

**ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL OF SCIENCE**  
**ENGINEERING AND TECHNOLOGY**

**EXPERIMENTS FOR DESIGN AND OPTIMIZATION OF THIN SHELL  
STRUCTURES**

**M.Sc. THESIS**

**Erenalp SALTIK**

**Department of Informatics**

**Architectural Design Computing Programme**

**JUNE 2018**



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**İSTANBUL TEKNİK ÜNİVERSİTESİ ★ FEN BİLİMLERİ ENSTİTÜSÜ**

**İNCE KABUK STRÜKTÜRLERİN TASARIMI VE OPTİMİZASYONU  
ÜZERİNE DENEYLER**



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*To my beloved family,*



## **FOREWORD**

Firstly, I would like to thank my thesis advisor Sema ALAÇAM for endless assistance and guidance.

Then, I owe a special gratitude to my mother Gülsen SALTIK, and my father Tanzer SALTIK. I couldn't become what I am without their endless support. Also, I am very grateful to my sister Tulça SALTIK for emotional support throughout the years.



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## ABBREVIATIONS

<b>CAD</b>	: Computer Aided Design
<b>CAM</b>	: Computer Aided Manufacturing
<b>DR</b>	: Dynamic Relaxation
<b>DOF</b>	: Degree of Freedom
<b>FDM</b>	: Force Density Method
<b>FEA</b>	: Finite Element Analysis
<b>IASS</b>	: International Association for Shell Structures
<b>NURBS</b>	: Non Uniform Rational B-spline
<b>PS</b>	: Particle-Spring System
<b>TNA</b>	: Thrust Network Analysis



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## **EXPERIMENTS FOR DESIGN AND OPTIMIZATION OF THIN SHELL STRUCTURES**

### **SUMMARY**

Shell structures have been a focus of interest for many years by architects and engineers because of aesthetic concerns and their ability to cover large spans. Finding the ideal form for shell structures can be traced back to Robert Hooke, who has examined the ideal arch form by gravity-impacting chains. Inspired by Hooke's work and Catalan vaults, Gaudi has carried out experiments on finding ideal forms with chains in three dimensions.

With the development of the reinforced concrete, the Spanish architect Felix Candela was able to reveal new and revolutionary shell experiments. Candela, who usually used hyperbolic paraboloid in shells, has reached to a thickness of 4 cm.

Heinz Isler, who also saw and was impressed by Candela's work, further advanced form finding experiments. In order to reach the ideal form, the fabric which is covered with plastic is hanged from the support points so that they can be shaped by the force of gravity. Later on, these forms are reversed and large openings passed through the forms, which only work with compression and carry their own weight.

Another pioneer in the area is the Frei Otto. Otto has also done gravitational form finding experiments using cable networks. In some experiments he used soap bubbles to reach minimal surfaces. He has worked on suspension systems and wood grid shells in the following years.

Developments in the aviation and car manufacturing sector have facilitated mathematical identification of the forms and their transfer to the computer environment. Computer-assisted form finding methods have progressed, parallel to this situation. These methods are also described in the scope of the study. These methods are described as Force Density Method (FDM), Thrust Network Analysis (TNA), Dynamic Relaxation (DR) and Particle-Spring Method (PS). FDM is a form-finding method that Frei Otto and his team use more widely. It is more suitable for membrane structures. TNA is a form finding method more suitable for masonry and concrete shells developed by Block Research Group. This method is also independent of the material and it allows to see and intervene the process because it is a geometric method. DR, on the other hand, is a form finding method used mostly in the design of wooden gridshell structures. In this method, connection points are very important and physical modeling as well as complex calculations must be done in order to be successful. PS is a computerized version of the chain method developed by Hooke and Gaudi. It aims to find an equilibrium between the elements by dividing them into nodes and springs. It is a convenient way to produce quick form alternatives.

Gaussian curvature is an important criterion that determines the production and durability of surfaces. It is determined by the state of the main curves over the surfaces. If the curvatures of these curves are in the same direction they are positive, if in different direction they are negative. If one of the curves is flat, the Gaussian curvature is 0 and these surfaces are single curved surfaces. If this curvature is different from zero, it is a double curved surface. If surfaces can be formed with straight lines, they become ruled surfaces. All single curved surfaces are such surfaces. Surfaces such as hyperbolic paraboloid are also such surfaces from double curvature surfaces, and architects have always been interested in them, because the ease of production of these types of ruled surfaces. There are also free-form surfaces defined by NURBS curves that require special production, and special optimization is required for their production.

Within the scope of the study, three shell structures were designed by different base surfaces, and after they were analyzed, openings were extracted on them and weights were reduced and form alternatives were formed. The resulting forms were analyzed again and the structures were stayed in the structural safe zone. Thus, a process for the formation of shell structures has been proposed.

## İNCE KABUK STRÜKTÜRLERİN TASARIMI VE OPTİMİZASYONU ÜZERİNE DENEYLER

### ÖZET

Kabuk strüktürler estetik kaygılar ve büyük açıklıklıkları gecebilmeleri sebebiyle mimarlar ve mühendisler tarafından uzun yillardır ilgi odağı olmuşlardır. Bu strüktürler için ideal formu bulma çalışmaları zincirleri asıp yerçekimi etkisiyle ideal kemer formunu bulmaya çalışan Robert Hooke'a kadar götürülebilir. Hooke'un çalışmalarını ve Katalan tonozlarını gören Gaudi, zincirlerle ideal form bulma deneylerini üç boyuta taşımıştır.

Betonarmenin gelişmesiyle birlikte İspanyol mimar Felix Candela yeni ve devrimci kabuk denemeleri ortaya koyabilmiştir. Kabuklarda genelde hiperbolik paraboloid kullanan Candela, 4 cm'lik kalınlığa kadar düşebilmiştir.

Candela'nın çalışmalarını da gören ve bundan da etkilenen İsviçreli Heinz Isler, form bulma deneylerini daha da ileriye taşımıştır. İdeal forma ulaşmak için plastikle kapladığı kumaşları destek noktalarından asarak yerçekimi kuvvetiyle şekillenmelerini sağlamıştır. Daha sonra bu formları ters çevirerek sadece basınç kuvvetiyle çalışan ve kendi ağırlığını taşıyan formlarla büyük açıklıklar geçmeyi başarmıştır.

Alandaki bir diğer öncü ise Alman Frei Otto'dur. Otto da yerçekimi ile form bulma işlemini kablo ağlar kullanarak yapmıştır. Bazı deneylerinde minimal yüzeylere ulaşmak için sabun köpüklerini kullanmıştır. Asma germe sistemler ve ilerleyen yıllarda ahşap grid kabuklar üzerinde çalışmıştır.

Havacılık ve araba üretim sektöründeki gelişmeler formların matematiksel olarak tanımlanmasını ve bunların bilgisayar ortamına aktarılmasını kolaylaştırmıştır. Bununla birlikte bilgisayar destekli form bulma yöntemleri de ilerlemiştir. Çalışma kapsamında bu metodlar da anlatılmıştır. Bu metodların çalışma kapsamında anlatılanları Force Density Method(FDM), Thrust Network Analysis (TNA), Dynamic Relaxation (DR) ve Particle-Spring Method (PS) olarak tanımlanmıştır. FDM daha çok Frei Otto ve ekibinin kullandığı, malzemeden bağımsız olan bir form bulma yöntemidir. Membran strüktürler için daha uygundur. TNA Block Research Group tarafından geliştirilen yığma ve beton kabuklar için daha uygun olan bir form bulma yöntemidir. Bu yöntem de malzemeden bağımsızdır ve geometrik bir yöntem olduğu için süreci görmeye ve müdahale etmeye olanak sağlar. DR ise daha çok ahşap gridshell strüktürlerin tasarımında kullanılan bir form bulma yöntemidir. Bu yöntemde bağlantı noktaları çok önem taşır ve başarılı olabilmesi için karmaşık hesapların yanında fiziksel modellemenin de yapılması gereklidir. PS ise Hooke ve Gaudi'nin geliştirdiği asılmış zincir yönteminin bilgisayar ortamına aktarılmış halidir. Formu oluşturan elemanları noktalar ve yaylara bölerek bunlar arasında bir denge kurmayı amaçlar. Hızlı form alternatifleri üretmek için uygun bir yöntemdir.

Gauss eğriliği yüzeylerin üretimi ve dayanıklılığını belirleyen önemli bir kriterdir. Yüzeylerin üzerindeki ana eğrilerin durumuna göre belirlenir. Eğer bu eğrilerin

egrilikleri aynı yönde ise pozitif, farklı yönde ise negatif Gauss eğriliği olur. Eğer eğrilerden birisi düz ise Gauss eğriliği 0 olur ve bu yüzeyler tek eğrilikli yüzeylerdir. Eğer bu eğrilik sıfırdan farklı ise çift eğrilikli yüzeylerdir. Eğer yüzeyler düz çizgilerle oluşturulabiliyorsa ruled yüzeyler olurlar. Tek eğrilikli yüzeylerin hepsi böyle yüzeylerdir. Çift eğrilikli yüzeylerden de hiperbolik paraboloid gibi yüzeyler bu tür yüzeylerdir ve üretim kolaylığı açısından mimarların her zaman ilgisini çekmiştir. Bir de özel üretim gerektiren NURBS eğrileriyle tanımlanan serbest formlu yüzeyler vardır ve bunların üretimi için özel bir optimizasyon gereklidir.

Çalışmanın deneyler kısmında ilk olarak kabuk strüktürün form üretimi üzerine denemeler yapılmıştır. 20x20 metrelük bir alanı kaplayan üç farklı kabuk tasarlanmıştır. Formların tasarımında TNA metodu ve RhinoVault eklentisi kullanılmıştır. Bu metodun önemi malzemeden bağımsız bir form bulma yöntemi olması ve süreci görsel olarak kontrol etmeye olanak sağlamasıdır. Rastgele bir yüzey oluşturup onu optimize etmeye çalışmak yerine bu metod kullanılarak tasarımın erken aşamalarında kabuk strüktür gerekliliklerine uygun bir form tasarlamak amaçlanmıştır. Öncelikle kabukları oluşturmayı sağlayacak iki boyutlu temel yüzeyler tasarlanmıştır. Bu yüzeyler farklı sayıda destek noktaları ve açıklıklar içermektedir. Bu yüzeylerin oluşturulma biçimleri hesaplama zamanlarını ve üç boyutta dengeye ulaşılmasını etkilemektedir. Bu yüzeylerden form diyagramları oluşturulmuştur. Form diyagramlarından da kuvvet diyagramları oluşturulmuştur. Bu metod geometrik bir form bulma yöntemi olduğu için iki diyagram birbirleriyle etkileşim halindedir. Kuvvet diyagramları oluşturduğu zaman açıklıklar, destek noktaları ve çok yük binen kısımlar okunabilmektedir. Daha sonraki aşamalarda dengeye ulaşmayı kolaylaştırmak için kuvvet diyagramındaki sorunlu noktalar bulunup müdahale edilmelidir. Bu müdahaleler yapıldıktan sonra yatay denge bulunmaya çalışılır. Bu sırada kabuk formu iki boyutta olumaya başlar. Eğer dengeye ulaşamıyorsa form veya kuvvet diyagramında rahatlatma yapılması gereklidir, ve dengeye ulaşana kadar bu süreç tekrarlanır. Dengeye ulaşmak için eklenti yüze bağlı olan yer değiştirmeyi minimumda tutmaya çalışır. Eğer yatay dengeye ulaşıldıysa düşey dengeye de ulaşılabilir. Bu durumda üç boyutlu kabuk form şekillenmiş olur. Bu formun üzerinde malzemeden bağımsız olarak yüze bağlı yer değiştirme renklerle gösterilebilir. Bu renklendirme bize bir ön bilgi verebilir fakat daha detaylı strüktürel analiz için formlar Karamba eklentisine aktarılmıştır.

Karamba bir Sonlu Elemanlar Analizi eklentisidir. Mühendislikte kullanılan daha detaylı programlar olmasına rağmen mimarların iş akışına ve bilgi düzeyine uygun bir eklentidir. Üretilen formlar bu eklentide strüktürel modele çevrilmiştir. Analizin yapılabilmesi için bazı kısıtlamalar belirlenmiştir. Öncelikle destek noktaları tanımlanmış ve bunların uzayda hareket edebileceği düzlemler seçilmiştir. Daha sonra kabuğa etkiyen yük olarak yerçekimi seçilmiştir. Kabuk strüktürlerde ana yük strüktürün kendi ağırlığı dolayısıyla yerçekimidir. Yapının kesiti 5 cm kalınlığında kabuk strüktür olarak seçilmiş, malzeme olarak C25/C30 beton kullanılmıştır. Analiz sonuçlarını doğru değerlendirebilmek adına standart renk skalası eklenmiştir. Sonuçlarda bir çok veri elde edilmesine rağmen bu çalışma kapsamında yüze bağlı deformasyona odaklanılmıştır. Bu deformasyon görselleştirilmiş ve yük dolayısıyla çok yer değiştiren bölgeler gözlemlenmiştir. Bu bölgeler çalışmanın bir sonraki aşamasına temel oluşturmuştur. Ayrıca kuvvet akış çizgileri gösterilerek kabuk strüktürün yükü ne şekilde destek noktalarına ettiği de gözlemlenebilmiştir.

Çalışmanın son kısmında strüktürün ağırlığını azaltmak için kabuk yüzeyinde eksiltmeler yapılmıştır. Bu eksiltmeler iki farklı biçimde uygulanmıştır. Birinci

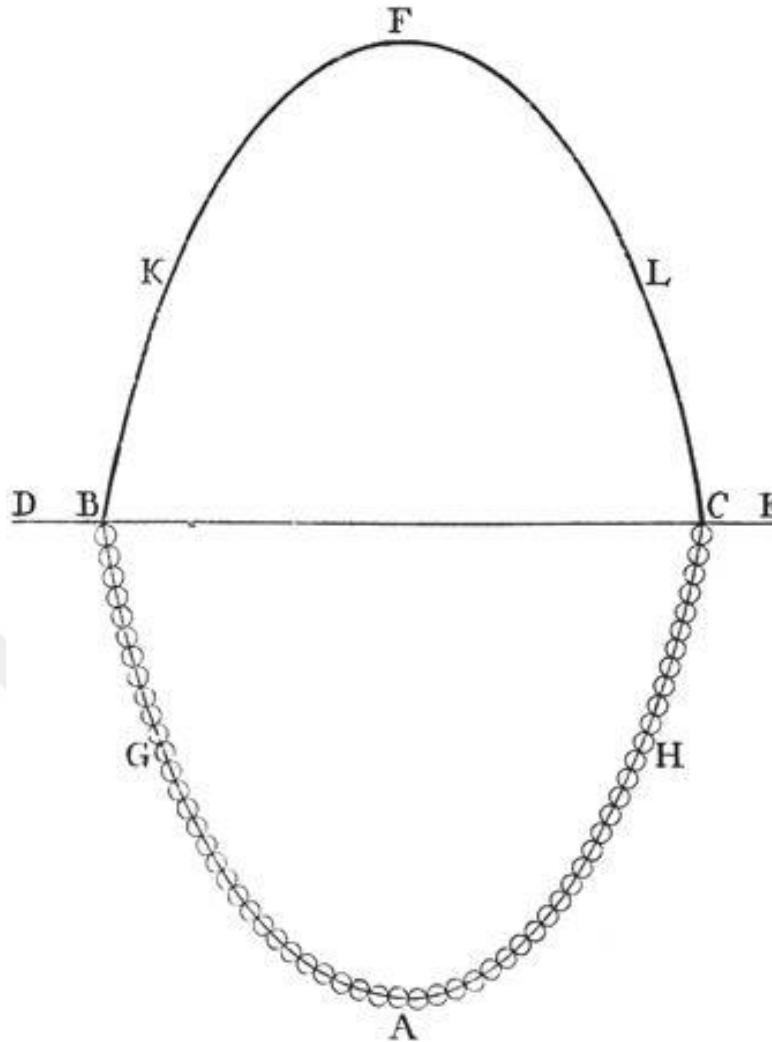
yöntemde boşluklu bir yüzey olan tafoni imajı temel yüzey üzerine oturtulmuştur. Karamba'da yapılan analizde az deformasyona uğrayan bölgeler seçilip tafoni imajındaki parlaklık değerlerine göre çapı belirlenen çemberler oluşturulmuştur. Daha sonra bu çemberler silindir haline getirilerek kabuk yüzeyinden çıkartılmıştır. İkinci yöntemde ise yine aynı süreç kullanılmış fakat bu sefer çok deformasyona uğrayan bölgeler temel alınarak o bölgelerde yoğunlaşan bir Voronoi strüktürü oluşturulmuştur. Böylece çok deformasyona uğrayan noktalarda daha yoğun, az deformasyona uğrayan noktalarda daha boşluklu bir yapı ortaya çıkmıştır. Bu iki kabuk da tekrar Karamba'da analiz edilmiş ve deformasyon değerlerinin çok fazla değişmediği görülmüştür. Fakat boşlukların kenarlarında stres yükü artmıştır. Ayrıca kuvvet akışı çizgilerinin de buna göre değiştiği gözlemlenmiştir.

