ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL

EXTENSIONS OF Z-FUZZY NUMBERS AND NOVEL MULTI CRITERIA DECISION MAKING MODELS

Ph.D. THESIS

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ISTANBUL TEKNİK ÜNİVERSİTESİ ★ LİSANSÜSTÜ EĞİTİM ENSTİTÜSÜ

Z-BULANIK SAYILARIN UZANTILARI VE YENİ ÇOK KRİTERLİ KARAR VERME MODELLERİ

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To my lovely family,

FOREWORD

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ABBREVIATIONS

AHP	**:** Analytic Hierarchy Process
ANP	**:** Analytic Network Process
BWM	**:** Best Worst Method
CoCoSo	**:** Combined Compromise Solution
CODAS	**:** COmbinative Distance-based Assessment
COPRAS	**:** COmplex PRoportional Assessment
CR	**:** Consistency Ratio
DEA	**:** Data Envelopment Analysis
DEMATEL	**:** The Decision Making Trial and Evaluation Laboratory
DFSs	**:** Decomposed Fuzzy Sets
DM	**:** Decision Maker
DWAM	**:** Decomposed Weighted Arithmetic Mean
DWGM	**:** Decomposed Weighted Geometric Mean
EDAS	**:** Evaluation based on Distance from Average Solution
ELECTRE	**:** ELimination Et Choix Traduisant la REalité
GRA	**:** Grey Relational Analysis
GSS	**:** Green Supplier Selection
IVSFSs	**:** Interval-Valued Spherical Fuzzy Sets
IVSWAM	**:** Interval-Valued Spherical Weighted Arithmetic Mean
IVSWGM	**:** Interval-Valued Spherical Weighted Geometric Mean
MABAC	**:** Multi-Attributive Border Approximation area Comparison
MACBETH	**:** Measuring Attractiveness by a Categorical Based Evaluation
MAIRCA	**:** MultiAtributive Ideal-Real Comparative Analysis
MARCOS	**:** Measurement of Alternatives and Ranking according to COmpromise Solution
MCDM	**:** Multi Criteria Decision Making
MULTIMOORA	**:** Multi-Objective Optimization by Ratio Analysis Plus Full Multiplicative Form
ORESTE	**:** Organisation, rangement et Synthese de donnees relarionnelles, in French
PFSs	**:** Picture Fuzzy Sets
PFWAM	**:** Picture Fuzzy Weighted Arithmetic Mean
PFWGM	**:** Picture Fuzzy Weighted Geometric Mean
PROMETHEE	**:** Preference Ranking Organization Method for Enrichment Evaluation
SAW	**:** Simple Additive Weighting
SFSs	**:** Spherical Fuzzy Sets
SFWAM	**:** Spherical Fuzzy Weighted Arithmetic Mean
SFWGM	**:** Spherical Fuzzy Weighted Geometric Mean
SVSFSs	**:** Single-Valued Spherical Fuzzy Sets
TCLS	**:** Transfer Center Location Selection
TODIM	**:** An acronym in Portuguese for iterative multicriteria decision making

TOPSIS **:** Technique for Order Preference by Similarity to an Ideal Solution
VIKOR **:** Višekriterijumsko kompromisno rangiranje Resenje
WASPAS **:** Weighted Aggregated Sum Product Assessment
WPM **:** Weighted Product Method
WSM **:** Weighted Sum Method

SYMBOLS

$\widetilde{\boldsymbol{D}}_{\boldsymbol{DF_W}}^{\boldsymbol{Agg}}$	: Weighted aggregated decomposed fuzzy decision matrix
$\widetilde{\boldsymbol{D}}_{\boldsymbol{PF_W}}^{\boldsymbol{Agg}}$	: Weighted aggregated picture fuzzy decision matrix
$\widetilde{\boldsymbol{D}}_{\boldsymbol{DF}}^{\boldsymbol{Agg}}$	: Aggregated decomposed fuzzy decision matrix
$\widetilde{\boldsymbol{D}}_{\boldsymbol{DZF}}$	: Decomposed fuzzy Z-decision matrix
$\widetilde{\boldsymbol{D}}_{\boldsymbol{DZF}}^{\boldsymbol{Agg}}$	: Aggregated decomposed fuzzy Z-decision matrix
$\widetilde{\boldsymbol{D}}_{\boldsymbol{PF}}^{\boldsymbol{Agg}}$	: Aggregated picture fuzzy decision matrix
$\widetilde{\boldsymbol{D}}_{\boldsymbol{PFZ}}$	: Picture fuzzy Z-decision matrix
$\widetilde{\boldsymbol{D}}_{\boldsymbol{PFZ}}^{\boldsymbol{Agg}}$	: Aggregated picture fuzzy Z-decision matrix
$\widetilde{\boldsymbol{Z}}'$	: Ordinary fuzzy number converted from Z-number
$\widetilde{\boldsymbol{Z}}_{\boldsymbol{DF}}(\widetilde{\boldsymbol{A}}, \widetilde{\boldsymbol{R}})$	: Decomposed fuzzy Z-number
$\widetilde{\boldsymbol{Z}}'_{\boldsymbol{DF}}$	: Decomposed fuzzy number converted from decomposed fuzzy Z-number
$\widetilde{\boldsymbol{Z}}_{\boldsymbol{PF}}(\widetilde{\boldsymbol{A}}, \widetilde{\boldsymbol{R}})$	: Picture fuzzy Z-number
$\widetilde{\boldsymbol{Z}}'_{\boldsymbol{PF}}$	: Picture fuzzy number converted from picture fuzzy Z-number
$\boldsymbol{C}_{\boldsymbol{i}}^{*}$	: Closeness coefficient
$\boldsymbol{NIS_{DF}}$	: Decomposed fuzzy negative ideal solution
$\boldsymbol{NIS_{PF}}$	: Picture fuzzy negative ideal solution
$\widetilde{\boldsymbol{PFM}}$	: Picture fuzzy mean vector
$\boldsymbol{PIS_{DF}}$	: Decomposed fuzzy positive ideal solution
$\boldsymbol{PIS_{PF}}$	: Picture fuzzy positive ideal solution
$\boldsymbol{S}^{-}$	: Negative separation measure
$\boldsymbol{S}^{*}$	: Positive separation measure
$\boldsymbol{w_j}$	: Weight of j^{th} criterion
$\boldsymbol{Z}(\widetilde{\boldsymbol{A}}, \widetilde{\boldsymbol{R}})$	: Z-number
$\boldsymbol{def}\widetilde{\boldsymbol{R}}$	: Defuzzified reliability function
$\boldsymbol{\eta}$	: Neutral membership degree
$\boldsymbol{\mu}$	: Membership degree
$\boldsymbol{\vartheta}$	: Non-membership degree

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EXTENSIONS OF Z-FUZZY NUMBERS AND NOVEL MULTI CRITERIA DECISION MAKING MODELS

SUMMARY

The ordinary fuzzy sets are based on the fact that the belonging of an element to a set can take values between 0 and 1, and they emerged due to the incapability of classical sets to describe uncertainty in human thought. After fuzzy sets were introduced to the literature, it began to propose more than one parameter to define uncertainty. For example, while ordinary fuzzy sets use only membership functions, intuitionistic and Pythagorean fuzzy sets use membership and non-membership functions; neutrosophic, picture, and spherical fuzzy sets use membership, non-membership and indeterminacy functions. Although all these fuzzy sets have different properties and conditions for defining uncertainty, they are unable to define the reliability degrees of judgments.

Z-fuzzy numbers allow judgments to be defined not only with a restriction function but also with their reliability degrees. In this thesis, extensions of Z-numbers have been proposed to the literature by integrating fuzzy set extensions with Z-numbers. Thus, novel Z-numbers have been presented to the literature for defining uncertainty, and fuzzy sets have been given the ability to represent reliability under their own properties and conditions. In addition, multi-criteria decision-making (MCDM) methods have been expanded by using ordinary fuzzy Z-numbers and these new fuzzy Z-numbers. Thus, new Z-fuzzy MCDM methods have been introduced to the literature. For this purpose, in the first three chapters, new Z-fuzzy MCDM methods are presented such as Z-CODAS, Z-AHP and Z-EDAS methods. In other three chapters, decomposed fuzzy Z-numbers, picture fuzzy Z-numbers, and interval-valued spherical fuzzy Z-numbers have been developed and integrated with different MCDM methods. Each chapter is summarized below:

In Chapter 2, the COmbinative Distance-based ASsessment (CODAS) method, which is a method based on Euclidean and Taxicab distances, is expanded with Z-numbers and introduced to the literature. The proposed Z-CODAS method has been applied to the supplier selection problem. For this purpose, firstly, decision criteria are weighed based on Z-pairwise comparison matrices. Then, the obtained criteria weights are integrated into the Z-CODAS method and used to rank alternative suppliers. The obtained results are compared with the ordinary fuzzy simple additive weighting (SAW) method.

Chapter 3 presents a multi-experts MCDM method for evaluating social sustainable development factors. The proposed approach integrates Z-numbers and AHP method and may guide many sustainable development researches. In this study, Z-numbers have been used for the first time to evaluate social sustainable development factors. In addition, the other contribution of the study is presenting the Z-AHP method with multi-experts which can be useful for the solution of many MCDM problems containing uncertainty. The proposed Z-AHP method allows pairwise evaluations to be represented with their reliability degrees and integrated into the calculations.

Chapter 4 extends the Evaluation based on Distance from Average Solution (EDAS) method to the Z-EDAS method. In this chapter, a decision making methodology is proposed by the integration of Z-AHP method and Z-EDAS method. The practicality of the proposed methodology is presented with an application on wind turbine selection problem. The comparative analysis conducted with Z-TOPSIS method demonstrates that the usefulness and competitiveness of the proposed methodology are provided. The results show that proposed methodology can both represent decision makers' judgments extensively, and reveal a logical ranking results related to alternatives by the usage of reliability information.

In Chapter 5, decomposed fuzzy Z-numbers, which are the integration of decomposed fuzzy sets (DFSs) and Z-numbers, are introduced to model functional and dysfunctional judgments in a reliable decision environment. Collecting judgments under the circumtances of Z-numbers from experts using functional and dysfunctional questions can provide more consistent and reliable decision environment. In this chapter, a new decomposed fuzzy Z-linguistic scale and defuzzification formula are introduced. Then, decomposed fuzzy Z-TOPSIS method is developed for the solutions of MCDM problems under uncertainty. An application on transfer center location selection for a private cargo company in Marmara Region of Turkey is presented. The effect of the reliability parameter on the results is analyzed.

Chapter 6 presents a decision methodology that integrates the picture fuzzy Z-AHP (PF Z-AHP) method for weighting criteria and a novel PF Z-TOPSIS method for ranking the alternatives. Although the picture fuzzy TOPSIS methods are used to model decision makers' hesitancy in their evaluations, adding reliability degrees to these evaluations can provide better solutions and reliable decision environments for real-life applications. In order to analyze the utility of the proposed PF Z-AHP&TOPSIS methodology, it is applied for solar energy panel selection problem. The sensitivity and comparative analyses are also performed to analyze given decisions and the effects of Z-numbers on the results.

In Chapter 7, a new interval-valued spherical fuzzy (IVSF) Z-number is developed combining the ability of SFSs to allow the assignment of membership degrees in a wider domain with the ability of Z-numbers to represent reliability. In addition, a novel Interval-valued Spherical Fuzzy Z-Analytic Hierarchy Process (IVSF Z-AHP) is proposed by integrating the IVSF Z-numbers and AHP method. Then, a new IVSF Z linguistic scale and a new defuzzification formula are proposed. The proposed IVSF Z-AHP method is applied for green supplier selection problem to show the practicality and applicability of the method. Comparative analysis and sensitivity analysis show the necessity of reliability information in decision making.

In summary, in this thesis, new extensions of Z-numbers and new fuzzy MCDM methods integrated with these extensions are proposed to the literature. Then, the proposed method and methodologies have been applied to various decision-making problems to demonstrate their practicality. In order to show the importance of reliability information, this information has been ignored and the problems have been resolved with the same data and it has been investigated whether the rankings of the alternatives changed. The results and the analyzes provide evidence that reliability information has the potential to change the rankings of alternatives. Especially when the reliability degrees of experts' judgments are wanted to be considered in the decisions, managers or practitioners can use the proposed approaches in this thesis to produce more reliable and meaningful solutions to their problems. In further researches, many different extensions of Z-numbers can be developed and compared with the results of the methods proposed in this thesis.

Z-BULANIK SAYILARIN UZANTILARI VE YENİ ÇOK KRİTERLİ KARAR VERME MODELLERİ

ÖZET

Bulanık kümeler, klasik kümelerin insan düşüncesindeki belirsizliği tanımlamadaki yetersizliği nedeniyle ortaya çıkmış olup, bir elemanın bir kümeye aitliği 0 ile 1 arasında değişen değerler alabilmesi temeline dayanır. Bulanık kümeler literatüre tanıtıldıktan sonra belirsizliği tanımlamada birden fazla parametreye ihtiyaç duyulmaya başlanmıştır. Örneğin, sıradan bulanık kümeler sadece üye olma fonksiyonu kullanırken sezgisel (intuitionistic) ve Pisagor bulanık kümeler üye olma ve olmama fonksiyonunu, nötrosofik (Neutrosophic), resim (Picture) ve küresel (spherical) bulanık kümeler üye olma, üye olmama ve kararsızlık/tanımsızlık fonksiyonlarını kullanmaktadır. Ortaya atılan tüm bu bulanık kümeler kendi içerisinde belirsizliği tanımlamada farklı özellik ve koşullara sahip olsa da sahip oldukları parametreler yargılara ilişkin güvenilirlik derecesini tanımlamaktan yoksundur.

Z-bulanık sayılar yargıların sadece sınırlayıcı bir fonksiyonla değil aynı zamanda ona ilişkin güvenilirlik derecesi ile birlikte tanımlanmasına olanak sağlamaktadır. Bu tezde, sıradan bulanık kümelerin uzantıları Z-bulanık sayılarla entegre edilerek, literatüre Z-bulanık sayıların uzantıları ortaya atılmıştır. Böylece, belirsizliği tanımlamada hem bulanık kümelere kendi özellik ve koşulları altında güvenilirliği de temsil etme kabiliyeti kazandırılmış, hem de yeni Z-bulanık sayılar ortaya konmuştur. Buna ek olarak, birden fazla alternatif ve kriterin olması durumunda kullanılan çok kriterli karar verme (ÇKKV) yöntemleri, ortaya atılan yeni Z-bulanık sayılarla genişletilerek literatüre yeni bulanık ÇKKV yöntemleri tanıtılmıştır. Bu amaç doğrultusunda bu tezin kapsamını oluşturan ilk üç bölümde Z-CODAS, Z-EDAS, Z-AHP gibi sıradan bulanık Z-sayılarla entegre edilmiş yeni bulanık ÇKKV yöntemleri sunulmuştur. Diğer üç bölümde ise sırasıyla ayrıştırılmış (decomposed) bulanık Z-sayılar, resim bulanık Z-sayılar, aralık değerli küresel bulanık Z-sayılar geliştirilmiş ve çeşitli ÇKKV yöntemlerine entegre edilmiştir. Her bir bölümde yapılan çalışma aşağıda özetlenmiştir:

Bölüm 2'de Öklid ve Taxicab uzaklıkların dayalı bir yöntem olan "Birleştirilebilir Uzaklık Tabanlı Değerlendirme - COmbinative Distance-based ASsessment" (CODAS) yöntemi sıradan bulanık Z-sayılarla genişletilerek literatüre kazandırılmıştır. Önerilen Z-CODAS yöntemi tedarikçi seçimi problemi için uygulanmıştır. Bu amaçla ilk olarak değerlendirme kriterleri Z-bulanık ikili karşılaştırma matrislerine dayalı olarak elde edilmiştir. Daha sonra elde edilen kriter ağırlıkları Z-CODAS yöntemine entegre edilerek alternatif tedarikçilerin sıralanmasında kullanılmıştır. Elde edilen sonuçlar, önerilen yöntemin doğrulamasını ve geçerliliğini analiz etmek için karşılaştırma analizinde sıradan bulanık "Basit Toplamsal Ağırlıklandırma –Simple Additive Weighting" (SAW) yöntemiyle karşılaştırılmıştır.

Bölüm 3, sosyal sürdürülebilir kalkınma faktörlerinin değerlendirilmesi için çok uzmanlı ÇKKV yaklaşımı sunmaktadır. Z-sayıları ve bulanık Analitik Hiyerarşi

Proses'i (AHP) birleştiren önerilen model, birçok sürdürülebilir kalkınma araştırmasına rehberlik edebilecek sosyal sürdürülebilir kalkınma faktörlerinin ağırlıklandırılmasına ve derecelendirilmesine olanak sağlamaktadır. Z-sayıların sosyal sürdürülebilir kalkınma faktörlerinin ağırlıklandırılması kararında ilk kez kullanılmasının yanı sıra, çalışmanın bir diğer katkısı da birçok problem ve uygulamada faydalı olabilecek Z-sayılarla entegre edilen AHP yönteminin çok uzmanla sunulmasıdır. Z-sayılarla bütünleşik AHP yönteminin en önemli avantajı karar vericilerin güven derecesinin hesaplamalara dahil edilmesine olanak sağlamasıdır.

Bölüm 4, bulanık dilsel ifadelerin temsil yeteneğini güçlendirmek için Z-sayıları kullanarak "Ortalama Çözüm Uzaklığına Dayalı Değerlendirme - Evaluation based on Distance from Average Solution" (EDAS) yöntemini genişletmektedir. Bu bölümde, bulanık ÇKKV problemlerinin çözümü için, kriter ağırlıklarının belirlenmesi için Z-AHP yöntemi ve en iyi alternatifin seçimi için Z-EDAS yöntemi kullanılarak bir karar verme metodolojisi önerilmiştir. Çalışmanın asıl katkısı, yöneticilere belirsiz ve kesin olmayan veriler altında, bu verilerin güvenilirliğini de dikkate alan ÇKKV tabanlı bir karar destek aracı sunmaktır. Önerilen modelin uygulanabilirliği, en iyi rüzgar türbininin seçimini amaçlayan rüzgar enerjisi yatırım problemi ile gösterilmiştir. Önerilen metodolojinin etkinliği ve rekabetçiliği, Z-TOPSIS yöntemi ile karşılaştırmalı bir analiz yapılarak ortaya konmuştur. Sonuçlar, önerilen metodolojinin yalnızca uzmanların değerlendirme bilgilerini kapsamlı bir şekilde temsil etmekle kalmayıp, aynı zamanda güvenilirlik bilgilerini kullanarak rüzgâr türbini alternatifleriyle ilgili mantıksal ve tutarlı bir sırayı ortaya çıkardığını göstermektedir.

Bölüm 5'te, Z-sayılar ile entegre edilen ayrıştırılmış bulanık kümeler (decomposed fuzzy sets) olan ayrıştırılmış bulanık Z-sayılar, güvenilir bir karar ortamında işlevsel ve işlevsel olmayan yargıları modellemek üzere tanıtılmaktadır. Karar vericilerden yargıların işlevsel ve işlevsel olmayan sorulara dayalı olarak hem bulanık kısıtlamalar hem de onların bulanık güvenilirlikleri ile toplanması, uygulamada daha tutarlı ve güvenilir yargıların elde edilmesini sağlamaktadır. Bu bölümde nihai çözüme ulaşmak için yeni bir ayrıştırılmış bulanık Z-dilsel ölçek ve durulaştırma formülü tanıtılmıştır. Daha sonra belirsizlik altında ÇKKV problemlerinin çözümünde kullanılmak üzere ayrıştırılmış bulanık Z-TOPSIS yöntemi geliştirilmiştir. Önerilen yöntem Türkiye'nin Marmara Bölgesi'ndeki özel bir kargo firmasının transfer merkezi yeri seçimi için uygulamıştır. Güvenilirlik parametresinin verilen kararlara etkisi analiz edilmiştir.

Bölüm 6, ikili karşılaştırmalara dayanan resim bulanık Z-AHP (PF Z-AHP) yöntemini ve alternatifleri sıralamak için güvenilirlik bilgilerini içeren yeni bir PF Z-TOPSIS yöntemini birleştiren bir karar destek aracı sunmaktadır. TOPSIS yönteminin resim bulanık uzantıları, uzmanların yargılarındaki belirsizliği modellemek için kullanılsa da, gerçek hayattaki karar problemlerine daha iyi çözümler ve güvenilir bir karar ortamı sağlamak için bu yargılara güvenilirlik derecelerinin eklenmesi gerekmektedir. Önerilen PF Z-AHP&TOPSIS metodolojisi, metodolojinin uygulanabilirliğini ve üstünlüğünü analiz etmek için güneş enerjisi paneli seçim problemine uygulanmıştır. Duyarlılık analizi, önerilen metodolojinin sağlamlığını göstermektedir. Karşılaştırma analizi, Z-sayıları tarafından sunulan güvenilirlik fonksiyonlarının sonuçları etkileyebileceğini göstermiştir.

Bölüm 7'de, küresel bulanık kümelerin daha geniş bir alanda üyelik dereceleri atanmasına izin verme yeteneğini Z-sayıların güvenilirliği temsil etme yeteneği ile birleştiren yeni bir aralık değerli küresel bulanık Z-sayısı geliştirilmiştir. Buna ek olarak, aralık değerli küresel bulanık Z-sayılar ile AHP yönteminin entegre edilmesiyle yeni bir Aralık Değerli Küresel Bulanık Z-AHP (IVSF Z-AHP) yöntemi önerilmiştir. Ayrıca, karar vericilerin ikili karşılaştırma matrisleri için objektif ve tutarlı bir puanlama sağlayan yeni bir IVSF Z-dilsel ölçek ve yeni bir durulaştırma formülü önerilmiştir. Önerilen metodolojinin uygulanabilirliğini göstermek amacıyla yeşil tedarikçi seçimi problemine yönelik bir uygulama yapılmıştır. Karar vermede güvenilirlik bilgisinin gerekliliğini açıkça göstermek için karşılaştırmalı analiz ve duyarlılık analizine yer verilmiştir. Sonuçlar, önerilen yöntemin güvenilirlik ve tereddüt bilgisi ile en iyi alternatifi belirlemede oldukça etkili olduğunu göstermektedir.

Özet olarak, bu tezde, literatüre Z-bulanık sayıların yeni uzantıları ve bu uzantılarla entegre yeni bulanık ÇKKV yöntemleri tanıtılmıştır. Ardından, önerilen metot ve metodolojiler pratikteki uygulanabilirliklerini gösterebilmek için çeşitli karar verme problemlerine uygulanmıştır. Güvenilirlik bilgisinin önemini kanıtlamak amacıyla bu bilgi ihmal edilerek aynı verilerle problemler yeniden çözülmüş ve alternatiflerin sıralamalarının değişip değişmediği araştırılmıştır. Elde edilen sonuçlar ve yapılan analizler, güvenilirlik bilgisinin alternatiflerin sıralamalarını değiştirebilme potansiyeline sahip olduğuna dair kanıtlar sunmaktadır. Özellikle alınacak kararlarda karar vericilerin yargılarına olan güvenilirlik derecesi dikkate alınmak istendiğinde yöneticiler veya uygulayıcılar, problemlerine daha güvenilir ve anlamlı çözümler üretmek için bu tezde önerilen ÇKKV yaklaşımlarını kullanabilirler. Gelecek yıllarda, Z-bulanık sayıların çok daha farklı uzantıları ortaya atılabilir ve bu tezde önerilen yöntemlerin sonuçları ile karşılaştırılabilir.

1. INTRODUCTION

Making decisions is a necessary part of every moment of our lives. During the decision-making process, we express personal ideas and comments via linguistic terms based on our knowledge and experiences. These linguistic terms expressing decision-makers' opinions on a subject contain uncertainty and vagueness by nature of human thoughts. Expressions like "not very clear," "likely", etc. which are widely used in daily or business life, reveal the doubt and vagueness in human thought. Due to the inability of classical sets for modeling this ambiguity, Zadeh (1965) introduced fuzzy set theory. The fuzzy set theory provides mathematical modeling of these vague preferences into the solution process.

Decisions-making is a very simple process when alternatives are evaluated under only one criterion. However, real-life problems may have different degrees of difficulty, and they require the evaluation of alternatives according to several criteria, including conflict and relationships among them. Thus, problems are getting complicated and they need to be evaluated with more comprehensive methods. Multi criteria decision-making (MCDM) methods are commonly used in solving complex decision problems.

To be able to better model people's uncertain subjective evaluations, in recent years, MCDM methods have been expanded by several fuzzy set extensions such as type-2 fuzzy sets, intuitionistic fuzzy sets, hesitant fuzzy sets, Pythagorean fuzzy sets, neutrosophic sets, etc. These fuzzy sets only represent the restriction of ambiguous judgments and do not consider their reliability. However, decision makers may not be absolutely certain about their judgments. Even a doctor's probability of making a correct diagnosis is not 100% (Xian et al., 2019). For example, a doctor can say "you likely have ulcer" to his/her patient. In order to confirm this diagnosis, tests and investigations can be applied in the medical area. However, in many fields that require decision-making, subjective evaluations cannot be verified by that way. Furthermore, when quantitative data are employed in decision-making, these data are treated to be exactly accurate since the sources' reliability level is not asked. However, it would not be accurate to accept the numerical data with 100% certainty due to the concept of time and measurement accuracy factors, etc. In order to model the possible variations that may arise in numerical data, different extensions of fuzzy set theory can be used. However, when linguistic data is used in decision process, it would be most logical to explicitly ask decision-makers' reliability level of their evaluations. Consequently, when linguistic terms representing subjective judgments are used in decision models, it is clear that restrictive information needs to be integrated with reliability information. Z-numbers introduced by Zadeh (2011) allow these judgments to be represented by fuzzy restrictions and their fuzzy reliability. Thus, Z-numbers provide us making computations with numbers that are not absolutely (100%) reliable. In this thesis, new extensions of Z-numbers that integrate reliability information and different fuzzy sets have been introduced in order to better model humans' complex thinking structures. Then, different MCDM methods have been expanded using these extensions to be used for the solutions of real-life problems containing imprecise and ambiguous judgments.

1.1 Purpose of Thesis

The main objective of this thesis is to develop new extensions of Z-numbers such as picture fuzzy Z-numbers, interval-valued spherical fuzzy Z-numbers, decomposed fuzzy Z-numbers. In addition, another purpose of this thesis is to integreate these new Z-numbers with MCDM methods in order to model linguistic judgments of decision makers in a more reliable decision environment. Thus, different MCDM methods have been expanded using ordinary fuzzy Z-numbers and the proposed Z-numbers. The proposed method and methodologies have been applied to different decision making problems to show their practicalities and utilities.

1.2 Impact

The papers related to the purpose of the thesis have been published in various international journals. The articles published within the scope of this thesis are as follows:

- Tüysüz, N., & Kahraman, C. (2020). CODAS method using Z-fuzzy numbers. *Journal of Intelligent & Fuzzy Systems*, *38*(2), 1649-1662.

- Tüysüz, N., & Kahraman, C. (2020). Evaluating social sustainable development factors using multi-experts Z-fuzzy AHP. *Journal of Intelligent & Fuzzy Systems*, *39*(5), 6181-6192.
- Tüysüz, N., & Kahraman, C. (2023). A Novel Z-Fuzzy AHP&EDAS Methodology and Its Application to Wind Turbine Selection. *Informatica*, 34(4), 847-880.
- Tüysüz, N., & Kahraman, C. (2024). A Novel Decomposed Z-Fuzzy TOPSIS Method with Functional and Dysfunctional Judgments: An Applicatıon to Transfer Center Location Selection. *Engineering Applications of Artificial Intelligence*, 127, 107221.
- Tüysüz, N., & Kahraman, C. (2023). An Integrated Picture Fuzzy Z-AHP & TOPSIS Methodology: Application to Solar Panel Selection. *Applied Soft Computing*, 110951.
- Tüysüz, N., & Kahraman, C. (2024). Interval-Valued Spherical Fuzzy Z-AHP Method based on Reliability of Judgments: Green Supplier Selection. *Journal of Multiple-Valued Logic & Soft Computing. (Accepted).*

Published papers in proceedings:

- Yildiz, N., & Kahraman, C. (2020). Evaluation of social sustainable development factors using Buckley's fuzzy AHP based on Z-numbers. In *Intelligent and Fuzzy Techniques in Big Data Analytics and Decision Making: Proceedings of the INFUS 2019 Conference, Istanbul, Turkey, July 23-25, 2019* (pp. 770-778). Springer International Publishing.
- Tüysüz, N., & Kahraman, C. (2023, August). Picture Fuzzy Z-AHP: Application to Panel Selection of Solar Energy. In *International Conference on Intelligent and Fuzzy Systems* (pp. 337-345). Cham: Springer Nature Switzerland.

2. CODAS METHOD USING Z-FUZZY NUMBERS[1]

COmbinative Distance-based ASsessment (CODAS) method was proposed by Ghorabaee et al. (2016) to be used for MCDM problems. According to this method, the best alternative is assessed by considering Euclidean and Taxicab distances from the negative ideal solution. If the incremental Euclidean distances between two alternatives are sufficiently large, a total distance is calculated taking into account the Taxicab distances. The best alternative is the alternative which has farthest total distance from the negative ideal solution.

There are few publications on CODAS method in the literature. Fuzzy CODAS methods can consider linguistic evaluations by transforming them into numerical values. Table 2.1 lists the crisp CODAS papers published in the literature.

Table 2.1 : A literature review on crisp CODAS

Year	Authors	Application area
2016a	Keshavarz Ghorabaee, Zavadskas, Turskis, & Antucheviciene	Most appropriate robot selection
2018a	Badi, Abdulshahed, & Shetwan	Supplier selection in Libia
2018	Mathew and Sahu	Material handling equipment selection
2018b	Badi, Ballem, Shetwan,	Site selection of desalination plant in Libya

Each of the MCDM methods in the literature has their own advantages and disadvantages. The CODAS uses the Euclidean distance as the primary measure of assessment while Taxicab distance is used as the secondary measure of assessment when Euclidean distances of two alternatives are very close to each other. The closeness degree between two Euclidean distances is determined by a threshold parameter. CODAS has been integrated with different fuzzy extensions in a few fields of application in recent years, as given in Table 2.2. Few publications on fuzzy CODAS method are summarized in Table 2.2.

[1] This chapter is based on the paper "Tüysüz, N., & Kahraman, C. (2020). CODAS method using Z-fuzzy numbers. Journal of Intelligent & Fuzzy Systems, 38(2), 1649-1662."

In Figure 2.1, the subject areas of the papers on CODAS method are illustrated. Business, management and accounting, computer science, and economics, econometrics and finance are the top three areas with 75% while mathematics and decision sciences share the second and third ranks with 16.7% and 8.3%, respectively.

Table 2.2 : A literature review on fuzzy CODAS

Year	Authors	Extension of CODAS	Application area
2017	Keshavarz Ghorabaee, Amiri, E.K., R., & Antucheviciene	Fuzzy CODAS	Market segment evaluation
2018	Panchal, Chatterjee, Shukla, Choudhury, & Tamosaitiene	Fuzzy CODAS	Maintenance decision problem
2018	Ren	IF-CODAS	Energy storage technologies for promoting the development of renewable energy
2018	Peng & Garg	Interval valued fuzzy soft CODAS	A case study in mine emergency decision making
2018	Bolturk	Pythagorean fuzzy CODAS	Supplier selection
2018	Bolturk & Kahraman	Interval-valued intuitionistic fuzzy CODAS	Wave energy facility location selection
2018	Pamučar, D., Badi, I., Sanja, K., Obradović, R.	Linguistic Neutrosophic CODAS	Optimal Power-Generation Technology Selection (PGT) in Libya.

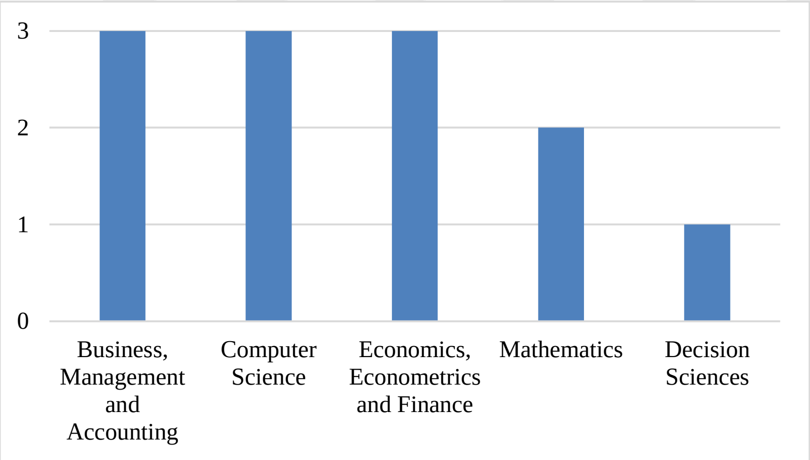

Figure 2.1 : Subject areas of the CODAS papers.

As it is seen from Table 2.2, the crisp CODAS method has been extended to several extensions of ordinary fuzzy sets under uncertainty. One of the possible extensions is to extend the crisp CODAS method by using Z-fuzzy numbers under uncertainty. Z-fuzzy numbers are defined by both a restriction function and a reliability function, each having its own membership function. Z-fuzzy numbers have been employed in the development of fuzzy extensions of several MCDM methods such as Z-fuzzy AHP and Z-fuzzy TOPSIS.

Kang et al. (2012b) proposed a linguistic MCDM method using Z-fuzzy numbers and applied it for a vehicle selection problem. Azadeh et al. (2013) proposed an AHP method using Z-fuzzy numbers. They determined the weights of criteria to assess the performance of universities. Sahrom and Dom (2015) integrated AHP and DEA method for the risk assesment problem. AHP method is used to determine the weights of criteria and Z-fuzzy DEA method is used for ranking the risk priority of 20 bridge structures. They used Kang et al. (2012a)'s approach for converting Z-fuzzy numbers to classical fuzzy numbers in the Z-fuzzy DEA method. Yaakob and Gegov (2015) modificated TOPSIS method using Z-fuzzy numbers by expanding a fuzzy rule based approach in MCDM. They showed that the proposed approach is a successfully applicable method to express vagueness in decision making. Azadeh and Kokabi (2016) proposed a new DEA method using Z- fuzzy numbers for the portfolio selection problem. They transformed Z-fuzzy DEA method to linear possibility programming and obtained a crisp linear programming model using α-cut approach. Sadi-Nezhad and Sotoudeh-Anvari (2016) proposed a new DEA using Z-fuzzy numbers. Decision makers indicate the opinion with linguistic terms. They used trapezoidal and triangular fuzzy numbers for the first and second components of Z-fuzzy numbers, respectively. They indicated that the DEA with Z-fuzzy data can be effectively used for solution of real-world problems. Yaakob and Gegov (2016) presented a modified TOPSIS method using Z-fuzzy numbers which is called Z-TOPSIS. They applied Z-TOPSIS algorithm for stock selection problem and showed its effectiveness. Peng and Wang (2017) introduced hesitant uncertain linguistic Z-fuzzy numbers (HULZNs) for MCDM problems under uncertainty. They extended VIKOR method using HULZNs and applied for ERP selection problem. Khalif et al. (2017a) presented a fuzzy similarity based TOPSIS method using Z-fuzzy numbers and applied it for performance assessment problem. Khalif et al. (2017b) presented a hybrid fuzzy MCDM model using z-fuzzy numbers and applied it to select the most appropriate staff in recruitment. Wang et al. (2017) extended TODIM method with Choquet integral using Z-fuzzy numbers. They used it in the evaluation of medical inquiry applications. Karthika and Sudha (2018) applied F-AHP method using Z-fuzzy numbers for risk assessment and they decided the best safety measure for the disease. They used triangular fuzzy numbers for the components of Z-fuzzy numbers. Then they added the second component to first component using centroid method. Forghani et al. (2018) proposed a supplier selection model for pharmaceutical companies using Principal component analysis (PCA), Z-TOPSIS and MILP. PCA method is used to reduce the number of supplier selection criteria. Importance value of each supplier is obtained using Z-TOPSIS method. They finally used these values for the mixed integer linear programming model. Chatterjee and Kar (2018) proposed COPRAS method using Z-fuzzy numbers for renewable energy selection. Aboutorab et al. (2018) improved the best-worst method using Z-fuzzy numbers in order to overcome the uncertain expressions. Peng and Wang (2018) extended Z-fuzzy MULTIMOORA method to handle multi criteria group decision making problems. They used it in the evaluation of potential areas of air pollution. Shen and Wang (2018) proposed a modified VIKOR method using Z-fuzzy numbers. They applied it in the selection of economic development plan.

In the extensions of ordinary fuzzy sets such as type-2 fuzzy sets, hesitant fuzzy sets and intuitionistic fuzzy sets, decision makers try to reflect the uncertainty in their mind through membership functions. In type-2 fuzzy sets, three dimensional membership functions are used. In hesitant fuzzy sets, more than one membership degrees can be assigned for a certain x value. In intuitionistic fuzzy sets, decision makers' hesitancy depends on the sum of membership and non-membership degrees, providing that this sum is equal to at most 1.

Alternatively, Z-fuzzy numbers can take into account the uncertainty in decision makers' mind through a reliability function, which express how confident they are about their evaluations. Z-fuzzy numbers have been very popular after they are introduced by Zadeh (2011). Figure 2.2 illustrates the frequencies of Z-fuzzy number publications with respect to the years. There is a clear acceleration in the frequencies of Z-fuzzy number publications after the year 2014.

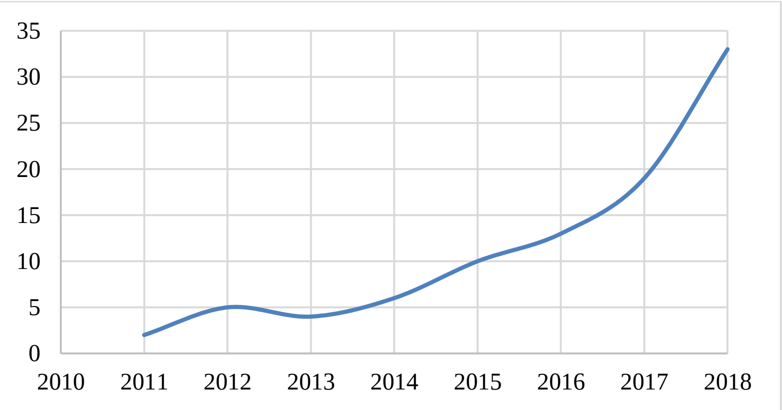

Figure 2.2 : Frequencies of Z-fuzzy publications with respect to the years.

Figure 2.3 shows the frequencies of Z-fuzzy number publications with respect to their subject areas. The top three subject areas that Z-fuzzy numbers are used are computer science, mathematics, and engineering, respectively.

The organization of the paper is as follows. In Section 2.1, Z-fuzzy numbers are explained in detail. In Section 2.2, the classical CODAS and proposed Z-fuzzy CODAS methods are presented. In Section 2.3, an application is presented for a supplier selection problem. In Section 2.4, a comparative analysis is performed with fuzzy simple additive weighting method. Finally, conclusions are given.

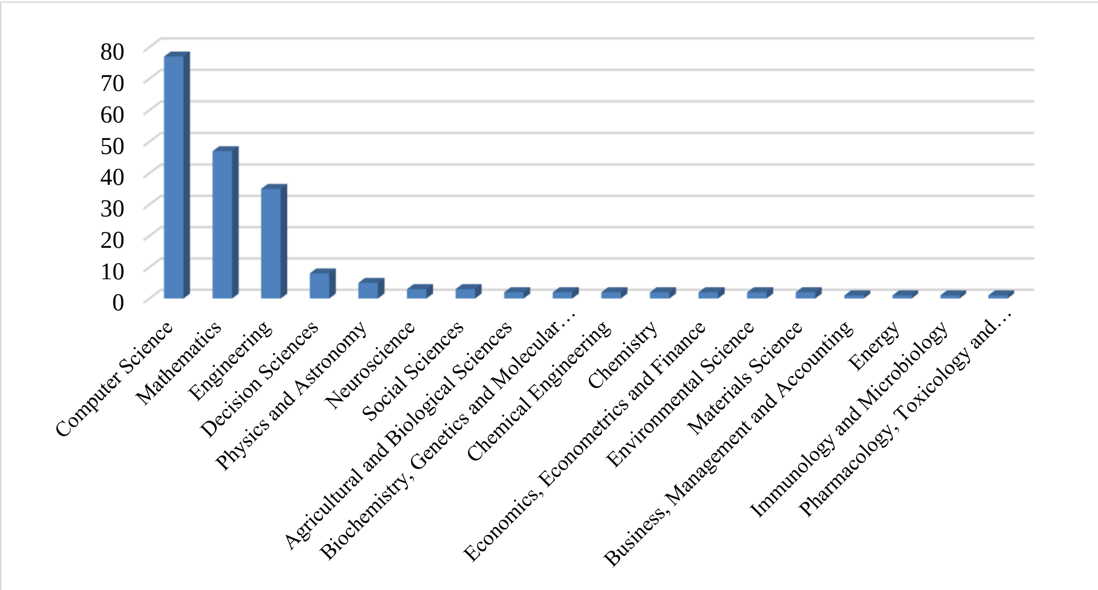

Figure 2.3 : Frequencies of Z-fuzzy publications with respect to the years.

2.1 Z-Fuzzy Numbers

Zadeh (2011) introduced the Z-fuzzy numbers to the literature in 2011. A Z-fuzzy number is an ordered pair of fuzzy numbers, Z$(\tilde{A},\tilde{B})$ as given in Figure 2.4. The first component $\tilde{A}$ is a restriction function whereas the second component $\tilde{B}$ is a measure of reliability for the first component.

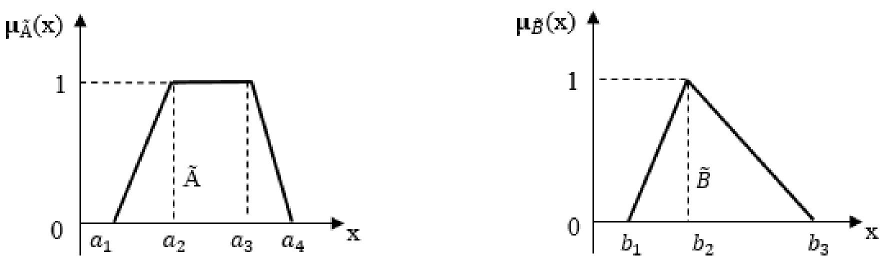

Figure 2.4 : A simple Z-fuzzy number, $Z(\tilde{A},\tilde{B})$.

The concept of a Z-fuzzy number is intended to provide a basis for computation with ordinary fuzzy numbers which are not reliable.

Definition 2.1. Let a fuzzy set A be defined on a universe X may be given as: $\mathrm{A} = \{\langle \mathrm{x}, \mu_{\mathrm{A}}(\mathrm{x})\rangle \mid \mathrm{x} \epsilon \mathrm{X}\}$ where $\mu_{\mathrm{A}}: \mathrm{X} \rightarrow [0,1]$ is the membership function A. The membership value $\mu_{\mathrm{A}}(\mathrm{x})$ describes the degree of belongingness of $\mathrm{x} \in \mathrm{X}$ in A. The Fuzzy Expectation of a fuzzy set is denoted as:

$$E_A(x) = \int_x \; x\mu_A(x)\, dx \tag{2.1}$$

which is not the same as the meaning of the Expectation of Probability Space. It can be considered as the Information Strength supporting the fuzzy set A.

Definition 2.2: Converting Z-fuzzy number to Regular Fuzzy Number

Consider a Z-fuzzy number $Z = (\widetilde{A}, \widetilde{R})$, which is described by Figure 2.4. The figure on the left is the part of restriction, and the figure on the right is the part of reliability. Let $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x)\rangle | \mu(x) \in [0,1]\}$ and $\tilde{R} = \{\langle x, \mu_{\tilde{R}}(x)\rangle | \mu(x) \in [0,1]\}$, $\mu_{\tilde{A}}(x)$ is a trapezoidal membership function, $\mu_{\tilde{R}}(x)$ is a triangular membership function.

(1) Convert the second part (reliability) into a crisp number.

$$\alpha = \frac{\int x\mu_{\tilde{R}}(x)\, dx}{\int \mu_{\tilde{R}}(x)\, dx} \tag{2.2}$$

where $\int$ denotes an algebraic integration.

Alternatively, the defuzzification equation $(a_1 + 2 * a_2 + 2 * a_3 + a_4)/6$ for symmetrical trapezoidal fuzzy numbers and $(a_1 + 2 * a_2 + a_3)/4$ for symmetrical triangular fuzzy numbers can be used.

(2) Add the weight of the second part (reliability) to the first part (restriction). The weighted Z-fuzzy number can be denoted as

$$\tilde{Z}^{\alpha} = \{\langle x, \mu_{\tilde{A}^{\alpha}}(x)\rangle | \mu_{\tilde{A}^{\alpha}}(x) = \alpha\mu_{\tilde{A}}(x), \mu(x) \in [0,1]\} \tag{2.3}$$

(3) Convert the Z fuzzy number (weighted restriction) to ordinary fuzzy number. The ordinary fuzzy set can be denoted as in Eq. (2.4)

$$\tilde{Z}' = \left\{\langle x, \mu_{\tilde{Z}'}(x)\rangle \middle| \mu_{\tilde{Z}'}(x) = \mu_{\tilde{A}}\left(\frac{x}{\sqrt{\alpha}}\right), \mu(x) \in [0,1]\right\} \tag{2.4}$$

$\tilde{Z}'$ has the same Fuzzy Expectation with $\tilde{Z}^{\alpha}$, and they are equal with respect to Fuzzy Expectation, which can be denoted by Figure 2.5.

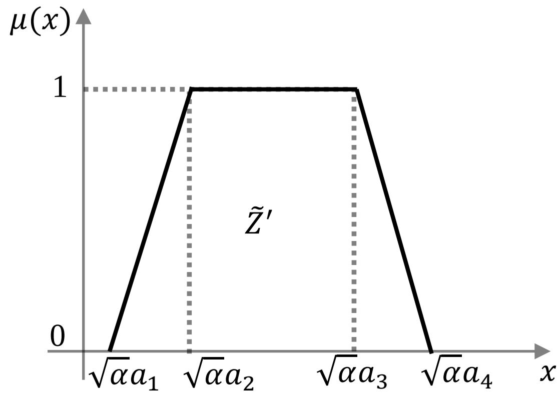

Figure 2.5 : Ordinary fuzzy number converted from Z-fuzzy number.

(4) If the restriction function and reliability function are defined as in Figure 2.6 (their heights may be any value between 0 and 1), the calculations are modified as follows:

Let $\tilde{A}_\delta = \{\langle x, (\mu_{\tilde{A}}(x);\ \delta)\rangle | \mu(x) \in [0,1]\}$ and $\tilde{R}_\beta = \{\langle x, (\mu_{\tilde{R}}(x);\ \beta)\rangle | \mu(x) \in [0,1]\}$, $\mu_{\tilde{A}}^{\delta}(x)$ is a trapezoidal membership function, $\mu_{\tilde{R}}^{\beta}(x)$ is a triangular membership function.

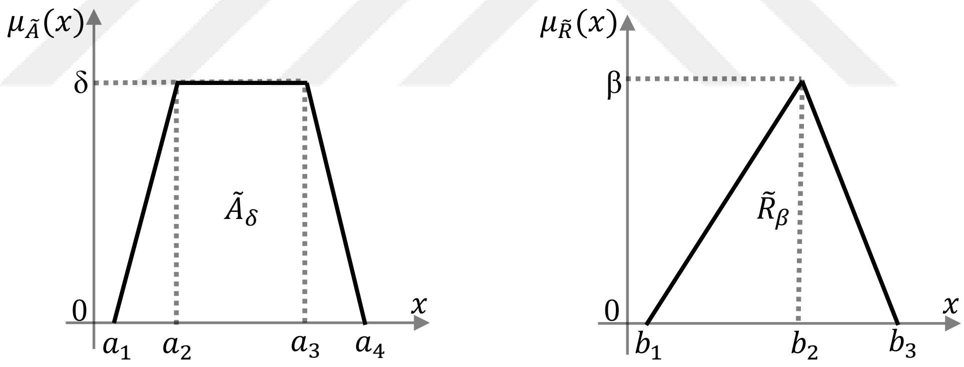

Figure 2.6 : A simple $\tilde{Z}_{\delta,\beta}$ number, $\tilde{Z}_{\delta,\beta} = (\tilde{A}_\delta, \tilde{R}_\beta)$

In this case, restriction and reliability functions are defined as in Eqs. (2.5-2.6), respectively. The reliability membership function in Eq. (2.6) is substituted into the defuzzification formula (Eq. (2.2)); so that, Eq. (2.7) is obtained.

$$\mu_{\tilde{A}}^{\delta}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}\ \delta\,, if a_1 \le x \le a_2 \\ \delta, \quad if a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3}\ \delta, if a_3 \le x \le a_4 \\ 0, \quad Otherwise \end{cases} \tag{2.5}$$

$$\mu_{\tilde{R}}^{\beta}(x) = \begin{cases} \frac{x-b_1}{b_2-b_1}\ \beta\ , if b_1 \le x \le b_2 \\ \frac{b_3-x}{b_3-b_2}\ \beta\ , if b_2 \le x \le b_3 \\ 0, \qquad Otherwise \end{cases} \tag{2.6}$$

Thus, we have

$$\sqrt{\alpha} = \sqrt{\frac{\int x\boldsymbol{\mu}_{\tilde{R}}^{\boldsymbol{\beta}}(x)\,dx}{\int \boldsymbol{\mu}_{\tilde{R}}^{\boldsymbol{\beta}}(x)\,dx}} \tag{2.7}$$

Then, the weighted $\tilde{Z}_{\delta,\beta}$ number can be denoted as in Eq. (2.8).

$$\tilde{Z}_{\delta,\beta}^{\alpha} = \left\{ \langle x, \mu_{\tilde{A}^{\alpha}}^{\delta}(x) \rangle \middle| \mu_{\tilde{A}^{\alpha}}^{\delta}(x) = \frac{\int x\mu_{\tilde{R}}^{\beta}(x)\,dx}{\int \mu_{\tilde{R}}^{\beta}(x)\,dx} \mu_{\tilde{A}}^{\delta}(x), \mu(x) \in [0,1] \right\} \tag{2.8}$$

The ordinary fuzzy number converted from Z-fuzzy number can be given as in Eq. (2.9).

$$\tilde{Z}'_{\delta,\beta} = \left\{ \langle x, \mu_{\tilde{Z}'}^{\delta}(x) \rangle \middle| \mu_{\tilde{Z}'}^{\delta}(x) = \mu_{\tilde{A}}^{\delta}\left(x\ \frac{\int \mu_{\tilde{R}}^{\beta}(x)\,dx}{\int x\mu_{\tilde{R}}^{\beta}(x)\,dx} \right),\ \mu(x) \in [0,1] \right\} \tag{2.9}$$

2.2 Classical CODAS and Z-Fuzzy CODAS

2.2.1 Classical CODAS

Step 1. Construct the decision-making matrix (X), shown as in Eq. (2.10).

$$X = [x_{ij}]_{n\times m} = \begin{array}{c} \\ \boldsymbol{A_1} \\ \boldsymbol{A_2} \\ \vdots \\ \boldsymbol{A_n} \end{array} \begin{array}{ccccc} \boldsymbol{C1} & \boldsymbol{C2} & \boldsymbol{C3} & \cdots & \boldsymbol{C_m} \\ x_{11} & x_{12} & x_{13} & \cdots & x_{1m} \\ x_{21} & x_{22} & x_{23} & \cdots & x_{2m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & x_{n3} & \cdots & x_{nm} \end{array} \tag{2.10}$$

where x_{ij} ($x_{ij} \ge 0$) denotes the performance value of ith alternative on jth criterion

($i \in \{1,2,\dots,n\}$ and $j \in \{1,2,\dots,m\}$).

Step 2. Calculate the normalized decision matrix. Performance values are calculated using linear normalization as in Eq. (2.11).

$$n_{ij} = \begin{cases} \frac{x_{ij}}{\max_i x_{ij}} & if\ j \in N_b \\ \frac{\min_i x_{ij}}{x_{ij}} & if\ j \in N_c \end{cases} \tag{2.11}$$

where N_b and N_c represent the benefit and cost criteria, respectively.

Step 3. Calculate the weights of the criteria by using Saaty's pairwise comparison matrix as in Eq. (2.12).

$$W = [w]_{m\times m} = \begin{bmatrix} 1 & \frac{\mathrm{w}_1}{\mathrm{w}_2} & \frac{\mathrm{w}_1}{\mathrm{w}_3} & \frac{\mathrm{w}_1}{\mathrm{w}_4} & \cdots & \frac{\mathrm{w}_1}{\mathrm{w}_\mathrm{m}} \\ \frac{\mathrm{w}_2}{\mathrm{w}_1} & 1 & \frac{\mathrm{w}_2}{\mathrm{w}_3} & \frac{\mathrm{w}_2}{\mathrm{w}_4} & \cdots & \frac{\mathrm{w}_2}{\mathrm{w}_\mathrm{m}} \\ \frac{\mathrm{w}_3}{\mathrm{w}_1} & \frac{\mathrm{w}_3}{\mathrm{w}_2} & 1 & \frac{\mathrm{w}_3}{\mathrm{w}_3} & \cdots & \frac{\mathrm{w}_3}{\mathrm{w}_\mathrm{m}} \\ \frac{\mathrm{w}_4}{\mathrm{w}_1} & \frac{\mathrm{w}_4}{\mathrm{w}_2} & \frac{\mathrm{w}_4}{\mathrm{w}_3} & 1 & \cdots & \frac{\mathrm{w}_4}{\mathrm{w}_\mathrm{m}} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\mathrm{w}_\mathrm{m}}{\mathrm{w}_1} & \frac{\mathrm{w}_\mathrm{m}}{\mathrm{w}_2} & \frac{\mathrm{w}_\mathrm{m}}{\mathrm{w}_3} & \frac{\mathrm{w}_\mathrm{m}}{\mathrm{w}_4} & \cdots & 1 \end{bmatrix} \tag{2.12}$$

From Eq. (2.12), we obtain $W = (w_{C1}, w_{C2}, w_{C3}, \dots, w_{Cm})$.

Step 4. Calculate the weighted normalized decision matrix as in Eq. (2.13).

$$r_{ij} = w_j n_{ij} \tag{2.13}$$

where w_j ($0 < w_j < 1$) denotes the weight of jth criterion, and $\sum_{j=1}^{m} w_j = 1$

Step 5. Determine the negative-ideal solution as in Eqs. (2.14) and (2.15).

$$ns = \left[ns_j\right]_{1\times m} \tag{2.14}$$

$$ns_j = \min_i r_{ij} \tag{2.15}$$

Step 6. Calculate the Euclidean and Taxicab distances of alternatives from the negative-ideal solution, shown as in Eqs. (2.16) and (2.17).

$$E_i = \sqrt{\sum_{j=1}^{m} (r_{ij} - ns_j)^2} \tag{2.16}$$

$$T_i = \sum_{j=1}^{m} \left| r_{ij} - ns_j \right| \tag{2.17}$$

Step 7. Construct the relative assessment matrix, shown as in Eqs. (2.18) and (2.19).

$$Ra = [h_{ik}]_{n\times n} \tag{2.18}$$

$$h_{ik} = (E_i - E_k) + (\psi(E_i - E_k) \times (T_i - T_k)) \tag{2.19}$$

where $k \in \{1,2,\dots,n\}$ and ψ denotes a threshold function to recognize the equality of the Euclidean distances of two alternatives and is defined as in Eq. (2.20).

$$\psi(x) = \begin{cases} 1 & if \quad |x| \geq \tau \\ 0 & if \quad |x| < \tau \end{cases} \tag{2.20}$$

where τ is a threshold parameter set by decision-makers. It is recommended to set this parameter at a value between 0.01 and 0.05. If the difference between Euclidean distances of two alternatives is larger than τ, these two alternatives are also compared by Taxicab distance. In this study, we set τ=0.02 in the application section.

Step 8. Calculate the assessment score of each alternative, shown as in Eq. (2.21).

$$H_i = \sum_{k=1}^{n} h_{ik} \tag{2.21}$$

Step 9. Rank the alternatives according to assessment score (H_i). The alternative which has the highest H_i is the best choice among the alternatives.

2.2.2 Z-Fuzzy CODAS

Step 1. Construct the Z-fuzzy decision matrices as given by Eq. (2.22) and aggregate them using the geometric mean operation.

$$\widetilde{D}_Z = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \begin{array}{ccccc} C1 & C2 & C3 & \cdots & C_m \\ Z_{11}(\tilde{A},\tilde{B}) & Z_{12}(\tilde{A},\tilde{B}) & Z_{13}(\tilde{A},\tilde{B}) & \cdots & Z_{1m}(\tilde{A},\tilde{B}) \\ Z_{21}(\tilde{A},\tilde{B}) & Z_{22}(\tilde{A},\tilde{B}) & Z_{23}(\tilde{A},\tilde{B}) & \cdots & Z_{2m}(\tilde{A},\tilde{B}) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ Z_{n1}(\tilde{A},\tilde{B}) & Z_{n2}(\tilde{A},\tilde{B}) & Z_{n3}(\tilde{A},\tilde{B}) & \cdots & Z_{nm}(\tilde{A},\tilde{B}) \end{array} \tag{2.22}$$

where $Z_{11}(\tilde{A},\tilde{B})$ $(Z_{11}(\tilde{A},\tilde{B}) \geq 0)$ denotes the Z-fuzzy performance value of ith alternative on jth criterion ($i \in \{1,2,\dots,n\}$ and $j \in \{1,2,\dots,m\}$).

Tables 2.3 and 2.4 can be used for the restriction scale and reliability scale, respectively.

Table 2.3 : Restriction Scale

Linguistic Terms	Abbr.	Z-fuzzy restriction function
Very Poor	VP	(1/4,1/2,1/2,1;1)
Poor	P	(1/2,1,1,3;1)
Medium Poor	MP	(1,3,3,5;1)
Fair	F	(3,5,5,7;1)
Medium good	MG	(5,7,7,9;1)
Good	G	(7,9,9,10;1)
Very good	VG	(9,10,10,10;1)

Table 2.4 : Reliability Scale

Linguistic Reliability	Abbr.	Triangular Z-fuzzy reliability function
Certainly Reliable	CR	(1, 1, 1; 1)
Very Strongly Reliable	VSR	(0.8, 0.9, 1; 1)
Strongly Reliable	SR	(0.7, 0.8, 0.9; 1)
Very Highly Reliable	VHR	(0.6, 0.7, 0.8; 1)
Highly Reliable	HR	(0.5, 0.6, 0.7; 1)
Fairly Reliable	FR	(0.4, 0.5, 0.6; 1)
Weakly Reliable	WR	(0.3, 0.4, 0.5; 1)
Very Weakly Reliable	VWR	(0.2, 0.3, 0.4; 1)
Strongly Unreliable	SU	(0.1, 0.2, 0.3; 1)
Absolutely Unreliable	AU	(0, 0.1, 0.2; 1)

Step 2. Convert Z-fuzzy performance values to regular fuzzy numbers $\tilde{Z}'$ using Eq. (2.4). Then, Eq. (2.22) becomes Eq. (2.23):

$$\widetilde{D}_{\tilde{Z}'} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \begin{array}{ccccc} C1 & C2 & C3 & \cdots & C_m \\ \tilde{Z}'_{11} & \tilde{Z}'_{12} & \tilde{Z}'_{13} & \cdots & \tilde{Z}'_{1m} \\ \tilde{Z}'_{21} & \tilde{Z}'_{22} & \tilde{Z}'_{23} & \cdots & \tilde{Z}'_{2m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{Z}'_{n1} & \tilde{Z}'_{n2} & \tilde{Z}'_{n3} & \cdots & \tilde{Z}'_{nm} \end{array} \tag{2.23}$$

where $\tilde{Z}'_{ij} = \left(z'_{ija}, z'_{ijb}, z'_{ijc}, z'_{ijd}\right)$, i=1, 2,…,n and j=1,2,.., m.

Step 3. Calculate the normalized $\widetilde{Z}'$-fuzzy decision matrix. Performance values are calculated using linear normalization as in Eqs. (2.24)-(2.27).

$$\tilde{n}_{ij} = \begin{cases} \dfrac{\tilde{Z}'_{ij}}{z'_{ijd^*}} & if\ j \in N_b \\ \dfrac{z'_{ija^-}}{\tilde{Z}'_{ij}} & if\ j \in N_c \end{cases} \tag{2.24}$$

where

$$\tilde{n}_{ij} = \begin{array}{ll} \left(\dfrac{z'_{ija}}{z'_{ijd^*}}, \dfrac{z'_{ijb},}{z'_{ijd^*}}, \dfrac{z'_{ijc}}{z'_{ijd^*}}, \dfrac{z'_{ijd}}{z'_{ijd^*}}\right) & if\ j \in N_b \\ \left(\dfrac{z'_{ija^-}}{z'_{ijd}}, \dfrac{z'_{ija^-}}{z'_{ijc}}, \dfrac{z'_{ija^-}}{z'_{ijb}}, \dfrac{z'_{ija^-}}{z'_{ija}}\right) & if\ j \in N_c \end{array} \tag{2.25}$$

$$z'_{ijd} * = \max_i z'_{ijd}\ of\ \tilde{Z}'_{ij}s \qquad j \in N_b \tag{2.26}$$

$$z'_{ija^-} = \min_i z'_{ija}\ of\ \tilde{Z}'_{ij}s \qquad j \in N_c \tag{2.27}$$

where N_b and N_c represent the sets of benefit and cost criteria, respectively.

Step 3. Calculate the criteria weights by applying Buckley's (1985) fuzzy pairwise comparison procedure as follows:

Step 3.1. Fill in the pairwise comparison matrix for calculating the Z-fuzzy weights of the criteria by using the scales given in Table 2.4 and Table 2.5.

Table 2.5 : Linguistic scale for weighing the criteria

Linguistic Terms	Abbr.	Restriction function
Absolutely Important	AI	(7,9,9)
Strongly Important	SI	(5,7,9)
Highly Important	HI	(3,5,7)
Weakly Important	WI	(1,3,5)
Equally Important	EI	(1,1,1)

Step 3.2. Convert Z-fuzzy evaluations to ordinary fuzzy evaluations by using Eq. (2.4).

Step 3.3. Calculate the geometric mean for each parameter of $\tilde{a}_{ij}$ in the m dimensional pairwise comparison matrix. Thus, $m \times m$ matrix is converted to $m \times 1$ matrix.

Step 3.4. Sum the values of each parameter in the column to normalize the values in $m \times 1$ matrix.

Step 3.5. Apply fuzzy division operation to get the normalized weights vector.

Step 3.6. Defuzzify the normalized weights vector using the center of gravity method given by Eq. (2.2).

Step 3.7. Normalize the weights so that their sum is equal to 1.

Step 4. Calculate the weighted normalized decision matrix by using Eq. (2.28).

$$\tilde{r}_{ij} = w_j \, \tilde{n}_{ij} \tag{2.28}$$

where w_j ($0 < w_j < 1$) denotes the weight of jth criterion, and $\sum_{j=1}^{m} w_j = 1$.

