

PARALLEL DECODABLE CHANNEL CODING IMPLEMENTED ON
A MIMO TESTBED

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ABSTRACT

PARALLEL DECODABLE CHANNEL CODING IMPLEMENTED ON A MIMO TESTBED

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This thesis considers the real-time implementation phases of a multiple-input multiple-output (MIMO) wireless communication system. The parts which are related to the implementation detail the blocks realized on a field programmable gate array (FPGA) board and define the connections between these blocks and typical radio frequency front-end modules assisting the wireless communication. Two sides of the implemented communication testbed are discussed separately as the transmitter and the receiver parts. In addition to usual building blocks of the transmitter and the receiver blocks, a special type of iterative parallelized decoding architecture has also been implemented on the testbed to demonstrate its potential in low-latency communication systems. In addition to practical aspects, this thesis also presents theoretical findings for an improved version of the built system using analytical tools and simulation results for possible extensions to orthogonal frequency division multiplexing (OFDM).

Keywords: Wireless Communication, MIMO, OFDM, Iterative Decoding, FPGA

ÖZ

BİR MIMO HABERLEŞME TEST DÜZENEGİ ÜZERİNE KURULMUŞ PARALEL ÇÖZÜLEBİLİR KANAL KODLAMA

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Bu çalışmada gerçek zamanlı bir çok-girdili çok-çıkıtlı (MIMO) telsiz haberleşme sisteminin hayata geçirilme evreleri ele alınmıştır. Uygulamaya yönelik kısımlarda, yerinde programlanabilir geçit dizisi (FPGA) kartı üzerinde gerçekleştirilen bloklar ve blokların alışıldık radyo frekansı ön bölüm birimleri ile bağlantısı ayrıntılarıyla anlatılmıştır. Kurulan test düzeneğinin alıcı ve verici parçaları ayrı ayrı tartışılmıştır. Alıcı ve vericideki alışlagelmiş yapı blokları dışında, düşük gecikmeli haberleşme sistemlerindeki kullanım olanaklarını gösterebilmek için özel bir yapıdaki paralelleştirilmiş döngülü kod çözme mimarisi test düzeneği üzerinde gerçekleştirilmiştir. Uygulamaya yönelik kısımlarla birlikte, kurulan sistemin dikgen sıklık bölümlemeli çoklama (OFDM) yöntemine uyarlanarak geliştirilmesi için çözümsel ve benzetimsel sonuçları kullanan teorik bulgular da bu tezde sunulmuştur.

Anahtar sözcükler: Telsiz Haberleşme, MIMO, OFDM, Döngülü Kod Çözme, FPGA

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CHAPTER 1

INTRODUCTION AND MOTIVATION

Although receive diversity enabled by multiple antennas at the receiver side (multi-output) has been common for a very long time, utilization of multiple antennas at the transmitter side (multi-input) is quite new in communications. Multiple-input multiple-output (MIMO) wireless systems have been the center of attention in the last decade following the pioneering work in [1], where the information theoretic capacity improvements provided by the use of MIMO systems became apparent. In order to manage the increasing data rate demand of lately emerging real-time applications, spatial multiplexing has been one of the promising solutions. Moreover, spatial diversity created by multiple-antenna systems is also used for improved signal reliability and reduced bit error rates.

High data rate demand of recent applications presented another problem in wireless systems. Due to increased bandwidth usage many communication systems have become more prone to selective frequency response of the transmission medium. Consequently, multicarrier transmission methods have become more popular. Orthogonal frequency division multiplexing (OFDM) is currently one of the most utilized multicarrier transmission techniques and has been the underlying element in IEEE 802.11a/g based wireless LANs, DVB terrestrial digital TV systems, IEEE 802.16 based WiMAX, etc.

In many recent research and development projects in wireless communications, joining two strong techniques (MIMO and OFDM) high data rates have been achieved [32]. In these systems, multipath propagation and frequency selective channel effects are remedied by the use of OFDM. Furthermore,

multiple usage of the same frequency bands through spatial multiplexing supply extra data rate increase with respect to single-antenna counterparts. The focus of this thesis is mainly on explaining the implementation steps of a wireless communication testbed which has been initialized to experiment newly developed communication techniques as well as the techniques currently known. Main goals include building the single-input single-output (SISO) system with all the digital signal processing modules realized on a flexible hardware, expanding the receiver side to utilize multiple antennas for improved signal quality, supporting the transmitter side with extra antennas for creating transmitter diversity, improving the MIMO system so that it can support transmission of data substreams using OFDM.

Throughout this thesis, there are three main subjects that are discussed. The initial emphasis is given to development stages of a SISO system on a field programmable gate array (FPGA) and afterwards the improvement obtained by addition of another receive antenna to the system is discussed. Subsequently, in the second part, a parallelized decoder structure for reducing the latency introduced by iterative decoding is studied and implementation results are given. Finally, in the third part, the optimal power adaptation method in OFDM and MIMO systems is inspected with regard to the peak-to-average power ratio (PAPR) problem. In addition, a suboptimal power adaptation scheme is developed and analyzed.

Signal processing required for detecting the transmitted data that travels in a noisy communication medium is realized on an FPGA. Two main building blocks of the wireless testbed, transmitter and receiver modules, are developed on two separate FPGA boards which are connected to high speed digital-to-analog and analog-to-digital converters, respectively. Over-the-counter radio frequency transmission/reception modules serve as the means for accessing the wireless medium. The transmitter structure is composed of three blocks: pseudo-random data generator, pulse shaping filter, and the in-phase/quadrature (I/Q) modulator. The receiver structure, on the other hand, is more sophisticated and includes various building blocks. The

received and sampled signals are I/Q demodulated and lowpass filtered initially. After a down-sampling operation, the signals on I and Q branches are matched filtered. Moreover, packet, bit, and frequency synchronization modules directly affect these blocks and the signal on which bit detection is carried out.

In 1993, it was shown that bit error rates can be improved by concatenating simpler codes in parallel and decoding the received signal iteratively [17]. In [14, 13], it was observed that the serial concatenation of codes may yield even better bit error rate performance under specific conditions. However, both of the serial and the parallel concatenated code decoders suffer from increased decoding latency due to many iterations. Therefore, we utilize parallelized encoder/decoder structures for simultaneously decoding substreams of data. We concentrate on serial concatenation of two codes as described in [10]. The data throughput for this implementation is shown to support over 4 Mbps data rate on a SISO system.

In addition to practical aspects of the testbed development, we questioned the enhancement in the PAPR of MIMO and OFDM systems under optimal power allocation techniques. The suggested equal power allocation technique is shown to provide comparable channel capacity to the optimal method when the PAPR performance of two techniques are also taken into consideration.

CHAPTER 2

IMPLEMENTATION OF THE WIRELESS TESTBED

This thesis work was predominantly routed by a research project¹ which has been funded by TUBİTAK. The aim of the project is to implement a broadband wireless communication system exploiting well established ideas such as multicarrier modulation and antenna diversity/multiplexing and to further procure new theoretical findings in accordance with observations on the implemented system. We successfully built up the first version of the operational testbed and will summarize the implementation under four main subtitles. Initially, we will elaborate on the hardware we made use of. Following the elaboration on hardware, the software programs used in developing the codes and simulating them will be named. Then, the transmitter and the receiver parts will be functionally characterized. Finalizing the operation essentials of the conventional single antenna setup, we will present the multiple-antenna system and its performance measures.

2.1 Testbed Specifications

Prior to the details of the testbed implementation, some figures for the overall specifications of it will be given. The RF front-end modules are designed for baseband video signal transmission by using FM modulation. Accordingly, the transmitted signal bandwidth is allowed to be at most 5 MHz.

¹TUBİTAK Kariyer (104E027) project entitled “Yüksek Başarımli Gezin Haberleşme: Çarpım Kodları Kullanarak Ortak Kanal Kestirimi ve Kodlama” and supervised by Ali Özgür Yılmaz

The carrier frequency of FM modulator is 2.4 GHz. In our system, the available 5 MHz bandwidth is not used, but rather 1.35 MHz bandwidth around an intermediate frequency of 3 MHz is utilized. In the transmitter side, before the RF transmitter 12 bit digital words in the FPGA are converted to analog. Digital-to-analog converter can conduct conversion of 2 different digital words to analog simultaneously. Digital-to-analog conversion can be performed with a maximum rate of 160 Msps. In particular, the sampling rate used is 24 Msps. Similarly, baseband analog signal received from the RF receiver is converted to 12 bit digital words with a rate of 24 Msps. It should be noted that analog-to-digital converter can conduct conversion of analog signals coming from 4 different channels into 12 bit digital words. In addition, it can support conversion rates upto 65 Msps. The 12 bit digital words are transmitted to the FPGA via synchronous serial transmission. The serial port interface that is used for debugging the digital filters on FPGA is capable of transmitting 115.2 Kbps to the computer.

Currently, the implementation supports wireless communication using QPSK modulation at a rate of 2 Mbps. Our final goal at the end of the TUBİTAK funded project is to support data rates upto at least 10 Mbps with MIMO-OFDM system and adaptive modulation methods.

2.2 Hardware Specifications

We carried out the experiments on our wireless testbed in the Telecommunications Laboratory of the Electrical and Electronics Engineering Department. In addition to the commonplace test and measurement equipment like the function generators, oscilloscopes, spectrum analyzers, etc., we utilized some specialized evaluation boards crafted for high-speed telecommunication applications. This section explains the physical building blocks of our testbed.

2.2.1 ML-310 FPGA Board

Field Programmable Gate Arrays (FPGAs) are programmable logic devices². They are composed of *configurable logic blocks* (CLBs) which are connected via modifiable switches. Most of the FPGAs may be reprogrammed many times due to their static random access memory (SRAM) based structures. Moreover, they gained an increasing interest in the last two decades due to their potential in executing parallel processes simultaneously. CLBs (denominated as slices by Xilinx³ as well) include the most elementary structures like flip-flops, multiplexers, lookup tables (LUT), etc. A slice is composed of two four-input LUTs, six various size multiplexers, and two flip-flops as shown in Figure 2.1.

The LUTs are capable of realizing all possible functions of at most 9 binary variables (inputs) when used together with a second stage 3-input LUT. The results of the LUTs may be multiplexed to the flip-flop(s) in case of a need for a register for storing the result of the function. Other than CLBs, some secondary structures may exist on FPGAs for improving the performance. As an example, the hardware designer may require large arrays of registers for storing data or implementing the taps of a digital filter, which makes existence of some Random Access Memory (RAM) modules compulsory together with CLBs. Moreover, design of relatively large multiplexers using simply CLBs will slow down the operation of them unavoidably, due to large routing delays between many CLBs especially for massive implementations. Therefore, large dedicated multiplexers serve as a means of improving hardware performance. In addition to these, many recent FPGA chips include dedicated binary multipliers for fast multiplication, clock management modules for diminishing skew effects on the clock signal(s) and synthesizing various

²Unlike microprocessors, FPGAs can be reconfigured repetitively many times and for many different functionalities.

³For more information on this well-known FPGA manufacturer see www.xilinx.com/.

frequencies, and even previously programmed microprocessors for embedded designs.

Our evaluation board was Xilinx ML310 Embedded Development Platform. This FPGA board is equipped with numerous assets to capture all requirements of embedded design process. Primarily, it features a XC2VP30-FF896 Xilinx FPGA chip, whose basic specifications are given in Table 2.1. The PowerPC microprocessors are based on Harward Architecture and developed by IBM. Two instances of PowerPC are programmed onto the FPGA chip in read-only mode during production. In our project, they are expected to be responsible for execution of sequential-type jobs and calculations involving floating points or functions that are hard to implement on an FPGA (like precise calculation of logarithmic functions) in the following development phases. They are capable of operating at 300 MHz clock frequency and connected to the programmable part of the FPGA directly. The ap-

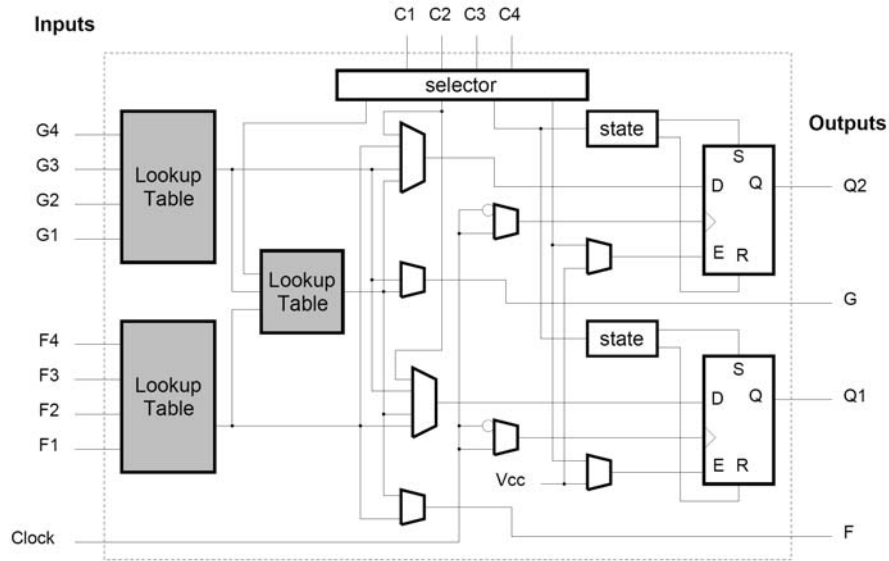


Figure 2.1: Slice structure for Xilinx FPGAs.

Table 2.1: Specifications of XC2VP30-FF896 FPGA chip

Structure	Count	Explanations
Logic Cells	30,816	Lookup tables and flip-flops for implementing logic
PPC405	2	IBM PowerPC microprocessors for sequential code execution
MGTs	8	Very high speed serial input/output interfaces
BRAM(kb)	2,448	Variable size RAM blocks for medium size storage
Xtreme Multipliers	136	18-bit by 18-bit fast multiplier blocks

plication and the network layer protocols are the essential usage areas for these processors. The BRAM (Block RAM) modules are true dual-port⁴ RAMs, which can be concatenated vertically and/or horizontally in order to obtain RAMs larger than the default size of “1024 by 18-bit”. In total, 136 Block RAM modules take place in the chip. The MGTs in Table 2.1 stand for Multi-Gigabit Transceivers and connect the chip to the outside world through 3.125 Gbps serial data lines. However, we made use of another type of serial interface to provide the communication between the analog-to-digital converter and the FPGA. This connection type is Low Voltage Differential Signaling (LVDS) and widely used for serial communication applications with high data rate (around 1 Gbps). Details of operation for LVDS connection used in our setup are given in Sections 2.2.3 and 2.5.1.

Besides the FPGA chip, ML310 comes with several other interfaces and properties. A concise list of them is given below.

- **DDR-RAM sockets filled with 256 MB DDR DIMM type RAM:** Applications that generate the data to be sent through our

⁴Dual-port RAMs can be written(read) to(from) two distinct addresses concurrently.

testbed may utilize this memory resource. In some specific cases programmable part of the FPGA may also use this memory. They are accessed slower than Block RAMs, hence usually not preferred unless the storage requirement is very high.

- **512 MB CompactFlash card:** It acts like a harddisk on which the PowerPC processors can write calculation results or data captured from the physical layer of the receiver. Moreover, there exist two pre-installed operating systems (*MonteVista Linux 3.1* and *VxWorks Tornado 2.2*), which provide many tools to be used over PowerPC processors.
- **High Speed Personality Module Connectors:** There are two Z-DOK connectors on ML310 that serve as the means of communication between other boards and ML310. Various voltage levels are supported over almost a hundred input/output (I/O) ports and the voltage levels are software configurable.
- **RS-232 Port with Direct FPGA connection:** In our testbed, this connection is helpful whenever the user wants to analyze the output of a module (like a filter) on FPGA. We managed to send data generated on FPGA to a computer running MATLAB in order to debug the various blocks operating on FPGA.
- **ALi South Bridge:** This is an I/O controller chip and arbitrates the I/O requests of peripheral units on the ML310 board for an organized communication with processors. These peripheral units include two IDE units (harddisks, CD/DVD-ROM drives), two USB units, audio codec chip, two RS-232 ports, and one parallel port. Together with two PCI connectors and an Ethernet controller, the data bus of ALi South Bridge is connected to Peripheral Control Interface (PCI) Bridge of the FPGA.

2.2.2 Digital-To-Analog Converter, AD9773

The transmitter part converts the digital data constructed on FPGA into analog form using a digital-to-analog converter (DAC) chip, AD9773⁵. This is a 12-bit resolution converter chip designed by Analog Devices. It is capable of converting 160 Msps (mega-samples per second) and has two 12-bit input ports. According to the desired operation, these two inputs may behave as the digital input for two completely different analog signals or as the in-phase and the quadrature-phase components of a complex signal for direct *intermediate frequency* (IF) transmission. Through a serial port interface, the unit can be programmed to select this and many more operation modes. It is possible to select the IF frequency as the fractions ($1/2$, $1/4$, or $1/8$) of the supplied oscillator input frequency, turn on interpolated data conversion, and specify the voltage (hence, signal power) level at the analog outputs precisely. The evaluation board of this chip is easily operated after making power connections, the signal output connections through SMA (subminiature versionA) connectors, and plugging/unplugging a few jumper connections on the board. The outputs of this DAC board are AC-coupled, hence do not pass DC-signals.

2.2.3 Analog-To-Digital Converter, AD9229

AD9229⁶ is another chip designed for conversion of analog signals to digital ones by Analog devices. This analog-to-digital converter (ADC) operates at sampling frequencies in the range from 10 Msps to 65 Msps and supply its digital output in 12-bit offset binary⁷ format. There are four distinct con-

⁵For detailed information, see <http://www.analog.com/en/prod/0,,AD9773,00.html>

⁶For more information, see <http://www.analog.com/en/prod/0,2877,AD9229,00.html>

⁷Offset binary represents the most negative number with all zeros, and the most positive number with all ones, hence conversion to 2's complement binary format is as simple as inverting the first bit in the offset binary representation

version units; therefore four distinct digital outputs on a single chip. The digital outputs of the ADC conform to the ANSI-644 LVDS standard. The differential output signals swing within a 375 mV peak-to-peak range and require 100Ω termination at the receiver side. Together with the LVDS data outputs, two LVDS clock signals are also supplied. The frame clock output designates the start of new 12-bit conversion result at each rising edge. It is the same as the sampling frequency. In comparison, the data clock is six times faster than the sampling clock, since at both the rising- and the falling-edges of this data clock one data bit is given as output. As a consequence of this serial output interface, the digital signal output frequency is twelve times faster than the sampling frequency. During the design phases of the interconnection board for interfacing ML310 board with ADC and DAC boards, the signal integrity of these LVDS lines posed a great problem for us due to three main reasons:

- The LVDS line pairs need to have 100Ω impedance for minimum power loss.
- All of the LVDS data line pairs and the differential clock pairs must be of very similar lengths for diminishing the delay differences imposed on different signals. To demonstrate with an example, at 65 MHz sampling rate, the delay difference between any two lines should be kept below 640 picoseconds.
- Different LVDS signal pairs should be kept at a relatively large distance and their paths should be as smooth as possible (avoiding sharp corners) to keep signal interference between the lines at a minimum.

To minimize crosstalk, signal integrity, and electromagnetic compatibility (EMC) problems, we made use of several simulation softwares like Advanced Design System (ADS) and Hyperlynx, which are also mentioned in Section 2.3.3. As a final point, the analog inputs of the ADC board are AC-coupled as in the case of analog outputs of the DAC board.

2.2.4 RF Transmitter and Receiver Modules

The transmitter and the receiver modules are responsible for up-conversion of the IF signal to radio frequency (RF) level and down-conversion of the RF signal to IF level respectively. They are manufactured by UDEA (an Ankara based communication technologies company) and designed to operate at one of four selectable frequency intervals between 2.4 GHz and 2.5 GHz. The transmitter module delivers 18 dBm (64 mW) power to the channel and is capable of transmitting signals in the (-5 MHz, 5 MHz) band. One of the main disadvantages we had in the first version of our testbed was that this transmitter and receiver pair used frequency modulation (FM) technique being designed to deliver analog video/audio data from a source to a TV unit. Consequently, we initiated the design of the second version of the testbed, which is expected to remedy this problem using amplitude modulation (AM) for transmission. More details on this second version can be found in Chapter 5. The receiver module has -85 dBm signal sensitivity and also gives an analog *received signal strength indicator* (RSSI) output that can be used to determine the instantaneous effect of the automatic gain control on the received signal. Both the receiver and the transmitter modules accept the 1 V_{p-p} signal level for their data input and output ports. This voltage level is in accordance with the default voltage levels of the ADC and the DAC boards as well.

2.3 Software Used for Implementation, Debugging and Simulation

We made use of several application software tools in order to design, simulate, synthesize, and program FPGA; develop and simulate algorithms to be used on our testbed; design and analyze printed circuit boards (PCB) for new extensions to our testbed hardware. This section summarizes the most frequently used ones.

2.3.1 MATLAB v7.0

MATLAB was used in various phases of the development of our testbed. To start with, it is utilized for implementing the filters and sine lookup table to be used in our design. We generated and operated all of the receiver and the transmitter blocks initially within MATLAB as stated in Sections 2.4 and 2.5. Secondly, MATLAB functioned as a debug environment for the codes programmed onto the programmable device (FPGA). We verified the results of each newly developed block through an interface which first writes the results of that block onto Block RAMs and then delivers these results using the serial port of the FPGA board. Another code, in MATLAB, was responsible for listening to the serial port of the computer, converting received data into real numbers, and plotting the desired curves for assuring us that the block on the programmable device operates in the same manner as the corresponding block in MATLAB. Also, as it is underlined in Section 3.6.1, the PDSCC (parallel decodable serially concatenated convolutional code) decoder was developed in MATLAB, while it was being coded for FPGA implementation. MATLAB environment presented a fast verification method for decoding calculations obtained from FPGA simulation and operation results. It reduced the time required for debug process considerably. In addition to all these, we tested our suggestions for suboptimal power allocation methods (see Chapter 4) via the simulation codes conducted under MATLAB.

2.3.2 Xilinx ISE Webpack Edition v9.1

Nearly all of the hardware implementations are coded using a hardware description language that is known as VHDL. Xilinx ISE was the development environment for these codes. The initial phase of development is usually named as the *synthesis* part. In the synthesis phase, Xilinx ISE first checks the syntax of the code, then generates a register level description of the de-

signed circuit, and produces a netlist⁸ of the design that will be taken as an input by the following phase (called *implementation*). Before the second phase of Xilinx ISE is invoked, the user may wish to declare some physical requirements that the final design will satisfy. Hence, it is possible to enter the timing constraints (like imposing the minimum operating clock frequency, or dictating all of the register outputs to be ready in less than a given amount of time before the rising-edge of the clock), the area constraints (as a good example, stating that the overall design should utilize less than a given percentage of the programmable device), and some more complicated constraints like the operation temperature of the hardware. In the implementation phase, the final physical structure is formed after placing and routing the building units (registers, multiplexers, multipliers, RAM blocks, ...) and connecting the input and the output pins of the outermost block in the design. The pin assignments for interaction with the outer world is also highly flexible. Furthermore, in some special cases, user may prefer placing⁹ a subset or all of the components in a design manually instead of an automated design procedure. Xilinx ISE allows this manual configuration through location constraints. The final phase realized by Xilinx ISE is the programming and verification of the programmable device through a special download cable.

In connection with the embedded design process (which includes operation of the PowerPC cores on the device as well), *Xilinx Platform Studio and the Embedded Development Kit* is preferred, which will not be detailed here.

⁸Netlists convey the connectivity information between the registers and other design units.

⁹An example for such a case and solution procedure is given in Section 2.6.2

2.3.3 Printed Circuit Board Design and Simulation Softwares

Basic resource for drawing an interface board between the FPGA board and the peripheral boards (DAC and ADC boards) was PCAD 2004. This software is such a sophisticated tool that many complex circuits with two or more layers can be conveniently drawn. It has a trial version that we made use of and it was fortunate for us that one of the students in Telecommunications Laboratory had learnt numerous shortcuts and tricks of this software during his summer practice. This software is also capable of routing the lines between units automatically, which helped us during the routing phase of lines that were to carry relatively slow frequency signals. For improved signal integrity on critical lines (like LVDS pairs described in Section 2.2.3), we initially defined the line and the ground plane characteristics for matching 100Ω impedance. For this purpose, Hyperlynx free trial software is utilized. This software calculated the proper distance between LVDS pairs, the required thickness of them, and the desired clearance between an LVDS line and the ground plane that surrounds it. All of these values are obtained according to the material types to be used for manufacturing the lines and the insulator layer between the top and the bottom layers of our two-layer board design. At this point, invaluable support and guidance of friends from Electromagnetic Waves and Microwave Techniques Group directed us to simulate the potentially problematic parts in the Advanced Design Systems environment. After some final arrangements, we had our PCB prepared in Delron¹⁰. The results were satisfactory in terms of signal integrity on the interface board and we made use of this board until the preparation of this thesis report.

¹⁰Delron is a well-known PCB printing company located in Manisa, Turkey.

2.3.4 Labview Interface for Programming AD9773 Board

This interface is simply designed for accessing programmable properties of our DAC board and distributed freely by Analog Devices. Other than setting IF signal's center frequency and interpolation features, we simulated various signal-to-noise (SNR) ratio scenarios using the coarse and the fine gain adjustment registers for the output signal level.

2.4 Transmitter Structure

Since we have given all the basic information about the hardware and the software that constitute the underlying structure for our testbed implementation, we can continue with the implementation phases. Initial attention will be given to the transmitter hardware we built up on the ML310 board. Basic building blocks of the transmitter is going to be followed by the receiver architecture. Figure 2.2 highlights the transmitter structure via a block diagram.

2.4.1 Pseudo-random Data Generation

In many applications random data generation is indispensable. Cryptographic works, statistical experiments, telecommunications applications (as an example, Gold codes in CDMA system), and simulation softwares are some areas of interest for people in search of good random-like sequences. For most of the cases it is impossible to create a true random number generator on a computer, since any outside factor that has an effect on the output of generator will make the generator non-random. Although some methods to generate true random numbers and test their randomness on specific hardwares (like FPGAs) have been proposed [2], most of the currently utilized random number generation methods depend on generating numbers accord-

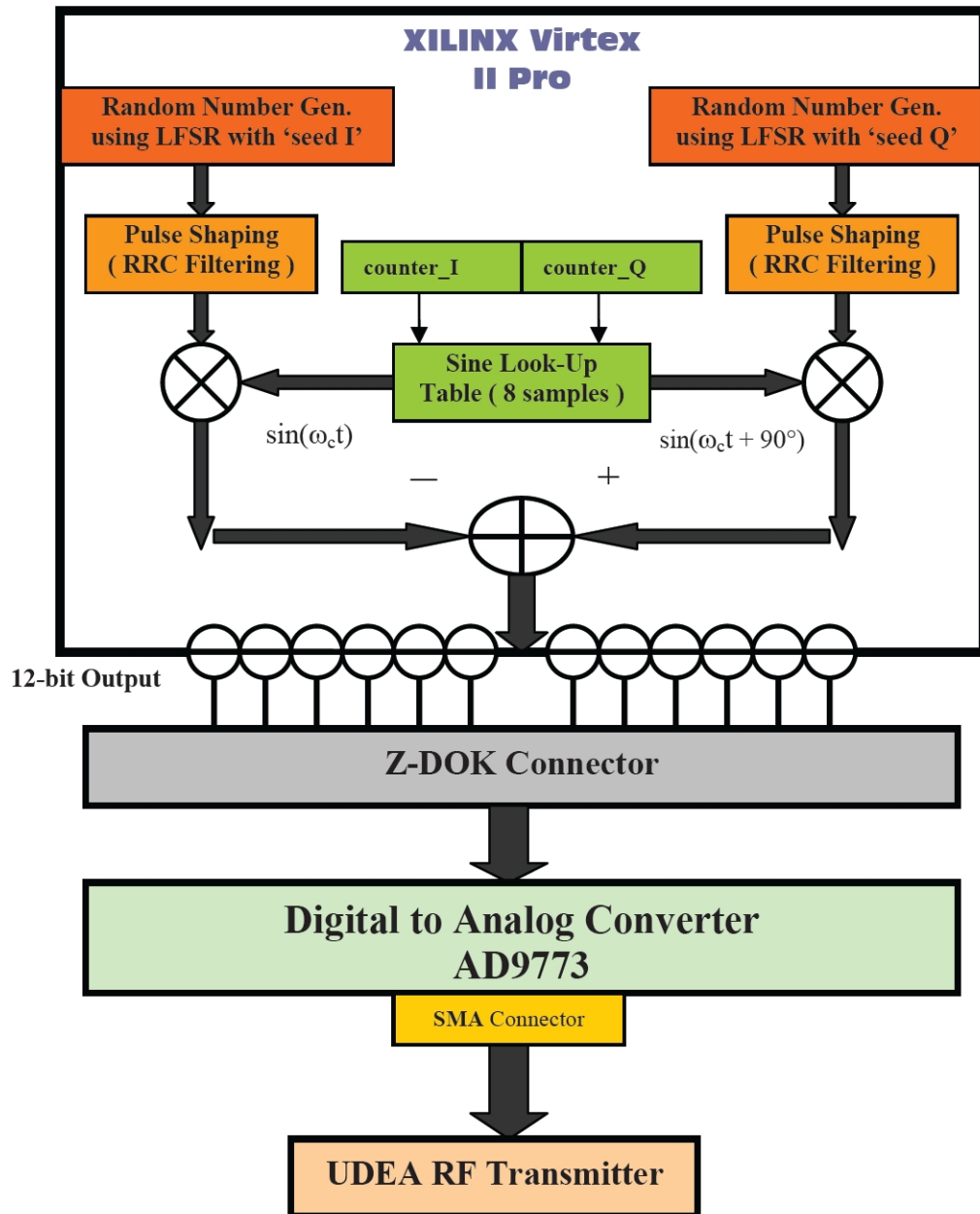


Figure 2.2: Transmission of QPSK modulated data is shown.

ing to a deterministic procedure. For this deterministic procedure, once the initial state (or also known as *seed*) of the generator and the procedure are given all the data to be created in next steps is also known. However, for such pseudo-random number generators, it usually takes a very long time to repeat the generated data that in any small interval of observation, the generated numbers may resemble a truly random sequences. In our case pseudo-random data generation was important for two reasons:

- If we use a series of bits repeating themselves in short intervals, the power spectrum of the output of the transmitter will possess an impulsive character, hence the frequency selectivity of the transmission medium (if any) will not be clearly observed at the receiver side. On the other hand the power spectrum density (PSD) for a pseudo-random sequence will resemble the one of a truly random sequence and will not be impulsive as given in Section 2.4.2.
- It is almost always true that a code working for a set of possible inputs may fail in others. Therefore, it is wiser to test any code with as many different input combinations as possible. Determination of fault-free operation for our transmitter block will be much reliable when a pseudo-random bit sequence is utilized as well.

There are various methods for pseudo-random bit generation. One of the best known techniques is using *linear feedback shift register* (LFSR). An LFSR is called linear because it consists of only xor (exclusive-or) operators, which are linear operators for binary variables. Moreover, it also has a feedback structure that shifts a function of a set of its state bits (registers) back to its input as it is given in Figure 2.3 for an example LFSR. The initial state of the LFSR is called the *seed* and it deterministically gives whole output bit sequence. In Figures 2.3, 2.4, 2.5, and 2.6, an example operation cycle is given for a 16-bit register LFSR. The bits shown shaded are called the taps of the LFSR and they are the bits summed (in binary sense) to obtain the next input. Once the next input is obtained, we can shift the register to the

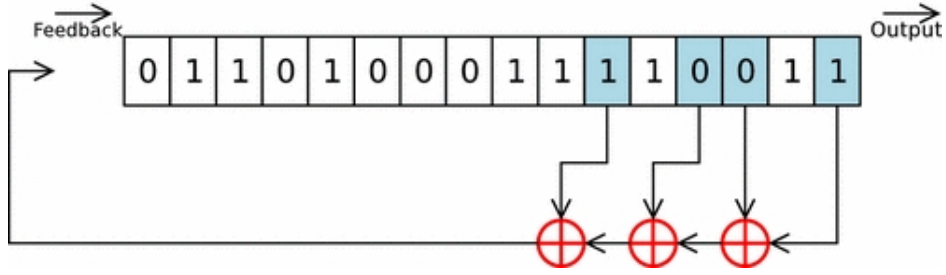


Figure 2.3: An LFSR with seed: 0110100011110011.

right and the rightmost bit yields the output whereas the input enters to the LFSR from the left as in Figure 2.6.

