ELECTRICITY PRICE FORECASTING USING HYBRID TIME SERIES MODELS

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submitted by **BÜŞRA TAŞ** in partial fulfillment of the requirements for the degree of **Master of Science in Statistics Department, Middle East Technical University** by,

Prof. Dr. Halil Kalıpçılar _____
Dean, Graduate School of **Natural and Applied Sciences**

Prof. Dr. Ayşen Dener Akkaya _____
Head of Department, **Statistics**

Assoc. Prof. Dr. Ceylan Yozgatlıgil _____
Supervisor, **Deparment of Statistics, METU**

Examining Committee Members:

Assoc. Prof. Dr. Özlem İlk Dağ
Deparment of Statistics, METU _____

Assoc. Prof. Dr. Ceylan Yozgatlıgil
Deparment of Statistics, METU _____

Prof. Dr. Ömür Uğur
Institute of Applied Mathematics, METU _____

Assoc. Prof. Dr. Seher Nur Sülkü
Deparment of Econometrics, Hacı Bayram Veli University _____

Assoc. Prof. Dr. Könül Bayramoğlu Kavlak
Department of Actuarial Science, Hacettepe University _____

Date: _____

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last Name: BÜŞRA TAŞ

Signature :

ABSTRACT

ELECTRICITY PRICE FORECASTING USING HYBRID TIME SERIES MODELS

Taş, Büşra
M.S., Department of Statistics
Supervisor : Assoc. Prof. Dr. Ceylan Yozgatlıgil

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Accurate forecasting of hourly electricity price is very important in a competitive market. Decision makers highly benefit from accurate forecasting. Because electricity cannot be stored, shocks to demand or supply affect the electricity prices. As a result, electricity prices show high volatility. Additionally, it may have multiple levels of seasonality. Therefore, forecasting with conventional methods is very difficult.

In this study, hybrid models are constructed with Seasonal Autoregressive Integrated Moving Average (SARIMA), TBATS and Neural Network models for the analysis of hourly electricity prices in Turkey. Time series can contain both linear and nonlinear patterns. Thus, using a hybrid model can give better results in forecasting. Both linear and nonlinear parts of the time series can be modeled by this approach. While SARIMA model and TBATS model are used to capture the linear behavior of the electricity price series. Neural Network is used to model the nonlinearity in the series. Electricity demand is used as exogenous variable. Different combinations of hybrid

models and individual models are compared in terms of forecasting performance. The results indicate that mostly hybrid models outperform the individual models in one-week ahead and one-day ahead forecasting.

Keywords: Electricity Price Forecasting, Hybrid Method, Artificial Neural Network, NARX, Time Series

ÖZ

HİBRİT ZAMAN SERİSİ MODELLERİYLE ELEKTRİK FİYAT TAHMİNİ

Taş, Büşra
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Tez Yöneticisi : Doç. Dr. Ceylan Yozgatlıgil

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Rekabetçi piyasalarda saatlik elektrik fiyatlarının doğru öngörülmesi çok önemlidir. Karar vericiler doğru öngörülerden oldukça yararlanmaktadır. Elektrik depolanamadığı için talep ve arzda meydana gelen şoklar elektrik fiyatlarını etkilemektedir. Sonuç olarak elektrik fiyatları yüksek dalgalanma göstermektedir. Buna ek olarak birden fazla mevsimselliğe sahip olabilir. Bu sebeple geleneksel yöntemlerle öngörülerin doğru bir şekilde elde edilmesi güçtür.

Bu çalışmada, Türkiye'deki saatlik elektrik fiyatlarının modellenmesi için SARIMA, TBATS ve yapay sinir ağları modelleri kullanılarak hibrit modeller oluşturulmuştur. Zaman serilerinde hem doğrusal hem de doğrusal olmayan yapılar bulunabilir. Bu nedenle, hibrit modeller öngörüde daha iyi sonuçlar verebilir. SARIMA ve TBATS modeller elektrik fiyatlarının doğrusal hareketini yakalamak için kullanılırken, serideki doğrusal olmayan yapıyı modellemek için sinir ağları kullanılmıştır. Elektrik talebi dışsal değişken olarak kullanılmıştır. Hibrit modellerin farklı kombinasyonları

ve tekli modeller öngörü performansına göre karşılaştırılmıştır. Sonuçlara göre bir haftalık ve bir günlük öngörülerde hibrit modeller çoğunlukla tekli modellere göre daha iyi performans göstermiştir.

Anahtar Kelimeler: Elektrik Fiyat Tahmini, Hibrit Metot, Yapay Sinir Ağları, NARX, Zaman Serileri

To my family

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LIST OF ABBREVIATIONS

ACF	Autocorrelation Function
AICc	corrected Akaike Information Criterion
ANN	Artificial Neural Network
ARMA	Autoregressive Moving Average
ARMAX	Autoregressive Moving Average with Explanatory Variable
BIC	Bayesian Information Criterion
DSHW	Double Seasonal Holt Winters
EPF	Electricity Price Forecasting
GARCH	Generalized Autoregressive Conditional Heteroskedasticity
MAE	Mean Absolute Error
MAPE	Mean Absolute Percent Error
MPE	Mean Percentage Error
MSE	Mean Squared Error
NARX	Nonlinear Autoregressive Exogenous
PACF	Partial Autocorrelation Function
RMSE	Root Mean Square Error
SARFIMA	Seasonal Autoregressive Fractionally Integrated Moving Average
SARIMA	Seasonal Autoregressive Integrated Moving Average
SVM	Support Vector Machine
TBATS	Trigonometric Seasonal, Box-Cox Transformation, ARMA residuals, Trend, Seasonality

CHAPTER 1

INTRODUCTION

In many countries all around the world, the electric power industry is converting to a competitive market. Since 1980, many reforms have been made to reduce the government control in many sectors. Most of the major economies started liberalization of energy market in the early 1990s by some deregulations. The first step of these reforms is separation of some functions of electricity market. These functions are generation, transmission, distribution and marketing of electricity. These functions need to be carried out by private firms to ensure competition. Nowadays, electricity is traded according to market rules by contracts in many countries.

Electricity is a commodity that cannot be stored. This means that it is necessary to have a balance between production (supply) and consumption (demand). Demand shows high variability since it depends on weather, business activities, etc. Additionally, variation of supply depends on demand. Therefore, spot prices show seasonality at multiple levels and high volatility. There are also unexpected excessive changes in the price which are called spikes or jumps. These features make forecasting very difficult with conventional methods. There are various studies on electricity price forecasting in the literature.

According to Weron [1], there are five categories of electricity price forecasting methods.

- Multi-agent models are multi-agent simulation, equilibrium and game theoretic models.

- Fundamental models define the price characteristics by modeling the effects of some significant factors on the electricity price.
- Reduced-form models are quantitative, stochastic models which define statistical properties over time.
- Statistical models are application of statistical techniques on electricity price.
- Computational intelligence models are artificial intelligence-based, non-parametric, non-linear statistical models.

There are many studies that use hybrid methods which combine methods from these groups. In this study, hybrid methodology is adopted for electricity price forecasting by using statistical models and computational intelligence models. Time series can contain both linear and nonlinear patterns. Generally, statistical models are not very good at capturing nonlinear structure in the series while computational intelligence models are not good at modelling linear part. Besides that using only a computational intelligence model like neural network model can cause overfitting problem. Thus, combining these two types of models by using a hybrid model can give better results in forecasting. A taxonomy of electricity price modelling methods can be seen in Figure 1.1.

The motivation behind this study is to find the method that has the best forecasting performance for electricity prices in Turkey. The electricity consumption of the previous day is considered to have an effect on electricity prices of current day. Therefore, electricity consumption of the previous day is taken as electricity demand which is included as an exogenous variable to the modelling. First aim of this study is to construct a hybrid model for an hourly series with an exogenous variable. This aim is fulfilled by conducting several hybrid models with different combinations of various methods. Second aim is to compare forecast performances of different hybrid models and also individual models. The comparison is made according to one-week ahead forecast and one-day ahead forecast. The results are similar for both cases. The contribution of this study to the literature is that as far as our knowledge it is the first application of hybrid method to forecast hourly electricity prices in Turkey.

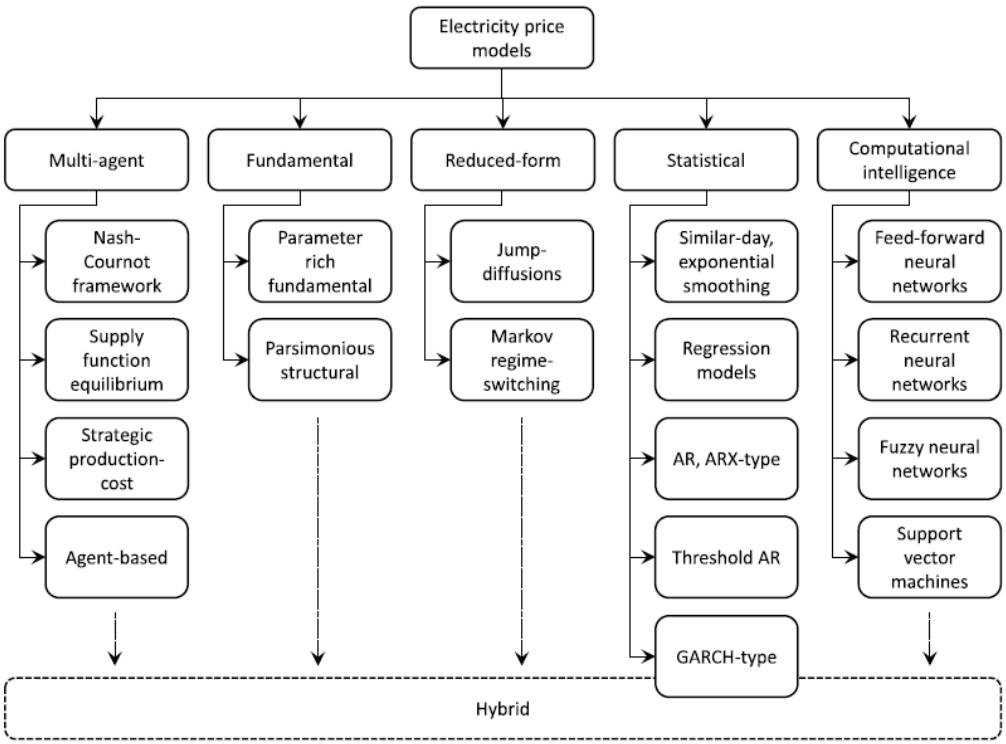

Figure 1.1: A taxonomy of electricity price modelling methods [1]

This thesis has mainly six sections. The next chapter is about electricity markets. It presents the characteristics of an electricity market. Then it summarizes the liberalization movements of electricity markets in the world and in Turkey. In chapter 3, literature research on electricity price forecasting is presented. The first 3 parts cover the studies around the world while $4th$ part covers the studies in Turkey. The first section reviews traditional time series models. The second section includes computational intelligence models in the literature. The third section shows studies on hybrid models. The last section reviews electricity price forecasting studies in Turkish electricity market. In chapter 4, methodology behind this study is explained. The first section shows AR-type and ARX-type models. The second section presents some of the exponential smoothing methods. The third section presents artificial neural network models. Last section of this chapter shows the hybrid methodology which is the main method of this thesis. Chapter 5 presents the modeling of one-day ahead and one-week ahead forecasting of Turkish electricity prices. This chapter consists of two sections which are hybrid models and individual models. Chapter 6 is devoted to

conclusion and future work. Appendix A contains time series plots of actual values and predicted values and table of RMSE values for prediction performances of all models.

CHAPTER 2

ELECTRICITY MARKET

Electricity is a form of energy which can be used in a wide range of application areas such as heat, light and power [2]. Additionally, it can be generated from other sources such as coal, natural gas, oil, nuclear power etc.

Before the deregulation process in the electricity markets, regulators fixed prices as a function of generation, transmission and distribution costs and so there were not much uncertainty in prices [3]. After the deregulation process, prices are determined by supply and demand. Suppliers buy energy from generators and sell it to the customers according to demand. Therefore, prices are determined hourly by intersection of total demand and supply in the market pool. Indirectly, electricity can be stored via hydroelectric schemes or storage of generator fuel. However, it cannot be physically stored directly, so production and consumption of electricity should be balanced. Therefore, shocks to demand or supply have an impact on electricity spot prices. Due to non-storability of the electricity, prices show extremely high volatility in all markets, and this makes electricity price forecasting challenging.

Demand and supply have significant roles in the volatility. Since electricity is a needed commodity, electricity demand is very price-inelastic. Price-inelastic demand means that even if the price of electricity increase excessively, demand for electricity will decrease a small amount. In this study, electricity demand is used as exogenous factor in modeling the electricity price. It is also difficult to model the unanticipated extreme changes in prices which is called spikes or jumps. After the jumps, prices have a tendency to move back to its prior value rapidly. This is another characteristic

of the electricity price series which is called mean reversion.

Electricity demand highly depends on economic activities, weather conditions and amount of sunlight. These factors create seasonal behaviors in the prices. There can be intra-daily, weekly and intra-year seasonality. The existence of multiple levels of seasonality is another challenge in the modelling.

In short, the general characteristics of electricity price series in competitive markets are [4]:

- High volatility
- Multiple seasonality (daily, weekly seasonality)
- Calendar effect (holidays)
- Spikes
- Nonconstant mean and variance
- High frequency

2.1 Liberalization of Electricity Markets

Operations in the electricity markets are generation, wholesale, transmission and distribution [5]. Generation of the electricity can be made in two ways that are by fuel fired power plants or by using renewable energy sources. Wholesale is the sale of electricity in large quantities and at low prices, to customers who are allowed to make bilateral contracts with the suppliers. Transportation of electricity from power plants to substations is called transmission. Distribution is the sale and distribution of electricity to users. In vertically integrated electricity markets; generation, transmission and distribution are managed by the same state-owned company. In deregulated electricity markets, these operations are operated by private companies yet transmission is generally performed by the state owned company.

Liberalization is the splitting the generation, transmission and distribution of electricity to make the market competitive. This way generation and transmission will be improved and prices will be determined by the demand-supply balance. Since the 1980s, liberalization of electricity markets has started worldwide. In these reforms, common idea was privatizing state monopolies and decreasing government controls [6].

Liberalization of the electricity market started with Chile in 1982. This reform was based on the idea of splitting generation and distribution companies. In 1990, electricity market of England and Wales is reorganized. Then, these reforms followed by the opening of Nordic market in 1992. The number of liberalized electricity markets is increasing around the world, especially in European countries [7].

2.1.1 Turkish Electricity Market

The financial resources were limited in Turkey after World War 1 [8]. Therefore, investments cannot be made in the electricity sector and electricity was traded from foreign companies. Since the early 1960s, electricity sector has grown as a result of rapid industrialization [9]. In 1963, the Ministry of Energy and Natural Resources (MENR) of Turkey was founded. In 1970, the Turkish Electricity Authority (TEK) was established as an integrated monopoly by combining all electricity activities except distribution, which was assigned to municipalities until 1982 [10]. TEK had the monopoly power until 1984. There have been important changes in monopolistic electricity system of Turkey.

In 1984, private entities were allowed to take in charge of generation, transmission and distribution activities of electricity. Three different financial models were tried to fulfill investment needs of electricity sector. However, these models were in need of a guarantee from Treasury [10].

Turkey started to restructure its electricity market in the early 2000 with the introduction of the first Electricity Market Law. The law was revised in 2012, 2013 and 2016. With the deregulations, the aim is to ensure a transparent and competitive industry .

Energy Market Regulatory Authority (EMRA; EPDK in Turkish) was established to control market players and regulate market rules. In 1993, TEK was separated into two companies: the Turkish Electricity Generation Transmission Company (TEAS) was responsible of generation and transmission, and the Turkish Electricity Distribution Co. (TEDAS) was responsible for distribution and retail sale activities. In 2001, TEAS was further separated into three companies, Electricity Generation Company (EÜA), Turkish Wholesale Company (TETA) and Turkish Electricity Transmission Company (TEA) which were responsible for generation, trading and transmission respectively [9]. Energy Market Regulatory Authority (EPDK) was also established to control market players and regulate market rules [11].

After liberalization, participants could sell and buy electricity in a liberal market environment [11]. Nowadays, the price of electricity is settled in the balancing and settlement market or by bilateral contracts between parties [12]. TETAS is responsible for purchasing the energy within the provisions of vesting contracts between the government and private parties. Since 2015, the wholesale electricity market in Turkey has been operated by Energy Exchange Istanbul (EXIST) [13]. There are two electricity spot markets which are day-ahead and intra-day markets. A daily double-sided blind auction is held in the day-ahead market under the principle of uniform pricing. On the other hand, intra-day market is operated under continuous trading mechanism. Since spot market has an impact on electricity prices, forecasting is very important for market participants.

CHAPTER 3

LITERATURE REVIEW

Many studies have been carried out on spot electricity price forecasting and electricity demand modeling after the market deregulation. Since electricity cannot be stored, there should be a balance between production and consumption. This lead to some specific price dynamics such as daily, weekly and annual seasonality and sudden price spikes. Therefore, electricity price forecasting is very important in a competitive market. Market participants can obtain huge financial gains with accurate forecasting.

Hahn et al. [14] classify forecasting methods into two categories: classical time series and regression methods and artificial intelligence and computational intelligence methods. There are also hybrid models which is the combination of two or more different categories.

Most of the studies mentioned in this chapter is the summary of work of Weron [1] who already reviewed most of the studies on electricity price forecasting (EPF) from 1989 to 2013. Studies published after 2013 are also mentioned.

3.1 Classical Time Series and Regression Methods

Time series and regression methods predict the current price from previous values of price and explanatory variables and also current value of explanatory variable by using a mathematical relation. They usually have limited power to model nonlinear behavior of electricity prices.

3.1.1 Regression Models

In multiple regression, the aim is to learn the relationships between input variables and a target variable. In EPF, lagged electricity prices are included as regressors.

Kim et al. [15] use wavelet decomposition with multiple regression. The regression coefficients are found by using the wavelet decomposition detail series and the predicted demand. Then, one-day ahead price forecast is obtained. Conejo, Contreras et al. [16] used a similar method for hourly electricity price. Schmutz and Elkuch [17] also apply multiple regression for price forecasting.

Koopman, Ooms, and Carnero [18] forecast daily electricity spot prices by using seasonal periodic regression models with ARIMA, ARFIMA and GARCH disturbances. Karakatsani and Bunn [19] use data from British market to conduct a regression model for half-hourly load periods, and compare its day-ahead forecasting performance to regime-switching regression models. Bordignon et al. [20] utilize linear regression and time-varying parameter regression models with different forecast combinations.

Azadeh et al. [21]) introduce a method which changes between the predictions of different models according to some rules, and make long term forecasting. Jonsson et al. [22] propose a methodology which focuses on the effect of the predicted system load and wind power generation.

3.1.2 AR-type Time Series Models

In an ARMA(p, q) model, the current value of X_t depends on p past values of X and q previous values of the noise. AR-type models are the most basic time series models for forecasting.

Cuaresma et al. [23] use data from German European Energy Exchange market to apply ARMA processes for short-run EPF. In a related study, Weron and Misiorek [24] utilize different autoregressive models to forecast prices in the California market.

Misiorek et al. [25] use an AR model to compare with more complex models in the California market. Jonsson et al. [22] also use a simple AR model for short-run EPF.