Bu çalışma kapsamında tasarımin erken aşamalarında strüktürel kriterleri göz önüne almanın tasarımin ileriki aşamalarında kolaylık sağladığı görülmüştür. Rastgele formlar oluşturup onları bir kabuk strüktür haline getirmeye çalışmak yerine bu tür bir süreç izlenmesinin tasarıma etkileri gözlemlenebilmiştir. Ayrıca kabuk tasarıımı için bir süreç önerilerek bu süreç kullanılarak sonsuz sayıda varyasyon üretilebilmesinin mümkün olduğu gösterilmiştir.



## 1. INTRODUCTION

Covering large spans has always been a source of interest for architects and engineers. The main problem when covering these spans was that transferring the load of the shell to the supports in order to avoid collapse. This covering process, which started with the dome in the first eras, later evolved arches and vaults. Besides, vaults with more complicated geometry began to emerge in the Gothic era. Over time mathematicians and physicists began to gain interest of ideal form and structural performance. In such structures, the main goal is to achieve minimum bending by compression-only loads. In the late 17th century, Robert Hooke published his work on finding the ideal form (Figure 1.1). According to him, a hanging chain would be free of bending and pure tension. When we turn this chain upside down, we will get free of bending and pure compression arch. This idea has been influential on the emergence of funicular structures. Funicular structures are described as tension-only or compression-only structures. Parallel to these developments, masonry domes and vaults craftsmanship continued to evolve. These masonry vaults were also forming a thrust network similar to hanging chain. Gaudi, aware of the developments in mathematics and Catalan masonry, carried the hanging chain method to the third dimension (Burry&Burry, 2010). He formed the vaults with the loads he put on the chain network he had created. The emergence of non-Euclidian geometry and the development of concrete became a giant leap in design of shell structures. Architects began to be able to mathematically define the complex forms they designed. The hyperbolic paraboloid and its derivatives has been utilized by Felix Candela to achieve more complex forms. Heinz Isler used hanging fabrics in his own experiments. Frei Otto even made experiments with soap bubbles, contributing to the development of shell structures. Computers led calculation of complex forms, therefore definition and production of free form shells became possible. Shell structures are still an important issue and offer the designer almost endless variations. Effective shells can be achieved with proper form selection and analysis in early stage.



**Figure 1.1 :** Hooke's chain (Poleni, 1748).

## 1.1 Purpose of Thesis

This research aims to design different shell structures using form finding methods with the help of various computational tools in the earlier phases of design process. Further to this, optimizing shell structures for more lightweight structures by extracting some parts of them with different approaches.

### 1.1.1 Scope

The scope of this thesis includes basic geometric definitions describing the shells in first chapter. Then, different applications in architecture are explained with chosen architects and buildings in chapter two. After that the methods for form finding been introduced. In the last chapter different experiments are made for designing three shells for understanding the thin shell design. After that different extractions has been made based on displacement.

### **1.1.2 Motivation**

The motivation behind this study is to broaden the knowledge of mathematics behind the architecture, and exploring the new ways to produce forms and observe the effects of different structural criteria in early design stage while designing it.





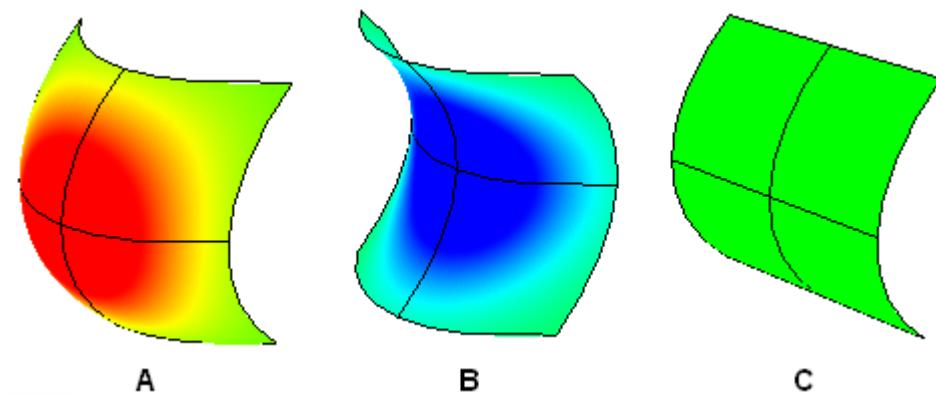
## 2. CURVATURE OF SURFACES

Curve curvature is described by deviation of curve from a straight line. In a surface, there are two principal curvatures, named as  $k_1$  and  $k_2$ . Mean curvature and Gaussian curvature are calculated by these principal curvatures.

Mean curvature is the arithmetic mean of  $k_1$  and  $k_2$  ( $K=(k_1+k_2)/2$ ). If the mean curvature is 0, the surface is called minimal surface. The pressure to a surface related to mean curvature, so structural performance and mean curvature is related (Shelden, 2002).

Gaussian curvature is the multiplication of principal curvatures ( $K=k_1.k_2$ ). According to Pottmann(2007), there are four conditions according to this calculation (Figure 2.1):

1. If  $k_1.k_2$  is positive, the point is elliptic point.
2. If  $k_1.k_2$  is negative, the point is hyperbolic point.
3. If  $k_1.k_2$  is 0, and one of them is not equal to 0, it is called parabolic point.
4. If  $k_1.k_2$  is 0, and both of them are equal to 0, the point is a flat point.

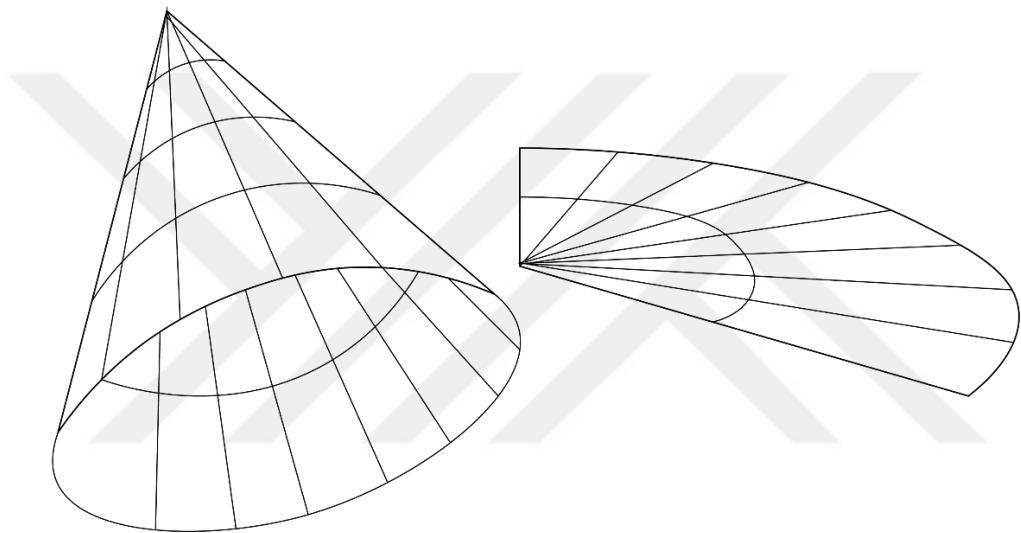


**Figure 2.1 :** Gaussian curvature (Url-1).

### 2.1 Developable Surfaces

If a surfaces's Gaussian curvature is 0, this surface called developable surface. The other name of this type of surfaces are single curved surfaces. Cones and cylinders are some examples of single curved surfaces (Pottmann, 2007). Developable surfaces have

very significant role in architecture because they can be unfolded to a plane without deformation (Figure 2.2). Developable surfaces are practical in manufacturing and fabrication since forms can be constructed by bending flat sheets of flexible materials such as: metal, cardboard, plywood etc. Additionally, these surfaces contain a set of straight lines which also simplify the structure's construction (linear beams). It is a great advantage for making a shape with only twisting and bending metal sheets. Many architects, designers and artists use developable surfaces in their work. For example, Frank Gehry often studies geometry using physical models composed of flexible sheet materials to developable geometries for his buildings.

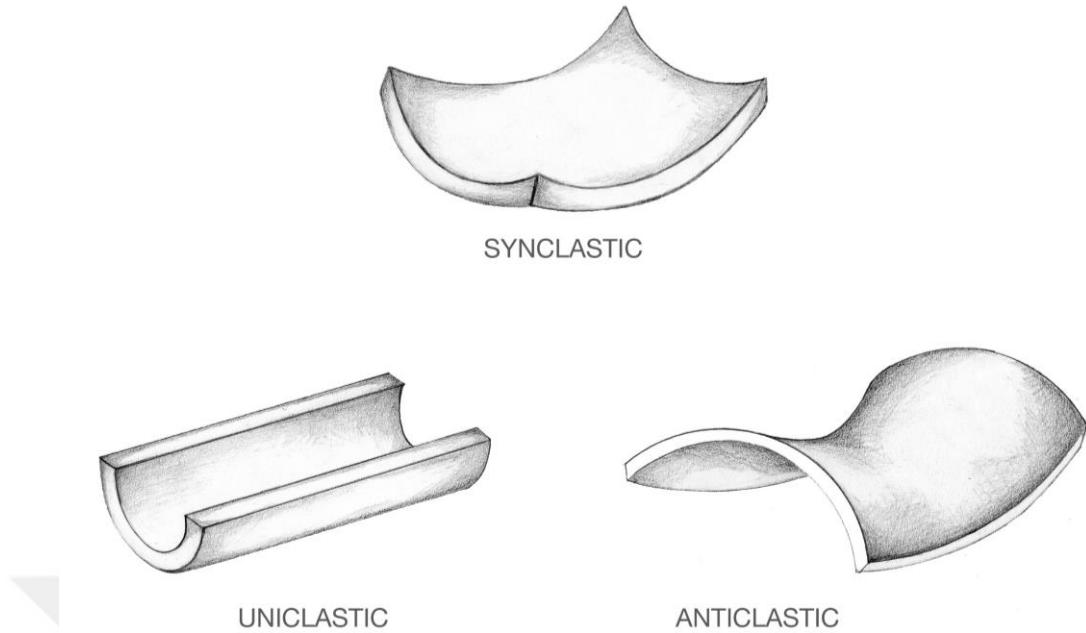


**Figure 2.2 :** Developable surface Cone

## 2.2 Non-developable Surfaces

Non-developable surfaces are surfaces that Gaussian curvature is different than 0. If Gaussian curvature is positive, this surface called synclastic. If Gaussian curvature is negative, this surface called anticlastic. Ball and dome are examples of synclastic, and cooling tower and saddle are examples of anticlastic (Tedeschi, 2014) (Figure 2.3).

Non-developable surfaces are also called double-curved surfaces and they used in architecture because of their structural stiffness. Due to these features, they are also widely used in shell structure design. Felix Candela, Heinz Isler and Frei Otto have always used non-developable surfaces. In particular, Felix Candela focuses on a special non-developable surface called hyperbolic paraboloid and its derivatives.



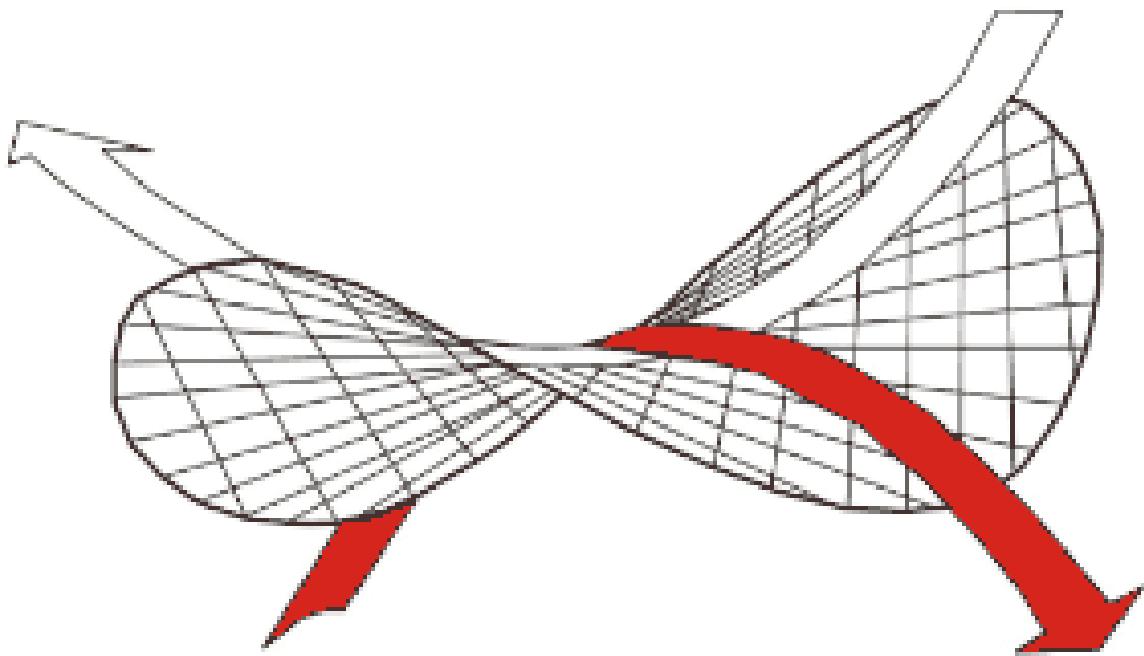
**Figure 2.3 :** Synclastic and anticlastic surfaces (Pender, 2009).

### 2.3 Ruled Surfaces

Ruled surfaces are special surfaces which we can draw a straight line throughout the surface (Burry&Burry, 2010). Ruled surfaces can be constructed by linear elements like pipes and boards, therefore they are used in architecture so many times. They can be described with mathematical formulas, so their compression and tension can be calculated manually.

All of the developable surfaces are part of ruled surfaces. But, ruled surfaces can be also non-developable. Hyperbolic paraboloids and hyperboloids are examples of non-developable ruled surfaces (Figure 2.4).

Conoid, hyperbolic paraboloid and helicoid can be described as ruled surface examples. Ruled surfaces are very common among architects because they can be produced using planar elements. It is always advantageous to have ruled surfaces when making formworks or dividing into shell to small elements for fabrication. Gaudi used ruled surfaces in most of his designs. This allowed the continue to construction of Sagrada Familia even after his death. Candela also reduced the cost and increased the feasibility thanks to the ruled surfaces and was able to fabricate a lot of shells.



**Figure 2.4 :** Ruled, double-curved hypar (Url-2).

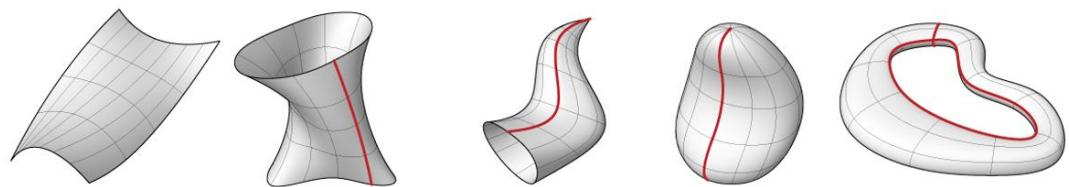
## 2.4 Freeform Surfaces

Freeform surfaces are surfaces different than classical surfaces. They can be classified as Bezier, B-spline, Non-Uniform Rational B-Splines (NURBS) or subdivision surfaces by their definition (Figure 2.5). Freeform surfaces used in architecture from early years by manually, but with the help of developments in mathematics related to car and aeronautical industries, they spread in computer aided architecture and manufacturing.

Their calculations are hard, because they are complex shapes, but they have been widely used in shell designs. Freeform surfaces have aesthetic quality and they can be produced by discrete elements with meshes or are fabricated by NURBS definitions.

