ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL OF SCIENCE ENGINEERING AND TECHNOLOGY

COMPUTATIONAL DESIGN IN THE MEDIEVAL SELJUK ART: LEARNING FROM THE HANDS-ON CREATIVE TRADITIONS OF GEOMETRIC PATTERNS

Ph.D. THESIS

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ISTANBUL TEKNİK ÜNİVERSİTESİ ★ FEN BİLİMLERİ ENSTİTÜSÜ

ORTA ÇAĞ SELÇUKLU SANATI'NDA HESAPLAMALI TASARIM: GEOMETRİK DESENLERİN ELLE YAPIM SÜREÇLERİNDEKİ YARATICI GELENEKLERDEN ÖĞRENMEK

DOKTORA TEZİ

Sibel Yasemin ÖZGAN
(523122005)

Bilişim Anabilim Dalı

Mimari Tasarımda Bilişim Programı

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Sibel Yasemin Özgan, a Ph.D. student of ITU Graduate School of Science Engineering and Technology student ID 523122005, successfully defended the thesis/dissertation entitled "COMPUTATIONAL DESIGN IN THE MEDIEVAL SELJUK ART: LEARNING FROM THE HANDS-ON CREATIVE TRADITIONS OF GEOMETRIC PATTERNS" which she prepared after fulfilling the requirements specified in the associated legislations, before the jury whose signatures are below.

Thesis Advisor :	**Prof. Dr. Mine ÖZKAR** Istanbul Technical University	.....
Jury Members :	**Prof. Dr. Gülen ÇAĞDAŞ** Istanbul Technical University	.....
	Asst. Prof. Dr. Ethem GÜRER Istanbul Technical University	.....
	Asst. Prof. Dr Elif KENDİR BERAHA Istanbul Bilgi University	.....
	Asst. Prof.Dr. Ufuk SOYÖZ Kadir Has University	.....

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To my parents, Christine and Ramazan

FOREWORD

"*I see here a three-dimensional solid and it has some tile remains on it"* were my very premature observations that led at the end to the discoveries, which ground the fundamentals of this dissertation. There are not many who appreciate *seeing* as much as Prof. Dr. Mine Özkar does, and I am indebted for her continuous attention and encouragement, even at times when I conveyed such ambiguous and simple words to her. She has provided me with immense inspiration, caring and support throughout my studies. Her thoughtful feedback and brilliance throughout the years continue to support me with direction.

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Last but not least, my biggest gratitude goes to my family. My brother helped a lot in various technical matters and I especially thank his children Zeliş and Alya for making me laugh and bringing joy into my life. My parents provided an immense material and moral support throughout the years. They did their best to facilitate the dissertation journey. My mother literally carried all these heavy books around for me, while my father got in contact with several people to ensure I could get all necessary information and documents. Thousands of their insights and exchanges throughout my life have shaped how I see the world. Nevertheless, most of all, I am grateful to my parents for clarifying the real significance of *seeing* by quoting repeatedly the recognized words of Henry David Thoreau: "*It's not what you look at that matters, it's what you see"*. I dedicate this work to them.

July 2018

Sibel Yasemin ÖZGAN
(Interior Architect)

TABLE OF CONTENTS

Page

LIST OF FIGURES

Page

COMPUTATIONAL DESIGN IN THE MEDIEVAL SELJUK ART: LEARNING FROM THE HANDS-ON CREATIVE TRADITIONS OF GEOMETRIC PATTERNS

SUMMARY

Historical examples of design production provide valuable references for non-digital calculations past. Departing from a fundamental idea that historical design processes can initiate new insights and knowledge to the design computation field, this research investigates the hands-on creative traditions of the medieval Seljuk artisan. Seljuk art encompasses the renowned visual tradition of the medieval Islamic art that is commonly cited for the geometric patterns that consist of polygons, stars and lines. The lace-like manifestation and geometrical rules beneath the patterns gained a huge attention, especially by mathematicians. Many authors have attempted to decipher the complicated geometrical principles of setting out the pattern designs. Yet, such studies generally approach patterns as end products and do not deliberate upon the design processes. The design process, in contrast, embraces an embodied knowledge that relate to material and making that impinge on design decisions. Throughout the study, due attention is given to material processes of how these designs are constructed as they are not mere form exercises from a design point of view.

Artisans working under the patronage of Seljuks in Anatolia incorporated geometrical patterns to their buildings. Hence, the making of the geometrical patterns do not merely depend on visual transformations, but rather relate to the spatial qualities of the space and the architectural form. The builders and artisans of the time and region approached their tasks in a holistic manner, understanding the relations between shapes in two and three-dimensions, and across different scales. The investigations in the thesis revolve mainly on a certain medieval master builder named Muhammad al-Tusi, who owned a ceramic atelier in Konya, a central city in modern Turkey. The craftsmen working in this atelier were using a particular material, namely tile-mosaic through which they materialized geometrical patterns in tremendous diversity. This material appeared first at that time in thirteenth century Anatolia and Tusi's principal undertaking was to transfer the existing geometric pallet to this new material. The sophistication of the holistic and consistent undertaking in designing the form of the building as well as in the application of a geometry of polygons and three-dimensional forms in the tradition of the Tusi Atelier provides an insight to the design knowledge, method and thinking available at that time and in that place. The investigations establish that creative design decisions cannot merely explained with visual transformations. Craftsmanship relies on continuous involvement and the potentials provided by the new material and the knowledge gained from the hand and the eye shapes the designs.

The design decisions that arise during medieval artisans' processes are still relevant from a contemporary viewpoint. Today, the inflexibility in the ways that design processes integrate technology contrasts the Seljuk artisans' interest in forming novel geometric patterns that take spatial relations into account as well. Current computational design approaches make generally use of predetermined representations that are embedded in computer-based design tools. Conversely, medieval builder's analog computations that involve the geometrical pattern adaptation to a wide variety of geometrical forms and architectural surfaces plays into creativity in much more flexible ways than those usually attributed in technology. Nonetheless, that does not mean that rule-based approaches cannot aid design processes in more suitable ways. Tusi's designs confirm that initial rules can be applied in different ways, can be comparative tools to understand what repeats and what changes in design. Although the word rule comes into conflict with creative processes to some extent, rule-based systems can operate as inventive thinking tools in design. In reflective analysis, rules benefit the designer as live documentation of possibly infinite yet interconnected alternatives of manifestations and rule-based approach to design holds potential in talking about design creatively and, with awareness.

Computational design theories that use abstractions instead of deterministic ways develop formal ways that hold potentials to aid creative processes in appropriate ways. The dissertation encourages designers to use computational tools and methods. Yet, designers should not serve as the passive witnesses to the development of computational tools, rather they should search for ways to expand existing implementations by active participation.

ORTA ÇAĞ SELÇUKLU SANATI'NDA HESAPLAMALI TASARIM: GEOMETRİK DESENLERİN ELLE YAPIM SÜREÇLERİNDEKİ YARATICI GELENEKLERDEN ÖĞRENMEK

ÖZET

Geçmiş çağlara ait tasarım pratiklerinin ve kültürlerinin araştırılmasının, günümüzde hesaplamalı tasarım kuramına ışık tutacağı düşüncesi üzerinden yola çıkan bu araştırma, orta çağ Selçuklu zanaatkârının yaratıcı süreçlerini geometrik desen tasarımları üzerinden incelemektedir. Çokgen, yıldız, kırık ve düz çizgilerden oluşan ve zengin geometrik strüktürler üzerinden şekillenen geometrik desenler, İslam sanatının kendine has ve özelleşmiş bir görsel geleneğe sahip olmasında önemli bir rol oynamıştır. Orta çağ İslam sanat ve mimarisinin temel unsurlarından olan bu süslemelerin altyapılarında bulunan geometrik kurallar hakkında birçok araştırma bulunmakta, fakat bu araştırmalar genel itibari ile yalnızca sonuç ürüne odaklanmakta ve tasarım süreçlerine dair çok fazla bilgi içermemektedir. Bunun yanı sıra, Anadolu'da üretilmiş ve bu coğrafyaya özgün desenler ve malzeme teknikleri ne yazık ki hak ettikleri kadar kapsamlı olarak incelenmemiştir.

Öte yandan, geometrik desenlerin Anadolu coğrafyasında bu denli çeşitli olması, desenlerin salt geometri bilgisinin görsel uygulamaları değil de deseni üreten sanatçıların farklı malzeme ve zanaat teknikleriyle çeşitlendirebildiği tasarım ürünleri olduklarını ortaya koymaktadır. Literatürde var olan araştırmaların aksine, bu tez geometrik desenlerin hesaplamalı tasarım ve yapım süreçlerine odaklanmakta, geometri bilgisinin, kendi dönemi içerisinde yeni geliştirilmiş olan malzeme tekniğinden doğan tasarım problemleri sebebiyle ne şekilde yorumlanmış olabileceğini incelemektir. Tez kapsamında yapılan çalışmalar, desenleri tasarlayan sanatçıların malzeme üzerine düşündüklerini ve malzemeye bağlı olarak gelişen yapım teknikleriyle geometrik desenleri çeşitlendirdiklerini göstermektedir. Bu anlamda malzeme ve yapım bilgisinin, geometrik desenlerin hesaplamalı tasarım süreçlerini nasıl ve hangi şekillerde etkilediğini anlamak mümkün olmaktadır.

Yapılan araştırmalar Türkçe sanat tarihi literatüründe ismi Mehmet olarak geçen Tus'lu bir yapı ustasına, Muhammad al-Tusi'ye, ve ona atfedilen eserlere odaklanmıştır. Literatürde yer alan bilgiler ışığında, Tus'lu yapı ustasının yalnızca geometrik süslemelerden değil, aynı zamanda mimariden ve farklı yapı paydaşları tarafından sürdürülen tasarım süreçlerinin uyumlu çalışmasından da sorumlu olduğu anlaşılmaktadır. Orta çağ zanaatkarlarının tasarımlarını her zaman önceden planlanmış şemalar üzerinden değil de çoğu zaman da elde yapım süreçleri esnasında ortaya çıkan dokunma, tutma görme gibi fiziksel deneyimler ışığında şekillendirdikleri bilinmektedir. Hesaplamalı tasarım alanında gerçekleştirilen çalışmaların amaçları

arasında pratikle öğrenilen becerilerin ve tasarım kararlarının modern teknolojiye ne şekilde aktarılabileceği üzerine çeşitli çalışmalar yapılmıştır. Çalışmaların merkezine Gotik çağın yapı temellerini ve yapı ustalarını alan araştırmalardan başarılı sonuçlar elde edilmiş, elde edilen verilerin dijital fabrikasyon teknolojilerine aktarılmasıyla ilginç üretimler gerçekleştirilmiştir. Bu tezin esas amacı ise Tus'lu ustanın elde yaptığı hesaplamalı tasarım sürecini, özelleşmiş bazı eserleri üzerinde yapılan detaylı çalışmalarla anlamak ve bu süreçten günümüz hesaplamalı tasarım modelleri için paydalar çıkarmak olmuştur.

Tus'lu ustanın yönettiği ve onüçüncü yüzyılda Konya'da faaliyet gösteren çini atölyesinde çalışan zanaatkarlar, on ikinci yüzyılın sonu ve on üçüncü yüzyılın başında İç Anadolu topraklarında ortaya çıkmış kendine has ve benzersiz bir malzeme olan çini mozaik malzemeyi geometrik desen üretimlerinde kullanmışlardır. Selçukluların altın çağında, komşu İran'dan Anadolu topraklarına göç eden seramik zanaatkarları tarafından geliştirilen bu yenilikçi malzeme, geometrik desenlerin üslup değişiminde büyük rol oynamıştır. O dönemin zanaatkarları ve sanatçıları, farklı coğrafyalarda ve ölçeklerde sürdürülen projeler arasında tasarım ve yapım bilgisini bilinçli bir şekilde karşılaştırmış, uyarlamış ve yeniden uygulamışlardır. Nesiller ve coğrafyalar boyunca aktarılan yapım bilgisi yalnızca görsel desen tasarımını değil, malzemeyi, zanaat tekniklerini ve geometri bilgisini de içermiştir. İran Büyük Selçuklu topraklarından Anadolu'ya yerleşen seramik ustaları, İran'ın geleneksel tuğla mimarisine uyguladıkları seramik zanaatını, Anadolu'nun geleneksel malzemesi olan taş işçiliğine uyarlamak istemiş ve sürdürdükleri deneysel çalışmalar sonucunda çini mozaik malzemeyi geliştirmişlerdir.

Yapı faaliyetlerinin her evresinden ve tüm bireyler arasındaki koordinasyondan sorumlu olan Tus'lu ustanın temel tasarım problemlerinden biri ise o zamana kadar farklı malzemeler üzerinde uygulanmış olan mevcut geometrik tasarım paletini bu yeni malzemeye aktarmak olmuştur. Tezde yapılan araştırmalar, zanaatkarın somut el pratikleri esnasında yeni malzeme ile kurduğu diyaloğu anlamak üzerine kurgulanmıştır. Başka atölyelere atfedilen çalışmalarla yapılan karşılaştırmalar sonucunda, Tus'lu ustanın malzeme potansiyellerini göz önüne alarak tasarımlarını şekillendirdiği anlaşılmaktadır. Malzemenin getirdiği ve Tus'lu usta tarafından uygulanan temel yeniliklerin başında, desene ait olmayan ek çizgilerle desen tasarımının herhangi bir kesintiye uğramaksızın üretilebilmesi bulunmaktadır. Diğer zanaat türlerinde bulunmayan bu özellik, desenlerin sonsuzluk imgesini bozmayacak şekilde üretilebilmelerine olanak sağlamıştır. Bu ayırıcı özellik, Tus'lu ustanın benzer paydaşları arasındaki bireyselliğini ve farklılığını ortaya koymaktadır. Zanaat faaliyetleri, yeni malzeme tarafından sağlanan potansiyeller ve el işçiliği sırasında elde edilen bilgiler zanaatkarların tasarım kararlarını etkilemiş ve böylelikle tasarımları şekillendirmiştir.

Tez kapsamında o dönemde faaliyet gösteren zanaatkarların çoğunlukla iki boyutlu düzlemlerde uyguladıkları desen tasarımlarını malzeme ve yapım teknikleri ışığında farklı eğrisel yüzeylere ve üç boyutlu formlara nasıl uyarladıkları da incelenmiştir. Muhammad al-Tusi ustanın atölyesinde çalışan zanaatkarlar, tasarımları bütünsel bir şekilde ele alarak, iki ve üç boyutlu şekiller ve farklı ölçekler arasındaki ilişkileri de kurgulamışlardır. Yapılan incelemeler, sanatçıların mekânsal nitelikleri ve mimari biçimleri de göz önüne aldığını göstermektedir. Dolayısıyla, geometrik örüntülerin oluşumu ve tasarım süreçleri yalnızca görsel geometrik kurallar üzerinden tam anlamıyla açıklanamamaktadır. Biçime etki eden bu kuralların yanı sıra, desenlerin üretiminde kullanılan farklı malzemeler ve yapım teknikleri, sanatçıların desenleri ele

alış biçimlerini değiştirerek görsel tasarımların çeşitlenmesine sebep olmuştur. Malzemeyi özümsemek ve getirilerini keşfetmek desen tasarımını farklı şekilde yönlendirmeyi ve değiştirmeyi mümkün kılmıştır.

Bu araştırma ışığında ortaya çıkan malzeme ve yapıma dair tasarım bilgileri, günümüzde ne yazık ki kaybolmuş olan tasarım ve zanaata dair yapım bilgilerinin yeniden yorumlanmasına ve hesaplamalı tasarım araçlarıyla yeniden üretilebilir hale gelmesine olanak sağlayacak bir teorik model ortaya koymaktadır. Böylelikle, Anadolu Selçuklu kültürüne ait bilgilerin eski örneklerden salt kopyalanarak birebir aynı şekilde üretilmelerinden ziyade, güncel bir tasarım bilgisi olarak üretilmesi ve çeşitlendirilmesi mümkün olabilecek, böylelikle kültürel zenginliklerimizin sürdürülmesine katkı sağlanabilecektir.

Orta çağ sanatçılarının geometrik desen yapım süreçleri sırasında ortaya çıkan tasarım kararları, çağdaş bir bakış açısından hala geçerlidir. Tusi'nin Atölyesi'nin geleneğinde iki boyutlu çokgen geometrisinin farklı üç boyutlu formlara ve mimari yüzeylere uygulanmasında ele alınan bütünsel yaklaşım, tasarım bilgisi ve yöntemi günümüz hesaplamalı tasarım uygulamalarına farklı bir akış açısı kazandırmaktadır.

Günümüz uygulamalarında farklı teknolojik araçlar ile gerçekleştirilen tasarım uygulamaları, Selçuklu zanaatkârlarının mekânsal ilişkileri de göz önünde bulunduran yeni geometrik desenler oluşturma konusundaki yaratıcı yaklaşımına ters düşmektedir. Mevcut hesaplamalı tasarım yaklaşımları, genellikle bilgisayar tabanlı tasarım araçlarına yerleştirilmiş olan, bu sebeple de önceden belirlenmiş belirli temsilleri kullanır. Tersine, orta çağ sanatçısının, desenleri çok çeşitli üç boyutlu formlara ve mimari yüzeylere adaptasyonunu içeren elde yaptığı hesaplamalı süreçleri geometrik desen tasarımlarını genellikle teknolojiye atfedilenlerden çok daha esnek yollarla yaratıcılığa dönüştürmüştür.

Bununla birlikte, bu kısıtlı durum kural tabanlı yaklaşımların tasarım süreçlerine daha uygun şekillerde yardımcı olamayacakları anlamına gelmemelidir. Kural sözcüğü yaratıcılıkla ters düşüyor gibi görünse de kural tabanlı sistemler tasarımda yaratıcı düşünme araçları olarak çalışabilir. Bu anlamda Tusi'nin tasarımları, başlangıç kurallarının farklı şekillerde uygulanabileceğini, kuralların tasarımda hangi tekrarların ve hangi değişimlerin olduğunu anlamak için karşılaştırmalı ve esnek araçlar olarak kullanılabileceklerini doğrulamaktadır.

Tez, tasarımcıları, hesaplama araçlarını ve yöntemlerini kullanmaya teşvik eder, fakat söz konusu teknoloji olduğunda üzerinde çok sorgulamadan uygulanan mekanik tasarım pratiklerine karşı çıkar. Deterministik biçimler yerine soyutlamaları kullanan hesaplamalı tasarım teorileri ve metotları, yaratıcı süreçleri geliştirmede yardımcı olma potansiyeli taşıyan tanımlanabilen modeller geliştirir. Teknolojiyi sorgulamak, bilgisayar programlarının sınırları yerine tasarım kriterleri çerçevesinde onun potansiyellerini araştırmak ve kullanım tarzı hakkında fikir sahibi olmak gerekmektedir. Tasarımcılar hesaplamalı tasarım araçlarının geliştirilmesinde pasif tanıklar olarak hizmet etmemeli, aksine mevcut teknolojik araçları aktif katılımla geliştirmenin ve genişletmenin yollarını aramalıdırlar.

1. INTRODUCTION: A COMPUTATIONAL UNDERSTANDING OF THE KNOW-HOW IN MEDIEVAL ISLAMIC ART

1.1 Computation and Medieval Islamic Art

This study bears a title that brings the words *computation* and *medieval Seljuk art* together, triggering cognitively a conflict as *computation* is commonly misinterpreted, due to its inevitable association with the usage of computers. Nevertheless, the etymological roots of *computation,* meaning to reckon things together, originate actually from Latin and its first usage as a word dates to fifteenth century, an era when computer technologies did even not come into use (Yalınay Çinici, 2012, p. 12). In his book "Algorithmic Architecture", Kostas Terzidis (2006) starts a discussion and explains how computation is often misunderstood and confused with the term *computerization*:

> While computation is the procedure of calculating, i.e. determining something by mathematical or logical methods, computerization is the act of entering, processing, or storing information in a computer or a computer system. Computerization is about automation, mechanization, digitization, and conversion. Generally, it involves the digitization of entities or processes that are preconceived, predetermined, and well defined. In contrast, computation is about the exploration of indeterminate, vague, unclear, and often ill-defined processes; because of its exploratory nature, computation aims at emulating or extending the human intellect. It is about rationalization, reasoning, logic, algorithm, deduction, induction, extrapolation, exploration, and estimation. In its manifold implications, it involves problem solving, mental structures, cognition, simulation, and rule-based intelligence, to name a few. (pp. 57-58)

Computation, described merely as any type of calculation, is different than the use of computers with little insight to how they work. It involves reasoning, hence a deeper understanding of the task at hand. As such, computation resembles the act of design, yet there are still ambiguous matters embedded in design activities, which we are still not reasonably competent to formalize. Designing is obviously a multifaceted activity and cannot be defined just as a straight- forward mechanism. Yet, we ought to also question whether computable problems exist in design or not. Nigel Cross's recognized observations on the issue may guide us through this debate:

> We have come to realize that we do not have to turn design into an imitation of science, nor do we have to treat design as a mysterious, ineffable art. We recognize that design has its own distinct intellectual culture; its own designerly "things to know, ways of knowing them, and ways of finding out about them. (Cross, 1999, p. 7)

Considering courses of action in design sorts of computation as well, this study attempts to untie various parameters that alter medieval Seljuk artisans' computational design processes. In order to do a centralized inquiry, the dissertation investigates particular geometric pattern designs that are attributed to the legacy of a specific medieval master builder namely Muhammad al Tusi and the craftsmen that worked in his ceramic workshop. This workshop, referred to Tusi atelier in the dissertation was active in thirteenth century central Anatolia during the reign of Seljuks.

How do designers design? A clear-cut and complete explanation to the question is not within reach. Yet, the question arises as to whether we can distinguish certain parameters that alter a design process, factors such as visual transformations, material properties or issues that are associated with making- tools and techniques to mention but a few. The investigations on medieval craftsmen's processes in this study looks upon various matters that initiate apparent changes in the making of geometric pattern designs. An understanding on such lucid design parameters initiates a transferrable knowledge of computation, hence aids to reflect upon current debates on computational design paradigms.

Computational theories and technologies had a profound impact on the traditional understanding of design and creativity. They altered the way designers work and consequently, think and approach design problems. Hence, the digital age has witnessed a drastic shift in design theories and methods. These new perspectives in design methods engendered novel techniques and resulted in an expanding usage of computational tools in design practices. Nevertheless, the adaptation of computational design tools into design practices has brought up open questions about their role in creative processes. The execution of certain design actions in the strict mechanism of computer programs requires well understood and defined descriptions of the imminent activity. Thus, incorporating the working logic of a computer that lay on combinatorial problem solving into creative processes has created skepticism among designers, who are not proficient in formalizing their thoughts and actions by certain rule definitions and in appropriate formalism. Whether design thinking is reducible to precise

algorithms have become of concern in many philosophical matters. Furthermore, some of the computer implementations ended up being repetitive and not necessarily very creative, which similarly caused such concerns to increase.

Computational models with a great capacity of calculation undoubtedly simplified our daily lives and solved many our problems we were not even alone capable of tackling around. Furthermore, the step towards the algorithmic description of the design made some of the prosperous projects from the twenty-first century feasible. Mitchell (1993) writes that:

> Any successful attempt to describe the mechanism of some 'creative' design activity will have the immediate effect of redefining that activity as 'noncreative'. The more success we have, the more we can be accused of dealing only with the noncreative aspects of design.

As designers, instead of a sole criticism on the available computer applications and opposing against the limited nature of computers, we need to bring forward solutions on how computation could fit more into our creative working. Therefore, we need to learn to describe our design processes. We require an understanding on how much of our work can be depicted in appropriate formalism and try to distinguish how computational models can aid our creative processes, perhaps best depicted in Knight and Stiny's (2001) words: "Computation and creativity should be mutually supportive in the design process" (p.355).

What kind of functions can be computed? is not only a kind of inquiry, design specialists seek out the answer for. It has acted as the key question of computational theory from the very outset. The computational theory and digital computers with a continuously developing technology were the inventions of many creative minds, pioneers who questioned how particular problems from daily life could be solved on a model of computation. We remember these pioneers with their scientific contributions and rarely give attention to their creative thinking and their resemblance to designers. By helping in the creation, development and imagining of what computers and electronics could do, these creators unquestionably inquired issues akin to designer tasks. Just as the computational design methods, models and theories of computation are also constantly shifting over time. Instead of focusing on developing programs with strict algorithms, computer scientists pursue the solution for more complex problems. They endeavor to create more ambiguous programs, closer to the uncertainty in the

nature of design thinking. Noticing this potential of *variability of* computer algorithms can ensure more efficient adaptations of computer tools into design processes.

Historical examples of design production provide valuable references for non-digital calculations past. Awareness of how computation is involved in creative acts is obtainable thru in-depth research on certain design processes, especially on instances of hands-on computation that are fulfilled manually, rather than using a digital computer. That computation is embedded in design and computational design is not always a restriction are provable by analyzing such analog processes, principally because those designers were not restricted to pre-defined computer programs. They computed manually the design problems they faced. In this regard, the visual tradition of geometric patterns in Seljuk architecture represents a great example of such inquiries. The design and construction of geometric patterns on all kind of surfaces depend on certain geometrical rules. Thus, it is feasible to follow the computational processes beneath different design decisions. Through such an investigation, we can compare available processes executed in computer environments with by-hand practices. Significant knowledge is embedded in these design processes.

Medieval Islamic patterns are still triggers of fascination with their lace-like appearance and complicated geometrical rules behind. Out of geometrical shapes, medieval craftsmen developed a distinctive art form, which embraces a recognizable manifestation. There are many aspects to learn upon the making of the Seljuk patterns. Obviously, they encompass an aesthetic aspect as ornaments. Yet, the information entailed in these patterns are far more extensive than mere copies of the polygon and star shapes.

1.2 A Contemporary Culture of Ornaments

The visual appearance of ornaments is based on the production of intricate diapers on a tessellated geometry. A comprehension of base geometrical transformations is open for new possibilities in pattern designs. The most acknowledged instances for such explorations are probably observable throughout M.C.Escher's works. That Escher copied the Alhambra patterns on his visits (Piller et al, 2013) helped him in developing his own pattern sketches to study shape combinations (Figure 1.1). He was captivated by the traditional artists' ability to create such complex geometric relations. As he read on crystallography and conversed with mathematicians, he started creating his own

ideas (Schattschneider, 1990, 2010) for similar shape relations (Özgan and Özkar, 2014a).

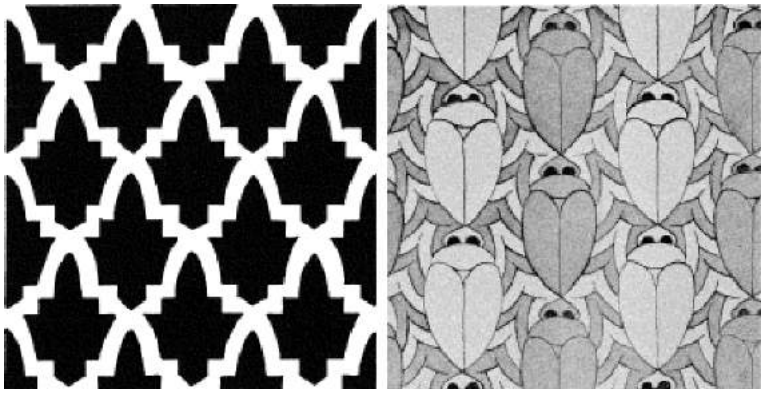

Figure 1.1 : Left: Moorish tiles from Alhambra. Right: Escher's drawings with beetles (Stevens, 1981, p. 205).

Ornament designs have been an inspirable element of art and design since the prehistoric era. Owen Jones's (1856) Grammar of Ornament, was the first to show systematically all the richness of patterns across many geographies and cultures. The London-based architect and designer Jones was one of the most influential design theorists of the nineteenth century. Washburn and Crowe (1988), Stevens (1981), Christie (1910) and Trilling (2001) are among other explorers, who investigated many aspects of distinctive pattern designs.

A right perception on tessellations and pattern symmetries explored throughout medieval designs are open for various geometric designs that can be easily created in available computer applications. There are a wide variety of computer programs that allow the generation of diverse pattern designs. Usually the algorithms behind these are based on generative tiling that cover the plane systematically and the results cover a variety of instances in art and design. Among many examples, *iOrnament*, a drawing app designed by Jürgen Richter Gebert, a mathematics professor at the Technical University of Munich, allows users to create original patterns based on seventeen principal classes of ornaments (Richter-Gebert, 2012). *Girih* by Stefan Hintz is another mobile application that establish geometric designs by joining multiple shapes edge to edge (Hintz, 2015).

Applications as such admit efficient implementation and create a wide variety of designs that include Islamic star patterns, yet there are additional approaches especially created to construct medieval Islamic patterns. One of these, *Engare* is an interactive

game created by the Iranian game designer Mahdi Bahrami. Released with the motto *"*a game about motion and geometry*", Engare* provides its users with an interactive drawing tool that illustrates the compass and straightedge practice (Figure 1.2). Users have to place moving and rotating shapes, lines and dots into correct positions to acquire different pattern designs (Bahrami, 2017).

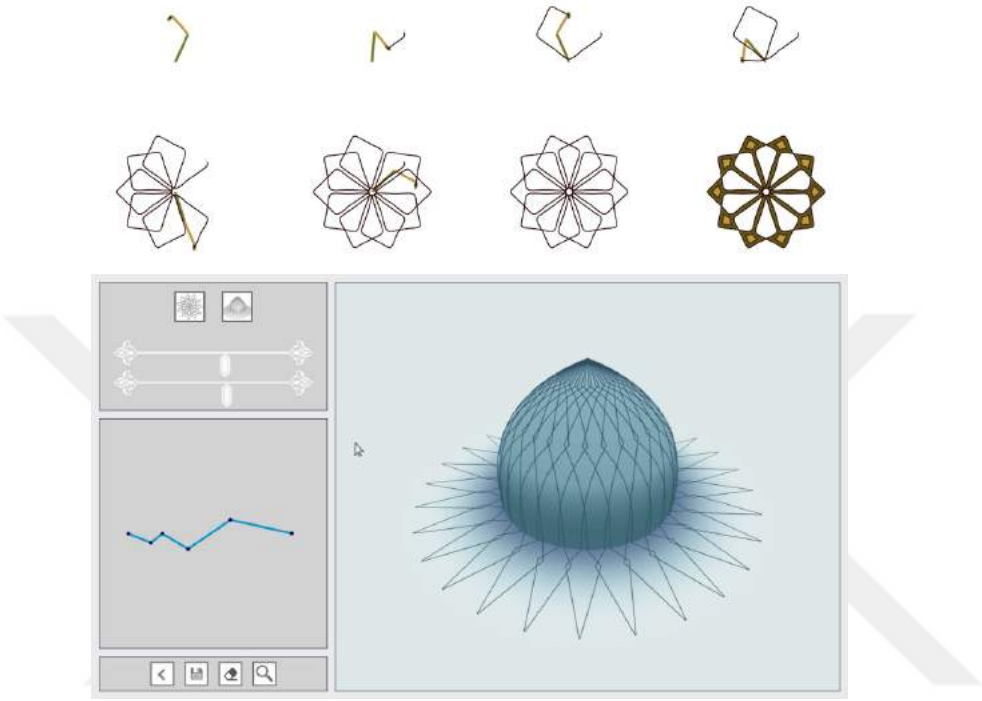

Figure 1.2 : Above: Interactive drawing sequence illustrated in Engare's interface. Below: A sample screenshot from the program (Bahrami, 2017).

On the other hand, various research such as Cenani and Cağdaş (2006) and Çolakoğlu et al. (2008) introduce formal descriptions to create variations of Islamic patterns. These algorithms are based on shape grammars, a computational theory that introduces a visual calculation method with shapes arranged through a set of rules (Stiny, 1980, 2006; Stiny and Gips, 1971). In an expanded study Keles et al. (2010, 2012) provide a platform for dynamic shape perception and synthesis that is valuable for design processes. Their technical framework implements part relations in shapes by staying true to their continuous nature. Their approach considers "perceived wholes rather than defined categories of shapes such as line or plane".

All the above-referred approaches provide valuable implementations and academic discussions, whereas Craig Kaplan has developed the most accessible open-source computer program that constructs the largest number of Islamic patterns. This program, entitled *Taprats* is built upon one of the historically used pattern drawing

systematics that are explained in detail in Chapter II "Geometric Designs in Medieval Anatolia". This pattern drawing system, named as Hankin's Polygons in Contact approach, uses a grid of polygons as the generative schema for pattern designs (Kaplan, 2000a, 2000b, 2002, 2005, 2017).

Computer programs as such allocate to create new designs, as they allow users to explore the logic behind geometric patterns. Yet, a design inclusive of an awareness on how these patterns can really fit a certain three-dimensional geometry or the spatial qualities of a room does still require a further understanding that can be learned from Tusi atelier's designs examined in the dissertation as well.

1.3 Complementary Bonds Between Forms and Geometric Patterns

For centuries, ornaments have adorned surfaces of buildings. In *Stones of Venice*, John Ruskin (1890) writes on the aesthetical captivation as following:

> No architecture is so haughty as that which is simple; which refuses to address the eye, except in a few clear and forceful lines; which implies in offering so little to our regards, that all it has offered perfect; and disdains, whether by the complexity or attractiveness of its features, to embarrass our investigation, or betray us into delight. (p.207)

Ornaments vanished in the twentieth century as the western architecture was deprived of any pattern design. This decline was clearly manifested by the famous architect Adolf Loos (1998) with the eminent caption "ornament is crime". Reasons behind this regression is only plausible by reading a comprehensive history of attitudes towards ornament. The many degrees of approbation and condemnation in the design history is documented and discussed broadly by Gombrich (1980) but lie far beyond the scope of the current study. Nevertheless, as Spuybroek (2016, p. 54) suggests, one of the fundamental causes in the disappearance of ornaments was "the fatal disconnection of matter and form".

The contemporary age is attesting once again a prevalent practice of pattern designs on buildings. This is mainly triggered by the growing adaptation of computational design tools in design practices. In *Ornament: The Politics of Architecture and Subjectivity*, Antoine Picon (2014) discusses the contemporary conception of ornament and relates the reappearance with the advancement of digital technology. *Coded ornament*, *dynamic ornament*, *ornamentalism*, *ornamatics* or *digital nouveau*

are some of the many brand-new terms associated with the new fashion of using ornaments (Balık & Allmer, 2016, p. 163).

Computer environments enable designers to create elaborate patterns and adapt these easily to building surfaces. Yet, apart from some rare examples, there is still a misinterpretation and misconception of the relation between the form and the ornament. Spuybroek (2016, p. 53) debates "Today we so completely and fundamentally lack techniques of ornamentation that we must conceptualize it in full before we can even begin to reconsider it". An inquiry on medieval craftsmen's processes provide an insight on how ornament can really complete the architectonic and is formed with the shape. These craftsmen sought after a complementary bond between the form and pattern. Compared to the present-day architects, they were not restricted to some of the straightforward operations on the computer, yet computed the many possibilities by their hands.

In this study, we examine the geometric designs of medieval craftsmen to explore how their surface tessellation and pattern mapping strategies differ from existing computer applications. Contemporary computer implementations that transfer two-dimensional patterns onto three-dimensional geometry rely on algorithms that first systematically parameterize surfaces and then map the texture in accordance with surface subdivisions. These implementations are used in computer graphics for digital modelling, rendering and for computer-aided manufacturing purposes as well. Like in a two-dimensional design, the three-dimensional surface is split with a set of tiling at the beginning. The parts of a continuous geometric design are placed upon these tiles. The surface division methodology can differ as in medieval artisan's processes and applied in various forms, but the logic behind remains alike.

Tiling a complicated surface with a geometric design is still a debated and challenging task for computer specialists. A wide variety of studies (Akleman et al., 2009; Akleman et al., 2005; Peng et al., 2018; Tarini et al., 2004) propose several methods for accurate surface subdivisions. In one study as such, Kaplan and Salesin (2004) describe a tiling-based approach to adapt two-dimensional geometric designs onto the surfaces of a sphere and the hyperbolic plane. In a further research, Kaplan (2009) describes a methodology for covering more complicated arbitrary surfaces seamlessly with repeating decorative patterns that even include traditional Islamic pattern designs.

Methods that map two-dimensional geometric patterns onto three-dimensional surfaces are implemented in computer aided modelling tools as well and designers use such tools to enhance their creative projects. Nevertheless, mapping processes are usually made with certain program implementations that cause inevitable distortions as geometric patterns follow curvatures of the forms they are projected onto. As the investigations on Tusi atelier's works in this dissertation show, such distortions or in other words, mismatches between the form and the geometric pattern are avoided by the medieval Seljuk craftsmen. Some of the medieval polyhedra examples that are subjects for examination in this study (in Chapter V "The Form and Matter: The Computation" to be exact) are adorned with unique pattern designs that follow the symmetries of the form and consequently rightfully adapt to the shape.

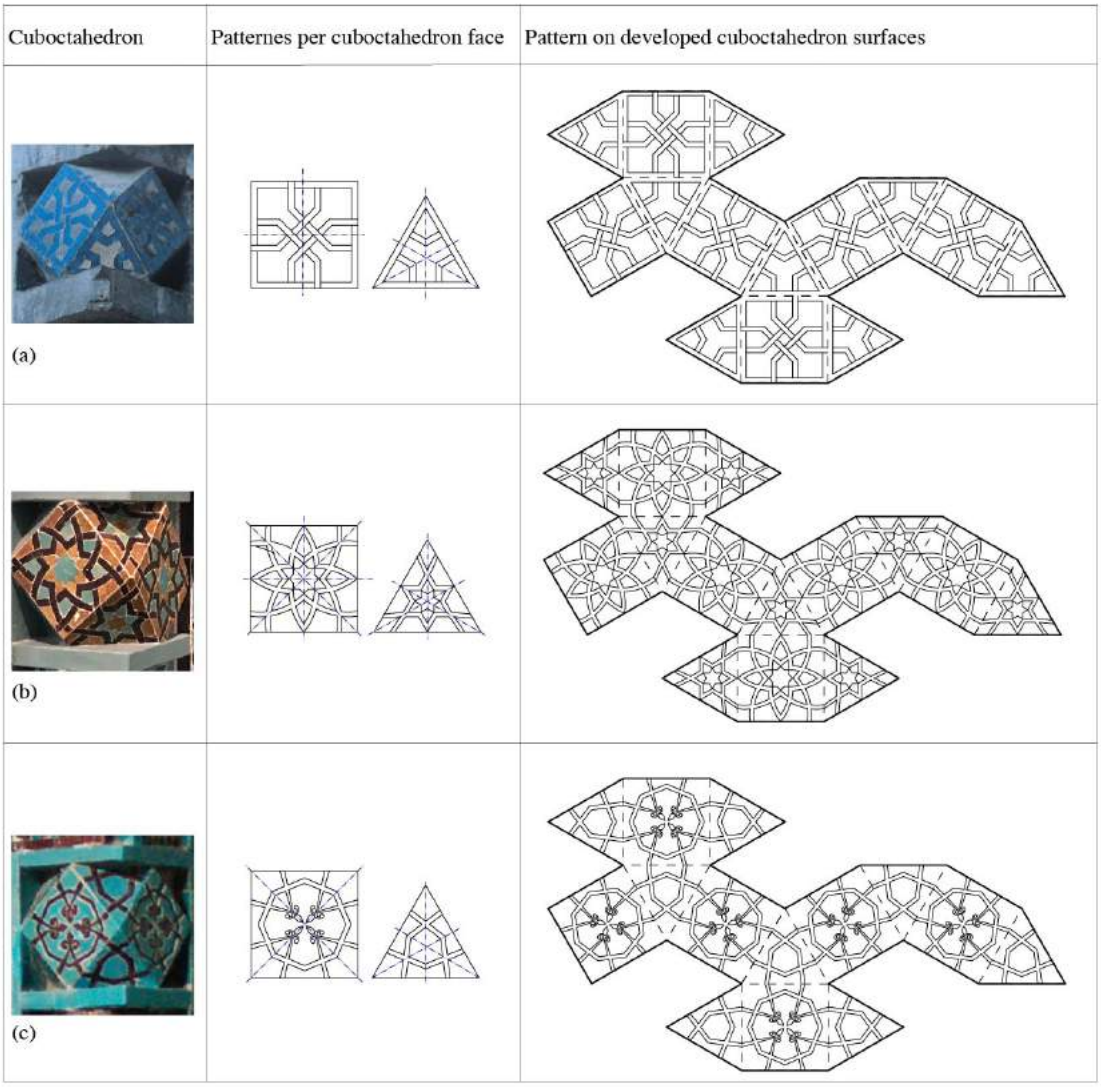

Figure 1.3 : Medieval patterned cuboctahedra examples that are examined in "Chapter V: The Form and Matter: The Computation".

Processes conducted in computer environments require further detailing and user manipulations, if one intends to create a geometric pattern that precisely adapt to the three-dimensional form as in the same way as in medieval craftsmen's hands-on approaches. Yet, designers occasionally tend to use premeditated computer implications that cause automated design processes and result in the aforementioned fatal connection between the form and matter.

How the medieval artisan's by-hand computations contrast current designer attitudes towards pattern designs came in sight in a design workshop on 2D and 3D Pattern Transformations called "Polyhedral Tessellations"[1]. Participants were introduced to three-dimensional polyhedral geometries and basic notions such as symmetry rules and groups, and the transition process between different dimensions. At the beginning, participants were provided with various two-dimensional pattern designs to explore the symmetry notations and the visual rules behind these patterns. Secondly, they acquired polyhedral geometries both as digital models and paper models for hands-on exploration. The main design task consisted of developing and manipulating three-dimensional models by placing patterns onto three-dimensional geometries. Participants used analog tools (such as paper cutter) and as well as digital tools to explore innovative geometries. Final models were produced with a 3D printer. One of the participants used only digital tools and designed a wide variety of different polyhedra adorned with geometric patterns. Nevertheless, this automated design process created complicated pattern designs on different solids that did not match the symmetries of individual polyhedra faces and consequently caused incompatible views on three-dimensional forms. (Figure 1.4).

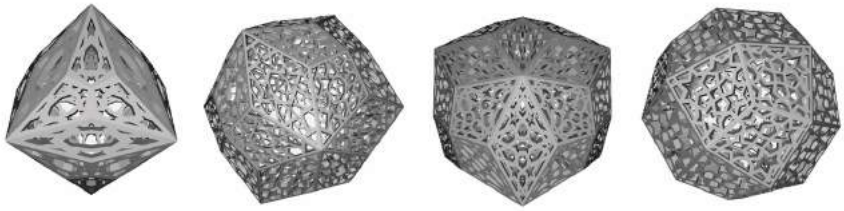

Figure 1.4 : Same geometric pattern mapped on the surfaces of different polyhedra by a participant of the *Polyhedral Tessellations* workshop.

[1]The workshop was conducted by Sibel Yasemin Özgan and Begüm Hamzaoğlu on 10th of July as part of the international CAADFutures 2017 conference The conference was organized by Istanbul Technical University, Institute of Science, Engineering and Technology, Department of Informatics, Architectural Design Computing Graduate Program.

The projection methods implemented into three-dimensional modelling programs, such as the one used by the workshop participant, are straightforward processes that map prearranged geometric patterns onto different surfaces. Figure 1.5 illustrates one of the participant's designs, a joined truncated octahedron embellished with a rectangular repeat unit cut out of a sample Seljuk geometric design. On the other hand, Seljuk artisan's hands-on computations that are based on geometrical rules as well differ as rules alter and follow designer's intention for creating a complementary bond between the form and the geometric design. Such insights learned from the inquiries on Seljuk designs can aid contemporary designer's ways of using computer tools.

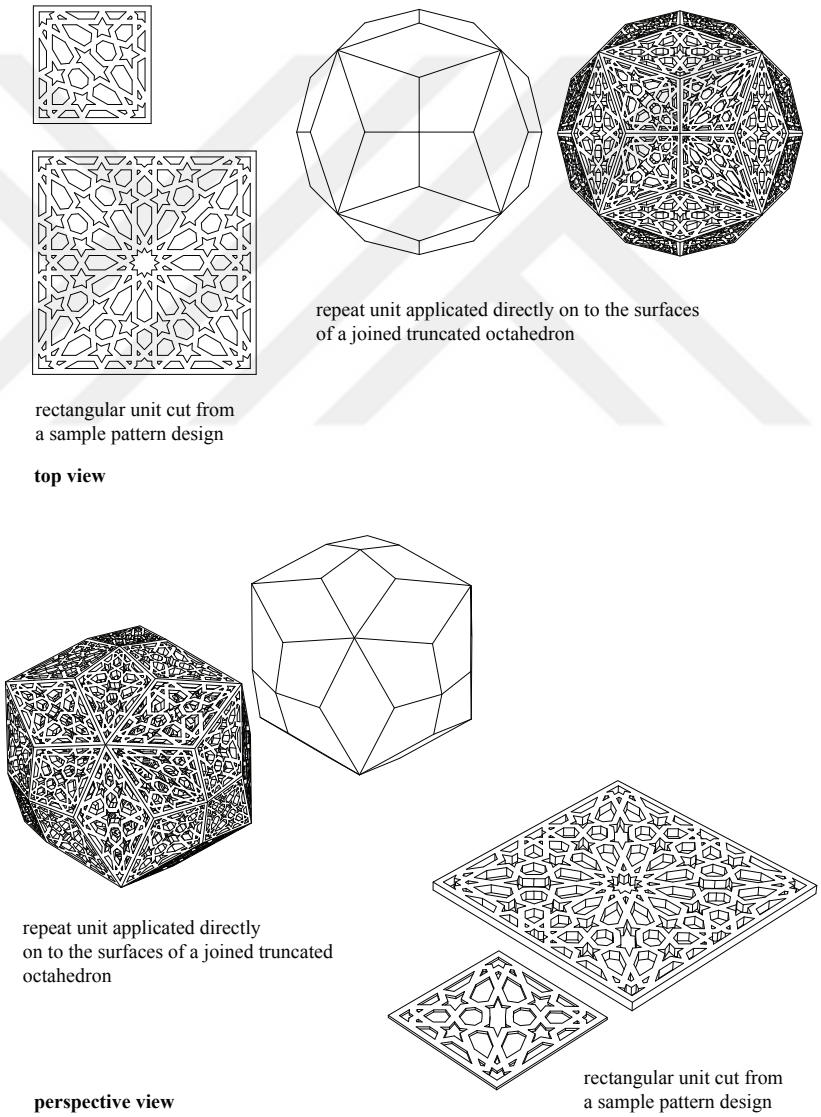

Figure 1.5 : Joined truncated octahedron adorned with a rectangular unit cut from a sample Seljuk geometric design. The Design was produced as part of the *Polyhedral Tessellations* workshop.

1.4 Inventiveness Stimulated by Tessellation and Symmetry

Apart from above mentioned insights, an understanding of tessellations is not merely obligatory for decorative and visual purposes. The rules beneath these designs act still as a source of inspiration for scientist from different fields of study as well. Perhaps one of the most noteworthy instances is the metamaterials developed by a group of scientists from the McGill University in Montreal (Figure 1.6). Researchers created auxetics with counterintuitive properties out of the geometry seen in the Islamic pattern designs (J. Webb, 2016). Similar geometrical relations (Figure 1.7) were observed by Gerd Schneider, an architect who produced an exhaustive survey of Seljuk geometric patterns in Anatolia by free-hand drawings (Schneider, 1980).

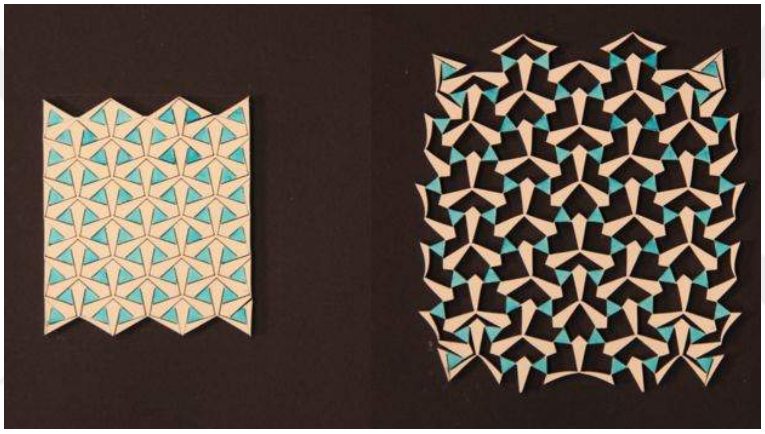

Figure 1.6 : Auxetics inspired by Islamic patterns (J. Webb, 2016).

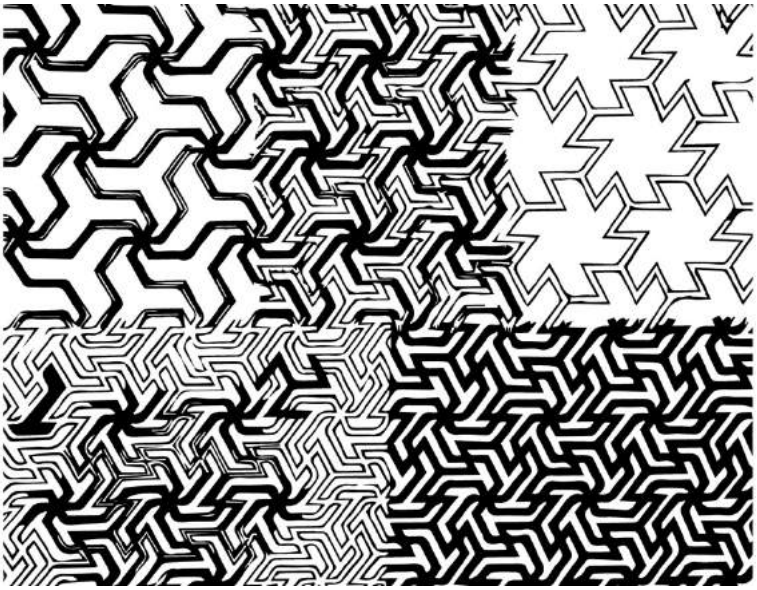

Figure 1.7 : Some Anatolian geometric patterns grouped after their geometrical similarities by Schneider (1980, p. 77).

1.5 Medieval Ways of Computational Making and Computing with Materials

Long before the digital era, mass production and industrialization drove designers and artisans from the work bench. Apart from outstanding examples, mechanical age resulted mostly in uninformed and repetitive designs. William Morris, a protagonist of ornaments, was one of the main figures in the Arts & Crafts movement and spent much of his life fighting against mass-production. Excited about the many possibilities machines offered, the Victorian era praised the Industrial revolution, hence Morris was neglected by many design authorities. Machine control was supposed to facilitate production phases for designers, yet it occasioned with copying the same product over and over again, without thinking in a proper creative context, eventually proving Morris was right. This was caused primarily by lack of adequate knowledge about machine operations.

The digital era has presented a great potential to deal with this problematic situation. Now, as we stand in the digital era, we are witnessing a shift towards an increasing usage of computational design methods and tools. With their enormous calculation and production capacity, digital technologies accelerate our design processes and give at the same time the power to easily operate on our design models. We use digital fabrication for manufacturing processes and have easily access to computer operations. Thus, the machine has become an internal part of the whole design process. Mario Carpo (2011, p. 10) argues that the long duration of historical time, the age of mass-produced, standardized, mechanical and identical copies came to an end with the rise of digital technologies as they give the option of unlimited visual variability. According to Carpo, everything digital is variable. We are capable of easily manipulating our designs in computer environments and send the virtual data to digital fabrication technologies, which eventually materialize our designs. Thus, as designers we are involved in all parts of a design process: from visual design to the far end of production.

The *variable* that Carpo (2011) allocate to digital technologies is observable in Tusi Atelier's designs. The hands-on craft tradition is based upon certain rules. Still, rules change in compliance with creative decisions. Rules do not limit the creative process, rather aid the craftsman during the process. Hence, even if they are based initially on the same visual geometrical rule, final design products vary as rules alter.

Since they provide an interactive involvement during the design process, digital modelling and fabrication tools are worshipped by many authorities as a new way of craftsmanship. Conventional craft practices have witnessed a decrease because of the long-term job tenure. However, the digital era can change this by redefining the craftsmanship and altering it to evolve into rapider processes. Digital technologies can strengthen certain skills and produce things, which are not easily made in human hands.

In a more traditional sense, a craft practice is informed by the physical properties of a material and is advanced through the skillful usage of the tool. Such properties of making correspond to a physical reality. However, the digital environment is not tangible. There is the lack of haptic dimensions of making on the computer screen and therefore, an uninformed relation between a digital model and the end product appears. This connection can only be built on the understanding of both the ways of making and also the technology. In *The Craftsman*, Richard Sennett (2008) considers craftwork very broadly, ranging from the very traditional craft practices even to unexpected areas such as computer programming. Arguing that "making is thinking*",* Sennett gathers various cases, in which we see how the work of the hand can inform the work of the mind.

There has been a recent interest to develop new, digital and computational ways of making, both theoretically and practically. The lack of the tangible in the digital environment has led researchers to investigate broadly traditional crafting practices to advance current computational theories and methods. The phrase "digital craft", introduced by McCullough (1998) stands for a fusion of made-by-hand and made-by-machine methods. Many forms of by-hand crafts were surveyed to merge these with digital technologies to initiate eventually new forms of digital crafts (Knight, 2018).

Similarly, making became a widely debated issue in current design theories. Since its introduction by Stiny and Gips (1971), Shape has been one of the most prosperous computational theories close to the ambiguous nature of design. In *Shape: Talking About Seeing and Doing*, Stiny (2006) developed a formalized way of designing by shape grammars. Stiny's writings (1980, 2006) are indicative of the comparative and creative use of rules through visual schemas. In 2014, Knight and Stiny, expanded the shape theory from a visual computation methodology towards making grammars to include computing or making things (Knight and Stiny, 2014). This approach deals

with several types of making actions on different scales: from a drawing on a paper or an image production on the computer screen to a building construction. Diverse research on physical model making, crafts like weaving (Muslimin, 2010) , and further art and design practices (Gürsoy and Özkar, 2015a; Knight, 2018) have examined the act of making on a model of computation.

Design practices are informed by certain parameters: materials and tools among others. Material has always played an important role in creative making processes. Exploring the potentials of materials has evoked new forms in the history of design and architecture. One of the best hands-on examples is probably Frei Otto's experiments conducted with soap in order to explore curvilinear surfaces, which reduced eventually the surface tension (Otto and Rasch, 1995). The digital age offers ways for relocating information related to materials into digital environments. Some studies of Oxman (2010a, 2010b), Gramazio and Kohler (2008) and Menges (2010) are relevant examples which stress the notion of material computation. In these studies, data related to material properties, organizations and behaviors is assigned to the computer environment. In an alternative approach, Gürsoy and Özkar (2015b) expand material computation to include hands-on practices as well. Using making grammars, they offer ways to explicitly include material manipulation in a computational formalism.

Tusi atelier's designs provide insights on how issues related to making, such as material properties, craft technique and tool usage affect design decisions, hence the hands-on computation. Medieval pattern designs are full of tacit as well as of uttered reference. In a more observable way, geometrical rules embedded in pattern designs provide an insight on visual transformations. Thus, there has been an enormous interest to examine geometrical operations behind medieval Islamic patterns, especially by mathematicians. The main intend in such studies is to illustrate possible visual production methodologies for the patterns. These works, focus on the complex geometric rules beneath patterns, approach designs as mere outcomes and usually do not examine design processes. Yet, the process is not informed only by visual transformations. Studies with a focus on the Turkish culture, art and Anatolian geography (Arseven, 1950; Aslanapa, 1972; Bakırer, 1981; Demiriz, 2017; Mülayim, 1982; Ögel, 1986, 1987, 1994; Öney, 1992; Peker, 1998), show how some geometric patterns in Anatolia differ from any other examples and were unique for the society

and culture they were produced in. The fundamental reasons for such a distinctiveness are exceptional materials and craft techniques used for the making of these designs.

The focal point of the investigations in the dissertation, craftsmen working in Tusi atelier, use a certain material and the associated crafting technique. This material is tile-mosaic, a novel technique initially developed in Anatolia but seen across other geographies later on. Tile mosaic bears unique properties that are not seen in other renowned techniques such as stone carving or ornamental brickwork. Examinations on Tusi atelier's works illustrate how material properties affect design decisions, hence alter final designs.

The craft technique of tile-mosaic requires an explicit understanding of all shapes that form a specific geometric design. Such a comprehension on singular shapes can be discarded in other craftworks, therefore tile-mosaic demands harder labor compared to other crafts. Yet, it is inclusive of advantages as well. Operating with single shapes provides flexibility in design as it allows craftsmen to adapt and alter their designs. Investigations show that this fundamental property of the material was acknowledged by the Tusi atelier.

Based on a detailed investigation on the design processes and outcomes of Tusi atelier's works, the study examines how material and crafting skills shape design decisions and outputs. The inquiries show that craftsmen transform visual rules in accordance with the properties of the new material, consequently their hands-on practice. Making became an integral part of their hands-on computational designs.

1.6 Learning from the Medieval Master Builder

Additional attention is given to the workshop owner Muhammad Al-Tusi, who is also the medieval master builder in charge for the construction. Medieval master builders or architects as their equivalents in modernity, are the main figures responsible for the appropriate merge between the form and the material. It is up to the architect's hands and creativity, whether a material becomes an integral part of the building or not. In his inquiry on the material change in architecture, Ákos (2017, p. 10) quotes Ludwig Mies van der Rohe, one of the key figures in modern architecture: "Whether we build vertically or horizontally, with steel and glass, says nothing about the value of this building… But it is precisely this question of value that is decisive". How an unusual

characteristic of a certain material can cause huge differences is perhaps best observable in Alvar Aalto's Baker House, built in 1946-1949. Dark and red rustic bricks shape up the curvy wall from the student dormitory in Massachusetts Institute of Technology. The bricks are not selected and arranged in a usual way though, highlighting the ''honesty'' of his brickmaking, Aalto includes distorted, over burned and twisted bricks on the wall surface (Figure 1.8). Aalto's vision and choice creates an unusual perception. Here's how Sennett describes the rare visual effect on the façade: "We are thus disposed to think about what brick is - a reflection on the material that might not come to us were all the bricks imperturbably, uniformly perfect." (Sennett, 2008, pp. 143–144).

Figure 1.8 : The visual variety formed through the unusual selection of bricks in Alvar Aalto's Baker House in MIT (Photo from Pilsbury(2010)).

Investigations on material computation and computational making that bear similarities with Aalto's visions had also profound practical reflections on the building scale. Envisioning material properties or material techniques can aid thought-through parametric design strategies. As an example, Studio Admun, an Iranian architecture office designed a building façade in Tehran, Iran by merging parametric design tools with traditional brick construction techniques (Figure 1.9). Due to limited budget, Studio Admun searched for a new construction technique that eliminates mortar and attaches bricks onto a string instead. In the final design, various degrees of opening done by rotating bricks disregard the necessity of mortar and ensures a textured outlook. The surface geometry was digitally modelled by using a script while individual bricks were manufactured by hand. Hence, designers required a precise coordination between the digital modelling and the handmade manufacturing processes ("Cloaked in Bricks," 2016; Stoughton, 2016).

Figure 1.9 : Parametric brick façade in Tehran, Iran. Design by Studio Admun ("Cloaked in Bricks," 2016).

Aiming for an explanation for his perception of architecture, Aalto ones quoted his renowned friend Frank Lloyd Wright who opened a lecture with these words:

> Ladies and gentlemen, do you know what a brick is? It is a small, worthless, ordinary thing which costs eleven cents but has a peculiar quality. Give me this brick and it immediately becomes worth its weight in gold. (quoted in Moravánszky, 2017, p. 9)

Apart from being a remarkable architect, Wright was after all a visionary, who redefined the relation between the form and function to become one. Wright was not against patterns but his definition on ornament highlights the proper relation between the form and the ornament:

> Not all organic architecture has ornament, but when used, it is developed as an integral part of the material, not applied. Examples are patterns cast in concrete or carved in stone, leaded glass panels, and tile or glass mosaics. (Wright, 1939)

In the medieval Islamic world, creating a proper relation between the ornament and the building was one of the many tasks of the master builder. Similar to craft practices, the concept of master builder became a widely debated issue in the digital era. Protagonists of the digital turn in architecture praise computational technologies for allowing architects to control all aspects of their work. Thus, ensuring them to be heavily involved not only in the design process, but as well in the construction. Putting architects into the role of the twenty-first century version of medieval master builders, Kolarevic (2003) discusses how separate professional realms of architecture, engineering and construction can be merged into digital collaborative enterprises. That kind of involvement is usually allied to the Gothic architecture, in which the form taking processes are largely informed during material manipulations. In Gothic, master builders direct the work of the stone masons and architectural elements are shaped in compliance with the stereotomy of the stone. In the light of the discussions on the

twenty-first century master builders, revitalizing historical figures became one of the most esteemed issues in current design theories.

One of such research is concentrated on Antoni Gaudi, a visionary Spanish architect from the nineteenth century. Continuous research conducted on Gaudi's ways of making aided the completion of the unfinished La Sagrada Familia church in Barcelona (Figure 1.10 and Figure 1.11). Master builder's unexpected death left the construction unfinished, as the complex and exceptional form was not straightforwardly built. Gaudi invented methods of modelling catenary curves, which consequently resulted in a rigorously engineered building of complex organic geometric forms. In order to complete the construction and deal with the highly sculptural building, Mark Burry and his team studied thoroughly Gaudi's oeuvre and developed new computational techniques. While they were encoding the geometric code beneath the form and building a digital model, they tested the possibilities of the structure with plaster models (Figure 1.12) at the same time (Burry, 1993, 2005; Burry et al., 2001).

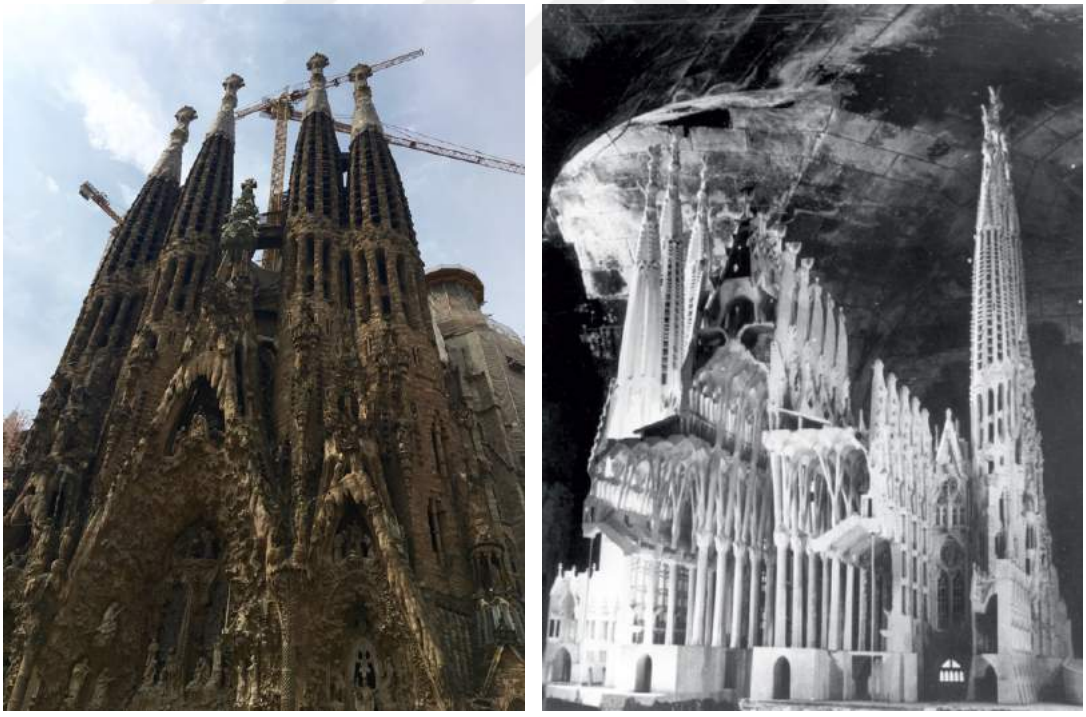

Figure 1.10 : Left: Gaudi's La Sagrada Familia Church in Barcelona, still under construction. Right: 1:25 scale model by Gaudi, restored after being smashed during the 1936–9 Spanish Civil War (Model Photo from Burry (2011, p. 96)).

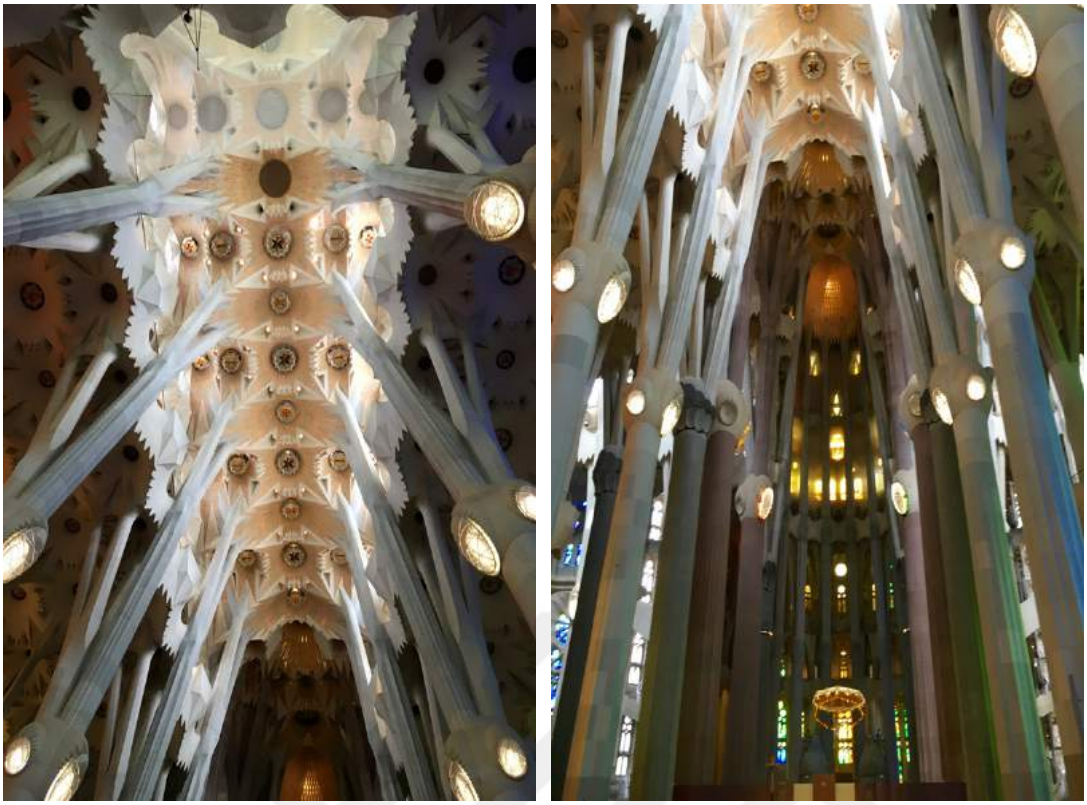

Figure 1.11 : Interior views of Gaudi's La Sagrada Familia Church in Barcelona.

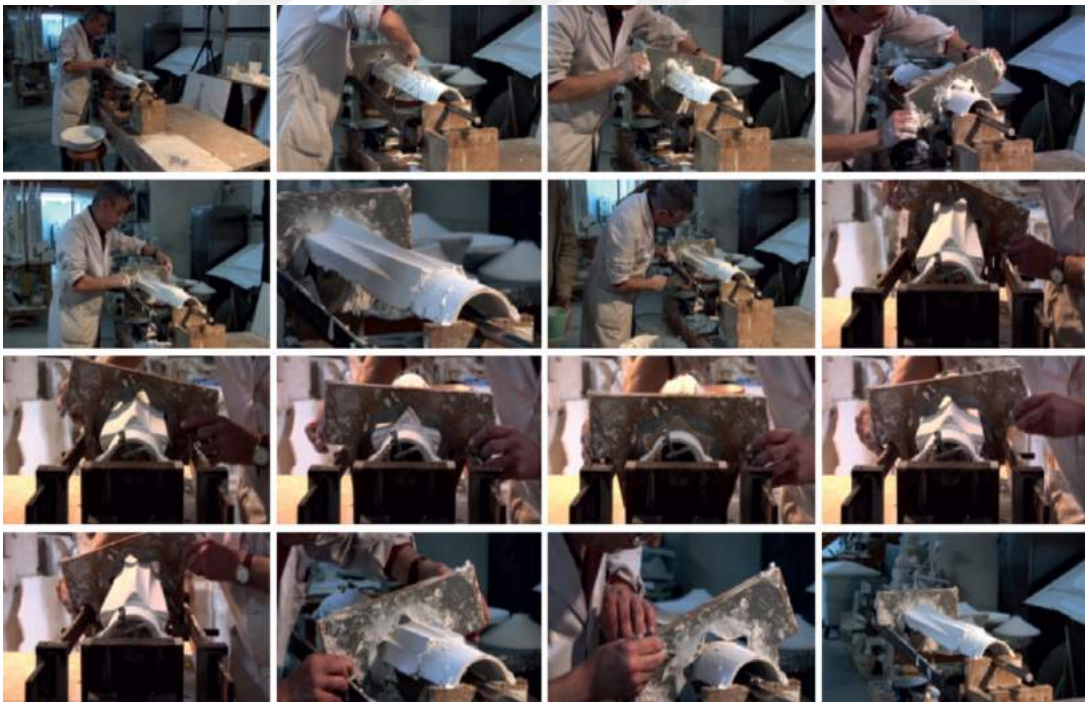

Figure 1.12 : Analogue experiments with gypsum plaster were conducted for making the columns from Antoni Gaudí's Sagrada Família church, Barcelona (Photo taken from Burry (2011, p. 104)).

Similarly, with the motto "Learning from the past to design a better future" Philippe Block (2017) and his team designed a novel structure for the Venice Architecture Biennale in 2016 (Figure 1.13). They merged some methods from the Gothic architecture with new technologies and fabrication techniques, to form a freeform stone shell which they call Armadillo Vault (Figure 1.14). They described their attempt: "rather than being romantic attempts at revival for revival's sake, these structures draw upon historical examples and "lost" techniques that have been reinvigorated and adapted for current technological and fabricational processes." (Block et al., 2017, p. 7).

Figure 1.13 : The Armadillo Vault, a project by Block research group (Block et al., 2017, pp. 286–287).

Apart from such practical inquiries, in a similar attempt, Spuybroek (2016) developed a new 'digital Gothic' ontology, in which he reinterpreted John Ruskin's concepts from a digital perspective and adapted his ideas to fit into the contemporary age. With an intention to transform a historical Ruskin into a theory of design, Spuybroek discusses:

> I think that if there is one thing we can learn from John Ruskin, it is that each age must find its own way to beauty, and in our case this means finding our way back to beauty, since we have seem to lost sight of it completely. (Spuybroek, 2016, p. xviii)

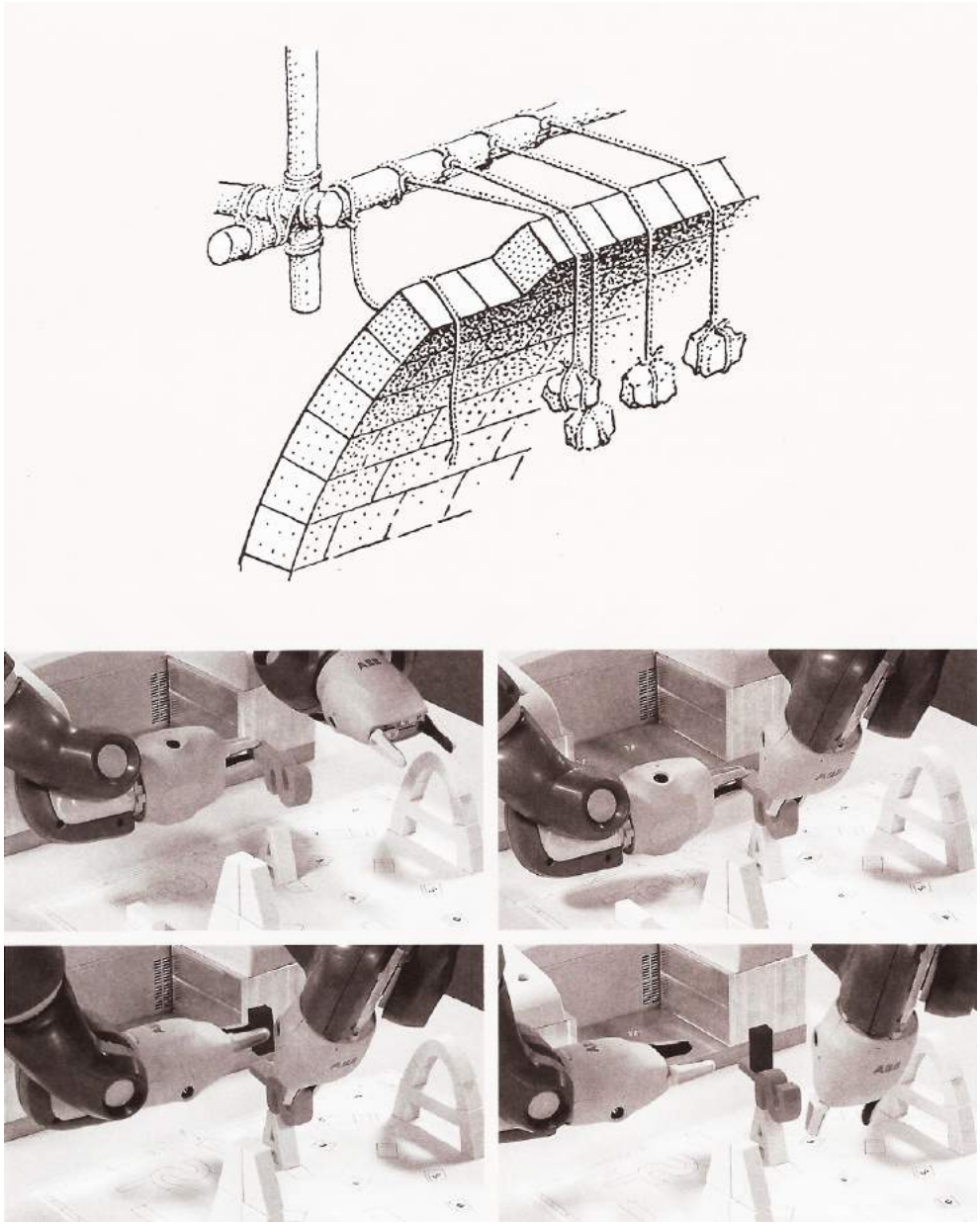

Figure 1.14 : Block research group reinterpreted Gothic builders' system of ropes and counterweight. They adapted this system to robotic assembly (Block et al., 2017, p. 94).

The central figure in this study, Muhammad Al Tusi, does not seek for complicated structural properties by an inquiry in form-finding. Yet, how he alters geometrical rules, consequently pattern designs provide assignable knowledge for current computational design practices. Physical properties of the new material, as well as the surface geometry and spatial relations affect his design decisions. In the dissertation, we analyze designs attributed to him and try to understand how he comes up with novel solutions and pattern designs when he faces a design problem that is related to the construction.

1.7 Design of the Study

1.7.1 Motivation and research questions

1.7.1.1 Material and making as formalizable attributes

This thesis departs principally from the idea that medieval geometric art can introduce new insights and knowledge to the design computation field. Several studies illustrate the geometrical principles and generative methodology beneath geometric pattern designs (Bonner and Kaplan, 2017; Bourgoin, 1973; Broug, 2008; Critchlow, 1976; Hankin, 1905, 1925b; Sutton, 2011; David Wade, 1976). These studies largely concentrate on specific patterns that are the final design products and do not take their creative design processes into consideration. The generative construction techniques that appear in the literature only explain how a geometric pattern is drawn on a paper. Nonetheless, as soon as the craftsman strives for the application onto the building surface, the geometric pattern has to come out of the paper and transform into a material thing. This process entails reasoning, a dialogue between the diverse partakers of a design process: materials, forms, haptic properties, spatial dimensions and so on. Hence, the maker computes with such attributes, not merely with visual properties.

Studies in art history that are specific to the Anatolian geography (Aslanapa, 1972; Meinecke, 1976a, 1976b; Mülayim, 1982; Ögel, 1987, 1994) show how some of the pattern designs demonstrate explicit characteristic in the geography they were produced in. Designers form novel designs after a valuable reasoning process. Available approaches are capable of enlighten the geometrical rules behind visual transformations, therefore the visual calculation processes. Yet, they lack of a general theory to explain remaining design problems. These studies do not consider how issues related to hands-on making activities, such as the nature of the material or the craft practice affect pattern designs. The main intend of the current study, in contrast, is to explore designer processes, not only the end products. We try to figure out how designers develop their designs in light of several design attributes and how pattern designs evolve in line with making.

The pattern design investigations are focused (but not limited) to a specific time period, thirteenth century Anatolia and to an explicit material and crafting technique above all. We concentrate especially to the pattern designs of Tusi atelier, with the basic

intention to understand their working and designing principles, in other words, their hands-on calculation processes informed by the material and the craft technique. The craftsmen in this atelier use a specific material, tile mosaic, which evidently was developed as a new crafting technique and was unique for Central Anatolia for the century. In close examinations of the Tusi atelier's techniques, and with reference to the knowledge of geometry and materials of the time and sustained today, our main intention is to reveal a logic of design (also preferred to as computation) that crosses across making and geometry of form. This logic of design is a valuable part of the local and global traditions and knowledge.

1.7.1.2 Learning for effective restorations

In a practical sense, such investigations can aid restoration projects and help professionals to understand how pattern designs that are usually discarded during renovations were made. Available computer implementations that can partially aid restorations are useful as they can be used to create patterns on papers. Yet, the investigations in the study show that designs on a paper are not sufficient to acquire designs alone, as making requires further know-how. The dissertation explores how quests for material and making was an integral part of medieval design processes. Understanding as such is useful for practical applications, especially for restoration purposes.

Lack of understanding of geometric patterns causes unsuccessful renovations, such as the one that was recently conducted on the cenotaphs from the Tomb of Keykavus inside Sifaiye Madrasa, Sivas (1217- 1220 A.D.) (Figure 1.15). Contemporary software packages are useful to get the rightful pattern on a paper. Nevertheless, the craftsperson, who has to materialize the design, would still have important questions related to making in mind. Medieval craftsmen understood how the material worked, not only the geometrical principles that are useful to arrange the pattern in an accurate way. Reverse-engineering on Tusi Atelier's designs enhance our knowledge of the craft, hence the making process can be formalized and transferred to the computer.

Figure 1.15 : Left: The original pattern on the cenotaph in Keykavus Tomb, Sivas (1217-1220 A.D.) (Photo R.Arık and O. Arık (2007, p. 52)).
Right: Recent restoration caused an unusual decoration (Photo Bursalı (2015)).

1.7.1.3 Learning for the contemporary design culture

The investigations in this dissertation explore the possible know-how that was transferred from medieval masters to their apprentices. Working with the same patterns and materials for decades, *repetition* to put in different way, craftsmen get into sensible and functional conservations with rules during the design processes. Hence, they develop ways to get various designs from the initial visual rules. Medieval design processes encompass valuable knowledge that entails *variability* achieved not only with visual rules, but also with materials and craft properties. Repetition and variability are some of the key debates in current computational design discussions. For that reason, the knowledge gained from the inquiries through the thesis are valuable both for professional design practices and the design studio.

Today, especially in the design studio, we encourage future designers to understand and use computer tools in accurate manners. That is, not being limited by premediated implications, but rather develop ways to fit computer usage to creative processes. Sufficient usage of computers for design is only possible by understanding the task at hand and finding ways to formalize design actions. In this dissertation, we explore and formalize how medieval craftsmen alter rules to suit their design decisions. Rules do not embrace any restriction, if anything they are creative tools for new designs.

Furthermore, the thesis explores how material and making change design decisions. The insights learned can aid current computational design paradigms that explore the effects of materials and craft practices on designs, as they look for ways to formalize attributes as such.

1.7.1.4 Medieval ways of computational design

Another principal objective of the study is to demonstrate a comparison of production and design processes across traditional and contemporary technologies in order to inform processes today based on history.

Studies on Islamic geometric art explores the systematic behind two-dimensional pattern arrangements. These patterns do mostly cover planar surfaces. Yet, patterns on several building elements, such as vaults, columns, domes or muqarnas show that medieval artisans were dealing also with pattern covering on three-dimensional surfaces. Such matters imitate design inquiries on spatial relation, which is usually coordinated by a master builder. The subject in our quest, Muhammad Al Tusi, was the chief person responsible for the appropriate coordination between the ornament and the form. These refer to an understanding on surface tessellations, still one of the key issues in computational design practices. We study particularly different geometrical shapes of architectural elements to see how patterns fit the form and discover how matter goes along with the form.

We are aware of the new, and continuously developing possibilities and directions in design. The intend is not to execute mere copies of existing pattern designs nor talking about only the geometrical rules and construction techniques behind them. Rather, we aim to learn by investigating master builder's ways of designing. We learn much from the studies we conduct and therefore, we are able to reflect on to the current state of computational design tools. The studies show how the hands-on calculation processes differ from the straight-forward processes done on available computer programs

With the new insights learned from medieval artisan's ways of designing, we aim to develop an idea of how designers should approach computational design methods and tools. The study aims to encourage designers to adapt computational tools into their creative processes followed by an understanding on what their real intentions are. We suggest that computation is not always a restriction, as we can produce flexible series of commands to aid our creative processes.

1.7.2 Method of Investigation

The investigations and discussions on the current study bonds a connection between different studies that appear in *history, history of art* and *architecture* literature (R. Arık and Arık, 2007, p. 74; Aslanapa, 1972a; Bakırer, 1976, 1981, 1999; Canby et al.,

2016; Erdemir, 2002; Hillenbrand, 1999; Jacobsthal, 1899; Konyalı, 1964, p. 896; Meinecke, 1976a, 1976b, 1968; Mülayim, 1981, 1982; Ögel, 1986, 1987, 1994; Öney, 1992; Öney and Çobanlı, 2007; Otto-Dorn, 1957; Peacock, 2010; Peker, 1996a, 1998, 2006; Yetkin, 1986, p. 185), the literature on the *geometrical principles* and *constructive methodology behind pattern designs* (Bonner, 2017; Broug, 2008; Critchlow, 1976; Cromwell, 2009; Grünbaum and Shephard, 1992; Sutton, 2011) , diverse literature on *computer implementations of pattern designs* (Kaplan, 2000b, 2000a, 2002) and various research on the diverse discussion on *computational design* (Gürsoy, 2017; Gürsoy and Özkar, 2015a, 2015b; Keles et al., 2012; Knight and Stiny, 2001, 2014; Özkar, 2014; Stiny, 1980, 2006). The thesis founds its main arguments with a holistic overview on such diverse studies and entails a detailed investigation on the hands-on computational processes that encompass both visual computation with geometrical principles and computation with other issues related to historical making principles. Hence, provides a reflection on current restoration processes, computational design debates and contemporary technologies.

This study is partially based on the TÜBİTAK (the Scientific and Technological Research Council of Turkey) 114283 project entitled "A Computational Analysis of the Design Processes Behind Two-Dimensional Seljukid Geometric Patterns in Anatolia" (2014 – 2016). Prof. Dr. Mine Özkar was the lead researcher of this founded research project and the project was undertaken by the project members: researcher Dr. Aslıhan Erkmen Birkandan and assisting graduate and undergraduate students: Ezgi Baştuğ, Bahar Akgün, Begüm Hamzaoğlu, Baran Yıldız and Sibel Yasemin Özgan. The main intention was to follow the developmental design processes of medieval geometric patterns in Anatolia. The project initially started with comprehensive field surveys of the twelfth and thirteenth century Anatolian Seljuk monuments from seven different cities in modern Turkey: Amasya, Tercan (Erzincan), Kayseri, Konya, Kütahya, Sivas and Tokat. During these expeditions, we conducted in situ analysis and documented geometric patterns on several architectural monuments with photographs. The formal constructions of the selected patterns, some of which also appear in this dissertation, were investigated to develop their design grammars for building computational models. In order to do so, the global and local rules and schemas beneath the geometric patterns were identified by using diverse drawing and modelling techniques. Furthermore, in order to understand how the

geometric construction of the designs and the making of these patterns correlate, making grammars for some special patterns were illustrated as well through comparative analysis. Some of the studies are documented in Akgün (2016); Baştuğ (2015); Hamzaoğlu (2017); Hamzaoğlu and Özkar (2016a, 2016b) and Özgan and Özkar (2017).

There are different generative methods that can be used to draw the geometric designs with specific geometrical principles and rules. The pattern examples that are documented as part of the research project are analyzed and illustrated on the circular grid technique, which is the most acknowledged technique (Bakırer, 1981, 1999; Broug, 2008; Critchlow, 1976; Sutton, 2011; Wade, 1976) amongst others. The circular grid that is used to create diverse pattern designs is built on combinations of repeated circles, overlapped and interlaced.

Nonetheless, the thesis concentrates on how making affects pattern designs and therefore introduces other possible generative techniques in order to accomplish a comparative analysis on the possible construction methodologies for pattern designs.

The patterns recorded within the TÜBİTAK project are from the aforementioned different regions in Anatolia and are applied on diverse material and crafting techniques. With the intention of achieving an insight on the possible effects of materials and making on design processes, the primary investigations in this dissertation center the focus on a specific material, namely tile mosaic. This study is not a history of art dissertation and the discussions concerning the history, such as stylistic influences or exact dates of the historical monuments are deliberately left out of the scope. Nevertheless, to center the investigations on a specific material tradition, a comprehensive literature survey on history and history of art is conducted. That tile-mosaic was unique and first appeared in Anatolia in the thirteenth century, is a widely accepted fact (R. Arık and Arık, 2007; Aslanapa, 1972b; Konyalı, 1964; Meinecke, 1976a, 1976b; Mülayim, 1982; Ögel, 1994; Öney, 1992; Öney and Çobanlı, 2007; Otto-Dorn, 1957; Yetkin, 1986) addressed the investigations on geometric patterns materialized with this technique. Likewise, an integrated design inquiry inclusive of form and process is accomplished by placing the ceramic workshop of Muhammad Al Tusi in Konya region, onto the main subject in this study. This kind of focus is essential to speculate on design processes, as they are the products of the same creative minds and hands. It is not frequent that Seljuk master builders left signatures on monument

facades. Yet, Muhammad al- Tusi's name appear on the Sırçalı Madrasa (1242-1243 A.D.) and his existence and contributions to the stylistic alteration is widely acknowledged as well (R. Arık and Arık, 2007, p. 74; Erdemir, 2002; Konyalı, 1964, p. 896; Meinecke, 1968, 1976a, pp. 35–45; Öney and Çobanlı, 2007, p. 44; Yetkin, 1986, p. 185).

The investigations are centered on this particular atelier, yet not limited only by their works. Geometric designs on diverse building materials, especially brick and stone are considered and analyzed for comparison. The inquiries on the effects of the new material are informed for the most part by history of art literature that explore especially material and making processes to define Anatolian Seljuk artistic traditions. Among these, the observations of Meinecke (1968, 1976a, 1976b), a German art historian specialized in Islamic Art presents the principal knowledge on the tile mosaic material and its effects on the geometric design traditions. Travelling widely in Anatolia and Turkic Central Asia, Meinecke completed a dissertation called Faience decoration of Seljuk religious buildings in Asia Minor (*Fayencedekorationen Seldschukischer Sakralbauten in Kleinasien*) and clarified distinctively the differences between severe craft techniques that are based on glaze. Likewise Jacobstahl (1899) provides a rare approach to the geometric designs and explores the making marks and methodologies for the geometric brickwork tradition, a technique used prior to the tile mosaic. Due to the loss of valuable information and maker mark's on the buildings, elder photographs in art history literature (R. Arık and Arık, 2007; Mülayim, 1982; Öney, 1992; Öney and Çobanlı, 2007; Yetkin, 1986) are examined to spot clues for making. Schneider (1980, 1989), who worked with Kurt Erdmann, a renowned historian specialized in Anatolian Seljuk art and architecture documented all the drawings he saw during his trip to various cities in Turkey and produced free-hand drawings of a large collection of geometric patterns. Some of the patterns illustrated by Schneider are unfortunately lost at this time. Yet, his drawings provide valuable reference to these lost instances. In these drawings, he also explored several interesting properties of the pattern designs, such as the shape relations or irregularities inside the geometric designs. These observations provide valuable information to this study and are used in several cases for comparative investigations.

How quests concerning material properties and craft techniques change the appearance and making of geometric designs are examined with two dimensional drawings. These

drawings are based on different visual construction techniques beneath geometric patterns that appear in literature (Bakırer, 1981; Bonner, 2017; Bourgoin, 1973; Broug, 2008; Critchlow, 1976; Hankin, 1905, 1925a; Kaplan, 2005; A. J. Lee, 1987).

The subjects of the examinations, the works and artifacts, are selected because of many reasons that shall be exposed during the investigations. We can categorize them here according to their historical order [2], yet they appear not necessarily in a historical order during the diverse inquiries present in the dissertation:

Muhammad al Tusi was an immigrant originally from Tus, a city in the Khorasan Province in Iran. The Seljuks in Khorasan were trained in geometric brickwork technique and applied pattern designs by arranging the separate brick units upon a geometric schema. Immigrant ceramic artisans like Tusi had to transfer the existing geometric pallet on to the new tile-mosaic. Therefore, we examine some of the previous pattern designs on different artifacts from preceding cultures, especially from Iran, Khorasan and Azerbaijan in order to understand how patterns were adapted to the new craft.