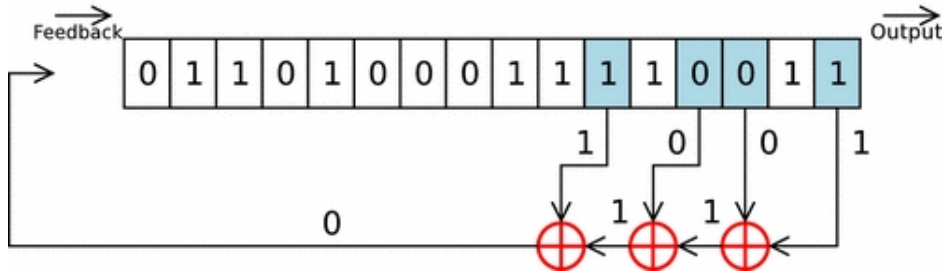


Figure 2.4: The 16th, 14th, 13th, and the 11th bits (taps) are summed.

Since the possible number of states of an LFSR is limited to 2^n (n is the number of registers in the LFSR) at most, every LFSR repeats itself within a fixed interval. However, when this interval is maximum, i.e. $2^n - 1$, where the all zero state is substituted, such an LFSR is called as *maximal*. For example Figure 2.3 demonstrates a maximal LFSR¹¹. It is fairly simple to

¹¹Proving that a given LFSR is maximal requires showing the polynomial corresponding to its taps (the bits which affect the input) is primitive. For details, see the book “Shift Register Sequences” by Solomon Golomb.

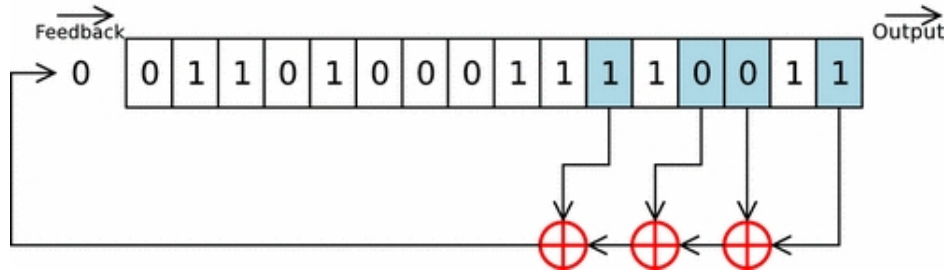


Figure 2.5: The input bit is found as 0.

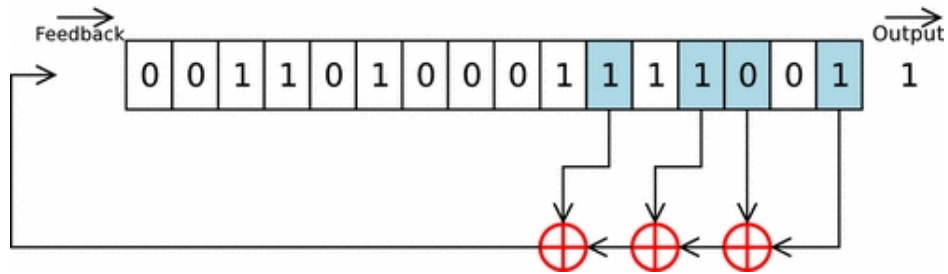


Figure 2.6: Right shift operation is carried out. Output bit is 1.

implement the LFSR structure on FPGA. The following code segment shows the implemented LFSR in VHDL:

```

... (Signal declarations follow:)
signal lfsr_input : std_logic;
signal generator_reg : std_logic_vector(31 downto 0);
... (LFSR input is updated using the desired taps:)
lfsr_input <= generator_reg(0) xor generator_reg(10) xor
               generator_reg(30) xor generator_reg(31);
... (At the next rising-edge of the clock, LFSR is right-shifted:)
elsif(rising_edge(clock)) then

```

```
generator_reg <= lfsr_input & generator_reg(0 to 30);
... (Remaining part of the code follows)
```

As seen in the example code segment, we used a 32-bit LFSR, which is maximal. Hence, we generated a pseudonoise (PN) sequence that repeats itself after $2^{32} - 1$ cycles. Our data rate was 1 Mbps, which corresponds to a repetition interval of approximately 4000 seconds. This figure was enough to observe a PSD that is quite similar to the PSD of rectangular pulse shaped BPSK.

2.4.2 Pulse Shaping

During the implementation of the pulse shaping filter for the transmitter, we considered two main points:

- **No intersymbol interference (ISI) at the sampling instants:**

For a pulse shape $x(t)$, k being any sampling instance, the no ISI condition can be defined as

$$x(t = kT) = \begin{cases} 1 & \text{if } k = 0 \\ 0 & \text{if } k \neq 0 \end{cases} . \quad (2.1)$$

According to the *Nyquist pulse-shaping criterion* [3], the necessary and sufficient condition for $X(f)$ is given as

$$\sum_{m=-\infty}^{\infty} X(f + m/T) = T \quad (2.2)$$

- **Bandlimited frequency response:** In wireless communications, limiting the bandwidth of a baseband signal is crucial to avoid aliasing during the IF up-conversion and to suppress out-of-band radiation. However, a signal limited in frequency domain is unlimited in time domain. For that reason, the signal to be implemented should be *nearly limited* in frequency for realization of a *nearly unlimited* time domain representation.

There are many pulse shapes satisfying the first condition noted above. These pulse shapes constitute a class called Nyquist-I pulse. The smallest bandwidth for these pulses is satisfied by the *sinc* function:

$$x(t) = \frac{\sin(\pi t/T)}{\pi t/T}, \quad (2.3)$$

where T is the symbol duration. In contrast to its bandwidth advantage, sinc function has a slow decay rate and sensitive to *timing phase errors*. As a result, it is usually very hard to approximate the sinc function in time domain and in case of timing errors, the ISI term in the received signal may increase indefinitely. For these two reasons, we decided to use another well-known signal for pulse shaping the symbols to be sent. This pulse shape is known as *raised cosine* (RC) and it is usually defined using a parameter named as the roll-off factor, β . The normalized frequency domain representation for RC is given by

$$X(f) = \begin{cases} 1 & |f| \leq \frac{1-\beta}{2T} \\ \frac{1}{2} \left[1 + \cos \left(\frac{\pi T}{\beta} \left[|f| - \frac{1-\beta}{2T} \right] \right) \right] & \frac{1-\beta}{2T} < |f| \leq \frac{1+\beta}{2T} \\ 0 & |f| > \frac{1+\beta}{2T} \end{cases} \quad (2.4)$$

Since the representation in Eqn. 2.4 is limited in frequency domain, the corresponding time domain pulse is also unlimited just as in the case of sinc pulse. Nonetheless, RC pulse shape decay rate is proportional to $(1/t^3)$ for $\beta > 0$ case¹², whereas sinc function only decays with a $(1/t)$ rate. In this sense RC pulse shape can be approximated more easily and it has higher immunity to timing phase errors. The bandwidth occupied by RC filter is

$$BW_{RRC} = \frac{1}{2T}(1 + \beta). \quad (2.5)$$

In our application, since the receiver had to match the transmitter pulse shaping filter and the overall response had to represent an RC filter, we used the *root raised cosine* (RRC) filtering at both ends. The frequency response

¹²For $\beta = 0$ RC pulse shape becomes nothing but the sinc pulse shape.

of the RRC filter is just the square-root of the RC response given in Eqn. 2.4. Our design choice for the roll-off factor was **0.35** due to the effective sampling frequency at the receiver side (see Section 2.5.3).

During the implementation of pulse shaping (which applied on the pseudo-random output of the LFSR in Section 2.4.1), we initially generated the desired impulse response of the RRC filter in MATLAB. Then, the filter taps are normalized and quantized in 6-bit 2's complement signed format. The exact RRC pulse shape together with the digital approximate (that is stored in read-only memory modules of FPGA) is shown in Figure 2.7. The exact pulse is scaled such that the peak value in the middle is equal to the largest positive integer (in our case, for 6-bit signed representation, it is 31) for better visualization of the approximation errors. Moreover, due to 24 MHz sampling rate of digital-to-analog converter (DAC), and selected 1 Mbps data rate, the oversampling rate for the RRC filter would be an integer between 1 and 24. Figure 2.7 is drawn with the assumption that approximate filter's oversampling rate is 6 and a given sample is constant for 4 time steps in 24 MHz sampled domain. Having a higher oversampling rate means storing more intermediate samples from exact waveform and better approximation of RRC pulse. Finally, we divided a symbol time into six intervals, which corresponds to oversampling rate of 6. In each interval a distinct value sampled from the quantized exact waveform is used for realizing the convolution operation with the RRC pulse shape. With 6-bit representation the quantized values for RRC become zero outside the ± 3 symbol interval and 19 symbols (6 for each 3 symbol intervals and a single midpoint) are stored making use of the symmetry of the RRC pulse shape around its peak (midpoint) value. The convolution operation is, then, summation of 7 of these samples or their negated versions according to the values of the symbols to be sent, when oversampling is taken into account. In our case, for QPSK modulated symbols, we directly add the sample for a 1 to be sent and add the negated sample for a 0 to be sent in both in-phase and quadrature-phase branches.

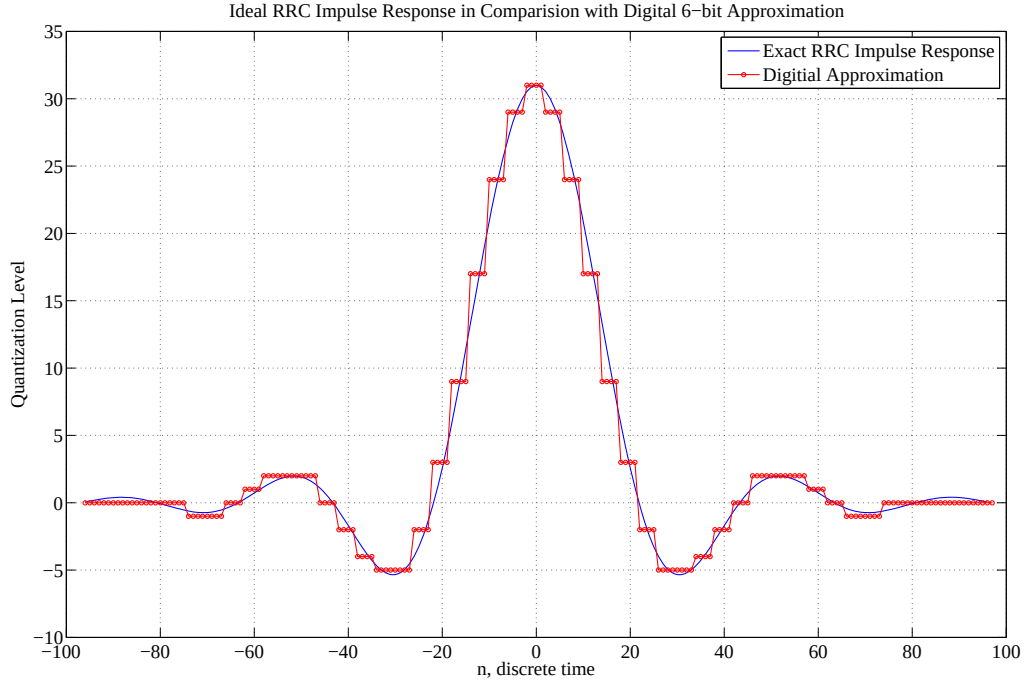


Figure 2.7: Digital RRC filter closely approximates the exact waveform within ± 3 symbol interval.

The RRC pulse shape (and its approximation) is not causal as seen in Figure 2.7. This results in the requirement that at least 3 past symbols sent should also be kept for adding their pulses' tails to the current symbols pulse shape. We solved this issue simply by taking the 29^{th} bit of the LFSR as the current bit to be transmitted and taking the 30^{th} , 31^{st} , and 32^{nd} bits as the past bits. In accordance with that idea, the 28^{th} , 27^{th} , and 26^{th} ones are the next bits effecting the current bit. Applying this filter on the random sequence of BPSK symbols, we obtained the time and frequency domain signals as in Figures 2.8 and 2.9. The quantization levels (steps) for the time domain representation are obvious. If we take the PSD of the filter output into consideration, the first thing to note is that occupied bandwidth is very close to 0.675 MHz, which is also the expected bandwidth found from

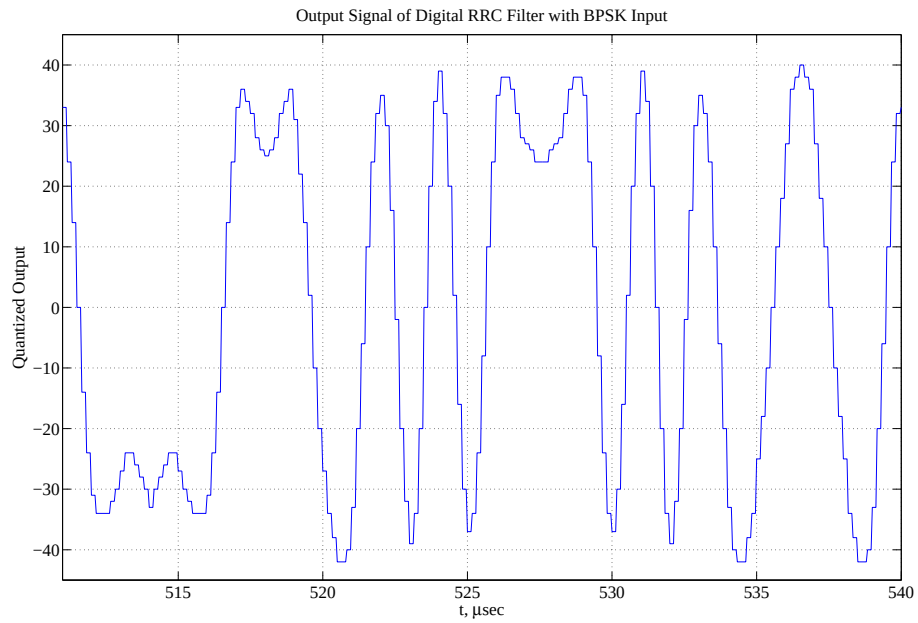


Figure 2.8: Digital RRC Filter Output

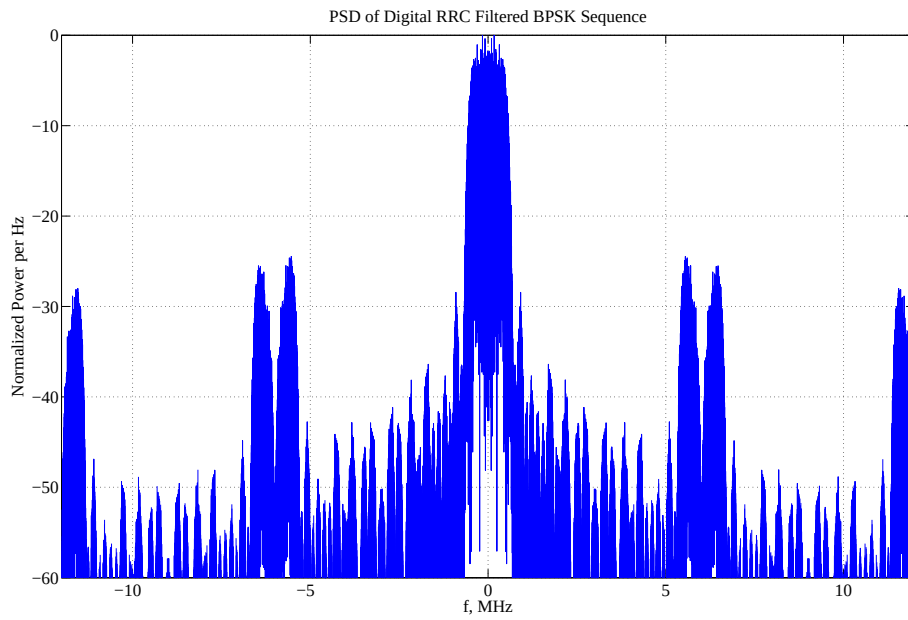


Figure 2.9: The PSD of the digital RRC filter output.

Eqn. 2.5 with $\beta = 0.35$. Secondly, there are *harmonic signals* at $f=6$ MHz and $f=12$ MHz distorted by a sinc type multiplicative signal. The signal around $f=6$ MHz is approximately 25 dB weaker than the baseband signal; therefore was observed to have limited effect during the IF conversion described in Section 2.4.3. A lowpass filter placed before the I/Q modulation stage may improve the performance further.

2.4.3 I/Q Modulation

QPSK modulation is performed to transmit data after RRC pulse shaping for BPSK modulated data. In order to generate two bit streams, we used two LFSRs with different seeds. We quantized single period of a 3 MHz sine signal in MATLAB and formed an 8-bit resolution sine lookup table (with 8 entries) from this quantized data in VHDL. We synthesized a free-running counter for accessing this table to form a digital sine signal and used quarter-length (90°) shifted version of that counter to form the cosine signal. After multiplying the in-phase and quadrature-phase RRC shaped signals with cosine and sine signals, the difference of the products were ready to be given to DAC board. Figure 2.10 demonstrates the PSD for the IF QPSK signal taken as input by the DAC. Obviously, the baseband RRC type frequency response is carried to the IF, which is 3 MHz in our implementation. The effect of the harmonic signal that was present before the conversion is negligible. The IF signal given to the DAC is converted into analog form and transmitted after an RF up-conversion.

2.5 Receiver Structure

The receiver side, in fact, carries out similar operations to the transmitter, however in the reverse order. Mainly, initial steps constitute RF to IF *down-conversion*, *sampling* of the IF signal, *I/Q demodulation*, and *low-pass filtering* to obtain the baseband equivalent signal. Following these steps,

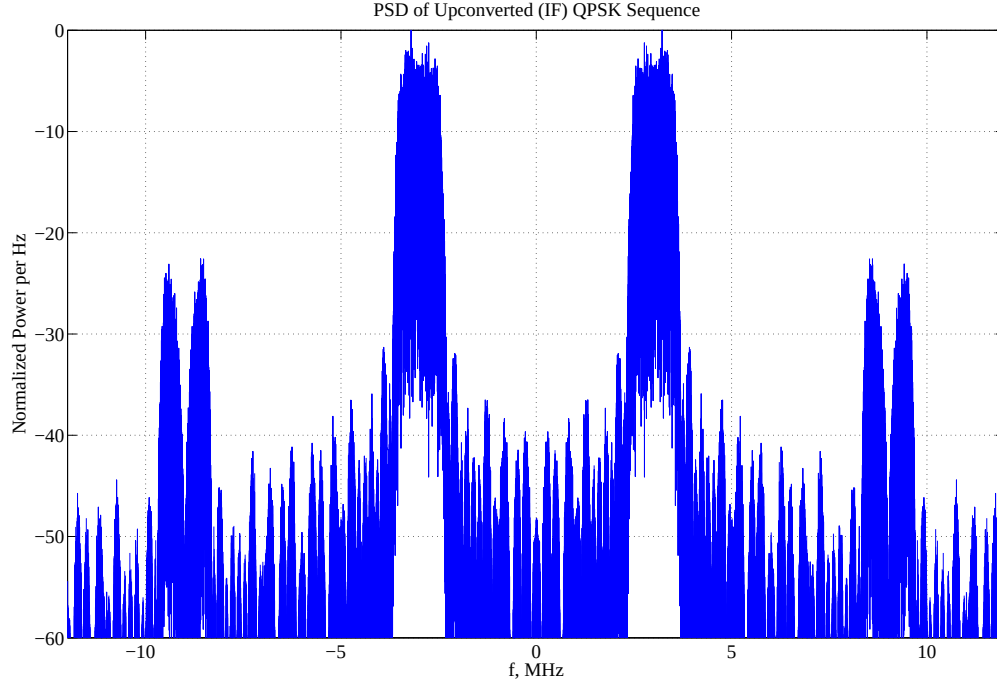


Figure 2.10: The PSD of the digital I/Q modulation output.

RRC matched filtering is applied and finally, the symbols are *detected*. Other than these steps, some auxiliary steps should also be implemented. Firstly, the start for the group of bits, which is called a packet, must be detected. The optimal detection points for bits needs to be found and effects of the channel and the frequency offsets should be estimated. It is common to most telecommunications systems that the receiver structure is much more sophisticated than the transmitter. In accordance with this trend, our receiver setup (with all the blocks stated above) is also almost six times more complicated than the transmitter in terms of the logic area usage on the FPGA chip. Figure 2.11 demonstrates the complexity of the receiver side better.

Now, we will explain in detail the receiver blocks one by one in the order of their operation on the received data.

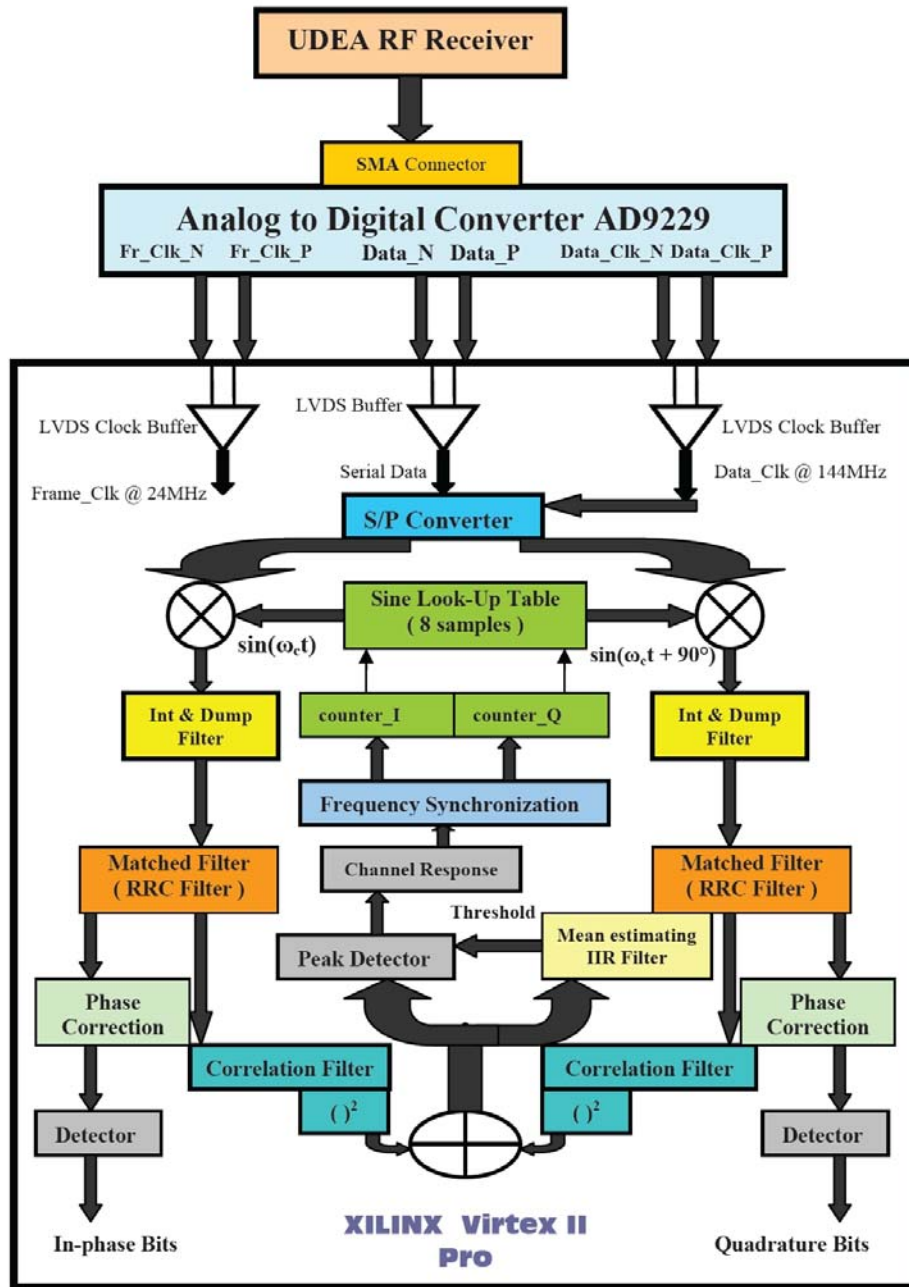


Figure 2.11: Reception, processing and detection tasks carried out in the receiver side are summarized.

2.5.1 Serial to Parallel (LVDS to CMOS) Conversion

We described in Section 2.2.3 that the 12-bit samples of the analog data are given in serial *LVDS pairs* together with *data* and *frame clocks* by the ADC board. After being conveyed to the FPGA board through an interface board, this serial data bits needs to be converted into 12-bit parallel data. This posed one of the biggest problems we faced, since not only the serial to parallel (S/P) conversion requires adequate usage of two different frequency clock signals, but also it has to be done by a circuit with a very low combinational delay. We aimed to design the simplest possible S/P converter that can sample the received serial bits at the rising- and the falling-edges of the data clock. Although the serial LVDS data rate in our case ($6 \times 24 = 144$ MHz) was moderate when compared with the upper limit of the ADC (nearly 400 MHz), the design was still demanding. The initial phase was to convert the obtained LVDS pair into a single ended CMOS signal that can be used by our design. The LVDS buffers and the LVDS clock buffers located on some special parts of the FPGA were the solution for voltage level conversion. Following this, we separated the S/P conversion block from the other design units and put some timing constraints so that all the S/P conversion related logic should satisfy 144 MHz operating frequency. These precautions made Xilinx ISE synthesize and implement a logic that meets our timing requirements. To obtain the first results, the parallelized data is tested by setting the ADC to transmit some factory-coded *test patterns* without actually sampling the received signals. Then, a logic block for converting the binary offset (see Section 2.2.3) representation into 2's complement had been written. Finally, assuring the correct operation of the S/P converter we used this structure until the realization of multiple receiver antenna system. The multi-antenna system is more complicated with timing constraints becoming harder to meet. Therefore, an improved version of this S/P converter is given in Section 2.6.2.

2.5.2 I/Q Demodulation

The sine lookup table discussed in Section 2.4.3 is used for I/Q demodulation with a slight difference in content and operation. We added some intermediate values to sine samples in order to increase time domain resolution of the representation. The overall table had 256 8-bit samples from a single period of a sine function. This method has barely any effect on the performance when sine lookup table is directly accessed with 32-step increments¹³ in the counter that is used for accessing the table. However, a feedback signal coming from the *frequency synchronization block* (see Section 2.5.8) orders the local oscillator signals to shift the frequency of the sine and cosine signals in small amounts for frequency offset mitigation. Hence, having a sine table with higher resolution in time allows this frequency correction operation to be more precise.

The I/Q demodulation operation involves multiplication of the received signal samples with the digitally generated sine and cosine signals. From this point on, we have two branches on which we will apply the identical filtering operations. (Hereafter one of these branches will be referenced as *I* for in-phase component of the received signal and the other one as *Q* for quadrature-phase component.) Since multiplication with a sinusoidal signal creates images of the desired baseband signal at twice the modulation frequency, both I and Q branches seem distorted at the output of the I/Q demodulator in Figure 2.12. In addition to the effect of high frequency signals, the power levels of I and Q branches are different. This is mainly due to the channel response multiplying the baseband equivalent complex (QPSK) signal. Such a multiplication modifies both the magnitude and the phase of a QPSK signal. With phase distortion, if the transmitted signal is $(1 + j)$ and the phase of the channel response is 45° , the corresponding I/Q demodulator output will be on the imaginary axis with zero power in the I branch. Since

¹³32-step increments in a 256-entry sine lookup table with a clock frequency of 24 MHz corresponds to generating 3 MHz sinusoidal signals just as in the transmitter side.

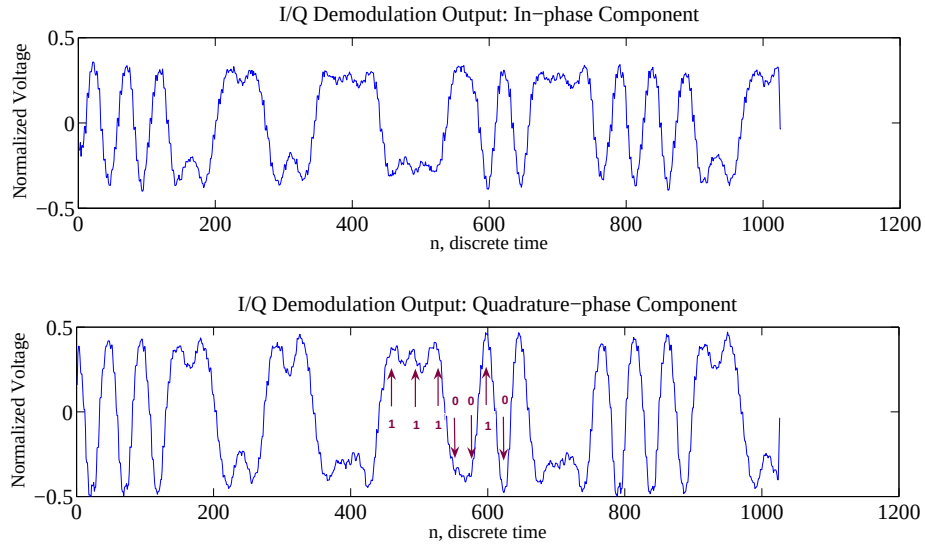


Figure 2.12: I and Q branch outputs after I/Q demodulation.

the phase correction to remedy this problem is given in Section 2.5.6, we will point out a final detail in Figure 2.12. In spite of the deteriorations at the output of the I/Q demodulator due to noise, the *7-bit Barker code sequence*¹⁴ is still observable with RRC pulse shaping at the Q branch output as shown by arrows.

2.5.3 Integrate and Dump Filtering

Having two identical implementations of this filter for handling I and Q branches simultaneously, we had the following reasonings for having them in the system:

- After demodulating the received signal, I and Q branches have high frequency terms as well as the desired baseband terms. A low pass

¹⁴7-bit Barker sequence is 1,1,1,0,0,1,0.

filtering is desired to cancel these high frequency terms out.

- Since the signal is sampled at 24 MHz, we have 24 samples for each bit to be decoded. Even if we take the *excess bandwidth* (resulting from the usage of RRC filter with non-zero β value) into account, the sampling rate is still much higher than the *Nyquist frequency*. The finite impulse response (FIR) RRC matched filter should have $(6 \times 24 + 1 = 145)$ taps, which is very costly to implement.

The integrate and dump filters (IDFs) accumulate¹⁵ their discrete inputs by summing them up for a defined interval and at the end of this interval gives a single output for all the summed up signals. Hence, being an averaging filter it behaves as a lowpass filter eliminating the high frequency terms. Moreover, it decreases the number of samples per bit at its output by down-sampling the input data. Our IDF realization sums 8 consecutive samples¹⁶ and outputs the average of them before accumulating next 8 samples. Following the IDF, the sampling frequency is decreased by 8 and becomes 3 MHz, which diminishes the logic area that the next stage filters consume. The effect of the IDF filter can be better understood by observing the Figures 2.13, 2.14, and 2.15. In Figure 2.13, the effect of eliminating the high frequency terms at the output of the I/Q demodulator is noted as having smoother curve in comparison with the results in Figure 2.12. Figure 2.14 proves us the existence of high frequency term at 6 MHz having a comparable power with the desired baseband signal for the I branch. Moving to the Figure 2.15, we can easily discriminate the desired RRC shaped baseband frequency response at the output of the IDF for the I branch.

¹⁵Correspond to integrating in continuous time.

¹⁶In fact, the frequency response of an IDF is sinc type (not a lowpass type) and the first null is at 6 MHz for this case.