Heinz Isler and Frei Otto tried to built freeform shapes that they designed by physical models. These forms can be produced by making physical models at different scales and analyzing them continuously. However, complex mathematical definitions of freeform surfaces extend the computation time. As a result, the analysis becomes complicated and optimization has to be made by experts in order to be able to fabricate these types of surfaces. All of these factors are increasing total cost and construction time. With the development of computers, communication with production tools has

increased. Complex gridshells and freeform concrete shells are now being produced. Moreover, 6-axis robots and 3d printers enhanced the fabrication of such complex geometries.



**Figure 2.5 :** Free-formed NURBS Surfaces (Url-3).

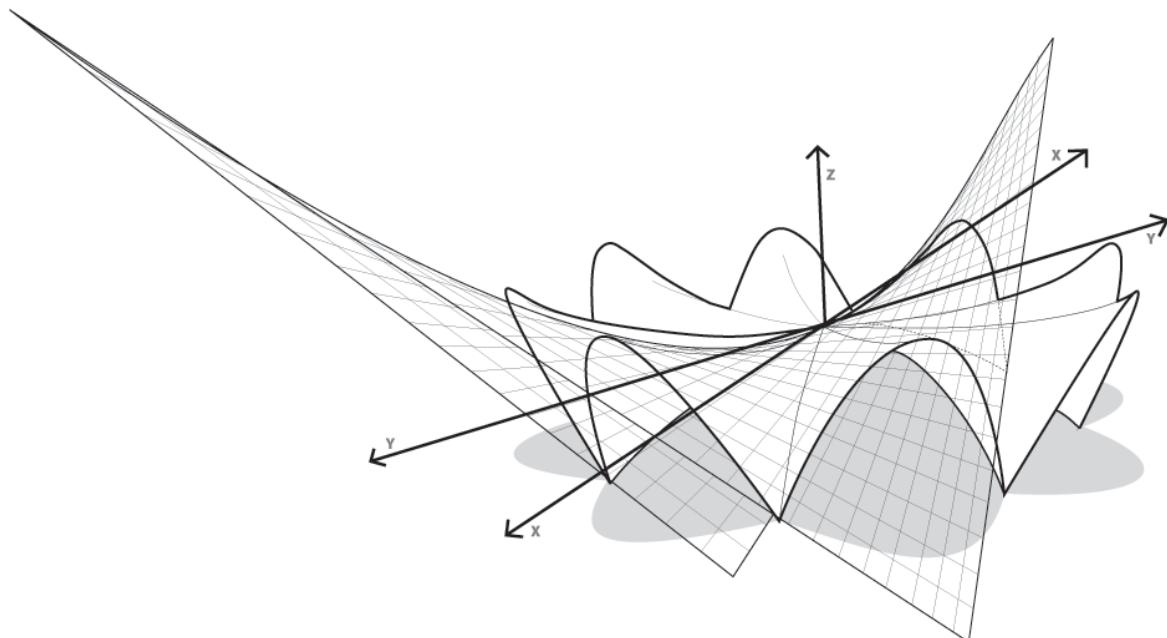




### 3. EXAMPLES OF SHELL STRUCTURES

Shell structures are curved surfaces in three dimension, that one dimension is very small than other two. Shells do not change their forms under external loads significantly. These structures transfer external loads from their surfaces to supports with membrane stresses. Membrane stresses and might be compression, or a combination of compression and tension. The word 'membrane' might suggest a film or fabric that can only carry tension, but the compressive stresses in a steel, concrete or masonry shell are still called membrane stresses. Shell structures try to carry these loads without buckling. For shells, the main load is typically the dead load, most often being its self-weight.

Shell structures can be one continuous surface or can be constructed by discrete elements forming that surface. Freeform shells can be designed digitally and then described by NURBS. But classic shell shapes like hyperboloid or hyperbolic paraboloid are chosen for structural performance and fabrication purposes (Figure 3.1).



**Figure 3.1 :** Los Manantiales Restaurant by Candela (Url-4).

### 3.1 Precedents of Thin Shell Structures

Gothic vaults can be considered early examples of shells. The methods developed by the masters of masonry have attracted Gaudí's interest as it contains an approach similar to Hooke's inverted chain theory. Antonio Gaudí then began experimenting with funicular vaults and hanging chain systems to combine these approaches (Tomlow, 2011). Ruled surfaces are also focused by him for ease of production. With the spread of usage of reinforced concrete, Felix Candela became a revolutionary designer for shells. For ease of production, he also preferred ruled double curved surfaces in certain shapes. Influenced by Candela's works, Heinz Isler began experimenting with hanging fabrics to find the form. Frei Otto developed these experiments and opened the way to tensile membrane structures. Although the pioneering names such as Eduardo Torroja, Eladio Dieste and Pierluigi Nervi are in the design of shell structures, this research focuses on the work of Felix Candela, Heinz Isler and Frei Otto.

#### 3.1.1 Felix Candela

Candela was a Spanish architect who educated in Madrid, and moved to Mexico because of Civil War in Spain. He always interested in mathematics and geometry but he couldn't have a chance to built a shell until late 1940s.

After some minor works, he built his first important shell for a laboratory in UNAM. He used hyperbolic paraboloid (hypar) for shell, because this form has a double curvature for stiffness, but also can be produced by straight elements. Edges of hypar can be curved or straight, and he used both of them and their variations in his important buildings. He chose doubly curved shells in his projects to eliminate bending, and use compression stress only, for stability. He insisted on designing structures that correspond directly to how they transfer forces, rather than shaping the form arbitrarily and then trying to fit shells or surfaces to conform. He tend to emphasize design as he stated in his biography by Faber (1963): "Science goes on analyzing... but art, the synthetic process, pools many things together so as to get the complete vision."

According to Garlock&Billington (2014), in construction phase, he placed straight form boards first, then add steel bars for reinforcing, and pour the concrete lastly. He was able to reach 4 cm thickness in his shells, but used reinforcement for preventing cracks. His most important works are Los Manantiales by combination of curved edged hypars, and Milagrosa Church by combination of straight edge hypars, which called umbrella form (Figure 3.2).



**Figure 3.2 :** Umbrella form by Candela (Url-5).

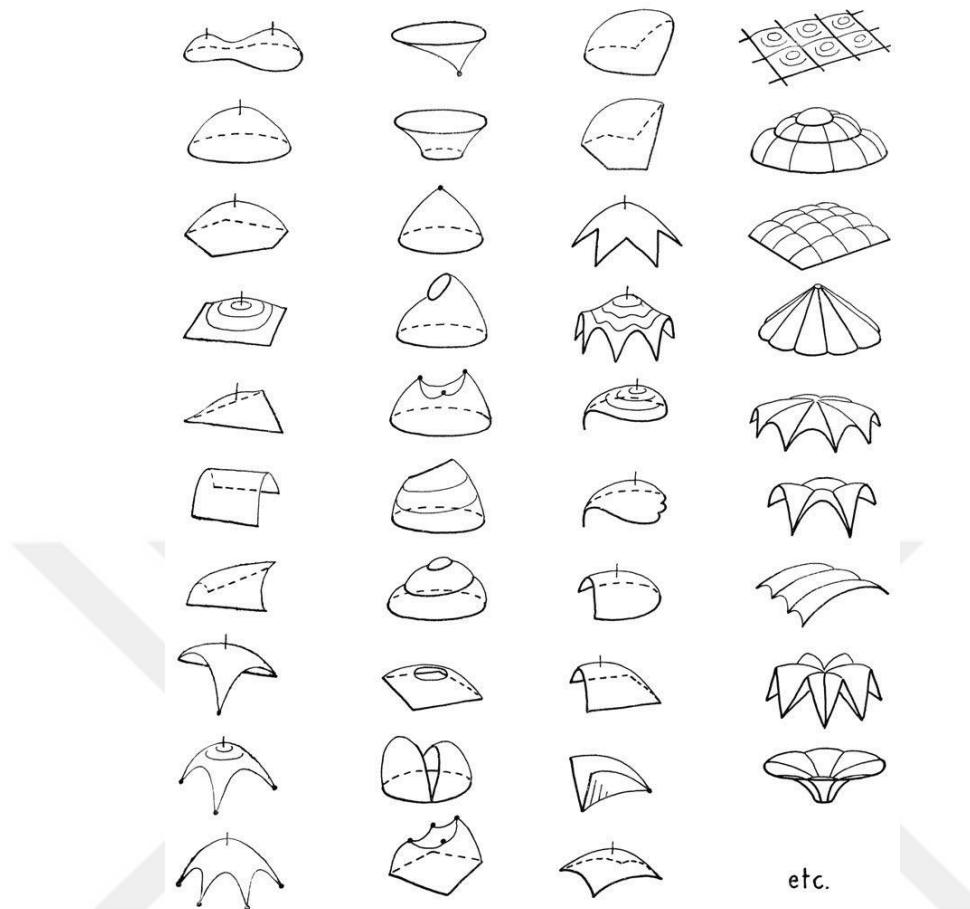
### 3.1.2 Heinz Isler

Isler was a Swiss civil engineer who was graduated from ETH Zurich. Despite of being a civil engineer, he always interested art, and had an excellent drawing skills. After finishing his military service, he firstly took attention of the world at 1959 IASS congress. His presentation about new shapes for shells created discussions and impressions.

He envisioned three ways of making shells:

1. By pouring concrete to free formed hill, then excavating it.
2. By making pressure to membranes.
3. By hanging membranes and reversing them.

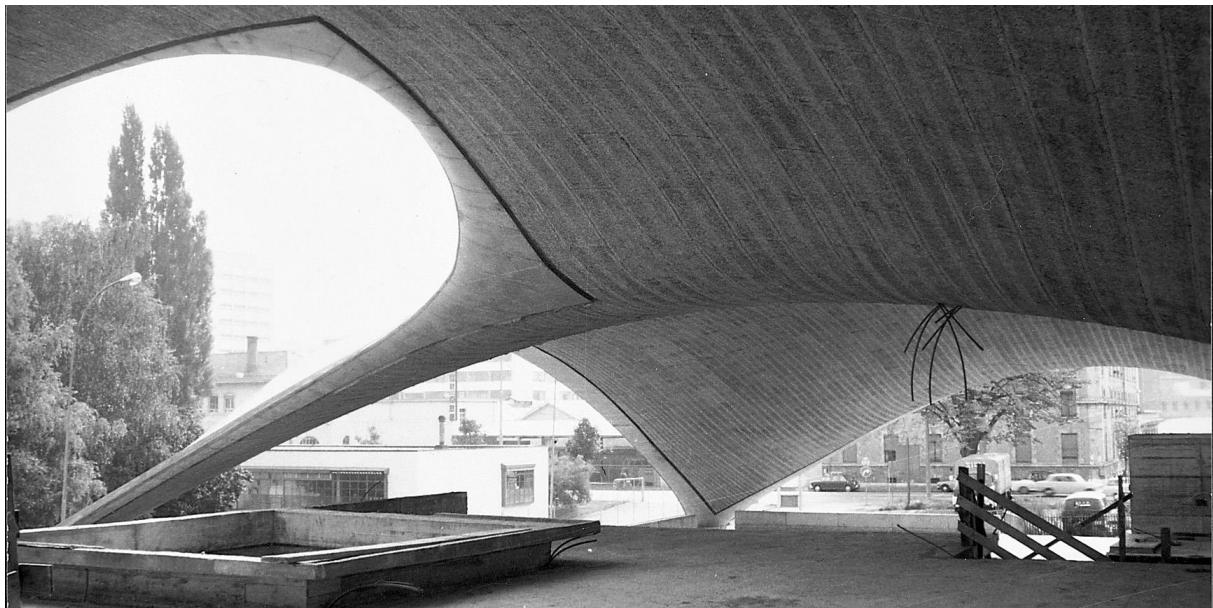
In his paper he presented a figure including 39 possible shapes and finished with “etc.” (Isler, 1960). It means the shell have endless possible form variations (Figure 3.3). It made an great impact on other designers like Torroja and Arup. Because Heinz Isler was a designer whose free-form shells are of a shape that cannot be defined by simple geometric formulae as they have continuously varying double curvature across the whole surface and they obey physical laws (Chilton, 2000).



**Figure 3.3 :** Form variations (Isler, 1960).

His shells were thicker than Candela's, because they are mostly in Switzerland's harsh conditions. In his first works he used pneumatic forms, but then he started to make concrete thin-shell structures by hanging membranes in 1960s. His main method was covering a cloth with a plastic, then hang the cloth upside-down from its corners. With the help of gravity, membrane find its form with pure tension. Then, when he reversed the membrane, the form was producing pure compression (Garlock&Billington, 2014).

He used metal formwork and curved wooden parts, and inserted flat fibreboards and reinforcing bars before pouring concrete for thin shell structure. He removed the falsework and wooden parts to use in other projects, but fibreboards were stayed because of insulation needs in Switzerland's climate. He designed too a great number of shells during his lifetime, but arguably his most important works were Deitingen service station and Sicli factory building in Switzerland (Figure 3.4).



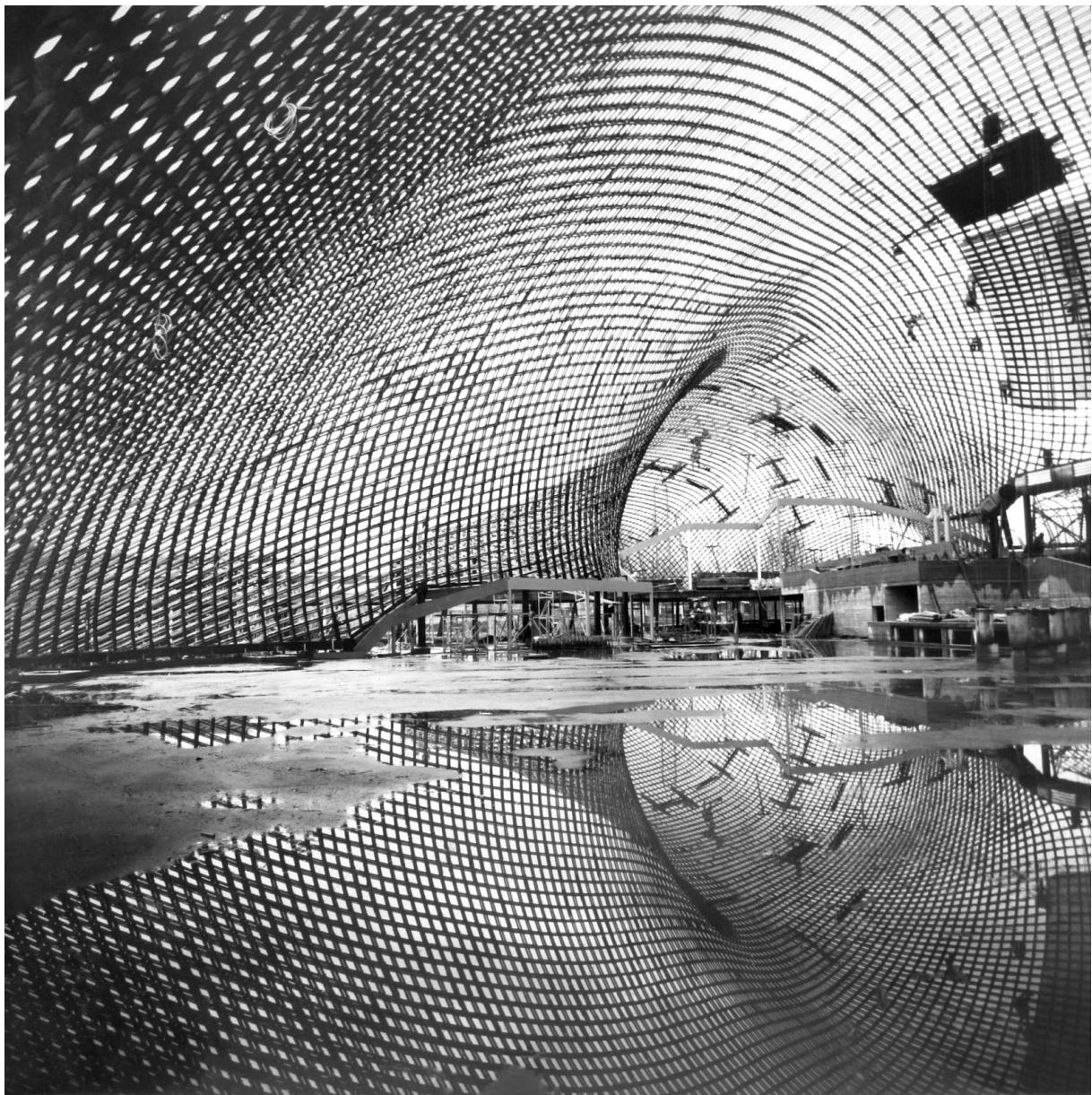
**Figure 3.4 :** Sicli Factory by Isler (Chilton, 2000).

