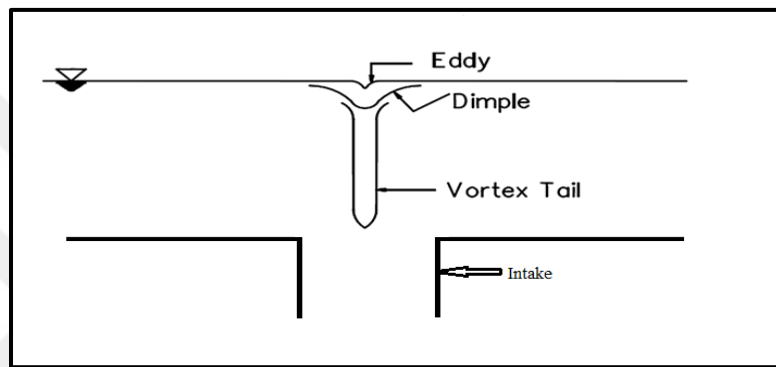


Figure 3 : General Appearance of Eddy, Dimple and Vortex Tail [15]

Figure 4 shows that researchers [18 –24] have put different types of vortices into six groups based on how they look. These groups are Type 1 flow has a slight rotation of the flow at the top of the water. When Type 2 happens, the water surface moves and there is also a dimple there. In Type 3, the flow turns around and goes to the entrance. In Type 4, debris is pulled into the entry. In Type 5, air bubbles are drawn in from the top of the water and brought down to the intake. Type 6 has the most violent surface flow, this kind of vortex is the strongest and most dangerous because it pulls air into the intake by making continuous air cores.

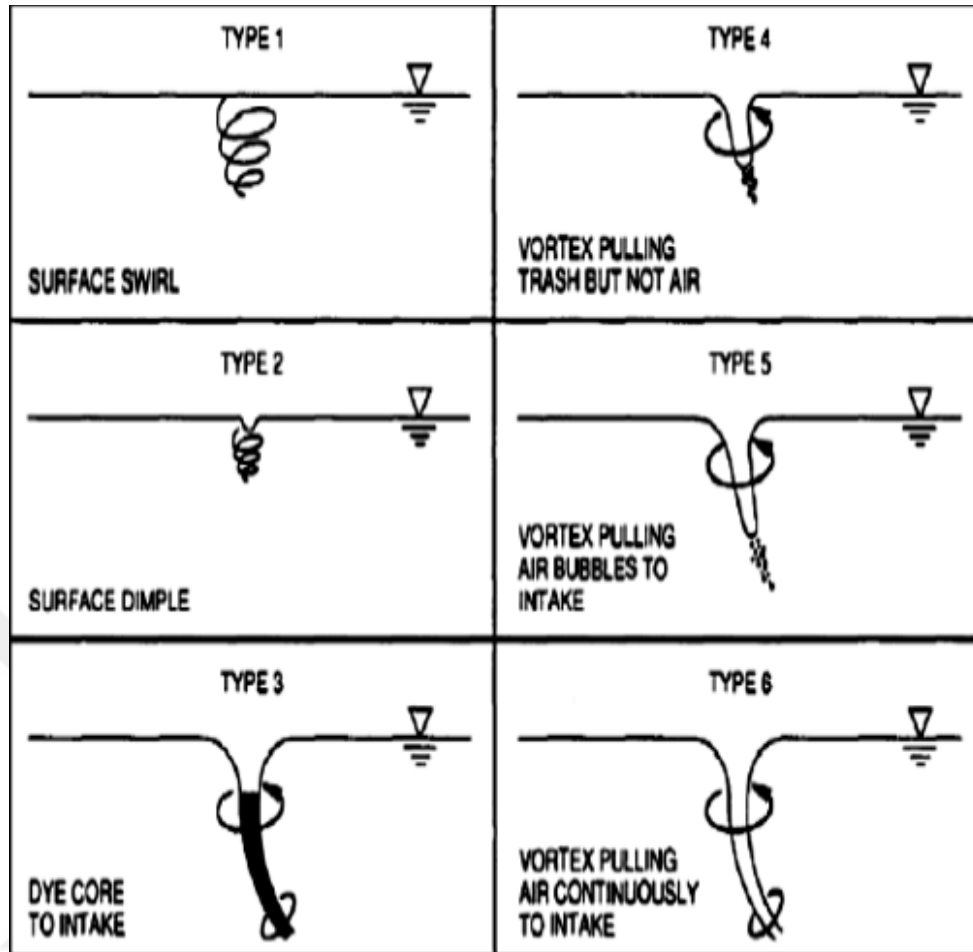


Figure 4 : Vortex Types [16]

1.7 THE IMPORTANT FACTORS TO PREDICT VORTEX BEHAVIOR

The important factors to predict vortex behavior are as follows.

1. The intake submergence.
2. Geometry of the reservoir and intake.
3. The intake Froude number.

1.7.1 The Intake Submergence

As shown in Figure 5, the vertical distance between the intake and the free surface is known as the submergence. Air-entraining vortices begin to form on the open surface when this submergence drops to a crucial level, also known as "critical submergence". According to experts [25–33], for an intake to be efficient, the

submergence must be sufficiently large to avoid creating air-entraining vortices that run from the free surface down to the intake entrance.

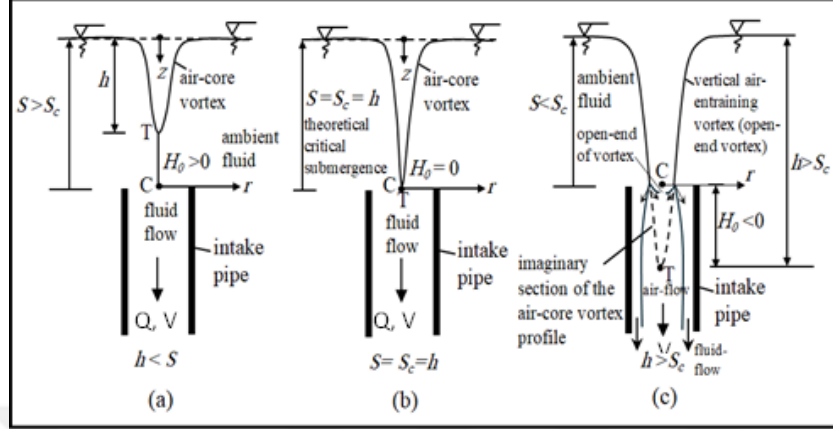


Figure 5 : Air-core Vortex Types [33]

1.7.2 Geometry of the Reservoir and Intake

A reservoir's geometry and intake significantly influence vortex behavior, which can impact the efficiency and safety of hydraulic structures behavior as mentioned by researchers in [34-37]. Vortex formation is a complex phenomenon affected by various factors, including reservoir asymmetry, intake design, and flow conditions. Understanding these influences is crucial for predicting and mitigating vortex-related issues in water intake systems. The following details the impact of reservoir and intake geometry on vortex.

1.7.2.1 Reservoir Geometry

- **Asymmetry and Blockages:** Asymmetrical flow approaching the intake, such as side wall boundary blockages, can significantly increase the critical submergence depth that is the depth at which air enters the intake due to vortex formation. Even a 10% side blockage can double this depth as mentioned by researchers [38].
- **Bottom Unevenness:** Blockages on the reservoir bottom can increase the critical submergence depth by 10-25% at low and high Froude numbers. More substantial blockages can increase this effect by up to 60% as mentioned in [38].

1.7.2.2 Intake Design

- Head Wall Slope: The slope of the intake head wall affects vortex stability. A vertical head wall can reduce vortex strength and critical submergence depth, acting as an anti-vortex structure as mentioned by researchers in [38].
- Intake Shape or Geometry and Configuration: The shape and configuration of the intake, such as the presence of side curtain walls, influence vortex formation. Numerical models have shown that altering these structures can mitigate vortex effects [39].

1.7.2.3 Flow Patterns and Vortex Characteristics

- Funnel and Spiral Patterns: Numerical simulations have identified funnel-shaped flow patterns and spiral vortex motions from the water surface towards the intake, which are critical in understanding vortex dynamics as mentioned in [40].
- Free Surface Vortices: Axially stretched free-surface vortices can occur under specific conditions, affecting mechanical components like turbine blades. A semi-empirical model helps to predict the vortex characteristics based on approach flow and intake geometry as mentioned in [41].

While the geometry of the reservoir and intake plays a crucial role in vortex behavior, other factors, such as flow velocity and liquid properties, also contribute to vortex formation. Additionally, anti-vortex structures and devices like floating rafts can mitigate vortex effects, enhancing the performance and safety of water intake systems as stated in [39].

1.7.3 The Intake Froude Number

The Froude number is defined later in chapter III .

1.8 VORTEX CHARACTERISTICS

Some of the fundamental components required to comprehend how vortices behave in fluid dynamics include the radial and tangential velocity distributions, changes in the radius of the vortex core, the water surface form, and the circulation distribution. These parameters are essential for predicting the behavior of vortices in many applications, such as hydraulic machines and meteorological phenomena as

mentioned in the literature presented in [32] and [42- 45]. The followings are the definitions of these factors:-

- 1) The radial velocity is the component of the fluid velocity that points directly towards the central axis (or center of rotation) of the air-core vortex.
- 2) The tangential velocity also known as azimuthal or circumferential velocity, that is the component of the fluid velocity perpendicular to the radial direction and lies along the circular path about the vertical central axis of the vortex.
- 3) The vortex core radius is the radial distance from the vortex center to the point of maximum tangential velocity or where the velocity distribution moves from solid-body rotation, which has a linear increase in velocity with radius, to potential flow, where velocity decreases with increasing radius.
- 4) Circulation measures the whole rotation or vorticity within a flow and is defined as the line integral of the velocity field around a closed curve. In fluid dynamics, analyzing and predicting the behavior of vortex and boundary layers necessitates an understanding of circulation distribution.
- 5) The water surface profile refers to the shape of the water surface along the air-core vortex.

1.9 PREVENTION OF VORTICES

To minimize the cavitation in fluid systems, it is very important to work on the elimination of the vortex. Stopping the formation of the vortex has been studied by a set of researchers with different approaches ranging from the use of anti-vortex devices to the flow improvement techniques as shown in the studies [46-49]. This description will cover the aforementioned methods: anti-vortex devices, flow lines extension, and improvement in the flow conditions.

1.9.1 Extending Flow Lines Between the Free Surface and Intake

Make the flow wider between the surface and the intake facilitates smooth and uninterrupted flow of water from the surface to the intake. This action is necessary to reduce turbulence, preventing vortex formation, and improving the efficiency of the piping system.

1.9.2 Improvement Flow Conditions

It is important to improve the flow conditions by modifying the flow path utilizing modification of piping layouts and the adjustment of component orientations to minimize obstructions and optimize flow, thereby reducing turbulence and pressure losses at bends, junctions, and transitions to avoid vortex formation. In addition, inefficient operation due to flooding or under-flooding can be avoided by adjusting flow rates to better match the system capacity. Managing fluid properties such as viscosity, density and temperature helps improve the flow behavior, while regular maintenance and cleaning removes debris, and sediment, that may disrupt flow and impair performance.

1.9.3 Using Some Anti-Vortex Devices to Prevent Asymmetric Flow Conditions

For eliminating the formation of vortices in the fluid field in hydraulic systems, mechanical or structural components are designed . Vortices generate when fluid flow creates a swirling motion that causes many problems, such as air entrainment, uneven flow distribution, or cavitation. Anti-vortex devices help to save smooth and uniform flow conditions, prevent equipment damage, and ensure operational efficiency. These devices are commonly used in numerous applications, such as water treatment facilities, swimming pools, industrial tanks, and hydraulic systems. Researchers [50-56] list many types of anti-vortex devices. Some of these devices are shown in Figures 6, 7, 8, and 9.

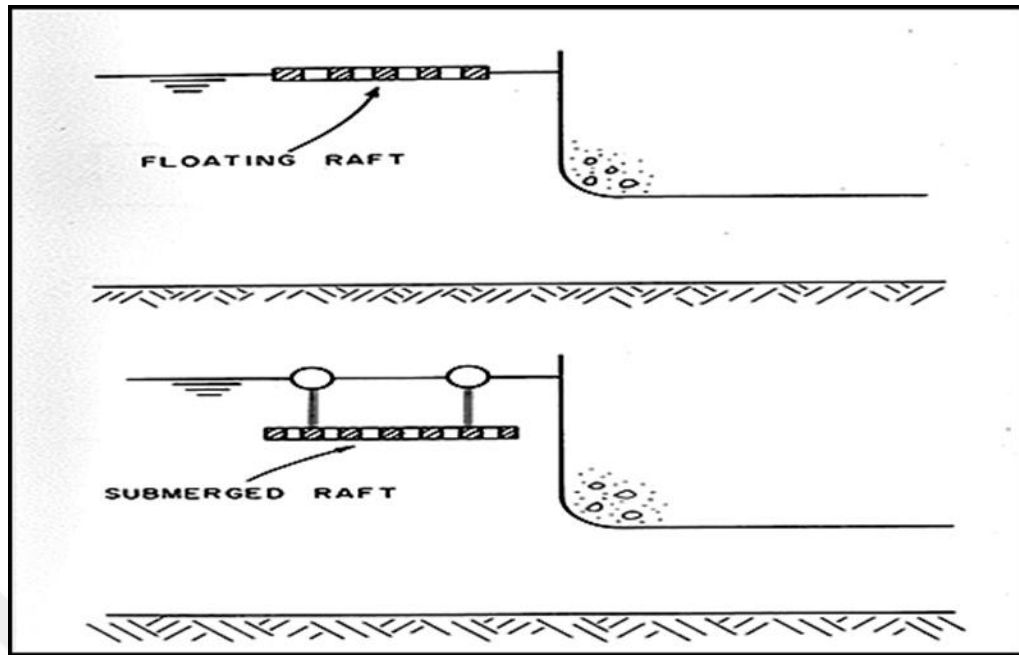


Figure 6: Floating and Submerged Raft Vortex Suppressors [52]

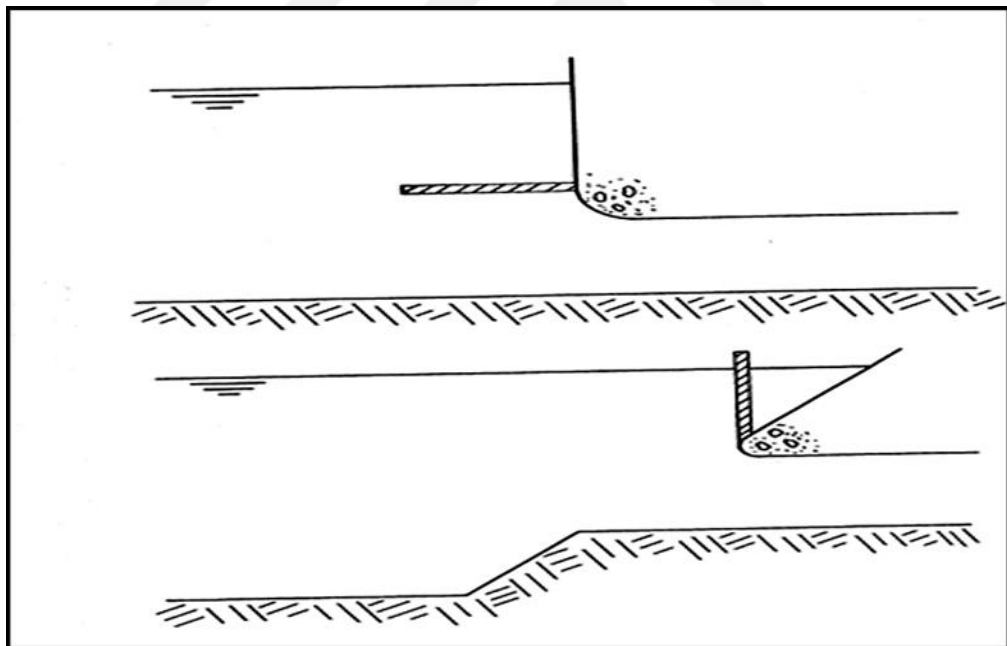


Figure7: Extended Plates Used to Dissipate Vortices[52]

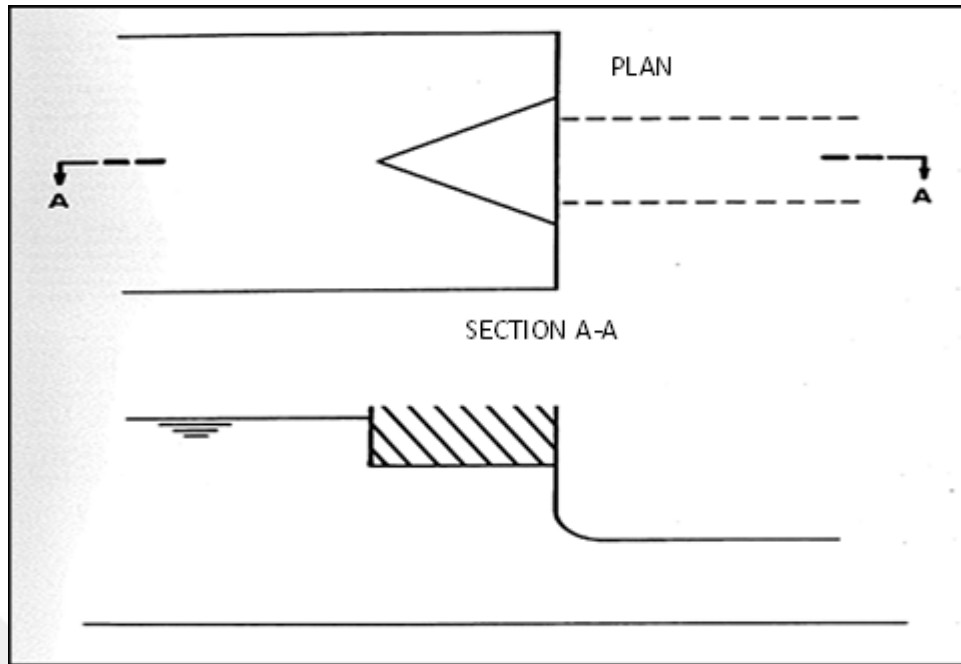


Figure 8:Wedge Type Vortex Suppressor[52]

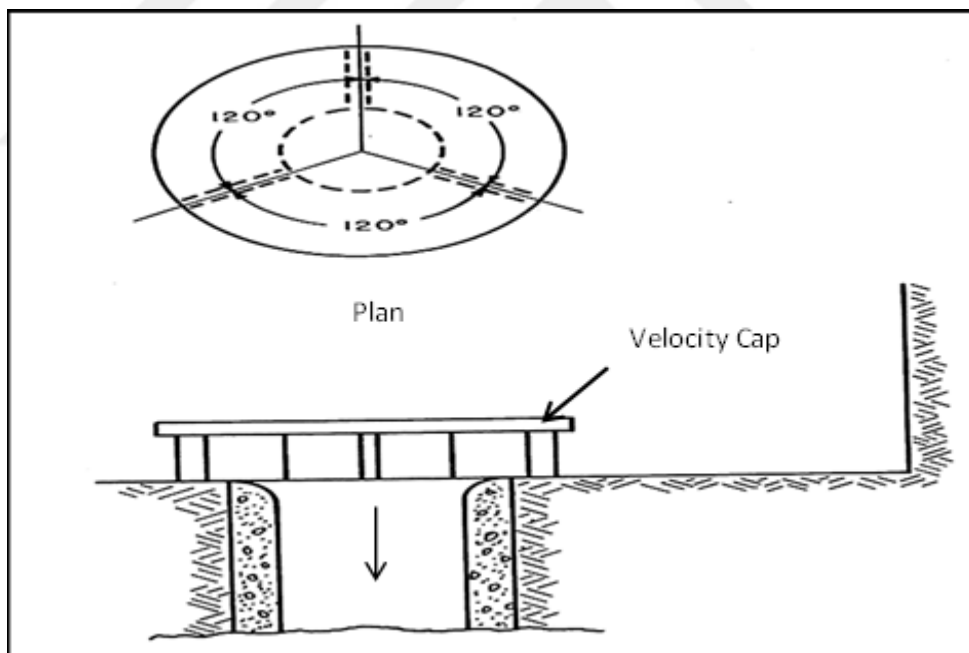


Figure 9:Covered Intake Type Vortex Suppressor[52]

1.10 AIR- ENTRAINING VORTICES AT INTAKES

A single-phase flow becomes a two-phase flow due to the entrained air. Air begins to enter an intake if its submergence is less than a certain limit value, known as critical submergence. According to studies [57–59], critical submergence is thus a crucial characteristic about intake structures. According to researchers [51] and [60], an intake's efficiency may drop by 15% for every 1% of entrained air.

1.11 CHARACTERISTICS OF AIR- ENTRAINING VORTICES

Air entraining vortex is mainly characterized by the following parameters that are follows [61] .

1.11.1 Inlet Direction Relative to the Mean Canal Flow

This parameter refers to the angle or orientation at which the intake is positioned relative to the direction of the main flow in the canal .

1.11.2 Ratio of Intake Velocity to Mean Flow Velocity

This is the ratio of the velocity of the fluid entering the intake (intake velocity) to the average velocity in the canal flow (mean flow velocity of the incoming canal flow).

A higher ratio can indicate a stronger tendency for vortex formation. A higher intake velocity can create a significant pressure drop and generate swirling motion.

1.11.3 Upstream Circulation of the Mean Flow

Circulation refers to the presence of rotational motion or angular momentum in the fluid upstream of the intake. It is influenced by factors such as upstream obstructions, bends, and overall flow pattern.

1.11.4 Intake Froude number

It is a dimensionless parameter that compares the flow's inertia force to the gravitational force acting on the fluid. A higher Froude number indicates a stronger potential for vortex formation due to the dominance of the inertial forces over gravitational force, the mathematical formula that represents Froude number is given in Chapter III.

CHAPTER II

LITERATURE REVIEW

2.1 INTRODUCTION

The study of vertical air-entraining vortices in hydraulic systems has attracted a great deal of scholarly attention due to their great relevance in hydraulic engineering in relating the design and operational performance of hydraulic systems performance by mechanical damage, and understanding of the underlying mechanisms of vortex formation and prediction of parameters is needed to optimize intake systems and ensure reliable operational efficiency. Through this review various studies conducted to scrutinize formation, dynamics and mitigation strategies at an intake are discussed. This will provide an overview of experimental, analytical and numerical methods, focusing on important findings and progress in prediction and on the vortex event. This chapter discusses the effects of various parameters, such as submergence depth, flow rate, and intake geometry.

2.2 APPROACHES FOR STUDYING VORTEX PHENOMENON

There are three approaches for studying vortex phenomenon as researchers outlined [62] :

1. The scale effects and the vortices under various hydraulic and geometric conditions were considered experimentally.
2. Validation of the numerical solution for the flow domain by comparing it with the results that are approved by the experimental data .
3. Analytical (mathematical) solution of fluid dynamics equations that predict how vortices form and behave at water intake structures.

2.3 RELATED STUDIES

The hydraulic factors that influence the formation of air-entraining vortices at horizontal intakes are investigated in this experimental study in [63]. especially when approach flow-induced circulation is not occurring. In their studies, the researchers used six different intake pipe size, methodically varied the flow discharge, and used adjustable approach channel side walls to discover critical submergences. The experiment consists of a large rectangular reservoir with dimensions of $3.1 \text{ m} \times 3.1 \text{ m} \times 2.2 \text{ m}$. With a diaphragm slab supplying consistent flow conditions, it consists of three sections: an input portion, an active reservoir section, and a dead-volume part. For dimensionless critical submergence, empirical equations were developed as functions of the important hydraulic parameters, such as the Weber, Reynolds, and Froude numbers. The findings showed that significant boundary blockage affects vortex formation at modest sidewall clearances and that critical submergence rises with these parameters. Researchers found that the impact of this parameter on vortex generation becomes insignificant beyond a specific sidewall clearance limit. Furthermore, the study examined the scale effects, which showed that correction factors are required for model-based predictions used in real-world applications. The empirical equations showed good agreement with previous researches and were checked against available data.

The aim of the study [64] was to investigate the occurrence of vortices at two vertical pump intakes with an emphasis on the impact of suction velocity and submergence depth on vortex formation. In order to model and characterize flow behavior, a numerical simulation method utilizing Computational Fluid Dynamics (CFD) and ANSYS Fluent 19.0. Researchers used the SST $k-\omega$ turbulence model to simulate vortex generation under five distinct suction velocities ($V = 0.75, 1, 1.5, 2, 3 \text{ m/s}$) and five different submergence depths ($S = 1D, 1.25D, 1.5D, 1.75D, 2D$). The results indicated that when the submergence depth is at least 1.5 times the bell mouth diameter ($S/D > 1.5$), air-entraining vortices do not form. The pulling air vortex appeared clearly when the submergence became 1.25 from the bell mouth diameter and the suction velocity was equal to or greater than 2 m/s . While more obvious vortex formation occurred, the submergence depth is equal to the bell diameter $S/D = 1.0$ when the Froude number is beyond 0.77. Moreover, when the distance between

the sidewall and intake center was at least twice the bell mouth diameter ($c/D = 2$), subsurface vortices were insignificant. The study's findings indicate that preventing vortex formation and optimizing intake location are crucial for enhancing pump efficiency, minimizing air entrainment, and protecting structures from damage.

The essential submerge depth of an intake located in an open channel flow was of interest to [65]. The Rankine half body of revolution served as a paradigm for the investigation, which was grounded in theoretical and experimental research. When the upper border of the Rankine half body of revolution reaches the water surface above the intake center, a critical submerged depth is noted. According to their findings, the radius of an imaginary critical spherical sink surface (CSSS) with a radial velocity equal to half of the incoming uniform canal flow velocity is equivalent to the critical submergence. They found that this theoretical agrees well with the experimental results. This is a practical way to predict the critical submergence depth in real intake. This analysis is particularly useful for optimizing intake design by balancing conditions of flow and submergence requirements and enabling the engineers to address operational problems relating to air- entrainment at intakes.

Researchers experimentally studied the bathtub vortex in Reference [66], where water drains through a tiny central hole to create a continuous free-surface flow in a spinning cylindrical container. A 40 cm diameter cylinder was part of the setup, and it was placed within a square reservoir with a 5 mm opening above the horizontal bottom to let water in. Fluid outflow was assisted by a circular drain hole in the middle with a radius of 1 mm. The free surface profile and velocity field are experimentally measured at rotation rates of 6, 12, and 18 rpm. The objective was to analyze the flow structure, velocity profiles, and free surface deformation, comparing observations with Ekman boundary layer theory and Lundgren's vortex model. Using flow visualization techniques, including fluorescent dye injection and particle tracking, the study captured key vortex characteristics, such as the geostrophic bulk region, bottom Ekman boundary layer, surface boundary layer, and a central vortex core with strong axial down-flow. Results showed that while nonlinear effects became more noticeable close to the vortex core, velocity distribution in the bulk matched linear Ekman theory. As the rotation rate increased, the surface depression

deepened, and at critical speeds (~ 22 rpm), tip instabilities and bubble detachment led to the formation of an air-filled core extending through the drain hole. Numerical simulations of Lundgren's model accurately predicted the surface profile and azimuthal velocity once surface tension effects were accounted for, addressing previous overestimations of the depression depth. The study concluded that while linear Ekman theory effectively describes bulk flow, a complete understanding of the bathtub vortex requires consideration of nonlinear effects, boundary layer interactions, and surface tension, especially in applications such as drainage systems, intake structures, and geophysical fluid flows.

The goal of the research in reference [1] was to investigate the effects of boundary friction and dimensionless factors on air-entraining vortices and the critical submergence of intakes under cross-flow and still water circumstances without circulation. Therefore, in order to investigate the connections between the Froude, Reynolds, and Weber numbers, researchers conducted tests in a horizontal rectangular channel that was 50 cm wide. According to their findings, the behavior of these vortices is greatly influenced by limit values, which are dependent on flow and geometrical parameters. Higher limit values are associated with slower cross-flow velocities. Furthermore, the research demonstrates that boundary friction, especially when the boundary coincides with the vortex center, has a more significant effect on air-entraining vortices when it occurs close to the water's surface. The researchers also found that the ratio of intake to approach flow velocities affects the frictional effects of boundary sections at the free surface. Crucially, they came to the conclusion that the frictional impact on the critical submergence and the air-entraining vortex is insignificant if the vertical clearance from the top edge of a submerged boundary to the free surface is more than 0.1 times the critical submergence.

Particle image velocimetry (PIV) was used in reference [67] to experimentally analyze the flow field and structure of unforced bathtub vortices. The researchers created the Gas Entrainment Test Section (GETS) facility, which had a $700 \text{ mm} \times 500 \text{ mm} \times 500 \text{ mm}$ test tank with clear sides designed to ensure symmetrical flow and avoid forced rotation. Three horizontal planes were used to determine velocity fields associated with the free surface vortex during tests

conducted at various Reynolds numbers between 2400 and 11000. However, Rankine's flow solution only provided a good approximation of the tangential velocity component close to the free surface. The results of this study indicate that unforced vortices close to the free surface can be well described by the potential flow approximation, and the observed radial and tangential velocity profile behaviors may be used as boundary conditions to create more accurate analytical vortex models. Additionally, the results serve as a useful guide for confirming analytical and numerical models of free surface vortices. Further investigation into the effects of induced rotation on vortex structure is being conducted by comparing forced and unforced vortex flow fields.

Using both theoretical and experimental techniques, researchers in reference [32] examined the properties of free surface vortices at hydraulic intakes. Water entered tangentially by four rectangular inlets and departed through a central vertical intake in a cylindrical Perspex tank measuring 52 cm in radius and 45 cm in height. Particle image velocimetry (PIV) was used to assess the velocity distribution of the free surface vortex and collect data on circulation distributions, water surface profiles, vortex core radius change, and radial and tangential velocity distributions. Based on the results, an empirical model was created that describes key vortex characteristics, including water surface profile equations, vortex core radius, and tangential velocity. Moreover, dimensionless equation for critical submergence at the hydraulic intake was developed in terms of the circulation and Froude numbers only. This equation agreed with both existing theoretical models and experimental data. By improving knowledge of vortex behavior and offering a solid framework for forecasting vortex properties in hydraulic systems, these methods help to design and manage hydraulic intake facilities to reduce the occurrence of the vortex.

At Hamburg University of Technology, researchers [68] have explored how strong momentum influences the formation of large vortices in the gas core of a cylindrical tank. They set up an experiment using a large reservoir that was 4 meters wide and high, with a capacity of 50 cubic meters of water filled to different levels. A measuring rod they used to accurately measure the height of the water. The tank had 24 round glass windows arranged in three rows around it, which allowed them to see inside. They placed a camera at one window to capture the gas core's length and

used a bright 1000-W spotlight at another window to help with visibility. The team conducted particle image velocimetry (PIV) measurements at a submergence depth of 1.5 meters and a Froude number of 0.5. Results show a sigmoid dependency of gas-core length on the Froude number, leading to an equation for critical Froude number determination. Vortex suppressors, especially cross- and cross-baffle-shaped designs, effectively prevent gas entrainment and simplify implementation. Particle image velocimetry (PIV) measurements confirm velocity fields, enhancing safety with a high-power LED system. PIV results validate numerical simulations and align well with the improved Burgers-Rott vortex model by Ito et al.

Using both numerical and physical (experimental) models, the researchers [69] simulated vortex flows, particularly the air core vortices near the power plant's inlets. by comparing the Euler and Lagrangian methods to assess their computational effectiveness and correctness. The Eulerian technique used the finite volume scheme, whereas the Lagrangian approach employed smoothed particle hydrodynamics (SPH). A cylindrical vortex tank composed of borosilicate glass measuring 40 cm in height and 29.4 cm in inner diameter makes up the experimental setup. At the bottom of the container, the intake diameter measures 3.5 cm. Particle image velocimetry (PIV) was used to measure the velocity field as the discharge flow was pushed from the inlet towards the outlets. The results confirmed that while both methods can model vortex structures, the Eulerian approach showed superior performance, especially for large-scale quantitative analyses aimed at reducing air drift and improving system efficiency. The Eulerian method showed better accuracy in predicting the tangential and radial velocity distributions, with errors of 7.4% compared to 23.3% for the Lagrangian approach. In conclusion, while both methods successfully simulated steady vortex flow, the Eulerian approach was more efficient and reliable due to its lower computational cost and greater accuracy.

In the reference [62] , the paper presents an analytical model to examine free surface vortex formation at vertical intakes, utilizing a spiral streamline approach. The momentum and continuity equations were integrated within a vortex flow domain to develop a comprehensive understanding of the phenomenon. By fully solving the three-dimensional flow field with an air-core vortex, the study derived a critical submerged depth equation, essential for mitigating vortex-induced issues,

using non-dimensional parameters. Furthermore, the empirical equations were used to illustrate the water surface profile and air-core formation. One of the study's key conclusions is that the formation of an air-core vortex causes the pressure at the intake entry to decrease to atmospheric levels, which helps formulate the critical submerged depth equation. The suggested model showed high accuracy, with deviations in water surface profile predictions of about 20%, when tested against experimental data and previous analytical investigations. Comparing the spiral model to the well-known Odgaard (1986) [70] model showed that it more closely matches the experimental findings. Figures 10 and 11 illustrate the vortex structure. Figure 10 shows spiral vortices in both schematically and in real-spiral vortex, while to prevent air entrainment from surface vortices, a minimum submerged depth of the intake, referred to as the critical submerged depth S_c is advised in Figure 11(a). Submerged depth is defined as the vertical distance from the water surface to the intake center level as illustrate in Figure 11(b) . In order to avoid problems like air entrainment, energy dissipation, and structural vibrations, it is crucial to maintain a sufficient submerged depth. This study provides hydraulic engineers with important information for optimizing intake designs.