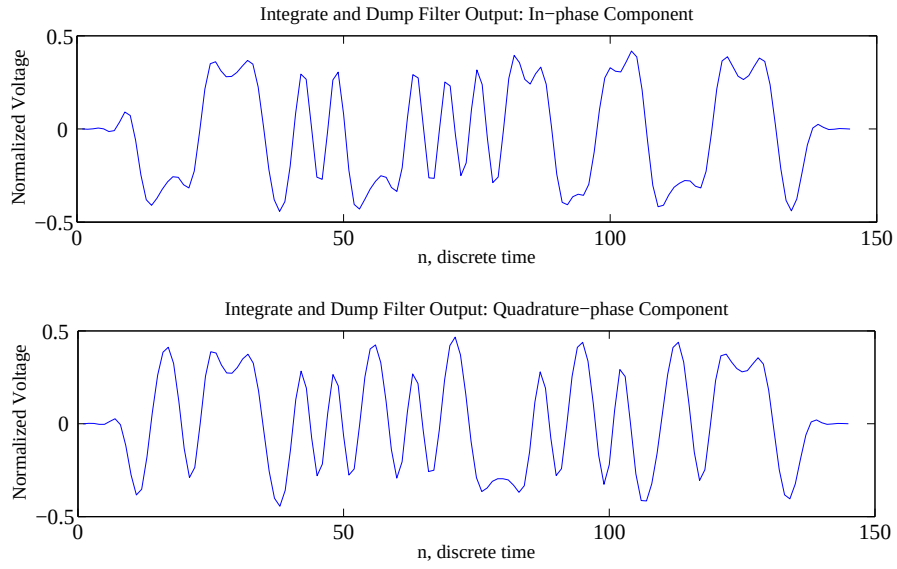


Figure 2.13: I and Q branch outputs after IDF.

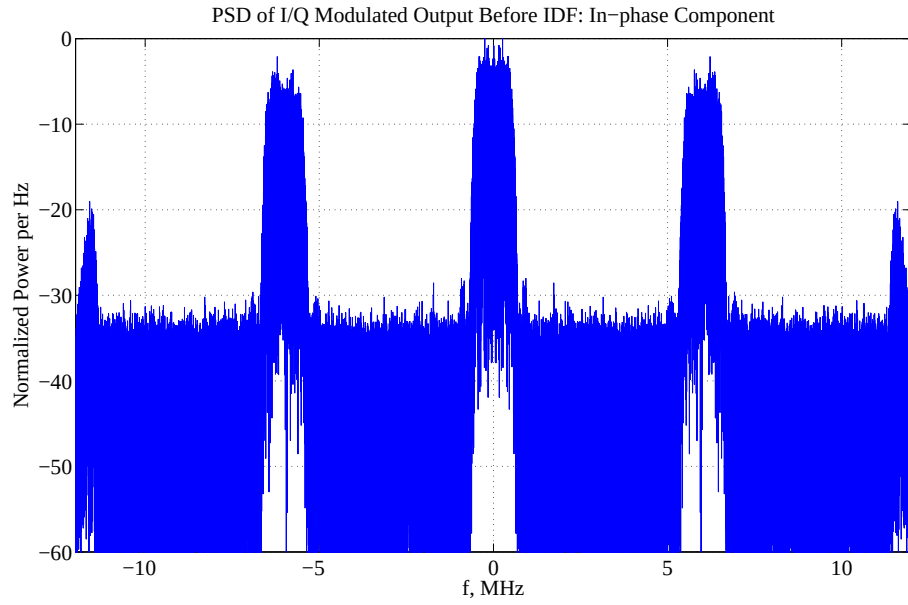


Figure 2.14: PSD of I branch includes high frequency term at 6 MHz.

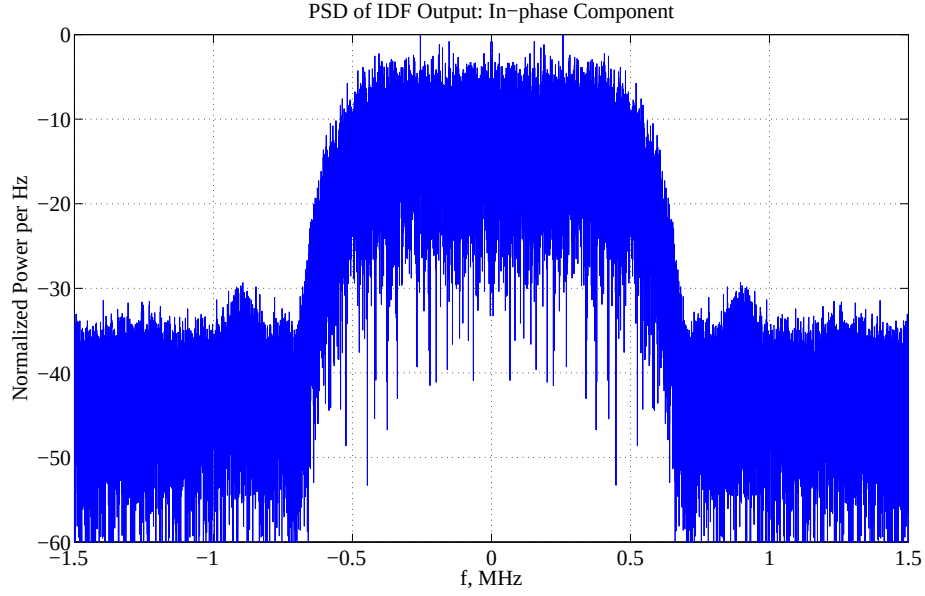


Figure 2.15: RRC frequency response with $BW=0.675$ MHz is clearly observed.

2.5.4 Matched Filtering

The matched filter is exactly the same pulse shaping filter used as the transmitter side, however since the sampling frequency at the output of the IDF is 3 MHz, an RRC filter with oversampling rate of 6 can not be realized for the receiver. Therefore, the oversampling rate for the matched filter is 3, which corresponds to building up an FIR filter with 19 taps under the assumption that only the symbols at a distance less than or equal to 3 are effective on the current symbol. A filter with 19 taps requires 19 multiplications and 18 additions for evaluation of an output at each instance. In order to avoid slowing down of this operation, we used two approaches while coding the matched filter:

- Multiplication operation is usually more time-consuming than simple shifting operations and there are only limited number of fast multipli-

ers on our board as stated in Section 2.2.1. Therefore, we preferred arithmetically shifting the inputs to the left to multiplying them with tap weights. For example, multiplication by 5 is implemented as adding the input value to its 2 times arithmetically left-shifted version.

- Although we removed away the multipliers from our RRC design, still we had many additions, even more than what we would have had when we used multiplication. If all of the additions were to be carried out in single clock time, this would result in a very high combinational delay that the RRC filter would be useless. At this point, the *pipelining* was thought to be the key for decreasing combinational delay of the matched filter. With pipelining it is possible to evaluate the results of subsets of all the additions separately and joining these intermediate results to evaluate the final result by either using another pipelining stage(s) or directly adding them up. As expected, pipelining comes with its own cost, excessive storage area used by the intermediate results in different pipelining stages. In contrast to the excessive multiplier usage and high combinational delay issues, this problem is fairly acceptable especially when a good pipelining design taking care of the structure of the filter is done. After designing the matched filter, we plotted the experimental data taken from FPGA in MATLAB, which is given in Figure 2.16.

2.5.5 Packet Synchronization and Correlation Filtering

For detecting the existence and determining the beginning of data packets we add a *pilot symbol sequence* at the start of each packet. Although various kinds of pilot symbols may be used for this purpose, we mainly concentrated on a specific class of sequences that are also utilized in practice [5]. We appended 7-bit and 13-bit Barker codes as the prefix of transmitted packets of length 512 or 1024 bits. The Barker codes are known for their low auto-correlation sidelobe properties. The 7-bit packet prefix is (1,1,1,0,0,1,0) and

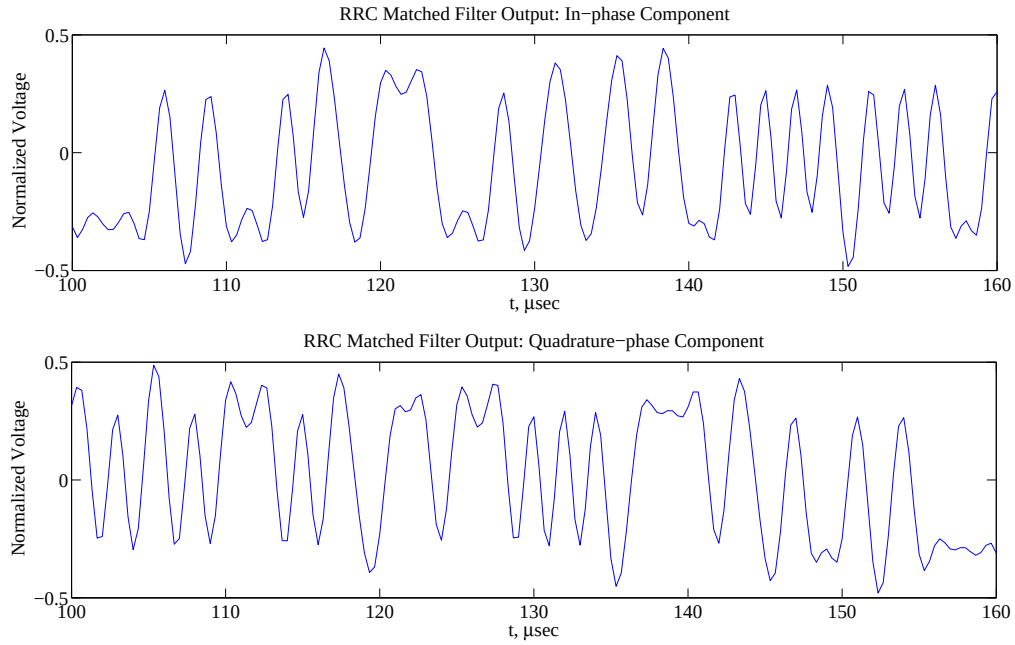


Figure 2.16: Power in I and Q branches are quite close to each other, which indicates that instantaneous phase of channel response is close to a multiple of $\pi/4$ and it effects the transmitted signals via a rotation of $k\pi/4$ radians.

the 13-bit sequence is (1,1,1,1,1,0,0,1,1,0,1,0,1) with possible reversed and negated combinations. The RRC shaped 13-bit Barker sequence is plotted in Figure 2.17 ideally in MATLAB.

The packet synchronizer uses a *correlation filter*, which shifts over the received signal with a time uncertainty step of $(T/3)$, where T is the symbol duration, and at each step (chip interval) it correlates the received signal with the known synchronization sequence. In our case the known synchronization sequence is the Barker sequence used. Therefore, the Barker sequence taps (+1 for bit values of 1 and -1 for bit values of 0) advance over the received signal in time steps of $\frac{1}{3}\mu\text{sec}$ in order to detect a high correlation point. There are two such correlation filters, one for I branch and the other for the

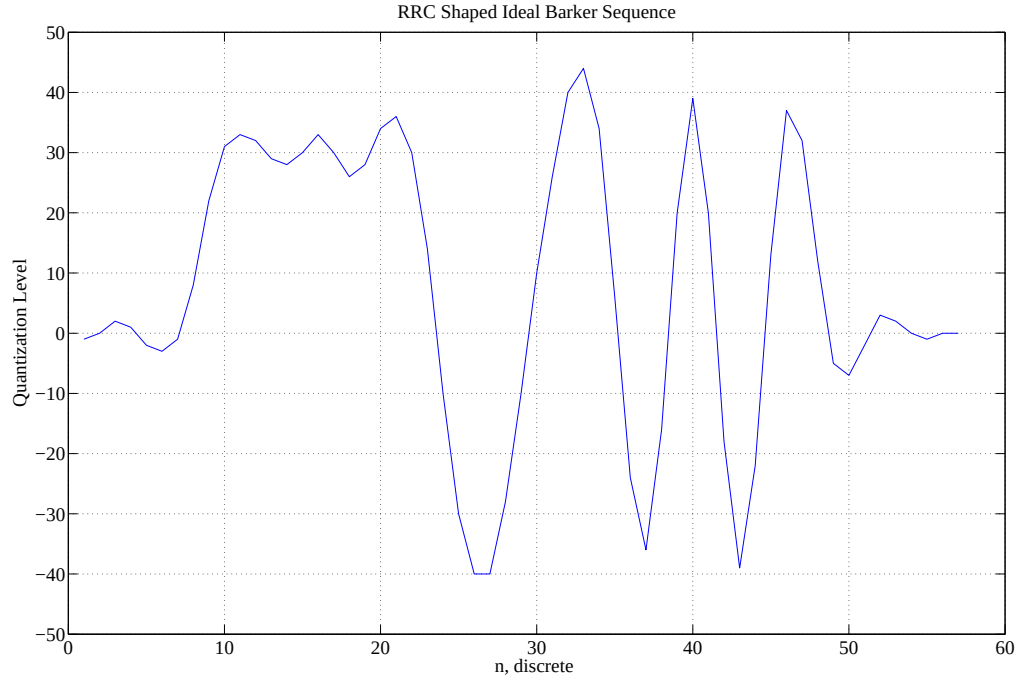


Figure 2.17: MATLAB plot of the 13-bit Barker code.

Q branch. The ideal method for detecting a new packet requires the joint detection over two branches. Since we transmit the same Barker sequence over two branches, the packet synchronization module simply finds the squares of the correlation filters and sums these results¹⁷, which is given as,

$$b_{total}[n] = b_I^2[n] + b_Q^2[n]. \quad (2.6)$$

In Eqn. 2.6, $b_I[n]$ and $b_Q[n]$ denote the outputs of the correlation filters of the I and the Q branches respectively. Figure 2.18 gives an idea about the *measure* ($b_{total}[n]$) that is used for determining the start of a new packet. In Figure 2.18, the first normalized peak value is found to be 0.4594, while the sidelobe value with `datatip` is 0.00421. Hence, the ratio of the peak-

¹⁷This method is ideal for the case that noise in the received signal is complex AWGN.

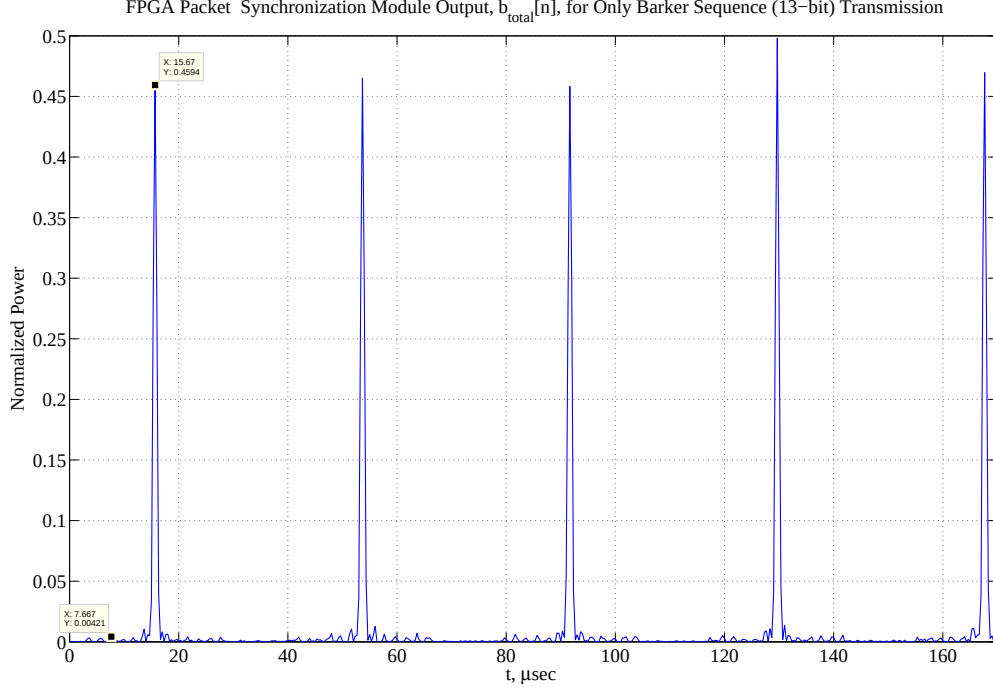


Figure 2.18: Peak points showing the start of data packets and the small sidelobes are clearly distinguishable.

to-sidelobe is nearly 109, which is a bit smaller than the ideal ratio¹⁸ for $b_{total}[n]$ with no noise in the channel. In addition to this, the peak values for consecutive Barker sequences slightly differ from each other. As a result, it is usually impractical to set a constant limit for detection of the packet start peaks. On the contrary, the threshold in our system is calculated according to two parameters: the expected value of the noise power and the estimated signal power. The expectation for noise power is obtained by lowpass filtering the correlation power output, $b_{total}[n]$, of the packet synchronization module. This lowpass filter is an infinite impulse response (IIR) filter whose update

¹⁸Ideal peak-to-sidelobe ratio is the square of bits used in the sequence, hence 169 for this example.

equation is given as,

$$m_{noise}[n] = \begin{cases} \frac{k-1}{k}m_{noise}[n-1] + \frac{1}{k}b_{total}[n-1] & \text{if not in a packet} \\ m_{noise}[n-1] & \text{if in a packet} \end{cases} \quad (2.7)$$

According to Eqn. 2.7, the expectation of the noise signal power is updated (calculated) only if the system is waiting for a packet arrival. If a packet is already being sampled or the leading sidelobes of the correlation power output has arrived, due to possibly very high signal power, the noise power update is paused until the end of the current packet¹⁹. Another important topic is the selection of the k value in the update equation. With smaller values of k , the filter tend to approximate the noise power using less number of past values. However, very large values also has a disadvantage, since the input ($b_{total}[n-1]$) to the filter is divided by larger numbers which may result in loss of data in the least significant bits or even the input may be totally nulled. After some *trial and error*, we decided to use $k = 64$.

The signal power estimation is based on the detection of the levels of the leading sidelobes of the correlation power output. The higher the sidelobes, the higher should be the threshold that $b_{total}[n]$ surpass. Moreover, any point should be a peak (maximum point) in order to be classified as a starting instance for a packet. This is handled by storing the past value and also a future value of $b_{total}[n]$ so that any point over the threshold can also be compared with its neighboring points. (As an example, in Figure 2.18, the point marked with the datatip around $t = 16 \mu sec$ is larger than the points on its left and right, hence is a maximum point.) Then, a maximum point higher than the *adaptively determined* threshold value marks the start of a new packet, which triggers the channel response estimation and bit sampling procedures.

¹⁹Our transmitter structure guarantees that there will be empty periods between packets so that no data will be transmitted in these periods.

2.5.6 Channel Response Estimation and Phase Correction

In most practical systems it is crucial for the receiver to estimate the effects of the medium that the received data is transmitted in. This estimation is commonly known as *channel response estimation*. In our receiver structure, if we denote the received signal as $y[n]$, the channel response as $h[n]$, and the noise sample as $w[n]$ at the instant n , the relation between transmitted symbol, $x[n]$, and $y[n]$ can be written as,

$$y[n] = h[n]x[n] + w[n]. \quad (2.8)$$

In Eqn. 2.8, the most important assumption is that channel response can be approximated by a single constant complex number. Therefore, the channel is assumed to be frequency non-selective (flat-fading or ISI-free). Moreover, if we assume that the channel response, $h[n]$, is constant also within the packet length (which is acceptable for packet duration of 512 μ sec in most indoor mobility cases), then estimating the value of $h[n]$ at the start of a packet (hence dropping the discrete time index), channel equalization is performed by,

$$\tilde{y}[n] = h^*y[n] \quad (2.9)$$

$$= |h|^2 x[n] + h^*w[n]. \quad (2.10)$$

The operation in Eqn. 2.9 involves complex multiplication of the received complex signal with another complex number's conjugate. On FPGA, this can be realized as in the following code segment,

```
... (Signal declarations and other parts that precede are removed.)
if( (corr_power_1past >= threshold) and
    (corr_power_1past >= corr_power) and
    (corr_power_1past >= corr_power_2past)) then
    ch_resp_I <= barker_corr_I;
```

```

ch_resp_Q <= barker_corr_Q;
... (The codes in between are removed.)
I_corrected <= (rrc_I - rrc_Q) * ch_resp_I +
               (rrc_I + rrc_Q) * ch_resp_Q;
Q_corrected <= (rrc_I + rrc_Q) * ch_resp_I -
               (rrc_I - rrc_Q) * ch_resp_Q;
... (Remaining part of the code follows.)

```

The initial *if statement* in the code searches for the packet synchronization point as described in Section 2.5.5. However, channel response value obtained in this code uses directly the correlation filters' outputs at the packet initialization instance. In fact, since the transmitter side sends the same Barker sequence from both I and Q branches, the found channel response corresponds to the effect of channel on complex symbol $x[n] = 1 + j$ instead of simply $x[n] = 1$. Therefore, we have to divide the found $h[n]$ value by $(1 + j)$ in order to carry out the operation described in Eqn. 2.9. The remaining parts of the above code segment takes care of this issue to obtain the corrected²⁰ I and Q values for $x[n]$. For QPSK (and BPSK) modulation only the received signal's phase is important. There is no need for correcting the magnitude of the symbols to be detected unless non-constant amplitude modulation techniques are used. Therefore, the $(|h|^2 x[n])$ term in Eqn. 2.10 was left as it is before detecting the QPSK symbols.

Obtaining the channel response values for consecutive packets, we plot these complex numbers in the complex domain to observe their characteristics. The *scatterplot* for the estimated experimental $h[n]$ values (channel is the transmission medium, that is air, for all of the experiments) are given in Figure 2.19. The Figure 2.19 is drawn by estimating 32 consecutive channel response values at five different time intervals, while the transmitter and the

²⁰The correction at this point is only for the channel phase subtraction, no magnitude correction is carried out.

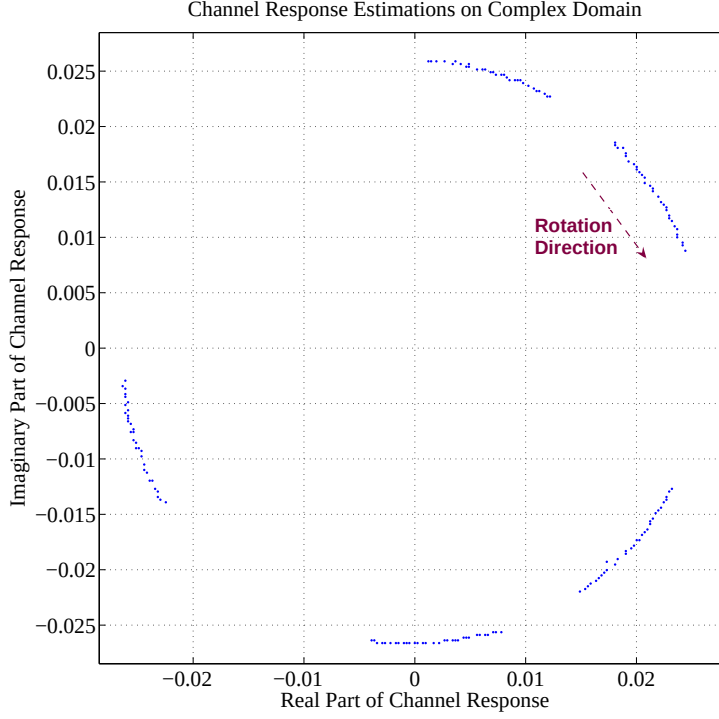


Figure 2.19: The phase of the channel response varies fast.

receiver antennas are in *line of sight* (LOS). Each one of these five groups span 1 msec intervals. Surprisingly, although the magnitude of the estimates stays nearly constant, the phase varies in a relatively fast pace so that the channel response rotates almost by $\frac{\pi}{6}$ radians within 1 msec. This would mean that for symbol time of 1 μ sec and packet length of 1500 bits, a symbol value of $(1 + j)$ becomes $\sqrt{2}$ (with no imaginary component at all) at the end of the packet, which will result in false detections even if no noise is present in the received signal. Hence, it is almost impossible to transmit packets longer than 1000 bits by estimating the channel response only at the start of the packet or without finding the reason for the rotation in the channel response. The solution for avoiding this rotation is given in Section 2.5.8.

2.5.7 Bit Synchronization

Bit synchronization procedure is totally dependent on the determination of the starting point of a packet on our testbed. Since the oversampling rate for the corrected I and Q branch RC shaped signals is three (see Section 2.5.3), we have only three candidates for being the optimal sampling point. In fact the optimal sampling point for the RC pulse shape is the one we need to find. The sampling point is chosen as the one having the largest Barker correlation output. Once the first optimal sampling point is found the following optimal points can be chosen by jumps of three (oversampling rate) samples from the initial optimal point. This optimal sampling routine is demonstrated on the real data from our testbed, in Figure 2.20. As seen in Figure 2.20, some of

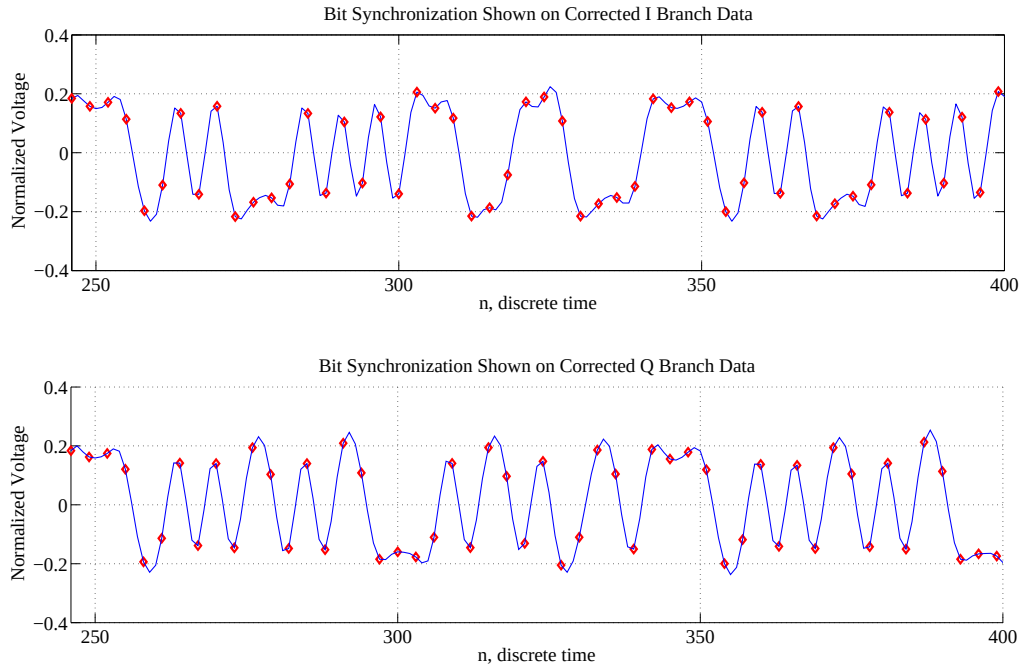


Figure 2.20: Diamonds represent the optimal sampling points obtained by the testbed.

the sampling points coincide with the optimal RC sampling points very well, whereas some others (like the 6th sample of Q branch) do not. This is mainly due to two reasons:

- Since the oversampling rate is only three, there is still a high chance that all the three points misses the optimal point.
- The rotation of the channel response described in Section 2.5.6 results in phase offsets dynamically changing the real sampling points throughout the packet, while our receiver decides on the optimal points for whole packet once only at the beginning of it.

The solution to the first reason has a high cost associated with it, since increasing the sampling rate would require all the filters, following the IDF, to have more input signals to be processed at an instant. (See Section 2.5.3 for details.) Therefore, we mainly concentrated on the second reason for non-optimal sampling in Section 2.5.8.

2.5.8 Frequency Synchronization and Bit Detection

When we analyzed the reasons for channel response rotation described in Section 2.5.6, we observed that the rotation direction of the channel response changes from *clockwise* to *counterclockwise* when we exchange the crystal oscillators used at the receiver and the transmitter sides. After that, we carefully investigated the frequency offset between these two *roughly* 24 MHz oscillators to find a frequency difference of nearly 640 Hz. This offset corresponds to a $(640/8 = 80 \text{ Hz})$ difference at 3 MHz, which is the carrier frequency for our IF signals. If we model that IF offset as a modulating sinusoidal signal with period $((80 \text{ Hz})^{-1} = 12.5) \text{ msec}$, this would result in a channel rotation of $(2\pi \frac{(1 \text{ msec})}{(12.5 \text{ msec})}) = (\frac{2\pi}{12.5})$ in 1 msec that is very close to the rotation observed in Figure 2.19.

The larger frequency offsets between the IF frequencies generated on two sides of our testbed result in faster rotations of the channel response esti-

mates. As a result, the expected value of the rotation angles between consecutive channel response estimates gives a metric for frequency mismatch. Therefore, we desire a mechanism for calculating the phase of the channel response estimates and for averaging the phase differences over many estimation results.

Coordinate rotation digital computer (CORDIC) [6] is an algorithm especially used for calculating trigonometric, hyperbolic, logarithmic functions, or division and square root operations, where no or limited multiplication units exist. A CORDIC-based algorithm makes use of only additions, arithmetic shift operations, and a fairly small size lookup table. It tries to converge to the correct result by *successive approximation* iterations. The resolution of the final result (so the number of iterations to be made) depends on the size and the resolution of the lookup table utilized. In our case, we used a CORDIC-based algorithm to calculate the phase of each channel response estimate. The lookup table, for this implementation, is filled with digital representations of various arctangent values. There are eight 8-bit representations: $\arctan(1)$, $\arctan(1/2)$, $\dots$, $\arctan(1/128)$. In the first step algorithm starts with assuming the phase to be calculated is 0, π , or $-\pi$ radians according to the quadrant of the channel response and modifies the channel response. In the second step it adds/subtracts $\arctan(1) = \pi/4$, the first lookup table entry, from the initial guess by observing the sign of the modified channel response and modifies the channel response once more. Continuing in this fashion the algorithm adds/subtracts smaller and smaller angles at each step and obtains a result with an error on the order of the last lookup table entry after the 8th step. If we convert the last entry of the lookup table into degrees, the maximum error in phase calculation is only 0.45° with 8 iterations that would be done in $8 \mu\text{sec}$ after start of the packet is detected.

Final point in frequency synchronization is obtaining an expected (average) value for frequency offset. This is accomplished by using a simple IIR filter that bare resemblance to the IIR filter in Eqn. 2.7, but with $k = 8$.

This filter operates only once for each packet. The output of this filter gives us information about the sign and the magnitude of the frequency difference between the transmitter and receiver. These two parameters modify the frequency of the generated IF demodulation signals. As explained in Section 2.5.2, the sine and the cosine signals are generated using a 256-entry table, which includes the samples from a period of a sine function. If no frequency difference is present between sides, this table is accessed by 32-step increments at each clock cycle to synthesize 3 MHz signals. Whenever the frequency offset results in an output signal of *non-zero magnitude* for the IIR filter of frequency synchronization module, this 32-entry step is altered to 31- or 33-entry steps in order to accelerate/decelerate the receiver sides signals. A 31-entry jump is used for accelerating the sine and cosine signals, whereas a 33-entry jump slows down these signals. Since these short/long (abnormal) steps are used only once in hundreds (or even thousands) of normal steps, the resulting sine and cosine signals do not deteriorate much, but they are only slightly frequency shifted. The counter which determines the period of abnormal steps is adaptively updated according to the newly obtained phase differences from consecutive channel response estimates, which completes the feedback loop. To manifest the tremendous effect of frequency synchronization block on the recorded experimental channel response estimates, we implemented a test scenario, in which 1024 packets each with 512 QPSK symbols are transmitted. The channel response estimates without and with the digital phase-locked loop-like (D-PLL-like) structure are given in Figure 2.21. It is obvious that the channel response do not change much between consecutive packets, according to Figure 2.21. The channel response rotates over itself completing many tours when the frequency synchronization unit is non-operational (left-hand side). However, when it is operated, the rotation for 1024 packets is almost 42° , which gives an average rotation of 0.041° between two consecutive packets. Let us, finally, demonstrate the effect of frequency synchronization, this time, for the QPSK symbols to be detected. The presented data on Figure 2.22 is experimental data, which is

acquired from the digital bit detector on FPGA, using the serial port communication. In Figure 2.22, QPSK symbol constellation diagram is plotted for the case when the frequency synchronization block on FPGA is deactivated (shown on the left-hand side) and the case when it is activated (shown on the right-hand side). Both of the experiments are carried out using single 1024-symbol packet with previously known content and QPSK modulation. The bit detection is done on I and Q branches separately, since signal levels on these two branches are independent. For both branches, if the signal is negative then a 0 is decided; otherwise a 1 is decided. The performance measure for this single input single output (SISO) system is given in comparison with single input multiple output (SIMO) system in Section 2.6.3.