### 3.1.3 Frei Otto

Frei Otto was a German architect studied in Berlin. He was a pilot in World War II, and interested in planes since childhood. This curiosity give him opportunity to observe aerodynamic forces. His visit in United States affected him and started a doctoral thesis about suspended roofs.

After the Berlin years he founded Institute for Lightweight Structures in Stuttgart. He mainly made his experiments of tensile structures with physical models. He began using models in the 1950s as the only way of establishing the form of three-dimensional, membrane and cable-net structures whose final geometry could not be determined using analytical methods. He used similar methods of Isler like hanging and reversing to produce double curved surfaces, but he used cable nets instead of cloths (Glaeser, 1972). For models, he used elastic sheets whose surface tension depends on the strain; and nets whose surface tension arises partly from the elastic extension of fibres, and partly from shear deformations of the net. He even used soap bubbles, which have a constant surface tension, in his models to understand surface tension and form making (Addis, 2013). After finding the equilibrium geometry of the tensile structure, it was then possible to use analytical methods to determine in-plane stresses and forces at the boundary supports. First important example of these type of structures was 1967 German Pavilion designed by him.

In his later years he also started to investigate timber gridshells. Because a funicular shape for a gridshell is more easily obtained from a hanging model comparing the old timber methods. Mannheim Multihalle was a perfect example of this with a multilayered timber grid system resting on an edge beam at the perimeter the shell (Figure 3.5). He always tried alternate solutions for covering large areas, and his experiments made great contribution to design of shells. These observations led Frei Otto, after years of experimental and analytical studies, to formulate his theory of minimal surfaces. Minimal surfaces, if correctly determined, generate the smallest possible surfaces within given curvilinear boundaries.

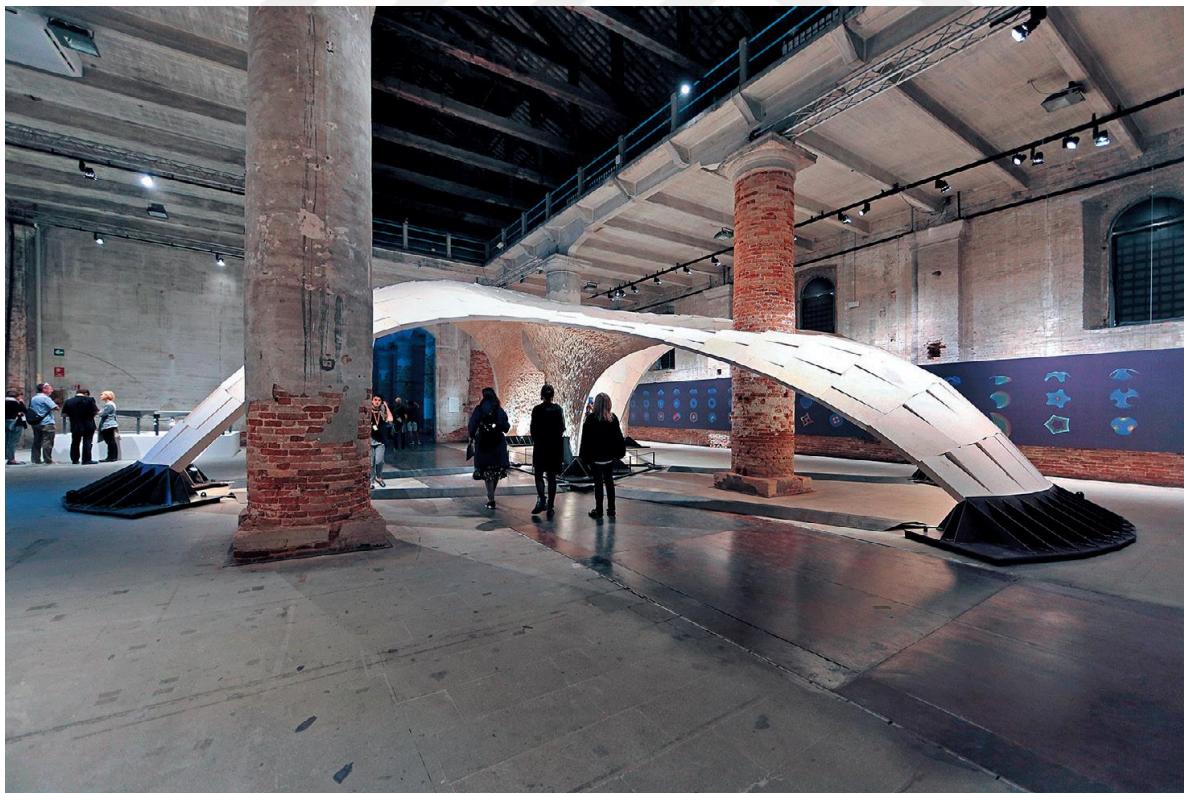


**Figure 3.5 :** Roof of Multihalle (Url-6).

### 3.2 Contemporary Designers Using Advanced Computational Methods

We can assume several architects like Philippe Block, Toyo Ito, Sanaa, Shigeru Ban as contemporary shell designers.

One of the contemporary examples might be Block Research Group in ETH Zürich. Their researchs focuses on several core areas, including analysis of masonry structures, graphical analysis and design methods, computational form finding and structural design, discrete element assemblies, and fabrication and construction technologies. They developed Thrust Network Analysis (TNA) method by inspired from masonry vaults. In their researchs, they try to expand TNA method by fabricating 1:1 prototypes of different shell designs (Figure 3.6). Moreover, the group analyze their fabrications for potential collapses. A computational approach is seen from the beginning to the final production of their designs. Because according to Van Mele et al. (2016), the design of an unreinforced, discrete, cut-stone, dry-set vault with complex geometry is a complicated process that requires an integrated computational setup that from the early design and form-finding stages accounts for structural and architectural considerations, and fabrication and assembly constraints.



**Figure 3.6 :** Armadillo Vault, Block Research Group (Block et al., 2016).

The advances in architectural geometry and took attention of many contemporary Japanese architects and engineers. One of the significant examples is Shigeru Ban. He collaborated with Frei Otto for Japanese Pavilion in Expo 2000 Hannover, and then he continued to use freeformed timber gridshells and membrane structures in his projects (Chilton and Tang, 2017).

Another Japanese architect that use freeform shells is Toyo Ito. Toyo Ito is using the shells in his new projects. His crematorium design collaborated with Sasaki is an excellent sample of free formed shell design with computational methods (Figure 3.7). In the crematorium the dominant element was the undulating roof, a concrete shell that appears to have the character and lightness of a piece of fabric fluttering in the breeze. Mutsuro Sasaki, who contributed his methodology of structural optimization by applying repeated passes of a refinement algorithm. This enabled an initial approximation of the desired form to approach its most structurally efficient shape after a number of iterations (Turnbull, 2012). This approach gave freedom both to structural engineers and to architects.



**Figure 3.7 :** Contemporary Shell Example, Toyo Ito (Url-7).

SANAA also used a freeform shell in their Rolex Learning Centre project. Also, their art museum project in Teshima is built with method proposed by Isler. They shaped to earth, poured the concrete, then excavated the earth to reach to final form. Nishizawa (2011) stated that extensive structural calculations and material analyses were necessary to restrict the arc of the dome to a height of four and a half meters and the thickness of the shell to 25 cm. The form was implemented by precisely recreating the topography of a hill, based on 3,500 points of measurement.

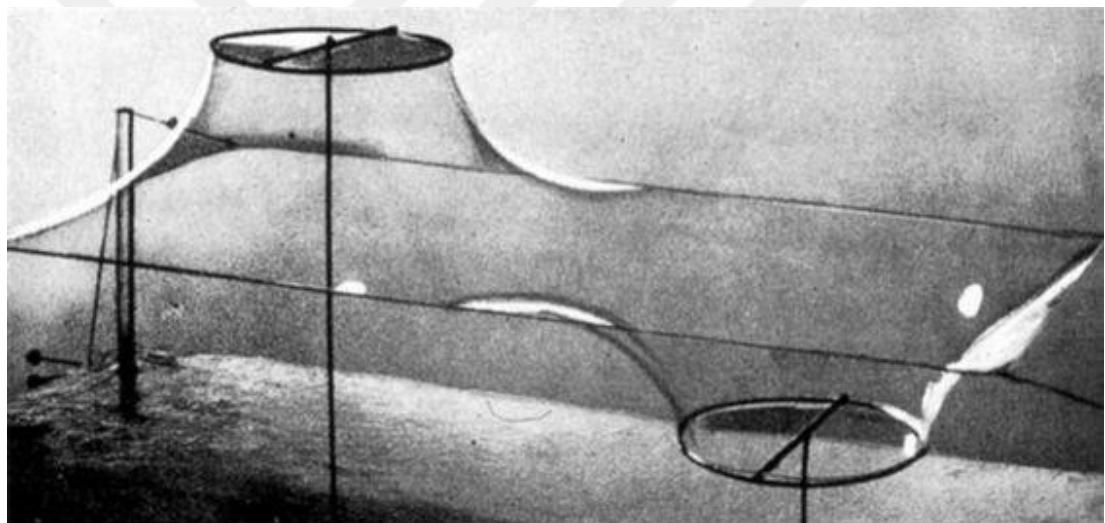
In addition to many architects and engineers, architectural schools are also producing full-scale shell structure prototypes to learn computational design and fabrication methods.





#### 4. METHODS OF FORM FINDING

Form finding is the process of finding a form based on computer simulations or physical models (Figure 4.1). In form finding process, parameters are controlled to reach an optimum geometry under the certain loads. For shells, the load is most often the self-weight of the shell. Some of the parameters of form finding can be supports, loads, boundaries, topology and internal forces. Form finding is a continuous process with a starting geometry then updated by feedback from physical or computational simulations.



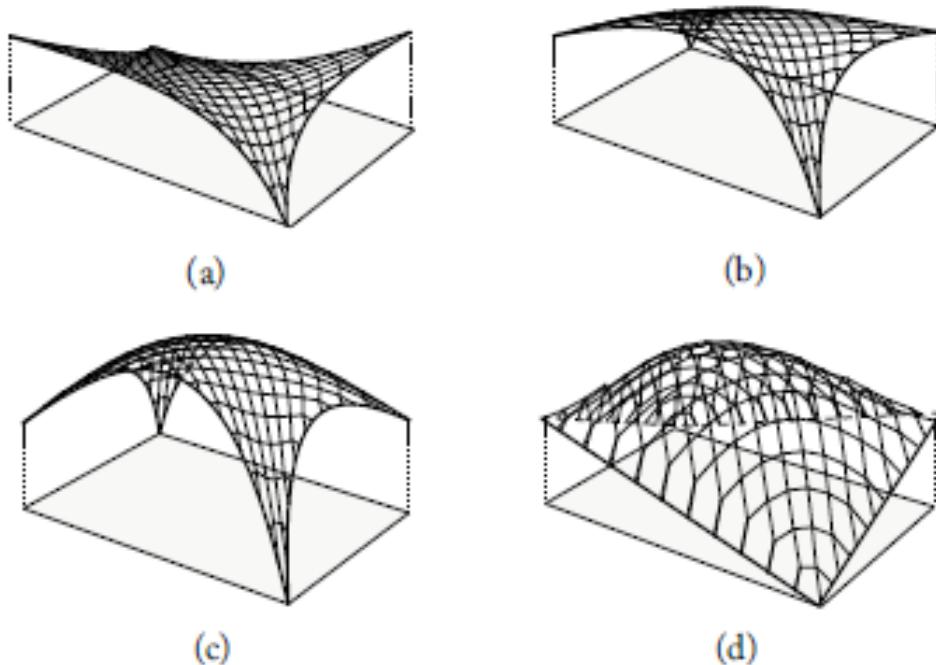
**Figure 4.1 :** Form finding with soap bubbles (Url-8).

After form finding, the optimization can be made by different objectives. These objectives can be weight, material, and deflection minimization and stiffness maximization. Furthermore, this optimization could be shape optimization, topology optimization and sizing optimization. In the scope of this study, form optimization experiments will be made with weight reduction by openings.

#### 4.1 Force Density Method (FDM)

The use of ‘force densities’ presents an approach for the rapid generation of feasible shapes for prestressed and (inverted) hanging structures (Figure 4.2). This method allows, especially in the early stages of a new project, the instant exploration of large numbers of alternative, feasible solutions. It is also known as the ‘Stuttgart direct approach’. It has been applied to the design of many built structures, particularly to tensioned roofs, but also to the timber shell roofs. The advantage of using force densities is that they do not require any information about the material for the later realization of the design. As we are dealing with non-materialized equilibrium shapes, no limitations with respect to material laws exist. The materialization follows in a second step. When introducing material, we may choose (independently for each bar in the net) the material, without changing the shape created with force densities.

Force density is the tension coefficient and it is the force ratio over a cable. It is mostly used in hanging structures, and allows to find many feasible alternatives in early design phase. It is used by Frei Otto’s Stuttgart group in many of their designs. It is independent from material properties, therefore very suitable for early design stages. It aims to reach statical equilibrium by applying tension and the resulted forms are mostly doubly curved (Linkwitz, 1999).



**Figure 4.2 :** Force Density Method (Linkwitz, 2014)

The method, originally developed for cable nets, is, to this day, very common in the design practice of tensioned membrane roofs. By introducing loads, it also allows the form finding of synclastic structures, highly suitable for efficient shell structures. Because the method is entirely independent of material properties, two interesting opportunities arise. First, resulting designs can be materialized arbitrarily, giving the initial lengths of the network in undeformed state, without affecting the final shape. Second, one can simply multiply the loads to any realistic value, and then calculate the internal force distribution, again without changing the geometry. It is mostly used in cable nets, but can be used in other materials, because of independence from material.

## 4.2 Thrust Network Analysis (TNA)

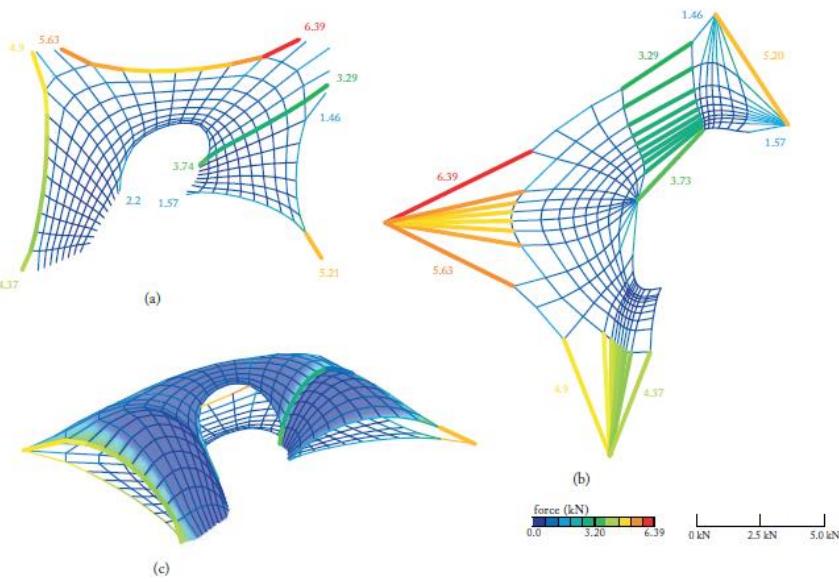
Thrust Network Analysis is mostly suitable for funicular shells. It is developed and implemented in ETH Zurich Block Research Group. This method was inspired by the similarities between hanging chains and thrust lines of masonry structures. Weights, proportional to the self-weight of each stone piece (voussoir), of an arch are applied on the vertical lines of action through their centroids, to a hanging string. When inverted, it produces a thrust line that fits within the arch's geometry. This compression funicular can be used to show a possible compression-only equilibrium of the arch.

For two-dimensional problems, graphics statics can be used instead of a hanging model. It allows finding the form of possible funicular shapes for given loads, but at the same time also the magnitude of the forces in them. The geometry of the structure, represented here by the funicular polygon, is named the form diagram. The magnitude of force in each element of the form diagram is simply known by measuring the length of the corresponding, parallel element in the force diagram, which is drawn to scale. The geometrical and topological relationship between form and force diagram is called reciprocal. Unfortunately, graphic statics is practically limited to two-dimensional problems. Graphic statics can be used to generate thrust lines, which, when fitted within the masonry structure, visualize possible compressive 'flow of forces' through the structure (Rippmann et al., 2012). These thrust lines can be used for three-dimensional problems with creating thrust network.