Haldrup and Nielsen [26] use seasonal ARFIMA models to forecast the Nord Pool market prices. Lagarto et al. [27] forecast the one-day ahead Spanish market prices by using an ARIMA model applied to the conjectural variations of the firms. They conclude that it is slightly better than a standard ARIMA model.

Amjady and Hemmati [28]; Che and Wang [29]; Cruz et al. [30]; and Tan et al. [31] also use AR-type models in EPF. In these papers, they compare the performance of more complicated models or hybrid models including neural networks, support vector machines or GARCH components.

3.1.3 ARX-type Time Series Models

There are also external factors that affect electricity price which are called exogenous variables. Time series models with exogenous variables can be used to discover the relationship between prices and other significant variables. These models can be considered as generalizations of existing models. For example, ARX, ARMAX, ARIMAX and SARIMAX are extended versions of AR, ARMA, ARIMA and SARIMA, respectively. In EPF, there are a lot of studies with time series models with exogenous variables.

Nogales et al. [4] call ARMAX model transfer function and ARX model dynamic function and use them for forecasting hourly prices in California and Spain. The results are significantly better than models proposed by Contreras et al. [32]. Nogales et al. [33] use the same methods for a different series. Conejo, Contreras et al. [16] use different models to compare for short term EPF: three time series models, namely a wavelet multivariate regression technique, and a multilayer perceptron (MLP) with one hidden layer.

Weron and Misiorek [24] and Misiorek et al. [25] employ 24 ARX models, one for each hour of the day, by using load forecast as exogenous variable and three dummies

for weekly seasonality. They found that these models have a better performance than a single ARIMA model for all hours [32], and have a worse performance than the transfer function and dynamic regression models [4].

Knittel and Roberts [34] use different models for one-day ahead price forecasting in the California market, a seasonal ARMA model with the temperature, squared temperature and cubed temperature as explanatory variables is one of these models.

Zareipour et al. [35] find 3 hours ahead and 24 hours ahead forecast of Ontario energy price. The forecast performances of ARMAX and ARX models are compared with ARIMA models.

Weron and Misiorek [36] use various time series models for one-day ahead EPF using hourly spot prices and loads in California and hourly spot prices and air temperatures from Nordic market. These models are AR models which are spike preprocessed, threshold, semiparametric and mean-reverting jump diffusions. They found that models with the electricity load as the explanatory variable usually have better performance than only price models. Additionally, semiparametric models, and the smoothed nonparametric ARX model, perform better than other models and they perform well under different market conditions.

Lira et al. [37] make one-day ahead EPF for Colombian market by using a fuzzy logic model and ARMAX models which identified by a Kalman filter. The input variables in the models are reservoir levels and load. Cruz et al. [30] compare the accuracies of various models: SARIMA, double seasonal exponential smoothing, dynamic regression and a feedforward neural network, and conclude that their accuracies can be increased significantly by including the system operators wind generation forecasts.

Kristiansen [38] changes the model of Weron and Misiorek [36] to involve Nordic demand and Danish wind power as explanatory variables and predicts prices together for all hours. Caihong and Wenheng [39] introduce a new method for the system identification of multi-input, single output ARMAX models using the CPSO algorithm on California market. Bordignon et al. [20] use an ARMAX model in the analysis with different forecast combinations.

3.1.4 Exponential Smoothing

Exponential smoothing is commonly used in electricity load forecasting, however there are not many studies on EPF.

THETA method is proposed by Assimakopoulos and Nikolopoulos [40] which is a variant of exponential smoothing method. The method was performed in the M3 forecasting competition (Makridakis and Hibon, [41]).

Cruz et al. [30] use double seasonal exponential smoothing to compare with more sophisticated models in Spanish market. They found that exponential smoothing has a slightly better performance than ARIMA for hourly electricity prices. On the other hand, these models perform worse than dynamic regression models or neural network models.

3.1.5 Threshold Autoregressive Models

Threshold Autoregressive (TAR) model is proposed by Tong and Lim [42] which assumes that the regime can be determined by an observable variable relative to a threshold value. Exogenous variables can be included by extending TAR model to TARX model. Besides, some of the nonlinear time series models are Self Exciting TAR (SETAR) model, Smooth Transition AR (STAR) model and Logistic STAR (LSTAR) model.

Robinson [43] builds an LSTAR model to electricity prices in the Great Britain market, and shows that its performance is much better than linear autoregressive alternatives. Stevenson [44] adjusts AR and TAR processes to wavelet filtered half-hourly series and concludes that the forecast performance of TAR model surpasses the AR model. Rambharat et al. [45] offer a SETAR model with temperature as input variable and a gamma distributed jump component for the daily data from Pennsylvania. The model is estimated by using Markov chain Monte Carlo method.

Weron and Misiorek [46] use California market data to conduct TAR and TARX mod-

els with electricity load as the exogenous variable. The last hour of previous day is used as threshold variable in the TARX models. A multi-step optimization procedure is used to estimate threshold levels of every hour. In another study, Misiorek et al. [25] use an expanded range of threshold variables for testing. As a result, they found that using the difference between the mean prices for yesterday and eight days ago as threshold variable gives better forecasting performance. Then, Weron and Misiorek [36] use the same TARX models for Nord Pool data. They conclude that mean errors of TARX models are the worst in a more regular period, there is a great difference between the actual price and predicted value.

Chen and Bunn [47] use logistic smooth transition regression to test that electricity spot price dynamics show a pattern of varying intra-day nonlinear functions of other significant variables. They built different models for different periods of the day which are off-peak, morning peak and evening peak.

3.1.6 Heteroskedasticity and GARCH-type models

In the linear AR and ARX-type models, homoscedasticity (constant variance) assumption must be satisfied. However, electricity spot prices show nonlinear dynamics which cause heteroscedasticity. Engle [48] proposed the Autoregressive Conditional Heteroskedastic (ARCH) for the problem of heteroscedasticity and Bollerslev [49] extended this model and propose the Generalized Autoregressive Conditional Heteroskedastic (GARCH) model. In EPF, GARCH model is generally used with an AR-type model where the residuals of the regression part are modeled with a GARCH process.

In the study of Knittel and Roberts [34], an AR-EGARCH model is used for EPF in California. They conclude that this model is better than other models for crisis period, but it has the worst performance for pre-crisis period. Garcia et al. [50] deduce that ARIMA-GARCH performs better than an ARIMA model when there is high volatility and price spikes.

Diongue et al. [51] use prices from Germany to examine conditional mean and con-

ditional variance forecasts by a dynamic model with a k-factor GIGARCH process. This model has a better forecasting performance than SARIMA-GARCH model.

Karakatsani and Bunn [52] use three modeling approaches and one of the conclusions is that allowing for the time-varying responses of prices to market fundamentals can give more accurate volatility estimates than an explicit GARCH model.

Tan et al. [31] apply a wavelet decomposition to price series in the Spanish and PJM electricity markets, then predict each subseries by an ARIMA-GARCH model or a GARCH model.

Gianfreda and Grossi [53] use the Italian power market data to examine the effect of technologies, market concentration, congestions and volumes on price dynamics by applying the model of Koopman et al. [18] which uses periodic extensions of dynamic long-memory regression models with autoregressive conditional heteroscedastic errors. They conclude that the models perform better when these factors are taken into account. Huurman et al. [54] use GARCH-type time-varying volatility models to forecast Scandinavian electricity prices. The results show that models extended with weather forecasts perform better.

3.2 Computational (Artificial) Intelligence Models

Computational intelligence methods are computational methods that have been designed to handle the problems which traditional methods cannot solve effectively. They are considered as intelligent because of their ability to adapt to complex dynamic systems. The main categories are artificial neural networks, fuzzy systems and support vector machines.

Artificial neural networks can be classified into two groups given in Figure 3.1, namely feed-forward neural networks and recurrent neural networks which will be explained in detail in the next chapter.

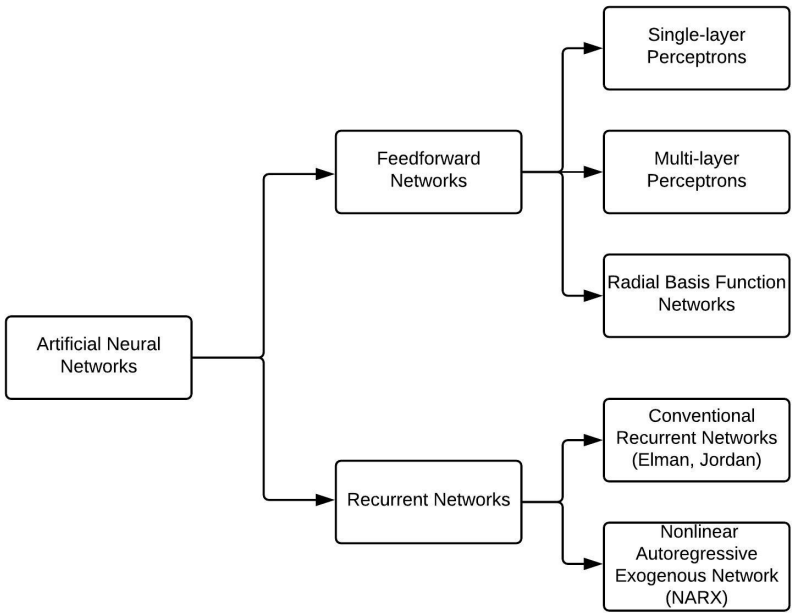

Figure 3.1: A taxonomy of artificial neural networks that are most commonly used in EPF

3.2.1 Feed-forward Neural Networks

Feed-forward neural network is an example of static neural networks which obtains output directly from the input [55].

Aggarwal et al. [56] use back-propagation training algorithm for the MLP in EPF applications. Another training algorithm is Levenberg-Marquardt algorithm, there are several applications in EPF such as Catalão et al. [57]; Pindoriya et al. [58]; and Rodriguez and Anders [59]. Pao [60] uses a generalized delta learning rule. Zhang and Luh [61] apply neural networks using decoupled extended Kalman filter as an integrated adaptive learning and CI estimation method with a modified U-D factorization method.

There are more complicated methods; for example, Gareta et al. [62] apply a combination of univariate MLP networks where different networks forecast maximum, minimum and medium values of the electricity price. Hu et al. [63] use a market concentration index as an exogenous variable in MLP and conclude that it has an effect

on forecasts. Besides that, MLP network has been used by Chen et al. [64]; Cruz et al. [30]; Garcia-Ascanio and Mate [65]; Gareta et al. [62]; Mandal et al. [66]; Pindoriya et al. [58]; and Yamin et al. [67]. Additionally, RBF network has been applied by Guo and Luh [68]; Lin, Gow, and Tsai [69]; Pindoriya et al. [58]; and Yao et al. [70].

Amjady and Keynia [28] use a multi-layer perceptron model where the numbers of hidden and input units are specified by an iterative method. Chaâbane [71] build a model to predict the residuals of an ARFIMA model by using a MLP with past prices as inputs. Huang et al. [72] use a particle swarm optimization to optimize the network structure. Guo and Luh [68] propose a procedure with MLP and RBF network.

3.2.2 Recurrent Neural Networks

Recurrent neural network is an example of dynamic neural networks which obtains the output by using the current input, and the previous inputs, outputs, and/or hidden states of the network [55]. Elman neural network is a popular recurrent neural network which consists of an input layer, a context layer, a hidden layer and an output layer. Gradient algorithm can be used to train recurrent networks.

Anbazhagan and Kumarappan [73] obtain short-term price forecasts in the Spanish market by using Elman networks. They conclude that this network performs better than ARIMA, wavelet-ARIMA, MLP, fuzzy ANN and wavelet-ARIMA-RBF networks. However, simple recurrent networks have a poor performance for long run forecast.

Lin, Horne, Tino, and Giles [74] proposed nonlinear autoregressive models with exogenous inputs (NARX) to solve this problem. NARX Neural Network will be explained in detail in the next chapter. Andalib and Atry [75] apply a NARX model to forecast hourly Ontario energy prices where demand is used as explanatory variable.

3.2.3 Fuzzy Neural Networks

Fuzzy logic is a mathematical logic in which instead of an input taking a value of 0 or 1, it can possess a degree of truth anywhere between 0 and 1. Fuzzy neural networks (FNN) combine ANNs with fuzzy logic [76], [77].

Hong and Hsiao [78] apply fuzzy logic to EPF by using fuzzy-c-means to categorize historical data into three clusters and employ a recurrent network for forecasting. Vahidinasab et al. [79] utilize a MLP for price forecasting in a similar method. Rodriguez and Anders [59] construct an adaptive-network-based fuzzy inference system (ANFIS) and they show that their model have a better performance than a MLP. Amjady [80] built a FNN model in Spanish electricity market and show that their method performs better than ARIMA, wavelet-ARIMA, MLP or a RBF network.

Meng et al. [81] train a RBF network by using fuzzy-c-means. Azadeh et al. [21] propose an integrated, multistep algorithm which combines three ANNs, seven fuzzy regressions and one standard regression model to obtain long-term EPF. The algorithm changes between the predictions of the different methods. It is concluded that the standard and fuzzy regressions have better performance than ANN.

3.2.4 Support Vector Machines

The support vector machine (SVM) is a supervised learning method used for classification, regression and outlier detection. SVM produces nonlinear boundaries by building a linear boundary in a large, transformed version of the feature space (Hastie [82]).

Sansom et al. [83] compare SVM to MLP with the same inputs, and results show that the SVM perform better in forecasting. Zhao et al. [84] use a SVM model to forecast the electricity price.

Fan et al. [85] and Niu et al. [86] use Self-organizing maps (SOM) classifiers to cluster hourly electricity price, then employ SVM to predict the prices in each group.

3.3 Hybrid Models

Hybrid models divide the data into two parts. The first part is modelled by traditional methods such as regression or time series models. Then, the errors are modelled by using computational intelligence models such as artificial neural networks or fuzzy logic models. Hybrid methodology is explained in Chapter 4.6 in detail. In this thesis, hybrid methodology is adopted as forecasting method since its promising results in electricity markets.

Conejo et al. [16] introduce a wavelet-ARIMA technique for one-day ahead prediction of electricity price for Spanish market. They conclude that the performance of the wavelet-ARIMA is better than that of an ARIMA process. Similarly, Shafie-Khah et al. [87] find a hybrid method to forecast one-day ahead prices. In this model, a wavelet transform gives a time series, an ARIMA model is utilized to obtain a linear forecast, and after that a radial basis function network is employed to fix the estimation error of the wavelet-ARIMA forecast.

Yan and Chowdhury [88] propose a hybrid mid-term EPF model integrating a least squares support vector machine (LSSVM) and ARMAX models. The model has better forecasting accuracy compared to only LSSVM model.

Gonzalez et al. [89] test the performances of two hybrid models for predicting the one-day ahead spot electricity prices. The first model is a hybrid method which uses a supply stack modeling with an econometric model. The second model is an extended version of the first one which involves logistic smooth transition regression (LSTR) to represent regime-switching for periods of structural change. The forecasting performances of these models are better than non-hybrid models.

Wu and Shahidehpour [90] apply a hybrid ARMAX-GARCH adaptive wavelet neural network model in the PJM market. The ARMAX model is employed to model the linear relationship between the price and electricity load; the GARCH model is applied to explain the heteroskedastic character of residuals; and the wavelet neural network is utilized to show the nonlinear effect of load on electricity prices.

Amjady and Hemmati [28] propose a hybrid model with an application in Spanish market, they show that the method is better than a standard ARIMA model, a wavelet-ARIMA model or a fuzzy ANN.

For the RBF network, Jain et al. [91], Rutkowski [77] use a hybrid learning algorithm which is a combination of a supervised algorithm and an unsupervised algorithm. Shafie-Khah et al. [87] build a hybrid wavelet-ARIMA-RBF network where a RBF network adjusts the estimation error of the wavelet-ARIMA forecast.

In most of the cases, RBF and MLP are used as a part of hybrid models or used to compare with more complicated methods. Gonzalez et al. [92] use MLP in a hybrid hidden Markov model. Mori and Awata [93] use regression trees with RBF networks to find one-step ahead price forecasts. Keynia and Amjady [94] employ a hybrid MLP network model for forecasting PJM data and compare to other MLP models.

Sharma and Srinivasan [95] use a hybrid model which involves a Fitz-Hugh Nagumo model with an Elman network and a feed-forward ANN for forecasting in Australia, Ontario, Spain and California markets. Fitz-Hugh Nagumo model is a dynamic model which is used to model spiking.

Catalão et al. [96] offer a hybrid method that uses a wavelet transform, particle swarm optimization and an adaptive-network-based fuzzy inference system.

Che and Wang [97] use support vector regression and ARIMA models to build a hybrid model called SVRARIMA. They conclude that this hybrid model performs better than some of the ANN models and ARIMA models. Yan and Chowdhury [98] use PJM data to build a hybrid mid-term EPF model consists of least-squares SVM and ARMAX models. Chaâbane [99] construct a hybrid model in Nord Pool market, which uses the features of ARFIMA and least-squares SVM, and they show that it performs better than individual models.

3.4 EPF Studies in Turkey

After the commencement of liberalization process in Turkey, the number of studies on EPF has increased significantly. The reason of this is the importance of making an accurate forecast for electricity prices in a competitive market.

Özmen et al. [100] propose a modelling approach to forecast next-days electricity price in each period which are day, peak, and night. They conclude that CMARS and RCMARS techniques perform better than dynamic regression.

Ünlü [5] examines electricity price as a univariate stochastics process and also examines with temperature as a two-dimensional stochastics process.

Kölmek and Navruz [11] conduct simulation studies about price modeling via artificial neural networks for forecasting one-day ahead electricity price. The selected ANNs performance is compared with a time series model.

Zakeri [8] builds a time series model to obtain short-term forecasts of hourly electricity prices using multiple regression method. Lagged price values, demand and dummy variables for Saturdays and Sundays are employed as exogenous variables.

Gökgöz and Filiz [101] use ANN for one-day ahead electricity price forecasting in Turkey. They use different training algorithms, number of neurons and transfer functions to create different neural network models.

Benli [102] compares nineteen forecasting methods including Double exponential smoothing and ARIMA models for four various electricity tariffs pricing (monochromic, day, peak and night) in Turkey. He concludes that Holt Winters exponential smoothing model outperforms for the time period 2011 to 2014.

Yıldız et al. [103] use an ANN model based on feed forward back propagation approach for one-day ahead price forecasting by using a series of four years period.

Ugurlu et al. [104] utilize Recurrent Neural Networks for Turkish one-day ahead electricity market. The results indicate that 3-layered Gated Recurrent Units (GRU)

have better performance than other neural network models and statistical methods.

Ugurlu et al. [105] use hourly electricity prices in Turkey to make one-day ahead predictions by using various univariate models. They conclude that the SARIMA model is successful under the given conditions.

CHAPTER 4

METHODOLOGY

4.1 AR-type Models

Time series models use past values of variables in order to predict their future values. In forecasting, the reliability of the model should be based on performance of test sample [106].

The time series model using Box-Jenkins approach has been proposed by Box and Jenkins [107] based on works of Yule [108] and Wold [109]. Autoregressive Moving Average (ARMA) models are estimated with this approach. ARMA models are time series models that are widely used in time series forecasting.