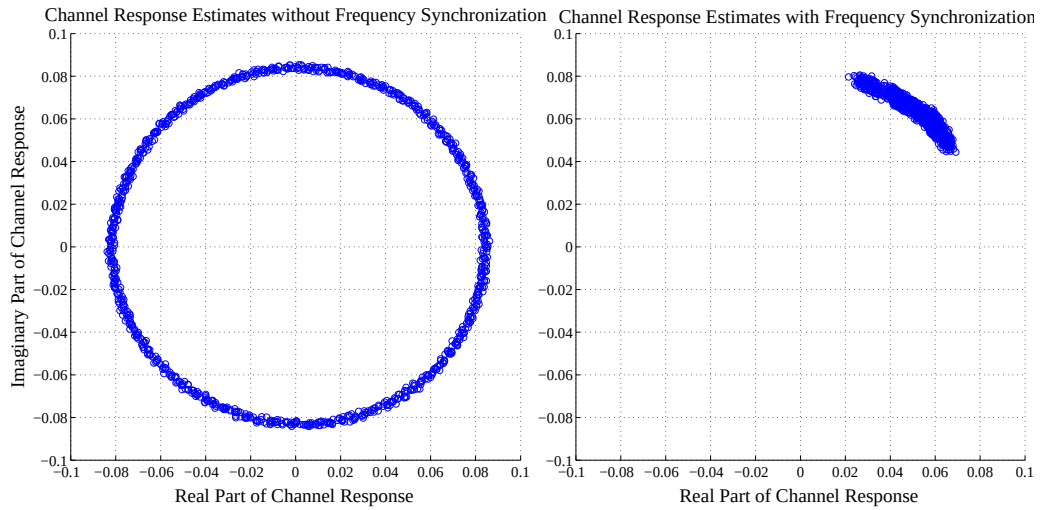


Figure 2.21: The phase of the channel response estimates changes very slowly in case of frequency synchronization.

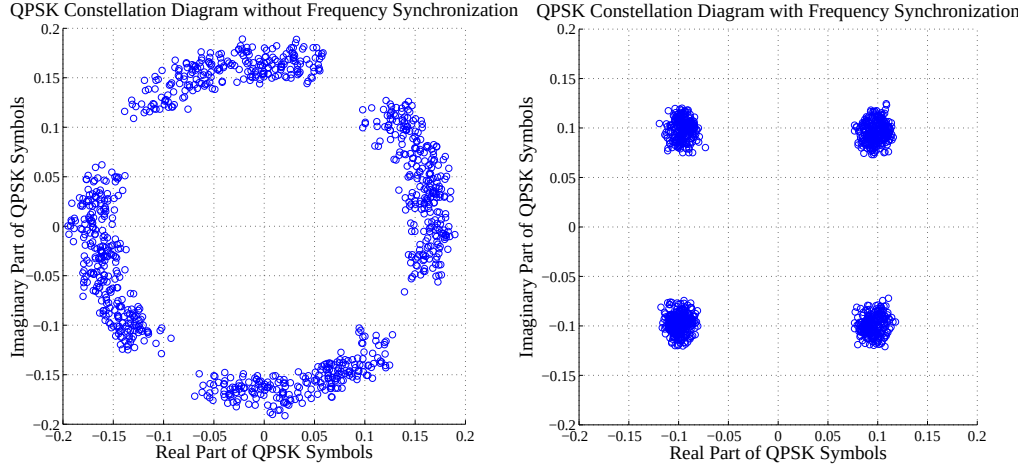


Figure 2.22: Many symbols would be incorrectly detected if no frequency synchronization were utilized.

2.6 SIMO System Setup

Once all of the blocks of SISO testbed are developed and verified, we set the goal of operating multiple-antenna cases for receiver, then transmitter, and finally both sides. In the following sections, there will be information on the two receiver antenna structure.

2.6.1 Maximal Ratio Combining for Two Receive Antenna System

One of the frequently faced problems for the wireless systems is *fading*. The main reason for fading in wireless systems is the *multipath* phenomena. Usually, the transmitter and the receiver are in an environment consisting of many reflectors and scatterers such that the transmitted signal reaches the destination following many paths. Each path has its own attenuation and phase-shifts and the receiver observes a superposition of these signals from

multiple paths. At some point in space, these signals may add constructively, whereas within a small distance, at an other point they may add destructively. This results in wild variations in the received signal power, hence the system SNR. When the phases of these signals are assumed to be uniformly distributed in $(0, 2\pi)$ and independent from each other, the magnitude of the superposition result (received signal) is Rayleigh distributed [7], which is given as,

$$p_Z(z) = \frac{z}{\sigma^2} \exp\left(-\frac{z^2}{2\sigma^2}\right), \text{ for } z \geq 0, \quad (2.11)$$

where z is the amplitude of the received signal and σ^2 is the variance of the received signal's real and imaginary components. The power of received signal is exponentially distributed and given as,

$$p_X(x) = \frac{1}{2\sigma^2} \exp\left(-\frac{x}{2\sigma^2}\right), \text{ for } x \geq 0, \quad (2.12)$$

where x denotes the power of the received signal. Hence, with moderate average channel SNR ($2\sigma^2$) values, there is a good probability that the *instantaneous channel SNR* is below a threshold level under which the wireless communication system of interest will be unable to support reliable communication. In such a case, the channel is said to be in *deep fade* and communication stops until the received power exceeds the threshold value.

The usual way to fight fading in wireless channels is creating some type of *diversity* in one or more of the communication domains: frequency, time, space, ... The probability of having all of the diversity elements experience deep fade at the same time is much smaller than the probability of a single element facing a deep fade, if the signals for all these diversity elements are independent. Therefore, with diversity, the reliable communication between the transmitter and the receiver is supported unless all of the diversity elements fail. We, while building our testbed, used the *spatial domain* for obtaining diversity gain and anticipated that if at least one of the spatial diversity elements (receiver antennas) could survive, the communication would not fail due to other elements.

In our *single input multiple output* (SIMO) system, we used two receiver antennas in order to create spatial diversity. In general, assuming that all the M_r receiver antennas are placed at such distances from each other that they observe independent fading, there are many ways of combining the received signals from different antennas. The combined signal is a weighted sum of the received signals. In Figure 2.23, $s(t)$ denotes the transmitted signal, h_1 ,

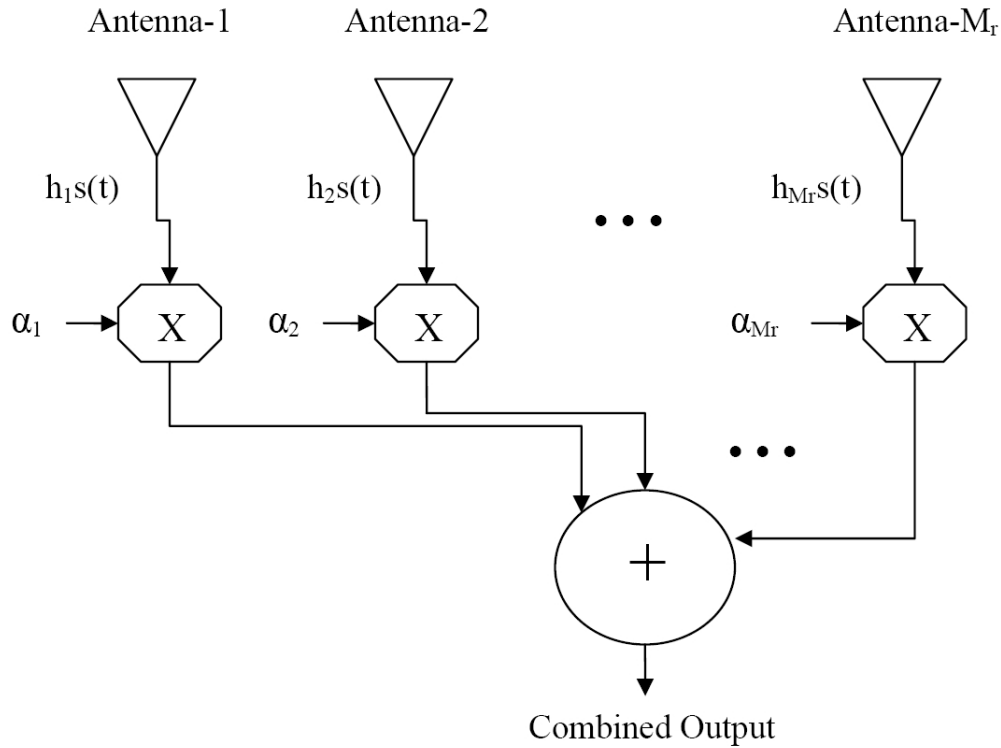


Figure 2.23: Combined output is a linear combination of the received signals.

$h_2, \dots, h_{M_r}$ denote the complex channel gains for paths from the transmitter antenna to the receiver antennas, and $\alpha_1, \alpha_2, \dots, \alpha_{M_r}$ the complex weighting factors that the combiner apply to each of the received signals. In the case

that more than one of the signals are weighted by non-zero α_i values, these α_i values must be selected in a way that the weighted signals should not add up destructively under any realizations of h_i values. Therefore, the selection of the phases for non-zero α_i 's require,

$$\alpha_i = a_i e^{-j\phi_i}, \quad (2.13)$$

where ϕ_i is the phase of the i^{th} channel response ($h_i = r_i e^{j\phi_i}$) and a_i 's are real.

The optimal way of combining the diversity branches is named as the *maximal ratio combining* (MRC) [8]. With MRC, the a_i values are selected in order to emphasize the signals with higher SNR values and to weight the signals having lower SNR values with smaller factors. This approach achieves the maximum SNR for the combined output signal if a_i values are chosen as,

$$a_i^2 = r_i^2 / N_i, \quad i = 1, 2, \dots, M_r. \quad (2.14)$$

In Eqn. 2.14, N_i is the noise power in the i^{th} channel and therefore $\gamma_i = r_i^2 / N_i$ represents the SNR for this channel. Then, the maximum total SNR for the combined output is given as,

$$\gamma_{total} = \frac{\left(\sum_{i=1}^{M_r} (a_i e^{-j\phi_i}) (r_i e^{j\phi_i}) \right)^2}{\sum_{i=1}^{M_r} |a_i e^{-j\phi_i}|^2 N_i}, \quad (2.15)$$

$$= \frac{\left(\sum_{i=1}^{M_r} a_i r_i \right)^2}{\sum_{i=1}^{M_r} a_i^2 N_i}. \quad (2.16)$$

If we place optimal a_i values given in Eqn. 2.14, into Eqn. 2.16, and assume that the N_i values for all channels are the same and equal to N_0 , the total

SNR is given as,

$$\gamma_{total} = \frac{\frac{1}{N_0} \left(\sum_{i=1}^{M_r} r_i^2 \right)^2}{\sum_{i=1}^{M_r} \frac{r_i^2}{N_0} N_0}, \quad (2.17)$$

$$= \sum_{i=1}^{M_r} r_i^2 / N_0, \quad (2.18)$$

$$= \sum_{i=1}^{M_r} \gamma_i. \quad (2.19)$$

Hence, the maximum total SNR is obtained as the sum of SNR values for all receiver antennas when we utilize MRC as our combining scenario. For the two-antenna testbed, simply assuming noise power is identical for two receiver antennas, we carried out the same operation for combining the received signals. Therefore, we detected the QPSK symbols using $\tilde{y}_{total}$, which is found as,

$$\tilde{y}_{total}[n] = h_1^* (y_1[n]) + h_2^* (y_2[n]), \quad (2.20)$$

$$= h_1^* (h_1 x[n] + w_1[n]) + h_2^* (h_2 x[n] + w_2[n]), \quad (2.21)$$

$$= (|h_1|^2 + |h_2|^2) x[n] + h_1^* w_1[n] + h_2^* w_2[n], \quad (2.22)$$

where $x[n]$ is the symbol transmitted, $w_1[n]$ and $w_2[n]$ are the noise samples for receiver antennas, h_1 and h_2 are the estimated channel responses (both channels are again assumed to be frequency non-selective as in Section 2.5.6) for two receiver paths. This shows us that the total SNR is just the sum of SNRs and for any instantaneous pair of γ_1 and γ_2 values the performance of the MRC system will be better than corresponding single receiver antenna systems.

2.6.2 Implementation Issues

Although, the approach for implementing the SIMO system from the basic blocks of SISO system is obvious, these issues should be handled carefully for proper operation:

- **The orientation of the receiver antennas:** In order to gain from diversity of the SIMO system to the fullest extent, the receiver antennas need to be spaced sufficiently away from each other. The figure for the distance is accepted as 0.4λ in most practical applications [8], where λ , the signal wavelength, is 12.5 cm for our RF signal at 2.4 GHz.
- **Packet and frequency synchronization:** The packet synchronization and the frequency synchronization modules should be operated using both of the received signals for minimum packet miss rate and best frequency offset estimation. In our testbed we always used only one of the signals for this purpose; however we utilized the one with line of sight (LOS) to compensate for the non-optimality. In a more realistic scenario, the correlation power outputs²¹ corresponding to the signals received from two antennas must be summed for detecting the peak at the start of a packet. Moreover, the frequency offset estimation should be based on a weighted average of the phase differences of the channel responses, h_1 and h_2 .
- **Xilinx ISE global buffer inference:** The SISO system receiver includes only 12 registers for serial-to-parallel (S/P) conversion of single ADC²² output. Therefore, there are only 12 units clocked by the data clock output of ADC. For SIMO case, due to S/P conversion of two separate ADC outputs, the number of registers connected to the data clock doubles. Unfortunately, Xilinx ISE infers *clock buffers* only for clock signals that have fanout larger than 16. While the SISO system uses only an LVDS clock buffer for LVDS-to-CMOS conversion, for the SIMO system another CMOS clock buffer is appended to the output of the LVDS clock buffer. This addition of the new clock buffer also adds

²¹See Section 2.5.5 for definition.

²²Serial LVDS data output of the analog-to-digital converter in our system is parallelized using the data clock output of the same module

a routing and buffering delay to the clock signal leading to unexpected S/P conversion results. On the other hand, Xilinx ISE offers other tools for solving these type of timing problems. To start with, forcing all of the registers (connected to data clock) to be placed in a close proximity of the CMOS clock buffer output guarantees all registers to be clocked almost at the same instance. These type of constraints are known as *placement* or more specifically *location constraints*. Other than the registers, it is also possible to place the CMOS clock buffers in a desired area, but with more restrictions. Secondly, there are 8 *digital clock management* (DCM) on our FPGA chip. A DCM module is capable of synthesizing integer or fractional multiples of a given clock signal, phase-shifting a signal by a user-defined amount, and compensating the clock delays in the clock distribution network. This final property, made us remove the delay caused by the CMOS clock buffer, since the synthesized DCM module adaptively removed any routing or buffering delay on its output signal via a feedback clock connected as an input. This feedback signal observes all delays that a clock signal without a DCM module would observe, and with the help of a *delay-locked loop* (DLL) mechanism, DCM module compares its original and feedback clock inputs to generate a compensated output clock signal with theoretically no delay (See Figure 2.24). This zero-delay clock output had been the key for operating the SIMO system with two receiver antennas.

- **Digital output offsets for ADC:** This issue is minor when compared to previous ones; on the other hand, it still may be important for special experiments. As with the most ADCs, AD9229 also gives non-zero digital outputs when all of its analog inputs are connected to the ground. Moreover, this offset has different values for distinct sampling channels and may even be positive for one channel, while it is negative for another one. A simple offset correction circuit should be

implemented on FPGA for each channel.

2.6.3 Mobile Receiver Tests and Results

For testing the performance increase proposed by the SIMO system and MRC method, we developed an experiment scenario. Initially, we placed the transmitter module behind some laboratory equipment to prevent a very strong signal reception through the LOS component. In order to further decrease the SNR, we kept the receiver modules behind some other equipment. Moreover, for observing different fading effects we slowly moved the receiver antennas in different directions. The transmitter side sent 1024 packets each with 1024 bits (512 *known* QPSK symbols) and the receiver side recorded the number of bit errors for each packet and for three different methods separately. The first method was using only the first antenna, the second one chose the second antenna as its source of symbol detection, and finally, the third method utilized MRC to combine the signals from both antennas and detected the symbols. Figures 2.25 and 2.26 give the results for this experiment. Figure 2.25 demonstrates the dynamic performance of two single antenna systems. A very high correlation between the estimated instantaneous channel SNR (top-left and

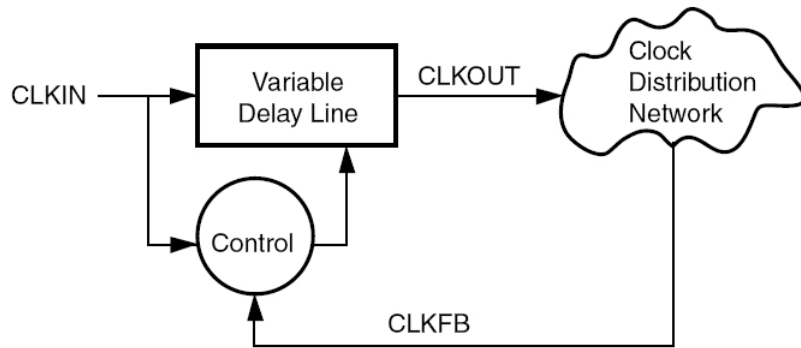


Figure 2.24: Delay-locked loop in a DCM eliminates clock delays.

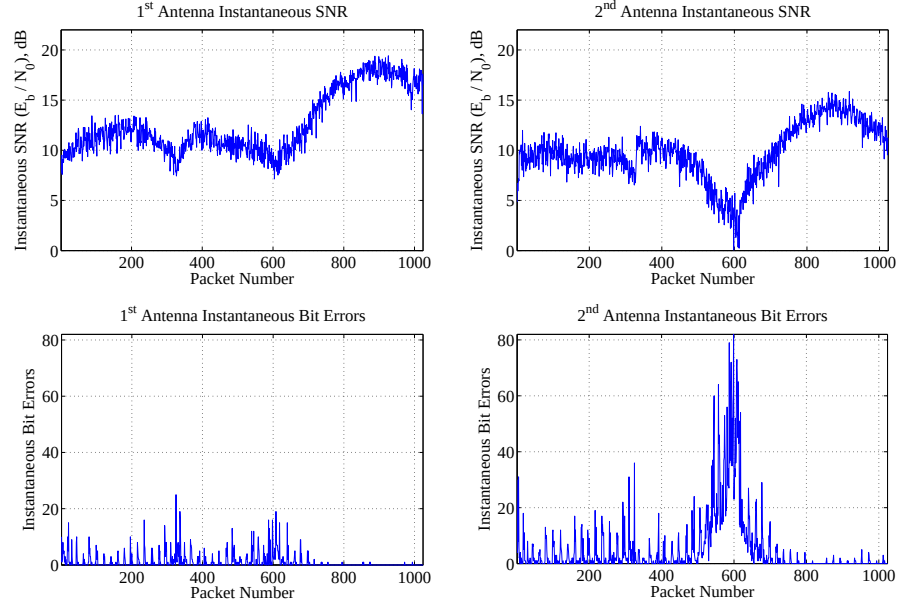


Figure 2.25: A deep fade results in increased bit errors in the bottom figures.

top-right figures) and the instantaneous bit error rate²³ (BER) (bottom-left and bottom-right figures) is trivially observed. Moreover, rapid variations in the received SNR are observed for both antennas. Around the 600th packet, the second antenna is in a deep fade and the BERs are very close to 0.1 for these packets.

Fulfilling the expectations, Figure 2.26 proves the strength of two-antenna system with MRC. The BER values for SIMO system never exceeds the BERs of two single antenna detectors for the same packets. Moreover, since the SNR values of the two received signals are summed for MRC, the SIMO system does not observe a deep fade at any instance. As a result, using MRC together with spatial diversity improved the performance of our testbed definitely.

²³Instantaneous BER stands for the number of bit errors in a given packet at any instant.

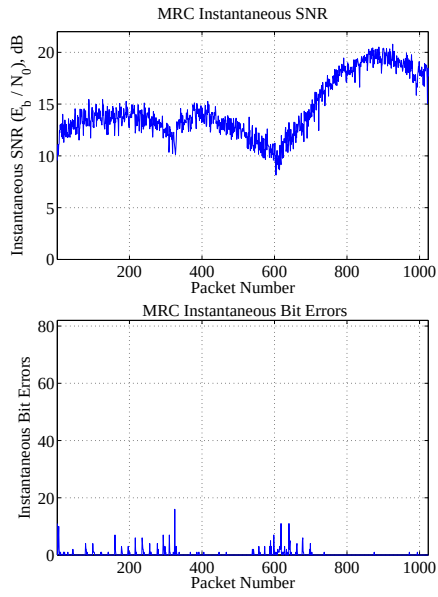


Figure 2.26: A deep fade (hence large number of bit errors) probability for two-antenna system is very low.

CHAPTER 3

PDSCCC ENCODER/DECODER

Channel coding can be defined as adding some *redundant bits* to a sequence of bits that will be conveyed to a receiver, so that the discrepancies caused by the noisy transmission medium can be mitigated. Also known as forward error correction, channel coding aims the receiver side to correct the erroneous bits as much as possible. Related with the nature of the transmission medium used, the coding technique to be used may be highly application specific. For this reason, various types of coding techniques have been offered for channels with bursts of errors, or with continuous thermal noise, or with varying fading effects [11]. In the algebraic coding theoretical sense, we can divide the codes into two families:

- **Block Codes:** Some examples include repetition codes, cyclic codes, Reed Solomon codes, and BCH codes. They operate on *fixed-size* blocks of bits (or symbols). Since not directly utilized in the scope of this thesis research, block codes will not be detailed here.
- **Convolutional Codes:** Unlike block codes, they work on arbitrary size bit (or symbol) sequences. They have encoders and decoders that are relatively easier to implement when compared to block codes. More information about their encoder and decoder structures is given in Sections 3.1 and 3.4.

One of the crucial steps in completion of our wireless testbed was including appropriate encoding/decoding structures enabling efficient forward error

correction. Our main considerations on selection of appropriate coding technique included adequate hardware size to be implemented on our FPGA, considerably small decoding time, easily scalable structure, and, certainly, good performance within the SNR region of interest. Naturally, each one of these listed requirements are in contradiction with the remaining ones and a careful trade-off between them had to be made. After a careful investigation in the literature we decided to implement a structure (PDSCCC) suggested by one of the recent PhD. dissertations [10]. The reasons for selecting the PDSCCC codes is clarified in the following sections together with essential definitions; however, in the broadest sense, a *parallel decodable serially concatenated convolutional code* (PDSCCC) encoder is a hardware or software which encodes its input bits making use of two (or more) *groups* of convolutional encoders, where each group in itself operates in *parallel*. As expected, a PDSCCC decoder carries out the reverse operation (decoding of the received signals) within groups of parallel decoders in order to obtain the uncoded bits back.

3.1 Convolutional Encoding

Convolutional encoding involves generating the output bits for an arbitrary length of sequence of input bits according to a defined state transition mechanism. Convolutional encoders are of very simple structure and can be seen as finite state machines which produce n output bits for given k input bits according to their states and input bits at that instant. Here, k/n is called the code rate of the convolutional code. There are two main building blocks of a convolutional encoder: shift registers and binary addition operators (XOR operators). The number of registers in each *shift register block* for a convolutional code is usually shown by m_i and the constraint length for a convolutional encoder is defined as the $\max(m_i + 1)$ ¹ [11]. Another param-

¹Note that there is one other definition which multiplies $\max(m_i + 1)$ by the number of output bits, n .

ter for convolutional codes is the *minimum free distance*, d_{free} , which is the minimum Hamming distance between any two output sequences. Figure 3.1 demonstrates a convolutional encoder with single shift register block consisting of two registers (shown by D, the delay elements), $m_1 = 2$, one input information bit, u , and two corresponding coded output bits, c_1 and c_2 . This encoder has 4 states, its constraint length is equal to three and its minimum free distance is 5 [12]. The state diagram corresponding to the same encoder is given in Figure 3.2. In addition to the state diagram representation,

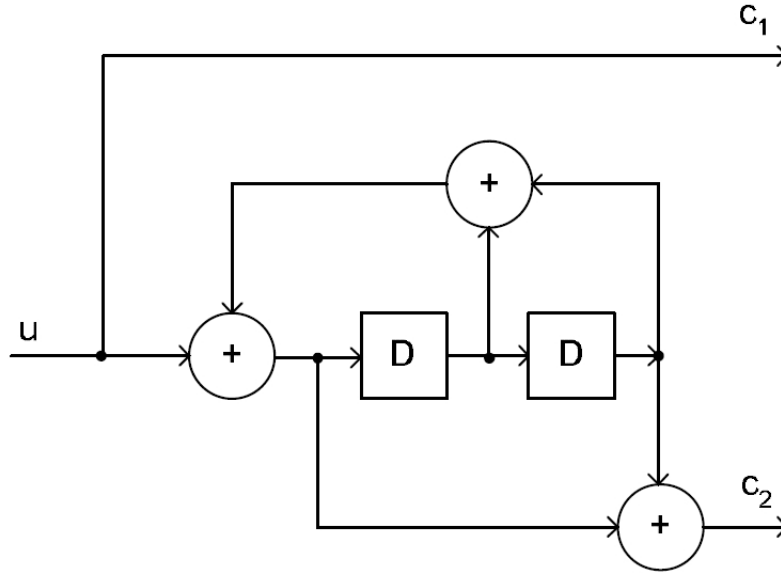


Figure 3.1: A Rate 1/2 Convolutional Encoder.

there is one more representation method for convolutional codes which includes time information in addition to the information present in Figure 3.2.

This representation is known as the trellis diagram representation² and the trellis diagram representation for the rate 1/2 code in Figure 3.1 is given in Figure 3.3 with the assumption that initial state and the final state for the encoder is S0 (shown in Figure 3.2). It is quite common to make the convolutional encoders start at state S0, and if we also force the last state to be S0 (as in Figure 3.3) with appropriate extra input bits, this approach is called *trellis termination(ending)*. We have used trellis termination for all encoding processes defined in this thesis. The following VHDL code segments show how easily re-configurable state and output tables can be constructed for designing the convolutional encoder in Figure 3.1.

```

...(Firstly define basic properties of the convolutional encoder:
number of states, input bits, and output bits.)
constant states : integer := 4;
constant input_bits : integer := 1;
constant output_bits : integer := 2;
...(Define a state transition table which is a two dimensional
matrix with one row for each state and one column for each input
combination3. The entries are the next states when a row (cur-
rent state) and a column (input bit combination, here 0 or 1) are
given.)
type state_matrix2D is ARRAY
    (0 TO states-1, 0 TO 2**input_bits-1)
    OF integer range 0 TO states-1;

```

²Trellis diagram representation is especially helpful for decoding algorithms that utilize state transition probabilities.

³In VHDL, (2**input_bits-1) is equivalent to (2^k - 1), k being the number of input bits.

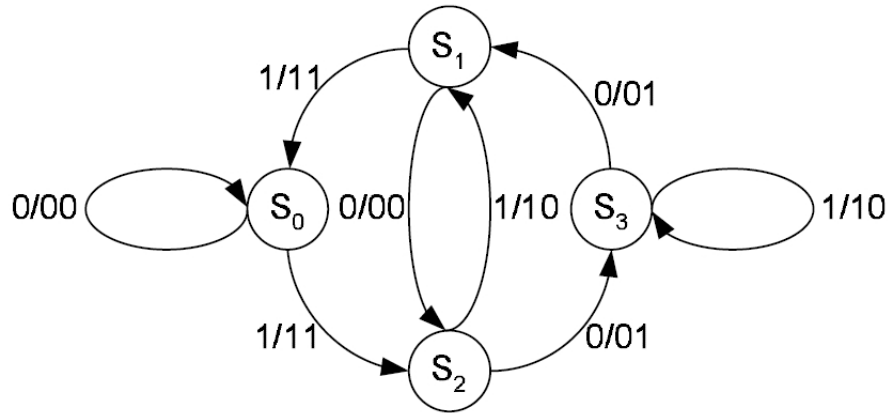


Figure 3.2: The input bit and the output bits for transition from one state to other is shown on transition arrows in I/OO format. Here, S_i corresponds to the octal representation of i for the registers in the encoder, as an example S_1 stands for the register values 0 and 1 from left to right.

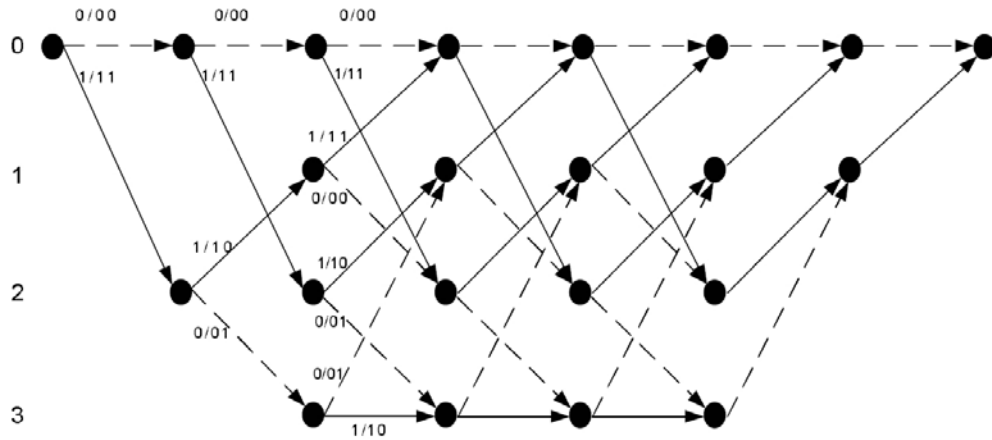


Figure 3.3: The state of the encoder turns back to 0^{th} state (S_0) after encoding this 7-bit input sequence.

... (Similarly define an output table of two dimensions, where the entries are encoded bits for a given current state and a given input combination.)

type output_matrix2D is ARRAY

(0 TO states-1, 0 TO 2*input_bits-1)

OF std_logic_vector(output_bits-1 downto 0);

... (Now, instantiate (create) one instance for each of the tables defined above. These instantiations will be directly used in the convolutional encoder and also the decoder.)

constant state_table : state_matrix2D := ((0,2),

(2,0),

(3,1),

(1,3));

constant out_table : output_matrix2D := ("00","11"),

("00","11"),

("01","10"),

("01","10"));

... (Remaining part of the code follows.)

With these definitions very flexible encoder and decoder structures can be implemented. Once a different encoder with different state transitions and corresponding outputs is desired to be defined, the only change will be made in these *constant definitions*. Especially for decoder structure, which is highly complicated as given in Section 3.4, this improves the readability and the reconfigurability of the design.

Convolutional encoders are classified with respect to the ways of generation of output bits. A convolutional encoder is said to be recursive whenever an output bit effects the following states (hence output bits) through a feed-

back mechanism. Moreover, if the input bits of a convolutional decoder are directly given as a part of the output bits, such encoders are called systematic. In the light of these definitions, the convolutional encoder in Figure 3.1 is both recursive and systematic. The code defined by this encoder is a *maximal* minimum free distance code for the codes of constraint length three [11], and will be the basis of encoding and decoding procedures in the subsequent sections. Hereafter, we will refer to this code (1,5/7) code in relation to its generator polynomial written in octal form.

3.2 Serial Concatenation of Convolutional Codes

In the search of codes that can achieve error-rate performance close to the limits defined by Shannon with appropriate decoding complexity, many theorists tried to combine the best properties of different codes in a single code. In one of the initial trials [16], concatenation of a Reed Solomon code with a simple convolutional code gave rise to various descendants with some other forms of concatenation. A form of serial concatenation of two convolutional codes with an *interleaver* structure in between is suggested in [15]. In 1993, a new class of codes with parallel concatenation of convolutional codes connected by a specially designed interleaver was introduced [17]. These parallel concatenated convolutional codes (PCCCs) are also called *turbo codes*. Subsequent research carried on the *serially concatenated convolutional codes* (SCCCs) provided some similarities of these codes with the PCCCs. In addition to the analogy between these two types, under some special conditions SCCCs are proved to have better bit error rate (BER) performance than the same rate PCCCs. In [13], SCCCs outperform PCCC counterparts under many scenarios for frequency-nonselective Rayleigh fading channels. Also in [14], it was found that SCCCs may have smaller changes in their slope of bit error curves than the PCCCs, which makes PCCCs reach their *error floor*

before the SCCCs. This property of SCCCs may be utilized where extremely low BER values are needed.