Since only vertical loads are considered in TNA, the equilibrium of the horizontal force components (thrusts) in the thrust network can be computed independently of the chosen external loading. As Block(2009) stated, this allows splitting the form-finding

process in two steps: solving for an equilibrium of the horizontal thrusts first, and then solving for the heights of the nodes of the thrust network, based on the external vertical loads, the given boundary conditions, and the obtained horizontal equilibrium. Force diagram and form relationship can be observed during design phase in this method (Figure 4.3). Form diagram is generated by starting NURBS shape, then force diagram is created. The force diagram can be manipulated and it reaches to horizontal equilibrium. After this, the vertical equilibrium has reached and the vault form is generated.

The key strategy in TNA is to give the designer direct control over the distribution of the thrusts in the system. The designer can choose these horizontal forces within the geometric constraints of the reciprocal relationship between form and force diagram. As in graphic statics, both form and force can be manipulated to determine the equilibrium shape (Rippmann and Block, 2013). The intuitive force diagrams allow the designer to visually and explicitly distribute internal forces that define the three-dimensional equilibrium shape.



**Figure 4.3 :** Thrust Network Analysis (Block et al., 2014)

### 4.3 Dynamic Relaxation (DR)

Dynamic Relaxation is mostly appropriate for gridshell structures. A gridshell is essentially a shell with its structure concentrated into individual members in a relatively fine grid compared to the overall dimensions of the structure. The members may be short in length and only pass from node to node, or they may be continuous,

crossing each other at the nodes. The grid may have more than one layer, but the overall thickness of the shell is small compared to the overall span.

Gridshells can be made by prefabricated curved members or straight elements (Figure 4.4). The method of dynamic relaxation can be used for the form finding of either. This technique observes the deformation of structure under loads through time (Adriaenssens and Barnes, 2001). The structural action of gridshell structures is so complex that even today with powerful and affordable computers, there is still a place for physical model testing. The most rudimentary physical model can give more accurate predictions of deflections and buckling load than hand calculations. Because of the complex interaction between membrane and bending action, the prediction of buckling loads by hand calculations is effectively impossible. However, the deflections and buckling load from a physical model can be scaled using dimensional analysis.



**Figure 4.4 :** Murinsel gridshell (Adriaenssens et al., 2014)

The method can give many variations of shape for different loads and supports with paneling. Gridshells have been made from aluminium, concrete, steel, wood, bamboo and composite materials. Each material has advantages and disadvantages regarding strength, ductility, stiffness, cost, weight and durability. Connection and cladding design also inform the material choice. Because of this material selection, cladding

and connections are important in gridshell design. The joints and nodes of gridshells could be very complex, and must be calculated very carefully.

#### **4.4 Particle-Spring Method (PS)**

Particle-Spring method is widely used in architecture including Gaudi and Isler because of suitability of shape generation of form-active and form-passive structures. This method is a computational approach simulate hanging or pretensioned chains and grids. This approach adopts subdivision surfaces for parameterization and particle-spring systems for form finding. For doing this, the method uses high-poly meshes with subdivision (Figure 4.5). Because subdivision offers the designer better control of the topology.

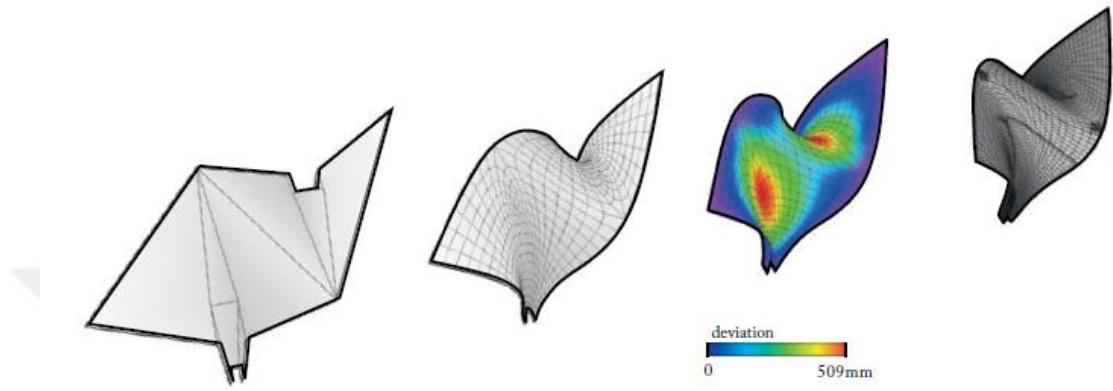
The principal purpose of the particle-spring method is to find structures in static equilibrium. This objective is achieved by defining the topology of a particle spring network with loads on the particles, the masses of the particles, the stiffnesses and lengths of the springs, and then by attempting to equalize the sum of all forces in this system. With these method, surfaces become lines and points. With loads on nodes and reaction of springs balanced in a shape (Ahlquist and Menges, 2013).

According to Ochsendorf and Kilian (2005), the following are essential assumptions made within a PS form-finding framework:

- Surfaces are discretized into points and lines. The points are nodes with mass and the lines are springs connecting them.
- Upon applying forces, each node is either free to move or fixed in each direction (between zero and three degrees of freedom, corresponding to between three and zero orthogonal reaction forces).
- The internal forces (exerted by springs connecting the node to other nodes) and external forces (gravity and applied loads) act on the nodes. The result of all such interactions between nodes and springs iteratively leads to a balance of forces on each node and overall to an equilibrium shape.

This type of simulation usually produces anticlastic geometries, so negative Gaussian curvature appears everywhere. It is suited for quick exploration of designs in CAD environment.

Grasshopper plugin Kangaroo by Daniel Piker is an example of extensions using the particle-spring method. The add-on, which is a physics simulator, allows you to find the form and see the effects in real time using the particle-spring method. Although it is good for early design phases, it has disadvantages because it can not provide detailed control in the future stages.



**Figure 4.5 :** Particle-spring method (Booshan et al., 2014)

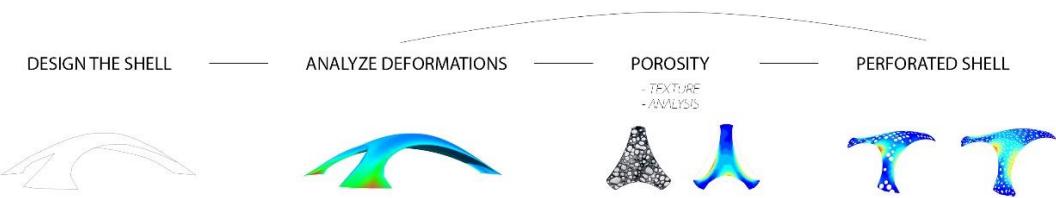


## 5. EXPERIMENTS FOR DESIGN AND OPTIMIZATION

Thin shell design is a very comprehensive issue. To reach to optimal solutions, the limitations and goals should be set. In the scope of this study, different types of geometries will be experimented to achieve an efficient shell design.

### 5.1 Aim and Scope

The purpose of the study is to observe effects of different designs of structure then optimize it with the feedback from computational tools. The scope of this thesis includes basic geometric definitions describing the shells in the first chapter. Then, different applications in architecture are explained with chosen architects and buildings in chapter two. After that the methods for finding a form been introduced. In this chapter different experiments are made to get a better understanding and optimize the thin shell design (Figure 5.1).

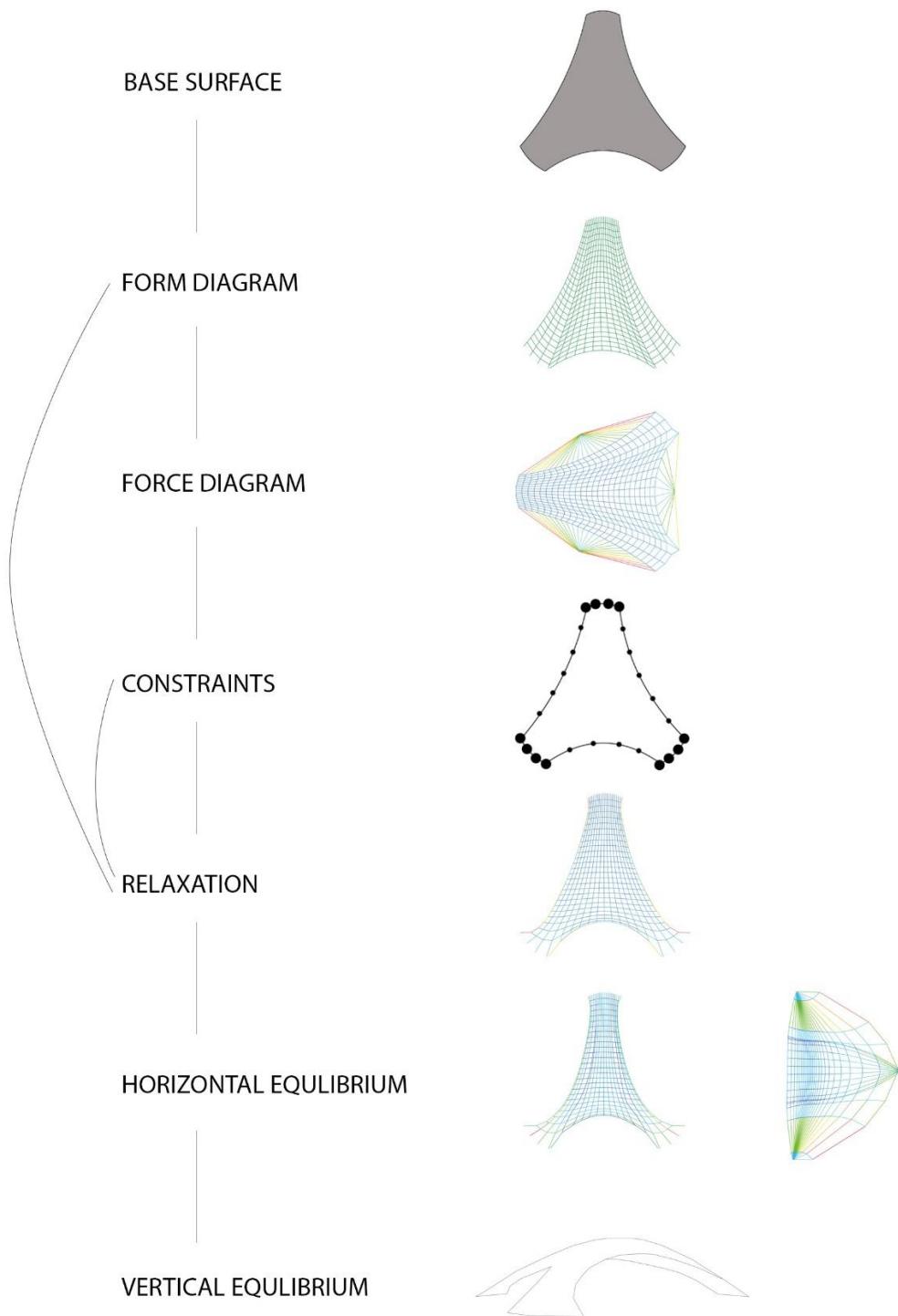


**Figure 5.1 :** Workflow of experiments

### 5.2 Form Finding of Shells

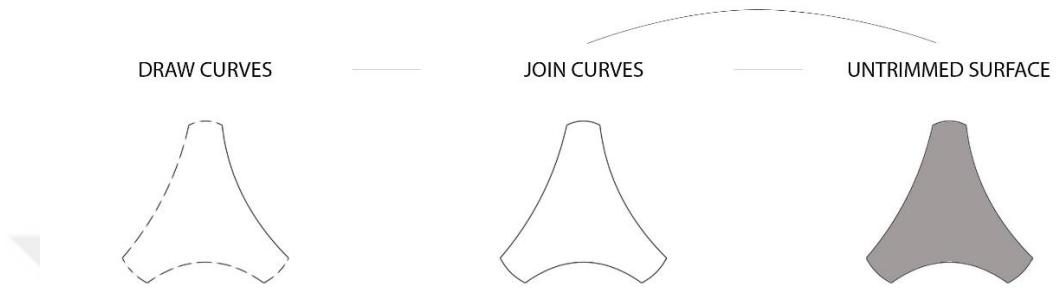
An area of 20x20 m was determined for the formation of the forms. These forms were considered to be shells with different properties covering the same area. The Thrust Network Analysis (TNA) method was used to design the shells (Figure 5.2). The use of this method is a method of finding a geometric form independently of the intended material. It examines the relationship between form and force and tries to find a horizontal and vertical balance between them. The most important factor in shell design is to create only compression surfaces. Tension and bending are avoided. This

method is suitable for geometrical properties in the form finding stage rather than trying to fix pre-designed surfaces. In this method, it is more advantageous to use concrete and masonry shells instead of wooden and metal gridshells due to the structure of material.



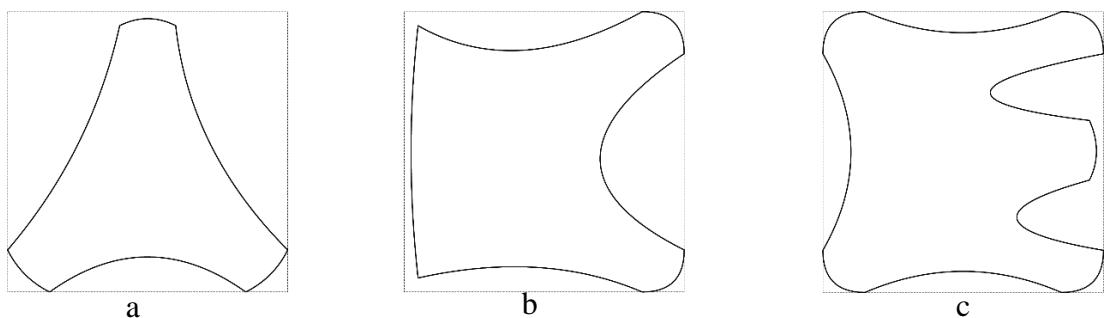
**Figure 5.2 :** TNA method process

To implement this method, the Rhino Vault plugin written by Matthias Rippmann, a student of Phillippe Block who developed the method, was used. In order to create surfaces in RhinoVault, form diagrams need to be created first. Form diagrams are two shapes, normal and triangulated. While normal form diagrams require untrimmed surfaces, triangulated form diagrams can be created from each surface (Figure 5.3). A normal form diagram has been created in order to facilitate later processing.



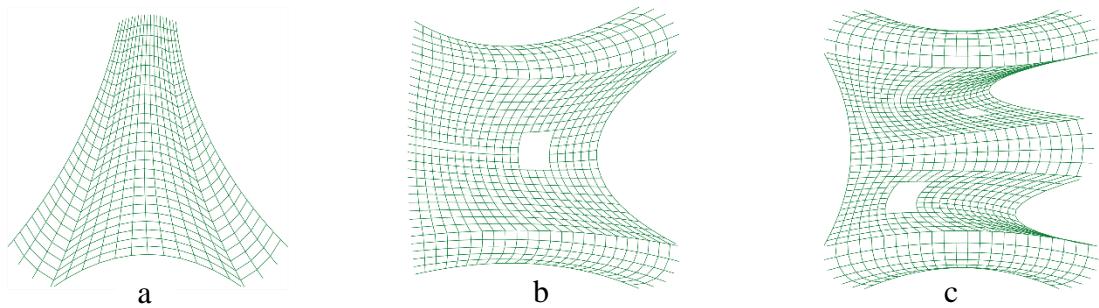
**Figure 5.3 :** Base surface creation

In order to be able to produce 3 forms to be examined within the scope of the thesis, firstly the base surfaces (a), (b) constituting these are produced (Figure 5.4). The surface of the first form was started with a rectangle and the surface was rebuilt to reach the desired surface and played with the control points of the open areas. When the surface of the second form is formed, a major opening is left in the middle and an untrimmed surface is formed by using the edges of the first surface and the edges of the circumference. When the third surface is being formed, the surface is formed by



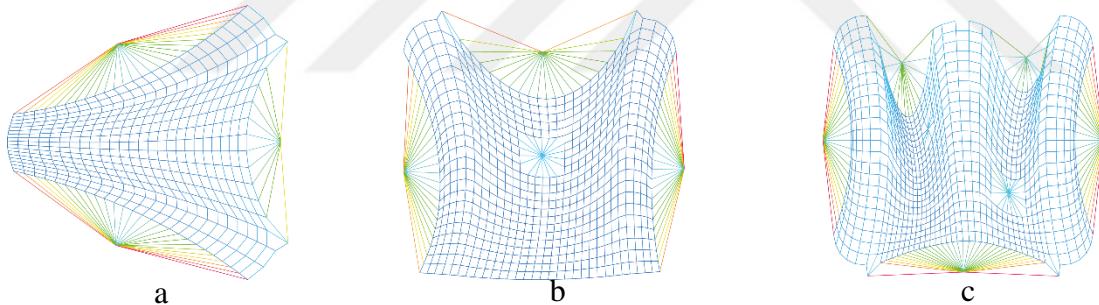
**Figure 5.4 :** Base surfaces for shell design

drawing and joining the different curves according to the desired support points and openings. The subdivisions of the surfaces can be adjusted manually while forming the form diagrams from these base surfaces. High subdivisions extend the calculation time while giving smoother results. After this step, estimated openings and support points are selected. This results in a form diagram (Figure 5.5).