Step 5. Determine the negative-ideal solution as in Eqs. (2.29) and (2.30). The negative ideal solutions of each criterion are determined based on the defuzzification equations mentioned in Definition (2.3).

$$\widetilde{ns} = \left[\widetilde{ns}_j\right]_{1\times m} \tag{2.29}$$

$$\widetilde{ns}_j = \min_i \tilde{r}_{ij} \tag{2.30}$$

Step 6. Calculate the Euclidean and Taxicab distances of alternatives from the negative-ideal solution, shown as in Eqs. (2.31) and (2.32).

$$E_i = \sqrt{\sum_{j=1}^{m} \frac{1}{4}\left[\left(\tilde{r}_{ija} - \widetilde{ns}_{jd}\right)^2 + \left(\tilde{r}_{ijb} - \widetilde{ns}_{jc}\right)^2 + \left(\tilde{r}_{ijc} - \widetilde{ns}_{jb}\right)^2 + \left(\tilde{r}_{ijd} - \widetilde{ns}_{ja}\right)^2\right]} \tag{2.31}$$

$$T_i = \sum_{j=1}^{m} \left\{\frac{1}{4}\left(\left|\tilde{r}_{ija} - \widetilde{ns}_{ja}\right| + \left|\tilde{r}_{ijb} - \widetilde{ns}_{jb}\right| + \left|\tilde{r}_{ijc} - \widetilde{ns}_{jc}\right| + \left|\tilde{r}_{ijd} - \widetilde{ns}_{jd}\right|\right)\right\} \tag{2.32}$$

Step 7. Construct the relative assessment matrix, shown as in Eqs. (2.33) and (2.34):

$$Ra = [h_{ik}]_{n\times n} \tag{2.33}$$

$$h_{ik} = (E_i - E_k) + (\psi(E_i - E_k) \times (T_i - T_k)) \tag{2.34}$$

where $k \in \{1,2, \dots, n\}$ and ψ denotes a threshold function to recognize the equality of the Euclidean distances of two alternatives, and is defined as in Eq. (2.35):

$$\psi(x) = \begin{cases} 1 & if \quad |x| \geq \tau \\ 0 & if \quad |x| < \tau \end{cases} \tag{2.35}$$

where τ is a threshold parameter set by decision-makers. It is recommended to set this parameter at a value between 0.01 and 0.05. If the difference between Euclidean distances of two alternatives is larger than τ, these two alternatives are also compared by Taxicab distance. In this study, we set τ=0.02 in the application section.

Step 8. Calculate the assessment score of each alternative, shown as in Eq. (2.36):

$$H_i = \sum_{k=1}^{n} h_{ik} \tag{2.36}$$

Step 9. Rank the alternatives according to assessment score (H_i). The alternative which has the highest H_i is the best choice among the alternatives.

2.3 Application

Let us consider a supplier selection problem with three criteria (C1, C2 and C3) and three alternatives (A1, A2 and A3). Three decision makers (DMs) evaluated the alternatives using the criteria *quality* (C1), *price* (C2), and *delivery time* (C3). We now apply the steps of the model as described in Section 2.2.2.

Step 1. First, we collect Z-fuzzy evaluations for alternatives from decision makers. Tables 2.6-2.8 present Z-fuzzy evaluation matrices for alternatives according to the determined criteria.

Table 2.6 : DM 1's evaluations

Criteria	Alternatives	Evaluations	Restriction	Reliability
C1	A1	G,SR	(7, 9, 9, 10)	(0.7, 0.8, 0.9)
	A2	P,WR	(0.5, 1, 1, 3)	(0.3, 0.4, 0.5)
	A3	MP,HR	(1, 3, 3, 5)	(0.5, 0.6, 0.7)
C2	A1	P,VHR	(0.5, 1, 1, 3)	(0.6, 0.7, 0.8)
	A2	VG,SU	(9, 10, 10, 10)	(0.1, 0.2, 0.3)
	A3	G,AR	(7, 9, 9, 10)	(0.8, 0.9, 1)
C3	A1	MP,AU	(1, 3, 3, 5)	(0, 0.1, 0.2)
	A2	G,FR	(7, 9, 9, 10)	(0.4, 0.5, 0.6)
	A3	VG,SR	(9, 10, 10, 10)	(0.7, 0.8, 0.9)

Table 2.7 : DM 2's evaluations

Criteria	Alternatives	Evaluations	Restriction	Reliability
C1	A1	P, HR	(0.5, 1, 1, 3)	(0.5, 0.6, 0.7)
	A2	G, VWR	(7, 9, 9, 10)	(0.2, 0.3, 0.4)
	A3	G, SR	(7, 9, 9, 10)	(0.7, 0.8, 0.9)
C2	A1	MG, FR	(5, 7, 7, 9)	(0.4, 0.5, 0.6)
	A2	F, HR	(3, 5, 5, 7)	(0.5, 0.6, 0.7)
	A3	MG, AU	(5, 7, 7, 9)	(0, 0.1, 0.2)
C3	A1	P, WR	(0.5, 1, 1, 3)	(0.3, 0.4, 0.5)
	A2	VG, HR	(9, 10, 10, 10)	(0.5, 0.6, 0.7)
	A3	VP, FR	(0.25, 0.5, 0.5, 1)	(0.4, 0.5, 0.6)

Table 2.8 : DM 3's evaluations

Criteria	Alternatives	Evaluations	Restriction	Reliability
C1	A1	F, SR	(3, 5, 5, 7)	(0.7, 0.8, 0.9)
	A2	MP, HR	(1, 3, 3, 5)	(0.5, 0.6, 0.7)
	A3	MG, SR	(5, 7, 7, 9)	(0.7, 0.8, 0.9)
C2	A1	MG, HR	(5, 7, 7, 9)	(0.5, 0.6, 0.7)
	A2	G, WR	(7, 9, 9, 10)	(0.3, 0.4, 0.5)
	A3	VP, VHR	(0.25, 0.5, 0.5, 1)	(0.6, 0.7, 0.8)
C3	A1	F, HR	(3, 5, 5, 7)	(0.5, 0.6, 0.7)
	A2	MG, HR	(5, 7, 7, 9)	(0.5, 0.6, 0.7)
	A3	P, SR	(0.5, 1, 1, 3)	(0.7, 0.8, 0.9)

The aggregated decision matrix is obtained as in Table 2.9 using the geometric mean operation.

Step 2. We convert Z-fuzzy numbers to ordinary fuzzy numbers using Eq. (2.3). The obtained ordinary fuzzy numbers are shown in Table 2.10.

Table 2.9 : Aggregated decision matrix

Criteria	Alternatives	Restriction	Reliability
C1	A1	(2.190, 3.557, 3.557, 5.944)	(0.626, 0.727, 0.828)
	A2	(1.518, 3.000, 3.000, 5.313)	(0.311, 0.416, 0.519)
	A3	(3.271, 5.739, 5.739, 7.663)	(0.626, 0.727, 0.828)
C2	A1	(2.321, 3.659, 3.659, 6.240)	(0.493, 0.594, 0.695)
	A2	(5.739, 7.663, 7.663, 8.879)	(0.247, 0.363, 0.472)
	A3	(2.061, 3.158, 3.158, 4.481)	(0.000, 0.398, 0.543)
C3	A1	(1.145, 2.466, 2.466, 4.718)	(0.000, 0.288, 0.412)
	A2	(6.804, 8.573, 8.573, 9.655)	(0.464, 0.565, 0.665)
	A3	(1.040, 1.710, 1.710, 3.107)	(0.581, 0.684, 0.786)

Table 2.10 : Ordinary fuzzy numbers converted from Z-fuzzy numbers

Alter-natives	C1	C2	C3
A1	(1.867, 3.032, 3.032, 5.068)	(1.789, 2.821, 2.821, 4.811)	(0.615, 1.325, 1.325, 2.534)
A2	(0.979, 1.935, 1.935, 3.427)	(3.460, 4.620, 4.620, 5.353)	(5.113, 6.442, 6.442, 7.255)
A3	(2.789, 4.893, 4.893, 6.533)	(1.300, 1.992, 1.992, 2.827)	(0.860, 1.414, 1.414, 2.570)

Step 3. We normalize ordinary fuzzy evaluation values of alternatives by using Eqs. (2.24)-(2.27). Thus, Table 2.11 is obtained. In this application, price (C2) and delivery time (C3) are cost criteria while quality (C1) is benefit criterion. However, the assigned scores are in terms of benefit. Hence, the normalization type of benefit criteria is applied to all the criteria.

Table 2.11 : Normalized decision matrix

Alter natives	C1	C2	C3
A1	(0.286, 0.464, 0.464, 0.776)	(0.334, 0.527, 0.527, 0.899)	(0.085, 0.183, 0.183, 0.349)
A2	(0.150, 0.296, 0.296, 0.525)	(0.646, 0.863, 0.863, 1.000)	(0.705, 0.888, 0.888, 1.000)
A3	(0.427, 0.749, 0.749, 1.000)	(0.243, 0.372, 0.372, 0.528)	(0.119, 0.195, 0.195, 0.354)

For weighing criteria, we apply steps 3.1-3.7.

Step 3.1. We construct Z-fuzzy pairwise comparison matrix for the criteria. Three decision makers compromised on the evaluation of the criteria weights as given in Table 2.12. The corresponding numerical scale is used to construct numerical Z-fuzzy pairwise comparison matrix as given in Table 2.13.

Table 2.12 : Pairwise comparisons of the criteria using Z-fuzzy numbers

	C1	C2	C3
C1	EI, CR	HI, HR	WI, VSR
C2	1/HI, HR	EI, CR	1/HI, VHR
C3	1/WI, VSR	HI, VHR	EI, CR

Table 2.13 : Z-fuzzy pairwise comparison matrix for criteria

	C1		C2		C3	
	Restriction	Reliability	Restriction	Reliability	Restriction	Reliability
C1	(1, 1, 1)	(1, 1, 1)	(3, 5, 7)	(0.5, 0.6, 0.7)	(1, 3, 5)	(0.8, 0.9, 1.0)
C2	(0.1, 0.2, 0.3)	(0.5, 0.6, 0.7)	(1, 1, 1)	(1, 1, 1)	(0.1, 0.2, 0.3)	(0.6, 0.7, 0.8)
C3	(0.2, 0.3, 1.0)	(0.8, 0.9, 1.0)	(3, 5, 7)	(0.6, 0.7, 0.8)	(1, 1, 1)	(1, 1, 1)

Step 3.2. We convert Z-fuzzy evaluation values of criteria to ordinary fuzzy numbers by using Eq. (2.3). Table 2.14 shows the converted ordinary fuzzy values of criteria.

Table 2.14 : Ordinary fuzzy pairwise comparison matrix for criteria

	C1	C2	C3
C1	(1, 1, 1)	(2.32, 3.87, 5.42)	(0.95, 2.85, 4.74)
C2	(0.11, 0.15, 0.26)	(1, 1, 1)	(0.12, 0.17, 0.28)
C3	(0.19, 0.32, 0.95)	(2.51, 4.18, 5.86)	(1, 1, 1)

Step 3.3. We calculate the geometric mean for each parameter of Z-fuzzy numbers. The obtained results are given in Table 2.15.

Table 2.15 : Geometric mean of ordinary fuzzy pairwise comparison matrices

Criteria	Geometric means of each parameter of a_{ij}
C1	(1.301, 2.226, 2.952)
C2	(0.236, 0.296, 0.416)
C3	(0.781, 1.098, 1.771)
Total	(2.319, 3.619, 5.139)

For instance, the weight of C1 is calculated as follows.

$$w_1 = [(1.301/5.139\,, 2.226/3.619\,, 2.952/2.319)] = [0.253, 0.615, 1.273]$$

Steps 3.4-3.7. After step 3.3, we obtain *m*1* fuzzy matrix. We calculate the sum of values and normalized according to fuzzy division rules. Then, we defuzzify the fuzzy numbers and normalize them to obtain the weights of criteria. The obtained values are given in Table 2.16.

Table 2.16 : Relative fuzzy weights and crisp weights of each criterion

Criteria	Fuzzy weigths	Defuzzification	Crisp weights
C1	(0.253, 0.615, 1.273)	0.689	0.590
C2	(0.046, 0.082, 0.179)	0.097	0.083
C3	(0.152, 0.303, 0.764)	0.381	0.326
	Total:	1.167	1.000

Step 4 and Step 5. We calculate weighted normalized decision matrix as in Eq. (2.28) and determine the negative ideal solutions by applying Eqs. (2.29) and (2.30). In this step, the weights obtained at the end of Step 3 are used, then Table 2.17 is obtained.

Table 2.17 : Weighted normalized decision matrix and negative ideal solutions

	C1	C2	C3
A1	(0.169, 0.274, 0.274, 0.458)	(0.028, 0.044, 0.044, 0.075)	(0.028, 0.060, 0.060, 0.114)
A2	(0.089, 0.175, 0.175, 0.310)	(0.054, 0.072, 0.072, 0.083)	(0.230, 0.290, 0.290, 0.326)
A3	(0.252, 0.442, 0.442, 0.590)	(0.020, 0.031, 0.031, 0.044)	(0.039, 0.064, 0.064, 0.116)
NIS	(0.089, 0.175, 0.175, 0.310)	(0.020, 0.031, 0.031, 0.044)	(0.028, 0.060, 0.060, 0.114)

Step 6. Euclidean and Taxicab distances are calculated by using Eqs. (2.31) and (2.32). Table 2.18 presents the calculated values.

Table 2.18 : Euclidean and Taxicab distances of alternatives

	Euclidean Distance	Rank	Taxicab Distance	Rank
A1	0.2205	3	0.2445	3
A2	0.2801	2	0.3680	1
A3	0.3212	1	0.3283	2

Step 7-Step 9. In these steps we calculate the relative assessment matrix and the assessment scores of alternatives by using Eqs. (2.33)-(2.36). Finally, we rank the alternatives. Table 2.19 presents the calculated results.

Table 2.19 : The relative assessment matrix and the assessment scores of alternatives

	A1	A2	A3	Total H_i	Rank
A1	0	-0.183	-0.184	-0.367	3
A2	0.183	0	-0.001	0.181	2
A3	0.184	0.001	0	0.186	1

Supplier A3 is determined as the best alternative. A1 is the worst supplier among three alternatives. A3 is the best alternative according to Euclidean distance while A2 is the best alternative according to taxicab distance (see Table 2.18). CODAS method considers both distances to rank the alternatives. This shows the advantage of the CODAS method, which allows both distances to be considered.

2.4 Comparative Analysis Using Simple Additive Weighting

We compared the proposed method with the fuzzy simple additive weighting (fuzzy SAW) method. Initial decision matrix of fuzzy SAW method is constructed by multiplying restriction and reliability values of DMs' evaluations. Thus, the expected values of each evaluation are obtained and shown in Table 2.20.

Table 2.20 : Expected values of DMs' evaluations

Criteria	Alternatives	DM1	DM2	DM3
C1	A1	(4.9, 7.2, 7.2, 9.0)	(0.3, 0.6, 0.6, 2.1)	(2.1, 4.0, 4.0, 6.3)
	A2	(0.2, 0.4, 0.4, 1.5)	(1.4, 2.7, 2.7, 4.0)	(0.5, 1.8, 1.8, 3.5)
	A3	(0.5, 1.8, 1.8, 3.5)	(4.9, 7.2, 7.2, 9.0)	(3.5, 5.6, 5.6, 8.1)
C2	A1	(0.3, 0.7, 0.7, 2.4)	(2.0, 3.5, 3.5, 5.4)	(2.5, 4.2, 4.2, 6.3)
	A2	(0.9, 2.0, 2.0, 3.0)	(1.5, 3.0, 3.0, 4.9)	(2.1, 3.6, 3.6, 5.0)
	A3	(5.6, 8.1, 8.1, 10.0)	(0.0, 0.7, 0.7, 1.8)	(0.2, 0.4, 0.4, 0.8)
C3	A1	(0.0, 0.3, 0.3, 1.0)	(0.2, 0.4, 0.4, 1.5)	(1.5, 3.0, 3.0, 4.9)
	A2	(2.8, 4.5, 4.5, 6.0)	(4.5, 6.0, 6.0, 7.0)	(2.5, 4.2, 4.2, 6.3)
	A3	(6.3, 8.0, 8.0, 9.0)	(0.1, 0.3, 0.3, 0.6)	(0.4, 0.8, 0.8, 2.7)

We calculate the average of the expected values of evaluations to aggregate DMs' decision. Table 2.21 shows the obtained values.

Table 2.21 : Average of the DMs' evaluations

Alternatives	C1	C2	C3
A1	(2.417, 3.933, 3.933, 5.800)	(1.600, 2.800, 2.800, 4.700)	(0.550, 1.233, 1.233, 2.467)
A2	(0.683, 1.633, 1.633, 3.000)	(1.500, 2.867, 2.867, 4.300)	(3.267, 4.900, 4.900, 6.433)
A3	(2.967, 4.867, 4.867, 6.867)	(1.917, 3.050, 3.050, 4.200)	(2.250, 3.017, 3.017, 4.100)

We normalize the values of Table 2.21 using Eqs. (2.24)-(2.27) and obtain the normalized decision matrix as given in Table 2.22.

Table 2.22 : Normalized decision matrix

Alternatives	C1	C2	C3
A1	(0.352, 0.573, 0.573, 0.845)	(0.340, 0.596, 0.596, 1.000)	(0.085, 0.192, 0.192, 0.383)
A2	(0.100, 0.238, 0.238, 0.437)	(0.319, 0.610, 0.610, 0.915)	(0.508, 0.762, 0.762, 1.000)
A3	(0.432, 0.709, 0.709, 1.000)	(0.408, 0.649, 0.649, 0.894)	(0.350, 0.469, 0.469, 0.637)

We calculate weighted normalized decision matrix by multiplying the normalized values with the weights of criteria and Table 2.23 is obtained.

Table 2.23 : Weighted normalized decision matrix

Alternatives	C1	C2	C3
A1	(0.208, 0.338, 0.338, 0.499)	(0.028, 0.050, 0.050, 0.083)	(0.028, 0.063, 0.063, 0.125)
A2	(0.059, 0.140, 0.140, 0.258)	(0.027, 0.051, 0.051, 0.076)	(0.166, 0.248, 0.248, 0.326)
A3	(0.255, 0.419, 0.419, 0.590)	(0.034, 0.054, 0.054, 0.074)	(0.114, 0.153, 0.153, 0.208)

We obtain the final score of each alternative by summing the values in each row. Then we defuzzify final scores to obtain crisp values as mentioned in Definition 2.2. Finally, we rank the alternatives according to the defuzzified values as seen in Table 2.24.

Table 2.24 : Final score, defuzzified score and rank of alternatives

Alternatives	Final score	Defuzzified score	Rank
A1	(0.264, 0.450, 0.450, 0.707)	0.344	2
A2	(0.251, 0.440, 0.440, 0.660)	0.335	3
A3	(0.403, 0.626, 0.626, 0.873)	0.484	1

Comparison of two methods is given in Table 2.25. A3 is the first ranking among the alternatives in both methods. The ranking of the A1 and A2 alternatives in two methods differs. Thus, we are more confident that A3 is the best alternative.

Table 2.25 : Comparison of fuzzy SAW and Z-CODAS methods

Alternatives	Fuzzy SAW	Z-fuzzy CODAS
A1	2	3
A2	3	2
A3	1	1

2.5 Conclusions

CODAS method has been very popular after its introduction to the literature. After ordinary fuzzy CODAS method was developed by Ghorabaee et al. (2016), its fuzzy extensions such as type-2 fuzzy CODAS, intuitionistic fuzzy CODAS, hesitant fuzzy

CODAS, Pythagorean fuzzy CODAS, and neutrosphic CODAS have been proposed by several researchers. Extensions of multicriteria decision making methods by using Z-fuzzy numbers are relatively new when compared with these MCDM methods. CODAS method has been extended by using Z-fuzzy numbers and applied to a supplier selection problem. A comparative analysis has also been presented. The application and comparative analysis showed that the proposed Z-fuzzy CODAS method yields meaningful results. DMs could incorporate their opinions related to the reliability of a determined membership function.

For further research, we suggest LR type Z-fuzzy numbers to be used in CODAS method for comparative analyses. Nonlinear membership functions may provide a more realistic approach to the method.

3. EVALUATING SOCIAL SUSTAINABLE DEVELOPMENT FACTORS USING MULTI-EXPERTS Z-FUZZY AHP[2]

Sustainable development is defined as «the development that meets the needs of the present without compromising the ability of future generations to meet their own needs.» (Brundtland et al., 1987). One of the most important concerns of people in our globalizing world is leaving a livable world for future generations. Everyone is getting more conscious about this issue. The importance of the issue is not only related to the environmental concern but also related sociological and economic development and the factors affecting them.

Today, most of the people live in cities, which are centers where social, economic and environmental activities that people require to survive are gathered (Dempsey et al., 2011; Phillis et al., 2017; Zinatizadeh et al., 2017). Therefore, sustainable development is examined in the literature from three aspects which are economic, social and environmental. Economic aspect of sustainable development is related to the protection of capital and the prevention of its deterioration (Goodland, 2002). Environmental aspect is to take care health of the ecosystems while meeting the needs of people to sustain existence (Morelli, 2011). Social sustainability is a human and community oriented aspect and social sustainable development corresponds to a concept that works to make the system sustainable in social and cultural life welfare such as education, health, demography etc. that are mentioned Table 3.4.

Sustainability is an interrelated concept which means that the sustainability of cities directly affects the sustainability of countries, regions and the world. Therefore, it is important to identify the factors that affect the sustainability of cities to be able to better analyze the issue (Zhang et al., 2016).

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[2] This chapter is based on the paper "Tüysüz, N., & Kahraman, C. (2020). Evaluating social sustainable development factors using multi-experts Z-fuzzy AHP. Journal of Intelligent & Fuzzy Systems, 39(5), 6181-6192."

Social sustainability is relatively new and thus is less researched in the literature when compared to the other dimensions of sustainability. Due to this reason, social sustainable development factors have not been addressed as much as the other ones (Hediger, 2000; Murphy, 2012), which is one of the main motivations of this study.

This study considers the social sustainable development and in order to evaluate the sustainability of cities, countries or regions, it is necessary to identify the factors that affect sustainability and to evaluate them subjectively based on the ideas of experts. Therefore, the choice of sustainability factors is an important issue affecting the performance of cities' sustainability (Zhang et al., 2016). Since the evaluation of social sustainable development handled in this study contains many factors to be considered, they should be evaluated with multi-criteria decision making (MCDM) techniques. MCDM methods provide the necessary tools for analyzing the multi-dimensionality of such problems. There are many developed MCDM methods in literature and AHP is the most widely used one which enables to determine importance levels of factors based on the relative evaluations of decision-makers.

Although the classical AHP method which was proposed by Saaty (1980) uses crisp numbers in expressing decision-makers' evaluations, different fuzzy versions of the method have also emerged to be able to better reflect uncertainty in subjective evaluations. In this study, the AHP method is extended by using Z-fuzzy numbers which include both vague evaluations of decision-makers together with their degree of confidence. The most important contribution of this study to the literature is the use of Z-fuzzy numbers integrated AHP method in evaluating the social sustainability on a group decision making basis, which we think that helps the better understanding and analyzing the issue.

The rest of the paper is as follows. In section 3.1, literature review on fuzzy AHP that proposes new approaches with various fuzzy sets is given. In section 3.2, preliminaries of Z-fuzzy numbers are explained. In section 3.3, the algorithm of proposed method, multi-experts Z-Fuzzy AHP method, is given. In section 3.4, an application on evaluating sustainable development factors is performed to display the applicability of the method. In section 3.5, sensitivity analysis is applied to examine the results under different reliabilities of experts’ evaluations. Finally, concluding remarks are presented.

3.1 Literature Review

Since AHP is the most widely used multi-criteria decision making method (Tüysüz, 2018), there have been developed many fuzzy extensions of the method. The summary of the important extensions of the AHP method is presented in Table 3.1.

Table 3.1 : A literature review on fuzzy extensions of AHP.

Year	Author	Extension of AHP
1983	Laarhoven and Pedrycz	Ordinary fuzzy AHP
1985	Buckley	Ordinary fuzzy AHP
1996	Chang	Ordinary fuzzy AHP
2009	Sadiq and Tesfamariam	Intuitionistic fuzzy AHP
2009	Abdullah et al.	Intuitionistic fuzzy AHP
2011	Wang et al.	Intuitionistic fuzzy AHP
2011	Zhang et al.	Intuitionistic fuzzy AHP
2012	Feng et al.	Intuitionistic fuzzy AHP
2014	Kaur	Intuitionistic fuzzy AHP
2015	Dutta and Guha	Intuitionistic fuzzy AHP
2015	Keshavarzfard and Makui	Intuitionistic fuzzy AHP
2016	Deepika and Kannan	Intuitionistic fuzzy AHP
2016	Tavana et al.	Intuitionistic fuzzy AHP
2016b	Abdullah and Najib	Intuitionistic fuzzy AHP
2013	Wu et al.	Interval valued intuitionistic fuzzy AHP
2016a	Abdullah and Najib	Interval-valued intuitionistic fuzzy AHP
2015	Onar et al.	Interval valued intuitionistic fuzzy AHP
2015	Fahmi et al.	Interval valued intuitionistic fuzzy AHP
2016	Kahraman et al.	Interval valued intuitionistic fuzzy AHP
2014	Kahraman et al.	Interval valued type-2 fuzzy AHP
2014	Zhu & Xu	Hesitant fuzzy AHP
2015	Oztaysi et al.	Hesitant fuzzy AHP
2017	Tüysüz and Şimşek	Hesitant fuzzy linguistic term sets-based AHP
2017	Kahraman et al.	Hesitant fuzzy linguistic AHP
2016	Ren et al.	Group intuitionistic multiplicative AHP
2016	Radwan et al.	Hybrid neutrosophic AHP
2017	Abdel-Basset et al.	Neutrosophic AHP
2018	Ilbahar et al.	Pythagorean fuzzy AHP
2019	Karasan et al.	Pythagorean fuzzy AHP
2013	Azadeh et al.	Z-fuzzy AHP
2018	Kahraman and Otay	Z-fuzzy AHP
2020	Kutlu Gundogdu and Kahraman	Spherical fuzzy AHP

Ordinary fuzzy AHP is a weighting and selection method that uses a crisp linguistic scale that can reflect decision-makers' uncertain judgements in their evaluations. This linguistic scale can be defined by using different fuzzy numbers such as triangular, trapezoidal numbers etc. While these fuzzy numbers include only the restrictions of decision makers' evaluations, Z-fuzzy numbers that are used in this study include the confidence level of decision makers’ evaluations. Assessments made with other fuzzy numbers can be considered as exactly confident evaluations made with Z-fuzzy numbers. But in real life, it can be unusual to expect decision-makers to be absolutely sure of their assessment when making an evaluation. Since Z-fuzzy numbers contain all degrees of confidence, it can be said that Z-fuzzy numbers are better in expressing uncertainty than the other fuzzy numbers. Table 3.1 shows the historical review of fuzzy AHP from ordinary fuzzy AHP to the latest fuzzy extensions of AHP.

As it can be seen from Table 3.1, Z-Fuzzy AHP, Pythagorean fuzzy AHP and Spherical fuzzy AHP extensions are relatively new studies. Our study proposes a new Z-Fuzzy extension of AHP. In coming sections, the preliminary information related to the methodology will be presented in detail.

3.2 Z-Fuzzy Numbers

Zadeh (2011) introduced the concept of Z-fuzzy number which can be defined as a sequence of fuzzy number pairs (A, B) where *A* is the restriction function and *B* is the second component for measuring the reliability of *A*. The Z-fuzzy number allows making calculations the numbers that are not completely reliable. It can be used to reflect the expression about a variable together with its precision level. In other words, *A* is the value of the variable and *B* is the precision or probability of that value.

Definition 3.1. Let a Z-fuzzy number is defined as $Z = (\widetilde{A}, \widetilde{R})$ which is sequenced pair of fuzzy numbers. The first component $\widetilde{A}$ represent the restriction of real-valued vague variable *X* while the second component $\tilde{R}$ is a measure of reliability for $\widetilde{A}$. A simple Z-fuzzy number can be defined as in Figure 3.1.

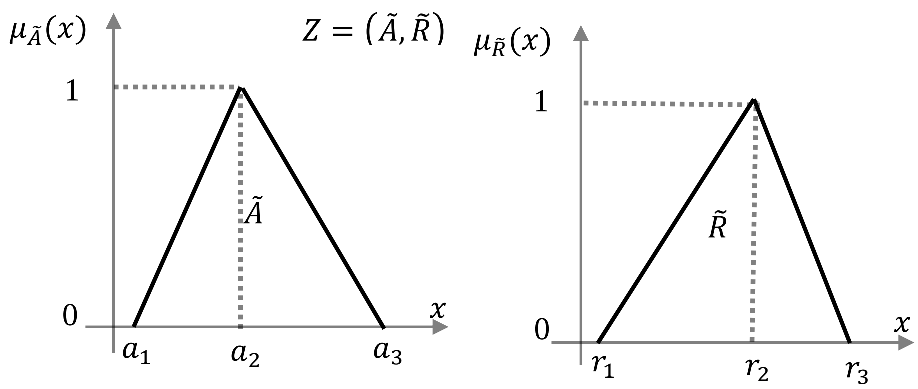

Figure 3.1 : A simple Z-fuzzy number.

Definition 3.2. Let a fuzzy set $\widetilde{A}$ which is defined on a universe *X*. Then, $\widetilde{A} = \{\langle x, \mu_A(x)\rangle \mid x \epsilon X\}$ where $\mu_{\widetilde{A}}: X \rightarrow [0,1]$ is the membership function $\widetilde{A}$ describing the belongingness degree of x ∈ X in $\widetilde{\mathrm{A}}$. The Fuzzy Expectation of a fuzzy set is defined as follows (Puri and Ralescu, 1986).

$$E_{\widetilde{\mathrm{A}}}(x) = \int_x x\mu_{\widetilde{\mathrm{A}}}(x)\,dx \tag{3.1}$$

which should not be considered as probabilistic expectation. Table 3.2 presents the linguistic scale for restriction function while Table 3.3 gives the reliability scale.

Definition 3.3: Converting Z-Fuzzy Number to Regular Fuzzy Number

Consider a Z-fuzzy number $Z = (\widetilde{A}, \tilde{R})$, which is shown as in Figure 3.1. The left side shows the restriction function, and the right side shows the reliability function. Let $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x)\rangle | \mu(x) \in [0,1]\}$ and $\tilde{R} = \{\langle x, \mu_{\tilde{R}}(x)\rangle | \mu(x) \in [0,1]\}$, where $\mu_{\tilde{A}}(x)$ and $\mu_{\tilde{R}}(x)$ are triangular membership functions (Kang et al., 2012a).

(1) Convert the second part ($\tilde{R}$) of Z-fuzzy number into a crisp number using Eq. (3.2).

$$\alpha = \frac{\int x\mu_{\tilde{R}}(x)\,dx}{\int \mu_{\tilde{R}}(x)\,dx} \tag{3.2}$$

where $\int$ denotes an algebraic integration.

(2) Add the weight of the reliability part to the restriction part and obtain the weighted Z-fuzzy number which can be denoted as

$$\tilde{Z}^{\alpha} = \{\langle x, \mu_{\tilde{A}^{\alpha}}(x)\rangle | \mu_{\tilde{A}^{\alpha}}(x) = \alpha\mu_{\tilde{A}}(x), \mu(x) \in [0,1]\} \tag{3.3}$$

(3) Convert the weighted restriction part (irregular fuzzy number) to ordinary fuzzy number. The ordinary fuzzy set can be denoted as $\tilde{Z}' = \left\{\langle x, \mu_{\tilde{Z}'}(x)\rangle \middle| \mu_{\tilde{Z}'}(x) = \mu_{\tilde{A}}\left(\frac{x}{\sqrt{\alpha}}\right), \mu(x) \in [0,1]\right\}$. $\tilde{Z}'$ has the same Fuzzy Expectation with $\tilde{Z}^{\alpha}$, and they are equal with respect to Fuzzy Expectation, which can be denoted by Figure 3.2.

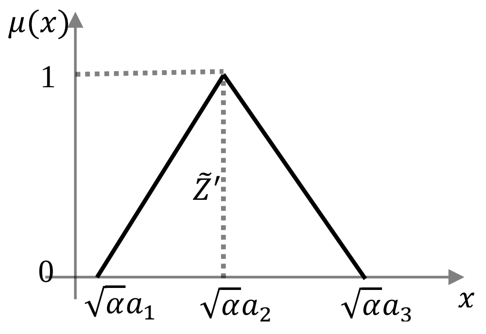

Figure 3.2 : Ordinary fuzzy number converted from Z-fuzzy number.

3.3 Proposed Method: Multi-Experts Z-Fuzzy AHP

The steps of the proposed multi-experts Z-fuzzy AHP method are presented in the following.

Step 1. Determine the hierarchy of the decision problem as illustrated in Figure 3.3.

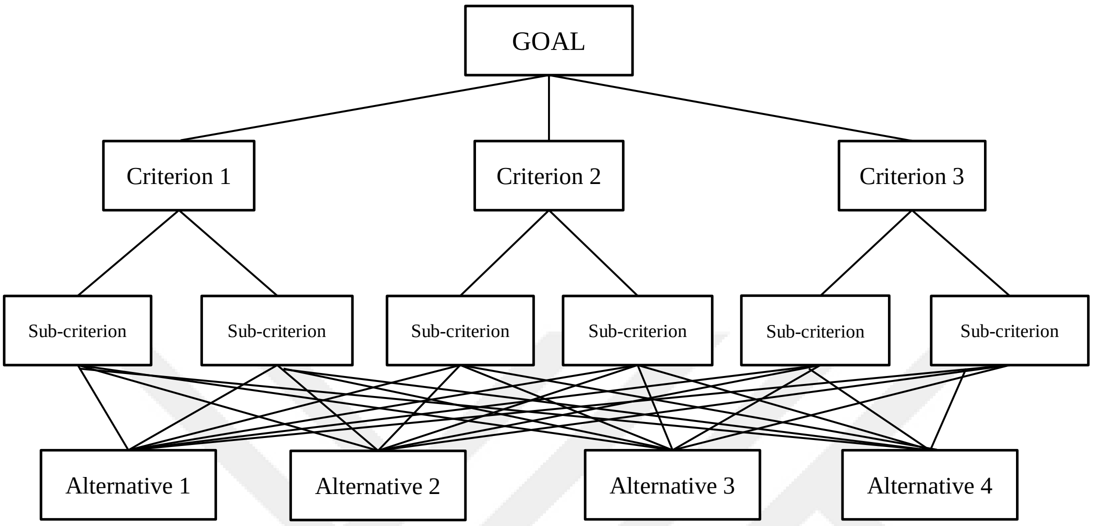

Figure 3.3 : Hierarchical structure.

Step 2. Collect the linguistic assessments of each expert for pairwise comparisons between criteria, sub-criteria, and alternatives by using questionnaires and triangular Z-fuzzy restriction and reliability scales given in Tables 3.2-3.3.

Let each expert (Ei) assign an independent evaluation for any pairwise comparison as follows:

$$Z^{Ei} = (\tilde{A}, \tilde{R}) = \left((a_1^{Ei}, a_2^{Ei}, a_3^{Ei}), (r_1^{Ei}, r_2^{Ei}, r_3^{Ei}) \right) \tag{3.4}$$

Tables 3.2 and 3.3 can be used for the restriction scale and reliability scale, respectively.

Table 3.2 : Z-fuzzy Restriction Scale for pairwise comparisons.

Linguistic Scale	Abbr.	Restriction function
Equally Important	EI	(1,1,1;1)
Slightly Important	SLI	(1,1,3;1)
Moderately Important	MI	(1,3,5;1)
Strongly Important	STI	(3,5,7;1)
Very Strongly Important	VSTI	(5,7,9;1)
Certainly Important	CI	(7,9,10;1)
Absolutely Important	AI	(9,10,10;1)

Table 3.3 : Reliability Scale for Triangular Z-fuzzy numbers.

Linguistic Scale	Abbr.	Reliability function
Absolutely Reliable	AR	(0.8,0.9,1;1)
Strongly Reliable	SR	(0.7,0.8,0.9;1)
Very Highly Reliable	VHR	(0.6,0.7,0.8;1)
Highly Reliable	HR	(0.5,0.6,0.7;1)
Fairly Reliable	FR	(0.4,0.5,0.6;1)
Weakly Reliable	WR	(0.3,0.4,0.5;1)
Very Weakly Reliable	VWR	(0.2,0.3,0.4;1)
Strongly Unreliable	SU	(0.1,0.2,0.3;1),
Absolutely Unreliable	AU	(0,0.1,0.2;1)

Step 2.1. Check the consistency of the fuzzy pairwise comparison matrix judged by each expert. Defuzzify fuzzy numbers in the decision matrix using Eq. (3.2) or alternatively, the defuzzification equation $(a_1 + 2 * a_2 + a_3)/4$ for symmetrical triangular fuzzy numbers and obtain crisp decision matrix. Then apply Saaty's (1996) consistency check procedure. If this matrix is consistent, then experts' evaluations can be implied consistent.

Step 3. Aggregate the restriction and reliability functions of experts' evaluations using geometric mean for each hierarchy level**.** Assume three experts assign the following terms:

$$Z^{E1} = \left(\tilde{A}, \tilde{R}\right) = \left((a_1^{E1}, a_2^{E1}, a_3^{E1}), (r_1^{E1}, r_2^{E1}, r_3^{E1})\right)$$

$$Z^{E2} = \left(\tilde{A}, \tilde{R}\right) = \left((a_1^{E2}, a_2^{E2}, a_3^{E2}), (r_1^{E2}, r_2^{E2}, r_3^{E2})\right)$$

$$Z^{E3} = \left(\tilde{A}, \tilde{R}\right) = \left((a_1^{E3}, a_2^{E3}, a_3^{E3}), (r_1^{E3}, r_2^{E3}, r_3^{E3})\right)$$

Aggregation of these three experts' opinions is realized by using geometric mean:

$$Z^{Agg} = \begin{pmatrix} \left(\sqrt[3]{a_1^{E1} * a_1^{E2} * a_1^{E3}}, \sqrt[3]{a_2^{E1} * a_2^{E2} * a_2^{E3}}, \sqrt[3]{a_3^{E1} * a_3^{E2} * a_3^{E3}}\right), \\ \left(\sqrt[3]{r_1^{E1} * r_1^{E2} * r_1^{E3}}, \sqrt[3]{r_2^{E1} * r_2^{E2} * r_2^{E3}}, \sqrt[3]{r_3^{E1} * r_3^{E2} * r_3^{E3}}\right) \end{pmatrix} \tag{3.5}$$

Step 4. Convert the aggregated Z-fuzzy comparison matrix to ordinary fuzzy number using Definition 3.3 and Eqs. (3.6-3.7).

$$\alpha_j = \frac{\left(\sqrt[3]{r_1^{E1} * r_1^{E2} * r_1^{E3}} + 2 * \sqrt[3]{r_2^{E1} * r_2^{E2} * r_2^{E3}} + \sqrt[3]{r_3^{E1} * r_3^{E2} * r_3^{E3}}\right)}{4} \tag{3.6}$$

Now, Z-fuzzy number can be transformed to ordinary fuzzy number through Eq. (3.7).

$$\tilde{A} = \left(\sqrt[3]{a_1^{E1} * a_1^{E2} * a_1^{E3}}\sqrt{\alpha_j}, \sqrt[3]{a_2^{E1} * a_2^{E2} * a_2^{E3}}\sqrt{\alpha_j}, \sqrt[3]{a_3^{E1} * a_3^{E2} * a_3^{E3}}\sqrt{\alpha_j}\right) \tag{3.7}$$

Step 5. Calculate the geometric mean (*GM*) for each parameter of ordinary fuzzy comparison matrix and obtain $n \times 1$ matrix converted from $n \times n$ matrix as shown in Eq. (3.8). In this step, Buckley (1985)'s ordinary fuzzy AHP method is applied.

$$GM = \begin{pmatrix} \sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j1}^{E1} * a_{j1}^{E2} * a_{j1}^{E3}}\sqrt{\alpha_j}\right)}, \\ \sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j2}^{E1} * a_{j2}^{E2} * a_{j2}^{E3}}\sqrt{\alpha_j}\right)}, \\ \sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j3}^{E1} * a_{j3}^{E2} * a_{j3}^{E3}}\sqrt{\alpha_j}\right)} \end{pmatrix}, \text{j=1,2,…,n} \tag{3.8}$$

Step 6. Calculate the total values of each parameter in the column to normalize the values in $n \times 1$ matrix as shown in Eq (3.9).

$$\begin{pmatrix} \sum_{j=1}^{n}\left(\sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j1}^{E1} * a_{j1}^{E2} * a_{j1}^{E3}}\sqrt{\alpha_j}\right)}\right), \sum_{j=1}^{n}\left(\sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j2}^{E1} * a_{j2}^{E2} * a_{j2}^{E3}}\sqrt{\alpha_j}\right)}\right), \\ \sum_{j=1}^{n}\left(\sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j3}^{E1} * a_{j3}^{E2} * a_{j3}^{E3}}\sqrt{\alpha_j}\right)}\right) \end{pmatrix} \tag{3.9}$$

Step 7. Apply fuzzy division operation to get the normalized weights vector as shown in Eq (3.10).

$$\begin{pmatrix} \dfrac{\sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j1}^{E1}*a_{j1}^{E2}*a_{j1}^{E3}}\sqrt{\alpha_j}\right)}}{\sum_{j=1}^{n}\sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j3}^{E1}*a_{j3}^{E2}*a_{j3}^{E3}}\sqrt{\alpha_j}\right)}}, \dfrac{\sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j2}^{E1}*a_{j2}^{E2}*a_{j2}^{E3}}\sqrt{\alpha_j}\right)}}{\sum_{j=1}^{n}\sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j2}^{E1}*a_{j2}^{E2}*a_{j2}^{E3}}\sqrt{\alpha_j}\right)}}, \\ \dfrac{\sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j3}^{E1}*a_{j3}^{E2}*a_{j3}^{E3}}\sqrt{\alpha_j}\right)}}{\sum_{j=1}^{n}\sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j1}^{E1}*a_{j1}^{E2}*a_{j1}^{E3}}\sqrt{\alpha_j}\right)}} \end{pmatrix} \tag{3.10}$$

Step 8. Defuzzify the normalized weights vector using the method given by Eq. (3.11).

$$\alpha_j = \frac{\begin{pmatrix} \dfrac{\sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j1}^{E1}*a_{j1}^{E2}*a_{j1}^{E3}}\sqrt{\alpha_j}\right)}}{\sum_{j=1}^{n}\sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j3}^{E1}*a_{j3}^{E2}*a_{j3}^{E3}}\sqrt{\alpha_j}\right)}} + 2 * \dfrac{\sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j2}^{E1}*a_{j2}^{E2}*a_{j2}^{E3}}\sqrt{\alpha_j}\right)}}{\sum_{j=1}^{n}\sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j2}^{E1}*a_{j2}^{E2}*a_{j2}^{E3}}\sqrt{\alpha_j}\right)}} \\ + \dfrac{\sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j3}^{E1}*a_{j3}^{E2}*a_{j3}^{E3}}\sqrt{\alpha_j}\right)}}{\sum_{j=1}^{n}\sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j1}^{E1}*a_{j1}^{E2}*a_{j1}^{E3}}\sqrt{\alpha_j}\right)}} \end{pmatrix}}{4} \tag{3.11}$$

Step 9. Normalize the weights so that their sum is equal to 1 as shown in Eq (3.12). Thus, the priorities of the elements in the pairwise comparison matrix are obtained.

$$\frac{\dfrac{\left(\dfrac{\sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j1}^{E1}*a_{j1}^{E2}*a_{j1}^{E3}}\sqrt{\alpha_j}\right)}}{\sum_{j=1}^{n}\sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j3}^{E1}*a_{j3}^{E2}*a_{j3}^{E3}}\sqrt{\alpha_j}\right)}}+2*\dfrac{\sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j2}^{E1}*a_{j2}^{E2}*a_{j2}^{E3}}\sqrt{\alpha_j}\right)}}{\sum_{j=1}^{n}\sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j2}^{E1}*a_{j2}^{E2}*a_{j2}^{E3}}\sqrt{\alpha_j}\right)}}+\dfrac{\sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j3}^{E1}*a_{j3}^{E2}*a_{j3}^{E3}}\sqrt{\alpha_j}\right)}}{\sum_{j=1}^{n}\sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j1}^{E1}*a_{j1}^{E2}*a_{j1}^{E3}}\sqrt{\alpha_j}\right)}}\right)}{4}}{\sum_{j=1}^{n}\dfrac{\left(\dfrac{\sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j1}^{E1}*a_{j1}^{E2}*a_{j1}^{E3}}\sqrt{\alpha_j}\right)}}{\sum_{j=1}^{n}\sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j3}^{E1}*a_{j3}^{E2}*a_{j3}^{E3}}\sqrt{\alpha_j}\right)}}+2*\dfrac{\sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j2}^{E1}*a_{j2}^{E2}*a_{j2}^{E3}}\sqrt{\alpha_j}\right)}}{\sum_{j=1}^{n}\sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j2}^{E1}*a_{j2}^{E2}*a_{j2}^{E3}}\sqrt{\alpha_j}\right)}}+\dfrac{\sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j3}^{E1}*a_{j3}^{E2}*a_{j3}^{E3}}\sqrt{\alpha_j}\right)}}{\sum_{j=1}^{n}\sqrt[n]{\prod_{j=1}^{n}\left(\sqrt[3]{a_{j1}^{E1}*a_{j1}^{E2}*a_{j1}^{E3}}\sqrt{\alpha_j}\right)}}\right)}{4}} \tag{3.12}$$

Step 10. Apply Steps (3-9) for the other Z-fuzzy pairwise comparison matrices.

Step 11. Combine all the weights vectors to determine the best alternative as in classical AHP.

3.4 Application: Evaluation of Social Sustainable Development Factors

Since social sustainable factors affect the performance of sustainability development for cities, countries or regions, the choice of these factors is important issue. These factors were obtained to be evaluated by considering the literature. Since each factor is not of the same importance for social sustainable development, the importance weights of these factors should be calculated. For this purpose, in this study factors were weighted by experts. Factors and their explanations are given in Table 3.4.

Step 1. First, the hierarchy of the decision problem is determined. In this application, there are 6 main social sustainable development factors which are given in Table 3.4: demography (F1), health (F2), security (F3), education (F4), traffic (F5), culture (F6) and three experts (Es) to evaluate these factors.

Table 3.4 : Social sustainable development factors and their explanations.

Factors	Explanations	References
Demography	Population density, Population growth ratio, infant mortality ratio etc.	Phillis et al. (2017), Zinatizadeh et al. (2017), Tanguay et al. (2010), Moussiopoulos et al. (2010), Michael et al. (2014), Mascarenhas et. (2015), King (2016), Gazibey et al. (2014), Saraç and Alptekin (2017).
Health	Number of doctors, hospital bed per capita, health expenditures etc.	Phillis et al. (2017), Zinatizadeh et al. (2017), Zhang et al. (2016), Shen et al. (2011), Moussiopoulos et al. (2010), Mascarenhas et. (2015), King (2016), Gazibey et al. (2014), Saraç and Alptekin (2017).
Security	Crime rate, number of police stations per capita	Phillis et al. (2017), Zinatizadeh et al. (2017), Tanguay et al. (2010), Shen et al. (2011), Michael et al. (2014), Mascarenhas et. (2015), Gazibey et al. (2014), Saraç and Alptekin (2017).
Education	Literacy rate, number of students per teacher, graduation rate	Phillis et al. (2017), Zinatizadeh et al. (2017), Tanguay et al. (2010), Shen et al. (2011), Moussiopoulos et al. (2010), Braulio-Gonzalo et al. (2015), Mascarenhas et. (2015), King (2016), Gazibey et al. (2014), Saraç and Alptekin (2017).
Traffic	Number of traffic accident, number of fatal accidents	Zinatizadeh et al. (2017), Shen et al. (2011), Moussiopoulos et al. (2010), Gazibey et al. (2014), Saraç and Alptekin (2017).
Culture	Number of books per capita, number of cultural centers per capita	Zinatizadeh et al. (2017), Zhang et al. (2016), Shen et al. (2011), Mascarenhas et. (2015), Gazibey et al. (2014), Saraç and Alptekin (2017).

Step 2. Three experts compare the 6 main factors with each other using Z-fuzzy evaluation scales given in Tables 3.2-3.3. Tables 3.5-3.7 show Z-fuzzy evaluation matrices for the determined factors.

Step 2.1. Consistency checks were performed for three experts and consistency ratios were obtained as 0.089, 0.093 and 0.095 respectively. The calculations were continued since the assessments were consistent.

Table 3.5 : E1's evaluations.

Social Sustainable Development Factors	F1	F2	F3	F4	F5	F6
F1	(EI)	(1/STI, FR)	(1/MI, AR)	(1/VSTI, VWR)	(SLI, SR)	(MI, HR)
F2	(STI, FR)	(EI)	(SLI, VHR)	(SLI, HR)	(VSTI, AR)	(CI, SR)
F3	(MI, AR)	(1/SLI, VHR)	(EI)	(1/SLI, SR)	(MI, HR)	(STI, VHR)
F4	(VSTI, VWR)	(1/SLI, HR)	(SLI, SR)	(EI)	(VSTI, FR)	(CI, WR)
F5	(1/SLI, SR)	(1/VSTI, AR)	(1/MI, HR)	(1/VSTI, FR)	(EI)	(SLI, VWR)
F6	(1/MI, HR)	(1/CI, SR)	(1/STI, VHR)	(1/CI, WR)	(1/SLI, VWR)	(EI)

Table 3.6 : E2's evaluations.

Social Sustainable Development Factors	F1	F2	F3	F4	F5	F6
F1	(EI)	(1/VSTI, VHR)	(1/STI, AR)	(1/CI, WR)	(MI, FR)	(SLI, WR)
F2	(VSTI, VHR)	(EI)	(SLI, FR)	(1/SLI, VHR)	(CI, AR)	(VSTI, SR)
F3	(STI, AR)	(1/SLI, FR)	(EI)	(1/STI, FR)	(STI, WR)	(STI, WR)
F4	(CI, WR)	(SLI, VHR)	(STI, FR)	(EI)	(AI, SR)	(CI, VHR)
F5	(1/MI, FR)	(1/CI, AR)	(1/STI, WR)	(1/AI, SR)	(EI)	(1/MI, HR)
F6	(1/SLI, WR)	(1/VSTI, SR)	(1/STI, WR)	(1/CI, VHR)	(MI, HR)	(EI)

Table 3.7 : E3's evaluations.

Social Sustainable Development Factors	F1	F2	F3	F4	F5	F6
F1	(EI)	(1/VSTI, HR)	(1/CI, FR)	(1/VSTI, SR)	(1/MI, WR)	(1/STI, VHR)
F2	(VSTI, HR)	(EI)	(1/SLI, HR)	(EI, WR)	(STI, VHR)	(STI, SR)
F3	(CI, FR)	(SLI, HR)	(EI)	(SLI, VHR)	(STI, WR)	(VSTI, HR)
F4	(VSTI, SR)	(EI, WR)	(1/SLI, VHR)	(EI)	(MI, HR)	(STI, WR)
F5	(MI, WR)	(1/STI, VHR)	(1/STI, WR)	(1/MI, HR)	(EI)	(1/SLI, SR)
F6	(STI, VHR)	(1/STI, SR)	(1/VSTI, HR)	(1/STI, WR)	(SLI, SR)	(EI)

Step 3. Geometric means are calculated to aggregate the restriction and reliability functions of experts' evaluations using Eq. (3.5). Aggregated Z-fuzzy decision matrix is shown in Table 3.8.

Step 4. After aggregation calculation, obtained matrix is converted to ordinary fuzzy number using Definition 3.3. Table 3.9 shows the ordinary fuzzy decision matrix.

Table 3.8 : Aggregated Z-fuzzy decision matrix.

	F1	F2	F3
F1	((1, 1, 1), (1, 1, 1))	((0.12, 0.16, 0.24), (0.49, 0.59, 0.7))	((0.14, 0.19, 0.33), (0.63, 0.74, 0.84))
F2	((4.22, 6.26, 8.28), (0.49, 0.59, 0.7))	((1, 1, 1), (1, 1, 1))	((0.69, 1, 2.08), (0.49, 0.59, 0.7))
F3	((3, 5.13, 7.05), (0.63, 0.74, 0.84))	((0.481, 1, 1.442), (0.49, 0.59, 0.6952))	((1, 1, 1), (1, 1, 1))
F4	((5.59, 7.61, 9.32), (0.35, 0.46, 0.56))	((0.69, 1, 1.44), (0.45, 0.55, 0.65))	((1, 1.71, 2.76), (0.55, 0.65, 0.76))
F5	((0.41, 1, 1.71), (0.44, 0.54, 0.65))	((0.12, 0.15, 0.21), (0.73, 0.83, 0.93))	((0.16, 0.24, 0.48), (0.36, 0.46, 0.56))
F6	((0.58, 1.19, 1.91), (0.45, 0.55, 0.65))	((0.12, 0.15, 0.21), (0.7, 0.8, 0.9))	((0.13, 0.18, 0.28), (0.45, 0.55, 0.65))

	F4	F5	F6
F1	((0.11, 0.13, 0.18), (0.35, 0.46, 0.56))	((0.58, 1, 2.47), (0.44, 0.54, 0.65))	((0.52, 0.84, 1.71), (0.45, 0.55, 0.65))
F2	((0.69, 1, 1.44), (0.45, 0.55, 0.65))	((4.72, 6.8, 8.57), (0.73, 0.83, 0.93))	((4.72, 6.8, 8.57), (0.7, 0.8, 0.9))
F3	((0.36, 0.58, 1), (0.55, 0.65, 0.76))	((2.08, 4.22, 6.26), (0.36, 0.46, 0.56))	((3.56, 5.59, 7.61), (0.45, 0.55, 0.65))
F4	((1, 1, 1), (1, 1, 1))	((3.56, 5.94, 7.66), (0.52, 0.62, 0.72))	((5.28, 7.4, 8.88), (0.38, 0.48, 0.58))
F5	((0.13, 0.17, 0.28), (0.52, 0.62, 0.72))	((1, 1, 1), (1, 1, 1))	((0.41, 0.69, 1.44), (0.41, 0.52, 0.63))
F6	((0.11, 0.14, 0.19), (0.38, 0.48, 0.58))	((0.69, 1.44, 2.47), (0.41, 0.52, 0.63))	((1, 1, 1), (1, 1, 1))

Table 3.9 : Ordinary fuzzy decision matrix.

	F1	F2	F3	F4	F5	F6
F1	(1, 1, 1)	(0.09, 0.12, 0.18)	(0.12, 0.17, 0.29)	(0.07, 0.09, 0.12)	(0.43, 0.74, 1.82)	(0.39, 0.63, 1.27)
F2	(3.25, 4.82, 6.38)	(1, 1, 1)	(0.53, 0.77, 1.6)	(0.51, 0.74, 1.07)	(4.29, 6.19, 7.8)	(4.22, 6.09, 7.67)
F3	(2.58, 4.41, 6.06)	(0.37, 0.77, 1.11)	(1, 1, 1)	(0.29, 0.47, 0.81)	(1.41, 2.85, 4.23)	(2.64, 4.15, 5.65)
F4	(3.78, 5.15, 6.3)	(0.51, 0.74, 1.07)	(0.81, 1.38, 2.23)	(1, 1, 1)	(2.8, 4.69, 6.04)	(3.66, 5.14, 6.16)
F5	(0.3, 0.74, 1.26)	(0.11, 0.13, 0.19)	(0.11, 0.16, 0.33)	(0.1, 0.13, 0.22)	(1, 1, 1)	(0.29, 0.5, 1.04)
F6	(0.43, 0.88, 1.42)	(0.1, 0.13, 0.19)	(0.1, 0.13, 0.21)	(0.08, 0.09, 0.13)	(0.5, 1.04, 1.78)	(1, 1, 1)

Steps 5 and 6. Geometric means for each parameter of ordinary fuzzy decision matrix are calculated as in Buckley's ordinary fuzzy AHP method and Table 3.10 is obtained.

The values of each column in Table 3.10 are summed to use in normalization procedure.

Table 3.10 : Geometric means of the ordinary fuzzy evaluations.

Factors	Geometric means
F1	(0.227, 0.308, 0.494)
F2	(1.591, 2.169, 2.947)
F3	(1.007, 1.634, 2.252)
F4	(1.590, 2.242, 2.871)
F5	(0.217, 0.319, 0.513)
F6	(0.236, 0.338, 0.486)
Total	(4.868, 7.010, 9.564)

Step 7. Fuzzy division operation is applied using Eq. (3.10) to normalize the values in Table 3.10 and Table 3.11 is obtained.

Table 3.11 : Normalized values calculated from Table 3.10.

Factors	Normalized values
F1	(0.024, 0.044, 0.102)
F2	(0.166, 0.309, 0.605)
F3	(0.105, 0.233, 0.463)
F4	(0.166, 0.320, 0.590)
F5	(0.023, 0.045, 0.105)
F6	(0.025, 0.048, 0.100)

Step 8. Defuzzification process is applied for the normalized values in Table 3.11 by using Eq. (3.11). The defuzzified values are given in Table 3.12.

Table 3.12 : Defuzzified values calculated from Table 3.11.

Factors	Defuzzified values
F1	0.053
F2	0.348
F3	0.259
F4	0.349
F5	0.055
F6	0.055
Total	1.118

Step 9. The final crisp weights of the factors are obtained to provide that the sum of the weights is equal to 1.0. Table 3.13 shows the final factor weights and the ranking of the factors.

Table 3.13 : Final importance weights of the factors.

Factors		Final weights	Ranking
F1	Demography	0.0476	6
F2	Health	0.3108	2
F3	Security	0.2312	3
F4	Education	0.3120	1
F5	Traffic	0.0490	5
F6	Culture	0.0494	4

Thus, the most important factor is determined as education factor with a weight of 0.3120. Health factor follows it with a weight of 0.3108.

3.5 Sensitivity Analysis

Sensitivity analysis was performed in order to see how the changes in the reliability degree of experts affect the results. Each time in the sensitivity analysis, only one of the experts reliability degrees were reduced and increased while the other experts' reliability degrees' were kept constant. In addition, the reliabilities of all three experts were changed. Totally 8 cases were conducted and results are presented in Tables 3.14-3.17.

In the first case, all restriction values of the first expert were kept constant and their reliabilities were reduced by only one level. Assessments of other experts' reliabilities were not changed. This procedure was applied to all experts' evaluations in cases 1-3. Finally, the restriction values of all experts were kept constant and all of their reliabilities were reduced by one level in case 4.

In the cases 5-8, instead of the reductions in the first 4 cases, a level increase was made based on the present case. For example, in case 7, all restriction values of the Expert 3 were kept constant and their reliabilities were increased by only one level.

Table 3.14 : Reliability reduction, present case and reliability increase for only Expert 1.

Only Expert-1						
Factors	Reliability Reduction (Case 1)		Present Case		Reliability Increase (Case 5)	
	Weights	Rank	Weights	Rank	Weights	Rank
F1	0.04749	6	0.04763	6	0.04781	6
F2	0.31223	1	0.31081	2	0.31140	2
F3	0.23268	3	0.23116	3	0.23105	3
F4	0.30964	2	0.31202	1	0.31364	1
F5	0.04882	5	0.04896	5	0.04864	5
F6	0.04914	4	0.04942	4	0.04747	4

Table 3.15 : Reliability reduction, present case and reliability increase for only Expert 2.

Only Expert-2						
Factors	Reliability Reduction (Case 2)		Present Case		Reliability Increase (Case 6)	
	Weights	Rank	Weights	Rank	Weights	Rank
F1	0.04747	6	0.04763	6	0.04768	6
F2	0.31231	1	0.31081	2	0.30947	2
F3	0.22993	3	0.23116	3	0.23175	3
F4	0.31203	2	0.31202	1	0.31260	1
F5	0.04897	5	0.04896	5	0.04890	5
F6	0.04929	4	0.04942	4	0.04960	4

Table 3.16 : Reliability reduction, present case and reliability increase for only Expert 3.

Factors	Only Expert-3					
	Reliability Reduction (Case 3)		Present Case		Reliability Increase (Case 7)	
	Weights	Rank	Weights	Rank	Weights	Rank
F1	0.04765	6	0.04763	6	0.04761	6
F2	0.31131	2	0.31081	2	0.31047	2
F3	0.23097	3	0.23116	3	0.23132	3
F4	0.31160	1	0.31202	1	0.31227	1
F5	0.04889	5	0.04896	5	0.04900	5
F6	0.04957	4	0.04942	4	0.04932	4

Table 3.17 : Reliability reduction, present case and reliability increase for all experts.

Factors	All Experts					
	Reliability Reduction (Case 4)		Present Case		Reliability Increase (Case 8)	
	Weights	Rank	Weights	Rank	Weights	Rank
F1	0.04736	6	0.04763	6	0.04785	6
F2	0.31422	1	0.31081	2	0.30972	2
F3	0.23124	3	0.23116	3	0.23180	3
F4	0.30926	2	0.31202	1	0.31448	1
F5	0.04877	5	0.04896	5	0.04861	5
F6	0.04916	4	0.04942	4	0.04754	4

When results of the one level reduction in reliability are examined, it is seen that the ranking of health and education factor, which are in the first and second place, replaced with each other while the ranking of other factors remains the same in first 4 cases except case 3. This result shows that the rankings are sensitive to the degree of reliability of the experts' evaluations, indicating the importance of Z-fuzzy numbers.

When results of the one level increase in reliability are examined, it is seen that the ranking of all factors are the same. This result shows that the further increase of the reliability level of the experts does not change the results.

When the reductions, current situation and increases in reliability levels were examined together on the basis of experts, it was observed that increasing or decreasing the reliability of Expert 3 by one level does not change the results. Reduction and increases in the reliability level performed by only Expert 1, only Expert 2 and all experts cause changes in the rankings in the same way. Therefore, it can be concluded that the evaluations of Expert 1 and Expert 2 are more effective on the results. This suggests that experts may have different weights in decision-making. Therefore, future studies may focus on this issue.

3.6 Conclusion

Sustainable development has become one of the most important issues in globalizing world. How to make cities, regions and the world livable has become very important for people because of increasing world population, changing living conditions, technological and industrial developments, increasing globalization etc. The significant increase in the urbanization ratio caused by these developments affects the economic, social and environmental conditions in the cities. This conditions have become known worldwide and everyone is more conscious about leaving a more sustainable world to future generations (Zinatizadeh et al., 2017). The sustainability of the world is directly proportional to the sustainability of the countries, regions and cities.

In the literature, most studies on this subject are covered by qualitative research and there are few studies in the field of social sustainable development. This study handles the problem quantitatively which is one of the significant contribution of the study. This study presents a multi-experts MCDM approach for evaluating social sustainable development factors. This approach integrates Z-fuzzy number and Buckley's fuzzy AHP. Since the proposed model enables to weight and rank social sustainable development factors, the results can give guidance to many sustainable development researches.

The importance of this study is the first usage of the Z-fuzzy number for the weighting decision of social sustainable development factors. Another contribution is proposing the Z-fuzzy number integrated AHP method with multi-experts which can be useful in many problems and applications containing uncertainty. This study is also important because there are very few studies on Z-fuzzy AHP method in the literature. Besides, the Z-fuzzy number integrated AHP method allows the experts to include the degree of confidence of decision makers to the calculations, while the other fuzzy AHP methods do not.

The proposed approach is successfully applied for weighting social sustainable development factors based on the experts' evaluations. When looked at the application results, education factor is the first in the ranking with the weight of 0.3120 and demography is the last in the ranking with the weight of 0.0476.

In this study, a sensitivity analysis was conducted to examine the effect of reliability degrees on the results. The level of reliability of the evaluations made with Z-fuzzy numbers has been gradually changed, and thus whether the degree of confidence of the experts has made changes on the results has been examined. It is concluded from the change in the sensitivity analyzes results that it is important not only the evaluations of the decision makers but also how confident they are with their evaluations. This result also reveals the importance of Z-fuzzy numbers.

For further research, weights of factors obtained from this study can be used in future sustainability assessment studies. It can also be compared with the results obtained by combining Z-fuzzy numbers with other MCDM methods. In addition, this study can be extended with the presented approach for weighting of all sustainable development factors.