Essentially, an SCCC is composed of two convolutional codes. The encoder encoding the information bits initially is called the outer encoder. The second one that encodes the output of the outer encoder is named as the inner encoder. Inner encoders usually take their input in an interleaved fashion instead of directly reading the outputs of the outer encoders. The structure of a SCCC is presented in Figure 3.4. Effectively, the inner and outer en-

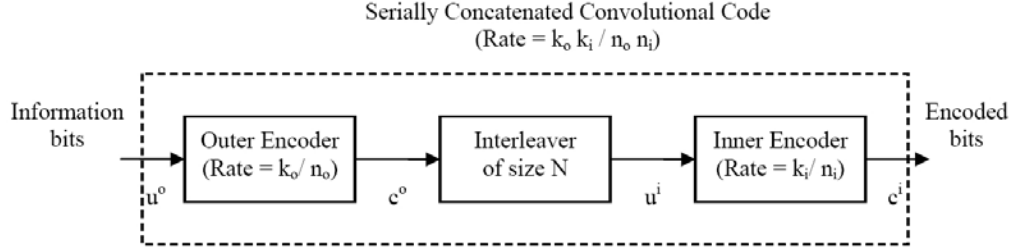


Figure 3.4: Structure of a serially concatenated convolutional code.

coders may have different coding parameters, and the overall rate for SCCC is the multiplication of their rates. The interleaver size, N , is determined by the number of information bits (u^o) and the rate of the outer encoder (k_o/n_o) ($N = \text{length}(u^o) \frac{k_o}{n_o}$). The reasons and details for interleaver implementation will be given in Section 3.2.1.

3.2.1 Interleaving

Interleaving of a given data sequence requires moving each data element into another place in the sequence such that there is a one-to-one mapping between the initial and the final places. As described in Section 2.6.1, wireless channels are usually prone to fading problems, which may result in consecu-

tive bit errors in the reception. Appropriately designed interleavers distribute the *elements in error* in great distances from each other so that the errors can be seen as independent when observed from the output of the interleaver. Moreover, they are capable of distributing the errors occurred in the first stage of a decoding process onto many data elements that will be processed by the next stages. Furthermore, interleaving can also be utilized to reduce the number of *low weight* codewords and increase the minimum free distance [10].

There are various types of interleavers that are used under different conditions. Block interleavers, convolutional interleavers, random interleavers, code-matched interleavers are some examples to be given. Our main interest is on the S-random interleavers for the design of interleaver that will be utilized in both the encoder and the decoder structures. An S-random interleaver maps its *input place* order to an *output place* order according to two rules:

- Firstly, all the mappings are determined randomly, with equal chance of selection for each output place order.
- The randomly selected order is accepted only if it is in a distance *greater* than S for all of the *S previously selected orders*. Otherwise, it is not accepted and a *new random order* is generated, until this condition is satisfied.

The parameter S is a predetermined integer and it usually satisfies $S \leq \sqrt{K/2}$, K being the interleaver size [18]. S-random interleavers have good spreading properties when compared to other types and provide good BER performance when used with convolutional codes. Its implementation details in our testbed are listed in Section 3.3.1 following the specific parallelized encoder structure.

3.3 Parallel Encoder Structure

The reason behind parallelization is to decrease the delay incurred by iterative decoding procedure of the serially (or parallel) concatenated decoders. Following the rules defined in [10], we aimed to implement a special serially concatenated convolutional code (SCCC) that can be encoded and especially decoded with reduced latency. The encoder for such a code is known as parallel decodable serially concatenated convolutional code (PDSCCC) encoder. Both the outer and the inner encoders of a PDSCCC encoder include more than one convolutional encoders that operate on parallelized information bits simultaneously. Figure 3.5 demonstrates a PDSCCC encoder in its most general form. The information bits to be encoded are first given to the outer

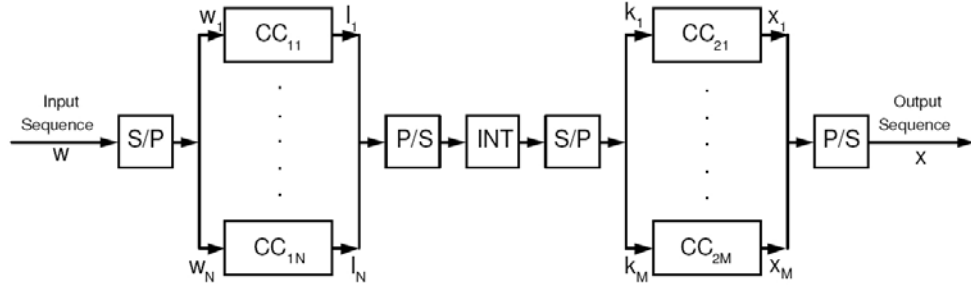


Figure 3.5: Parallel convolutional codes (CCs) operate at the same time for the outer and the inner encoders.

encoder after being converted into N parallel substreams. N , here, is the number of convolutional encoders that make up the outer encoder. These N encoders may correspond to different convolutional codes without any restrictions. $CC_{1,1}$ denotes the first encoder of the outer parallel encoder, while $CC_{1,N}$ shows the last encoder. The output of the outer encoder stems from N distinct substreams, hence it requires a parallel-to-serial (P/S) conversion

operation before the interleaving operation. Following the interleaver, whose implementation rules will be given in Section 3.3.1, the data bits have to be converted into M parallel substreams (k_1 to k_M) which will be encoded by M convolutional encoders of the inner parallel encoder. The structure for the parallel inner encoder is analogous to the outer one.

In the testbed, the numbers N and M are both chosen as 4 for initial experiments. The length of the information bit sequence (length of W in Figure 3.5) is taken as 128 in order to keep the final encoded bit length (length of X) within the limits of the testbed packet length. All of the convolutional encoders are selected to encode their input bits using the (1,5/7) code defined in Section 3.1. According to these assumptions, the input sequence for $CC_{1,i}$ is 32 and with two trellis termination bits, each $CC_{1,i}$ yields $2 \times (32 + 2) = 68$ encoded bits. Therefore, the interleaver is required to have length $4 \times 68 = 272$, which is a small number when we compare the performance of serially concatenated encoders with different interleaver sizes [10]. This issue is discussed in Chapter 5. Another point in relation to interleaving is that this operation should not take up extra time and extra storage area for FPGA implementation. In other words, the encoders within the inner parallel encoder should read their input in an interleaved fashion from the output substreams of the outer parallel encoder directly. As a result, each inner level encoder ($CC_{2,1}$ to $CC_{2,4}$) should keep one table describing the substream number and the order of the bit in that substream for each bit it will encode. Keeping such tables for the inner level encoders, we can combine the tasks P/S, interleaving, and S/P in one process. This idea comes with its cost in the implementation as expected. First of all, we have to keep 4 68-entry tables that store the addresses revealing how the inner level encoders will take their inputs. Moreover, we have implemented the outer and the inner decoder using the same hardware resources since they are identical in nature (both have 4 identical (1,5/7) convolutional encoders) and they operate in a well-defined order (one is non-operational when the other one is encoding data). This reuse of encoders diminished the logic area require-

ment. However, when we imposed an interleaved reading task for the inner encoder, the similarities between the two structures decreased leading to extra control logic for handling both interleaved and non-interleaved reading operations on a single architecture. The interleaved reading process involves one extra clock cycle for firstly grasping the address of the data to be read. This extra delay should also be treated with great care in the design of the PDSCCC decoder which is detailed in Section 3.5.2.

Each inner level encoder output consists of $2 \times (68 + 2) = 140$ encoded bits, which add up to 560 bits for $M = 4$ encoders. Finally, these 560 bits are stored in a packet which is initiated by the 13-bit Barker sequence for synchronization and channel equalization purposes.

3.3.1 Memory Collision-Free Interleavers

The interleaver structure introduced in Section 3.2.1 is further detailed in this chapter. The basic structure for our implementation of PDSCCC interleaver depends on the S-random interleavers. However, a subclass of this type of interleavers is to be defined for efficient utilization in the PDSCCC encoder and the PDSCCC decoder. From the viewpoint of design rules for S-random interleavers, it is very likely that two or more inner level encoders (trying to read the encoded outputs of a *single* outer level encoder at the same time) will lead to memory collisions. There is no rule for S-random interleaver generation that will restrict the usage of the same output RAM of an outer convolutional encoder by more than one inner convolutional encoders. Figure 3.6 shows how an S-random interleaver causes memory collision in the first encoding cycle of the inner encoder. The address tables for the inner level encoders are shown in the middle. For example, the first inner level encoder ($CC_{2,1}$) will read the 1st output bit of the first outer level encoder ($CC_{1,1}$) initially, then it will read the 1st output bit of the second outer level encoder (this address is 5), then the 1st output bit of the third outer level encoder (its address is 9) and finally the 1st output bit of the fourth outer

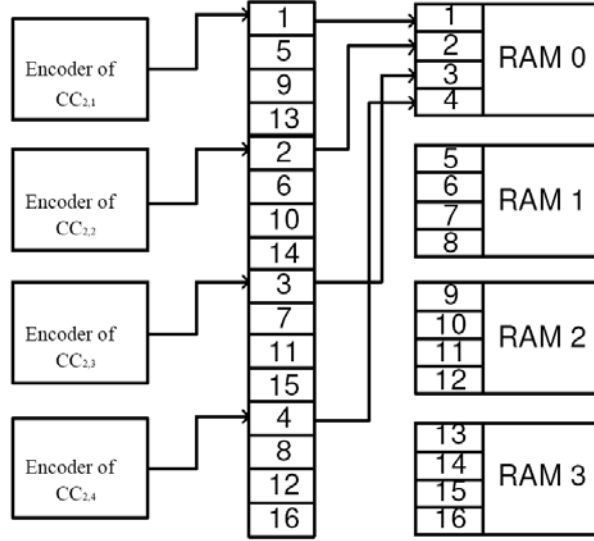


Figure 3.6: All of the 4 inner level decoders try to read from RAM0 in the first encoding interval.

level encoder. Similarly, the second inner level encoder will read the 2^{nd} output bit of the first outer level encoder initially. Hence, both the first and the second inner level encoders will try to access the same RAM, namely RAM0, which stores the encoded outputs of the first outer level encoder. In addition to these, the third and the fourth inner level encoders will also try to read from RAM0. Although the Block RAM resources on ML310 FPGA board are capable of being read from two distinct addresses at the same time, four accesses at the same time can not be handled.

Therefore, it is desired to define a new subclass of S-random interleavers, which are named as *row-column S-random* (RCS-random) interleavers in [10], so that memory collisions during interleaved reading can be avoided. An RCS-random interleaver is made up of many smaller S-random interleavers working on different parts of the whole data to be interleaved. Firstly, the data in each RAM (that is each output sequence of outer level encoders) are

S-random interleaved, which corresponds to interleaving the rows of the whole data. In our case, this means building 4 distinct S-random interleavers, all of which are of size 68. This row interleaving is followed by interleaving within the same addresses of RAMs (column interleaving). In column interleaving, the data bits in the first addresses of ($N = 4$) RAMs are interleaved by an S-random interleaver⁴, the ones in the second addresses by another S-random interleaver, and so forth for all the 68 addresses. It is easy to prove that such an interleaving procedure will not cause memory collisions by just analyzing the resulting interleavers after row interleaving and after column interleaving separately:

- If the given structure is memory collision-free, just applying row interleaving to each row separately will not cause memory collisions to appear, since simply arranging rows (RAM0, RAM1, ...) within themselves results in all the inner level encoders reading from distinct RAMs for all encoding instances. ($CC_{2,1}$ from the first RAM, $CC_{2,2}$ from the second RAM, ...).
- If the given structure is memory collision-free (it is true for the output of row interleaving operation), just applying column interleaving to each address of RAMs, will not alter the one-to-one mapping property between the inner level encoders and the RAMs, but only change the one-to-one mapping to another one-to-one mapping.

These steps are exemplified in Figure 3.7, where an RSC-random interleaver is obtained at the bottom from the ordered table in the top-left corner. The operation of this RCS-random interleaver at the first interleaving instant is shown in Figure 3.8. At the first encoding interval each $CC_{2,i}$ reaches a distinct RAM and this is true for all remaining encoding intervals as well. In order to implement an RCS-random interleaver of size $4 \times 68 = 272$, we initially prepared 4 S-random interleavers of size 68 in MATLAB. In

⁴In fact, with $N = 4$, S-random interleaver reduces to purely random interleaver.

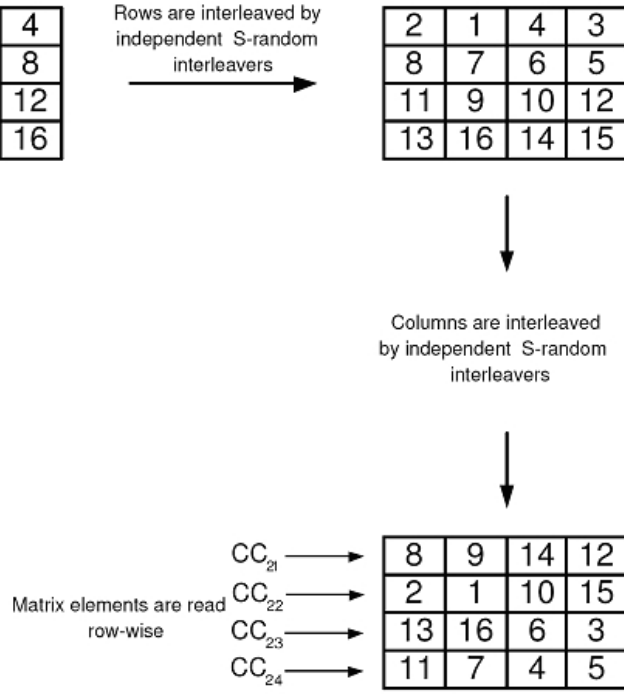


Figure 3.7: Firstly the rows are interleaved, then the columns [10].

this step, S was taken as 5. In the second step we built up 68 S-random interleavers of size 4 in a similar fashion, with S is equal to 1. Then, a conversion code for translating these interleaved address information from integers to binary representation was written. This code gave the addresses of the interleaved bits within a RAM together with the selected RAM number for that interleaving. The convention for addressing is : “The first two bits in the interleaving table give the RAM number from which data bit is to be read. Remaining 7 bits are used for addressing within the RAM selected by the first two bits”. The storage of these addresses is realized by using 4 Block RAMs as read-only memories (ROM) with 68 entries of 9-bit data width. As the final step of PDSCCC encoder implementation, the encoded bits of length 560 are compared with the output of PDSCCC encoder written

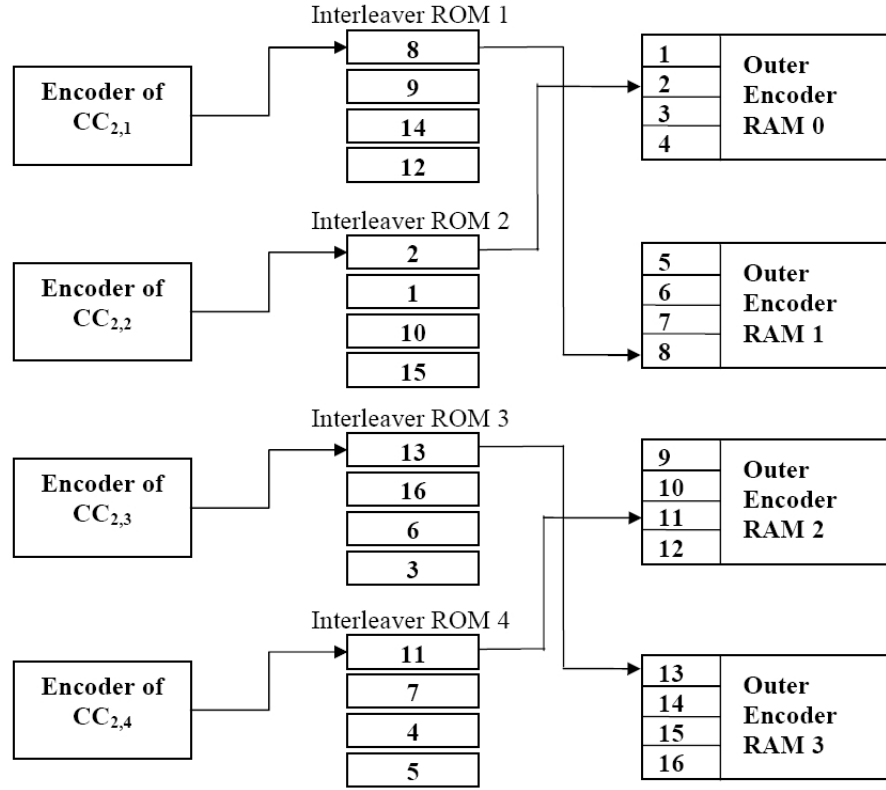


Figure 3.8: At each time step, each encoder reads from a distinct RAM.

in MATLAB. Table 3.1 presents some figures for hardware complexity of the implemented PDSCCC encoder. It is possible to neglect the area used by the encoder, however the Block RAM usage may be important in case of a larger PDSCCC encoder design with more convolutional encoders in the inner and/or outer encoders.

3.4 Marginal a Posteriori Decoding

Two types of decoding algorithms are very popular for convolutional codes. The first type of algorithms find a codeword, $\hat{\vec{v}}$, for a transmitted codeword,

Table 3.1: PDSCCC Encoder Synthesis Results

Unit Name	Usage Count and Percentage
Number of Slices	282(2%)
Number of Slice Flip Flops	138(0.5%)
Number of 4 input LUTs	515(1.5%)
Number of BRAMs	5(3.5%)
Maximum Operating Frequency	202.192 MHz

$\vec{v}$, and the received sequence, $\vec{r}$, such that the likelihood function is maximized. Hence, they are called as the maximum likelihood (ML) decoding algorithms. Viterbi algorithm is an example of ML algorithms, which can find the most likely sequence (or path) with only linear decoding time increase as the sequence length increases. In this sense, Viterbi algorithm is word error rate (WER) minimizing ($P(\hat{\vec{v}} \neq \vec{v} \mid \vec{r})$ is minimized). The second type of popular algorithms aim to minimize the bit error rate (BER) through maximization of the marginal a posteriori (MAP) probabilities for the transmitted information bit u_l , $P(\hat{u}_l = u_l \mid \vec{r})$, where $\hat{u}_l$ is the decoded information bit. They are named as MAP algorithms and the best known example to MAP algorithms is the BCJR algorithm [4]. When the probabilities for having $u_l = 1$ and $u_l = -1$ are equal, two algorithms are equivalent and Viterbi algorithm is preferable due to its lower complexity. However, when the bit probabilities are different BCJR algorithm performs better than Viterbi algorithm. Good examples for having unequal bit probabilities can be iterative decoding algorithms. During the iteration steps the likelihood assigned to each bit is updated and next steps use unequal bit likelihoods in general. Since PDSCCC decoder use iteration for achieving lower BER values, it is inevitable for PDSCCC decoder structure to utilize BCJR algorithm.

Without deriving the equalities, simply the variables used in BCJR algorithm will be defined here. The detailed derivations are presented in [12].

We will start with the definition for the a posteriori log-likelihoods (LL) of the information bits. LL values are defined as,

$$LL(u_l) \equiv \left[\frac{P(u_l = 1 \mid \vec{r})}{P(u_l = -1 \mid \vec{r})} \right]. \quad (3.1)$$

Then, a bit is decided as 1 whenever LL value corresponding to it is positive after the last iteration step. Otherwise, it is decided as a -1 . The definition for the α metric, which is obtained by a *forward* iteration on the received sequence, is given as,

$$\alpha_l(s') \equiv p(s', \vec{r}_{t < l}). \quad (3.2)$$

Therefore, α metric can be defined as the probability of being at state s' and having a received sequence $\vec{r}_{t < l}$ upto time l . Similarly, γ (branch) metric is defined for the probability of having a state transition from s' to state s at time l with received sequence value of r_l given s' . Hence, γ metric is given as,

$$\gamma_l(s', s) \equiv p(s, \vec{r}_l \mid s'). \quad (3.3)$$

The backward metric, which is calculated over the received sequence in backwards direction, is known as β metric and defined as,

$$\beta_{l+1}(s) \equiv p(\vec{r}_{t > l} \mid s). \quad (3.4)$$

After some steps on α and β metric it is easily found that both metrics' calculations can be written in recursive equations as,

$$\alpha_{l+1}(s) = \sum_{s' \in \sigma_l} \gamma_l(s', s) \alpha_l(s'), \quad (3.5)$$

$$\beta_l(s') = \sum_{s \in \sigma_{l+1}} \gamma_l(s', s) \beta_{l+1}(s). \quad (3.6)$$

In Eqns. 3.2 and 3.4, the sets σ_l and σ_{l+1} denote the sets all possible states from (to) which a transition is possible at time l ($l + 1$). For our PDSCCC decoder, since all of the (1,5/7) convolutional codes have 4 states, for obtaining a new α or β metric we have to carry out 4 multiplications and a

summation of these results. The basic steps for these recursive calculations are related to the S0 state assumption for the initial state of all encoders and the trellis termination that identifies the final state as S0. The initial assignments for α and β metrics are given as,

$$\alpha_0(s) = \begin{cases} 1 & s = 0 \\ 0 & s \neq 0 \end{cases}, \quad (3.7)$$

$$\beta_K(s) = \begin{cases} 1 & s = 0 \\ 0 & s \neq 0 \end{cases}. \quad (3.8)$$

In Eqn. 3.8, K denotes the number of information bits to be decoded by the BCJR decoder. If the γ metric given in Eqn 3.3 is analyzed with the assumption that noise signal in the received signal is additive white Gaussian, it can be simplified in terms of channel reliability factor (L_c) and a priori bit probability ($L_a(u_l)$) as,

$$\gamma_l(s', s) = e^{u_l L_a(u_l)/2} e^{(L_c/2)(\vec{r}_l \cdot \vec{v}_l)}. \quad (3.9)$$

In Eqn. 3.9, we have two factors determining the branch metric. One is the a priori bit probability, which is found by the previous iteration step⁵. The other one is the inner product of received vector at time l , $\vec{r}_l$, and output vector for the transition from state s' to state s , $\vec{v}_l$. This inner product result is scaled by $L_c/2 = 2E_s/N_0$, which corresponds to multiplying it by a larger number when the symbol SNR is high. Therefore, with BCJR algorithm, observations are trusted more when the SNR is high, otherwise a priori values are utilized more in finding the updated branch metrics (and also updated LL values). This requires estimation of the received SNR to be used in the PDSCC decoder. In Section 2.5.5, a method for estimating the signal power and the noise power is given. Using that method the estimated SNR may be multiplied by $2\vec{r}_l$ for removing the multiplication operation from the PDSCC decoder. However, simply that removal is not enough, there are

⁵If no previous steps exist, $L_a(u_l)$ is initialized as 0 for all $l = 0, 1, \dots, K - 1$.

many multiplication operations in Eqns. 3.5, 3.6, and 3.9. In order to simplify these calculations, log-domain metrics can be defined as,

$$\gamma_l^*(s', s) \equiv \ln \gamma_l(s', s) = u_l L_a(u_l)/2 + (L_c/2)(\vec{r}_l \cdot \vec{v}_l), \quad (3.10)$$

$$\alpha_{l+1}^*(s) \equiv \ln \alpha_{l+1}(s) = \ln \sum_{s' \in \sigma_l} e^{\gamma_l^*(s', s) + \alpha_l^*(s')}, \quad (3.11)$$

$$\beta_l^*(s') \equiv \ln \beta_l(s') = \ln \sum_{s \in \sigma_{l+1}} e^{\gamma_l^*(s', s) + \beta_{l+1}^*(s)}. \quad (3.12)$$

Now the metric calculation for γ is greatly simplified, whereas the calculations of α and β metrics still involve “ln” operation over summation of 4 elements, since convolutional encoders have 4 states in our case. This complication can be solved by using the identity,

$$\max^*(x, y) \equiv \ln(e^x + e^y) = \max(x, y) + \ln(1 + e^{-|x-y|}). \quad (3.13)$$

With $\max^*$ operator instead of implementing a two-dimensional lookup table for approximating ln function of two variables x and y , a single dimension table using the absolute difference of x and y , and a maximization operator is implemented. Moreover, $\max^*$ can be defined for more than two operands as,

$$\max^*(x_1, x_2, \dots, x_p) = \max^*(x_1, \max^*(x_2, x_3, \dots, x_p)) \quad (3.14)$$

$\vdots$

$$= \max^*(x_1, \max^*(x_2, \dots, \max^*(x_{p-1}, x_p) \dots)) \quad (3.15)$$

Using Eqn. 3.14, we can rewrite forward and backward metric calculations as,

$$\alpha_{l+1}^*(s) = \max_{s' \in \{S0, S1, S2, S3\}}^* [\gamma_l^*(s', s) + \alpha_l^*(s')], \quad (3.16)$$

$$\alpha_0^*(s) = \begin{cases} 0 & s = S0 \\ -\infty & s \neq S0 \end{cases}, \quad (3.17)$$

$$\beta_l^*(s') = \max_{s \in \{S0, S1, S2, S3\}}^* [\gamma_l^*(s', s) + \beta_{l+1}^*(s)], \quad (3.18)$$

$$\beta_K^*(s) = \begin{cases} 0 & s = S0 \\ -\infty & s \neq S0 \end{cases}. \quad (3.19)$$

The $\max^*$ operations in Eqns. 3.16 and 3.18 can be implemented by firstly taking two groups of two operands into two $\max^*$ operators with the definition in Eqn. 3.13, then applying the same $\max^*$ operator on the results of the first stage. For this way of metric calculation, three $\max^*$ operators are required for single $\alpha_l^*(s)$ or $\beta_l^*(s)$ value. Since there are 4 states, in total $(4 \times 2 \times 3 = 24)$ $\max^*$ operators need to be implemented. This may be the bottleneck for optimizing the logic area utilized by the PDSCCC decoder. The solution proposed for $\max^*$ implementation is given in Section 3.5.5.

This version of the BCJR algorithm (calculating the metrics in the log-domain) is also known as *log-MAP algorithm*. The final expression for PDSCCC implementation is on the LL values for information bits and given as,

$$LL(u_l) = \ln \left[\sum_{(s',s) \in \delta^+} e^{\beta_{l+1}^*(s) + \gamma_l^*(s',s) + \alpha_l^*(s')} \right] \quad (3.20)$$

$$\begin{aligned} & - \ln \left[\sum_{(s',s) \in \delta^-} e^{\beta_{l+1}^*(s) + \gamma_l^*(s',s) + \alpha_l^*(s')} \right], \\ & = \max_{(s',s) \in \delta^+} [\beta_{l+1}^*(s) + \gamma_l^*(s',s) + \alpha_l^*(s')] \quad (3.21) \\ & - \max_{(s',s) \in \delta^-} [\beta_{l+1}^*(s) + \gamma_l^*(s',s) + \alpha_l^*(s')]. \end{aligned}$$

The δ^+ and δ^- given in Eqns. 3.20 and 3.21 denote the sets of transitions made by an information bit 1 and -1 respectively. Consequently, for calculating a single LL value of an information bit two 4-input $\max^*$ operators, or equally 6 two-input usual $\max^*$ operators are required in a single MAP decoder.

3.5 Fast PDSCCC Decoder

The decoding algorithm to be used as the basis for construction of PDSCCC decoder is given with its essentials in Section 3.4. Now, in this section, more

implementation details about the MAP decoder and interconnection rules for inner and outer decoder structures will be explained.

Firstly, the decoder structure for a serially concatenated code is composed of four main building blocks. It includes an inner decoder (corresponding to the inner encoder in serial concatenation), an outer decoder (corresponds to the outer encoder), an interleaver with the same properties as in the serially concatenated encoder part, and a deinterleaver which supports the feedback between the outer decoder and the inner decoder for iterating over the received sequence many times. These main building blocks together with some additional blocks are shown in Figure 3.9. The decoder of a serially concate-

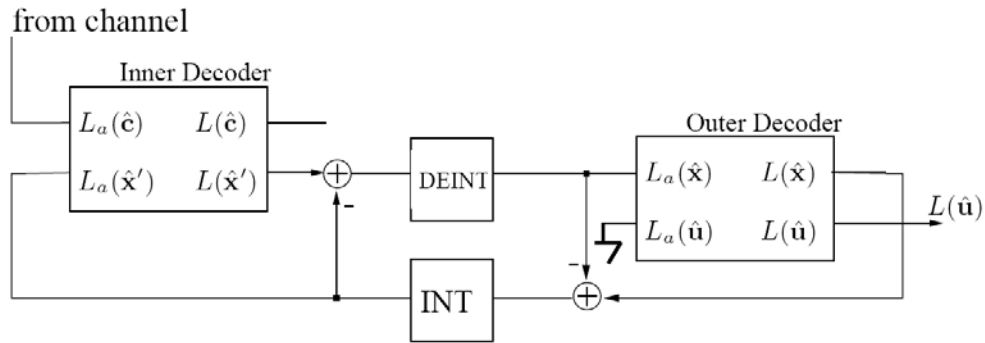


Figure 3.9: Interleaver and the corresponding deinterleaver connect the inner and outer decoders.

nated convolutional code (SCCC) resembles that of a parallel concatenated convolutional code (turbo code). The main distinctions between two types of decoders lie in two points:

- The way outer decoder of a SCCC decoder takes its channel information and the a priori information is different from the way of lower and upper decoders of a turbo decoder. The outer decoder does not directly take its channel information from the channel, but this input is

supplied by the inner decoder as the log-likelihood (LL) values (for x in Figure 3.9). Also the a priori information of the uncoded information bits (u) are always *zero* for outer decoder, since it has no knowledge on these uncoded bits. It also gives its *coded* output estimates ($L(\hat{x})$) to the inner decoder since these are the uncoded bit LL values for the inner decoder.

- Both the inner and the outer decoders subtract only the information taken from the other decoder from their estimated outputs. This is not the case for turbo decoding where both of the decoders subtract the channel information from their estimates, too. Here only one of the decoders (inner decoder) directly observe the channel and this will not result in divergence of the metrics due to accumulation of channel information.

The construction of the parallelized SCCC (PDSCCC) decoder is similar to the construction of PDSCCC encoder from the SCCC encoder structure. An illustrative diagram for a PDSCCC decoder is provided in Figure 3.10. As with the PDSCCC encoder, for PDSCCC decoder the number of sub-

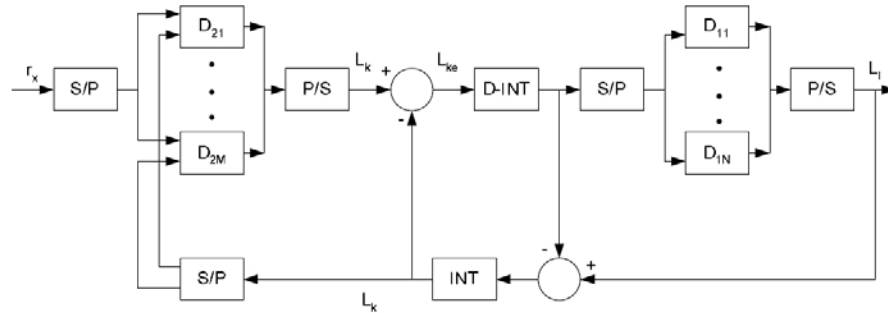


Figure 3.10: Parallelized convolutional decoders speed up the decoding operation for both the inner and the outer decoding [10].

decoders in both the inner and the outer decoders is taken as 4. Therefore, each encoder in the inner encoder will take $(560/4 = 140)$ bits of received sequences. They will provide output estimates for $(140/2 - 2 = 68)$ bits without any need for estimating the trellis ending bits used in the inner encoder of PDSCCC encoder. After being deinterleaved, these (4×68) -bit LL values will be given to the outer encoder. Outer encoder will provide not only the LL values for $(68/2 - 2 = 32)$ uncoded information bits, but also the a priori information for 4×68 bits to the inner decoder back, which will read these data in an interleaved fashion. This iteration is carried out for a pre-defined or adaptively chosen number of times and only then will the hard decisions for the uncoded bits be given using the uncoded information bit LL value output of the outer decoder. The following sections will provide information about critical parts of PDSCCC implementation on FPGA and list a number of improvements for faster operation and reduced hardware size.