The ARMA model is denoted as ARMA(p,q) which is given as Equation 4.1.

$$\dot{y}_t - \phi_1 \dot{y}_{t-1} - ... - \phi_p \dot{y}_{t-p} = \epsilon_t + \theta_1 \epsilon_{t-1} + ... + \theta_q \epsilon_{t-q} \tag{4.1}$$

where $\dot{y}_t = y_t - \mu$ and μ is the process mean. In Equation 4.1, $\dot{y}_t$ is a stationary series. Coefficients ϕ and θ are autoregressive and moving average parameters respectively. Orders of autoregressive and moving average parameters are respectively p and q. Random errors, ϵ_t , are assumed to be independently and identically distributed white noise process with a mean of zero and a constant variance of σ^2 which is generally denoted as WN$(0, \sigma^2)$. There is no common factor between autoregressive part $(1 - \phi_1 B - \phi_2 B^2 - ... - \phi_p B^p)$ and moving average part $(1 + \theta_1 B + \theta_2 B^2 + ... + \theta_q B^q)$ where B is backshift operator, defined by $By_t = y_{t-1}$. These polynomials can be

represented by $\phi(B)$ and $\theta(B)$.

Thus ARMA(p, q) model can also be denoted as Equation 4.2,

$$\phi(B)\dot{y}_t = \theta(B)\epsilon_t. \tag{4.2}$$

When $q = 0$, then equation becomes an AR(p) model. If $p = 0$, the model becomes MA(q) model.

4.1.1 Autoregressive Integrated Moving Average (ARIMA) Models

ARMA, AR and MA models are only applicable for stationary time series. However, most of the real life examples of price series are not stationary. If the series become stationary by taking difference, the autoregressive integrated moving average (ARIMA) model is implemented. The ARIMA model is denoted as ARIMA(p, d, q) which is given as Equation 4.3.

$$\phi(B)(1 - B)^d\dot{y}_t = \theta(B)\epsilon_t \tag{4.3}$$

where d is the dth difference operator. If there is no need to take difference then d is equal to 0 and ARIMA model can be called ARMA model.

4.1.2 Seasonal Autoregressive Integrated Moving Average (SARIMA) Models

When there is a stochastic seasonal periodicity within the series, SARIMA models are used to describe the behavior of the series. The SARIMA model is denoted as SARIMA$(p, d, q)(P, D, Q)s$ which is given as Equation 4.4.

$$\phi(B)\Phi(B^s)(1 - B)^d(1 - B^s)^D\dot{y}_t = \theta(B)\Theta(B^s)\epsilon_t \tag{4.4}$$

where B is the backshift operator, defined by $B^s y_t = y_{t-s}$. Coefficients ϕ and θ

are autoregressive and moving average parameters, respectively. Coefficients Φ and Θ are seasonal autoregressive and seasonal moving average parameters, respectively. Orders of nonseasonal autoregressive and moving average parameters are p and q, and orders of the seasonal autoregressive and moving average parameters are P and Q, respectively; d and D denote nonseasonal and seasonal differences, respectively. Random errors, ϵ_t , are assumed to be independently and identically distributed with a mean of zero and a constant variance of σ^2.

By taking $\bar{y}_t = (1 - B)^d(1 - B^s)^D y_t$, seasonal unit root is removed so SARIMA model is converted to SARMA. Thus, estimation of ARIMA and Seasonal ARIMA models is similar to estimation of ARMA model.

Box-Jenkins approach which is used to model ARIMA models was described in the book by George Box and Gwilym Jenkins in 1970 [107]. The steps of Box-Jenkins modelling approach are as follows:

1. Data Preparation
   Transformation and differencing are made in this part. Power transformations are applied to stabilize the variance. Then, series is differenced when there is a stochastic trend. Differencing is taking the difference between consecutive observations or between observations according to seasonal period. Differencing is done to make the data stationary. Stationarity is necessary in building an ARIMA model that will be used for forecasting. There are stationarity tests such as KPSS test or unit root tests such as ADF test to decide on the existence of a trend in the series.

2. Model Selection
   ACF-PACF graphs of transformed and differenced series are used to identify potential ARIMA models. It is possible to identify one or several potential models for the given time series by using the autocorrelation function and the partial autocorrelation function of the realizations. Additionally, model selection tools such as Akaike's Information Criterion and Bayesian Information Criterion are used to select the best model.

3. Parameter Estimation

   After tentative model is specified, values of model coefficients are estimated mostly using maximum likelihood estimation (MLE) technique.

4. Model Checking

   The last step of model building is the diagnostic checking of model adequacy. The assumptions about the errors of the estimated model are checked in this step of the process. If some assumptions are not satisfied, the model is not adequate. Then, it is necessary to return to model selection step to select a better tentative model.

5. Forecasting

   The future values of the series are predicted by using the model selected. The minimum MSE forecasts are used. Since ARMA type models are stochastic models, forecasts tend to converge to the process mean when forecast period is far from the forecast origin. Therefore, short term forecasts give good results.

4.2 ARX-type Models

The SARIMA process only use the previous values of price and the error to predict the future values. However, there are also external factors that may affect electricity price which are called exogenous variables. The ARIMA model is extended into ARIMA model with exogenous variable X, called ARIMAX(p, d, q).

The autoregressive moving average model with exogenous variable, ARIMAX(p, d, q) can be represented by

$$\phi(B)(1-B)^d \dot{y}_t = \beta X_t + \theta(B)\epsilon_t \tag{4.5}$$

where X_t is the previous day's demand of electricity and β is its coefficient. The least square estimation and maximum likelihood estimation methods can be used for the estimation of ARX-type models. Besides that, other estimation methods can be used.

When there is seasonality in the series, Seasonal ARIMAX (SARIMAX) model is employed to capture seasonality.

4.3 GARCH Models

In ARIMA estimation, constant variance assumption may not hold. The models where this assumption does not hold is called heteroskedastic. A variance that changes over time has implications for the validity and efficiency of statistical inference about parameters [110].

The Autoregressive Conditional Heteroskedastic (ARCH) class of models was introduced by Engle [48]. He showed that mean and variance of the series can be modelled simultaneously. The ARCH(q) model takes the conditional variance as time dependent, so the conditional variance is denoted by an autoregressive process, which is a weighted sum of squared previous observations:

$$\hat{\epsilon}_t^2 = \alpha_0 + \alpha_1 \hat{\epsilon}_{t-1}^2 + \alpha_2 \hat{\epsilon}_{t-2}^2 + ... + \alpha_q \hat{\epsilon}_{t-q}^2 + v_t \tag{4.6}$$

where v_t is a white-noise process.

Bollerslev [49] extended Engles [48] ARCH model by developing a method that lets the conditional variance to be an ARMA process. GARCH stands for Generalized Autoregressive Conditional Heteroskedasticity while the ARIMA models are aimed at modeling and forecasting the changing price itself, GARCH process can measure the implied volatility of a time series due to price spikes [111], [50].

Let the squared error process be

$$\epsilon_t^2 = v_t^2 h_t \tag{4.7}$$

where $\sigma_v^2 = 1$ is basically a white noise process and

$$h_t = c + \sum_{i=1}^{p} \alpha_i h_{t-i} + \sum_{i=1}^{q} \beta_i \epsilon_{t-i}^2 \tag{4.8}$$

gives the conditional variance of ϵ_t which is an ARMA process. GARCH(p, q) model becomes ARCH(q) model when $p = 0$.

ARCH and GARCH models have become very popular in that they enable the researcher to estimate the variance of a series at a point in time [112]. GARCH models can be combined with an AR-type model, it is called a (S)AR(IMA)-GARCH model, where the residuals of the regression part are modelled with a GARCH process.

In this study, the SARIMAX-GARCH model is used in Model 5, where the residuals of the regression part are modelled further with a GARCH process.

4.4 Exponential Smoothing Models

Exponential Smoothing is a deterministic forecasting method which was developed in the late 1950s by the works of Brown [113], Holt [114] and Winters [115]. Forecasts are weighted averages of past observations. As observations get older, weights decrease exponentially [116].

The method of simple exponential smoothing, Brown [113], takes the forecast for the previous period and adjusts it using the forecast error. The forecast for the next period is

$$\hat{y}_{t+1} = \alpha y_t + (1 - \alpha)\hat{y}_t \tag{4.9}$$

where α is a smoothing parameter between 0 and 1. The most recent observation y_t is weighted with a weight value α, and the most recent forecast $\hat{y}_t$ is weighted with a weight of $1 - \alpha$.

4.4.1 Holt Winters Exponential Smoothing Method

Holt Winters is an exponential smoothing method which is designed for series with trend and seasonality. Holt Winters method is based on three smoothing equations: one for the level, one for trend, and one for seasonality. Taylor [117] extended the linear version of the Holt-Winters method to incorporate a second seasonal compo-

nent and develops the following double seasonal Holt Winters exponential smoothing method.

$$y_t = l_{t-1} + b_{t-1} + s_t^{(1)} + s_t^{(2)} + d_t \tag{4.10a}$$

$$l_t = l_{t-1} + b_{t-1} + \alpha d_t \tag{4.10b}$$

$$b_t = b_{t-1} + \beta d_t \tag{4.10c}$$

$$s_t^{(1)} = s_{t-m_1}^{(1)} + \gamma_1 d_t \tag{4.10d}$$

$$s_t^{(2)} = s_{t-m_2}^{(2)} + \gamma_2 d_t \tag{4.10e}$$

where m_1 and m_2 are periods of the seasonal cycles and d_t is a white noise random variable representing the prediction error. l_t and b_t show the level and trend components of the series at time t respectively, and $s_t^{(i)}$ denotes the ith seasonal component at time t. α, β, γ_1, γ_2 are the smoothing parameters ranging over [0,1].

4.4.2 Trigonometric Seasonal Models

TBATS (Trigonometric Seasonal, Box-Cox Transformation, ARMA residuals, Trend and Seasonality) Model is proposed by De Livera et al. [118]. The Equation 4.10 can be extended to Equation 4.11 to include a Box-Cox transformation, ARMA errors and trigonometric seasonal patterns as follows. This method allows to capture more than two seasonal periodicities within the series.

The algorithm that is used in the TBATS modelling is shown in Equation 4.11.

$$y_t^{(w)} = \begin{cases} \dfrac{y_t^w - 1}{w} & w \neq 0, \\ logy_t & w = 0, \end{cases} \tag{4.11a}$$

$$y_t^{(w)} = l_{t-1} + \phi b_{t-1} + \sum_{i=1}^{T} s_{t-m_i}^{(i)} + d_t \tag{4.11b}$$

$$l_t = l_{t-1} + \phi b_{t-1} + \alpha d_t \tag{4.11c}$$

$$b_t = (1-\phi)b + \phi b_{t-1} + \beta d_t \tag{4.11d}$$

$$s_t^{(i)} = s_{t-m_i}^{(i)} + \gamma_i d_t \tag{4.11e}$$

$$d_t = \sum_{i=1}^{p} \varphi_i d_{t-i} + \sum_{i=1}^{q} \theta_i \epsilon_{t-i} + \epsilon_t \tag{4.11f}$$

where $m_1, ..., m_T$ show the seasonal periods, l_t is the local level in period t, b is the long-run trend, b_t is the short-run trend in period t, $s_t^{(i)}$ denotes the ith seasonal component at time t, d_t represents an ARMA(p, q) process and ϵ_t is a white noise process with zero mean and constant variance σ^2. α, β, γ_i for $i = 1, ..., T$ are smoothing parameters, ranging in [0,1].

Trigonometric seasonal components is represented in the Equation 4.12.

$$s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)} \tag{4.12a}$$

$$s_{j,t}^{(i)} = s_{j,t-1}^{(i)} cos\lambda_j^{(i)} + s_{j,t-1}^{*(i)} sin\lambda_j^{(i)} + \gamma_1^{(i)} d_t \tag{4.12b}$$

$$s_{j,t}^{*(i)} = -s_{j,t-1} sin\lambda_j^{(i)} + s_{j,t-1}^{*(i)} cos\lambda_j^{(i)} + \gamma_2^{(i)} d_t \tag{4.12c}$$

where $\gamma_1^{(i)}$ and $\gamma_2^{(i)}$ are the smoothing parameters. $\lambda_j^{(i)} = 2\pi j/m_i$. $s_{j,t}^{(i)}$ is the stochastic level of the ith seasonal component, the stochastic growth in the level of the ith seasonal component that is necessary to define the change in the seasonal component over time by $s_{j,t}^{*(i)}$. For even m_i values, $k_i = m_i/2$ and for odd m_i values $k_i = (m_i - 1)/2$ where k_i is the number of harmonics that is needed for the ith seasonal component.

4.5 Artificial Neural Networks (ANN)

Learning is described as ability of improving behavior through studying on experiences [119]. A learning algorithm is a technique used to work on data to get convenient patterns for application in a new situation. The learning problems can be categorized as supervised and unsupervised learning. In supervised learning, the purpose is to predict the value of an outcome based on input measures; in unsupervised learning, the aim is to identify the relationships among input measures [82]. Supervised learning uses classification and regression techniques to build predictive models while unsupervised learning mostly uses clustering.

The artificial neural network is a supervised learning technique which was proposed for pattern recognition purposes [120]. Yet, it can be used as regression as well. ANN is developed as generalizations of mathematical models of human neural biology, the similarity can be seen in Figure 4.1.

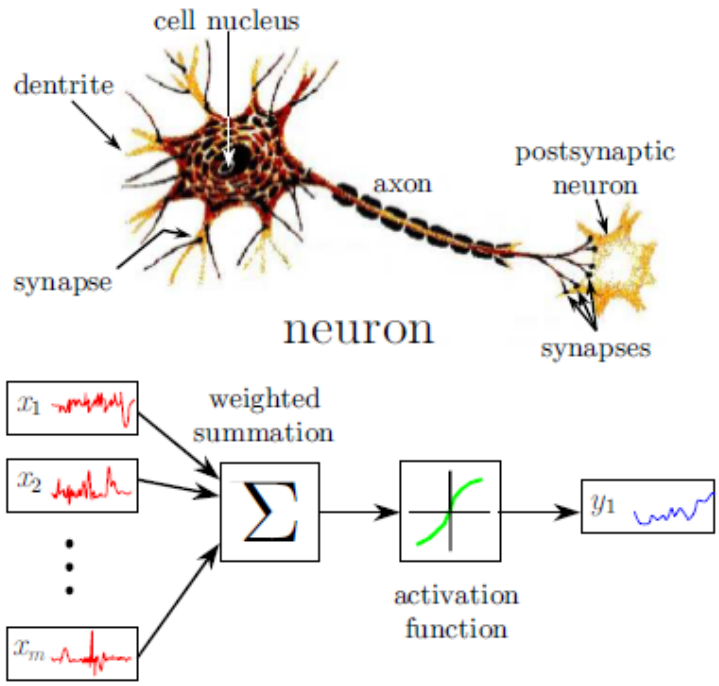

Figure 4.1: A biological neuron and an artificial neural network [121]

The network model is mostly determined by the characteristics of the data and no prior assumption is needed to build the model [122]. In the network, units are con-

nected with links and each link has a weight. It captures complicated relationships between input and output information with the network structure. A unit (node) gets inputs, multiplies by a numeric weight, adds bias value, forms an output by an activation function and it learns by adjusting the weights. The purpose of adding bias is adjusting the threshold of activation function.

The relationship between the output and inputs for single hidden layer feedforward network model has the following representation:

$$y_t = \alpha_0 + \sum_{j=1}^{q} \alpha_j f(\beta_{0j} + \sum_{i=1}^{p} \beta_{ij} y_{t-i}) + \epsilon_t \tag{4.13}$$

where $i = 0, 1, 2, ..., p$; $j = 1, 2, ..., q$; y_t is output, $y_{t-1}, ..., y_{t-p}$ are inputs, α_j and β_{ij} are model parameters which are connection weights; p is the number of input nodes and q is the number of hidden nodes; f is the hidden layer activation function which is generally sigmoid (logistic) function that is given in Equation 4.14. Activation function is required to bring nonlinearity into the network. The sigmoid function converts the real valued numbers to numbers in the range between 0 and 1.

$$f(x) = \frac{1}{1 + exp(-x)} \tag{4.14}$$

The transformation happens according to activation function in hidden and output layers. For training, back-propagation algorithm is widely used which is based on gradient descent. The training set put into the input layer, output values are obtained and compared with actual values. The weights are changed according to error which is the difference of estimated value and actual value. The aim is to find a set of weights that minimizes error (cost). Learning cycle can be seen in Figure 4.2, each cycle is called epoch. The weights are modified until convergence is reached. This process is known as a back propagation which mainly uses chain rule. As a result, the future values are predicted from the past values by the neural network model.

There are two major types of neural networks: dynamic and static [55]. In static neural networks, output is calculated directly from the input. In a feedforward neural

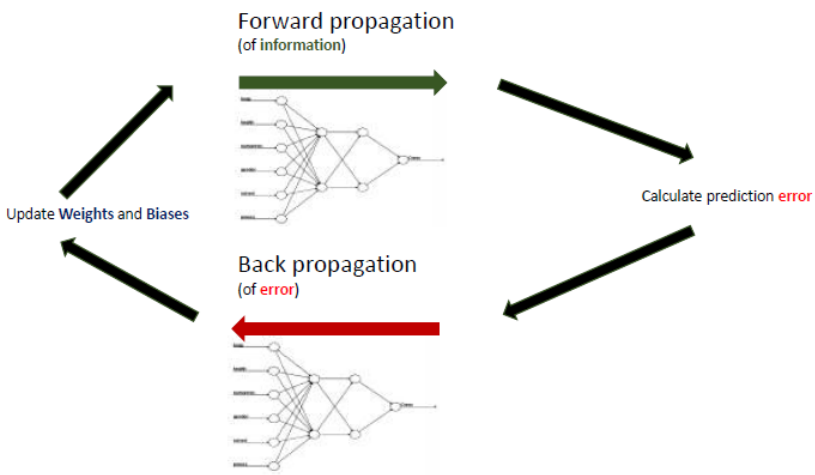

Figure 4.2: Neural network learning cycle [55]

network, the process follows a single direction which is from input to output. There is no feedback component which can be seen in Figure 4.3. On the other hand, the output depends on the current and previous inputs, outputs, and hidden states in a dynamic neural network. Recurrent neural network model is an example of a dynamic network as can be seen in Figure 4.4.