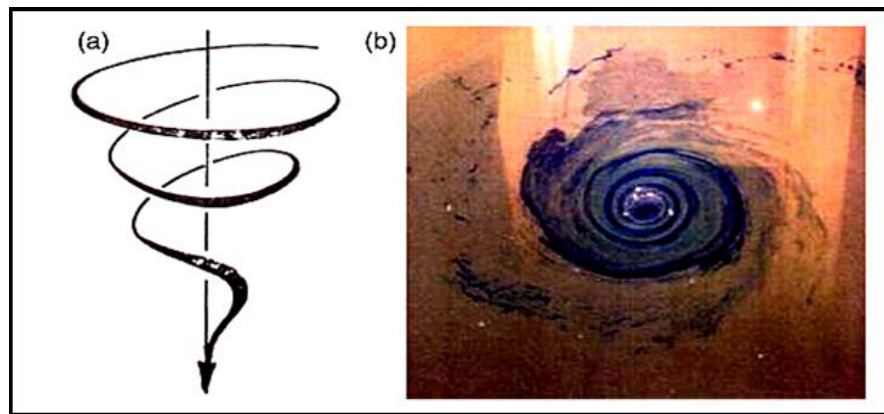


Figure 10: Spiral Streamlines in a Vortex (a) from [71] and (b) from [72], as mentioned in [62]

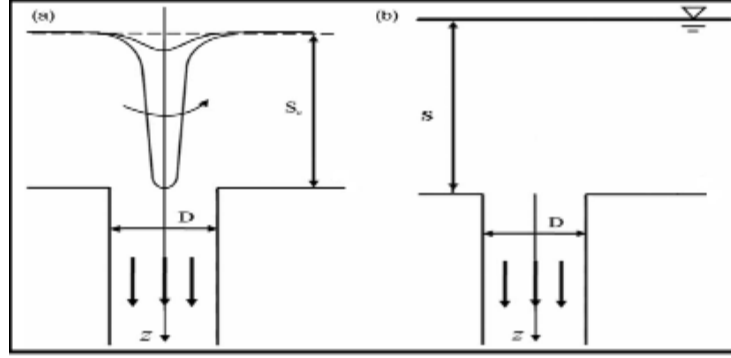


Figure 11: Schematic View of an Intake and Definitions
(a) With an Air Core Vortex (b) Without a Vortex as Mentioned in [62]

The study [33] aimed to improve existing semi-empirical formulas for predicting vertical air core vortices, addressing limitations such as reliance on unknown parameters, laborious-intensive measurements, and inapplicability to air-entrained vortices. The authors examined two widely used formulas Vatistas et al. [73] and Hite and Mih [74] and modified them to make them more practical by introducing new parameters based on the ratio of vortex height to inlet immersion. The improved formulas eliminated the need for local viscous core radius measurements, replacing them with a more accessible half-depth radius. The study used experimental data from previous research, including Sun and Liu [32], to validate the modified formulas, demonstrating their accuracy in predicting vortices for both air-entrained and non-air-entrained vortices with and without inlet. The results confirmed that the improved formulas provide better agreement with experimental data, requiring fewer arbitrary adjustments, and making them more suitable for practical engineering applications.

CHAPTER III

DIMENSIONAL ANALYSIS FOR AN AIR-ENTRAINING VORTEX

3.1 INTRODUCTION

The air-core vortex is a very complex phenomenon due to which it is very difficult to find a governing equation relating to its flow characteristics (especially for its profile). As presented later in Chapter IV , there can be a semi-empirical expression for the profile of the air core vortex. In brief, it is not directly possible to find the function relationships among the variables relating to the profile of an air-entraining vortex. The easiest way to find relations among the variables is to utilize the dimension analysis. Dimensional analysis arranges the variables in dimensionless groups and it causes a great amount of reduction in the number of both groups and experiments. A problem may or may not be theoretically solved. A dimensional analysis can be used, especially before conducting the experiments. After obtaining test data, one can draw graphs and find or show the relationships among the dimensionless groups.

3.1.1 Application of the Buckingham Pi -Theorem

The Buckingham Pi Theorem serves as a fundamental instrument in dimensional analysis. If there are “n” variables and “m” fundamental dimensions (usually mass, length, and time), the number of the dimensionless groups is equal to $(n - m)$ that are designed as Pi terms.

3.1.2 Formation of the Dimensionless Groups

Utilizing the Buckingham Pi Theorem, amalgamating the identified variables to construct dimensionless groups (also referred to as Pi terms). These groups should be independent of any units. These dimensionless groups facilitate the generalization of flow behavior and are instrumental in comprehending vortex dynamics

3.1.3 Interpretation of the Results

After deriving the dimensionless parameters or groups, their physical implications and the manner in which they govern the dynamics of the system can be discussed and evaluated.

One can establish dimensionless parameters that allow the prediction and evaluation of vortex-behavior. Thus considering the scale of the model experimental results can be applied to the real structures (prototype).

3.2 DIMENSIONAL ANALYSIS

In general, the influencing factors and the functional connection involving the profile "z" of a vertical air-entraining vortex at a vertically running downhill pipe intake (not required, but assuming it is situated in a rectangular canal or sump) may be expressed as

$$z = f(S, V, U_{\infty}, \rho, \nu, \Gamma, \zeta, g, D, r, c, b_1, b_2, \ell) \quad (3.1)$$

$$z = h - (H_r - H_0) = \text{vertical downward axis} \quad (3.2)$$

The main variables in Eq.(3) are as follows below, (Figure 12 shows the main variables of the air-core vortex).

b_1 : distance (clearance) of the right side wall of the canal to the intake center.

b_2 : distance (clearance) of the left side wall of the canal to the intake center.

c : Vertical distance of the bottom (bed) boundary to the intake center (bottom clearance).

D : Internal diameter of the intake.

g : Acceleration due to gravity.

h : Total vertical height (depth) of the air-core vortex profile.

H_0 : the air-core vortex's lower closed end tip's vertical distance from the intake entrance.

H_r : The vertical distance of the profile of the air-core vortex at r .

ℓ : Distance of the end (rear) wall (gate) to the intake center.

r : Radial distance from the vertical axis of the vortex (vertical intake).

S : Submergence of the intake.

U_{∞} : Incoming uniform canal flow velocity.

V : Average intake velocity.

z : A vertical axis that descends from the horizontal free-surface along the air-core vortex's vertical central axis (air-core vortex profile)

Γ : Imposed circulation.

ρ : Fluid density (water density).

σ : Surface tension of fluid (water).

ν : Kinematic viscosity of fluid (water).

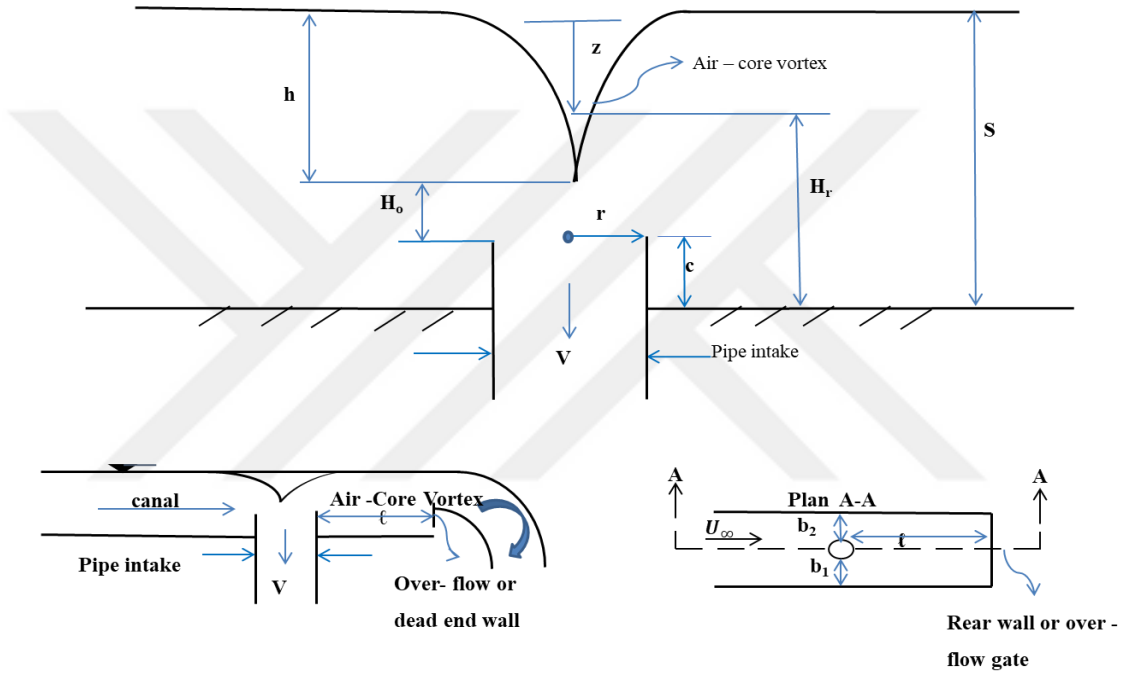


Figure 12: Simple Illustration of the Main Variables of the Air-core Vortex

The dimensional analysis considering Eq. (3.1) is conducted in Appendix I. The functional relationship between the dimensionless variables in Eq. (3.1) is obtained as :-

$$\frac{z}{D} = f_2 \left(\frac{V}{U_{\infty}}, \frac{S}{D}, \frac{r}{D}, \frac{c}{D}, \frac{b_1}{D}, \frac{b_2}{D}, \frac{\ell}{D}, F, K, Re, W \right) \quad (3.3)$$

$$F = \frac{V}{\sqrt{gD}}, \text{ (Intake Froude number)} \quad (3.4)$$

$$K = \frac{\Gamma}{(VD)}, \text{ (Circulation number)} \quad (3.5)$$

$$Re = \frac{VD}{\nu}, \text{ (Intake Reynolds number)} \quad (3.6)$$

$$W = \frac{\rho DV^2}{\sigma}, \text{ (Intake Weber number)} \quad (3.7)$$

Where:-

- The Froude number: is the ratio of the inertia forces to the gravity forces .
- The Reynolds number: it represents the ratio of the inertia forces to viscous forces.
- The Weber number : is the ratio of the inertia forces to the surface tension forces.

A boundary has two kinds of effects on "z" or air-entraining vortex. They are:

- i) Blockage effect on the ambient flow entrainment to the vortex-flow.
- ii) viscosity-friction effect.

Since friction dissipates the energy (velocity and hence circulation) of the vortex flow (especially the boundary segments across the vortex profile at the free surface) it very significantly reduces the air-core vortex development.

The air-core vortex profile is influenced by several characteristics, according to Eq. (3.3) (as mentioned in Chapter I). It is quite challenging to determine a link between " $\frac{z}{D}$ " and several dimensionless factors in Eq. (3.3) either graphically or analytically.

To overcome this difficulty, the following method is utilized.

For $z = h$, at the lower closed – end tip, $r = 0$, for which Eq. (3.3) results in:

$$\frac{h}{D} = f_3\left(\frac{V}{U_\infty}, \frac{S}{D}, \frac{c}{D}, \frac{b_1}{D}, \frac{b_2}{D}, \frac{\ell}{D}, F, K, Re, W\right) \quad (3.8)$$

Note that for a given flow and geometrical condition the parameters $\frac{V}{U_\infty}, \frac{S}{D}, \frac{c}{D}, \frac{b_1}{D}, \frac{b_2}{D}, \frac{\ell}{D}, F, K, Re$ and W are constant (independent of z) across whole height "h" of the vortex .

Thus, considering Eqs. (3.3) and (3.8), it is appropriate to rewrite Eq. (3.3) that is follows

$$\frac{z}{D} = f_4\left(\frac{h}{D}, \frac{r}{D}\right) \quad (3.9)$$

As may be seen, compared to Eqs. (3.3) or (3.8), the profile of the air-core vortex is much simplified in Eq. (3.9). The whole "z," "h," and "r" for a non-air-entraining vortex (air-core vortex in suspension) are measured and perhaps known.

Therefore, the plot of the whole profile of a non-air-entraining vortex (where the air-core vortex is in suspension) may easily be covered by Eq. (3.9). However, for an air-entraining vortex, "z," "h," and "r" for the imaginary portion of the air-entraining vortex downstream of the intake are unknown and not quantifiable since the bottom part of the air-core vortex is hypothetical.

As a result, the plot of an air-entraining vortex's whole profile cannot be made using Eq. (3.9). The primary challenge in applying Eq. (3.9) is this point. Eq. (3.9) must be changed in the manner described below to overcome this problem.

As it is presented a little later the most common available semi-empirical expressions relating to the profile of an air-core vortex ([73] and [74]) the profile of the air-core vortex is derived in terms of "h", "S" and "r_m". Here, r_m stands for the air-core vortex's local viscous-core radius, which is also known as the outer limit radius of the viscous area or the radius at the maximum tangential velocity that separates the outer inviscid zone from the inner. Since r_m varies across the whole height or h and requires so many measurements it is difficult to work with r_m.

Due to this reason, [33] have used r_{mid} in lieu of r_m where r_{mid} is the air-core's radius at the air-core vortex profile's half-depth (r_{mid} = r at z = h/2). Doing so, the equation of the profile of an air-core vortex is considerably simplified.

Hence, one can utilize h, S and r_{mid} in lieu of D in z/D, h/D and r/D, respectively in Eq. (3.9), Thereby Eq. (3.9) can be rewritten as

$$\frac{z}{h} = f_5\left(\frac{h}{S}, \frac{r}{r_{mid}}\right) \quad (3.10)$$

or,

$$\frac{z}{h} = f_6\left(\frac{h}{S}, \frac{r}{D}\right) \quad (3.11)$$

Considering Eqs.(3.9) -(3.11) the available test data relating to air – core vortices (for both non-air-entraining, air-core vortices and air-entraining vortices) in the studies by [32, 66-69, 75-77], (more details of these studies are given in Appendixes III and IV), the graphs relating to the profiles of the air core vortex are presented in Figures (13-16).

Figures (13-16) indicate that there is no good correlation among z/h , h/S , r/D or r/r_{mid} . This is because every available test data are obtained in different flow and geometrical conditions.



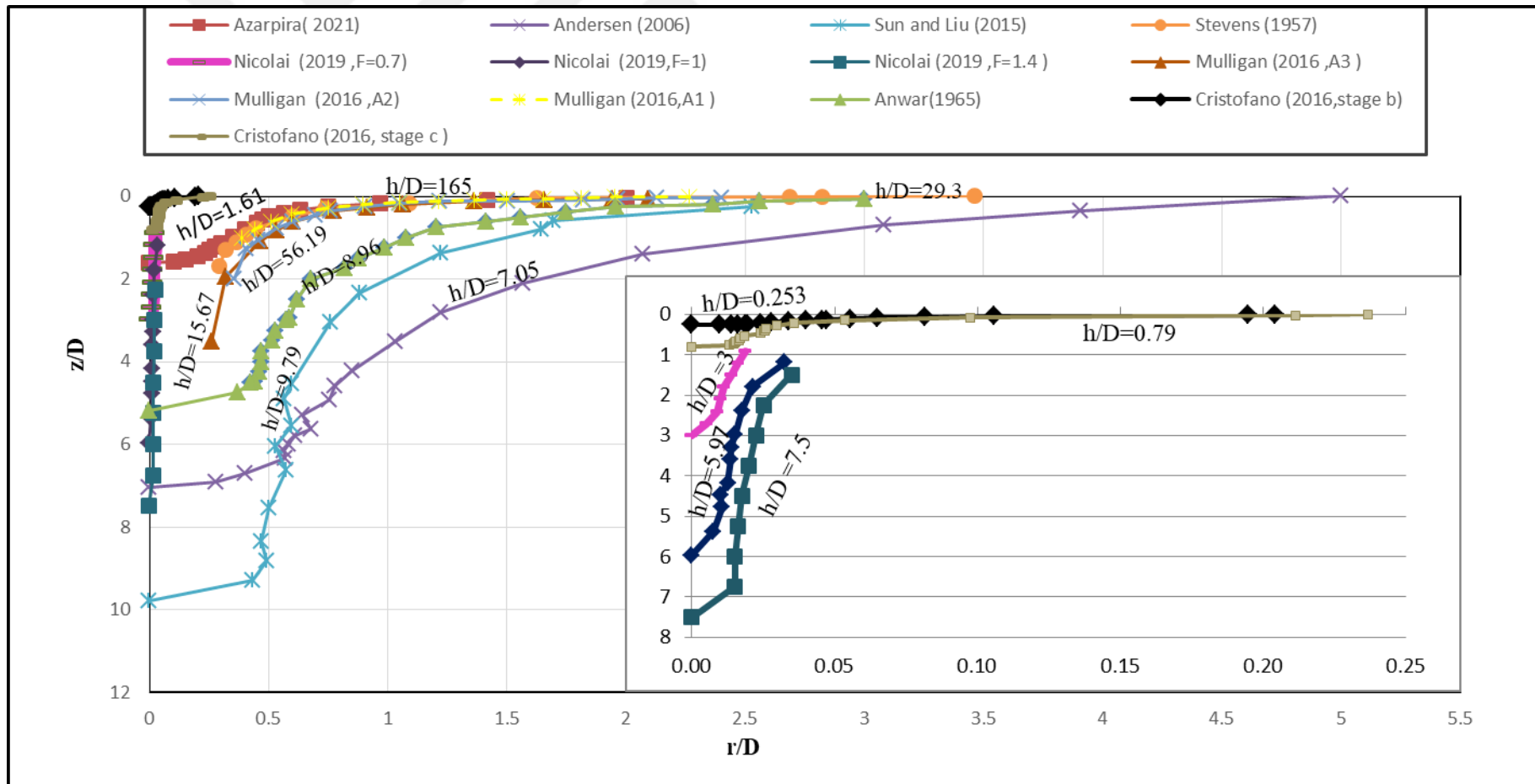


Figure 13: Profile of Air -core Vortices in Accordance with the Test Data Relating to z/D , r/D and the Curve Representing h/D Value.

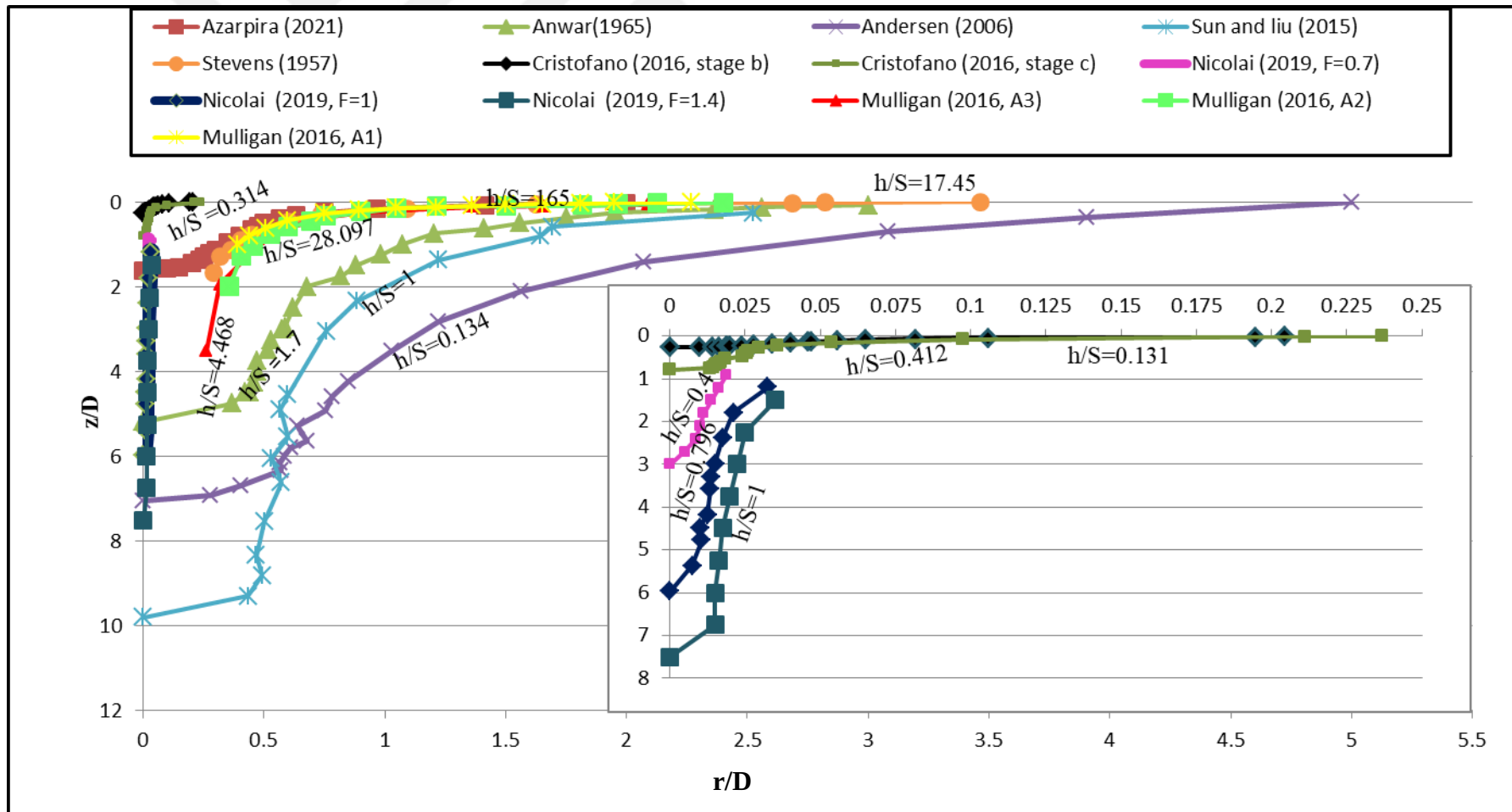


Figure 14:Profile of Air -core Vortices in Accordance with the Test Data Relating to z/D , r/D and the Curve Representing h/S Value.

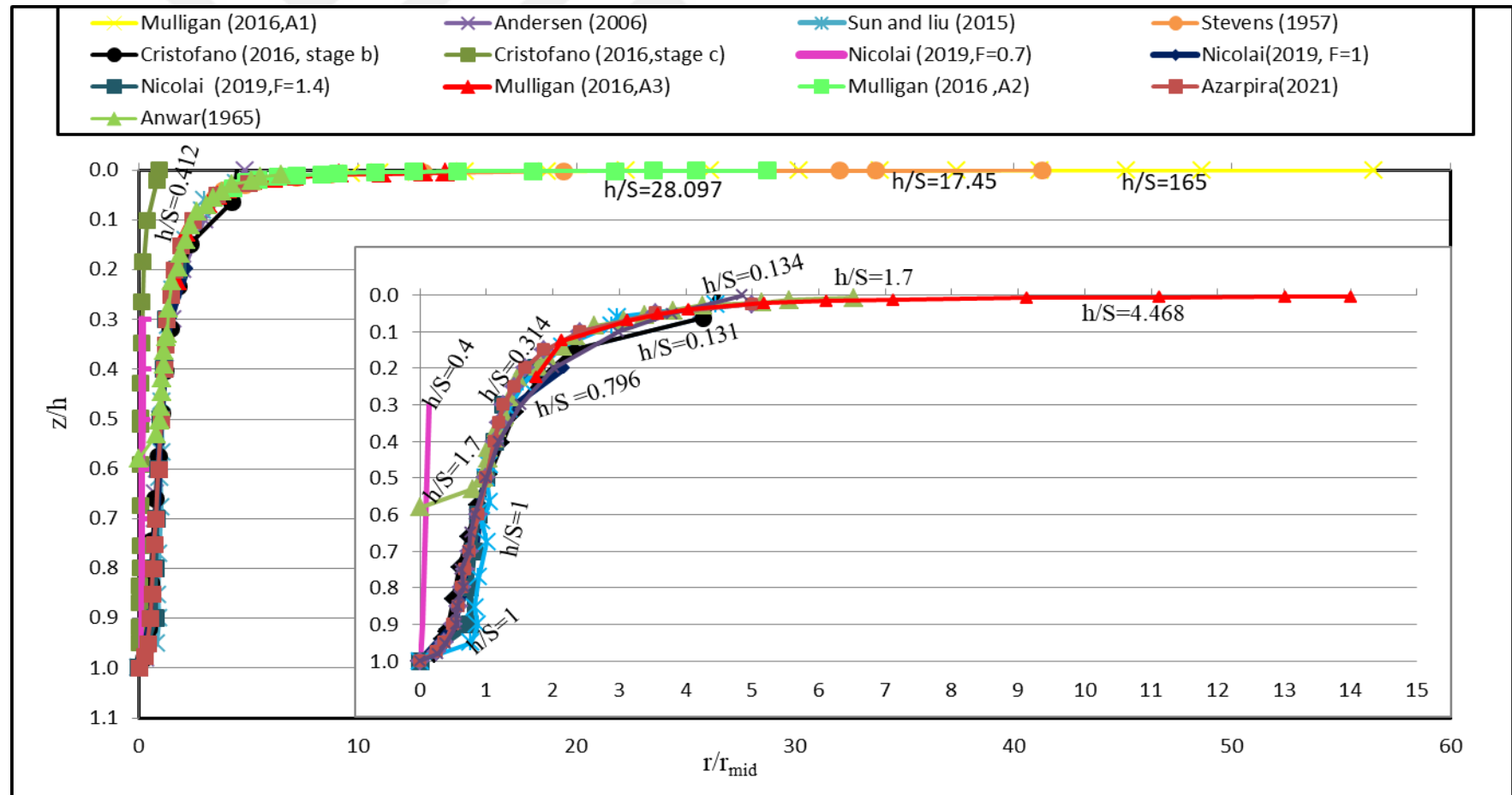


Figure 15:Profile of Air -core Vortices in Accordance with the Test Data Relating to r/r_{mid} , z/ h and the Curve Representing h/S Value

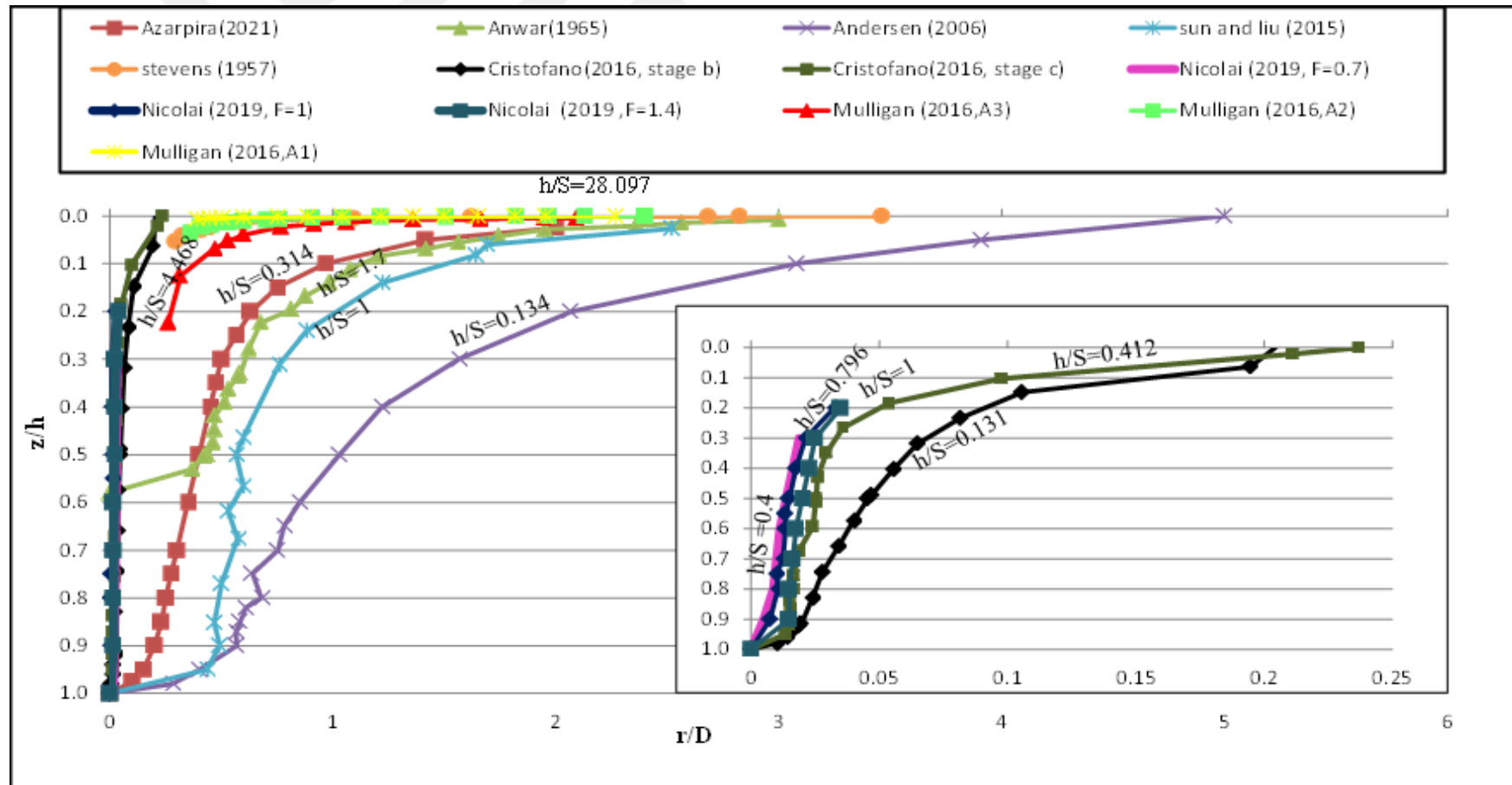


Figure 16: Profile of Air -core Vortices in Accordance with the Test Data Relating to z/h , r/D and the Curve Representing h/S Value.

CHAPTER IV

METHODOLOGY

4.1 THEORETICAL CONSIDERATIONS

Since the flow in an air-core vortex is a very complex process (that depends largely on flow and geometric conditions) it is very difficult to obtain exactly one theoretical general formula for the air core vortex profile (no specific one exists). Generic formulas are derived from experiments for air-core vortex or air-entraining vortex profile. Few semi-empirical case study-based formulas are available but they may work for the flow and geometric conditions under which they are obtained. Numerical methods and solutions are available to predict the profile of the air core vortex. For the air-entraining vortex, there is an empirical formula for a non-air-entraining vortex (an air-core vortex in suspension for which "h" is quantifiable) even though "h" is not measurable.

The formula pertaining to the profile of a vertical non-air-entraining vortex ($S \geq S_c$, air-core vortex in suspension), created by [33] is updated in the current work to approximate and forecast the profile of a vertical air-entraining vortex. Here's what it is.

In [33], they have improved the available semi-empirical formula relating to the profile of a vertical non-air-entraining vortex ($S \geq S_c$, air-core vortex in suspension) given in the studies by [74] {also [32], Eqs. (9) and (14); [78], Eq. (3)}, and {[73], Eq. (8)}.

The improved formula relating to the profile of a vertical non-air-entraining vortex in the study by ([33], Eq. 8)) , in conjunction with the formula given by [74] [also [32], [78], is as follows .

$$\frac{z}{h} = 1 - \frac{2}{\pi} \arctan(\alpha \bar{R}^2) \quad (4.1 \text{ a})$$

$$\alpha = 1 \quad \left(\frac{z}{h} \leq 0.5\right) \quad (4.1 \text{ b})$$

$$\alpha = \frac{(1 - \alpha_{LBP})}{\left[1 - \left(\frac{z}{h}\right)_{BP}\right]} \left(1 - \frac{z}{h}\right) + \alpha_{LBP} = 2 \left(1 - \frac{z}{h}\right) + \alpha_{LBP} \quad \left(\text{for } \frac{z}{h} \geq 0.5\right) \quad (4.1 c)$$

Where, α is a parameter; $\bar{R} = r/r_{mid}$ is the dimensionless horizontal radial distance from the vertical centerline of the vertical intake entrance or vertical air-core vortex depression; and α_{LBP} is the magnitude of α at $z/h = 1$ as it is explained a little later.

The variation of α with z/h and h/S is given in ([33], Figure 3) that is replicated in Figure 17. In this figure the slopes of the linear variation lines of α for whole magnitudes of h/S is -2.

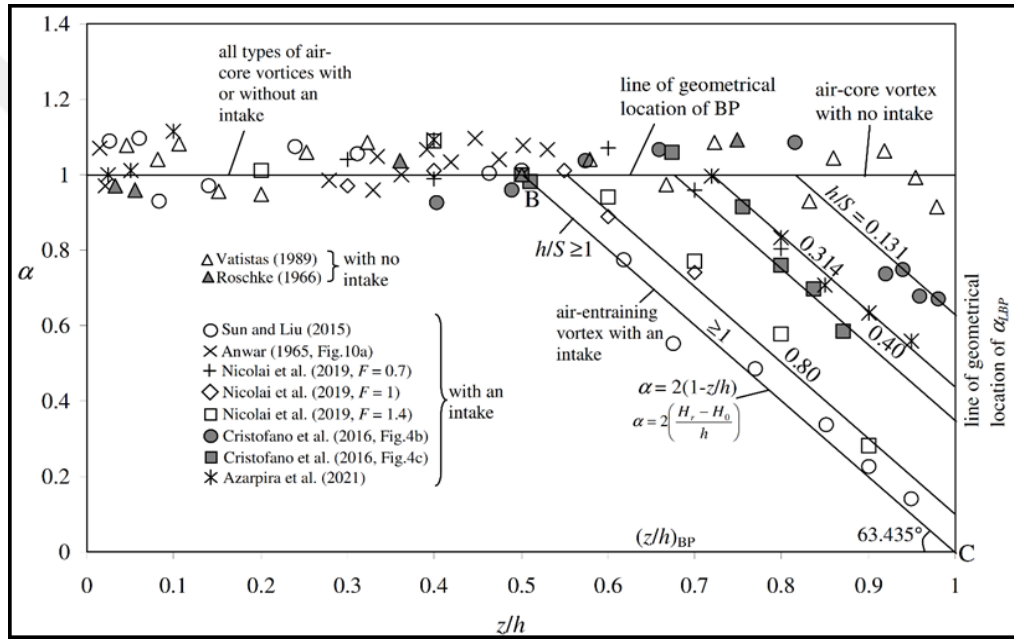


Figure 17: Variation of α with z/h [33]

It is indicated that in Figure 17 there is a point at which the slope of α suddenly changes from 0 to -2. This point is called the “Break-point (BP)”. Let denote $(z/h)_{BP} = z/h$ for its corresponding BP. The intersection of a linear line drawn from BP with a slope of -2 with the vertical line of $z/h = 1$ is $\alpha = \alpha_{LBP}$ (subscript LBP). The geometric location of α_{LBP} is the vertical line of $z/h = 1$ (Figure 17). Notice that for each BP, there exists a pair of $(z/h)_{BP}$ and $\alpha = \alpha_{LBP}$ corresponding to each other. The variations of $(z/h)_{BP}$ and α_{LBP} are given in the study by ([33], Figure 5) that are replicated in Figure 18.

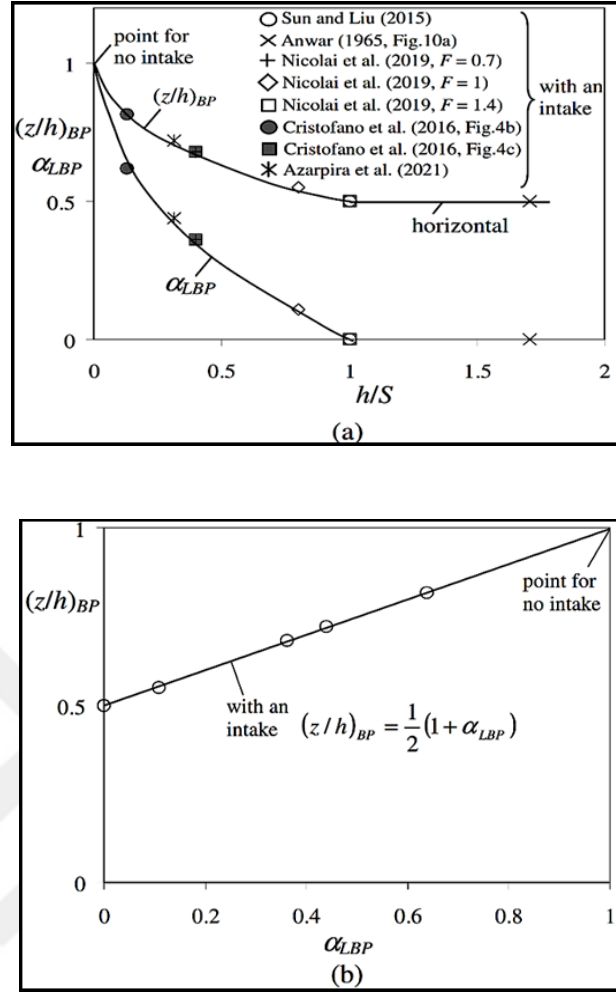


Figure 18: Variations of: (a) $(z/h)_{BP}$ and α_{LBP} with h/S (b) α_{LBP} with $(z/h)_{BP}$ [33]

The researchers in ([33], Figure 3) have considered that for an air-entraining vortex ($S < Sc$ or $h/S > 1$) the linear line of BC of the variation of α for $h/S \geq 1$ (Figure 17), the criteria of $z/h \leq 0.5$ and $z/h \geq 0.5$ in Eq. (4.1) are also valid because, in their solved-examples based on the test data relating to the air-entraining vortices in the study by ([75], Figure 10 a and b) the magnitudes of α_{LBP} and $(z/h)_{BP}$ are by chance close to 0 and 0.5, respectively due to which they got such an impression.