3.5.1 Simultaneous Calculation of Alpha and Beta Metric Values

If the branch metric calculations in Eqns. 3.16 and 3.18 are considered, two operations, i.e., calculation of α and β metrics, are independent from each other so that simultaneous evaluation of α metric values in forward direction and β metric values in backward direction is possible. This will bring an advantage in decoding latency only if the LL values are determined during these calculations, because there are currently known algorithms which yield the LL values after making two sweeps over the received vector [19, 20]. The idea for obtaining the LL values with just one sweep depends on calculating the LL values at the first possible instant. As an example, assuming LL values for 70 information bits will be found, consider that the first metric values, $\alpha_0^*(s)$ and $\beta_{70}^*(s)$, are written to corresponding places at time 0 as in Figure 3.11. In the following cycle using the $\alpha_0^*(s)$ and the $\gamma_0^*(s)$ values

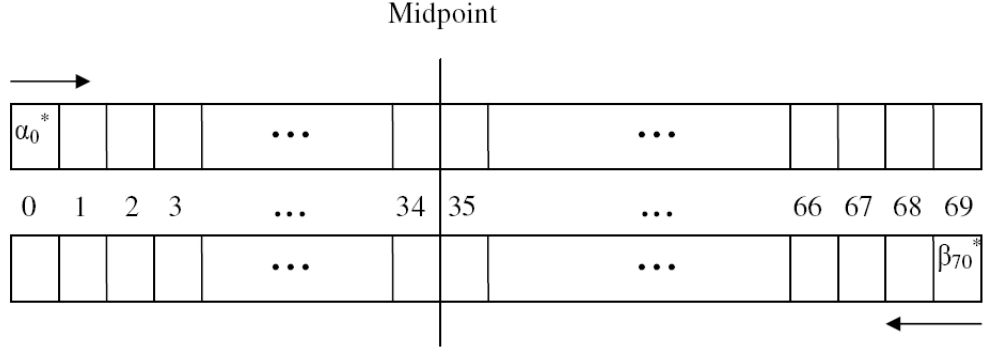


Figure 3.11: α and β metric RAMs are initialized at time 0.

$\alpha_1^*(s)$ values will be obtained for each state s . Similarly, $\beta_{69}^*(s)$ values will be calculated and written to next place, 69th place, in β metric RAMs as in Figure 3.12. In the next cycles, both of the metrics will be calculated in the

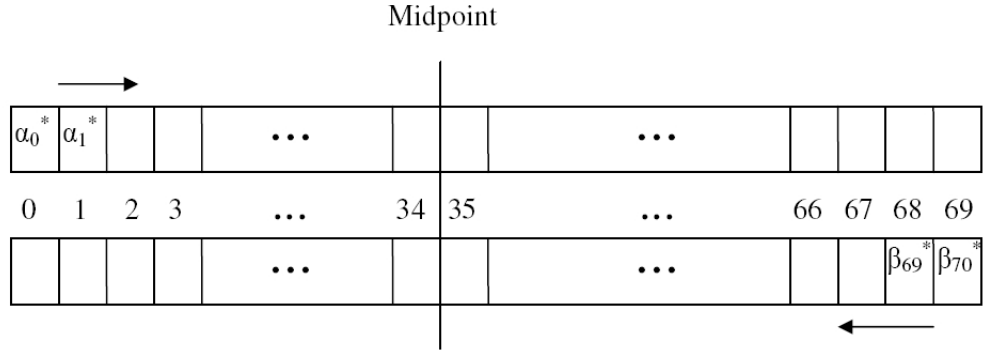


Figure 3.12: Metric calculations continues at time 1 with second values.

directions shown by arrows. Remembering Eqn. 3.21 for LL value calculation, for finding $LL(u_l)$ the required metrics are $\alpha_l^*(s)$, $\gamma_l^*(s)$, and $\beta_{l+1}^*(s)$. Hence, the earliest calculation of an LL value is possible at time 35, after the α and

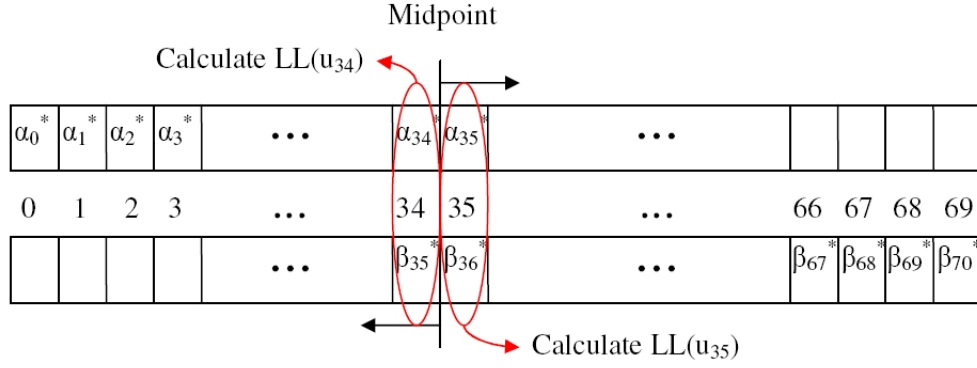


Figure 3.13: At time 35, first LL values are ready to be calculated.

β metric calculations meet at the midpoint. In fact, at this instant two LL values, namely $LL(u_{35})$ and $LL(u_{34})$, are calculated. This first LL calculation instant is given in Figure 3.13. Following this cycle two LL values are found in each cycle and at the end after 70 cycles, with just one sweep over the received vector, all LL values are calculated as in Figure 3.14 Especially, for

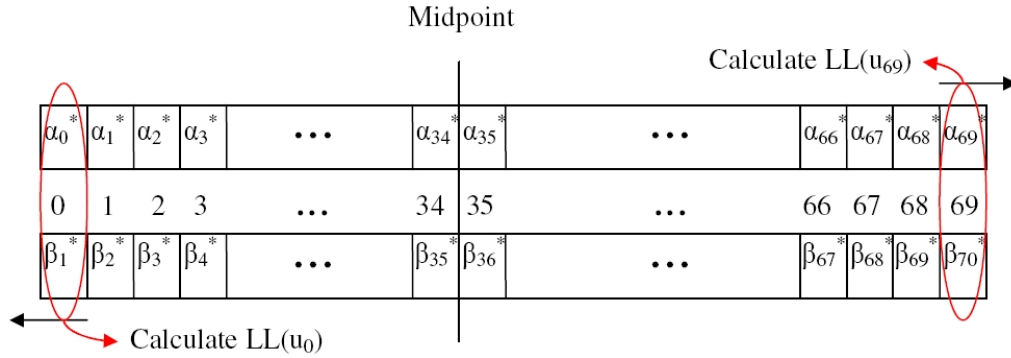


Figure 3.14: Finally, after time 69, all LL values are obtained.

iterative decoding algorithms decoder latency is the bottleneck for increasing

the iteration number. Our implementation improves the decoding latency of each iteration step by halving it.

3.5.2 Interleaving and Deinterleaving Operations

As it is given in Figure 3.10, various S/P, P/S converters, subtractions for obtaining extrinsic information, and interleaving/deinterleaving operations exist in a PDSCC decoder. In Section 3.3, the interleaving operation is defined such that the inner encoder reads from substreams of the outer encoder output so that P/S converter, interleaver, and S/P converter are joined in a single structure with just one cycle delay. The same interleaver is used for reading the LL outputs (extrinsic information) of the outer decoder as well. However, a deinterleaver which combines the jobs of P/S converter, deinterleaver, and S/P converter in Figure 3.10 is still required. Implementing new address (deinterleaving) tables for each output level decoder to read its input from the output substreams of the inner level decoders will be very costly in terms of Block RAM usage. Just as in the case of interleaver tables, 4 separate ROMs will be made out of Block RAMs. In fact, there is no need for deinterleaver tables if we can observe this operation from the point of inner decoder. An inner level convolutional decoder, knowing which address it has read its a priori information from, can write its extrinsic information output (after subtraction) LL value just to the same address. This operation force all inner level decoders to maintain last reading addresses from the outputs of the outer level decoders and writing its LL result to the same address in already a deinterleaved fashion. Therefore, the deinterleaving job is also given to the inner decoder, while the outer decoder reads and writes in a sequential manner. It must be emphasized that since we have simultaneous calculation of α and β metrics as described in Section 3.5.1, two LL values are written to LL value RAMs at each instance once the midpoint of the data is passed. This is easily handled by defining LL RAMs as dual port Block RAMs, however, the γ calculation described in Eqn. 3.10 requires reading of

two consecutive received values (for one information bit two encoded bits are sent). When the simultaneous calculation of forward and backward metrics is also considered, at any instant outer decoder requires to read 4 extrinsic information values (from the output of the inner decoder) at the same time. It may be proposed that having two consecutive extrinsic information in the same word (address) of the inner decoder output RAMs is enough for solution. On the contrary, these consecutive extrinsic information values are written at different times by the inner decoder since they are not consecutively found. The only solution is to keep the extrinsic information values of the information bits and their parity bits in separate RAMs for the outer decoder input. In total, 8 Block RAMs of size 34-word⁶ are desired. This problem does not exist for the outer decoder output since it writes its output LL values (a-priori information for inner encoder) sequentially and inner decoder just reads these values (in an interleaved fashion) with no write operations. The memory requirement at the output of the outer decoder is 4 Block RAMs of size 34-word⁷. The Figure 3.15 demonstrates the placement of these LL value storage RAMs.

3.5.3 Re-usage of Parallel MAP Decoders

Due to serial concatenation, the inner and the outer decoders operate in succession. Inner decoder does not operate on the received data while the outer decoder is processing the received data, and vice versa. This is a great opportunity for reducing the logic area utilized by the PDSCCC decoder. However, this reusable architecture has its own problems which should be handled separately:

- The reading and writing of LL values for the inner and the outer decoders differ. Inner decoder can always read and write with one clock

⁶Each word is a single LL value.

⁷Here, each word is a combination of two LL values.

LL values. Since the input lengths are different the midpoints (after which LL calculation will begin) are also different for two decoders.

Therefore, it is clear that reusing the modules for implementing both the inner and the outer decoders increases complexity of the PDSCCC decoder design, although a great size reduction is obtained. All convolutional decoders must know whether they operate as an inner decoder or an outer decoder constituent block. When combined with the interleaving/deinterleaving and information subtraction procedures, reusing modules results in highly complicated control logic modules. Consequently, the maximum combinational delay of the PDSCCC decoder increases leading to lower operating frequencies. Although our final operating frequency is above the desired levels (see Section 3.7.1 for exact values), some optimization methods for further improving the clock frequency of PDSCCC decoder is given Chapter 5.

3.5.4 Alpha, Beta, Gamma Metric Size Selection and Metric Storage Allocation

The α , β , γ metrics, and LL values are represented by fixed point numbers on FPGA for simplified operations in 2's complement logic. It is easy to see that there is a trade-off between the metric size (number of bits used representing metrics) and the hardware complexity of the PDSCCC decoder, which consists of 4 BCJR decoders in our case. It is shown in [21] that using 8-bit quantization for representing the metric values in log-MAP algorithm yields such a good performance that almost no SNR loss is incurred with respect to the floating point utilization for the metrics. It is also observed in [21] that 4-bit quantization results in an SNR loss of approximately 0.1dB for most simulation cases. Hence, we decided to have a PDSCCC decoder with parametrizable metric size and a parametrization interval of 4-bit to 8-bit quantization. Moreover, the ideal metric size is determined as 6-bit when the hardware size, operation frequency, and SNR loss are considered as the constraints of the system. 8-bit quantization is an upper limit when

the PDSCCC decoder size is constrained to fit into the used ML310 FPGA board. Especially, when the multiple-antenna system with more than two receiver antennas is the destination of future testbed implementation, some optimization will be required for this 8-bit quantized architecture.

Each inner level decoder utilizes single Block RAM for storing both of the α and the β metrics. This optimization is possible only with simultaneous calculation of two metrics. Once the middle point for calculation of the metrics is left behind LL values can be calculated, thus there is no more need for storing new α and β values. Newly found values will be directly used in the next clock cycle for LL value calculation. In this way, the α and the β metric storage RAMs shown in Figure 3.11 can be merged into a single RAM, in which the first half is reserved for storing $(\alpha_0^*, \dots, \alpha_{34}^*)$ and the second half is reserved for $(\beta_{36}^*, \dots, \beta_{70}^*)$. This optimization is novel to the knowledge of the author of this thesis and it reduces the total RAM usage by the number of parallel decoders in the inner decoder, which results in saving 4 Block RAMs (or more for metric sizes larger than 4-bit). Since the outer level decoders are physically the same decoders as the inner level decoders, no more RAM utilization is required for the outer decoder. The $\alpha_l(s = S0)$, $\alpha_l(s = S1)$, $\alpha_l(s = S2)$, and $\alpha_l(s = S3)$ values are stored in the l^{th} address of a Block RAM, whose size is parametrically tuned for housing 4 α (or equally β) values next to each other.

Both the inner and the outer level decoders have to store γ metrics for all possible transitions at any instant. With a 1/2-rate and 4-state code, this corresponds to storing 8 possible transitions for $\gamma_l(s', s)$. Due to this large number and some other practical implementation issues, log-likelihoods for the transitions made as a result of an information bit 1 are stored in a separate RAM than the ones as a result of an information bit -1 . Therefore, single decoder uses two Block RAMs for the storage of γ metrics. Similar to α - β storage 4 γ values for same information bit are kept in a single word of a Block RAM, occupying the same address.

3.5.5 Max^{*} Approximation Method

The max^{*} expression given in Eqn. 3.13 involves two terms: the maximization term ($\max(x, y)$) and the nonlinear correction function $f_c(|x - y|) = \ln(1 + e^{-|x-y|})$. The maximum finding operator is fairly easy for FPGA implementation, however $f_c(|x - y|)$ is the problematic part in max^{*} operator implementation. In [19], four methods of approximating the correction function are listed and a BER performance comparison is made. These methods, three of which are shown in Figure 3.16, are:

- **Max-log-MAP approximation:** This is the simplest method which totally neglects the effects of the correction function. In other words, ($f_c(|x - y|) = 0$) is accepted for all (x, y) pairs. Therefore, it is not shown in Figure 3.16. It leads to worst BER performance as expected.
- **Constant log-MAP approximation:** This algorithm takes the correction function as a constant value for values of $|x - y|$ smaller than a given value and as zero elsewhere.
- **Linear log-MAP approximation:** The correction function is approximated by a piece-wise linear function in this approach. The approximation function is given as,

$$f_c(|x - y|) \approx \begin{cases} 0 & \text{if } |x - y| > T \\ a(|x - y| - T) & \text{if } |x - y| \leq T \end{cases} . \quad (3.22)$$

Minimization of mean-square error for this approximation yields $a = -0.24904$ and $T = 2.5068$. This is one of the most common methods in implementation of max^{*} operator.

- **Lookup table approximation:** The performance of this method is heavily dependent on the depth and the width of the lookup table used. The Figure 3.16 demonstrates an approximation over 30 samples evenly spaced in the significant part of the exact correction function. It closely approximates the desired shape, however implementation cost

of a such a large lookup table is high and it is not preferable in case many instances of $\max^*$ operator exist.

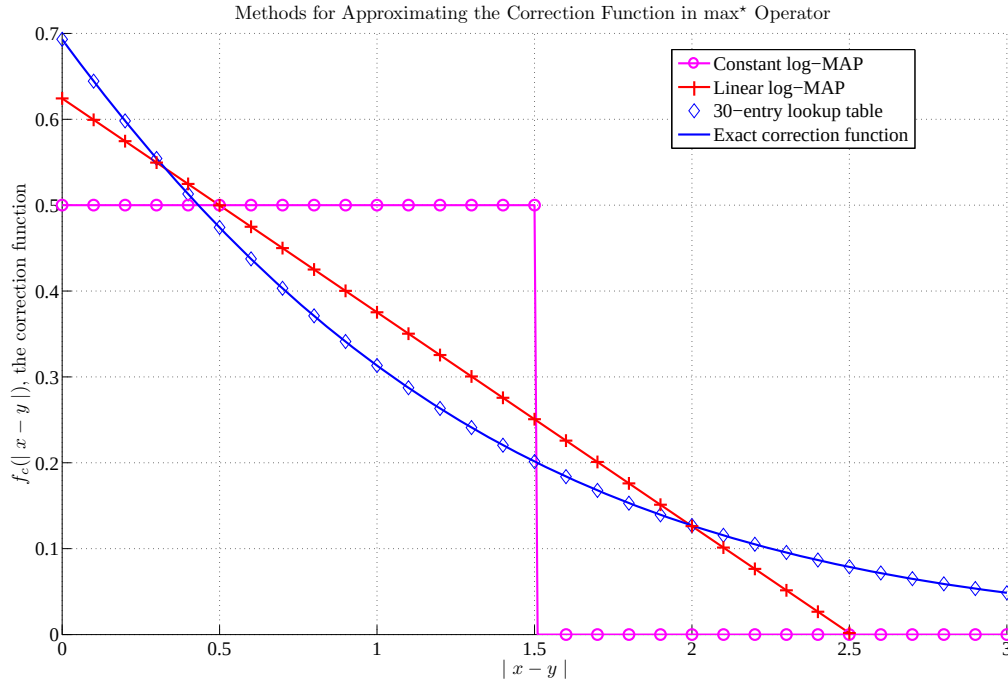


Figure 3.16: Different methods exist for approximating the correction function.

Using 6-bit metrics, linear log-MAP algorithm requires $\max^*$ units utilizing only 31 slices, whereas the lookup table approach would require 60 flip-flops (that is 30 slices) only for implementing the ROM of 10 samples from the correction function, where each sample would have 6 bits. As a result, our design choice is linear approximation method for the log-MAP algorithm.

3.6 PDSCCC Decoder Simulations

This section will present the methods and the results of extensive simulations made on the hardware implementation of the PDSCCC decoder. Two separate simulation fields were used in conjunction for stricter debugging purposes. Initially, the code written VHDL was compiled and tested for basic timing constraints in ModelSim. Following this basic ModelSim simulation and timing error corrections, a MATLAB code was written in order to generate a realistic received vector under various conditions like different SNR levels and different quantization resolutions. This received vector data was converted into VHDL representation and written on the ROM blocks placed in the *testbench* module of the PDSCCC decoder. In this way, operation of different versions of PDSCCC decoder on the same received data and quick performance comparisons were made possible. Finally, a MATLAB code for simulating all blocks of the proposed PDSCCC structure in MATLAB environment was written. This code and the ModelSim simulation was operated concurrently for detecting possible errors in hardware implementation of the PDSCCC decoder.

3.6.1 MATLAB Implementation

In VHDL, whole code of PDSCCC decoder depends on 2's complement arithmetic operations on fixed point representations. On the other hand, floating point is more frequently used representation within MATLAB environment. In order to simulate the operation of such a VHDL code in MATLAB, the Fixed Point Toolbox should be exploited. In our VHDL implementation, 6-bit representation is the final selection for size of the metric and the LL values. Similarly, the MATLAB implementation includes all arithmetic operations defined on 6-bit signed variables. Exactly the same algorithms that described in previous sections are also written in MATLAB in addition to generation of the received sequence with adequate SNR levels and analog-to-digital conversion operations. The MATLAB implementation proved to be

useful especially when we had to check hundreds of signals while probing for possible erroneous parts.

3.6.2 VHDL Implementation

During the simulations executed in ModelSim, usually testbench structures are utilized for supplying input data to the simulated module at the correct instances. Then outputs and intermediate signal assignments are observed for assuring proper operation. The testbench module of the PDSCCC decoder consists of an initialization PDSCCC decoder and 4 dual-port Block RAMs (in fact ROMs) for supplying the MATLAB generated noisy received sequences for the inner level decoders. During the simulations using this testbench structure many critical corrections and modifications are performed. Some of these critical findings are given next.

- **Clipping addition and subtraction operations:** The 6-bit representation for metrics and LL values is extremely limiting for all arithmetical operations. Especially, when we take the iterative additions while obtaining α and β metrics into account, it is very likely that some *overflows* will happen during these operations. Naturally, this enforces the designer to implement somewhat specialized addition and subtraction modules. Such modules should sense the possible overflows beforehand and clip the results to the closest extreme values of the representation. The following code segment shows how such a clipping addition can be realized:

```
... (Firstly, some constants to be used are defined.)
constant m_size : integer := 6;
constant minus_inf : signed :=
    conv_signed(-(2**(m_size - 1)), m_size);
constant plus_inf : signed :=
```

```

conv_signed(2**( $m\_size - 1$ ) - 1,  $m\_size$ );
... (Then, define the input and outputs of the module. Their
lengths are equal to metric size ( $m\_size$ ).)
entity clipping_sum is
    Port (inp1 : in SIGNED ( $m\_size - 1$  downto 0);
          inp2 : in SIGNED ( $m\_size - 1$  downto 0);
          clipped_out : out SIGNED ( $m\_size - 1$  downto 0));
end clipping_sum;
... (Define the signals used in the module.)
architecture Behavioral of ab_clipping_sum is
    signal mid_sum : SIGNED ( $m\_size$  downto 0);
    signal ovf_bit : std_logic;
    ... (Sum the operands in a larger register ( $mid\_sum$ ) and
check for the overflow bit.)
begin
    mid_sum <= (inp1( $m\_size - 1$ ) & inp1) +
               (inp2( $m\_size - 1$ ) & inp2);
    ovf_bit <= mid_sum( $m\_size$ ) xor mid_sum( $m\_size - 1$ );
    clipped_out <= (mid_sum( $m\_size - 1$  downto 0))
                   when (ovf_bit = '0')
                   else (minus_inf)
                   when (mid_sum( $m\_size$ ) = '1')
                   else (plus_inf);
end Behavioral;

```

For clipping the sum of two operands when necessary, the method is handling the addition operation in a register which is 1 bit larger than

the operands and then checking whether its sign bit is identical to its second significant bit (an XOR operator is used for this). If the response is positive, then the result can fit into the limits of the representation; otherwise, a minus infinity or plus infinity must be multiplexed to the output in order to avoid overflow. The minus infinity is represented by -32 and plus infinity by $+31$ in case of 6-bit 2's complement arithmetic. For all arithmetic operations in PDSCCC decoder, the variables are strictly kept within these limits.

- Negation of the most negative number: Whenever a variable is to be negated, special care should be taken to check whether it is minus infinity (-32). The reason for this extra control is that when the most negative number is negated in 2's complement arithmetic, the result is again the most negative number (2's complement of "100000" is also "100000"). Therefore, the negation operator should map minus infinity to plus infinity.
- Obtained information subtraction: Both the inner and the outer decoders require subtraction of the information given by the other decoder (here, it is called the *obtained information*) so that they can pass to the other decoder only the extrinsic information they found. However, this subtraction operation is not straightforward due to the limited arithmetic range utilized. As an example, if the LL value of a bit found by the inner decoder is minus infinity, this means that the inner decoder is almost certain about this bit being a -1 . After it passes this information to the outer decoder, if the outer decoder decides -30 as the LL value of this bit before the obtained information subtraction, the result following the subtraction would be $+1$ (due to negation of minus infinity as defined above). Although the outer decoder anticipates this bit as a very strong -1 , after subtraction it would be a very weak $+1$ bit. For avoiding such situations some rules are defined on the obtained information subtraction. Firstly, if the operands entering the subtraction oper-

ation have opposite signs, then realize the subtraction as it is. Hence, $((+10) - (-12) = (+22))$ and $((+10) - (-32) = (+31))$ with proper clipping. Secondly, if the signs are the same, but none of the operands is equal to plus or minus infinity, again carry out normal subtraction, which requires $((+10) - (+12) = (-2))$ and $((-10) - (-25) = (+15))$. Finally, when the signs are the same if at least one of the operands is equal to extreme values, then do not subtract the obtained information, just give the result as it is. Therefore, $((+10) - (+31) = (+10))$ and $((-32) - (-12) = (-32))$. This way of controlled obtained information subtraction is more suitable for obeying the rules of arithmetic operations that include operands equal to plus or minus infinity.

- Channel reliability factor usage: In Eqn. 3.10, γ metric calculation requires a multiplication of the received sequence by L_c , which is named as the channel reliability factor. Although the precise approximation of this factor is not crucial for turbo decoding as shown in [22, 23], still the basic performance improvement offered by the BCJR algorithm depends on the proper usage of the channel SNR. As described in Section 2.5.5, testbed implementation is capable of estimating the received SNR. However, the important question is based on how this estimate will be mapped onto the 6-bit representation of the metrics and LL values. We only know -32 stands for minus infinity and 31 for plus infinity, no clue seems to be present for the meaning of the other values. One way to overcome this complication is to use the $\ln$ function in the correction function as a reference value. The maximum value of the correction function ($f_c(|x - y|) = \ln(1 + e^{-|x-y|})$) is $(\ln(2) \approx 0.6931)$. If we represent this maximum value by a suitable value in our fixed point arithmetic, then the SNR information can also be mapped to our fixed point world using linear or nonlinear methods. For 6-bit representation, the selected mapping provides 3 as the maximum value of the correction function and uses a linear method to scale the received SNR

value accordingly. In such a mapping the maximum difference between two probability values ($31 - (-32) = 63$) corresponds to a multiplicative difference of $e^{\frac{63 \ln(2)}{3}} = 2.1 \times 10^6$, which is enough for representing small probability values in many cases.

- **Metric Normalizations:** The α and β metrics denote probability values for states and the received sequences in the forward and backward directions. Thus, at any instant the sum of the probabilities of in relation with all states should be equal to 1. This can also be interpreted as the natural logarithm of the sum of probabilities attached to all states should be 0. Assuming we have 4 states, at any time l , this condition is given for α metrics as,

$$\begin{aligned}
& \ln(\alpha_l(s = S0) + \alpha_l(s = S1) + \alpha_l(s = S2) + \alpha_l(s = S3)) = 0, \\
& \Rightarrow \ln(e^{\ln(\alpha_l(s=S0))} + e^{\ln(\alpha_l(s=S1))} + e^{\ln(\alpha_l(s=S2))} + e^{\ln(\alpha_l(s=S3))}) = 0, \\
& \Rightarrow \ln(e^{\alpha_l^*(s=S0)} + e^{\alpha_l^*(s=S1)} + e^{\alpha_l^*(s=S2)} + e^{\alpha_l^*(s=S3)}) = 0, \\
& \Rightarrow \max^*(\alpha_l^*(s = S0), \alpha_l^*(s = S1), \alpha_l^*(s = S2), \alpha_l^*(s = S3)) = 0. \quad (3.23)
\end{aligned}$$

According to Eqn. 3.23, a normalization operation on the obtained α or β metrics can be applied in order to remove any bias from the probabilities. The bias may exist as a result of fixed point approximations or unnormalized γ metrics. The correction method is to operate on these log-likelihood metrics using $\max^*$ operators, and subtract the final $\max^*$ output from the previously obtained metrics.

3.6.3 Importance of Metric Size and Iteration Number

As described in 3.5.4, the selected metric size for PDSCCC implementation is 6. This section will clarify some points in this choice by presenting some simulation results based on the BER performance of the PDSCCC decoder under various bit resolutions for metrics. The Table 3.2 demonstrates the incorrectly decoded bits at all iteration steps of PDSCCC decoders with

4-bit, 6-bit, and 8-bit metric sizes. For generating the data on Table 3.2 a 560-bit packet is simulated with data bit SNR, $E_b/N_0 = 5.21dB$, which corresponds to a coded bit SNR, $E_s/N_0 = -0.81dB$. The bits in the packet are selected as all zeros, with out any loss of generality since the code used is based on convolutional codes and these codes are linear. The interleaver type used in simulations is purely random (not RCS-random) and in all three cases the same purely random interleaver generated prior to simulations is utilized. Furthermore, for all three experiments, the received sequences are multiplied by the distinct optimal gain factors for maximum *signal-to-distortion ratio* (SDR). The optimum gain factors for different bit resolutions and SNR levels are drawn in [21]. The same Gaussian noise samples are added onto the coded BPSK symbols for all three metric sizes. The shown number of bits in error are out of 272 bits for inner decoder output and out of 128 bits for the outer decoder output. From this sample of bit errors it can be deduced that 8-bit decoder reaches to zero output errors (at the output of the outer decoder which gives the final decisions) with less number of iterations. It

Table 3.2: Iteration Steps for Three Types of PDSCCC Decoders

	8-bit Metrics (Bits in Error)		6-bit Metrics (Bits in Error)		4-bit Metrics (Bits in Error)	
Iteration Step	Inner	Outer	Inner	Outer	Inner	Outer
1	34	12	33	18	53	42
2	11	1	16	3	31	24
3	0	0	4	0	23	15
4	0	0	0	0	24	17
5	0	0	0	0	23	6
6	0	0	0	0	7	1
7	0	0	0	0	2	0
8	0	0	0	0	1	0

achieves decoding all the information bits correctly just after two iteration steps, where as this number is three for the 6-bit decoder, and 7 for the 4-bit decoder. Moreover, the inner decoder of the 4-bit PDSCCC decoder can not give correct soft information about one of its output bits even after 8 steps. It must be also noted that the $5.21dB$ SNR value is a fairly high level for such iterative decoders, hence 4-bit decoder may easily decide some bits incorrectly at slightly lower SNR levels.

In most cases for parallel and serially concatenated convolutional codes, 8 to 16 iteration steps are suitable. Although having more iterations yields better BER performance, a very high decoding latency is the sideeffect. In our implementation, the number of iterations was left as an adaptively updated parameter that may be varied according to channel conditions. However, in most of the trials, for keeping the decoding latency low it was limited by 8.

3.7 PDSCCC Decoder Hardware

The synthesis and implementation phase results for hardware implementation of the PDSCCC decoder are given in the following sections.

3.7.1 Xilinx ISE Synthesis

This section will present the logic area utilization and maximum operating frequency statistics of the PDSCCC decoder and its building block BCJR decoder. These statistics are approximated in the Xilinx ISE's synthesis phase. The goal of optimization is set as area optimization and the optimization effort is at normal level. Table 3.3 gives information about the BCJR decoder. The BCJR decoder is capable of working at 35.370 MHz and it is approximately covering 14 % of the FPGA area. Therefore, it is possible to use 6 of these decoders in a single antenna design (other units utilize nearly 10 percent of the whole area), or 4 of them in a two-antenna receiver system. The synthesis results of the PDSCCC decoder with 4 parallelized decoders is detailed

Table 3.3: Single BCJR Decoder Synthesis Results

Unit Name	Usage Count and Percentage
Number of Slices	1928(14%)
Number of Slice Flip Flops	134(0.5%)
Number of 4 input LUTs	3480(12.5%)
Number of BRAMs	6(4.5%)
Maximum Operating Frequency	35.370 MHz

Table 3.4: PDSCCC Decoder Synthesis Results

Unit Name	Usage Count and Percentage
Number of Slices	8899(64%)
Number of Slice Flip Flops	1169(4%)
Number of 4 input LUTs	15888(58%)
Number of BRAMs	40(29%)
Maximum Operating Frequency	31.251 MHz

in Table 3.4. The PDSCCC decoder implementation utilizes an area which is larger than the half area of XC2VP30 FPGA chip. Some area optimizations are possible with corresponding performance degradations. As an example, it is possible to construct the $\max^*$ operator using the max-log-MAP approximation approach defined in Section 3.5.5. This brings an SNR loss around $0.5dB$ in case of parallel concatenated codes as given in [19]. Moreover, we can also decrease the resolution of the metrics, which will result in an SNR degradation of nearly $0.1dB$ for metric size change from 6-bit to 4-bit. The synthesis results for 4-bit linear-log-MAP based PDSCCC implementation is given in Table 3.5. If we also use the max-log-MAP algorithm together with the 4-bit metrics, the corresponding PDSCCC has the utilization figures presented in Table 3.6.