4. A NOVEL Z-FUZZY AHP&EDAS METHODOLOGY AND ITS APPLICATION TO WIND TURBINE SELECTION[3]

We face decision-making processes at every moment of our lives. In the decision-making process, people express their knowledge and thoughts via their personal opinions and comments. Decision makers (DMs) often use expressions containing doubt and uncertainty in their judgments. Expressions such as “not very clear”, “likely” etc. show the uncertainty of human thought and are frequently used in daily or business life. Zadeh (1965) introduced fuzzy set theory in order to model this ambiguity and subjectivity of human judgments and to use linguistic terms in the decision-making process. Thus, fuzzy set theory enables DMs to incorporate their uncertain information in the decision model.

DMs who have knowledge and experience are often not exactly sure of their assessments when they are making a decision. The probability of correct diagnosis of even a doctor is not one hundred percent (Xian et al., 2019). For example, one doctor can say “you likely have anemia”. In the medical world, tests and investigations can be performed to confirm this diagnosis. However, in many fields that need decision-making, subjective judgments cannot be confirmed by that way. Moreover, when quantitative data are used in decision making, they are treated to be exactly accurate since the sources' reliability level is not questioned. However, it would not be correct to assume the numerical data with 100% certainty due to factors such as the concept of time and measurement accuracy. The possible variations that may occur in numerical data can be modeled with different extensions of fuzzy set theory. However, when qualitative data consisting of uncertain judgments is used in decision making, it would be most logical to explicitly ask people about their confidence level in their judgments.

[3] This chapter is based on the paper “Tüysüz, N., & Kahraman, C. (2023). A Novel Z-Fuzzy AHP&EDAS Methodology and Its Application to Wind Turbine Selection. *Informatica*, 1-34.”

In these cases, the reliability of the experts' fuzzy judgments must be considered and incorporated to the decision model. As a result, it is clear that restrictive information must be integrated with reliability information when especially linguistic expressions which represent subjective judgments are employed in the decision model.

After the introduction of fuzzy set theory, fuzzy versions of classical MCDM methods have emerged to capture the DMs' uncertain expressions (Chatterjee et al., 2018a). These methods have been expanded by ordinary fuzzy sets and their several extensions, such as type-2 fuzzy sets, intuitionistic fuzzy sets, hesitant fuzzy sets, Pythagorean fuzzy sets, and neutrosophic sets, to find the best representation of human thinking structure. Although the extensions of fuzzy sets are highly beneficial and skilled to vague information, their capabilities are limited to represent the reliability of the assigned fuzzy data. In order to overcome this limitation and to reach more accurate and effective results, reliability information must be incorporated into the decision processes.

Z-fuzzy numbers have been proposed by Zadeh (2011) in order to deal with the vagueness and impreciseness of membership functions by incorporating a reliability function to the evaluation system as a complementary element. This can be commented as a similar effort by Zadeh to his type-2 fuzzy sets for preventing the criticisms that membership functions themselves are not fuzzy. Thus, the requirement of reliability information in the decision-making can be satisfied by the use of Z-fuzzy numbers. Z-fuzzy numbers reflect the uncertainty in DMs' mind through a reliability function, which express how confident they are about their evaluations. In the doctor example, whereas the word "anemia" represents restrictive information, the word "likely" represents reliability information.

Evaluation Based on Distance from Average Solution (EDAS) is one of the recently developed MCDM methods. The EDAS method has been integrated with various fuzzy set extensions to better define the DMs' uncertain judgments. However, these versions of the EDAS method such as intuitionistic fuzzy EDAS or picture fuzzy EDAS do not fully include the reliability information. To the best knowledge of the authors, the EDAS method has not been extended with Z-fuzzy numbers by any researcher. In the literature, there is only one paper trying to use linguistic Z-numbers in EDAS method, different from our study, for quality function deployment (Mao et al., 2021). In this study, EDAS method is extended to Z-fuzzy EDAS method using ordinary Z-fuzzy numbers to strengthen the reliability degree of the given decisions.

Main objectives of the study are as follows:

i. The first aim of the study is to extend the traditional EDAS method to Z-fuzzy EDAS for the solution of MCDM problems under vagueness and impreciseness, which takes the reliability of the experts' data into account.
ii. The second aim of this study is to integrate Z-fuzzy AHP method with Z-fuzzy EDAS method in order to use the criteria weights obtained from AHP in the Z-fuzzy EDAS method for ranking the alternatives.
iii. The proposed methodology is applied to a wind turbine technology selection problem to present its practicality and efficiency. A comparative analysis is performed by using the same data with the Z-fuzzy TOPSIS method.

This study contributes to the literature in four aspects:

i. First, a novel Z-fuzzy EDAS have been developed for the first time by formulating it step by step using Z-fuzzy numbers. Thus, the literature gap on Z-fuzzy MCDM methods will be filled.
ii. Second, to the best of our knowledge, it has not been developed a methodology integrating Z-fuzzy numbers and AHP & EDAS methods.
iii. Third, all steps of the Z-fuzzy EDAS method have been performed by Z-fuzzy numbers which prevents the loss of information existing in the fuzzy data.
iv. Finally, the proposed approach has been applied to a renewable energy problem in the literature illustrating how to use the proposed methodology step by step.

The rest of the paper is organized as follows. Section 4.1 presents a literature review on EDAS and Z-fuzzy MCDM. Section 4.2 includes the preliminaries of Z-fuzzy numbers. Section 4.3 presents the proposed Z-fuzzy AHP method and Section 4.4 gives the steps of the proposed Z-fuzzy EDAS method. Section 4.5 presents the application on wind turbine technology selection. Section 4.6 gives a comparative analysis using Z-fuzzy AHP&TOPSIS methodology. The last section presents the conclusions and future research directions.

4.1 Literature review on EDAS and Z-fuzzy MCDM

Decision making problems arise when there is a need for comparison or selection from a set of alternatives, taking into account impact of multiple conflicting criteria. For this purpose, various MCDM methods are constructed to determine the best alternative with respect to all relevant criteria (Chatterjee et al., 2018b). Decisions taken in daily life or business life may have different degrees of difficulty due to the factors such as the considered criteria, the relationship between them and the number of alternatives. However, when DMs need to evaluate the alternatives by considering many criteria; many factors such as the number of criteria and alternatives, criteria weights and conflicts between criteria further complicate the problem and need to be evaluated with more comprehensive methods. Therefore, MCDM methods are used in order to get more accurate decisions in solving more complex decision problems.

EDAS method has been introduced to the literature by Keshavarz Ghorabaee et al. (2015) as a MCDM method. It is based on the measurement of the positive and negative distances from the average solution rather than calculating the negative ideal solution (NIS) and positive ideal solution (PIS) as in TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) (Chatterjee and Kar, 2016) and VIKOR (Vise Kriterijumska Optimizacija I Kompromisno Resenje) methods. Thus, unlike the TOPSIS and VIKOR methods, EDAS offers a solution based on how far the alternatives are from the average solution instead of PIS and NIS.

After the introduction of EDAS method to the literature, it has been used in many application areas such as supplier selection, project selection, personnel selection, material selection and drug selection. Due to the fact that fuzzy set theory in decision making better defines human thoughts, various fuzzy extensions of EDAS method have been used more frequently than classical EDAS method in the literature. Table 4.1 presents the classical, stochastic, neutrosophic, and fuzzy EDAS papers published in the literature and their application areas in historical order.

Table 4.1 shows that the classical EDAS method has been developed by many extensions of ordinary fuzzy sets such as type-2 fuzzy sets, intuitionistic fuzzy sets and hesitant fuzzy sets. However, since it was only put forward in 2015, there is still a gap in the literature about the method and its usage areas.

Since the fuzzy versions of the EDAS method proposed so far do not fully reflect the reliability information, another possible extension of the classical EDAS method is realized in this study through Z-fuzzy numbers, which represent the natural language with better descriptive ability. Thus, apart from the fuzzy extensions in Table 4.1, the EDAS method has been extended with Z-fuzzy numbers, which are composed of trapezoidal restriction function and triangular fuzzy reliability function.

Table 4.1 : Papers in the literature on EDAS method.

Year	Authors	Extension of EDAS	Application area
2015	Keshavarz Ghorabaee et al.	Crisp EDAS	Inventory classification
2016b	Keshavarz Ghorabaee et al.	Fuzzy EDAS	Supplier selection
2017	Kahraman et al.	Intuitionistic EDAS	Solid waste disposal site selection
2017a	Keshavarz Ghorabaee et al.	Stochastic EDAS	Performance evaluation of bank branches
2017	Stanujkic et al.	Interval grey valued EDAS	Contractor selection
2017b	Keshavarz Ghorabaee et al.	Interval type-2 fuzzy EDAS	Supplier selection with respect to environmental criteria
2017c	Keshavarz Ghorabaee et al.	Interval type-2 fuzzy EDAS	Evaluation of subcontractors
2017	Peng and Liu	Single valued neutrosophic EDAS	Evaluation of software development project
2018	Stević et al.	Fuzzy EDAS	Carpenter manufacturer selection
2018	Feng et al.	Hesitant fuzzy EDAS	Project selection
2018c	Chatterjee et al.	Crisp EDAS	Material selection
2018	Keshavarz Ghorabaee et al.	Dynamic fuzzy EDAS	Evaluation of subcontractors
2018	Karabasevic et al.	Crisp EDAS	Personnel Selection
2018	Liang et al.	Integrated EDAS-ELECTRE method	Cleaner Production Evaluation
2018	Ilieva	Interval type-2 fuzzy EDAS	An illustrative example
2018	Karaşan and Kahraman	Interval-valued neutrosophic EDAS	Prioritization of the united nations national sustainable development goals
2018	Gündoğdu et al.	Hesitant fuzzy EDAS	Hospital selection
2019	Karaşan et al.	Interval-valued neutrosophic EDAS	Ranking of social responsibility projects
2019b	Zhang et al.	Picture 2-tuple linguistic EDAS	Green supplier selection
2019	Schitea et al.	Intuitionistic EDAS	Selection of hydrogen collection site
2019	Kundakcı	Crisp EDAS	Steam boiler selection
2019	Wang et al.	2-tuple linguistic neutrosophic EDAS	Safety assessment of construction project
2019	Stević et al.	Fuzzy EDAS	Supplier selection
2020	Yanmaz et al.	Interval-valued Pythagorean Fuzzy EDAS	Car selection
2020	Han and Wei	Neutrosophic EDAS	Investment evaluation
2020	Liang	Intuitionistic Fuzzy EDAS	Selection of energy-saving design projects
2020	He et al.	Pythagorean 2-tuple linguistic sets based EDAS	Construction project selection
2020	Darko and Liang	q-rang orthopair fuzzy EDAS	Mobile payment platform selection
2020	Li et al.	q-rung orthopair fuzzy EDAS	Refrigerator selection
2020	Mishra et al.	Intuitionistic fuzzy EDAS	Disposal method selection
2020	Tolga and Basar	Fuzzy EDAS	Hydroponic system evaluation
2021	Wei et al.	Probabilistic EDAS	Supplier selection
2021	Chinram et al.	Intuitionistic fuzzy EDAS	Geographical site selection for construction
2021	Özçelik and Nalkıran	Trapezoidal bipolar Fuzzy numbers based EDAS	Medical device selection

Table 4.1 (Continued): Papers in the literature on EDAS method.

Year	Authors	Extension of EDAS	Application area
2021	Jana and Pal	Bipolar fuzzy EDAS	Construction company selection
2021	Mao et al.	Z-fuzzy EDAS	Ranking of engineering characteristics in quality function deployment
2022	Mitra	Crisp EDAS	Selection of cotton fabric
2022	Batool et al.	EDAS method under Pythagorean probabilistic hesitant fuzzy information	Drug selection for coronavirus disease
2022	Garg and Sharaf	Spherical fuzzy EDAS	Supplier selection and industrial robot selection
2022	Mishra et al.	Fermatean fuzzy EDAS	Evaluation of sustainable third-party reverse logistics providers
2022	Naz et al.	2-tuple linguistic T-spherical fuzzy EDAS	Selecting of the best COVID-19 vaccine
2023	Liao et al.	Probabilistic hesitant fuzzy EDAS	Evaluation of the commercial vehicles and green suppliers
2022	Demircan and Acarbay	Neutrosophic fuzzy EDAS	Vendor selection
2022	Rogulj et al.	Intuitionistic fuzzy EDAS	Prioritization of historic bridges
2022	Huang et al.	2-tuple spherical linguistic EDAS	Selection of the optimal emergency response solution
2022	Polat and Bayhan	Fuzzy EDAS	Supplier selection
2022	Su et al.	Probabilistic uncertain linguistic EDAS	Green finance evaluation of enterprises
2023a	Akram et al.	Linguistic Pythagorean fuzzy EDAS	Selection of waste management technique

Table 4.2 : A literature review on MCDM studies using Z-fuzzy numbers.

Year	Authors	MCDM method used Z-fuzzy number	Application areas
2012b	Kang et al.	A proposed approach	Vehicle selection
2013	Azadeh et al.	AHP	Weighing the performance evaluation factors of universities
2014	Xiao	A proposed approach	Evaluation of cloths
2015	Sahrom and Dom	AHP and DEA	Risk assessment
2015	Yaakob and Gegov	TOPSIS	Stock selection
2016	Azadeh and Kokabi	DEA	Portfoilo selection
2016	Sadi-Nezhad and Sotoudeh-Anvari	DEA	Efficiency assessment
2016	Yaakob and Gegov	TOPSIS	Stock selection
2017	Peng and Wang	A proposed approach	ERP selection
2017a	Khalif et al.	TOPSIS	Performance assessment
2017b	Khalif et al.	TOPSIS	Staff selection
2017	Wang et al.	TODIM	Evaluation of medical inquiry applications
2018	Karthika and Sudha	AHP	Risk assessment
2018	Forghani et al.	TOPSIS	Supplier selection
2018	Chatterjee and Kar	COPRAS	Renewable energy selection
2018	Aboutorab et al.	Best-worst method	Supplier development problem
2018	Peng and Wang	MULTIMOORA	Evaluation of potential areas of air pollution
2018	Shen and Wang	VIKOR	Selection of economic development plan

Table 4.2 (continued): A literature review on MCDM studies using Z-fuzzy numbers.

Year	Authors	MCDM method used Z-fuzzy number	Application areas
2018	Akbarian Saravi et al.	DEA	Evaluation of biomass power plants location
2018	Kahraman and Otay	AHP	Power plant location selection
2019	Gardashova	TOPSIS	Vehicle selection
2019	Wang and Mao	TOPSIS	Supplier selection
2019	Xian et al.	TOPSIS	Numerical examples on investment and medical diagnosis
2019	Kahraman et al.	AHP	Evaluation of law offices
2019	Krohling et al.	TODIM and TOPSIS	Case studies from literature
2019	Shen et al.	MABAC	Selection of economy development program
2020	Yildiz and Kahraman	AHP	Prioritization of social sustainable development factors
2020	Qiao et al.	PROMETHEE	Travel plan selection
2020	Das et al.	VIKOR	Prioritazing risk of hazards for crane operations.
2020	Jiang et al.	DEMATEL	Hospital performance measurement
2020	Mohtashami and Ghiasvand	DEA	Evaluation of banks and financial institutes
2020a	Liu et al.	ANP and TODIM	Evaluation of suppliers for the nuclear power industry
2020	Tüysüz and Kahraman	AHP	Evaluation of social sustainable development factors
2020	Tüysüz and Kahraman	CODAS	Supplier selection
2021	Akhavein et al.	DEMATEL and VIKOR	Evaluation of projects
2021	Zhu and Hu	DEMATEL	Evaluation of sustainable value propositions for smart product-service systems
2021	Wang et al.	DEMATEL	Evaluation of human error probability for cargo loading operations.
2021	Mao et al.	EDAS	Ranking of engineering characteristics in quality function deployment
2021	Sergi and Ucal Sari	AHP and WASPAS	Evaluation of public services
2021	Karaşan et al.	DEMATEL	Blockchain risk assessment
2022	Peng et al.	MULTIMOORA	Hotel selection
2022	İlbahar et al.	DEMATEL and VIKOR	Evaluation of hydrogen energy storage systems
2022	Sari and Tüysüz	AHP and TOPSIS	Covid-19 risk assessment of occupations
2022	Liu et al.	ELECTRE II	Selection of logistics provider
2022	Rahmati et al.	SWARA and WASPAS	Prioritization of financial risk factors
2023	Gai et al.	MULTIMOORA	Green supplier selection
2022	RezaHoseini et al.	AHP and DEA	Performance evaluation of sustainable projects
2023	Božanić et al.	MABAC	Selection of the best contingency strategy

After Z-fuzzy numbers were introduced to the literature, they have been integrated with several MCDM methods such as AHP (Azadeh et al., 2013; Sergi and Ucal Sari, 2021; Tüysüz and Kahraman, 2020; Kahraman and Otay, 2018), TOPSIS (Krohling et al., 2019), VIKOR (Shen and Wang, 2018), and WASPAS (Sergi and Ucal Sari, 2021). Table 4.2 presents the Z-fuzzy number integrated MCDM methods based on their publication years.

As can be seen in Table 4.2, Z-fuzzy numbers are integrated with different MCDM methods and found different application areas. However, there is still a significant literature gap regarding the combined use of Z-fuzzy numbers and MCDM methods. This study contributes to fill this literature gap by integrating the EDAS method with Z-fuzzy numbers.

4.2 Z-Fuzzy Numbers: Preliminaries

DMs are often not 100% confident in their assignments for membership degrees. Hence, in addition to assigning a membership degree/function $\mu_{\tilde{A}}(x)$, it makes sense to also assign a reliability degree $\mu_{\tilde{B}}(x)$ so that DMs can reflect their confidence to the membership. The corresponding pairs ($\mu_{\tilde{A}}(x)$, $\mu_{\tilde{B}}(x)$) is known as a Z-fuzzy number which introduced by Zadeh (2011).

A Z-fuzzy number is an ordered pair of fuzzy numbers, $Z(\tilde{A}, \tilde{B})$ as given in Figure 4.1. The first component $\tilde{A}$ is a restriction function whereas the second component $\tilde{B}$ is a measure of reliability for the first component.

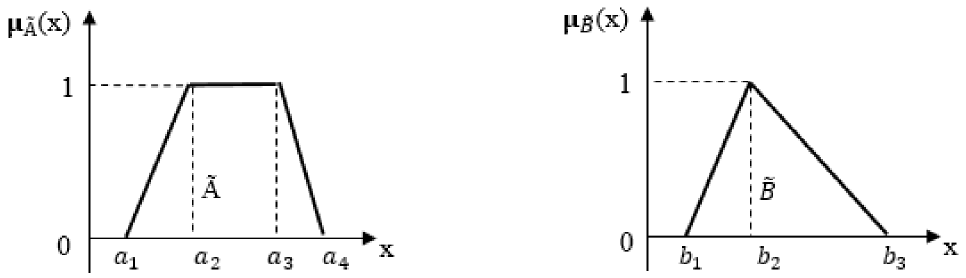

Figure 4.1 : A simple Z-fuzzy number, $Z(\tilde{A}, \tilde{B})$

The concept of a Z-fuzzy number is intended to provide a basis for computation with ordinary fuzzy numbers which are not reliable.

Definition 4.1. Let a fuzzy set $\tilde{A}$ be defined on a universe X may be given as: $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x)\rangle \mid x \epsilon X\}$ where $\mu_{\tilde{A}}: X \to [0,1]$ is the membership function $\tilde{A}$. The membership value $\mu_{\tilde{A}}(x)$ describes the degree of belongingness of $x \in X$ in $\tilde{A}$. The Fuzzy Expectation of a fuzzy set is given in Eq. (4.1).

$$E_A(x) = \int_x x\mu_A(x)\,dx \tag{4.1}$$

which is not the Expectation of Probability Space.

Definition 4.2: Converting Z-fuzzy number to Regular Fuzzy Number (Kang et al., 2012a)

Consider a Z-fuzzy number $Z = (\widetilde{A}, \tilde{B})$, which is described by Figure 4.1. The figure on the left is the part of restriction, and the figure on the right is the part of reliability. Let $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x)\rangle | \mu(x) \in [0,1]\}$ and $\tilde{B} = \{\langle x, \mu_{\tilde{B}}(x)\rangle | \mu(x) \in [0,1]\}$, $\mu_{\tilde{A}}(x)$ is a trapezoidal membership function, $\mu_{\tilde{B}}(x)$ is a triangular membership function.

(1) Convert the reliability function into a crisp number using Eq. (4.2).

$$\alpha = \frac{\int x\mu_{\tilde{B}}(x)\,dx}{\int \mu_{\tilde{B}}(x)\,dx} \tag{4.2}$$

where $\int$ denotes an algebraic integration.

Alternatively, the defuzzification equation $(a_1 + 2 * a_2 + 2 * a_3 + a_4)/6$ for symmetrical trapezoidal fuzzy numbers and $(a_1 + 2 * a_2 + a_3)/4$ for symmetrical triangular fuzzy numbers can be used.

(2) Weigh the restriction function with the crisp value of the reliability function (α). The weighted restriction number is denoted in Eq. (4.3).

$$\tilde{Z}^\alpha = \{\langle x, \mu_{\tilde{A}^\alpha}(x)\rangle | \mu_{\tilde{A}^\alpha}(x) = \alpha\mu_{\tilde{A}}(x), \mu(x) \in [0,1]\} \tag{4.3}$$

(3) Convert the weighted restriction number to ordinary fuzzy number using Eq. (4.4).

$$\tilde{Z}' = \left\{\langle x, \mu_{\tilde{Z}'}(x)\rangle \middle| \mu_{\tilde{Z}'}(x) = \mu_{\tilde{A}}\left(\frac{x}{\sqrt{\alpha}}\right), \mu(x) \in [0,1]\right\} \tag{4.4}$$

$\tilde{Z}'$ has the same Fuzzy Expectation with $\tilde{Z}^\alpha$, and they are equal with respect to Fuzzy Expectation, which can be denoted by Figure 4.2.

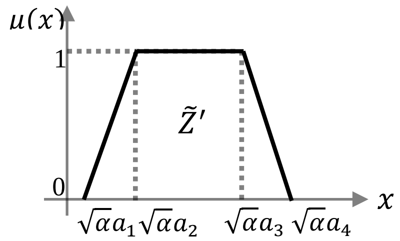

Figure 4.2 : Ordinary fuzzy number converted from Z-fuzzy number.

If the restriction function and reliability function are defined as in Figure 4.3, the calculations are modified as follows:

Let $\tilde{A}_\delta = \{\langle x, (\mu_{\tilde{A}}(x);\ \delta)\rangle | \mu(x) \in [0,1]\}$ and $\tilde{B}_\beta = \{\langle x, (\mu_{\tilde{B}}(x);\ \beta)\rangle | \mu(x) \in [0,1]\}$, $\mu_{\tilde{A}}^{\delta}(x)$ is a trapezoidal membership function, $\mu_{\tilde{B}}^{\beta}(x)$ is a triangular membership function.

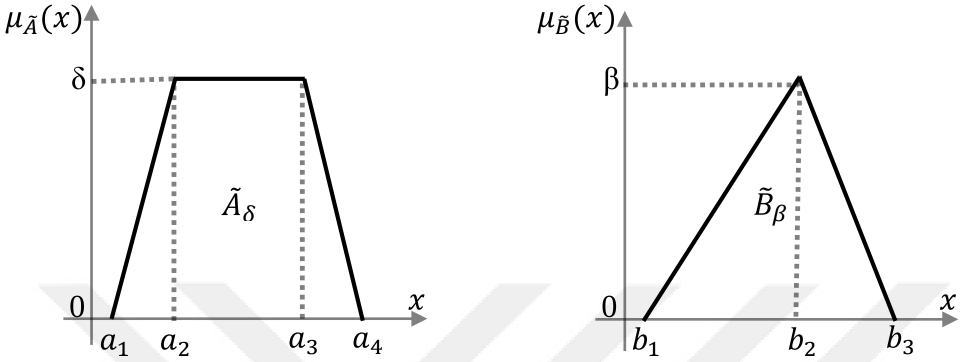

Figure 4.3 : A simple $\tilde{Z}_{\delta,\beta}$ number, $\tilde{Z}_{\delta,\beta} = \left(\tilde{A}_\delta, \tilde{B}_\beta\right)$

In this case, restriction and reliability functions are given in Eqs. (4.5-4.6), respectively. The reliability membership function in Eq. (4.6) is substituted into the defuzzification formula (Eq. (4.2)); so that, Eq. (4.7) is obtained.

$$\mu_{\tilde{A}}^{\delta}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}\ \delta\ , if\, a_1 \le x \le a_2 \\ \delta, \qquad if\, a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3}\ \delta, if\, a_3 \le x \le a_4 \\ 0, \qquad Otherwise \end{cases} \tag{4.5}$$

$$\mu_{\tilde{B}}^{\beta}(x) = \begin{cases} \frac{x-b_1}{b_2-b_1}\ \beta\ , if\, b_1 \le x \le b_2 \\ \frac{b_3-x}{b_3-b_2}\ \beta\ , if\, b_2 \le x \le b_3 \\ 0, \qquad Otherwise \end{cases} \tag{4.6}$$

Thus, we have

$$\sqrt{\alpha} = \sqrt{\frac{\int x \mu_{\tilde{B}}^{\beta}(x)\, dx}{\int \mu_{\tilde{B}}^{\beta}(x)\, dx}} \tag{4.7}$$

Then, the weighted $\tilde{Z}_{\delta,\beta}$ number can be denoted as in Eq.(4.8).

$$\tilde{Z}_{\delta,\beta}^{\alpha} = \left\{ \langle x, \mu_{\tilde{A}^{\alpha}}^{\delta}(x)\rangle \middle| \mu_{\tilde{A}^{\alpha}}^{\delta}(x) = \frac{\int x \mu_{\tilde{B}}^{\beta}(x)\, dx}{\int \mu_{\tilde{B}}^{\beta}(x)\, dx} \mu_{\tilde{A}}^{\delta}(x), \mu(x) \in [0,1] \right\} \tag{4.8}$$

The ordinary fuzzy number converted from Z-fuzzy number can be given as in Eq. (4.9)

$$\tilde{Z}'_{\delta,\beta} = \left\{ \langle x, \mu^{\delta}_{\tilde{Z}'}(x) \rangle \middle| \mu^{\delta}_{\tilde{Z}'}(x) = \mu^{\delta}_{\tilde{A}}\left(x \; \frac{\int \mu^{\beta}_{\tilde{B}}(x)\,dx}{\int x\mu^{\beta}_{\tilde{B}}(x)\,dx} \right),\; \mu(x) \in [0,1] \right\} \tag{4.9}$$

4.3 Z-Fuzzy AHP

The AHP method is one of the most widely used MCDM methods to calculate the criteria weights and there are several versions of it (Chatterjee and Kar, 2017). Due to the nature, it is usual for DMs to have hesitation while making pairwise comparisons, and in these situations, it is expected that they will not be absolutely sure about their evaluations. These preferences can be included in the decision methods by modeling the DMs' thinking structure under the concept of Z-fuzzy numbers. Therefore, in this study, to obtain criteria weights, it is suggested to collect DMs' judgments using Z-fuzzy numbers integrated AHP method rather than commonly used fuzzy versions of AHP method.

To calculate criteria weights, the steps of the Z-fuzzy AHP method are presented in the following:

Step 1. Determine the criteria set of the decision problem. Figure 4.4 can be used to establish the hierarchical structure of goal, main criteria and sub-criteria. Level 1 of the hierarchy represents a goal whereas Level 2 and Level 3 are composed of main-criteria and sub-criteria, respectively.

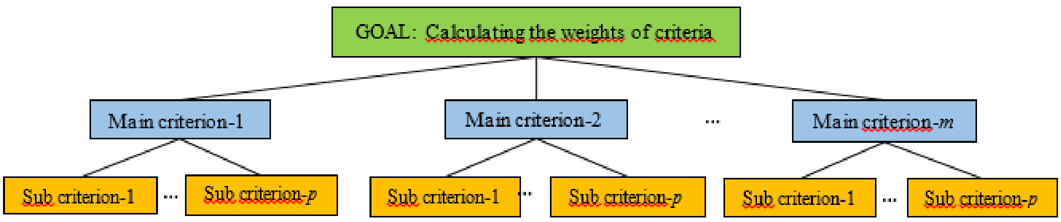

Figure 4.4 : Hierarchical structure for criteria.

Step 2. Determine the linguistic terms and their corresponding Z-fuzzy restriction and reliability numbers. Collect the linguistic pairwise comparison evaluations from each DM for the main criteria and sub-criteria by using questionnaires. Then, Z-fuzzy pairwise comparison matrices are constructed based on these evaluations. Each DM can use Z-fuzzy linguistic scales given in Tables 4.3-4.4 for his/her assessments, respectively.

Let each decision maker (DM_k) assign an independent assessment for any pairwise comparison as shown in Eq. (4.10):

$$Z^{DMk} = (\tilde{A}, \tilde{B}) = \left((a_1^{DMk}, a_2^{DMk}, a_3^{DMk}), (b_1^{DMk}, b_2^{DMk}, b_3^{DMk})\right) \tag{4.10}$$

Table 4.3 : Triangular Restriction Scale for pairwise comparisons of criteria.

Linguistic Terms	Abbr.	Restriction function
Equally Important	EI	(1,1,1;1)
Slightly Important	SLI	(1,1,3;1)
Moderately Important	MI	(1,3,5;1)
Strongly Important	STI	(3,5,7;1)
Very Strongly Important	VSTI	(5,7,9;1)
Certainly Important	CI	(7,9,10;1)
Absolutely Important	AI	(9,10,10;1)

Table 4.4 : Triangular Reliability Scale.

Linguistic Terms	Abbr.	Reliability function
Certainly Reliable	CR	(1,1,1;1)
Very Strongly Reliable	VSR	(0.8,0.9,1;1)
Strongly Reliable	SR	(0.7,0.8,0.9;1)
Very Highly Reliable	VHR	(0.6,0.7,0.8;1)
Highly Reliable	HR	(0.5,0.6,0.7;1)
Fairly Reliable	FR	(0.4,0.5,0.6;1)
Weakly Reliable	WR	(0.3,0.4,0.5;1)
Very Weakly Reliable	VWR	(0.2,0.3,0.4;1)
Strongly Unreliable	SU	(0.1,0.2,0.3;1),
Absolutely Unreliable	AU	(0,0.1,0.2;1)

Step 3. Calculate the consistency ratio (CR) of each Z-fuzzy pairwise comparison matrix obtained by the DMs' assessments. Defuzzify the restriction functions of Z-fuzzy numbers in the pairwise comparison matrix using Eq. (4.2) and obtain the crisp pairwise comparison matrix. Apply Saaty's classical consistency procedure and check if CR is less than 0.1, which is accepted as the consistency limit in the literature (Saaty, 1980).

Step 4. Apply the aggregation procedure for DMs' Z-fuzzy assessments. Each element of restriction and reliability functions of Z-fuzzy assessments is aggregated by using geometric mean and one Z-fuzzy decision matrix is obtained.

Assume three DMs assign the following terms:

$$\tilde{Z}^{DM1} = (\tilde{A}, \tilde{B}) = \left((a_1^{DM1}, a_2^{DM1}, a_3^{DM1}), (b_1^{DM1}, b_2^{DM1}, b_3^{DM1})\right)$$

$$\tilde{Z}^{DM2} = (\tilde{A}, \tilde{B}) = \left((a_1^{DM2}, a_2^{DM2}, a_3^{DM2}), (b_1^{DM2}, b_2^{DM2}, b_3^{DM2})\right)$$

$$\tilde{Z}^{DM3} = (\tilde{A}, \tilde{B}) = \left((a_1^{DM3}, a_2^{DM3}, a_3^{DM3}), (b_1^{DM3}, b_2^{DM3}, b_3^{DM3})\right)$$

Aggregation of these three DMs' assessments is made by using the geometric mean operator given in Eqs. (4.11-4.12):

$$\tilde{Z}^{Agg} = (\tilde{A}^{Agg}, \tilde{B}^{Agg}) = \begin{bmatrix} \tilde{c}_{11} & \tilde{c}_{12} & \cdots & \tilde{c}_{1m} \\ \tilde{c}_{21} & \tilde{c}_{22} & \cdots & \tilde{c}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{c}_{m1} & \tilde{c}_{m2} & \cdots & \tilde{c}_{mm} \end{bmatrix} \tag{4.11}$$

where

$$\tilde{c}_{ij} = \left(\begin{array}{c} \left(\sqrt[3]{a_{1,ij}^{DM1} * a_{1,ij}^{DM2} * a_{1,ij}^{DM3}}, \sqrt[3]{a_{2,ij}^{DM1} * a_{2,ij}^{DM2} * a_{2,ij}^{DM3}}, \sqrt[3]{a_{3,ij}^{DM1} * a_{3,ij}^{DM2} * a_{3,ij}^{DM3}} \right), \\ \left(\sqrt[3]{b_{1,ij}^{DM1} * b_{1,ij}^{DM2} * b_{1,ij}^{DM3}}, \sqrt[3]{b_{2,ij}^{DM1} * b_{2,ij}^{DM2} * b_{2,ij}^{DM3}}, \sqrt[3]{b_{3,ij}^{DM1} * b_{3,ij}^{DM2} * b_{3,ij}^{DM3}} \right) \end{array} \right)$$

i=1, 2, …, m;   j=1, 2, …, m. (4.12)

Step 5. Calculate the alpha (α) from the reliability components of the aggregated pairwise comparison matrix by using Eq. (4.13). The reciprocal reliability values are the multiplicative inverse of the calculated α values.

$$\alpha_{ij} = \frac{\left(\sqrt[3]{b_{1,ij}^{DM1} * b_{1,ij}^{DM2} * b_{1,ij}^{DM3}} + 2 * \sqrt[3]{b_{2,ij}^{DM1} * b_{2,ij}^{DM2} * b_{2,ij}^{DM3}} + \sqrt[3]{b_{3,ij}^{DM1} * b_{3,ij}^{DM2} * b_{3,ij}^{DM3}} \right)}{4}$$

i=1, 2, …, m;   j=1, 2, …, m. (4.13)

Step 6. Convert the Z-fuzzy numbers ($\tilde{Z}^{Agg}$) to ordinary fuzzy numbers ($\tilde{O}$) using the matrix obtained in Step 5 by using Eqs. (4.14) and (4.15).

$$\tilde{O} = \begin{bmatrix} \tilde{o}_{11} & \tilde{o}_{12} & \cdots & \tilde{o}_{1m} \\ \tilde{o}_{21} & \tilde{o}_{22} & \cdots & \tilde{o}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{o}_{m1} & \tilde{o}_{m2} & \cdots & \tilde{o}_{mm} \end{bmatrix} \tag{4.14}$$

where

$$\tilde{o}_{ij} = \left(\begin{array}{c} \sqrt[3]{a_{1,ij}^{DM1} * a_{1,ij}^{DM2} * a_{1,ij}^{DM3}} \sqrt{\alpha_{ij}}, \sqrt[3]{a_{2,ij}^{DM1} * a_{2,ij}^{DM2} * a_{2,ij}^{DM3}} \sqrt{\alpha_{ij}}, \\ \sqrt[3]{a_{3,ij}^{DM1} * a_{3,ij}^{DM2} * a_{3,ij}^{DM3}} \sqrt{\alpha_{ij}} \end{array} \right) \tag{4.15}$$

Step 7. Apply the ordinary fuzzy AHP method using Buckley's method (Buckley, 1985).

Step 7.1. Calculate the geometric mean vector ($\widetilde{GM}$) whose elements are given in Eqs. (4.16-4.17). Thus, $m \times 1$ matrix is obtained from $m \times m$ matrix.

$$\widetilde{GM} = \begin{bmatrix} \tilde{g}_{11} \\ \tilde{g}_{21} \\ \vdots \\ \tilde{g}_{m1} \end{bmatrix} \tag{4.16}$$

where

$$\tilde{g}_{i1} = \left(\sqrt[m]{\prod_{j=1}^{m} \left(\sqrt[3]{a_{1,ij}^{DM1} * a_{1,ij}^{DM2} * a_{1,ij}^{DM3}} \sqrt{\alpha_{ij}} \right)}, \sqrt[m]{\prod_{j=1}^{m} \left(\sqrt[3]{a_{2,ij}^{DM1} * a_{2,ij}^{DM2} * a_{2,ij}^{DM3}} \sqrt{\alpha_{ij}} \right)}, \sqrt[m]{\prod_{j=1}^{m} \left(\sqrt[3]{a_{3,ij}^{DM1} * a_{3,ij}^{DM2} * a_{3,ij}^{DM3}} \sqrt{\alpha_{ij}} \right)} \right)$$

i=1,2,…,m. (4.17)

Step 7.2. Sum the values in $\widetilde{GM}$ vector using Eq. (4.18).

$$\tilde{S} = \begin{pmatrix} \sum_{i=1}^{m} \left(\sqrt[m]{\prod_{j=1}^{m} \left(\sqrt[3]{a_{1,ij}^{DM1} * a_{1,ij}^{DM2} * a_{1,ij}^{DM3}} \sqrt{\alpha_{ij}} \right)} \right), \\ \sum_{i=1}^{m} \left(\sqrt[m]{\prod_{j=1}^{m} \left(\sqrt[3]{a_{2,ij}^{DM1} * a_{2,ij}^{DM2} * a_{2,ij}^{DM3}} \sqrt{\alpha_{ij}} \right)} \right), \\ \sum_{i=1}^{m} \left(\sqrt[m]{\prod_{j=1}^{m} \left(\sqrt[3]{a_{3,ij}^{DM1} * a_{3,ij}^{DM2} * a_{3,ij}^{DM3}} \sqrt{\alpha_{ij}} \right)} \right) \end{pmatrix} \tag{4.18}$$

Step 7.3. Apply fuzzy division operation to obtain relative fuzzy weights vector ($\tilde{R}$) of criteria as given in Eqs. (4.19-4.20).

$$\tilde{R} = \begin{bmatrix} \tilde{r}_{11} \\ \tilde{r}_{21} \\ \vdots \\ \tilde{r}_{m1} \end{bmatrix} = \begin{bmatrix} \tilde{g}_{11}/\tilde{S} \\ \tilde{g}_{21}/\tilde{S} \\ \vdots \\ \tilde{g}_{m1}/\tilde{S} \end{bmatrix} \tag{4.19}$$

where

$$\tilde{r}_{i1} = \left(\frac{\sqrt[m]{\prod_{j=1}^{m} \left(\sqrt[3]{a_{1,ij}^{DM1} * a_{1,ij}^{DM2} * a_{1,ij}^{DM3}} \sqrt{\alpha_{ij}} \right)}}{\sum_{i=1}^{m} \sqrt[m]{\prod_{j=1}^{m} \left(\sqrt[3]{a_{3,ij}^{DM1} * a_{3,ij}^{DM2} * a_{3,ij}^{DM3}} \sqrt{\alpha_{ij}} \right)}}, \frac{\sqrt[m]{\prod_{j=1}^{m} \left(\sqrt[3]{a_{2,ij}^{DM1} * a_{2,ij}^{DM2} * a_{2,ij}^{DM3}} \sqrt{\alpha_{ij}} \right)}}{\sum_{i=1}^{m} \sqrt[m]{\prod_{j=1}^{m} \left(\sqrt[3]{a_{2,ij}^{DM1} * a_{2,ij}^{DM2} * a_{2,ij}^{DM3}} \sqrt{\alpha_{ij}} \right)}}, \frac{\sqrt[m]{\prod_{j=1}^{m} \left(\sqrt[3]{a_{3,ij}^{DM1} * a_{3,ij}^{DM2} * a_{3,ij}^{DM3}} \sqrt{\alpha_{ij}} \right)}}{\sum_{i=1}^{m} \sqrt[m]{\prod_{j=1}^{m} \left(\sqrt[3]{a_{1,ij}^{DM1} * a_{1,ij}^{DM2} * a_{1,ij}^{DM3}} \sqrt{\alpha_{ij}} \right)}} \right)$$

i=1,2,…,m. (4.20)

Step 7.4. Defuzzify the relative fuzzy weights vector ($\tilde{R}$) using Eq. (4.21).

$$d_j = \frac{\left(\frac{\sqrt[m]{\prod_{j=1}^{m}\left(\sqrt[3]{a_{1,ij}^{DM1} * a_{1,ij}^{DM2} * a_{1,ij}^{DM3}}\sqrt{\alpha_{ij}}\right)}}{\sum_{i=1}^{m}\sqrt[m]{\prod_{j=1}^{m}\left(\sqrt[3]{a_{3,ij}^{DM1} * a_{3,ij}^{DM2} * a_{3,ij}^{DM3}}\sqrt{\alpha_{ij}}\right)}} + 2 * \frac{\sqrt[m]{\prod_{j=1}^{m}\left(\sqrt[3]{a_{2,ij}^{DM1} * a_{2,ij}^{DM2} * a_{2,ij}^{DM3}}\sqrt{\alpha_{ij}}\right)}}{\sum_{i=1}^{m}\sqrt[m]{\prod_{j=1}^{m}\left(\sqrt[3]{a_{2,ij}^{DM1} * a_{2,ij}^{DM2} * a_{2,ij}^{DM3}}\sqrt{\alpha_{ij}}\right)}} + \frac{\sqrt[m]{\prod_{j=1}^{m}\left(\sqrt[3]{a_{3,ij}^{DM1} * a_{3,ij}^{DM2} * a_{3,ij}^{DM3}}\sqrt{\alpha_{ij}}\right)}}{\sum_{i=1}^{m}\sqrt[m]{\prod_{j=1}^{m}\left(\sqrt[3]{a_{1,ij}^{DM1} * a_{1,ij}^{DM2} * a_{1,ij}^{DM3}}\sqrt{\alpha_{ij}}\right)}} \right)}{4}$$

j=1,2,…,m. (4.21)

Step 7.5. Normalize the defuzzified weights to satisfy $\sum w_j = 1$ using Eq. (4.22). Thus, the weights of the criteria are obtained as crisp values.

$$w_j = \frac{d_j}{\sum_{j=1}^{m} d_j} \qquad \text{j=1,2,…,m.} \tag{4.22}$$

Step 8. Apply Steps (3-7) for the other Z-fuzzy pairwise comparison matrices of DMs for the sub-criteria under each main criterion and obtain the weight of each sub-criterion $\acute{j}$, $\acute{j} = 1,2,\ldots,p$.

$w_{j\acute{j}}$ where $j = 1,2,\ldots,m$ and $\acute{j} = 1,2,\ldots,p$ for each j.

Step 9. Combine the local sub-criteria weights ($w_{j\acute{j}}$) and main criteria weights (w_j) in order to obtain global criteria weights ($w_{j\acute{j}}^{G}$) as in Eq. (4.23).

$$w_{j\acute{j}}^{G} = w_j * w_{j\acute{j}}$$

$$j = 1,2,\ldots,m \text{ and } \acute{j} = 1,2,\ldots,p \text{ for each } j. \tag{4.23}$$

4.4 Z-Fuzzy EDAS

The first fuzzy EDAS method is introduced by Keshavarz Ghorabaee et al. (2016b) for the solution of MCDM problems under uncertainty. It is integrated with various fuzzy set extensions to model the vagueness and impreciseness. In this study, due to the fact that these extensions cannot completely combine the reliability information with the EDAS method, it is extended to Z-fuzzy EDAS method by using ordinary Z-fuzzy numbers. This method allows to define the DMs' preferences over the alternatives with their degree of confidence, which creates a more comprehensive and flexible decision-making environment. Z-Fuzzy EDAS method is presented as follows:

Step 1. Determine the evaluation criteria $C = (C_1, C_2, \dots C_m)$ and alternatives $A = (A_1, A_2, \dots A_n)$ for the decision problem.

Step 2. Construct the fuzzy decision matrix ($\widetilde{D}$) using Z-fuzzy numbers, shown as in Eq. (4.24):

$$\widetilde{D} = \left[\tilde{x}_{ij}\right]_{n\times m} = \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \dots & \tilde{x}_{1m} \\ \tilde{x}_{21} & \tilde{x}_{22} & \dots & \tilde{x}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{n1} & \tilde{x}_{n2} & \dots & \tilde{x}_{nm} \end{bmatrix} \tag{4.24}$$

where $\tilde{x}_{ij} \geq 0$ and it denotes the Z-fuzzy performance value of ith alternative on jth criterion ($i \in \{1,2,\dots,n\}$ and $j \in \{1,2,\dots,m\}$).

Z-fuzzy linguistic restriction scale presented in Table 4.5 and the reliability scale in Table 4.4 are used for DMs' assessments in the decision matrix.

Table 4.5 : Z-fuzzy Restriction Scale for evaluation of alternatives.

Linguistic Terms	Abbr.	Restriction function
Very Poor	VP	(1/4,1/2,1/2,1;1)
Poor	P	(1/2,1,1,3;1)
Medium Poor	MP	(1,3,3,5;1)
Fair	F	(3,5,5,7;1)
Medium Good	MG	(5,7,7,9;1)
Good	G	(7,9,9,10;1)
Very Good	VG	(9,10,10,10;1)

Step 3. Aggregate the Z-fuzzy evaluation matrices of all DMs. Aggregation of three DMs' assessments is made by using the geometric mean given in Eqs. (4.25-4.26):

$$\tilde{Z}_{\widetilde{D}}^{Agg} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \dots & \tilde{x}_{1m} \\ \tilde{x}_{21} & \tilde{x}_{22} & \dots & \tilde{x}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{n1} & \tilde{x}_{n2} & \dots & \tilde{x}_{nm} \end{bmatrix} \tag{4.25}$$

where

$$\tilde{x}_{ij} = \left(\begin{array}{c} \left(\sqrt[3]{a_{1,ij}^{DM1} * a_{1,ij}^{DM2} * a_{1,ij}^{DM3}}, \sqrt[3]{a_{2,ij}^{DM1} * a_{2,ij}^{DM2} * a_{2,ij}^{DM3}}, \sqrt[3]{a_{3,ij}^{DM1} * a_{3,ij}^{DM2} * a_{3,ij}^{DM3}}\right), \\ \left(\sqrt[3]{b_{1,ij}^{DM1} * b_{1,ij}^{DM2} * b_{1,ij}^{DM3}}, \sqrt[3]{b_{2,ij}^{DM1} * b_{2,ij}^{DM2} * b_{2,ij}^{DM3}}, \sqrt[3]{b_{3,ij}^{DM1} * b_{3,ij}^{DM2} * b_{3,ij}^{DM3}}\right) \end{array} \right)$$

i=1, 2, …, n;   j=1, 2, …, m. (4.26)

Step 4. Calculate the Z-fuzzy average values ($\widetilde{AV}$) by using Eqs. (4.27-4.28).

$$\widetilde{AV} = \left[\widetilde{AV}_j\right]_{1\times m} = \left[\widetilde{AV}_1 \quad \widetilde{AV}_2 \quad \dots \quad \widetilde{AV}_j\right] \tag{4.27}$$

$$\widetilde{AV}_j = \frac{\sum_{i=1}^{n} \tilde{x}_{ij}}{n} \qquad \forall j \quad \text{j=1, 2, …, m.} \tag{4.28}$$

Step 5. Calculate the Z-fuzzy positive distance from average ($\widetilde{PDA}$) and Z-fuzzy negative distance from average ($\widetilde{NDA}$) for each alternative by employing Eqs. (4.29-4.32).

$$\widetilde{PDA} = \left[\widetilde{PDA}_{ij}\right]_{n\times m} \tag{4.29}$$

$$\widetilde{NDA} = \left[\widetilde{NDA}_{ij}\right]_{n\times m} \tag{4.30}$$

$$\begin{cases} \widetilde{PDA}_{ij} = \frac{\max(0,(\tilde{x}_{ij}-\widetilde{AV}_j))}{\widetilde{AV}_j} \\ \widetilde{NDA}_{ij} = \frac{\max(0,(\widetilde{AV}_j-\tilde{x}_{ij}))}{\widetilde{AV}_j} \end{cases} \quad \text{for benefit criteria} \tag{4.31}$$

$$\begin{cases} \widetilde{PDA}_{ij} = \frac{\max(0,(\widetilde{AV}_j-\tilde{x}_{ij}))}{\widetilde{AV}_j} \\ \widetilde{NDA}_{ij} = \frac{\max(0,(\tilde{x}_{ij}-\widetilde{AV}_j))}{\widetilde{AV}_j} \end{cases} \quad \text{for cost criteria} \tag{4.32}$$

where $\widetilde{PDA}_{ij}$ and $\widetilde{NDA}_{ij}$ represent the Z-fuzzy positive and negative distances from average value of i^{th} alternative according to j^{th} criterion, respectively.

To determine $\max(0,(\tilde{x}_{ij} - \widetilde{AV}_j))$, Z-fuzzy numbers are defuzzified as in Eqs. (4.33-4.34) and compared with each other.

$$a_j = \frac{(a_{1,ij}+2*a_{2,ij}+2*a_{3,ij}+a_{4,ij})}{6} \quad \forall j \text{ for restriction function} \tag{4.33}$$

$$b_j = \frac{(b_{1,ij}+2*b_{2,ij}+b_{3,ij})}{4} \quad \forall j \text{ for reliability function} \tag{4.34}$$

After determining the $\max(0,(\tilde{x}_{ij} - \widetilde{AV}_j))$, we still continue with Z-fuzzy numbers. Then, $\max(0,(\tilde{x}_{ij} - \widetilde{AV}_j))$ is divided by $\widetilde{AV}_j$ using Z-fuzzy numbers.

Step 6. Use the criteria weights obtained by Z-fuzzy AHP method in Section 4.3 and calculate the weighted summation of $\widetilde{PDA}$ and $\widetilde{NDA}$ shown as in Eqs. (4.35-4.36).

$$\widetilde{SP}_i = \sum_{j=1}^{m} w_j * \widetilde{PDA}_{ij} \tag{4.35}$$

$$\widetilde{SN}_i = \sum_{j=1}^{m} w_j * \widetilde{NDA}_{ij} \tag{4.36}$$

where $w_j = (w_1, w_2, \dots, w_m)$ and it is the weight of j^{th} criterion.

w_j ($0 < w_j < 1$) denotes the weight of j^{th} criterion and $\sum_{j=1}^{m} w_j = 1$.

Step 7. Transform the obtained Z-fuzzy $\widetilde{SP}_i$ and $\widetilde{SN}_i$ values to positive values if there is any negative value among them for all alternatives shown as in Eqs. (4.37-4.40). Thus, we obtain the shifted $\widetilde{SP}_i$ and $\widetilde{SN}_i$ values, $\widetilde{SSP}_i$ and $\widetilde{SSN}_i$, respectively.

For restriction function:

$$\widetilde{SSP}_i^{Res} = \widetilde{SP}_i^{Res} + max_i \left|\left(\widetilde{SP}_{i_{a_1}}^{Res}\right)\right|, \text{ if any } a_1 < 0. \qquad (4.37)$$

$$\widetilde{SSN}_i^{Res} = \widetilde{SN}_i^{Res} + max_i \left|\left(\widetilde{SN}_{i_{a_1}}^{Res}\right)\right|, \text{ if any } a_1 < 0. \qquad (4.38)$$

For reliability function:

$$\widetilde{SSP}_i^{Rel} = \widetilde{SP}_i^{Rel} + max_i \left|\left(\widetilde{SP}_{i_{b_1}}^{Rel}\right)\right|, \text{ if any } b_1 < 0. \qquad (4.39)$$

$$\widetilde{SSN}_i^{Rel} = \widetilde{SN}_i^{Rel} + max_i \left|\left(\widetilde{SN}_{i_{b_1}}^{Rel}\right)\right|, \text{ if any } b_1 < 0. \qquad (4.40)$$

Step 8. Normalize the Z-fuzzy $\widetilde{SSP}_i$ and $\widetilde{SSN}_i$ values by using Eqs. (4.41-4.44).

for restriction function

$$\widetilde{NSP}_{i_a}^{Res} = \left(\frac{\widetilde{SSP}_{i_{a_1}}}{\max_i(\widetilde{SP}_i^{Res})}, \frac{\widetilde{SSP}_{i_{a_2}}}{\max_i(\widetilde{SP}_i^{Res})}, \frac{\widetilde{SSP}_{i_{a_3}}}{\max_i(\widetilde{SP}_i^{Res})}, \frac{\widetilde{SSP}_{i_{a_4}}}{\max_i(\widetilde{SP}_i^{Res})}\right) \qquad (4.41)$$

and

$$\widetilde{NSN}_{i_a}^{Res} = (1,1,1,1) - \left(\frac{\widetilde{SSN}_{i_{a_4}}}{\max_i(\widetilde{SN}_i^{Res})}, \frac{\widetilde{SSN}_{i_{a_3}}}{\max_i(\widetilde{SN}_i^{Res})}, \frac{\widetilde{SSN}_{i_{a_2}}}{\max_i(\widetilde{SN}_i^{Res})}, \frac{\widetilde{SSN}_{i_{a_1}}}{\max_i(\widetilde{SN}_i^{Res})}\right) \qquad (4.42)$$

for reliability function

$$\widetilde{NSP}_{i_b}^{Rel} = \left(\frac{\widetilde{SSP}_{i_{b_1}}}{\max_i(\widetilde{SP}_i^{Rel})}, \frac{\widetilde{SSP}_{i_{b_2}}}{\max_i(\widetilde{SP}_i^{Rel})}, \frac{\widetilde{SSP}_{i_{b_3}}}{\max_i(\widetilde{SP}_i^{Rel})}\right) \qquad (4.43)$$

and

$$\widetilde{NSN}_{i_b}^{Rel} = (1,1,1) - \left(\frac{\widetilde{SSN}_{i_{b_3}}}{\max_i(\widetilde{SN}_i^{Rel})}, \frac{\widetilde{SSN}_{i_{b_2}}}{\max_i(\widetilde{SN}_i^{Rel})}, \frac{\widetilde{SSN}_{i_{b_1}}}{\max_i(\widetilde{SN}_i^{Rel})}\right) \qquad (4.44)$$

Step 9. Calculate the Z-fuzzy appraisal score ($\widetilde{AS}_i = (AS_{i_a}^{Res}, AS_{i_b}^{Rel})$) of alternatives, as shown in Eqs. (4.45-4.46):

$$AS_{i_a}^{Res} = \frac{1}{2}\left(\widetilde{NSP}_{i_a}^{Res} + \widetilde{NSN}_{i_a}^{Res}\right) \qquad (4.45)$$

$$AS_{i_b}^{Rel} = \frac{1}{2}\left(\widetilde{NSP}_{i_b}^{Rel} + \widetilde{NSN}_{i_b}^{Rel}\right) \qquad (4.46)$$

Step 10. Convert the Z-fuzzy $\widetilde{AS}_i$ to ordinary fuzzy number using Definition 4.2.

Step 11. Transform the ordinary fuzzy $\widetilde{AS}_i$ to a crisp number using Eq. (4.2).

Step 12. Rank the alternatives according to the decreasing values of crisp AS_i. The alternative which has the highest AS_i is the best choice among the alternatives.

Figure 4.5 shows the flowchart of the methodology which integrates Z-fuzzy AHP and Z-fuzzy EDAS methods. The proposed methodology aims at finding the weights of the criteria to be used in wind turbine selection (Z-fuzzy AHP) and also ranking the alternatives (Z-fuzzy EDAS) according to these criteria.

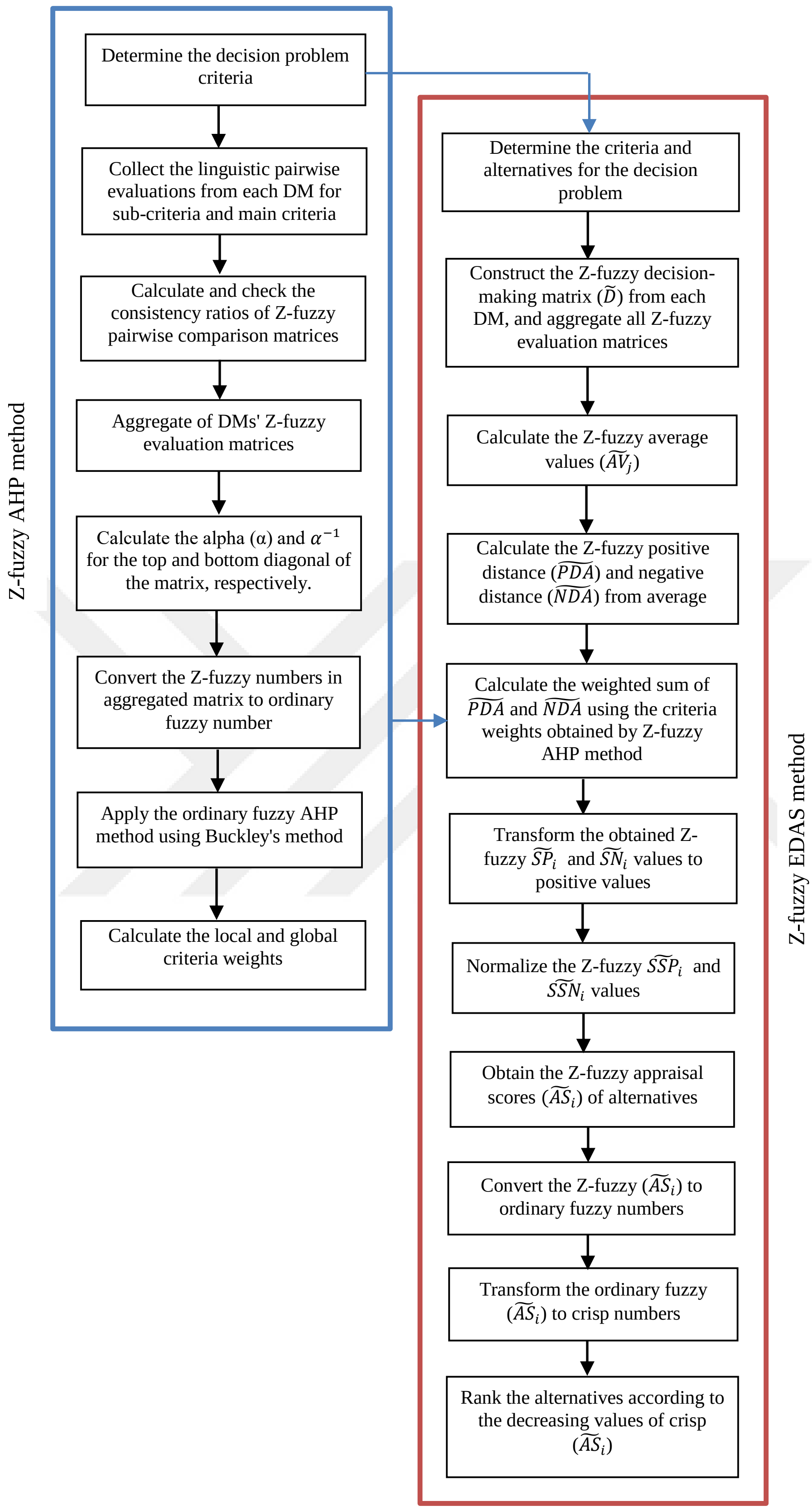

Figure 4.5 : Proposed Z-fuzzy AHP&EDAS methodology.

4.5 Application

Wind power is one of the fastest growing renewable energy alternatives. Due to the increasing energy demand, investments toward renewable energy sources are getting more importance day by day. Wind energy is the most widely used renewable energy source in Turkey (Kahraman and Kaya, 2010). According to the March 2022 TEİAŞ (Turkish Electricity Transmission Corporation) report, there are 355 wind power plants, and approximately 10861 megawatts of energy are produced from the wind in Turkey (TEİAŞ, 2022). In order to produce energy efficiently from the wind, the turbine characteristics of the power plant to be established have great importance. Therefore, the selection of wind turbines in a wind energy investment is extremely important for investors. There are many types of wind turbines according to their characteristics. In order to produce energy efficiently from the wind, the right wind turbine should be selected by the DMs according to the wind characteristics of the region to be established. In addition, the problem should be considered as a MCDM problem since many factors should be evaluated together in wind turbine selection. The MCDM studies of wind turbine selection in the literature are quite limited (Supciller and Toprak, 2020). Studies related to wind turbine selection can be found in Supciller and Toprak (2020) and Pang et al. (2021).

The proposed Z-fuzzy AHP&EDAS methodology is applied for the selection of the best alternative among wind turbines in the Aegean region of Turkey. For this purpose, in Step 1, the alternatives and criteria have been determined. There are five wind turbine alternatives represented by A1, A2, A3, A4 and A5 and six criteria which are reliability (C1), technical characteristics (C2), performance (C3), cost factors (C4), availability (C5) and maintenance (C6) (Cevik Onar et al., 2015). In Step 2, decision matrices have been constructed by three DMs using the linguistic terms given in Tables 4.4 and 4.5. Three DMs' pairwise comparison matrices for the criteria are presented in Tables 4.6-4.8.

Table 4.6 : Pairwise comparisons of the criteria by DM1.

DM1	C1	C2	C3	C4	C5	C6
C1	(EI, CR)	(CI, VSR)	(STI, HR)	(SLI, VSR)	(VSTI, VHR)	(CI, VSR)
C2	(1/CI, VSR)	(EI, CR)	(1/MI, SR)	(1/VSTI, FR)	(1/MI, SR)	(MI, FR)
C3	(1/STI, HR)	(MI, SR)	(EI, CR)	(1/MI, VHR)	(SLI, VSR)	(STI, VHR)
C4	(1/SLI, VSR)	(VSTI, FR)	(MI, VHR)	(EI, CR)	(STI, FR)	(CI, VSR)
C5	(1/VSTI, VHR)	(MI, SR)	(1/SLI, VSR)	(1/STI, FR)	(EI, CR)	(STI, WR)
C6	(1/CI, VSR)	(1/MI, FR)	(1/STI, VHR)	(1/CI, VSR)	(1/STI, WR)	(EI, CR)

λ_{max} = 6.6085, Consistency index (CI) = 0.1216, Consistency ratio (CR) = 0.097

Table 4.7 : Pairwise comparisons of the criteria by DM2.

DM2	C1	C2	C3	C4	C5	C6
C1	(EI, CR)	(VSTI, VHR)	(MI, FR)	(EI, SR)	(STI, SR)	(VSTI, HR)
C2	(1/VSTI, VHR)	(EI, CR)	(1/STI, VHR)	(1/CI, HR)	(1/SLI, VSR)	(SLI, VSR)
C3	(1/MI, FR)	(STI, VHR)	(EI, CR)	(1/STI, FR)	(MI, FR)	(MI, HR)
C4	(EI, SR)	(CI, HR)	(STI, FR)	(EI, CR)	(VSTI, VHR)	(CI, VHR)
C5	(1/STI, SR)	(SLI, VSR)	(1/MI, FR)	(1/VSTI, VHR)	(EI, CR)	(MI, FR)
C6	(1/VSTI, HR)	(1/SLI, VSR)	(1/MI, HR)	(1/CI, VHR)	(1/MI, FR)	(EI, CR)

λ_{max} = 6.5761, Consistency index (CI) = 0.1152, Consistency ratio (CR) = 0.092

Table 4.8 : Pairwise comparisons of the criteria by DM3.

DM3	C1	C2	C3	C4	C5	C6
C1	(EI, CR)	(AI, SR)	(VSTI, VHR)	(MI, WR)	(VSTI, VHR)	(STI, HR)
C2	(1/AI, SR)	(EI, CR)	(1/SLI, SR)	(1/STI, VHR)	(EI, VSR)	(1/SLI, SR)
C3	(1/VSTI, VHR)	(SLI, SR)	(EI, CR)	(1/MI, VSR)	(MI, FR)	(SLI, FR)
C4	(1/MI, WR)	(STI, VHR)	(MI, VSR)	(EI, CR)	(CI, HR)	(VSTI, HR)
C5	(1/VSTI, VHR)	(EI, VSR)	(1/MI, FR)	(1/CI, HR)	(EI, CR)	(1/SLI, VSR)
C6	(1/STI, HR)	(SLI, SR)	(1/SLI, FR)	(1/VSTI, HR)	(SLI, VSR)	(EI, CR)

λ_{max} = 6.5962, Consistency index (CI) = 0.1192, Consistency ratio (CR) = 0.095

Applying the Z-fuzzy AHP method in Section 4.3 the criteria weights have been obtained as in Table 4.9.