In reality; for an air-entraining vortex ($S < Sc$ or $h/S > 1$) the criteria of $z/h \leq (z/h)_{BP}$ and $z/h \geq (z/h)_{BP}$ should be used in lieu of $z/h \leq 0.5$ and $z/h \geq 0.5$, respectively in both Eq. (4.1) and Figure (17) in which the linear line of BC of the variation of α should not be valid for an air entraining vortex ($h/S > 1$).

Therefore, the parameter of $h/S = 1$ should be written in lieu of $h/S \geq 1$ for the linear line of BC indicated in Figure 17.

For a vertical air-entraining vortex (for a given $h/S > 1$) Eq. (4.1) and the slope “-2” of the linear line section of the variation of α are the same as that for a non-air-entraining vortex ($h/S \leq 1$). Therefore, for a given $h/S > 1$ the linear line section of variation of α for an air-entraining vortex can be drawn parallel to that for a non-air-entraining vortex (i.e., line of BC for $h/S = 1$ in Figure 17). If it is done so a graph similar to the sketch shown in Figure 19 is to be obtained.

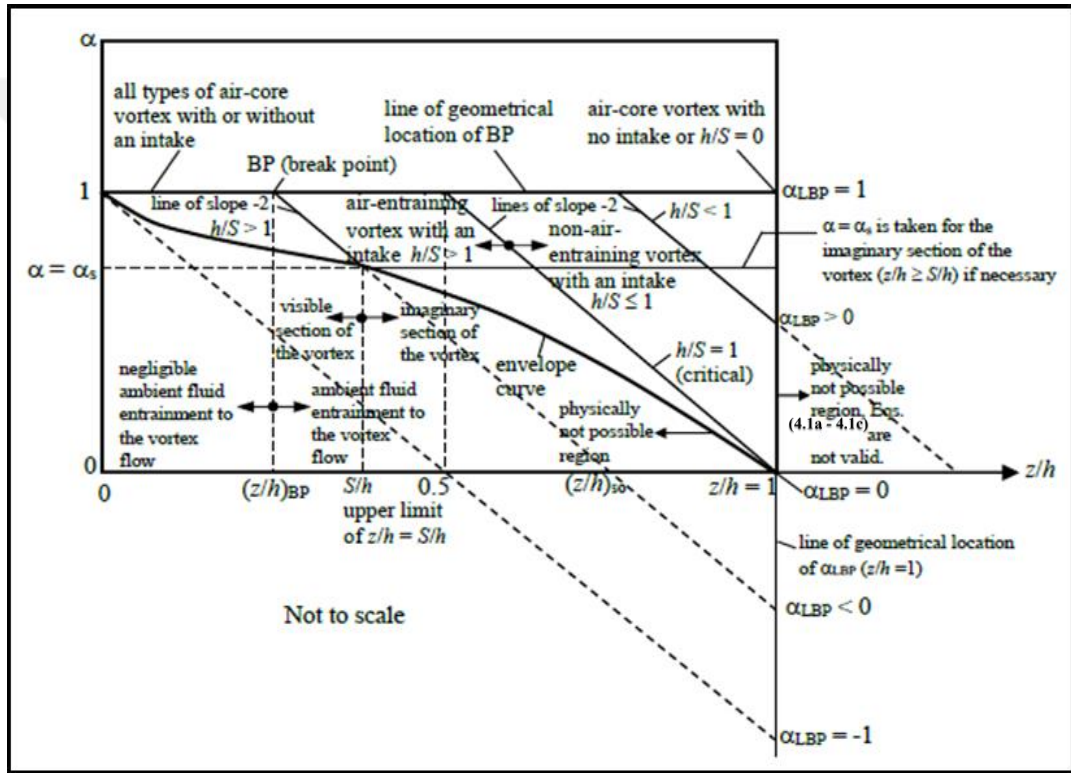


Figure 19: Comparison of Linear Variation of α for Air-entraining and Non-Air-entraining Vortices.

The main focus of the present study is (by considering the available test data) to provide a graph-chart as indicated in Figure 19 that can be useful for the prediction of the profile of a vertical air-entraining vortex with an intake in practice.

The linear line section of variation of α intersects both the horizontal axis of z/h (or line of $\alpha = 0$) at $z/h = (z/h)_{so}$ (subscript “so”) and the vertical line of $z/h =$

1 at $\alpha = \alpha_{LBP}$ that has to have a negative magnitude as indicated in Figure 19, Considering Figure 19 the following cases may be exist .



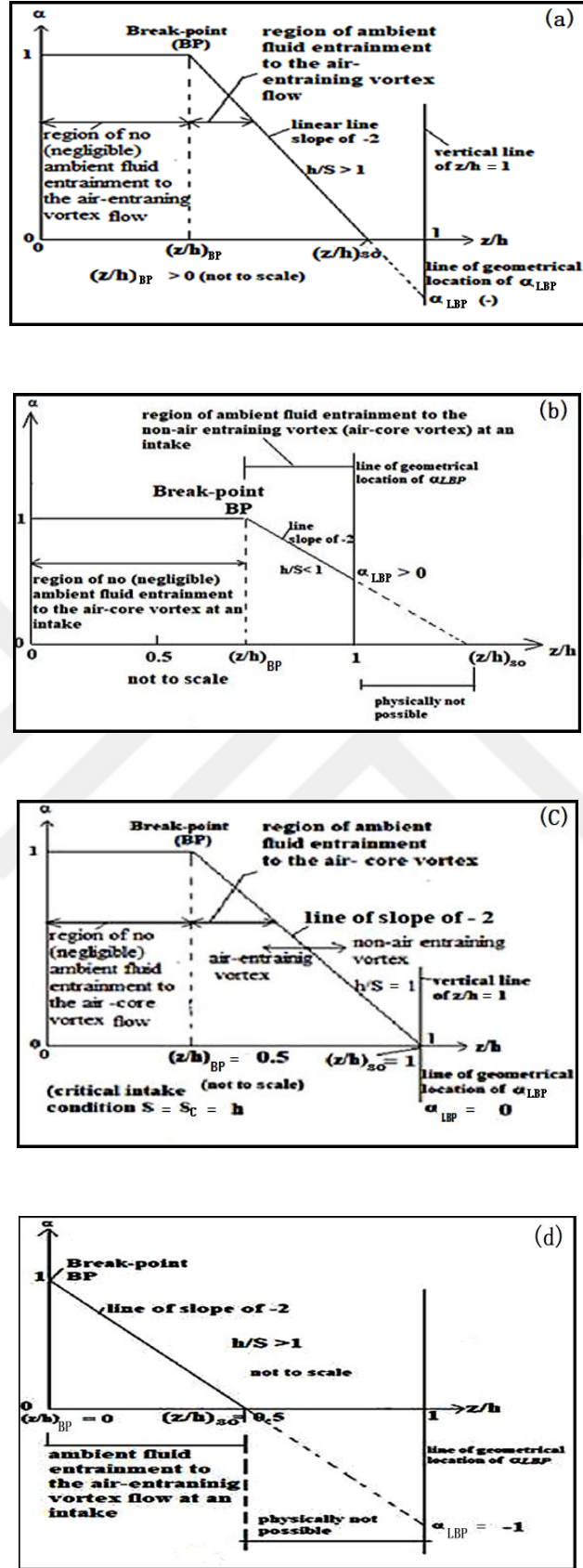


Figure 20: Variation of α with Respect to z/h : Intersection Points and Cases

As indicated in Figures 19 or 20, for every set of $(z/h)_{BP}$, α_{LBP} and $h/S > 1$ there exists a corresponding $(z/h)_{SO}$. Note that α has to physically be always positive ($\alpha \geq 0$). But, α_{LBP} is always negative ($\alpha_{LBP} < 0$) for an air-entraining vortex; and α_{LBP} is positive ($\alpha_{LBP} > 0$) for non an air-entraining vortex ($h/S \leq 1$) (Figure 17).

Since there is no ambient fluid entrainment to the vortex flow ($z/h > S/h$) at downstream of the intake the linear variation line of slope of -2 of the α indicated in Figure 19 becomes invalid for the imaginary section of the vortex profile at downstream of the intake. Therefore, for a visible vertical air-entraining vortex section ($h/S > 1$) in conjunction with Eq. (4.1) cannot physically take a magnitude greater than $z = S$.

That means $z = S$ or $z/h = S/h$ is the upper limit for z . Hence, for the visible vertical air-entraining vortex section ($h/S > 1$) Eq. (4.1) is valid for $0 \leq z/h \leq S/h$. Due to this reason Eq. (4.1) is not applicable in the “physically impossible regions” indicated in Figure 19. The limits of the validity of Eq. (4.1) for an air-entraining vortex should be an “envelope-curve” as presented in Figure 19.

The relationship between $(z/h)_{BP}$ and α_{LBP} is available in the study by ([33] , Figure 5b) that is as follows

$$\left(\frac{z}{h}\right)_{BP} = \frac{1}{2}(1 + \alpha_{LBP}) \quad (4.2)$$

Let $\alpha = \alpha_s$ corresponding to $z = S$ or $z/h = S/h$ (Figure 19). For $z/h = S/h$, Eq. (4.1) and Eq. (4.2) give

$$\alpha_s = 2 \left(1 - \frac{S}{h}\right) + \alpha_{LBP} = \left\{1 - 2 \left[\frac{S}{h} - \left(\frac{z}{h}\right)_{BP}\right]\right\} \quad (\text{for } z/h = S/h) \quad (4.3 \text{ a})$$

$$\left(\frac{z}{h}\right)_{BP} = \left(\frac{S}{h} - \frac{1}{2}\right) + \frac{\alpha_s}{2} \quad (4.3 \text{ b})$$

$$\alpha = 2 \left(\frac{S}{h} - \frac{z}{h}\right) + \alpha_s \quad [\text{for } (z/h)_{BP} \leq z/h \leq S/h] \quad (4.3 \text{ c})$$

As stated earlier since there is no ambient fluid entrainment to the vortex flow ($z/h > S/h$) at downstream of the intake the linear variation line of slope of -2 of the α indicated in Figure 19 becomes invalid for the imaginary section of the vortex profile at downstream of the intake (invisible imaginary section of the air-entraining vortex). Therefore, across whole height the invisible imaginary vertical air-entraining vortex

section the magnitude of α does not change thereby should remain the same and identical as the α_s ($\alpha = \alpha_s = \text{constant}$) as indicated in Figure 19. Therefore, it is proper to consider $\alpha = \alpha_s$ for $z/h > S/h$. In reality no need to indicate the profile of the invisible imaginary section of the air-entraining vortex (if wanted it can be indicated) that is not considered in the figures indicated in the present study.

Considering whole these points or constraints for the vertical air-entraining vortex section ($h/S > 1$) Eq. (4.1) is rewritten as

$$\frac{z}{h} = 1 - \frac{2}{\pi} \arctan(\alpha \bar{R}^2) \quad (4.4 \text{ a})$$

$$\alpha = 1 \quad [\text{for } (z/h)_{BP} \geq z/h] \quad (4.4 \text{ b})$$

$$\alpha = 2 \left(1 - \frac{z}{h}\right) + \alpha_{IBP} = 2 \left(\frac{S}{h} - \frac{z}{h}\right) + \alpha_s \quad [\text{for } (z/h)_{BP} \leq z/h \leq S/h] \quad (4.4 \text{ c})$$

$$\alpha = \alpha_s = \text{constant } (z/h > S/h \text{ that is for the imaginary section of the vortex if wanted}) \quad (4.4 \text{ d})$$

From Eqs. (4.4 c) and (4.1) or for $\alpha = 0$, $z/h = (z/h)_{SO}$ thus Eq. (4.4 c) gives

$$\left(\frac{z}{h}\right)_{SO} = \left(1 + \frac{1}{2} \alpha_{IBP}\right) = \left[0,5 + \left(\frac{z}{h}\right)_{BP}\right] \quad (4.5)$$

For an air-entraining vortex ($h > S = S_c$ or $h/S > 1$), the lower closed end tip point T (Figure 5c) of the air-core vortex (H_0 is negative; Figure 5c) is an imaginary point at some downstream-distance of the intake due to which “h” is not measurable (h is unknown). It is difficult to know the magnitude of h/S in advance. Therefore, one cannot know which linear line section of the variation of α belongs to the magnitude of h/S under consideration. To predict the profile of a vertical air-entraining vortex a trial-and error solution procedure is to be used. As it is explained a little later, the trial-and error solution procedure requires a chart (similar to Figure 19) indicating the variation of $(z/h)_{BP}$ (or α_{IBP}) with h/S to be provided by considering the available test data that is as follow.

The following flowchart in Figure 21 illustrates a systematic, trial-and-error approach designed to characterize a vertical air entraining vortex. This flowchart systematically guides the user through a series of iterative stages, where initial hypotheses are progressively refined based on derived results and experimental verification.

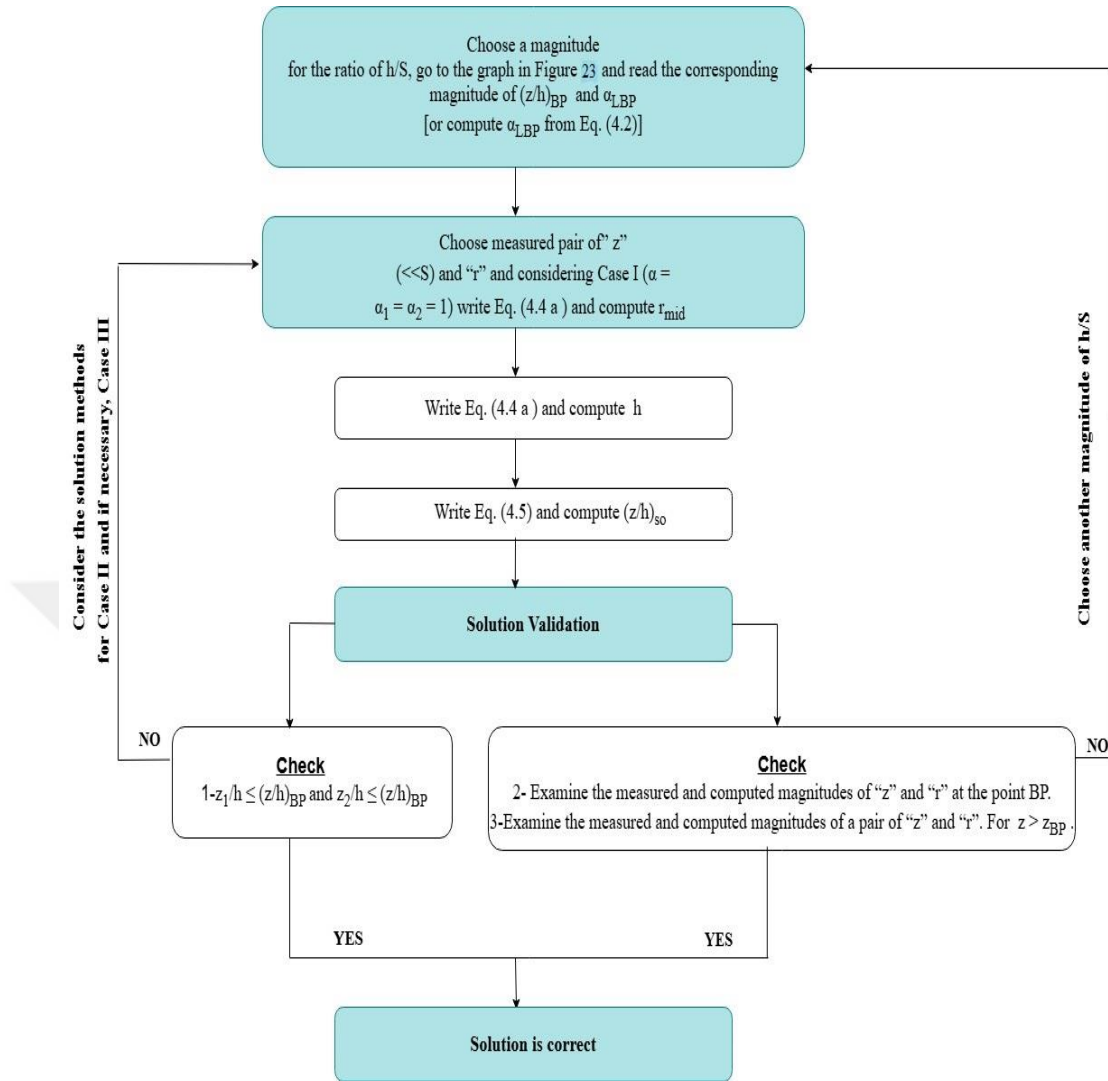


Figure 21:Flowchart for the Trial and Error Solution Procedure for the Profile of A Vertical Air-entraining Vortex

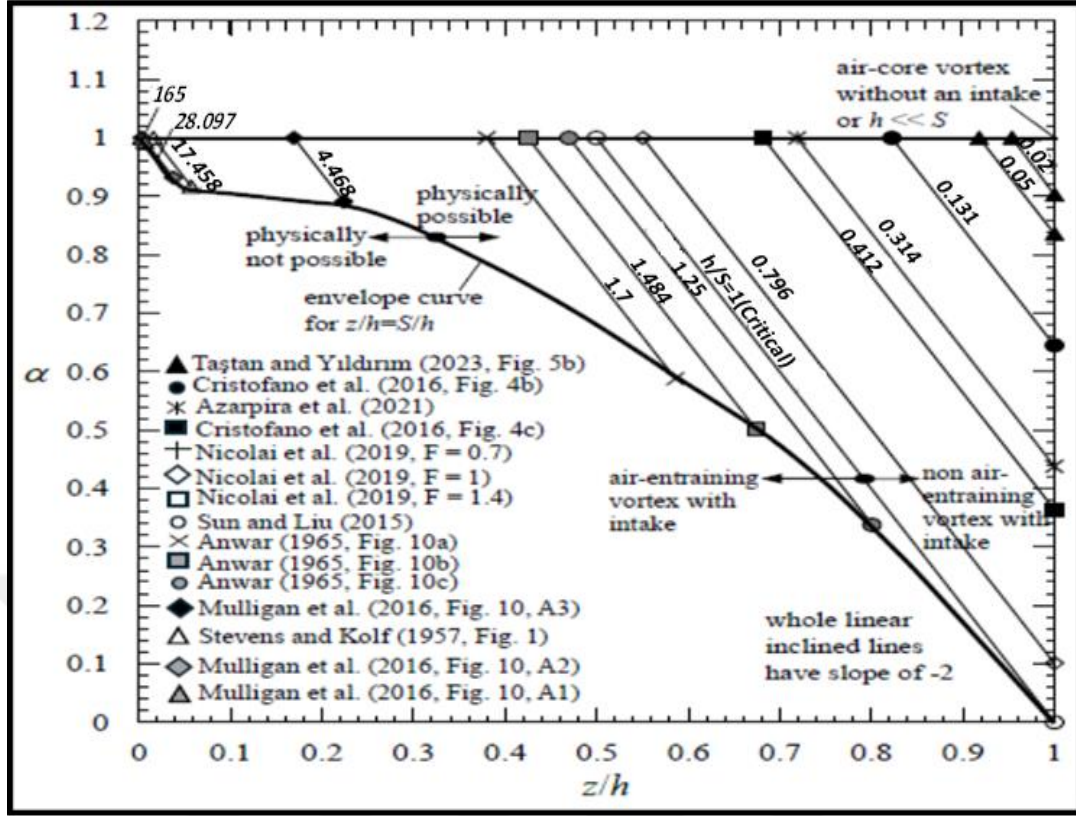


Figure 22: Predicted Variation of α with h/S and z/h Basing on the Available Test Data

The variation of $(z/h)_{BP}$ with h/S for the vertical non-air-entraining vortices ($h/S \leq 1$, air-core vortex in suspension) given in the study by ([33], Fig. 5), and the variation of $(z/h)_{BP}$ with h/S for the vertical air-entraining vortices ($h/S > 1$) predicted by means of Eq. (4.4a) in the present study in conjunction with the available test data in the studies by ([77], Figure 1), ([76], Figure 10, test cases A1, A2 and A3), and ([75], Figure 10) are indicated in Figure 22 (the solution procedure is provided a little later). Typical solved examples by considering the available test data are presented in Appendix II. Considering Eqs. (4.3) - (4.5) and the above explanations the variations of α , $(z/h)_{BP}$ and α_{IBP} with $h/S > 1$ and z/h are indicated in Figure 23.

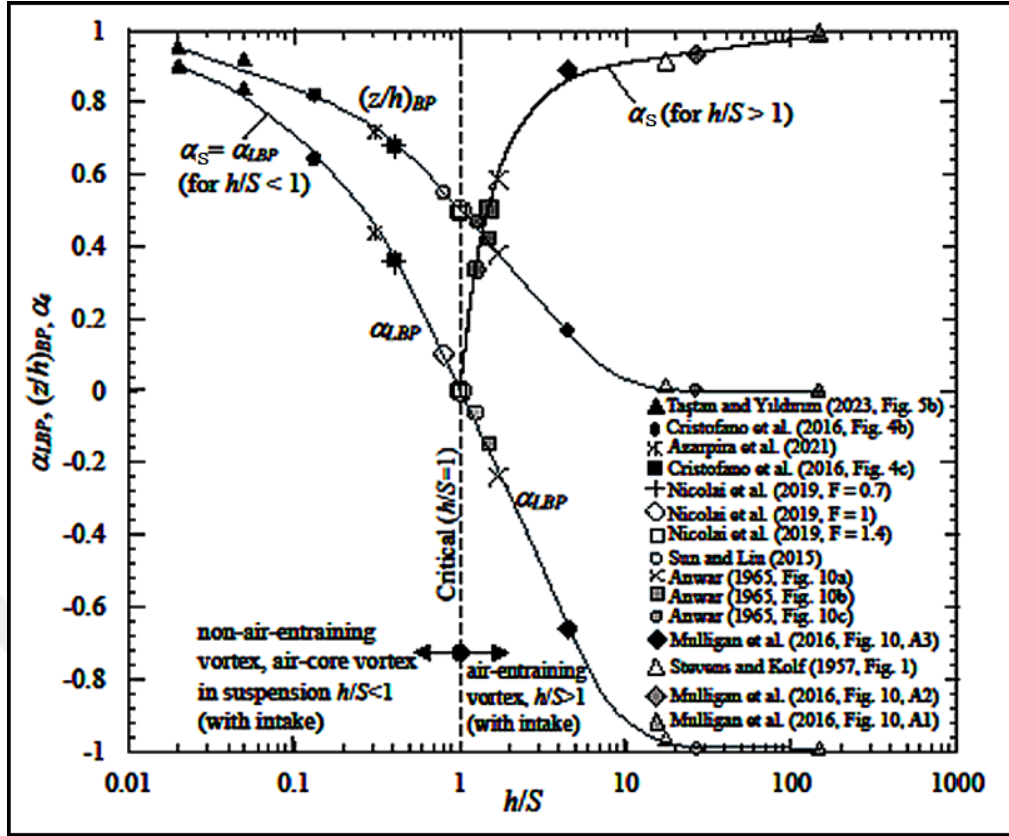


Figure 23: Variations of: (a) $(z/h)_{BP}$ and α_{BP} with h/S (b) α_{BP} with $(z/h)_{BP}$

The results of Eq. (4.4a) are compared with the available test data relating to the profile of vertical air-entraining vortices in the studies ([77] Figure 1, [75] Figure 10) and ([76], Figure 10, A1, A2 and A3) as indicated in Figures. 24 and 25. Solved examples of (4.4a) for these available studies are provided in Appendix II.

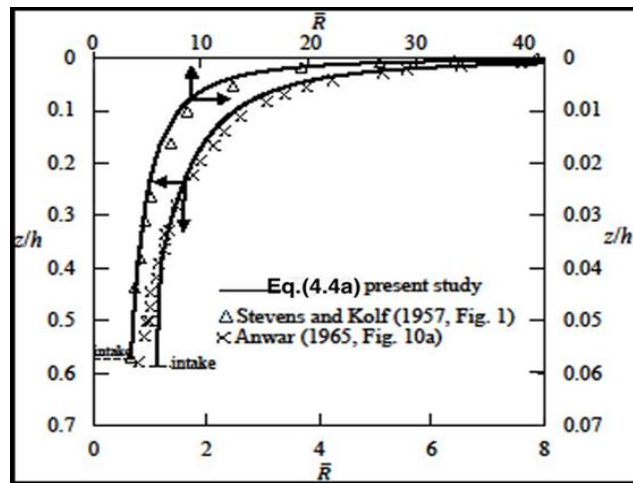


Figure 24: Comparison of Eq. (4.4a) with Test Data ([77],[75])

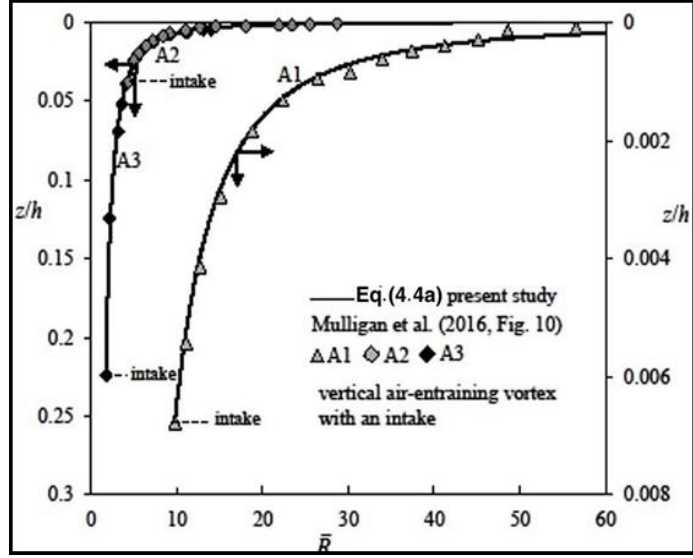


Figure 25:Comparison of Eq. (4.4a) with Test Data[76]

Figures (24) and (25), indicate that the difference (error) between the results of Eq. (4.4a) and the test data in the very upper section of the vortex profile (small magnitudes of z) are considerably large, that is mainly due to the reason that is as follows. Point gauges are used to acquire test data related to the vortex profile. On the free surface, there are surface tension effects as well as inevitable and unstable undulations and ripples (inertia waves). Water clings to the point gauge's tip when it approaches (just touches) the free surface because of these causes. As a result, the real and measured data on the very top part of the vortex profile (small magnitudes of z) really vary significantly.

4.2 DISCUSSION RELATING TO THE FIGURES (22-25)

Figures (22-25) indicates that the magnitude of α for an air-core vortex with an intake linearly varies with linear-lines of slope of -2 for all ranges of $(z/h)_{Bp} \leq z/h \leq S/h$ (indicating the lower section of the vortex). The magnitude of α for an air-core vortex with an intake is a constant ($\alpha = 1$) for all ranges of $0 \leq z/h \leq (z/h)_{Bp}$ (indicating the upper section of the air-core vortex).

4.3 THE REASONS FOR THE VARIATIONS OF Δ ARE AS FOLLOWS

- i. The streamline of geometrical location of the particles of maximum velocities towards the intake at each level makes the central core axis-line or path of the swirling vortex filament as a subsurface impinging jet type of flow extending from the free surface to the intake [78]. The magnitude of the velocity of the jet flow increases from a very low (minimum) magnitude at the free surface to a very high magnitude (maximum) at the intake entrance. As the subsurface impinging jet flow continues from the free surface to the intake due to the shear stress between the jet fluid flow and the ambient fluid the jet flow has to drag (entrain) the ambient fluid along with it towards the intake. This subsurface impinging jet flow is a self-developed main mechanism for the entrainment of the ambient fluid to the air-core vortex with an intake (note that this subsurface impinging jet flow mechanism is not present in the case of a vortex with no intake). The magnitude of α (in reality the maximum radius of the viscous core " r_m ") is closely linked to the properties and quantity of the ambient fluid entrained to the vortex flow at an intake. The reason is that the entrained an ambient fluid establishes the amount of total vorticity (thus r_m or α) that is harvested or inherited by this flow causes increased in the strength of circulation that stimulates the occurrence of the air-core vortex
- ii. As stated in the study by [78] at the point of arrival on the very path-way axis of the vortex, the ambient fluid particle entrained to the vortex flow inputs the total of the imposed, induced, or loaded vortices, irregularities, Coriolis force (Earth's rotation), non-uniformities or disturbances present within the ambient fluid, and perturbations and rotations that have been inherited along its entire path to the swirling vortex filament. Later, this particle produces even more disruptions as it continues on its whirling course to the intake by clashing with the entrained particles from the top levels. The circulation increases at the lower levels as a result of instabilities and rotations for those particles that reach the swirling vortex filament below the level of its point of entry in the swirling vortex area. This should be the cause of the rise in circulation amplitude at each level along the air-core vortex's or the swirling vortex's vertical axis (path-way) when one passes through the intake from the free-surface. At and along the vortex's path-way, any vortex and disturbance

discovered in the surrounding fluid region that provides fluid flow to the vortex's path-way is carried and increased. This suggests that the free-surface above the intake and the intake level are where the circulation reaches its minimal and highest magnitudes, respectively. Actually, the test results from ([32] Figure 7) show that circulation rises as one moves from the free-surface to the intake along the vortex flow.

It is clear that since the velocity of the jet flow (in reality the velocity gradient) at the upper section of the vortex with an intake $[z/h \leq (z/h)_{BP}]$ is very low (very low shear stress) the jet flow cannot drag and entrain a considerable amount of ambient fluid to the vortex flow. This is the main reason why the amount of the entrained ambient fluid is negligible and the magnitude of α remains constant ($\alpha = 1$, negligible change in α) in the upper section of the vortex $[z/h \leq (z/h)_{BP}]$ for which the vortex is almost the same as the vortex with no intake where as if there exists no impinging jet flow mechanism to entrain the ambient flow to the vortex flow (no ambient fluid entrainment) as explained a little later.

In the lower section of the vortex $[z/h > (z/h)_{BP}]$ the velocity of the jet flow increases towards the intake and it attains its maximum magnitude at the intake. The fast jet flow (means high velocity gradient and high shear stress) at lower sections $[z/h > (z/h)_{BP}]$ of the vortex drags and entrains considerable amount of ambient fluid to the vortex flow due to which r_m increases and thereby α linearly decreases in the region of $z/h > (z/h)_{BP}$ as indicated in Figure 20.

In the case of a non-air-entraining vortex ($h/S \leq 1$ the air-core vortex is in suspension) without an intake, α is a constant ($\alpha = 1$) for whole ranges of $0 \leq z/h \leq 1$ (Figure 17). This is because there exists no mechanism of subsurface impinging jet flow to drag and entrain the ambient fluid to the air-core vortex with no intake. Thereby, there is no considerable rate of ambient fluid flow entering along the height of the vortex due to which no increment occurs in vorticity or r_m or no decrement in α .

The case of $h/S \approx 0$ [that means an air-core vortex with an intake of very large submergence ($h \ll S$)] is similar to the case of a non-air-entraining vortex (air-core vortex in suspension) without an intake for which α remains constant ($\alpha = 1$) for

whole ranges of $0 \leq z/h \leq 1$. Since $S \gg h$, the effect of the intake flow on the ambient fluid entrainment to the vortex flow is negligible.

In the case of $(z/h)_{BP} = 0$ [for which Eqs. (4.2) and (4.5) results in $\alpha_{LBP} = -1$ and $(z/h)_{so} = 0.5$ (Figures 19 and 20)] whole ambient fluid flow over the intake level [across whole height or submergence of the intake from the horizontal free-surface level to the intake level $(z/h)_{BP} = 0 \leq z/h \leq S/h$] participates and enters the vortex flow due to which the strength of vortex becomes very large and the vortex is seen as a very strong air-entraining vortex. Therefore, the case of $(z/h)_{BP} = 0$ is the initiation of the occurrence of the entrainment of whole ambient fluid above the intake level across and along whole height of the vortex or submergence as it is indicated in Figure 17 .

4.4 THE REASONS FOR DIFFERENCES BETWEEN THE TEST DATA AND PRESENT STUDY

The available test data for the measurements of vortex profile were obtained by means of point gauges. As the tip of the point gauge is approached (just to touch) the free-surface, water clings to the tip of the point gauge. A tiny reading-error in the height (z) of the section of the vortex profile asymptotic to the top horizontal water-surface line causes a considerable deviation in the magnitude of r . Similar statements are also especially valid for the lower and extremely steep section of the air-core vortex. Therefore, in the selection of the magnitude of z to be used in the trial-and-error solution procedure it is advised not to prefer the very small and large magnitudes of z . For this purpose, one should refer to the useful advices and explanations provided in { [33] , “Discussion” section}.

4.5 TRIAL-AND-ERROR SOLUTION PROCEDURE FOR THE PROFILE OF A VERTICAL AIR-ENTRAINING VORTEX

For an air-entraining vortex “ h ” is not measurable. Also, z and r are not measurable for $z > S$. But S , z and r (for $z \leq S$) are measurable. To predict the profile of an air-entraining vortex along with Figure 23 at least two pairs of measured magnitudes of z and r (preferably in the very upper section of the vortex) are needed.

In Eq. (4.4 a) there are three unknowns (α , $\bar{R}$ and h) due to which Eq. (4.4 a) may be solved for z/h by a trial-and error solution procedure for which data at least at two z and their corresponding r across S of the intake are needed in order to check and examine the solution. There may be more than one pair of $\alpha\bar{R}^2$ and h satisfying Eq. (4.4 a). Since it is an air-entraining vortex, only the pairs having $h > S$ have to be considered.

As stated earlier the main focus of the present study is to obtain a simple chart(s) [indicating the variations of $(z/h)_{BP}$ and α_{LBP} with h/S] to be used in practical design relating to the profile a vertical air-entraining vortex occurring at a vertically-flowing downward intake. To obtain the variations of $(z/h)_{BP}$ and α_{LBP} with h/S it is necessary to predict the magnitude of the not measurable “ h ” for which that at least two pairs of measured test data relating to “ z ” and “ r ” corresponding to each other. For this purpose, let consider two pairs of measured $z = z_1$ for which $r = r_1$, and $z = z_2$ for which $r = r_2$ [that means pairs of $(z; r) = (z_1; r_1)$ and $(z_2; r_2)$] in such a way that $z_1 < z_2$ (from large to small).