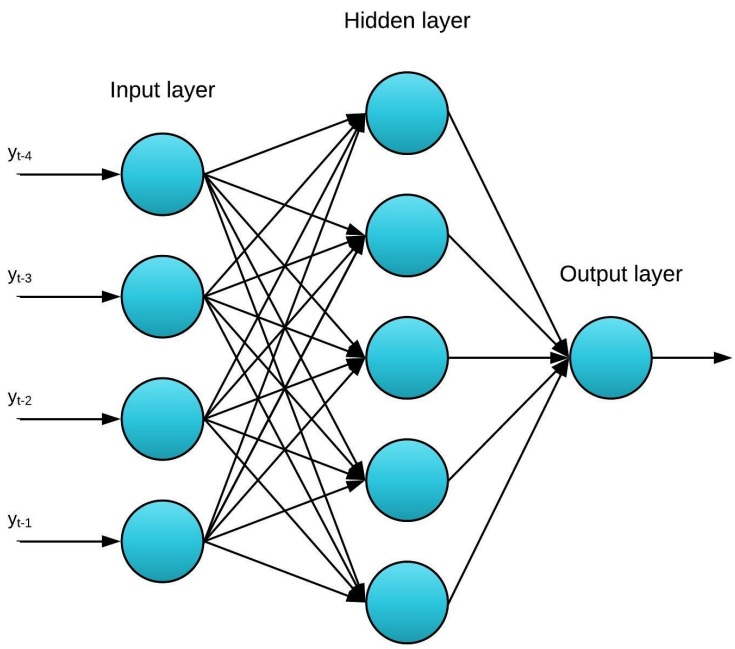

Figure 4.3: Example of three-layered feed-forward neural network

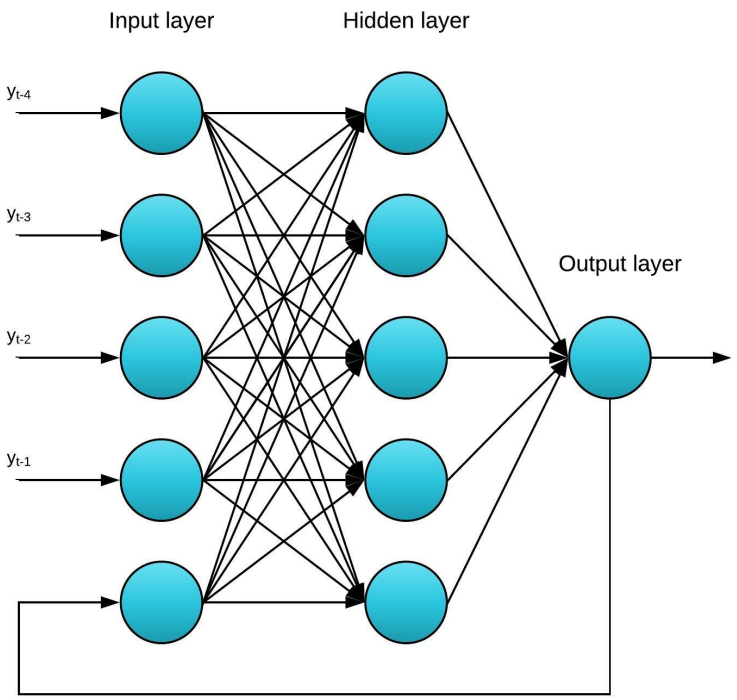

Figure 4.4: Example of three-layered recurrent neural network

Feedforward neural network models are used to make one-step-ahead prediction of a time series. For making multi-step-ahead prediction, the model's output should be fed back to the input regressor for a fixed number of time steps, so recurrent neural network should be used. Recurrent neural networks are also useful to show nonlinear dynamical mappings, which is widely found in nonlinear time series prediction [123].

4.5.1 Nonlinear Autoregressive with Exogenous Inputs (NARX)

Discrete time nonlinear systems can be modelled by the nonlinear autoregressive moving average with exogenous inputs (NARMAX) model [124]. Chen et al. [125] build NARMAX model by using neural network. The method used in this study is non-linear autoregressive exogenous (NARX) neural network model which is a recurrent dynamic neural network model. In feedback dynamic neural networks, the output of the network depends on current input, previous inputs and outputs of the network. NARX model is based on ARX-type models, so it allows exogenous inputs

in the network. In this type of network, future values of a time series y_t can be predicted from past values of that time series and past values of another time series x_t. Therefore, it is a useful method for time series prediction.

NARX model can be written as

$$y_t = f[y_{t-1}, y_{t-2}, ..., y_{t-d_y}, x_{t-1}, x_{t-2}, ..., x_{t-d_x}] \tag{4.15}$$

where y and x are outputs and inputs of the model respectively and d_y and d_x are output and input delays of the model.

NARX NN models are useful for modelling long term dependencies in time series data since the time delays captured by d_y and d_x. In this study, feedback dynamic neural network is used which depends on previous inputs and outputs of the series.

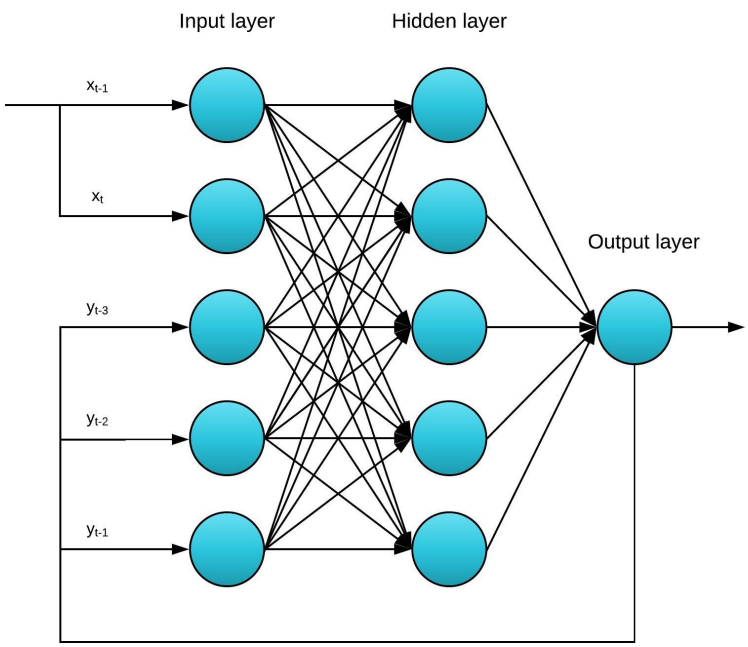

Figure 4.5: Example of NARX neural network

4.6 Hybrid Methods

The methods mentioned in this chapter are successful at different circumstances. For instance, ARIMA models are good at modelling linear time series, however they are not very good at capturing nonlinear structure. In real life problems, it is not easy to know the characteristics of data structure completely. Therefore, using hybrid methodology which has both linear and nonlinear modelling capabilities will be a better choice for modelling series like electricity price.

Zhang [122] states that a time series can be considered as a combination of a linear autocorrelation structure and nonlinear component as follows

$$y_t = L_t + N_t \tag{4.16}$$

where linear component is represented by L_t and nonlinear component is shown by N_t. These components should be estimated from the series. First, linear component will be modelled by ARIMA or TBATS. Then, the residuals from these models will have only the nonlinear relationship. The residuals can be obtained by taking difference of actual values and predicted values as in Equation 4.17.

$$e_t = y_t - \hat{L}_t \tag{4.17}$$

where e_t represents the residual of linear model at time t and $\hat{L}_t$ is the estimated value at time t. To be an adequate linear model, no linear correlation structure should be left in its residuals. To find the nonlinear relationship, residuals can be modelled by ANN.

$$e_t = f(e_{t-1}, e_{t-2}, ..., e_{t-n}) + \epsilon_t \tag{4.18}$$

where f is transformation function in ANN and ϵ_t is random error. The forecast from ANN and forecast from the first model are combined to obtain forecast of the series y_t which is denoted by

$$\hat{y}_t = \hat{L}_t + \hat{N}_t. \tag{4.19}$$

CHAPTER 5

ANALYSIS

In this study, price forecasts are obtained by using hybrid methodology which is explained in section 4.6. Different hybrid models are constructed by using methods which are discussed in the previous chapter. For predicting the linear part of the model, SARIMA and TBATS models are used. For nonlinear part, NARX Neural Network, NAR Neural Network methods are used. Hybrid models are built with combination of these models. Additionally, exogenous variable is included at different stages for different hybrid models. These models are compared according to forecast performances. In addition, individual models which are SARIMA, TBATS and NN are also compared with hybrid models. Time series plots of predicted values and actual values and table of prediction performances for training set can be found in Appendix A. Linear modelling is implemented in R and NN model is built using Neural Network Time Series Tool in MATLAB. There are packages available for neural network modelling in R, however it is more convenient to use MATLAB for this part of the analysis. It provides many options for training and it is easier to add exogenous variables in MATLAB.

Short run and medium run forecasts are obtained by using these models. So, there are two results from each model. Short run forecast is 24 hours (1 day) ahead prediction and medium run forecast is 168 hours (1 week) ahead prediction. To act fast in the market, accurate short run forecasting is very important. One may consider daily or hourly forecasting than weekly forecasting.

RMSE and MAPE are used to compare the model performances. These are most

commonly used accuracy measures in time series analysis.

5.1 Data Description

The data set used in this study is hourly electricity price (in TL/MWh) and hourly electricity demand (in MWh) of Turkey. Series is starting from 1st January 2012 to 15th January 2018. This time period is equivalent to 52944 hours. There are multiple seasonality (daily, weekly, monthly) in the price series. However, only daily and weekly seasonality have significant effects in the models built. So, daily and weekly seasonal periods are included in all the models in this study.

At first, we have a look at data structure. We have two variables which are hourly electricity price and demand.

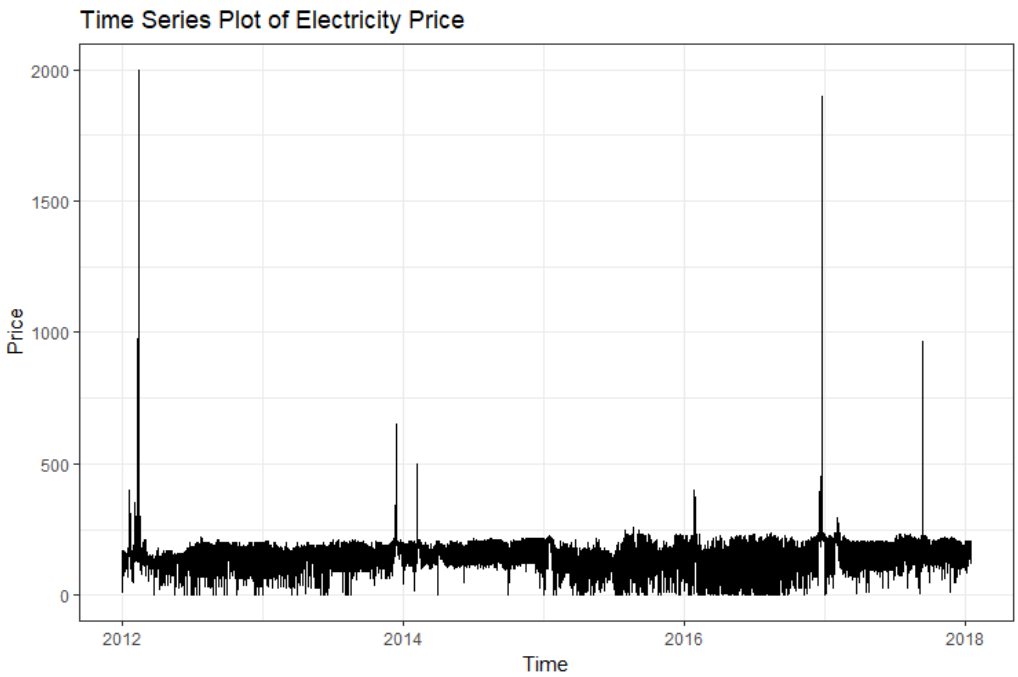

Figure 5.1: Time Series Plot of Electricity Price

From the time series plots in Figures 5.1, 5.2, it can be seen that there are some significant spikes at both variables. The reason of huge spikes in February 2012 and December 2016 in Figure 5.1 is supply shortage of natural gas to power plants. Petroleum Pipeline Company (BOTAŞ in Turkish) cut off 90% of natural gas supply

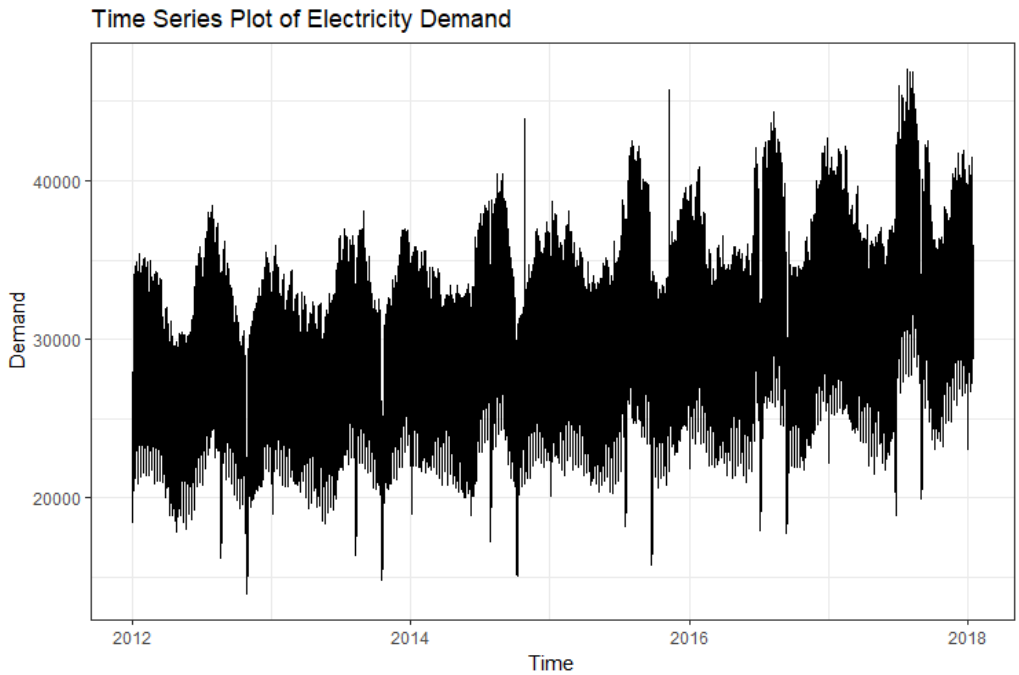

Figure 5.2: Time Series Plot of Electricity Demand

to power plants, therefore electricity spot prices increased up to 1200%. An interesting fact about the price series is that minimum price for electricity is zero. Sometimes electricity was traded for free since it cannot be stored. The variance of price is very high. Generally, there is high volatility in electricity prices. Demand variable has a repeated pattern over time. In addition, there is a slightly increasing trend.

Linearity of the price and demand are tested by Teraesvirta's neural network linearity test which uses a Taylor series expansion of the activation function to find a suitable test statistic [126]. The null hypothesis of linearity in mean is rejected with p-values smaller than 2.2×10^{-16} for both demand and price series with a significance level of 0.05. Therefore, it is concluded that they are both nonlinear.

5.2 Demand Forecast

To make a prediction about electricity prices, we should predict the future values of demand which is the exogenous variable in all models. Double Seasonal Holt Winters Exponential Smoothing Method gives the best forecast performance among

other methods such as SARIMA, TBATS and NAR NN. It can be seen from Figure 5.3 and Figure 5.4 that the actual values and forecast values are very close to each other at most of the time points.

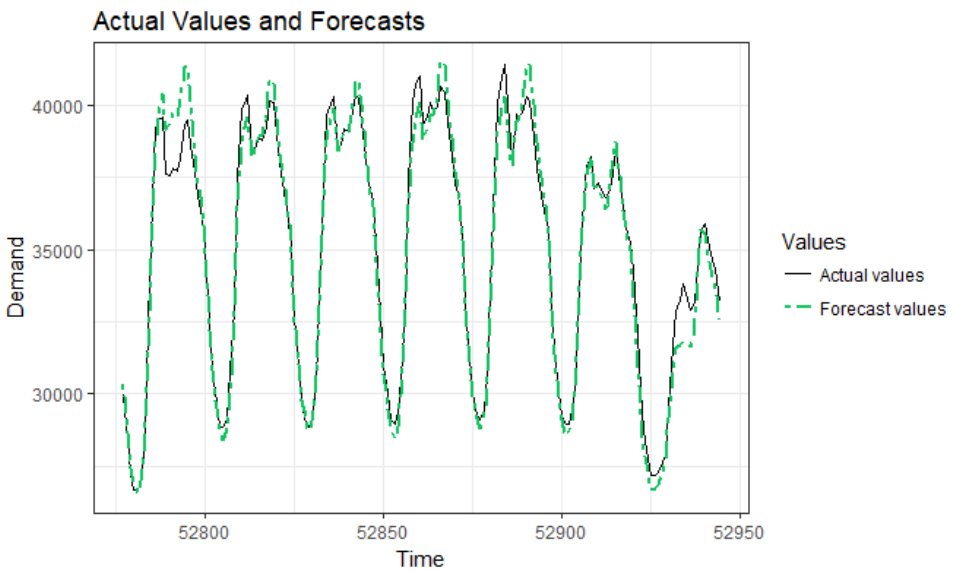

Figure 5.3: Time Series Plot of Actual Values and One-week Ahead Forecast Values of Demand

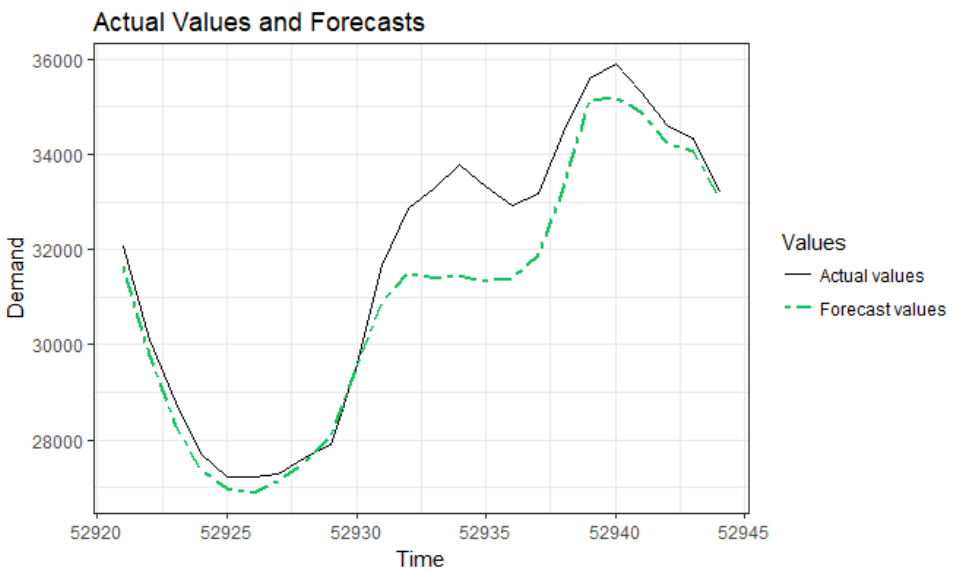

Figure 5.4: Time Series Plot of Actual Values and One-day Ahead Forecast Values of Demand

5.3 Hybrid Models

5.3.1 Model 1: SARIMA and NARX

Algorithm that is used in Model 1 for electricity price forecasting is as follows:

- Step 1. A model with dummy variables is constructed to eliminate weekly seasonality. Since SARIMA models in R can only have one seasonal period, it is required to examine one type of seasonality with dummy variables.
- Step 2. SARIMA model is constructed with the residuals of the model in step 1.
- Step 3. NARX neural network model is used for modelling the residuals of the model in step 2 and demand is used as input variable.
- Step 4. Forecasts for demand is obtained by DSHW exponential smoothing method.
- Step 5. Forecasts are combined with the hybrid model by using the forecasts from DSHW. Forecast values from SARIMA model, forecast values from NN model and coefficients of model with dummy variables are added together.