Table 4.9 : Criteria weights obtained by Z-fuzzy AHP method.

Reliability	Technical char.	Performance	Cost factors	Availability	Maintenance
0.353	0.046	0.118	0.355	0.074	0.053

After the DMs have compared the criteria, the evaluations of the alternatives according to the criteria have been collected. Tables 4.10-4.12 show the Z-fuzzy decision matrices including the linguistic evaluations of three DMs.

Table 4.10 : Z-fuzzy decision matrix of DM1.

	C1	C2	C3	C4	C5	C6
A1	(MG, SR)	(VP, HR)	(VG, SR)	(F, HR)	(MG, FR)	(P, SR)
A2	(VG, FR)	(F, VHR)	(P, SU)	(G, VHR)	(P, WR)	(VG, SR)
A3	(MG, HR)	(MG, HR)	(G, HR)	(VG, FR)	(MP, SU)	(G, HR)
A4	(G, HR)	(G, SR)	(F, WR)	(P, SR)	(VG, HR)	(F, SU)
A5	(P, SR)	(VG, HR)	(VP, FR)	(G, HR)	(MG, HR)	(VG, HR)

Table 4.11 : Z-fuzzy decision matrix of DM2.

	C1	C2	C3	C4	C5	C6
A1	(F, VHR)	(MP, VHR)	(MG, HR)	(G, SR)	(MG, SR)	(F, FR)
A2	(G, SR)	(G, WR)	(F, VWR)	(G, WR)	(P, HR)	(G, FR)
A3	(MP, SU)	(G, VSR)	(VG, FR)	(G, HR)	(G, HR)	(MG, SR)
A4	(VG, FR)	(VG, HR)	(G, HR)	(VP, HR)	(VG, SU)	(G, HR)
A5	(F, HR)	(G, SR)	(P, HR)	(MG, FR)	(MG, VHR)	(G, VSR)

Table 4.12 : Z-fuzzy decision matrix of DM3.

	C1	C2	C3	C4	C5	C6
A1	(MP, HR)	(F, SR)	(G, FR)	(MG, SU)	(G,SU)	(MP, HR)
A2	(MG, WR)	(MG, FR)	(MP, HR)	(VG, FR)	(VP, VHR)	(VG, VHR)
A3	(G, FR)	(MG, SR)	(G, SR)	(G, SR)	(F, SU)	(F, WR)
A4	(F, HR)	(VG, FR)	(MG, FR)	(P, WR)	(G, SR)	(MG, SR)
A5	(MP, VHR)	(G, FR)	(F, CR)	(MG, SU)	(F, HR)	(G, WR)

In Step 3, the individual evaluations of DMs are aggregated by using geometric mean method given by Eqs. (4.25-4.26). The obtained aggregated matrix is presented in Table 4.13.

Table 4.13 : Aggregated evaluations of wind turbines.

	Criteria	Z-fuzzy aggregated evaluations
A1	Reliability	((2.47,4.72, 4.72, 6.80), (0.59, 0.70, 0.80))
	Technical characteristics	((0.91, 1.96, 1.96, 3.27), (0.59, 0.70, 0.80))
	Performance	((6.80, 8.57, 8.57, 9.65), (0.52, 0.62, 0.72))
	Cost factors	((4.72, 6.80, 6.80, 8.57), (0.33, 0.46, 0.57))
	Availability	((5.59, 7.61, 7.61, 9.32), (0.30, 0.43, 0.55))
	Maintenance	((1.14, 2.47, 2.47, 4.72), (0.52, 0.62, 0.72))
A2	Reliability	((6.80, 8.57, 8.57, 9.65), (0.44, 0.54, 0.65))
	Technical characteristics	((4.72, 6.80, 6.80, 8.57), (0.42, 0.52, 0.62))
	Performance	((1.14, 2.47, 2.47, 4.72), (0.22, 0.33, 0.44))
	Cost factors	((7.61, 9.32, 9.32, 10.00), (0.42, 0.52, 0.62))
	Availability	((0.40, 0.79, 0.79, 2.08), (0.45, 0.55, 0.65))
	Maintenance	((8.28, 9.65, 9.65, 10.00), (0.55, 0.65, 0.76))
A3	Reliability	((3.27, 5.74, 5.74, 7.66), (0.27, 0.39, 0.50))
	Technical characteristics	((5.59, 7.61, 7.61, 9.32), (0.63, 0.73, 0.83))
	Performance	((7.61, 9.32, 9.32, 10.00), (0.52, 0.62, 0.72))
	Cost factors	((7.61, 9.32, 9.32, 10.00), (0.52, 0.62, 0.72))
	Availability	((2.76, 5.13, 5.13, 7.05), (0.17, 0.29, 0.40))
	Maintenance	((4.72, 6.80, 6.80, 8.57), (0.47, 0.58, 0.68))
A4	Reliability	((5.74, 7.66, 7.66, 8.88), (0.46, 0.56, 0.66)
	Technical characteristics	((8.28, 9.65, 9.65, 10.00), (0.52, 0.62, 0.72))
	Performance	((4.72, 6.80, 6.80, 8.57), (0.39, 0.49, 0.59))
	Cost factors	((0.40, 0.79, 0.79, 2.08), (0.47, 0.58, 0.68))
	Availability	((8.28, 9.65, 9.65, 10.00), (0.33, 0.46, 0.57))
	Maintenance	((4.72, 6.80, 6.80, 8.57), (0.33, 0.46, 0.57))
A5	Reliability	((1.14, 2.47, 2.47, 4.72), (0.59, 0.70, 0.80))
	Technical characteristics	((7.61, 9.32, 9.32, 10.00), (0.52, 0.62, 0.72))
	Performance	((0.72, 1.36, 1.36, 2.76), (0.54, 0.65, 0.75))
	Cost factors	((5.59, 7.61, 7.61, 9.32), (0.27, 0.39, 0.50))
	Availability	((4.22, 6.26, 6.26, 8.28), (0.53, 0.63, 0.73))
	Maintenance	((7.61, 9.32, 9.32, 10.00), (0.47, 0.58, 0.68))

In Step 4, using the aggregated evaluations and Eqs. (4.27-4.28), the Z-fuzzy average values are calculated for both the restriction and reliability functions separately, and the resulting values are shown in Table 4.14.

Table 4.14 : Z-fuzzy average values.

Criteria	Z-fuzzy average values
Reliability	((3.88, 5.83, 5.83, 7.54), (0.47, 0.58, 0,68))
Technical characteristics	((5.42, 7.07, 7.07, 8.23), (0.53, 0.64, 0.74))
Performance	((4.2, 5.7, 5.7, 7.14), (0.44, 0.54, 0.65))
Cost factors	((5.19, 6.77, 6.77, 7.99), (0.4, 0.51, 0.62))
Availability	((4.25, 5.89, 5.89, 7.35), (0.36, 0.47, 0.58))
Maintenance	((5.29, 7.01, 7.01, 8.37), (0.47, 0.58, 0.68))

In Step 5, Z-fuzzy $\widetilde{PDA}$ and $\widetilde{NDA}$ values are obtained for each alternative using Eqs. (4.29-4.34) and they are shown in Tables 4.15-4.16, respectively.

Table 4.15 : Z-fuzzy $\widetilde{PDA}$ values.

	Criteria	Z-fuzzy $\widetilde{PDA}$ values
A1	Reliability	((0, 0, 0, 0), (-0.127, 0.203, 0.684))
	Technical characteristics	((0, 0, 0, 0), (-0.195, 0.092, 0.488))
	Performance	((-0.047, 0.503, 0.503, 1.299), (-0.196, 0.145, 0.652))
	Cost factors	((0, 0, 0, 0), (-0.279, 0.108, 0.73))
	Availability	((-0.238, 0.292, 0.292, 1.194), (0, 0, 0))
	Maintenance	((0.069, 0.648, 0.648, 1.365), (0, 0, 0))
A2	Reliability	((-0.098, 0.47, 0.47, 1.485), (0, 0, 0))
	Technical characteristics	((0, 0, 0, 0), (0, 0, 0))
	Performance	((0, 0, 0, 0), (0, 0, 0))
	Cost factors	((0, 0, 0, 0), (0, 0, 0))
	Availability	((0, 0, 0, 0), (-0.228, 0.169, 0.836))
	Maintenance	((0, 0, 0, 0), (0, 0, 0))
A3	Reliability	((0, 0, 0, 0), (0, 0, 0))
	Technical characteristics	((-0.321, 0.077, 0.077, 0.719), (-0.152, 0.141, 0.547))
	Performance	((0.066, 0.634, 0.634, 1.381), (-0.196, 0.145, 0.652))
	Cost factors	((0, 0, 0, 0), (0, 0, 0))
	Availability	((0, 0, 0, 0), (0, 0, 0))
	Maintenance	((-0.392, 0.029, 0.029, 0.69), (-0.311, 0.001, 0.45))
A4	Reliability	((-0.239, 0.314, 0.314, 1.285), (0, 0, 0))
	Technical characteristics	((0.005, 0.366, 0.366, 0.844), (0, 0, 0))
	Performance	((-0.339, 0.193, 0.193, 1.041), (0, 0, 0))
	Cost factors	((0.389, 0.883, 0.883, 1.465), (0, 0, 0))
	Availability	((0.127, 0.639, 0.639, 1.354), (0, 0, 0))
	Maintenance	((-0.392, 0.029, 0.029, 0.69), (-0.155, 0.207, 0.759))
A5	Reliability	((0, 0, 0, 0), (-0.127, 0.203, 0.684))
	Technical characteristics	((-0.075, 0.318, 0.318, 0.844), (0, 0, 0))
	Performance	((0, 0, 0, 0), (-0.159, 0.191, 0.711))
	Cost factors	((0, 0, 0, 0), (-0.162, 0.237, 0.869))
	Availability	((-0.426, 0.062, 0.062, 0.948), (-0.085, 0.338, 1.054))
	Maintenance	((0, 0, 0, 0), (-0.311, 0.001, 0.45))

Table 4.16 : Z-fuzzy $\widetilde{NDA}$ values.

	Criteria	Z-fuzzy $\widetilde{NDA}$ values
A1	Reliability	((-0.387, 0.191, 0.191, 1.307), (-0.475, -0.203, 0.183))
	Technical characteristics	((0.261, 0.723, 0.723, 1.351), (-0.353, -0.092, 0.269))
	Performance	((0, 0, 0, 0), (0, 0, 0))
	Cost factors	((-0.41, 0.005, 0.005, 0.653), (-0.472, -0.108, 0.431))
	Availability	((0, 0, 0, 0), (0, 0, 0))
	Maintenance	((0, 0, 0, 0), (0, 0, 0))
A2	Reliability	((0, 0, 0, 0), (0, 0, 0))
	Technical characteristics	((-0.383, 0.038, 0.038, 0.648), (-0.117, 0.185, 0.602))
	Performance	((-0.073, 0.568, 0.568, 1.428), (0, 0.391, 0.983))
	Cost factors	((-0.048, 0.377, 0.377, 0.928), (-0.329, 0.011, 0.549))
	Availability	((0.295, 0.865, 0.865, 1.635), (-0.513, -0.169, 0.372))
	Maintenance	((-0.011, 0.377, 0.377, 0.889), (-0.192, 0.133, 0.614))
A3	Reliability	((-0.501, 0.016, 0.016, 1.1), (-0.042, 0.323, 0.867))
	Technical characteristics	((0, 0, 0, 0), (0, 0, 0))
	Performance	((0, 0, 0, 0), (0, 0, 0))
	Cost factors	((-0.048, 0.377, 0.377, 0.928), (-0.163, 0.21, 0.803))
	Availability	((-0.381, 0.129, 0.129, 1.079), (-0.072, 0.389, 1.15))
	Maintenance	((0, 0, 0, 0), (0, 0, 0))
A4	Reliability	((0, 0, 0, 0), (0, 0, 0))
	Technical characteristics	((0, 0, 0, 0), (0, 0, 0))
	Performance	((0, 0, 0, 0), (0, 0, 0))
	Cost factors	((0, 0, 0, 0), (0, 0, 0))
	Availability	((0, 0, 0, 0), (0, 0, 0))
	Maintenance	((0, 0, 0, 0), (0, 0, 0))
A5	Reliability	((-0.11, 0.577, 0.577, 1.647), (-0.475, -0.203, 0.183))
	Technical characteristics	((0, 0, 0, 0), (0, 0, 0))
	Performance	((0.202, 0.762, 0.762, 1.529), (-0.482, -0.191, 0.234))
	Cost factors	((-0.3, 0.124, 0.124, 0.797), (-0.562, -0.237, 0.25))
	Availability	((0, 0, 0, 0), (0, 0, 0))
	Maintenance	((-0.091, 0.33, 0.33, 0.889), (-0.309, -0.001, 0.453))

In Step 6, the criteria weights obtained in Section 4.3 by using Z-fuzzy AHP method are employed to find $\widetilde{SP}_i$ and $\widetilde{SN}_i$ values. They are given in Tables 4.17-4.18, respectively.

Table 4.17 : $\widetilde{SP}$ values for each alternative.

	Z-fuzzy $\widetilde{SP}$ values
A1	((-0.02, 0.115, 0.115, 0.314), (-0.176, 0.132, 0.601))
A2	((-0.035, 0.166, 0.166, 0.525), (-0.017, 0.013, 0.062))
A3	((-0.028, 0.08, 0.08, 0.233), (-0.046, 0.024, 0.126))
A4	((0.003, 0.513, 0.513, 1.274), (-0.008, 0.011, 0.04))
A5	((-0.035, 0.019, 0.019, 0.11), (-0.144, 0.204, 0.737))

Table 4.18 : $\widetilde{SN}$ values for each alternative.

	Z-fuzzy $\widetilde{SN}$ values
A1	((-0.27, 0.103, 0.103, 0.756), (-0.352, -0.114, 0.23))
A2	((-0.022, 0.287, 0.287, 0.697), (-0.171, 0.053, 0.399))
A3	((-0.222, 0.149, 0.149, 0.799), (-0.078, 0.218, 0.677))
A4	((0, 0, 0, 0), (0, 0, 0))
A5	((-0.127, 0.355, 0.355, 1.093), (-0.441, -0.179, 0.205))

In Step 7, $\widetilde{SSP}_i$ and $\widetilde{SSN}_i$ values are calculated by Eqs. (4.37-4.40) and presented in Tables 4.19 and 4.20, respectively.

Table 4.19 : $\widetilde{SSP}$ values for each alternative.

	Z-fuzzy $\widetilde{SSP}$ values
A1	((0.015, 0.150, 0.150, 0.349), (0, 0.308, 0.777))
A2	((0, 0.201, 0.201, 0.560), (0.159, 0.189, 0.238))
A3	((0.007, 0.115, 0.115, 0.268), (0.130, 0.200, 0.302))
A4	((0.038, 0.549, 0.549, 1.309), (0.168, 0.187, 0.216))
A5	((0, 0.055, 0.055, 0.145), (0.032, 0.380, 0.913))

Table 4.20 : $\widetilde{SSN}$ values for each alternative.

	Z-fuzzy $\widetilde{SSN}$ values
A1	((0, 0.373, 0.373, 1.027), (0.089, 0.326, 0.671))
A2	((0.248, 0.557, 0.557, 0.967), (0.270, 0.494, 0.840))
A3	((0.048, 0.419, 0.419, 1.069), (0.363, 0.658, 1.118))
A4	((0.270, 0.270, 0.270, 0.270), (0.441, 0.441, 0.441))
A5	((0.144, 0.626, 0.626, 1.363), (0, 0.262, 0.646))

In Step 8, Z-fuzzy $\widetilde{SSP}_i$ and $\widetilde{SSN}_i$ values are normalized for both restriction and reliability functions separately by using Eqs. (4.41-4.44). The obtained $\widetilde{NSP}_i$ and $\widetilde{NSN}_i$ values are given in Tables 4.21-4.22, respectively.

Table 4.21 : $\widetilde{NSP}$ values for each alternative.

	Z-fuzzy $\widetilde{NSP}$ values
A1	((0.012, 0.115, 0.115, 0.267), (0, 0.337, 0.851))
A2	((0, 0.154, 0.154, 0.428), (0.174, 0.207, 0.261))
A3	((0.006, 0.088, 0.088, 0.205), (0.142, 0.219, 0.331))
A4	((0.029, 0.419, 0.419, 1), (0.184, 0.205, 0.237))
A5	((0, 0.042, 0.042, 0.11), (0.035, 0.416, 1))

Table 4.22 : $\widetilde{NSN}$ values for each alternative.

	Z-fuzzy $\widetilde{NSN}$ values
A1	((0.247, 0.726, 0.726, 1), (0.4, 0.708, 0.921))
A2	((0.291, 0.591, 0.591, 0.818), (0.249, 0.558, 0.758))
A3	((0.216, 0.692, 0.692, 0.965), (0, 0.411, 0.675))
A4	((0.802, 0.802, 0.802, 0.802), (0.606, 0.606, 0.606))
A5	((0, 0.541, 0.541, 0.895), (0.422, 0.766, 1))

In Step 9, Z-fuzzy $\widetilde{AS}_i$ values for all alternatives are calculated by Eqs. (4.45-4.46) and obtained values are given in Table 4.23.

Table 4.23 : $\widetilde{AS}_i$ values for each alternative

	Z-fuzzy $\widetilde{AS}_i$ values
A1	((0.129, 0.421, 0.421, 0.633), (0.200, 0.522, 0.886))
A2	((0.145, 0.373, 0.373, 0.623), (0.211, 0.382, 0.51))
A3	((0.111, 0.390, 0.390, 0.585), (0.071, 0.315, 0.503))
A4	((0.415, 0.610, 0.610, 0.901), (0.395, 0.405, 0.421))
A5	((0, 0.291, 0.291, 0.503), (0.229, 0.591, 1))

In Step 10, Z-fuzzy $\widetilde{AS}_i$ values are converted to ordinary fuzzy numbers using Definition 4.2. The obtained trapezoidal fuzzy numbers are shown in Table 4.24.

Table 4.24 : Trapezoidal fuzzy $\widetilde{AS}_i$ values converted from Z-fuzzy $\widetilde{AS}_i$.

	Trapezoidal fuzzy $\widetilde{AS}_i$ values of alternatives
A1	(0.094, 0.307, 0.307, 0.462)
A2	(0.089, 0.227, 0.227, 0.380)
A3	(0.061, 0.214, 0.214, 0.321)
A4	(0.265, 0.389, 0.389, 0.574)
A5	(0, 0.226, 0.226, 0.39)

In Step 11, trapezoidal fuzzy $\widetilde{AS}_i$ values are transformed to crisp numbers using Eq. (4.2). In Step 12, alternatives are ranked according to the decreasing values of crisp AS_i. Crisp AS_i values and ranking of the alternatives are presented in Table 4.25. A1 which has the highest AS_i is the best choice among five alternatives. Based on the computed AS_i values, the ranking of the alternatives is A4>A1>A2>A5>A3. These results show that alternative A4 is the best choice among the wind turbine alternatives according to the determined criteria.

Table 4.25 : Crisp AS_i values.

Alternative	Crisp AS_i
A1	0.2926
A2	0.2306
A3	0.2024
A4	0.4044
A5	0.2106

In order to investigate the importance of reliability information, the reliability judgments regarding all DMs' evaluations have been accepted as "*certainly reliable*" when applying the Z-fuzzy EDAS method without changing the criteria weights. Then, Z-fuzzy EDAS method has been re-applied. The obtained AS_i values are presented in the Table 4.26.

Table 4.26 : Crisp AS_i values (DMs' reliability judgments accepted as (1,1,1)).

Alternative	Crisp AS_i
A1	0.2431
A2	0.2751
A3	0.3071
A4	0.5332
A5	0.2220

According to these results, when the reliability information is neglected (accepted as (1,1,1) for all evaluations), the ranking of all alternatives except for the alternatives A4 and A2 has changed. A4 alternative has been found as the best alternative again. Although the best alternative does not change, this difference shows that the reliability information should not be neglected. The fact that the ranking of the best alternative (A4) remains the same can be interpreted as the DMs stated their restriction judgments quite dominantly when comparing the alternative A4 with the other alternatives.

Similarly, while the Z-fuzzy AHP method has been applied to find the criteria weights, the reliability information has been accepted as "*certainly reliable*", and the criteria weights have been recalculated. The obtained criteria weights are presented in Table 4.27.

Table 4.27 : Criteria weights obtained by Z-fuzzy AHP method (DMs' reliability judgements accepted as (1,1,1)).

Reliability	Technical char.	Performance	Cost factors	Availability	Maintenance
0.396	0.049	0.119	0.328	0.066	0.042

Table 4.27 shows that the ranking of cost factor and reliability factor, which are in the first two rankings, have changed when compared to previous results (Table 4.9). Among the six criteria, only the rankings of the *performance* and *availability* factors have not changed. These results support the obtained result regarding the importance of reliability information as in the EDAS method.

4.6 Comparative Analysis

To compare the results, the Z-fuzzy TOPSIS methodology proposed by Yaakob and Gegov (2016) is used. Z-fuzzy TOPSIS is one of the first fuzzy extensions which is performed by Z-fuzzy numbers in MCDM methodology. TOPSIS method was developed by Yoon and Hwang (1981). It is one of the most commonly used MCDM methodology by researchers in the literature. TOPSIS method allows to reach the solution by using the distances of the alternatives from the positive and negative ideal solutions.

Z-fuzzy TOPSIS methodology consists of the following steps; (i) construction of Z-fuzzy decision matrix, (ii) conversion of Z-fuzzy numbers to ordinary fuzzy numbers, (iii) normalization procedure, (iv) weighting the normalized decision matrix, (v) calculation of distances from positive and negative ideal solutions, and (vi) calculation of closeness coefficients (Yaakob and Gegov, 2016).

Table 4.28 presents the results of Z-fuzzy AHP&TOPSIS methodology and it shows the distances from positive and negative ideal solutions (d^* and d^-), and closeness coefficients (CC*), respectively. Based on the computed CC* values, the ranking of the alternatives is obtained as A4>A1>A2>A3>A5.

Table 4.28 : Results of Z-fuzzy TOPSIS methodology.

	d^*	d^-	CC*
A1	5.5530	1.4140	0.2030
A2	5.5551	1.3976	0.2010
A3	5.5572	1.3945	0.2006
A4	5.4034	1.5714	0.2253
A5	5.6267	1.3523	0.1938

According to the results obtained by the Z-fuzzy TOPSIS method, the ranking of the alternatives except alternatives 3 and 5 is the same as the methodology proposed in this study. The comparison of the rankings can be seen in Table 4.29.

Table 4.29 : Comparison of Z-fuzzy EDAS and Z-fuzzy TOPSIS.

Alternatives	Ranking of Z-fuzzy EDAS	Ranking of Z-fuzzy TOPSIS
A1	2	2
A2	3	3
A3	5	4
A4	1	1
A5	4	5

EDAS method considers the positive and negative distances from the average solution rather than calculating the negative and positive ideal solutions as in TOPSIS method. According to the results of both methods, the closeness coefficients in Z-fuzzy TOPSIS are composed of quite closer values whereas appraisal scores in Z-fuzzy EDAS indicate larger differences between alternatives. In general, it can be concluded that the proposed method is consistent since the rankings of two methods are quite similar. The only difference is between alternatives A3 and A5. The first three best alternatives are the same in both methods.

As a result of the comparative analysis, obtaining similar results with the Z-fuzzy TOPSIS method shows the consistency and competitiveness of the proposed method.

4.7 Conclusion

Extensions of ordinary fuzzy sets are quite successful in modeling the uncertainty in the decision-making process. However, they do not exactly represent the reliability information inherent in the solutions. The reliability information of the evaluations is very important as it can have significant impacts on the obtained results. The Z-fuzzy numbers introduced by Zadeh (2011) allow the reliability of the DMs' judgments to be included in the decision models. In this study, a novel Z-fuzzy EDAS method is introduced to the literature. Then, an integrated usage of Z-fuzzy AHP and Z-fuzzy EDAS method is proposed to the field for the first time to deal with uncertain expressions of DMs in real life decision making problems. The inclusion of the reliability information of the DMs in the decision model makes the decision making process more realistic in both daily and business decisions as in the case of renewable energy investment decisions.

The importance of renewable energy sources has increased considerably with the concern of leaving a sustainable world to future generations in recent years. In this study, the selection of a suitable wind turbine problem has been handled by considering the multiple factors affecting the decision. Criteria weights to be used in alternative selection have been calculated by using Z-fuzzy AHP method which has also been integrated to Z-fuzzy EDAS method. Z-fuzzy numbers integrated AHP method offers a more realistic solution by reflecting the DMs' hesitancy in pairwise comparisons to the proposed Z-fuzzy AHP&EDAS methodology. After defining the criteria weights, three DMs have evaluated the five alternatives using Z-fuzzy EDAS method. All the DMs' evaluations have been expressed by Z-fuzzy numbers in both methods, and all steps of the Z-fuzzy EDAS method have been performed by Z-fuzzy numbers. The proposed methodology allows DMs to express both restriction and reliability information about criteria and alternatives. In order to show the effects of reliability component on the decision system, the reliability information of all evaluations have been made "*certainly reliable*" and the calculations have been re-performed, then the results have compared with the proposed method. It is concluded from this analysis that the difference in the ranking results displays the importance of consideration of the reliability information. Therefore, the proposed methodology offers a more reliable evaluation system to DMs, including their degree of confidence to their assessments.

In order to show the robustness and stability of the proposed method, the obtained results have been compared with the results of the Z-Fuzzy AHP&TOPSIS methodology. It can be stated that the suggested methodology is an effective and useful method for researchers who want to decide based on distances from average solution rather than the distance from positive and negative ideal solutions. For further research, other MCDM approaches integrated with Z-fuzzy numbers can be used and compared with the results of this paper.

Although there are many fuzzy versions of the AHP method in the literature, its integration with Z-fuzzy numbers is limited. This research gap in the literature can be filled with the more application of Z-fuzzy AHP method, then importance and advantages of Z-fuzzy numbers can be further analyzed. In addition, other fuzzy set extensions such as fermatean fuzzy sets or picture fuzzy sets can be used in the improvement of Z-fuzzy numbers. Then, in future research, it can be suggested to combine these extensions of Z-fuzzy numbers with different MCDM methods to expand the related literature.

5. A NOVEL DECOMPOSED Z-FUZZY TOPSIS METHOD WITH FUNCTIONAL AND DYSFUNCTIONAL JUDGMENTS: AN APPLICATION TO TRANSFER CENTER LOCATION SELECTION[4]

Decision making can be defined as the process of choosing the most preferable option among the multiple alternatives. Decision-makers or experts use multi-criteria decision making (MCDM) approaches to assess the overall performance of all alternatives with respect to the criteria set of the problem. In this process, although the decision makers have knowledge and experience, it is inevitable that the statements they give about the problem contain uncertainty and vagueness. Since classical sets are insufficient to capture the uncertainty in these linguistic expressions, Zadeh (1965) introduced the fuzzy set theory to model the impreciseness and vagueness in them (Ren et al., 2020; Akram et al., 2023). After the introduction of ordinary fuzzy sets, several extensions (see Table 5.1) have been proposed and commonly used with MCDM methods to both inclusively model linguistic expressions and obtain more reasonable solutions. These fuzzy sets have some different conditions in which the membership degrees of an element to a set are defined as in Table 5.1. Although the increasing number of parameters and difficulties of operations may cause complexity in terms of their applicability, they are important for finding a better representation of subjectivity in human thoughts. Human's complex decision-making mechanisms can be better represented by fuzzy set extensions that contain more parameters, each representing a separate dimension of human thoughts.

However, these circumstances are not sufficient to provide general information for modeling natural language since they cannot present the reliability information, which has a significant impact on the final result in the decision-making process.

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[4] This chapter is based on the paper "Tüysüz, N., & Kahraman, C. (2024). A novel decomposed Z-fuzzy TOPSIS method with functional and dysfunctional judgments: An application to transfer center location selection. *Engineering Applications of Artificial Intelligence*, 127, 107221."

Table 5.1 : Extensions of ordinary fuzzy sets

Fuzzy sets defined by		
Membership functions (μ)	**Membership (μ) and non-membership (ϑ) functions**	**Membership (μ), non-membership (ϑ) and hesitancy (π) functions**
Ordinary fuzzy sets (Zadeh, 1965) $(0 \leq \mu \leq 1)$	Intuitionistic Fuzzy Sets (Atanassov, 1986) $(0 \leq \mu + \vartheta \leq 1)$	Neutrosophic Fuzzy Sets (Smarandache, 1999) $(0 \leq \mu + \vartheta + \pi \leq 3)$
Type-2 Fuzzy Sets (Zadeh, 1975) $(0 \leq \mu \leq 1)$	Pythagorean Fuzzy Sets (Yager, 2013) $(0 \leq \mu^2 + \vartheta^2 \leq 1)$	Picture Fuzzy Sets (Cuong, 2014) $(0 \leq \mu + \vartheta + \pi \leq 1)$
Interval valued fuzzy sets (Zadeh, 1975; Sambuc, 1975; Jahn, 1975; Grattan-Guinness, 1976) $(0 \leq \mu \leq 1)$	Fermatean Fuzzy Sets (Senapati and Yager, 2019) $(0 \leq \mu^3 + \vartheta^3 \leq 1)$	Spherical Fuzzy Sets (Kahraman and Kutlu Gündoğdu, 2018) $(0 \leq \mu^2 + \vartheta^2 + \pi^2 \leq 1)$
Fuzzy Multisets (Yager, 1986) $(0 \leq \mu \leq 1)$	q-Rung Orthopair Fuzzy Sets (Yager, 2016) $(0 \leq \mu^q + \vartheta^q \leq 1)$	t-Spherical Fuzzy Sets (Mahmood et al., 2019) $(0 \leq \mu^t + \vartheta^t + \pi^t \leq 1)$
Nonstationary Fuzzy Sets (Garibaldi and Ozen, 2007) $(0 \leq \mu \leq 1)$	Circular Intuitionistic Fuzzy Sets with a radius *r* (Atanassov, 2020) $(0 \leq \mu + \vartheta \leq 1)$	
Hesitant fuzzy sets (Torra, 2010) $(0 \leq \mu \leq 1)$	Decomposed Fuzzy Sets (Cebi et al., 2022) $(0 \leq \mu^{\mathcal{O}} + \vartheta^{\mathcal{O}} + \mu^{\mathcal{P}} + \vartheta^{\mathcal{P}} \leq 2)$	

Decomposed fuzzy sets (DFSs) proposed by Cebi et al. (2022) are one of the latest extensions of intuitionistic fuzzy sets, and they consider functional and dysfunctional points of view in order to check decision makers' consistency in their judgments, which are different from existing extensions of fuzzy sets. DFSs suggest to collect the decision makers' judgments with both positive and negative questions rather than collecting them with unidirectional (only positive) questions. Functional and dysfunctional sets in DFSs represent these judgments through positive and negative questions. Thus, DFSs define decision-makers' expressions not only from positive but also from negative perspectives. DFSs, like other fuzzy set extensions, do not cover reliability information in the evaluation. For this purpose, it is aimed to integrate Z-fuzzy numbers and DFSs in this paper. Thus, multi-criteria decision problems will be solved with reliable and consistent judgments. Decomposed Z-fuzzy numbers allow us to make evaluations through functional and dysfunctional questions and to express the reliability degree of evaluations in a positive and negative circumstances.

With the introduction of Z-fuzzy numbers by Zadeh (2011), it has been possible to represent an object's membership to a set together with its reliability degree. DMs can give their opinions in a form "*Z*= (*A*, *R*)" where *A* is a restrictive membership degree and *R* is the reliability degree of *A*. The DMs' reliability judgments (*R*) within Z-fuzzy numbers provide an adjunctional and complementary element to construct a systematic approach for decision making. Z-fuzzy numbers cover other fuzzy numbers by defining fuzzy reliability in addition to fuzzy restriction, and in fact, other fuzzy numbers are only fuzzy restrictions assumed that they are 100% reliable. Therefore, Z-fuzzy numbers are more sensible and useful than existing fuzzy numbers in cases where the certainty of fuzzy restriction is under 100%. The Z-fuzzy numbers have been progressively investigated in recent years, and they are employed in many MCDM methods. Combining Z-fuzzy numbers with MCDM methods such as VIKOR (Shen and Wang, 2018; Das et al., 2020), TOPSIS (Yaakob and Gegov, 2016; Xian et al., 2019; Rathore et al., 2021), CODAS (Tüysüz and Kahraman, 2020a) and AHP (Tüysüz and Kahraman, 2020b) investigates the effects of reliability information on decision making and reveals its advantages and necessity. In addition to these studies, new extensions of Z-fuzzy numbers have been introduced to the literature by integrating with different fuzzy set approaches such as neutrosophic Z-fuzzy numbers (Du et al., 2021), intuitionistic fuzzy Z-numbers (Sari and Kahraman, 2020), hesitant uncertain linguistic Z-numbers (Peng and Wang, 2017; Ren et al., 2020; Peng et al., 2021), orthopair Z-fuzzy numbers (Zhao and Ye, 2021), interval type-2 Z-fuzzy numbers (Sari and Tüysüz, 2022) and spherical Z-fuzzy numbers (Alkan and Kahraman, 2022). To enhance the representation capability of human judgments in DFSs and to enable the consideration of reliability information in MCDM problems under uncertainty, we propose a new decomposed Z-fuzzy number (DF Z-number), which can produce more comprehensive solutions since it combines the abilities of DFSs and Z-fuzzy numbers. The distinguishing feature of DF Z-numbers is their broader and multidirectional framework, which encompasses the reliability component in addition to the membership and non-membership degrees of the functional and dysfunctional points of view. The proposed DF Z-numbers dominate the ordinary or intuitionistic fuzzy numbers in the literature as they cover both the thinking structure of people and the reliability information about judgments, including satisfaction and dissatisfaction degrees.

Due to the reasons mentioned above, it can be suggested that decision-making approaches should incorporate the reliability information into the solution process. In this paper, we propose a novel decomposed Z-fuzzy TOPSIS (DF Z-TOPSIS) method using the developed DF Z-numbers. We also propose a DF Z-linguistic scale and a new defuzzification formula for DFSs to reach a crisp solution at the end. The proposed methodology transforms the DMs' opinions to interpretable and understandable values in the light of the developed defuzzification formula. The proposed DF Z-TOPSIS method has a more generalized structure than ordinary fuzzy TOPSIS methods in the literature as it possesses both functional and dysfunctional viewpoints and combines reliability information with membership and non-membership degrees. Figure 5.1 presents the general structure of the proposed method. Linguistic data are represented by DFSs and Z-numbers. Thus, the fuzzified linguistic data become the input for the proposed DF Z-TOPSIS method. The fuzzy outputs of the method are defuzzified to be able to rank the considered alternatives.

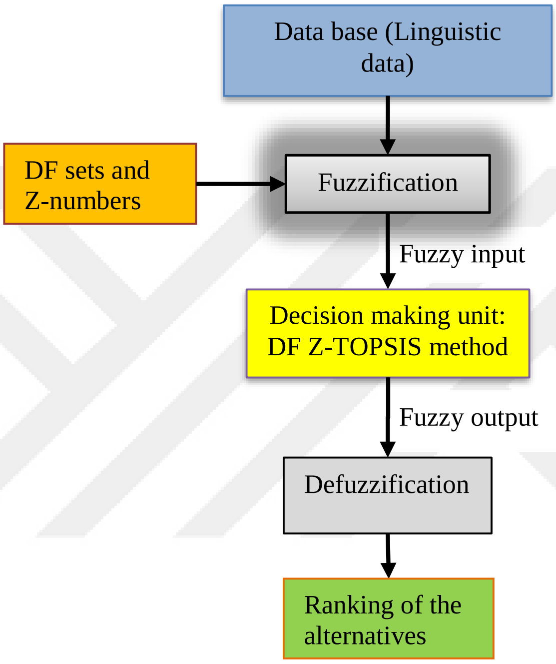

Figure 5.1 : General structure of proposed decision-making method.

There are four different contributions making this study original.

i. A new decomposed Z-fuzzy number is firstly proposed to the literature to represent the uncertain expressions more inclusively and with a multidirectional framework. A DF Z-number includes both the vague preferences and their reliability under functional and dysfunctional viewpoints.

ii. A novel DF Z-TOPSIS method is developed to reinforce the natural language representation capability of fuzzy MCDM models. It can effectively reflect the reliability degree of expressions, and provide more credible and inclusive results than the ordinary or intuitionistic fuzzy TOPSIS methods.

iii. An application for selecting the best transfer center location for a private cargo company is first time presented to measure the practicality, capability and efficiency of the proposed method.

iv. A new DF Z-linguistic scale for the DMs' evaluations and the new defuzzification formula are proposed to the literature. All evaluations in the problem are given by DF Z linguistic terms.

The organization of the paper is as follows. In Section 5.1, a literature review on Z-fuzzy TOPSIS method is presented. In Section 5.2, the preliminaries on ordinary Z-fuzzy numbers, decomposed fuzzy sets and decomposed Z-fuzzy numbers are presented. In Section 5.3, the proposed DF Z-TOPSIS method is presented step by step. In Section 5.4, an application on transfer center location selection is given to show the practicality of the proposed method. Finally, the results and the conclusions are discussed in the last section.

5.1 Literature Review on Z-Fuzzy TOPSIS Method

In order to investigate the effect of reliability information on decision methods, many Z-fuzzy MCDM studies have been developed in the literature. Figure 5.2 illustrates the Z-fuzzy MCDM methods or methodologies in the literature.

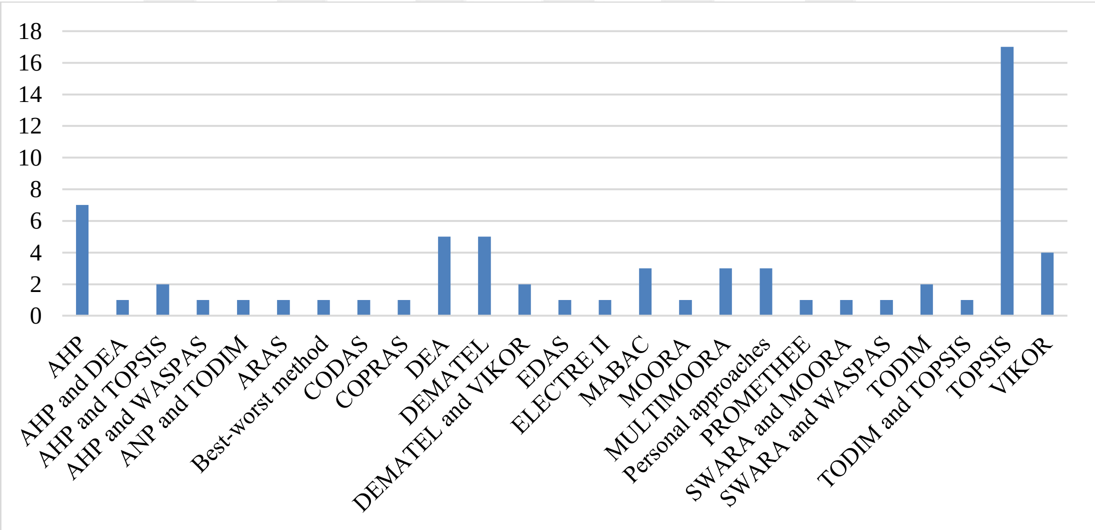

Figure 5.2 : Z-fuzzy MCDM methods or methodologies in the literature.

It can be stated that TOPSIS method is the most integrated MCDM method with Z-fuzzy numbers among other methods. Then, the AHP method follows it. The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method is first introduced by Yoon and Hwang (1981) for the solution of MCDM problems. It ranks the alternatives with respect to the distances from negative and positive ideal solutions. It is one of the most efficient decision making methods to solve MCDM problems (Xian et al., 2019; Haktanır and Kahraman, 2022). Table 5.2 presents the Z-fuzzy TOPSIS methods in the literature. Although there are many extensions of ordinary fuzzy sets in the literature, the integration of new Z-fuzzy extensions and TOPSIS method is limited, as it can be seen in Table 5.2.

Table 5.2 : Literature review on Z-fuzzy TOPSIS.

Year	Authors	Extension of TOPSIS	Application area
2016	Zamri et al.	Ordinary Z-fuzzy TOPSIS	Evaluation of the causes of accidents in the construction industry
2016	Yaakob and Gegov	Ordinary Z-fuzzy TOPSIS	Stock selection problem
2017	Ku Khalif et al.	Ordinary Z-fuzzy TOPSIS	Staff recruitment
2018	Forghani et al.	Ordinary Z-fuzzy TOPSIS	Supplier selection
2019	Gardashova	Ordinary Z-fuzzy TOPSIS	Vehicle selection
2019	Krohling et al.	Ordinary Z-fuzzy TOPSIS	Two case studies (vehicle choice and clothing evaluation)
2019	Zamri and Ibrahim	Ordinary Z-fuzzy TOPSIS	Numerical example
2019	Xian et al.	TOPSIS under intuitionistic Z-linguistic terms	Numerical examples on investment and medical diagnosis
2019	Wang and Mao	Ordinary Z-fuzzy TOPSIS	Supplier selection
2019	Ahmad et al.	Ordinary Z-fuzzy TOPSIS	Supplier selection
2020	Peng et al.	Ordinary Z-fuzzy TOPSIS	Nuclear power plant location selection
2020	Tao and Xiao	TOPSIS under Z-linguistic terms	Supplier selection
2021	Liu et al.	Ordinary Z-fuzzy TOPSIS	Conceptual design evaluation
2021	Rathore et al.	Ordinary Z-fuzzy TOPSIS	Ranking of renewable energy sources
2022	Sari and Tüysüz	Interval-valued type-2 Z-fuzzy TOPSIS	Risk evaluation of occupations
2022	Cheng et al.	Ordinary Z-fuzzy TOPSIS	Supplier selection
2022	Haktanır and Kahraman	Intuitionistic Z-fuzzy TOPSIS	Hydrogen storage technology selection

It can be deduced from Table 5.2 that ordinary Z-fuzzy numbers are the most preferred concept for the integration of TOPSIS method with Z-fuzzy numbers. In addition, there are only few methods proposed to integrate other Z-fuzzy extensions with TOPSIS method, which can be seen from Figure 5.3.

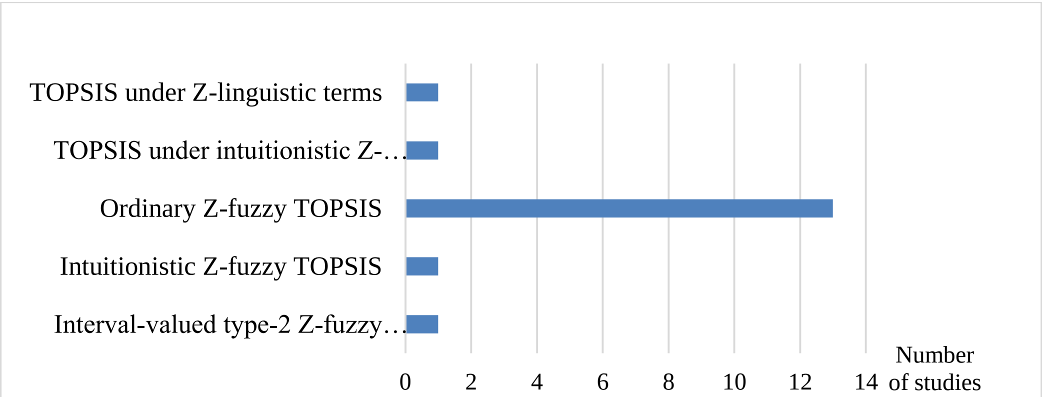

Figure 5.3 : Z-fuzzy extensions of TOPSIS method in the literature.

This literature review shows that the TOPSIS method is a very effective method based on the distances to positive and negative ideal solutions and it is very easy to implement and integrate with other approaches. Therefore, this research has been conducted on the basis of the TOPSIS method as a decision making tool.

The above literature review also shows that fuzzy extensions in the literature do not test the accuracy of the membership degrees to be assigned by decision makers. With functional and dysfunctional questions, it should be checked whether the decision maker can assign membership values consciously and correctly. The fuzzy set extension that handles this check is decomposed fuzzy sets introduced by Cebi et al (2022). Integrating DFSs that check consistency and Z-fuzzy numbers that measure the reliability of judgments will significantly increase the consistency and reliability of the MCDM methods.

5.2 Preliminaries

In this section, we firstly summarize some basic concepts related to the ordinary Z-fuzzy numbers and decomposed fuzzy sets. Then, we develop decomposed Z-fuzzy numbers in Section 5.2.3.

5.2.1 Ordinary Z-fuzzy numbers

Zadeh (2011) introduced the Z-fuzzy numbers to the literature as a generalization of other numbers. A Z-fuzzy number is an ordered pair of fuzzy numbers, $Z(\tilde{A}, \tilde{R})$ as given in Figure 5.4. The first component $\tilde{A}$ is a restriction function whereas the second component $\tilde{R}$ is a measure of reliability for the first component.

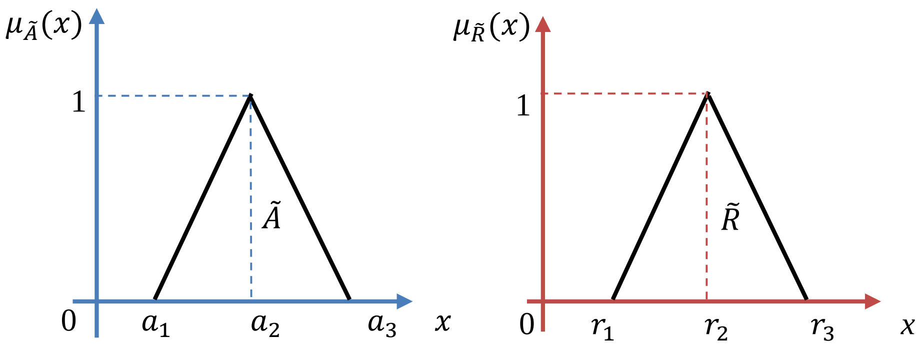

Figure 5.4 : A simple Z-fuzzy number, $Z(\tilde{A}, \tilde{R})$.

The concept of a Z-fuzzy number enables us to compute with ordinary fuzzy numbers together with their reliability information.

Definition 5.1: Converting Z-fuzzy number to regular fuzzy number (Kang et al., 2012a)

Consider a Z-fuzzy number $Z = (\widetilde{A}, \widetilde{R})$, which is described by Figure 5.4. The figure on the left is the part of restriction, and the figure on the right is the part of reliability. Let $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x)\rangle | \mu(x) \in [0,1]\}$ and $\tilde{R} = \{\langle x, \mu_{\tilde{R}}(x)\rangle | \mu(x) \in [0,1]\}$, $\mu_{\tilde{A}}(x)$ and $\mu_{\tilde{R}}(x)$ are the triangular membership functions. To convert the second part (reliability) into a crisp number, Eq. (5.1) is used. α is the center of gravity value of a fuzzy number.

$$\alpha = \frac{\int x\mu_{\tilde{R}}(x)\, dx}{\int \mu_{\tilde{R}}(x)\, dx} \tag{5.1}$$

where $\int$ denotes an algebraic integration.

Figure 5.5 illustrates the weighted restriction function by the reliability function.

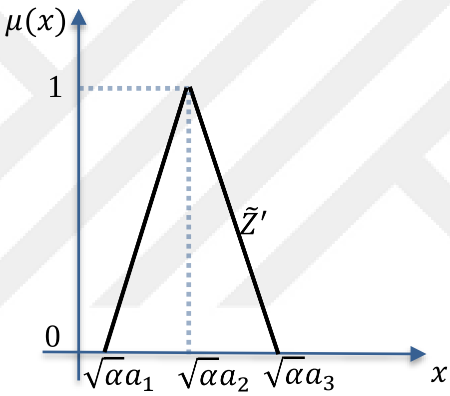

Figure 5.5 : Ordinary fuzzy number ($\tilde{Z}'$) converted from Z-fuzzy number.

5.2.2 Decomposed fuzzy sets

In this section, we present brief definitions for some basic concepts on Decomposed Fuzzy Sets (DFSs), which are proposed by Cebi et al. (2022).

Definition 5.2. Let X be a universe of discourse. A Decomposed Fuzzy Set (DFS) $\tilde{A}$ on X is defined as $\tilde{A} = \left\{\left\langle \mathrm{x}, \left(\mathcal{O}\left(\mu_{\tilde{A}}^{\mathcal{O}}(\mathrm{x}), \vartheta_{\tilde{A}}^{\mathcal{O}}(\mathrm{x})\right), \mathcal{P}\left(\mu_{\tilde{A}}^{\mathcal{P}}(\mathrm{x}), \vartheta_{\tilde{A}}^{\mathcal{P}}(\mathrm{x})\right)\right)\right| \mathrm{x} \in \mathrm{X}\right\}$, where the functions $\mu_{\tilde{A}}(\mathrm{x})\colon \mathrm{X} \to [0,1]$ and $\vartheta_{\tilde{A}}(\mathrm{x})\colon \mathrm{X} \to [0,1]$ denote the degrees of membership and non-membership of x to $\mathcal{O}$ (functional set) and $\mathcal{P}$ (dysfunctional set), respectively. DFSs satisfy the conditions $0 \le \mu_{\tilde{A}}^{\mathcal{O}}(\mathrm{x}) + \vartheta_{\tilde{A}}^{\mathcal{O}}(\mathrm{x}) \le 1$, $0 \le \mu_{\tilde{A}}^{\mathcal{P}}(\mathrm{x}) + \vartheta_{\tilde{A}}^{\mathcal{P}}(\mathrm{x}) \le 1$, and inconsistency in the judgment is $\mathcal{J}^{A} = 1 - \left(\mu_{\tilde{A}}^{\mathcal{O}}(\mathrm{x}) + \vartheta_{\tilde{A}}^{\mathcal{O}}(\mathrm{x}) + \mu_{\tilde{A}}^{\mathcal{P}}(\mathrm{x}) + \vartheta_{\tilde{A}}^{\mathcal{P}}(\mathrm{x})\right)$ where $-1 \le \mathcal{J}^{A} \le 1$ and $0 \le \mu_{\tilde{A}}^{\mathcal{O}}(\mathrm{x}) + \vartheta_{\tilde{A}}^{\mathcal{O}}(\mathrm{x}) + \mu_{\tilde{A}}^{\mathcal{P}}(\mathrm{x}) + \vartheta_{\tilde{A}}^{\mathcal{P}}(\mathrm{x}) \le 2$ (Cebi et al., 2022).

Definition 5.3. Decomposed fuzzy number (DFN): A DFN $\tilde{A}$ is defined as follows:

i) a decomposed fuzzy subset of the real line

ii) normal, i.e., there is any $x_0 \in R$ such that $\mu_{\tilde{A}}(x_0)$=1 (so $\vartheta_{\tilde{A}}(x_0)$=0)

iii) a convex set for the membership function

$$\left(\mathcal{O}\left(\mu_{\tilde{A}}^{\mathcal{O}}(\lambda x_1 + (1-\lambda)x_2)\right), \mathcal{P}\left(\mu_{\tilde{A}}^{\mathcal{P}}(\lambda x_1 + (1-\lambda)x_2)\right)\right) \geq$$

$$\min\left(\left(\mathcal{O}\left(\mu_{\tilde{A}}^{\mathcal{O}}(\mathrm{x}_1)\right), \mathcal{P}\left(\mu_{\tilde{A}}^{\mathcal{P}}(\mathrm{x}_1)\right)\right), \left(\mathcal{O}\left(\mu_{\tilde{A}}^{\mathcal{O}}(\mathrm{x}_2)\right), \mathcal{P}\left(\mu_{\tilde{A}}^{\mathcal{P}}(\mathrm{x}_2)\right)\right)\right) \ \forall x_1, x_2 \in R, \lambda \in [0,1]$$

iv) a concave set for the non-membership function

$$\left(\mathcal{O}\left(\vartheta_{\tilde{A}}^{\mathcal{O}}(\lambda x_1 + (1-\lambda)x_2)\right), \mathcal{P}\left(\vartheta_{\tilde{A}}^{\mathcal{O}}(\lambda x_1 + (1-\lambda)x_2)\right)\right) \leq$$

$$\max\left(\left(\mathcal{O}\left(\vartheta_{\tilde{A}}^{\mathcal{O}}(\mathrm{x}_1)\right), \mathcal{P}\left(\vartheta_{\tilde{A}}^{\mathcal{P}}(\mathrm{x}_1)\right)\right), \left(\mathcal{O}\left(\vartheta_{\tilde{A}}^{\mathcal{O}}(\mathrm{x}_2)\right), \mathcal{P}\left(\vartheta_{\tilde{A}}^{\mathcal{P}}(\mathrm{x}_2)\right)\right)\right) \ \forall x_1, x_2 \in R, \lambda \in [0,1]$$

For a detailed proof of the above conditions, Mitchell (2006) can be examined.

Definition 5.4. For two DFS $\tilde{A}$ and $\tilde{B}$, some set operations are defined by Eqs. (5.2-5.6) (Cebi et al., 2022):

The complement $\mathrm{c}(\tilde{A})$ of $\tilde{A}$ is defined by Eq. (5.2).

$$\mathrm{c}(\tilde{A}) = \bar{\tilde{A}} = \left\{x, \mathcal{P}^A\left(\vartheta_{\tilde{A}}^{\mathcal{P}}(\mathrm{x}), \mu_{\tilde{A}}^{\mathcal{P}}(\mathrm{x})\right), \mathcal{O}^A\left(\vartheta_{\tilde{A}}^{\mathcal{O}}(\mathrm{x}), \mu_{\tilde{A}}^{\mathcal{O}}(\mathrm{x})\right) | x \in X\right\} \tag{5.2}$$

The superset, equality, union and intersection operations are given by Eqs. (5.3-5.6), respectively.

$$\tilde{A} \subset \tilde{B} = \begin{Bmatrix} \forall x \in X, \mu_{\tilde{A}}^{\mathcal{O}}(x) < \mu_{\tilde{B}}^{\mathcal{O}}(x), \vartheta_{\tilde{A}}^{\mathcal{O}}(x) > \vartheta_{\tilde{B}}^{\mathcal{O}}(x), \\ \mu_{\tilde{A}}^{\mathcal{P}}(x) < \mu_{\tilde{B}}^{\mathcal{P}}(x), \vartheta_{\tilde{A}}^{\mathcal{P}}(x) > \vartheta_{\tilde{B}}^{\mathcal{P}}(x) \end{Bmatrix} \tag{5.3}$$

$$\tilde{A} = \tilde{B} \leftrightarrow \begin{Bmatrix} \forall x \in X, \mu_{\tilde{A}}^{\mathcal{O}}(x) = \mu_{\tilde{B}}^{\mathcal{O}}(x), \vartheta_{\tilde{A}}^{\mathcal{O}}(x) = \vartheta_{\tilde{B}}^{\mathcal{O}}(x), \\ \mu_{\tilde{A}}^{\mathcal{P}}(x) = \mu_{\tilde{B}}^{\mathcal{P}}(x), \vartheta_{\tilde{A}}^{\mathcal{P}}(x) = \vartheta_{\tilde{B}}^{\mathcal{P}}(x) \end{Bmatrix} \tag{5.4}$$

$$\tilde{A} \cup \tilde{B} = \begin{Bmatrix} \forall x \in X, max\left(\mu_{\tilde{A}}^{\mathcal{O}}(x), \mu_{\tilde{B}}^{\mathcal{O}}(x)\right), min\left(\vartheta_{\tilde{A}}^{\mathcal{O}}(x), \vartheta_{\tilde{B}}^{\mathcal{O}}(x)\right) \\ min\left(\mu_{\tilde{A}}^{\mathcal{P}}(x), \mu_{\tilde{B}}^{\mathcal{P}}(x)\right), max\left(\vartheta_{\tilde{A}}^{\mathcal{P}}(x), \vartheta_{\tilde{B}}^{\mathcal{P}}(x)\right) \end{Bmatrix} \tag{5.5}$$

$$\tilde{A} \cap \tilde{B} = \begin{Bmatrix} \forall x \in X, min\left(\mu_{\tilde{A}}^{\mathcal{O}}(x), \mu_{\tilde{B}}^{\mathcal{O}}(x)\right), max\left(\vartheta_{\tilde{A}}^{\mathcal{O}}(x), \vartheta_{\tilde{B}}^{\mathcal{O}}(x)\right) \\ max\left(\mu_{\tilde{A}}^{\mathcal{P}}(x), \mu_{\tilde{B}}^{\mathcal{P}}(x)\right), min\left(\vartheta_{\tilde{A}}^{\mathcal{P}}(x), \vartheta_{\tilde{B}}^{\mathcal{P}}(x)\right) \end{Bmatrix} \tag{5.6}$$

Definition 5.5. Let $\tilde{\alpha} = \{\mathcal{O}(a,b), \mathcal{P}(c,d)\}$ is a DFN, where $a = \mu_{\tilde{A}}^{\mathcal{O}}(\mathrm{x}), b = \vartheta_{\tilde{A}}^{\mathcal{O}}(\mathrm{x}), c = \mu_{\tilde{A}}^{\mathcal{P}}(\mathrm{x}), d = \vartheta_{\tilde{A}}^{\mathcal{P}}(\mathrm{x})$. Consider two DFNs $\tilde{\alpha}_1 = \{\mathcal{O}(a_1,b_1), \mathcal{P}(c_1,d_1)\}$ and

$\tilde{\alpha}_2 = \{\mathcal{O}(a_2, b_2), \mathcal{P}(c_2, d_2)\}$. Basic operations are given by Eqs. (5.7-5.10) (Cebi et al., 2022):

Addition:

$$\tilde{\alpha}_1 \oplus \tilde{\alpha}_2 =$$

$$\begin{cases} \left\{\mathcal{O}\left(\frac{a_1+a_2-2a_1a_2}{1-a_1a_2}, \frac{b_1b_2}{b_1+b_2-b_1b_2}\right), \mathcal{P}(c_1c_2, d_1+d_2-d_1d_2)\right\}, a_{\mathrm{i}}, b_i \in (0,1) \\ \{\mathcal{O}(1,0), \mathcal{P}(c_1c_2, d_1+d_2-d_1d_2)\},\ a_1 = a_2 = 1 \textit{ and } b_1 = b_2 = 0 \end{cases} \quad (5.7)$$

Multiplication by a scalar λ:

$$\lambda \cdot \tilde{\alpha} =$$

$$\left\{\mathcal{O}\left(\frac{\lambda a}{(\lambda-1)a+1}, \frac{b}{\lambda-(\lambda-1)b}\right), \mathcal{P}\left(c^{\lambda}, \left(1-(1-d)^{\lambda}\right)\right)\right\} \text{ for } \lambda > 0 \quad (5.8)$$

Multiplication:

$$\tilde{\alpha}_1 \otimes \tilde{\alpha}_2 =$$

$$\begin{cases} \left\{\mathcal{O}(a_1a_2,\ b_1+b_2-b_1b_2), \mathcal{P}\left(\frac{c_1+c_2-2c_1c_2}{1-c_1c_2}, \frac{d_1d_2}{d_1+d_2-d_1d_2}\right)\right\}, c_{\mathrm{i}}, d_i \in (0,1) \\ \{\mathcal{O}(a_1a_2,\ b_1+b_2-b_1b_2), \mathcal{P}(0,1)\}, c_1 = c_2 = 0 \textit{ and } d_1 = d_2 = 1 \end{cases} \quad (5.9)$$

λ^{th} power of $\tilde{\alpha}$: $\lambda > 0$

$$\tilde{\alpha}^{\lambda} =$$

$$\left\{\mathcal{O}\left(a^{\lambda}, \left(1-(1-b)^{\lambda}\right)\right), \mathcal{P}\left(\frac{c}{\lambda-(\lambda-1)c}, \frac{\lambda d}{(\lambda-1)d+1}\right)\right\} \text{ for } \lambda > 0 \quad (5.10)$$

Definition 5.6: Let $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ are two DFNs. Then some properties are defined by Eqs. (5.11-5.16) (Cebi et al., 2022).

$$\tilde{\alpha}_1 \oplus \tilde{\alpha}_2 = \tilde{\alpha}_2 \oplus \tilde{\alpha}_1 \quad (5.11)$$

$$\tilde{\alpha}_1 \otimes \tilde{\alpha}_2 = \tilde{\alpha}_2 \otimes \tilde{\alpha}_1 \quad (5.12)$$

$$\lambda(\tilde{\alpha}_1 \oplus \tilde{\alpha}_2) = \lambda \cdot \tilde{\alpha}_1 \oplus \lambda \cdot \tilde{\alpha}_2 \quad (5.13)$$

$$(\tilde{\alpha}_1 \otimes \tilde{\alpha}_2)^{\lambda} = \tilde{\alpha}_1^{\lambda} \otimes \tilde{\alpha}_2^{\lambda} \quad (5.14)$$

$$\lambda_1 \cdot \tilde{\alpha} \oplus \lambda_2 \cdot \tilde{\alpha} = (\lambda_1 + \lambda_2) \cdot \tilde{\alpha} \quad (5.15)$$

$$\tilde{\alpha}^{\lambda_1} \otimes \tilde{\alpha}^{\lambda_2} = \tilde{\alpha}^{\lambda_1+\lambda_2} \quad (5.16)$$

where $\lambda, \lambda_1, \lambda_2 \geq 0$.

Definition 5.7: Let $\tilde{\alpha}_i = \{\mathcal{O}(a_i, b_i), \mathcal{P}(c_i, d_i)\}$ be a collection of Decomposed Weighted Arithmetic Mean (DWAM) with respect to, $\lambda_i = (\lambda_1, \lambda_2 \ldots\ldots, \lambda_n); \lambda_i \in [0,1]$, and $\sum_{i=1}^{n} \lambda_i = 1$, DWAM is given by Eq. (5.17) (Cebi et al., 2022).

$$DWAM(\tilde{\alpha}_1, \tilde{\alpha}_2 \ldots\ldots, \tilde{\alpha}_n) = \lambda_1 \cdot \tilde{\alpha}_1 \oplus \lambda_2 \cdot \tilde{\alpha}_2 \oplus \ldots\ldots \oplus \lambda_n \cdot \tilde{\alpha}_n =$$

$$\begin{Bmatrix} \mathcal{O}\left(\frac{\sum_{i=1}^{n} \lambda_i a_i}{1+\sum_{i=1}^{n}\left(\lambda_i a_i - \frac{a_i}{n}\right)}, \frac{\prod_{i=1}^{n} b_i}{\sum_{i=1}^{n} b_i^{n-1} \lambda_i (1-b_i) + \prod_{i=1}^{n} b_i}\right), \\ \mathcal{P}\left(\left(1 - \prod_{i=1}^{n}(1-c_i)^{\lambda}\right), \prod_{i=1}^{n} d_i^{\lambda}\right) \end{Bmatrix} \tag{5.17}$$

Definition 5.8. Let $\tilde{\alpha}_i = \{\mathcal{O}(a_i, b_i), \mathcal{P}(c_i, d_i)\}$ be a collection of Decomposed Weighted Geometric Mean (DWGM) with respect to, $\lambda_i = (\lambda_1, \lambda_2 \ldots\ldots, \lambda_n); \lambda_i \in [0,1]$ and $\sum_{i=1}^{n} \lambda_i = 1$, DWGM is defined by Eq. (5.18) (Cebi et al., 2022).

$$DWGM(\tilde{\alpha}_1, \tilde{\alpha}_2 \ldots\ldots, \tilde{\alpha}_n) = \tilde{\alpha}_1^{\lambda_1} \otimes \tilde{\alpha}_2^{\lambda_2} \otimes \ldots\ldots \otimes \tilde{\alpha}_n^{\lambda_n} =$$

$$\begin{Bmatrix} \mathcal{O}\left(\prod_{i=1}^{n} a_i^{\lambda}, \left(1 - \prod_{i=1}^{n}(1-b_i)^{\lambda}\right)\right), \\ \mathcal{P}\left(\frac{\prod_{i=1}^{n} c_i}{\sum_{i=1}^{n} c_i^{n-1} \lambda_i (1-c_i) + \prod_{i=1}^{n} c_i}, \frac{\sum_{i=1}^{n} \lambda_i d_i}{1+\sum_{i=1}^{n}\left(\lambda_i d_i - \frac{d_i}{n}\right)}\right) \end{Bmatrix} \tag{5.18}$$

A novel defuzzification formula and a novel Euclidean distance for DFSs are proposed in Definitions 5.9 and 5.10, respectively.