For a quick short iterative solution, it is preferable that both z_1 and z_2 should be belong to the very upper section (at upstream of the considerable curvature-section) of the air-entraining vortex.

Considering α_1 , α_2 , z_1 , z_2 and $(z/h)_{BP}$ there can be three possible cases that are herein called the Case I, Case II and Case III as indicated in Figure 26 (a-c).

Computation procedure for an air-entraining vortex profile is as follows.

- 1) Choose a magnitude for the ratio of h/S , go to the graph in Figure 23 and read the corresponding magnitude of $(z/h)_{BP}$ and α_{LBP} [or compute α_{LBP} from Eq. (4.2)].
- 2) For a measured pair of “ z ” ($< S$) and “ r ” write Eq. (4.4 a) and compute r_{mid} (if the solution is not converging consider the solution methods for Case II and if necessary, Case III explained in Figure 26).
- 3) For the other pair of measured “ z ” ($< S$) and “ r ” write Eq. (4.4 a) and check it whether or not it is satisfied (if it is not satisfied choose another magnitude for h/S and repeat the solution). If it is satisfied compute “ h ” (since S is known compute “ h ” from the chosen ratio of h/S in the step 1).
- 4) From Eq. (4.5) compute $(z/h)_{so}$.

- 5) The profile of the air-core vortex for $0 \leq z/h \leq S/h$ can be predicted by means of Eq. (4.4 a).

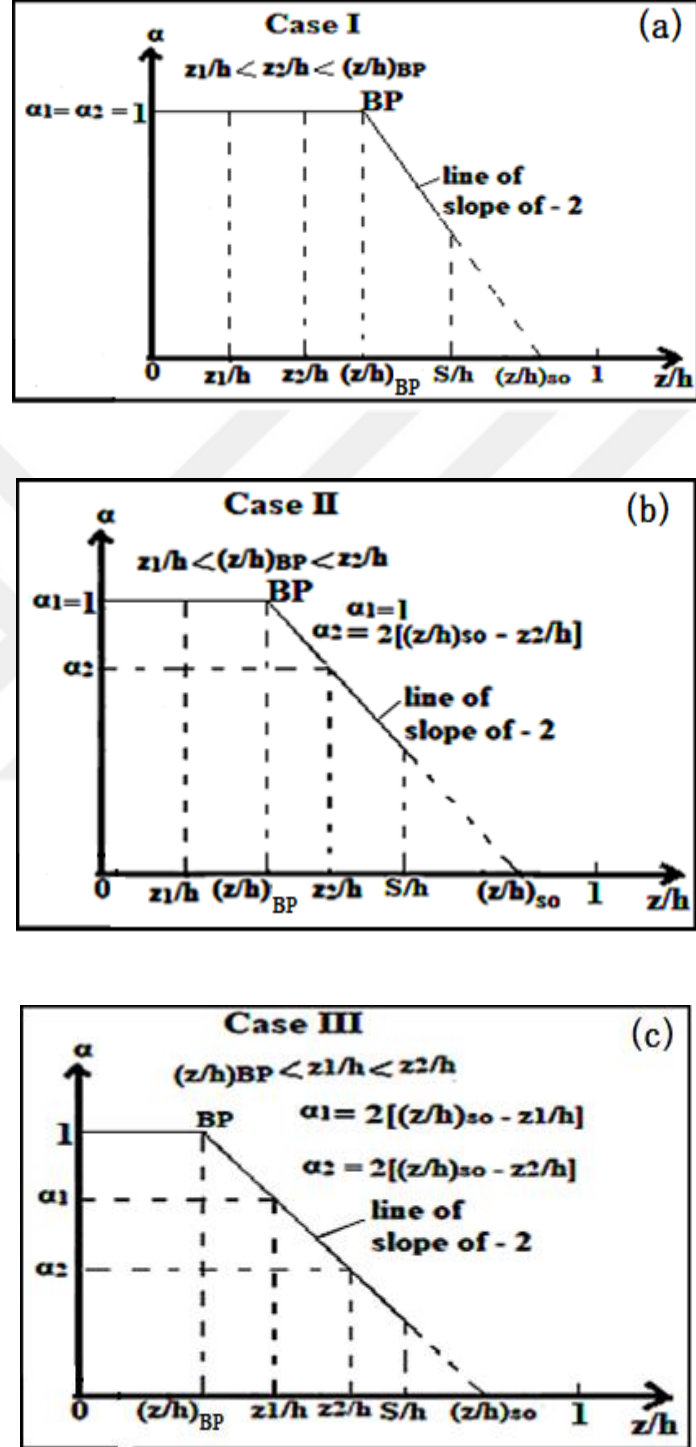


Figure 26: Classification of Cases I, II, and III Based on α_1 , α_2 , z_1 , z_2 , and $(z/h)_{BP}$

➤ For the Case I: $\alpha_1 = \alpha_2 = 1$; From Eq. (4.4 a)

$$\frac{z_1}{z_2} = \frac{1 - \frac{2}{\pi} \arctan \left(\frac{r_1}{r_{mid}} \right)^2}{1 - \frac{2}{\pi} \arctan \left(\frac{r_2}{r_{mid}} \right)^2} \quad (4.6)$$

from which “ r_{mid} ” and from Eq. (4.4a) “ h ” can be obtained.

➤ For the Case II: $\alpha_1 = 1$; from Figure 26 (b) get

$$\alpha_2 = 2[(z/h)_{so} - z_2/h] = \left[2 \left(1 - \frac{z_2}{h} \right) + \alpha_{LBP} \right] \quad (4.7 a)$$

$$\begin{aligned} \frac{z_1}{z_2} &= \frac{1 - \frac{2}{\pi} \arctan \left(\frac{r_1}{r_{mid}} \right)^2}{1 - \frac{2}{\pi} \arctan \left[\alpha_2 \left(\frac{r_2}{r_{mid}} \right)^2 \right]} \\ &= \frac{1 - \frac{2}{\pi} \arctan \left(\frac{r_1}{r_{mid}} \right)^2}{1 - \frac{2}{\pi} \arctan \left\{ \left[2 \left(1 - \frac{z_2}{h} \right) + \alpha_{LBP} \right] \left(\frac{r_2}{r_{mid}} \right)^2 \right\}} \end{aligned} \quad (4.7 b)$$

➤ For the Case III from Figure 26(c) get

$$\alpha_1 = 2[(z/h)_{so} - z_1/h] = \left[2 \left(1 - \frac{z_1}{h} \right) + \alpha_{LBP} \right] \quad (4.8 a)$$

$$\alpha_2 = 2[(z/h)_{so} - z_2/h] = \left[2 \left(1 - \frac{z_2}{h} \right) + \alpha_{LBP} \right] \quad (4.8 b)$$

$$\begin{aligned} \frac{z_1}{z_2} &= \frac{1 - \frac{2}{\pi} \arctan \left[\alpha_1 \left(\frac{r_1}{r_{mid}} \right)^2 \right]}{1 - \frac{2}{\pi} \arctan \left[\alpha_2 \left(\frac{r_2}{r_{mid}} \right)^2 \right]} \\ &= \frac{1 - \frac{2}{\pi} \arctan \left\{ \left[2 \left(1 - \frac{z_1}{h} \right) + \alpha_{LBP} \right] \left(\frac{r_1}{r_{mid}} \right)^2 \right\}}{1 - \frac{2}{\pi} \arctan \left\{ \left[2 \left(1 - \frac{z_2}{h} \right) + \alpha_{LBP} \right] \left(\frac{r_2}{r_{mid}} \right)^2 \right\}} \end{aligned} \quad (4.8 c)$$

From Eqs. (4.6- 4.8) “ r_{mid} ” and from Eq. (4.4 a) “ h ” can be found (solution by means of the Case III may be a long laborious one).

In reality it is advisable to start the trial-and error solution with relatively small z_1 and z_2 [both of them at upstream of the considerable curvature section of the vortex and smaller than $(z/h)_{BP}$]. This means start the solution with the Case I (so that the solution may be short) and then (if necessary) continue with the Cases II or Case III. Note that for a weak vertical air-entraining vortex with an intake generally Case I type is present and the solution is relatively easy. For a very extremely strong vertical air-entraining vortex with an intake Case II or Case III may be present and the solution is a bit long laborious iterative solution.

4.6 APPLICATION OF EQ. (4.4 A) FOR A VERTICAL AIR-ENTRAINING VORTEX WITH AN INTAKE

There are a few available studies relating to the test-measurements of the profile of a vertically flowing downward air-entraining vortex. Test data for the profile of a vertical air-entraining vortex ($h/S > 1$) with a centrally sited circular-intake(s) of $D = 1.6, 2.4$ and 3 cm leveled with bottom of the tank (a cylindrical vessel with internal diameter of 24.5 cm) with $S = 18$ cm are available in the study by ([79], Table 1 and Figure 4.a). In their tests a circulation is imposed to the ambient fluid within the cylindrical vessel. Herein the test data for $D = 3$ cm is considered (note that data for $z \leq 7$ cm are taken from Table.1, and the test data for $z = S = 18$ cm is taken from Figure (4.a) since this data is not available in Table. 1 in their study).

To prevent this study from being excessively long, herein only the solution of the final trial is given.

Let us choose $h/S = 3.7$ or $h = 3.7 \times 18 \text{ cm} = 66.6 \text{ cm}$ (they will be checked at the end of the solution). For $h/S = 3.7$ from (Figure 23) read both $\alpha_{LB} = - 0.58$ and $(z/h)_{BP} = 0.24$ or $z = z_{BP} = 0.24 \times 66.6 = 15.984 \text{ cm}$ [if wanted from Eq. 4.5 get $(z/h)_{so} = 0.74$]. Consider the data pairs of $z = z_1 = 3 \text{ cm}$ ($< z_{BP} = 15.984 \text{ cm}$) and $z = z_2 = 3.7 \text{ cm}$ ($< z_{BP} = 15.984 \text{ cm}$) for which $r = r_1 = 3 \text{ cm}$ and $r = r_2 = 2.7 \text{ cm}$, respectively [79], from Table. 15 in Appendix IV). Since $z_1 = 3 \text{ cm} < z_{BP} = 15.984 \text{ cm}$ and $z_2 = 3.7 \text{ cm} < z_{BP} = 15.984 \text{ cm}$ one can take $\alpha = \alpha_1 = 1$ and $\alpha = \alpha_2 = 1$ corresponding to $z_1 = 3 \text{ cm}$ and $z = 3.7 \text{ cm}$, respectively.

Write Eq. (4.4 a) for both $z_1/h = 3/66.6 = 0.045045$ and $z_2/h = 3.7/66.6 = 0.055555$ and divide them side by side then get

$$\frac{0.045045}{0.055555} = 0.81081 = \frac{1 - \frac{2}{\pi} \arctan\left(\frac{3}{r_{mid}}\right)^2}{1 - \frac{2}{\pi} \arctan\left(\frac{2.7}{r_{mid}}\right)^2}$$

By a trial-and-error solution procedure this equality gives $r_{mid} = 0.812$ cm (acceptably satisfied, 0.68 % error acceptable that may be attributed to the readings from the graphs and measurement errors in the tests).

Let us check the solution for the pair of test data of $z = z_3 = S = 18$ cm and $r = 1.30$ cm, $z/h = z_3/h = S/h = 18/66.6 = 0.27$ ([79], Appendix IV- Table 15).

Since $z/h = z_3/h = S/h = 18/66.6 = 0.27 > (z/h)_{BP} = 0.24$ (that means $\alpha \neq 1$) it is necessary to use $\alpha = [2(1 - z/h) + \alpha_{LB}]$ in Eq. (4.4 a). Thereby,

$$\frac{z_3}{h} = \frac{S}{h} = \frac{18}{66.6} = 0.27 = 1 - \frac{2}{\pi} \arctan\left\{[2(1 - 0.27) - 0.58] \left(\frac{1.3}{0.812}\right)^2\right\} = 0.26566$$

(acceptably satisfied, 1.6 % error acceptable that may be attributed to the readings from the graphs and measurement errors in the tests).

This indicates that the chosen magnitude of $h/S = 3.7$ or $h = 66.6$ cm, and $\alpha_{LB} = -0.58$ and $(z/h)_{BP} = 0.24$ are correct. Considering the magnitudes of these parameters the results of Eqs. (4.4 - 4.5) are compared with the measured profile of the vertical air-entraining vortex (for $z/h \leq S/h = 18/66.6 = 0.27$) in the study by ([79], Appendix IV- Table 15, $D = 3$ cm) in Figure 27.

Note that for $z/h \leq (z/h)_{BP}$ ($= 0.24$) and for $z/h > (z/h)_{BP}$ ($= 0.24$) the magnitudes of $\alpha = 1$ and $\alpha = \left[2\left(1 - \frac{z}{h}\right) - 0.58\right]$ are used, respectively in Eq. (4.4 a). Figure 27 indicates that Eq. (4.4a) is in good agreement with the test data.

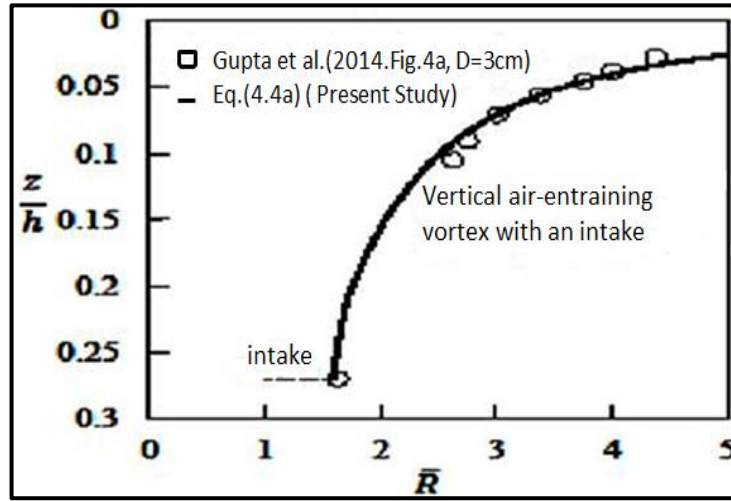


Figure 27:Comparison of Eq. (4.4 a) with Test Data Relating to the Profile of a Vertical Air-entraining Vortex with an Intake.

CHAPTER V

STUDY FINIDINGS

5.1 EXAMPLE I

For the available test data for the profile of an air-entraining vortex given in the study by ([77] ,Figure 1; $S = 17.069$ cm, intake diameter $D = 10.16$ cm). This vortex is a very strong vertical air-entraining vortex with an intake.

The summary of the solution is as follows.

$h = 298$ cm, $r_{mid} = 0.8515$, $(z/h)_{BP} = 0.01575$, $\alpha_{LBP} = -0.9685$, $(z/h)_{so} = 0.51575$ and $h/S = 17.458$, and $H_o = -(298 - 17.069) = -280.931$ cm (more details in Appendix II). These results are plotted in Figure 23 and indicated as [77].

Considering the magnitudes of these parameters the results of this equality [or Eq. (4.4 a)] are compared with the measured profile of a vertical air-entraining vortex given in the study ([77] , Figure1) in Figure 24 . Figures 24 and 28 indicate that Eq. (4.4a) is in good agreement with the test data (Appendix IV- Table16)

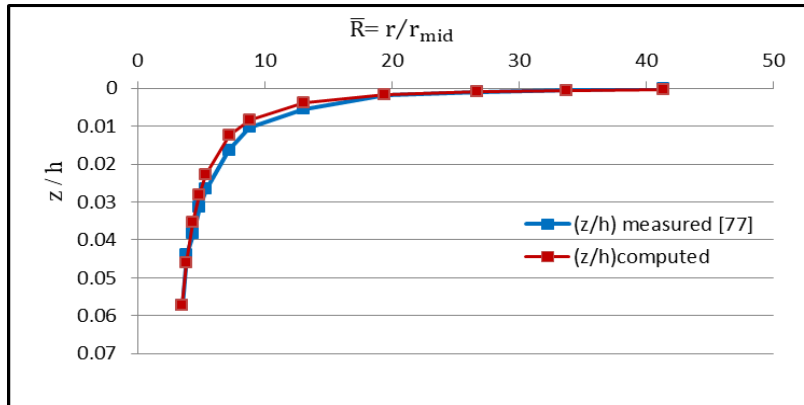


Figure 28: Comparison Between the Measured (z/h) [77] and (z/h) Computed by Eq.(4.4a)

5.2 EXAMPLE II

Test data relating to the profile of a typical air-entraining vortex is available in the study ([75], Figure 10a) , $D = 10.16$ cm, $S = 53.34$ cm) (this figure is also available as Fig. 3.2a in page 40 [11]).

The summary of the solution is as follows.

$h = 91$ cm; $(z/h)_{BP} = 0.38$; $\alpha_{LBP} = -0.24$; $(z/h)_{so} = 0.88$ and $h/S = 91 / 53.34 = 1.706$

If wanted, $(h-S) = -H_0$ that gives $H_0 = -(91-53.34) = -37.66$ cm (more details in Appendix II).

Considering the magnitudes of these parameters the results of Eq. (4.4 a) are compared with the measured profile of a vertical air-entraining vortex given in the study [75] in Figure 24. Figures 24 and 29 indicate that Eq. (4.4a) is in good agreement with the test data (Appendix IV -Table 17).

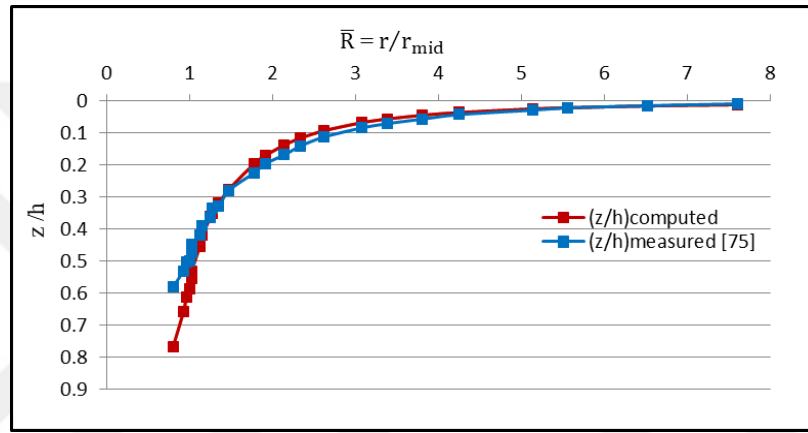


Figure 29: Comparison Between the Measured (z/h) ([75], Fig. 10-a) and (z/h) Computed by Eq.(4.4a)

5.3 EXAMPLE III

Test data relating to the profile of a typical air-entraining vortex is available in the study by Anwar ([75] Fig. 10b, $D = 6.604$ cm, $S = 50.80$ cm) (this Figure is also available as Figure 3.2 a in page 40 of the book [11])

The summary of the solution is as follows.

$h = 77$ cm, $(z/h)_{BP} = 0.425$, $\alpha_{LBP} = -0.15$, $(z/h)_{so} = 0.925$, and $h/S = 77 / 50.8 = 1.516$ or $S/h = 0.6597$, $H_0 = -(77 - 50.8) = -26.2$ cm (more details in Appendix II).

Considering the magnitudes of these parameters the results of Eq. (4.4 a) are compared with the measured profile of a vertical air-entraining vortex given in the study by [75] in Figure 30. Figure 30 indicates that Eq. (4.4 a) is in good agreement with the test data. (Appendix IV-Table 18).

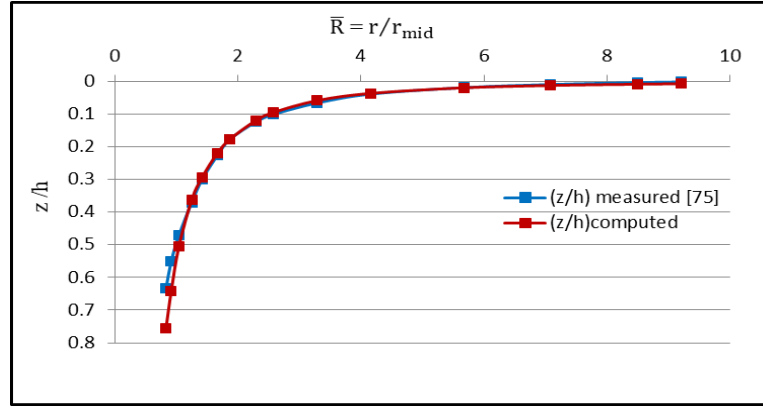


Figure 30:Comparison Between the Measured (z/h) ([75], Fig.10-b) and (z/h) Computed by Eq.(4.4 a)

5.4 EXAMPLE IV

Test data relating to the profile of a typical air-entraining vortex is available in the study ([75] Figure 10c , $D = 2.2225$ cm, $S = 30$ cm) (this figure is also available as Figure 3.2a in page 40 of the book [11]).

The summary of the solution is as follows.

$h = 37.7$ cm, $(z/h)_{BP} = 0.47$, $\alpha_{LBP} = -0.06$, $(z/h)_{so} = 0.97$ and $h/S = 37.7 / 30 = 1.257$ or $S/h = 0.796$, $(h-S) = -H_0$ that gives $H_0 = -(37.7 - 30) = -7.7$ cm (more details in Appendix II).

Considering the magnitudes of these parameters the results of Eq. (4.4 a) are compared with the measured profile of a vertical air-entraining vortex given in [75] in Figure 31. Figure 31 indicates that Eq. (4.4 a) is in good agreement with the test data (Appendix IV- Table 19).

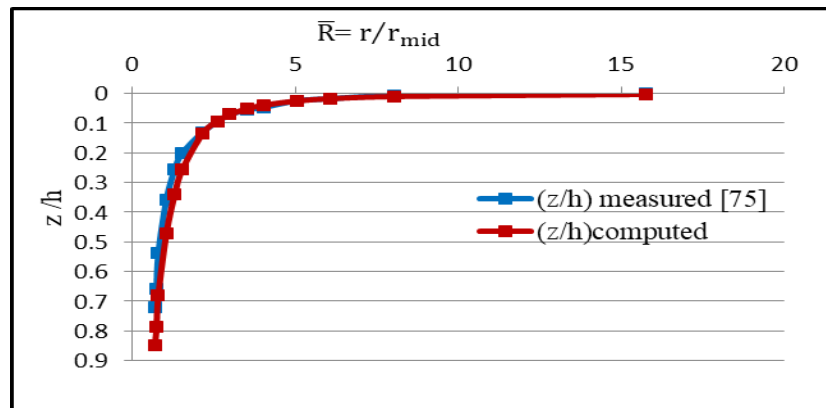


Figure 31:Comparison Between the Measured (z/h) ([75] , Fig.10-c) and (z/h) Computed by Eq.(4.4a)

5.5 EXAMPLE V

Let us consider the test data relating to the profile of an air-entraining vortex ([76], Figure 10- A1, $S/D = 1$, $D = 67$ mm, test case) in which $S = 6.7$ cm (note that “h” in their study is equal to “S” in the present study). This vortex is an extremely strong air-entraining vertical vortex with an intake.

The summary of the solution is as follows.

$h = 1106$ cm , $r_{mid} = 0.2688$ cm , $h/S = 1106 / 6.7 \approx 165.075$, $\alpha_{LBP} = - 0.9938$, $(z/h)_{BP} = 0.0031$, $\alpha = 1$, $(z/h)_{so} = 0.5031$, $z_{so} = 0.5031 \times 1106 = 556.43$ cm, $H_0 = - (h - S) = - (1106 - 6.7) = - 1099.3$ cm (more details in Appendix II).

Considering the magnitudes of these parameters the results of this equality [or Eq. (4.4 a)] are compared with the measured profile of a vertical air-entraining vortex given in the study by ([76] Figure 10- test case A1, $S/D = 1$) in Figure 25. Figures 25 and 32 indicate that Eq. (4.4 a) is in good agreement with the test data (Appendix IV-Table 20).

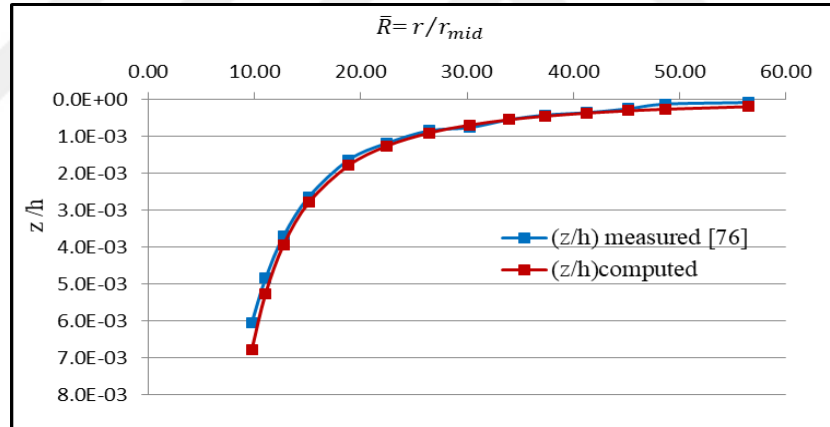


Figure 32: Comparison Between the Measured (z/h) ([76], Test Case A1) and (z/h) Computed by Eq.(4.4a)

5.6 EXAMPLE VI

Let us consider the test data relating to the profile of an air-entraining vortex in ([76] Figure 10- test case A2, $S/D = 2$, $D = 67$ mm) in which $S = 13.4$ cm

The summary of the solution is as follows.

$h = 376.5$ cm, $r_{mid} = 0.5595$ cm, $h/S = 376.5/13.4 = 28.097$, $\alpha_{LBP} = - 0.9926$, $(z/h)_{BP} = 0.0037$, $z_{BP} = 1.39305$ cm, $(z/h)_{so} = 0.5 + (z/h)_{BP} = 0.5 + 0.0037 = 0.5037$,

$z_{so} = 0.5037 \times 376.5 = 189.64$ cm, $H_0 = -(h-S) = -(376.5 - 13.4) = -363.1$ cm (more details in Appendix II).

Considering the magnitudes of these parameters the results of this equality [or Eq. (4.4a)] are compared with the measured profile of a vertical air-entraining vortex given in ([76] Figure 10- test case A2, $S/D = 2$, $D = 67$ mm) in Figure 25. Figures 25 and 33 indicate that Eq. (4.4 a) is in good agreement with the test data (Appendix IV- Table 21) .

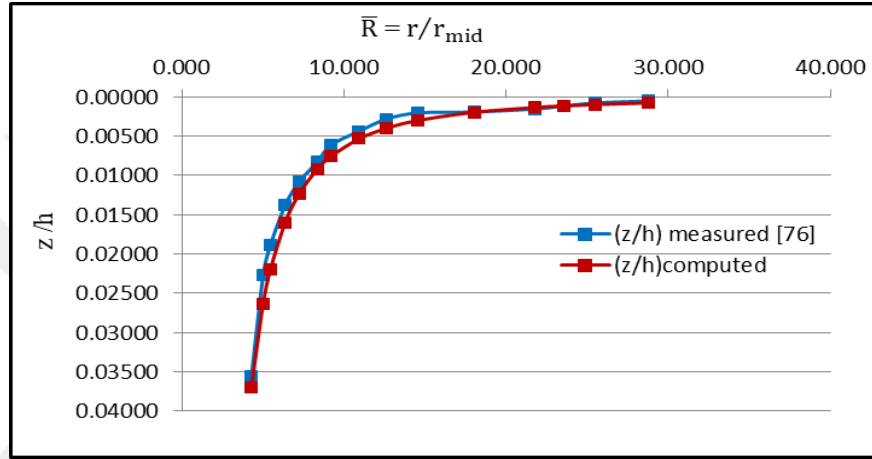


Figure 33:Comparison Between the Measured (z/h) ([76], Test Case A2) and (z/h) Computed by Eq.(4.4a)

5.7 EXAMPLE VII

Let us consider the Case I (Figure 26a): the test data relating to the profile of an air-entraining vortex in ([76] Figure 10- test case A3, $S/D = 3.5$, $D = 67$ mm) in which $S = 23.5$ cm.

The summary of the solution is as follows.

$h = 105$ cm, $r_{mid} = 1$ cm, $h/S = 105/23.5 = 4.468$, $\alpha_{LBP} = -0.66125$, $(z/h)_{BP} = 0.1694$, $z_{BP} = 17.787$ cm, $(z/h)_{so} = 0.5 + (z/h)_{BP} = 0.5 + 0.1694 = 0.6694$ cm, $z_{so} = 0.6694 \times 105 = 70.287$ cm, $H_0 = -(h - S) = -(105 - 23.5) = -81.5$ cm (more details in Appendix II).

Considering the magnitudes of these parameters the results of this equality [or Eq. (4.4a)] are compared with the measured profile of a vertical air-entraining vortex given in the study by ([76], Figure 10- test case A3, $S/D = 3.5$, $D = 6.7$ cm) in

Figure 25. Figures 25 and 34 indicate that Eq. (4.4 a) is in good agreement with the test data (Appendix IV- Table 22).

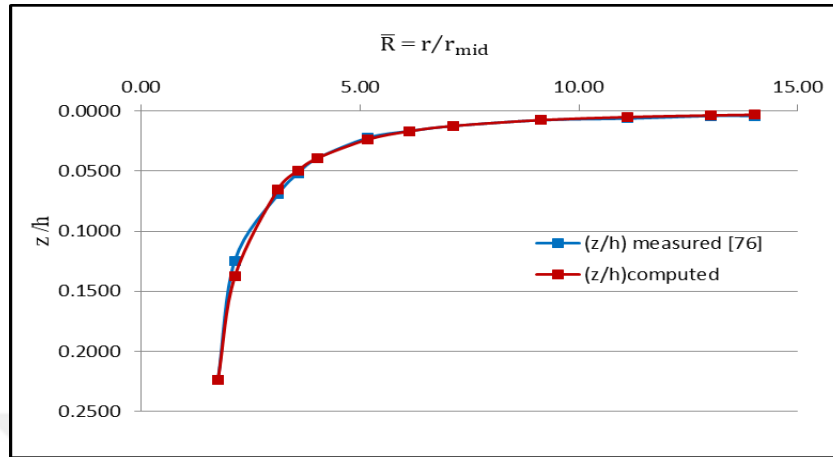


Figure 34: Comparison Between the Measured (z/h) ([76] , Test Case A3) and (z/h) Computed by Eq.(4.4 a)

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 CONCLUSIONS

The results of this investigation are as follows.

- For the profile of a vertical air-entraining vortex at a vertically flowing downward intake, a simple chart and formula were supplied.
- For an air-entraining vortex with a vertically flowing downward intake and known ratio of h/S , one can read the corresponding magnitudes of $(z/h)_{BP}$ and α_{LBP} in Figures 20 and 23, and then evaluate the profile of the vertical air-entraining vortex by means of Eq. (4.4 a).

Regarding the profiles of the vertical air-entraining vortices at vertically flowing downward intakes, the current study's findings were determined to be in high consistency with the test data that was available.

6.2 RECOMMENDATIONS

The following recommendations for the future studies may be as follows

- 1- Different empirical formulas may be developed or available other empirical formulas may be examined for the predication of the profiles of vertical air entraining vortices.
- 2- It is better to have test data relating to the profile of the vortex obtaining by means of optical devices rather than the point- gage type instruments that give large measurement error due to the surface tension or capillarity effect especially in top asymptotic profile section of the vortex.