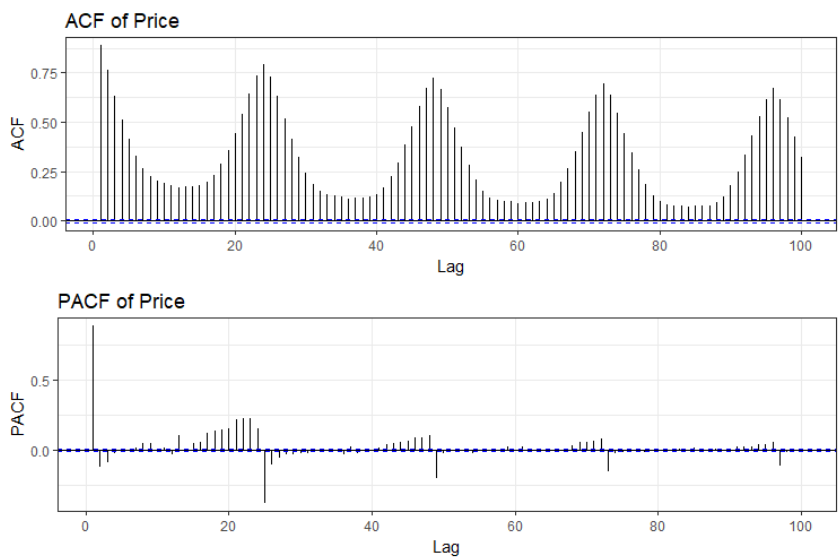

Figure 5.5: ACF and PACF Plots of Price

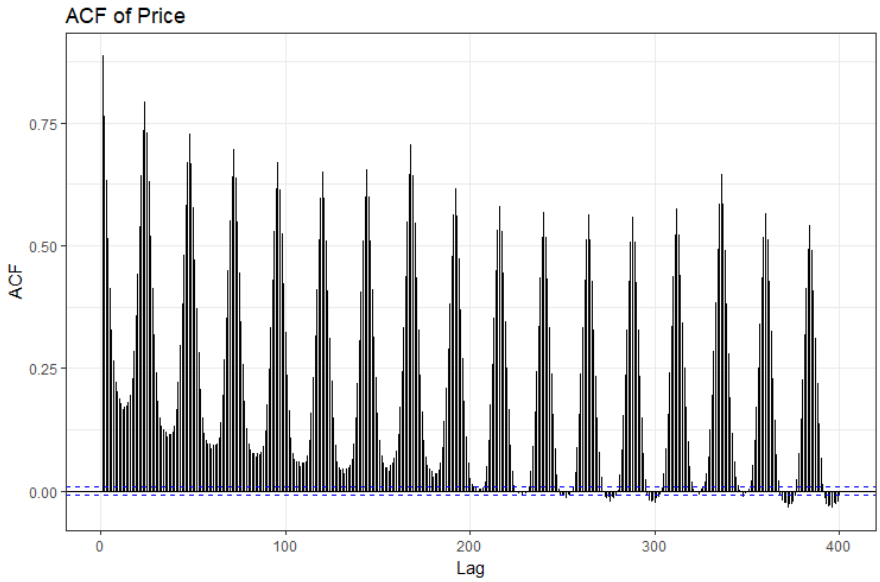

Figure 5.6: ACF Plot of Price

Price variable does not look stationary from ACF-PACF plots in Figure 5.5. There are both hourly and weekly seasonality. Weekly seasonality can be seen from the peaks at times of 168 at Figure 5.6 which shows ACF for longer lags than the Figure 5.5. Dummy variables are made for days of the week. A model is constructed with price as response, dummy variables as covariates without an intercept term. Thus, weekly seasonality is captured by means of daily dummies.

Table 5.1: Coefficients, Standard Errors and p-values of model with dummy variables

	Coefficients	S.E.	P-value
Mon	151.5541	0.5860	<2e-16
Tues	155.7753	0.5860	<2e-16
Wed	155.2993	0.5860	<2e-16
Thur	156.5983	0.5860	<2e-16
Fri	156.0184	0.5860	<2e-16
Sat	150.3039	0.5860	<2e-16
Sun	131.9824	0.5851	<2e-16

All dummy variables look significant in Table 5.1. Estimated dummy model is represented in Equation 5.1. Then we examine the residuals of this model.

$$\begin{aligned}\hat{y}_t =& 151.55Mon + 155.77Tues + 155.29Wed + 156.59Thur \\ &+ 156.02Fri + 150.3Sat + 131.98Sun\end{aligned} \tag{5.1}$$

Stationarity of the residuals is tested by KPSS test by rejecting the null hypothesis for level stationarity with a p-value equals to 0.01 which is smaller than significance level 0.05. Therefore, residuals are not stationary according to KPSS test. Additionally, R code *ndiffs* gives the number of differences required to make the series stationary and it states that one regular difference should be taken to obtain a stationary process. Therefore, a regular difference is taken. Then, stationarity is checked again.

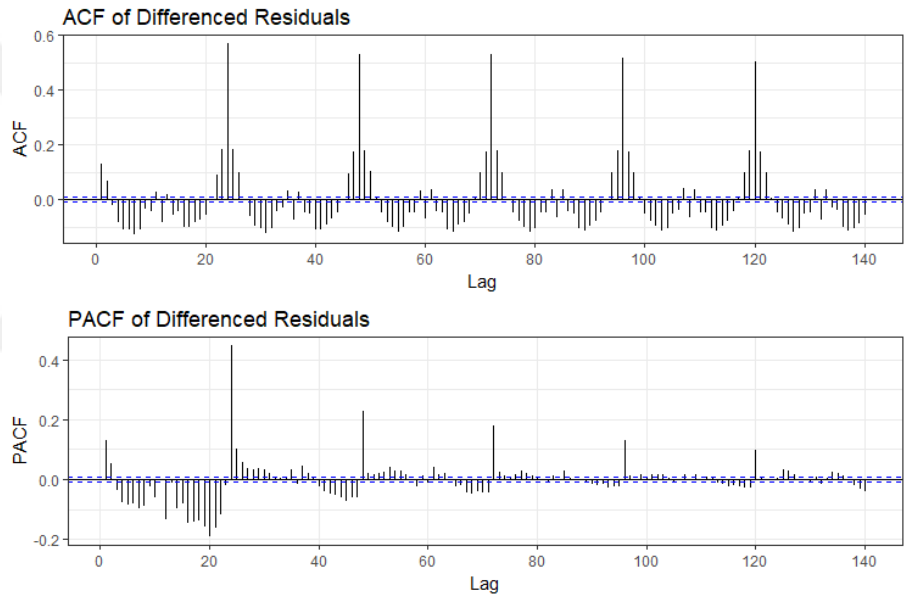

Figure 5.7: ACF and PACF Plots of Differenced Residuals

In Figure 5.7 which shows ACF-PACF plots of the differenced residuals, it looks like there is seasonality. From the result of *nsdiffs* which gives the number of seasonal differences required to make the series stationary, one can conclude that a seasonal difference should be taken.

After taking the seasonal difference, it can be concluded that there is no need for taking difference again by looking at the results of *ndiffs* and *nsdiffs* codes in R.

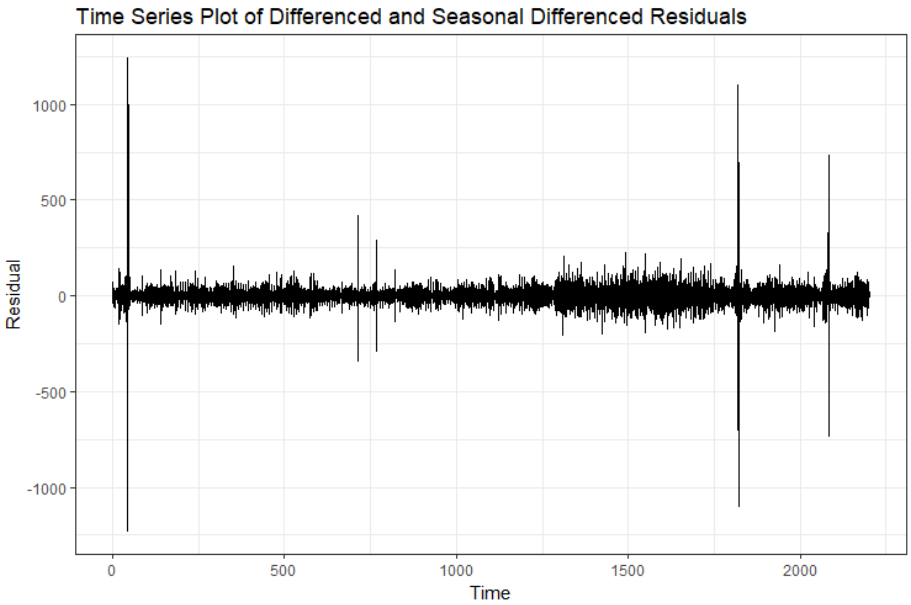

Figure 5.8: Time Series Plot of of Differenced and Seasonal Differenced Residuals

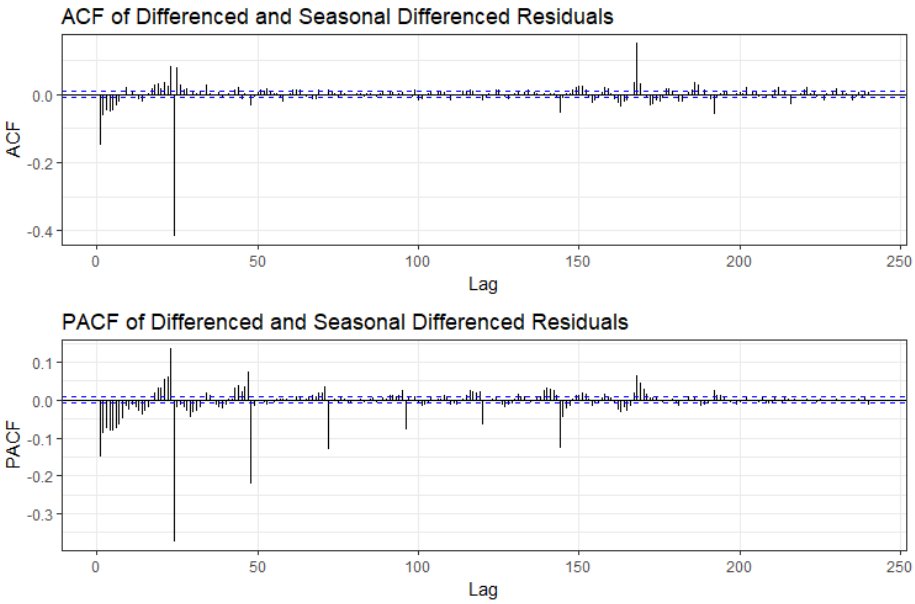

Figure 5.9: ACF and PACF Plots of Differenced and Seasonal Differenced Residuals

Differenced series have zero mean and there are some spikes which can be seen in Figure 5.8. However, it can be concluded that series looks stationary after taking regular and seasonal difference by looking at Figure 5.9 and the results of *ndiffs* and *nsdiffs* codes in R.

So, we can fit a SARIMA model with differenced residuals of the first model. Orders are decided by looking at ACF-PACF plots given in Figure 5.9. Significance of the lags are checked by fitting models with different orders. The best model having the smallest BIC is obtained as

$$
\begin{aligned}
&(1 + 0.232 - 0.137B^2 - 0.206B^3 - 0.314B^4)(1 - 0.072B^{24} - 0.056B^{48} - 0.044B^{72}) \\
&(1 - B)(1 - B^{24})y_t = (1 - 0.015B - 0.409B^2 - 0.262B^3 - 0.277B^4) \\
&(1 - 0.932B^{24})\epsilon_t
\end{aligned}
\tag{5.2}
$$

Table 5.2: Coefficients and Standard Errors of SARIMA Model

	Coefficients	S.E.
ar1	-0.232	0.0510
ar2	0.1371	0.0330
ar3	0.2062	0.0397
ar4	0.3139	0.0292
ma1	-0.0152	0.0520
ma2	-0.4090	0.0424
ma3	-0.2622	0.0446
ma4	-0.2774	0.0380
sar1	0.0725	0.0052
sar2	0.0563	0.0050
sar3	0.0444	0.0049
sma1	-0.9322	0.0026

Then, we check the assumptions of residuals of the SARIMA model.

First, normality assumption is checked. P-value of the Jarque-Bera Normality Test equals to 2.2×10^{-16} which is smaller than $\alpha = 0.05$. So, null hypothesis which states that residuals are normally distributed is rejected. Therefore, it can be concluded that residuals are not normally distributed.

Then, correlation of residuals is checked by using Box Pierce Test for correlation. P-value of this test is 0.3914 which is greater than $\alpha = 0.05$. This means that null hypothesis which says that residuals are not correlated is failed to reject. Thus, this assumption is satisfied.

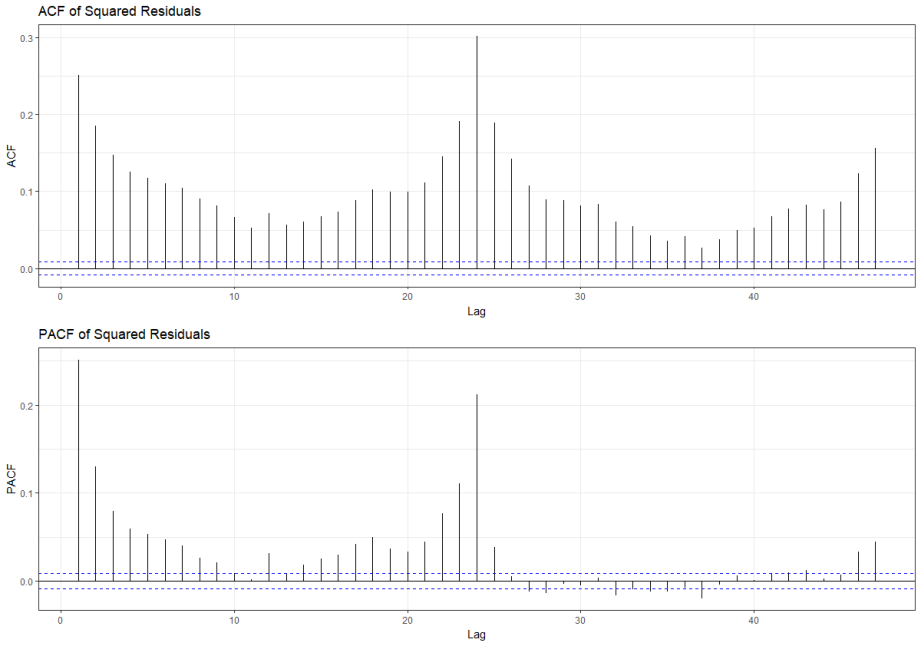

Figure 5.10: ACF-PACF Plots of Squared Residuals of SARIMA Model

ACF- PACF plots of squared residuals given in Figure 5.10 show that there are some lags that are out of white noise band. So, there is heteroscedasticity. Therefore, it can be concluded that some assumptions about the residuals are not satisfied. In the beginning of modelling process, Box-Cox transformation is used to stabilize the variance. However, it does not solve the problems about assumptions. So, Box-Cox transformation step is excluded from the analysis. Because of the high variation in the series, residuals follow a heavy tailed distribution. So, there is nonlinearity in the residuals that we will try to explain this by NN model.

NARX NN model is used to model the residuals of SARIMA model. In NARX NN Levenberg-Marquardt algorithm is used for training. Mean squared error (MSE) is used to find the best fit. The network for this model can be seen in Figure 5.11. For forecasting purpose, closed-loop network is used which can be seen in Figure 5.12. For other models in this study, closed-loop network will not be shown since the

delays are the same with the open-loop ones. In this network, 24 is the number of delays for both input and target variable and number of hidden layers is 10. These values are decided by trying different numbers and comparing the performances. In these networks, w denotes weights and b denotes bias term. Moreover, x is demand variable and y shows the residuals of SARIMA model in Equation 5.2.

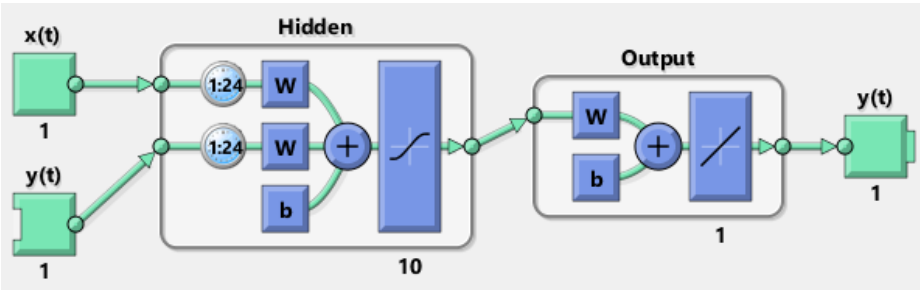

Figure 5.11: NARX Neural Network for Model 1

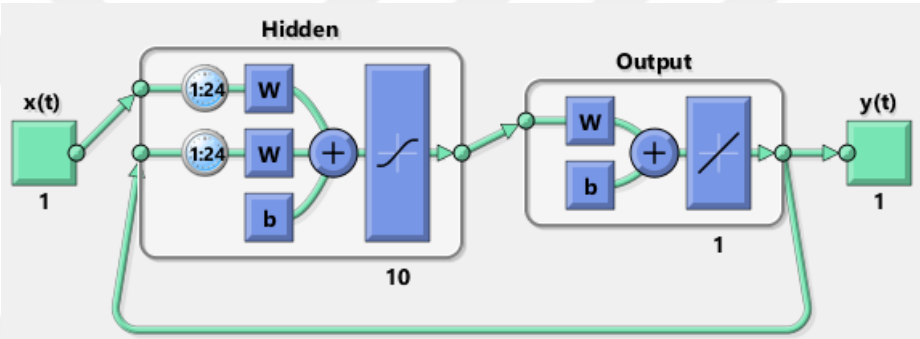

Figure 5.12: Closed-loop NARX Neural Network for Model 1

In the final step of Model 1, forecast from SARIMA model, forecast from NARX NN model and coefficients of the dummy model are combined to obtain the final forecast values for the electricity price. All these processes are performed for one-week ahead prediction and one-day ahead prediction.