These monuments are:

- Ibn Kutaijir Mausoleum in Azerbaijan, 1161-1162 A.D.

- Mu'mine Khatun mausoleum in Nakhchivan, Azerbaijan, 1186 A.D.

- Gunbadh-i Qabud (The Blue Tomb) in Maragha, 1197 A.D.

Similarly, some brickwork examples from Anatolia, completed by old Iranian-Azerbaijanian master builders are taken into consideration:

- Keykavus Tomb, in Şifaiya Madrasa, 1205 A.D.

- Gök Madrasa Mosque in Amasya, 1266 A.D.

Additionally, earlier and subsequent examples of tile mosaic are investigated to understand how Tusi atelier's designs differ from others. This helps to follow the distinct ways of designing throughout several works and to see how the designs change

[2] Note that some of the dates are controversial with the predictions of different art historians. Dates are chosen according to Arık (2007, pp. 25–37).

through time while craftsmen get more used to the decoration with the new material. Some monuments are particularly important to see the differences of the material applications:

- Ulu Mosque in Akşehir, 1213 A.D. is one of the earliest surviving monuments, which expose some of the premature examples of tile mosaic decoration.

- Ulu Mosque in Malatya, 1247 A.D. attributed to another artisan [3], presents a stylistic change between brick ornamentation and tile mosaic. Apart from geometric brickwork, tile-mosaic plates adorn some parts of the surfaces. Because of the similarities in pattern designs, Meinecke (1976a, pp. 47–49) suggests that the craftsmen responsible for this monument might worked in a close connection with the Tusi atelier.

- Gök Madrasa in Tokat, 1270 A.D.

Works that are attributed to the Tusi Atelier are:

- A building part in the Alaaddin Mosque Complex, in Konya, 1235 A.D.

- Sırçalı Madrasa in Konya. 1242-43 A.D.

- The Anatolian Seljuk Mihrab inside the Ottoman Mısri Mosque built in Afyon (1250 A.D.), was not one of the excursion spots in the TÜBİTAK founded research project. Yet, the city was visited as well and the Mısri Mosque mihrab is analyzed in-situ and documented with photographs.

- Karatay Madrasa 1251-1253 A.D.

The crafting tradition lasted for many years, even after Tusi's epoch, until the Mongol invasion caused artisans to escape the lands. Some tile mosaic examples are from the same area but from a latter period, where subsequent dynasties inherited control of central Anatolia from the Seljuks. Nonetheless, stylistic similarities suggest that some principalities continued the artistic tradition and they present unique examples of tile mosaic as well. This works largely maintain the design legacy of the Tusi Atelier. Thus, we may infer that it was either done by Tusi's students or influenced mostly by Tusi's works.

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[3] An inscription on the building façade reveals the name Yaqub b.Abi Bakr al-Malati ((Meinecke, 1976a, pp. 47–49).

These are:

- Gömeç Hatun Tomb in Konya 1270 A.D., Karamanid Dynasty

- Beyhekim Mosque in Konya 1270-80 A.D., Karamanid Dynasty

- Sahib Ata Kulliye (Complex) that consist of a Mosque (1258 A.D.) a Hankah (Dervish Convent) (1279 A.D.) and Tomb (1277 A.D.) built in Konya, Karamanid Dynasty

- Eşrefoğlu Kulliye (Complex) that consist of a Mosque (1296-1297 A.D.) and Tomb (1301-1302 A.D.) in Beyşehir, Eşrefoğulları Dynastie

The thesis is conducted as an experimental investigation on the computational design processes that are informed by the material and architectural geometry. The principal subject of investigation in the thesis, the craftsmen from Tusi atelier produce various patterns from same principles, work and do experiments with different forms. In order to get a better understanding of their making approach towards their designs informed by the tile mosaic material and the surface geometry, three main design instances and issues are examined in more detail in Chapter 5. These designs are:

- ***The complex geometric design on the Karatay Iwan***. This design consists of rectangular repeat units that inherit five-fold symmetries. Hence, multiplying the repeating units many times results in an irregular geometry and pattern design, which proves that the craftsmen improvised their design by changing the geometrical rules for making. This improvisation is done by noticing material potentials, telling us that material understanding played an important role in the production of the geometric designs. The analyses are done with two-dimensional pattern drawings completed using Autodesk's AutoCAD and Adobe's Illustrator.

- ***Geometric patterns on polyhedral surfaces.*** The design examples investigated in detail in this section examines the relation between the two-dimensional pattern designs and the three-dimensional forms by an examination on various polyhedra. The investigations show that craftsmen had an informal understanding of both three-dimensional solids and two-dimensional geometric patterns. The investigations are based on both two-dimensional drawings completed using Autodesk's AutoCAD and Adobe's Illustrator and three-dimensional drawings done with Rhino, a three-dimensional modelling program.

- ***Tile Mosaic Covered Dome from the Karatay Madrasa.*** With the primary intention of understanding the generative methodology and the geometrical rules beneath the domical pattern design, the pattern drawing is completed by using Rhino, a 3D modelling program and Grasshopper, an algorithmic design tool integrated into Rhino. In our investigation, we initially documented the dome and the geometrical pattern on its surface with a 3D laser scanner. The documentation is completed by ZEMASTEK, a construction and restoration firm led by F. Doğan Tekin. The data acquired from the 3D laser scanner is used to inform and compare the algorithmic three-dimensional model with the actual design.

1.7.3 The structure of the thesis

The present chapter "Introduction: A Computational Understanding of the Know-How in Medieval Islamic Art" introduces the theoretical and technical framework of the thesis and discusses how design computation field is related to medieval Seljuk art.

In Chapter II "Geometric Designs in Medieval Anatolia" a short summary of Anatolian Seljuk history and their artistic and architectural traditions are provided. Like other dynasties in the Islamic world, The Seljuk's in Anatolia embellished building surfaces with geometric pattern designs. Nevertheless, Anatolian Seljuks created peculiar designs that were specific to the geography. Previous to the descriptive brief on the Islamic geometric art, an overview of the history depicts an understanding on Anatolian Seljuk's unique artistic conventions that differ from other reigns. The geometrical background behind pattern designs together with possible generative construction methods are described in this section as well. Finally, a short outline of extant historical documents and the historical relation among mathematicians and artisans are explained in detail.

Chapter III "New Material and New Ways of Making: The Tile Mosaic" concentrates on the new material and explicates initially the historical background of the new material. It explains under which circumstances the tradition arose and how the material was invented and used by immigrant Persian artisans like Muhammad al-Tusi. The features of the extinct and unique thirteenth century craft are portrayed thoroughly by examining the step by step production processes of similar contemporary crafts.

Chapter IV "The Master Craftsman and The Means to the Craft: The Legacy of Muhammad Al-Tusi" introduces the medieval master builder Muhammad al-Tusi,

who was responsible of the building designs and the coordination between the geometrical design and the architecture. The chapter concentrates on the craft process and explains how making differs from the visual designs on the paper and requires additional understanding to apply geometric designs. The chapter explores Tusi's design tendencies and distinctness through comparative analyses on designs that ascribed to other master builders.

Chapter V "The Form and Matter: The Computation" explores the construction process and master builder's primary role by discussing how patterns were applied to diverse surfaces. The chapter concentrates on peculiar designs and examines how Tusi built the relation between the form and matter. Maker's marks and the indivisible parts of any craftsmanship such as improvisations, failures or inconsistencies are extensively discussed in this chapter. The special designs explored more in detail are a complicated iwan pattern from Karatay Madrasa (1252-1253 A.D.), several polyhedron designs from various monuments and finally the domical pattern from the Karatay Madrasa.

Chapter VI "Conclusion" provides a summary on *how historical ancestors guide us.* The chapter discusses the insights that are learned from the previous chapters and ends the thesis discussion with the statement, like in the modern world, materials and making played an essential role in designs. Additionally, even craftsmen used certain principal rules for their designs, they were not restricted by them and rules aided their creative processes as they can do it now if designers learn the language of the computer.

2. GEOMETRIC DESIGNS IN MEDIEVAL ANATOLIA

Medieval Islamic world was formed by different countries inhabited by various ethnic societies. Nations that are cited under the Islamic world spread out to a large geography that stretched from India to Spain. These societies were ruled by Muslim majorities, yet they had their very different kinds of social organization and featured unalike religious customs. Diverse cultures shared artistic traditions, especially geometric pattern designs as well, still they add their own identity to artworks they produced. Chief attention in this thesis is given to the thirteenth century Anatolia, in which a master ceramic craftsman, by the name of Muhammad Al Tusi, carried pattern design activities under the Anatolian Seljuk patronage in the then-capital city Konya. The current chapter provides a brief overview on the architecture of Seljuk Sultanate in Anatolia and portrays fundamental aspects of geometric patterns.

Artistic and architectural traditions of Anatolian Seljuks were influenced partially by their predecessors. Nevertheless, Anatolian Seljuks created their own design conventions and built a unique and multifaceted artistic culture (Ögel, 1987, 1994; Peker, 1996a, 2008, 2009). Additionally, Anatolia was a home to designer immigrants- including Muhammad al Tusi, whose origin was Tus, a city in Khorasan Province of Iran, a land governed by the Great Seljuk empire. New comers as such brought their own cultures and traditions to the land and combined these with Anatolian customs , hence gave shape to novel artistic practices. An insight on the uniqueness of their design culture is achievable by following their historical roots.

The medieval Seljuks, named after their dynastic ancestor- a chief named Seljuk b. Duqaq, were initially a Turkish nomadic group living in Eurasian steppes (Peacock, 2010, p. 1). They founded an Empire that at its height expanded its realm from the borders of modern western China to the eastern Mediterranean between the eleventh and fourteenth centuries (Figure 2.1). They laid the foundations of the modern Turkey accompanied by the substantial Turkish-speaking populations that exist today in Iran, Iraq and Caucasus. The age of the Seljuks refers initially to the Great Seljuk Empire, that lasted over a century from about 1040 to 1157. Established initially in Khorasan, the lands governed by the first descendants were primarily Central Asia, Iran, Iraq and

a part of Syria. Subsequent to the collapse of the Great Seljuk Empire, dynasties within the Great Seljuks continued their reign in parts of Syria and northern Iraq until the thirteenth century. The Seljuks eventually reached the Anatolian territories and following the defeat of the Byzantines at Manzikert (Malazgirt in Turkish) in 1071, they inhabited these new lands. The Seljuk successors ruled in Anatolia up to the fourteenth, even in some parts until the early fifteenth century. The Anatolian domain of the Seljuk Turks, preside over the lands from about 1081 to 1307 is known as the Seljuk Sultanate in Anatolia and sometimes called the Seljuk Sultanate of Rum, referring to the Roman Byzantine past of the Seljuk territories (Figure 2.2). Seljuk Sultanate in Anatolia, in the center for this study, was the most significant among the Seljuk successor states (Canby et al., 2016, pp. 1–4).

The age of the Seljuks is commemorated for remarkable social, religious, as well as artistic change. Provincial courts alongside middle-class consumers invested and consequently stimulated the artistic production. This was an asset, which eventually assigned a period of an immense innovation in art and architecture. The Seljuks did therefore not only influenced the politics and demographics of the Islamic world. Besides, they shaped the strong cultural and artistic legacy in other Islamic Monarchies (Canby et al., 2016, p. 4).

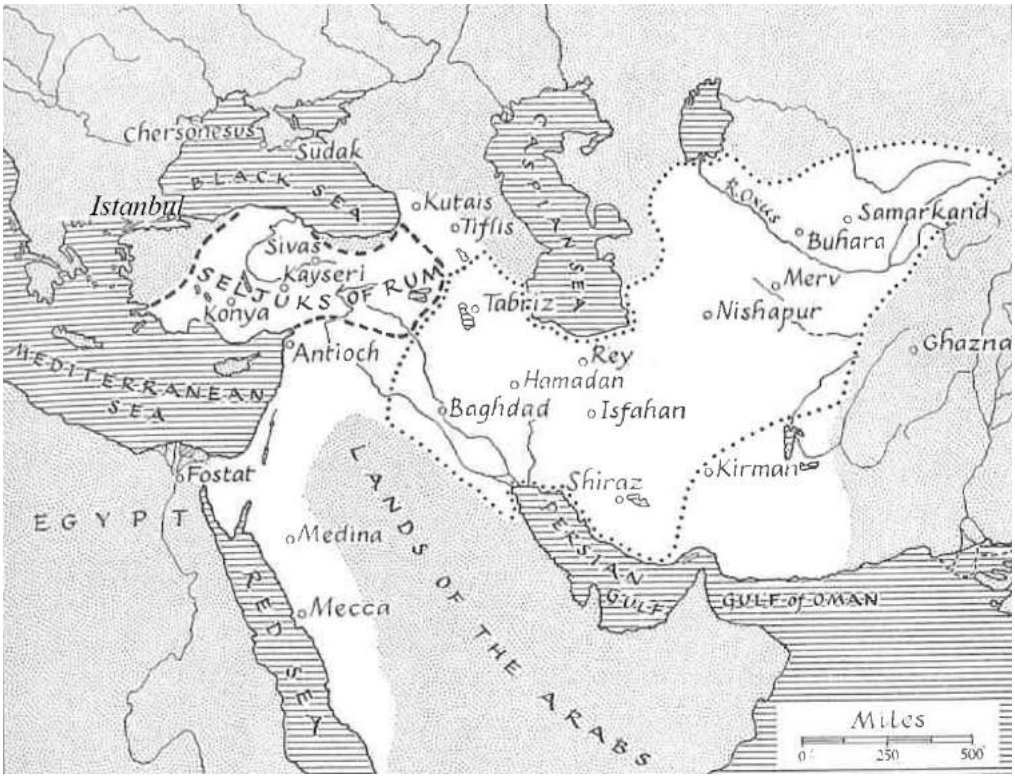


Figure 2.1 : The territories of the Great Seljuks (Rice, 1961, p. 33).

Figure 2.2 : The territories of the Seljuks in Anatolia (Rice, 1961, p. 47).

Seljuks produced countless masterpieces of art and architecture in the lands they governed, a vast area from Hindu Kush to eastern Anatolia and from Central Asia to the Persian Gulf. Thus, base materials used for the architecture is assorted among geographies. Some of the most prominent edifices of this period were obviously religious buildings, such as the monumental mosque complex in Isfahan, Iran (Figure 2.3). The building is the result of continual constructions that include additions and renovations, yet most important parts were constructed during the reign of the Seljuk Sultan Meliksah, between 1072-1092 A.D. The unusual plan schema of this colossal monument guided the mosque architecture in the Islamic lands. The building typology was reinterpreted and transferred to stone in Anatolia (Altun, 1988). Perhaps one of the most acknowledged and earliest instances is the Great Mosque in Divriği - Sivas province in central eastern Turkey (Figure 2.4). This masterpiece of Islamic architecture, included in the UNESCO World heritage list, was founded by the Mengujek dynasty in 1228-29 A.D (Kuban et al., pp. 15–20). As put forward by some scholars, the sophisticated technique of vault construction and the architectonic formed as a decorative sculpture, bears a resemblance to Gothic and Baroque architectures of the west (Aslanapa, 1972).

Figure 2.3 : Masjid-i Cami built in Isfahan is a prominent edifice of the Seljuk rule in Iran (From the Aga Khan Visual Archive (2017)).

Figure 2.4 : The Great Mosque in Divriği, Sivas-Turkey, 1228-29 A.D.

The Seljuk era is renowned for other building typologies as well. Predominantly for Madrasa, a place where the state's administrators and religious scholars were trained. Seljuks promoted learning, hence they put a great effort to build a network of madrasas. The first appearance of the madrasa is dated back to eleventh century, yet best surviving examples are those found in Anatolia (Altun, 1988, pp. 80–83). These buildings featured sculptural entrance portals and impressive interior spaces (Figure 2.5, Figure 2.6 and Figure 2.7).

Figure 2.5 : Cifte Minareli Madrasa in Sivas, Turkey, 1271 A.D. (Left: Photo credits Ezgi Baştuğ. Right: Drawing by Alexandre M. Raymond (in Ertuğ et al., 1991)).

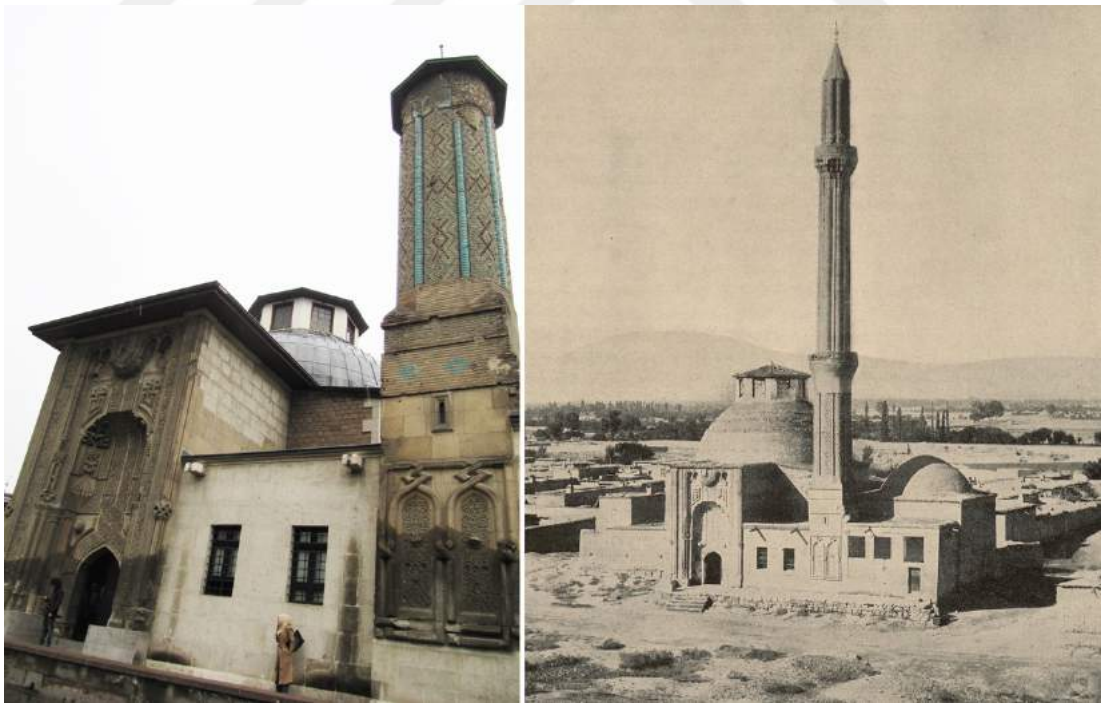


Figure 2.6 : Left: Current view from the Ince Minareli Madrasa in Konya, Turkey, 1254 A.D. Right: Photo before the lightning that destroyed the monumental minaret (Photo from Sarre (1901a, p. 134)).

Figure 2.7 : Ince Minareli Madrasa in Konya is renowned for the sculptural entrance and the interior space is roofed with a magnificent brickwork dome.

Another significant type is the caravanserai, an inn for the merchants travelling across long distances. Seljuk's pursuit for evolvement in trade and commerce ensured an immense heritage of caravanserais, especially in Anatolian territories. This building typology is named as *Han* in Anatolia and the complete number of Hans exceed hundred (Altun, 1988). Among different individual surveys on madrasas, a German art historian, Kurt Erdmann (1961) realized a comprehensive study on the Anatolian caravansaries. Anatolian Seljuk caravansaries, such as the Sultan Han near Kayseri built in 1233-1237 A.D. (Figure 2.8), are distinguished because of the stonemasonry accomplished expressly on the monuments.

Figure 2.8 : Tuzhisarı Sultanhanı near Kayseri, Turkey, 1233-1237 A.D.

The architecture of this period is characterized by memorial tombs as well. The idea of building epic mausoleums was first introduced to the Islamic Heritage by Turks. This building typology was developed considerably under the Seljuk patronage. Mausoleums have different typologies and they are named either as Kümbet (cupola) or Türbe (tomb) in art terminology (Altun, 1988, p. 84). The architecture comes correspondingly in different forms including octagonal, cylindrical and square shapes.

Mausoleums are either topped with a dome, which are mainly found in Persian examples, (Figure 2.9) or conical roofs seen especially in Anatolia (Figure 2.10) (Saoud, 2003).

Figure 2.9 : Seljuk twin tombs in Kharraqhan, Qazvin province, Iran, 1067 A.D. (Photo from Wikimedia Commons (2004)).

Figure 2.10 : Left: Döner Kümbet, 1275 A.D. Right: Çifte Kümbet, 1247-1248 A.D. Both in Kayseri, Turkey.

Although very rare examples have survived and that not completely, excavations show that, like any other empire, Seljuks built palaces and kiosks as monumental examples

of civil architecture. Only few remnants exist, including the palaces in Merv and Rey from the Great Seljuk era and two major palaces in Anatolia: Keykubadiye palace near Kayseri and Kubadabad on the southwestern shores of Lake Beyşehir (Altun, 1988, pp. 89–91). Ruins uncovered during the excavations in Kubadabad palace show that the fortress was a huge complex that comprised various buildings (R. Arık, 2014).

Remnant examples of Seljuk architecture show exceptional building characteristic and spatial proportions and continue to be sources of inspiration. Yet, these monuments are also accredited for another distinctive feature, for decorations that adorn buildings and artworks. In addition to the calligraphic art and arabesques (floral patterns), geometric patterns seen in medieval Islamic Art were inseparable parts of Seljuk art and architecture as well.

2.1 Islamic Geometric Art

Medieval Islamic art is commonly cited for the geometric patterns that consist of polygons, stars and lines. Although most prominently on building surfaces, these intricate patterns also appear in further artworks such as book art, carpets, pottery, furniture, metal wares and woodwork. There is a strong belief that the visual tradition of geometric patterns was developed because of the restriction in figurative art in Islam. Yet, from the very early periods there are examples of figurative motifs found in Islamic artworks. However, none of those gained as much attention as geometric patterns. A travel across the globe shows that many other cultures throughout the history admired and used geometric patterns in their works of art and design. Nevertheless, none of these cultures probably reached up to the intricacy level as we see in the medieval Islamic art. Bonner (2016, pp. 56–57) observes that Pre-Islamic cultures of Ancient Greeks, Byzantium, Coptic Egypt and Sassanid Persia all used pattern designs. Then again, in the earlier Hellenic art, star patterns are exercised in a more independent fashion, not within an interconnected network. This intricate arrangement, where polygon or star motifs become an integral part of the design is what creates a unified whole, an image of infinity that was practiced by the medieval Islamic artisans on several kinds of objets d'art.

The exact origins of Islamic geometric designs are impossible to ascertain systematically. Yet, surviving examples expose a progressive evolution starting from

very simple compositions to become more intricate compositions (Bonner, 2016, pp. 56–57).

One of the many reasons for this advancement may have been continuous research and developments in both mathematics and astronomy, conducted during the epoch. Since, the creation and application of these lace-like patterns onto all kinds of surfaces was very much dependent on geometric rules utilized in order to create unique designs.

2.1.1 Geometrical background

Knight's (1998, p. 306) very basic explanation of a pattern "a set of spatial elements- points, lines, planes or volumes- in two or three dimensions", enlightens as well its relation to geometry. Both geometry and pattern communicate with a similar vocabulary, mainly shapes in various forms. Geometry is a kind of form language concerned with the measurement and angular properties of shapes and their relationships among the entities (Williams, 1979, p. 17). Thus, geometry provides a common language that bridges the disciplines of art, architecture and natural sciences (Kappraff, 1992). Patterns as design products are based on some rules that are formalizable through geometrical concepts (Kappraff, 2001, p. XXIII). The visual progression and stylistic diversity in geometric art is very much depended on the practice of such rules. Applying rules for aesthetic reasons was not only intrinsic to pattern designs in the medieval world. For instance, Abu 'Ali Muhammad ibn Muqlah, a well-known calligrapher, developed a rule-based system of calligraphic proportions to employ on calligraphic art in the tenth century (Bonner and Kaplan, 2017, p. 2). Thus, working with rules was a familiar concept to the medieval artisan.

The formalization attempts to study ornament designs are based mainly on the concept of symmetry. The appreciation of symmetric patterns roots back to the beginnings of humanity, yet the symmetry theory in mathematics, as a descriptive way to formalize patterns, is relatively new. In a simple descriptive way, symmetry is a kind of transformation, which maps a shape of the object onto itself, leaving that object unchanged. In geometry, a continuous pattern design, which decorates a surface by the regular repetition of a unit, is examined by symmetry operations presented in its structure. These transformations, which extend the same shape by multiple copies, are rotations, reflections, glide reflections and translations. The classification of a pattern design is based on symmetry groups, which involve a combination of these isometric

transformations both in two and three dimensions (Knight, 1998, p. 305). As explained in Grünbaum and Shephard (1987, p.26), these transformations are:

Rotation: A geometric design has n-fold symmetry about a fixed point if, when rotated 360/n degrees or 2/n radians, the resulting image maps the pattern onto itself (Figure 2.11 (a)).

Translation: A geometric design has translational symmetry, if a shift or slide in a given direction through a given distance does not change the initial shape in terms of size and the shape (Figure 2.11 (b)).

Reflection: Called also line symmetry or mirror symmetry, is a type of symmetry in which the operation results with a mirror image of the initial shape (Figure 2.11 (c)).

Glide reflection: The operation includes both reflection and translation of the same object (Figure 2.11 (d)) (Grünbaum and Shephard, 1987, p. 26).

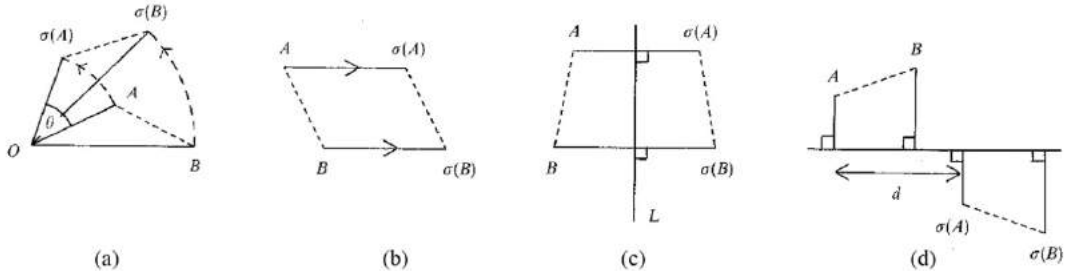


Figure 2.11 : Four symmetry operations of plane isometry: (a) rotation (b) translation (c) reflection (d) glide reflection (Diagrams from Grünbaum and Shephard (1987, p. 26).

Mathematical methods, especially group theory can be brought to bear on the analysis and synthesis of geometric designs. The illustration in Figure 2.12 shows different symmetries embedded in a simple geometric design. The classification of this pattern can be done through an understanding on wallpaper groups. There are seventeen different ways to fill a two-dimensional plane and the design in this illustration is classified as p6m type, one of the most frequently used type for pattern designs (Abas & Salman, 2007, pp. 46–47).

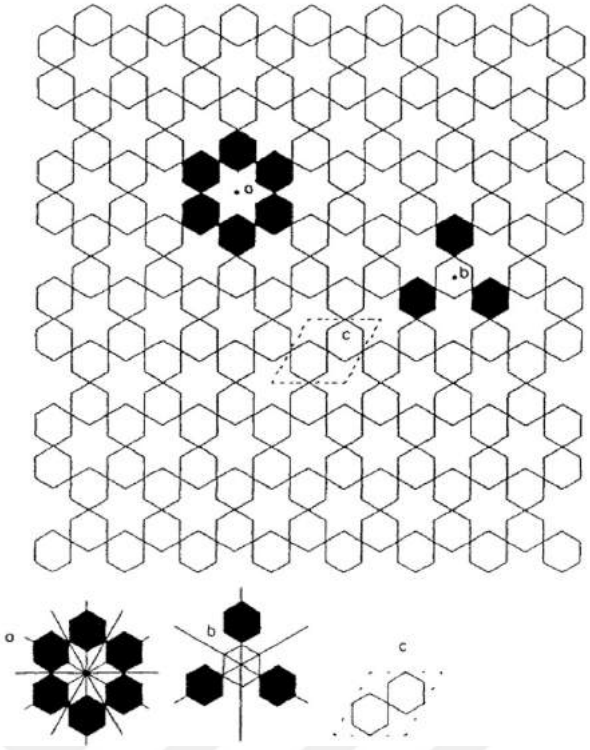


Figure 2.12 : A p6m type pattern (a) 6-fold center of rotation (b) 3-fold of rotation (c) minimum unit cell to construct the entire pattern out of multiple copies (Abas and Salman, 2007, p. 47).

Comprehensive studies such as Britton (2000), Farmer (1991), Grünbaum and Shephard (1987), Rosen (1995) and Washburn & Crowe (1988) provide a deeper understanding on the concepts of overall symmetry and groups. Furthermore, studies that concentrate especially on Islamic patterns (Abas and Salman, 2007; Bonner, 2017, pp. 183–187; Grünbaum and Shephard, 1992; Kaplan, 2002; Makovicky, 2016b) or as in Schattschneider's (1990) case, on Escher designs, are useful to understand the correlation between symmetry theory and pattern designs. Yet, the center of attention in this study is basically artisans' design approaches and processes. Hence, an unformal understanding of basic transformations is sufficient to follow the basic descriptions presented throughout this study. Grünbaum et al. (1986, p. 649) go along with these thoughts:

> We--mathematicians and some other scientists--may find it convenient and useful to interpret regularity of a pattern in terms of its group of symmetry (or color symmetry, etc.). In this way we can apply the results of algebra and other mathematical disciplines to the study of such patterns. However, it could be argued that this is not the concept of regularity that artisans (Moorish or any other) had in mind as they were creating their art. In fact, until a century or so ago, even to mathematician's regularity of mathematical objects had a completely different meaning. (p.649)

Mathematical approaches classify patterns usually on a count of their symmetry groups, yet there are other informal ways to categorize designs. These classifications usually consider the character of the main motifs that are embedded in pattern designs. For instance, a star or rosette motif is identified by counting the rays or the petal shaped elements that surround it. Similarly, if there are only visible polygons in an ornament, then the side number of the main polygons classify the design. As an example, illustrations in Figure 2.13 depict two different patterns: The one above is a rosette and the one below is a star motif. They have both eight petals that surround their centers and display eight-fold rotational symmetry.

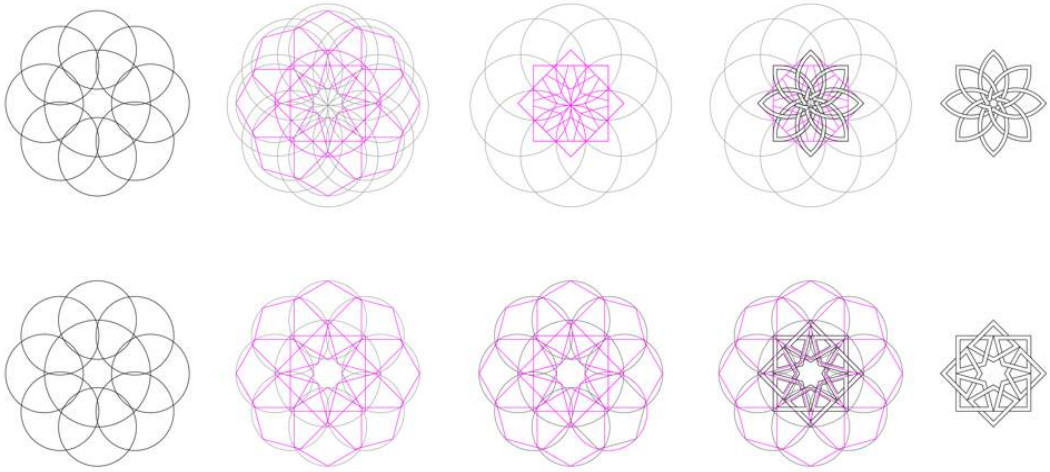


Figure 2.13 : Step-by step construction of two different eight-pointed motifs (above a rosette and below a star) (Drawings by Begüm Hamzaoğlu produced for the TÜBİTAK project 114K283).

Figure 2.14 presents the geometric family tree proposed by Broug (2008, p. 21). Broug's categorization is based on the division of a circle into its equal parts to create a given geometric pattern. This is also credited as the rotational symmetry embedded in the motif. For example, a six-pointed star has a six-fold symmetry and is based on the division of a circle into six equal parts. Thus, it belongs to the family of six in Broug's family tree (2008, p. 21). Most pattern designs can be classified under this family tree. Nonetheless, some of the complicated patterns belong to neither of these groups and therefore cannot be categorized under the branches.

In another approach, Bourgoin (1973) classifies patterns based on their polygonal nature of repeating units. In Bourgoin's (1973) approach, hexagonal patterns refer to the patterns with six-fold symmetry, while the dodecagonal group contains motifs with twelve-fold symmetries. Both Broug's (2008) and Bourgoin's (1973) categories are

not sufficient to label all pattern designs in an appropriate fashion. On the other hand, Lee (1987) and Bonner (2017) present more systematic taxonomies of pattern designs by examining their mathematical structures more explicitly. Lee (1987) introduces definitions of star and rosette motifs by exploring their construction principles and classifies different types. For instance, in Lee's (1987) grouping, type I refers to simple stars while type II encompasses patterns with rosettes (Cromwell, 2017, pp. 3–4). Bonner's (2017) classifications are based on generative polygonal tessellations and line slopes that form various pattern designs.

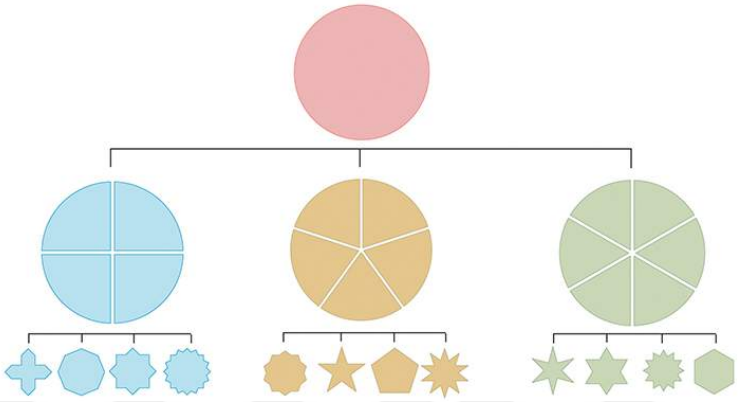


Figure 2.14 : "The family tree of Islamic geometric design" as proposed by Broug (2008, p. 21). Patterns are labeled respectively as fourfold, fivefold or sixfold designs.

Apart from classifications based on geometrical properties, there are also other ways for pattern groupings. Art historians like Demiriz (2017) tend to label designs under different surface sections on which patterns appear. Medallions or border pattern are some example categories under such taxonomies. With his designer perspective, Schneider (1980) identifies broader categories such as ornaments with calligraphy, brick patterns, domical brick patterns, spiral patterns, swastikas, border patterns, star patterns, pentagonal patterns, patterns on hemispherical domes etc. Likewise, Mülayim (1982) examines patterns from a wider scope by considering their base materials and geographies they were produced in.

2.1.2 Visual design methods

Geometry constituted obviously an important key to the development seen in the diverse and intricate nature of pattern designs. Broug (2008, p. 15) writes: "A geometric composition is the product of both rules and creativity. The basic rules are the same for every composition, but the creativity is the input of the individual. The

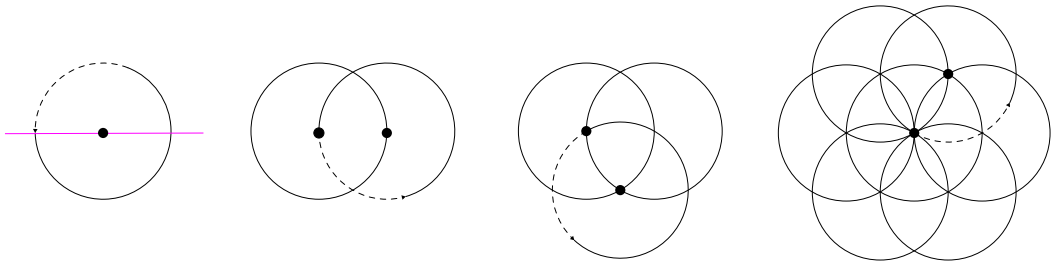
rules are an integral part of geometric design and paradoxically, without such strict rules creativity could not flourish". The means of a rule-based system is beneath the patterns is clear, yet there is not enough evidence about design processes, particularly on how the generative methodology worked.

The generative approach behind pattern designs in Islamic art and architecture has attracted many scholars from different disciplines. In his famous book *The Grammar of Ornament,* Oven Jones (1856) had already provided some examples of Islamic art together with other culture-specific ornament designs. Yet, Bourgoin (1973) was the first to publish a comprehensive catalog concentrated only on Islamic geometric designs. Since then, numerous inquiries are conducted to represent geometrical rule definitions behind diverse pattern designs. Since there are not many extant medieval resources on the making of visual designs, different approaches were developed for visual construction methods behind patterns. Except in particular cases, authors of such studies commonly acknowledge and favor one of many possible methodologies and illustrate pattern drawings by using these. These diverse techniques can be classified under four different categories based on the grids used for pattern generations: The circular grid, generative polygonal tessellations, the isometric and orthographic grid and the radial grid.

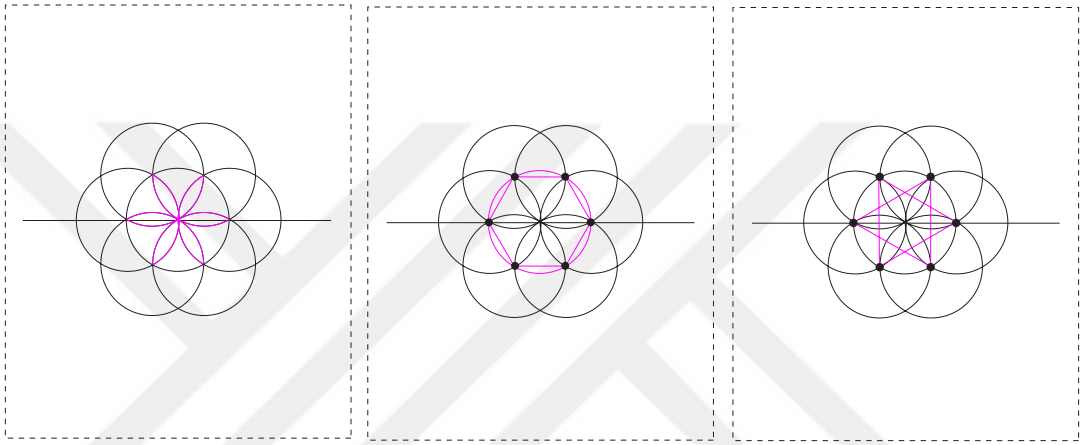
2.1.2.1 The circular grid

Established research demonstrates the geometric principles underlying these unique designs created using the compass and the straightedge, building on combinations of repeated circles, overlapped and interlaced (Bakırer, 1981, 1999; Broug, 2008; Critchlow, 1976; Sutton, 2011; Wade, 1976). This is the most popular and acknowledged generative design methodology, yet there is not a specified name for the methodology. Bonner (2017) labels this method both as the point-joining or the graph-paper technique. The method starts by drawing a horizontal line using a straightedge and then placing a point on the line to construct a circle with a compass. The opening degree of the compass gives the radius of the first circle and is fixed during the drawing process to get the forthcoming identical circles. Intersection points appear when the horizontal line meets the circle and these are used as new compass points to center further circles, at the very end a regular grid of overlapping circles is

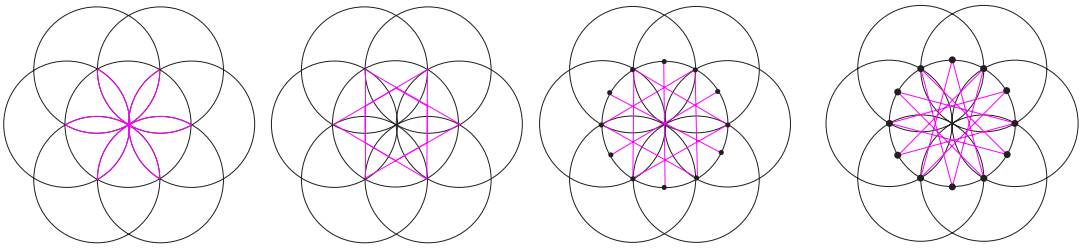
obtained (Figure 2.15). This circular grid already forms alone some of the very basic designs.



Steps to draw a circular grid



Various shapes on the same circular grid



Steps to draw a twelve-pointed star

Figure 2.15 : Constructing step by step a regular grid of circles using a compass and straightedge. Black dots on circle centers indicate the right place for the compass point at each step. Diverse motifs are derived from the same circular grid (After the illustrations in Met Museum's *Islamic Art and Geometric Design: Activities for learning* (2004)).

By drafting a network of zigzagging lines on the circular tessellation, it is possible to construct the star and polygon patterns- the foundation of the overall ornament design.

Construction lines create a network of various segments and by picking up from the many possible intersection points and connecting these through new lines, a geometric design emerges (Figure 2.16). Many of the intricate pattern designs are easily drawn with this method, based simply on the compass and straightedge handling. The obscurity is up to the artisan's curiosity. As portrayed by Özkar (2014, p. 52), a variety of designs emerge when the "artisan's eye connects intersection points with new lines".

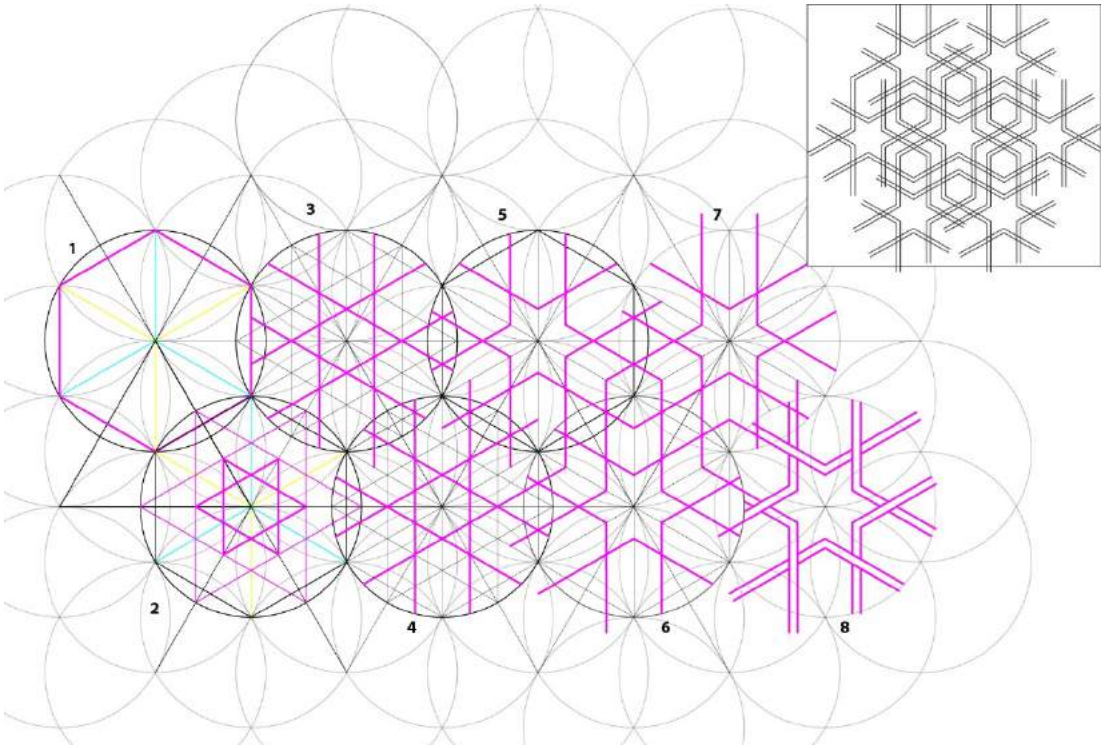


Figure 2.16 : Illustration of a panel from Ebul Kasım Ali Tomb, Tokat, Anatolia, 1233 A.D. (Drawing by Ezgi Baştuğ, using Bakırer's (1981) method, produced for the TÜBİTAK project 114K283).

Various arrays of tangential and interlaced circles of the same radius establish a grid in the forms of three primary shapes: the triangle, the hexagon and the square (Critchlow, 1976, p. 7). After the formation of a specific grid, circles and lines can be used again to connect the intersection points, resulting in various compositions (Figure 2.17 and Figure 2.18).

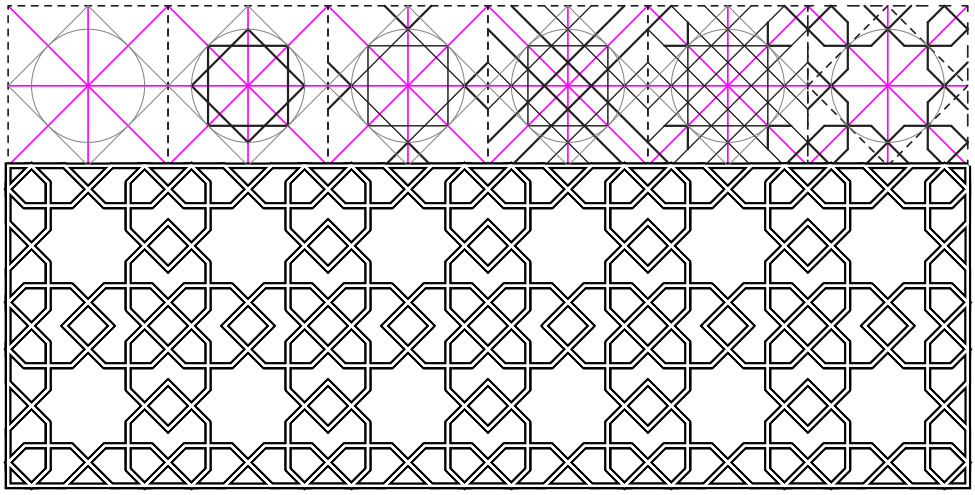


Figure 2.17 : Pattern design built by dividing circles and connecting intersection points with lines (adapted from Bonner (2017, p. 212).

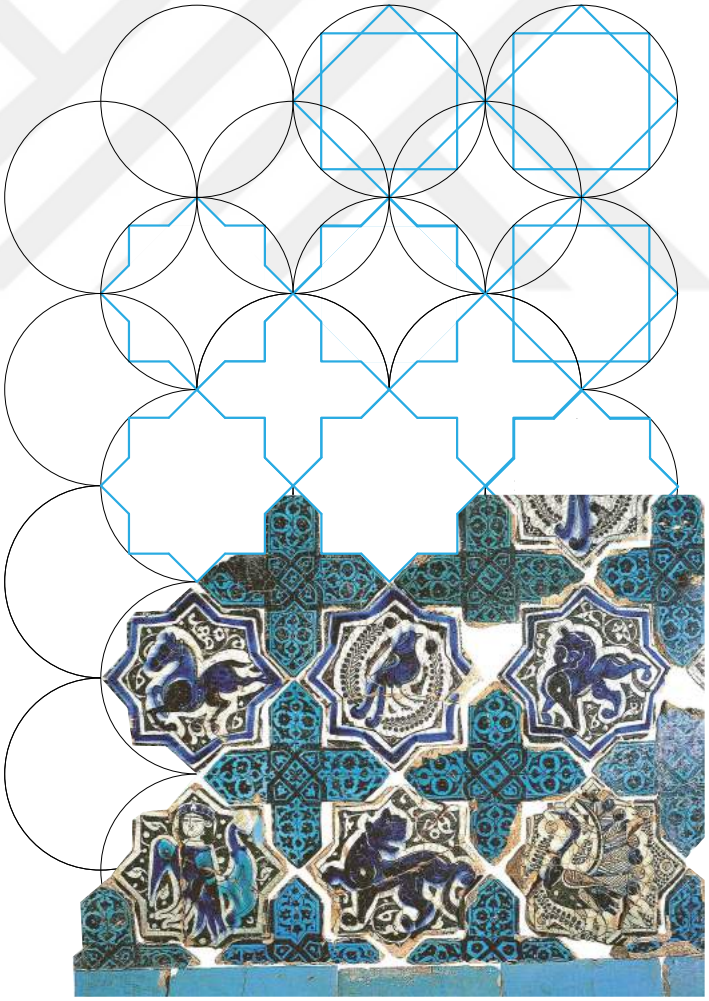


Figure 2.18 : Circular grid behind wall tiles from the Kubadabad palace in Konya, (a photograph from R. Arık and O. Arık (2007, p. 294) is juxtaposed to the illustration).

The initial circle grid forms further polygonal tessellations that can work as invisible foundations for pattern designs (Figure 2.19). Polygons as such work as the base for the following alternative method.

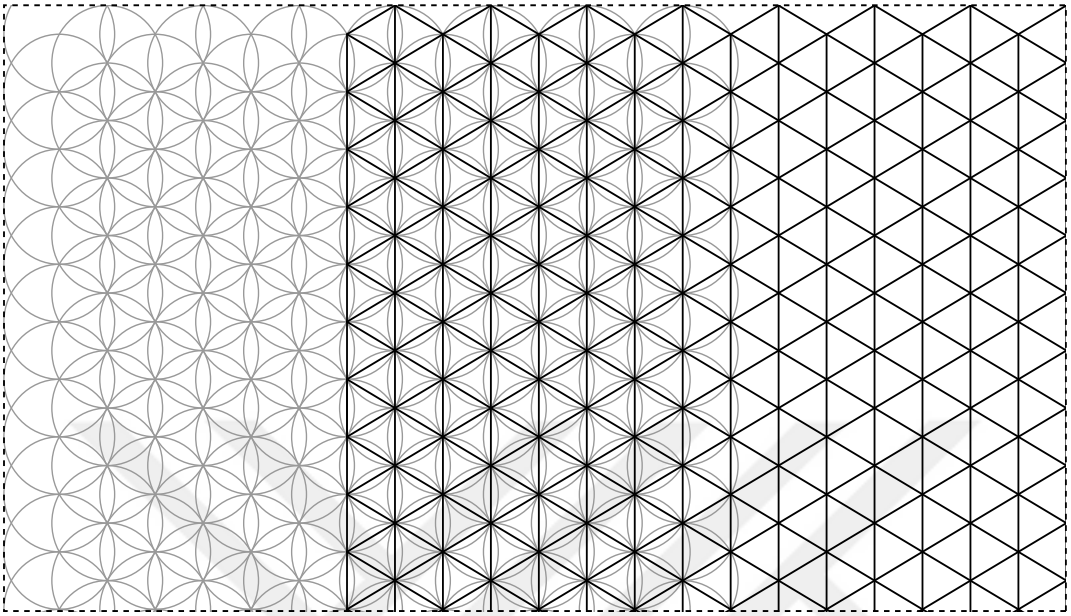


Figure 2.19 : Triangular tessellation is drawable from a circular grid.

2.1.2.2 Generative polygonal tessellations

Principles behind polygonal tessellations are based on the tiling theory in geometry. A tiling is the division of an infinite space into a number of distinct shapes that cover the plane without any overlap. There are infinite ways to cover a plane and most of these are examined in the renowned book *Tilings and Patterns* by Grünbaum and Shephard (1987). These tessellations work as the geometrical substructure that derive many pattern designs. Similar to one of the possible approaches in the previous technique, this method is based on pre-determined polygonal tessellations. Yet, the main difference in this method is that the circular grid vanishes, and instead only specific points link the lines of the pattern. These points, located on each side of each polygon, determine how eventually the pattern lines are connected. Two lines are drawn through all these centers, they cross each other and continue till they meet other lines of similar origin. Therefore, the technique is based on previously determined contact angles, which are the angles between motif edges and polygon edges they emanate from (Figure 2.20) (Kaplan, 2002, pp. 50–51).

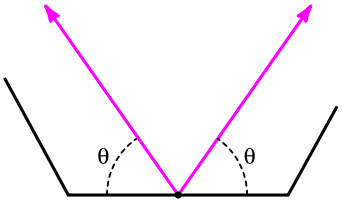


Figure 2.20 : The contact angle between the base polygon and the pattern line determines the final design (Kaplan, 2002, p. 52).

Once the pattern design is complete, the generative polygonal tessellation is discarded (Figure 2.21). Figure 2.22 illustrates the same design in Figure 2.18, this time on a polygonal tessellation.

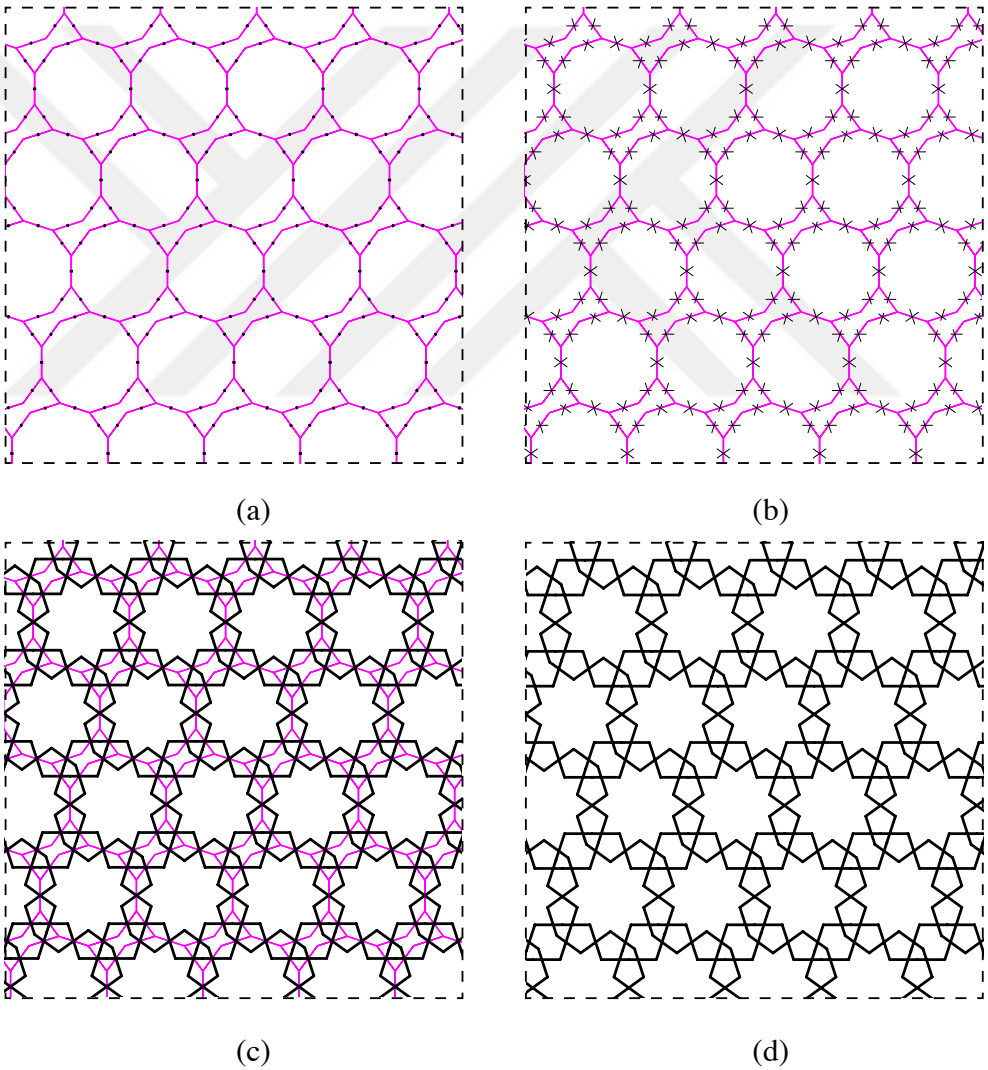


Figure 2.21 : The polygonal technique (a) a sample polygonal tiling (b) points to be connected on each polygon line (c) geometric design juxtaposed on the tessellation (d) pattern design without the polygonal substructure (adapted from Kaplan (2002, p. 53)).

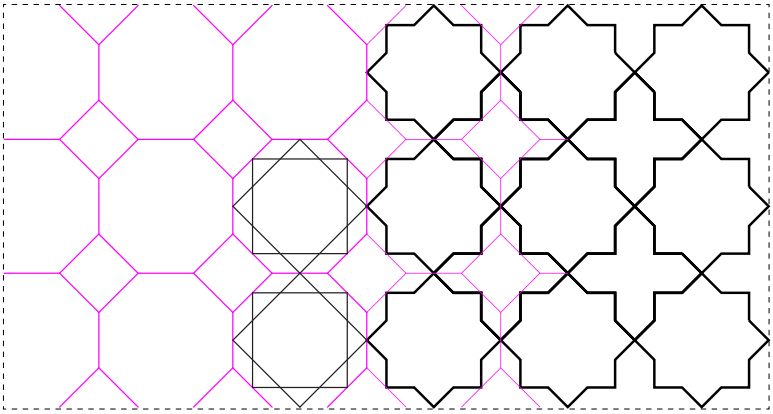


Figure 2.22 : Same pattern from Figure 2.18, this time constructed with the polygonal technique.

Possibilities for creating ornament designs are endless and from the existing designs, we can conclude that medieval artisans were using a large collection of polygonal tessellations. There are only three regular tessellations of the plane by congruent tilings: a tessellation with triangles, a tessellation with squares or a tiling with hexagons. Yet, apart from these, established research demonstrate that many other tessellations were used beneath Islamic geometric designs and the variation in contact angles resulted in numerous instances. Figure 2.23 illustrates three different designs that are derived from the same polygonal tessellation. Contact angles differ and consequently create dissimilar pattern designs.

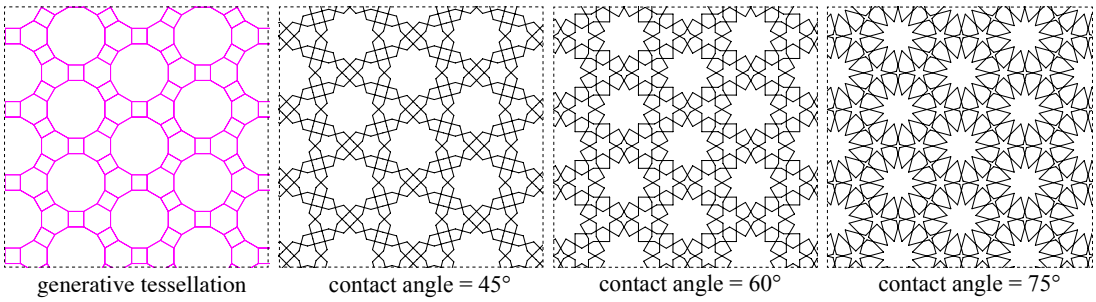


Figure 2.23 : Diverse pattern designs emerge when the line angles change on a tiling formed with squares, hexagons and dodecagons (Kaplan, 2002, p. 54).

This method is called as well *Polygon's in Contact* (Kaplan, 2000a) or *Hankin Method*, named after Ernest Hanbury Hankin (1905; 1925b; 1925a), the first person to publish this construction method. Originally a bacteriologist, Hankin was working in India during the nineteenth century and took a considerable interest in Mughal geometric patterns. Apart from his professional studies on biology, he also published his observations on pattern making in India that led him to develop this method (Bonner,

2017, p. 192). Hankin's studies influenced mathematicians and computer scientists, especially Craig Kaplan (2000a; 2002; 2005), who developed computational algorithms based on this technique. Apart from that, many authors have related Islamic geometric patterns to configurations of polygons. Especially Emil Makovicky (1992; 2016b; 2000), Cromwell (2009; 2010a; 2010b;2013a; 2016), Lu and Steinhardt (2007) and Sarhangi (2012b; 2012a) illustrated complicated patterns designs that inherit five-fold symmetry on polygonal tessellations. While these studies mostly concentrated on particular symmetries and cases, Jay Bonner (2000; 2016; 2017) provided a comprehensive study on the polygonal technique. Depending on their geometrical configuration, Bonner (2016, pp. 64–65, 2017) identifies five different design systems: "the system of regular polygons, the four-fold system A, the four-fold system B, the five-fold system and the seven-fold system". Different strategies are used for extracting geometric designs from underlying polygonal tessellations. Thus, Bonner proposes four standard techniques based on how the pattern lines are formed. Bonner entitles these techniques as *acute, median, obtuse* and *two-point* (Figure 2.24).

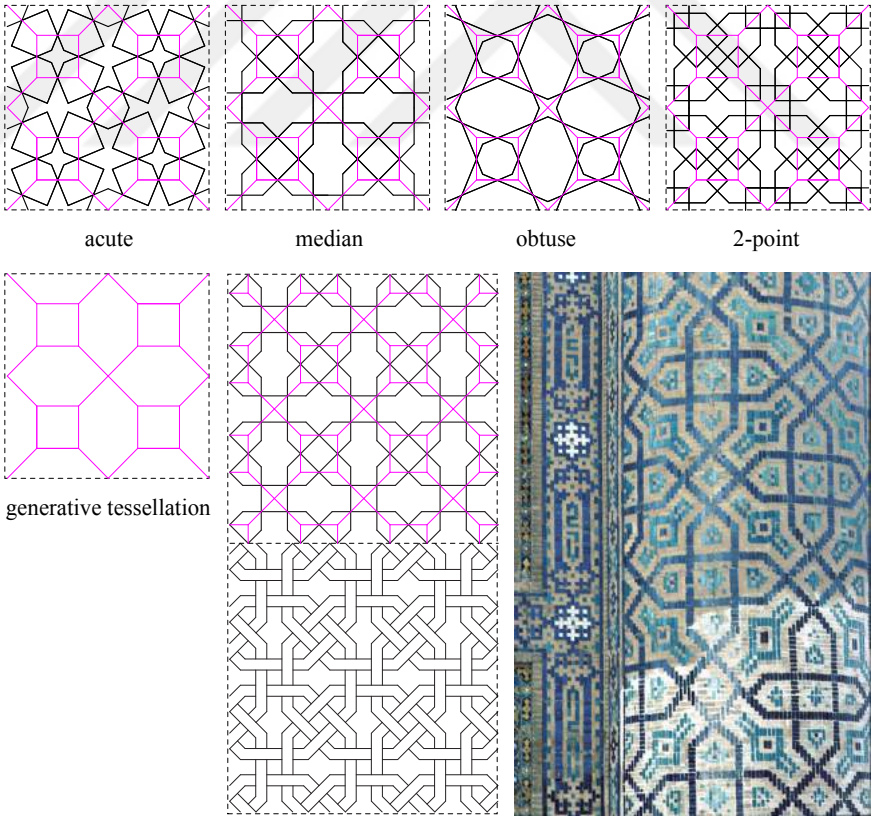


Figure 2.24 : Bonner's four standard techniques applied to the same generative tessellation (Drawings adapted from Bonner (2016, p. 64, 2017, p. 266)).

Figure 2.25 illustrates nine different designs constructed with Bonner's (2017) two-point technique on a regular hexagonal tessellation. Based on some of the available pattern scrolls and treatises, Bonner (2017, p. 201) suggests that the historical significance of this technique is supported by a preponderance of evidence. In his opinion, some of the geometric patterns expose greater complexity and are only practicable with this generative method. Nevertheless, such a statement is debatable, since there could be other possible ways to generate designs that we are not still aware of.

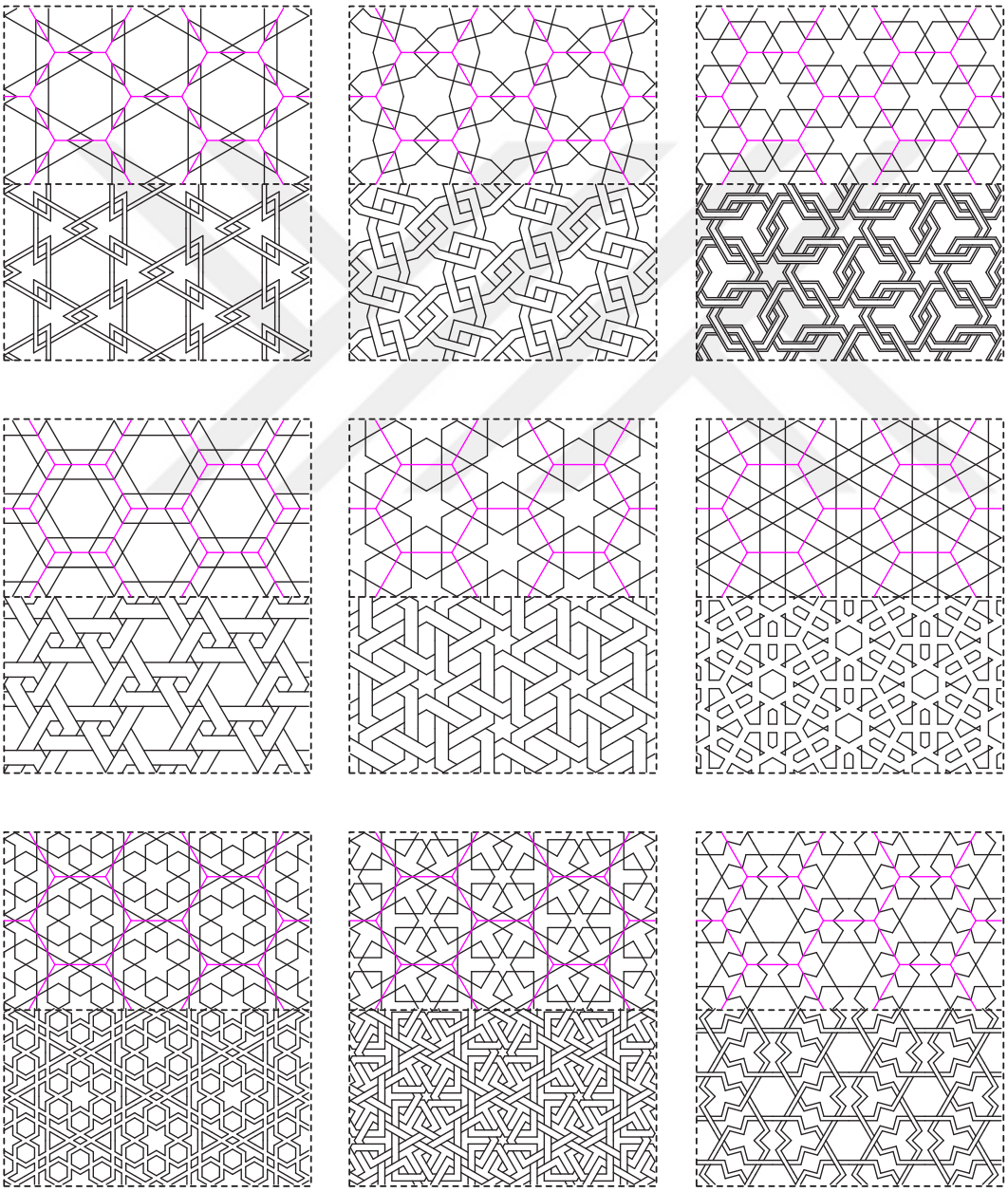
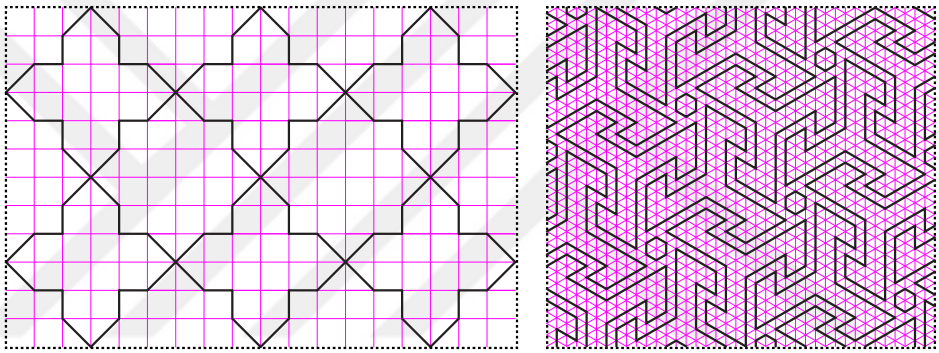


Figure 2.25 : Nine different designs based on the same hexagonal tiling (adapted from Bonner (2017, p. 233).

2.1.2.3 Orthogonal or isometric grids

Some of the pattern designs are drawable on either an isometric grid – a grid of triangles- or an orthogonal grid- a grid of squares- and therefore make use of such regular geometric grids. A grid system can become handy during design explorations, yet it requires adjustment to the proportions of the design and a precise design without any distortion (Bonner, 2017, pp. 214–216). Consequently, dimensions of the patterns that are drawn on these regular grids are optimized and adjusted to fit onto the regular grid structure, therefore differ from the results produced with other techniques. As an example, the pattern design in Figure 2.18 presented together with the Kubadabad tiles is principally the same as in the Figure 2.26 (a). Yet, motifs inside the design (such as the cross motif) in Figure 2.26 (a)) are slightly thicker than preceding examples.



(a) (b)

Figure 2.26 : (a) Same pattern in Figures 2.18 and 2.21 on a square grid. Motifs are optimized to fit the grid (b) a geometric design drawn on an isometric grid (Drawings after Bonner (2017)).

Patterns including square *kufi,* a particular calligraphic style with an orthogonal nature, are created easily by using a grid of squares (Bonner, 2017, pp. 214–216). A sample construction manual for kufi designs is included in the *Topkapı scroll*, a historic treatise broadly explained in the following section. The grid method is preferred for current cut-tile practices in Morocco as well (Abas and Salman, 2007; Castera, 1999; Sutton, 2011). Similarly, Sarhangi (2005; 2012a) proposes a modularity approach for some mosaic designs. Yet, there is no evidence that such a methodology was historically used for the making of more complicated patterns. Nevertheless, Özdural's (1991) analysis on the stalactites in Buruciye Madrasa, Sivas, Turkey 1271 A.D. show that the design was based on a modular orthogonal system. Therefore, we can assume that the medieval artisan was not inexperienced in using grids for constructive purposes.

2.1.2.4 Radial grid method

There are different approaches based on the extension of radii diverging from geometrical centers of the primary motif in a particular design. These create sorts of radii matrices for making patterns and the method was traditionally used, moreover is corroborated in historical treatises (more on historical scrolls are explained in the following section). Contemporary Iranian artisans inherited this technique from their ancestors as well (Kasraei et al.,2016, p. 313).

Sarhangi (2012a, pp. 350–351, 2012b, pp. 166–167) tracks traditional methods by studying extant historical scrolls and illustrates the constructive sequence. The method starts by locating the first point and the division of a right angle divided into congruent angles where the radii are located. Later, an arbitrary point at the opposite site is detected to create a rectangle. Radii are placed onto that new radial. Intersections of the rays and perpendicular lines create nodes that are used to connect pattern lines, hence the pattern design is drawn directly without any underlying tessellation (Figure 2.27). The method creates a rectangular repeat unit that works as the basic part of a certain pattern design. This fundamental repeat unit is then replicated several times to cover a large surface.

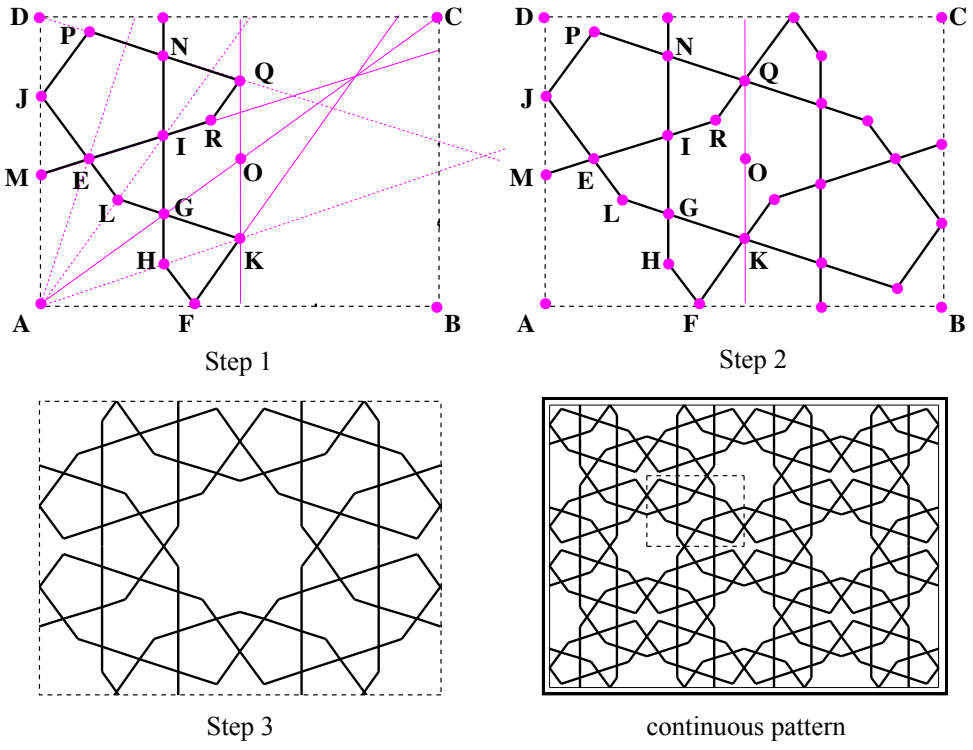


Figure 2.27 : Construction of a rectangular repeat unit from a radial grid. Multiple copies of the repeat unit forms a continuous pattern (Sarhangi, 2012b, p. 166).

The approach of using a repeat unit is essential to notice, as it is explained more in detail in the following Chapters 4 and 5, the method was used evidently by Muhammad al Tusi's atelier as well. Especially in "the complex geometric design on the Karatay Iwan", from the Karatay Madrasa in Konya, we can find indications of a rectangular repeat unit that display five-fold symmetry (explained more in detail in section 5.4)).

There are other approaches that use radii, nonetheless the construction sequence differs slightly. Bonner (2016; 2017) illustrates a similar approach that he calls "extended parallel radii method" and labels complicated patterns drawn with this method as "non-systematic designs". Out of radii connections, Bonner (2016; 2017) draws complicated polygonal tessellations from which pattern designs emanate. Hence, he links the radii grid to the abovementioned drawing systematic of generative polygonal tessellations (section 2.1.2.2). The illustration in Figure 2.28 illustrates the step by step sequence for Bonner's extended parallel radii method.

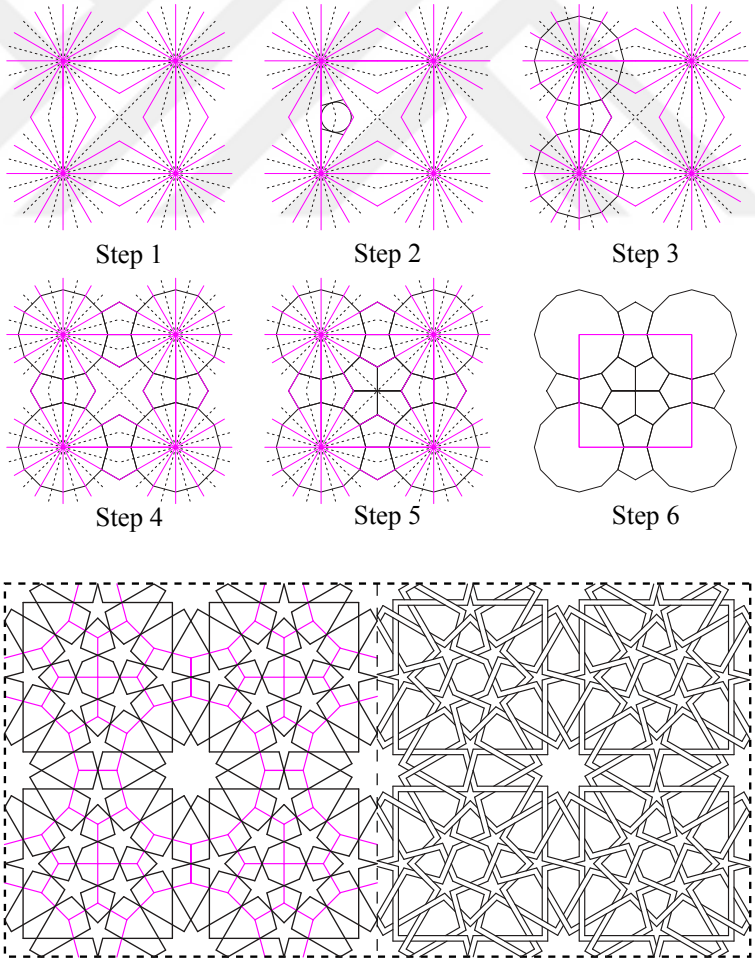


Figure 2.28 : Step-by-step radii matrix construction to draw a polygonal tessellation (Illustrations adapted from Bonner (2016, pp. 82–83).

In Bonner's (2016; 2017) method, initially, 24 radii are placed at each vertex of the square repeat unit in Step 1. Step 2 draws a circle that is tangent to the non- dashed radii. This circle establishes edges of dodecagons and the separating pentagon. Dodecagons and pentagons are completed in Step 3. These polygons are rotated four times throughout the square repeat unit in Step 4. Four clustered pentagons emerge at the center of the repeat in Step 5. The tessellation is completed in Step 6. This generative tessellation can then be used to create intricate geometric designs (Figure 2.28).

Cromwell (2013) uses a similar constructive technique based on radial grid approach as well (Figure 2.29). In this technique, star centers are the fundamental starting point for the radial grid construction. It is coherent with the statement of A. J. Lee (1987, p. 184) "The symmetrical properties of any regular n-pointed star motif or n-sided regular polygon (n being any whole number greater than two) can be represented as a system of 2n radii diverging from the geometrical center of that motif". Once the radial grid is constructed, Cromwell (2013) places smaller circles between the radii and star centers.

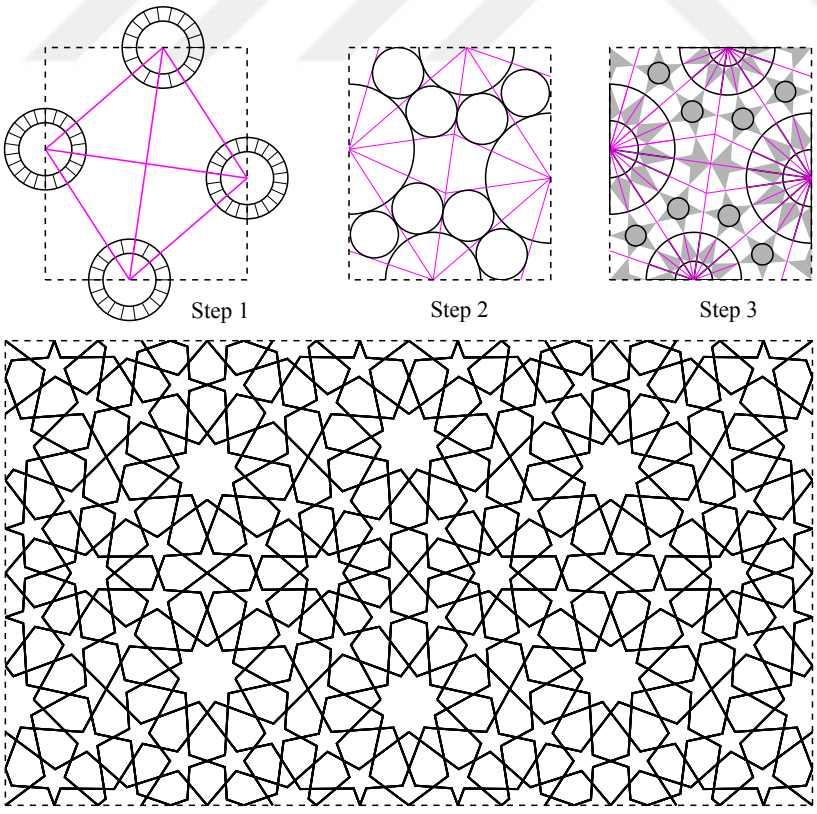


Figure 2.29 : Cromwell's (2013, p. 25) method to create a pattern that contain regular 11-pointed stars and irregular 9-pointed stars.

Cromwell's method, is "developed from simple rules based on good design principles rather than the mathematics of symmetry and involves only simple geometric constructions (angle bisection) and approximations (angle trisection, construction of regular polygons)" (Cromwell, 2013, p. 30). Radii approaches become especially important for the making of complicated geometric designs that include unusual combinations of stars or polygons. Such methods are used to construct complex arrangements, in which symmetries of the primary stars are incompatible with each other (Cromwell, 2013, p. 30).

2.2 Extant Historical Documents

There are several resources on geometrical works in medieval Islamic world, yet historical documents related to creative processes, especially for geometric designs are quite few. These are as following:

1. A tenth century treatise preserved as five different Arabic Manuscripts by mathematician-astronomer Abu'l-Wafa' al-Buzjani (ca. 940-998) ***Kitab Fī mā yaḥtaj ilayhi al-ṣāni' min al-a'māl al-handasiyya (On the Geometric Constructions Necessary for the Artisan*)** (Figure 2.30). Al-Buzjani was born in Buzajan, a city in Khorasan, Iran. Previously educated by his uncles in mathematics, he then moved to Baghdad in his twenties. He is one of the greatest Muslim mathematicians and was given the title *Mohandes* denoting the most knowledgeable and skillful geometer (Sarhangi, 2012a, p. 348). Al-Buzjani has written several books on mathematics and astronomy including commentaries to Euclid, Diaphantus and el-Khwarizmi. Al-Buzjani's chief contribution was in the development of trigonometry (Özdural, 1991, p. 61). Among his other works, *On the Geometric Constructions Necessary for the Artisan* is especially important as it indicates an interaction between designer-architects and mathematicians. The treatise that is divided into eleven chapters contains a collection of 171 problems of geometry (Raynaud, 2012, p. 35). The treatise does not include any pattern designs. Nonetheless, it illustrates and describes compass and straightedge constructions, upon which pattern designs are formed.

Figure 2.30 : Sample page from copy of the text of al-Buzjani at the Süleymaniye Library in İstanbul, Turkey.

2. ***Fi taddkhul al- ashkal al-mutashdbiha aw al-mutawdfiqa (On interlocking similar and corresponding figures),*** referred to hereafter as ***Interlocking figures*** is an anonymous document appended to one of the Persian translations of al-Buzjani's aforementioned treatise (Sarhangi, 2012a, p. 348). The only copy is located in MS Persan 169 in the Bibliotheque Nationale, Paris. Among the surviving documents, *Interlocking figures* is the only known practical manual that provides how-to instructions for drawing patterns (Necipoğlu, 1995, p. 133). The drawings of ornamental geometry are accompanied by explanatory texts in Persian and the document includes twenty-five works on mathematical subjects, mainly practical geometry (Özdural, 2010, p. 191). Through a comprehensive study Özdural (2010) suggest that it is "the documentation of a strong collaboration between mathematicians and artisans" and was written by a mathematician/craftsman in the thirteenth century. However, there are other opinions that identify the author as the fifteenth century Persian mathematician Abul-Es-hagh Koobnani (Sarhangi, 2012a, p. 348). The importance of the document lies that it proves an intimate link between the practical geometry promoted in Bagdad during the tenth-century and subsequent Persian traditions (Necipoğlu, 1995, p. 138) (Figure 2.31).

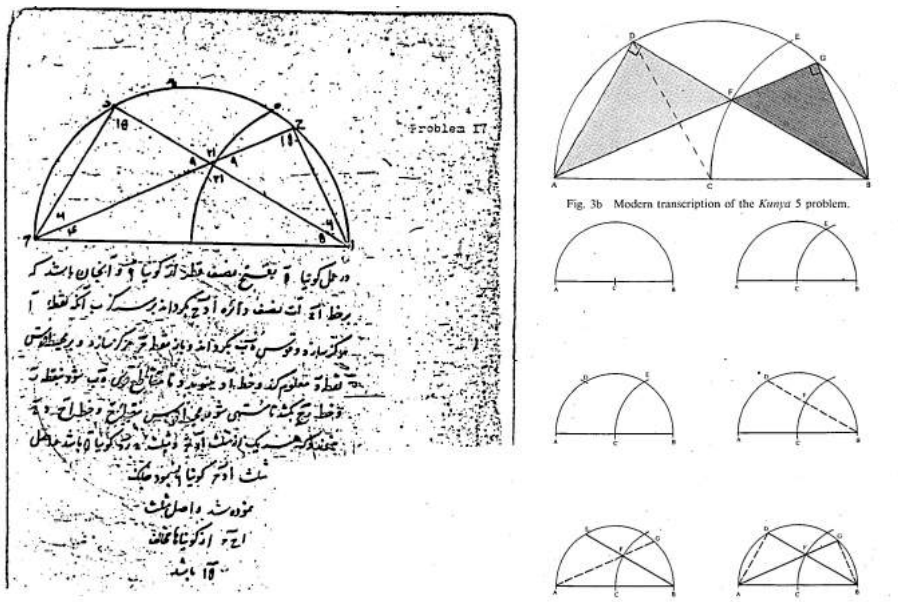


Figure 2.31 : A geometrical problem from *Interlocking Figures* and a modern transcription of the problem (Chorbachi and Loeb, 1992, pp. 288–289).

3. The ***Topkapı Scroll,*** dated to fifteenth century is a document created by Persian master builders, who worked under the Timurid dynasty. The scroll is preserved in the collection of the Topkapı palace museum and was published extensively by Gülru Necipoğlu in 1995. The wide scroll, 29.5 meters long and 33-34 meters high, is a typical tumar, fixed at one end to a wooden rod and the other end to a leather flap. Best-preserved example of its kind, Topkapı Scroll contains 114 drawings in square and rectangular frames. These drawings compile a rich repertory of geometric drawings for wall surfaces and vaults. Geometric patterns for diverse materials appear side by side and are drawn in ink and dye. Schematized drawings, especially polygonal tiles that appear by red dotted lines beneath pattern designs indicate that it worked as a sort of guide book for the designers (Necipoğlu, 1995) (Figure 2.32).

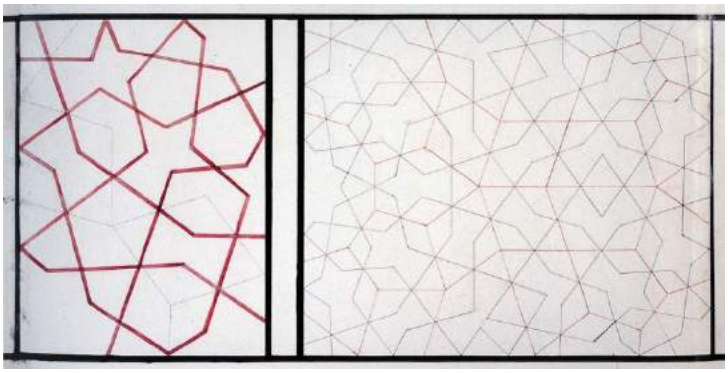


Figure 2.32 : A drawing from the Topkapı Scroll (Wade, 2018).

4. **Tashkent Scrolls** are sixteenth-seventeenth century documents (38 cm x 160 cm) attributed to practicing master masons in 16th century Bukhara, a city located in the Greater Khorasan of Sassanid and medieval Persian periods. The scrolls are currently located in the collection of Oriental studies in Tashkent, Uzbekistan. The designs in the scrolls are executed with simple drafting instruments and consist of squared ground plans, geometric and calligraphic patterns, and ground projections of muqarnas or arch-net vaults (Bodner, 2012; Necipoğlu, 1995, pp. 9–14; Sarhangi, 2012a, p. 349).

5. Scrolls in the **Mirza Akbar** collection preserved at the Victoria & Albert Museum are attributed to the nineteenth century Qajar state architect Mirza Akbar. Two complete scrolls and 236 smaller designs cut from other scrolls are held in the museum collection (Necipoğlu, 1995, pp. 14–15). Necipoglu (1995, pp. 14–15) suggests that these designs prove that contemporary Iranian masons kept the tradition of drafting scrolls with architectural drawings (Figure 2.33).

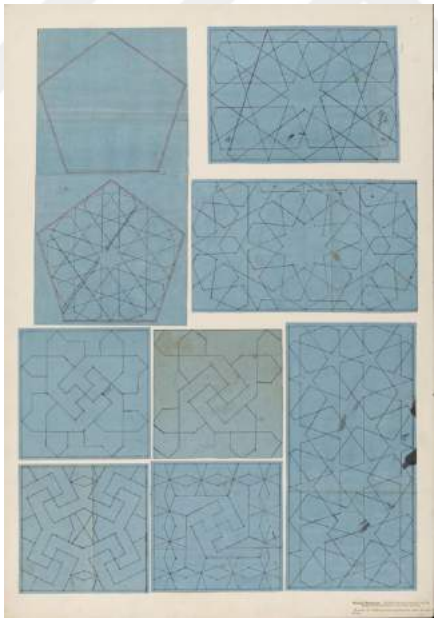


Figure 2.33 : Sample pages from the Mirza Akbar Scroll (from the Victoria & Albert Museum's online collections ("Victoria and Albert (V&A) Museum Online Collections," n.d.).

2.3 The Mathematical Knowledge and The Connection Between Design Practice

Crafting a pattern onto an original building surface initiates some design problems which were then solved to some extent by use of theoretical knowledge in geometry.

During the demographic, cultural and economic deterioration of the Medieval West, Islamic lands prospered in scientific explorations. A greater part of the scientific texts then was on mathematics. Furthermore, texts on astronomy, optics, law, and linguistics provided new insights into mathematics yielding to major explorations in the field of geometry during that period (Scriba and Schreiber, 2015, pp. 171–189).