According to all these timing results, the PDSCCC decoder is capable of

Table 3.5: PDSCCC Decoder Synthesis Results(4-bit Metrics)

Unit Name	Usage Count and Percentage
Number of Slices	6899(50%)
Number of Slice Flip Flops	961(3%)
Number of 4 input LUTs	12046(43%)
Number of BRAMs	28(20%)
Maximum Operating Frequency	33.649 MHz

Table 3.6: PDSCCC Decoder Synthesis Results(4-bit and Max-log-MAP)

Unit Name	Usage Count and Percentage
Number of Slices	4159(30%)
Number of Slice Flip Flops	961(3%)
Number of 4 input LUTs	7355(26%)
Number of BRAMs	28(20%)
Maximum Operating Frequency	38.551 MHz

supplying a data throughput of 4.167 Mbps in the highest BER performance case, that is for 6-bit metrics and linear-log-MAP. (Each iteration takes 120 cycles and for 8 iterations 128 uncoded data bits are decided at 31.255 clock frequency.) For lower performance case given in Tables 3.5 and 3.6, the throughputs are 4.487 Mbps and 5.140 Mbps respectively. These figure are quite higher than some recent implementations of iterative approaches [20]. When the PDSCCC decoder is operated using the 24 Mhz clock source of the other receiver modules, it still has a throughput of 3.2 Mbps.

3.7.2 Optimization and Implementation Issues

The PDSCCC decoder is a relatively huge hardware module especially when the metric size is selected as 6 and linear-log-MAP algorithm is used. In order to successfully implement and operate large designs some parameters

of Xilinx ISE should be set correctly. Firstly, within the Xilinx Constraints Editor the clock signal should be defined and corresponding clock period constraint must be set according to the operating frequency. In order to assure that all calculation results are assigned to corresponding registers correctly, the *pad to setup* constraint should be set as at most half the period of the clock signal. This constraint guarantees that any input signal from the outside world observes a combinational delay at most at the required length. Similarly, the *clock to pad* constraint determines the maximum combinational delay for a signal traveling from the output of a register to an output pad and it is usually set as the half clock period as well. In addition to these, some advanced implementation parameters should be tuned for expected circuit behavior. These parameters are set from the properties dialog box of the “Implement Design” phase. Map effort level is required to be set as high and the optimization strategy as balanced between the area and speed optimizations. Place and route effort level must also be set as high and multi-pass place and route choice may be selected as well in order to avoid placement and routing problems in large designs.

CHAPTER 4

THE PAPR PROBLEM IN WATER-FILLING AND A SUBOPTIMAL WATER-FILLING ALGORITHM

The Chapters 2 and 3 are concentrated on implementation aspects of a complete wireless communication testbed with all related RF transmission modules, signal processing blocks, and coding schemes used. This chapter, on the other hand, is based on algorithms developed for signal transmission in the next step of the testbed implementation with orthogonal frequency division multiplexing (OFDM) and multiple input multiple output (MIMO) methods. Few key points have been described about the MIMO perspective of the testbed development in Chapter 2. Also no details on the OFDM implementation has been given since the fast Fourier transform (FFT) algorithm (which is the basis for OFDM transmission) was under development during the submission of this thesis. However, we carried out many simulations and derived some analytical results for the possible algorithms that had potential usage in the final MIMO-OFDM system. The essential goal of the algorithms tested is decreasing a parameter, known as *peak-to-average-power ratio* (PAPR), in MIMO-OFDM testbed transmitter output.

4.1 Definition of PAPR

The peak-to-average-power ratio (PAPR) for a stochastic discrete time process $x[n]$ is given as,

$$\text{PAPR} = \frac{\max_n |x[n]|^2}{E_x \{|x[n]|^2\}} \quad (4.1)$$

According to the definition of PAPR, some simple examples can be given to demonstrate the PAPR for some schemes. The PAPR of a system with PSK modulated output symbols is trivially 1 due to constant amplitude at the output. In opposition to the PSK modulation scheme, with square M-QAM modulations ($M > 4$), the PAPR is always greater than 1, since the peak power is associated with the symbols on the corners that have larger magnitude than the other symbols. Also with multicarrier transmission schemes, signals may add up constructively at some time instances while adding up destructively at other instances, which may lead to a large dynamic range for the output signal power, so a high PAPR.

Most of the power amplifiers at the last stage of the wireless transmitter modules have nonlinear input vs. output power characteristics. Some models for this nonlinearity are suggested in [24, 25]. According to the solid state amplifier (SSA) model, as an example, the power characteristics of an amplifier (with smoothness factor of 1, and saturation level of 1 Watt) is given in Figure 4.1. It is obvious that when we use the amplifier in the linear region we can transmit the desired signal without any power change. However, close to the saturation region there is considerable degradation in the transmitted signal power. This may result in severe signal distortion ([28]) unless PAPR of the transmitted signal is reduced or the transmitted signal power is reduced so that the peak power will also be close to the linear region, which is known as *backoff*. Applying high backoff to the transmitted signal has a disadvantage of using the power amplifier in its low-efficiency region for most of the time.

In most cases simply defining and using the maximum PAPR value as a system performance parameter is practically insignificant. Considering the very low probability of having the maximum possible PAPR value, in most systems the cumulative distribution function of the PAPR is the performance criteria for transmitted signal. Figure 4.2 demonstrates a comparison of two systems in terms of the PAPR distribution of their output signals. These two systems are arbitrary, but with the corresponding given PAPR distributions.

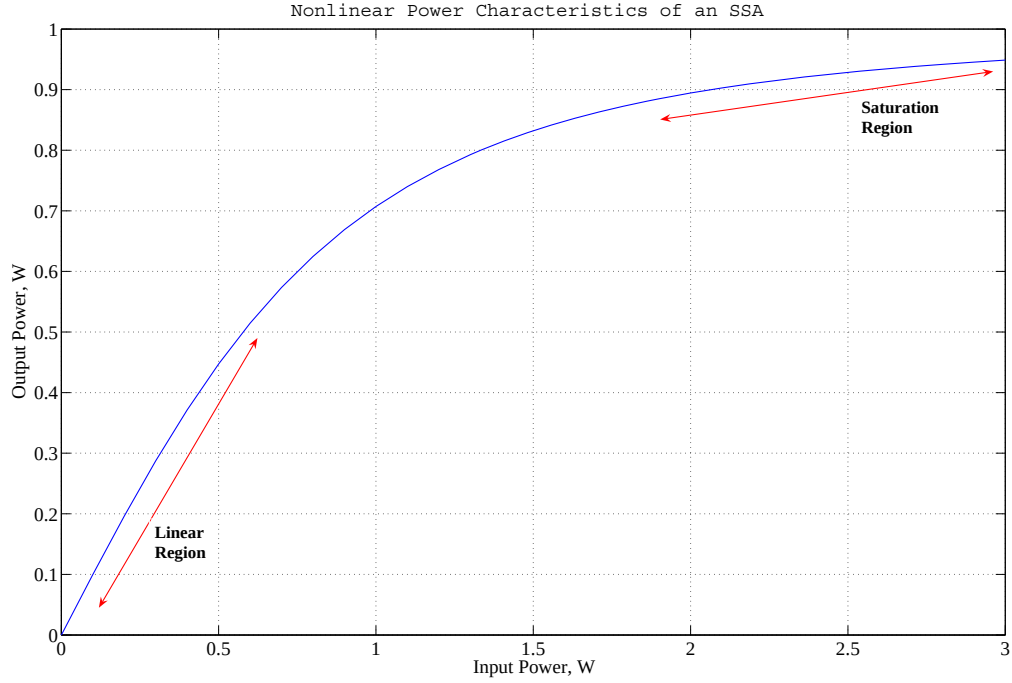


Figure 4.1: The linear and the saturation regions of the power amplifier are clearly distinguishable.

Several conclusions can be drawn from Figure 4.2. Firstly, we can compare two systems by setting a probability value of interest under which we assume the probability of having corresponding $PAPR_0$ value is negligible. Assuming this limit is set at 10^{-4} for a specific application such that in one symbol out of 10000, we observe a problem due to high signal level. Therefore, in order to avoid such problems we can backoff the average signal power level by the corresponding $PAPR_0$ value. This value is nearly $9.5dB$ for the first system whereas it is almost $11dB$ for the second. Hence the designer of the second system should decrease its average power level by $1.5dB$ more than the first system's designer so that both systems can operate with same level of degradation on the output signal. This power reduction will pose a disadvantage to the designer of the second system due to decreased SNR and

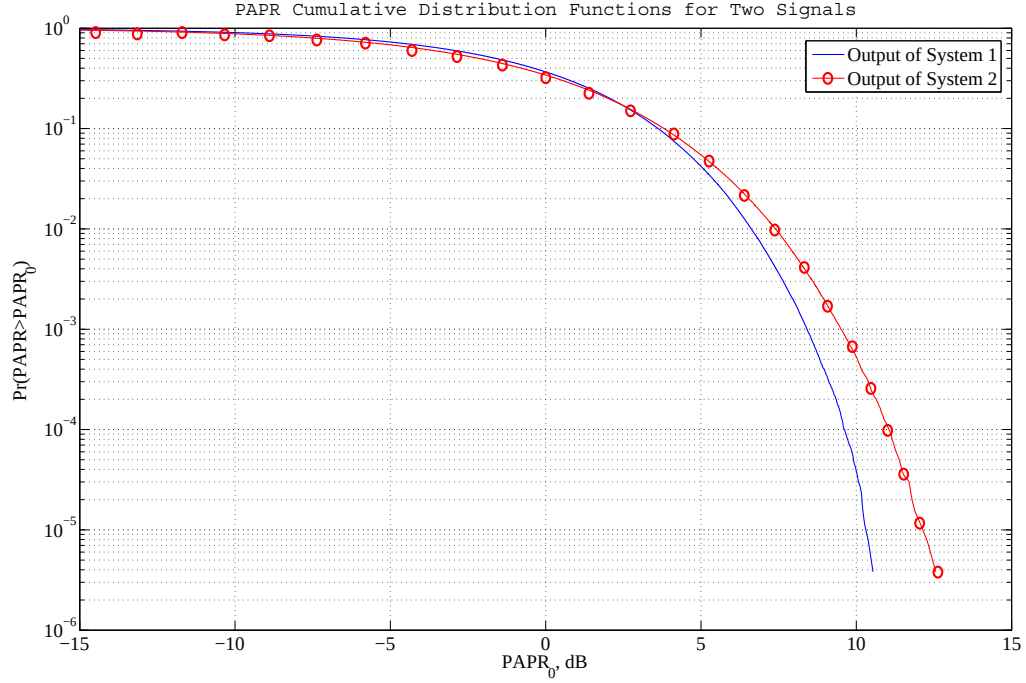


Figure 4.2: System 1 has better PAPR distribution property with lower probabilities of surpassing a given level of $PAPR_0$ value.

hence the decreased available channel capacity. Secondly, we can compare two systems' probabilities of having a specific $PAPR_0$ value. As an example, the first system will have $10dB$ instantaneous PAPR with the probability of 4×10^{-5} and the second system with the probability of 5×10^{-4} . If no backoff is applied and the critical signal distortion level is set as $PAPR_0 = 10dB$, the second system will roughly have its symbols degraded with 10 times higher probability than the first one.

4.2 PAPR Reduction Methods in Literature

Since PAPR is especially an intrinsic and crucial problem for multicarrier systems, techniques for reducing PAPR are diversified with the increasing

interest in multicarrier systems in recent years. Although a detailed analysis of all of the methods will not be given, some of the frequently used ones will be named here [26].

- **Amplitude Clipping and Filtering:** This technique clips the signals over a given level to the desired output level. This clipping operation usually generates some noise in the bandwidth of the signal (in-band distortion) and also some noise out of the signal bandwidth (out-of-band radiation). In order to suppress the out-of-band radiation after clipping, filtering is applied to the signal which causes some additional PAPR problems. Thus, an iterative clipping and filtering stages are suggested which result in excessive processing delays.
- **Coding:** This class of techniques is large with a lot of subclasses that suggest reduction for distinct modulation schemes. One idea is to select such codewords for transmission such that the overall combined effect of the codewords results in decreased PAPR levels. This approach requires large lookup tables and finding good codes that reduce PAPR. In [27], for an OFDM system with MPSK modulation the optimum lengths for Golay complementary codes are given. Using these sequences of codes the PAPR level of the output signal is kept at desired levels deterministically. In most cases, the coding for PAPR reduction suffers from bandwidth efficiency loss due to very low coding rates.
- **Selected Mapping:** Preparing multiple versions of the data to be transmitted, the one with the lowest PAPR value is selected in this technique. As an example for an OFDM system, many FFT's are obtained for one data sequence and transmitter selects the optimum one in terms of PAPR. Obviously this method is problematic when sufficiently enough representations for the same signal is to be generated. Hardware resources may be the bottleneck in implementation of selected mapping technique.

4.3 PAPR Reduction in OFDM systems

Due to increased data rate requirements of modern applications the bandwidth utilized for wireless communications has increased in the last decade. The increased bandwidth usually comes with a problem to be remedied for reliable communication. This problem is known as the intersymbol interference and mainly caused by the selective response of the channel within the communication bandwidth. To fight with this frequency selective response, a method is to split the bandwidth utilized into subbands such that on each subband the gain of the frequency selective channel is almost constant. Then the data to be transmitted is also divided into substreams each of which has only a fraction of the total data rate. Furthermore, the subcarrier frequencies to carry these substreams are selected such that each modulated substream is orthogonal to others in frequency domain. This type of modulation is known as the orthogonal frequency division multiplexing (OFDM) and has been used in many wideband wireless communication systems, such as IEEE 802.11a/g based wireless LANs, DVB terrestrial digital TV systems, IEEE 802.16 based WiMAX.

4.3.1 Definition of OFDM Signal

An OFDM signal can be defined as the time domain representation of the data signal that is constructed in the frequency domain by assigning N substreams of data to orthogonally spaced N subcarriers. This operation can be implemented as an inverse fast Fourier transform (IFFT) operation [29]. The substreams may be modulated adaptively in the frequency domain before IFFT as shown in Figure 4.3¹. In order to mitigate the effects of multipath fading signal reception, a *cyclic prefix* is usually copied from the above defined time domain signal's end and attached to its head. This way the synchronization mismatches are avoided, the orthogonality between the sub-

¹Figure is taken from <http://en.wikipedia.org/wiki/OFDM>

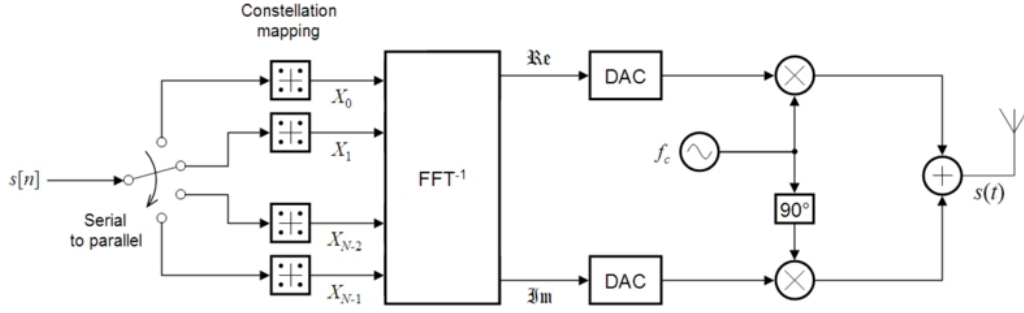


Figure 4.3: The IFFT based OFDM transmitter shown has N subcarriers.

carriers is preserved and consequently a simpler receiver design with an FFT operation is possible [8]. Therefore, many resources accept that an OFDM symbol includes the cyclic prefix part in addition to the IFFT of the data in frequency domain.

The discrete time domain representation for OFDM signal with N subcarriers is given as,

$$x[n] = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} X[i] e^{j2\pi in/N} \quad (4.2)$$

When we observe the distribution of the in-phase and quadrature-phase components of the OFDM signal, when N is a large number (due to Central Limit Theorem) these components resemble Gaussian noise samples. This Gaussian distribution assumption is practically valid for most applications when $N \geq 64$ [8]. This is exemplified in Figure 4.4, where 8192 OFDM symbols for a 64-subcarrier system are generated in MATLAB and the histogram of the real part of the resulting signal is drawn. As a result of the Gaussian-like distribution, inherently OFDM signals have high PAPR. This can be easily observed on Eqn. 4.2 as well. The complex frequency domain signals $X[i]$ are multiplied by subcarriers with different phases. Their sum ($x[n]$) at any time n may be very large due to possible constructive superposition at that time instance.

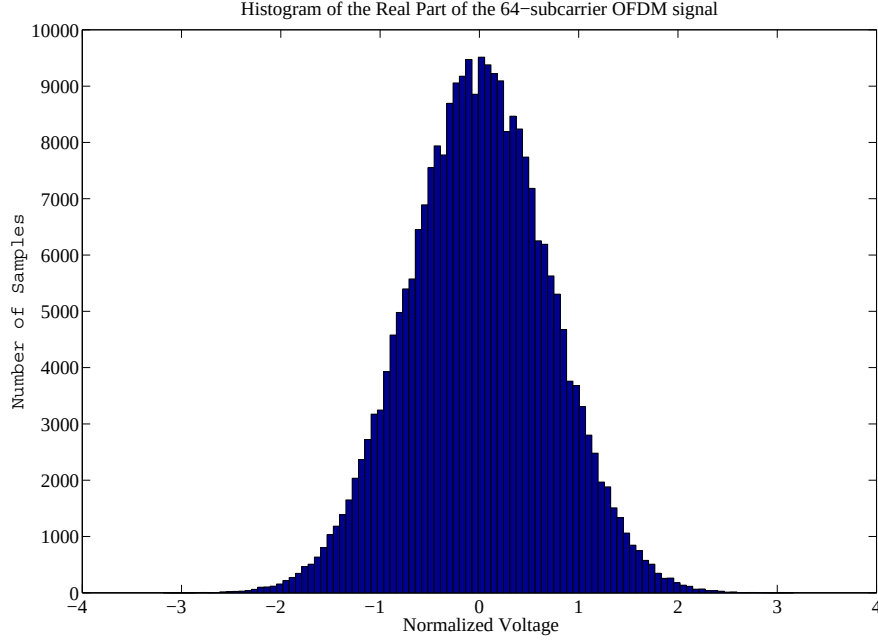


Figure 4.4: Gaussian characteristics of the histogram is obvious for real part of the OFDM signal.

4.3.2 PAPR Problem with Optimal Power Allocation in Frequency Domain

For time varying frequency selective channels, the optimal algorithm for achieving channel capacity is the two-dimensional water-filling algorithm [8]. This algorithm allocates distinct power levels to each frequency at any time according to the instantaneous SNR level of that frequency. The frequencies with higher SNR levels receive higher power, while the worse frequencies obtain less or no power according to a predefined threshold. A simple example (using only one-dimensional water-filling in frequency domain at an instant) for demonstrating the water-filling operation is given in Figure 4.5. The γ_0 value shown in Figure 4.5 is the threshold, whose reciprocal will determine

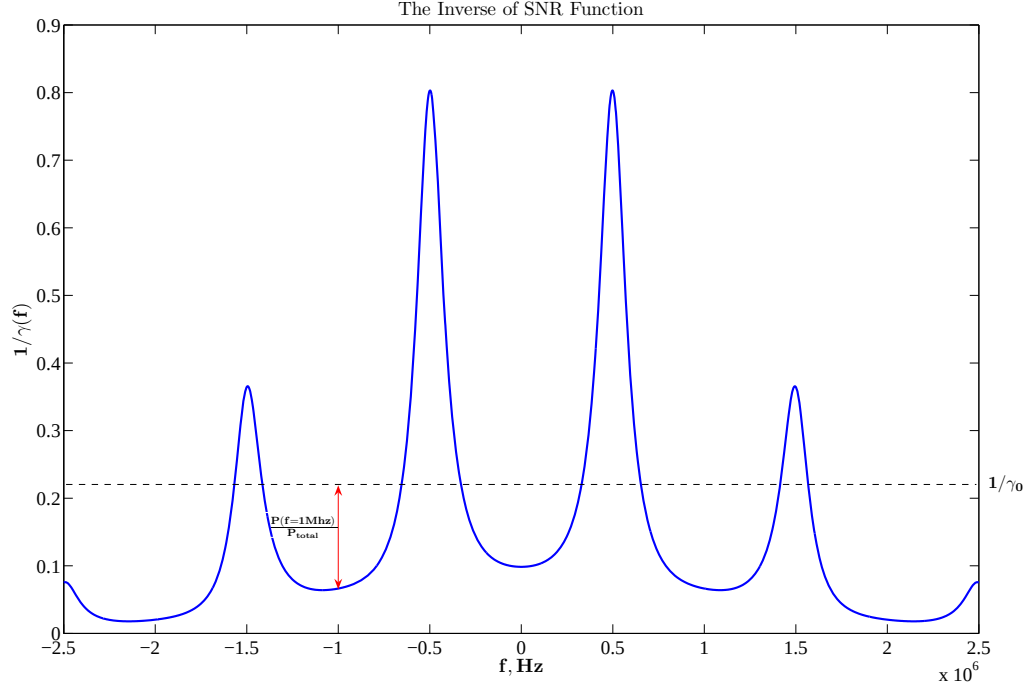


Figure 4.5: The inverse of the SNR function is used for allocating the power to frequencies.

the power to be allocated to various frequencies. The total power to be distributed is denoted by P_{total} and the power allocation, $P(f, t)$, to frequency f at time t is given as,

$$\frac{P(f, t)}{P_{total}} = \begin{cases} 1/\gamma_0 - 1/\gamma(f, t) & \gamma(f, t) \geq \gamma_0 \\ 0 & \gamma(f, t) < \gamma_0 \end{cases}. \quad (4.3)$$

In Eqn. 4.3, $\gamma(f, t)$ is used for the instantaneous SNR at frequency f at time t . A very similar algorithm for optimally distributing a given power to frequency-selective block fading channels can also be obtained. This algorithm also applies to OFDM subcarrier power distribution, since within the frequency subbands the frequency response of the channel is assumed to follow block fading in frequency domain for OFDM. Figure 4.6 shows power

allocation on block fading frequency bands. For OFDM optimal power dis-

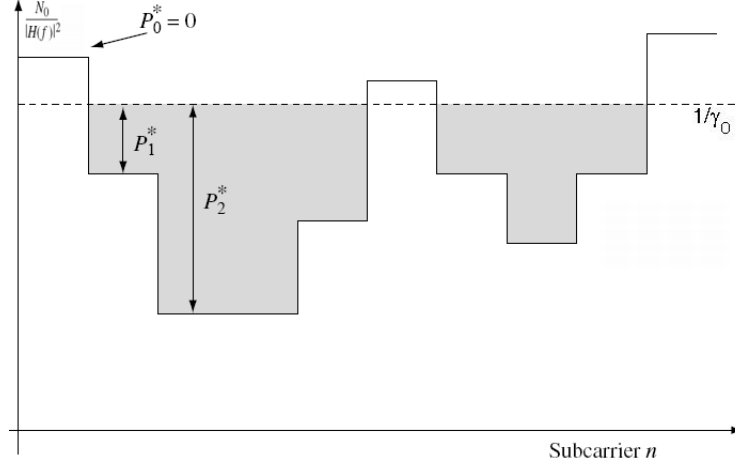


Figure 4.6: The subcarrier number 0 is not allocated any power since it is below the threshold [7].

tribution, the power allocated to the subcarriers is defined as,

$$\frac{P_j}{P_{total}} = \begin{cases} 1/\gamma_0 - 1/\gamma_j & \gamma_j \geq \gamma_0 \\ 0 & \gamma_j < \gamma_0 \end{cases}, j = 0, 1, \dots, N-1. \quad (4.4)$$

This power allocation method is optimal in the sense that it achieves the channel capacity which is given as,

$$C = \sum_{j:\gamma_j \geq \gamma_0} B \log_2\left(\frac{\gamma_j}{\gamma_0}\right). \quad (4.5)$$

In Eqn. 4.5, B denotes the allocated bandwidth for communication. Although the channel capacity is satisfied with the given constraints, there is no constraint on the PAPR of the output signal after this power allocation scheme. With the motivation of including a PAPR constraint into the power allocation scheme, we suggested a suboptimal algorithm for OFDM systems. This

method is simple to apply when compared to the water-filling algorithm and depends on distributing the given total power in equal amounts to a selected number of subcarriers. The following section summarizes our analytical findings on the PAPR of the OFDM signal when water-filling is utilized. It further compares the optimal (water-filling) scheme and the scheme where all the subcarriers are allocated equal power. Then, the next section compares the suggested suboptimal scheme (allocating equal power to a selected number of subcarriers) and the water-filling.

4.3.3 Comparison of Non-adaptive Scheme with Water-filling

The cumulative probability distribution function of an OFDM signal with N subcarriers is given in [8] as,

$$F_{\lambda}(\lambda) = Pr(\max_{i=0,1,\dots,N-1} |x[i]|^2 \leq \lambda), \quad (4.6)$$

$$= (1 - e^{-\lambda})^N. \quad (4.7)$$

However, with two-dimensional water-filling at some instances most of the SNR levels of the subcarriers may be lower than the threshold level. Then, the power is saved for better channel SNR levels in future transmissions, which will result in significantly varying PAPR for the power adapted OFDM signal. Therefore, this requires a new analysis for determination of cumulative probability distribution function with water-filling. In [30], the instantaneous PAPR corresponding to an instantaneous SNR vector, $\vec{\gamma} = \{\gamma_0, \gamma_1, \dots, \gamma_{N-1}\}$, is found as,

$$F_{\lambda(\vec{\gamma})}(\lambda(\vec{\gamma})) = (1 - e^{-\lambda/\sigma_{\vec{\gamma}}^2})^N, \quad (4.8)$$

where $\sigma_{\vec{\gamma}}^2$ is the random variance of the channel SNR vector, $\vec{\gamma}$. This random variance is given as,

$$\sigma_{\vec{\gamma}}^2 = E\left\{\frac{1}{N} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \sqrt{P_i(\gamma_i)} \sqrt{P_j(\gamma_j)} X[i] X^*[j] e^{j2\pi(i-j)n/N}\right\}, \quad (4.9)$$

$$= \frac{1}{N} \sum_{i=0}^{N-1} P_i(\gamma_i). \quad (4.10)$$

Then the cumulative probability distribution function for the PAPR is found after taking expectation of the expression in Eqn. 4.8 over all realizations of the channel SNR vector as,

$$F_{\lambda}(\lambda) = \int \cdots \int f_{\vec{\gamma}}(\gamma_0, \dots, \gamma_{N-1}) (1 - e^{-\lambda/\sigma_{\vec{\gamma}}^2})^N d\gamma_0 \dots d\gamma_{N-1}. \quad (4.11)$$

For the following comparisons this analytical result is evaluated using the Monte Carlo simulation method. Moreover, for all the water-filling and sub-optimal method simulation results are obtained by generating 100000 channel vectors with zero-mean Gaussian vector elements that have variance 1. The channel capacity comparisons are carried out by generating 10000 channel vectors for each average total SNR level.

Figure 4.7 presents the capacity of optimal power allocation algorithm with the non-adaptive (suggested equal power allocation) algorithm. It is easy to observe that water-filling algorithm degrades to equal power allocation method in the high SNR values. Around 10dB SNR value (at a capacity of 2.5 *bits/sec/Hz*) the SNR loss due to equal power allocation is 2.5dB. Around 0dB SNR the loss is approximately 5dB and around 30dB it is less than 0.5dB. Figure 4.8 shows the PAPR performance of two methods at an average SNR of 0dB. Moreover, it also demonstrates the analytical result curve in a close proximity of the 2-D water-filling method's curve. The SNR loss due to the excess backoff should be done by the water-filling method is slightly above 2dB at PAPR probability of 10^{-4} .

Figures 4.9 and 4.10 show the comparison results for two methods at average SNR levels 10dB and 30dB respectively. For 10dB average SNR,

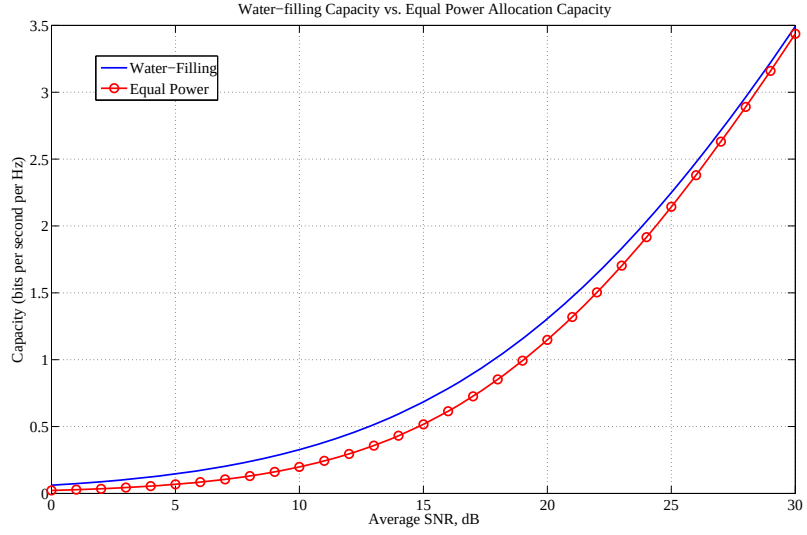


Figure 4.7: Channel capacity of two methods are close for high SNR values.

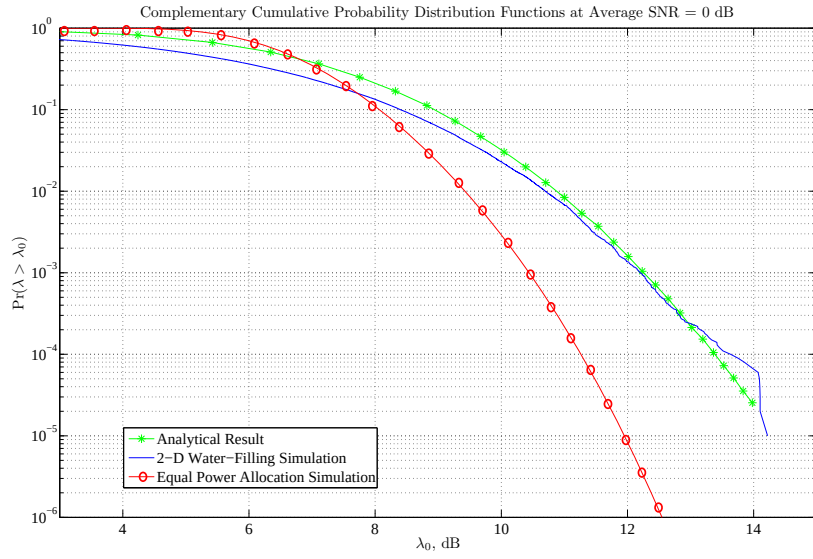


Figure 4.8: The non-adaptive method offers better PAPR distribution.

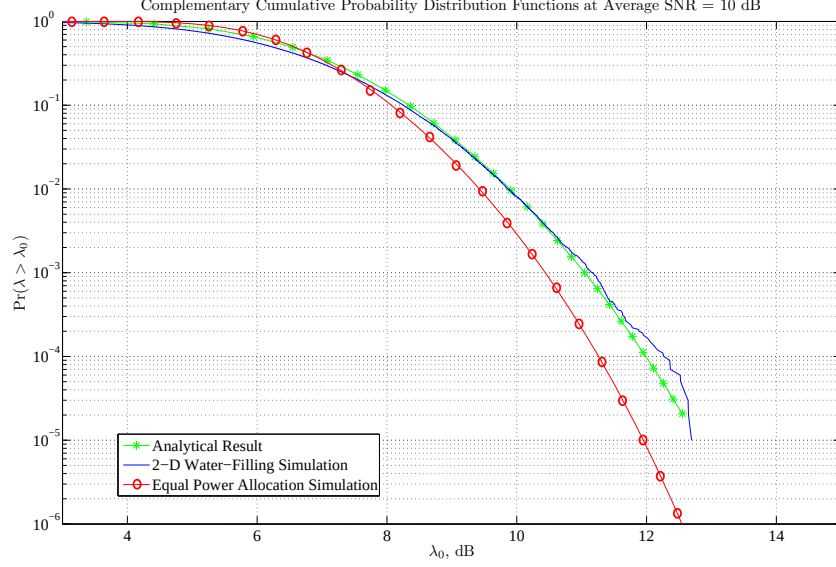


Figure 4.9: The analytical PAPR result is in accordance with the simulated water-filling PAPR curve.

the loss in water-filling algorithm is nearly $1dB$, and for $30dB$ the PAPR distributions of two methods are almost identical. As a result, the channel capacity gain obtained by the optimal water-filling algorithm is not very high when the nonlinear characteristics of power amplifiers, which are effected by high PAPR values, is taken into account. The simple equal power allocation method can perform with only $2.5 - 1 = 1.5dB$ performance degradation at $10dB$ average SNR. Furthermore, the analytical PAPR distribution suggested by [30] strictly follows the simulation results at $10dB$ and higher SNR values. This is mainly due to the fact that at very low SNR values the water-filling method uses only a few subcarriers and they may not add up to Gaussian distributed signals leading to invalidity of the Gaussian assumption in the derivation.