Definition 5.9. Let $\tilde{A}$ be a DFN. Its defuzzification formula $Def_{\tilde{A}}$ is defined by Eq. (5.19):

$$Def_{\tilde{A}} = 0.5 + \sqrt{\frac{\mu_{\tilde{A}}^{\mathcal{O}}(x) + \vartheta_{\tilde{A}}^{\mathcal{P}}(x)}{8}} - \sqrt{\frac{\vartheta_{\tilde{A}}^{\mathcal{O}}(x) + \mu_{\tilde{A}}^{\mathcal{P}}(x)}{8}} \tag{5.19}$$

Definition 5.10. Let $\tilde{a} = \left\{\mathcal{O}\left(\mu_{\tilde{a}}^{\mathcal{O}}(x), \vartheta_{\tilde{a}}^{\mathcal{O}}(x)\right), \mathcal{P}\left(\mu_{\tilde{a}}^{\mathcal{P}}(x), \vartheta_{\tilde{a}}^{\mathcal{P}}(x)\right)\right\}$ and $\tilde{b} = \left\{\mathcal{O}\left(\mu_{\tilde{b}}^{\mathcal{O}}(x), \vartheta_{\tilde{b}}^{\mathcal{O}}(x)\right), \mathcal{P}\left(\mu_{\tilde{b}}^{\mathcal{P}}(x), \vartheta_{\tilde{b}}^{\mathcal{P}}(x)\right)\right\}$ be two DFNs and then Euclidean distance of $\tilde{a}$ and $\tilde{b}$ is given by Eq. (5.20):

$$E_{DF}(\tilde{a}, \tilde{b}) = \begin{Bmatrix} \mathcal{O}\left(\sqrt{\frac{1}{2n}\sum_{i=1}^{n}\left[\left(\left(\mu_{\tilde{a}}^{\mathcal{O}}(x) - \mu_{\tilde{b}}^{\mathcal{O}}(x)\right)^2 + \left(\vartheta_{\tilde{a}}^{\mathcal{O}}(x) - \vartheta_{\tilde{b}}^{\mathcal{O}}(x)\right)^2\right)\right]}\right), \\ \mathcal{P}\left(\sqrt{\frac{1}{2n}\sum_{i=1}^{n}\left[\left(\left(\mu_{\tilde{a}}^{\mathcal{P}}(x) - \mu_{\tilde{b}}^{\mathcal{P}}(x)\right)^2 + \left(\vartheta_{\tilde{a}}^{\mathcal{P}}(x) - \vartheta_{\tilde{b}}^{\mathcal{P}}(x)\right)^2\right)\right]}\right) \end{Bmatrix} \tag{5.20}$$

5.2.3 Decomposed Z-fuzzy numbers

Decomposed Z-fuzzy numbers are developed to define decomposed fuzzy restrictions with their decomposed fuzzy reliabilities. Collecting judgments with both their fuzzy restrictions and fuzzy reliabilities from decision makers based on positive (functional) and negative (dysfunctional) questions make them consistent and reliable judgments. Decomposed Z-fuzzy numbers can remove the inconsistent assignment of membership and non-membership degrees, which is the main motivation of this study. Although the computational complexity of the decomposed Z-fuzzy numbers makes the method difficult to extend and apply, they can reveal significant effects to find out more meaningful and more realistic results than ordinary fuzzy approaches.

Decomposed Z-fuzzy number $\tilde{Z}_{DF}(\tilde{A},\tilde{R})$ is an ordered pair fuzzy number in which its fuzzy restriction and fuzzy reliability functions are denoted by decomposed fuzzy numbers. Let the decomposed fuzzy restriction function $\tilde{A} = \left(\mathcal{O}\left(\mu_{\tilde{A}}^{\mathcal{O}}(x), \vartheta_{\tilde{A}}^{\mathcal{O}}(x)\right), \mathcal{P}\left(\mu_{\tilde{A}}^{\mathcal{P}}(x), \vartheta_{\tilde{A}}^{\mathcal{P}}(x)\right)\right)$ and the decomposed fuzzy reliability function $\tilde{R} = \left(\mathcal{O}\left(\mu_{\tilde{R}}^{\mathcal{O}}(x), \vartheta_{\tilde{R}}^{\mathcal{O}}(x)\right), \mathcal{P}\left(\mu_{\tilde{R}}^{\mathcal{P}}(x), \vartheta_{\tilde{R}}^{\mathcal{P}}(x)\right)\right)$, whose membership and non-membership degrees of $\mathcal{O}$ and $\mathcal{P}$ are represented by triangular fuzzy numbers. A general representation of decomposed Z-fuzzy number is given in Eq. (5.21).

$$\tilde{Z}_{DF}(\tilde{A},\tilde{R}) = \begin{pmatrix} \left(\mathcal{O}\left(\mu_{\tilde{A}}^{\mathcal{O}}(x), \vartheta_{\tilde{A}}^{\mathcal{O}}(x)\right), \mathcal{P}\left(\mu_{\tilde{A}}^{\mathcal{P}}(x), \vartheta_{\tilde{A}}^{\mathcal{P}}(x)\right)\right), \\ \left(\mathcal{O}\left(\mu_{\tilde{R}}^{\mathcal{O}}(x), \vartheta_{\tilde{R}}^{\mathcal{O}}(x)\right), \mathcal{P}\left(\mu_{\tilde{R}}^{\mathcal{P}}(x), \vartheta_{\tilde{R}}^{\mathcal{P}}(x)\right)\right) \end{pmatrix} \tag{5.21}$$

where $\mu_{\tilde{A}}(x): X \to [0,1], \vartheta_{\tilde{A}}(x): X \to [0,1]$ are the degrees of membership and non-membership of x to $\mathcal{O}$ and $\mathcal{P}$ for restriction function; $\mu_{\tilde{R}}(x): X \to [0,1], \vartheta_{\tilde{R}}(x): X \to [0,1]$ are the degrees of membership and non-membership of x to $\mathcal{O}$ and $\mathcal{P}$ for reliability function satisfying the conditions $0 \le \mu_{\tilde{A}}^{\mathcal{O}}(x) + \vartheta_{\tilde{A}}^{\mathcal{O}}(x) \le 1$, $0 \le \mu_{\tilde{A}}^{\mathcal{P}}(x) + \vartheta_{\tilde{A}}^{\mathcal{P}}(x) \le 1$, $0 \le \mu_{\tilde{R}}^{\mathcal{O}}(x) + \vartheta_{\tilde{R}}^{\mathcal{O}}(x) \le 1$ and $0 \le \mu_{\tilde{R}}^{\mathcal{P}}(x) + \vartheta_{\tilde{R}}^{\mathcal{P}}(x) \le 1$. Inconsistencies in the judgments are computed by $\mathcal{J}^{\tilde{A}} = 1 - \left(\mu_{\tilde{A}}^{\mathcal{O}}(x) + \vartheta_{\tilde{A}}^{\mathcal{O}}(x) + \mu_{\tilde{A}}^{\mathcal{P}}(x) + \vartheta_{\tilde{A}}^{\mathcal{P}}(x)\right)$ and $\mathcal{J}^{\tilde{R}} = 1 - \left(\mu_{\tilde{R}}^{\mathcal{O}}(x) + \vartheta_{\tilde{R}}^{\mathcal{O}}(x) + \mu_{\tilde{R}}^{\mathcal{P}}(x) + \vartheta_{\tilde{R}}^{\mathcal{P}}(x)\right)$, where $-1 \le \mathcal{J}^{\tilde{A}} \le 1$ and $1 \le \mathcal{J}^{\tilde{R}} \le 1$, $0 \le \mu_{\tilde{A}}^{\mathcal{O}}(x) + \vartheta_{\tilde{A}}^{\mathcal{O}}(x) + \mu_{\tilde{A}}^{\mathcal{P}}(x) + \vartheta_{\tilde{A}}^{\mathcal{P}}(x) \le 2$ and $0 \le \mu_{\tilde{R}}^{\mathcal{O}}(x) + \vartheta_{\tilde{R}}^{\mathcal{O}}(x) + \mu_{\tilde{R}}^{\mathcal{P}}(x) + \vartheta_{\tilde{R}}^{\mathcal{P}}(x) \le 2$.

A decomposed triangular Z-fuzzy number is represented in Figure 5.6.

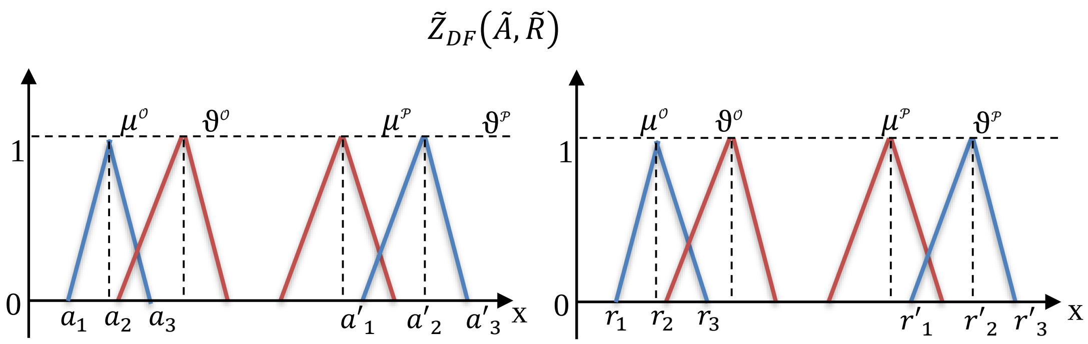

Figure 5.6 : A decomposed triangular Z-fuzzy number, $\tilde{Z}_{DF}(\tilde{A},\tilde{R})$.

Definition 5.11: Transformation from a decomposed Z-fuzzy number to a decomposed fuzzy number

A decomposed Z-fuzzy number $\tilde{Z}_{DF}(\tilde{A},\tilde{R})$ with triangular fuzzy membership degrees is given by Eq. (5.22):

$$\tilde{Z}_{DF}(\tilde{A},\tilde{R}) =$$

$$\left(\begin{array}{c}\left(\begin{array}{l}\mathcal{O}_{\tilde{A}}\left(\left(\mu^{\mathcal{O}}_{AL}(x),\mu^{\mathcal{O}}_{AM}(x),\mu^{\mathcal{O}}_{AU}(x)\right),\left(\vartheta^{\mathcal{O}}_{AL}(x),\vartheta^{\mathcal{O}}_{AM}(x),\vartheta^{\mathcal{O}}_{AU}(x)\right)\right),\\ \mathcal{P}_{\tilde{A}}\left(\left(\mu^{\mathcal{P}}_{AL}(x),\mu^{\mathcal{P}}_{AM}(x),\mu^{\mathcal{P}}_{AU}(x)\right),\left(\vartheta^{\mathcal{P}}_{AL}(x),\vartheta^{\mathcal{P}}_{AM}(x),\vartheta^{\mathcal{P}}_{AU}(x)\right)\right)\end{array}\right),\\ \left(\begin{array}{l}\mathcal{O}_{\tilde{R}}\left(\left(\mu^{\mathcal{O}}_{RL}(x),\mu^{\mathcal{O}}_{RM}(x),\mu^{\mathcal{O}}_{RU}(x)\right),\left(\vartheta^{\mathcal{O}}_{RL}(x),\vartheta^{\mathcal{O}}_{RM}(x),\vartheta^{\mathcal{O}}_{RU}(x)\right)\right),\\ \mathcal{P}_{\tilde{R}}\left(\left(\mu^{\mathcal{P}}_{RL}(x),\mu^{\mathcal{P}}_{RM}(x),\mu^{\mathcal{P}}_{RU}(x)\right),\left(\vartheta^{\mathcal{P}}_{RL}(x),\vartheta^{\mathcal{P}}_{RM}(x),\vartheta^{\mathcal{P}}_{RU}(x)\right)\right)\end{array}\right)\end{array}\right) \quad (5.22)$$

where $\mu_{AL}(x)$, $\mu_{AM}(x)$, $\mu_{AU}(x)$, $\vartheta_{AL}(x)$, $\vartheta_{AM}(x)$, and $\vartheta_{AU}(x)$ of the restriction function; and $\mu_{RL}(x)$, $\mu_{RM}(x)$, $\mu_{RU}(x)$, $\vartheta_{RL}(x)$, $\vartheta_{RM}(x)$, and $\vartheta_{RU}(x)$ of the reliability function represent the lower, middle, and upper values of the judgments for $\mathcal{O}$ and $\mathcal{P}$, respectively.

The reliability function ($\tilde{R}$) can be transformed into a crisp number (α) using Eq. (5.23), which is derived from Definition 5.9, and this crisp α is used for weighing restriction part ($\tilde{A}$) as in Eq. (5.24). Alternatively, to continue operations with DF reliability, $\tilde{R}$, DF restriction function is multiplied by the square root of DF reliability function ($\tilde{\alpha}$) as in Eq. (5.25).

$$Def_{\tilde{R}} = \alpha =$$

$$0.5 + \frac{\sqrt{\mu^{\mathcal{O}}_{RL}(x)+\mu^{\mathcal{O}}_{RU}(x)+\vartheta^{\mathcal{P}}_{RL}(x)+\vartheta^{\mathcal{P}}_{RU}(x)}}{4} - \frac{\sqrt{\vartheta^{\mathcal{O}}_{RL}(x)+\vartheta^{\mathcal{O}}_{RU}(x)+\mu^{\mathcal{P}}_{RL}(x)+\mu^{\mathcal{P}}_{RU}(x)}}{4} \quad (5.23)$$

$$\tilde{Z}_{DF}^{\alpha} = \{\langle x, \mu_{\tilde{A}^{\alpha}}(x)\rangle | \mu_{\tilde{A}^{\alpha}}(x) = \alpha\mu_{\tilde{A}}(x), \mu(x) \in [0,1]\} \quad (5.24)$$

$$\tilde{Z}_{DF}^{\tilde{\alpha}} = \{\langle x, \mu_{\tilde{A}^{\tilde{\alpha}}}(x)\rangle | \mu_{\tilde{A}^{\tilde{\alpha}}}(x) = \tilde{\alpha}\mu_{\tilde{A}}(x), \mu(x) \in [0,1]\} \quad (5.25)$$

The weighted restriction function is transformed to regular decomposed triangular fuzzy number $\tilde{Z}'_{DF}$ which is presented in Figure 5.7 using Eq. (5.26):

$$\tilde{Z}'_{DF} = \left\{\langle x, \mu_{\tilde{Z}'_{DF}}(x)\rangle \middle| \mu_{\tilde{Z}'_{DF}}(x) = \mu_{\tilde{A}}\left(\frac{x}{\sqrt{\alpha}}\right), \mu(x) \in [0,1]\right\} \quad (5.26)$$

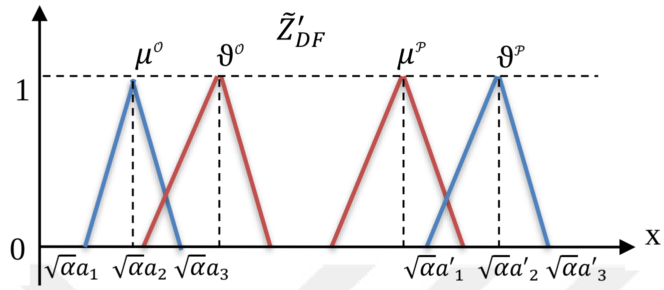

Figure 5.7 : Weighted decomposed triangular Z-fuzzy number.

5.3 Decomposed Z-Fuzzy TOPSIS

In this section, the TOPSIS method is extended to Decomposed Z-Fuzzy TOPSIS (DF Z-TOPSIS) method. Figure 5.8 presents the flowchart of the proposed method.

Step 1. Define the criteria set $C = \{C_1, C_2, C_3, \ldots, C_m\}$ and alternative set $A = \{A_1, A_2, A_3, \ldots A_n\}$. The decomposed Z-fuzzy decision matrix ($\tilde{D}_{DZF} = (\tilde{d}_{ij})_{n\times m}$) is constructed as in Eq. (5.27).

$$\tilde{D}_{DZF} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \ldots & C_m \end{array} \\ \begin{bmatrix} \tilde{d}_{11} & \tilde{d}_{12} & \ldots & \tilde{d}_{1m} \\ \tilde{d}_{21} & \tilde{d}_{22} & \ldots & \tilde{d}_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{d}_{n1} & \tilde{d}_{n2} & \ldots & \tilde{d}_{nm} \end{bmatrix} \end{array} \quad (5.27)$$

where $i = 1,2,3 \ldots, n$, $j = 1,2,3 \ldots, m$ and;

$\tilde{d}_{ij}$ is the DMs' decomposed Z-fuzzy assessments and defined in Eq. (5.28).

$$\tilde{d}_{ij} = (\tilde{A}, \tilde{R}) = \left(\begin{pmatrix} \mathcal{O}_{\tilde{A}}\left(\left(\mu^{\mathcal{O}}_{AL}(x), \mu^{\mathcal{O}}_{AM}(x), \mu^{\mathcal{O}}_{AU}(x)\right), \left(\vartheta^{\mathcal{O}}_{AL}(x), \vartheta^{\mathcal{O}}_{AM}(x), \vartheta^{\mathcal{O}}_{AU}(x)\right)\right), \\ \mathcal{P}_{\tilde{A}}\left(\left(\mu^{\mathcal{P}}_{AL}(x), \mu^{\mathcal{P}}_{AM}(x), \mu^{\mathcal{P}}_{AU}(x)\right), \left(\vartheta^{\mathcal{P}}_{AL}(x), \vartheta^{\mathcal{P}}_{AM}(x), \vartheta^{\mathcal{P}}_{AU}(x)\right)\right) \end{pmatrix}, \begin{pmatrix} \mathcal{O}_{\tilde{R}}\left(\left(\mu^{\mathcal{O}}_{RL}(x), \mu^{\mathcal{O}}_{RM}(x), \mu^{\mathcal{O}}_{RU}(x)\right), \left(\vartheta^{\mathcal{O}}_{RL}(x), \vartheta^{\mathcal{O}}_{RM}(x), \vartheta^{\mathcal{O}}_{RU}(x)\right)\right), \\ \mathcal{P}_{\tilde{R}}\left(\left(\mu^{\mathcal{P}}_{RL}(x), \mu^{\mathcal{P}}_{RM}(x), \mu^{\mathcal{P}}_{RU}(x)\right), \left(\vartheta^{\mathcal{P}}_{RL}(x), \vartheta^{\mathcal{P}}_{RM}(x), \vartheta^{\mathcal{P}}_{RU}(x)\right)\right) \end{pmatrix}\right)$$

(5.28)

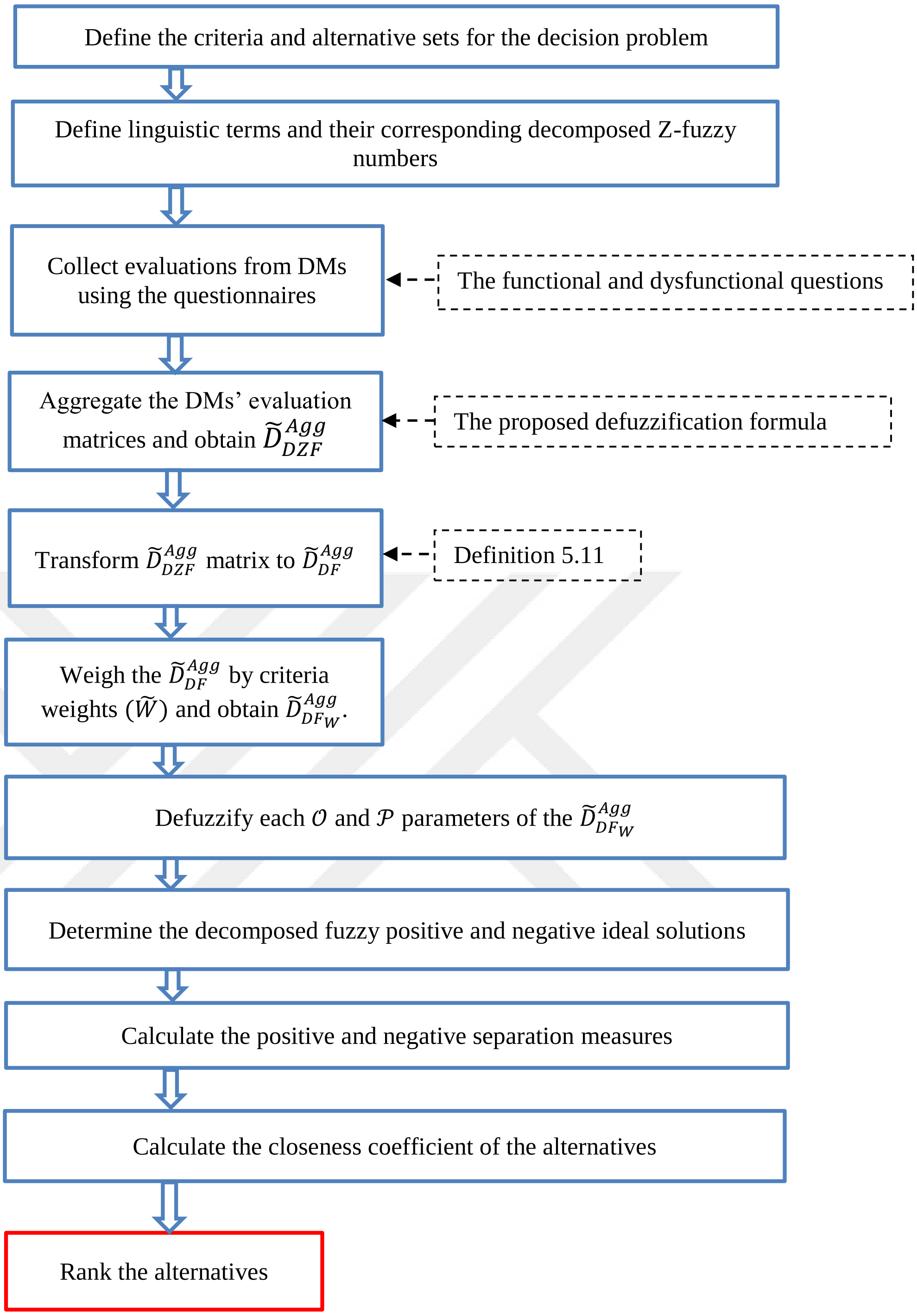

Figure 5.8 : Flowchart of the proposed DF Z-TOPSIS method.

Step 2. Collect the decision matrices from K DMs using the decomposed Z-fuzzy scales given in Table 5.3. For this purpose, the functional and dysfunctional questions are asked each DM.

Table 5.3 : Decomposed Z-fuzzy restriction and reliability scale.

Linguistic Terms for Restriction	Linguistic Terms for Reliability	μ	ϑ
Absolutely High (AH)	Absolutely High Reliable (AHR)	(0.7, 0.8, 0.9)	(0, 0.05, 0.1)
Very High (VH)	Very High Reliable (VHR)	(0.6, 0.7, 0.8)	(0, 0.1, 0.2)
High (H)	High Reliable (HR)	(0.5, 0.6, 0.7)	(0.1, 0.2, 0.3)
Medium (M)	Reliable (R)	(0.4, 0.5, 0.6)	(0.2, 0.3, 0.4)
Low (L)	Low Reliable (LR)	(0.3, 0.4, 0.5)	(0.3, 0.4, 0.5)
Very Low (VL)	Very Low Reliable (VLR)	(0.2, 0.3, 0.4)	(0.4, 0.5, 0.6)
Absolutely Low (AL)	Absolutely Low Reliable (ALR)	(0.1, 0.2, 0.3)	(0.5, 0.55, 0.6)

Step 3. Aggregate the K decision matrices $\widetilde{D}_{DZF}$s of DMs using DWAM or DWGM operators given in Eqs. (5.17) and (5.18), respectively. In this study, we use DWGM operator and obtain the aggregated $\widetilde{D}_{DZF}$ ($\widetilde{D}_{DZF}^{Agg}$) using Eqs. (5.29-5.30). $\lambda_k = (\lambda_1, \lambda_2 \ldots\ldots, \lambda_K)$; $\lambda_k \in [0,1]$ and $\sum_{k=1}^{K} \lambda_k = 1$, where λ_k is the weight of k^{th} DM.

$$\widetilde{D}_{DZF}^{Agg} = \begin{bmatrix} \tilde{c}_{11} & \tilde{c}_{12} & \cdots & \tilde{c}_{1m} \\ \tilde{c}_{21} & \tilde{c}_{22} & \cdots & \tilde{c}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{c}_{i1} & \tilde{c}_{i2} & \tilde{c}_{ij} & \tilde{c}_{im} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{c}_{n1} & \tilde{c}_{n2} & \cdots & \tilde{c}_{nm} \end{bmatrix} \tag{5.29}$$

where $\tilde{c}_{ij} = (\tilde{A}, \tilde{R}) = \left(\left(\tilde{A}_1^{\lambda_1} \otimes \tilde{A}_2^{\lambda_2} \otimes \ldots \otimes \tilde{A}_K^{\lambda_K} \right), \left(\tilde{R}_1^{\lambda_1} \otimes \tilde{R}_2^{\lambda_2} \otimes \ldots \otimes \tilde{R}_K^{\lambda_K} \right) \right)$ and

$$\tilde{A} = \left\{ \begin{array}{l} \mathcal{O}_{\tilde{A}} \left(\begin{array}{c} \left(\prod_{k=1}^{K} \mu_{AL}^{\mathcal{O}}(x_{ij})_k^{\lambda_k}, \prod_{k=1}^{K} \mu_{AM}^{\mathcal{O}}(x_{ij})_k^{\lambda_k}, \prod_{k=1}^{K} \mu_{AU}^{\mathcal{O}}(x_{ij})_k^{\lambda_k} \right), \\ \left(1 - \prod_{k=1}^{K} \left(1 - \vartheta_{AL}^{\mathcal{O}}(x_{ij})_k\right)^{\lambda_k}, 1 - \prod_{k=1}^{K} \left(1 - \vartheta_{AM}^{\mathcal{O}}(x_{ij})_k\right)^{\lambda_k}, 1 - \prod_{k=1}^{K} \left(1 - \vartheta_{AU}^{\mathcal{O}}(x_{ij})_k\right)^{\lambda_k} \right) \end{array} \right), \\ \mathcal{P}_{\tilde{A}} \left(\begin{array}{c} \left(\begin{array}{c} \dfrac{\prod_{k=1}^{K} \mu_{AL}^{\mathcal{P}}(x_{ij})_k}{\sum_{k=1}^{K} \mu_{AL}^{\mathcal{P}}(x_{ij})_k^{n-1} \lambda_k \left(1 - \mu_{AL}^{\mathcal{P}}(x_{ij})_k\right) + \prod_{k=1}^{K} \mu_{AL}^{\mathcal{P}}(x_{ij})_k}, \\ \dfrac{\prod_{k=1}^{K} \mu_{AM}^{\mathcal{P}}(x_{ij})_k}{\sum_{k=1}^{K} \mu_{AM}^{\mathcal{P}}(x_{ij})_k^{n-1} \lambda_k \left(1 - \mu_{AM}^{\mathcal{P}}(x_{ij})_k\right) + \prod_{k=1}^{K} \mu_{AM}^{\mathcal{P}}(x_{ij})_k}, \\ \dfrac{\prod_{k=1}^{K} \mu_{Au}^{\mathcal{P}}(x_{ij})_k}{\sum_{k=1}^{K} \mu_{AU}^{\mathcal{P}}(x_{ij})_k^{n-1} \lambda_k \left(1 - \mu_{AU}^{\mathcal{P}}(x_{ij})_k\right) + \prod_{k=1}^{K} \mu_{AU}^{\mathcal{P}}(x_{ij})_k} \end{array} \right), \\ \left(\begin{array}{c} \dfrac{\sum_{k=1}^{K} \lambda_k \vartheta_{AL}^{\mathcal{P}}(x_{ij})_k}{1 + \sum_{k=1}^{K} \left(\lambda_k \vartheta_{AL}^{\mathcal{P}}(x_{ij})_k - \dfrac{\vartheta_{AL}^{\mathcal{P}}(x_{ij})_k}{n} \right)}, \\ \dfrac{\sum_{k=1}^{K} \lambda_k \vartheta_{AM}^{\mathcal{P}}(x_{ij})}{1 + \sum_{k=1}^{K} \left(\lambda_k \vartheta_{AM}^{\mathcal{P}}(x_{ij})_k - \dfrac{\vartheta_{AM}^{\mathcal{P}}(x_{ij})_k}{n} \right)}, \dfrac{\sum_{k=1}^{K} \lambda_k \vartheta_{AU}^{\mathcal{P}}(x_{ij})_k}{1 + \sum_{k=1}^{K} \left(\lambda_k \vartheta_{AU}^{\mathcal{P}}(x_{ij})_k - \dfrac{\vartheta_{AU}^{\mathcal{P}}(x_{ij})_k}{n} \right)} \end{array} \right) \end{array} \right) \end{array} \right\}$$

$$
\tilde{R} = \left\{ \begin{array}{c} \mathcal{O}_{\tilde{R}} \left(\begin{array}{c} \left(\prod_{k=1}^{K} \mu_{RL}^{\mathcal{O}}\left(x_{ij}\right)_k^{\lambda_k}, \prod_{k=1}^{K} \mu_{RM}^{\mathcal{O}}\left(x_{ij}\right)_k^{\lambda_k}, \prod_{k=1}^{K} \mu_{RU}^{\mathcal{O}}\left(x_{ij}\right)_k^{\lambda_k} \right), \\ \left(1 - \prod_{k=1}^{K} \left(1 - \vartheta_{RL}^{\mathcal{O}}\left(x_{ij}\right)_k\right)^{\lambda_k}, 1 - \prod_{k=1}^{K} \left(1 - \vartheta_{RM}^{\mathcal{O}}\left(x_{ij}\right)_k\right)^{\lambda_k}, 1 - \prod_{k=1}^{K} \left(1 - \vartheta_{RU}^{\mathcal{O}}\left(x_{ij}\right)_k\right)^{\lambda_k} \right) \end{array} \right), \\ \mathcal{P}_{\tilde{R}} \left(\begin{array}{c} \left(\begin{array}{c} \frac{\prod_{k=1}^{K} \mu_{RL}^{\mathcal{P}}\left(x_{ij}\right)_k}{\sum_{k=1}^{K} \mu_{RL}^{\mathcal{P}}\left(x_{ij}\right)_k^{n-1} \lambda_k \left(1 - \mu_{RL}^{\mathcal{P}}\left(x_{ij}\right)_k\right) + \prod_{k=1}^{K} \mu_{RL}^{\mathcal{P}}\left(x_{ij}\right)_k}, \\ \frac{\prod_{k=1}^{K} \mu_{RM}^{\mathcal{P}}\left(x_{ij}\right)_k}{\sum_{k=1}^{K} \mu_{RM}^{\mathcal{P}}\left(x_{ij}\right)_k^{n-1} \lambda_k \left(1 - \mu_{RM}^{\mathcal{P}}\left(x_{ij}\right)_k\right) + \prod_{k=1}^{K} \mu_{RM}^{\mathcal{P}}\left(x_{ij}\right)_k}, \\ \frac{\prod_{k=1}^{K} \mu_{RU}^{\mathcal{P}}\left(x_{ij}\right)_k}{\sum_{k=1}^{K} \mu_{RU}^{\mathcal{P}}\left(x_{ij}\right)_k^{n-1} \lambda_k \left(1 - \mu_{RU}^{\mathcal{P}}\left(x_{ij}\right)_k\right) + \prod_{k=1}^{K} \mu_{RU}^{\mathcal{P}}\left(x_{ij}\right)_k} \end{array} \right), \\ \left(\begin{array}{c} \frac{\sum_{k=1}^{K} \lambda_k \vartheta_{RL}^{\mathcal{P}}\left(x_{ij}\right)_k}{1 + \sum_{k=1}^{K} \left(\lambda_k \vartheta_{RL}^{\mathcal{P}}\left(x_{ij}\right)_k - \frac{\vartheta_{RL}^{\mathcal{P}}\left(x_{ij}\right)_k}{n} \right)}, \frac{\sum_{k=1}^{K} \lambda_k \vartheta_{RM}^{\mathcal{P}}\left(x_{ij}\right)}{1 + \sum_{k=1}^{K} \left(\lambda_k \vartheta_{RM}^{\mathcal{P}}\left(x_{ij}\right)_k - \frac{\vartheta_{RM}^{\mathcal{P}}\left(x_{ij}\right)_k}{n} \right)}, \\ \frac{\sum_{k=1}^{K} \lambda_k \vartheta_{RU}^{\mathcal{P}}\left(x_{ij}\right)_k}{1 + \sum_{k=1}^{K} \left(\lambda_k \vartheta_{RU}^{\mathcal{P}}\left(x_{ij}\right)_k - \frac{\vartheta_{RU}^{\mathcal{P}}\left(x_{ij}\right)_k}{n} \right)} \end{array} \right) \end{array} \right) \end{array} \right\}
$$

i=1, 2, …, n. j=1, 2, …, m. (5.30)

As an explanatory symbol in Eq. (5.30), $\mu_{AL}^{\mathcal{P}}\left(x_{ij}\right)_k$ and $\mu_{RL}^{\mathcal{P}}\left(x_{ij}\right)_k$ represents the pessimistic membership degree of kth DM's assessment for alternative *i* with respect to criterion *j* in the restriction function and the same for the reliability function, respectively. The other symbols have the similar explanations.

Step 4. Integrate the aggregated decomposed fuzzy reliability function to the aggregated decomposed fuzzy restriction function using Definition (5.11) and thus obtain $\tilde{D}_{DF}^{Agg}$.

Step 5. Weigh the $\tilde{D}_{DF}^{Agg}$ by criteria weights ($\tilde{W}$) and obtain $\tilde{D}_{DF_W}^{Agg}$. $\tilde{W} = \left\{\tilde{w}_1, \tilde{w}_2, \tilde{w}_3, \dots, \tilde{w}_j\right\}$ is the weight vector satisfying the conditions $0 \le \tilde{w}_j \le 1$ and $\sum_{j=1}^{m} \tilde{w}_j = 1$.

$$
\tilde{D}_{DF_W}^{Agg} = \tilde{D}_{DF_{ij}}^{Agg} * \tilde{w}_j \tag{5.31}
$$

where

$$
\tilde{w}_j =
$$

$$\left\{\left\langle x, \left(\begin{array}{c}\left(\mathcal{O}_{\widetilde{W}}\left(\left(\mu^{\mathcal{O}}_{WL}(x_j), \mu^{\mathcal{O}}_{WM}(x_j), \mu^{\mathcal{O}}_{WU}(x_j)\right), \left(\vartheta^{\mathcal{O}}_{WL}(x_j), \vartheta^{\mathcal{O}}_{WM}(x_j), \vartheta^{\mathcal{O}}_{WU}(x_j)\right)\right)\right), \\ \left(\mathcal{P}_{\widetilde{W}}\left(\left(\mu^{\mathcal{P}}_{WL}(x_j), \mu^{\mathcal{P}}_{WM}(x_j), \mu^{\mathcal{P}}_{WU}(x_j)\right), \left(\vartheta^{\mathcal{P}}_{WL}(x_j), \vartheta^{\mathcal{P}}_{WM}(x_j), \vartheta^{\mathcal{P}}_{WU}(x_j)\right)\right)\right)\end{array}\right)\right\rangle \middle| x_j \in X\right\} \tag{5.32}$$

and

$$\widetilde{D}^{Agg}_{DF_W} = \left\{\begin{array}{c}\mathcal{O}\left(\begin{array}{c}\left(\mu^{\mathcal{O}}_{AL}(x_{ij})\mu^{\mathcal{O}}_{WL}(x_j), \mu^{\mathcal{O}}_{AM}(x_{ij})\mu^{\mathcal{O}}_{WM}(x_j), \mu^{\mathcal{O}}_{AU}(x_{ij})\mu^{\mathcal{O}}_{WU}(x_j)\right), \\ \left(\begin{array}{c}\vartheta^{\mathcal{O}}_{AL}(x_{ij}) + \vartheta^{\mathcal{O}}_{WL}(x_j) - \vartheta^{\mathcal{O}}_{AL}(x_{ij})\vartheta^{\mathcal{O}}_{WL}(x_j), \\ \vartheta^{\mathcal{O}}_{AM}(x_{ij}) + \vartheta^{\mathcal{O}}_{WM}(x_j) - \vartheta^{\mathcal{O}}_{AM}(x_{ij})\vartheta^{\mathcal{O}}_{WM}(x_j), \\ \vartheta^{\mathcal{O}}_{AU}(x_{ij}) + \vartheta^{\mathcal{O}}_{WU}(x_j) - \vartheta^{\mathcal{O}}_{AU}(x_{ij})\vartheta^{\mathcal{O}}_{WU}(x_j)\end{array}\right)\end{array}\right), \\ \mathcal{P}\left(\begin{array}{c}\frac{\mu^{\mathcal{P}}_{AL}(x_{ij})+\mu^{\mathcal{P}}_{WL}(x_j)-2\mu^{\mathcal{P}}_{AL}(x_{ij})\mu^{\mathcal{P}}_{WL}(x_j)}{1-\mu^{\mathcal{P}}_{AL}(x_{ij})\mu^{\mathcal{P}}_{WL}(x_j)}, \frac{\mu^{\mathcal{P}}_{AM}(x_{ij})+\mu^{\mathcal{P}}_{WM}(x_j)-2\mu^{\mathcal{P}}_{AM}(x_{ij})\mu^{\mathcal{P}}_{WM}(x_j)}{1-\mu^{\mathcal{P}}_{AM}(x_{ij})\mu^{\mathcal{P}}_{WM}(x_j)}, \\ \frac{\mu^{\mathcal{P}}_{AU}(x_{ij})+\mu^{\mathcal{P}}_{WU}(x_j)-2\mu^{\mathcal{P}}_{AU}(x_{ij})\mu^{\mathcal{P}}_{WU}(x_j)}{1-\mu^{\mathcal{P}}_{AU}(x_{ij})\mu^{\mathcal{P}}_{WU}(x_j)} \\ \frac{\vartheta^{\mathcal{P}}_{AL}(x_{ij})\vartheta^{\mathcal{P}}_{WL}(x_j)}{\vartheta^{\mathcal{P}}_{AL}(x_{ij})+\vartheta^{\mathcal{P}}_{WL}(x_j)-\vartheta^{\mathcal{P}}_{AL}(x_{ij})\vartheta^{\mathcal{P}}_{WL}(x_j)}, \frac{\vartheta^{\mathcal{P}}_{AM}(x)\vartheta^{\mathcal{P}}_{WM}(x_j)}{\vartheta^{\mathcal{P}}_{AM}(x_{ij})+\vartheta^{\mathcal{P}}_{WM}(x_j)-\vartheta^{\mathcal{P}}_{AM}(x_{ij})\vartheta^{\mathcal{P}}_{WM}(x_j)}, \\ \frac{\vartheta^{\mathcal{P}}_{AU}(x_{ij})\vartheta^{\mathcal{P}}_{WU}(x_j)}{\vartheta^{\mathcal{P}}_{AU}(x_{ij})+\vartheta^{\mathcal{P}}_{WU}(x_j)-\vartheta^{\mathcal{P}}_{AU}(x_{ij})\vartheta^{\mathcal{P}}_{WU}(x_j)}\end{array}\right)\end{array}\right\} \tag{5.33}$$

Criteria weights can be crisp or decomposed fuzzy number. If the $\widetilde{D}^{Agg}_{DF}$ is to be multiplied by the crisp weights, then Eq. (5.8) is used. In this step, Tüysüz and Kahraman (2020b)'s Z-AHP method can be used to obtain criteria weights.

Step 6. Defuzzify each $\mathcal{O}$ and $\mathcal{P}$ parameters of the $\widetilde{D}^{Agg}_{DF_W}$ by using the defuzzification equation $(a_1 + 2 * a_2 + 2 * a_3 + a_4)/6$ for symmetrical trapezoidal fuzzy numbers and $(a_1 + 2 * a_2 + a_3)/4$ for symmetrical triangular fuzzy numbers. $Def(\widetilde{D}^{Agg}_{DF_W})$ matrix is obtained.

Step 7. Determine the decomposed fuzzy positive ideal solution (PIS_{DF}) and decomposed fuzzy negative ideal solution (NIS_{DF}).

In the literature, positive ideal and negative ideal solutions are determined as the maximum or minimum values that the alternatives have according to the criteria, or directly as extreme values of 0 or 1. In this study, PIS_{DF} and NIS_{DF} values are determined by the corresponding values of DFNs rather than accepting them as 1 or 0 in the decomposed fuzzy form.

The set of criteria consists of two groups as the benefit C_B and the cost C_C criteria. According to the criteria type, PIS_{DF} and NIS_{DF} are defined by Eqs. (5.34-5.37).

$$PIS_{DF} = p_j = \left(\mathcal{O}\left(\mu^{\mathcal{O}*}(\mathrm{x}_j), \vartheta^{\mathcal{O}*}(\mathrm{x}_j)\right), \mathcal{P}\left(\mu^{\mathcal{P}*}(\mathrm{x}_j), \vartheta^{\mathcal{P}*}(\mathrm{x}_j)\right)\right) \tag{5.34}$$

$$NIS_{DF} = n_j = \left(\mathcal{O}\left(\mu^{\mathcal{O}-}(\mathrm{x}_j), \vartheta^{\mathcal{O}-}(\mathrm{x}_j)\right), \mathcal{P}\left(\mu^{\mathcal{P}-}(\mathrm{x}_j), \vartheta^{\mathcal{P}-}(\mathrm{x}_j)\right)\right) \tag{5.35}$$

where

$$p_j = \left\{\max_{1\le i\le n}\left(\left(\mathcal{O}\left(\mu^{\mathcal{O}}(\mathrm{x}_{ij}), \vartheta^{\mathcal{O}}(\mathrm{x}_{ij})\right), \mathcal{P}\left(\mu^{\mathcal{P}}(\mathrm{x}_{ij}), \vartheta^{\mathcal{P}}(\mathrm{x}_{ij})\right)\right)\middle| C_j \in C_B\right)\right\},$$

$$\left\{\min_{1\le i\le n}\left(\left(\mathcal{O}\left(\mu^{\mathcal{O}}(\mathrm{x}_{ij}), \vartheta^{\mathcal{O}}(\mathrm{x}_{ij})\right), \mathcal{P}\left(\mu^{\mathcal{P}}(\mathrm{x}_{ij}), \vartheta^{\mathcal{P}}(\mathrm{x}_{ij})\right)\right)\middle| C_j \in C_C\right)\right\} \tag{5.36}$$

$$n_j = \left\{\min_{1\le i\le n}\left(\left(\mathcal{O}\left(\mu^{\mathcal{O}}(\mathrm{x}_{ij}), \vartheta^{\mathcal{O}}(\mathrm{x}_{ij})\right), \mathcal{P}\left(\mu^{\mathcal{P}}(\mathrm{x}_{ij}), \vartheta^{\mathcal{P}}(\mathrm{x}_{ij})\right)\right)\middle| C_j \in C_B\right)\right\},$$

$$\left\{\max_{1\le i\le n}\left(\left(\mathcal{O}\left(\mu^{\mathcal{O}}(\mathrm{x}_{ij}), \vartheta^{\mathcal{O}}(\mathrm{x}_{ij})\right), \mathcal{P}\left(\mu^{\mathcal{P}}(\mathrm{x}_{ij}), \vartheta^{\mathcal{P}}(\mathrm{x}_{ij})\right)\right)\middle| C_j \in C_C\right)\right\} \tag{5.37}$$

To determine p_j and n_j for each criterion, decomposed fuzzy values of alternatives are defuzzified $\left(Def(\widetilde{D}_{DF_W}^{Agg})\right)$ using Eq. (5.23). The *max* and *min* values of the $Def(\widetilde{D}_{DF_W}^{Agg})$ help us to identify p_j and n_j. Thus, we continue with decomposed fuzzy values.

Step 8. Calculate the positive and negative separation measures (S^* *and* S^-).

The separation measures of alternatives can be calculated by Euclidean distance, Hamming distance, etc. In this study, the Euclidean distance is proposed to measure distance between two decomposed fuzzy numbers in Definition 5.10. Eq. (5.38) and Eq. (5.39) can be used to determine the separation measures, $S^*$and S^-, relating to each alternative from PIS_{DF} and NIS_{DF}, respectively.

$$S_i^* = \sqrt{\frac{1}{2m}\sum_{j=1}^{m}\left[\left(\begin{array}{c}\left(\mu^{\mathcal{O}}_{\widetilde{D}_{DF_W}^{Agg}}(x_{ij}) - \mu^{\mathcal{O}}_{PIS_{DF}}(x_j)\right)^2 + \left(\vartheta^{\mathcal{O}}_{\widetilde{D}_{DF_W}^{Agg}}(x_{ij}) - \vartheta^{\mathcal{O}}_{PIS_{DF}}(x_j)\right)^2 \\ + \left(\mu^{\mathcal{P}}_{\widetilde{D}_{DF_W}^{Agg}}(x_{ij}) - \mu^{\mathcal{P}}_{PIS_{DF}}(x_j)\right)^2 + \left(\vartheta^{\mathcal{P}}_{\widetilde{D}_{DF_W}^{Agg}}(x_{ij}) - \vartheta^{\mathcal{P}}_{PIS_{DF}}(x_j)\right)\end{array}\right)\right]}$$

i=1,2,…,n (5.38)

$$S_i^- =$$

$$\sqrt{\frac{1}{2m}\sum_{j=1}^{m}\left[\left(\begin{array}{c}\left(\mu^{\mathcal{O}}_{\tilde{D}^{Agg}_{DF_W}}(x_{ij})-\mu^{\mathcal{O}}_{NIS_{DF}}(x_j)\right)^2+\left(\vartheta^{\mathcal{O}}_{\tilde{D}^{Agg}_{DF_W}}(x_{ij})-\vartheta^{\mathcal{O}}_{NIS_{DF}}(x_j)\right)^2\\+\left(\mu^{\mathcal{P}}_{\tilde{D}^{Agg}_{DF_W}}(x_{ij})-\mu^{\mathcal{P}}_{NIS_{DF}}(x_j)\right)^2+\left(\vartheta^{\mathcal{P}}_{\tilde{D}^{Agg}_{DF_W}}(x_{ij})-\vartheta^{\mathcal{P}}_{NIS_{DF}}(x_j)\right)\end{array}\right)\right]}$$

i=1,2,…,n (5.39)

Step 9. Calculate the closeness coefficient C_i^* of the alternatives using Eq. (5.40). Then, rank the alternatives according to the descending order of C_i^*.

$$C_i^* = \frac{S_i^-}{S_i^- + S_i^*} \tag{5.40}$$

where $0 \leq C_i^* \leq 1$.

5.4 Application

In this section, the proposed methodology is applied to distribution center location selection problem for a cargo company in Marmara Region of Turkey. The problem definition is first presented and then, its solution is given step by step based on the proposed methodology. Finally, comparative and sensitivity analyses are performed.

5.4.1 Problem definition

Facility location selection can be defined as the selection of the region or piece of land where a facility will be established or placed. The transfer center location selection (TCLS) is a special case of the facility location selection problem and it is important for the companies to construct the logistics network structure effectively. Companies require to position their transfer centers in the right places in order to maintain their existence in the increasing competitive conditions. TCLS is strategically important as it is one of the long-term investments, and it is affected by many factors such as transportation features, workforce opportunities, installation and operating costs, and the proximity to customers. All these factors make difficult to choose among more than one transfer center and complicated to manage the logistics networks for the companies. A multi criteria decision making model that simultaneously meets these contradictory factors is necessary for the selection of suitable transfer center location. Therefore, in this study, we present the application of the proposed DF Z-TOPSIS method, which has the ability to include DMs' imprecise and uncertain assessments and their reliabilities to their assessments for transfer center location selection.

Since the outbreak of the coronavirus pandemic, the number of products distributed by cargo companies has increased. Therefore, the new cargo companies have been opened and many private cargo companies have increased the number of their facilities in order to meet their increasing demand. With this increase, distribution networks have branched and new transfer centers have to be constructed for some new companies.

In this study, a private cargo company which was established after the coronavirus pandemic wants to open a new transfer center in Marmara Region in order to increase their profitability and make themselves advantageous in the competitive environment. The management group wants to choose the best location among the five alternative transfer center locations determined as A1-Çorlu, A2-Silivri, A3-Sultanbeyli, A4-Osmangazi and A5-Izmit illustrated in Figure 5.9.

Figure 5.9 : Alternative locations for opening a new transfer center.

5.4.2 Problem solution

To find the best location, we first have conducted a literature review and identified seven criteria given in Table 5.4.

Table 5.4 : Criteria set of the problem.

Criteria		Criteria type: max (benefit) / min (cost)	Explanations	References
Costs	C1	min	Land, building and utility costs, rent rates, taxes, etc.	Nong (2022), Awasthi et al. (2011), Agrebi and Abed (2021), Keshavarz-Ghorabaee (2021), He et al. (2017)
Proximity to transportation ports	C2	max	Closeness from airports, railways and other transportation ports	Nong (2022), Awasthi et al. (2011), Agrebi and Abed (2021), Keshavarz-Ghorabaee (2021), He et al. (2017)
Traffic Flow	C3	min	Average traffic volume	He et al. (2017)
Proximity to markets, suppliers and other services	C4	max	Distance from markets and suppliers	Nong (2022), Awasthi et al. (2011), Agrebi and Abed (2021), Keshavarz-Ghorabaee (2021)
Workforce availability	C5	max	Characteristics of local demographics	Nong (2022), Awasthi et al. (2011), Keshavarz-Ghorabaee (2021)
Service characteristics	C6	max	Service level, delivery flexibility, quality etc.	Nong (2022), Awasthi et al. (2011), Agrebi and Abed (2021), Keshavarz-Ghorabaee (2021), He et al. (2017)
Environmental conditions	C7	max	Ecological landscape, climate conditions	Bennani et al. (2022), He et al. (2017)

Then, in Step 2, in order to fill the decision matrix, questionnaires have been applied to the DMs. For instance, to evaluate the alternative *A1-Çorlu* according to the criterion *C1-costs*, following questions have been asked. There are four DMs in the evaluation group who can answer these questions by using the linguistic terms in Table 5.3. Equal weights were given to DMs since the four DMs had similar experiences and similar training. The DMs are two professors from the industrial engineering department of a university and two professors from the logistics department of another university in Istanbul. When we check the consensus level of their judgments, we have seen that their judgments are more or less the same.

for restriction function:

(functional): What do you think about the maximum cost of a transfer center at Çorlu?

DM1's assessment is $\mathcal{O}(VL)$.

(dysfunctional): What do you think about the minimum cost of a transfer center at Çorlu?

DM1's assessment is $\mathcal{P}(H)$.

for reliability function:

(functional): How sure are you about your assessment?

DM1's assessment is $\mathcal{O}(HR)$.

(dysfunctional): How unsure are you about your assessment?

DM1's assessment is $\mathcal{P}(LR)$.

Thus, the complete DF Z-number obtained from the given answers is $\tilde{Z}_{DFS}(\tilde{A},\tilde{R}) = \left(\left(\mathcal{O}(VL),\mathcal{P}(H)\right),\left(\mathcal{O}(HR),\mathcal{P}(LR)\right)\right)$ and can be seen in Table 5.5.

Decision matrices of four DMs are presented in Tables 5.5-5.8.

Table 5.5 : DM_1's decomposed Z-fuzzy evaluations.

	C1	C2	C3	C4	C5	C6	C7
A1	**((VL, H), (HR, LR))**	((VH, L), (LR, VHR))	((L, M), (HR, LR))	((AL, H), (HR, LR))	((H, VL), (LR, AHR))	((VL, H), (LR, VHR))	((L, H), (R, LR))
A2	((AH, L), (VLR, HR))	((VL, M), (AHR, LR))	((VH, L), (LR, R))	((L, M), (R, HR))	((VL, H), (VHR, LR))	((AH, L), (LR, HR))	((M, VH), (AHR, LR))
A3	((M, H), (VHR, R))	((L, AL), (R, LR))	((VH, AL), (VHR, LR))	((H, VL), (VLR,AHR))	((AH, VL), (HR, LR))	((H, AL), (VHR, LR))	((H, L), (R, ALR))
A4	((VL, H), (VLR,VHR))	((H, VL), (VHR,VLR))	((L, VH), (HR, ALR))	((M, H), (ALR, R))	((H, VL), (HR, LR))	((VH, VL), (HR, LR))	((H, VL), (LR, HR))
A5	((L, AH), (LR, HR))	((VH, VL), (HR, LR))	((H, L), (LR, VHR))	((AH, VL), (HR, LR))	((VL, H), (ALR, HR))	((H, L), (HR, LR))	((AH, L), (HR, R))

Table 5.6 : DM_2's decomposed Z-fuzzy evaluations.

	C1	C2	C3	C4	C5	C6	C7
A1	((AH, L), (HR, R))	((H, VL), (R, LR))	((VL, H), (R, LR))	((VL, AH), (LR, HR))	((VH, AL), (LR,AHR))	((H, VL), (LR, HR))	((VL,AH), (LR,AHR))
A2	((VH, VL), (R, LR))	((L, VH), (AHR,LR))	((H, AL), (VHR, LR))	((H, L), (HR,ALR))	((L, VH), (R, HR))	((VH, L), (HR,ALR))	((L, M), (AHR,LR))
A3	((H, VL), (HR, VLR))	((VL, M), (LR,VHR))	((VH, L), (HR,ALR))	((H, AL), (R, LR))	((VH, VL), (HR, LR))	((AH, L), (HR, LR))	((H, VL), (R, LR))
A4	((AH, VL), (VLR,AHR))	((AH, AL), (VLR, HR))	((VH, AL), (LR, HR))	((L, AH), (HR, VLR))	((M, L), (LR,AHR))	((H, AL), (VLR,VHR))	((VH, AL), (HR,VLR))
A5	((VL, H), (LR, R))	((H, AL), (HR, LR))	((H, AL), (HR, LR))	((M, AL), (VLR, HR))	((L, VH), (ALR, HR))	((H, VL), (LR, HR))	((VH, L), (LR,AHR))

Table 5.7 : DM_3's decomposed Z-fuzzy evaluations.

	C1	C2	C3	C4	C5	C6	C7
A1	((VH, L), (AHR,VLR))	((AH, VL), (LR, VHR))	((L, VH), (ALR, HR))	((L, VH), (HR, LR))	((AH, VL), (VHR,LR))	((VH, AL), (VLR,HR))	((AL, VH), (AHR,LR))
A2	((H, M), (LR, VHR))	((AL, VH), (HR, LR))	((AH, VL), (VHR,ALR))	((VL, AH), (HR, LR))	((H, M), (VLR,HR))	((H, VL), (HR, ALR))	((L, H), (LR, VHR))
A3	((AL, VH), (VHR, LR))	((H, VL), (VHR, LR))	((AL, H), (HR, LR))	((H, VL), (LR, VHR))	((VL, H), (LR, R))	((L, VH), (LR, HR))	((H, VL), (VHR,LR))
A4	((L, H), (LR,VHR))	((VH, L), (LR, AHR))	((VH, L), (LR, AHR))	((VL, H), (HR, LR))	((L, VH), (ALR,VHR))	((VH, AL), (VLR,HR))	((H, L), (HR, R))
A5	((VL, M), (R, VLR))	((H, AL), (VLR, HR))	((H, AL), (HR, LR))	((H, VL), (R, VLR))	((AL, VH), (HR, VLR))	((AH, L), (LR, R))	((M, VL), (ALR,HR))

Table 5.8 : DM_4's decomposed Z-fuzzy evaluations.

	C1	C2	C3	C4	C5	C6	C7
A1	((L, VH), (VLR, HR))	((AH, VL), (HR, LR))	((AL, VH), (VHR, LR))	((L, VH), (HR,VLR))	((H, VL), (HR, LR))	((VH, AL), (LR, VHR))	((VL, H), (HR, R))
A2	((H, VL), (HR, LR))	((VH, L), (LR, AHR))	((AH, AL), (LR, AHR))	((VH, L), (LR,AHR))	((L, H), (VLR,HR))	((H, L), (VHR, LR))	((AH, L), (VLR,HR))
A3	((L, HR), (VHR,ALR))	((H, AL), (R, LR))	((VL, AH), (HR, LR))	((H, AL), (VHR,LR))	((AL, VH), (HR,ALR))	((AH, L), (HR, LR))	((H, AL), (HR, LR))
A4	((VH, AL), (HR, LR))	((VH, L), (VLR,AHR))	((VL, H), (VHR,ALR))	((L, VH), (VLR,HR))	((H, VL), (LR, HR))	((AH, VL), (HR, VLR))	((H, L), (LR,VHR))
A5	((VL, VH), (LR, VHR))	((L, VH), (HR, LR))	((VH, L), (VLR,VHR))	((VH, L), (HR,VLR))	((VL, H), (VHR,LR))	((VH, L), (AHR, LR))	((VL, H), (R, LR))

In Step 3, the decision matrices collected from four DMs are aggregated by DWGM operator using Eqs. (5.29-5.30). We assigned equal weights to the DMs based on their experiences in this computation. Aggregated decomposed triangular Z-fuzzy evaluations are presented in Table 5.9.

A calculation example for preferences of A1 with respect to C1 is presented to make clear how the values are obtained for the bold numbers (((**0.40**, 0.51, 0.62), (**0.19**, 0.29, 0.38)), ((**0.37**, 0.50, 0.63), (**0.18**, 0.28, 0.38))) in Table 5.9.

The preferences of four DMs and their corresponding decomposed fuzzy numbers are given as follows:

DM_1 = ((VL, H), (HR, LR)) = ((0.2, 0.3, 0.4), (0.4, 0.5, 0.6)), ((0.5, 0.6, 0.7), (0.1, 0.2, 0.3))

DM_2 = ((AH, L), (HR, R)) = ((0.7, 0.8, 0.9), (0, 0.05, 0.1)), ((0.3, 0.4, 0.5), (0.3, 0.4, 0.5))

DM_3 = ((VH, L), (AHR, VLR))= ((0.6, 0.7, 0.8), (0, 0.1, 0.2)), ((0.3, 0.4, 0.5), (0.3, 0.4, 0.5))

DM_4 = ((L, VH), (VLR, HR))= ((0.3, 0.4, 0.5), (0.3, 0.4, 0.5)), ((0.6, 0.7, 0.8), (0, 0.1, 0.2))

$$0.20^{\frac{1}{4}} * 0.70^{\frac{1}{4}} * 0.60^{\frac{1}{4}} * 0.30^{\frac{1}{4}} = 0.40$$

$$1 - \left((1-0.40)^{\frac{1}{4}}\right) * \left((1-0.00)^{\frac{1}{4}}\right) * \left((1-0.00)^{\frac{1}{4}}\right) * \left((1-0.30)^{\frac{1}{4}}\right) = 0.19$$

$$\frac{0.50 * 0.30 * 0.30 * 0.60}{\begin{pmatrix} 0.5^3 * \frac{1}{4} * (1-0.5) + 0.3^3 * \frac{1}{4} * (1-0.3) + 0.30^3 * \frac{1}{4} * (1-0.3) \\ +0.6^3 * \frac{1}{4} * (1-0.6) + 0.5 * 0.3 * 0.3 * 0.6 \end{pmatrix}} = 0.37$$

$$\frac{\frac{1}{4} * 0.10 + \frac{1}{4} * 0.30 + \frac{1}{4} * 0.30 + \frac{1}{4} * 0.0}{1 + \frac{1}{4} * 0.10 + \frac{1}{4} * 0.30 + \frac{1}{4} * 0.30 + \frac{1}{4} * 0.0 - \frac{0.10}{4} + \frac{0.30}{4} + \frac{0.30}{4} + \frac{0.0}{4}} = 0.18$$

Table 5.9 : Aggregated decomposed triangular Z-fuzzy restrictions and reliabilities.