The Islamic Golden Age, traditionally dated from the 8th century to the 13th century, was the historical period when Islamic scholars translated Elements along with the primary works of Pythagoras, Plato, Aristotle, Archimedes, Galen, Ptolemy and others. In Elements, Euclid deduced the principles of what is now called Euclidean geometry. Besides plane geometry, Euclid also wrote on perspective, conic sections, tiling, number theory, optic and spherical geometry in relation with the positioning of stars. Most of the medieval mathematicians cross-examined Euclid's works. Apart from ancient Greek, scholars translated Indian, Assyrian and Iranian knowledge into Arabic and conducted their own investigations resulting in novel scientific findings and developments (Scriba and Schneider, 2013). Together with the original Ancient Greek texts, advanced Arabic science would open the way for the scientific revolution in the west of seventeenth century (Freely, 2010). We owe many concepts that we use in modern world, such as the decimal numbering system, the creation of algebra, important discoveries in plane and spherical trigonometry to famous medieval Muslim scientist (Gericke, 2014, p. 214). Ibn Musa Al-Kharizmi who was the father of algebra (8^{th}-9^{th} century), Abu Yusuf Yaqub Ibn Ishaq Al-Kindi (9^{th} century), Abu'l-Wafa' al-Buzjani (10^{th} century) Abu l-Rayhan al-Biruni (10th-11th century), the famous poet Omar Khayyam (12^{th} century), and the Persian astronomer Nasir Al-Din Al-Tusi (13^{th} century) were amongst a series of mathematicians and astronomers who have their credit for several innovations (Berggren, 2003).

For many years, scholars have discussed whether it is possible for the artisans to acquire the necessary mathematical knowledge during the practical application of the pattern designs. Sarhangi (2008, p. 523) highlights that some specific design ideas necessitated the collaboration of artisans and mathematicians. Even if there are not many texts that survived from the said period and geography, the aforementioned 10th century treatise by the mathematician-astronomer Abu'l-Wafa' al-Buzjani (ca. 940-998) *On the Geometric Constructions Necessary for the Artisan*, a document on

geometry written especially for architect-artisans, is a key reference and evidence for the mathematician's involvement in apparent design problems.

Through his studies, Özdural (1995, 1998, 2000, 2010) suggests an acknowledged relation between the arts of building and mathematics. Geometricians developed visual instructions to offer knowledge and strategies to artisans to simplify geometrical challenges. When facing a new design problem (e.g. a new architectural form or the application of a new material), artisans had to expand practical knowledge in order to deal with that specific problem. These types of problems may lead the mathematicians to search for a theoretical understanding of that type of an issue. With reference to Omar Khayyam's writings, Özdural (1995, p. 55) reports that mathematicians and artisans collaborated through special meetings and discussed several design problems. These gatherings involved a two-way interaction where mathematicians visually demonstrated the fundamental principles of geometry and the artisans expanded these illustrations with new ideas. For instance, a visual proof of the Pythagoras' theorem by al-Buzjani would provide the basis for novel compositions that artisans created utilizing part relations of the shapes in the proof (Figure 2.33).

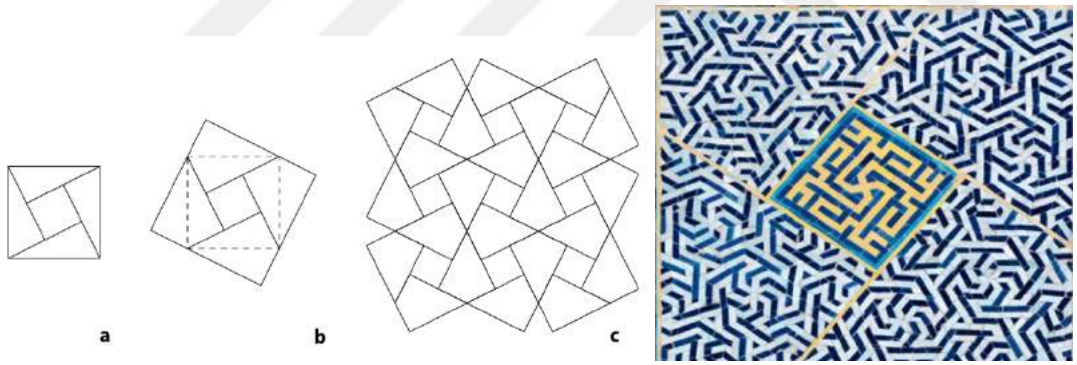


Figure 2.34 : The visual proof of the Pythagoras' theorem that provide the basis for novel compositions and a tile panel based on the illustration, from the Friday Mosque in Isfahan, Iran (Photo from Makovicky (2016b, p. 89).

These meetings, called *conversazioni* in Özdural (1995), were apparently not only gatherings, where mathematicians gave only instructions, but rather platforms where mathematicians had to come up with solutions to new problems created by designers. Omar Khayyam described the creative thinking and these type of designer problems as "wilderness" and stated that he would never come up with such ideas. Collaborations between mathematicians and artisans concerning the field of architecture opened up the way for creative designs. (Özdural, 1995). Özdural (2000,

p. 701) writes: "that mathematicians taught geometry to artisans by means of cut-and-paste methods and of geometrical figures that had the potential of being used for ornamental purposes".

The number of surviving historical documents are quite few, yet along with the numerous studies on Islamic geometric designs, they prove that geometric instructions or illustrations aided craftsman during the pattern making processes. Similarly, from the many complicated pattern designs we may infer that mathematician's involvement during creative processes was constant.

The geometrical knowledge in ancient Islamic world was not only about plane geometry, geometrical theorems concerning spherical geometry and spherical trigonometry were also practiced extensively in medieval era. Practical applications of such geometries for the making of polyhedron surfaces or domical pattern designs will be discussed extensively through the study, especially in Chapter V. Figure 2.35 illustrates a timeline of the principal research accomplished during the medieval Islamic era.

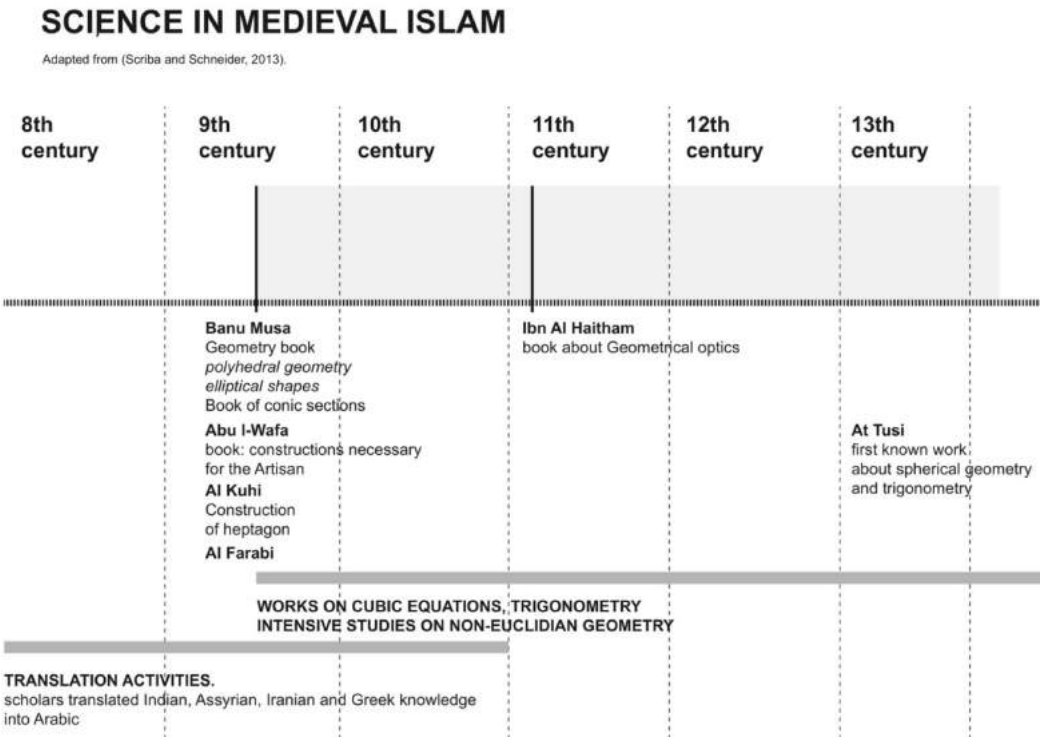


Figure 2.35 : Science in Medieval Islam (Adapted from Scriba & Schneider (2013)).

Based on his investigations, Özdural (1995, 1998, 2000, 2010) suggests that geometric designs were not sample products of only a compass and ruler practice, but rather products of continuous team work of mathematicians and artisans. Referring to

mathematicians involvement, Özdural writes “Ornamental patterns based on cubic equations or conic sections, bear further witness to their association, since these powerful conceptual tools were available only to mathematicians at the time” (Özdural, 1998, p. 701).

A famous miniature painting from the sixteenth century portrays Taqi al-Din b. Maruf and his colleagues, a group of astronomers, at the Ottoman observatory in Istanbul (Figure 2.36). Apart from the figure second to the left that stands behind and works with a compass and straightedge, other astronomers in the picture hold and calculate with many further instruments. There is no apparent designer among the astronomers. Yet, Özdural’s expression make us wonder whether these devices were also used in practical architectural problems in the thirteenth century Anatolia. In the following chapters, we search through many design examples and try to shed on light on to which degree compass & rulers are sufficient for accomplishing the designs.

Figure 2.36 : A sixteenth century miniature painting that depicts astronomers working in the Ottoman Observatory in Istanbul (from Scriba & Schreiber (2010, p. 178)).

3. NEW MATERIAL AND NEW WAYS OF MAKING: THE TILE MOSAIC

The artistic culture of geometric patterns unquestionably had an effect on the appearance of the medieval Islamic architecture. While these patterns decorated all types of building facades, diverse materials and crafting techniques resulted in the foundation of a rich variety of architectural examples across different geographies and cultures. Islamic ornaments were replicated in any scale and transferred across various materials. For the purpose of the current study, we center our attention to a material, tile mosaic, that was practiced by Muhammad al Tusi's atelier. This unique material first appeared in Anatolian ground in thirteenth and fourteenth centuries, during the Seljuk reign. This chapter presents a brief history on material's development and fundamental characteristics of the craft.

3.1 Historical Background

Individual craft ateliers and designers played an important role in the structural and stylistic alteration of the Rum Seljuk design and architecture. After the battle of Manzikert (Malazgirt) and the Seljuk's settling into the Anatolian territories, artisans and architects came into the lands and started the first building activities. The first political capital of the Rum Seljuk was Iznik (Nicea), over time Seljuks marched till Konya, a city in central Turkey. The conquest of Konya in 1189 from Friedrich

Barbarossa set out stabilization in Seljuk political life. This equilibrium served as the basis for increasing activities in architecture, arts and crafts. City walls were constructed in Sinop (1215), Antalya (1216), Alanya (1226) and Konya. Trading routes along with Caravansaries were established and Imperial residences, where artistic activities peak, were being built. As the result of new building plans, artists immigrated from neighboring countries, such as northwest Iran, Azerbaijan, Iraq and Syria. Mongol attacks under the leadership of Cengiz Han gave rise to artisan migrations. Skilled workers, even from more distant places as Turkistan and Khorasan came to Anatolia. Consequently, new artistic traditions were established and new architectural forms were generated. Thus, the first examples of Seljuk Architecture built in Asia Minor emerged under the influence of surrounding countries. The forms characterized with Syria inspired the architecture in Hasankeyf (1098-1231), Harput (1185-1233) and Mardin (1104-1407); Buildings in Erzurum, Erzincan, Kemah and Divriği were influenced by the designs seen in Georgian and Armenian architecture; Brick Structures from Azerbaijan shaped buildings in Niksar, Malatya, Kayseri and Sivas (Meinecke, 1976a, pp. 5–19).

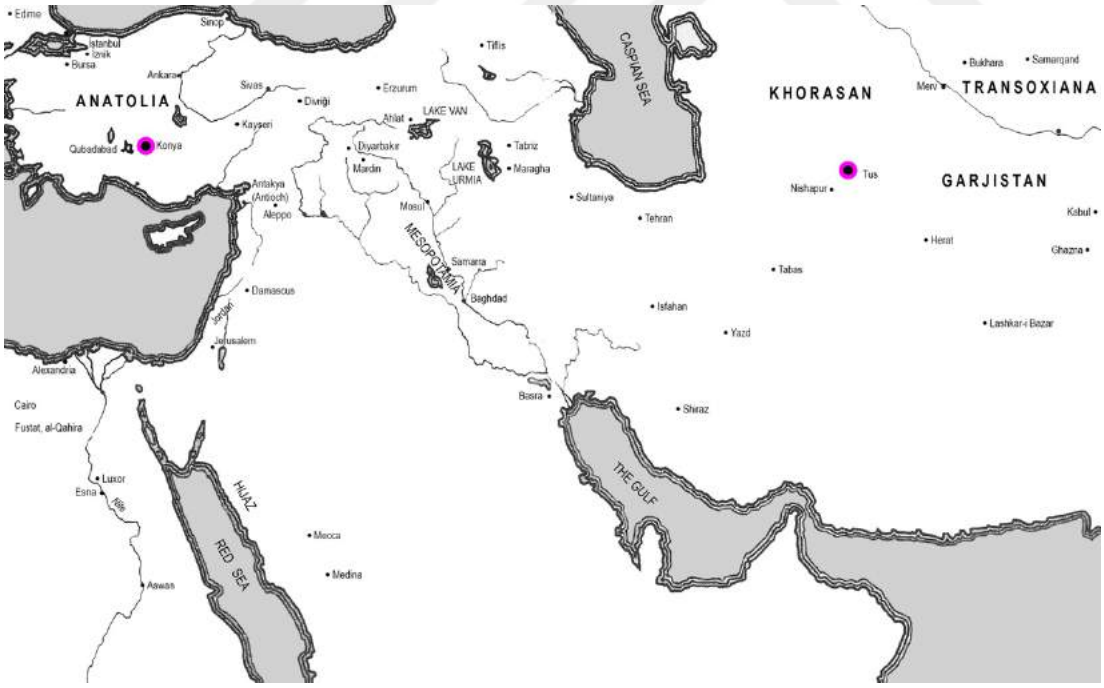


Figure 3.1 : Anatolia and its neighboring cities to the east. Master builder Muhammad al-Tusi was originally from Tus, Khorasan and immigrated to Konya during Anatolian Seljuk period. (map after Hillenbrand (1999, pp. 6–7)).

Konya, a major city in the middle of the Anatolian-Plateau, took architectural and decorative traditions mainly from Iranian art (Meinecke, 1976a, pp. 5–47). The

newfound capital witnessed particularly a great effort in building activities. It became a fundamental city, where the palace for the empire and some of the most significant masterpieces of the Anatolian Seljuks were being built. Anatolian Seljuks were renowned for their tolerance towards other beliefs. They decorated their buildings and castle walls with sculptures inherited from the Byzantium empire (Sarre, 1967, pp. 1–2). In his hermeneutical studies Redford (1993, 2000) interprets such tendencies as the will to forge a bond with former emperors of Anatolia and an expression of monarchy (Peker, 2009, pp. 70 & 80).

Unfortunately, due to the loss of many monuments, it is not possible to see all of the artistic tradition dominated every part of the city at that time. Yet, historical drawings of the city by two French travelers, Charles Texier and Léon de Laborde illustrate partially the ambience (Figure 3.2). Texier and Laborde, both witnessed now extinct remains of some major Seljuk monuments, a greater part of the palace and the city walls among others, during their travels to Konya in early nineteenth century.

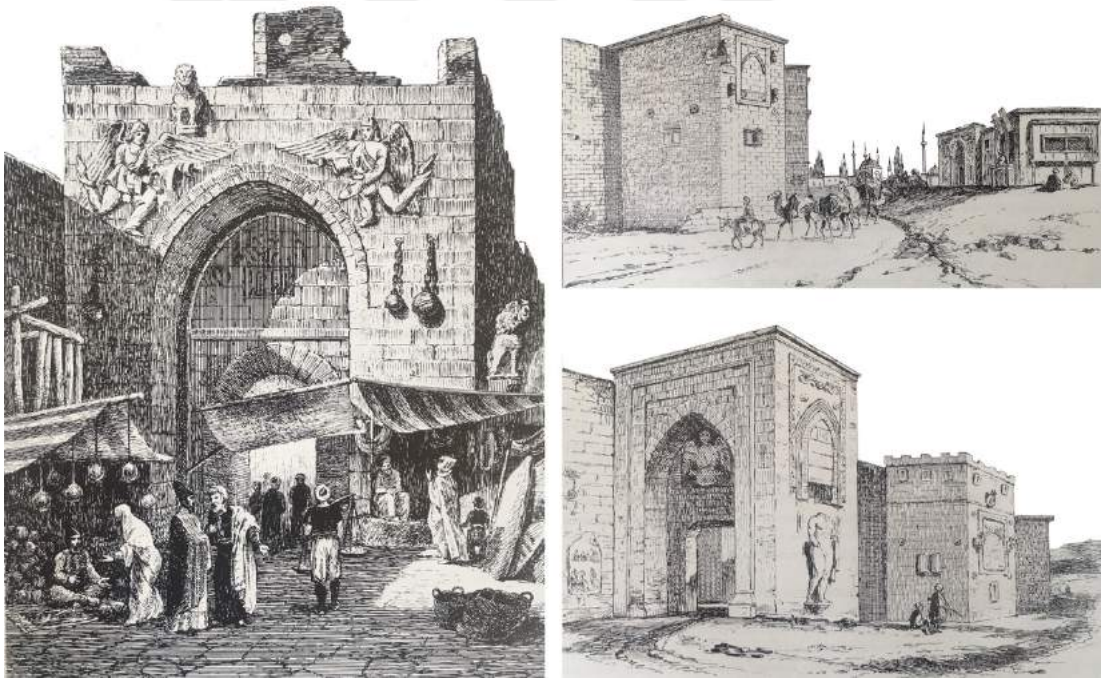


Figure 3.2 : Historical drawings of the once Seljuk capital Konya.
Left: The bazaar gateway (1220-1221 A.D.) by Charles Texier's, 1828.
Right*:* City walls (1219-1236 A.D.) by Léon de Laborde, 1825 (both drawings from Sarre (1967, p. VIII & 2)).

The uniqueness of the Anatolian Seljuk monuments was as well as the result of an exclusive material tradition created predominantly by ceramic artisans with Persian origins. Governing the Persian lands, the predecessors of the Rum Seljuk formerly

became a Persianate society and therefore adopted for the most part Perso – Islamic tradition, established designs, materials and techniques in their monuments (Yalman, 2001). Peker (2009, p. 69) discusses the multifaceted cultural identity and explains how both Persian and Central Asian Turkic traditions influenced the thirteenth century Anatolian art and architecture.

Iran, the main land of the Great Seljuk empire, was one of the biggest cultural hubs in Seljuk history. Researchers interpret the Iranian Seljuk period as the Renaissance epoch (R. Arık and Arık, 2007, p. 32). Pope (1981) portrays the Seljuk Architecture in ancient Iran as "noble and powerful and structurally sophisticated". The progress in Iranian architecture with unique building plans and unusual forms influenced the entire Islamic World. (Altun, 1988, p. 16).

The Seljuk architecture in Iran was built as large brick structures, hence geometric pattern decorations on the monuments were formed by diverse compositions of bricks (Figure 3.3). This tradition is often referred in literature as ornamental or geometric brickwork and the Persian term for this tradition is *banna-i* (Bonner, 2017, p. 579) Occasionally, ceramic artists covered the bricks with an additional layer of glaze and combined these with plain units to create a multi colored geometric decoration. This glaze based material technique is designated by art historians (Çeken, 2007, pp. 16–17; Meinecke, 1976a, pp. 163–164; Öney & Çobanlı, 2007; Ş. Yetkin, 1986, pp. 159–160) as *brick mosaic* (another technique not to be confused with tile mosaic).

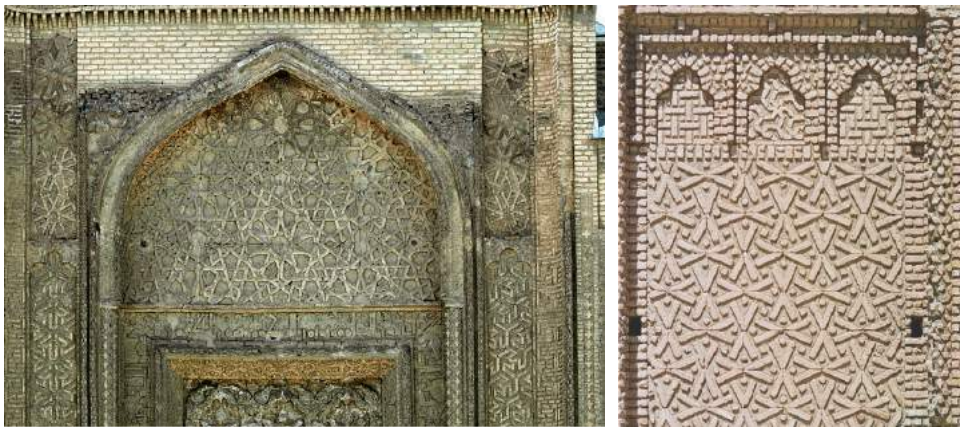


Figure 3.3 : Ornamental brickwork from Left: Gunbad-i 'Alaviyan in Hamadan, Iran, twelfth century A.D. (Photo from Bonner (2017, p. 49) Right: Kharraqhan tower in Iran eleventh century A.D. (Photo from Makovicky (2016b, p. 44)).

On the other hand, Seljuk art was formed by the mixture of many distinctive traditions. Along with the Perso-Islamic tradition, a strong Byzantine and Armenian Christian

heritage was built in Anatolia for years (Yalman, 2001). The Seljuk conquerors adopted stone masonry from former Byzantine and Armenian Christian traditions and it became the characteristic material for architecture. Thus, the craftsmen of pottery and ceramics from Iran confronted a constructional problem: They required a new technique for combining the stone structures and the glaze. Consequently, ceramic artisans conducted a great effort to create new techniques during the twelfth and thirteenth centuries (Meinecke, 1976a, pp. 5–47).

The technical and aesthetical development of the Seljuk architecture, in which ceramics stands as a main decorative architectural element, was the result of conscious labor and experiments [4] (Erdoğan, 1989). The introduction of any new technique brought the difficulty of transferring the existing geometric design pallet to that new technique. At the end of the twelfth century, geometric patterns seen across Azerbaijan and Iran in ornamental brickwork tradition were applied to the cut-stone architecture, a tradition seen for years in Anatolia. However, when the newcomer ceramic artisans tried to inlay ceramic pieces inside the wholes of the cut stone facade, they faced a making problem: The technique was demanding and required a lot of time and effort. Therefore, this practice did not become widespread and is seen only in quite few examples. Subsequently, Seljuk master builders developed a new structural system of an architectural form built upon the combination of brick and cut stone materials. Basic structural elements were made from cut-stone and an additional brickwork wall was attached to the front as the entrance façade. Geometric decorations were mainly applied to this brick wall and bricks with an additional glazed layer caused a parti-colored decoration (Meinecke, 1976a, pp. 5–47). The Şifaiye Madrasa in Sivas, built in 1217-18, is one of the first architectural examples of this structural combination. Figure 3.4 shows one of the entrance portals – the gateway for Tomb of Keykavus, located inside the Şifaiye Madrasa. The geometric ornamentation of the entire portal is attributed to an Iranian Azerbaijani artisan (Meinecke, 1976a, pp. 19–26). Typical to the architecture of Iran Seljuk Dynasty, the adornment is completed with bricks that are arranged to build an interlaced pattern and held together with plaster. Some of these pieces, cut in straight lines due to the technique, are covered with a layer of glaze. Red

[4] Glazed brick, faience tiling, reliefed faience tiling, saray faience, underglaze tiles, lustre technique and the minai technique and tile mosaic are the primary ceramic based techniques (Öney, 1992) .

and blue glazed bricks are arranged to form geometrical patterns and decorate the portico.

Figure 3.4 : Left: Entrance portal of the Keykavus Tomb, in Şifaiye Madrasa, 1217-1218 A.D. (Photo Credits Right: Aslıhan Erkmen). Right: Close-up before the recent restoration (ca. 1960) (Photo taken from the "Aga Khan Visual Library" (n.d.)). The structural element is a combination of cut-stone and brick.

Nevertheless, this new structural system required a demanding construction and similarly, the brick mosaic was only applicable to the exterior surfaces (Erdoğan, 1989). Subsequently, master craftsmen of ceramics required a new mounting technique for the direct application onto stone surfaces of interior spaces. Consequently, migrant Iranian masters eventually created their own and unique style of ***tile mosaic*** *(also called faience mosaic or cut-tile mosaic)* in the beginning of the thirteenth century (Meinecke, 1976a, p. 27). They married this novel material to the dominant stone tradition. Building shells included sculptural stone portal facades carved in deep relief, while the interior was covered with tile mosaic (Figure 3.5 and Figure 3.6). Using tile mosaic method, artisans created a completely different style, which was at that time unique for Central Anatolia (Meinecke, 1976a, 1976b; Schneider, 1980). This is the region which form the focus in the dissertation.

Figure 3.5 : The stone portal from Karatay Madrasa, Konya, Turkey, 1251-1253 A.D. Left: Drawing by Alexandre M. Raymond (in Ertuğ et al., 1991)).

Figure 3.6 : Interior view from the Karatay Madrasa in Konya, Turkey, 1251 - 1253 A.D. Tile-mosaic decoration adorns different surfaces.

3.2 The Crafting Technique

The tile mosaic tradition was unique and helped Seljuks of Anatolia to build their own artistic character (R. Arık & Arık, 2007; Aslanapa, 1972a; Erdoğan, 1989; Öney and Çobanlı, 2007; Otto-Dorn, 1957; Yetkin, 1986). Unfortunately, the material became extinct and the craft is not practiced any longer in Anatolia. Yet, comparable techniques akin to tile-mosaic tells us how the technique was applied historically.

Figure 3.7 : Close ups from the tile-mosaic decoration in Sırçalı Madrasa in Konya, Turkey, 1242-1243 A.D.

Even during the Seljuk reign in Anatolia, tile-mosaic influenced and altered the stylistic tradition in the neighboring countries. After the defeat at the battle of Kösedağ (1243) by the Ilkhanids (the Mongol dynasty ruling in Iran), the Seljuks in Anatolia faced a power loss causing political instability. This slowed arty traditions and artisans from Konya left eventually Anatolian territories. They partly migrated to eastern lands. The descendants of the tile mosaic became as well prominent under the auspices of the Muzaffarids and Kartids. Similar crafts appear in some of the latter monuments from Khorasan and Iran, almost a century later, (Bonner, 2017, p. 69). The Persian term for cut-tile mosaic is *muarraq* [5] (Sarhangi, 2012a) and the tradition in Iran was practiced for many years. In *A Survey of Persian Art,* Pope (1981, pp. 1326–1334) and similarly Erdmann (1963) in *Neue Arbeiten zur Türkischen Keramik* (New Works on Turkish Ceramics) report their observations on the making of similar cut-tile mosaic practices. Compared to the examples in Anatolia, subsequent artworks in other countries entail many more colors. Apart from the advances in the craft tradition, technical properties of the material were developed in a way that eventually artisans could use a variety of colors. Especially Kashan is particularly renowned for similar faience decorations. A historical document dated to A.D. 1301 enlightens technical details and chemical properties of the faience plates from Kashan (Ritter et al., 1935). In this treatise, the medieval author gives precise technical details about the faience. Chemical properties,

[5] The term is written as *muarak* in Bonner (2017, p.580).

the right amount of the mixtures and firing temperatures are important details to get accurate colors. Thus, material production relies on the work of a scientist.

Another similar crafting tradition of tile mosaic is still practiced today in Morocco. This tradition is called *zellij* and is also known as Moorish tiles, zellige, zillij, or Moroccan mosaic. Nevertheless, the tradition is best known for its preceding examples from the Andalusian Islamic period. Especially various geometric art in the famous palace of Alhambra, a fortress complex built in Granada, Spain is decorated with this particular material. (Figure 3.8).

Figure 3.8 : Zellij panels from Alhambra, Granada in Spain (Wade, 2018).

Morocco inherited the zellij technique after the surrender of Granada in 1492. Skills and the knowledge of this old tradition is maintained and still applied in Moorish art (Castera, 1999, p. 85). In his comprehensive work Castera (1999) explains the craft in precise details, which is also helpful to understand the making of the former tile-mosaic material. Figure 3.9 shows the step by step production process of the Moroccan tiles.

In the cut-tile mosaic method, the craftsman kneads first a special mixture of clay, shapes the mud into square plates and left sheets to dry in the sun. These ceramic tiles are later enameled with a layer of glaze, both the clay mixture and the glaze differ in accordance with the desired color. Enameled tiles are put vertically into a special oven. After the baking process, tiles are collected, sorted and sent for cutting. Based on a chosen geometric pattern, all individual shapes inside the design are drawn to initial square tiles with a stencil by using a template (Ş. Yetkin, 1986, pp. 159–165).

Figure 3.9 : Production steps of Moorish tiles. (Photos, “Moroccan-tiles”, 2017).

Later, the master cutter can cut all the mosaic pieces from these monochrome tile plates. The cutting is executed with a small peen hammer (Figure 3.10). Each individual piece is slightly tapered in a beveled style, therefore not completely perpendicular in the side view (Ş. Yetkin, 1986, pp. 159–165). Colored and glazed mosaics are then assembled and joined together according to the predetermined design (Pope, 1981, pp. 1326–1331). The non-vertical section of the pieces ensures a seamless view between the pieces, as fragments interlock well.

Figure 3.10 : Individual zellij pieces are cut from monochrome square tiles (Photos Left: Aga Khan Visual Archive (2017). Right: Castera (1999)).

Figure 3.11 : The cutting results in a collection of individual zellij pieces, which are then filled into baskets or bags (Building the Moroccan Court, 2013).

The most significant property of the craft is that the labor is directly completed in the construction site, not in the atelier (Ş. Yetkin, 1986, p. 163). Previous to the application on to the building surface, tile mosaic pieces are put together to form larger plates. The assemblage is done upside down on a surface. This surface alters in conjunction with the base geometry of the desired architectural form. For a plane surface, mosaic pieces are either assembled on a flat panel or directly on the floor. Nevertheless, if the facade is curved, pieces are put together on a mold that is formed as the negative of the architectural surface. In either case, the craftsman has to recognize the back view of the geometrical design, so that he can accurately arrange pieces and form precise plates (Figures 3.12 and 3.13).

Figure 3.12 : Both The back and front views of the same zellij arrangement (Pictures adapted from Meyer (2010)).

Figure 3.13 : Zellij craftsmen execute reverse assemblage to form larger plates (Photos, left: Castera (1999) and right: Fremson (2011)).

Individual pieces are bound together with a layer of mortar. Previous to the pouring of mortar, calibration is essential for some exclusive reasons: First, the mortar that bind the shapes should not be obvious to the viewer and pieces should be strictly bond together, so that a clean panel is obtainable (Figure 3.14 and 3.15).

Figure 3.14 : Careful reverse calibration of the tesserae to gather an even surface (Kennedy, 2011).

Figure 3.15 : Ones the mortar is poured, a larger cut tile panel is obtained (Campbell, 2011).

> each individual piece was taken with the same amount of love and care, so that the life was brought into this one green hexagon on the same level that the person had it in mind to bring the life to the whole thing and that's really… that's the key of what is going on in any of these things. There is no part of it. That doesn't have that sort of deep infusion of life into it. Because that is a different process completely from the one that's normally used to make a building today and that's where the whole problem arises because it's very difficult to do that under the circumstances where it's not assumed that, that's what the point is. When you recognize that that is the point and then you have to find ways of doing that in our time. Then life gets interesting. (Christopher Alexander in a documentary film by Landy (1990))

These lines belong to Christopher Alexander, a visionary architect and the author of many seminal works, most prominently *A Pattern Language: Towns, Buildings and Construction* (1977) and *Notes on the Synthesis of Form* (1973). In *Places for the Soul: The Architecture of Christopher Alexander*, a documentary film about the architect,

Alexander holds a small fragment of a zellij panel from Alhambra, while he is expressing enthusiastically these words. In the entire documentary, Alexander deliberates architecture as a process founded upon feelings and provides examples in different scales to strengthen his point. The reason for Alexander's eagerness for the zellij panel is clear once the enormous effort behind such a design is understood.

Figure 3.16 : Christopher Alexander, valuing the craftsmanship done for a zellij panel from Alhambra (Screenshots from Landy's (1990) documentary).

Alexander is one of the key figures, whose writings had an undeniable impact on computational design paradigm. In his writings, Alexander valued the role of rules in design and argued that mathematics "can become a very powerful tool indeed if it is used to explore the conceptual order and pattern which a problem presents to its designer" (Alexander, 1973, pp. 6–7).

Together with his co-authors Sarah Ishikawa and Murray Silverstein, Alexander introduced a new attitude to architecture and planning by depicting different patterns as forms of a design technique in *A Pattern Language: Towns, Buildings and Construction* (1977). Their book, as they highlight: "provides the theory and instructions for the use of the language" (1977, p. IX). Their language is formed by patterns that are comprised of rules and guidelines. Patterns described by the authors differ visually from geometrical patterns seen in medieval Islamic art. Nonetheless, they share the same property as they all encompass rules and principles and work as practical and stimulating tools for design.

Alexander's works are essential to decipher the accurate role of rule-based systems in design. In 1967, in another work "*The Question of Computer in Design*", Alexander positioned the role of the computer in design with following words:

> A digital computer is, essentially, the same as a huge army of clerks, equipped with rule books, pencil and paper, all stupid and entirely without initiative, but able to follow exactly millions

> of precisely defined operations… In asking how the computer might be applied to architectural design, we must, therefore, ask ourselves what problems we know of in design that could be solved by such an army of clerks… At the moment, there are very few of such problems. (Cited in Frazer (2011, p. 154)

Alexander's point of view explicitly describes the reasons why an understanding on hands-on craft practices such as the making of the Alhambra panel are important for the digital realm. By-hand making processes entail valuable knowledge about the computation in design and can provide transferrable information for computational design tools and theories. The forthcoming chapters examine various aspects of Tusi atelier's geometric design making with tile-mosaic and illustrates how making and material is an integral part of a computational design process.

4. THE MASTER CRAFTSMAN AND THE MEANS TO THE CRAFT: THE LEGACY OF MUHAMMAD AL-TUSI

"I made this ornamentation which does not exist elsewhere in the world; I will not last but it will last as a souvenier." (Muhammad al-Tusi, 13th century)

Mastering a craft is the outcome of working hard on a material and technique, so hard that one ends up with understanding all the potentials and every single detail that accompanies it. Craftsmanship requires obviously patience, but predominantly the driving force behind an admirable artwork is the irresistible will to make a delicate work. As Sennett (2008) writes "nearly anyone can become a good craftsman" as long as they possess this will and ready to devote their lives to tremendous effort.

In *The Craftsman,* Sennett (2008) reflects upon diverse craftsmanship by looking from a rather unusual perspective, in which he even regards computer programming as a craft practice. Approached from a contemporary viewpoint, Sennett's manifestation implies to computational design practitioners as well. Designers can enhance their computational design practices, namely their craft, by forming a meaningful bond between creativity and computation during their design process. Hence, they have to find ways of formalizing their creative acts inclusive of all parameters that accompany a design act, so they work with computer algorithms in appropriate ways.

The monuments in medieval Islamic world were embellished with great craftsmanship. While exact symmetrical building proportions seen in the architecture necessitated already a good knowledge in geometry, the geometric pattern tradition required even trickier calculations. Thus, the craftsmen, who were responsible for intricate artworks, had to deal a lot with geometry. Apart from the skills they already had to focus on, how could they possibly gain the necessary knowledge in geometry and accomplished tough geometrical problems? The key for such a challenge is obtainable in Özdural (1998, p. 709) who quotes Ibn-i Khaldun:

> In view of its origin, carpentry needs a good deal of geometry of all kinds. It requires either a general or specialized knowledge of proportion and measurement, in order to bring the forms (of things) from potentiality to actuality in the proper manner, and for the knowledge of proportions one must have recourse to the geometrician. (p.709)

Ibn-i Khaldun is celebrated as the greatest Arab historian, who wrote *the Muqaddimah* in the 14th century. The book is a comprehensive history, which encompasses issues in sociology, politics, education, economy and urban life. In the final chapter, Ibn-i Khaldun writes extensively about the crafts, and the sciences. Both are reflected as conditions and consequences of urban life and therefore, indispensable for the understanding of history (Khaldun, 1967). Özdural (1998, p. 709) suggests that Khaldun's quoted words on carpentry reveals how geometricians were involved in craft practices or even participated in the architecture.

We understand from such references that mathematicians aided the craftsmen through their works, yet there was still too much work that the craftsmen had to accomplish. There was a great labor force undertaken by several men and an extensive organization of work was necessary. McClary (2015, pp. 259–272) studies broadly diverse roles involved in the extraction, manufacture and combination of several building materials. McClary lists the processes from mining and preparing the material to production of glazed tiles, and final designs. The processes that require the highest level of skills range from more technical matters such as preparing glaze mixtures or managing firing temperature to more perquisite craftsmanship such as cutting the tiles or painting underglaze designs (McClary, 2015, p. 265).

Precise details of the work and the role divisions are inaccessible due to the lack of historical references, yet we may infer from the craft details examined in the previous chapter, that the work involved was no less then these assumptions. Obviously, as it is still valid in the modern world, a person was in charge for the organization of all the craftsmen involved in the work. This person was the **master builder**, the owner of the craft atelier. In his pursuit for the possible involvement of the famous mathematician Omar Khayyam in the Friday Mosque of Isfahan, Özdural (1998, p. 712) cites the mathematician al-Isfizari from the late 11th – early 12th century and explains the hierarchy in a participatory design process:

> Geometry is the basis for architecture; that is why the geometer with his science constitutes the foundation. He is followed by the **master builder** who in turn is followed by the wage laborer (bricklayer). The geometer commands the second (i.e., master builder) and the master

> builder commands the wage laborer, while the wage laborer busies himself with water and clay. (p.712)

Master builders were central figures who lead the construction projects and therefore were responsible for the whole design. Figure 4.1 presents three examples of tile-mosaic panels. Each artwork is dated to a different time and is assigned to diverse atelier. The first panel (Figure 4.1(a)) is from the Ulu Mosque Mihrab in Akşehir, Turkey, built in 1213. A.D. This artwork is one of the earliest examples of the tile mosaic craft in Anatolia. Only pattern lines are cut from initial glazed tiles and intermediate areas are filled with mortar. Thus, there is a loose appearance between mosaic pieces. This example is important as it gives an idea how the making of tile mosaic evolved through time. The artisan responsible for this earlier work was probably inexperienced in the craft and did not master all details, hence was not aware of material's behavior. There are two different colored tiles (turquise and dark purple), which would allow to compose the whole pattern by only cut-tiles. Rather, the artisan cuts only the lines and the independent star motifs, unlike in the subsequent example, in which all shapes are cut from glazed tiles, hence interlock flawlessly (Figure 4.1 (b)). This work is from the Sırçalı Madrasa (1242-1243 A.D.) and is attributed to the ceramic ateliers of Muhammad al-Tusi [6], henceforth referred as Tusi Atelier. The cut tile examples in these works present an exquisite craftsmanship, the end result of the tremendous effort given to the art of making.

The last example (Figure 4.1 (c)) is from the Mihrab of Tahir ile Zühre Mescid in Konya, 1280 A.D. It is the latest among the three instances. There are small cut-tile pieces on the facade. Yet, most of the decoration is completed with large monochrome tiles. The cut-tile craft required much precision and consequently a great amount of time and budget. Therefore, most of the subsequent craftsmen decided to use larger panels to facilitate the making process, resulting in more effortless designs (Meinecke, 1976a, p. 71). The main intention in the present chapter is to discuss why the works of the Tusi Atelier seems more intricate than other examples and whether such distinctions in the making are formalizable.

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[6] The complete name is cited as *Muhammed B. Muhammed B. U-tman Al-Banna' At-Tusi* in Meinecke (1968) and *Amel-i Muhammed ibn Osman el-Bennâ el-Tusî* in Yetkin(1986, p.185) we use the shorter version Muhammad al-Tusi from Canby et al. (2016).

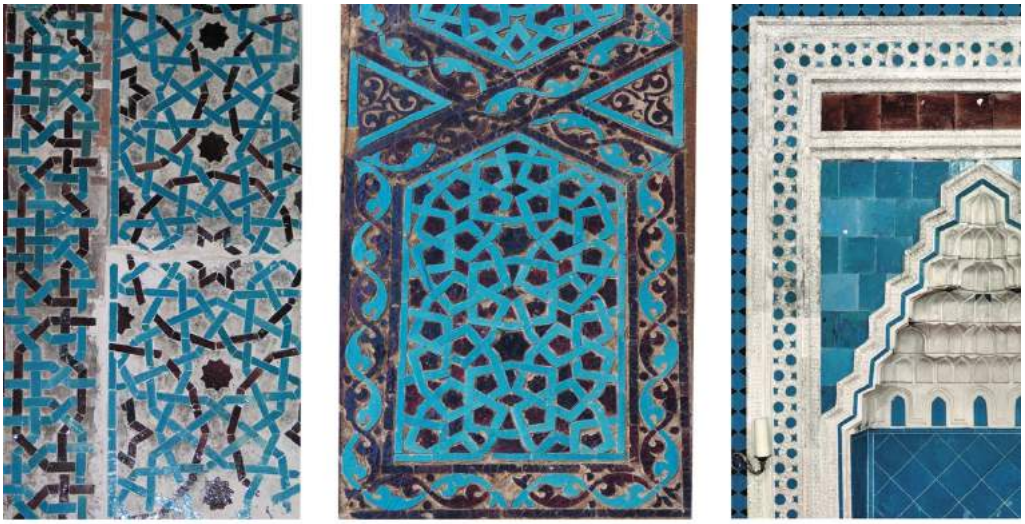


Figure 4.1 : (a) Mihrab from Ulu Mosque in Akşehir (1213 A.D) exhibits one of the first examples of tile mosaic decoration (Aladağ, 2013, p. 126) (b) Sırçalı Madrasa, Konya (1242-43 A.D.) is attributed to Tusi atelier (c) Tahir ile Zühre Mescid, 1280 A.D. ("Heritage of the Great Seljuks," 2018).

4.1 The Ceramic Atelier of Muhammad Al-Tusi (Tusi Atelier)

Muhammad al-Tusi was originally from Tus, a city in Khorasan Province in Iran. Khorasan is a historically important province because of many aspects. First, the Seljuks had started their reign from Khorasan at the very beginning and the empire seemed as much Khorasani as Turkish. Furthermore, many prominent figures in the Seljuk History originated from Khorasan. The famous vizier and bureaucrat Nizam al Mulk, who wrote the Siyasatnama (book of governance) was originally from Khorasan (Canby et al., 2016, pp. 8–10). Similarly, the renowned mathematician al-Buzjani who moved later to Baghdad, was actually born in Khorasan as well (Özdural, 1991, p. 61). Medieval Historian Abu Nasr al-Sarraj (d. 988), the famous philosopher Abū Ḥāmid al-Ghazālī (1058–1111) and the Persian scholar Nāṣir al-Dīn al- Ṭūsī (1201–1274) were amongst the other notable residents of Tus (Esposito, 2003). Above all, together with Persia, Khorasan was famous for its magnificent pieces of ornamental brickwork, which exhibit the earliest examples of many complicated designs (Bonner, 2016, p. 95).

Master craftsman Muhammad al-Tusi did not invent the tile mosaic material technique. Yet, he did enormous contributions to the development of this crafting tradition. Meinecke (1968, pp. 75–80) suggest that he owned a ceramic atelier, whose designers

worked for many artifacts in the region. This atelier is famous for some unique works of the tile mosaic decoration in Central Anatolia, especially in the Konya region.

Muhammd al Tusi's name first appears on the façade of the Sırçalı Madrasa, built in 1242-1243 A.D. (Figure 4.2 and Figure 4.3). In the Anatolian culture, *"Sırça"* denotates faience and because of its intricate tile decoration, the monument is named by the folk *Sırçalı*, meaning overlaid with Sırça.

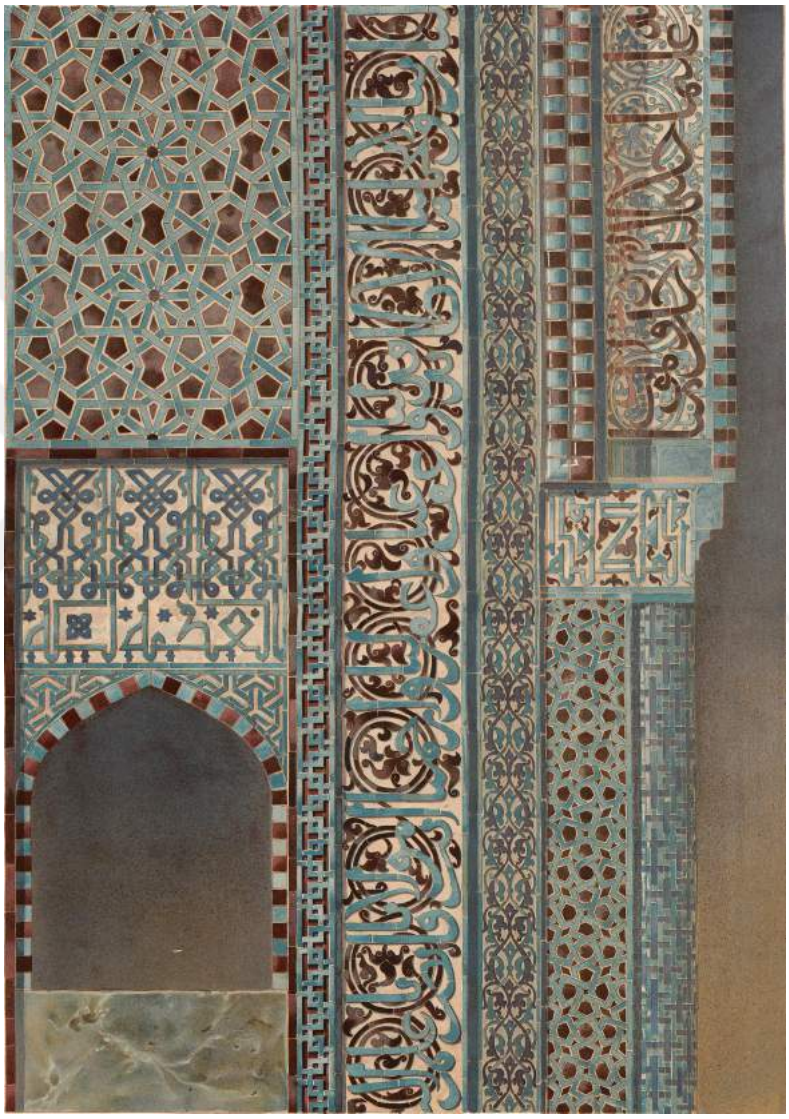


Figure 4.2 : G.Krecker's drawing that partially depicts the Sırçalı Madrasa (in Sarre (1901b)).

Two opposing hexagonal pendants were placed onto the walls of this extraordinary monument: The inscription on the right side indicates "I made this ornamentation which does not exist elsewhere in the world; I will not last but it will last as a souvenir.", while "Work of Muhammad B. Muhammad B.'Uthman, master-builder

from Tus" is written on the left side (Canby et al., 2016, p. 264). Essential is to notice that Muhammad al-Tusi describes himself as the *master builder* of the building. Such an indication is really rare for the time (Meinecke, 1968; Ş. Yetkin, 1986) Due to the master builder notion by the artisan, Meinecke (1968, pp. 75–80) suggests that Tusi was not only responsible for the tile decoration, but had also a voice on the overall architecture of the buildings he was involved in. He was responsible for all of the techniques used for decoration and as well as the construction.

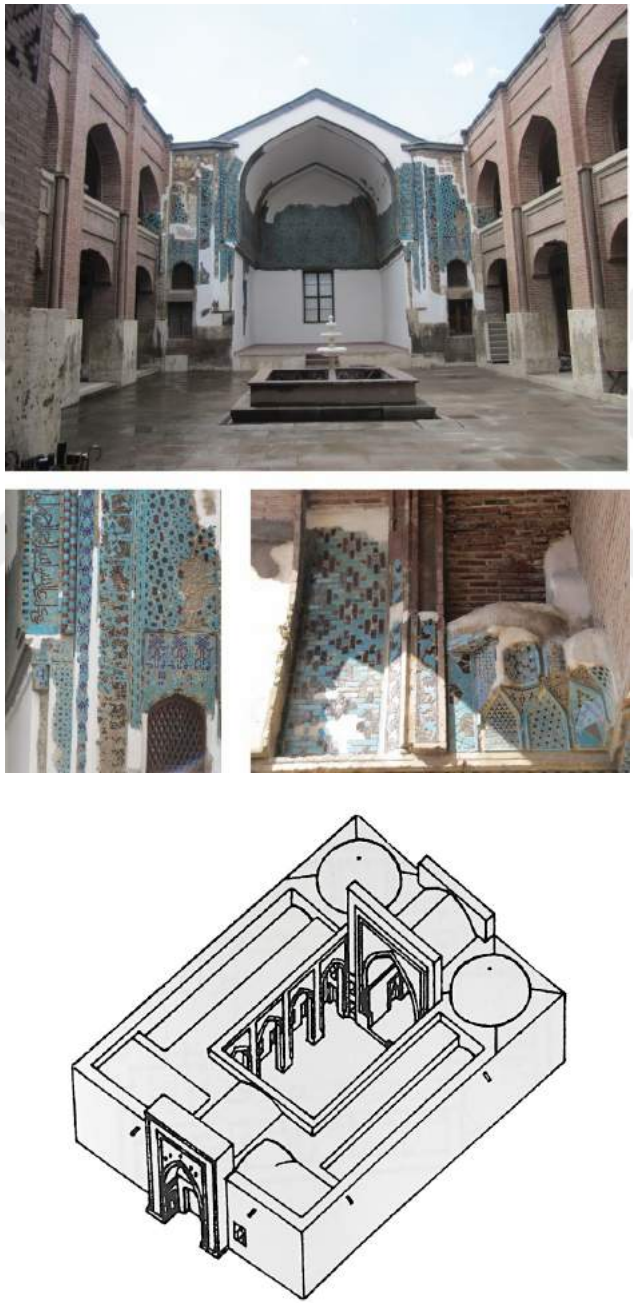


Figure 4.3 : Sırçalı Madrasa and close-up of the tile mosaic decoration attributed to Tusi Atelier. (Photo Credits: Ezgi Baştuğ and Sibel Yasemin Özgan) below Schneider's perspective drawing (in Schneider et al., 2000, p. 119).

Meinecke (1968, pp. 75–80) proposes that 47 sacred buildings dated between 1235-1300 A.D. were made either by him or his atelier. Diverse building decorations are assigned to a Tusi's workshop based on stylistic or technical similarities between the Sırçalı Madrasa and other monuments. Figure 4.4 display two different pattern decorations from the Sırçalı Madrasa and Karatay Madrasa, another monument from the Konya region built around 1251-1253 A.D. Unquestionably, these patterns have same visual characteristics and the close building dates indicate that it was accomplished by Tusi Atelier as well.

Figure 4.4 : Above: Konya, Sırçalı Madrasa, interlaced star pattern on the iwan (Photo Credits: Ezgi Baştuğ). Below: Decoration on the walls of Karatay Madrasa 1251 (painting from (Sarre, 1901b)). Stylistic similarities are apparent and suggest that Karatay Madrasa was a work of Tusi Atelier as well.

Meinecke (1976) attributes the decoration of Karatay Madrasa A.D. 1251-1253, a part of Alaaddin Mosque A.D. 1235 [7] ,both in Konya, and the Mihrab of Mısri Mosque in Afyon, 1250 A.D. to the Tusi Atelier. Therefore, these monuments are particularly important in the conducted studies. Schneider (1980) stresses the similarities in between the Sırçalı Madrasa and Sahib Ata complex (Tomb and Mosque) and suggests that craftsmens from the Tusi atelier may have worked for this building as well. Without getting involved into such a debate, whether specific decorations were made by the Tusi Atelier or another atelier, we analyze some of the decorations from the Sahib Ata complex, in the following chapter. Similarly, other patterns dated even to a much latter time and even another dynasty or much earlier monuments exposing typical Persian tradition are subjects for our examinations.

4.2 Works of Tusi Atelier

4.2.1 Blueprints: Designs on the paper

Chapter two presented diverse generative ways for pattern designs. It is hard to separate all the given design methodology as they are closely related and driven from the same geometrical description of building a regular polygon with a compass and a ruler. Furthermore, there is simply not enough evidence to be sure which one was used on a particular design. Many of the less complicated patterns can be generated from more than one technique.

The circular grid method is the most acknowledged and promoted technique in the literature. Considering the instructions given in Būzjānī's (940) treatise for drawing regular polygons, it is reasonable to think that craftsmen worked on the same methodology for designing their patterns. However, the only tangible evidence for this method is the anonymous Persian treatise of *Interlocking figures* [8], which gives instructions for patterns ranging from simple ones to more complex ones that encompass even conic sections. The illustrated directions for only simple patterns

[7] Alaadin Mosque is a complex, built initially much earlier. The part where Tusi's work stand was done in A.D. 1235.
[8] Original name is *Fi taddkhul al- ashkal al-mutashdbiha aw al-mutawdfiqa* (On interlocking similar and corresponding figures)

resemble the circular grid method. Moreover, there is still debate among scholars, whether this treatise was intended to give instructions to artisans or was just a document only used among curious mathematicians, who were analyzing the underlying geometrical principles seen in their traditional art (Bonner, 2017, p. 204). Through an extensive survey Bonner (2017, p. xiii) writes three main problems about the circular grid technique:

> (1) It does not lend itself to creating original designs, but is primarily useful in recreating existing patterns in a step-by-step fashion
>
> (2) it is impractical for recreating complex patterns with multiple regions of local symmetry
>
> (3) the step-by-step construction sequence of each specific pattern must be individually memorized or kept in documentary form. (Bonner, 2017, p. 204)

Generative polygonal tessellations, on the other hand, was witnessed by Hankin (1905, 1925b, 1925a) and construction lines indicated by red-dotted lines in *the Topkapı Scroll* stand as smooth proofs for its usage. Nevertheless, it should be noticed that Hankin was travelling around 1920's through India and Topkapı Scroll is dated to fifteenth century, still centuries later then the Anatolian Seljuks. The only factor, which perhaps strengthens the idea of the widespread usage of generative polygonal tessellations is that not all of the patterns are drawable with the circular grid method. Except from the really complicated ones, different methods are employable to form the same patterns (Figure 4.5). Thus, we cannot come to a real conclusion on which of the design approach was really utilized for a specific pattern design.

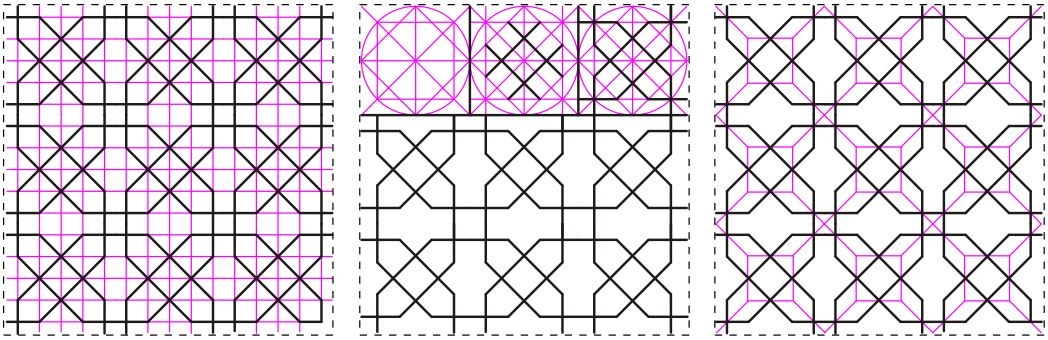


Figure 4.5 : Same pattern constructed with three different methods (a) The Grid Method (b) The Point-joining technique (c) The Polygonal technique (adapted from Bonner (2016, p. 62)).

From a designer's perspective, there are no real differences between the proposed generative methodologies. Artisans, who were trained in the point joining-technique

could easily facilitate their designs by transforming the circular grid to a system of repeating polygons and using those as templates for their upcoming drawings. It is hard to believe that artisans were building the circular grid all over again in each of their creative attempts. Rather, it is reasonable to assume that after a certain point, they used more practical ways to build their own designs. As much as it is open for many creative ideas as discussed by Özkar (2014), the circular grid can become also impractical as soon as many lines emerge on the canvas.

We may refer to the illustrations of Gerd Schneider (1980, 1989) as good examples for creative processes of a designer. In the late 70's, the German architect was working together with Kurt Erdmann, an acknowledged historian specialized in Anatolian Seljuk art and architecture. Schneider documented all the drawings he saw during his trip to various cities in Turkey and produced free-hand drawings of patterns. In these drawings, he does not use pre-defined rule-systematic, rather he looks for similarities between primary shapes inside different patterns and categorizes designs accordingly (Figure 4.6).

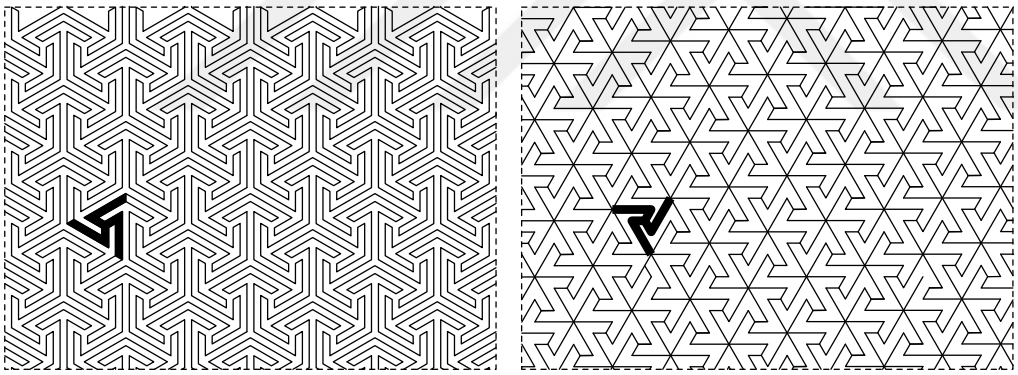


Figure 4.6 : Schneider's (1980) patterns number 51 and 54. Schneider draws attention to similar shapes he observes inside both patterns (Drawings adapted from Schneider (1980)).

A related experimental study by Baştuğ (2015) shows partially how reasoning can alter designers' ways to develop novel geometric designs. In the experiment, Baştuğ starts by showing subjects existing patterns from a Seljuk monument in Anatolia. The subjects are then asked to produce new designs from the shapes they see. The results are executed mostly on the combinations of these shapes and do not match up with the methodology seen in traditional patterns. This experiment is conducted with a limited number of participants and the results cannot be referred to whole populations. Yet, even with a limited research on designers as such, one debates whether medieval

artisans observed similar relationships, repetitions or any hierarchies among the shapes inside their designs or not.

Most reliable sources for pattern designs are unquestionably the buildings themselves and unfortunately, we cannot tell how the design on the paper was handled. On the other hand, some monuments represent physical evidences on how the geometric design was applied to diverse surfaces.

4.2.2 Making

Incorporating patterns to buildings was an integral part of the architectural design in medieval Anatolia. A visual design on a paper is not alone enough to apply a given pattern onto a building surface. Features such as surface geometry, physical properties of a given material or the crafting technique alone require further considerations. The main idea of this section is to discuss whether making requires additional rules during the physical construction of a specific geometric design onto a building surface. How making rules alter the appearance of a given design and how they affect the overall design decisions are other topics for examination.

Prior to an evaluation on Tusi ateliers works, we require a deeper understanding on the conceivable consequences of possible design decisions.

4.2.2.1 The distinction between visual rules and making rules

The final appearance of a certain pattern differs because of diverse parameters. Once a geometric design draft is applied onto a surface, it becomes a tangible thing. Consequently, quests about a material are of the primary importance and they can alter the generative methodology of a design. Thus, the application differs based on the crafting techniques (Figure 4.7).

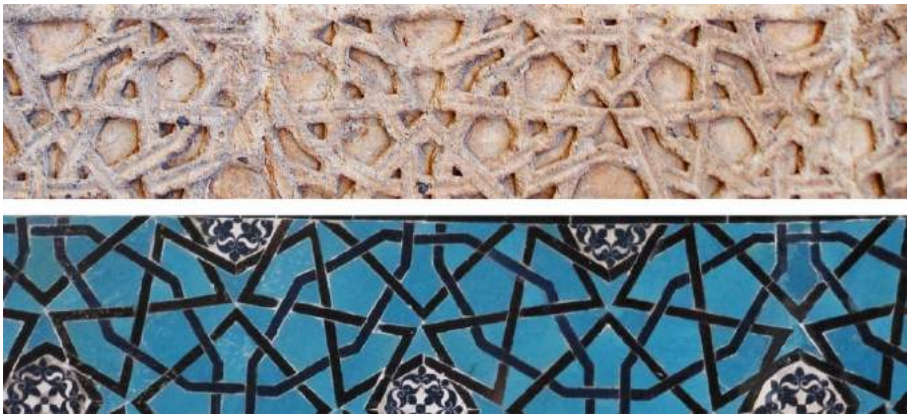


Figure 4.7 : The same pattern design (arranged here symmetrically placed along a horizontal axes) applied to above stone and below tile-mosaic, both from Sahib Ata Kulliye in Konya, Turkey, (1258-1283 A.D.).

Some of the most acknowledged crafting techniques from the Anatolian Seljuks are based both on carving: stone masonry and wood-works. In these techniques, pattern outlines are first drawn on the material surface, then remaining areas between pattern lines are carved. Thus, a multi-layered texture on the surface appears: The pattern lines stand slightly higher than the other parts of the design and the material is chiseled at varying depths. In modern day, traditions continue and even if few, we can still observe craftsman at work in various cities in Anatolia, carving the stone when soft (before being exposed to air for a time period) with a hand tool (Kendir-Beraha, 2014). Similarly, Moroccan craftsmen still embellish buildings, especially traditional building courtyards with gypsum plasters, on which they carve ornamental designs (P. Webb, 2015).

The final visual appearances of the patterns in such crafts depend highly on the carving tool handling. The carving requires a great deal of skills and patience and is executed with several types of metallic tools. Facts just as how deep or how much the areas are carved effects the overall result. As an example, in their research, Hamzaoğlu and Özkar (2016b) show how one uniform visual design is materialized differently in conjunction with the tool usage on separate surfaces. They demonstrate three different geometric designs, two applied onto the stone surfaces of the thirteenth century Seljuk Hospital in Amasya, Turkey and one to a brick monument in Qazvin, Iran . These patterns share the same visual schema, yet their making, in terms of how much the stone is carved or the quantity of the mortar between the bricks, alter the final appearances of the designs (Hamzaoğlu & Özkar, 2016b).

Craftsmen carve the geometric design directly onto the surface. Consequently, the first step is to draw pattern outlines or in some cases axes of symmetry and measuring points onto the surface (Figure 4.8). Later, they carve the surface by eliminating these guides. In her research, Kendir (2014) converses with stonemasons, who still maintain the craft tradition, these masons usually work with 1:1 scale cardboards while drawing the design onto a surface. In some rare medieval examples, it is still possible to observe some of the pattern outlines on the stone surfaces. McClary (2015, p. 163) documents curvy lines on the stone from the Citadel Mosque in Divriği, while Bakırer (1999, p. 43) monitors a graffiti incised on the stone block of the Divriği Mosque, both monuments are from Sivas, Anatolia.

Figure 4.8 : Prior to carving, the pattern is drawn onto the gypsum surface by using a template (Photos from Fremson (2011)).

Yet, if the pattern decoration is not directly applied on to the surface as in the carving technique, additional rules for making become necessary. This is the case as in the ornamental brickwork technique. Already mentioned before, prior to immigrating to the Anatolian territories and mastering the fresh tile mosaic material, Persian or Khorasani ceramic masters like Muhammad al-Tusi were trained in the banna-i technique (ornamental brickwork). In this typical Persian technique, interlocked bricks are applied to the building surface as a sort of decorative cladding and some of these bricks are covered with a layer of glaze. Hence, like in any case of a cladding, a quest for surface handling arises. Subsequently, in order to manage the cladding in a more practical way, a method for the regular division of the surface is necessary.

Remaining architectural examples provide visual clues for how medieval master builders applicated the decoration program onto building surfaces. Such clues became non-observable after restoration works, however some old literature fortunately guides us through the making methodologies in medieval times. The work of Jacobstahl (1899) demonstrates that the geometrical brickwork was handled through the regular division of the surfaces. He illustrated some of his observations during his visits to

Azerbaijan. In banna-i, individual bricks are assembled in separate rectangular or triangular wooden boxes. Thus, at the beginning, the visual design is divided into recurring polygons, which also inform the shapes of these boxes (Figure 4.9). Individual bricks are arranged on top of a two-dimensional pattern draft and form the geometrical design when they get into connection. Therefore, the thickness of the lines inside the pattern is equal to the width of the bricks. Intermediate areas are filled with mortar, which bond the bricks together (Jacobsthal, 1899, p. 25).

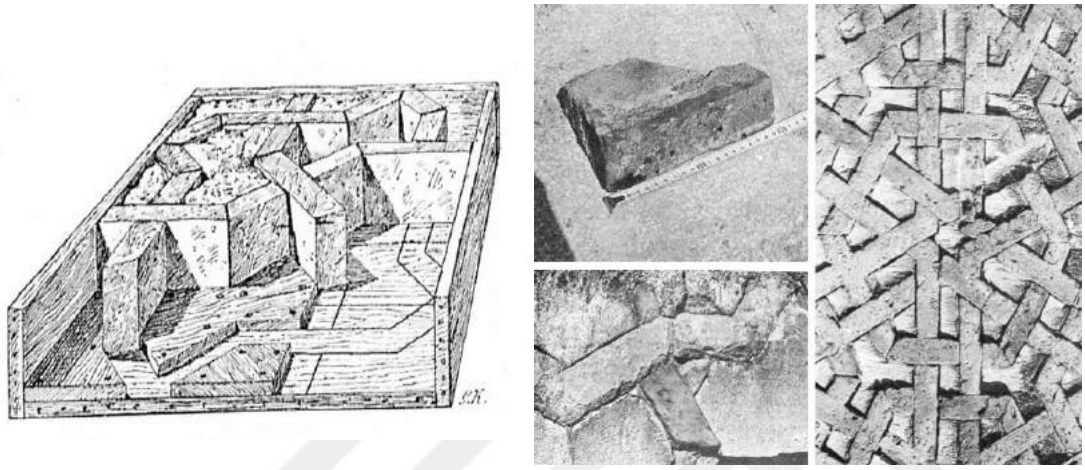
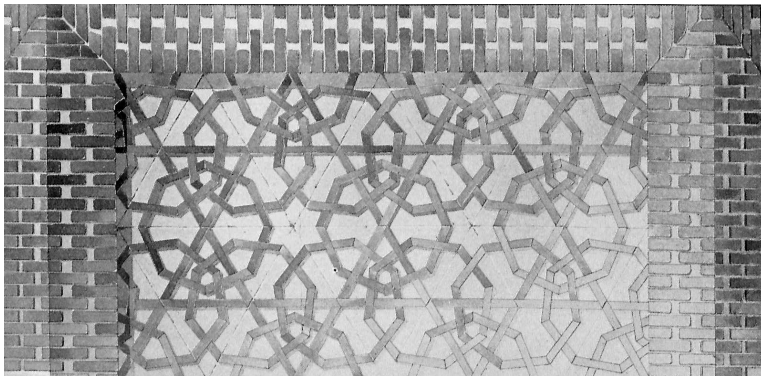


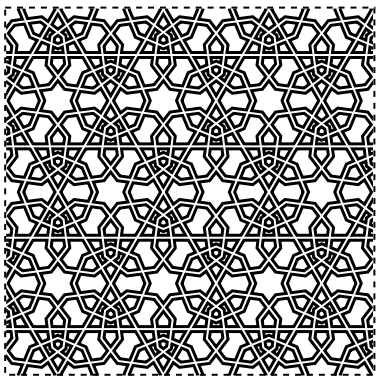
Figure 4.9 : Left: Jaconstahl's (1899, p. 25) illustration of the rectangular brick box arrangement in Ibn Kutaijir, Aserbaijan. Right: Close-ups from brick decoration (Photo credits Ömür Bakırer).

The entire making of the surface decoration is based on multiple copies of the repeating unit and its shape differs in accordance with the generative polygonal tessellation behind the pattern design. Some of Jacobstahl's (1899) drawings presented in Figure 4.10 show two different patterns from the Ibn Kutaijir Mausoleum in Azerbaijan. In these examples that appear on the same monument, panels are separated into triangular (Figure 4.10, panel 1) and rectangular (Figure 4.10, panel 2) repeating units.

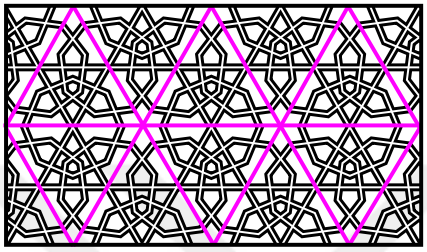
Jacobstahl (1899) suggests that the gaps between individual polygonal plates were originally hidden right after they were assembled on the surface, but divisions became sharper over time and therefore became observable. Such partition borders are partly visible around Anatolia as well. Likewise, the ones disappeared after restorations are readable in elder photos such as the one from the Gök Madrasa Mosque in Amasya, (1266-1267 A.D), found in Öney (1992). The partitions of the design are illustrated together with the picture in Figure 4.11.



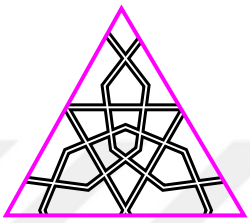
Jacobstahl's (1899) drawing



continuous pattern

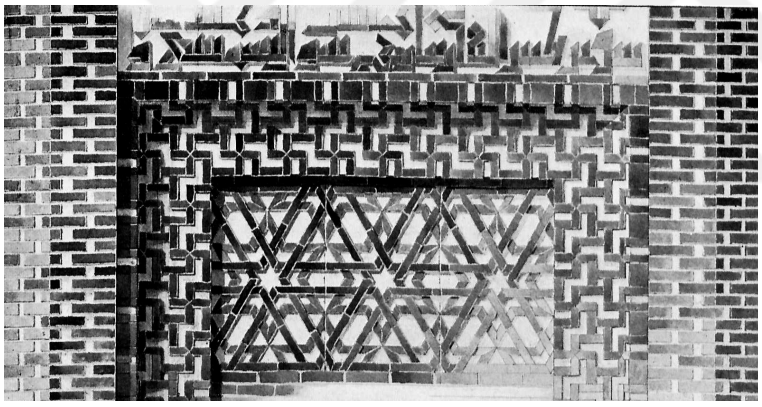


panel division

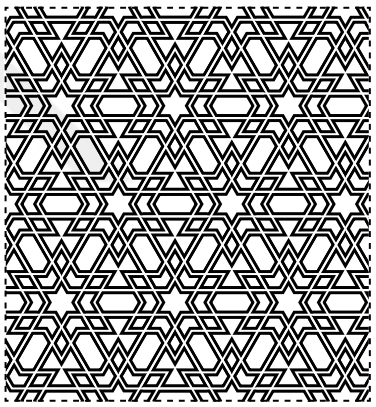


triangular unit

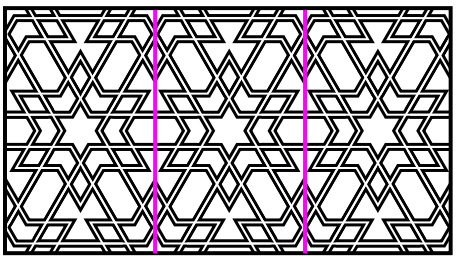
PANEL 1



Jacobstahl's (1899) drawing



continuous pattern

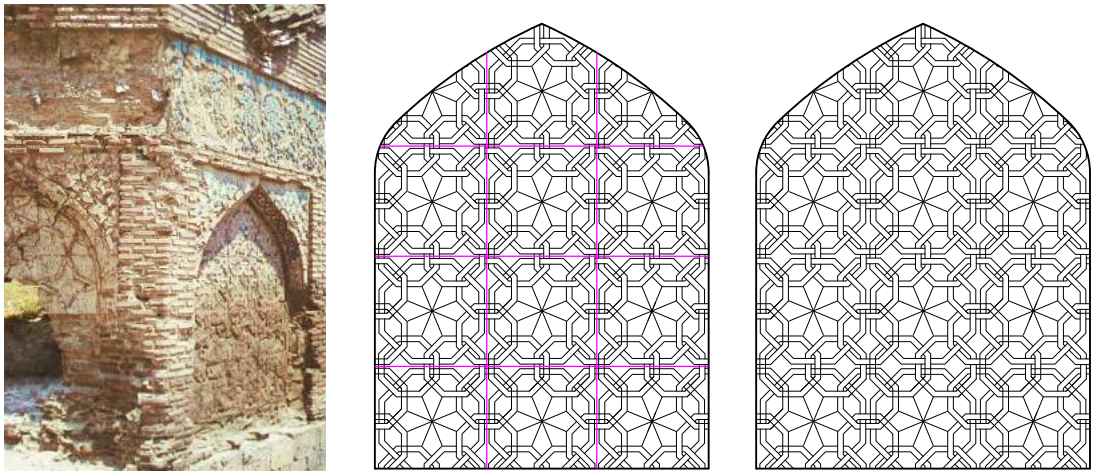


panel division

rectangular unit

PANEL 2

Figure 4.10 : The making of brickwork is based on the systematical division of the surface. Above: Panel 1 is divided into triangular brickwork. Below: Panel 2 is divided into rectangular brickwork. Both designs are from the Ibn Kutaijir Mausoleum in Azerbaijan, 1161 A.D.



(a) (b) (c)

Figure 4.11 : (a) Picture from Gök Madrasa Mosque, Amasya, Turkey, 1266-1267 A.D. (Photo from Öney (Öney, 1992)). (b) Clear square divisions are visible on the surface. (c) Pattern without divisions.

The methodological division of the surface facilitates the process, yet it requires another type of calculation at the same time: the proper division of the pattern. The proper division here refers to a systematic, which is not necessarily the same as in the generative polygonal tessellation method (explained in section 2.1.2.2). In the polygonal technique, the primary polygon is sometimes a hexagon, but in the sense of the application, a hexagon is difficult to assemble. Thus, the craftsmen divide the hexagon to smaller units, like a triangle or alternatively splits the whole design to a rectangular repeating unit. In the case presented in Figure 4.12, both drawings are based on six-fold symmetry, yet the systematical divisions are not handled with hexagonal panels. Figure 4.12 illustrates the second panel in Figure 4.10. Only, this time the polygonal technique behind the pattern design, together with the primary hexagonal modules that are used for the generative drawings on a paper is compared to the panel division based on the rectangular unit. Hexagonal panels might have been used for making as well, nevertheless there is no evidence for such an approach in the literature.

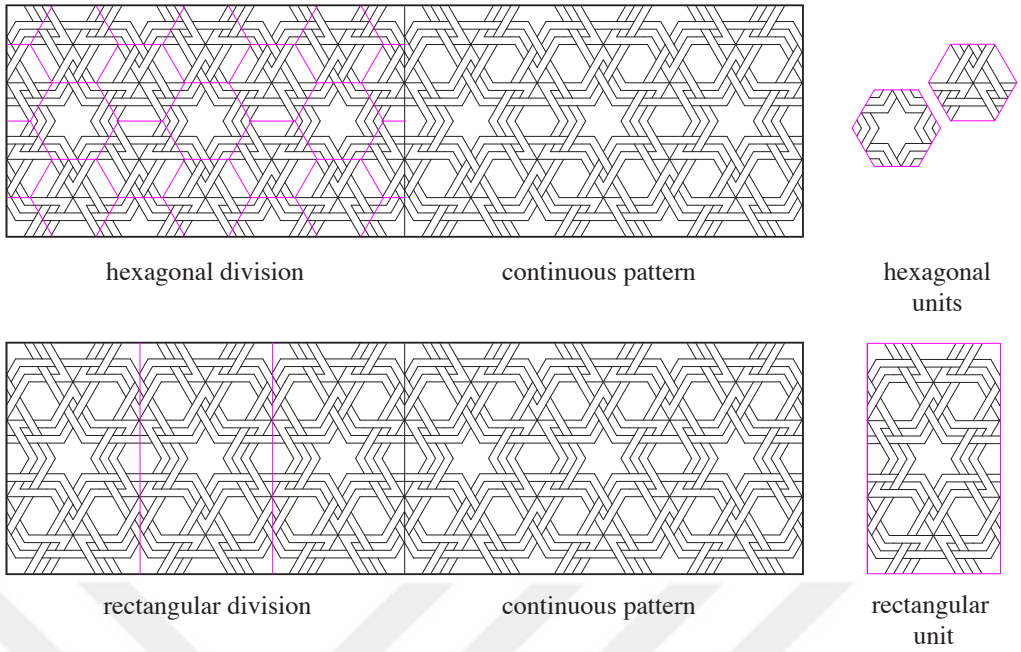


Figure 4.12 : One of the panels from Ibn Kutaijir Mausoleum in Azerbaijan, 1161 A.D. Above: The visual design on the paper is based on hexagonal tiling (After Bonner(2017)). Below: The actual making is based on rectangular units.

Similarly, it is even more challenging to materialize circular boxes. Consequently, a pattern designed on a circular grid at the very beginning, like in the polygonal technique, requires a further thought during the realization phase. The illustrations in Figure 4.13 show how visual rules differ from the making rules. In this example from Gök Madrasa in Amasya, the pattern is depicted on a circular grid system as proposed by Bakırer (1981). However, visible joints on the brickwork show that the pattern was handled by using triangular units. While we can't be sure about the underlying generative design process on the paper, the making process, based on the brick assemblage with repeating triangular wooden boxes is evident.

Now that the whole visual design is split to one main element – a triangular bounding box (Figure 4.13 c), it is possible to get a same scale seamless pattern on different surfaces. The making becomes pretty straightforward for similar buildings. The brick workers have just to take one triangular template and can then decorate other surfaces with the same visual design. It is conceivable that over time, craftsmen operated with such a bottom-up approach, rather than dividing the same design over and over again.

Such observations are also analogue to the descriptions from *The Muqaddimah*, written in 1377 by Ibn-i Khaldun who states:

"... (The material) may be divided either into identically shaped or differently shaped pieces. These pieces are arranged in whatever symmetrical figures and arrangements are being utilized by the (various artisans) and set into the quicklime (with which the walls have been covered). Thus, the walls come to look like colorful flower beds". (Khaldun, 1967)

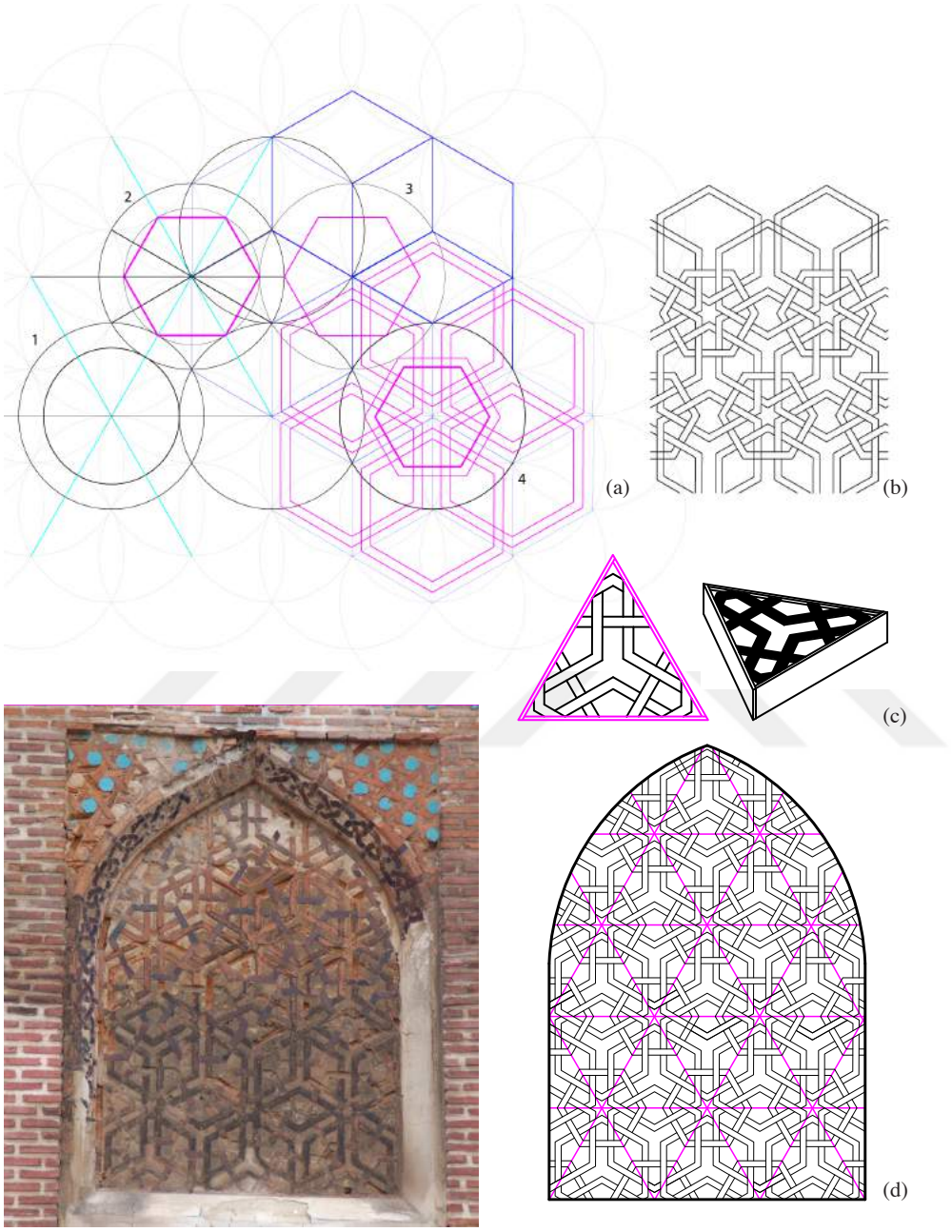


Figure 4.13 : a. (a) Pattern produced from a circular grid, from Gök Madrasa Mosque in Amasya, 1266-1267 A.D. (Drawing credits: Ezgi Baştuğ for the TÜBİTAK project 114K283) (b) Seamless pattern (c) Triangular brickwork assemblage units (d) Visible brickwork arrangement on the façade.

This type of making based constructional approaches were perhaps developed over time. Some of the illustrations in the Topkapı Scroll provide evidence for such a hypothesis. This scroll from a Turkmenistan- Iran origin, demonstrates geometric

design principles, which are not suitable for all types of crafts. Necipoğlu (1995) highlights that some of the designs are not applicable on stone. It is also noticeable that even the illustration on the geometrical brickwork in the scroll do not match the systematic of preceding examples from the Seljuk period. Illustrations and likewise newer brick buildings expose that over time, individual brick sizes became standardized and the whole geometric design was fixed to match a uniform orthogonal grid system (Figure 4.14).

Figure 4.14 : Pattern number 51 from the Topkapı Scroll, a Naskhi inscription for Banna-i technique. Bricks fit to standard dimensions and scaling and the design matches a square grid (Necipoğlu, 1995, pp. 273 and 314).

This type of inquiries, which cause the visual design to be evaluated once again, require further comprehension of the material and the craft in demand. Hence, we may infer that all craftsmen working in a tile mosaic atelier probably struggled at the beginning because of a principal issue: They were trained in the brickwork (banna-i) technique and had to apply the geometric tradition, they were used to build with brick mosaics, with tile mosaic. Thus, they required a proper understanding for the making of the new tile mosaic material.

4.2.3 The principles of the making of the tile mosaic

The first noticeable difference of tile mosaic from other crafts is that every individual shape in the geometric arrangement demands an explicit recognition by the tile mosaic artisan. In banna-i, interlocked bricks are following pattern outlines, the amidst shapes are filled with mortar, which is adjustable and requires not a perfect calculation of the filling quantity. Similarly, in the stone carving, the pattern design is shaped by the controlled removal of the stone and the heights of the intermediate shapes are adjustable.

The approach of tile mosaic craftsmen on the other hand, is not the same. As it is previously discussed, the tile mosaic master has to cut all shapes inside the pattern from initial enameled tiles very precisely, to prevent any blank spaces when all pieces are put together. The difference from carving or brick interlocking technique is that each shape inside the overall pattern design becomes unvaryingly important. After all the cutting is done, several pieces appear and they need to come together in a correct way to build the pattern (Figure 4.15). Because of the nature of the puzzle-like tile mosaic material and technique, not only overall pattern outlines, but also all the shape relations inside the pattern become notable and the craftsman has to recognize and evaluate all of these shape relations between single pieces.

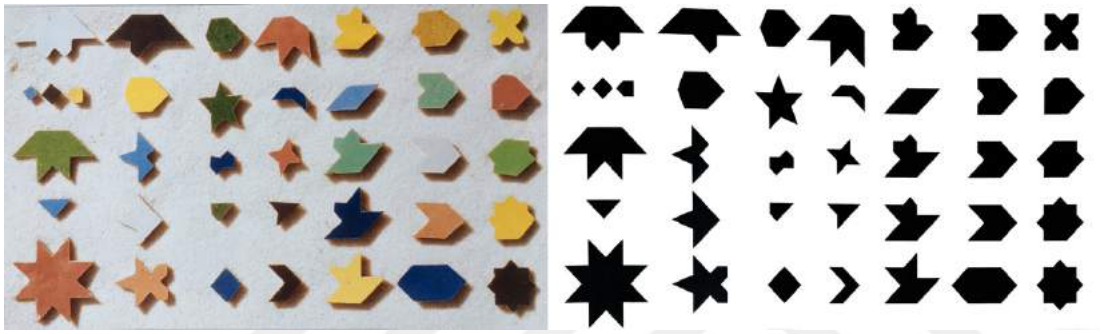


Figure 4.15 : Individual pieces from a rather less complicated zellij design (Pictures from Castera (2016, pp. 114–115)).

The aforementioned brickwork examples have already shown that quests on making alter how the pattern is planned at the very beginning. It is plausible to think that working with shapes resulted in new design approaches for geometric pattern making. Figure 4.16 illustrates two different Turkish triangles [9] from the Alaaddin Mosque in Konya. The design is attributed to the Tusi atelier and valued as the first surviving design of Muhammad al-Tusi by Meinecke (1976b, pp. 212–223). The general view of the dome and Turkish triangles are seen in Figure 4.17. As much as visual rules for the pattern designs are not identical, shapes that form the pattern resemble. It is assumable that artisans used the same cut mosaic pieces for the second panel once they completed the initial panel.

[9] Turkish triangle is the rationalization of a sort of pendentive which structures the passage from the square plan to dome

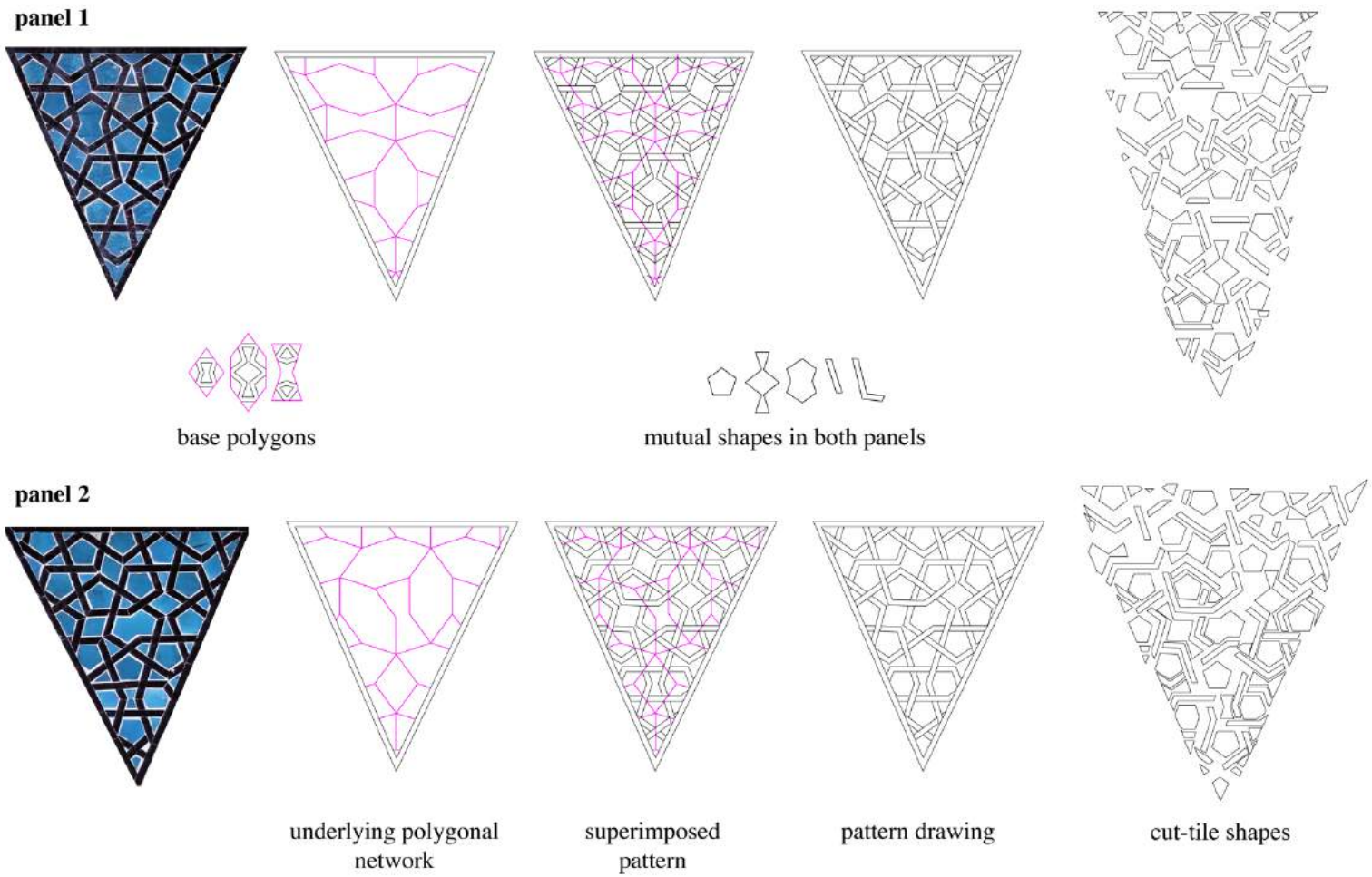


Figure 4.16 : Two Turkish triangles from the Alaaddin Mosque, Konya, 1235 A.D. (Patterns generated with girih tiles that appear in Lu and Steinhardt (2007)).