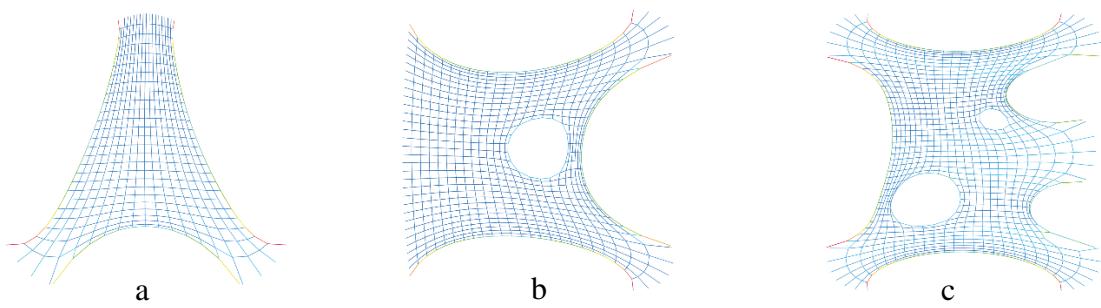
**Figure 5.5 :** Form diagrams of base surfaces

Once the form diagram is created, it is possible to edit it. In particular, adding and subtracting to areas where openings and support points are present is important to achieve desired results. It is advantageous to anticipate problems that may arise especially when complex forms are being created and to intervene in the diagram. After the form diagram is created, a force diagram is created accordingly (Figure 5.6). Form and force diagrams are dual diagrams. The force diagram consists of taking the barycenter of the gaps in the form diagram and is rotated 90 degrees. Since it is a geometric calculation method, this rotation will facilitate later stages.

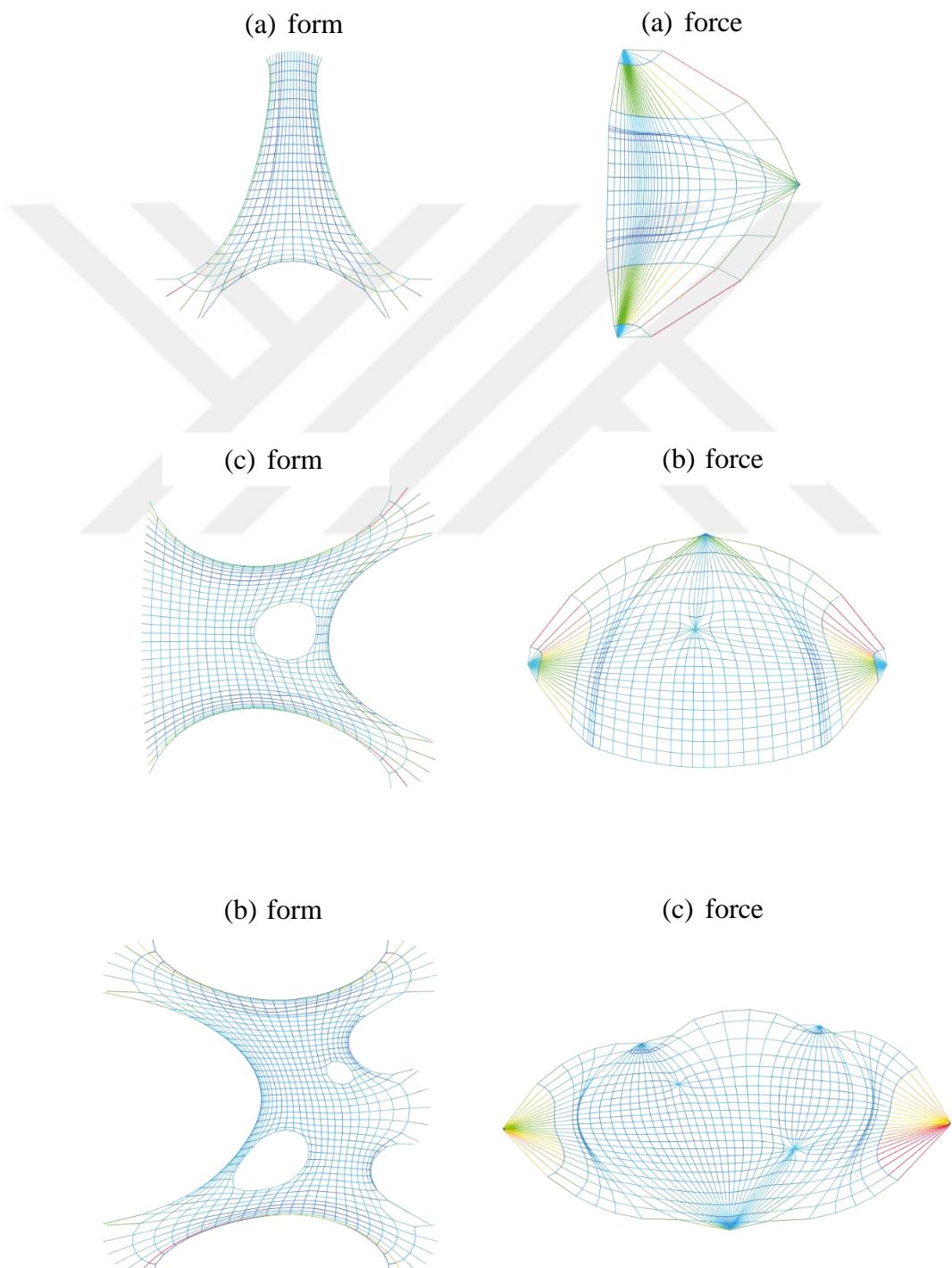


**Figure 5.6 :** Reciprocal force diagrams of form diagrams

Deviation angles also occur when the force diagram is created. Already making these calculations is to achieve a balance by minimizing the aim of devolution. Thanks to this balance, only the compression area can be reached. After these two diagrams are constructed, horizontal equilibrium is found. Horizontal equilibrium may not always be found. When not available, you may need to go back to the previous step and make adjustments in the form or force diagram. Or, if it is desired to load more places in certain places, it can be stretched on the force diagram. When equilibrium is not found, a method is to relax the form or force diagram (Figure 5.7). By determining the weights of unwanted points and edges, the remaining parts can be relaxed for a more regular shape. This facilitates reaching the horizontal equilibrium (Figure 5.8).

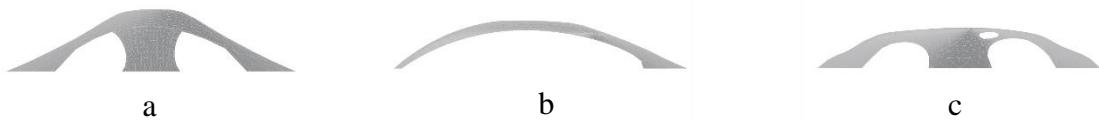


**Figure 5.7 :** Form diagrams after relaxation



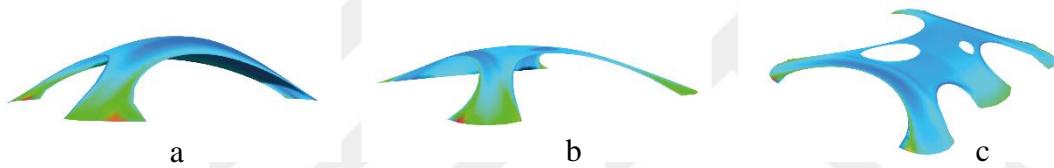
**Figure 5.8 :** Form and force diagrams after horizontal equilibrium

Modifications may be made if the openings and supports are considered not to be correctly detected after equilibrium. If equilibrium is reached, vertical equilibrium can be established (Figure 5.9). At the end of the vertical equilibrium, a shell is formed which will form a reference to other works (Figure 5.10).



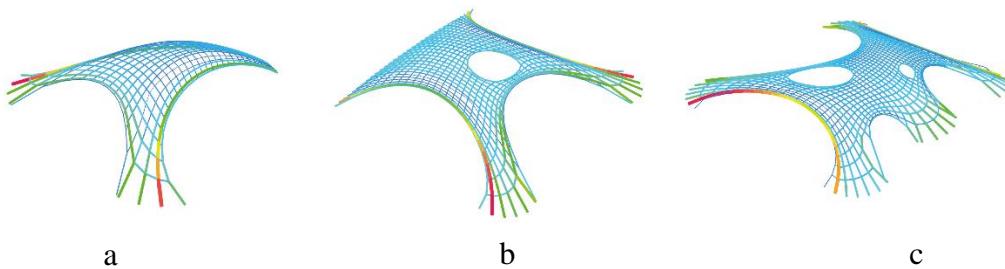
**Figure 5.9 :** Orthogonal views of designed shells

This crust can be colored if desired, and then the overburden areas can be seen, which will be done later. Or it can be displayed with colored and enlarged pipes according to the loads (Figure 5.11). In addition, dead load can be shown at the end of the load.

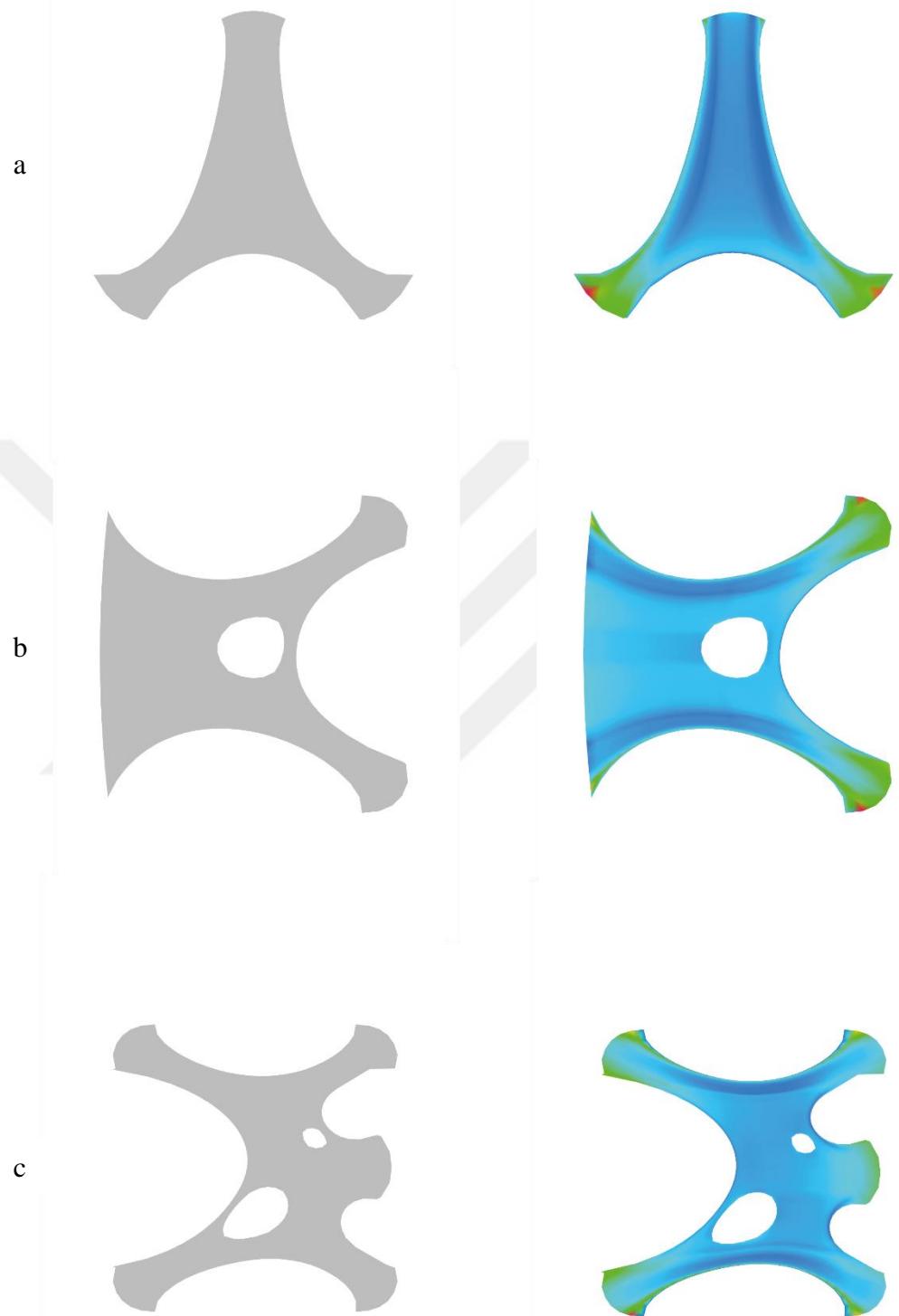


**Figure 5.10 :** Perspective views of designed shells

After finding the forms, the shells will be analyzed in Karamba with Finite Elements Method (FEA) to check the deformation and take data for continuing part of the study (Figure 5.12).



**Figure 5.11 :** Axial forces represented by pipes



**Figure 5.12 :** Top views of designed shells

### 5.3 Analyzing Forms

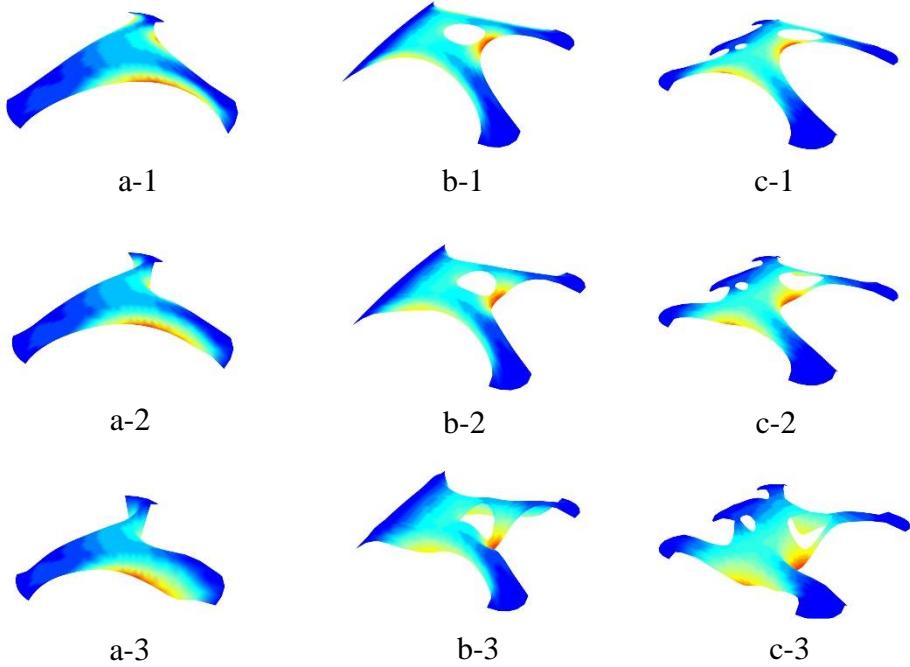
In this phase, Finite Element Analysis (FEA) method is used to check the forms created with RhinoVault and to obtain different data that will work in the future. Finite Element Analysis is a method predominantly used in engineering, which is based on analyzing these parts by separating smaller parts from the main surface. Since the geometry created in RhinoVault is a mesh, it is suitable to be analyzed with this method. Although there are more detailed Finite Element Analysis programs for engineers, Karamba, a grasshopper plugin, was used (Figure 5.13). Since it is a designed shell, it is necessary to first convert it to a meshtoshell. In this phase, geometrical model is translated into a structural model. The mesh is translated as vertices point while the edges are identified as an element. In order to perform the analysis, a Finite Element Model must first be assembled. Here, point and element components come from the mesh. Support points must then be specified. When supporting points are determined, vertices with mesh Z-coordinate 0 are selected. The Degree of Freedom (DoF) of these points should also be determined when supporting support is set. The DOFs for this node represent the possible movement of this point due to the loading of the structure. In the real world, a point can move in 6 different directions, translation in X, Y, and Z, rotation in X, Y, Z. These are denoted as Tx, Ty, Tz, Rx, Ry, Rz. Since this is a shell structure, it is allowed to act in every direction.