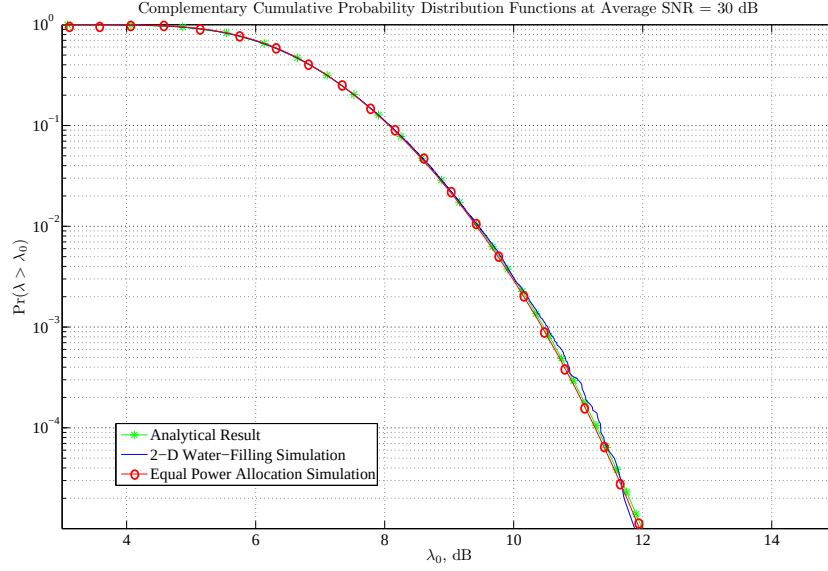


Figure 4.10: Water-filling converges to equal power distribution.

4.3.4 Comparison of Subcarrier Selection Scheme with Water-filling

In implementation of the power adaptation schemes, transmitter should be fed back by the appropriate channel state information (like the SNR of each subcarrier). The water-filling algorithm also requires information about the power to be allocated to each subcarrier and this feedback may result in excessive utilization of the feedback channel. In order to reduce the time for feedback, limited rate feedback may be preferred. In our scheme, instead of sending the quantized subcarrier SNR values back to the transmitter, only one bit per subcarrier is transmitted to reveal whether the corresponding subcarrier is allocated power or not. Therefore, the channel is used for shorter intervals during each feedback event. This section presents the capacity curves for various number of subcarriers (with highest SNR values) selected to carry signal and it compares them with the optimal method. (In Appendices closed

form expression for capacity of arbitrary number of selected subcarriers is derived.) Figure 4.11 gives this comparison. In Figure 4.11 it is clearly seen

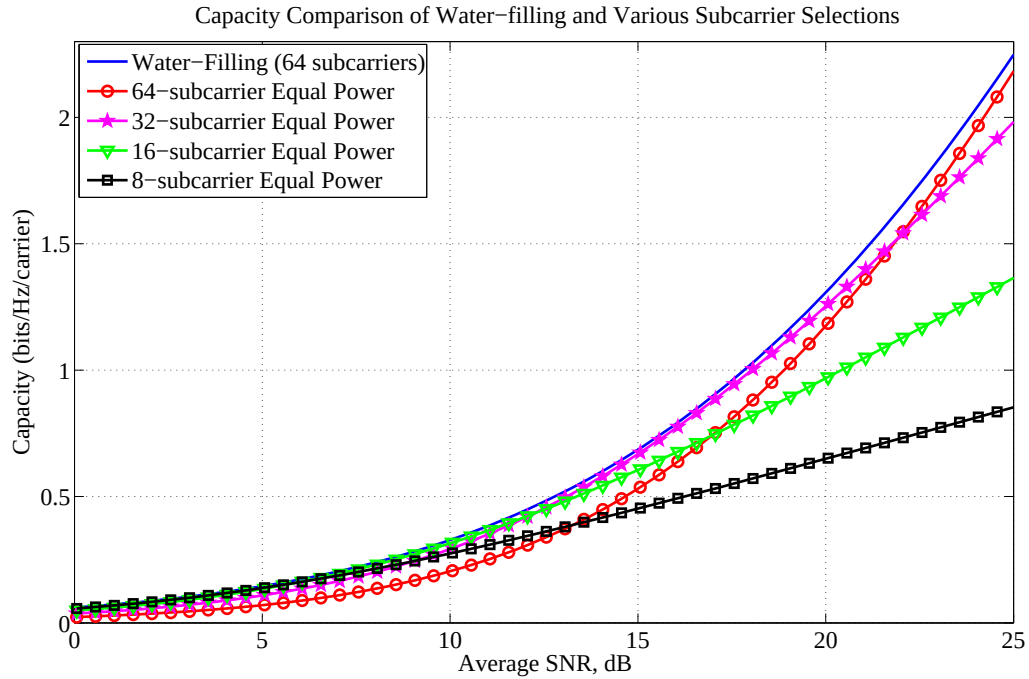


Figure 4.11: For different number of subcarriers capacity curves are given.

that at lower SNRs low subcarrier numbers yield higher capacity than the high subcarrier numbers. Moreover, the 8-subcarrier suboptimal method has almost a $2dB$ loss around the average SNR value of $10dB$. Figure 4.12 demonstrates the PAPR cumulative probability distribution functions for optimal and suboptimal power allocation techniques. Average SNR is $10 dB$ for this simulation and all of the curves are drawn using approximately 8 million QPSK symbols. For the subcarrier selection method, given number of subcarriers with highest SNR values are allocated equal amounts of power with a total power constraint. Obviously, the PAPR performance of 8-subcarrier

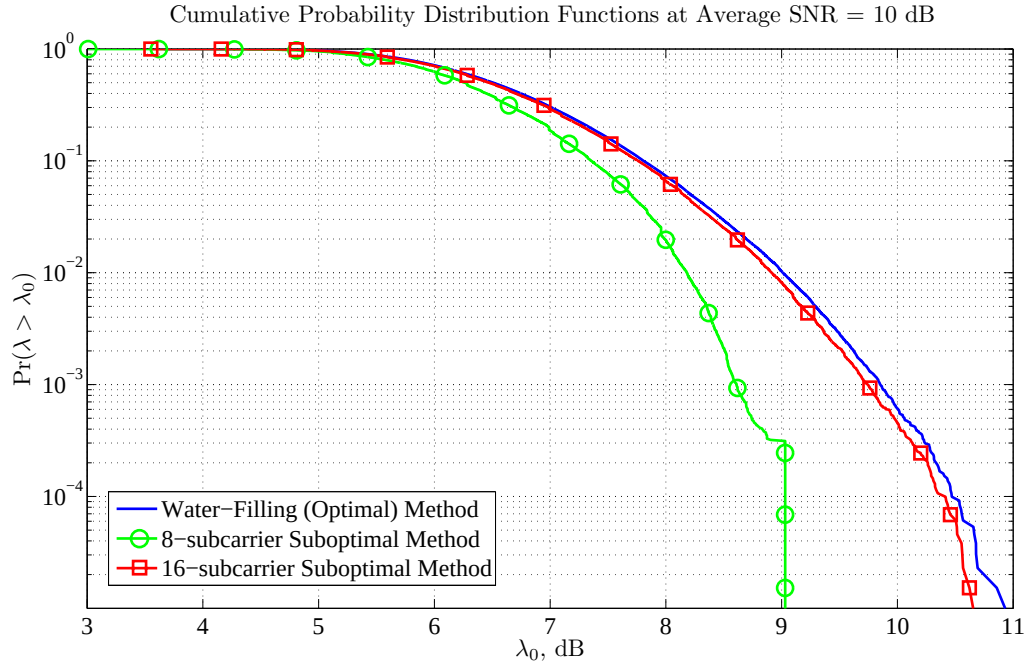


Figure 4.12: 16-subcarrier selection method has much worse PAPR performance than the 8-subcarrier selection method.

suboptimal method is better than the optimal water-filling method and the 16-subcarrier suboptimal method. Moreover, 8-subcarrier selection has 1.5dB PAPR advantage which leads to only a $2\text{dB} - 1.5\text{dB} = 0.5\text{dB}$ gain for the optimal method in the overall comparison. This means a negligible capacity loss is incurred in the suggested method which greatly reduces the feedback rate required.

4.4 PAPR Reduction in MIMO systems

Quite similar to sending data signals as substreams distributed in the frequency domain, it is also possible to use the spatial domain to create new independent substreams for either sending independent data or creating di-

versity. Multiple-input multiple-output (MIMO) systems, in this sense, bare resemblance to OFDM systems, with the distinction that the methods used for achieving channel capacity differ. Consequently, it may be interesting to analyze the PAPR properties of MIMO systems under different scenarios.

4.4.1 Overview of MIMO systems

Figure 4.13 shows the MIMO system in its most general form with N_t transmit antennas and N_r receive antennas (called $N_t \times N_r$ MIMO system). Within the MIMO wireless channel there are signal paths from all transmitter antennas to all receiver antennas and for transmission various space-time coding methods are possible. A MIMO wireless system is usually generalized by a

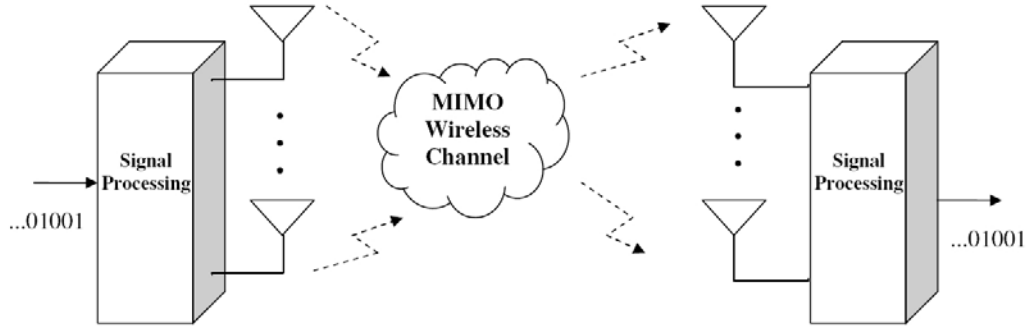


Figure 4.13: Many paths exist from the transmit antennas to the receive antennas in a MIMO wireless channel.

matrix-vector equation that is given as,

$$\mathbf{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_{N_r} \end{bmatrix} = \mathbf{H}\mathbf{x} + \mathbf{n} = \begin{bmatrix} h_{11} & \dots & h_{1N_t} \\ \vdots & \ddots & \vdots \\ h_{N_r1} & \dots & h_{N_rN_t} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_{N_t} \end{bmatrix} + \begin{bmatrix} n_1 \\ \vdots \\ n_{N_r} \end{bmatrix}. \quad (4.12)$$

The transmitted vector, $\mathbf{x}$, is an N_t dimensional vector whose elements are the transmitted signals from the antennas. $\mathbf{H}$ represents the channel gain matrix, where the entry h_{ij} (in general complex) is the fading parameter for the channel between the j^{th} transmit antenna and the i^{th} receive antenna. n is the complex Gaussian noise vector representing the noise added to signals at each receive antenna. Finally, y is the received vector with each entry corresponding to a signal at each receive antenna.

For any $N_r \times N_t$ complex matrix, $\mathbf{H}$, there exists a method called singular value decomposition (SVD) that decomposes $\mathbf{H}$ into *unitary* $N_r \times N_r$ $\mathbf{U}$ matrix, *unitary* $N_t \times N_t$ $\mathbf{V}$ matrix, and diagonal $N_r \times N_t$ $\mathbf{\Sigma}$ matrix as,

$$\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H \quad (4.13)$$

Using this decomposition if we modify the vector $\mathbf{x}$ and transmit $\tilde{\mathbf{x}} = \mathbf{V}\mathbf{x}$, and similarly shape the received vector $\mathbf{y}$ as $\tilde{\mathbf{y}} = \mathbf{U}^H\mathbf{y}$, the modified receive vector can be given as,

$$\tilde{\mathbf{y}} = \mathbf{U}^H \mathbf{y}, \quad (4.14)$$

$$= \mathbf{U}^H (\mathbf{H}\tilde{\mathbf{x}} + \mathbf{n}), \quad (4.15)$$

$$= \mathbf{U}^H (\mathbf{U}\mathbf{\Sigma}\mathbf{V}^H \mathbf{V}\mathbf{x} + \mathbf{n}), \quad (4.16)$$

$$= \mathbf{\Sigma}\mathbf{x} + \tilde{\mathbf{n}}, \quad (4.17)$$

where $\tilde{\mathbf{n}} = \mathbf{U}^H\mathbf{n}$ has the same distribution as $\mathbf{n}$, since $\mathbf{U}$ (and also $\mathbf{U}^H$) is unitary. Then, after applying these shaping transforms onto the signal to be sent and the signal that is received, we obtain $\min\{N_t, N_r\}$ parallelized channels with distinct signal powers given in diagonals of $\mathbf{\Sigma}$. The optimal power allocation on these independent channels is also water-filling (see [8]) as in the case of independent subcarriers of an OFDM signal.

4.4.2 PAPR Problem with Optimal Power Allocation in Spatial Domain

In this section, similar to the results given in Section 4.3.3 for OFDM system, the capacity and PAPR comparisons for different average SNR levels will be given. As it is shown in [31], for fast Rayleigh fading MIMO systems there is slight difference between the capacities achieved by one-dimensional (over spatial dimension) and two-dimensional (over both spatial and time dimensions) water-filling. Therefore, in the following simulations the water-filling method used is one-dimensional. Moreover, $N_r = N_t = 4$ is the general assumption for all simulations.

The channel capacity achieved by water-filling and equal power allocation methods are compared in Figure 4.14. The SNR loss of the non-adaptive

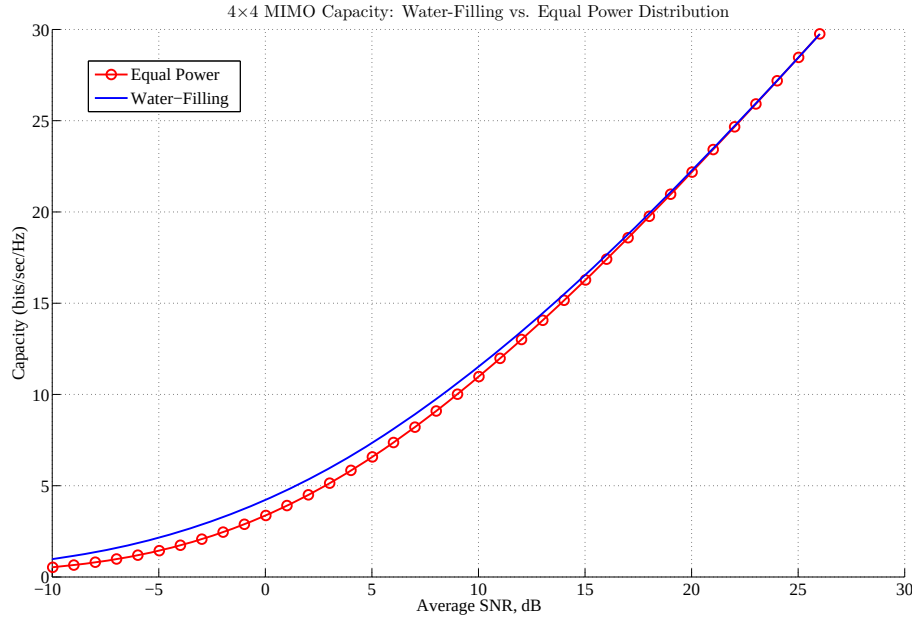


Figure 4.14: The loss due to equal power allocation is negligible even for moderate SNR levels.

method is close to $2dB$ around $0dB$ average SNR, $0.5dB$ around $10dB$ average SNR, and negligibly small around $25dB$ average SNR. Figure 4.15 demonstrates the PAPR distributions for two methods at $0dB$ average SNR. For

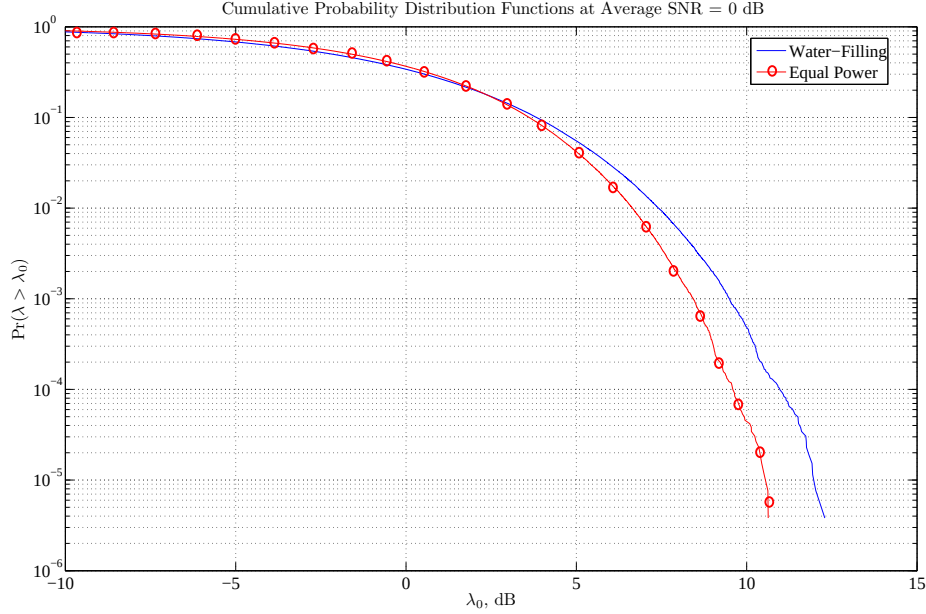


Figure 4.15: Equal power allocation is much less prone to PAPR problems.

$0dB$ average SNR and a fixed PAPR probability of 10^{-4} , the extra backoff for the water-filling power allocation method is almost $1.5dB$ when compared with the equal power allocation method. Hence the overall gain for the optimal method is close to a meager $0.5dB$. Figure 4.16 is drawn for $10dB$ average SNR. When we set the PAPR probability at 10^{-4} , the loss of the water-filling method is nearly $0.5dB$, which makes it comparable with the non-adaptive method in terms of channel capacity. Finally, Figure 4.17 shows us that two methods converge to each other when the average SNR is $25dB$.

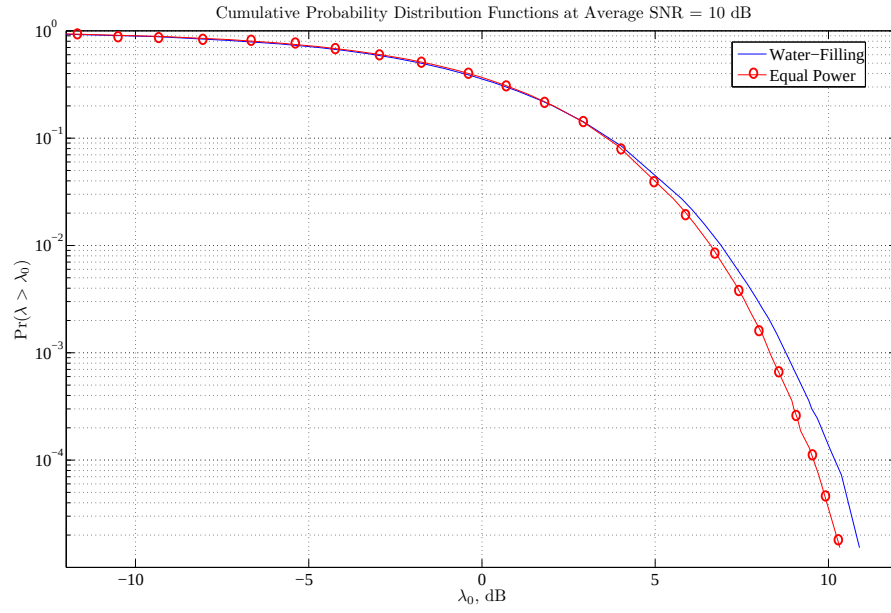


Figure 4.16: An observable PAPR performance difference still exists between two methods.

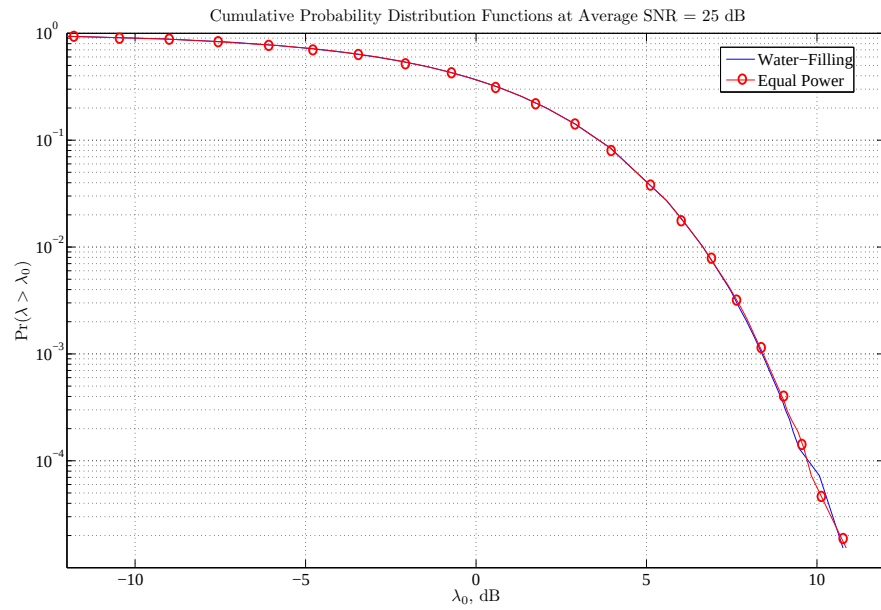


Figure 4.17: PAPR problem effects two systems almost identically.

CHAPTER 5

CONCLUSIONS AND FUTURE WORK

There are three main themes of this thesis:

- The SISO and SIMO wireless testbed structures are described, corresponding filtering operations and the symbol detection results are discussed. The maximal ratio combining method applied on the SIMO system is shown to achieve much better bit error rates than the SISO system, which encourages further studies with the MIMO approach.
- A parallelized serially concatenated convolutional code structure is defined and its fixed point implementation based on the linear-log-MAP algorithm is detailed. Various parameters effective on the performance of the corresponding decoder are presented and final data throughput for each major case is listed.
- The effect of optimal power allocation technique for OFDM and MIMO systems on the PAPR of the output signal are investigated. The results for the comparison of this algorithm with the simplest equal power allocation technique are given and the gain obtained by using the optimal technique is shown to be small for especially 4×4 MIMO system. Moreover, a closed form expression for the capacity of a suboptimal subcarrier selection method is derived (see Appendices).

In addition to the achievements described in this thesis, some improvements to our testbed are set as future goals. These goals can be itemized as follows:

- In order to optimize the system performance, the frequency modulating (FM) RF transmitter/receiver modules described in Section 2.2.4 have to be replaced with new transceiver modules using amplitude modulation (AM). This replacement has forced us to design a new board that uses the desired modulation and supports up to 4 receive and 4 transmit antennas. The new testbed is also expected to operate FFT modules for realizing OFDM. Moreover, the optimal adaptive power allocation methods and their suboptimal counterparts for both MIMO and OFDM can also be experimented on this second version of the testbed.
- For further improving the data throughput of the iterative decoder architecture discussed in Section 3.5, the number of the parallel decoders in the inner and/or the outer decoder can be increased. Moreover, some pipelining stages may be inserted into metric and log-likelihood calculations in order to improve the operating clock frequency.
- The serial port connection of the FPGA board with the computer may be upgraded to support USB. This may decrease the data acquisition time and provide real-time data acquisition for an in-depth analysis of the wireless channel properties .
- Rate adaptation over various spatial or frequency dimensions may be implemented with the addition of higher rate modulation schemes to the current QPSK modulation.

In addition to equal power allocation method for MIMO and OFDM systems, we analyzed the achievable capacity for suboptimal algorithms utilized during the distribution of the power over antennas and/or subcarriers. We mainly concentrated on selection of a predetermined number of the possible antennas and allocating the power in equal amounts to these antennas in MIMO systems. The capacity obtained by such a method requires analyzing the random eigenvalue distributions for the channel response matrix. After

some detailed derivations that include second order integral approximations for the capacity expressions we could not manage to obtain a convenient solution to the problem due to mathematical intricacies involved. This is also a problem that remains to be solved in near future.

The observation of PAPR and capacity comparisons under more realistic conditions may also be an interesting extension to the problems described here. The correlation of the subcarriers in the OFDM system and the correlation of the receive antennas in the MIMO system would be nice starting points. The bit error rate performance of the competing methods, with all the coding and modulation schemes included as the system parameters, may also be investigated. Furthermore, realistic nonlinear power amplifier models may also be added in this comparisons.

APPENDICES

Analytical Capacity of Suboptimal Water-Filling Algorithm for OFDM

The instantaneous capacity for a Rayleigh Fading channel is given as,

$$C = B \log_2(1 + \gamma), \quad (\text{A-1})$$

where γ is the instantaneous SNR value and B is the bandwidth of the channel. The distribution of γ is exponential and given as,

$$f_\gamma(\gamma) = \left(\frac{1}{\gamma_0}\right) e^{-\frac{\gamma}{\gamma_0}}, \quad (\text{A-2})$$

where γ_0 is the expected value of γ . Then the expected channel capacity for a Rayleigh fading channel is given as,

$$E_\gamma[C] = \int_0^\infty B \log_2(1 + \gamma) f_\gamma(\gamma) d\gamma, \quad (\text{A-3})$$

$$= \frac{B}{\ln 2} \int_0^\infty \ln(1 + \gamma) \left(\frac{1}{\gamma_0}\right) e^{-\frac{\gamma}{\gamma_0}} d\gamma, \quad (\text{A-4})$$

$$= \frac{B}{\ln 2} e^{1/\gamma_0} E_1(1/\gamma_0). \quad (\text{A-5})$$

In Eqn. A-5, $E_1(x)$ is the E_n function (with $n = 1$) whose definition is given as,

$$E_n(x) \equiv \int_1^\infty \frac{1}{\gamma^n} e^{-x\gamma} d\gamma. \quad (\text{A-6})$$

For obtaining the capacity of a number of selected independent subcarriers' total capacity the following properties will be utilized:

- **Property-1:**

$$\int_0^\infty \ln(1 + \gamma) \left(\frac{1}{\gamma_0}\right) e^{-\frac{k\gamma}{\gamma_0}} d\gamma = \frac{e^{k/\gamma_0}}{k} E_1\left(\frac{k}{\gamma_0}\right). \quad (\text{A-7})$$

- **Property-2:** For n independently distributed random variables, the probability distribution function of the i^{th} smallest variable for ordered statistics is given as,

$$f_{\gamma_{(i)}}(\gamma) = \frac{n!}{(i-1)! \cdot (n-i)!} (F_\gamma(\gamma))^{i-1} (1 - F_\gamma(\gamma))^{n-i} f_\gamma(\gamma), \quad (\text{A-8})$$

$$= i \cdot \binom{n}{i} (F_\gamma(\gamma))^{i-1} (1 - F_\gamma(\gamma))^{n-i} f_\gamma(\gamma). \quad (\text{A-9})$$

Problem-1: What is the capacity when always the best channel (that is $\gamma_{(n)}$) is allocated the whole power?

Solution: Using Property-2, the expected value for the capacity of the highest SNR channel is given as,

$$E[\log_2(1 + \gamma_{(n)})] = \frac{B}{\ln 2} \int_0^\infty \ln(1 + \gamma) \frac{n!}{(n-1)! \cdot 0!} (1 - e^{-\frac{\gamma}{\gamma_0}})^{n-1} \left(\frac{1}{\gamma_0} \right) e^{-\frac{\gamma}{\gamma_0}} d\gamma, \quad (\text{A-10})$$

$$= \frac{B}{\ln 2} \int_0^\infty \ln(1 + \gamma) \frac{n!}{(n-1)! \cdot 0!} \cdot \left(\sum_{j=0}^{n-1} \binom{n-1}{j} (-1)^j e^{-\frac{j\gamma}{\gamma_0}} \right) \left(\frac{1}{\gamma_0} \right) e^{-\frac{\gamma}{\gamma_0}} d\gamma, \quad (\text{A-11})$$

$$= \frac{B}{\ln 2} n \sum_{j=0}^{n-1} \binom{n-1}{j} (-1)^j \int_0^\infty \ln(1 + \gamma) \cdot \left(\frac{1}{\gamma_0} \right) e^{-\frac{(1+j)\gamma}{\gamma_0}} d\gamma. \quad (\text{A-12})$$

Now, if we apply the Property-1 on Eqn. A-12, the capacity is given as,

$$E[\log_2(1 + \gamma_{(n)})] = \frac{B}{\ln 2} n \sum_{j=0}^{n-1} \binom{n-1}{j} (-1)^j \frac{1}{1+j} e^{\frac{1+j}{\gamma_0}} E_1 \left(\frac{1+j}{\gamma_0} \right). \quad (\text{A-13})$$

Problem-2: What is the capacity when the k best channels among n independent Rayleigh channels are allocated equal power?

Solution: We need to find $E[\log 2(1 + \gamma_{(t)})]$ for $t = n - k + 1, \dots, n$ and

sum the results for the total capacity. The capacity of an individual channel can be obtained using the Properties-1 and 2 as,

$$E[\log_2(1 + \gamma_{(t)})] = \frac{B}{\ln 2} \int_0^\infty \ln(1 + \gamma) t \binom{n}{t} (1 - e^{-\frac{\gamma}{\gamma_0}})^{t-1} \cdot (e^{-\frac{\gamma}{\gamma_0}})^{n-t} \left(\frac{1}{\gamma_0}\right) e^{-\frac{\gamma}{\gamma_0}} d\gamma, \quad (\text{A-14})$$

$$= \frac{B}{\ln 2} \int_0^\infty \ln(1 + \gamma) t \binom{n}{t} \left(\sum_{j=0}^{t-1} \binom{t-1}{j} (-1)^j e^{-\frac{j\gamma}{\gamma_0}} \right) \cdot (e^{-\frac{\gamma}{\gamma_0}})^{n-t} \left(\frac{1}{\gamma_0}\right) e^{-\frac{\gamma}{\gamma_0}} d\gamma, \quad (\text{A-15})$$

$$= \frac{B}{\ln 2} t \binom{n}{t} \sum_{j=0}^{t-1} \binom{t-1}{j} (-1)^j \int_0^\infty \ln(1 + \gamma) e^{-\frac{j\gamma}{\gamma_0}} \cdot e^{-\frac{(n-t)\gamma}{\gamma_0}} \left(\frac{1}{\gamma_0}\right) (e^{-\frac{\gamma}{\gamma_0}})^{n-t} d\gamma, \quad (\text{A-16})$$

$$= \frac{B}{\ln 2} t \binom{n}{t} \sum_{j=0}^{t-1} \binom{t-1}{j} (-1)^j \frac{1}{j + n - t + 1} \cdot e^{\frac{j+n-t+1}{\gamma_0}} E_1\left(\frac{j + n - t + 1}{\gamma_0}\right). \quad (\text{A-17})$$

Finally, using Eqn. A-17, the capacity for suboptimal subcarrier selection method can be found as,

$$E \left[\sum_{t=n-k+1}^n \log_2(1 + \gamma_{(t)}) \right] = \sum_{t=n-k+1}^n E[\log_2(1 + \gamma_{(t)})]. \quad (\text{A-18})$$

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