Figure 4.17 : Turkish triangle panels from Alaaddin Mosque, Konya, attributed to Tusi Atelier, 1235 A.D.

As a matter of fact, more contemporary Moroccan zellij designs are based on the utilization of sets of characteristic shapes. Mallems, designers of the zellij patterns, create their designs on a square grid. They do not draw the entire design on a paper, rather they measure rational relations between shapes. They sketch individual forms on a square graph paper to calculate the right measurement of individual shapes before they start cutting square tiles (Sutton, 2011, p. 34). This method has apparently evolved from experience (Abas and Salman, 2007, pp. 24–26). Such an approach is similar to brickwork patterns depicted on a square grid in the Topkapı Scroll (such as the one in Figure 4.14), as rather than the entire design, dimensions of individual units are calculated in both examples.

Similarly, Jean Marc Castera (1999), an artist with a background in mathematics, developed a design approach consistent with the physical construction of zellij. In his comprehensive book on the decorative art in Morocco, he introduces this new technique. In his approach, there are basic shapes which are put on a structural skeleton. Graphical transformations, as he suggests, allow the emergence of a large set of designs with increasing complexity (Castera, 1999). There are also other approaches

that are based on the nature of the modularity seen in similar cut tile-based material techniques. Sarhangi (2005) proposes such a technique for Persian mosaic decorations. In Sarhangi's method, the designer uses a set of modules for exploring new patterns.

Despite the fact that there is no historical evidence for such shape-based techniques, it is conceivable that medieval artisans created novel compositions by utilizing part relations of individual shapes inside geometric patterns. Nevertheless, even if they did not notice such potentials, the nature of the tile-mosaic material entails a recognition of all pieces cut for the design.

An exquisite craftmanship is not conceivable without an extensive comprehension on material and the necessary skills. Obviously, such attributes are developed through experience. For instance, if the shape relations in tile-mosaic designs are not handled skillful enough, the craftsmanship fails and would not seem as decent as in Tusi's works. Most of the unsuccessful restoration projects are caused by the lack of an extensive work on pattern designs. As an example, Figure 4.18 shows details of a mihrab from the Mısri Mosque in Afyon, A.D. 1250. The Seljuk Mihrab was moved and replaced later into the Ottoman Mısri mosque (Meinecke, 1976b, p. 5). The new placement of the mihrab did not match the original height of the Mihrab. Consequently, the original mihrab was shrink at some point to fit its new location. However, this process was not done according to a proper plan. Consequently, pieces are not located in their original places. A closer look shows the failures on the pattern arrangement, which is caused by the incorrect gathering of mosaic pieces. In the restructured design of the mihrab, the eye is distracted from the irregularities of single mosaic pieces. In the close-up from Figure 4.18 (Picture on the right side), both panels are originally embellished with the same geometric pattern, yet tile-mosaic pieces are misplaced and both panels are re-arranged inaccurately.

In his extensive survey, O'Kane (2012) compares several works on stone dome surfaces and discusses a similar situation for stone carving craft. He draws attention to discontinuities among carved pattern lines. Apparently, if the master mason chooses carving a pattern on a finished building facade, the decoration appears more sophisticated. Conversely, if he engraves on individual building blocks rather than the complete surface, broken lines are more likely to emerge.

Figure 4.18 : Left: Modified design of a Seljuk mihrab (originally dated 1250 A.D.) placed in the Ottoman Mısri Mosque, Afyon, Turkey. Right: Mistakes and inaccuracies are visible.

The intricacy level of a design is up to the artisan and this is valid for every kind of craftsmanship. The craftsman has to master all details and plan everything in advance, for a more refined visual appearance. Disproportions become non-avoidable once the designer does not evaluate the material sufficiently well.

4.2.4 Key concepts of making

4.2.4.1 Interlaced lines

One of the important key concepts in medieval geometric design is *interlaced lines*, sometimes called interweaved or interweaving lines as well. Lines that form pattern designs are usually embellished with interlaces to further the sophisticated appearance. Pattern lines continue through above and below configurations in regular alteration. These zigzagging pattern lines are going up and down, thus create a visual effect as if the pattern is weaved on a textile surface. This tradition is not exclusive for the tile mosaic and was practiced with stone and brick as well. Yet, it is essential to notice that interlaced lines not only cause a stylistic alteration in the outlook. The “one over-one under” sequence change the overall symmetry rules as broadly investigated by

Grünbaum et al.(1986) and Makovicky (2016b), and consequently transform the making of tile-mosaic plates. Individual tile mosaic pieces that form the lines are cut in a specific way, so that alternating shapes can form the interlaced appearance. As an example, there are three different intermediate shapes, while two unalike shapes form the interlaced line effect in the geometric design illustrated in Figure 4.19.

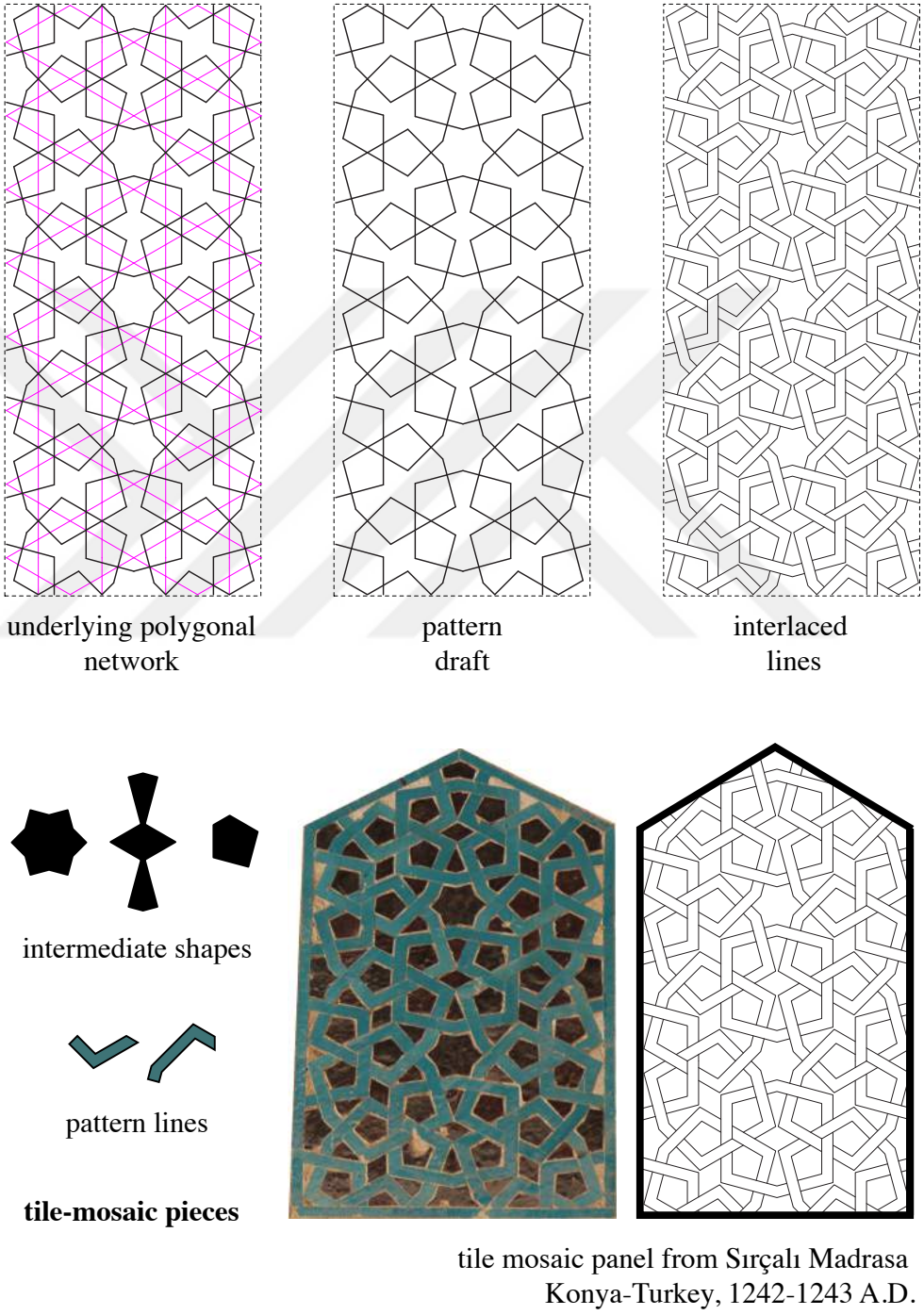


Figure 4.19 : Tile mosaic panel from Sırçalı madrasa. Notice how the appearance changes when the lines embellish with interlaces.

4.2.4.2 Uninterrupted patterns

A pattern, built upon the recurrence of only two or three shapes is easier to make. Muqarnas surfaces of mihrabs are usually covered with such designs (Figure 4.20). Figure 4.21 demonstrates the making approach for a design on the Mısri Mosque mihrab from Afyon, Turkey, 1250 A.D. Primary shapes, a pentagon and a bowtie, are cut from initial square tiles that are glazed with either turquoise or dark purple color. Later, these individual shapes are organized in a regular way to form the geometric design.

Figure 4.20 : Left: Close-up from the muqarnas in the Eşrefoğlu Mosque, Beyşehir, A.D. 1297-1300. Right- above*:* Muqarnas surfaces in Sırçalı Madrasa, Konya, A.D. 1242-43, (Photo Credit Ezgi Baştuğ). Right, below: Muqarnas surfaces from Mısri Mosque, Afyon, A.D. 1250.

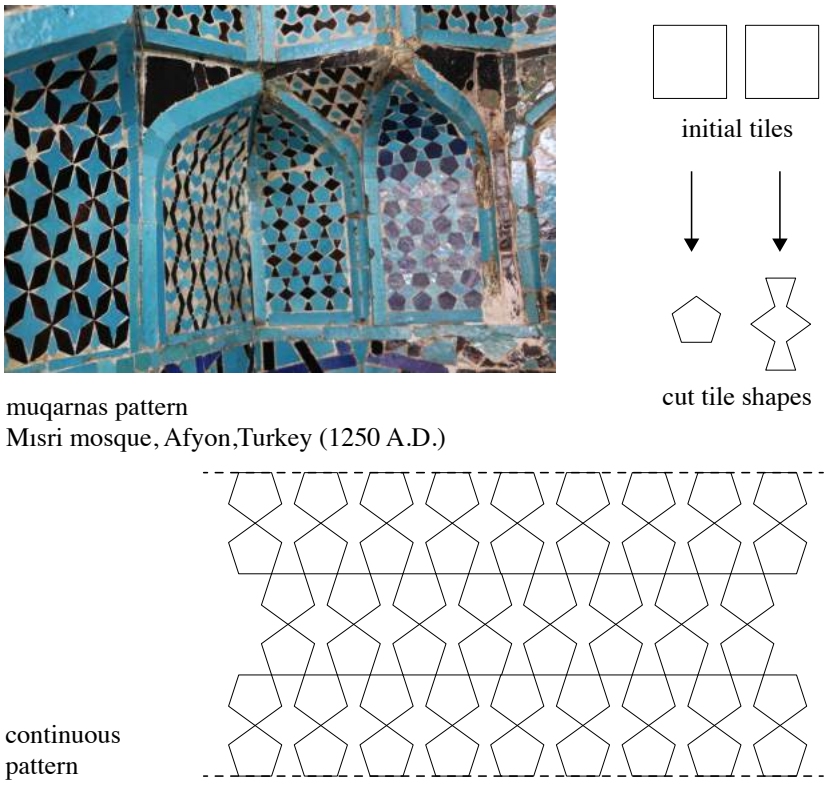


Figure 4.21 : The pattern is formed by the iteration of two alternate shapes.

Nevertheless, building a more complex pattern arrangement is more challenging. The same as in the above referred muqarnas example, the easier way to build such an ornament is to divide the pattern into repeating individual shapes, so that only few different pieces appear after the cutting process (Figure 4.22), Yet, designs are constructed without interlaced lines in such examples, only intermediate shapes form the designs.

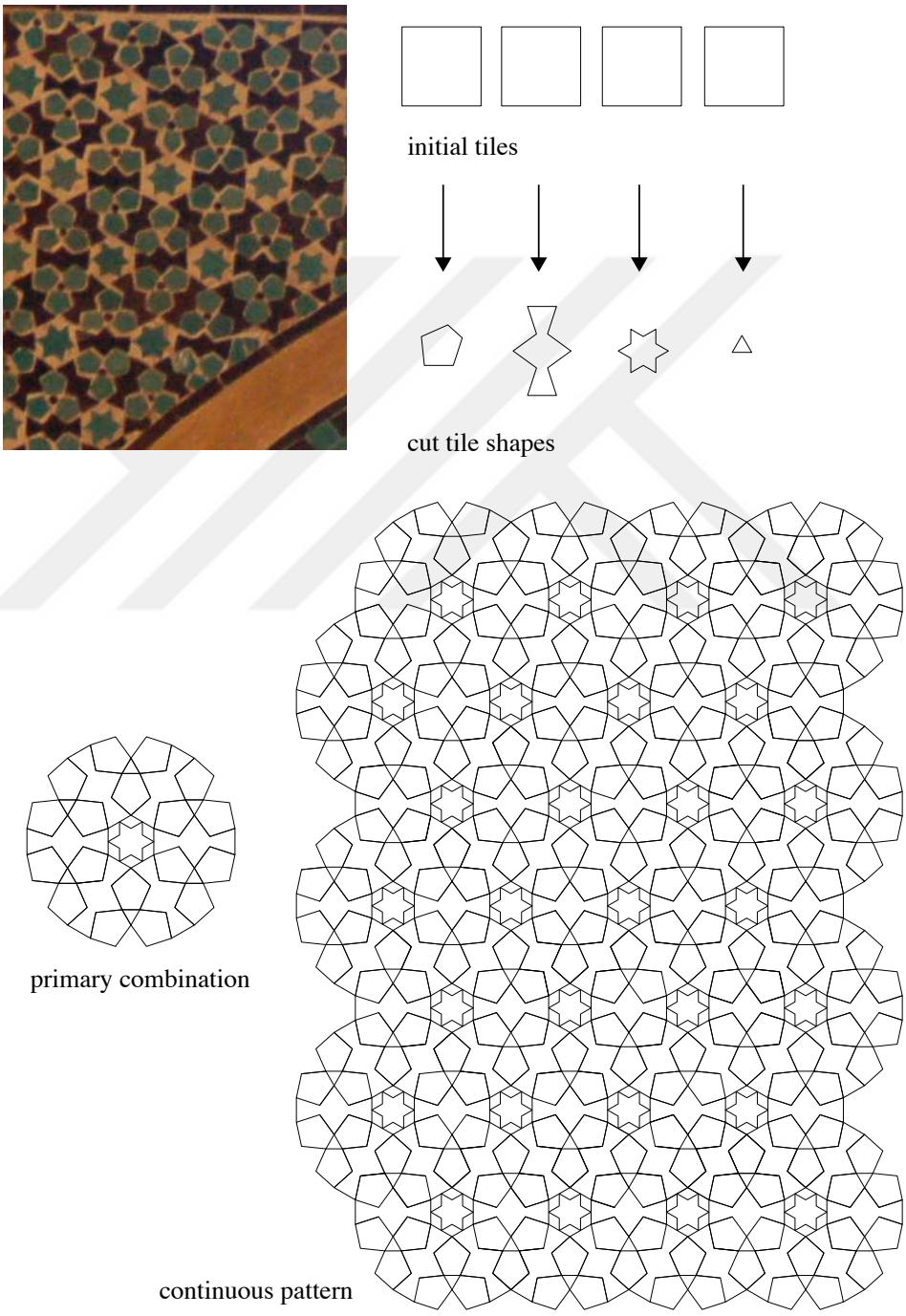


Figure 4.22 : Tile mosaic panel from Sahib Ata Hankah Konya, 1279 A.D. The pattern is divided into four recurring shapes.

Designs, where lines are embellished with interlaces require a more sophisticated approach. In such a design, several mosaic pieces emerge after the cutting process. The easier way to construct such a pattern is to divide the design into repeating units and enlarging these until the intended panel dimension is acquired. A panel from Malatya Ulu Mosque, a work done by another ceramic master, Yaqub b. Abi Bakr al Malati (Meinecke, 1976a, pp. 47–49) is completed with such an approach. Borders of repeating hexagons are visible. These hexagons also reveal the polygonal tessellation behind the pattern. The overall design is created by combining all mosaic pieces into one repeating polygonal shape, in this case a hexagon that is then copied multiple times (Figure 4.23).

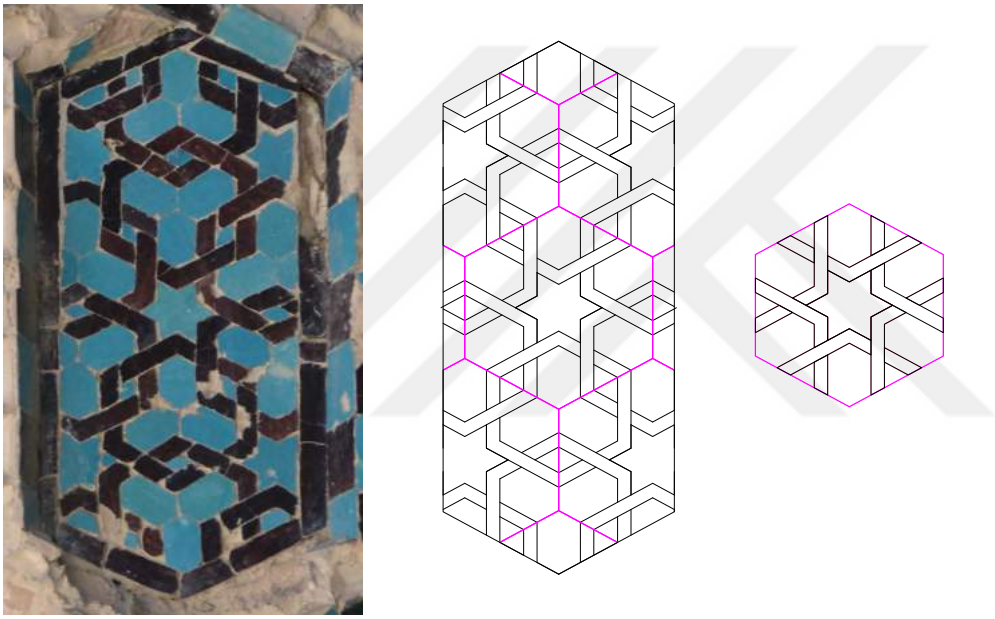


Figure 4.23 : Tile mosaic panel from Ulu Mosque in Malatya, Turkey (Left: Photo from Cambaz (2016, p. 396)) – Borders of repeating hexagons are observable. Colored lines illustrate hexagonal borders.

The approach of Yaqub b. Abi Bakr al Malati (henceforth referred as Malati) requires the intuitive comprehension of symmetry rules of any given pattern design, since the overall design is split into repetitive polygons. Such a methodology is similar to the aforementioned brickwork technique (such as the one illustrated in Figure 4.13).

Master craftsman Muhammad al Tusi's method to build patterns as such differs slightly from Malati's approach. A visual comparison between pattern designs illustrate this contrast. Figure 4.24 is a panel from Sırçalı Madrasa (1242-1243 A.D.),

the geometrical rule of this particular pattern is similar to the one from the panel demonstrated in Figure 4.23. The generative construction based on a circular grid is illustrated in Figure 4.25. The polygonal tessellation behind the pattern in the Sırçalı Madrasa consists of repetitive hexagons as well. Yet, we are not able to see any visible divisions of repeating polygons that would assist the making process. Unlike in the example from Malatya, there are no extra visible lines in the pattern from the Sırçalı Madrasa. Therefore, the pattern stays ***uninterrupted*** inside the surface borders.

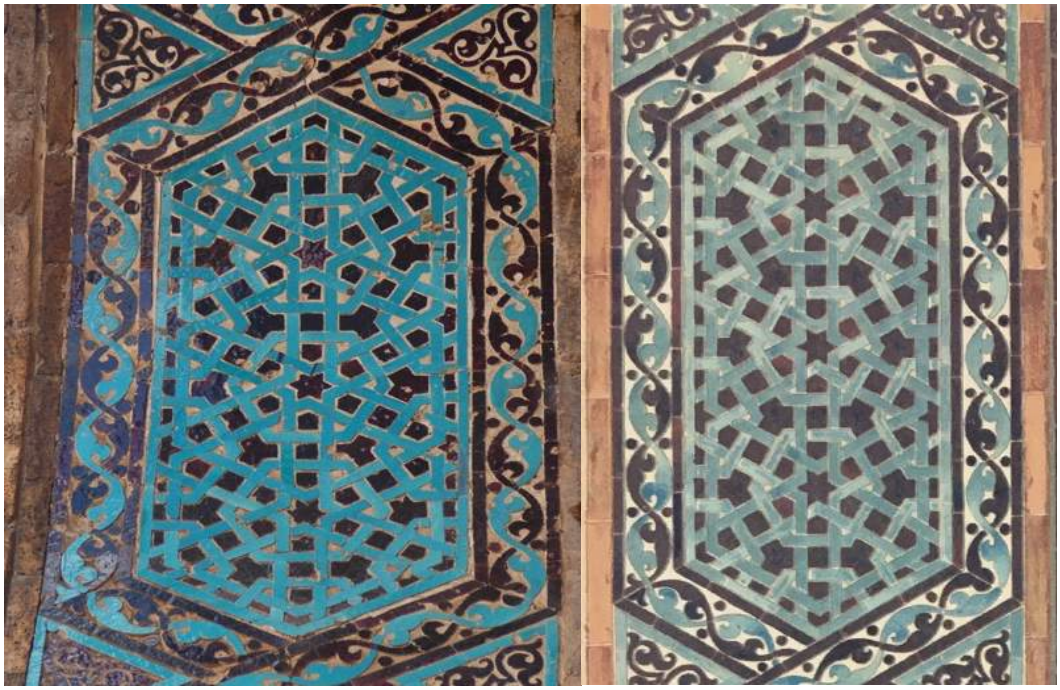
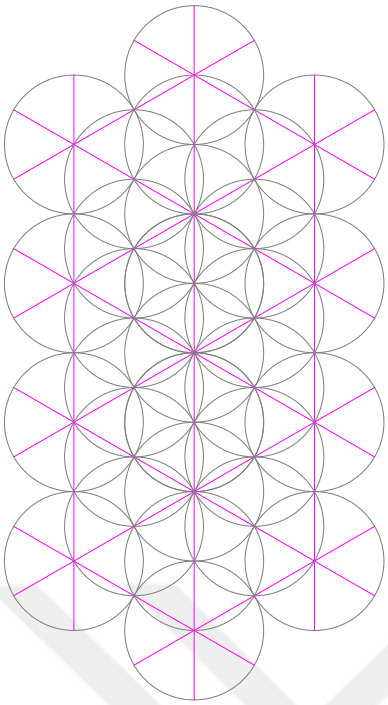


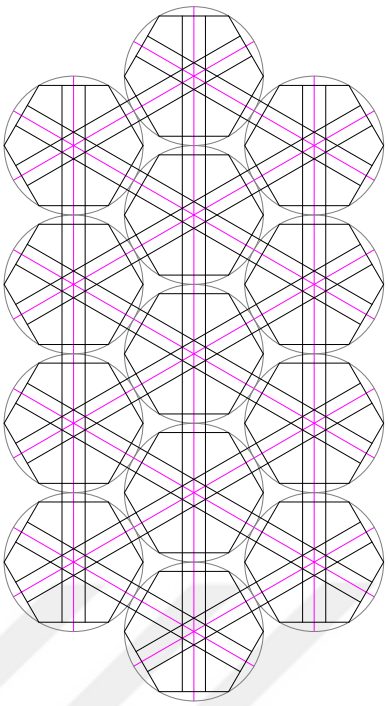
Figure 4.24 : A panel from Sırçalı Madrasa, Konya, 1242-1243 A.D. The geometric design is similar to the one in Figure 4.26 (Right: Photo credit Ezgi Baştuğ. Left: illustration from Sarre (1901b)).

Uninterrupted Pattern is the key term in this study. Informally, a pattern is uninterrupted, whenever the line continuity inside the ornament appears to be never-ending. Starting to observe from a certain part of the pattern, the eyes of the beholder can trace pattern lines in a loop, hence is triggered by an image of perpetuity. In other words, an uninterrupted pattern is an ornament arrangement that is not interfered through an extra line. The extra line here refers to a stroke, which does not belong to the original contours of the pattern design.

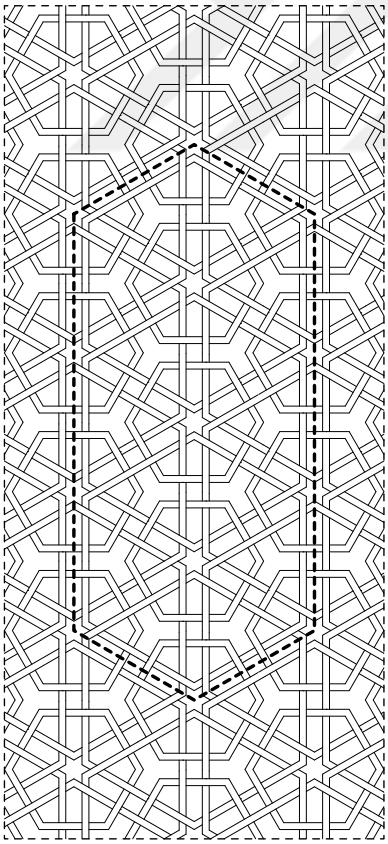
Various reasons cause a pattern to get interrupted and the interruption interferes our visual perception by disrupting the pattern continuity. As previously explained, pattern designers omit borders of polygonal tessellations after a generative process, hence observers are not able to easily recognize the geometrical rule behind a given design.



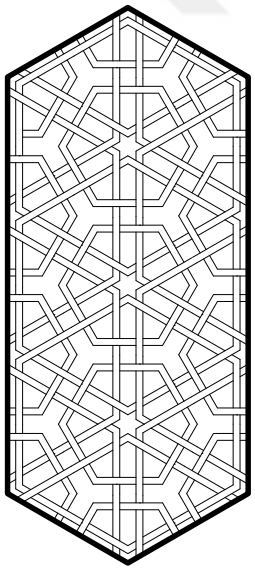
generative circular grid



hexagonal tessellation



continuous pattern



panel from Sırçalı Madrasa

Figure 4.25 : Rule-based drawing of the pattern from Figure 4.24, hexagonal tessellation is non-observable on the actual tile-mosaic panel.

Nevertheless, in case polygonal tessellation in the background is not eliminated, actual pattern lines in the foreground are interrupted by the borders of generative polygons (Figure 4.26).

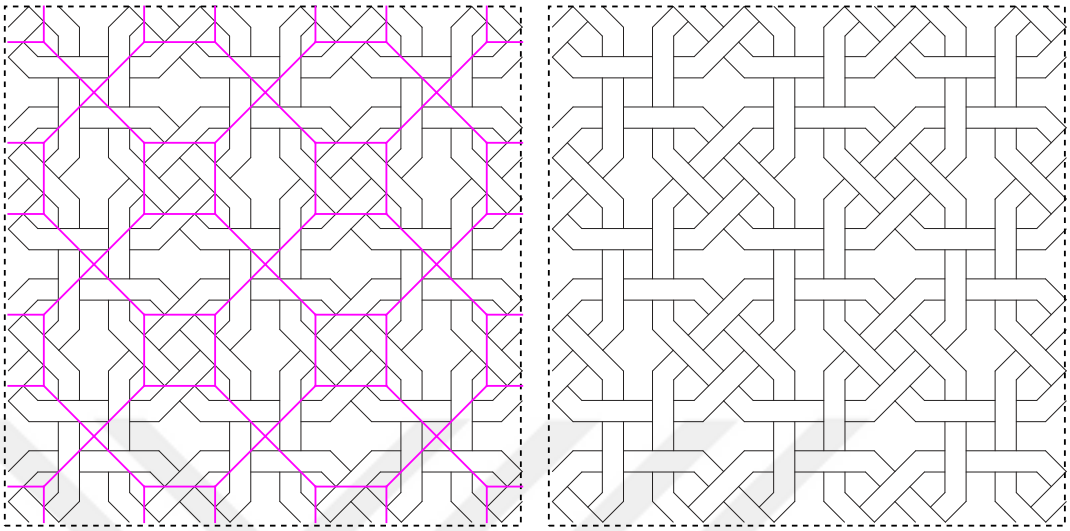


Figure 4.26 : The pattern stays uninterrupted when the polygonal tessellation is eliminated.

Similarly, an extra line appears usually because the surface has to end at some point. When a pattern is applied on a two-dimensional planar surface, the interruption occurs where the surface ends - in other words, on the surface border. However, there could be other reasons why a line of a visual interruption is necessary. Material dimensions are one of the main reasons why we have to see a contour inside any pattern design.

Ceramic tiles, frequently used in the modern world, are familiar examples. The application of the tiles results in joints between neighboring units, consequently interruption strokes appear inside a continuous pattern. Apart from the tile mosaic tradition, Seljuk artisans worked with ceramic tiles for decoration purposes as well. For instance, Figure 4.26 presents a fragment of the hexagonal ceramic decoration inside the Karatay Madrasa, A.D. 1251-1253, another work attributed to the Tusi atelier. There are only small fragments from the painting left, however the original appearance of the design is found in Sarre's(1901b) work. This time, patterns are gilded with gold, and shapes are not cut individually as in the tile mosaic. Hexagonal tile borders occur as interruption lines and break the visual continuity. Subsequently, the nature of this technique does not allow Tusi to create an uninterrupted pattern this time.

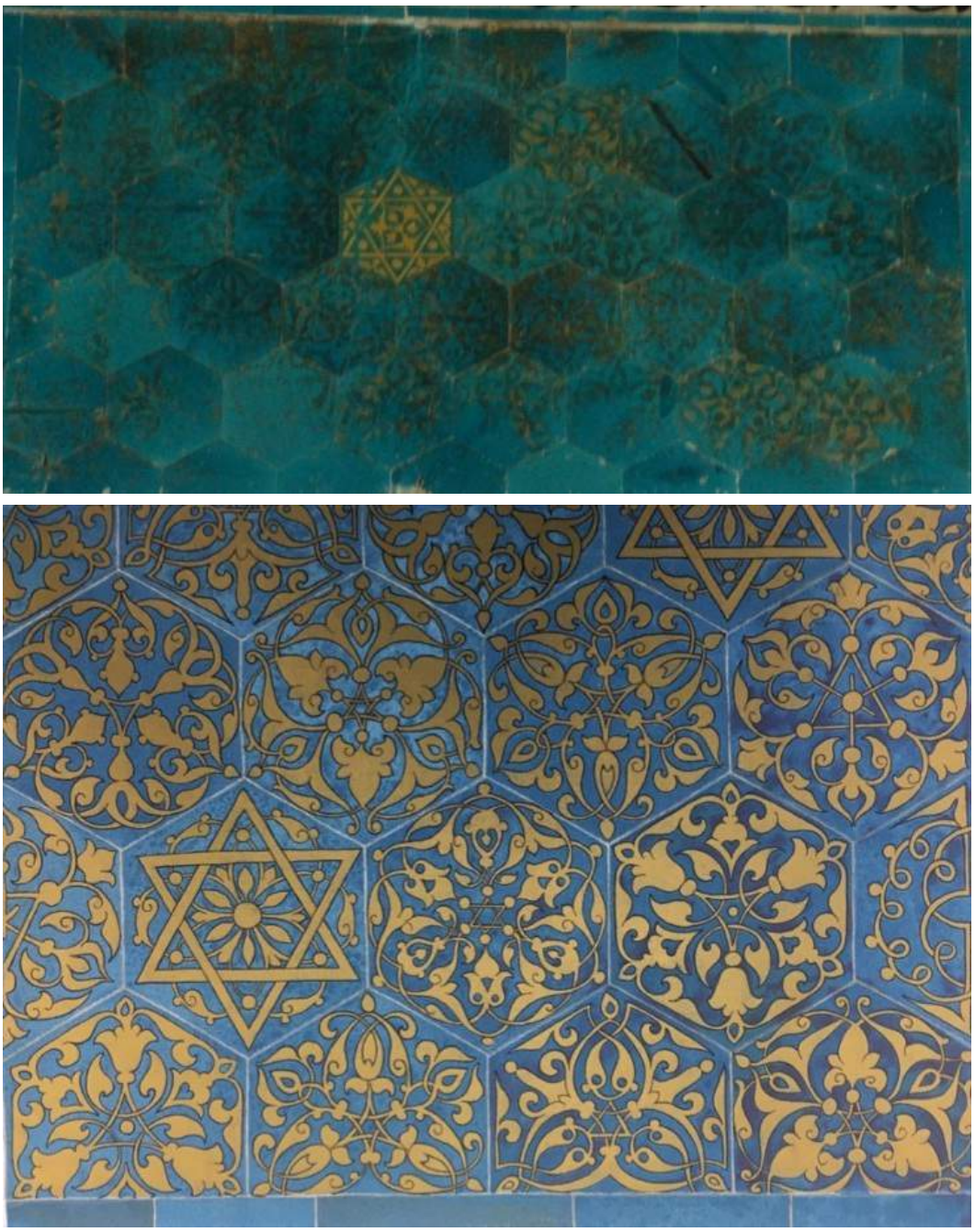
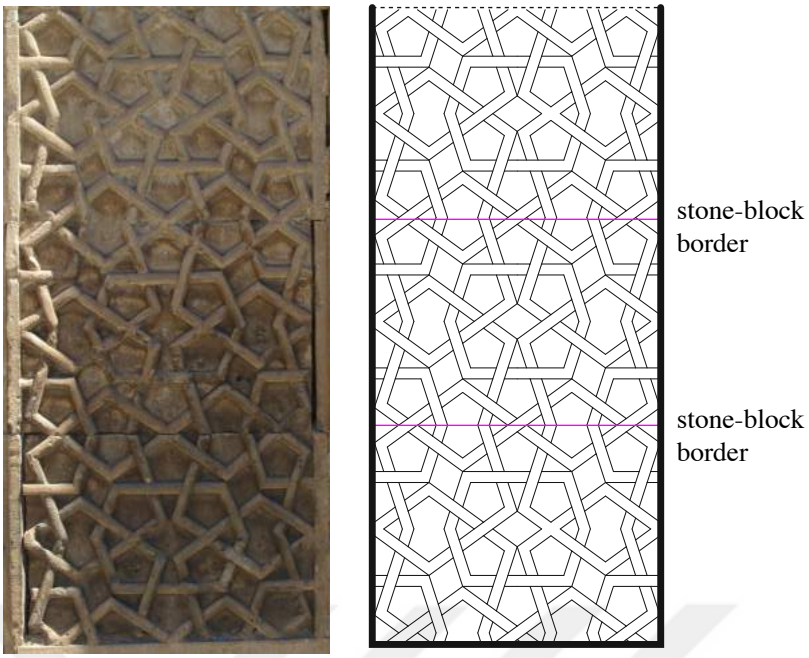


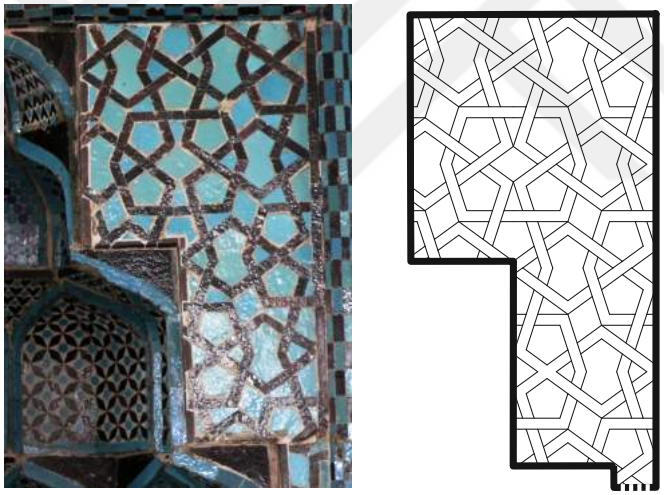
Figure 4.27 : Tile Decoration on Karatay Madrasa walls (1251-1253 A.D.) Individual motifs are painted onto hexagonal ceramic tiles. Borders of the ceramics hinder an uninterrupted pattern view (Above: Photo Credit Ezgi Baştuğ. Below earlier drawing by Krecker in Sarre (1901b).

Compared to the nature of a cladding material, carving would be ideal to create uninterrupted patterns. Yet, specific sizes of the material production, as in the case of stone blocks, hinder the uninterrupted appearance. The usage potential of tile mosaic material stands right in "the uninterrupted pattern" key concept, when precisely planned, it is possible to have a design without any extra contour (Figure 4.28).



pattern on stone from Mama Hatun Mausoleum, Tercan (1200 A.D.)

pattern interrupted by material borders



pattern on tile-mosaic panel from Mısri Mosque, Afyon (1250 A.D.)

uninterrupted pattern

Figure 4.28 : Same pattern design applied with different material techniques. Above: Stone façade from Mama Hatun Mausoleum, Tercan (Photo Credit Mine Özkar). Below: Tile mosaic panel from Mısri Mosque, Afyon.

Pattern lines alter directions as they adjust angles in order to connect with further lines in some cases. Nevertheless, such a process is not defined as interrupted formations throughout this study. Lines change usually directions on curved surfaces so that lines on altered curvature can link accurately. This might be inferred as craftsman's trick to get a continuous view and cheats as such go along with the infinite image. Hence, such

adaptations are interpreted in the study as uninterrupted patterns as well. Figure 4.29 illustrates one of such designs. The illustration depicts a partial section of the Karatay Madrasa dome that is examined in detail in the following chapter. In the dome pattern, lines between individual star motifs change directions to link in different ways, consequently intermediate areas are filled with various designs. Lines do not follow a smooth path as they exhibit altered angles on different curvature degrees. Yet this design is accounted for an uninterrupted pattern as well, since it does not embrace any extra stroke in the background.

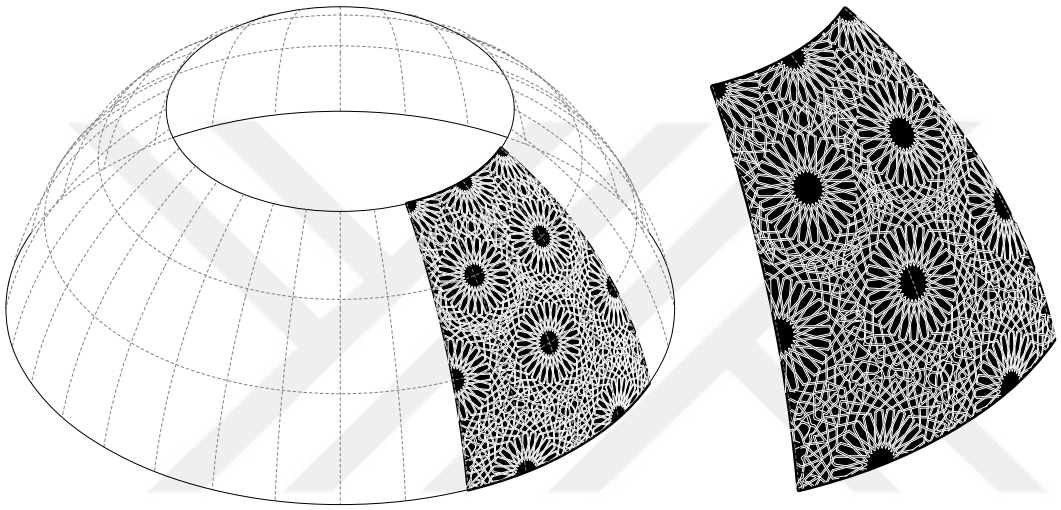


Figure 4.29 : The illustration shows a part of the dome pattern in the Karatay Madrasa, Konya (more detail in Chapter V). Pattern lines change directions, yet the uninterrupted appearance remains.

The will to create an infinite image was not specific to the cut tile material, this pursuit is perceptible in brickwork monuments as well. As it was previously discussed, masons were covering the gap between brick units with an extra layer of mortar. Occasionally, they even applied mortar imprints to decorate the filling with ornaments (McClary, 2015, pp. 222–223; Meinecke, 1976a, p. 167; Sarre, 1901a, p. 13). Embellishments as such point craftsmen's awareness of material borders. Borders break the uninterrupted pattern image in case they are not covered. Hence, craftsmen's intention for creating mortar imprints is perhaps to alter observers' perception by hiding borders with a different approach while ensuring the continuous pattern image. As an example, Figure 4.30 illustrates details of a panel from Mu'mine Khatun mausoleum in Nakhchivan, Azerbaijan, 1186 A.D. Primarily observed by Jacobstahl (1899) and then by Sarre (1901a), the pattern design is formed by rectangular repetitive brickwork units. Four

units construct a twelve-pointed star that is the primary motif of the entire design. The filing is embellished with small free hand modelled ornamentation. Such details are consistent with Khaldun's historical statements on the architecture in Spain and North Africa, where decorative motifs as such were engraved into the mortar (Sarre quotes Ibn-i Khaldun in Denkmäler persischer Baukunst (1901a, p. 13)).

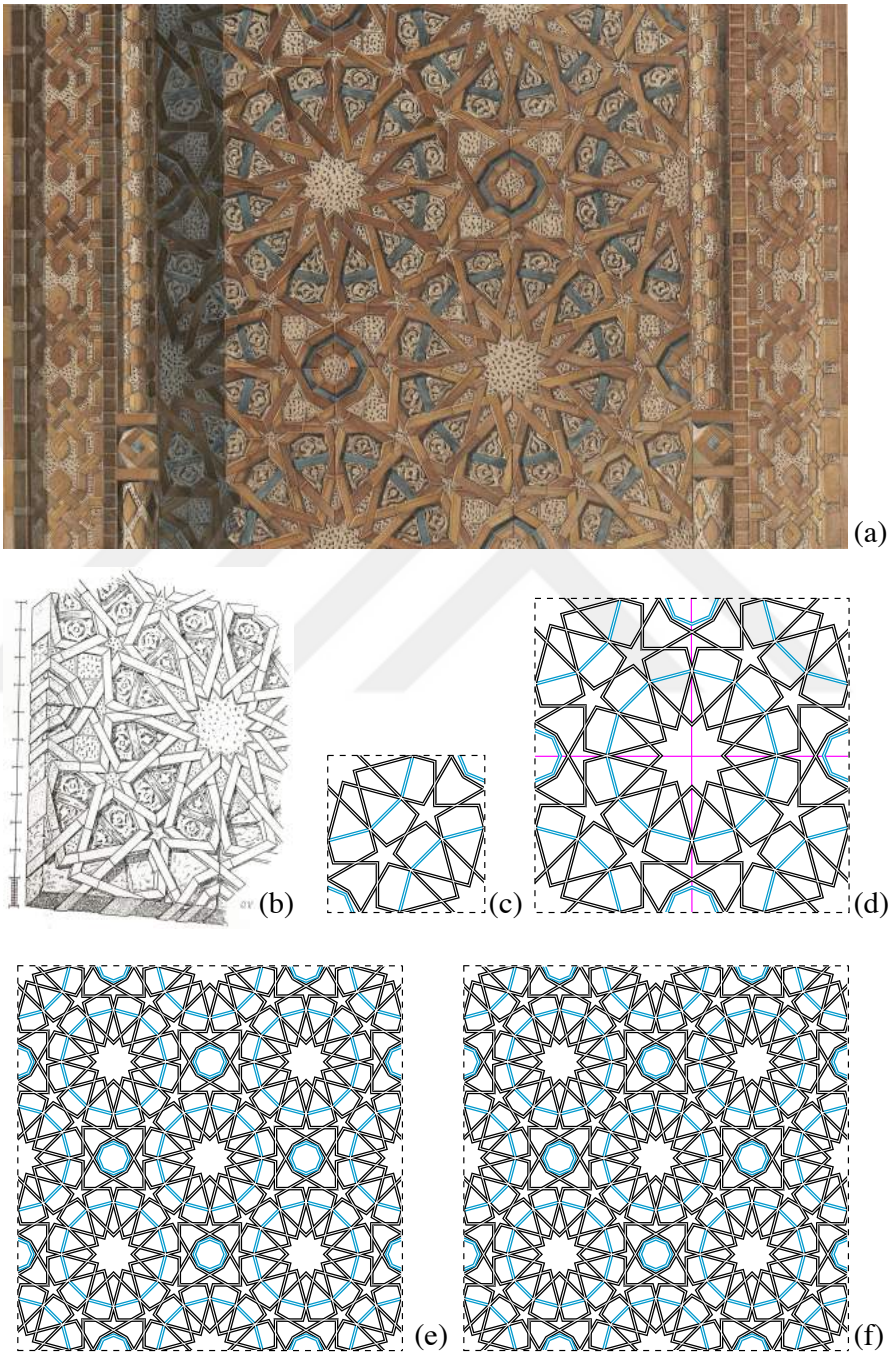


Figure 4.30 : (a) Pattern design from Mu'mine Khatun mausoleum in Nakhchivan, Azerbaijan, 1186 A.D. (drawing from (Sarre, 1901b)) (b) Methodological division of the brickwork as observed by Jacobstahl (1899, p. 25) (c) One brickwork unit (d) Four brickwork units compose the twelve-pointed star (e) Pattern design superimposed on the square tessellation (f) Uninterrupted pattern.

The innovation of cutting shapes individually provides a stylistic alteration in brickwork (Meinecke, 1976a). Cut-tile pieces are laid upon the fillings, which eventually conceals the joints and provides a partially uninterrupted pattern. Thus, mortar decorations become excessive. An example of this technique is seen in the Ulu Mosque from Malatya, Turkey (Figure 4.31). Both panels in Figure 4.31 are crafted with the same geometric design, only glaze colors and pattern sections differ. The one on the right (labeled as panel 2) is one part of the panel on the left side (labelled as panel 1), when split into two equal fragments. The uniformity of brick dimensions suggest that both designs were made with the same brickwork template. Continuous division lines between the brickwork are observable especially in elder photos (like in a photo from Öney & Çobanlı (2007, p. 42)). However, cut-tiles in intermediate areas prevent divisions to appear through the entire pattern. Thus, it is hard to speculate about the methodological brickwork assemblage, as borders remain hidden.

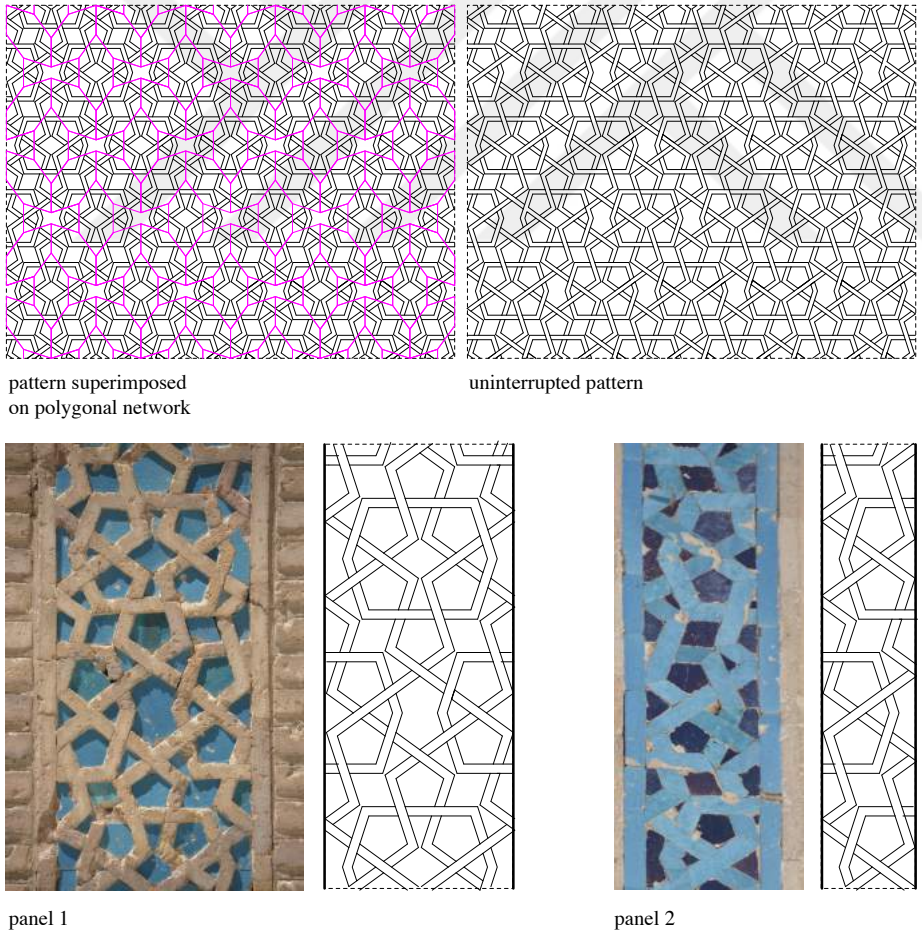


Figure 4.31 : Two different panels with the same geometrical design from Ulu Mosque in Malatya. Cut-tiles ensuring the uninterrupted pattern image are placed inside the brickwork (photos from Cambaz (2016, pp. 9–10) drawings based on girih tiles that appear in Lu and Steinhardt (2007)).

The key to an uninterrupted pattern is coherent with the physical construction of the material, they both require the perception of all individual shapes in a given pattern. Since the puzzle like nature of the cut tile marks all shape relations important, such intentions entail further understanding of the geometric designs. Figure 4.32 encompasses the pattern from Figure 4.23, the one attributed to Malati. Supposed that Malati sought for an uninterrupted pattern image as in Tusi's approach, his initial goal would be to find out on which shapes the interruption occur. After recognizing all single shapes, he would create a more *organic assemblage* that is not formed as a clear-cut hexagon, but rather a shape grouping without sharp edges. Such a methodology reflects Tusi atelier's approach for making patterns with cut tile mosaic.

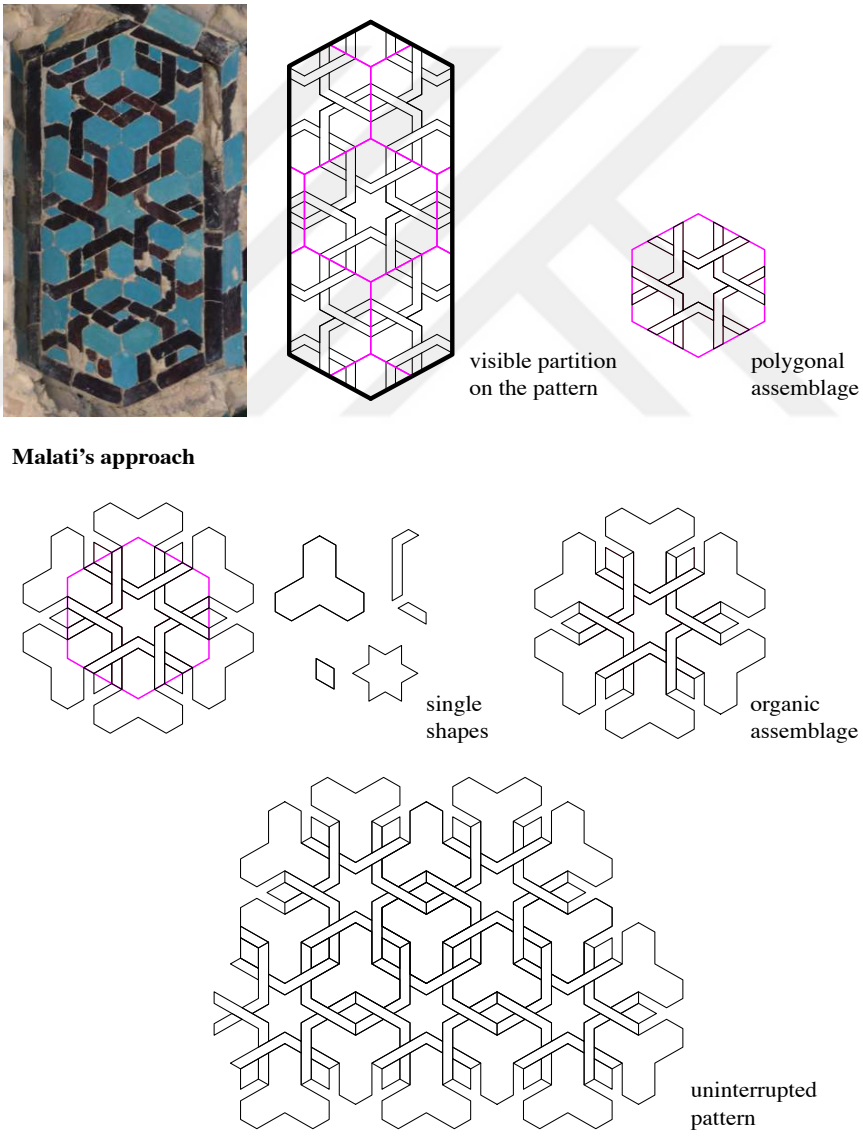


Figure 4.32 : Tusi's uninterrupted pattern approach compared with Malati's approach to construct the panel from Malatya Ulu Mosque.

Distinguishing all individual shapes get even more trickier whenever the geometric design gets more complicated. Hence, spotting out the pattern rules and making becomes more challenging. The panorama of an uninterrupted pattern that spread to a broader area is more perceptible. We may envision this by looking at a comparison between Malati's and Tusi's approaches on the same pattern as demonstrated in Figure 4.33. This pattern entails repeating triangles and squares. Just like in the first drawing in the figure, If Malati had to apply the design on to a building surface, border lines of the polygons would be noticeable by the observer. Tusi's endeavor for an uninterrupted pattern, on the other hand, requires a comprehensive pre-study of the geometric pattern. This is obviously the job of the master builder, in our case, Muhammad Al Tusi. Following chapters explain how the will for an uninterrupted pattern alters the hands-on computational making processes of the craftsmen.

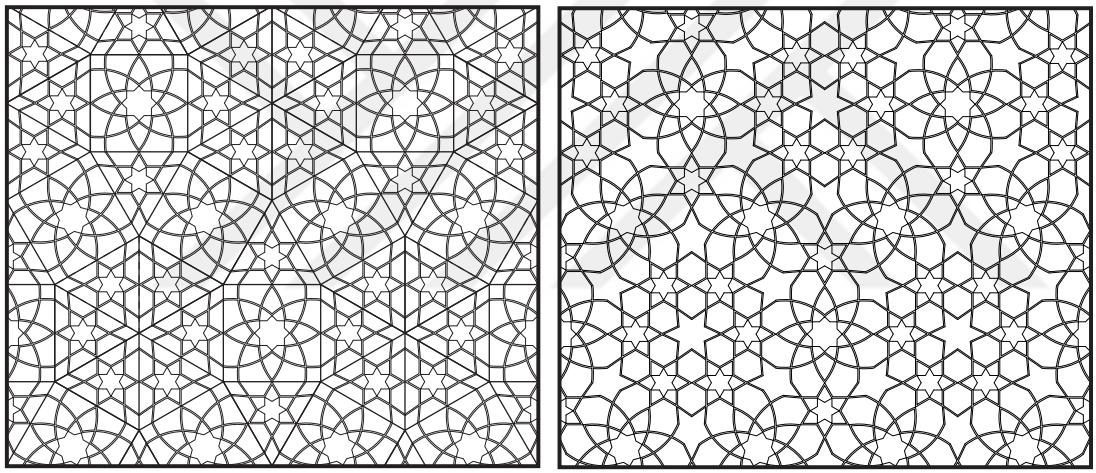


Figure 4.33 : A hypothetical pattern design, which consists of repeating triangles and squares. If it was applied by Malati, polygons would be visible to eyes as well.

5. CHAPTER V. THE FORM AND MATTER: THE COMPUTATION

5.1 Material Application and Construction Details

Material properties and the craft technique of the tile mosaic, together with some of the initial observations of Tusi Atelier's approach are still insufficient to understand the complete construction process. In previous chapters, we acquired the primary methodology for fabricating individual pieces and bonding them to form larger panels.

Once craftsmen produce these panels, they connect them to the intended surface again with mortar. Thus, the overall decorative cladding thickness, associated with the construction, consists of the width of tile plates plus a bonding mortar layer. Mosaic decorations do not cover the whole façade in all cases, consequently empty spaces must already be planned during the wall construction, since these panels are placed into these openings. ***Hence, the whole process requires a precise coordination between the architecture and the geometric design program.*** It becomes more and more challenging, the larger and comprehensive the glaze decoration is planned (Meinecke, 1976a). As an example, the picture on the right in Figure 5.1 depicts a part of the iwan surface from the Sırçalı Madrasa in Konya (1242-1243 A.D.). The surface is not covered entirely with tile mosaic, rather three individual panels are placed into the openings of the surface. Thus, the craftsman Tusi plans the holes in advance, before the construction starts.

Figure 5.1 : Three individual panels are placed separately into the surface openings (depicted in black) from Sırçalı Madrasa, Konya, 1242-1243 A.D.

Figure 5.2 : Building of the great Mosque in Samarkand. All partakers of a construction process are coordinated by the master-builder (Illustrations of Timurids in the *Zafarnama* of Sultan Husayn or 'Garrett Zafarnama', John Work Garrett Library John Hopkins University, Baltimore, Maryland. (Photo "Building of the Great Mosque in Samarkand" (n.d.)).

In order to facilitate the application to intricate surfaces, such as curved iwan facades, portal profiles or prayer niches, a rough wall of rubble stones is constructed at the beginning. This wall core then receives a cladding, in a form of vertically arranged square or rectangular flat bricks fixed with a layer of mortar. Individual plates adorned with geometric pattern decorations are connected to this cladding (Figure 5.3). This structural technique ensures also the adjustment of decorative parts to curvilinear surfaces. (Meinecke, 1976a, pp. 169–171)

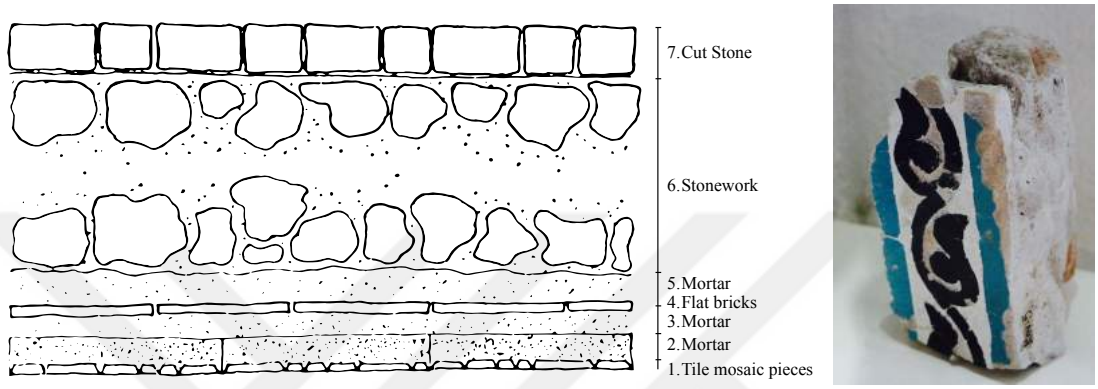


Figure 5.3 : Sample section of a tile mosaic covered wall (Illustration adapted from German in Meinecke (1976a, pp. 169–171)).

5.2 Patterns and Surfaces

Patterns were often adapted to varying kinds of continuous surfaces, curved or flat, and designed with the boundary of this surface in mind. Nevertheless, why artisans choose particular designs to decorate specific surfaces remains unclear. Therefore, we cannot verify why master builder Tusi, the person in charge for the entire construction process in our case, choose certain pattern designs. On the other hand, aforementioned historical documents attest that craftsmen inherited pattern books. Likewise, Meinecke (1976a, 1976b) compares the designs and suggests that patterns seen in Anatolian territories were largely inherited from the Iranian tradition and repeat across the geography. Also, through a comprehensive study by Bonner (2017), we are able to see how similar patterns are scattered among countries. We may infer from these, that Tusi was a pattern book holder as well and created his designs based on the patterns in that guidebook.

A pattern on a book page was by far tinier compared to the colossal Seljuk monumental facades. Cut-tile panels examined previously were rather small compared to surface dimensions. Yet, in some cases, even the whole massive surface was covered with tile-

mosaic panels. Therefore, Tusi had to find his way out for enlarging the pattern to match the desired surface. As mentioned in the previous chapter, the convenient way for executing such a task is the reduction of the pattern into recurring polygons. This procedure was only possible by a deep inquiry in the symmetrical rules behind the ornament. Evidently, design manuals in master builder's possession exposed basic construction rules for given patterns. However, to facilitate the construction process, the master builder had to form even larger panels to fit optimally the dimensions of the surfaces. Literature suggests that medieval artisans were using rectangular repeat units, which were more convenient for practical application (Bonner, 2017, p. 162). This is coherent with an example Moroccan zellij practice seen in Figure 5.4.

Figure 5.4 : Moroccon craftsmen installing rectangular cut tile panels (Images Sanders (n.d.)).

Rectangular repeating units were used by the artisans from Anatolian Seljuks as well. The Seljuks of Anatolia were the first to adorn complete mihrab surfaces with tile-mosaic (Bakırer, 1976, p. 32). First examples of tile-mosaic, for instance the Ulu Mosque Mihrab in Aksehir, Konya, 1213 A.D. were built by dividing the surface into repeating rectangular panels as well. Unfortunately, latest restorations have concealed some of the original making traces, but former pictures show rectangular divisions and research (Meinecke, 1976b, p. 27) write on the visible seams as well (Figure 5.5).

The overall height of the Akşehir Ulu Mosque mihrab is five meters and it is enclosed with two pattern borders: the outer one, 37 centimeters in width and the inner one, 60 centimeters across (Bakırer, 1976, p. 136). The wider border exposes a complex geometric design built up with twelve pointed stars and nonagons. There are several ways to generate the design on a paper. One alternative polygonal technique as proposed by Bonner (2017, p. 417) is illustrated in Figure 5.5. The pattern can be split into smaller hexagonal repeat units, yet for the constructive purposes, rectangular panels are preferred.

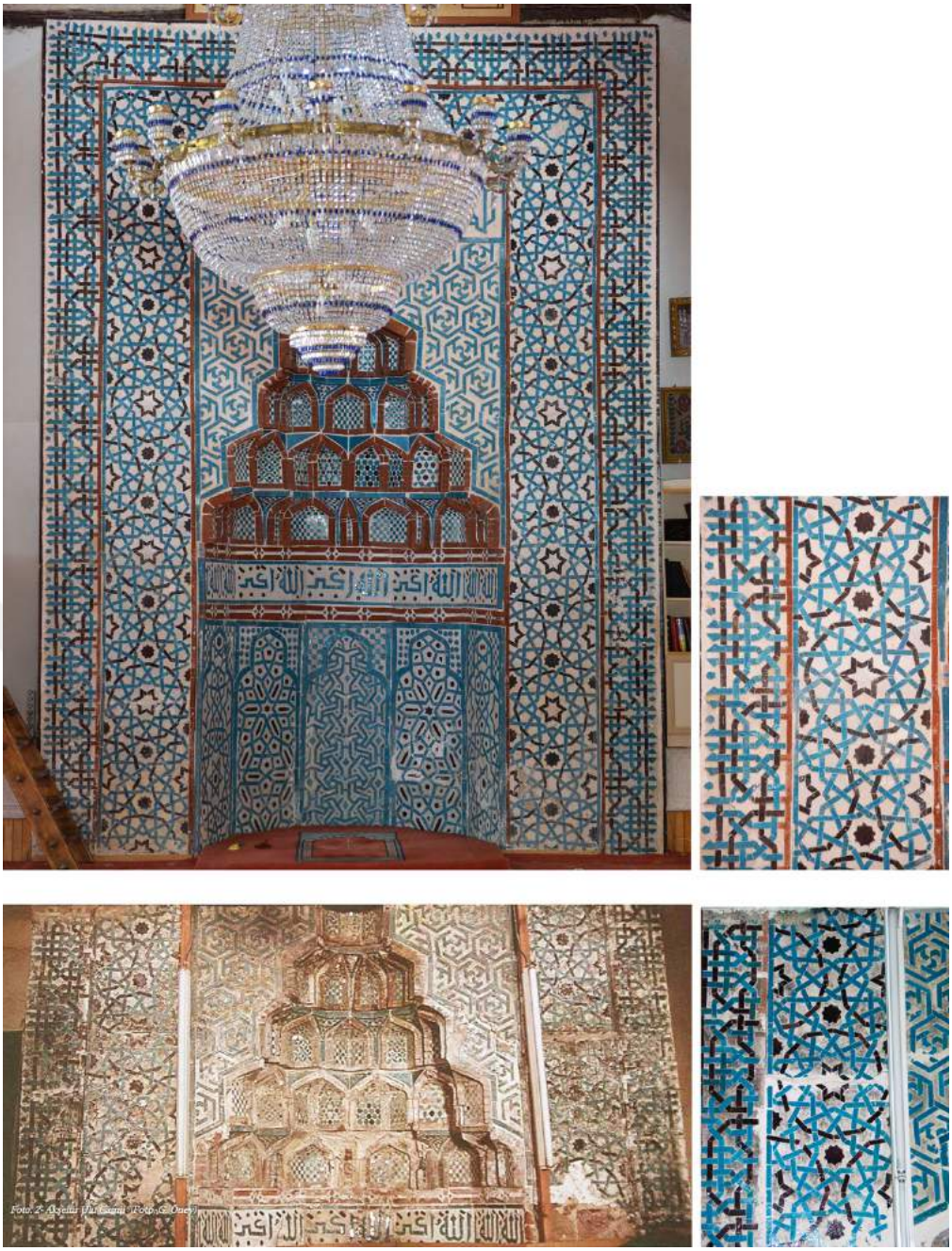
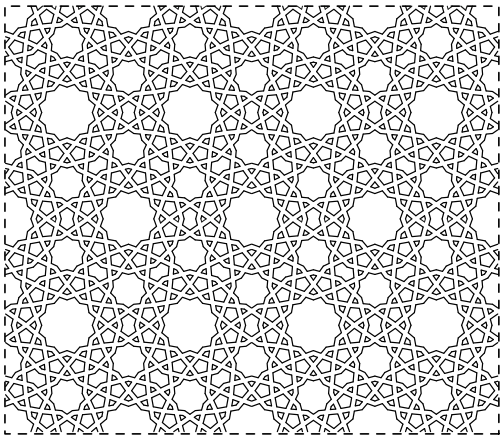
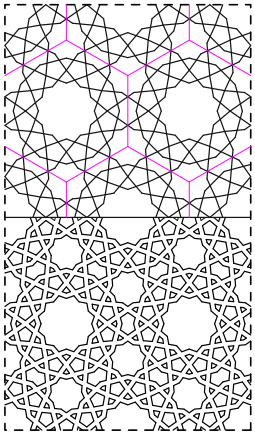


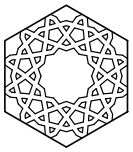
Figure 5.5 : The Ulu mosque mihrab in Akşehir, 1213 A.D. Above: Visible seam on the lower left after restoration (Photo from Cambaz (2016, pp. 351–353)). Below: Photos prior to the latest restoration expose clear seams on the entire surface (Photos Left: Öney & Çobanlı (2007, p. 33) Right: Aladağ (2013, p. 117)).

Dividing the pattern into repetitive rectangular units facilitated the construction, since it created modular panels for mounting. Nonetheless, Tusi's desire for building uninterrupted patterns forced him to cover panel seams, which required again an understanding of all the petite forms within the geometric designs. The aforementioned shape-based division, which is not strictly polygonal, but rather in more organic forms, allowed craftsmen to form larger panels with uninterrupted patterns.



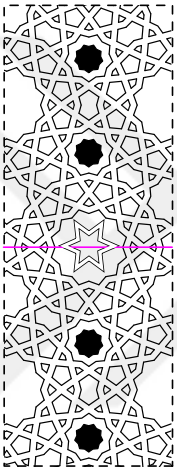
uninterrupted pattern

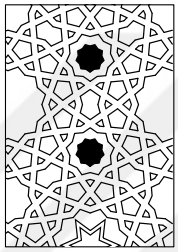




repeat unit

hexagonal tessellation





rectangular panel

rectangular division
on the mihrab surface (Photo Aladağ (2013, p. 117))

visible seams
(Photo Öney & Çobanlı (2007, p.33))

Figure 5.6 : Generative drawing method based on hexagonal tessellation vs. the actual making behind the pattern design from Akşehir Ulu Mosque mihrab. Visible seams suggest that the making is based on repetitive rectangular units.

Figure 5.7 illustrates a sample geometric pattern that is materialized with cut-tile across different monuments. The visual construction based on the circular grid is shown more in detail in Figure 5.8. The geometric design is the same in all examples from Figure 5.7, nevertheless diverse sections from the pattern are selected to adorn these unalike surfaces. The geometric design from the Karatay madrasa and the Eşrefoğlu mosque are one-half of the pattern from the Sahib Ata Hangah. Large surface dimensions in all examples require regular partition of the cladding, since physical restrictions do not allow mounting the entire cladding at once. Hence, the craftsman requires a systematic division of the geometric pattern. As in Tusi's approach inclusive of extra effort to hide any visible seam, spotting all shapes inside the design allows an organic form of assemblage that can then be employed to create the uninterrupted pattern appearance.

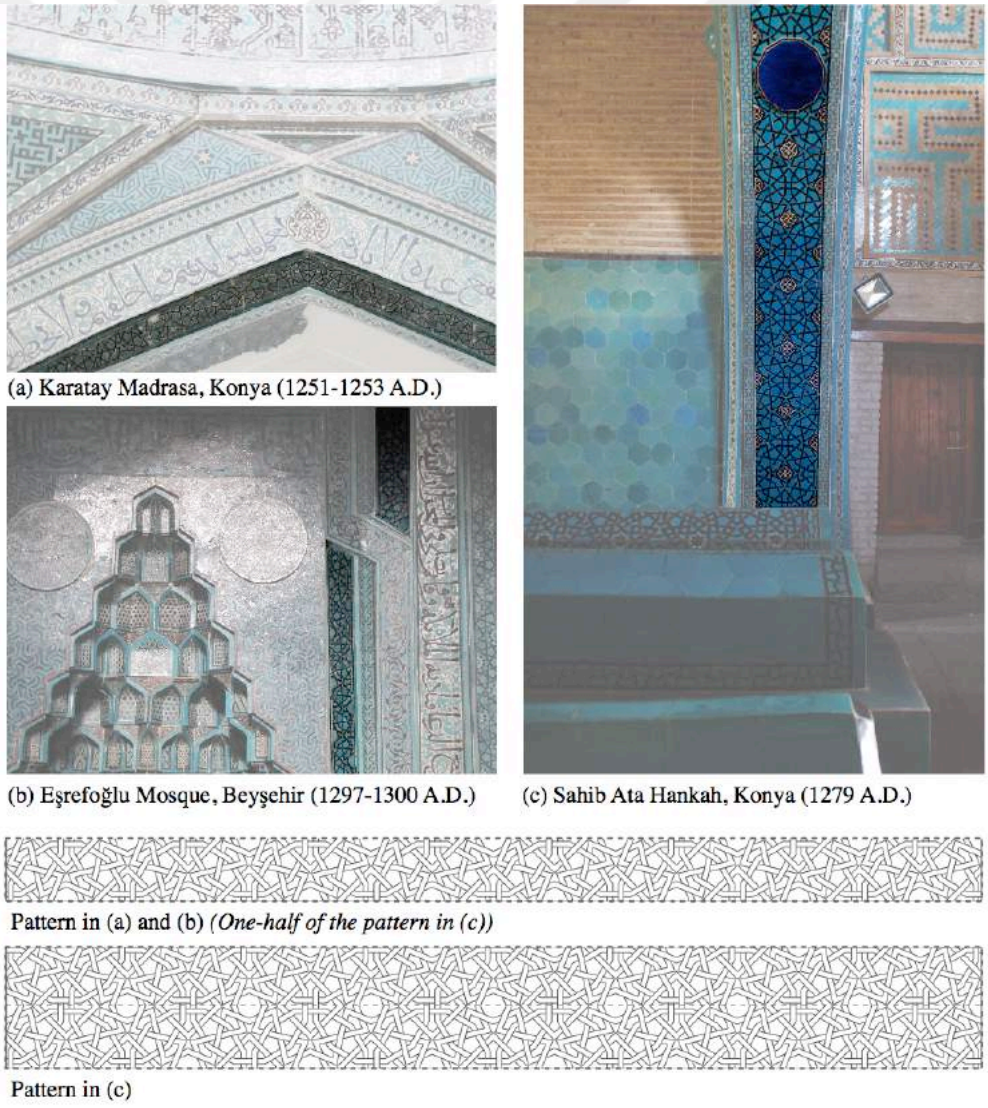


Figure 5.7 : Same geometric design on diverse surfaces.

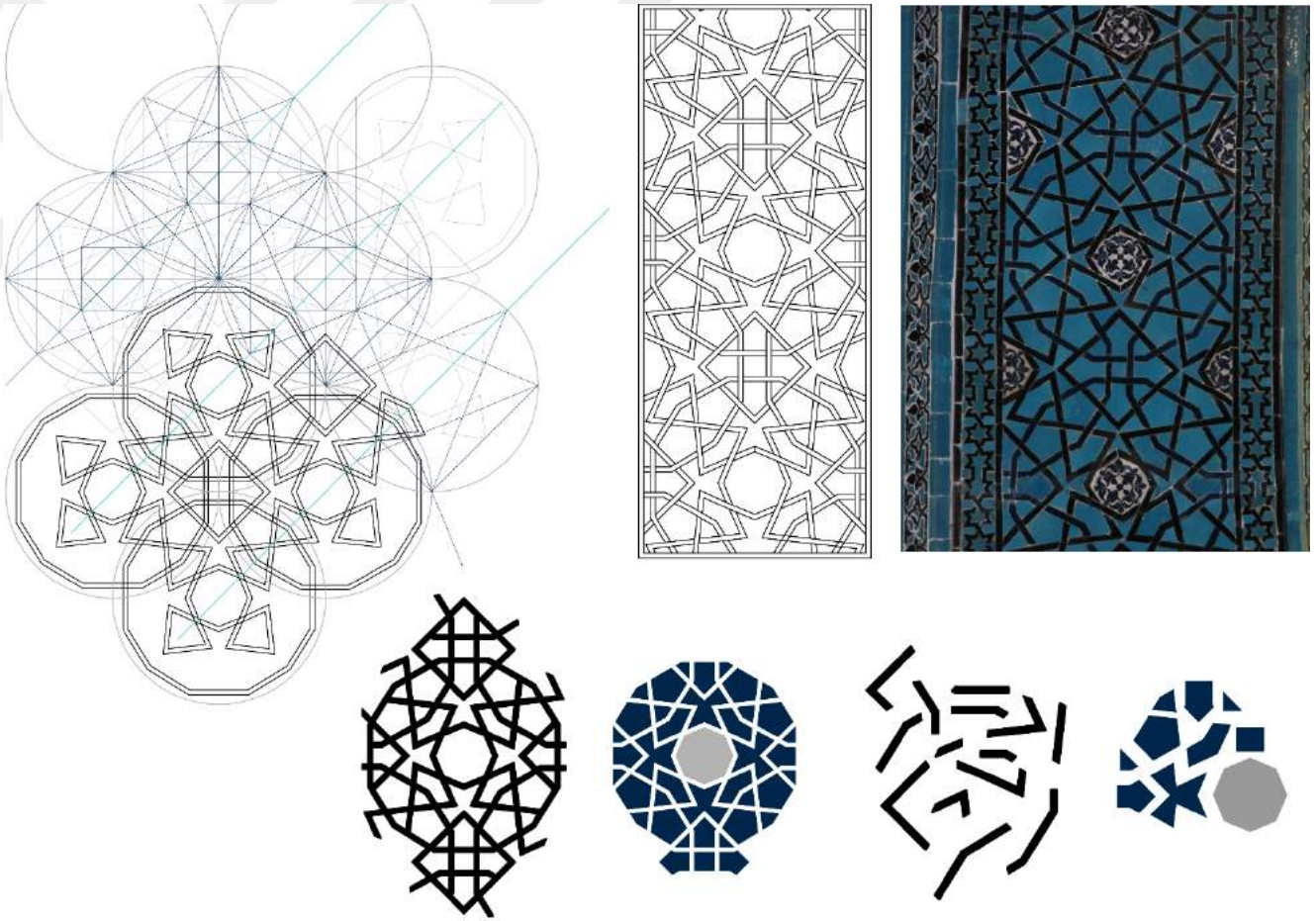


Figure 5.8 : The pattern from the Sahib Ata Hangah and basic units for an organic assemblage (a shape-based division) of a repeating unit.

5.3 Imperfection and Improvisation

A maker's mark is not always an obvious one, as in the case where visible seams appear between tile mosaic plates. Yet, there are other hints that reveal the making process. Especially, in the case of geometric designs, one small mistake can cause failure on the entire design. Consequently, some imperfections appear on the visual appearance, when examined precisely. One such example is illustrated in Figure 5.8. The pattern in Gök Madrasa, Tokat includes tenfold rosettes that are arranged in a systematical way. Visible borders of rectangular divisions appear on the façade. Yet, even if there were none, the inadequacy on the design proves that the design is split gradually.

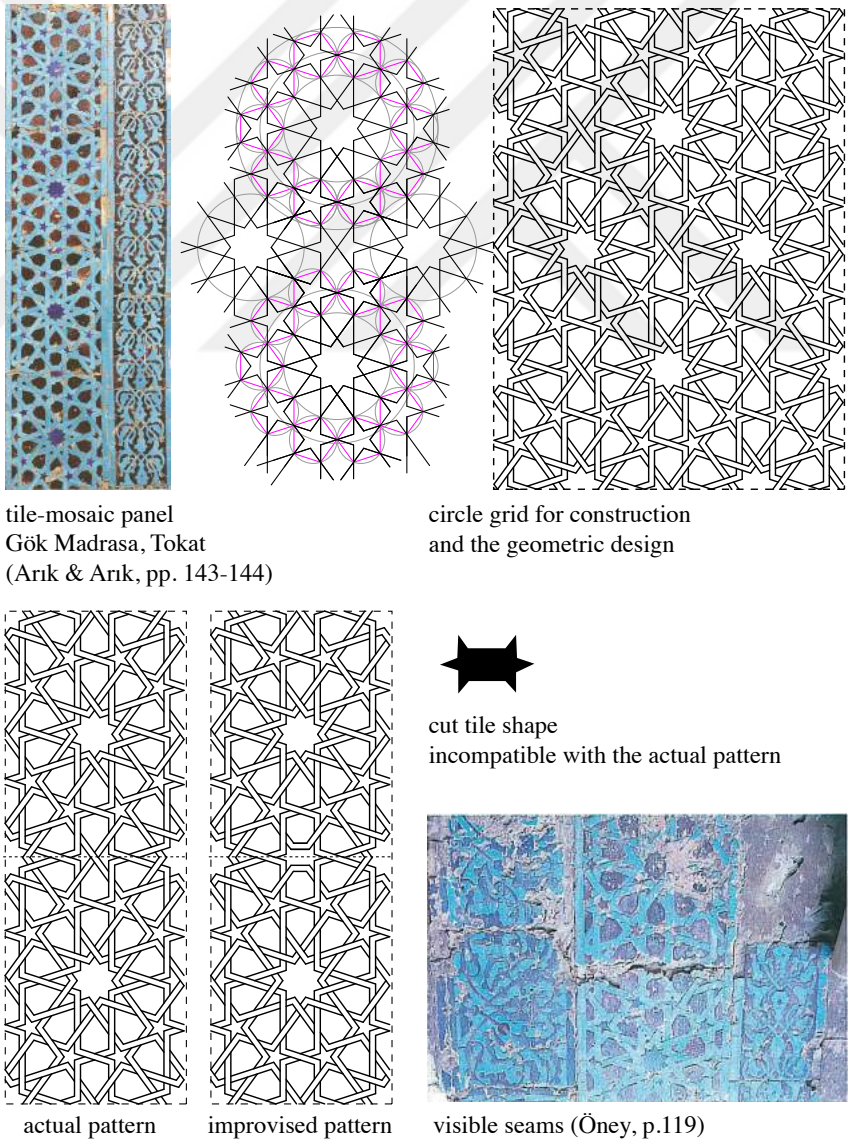


Figure 5.9 : Improvised pattern design from the Gök Madrasa in Tokat (1270 A.D.).

Making mistakes is an inseparable part of any craftsmanship. John Ruskin, a protagonist of imperfections writes:

> *... imperfection is in some sort essential to all that we know of life. It is the sign of life in a mortal body, that is to say, of a state of progress and change. Nothing that lives is, or can be, rigidly perfect; part of it is decaying, part nascent. The foxglove blossom, --a third part bud, a third part past, a third part in full bloom,--is a type of the life of this world. And in all things that live there are certain irregularities and deficiencies which are not only signs of life, but sources of beauty. All admit irregularity as they imply change; and to banish imperfection is to destroy expression, to check exertion, to paralyze vitality.* (Ruskin, 1890)

Repetitive partition of the geometric pattern that is based on organic modules provides an uninterrupted pattern outlook. However, in some cases, even with the shape-based assemblage, the perfect view of a geometric pattern that does not contain any inaccuracy was not obtainable. Frequently, tricky surfaces caused inconsistencies on patterns. As an example, some of the mihrab borders in the Eşrefoğlu Mosque in Beyşehir (1297-1300 A.D.) follow a zigzag path (Figure 5.10). One of these patterns that match with the design in Figure 5.7 exposes irregularities where the surface folds. Figure 5.11 compares the actual pattern design following a rule-based sequence with the improvised pattern on the mihrab border.

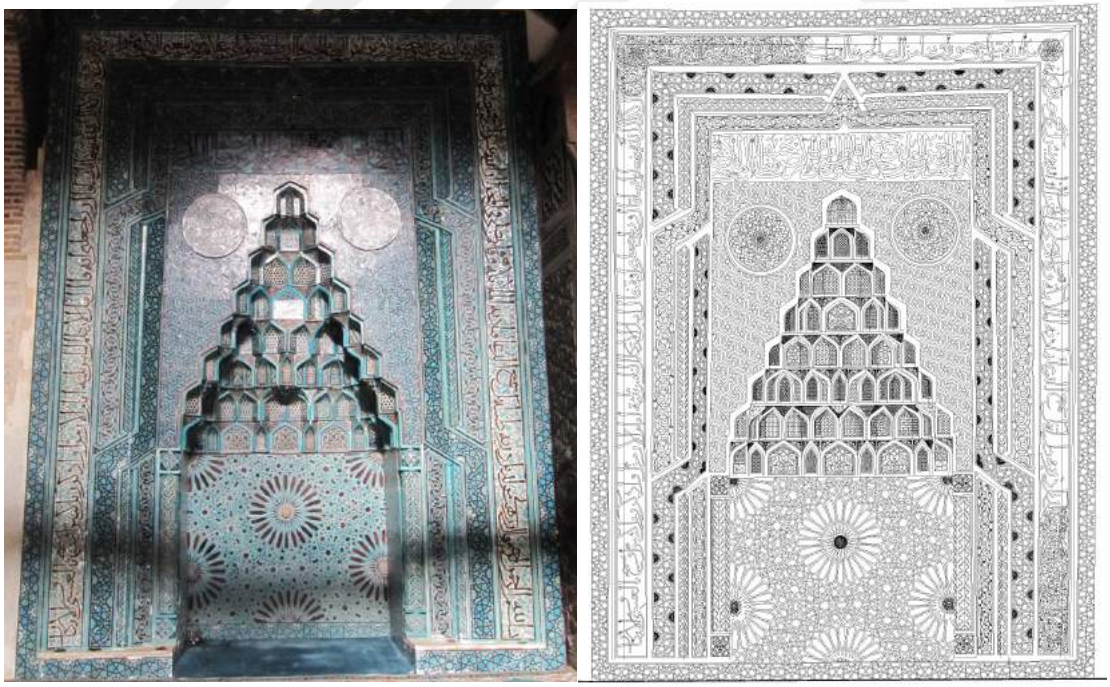


Figure 5.10 : The tile-mosaic covered mihrab from Eşrefoğlu Mosque in Beyşehir, Konya, built between 1297-1300 A.D. (Right: Building survey and drawing by M. Argun Kocadağıstan).

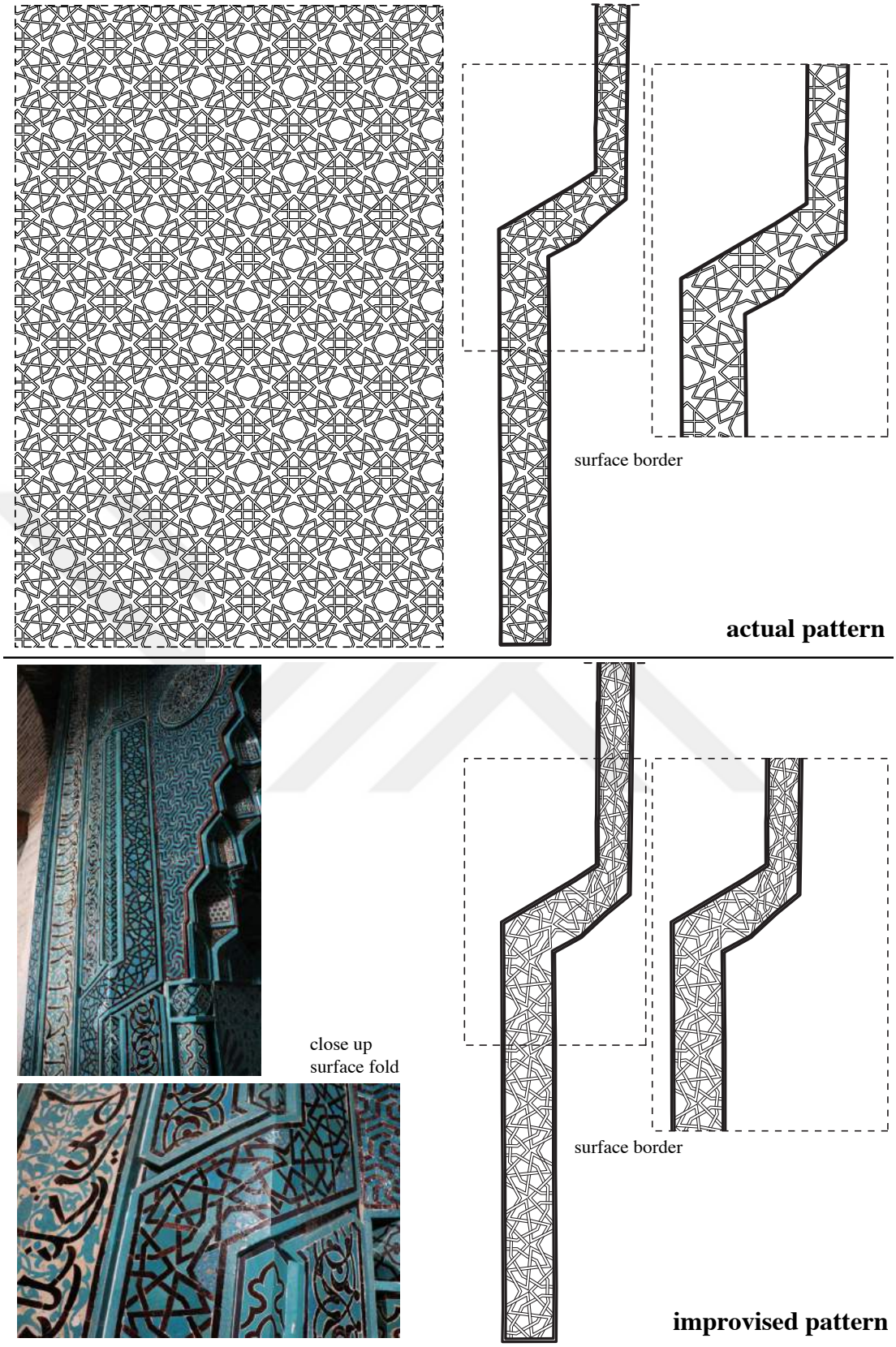
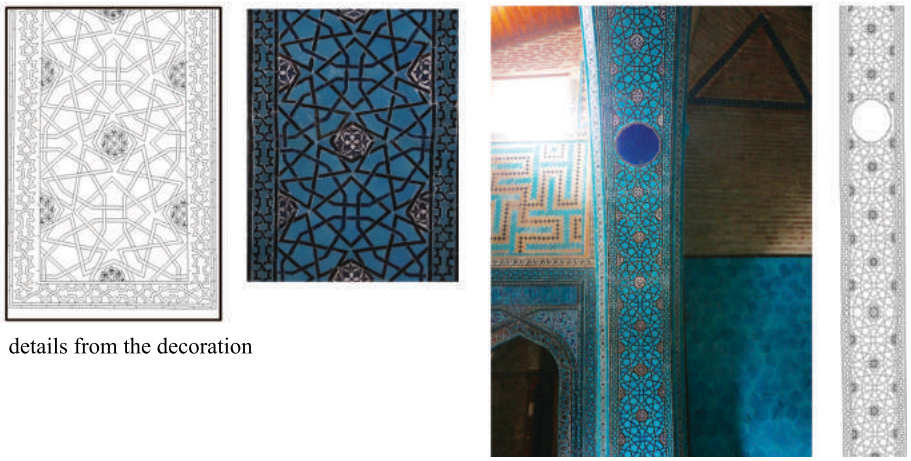


Figure 5.11 : Above: Pattern design flow on a regular basis. Below: Improvised pattern exposes irregularities on the zigzagging surface in Eşrefoğlu Mosque Beyşehir, Konya 1297-1300 A.D. (Below: Illustrations based on the Building survey and drawing by M. Argun Kocadağıstan).