Table 5.3: Accuracy measures for one-week ahead and one-day ahead forecasts of Model 1

Forecast	ME	MSE	RMSE	MAE	MPE	MAPE
One-week ahead	1.588	225.767	15.025	11.491	0.4185	6.545
One-day ahead	5.849	422.222	20.548	16.971	2.532	9.905

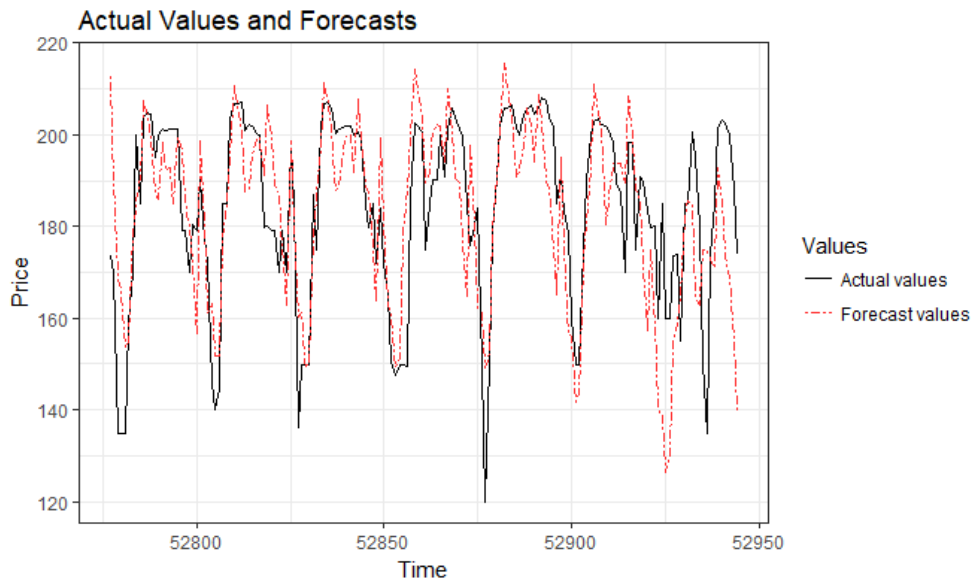

Figure 5.13: Time Series Plot of Actual Values and One-week Ahead Forecast Values of Model 1

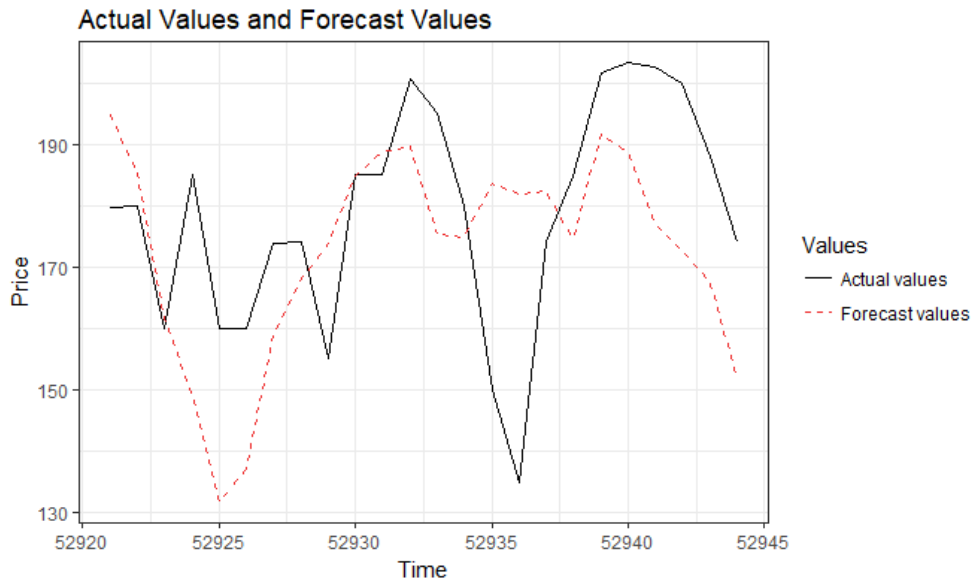

Figure 5.14: Time Series Plot of Actual Values and One-day Ahead Forecast Values of Model 1

Figure 5.13 and Figure 5.14 show the actual values and forecast values of electricity price for 168 hours and 24 hours, respectively. It can be concluded that actual values and forecast values are close to each other. For one-week ahead forecast in Figure 5.13, the model captures the behavior very well at most of the times. In the final 24

hours, the real series has an unexpected movement, so most of the models are not very successful at capturing this behavior. Therefore, comparing one-day ahead forecast performances will be a good measure to see which model is better at modelling the unexpected behaviors in the series. For one-week ahead forecast in Figure 5.14, at some points forecast values show a different pattern from the actual values yet it is the best daily forecast among all models.

5.3.2 Model 2: SARIMAX and NAR NN

Algorithm that is used in model 2 for electricity price forecasting is as follows:

- Step 1. A model with dummy variables is constructed to eliminate weekly seasonality.
- Step 2. SARIMA model of residuals of the model in step 1 is built with demand as an exogenous variable.
- Step 3. Forecasts for demand is obtained by DSHW exponential smoothing method.
- Step 4. NAR neural network model is used for modeling the residuals of the model in step 2.
- Step 5. Forecasts are combined with the hybrid model by using the forecasts from DSHW. Forecast values from SARIMA model, forecast values from NN model and coefficients of model with dummy variables are added together.

The difference between first and second hybrid models are the inclusion of exogenous variable. In the second one, demand is included in the linear part, so the SARIMA model becomes SARIMAX model. Again, weekly seasonality will be eliminated by the same dummy model which is constructed with price as response, dummy variables as covariates without an intercept term. Estimated dummy model is shown in the following equation. Then, a SARIMAX model will be built by the residuals of this model.

$$\begin{aligned}\hat{y}_t =& 151.55Mon + 155.77Tues + 155.29Wed + 156.59Thur \\ &+ 156.02Fri + 150.3Sat + 131.98Sun\end{aligned} \tag{5.3}$$

Table 5.4: Coefficients and Standard Errors of SARIMAX Model

	Coefficients	S.E.
ar1	-0.0555	0.0644
ar2	0.1225	0.0451
ar3	0.1454	0.0394
ar4	0.3030	0.0287
ma1	-0.2109	0.0651
ma2	-0.3502	0.0604
ma3	-0.1553	0.0435
ma4	-0.2614	0.0351
sar1	0.0802	0.0051
sar2	0.0659	0.0050
sar3	0.0487	0.0049
sma1	-0.9423	0.0024
demand	0.0049	0.0001

SARIMAX$(4,1,4)(3,1,1)_{24}$ model can also be shown as follows

$$\begin{aligned}&(1 + 0.055B - 0.1225B^2 - 0.145B^3 - 0.303B^4)(1 - 0.080B^{24} - 0.066B^{48} \\ &- 0.049B^{72})(1 - B)(1 - B^{24})y_t = (1 - 0.211B - 0.350B^2 - 0.155B^3 - 0.261B^4) \\ &(1 - 0.942B^{24})\epsilon_t + 0.005x_t\end{aligned} \tag{5.4}$$

Assumptions of residuals of SARIMAX model will be checked. The results of the diagnostic checking are same with the SARIMA model in the previous hybrid model.

In Jarque-Bera Normality Test, the null hypothesis that residuals are normally distributed is rejected with a p-value equals to 2.2×10^{-16} at 0.05 significance level. In Box Pierce Test, the null hypothesis which states that residuals are not correlated is failed to reject with a p-value 0.2928 which is greater than 0.05 significance level. ACF-PACF plots of squared residuals show that there is heteroscedasticity. Again, a NN model will be used in following stages to try to explain nonlinearity in the residuals.

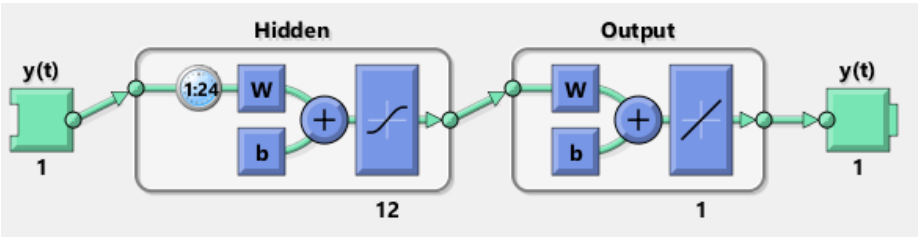

Figure 5.15: NAR Neural Network for Model 2

In this model, demand is not included in the neural network because it is included in the linear part. Therefore, Nonlinear Autoregressive (NAR) Neural Network model is used in the nonlinear part to model residuals of SARIMAX model. In NAR NN, Levenberg-Marquardt algorithm is used for training and mean squared error (MSE) is used to find the best fit. The network for this model can be seen in Figure 5.15. In this networks, 24 is the number of delays and number of hidden layers is 12. These values are decided by trying different numbers and comparing their performances.

Table 5.5: Accuracy measures for one-week ahead and one-day ahead forecasts of Model 2

Forecast	ME	MSE	RMSE	MAE	MPE	MAPE
One-week ahead	-3.122	302.303	17.387	13.691	-2.005	7.733
One-day ahead	24.647	917.168	30.285	26.844	13.153	14.633

The second hybrid model is also good at capturing general movement of the series. However, forecast values are greater than actual values at most of the time points in the series which can be seen in Figure 5.16. Therefore, this makes the RMSE and

MAPE values greater than the previous model. There is also a significant decrease in the performance of one-day ahead prediction in the Model 2 when comparing with Model 1 which also can be seen in Figure 5.17.

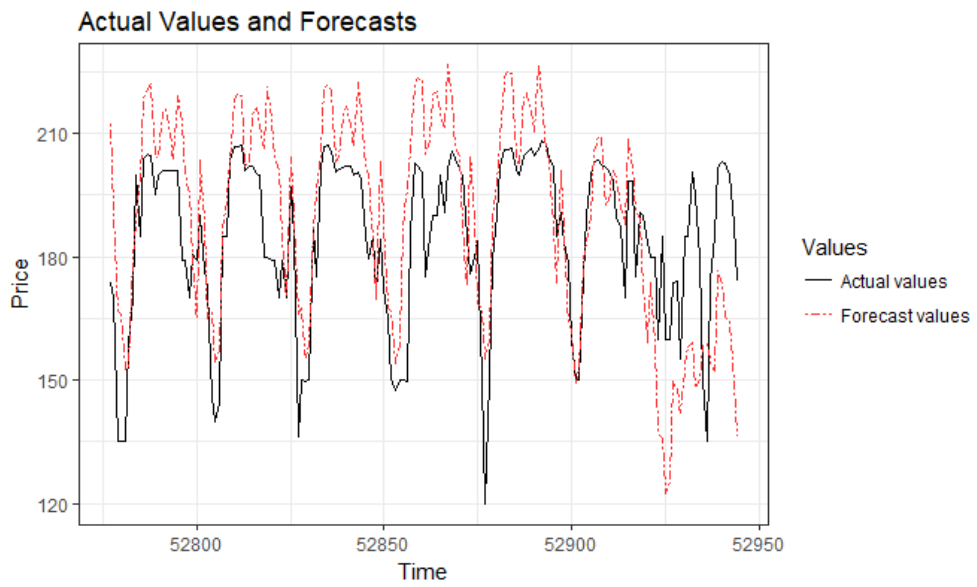

Figure 5.16: Time Series Plot of Actual Values and One-week Ahead Forecast Values of Model 2

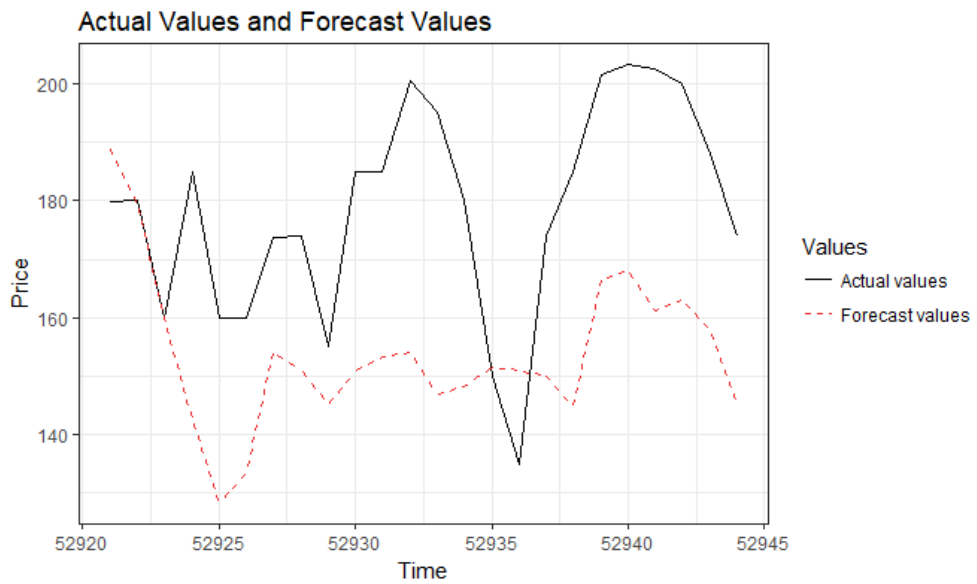

Figure 5.17: Time Series Plot of Actual Values and One-day Ahead Forecast Values of Model 2

5.3.3 Model 3: TBATS and NARX NN

Algorithm that is used in model 3 for electricity price forecasting is as follows:

- Step 1. A TBATS model is conducted for electricity price with daily and weekly seasonality
- Step 2. NARX neural network model is used for modeling the residuals of the TBATS model and demand is used as input variable.
- Step 3. Forecasts for demand is obtained by DSHW exponential smoothing method.
- Step 4. Forecasts are combined with the hybrid model by using the forecasts from DSHW. Forecast values from TBATS model and forecast values from NN model are added together.

In the third hybrid model, TBATS model is used in linear part instead of SARIMA model. In TBATS, multiple levels of seasonality can be included in the model.So, there is no need to make a model with dummy variables. As a result, number of steps in the algorithm decreases. Additionally, there is no assumption about the errors of the model.

msts (Multi-Seasonal Time Series) command in R is used for reading time series with multiple seasonality [127]. In this study, the seasonal periods are 24 and 168 (7*24) because we have hourly data.

TBATS model that is used in this hybrid model is TBATS(1, 0,0, 0.82, <24,5>, <168,4>). First value is Box-Cox parameter which is 1, so it means that there is no need to make a Box-Cox transformation. The error is modelled as an ARMA$(0, 0)$ process and $phi = 0.82$ is the damping parameter. The number of Fourier terms used for daily seasonal period is 5 and the number of Fourier terms used for weekly seasonal period is 4 in this model.

Smoothing parameters of this model are:

Alpha: 0.7757752

Beta: -0.1572273

Gamma-1 Values: 0.007573274 0.001725114

Gamma-2 Values: -0.0004319049 -0.002426854

Then, NARX NN model is used to model the residuals of TBATS model. In NARX NN Levenberg-Marquardt algorithm is used for training. Mean squared error (MSE) is used to find the best fit. The network for this model can be seen in Figure 5.18. In this networks, 24 is the number of delays for both input and target variable and number of hidden layers is 10. These values are decided by trying different numbers and comparing the performances.

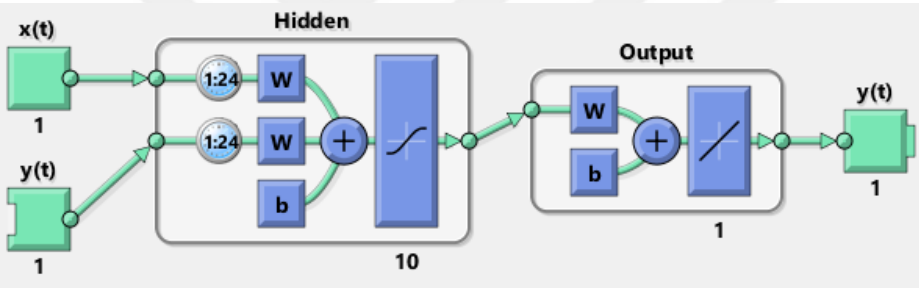

Figure 5.18: NARX Neural Network for Model 3

Table 5.6: Accuracy measures for one-week ahead and one-day ahead forecasts of Model 3

Forecast	ME	MSE	RMSE	MAE	MPE	MAPE
One-week ahead	14.485	468.645	21.648	18.381	7.554	10.023
One-day ahead	12.559	555.738	23.574	19.990	6.253	11.419

For one-week ahead forecast in Figure 5.19, it can be seen that the predicted values are smaller than real values. The performance of this model is worse than Model 1 and Model 2. However, for one-day ahead prediction, it is better than the Model 2.

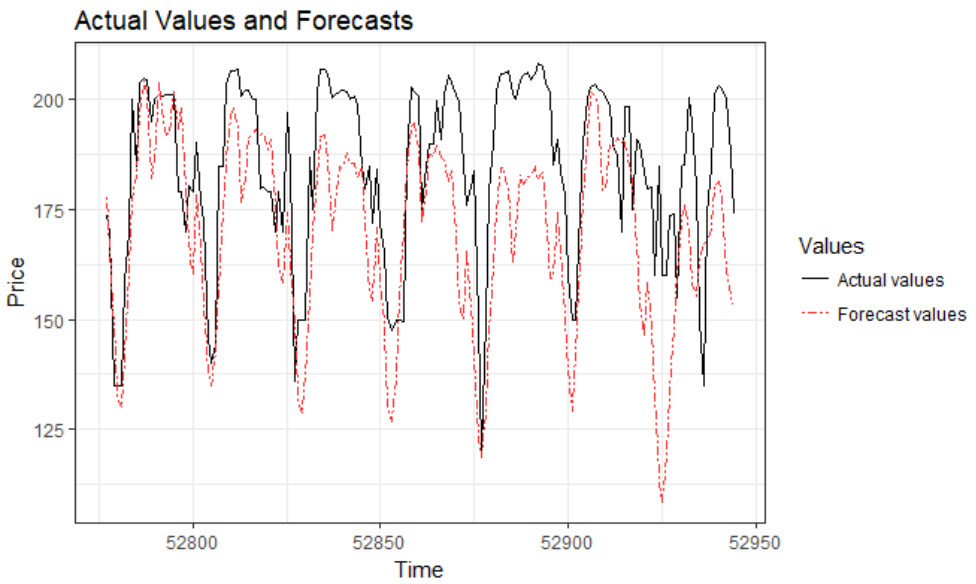

Figure 5.19: Time Series Plot of Actual Values and One-week Ahead Forecast Values of Model 3

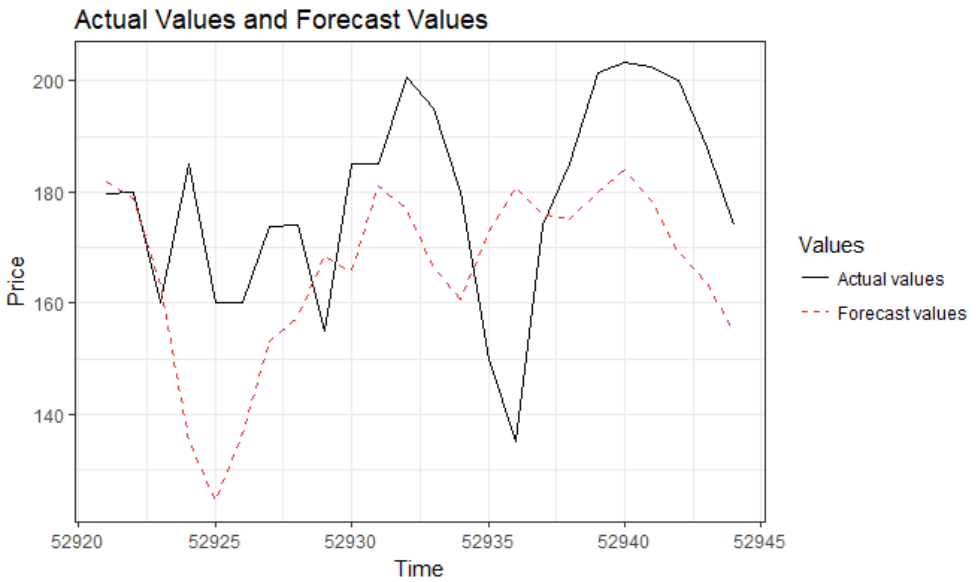

Figure 5.20: Time Series Plot of Actual Values and One-day Ahead Forecast Values of Model 3

5.3.4 Model 4: TBATS with regressor and NAR NN

Algorithm that is used in model 4 for electricity price forecasting is as follows:

- Step 1. A TBATS model with demand as exogenous variable is conducted for electricity price with daily and weekly seasonality.
- Step 2. Forecasts for demand is obtained by DSHW exponential smoothing method.
- Step 3. Neural network model is used for modeling the residuals of the TBATS model.
- Step 4. Forecasts are combined with the hybrid model by using the forecasts from DSHW. Forecast values from TBATS model and forecast values from NN model are added together.

It will be tested that how will be the effect of including the exogenous variable in linear part for TBATS and NN hybrid model. In R, *tbats* command ignores the exogenous variable, so *auto.arima* command is used with Fourier terms as additional covariates [128]. Daily and weekly seasonality are added to TBATS model with demand as regressor. The number of Fourier terms are selected by minimizing the AICc and BIC. The order of the ARIMA model is also selected by minimizing the AICc in the auto.arima function.