	C1		C2		C3		C4	
	Restriction	Reliability	Restriction	Reliability	Restriction	Reliability	Restriction	Reliability
A1	**(((0.4,** 0.51, 0.62), **(0.19,** 0.29, 0.38)), **((0.37,** 0.5, 0.63**),** **(0.18,** 0.28, 0.38)))	(((0.43, 0.54, 0.65), (0.17, 0.26, 0.35)), ((0.28, 0.41, 0.54), (0.25, 0.35, 0.45)))	(((0.21, 0.31, 0.42), (0.38, 0.47, 0.55)), ((0.51, 0.62, 0.73), (0.08, 0.18, 0.28)))	(((0.33, 0.45, 0.56), (0.23, 0.31, 0.39)), ((0.31, 0.43, 0.55), (0.25, 0.35, 0.45)))	(((0.21, 0.31, 0.42), (0.38, 0.47, 0.55)), ((0.6, 0.7, 0.81), (0.03, 0.11, 0.2)))	(((0.44, 0.54, 0.64), (0.15, 0.26, 0.36)), ((0.25, 0.39, 0.51), (0.28, 0.38, 0.48)))	(((0.62, 0.72, 0.82), (0.03, 0.1, 0.18)), ((0.2, 0.31, 0.42), (0.38, 0.48, 0.58)))	(((0.37, 0.47, 0.57), (0.23, 0.33, 0.43)), ((0.38, 0.53, 0.66), (0.15, 0.25, 0.35)))
A2	(((0.57, 0.67, 0.77), (0.05, 0.14, 0.23)), ((0.28, 0.41, 0.54), (0.25, 0.35, 0.45)))	(((0.33, 0.44, 0.54), (0.26, 0.36, 0.46)), ((0.37, 0.5, 0.63), (0.18, 0.28, 0.38)))	(((0.62, 0.72, 0.82), (0.03, 0.1, 0.18)), ((0.12, 0.32, 0.53), (0.3, 0.38, 0.45)))	(((0.42, 0.53, 0.63), (0.16, 0.27, 0.37)), ((0.17, 0.38, 0.57), (0.25, 0.33, 0.4)))	(((0.37, 0.47, 0.58), (0.22, 0.32, 0.42)), ((0.45, 0.59, 0.73), (0.13, 0.21, 0.3)))	(((0.42, 0.52, 0.62), (0.18, 0.28, 0.38)), ((0.18, 0.4, 0.6), (0.23, 0.3, 0.38)))	(((0.24, 0.36, 0.47), (0.32, 0.41, 0.5)), ((0.54, 0.65, 0.76), (0.05, 0.15, 0.25)))	(((0.52, 0.63, 0.73), (0.11, 0.19, 0.27)), ((0.32, 0.48, 0.63), (0.23, 0.31, 0.4)))
A3	(((0.28, 0.39, 0.5), (0.29, 0.38, 0.46)), ((0.29, 0.45, 0.59), (0.2, 0.3, 0.4)))	(((0.57, 0.67, 0.77), (0.03, 0.13, 0.23)), ((0.13, 0.28, 0.41), (0.35, 0.44, 0.53)))	(((0.29, 0.41, 0.53), (0.26, 0.35, 0.43)), ((0.12, 0.28, 0.43), (0.33, 0.41, 0.5)))	(((0.52, 0.62, 0.72), (0.08, 0.18, 0.28)), ((0.16, 0.3, 0.42), (0.35, 0.44, 0.53)))	(((0.5, 0.6, 0.7), (0.1, 0.2, 0.3)), ((0.1, 0.25, 0.4), (0.35, 0.44, 0.53)))	(((0.35, 0.45, 0.56), (0.24, 0.34, 0.44)), ((0.4, 0.56, 0.71), (0.15, 0.24, 0.33)))	(((0.35, 0.46, 0.56), (0.24, 0.34, 0.44)), ((0.13, 0.29, 0.45), (0.3, 0.39, 0.48)))	(((0.41, 0.51, 0.62), (0.18, 0.28, 0.38)), ((0.31, 0.45, 0.58), (0.23, 0.33, 0.43)))
A4	(((0.4, 0.51, 0.62), (0.19, 0.29, 0.38)), ((0.36, 0.51, 0.64), (0.15, 0.25, 0.35)))	(((0.28, 0.38, 0.49), (0.31, 0.41, 0.51)), ((0.51, 0.64, 0.77), (0.08, 0.16, 0.25)))	(((0.38, 0.49, 0.6), (0.19, 0.3, 0.4)), ((0.11, 0.29, 0.46), (0.3, 0.39, 0.48)))	(((0.41, 0.51, 0.61), (0.19, 0.29, 0.39)), ((0.08, 0.28, 0.51), (0.28, 0.34, 0.4)))	(((0.29, 0.39, 0.49), (0.3, 0.4, 0.51)), ((0.46, 0.6, 0.72), (0.13, 0.21, 0.3)))	(((0.27, 0.38, 0.49), (0.3, 0.38, 0.47)), ((0.28, 0.41, 0.54), (0.25, 0.35, 0.45)))	(((0.6, 0.7, 0.8), (0.03, 0.11, 0.2)), ((0.11, 0.29, 0.46), (0.3, 0.39, 0.48)))	(((0.29, 0.4, 0.5), (0.29, 0.39, 0.5)), ((0.42, 0.6, 0.76), (0.13, 0.2, 0.28)))
A5	(((0.22, 0.32, 0.42), (0.38, 0.48, 0.58)), ((0.35, 0.52, 0.67), (0.18, 0.26, 0.35)))	(((0.32, 0.42, 0.52), (0.28, 0.38, 0.48)), ((0.33, 0.48, 0.62), (0.18, 0.28, 0.38)))	(((0.52, 0.62, 0.72), (0.08, 0.18, 0.28)), ((0.06, 0.23, 0.42), (0.33, 0.4, 0.48)))	(((0.35, 0.46, 0.56), (0.24, 0.34, 0.44)), ((0.38, 0.53, 0.66), (0.15, 0.25, 0.35)))	(((0.54, 0.64, 0.74), (0.08, 0.17, 0.26)), ((0.09, 0.26, 0.44), (0.33, 0.41, 0.5)))	(((0.38, 0.48, 0.59), (0.21, 0.31, 0.41)), ((0.2, 0.35, 0.48), (0.3, 0.4, 0.5)))	(((0.46, 0.56, 0.67), (0.13, 0.23, 0.33)), ((0.08, 0.22, 0.34), (0.43, 0.5, 0.58)))	(((0.4, 0.5, 0.61), (0.19, 0.29, 0.39)), ((0.31, 0.43, 0.55), (0.25, 0.35, 0.45)))

Table 5.9 (continued): Aggregated decomposed triangular Z-fuzzy restrictions and reliabilities.

	C5		C6		C7	
	Restriction	Reliability	Restriction	Reliability	Restriction	Reliability
A1	(((0.57, 0.67, 0.77), (0.05, 0.14, 0.23)), ((0.14, 0.26, 0.36), (0.43, 0.51, 0.6)))	(((0.41, 0.51, 0.61), (0.19, 0.29, 0.39)), ((0.42, 0.59, 0.75), (0.15, 0.23, 0.3)))	(((0.44, 0.54, 0.65), (0.14, 0.25, 0.35)), ((0.05, 0.2, 0.36), (0.38, 0.45, 0.53)))	(((0.27, 0.37, 0.47), (0.33, 0.43, 0.53)), ((0.55, 0.65, 0.75), (0.05, 0.15, 0.25)))	(((0.19, 0.29, 0.39), (0.4, 0.49, 0.58)), ((0.57, 0.68, 0.79), (0.05, 0.14, 0.23)))	(((0.45, 0.56, 0.66), (0.16, 0.25, 0.34)), ((0.36, 0.51, 0.66), (0.2, 0.29, 0.38)))
A2	(((0.31, 0.41, 0.51), (0.28, 0.38, 0.49)), ((0.41, 0.54, 0.65), (0.15, 0.25, 0.35)))	(((0.31, 0.42, 0.53), (0.27, 0.37, 0.47)), ((0.42, 0.54, 0.65), (0.15, 0.25, 0.35)))	(((0.57, 0.67, 0.77), (0.05, 0.14, 0.23)), ((0.25, 0.39, 0.51), (0.28, 0.38, 0.48)))	(((0.46, 0.56, 0.67), (0.13, 0.23, 0.33)), ((0.07, 0.22, 0.38), (0.35, 0.43, 0.5)))	(((0.4, 0.5, 0.61), (0.21, 0.3, 0.39)), ((0.54, 0.65, 0.77), (0.08, 0.16, 0.25)))	(((0.41, 0.53, 0.63), (0.19, 0.28, 0.37)), ((0.37, 0.5, 0.63), (0.18, 0.28, 0.38)))
A3	(((0.3, 0.43, 0.54), (0.26, 0.34, 0.42)), ((0.1, 0.25, 0.4), (0.35, 0.44, 0.53)))	(((0.44, 0.54, 0.64), (0.15, 0.26, 0.36)), ((0.16, 0.31, 0.44), (0.33, 0.41, 0.5)))	(((0.52, 0.63, 0.73), (0.11, 0.19, 0.27)), ((0.19, 0.42, 0.63), (0.2, 0.28, 0.35)))	(((0.46, 0.56, 0.67), (0.13, 0.23, 0.33)), ((0.31, 0.43, 0.55), (0.25, 0.35, 0.45)))	(((0.5, 0.6, 0.7), (0.1, 0.2, 0.3)), ((0.2, 0.35, 0.48), (0.3, 0.4, 0.5)))	(((0.47, 0.57, 0.67), (0.13, 0.23, 0.33)), ((0.16, 0.3, 0.42), (0.35, 0.44, 0.53)))
A4	(((0.42, 0.52, 0.62), (0.18, 0.28, 0.38)), ((0.29, 0.45, 0.59), (0.2, 0.3, 0.4)))	(((0.26, 0.37, 0.48), (0.31, 0.4, 0.49)), ((0.48, 0.62, 0.75), (0.1, 0.19, 0.28)))	(((0.6, 0.7, 0.8), (0.03, 0.11, 0.2)), ((0.05, 0.22, 0.47), (0.35, 0.41, 0.48)))	(((0.32, 0.42, 0.53), (0.27, 0.37, 0.47)), ((0.29, 0.45, 0.59), (0.2, 0.3, 0.4)))	(((0.52, 0.62, 0.72), (0.08, 0.18, 0.28)), ((0.12, 0.28, 0.43), (0.33, 0.41, 0.5)))	(((0.39, 0.49, 0.59), (0.21, 0.31, 0.41)), ((0.33, 0.48, 0.62), (0.18, 0.28, 0.38)))
A5	(((0.19, 0.29, 0.39), (0.4, 0.49, 0.58)), ((0.37, 0.53, 0.67), (0.13, 0.23, 0.33)))	(((0.23, 0.36, 0.47), (0.31, 0.38, 0.45)), ((0.29, 0.43, 0.56), (0.23, 0.33, 0.43)))	(((0.57, 0.67, 0.77), (0.05, 0.14, 0.23)), ((0.25, 0.4, 0.55), (0.25, 0.35, 0.45)))	(((0.42, 0.53, 0.63), (0.19, 0.28, 0.37)), ((0.34, 0.46, 0.57), (0.23, 0.33, 0.43)))	(((0.43, 0.54, 0.64), (0.17, 0.26, 0.36)), ((0.22, 0.33, 0.44), (0.35, 0.45, 0.55)))	(((0.28, 0.39, 0.5), (0.29, 0.38, 0.46)), ((0.43, 0.57, 0.7), (0.15, 0.24, 0.33)))

In Step 4, reliability components of the evaluations have been integrated to the restriction component using Definition 5.11. Decomposed triangular fuzzy restriction values integrated with their DF reliabilities are given in Table 5.10.

In this step, to be able to continue with decomposed fuzzy reliabilities, square root of reliability part of the preference A1 with respect to C1 ((((**0.43**, 0.54, 0.65), (**0.17**, 0.26, 0.35)), ((**0.28**, 0.41, 0.54), (**0.25**, 0.35, 0.45))) is calculated as follows:

$$0.43^{\frac{1}{2}} = 0.66,$$

$$1 - (1 - 0.17)^{\frac{1}{2}} = 0.09$$

$$0.28/\left(\frac{1}{2} - \left(\frac{1}{2} - 1\right) * 0.28\right) = 0.43,$$

$$\frac{\left(\frac{1}{2} * 0.25\right)}{\left(\left(\frac{1}{2} - 1\right) * 0.25 + 1\right)} = 0.14$$

Then, fuzzy restrictions are multiplied by square root of fuzzy reliabilities as follows:

$$0.40 * 0.66 = 0.26,$$

$$0.19 + 0.09 - 0.19 * 0.09 = 0.26$$

$$\frac{(0.37 + 0.43 - 2 * 0.37 * 0.43)}{(1 - 0.37 * 0.43)} = 0.57$$

$$\frac{0.18 * 0.14}{0.18 * 0.14 - 0.18 * 0.14} = 0.09$$

Thus, the bold values in Table 5.10 are obtained.

Table 5.10 : Decomposed triangular fuzzy restrictions integrated with their DF reliabilities.

	C1	C2	C3	C4	C5	C6	C7
A1	(((**0.26**, 0.37, 0.5), (**0.26**, 0.39, 0.5)), ((**0.57**, 0.71, 0.8), (**0.09**, 0.14, 0.2)))	(((0.37, 0.49, 0.62), (0.14, 0.27, 0.38)), ((0.6, 0.73, 0.82), (0.07, 0.12, 0.18)))	(((0.12, 0.21, 0.31), (0.46, 0.56, 0.65)), ((0.66, 0.76, 0.84), (0.05, 0.11, 0.16)))	(((0.14, 0.23, 0.33), (0.43, 0.54, 0.64)), ((0.68, 0.78, 0.86), (0.02, 0.08, 0.14)))	(((0.36, 0.48, 0.6), (0.14, 0.27, 0.4)), ((0.62, 0.76, 0.87), (0.07, 0.11, 0.16)))	(((0.23, 0.33, 0.45), (0.3, 0.43, 0.55)), ((0.71, 0.8, 0.87), (0.02, 0.07, 0.13)))	(((0.13, 0.22, 0.32), (0.45, 0.56, 0.66)), ((0.71, 0.81, 0.88), (0.04, 0.08, 0.13)))
A2	(((0.33, 0.44, 0.57), (0.18, 0.31, 0.43)), ((0.61, 0.73, 0.82), (0.07, 0.12, 0.18)))	(((0.18, 0.28, 0.4), (0.36, 0.47, 0.57)), ((0.68, 0.79, 0.87), (0.04, 0.09, 0.14)))	(((0.4, 0.52, 0.65), (0.11, 0.23, 0.35)), ((0.35, 0.63, 0.79), (0.11, 0.15, 0.19)))	(((0.24, 0.34, 0.46), (0.29, 0.42, 0.54)), ((0.56, 0.74, 0.85), (0.07, 0.11, 0.15)))	(((0.17, 0.27, 0.37), (0.39, 0.51, 0.63)), ((0.68, 0.78, 0.85), (0.06, 0.1, 0.15)))	(((0.39, 0.5, 0.63), (0.12, 0.25, 0.37)), ((0.33, 0.54, 0.7), (0.14, 0.19, 0.24)))	(((0.26, 0.36, 0.48), (0.29, 0.41, 0.52)), ((0.7, 0.8, 0.87), (0.04, 0.09, 0.14)))
A3	(((0.21, 0.32, 0.44), (0.3, 0.42, 0.53)), ((0.42, 0.61, 0.74), (0.11, 0.17, 0.23)))	(((0.22, 0.33, 0.44), (0.31, 0.44, 0.56)), ((0.51, 0.67, 0.78), (0.1, 0.15, 0.21)))	(((0.21, 0.33, 0.45), (0.29, 0.41, 0.52)), ((0.34, 0.55, 0.69), (0.15, 0.2, 0.26)))	(((0.29, 0.4, 0.52), (0.21, 0.35, 0.48)), ((0.59, 0.74, 0.85), (0.07, 0.11, 0.17)))	(((0.2, 0.32, 0.43), (0.32, 0.43, 0.53)), ((0.32, 0.55, 0.69), (0.14, 0.19, 0.26)))	(((0.35, 0.47, 0.6), (0.17, 0.29, 0.4)), ((0.53, 0.69, 0.8), (0.09, 0.14, 0.19)))	(((0.34, 0.45, 0.57), (0.16, 0.3, 0.43)), ((0.39, 0.58, 0.7), (0.14, 0.2, 0.26)))
A4	(((0.21, 0.32, 0.43), (0.33, 0.45, 0.57)), ((0.72, 0.82, 0.9), (0.03, 0.07, 0.11)))	(((0.32, 0.44, 0.57), (0.18, 0.31, 0.43)), ((0.61, 0.77, 0.88), (0.06, 0.09, 0.14)))	(((0.24, 0.35, 0.47), (0.27, 0.41, 0.53)), ((0.23, 0.54, 0.75), (0.12, 0.15, 0.2)))	(((0.15, 0.24, 0.35), (0.42, 0.53, 0.64)), ((0.62, 0.74, 0.83), (0.07, 0.12, 0.17)))	(((0.21, 0.32, 0.43), (0.32, 0.44, 0.56)), ((0.69, 0.8, 0.88), (0.04, 0.08, 0.13)))	(((0.34, 0.45, 0.58), (0.17, 0.3, 0.42)), ((0.47, 0.66, 0.79), (0.09, 0.14, 0.2)))	(((0.33, 0.44, 0.56), (0.18, 0.31, 0.44)), ((0.53, 0.69, 0.8), (0.08, 0.13, 0.19)))
A5	(((0.13, 0.21, 0.31), (0.47, 0.59, 0.69)), ((0.6, 0.75, 0.84), (0.07, 0.11, 0.16)))	(((0.29, 0.4, 0.52), (0.22, 0.35, 0.48)), ((0.5, 0.64, 0.75), (0.12, 0.18, 0.24)))	(((0.31, 0.42, 0.54), (0.19, 0.33, 0.46)), ((0.56, 0.72, 0.82), (0.07, 0.12, 0.17)))	(((0.33, 0.44, 0.57), (0.18, 0.31, 0.43)), ((0.38, 0.58, 0.72), (0.13, 0.18, 0.25)))	(((0.09, 0.17, 0.27), (0.51, 0.6, 0.69)), ((0.58, 0.72, 0.82), (0.07, 0.12, 0.17)))	(((0.37, 0.49, 0.61), (0.14, 0.27, 0.39)), ((0.58, 0.7, 0.79), (0.09, 0.14, 0.2)))	(((0.23, 0.34, 0.46), (0.3, 0.42, 0.53)), ((0.64, 0.76, 0.84), (0.07, 0.12, 0.17)))

In Step 5, aggregated decomposed triangular fuzzy restrictions are weighted by criteria weights. In this section, we have asked to DMs for weighing the criteria using Z-fuzzy AHP method given in Tüysüz and Kahraman (2020b). We have obtained the criteria weights as shown in Table 5.11. Then, we have multiplied the values coming from Step 4 with the criteria weights using scalar multiplication operation given in Eq. (5.8) and obtained the weighted decision matrix as given in Table 5.12.

Table 5.11 : Criteria weights.

	Criteria	Weights
C1	Costs	0.213
C2	Proximity to transportation ports	0.149
C3	Traffic Flow	0.077
C4	Proximity to markets, suppliers and other services	0.235
C5	Workforce availability	0.109
C6	Service characteristics	0.072
C7	Environmental conditions	0.145

A calculation example is presented to make clear how the bold values are obtained in Table 5.12.

$$\frac{0.26*0.213}{(0.213-1)*0.26+1} = 0.07, \qquad \frac{0.26}{0.213-(0.213-1)*0.26} = 0.63$$

$$0.57^{0.213} = 0.89, \qquad 1-(1-0.09)^{0.213} = 0.02$$

In step 6, we have defuzzified each $\mathcal{O}$ and $\mathcal{P}$ parameters obtained in Step 5. In Step 7, we have determined PIS_{DF} and NIS_{DF} using Eqs. (5.34-5.37). Defuzzified decomposed fuzzy values and PIS_{DF} and NIS_{DF} of them are shown in Table 5.13.

A calculation example is presented to make clear how the bold values are obtained in Table 5.13.

$$\frac{(0.07+2*0.11+0.17)}{4} = 0.12, \quad \frac{(0.63+2*0.75+0.83)}{4} = 0.74$$

$$\frac{(0.89+2*0.93+0.95)}{4} = 0.92, \quad \frac{(0.02+2*0.03+0.05)}{4} = 0.03$$

In order to define PIS_{DF} and NIS_{DF} values in Table 5.13, we calculate the defuzzified values of decomposed fuzzy numbers in Table 5.12 using Eq. (5.23). Defuzzified value of (((0.07, 0.11, 0.17), (0.63, 0.75, 0.83)), ((0.89, 0.93, 0.95), (0.02, 0.03, 0.05))) in Table 5.12 is calculated as follows:

$$0.5 + \frac{\sqrt{0.07+0.17+0.02+0.05}}{4} - \frac{\sqrt{0.63+0.83+0.89+0.95}}{4} = 0.18$$

Table 5.12 : Weighted decision matrix.

	C1	C2	C3	C4	C5	C6	C7
A1	(((**0.07**, 0.11, 0.17), (**0.63**, 0.75, 0.83)), ((**0.89**, 0.93, 0.95), (**0.02**, 0.03, 0.05)))	(((0.08, 0.17, 0.26), (0.53, 0.71, 0.8)), ((0.93, 0.95, 0.97), (0.01, 0.02, 0.03)))	(((0.01, 0.02, 0.03), (0.92, 0.94, 0.96)), ((0.97, 0.98, 0.99), (0, 0.01, 0.01)))	(((0.04, 0.07, 0.11), (0.76, 0.83, 0.88)), ((0.91, 0.94, 0.97), (0.01, 0.02, 0.03)))	(((0.06, 0.09, 0.14), (0.61, 0.77, 0.86)), ((0.95, 0.97, 0.98), (0.01, 0.01, 0.02)))	(((0.02, 0.03, 0.05), (0.85, 0.91, 0.95)), ((0.98, 0.98, 0.99), (0, 0.01, 0.01)))	(((0.02, 0.04, 0.06), (0.85, 0.9, 0.93)), ((0.95, 0.97, 0.98), (0.01, 0.01, 0.02)))
A2	(((0.09, 0.14, 0.22), (0.51, 0.68, 0.78)), ((0.9, 0.94, 0.96), (0.02, 0.03, 0.04)))	(((0.03, 0.08, 0.12), (0.79, 0.85, 0.9)), ((0.94, 0.97, 0.98), (0.01, 0.01, 0.02)))	(((0.05, 0.08, 0.13), (0.62, 0.8, 0.87)), ((0.92, 0.96, 0.98), (0.01, 0.01, 0.02)))	(((0.07, 0.11, 0.16), (0.63, 0.76, 0.84)), ((0.87, 0.93, 0.96), (0.02, 0.03, 0.04)))	(((0.02, 0.04, 0.06), (0.85, 0.91, 0.94)), ((0.96, 0.97, 0.98), (0.01, 0.01, 0.02)))	(((0.04, 0.07, 0.11), (0.65, 0.82, 0.89)), ((0.92, 0.96, 0.97), (0.01, 0.01, 0.02)))	(((0.05, 0.08, 0.12), (0.74, 0.82, 0.88)), ((0.95, 0.97, 0.98), (0.01, 0.01, 0.02)))
A3	(((0.05, 0.09, 0.14), (0.67, 0.77, 0.84)), ((0.83, 0.9, 0.94), (0.03, 0.04, 0.05)))	(((0.04, 0.09, 0.14), (0.75, 0.84, 0.9)), ((0.91, 0.94, 0.96), (0.02, 0.02, 0.03)))	(((0.02, 0.04, 0.06), (0.84, 0.9, 0.93)), ((0.92, 0.96, 0.97), (0.01, 0.02, 0.02)))	(((0.09, 0.14, 0.2), (0.54, 0.7, 0.8)), ((0.88, 0.93, 0.96), (0.02, 0.03, 0.04)))	(((0.03, 0.05, 0.08), (0.81, 0.87, 0.91)), ((0.88, 0.94, 0.96), (0.02, 0.02, 0.03)))	(((0.04, 0.06, 0.1), (0.74, 0.85, 0.9)), ((0.96, 0.97, 0.98), (0.01, 0.01, 0.01)))	(((0.07, 0.11, 0.16), (0.57, 0.74, 0.84)), ((0.87, 0.92, 0.95), (0.02, 0.03, 0.04)))
A4	(((0.05, 0.09, 0.14), (0.7, 0.8, 0.86)), ((0.93, 0.96, 0.98), (0.01, 0.02, 0.03)))	(((0.07, 0.14, 0.22), (0.6, 0.75, 0.84)), ((0.93, 0.96, 0.98), (0.01, 0.01, 0.02)))	(((0.02, 0.04, 0.06), (0.83, 0.9, 0.94)), ((0.89, 0.95, 0.98), (0.01, 0.01, 0.02)))	(((0.04, 0.07, 0.11), (0.75, 0.83, 0.88)), ((0.89, 0.93, 0.96), (0.02, 0.03, 0.04)))	(((0.03, 0.05, 0.08), (0.81, 0.88, 0.92)), ((0.96, 0.98, 0.99), (0, 0.01, 0.01)))	(((0.03, 0.06, 0.09), (0.73, 0.85, 0.91)), ((0.95, 0.97, 0.98), (0.01, 0.01, 0.02)))	(((0.07, 0.1, 0.15), (0.6, 0.76, 0.85)), ((0.91, 0.95, 0.97), (0.01, 0.02, 0.03)))
A5	(((0.03, 0.05, 0.09), (0.81, 0.87, 0.91)), ((0.9, 0.94, 0.96), (0.01, 0.02, 0.04)))	(((0.06, 0.12, 0.19), (0.65, 0.79, 0.86)), ((0.9, 0.94, 0.96), (0.02, 0.03, 0.04)))	(((0.03, 0.05, 0.08), (0.76, 0.87, 0.92)), ((0.96, 0.97, 0.99), (0.01, 0.01, 0.01)))	(((0.1, 0.16, 0.24), (0.49, 0.66, 0.76)), ((0.8, 0.88, 0.93), (0.03, 0.05, 0.07)))	(((0.01, 0.02, 0.04), (0.9, 0.93, 0.95)), ((0.94, 0.97, 0.98), (0.01, 0.01, 0.02)))	(((0.04, 0.06, 0.1), (0.7, 0.84, 0.9)), ((0.96, 0.98, 0.98), (0.01, 0.01, 0.02)))	(((0.04, 0.07, 0.11), (0.75, 0.83, 0.88)), ((0.94, 0.96, 0.98), (0.01, 0.02, 0.03)))

Table 5.13 : Defuzzified decomposed fuzzy values and their PIS_{DF} and NIS_{DF} values.

	C1-Costs	C2-Proximity to transportation ports	C3-Traffic Flow	C4-Proximity to markets, suppliers and other services	C5-Workforce availability	C6-Service characteristics	C7-Environmental conditions
A1	**((0.12, 0.74), (0.92, 0.03))**	((0.17, 0.69), (0.95, 0.02))	((0.02, 0.94), (0.98, 0.01))	((0.07, 0.83), (0.94, 0.02))	((0.1, 0.75), (0.97, 0.01))	((0.04, 0.91), (0.98, 0.01))	((0.04, 0.89), (0.97, 0.01))
A2	((0.15, 0.66), (0.93, 0.03))	((0.08, 0.85), (0.96, 0.01))	((0.08, 0.77), (0.96, 0.01))	((0.11, 0.75), (0.92, 0.03))	((0.04, 0.9), (0.97, 0.01))	((0.07, 0.80), (0.95, 0.01))	((0.08, 0.82), (0.97, 0.01))
A3	((0.1, 0.76), (0.89, 0.04))	((0.09, 0.83), (0.94, 0.02))	((0.04, 0.89), (0.95, 0.02))	((0.14, 0.68), (0.93, 0.03))	((0.05, 0.87), (0.93, 0.02))	((0.063, 0.84), (0.97, 0.01))	((0.11, 0.72), (0.92, 0.03))
A4	((0.09, 0.79), (0.96, 0.02))	((0.14, 0.73), (0.96, 0.01))	((0.04, 0.89), (0.94, 0.01))	((0.07, 0.82), (0.93, 0.03))	((0.05, 0.87), (0.97, 0.01))	((0.06, 0.84), (0.97, 0.01))	((0.11, 0.74), (0.94, 0.02))
A5	((0.06, 0.86), (0.94, 0.02))	((0.12, 0.77), (0.93, 0.03))	((0.06, 0.85), (0.97, 0.01))	((0.16, 0.64), (0.87, 0.05))	((0.02, 0.93), (0.96, 0.01))	((0.07, 0.82), (0.97, 0.01))	((0.07, 0.82), (0.96, 0.02))
PIS	((0.06, 0.86), (0.94, 0.03))	((0.17, 0.69), (0.95, 0.02))	((0.02, 0.94), (0.98, 0.01))	((0.16, 0.64), (0.87, 0.05))	((0.1, 0.75), (0.97, 0.01))	((0.07, 0.8), (0.95, 0.01))	((0.11, 0.72), (0.92, 0.03))
NIS	((0.15, 0.66), (0.93, 0.03))	((0.08, 0.85), (0.96, 0.01))	((0.08, 0.77), (0.96, 0.01))	((0.07, 0.83), (0.94, 0.02))	((0.02, 0.93), (0.96, 0.01))	((0.04, 0.91), (0.98, 0.01))	((0.04, 0.89), (0.97, 0.01))

A1-Çorlu, A2-Silivri, A3-Sultanbeyli, A4-Osmangazi, A5-Izmit

In Step 8, S^{*} and S^{-} have been determined by Eqs. (5.38-5.39).

In Step 9, the C_i^{*} of the alternatives are calculated using Eq. (5.40). Final ranking results are given in Table 5.14.

Table 5.14 : S^{*}, $S^{-}$and C_i^{*}values of alternatives and rankings.

Alternatives		**S***	**S-**	**C***	Rank
A1	Çorlu	0.093	0.089	0.488	4th
A2	Silivri	0.111	0.048	0.303	5th
A3	Sultanbeyli	0.070	0.088	0.557	2nd
A4	Osmangazi	0.075	0.081	0.518	3rd
A5	Izmit	0.071	0.097	0.579	1st

According to the obtained results, the overall ranking is Izmit>Sultanbeyli>Osmangazi>Çorlu>Silivri. Since the location selection is strategically important for the company and the investment requires large costs and long term risks, the reliabilities of the assigned restriction functions should be seriously taken into account. This analysis is presented in sub-section 5.4.3. We also applied DWAM operator to our problem and obtained a similar ranking A5>A3>A2>A1>A4 whereas it is A5>A3>A4>A1>A2. The first two best alternatives are same.

5.4.3 Utility of reliability component on results

In order to assess the validity of the proposed DF Z-TOPSIS methodology and investigate the importance of reliability information, we apply the decomposed fuzzy TOPSIS (DF-TOPSIS) method to the problem presented in Section 5.4 using decomposed fuzzy linguistics, from which Z-fuzzy terms are removed from DF Z linguistics. Steps of the decomposed fuzzy TOPSIS method are summarized below:

Step 1. Define the criteria and alternatives for the problem. To compare the final results of two methods, the same problem is used.

Step 2. Collect the decision matrices from DMs using the DF linguistic scale given in Table 5.15.

Table 5.15 : DF linguistic terms

Linguistic Terms	μ	ϑ
Absolutely High (AH)	0.8	0.05
Very High (VH)	0.7	0.1
High (H)	0.6	0.2
Medium (M)	0.5	0.3
Low (L)	0.4	0.4
Very Low (VL)	0.3	0.5
Absolutely Low (AL)	0.2	0.55

Step 3. Aggregate the DMs' decision matrix using Eq. (5.18).

Step 4. Weigh the aggregated decision matrix with criteria weights using Eq. (5.8). The same criteria weights are used to make comparison with DF-TOPSIS method.

Step 5. Determine the PIS_{DF} and NIS_{DF} values using Eqs. (5.34-5.37).

Step 6. S^{*} and S^{-} values are calculated using Eqs. (5.38-5.39).

Step 7. Calculate the C_i^* values of the alternatives using Eq. (5.40) and rank the alternatives.

Table 5.16 : Ranking results of DF-TOPSIS method.

Alternatives		S*	S-	C*	Rank
A1	Çorlu	0.162	0.195	0.546	4th
A2	Silivri	0.232	0.069	0.229	5th
A3	Sultanbeyli	0.136	0.186	0.578	3rd
A4	Osmangazi	0.126	0.189	0.600	1st
A5	Izmit	0.136	0.195	0.588	2nd

It can be concluded from the results presented in Table 5.16 that the ranking of alternatives is different from the DF Z-TOPSIS methodology which considers the reliabilities of the judgments. The most significant difference was seen in the best alternative which has been selected by the proposed methodology (DF Z-TOPSIS). Osmangazi has become the best alternative in the approach that does not involve reliability information (DF-TOPSIS). Overall ranking has changed as Osmangazi>Izmit>Sultanbeyli>Çorlu>Silivri.

We can concluded that if the reliability information had not been used, Osmangazi would have been chosen as the best alternative. When the reliability information is considered, the ranking of the alternatives becomes Izmit>Sultanbeyli>Osmangazi>Çorlu>Silivri, which shows that the use of reliability information has a significant effect on the results. The feedback received from the interviews with DMs also indicated the fact that it would be more accurate to find Izmit as the first alternative for the transfer center location compared to Osmangazi. This proves that the use of reliability information has played an important role in finding a more accurate solution to this problem. Therefore, the proposed DF Z-TOPSIS method may be a useful tool in conditions requiring linguistic assessments under uncertainty.

The application and utility analysis of reliability component shows that the proposed DF Z-TOPSIS method has the following advantages:

i. Integration of decomposed fuzzy sets (DFSs) with Z-numbers dominate the decomposed fuzzy sets. It models the descriptive property of linguistic preferences by adding the confidence and unconfidence degrees of assessments under the functional and dysfunctional viewpoints.
ii. Although the notational structure of DF Z-numbers may seem to be complex, they have succesfully been integrated by TOPSIS method. DF Z-numbers can reflect the whole thinking structure of decision makers by taking their judgments and the reliability of them with both positive and negative questions. Therefore, DF Z-TOPSIS methodology can create an active and operative decision environment to represent nature of human words.
iii. Obtaining different results in DF Z-TOPSIS and DF-TOPSIS methods shows that the reliability degrees of the evaluations can affect the managerial decisions. Therefore, it is important to include reliability information in decision structures.
iv. The proposed defuzzification formula for decomposed fuzzy sets can produce reasonable and objective corresponding numbers to achieve final solution. It also offers a simpler calculation structure than the existing score function in the literature.

Even the computational complexity is not low, the reliability and consistency of the experts' judgments compensate it. The effectiveness of the proposed method is quite high when compared with its disadvantages.

5.4.4 Comparative analysis

In this subsection, we compare our method DF Z-TOPSIS with crisp TOPSIS method. DMs' crisp evaluations based on the crisp scala AH: 70, VH: 60, H: 50, M: 40, L: 30, VL: 20 and AL: 10 are aggregated by using arithmetic average. The obtained aggregated matrix is given in Table 5.17.

Table 5.17 : Crisp aggregated decision matrix.

	C1	C2	C3	C4	C5	C6	C7
A1	45.00	62.50	22.50	22.50	57.50	47.50	20.00
A2	57.50	30.00	62.50	40.00	32.50	57.50	42.50
A3	32.50	37.50	37.50	50.00	40.00	55.00	50.00
A4	45.00	60.00	42.50	30.00	42.50	60.00	52.50
A5	22.50	47.50	52.50	55.00	20.00	57.50	47.50

The weighted decision matrix is obtained by using the criteria weights in Table 5.11 and given in Table 5.18. The distances of alternatives to PIS and NIS are given in Table 5.19. Based on the closeness to ideal solutions, C*, the ranking of the alternatives from the best to the worst is A5>A3>A4>A1>A2. Even we lost a lot of information while converting the data from fuzzy case to crisp case, the ranking is the same as our DF Z-TOPSIS method. This does not guarantee that the same ranking can be obtained in any similar conversion.

Table 5.18 : Weighted decision matrix.

	C1	C2	C3	C4	C5	C6	C7
A1	9.59	9.34	1.72	5.28	6.29	3.40	2.91
A2	12.25	4.48	4.78	9.39	3.55	4.12	6.18
A3	6.93	5.60	2.87	11.73	4.37	3.94	7.27
A4	9.59	8.97	3.25	7.04	4.65	4.30	7.63
A5	4.79	7.10	4.02	12.91	2.19	4.12	6.90
PIS	4.79	9.34	1.72	12.91	6.29	4.30	7.63
NIS	12.25	4.48	4.78	5.28	2.19	3.40	2.91

Table 5.19 : Distances to PIS and NIS and ranking of the alternatives.

	S*	S-	C*	Ranking
A1	10.21	7.54	0.425	4
A2	10.52	5.47	0.342	5
A3	5.01	9.95	0.665	2
A4	7.91	7.86	0.498	3
A5	5.26	11.73	0.690	1

A sensitivity analysis can show this possibility by changing some parameters. Table 5.20 shows the changed weights of the criteria. The new set of weights produced different ranking results in DF Z-TOPSIS and crisp TOPSIS methods as seen in Table 5.21.

Table 5.20 : Changed set of criteria weights.

	Criteria	Present weights	Changed weights
C1	Costs	0.213	0.41
C2	Proximity to transportation ports	0.149	0.05
C3	Traffic Flow	0.077	0.18
C4	Proximity to markets, suppliers and other services	0.235	0.03
C5	Workforce availability	0.109	0.21
C6	Service characteristics	0.072	0.07
C7	Environmental conditions	0.145	0.05

Table 5.21 : Comparative rankings.

	Alternatives	Ranking with DF Z-TOPSIS	Ranking with Crisp TOPSIS
A1	Çorlu	1	3
A2	Silivri	5	5
A3	Sultanbeyli	4	1
A4	Osmangazi	2	4
A5	Izmit	3	2

5.5 Conclusion

In this paper, DFSs are processed under the concept of Z-fuzzy numbers, then a novel decision-making method is constructed based on the proposed DF Z-numbers. There are several unique contributions of this paper. First, a new DF Z linguistic scale is developed for the usage in DMs' judgments. Then, a method of conversion from DF Z-numbers to their corresponding DFNs is proposed to the literature. Next, a new defuzzification formula is developed to reach the final crisp values of the DFNs. Finally, a novel DF Z-TOPSIS decision method is developed to solve MCDM problems.

DF Z-TOPSIS method was applied to the cargo transfer center selection problem to explore its practicality. All DMs' evaluations are collected in the form of DF Z-numbers. The ranking of the alternatives by the proposed method showed the usability and validity of the method. Then, the problem was compared with the DF-TOPSIS method and it was tested whether the DF Z-numbers in the proposed method make a significant difference on the results. Two methods gave different rankings of the cargo transfer center alternatives. Its reason is that DF Z-numbers offer more informative and reliable decision structure with the use of reliability component. Therefore, we verify that it is reasonable to integrate Z-fuzzy numbers with decomposed fuzzy sets in order to collect accurate data by asking functional and dysfunctional questions.

This study can guide researchers or practitioners who want to model the linguistic judgments of decision-makers not only from a positive point of view but also by asking negative questions. It fills any information gaps that may arise unilaterally by asking decision makers how unsure they are in their judgments, in addition to how sure they are in their judgments. For future research, DF Z-numbers can be extended to interval-valued DF Z-numbers to define the fuzziness in a broader framework. Since DFSs are quite new in the literature, they can be integrated with other MCDM methods such as VIKOR or ELECTRE in future studies and applied to other types of problems such as supplier selection, renewable energy selection or personnel selection. DF Z-numbers can be also used with these methods to develop new methodologies such as DF Z-AHP&VIKOR or DF Z-CRITIC&ELECTRE, and they can be compared with the results of this paper.

6. AN INTEGRATED PICTURE FUZZY Z-AHP & TOPSIS METHODOLOGY: APPLICATION TO SOLAR PANEL SELECTION[5]

Saaty's (1980) analytic hierarchy process (AHP) is a powerful multi criteria decision-making (MCDM) method and widely used since it has a simple algorithm and has an ability to consider both quantitative and qualitative factors (Tüysüz, 2018; Aliyev et al., 2020). Due to the nature of AHP method, experts need to make several pairwise comparisons to reach final result. It makes sense to develop an AHP method employing fuzzy numbers rather than precise numbers since experts naturally hesitate when doing pairwise comparisons. In order to enhance the pairwise comparisons' ability to describe uncertainty, AHP method is extended to different fuzzy AHP methods such as Pythagorean fuzzy AHP (Ilbahar et al., 2018), neutrosophic AHP (Radwan et al., 2016), hesitant fuzzy AHP (Mousavi et al., 2014; Öztaysi et al., 2015), intuitionistic fuzzy AHP (Sadiq and Tesfamariam, 2009), picture fuzzy AHP (Gündoğdu et al., 2021) and spherical fuzzy AHP (Kutlu Gündoğdu and Kahraman, 2020). TOPSIS method is another well-known MCDM method developed by Yoon and Hwang (1981) for complex decision making problems (Seker and Kahraman, 2022). In order to find better solutions for MCDM problems, several integrated fuzzy AHP and TOPSIS methods are developed using the extensions of fuzzy sets such as intuitionistic fuzzy sets (Karasan et al., 2018; Kahraman et al., 2018), neutrosophic sets (Junaid et al., 2019), Pythagorean fuzzy sets (Yucesan and Gul, 2020; Yildiz et al., 2020; Sarkar and Biswas, 2021; Çalık, 2021), hesitant fuzzy sets (Kumar et al., 2020; Beskese et al., 2020; Ayağ and Samanlioglu, 2021) and spherical fuzzy sets (Mathew et al., 2020; Jaller and Otay, 2020). Three dimensional (3D) fuzzy sets such as picture fuzzy sets, spherical fuzzy sets and neutrosophic sets have advantages in representing fuzziness compared to other fuzzy sets and provide more definition flexibility for membership parameters.

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[5] This chapter is based on the paper "Tüysüz, N., & Kahraman, C. (2023). An Integrated Picture Fuzzy Z-AHP & TOPSIS Methodology: Application to Solar Panel Selection. *Applied Soft Computing*, 110951."

Picture fuzzy sets (PFSs) were introduced by Cuong (2014), which are one of the 3D fuzzy sets that allow us to define situations where there is more than one answer, such as yes, no, rejection, and refusal. PFSs have relatively easier operation rules than other 3D fuzzy sets such as spherical fuzzy sets and t-spherical fuzzy sets (Haktanır and Kahraman, 2022). PFSs have a restriction function in a 1-tuple notational structure that the sum of membership, non-membership, and neutral membership degrees must be at most equal to 1. However, when dealing with real information, the fuzziness expressed by restriction functions is insufficient, and the reliability degree of information is very important (Ku Khalif et al., 2017). Z-numbers introduced by Zadeh (2011) allow the restriction function to be defined by its reliability degree. Z-numbers can model the data affected from different aspects, such as data accuracy degree, changes that may occur due to the concept of time, degree of reliability in subjective judgments (Tüysüz and Kahraman, 2023). Therefore, the concept of Z-numbers is a valuable idea for decision-makers to model this type of information that is not exactly reliable (reliability degree under 100%). A direction on when we should use the Z-numbers is presented in Figure 6.1.

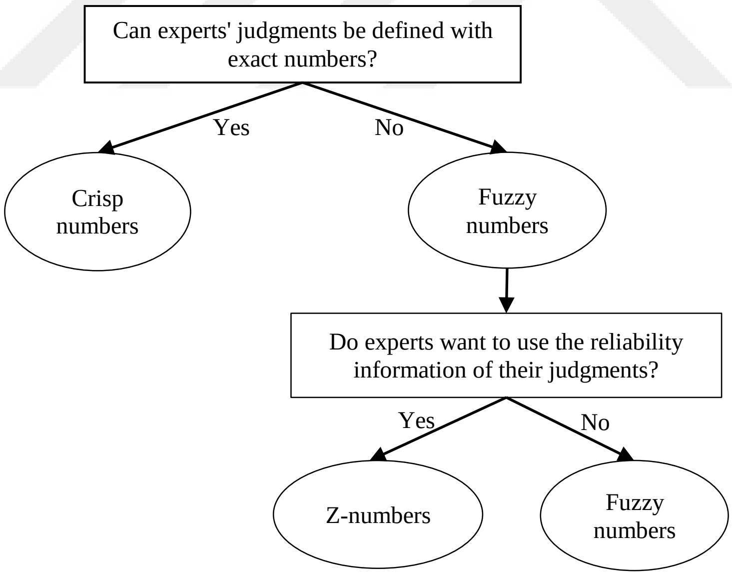

Figure 6.1 : A direction on the usage of Z-numbers

Figure 6.1 illustrates a recommendation for experts to directly use Z-numbers for the representation of reliability as an addition to fuzzy restriction functions. Combining the advantages of PFSs and Z-numbers to handle the uncertainties in decision systems, an integrated AHP&TOPSIS methodology with picture fuzzy Z-information is proposed for the solution of complex fuzzy MCDM problems in this paper.

The proposed methodology is applied to a multi-criteria solar energy panel selection problem involving vague and imprecise judgments of experts. Solar energy is one of the most important renewable energy sources, and there is considerable potential in Turkey. Turkey plans to convert this existing solar energy potential into electricity in the next years. According to Turkey's national energy plan (2022), the installed electricity capacity of 95.9 gigawatts (GW) in 2020 will reach 189.7 GW in 2035. Solar energy resources, which are 6.7 GW in installed power in 2020, will reach 52.9 GW by 2035. It is expected that the majority of the installed power until 2035 will be provided by solar and wind energy. In addition, solar energy will have the largest share of installed power in 2035. Considering the importance of solar energy in Turkey, solar energy investments and the demand for solar panels may increase in the next years and get more attention from researchers and organizations in Turkey. In the literature, studies related to evaluation problems in solar energy have often been conducted in four different areas: solar panel supplier selection (Wang and Tsai, 2018; Cao et al., 2019); solar plant site selection (Thongpun et al., 2017; Al Garni and Awasthi, 2017; Ozdemir and Sahin, 2018; Soydan, 2021; Wang et al., 2023; Almasad et al., 2023; Mian et al., 2023; Hassan et al., 2023; Khan et al., 2023; Hooshangi et al., 2023; Cattani, 2023); evaluation of solar panel selection criteria (Arman and Kundakcı, 2023) and solar energy panel selection (Rani et al., 2020). In this study, we focus on panel selection for solar energy investments, which is expected to attract great interest in Turkey due to its potential in the future. MCDM methods have been used in solar panel selection, such as TOPSIS (Yu, 2013), AHP (Balo and Şağbanşua, 2016), fuzzy AHP and classical TOPSIS (Sasikumar and Ayyappan, 2019), Pythagorean fuzzy SWARA–VIKOR (Rani et al., 2020), and entrophy TOPSIS (Kaur et al., 2023).

The main problem in the selection of solar energy panels is defining the solar panel selection criteria and obtaining the values of the alternatives according to the considered criteria in order to construct the decision matrix. Pairwise comparisons or evaluations are related to how much the experts are sure from their evaluations in addition to their knowledge and experience. Therefore, in this study, a solution proposal for solar panel selection is presented in a structure covering Z-numbers and picture fuzzy sets in an integrated manner. Thus, picture fuzzy sets reflect experts' preferences for the restriction function in a wider perspective while Z-numbers model the reliability degree of these preferences. Accordingly, we propose a novel fuzzy

AHP&TOPSIS methodology using PF Z-numbers to be used for the evaluation of solar energy panel alternatives in Turkey. The main contribution of our study is that it integrates Z-numbers with PFSs in the literature, creating a reliable three-dimensional decision environment and applies it to a real case study on solar panel selection in Turkey.

The rest of this study is organized as follows: Section 6.1 includes a literature review on the methods of PF-AHP, Z-AHP, PF-TOPSIS and Z-TOPSIS. Section 6.2 gives the preliminaries of PFSs, Z-numbers, and the proposed PF Z-numbers. Section 6.3 presents the integrated methodology including the PF Z-AHP and PF Z-TOPSIS methods step by step. Section 6.4 presents an application for the solar energy panel selection problem. Section 6.5 gives a comparative analysis with the PF-TOPSIS and Z-TOPSIS methods. Section 6.6 presents a sensitivity analysis. Finally, the last section summarizes the conclusions drawn from the study.

6.1 Literature Review

The literature is searched for four methods *picture fuzzy AHP (PF-AHP), Z-AHP*, *picture fuzzy TOPSIS (PF-TOPSIS)* and *Z-TOPSIS* given in Tables 6.1, 6.2, 6.3 and 6.4, respectively. In the literature, there are a limited number of PF-AHP methods given in Table 6.1.

Table 6.1 : Literature review on PF-AHP methods.

Year	Authors	Extension of PFSs	Application area
2021	Gündoğdu et al.	Single-valued PFSs	Public transport development problem
2021	Mahmood et al.	Interval-valued PFSs	Numerical example
2022	Ilderomi et al.	Single-valued PFSs	Ranking of flooding risks
2022	Bal and Ucal Sari	Single-valued PFSs	Evaluation of working areas
2023	Meshram et al.	Single-valued PFSs	Prioritization of watersheds
2023	Kaya	Single-valued PFSs	Supplier selection

As a limitation, the AHP papers in Table 6.1 using picture fuzzy sets do not consider the reliability of the assigned degrees of membership, neutral membership and non-membership. It can be drawn from Table 6.1 that more studies can be made for the development of AHP extensions by using triangular or trapezoidal PFSs. Our study both expands the literature on PF-AHP method and guides researchers methodologically with a new integration that includes the reliability information in addition to hesitancy of preferences existing in PFSs. Table 6.2 presents Z-AHP methods in the literature.

Table 6.2 : Literature review on Z-AHP methods.

Year	Authors	Type of fuzzy sets used with Z-numbers	Application area
2013	Azadeh et al.	Ordinary fuzzy sets	Evaluation of universities
2017	Zhang	Ordinary fuzzy sets	Geological risk evaluation
2018	Kahraman and Otay	Ordinary fuzzy sets	Location selection
2018	Karthika and Sudha	Ordinary fuzzy sets	Risk evaluation
2019	Kahraman et al.	Ordinary fuzzy sets	Performance evaluation
2020	Rafiee et al.	Ordinary fuzzy sets	Location selection
2020	Tüysüz and Kahraman	Ordinary fuzzy sets	Prioritization of social sustainable development factors
2020	Bobar et al.	Ordinary fuzzy sets	Evaluation of social media platforms
2021	Liu et al.	Ordinary fuzzy sets	Conceptual design evaluation
2021	Sergi and Ucal Sari	Ordinary fuzzy sets	Ranking of public services' digitalizations
2022	Alkan and Kahraman	Spherical fuzzy sets	Supplier selection
2022	Haktanır and Kahraman	Intuitionistic fuzzy sets	Technology selection
2022	Sari and Tüysüz	Interval type-2 fuzzy sets	Risk evaluation
2022	RezaHoseini et al.	Ordinary fuzzy sets	Performance evaluation
2022	Qendraj et al.	Ordinary fuzzy sets	Evaluation of UTAUT2 model
2023	Tüysüz and Kahraman	Ordinary fuzzy sets	Wind turbine selection

It can be concluded from Table 6.2 that there are many Z-AHP methods in the literature and most of these studies employ ordinary fuzzy Z-numbers. In addition, the usage of the fuzzy sets extensions in the Z-AHP methods has increased in recent years but not at a sufficient level yet. The fuzzy sets extensions such as fermatean fuzzy sets, neutrosophic sets and Pythagorean fuzzy sets have not yet been integrated with Z-AHP method. Each of these extensions handles the uncertainty using different parameters and different membership functions. For instance, while fermatean and Pythagorean fuzzy sets present two parameters for modeling uncertainty, neutrosophic sets presents three parameters for the same purpose. Considering the uncertainty level that decision makers may have in pairwise comparisons in the AHP method, it is important to develop new Z-AHP methods with different fuzzy sets extensions since they may provide different effects on the decision process. Therefore, the proposed PF Z-AHP method is utilized in this study to investigate the effects of reliability information on weighing evaluation criteria for solar energy panel selection problem.

Table 6.3 lists several PF-TOPSIS methods, and almost all studies use a different PFS such as bipolar, interval-valued and single-valued during their development. Although there are relatively more applications of PF-TOPSIS than PF-AHP in the literature, there are still opportunities to obtain extensions of the TOPSIS method with triangular or trapezoidal or LR-type picture fuzzy numbers. It can be also concluded from Table

6.3 that PF-TOPSIS method has not yet been extended with Z-numbers to PF Z-TOPSIS method, which makes this study unique in developing a 3D PF-TOPSIS method with reliability parameter.

Table 6.3 : Literature review on PF-TOPSIS methods.

Year	Authors	Extension of PFSs	Application area
2019	Zeng et al.	Linguistic PFSs	ERP system selection
2021a	Jin et al.	Covering-based PF rough sets	Risk evaluation
2021	Sindhu et al.	Bipolar PFSs	Recruitment evaluation
2022	Dhumras and Bajaj	Bi-parametric PFSs	Selection of hydrogen fuel cell technology
2022a	Kahraman et al.	Interval-valued PFSs	Evaluation of cloud service providers
2022	Bobin et al.	Interval-valued picture fuzzy hypersoft sets	Employee evaluation
2022b	Kahraman et al.	Interval-valued PFSs	Supplier selection
2023	Alkan and Kahraman	Interval-valued PFSs	Wind turbine selection
2023	Sun et al.	Single-valued PFSs	Selection of product design

Table 6.4 : Literature review on Z-TOPSIS methods.

Year	Authors	Type of fuzzy sets used with Z-numbers	Application area
2015	Yaakob and Gegov	Ordinary fuzzy sets	Stock selection
2019	Xian et al.	Intuitionistic fuzzy sets	Investment decision, medical diagnosis
2021	Yaakob et al.	Hesitant fuzzy sets	Stock selection
2022	Sari and Tüysüz	Interval type-2 fuzzy sets	Risk evaluation
2022	Haktanır and Kahraman	Intuitionistic fuzzy sets	Technology selection

Table 6.4 presents Z-TOPSIS methods in the literature. It is clearly seen that there are few Z-TOPSIS methods, which presents an open research area to propose new extensions of Z-TOPSIS methods. For instance, there is no study using spherical fuzzy sets, fermatean fuzzy sets or picture fuzzy sets together with Z-numbers in extending TOPSIS method. Especially, 3D fuzzy sets may relatively better represent human judgments with respect to fuzzy sets that are using one or two parameters. Usage of 3D fuzzy sets with Z-numbers may provide 3D reliable solutions for TOPSIS method to rank the alternatives according to the determined criteria. Therefore, in this study, a new fuzzy TOPSIS method using reliability information is proposed by the integration of Z-numbers and PFSs.

6.2 Preliminaries

In this section, we present some basic definitions on picture fuzzy sets (PFSs) and Z-numbers by Section 6.2.1 and Section 6.2.2, respectively.

6.2.1 Picture fuzzy sets

Definition 6.1. A PFS on a $\tilde{A}_P$ of the universe of discourse X is defined by $\tilde{A}_P = \{x, \mu_{\tilde{A}_P}(x), \eta_{\tilde{A}_P}(x), \nu_{\tilde{A}_P}(x) \mid x \in X\}$, where $\mu_{\tilde{A}_P}(x): X \to [0,1]$, $\eta_{\tilde{A}_P}(x): X \to [0,1]$ and $\nu_{\tilde{A}_P}(x): X \to [0,1]$ are the membership degree, neutral membership degree, and non-membership degree of x to $\tilde{A}_P$, respectively. All parameters must satisfy the condition $0 \leq \mu_{\tilde{A}_P}(x) + \eta_{\tilde{A}_P}(x) + \nu_{\tilde{A}_P}(x) \leq 1$. Then, $1 - \left(\mu_{\tilde{A}_P}(x) + \eta_{\tilde{A}_P}(x) + \nu_{\tilde{A}_P}(x)\right)$ is the refusal degree of x in X (Cuong, 2014; Cuong and Kreinovich, 2014).

Definition 6.2. The addition, fuzzy multiplication, scalar multiplication and exponentiation operations of PFSs are presented by Eqs. (6.1-6.4) (Wei, 2017; Wei et al., 2018).

$$\tilde{A}_P \oplus \tilde{B}_P = \{\mu_{\tilde{A}_P} + \mu_{\tilde{B}_P} - \mu_{\tilde{A}_P}\mu_{\tilde{B}_P}, \eta_{\tilde{A}_P}\eta_{\tilde{B}_P}, \nu_{\tilde{A}_P}\nu_{\tilde{B}_P}\} \tag{6.1}$$

$$\tilde{A}_s \otimes \tilde{B}_s = \{\mu_{\tilde{A}_P}\mu_{\tilde{B}_P}, \ \eta_{\tilde{A}_P} + \eta_{\tilde{B}_P} - \eta_{\tilde{A}_P}\eta_{\tilde{B}_P}, \nu_{\tilde{A}_P} + \nu_{\tilde{B}_P} - \nu_{\tilde{A}_P}\nu_{\tilde{B}_P}\} \tag{6.2}$$

$$k\tilde{A}_P = \left\{1 - \left(1 - \mu_{\tilde{A}_P}\right)^k, \eta_{\tilde{A}_P}^k, \nu_{\tilde{A}_P}^k\right\}; k > 0 \tag{6.3}$$

$$\tilde{A}_P^k = \left\{\mu_{\tilde{A}_P}^k, 1 - \left(1 - \eta_{\tilde{A}_P}\right)^k, 1 - \left(1 - \nu_{\tilde{A}_P}\right)^k\right\}; k > 0 \tag{6.4}$$

Definition 6.3. Let $\tilde{A}_P = (\mu_{\tilde{A}_P}, \eta_{\tilde{A}_P}, \nu_{\tilde{A}_P})$ be a picture fuzzy number (PFN). In order to calculate the corresponding crisp value of $\tilde{A}_P$, a modified defuzzification formula is presented by Eq. (6.5) (Son et al., 2017; Xu et al., 2019).

$$Def(\tilde{A}_P) = \mu_{\tilde{A}_P} + \frac{\eta_{\tilde{A}_P}}{2} - \delta * (\frac{\nu_{\tilde{A}_P}}{2}) \tag{6.5}$$

where δ is the net membership coefficient, $0 \leq \delta \leq 1$.

Definition 6.4. $w_1\tilde{A}_{P1} \oplus w_2\tilde{A}_{P2} \oplus \ldots \oplus w_n\tilde{A}_{Pn}$ be a collection of PFNs, where $w = (w_1, w_2, \ldots, w_n)$ is the weight vector; $w_j \in [0,1]$; $\sum_{j=1}^{n} w_j = 1$. Then, Picture Fuzzy Weighted Arithmetic Mean ($PFWAM$) operator is defined by Eq. (6.6).

$$PFWAM_w\left(\tilde{A}_{P1}, \tilde{A}_{P2}, \ldots, \tilde{A}_{Pn}\right) = w_1\tilde{A}_{P1} \oplus w_2\tilde{A}_{P2} \oplus \ldots \oplus w_n\tilde{A}_{Pn}$$

$$= \left\{1 - \prod_{j=1}^{n}\left(1 - \mu_{\tilde{A}_P}\right)^{w_j}, \prod_{j=1}^{n}\left(\eta_{\tilde{A}_P}\right)^{w_j}, \prod_{j=1}^{n}\left(\nu_{\tilde{A}_P}\right)^{w_j}\right\}$$

$$j = 1,2, \ldots, n \tag{6.6}$$

Definition 6.5. $\tilde{A}_{P1}^{w_1} \otimes \tilde{A}_{P2}^{w_2} \otimes \ldots \otimes \tilde{A}_{Pn}^{w_j}$ be a collection of PFNs, where $w = (w_1, w_2, \ldots, w_n)$ is the weight vector; $w_j \in [0,1]$; $\sum_{j=1}^{n} w_j = 1$. Then, Picture Fuzzy Weighted Geometric Mean ($PFWGM$) operator is defined by Eq. (6.7).

$$PFWGM_w(\tilde{A}_{P1}, \tilde{A}_{P2}, \ldots, \tilde{A}_{Pn}) = (\tilde{A}_{P1}^{w_1} \otimes \tilde{A}_{P2}^{w_2} \otimes \ldots \otimes \tilde{A}_{Pn}^{w_j})$$

$$= \{\prod_{j=1}^{n}(\mu_{\tilde{A}_P})^{w_j}, 1 - \prod_{j=1}^{n}(1 - \eta_{\tilde{A}_P})^{w_j}, 1 - \prod_{j=1}^{n}(1 - \nu_{\tilde{A}_P})^{w_j}\}$$

$$j = 1,2, \ldots, n \tag{6.7}$$

6.2.2 Z-numbers

Z-numbers $Z(\tilde{A}, \tilde{R})$ are introduced by Zadeh (2011) for representing both fuzzy restriction function ($\tilde{A}$) and the fuzzy reliability function ($\tilde{R}$) in defining the membership of an element to a set. Figure 6.2 shows a simple triangular fuzzy Z-number.

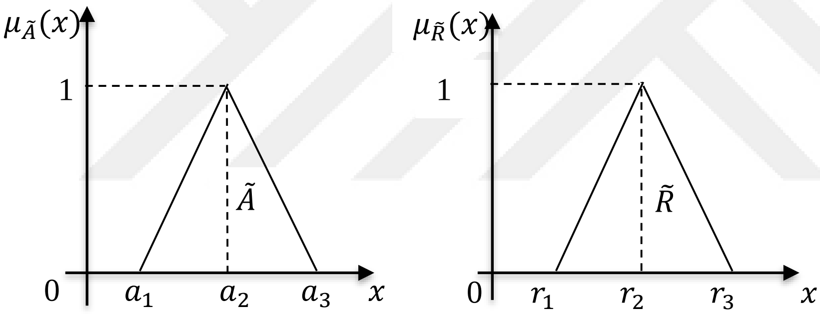

Figure 6.2 : A simple triangular fuzzy Z-number

Z-numbers allow us to create an adequate decision process when handling imprecise and vague information with its generalized structure (Aliev and Zeinalova, 2014). Detailed information on Z-numbers can be found in Tüysüz and Kahraman (2023).

6.2.3 Picture fuzzy Z-number

Picture fuzzy Z-number $\tilde{Z}_{PF}(\tilde{A}, \tilde{R})$ is a 2-tuple fuzzy number consisting of its restriction and reliability functions. Let $\tilde{A} = \{x, \mu_{\tilde{A}_P}(x), \eta_{\tilde{A}_P}(x), \nu_{\tilde{A}_P}(x) \mid x \in X\}$ is a picture fuzzy restriction function and $\tilde{R} = \{x, \mu_{\tilde{R}_P}(x), \eta_{\tilde{R}_P}(x), \nu_{\tilde{R}_P}(x) \mid x \in X\}$ is a picture fuzzy reliability function. A picture fuzzy Z-number is given by Eq. (6.8).

$$\tilde{Z}_{PF}(\tilde{A}, \tilde{R}) = \Big(\big(\mu_{\tilde{A}_P}(x), \eta_{\tilde{A}_P}(x), \nu_{\tilde{A}_P}(x)\big), \big(\mu_{\tilde{R}_P}(x), \eta_{\tilde{R}_P}(x), \nu_{\tilde{R}_P}(x)\big)\Big) \tag{6.8}$$

where $\mu_{\tilde{R}_P}(x), \eta_{\tilde{R}_P}(x), \nu_{\tilde{R}_P}(x): X \rightarrow [0,1]$ belong to the reliability function satisfying that $0 \leq \mu_{\tilde{R}_P}(x) + \eta_{\tilde{R}_P}(x) + \nu_{\tilde{R}_P}(x) \leq 1$.

Definition 6.6. Conversion from a PF Z-number to a PF number

Consider a PF Z-number $\tilde{Z}_{PF}(\widetilde{A}, \tilde{R})$, which integrates PF restriction and PF reliability functions. To obtain a PF number ($\tilde{Z}'_{PF}$) from PF Z-number, the following steps are applied.

(i) The reliability function ($\tilde{R}$) is converted to its corresponding crisp value (α) using Eq. (6.9).

$$Def_{\tilde{R}} = \alpha = 0.95 - 0.5 * \nu_{\tilde{R}_P} \tag{6.9}$$

(ii) PF restriction function is multiplied by the square root of defuzzified reliability function ($\sqrt{\alpha}$) as in Eq. (6.10).

$$\tilde{Z}'_{PF} = \left\{ \langle x; \mu_{\tilde{Z}'_{PF}}(x), \eta_{\tilde{Z}'_{PF}}(x), \nu_{\tilde{Z}'_{PF}}(x) \rangle \middle| \begin{pmatrix} 1 - \left(1 - \mu_{\tilde{A}}(x)\right)^{\sqrt{\alpha}}, \\ \eta_{\tilde{A}}(x)^{\sqrt{\alpha}}, \nu_{\tilde{A}}(x)^{\sqrt{\alpha}} \end{pmatrix}; \ \mu(x), \eta(x), \nu(x) \in [0,1] \right\} \tag{6.10}$$

(iii) Alternatively, to continue operations with PF reliability, $\tilde{R}$, PF restriction function is multiplied by the square root of PF reliability function ($\sqrt{\tilde{R}}$) as in Eq. (6.11).

$$\tilde{Z}'_{PF} = \left\{ \langle x; \mu_{\tilde{Z}'_{PF}}(x), \eta_{\tilde{Z}'_{PF}}(x), \nu_{\tilde{Z}'_{PF}}(x) \rangle \middle| \begin{pmatrix} \mu_{\tilde{A}_P}(x)\mu^{\frac{1}{2}}_{\tilde{R}_P}(x), \\ \eta_{\tilde{A}_P}(x) + \left(1 - \left(1 - \eta_{\tilde{R}_P}(x)\right)^{\frac{1}{2}}\right) \\ -\eta_{\tilde{A}_P}(x)\left(1 - \left(1 - \eta_{\tilde{R}_P}(x)\right)^{\frac{1}{2}}\right), \\ \nu_{\tilde{A}_P}(x) + \left(1 - \left(1 - \nu_{\tilde{R}_P}(x)\right)^{\frac{1}{2}}\right) \\ -\nu_{\tilde{A}_P}(x)\left(1 - \left(1 - \nu_{\tilde{R}_P}(x)\right)^{\frac{1}{2}}\right) \end{pmatrix}; \ \mu(x), \eta(x), \nu(x) \in [0,1] \right\} \tag{6.11}$$

6.3 Proposed Methodology: Picture Fuzzy Z-AHP&TOPSIS

In this section, the AHP method is extended by picture fuzzy Z-numbers for weighing the evaluation criteria and the TOPSIS method is extended by picture fuzzy Z-numbers for prioritizing the considered alternatives. The flowchart of this methodology is presented in Figure 6.3. Picture fuzzy Z-AHP method and Picture fuzzy Z-TOPSIS method are presented step by step in the following sub-sections.