Imperfections triggered by complicated surfaces are prevented when a thought-out relation between the surface and the geometric design is built. The same geometric design on a large iwan arch from the Sahib Ata Hankah (1279 A.D.) continues its path without any irregularities. Large surface dimensions necessitate a fragmental cladding in this case as well. Nonetheless, the master builder conceals any clues of the making methodology and hides the repetitive unit that might have been used by the craftsmen. Additional to the uninterrupted pattern outlook hiding any visible seam, any mistakes that give hint for the making process are non-observable. Yet, whether blue medallions that are placed upon the geometric design cover panel seams is debatable (Figure 5.12).

pattern on the large arch surface



details from the decoration

Figure 5.12 : Pattern from Figure 5.7 (Pattern in (c)) on the large arch from the Sahib Ata Hankah (1279 A.D.) continues without irregularities (Building survey and drawing by M. Argun Kocadağıstan).

Imperfections emerge sometimes as consequences of miscalculations, then again, they are occasionally craftsman's choices or "thought failures" as Ruskin (1891) calls

them. In *The Craftsman,* Sennett (2008, p. 134) gives examples of the adaptive irregularities in Roman brickwork, in which craftsmen cover imperfect joints behind the surface with extra decorative elements. Such intentions epitomize improvisation, one of the most effectual features of a successful workmanship. Although medieval Islamic artisans were working with geometric rules, they used this opportunity as well. In an extensive study, Cromwell (2017) spots such ingenuity in Seljuk brickwork, where stars that actually do not fit together are assembled in an irregular fashion. Unfitting and misaligned stars, do not matter as Cromwell (2017, p. 33) suggests, since "a pattern can be obtained by connecting the inner points of the stars" as well.

In the following investigations, we focus on some of Tusi atelier's distinctive designs, in which craftsmen improvise by noticing the new material's potentials and flexibilities.

5.4 The Complex Geometric Design on the Karatay Iwan

Improvising is sometimes inevitable when the intended geometric pattern or the target surface gets really complicated. Such an example is found on the curvilinear iwan surface from the Karatay Madrasa in Konya. We examine the complicated pattern arrangement on the Karatay Madrasa iwan (Fig. 5.13) to find out how craftsmen in Tusi Atelier handled it.

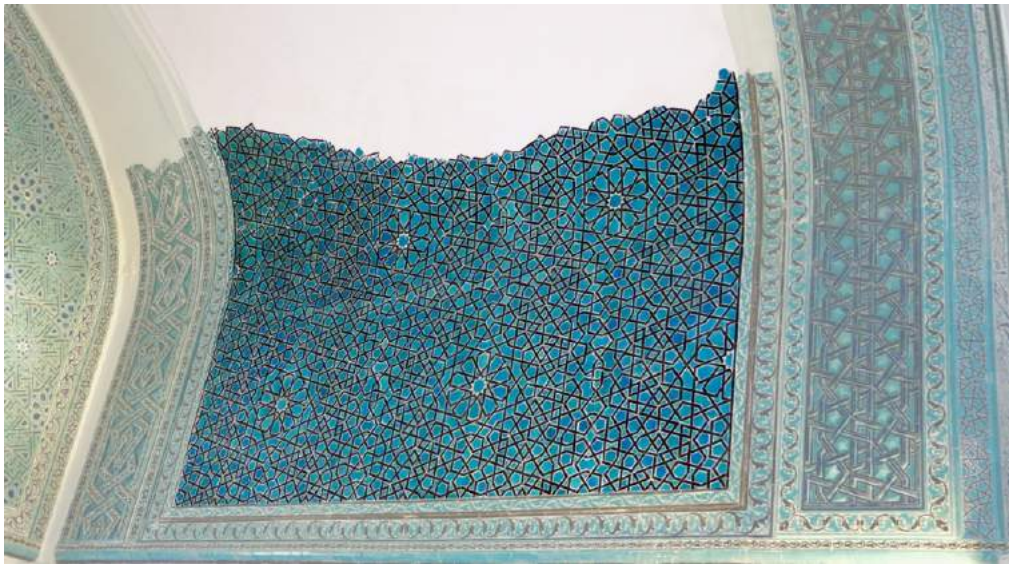


Figure 5.13 : Complicated geometric design on the Karatay Iwan, 1251-1253 A.D.

The Karatay Madrasa in Konya, built in 1251-1253 A.D., is considered as the apex of small-scale madrasa architecture in Seljuks of Anatolia. The entrance portal at the

south end of the east side alone is a masterpiece, a hallmark of Syrian influence. Geometric and floral ornaments are executed in relief on marble. The madrasa has a rectangular plan with a symmetrical organization that is dominated by a central hall. The interior space is adorned with an advanced decorative schema, mostly by glazed tiles and tile mosaic, but also with decorative brickwork. The open court is surmounted by a tile mosaic covered dome and the rooms, also enclosed with domes, are arranged around this courtyard. The vaulted iwan is at the one end of the main courtyard and leads to an open room of great height (Altun, 1988, p. 58). The decoration is attributed to the Tusi atelier (Figure 5.14, Figure 5.15 and Figure 5.16).

Figure 5.14 : A work attributed to Tusi atelier, Karatay Madrasa in Konya was built between 1251-1253 A.D.

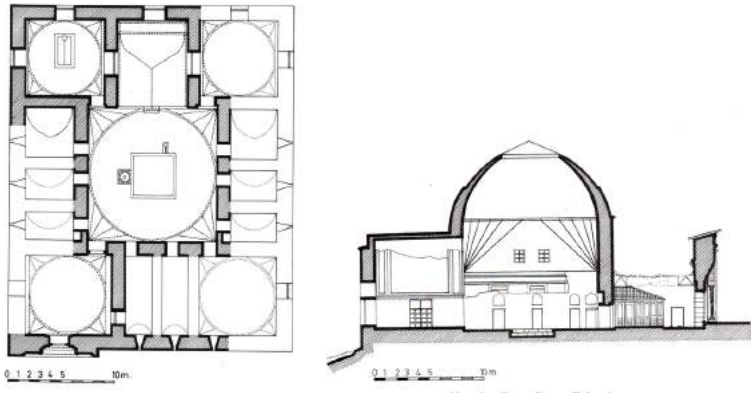


Figure 5.15 : The symmetrical plan and a horizontal section of the Karatay Madrasa in Konya (1251-1253 A.D.) (Yılmaz, Bilgicioğlu, Çelik and Boleken, 2018).

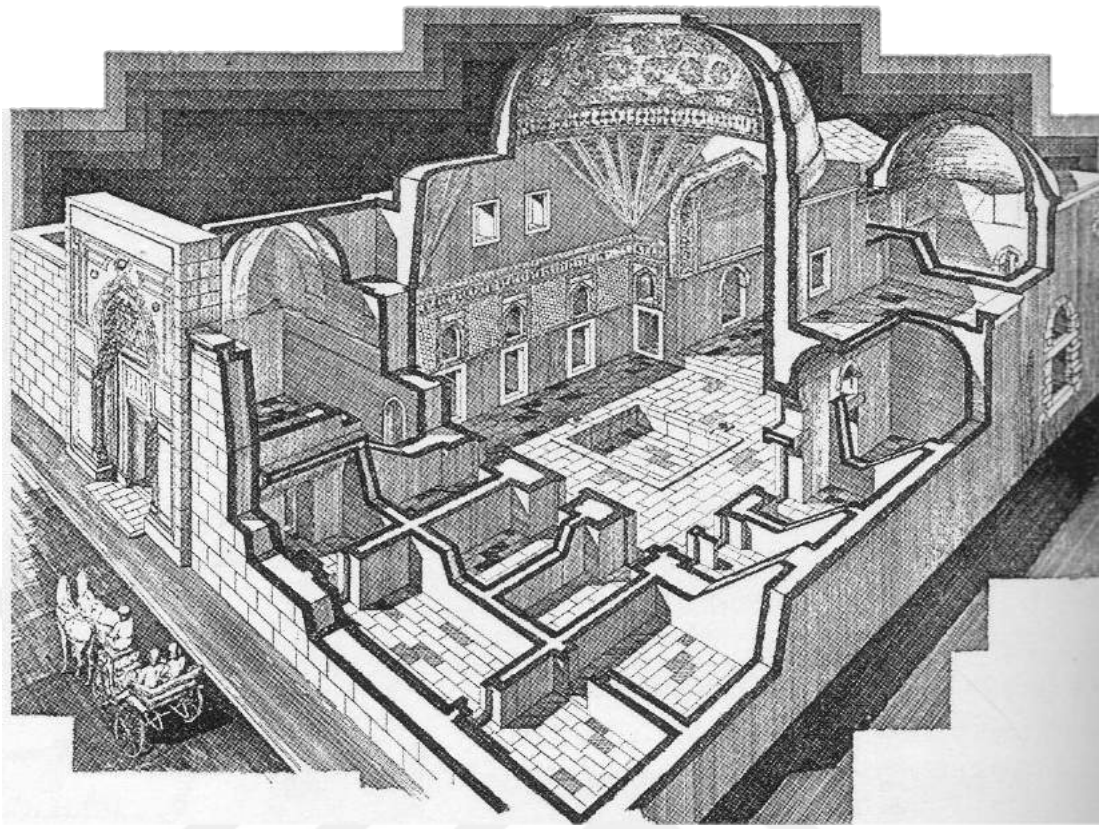


Figure 5.16 : Axonometric view from the Karatay Madrasa, Konya 1251-1253 A.D. (Drawing by M. Akok in Yılmaz et al. (2018)).

The complicated geometric design in question decorates the curved iwan surface at the one end of the courtyard. (Figure 5.17 and Figure 5.18). The adaptability to all kinds of nonlinear surfaces was one of the main advantages of the tile mosaic. Unlike larger glazed tiles, small mosaic pieces allowed the handling of curved surfaces (Figure 5.19). Nevertheless, the reverse assemblage of single pieces had to be completed on a curved mold. As soon as the pieces were assembled and formed as larger plates, they became a single module and lost their flexibility. Hence, previous to the application, a systematical surface division had to be planned in advance.

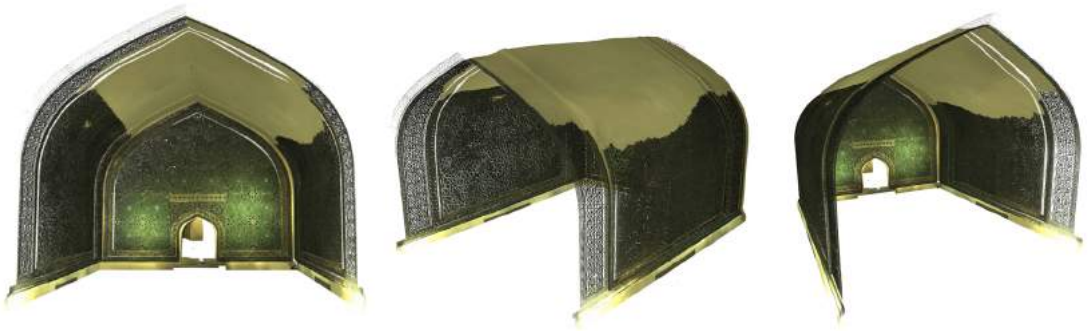


Figure 5.17 : The point-cloud model captured with a 3D laser scanner.

Figure 5.18 : The enclosed iwan, at one end of the dome in Karatay Madrasa, 1251-1253 A.D.

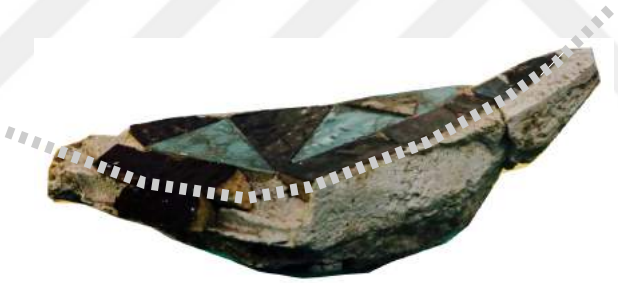


Figure 5.19 : One of the main advantages of the new material was its potential to decorate nonlinear surfaces. (The piece is on display in Ince Minareli Madrasa museum in Konya).

The complicated iwan pattern includes five-fold symmetry with pentagons. Unlike hexagons, rectangles or triangles, pentagons do not cover an entire flat surface alone (Figure 5.20). A pattern featuring regular pentagons requires additional polygons to cover the two-dimensional plane. Incidentally, a good number of continuous two-dimensional patterns with regular pentagons are not periodic. A periodic tiling is the division of the Euclidian plane with a regular symmetry. A tiling is periodic, if it contains at least two translations in non-parallel directions. That is, the pattern contains a translational unit, usually chosen to be a parallelogram with sides equal. In a periodic pattern, if we pick up a copy of this parallelogram, shift it in a certain direction and

put it to the new position, it matches up exactly with the original. Seljuks created many geometric designs that include periodic patterns with pentagons (M. Arık & Sancak, 2006) , one such attributed to Tusi atelier is illustrated in Figure 5.21. Patterns that cover the Euclidian plane in a non-repeating manner are labeled as aperiodic patterns (Kaplan, 2002, pp. 33–34)

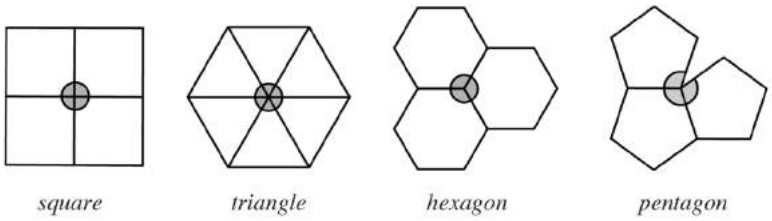


Figure 5.20 : Pentagons cannot fully cover alone the Euclidian plane.

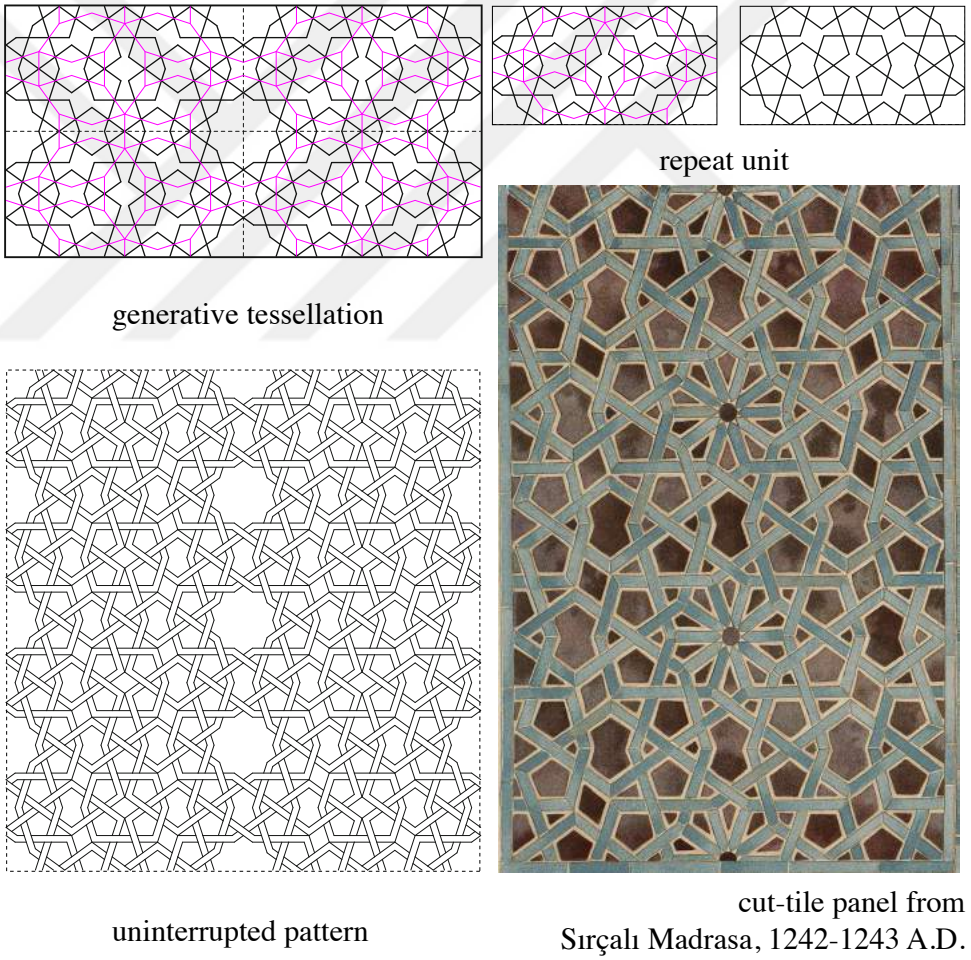


Figure 5.21 : A periodic pattern featuring ten-pointed stars by Tusi Atelier. The pattern can be split into a rectangular repeat unit.

5.4.1 Aperiodic Islamic patterns

Tiling the plane with shapes of fivefold symmetry has been a problem for centuries. Some notable attempts were made including those of Kepler, but it was the mathematician Roger Penrose who eventually introduced two aperiodic tiling in the mid 1970s (Further details on the subject are explained in Appendix A). The fact that some Islamic patterns share same features as Penrose tiling prompted many scientists to search after their generative rules (Ajlouni, 2012; Bier, 2012; Bonner, 2017; Cromwell, 2009; Lu & Steinhardt, 2007; Makovicky, 1992, 2007, 2016b, 2016a). A particularly complex Seljuk design from Gunbad-i Qabud (1196-97) in Maragha, Iran gained an enormous attention and served as focal point for many studies (Figure 5.22).

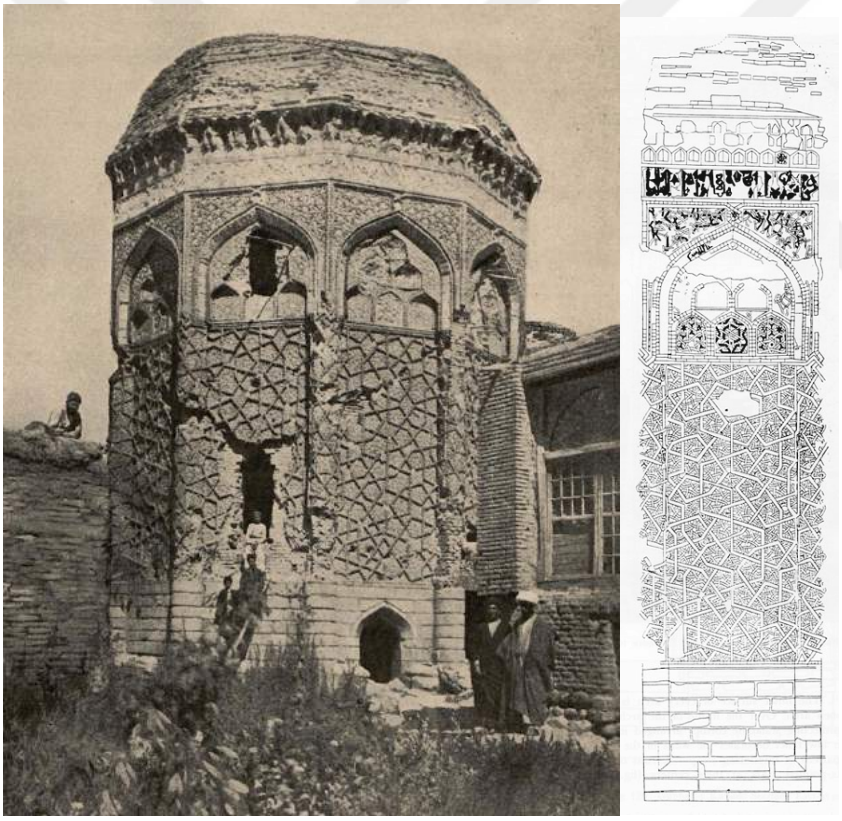


Figure 5.22 : Left: Gunbad-i Qabud (The Blue Tomb) (1196-97 A.D.), Maraga, Iran (Sarre, 1901a). Right: Drawing of one face (Bier, 2012).

The ornament executed in brickwork is complicated and does not expose a tessellation that can straightforwardly be constructed using a compass and straightedge. Makovicky (1992, p. 69), who overlaid Penrose-tilings with the geometric design in Gunbad-i Qabud, labelled the design as the first known quasiperiodic tiling in Islamic art (Makovicky, 2016a). Nonetheless, Gunbad-i Qabud gained the most widespread

interest through the study published by Lu and Steinhardt in 2007. Based on the drawings that indicate red dotted tile lines in Topkapı Scroll, Lu and Steinhardt (2007) suggested that fivefold patterns were constructed with a certain set of polygons, widely known as *girih tiles*. Girih tiles pop up out of the compass and straightedge construction of a ten-pointed star pattern. These tiles presented by Lu and Steinhardt are a decagon, a rhombus, a bowtie, and an elongated hexagon (Figure 5.23). *Girih* is a Farsi word, which means "knot" and the generative methodology behind girih tiles is actually based on the aforementioned generative polygonal tessellations (Chapter 2.1.2.2). Nevertheless, girih tiles do not have to cover the plane in a regular bases and create a regular tessellation for the generative design process. Hence, girih tiles can be arranged in a certain way in order to get a complicated pattern design as the one from Gunbad-i Qabud. Girih tiles gained the widest attention in Lu and Steinhardt (2007), since they provided the most accurate drawing technique with a certain set of tiles. Yet, Hankin (1905, 1925b) had illustrated some of the tiles before and Bonner (2003, p. 9) demonstrated some designs from the Topkapı Scroll that exposed the polygonal structure as well.

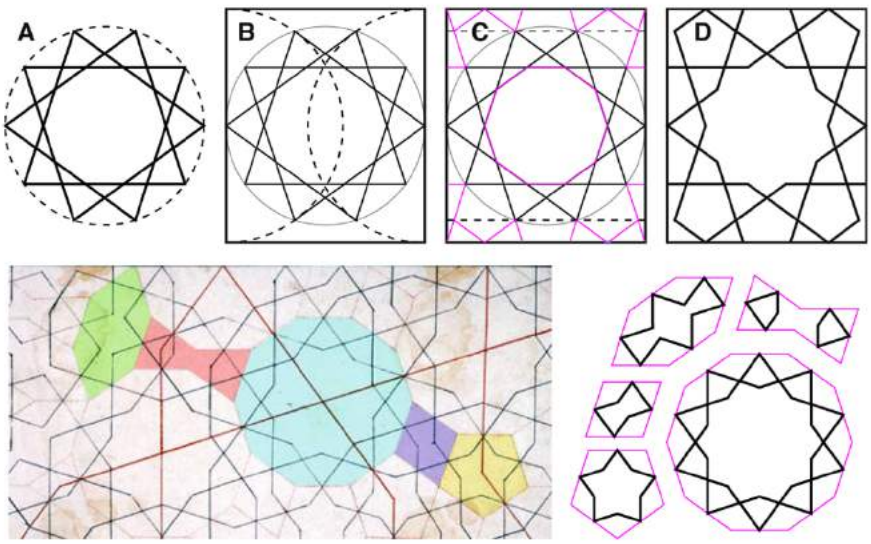
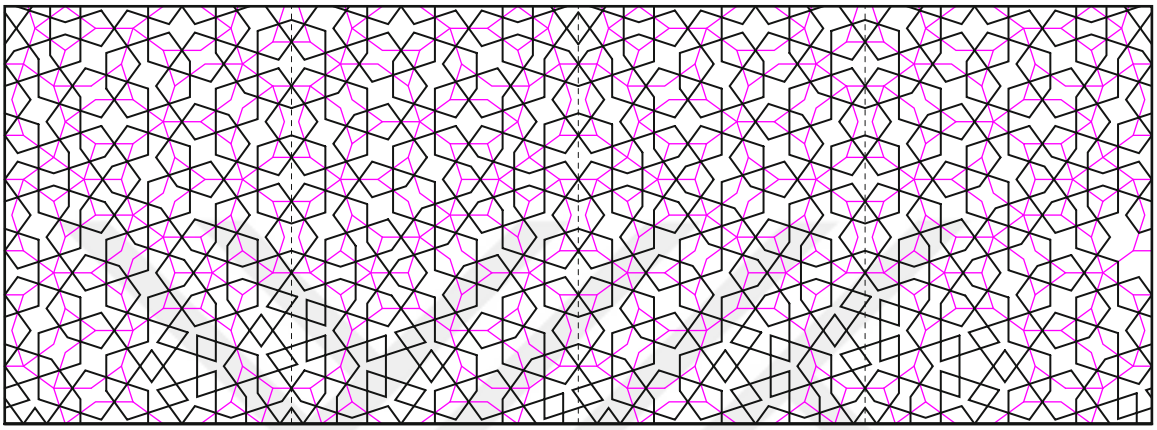


Figure 5.23 : Above: The compass and straightedge construction of a ten-pointed star leads the way to create the girih tiles. Below: The girih tiles as proposed by Lu and Steinhardt (2007): decagon, pentagon, hexagon, bowtie, and rhombus (After Lu and Steinhardt (2007, p. 1107).

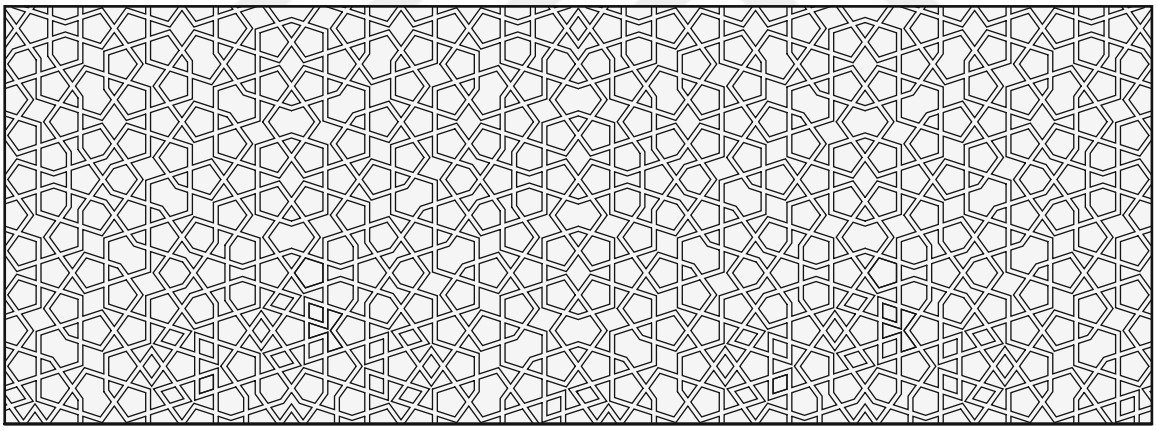
Girih tiles provide a straightforward construction technique. Figure 5.24 illustrates how the complicated pattern from Gunbad-i Qabud is constructed on the girih tiles. The usage of such irregular tiles are promoted as a "breakthrough" by Lu and

Steinhardt (2007) or a significant development in Islamic designs by Cromwell (2009). Cromwell (2009) writes:

> Regarding the tiles as the pieces of a jigsaw allows a less formal approach to composition. A design can be grown organically in an unplanned manner by continually attaching tiles to the boundary of a patch with a free choice among the possible extensions at each step. This new approach gave artists freedom and flexibility to assemble the tiles in novel ways and led to a new category of designs. (p.41)



polygonal tessellation with girih tiles



continuous pattern

Figure 5.24 : The geometric design in the Gunbad-i Qabud and its generative structure based on the girih tiles presented by Lu and Steinhardt (2007) (Drawing After Bonner (2017)).

Apart from the girih tiles that are presented by Lu and Steinhardt (2007), there are many irregular tiles upon which diverse pattern designs are derived (Figure 5.25). Yet, how artisans perceived the complete image of such complicated patterns remains still unclear (A related research is explained in Appendix A).

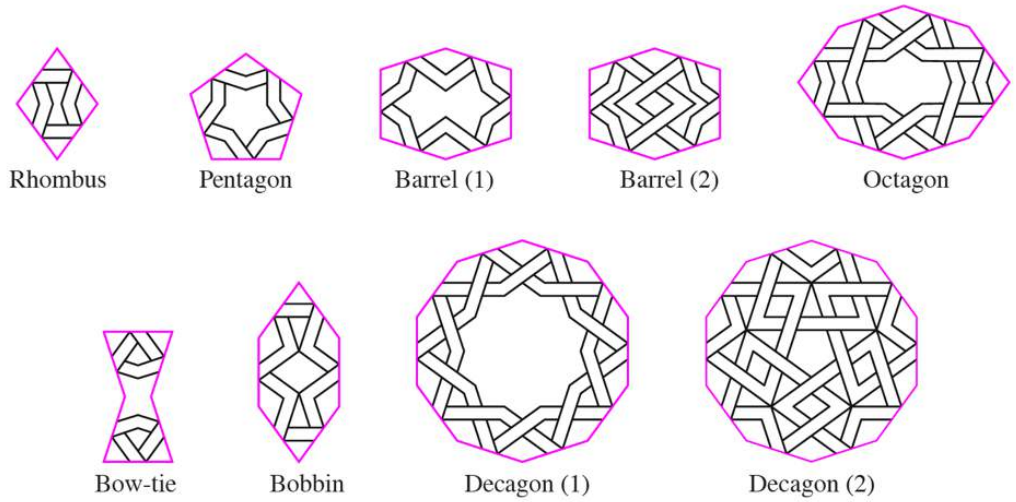


Figure 5.25 : A set of prototiles that are used to create Islamic geometric patterns (After Cromwell (2009, p. 41)).

The tomb tower in Maraga has a decagonal plan engaged by ten columns on each corner. The intricate geometric design continues across nine sides and also on the columns. Makovicky (1992, p. 69) describes it as "a grand polygonal ornamental net which envelopes the entire shaft of the building below the niches, without interruption at the corner pilasters.". There is a debate (Bonner, 2017; Cromwell, 2009) whether this complicated pattern is an example of an aperiodic geometric design as it has a repetitive structure in an overall image. The geometric design can be split into one rectangular repetitive unit, that continues in a mirrored fashion on the entire structure except on the entrance façade (Figure 5.26). Nonetheless, if the pattern would continue on the portal as well, the geometric design would not bear an uninterrupted view, as in its current situation. The even-numbered repeat by reflection of the repetitive unit on the entire surface would cause overlaps on the design, consequently result in an improvised pattern arrangement. Hence, the decagonal plan plus the exclusion of the entrance facade ensure the complexity in the design. The design in Gunbad-i Qabud has overall translational symmetry, yet the repetitive unit contains a very large amount of complicated geometric information (Bonner, 2017, p. 570). Because of this conflict between the overall systematic and the local complexity, aperiodicity remains a problematical issue in historical record. Penrose's remarks on the issue is as following: "strict periodicity seems to have played such a key role for these ancient geometric artists that quasisymmetric considerations would be unlikely" (in Bonner (2017, p. viii)).

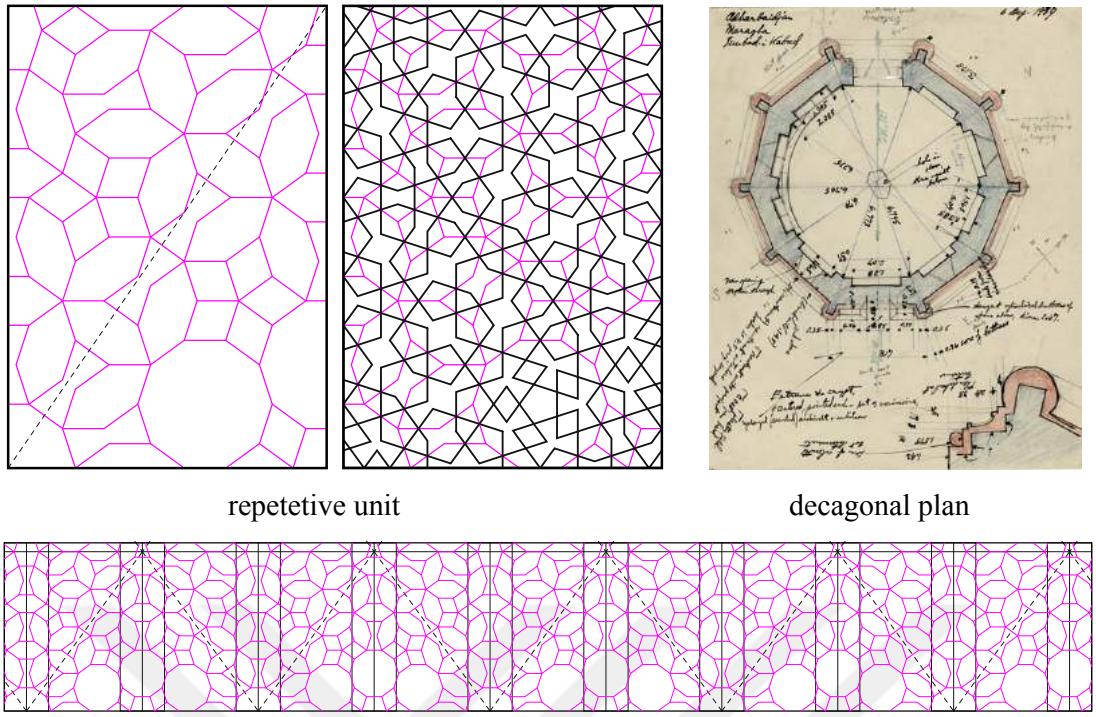


Figure 5.26 : The geometric design in Gunbad-i Qabud is built upon a repetitive unit that continues on nine facades and engaged columns. (Drawings after Bonner (2017) and decagonal plan from Bier (2012, p. 252)).

The repetitive panels expose locally a fivefold design. The flow on the columns enables the pattern to continue without any disruption and consequently, the rectangular unit is mirrored vertically in the neighboring surface. There is another significant property of the design, it exposes a secondary geometric design in each cell. Bonner (2003, 2017) labels such patterns as dual-level designs (Figure 5.27 and 5.28).

Figure 5.27 : The dual level design in Gunbad-i Qabud in Maraga, Iran (1196-97 A.D) (Photo from Mortel (2005)).

Figure 5.28 : Close ups of the dual level design in Gunbad-i Qabud in Maraga, Iran (1196-97 A.D). (Photos from Mortel (2005) and Archnet (2018)).

Figure 5.30 illustrates the secondary design more in detail. Aforementioned brickwork examples were formed in either triangular or square molds, yet the unusual geometric design in Gunbad-i Qabud cannot be divided into such regular translational units. There is no historical record on the possible surface division. Searching for visible joints on the brickwork, Sarre (1901a, p. 16) is not able to observe any divisions and highlights the complexity and spots "pentagons and irregular hexagons" as fundamental units for the generative system. From the brickwork making process and details explored in previous sections, we may infer that brickwork modules were formed as rather unusual polygonal shapes for the construction of this specific monument. Individual units were then separately applied to the surface. In fact, some of these modules that adhere rather uncommon shapes have certain names in Persian art (Sarhangi, 2012a, p. 350): The quadrilateral tile "Torange", the pentagonal tile "Pange", the concave octagonal tile "Shesh Band", the bow tie tile "Sormeh Dan" (Figure 5.30 (b) and (c)).

Individual units that wrap column edges show slight proportional distortions as they match the curvature. Hence, craftsmen ensure the continuous effect of the geometric design (Figure 5.29).

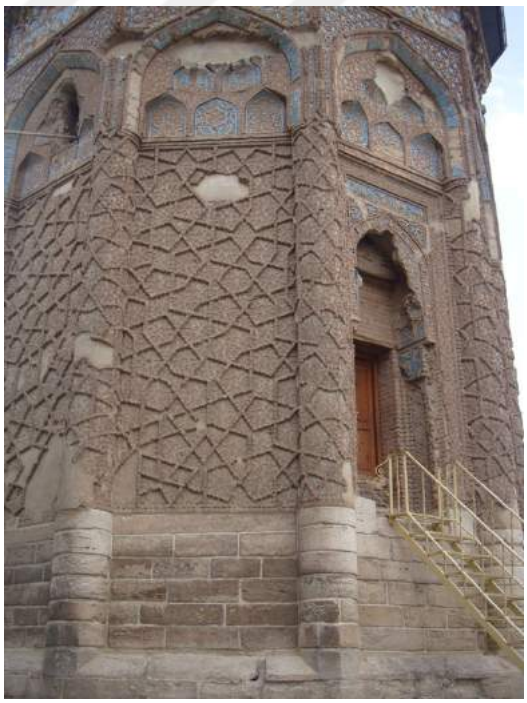


Figure 5.29 : Individual modules for the column curvature Gunbad-i Qabud in Maraga, Iran (1196-97 A.D). (Photo Archnet (2018)).

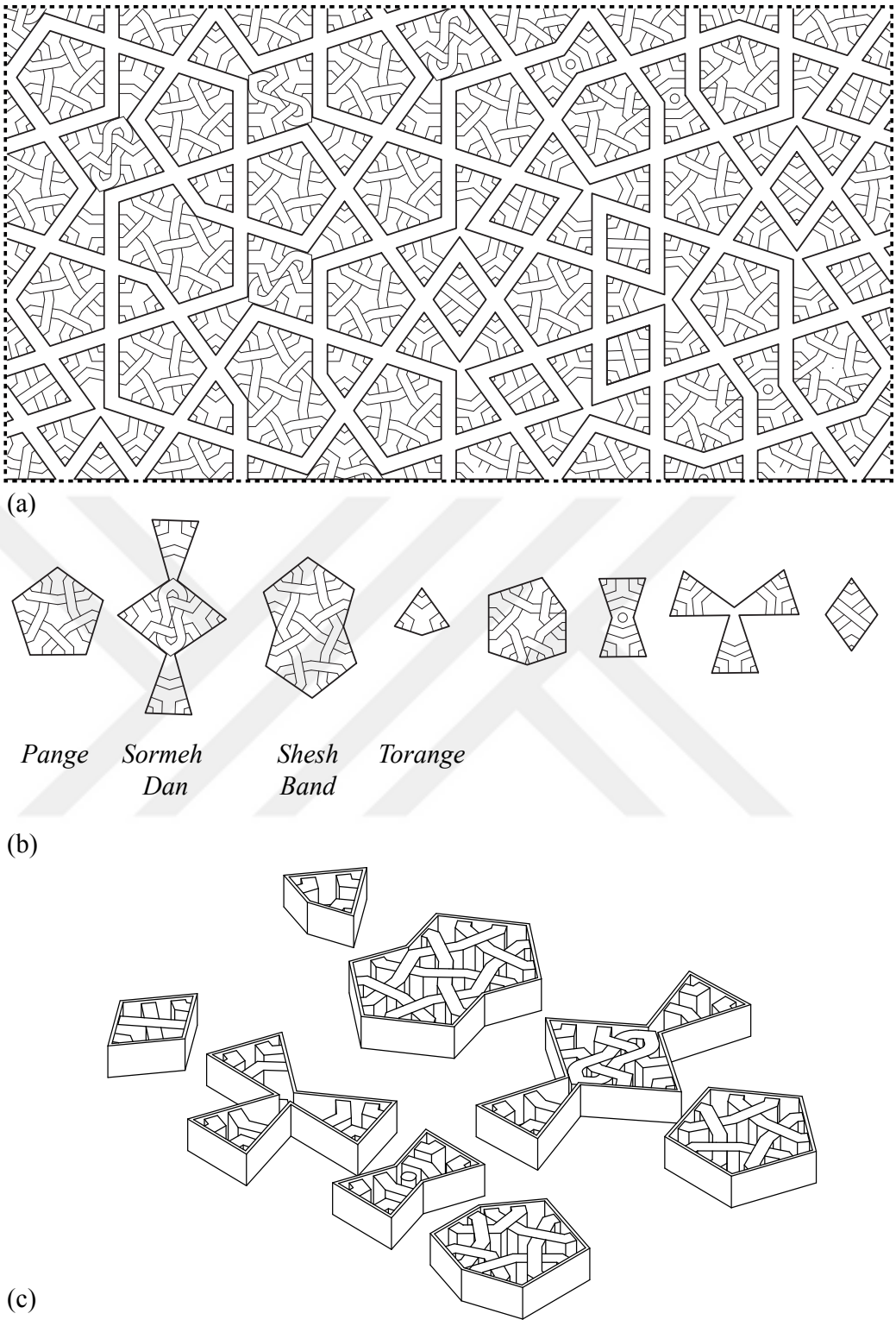


Figure 5.30 : (a) The secondary (dual-level) geometric design inside the pattern from the Gunbad-i Qabud in Maraga (Drawing after Bonner (2017, p. 201). (b) Some of the modules have certain Persian names. (c) Individual brickwork modules that might have been prepared for making.

5.4.2 The iwan pattern

The iwan pattern from the Karatay Madrasa largely copies the geometric design from Gunbad-i Qabud but involves a ten-fold rosette at the center (Figure 5.31). The complexity of the pattern has attracted several researchers (Cromwell, 2009; Makovicky, 2016a; Rigby, 2005), who tried to explain the rules behind the geometric construction. Makovicky (2016a, pp. 7–8) highlights that the pattern "is distinguished by a rich use of fivefold rotation-symmetric fill of the 10-fold stars and a frequent rotation-symmetric fill of selected pentagons".

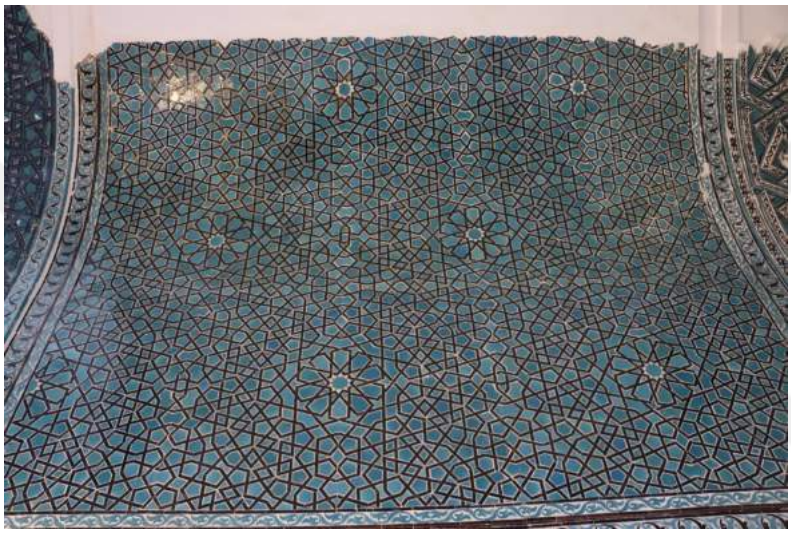


Figure 5.31 : The complicated pattern design on the iwan from the Karatay Madrasa.

Rigby (2005) proposes an alternative polygonal tessellation to build the pattern (Figure 5.32), yet the pattern can partly be constructed by using aforementioned girih tiles. Nevertheless, the tenfold rosette inside the iwan pattern requires an additional polygon to fit in between the girih tiles in Gunbad-i Qabud (Figure 5.33).

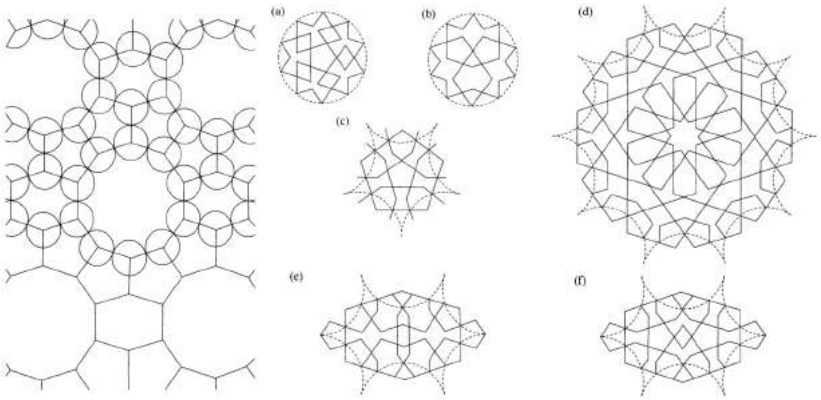


Figure 5.32 : The underlying polygonal tessellation behind the Karatay pattern as proposed by Rigby (2005) and individual motifs placed onto the tessellation.

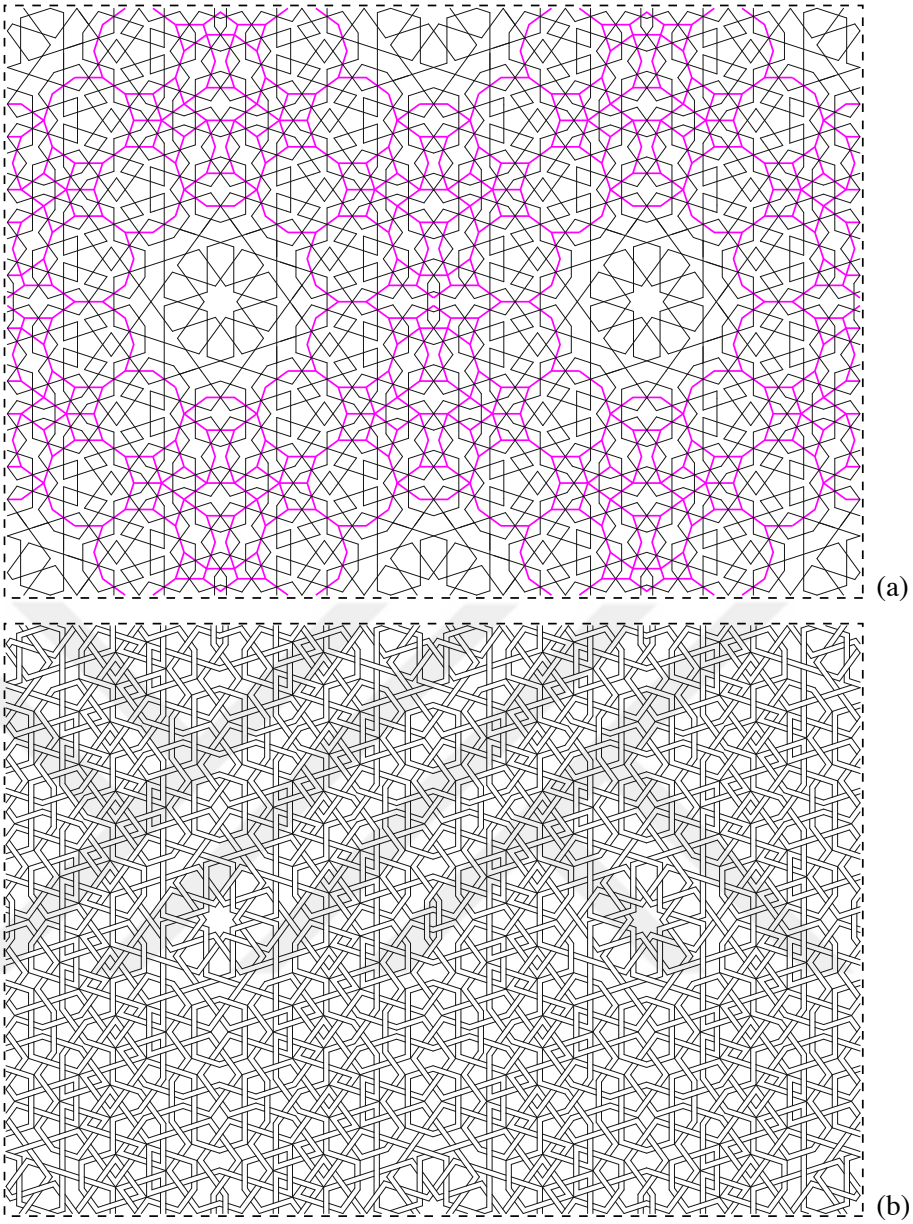


Figure 5.33 : (a) Generative process based on a polygonal tessellation by girih tiles. (b) Uninterrupted pattern (Drawings after Bonner (2017)).

The geometric design is built with a rectangular repeat unit, hence expose translational symmetry. Centers of the rosette motifs that stand in the center of the figure and in the top-left corner are diagonally opposite corners of this repeat unit (Cromwell, 2009, p. 42). In the draft pattern without line embellishments, this repeat unit can also be split into four smaller units that repeat by reflection (Figure 5.34 (a) and (b)). Nevertheless, if the mirrored multiplication process is done with a rectangular unit with interlaced lines, the one above-one under sequence is not acquirable (Figure 5.34 (c)). Hence, we are able to tell that the making is based on the copies of a panel inclusive of the larger translational unit (Figure 5.34 (e)).

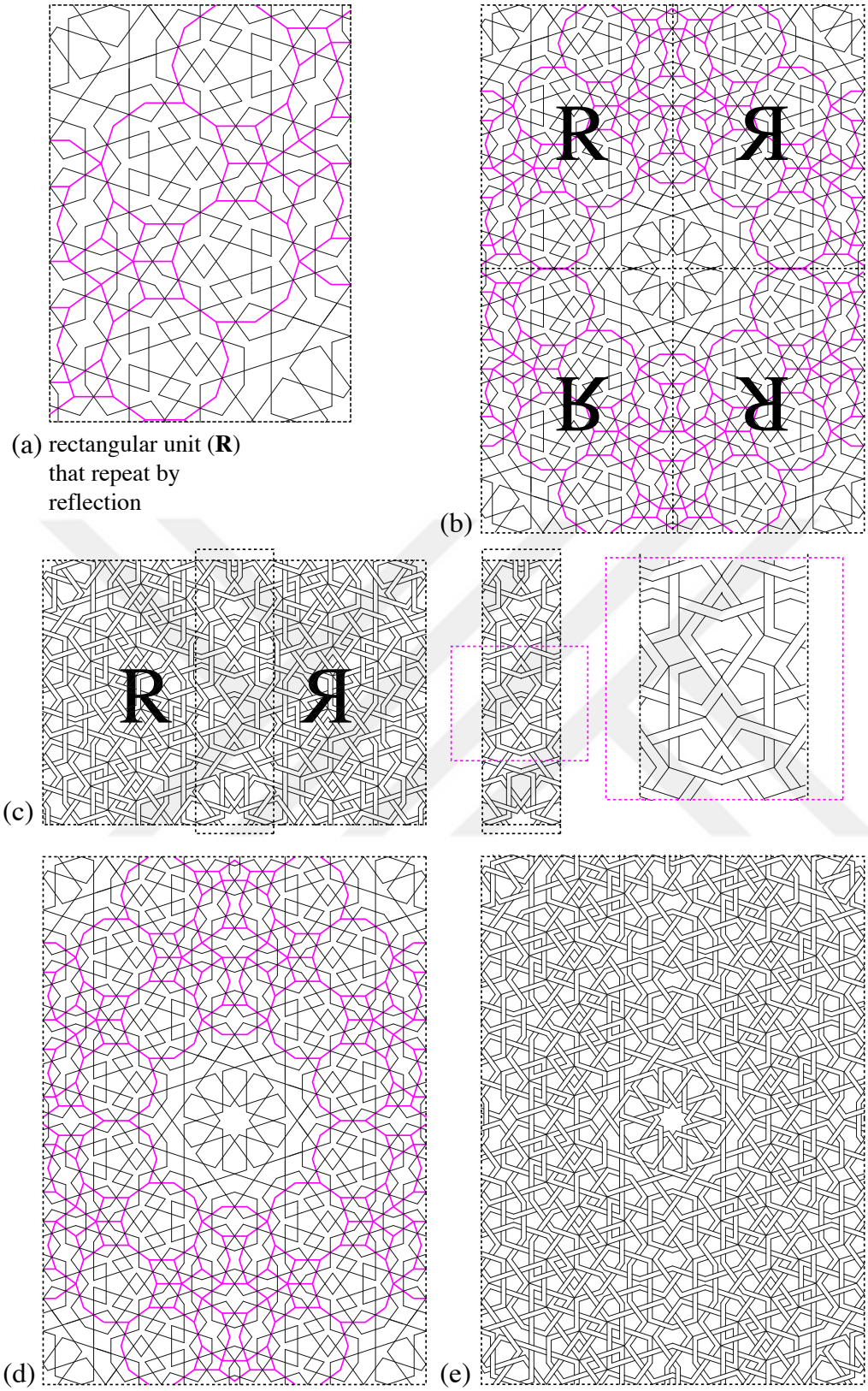


Figure 5.34 : (a) Rectangular unit that repeat by reflection. (b) The translational unit consists of four units with line drafts. (c) The reflection of the smaller unit with interlaced lines disrupts the one above-one under sequence. (d)Rectangular translational unit with pattern draft. (e) Panel used for making.

Since polygonal modules inside the rectangle unit bear a non-periodic order, a shifted copy cannot match the original. Therefore, unlike in the repetitive unit from Gunbad-i Qabud, girih tiles do not perfectly complete the translational unit. Cromwell's (2009, 2014) caregul observations expose that anomalies appear within the pattern, where repeat units are connected. Hence, we are able to speculate on the making process that is based on the multiplies of the translational unit in Figure 5.34 (e). Nevertheless, we are not able to observe any visible seam on the surface. In order to ensure an uninterrupted design, Tusi atelier coveres panel joints with additional cut-tile, that originally do not belong to the geometric pattern. Hence, irregularities appear in the design (Figures 5.35).

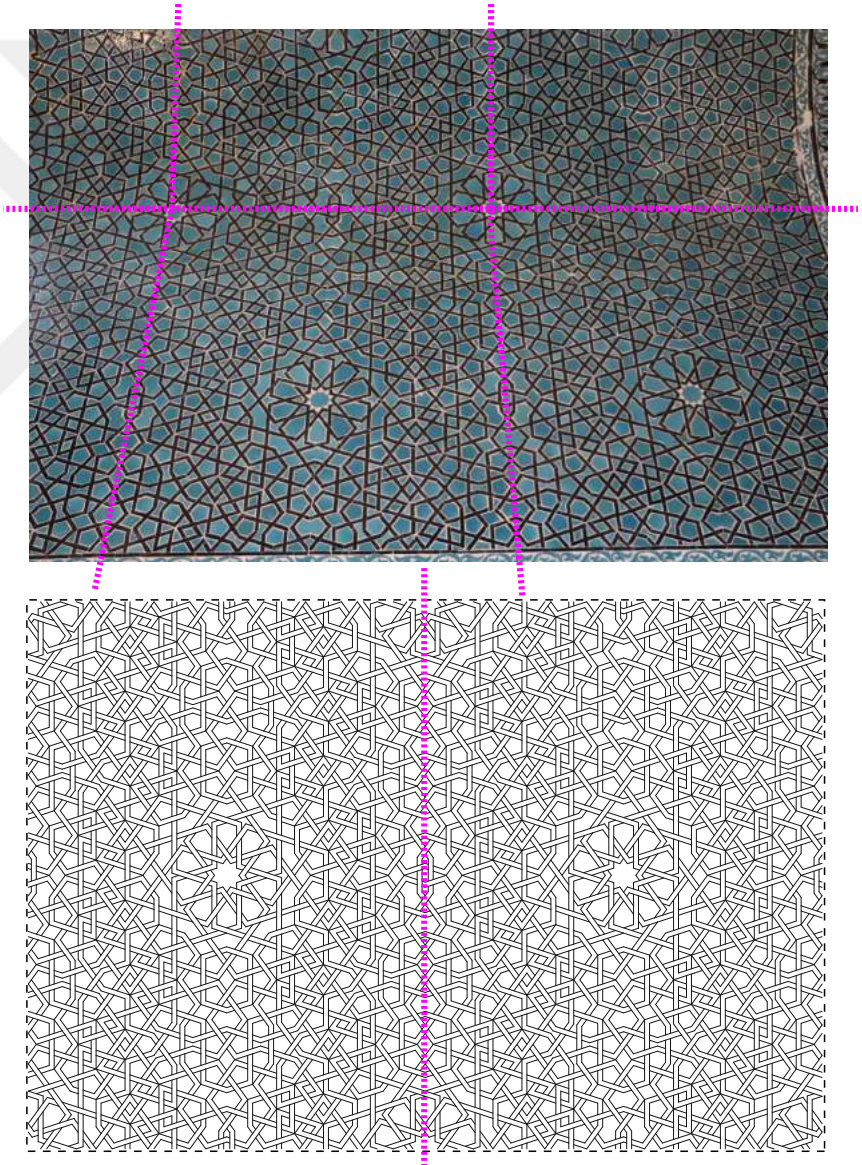


Figure 5.35 : Possible panel assemblage borders based on inconsistencies that appear in the design. Tusi atelier cover seams to ensure the uninterrupted view.

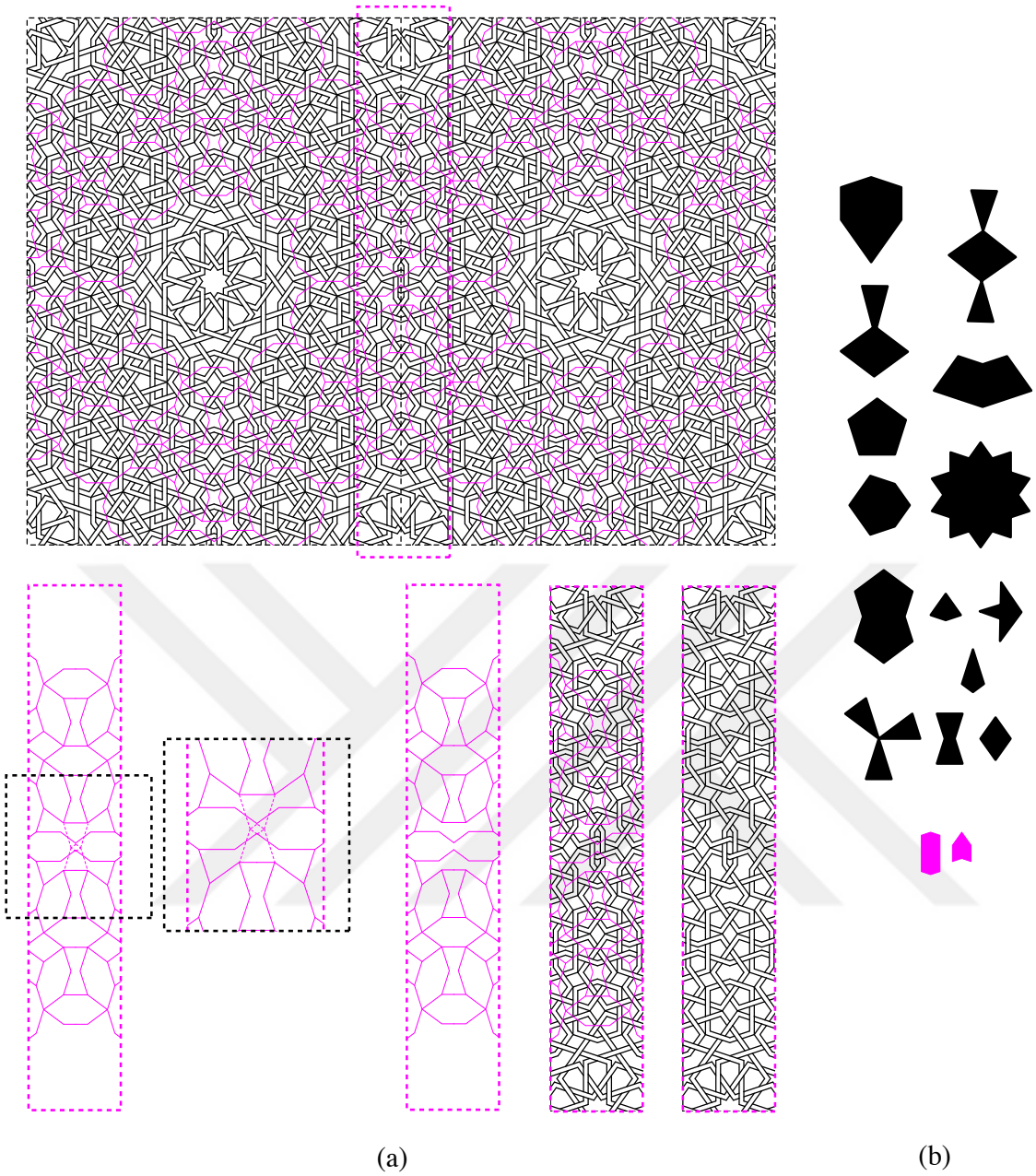


Figure 5.36 : (a) Overlaps on girih tiles mark the borders of the rectangular repeat unit that was used for making. (b) Cut-tile shapes between pattern lines. Seams are hidden by cutting extra cut tiles (marked in color) that do not belong to the actual pattern.

This particular case proves that Tusi Atelier took an extra effort to hide borders by playing with single mosaic pieces to get an uninterrupted pattern. *Cut* tile material allows to operate with single shapes and craftsmen are evidently aware of this potential and tend to improvise as they want to keep the uninterrupted appearance. Hence, this remarks that they *computed with the material.*

5.5 Geometric Patterns on Polyhedra

Several polyhedra examples in the Anatolian Seljuk art expose that medieval artisans were adorning these with pattern designs as well. The image of an uninterrupted pattern is never obtainable on a two-dimensional surface. A pattern on such a surface stretches only as far as the boundaries. Surfaces of a polyhedron, all connected, hold potential to further this appearance of uninterrupted pattern. However, the desired uninterrupted continuity also poses a mathematical challenge.

We analyze pattern covered polyhedra examples, with the purpose to understand whether the medieval artisan had a special understanding between the three-dimensional form and the pattern that adorns it. Parts of this study below are published in Özgan and Özkar (2017).

5.5.1 Patterning a polyhedron

Albeit sporadically, polyhedra can be seen in the medieval Islamic world as decorative elements of building facades and interiors (Further details on polyhedral geometry and the knowledge in Islamic world are explained in Appendix B). In most of the instances from thirteenth-century Anatolia, surfaces of polyhedra are ornamented with geometric patterns. While the making of a polyhedron with actual building materials already requires substantial knowledge of its geometry, these instances attest to the double challenge faced by the craftsmen and to their possibly integrated knowledge of polygonal and polyhedral geometry.

In the case of the polyhedron with patterns, the bounded surfaces are the polygon faces of the solid that are also connected as if a continuous surface. If the design of a polyhedron with patterns on its surfaces implies a thought-out relation between the pattern geometry and the polyhedral geometry, this involves for the craftsman, at least a visual/spatial, if not mathematical, comprehension of the geometry of the solid and that of a suitable tessellation in relation to it. Albrecht Dürer's Unterweysung der Messung ("On Teaching Measurement with Compass and Straightedge") (Dürer, 1525) is the earliest reference for drawing the developed surfaces (or nets) of polyhedra (O'Rourke, 2013, p. 77), establishing a link between two-dimensional geometry and the making of the solids (Malkevitch, 2013, p. 57). Practical applications suggest that mathematicians of the Islamic world already acquired such an understanding earlier than Dürer.

In order to keep the symmetry and not to interrupt the continuity of a two-dimensional pattern, its translation onto the surface of a three-dimensional object requires prior consideration of the symmetries of both the pattern and the object. This is easier when the three-dimensional object is a cylinder. However, it is a different matter for polyhedra, where the right angles between adjacent faces of rectangular prisms may cause interruptions in the patterns. Still, most Platonic, Archimedean, Johnson or Catalan solids hold potential surfaces for uninterrupted tessellations.

Many examples in medieval Seljuk era indicate that artisans had an awareness on the relation between the three-dimensional form and the pattern design that decorates it. Some examples of pattern covered polyhedra from the 13th century Islamic Architecture are four pairs of cuboctahedra from the Akhan Caravanserai in Denizli, the Şifaiye Madrasa in Sivas, the Eşrefoğlu Mosque in Beyşehir (all in modern Turkey) and the Beyhekim Mihrab currently in Pergamon Museum in Berlin (Figure 5.37).

Figure 5.37 : Some of the patterned cuboctahedron examples from Anatolian Seljuk Period: (a) Şifaiye Madrasa, 1217-1220, from Sivas (Photo from Ögel, 1994). (b) Akhan Caravanserai, 1253-1254, from Denizli (Photo from Ögel, 1994). (c) Beyhekim Mosque 1270-80, originally from Konya, currently in Berlin Museum of Islamic Art. (d) Eşrefoğlu Mosque 1296-1300, from Beyşehir- Konya.

Continuous and fine pattern arrangements on the four cited pairs of cuboctahedra indicate that the artisans of the period had the necessary geometrical knowledge of the cuboctahedron. The planar development of a cuboctahedron and its befitting pattern differ in depiction from its three-dimensional geometry as the adjoining angles between the edges change when the form is unfolded. In our study, we first examine ways to match the three-dimensional form and the uninterruptedness of the pattern in the examples. Each pattern is different in design, yet, the interlocking symmetry axes are preserved.

The cuboctahedron was a common form used as column capitals in Medieval Anatolia. This Archimedean solid, named as dymaxion by Buckminster Fuller, is a quasi-regular polyhedron and has six square and eight triangular faces. An uninterrupted pattern for

covering the faces of a cuboctahedron is to match a semi-regular tessellation built up with triangles and squares. Thus, the development of a cuboctahedron on the two-dimensional plane requires a pattern arrangement involving both threefold and fourfold symmetries (Figure 5.38).

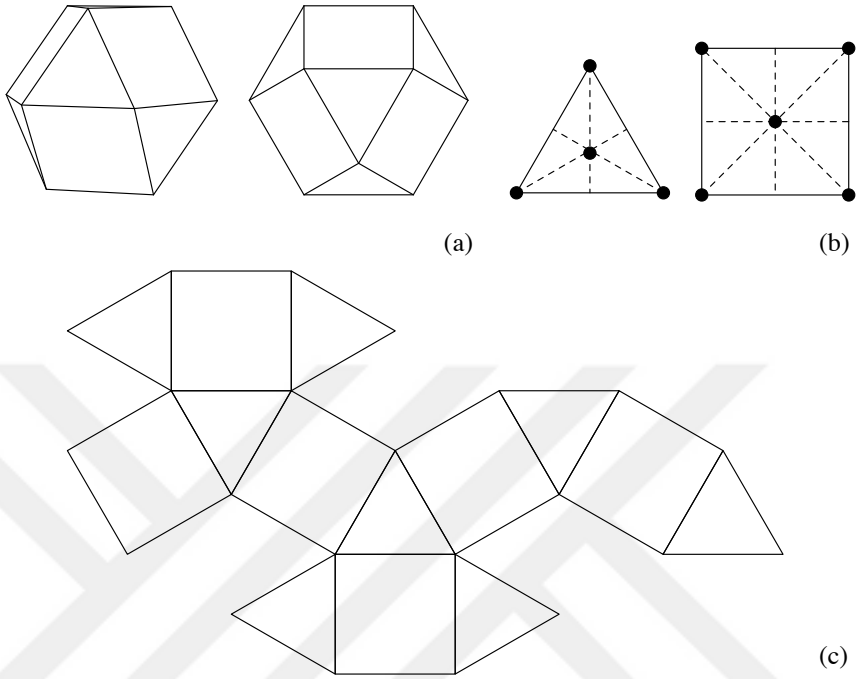
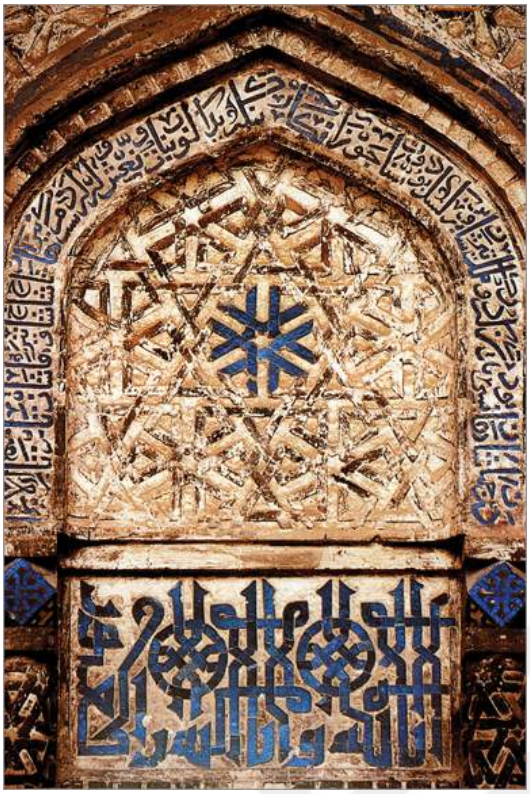


Figure 5.38 : a. (a) The cuboctahedron. (b) Triangle and square surfaces and their corresponding symmetry axes. (c) The plane development of the solid.

The cuboctahedron illustrated in detail in Figure 5.39 is one of the earliest examples of a tiling covered cuboctahedron found in Anatolia. The solid stands on the portal of the Şifaiye Madrasa in Sivas. The colored pattern lines in this example are cut out of glazed bricks, a material used prior to tile-mosaic.

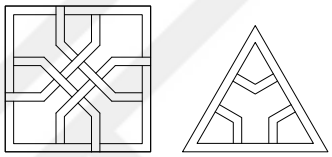
Like other tile mosaic artisans, Tusi atelier followed in this tradition of patterning cuboctahedrons. Figure 5.40 illustrates an example attributed to Tusi atelier (Meinecke, 1976b, p. 53) The mihrab is from Sırçalı Mescit, a small mosque in Konya, 1240 A.D. The patterns are abstract geometric shapes that fit into the symmetry of each face of the cuboctahedra.

Cuboctahedron patterns appear also in subsequent tile mosaic examples. In these examples, the assembly is premeditated based on an understanding of the development of the polyhedron on a planar surface. The designs also evolve into more intricate, curved and floral patterns on surfaces on the Beyhekim (1270 A.D.) (Figure 5.41) and Eşrefoğlu (1297-1300 A.D.) cuboctahedra (Figure 5.42).



(a)

(b)



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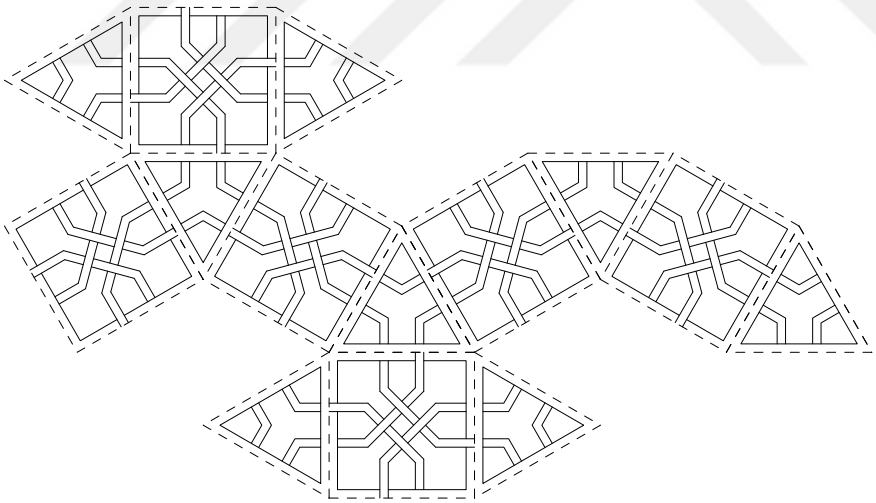
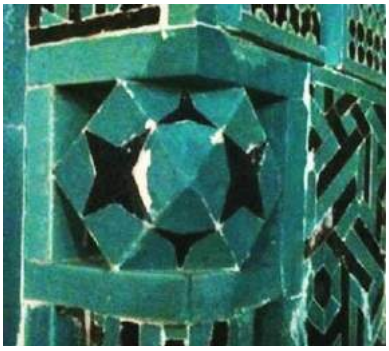


(d)

Figure 5.39 : (a) The portal decoration on the Şifaiye Madrasa in Sivas, Turkey (1217-1220 A.D.) (Photos from ("Anadolu Selçuklu Mimarisi," 2012) (b) The cuboctahedron column capital in detail (Ögel, 1994, p. 67). (c) Patterns per cuboctahedron face. (d) Pattern design on developed cuboctahedron surfaces.

(a)



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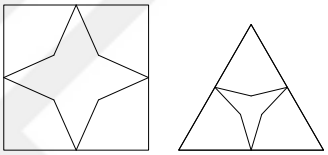

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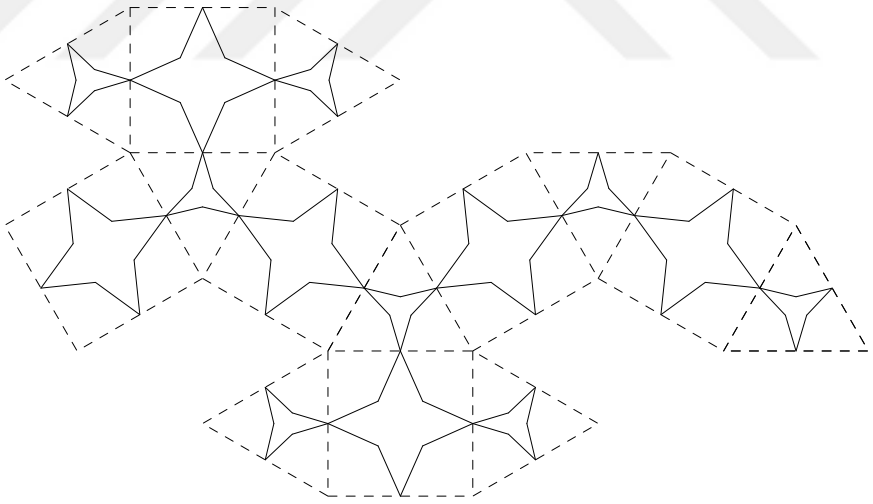
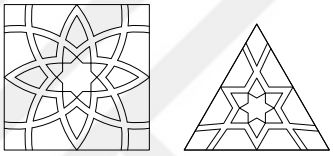


(d)

Figure 5.40 : (a) Mihrab from the Sırçalı Mescid, Konya, Turkey (1240 A.D.) attributed to Tusi Atelier (R. Arık & Arık, 2007, p. 98). (b) The cuboctahedron column capital in detail (Hisarlıgil & Bolak Hisarlıgil, 2018). (c) Patterns per cuboctahedron face. (d) Pattern design on developed cuboctahedron surfaces.

(a)

(b)



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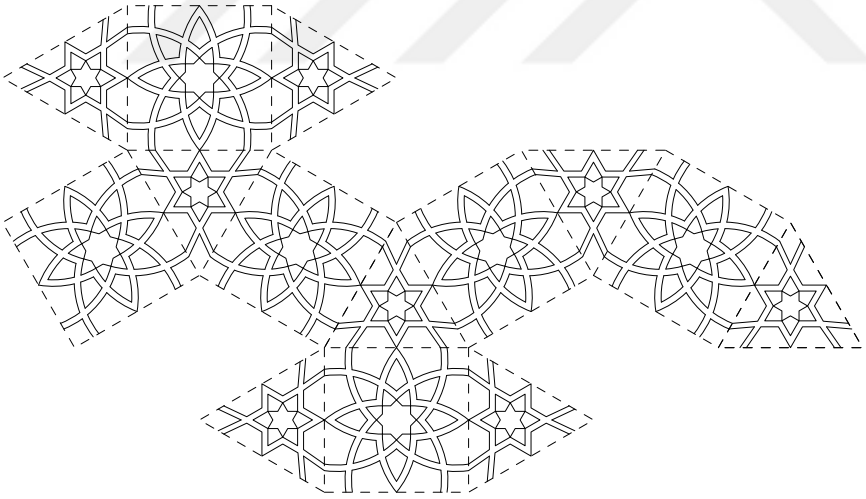
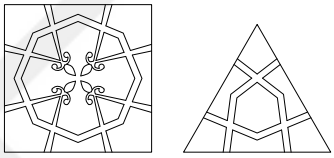


(d)

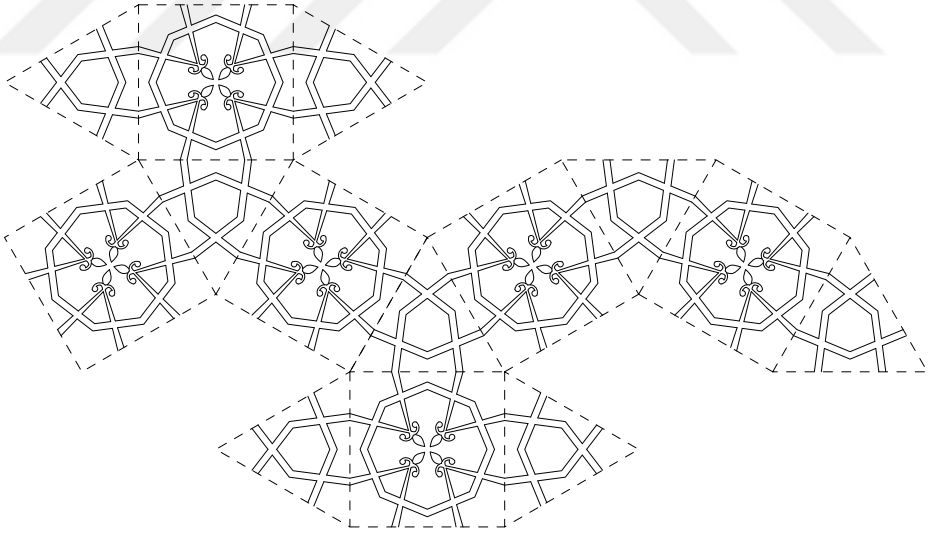
Figure 5.41 : (a) Mihrab from the Beyhekim Mosque (1270-1280 A.D.) (currently in Berlin Museum of Islamic Art). (b) The cuboctahedron column capital in detail. (c) Patterns per cuboctahedron face. (d) Pattern design on developed cuboctahedron surfaces.

(a)

(b)



(c)



(d)

Figure 5.42 : Mihrab from the Eşrefoğlu Mosque, Beyşehir, Turkey (1297-1300 A.D.). (b) The cuboctahedron column capital in detail. (c) Patterns per cuboctahedron face. (d) Pattern design on developed cuboctahedron surfaces.

The plane developments of cuboctahedra demonstrate the relations of the square and equilateral triangle faces and the patterns inside. The polygonal faces of the cuboctahedron developed on a plane surface correspond to parts of a continuous extension of geometric designs. For a generic cuboctahedron, one is able to combine individual faces to develop a wide variety of uninterrupted ornament designs on a planar surface. The pattern design in Figure 5.43 demonstrates such a design. This connection is significant in understanding that the artisan may have been aware of the relation between the two approaches to the construction of similar geometric patterns, between a tiled pattern and a pattern constructed out of a grid of interlocking circles and polygons. Nevertheless, in this case, the design on each face is a premeditated singular composition rather than a part of a pattern emerging from interlocking shapes. The continuity effect is still sustained on the plane, hence all around the polyhedron. The advantage of this approach lies in the handling of individual mosaic tile pieces and applying them onto stone surfaces as one whole shape.

Cuboctahedron examples are common in Seljuk architecture. Yet, Tusi atelier did also work with other geometries. An exceptional relief decorated geometric design is found next to the afore examined iwan pattern in Karatay Madrasa, Konya. The geometric pattern is adorned with relief that create a unique transition between three and two dimensions (Figure 5.44).

There are two instances of dodecahedron column capitals found in Anatolia. The first one is a restorated form in the Tomb of Gömeç Hatun [10] . The originality of the form is therefore questionable. The only original dodecahedron is found in the mihrab from the Mısri Mosque in Afyon and is attributed to Tusi Atelier by Meinecke (1976a, 1976b) (Figure 5.45). Unlike the cuboctahedron examples, the dodecahedron in the Mısri Mosque does not expose a pattern design. Still, individual pentagonal shapes are cut from the initial glazed tiles and are then applied on to the three-dimensional form.

[10] The restorated form is investigated more in detail in the conclusion chapter.

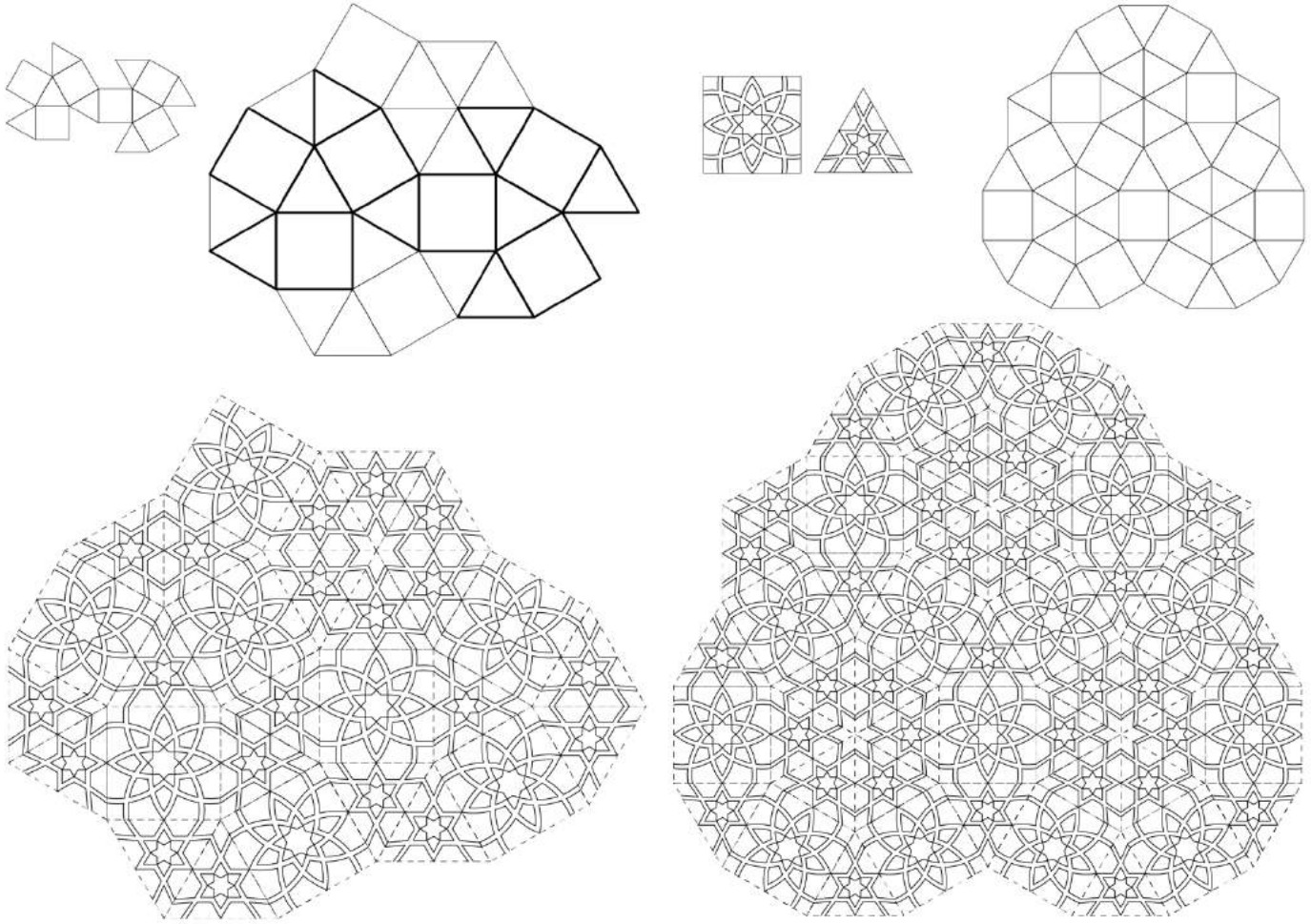


Figure 5.43 : The plane development of a cuboctahedron is extendable through its basic units and it is possible to combine a wide variety of tessellations with the polygonal surfaces.

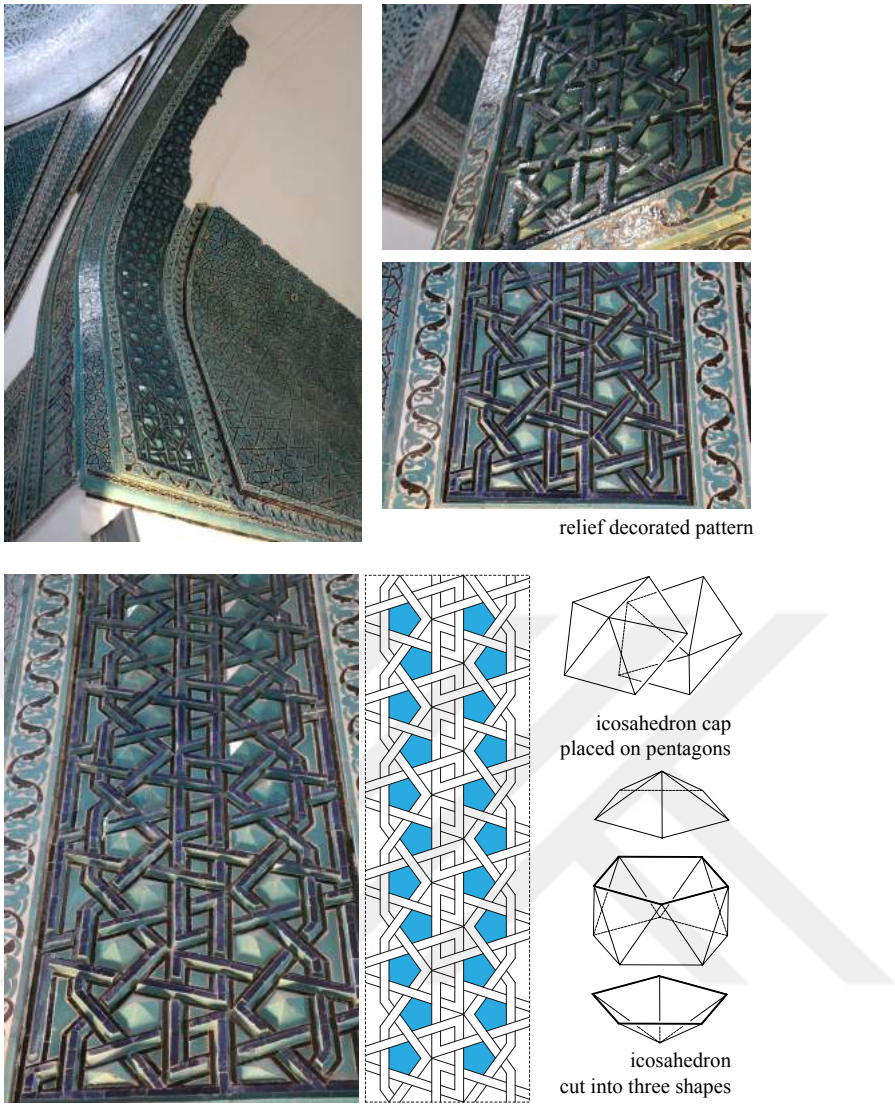


Figure 5.44 : The relief design in Karatay Madrasa, Konya, 1251-1253 A.D.

Figure 5.45 : The Mihrab of the Afyon Mısri mosque and the dodecahedron on top of the engaged column.

Designing and building a continuous pattern for a polyhedron both require a comprehension of the three-dimensional geometry of the solid as well as the symmetry axes of each individual surface. The pattern designs on various cuboctahedra and techniques applied in constructing polyhedra out of glazed tiles show that artisans of the medieval Anatolia possessed mathematical knowledge of solids and polygons together. Just as the patterns on larger architectural surfaces adhered to the borders, these designs took note of the edges of the polyhedra and how the faces connected, what angle they connected at, and their symmetry group. Edges connected continuing lines and allowed for spatially perceiving the visual field of continuity. There is an informed translation of the two-dimensional geometry to a three-dimensional one. The builders and artisans of the time and region approached their tasks in a holistic manner, understanding the relations between shapes in two and three- dimensions, and across different scales. There is no real evidence for why artisans choose specific pattern designs for particular surfaces. Yet, these inquiries on polyhedra show they possessed an enthusiasm for creating a close relation between the three-dimensional form and the pattern that adorn the surfaces.