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APPENDICES

APPENDIX I: THE DETAILS OF DIMENSIONAL ANALYSIS

Considering Eq.(3.1), $z = f(S, V, U_\infty, \rho, \nu, \Gamma, \sigma, g, D, r, c, b_1, b_2, \ell)$, herein the details of dimensional analysis

- Dimensional Matrix

	z	S	V	U_∞	ρ	ν	Γ	σ	g	D	r	c	b_1	b_2	ℓ
	L	L	LT^{-1}	LT^{-1}	ML^{-3}	L^2T^{-1}	L^2T^{-1}	MT^{-2}	LT^{-2}	L	L	L	L	L	L
M					1			1							
L	1	1	1	1	-3	2	2		1	1	1	1	1	1	1
T			-1	-1		-1	-1	-2	-2						

- The rank of the dimensional matrix = 3.
- Number of dimensionless parameters = $15-3 = 12$ (Π - Groups).
- Repeating Variables: [ρ (fluid property), V (flow property), D (geometric property)].

$$\Pi_1 = \rho^{x_1} \nu^{y_1} D^{z_1} z$$

$$M^0 L^0 T^0 = (M L^{-3})^{x_1} (L T^{-1})^{y_1} (L)^{z_1} (L)$$

$$M: 0 \rightarrow 0 = x_1 \rightarrow x_1 = 0$$

$$L: 0 \rightarrow 0 = -3x_1 + y_1 + z_1 + 1$$

$$T: 0 \rightarrow 0 = 0 = -y_1 \rightarrow y_1 = 0$$

$$\text{Then } 0 = -3x_1 + y_1 + z_1 + 1 \rightarrow 0 = -3(0) + 0 + z_1 + 1 \rightarrow z_1 = -1$$

$$\Pi_1 = \rho^0 \nu^0 D^{-1} z$$

$$\Pi_1 = \frac{z}{D}$$

$$\Pi_2 = \rho^{x_2} V^{y_2} D^{z_2} S$$

$$M^0 L^0 T^0 = (M L^{-3})^{x_2} (L T^{-1})^{y_2} (L)^{z_2} (L)$$

$$M: 0 \rightarrow 0 = x_2 \rightarrow x_2 = 0$$

$$L: 0 \rightarrow 0 = -3x_2 + y_2 + z_2 + 1$$

$$T: 0 \rightarrow 0 = 0 = -y_2 \rightarrow y_2 = 0$$

$$\text{Then } 0 = -3x_2 + y_2 + z_2 + 1 \rightarrow 0 = -3(0) + 0 + z_2 + 1 \rightarrow z_2 = -1$$

$$\Pi_2 = \rho^0 V^0 D^{-1} S$$

$$\Pi_2 = \frac{S}{D}$$

$$\Pi_3 = \rho^{x_3} V^{y_3} D^{z_3} U_\infty$$

$$M^0 L^0 T^0 = (M L^{-3})^{x_3} (L T^{-1})^{y_3} (L)^{z_3} (L T^{-1})$$

$$M: 0 \rightarrow 0 = x_3 \rightarrow x_3 = 0$$

$$L: 0 \rightarrow 0 = -3x_3 + y_3 + z_3 + 1$$

$$T: 0 \rightarrow 0 = 0 = -y_3 - 1 \rightarrow y_3 = -1$$

$$\text{Then } 0 = -3x_3 + y_3 + z_3 + 1 \rightarrow 0 = -3(0) - 1 + z_3 + 1 \rightarrow z_3 = 0$$

$$\Pi_3 = \rho^0 V^{-1} D^0 U_\infty$$

$$\Pi_3 = \frac{U_\infty}{V} \text{ or } \Pi_3 = \frac{V}{U_\infty}$$

$$\Pi_4 = \rho^{x_4} V^{y_4} D^{z_4} v$$

$$M^0 L^0 T^0 = (M L^{-3})^{x_4} (L T^{-1})^{y_4} (L)^{z_4} (L^2 T^{-1})$$

$$M: 0 \rightarrow 0 = x_4 \rightarrow x_4 = 0$$

$$L: 0 \rightarrow 0 = -3x_4 + y_4 + z_4 + 2$$

$$T: 0 \rightarrow 0 = 0 = -y_4 - 1 \rightarrow y_4 = -1$$

$$\text{Then } 0 = -3x_4 + y_4 + z_4 + 2 \rightarrow 0 = -3(0) - 1 + z_4 + 2 \rightarrow z_4 = -1$$

$$\Pi_4 = \rho^0 V^{-1} D^{-1} v$$

$$\Pi_4 = \frac{v}{VD} \text{ or } \Pi_4 = \frac{VD}{v} = \text{Re (Reynolds number)}$$

$$\Pi_5 = \rho^{x_5} V^{y_5} D^{z_5} \Gamma$$

$$M^0 L^0 T^0 = (M L^{-3})^{x_5} (L T^{-1})^{y_5} (L)^{z_5} (L^2 T^{-1})$$

$$M: 0 \rightarrow 0 = x_5 \rightarrow x_5 = 0$$

$$L: 0 \rightarrow 0 = -3x_5 + y_5 + z_5 + 2$$

$$T: 0 \rightarrow 0 = 0 = -y_5 - 1 \rightarrow y_5 = -1$$

$$\text{Then } 0 = -3x_5 + y_5 + z_5 + 2 \rightarrow 0 = -3(0) + (-1) + z_5 + 2 \rightarrow z_5 = -1$$

$$\Pi_5 = \rho^0 V^{-1} D^{-1} \Gamma$$

$$\Pi_5 = \frac{\Gamma}{VD} = K = \text{Circulation number (Kolf number)}$$

$$\Pi_6 = \rho^{x_6} V^{y_6} D^{z_6} \sigma$$

$$M^0 L^0 T^0 = (M L^{-3})^{x_6} (L T^{-1})^{y_6} (L)^{z_6} (M T^{-2})$$

$$M: 0 \rightarrow 0 = x_6 + 1 \rightarrow x_6 = -1$$

$$L: 0 \rightarrow 0 = -3x_6 + y_6 + z_6$$

$$T: 0 \rightarrow 0 = 0 = -y_6 - 2 \rightarrow y_6 = -2$$

$$\text{Then } 0 = -3x_6 + y_6 + z_6 \rightarrow 0 = -3(-1) + (-2) + z_6 \rightarrow z_6 = -1$$

$$\Pi_6 = \rho^{-1} V^{-2} D^{-1} \sigma$$

$$\Pi_6 = \frac{\sigma}{\rho V^2 D} = W \text{ (Weber number)}$$

$$\Pi_7 = \rho^{x_7} V^{y_7} D^{z_7} g$$

$$M^0 L^0 T^0 = (M L^{-3})^{x_7} (L T^{-1})^{y_7} (L)^{z_7} (L T^{-2})$$

$$M: 0 \rightarrow 0 = x_7 \rightarrow x_7 = 0$$

$$L: 0 \rightarrow 0 = -3x_7 + y_7 + z_7 + 1$$

$$T: 0 \rightarrow 0 = 0 = -y_7 - 2 \rightarrow y_7 = -2$$

$$\text{Then } 0 = -3x_7 + y_7 + z_7 + 1 \rightarrow 0 = -3(0) + (-2) + z_7 + 1 \rightarrow z_7 = 1$$

$$\Pi_7 = \rho^0 V^{-2} D^{-1} g$$

$$\Pi_7 = \frac{Dg}{V^2} = \frac{1}{Fr^2} \text{ or } \Pi_7 = Fr = \frac{V}{\sqrt{gD}} = \text{Intake Froude number}$$

$$\Pi_8 = \rho^{x_8} V^{y_8} D^{z_8} r$$

$$M^0 L^0 T^0 = (M L^{-3})^{x_8} (L T^{-1})^{y_8} (L)^{z_8} (L)$$

$$M: 0 \rightarrow 0 = x_8 \rightarrow x_8 = 0$$

$$L: 0 \rightarrow 0 = -3x_8 + y_8 + z_8 + 1$$

$$T: 0 \rightarrow 0 = -y_8 \rightarrow y_8 = 0$$

$$\text{Then } 0 = -3x_8 + y_8 + z_8 + 1 \rightarrow 0 = -3(0) + 0 + z_8 + 1 \rightarrow z_8 = -1$$

$$\Pi_8 = \rho^0 V^0 D^{-1} r$$

$$\Pi_8 = \frac{r}{D}$$

$$\prod_9 = \rho^{x_9} V^{y_9} D^{z_9} c$$

$$M^0 L^0 T^0 = (M L^{-3})^{x_9} (L T^{-1})^{y_9} (L)^{z_9} (L)$$

$$M: 0 \rightarrow 0 = x_9 \rightarrow x_9 = 0$$

$$L: 0 \rightarrow 0 = -3x_9 + y_9 + z_9 + 1$$

$$T: 0 \rightarrow 0 = -y_9 \rightarrow y_9 = 0$$

$$\text{Then } 0 = -3x_9 + y_9 + z_9 + 1 \rightarrow 0 = -3(0) + 0 + z_9 + 1 \rightarrow z_9 = -1$$

$$\prod_9 = \rho^0 V^0 D^{-1} c$$

$$\prod_9 = \frac{c}{D}$$

$$\prod_{10} = \rho^{x_{10}} V^{y_{10}} D^{z_{10}} b_1$$

$$M^0 L^0 T^0 = (M L^{-3})^{x_{10}} (L T^{-1})^{y_{10}} (L)^{z_{10}} (L)$$

$$M: 0 \rightarrow 0 = x_{10} \rightarrow x_{10} = 0$$

$$L: 0 \rightarrow 0 = -3x_{10} + y_{10} + z_{10} + 1$$

$$T: 0 \rightarrow 0 = -y_{10} \rightarrow y_{10} = 0$$

$$\text{Then } 0 = -3x_{10} + y_{10} + z_{10} + 1 \rightarrow 0 = -3(0) + 0 + z_{10} + 1 \rightarrow z_{10} = -1$$

$$\prod_{10} = \rho^0 V^0 D^{-1} b_1$$

$$\prod_{10} = \frac{b_1}{D}$$

$$\prod_{11} = \rho^{x_{11}} V^{y_{11}} D^{z_{11}} b_2$$

$$M^0 L^0 T^0 = (M L^{-3})^{x_{11}} (L T^{-1})^{y_{11}} (L)^{z_{11}} (L)$$

$$M: 0 \rightarrow 0 = x_{11} \rightarrow x_{11} = 0$$

$$L: 0 \rightarrow 0 = -3x_{11} + y_{11} + z_{11} + 1$$

$$T: 0 \rightarrow 0 = -y_{11} \rightarrow y_{11} = 0$$

$$\text{Then } 0 = -3x_{11} + y_{11} + z_{11} + 1 \rightarrow 0 = -3(0) + 0 + z_{11} + 1 \rightarrow z_{11} = -1$$

$$\prod_{11} = \rho^0 V^0 D^{-1} b_2$$

$$\prod_{11} = \frac{b_2}{D}$$

$$\prod_{12} = \rho^{x_{12}} V^{y_{12}} D^{z_{12}} \ell$$

$$M^0 L^0 T^0 = (M L^{-3})^{x_{12}} (L T^{-1})^{y_{12}} (L)^{z_{12}} (L)$$

$$M: 0 \rightarrow 0 = x_{12} \rightarrow x_{12} = 0$$

$$L: 0 \rightarrow 0 = -3x_{12} + y_{12} + z_{12} + 1$$

$$T: 0 \rightarrow 0 = -y_{12} \rightarrow y_{12} = 0$$

$$\text{Then } 0 = -3x_{12} + y_{12} + z_{12} + 1 \rightarrow 0 = -3(0) + 0 + z_{12} + 1 \rightarrow z_{12} = -1$$

$$\Pi_{12} = \rho^0 V^0 D^{-1} \ell$$

$$\Pi_{12} = \frac{\ell}{D}$$



APPENDIX II: APPLICATIONS OF EQ. (4.4a) FOR VERTICAL AIR-ENTRAINING VORTICES

Examine the sample solutions given below.

1- Example I

Consider the available test data for the profile of an air-entraining vortex given in the study by ([77] ,Figure 1; $S = 17.069$ cm, intake diameter $D = 10.16$ cm). This vortex is a very strong vertical air-entraining vortex with an intake.

1st trial: Let start the solution by considering the Case I (Fig. 26 a) and select or choose the data of $z = z_1 = 0.266$ cm $\ll S$ so that $z/h = z_1/h \leq (z/h)_{BP}$ is highly possible so as to have a high possibility of constant magnitude of $\alpha = 1$ in Figure 17 [the magnitude of α will be taken as $\alpha = 1$ constant-horizontal section of the graph in (Figure 17 or Figure 26(a -Case I) or in Eq. (4.4 a)]. This point is to be checked at the end of the solution. For $z = z_1 = 0.266$ cm the measured $r = r_1 = 22.73$ cm (Appendix IV-Table16). Hence, From Eq. (4.4 a); 0.266 cm $= [(1 - (2/\pi) \arctan(\alpha \bar{R}^2))] h$. Note that there can be several pairs of $\alpha \bar{R}^2$ and h satisfying 0.266 cm $= [(1 - (2/\pi) \arctan(\alpha \bar{R}^2))] h$; since it is an air-entraining vortex, only the pairs having $h > S (= 17.069$ cm) have to be considered. As a second data of z , let us choose $z = z_2 = 0.503$ cm which is also much smaller than $S = 17.069$ cm so that $z/h = z_2/h \leq (z/h)_{BP}$ is highly possible so as to have a high possibility to take and use $\alpha = 1$ in Figure 17 or Eq. (4.4 a). For $z_2 = 0.503$ cm the measured $r = r_2 = 16.53$ cm (Appendix IV-Table16). Most likely for both z_1/h and z_2/h an identical constant magnitude of $\alpha = \alpha_1 = \alpha_2 = 1$ in Figure 17 may be used. If Eq. (4.4 a) is separately written for $z = 0.266$ cm and 0.503 cm and divide them side by side (thereby h is eliminated) it results in

$$\frac{0.266}{0.503} = 0.5288 = \frac{1 - \frac{2}{\pi} \arctan\left(\frac{22.73}{r_{mid}}\right)^2}{1 - \frac{2}{\pi} \arctan\left(\frac{16.53}{r_{mid}}\right)^2}$$

By a trial- and-error solution procedure this equality gives $r_{\text{mid}} = 0.8515$ cm. Hence, from $z_1 = 0.266$ cm $= \left[1 - \frac{2}{\pi} \arctan \left(\frac{22.73}{0.8515} \right)^2 \right] h$ get $h = 297.736 \approx 298$ cm; or, $z_2 = 0.503$ cm $= \left[1 - \frac{2}{\pi} \arctan \left(\frac{16.53}{0.8515} \right)^2 \right] h$ get $h = 297.759 \approx 298$ cm. Thus $h/S = 298 / 17.069 \approx 17.46$ or $S/h = 0.05727$

Let us find α_{LBP} , the largest measured test data related to z in the study ([77], Figure1) is $z = S = 17.069$ cm for which the measured $r = 2.96$ cm for which most likely $\alpha \neq 1$ (since it is a very strong vortex). Therefore,

$$\frac{z}{h} = \frac{17.069}{298} = 0.057278 = 1 - \frac{2}{\pi} \arctan \left\{ [2(1 - 0.057278) + \alpha_{\text{LBP}}] \left(\frac{2.96}{0.8515} \right)^2 \right\}$$

By a trial-and-error solution procedure this equality gives $\alpha_{\text{LBP}} = -0.9685$. Thereby $\left(\frac{z}{h} \right)_{\text{BP}} = \frac{1}{2} (1 + \alpha_{\text{LBP}}) = \frac{1}{2} (1 - 0.9685) = 0.01575 > 0$ (OK); and from Eq. (4.5) get $(z/h)_{\text{so}} = (0.5 + 0.01575) = 0.51575$.

Let us examine the solution (whether or the solution is correct).

- i) **1st check:** Examine whether or not both $z_1/h \leq (z/h)_{\text{BP}}$ and $z_2/h \leq (z/h)_{\text{BP}}$ [whether or not the assumption of identical $\alpha = 1$ (for both z_1/h and z_2/h) made at the beginning of the solution is correct]. Since both $z_1/h = 0.266 / 298 = 0.0008926 < (z/h)_{\text{BP}} = 0.01575$ (OK) and $z_2/h = 0.503 / 298 = 0.0016879 < (z/h)_{\text{BP}} = 0.01575$ (OK) it indicates that the solution is correct (that means Case I is valid, $\alpha = \alpha_1 = \alpha_2 = 1$ can be taken for both $z_1 = 0.266$ cm and $z_2 = 0.503$ cm as it is taken at beginning of the solution).
- ii) **2nd check:** Examine the measured and computed magnitudes of “ z ” and “ r ” at the point BP. Note that at the point BP the magnitude of $\alpha = 1$ and $z/h = (z/h)_{\text{BP}}$ for which Eq. (4.4 a) is written as $\left(\frac{z}{h} \right)_{\text{BP}} = 1 - \frac{2}{\pi} \arctan \bar{R}^2$, since both $(z/h)_{\text{BP}}$ and $\bar{R}$ are not known it requires a trial and error solution procedure (to prevent this paper from being excessively long, only the solution of the final trial is given). Since $(z/h)_{\text{BP}} = 0.01575$, hence $(z/298)_{\text{BP}} = 0.01575$ thereby get $z_{\text{BP}} = 0.01575 \times 298 = 4.6935$ cm for which the measured $r = 6.052$ cm ([77] Figure 1) , also $0.01575 = 1 - \frac{2}{\pi} \arctan \bar{R}^2$ that $\bar{R} = 6.36$ from which

$\bar{R} = r / r_{\text{mid}} = 6.36 = r / 0.8515 \text{ cm}$ and get $r = 5.4156 \text{ cm}$ (computed) . Since $r = 5.4156 \text{ cm}$ (computed) $\approx r = 6.052 \text{ cm}$ (measured) that means the magnitude of $(z/h)_{\text{BP}} = 0.01575$ is acceptably correct (10.5 % error in “r” may be attributed to the readings from the graphs and the measurement errors in the test relating to “z” and “r”).

- iii) **3rd check:** Examine the measured and computed magnitudes of a pair of “z” and “r”. For example, let consider the test data for $z = 4.877 \text{ cm}$ ($> z_{\text{BP}} = 4.6935 \text{ cm}$) for which the measured $r = 6.11 \text{ cm}$ ([77], Figure 1), $\frac{z}{h} = \frac{4.877}{298} = 0.0163657 = 1 - \frac{2}{\pi} \arctan \left\{ [2(1 - 0.0163657) - 0.9685] \left(\frac{r}{0.8515} \right)^2 \right\}$ that gives the computed $r = 6.2408 \text{ cm}$. Since the (measured) $r = 6.11 \text{ cm} \approx$ (computed) $r = 6.2408 \text{ cm}$ (acceptable) indicating that the solution is correct (2.1 % error in “r” may be attributed to the readings from the graphs, measurement errors in the test relating to “r” and “z”).

These results indicate that the solution is correct. The summary of the solution is as follows.

$h = 298 \text{ cm}$, $r_{\text{mid}} = 0.8515$, $(z/h)_{\text{BP}} = 0.01575$, $\alpha_{\text{LBP}} = -0.9685$, $(z/h)_{\text{so}} = 0.51575$ and $h/S = 17.458$, and $H_o = -(298 - 17.069) = -280.931 \text{ cm}$. These results are plotted in Figure 24 and indicated as [77] .

2- Example II

Test data relating to the profile of a typical air-entraining vortex is available in the study ([75], Figure 10a) , $D = 10.16 \text{ cm}$, $S = 53.34 \text{ cm}$) (this figure is also available as Fig. 3.2a in page 40 [11]). [33] have chosen $z = z_1 = 2.54 \text{ cm}$ and $z = z_2 = 20.32 \text{ cm}$ (by taking $\alpha = \alpha_1 = \alpha_2 = 1$) for which the measured $r = 24.014 \text{ cm}$ and 8.313 cm , respectively (Appendix IV- Table 17), and predicted $h = 91 \text{ cm}$ and $r_{\text{mid}} = 4.674 \text{ cm}$. For $h/2 = 91/2 = 45.5 \text{ cm}$ ([75], Figure 10a).

To examine the solution and find the magnitudes of the necessary parameters of $(z/h)_{\text{BP}}$, α_{LBP} and $(z/h)_{\text{so}}$ a trial and error solution procedure is to be conducted.

To prevent this paper from being excessively long, only the solution of the final trial is given herein. That is as follows.

Let us choose $(z/h)_{BP} = 0.38$ hence $(z/91)_{BP} = 0.38$ and get $z_{BP} = 34.58$ cm. From Eqs. (4.2) and (4.5) get $\alpha_{LBP} = [2(z/h)_{BP} - 1] = (2 \times 0.38 - 1) = -0.24$, and $(z/h)_{so} = [1/2 + (z/h)_{BP}] = (0.5 + 0.38) = 0.88$, or $z = z_{so} = 0.88 \times 91 = 80.08$ cm.

Let us examine the solution (check whether or the solution is correct).

- i) **1st check:** Check whether or not both $z_1/h \leq (z/h)_{BP}$ and $z_2/h \leq (z/h)_{BP}$ [whether or not the assumption of identical $\alpha = 1$ (for both z_1/h and z_2/h) made at the beginning of the solution is correct]. Since both $z_1/h = (2.54 / 91) = 0.0279 < 0.38$ (OK) and $z_2/h = (20.32 / 91) = 0.223 < 0.38$ (OK) it indicates that the solution is correct (that means $\alpha = 1$ can be taken for both $z_1 = 2.54$ cm and $z_2 = 20.32$ cm at beginning of the solution).
- ii) **2nd check:** Examine the measured and computed magnitudes of “z” and “r” at the point BP. At the point BP the magnitude of $\alpha = 1$ and $z/h = (z/h)_{BP}$ for which Eq. (4.4 a) is written as $\left(\frac{z}{h}\right)_{BP} = 1 - \frac{2}{\pi} \arctan \bar{R}^2$, since both $(z/h)_{BP}$ and $\bar{R}$ are not known it requires a trial and error solution procedure (to prevent this paper from being excessively long, only the solution of the final trial is given). For $z_{BP} = 34.58$ cm the measured $r = 5.696$ cm ([75], Figure 10a), also $0.38 = 1 - \frac{2}{\pi} \arctan \bar{R}^2$ that gives $\bar{R} = 1.216$ from which $\bar{R} = r/r_{mid} = 1.216$ cm = $r / 4.674$ cm and get $r = 5.68$ cm (computed). Since $r = 5.68$ cm (computed) $\approx r = 5.696$ cm (measured) (OK) that means the magnitude of $(z/h)_{BP} = 0.38$ is correct.
- iii) **3rd check:** Examine the measured and computed magnitudes of a pair of “z” and “r”. For example, let consider the test data for $z = 38.1$ cm (38.1 cm $> z_{BP} = 34.58$ cm) for which the measured $r = 5.232$ cm ([75] Figure 10a). $\frac{z}{h} = \frac{38.1}{91} = 0.418 = 1 - \frac{2}{\pi} \arctan \left\{ [2(1 - 0.418) - 0.24] \left(\frac{r}{4.674} \right)^2 \right\}$ that gives the computed $r = 6.019$ cm. Since the (measured) $r = 5.232$ cm is a little smaller than (computed) $r = 6.019$ cm indicating that the solution is correct (15 % error in “r” may be attributed to the readings from the graphs, measurement errors in the test relating to “r” and “z”).

Hence; $h = 91$ cm; $(z/h)_{BP} = 0.38$; $\alpha_{LBP} = -0.24$; $(z/h)_{so} = 0.88$ and $h/S = 91 / 53.34 = 1.706$ are correct.

The magnitudes of $(z/h)_{BP} = 0.38$; $\alpha_{LBP} = -0.24$; $(z/h)_{so} = 0.88$, and $h/S = 1.706$ are plotted in Figures (17) , (24) and indicated in [75]. If wanted, $(h-S) = -H_0$ that gives $H_0 = -(91-53.34) = -37.66$ cm.

3- Example III

Test data relating to the profile of a typical air-entraining vortex is available in the study by Anwar ([75] Fig. 10b , $D = 6.604$ cm, $S = 50.80$ cm) (this Figure is also available as Figure 3.2 a in page 40 of the book [11]). [33] have chosen $z = z_1 = 13.597$ cm and $z = z_2 = 36.307$ cm (by taking $\alpha = \alpha_1 = \alpha_2 = 1$) for which the measured $r = 6.723$ cm and 3.734 cm, respectively (Appendix IV -Table18), and predicted $h = 77$ cm and $r_{mid} = 3.586$ cm.

To prevent this paper from being excessively long, only the solution of the final trial is given herein. That is as follows.

Let us choose $(z/h)_{BP} = 0.425$ hence $(z/77)_{BP} = 0.425$ and get $z_{BP} = 32.725$ cm for which the measured $r = 4.064$ cm ([75], Figure 10b).

From Eqs. (4.2) - (4.5), get $\alpha_{LBP} = [2(z/h)_{BP} - 1] = (2 \times 0.425 - 1) = -0.15$, and $(z/h)_{so} = [0.5 + (z/h)_{BP}] = (0.5 + 0.425) = 0.925$, or $z = z_{so} = 0.925 \times 77 = 71.225$ cm.

Let us examine the solution (check whether or the solution is correct).

- i) **1st check:** Check whether or not both $z_1/h \leq (z/h)_{BP}$ and $z_2/h \leq (z/h)_{BP}$ [whether or not the assumption of identical $\alpha = \alpha_1 = \alpha_2 = 1$ (for both z_1/h and z_2/h) made by [33] to predict $h = 77$ cm is correct]. Since both $z_1/h = (13.597/77) = 0.1765 < 0.425$ (OK) and $z_2/h = (36.307/77) = 0.4715 > 0.425$ (not OK) but acceptable (because the magnitude of $\alpha = \alpha_2$ for $z/h = 0.4715$ is also very close to $\alpha_2 = 1$). It indicates that $h = 77$ cm predicted by [33] is correct.
- ii) **2nd check:** Examine the measured and computed magnitudes of “z” and “r” at the point BP. At the point BP the magnitude of $\alpha = 1$ and $z/h = (z/h)_{BP}$ for which Eq. (4.4 a) is written as $\left(\frac{z}{h}\right)_{BP} = 1 - \frac{2}{\pi} \arctan \bar{R}^2$, since both $(z/h)_{BP}$ and $\bar{R}$ are not known it requires a trial and error solution procedure (to prevent this paper from being excessively long, only the solution of the final trial is given). For $z_{BP} = 32.725$ cm the measured $r = 4.064$ cm ([75], Figure 10b), also $0.425 = 1 - \frac{2}{\pi} \arctan \bar{R}^2$ that gives $\bar{R} = 1.126603$ from which $\bar{R} =$

$r/r_{\text{mid}} = 1.126603 \text{ cm} = r / 3.586 \text{ cm}$ and get $r = 4.04 \text{ cm}$ (computed). Since $r = 4.04 \text{ cm}$ (computed) $\approx r = 4.064 \text{ cm}$ (measured) [Error % = 0.59%] (OK) that means the magnitude of $(z/h)_{\text{BP}} = 0.425$ is correct.

- iii) **3rd check:** Examine the measured and computed magnitudes of a pair of “z” and “r”. For example, let consider the test data for $z = 48.707 \text{ cm}$ ($48.707 \text{ cm} > z_{\text{BP}} = 32.725 \text{ cm}$) for which the measured $r = 2.987 \text{ cm}$ ([74], Figure 10b). $\frac{z}{h} = \frac{48.707}{77} = 0.6325584 = 1 - \frac{2}{\pi} \arctan\{[2(1 - 0.6325584) - 0.15] (r/3.586)^2\}$ that gives the computed $r = 3.783 \text{ cm}$. Since the (measured) $r = 2.987 \text{ cm} \approx$ (computed) $r = 3.783 \text{ cm}$ indicating that the solution is correct (26.6% error in “r” may be attributed to the readings from the graphs, measurement errors in the test relating to “r” and “z”).

Hence; $h = 77 \text{ cm}$, $(z/h)_{\text{BP}} = 0.425$, $\alpha_{\text{LBP}} = -0.15$, $(z/h)_{\text{so}} = 0.925$, and $h/S = 77 / 50.8 = 1.516$ or $S/h = 0.6597$ are correct.

The magnitudes of $(z/h)_{\text{BP}} = 0.425$, $\alpha_{\text{LBP}} = -0.15$, $(z/h)_{\text{so}} = 0.925$, and $h/S = 1.516$ or $S/h = 0.6597$ are plotted in Figure (17) , (20) and indicated in [75]. If wanted, $(h-S) = -H_0$ that gives $H_0 = -(77 - 50.8) = -26.2 \text{ cm}$.

4- Example IV

Test data relating to the profile of a typical air-entraining vortex is available in the study ([75] Figure 10c , $D = 2.2225 \text{ cm}$, $S = 30 \text{ cm}$) (this figure is also available as Figure 3.2a in page 40 of the book [11]) . Let select the data of $z = z_1 = 0.651 \text{ cm}$ which is much smaller than $S = 30 \text{ cm}$ so that $z/h \leq (z/h)_{\text{BP}}$ is highly possible so as to have high possibility of $\alpha =$ a constant horizontal section of the graph ($\alpha = \alpha_1 = 1$) in Figures 17 and 20 or Eq. (4.4 a). Hence, from Eq. (4.4a), $0.651 \text{ cm} = [(1-(2/\pi) \arctan \alpha \bar{R}^2) h]$ that may be satisfied by several pairs of $\alpha \bar{R}^2$ and h . For an air-entraining vortex, only the pairs having $h > S$ ($= 30 \text{ cm}$) have to be considered.

As a second data of z , let us choose $z = z_2 = 2.67 \text{ cm}$ which is much smaller than $S = 30 \text{ cm}$ so that $z_2/h \leq (z/h)_{\text{BP}}$ so as to have high possibility of $\alpha =$ a constant horizontal section of the graph ($\alpha = \alpha_2 = 1$) in Figures. 17 and 20 or Eq. (4.4 a) (note

that this point is to be checked at the end of the solution). In this way for both z_1/h and z_2/h an identical magnitude of α in Figures 17 and 20 may be used; $\alpha = 1$). From the study by ([75], Figure 10c), for $z = 0.651$ cm and 2.67 cm, read the measured magnitudes of $r = r_1 = 7.826$ cm and $r = r_2 = 3.86$ cm, respectively (Appendix IV-Table 19,). If Eq. (4.4 a) is separately written for $z = 0.651$ cm and 2.67 cm and, divide them side by side (thereby h is eliminated) it results in

$$\frac{0.651}{2.67} = \frac{1 - \frac{2}{\pi} \arctan \left[\frac{1}{r_{mid}^2} (7.826)^2 \right]}{1 - \frac{2}{\pi} \arctan \left[\frac{1}{r_{mid}^2} (3.86)^2 \right]}$$

By trial- and-error solution procedure this equality gives $r_{mid} = 1.29$ cm.

Hence, from

$$z = 0.651 = \left\{ 1 - \frac{2}{\pi} \arctan \left[\frac{1}{r_{mid}^2} (7.826)^2 \right] \right\} h; \text{ get } h = 37.7 \text{ cm (computed).}$$

For $h/2 = 37.7/2 = 18.85$ cm from ([75], Figure 10c) one reads the measured $r_{mid} = 1.0922$ cm that is acceptably close to the computed $r = r_{mid} = 1.29$ cm (OK). Thus, the solution (predicted $h = 37.7$ cm) may be correct that is to be checked a little later.

To examine the solution and find the magnitudes of the necessary parameters of $(z/h)_{BP}$, α_{LBP} and $(z/h)_{SO}$ a trial and error solution procedure is to be conducted.

To prevent this paper from being excessively long, only the solution of the final trial is given herein. That is as follows.

Let us choose $(z/h)_{BP} = 0.47$ hence $(z/37.7)_{BP} = 0.47$ and get $z_{BP} = 17.719$ cm for which the measured $r = 1.029$ cm ([75], Figure 10c).

From Eqs. (4.2) and (4.5) get $\alpha_{LBP} = [2(z/h)_{BP} - 1] = (2 \times 0.47 - 1) = -0.06$, and $(z/h)_{SO} = [0.5 + (z/h)_{BP}] = (0.5 + 0.47) = 0.97$, or $z = z_{SO} = 0.97 \times 37.7 = 36.569$ cm.

Let us examine the solution (check whether or the solution is correct).

- i) **1st check:** Check whether or not both $z_1/h \leq (z/h)_{BP}$ and $z_2/h \leq (z/h)_{BP}$ [whether or not the assumption of identical $\alpha = \alpha_1 = \alpha_2 = 1$ (for both z_1/h and z_2/h) made by [33] to predict $h = 37.7$ cm is correct]. Since both $z_1/h = (0.651/37.7) = 0.0173 < 0.47$ (OK) and $z_2/h = (2.67/37.7) = 0.0708 < 0.47$ (OK). It indicates that $h = 37.7$ cm is correct.

ii) **2nd check:** Examine the measured and computed magnitudes of “z” and “r” at the point BP. At the point BP the magnitude of $\alpha = 1$ and $z/h = (z/h)_{BP}$ for which Eq. (4.4 a) is written as $\left(\frac{z}{h}\right)_{BP} = 1 - \frac{2}{\pi} \arctan \bar{R}^2$, since both $(z/h)_{BP}$ and $\bar{R}$ are not known it requires a trial and error solution procedure (to prevent this paper from being excessively long, only the solution of the final trial is given). For $z_{BP} = 17.719$ cm the measured $r = 1.029$ cm ([75], Figure 10c), also $0.47 = 1 - \frac{2}{\pi} \arctan \bar{R}^2$ that gives $\bar{R} = 0.8527$ from which $\bar{R} = r/r_{mid} = 0.8527$ cm = $r/1.29$ cm and get $r = 1.1$ cm (computed). Since $r = 1.1$ cm (computed) $\approx r = 1.029$ cm (measured) (OK) that means the magnitude of $(z/h)_{BP} = 0.47$ and $h = 37.7$ cm are correct.

iii) **3rd check:** Examine the measured and computed magnitudes of a pair of “z” and “r”. For example, let consider the test data for $z = 24.748$ cm (24.748 cm $> z_{BP} = 17.719$ cm) for which the measured $r = 0.961$ cm ([75], Figure 10c). $\frac{z}{h} = \frac{24.748}{37.7} = 0.656445 = 1 - \frac{2}{\pi} \arctan \left\{ [2(1 - 0.656445) - 0.06] \left(\frac{r}{1.29} \right)^2 \right\}$ that gives the computed $r = 1.26$ cm. Since the (measured) $r = 0.961$ cm is a bit lower than (computed) $r = 1.26$ cm (31% error in “r” may be attributed to the readings from the graphs, measurement errors in the test relating to “r” and “z”).

Hence, $h = 37.7$ cm, $(z/h)_{BP} = 0.47$, $\alpha_{LBP} = -0.06$, $(z/h)_{so} = 0.97$ and $h/S = 37.7 / 30 = 1.257$ or $S/h = 0.796$ are correct.

The magnitudes of $(z/h)_{BP} = 0.47$, $\alpha_{LBP} = -0.06$, $(z/h)_{so} = 0.97$, and $h/S = 1.257$ or $S/h = 0.796$ are plotted in Figures 17 and 20 and indicated in ([75], Figure 10c). If wanted, $(h-S) = -H_0$ that gives $H_0 = -(37.7 - 30) = -7.7$ cm.

5- Example V

Let us consider the test data relating to the profile of an air-entraining vortex ([76], Figure 10- A1, $S/D = 1$, $D = 67$ mm, test case) in which $S = 6.7$ cm (note that “h” in their study is equal to “S” in the present study). This vortex is an extremely strong air-entraining vertical vortex with an intake.

1st trial: Let start the solution by considering the Case I (Figure 26 c) , choose the data of $z = z_1 = 0.6117 \text{ cm} \ll S = 6.7 \text{ cm}$ so that $z/h = z_1/h \leq (z/h)_{BP}$ is highly possible so as to have a high possibility of constant magnitude of $\alpha = 1$ in Figure 17 [the magnitude of α will be taken as $\alpha = 1$ constant-horizontal section of the graph in Figures 17 and 20 or in Eq. (4.4 a)]. This point is to be checked at the end of the solution. For $z_1 = 0.6117 \text{ cm}$ the measured $r = r_1 = 9.12 \text{ cm}$. As a second data of z , let us choose $z = z_2 = 1.317 \text{ cm}$ which is much smaller than $S = 6.7 \text{ cm}$ (Appendix IV- Table 20) so that $z/h = z_2/h \leq (z/h)_{BP}$ is highly possible so as to have a high possibility to take (choose) and use $\alpha = 1$ in Figures 17 and 20 or Eq. (4.4 a). For $z_2 = 1.317 \text{ cm}$ the measured $r = r_2 = 6.023 \text{ cm}$. Most likely for both z_1/h and z_2/h an identical constant magnitude of $\alpha = 1$ in Figure 17 or Figure 20 may be used. If Eq. (4.4 a) is separately written for $z = 0.6117 \text{ cm}$ and 1.317 cm (considering $\alpha = 1$) and, divide them side by side (thereby h is eliminated) it results in

$$\frac{z_1}{z_2} = \frac{0.6117}{1.317} = 0.46446 = \frac{1 - \frac{2}{\pi} \arctan \left(\frac{9.12}{r_{mid}} \right)^2}{1 - \frac{2}{\pi} \arctan \left(\frac{6.023}{r_{mid}} \right)^2}$$

By a trial-and-error solution procedure this equality gives $r_{mid} = 0.2688 \text{ cm}$ (computed). Hence, $z_1 = 0.6117 \text{ cm} = \left[1 - \frac{2}{\pi} \arctan \left(\frac{9.12}{0.2688} \right)^2 \right] h$; get $h = 1106.08 \approx 1106 \text{ cm}$. Thus $h \approx 1106 \text{ cm}$ taken and $h/S = 1106/6.7 = 165.07$

Let us find α_{LBP} .