The selected TBATS model has 6 Fourier terms for daily seasonal period, and 1 Fourier term for weekly seasonal period. ARIMA model is selected as ARIMA(1,1,5).

Nonlinear Autoregressive (NAR) Neural Network model is used in nonlinear part to model residuals of TBATS with regressor model. In NAR NN, Levenberg-Marquardt algorithm is used for training and mean squared error (MSE) is used to find the best fit. The network for this model can be seen in Figure 5.21. In this networks, 24 is the number of delays and number of hidden layers is 9. These values are decided by trying different numbers and comparing their performances.

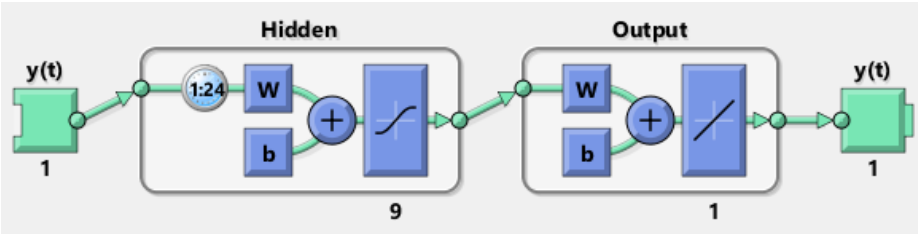

Figure 5.21: NAR Neural Network for Model 4

Table 5.7: Accuracy measures for one-week ahead and one-day ahead forecasts of Model 4

Forecast	ME	MSE	RMSE	MAE	MPE	MAPE
One-week ahead	1.989	610.995	24.718	19.799	1.362	10.945
One-day ahead	24.179	926.284	30.435	26.519	13.209	14.912

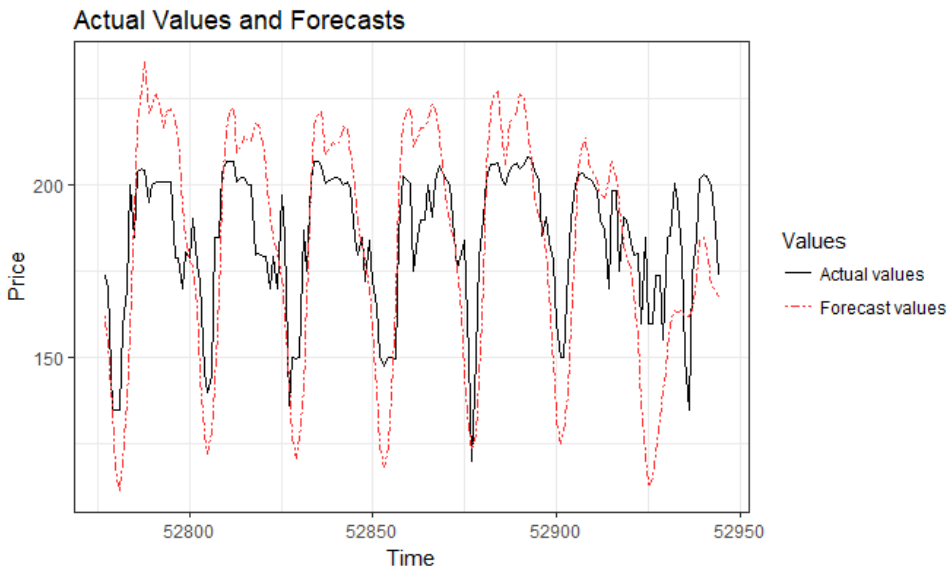

Figure 5.22: Time Series Plot of Actual Values and One-week Ahead Forecast Values of Model 4

For one-week ahead forecasting in Figure 5.22, forecast values and predicted values have a similar behaviour over time. However, it can be seen that the range of predicted values is greater than the range of actual values. Thus, it is not a very good model for one-week ahead forecast. Besides that, the results of one-day ahead forecasting which are given in Figure 5.23 are similar to Model 2.

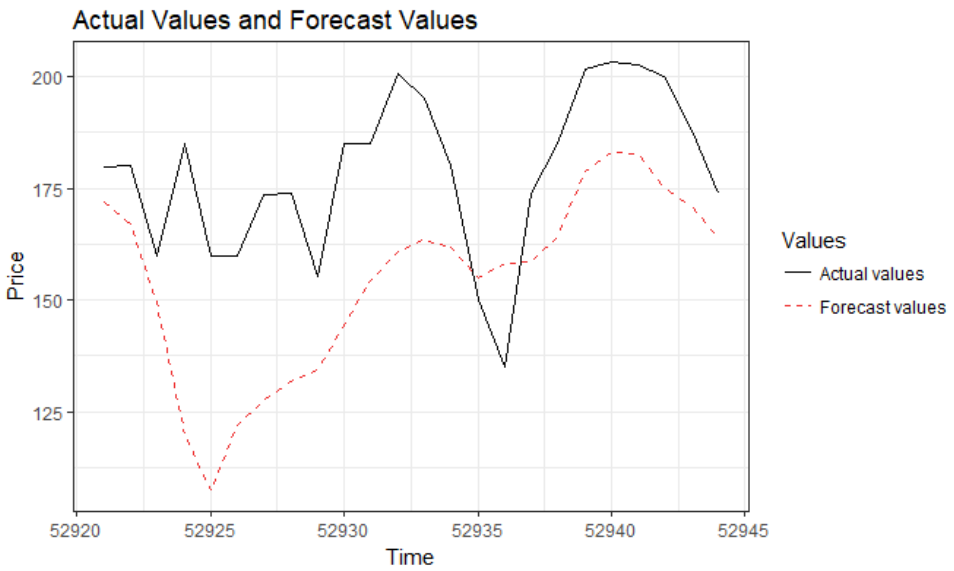

Figure 5.23: Time Series Plot of Actual Values and One-day Ahead Forecast Values of Model 4

5.4 Individual Models

Additionally, individual models are built to compare forecast performance with hybrid models, demand is also used as an exogenous variable in these models too.

5.4.1 Model 5: Seasonal Autoregressive Moving Average Model with Exogenous Input (SARIMAX)

Algorithm that is used in model 5 for electricity price forecasting is as follows:

- Step 1. A model with dummy variables are constructed to eliminate weekly seasonality.
- Step 2. SARIMAX model of residuals of the model in step 1 is built with demand as an exogenous variable.
- Step 3. Variance of the residuals are modelled with a GARCH Model. GARCH only affects the forecast prediction intervals when we use the standardized

residuals. Standardized residuals are obtained as subtracting conditional mean from the residuals and dividing by the square root of the conditional variance.

- Step 4. Forecasts for demand is obtained by DSHW exponential smoothing method.
- Step 5. Forecasts are obtained by combining with the forecasts from DSHW.

Same SARIMAX model in the model 2 is fitted to residuals of weekly dummy model which is SARIMAX$(4,1,4)(3,1,1)_{24}$. As aforementioned ACF-PACF plots of squared residuals show that there are some lags, which are out of white noise bands. GARCH model will be built to model variance of the residuals, since there is heteroscedasticity. GARCH(24,1) model is selected as the best model.

Table 5.8: Accuracy measures for one-week ahead and one-day ahead forecasts of Model 5

Forecast	ME	MSE	RMSE	MAE	MPE	MAPE
One-week ahead	-4.625	367.670	19.175	15.437	-2.773	8.658
One-day ahead	20.313	719.604	26.825	24.019	10.685	13.175

In Figure 5.24, generally the forecast values are close to actual values but there are some differences between forecast values and real values at higher values of price. In Figure 5.25, forecast values are smaller than actual values, so it is not a very good model when there is an abrupt change. SARIMA models give good results in forecasting when the series is linear. However, they are not easy to apply when assumptions are not satisfied.

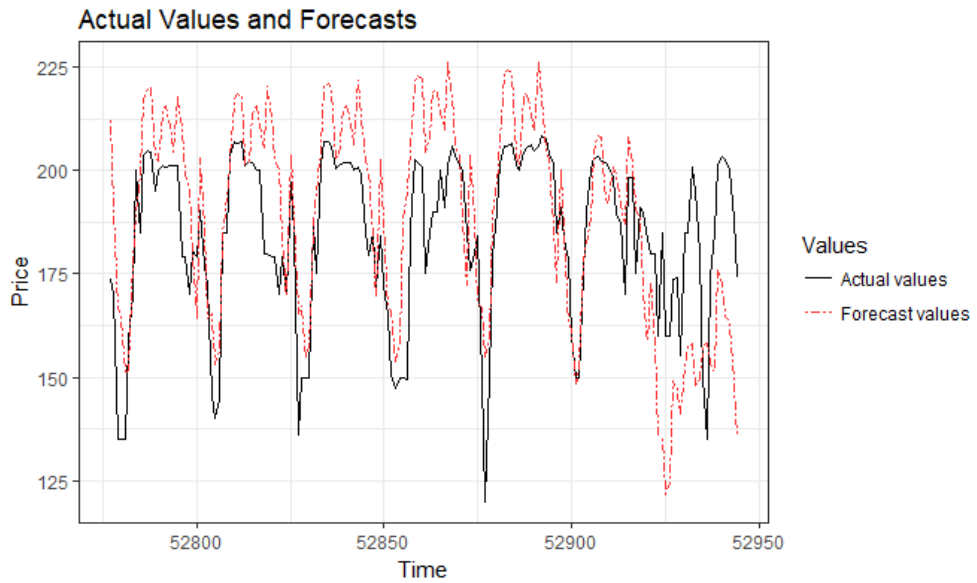

Figure 5.24: Time Series Plot of Actual Values and One-week Ahead Forecast Values of Model 5

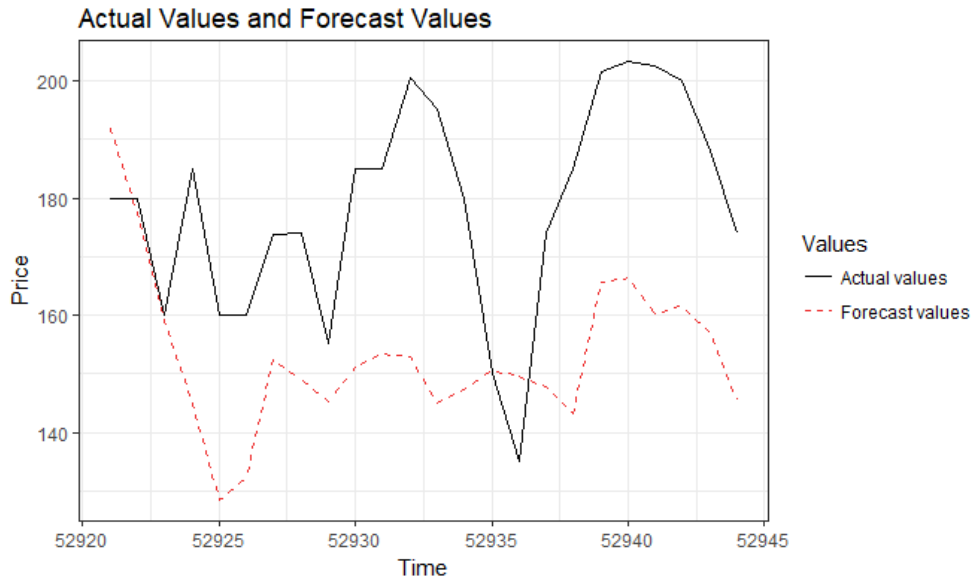

Figure 5.25: Time Series Plot of Actual Values and One-day Ahead Forecast Values of Model 5

5.4.2 Model 6: TBATS

Algorithm that is used in model 6 for electricity price forecasting is as follows:

- Step 1. A TBATS model is built with demand as an exogenous variable.
- Step 2. Forecasts for demand is obtained by DSHW exponential smoothing method.
- Step 3. Forecasts are obtained by combining with the forecasts from DSHW.

The TBATS model in the Model 4 is used solely. The number of Fourier terms and orders of the ARIMA model are selected by minimizing the AICc and BIC. The TBATS model has 6 Fourier terms for daily seasonality, 1 Fourier term for weekly seasonality and ARIMA model is ARIMA(1,1,5).

Table 5.9: Accuracy measures for one-week ahead and one-day ahead forecasts of Model 6

Forecast	ME	MSE	RMSE	MAE	MPE	MAPE
One-week ahead	2.635	606.980	24.637	19.737	1.734	10.935
One-day ahead	23.374	885.281	29.754	26.126	12.712	14.697

In Figure 5.26, predictions have the same data structure with the actual values but the range of predicted values is higher. Figure 5.27 indicates that Model 6 is not very successful at one-day ahead forecasting.

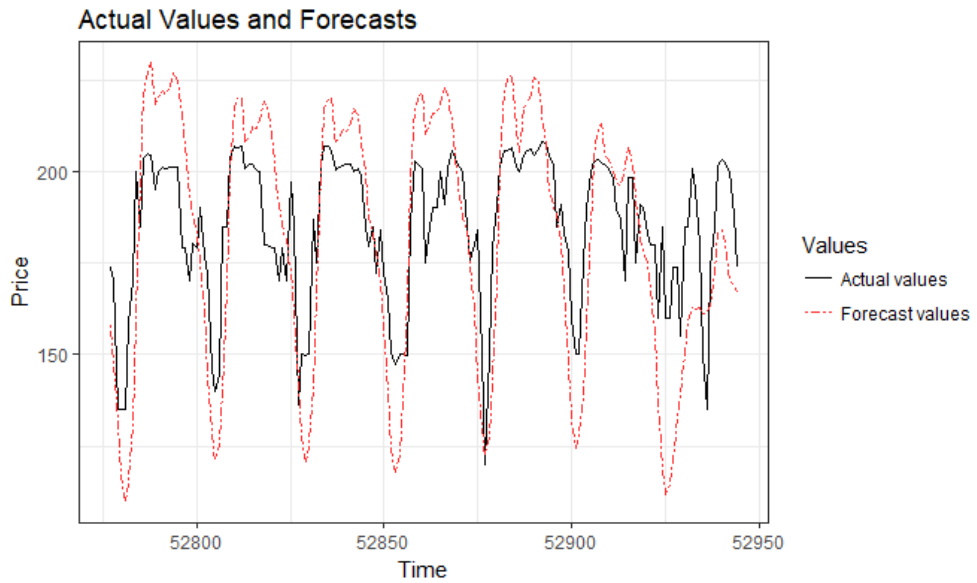

Figure 5.26: Time Series Plot of Actual Values and One-week Ahead Forecast Values of Model 6

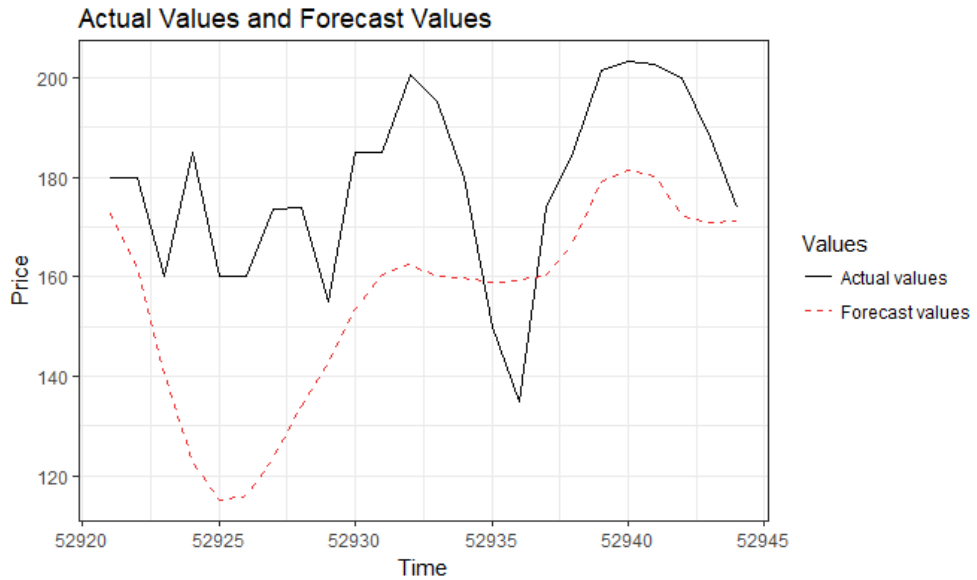

Figure 5.27: Time Series Plot of Actual Values and One-day Ahead Forecast Values of Model 6

5.4.3 Model 7: Nonlinear Autoregressive with Exogenous Input Neural Networks

Algorithm that is used in model 7 for electricity price forecasting is as follows:

- Step 1. A NARX Neural Network model is built with demand as an exogenous variable.
- Step 2. Forecasts for demand is obtained by DSHW exponential smoothing method.
- Step 3. Forecasts are obtained by combining with the forecasts from DSHW.

In the final model, only NARX NN model is used to forecast the electricity price. In NARX NN Levenberg-Marquardt algorithm is used for training. Mean squared error (MSE) is used to find the best fit. The network for this model can be seen in Figure 5.28. In this networks, 24 is the number of delays for both input and target variables and number of hidden layers is 10. These values are decided by trying different numbers and comparing the performances.

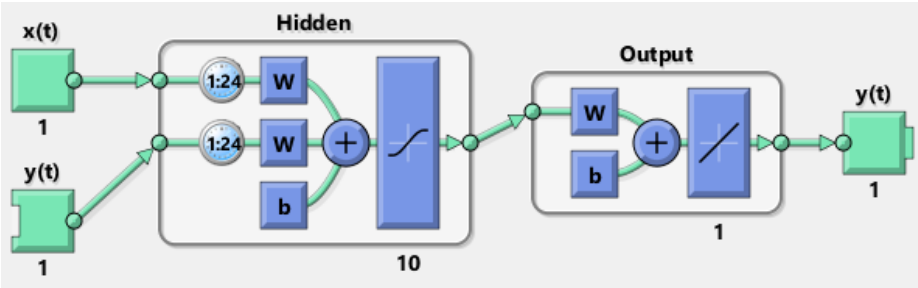

Figure 5.28: NARX Neural Network for Model 7

In Figure 5.29, forecast values are very close to the real values at most of the times. But at peak points, the forecast values are greater than real values. For one-day ahead forecasting, it performs better than most of the models in this study which can be seen in Figure 5.30. It can be concluded that NN models are good at series with sudden movements.