5.6 Tile mosaic covered dome from Karatay madrasa

The tradition of decorating the surfaces of dome structures were applied by the predecessors of Seljuk Sultanate in Anatolia long before the invention of the tile mosaic material. Some of the earlier examples constructed during the Great Seljuk dynasty in Iran reveal that master builders handled the spherical surface already as early as the 11th century. Patterned dome practice was followed by many artisans from different dynasties and geographies. Nearly all of these craftsmen pioneered the application of the geometric designs onto the surfaces of various domes. The Zangids and Ayyubids in Syria, the Nasrid and Christian Mude´jar artists in Spain, the Mamluks in Egypt, the Muzaffarids and Timurids in Persia and Central Asia, and the Mughals in India built examples of various dome surfaces with greater complexity (Bonner, 2016, p. 98)

In contrast to the numerous studies conducted on the geometric patterns on two-dimensional surfaces, there has been a limited number of researches about the spherical geometry in Islamic art. Spherical geometry is of interest to us because the geometrical patterns that adorn the dome surfaces rely on it. Some of the rare but

essential literature for this topic are investigations by Ashkan and Ahmad (2010;2012) Bonner (2016;2017) , Grabar (1990), Kasraei et al. (2016), Makovicky (2016b; 2000), O'Kane (2012), Peker (1996b) and Sarhangi (1999; 2006; 2008). Due to the lack of historical sources and tangible references on the application of dome patterns, researchers could only provide some assumptions on how designers applied geometrical ornament designs onto spherical surfaces.

There are only two survived dome examples the interior surface of which are covered with tile-mosaic in Anatolia and only one is attributed to the master craftsman Muhammad Al Tusi. That is the monumental dome from Karatay Madrasa built in Konya Province in 1251-1253 A.D. (Figure 5.46). The other one is located inside the Eşrefoğlu complex built in Beyşehir, Konya Province, in the small tomb of Süleyman Bey constructed in 1301-1302 A.D. (Figure 5.47). Nevertheless, as we have already seen previously, the artisans that worked for the Eşrefoğlu complex maintained the tradition of uninterrupted pattern of the Tusi atelier. Thus, this example provides valuable information on how the spherical geometry may have been handled by Tusi. The chief aim in our analysis is to uncover the methodological approach behind the design and construction of these dome surfaces.

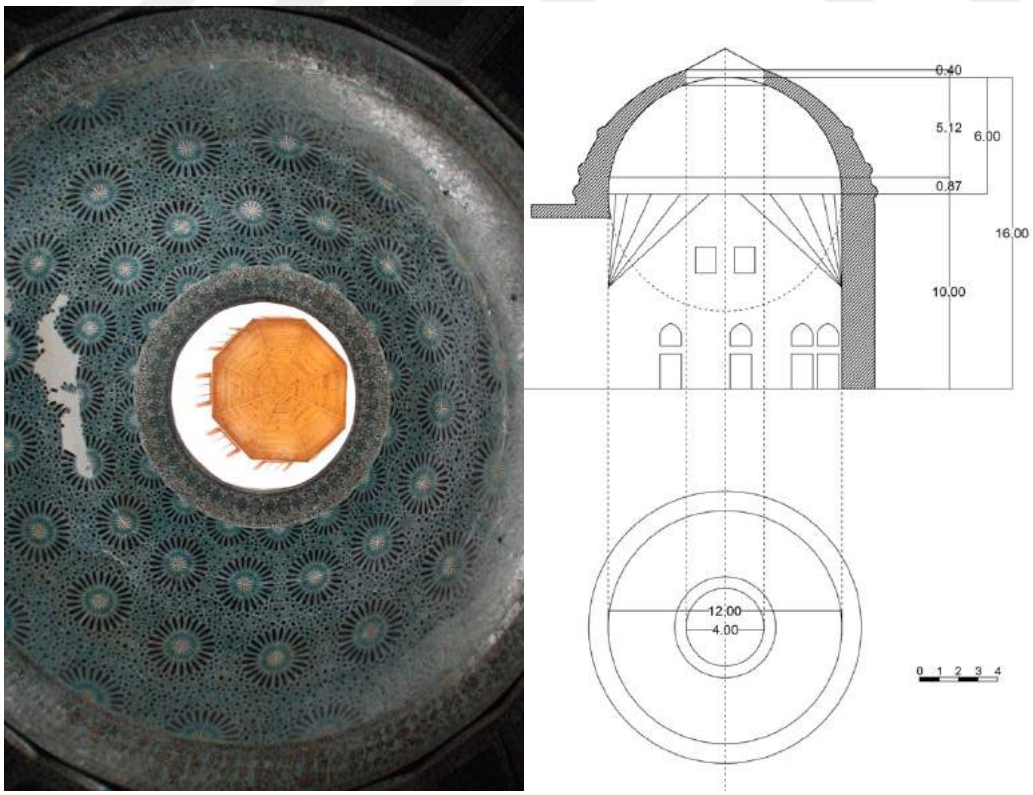


Figure 5.46 : The tile mosaic decorated dome in the Karatay Madrasa, Konya (1251-1253 A.D.) (Drawing after Mülayim (1981, p. 124)).

Figure 5.47 : The tile mosaic decorated dome in the Eşrefoğlu Tomb, Beyşehir-Konya (1301-1302). (Drawing after Akok (1976, p. 124)).

5.6.1 Works on spherical geometry in medieval Islamic world

The problems related to dome surfaces in architectural design were founded on the spherical geometry and more on the works on spherical geometry in medieval Islamic World can be found in Appendix C. Apart from the trigonometric descriptions in his other works, Al-Buzjani's aforementioned treatise *On the Geometric Constructions Necessary for the Artisan* deals with the problem of deconstructing a spherical surface into regular spherical polygons (Scriba & Schreiber, 2010, p. 175). Herein, Buzjani gives structural instructions for constructing convex regular and quasiregular spherical polyhedra (Figure 5.48). A spherical polyhedron refers in mathematics to the tiling of a spherical surface by great arcs into bounded regions named as spherical polygons.

Sarhangi (2006) reports that Buzjani's drawings for artisans were probably used as the basis to design both exterior and interior surfaces of a dome.

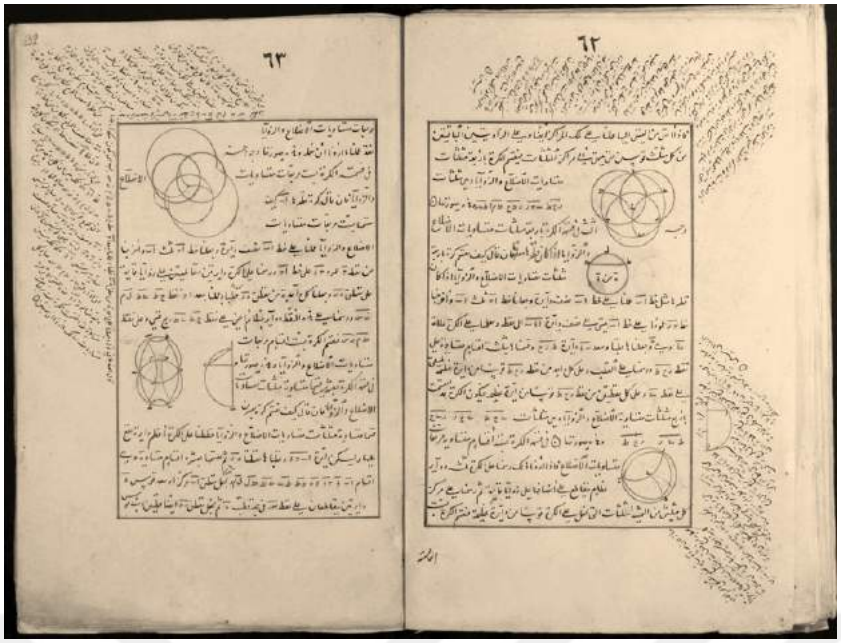


Figure 5.48 : Buzjani's (940) illustrations and descriptions for tiling of a sphere.

Buzjani's descriptions were enough to understand the basic notions of tiling the sphere. Nevertheless, it was Ghiyāth al-Dīn Jamshīd Mas'ūd al-Kāshī (14th-15th century), who gave precise method to calculate dome surfaces (Dold-Samplonius, 2003). Dold-Samplonius (1992, 2003) studies how al-Kashi measured several areas and volumes including arches, vaults, the *qubba* (the dome) and even the complicated surfaces of muqarnas. With a reference to the medieval Italian craft practices, in which artisans were paid according to the surface area, Dold-Samplonius (1992, p. 193) discusses that the same custom of estimating surface area and material amounts before construction may have existed in the medieval islamic world as well. Although, al-Kashi's descriptions are long after the constructions of the tile-mosaic domes in Anatolia, we can conclude from the material properties that the artisans in medieval Anatolia necessiated similar methods to estimate the surface area to get such intricate and precise dome decorations.

5.6.2 The craft process for dome patterning

As in two-dimensional surfaces, the compass, in some cases a rope, and straightedge practice can be used to tile the surface with a circular grid that ensure the application of various patterns on different surfaces, especially on stone. Prior to the carving, stone mason uses a rope that can be bend for modifying the circumference of the interlocked circles and consequently ensure a visual modification on the entire design to fit the

curvature. Hamzaoğlu and Özkar (2016a) illustrate such a hypothetical process both on a curved squinch and on a hemisphere. By using a rope, they draw a circular grid upon which the geometric design is generated. The rope can bend and consequently, its usage allows for small improvisations during the carving that at the end ensures the optimum visual accuracy for the design. Yet, such a process is only doable on small scale surfaces. Larger surfaces require a strategic division of the spherical surface. Furthermore, both brickwork and the tile mosaic technique, in which the materials are pre-fabricated and then applied as a sort of cladding require even more accurate calculations. Therefore, a further evaluation of the spherical surface is necessary. While the initial design of the geometric pattern may have been based on the compass and straightedge usage, the technical application must be based on a more accurate method in a larger scale. That is probably why Al-Farabi (ca. 933) declares In *Technical Geometry (Kitabu'l-Hiyali'r-Ruhanniye ve'l-Esrari't-Tabilyye fi Daku'iki'e-Hendesiyye)* that the technician cannot use the compass- straightedge method, since a precise calculation is not possible with this technique (Farabi, 1989, p. 45)

In order to approach the making process more systematically, the design process for applying a geometric design onto a dome surface can be divided into two main steps: First, ***the evaluation and accurate partition of the surface*** and second, ***applying the geometric design on the surface divisions***.

5.6.2.1 The evaluation and accurate partition of the dome surface

The handling of inner or outer facades of a dome starts initially with the appropriate tiling of the spherical surface. Medieval artisans pioneered two alternative methods for the spherical surface division: The first method is based on *tessellations of* ***vertical segments*** *on a spherical surface.* The second alternative method is *constructing a* ***spherical polyhedron*** *to tile the surface with polygonal panels* (Bonner, 2017; Sutton, 2011, pp. 52–53).

Method 1. Vertical segments

The first approach is based on the vertical divisions of the sphere (Figure 5.49). Bonner (2016, 2017) entitles the vertical repetitive parts as radial gore segments and suggests that this methodology became the historically preferred system for applying both geometric and floral patterns onto domical surfaces. The vertical tiling can be

processed in different degrees that results in a variety of possible segmentation numbers. Among many possibilities, Islamic artisans frequently favored 8-, 12-, and occasionally 16-fold radial divisions. In some rare cases 6-fold and even 24-fold segmentation were historically preferred. As a rule, the divisions follow the symmetry of the supporting chamber, from which the dome raises. Since most structures are based on a square floor plan, the segmentation numbers are almost always multiples of four (Bonner, 2017, p. 531).

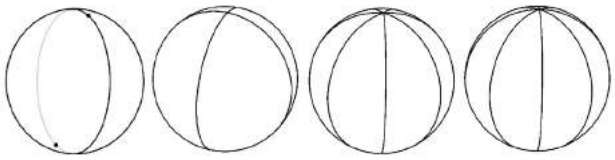


Figure 5.49 : Different degree vertical tiling of a sphere.

There are a variety of typologies and geometries of domes in Islamic architecture. The typologies that had gradually developed from the early Islamic epochs through to the late Islamic era alter across diverse dynasties. In their studies Ashkan & Ahmad (2009, 2010, 2012) examine the history, morphology and typologies of various Persian domes over historic era and illustrate some of their various typological features. Vertical tiling of the sphere is an appropriate methodology that can be used for both inner and outer surfaces of diverse dome typologies. Figure 5.50 shows one of the earlier Persian examples from the Masjid-I Jami in Ardistan (900-1199). The ornament of this dome repeats upon a sixteen-fold radial segmentation.

Figure 5.50 : The domical façade decoration of Masjid-i Jami' of Ardistan based on a sixteen fold vertical segmentation (Left: image from “Aga Khan Visual Archive,” 2017).

The vertical division of the spherical surface was a methodology that was documented by Hankin (1905) during his visits to India as well (Figure 5.51).

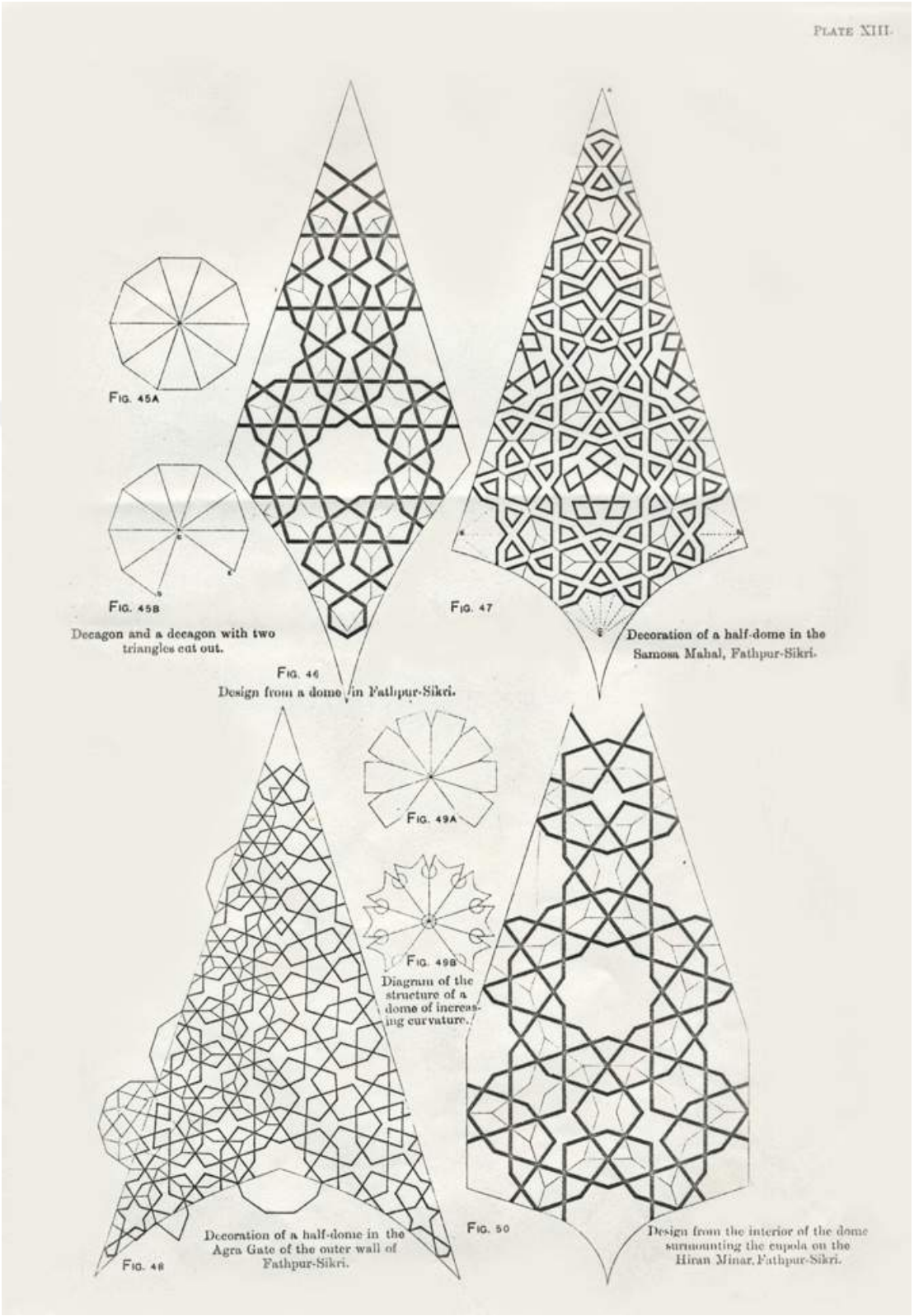


Figure 5.51 : Hankin's illustrations of domical patterns on vertical surface segments (Wade, 2018).

Similarly, the famous masonry domes in Cairo from the Mamluk dynasty were ornamented upon these systematic as well. Bonner (2017, p. 533) and Sutton (2011, pp. 52–53) both argue that the stone carved dome of Qaytbay mausoleum in Cairo, Egypt (1472-1474) was handled through a vertical partition, which at the end results in orange slice alike curved geometries. Sutton suggests a sixteen-fold division (Figure 5.52), while Bonner illustrates only eight radial segments. On the other hand, O'Kane (2012, p. 9) examines the mims and ashlars in detail and notices a twenty-sided division that results in alternating patterned segments that differ from each other. The geometric pattern incorporates arabesques (floral motif) that is combined with a geometric design. In Bonner's (2017, pp. 533–535) version with eight regular slices, each radial segment consists of a ring of half 10-pointed stars at the base of the dome, then by a ring of 9-pointed stars, followed by a ring of distorted 5-pointed stars, and one and two half of the arrays of a16-pointed star at the apex (Figure 5.52, illustration at the bottom).

Subsequent polychromatic cut-tile mosaic examples from Persia and Central Asia were based on the vertical division of the sphere as well. In his research Bonner (2017, pp. 536–538) demonstrates many examples. Among these are the interior dome of the mausoleum of Turabek-Khanym in Konye-Urgench, Turkmenistan (1370) and Safavid dome at the Aramgah-i Ni'mat Allah Vali in Mahan, Iran (1610) (Figure 5.53 and 5.54).

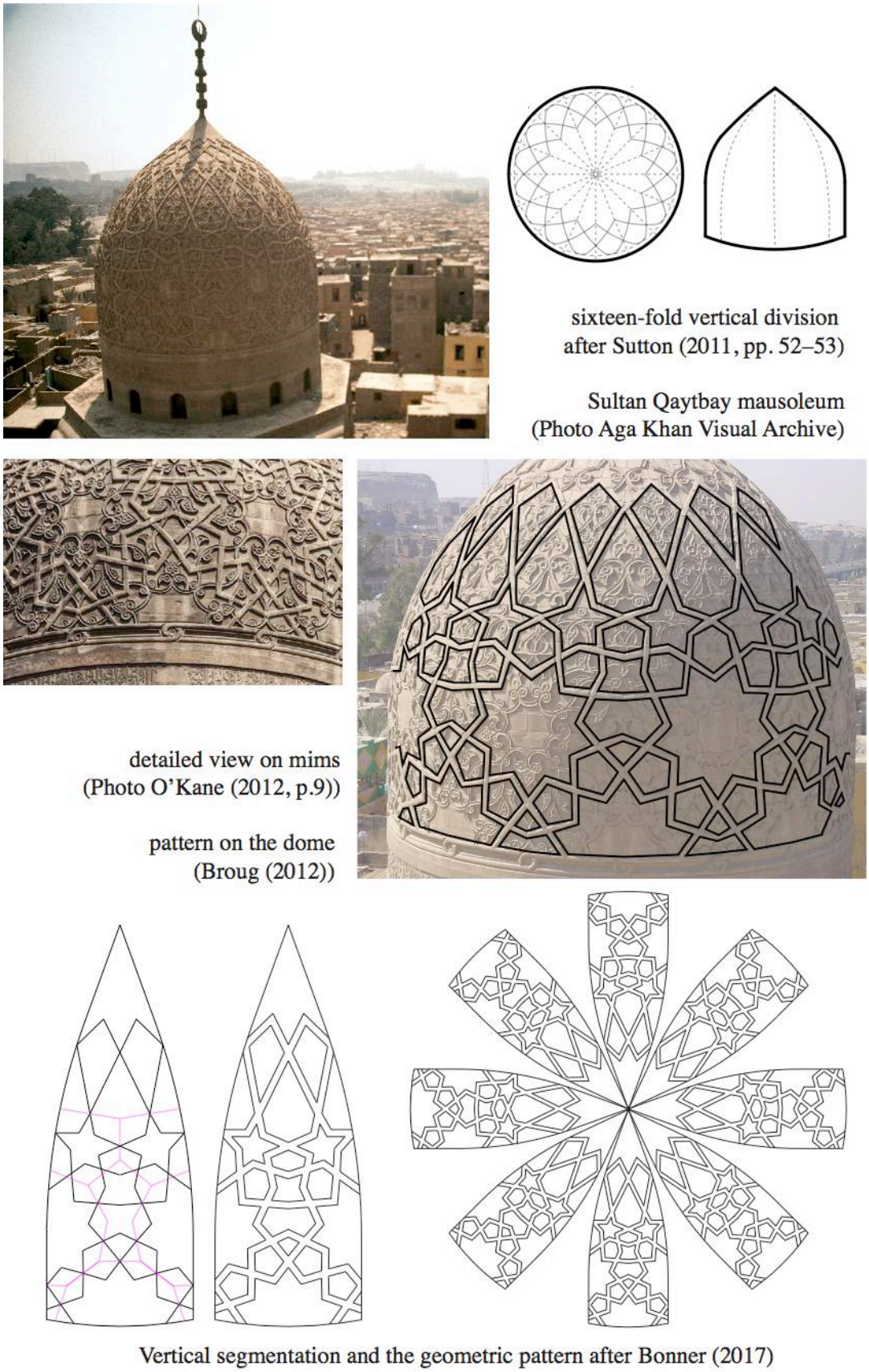


Figure 5.52 : The Dome of the Sultan Qaytbay mausoleum in the northern cemetery in Cairo (1472-1474).

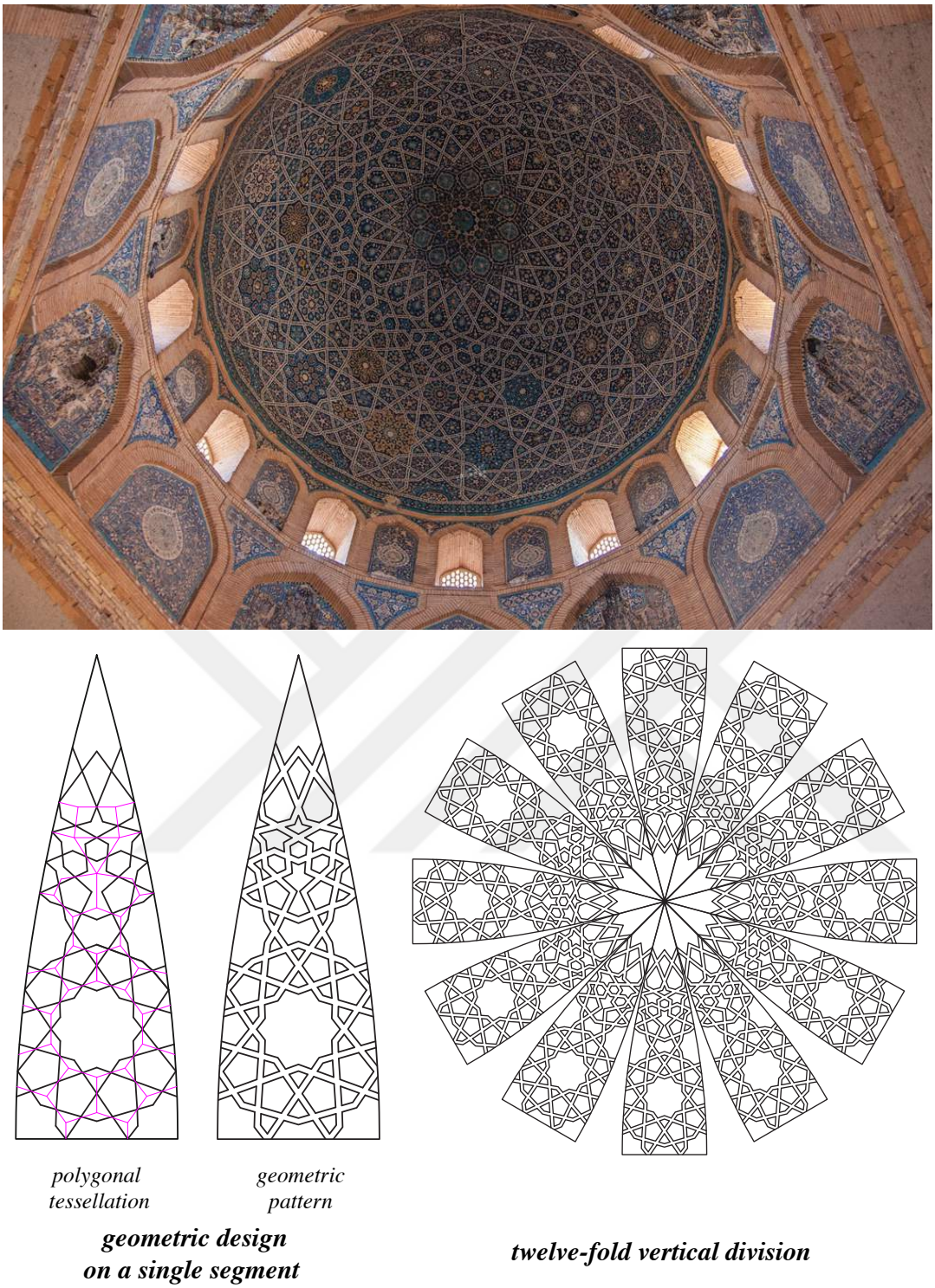


Figure 5.53 : The cut-tile mosaic dome from Turabek-Khanym mausoleum in Konye-Urgench, Turkmenistan (1370 A.D.) (Photo and drawings after Bonner (2017).

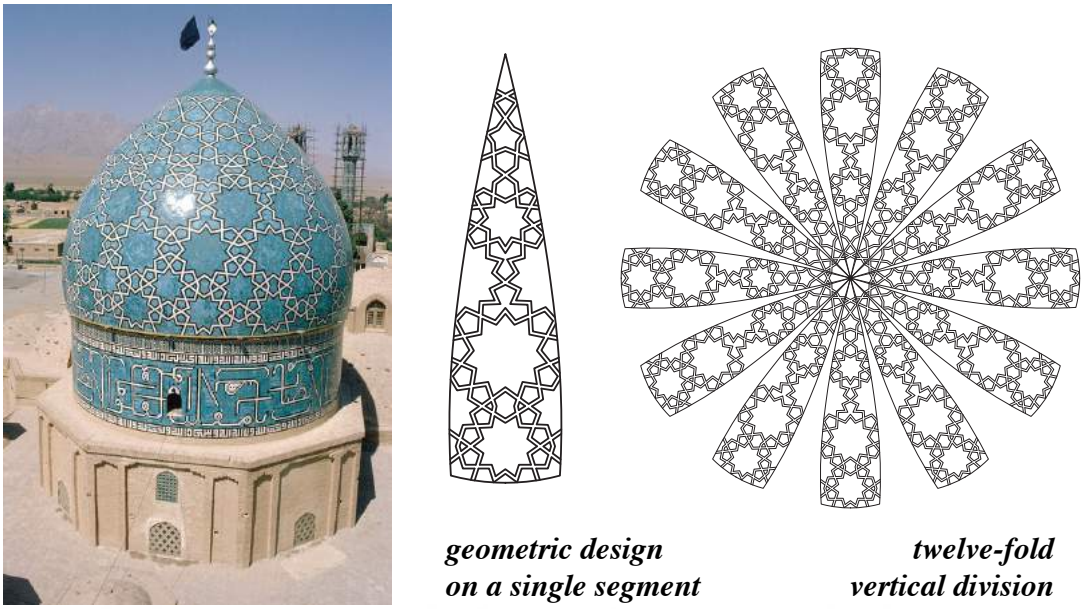


Figure 5.54 : The Safavid dome at the Aramgah-i Ni'mat Allah Vali in Mahan, Iran (Photo and drawings after Bonner (2017).

Arthur Pope's (1981) historical photos from the reconstruction of the Madar-ı Shah Madrasa Dome built in Isfahan Province of Iran in 1930 reveal how this method was used on similar Persian examples. The dome of the madrasa was built originally between 1706 and 1714. A photo prior to the restoration (Sarre, 1901b) shows a clear vertical cut that separated the fallen cladding parts. Thus, the reconstruction was probably completed faithfully in regard to the original methodology. In the historic photograph, the craftsmen decorate separate vertical strings by first drawing the partial geometric design on the surface. The cut-tile pieces are then arranged up-side down on this curved mold (Figure 5.56). The maker's mark- each individual slice of the dome- is visible in the present-day photographs that depict the general view of the dome (Figure 5.55).

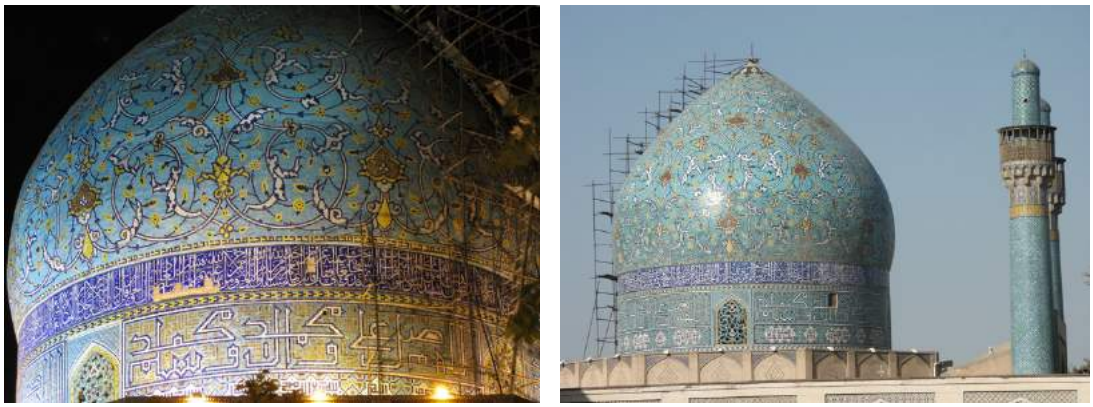


Figure 5.55 : Current view of the dome decoration from Madar-ı Shah Madrasa. Vertical slices are visible (Photos, "Panoramio Photo Sharing", n.d.).

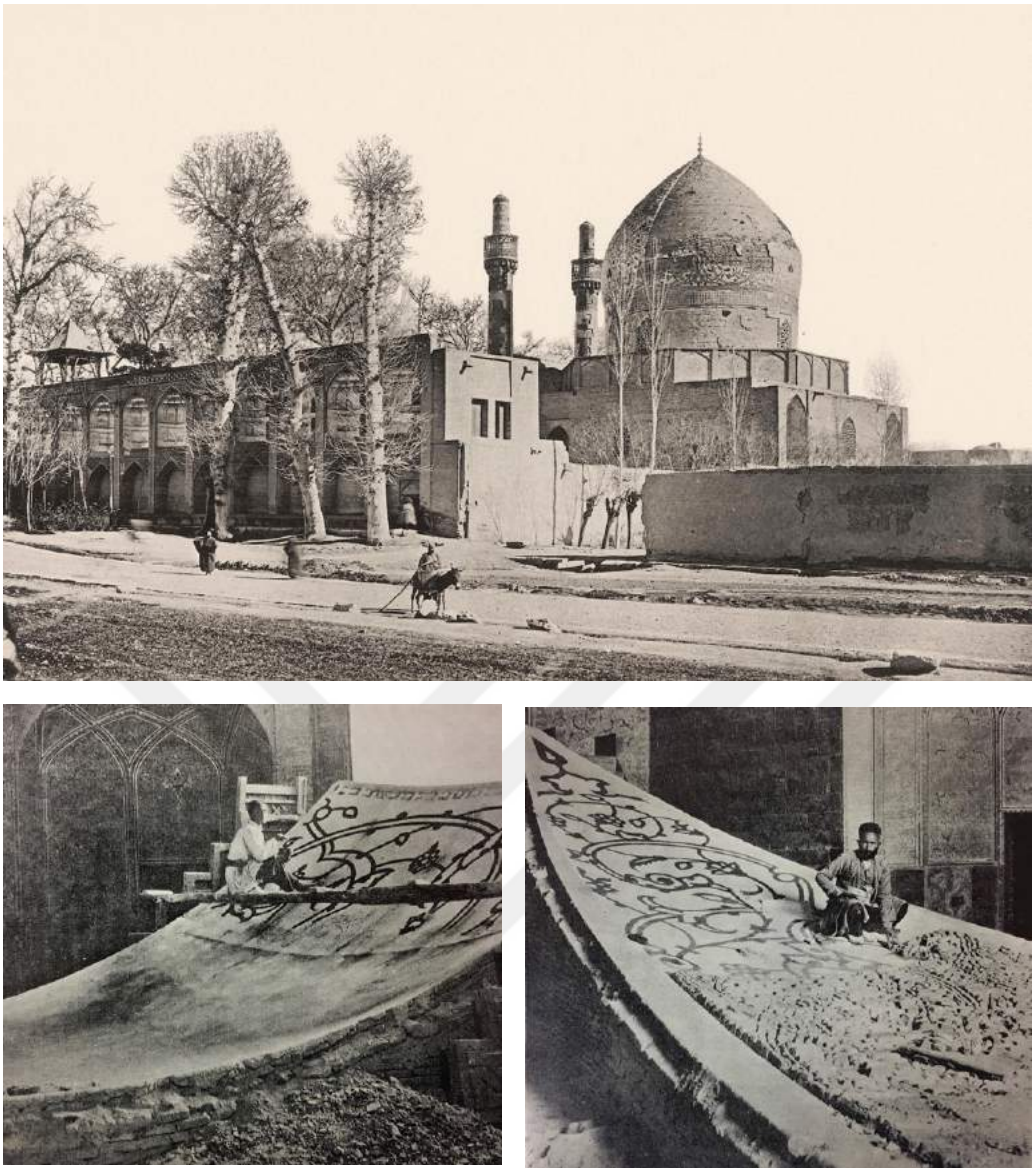


Figure 5.56 : Above: Madar-ı Shah Madrasa in Isfahan, Iran (1706-1714 A.D.) before the restoration in 1930 (Photo from Sarre (1901b)). Below: Two craftsmen decorate the individual slices of the spherical surface for the reconstruction. (Pope and Ackerman, 1981).

Method 2. Spherical polyhedra

Medieval artisans developed another approach for surface handling, which was rather infrequent compared to the previous method of vertical tessellations. The non-Euclidian geometric designs described as "the most geometrically interesting and visually arresting" by Bonner (2017, p. 537) employ spherical polyhedron geometry. A spherical polyhedron is set of arcs on the surface of a sphere that correspond to the projections of the edges of a polyhedron (Weisstein, 2018). Figure 5.57 illustrates the spherical symmetries of the five regular solids.

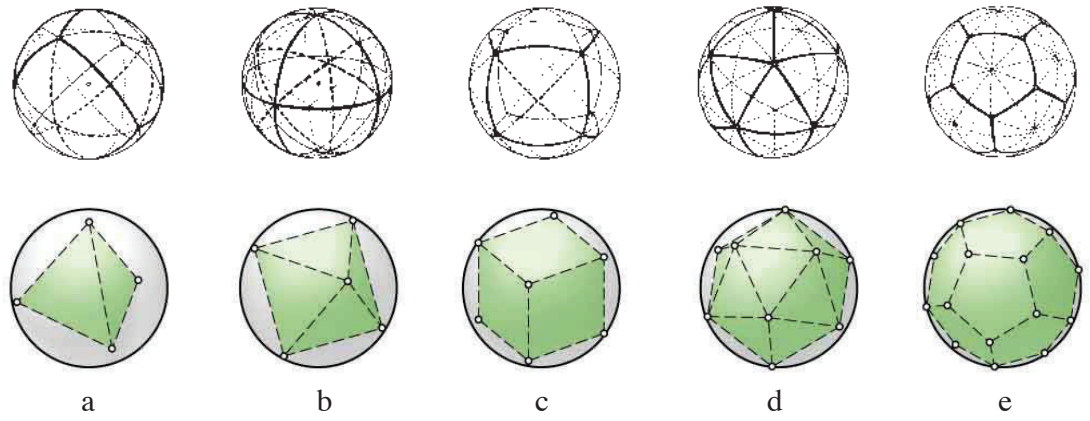


Figure 5.57 : Spherical polyhedra of the five platonic solids (a) Tetrahedron (b) Octahedron (c) Cube (d) Icosahedron and (e) Dodecahedron (adapted from Critchlow (1969, p. 35) and Pottmann (2007, p. 86)).

The first aparent historical usage of this method is observed in the north dome chamber of Terkan Khatun in the (Masjid-I Jami‘) Friday Mosque of Isfahan, Iran (1088-1089 A.D.). The geometric pattern arrangement on the dome uses a polyhedron as the base of its repetitive schema. The brickwork is arranged in fivefold rotational symmetry, so that the surface division clearly stands as a spherical dodecahedron (Figure 5.58). The foundations of this mosque complex dates back to the 8th century Seljuk period, but the northern dome was built later in 11th century. Özdural (1998, p. 708) stresses the exceptional precision of the construction. The ornamental brickwork of the later added chamber differs from earlier building parts in the mosque (Bonner, 2016, pp. 60–61). The fact that the brickwork on this famous dome is applied as a perfect spherical dodecahedron has led the famous French archeologist and art historian Oleg Grabar (1990, pp. 64–65) to assume that the renowned mathematician-astronomer Omar Khayyam, who lived in Isfahan at that time, was the designer of the chamber. Özdural (1998) has examined this hypothesis through a detailed investigation and stated that the proportional accuracy of the architectural geometry is evidence that Omar Khayyam designed the North Dome (Özdural, 1998, p. 711) In another research that concentrates principally on the geometric designs that embellish the surfaces of the same chamber that is enclosed with the North Dome, Bonner (2016, p. 101) highlights the uniqueness of the 5-fold geometric design on the dome surface as well. He discusses that this arrangement together with the seven neighboring geometric patterns placed within the eight recessed arches, stand as a notable advance in the historical

development of the ornamental tradition and Omar Khayyam might be responsible for this advancement.

Figure 5.58 : The north dome chamber of Terkan Khatun in the (Masjid-i Jami‘) Friday Mosque of Isfahan (Photo: “Wikimedia Commons", 2004).

Subsequent dynasties followed this pattern design tradition founded on an underlying polyhedron (Bonner, 2016). The examples produced by different cultures from the medieval Islamic world alter in terms of the materials used. Each material corresponds to another type of polyhedral geometry. The hemisphere-shaped stone surfaces seen occasionally in Anatolian Seljuk buildings are based on spherical polyhedron geometry as well. Figure 5.59 demonstrates such a geometric design that uses a spherical dodecahedron as its repetitive schema. The hemispherical stone stands on the entry portal of the Sahib Ata Mosque in Konya (Bonner, 2017, p. 539).

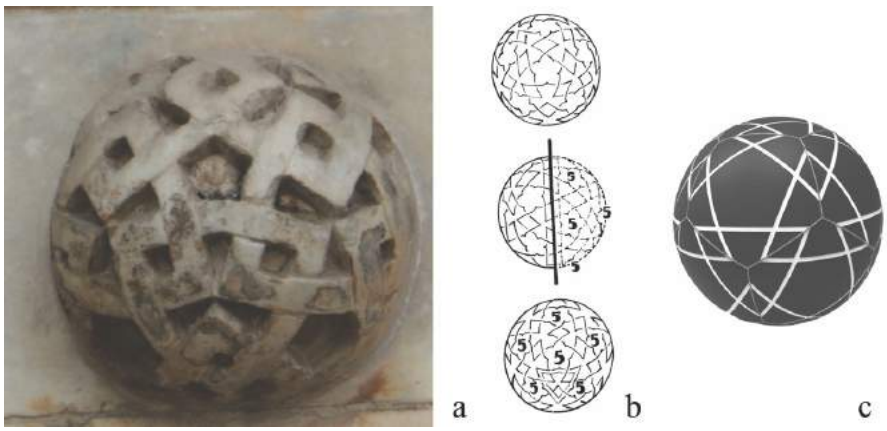


Figure 5.59 : (a) The hemispherical stone from Sahib Ata Mosque. (b) The geometric pattern (Schneider, 1980) and (c) Geometric design on a spherical dodecahedron (Bonner, 2017).

The scale of the hemispherical stone from Sahib Ata Mosque is much smaller compared to a dome surface, yet underlying geometrical principle for surface division is the same. Nevertheless, the making is based on stone carving, hence the craftsman responsible for the design can improvise on the pattern as he carves. On the other hand, a surface decoration based on a cladding system with panels requires more accurate calculation for surface divisions. As it was discussed in preceding chapters, as the base material change, the detailing level in the making alters correspondingly. While most of the carved or brickwork examples depend either on Platonic or Archimedean polyhedron, domes that are covered with a cladding, an additional layer of severe materials (ceramic, tile mosaic, wood parquet etc.) require more sophisticated geometries, consequently more subdivisions of the spherical surface. The geometries illustrated in Figure 5.60 present higher levels of tiling of spherical surfaces and corresponding polyhedra together with their surface development. Planning a spherical surface division based on such complicated geometries is more efficient for decorative cladding.

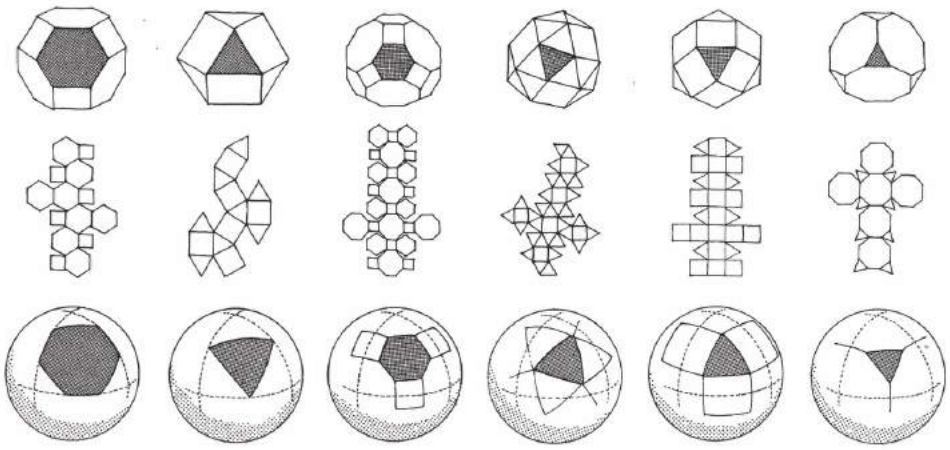


Figure 5.60 : Different subdivisions of the spherical surface, that tile the surface with polygons. From left to right truncated octahedron, cuboctahedron, truncated cuboctahedron, snub-cube, rhombicuboctahedron, truncated cube (Critchlow, 1969, p. 34).

In this sense, while a stone mason would be satisfied with a lower level degree of division (e.g. a spherical dodecahedron) tile mosaic artisans in the Seljuk Sultanate of Rum based their designs on complex polyhedron geometries. Obviously, the main aim of the tile mosaic artisan is to cover the surface without any apparent gaps. Such inquiries have parallels with the studies on map projections in geography and cartography. The aim in cartographic studies is to find a way for gathering the

minimum distortion of the earth surface when it is represented as a two-dimensional planar map, in other words a method for mapping the earth accurately. There are diverse methods for mapping the earth and transform its three-dimensional surface to create a flat map sheet. This mathematical transformation is commonly referred to as a map projection.

The most recognized projection is the stereographic projection that maps circles on the sphere to straight lines or circles on π and preserves angles. The making of the Astrolabe, a tool used in the medieval Islamic world to calculate the exact positions of the stars works also on stereographic projection principle (Berggren, 2003, p. 165). Stereographic projection maps circles on the sphere to straight lines or circles on π while preserving the angles (Figure 5.61, illustration on the left). Many authors refer to astrolabes as analogue computers and they can be used to solve any problems in spherical astronomy. Inherited from the ancient Greeks , early as in the 8th century, Ibrahim al-Fazari, an Arab mathematician and astronomer constructed the first astrolabe (Gericke, 2014, p. 197). Astrolabes had a significant role in astronomical studies, hence they were advanced through centuries (Figure 5.61, figures on the right). Among many other areas, astrolabes were also used to calculate building heights. However, they did not provide the most elegant solutions for all problems (Berggren, 2003, p. 173)

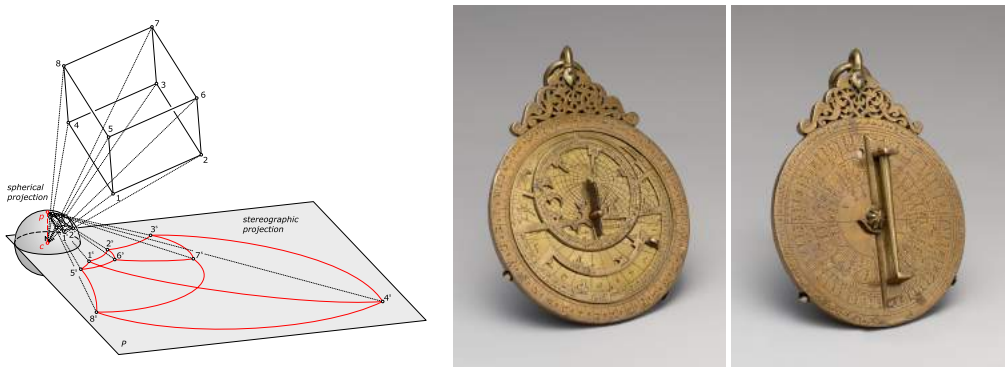


Figure 5.61 : Left: Spherical projection (Pottmann, 2007, p. 68). Right: A 13th century astrolabe by Umar ibn Yusuf ibn ‘Umar ibn ‘Ali ibn Rasul al-Muzaffari (MET Museum).

Another possibility of a map projection system is building a projection of a world map onto the surfaces of an unfolded polyhedron, similar to the second domical patterning method that is based on spherical polyhedron. The result can then be unfolded and flattened onto a flat surface. Such a method is similar to that used by the famous

architect Buckminster Fuller to create the Dymaxion map in 1943 (Figure 5.62). The quest on how a visual data can be reliably translated from the spherical surface onto a flat display with only one side and with minimum distortion was considered by Fuller a design problem that resulted in this particular project. In this cartographic accomplishment, the world map is projected onto the surface of an icosahedron that can be later unfolded to the triangular faces and flattened to two dimensions (Edmondson, 1987, pp. 263–265). In a relevant study Wijk (2008), proposes a new projection method, in which the globe is projected onto the surfaces of a myriahedron, a polyhedron with a very large number of faces. All these instances of map projections are based on the use of recursively subdivided polyhedra.

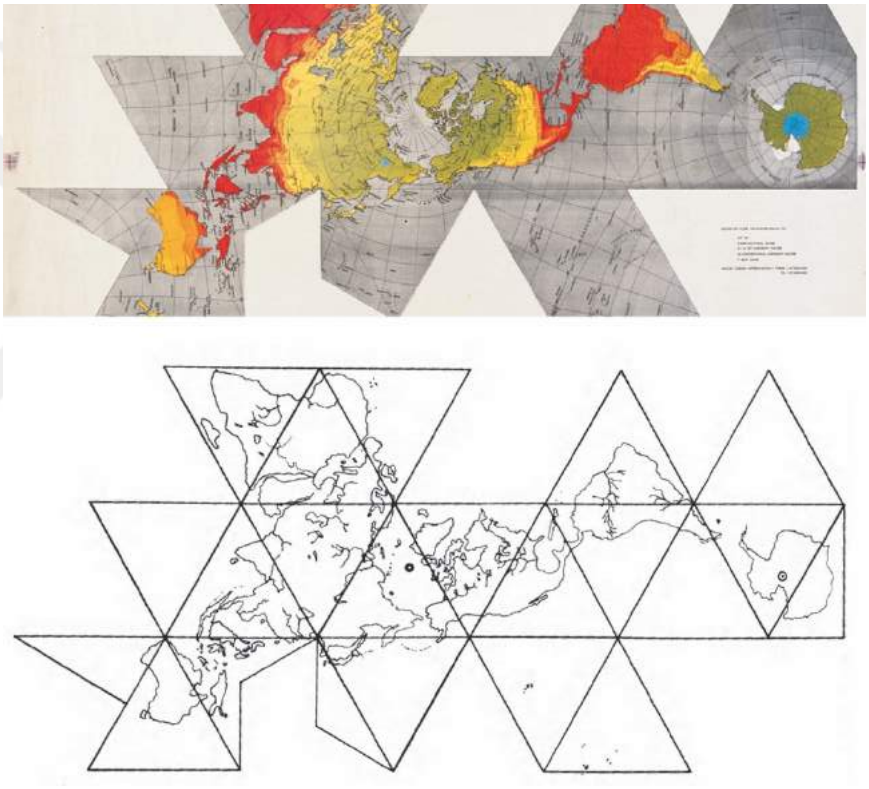


Figure 5.62 : The Dymaxion map patented by Buckminster Fuller (Photo "Buckminster Fuller Institute" (n.d.) and drawing Edmondson (1987, p. 265)).

A subsequent example, a Nasrid wooden dome in the Court of Lions in Alhambra (1354-91) in Granada, Spain is constructed on an intricate polyhedral geometry. Surface subdivisions are not visible inside the parquet wood cladding, yet Makovicky and Hach-Alí (2000) suggest that the geometric pattern is based on a distorted octacapped-truncated octahedron with two varieties of non-equilateral triangle. This geometry employs spherical projections of square and triangular faces and this complexity does not adhere to the geometry of either the Platonic or Archimedean

solids (Bonner, 2017, p. 544). Makovicky and Hach-Alí's (2000) assumptions of such a geometry is built on the existence of rare eleven-pointed stars that are the primary motifs of the pattern. The Nasrid designers probably used these infrequent eleven-fold stars to adapt the entire pattern to the polygonal division of the spherical surface (Figure 5.63).

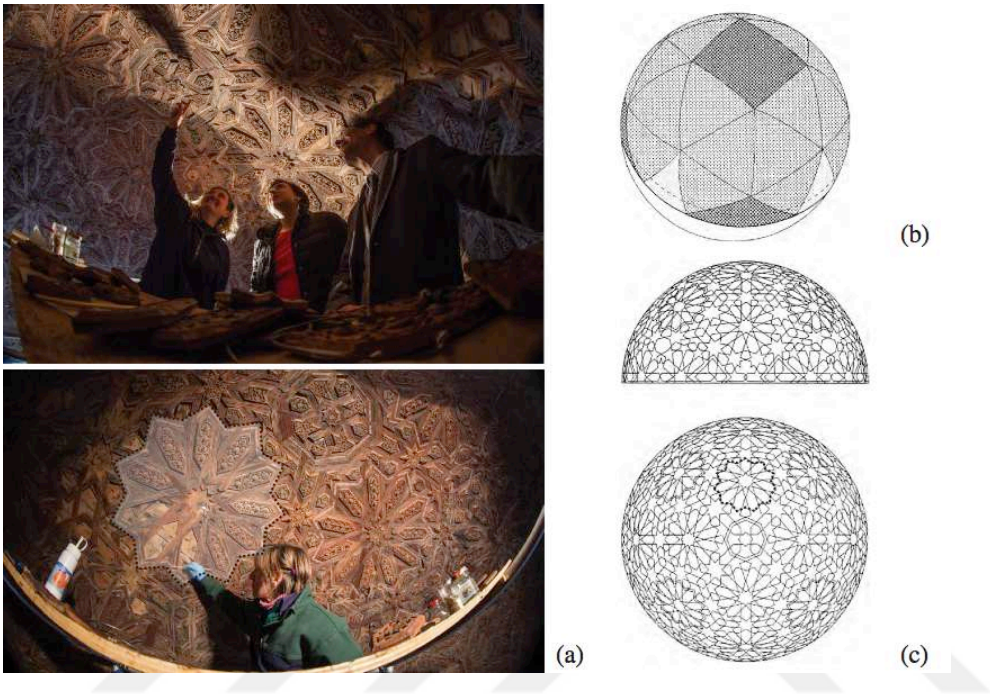


Figure 5.63 : (a) Dome of the Court of Lions in Alhambra (Photos from (Mora, 2015; Troyano, 2015). (b) The octacapped-truncated octahedron is the base generative geometry behind the pattern. (c) The dome pattern is inclusive of rare 11-fold rosettes (Drawings from Makovicky (2000)).

The tile mosaic craftsmen working in the Seljuk Sultanate in Anatolia used this technique as well. Just as the Nasrid parquet-layers in Alhambra, the more subdivision tile mosaic craftsmen acquired, the more exquisite the designs get. Although, the dimensions of the sphere circumference are different in both examples from the Karatay Madrasa and the Eşrefoğlu Tomb (Figure 5.64 and 5.65), the systematical division of the surface, consequently the spherical polyhedron geometry resembles.

Eşrefoğlu Dome demonstrates visible lines proving that the tiling of the surface was based on a spherical polyhedron with different polygonal faces. The division represents hexagonal and pentagonal tiles gathered together to form a half-sphere (Figure 5.64). Such divisions inside the geometric design cause an uninterrupted pattern that is avoided by Tusi atelier in the Karatay Dome. Nonetheless, from the tile mosaic plates that fell down and later fastened again after the restoration works,

Mülayim (1981, p. 120) observes the same structure on the Karatay Dome (Figure 5.65). Mülayim draws cut-tile panels that fell from the surface (Figure 5.68- left). Based on these examples, it is possible to claim that the Anatolian Seljuk designers used this method for the evaluation of both spherical surfaces. Sutton (2011, pp. 52–53) argues that there are not enough evidence to say that medieval artisans were aware of spherical polyhedron methodology, however, both examples are tangible proof to the practical knowledge.

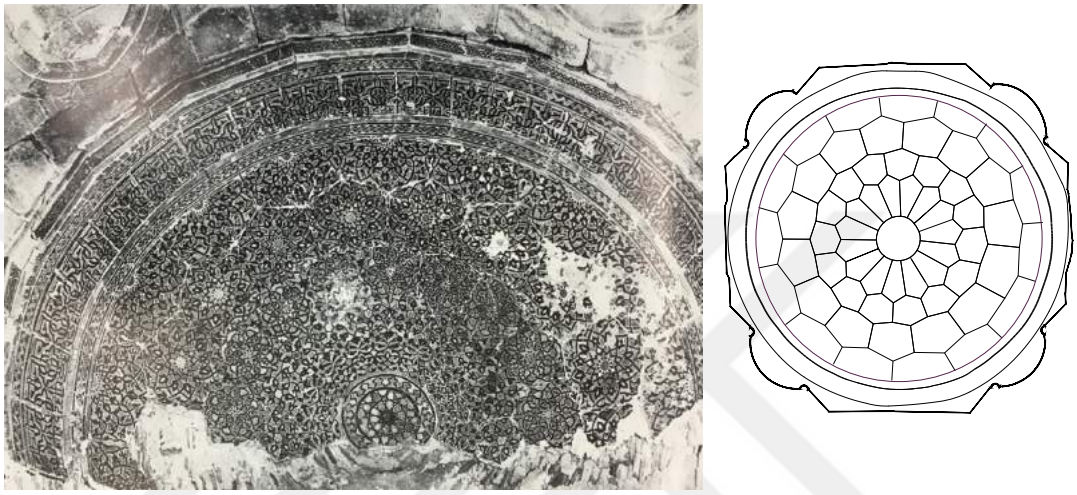


Figure 5.64 : Left: Visible polygonal lines on the Eşrefoğlu Dome Decoration (Photo from (Schneider, 1989)). Right: The surface division on the Eşrefoğlu Tomb (based on the latest building survey and drawings by architect M. Argun Kocadağıstan).

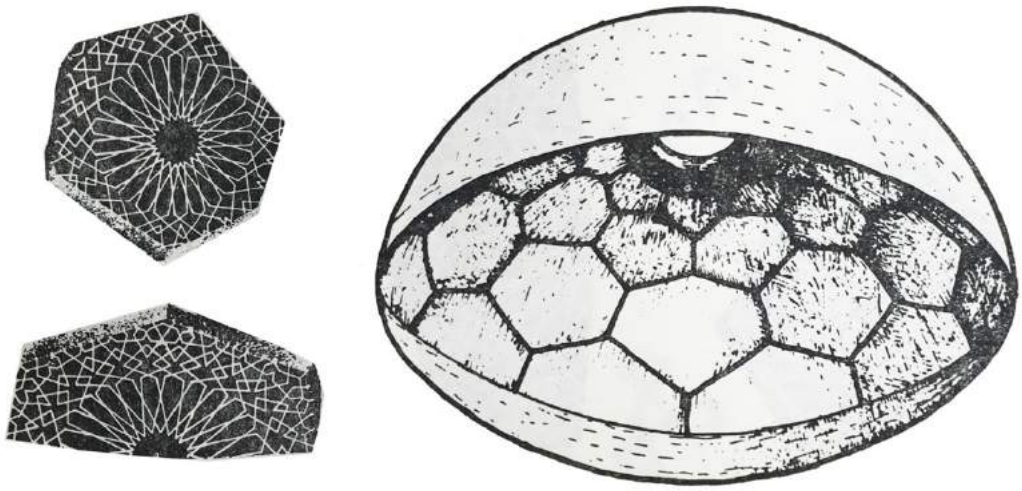


Figure 5.65 : Left: Diagrams of the fallen cut-tile panels from the Karatay Madrasa Dome. Right: Mülayim's structural drawing for the subdivision on Karatay Dome. (Both drawings are from Mülayim, who bases this simple sketches on observations (1981, p. 120)).

Step 2. The application of the geometric pattern

The systematical surface division of the dome that is either based on vertical segmentation or a spherical polyhedron is followed by the unfolding of the parts onto a two-dimensional surface. The pre-determined geometric design is then split into repetitive units that take their shapes in accordance with the divisions.

The greater part of the ornamented dome examples in Anatolian Seljuks followed the Persian tradition of brick ornamentation. In these examples, the pattern designs are created through different arrangements of interlocking bricks that are partially covered with a glaze layer. The Anatolian Seljuk brick domes expose either a whirling rosette that is curled to match a spherical surface or an arrangement of rotating bricks. Peker's (1996a, 1996b, 1998, 2006, 2009) hermeneutical studies confirm that cosmological meanings were attributed to Anatolian Seljuk architecture and the dome presented the celestial sphere- "a gate of earth and sky" (Peker, 2009, p. 80). Baer (1998, pp. 99–103) suggests that these ornaments convey two ideas: the first one alludes to the stellar firmament that creates a look of luminary bodies, while the other one presents the revolving movement of the world and the constant transform between day and night (Figure 5.66).

Figure 5.66 : Patterned brick domes from left to right: Sahib Ata Hankah, Konya (1283-93)- Eşrefoğlu Mosque,next to the Eşrefoğlu Tomb in Beyşehir- Konya (1297-1301)-Another vaulted room from Karatay Madrasa, Konya (1251-1253) and Ulu Mosque in Malatya (1247) (Malatya Dome photo credits MIT Aga Khan Visual Libraries, all drawings from Schneider (1980)).

Representative ideas as such can be constructed upon a basic geometric schema. Nonetheless, a geometric design that is shaped by dense arrangement of stars or arabesques requires more complicated geometrical substructures. Since a dome is a double-curved surface, the application is not straightforward as in a two-dimensional pattern design and distortions are necessary. The most conventional method is to create a geometric design that includes individual stars with different dimensions that follow a numeric sequence. In their study Kasraei et al. (2016), examine different instances and show how the point numbers of star polygons change in compliance with the curvature of the surface of a dome. The curvature of a dome changes from the spring point to the apex and decreasing the curvature of a dome's surface initiate a decrease in the number of points of star polygons (Figure 5.68). The floral pattern on the interior dome surface from the Eşrefoğlu Tomb is designed after this principal as well (Figure 5.67).

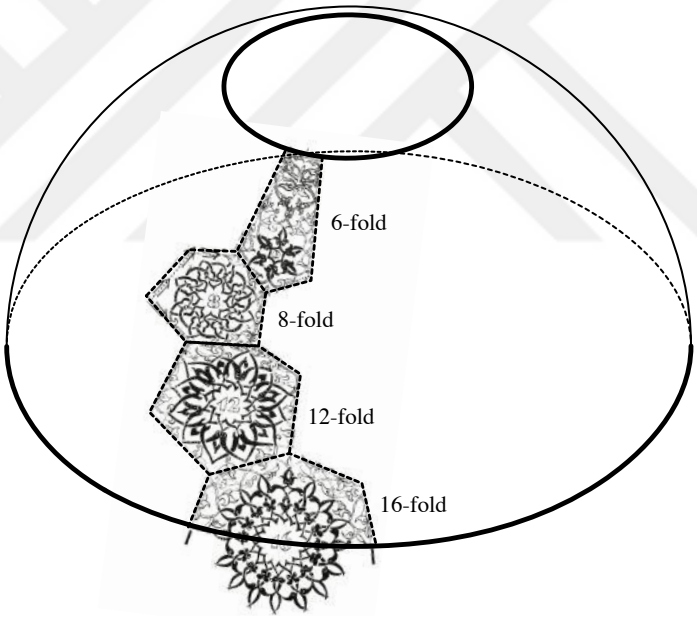


Figure 5.67 : The pattern on the Eşrefoğlu Dome. Individual floral motifs change in a numeric sequence and adapt to the curvature (Adapted from Schneider's (1989) drawing).

The Eşrefoğlu Tomb built in Beyşehir, Konya was renovated recently and absent part that is almost the one-half of the domical surface is completed with a painting that imitates the original design. The building survey based on a point cloud acquired with a 3D laser scanner [11] demonstrates clearly the relation between the geometric design

[11] The latest building survey iis completed by architect M. Argun Kocadağıstan.

and the surface division, hence the making approach (Figures 5.69 and 5.70). Based upon a complicated spherical polyhedron geometry, the entire surface is tiled by hexagonal and pentagonal panels that surround the central medallion on the apex. Individual tile mosaic pieces that form the geometric design are assembled as these polygonal shapes and applicated to the surface as larger panels.

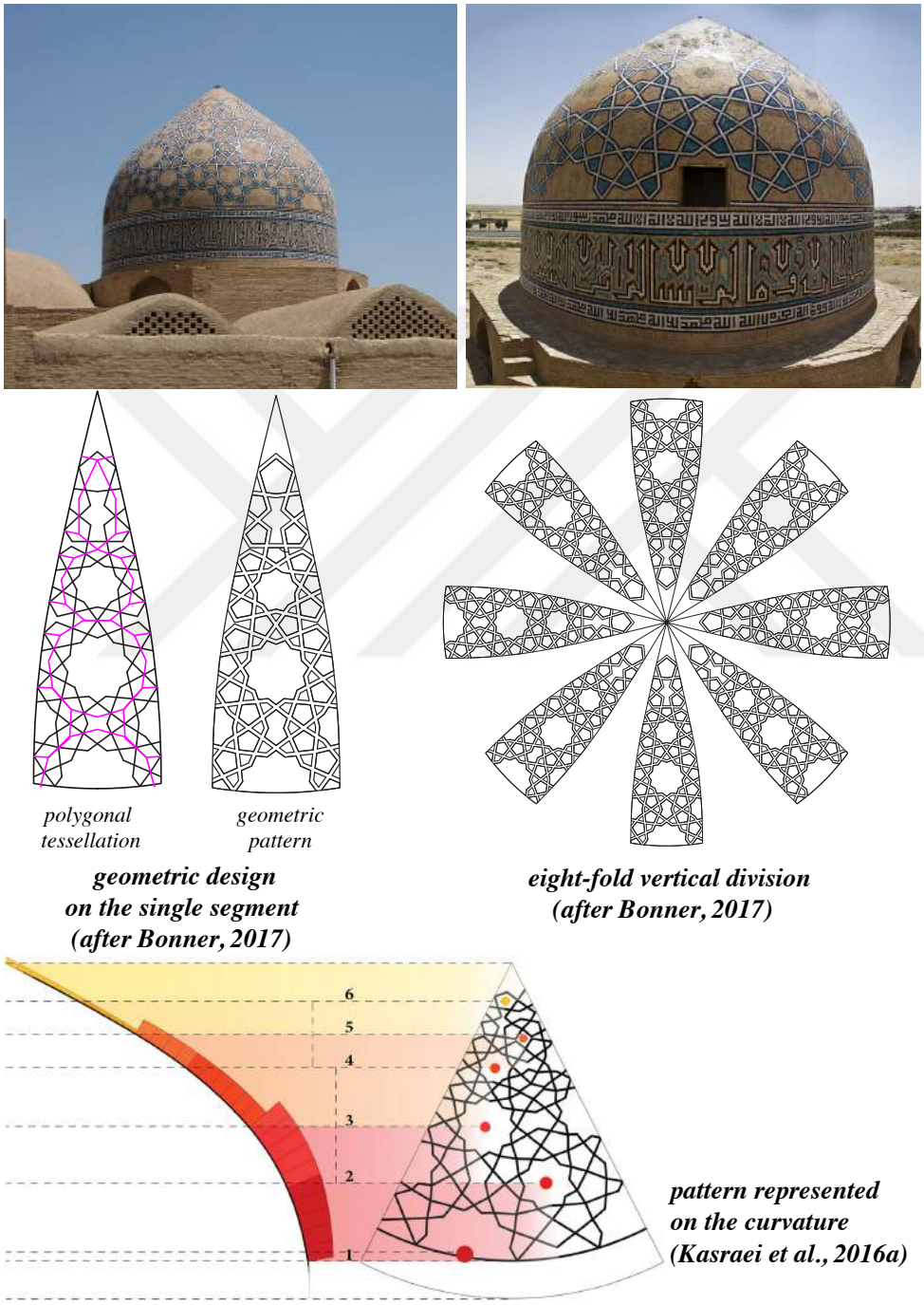


Figure 5.68 : The pattern on the Saveh Jame' Mosque dome in Iran (late 16th century) and the numeric sequence of the design along with the curvature (Both photos from Wikimedia Commons).

Figure 5.69 : Arabesque pattern design on the Eşrefoğlu dome, above with visible polygonal tiling (Drawings and building survey by architect M. Argun Kocadağıstan).

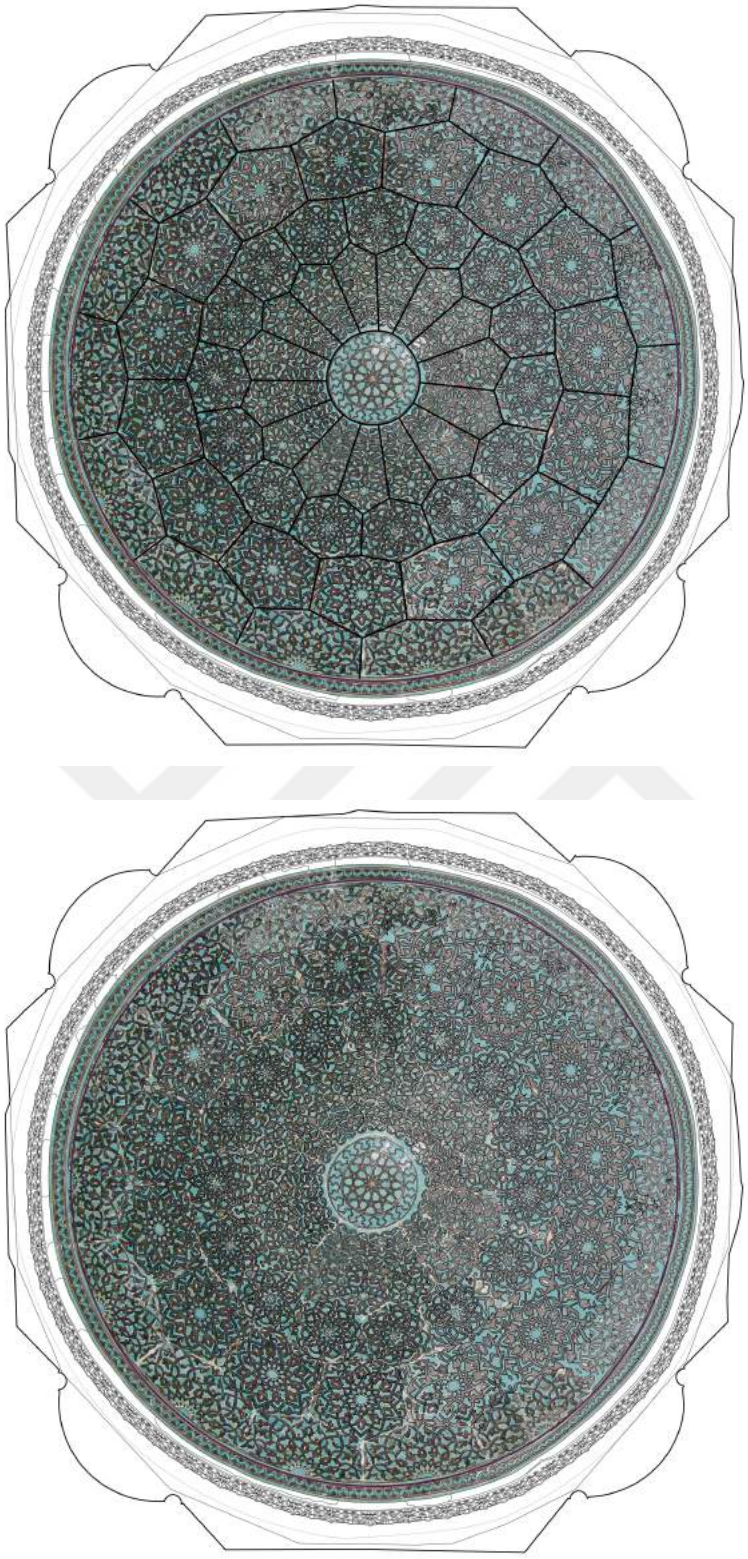


Figure 5.70 : Pattern design on the Eşrefoğlu dome juxtaposed on a color photograph, above the visible polygonal tiling (Drawings by architect M. Argun Kocadağıstan).

Arabesque patterns on each individual piece change in accordance with the dimensions of the polygonal slabs. The curved floral motifs that surround a ten-pointed star in the apex of the dome are based on 6-,8-,12- and 16- fold symmetries (Fig. 5.71). Figures 5.72 and 5.73 illustrate the orthographic view of the individual plates and their relation to the dome surface. The entire design is split into one repeating module that is based on four plates that surround the circular medallion on the apex. The orthographic drawings represent a distorted view of the plates. Yet, these individual cut-tile plates represent regular patterns in the perspective view.

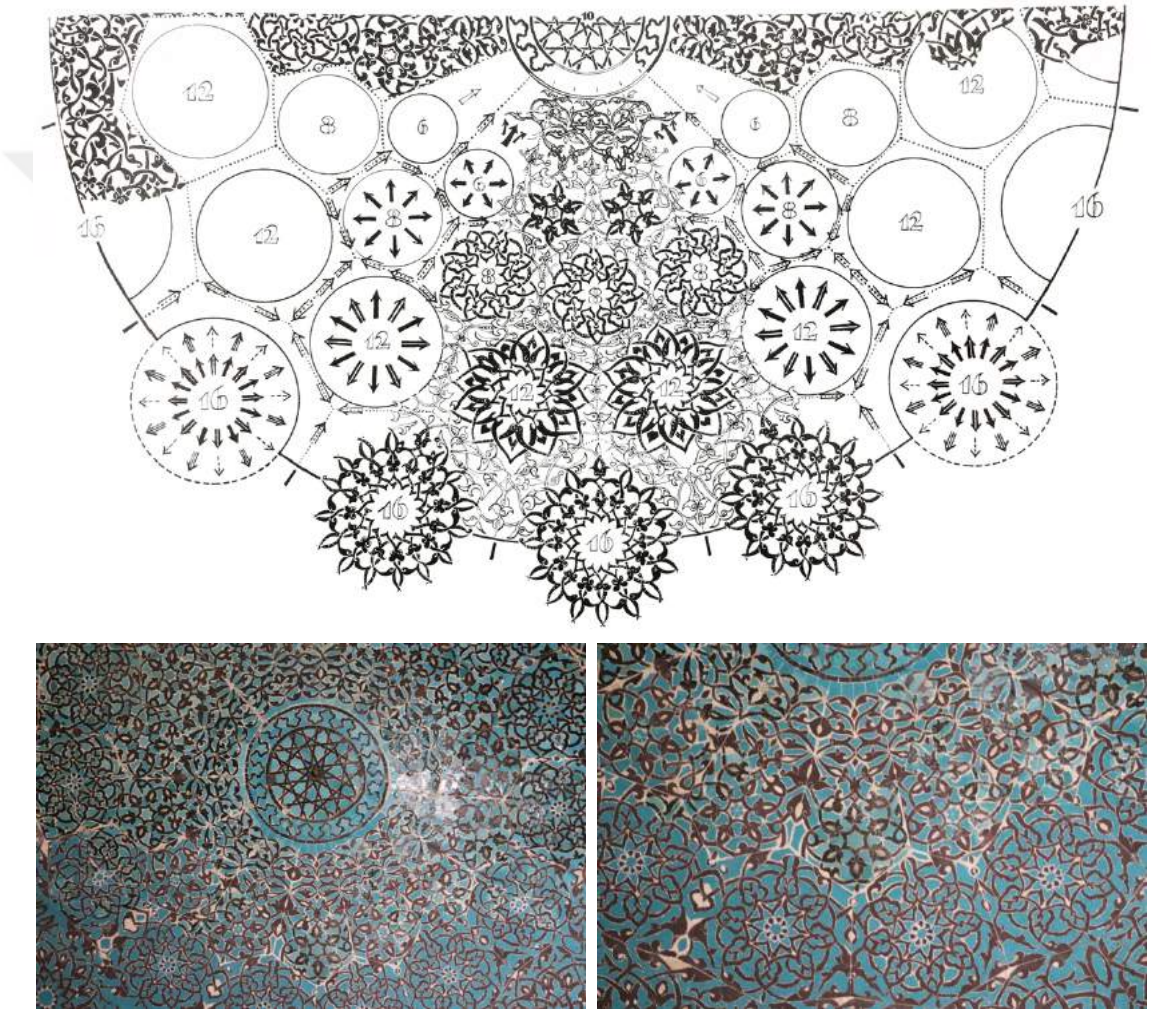


Figure 5.71 : Above: Individual patterns scattered on the Esrefoglu Dome (Drawing Schneider (1989)). Below: Close-ups from tile mosaic plates. 6-, 8- ,12- and 16-patterns stand at the center on each polygon.

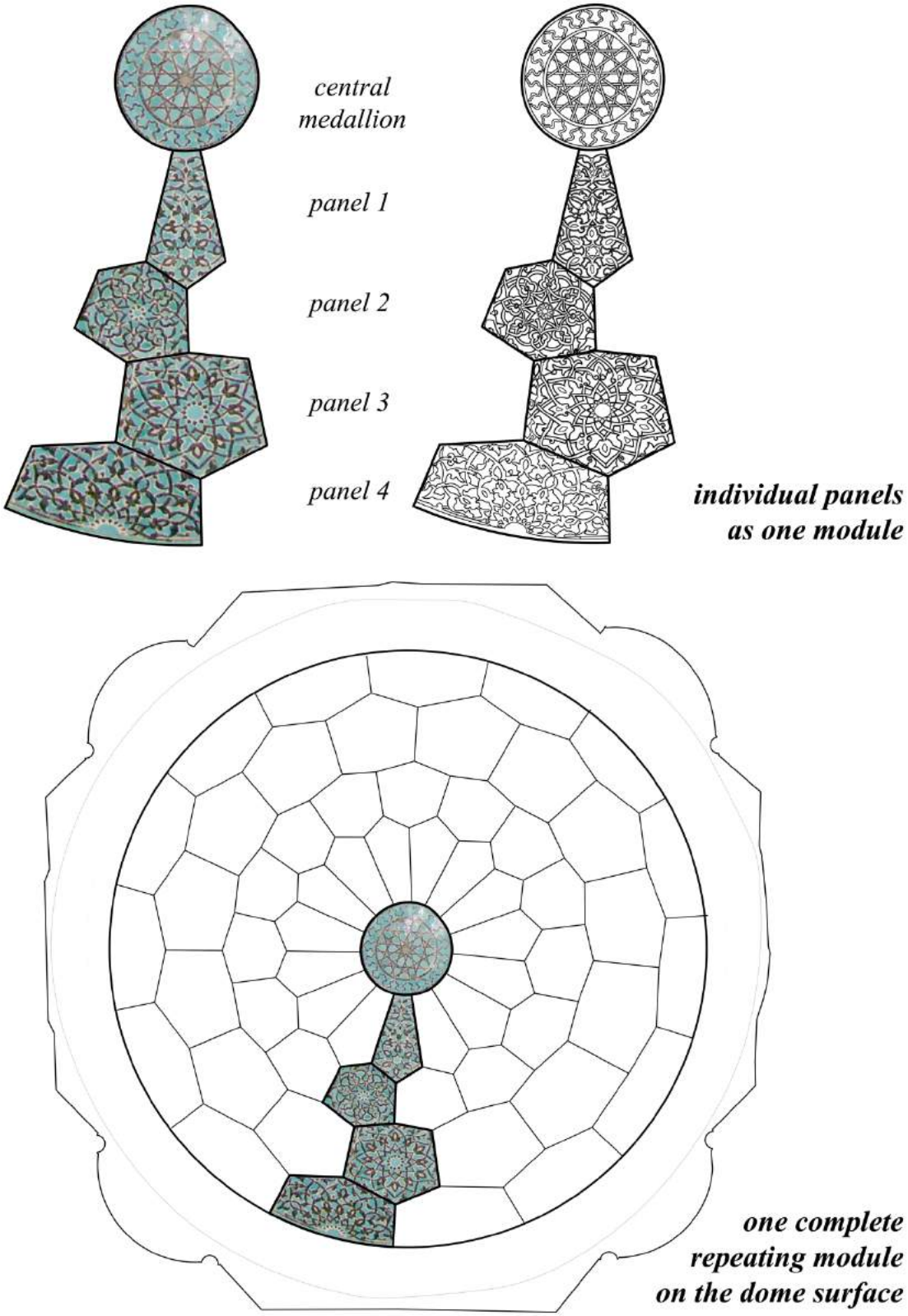


Figure 5.72 : Four different polygonal plates surround a central medallion adorned with a ten-pointed star on the Eşrefoğlu dome apex.

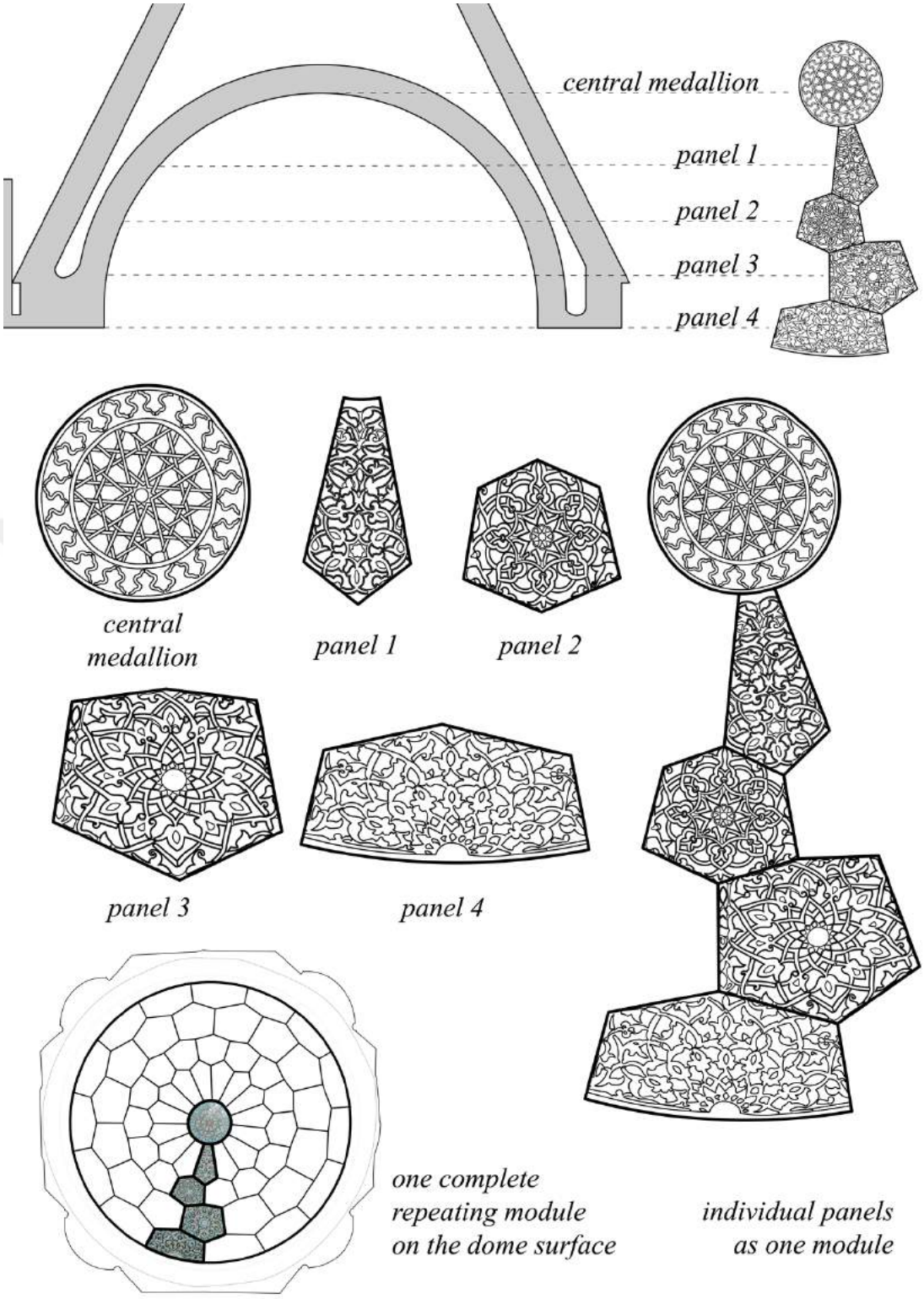


Figure 5.73 : Arabesque motifs on individual polygonal tiles from the Esrefoğlu dome.

The domical surface division, based on a complicated polyhedron geometry with hexagonal and pentagonal faces resemble in Karatay Madrasa and Eşrefoğlu Tomb. Yet, the geometric design that is placed upon the polygonal faces are addressed very differently. Tusi's approach towards the design problem for the Karatay Madrasa dome is quite unusual. The motifs that stand at each individual level does not follow a numeric sequence as in the aforementioned examples. Rather, these primary motifs are principally the same, while the line connections in intermediate areas change on different levels.

5.6.2.2 The domical geometric design in Karatay madrasa

The dome in the Karatay Madrasa is based on a square plan and the transition of the dome from the square court yard is supported by triangular pendentives that are famously known as the *Turkish triangle* (Figure 5.74). The entire geometric design on the dome is dominated by a single motif, a twenty-four-pointed star that is repeated on four different levels by sixteen multiples on the entire surface. At first glance, it appears as if the star dimensions change in accordance with the surface curvature, causing Mülayim (1981) to describe this pattern as a formation through the scaling of the primary motif. Yet, a closer look reveals that the geometric design is not completed as straight-forward as it seems. Careful observers such as Schneider (1980, pp. 134–135) realize that the primary motif is the same at each level, while the connecting lines and the shape relations change across the entire geometric design.

Figure 5.74 : The dome of the Karatay Madrasa (1251-1253 A.D.).

The common interpretation of the pattern is that it symbolizes a view of the sky surrounded with luminary bodies (Baer, 1998; Erdemir, 2001, p. 129; Mülayim, 1981; Ögel, 1994, pp. 91–92) (Figure 5.75). In his comprehensive study, Peker (2006) analysis the cosmological meaning behind the geometric design and explains the intended view. The eyes of an observer, who stands at the volumetric center perceives the star motifs as planets revolving around the celestial sphere and the opening on the spherical shape presents the doorway to heaven (Peker, 2006, pp. 36–37). The hermeneutical study on the domical pattern is outside the scope of this study. Yet, the inquiries on the pattern design that is presented in this study show that the Tusi atelier had a certain understanding of the formal relations between the architectural form and the geometric design.

Figure 5.75 : The domical pattern from the Karatay Madrasa is interpreted as a symbolization of the sky surrounded with luminary bodies.

There are only a small number of studies that evaluate meaning in Anatolian Seljuk architecture (Ögel, 1986, 1994; Peker, 1996b, 1996a, 2000, 2006, 2008, 2009; Redford, 1993, 2000) and consequently limited knowledge on how perception affect the geometrical rules behind the domical pattern designs. Nevertheless, domical decorations in the western architecture has been one of the primary subjects to various related studies. In *The Projective Cast* Robin Evans (2000, pp. 4–53) evaluates diverse examples from the ancient Roman, Renaissance and Baroque architectures and

explains how relation to the form shapes the ornament design on the dome surface. Figure 5.76 shows an example dome fresco from the sixteenth century. Evans (2000, pp. 20–22) explains how the heavenly sphere is represented by creating a hierarchy – "an undeviating order" between the figures that appear on the dome frescoes.

Figure 5.76 : Left: Choirs of Angels by G.P. Lomazzo from the Foppa Chapel, San Marco, in Milan (Image from Wikimedia Commons). Right: Evans's (2000, p. 21) diagram of dual centered organization implied by sixteen-century dome frescoes.

Symbolic meanings in Evans's domical fresco examples necessitate a thought-through formal relation between the figures. Nonetheless, that the Karatay pattern is a non-figurative geometrical pattern, manifests even a more complex hierarchical order between star motifs that generate the heavenly sphere image that is stressed by Peker (2006) when viewed from the floor level. Consequently, the generative process behind the construction of the entire geometric design is challenging.

Akin to the previously investigated examples of Tusi atelier's designs, there are not obvious polygonal slab marks, this proves that the design was based on a complicated polyhedral geometry. Nonetheless, aforementioned Mülayim's (1981, p. 120) observations illustrated in Figure 5.65 suggests that single mosaic pieces were arranged as hexagonal and pentagonal plates and then applied on to the surface. Erdemir (2001, p. 167) highlights the immense spherical diameter that is nearly twelve meters and the necessity of such a methodology to assist the application on to the domical surface. The measurement of the individual plates reach almost two meters at the root circle (Erdemir, 2001, p. 167). A photograph taken during the restoration works elucidate the colossal proportions of the geometric design (Figure 5.77). Furthermore, it gives an idea of the making process, as it shows individual tile mosaic pieces that were formed as polygonal plates that ensure an accurate application.

Figure 5.77 : A restaurateur working on the dome surface (Erdemir, 2001, p. 176).

In our study, we initially searched for certain maker's marks that could provide visual clues on how the making was approached. Tusi sought uninterrupted patterns, yet material dimensions require a cut at some point. The circular division lines visible with plain eye and those evidenced in detailed close-up photographs that depict the destroyed individual stars indicate that the primary motif might have been placed on to the surface in the very beginning (Figure 5.78 and Figure 5.79).

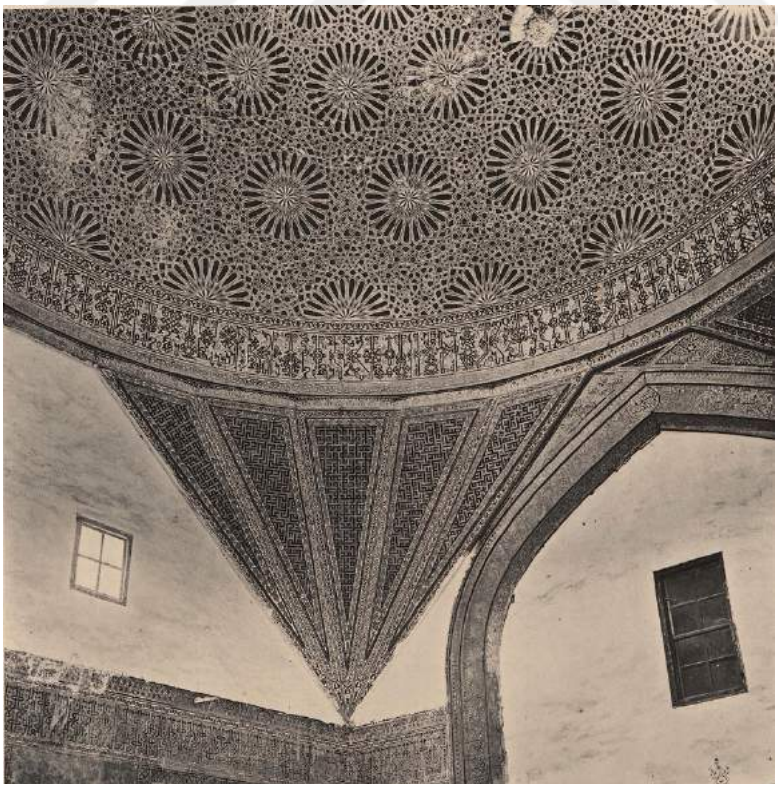


Figure 5.78 : A historical photo before the restoration depicts the fallen star centers (Sarre, 1901b).