The largest test data related to z in the study by ([76] Figure 10- test case A1, $S/D = 1$, $D = 67 \text{ mm}$), is $z = S = 6.7 \text{ cm}$ and the measured $r/D = 0.39$ or measured $r = 0.39 \times 6.7 = 2.613 \text{ cm}$ for which most likely $\alpha \neq 1$ (since it is a very strong vortex). Therefore,

$$\frac{z}{h} = \frac{6.7}{1106} = 0.0060579 = 1 - \frac{2}{\pi} \arctan \left\{ [2(1 - 0.0060579) + \alpha_{LBP}] \left(\frac{2.613}{0.2688} \right)^2 \right\}$$

By a trial-and-error solution procedure this equality gives $\alpha_{LBP} = -0.9938$. Thereby $\left(\frac{z}{h} \right)_{BP} = \frac{1}{2} (1 + \alpha_{LBP}) = \frac{1}{2} (1 - 0.9938) = 0.0031$; $(z/h)_{so} = [0.5 + (z/h)_{BP}] = (0.5 + 0.0031) = 0.5031$

Check the solution:

- i) **1st check:** Check whether or not both $z_1/h \leq (z/h)_{BP}$ and $z_2/h \leq (z/h)_{BP}$ [whether or not the assumption of identical $\alpha = 1$ (for both z_1/h and z_2/h) made at the beginning of the solution is correct]. Since both $z_1/h = 0.6117/1106 = 0.00055307 < (z/h)_{BP} = 0.0031$ (OK), and $z_2/h = 1.317/1106 = 0.00119077 < (z/h)_{BP} = 0.0031$ (OK) it indicates that the solution (to take $\alpha = 1$ at beginning of the solution) is correct.
- ii) **2nd check:** Examine the measured and computed magnitudes of “z” and “r” at the point BP. Note that at the point BP the magnitude of $\alpha = 1$ and $z/h = (z/h)_{BP}$ for which Eq. (4.4 a) is written as $\left(\frac{z}{h}\right)_{BP} = 1 - \frac{2}{\pi} \arctan \bar{R}^2$, since both $(z/h)_{BP}$ and $\bar{R}$ are not known it requires a trial and error solution procedure (to prevent this paper from being excessively long, only the solution of the final trial is given). Since $(z/h)_{BP} = 0.0031$, hence $(z/1106)_{BP} = 0.0031$ thereby get $z_{BP} = 0.0031 \times 1106 = 3.4286$ cm for which the measured $r = 3.953$ cm ([76] Figure 10- test case A1, S/D = 1, D = 67 mm), also $0.0031 = 1 - \frac{2}{\pi} \arctan \bar{R}^2$ that gives $\bar{R} = 14.323$ from which $\bar{R} = r/r_{mid} = 14.323 = r / 0.2688$ cm and get $r = 3.85$ cm (computed). Since $r = 3.85$ cm (computed) $\approx r = 3.953$ cm (measured) that means the magnitude of $(z/h)_{BP} = 0.0031$ is acceptably correct (2.6% error in “r” may be attributed to the readings from the graphs and the measurement errors in the test relating to “z” and “r”).
- iii) **3rd check:** Let us further examine the solution (whether or the solution is correct). For example, let consider the test data for $z = 4.094$ cm and measured $r = 3.419$ cm ([76] Figure 10- test case A1, S/D = 1, D = 67 mm). $z/h = 4.094/1106 = 0.0037016 > (z/h)_{BP} (= 0.0031)$ therefore $\frac{z}{h} = \frac{4.094}{1106} = 0.0037016 = 1 - \frac{2}{\pi} \arctan \left\{ [2(1 - 0.0037016) - 0.9938] \left(\frac{r}{0.2688} \right)^2 \right\}$ that gives the computed $r = 3.53$ cm. Since the (measured) $r = 3.419$ cm $\approx$ (computed) $r = 3.53$ cm (acceptable) indicating that the solution is correct (3.2 % error in “r” may be attributed to the readings from the graphs, measurement errors in the test relating to “r” and “h”).

The summary of the solution is as follows:

$h = 1106 \text{ cm}$, $r_{\text{mid}} = 0.2688 \text{ cm}$, $h/S = 1106 / 6.7 \approx 165.075$, $\alpha_{\text{LBP}} = - 0.9938$, $(z/h)_{\text{BP}} = 0.0031$, $\alpha = 1$, $(z/h)_{\text{so}} = 0.5031$, $z_{\text{so}} = 0.5031 \times 1106 = 556.43 \text{ cm}$, $H_0 = - (h - S) = - (1106 - 6.7) = - 1099.3 \text{ cm}$ (height of the imaginary section of the vortex).

For this example, the profile of the air-entraining vortex is as follows [Eq. (4.4 a), note that the term of r/r_{mid} is divided and multiplied by D]

$$\frac{z}{h} = \left\{ 1 - \frac{2}{\pi} \arctan \left[\left[2 \left(1 - \frac{z}{h} \right) - 0.9938 \right] \left(\frac{r}{D} \right)^2 \left(\frac{D}{0.2688} \right)^2 \right] \right\} \text{ (parameters are in cm).}$$

6- Example VI

Let us consider the test data relating to the profile of an air-entraining vortex in ([76] Figure 10- test case A2, $S/D = 2$, $D = 67 \text{ mm}$) in which $S = 13.4 \text{ cm}$ (note that “h” in their study is equal to S in the present study). Let choose the data of $z = z_1 = 0.432 \text{ cm} \ll S = 13.4 \text{ cm}$ so that $z/h = z_1/h \leq (z/h)_{\text{BP}}$ is highly possible so as to have a high possibility of constant magnitude of $\alpha = 1$ in Figure 17 or Figure 20 [the magnitude of α will be taken as $\alpha = 1$ constant-horizontal section of the graph in Figures 17 and 20 or in Eq. (4.4a). This point is to be checked at the end of the solution. For $z_1 = 0.432 \text{ cm}$ the measured $r = r_1 = 13.179 \text{ cm}$. As a second data of z , let us choose $z = z_2 = 0.721 \text{ cm}$ (Appendix IV-Table 21) which is also much smaller than $S = 13.4 \text{ cm}$ so that $z/h = z_2/h \leq (z/h)_{\text{BP}}$ is highly possible so as to have a high possibility to take and use $\alpha = 1$ in in Figures.17 and 20 or Eq. (4.4a). For $z_2 = 0.721 \text{ cm}$ the measured $r = r_2 = 10.084 \text{ cm}$. Most likely for both z_1/h and z_2/h an identical constant magnitude of $\alpha = 1$ in Figure 17 or Figure 20 may be used. If Eq. (4.4 a) is separately written for $z = 0.432 \text{ cm}$ and 0.721 cm (considering $\alpha = 1$) and, divide them side by side (thereby h is eliminated) it results in

$$\frac{0.432}{0.721} = 0.59916 = \frac{1 - \frac{2}{\pi} \arctan \left(\frac{13.179}{r_{\text{mid}}} \right)^2}{1 - \frac{2}{\pi} \arctan \left(\frac{10.084}{r_{\text{mid}}} \right)^2}$$

By a trial-and-error solution procedure this equality gives $r_{\text{mid}} = 0.5595 \text{ cm}$ (computed). Hence, from $z_1 = 0.432 \text{ cm} = \left[1 - \frac{2}{\pi} \arctan \left(\frac{13.179}{0.5595} \right)^2 \right] h$; get $h = 376.5 \text{ cm}$,Thus $h/S = 376.5/13.4 = 28.097$

Let us find α_{LBP} .

The largest test data related to z in ([76] Figure 10- test case A2, $S/D = 2$, $D = 67$ mm) is $z = 13.4$ cm and the measured $r = 2.398$ cm for which most likely $\alpha \neq 1$ (since it is a very strong vortex). Therefore,

$$\frac{z}{h} = \frac{13.4}{376.5} = 0.03559 = 1 - \frac{2}{\pi} \arctan \left\{ [2(1 - 0.03559) + \alpha_{LBP}] \left(\frac{2.398}{0.5595} \right)^2 \right\}$$

By a trial-and-error solution procedure this equality gives $\alpha_{LBP} = -0.9926$.

Thereby $\left(\frac{z}{h} \right)_{BP} = \frac{1}{2}(1 + \alpha_{LBP}) = \frac{1}{2}(1 - 0.9926) = 0.0037$

Let us examine the solution (whether or the solution is correct).

i) **1st check:** Check whether or not both $z_1/h \leq (z/h)_{BP}$ and $z_2/h \leq (z/h)_{BP}$ [whether or not the assumption of identical $\alpha = 1$ (for both z_1/h and z_2/h) made at the beginning of the solution is correct]. Since both $z_1/h = 0.432/376.5 = 0.0011471 < (z/h)_{BP} = 0.0037$ (OK) and $z_2/h = 0.721/376.5 = 0.001915 < (z/h)_{BP} = 0.0037$ (OK) it indicates that the solution is correct (that means $\alpha = 1$ can be taken for both $z_1 = 0.432$ cm and $z_2 = 0.721$ cm as at beginning of the solution).

ii) **2nd check:** $\left(\frac{z}{h} \right)_{BP} = 0.0037$ from which $z_{BP} = 0.0037 \times 376.5 = 1.39305$ cm for which the measured $r/D = 1.03$ or $r = 6.9$ cm ([76] Figure 10- test case A2, $S/D = 2$, $D = 67$ mm). $\left(\frac{z}{h} \right)_{BP} = 0.0037 = 1 - \frac{2}{\pi} \arctan \left(\frac{r}{0.5595} \right)^2$ gives the computed $r = 7.337$ cm. As it is seen r (measured) = 6.9 cm is acceptably close to (computed) $r = 7.337$ cm (the error is about 6.3 % that may be attributed to the errors due to both the readings of data from the graph and measurements (especially due to a point gage used in the tests) relating to the profile of the very extremely strong air-entraining vortex).

iii) **3rd check:** Let us further examine the solution (whether or the solution is correct). For example, let consider the test data for $z = 4.086$ cm and measured $r = 4.042$ cm ([76] Figure 10- test case A2, $S/D = 2$, $D = 67$ mm). $z/h = 4.08/376.5 = 0.0108366 > (z/h)_{BP} (= 0.0037)$, therefore

$$\frac{z}{h} = \frac{4.086}{376.5} = 0.01085 = 1 - \frac{2}{\pi} \arctan \left\{ [2(1 - 0.01085) - 0.9926] * \left(\frac{r}{0.5595} \right)^2 \right\}$$

That gives the computed $r = 4.10$ cm. Since the (measured) $r = 4.042$ cm $\approx$ (computed) $r = 4.1$ cm (acceptable) indicating that the solution is correct (1.4 % error in “r” may be attributed to the readings from the graphs, measurement errors in the test relating to “r” and “h”).

Thus, the summary of the solution is as follows.

$h = 376.5$ cm, $r_{\text{mid}} = 0.5595$ cm, $h/S = 376.5/13.4 = 28.097$, $\alpha_{\text{LBP}} = -0.9926$, $(z/h)_{\text{BP}} = 0.0037$, $z_{\text{BP}} = 1.39305$ cm, $(z/h)_{\text{so}} = 0.5 + (z/h)_{\text{BP}} = 0.5 + 0.0037 = 0.5037$, $z_{\text{so}} = 0.5037 \times 376.5 = 189.64$ cm, $H_0 = -(h-S) = -(376.5 - 13.4) = -363.1$ cm (height of the imaginary section of the vortex).

For this example, the profile of the air-entraining vortex is as follows [Eq. (4.4a), note that the term of r/r_{mid} is divided and multiplied by D]

$$\frac{z}{h} = \left\{ 1 - \frac{2}{\pi} \arctan \left[\left[2 \left(1 - \frac{z}{h} \right) - 0.9926 \right] \left(\frac{r}{D} \right)^2 \left(\frac{D}{0.5595} \right)^2 \right] \right\} \text{ (parameters are in cm).}$$

7- Example VII:

Let us consider the Case I (Figure 26a), the test data relating to the profile of an air-entraining vortex in ([76] Figure 10- test case A3, $S/D = 3.5$, $D = 67$ mm) in which $S = 23.5$ cm (note that “h” in their study is equal to S in the present study). Let choose the data of $z = z_1 = 1.324$ cm $\ll S = 23.5$ cm so that $z/h = z_1/h \leq (z/h)_{\text{BP}}$ is highly possible so as to have a high possibility of constant magnitude of $\alpha = 1$ in Figure 17 [the magnitude of α will be taken as $\alpha = 1$ constant-horizontal section of the graph in Figures 17 and 20 or in Eq. (4.4a)]. This point is to be checked at the end of the solution. For $z_1 = 1.324$ cm the measured $r = r_1 = 7.118$ cm. As a second data of z , let us choose $z = z_2 = 4.117$ cm (Appendix IV-Table 22) which is also much smaller than $S = 23.5$ cm so that $z/h = z_2/h \leq (z/h)_{\text{BP}}$ is highly possible so as to have a high possibility to take and use $\alpha = 1$ in Figures 17 and 20 or Eq. (4.4a). For $z_2 = 4.117$ cm the measured $r = r_2 = 4.033$ cm. Most likely for both z_1/h and z_2/h an identical constant magnitude of $\alpha = 1$ in Figures 17 and 20 may be used. If Eq. (4.4a) is separately written for $z = 1.324$ cm and 4.117 cm (considering $\alpha = 1$) and, divide them side by side (thereby h is eliminated) it results in

$$\frac{1.324}{4.117} = 0.3216 = \frac{1 - \frac{2}{\pi} \arctan\left(\frac{7.118}{r_{mid}}\right)^2}{1 - \frac{2}{\pi} \arctan\left(\frac{4.033}{r_{mid}}\right)^2}$$

By a trial-and-error solution procedure this equality gives $r_{mid} = 1$ cm (computed). Hence, from $z_1 = 1.324$ cm $= \left[1 - \frac{2}{\pi} \arctan\left(\frac{7.118}{1}\right)^2\right] h$; get $h \approx 105.38 \approx 105$ cm, or $z_2 = 4.117$ cm $= \left[1 - \frac{2}{\pi} \arctan\left(\frac{4.033}{1}\right)^2\right] h$; get $h \approx 105.31 \approx 105$ cm. Thus $h/S = 105 / 23.5 = 4.468$

Let us find α_{LBP} .

The largest test data related to z in ([76] Figure 10- test case A3, $S/D = 3.5$, $D = 67$ mm) is $z = 23.5$ cm and the measured $r = 1.749$ cm for which most likely $\alpha \neq 1$ (since it is a very strong vortex). Therefore,

$$\frac{z}{h} = \frac{23.5}{105} = 0.2238 = 1 - \frac{2}{\pi} \arctan\left\{[2(1 - 0.2238) + \alpha_{LBP}] \left(\frac{1.749}{1}\right)^2\right\}$$

By a trial-and-error solution procedure this equality gives $\alpha_{LBP} = -0.66125$. Thereby

$$\left(\frac{z}{h}\right)_{BP} = \frac{1}{2}(1 + \alpha_{LBP}) = \frac{1}{2}(1 - 0.66125) = 0.1694$$

Let us examine the solution (whether or the solution is correct).

- i) **1st check:** Check whether or not both $z_1/h \leq (z/h)_{BP}$ and $z_2/h \leq (z/h)_{BP}$ [whether or not the assumption of identical $\alpha = 1$ (for both z_1/h and z_2/h) made at the beginning of the solution is correct]. Since both $z_1/h = 1.324/105 = 0.0126 < (z/h)_{BP} = 0.1694$ (OK) and $z_2/h = 4.117/105 = 0.0392 < (z/h)_{BP} = 0.1694$ (OK) it indicates that the solution correct (the assumption of $\alpha=1$ taken at the beginning of the solution is correct).
- ii) **2nd check:** Check that $\left(\frac{z}{h}\right)_{BP} = 0.1694 = 1 - \frac{2}{\pi} \arctan\left(\frac{r}{1}\right)^2$ that gives $r = 1.915$ cm (computed) . For $z = z_{BP}$, $z_{BP} = 0.1694 \times 105 = 17.787$ cm for which the measured $r/D = 0.2777$ or $r = 0.2777 \times 6.7 = 1.861$ cm ([76], Figure 10- test case A3, $S/D = 3.5$, $D = 67$ mm). As it is seen r (measured) $= 1.861$ cm $\approx$ (computed) $r = 1.915$ cm (acceptable 2.9% error in “r” may be attributed to the readings from the graphs, measurement

errors in the test relating to “r” and “h”) indicating that the solution is correct .

- iii) **3rd check:** Let us further examine the solution (whether or the solution is correct). For example, let consider the test data for $z = 23.5$ cm and measured $r = 1.749$ cm ([76] Figure 10- test case A3 , $S/D = 2$, $D = 67$ mm). $z/h = 23.5/105 = 0.2238 > (z/h)_{BP} = 0.1694$, therefore

$$\frac{z}{h} = \frac{23.5}{105} = 0.2238 = 1 - \frac{2}{\pi} \arctan \left\{ [2(1 - 0.2238) - 0.66125] * \left(\frac{r}{1}\right)^2 \right\}$$

that gives the computed $r = 1.76$ cm. Since the (measured) $r = 1.749$ cm $\approx$ (computed) $r = 1.76$ cm (acceptable) indicating that the solution is correct (0.6 % error in “r” may be attributed to the readings from the graphs, measurement errors in the test relating to “r” and “h”).

Thus, the summary of the solution is as follows.

$h = 105$ cm, $r_{mid} = 1$ cm, $h/S = 105/23.5 = 4.468$, $\alpha_{LBP} = - 0.66125$, $(z/h)_{BP} = 0.1694$, $z_{BP} = 17.787$ cm, $(z/h)_{so} = 0.5 + (z/h)_{BP} = 0.5 + 0.1694 = 0.6694$ cm, $z_{so} = 0.6694 \times 105 = 70.287$ cm, $H_0 = - (h - S) = - (105 - 23.5) = - 81.5$ cm (height of the imaginary section of the vortex).

For this example, the profile of the air-entraining vortex is as follows [Eq. (4.4a), note that the term of r/r_{mid} is divided and multiplied by D]

$$\frac{z}{h} = \left\{ 1 - \frac{2}{\pi} \arctan \left[\left[2 \left(1 - \frac{z}{h} \right) - 0.66125 \right] \left(\frac{r}{D} \right)^2 \left(\frac{D}{1} \right)^2 \right] \right\} \text{ (parameters are in cm) .}$$

APPENDIX III :THE DETAILS OF AVAILABLE DATA RELATING TO AIR- CORE VORTEX IN EXPERIMENTAL STUDIES

1. COMPARSION BETWEEN THE LAGRANGAIN AND EULERIN APPROACH IN SIMULATION OF FREE SURFACE AIR- CORE VORTICES](MARYAM AZARPIRA ET AL. 2021) [69].

The information related to this study are submergence $S = 18$ cm, intake Diameter = 3.5 cm, $\Gamma = 0$ [69] at which the measured $h = 5.65$ cm, $h/S = 5.65/18 = 0.318$, $h/2 = 5.65/2 = 2.825$ cm, $r = r_{\text{mid}} = 1.4$ cm, $H_0 = 12.35$ cm, as illustrated in Figure 35, data relating to profile of the non-air-core vortices are measured from ([69], Figure 5, physical model) that is replicated in Figure 36.

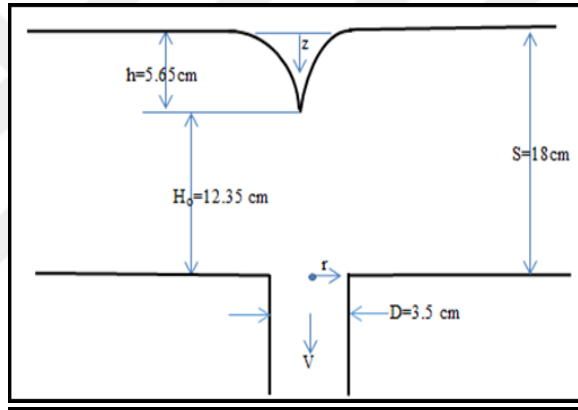


Figure 35 :Non-air Entraining Vortex (Air- core Vortex is in Suspension)

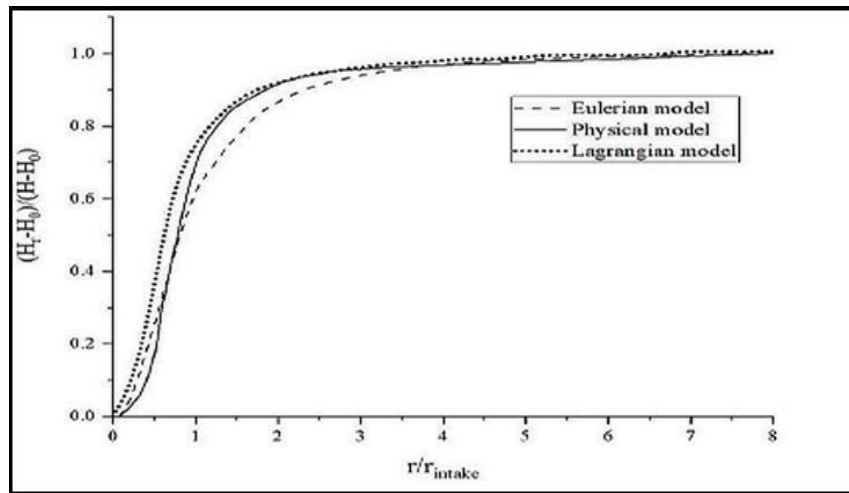


Figure 36: The Water Surface Profile Corresponding to the Physical Eulerian and Lagrangian Models ([69], Fig.5)

2. FLOW IN A FREE VORTEX (AN EXPERIMENTAL AND THEORETICAL STUDY OF THE FLOW CONDITIONS IN A STRONG STEADY VORTEX AT THE ENTRANCES OF AN OUTLET PIPE FROM A CIRCULAR TANK), (ANWAR .1965) [75].

The information related to this study are $Q = 0.74 \text{ CFS} = 0.02095 \text{ m}^3/\text{s}$, $D=10.16 \text{ cm}$, $\Gamma=1.069 \text{ m}^2/\text{s}$, $S = 21'' = 53.34 \text{ cm}$, $h = 35.827'' = 91 \text{ cm}$, and at $h/2 = 35.827''/2 = 17.914'' = 45.5 \text{ cm}$ [75] at which measured $r = r_{\text{mid}} = 1.84'' = 4.674 \text{ cm}$, $h/S = 91/53.4 = 1.706$, $H_0 = -37.66 \text{ cm}$ (height of the imaginary section of the vortex), as illustrated in Figure 37, data relating to the profile of the air-entraining vortices are measured in ([75] ,Figure 10-a) or ([11] , Figure 3.2a) that is replicated in Figure 38.

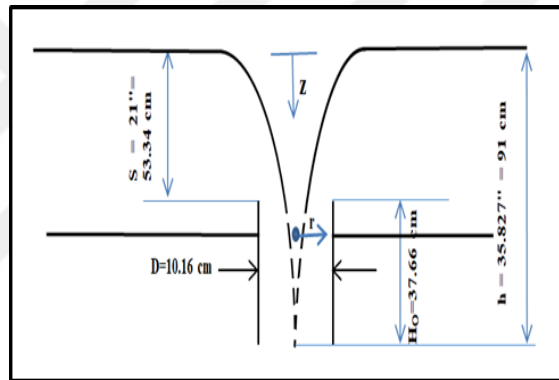


Figure 37: Air- entraining Vortex

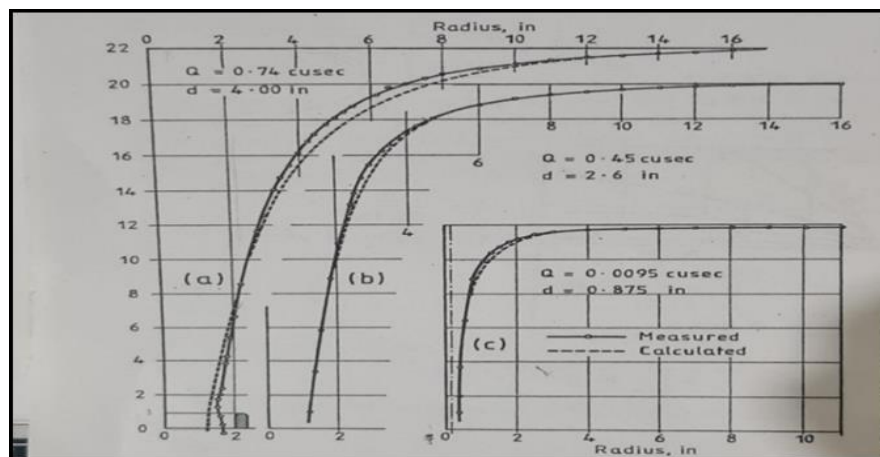


Figure 38: Measured and Calculated Profiles for (a) Large Outlet Pipe with High Swirl and Outlet Projecting into Intake. (b) Large Outlet Pipe with High Swirl and Outlet Level with Bottom of Tank (c) Small Outlet Pipe with Low Swirl and Outlet Projecting into Tank, ([75], Fig.10)

3. THE BATHTUB VORTEX IN A ROTATING CONTAINER (A. ANDERSEN ET AL. 2006) [66]

The information related to this study are submergence $S = 10.5$ cm, $h = (10.5 - 9.09) = 1.41$ cm, $h/S = 1.41/10.5 = 0.134$, $h/2 = 1.41/2 = 0.705$ cm, circulation imposed 12 rpm, $D = 0.2$ cm [66] at which the measured $r = r_{\text{mid}} = 0.206$ cm, $H_0 = 9.09$ cm, non-air-entraining vortex, as illustrated in Figure 39, data relating to the profile are measured and obtained in ([66] , Figure 13) that is replicated in Figure 40.

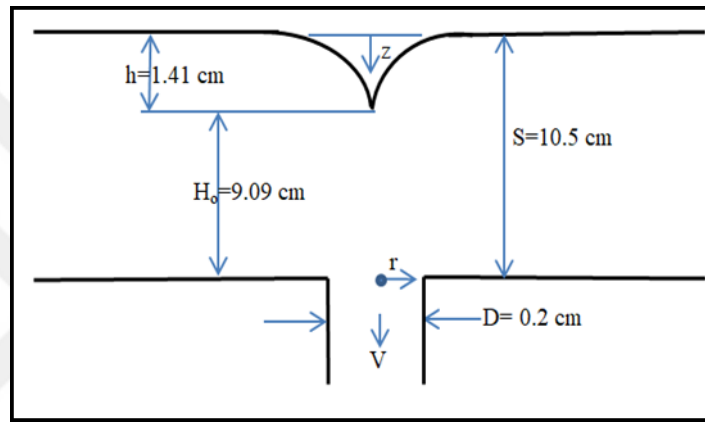


Figure 39: Non-air Entraining Vortex (Air- core Vortex is in Suspension)

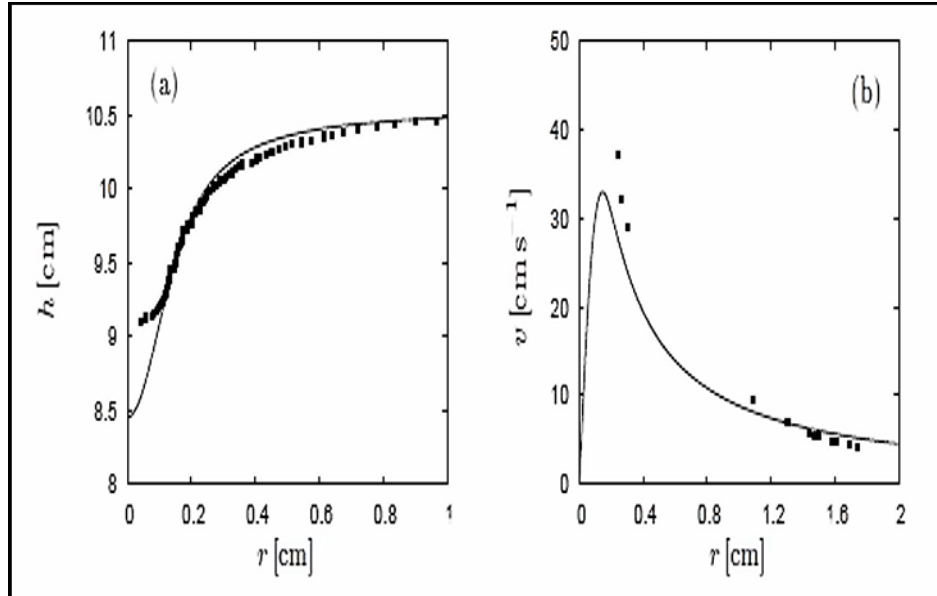


Figure 40 : Numerical Solution of Lundgren's differential Equations (Solid Lines) and Experimental Data (Squares) at 12 rpm. (a) the Height of the Free Surface and (b) the Azimuthal Velocity in the Co-rotating Reference Frame, ([66] , Fig. 13)

4. THEORETICAL AND EXPERIMENTAL STUDY ON THE VORTEX AT HYDRAULIC INTAKES (HONGLIANG SUN AND YAKUN LIU. 2015)[32].

The information related to this study are $D = 1.9 \text{ cm}$, $\Gamma = 0.0149 \text{ m}^2/\text{s}$, $h = S = 18.6 \text{ cm}$, $h/S = 1$ [32] at which the measured $r_{\text{mid}} = 1.08$, $H_0 = 0$, non-air-entraining vortex, as illustrated in Figure 41, data relating to the profile of the non-air-entraining vortices are measured and obtained in ([32], Figures 12 and 5) that are replicated in in Figures 42 and 43.

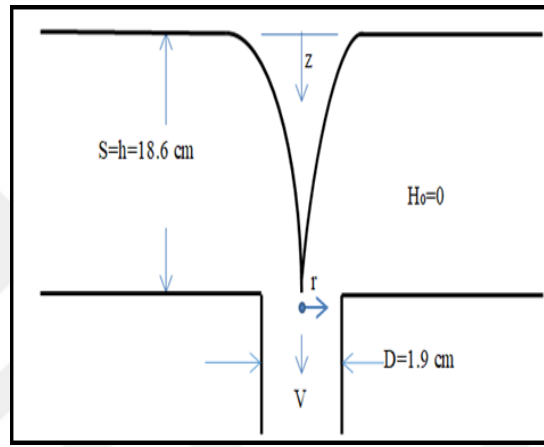


Figure 41: Non-air Entraining Vortex (Critical Submergence)

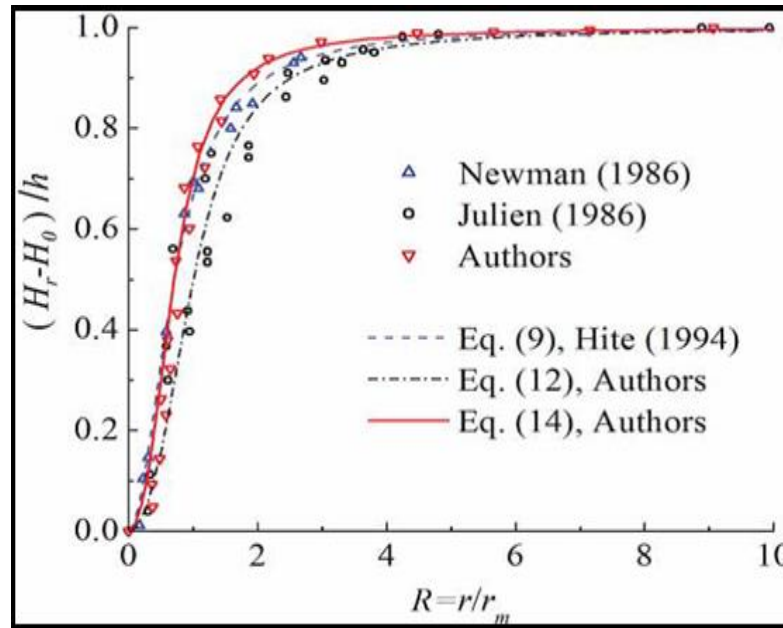


Figure 42: Comparison of the Vortex Water Surface Profiles; Present Study and Published Results, ([32], Fig.12)

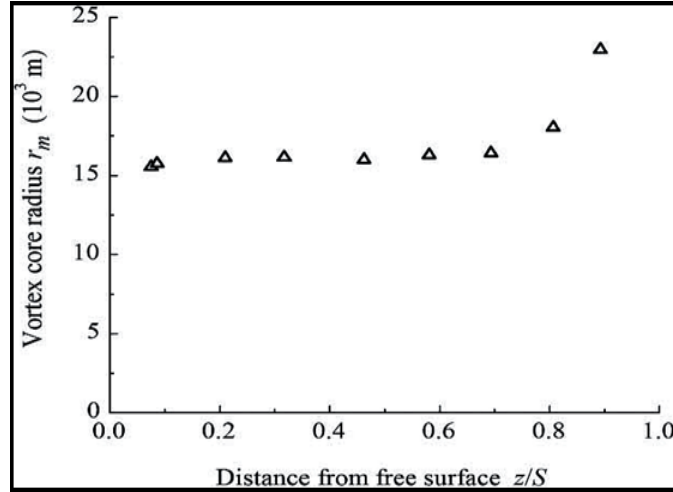


Figure 43: Vortex Core Radius Variation in the Axial Direction, ([32], Fig.5)

5. Vortex Flow Through Horizontal Orifices (J.C. Stevens. 1957) [77]

The information related to this study are submergence $S = 17.069$ cm, $D = 4'' = 10.16$ cm, imposed circulation $\Gamma \approx 0.396$ m²/s, air – entraining vortex[77] at which the measured $h = 298$ cm; $r_{mid} = 0.8515$, $H_0 = -280.931$ cm, as illustrated in Figure 44, data relating to the profile of the air-entraining vortices are measured and obtained in ([77] , Figure 1) that is replicated in Figure 45 .