LOADS	SUPPORTS	CROSS SECTION	MATERIAL	METHOD
Gravity (Self-weight)	Z=0 Tx, Ty, Tz, Rx, Ry, Rz	Shell	Concrete C25/30	Analyze Thl

**Figure 5.13 :** Selected constraints in Karamba

Cross sections of the shell forming parts should be determined. If it is a gridshell of smaller pieces, these parts can be selected from cross sections in different profiles. But since we designed a normal shell, it was chosen as a shell with a cross section thickness of 5 cm. At this stage, a material must be identified. C25 / C30 concrete was selected as material. From here, the properties of the material to affect their behavior against the load are taken. And finally, the loads that will affect the shell are selected. Since the main load in shells is self-weight, no point loads such as wind load, snow load etc.

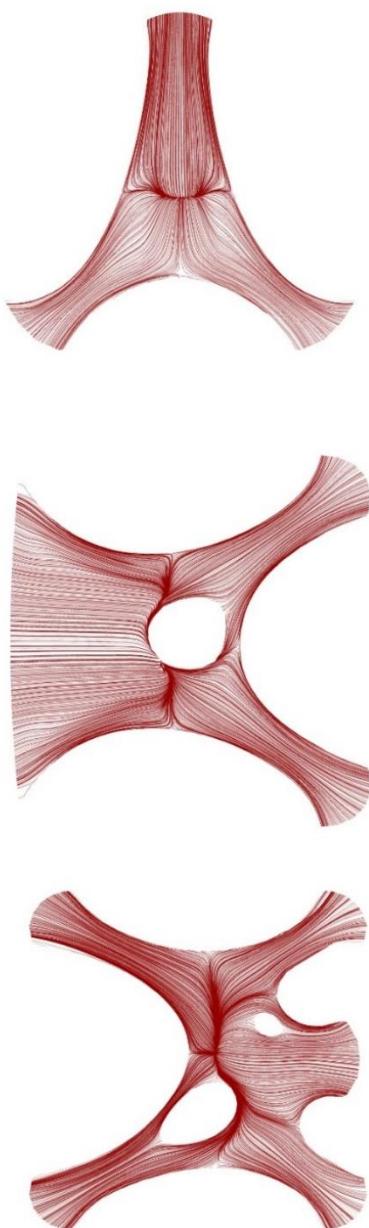
have been added. Also, these loads are not included in the account because it will be a shell that does not walk on it. Gravity is the burden involved in the account (Figure 5.14). Gravity is a dead load (static load). And the size is Unit Z.



**Figure 5.14 :** Deformation with different gravity in designed shells

The properties determined up to this stage are called boundary conditions. After that, Finite Element Analysis has been made. AnalyzeThI is used in analysis part. This means first order theory, and it is used in analysis like shell structures not involving bending forces. The analysis gives us maximum displacement, gravity load and elastic energy beside the model information. We can obtain various information by connecting different models of this model information. If we link ShellLineResults, we can see the Force Flow, Iso Lines, Principal Moment and Principal Stress lines. Force flow lines illustrate the load distribution in structures (Figure 5.15). In case you want to strengthen a structure with linear elements (e.g. fibers), you can align them with FF-lines to get the most effective layout. Iso Lines draws the contour lines. Principal Stress and Principal Moment lines are the main lines for stress and moment. And if we pass the Model View component, we can see the deformation here. Deformation can be viewed as exaggerated because it will be very low on a correctly installed model. Since the color scale here is not very suitable, a more suitable color

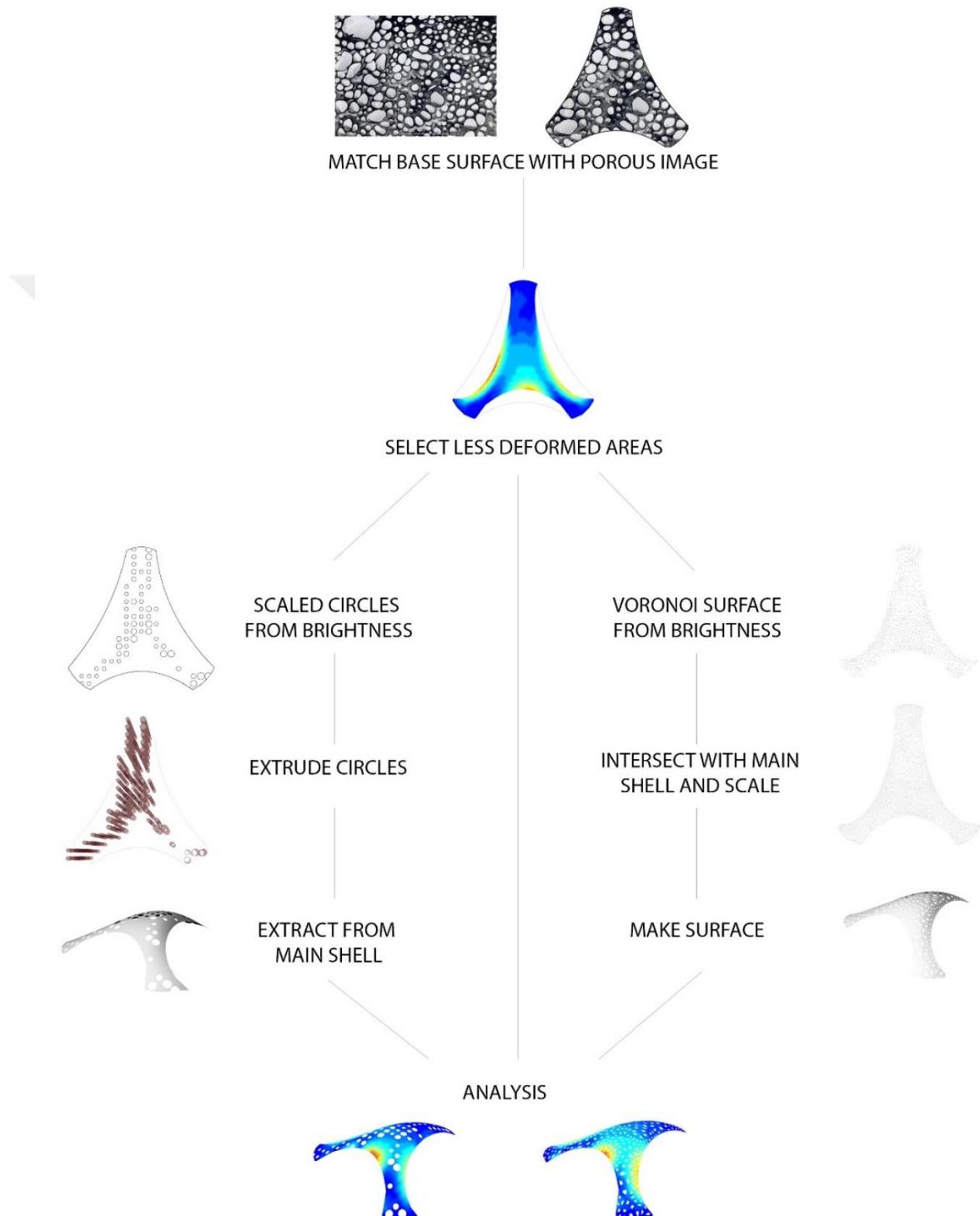
scale for Finite Element Analysis (FEA) is added manually. Places where the displacement is high are shown in red, places where there is little displacement are shown in blue. We can do visualizations if we pass that data to Shell View. The pre-emptive concepts here are Displacement, Utilization, Van Mises Stress and Principal Stress. Van Mises Stress, and accordingly utilization are stresses used in ductile materials such as steel and are not required for this analysis. What matters to us is displacement. From here we can see which region is exposed to the load and which will undergo deformation. We can show our results with the color legend. The resulting colored mesh has been the basis of our other work.



**Figure 5.15 :** Force flows of designed shell

#### 5.4 Form Optimization Based on Boolean Operations

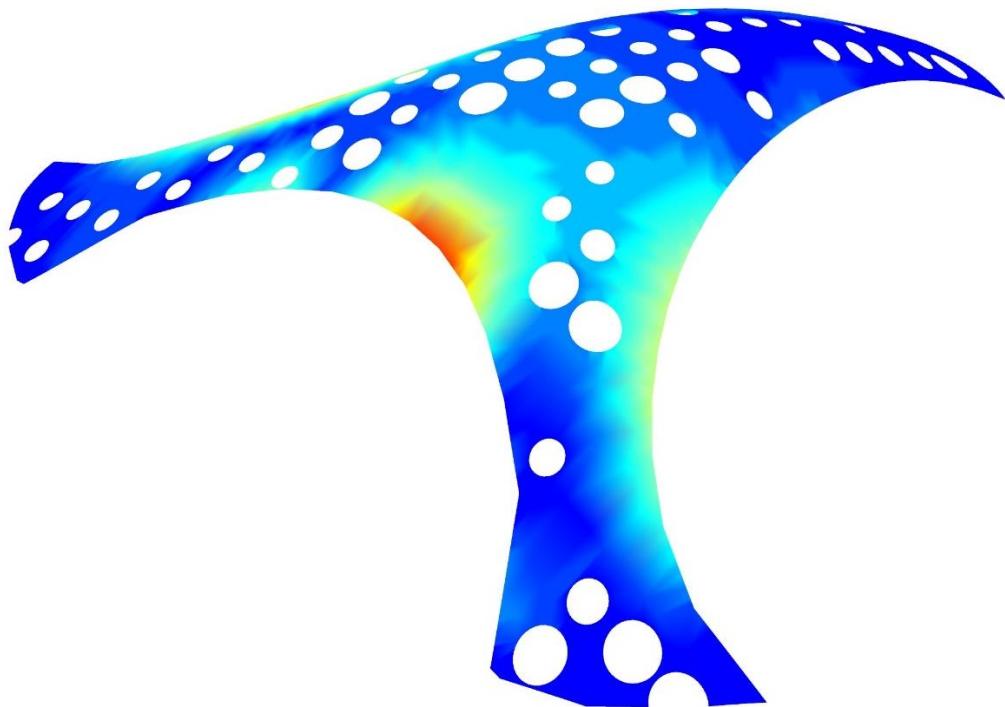
In this study, weight reduction with openings-based optimization was aimed. Two different methods have been applied to reach this goal. These methods have been tried and applied on the first designed shell (Figure 5.16).



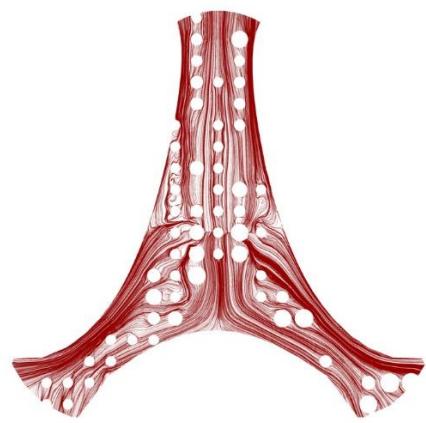
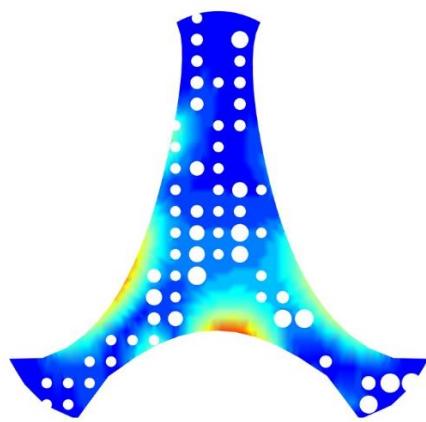
**Figure 5.16 :** Form optimization workflow

#### 5.4.1 Circle extraction

For the first openings-based optimization, the base surface is textured by selecting a tafoni photograph, which is a porous structure. The base surface was then reconstructed in the uv direction with a 20x20 points. From these points, blue dots with less deformation according to Finite Element Analysis (FEA) were selected with image sampler by hue values. The points corresponding to the gaps were selected from the selected points according to the image sampler and the porous texture brightness values. Then, circles were drawn according to the magnitudes of the brightness values of these pored areas. Circles were extruded in the Z direction and removed from the main form of the shell. New porous shell form was re-analyzed in Karamba (Figure 5.17). According to the results of the analysis, the deformation results are not changed much but it is observed that there is an increase of principal stress in the circumference of the opened pores (Figure 5.18). This process can be repeated until it reaches the desired form.



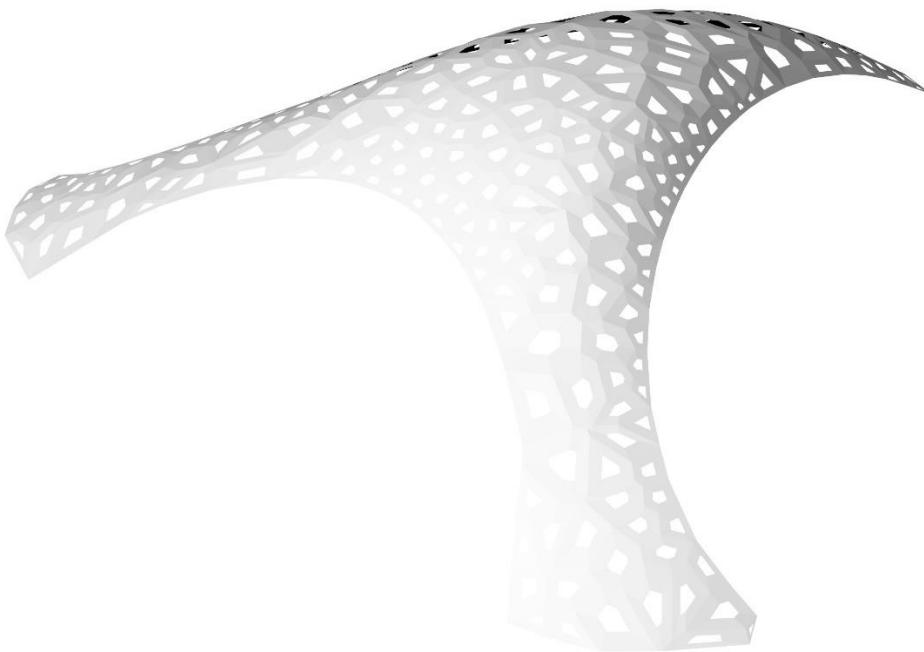
**Figure 5.17 :** Perspective view of porous shell



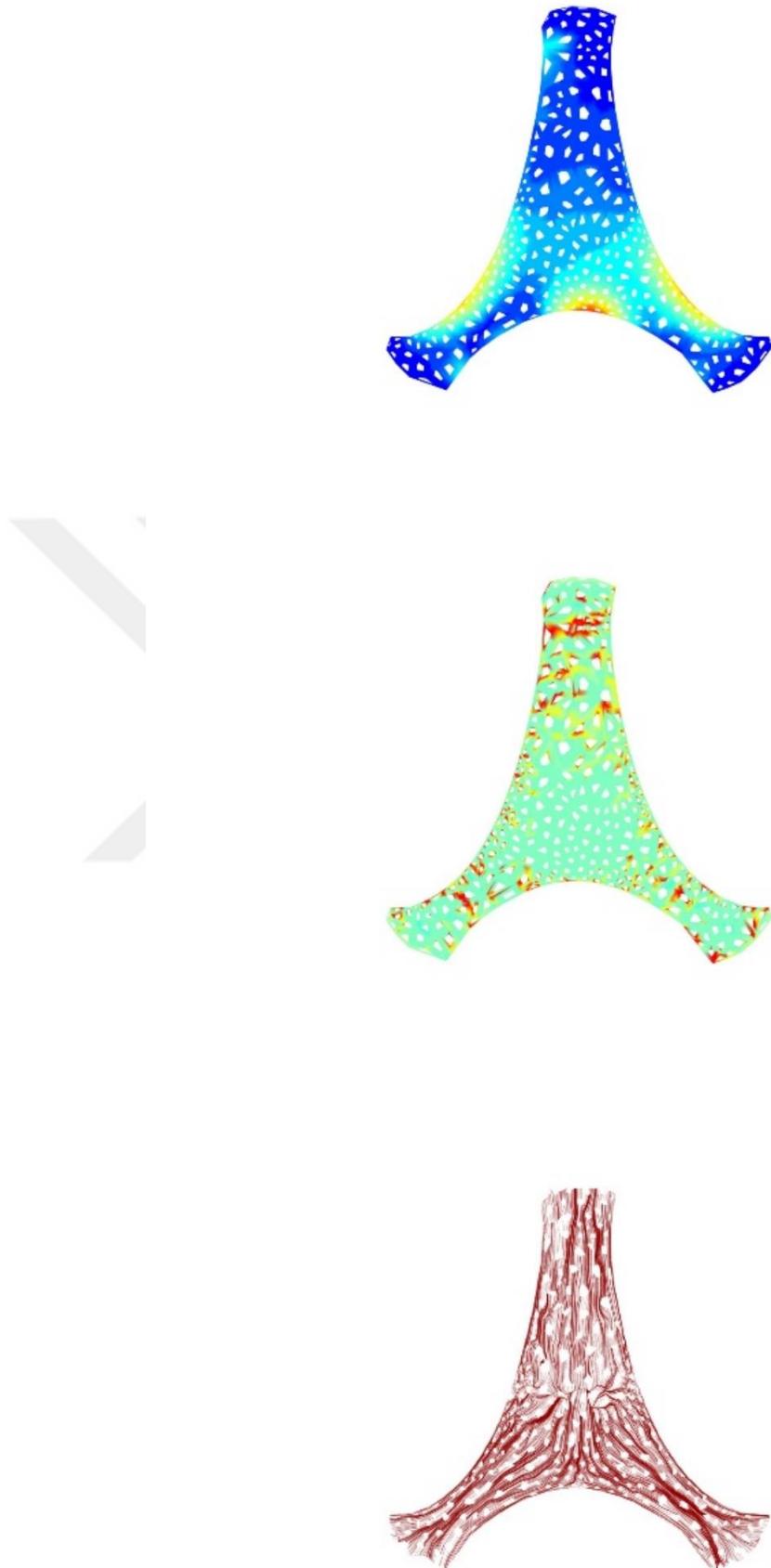
**Figure 5.18 :** Deformations, principal stresses and force flow