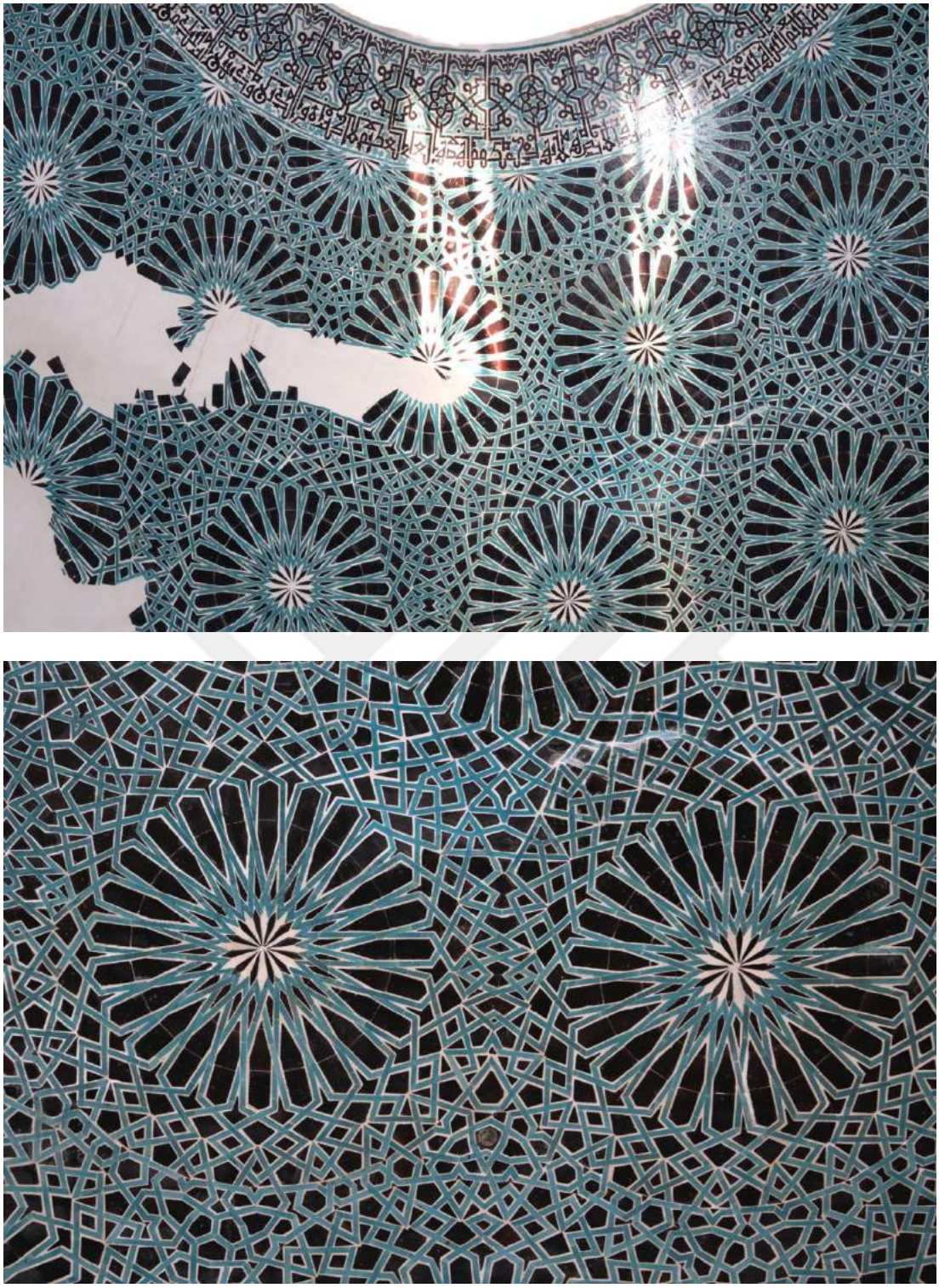


Figure 5.79 : Circular marks are visible inside the geometric pattern.

Based on this observation, in our investigation we attempted at generating the pattern by placing the star motifs on to the right positions. The drawing is completed using Rhino, a three-dimensional modelling program and an inclusive algorithmic design tool Grasshopper embedded in the program. We initially divide the surface into primary regions. There are four different horizontal levels, on which the centers of the

star motifs are located and these levels are not arranged concentrically. Similarly, each parallel level is divided into sixteen vertical areas and the star motifs are centered onto the intersections of horizontal and vertical axes. Nonetheless, the star centers alter, while the centers of the first and third levels are aligned vertically, the positions of the second and fourth levels are shifted (Figure 5.80).

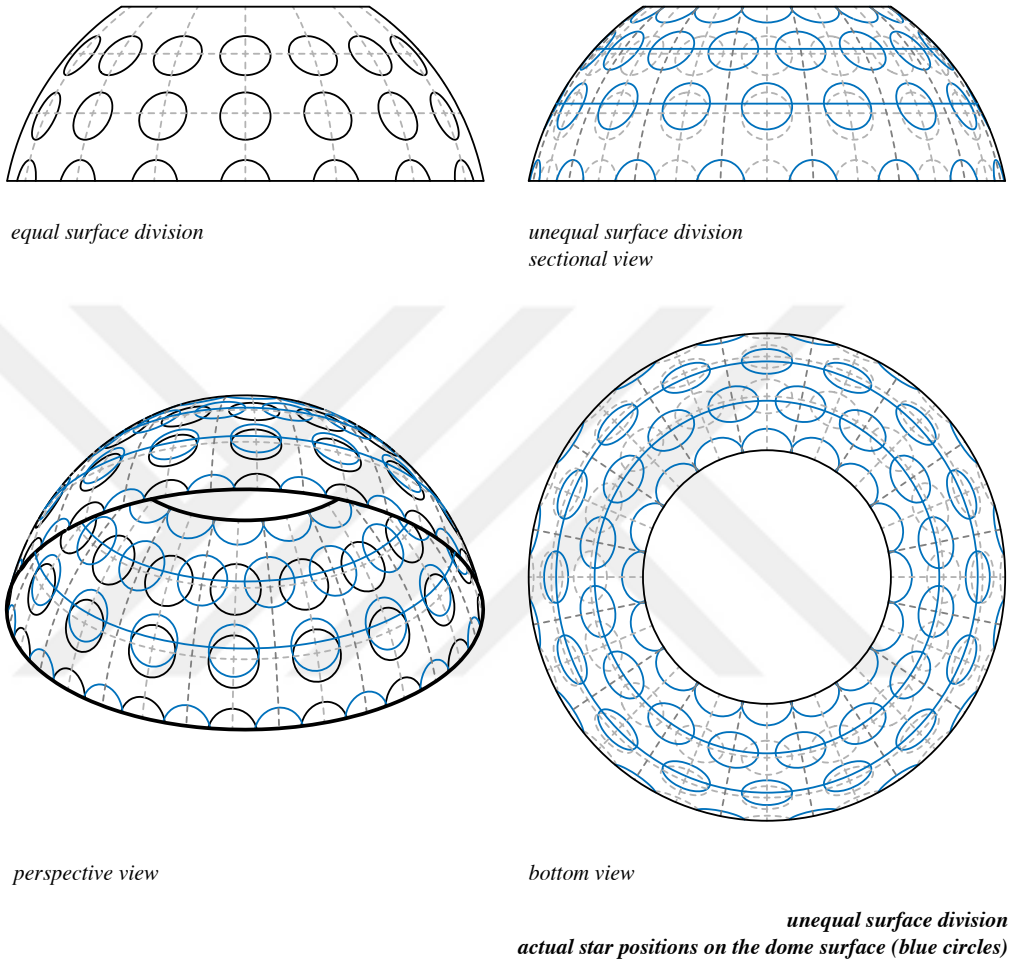


Figure 5.80 : The vertical and horizontal axes of the surface division and the placement of the star motifs scattered on the Karatay dome.

The primary unit that repeats at each level is a star motif that has twenty-four arms that connect once every two branches. Nevertheless, peripheral connections that are further links between star arms alter at each horizontal level. Figure 5.81 how the star motif alters, while different peripheral connections emerge at each horizontal level. Branches of the star motif bond further links in an ascending order. The different configuration of the motif at each horizontal level is illustrated in Figure 5.82.

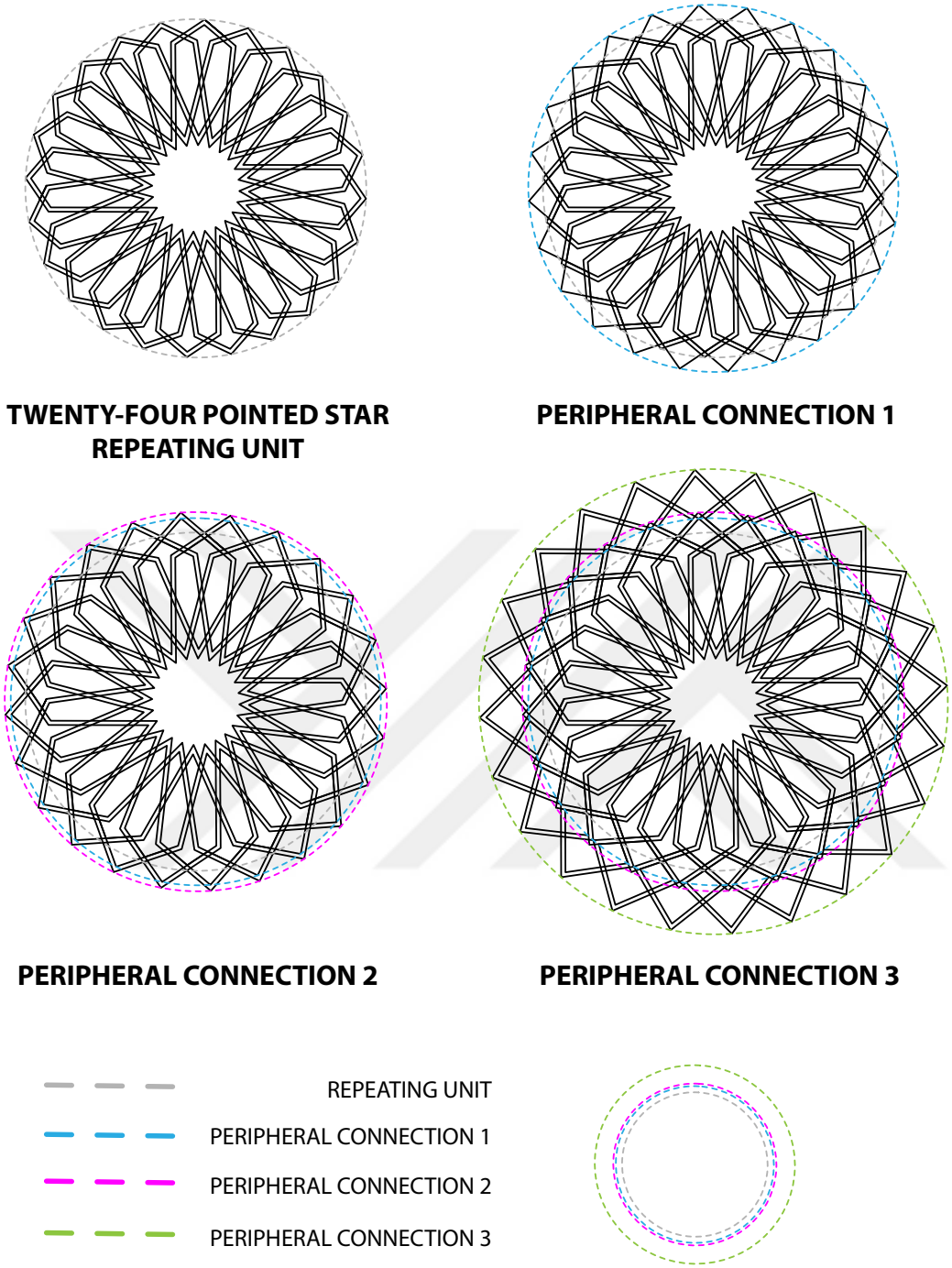


Figure 5.81 : The repeating unit is a twenty-four-fold star pattern and it alters while individual arms of the star bond further links at each horizontal level.

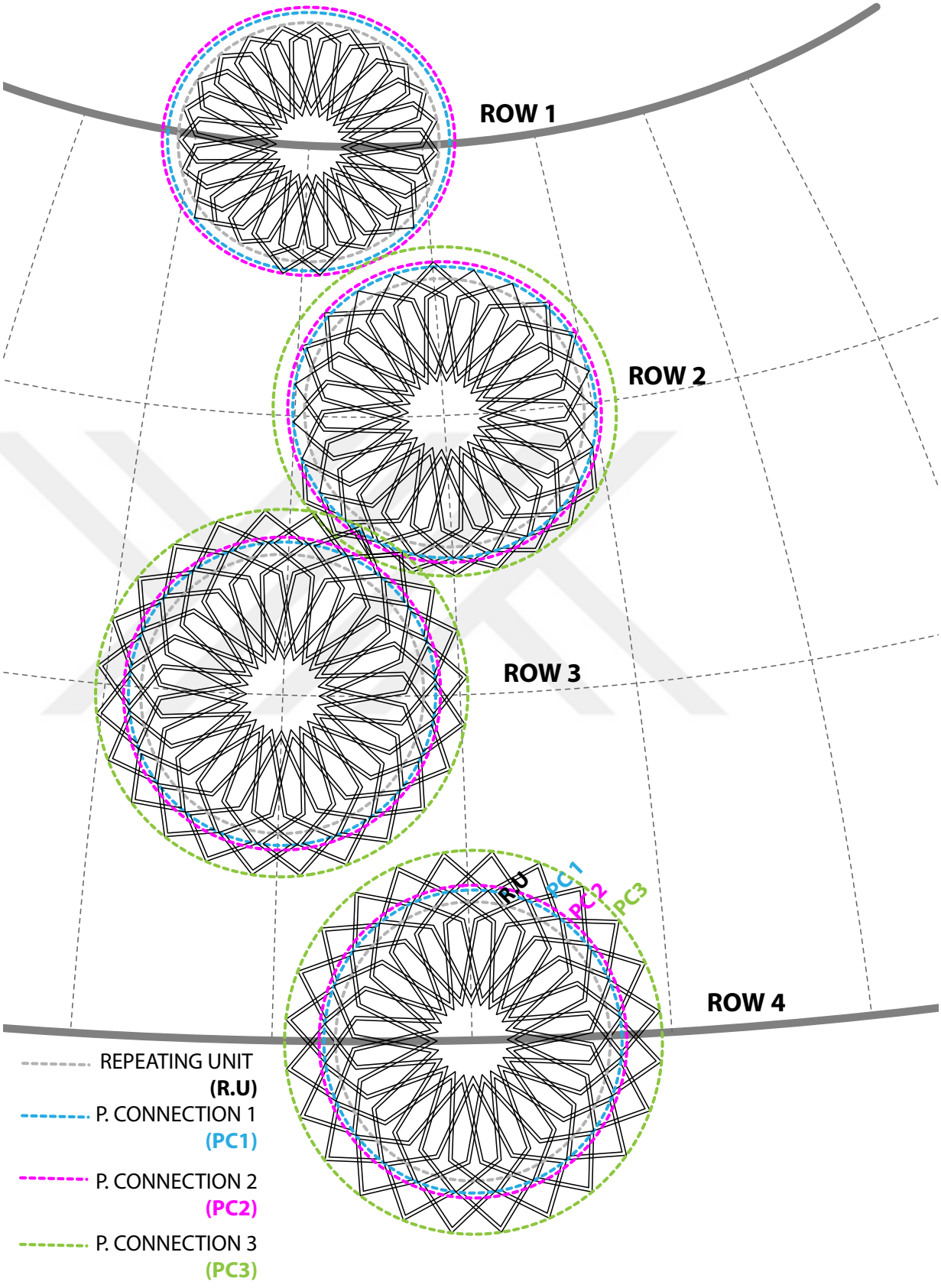


Figure 5.82 : The peripheral connection of the repeating unit changes at each level.

The star motifs on the first, third and fourth levels intersect with their vertical axes at the same spot- the halfway point of two branches. Nevertheless, the repeating unit on the second-row is rotated and lies on an axis off by 7,5 (360/48) degrees (Figure 5.82).

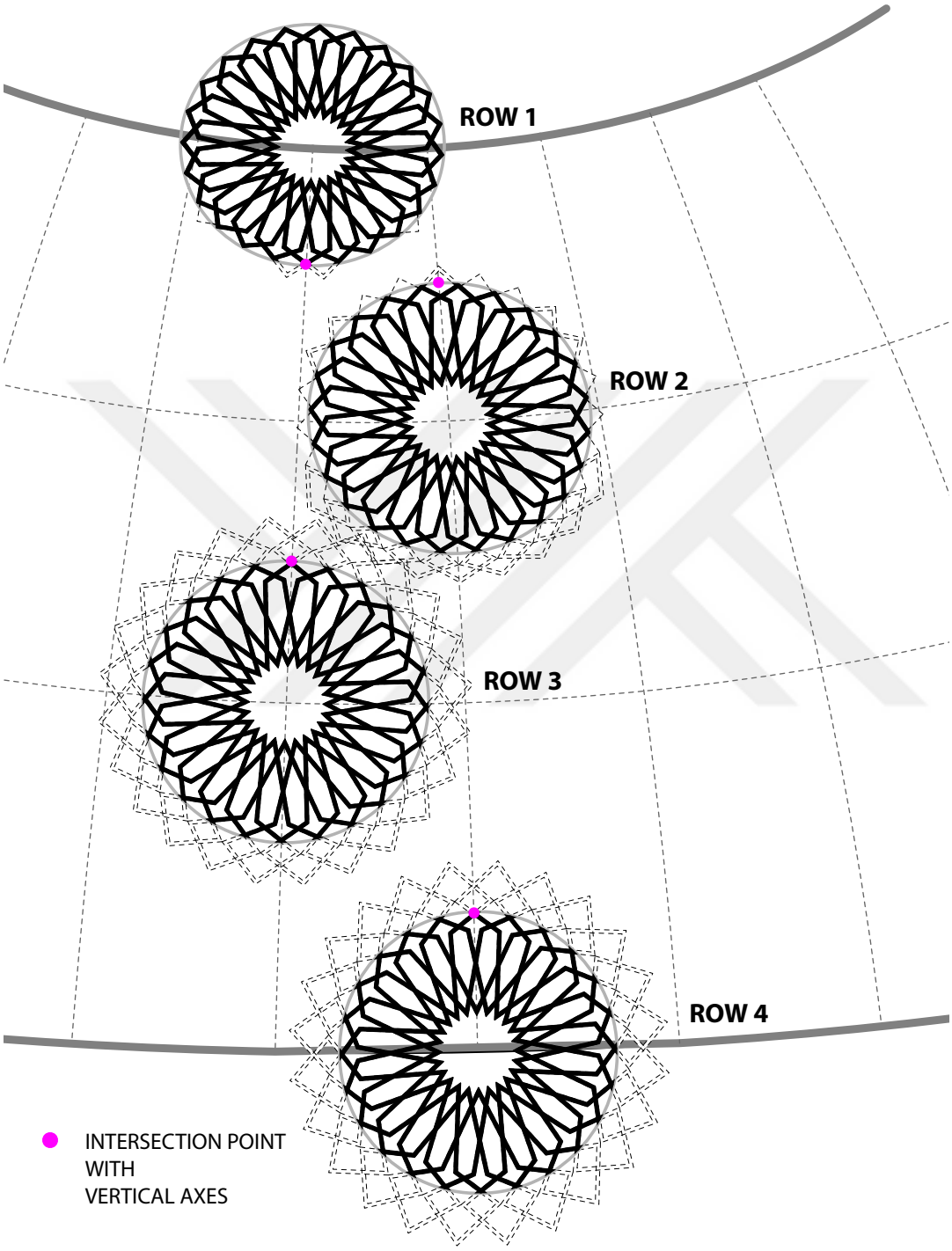


Figure 5.83 : The motif on the second level intersects with the vertical axe differently.

In our drawing, the diameter of the repeating star unit is determined by the first level star connections. The stars at the top-row successfully merge with each other, suggesting that the enclosing circles are nearly tangent. Our examination on the real time 3D laser scanning data suggests that the repeating star diameters change slightly across rows. This variance of the star scales could be caused by the different curvature degrees on each level. Sizes of the flat tile mosaic plates undoubtedly changed while craftsmen bend them to fit to the surface curvature (Figure 5.84).

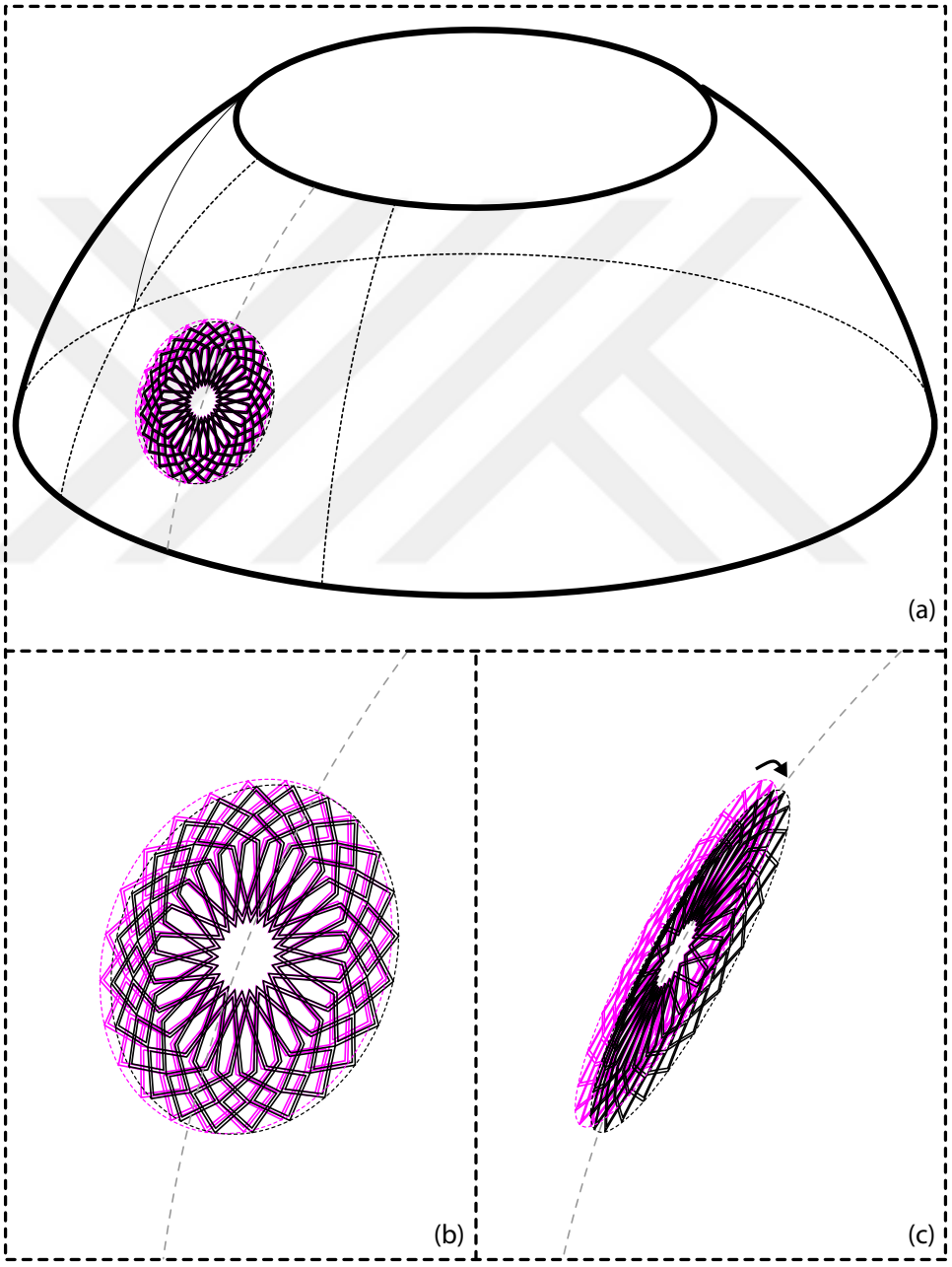


Figure 5.84 : The geometry changes when tile mosaic plates are bent to fit the spherical surface. (a) Flat tile-mosaic plate juxtaposed on a curved plate. (b) Close-up. (c) Side view. Colored lines present flat plates, curved plates are drawn in black.

After placing the star units based on the varying diameters, we are able to fill intermediate areas by connecting the stars. The stars drift apart increasingly starting from the uppermost level to the bottom of the cupola and the star connections alter at each row. As an example, Figure 5.85 illustrates how the intersecting star connections change in row 3 (illustrations for the other rows are presented in Appendix C). The ordinary connection of the stars that overlap is modified by the artisan. The adjustments create the locally adapted geometry of the design. This design is consistent throughout that row.

The complete drawing with different level connections are presented in Figure 5.86, Figure 5.87, Figure 5.88, Figure 5.89 and Figure 5.89 (further illustrations are present in Appendix C). Nonetheless, it should be noted that although star motifs are connected as in the same way in Tusi's approach, intermediate shapes appear different in sizes than from the original design. This dissimilarity is caused on account of different issues such as the variance in the surface geometry, drawings based on photographic observations, inaccurate star diameters and the restrictions of the computer program etc. These all suggest that an accurate design is only possible by measuring every single detail inside the pattern and the geometric design is contingent with the surface geometry.

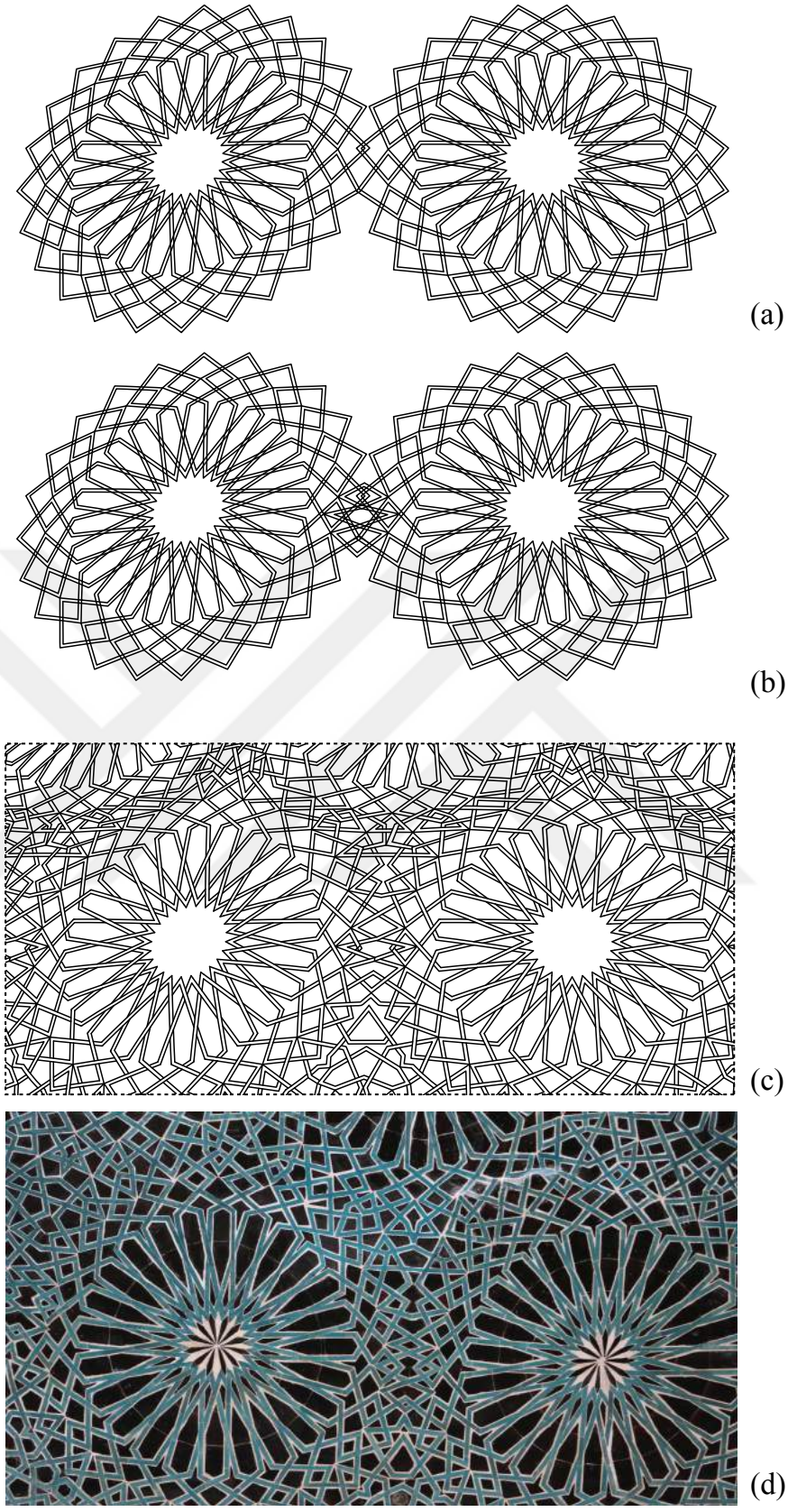


Figure 5.85 : Step by step alteration between the star connections in Row 3. (a) The ordinary connection of the stars. (b) Overlaps, that creates the locally geometry of the design. (c) The final design adapted by the artisan. (d) Close-up. This design is consistent throughout that row.

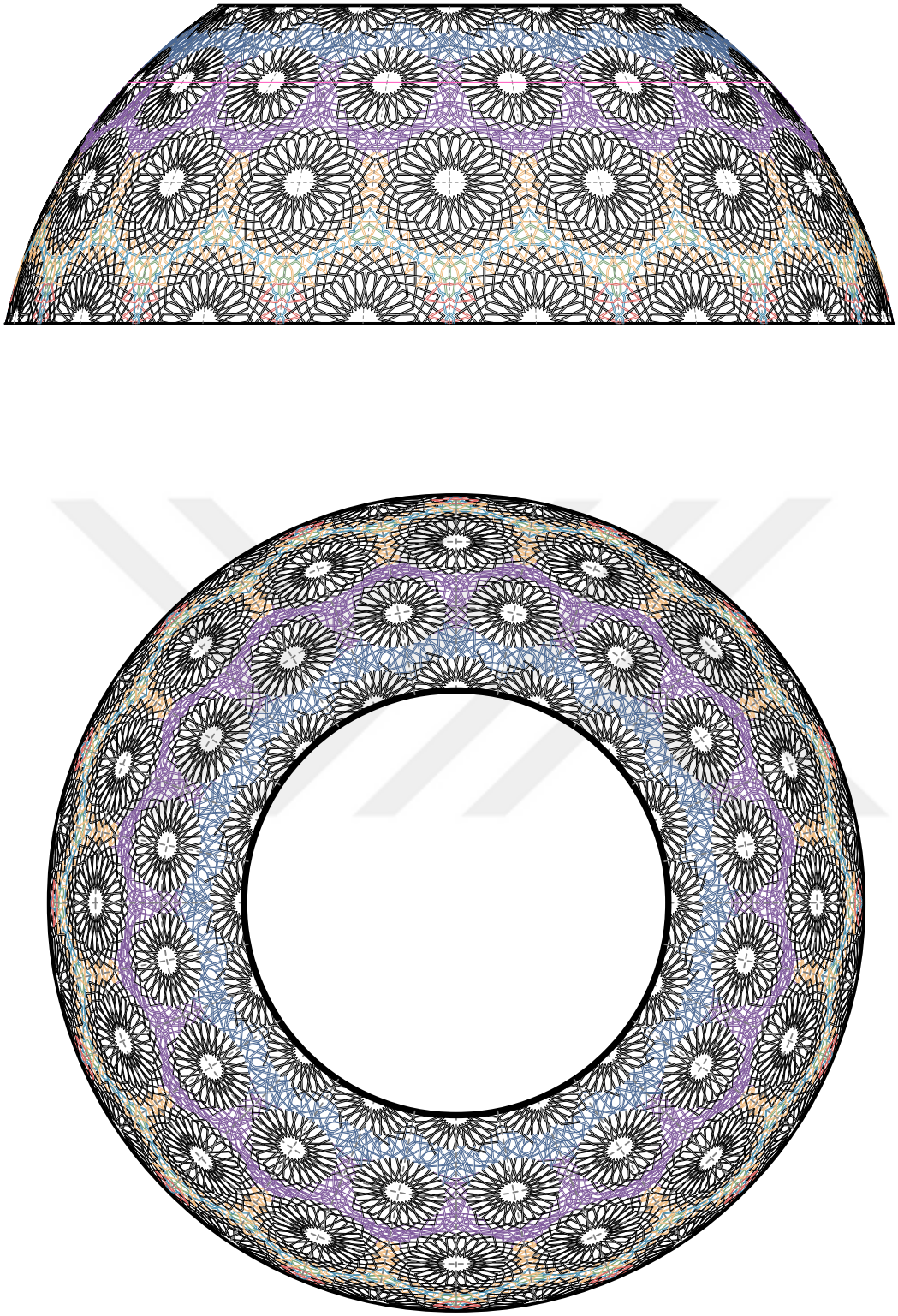


Figure 5.86 : Above: Sectional view of the pattern design. Below: Ceiling plan of the domical Karatay pattern.

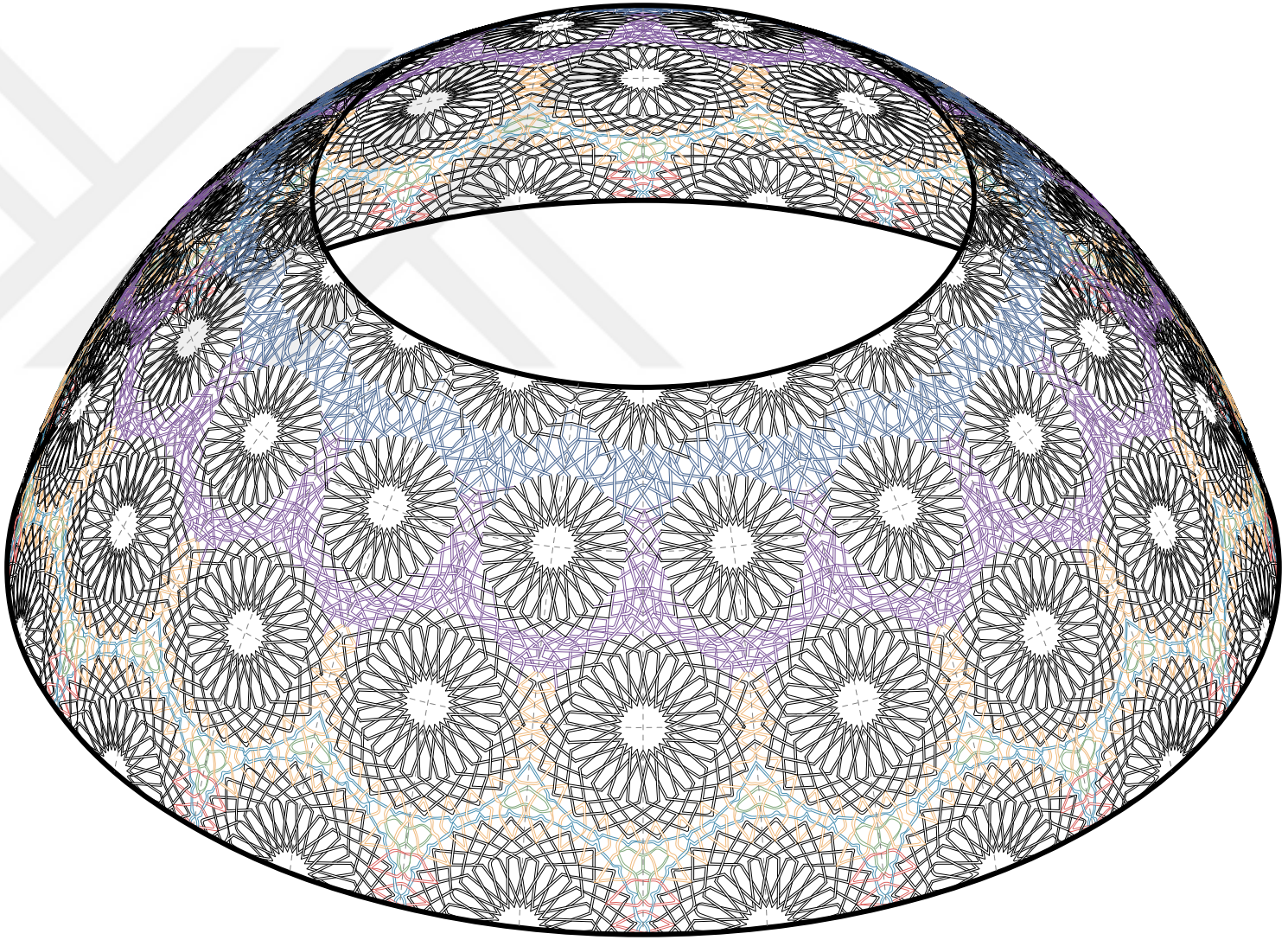


Figure 5.87 : The domical pattern on the Karatay Madrasa.

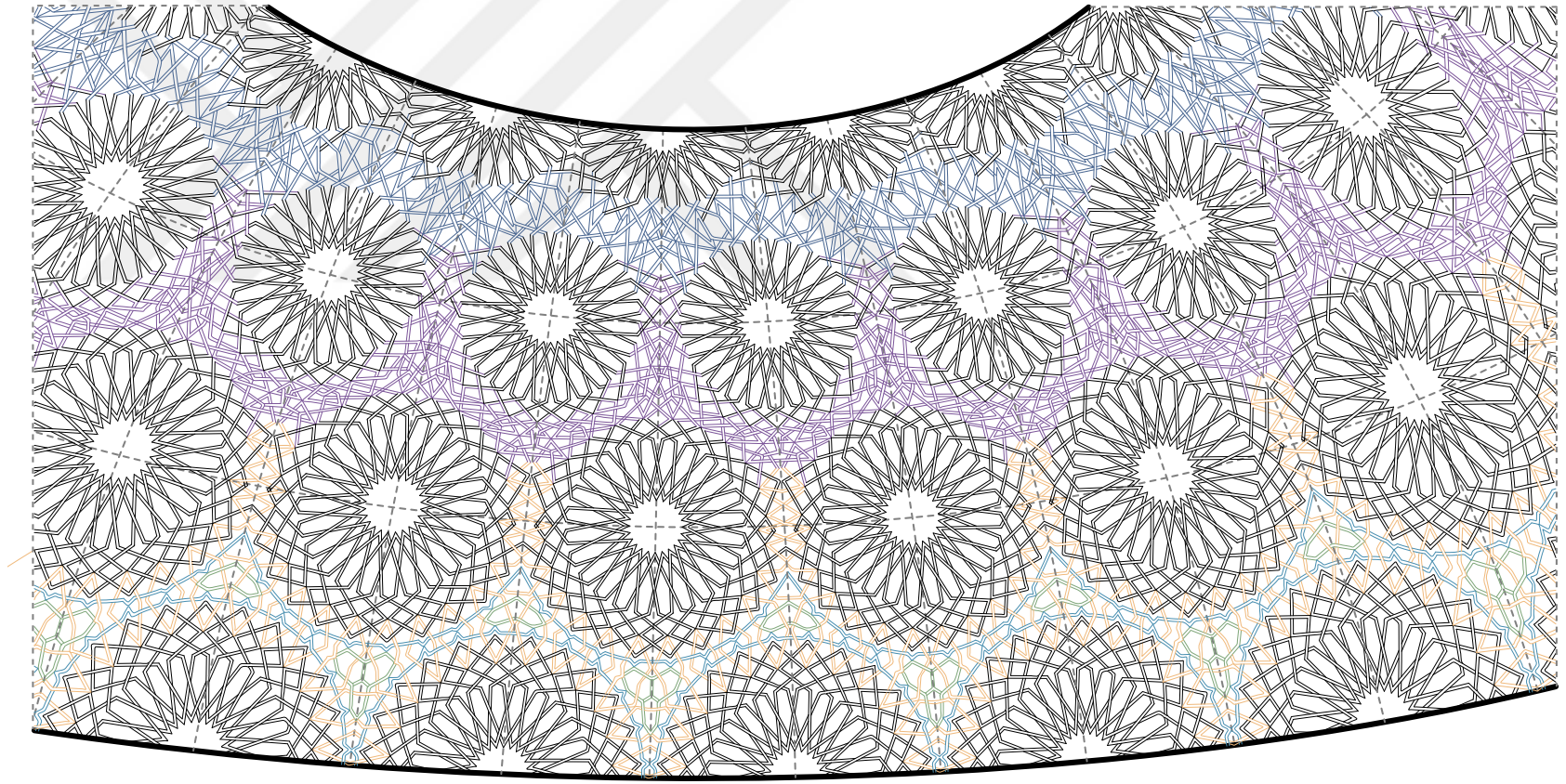


Figure 5.88 : The Karatay dome pattern. The 24-fold star pattern stands in the center of each individual piece, the connection lines and intermediate shapes differ on each horizontal level.

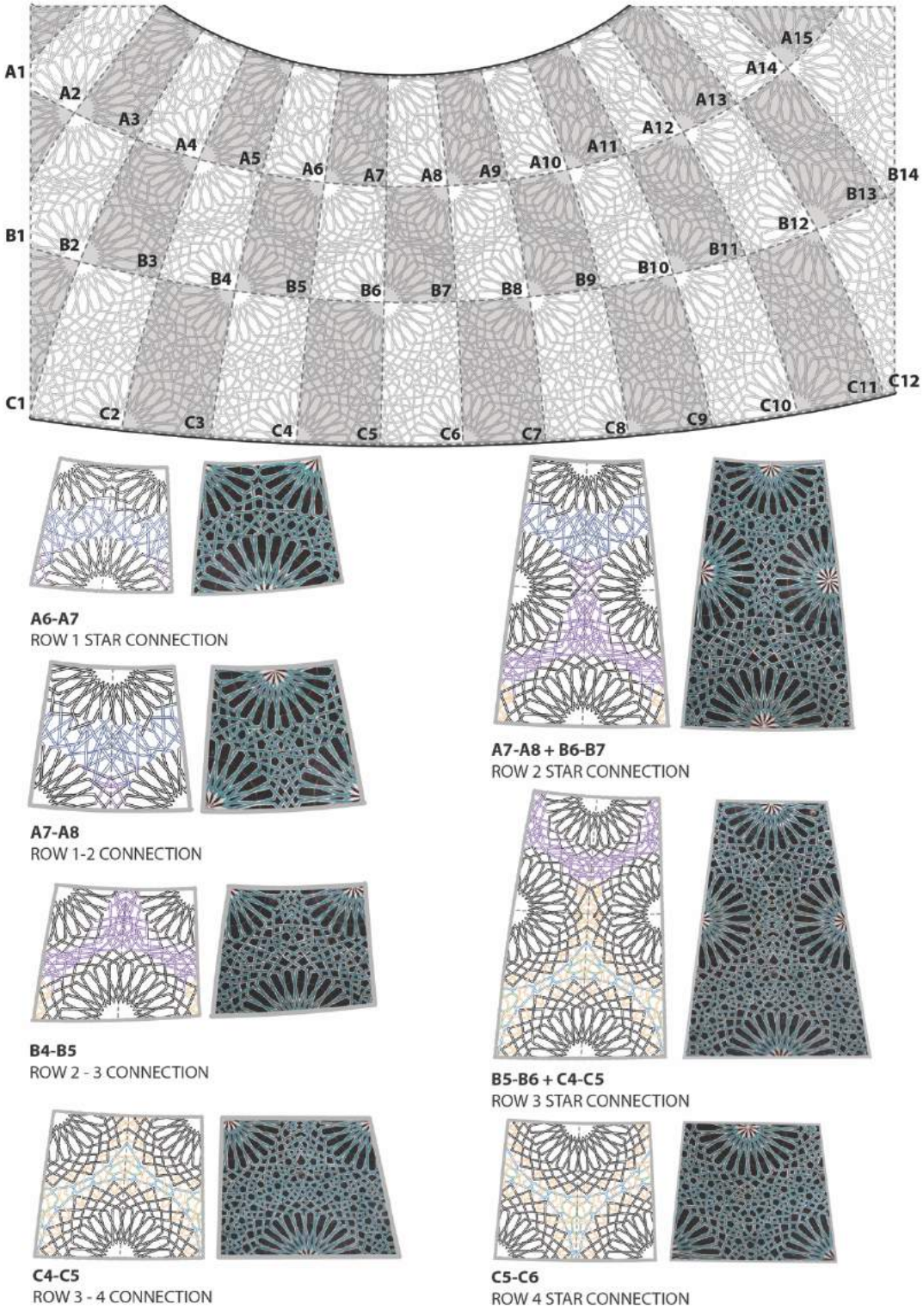


Figure 5.89 : Detailed view from the domical pattern at Karatay Madrasa. Various individual pieces appear between the twety-four pointed stars (Close-ups are presented in Appendix C).

During the making of this particular example, Tusi probably faced this design problem: He wanted to cover the whole surface with a geometric pattern. Yet, there was no previous examples of a dome that was covered completely with a cladding material such as the tile-mosaic. Thus, probably his guiding pattern book did not contain a suitable design for the spherical surface. Assumed that there was a similar design, he still had to figure out *how he could manipulate a sample design to fit the intended surface.* Nevertheless, adapting an existing two-dimensional geometric design to the spherical surface is not a straight-forward process. Just a small scaling on the primary motif would cause a great alteration on the entire geometric design, since individual motifs on the dome are linked closely together and affect each other. Hence, adapting a sample geometric design to the spherical surface is the principal challenge of master Tusi.

In 2011, curators from the Metropolitan Museum of Art (MET) in New York faced a related problem. They intended to build a gallery, which was supposed to look like a small-scale replica of a Moroccan Court yard. Curators, historians, designers and Moroccan craftsmen came together to design the newfound exhibition space. Skilled Moroccan Craftsmen were invited to craft ornaments all over the gallery. Different techniques and materials were used for ornament designs that adorned the surfaces of the exhibition space. They desired to have a panel created out of Moroccan mosaic tiles (zellij), a technique akin to the tile mosaic. Accordingly, the museum curators choose an existing pattern design from Alhambra in Granada, Spain to fit onto the walls of the new gallery. Nevertheless, the Alhambra pattern with large star motifs didn't match traditional Moroccan patterns that are inclusive of rather smaller stars.

Altering the geometric pattern, hence rules was a design challenge for the curators. One of the museum curators explains:

> If we were adapting the scale exactly, we would end up with really big stars all over our walls and we found out in general that the scale in Moroccan monuments are much smaller than in the Alhambra patterns. So, the challenge was how we were going to reduce the width of that right strap work to make it suitable for our skin. The incredible thing we learned is that, if you add a millimeter of width your white strap your stars become huge, if you take a millimeter a become small. It is incredible how much affect the width of the strap work has on the overall pattern.
>
> In the end, we were all in Morocco in the workshop trying desperately to make this work out… not able to do it on the computer, we tried painting on the wall, we tried printing it out in

> various stages**. In the end a designer took scissors and cut pieces of stars strap out and physically created a pattern to show us how the scale could be adapted, it was a very medieval solution.** ("Building the Moroccan Court," 2013)

The designer, who came up with the solution highlights the nature of the material and the importance of the relations between each separate piece: "when you look at the wall you see there are 70 different distinct pieces like a puzzle fitting to each other to create the design" ("Building the Moroccan Court," 2013). The pattern design in this MET Museum example was formed at the end by arranging and putting individually pieces together. According to Castera (1999, p. 206), today not all of the Moroccan craftsmen design the patterns by drawings, some of them use zellij pieces directly to form their designs.

This modern inquiry of the MET Museum curators shows the possible challenges the craftsmen had to deal with. The differences of the singular shapes inside the Karatay pattern reveal that artisans in the Tusi atelier might have created the domical pattern on site by combining different shapes together. Nevertheless, The MET gallery example involves only the matters of scaling and adapting the dimensions of a two-dimensional geometric design on to a smaller flat surface. On the other hand, the making of the Karatay dome pattern includes the transition of the three-dimensional surface onto a flat two-dimensional surface, a method necessary to fabricate the individual tesserae cut out of the square plates.

As in the other cases, the tile mosaic technique requires accurate calculations. The tiling master has to identify the exact number and dimensions of the tile mosaic pieces that he has to mount onto the surface. Then, he collects all these to form larger hexagonal and pentagonal panels that are the spherical polyhedron faces dividing the spherical surface. Due to the properties of the material, craftsmen have to adapt the plates to the surface curvature before the application on to the dome. As seen previously, for this, they use a formwork on which they assemble the pieces backwards and pour mortar to form larger curved polygonal plates.

The making process of the craft is already demanding, yet it gets even more challenging with a more complicated pattern design that relies on difficult geometrical rules. When the pattern gets too tricky to figure out backwards, like in the Domical Karatay pattern, then there might be two possible approaches: Drawing the whole pattern completely on the formwork or alternatively, defining basic relations inside the

pattern and drawing only axes of symmetry and star centers. In a contemporary zellij example demonstrated in Figure 5.89, the intricate pattern design is formed with stars in different sizes. The assemblage in this example is completed through a clear understanding of the relations between parts. The craftsmen draw only the symmetry axes and the star centers on to the mold.

Figure 5.90 : Above: Single pieces put on a reference image to understand shape relations. Below: Reverse assemblage aided by the symmetry axes, primary star centers are important to acquire the right location (Campbell, 2011).

There is a sample geometric pattern in Anatolia that might have worked as the foundational specimen for the design on the Karatay Dome. The exemplar pattern uses the same 24-fold star pattern as the primary motif and adorns the tile mosaic mihrab niche in the Eşrefoğlu Mosque in Beyşehir, close to Konya. The mosque is next to the Eşrefoğlu Turbe, where the other cut-tile domical pattern is. The mihrab is dated to 1301 A.D., roughly fifty years after the Karatay Madrasa. Yet, another pattern that is unfortunately unavailable at this time, reportedly adorned the surfaces of the Nalıncı Baba Tomb in Konya, dated 1251 A.D. Figure 5.90 illustrates Bonner's (2017) generative methodology based on polygonal tessellations to create the pattern.

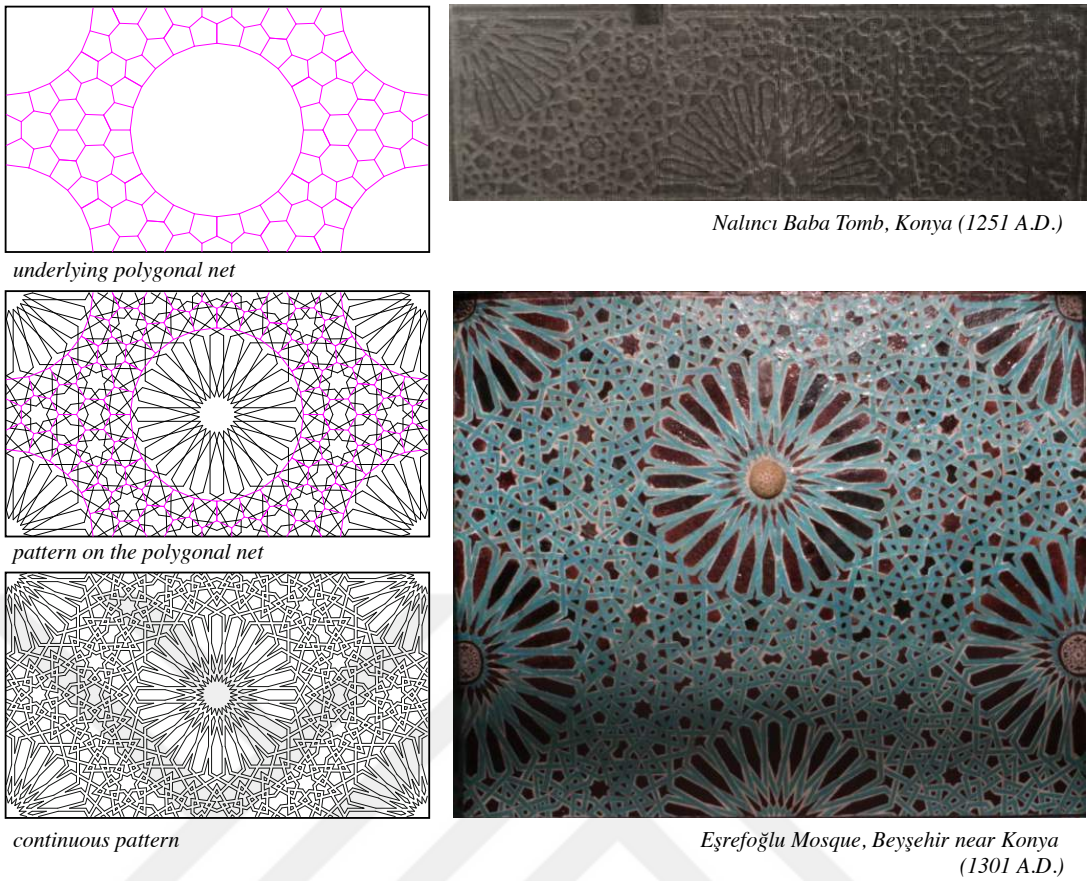
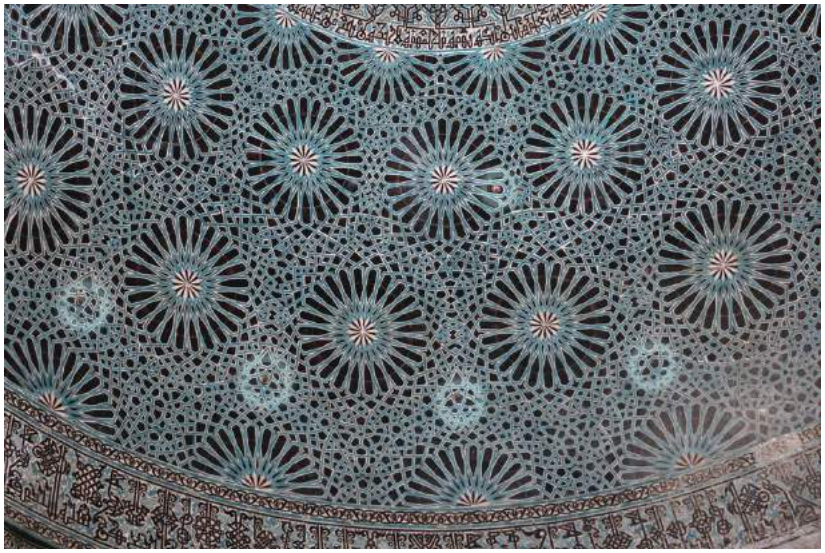
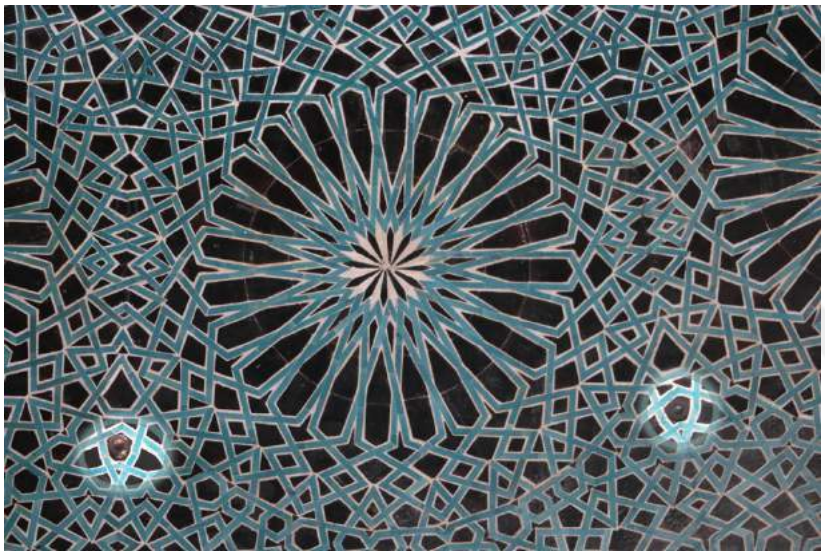


Figure 5.91 : A similar two-dimensional pattern with 24 pointed stars (Drawings adapted from Bonner (2017).

From the aforementioned central circular marks on the domical pattern we may infer that unlike Bonner's method that partly depict the main 24-pointed star on a hexagon, medieval artisans fixed the center of the circle that circumscribe individual motifs and then extended radii from these centers to specify intermediate connections. Later, they attached remaining individual shapes to form the larger hexagonal or pentagonal plates that systematically divide the surface. Another remarkable feature of the Karatay pattern is that metal studs appear over the tile pattern (Figure 5.91). The studs are located on two different levels. The lower stud level is adjusted between the third- and fourth-star rows and the individual metal studs are equally distanced. Yet, the metal studs are placed unevenly on the upper level and this axe almost match with the second star row from the top. Owing to the equally positioned lower level metal studs that coincide with the polygonal slab divisions, art historians (Erdemir, 2001, pp. 130–131; Mülayim, 1981, pp. 120–121; Ş. Yetkin, 1986, p. 67) suggest that these hobnails were used to fasten up tile mosaic plates on to the surface. Yet, the unevenly distributed metal studs that can shed light on to the making require still a review in more detail.





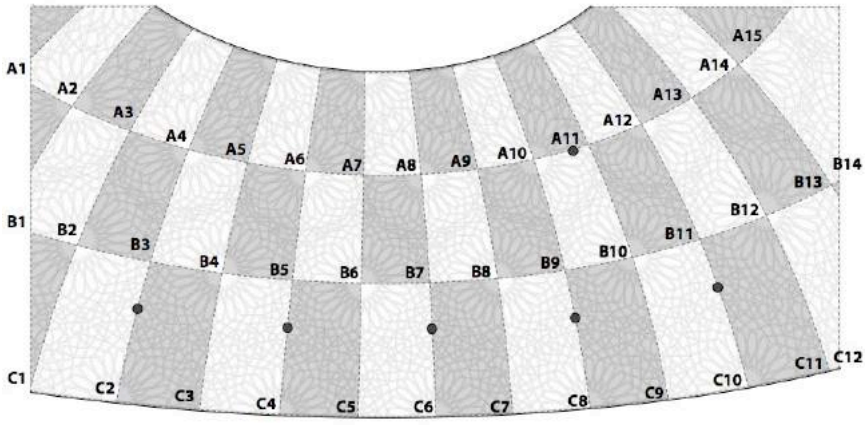


Figure 5.92 : Metal studs that probably fasten up polygonal plates onto the surface appear on the Karatay dome. Below: Studs are equally distanced in the lower level, yet they are unevenly distributed on the upper level (dots show some of the locations).

Preceding inquiries intend to understand Tusi's way of dealing with certain design tasks that include tricky surfaces, three dimensional forms and highly intricate geometric designs. From these studies we can sum up what we have learned as three principal observations:

- ***The complicated pattern on the iwan of the Karatay Madrasa.*** The irregularities inside the geometric design attest to Tusi atelier's computational design process informed by material properties. Craftsmen cover panel seams with additional tile-mosaic pieces that actually do not belong to a regular geometric design. They cut extra shapes from initial square tiles and adapt these to the design to ensure the uninterrupted pattern appearance. That is a remark on noticing the material ability and therefore, computing with material potentials.

- ***Patterned polyhedra.*** Geometric designs that cover the surfaces of diverse polyhedra follow the symmetries of the surface geometry. In order to get an accurate and continuous pattern design for the three-dimensional polyhedra surface, medieval craftsmen develop an intuitive geometrical knowledge of both two-dimensional pattern designs and three-dimensional solids.

- ***The geometric design on the interior dome surface from the Karatay Madrasa***. The pattern design adapts to the surface geometry. There is not an automated design approach, in which a certain pattern is adapted to a complicated surface directly without any modifications. Rather, to ensure a visual continuity designer from the Tusi atelier alter existing designs and change geometric design rules to adapt to the new surface. The rules aid for variations in designs and prove that they do not restrict the creative process.

6. CONCLUSION

6.1 How Does Heritage Guides Us?

This study undertakes computation differently than what is commonly understood from computerization, i.e. predetermined data structures, and investigates medieval artisans' reasoning processes perceiving these as hands-on computational designs that incorporate material and visual dynamics. The thesis argues that historical hands-on computation processes are inclusive of valuable reference and these can be used to develop insights for contemporary computational design approaches.

Centering inquiries on to the works and design processes of a certain medieval craft workshop, referred to Tusi Atelier in the dissertation, this study seeks for a partial answer to a fundamental question: *Why are material understanding and making of historical artifacts still relevant for the digital age?* The investigations on medieval geometric patterns demonstrate that computational design approaches cannot be merely explained by visual transformations, since the designs are informed by material and hands-on making processes. There is a significant chunk of knowledge that medieval craftsmen acquire during their design processes. The *computation* that equals designer's reasoning process is formed by diverse parameters that relate to making.

Additionally, the thesis demonstrates how computation that is inclusive of making and material understanding can go along with creativity. The craft tradition, which encompasses a series of rule-based actions, is still mainly formed through creative decisions. Rather than staying limited to deliberate rules and visual designs, the central figure of the investigations, master builder Muhammad al Tusi, enriches his design palette as he furthers existing rules, hence proves that the restrictions attributed to rules are just in the eye of the beholder. Rules enable sensible and functional conversations about the process and do not embrace any kind of constraint. Hence working with rules is not always a restriction, on the contrary they enhance creativity and work as helpful tools in design. The current chapter provides a summary of the basic concepts that

appear throughout the study, discusses the outcomes and reviews potentials for future works.

6.2 Medieval Craftsmen Undertake Hands-on Computation

The word computation is generally mistaken for the practices executed in the computer environment. Hence, computational design is deliberately abandoned by many design authorities as it is assumed to be restricted into pre-defined computer algorithms. Nevertheless, computation is basically the act of calculation, hence it resembles design that is, by its nature, an act where decisions get manifested, and constraints are defined. "Design computing does not have to be with computers" writes Özkar (2005) and indicates how basic design assignments are exercises of visual computing as well. Design relies on continuing involvement and reasoning and consists of ambiguities and uncertainties. Therefore, it is obviously not a straight-forward mechanism and an appropriate formalism of creativity is most likely unapproachable. Cross (1999) highlights the ill-defined nature of design problems and remarks: "trying to define or comprehensively to understand the problem (the scientists' approach) is quite likely to be fruitless in terms of generating an appropriate solution within a limited timescale" Yet, as much as ill-defined problems exist in it, design encompasses computable problems as well. Exposing the many degrees of the computable efforts and formalizing those are among the duties of designers.

Chapter I introduced a computational understanding of the know-how in the Medieval Islamic art. Historical examples that involve computation without computers, instead computation by eyes and hands, provide valuable reference to design computation's past. In this study, we investigated the hands-on and eyes-on making processes of medieval artisans from the medieval Seljuk Era.

Islamic art is renowned for the use of geometrical patterns that consist of intricate star and polygon motifs. As shown in Chapter II, the generation of these recognizable artistic signatures are founded on various geometrical principles. Therefore, we are able to follow the computational design decisions of the medieval craftsmen through the investigations in the study. Furthermore, as explained in Chapter II, Section 2.2, there is evidently the active involvement of the geometrician, who aid and design collectively with the artisan in a two-way interaction. Joining forces with the geometrician, medieval artisans acquired help when facing an inconvenience during

the design process. Such a collaboration is still valid in the digital era, since there are collaborative platforms, where designers work in partnership with engineers and scientists to enhance their designs. If designers can find ways for formalizing their actions, they can demand help from computer specialists, hence overcome the limited nature of premeditated algorithms. Collective work as in the age of Seljuks can enhance the culture of design computation.

6.3 Designs are Informed by Material and Visual Processes

Geometric patterns on surfaces of monumental Islamic architecture from the Medieval era are renowned for their lace-like appearance that is based on various geometric rules to construct the patterns. Various research that are summarized in Chapter II, demonstrate the geometric principles underlying these unique designs. Yet, such studies usually approach the patterns as end products and do not consider the design processes. On the contrary, a holistic overview on diverse literature that appear in this dissertation shows that mathematics, materials, crafts, traditions, geography, culture, architecture and many other impulses play an integrative role in the production of ornaments.

Geometric patterns were materialized on various media by different dynasties. Consequently, cultural identities had an immense influence on pattern designs. Among various historical civilizations that used geometric patterns, the study gives attention to the Seljuk Sultanate in Anatolia. As it is summarized in Chapter II, although they had deep historical roots that expand to a large geography, Seljuks built unique designs in Anatolian territories and produced distinctive pattern designs as well. There is a good number of researches on geometric patterns. Nonetheless, diverse research that demonstrate possible generative methods approach patterns usually as mere outcomes.

As an example, Jay Bonner (2017, p. 176), a specialist on Islamic geometric designs explains the design in Figure 6.1 (the same figure appears also in chapter V in Figure 5.91) as following:

> an exquisite design with 24-pointed stars in the vertices of the isometric grid and 7-pointed stars within the field: or 24s on triangle/7s in field. This pattern was executed in the carved stone relief of the portal at the Nalinci Baba tomb and madrasa in Konya, Turkey (1255-65), and in the cut-tile mosaic mihrab niche at the Esrefoglu Süleyman Bey mosque in Beysehir, Turkey (1296-97).

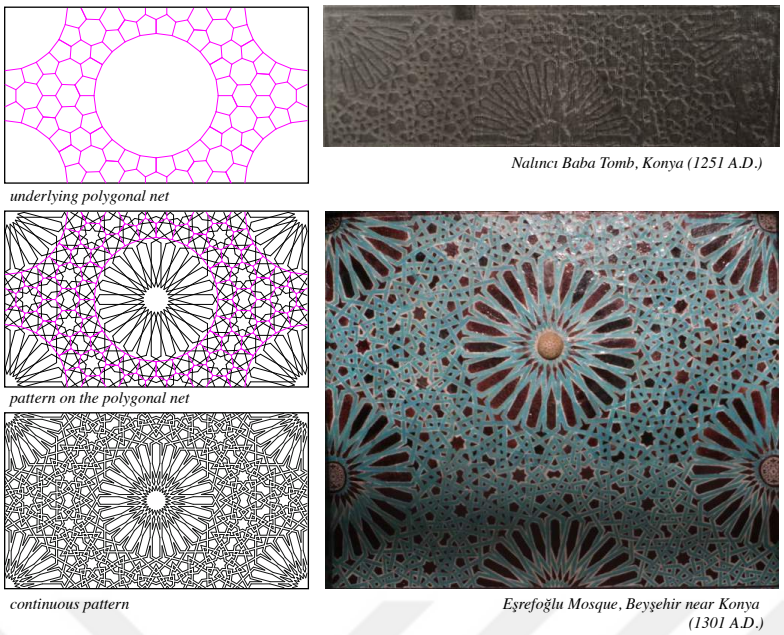


Figure 6.1 : Same visual design executed with two different materials. Drawing based on Bonner (2017).

Geometrical properties of the design are intricately given in Bonner's description, yet the fact that the same visual design appears in two different materials is only a small detail without any further explanation. However, as it is discussed fully in chapter IV, there is a distinction between visual rules and making rules and every single craft has peculiar properties. Hence, although they exhibit the same geometric pattern, the designs in Figure 6.1 are different and involve further computable details of making.

Patterns entail intricate geometrical rules, nonetheless once medieval artisans intend to materialize them, they become design problems. Hence, they are not mere visual products as designs are predominantly informed by the processes. The investigations in this thesis show that the making of the geometrical patterns on building surfaces cannot be reduced into simple visual rules. The design process is explorative and consequently informs the designer in various ways and that not only by visual transformations. The dissertation illustrates this statement by an intense investigation on a certain craft tradition and crafts person. Due attention is given to material processes of how these designs are constructed as they are not mere form exercises from a design point of view.

Seljuks valued artworks and implemented various forms of craftmanship to materialize pattern designs. Yet, one of these crafts, the craft of tile-mosaic that was developed evidently in Anatolian territories left a significant remark in art history. The focus on

the tile-mosaic material in the dissertation is based on significant statements in art history, such as the one by Aslanapa (1971) :

> Throughout the whole of the 13th century the greatest and most brilliant development of ceramics is to be seen in Anatolian Turkish art. From the outset ceramic decoration in architectural forms created by the Turks themselves displays a wealth of design and a technical maturity in advance of Persia. (p. 271)

Overviewed broadly in Chapter III, thirteenth century was one of the most active periods in the history of medieval Anatolia in terms of design activities. After settling into Anatolian territories and ensuring a political stabilization, Seljuks speeded up their artistic pursuits and started building activities. Seljuk Sultanate in Anatolia had already inherited an immense design knowledge from their predecessors, Great Seljuks in Persia. Medieval artisans who worked under the Seljuk Sultanate in Anatolia combined this knowledge with the skills learned from the new geography that was inhabited by Christian communities for many centuries. Many monumental buildings arose out of the inspiration that combined traditional Persian building materials and the stonemasonry mastered by Christians for years. Furthermore, immigrant designers from neighboring lands brought their own craft traditions and provided new insights into design.

Tile-mosaic, the fundamental subject of Chapter III, was a novel cladding material based on ceramic and the outcome of scientific experiments to merge stone and the glaze decoration. This material first appeared in Anatolia and was unique for that time. Chapter III offers the principal properties of tile mosaic and examines the craft in detail. Unlike in other craft practices, tile-mosaic is based on the assemblage of individual pieces (Figure 6.2) that are cut out of a monochrome square ceramic plate and then assembled together on a flat surface or a mold to form larger panels. Hence, the making requires an explicit recognition of all shape relations inside a geometric design. The craft remarks all individual pieces as equivalently important and this explicit material property requires a demanding work flow, unlike in other practices, in which intermediate shapes between pattern lines can be avoided.

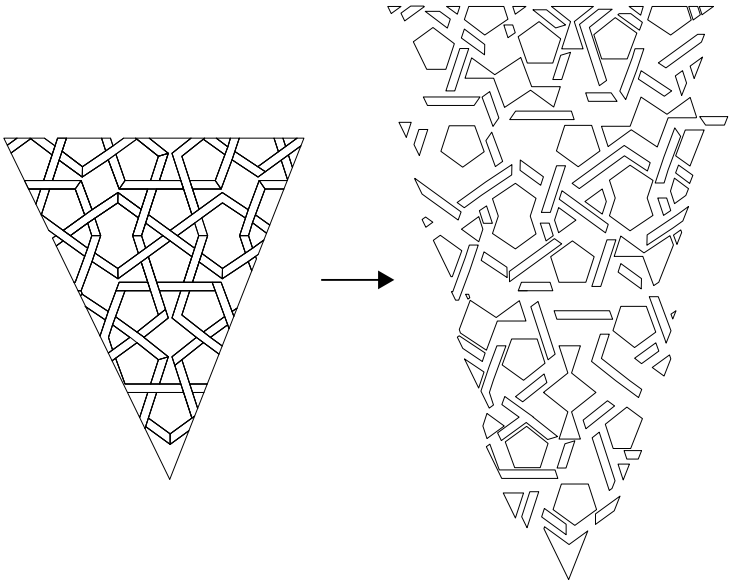
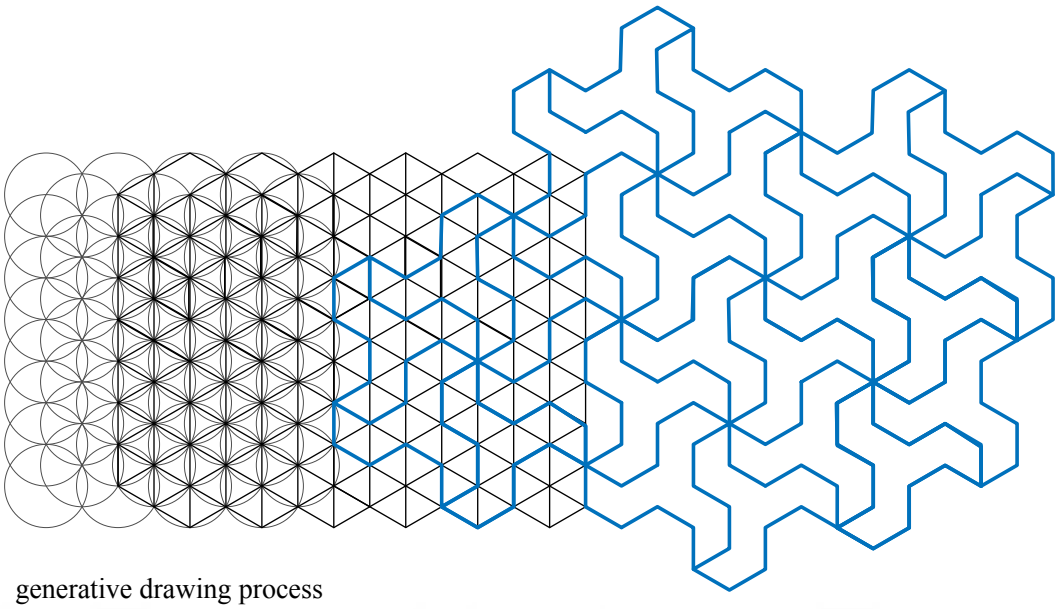


Figure 6.2 : Tile mosaic pieces are cut out of monochrome square tiles and assembled to form the geometric design. The generative method behind the pattern is examined in Chapter IV, Figure 4.16.

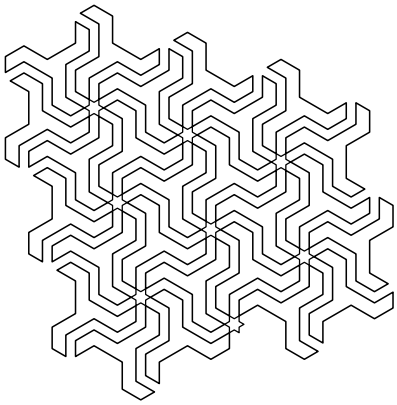
The tile-mosaic craft is demanding, yet it is inclusive of advantages as well. The manipulation of singular pieces is open to many design possibilities and the investigations on tile-mosaic designs present in Chapter IV and V illustrate how Seljuk artisans were computing with the tile mosaic as they recognized material properties. As an example, Figure 6.3 illustrates a tile mosaic design from the Eşrefoğlu Mosque built in Beyşehir. The base geometric pattern is generated with two alternative methods: the circular grid method and the isometric grid method (explained in Chapter II). This draft works as the base geometry for the design, nevertheless the final product is materialized by variating the shapes inside the pattern. A six-pointed star shape in turquoise color is placed upon the six-fold rotational centers. Two further shapes in turquoise and black colors are cut out of monochrome tile plates. The modifications on the primary shapes and colors are based upon design decisions, not geometrical properties. Hence, the final outcome is not merely formalizable by visual rules.

Another design that is illustrated in Figure 6.4 is based upon similar geometric principles from the preceding example. The draft patterns in both designs may be recognized as the same at first glance, nonetheless visual rules alter slightly and that is observable from the viewpoint of a designer. Schneider (1980), an architect who did the most comprehensive work on pattern designs of Seljuks of Anatolia, perceives visual alterations and similarities with the illustrations that are present in Figure 6.5.

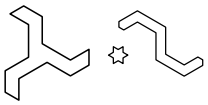


generative drawing process

tile mosaic panel from Eşrefoğlu Mosque, Beyşehir



final design



individual pieces
cut out of monochrome tiles

Figure 6.3 : Tile mosaic design from Eşrefoğlu Mosque built in Beyşehir.

tile mosaic panel from Sırçalı Mescid, Konya

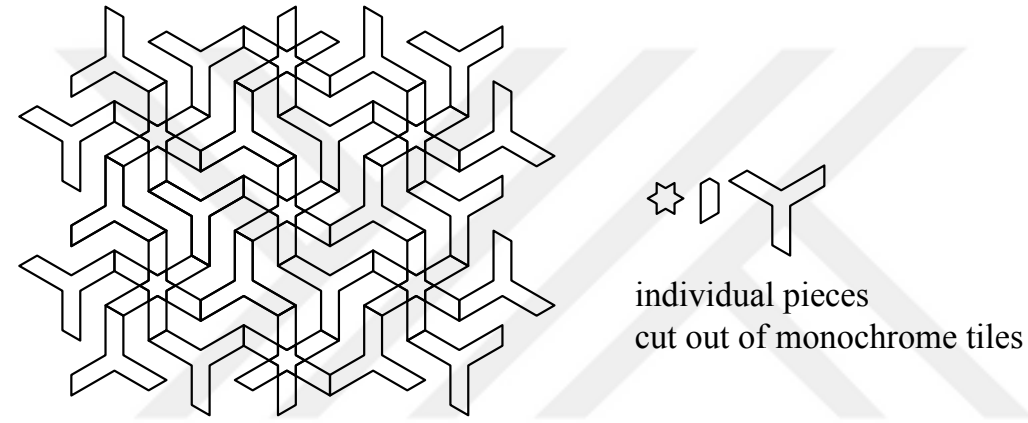


Figure 6.4 : Tile mosaic design from Sırçalı Mescid built in Konya. The design is attributed to the Tusi Atelier.

Figure 6.5 : Schneider (1980) compares the designs in Figure 6.3 and Figure 6.4 and searches for different shapes that appear on the geometric patterns.

Apart from the modifications in visual rules, the medieval artisan, who designed this sporadic instance develops a perquisite understanding of the material. The craftsman excludes some of the intermediate shapes from the pattern as he plays with individual tile-mosaic pieces and such an approach is exclusive to tile mosaic. Recognizing the

possibilities of the material enable to get a unique design as such. This design is attributed to the master builder Muhammad al Tusi and craftsmen who worked in his atelier. From this example and the ones that appear in Chapters III, IV and V, we can conclude that pattern designs are not merely formed by visual properties. The design process informs the medieval artisan in several ways and the computation is stimulated both by visual dynamics and further attributes that relate to making and material.

6.4 Excellent Craftsmanship Depends on the Rightful Understanding of Making

Tile mosaic practice entails many possibilities, yet not everyone conceives it. As broadly explained in Chapter IV, excellent craftmanship depends on years of hard work on a material and the technique in demand. Various instances of tile mosaic can be found in different places in Anatolia. Nevertheless, some of these, attributed to the master craftsman Muhammad al Tusi and his atelier (referred to Tusi atelier in the text) bear unique properties. Chapter IV explains why Muhammad al Tusi's designs appear more delicate compared to others. Initially, the chapter draws a picture of the historical settling in which Tusi operated. In order to illustrate the difference in making in Tusi atelier's approaches, the study explains how making rules are at variance with visual rules on a paper. Tusi Atelier's principal aim is to produce uninterrupted patterns, that don't interfere line connections by disturbing the visual continuity. Yet, an uninterrupted design is only acquirable by the rightful understanding of the tile mosaic material. Hence, Tusi operates with individual shapes and this allows for better designs that correctly fit spatial qualities as patterns express an infinite outlook. The symbolic meaning of patterns for discussions concerning infinity and divinity is deliberately left outside the scope of this thesis. It is assumed that only the uninterruptedness of the pattern yields the effect of infinity and the craftsmen use material potentials to create continuous patterns.

The investigations in Chapters IV and V inquire the dialogue that craftsmen working in the Tusi Atelier conduct with tile mosaic by explaining how material affects their design decisions. We examine designer processes and show how creative minds compute with their hands by touching and sensing the material, the architectural form and many other entities of making. The investigations on Tusi ateliers' designs show that material and making is an integral part of their pattern designs, hence their understanding of making gives the creative value to their designs.

Upon the insights of Tusi's ways of designs, Chapter V explains the construction process and examines how master builder, the person in charge, built the complementary bond between the form and the matter. Some of the outstanding design instances are studied more comprehensively in this chapter:

- *The complicated pattern design on the iwan surface in Karatay Madrasa*. The curved surface is adorned with a pattern design that is built with a repetitive rectangular unit. Nevertheless, the rectangular unit entails a pattern with five-fold symmetry. Consequently, multiplying the repetitive unit ends with an irregular design. Hence, strange shapes, that collide with the regular geometric pattern, appear on the design. These shapes are used by the craftsmen to hide panel borders and therefore prove that medieval craftsmen understand the potentials of the new material and the technique. Artisans operate with single shapes that entail many further possibilities for their creative tasks. The irregularities inside the geometric design attest to Tusi atelier's computational design process informed by material properties. That is a remark on noticing the material ability and therefore, computing with material potentials.

- *Geometric patterns on polyhedral shapes*. Several pattern designs that adorn the surfaces of polyhedra match the face symmetry of individual solids. Hence, geometric designs correctly fit the three-dimensional geometry and form complementary bonds with diverse solids. These instances of polyhedra prove that medieval craftsmen understood the relation between the three-dimensional form and developed an intuitive geometrical knowledge of both two-dimensional pattern designs and three-dimensional solids. They selected the patterns with that understanding in mind.

-*Dome pattern from the Karatay Madrasa.* Following the generative process behind this unique design, we show how medieval artisans approach the spatial properties of the room by constructing a contextual bond between the pattern and the architectural surface. The dome pattern consists of primary star motifs that are linked together in diverse fashion at each level. The design entails a play around with individual motifs and therefore presents the peak point of a creative process that benefits the most from material abilities.

The designs investigated in Chapter V show that the builders and artisans of the time and region approached their tasks in a holistic manner, understanding the relations

between shapes in two and three-dimensions, and across different scales. This ubiquity suggests a holistic undertaking of the architectural design of the monument and supports the transference of design vocabulary across two-dimensional and three-dimensional elements of the design. The sophistication of the holistic and consistent undertaking in designing the form of the building as well as in the application of a geometry of polygons and three-dimensional forms in the tradition of the Tusi Atelier provides an insight to the design knowledge, method and thinking available at that time and in that place (Özgan and Özkar, 2017).

6.5 Computation Goes Along with Creativity

The word rule comes into conflict with creative processes to some extent, yet rule-based systems can operate as inventive thinking tools in design. Rules can be created from scratch, can be applied in different ways visually, and moreover, can be comparative tools to understand what repeats and what changes in design. Tusi atelier's designs show that working with rules and templates does not necessarily limit a pursuit of variance, but on the contrary supports it.

Examined in detail in Chapters III, IV and V, the craft tradition of tile mosaic encompasses a series of rule-based actions, then again is still informed by artist's creative decisions and is therefore not limited to pre-defined rules, rather rules are flexible and changeable. Hence, conducted studies on artisans' approaches inform us how computational design does not necessarily have to be limited to predetermined decisions.

The nature of the puzzle-like tile mosaic material and technique requires a critical understanding of all the individual shapes inside a geometric design. Consequently, it alters the design perception of the medieval artisan. All examinations in the dissertation show how craftsmen calculate with these shape relations, which eventually alter their design decisions. Tusi's keenness in seeing shape relations and an ability in manipulating tile mosaic pieces leads to the unique dome design from the Karatay Madrasa. Tusi's ways of dealing with patterns provide valuable reference for the contemporary age.

Unlike Tusi's design approaches, available computer programs deal with pattern designs as unified wholes, thus straight-forward attempts on the computer environment

cannot recreate Tusi atelier's designs. The application of a certain pattern onto a flat or specifically onto a curvilinear geometry is usually done by distorting or scaling the entire pattern to fit the surface in the computer environment. This process is uncomplicated as it is easily executed with basic implementation in computer programs. The geometrical form in quest requires a suitable division and a mapping strategy that is compatible with the surface geometry as it can lead to visible fragmentation and distortions. Figure 6.6 demonstrates a glass vase with a geometric design that is inspired by a tile mosaic panel from Tusi's Karatay Madrasa in Konya. This design incorporates kufi script that appears on the primary octagonal motifs. The octagonal parts are directly applied on the amorph surface. Consequently, immense distortions appear on the surface geometry, both in the upper and lower parts.

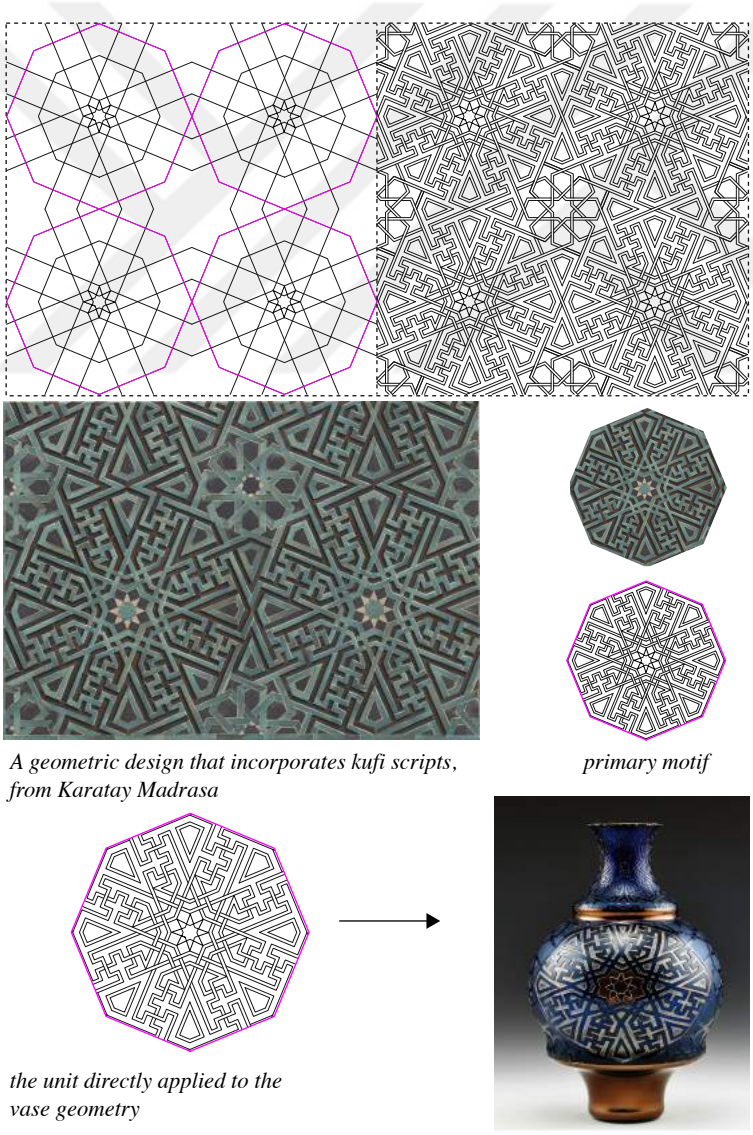


Figure 6.6 : A geometric design from Tusi's Karatay Madrasa used to decorate a glass vase (painting from Sarre (1901b), vase photo from "Paşabahçe Mağazaları").

Methods as such usually take a specific part of a pattern that changes dimensions when placed on different surfaces with diverse curvature degrees. Thus, distortions are inevitable. In this sense, existing computer approaches for tiling and patterning curvilinear surfaces differ from Tusi's approach that incorporates not only visual but material dynamics as well. As it is illustrated in the domical pattern example from the Karatay Madrasa (Chapter V), Tusi manipulates motifs individually to fit the surface. While existing computer applications tend to distort units of a pattern according to the parts of the mesh, Tusi does not manipulate the primary motif, but rather transforms the shape relations to get an intricate design. The different tendencies between Tusi's hands-on traditions and computer-based implementations are noticeable in many souvenir examples patterned with the help of diverse computer programs (Figure 6.7). In these instances, a part of the Karatay domical pattern is taken and applied directly to various forms. However, deformations are unavoidable and consequently these objects emerge as misinterpretations of the original design that is explored in detail in Chapter V.

Figure 6.7 : Houshold goods decorated inaccurately with the Karatay domical pattern ("Anadolu Kültürel Girişimcilik", n.d. ; "Paşabahçe Mağazaları", n.d.).

Nonetheless, just like Muhammad al Tusi did in the domical pattern design from the Karatay Madrasa, dealing with shape relations such as adding or extracting forms can create novel geometrical compositions that are difficult to attain in a straightforward manner in computer environments. The re-production of Tusi ateliers polyhedron patterns and the domical pattern cannot be executed directly with existing computer programs. Though that does not mean that such processes are not formalizable. One of the future aims is to describe Tusi's domical patterning methods in appropriate formalism.

The thesis embraces computational tools, since they entail many potentials. Yet, the thesis rails against the misuse of contemporary technologies dominated by restrictions. Technological tendencies towards computer aided-tools mostly rely on practices executed with predefined systems that not fit into the creative aspects at all times. Computer executes the tasks that are defined by the user and the main intend of the contemporary designer should be based on the proper use of the technology to aid the creative process. The difficult should be a positive entry in our design, challenging tasks stimulate us, whereas manipulation of straightforward methods cannot.

Technological tools become more powerful and sophisticated and if we want a proper adaptation into our creative processes, we have to understand their working logic. As remarked by Sennett (2008):

> The enlightened way to use a machine is to judge its powers, fashion its uses, in light of our own limits rather than the machine's potential. We should not compete against the machine. (pp. 105-106)

Programmers define new algorithms and useful interfaces that are not as complicated as they tend to be and machine language is no longer unapproachable. Muhammad Al Tusi's computational design approaches explored in the dissertation show how the master builder transforms rule-based systems in accordance with his creative design ideas. Akin to the approach of master Tusi, we need to adjust and customize tools and software, so they can truly aid the way we work and explore the expanded capacity of computational design tools.

6.6 Contributions for a Computational Design Culture Inclusive of Making

This dissertation brings a wide variety of literature on geometric art, art history, history, design and computational design fields together and examines how the design

process that is informed by material and visual dynamics influence geometric patterns. Various insights gained from the investigations in the dissertation shows that medieval Islamic Art does not merely entail aesthetical properties or geometrical order. As summarized above, key debates on contemporary computational design practices are conceived from pattern designs as well.