Table 5.10: Accuracy measures for one-week ahead and one-day ahead forecasts of Model 7

Forecast	ME	MSE	RMSE	MAE	MPE	MAPE
One-week ahead	4.016	292.878	17.114	13.602	1.970	7.559
One-day ahead	8.916	481.607	21.945	18.258	4.287	10.433

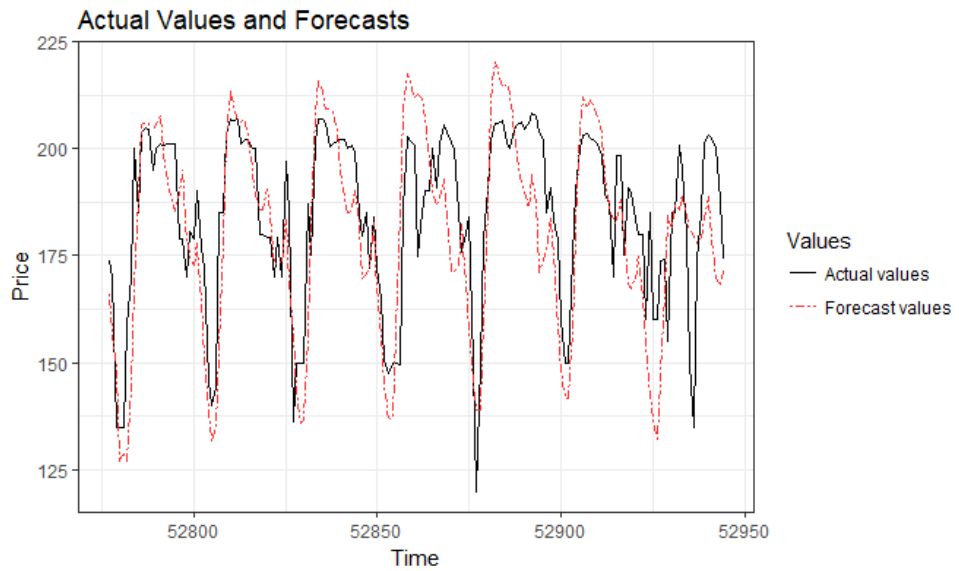

Figure 5.29: Time Series Plot of Actual Values and One-week Ahead Forecast Values of Model 7

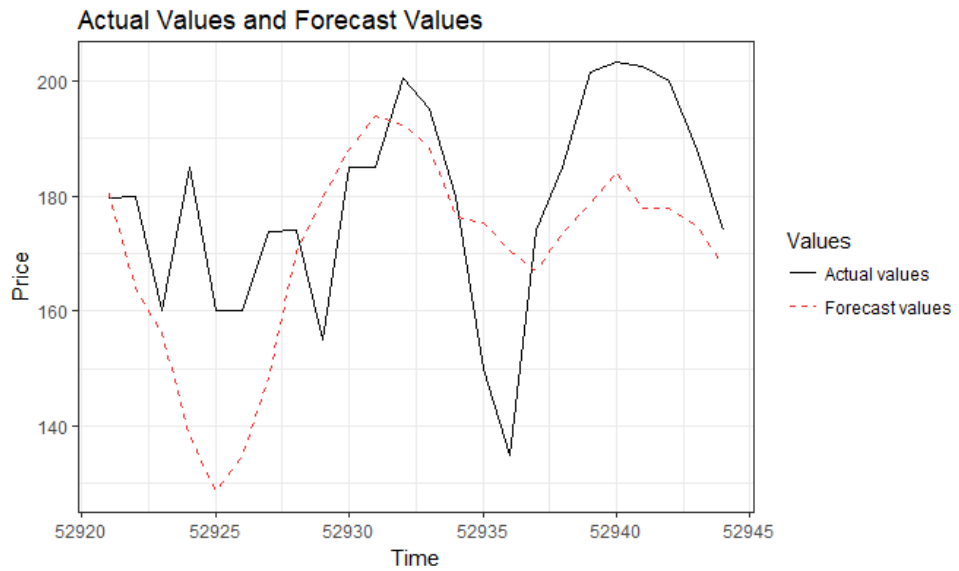

Figure 5.30: Time Series Plot of Actual Values and One-day Ahead Forecast Values of Model 7

5.5 Overall Results

One-week ahead and one-day ahead forecast performances of all models are shown in Table 5.11. RMSE and MAPE are used as accuracy measures.

Table 5.11: Forecast Performances

	Weekly Forecast		Daily Forecast	
MODEL	RMSE	MAPE	RMSE	MAPE
SARIMA-NARX NN	15.025	6.545	20.548	9.905
SARIMAX-NAR NN	17.387	7.733	30.285	14.633
TBATS-NARX NN	21.648	10.023	23.574	11.419
TBATS with reg- NAR NN	24.718	10.945	30.435	14.912
SARIMAX	19.175	8.658	26.825	13.175
TBATS with reg	24.637	10.935	29.754	14.697
NARX NN	17.114	7.559	21.946	10.433

Model 1 which is a hybrid SARIMA-NARX NN model outperforms other models in both one-week ahead and one-day ahead forecasting. Model 7 which is NARX NN model has the second best forecasting performance among all models. These results indicate that NARX NN model is a suitable method for time series like hourly electricity price. However, it is better to combine with a SARIMA model in a hybrid model.

Moreover, the results indicate that Model 1 is better than Model 2, and Model 3 is better than Model 4 in both one-week ahead and one-day ahead forecasting. This means that including exogenous variable demand in NN gives more accurate results.

CHAPTER 6

CONCLUSION

After the deregulations of Turkish electricity market in 2001, accurate electricity price forecasting has become a significant issue in the market. It is a difficult task to make accurate forecasts due to high volatility, multiple levels of seasonality and nonlinear relationships. As a result, there are numerous studies on electricity price forecasting.

In this thesis, several hybrid models are built by using SARIMA, TBATS and NN. In most cases, hybrid models outperform the individual ones. TBATS is more practical than SARIMA model, since multiple seasonality can be included in the R function and there is no need to check the assumptions for residuals. However, SARIMA models perform better for Turkish hourly electricity prices. It can be also concluded that among individual models, NARX NN has the best performance especially for one-day ahead forecast where an unexpected behavior has occurred.

Overall, hybrid model SARIMA-NARX NN has the best forecast performance for both one-week ahead and one-day ahead forecasting. This hybrid model outperforms its individual models that are SARIMAX and NARX NN. Another result is that including demand in nonlinear part of hybrid model as exogenous variable gives better result than including in linear part. The reason of this can be the nonlinearity in the demand series.

From the time series plots of actual values and forecast values, it can be deduced that some methods have better performance in the time periods that have high volatility. In the time series plots of weekly forecasts, it is clear that there is an unexpected movement in actual values in the last 24 hours. Therefore, results indicate that NARX

NN and SARIMA-NARX NN hybrid models have the most accurate forecasts when there is an unforeseen, nonlinear behavior.

In Appendix A, prediction performances of models for the train set is examined. A one week period has randomly chosen for time series plots in this chapter. According to RMSE values, hybrid models with TBATS have the best prediction performance for train set. However, main concern of this study is forecasting of future values. Therefore, comparison of the models is made according to performances for test set which is given in Table 5.11.

In conclusion, it is advisable to use SARIMA-NARX NN model for one-week ahead prediction. For forecasting oneday ahead or shorter time periods like one hour, it is recommended to employ NARX NN since it is more practical than a hybrid model and performance of NARX NN model is similar to SARIMA-NARX NN hybrid model.

For future studies, different models can be used in hybrid models. For instance, time series regression models and SARFIMA models are alternatives for SARIMA or TBATS. For the series in this study, SARFIMA models could give better results than SARIMA models because the series has long memory characteristics. However, there is no package for SARFIMA models in R, we could not have a chance to use SARFIMA models. Thus, an R package can be developed for SARFIMA models for future work. Instead of NN, threshold autoregressive (TAR) model, Self-Exciting Threshold AutoRegressive (SETAR) model, regime switching models and support vector regression can be used.

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APPENDIX A

PREDICTION PERFORMANCES

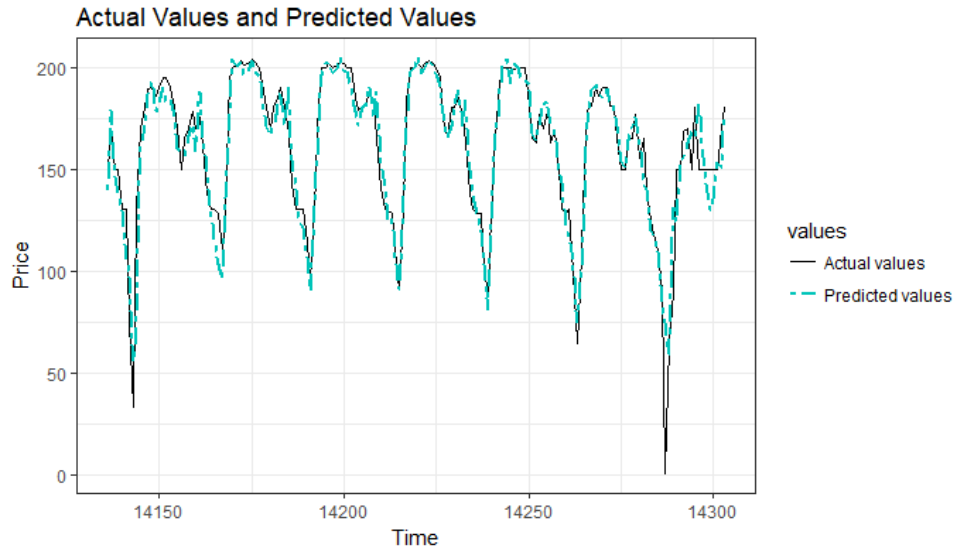

Figure A.1: Time Series Plot of Actual Values and Predicted Values from Model 1

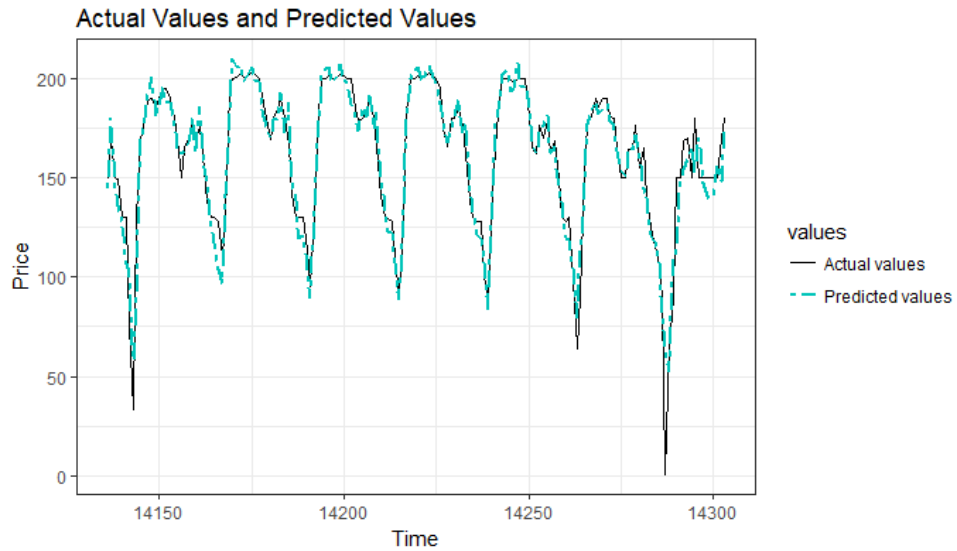

Figure A.2: Time Series Plot of Actual Values and Predicted Values from Model 2

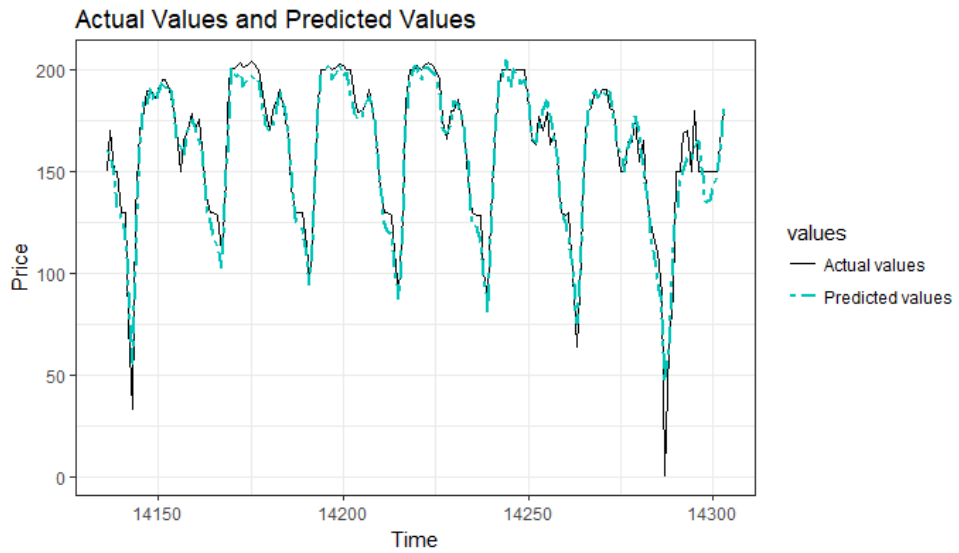

Figure A.3: Time Series Plot of Actual Values and Predicted Values from Model 3

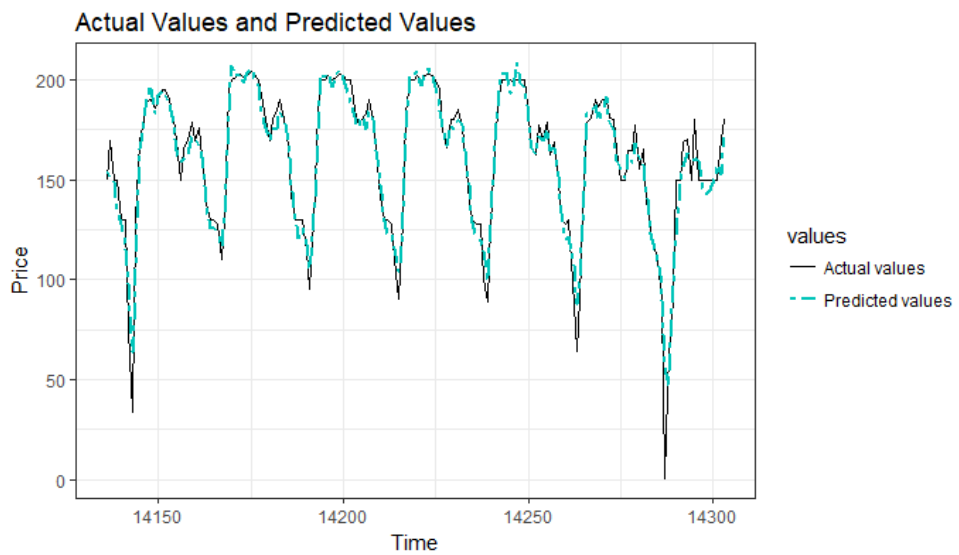

Figure A.4: Time Series Plot of Actual Values and Predicted Values from Model 4

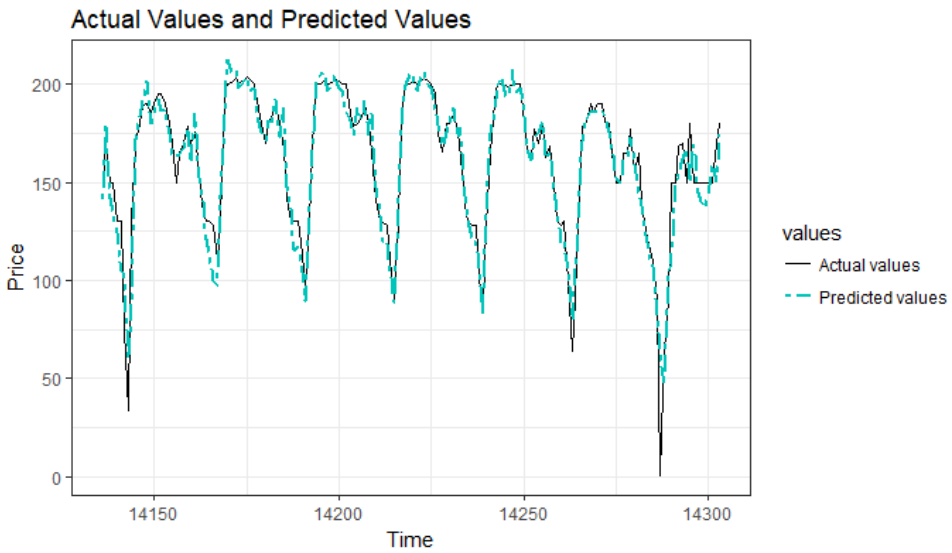

Figure A.5: Time Series Plot of Actual Values and Predicted Values from Model 5

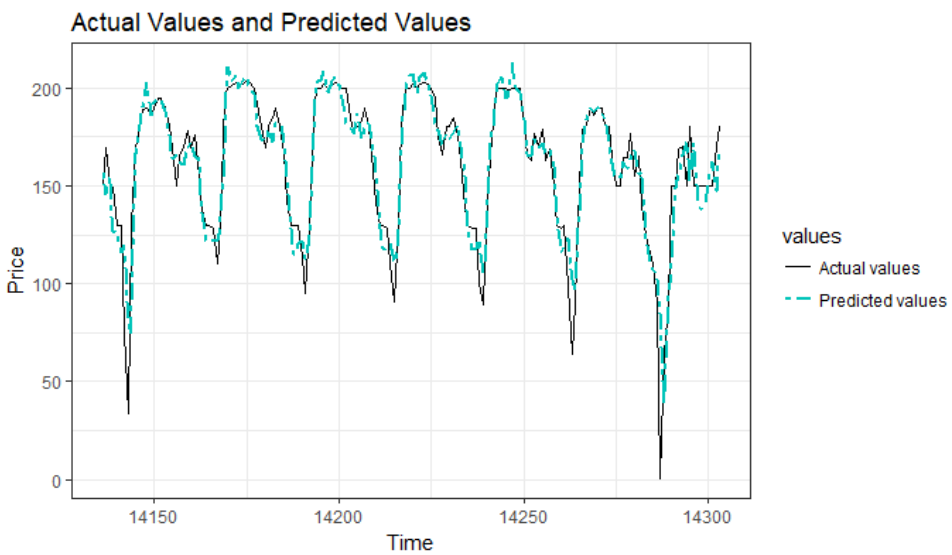

Figure A.6: Time Series Plot of Actual Values and Predicted Values from Model 6

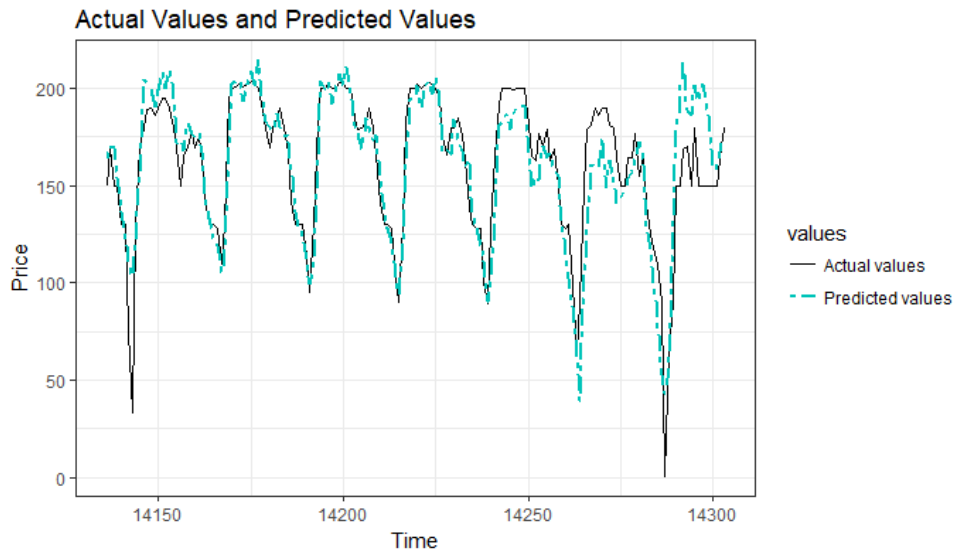

Figure A.7: Time Series Plot of Actual Values and Predicted Values from Model 7

Table A.1: Prediction Performances

MODEL	RMSE
SARIMA-NARX NN	23.224
SARIMAX-NAR NN	22.771
TBATS-NARX NN	18.193
TBATS with reg-NAR NN	18.877
SARIMAX	20.355
TBATS with reg.	21.859
NARX NN	40.846