### 5.4.2 Voronoi surface

For the second openings-based optimization, the base surface is textured by selecting a tafoni photograph which is a porous structure again. Then the shell geometry is reconstructed in 3d by points with populate geometry. From these points, the red dots that undergo more deformation according to the Finite Element Analysis (FEA) were selected with image sampler by hue values. In addition, points corresponding to the fillings according to the values of the porous texture brightness were selected from the image sampler. The two data are merged and the duplicated points are cleared. Then a 3d Voronoi is created based on these points. The purpose here is to improve the frequency of voronoi cells in the place where these points are. This voronoi texture is intersected with the main surface. From this intersection closed polylines were formed and scaled by 0.4. These polylines were lofted to form the surface. The resulting surface was separated into parts and again made into a mesh. The form which is too complex to enter the analysis has been simplified and rebuilt (Figure 5.19). New porous shell form was re-analyzed in Karamba with same parameters. According to the results of the analysis, the deformation results are not changed much but it is observed that there is an increase of principal stress in the circumference of the opened pores (Figure 5.20). This process can be repeated until it reaches the desired form.



**Figure 5.19 :** Voronoi surface



**Figure 5.20 :** Deformations, principal stresses and force flows

## 5.5 Findings and Outcomes

In this section, a method for creating forms was first selected. Though some experiments have been done with Kangaroo via Particle Spring method, Thrust Network Analysis method has been chosen as a result. The purpose of selecting this method is to be independent of the material and be able to intervene in the process with visual feedback. Such an approach is more advantageous in the form finding phase than trying to fix pre-designed surfaces. To create a form with this method, base surfaces must first be created. It has been observed that the failure to create proper base surfaces is problematic in the later stages of form finding. Furthermore, as the subdivision ratio increased, more smooth surfaces appeared but the calculation time increased. Determining the support and opening points while creating a clean form diagram is important for the future stages. It has been learned from experiments that some kinds of drawings or interventions will make it difficult to reach the horizontal equilibrium. Some predictions were made with this knowledge and they facilitated the reach to equilibrium with changes on the diagrams and relaxation. Incorrect interventions in the process lead to inability to reach horizontal and vertical equilibrium. In the early part of this study, many experiments have been made which can not reach to the equilibrium, and important points have been learned in the process. In fact, even this part of the experiment has shown that the interventions made in the early design stages give us an advantage over the later stages. After the formation of the form, the vertical and axial forces were color-coded independently from the material. These are simple facts and have been passed on to Karamba for more detailed analysis.

The main purpose of using Karamba is to observe the effects of the properties of the material. The Karamba was selected because the other phases of the design were suitable for the Grasshopper environment in which it was made in a way that the architects could understand. There are more Mesh and NURBS-based Finite Element Analysis programs but, in fact, most of them are more complex programs for engineers. The lack of expertise in Finite Element Analysis has been one of the most challenging issues when conducting experiments. A study can be done with the basic knowledge learned in the process but engineer support is needed for more extensive studies. In Karamba, the geometric model was first turned into a structural model. Constraints like loads, supports, cross section, material and analyze method must be

set for this. Loads different than gravity are not considered because the main load is self-weight in shell structures. While normally deformations are in millimeters, we can better visualize deformations by time with giving extreme values to gravity load. The results of the FEA analysis provides us lot of information but it is displacement that is important in this study. Extraction in areas with fewer displacement is planned at the beginning of the study.

In the last part of the study, it was aimed to decrease the weight by openings according to the porous textures taken from the images. In doing so, different ways have been tried like selecting the mesh vertices in 3d coordinates and apply Boolean operations, but the hardware has been inadequate due to complex calculations. Therefore, the method and the 3d model have been simplified and continued to work. At the beginning of the study, the goal was to design a process that would take structural features in early design phases into account when designing the shell instead of creating a random form and openings. As a result of this operation, it can be seen that the displacement is still not critical in the analysis of the structure with openings. By following this process in the early design phase, forms have been produced at later stages that can facilitate production and calculation rather than a randomly generated form. And a great number of shell variations can be produced using the same method.



## 6. CONCLUSION AND DISCUSSION

It has always been interesting to examine the methods and processes that make up the shell design process, which was initiated by Gothic vaults, and continued with Candela, Isler and Otto experiments. The works of the pioneers in this area has led to create methods of computational design and form finding along with the development of computers. The aim of this study is to understand and implement these processes.

In this study, the geometry of the surfaces was first investigated to understand the shell structures. Double curved surfaces are preferred in terms of their structural strength. Then the designs of the pioneers in this area and the methods of form finding have been examined. Criteria and applications of computational form finding methods after advancement in mathematics and computational tools have been described.

In the experiments, it is emphasized how the process of producing a shell structure using different computational tools can be used. In this study, first basic surfaces were created in Rhino, then form diagrams were created by TNA method using RhinoVault plugin, then force diagrams were created. These diagrams were intervened to find an equilibrium state. Openings, support points and diagrams have always been intervened and relaxed where necessary to find the balance. It is a process that repeats itself until it reaches the equilibrium state. In this process, the balance could not be reached at first, but in the later experiments, various form alternatives emerged while reaching the balance. And three different form alternatives covering same area were chosen. These alternatives were analyzed in Karamba by Finite Element Analysis to determine the parts exposed to the loads. As a result of this review, we observed that the displacement was at a low level, and the verification of the form finding method was established.

Later, extractions made in the structure through openings to reduce the weight and produce new form alternatives. The reductions were made by taking the brightness information of a pores on image of Tafoni. At the first extraction, the points which less deformed were selected and the circles were drawn by changing the diameters according to the brightness values of the pores in the Tafoni image, and these circles

were extruded. Extruded circles are extracted from the main structure by Boolean operations. In the second extraction, more deformed parts of the structure are joined together to intersect with the image data of the pores, and created a Voronoi shell with less openings at the very deformed points is obtained. These forms were then analyzed again and it was observed that there was no increase in deformations that would distort the structural balance of the shell. However, the force flow lines have changed and the stresses around the openings have increased.

In this process, some difficulties were encountered while working. Transitions between mesh and NURBS surfaces, long processing times of complex geometries, and lack of expertise in Finite Element Analysis has been a compelling part of this study. Nevertheless, these problems have been solved by simplifying methods and forms. The forms produced in the scope of this study should not be seen as final products. But this study has shown us that this method is applicable and produces form alternatives for shell structures.

According to the results of the analysis, the importance of proper shell design in early stages instead of designing a random surface is observed. Optimization is tried to be done in the form with openings made.

In further studies, process of form finding on shell structures and structural analysis can be deepened through implementing genetic algorithms. Loads, cross-sections and materials can be changed to find the desired optimal deformation value. Shell alternatives of different thicknesses or materials, which provide the same structural integrity, and covering the same area can be produced. The effects of other loads can also be examined before proceeding to the fabrication phase.

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## APPENDICES

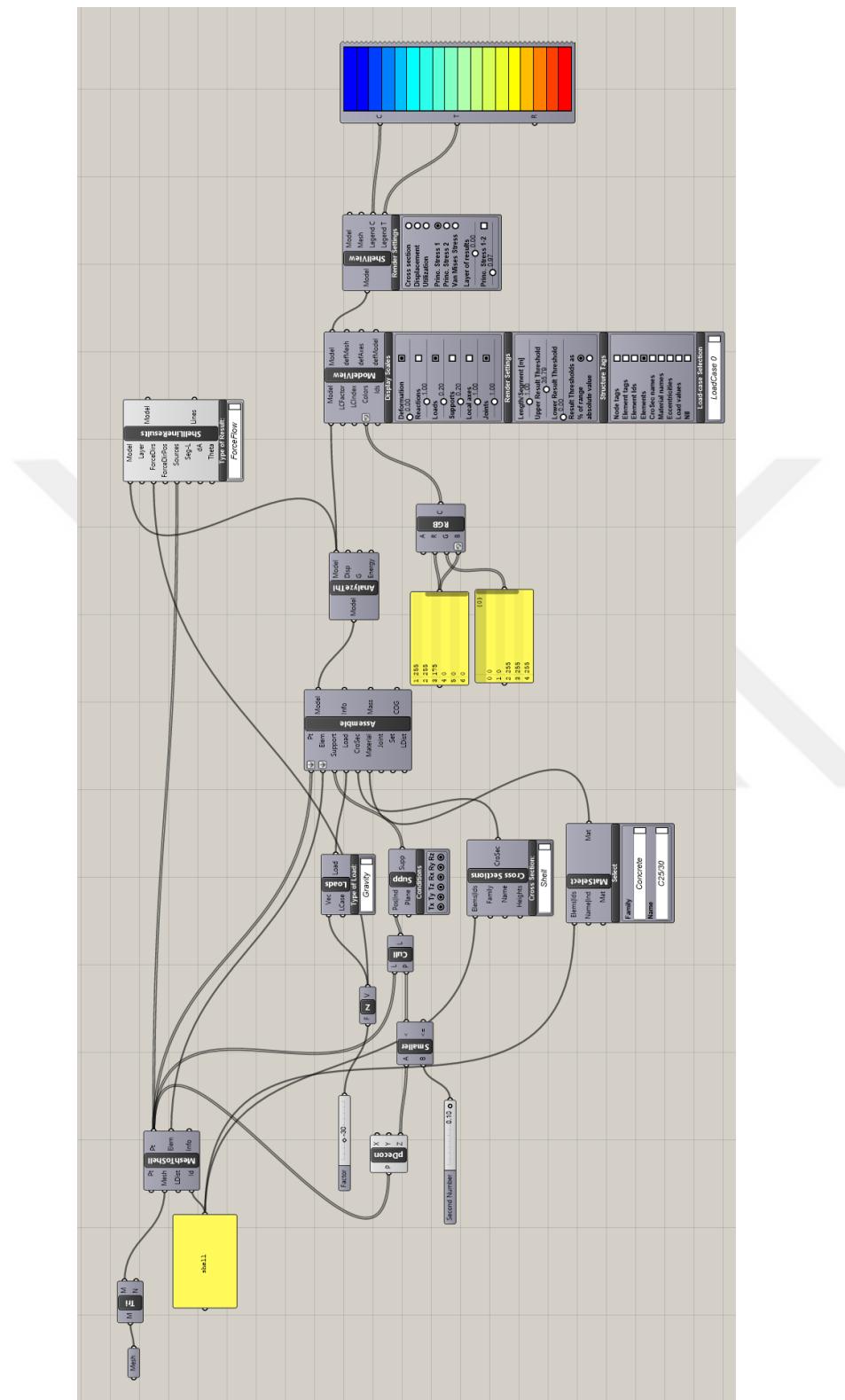
**APPENDIX A:** Karamba analysis algorithm

**APPENDIX B:** Circle extraction algorithm

**APPENDIX C:** Voronoi surface algorithm



## APPENDIX A



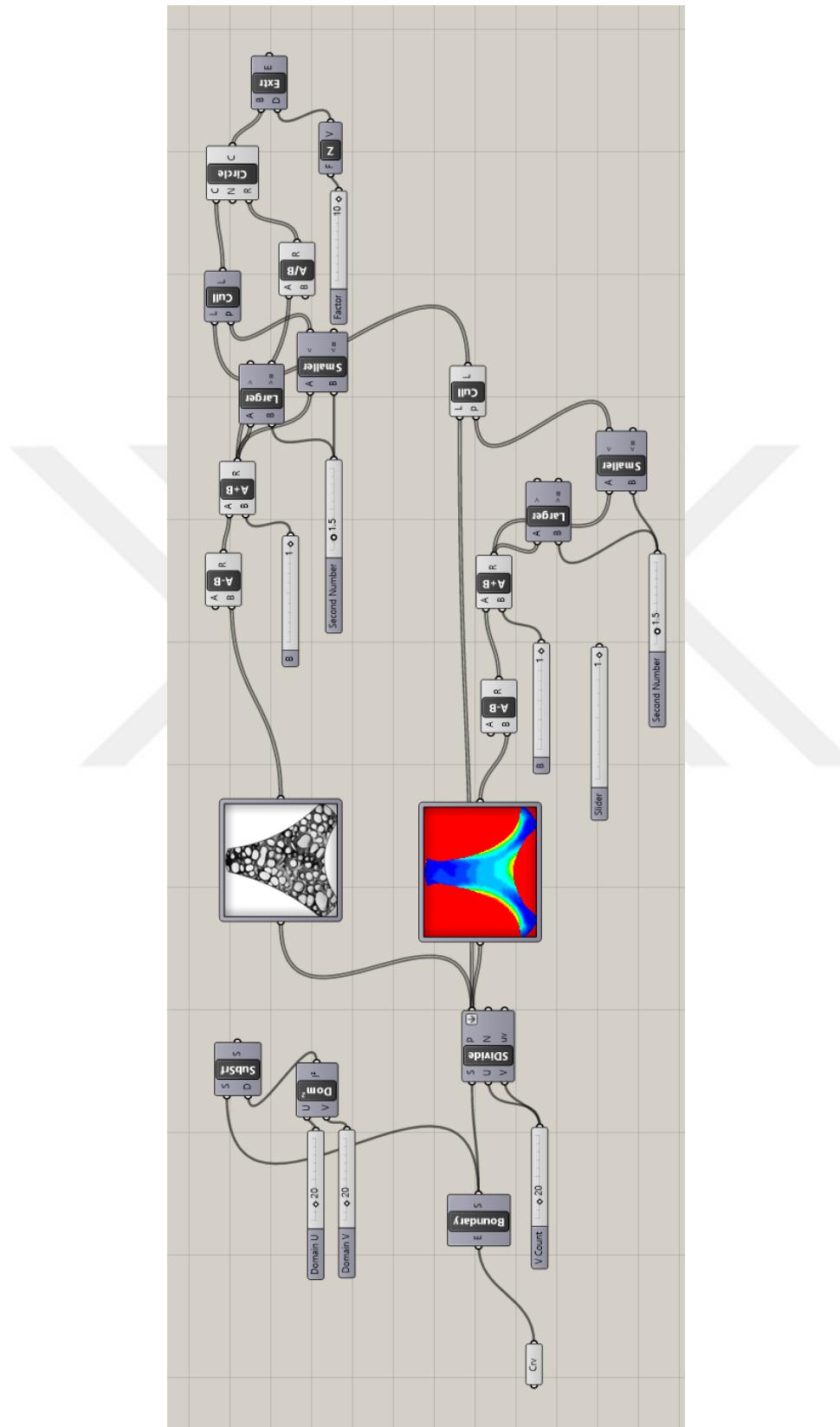
**Figure A.1 :** Karamba analysis algorithm

## APPENDIX B



**Figure B.1 :** Circle extraction algorithm

## APPENDIX C



**Figure C.1 :** Voronoi surface algorithm

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