Medieval builders were consciously comparing, adapting, reapplying a design know-how across decades and geographies, between projects and within different aspects of the same project. The design know-how included not only the visual design vocabulary but also materials, techniques and mathematical knowledge that allowed them to materialize their designs. The study explored the partial know-how of Tusi atelier by executing reverse engineering on various designs. Insights that are discovered from the medieval hands-on computational processes can serve both for practical as well as theoretical applications.

6.6.1 Completing the puzzle: Insights for current restoration works

Perceptions on medieval design processes is critical to reflect upon contemporary restoration works. Unfortunately, lack of understanding during restoration projects results in inappropriate applications and intense pre-investigations like those conducted throughout the current study could prevent such ill-treatments.

Historic preservation principles and approaches are a broad topic and that is out of the scope of the current study. However, our examinations in the thesis can aid restaurateurs and contemporary crafts people to understand how geometric designs were materialized.

From the various insights we learned in this study, we can sum up the design process as following:

1. A precise definition of the form and simultaneously a pattern choice from the pattern book, in other words the pattern design on the paper.
2. A strategy to divide periodically the geometric design and a proper scaling to adapt to the surface.
3. Adapting the pattern to the surface, imperfections etc. may arise during this process

4. The application that depends on the material technique. For tile-mosaic, individual pattern shapes are cut from the enameled square tiles and assembled to form larger panels.

How these insights aid a restoration process is discussed more in detail in the following example, the Tomb of Gömeç Hatun in Konya. Parts of this study below are published in Özgan and Özkar (2017).

6.6.1.1 Tomb of Gömeç Hatun in Konya

Tombs are a common building type in Seljuk architecture, but that of Gömec Hatun, dated 1270, is of a unique form (Figure 6.8). It is an iwan with a vaulted ceiling, more than eleven meters in height; the rectangular plan is about eight meters in width and nine meters in length.

The historic tomb has undergone restoration several times, most recently in 2008. The restored version features two irregularly plastered polyhedra (Figure 6.9) on top of two engaged columns of the front portal. These solids, although irregular in current form, seem to be dodecahedra with twelve pentagonal faces. Still, the source for this restitution is not clear. There is no written acknowledgement of any dodecahedra, in either in the reports written prior and posterior to the latest restoration of the tomb (Dazkırlı, 2008), or in the literature on the tomb. Historical references on ancient Seljuk tombs (Önkal, 1996, pp. 338–342; Sözen, 1968, pp. 184–188) highlight the unusual architecture of the monument, but they do not provide a detailed analysis on the column capitals. Earlier records on the tomb (Meinecke, 1976b, pp. 355–358; Uğur, 1937, pp. 567–570; S. K. Yetkin, 1961, pp. 357–360) and the restoration project report (Dazkırlı, 2008) present historical photos in which extremely damaged original capitals are visible. Although the details are difficult to decipher, the capitals allude to dodecahedral geometry, and the current restitution could have easily been based on these photographs. The aforementioned dodecahedron in the Afyon Mısri Mosque (Figure 5.45), attributed to the Tusi Atelier is the only existing dodecahedron geometry in Anatolia. Hence, precedents and a historical context present in Chapter V corroborate the assumption that a thirteenth-century construction of a dodecahedron was possible in Konya.

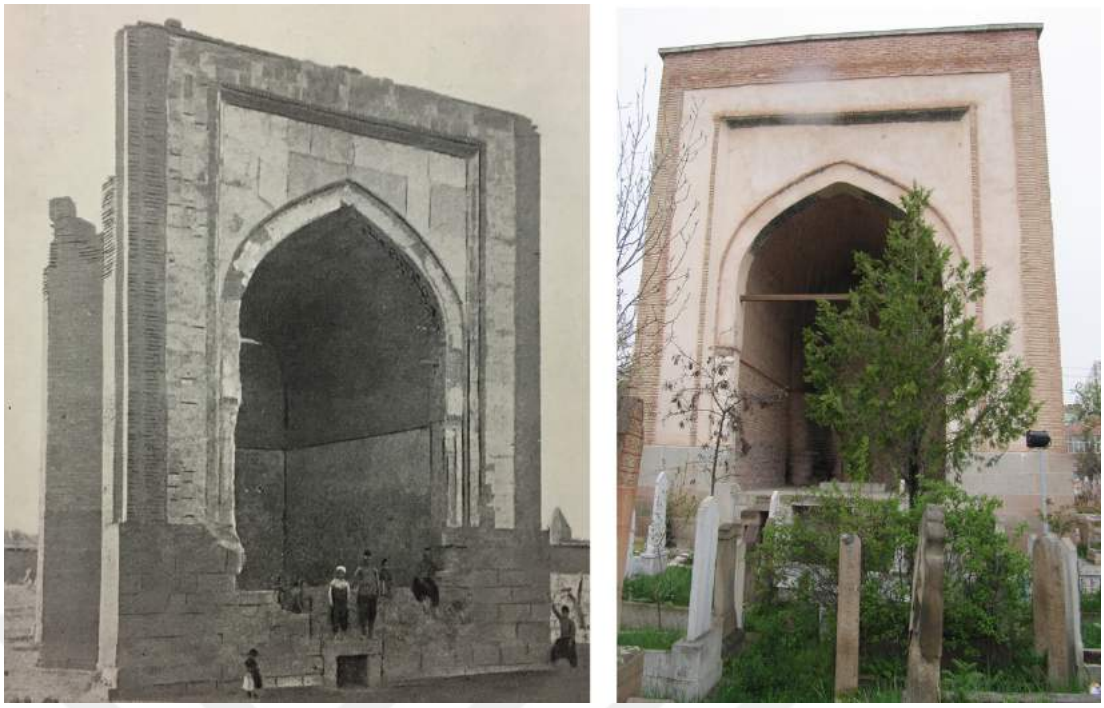


Figure 6.8 : The Tomb of Gömeç Hatun, Konya (1270 A.D.).

Figure 6.9 : The restorated dodecahedron column capital in Gömeç Hatun Tomb, Konya, 1270.

Furthermore, even with differences in material, there are stylistic similarities between the ornamental details of the Gömeç Hatun tomb and the Afyon Mısri Mosque. Stylistic features (Meinecke, 1976b, pp. 355–357) indicate that the legacy of the Tusi atelier may have been involved in the making of the tomb of Gömeç Hatun. This lends support to the hypothesis that the artisans who worked in the construction of the tomb of Gömeç Hatun could have had knowledge of and experience with dodecahedral geometry, as the atelier had worked with the solid before in the Mısri Mosque.

The restoration is a challenging task as it embraces:

- An inquiry on both a three-dimensional form and a search for a possible pattern that adorns the solid surfaces

- An atypical geometric design that does not follow regular rules. The atypical design is located on the adjacent two-dimensional surface of the three-dimensional form.

Making a dodecahedron

The conception and building of the polyhedron are a challenge in its own right for any artisan at the time. Sarhangi (2008, pp. 518–519) reports that Al-Būzjānī's illustrations translate spherical constructions of polyhedra, including pentagonal ones, to flat images. Once the cutter knows the dihedral angle, and hence the angle to taper the corners with, it is possible and straightforward to cut the edge-transitive dodecahedron out of a solid cube (Fig. 6.10).

Physically cutting the dodecahedron out of a solid cube requires supports to hold the irregular solids that emerge as subtraction continues. If the dihedral angle is not utilized, the artisan can at best estimate the angles and achieve a faulty irregular dodecahedron as in the restituted example in the tomb of Gömeç Hatun. From the hands-on making experience that we gained during the cutting, we can reflect on how the three-dimensional geometry can be cut out of a solid material via digital fabrication tools. If a robotic arm or a CNC (Computer Numerical Control) machine is used, a supportive platform is necessary to hold the form tight and prevent any irregularities as irregular solids emerge during the cutting (Figure 6.11).

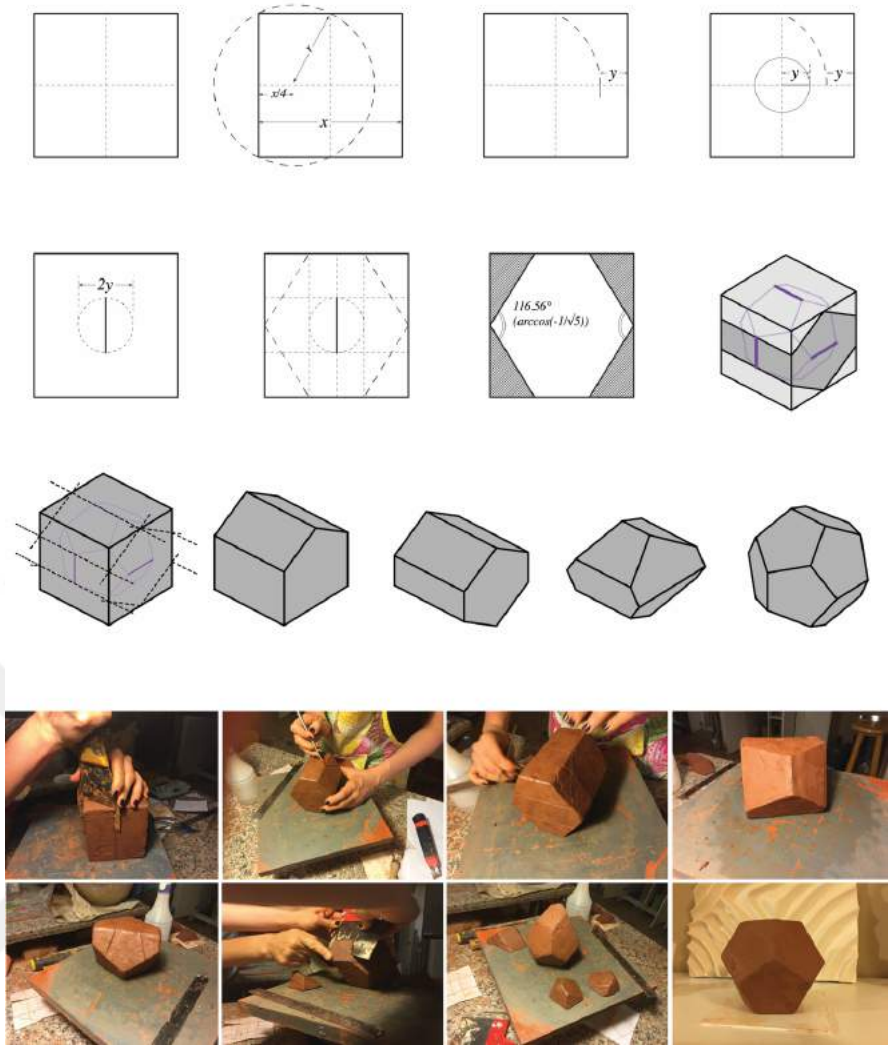


Figure 6.10 : Above: Instructional drawings to acquire the dihedral angle and mark the square with guides for cutting, the digital model of the step-by-step tapering of the edges of a solid cube to reveal a regular dodecahedron. Below: the actual making of a dodecahedron out of clay.

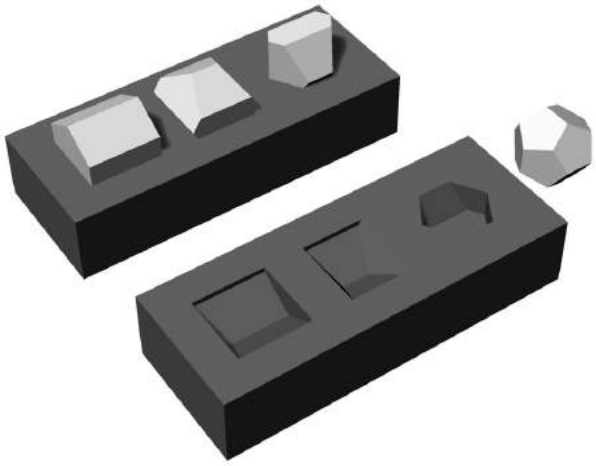


Figure 6.11 : The fabrication plate to hold the form during cutting.

A Geometric pattern for the dodecahedron

In the restored tomb of Gömeç Hatun, traces of tile mosaic remain on some parts of the polyhedron surfaces, revealing that each solid was once covered with a pattern. The turquoise and black coloring of the tiles and meticulous detailing, and that there are more than four different surfaces, albeit small, with the same geometric pattern repeating, provide evidence that they are likely to be original. Some records before the last restoration (Meinecke 1976b, p. 356; Sözen 1968, p.184-188) refer to traces of tile mosaic on the column capital surfaces as well.

The process of applying a seamless pattern on the faces of a dodecahedron is not straightforward. The regular dodecahedron is one of the five Platonic solids and has twelve regular pentagonal faces; the planar development of a dodecahedron leaves rhombus-shaped gaps in other shapes in between its pentagons (Fig. 6.12). A pattern featuring regular pentagons, as it was discussed in detail in Chapter V requires additional polygons to cover the two-dimensional plane that the surfaces of the solid are developed onto (Dürer, 1525, p. 69).

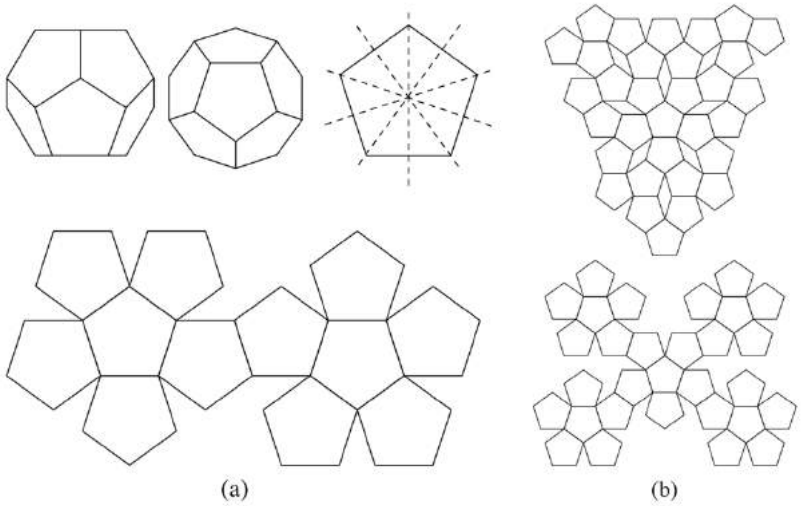


Figure 6.12 : The dodecahedron, the single pentagonal face with corresponding symmetry axes, and the planar development of the solid. (b) Pentagonal tessellations readapted from Dürer (Dürer, 1525, p. 69).

As previously mentioned, the Tusi Atelier practiced both periodic and aperiodic patterns with pentagons. The literature on non-periodic patterns such as Penrose-like tilings in Islamic Art (Cromwell, 2009, pp. 36–56; Lu and Steinhardt, 2007; Makovicky, 1992, pp. 67–86, 2016a, pp. 35–51) mostly suggests localized tiling systems and do not explain how and why artisans shift in methodology from constructing designs based on circular grids to those based on tiles. An investigation

of the dodecahedron and the search for a suitable tessellation on it may lead to the development of a continuous pattern with pentagons. The material properties and techniques of the tile mosaic application already require the planar development of the polyhedra.

In applying an Islamic geometric pattern on the faces of a dodecahedron, one approach is to first project of a periodic pattern onto a cube, and then translate it to the surfaces of the dodecahedron that cube is inscribed in. Most famously, this pattern can be Cairo tessellation (Thomas and Hann, 2008, pp. 101–103). Schattschneider and Walker (1987, pp. 26–27) provide an example to this approach by first covering a cube with Escher patterns based on Cairo tessellation. The shapes are then projected onto the surfaces of a dodecahedron that circumscribes the cube (Fig. 6.13).

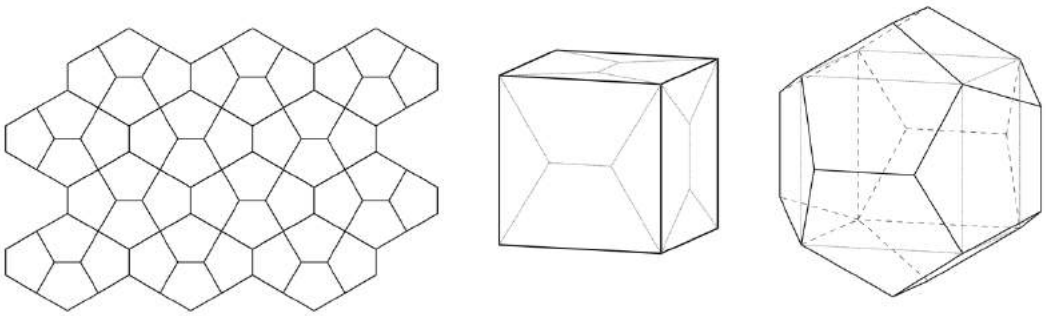


Figure 6.13 : Projecting a pattern onto a dodecahedron as adapted Schattschneider and Walker (1987, pp. 26–27). A Cairo tessellation covering the faces of a cube is then projected onto the surfaces of a dodecahedron circumscribing it.

Knowledge of polyhedral geometry is directly relevant to creating patterns with pentagonal symmetry. In recreating the possible pattern on the dodecahedra of the tomb of Gömec¸ Hatun, we initially relied on the remaining traces of tile mosaic on the surfaces of the portal adjacent to the polyhedra. Although there are no other examples known to us where the continuous patterns on the neighboring surfaces and the pattern on the faces of polyhedral column capitals match and we did not anticipate seeing the same design continue on the dodecahedron, we sought clues in these traces. Indeed, in whatever is left are found resembling polygons—irregular pentagons to be exact—on the surfaces of both the dodecahedron and the bordering pattern above it (Figure 6.14). Yet there is simply not enough evidence to draw conclusions about the whole pattern on the dodecahedron. With reference to the literature (Schneider, 1980, p. 117) and what is visible in the traces, we were able to complete a design for the

remaining patterns on the bordering surface above (Fig. 6.14a). Following Meinecke (1976b, pp. 355–358), Schneider (1980, p. 182) refers to Pattern 277 in his catalogue as one from the iwan of the tomb of Gömec ¸Hatun. This particular pattern adapts well to the traces around the dodecahedron but does not complete the picture. Corners are anomalies. Considering the design attempts by the Tusi atelier, we know that they adapt single pieces to fit onto a surface more accurately. That might have been the reason why these anomalies appear on the corners. Pattern 348 from Schneider (1980, p. 131) is also suitable in certain parts of the remaining tile mosaic. What we have is a combination of these two patterns (Figure 6.14b-d) Both 277 and 348 have pairs of irregular pentagons that group with a bowtie in motifs suggestive of Cairo tessellation.

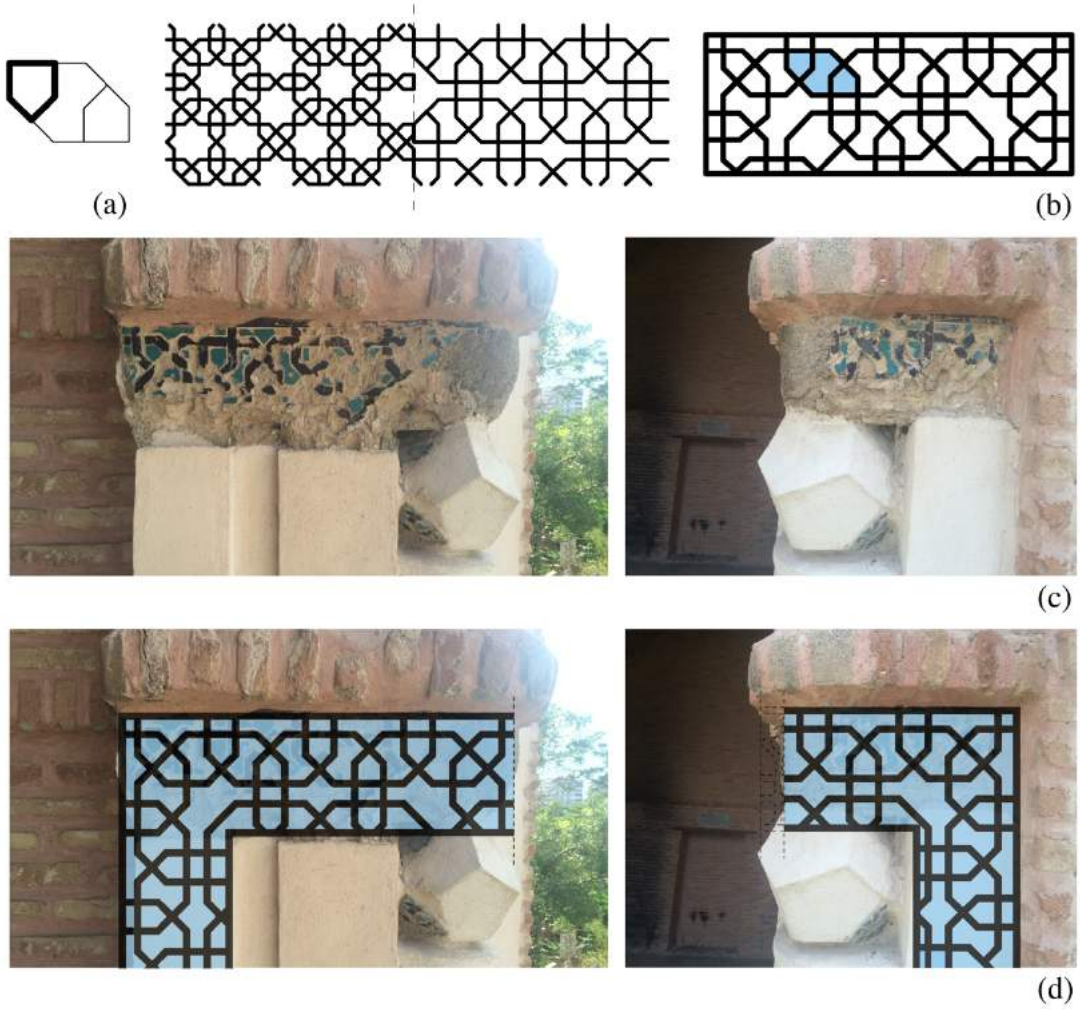


Figure 6.14 : Pattern design for the tiling based on the current condition. (a) Irregular pentagonal tiles are clearly visible inside the pattern traces. (b) Parts of Pattern 277 and Pattern 348, after (Schneider 1980: 117 and 131). (c) Current condition of the iwan. (d) Proposals of the authors juxtaposed on the images. Notice that the design has eightfold symmetry and does not match the fivefold symmetry axes of the pentagon faces of the dodecahedron.

Since this design does not follow a polygonal tessellation but rather embraces individual plays with shapes, it is not possible to straightforwardly create the entire design in an available computer program as further modifications are necessary. On the other hand, despite the similarity in visual vocabulary, it is not possible to extend this design seamlessly on the faces of the dodecahedron.

In an alternative approach, patterning a dodecahedron can basically involve twelve iterations of a design with fivefold symmetry, each fits onto one of the faces of the solid. The seemingly uninterrupted patterns from the cuboctahedra (Figure 6.15) cited earlier (Chapter V) provide a base for adaptations. In each of those patterns, line continuity was preserved over the edges of polyhedra. We adapt their threefold symmetry to fivefold symmetry. Specifically, we chose to adapt the designs on the triangular faces of the cuboctahedra, as the patterns on the square faces of the solids at Beyhekim (Figure 5.41) and Eşrefoğlu Mosques (Figure 5.42) display more refined, curved and floral characteristics unique to each design. Common to the patterns on the triangular faces in all three cuboctahedra is the central arrangement of groups of straight lines which can easily be reinterpreted when there are five sides instead of three (Figure 6.16).

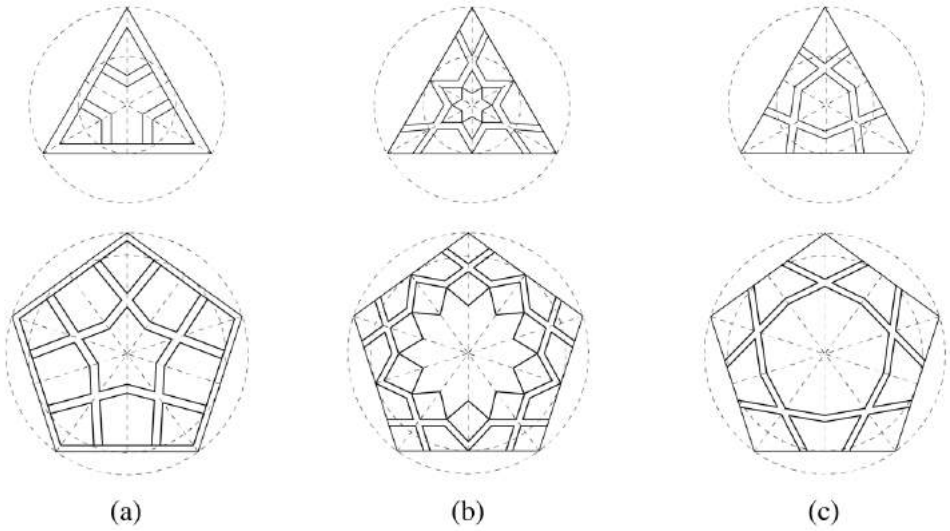


Figure 6.15 : Top row: Patterns from triangular faces of the cuboctahedra. Bottom row: proposed designs for the pentagon based on the styles of the cuboctahedra of (a) Şifaiya Madrasah Portal (Figure 5.29) ; (b) Beyhekim Mosque (Figure 5.41) ; (c) Eşrefoğlu Mosque (Figure 5.42).

Additionally, based on the visual clues remaining on the protected faces of the dodecahedron column capital, we complete two alternate designs (Fig. 6.16). The first

one, developed in Fig. 6.16a, turns inside out the motif shown in Fig. 6.15b. The second one, developed in Fig. 6.16b, includes the irregular pentagon from the bordering pattern from above but does not form a seamless tessellation. All five designs (Fig. 6.15 and Figure 6.16) are valid visually, as they are based on existing vocabularies of design, albeit for different polygons.

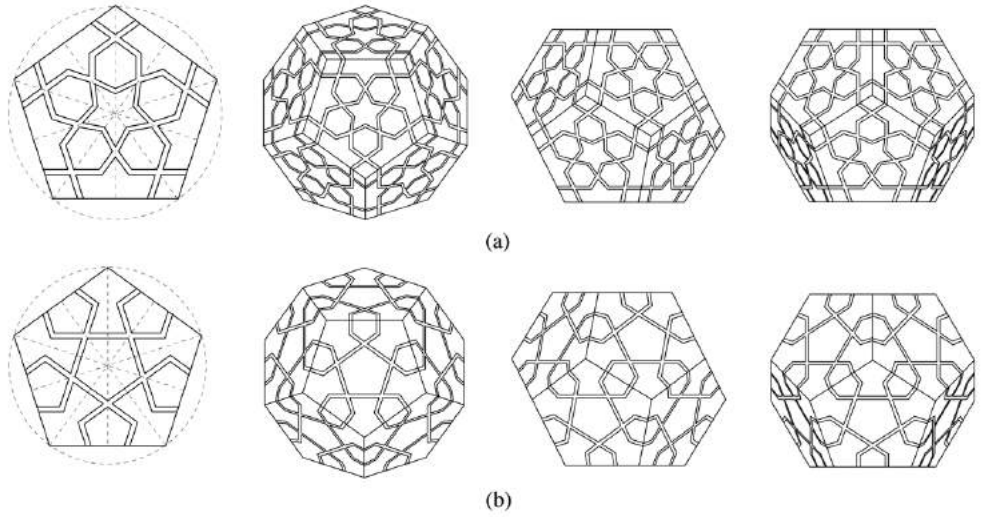


Figure 6.16 : Two alternate designs based on the visual clues left on the protected faces of the Gömeç Hatun dodecahedron column capital.

The design in Figure 6.16 (a) and Figure 6.17(a) embrace a shape that dominates the architecture of the tomb (Figure 6.18). As it was discussed in the Dome example from the Karatay Madrasa (Chapter V), architecture had an eminent effect on pattern designs. We may infer from these that compared to the designs we have developed in Figure 6.11 and Figure 6.12 (b), medieval craftsmen would prefer such a design to adorn the dodecahedron surfaces.

This particular case involves challenging design tasks such as the rightful construction of a three-dimensional form and the design of a proper pattern for the solid. Furthermore, the design on the adjacent surface requires a pattern design, that does not follow a regular tessellation but rather the surface geometry, thus the architecture of the tomb. Hence, a befitting pattern is obtained by the manipulation of singular shapes that are parts of what is left form the tile-mosaic remains. Such designs cannot be straightforwardly built without an understanding on medieval craft process. Nevertheless, these design tasks are secured by the insights learned in this study and can aid the rightful renovation.

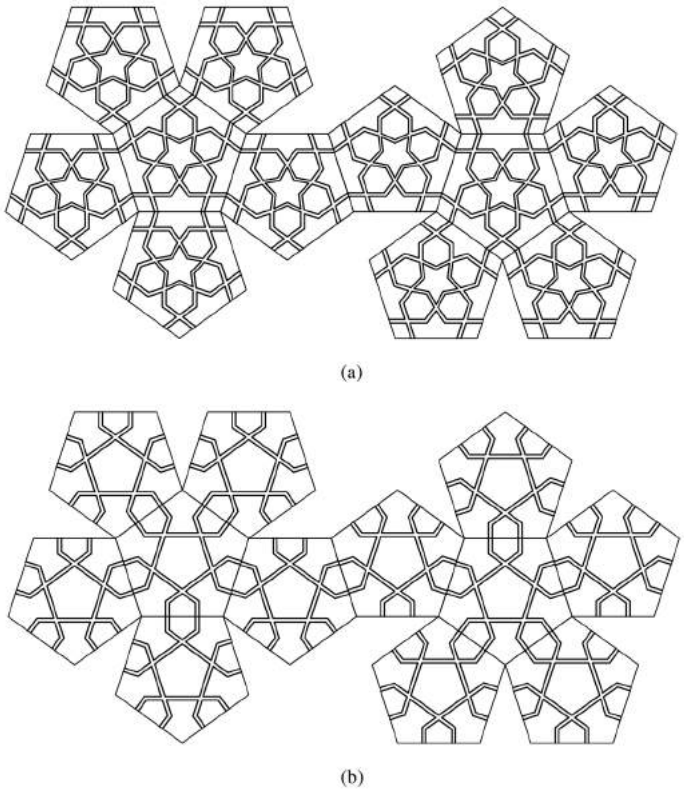


Figure 6.17 : The pattern designs from Fig. 6.16 developed on the dodecahedron net.

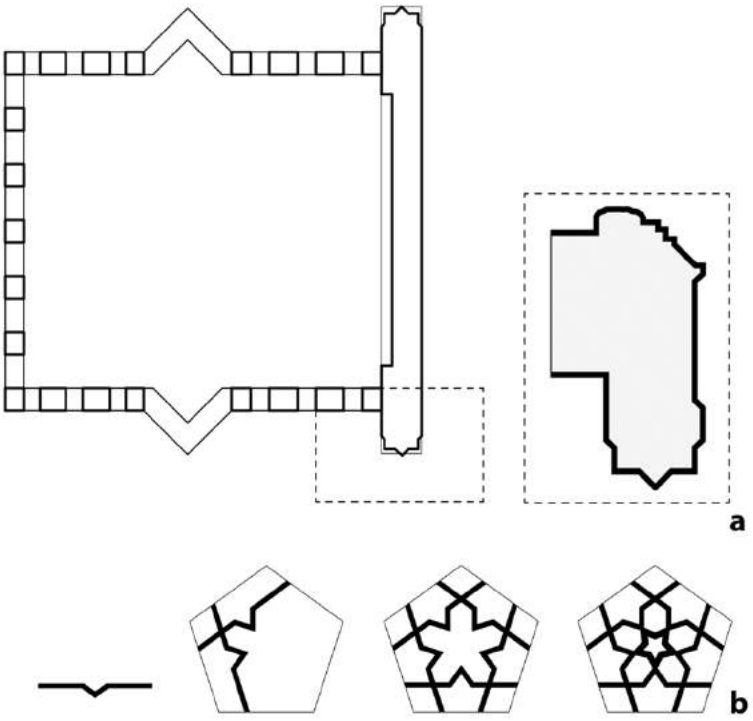


Figure 6.18 : (a) The building plan of the Tomb of Gömeç Hatun and detailed drawing of the back wall. (b) The shape dominating the architecture is visible in the five-pointed star pattern from our proposed dodecahedron design.

6.6.2 Theoretical contributions to the field of design computation

Computational design tools are worshipped by several authorities as means of a new way of digital craftsmanship. Nonetheless, traditional craft practices encompass interactive involvement with the material and the tool at hand and such attributes of making is not easily integrated into computer environments. Craftmanship involves multifaceted skills of making that are learned after many years of practice. Quoting the famous remark of Immanuel Kant "The hand is the window on to the mind" Sennett (2008) argues that we cannot understand the complete picture of a craft without adequately investigating the hand. The insights learned from medieval artisan's hands-on processes conform that making is important and as Gürsoy (2017, p. 111) debates it *"*should not be a discrete stage of design, but an integral part of it*"*. We need to discover ways to incorporate haptic dimensions into our computer aided design processes. One of the future aims is to explore and learn more from the ancient knowledge by tracing other design and construction processes. Medieval Seljuk designs embed an immense knowledge of material and making based processes that could provide many insights onto contemporary design computation fields such as computational making and material computation.

Contemporary debates on design computation are based on the inquiries whether rule-based systems restrict creativity or not. Tusi's design methodology could be useful to create new rule-based approaches and can provide insights to issues in computer graphics and geometric modelling that entail applications such as texture mapping, finite-element surface meshing, and clothing pattern generation. Furthermore, other subjects like surface divisions, which is an essential part of three-dimensional modelling programs and digital manufacturing processes can also benefit from Tusi ateliers creative processes.

Rule-based and grammar-based perspectives into design acknowledge repetitive acts that yield to creativity within design. Nevertheless, the emergence and comparative use of rules are part of the creative process. Medieval designs show how rules can work as comparative tools for variations in design. Medieval computational design processes provide a guide to playing creatively by both visual and making rules, rule-based approach to design holds potential in talking about design creatively and, with awareness. This insight is valuable today both for the design studio and professional

design practices, especially for the rightful integration of digital tools into design processes, as it exemplifies computational design processes that sticks to creativity.

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APPENDICES

APPENDIX A: Aperiodic patterns in Islamic art

Penrose Tiles

Roger Penrose (1979) developed the famous Penrose tiles out of the subdivision of a pentagon into smaller units and filling the gaps with other tiles. Wang, Robinson and Ammann are among other famous aperiodic tile sets and they are all named after their creators.

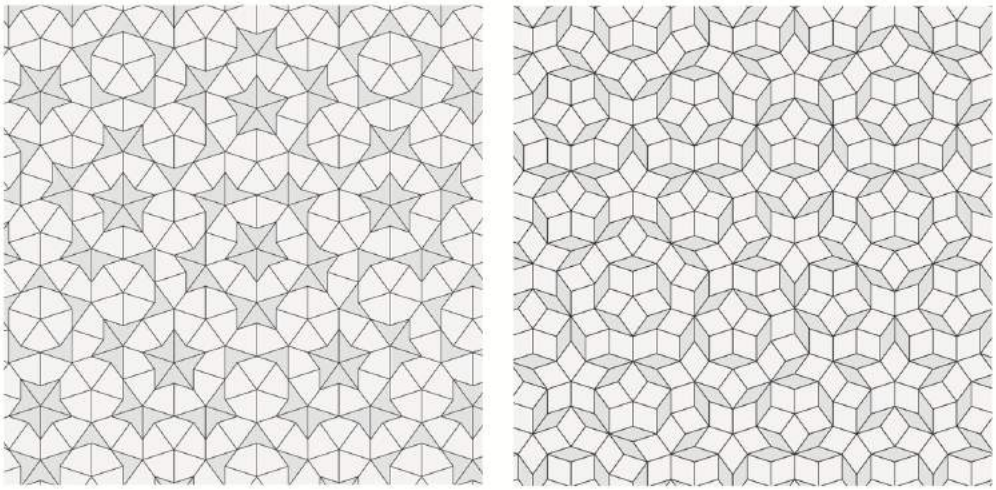


Figure A.1 : Two penrose tilings "kite and dart" (Kaplan, 2002, p. 34).

Five-fold Symmetry

Five-fold symmetry was labelled as forbidden symmetry among crystals until Dan Shechtman, a professor of material science, discovered a crystal with an atomic structure of five-fold symmetry. From then on, crystals with the forbidden five-fold symmetry are labeled as quasicrystals and their discovery brought Schechtman a Nobel prize in chemistry (Wade, 2011, p. 42). Shechtman captured the rotational symmetry in quasicrystals with an electron microscope that showed a diffraction pattern (Figure 5.22). One of the leading researchers that led to the discovery of the quasicrystals was Alan Mackay. Before quasicrystals, Mackay experimented in 1982 with Penrose mosaics by putting circles that represented atoms on tile intersections. He illustrated a theoretical structure that resembled Schechtman's actual diffraction pattern (Lidin, 2011) (Figure A.2). With these experiments, Mackay showed that despite its non-periodicity, the diffraction pattern of a Penrose tiling exhibits a long-range order.

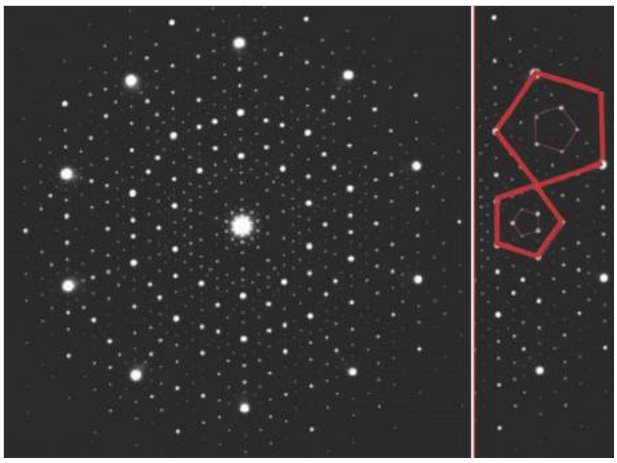


Figure A.2 : Electron diffraction panel from a quasicrystal. Perfect pentagons are present within the global structure (Lidin, 2011).

A Design Approach Based on Diffraction Patterns

Available structural models behind aperiodic Islamic pattern designs such as Lu and Steinhardt's (2007) depends heavily on local rules, yet Ajlouni (2012) proposed a new model that explains a global long-range order of Islamic aperiodic patterns. In this study, Ajlouni (2012) suggests a geometric structure that resembles the diffraction pattern of a quasicrystal and uses this as a sub-grid for the generative process of a geometric design. The continuous design is completed by putting single units on to the intersection points of the sub-gird. Yet, there is no evidence for the historical usage of such a methodology (Figure A.3).

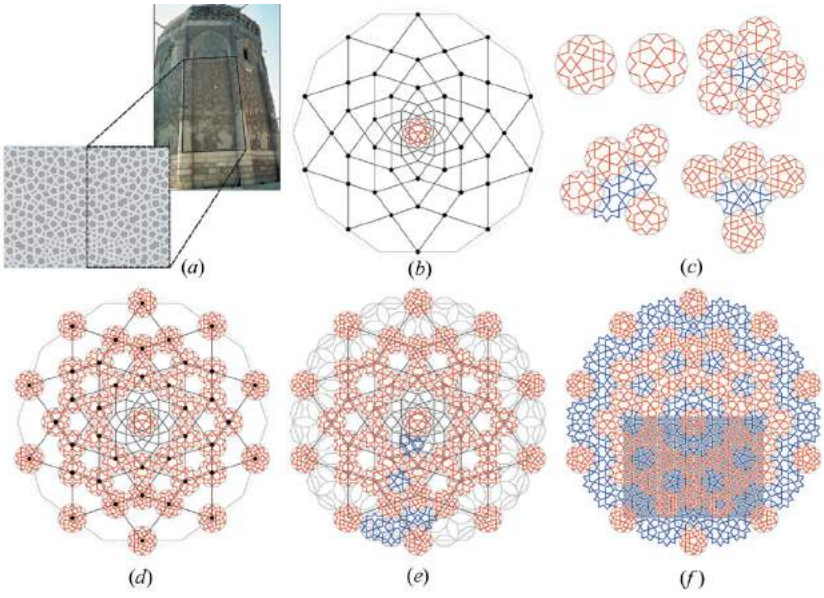


Figure A.3 : Ajlouni's (2012) method based on a diffraction pattern to construct the complicated pattern design in Gunbad-i Quabud.

APPENDIX B: Polyhedra in Islamic art

Polyhedral Geometry

In three-dimensional geometry, a polyhedron is a solid. It consists of flat polygonal surfaces usually joined at their straight edges. Polyhedra are one of the most intensely studied forms in history of mathematics. The first recorded theory of polyhedral geometry goes back to Euclid's book *Elements,* a treatise on mathematics and geometry, circa 300 BC. Euclid deduced the principles of what is now called Euclidean geometry and described the construction of the five regular polyhedra with congruent even faces. His reference to Plato's *Timeaus* led to the common depiction of these polyhedra as the *Platonic Solids* (Figure B.1). Platonic solids with their perfect symmetry and their exemplification of unity have been stimulating many and modifying the polygonal faces have resulted in the discovery of various intricate forms.

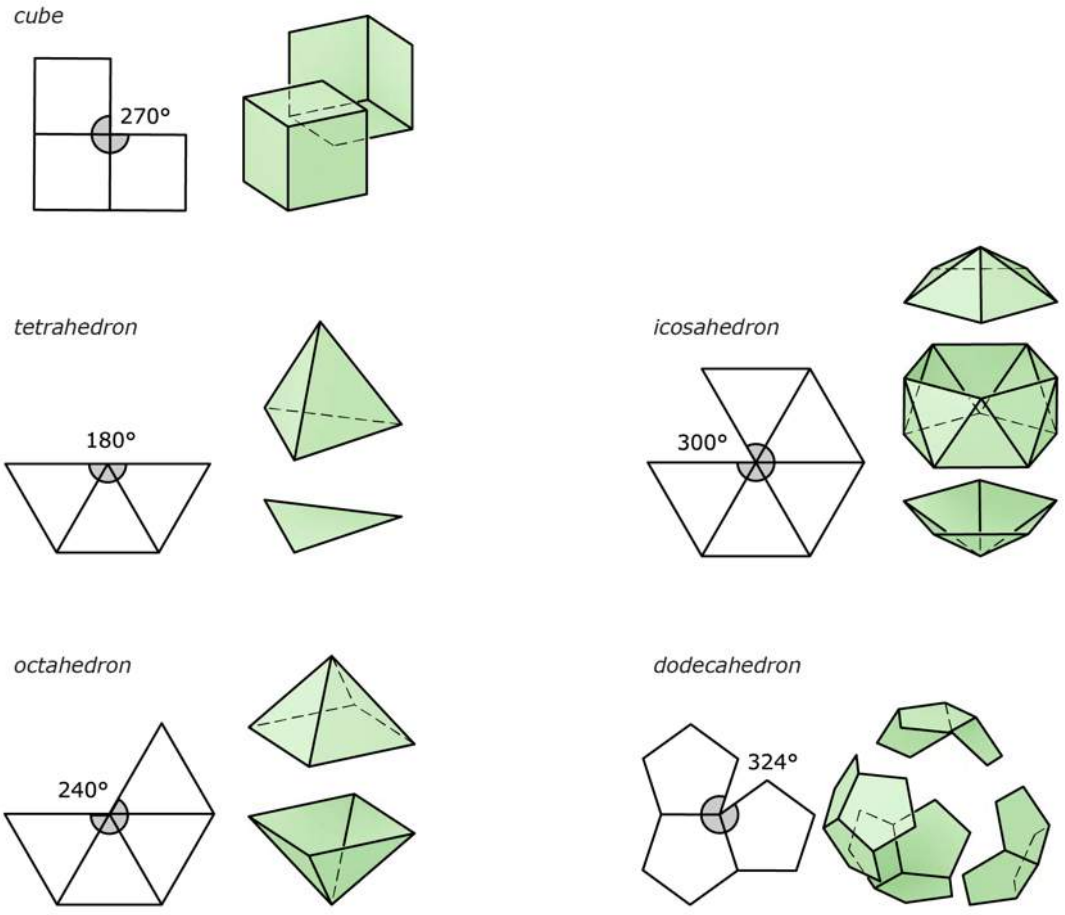


Figure B.1 : The five Platonic solids (Pottmann, 2007).

Polyhedra have been a matter for inquiry for both mathematicians and artisans. Archeological discoveries of carved stone objects from Scotland reveal that a conducive understanding of their geometry existed even in Neolithic periods (Lundy, 2010, p. 18). Renowned mathematicians such as Johannes Kepler and Marcus Wenniger studied the polyhedron while pioneers of art, such as Albrecht Dürer, M.C. Escher and Leonardo Da Vinci relied on mathematics in their works and especially investigated the polyhedral geometry. The concept and theory of the polyhedral geometry were also recognized among Islamic mathematicians.

Al-Kindi (801–873), the Banu Musa brothers (ninth c.), Thabit ibn Qurra (826–901), al-Khazin (900–971), Abū al-Wafā' Būzjānī (940–998), Abu Nasr Mansur Ibn Iraq (960–1036), Ibn al-Haytam (965–1040), and al-Biruni (973–1048) examined polyhedra extensively. Thabit ibn Qurra and Abū al-Wafā' Būzjānī have particularly studied and also geometrically illustrated these solids (Hisarlıgil & Bolak Hisarlıgil, 2018, p. 127) Abū al-Wafā' Būzjānī (ca 940-998) deals in the last chapter of his aforementioned treatise with the problem of deconstructing a spherical surface into regular spherical polygons. The inquiry of such a problem is connected to the construction of regular polyhedra. His manual of practical geometry describes the five platonic solids, but also a part of the thirteen semi-regular solids found by Archimedes (Scriba & Schreiber, 2015, p. 183).

Polyhedra in Islamic art

Several archeological items from the medieval Islamic world are evidence of artistic inquiry in Platonic solids and other polyhedra. An 11th century necklace of Iranian Seljuk origin, currently in the collection of the Metropolitan Museum of Art in New York City, is one example with dodecahedron gold beads. Other small polyhedron objects, on display in the collection of the Museum of Seljuk Civilisation in Kayseri, Turkey, are made out of bronze for use as decorative weights in trading and are a testament to the knowledge of constructing polyhedra by the cultures that produced the geometric patterns on architecture. Moreover, throughout architectural monuments from medieval Anatolia in the Seljuk period, polyhedra can be occasionally seen as part of architectural elements, mostly engaged column capitals and pedestals. Consistently, almost all of these are also unique instances with geometric patterns on the surfaces of the polyhedron. In their research Hisarlıgil and Bolak- Hisarlıgil (2018,

p. 126) visit twenty different Anatolian towns and declare that they record fifty-nine buildings, where they found various polyhedra examples. A part of the recorded polyhedra examples are dated to Ottoman period, yet most of them belong to the Seljuk Era.

Escher's Icosahedron Design

The Dutch graphic artist Escher is one of the few who worked on covering the surfaces of various solids with tessellations based on different symmetry groups. Escher was famous for his enthusiasm towards using complicated tessellations to create sophisticated pattern designs. His desire of an uninterrupted pattern design was never feasible on a two-dimensional plane. Thus, he started to experiment with paper polyhedron models to explore the geometry and befitting tessellations. In an early experiment on a paper rhombic dodecahedron, Escher tries to cover the polyhedron with one of his existing pattern designs and notices that the solid requires a special arrangement as the pattern continuity is disrupted on edges. He finally achieves his goal in a pattern covered icosahedron, a chocolate box designed in 1963 for a tin can manufacturer from the Netherlands. The pattern design on the icosahedron was based on one of his earlier prints (of sea shells and starfish) which he had to arrange in a different way to fit the icosahedral symmetry (Schattschneider & Walker, 1987)

In the last version of the pattern, a starfish with fivefold symmetry stands on each intersection points of the surfaces and the remaining triangle surface domain is covered with a threefold pattern made out of shells (Figure B.2).

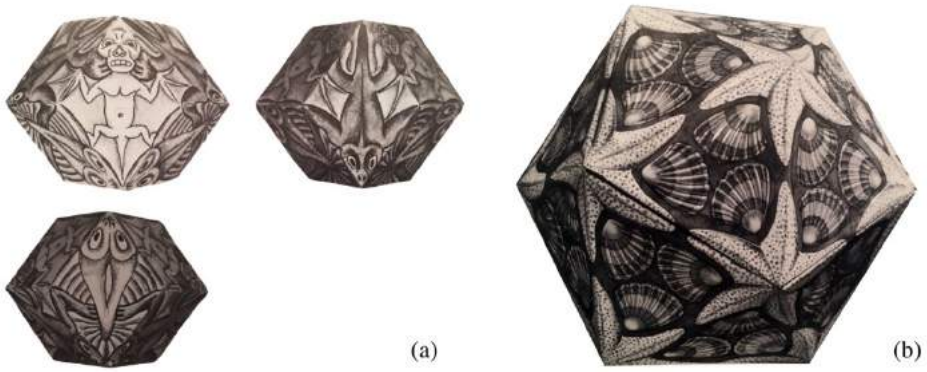


Figure B.2 : Escher's paper polyhedra models (a) Failed attempts with a rhombic dodecahedron (b) The pattern covered icosahedron chocolate box. (Schattschneider & Walker, 1987).

APPENDIX C: Spherical geometry and the dome pattern from Karatay Madrasa

Works on Spherical Geometry in Medieval Islamic World

Spherical geometry is "the study of figures on the surface of a sphere" (Weisstein, n.d.) and basic concepts in the plane geometry like points, angles or lines still exist on this three-dimensional geometry. Yet, the analogues of the straight lines on a plane are the great circles that are the intersections of the spherical surface with any plane that goes through its center. Among the many practical applications, mathematical geography and astronomy make the most usage of this geometry (Berggren, 2003, p. 157). Muslim mathematicians were remarkable astronomers as well and for many different purposes they had to deal with spherical geometry. Astronomical problems, some of which were related to religion like the calculation of prayer times or the accurate position of the qibla (the direction of the prayer to Mecca) drew forth to the many developments in descriptive geometry (Berggren, 2003, p. 127). Medieval Islamic astronomy was founded on the earlier records inherited from ancient India and Greek, works of Hipparchus, Ptolemy and Menelaus among others. These elder texts were addressing already some trigonometric methods, yet astronomers that lived around 8th-15th century A.D. in the Islamic world advanced those and gave their own descriptions. Al-Khwarizmi (c. 780 – c. 850) had already composed a sine table that included explanations. Yet, most remarkably Muslim astronomers extended sine and cosine, the two basic functions presented by the Indians, and described the complete six functions we use now in modern trigonometric functions. With his work *Zij al-Majisti,* Abu l-Wafa Al-Buzjani (10th century) is regarded as the first to have calculated and depicted the complete set of trigonometric functions. Nearly all of the Muslim astronomers composed trigonometric and astronomical handbooks that are known as *ziges* (Berggren, 2003, pp. 127–157; Scriba & Schreiber, 2015, pp. 185–186). Mathematicians in the Islamic era introduced trigonometric functions in spherical geometry as well. Amongst other, Habash al-Hasib (9th century), Abu l-Wafa Al-Buzjani (10th century), Abu Nasr Mansur ibn' Iraq (11th century) and Ibn Mu'adh al-Jayyani (11th century) contributed significantly to the development of spherical geometry (Berggren, 2003, pp. 173–174; Freely, 2010, p. 111). On the other hand, Mongol ruler Hulagu Khan had built an observatory in Maragha in Persia, for Nasır al-Din al-Tusi (13th century) who is recognised as the most significant oriental scholar for trigonometry. He dealt with spherical triangles as well and wrote the first independent treatise in trigonometry (Scriba & Schreiber, 2015, p. 188). Together with

his student Qut̤b al-Din al-Shirazi, al-Tusi developed an alternative model of Ptolemaic astronomy and explained in detail in his work *Tadhkira* (Berggren, 2014, p. 426).

The geometrical problems that relate to spherical geometry were mainly the calculation of circular arc sizes or angles on the spherical surface and related to practical applications of either the celestial sphere or the earth. Small models of the sphere similar to Ptolemy's armillary sphere that represent celestial longitudes and latitude with rings solved calculations for such problems. As early as in the 9th century Muslim astronomers recorded treatises that include descriptions of such devices (Figure 5.47) (Berggren, 2003, pp. 157–161).

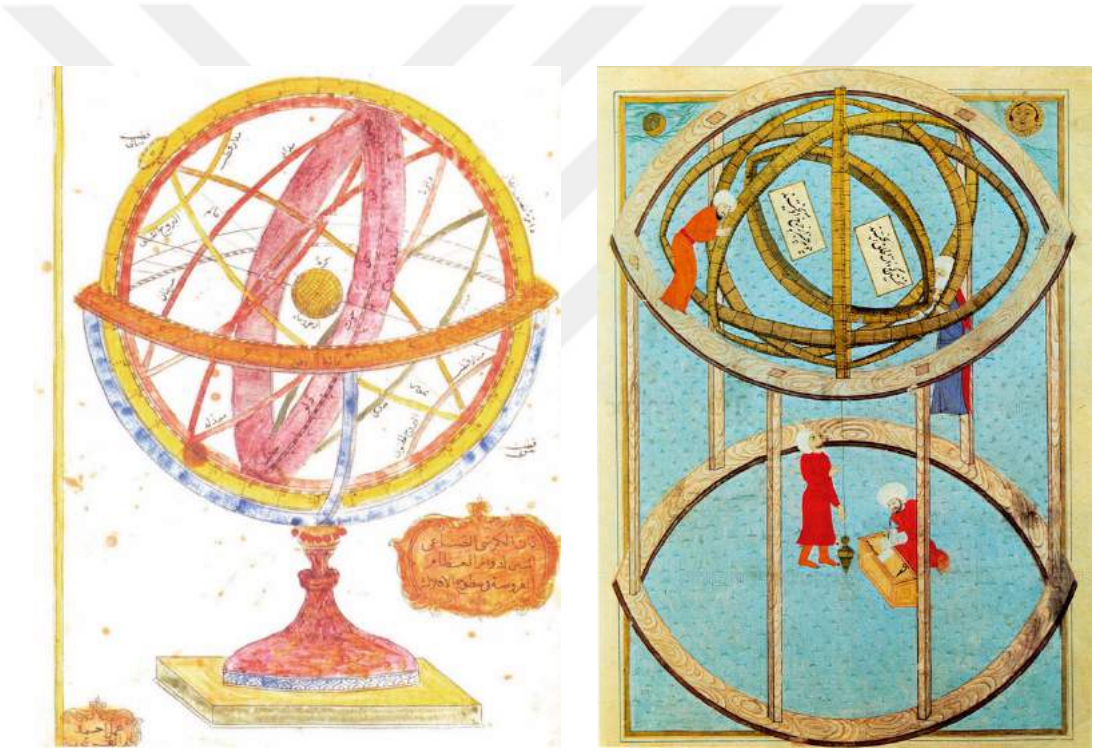


Figure C.1 : Left: The Armillary sphere from the first printed edition by Hajji Kalifahs Jihan Numa (mirror of the world)(Scriba & Schreiber, 2015, p. 186). Right: a sixteenth century Ottoman manuscript depicting a colossal armillary sphere ("Armillary sphere," n.d.).

Karatay Pattern

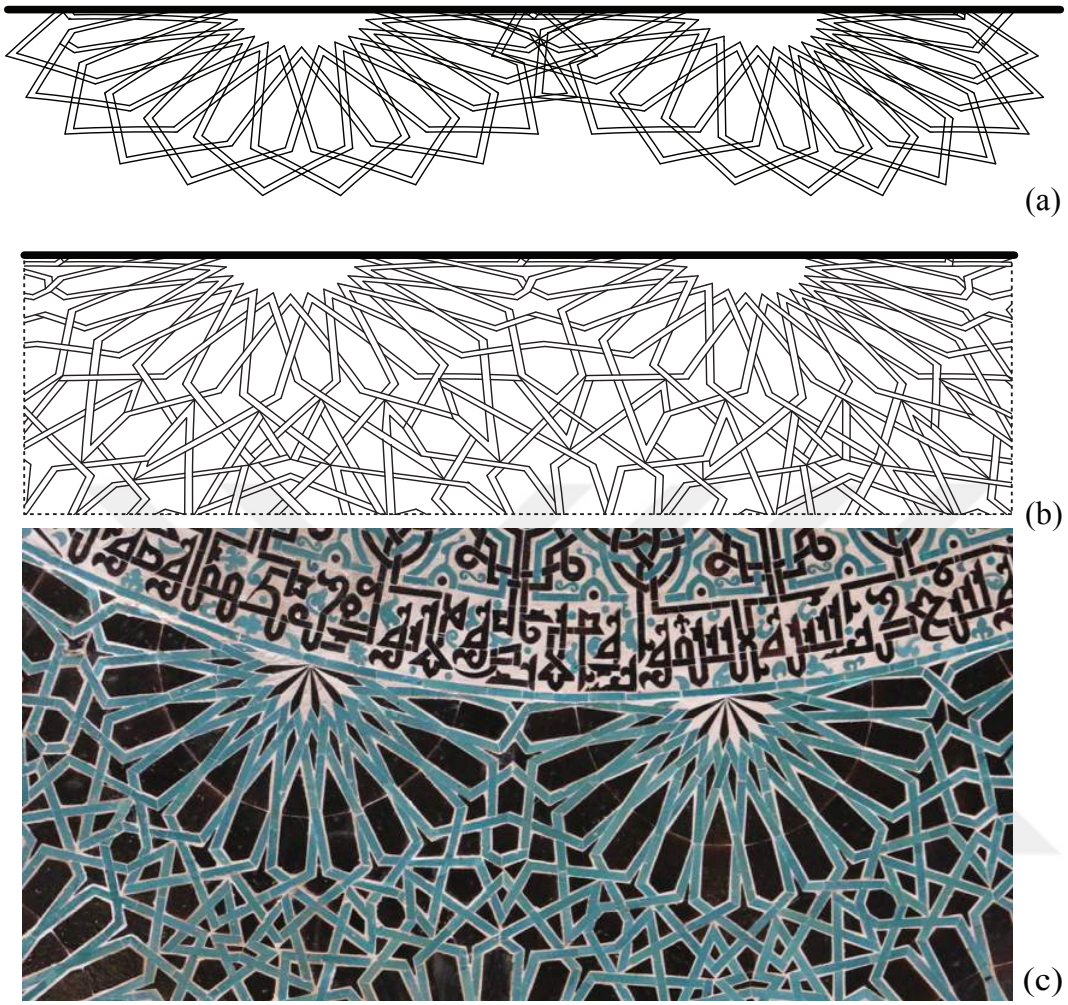


Figure C.2 : Step by step alteration between the star connections in Row 1. (a) The ordinary connection of the stars. (b) The final design adapted by the artisan. (c) Close-up. This design is consistent throughout that row.

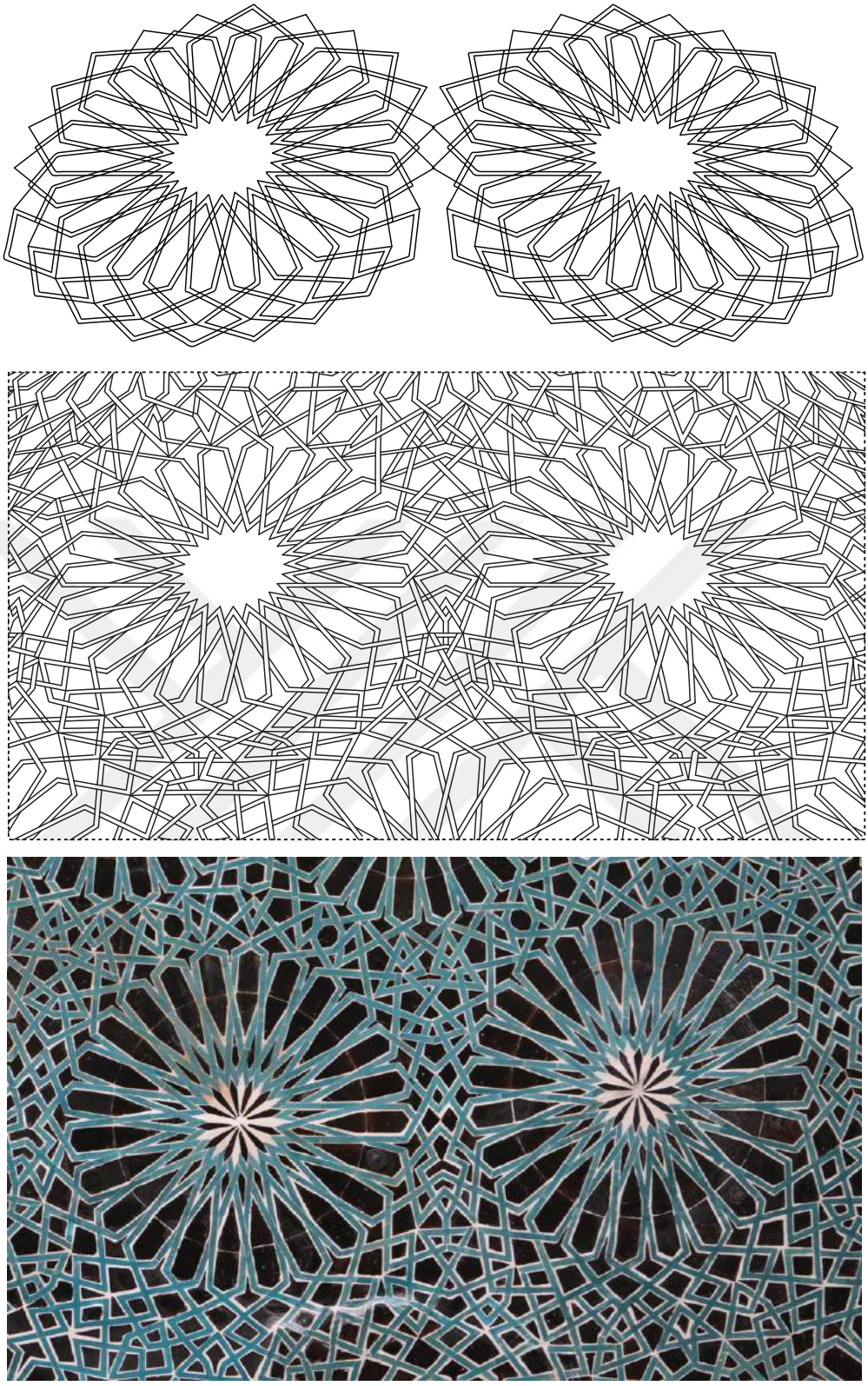


Figure C.3 : Step by step alteration between the star connections in Row 2. (a) The ordinary connection of the stars. (b) The final design adapted by the artisan. (c) Close-up. This design is consistent throughout that row.

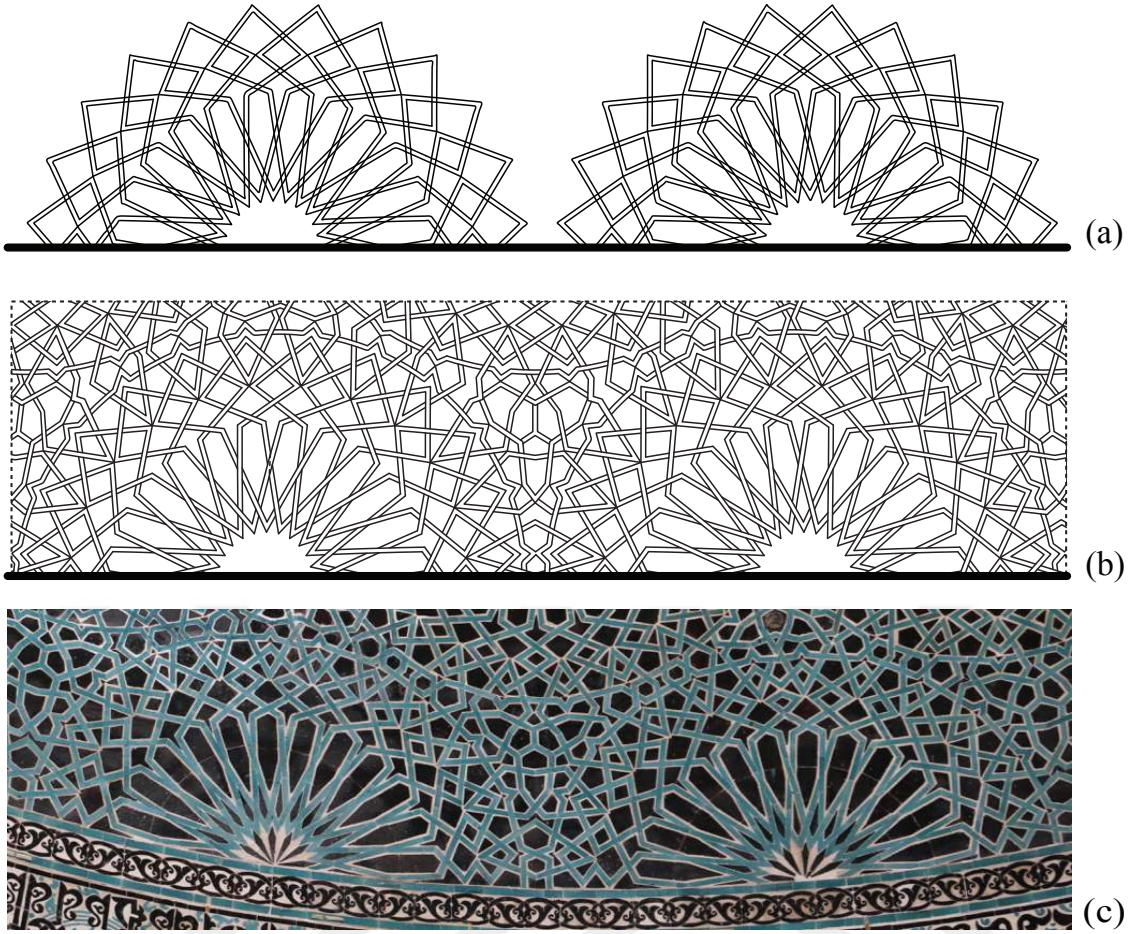


Figure C.4 : Step by step alteration between the star connections in Row 4. (a) The ordinary connection of the stars. (b) The final design adapted by the artisan. (c) Close-up. This design is consistent throughout that row.

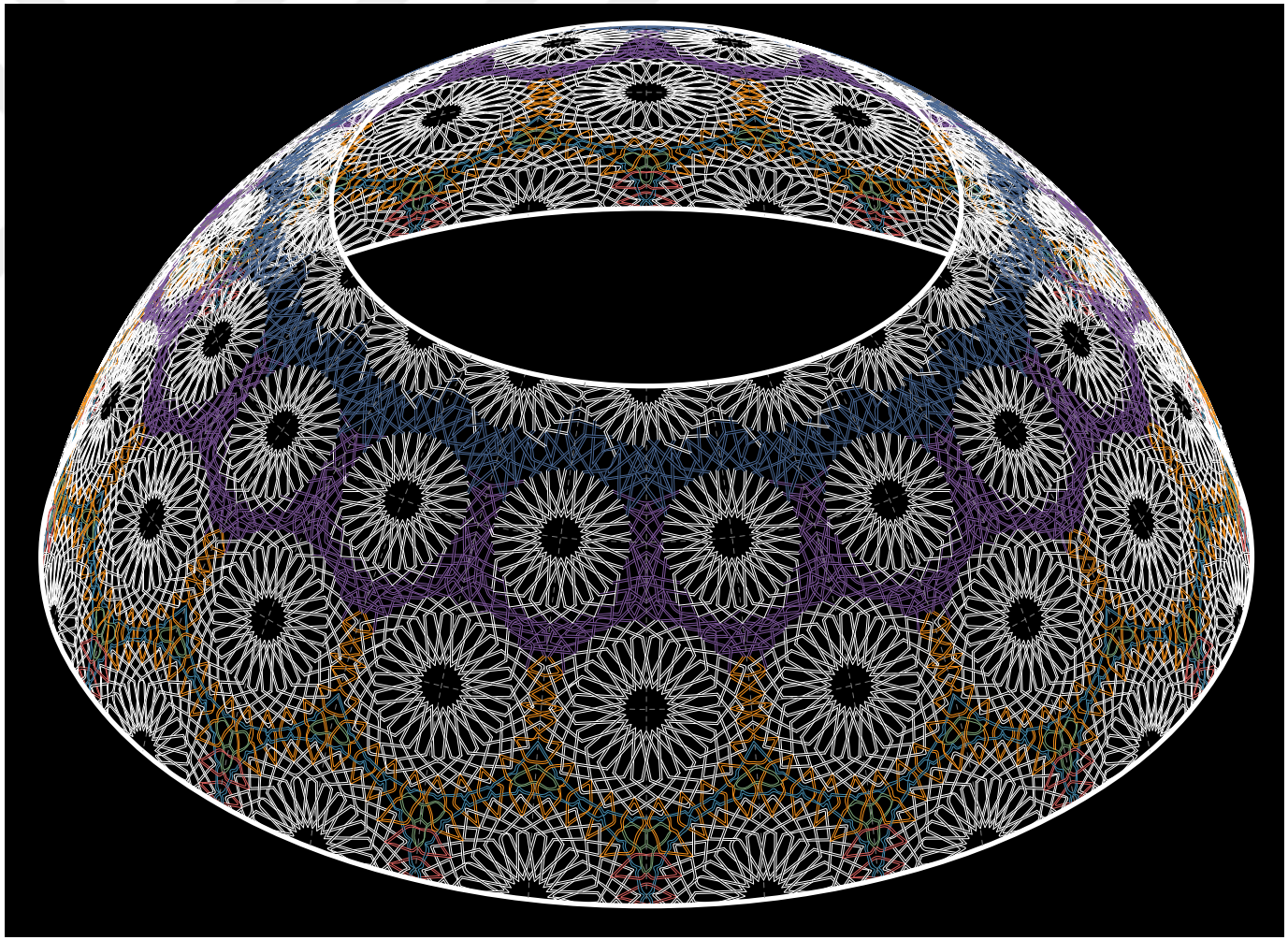


Figure C.5 : Perspective drawing of the Karatay dome pattern.

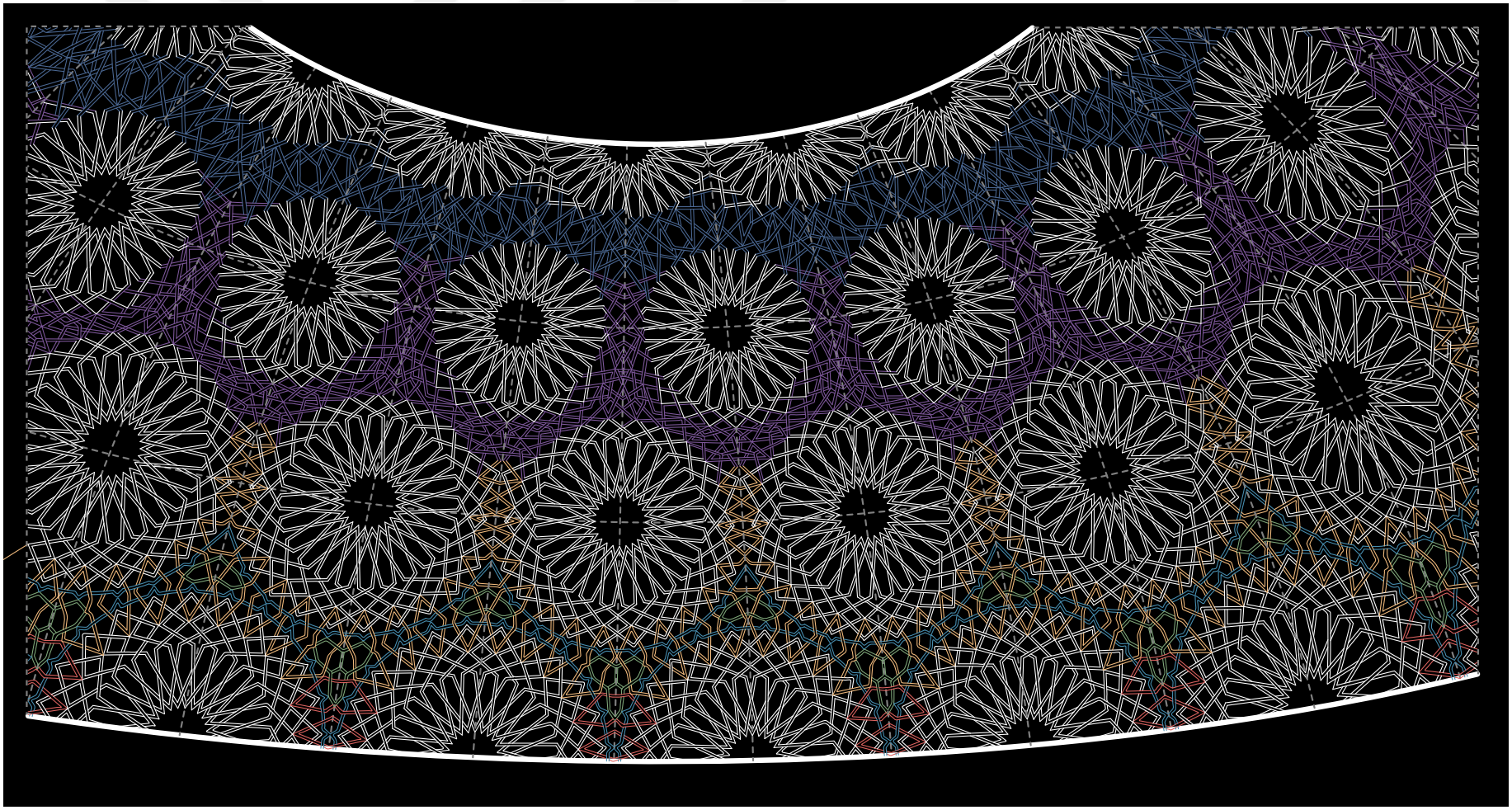


Figure C.6 : Close up of the Karatay dome pattern.

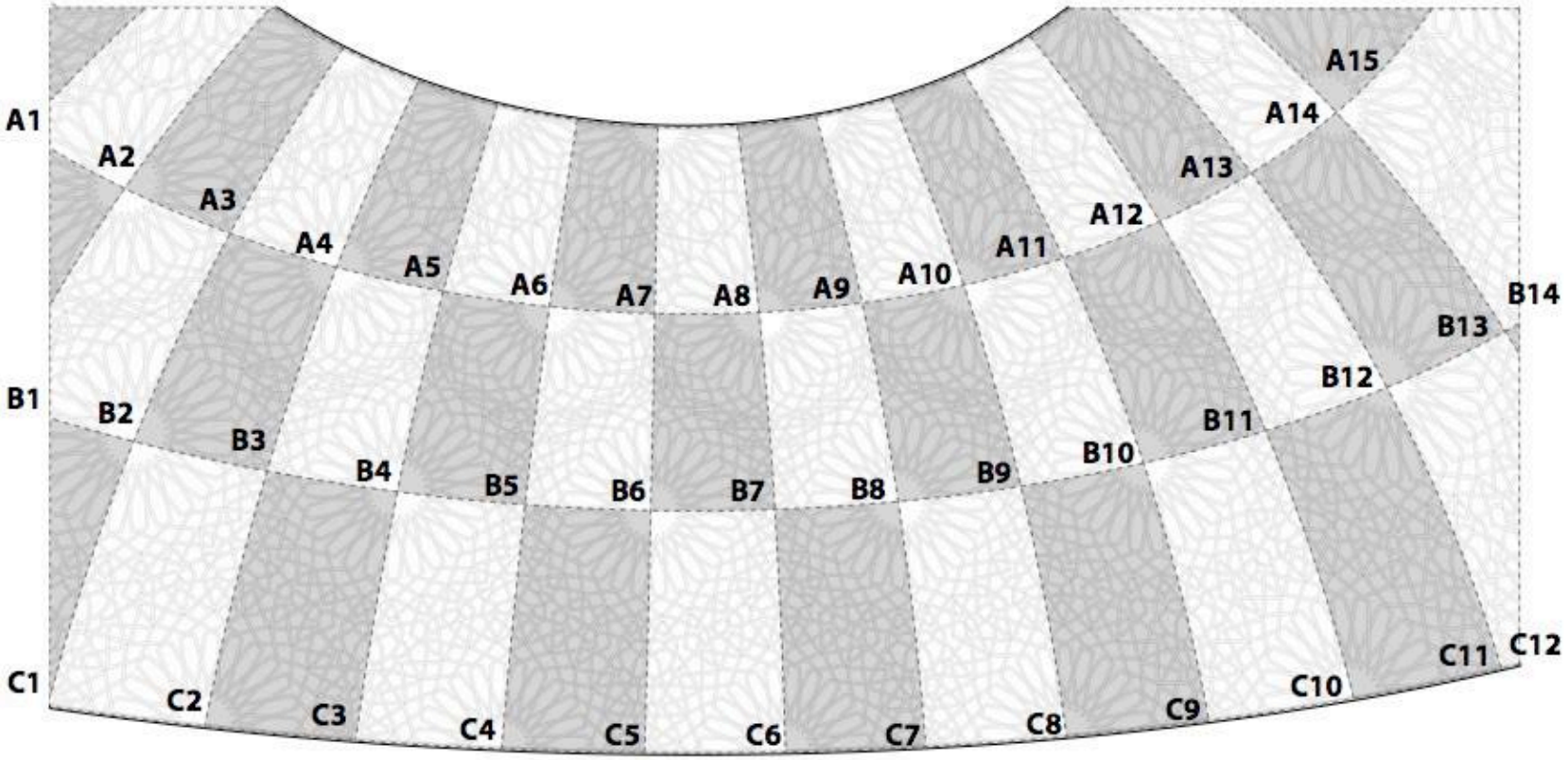


Figure C.7 : Close-up of the division diagram from Figure 5.89 and Figure 5.92.

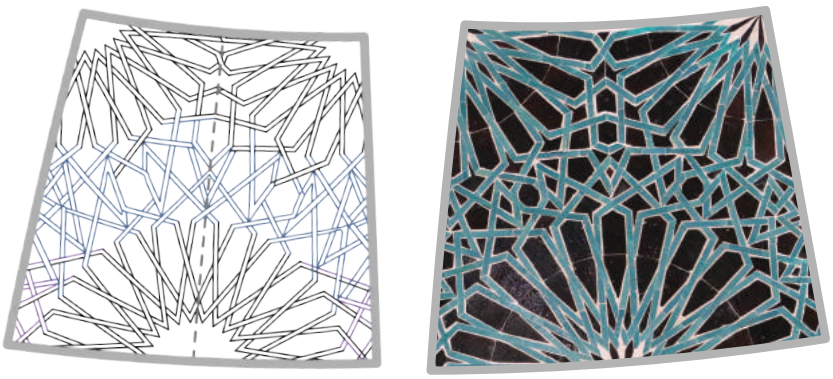


Figure C.8 : Star connections between sections A6 and A7 (Based on Figure C.7).

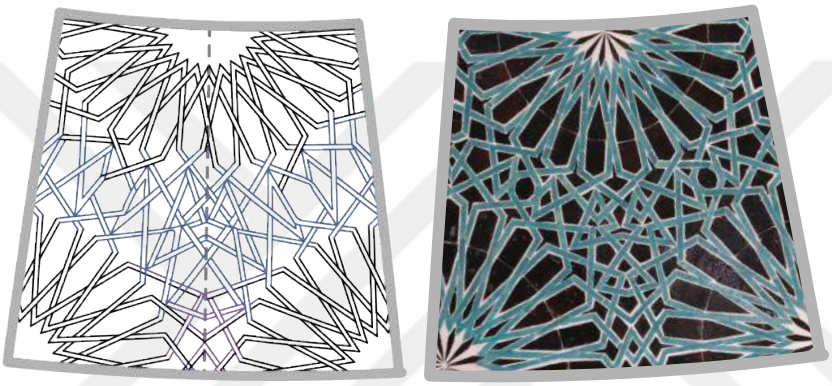


Figure C.9 : Star connections between sections A7 and A8 (Based on Figure C.7).

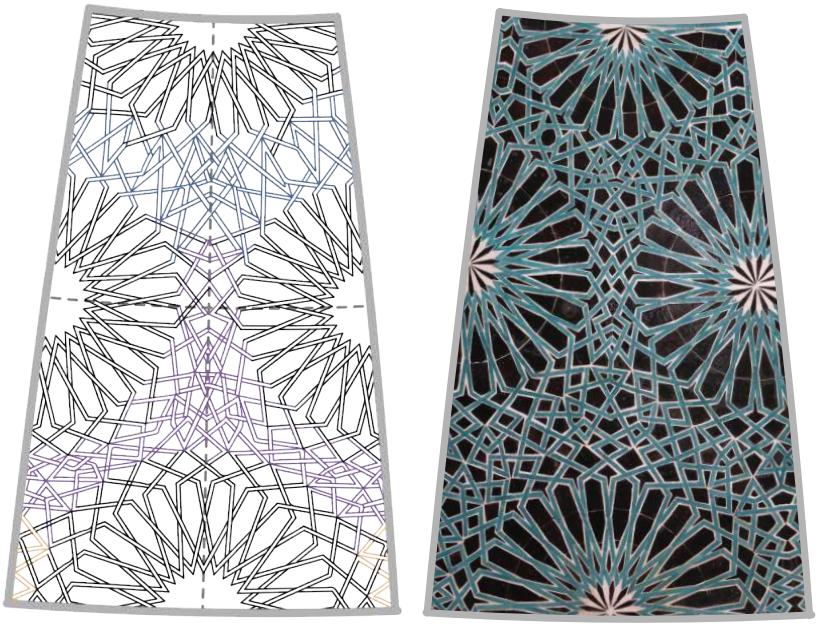


Figure C.10 : Star connections in Row 2 (A7-A8 plus B6-B7, based on Figure C.7).

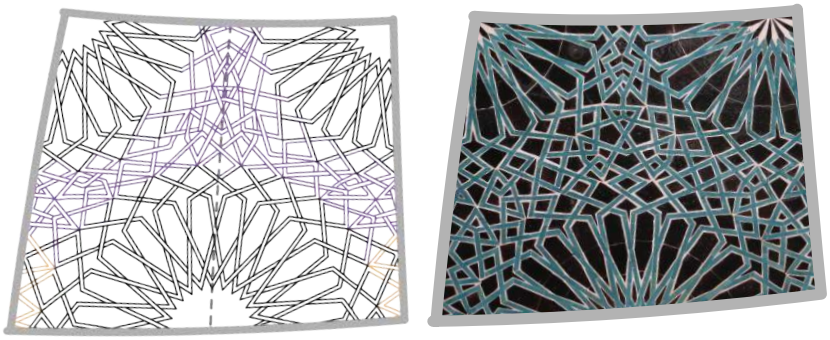


Figure C.11 : Star connections between sections B4 and B5 (Based on Figure C.7).

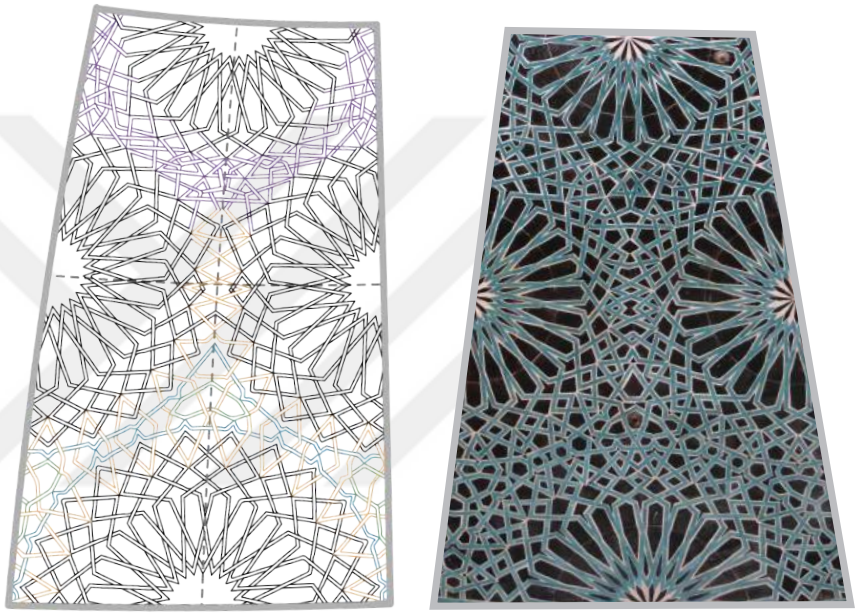


Figure C.12 : Star connections in Row 3 (B5-B6 plus C4-C5, based on Figure C.7).

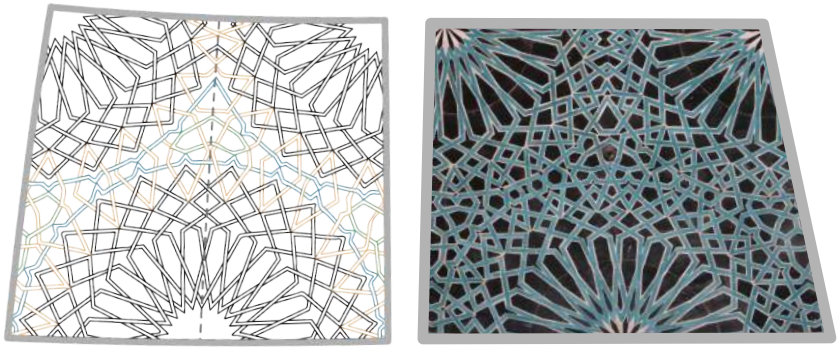


Figure C.13 : Star connections between sections C4 and C5 (Based on Figure C.7).

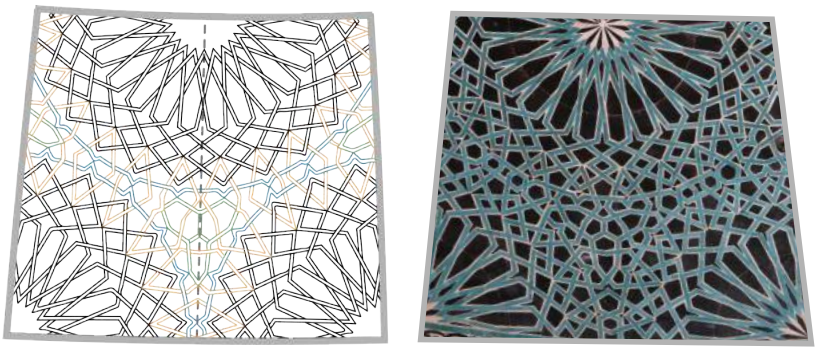
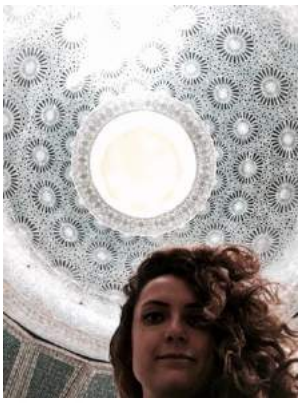


Figure C.14 : Star connections between sections C5 and C6 (Based on Figure C.7).

CURRICULUM VITAE



Name Surname : Sibel Yasemin ÖZGAN

Place and Date of Birth : 03.11.1985, Konya

E-Mail : sozgan@gmail.com

EDUCATION :

- **B.Sc.** : 2008, Istanbul Technical University, Faculty of Architecture, Department of Interior Architecture
- **M.Sc.** : 2012, Istanbul Technical University, Graduate School of Science Engineering and Technology, Department of Informatics, Architectural Design Computing Program

PROFESSIONAL EXPERIENCE AND REWARDS:

- (2012-2018) Research and teaching assistant, Istanbul Technical University Faculty of Architecture, Department of Interior Architecture.
- 2014 Young CAADRIA (Computer-Aided Architectural Design Research in Asia) Award.
- 2012-2018, National Scholarship Program for PhD Students, TUBITAK (The Scientic and Technological Research Council of Turkey).
- International Research Fellowship (Conducted for one year in Berlin, University of Fine Arts), TUBITAK (The Scientic and Technological Research Council of Turkey).
- Graduation award from the Rectorship Faculty and Department, Istanbul Technical University (Ranked First in her Graduating Class in Department of Interior Architecture).

PUBLICATIONS, PRESENTATIONS AND PATENTS ON THE THESIS:

- **Özgan, S.Y.** 2017: How Material and Making Alters Visual Computation: The Traditional Art of Tile Mosaic. International workshop (under EUSIPCO conference) - Creative Design and Advanced Manufacturing: An Emerging application area for Signals and Systems, September 2, 2017 Kos Island, Greece.

- **Özgan, S.Y.,** Özkar M. 2017. A Thirteenth-Century Dodecahedron in Central Anatolia: Geometric Patterns and Polyhedral Geometry, *Nexus Network Journal*, 19(2), 455-471. DOI 10.1007/s00004-017-0341-0.
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- **Özgan, S.Y.,** 2015: Geometric Patterns and Distorted Dodecahedrons, International Workshop - Computational Problems in Creative Design with Shapes, June 12-13, 2015 Massachusetts Institute of Technology, Cambridge, Massachusetts, USA.

OTHER PUBLICATIONS, PRESENTATIONS AND PATENTS:

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- **Özgan, S.Y.,** Demir Y. 2013: Use Of Augmented Reality Technologies In Cultural Heritage Sites; Virtu(Re)Al Yenikapi, National Conference - VII. Computational Design in Architecture Conference: Computational Design, Entropy, Creativity (7. Sayısal Tasarım Konferansı: Sayısal Tasarım, Entropi Yaratıcılık), June 27-28, 2013 Istanbul Technical University, Istanbul, Turkey.