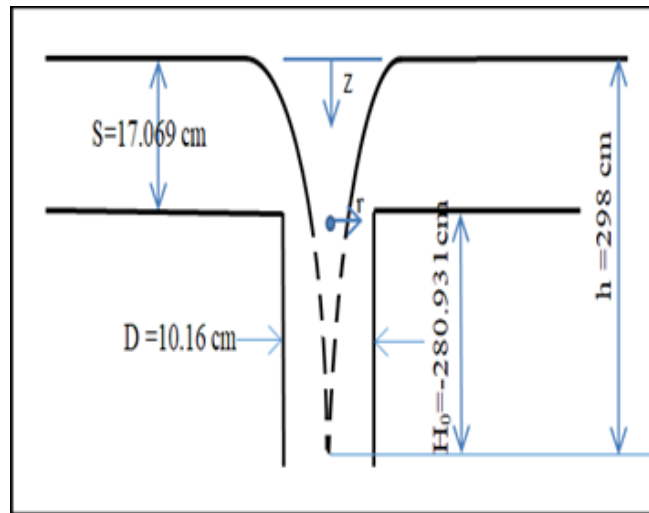


Figure 44: Air -entraining Vortex

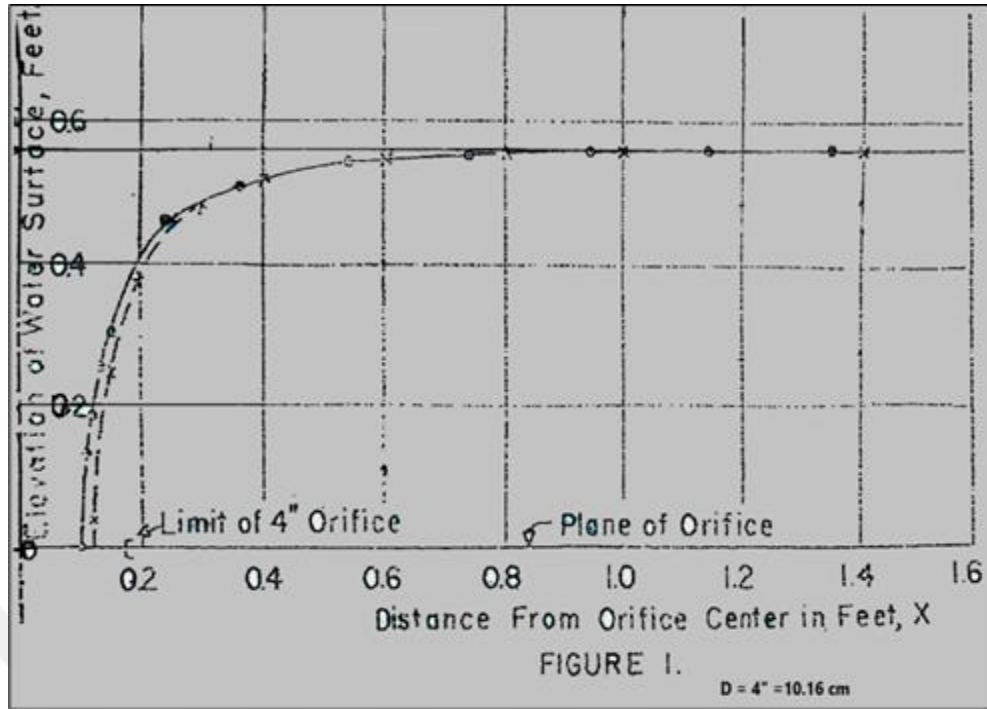


Figure 45: Comparison of Theoretical and Actual Water Surface profiles ([77], Fig.1)

6. INVESTIGATION ON BATHTUB VORTEX FLOW FIELD BY PARTICLE IMAGE VELOCIMETRY (CRISTOFANO ET AL. 2016)[67]

The data related to this study are $D = 2.6$ cm, $S = 50$ mm, No imposed circulation $\Gamma = 0$ [67] at which the measured h from photographs (b) and (c), $h = 6.58$ mm and 20.15 mm, respectively , $h/S = 0.131$ and 0.403 , non-air entraining vortex ([67], Figure 4) that is replicated in Figure 46.

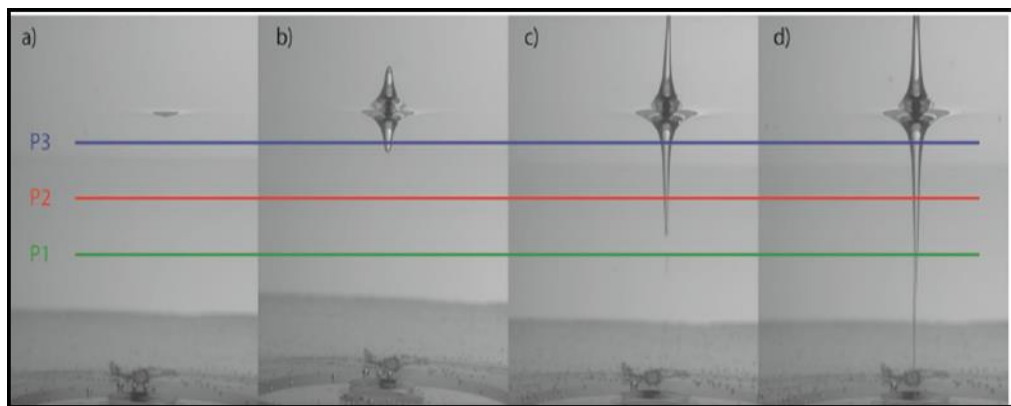


Figure 46: Photos of Free Surface Vortices: **a)** Stage 1, **b)** Stage 2, **c)** Stage 3, **d)** Stage 4 ([67], Fig.4)

- **First Case - (Stage b)**

$S = 50 \text{ mm} = 5 \text{ cm}$, $D = 2.6 \text{ cm}$; $\Gamma = 0$.

$h = 6.580 \text{ mm} = 0.658 \text{ cm}$ (photograph b)

$h/S = 0.131$, $H_0 = 4.342 \text{ cm}$, as illustrated in Figure 47.

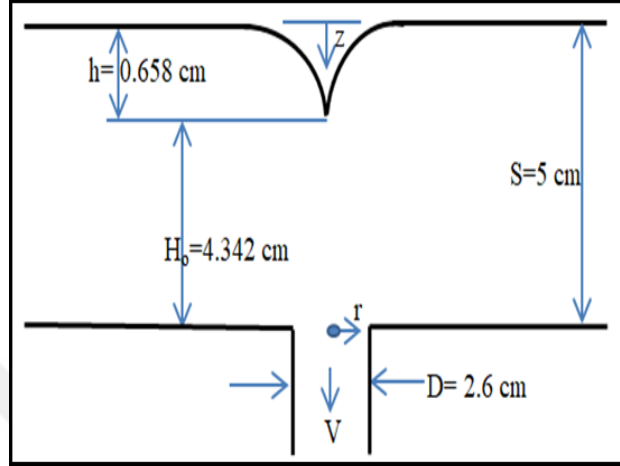


Figure 47: Non-air Entraining Vortex (Air- core Vortex is in Suspension) (Stage b)

- **Second Case - (Stage c)**

$S = 50 \text{ mm} = 5 \text{ cm}$, $D = 2.6 \text{ cm}$, no circulation imposed $\Gamma = 0$.

$h = 20.61519 \text{ mm} = 2.061519 \text{ cm}$ (photograph c)

$h/S = 0.412$, $H_0 = 2.938481 \text{ cm}$, as illustrated in Figure 48.

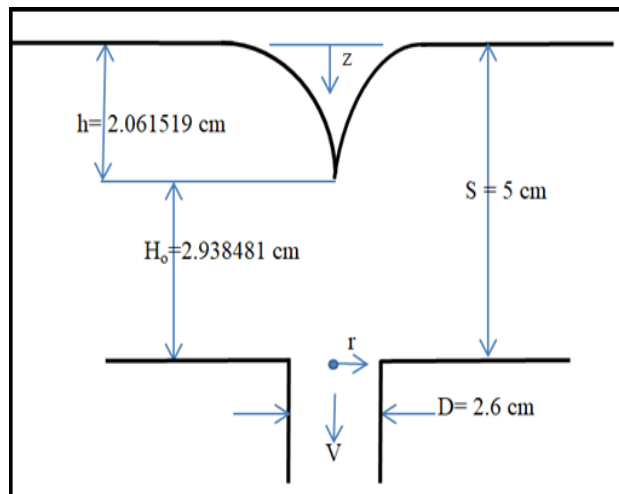


Figure 48: Non-air Entraining Vortex (Air- core Vortex is in Suspension) (Stage c)

7. Large-Scale Experiments on the Formation of Surface Vortices with and without Vortex Suppression (Nicolai et al. 2019) [68].

The information related to this study are $\Gamma = 0$, $D = 20$ cm, non-air entraining vortex and the data related of this study are measured and obtained in ([68], Figure 5) that is replicated in Figure 49, from the photographs of the air core vortices for the Froude number $F = 0.7$, 1 and 1.4 for which $S = 1.5$ m.

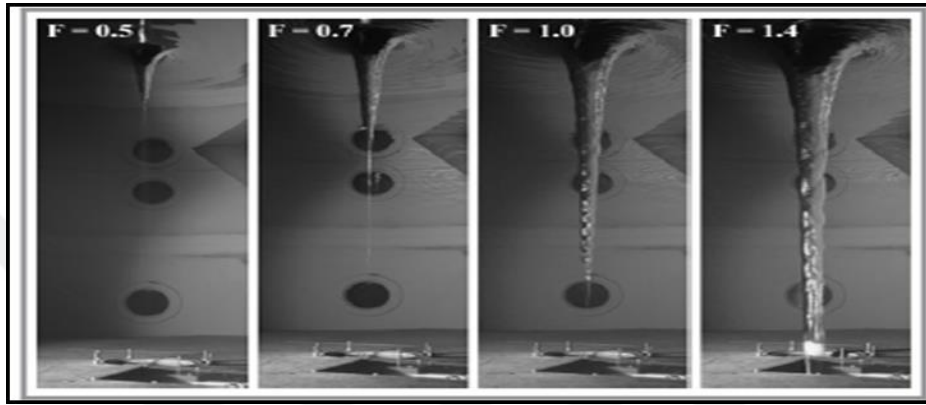


Figure 49: Photographs of the Gas-core Vortices for Experiments with Intermediate Floor and $S = 1.5$ m ([68], Fig.5)

- First case

$S = 1.5$ m

$h = 60$ cm (estimated in the photograph for $F = 0.7$)

$h/S = 0.40$, $D = 20$ cm, no imposed circulation $\Gamma = 0$, $H_0 = 90$ cm, as illustrated in Figure 50.

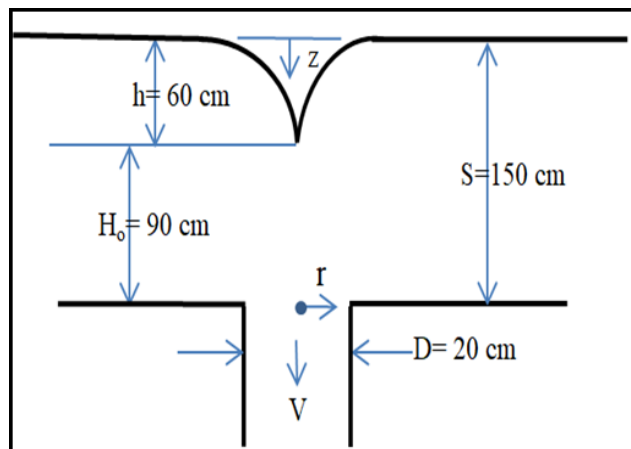


Figure 50: Non-air Entraining Vortex (Air- core Vortex is in Suspension) ($F = 0.7$)

- **Second case**

$$S = 1.5 \text{ m}$$

$$h = 119.4 \text{ cm (estimated in the photograph for } F = 1.0)$$

$h/S = 0.796$, $D = 20 \text{ cm}$, no imposed circulation $\Gamma = 0$, $H_0 = 30.6 \text{ cm}$, as illustrated in Figure 51.

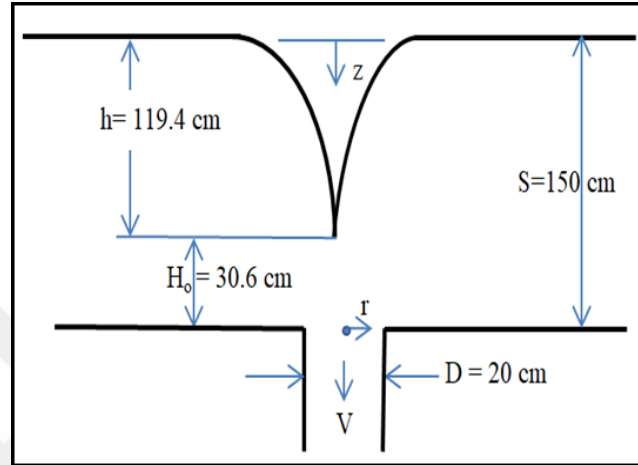


Figure 51: Non-air Entraining Vortex (Air- Core Vortex is in Suspension) ($F = 1$)

- **Third case**

$$S = 1.5 \text{ m}$$

$$h = 150 \text{ cm (estimated in the photograph for } F = 1.4)$$

$h/S = 1$, $D = 20 \text{ cm}$, no imposed circulation $\Gamma = 0$, $H_0 = 0$, as illustrate in Figure (52)

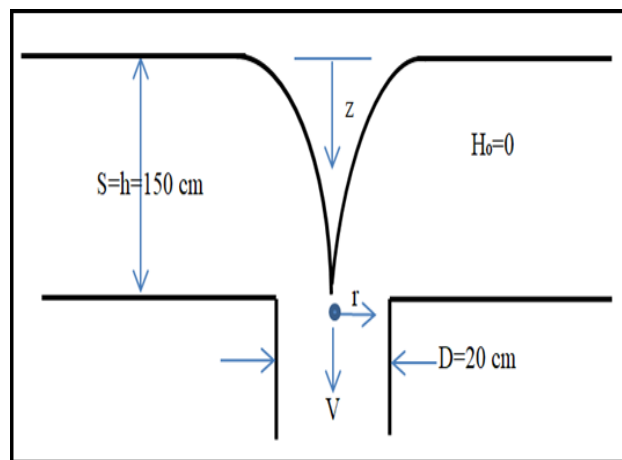


Figure 52: Air- entraining Vortex (Critical Submergence) ($F = 1.4$)

8. EXPERIMENTAL AND NUMERICAL MODELLING OF FREE-Surface Turbulent Flows in Full Air-Core Water Vortices (Mulligan et al. 2016) [76]

The data related of this study are measured and obtained in ([76] , Figure 10 (A1, A2, A3)- air –entraining vortices) that is replicated in Figure 53.

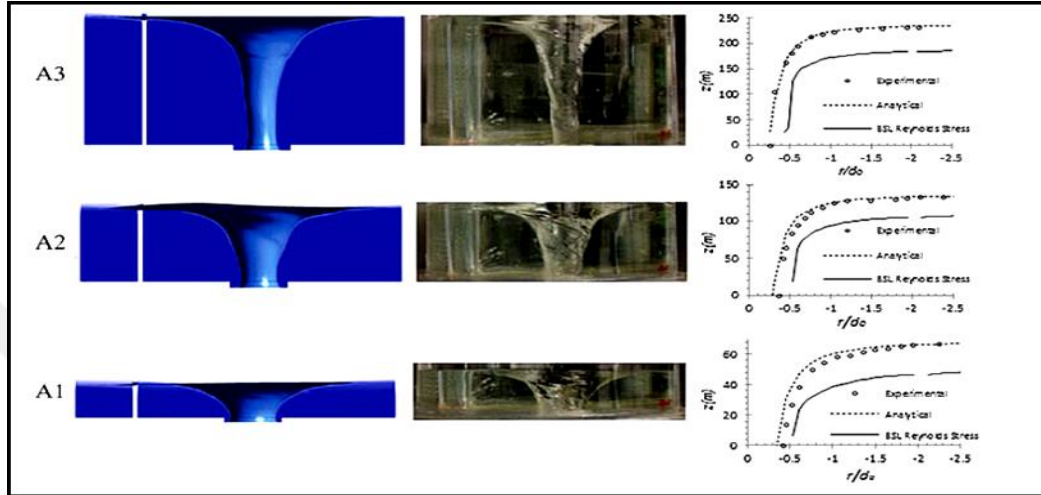


Figure 53: Summary of Comparison Between Numerical, Experimental and Analytical Models, Respectively, for Test Cases A1, A2 and A3 ([76] , Fig.10)

- First case

A3- large submergence $S = 23.5$ cm; $D = 6.7$ cm, $S/D = 3.5$ [76] , computed $h = 105$ cm; $r_{mid} = 1$ cm; $h/S = 105/23.5 = 4.468$; $H_0 = - 81.5$ cm (height of the imaginary section of the vortex), air-entraining vortex with very strong imposed circulation, as illustrated in Figure (54).

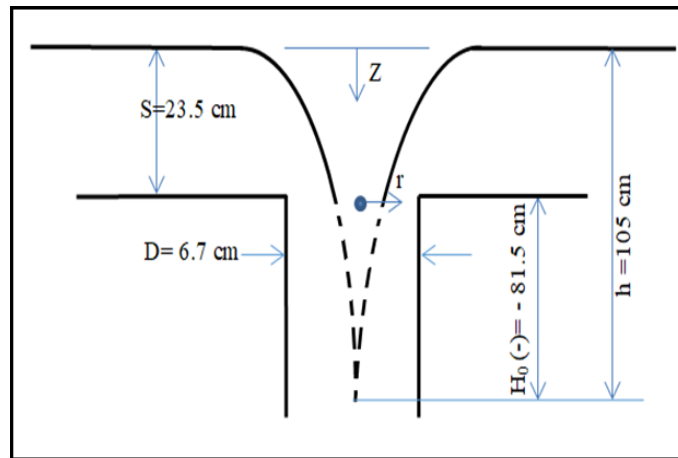


Figure 54: Air -entraining Vortex (A3- Large Submergence)

- **Second case**

A2- medium submergence $S = 13.4$ cm; $D = 6.7$ cm, $S/D = 2$ [76] , computed $h = 376.5$ cm; $r_{mid} = 0.5595$ cm; $h/S = 376.5/13.4 = 28.097$, $H_0 = -363.1$ cm, air-entraining vortex with very strong imposed circulation , as illustrated in Figure (55).

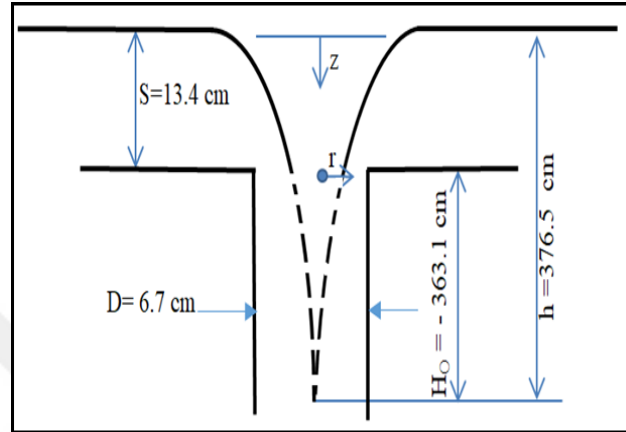


Figure 55: Air -entraining Vortex (A2- Medium Submergence)

- **Third case**

A1- small submergence $S = 6.7$ cm; $D = 6.7$ cm, $S/D = 1$ [76] , computed $h = 1106$ cm; $r_{mid} = 0.2688$ cm, $h/S = 1106 / 6.7 \approx 165.075$, $H_0 = -1099.3$ cm, air-entraining vortex with very strong imposed circulation , as illustrated in Figure (56).

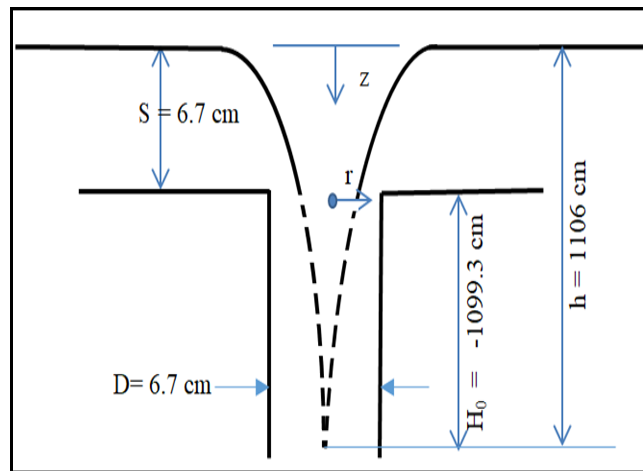


Figure 56: Non-air Entraining Vortex (A1- Small Submergence)

APPENDIX IV: TABLES

Table 1: Data Related to the "Comparison Between the Lagrangian and Eulerian Approaches for Simulating Free-Surface Air-Core Vortices" (Azarpira et al., 2021) [69].

z(cm)	h(cm)	z/h	(Hr-H0)/h	r/r _{intake}	r(cm)	D(cm)	S(cm)	r _{mid} (cm)	r/D	z/D	r/r _{mid}
0	5.65	0	1	∞	∞	3.5	18	1.4	∞	0	∞
0.14		0.025	0.98	4	7.0	3.5	18	1.4	2.00	0.04	5.0
0.28		0.05	0.95	2.83	5.0	3.5	18	1.4	1.42	0.08	3.5
0.57		0.1	0.90	1.93	3.4	3.5	18	1.4	0.97	0.16	2.4
0.85		0.15	0.85	1.50	2.6	3.5	18	1.4	0.75	0.24	1.9
1.13		0.2	0.80	1.27	2.2	3.5	18	1.4	0.63	0.32	1.6
1.41		0.25	0.75	1.13	2.0	3.5	18	1.4	0.57	0.40	1.4
1.70		0.3	0.70	1.00	1.8	3.5	18	1.4	0.50	0.48	1.3
1.98		0.35	0.65	0.95	1.7	3.5	18	1.4	0.48	0.57	1.2
2.26		0.4	0.60	0.90	1.6	3.5	18	1.4	0.45	0.65	1.1
2.83		0.5	0.50	0.80	1.4	3.5	18	1.4	0.40	0.81	1.0
3.39		0.6	0.40	0.70	1.2	3.5	18	1.4	0.35	0.97	0.9
3.96		0.7	0.30	0.60	1.1	3.5	18	1.4	0.30	1.13	0.8
4.24		0.75	0.25	0.55	1.0	3.5	18	1.4	0.28	1.21	0.7
4.52		0.8	0.20	0.50	0.9	3.5	18	1.4	0.25	1.29	0.6
4.80		0.85	0.15	0.47	0.8	3.5	18	1.4	0.23	1.37	0.6
5.09		0.9	0.10	0.40	0.7	3.5	18	1.4	0.20	1.45	0.5
5.37		0.95	0.05	0.30	0.5	3.5	18	1.4	0.15	1.53	0.4
5.51		0.975	0.03	0.20	0.4	3.5	18	1.4	0.10	1.57	0.3
5.65		1	0	0.00	0	3.5	18	1.4	0.00	1.61	0

Table 2A: Data Related to "An Experimental and Theoretical Study of Flow Conditions in a Strong Free Vortex at the Outlet of a Circular Tank" (Anwar, 1965) [75].

z(inch)	z(cm)	h(cm)	z/h	(Hr-H0)/h	r(inch)	r(cm)	D(cm)	S(cm)	r _{mid} (cm)	r/D	z/D	r/r _{mid}
0	0	91	0	1.00	∞	∞	10.16	53.34	4.674	∞	0	∞
0.29	0.75		0.01	0.99	14.00	35.560			4.674	3.50	0.074	7.61
0.50	1.27		0.01	0.99	12.00	30.480			4.674	3.00	0.125	6.52
0.75	1.91		0.02	0.98	10.24	25.997			4.674	2.56	0.188	5.56
1.00	2.54		0.03	0.97	9.45	24.014			4.674	2.36	0.250	5.14
1.50	3.81		0.04	0.96	7.82	19.873			4.674	1.96	0.375	4.25
2.00	5.08		0.06	0.94	7.00	17.780			4.674	1.75	0.500	3.80
2.50	6.35		0.07	0.93	6.24	15.837			4.674	1.56	0.625	3.39
3.00	7.62		0.08	0.92	5.65	14.351			4.674	1.41	0.750	3.07
4.00	10.16		0.11	0.89	4.82	12.250			4.674	1.21	1.000	2.62
5.00	12.70		0.14	0.86	4.30	10.930			4.674	1.08	1.250	2.34
6.00	15.24		0.17	0.83	3.94	10.005			4.674	0.98	1.500	2.14
7.00	17.78		0.20	0.80	3.52	8.928			4.674	0.88	1.750	1.91
8.00	20.32		0.22	0.78	3.27	8.313			4.674	0.82	2.000	1.78

Table 2B: Data Related to "Experimental and Theoretical Study of Flow Conditions in a Strong Free Vortex at the Outlet of a Circular Tank" (Anwar, 1965) [75].

z(inch)	z(cm)	h(cm)	z/h	(Hr-H0)/h	r (inch)	r(cm)	D(cm)	S(cm)	r _{mid} (cm)	r/D	z/D	r/r _{mid}
10.00	25.40	91	0.28	0.72	2.71	6.873	10.16	53.34	4.674	0.68	2.50	1.47
11.80	29.97		0.33	0.67	2.49	6.312			4.674	0.62	2.95	1.35
12.00	30.48		0.33	0.67	2.35	5.969			4.674	0.59	3.00	1.28
13.00	33.02		0.36	0.64	2.30	5.850			4.674	0.58	3.25	1.25
14.00	35.56		0.39	0.61	2.12	5.385			4.674	0.53	3.50	1.15
15.00	38.10		0.42	0.58	2.06	5.232			4.674	0.52	3.75	1.12
16.00	40.64		0.45	0.55	1.88	4.775			4.674	0.47	4.00	1.02
17.00	43.18		0.47	0.53	1.88	4.770			4.674	0.47	4.25	1.02
17.91	45.50		0.50	0.50	1.84	4.674			4.674	0.46	4.48	1.00
18.00	45.72		0.50	0.50	1.77	4.483			4.674	0.44	4.50	0.96
19.00	48.26		0.53	0.47	1.70	4.310			4.674	0.42	4.75	0.92
20.77	52.74		0.58	0.42	1.47	3.736			4.674	0.37	5.19	0.80

Table 3: Data Related to "Analysis of the Bathtub Vortex in a Rotating Container" (Andersen et al., 2006) [66].

z(cm)	z/h	(H_r-H₀)/h	r (cm)	D(cm)	S(cm)	r_{mid}(cm)	r/D	z/D	r/r_{mid}
0	0	1	1	0.2	10.5	0.206	5	0	4.85
0.071	0.05	0.95	0.781				3.91	0.355	3.79
0.141	0.1	0.90	0.616				3.08	0.705	2.99
0.282	0.2	0.80	0.414				2.07	1.410	2.01
0.423	0.3	0.70	0.313				1.57	2.115	1.52
0.564	0.4	0.60	0.244				1.22	2.820	1.18
0.705	0.5	0.50	0.206				1.03	3.525	1.00
0.846	0.6	0.40	0.170				0.85	4.230	0.83
0.916	0.65	0.35	0.156				0.78	4.580	0.76
0.987	0.7	0.30	0.151				0.76	4.935	0.73
1.056	0.75	0.25	0.128				0.64	5.280	0.62
1.128	0.8	0.20	0.136				0.68	5.640	0.66
1.157	0.821	0.18	0.122				0.61	5.785	0.59
1.198	0.85	0.15	0.117				0.59	5.990	0.57
1.229	0.872	0.13	0.113				0.57	6.145	0.55
1.269	0.9	0.10	0.114				0.57	6.345	0.55
1.34	0.95	0.05	0.081				0.40	6.700	0.39
1.382	0.98	0.02	0.056				0.28	6.910	0.27
1.41	1	0	0				0.00	7.050	0.00

Table 4: Data Related to "Theoretical and Experimental Analysis of Vortex Formation at Hydraulic Intakes" (Sun & Liu, 2015) [32].

z/h	h (cm)	z (cm)	$(H_r-H_0)/h$	$R= r/r_m$	r_m (cm) (from Fig.5)	r (cm)	D (cm)	S (cm)	r_{mid} (cm)	r/D	z/D	z/h	r/r_{mid}
0.026	18.6	0.48	0.974	4.444	1.60	4.800	1.9	18.6	1.08	2.526	0.255	0.026	4.444
0.060		1.12	0.940	2.978	1.60	3.216				1.693	0.587	0.060	2.978
0.082		1.53	0.918	2.889	1.60	3.120				1.642	0.803	0.082	2.889
0.140		2.60	0.860	2.148	1.60	2.320				1.221	1.371	0.140	2.148
0.240		4.46	0.764	1.556	1.60	1.680				0.884	2.349	0.240	1.556
0.312		5.80	0.688	1.333	1.60	1.440				0.758	3.054	0.312	1.333
0.464		8.63	0.536	1.052	1.60	1.136				0.598	4.542	0.464	1.052
0.500		9.30	0.500	0.993	1.60	1.072				0.564	4.895	0.500	0.993
0.566		10.53	0.434	1.052	1.60	1.136				0.598	5.541	0.566	1.052
0.618		11.49	0.382	0.933	1.60	1.008				0.531	6.050	0.618	0.933
0.676		12.57	0.328	1.007	1.60	1.088				0.573	6.618	0.676	1.007
0.770		14.32	0.235	0.881	1.67	0.952				0.501	7.538	0.770	0.881
0.852		15.85	0.147	0.825	1.98	0.891				0.469	8.341	0.852	0.825
0.900		16.74	0.100	0.866	2.46	0.935				0.492	8.811	0.900	0.866
0.950		17.67	0.050	0.765	2.95	0.826				0.435	9.300	0.950	0.765
1		18.60	0.000	0.000	0.00	0.000				0.000	9.789	1.000	0.000

Table 5: Data Related to "Analysis of Vortex Flow Through Horizontal Orifices" (Stevens, 1957) [77].

Eleva. H (cm)	z (cm)	h(cm)	z/h	(Hr-H0)/h	r(ft)	r (cm)	D(cm)	S(cm)	r _{mid} (cm)	r/D	z/D	r/r _{mid}
17.069	0	298	0.000	1	1.154	35.18	10.16	17.069	0.8515	3.463	0.000	41.32
16.982	0.174		0.001	0.999	0.941	28.68				2.823	0.017	33.68
16.783	0.26606		0.001	0.999	0.896	27.30				2.687	0.026	26.69
16.546	0.50308		0.002	0.998	0.542	16.53				1.627	0.050	19.41
15.414	1.655		0.006	0.994	0.363	11.08				1.090	0.163	13.01
14.02	3.049		0.010	0.990	0.245	7.47				0.735	0.300	8.77
12.192	4.877		0.016	0.984	0.200	6.11				0.601	0.480	7.18
9.144	7.925		0.027	0.973	0.149	4.55				0.448	0.780	5.34
7.75	9.319		0.031	0.969	0.135	4.12				0.406	0.917	4.84
5.66	11.409		0.038	0.962	0.121	3.69				0.363	1.123	4.34
4.006	13.063		0.044	0.956	0.107	3.26				0.321	1.286	3.83
0 (Intake level)	17.069		0.057	0.943	0.097	2.96				0.291	1.680	3.48

Table 6: Data Related to “Investigation of Bathtub Vortex Flow Field Using Particle Image Velocimetry” (Cristofano et al., 2016) / First Case - Stage b) [67].

$(H_r - H_0)$ (mm)	$(H_r - H_0)/h$	z/h	h (mm)	z (mm)	r (mm)	z (cm)	r (cm)	D (cm)	S (cm)	r_{mid} (cm)	r/D	z/D	r/r_{mid}
0	0	1.00	6.58	6.58	0	0.658	0	2.6	5	0.1188	0	0.253	0
0.130	0.020	0.98		6.45	0.256	0.645	0.026				0.010	0.248	0.215
0.263	0.040	0.96		6.32	0.362	0.632	0.036				0.014	0.243	0.305
0.395	0.060	0.94		6.19	0.423	0.619	0.042				0.016	0.238	0.356
0.526	0.080	0.92		6.05	0.492	0.605	0.049				0.019	0.233	0.414
0.561	0.085	0.91		6.02	0.512	0.602	0.051				0.020	0.232	0.431
1.121	0.170	0.83		5.46	0.622	0.546	0.062				0.024	0.210	0.524
1.682	0.256	0.74		4.90	0.723	0.490	0.072				0.028	0.188	0.609
2.242	0.341	0.66		4.34	0.886	0.434	0.089				0.034	0.167	0.745
2.803	0.426	0.57		3.78	1.037	0.378	0.104				0.040	0.145	0.873
3.290	0.500	0.50		3.29	1.188	0.329	0.119				0.046	0.127	1.000
3.363	0.511	0.49		3.22	1.222	0.322	0.122				0.047	0.124	1.029
3.924	0.596	0.40		2.66	1.440	0.266	0.144				0.055	0.102	1.213
4.484	0.681	0.32		2.10	1.687	0.210	0.169				0.065	0.081	1.420
5.045	0.767	0.23		1.54	2.124	0.154	0.212				0.082	0.059	1.788
5.605	0.852	0.15		0.98	2.752	0.098	0.275				0.106	0.038	2.317
6.166	0.937	0.06		0.41	5.061	0.041	0.506				0.195	0.016	4.260
6.580	1.000	0.00		0.00	5.314	0.000	0.531				0.204	0	4.473

Table 7: Data Related to "Investigation of Bathtub Vortex Flow Field Using Particle Image Velocimetry" (Cristofano et al., 2016 / Second Case - Stage c) [67].

$(H_r - H_0)$ (mm)	$(H_r - H_0)/h$	z/h	h (mm)	z (mm)	r (mm)	z (cm)	r (cm)	D (cm)	S (cm)	r_{mid} (cm)	r/D	z/D	r/r_{mid}
0	0	1	20.615	20.615	0	2.062	0	2.6	5	0.661	0	0.793	0.000
1.030	0.0500	0.950		19.585	0.342	1.959	0.034				0.013	0.753	0.052
1.682	0.0816	0.918		18.934	0.376	1.893	0.038				0.014	0.728	0.057
2.670	0.1295	0.870		17.945	0.392	1.795	0.039				0.015	0.690	0.059
3.363	0.1631	0.837		17.252	0.404	1.725	0.040				0.016	0.664	0.061
4.120	0.1999	0.800		16.495	0.432	1.650	0.043				0.017	0.634	0.065
5.045	0.2447	0.755		15.571	0.437	1.557	0.044				0.017	0.599	0.066
6.726	0.3263	0.674		13.889	0.482	1.389	0.048				0.019	0.534	0.073
8.408	0.4078	0.592		12.208	0.628	1.221	0.063				0.024	0.470	0.095
10.089	0.4894	0.511		10.526	0.656	1.053	0.066				0.025	0.405	0.099
10.313	0.5003	0.500		10.302	0.661	1.030	0.066				0.025	0.396	0.100
11.771	0.5710	0.429		8.845	0.673	0.884	0.067				0.026	0.340	0.102
13.452	0.6525	0.347		7.163	0.768	0.716	0.077				0.030	0.276	0.116
15.134	0.7341	0.266		5.482	0.930	0.548	0.093				0.036	0.211	0.141
16.815	0.8157	0.184		3.800	1.396	0.380	0.140				0.054	0.146	0.211
18.497	0.8972	0.103		2.119	2.532	0.212	0.253				0.097	0.081	0.383
20.178	0.9788	0.021		0.437	5.493	0.044	0.549				0.211	0.017	0.831
20.615	1	0		0	6.158	0	0.616				0.237	0.000	0.932

Table 8: Data Related to "Large-Scale Experiments on Surface Vortex Formation with and without Vortex Suppression" (Nicolai et al., 2019 / First Case / $F = 0.7$) [68].

$(H_r - H_0)/h$	z/h	$h(\text{cm})$	$z(\text{cm})$	$r(\text{mm})$	$r(\text{cm})$	$D(\text{cm})$	$S(\text{cm})$	$r_{\text{mid}}(\text{cm})$	r/D	z/D	r/r_{mid}
0	1	60	60	0	0	20	150	2.75	0	3	0
0.1	0.9		54	1	0.100				0.00500	2.7	0.036
0.2	0.8		48	1.75	0.175				0.00875	2.4	0.064
0.3	0.7		42	2	0.200				0.01000	2.1	0.073
0.4	0.6		36	2.25	0.225				0.01125	1.8	0.082
0.5	0.5		30	2.75	0.275				0.01375	1.5	0.100
0.6	0.4		24	3.25	0.325				0.01625	1.2	0.118
0.7	0.3		18	3.75	0.375				0.01875	0.9	0.136

Table 9: Data Related to "Large-Scale Experiments on Surface Vortex Formation with and without Vortex Suppression" (Nicolai et al., 2019 / Second Case / $F = 1$) [68].