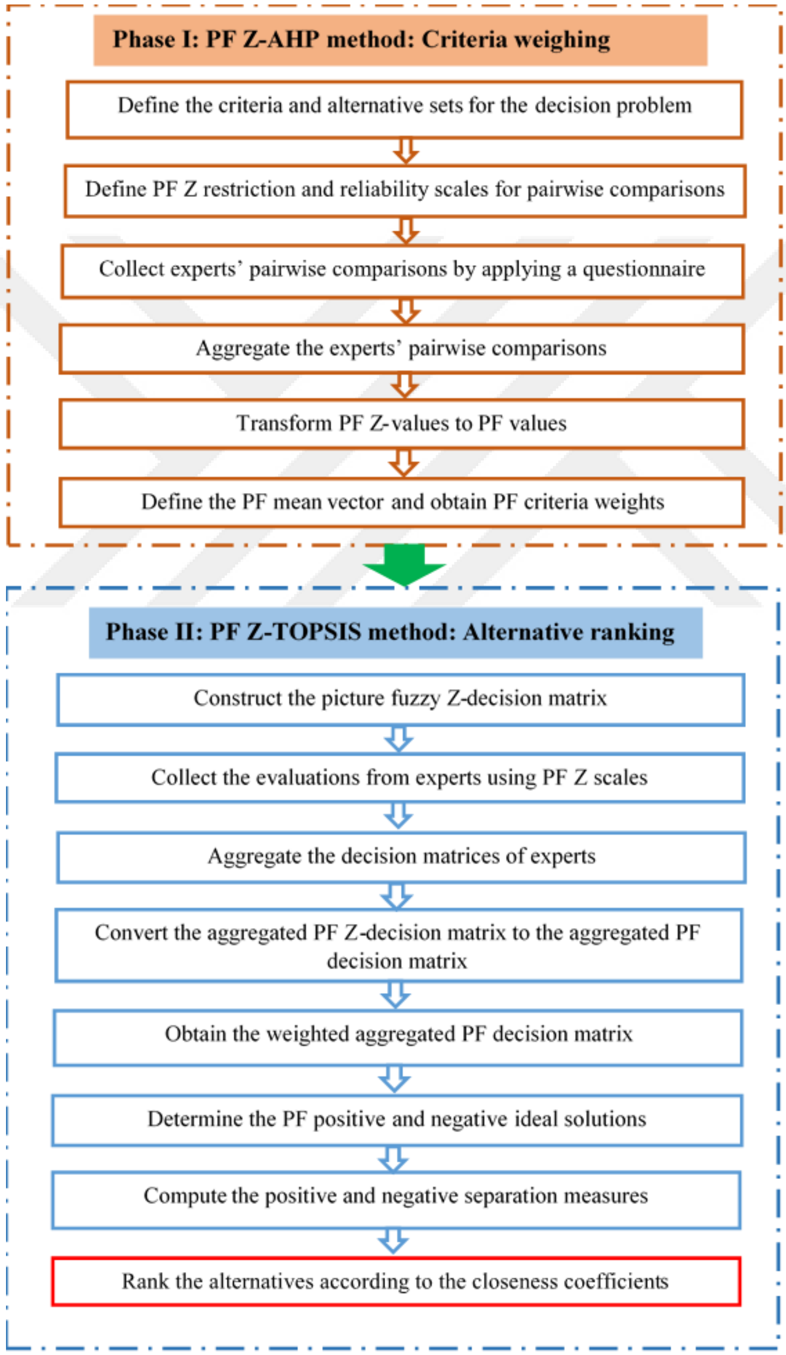

Figure 6.3 : Flowchart of the proposed PF Z-AHP&TOPSIS methodology.

6.3.1 Phase I: picture fuzzy Z-AHP method

In this section, we present the steps of the proposed Picture Fuzzy Z-AHP (PF Z-AHP) method in multi-expert decision environment.

Step 1. Determine the main-criteria (j=1,2,…,m), sub-criteria and alternative (i=1,2,…,n) sets of the decision problem.

Step 2. Define PF Z restriction and reliability scales for pairwise comparisons.

Table 6.5 : PF Z restriction and reliability scales for pairwise comparisons.

Linguistic Terms for Restriction	**Linguistic Terms for Reliability**	$(\boldsymbol{\mu}, \boldsymbol{\eta}, \boldsymbol{\nu})$
Absolutely High Importance (AHI)	Absolutely High Reliable (AHR)	(0.80, 0.05, 0.00)
Very High Importance (VHI)	Very High Reliable (VHR)	(0.70, 0.10, 0.10)
High Importance (HI)	High Reliable (HR)	(0.60, 0.15, 0.20)
Slightly High Importance (SHI)	Slightly High Reliable (SHR)	(0.50, 0.20, 0.30)
Equally Importance (EI)	Medium Reliable (MR)	(0.40, 0.20, 0.40)
Slightly Low Importance (SLI)	Slightly Low Reliable (SLR)	(0.30, 0.20, 0.50)
Low Importance (LI)	Low Reliable (LR)	(0.20, 0.15, 0.60)
Very Low Importance (VLI)	Very Low Reliable (VLR)	(0.10, 0.10, 0.70)
Absolutely Low Importance (ALI)	Absolutely Low Reliable (ALR)	(0.00, 0.05, 0.80)

Step 3. Collect experts' pairwise comparisons by applying a questionnaire using linguistic terms given in Table 6.5. Each of the three experts (k=1,2,3) presents his/her own pairwise comparison matrix as in Eqs. (6.12-6.13).

$$\tilde{Z}_k = \left[\left(\tilde{A}, \tilde{R}\right)_{ij}\right]_{n\times n} \quad \text{k=1,2,3.} \tag{6.12}$$

$$\tilde{Z}_k = \begin{bmatrix} \left(\tilde{A}, \tilde{R}\right)_{11} & \left(\tilde{A}, \tilde{R}\right)_{12} & \cdots & \left(\tilde{A}, \tilde{R}\right)_{1n} \\ \left(\tilde{A}, \tilde{R}\right)_{21} & \left(\tilde{A}, \tilde{R}\right)_{22} & \cdots & \left(\tilde{A}, \tilde{R}\right)_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \left(\tilde{A}, \tilde{R}\right)_{i1} & \left(\tilde{A}, \tilde{R}\right)_{i2} & \left(\tilde{A}, \tilde{R}\right)_{ij} & \left(\tilde{A}, \tilde{R}\right)_{in} \\ \vdots & \vdots & \ddots & \vdots \\ \left(\tilde{A}, \tilde{R}\right)_{n1} & \left(\tilde{A}, \tilde{R}\right)_{n2} & \cdots & \left(\tilde{A}, \tilde{R}\right)_{nn} \end{bmatrix}$$

i=1,2,…,n, j=1,2,…,n (6.13)

Step 4. Check the consistency ratio (CR) for each pairwise comparison matrix based on Saaty's traditional consistency calculation procedure using the corresponding crisp values of linguistic terms for restriction functions. Corresponding crisp values of PF restriction values can be calculated by Eq. (6.14). If CR>0.10, then a reevaluation of the pairwise comparisons is required.

$$Def_{\tilde{A}} = \begin{cases} 11 * \mu_{\tilde{A}_P} + 4 * \eta_{\tilde{A}_P} - 11 * \nu_{\tilde{A}_P} & , \ for \text{ AHI, VHI, HI, SHI, EI.} \\ \left| \dfrac{1}{11*\nu_{\tilde{A}_P} + 4*\eta_{\tilde{A}_P} - 11*\mu_{\tilde{A}_P}} \right| & , \ for \text{ SLI, LI, VLI, ALI.} \end{cases} \tag{6.14}$$

If the pairwise comparison matrices are consistent, they are aggregated using $PFWAM$ or $PFWGM$ operator given by Eq. (6.15) and Eq. (6.16), respectively.

$$PFWAM(\tilde{A}_{ij}) =$$
$$\left\{1 - \prod_{k=1}^{K}\left(1 - \mu_{\tilde{A}_{ij}}\right)^{w_k}, \prod_{k=1}^{K}\left(\eta_{\tilde{A}_{ij}}\right)^{w_k}, \prod_{k=1}^{K}\left(\nu_{\tilde{A}_{ij}}\right)^{w_k}\right\} \tag{6.15}$$

$$PFWGM(\tilde{A}_{ij}) =$$
$$\left\{\prod_{k=1}^{K}\left(\mu_{\tilde{A}_{ij}}\right)^{w_k}, 1 - \prod_{k=1}^{K}\left(1 - \eta_{\tilde{A}_{ij}}\right)^{w_k}, 1 - \prod_{k=1}^{K}\left(1 - \nu_{\tilde{A}_{ij}}\right)^{w_k}\right\} \tag{6.16}$$

where w_k is the weight of the k^{th} expert, and $w_k \in [0,1]$; $\sum_{k=1}^{K} w_k = 1$.

Eqs. (6.15) and (6.16) are used for both the restriction function $\tilde{A}_{ij}$ and the reliability function $\tilde{R}_{ij}$.

Step 5. Multiply PF restriction functions with the square root of PF reliability functions as given in Eq. (6.11). Thus, the aggregated PF Z-matrix is converted to a PF aggregated matrix.

Alternatively, the corresponding values of PF reliability functions ($\tilde{R}_{ij}$) are calculated by Eq. (6.9). Then, each restriction value is multiplied by square root of defuzzified PF reliability values ($def\tilde{R}_{ij}$) using Eq. (6.17). Thus, PF Z-values are transformed to PF values.

$$Z' = [c_{ij}]_{n\times n}$$
$$= \begin{bmatrix} \left(\tilde{A}_{11} \times \sqrt{def\tilde{R}_{11}}\right) & \left(\tilde{A}_{12} \times \sqrt{def\tilde{R}_{12}}\right) & \cdots & \left(\tilde{A}_{1n} \times \sqrt{def\tilde{R}_{1n}}\right) \\ \left(\tilde{A}_{21} \times \sqrt{def\tilde{R}_{12}}\right) & \left(\tilde{A}_{22} \times \sqrt{def\tilde{R}_{22}}\right) & \cdots & \left(\tilde{A}_{2n} \times \sqrt{def\tilde{R}_{2n}}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\tilde{A}_{i1} \times \sqrt{def\tilde{R}_{1i}}\right) & \left(\tilde{A}_{i2} \times \sqrt{def\tilde{R}_{2i}}\right) & \left(\tilde{A}_{ij} \times \sqrt{def\tilde{R}_{ij}}\right) & \left(\tilde{A}_{in} \times \sqrt{def\tilde{R}_{in}}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\tilde{A}_{n1} \times \sqrt{def\tilde{R}_{1n}}\right) & \left(\tilde{A}_{n2} \times \sqrt{def\tilde{R}_{2n}}\right) & \cdots & \left(\tilde{A}_{nn} \times \sqrt{def\tilde{R}_{nn}}\right) \end{bmatrix}$$

(6.17)

Step 6. Define the PF mean $\left(\widetilde{PFM}\right)$ vector using $PFWAM$ or $PFWGM$ operator given by Eqs. (6.18-6.19), respectively. Thus, $[c_{ij}]_{n\times n}$ matrix is transformed to $[m_{ij}]_{n\times 1}$ vector given by Eq. (6.20).

$$PFWAM(\widetilde{m}_{i1}) =$$

$$\left\{1 - \prod_{j=1}^{n}\left(1 - \mu_{\widetilde{m}_{ij}}\right)^{w_j}, \prod_{j=1}^{n}\left(\eta_{\widetilde{m}_{ij}}\right)^{w_j}, \prod_{j=1}^{n}\left(\nu_{\widetilde{m}_{ij}}\right)^{w_j}\right\} \tag{6.18}$$

$$PFWGM(\widetilde{m}_{i1}) =$$

$$\left\{\prod_{j=1}^{n}\left(\mu_{\widetilde{m}_{ij}}\right)^{w_j}, 1 - \prod_{j=1}^{n}\left(1 - \eta_{\widetilde{m}_{ij}}\right)^{w_j}, 1 - \prod_{j=1}^{n}\left(1 - \nu_{\widetilde{m}_{ij}}\right)^{w_j}\right\} \tag{6.19}$$

$$\widetilde{PFM} = \begin{bmatrix} \widetilde{m}_{11} \\ \widetilde{m}_{21} \\ \vdots \\ \widetilde{m}_{i1} \end{bmatrix}_{n\times 1} \tag{6.20}$$

where $\widetilde{m}_{ij} = (\mu_{\widetilde{m}_{ij}}, \eta_{\widetilde{m}_{ij}}, \nu_{\widetilde{m}_{ij}})$.

At the end of this step, we have PF criteria weights. To calculate crisp criteria weights, each PF values of criteria can be converted to corresponding crisp value using Eq. (6.14).

6.3.2 Phase II: picture fuzzy Z-TOPSIS method

Step 7. Construct the PF Z-decision matrix ($\widetilde{D}_{PFZ} = (\tilde{d}_{ij})_{n\times m}$) including criteria set $C = \{C_1, C_2, C_3, \dots, C_m\}$ and alternatives set $A = \{A_1, A_2, A_3, \dots A_n\}$ as in Eq. (6.21).

$$\widetilde{D}_{PFZ} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \dots & C_m \end{array} \\ \begin{bmatrix} \tilde{d}_{11} & \tilde{d}_{12} & \dots & \tilde{d}_{1m} \\ \tilde{d}_{21} & \tilde{d}_{22} & \dots & \tilde{d}_{2m} \\ \vdots & \vdots & \tilde{d}_{ij} & \vdots \\ \tilde{d}_{n1} & \tilde{d}_{n2} & \dots & \tilde{d}_{nm} \end{bmatrix} \end{array} \tag{6.21}$$

where $i = 1,2,3 \dots, n,\ j = 1,2,3 \dots, m$ and;

$\tilde{d}_{ij}$ is the experts' PF Z-evaluations and given by Eq. (6.22).

$$\tilde{d}_{ij} = (\tilde{A}, \tilde{R}) = \left(\left(\mu_{\tilde{A}}^{E_k}(x_{ij}), \eta_{\tilde{A}}^{E_k}(x_{ij}), \nu_{\tilde{A}}^{E_k}(x_{ij})\right), \left(\mu_{\tilde{R}}^{E_k}(x_{ij}), \eta_{\tilde{R}}^{E_k}(x_{ij}), \nu_{\tilde{R}}^{E_k}(x_{ij})\right)\right) \tag{6.22}$$

Step 8. Collect the evaluations from *k* experts using the PF Z-scales presented in Table 6.5. For this purpose, a questionnaire consisting of appropriate questions are applied to each expert.

Step 9. Aggregate the decision matrices $\widetilde{D}_{PFZ}$s of *k* experts using PFWAM or PFWGM operators given in Eqs. (6.15) and (6.16), respectively. Then, aggregated $\widetilde{D}_{PFZ}$ ($\widetilde{D}_{PFZ}^{Agg}$) is obtained using Eqs. (6.23-6.24).

$$\widetilde{D}_{PFZ}^{Agg} = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \dots & \tilde{a}_{1m} \\ \tilde{a}_{21} & \tilde{a}_{22} & \dots & \tilde{a}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{i1} & \tilde{a}_{i2} & \tilde{a}_{ij} & \tilde{a}_{im} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \dots & \tilde{a}_{nm} \end{bmatrix} \tag{6.23}$$

$$\tilde{a}_{ij} = (\tilde{A}, \tilde{R}) = \left(PFWAM_w\left(\tilde{A}_1^{\lambda_1}, \tilde{A}_2^{\lambda_2}, \dots, \tilde{A}_K^{\lambda_K}\right), PFWAM_w\left(\tilde{R}_1^{\lambda_1}, \tilde{R}_2^{\lambda_2}, \dots, \tilde{R}_K^{\lambda_K}\right)\right) \tag{6.24}$$

where λ_k is the weight of expert k, $\lambda_k = (\lambda_1, \lambda_2 \dots\dots, \lambda_K); \lambda_k \in [0,1]$ and $\sum_{k=1}^{K} \lambda_k = 1$.

Step 10. Convert the aggregated PF Z-decision matrix to the aggregated PF decision matrix using Definition (6.6) and thus obtain $\widetilde{D}_{PF}^{Agg}$.

Step 11. Multiply $\widetilde{D}_{PF}^{Agg}$ by PF criteria weights $(\widetilde{W})$ using Eq. (6.2). Thus, the weighted aggregated PF decision matrix $\widetilde{D}_{PF_W}^{Agg}$ is obtained using Eqs. (6.2) and (6.25).

$$\widetilde{D}_{PF_W}^{Agg} = \widetilde{D}_{PF_{ij}}^{Agg} * \widetilde{w}_j \tag{6.25}$$

where $\widetilde{w}_j = \{\langle x; \mu_A(x), \eta_A(x), \nu_A(x)\rangle | x_j \in X\}$ and $\widetilde{W} = \{\widetilde{w}_1, \widetilde{w}_2, \widetilde{w}_3, \dots, \widetilde{w}_j\}$ is the weight vector, $0 \le \widetilde{w}_j \le 1$ and $\sum_{j=1}^{m} \widetilde{w}_j = 1$.

Step 12. Calculate the corresponding values of the $\widetilde{D}_{DF_W}^{Agg}$ for each alternative using defuzzification formula given by Eq. (6.14). Thus, $Def\left(\widetilde{D}_{PF_W}^{Agg}\right)$ matrix is obtained.

Step 13. Determine the PF positive ideal solution (PIS_{PF}) and PF negative ideal solution (NIS_{PF}) using the defuzzified values of $\widetilde{D}_{PF_W}^{Agg}$ $\left(Def\left(\widetilde{D}_{PF_W}^{Agg}\right)\right)$ and Eqs. (6.26-6.29). PIS_{PF} and NIS_{PF} values are determined according to the *maximum* and *minimum* values of the $Def\left(\widetilde{D}_{PF_W}^{Agg}\right)$, respectively.

$$PIS_{PF} = p_j = \left(\mu_{\tilde{A}_P}^*(x_j), \eta_{\tilde{A}_P}^*(x_j), \nu_{\tilde{A}_P}^*(x_j)\right) \tag{6.26}$$

$$NIS_{PF} = n_j = \left(\mu_{\tilde{A}_P}^-(x_j), \eta_{\tilde{A}_P}^-(x_j), \nu_{\tilde{A}_P}^-(x_j)\right) \tag{6.27}$$

where

$$p_j = \left\{\max_{1 \le i \le n}\left(\left(\mu_{\tilde{A}_P}(x_{ij}), \eta_{\tilde{A}_P}(x_{ij}), \nu_{\tilde{A}_P}(x_{ij})\right) \middle| C_j \in C_B\right)\right\}$$

$$\left\{\min_{1 \le i \le n}\left(\left(\mu_{\tilde{A}_P}(x_{ij}), \eta_{\tilde{A}_P}(x_{ij}), \nu_{\tilde{A}_P}(x_{ij})\right) \middle| C_j \in C_C\right)\right\} \tag{6.28}$$

$$n_j = \left\{ \min_{1\le i\le n} \left(\left(\mu_{\tilde{A}_P}(x_{ij}), \eta_{\tilde{A}_P}(x_{ij}), \nu_{\tilde{A}_P}(x_{ij}) \right) \middle| C_j \in C_B \right) \right\},$$

$$\left\{ \max_{1\le i\le n} \left(\left(\mu_{\tilde{A}_P}(x_{ij}), \eta_{\tilde{A}_P}(x_{ij}), \nu_{\tilde{A}_P}(x_{ij}) \right) \middle| C_j \in C_C \right) \right\} \tag{6.29}$$

where C_B and C_C represent the benefit and cost criteria, respectively.

Step 14. Determine the positive and negative separation measures (S^* *and* S^-) using Eqs. (6.30) and (6.31), respectively.

$$S_i^* = \sqrt{\frac{1}{2m}\sum_{j=1}^{m}\left[\left(\begin{array}{c}\left(\mu_{\tilde{D}_{PF_W}^{Agg}}(x_{ij}) - \mu_{PIS_{PF}}(x_{ij})\right)^2 \\ + \left(\eta_{\tilde{D}_{PF_W}^{Agg}}(x_{ij}) - \eta_{PIS_{PF}}(x_{ij})\right)^2 + \left(\nu_{\tilde{D}_{PF_W}^{Agg}}(x_{ij}) - \nu_{PIS_{PF}}(x_{ij})\right)^2\end{array}\right)\right]}$$

i=1,2,…,n (6.30)

$$S_i^- = \sqrt{\frac{1}{2m}\sum_{j=1}^{m}\left[\left(\begin{array}{c}\left(\mu_{\tilde{D}_{PF_W}^{Agg}}(x_{ij}) - \mu_{NIS_{PF}}(x_{ij})\right)^2 \\ + \left(\eta_{\tilde{D}_{PF_W}^{Agg}}(x_{ij}) - \eta_{NIS_{PF}}(x_{ij})\right)^2 + \left(\nu_{\tilde{D}_{PF_W}^{Agg}}(x_{ij}) - \nu_{NIS_{PF}}(x_{ij})\right)^2\end{array}\right)\right]}$$

i=1,2,…,n (6.31)

Step 15. Rank the alternatives according to the closeness coefficient C_i^* of each alternative using Eq. (6.32). The highest C_i^* is referred to the best alternative.

$$C_i^* = \frac{S_i^-}{S_i^- + S_i^*} \tag{6.32}$$

where $0 \le C_i^* \le 1$.

6.4 Application

In this section, the proposed PF Z-AHP&TOPSIS methodology is applied for the solar energy panel selection problem. The objective is to reveal the practicality and superiority of the proposed methodology applying for solar energy investment decision under the Z-number based picture fuzzy information.

6.4.1 Problem definition

Solar energy is an important renewable energy source, and the demand for solar energy investments has been increasing because of environmental awareness and government policies. As of January 1, 2023, Turkey has made it mandatory for buildings to use renewable energy at a rate of at least 5% of their primary energy needs, within the concept of the "Buildings Producing Their Energy by Their Own". In addition, buildings with a construction area of 5000 m^2 or more are required to be built within the concept of " Buildings Producing Their Energy by Their Own". Considering that investments in solar energy will increase gradually owing to this policy of Turkish government, in this study, an application that can help companies to choose a solar energy panel for their energy investments is presented. For this problem, three academicians (E1, E2 and E3) who are experts on solar panels in Turkey are interviewed for the evaluation of five alternative solar panels (A1, A2, …, A5) considering eight evaluation criteria (C1, C2, …C8) determined by a literature review. The alternative solar panels are given in Figure 6.4.

Figure 6.4 : Solar energy panel alternatives.

The information regarding the alternative solar panels is as follows (Bagher et al., 2015).

A1-Monocrystalline silicon (mono-Si) solar panel: Monocrystalline solar panel, also known as single crystal panels, are made from a single crystal of pure silicon divided into several wafers. It is qualified to provide big energy in small space. The efficiencies of monocrystalline solar cells are more than 20%. Their installation possibilities with high initial costs are challenging and they require maintenance at regular periodic intervals. Monocrystalline silicon solar panels are adversely affected if the panel is partially shaded or covered with snow.

A2- Cadmium-telluride (CdTe) thin film solar panel: CdTe thin film solar panels use cadmium telluride in a thin film designed to produce electricity from sunlight. It has low installation cost among the other crystalline type panels. It typically has a short warranty period and its efficiency varies about 7%. Thin-film solar panels are suitable for locations where heavy and labor-intensive installation of crystalline silicon is not possible. For example, narrow spaces, areas that require flexible installation instead of rigid panels.

A3-Polycrystalline silicon (poly-Si) solar panel: Since the polycrystalline silicon panel consists of multiple crystals instead of a single silicon crystal, such panels are called polycrystalline. The efficiencies of polycrystalline solar cells are between 14% and 17%. Polycrystalline solar panels generally have lower price than monocrystalline type panels.

A4-Monocrystalline PERC solar panel: PERC solar panels are an evolution of the traditional monocrystalline cell. This technology adds a passivation layer to the back surface of the cell that improves its efficiency. PERC panels have an efficiency of over 25% and they have relatively high initial costs.

A5-Half cut monocrystalline solar panel: The half cut monocrystalline solar panel is divided into two parts, with the upper and lower parts working independently of each other. This structure provides less internal resistance, higher energy output and less efficiency loss in shading. Half-cut solar panels improve the overall output of solar energy. If there is frequent shading in the application area of the solar panels, half-cut solar panels perform much better. Its efficiency reaches up to about 20%.

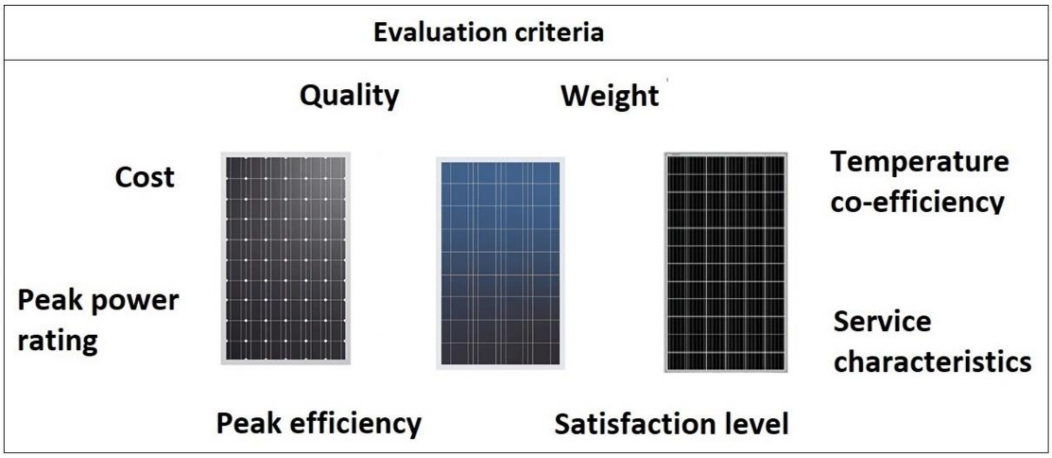

Figure 6.5 : Solar energy panel selection criteria.

The criteria set is determined based on the literature search and shown in Figure 6.5. Criteria, criteria types and their references are given in Table 6.6.

Table 6.6 : Criteria, criteria types and their references.

Criteria		Criteria types: max (benefit) / min (cost)	References
Cost	C1	min	Sasikumar and Ayyappan (2019), Balo and Şağbanşua (2016), Rani et al., (2020)
Quality	C2	max	Sasikumar and Ayyappan (2019), Sasikumar and Venkatachalam (2019)
Peak power rating	C3	max	Sasikumar and Ayyappan (2019), Rani et al., (2020), Sasikumar and Venkatachalam (2019).
Weight	C4	min	Sasikumar and Ayyappan (2019), Balo and Şağbanşua (2016), Rani et al., (2020), Sasikumar and Venkatachalam (2019)
Peak efficiency	C5	max	Sasikumar and Ayyappan (2019); Balo and Şağbanşua (2016), Rani et al., (2020), Sasikumar and Venkatachalam (2019)
Service characteristics	C6	max	Sasikumar and Ayyappan (2019), Balo and Şağbanşua (2016), Sasikumar and Venkatachalam (2019)
Satisfaction level	C7	max	Sasikumar and Ayyappan (2019), Balo and Şağbanşua (2016)
Temperature co-efficiency	C8	min	Sasikumar and Ayyappan (2019), Balo and Şağbanşua (2016), Sasikumar and Venkatachalam (2019)

C1-Cost: Solar panel costs include initial setup costs, regular maintenance costs, etc. The cost of a solar panel is affected by some features, such as the technical and physical characteristics of the panel, the material quality, and the maintenance period.

C2-Quality: The quality of the solar panel depends on the time it takes to use the energy it produces and the suitability of the materials used in its construction.

C3-Peak power rating: The peak power rating of a solar panel system refers to the maximum amount of electricity that can be produced under standard conditions (Rani et al., 2020).

C4-Weight: The weight of a solar panel is quite important when planning a solar energy system investment. Heavy solar panels make installation quite difficult, and the load that can be carried by the roof of a house, office, etc. is limited. A solar panel with a lower weight is better (Sasikumar and Ayyappan, 2019).

C5-Peak efficiency: The efficiency of a solar panel is the panel’s conversion rate of the energy from the sun into electricity. The efficiency of solar panels is generally around 15-24% on average. The solar panel's high efficiency means that it produces more power.

C6-Service characteristic: Service characteristics consist of warranty time, ease of maintenance and repair, etc. There are two different types of guarantees for solar panels: the material guarantee and the performance guarantee. Solar energy systems require regular maintenance and repair processes. A solar panel's easy maintenance and repair process provides a fast and effective elimination process of possible malfunctions.

C7-Satisfaction level: The satisfaction level is important for a solar energy system investment. It is beneficial to have an idea of whether the clients are satisfied or not with the related product and its customer service.

C8-Temperature co-efficiency: Solar panels heat up because they convert some of the light they receive from the sun into energy and some of it into heat. The heating of solar panels reduces energy efficiency and causes a decrease in performance. Therefore, a solar panel with a low temperature coefficient is better.

6.4.2 Problem data and solution

In Step 1, the problem is defined with the possible alternatives and criteria as given in Section 6.4.1. In Steps 2-3, pairwise comparisons for weighing the criteria are collected from experts via questionnaires, and three experts use PF Z-restriction and reliability scales in Table 6.5. The obtained pairwise comparisons are presented in Tables 6.7-6.9.

Table 6.7 : E1’s pairwise comparisons for criteria.

	C1	C2	C3	C4
C1	(EI, AHR)	(SHI, HR)	(SLI, VLR)	(HI, VHR)
C2	(SLI, HR)	(EI, AHR)	(LI, LR)	(SHI, LR)
C3	(SHI, VLR)	(HI, LR)	(EI, AHR)	(HI, AHR)
C4	(LI, VHR)	(SLI, LR)	(LI, AHR)	(EI, AHR)
C5	(SLI, SLR)	(SHI, HR)	(SLI, VHR)	(SHI, HR)
C6	(HI, VHR)	(VHI, LR)	(SHI, VHR)	(HI, VLR)
C7	(VHI, HR)	(AHI, VLR)	(SHI, LR)	(VHI, VHR)
C8	(VLI, HR)	(SLI, VHR)	(LI, LR)	(SLI, LR)
	C5	C6	C7	C8
C1	(SHI, SLR)	(LI, VHR)	(VLI, HR)	(VHI, HR)
C2	(SLI, HR)	(VLI, LR)	(ALI, VLR)	(SHI, VHR)
C3	(SHI, VHR)	(SLI, VHR)	(SLI, LR)	(HI, LR)
C4	(SLI, HR)	(LI, VLR)	(VLI, VHR)	(SHI, LR)
C5	(EI, AHR)	(VLI, HR)	(LI, HR)	(HI, SHR)
C6	(VHI, HR)	(EI, AHR)	(SLI, LR)	(VHI, HR)
C7	(HI, HR)	(SHI, LR)	(EI, AHR)	(AHI, VHR)
C8	(LI, SHR)	(VLI, HR)	(ALI, VHR)	(EI, AHR)
CR=0.097				

Table 6.8 : E2's pairwise comparisons for criteria.

	C1	C2	C3	C4
C1	(EI, AHR)	(HI, SHR)	(SLI, LR)	(SHI, HR)
C2	(LI, SHR)	(EI, AHR)	(SLI, SHR)	(SLI, MR)
C3	(SHI, LR)	(SHI, SHR)	(EI, AHR)	(SHI, VHR)
C4	(SLI, HR)	(SHI, MR)	(SLI, VHR)	(EI, AHR)
C5	(LI, AHR)	(SLI, LR)	(VLI, HR)	(SLI, VHR)
C6	(VHI, HR)	(HI, SHR)	(SHI, VHR)	(HI, VLR)
C7	(SHI, HR)	(VHI, LR)	(HI, VHR)	(HI, AHR)
C8	(LI, SHR)	(SLI, HR)	(ALI, VHR)	(SLI, HR)
	C5	C6	C7	C8
C1	(HI, AHR)	(VLI, HR)	(SLI, HR)	(HI, SHR)
C2	(SHI, LR)	(LI, SHR)	(VLI, LR)	(SHI, HR)
C3	(VHI, HR)	(SLI, VHR)	(LI, VHR)	(AHI, VHR)
C4	(SHI, VHR)	(LI, VLR)	(LI, AHR)	(SHI, HR)
C5	(EI, AHR)	(ALI, HR)	(ALI, HR)	(SHI, MR)
C6	(AHI, HR)	(EI, AHR)	(SHI, LR)	(VHI, HR)
C7	(AHI, HR)	(SLI, LR)	(EI, AHR)	(AHI, VHR)
C8	(SLI, MR)	(VLI, HR)	(ALI, VHR)	(EI, AHR)
CR=0.099				

Table 6.9 : E3's pairwise comparisons for criteria.

	C1	C2	C3	C4
C1	(EI, AHR)	**(SLI, VLR)**	(LI, LR)	(HI, VHR)
C2	(SHI, VLR)	(EI, AHR)	(SHI, HR)	(AHI, HR)
C3	(HI, LR)	(SLI, HR)	(EI, AHR)	(VHI, AHR)
C4	(LI, VHR)	(ALI, HR)	(VLI, AHR)	(EI, AHR)
C5	(VLI, SHR)	(ALI, HR)	(ALI, HR)	(SLI, LR)
C6	(SHI, HR)	(SLI, VLR)	(SLI, MR)	(AHI, VHR)
C7	(SLI, HR)	(LI, LR)	(SLI, SLR)	(SHI, HR)
C8	(SLI, MR)	(ALI, HR)	(VLI, AHR)	(SHI, LR)
	C5	C6	C7	C8
C1	(VHI, SHR)	(SLI, HR)	(SHI, HR)	(SHI, MR)
C2	(AHI, HR)	(SHI, VLR)	(HI, LR)	(AHI, HR)
C3	(AHI, HR)	(SHI, MR)	(SHI, SLR)	(VHI, AHR)
C4	(SHI, LR)	(ALI, VHR)	(SLI, HR)	(SLI, LR)
C5	(EI, AHR)	(VLI, AHR)	(LI, VHR)	(SLI, VLR)
C6	(VHI, AHR)	(EI, AHR)	(SHI, MR)	(HI, HR)
C7	(HI, VHR)	(SLI, MR)	(EI, AHR)	(SHI, HR)
C8	(SHI, VLR)	(LI, HR)	(SLI, HR)	(EI, AHR)
CR=0.095				

To compare C1 and C2, the question in the questionnaire is formed as follows:

For restriction ($\tilde{A}$): When considering solar energy panel selection, how important is the *cost* criterion (C1) compared to the *quality* criterion (C2)?

For reliability ($\tilde{R}$): How sure are you about your evaluation?

Expert 3 gives his/her opinion for C1 compared to C2 as "slightly low importance" and "very low reliable" for restriction and reliability functions, respectively. Thus, this evaluation is given in the following and indicated in bold in Table 6.9.

$\tilde{Z}_{PF}(\tilde{A}, \tilde{R}) = (SLI,\ VLR)$

$\tilde{Z}_{PF}(\tilde{A}, \tilde{R}) = ((0.30, 0.20, 0.50), (0.10, 0.10, 0.70))$

In Step 4, for all pairwise comparison matrices, PF restriction values are defuzzified using Eq. (6.14). Then, CR values of pairwise comparison matrices are calculated using defuzzified restriction values and shown in Tables 6.7-6.9. All CR values are in the consistency limits. Then, PF Z-pairwise comparison matrices are aggregated using $PFWGM$ operator given in Eq. (6.16).

In Step 5, the aggregated PF Z-pairwise comparison matrix obtained in Step 4 is converted to PF pairwise comparison matrix using Eq. (6.11). The aggregated PF pairwise comparison matrix is presented in Table 6.10.

Table 6.10 : Aggregated PF pairwise comparison matrix.

	C1	C2	C3	C4
C1	(0.36, 0.22, 0.40)	(0.25, 0.25, 0.51)	(0.10, 0.24, 0.72)	(0.46, 0.22, 0.29)
C2	(0.17, 0.25, 0.61)	(0.36, 0.22, 0.40)	(0.19, 0.25, 0.60)	(0.30, 0.23, 0.46)
C3	(0.21, 0.24, 0.56)	(0.28, 0.25, 0.49)	(0.36, 0.22, 0.40)	(0.52, 0.18, 0.22)
C4	(0.19, 0.22, 0.60)	(0.00, 0.23, 0.69)	(0.16, 0.18, 0.62)	(0.36, 0.22, 0.40)
C5	(0.13, 0.22, 0.67)	(0.00, 0.22, 0.67)	(0.00, 0.18, 0.72)	(0.24, 0.26, 0.55)
C6	(0.47, 0.21, 0.27)	(0.23, 0.22, 0.53)	(0.32, 0.26, 0.45)	(0.29, 0.16, 0.43)
C7	(0.37, 0.23, 0.39)	(0.19, 0.16, 0.57)	(0.26, 0.25, 0.51)	(0.50, 0.19, 0.25)
C8	(0.13, 0.23, 0.67)	(0.00, 0.21, 0.66)	(0.00, 0.15, 0.76)	(0.19, 0.26, 0.60)
	C5	C6	C7	C8
C1	(0.42, 0.22, 0.33)	(0.14, 0.21, 0.64)	(0.19, 0.23, 0.58)	(0.42, 0.23, 0.34)
C2	(0.32, 0.22, 0.44)	(0.10, 0.22, 0.71)	(0.00, 0.16, 0.78)	(0.46, 0.21, 0.28)
C3	(0.52, 0.18, 0.22)	(0.27, 0.26, 0.50)	(0.18, 0.25, 0.61)	(0.48, 0.15, 0.24)
C4	(0.28, 0.26, 0.49)	(0.00, 0.16, 0.79)	(0.15, 0.19, 0.63)	(0.23, 0.26, 0.56)
C5	(0.36, 0.22, 0.40)	(0.00, 0.14, 0.76)	(0.00, 0.18, 0.71)	(0.23, 0.26, 0.54)
C6	(0.59, 0.14, 0.13)	(0.36, 0.22, 0.40)	(0.21, 0.27, 0.58)	(0.52, 0.19, 0.23)
C7	(0.52, 0.18, 0.21)	(0.18, 0.27, 0.62)	(0.36, 0.22, 0.40)	(0.56, 0.16, 0.17)
C8	(0.16, 0.26, 0.63)	(0.10, 0.19, 0.70)	(0.00, 0.16, 0.75)	(0.36, 0.22, 0.40)

In Step 6, $\widetilde{PFM}$ vector is calculated using $PFWGM$ operator given in Eq. (6.19). Thus, we have PF criteria weights given in Table 6.11.

Table 6.11 : PF criteria weights.

Criteria	Weights
C1	(0.26, 0.05, 0.27)
C2	(0.13, 0.05, 0.34)
C3	(0.33, 0.05, 0.20)
C4	(0.07, 0.05, 0.38)
C5	(0.02, 0.04, 0.42)
C6	(0.35, 0.04, 0.17)
C7	(0.34, 0.04, 0.19)
C8	(0.02, 0.05, 0.44)

In Steps 7-8, three experts evaluate five alternatives according to eight criteria using Table 6.5. Thus, we have PF Z-decision matrices collected from three experts, which are presented in Tables 6.12-6.14.

Table 6.12 : E1's PF Z-decision matrix.

	C1	C2	C3	C4	C5	C6	C7	C8
A1	(LI, LR)	(SHI, HR)	(HI, SHR)	(HI, VHR)	(SHI, LR)	(LI, SLR)	(AHI, SHR)	(SLI, HR)
A2	(HI, VHR)	(LI, MR)	(SHI, HR)	(SLI, MR)	(VLI, HR)	(HI, VHR)	(SLI, HR)	(HI, VHR)
A3	(VLI, VLR)	(VLI, HR)	(HI, AHR)	(VHI, HR)	(LI, SLR)	(VHI, VHR)	(VHI, HR)	(VLI, HR)
A4	(SHI, LR)	(HI, SHR)	(SLI, LR)	(LI, SLR)	(SHI, HR)	(HI, SHR)	(HI, VHR)	(LI, SHR)
A5	(HI, AHR)	(VHI, HR)	(LI, VHR)	(SLI, HR)	(HI, HR)	(LI, VLR)	(HI, VHR)	(VHI, HR)

Table 6.13 : E2's PF Z-decision matrix.

	C1	C2	C3	C4	C5	C6	C7	C8
A1	(SLI, HR)	(HI, VHR)	(HI, HR)	(VHI, AHR)	(SLI, SLR)	(VLI, VHR)	(VHI, SHR)	(VLI, AHR)
A2	(HI, AHR)	(VLI, HR)	(VHI, VHR)	(LI, VLR)	(LI, HR)	(SHI, HR)	(LI, HR)	(VHI, MR)
A3	(VHI, HR)	(LI, LR)	(SHI, LR)	(SHI, LR)	(VLI, HR)	(HI, HR)	(AHI, VHR)	(LI, VHR)
A4	(VLI, LR)	(SHI, HR)	(VLI, SLR)	(SLI, LR)	(HI, SHR)	(VHI, AHR)	(SLI, LR)	(VLI, HR)
A5	(SHI, SLR)	(HI, AHR)	(SLI, HR)	(LI, HR)	(VHI, HR)	(VLI, HR)	(HI, HR)	(HI, MR)

Table 6.14 : E3's PF Z-decision matrix.

	C1	C2	C3	C4	C5	C6	C7	C8
A1	(SHI, VLR)	(HI, HR)	(SHI, LR)	(HI, HR)	(HI, AHR)	(SLI, VHR)	(SHI, HR)	(LI, HR)
A2	(VHI, HR)	(SLI, VLR)	(HI, VHR)	(SHI, VHR)	(ALI, HR)	(VHI, HR)	(VLI, LR)	(SLI, SHR)
A3	(HI, VHR)	(SHI, LR)	(SLI, LR)	(HI, HR)	(SLI, SHR)	(HI, VHR)	(LI, VHR)	(SLI, LR)
A4	(LI, LR)	(VHI, HR)	(LI, VHR)	(VLI, LR)	(VHI, VHR)	(LI, LR)	(VLI, HR)	(ALI, HR)
A5	(VHI, HR)	(HI, SHR)	(VLI, SHR)	(VLI, SLR)	(SHI, MR)	(SLI, MR)	(VHI, HR)	(SHI, SHR)

In Step 9, we aggregate three decision matrices using PFWGM operator by Eq. (6.16). In Step 10, the aggregated PF decision matrix is derived from the aggregated PF Z-decision matrix utilizing Eq. (6.11). Thus, we have $\widetilde{D}_{PF}^{Agg}$ matrix shown in Table 6.15.

Table 6.15 : Aggregated PF decision matrix $\widetilde{D}_{PF}^{Agg}$.

	C1	C2	C3	C4
A1	(0.15, 0.24, 0.65)	(0.45, 0.22, 0.30)	(0.35, 0.24, 0.40)	(0.53, 0.18, 0.21)
A2	(0.53, 0.18, 0.21)	(0.10, 0.22, 0.72)	(0.48, 0.20, 0.26)	(0.17, 0.24, 0.62)
A3	(0.20, 0.17, 0.54)	(0.12, 0.22, 0.69)	(0.25, 0.23, 0.52)	(0.38, 0.22, 0.37)
A4	(0.10, 0.22, 0.72)	(0.45, 0.23, 0.30)	(0.11, 0.22, 0.71)	(0.09, 0.23, 0.74)
A5	(0.43, 0.21, 0.32)	(0.50, 0.19, 0.24)	(0.14, 0.22, 0.65)	(0.13, 0.23, 0.68)
	C5	C6	C7	C8
A1	(0.27, 0.24, 0.50)	(0.13, 0.21, 0.66)	(0.48, 0.20, 0.27)	(0.15, 0.20, 0.64)
A2	(0.00, 0.17, 0.74)	(0.47, 0.21, 0.27)	(0.12, 0.22, 0.69)	(0.36, 0.23, 0.40)
A3	(0.12, 0.23, 0.68)	(0.52, 0.19, 0.23)	(0.39, 0.16, 0.34)	(0.12, 0.21, 0.68)
A4	(0.46, 0.22, 0.29)	(0.29, 0.19, 0.47)	(0.17, 0.21, 0.60)	(0.00, 0.18, 0.75)
A5	(0.43, 0.23, 0.32)	(0.10, 0.22, 0.72)	(0.50, 0.19, 0.24)	(0.42, 0.23, 0.34)

In Step 11, we multiply $\widetilde{D}_{PF}^{Agg}$ with PF criteria weights given in Table 6.11 obtained by PF Z-AHP method. Then, we calculate the weighted aggregated PF decision matrix $\widetilde{D}_{PF_W}^{Agg}$ using Eqs. (6.2) and (6.25). Table 6.16 shows the weighted aggregated PF decision matrix $\widetilde{D}_{PF_W}^{Agg}$.

Table 6.16 : Weighted aggregated PF decision matrix $\widetilde{D}_{PF_W}^{Agg}$.

	C1	C2	C3	C4
A1	(0.04, 0.28, 0.74)	(0.06, 0.26, 0.54)	(0.12, 0.28, 0.52)	(0.04, 0.22, 0.51)
A2	(0.14, 0.22, 0.42)	(0.01, 0.26, 0.81)	(0.16, 0.24, 0.40)	(0.01, 0.28, 0.76)
A3	(0.05, 0.21, 0.66)	(0.02, 0.26, 0.79)	(0.08, 0.27, 0.61)	(0.03, 0.25, 0.61)
A4	(0.03, 0.26, 0.80)	(0.06, 0.26, 0.54)	(0.04, 0.26, 0.76)	(0.01, 0.26, 0.84)
A5	(0.11, 0.25, 0.50)	(0.06, 0.23, 0.50)	(0.05, 0.26, 0.72)	(0.01, 0.26, 0.80)
PIS_{PF}	(0.03, 0.26, 0.80)	(0.06, 0.23, 0.50)	(0.16, 0.24, 0.40)	(0.01, 0.26, 0.84)
NIS_{PF}	(0.14, 0.22, 0.42)	(0.01, 0.26, 0.81)	(0.04, 0.26, 0.76)	(0.04, 0.22, 0.51)
	C5	C6	C7	C8
A1	(0.01, 0.28, 0.71)	(0.05, 0.25, 0.72)	(0.16, 0.24, 0.4)	(0.00, 0.24, 0.80)
A2	(0.00, 0.21, 0.85)	(0.17, 0.25, 0.4)	(0.04, 0.25, 0.75)	(0.01, 0.26, 0.66)
A3	(0.00, 0.27, 0.82)	(0.18, 0.22, 0.36)	(0.13, 0.19, 0.46)	(0.00, 0.25, 0.82)
A4	(0.01, 0.25, 0.59)	(0.1, 0.23, 0.56)	(0.06, 0.25, 0.67)	(0.00, 0.22, 0.86)
A5	(0.01, 0.26, 0.61)	(0.03, 0.25, 0.76)	(0.17, 0.23, 0.38)	(0.01, 0.27, 0.63)
PIS_{PF}	(0.01, 0.25, 0.59)	(0.18, 0.22, 0.36)	(0.17, 0.23, 0.38)	(0.00, 0.22, 0.86)
NIS_{PF}	(0.00, 0.21, 0.85)	(0.03, 0.25, 0.76)	(0.04, 0.25, 0.75)	(0.01, 0.27, 0.63)

In Steps 12-13, firstly, we defuzzify $\widetilde{D}_{PF_W}^{Agg}$ matrix using Eq. (6.14). Then, we determine PIS_{PF} and NIS_{PF} values according to defuzzified values and using Eqs. (6.26-6.29). If the criterion is the benefit type, its PIS_{PF} and NIS_{PF} values are the *maximum* and *minimum* values of the related criterion in $Def\left(\widetilde{D}_{PF_W}^{Agg}\right)$ matrix, respectively. If the criterion is the cost type, similar rule is valid. In Table 6.16, PIS_{PF} and NIS_{PF} values are given.

In Step 14, we determine S^* and S^- values for each alternative by Eqs. (6.30) and (6.31). In Step 15, we calculate C_i^* of alternatives by Eq. (6.32). S^*, $S^-$and C_i^* values of alternatives are presented in Table 6.17. Then, we rank the alternatives according to C_i^* values.

Table 6.17 : S^*, S^-, C_i^* values and ranking of alternatives.

	S^*	S^-	C_i^*	Ranking
A1	0.138	0.167	0.548	3
A2	0.181	0.151	0.454	5
A3	0.132	0.163	0.551	2
A4	0.135	0.181	0.573	1
A5	0.169	0.160	0.486	4

According to the obtained results presented in Table 6.17, A4-Monocrystalline PERC solar panel is the best alternative among the five solar panels. It has the highest C_i^* value with the value of 0.573. The overall ranking is determined as A4-*Monocrystalline PERC*>A3-*Polycrystalline silicon (poly-Si)*>A1-*Monocrystalline silicon (mono-Si)*>A5-*Half cut monocrystalline*>A2-*Cadmium telluride thin film.*

Although the proposed methodology is applied for solar energy panel selection, the approach allows for modifying criteria and alternative sets and gives an opportunity to apply for variety of different real life problems such as personnel selection, performance evaluation and supplier selection except solar energy panel selection. In such real-life problems involving subjective intense evaluation, the decision makers' complex mindset can be better modeled using the combination of picture fuzzy sets and Z-numbers, which allow to take 3D fuzzy reliability into account with a separate function in addition to 3D fuzzy restriction.

6.5 Comparative Analysis

In order to verify the suggested method and analyze the obtained results, the proposed methodology has been compared with the results of PF-TOPSIS and Z-TOPSIS methods. PF-TOPSIS method does not have the ability to represent the reliability degrees of decision makers' judgments expressed by 3D linguistic terms. Therefore, in the first analysis, in order to analyze the computational response generated by reliability function of the PF Z-numbers, the proposed methodology is compared with the PF-TOPSIS method by ignoring the reliability information based on the same criteria weights. For this purpose, the same linguistic scale and experts' data are used to eliminate all data effects. The main purpose of the first analysis is to observe only the differences caused by the reliability component. In the second analysis, in order to analyze the effects of PFSs in our proposed method, the Z-AHP & Z-TOPSIS methodology is implemented by using ordinary fuzzy Z-numbers. The Z-AHP & Z-TOPSIS methodology models human judgments using only ordinary triangular fuzzy membership functions of restriction and reliability components. This methodology represents the human thoughts without non-membership and neutral degrees when compared to our proposed PF Z-AHP&TOPSIS methodology. Thus, it was intended to demonstrate the differences between decision-makers' judgments represented by PF Z-numbers and ordinary fuzzy Z-numbers. For this purpose, criteria weights are calculated with the Z-AHP method developed by Tüysüz and Kahraman (2023). Comparison results of both PF Z-AHP & PF-TOPSIS methodology and Z-AHP & Z-TOPSIS methodology are presented in Table 6.18.

Table 6.18 : Comparison of ranking results.

Alternatives	Proposed method	PF Z-AHP PF-TOPSIS	Z-AHP Z-TOPSIS
A1	3	1	2
A2	5	5	5
A3	2	3	3
A4	1	2	1
A5	4	4	4

As demonstrated in Table 6.18, although the results of the proposed method are quite similar to the results of the Z-AHP & Z-TOPSIS methodology, the ranking obtained by the PF Z-AHP & PF-TOPSIS methodology is quite different. While the alternative A4 has been found as the best alternative in the proposed method and in the Z-AHP & Z-TOPSIS methodology, it has been ranked second in the PF Z-AHP & PF-TOPSIS methodology, which does not include reliability information. In addition, PF-TOPSIS method made it possible to easily distinguish the effect of Z-numbers by obtaining the alternative A1 as the best choice, unlike the other two methods.

The results of the proposed PF Z-AHP&TOPSIS methodology show that the best alternative has not changed when compared to the results of the Z-AHP & Z-TOPSIS methodology. The main difference in the results is that the overall ranking has slightly changed. This situation may have resulted from the experts' preferences in this application, and the fact that picture fuzzy sets have the effect of repositioning for some alternatives proves the importance of integrating picture fuzzy sets with Z-numbers. It can be interpreted from these comparisons that the proposed methodology provides a reliable decision process based on picture fuzzy information for experts.

6.6 Sensitivity Analysis

In order to measure the robustness of the proposed methodology, we perform a sensitivity analysis based on defuzzification of reliability functions in order to observe the importance of reliability component in picture fuzzy Z-data. In the application section, all calculations in this conversion process are realized without defuzzification. Then, PF Z-AHP&TOPSIS methodology have been resolved based on defuzzification of reliability functions. The obtained alternative rankings are given in Table 6.19.

Table 6.19 : S^*, S^-, C_i^* values and ranking of alternatives with defuzzification.

	S^*	S^-	C_i^*	Ranking
A1	0.159	0.217	0.577	2
A2	0.224	0.172	0.434	5
A3	0.163	0.191	0.539	3
A4	0.155	0.222	0.588	1
A5	0.204	0.200	0.496	4

When Tables 6.17 and 6.19 are analyzed together, it can be deduced that the best alternative A4 and the worst alternative A2 have the same ranks among five alternative solar panels. Only the second and third alternatives (A1 and A3) have replaced in the overall ranking. This result supports the strength of the proposed method.

Another sensitivity analysis has been performed by taking equal criteria weights. In this analysis, linguistic evaluations in the PF Z-AHP method have been accepted as "Equal Importance (EI), Medium Reliable (MR)" and the obtained PF criteria weights have been integrated into the PF Z-TOPSIS method. Then, the overall ranking has been obtained as A4>A1>A5>A3>A2. This result shows that the place of the first and the last alternatives in the ranking has not changed.

The other sensitivity analysis has been conducted to investigate the effects of aggregation operator types on the results of the proposed methodology. In our application, PFWGM operator is used in aggregation of pairwise and decision matrices of experts for both PF Z-AHP and PF Z-TOPSIS methods. For sensitivity analysis, PFWAM operator has been used to aggregate pairwise comparisons of three experts in the PF Z-AHP method, and the new criteria weights have been obtained. Then, in the PF Z-TOPSIS method, the decision matrices of three experts have been aggregated using PFWAM operator, and the aggregated decision matrix has been weighted using the new criteria weights obtained in the PF Z-AHP method aggregated by PFWAM operator. All models of PF Z-TOPSIS method have been used and four rankings have been obtained. In all cases in this analysis, PF Z-values have been converted to PF values without defuzzification. The obtained criteria weights and ranking results based on the utilization of the mentioned two aggregation operators are shown in Tables 6.20 and 6.21.

It can be concluded from Table 6.20 that some criteria weights remain approximately same while some fairly change. For this reason, it can be interpreted that the aggregation operators have an effect on the criteria weights in the PF Z-AHP method.

Table 6.20 : Criteria weights based on aggregation operators.

	PF Z-AHP method	
	PFWGM operator (obtained in application section)	PFWAM operator
C1	(0.26, 0.05, 0.27)	(0.29, 0.05, 0.24)
C2	(0.13, 0.05, 0.34)	(0.27, 0.04, 0.24)
C3	(0.33, 0.05, 0.20)	(0.36, 0.05, 0.16)
C4	(0.07, 0.05, 0.38)	(0.20, 0.04, 0.33)
C5	(0.02, 0.04, 0.42)	(0.17, 0.04, 0.36)
C6	(0.35, 0.04, 0.17)	(0.39, 0.04, 0.13)
C7	(0.34, 0.04, 0.19)	(0.38, 0.04, 0.13)
C8	(0.02, 0.05, 0.44)	(0.15, 0.04, 0.40)

In addition, in order to understand that the effects of the changes in criteria weights are significant or not, it is necessary to investigate if the ranking results of the alternatives are affected by this change. The obtained ranking results are given in Table 6.21 and Figure 6.6.

Table 6.21 : C_i^* values and alternative rankings based on aggregation operators.

	PF Z-AHP: PFWGM					PF Z-AHP: PFWAM			
	PF Z-TOPSIS: PFWGM (obtained in application section)		PF Z-TOPSIS: PFWAM			PF Z-TOPSIS: PFWGM		PF Z-TOPSIS: PFWAM	
	C_i^*	Ranking	C_i^*	Ranking		C_i^*	Ranking	C_i^*	Ranking
A1	0.548	3	0.566	3	A1	0.549	2	0.566	3
A2	0.454	5	0.412	5	A2	0.447	5	0.405	5
A3	0.551	2	0.581	2	A3	0.538	3	0.569	2
A4	0.573	1	0.612	1	A4	0.582	1	0.618	1
A5	0.486	4	0.477	4	A5	0.500	4	0.491	4

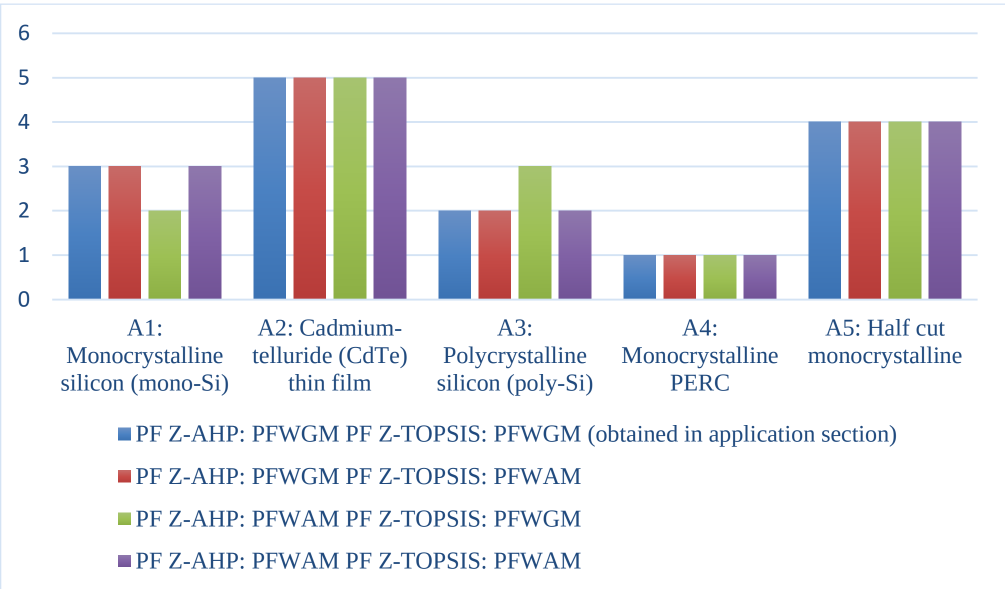

Figure 6.6 : Rankings of alternatives according to aggregation operators.

It can be clearly concluded from Table 6.21 that A4-*Monocrystalline PERC* is the best and A2-*Cadmium telluride thin film* is the worst solar panels in four cases. In addition, when the PFWAM operator is used in the PF Z-AHP method and the PFWGM operator in PF Z-TOPSIS method, minor changes have been obtained in the ranking results. These results demonstrate that the decision process is not sensitive enough to changes in the types of aggregation operators to affect the managerial decisions. In addition, although the use of the PFWAM operator in calculating the criteria weights has caused a slightly significant change in the criteria weights, it had little effect on the ranking results. Therefore, it can be commented that the robustness of the proposed methodology has been observed by sensitivity analysis.

6.7 Conclusions

The proposed methodology has the advantages of decision making with hesitancy and refusal parameters of picture fuzzy sets and reliability consideration of linguistic judgments of Z-numbers. This paper presented a novel hybrid methodology built on AHP and TOPSIS methods based on PF Z-numbers: PF Z-AHP method for determining the criteria weights and PF Z-TOPSIS method for selecting the best alternative. The proposed methodology successfully allowed specifying not only 3D picture fuzzy restriction functions but also their 3D picture fuzzy reliability functions. The methodology produced privileged decision results, which can be beneficial for experts who want to model 3D preferences under the favorable circumstances of Z-numbers. This study presented formulations to be used in converting PF numbers to crisp numbers and PF Z-numbers to PF numbers. The methodology also allowed to continue to operations with picture fuzzy sets without using any defuzzification.

In comparative analysis, the effects of ignoring reliability information and PF information on the results have been investigated by applying PF-TOPSIS and Z-TOPSIS, respectively. Ignoring the reliability information (PF-TOPSIS) significantly changed the results by placing the third alternative in the proposed method at the first rank while eliminating the PF information (Z-TOPSIS) has not changed the best and worst alternatives but slightly changed the overall ranking. These results summarize that both approaches using reliability information (PF Z-AHP&TOPSIS and Z-AHP&TOPSIS) produced similar results. Besides, quite different results have been obtained in the approach ignoring the reliability degrees (PF-TOPSIS). It can be interpreted that more trustworthy results can be obtained with the approaches consisting of fuzzy reliabilities in addition to fuzzy restrictions.

A sensitivity analysis conducted on a solar energy panel selection showed the robustness of the given decisions by the proposed methodology. It also showed that the picture fuzzy aggregation operators slightly changed the criteria weights but did not change the managerial decisions on alternative ranking. The monocrystalline PERC solar panel has generally been found as the best alternative, according to the results of both comparison analysis and sensitivity analysis. The reliability degrees of experts' 3D preferences created a substantial evaluation framework in obtaining these results.

As well as the proposed methodology provides a reliable decision support tool for decision makers, it has a quite mathematical complexity with the combination of picture fuzzy sets and Z-numbers. As in most MCDM approaches extended by different type of fuzzy sets, problem solving may become difficult when the number of criteria and alternatives increases. Nevertheless, since the decision makers' complex judgment system can be better represented by high level mathematical concepts, the mentioned disadvantages are compensated by this superiority of the proposed methodology.

For further research, other fuzzy set extensions, such as Fermatean fuzzy sets, can be used to extend Z-numbers. Then, integrated Fermatean fuzzy Z-MCDM methods such as Fermatean fuzzy Z-AHP&TOPSIS or Fermatean fuzzy Z-AHP&CODAS can be proposed to the literature to be compared with the results of this study. Another issue is to make the methodology ready for use through a software. Although MS Excel tools are enough to make the computations, we suggest that some specialized software using Python, C++ or Java can also be developed to facilitate the implementation of the proposed methodology.

7. INTERVAL-VALUED SPHERICAL FUZZY Z-AHP METHOD BASED ON RELIABILITY OF JUDGMENTS: GREEN SUPPLIER SELECTION[6]

Fuzzy set theory has been widely used since Zadeh (1965) introduced it. The fuzzy sets have been rapidly expanded to several new extensions such as type-2 fuzzy sets (Tolga et al., 2020; Castillo et al., 2022; Castillo et al., 2023), hesitant fuzzy sets, Pythagorean fuzzy sets, intuitionistic fuzzy sets, q-rung orthopair fuzzy sets, fermatean fuzzy sets and spherical fuzzy sets by different researchers in recent years. In this research, spherical fuzzy sets (SFSs) have been preferred because of their larger domain area and their consideration of experts' hesitancy as a separate parameter. Spherical fuzzy sets (SFSs), which have been introduced by Kahraman and Kutlu Gündoğdu (2018) to the literature, are one of the fuzzy sets put forward to represent the fuzziness with a wider domain. SFSs are proposed to the literature as an extension of picture fuzzy sets. SFSs have three parameters which are membership, non-membership and hesitancy degrees whose squared sum is at most equal to 1 whereas their sum has to be at most 1 in picture fuzzy sets. SFSs are superior to picture fuzzy sets as they offer a larger domain size than picture fuzzy sets to DMs. SFSs let decision makers assign those degrees from one eight of unit sphere, whereas picture fuzzy sets let them from a triangular prism. In order to better define and model the uncertainty, Zadeh (2011) introduced Z-fuzzy numbers, and states that Z-fuzzy numbers are the generalization of all other numbers (real numbers, interval numbers, fuzzy numbers, and random numbers) and mentions the importance of Z-fuzzy numbers by stating that the numbers expressed in areas such as decision analysis and economics are actually Z-fuzzy numbers. DMs cannot be 100% sure of their evaluations despite their knowledge and experience. However, people generally express their opinions as if they are 100% sure of them. A questionnaire can be used to learn how confident a DM is in his/her judgments using such linguistic terms as "weakly confident," "moderately confident," or "highly confident," which can be preferred by the DM.