$(H_r - H_o)/h$	z/h	$h(\text{cm})$	$z(\text{cm})$	$r(\text{mm})$	$r(\text{cm})$	$D(\text{cm})$	$S(\text{cm})$	$r_{\text{mid}}(\text{cm})$	r/D	z/D	r/r_{mid}
0	1	119.4	119.4	0	0	20	150	0.3	0	5.970	0
0.1	0.9		107.46	1.5	0.15				0.008	5.373	0.500
0.2	0.8		95.52	2.1	0.21				0.011	4.776	0.700
0.25	0.75		89.55	2	0.20				0.010	4.478	0.667
0.3	0.7		83.58	2.5	0.25				0.013	4.179	0.833
0.4	0.6		71.64	2.7	0.27				0.014	3.582	0.900
0.45	0.55		65.67	2.75	0.28				0.014	3.284	0.917
0.5	0.5		59.7	3	0.30				0.015	2.985	1.000
0.6	0.4		47.76	3.5	0.35				0.018	2.388	1.167
0.7	0.3		35.82	4.25	0.43				0.021	1.791	1.417
0.8	0.2		23.88	6.5	0.65				0.033	1.194	2.167

Table 10: Data Related to "Large-Scale Experiments on Surface Vortex Formation with and without Vortex Suppression" (Nicolai et al., 2019 / Third Case / $F = 1.4$) [68].

$(H_r - H_o)/h$	z/h	$h(\text{cm})$	$z(\text{cm})$	$r(\text{mm})$	$r(\text{cm})$	$D(\text{cm})$	$S(\text{cm})$	$r_{\text{mid}}(\text{cm})$	r/D	z/D	r/r_{mid}
0	1	150	150	0	0	20	150	0.4	0	7.50	0
0.1	0.9		135	3	0.300				0.0150	6.75	0.750
0.2	0.8		120	3	0.300				0.0150	6.00	0.750
0.3	0.7		105	3.25	0.325				0.0163	5.25	0.813
0.4	0.6		90	3.5	0.350				0.0175	4.50	0.875
0.5	0.5		75	4	0.400				0.0200	3.75	1.000
0.6	0.4		60	4.5	0.450				0.0225	3.00	1.125
0.7	0.3		45	5	0.500				0.0250	2.25	1.250
0.8	0.2		30	7	0.700				0.0350	1.50	1.750

Table 11: Data Related to "Experimental and Numerical Modeling of Free-Surface Turbulent Flows in Full Air-Core Water Vortices" (Mulligan, et al., 2016 / First Case - A3 - Large Submergence) [76].

z (cm)	r/do	r(cm)	D(cm)	S(cm)	r _{mid} (cm)	r/D	z/D	z/h	r/r _{mid}
0	∞	∞	6.7	23.5	1	∞	0	0	∞
0.441	2.0909	14.009				2.091	0.0658	0.0042	14.009
0.441	1.9430	13.018				1.943	0.0658	0.0042	13.018
0.664	1.6590	11.115				1.659	0.0991	0.0063	11.115
0.808	1.3630	9.132				1.363	0.1206	0.0077	9.132
1.324	1.0624	7.118				1.062	0.1976	0.0126	7.118
1.764	0.9140	6.124				0.914	0.2633	0.0168	6.124
2.353	0.7719	5.172				0.772	0.3512	0.0224	5.172
4.117	0.6019	4.033				0.602	0.6145	0.0392	4.033
5.440	0.5340	3.578				0.534	0.8119	0.0518	3.578
7.279	0.4660	3.122				0.466	1.0864	0.0693	3.122
13.088	0.3182	2.132				0.318	1.9534	0.1246	2.132
23.500	0.2610	1.749				0.261	3.5075	0.2238	1.749

Table 12: Data Related to "Experimental and Numerical Modeling of Free-Surface Turbulent Flows in Full Air-Core Water Vortices" (Mulligan et al., 2016 / Second Case - A2 - Medium Submergence) [76].

z (cm)	r/d ₀	r(cm)	h(cm)	D(cm)	S(cm)	r _{mid} (cm)	r/D	z/D	z/h	r/r _{mid}
0	∞	∞	376.5	6.7	13.4	0.5595	∞	0	0	∞
0.192	2.402	16.093					2.402	0.029	0.0005	28.76
0.288	2.130	14.271					2.130	0.043	0.0008	25.51
0.432	1.967	13.179					1.967	0.064	0.0011	23.55
0.577	1.820	12.194					1.820	0.086	0.0015	21.79
0.717	1.505	10.084					1.505	0.107	0.0019	18.02
0.769	1.217	8.154					1.217	0.115	0.0020	14.57
1.062	1.049	7.028					1.049	0.159	0.0028	12.56
1.643	0.904	6.057					0.904	0.245	0.0044	10.83
2.319	0.760	5.092					0.760	0.346	0.0062	9.10
3.125	0.696	4.663					0.696	0.466	0.0083	8.33
4.086	0.603	4.040					0.603	0.610	0.0109	7.22
5.192	0.532	3.564					0.532	0.775	0.0138	6.37
7.115	0.456	3.055					0.456	1.062	0.0189	5.46
8.557	0.407	2.727					0.407	1.277	0.0227	4.87
13.400	0.358	2.399					0.358	2	0.0356	4.29

Table 13: Data Related to "Experimental and Numerical Modeling of Free-Surface Turbulent Flows in Full Air-Core Water Vortices" (Mulligan et al., 2016 / Third Case - A1 - Small Submergence) [76].

z (cm)	r/d ₀	r(cm)	h(cm)	D(cm)	S(cm)	r _{mid} (cm)	r/D	z/D	z/h	r/r _{mid}
0	∞	∞	1106	6.7	6.7	0.2688	∞	0	0	∞
0.0941	2.267	15.189					2.267	0.014	8.50814E-05	56.506
0.1440	1.951	13.072					1.951	0.021	0.000130	48.630
0.2820	1.812	12.140					1.812	0.042	0.000255	45.165
0.4000	1.654	11.082					1.654	0.060	0.000362	41.227
0.4700	1.500	10.050					1.500	0.070	0.000425	37.388
0.6117	1.361	9.119					1.361	0.091	0.000553	33.924
0.8470	1.212	8.120					1.212	0.126	0.000766	30.210
0.9410	1.048	7.022					1.048	0.140	0.000851	26.122
1.3170	0.894	5.990					0.894	0.197	0.001191	22.283
1.8200	0.750	5.025					0.750	0.272	0.001646	18.694
2.9170	0.598	4.007					0.598	0.435	0.002637	14.906
4.0940	0.509	3.410					0.509	0.611	0.003702	12.687
5.3640	0.441	2.955					0.441	0.801	0.004850	10.992
6.7000	0.390	2.613					0.390	1	0.006058	9.721

Table 14A: Example for Determining the Profile of a `Vertical Air-Entraining Vortex

						Comment NAEV = not air-entraining vortex AEV = air-entraining vortex	
h (cm)	S (cm)	h/S	(z/h)_{BP}	α_{LBP}	(z/h)_{so}		
		0.000	1.0000	1.000	1.500	NAEV	vortex with no intake (or $h \ll S$ with an intake)
5.650	18.00	0.314	0.7190	0.438	1.219	NAEV	* Azarpira et al. (2021) with an intake
0.658	5.00	0.131	0.8225	0.645	1.322	NAEV	● Cristofano et al. (2016, Fig. 4b) with intake
2.065	5.00	0.412	0.6820	0.364	1.182	NAEV	■ Cristofano et al. (2016, Fig. 4c) with intake
60.000	150.00	0.400	0.6795	0.359	1.179	NAEV	+ Nicolai et al. (2019, $F = 0.7$) with an intake
119.40	150.00	0.796	0.5505	0.101	1.050	NAEV	◇ Nicolai et al. (2019, $F = 1$) with an intake
150.00	150.00	1.000	0.5000	0.000	1.000	NAEV	□ Nicolai et al. (2019, $F = 1.4$) with an intake
18.600	18.60	1.000	0.5000	0.000	1.000	NAEV	○ Sun and Liu (2015) with an intake (critical)
91.000	53.34	1.705	0.3800	- 0.240	0.880	AEV	× Anwar (1965, Fig. 10a) with an intake

Table 14B: Example for Determining the Profile of a Vertical Air-Entraining Vortex

h (cm)	S (cm)	h/S	$(z/h)_{BP}$	α_{LBP}	$(z/h)_{so}$	Comment	
						NAEV = not air-entraining vortex	AEV = air-entraining vortex
77	50.80	1.516	0.4250	- 0.150	0.925	AEV	■ Anwar (1965, Fig. 10b) with an intake
37.7	30.00	1.257	0.470	-0.060	0.970	AEV	☼ Anwar (1965, Fig. 10c) with an intake
298.00	17.069	17.45	0.0157	- 0.9685	0.5157	AEV	Δ Setevens and Kolf (1957, Fig. 1) with intake
105.00	23.50	4.468	0.1694	- 0.661	0.6694	AEV	◆ Mulligan et al. (2016, Fig. 10, A3) with intake
376.5	13.400	28.097	0.0037	- 0.9926	0.5037	AEV	▼ Mulligan et al. (2016, Fig. 10, A2) with intake
1106	6.700	165.075	0.0031	- 0.9938	0.5031	AEV	► Mulligan et al. (2016, Fig. 10, A1) with intake

Table 15: Comparison of Measured and Computed Test Results for (S = 18 cm, D = 3 cm, S/D = 6) to (Gupta et al., 2014) [79].

Test results (measured)							Computed Eq. (4.4 a)		Error (%)	Comment
Eleva H (cm)	z (cm)	z/D	r (cm)	r/D	$\bar{R} = r/r_{mid}$	z/h ($\times 10^{-2}$)	z/h($\times 10^{-2}$)	z (cm)		
16.20	1.80	0.600	3.50	1.166	4.310	2.702	3.423	2.280	26.660	?
15.50	2.50	0.833	3.20	1.066	3.941	3.753	4.093	2.726	9.050	Acceptable
15.00	3.00	1.000	3.00	1.000	3.695	4.504	4.656	3.101	3.354	Acceptable
14.30	3.70	1.233	2.70	0.900	3.325	5.555	5.742	3.824	3.361	Acceptable
13.30	4.70	1.566	2.40	0.800	2.956	7.057	7.256	4.832	2.816	Acceptable
12.00	6.00	2.000	2.20	0.733	2.709	9.009	8.619	5.741	4.324	Acceptable
11.00	7.00	2.333	2.10	0.700	2.586	10.510	9.448	6.292	10.107	Acceptable
0.0 Intake Level	18.00	6.000	1.30 read from Fig. 4a	0.433	1.601	27.027	26.581	17.703	1.65	Acceptable

Note: Computed parameters include h = 66.6 cm, $r_{mid} = 0.812$ cm, $(z/h)_{BP} = 0.24$, $\alpha_{LBP} = -0.58$, $(z/h)_{so} = 0.74$, $h/S = 3.7$

Table 16: Comparison of Measured and Computed Test Results for ($S = 17.069$ cm, $D = 4'' = 10.16$ cm, $S/D = 17.069/10.16 = 1.68$, Imposed Circulation $\Gamma \approx 0.396$ m²/s to (Stevens and Kolf 1957, Fig. 1) [77].

Test results (Measured)								(z/h) Computed			Error%	Comment
z(cm)	D(cm)	z/D	r(cm)	$\bar{R} = r/r_{mid}$	h(cm)	z/h	(z/h $\times 10^2$)	z/h	(z/h $\times 10^2$)	z		
0.000	10.16	0.0000	35.182	41.3177	298	0	0	0.000373	0.0373	0.111		
0.174		0.0171	28.677	33.6782	298	0.000583893	0.05839	0.000561	0.0561	0.167	3.87	Acceptable
0.266		0.0261	22.730	26.6940	298	0.000892819	0.08928	0.000893	0.0893	0.266	0.07	Acceptable
0.503		0.0495	16.530	19.4128	298	0.001688188	0.16882	0.001689	0.1689	0.503	0.06	Acceptable
1.655		0.1629	11.080	13.0123	298	0.005553691	0.55537	0.003760	0.3760	1.120	32.30	?
3.049		0.3001	7.469	8.7716	298	0.010231544	1.02315	0.008274	0.8274	2.466	19.14	Acceptable
4.877		0.4800	6.110	7.1756	298	0.016365772	1.63658	0.012378	1.2378	3.689	24.37	?
7.925		0.7800	4.550	5.3435	298	0.02659396	2.65940	0.022781	2.2781	6.789	14.34	Acceptable
9.319		0.9172	4.120	4.8385	298	0.031271812	3.12718	0.028046	2.8046	8.358	10.32	Acceptable
11.409		1.1229	3.692	4.3359	298	0.038285235	3.82852	0.035425	3.5425	10.557	7.47	Acceptable
13.063		1.2857	3.260	3.8285	298	0.04383557	4.38356	0.045937	4.5937	13.689	4.79	Acceptable
17.069 Intake Level		1.6800	2.960	3.4762	298	0.057278523	5.72785	0.057299	5.7299	17.075	0.04	Acceptable

Note: Computed parameters include $h = 298$ cm, $r_{mid} = 0.8515$, $(z/h)_{BP} = 0.01575$, $\alpha_{LBP} = -0.9685$, $(z/h)_{so} = 0.51575$, $h/S = 17.458$, and $H_o = -(298 - 17.069) = -280.931$ cm. A strong air-entraining vortex was observed.

Table 17A: Comparison of Measured and Computed Test Results for ($D = 4'' = 10.16$ cm, $Q = 0.74$ cfs = 0.02095 m³/s; $S = 21'' = 53.34$ cm; $h = 35.827'' = 91$ cm, $h/S = 91/53.34 = 1.706$, and at $h/2 = 35.827''/2 = 17.914'' = 45.5$ cm) to (Anwar 1965, Fig. 10a) [75].

Test results (Measured)								(z/h) Computed			Error%	Comment
z(cm)	D(cm)	z/D	r (cm)	$R = r/r_{mid}$	h(cm)	z/h	(z/h $\times 10^2$)	z/h	(z/h $\times 10^2$)	z		
0.747	10.16	0.0735	35.560	7.608	91	0.00821	0.821	0.01100	1.100	1.001	34.01	?
1.270		0.125	30.480	6.521		0.01396	1.396	0.01497	1.497	1.362	7.25	Acceptable
1.905		0.1875	25.997	5.562		0.02093	2.093	0.02057	2.057	1.872	1.73	Acceptable
2.540		0.25	24.014	5.138		0.02791	2.791	0.02410	2.410	2.193	13.65	Acceptable
3.810		0.375	19.873	4.252		0.04187	4.187	0.03518	3.518	3.201	15.98	Acceptable
5.080		0.5	17.780	3.804		0.05582	5.582	0.04392	4.392	3.997	21.32	?
6.350		0.625	15.837	3.388		0.06978	6.978	0.05531	5.531	5.033	20.73	?
7.620		0.75	14.351	3.070		0.08374	8.374	0.06728	6.728	6.122	19.66	?
10.160		1	12.250	2.621		0.11165	11.165	0.09203	9.203	8.374	17.57	Acceptable
12.700		1.25	10.930	2.338		0.13956	13.956	0.11515	11.515	10.479	17.49	Acceptable
15.240		1.5	10.005	2.141		0.16747	16.747	0.13679	13.679	12.448	18.32	Acceptable
17.780		1.75	8.928	1.910		0.19538	19.538	0.17030	17.030	15.497	12.84	Acceptable
20.320		2	8.313	1.779		0.22330	22.330	0.19491	19.491	17.736	12.71	Acceptable

Table 17B: Comparison of Measured and Computed Test Results for ($D = 4'' = 10.16$ cm, $Q = 0.74$ cfs = 0.02095 m³/s, $S = 21'' = 53.34$ cm, $h = 35.827'' = 91$ cm, $h/S = 91/53.34 = 1.706$, and at $h/2 = 35.827''/2 = 17.914'' = 45.5$ cm) to (Anwar 1965, Fig. 10a) [75].

Test results (Measured)								(z/h) Computed			Error%	Comment
z(cm)	D(cm)	z/D	r(cm)	$\bar{R} = r/r_{mid}$	h(cm)	z/h	z/h($\times 10^2$)	z/h	z/h($\times 10^2$)	z		
25.400	10.16	2.5	6.873	1.471	91	0.2791	27.91	0.276	27.6	25.09	1.2	Acceptable
29.972		2.95	6.312	1.350		0.3294	32.94	0.319	31.9	29.06	3.1	Acceptable
30.480		3	5.969	1.277		0.3349	33.49	0.350	35.0	31.87	4.5	Acceptable
33.020		3.25	5.850	1.252		0.3629	36.29	0.362	36.2	32.92	0.3	Acceptable
35.560		3.5	5.385	1.152		0.3908	39.08	0.418	41.8	38.01	6.9	Acceptable
38.100		3.75	5.232	1.119		0.4187	41.87	0.454	45.4	41.31	8.4	Acceptable
40.640		4	4.775	1.022		0.4466	44.66	0.532	53.2	48.39	19.1	Acceptable
43.180		4.25	4.770	1.021		0.4745	47.45	0.553	55.3	50.37	16.6	Acceptable
45.502		4.4785	4.674	1.000		0.5000	50.00	0.586	58.6	53.36	17.3	Acceptable
45.720		4.5	4.483	0.959		0.5024	50.24	0.613	61.3	55.82	22.1	?
48.260		4.75	4.310	0.922		0.5303	53.03	0.658	65.8	59.92	24.2	?
52.743		5.19125	3.736	0.799		0.5796	57.96	0.767	76.7	69.76	32.3	?

Note: Computed parameters include measured $r_{mid} = 1.84'' = 4.674$ cm, imposed circulation = $2\pi \times 264 \text{ in}^2 \text{ s}^{-1} = 1658 \text{ in}^2 \text{ s}^{-1} = 1.069 \text{ m}^2 \text{ s}^{-1}$, vertically flowing downward intake pipe projecting into tank for 7'', $h/S = 91/53.4 = 1.706$; $\alpha_{LBP} = -0.24$; $(z/h)_{BP} = 0.38$; $z_{BP} = 34.58$ cm; $(z/h)_{so} = 0.5 + (z/h)_{BP} = 0.5 + 0.38 = 0.88$; $z_{so} = 0.88 \times 91 = 80.08$ cm; $H_0 = -(h - S) = -(91 - 53.34) = -37.66$ cm A vertically flowing downward intake pipe projecting into the tank was used, and an air-entraining vortex was observed

Table 18A: Comparison of Measured and Computed Test Results for (D = 2.60" = 6.604 cm, Q = 0.45 cfs = 12.74 L/s, S = 20" = 50.8 cm , h=77 cm) to (Anwar 1965, Fig. 10b) [75].

Test results (Measured)								(z/h) Computed			Error%	Comment
z(cm)	D(cm)	z/D	r(cm)	$\bar{R} = r/r_{mid}$	h(cm)	z/h	z/h($\times 10^2$)	z/h	z/h($\times 10^2$)	z		
0.150	6.604	0.023	33.020	9.208	77	0.00195	0.195	0.0075	0.75	0.578	285.8	?
0.300		0.045	30.480	8.500		0.00389	0.389	0.0088	0.88	0.678	126.4	?
0.747		0.113	25.400	7.083		0.00970	0.970	0.0127	1.27	0.977	30.8	?
1.494		0.226	20.320	5.666		0.01940	1.940	0.0198	1.98	1.526	2.2	Acceptable
2.987		0.452	14.940	4.166		0.03879	3.879	0.0366	3.66	2.821	5.6	Acceptable
5.080		0.769	11.803	3.292		0.06597	6.597	0.0586	5.86	4.512	11.2	Acceptable
7.767		1.176	9.263	2.583		0.10087	10.087	0.0947	9.47	7.292	6.1	Acceptable
9.561		1.448	8.217	2.291		0.12416	12.416	0.1198	11.98	9.226	3.5	Acceptable
13.597		2.059	6.723	1.875		0.17658	17.658	0.1764	17.64	13.586	0.1	Acceptable
17.480		2.647	5.977	1.667		0.22702	22.702	0.2200	22.00	16.939	3.1	Acceptable

Table 18B: Comparison of Measured and Computed Test Results for ($D = 2.60'' = 6.604$ cm, $Q = 0.45$ cfs = 12.74 L/s, $S = 20'' = 50.8$ cm) to (Anwar 1965, Fig. 10b) [75].

Test results (Measured)								(z/h) Computed			Error%	Comment
z(cm)	D(cm)	z/D	r(cm)	$\bar{R} = r/r_{mid}$	h(cm)	z/h	z/h($\times 10^2$)	z/h	z/h($\times 10^2$)	z		
23.157	6.604	3.507	5.080	1.417	77	0.30074	30.074	0.2943	29.43	22.661	2.1	Acceptable
28.537		4.321	4.483	1.250		0.37061	37.061	0.3624	36.24	27.902	2.2	Acceptable
36.307		5.498	3.734	1.041		0.47152	47.152	0.5054	50.54	38.913	7.2	Acceptable
42.431		6.425	3.287	0.917		0.55105	55.105	0.6429	64.29	49.502	16.7	Acceptable
48.707		7.375	2.987	0.833		0.63256	63.256	0.7546	75.46	58.102	19.3	Acceptable

Note: Computed parameters include, $h = 77$ cm, $r_{mid} = 1.412'' = 3.586$ cm, imposed circulation $2\pi \times 192 \text{ in}^2/\text{s} = 0.7779 \text{ m}^2/\text{s}$, $h/S = 1.516$, $\alpha_{LBP} = -0.15$, $(z/h)_{BP} = 0.425$, $z_{BP} = 32.725$ cm, $(z/h)_{so} = 0.5 + (z/h)_{BP} = 0.925$, $z_{so} = 71.225$ cm, and $H_0 = -(h - S) = -(77 - 50.8) = -26.2$ cm. A vertically flowing downward intake pipe leveled with the bottom of the tank was used, and an air-entraining vortex was observed.

Table 19A: Comparison of Measured and Computed Test Results for ($D = 0.875'' = 2.22$ cm, $Q = 0.0095$ cfs, $S = (12'' - 0.189'') = 11.81'' = 30$ cm, $h = 37.7$ cm, and at $h/2 = 37.7/2 = 18.85$ cm, $h/S = 1.257$) to (Anwar 1965, Fig. 10c) [75].

Test results (Measured)								(z/h) Computed			Error%	Comment
z(cm)	D(cm)	z/D	r(cm)	$\bar{R} = r/r_{mid}$	h(cm)	z/h	z/h($\times 10^2$)	z/h	z/h($\times 10^2$)	z		
0	2.22	0	20.320	15.752	37.7	0	0	0.0026	0.26	0.10		
0.260		0.117	10.363	8.033		0.0069	0.69	0.0099	0.99	0.37	42.8	?
0.651		0.293	7.826	6.066		0.0173	1.73	0.0173	1.73	0.65	0.1	Acceptable
0.912		0.410	6.520	5.054		0.0242	2.42	0.0249	2.49	0.94	3.0	Acceptable
1.739		0.783	5.217	4.044		0.0461	4.61	0.0389	3.89	1.47	15.7	Acceptable
2.083		0.937	4.529	3.511		0.0552	5.52	0.0515	5.15	1.94	6.7	Acceptable
2.670		1.201	3.861	2.993		0.0708	7.1	0.0708	7.08	2.67	0.0	Acceptable
3.582		1.612	3.363	2.607		0.0950	9.50	0.0930	9.30	3.51	2.1	Acceptable

Table 19B: Comparison of Measured and Computed Test Results for ($D = 0.875'' = 2.22$ cm, $Q = 0.0095$ cfs, $S = (12'' - 0.189'') = 11.81'' = 30$ cm, $h = 37.7$ cm, and at $h/2 = 37.7/2 = 18.85$ cm, $h/S = 1.257$) to (Anwar 1965, Fig. 10c) [75].

Test results (Measured)								(z/h) Computed			Error%	Comment
z(cm)	D(cm)	z/D	r(cm)	$\bar{R} = r/r_{mid}$	h(cm)	z/h	z/h($\times 10^2$)	z/h	z/h($\times 10^2$)	z		
4.884	2.22	2.198	2.794	2.166	37.7	0.1296	12.96	0.1337	13.37	5.04	3.2	Acceptable
7.554		3.399	1.981	1.536		0.2004	20.04	0.2553	25.53	9.62	27.4	?
9.639		4.337	1.676	1.300		0.2557	25.57	0.3403	34.03	12.83	33.1	?
13.547		6.095	1.346	1.044		0.3593	35.93	0.4729	47.29	17.83	31.6	?
20.320		9.143	1.029	0.797		0.5390	53.90	0.6808	68.08	25.67	26.3	?
24.748		11.135	0.961	0.744		0.6564	65.64	0.7871	78.71	29.68	19.9	Acceptable
27.092		12.190	0.892	0.691		0.7186	71.86	0.8500	85.00	32.04	18.3	Acceptable

Note: Computed parameters include $D = 2.22$ cm, $Q = 0.0095$ cfs, $S = 30$ cm, $h = 37.7$ cm, $r_{mid} = 1.29$ cm, imposed circulation = $2\pi \times 53.4$ in²/s = 0.216 m²/s, $h/S = 1.257$, $\alpha_{LBP} = -0.06$, $(z/h)_{BP} = 0.47$, $z_{BP} = 17.719$ cm, $(z/h)_{so} = 0.5 + (z/h)_{BP} = 0.5 + 0.47 = 0.97$, $z_{so} = 36.569$ cm, and $H_0 = -(h - S) = -(37.7 - 30) = -7.7$ cm. A vertically flowing downward intake pipe projecting into the tank was used, and an air-entraining vortex was observed.

Table 20A: Comparison of Measured and Computed Test Results for (Small Submergence $S = 6.7$ cm, $D = 6.7$ cm, $S/D = 1$) to (Mulligan et al. 2016) [76].

Test results (Measured)								(z/h) Computed			Error %	Comment
z(cm)	D(cm)	z/D	r(cm)	$\bar{R} = r/r_{mid}$	h(cm)	z/h	z/h($\times 10^2$)	z/h	z/h($\times 10^2$)	z		
0.0941	6.7	0.0140	15.189	56.51	1106	8.5E-05	0.0085	0.00020	0.020	0.2205	134.34	?
0.1440		0.0215	13.070	48.62		1.3E-04	0.0130	0.00027	0.027	0.2978	106.82	?
0.2820		0.0421	12.141	45.17		2.5E-04	0.0255	0.00031	0.031	0.3451	22.39	?
0.4000		0.0597	11.080	41.22		3.6E-04	0.0362	0.00037	0.037	0.4144	3.60	Acceptable
0.4700		0.0701	10.050	37.39		4.2E-04	0.0425	0.00046	0.046	0.5037	7.17	Acceptable
0.6117		0.0913	9.120	33.93		5.5E-04	0.0553	0.00055	0.055	0.6117	0.01	Acceptable
0.8470		0.1264	8.123	30.22		7.7E-04	0.0766	0.00070	0.070	0.7710	8.97	Acceptable
0.9410		0.1404	7.100	26.41		8.5E-04	0.0851	0.00091	0.091	1.0092	7.25	Acceptable
1.3170		0.1966	6.023	22.41		1.2E-03	0.1191	0.00127	0.127	1.4024	6.48	Acceptable

Table 20B: Comparison of Measured and Computed Test Results for (Small Submergence $S = 6.7$ cm, $D = 6.7$ cm, $S/D = 1$) to (Mulligan et al. 2016) [76].

Test results (Measured)								(z/h) Computed			Error%	Comment
z(cm)	D(cm)	z/D	r(cm)	$\bar{R} = r/r_{mid}$	h(cm)	z/h	z/h($\times 10^2$)	z/h	z/h($\times 10^2$)	z		
1.8200	6.7	0.2716	5.060	18.82	1106	0.0016	0.16	0.00180	0.180	1.9870	9.17	Acceptable
2.9170		0.4354	4.060	15.10		0.0026	0.26	0.00279	0.279	3.0863	5.80	Acceptable
4.0940		0.6110	3.419	12.72		0.0037	0.37	0.00394	0.394	4.3573	6.43	Acceptable
5.3640		0.8006	2.960	11.01		0.0048	0.48	0.00527	0.527	5.8267	8.63	Acceptable
6.7000		1	2.613	9.72		0.0061	0.61	0.00678	0.678	7.4951	11.87	Acceptable

Note: Computed parameters include $h = 1106$ cm, $r_{mid} = 0.2688$ cm, $h/S \approx 165.075$, $\alpha_{LBP} = -0.9938$, $(z/h)_{BP} = 0.0031$, $z_{BP} = 3.4286$, $(z/h)_{so} = 0.5031$, $z_{so} = 556.43$ cm, and $H_o = -(h - S) = -(1106 - 6.7) = -1099.3$ cm. A strong air-entraining vortex with very strong imposed circulation was observed.

Table 21A: Comparison of Measured and Computed Test Results for (Medium Submergence $S = 13.4$ cm, $D = 6.7$ cm, $S/D = 2$) to (Mulligan et al. 2016, Fig. 10, A2) [76].

Test results (Measured)								(z/h) Computed			Error%	Comment
z(cm)	D(cm)	z/D	r(cm)	$\bar{R} = r/r_{mid}$	h(cm)	z/h	z/h($\times 10^2$)	z/h	z/h($\times 10^2$)	z		
0.192	6.7	0.0287	16.093	28.763	376.5	0.00051	0.051	0.00077	0.077	0.290	50.9	?
0.288		0.0430	14.270	25.505		0.00076	0.076	0.00098	0.098	0.368	27.9	?
0.432		0.0645	13.179	23.555		0.00115	0.115	0.00115	0.115	0.432	0.0	Acceptable
0.577		0.0861	12.194	21.794		0.00153	0.153	0.00134	0.134	0.505	12.5	Acceptable
0.721		0.1076	10.084	18.023		0.00192	0.192	0.00196	0.196	0.738	2.3	Acceptable
0.769		0.1148	8.154	14.574		0.00204	0.204	0.00300	0.300	1.128	46.7	?
1.062		0.1585	7.064	12.626		0.00282	0.282	0.00399	0.399	1.504	41.6	?
1.643		0.2452	6.117	10.933		0.00436	0.436	0.00533	0.53330	2.008	22.2	?

Table 21B: Comparison of Measured and Computed Test Results for (Medium Submergence $S = 13.4$ cm, $D = 6.7$ cm, $S/D = 2$) to(Mulligan et al. 2016, Fig. 10, A2) [76].

Test results (Measured)								(z/h) Computed			Error%	Comment
z(cm)	D(cm)	z/D	r(cm)	$\bar{R} = r/r_{mid}$	h(cm)	z/h	z/h($\times 10^2$)	z/h	z/h($\times 10^2$)	z		
2.319	6.7	0.3461	5.134	9.1761	376.5	0.00616	0.616	0.00760	0.760	2.861	23.4	?
3.125		0.4664	4.663	8.3342		0.00830	0.830	0.00925	0.925	3.483	11.4	Acceptable
4.086		0.6099	4.042	7.2243		0.01085	1.085	0.01237	1.237	4.659	14.0	Acceptable
5.192		0.7749	3.564	6.3700		0.01379	1.379	0.01601	1.601	6.027	16.1	Acceptable
7.115		1.0619	3.055	5.4602		0.01890	1.890	0.02201	2.201	8.288	16.5	Acceptable
8.557		1.2772	2.804	5.0116		0.02273	2.273	0.02633	2.633	9.915	15.9	Acceptable
13.4		2	2.398	4.2860		0.03559	3.559	0.03698	3.698	13.921	3.9	Acceptable

Note: Computed parameters include $h = 376.5$ cm, $r_{mid} = 0.5595$ cm, $h/S = 28.097$, $\alpha_{LBP} = -0.9926$, $(z/h)_{BP} = 0.0037$, $z_{BP} = 1.39305$ cm, $(z/h)_{s0} = 0.5037$, $z_{s0} = 0.5 + (z/h)_{BP} = 189.64$ cm, and $H_0 = -(h-S) = (376.5 - 13.4) = -363.1$ cm. A strong air-entraining vortex with very strong imposed circulation was observed

Table 22A: Comparison of Measured and Computed Test Results for (Large Submergence $S = 23.5$ cm, $D = 6.7$ cm, $S/D = 3.5$) to (Mulligan et al. 2016, Fig. 10, A3) [76].

Test results (Measured)								(z/h) Computed			Error%	Comment
z(cm)	D(cm)	z/D	r(cm)	$\bar{R} = r/r_{mid}$	h(cm)	z/h	z/h($\times 10^2$)	z/h	z/h($\times 10^2$)	z		
0.441	6.7	0.066	14.009	14.01	105	0.0042	0.42	0.00324	0.324	0.341	22.8	?
0.441		0.066	13.018	13.02		0.0042	0.42	0.00376	0.376	0.394	10.6	Acceptable
0.664		0.099	11.115	11.12		0.0063	0.63	0.00515	0.515	0.541	18.5	Acceptable
0.808		0.121	9.132	9.13		0.0077	0.77	0.00763	0.763	0.802	0.8	Acceptable
1.324		0.198	7.118	7.12		0.0126	1.26	0.01256	1.256	1.319	0.4	Acceptable
1.764		0.263	6.124	6.12		0.0168	1.68	0.01697	1.697	1.782	1.0	Acceptable
2.353		0.351	5.172	5.17		0.0224	2.24	0.02379	2.379	2.498	6.2	Acceptable

Table 22B: Comparison of Measured and Computed Test Results for (Large Submergence $S = 23.5$ cm, $D = 6.7$ cm, $S/D = 3.5$) to(Mulligan et al. 2016, Fig. 10, A3) [76].

Test results (Measured)								(z/h) Computed			Error%	Comment
z(cm)	D(cm)	z/D	r(cm)	$\bar{R} = r/r_{mid}$	h(cm)	z/h	z/h($\times 10^2$)	z/h	z/h($\times 10^2$)	z		
4.117	6.7	0.614	4.033	4.03	105	0.0392	3.92	0.03909	3.909	4.105	0.3	Acceptable
5.44		0.812	3.578	3.58		0.0518	5.18	0.04963	4.963	5.211	4.2	Acceptable
7.279		1.086	3.122	3.12		0.0693	6.93	0.06509	6.509	6.834	6.1	Acceptable
13.088		1.953	2.132	2.13		0.1246	12.46	0.13786	13.786	14.475	10.6	Acceptable
23.5		3.507	1.749	1.75		0.2238	22.38	0.22383	22.383	23.503	0.0	Acceptable

Note: Computed parameters $h = 105$ cm , $r_{mid} = 1$ cm, $h/S = 4.468$, $\alpha_{LBP} = -0.66125$, $(z/h)_{BP} = 0.1694$, $z_{BP} = 17.787$ cm, $(z/h)_{so} = 0.5 + (z/h)_{BP} = 0.5 + 0.1694 = 0.6694$, $z_{so} = 70.287$ cm, and $H_O = - (h - S) = - (105 - 23.5) = - 81.5$ cm (height of the imaginary section of the vortex). A strong air-entraining vortex with very strong imposed circulation was observed.