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[6] This chapter is based on the paper "Tüysüz N. & Kahraman C. (2024). Interval-valued spherical fuzzy Z-AHP method based on reliability of judgments: green supplier selection. *Journal of Multiple-Valued Logic & Soft Computing*. (Accepted)"

Alkan and Kahraman (2022a) propose spherical Z-fuzzy numbers to the literature by integrating SFSs with Z-fuzzy numbers, which have the ability to define uncertain expressions in a 2-tuple form representing both restriction and reliability degrees of preferences. This study forces decision makers to give a single value for each parameter. Expressing each parameter with an exact number may cause a loss of information in the representation of ambiguous expressions. This may lead to unexpected or wrong results for real life problems, especially if it is a decision problem that includes subjective and imprecise judgments of decision makers (DMs). Duleba et al. (2021) propose an interval-valued spherical fuzzy AHP method and apply it to the public transportation problem. The main contribution and originality of this study presents a wider assignment region for parameter values together with interval-valued membership functions by using spherical fuzzy sets. Besides, our study combines interval-valued SFSs with Z-fuzzy numbers in order to consider the reliability of the assigned interval-valued SFSs parameters for the first time. Another originality is to develop an interval-valued spherical Z-fuzzy AHP method.

With Zadeh's (2011) introduction of Z-fuzzy numbers, a research gap on their extensions has emerged and has started to attract the attention of researchers in recent years. Peng and Wang (2017) introduce hesitant uncertain linguistic Z-fuzzy numbers (HULZNs) for the solution of fuzzy MCDM problems. They extend VIKOR method using HULZNs and give an application for ERP selection problem. Xian et al. (2019) extend TOPSIS method using intuitionistic Z-linguistic sets and Minkowski distance. They present numerical examples for investment and medical diagnosis problems. Du et al. (2021) propose neutrosophic Z-fuzzy numbers. Ren et al. (2020) propose a decision-making approach that includes hesitant fuzzy linguistic information and Z-fuzzy numbers. They apply the method to the medicine selection problem for COVID-19 patients. Sari and Kahraman (2020) propose intuitionistic Z-fuzzy numbers to the literature for the first time. Zhao and Ye (2021) present the orthopair Z-fuzzy numbers and give some aggregation operations. Sari and Tüysüz (2022) propose a novel AHP & TOPSIS methodology under the information of interval type-2 Z-fuzzy numbers. Chen et al. (2022) introduce picture fuzzy Z-linguistic sets to create a more inclusive solution for decision making by adding reliability information to picture fuzzy sets. They improve VIKOR method using picture fuzzy Z-linguistic sets. Ashraf et al. (2023) extend Z-fuzzy numbers to Pythagorean fuzzy Z-numbers. They present an application on green supplier selection using the developed Pythagorean fuzzy Z-

EDAS method. Tüysüz and Kahraman (2023) propose a methodology that integrates the AHP and EDAS methods under the properties of Z-fuzzy numbers. All these studies aim to model imprecise and vague expressions mathematically in the most inclusive way.

"Supplier" can simply be defined as a person, company, or other entity providing goods or services to another person, company, or other entity. Supplier selection is a MCDM problem that includes selecting the right supplier among the alternatives by considering many criteria such as cost, quality, and delivery time related to providers. Green supplier selection (GSS) is a supplier selection problem that requires evaluation of many criteria not only related to providing minimum cost levels and high quality but also related to improving environmental performance, consuming fewer hazardous materials, less material and energy (Igarashi et al., 2013). The scarce resources in the world necessitate environmentally friendly practices in every field. GSS is a critical issue that needs to be addressed from an environmental point of view since the production of goods or services will continue as long as the world exists, and this is important for the continuity of a sustainable world. In addition, GSS is a problem that should be given importance by companies in order to support sustainability of the supply chain and to maintain their existence in competitive conditions in this sense, and it has a great effect on leaving a livable world to the future (Liou et al., 2021).

One of the main objectives of this study is to integrate Z-fuzzy numbers and interval-valued spherical fuzzy (IVSF) sets to find the best representation of uncertainty. Another aim of this study is to develop a novel IVSF Z-AHP method based on restriction and reliability of judgments for the solution of MCDM problems.

This study is original in three ways. First, there has not been a previous study that integrates Z-fuzzy numbers and IVSF numbers. This literature gap is clearly seen in Table 7.1. Second, AHP method has been firstly developed with IVSF Z-numbers. Finally, our proposed approach is also used for the first time for the GSS problem.

The organization of the remaining paper is presented as follows: Section 7.1 presents the literature review on fuzzy sets extensions and fuzzy GSS problems. Section 7.2 explains the evaluation criteria set of GSS problem based on the literature review. Section 7.3 includes preliminaries of single-valued SFSs, interval-valued SFSs and Z-fuzzy numbers. Section 7.4 presents the steps of the proposed IVSF Z-AHP method. Section 7.5 presents an application on GSS, which includes the problem definition and solution, as well as comparative and sensitivity analysis. The last section gives conclusive remarks and further research recommendations.

7.1 Literature Review

In this section, we first summarize the fuzzy sets extensions in Section 7.1.1. Then, a literature review of the extensions of fuzzy MCDM methods on GSS is presented in Section 7.1.2.

7.1.1 Fuzzy sets extensions

Extensions of ordinary fuzzy sets and their features are summarized in Table 7.1. We classify the literature review results based on the conditions of membership degrees and whether they are integrated with Z-fuzzy numbers.

Table 7.1 : Fuzzy sets extensions

Types of Fuzzy Extension	Developers	Parameters in Membership Functions	Conditions of Membership Degree	Integrated with Z-fuzzy numbers by now?
Ordinary fuzzy sets	Zadeh (1965)	Membership degree	Crisp number between 0 and 1	Yes
Type 2 fuzzy sets	Zadeh (1975)	3D Membership function	Fuzzy membership degrees	Yes
Interval-valued fuzzy sets	Sambuc, Jahn, Grattan Guiness, Zadeh (1975)	Interval-valued membership degree	Closed interval of membership degree	Yes
Intuitionistic fuzzy sets	Atanassov (1986)	Membership and non-membership degrees	Sum of degrees at most equal to 1	Yes
Neutrosophic sets	Smarandache (1999)	Truthiness, falsity and indeterminacy degrees	Sum of degrees at most equal to 3	Yes
Intuitionistic fuzzy sets of second type (Pythagorean fuzzy sets)	Atanassov (1999)	Membership and non-membership degrees	Sum of squared degrees at most equal to 1	Yes
Nonstationary fuzzy sets	Garibaldi and Ozen (2007)	Membership degree	Crisp number between 0 and 1	No
Hesitant fuzzy sets	Torra (2010)	More than one possible membership degrees	Crisp number between 0 and 1	Yes
Pythagorean fuzzy sets	Yager (2013)	Membership and non-membership degrees	Squared sum of degrees at most equal to 1	Yes
Picture fuzzy sets	Cuong (2014)	Positive, negative, neutral and refusal membership degrees	Sum of degrees at most equal to 1	Yes
q-rung orthopair fuzzy sets	Yager (2016)	Membership and non-membership degrees	Sum of the qth power of degrees at most equal to 1	Yes
Spherical fuzzy sets	Kahraman and Kutlu Gündoğdu (2018)	Membership, non-membership and hesitancy degrees	Squared sum of degrees at most equal to 1	Yes*
Fermatean fuzzy sets	Senapati and Yager (2019)	Membership and non-membership degrees	Sum of the 3rd power of degrees at most equal to 1	No

*For single valued spherical sets

Our literature review shows that Fermatean fuzzy sets and nonstationary fuzzy sets have not yet been integrated with Z-fuzzy numbers.

7.1.2 Fuzzy green supplier selection

The decision of GSS has attracted a lot of attention due to the concerns about leaving a more sustainable world to future generations in recent years. It is a complex process which requires the evaluation of various criteria related to suppliers' environmental characteristics. The main problem for both evaluating the criteria and selecting the green suppliers is that DMs usually express their evaluations and judgments by using natural linguistic terms.

The integration of fuzzy sets with MCDM methods has found a great place in GSS in recent years. The existing literature on GSS using fuzzy sets and their extensions with MCDM methods is quite extensive. In Table 7.2, a literature review of fuzzy extensions of MCDM methods on GSS between 2015 and 2023 is presented. All methods for GSS, such as mathematical programming, genetic algorithms, and MCDM methods are explained and listed in detail by Zhang et al. (2020). A detailed literature review of MCDM methods on GSS can be found in Schramm et al. (2020).

Table 7.2 : A literature review of the extensions of fuzzy MCDM methods on GSS

Year	Authors	Type of fuzzy sets	Selection Method
2015	Cao et al.	Intuitionistic fuzzy sets	TOPSIS
2016	Ghorabaee et al.	Interval type-2 fuzzy sets	WASPAS
2016	Sang and Liu	Interval type-2 fuzzy sets	TODIM
2017	Qin et al.	Interval type-2 fuzzy sets	TODIM
2018	Wang and Li	Q-rung orthopair fuzzy sets	TODIM
2018	Gitinavard et al.	Interval-valued hesitant fuzzy sets	ELECTRE
2018	Tian et al.	Intuitionistic fuzzy sets	TOPSIS
2018	Shi et al.	Interval-valued intuitionistic fuzzy linguistic sets	GRA & TOPSIS
2019	Memari et al.	Intuitionistic fuzzy sets	TOPSIS
2019a	Zhang et al.	Picture fuzzy sets	EDAS
2019a	Wu et al.	Interval-valued Pythagorean fuzzy sets	DEA
2019b	Wu et al.	Interval type-2 fuzzy sets	VIKOR
2019	Yu et al.	Interval-valued Pythagorean fuzzy sets	TOPSIS
2019	Meksavang et al.	Picture fuzzy sets	VIKOR
2019	Yucesan et al.	Interval type-2 fuzzy sets	TOPSIS
2019	Mishra et al.	Hesitant fuzzy sets	WASPAS
2019	Fan et al.	Pythagorean fuzzy sets	DEA
2019	Liu et al.	Q-rung interval-valued orthopair fuzzy sets	MULTIMOORA
2019	Liang et al.	Interval-valued 2-tuple fuzzy sets	TODIM
2019	Nie et al.	Interval-valued fuzzy linguistic sets	TODIM
2020	Wan et al.	Hesitant fuzzy sets	PROMETHEE

Table 7.2 (continued): A literature review of the extensions of fuzzy MCDM methods on GSS

2020	Kumari and Mishra	Intuitionistic fuzzy sets	COPRAS
2020	Xu et al.	Single-valued complex neutrosophic sets	EDAS
2020	Rouyendegh et al.	Intuitionistic fuzzy sets	TOPSIS
2020	Kilic and Yalcin	Intuitionistic fuzzy sets	TOPSIS
2020	Krishankumar et al.	Q-rung orthopair fuzzy sets	VIKOR
2020	Ghorabaee et al.	Fermatean fuzzy sets	WASPAS
2020	Tian et al.	Probabilistic hesitant fuzzy sets	TODIM
2020	Zhou and Chen	Pythagorean fuzzy sets	VIKOR
2021	Zhang et al.	Picture 2-tuple linguistic set	CODAS
2021	Sharaf and Khalil	Spherical fuzzy sets	TODIM
2021	Pınar et al.	Q-rung orthopair fuzzy sets	TOPSIS
2021	Kaur et al.	Pythagorean fuzzy sets	TODIM
2021	Celik et al.	Interval type-2 fuzzy sets	TODIM
2021	Çalık	Pythagorean fuzzy sets	TOPSIS
2021	Lu et al.	Picture fuzzy sets	COPRAS
2021	Sun and Cai	Single-valued neutrosophic sets	TOPSIS & GRA
2021	Mathew et al.	Interval-valued fermatean fuzzy sets	MABAC
2022	Wang et al.	Probabilistic dual hesitant fuzzy sets	BWM
2022	Gai et al.	Linguistic Z-fuzzy numbers	MULTIMOORA
2022	Unal and Temur	Spherical fuzzy sets	AHP
2022	Ecer	Interval type-2 fuzzy sets	AHP
2022	Giri et al.	Pythagorean fuzzy sets	DEMATEL
2023	Hajiaghaei-Keshteli et al.	Pythagorean fuzzy sets	TOPSIS
2023	Zeng et al.	Fermatean fuzzy sets	EDAS
2023	Zhou and Chen	Pythagorean fuzzy sets	TOPSIS
2023	Ashraf et al.	Pythagorean fuzzy Z-numbers	EDAS

Table 7.2 shows that many extensions of fuzzy sets are used for GSS problems. In addition, it is concluded from Table 7.2 that the interval-valued spherical Z-fuzzy AHP method has not been used for GSS problems in the literature, which makes our study unique and important to fill the gap in this research area.

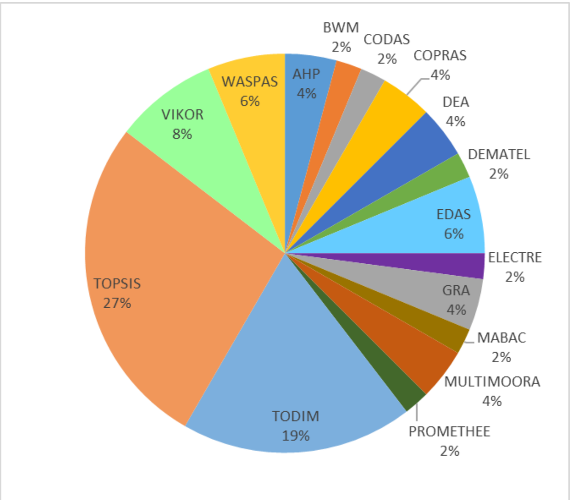


Figure 7.1 : Usage percentages of the fuzzy MCDM methods in GSS.

Figure 7.1 shows the usage percentages of the fuzzy MCDM methods in GSS problems. Based on these results, TOPSIS and TODIM methods are the most used MCDM methods for the solution of GSS problems. As Figure 7.1 indicates, fuzzy extensions of AHP method have also been rarely used for the solution of GSS problems. Our study fills a gap in this area.

7.2 Evaluation Criteria of Green Suppliers

GSS criteria vary according to which kind of supplier is selected. The criteria set discussed in this study include the criteria common to all GSS problems. The criteria and their explanations, which are presented in Table 7.3, have been determined by considering the literature.

Table 7.3 : Criteria set for GSS.

Main criterion	Sub-criterion	Code	Explanations	References
Economic features (C1)	Quality	C11	Conformity of goods to quality standards	Fallahpour et al., 2017; Gupta et al., 2019; Zhang et al., 2020; Rouyendegh et al., 2020; Rani et al., 2020; Yucesan et al., 2019
	Cost	C12	The total price of the goods to reflect to the buyer	Ecer, 2020; Fallahpour et al., 2017; Gupta et al., 2019; Zhang et al., 2020; Rouyendegh et al., 2020; Rani et al., 2020
	Delivery and services	C13	On-time delivery, ratio of ensuring delivery time, after sales service	Fallahpour et al., 2017; Gupta et al., 2019; Zhang et al., 2020; Rouyendegh et al., 2020
Environmental aspects (C2)	Environmentally friendly manufacturin	C21	The least harm to the environment during the production such as green design	Ecer, 2020; Fallahpour et al., 2017; Zhang et al., 2020; Rouyendegh et al., 2020
	Environmental pollution level	C22	Air emission level, hazardous waste, greenhouse gas emission	Gupta et al., 2019; Zhang et al., 2020; Rani et al., 2020
	Green technology	C23	Usage of min energy/material, reuse, recycle etc.	Ecer, 2020; Zhang et al., 2020; Rouyendegh et al., 2020; Khorasani, 2018; Phochanikorn
	Green logistic	C24	Use of environmental friendly transportation	Fallahpour et al., 2017; Zhang et al., 2020
Green competencies (C3)	Green image	C31	Supplier's reputation for green practices, supplier's green certification	Fallahpour et al., 2017; Gupta et al., 2019; Zhang et al., 2020; Rouyendegh et al., 2020; Phochanikorn and Tan, 2019
	Staff training	C32	Environmental trainings for employees	Ecer, 2020; Gupta et al., 2019; Zhang et al., 2020
	Competency management	C33	Green awareness and practices of management	Ecer, 2020; Fallahpour et al., 2017; Zhang et al., 2020; Rouyendegh et al., 2020; Yucesan et al., 2019

7.3 Spherical Fuzzy Sets

7.3.1 MCDM with spherical fuzzy sets

Spherical fuzzy sets (SFSs) are newly proposed fuzzy sets by Kahraman and Kutlu Gündogdu (2018) as an extension of picture fuzzy sets. SFSs include the degrees of membership, non-membership, and neutral membership of an object to a set. In SFSs, each parameter can be defined independently by the DMs between 0 and 1, provided that the squared sum of each parameter is at most 1, thus increasing the representation ability of uncertainty.

Many MCDM methods have been extended to their fuzzy versions by using spherical fuzzy sets as presented in Table 7.4. Z-fuzzy numbers can be employed in the development of these spherical fuzzy versions since they incorporate the reliability of the assigned fuzzy numbers in the decision-making process.

Table 7.4 : Spherical fuzzy sets in MCDM.

Year	Authors	Type of SFSs	Selection Method	Application
2019a	Kutlu Gündoğdu and Kahraman	SVSFSs	TOPSIS	Illustrative example
2019b	Kutlu Gundogdu and Kahraman	SVSFSs	WASPAS	Robot selection
2019c	Kutlu Gundogdu and Kahraman	IVSFSs	TOPSIS	3D printer selection
2019	Kutlu Gundogdu et al.	SVSFSs	VIKOR	Selection of waste disposal site
2019d	Kutlu Gundogdu and Kahraman	SVSFSs	CODAS	Illustrative example
2019e	Kutlu Gundogdu and Kahraman	SVSFSs	VIKOR	Warehouse site selection
2019	Kahraman et al.	SVSFSs	TOPSIS	Hospital location selection
2019f	Kutlu Gundogdu and Kahraman	SVSFSs	AHP	Industrial robot selection
2020	Kutlu Gundogdu and Kahraman	SVSFSs	AHP	Selection of renewable energy location
2020	Kutlu Gundogdu	SVSFSs	MULTIMOORA	Illustrative example
2020b	Liu et al.	Linguistic SFSs	MABAC & TODIM	Evaluation of shared bicycles
2020	Mathew et al.	SVSFSs	TOPSIS	Manufacturing system selection
2021a	Akram et al.	Complex SFSs	ELECTRE	Location selection
2021b	Akram et al.	Complex SFSs	TOPSIS	Selection of best water supply strategy
2021c	Akram et al.	Complex SFSs	VIKOR	Prioritizing of the advertisement aims on Facebook.
2021d	Akram et al.	Complex SF N-soft sets	VIKOR	Firm selection
2021	Gul and Ak	IVSFSs	TOPSIS	Evaluation of failure modes
2021	Sharaf and Khalil	SVSFSs	TODIM	Supplier selection

Table 7.4 (continued): Spherical fuzzy sets in MCDM.

Year	Authors	Type of SFSs	Selection Method	Application
2021	Duleba et al.	IVSFSs	AHP	Evaluation of public transportation development
2021b	Jin et al.	IVSFSs	ORESTE	Evaluation of key quality characteristics
2022	Hamal and Senvar	IVSFSs	MULTIMOORA	Ranking of financial ratios
2022	Candan and Cengiz Toklu	SVSFSs	GRA	Evaluation of countries' industrialization
2022	Unal and Temur	SVSFSs	AHP	Sustainable supplier selection
2022	Aydoğdu and Gül	IVSFSs	ARAS	3D printer selection
2022a	Menekşe and Camgöz Akdağ	SVSFSs	EDAS	Selection of distance education tool
2022b	Menekşe and Camgöz Akdağ	SVSFSs	ELECTRE	Ranking of units audit activity
2022	Kahraman et al.	SVSFSs	CRITIC	Prioritization of supplier selection criteria
2022a	Alkan and Kahraman	SF Z-numbers	AHP	Supplier selection
2022	Zahid et al.	Complex SFSs	ELECTRE	Selection of cadmium removal techniques
2022	Sangwan	IVSFSs	TOPSIS	Evaluation of cloud computing services
2022	Omerali and Kaya	IVSFSs	COPRAS	Evaluation of product lifecycle management applications
2022	Ghoushchi et al.	SVSFSs	CoCoSo	Assessment of wind turbine failure modes
2022	Erdoğan	IVSFSs	MAIRCA	Evaluation of agriculture technologies
2022	Monica and Sangwan	IVSFSs	TOPSIS	Evaluation of cloud computing services
2023a	Ghoushchi et al.	SVSFSs	MARCOS	Risk factor prioritization for roads
2023b	Ghoushchi et al.	SVSFSs	CODAS	Evaluation of clean energy barriers
2023	Aydoğdu et al.	Complex SFSs	TOPSIS	Ranking the objectives of advertisement on social web sites.
2023b	Akram et al.	SVSFSs	PROMETHEE	Hospital site selection
2023	Otay	IVSFSs	MULTIMOORA	Evaluation of tech-center locations
2023	Sharaf	SVSFSs	TOPSIS & VIKOR	Selection of warehouse location & evaluation of hydrogen storage systems
2023	Demircan	IVSFSs	AHP	Evaluation of digital business models.

Table 7.4 shows that spherical fuzzy sets are generally used as ordinary or interval-valued form in the literature. AHP and TOPSIS methods are the most commonly used MCDM methods integrated with spherical fuzzy sets.

7.3.2 Single-valued spherical fuzzy sets (SVSFSs)

In the following, we give the main definitions of single-valued spherical fuzzy sets (Kahraman and Kutlu Gündoğdu, 2018):

Definition 7.1. A single-valued spherical fuzzy set (SVSFS) of the universe X is given in Eq. (7.1) (Kahraman and Kutlu Gündoğdu, 2018; Kutlu Gündoğdu and Kahraman, 2019a; Mahmood et al., 2019).

$$\tilde{A}_s = \{x, \mu_{\tilde{A}_s}(x), \vartheta_{\tilde{A}_s}(x), \pi_{\tilde{A}_s}(x) \mid x \in X\} \tag{7.1}$$

where $\mu_{\tilde{A}_s}(x), \vartheta_{\tilde{A}_s}(x), \pi_{\tilde{A}_s}(x): X \rightarrow [0,1]$ are the degrees of membership, non-membership, and neutral membership (indeterminacy) of x to $\tilde{A}_S$, respectively, and

$$0 \leq \mu_{\tilde{A}_s}^2(x) + \vartheta_{\tilde{A}_s}^2(x) + \pi_{\tilde{A}_s}^2(x) \leq 1 \tag{7.2}$$

Then, $\sqrt{1 - \left[\mu_{\tilde{A}_s}^2(x) + \vartheta_{\tilde{A}_s}^2(x) + \pi_{\tilde{A}_s}^2(x)\right]}$ is defined as the refusal degree of x in X.

Definition 7.2. Let $\tilde{A}_s = (\mu_{A_s}, \vartheta_{A_s}, \pi_{A_s})$ and $\tilde{B}_s = (\mu_{B_s}, \vartheta_{B_s}, \pi_{B_s})$ be any two SFSs. The basic arithmetic operations of SFSs are given in Eqs. (7.3-7.6) (Kutlu Gündoğdu and Kahraman, 2019a).

$$\tilde{A}_s \oplus \tilde{B}_s = \left\{\sqrt{\mu_{\tilde{A}_s}^2 + \mu_{\tilde{B}_s}^2 - \mu_{\tilde{A}_s}^2\mu_{\tilde{B}_s}^2}, \vartheta_{\tilde{A}_s}\vartheta_{\tilde{B}_s}, \sqrt{(1-\mu_{\tilde{B}_s}^2)\pi_{\tilde{A}_s}^2 + (1-\mu_{\tilde{A}_s}^2)\pi_{\tilde{B}_s}^2 - \pi_{\tilde{A}_s}^2\pi_{\tilde{B}_s}^2}\right\} \tag{7.3}$$

$$\tilde{A}_s \otimes \tilde{B}_s = \left\{\mu_{\tilde{A}_s}\mu_{\tilde{B}_s}, \sqrt{\vartheta_{\tilde{A}_s}^2 + \vartheta_{\tilde{B}_s}^2 - \vartheta_{\tilde{A}_s}^2\vartheta_{\tilde{B}_s}^2}, \sqrt{(1-\vartheta_{\tilde{B}_s}^2)\pi_{\tilde{A}_s}^2 + (1-\vartheta_{\tilde{A}_s}^2)\pi_{\tilde{B}_s}^2 - \pi_{\tilde{A}_s}^2\pi_{\tilde{B}_s}^2}\right\} \tag{7.4}$$

$$k\tilde{A}_s = \left\{\sqrt{1-(1-\mu_{\tilde{A}_s}^2)^k}, \vartheta_{\tilde{A}_s}^k, \sqrt{(1-\mu_{\tilde{A}_s}^2)^k - (1-\mu_{\tilde{A}_s}^2 - \pi_{\tilde{A}_s}^2)^k}\right\}; k > 0 \tag{7.5}$$

$$\tilde{A}_s^k = \left\{\mu_{\tilde{A}_s}^k, \sqrt{1-(1-\vartheta_{\tilde{A}_s}^2)^k}, \sqrt{(1-\vartheta_{\tilde{A}_s}^2)^k - (1-\vartheta_{\tilde{A}_s}^2 - \pi_{\tilde{A}_s}^2)^k}\right\}; \quad k > 0 \tag{7.6}$$

Definition 7.3. For two SFSs $\tilde{A}_s = (\mu_{A_s}, \vartheta_{A_s}, \pi_{A_s})$ and $\tilde{B}_s = (\mu_{B_s}, \vartheta_{B_s}, \pi_{B_s})$, Eqs. (7.7-7.12) are valid for the conditions of k, k_1 and $k_2 \geq 0$ (Kutlu Gündoğdu and Kahraman, 2019a):

$$\tilde{A}_s \oplus \tilde{B}_s = \tilde{B}_s \oplus \tilde{A}_s \tag{7.7}$$

$$\tilde{A}_s \otimes \tilde{B}_s = \tilde{B}_s \otimes \tilde{A}_s \tag{7.8}$$

$$k(\tilde{A}_s \oplus \tilde{B}_s) = k\tilde{A}_s \oplus k\tilde{B}_{s_s} \tag{7.9}$$

$$k_1\tilde{A}_s \oplus k_2\tilde{A}_s = (k_1 + k_2)\tilde{A}_s \tag{7.10}$$

$$(\tilde{A}_s \otimes \tilde{B}_s)^k = \tilde{A}_s^k \otimes \tilde{B}_s^k \tag{7.11}$$

$$\tilde{A}_s^{k_1} \otimes \tilde{A}_s^{k_2} = \tilde{A}_s^{k_1+k_2} \tag{7.12}$$

Definition 7.4. To compare two spherical fuzzy numbers (SFNs) $\tilde{A}_s = (\mu_{A_s}, \vartheta_{A_s}, \pi_{A_s})$ and $\tilde{B}_s = (\mu_{B_s}, \vartheta_{B_s}, \pi_{B_s})$, score (SC) and accuracy (AC) functions are defined in Eqs. (7.13-7.14) (Kutlu Gündoğdu and Kahraman, 2019a):

$$SC(\tilde{A}_s) = (\mu_{\tilde{A}_s} - \pi_{\tilde{A}_s})^2 - (\vartheta_{\tilde{A}_s} - \pi_{\tilde{A}_s})^2 \tag{7.13}$$

$$AC(\tilde{A}_s) = \mu_{\tilde{A}_s}^2 + \vartheta_{\tilde{A}_s}^2 + \pi_{\tilde{A}_s}^2 \tag{7.14}$$

where $AC(\tilde{A}_s) \in [0, 1]$

Comparison rules of SFSs are given in the following (Kahraman and Kutlu Gündoğdu, 2018; Mahmood et al., 2019):

If $SC(\tilde{A}_s) > SC(\tilde{B}_s)$, then $\tilde{A}_s > \tilde{B}_s$;

If $SC(\tilde{A}_s) = SC(\tilde{B}_s)$ and $AC(\tilde{A}_s) > AC(\tilde{B}_s)$, then $\tilde{A}_s > \tilde{B}_s$;

If $SC(\tilde{A}_s) = SC(\tilde{B}_s)$, $AC(\tilde{A}_s) < AC(\tilde{B}_s)$, then $\tilde{A}_s < \tilde{B}_s$;

If $SC(\tilde{A}_s) = SC(\tilde{B}_s)$, $AC(\tilde{A}_s) = AC(\tilde{B}_s)$, then $\tilde{A}_s = \tilde{B}_s$.

Definition 7.5. SF Weighted Arithmetic Mean ($SFWAM$) with respect to, $w = (w_1, w_2, \ldots, w_n)$; $w_i \in [0,1]$; $\sum_{i=1}^{n} w_i = 1$, is defined in Eq. (7.15) (Kahraman and Kutlu Gündoğdu, 2019b).

$$SFWAM_w(\tilde{A}_{S1}, \tilde{A}_{S2}, \ldots, \tilde{A}_{Sn}) =$$
$$\left\{ \sqrt{1 - \prod_{i=1}^{n} \left(1 - \mu_{A_s}^2\right)^{w_i}}, \prod_{i=1}^{n} \vartheta_{A_s}^{w_i}, \sqrt{\prod_{i=1}^{n} (1 - \mu_{A_s}^2)^{w_i} - \prod_{i=1}^{n} (1 - \mu_{A_s}^2 - \pi_{A_s}^2)^{w_i}} \right\} \tag{7.15}$$

Definition 7.6. SF Weighted Geometric Mean ($SFWGM$) with respect to, $w = (w_1, w_2, \ldots, w_n)$; $w_i \in [0,1]$; $\sum_{i=1}^{n} w_i = 1$, is given in Eq. (7.16) (Kahraman and Kutlu Gundogdu, 2019b).

$$SFWGM_w(\tilde{A}_{S1}, \tilde{A}_{S2}, \ldots, \tilde{A}_{Sn}) = \tilde{A}_{S1}{}^{w_i} + \tilde{A}_{S2}{}^{w_i} + \cdots + \tilde{A}_{Sn}{}^{w_i}$$
$$= \left\{ \prod_{i=1}^{n} \mu_{A_s}^{w_i}, \sqrt{1 - \prod_{i=1}^{n} \left(1 - \vartheta_{A_s}^2\right)^{w_i}}, \sqrt{\prod_{i=1}^{n} (1 - \vartheta_{A_s}^2)^{w_i} - \prod_{i=1}^{n} (1 - \vartheta_{A_s}^2 - \pi_{A_s}^2)^{w_i}} \right\} \tag{7.16}$$

7.3.3 Interval-valued spherical fuzzy sets (IVSFSs)

IVSFSs provide a representation of uncertainty by allowing the values of parameters to be defined as intervals. IVSFSs have been proposed by Kutlu Gündoğdu and Kahraman (2019c). Membership, non-membership, and hesitancy degrees in IVSFSs are defined as intervals instead of single values. In this section, we give some definitions and mathematical operations of IVSFSs.

Definition 7.7: An IVSFS $\tilde{A}_S$ of the universe X is given by Eq. (7.17) (Kutlu Gündoğdu and Kahraman, 2019c).

$$\tilde{A}_S = \left\{ \left\langle x, \left(\left[\mu^L_{\tilde{A}_S}(x),\ \mu^U_{\tilde{A}_S}(x) \right], \left[\vartheta^L_{\tilde{A}_S}(x), \vartheta^U_{\tilde{A}_S}(x) \right], \left[\pi^L_{\tilde{A}_S}(x), \pi^U_{\tilde{A}_S}(x) \right] \right) \right| x \in X \right\} \quad (7.17)$$

where $0 \le \mu^L_{\tilde{A}_S}(x) \le \mu^U_{\tilde{A}_S}(x) \le 1$, $0 \le \vartheta^L_{\tilde{A}_S}(x) \le \vartheta^U_{\tilde{A}_S}(x) \le 1$, $0 \le \pi^L_{\tilde{A}_S}(x) \le \pi^U_{\tilde{A}_S}(x) \le 1$ and $0 \le \left(\mu^U_{\tilde{A}_S}(x)\right)^2 + \left(\vartheta^U_{\tilde{A}_S}(x)\right)^2 + \left(\pi^U_{\tilde{A}_S}(x)\right)^2 \le 1$.

For each $x \in X$, $\mu^U_{\tilde{A}_S}(x), \vartheta^U_{\tilde{A}_S}(x)$ and $\pi^U_{\tilde{A}_S}(x)$ denote the upper membership degree, upper non-membership degree and upper hesitancy degree of x to $\tilde{A}_S$, respectively. Similarly, for each $x \in X$, $\mu^L_{\tilde{A}_S}(x), \vartheta^L_{\tilde{A}_S}(x)$ and $\pi^L_{\tilde{A}_S}(x)$ denote the lower membership degree, lower non-membership degree and lower hesitancy degree of x to $\tilde{A}_S$, respectively. For each $x \in X$, if $\mu^L_{\tilde{A}_S}(x) = \mu^U_{\tilde{A}_S}(x)$, $\vartheta^L_{\tilde{A}_S}(x) = \vartheta^U_{\tilde{A}_S}(x)$, and $\pi^L_{\tilde{A}_S}(x) = \pi^U_{\tilde{A}_S}(x)$ then, IVSFS $\tilde{A}_S$ refers to as a single-valued SFS.

Definition 7.8: Let $\tilde{\alpha}_1 = \langle [a_1,\ b_1], [c_1, d_1], [e_1, f_1] \rangle$ and $\tilde{\alpha}_2 = \langle [a_2, b_2], [c_2, d_2], [e_2, f_2] \rangle$ be two IVSFSs. Basic arithmetic operations are given by Eqs. (7.18-7.21) (Kutlu Gündoğdu and Kahraman, 2019c).

$$\tilde{\alpha}_1 \oplus \tilde{\alpha}_2 = \left\{ \begin{array}{c} \left[\sqrt{a_1^2 + a_2^2 - a_1^2 a_2^2}, \sqrt{b_1^2 + b_2^2 - b_1^2 b_2^2} \right], [c_1 c_2, d_1 d_2], \\ \left[\sqrt{(1 - a_2^2)e_1^2 + (1 - a_1^2)e_2^2 - e_1^2 e_2^2}, \sqrt{(1 - b_2^2)f_1^2 + (1 - b_1^2)f_2^2 - f_1^2 f_2^2} \right] \end{array} \right\} \quad (7.18)$$

$$\tilde{\alpha}_1 \otimes \tilde{\alpha}_2 = \left\{ \begin{array}{c} [a_1 a_2, b_1 b_2], \left[\sqrt{c_1^2 + c_2^2 - c_1^2 c_2^2}, \sqrt{d_1^2 + d_2^2 - d_1^2 d_2^2} \right], \\ \left[1 - b_1^2 b_2^2 - (d_1^2 + d_2^2 - d_1^2 d_2^2), 1 - a_1^2 a_2^2 - (c_1^2 + c_2^2 - c_1^2 c_2^2) \right] \end{array} \right\} \quad (7.19)$$

Multiplication by a scalar; $k \ge 0$;

$$k \cdot \tilde{\alpha}_1 =$$

$$\left\{\begin{matrix} \left[\sqrt{1-(1-a_1^2)^k}, \sqrt{1-(1-b_1^2)^k}\right], [c_1^k, d_1^k], \\ \left[\sqrt{(1-a_1^2)^k-(1-a_1^2-e_1^2)^k}, \sqrt{(1-b_1^2)^k-(1-b_1^2-f_1^2)^k}\right] \end{matrix}\right\} \tag{7.20}$$

k^{th} Power of $\tilde{\alpha}$; $k \geq 0$;

$$\tilde{\alpha}_1^k =$$

$$\left\{\begin{matrix} [a_1^k, b_1^k], \left[\sqrt{1-(1-c_1^2)^k}, \sqrt{1-(1-d_1^2)^k}\right], \\ \left[\sqrt{(1-c_1^2)^k-(1-c_1^2-e_1^2)^k}, \sqrt{(1-d_1^2)^k-(1-d_1^2-f_1^2)^k}\right] \end{matrix}\right\} \tag{7.21}$$

Definition 7.9: Let $\tilde{\alpha}_j = \langle [a_j,\ b_j], [c_j, d_j], [e_j, f_j] \rangle$ be a collection of IVSFS with respect to $w_j = (w_1, w_2, \ldots, w_n)$; $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$. Interval-Valued Spherical Weighted Arithmetic Mean (IVSWAM) is presented in Eq. (7.22) (Kutlu Gündoğdu and Kahraman, 2019c).

$$IVSWAM_w(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n) = w_1 \cdot \tilde{\alpha}_1 \oplus\ w_2 \cdot \tilde{\alpha}_2 \oplus\ \ldots \oplus w_n \cdot \tilde{\alpha}_n$$
$$= \left\{\begin{matrix} \left[\sqrt{1-\prod_{j=1}^{n}\left(1-a_j^2\right)^{w_j}}, \sqrt{1-\prod_{j=1}^{n}\left(1-b_j^2\right)^{w_j}}\right], \\ \left[\prod_{j=1}^{n} c_j^{w_j}, \prod_{j=1}^{n} d_j^{w_j}\right], \\ \left[\begin{matrix} \sqrt{\prod_{j=1}^{n}\left(1-a_j^2\right)^{w_j}-\prod_{j=1}^{n}\left(1-a_j^2-e_j^2\right)^{w_j}}, \\ \sqrt{\prod_{j=1}^{n}\left(1-b_j^2\right)^{w_j}-\prod_{j=1}^{n}\left(1-b_j^2-f_j^2\right)^{w_j}} \end{matrix}\right] \end{matrix}\right\} \tag{7.22}$$

Definition 7.10: Let $\tilde{\alpha}_j = \langle [a_j,\ b_j], [c_j, d_j], [e_j, f_j] \rangle$ be a collection of IVSFS with respect to $w_j = (w_1, w_2, \ldots, w_n)$; $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$. Interval-Valued Spherical Weighted Geometric Mean (IVSWGM) is given in Eq. (7.23) (Kutlu Gündoğdu and Kahraman, 2019c).

$$IVSWGM_w(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n) = \alpha_1^{w_1} \otimes\ \alpha_2^{w_2} \otimes\ \ldots \otimes \alpha_n^{w_n}$$
$$= \left\{\begin{matrix} \left[\prod_{j=1}^{n} a_j^{w_j}, \prod_{j=1}^{n} b_j^{w_j}\right], \\ \left[\sqrt{1-\prod_{j=1}^{n}\left(1-c_j^2\right)^{w_j}}, \sqrt{1-\prod_{j=1}^{n}\left(1-d_j^2\right)^{w_j}}\right], \\ \left[\begin{matrix} \sqrt{\prod_{j=1}^{n}\left(1-c_j^2\right)^{w_j}-\prod_{j=1}^{n}\left(1-c_j^2-e_j^2\right)^{w_j}}, \\ \sqrt{\prod_{j=1}^{n}\left(1-d_j^2\right)^{w_j}-\prod_{j=1}^{n}\left(1-d_j^2-f_j^2\right)^{w_j}} \end{matrix}\right] \end{matrix}\right\} \tag{7.23}$$

Definition 7.11: The score function of an IVSF number ($\tilde{\alpha} = \langle[a,b],[c,d],[e,f]\rangle$) is given in Eq. (7.24) (Kutlu Gündoğdu and Kahraman, 2019c).

$$Score(\tilde{\alpha}) = S(\tilde{\alpha}) = \frac{a^2+b^2-c^2-d^2-\left(e/_2\right)^2-\left(f/_2\right)^2}{2} \tag{7.24}$$

where $Score(\tilde{\alpha}) = S(\tilde{\alpha}) \in [-1,+1]$. Obviously, the greater the $S(\tilde{\alpha})$, the larger the α. In particular, when $S(\tilde{\alpha}) = 1$ then $\tilde{\alpha} = \langle[1,1],[0,0],[0,0]\rangle$; when $S(\tilde{\alpha}) = -1$ then $\tilde{\alpha} = \langle[0,0],[1,1],[0,0]\rangle$.

Definition 7.12: The accuracy function of an IVSF number is given in Eq. (7.25) (Kutlu Gündoğdu and Kahraman, 2019c).

$$Accuracy(\tilde{\alpha}) = H(\tilde{\alpha}) = \frac{a^2+b^2+c^2+d^2+e^2+f^2}{2} \tag{7.25}$$

where $H(\tilde{\alpha}) \in [0,1]$.

Note that: $\tilde{\alpha}_1 < \tilde{\alpha}_2$ if and only if $S(\tilde{\alpha}_1) < S(\tilde{\alpha}_2)$ or $S(\tilde{\alpha}_1) = S(\tilde{\alpha}_2)$ and $H(\tilde{\alpha}_1) < H(\tilde{\alpha}_2)$

Definition 7.13: Let $\tilde{\alpha}_1 = \langle[a_1, b_1],[c_1,d_1],[e_1,f_1]\rangle$ and $\tilde{\alpha}_2 = \langle[a_2, b_2],[c_2,d_2],[e_2,f_2]\rangle$ be two IVSF numbers, the distance between $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ is given in Eq. (7.26) (Peng and Yang, 2016):

$$d(\tilde{\alpha}_1,\tilde{\alpha}_2) = \frac{1}{4}\begin{pmatrix} |a_1^2-a_2^2| + |b_1^2-b_2^2| + |c_1^2-c_2^2| \\ +|d_1^2-d_2^2| + |e_1^2-e_2^2| + |f_1^2-f_2^2| \end{pmatrix} \tag{7.26}$$

7.3.4 Z-fuzzy numbers

A Z-fuzzy number is a 2-tuple fuzzy number, $Z(\tilde{A},\tilde{R})$, consists of restriction function ($\tilde{A}$) and the reliability function ($\tilde{R}$) of it, as shown in Figure 7.2.

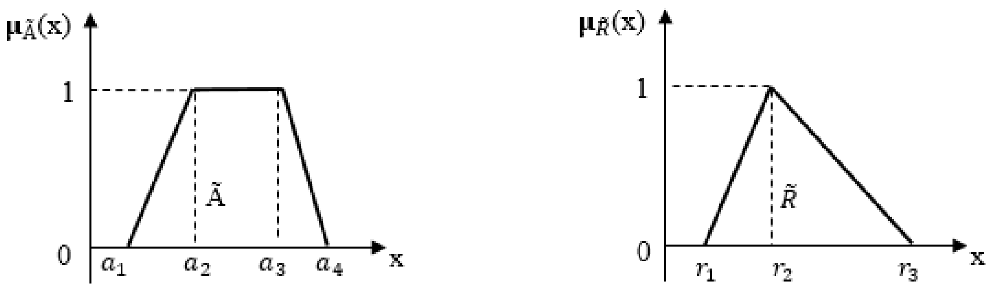


Figure 7.2 : A simple Z-fuzzy number, $Z(\tilde{A},\tilde{R})$.

Z-fuzzy numbers allow calculations with fuzzy numbers containing reliability information.

Due to the computational complexity of Z-fuzzy numbers, the conversion of Z-fuzzy numbers to ordinary fuzzy numbers is commonly used. Kang et al. (2012a) propose an approach to convert Z-fuzzy numbers to regular fuzzy numbers as given in Definition (7.14):

Definition 7.14: Let $Z = (\tilde{A}, \tilde{R})$ be a Z-fuzzy number as given in Figure 7.2 whose left side represents the trapezoidal fuzzy restriction function, and the right side represents the triangular fuzzy reliability function, where $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x)\rangle | \mu(x) \in [0,1]\}$ and $\tilde{R} = \{\langle x, \mu_{\tilde{R}}(x)\rangle | \mu(x) \in [0,1]\}$. $\mu_{\tilde{A}}(x)$ and $\mu_{\tilde{R}}(x)$ are the membership functions of restriction and reliability, respectively.

The fuzzy reliability function is converted to its corresponding crisp number using Eq. (7.27):

$$\alpha = \frac{\int x\mu_{\tilde{R}}(x)\, dx}{\int \mu_{\tilde{R}}(x)\, dx} \tag{7.27}$$

where $\int$ represents an algebraic integration.

The corresponding crisp value of reliability function is incorporated to the restriction function using Eq. (7.28) and the weighted restriction number ($\tilde{Z}^{\alpha}$) is obtained.

$$\tilde{Z}^{\alpha} = \{\langle x, \mu_{\tilde{A}^{\alpha}}(x)\rangle | \mu_{\tilde{A}^{\alpha}}(x) = \alpha\mu_{\tilde{A}}(x), \mu(x) \in [0,1]\} \tag{7.28}$$

The weighted restriction number is converted to ordinary fuzzy number by Eq. (7.29). The obtained ordinary fuzzy number is given in Figure 7.3.

$$\tilde{Z}' = \left\{\langle x, \mu_{\tilde{Z}'}(x)\rangle \middle| \mu_{\tilde{Z}'}(x) = \mu_{\tilde{A}}\left(\frac{x}{\sqrt{\alpha}}\right), \mu(x) \in [0,1]\right\} \tag{7.29}$$

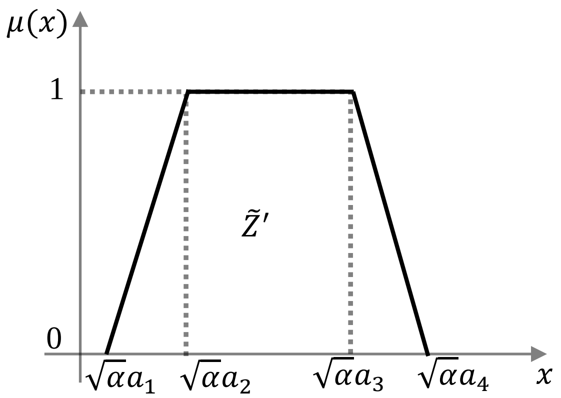


Figure 7.3 : Ordinary fuzzy number converted from Z-fuzzy number.

If the heights of restriction and reliability functions are any value between 0 and 1, then the modified calculations are given in Tüysüz and Kahraman (2023).

7.4 The Proposed Method: Interval-Valued Spherical Fuzzy Z-AHP

Steps of the interval-valued spherical fuzzy Z-AHP (IVSF Z-AHP) method are presented in this section. The flowchart of the proposed method is given in Figure 7.4.

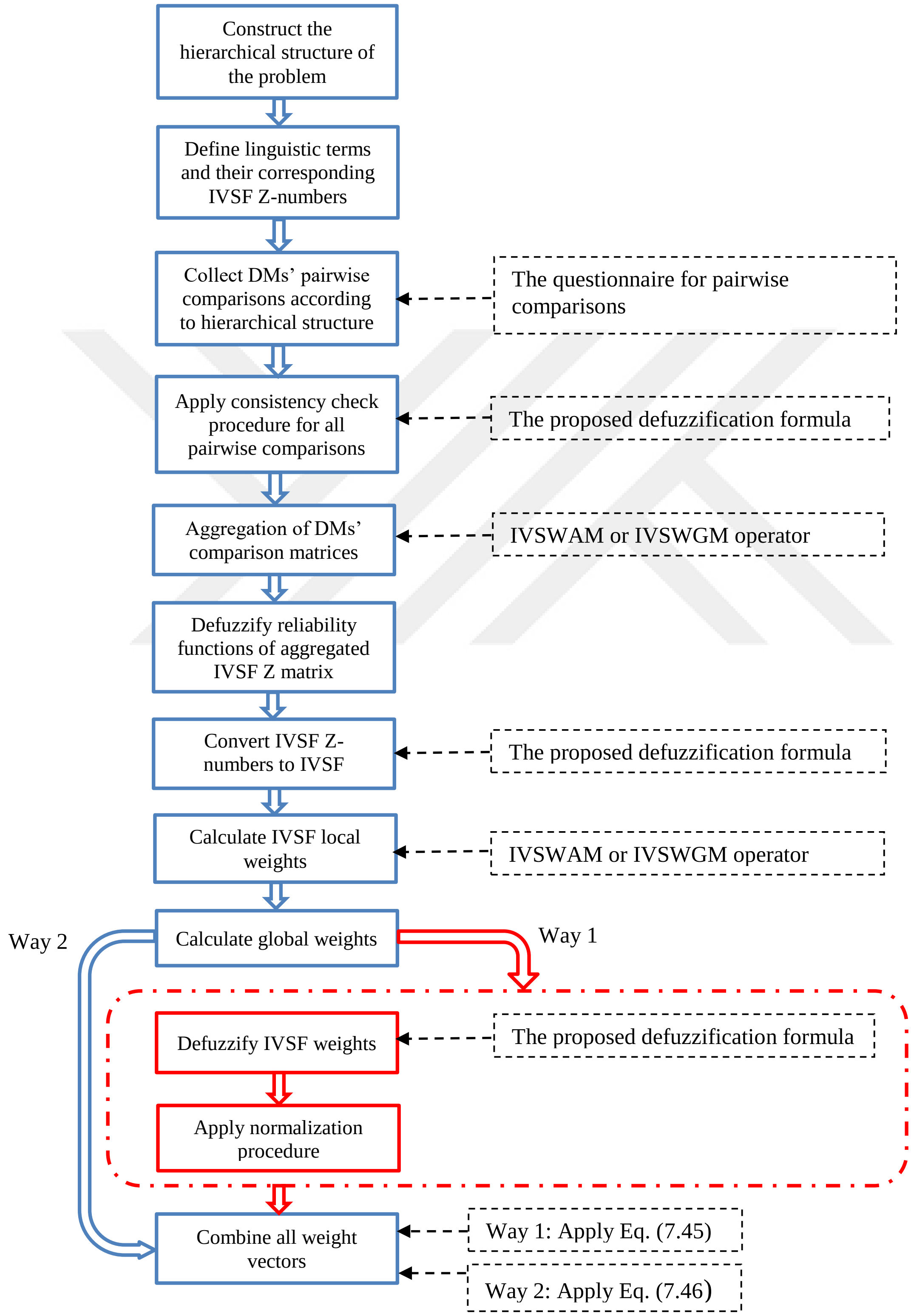


Figure 7.4 : Flowchart of the proposed IVSF Z-AHP method.

Step 1. The hierarchical structure consisting of goal, main-criteria (j=1,2,…,n), sub-criteria (s=1,2,…t) and alternatives (i=1,2,…,m) related to the decision problem is constructed as given in Figure 7.5.

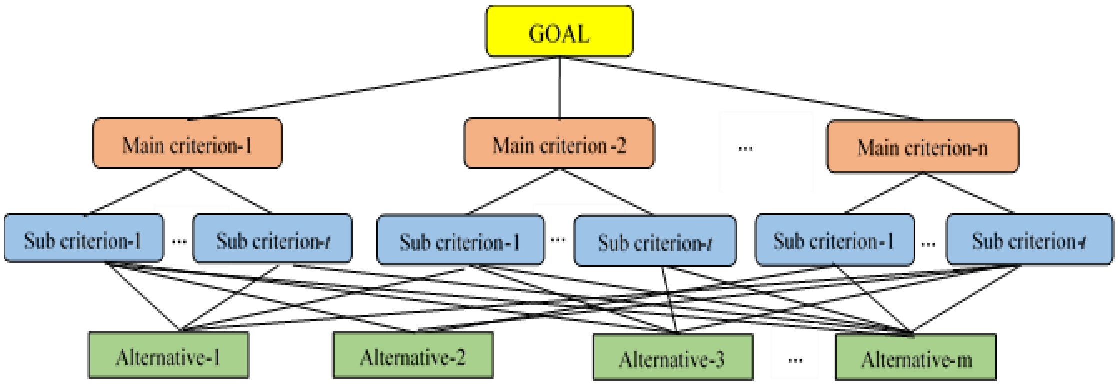


Figure 7.5 : Hierarchical structure for decision problem.

Step 2. Define the linguistic terms and their corresponding IVSF Z-numbers for both restriction and reliability functions that will be used in DMs' evaluations. In this step, we define new IVSF Z restriction and reliability scales as in Tables 7.5 and 7.6. Then, collect the pairwise comparisons from k (k =1,2,…,K) DMs using these linguistic expressions.

A linguistic term in Tables 7.5 and 7.6 can be represented by Eq. (7.30).

$$\tilde{Z}^{DM_k} = (\tilde{A}, \tilde{R}) = \begin{pmatrix} \left([\mu_{A,L}^{DM_k}, \mu_{A,U}^{DM_k}], [\vartheta_{A,L}^{DM_k}, \vartheta_{A,U}^{DM_k}], [\pi_{A,L}^{DM_k}, \pi_{A,U}^{DM_k}]\right), \\ \left([\mu_{R,L}^{DM_k}, \mu_{R,U}^{DM_k}], [\vartheta_{R,L}^{DM_k}, \vartheta_{R,U}^{DM_k}], [\pi_{R,L}^{DM_k}, \pi_{R,U}^{DM_k}]\right) \end{pmatrix} \tag{7.30}$$

where $\mu_{A,L}^{DM_k}, \vartheta_{A,L}^{DM_k}$ and $\pi_{A,L}^{DM_k}$ represent the lower membership degree, lower non-membership degree and lower hesitancy degree of x to $\tilde{A}$ for restriction function of k^{th} DM, respectively. $\mu_{R,L}^{DM_k}, \vartheta_{R,L}^{DM_k}$ and $\pi_{R,L}^{DM_k}$ represent the lower membership degree, lower non-membership degree and lower hesitancy degree of x to $\tilde{R}$ for reliability function of k^{th} DM. Other parameters represent upper bounds in both functions.

Table 7.5 : IVSF Z restriction scale for pairwise comparisons.

Linguistic Terms	Abbr.	$([\mu_L, \mu_U], [\vartheta_L, \vartheta_U], [\pi_L, \pi_U])$
Absolutely High Importance	AHI	([0.80, 0.95],[0.00, 0.15],[0.00, 0.15])
Very High Importance	VHI	([0.70, 0.85],[0.10, 0.25],[0.10, 0.25])
High Importance	HI	([0.60, 0.75],[0.20, 0.35],[0.20, 0.35])
Slightly High Importance	SHI	([0.50, 0.65],[0.30, 0.45],[0.30, 0.45])
Equally Importance	EI	([0.40, 0.55],[0.40, 0.55],[0.40, 0.55])
Slightly Low Importance	SLI	([0.30, 0.45],[0.50, 0.65],[0.30, 0.45])
Low Importance	LI	([0.20, 0.35],[0.60, 0.75],[0.20, 0.35])
Very Low Importance	VLI	([0.10, 0.25],[0.70, 0.85],[0.10, 0.25])
Absolutely Low Importance	ALI	([0.00, 0.15],[0.80, 0.95],[0.00, 0.15])

Table 7.6 : IVSF Z reliability scale for pairwise comparisons.

Linguistic Terms	Abbr.	$([\mu_L, \mu_U], [\vartheta_L, \vartheta_U], [\pi_L, \pi_U])$
Very Strongly Reliable	VSR	([0.80, 0.95],[0.00, 0.15],[0.00, 0.15])
Strongly Reliable	SR	([0.70, 0.85],[0.10, 0.25],[0.10, 0.25])
Very Highly Reliable	VHR	([0.60, 0.75],[0.20, 0.35],[0.20, 0.35])
Highly Reliable	HR	([0.50, 0.65],[0.30, 0.45],[0.30, 0.45])
Fairly Reliable	FR	([0.40, 0.55],[0.40, 0.55],[0.40, 0.55])
Weakly Reliable	WR	([0.30, 0.45],[0.50, 0.65],[0.30, 0.45])
Very Weakly Reliable	VWR	([0.20, 0.35],[0.60, 0.75],[0.20, 0.35])
Strongly Unreliable	SU	([0.10, 0.25],[0.70, 0.85],[0.10, 0.25])
Absolutely Unreliable	AU	([0.00, 0.15],[0.80, 0.95],[0.00, 0.15])

In Table 7.6, for instance, the expression "strongly reliable" means that DMs are strongly sure from their judgments. Similarly, the expression “weakly reliable” means that DMs are weakly sure from their judgments. The IVSF Z pairwise comparison matrix is shown in Table 7.7 for criteria.

Table 7.7 : IVSF Z pairwise comparison matrix for criteria.

	C_1	...	C_n
C_1	$\begin{pmatrix}([\mu^{DMk}_{A,L_{11}},\mu^{DMk}_{A,U_{11}}],[\vartheta^{DMk}_{A,L_{11}},\vartheta^{DMk}_{A,U_{11}}],[\pi^{DMk}_{A,L_{11}},\pi^{DMk}_{A,U_{11}}]),\\([\mu^{DMk}_{R,L_{11}},\mu^{DMk}_{R,U_{11}}],[\vartheta^{DMk}_{R,L_{11}},\vartheta^{DMk}_{R,U_{11}}],[\pi^{DMk}_{R,L_{11}},\pi^{DMk}_{R,U_{11}}])\end{pmatrix}$	...	$\begin{pmatrix}([\mu^{DMk}_{A,L_{1n}},\mu^{DMk}_{A,U_{1n}}],[\vartheta^{DMk}_{A,L_{1n}},\vartheta^{DMk}_{A,U_{1n}}],[\pi^{DMk}_{A,L_{1n}},\pi^{DMk}_{A,U_{1n}}]),\\([\mu^{DMk}_{R,L_{1n}},\mu^{DMk}_{R,U_{1n}}],[\vartheta^{DMk}_{R,L_{1n}},\vartheta^{DMk}_{R,U_{1n}}],[\pi^{DMk}_{R,L_{1n}},\pi^{DMk}_{R,U_{1n}}])\end{pmatrix}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
C_n	$\begin{pmatrix}([\mu^{DMk}_{A,L_{n1}},\mu^{DMk}_{A,U_{n1}}],[\vartheta^{DMk}_{A,L_{n1}},\vartheta^{DMk}_{A,U_{n1}}],[\pi^{DMk}_{A,L_{n1}},\pi^{DMk}_{A,U_{n1}}]),\\([\mu^{DMk}_{R,L_{n1}},\mu^{DMk}_{R,U_{n1}}],[\vartheta^{DMk}_{R,L_{n1}},\vartheta^{DMk}_{R,U_{n1}}],[\pi^{DMk}_{R,L_{n1}},\pi^{DMk}_{R,U_{n1}}])\end{pmatrix}$	...	$\begin{pmatrix}([\mu^{DMk}_{A,L_{nn}},\mu^{DMk}_{A,U_{nn}}],[\vartheta^{DMk}_{A,L_{nn}},\vartheta^{DMk}_{A,U_{nn}}],[\pi^{DMk}_{A,L_{nn}},\pi^{DMk}_{A,U_{nn}}]),\\([\mu^{DMk}_{R,L_{nn}},\mu^{DMk}_{R,U_{nn}}],[\vartheta^{DMk}_{R,L_{nn}},\vartheta^{DMk}_{R,U_{nn}}],[\pi^{DMk}_{R,L_{nn}},\pi^{DMk}_{R,U_{nn}}])\end{pmatrix}$

The IVSF Z pairwise comparison matrix for alternatives is shown in Table 7.8 with respect to each main criterion or each sub-criterion.

Table 7.8 : IVSF Z pairwise comparison matrix for alternatives.

	A_1	...	A_m
A_1	$\begin{pmatrix}([\mu^{DMk}_{A,L_{11}},\mu^{DMk}_{A,U_{11}}],[\vartheta^{DMk}_{A,L_{11}},\vartheta^{DMk}_{A,U_{11}}],[\pi^{DMk}_{A,L_{11}},\pi^{DMk}_{A,U_{11}}]),\\([\mu^{DMk}_{R,L_{11}},\mu^{DMk}_{R,U_{11}}],[\vartheta^{DMk}_{R,L_{11}},\vartheta^{DMk}_{R,U_{11}}],[\pi^{DMk}_{R,L_{11}},\pi^{DMk}_{R,U_{11}}])\end{pmatrix}$	...	$\begin{pmatrix}([\mu^{DMk}_{A,L_{1m}},\mu^{DMk}_{A,U_{1m}}],[\vartheta^{DMk}_{A,L_{1m}},\vartheta^{DMk}_{A,U_{1m}}],[\pi^{DMk}_{A,L_{1m}},\pi^{DMk}_{A,U_{1m}}]),\\([\mu^{DMk}_{R,L_{1m}},\mu^{DMk}_{R,U_{1m}}],[\nu^{DMk}_{R,L_{1m}},\nu^{DMk}_{R,U_{1m}}],[\pi^{DMk}_{R,L_{1m}},\pi^{DMk}_{R,U_{1m}}])\end{pmatrix}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
A_m	$\begin{pmatrix}([\mu^{DMk}_{A,L_{m1}},\mu^{DMk}_{A,U_{m1}}],[\vartheta^{DMk}_{A,L_{m1}},\vartheta^{DMk}_{A,U_{m1}}],[\pi^{DMk}_{A,L_{m1}},\pi^{DMk}_{A,U_{m1}}]),\\([\mu^{DMk}_{R,L_{m1}},\mu^{DMk}_{R,U_{m1}}],[\vartheta^{DMk}_{R,L_{m1}},\vartheta^{DMk}_{R,U_{m1}}],[\pi^{DMk}_{R,L_{m1}},\pi^{DMk}_{R,U_{m1}}])\end{pmatrix}$	...	$\begin{pmatrix}([\mu^{DMk}_{A,L_{mm}},\mu^{DMk}_{A,U_{mm}}],[\vartheta^{DMk}_{A,L_{mm}},\vartheta^{DMk}_{A,U_{mm}}],[\pi^{DMk}_{A,L_{mm}},\pi^{DMk}_{A,U_{mm}}]),\\([\mu^{DMk}_{R,L_{mm}},\mu^{DMk}_{R,U_{mm}}],[\vartheta^{DMk}_{R,L_{mm}},\vartheta^{DMk}_{R,U_{mm}}],[\pi^{DMk}_{R,L_{mm}},\pi^{DMk}_{R,U_{mm}}])\end{pmatrix}$

Step 3. For each DM’s pairwise comparison matrix, the consistency ratio (CR) is calculated. For this purpose, we propose new defuzzification functions as given in Eqs. (7.31-7.32) to obtain corresponding crisp values of restriction and reliability functions of IVSF Z-numbers. After the DMs’ pairwise comparisons are collected, for consistency measurement, restriction component of each judgment is transformed to its crisp value, and then Saaty’s (1980) classical consistency ratio (CR) is calculated for each pairwise comparison matrix. If CR is smaller than 0.1, the pairwise

comparison matrix is accepted within the allowable consistency limits. Otherwise, experts should reevaluate their pairwise comparisons.

Let an IVSF restriction number be $\tilde{\alpha} = \langle [a,b],[c,d],[e,f] \rangle$. The proposed defuzzification formula for this restriction function is given by Eq. (7.31):

$$Def_{\tilde{A}} = \begin{cases} 7*a^2+5*b^2-3*c^2-3*d^2-\left(e/_2\right)^2-\left(f/_2\right)^2, & for \text{ AHI, VHI, HI, SHI, EI.} \\ \left|\dfrac{1}{3*a^2+3*b^2-7*c^2-5*d^2-\left(e/_2\right)^2-\left(f/_2\right)^2}\right|, & for \text{ SLI, LI, VLI, ALI.} \end{cases} \quad (7.31)$$

Let an IVSF reliability number be $\tilde{\alpha} = \langle [a,b],[c,d],[e,f] \rangle$. The proposed defuzzification formula for this reliability function is given by Eq. (7.32):

$$Def_{\tilde{R}} = \begin{cases} \dfrac{\left(7*a^2+5*b^2-3*c^2-3*d^2-\left(e/_2\right)^2-\left(f/_2\right)^2\right)}{10}, & for \text{ VSR, SR, VHR, HR, FR.} \\ \left|\dfrac{1}{10*\left(3*a^2+3*b^2-7*c^2-5*d^2-\left(e/_2\right)^2-\left(f/_2\right)^2\right)}\right|, & for \text{ WR, VWR, SU, AU.} \end{cases} \quad (7.32)$$

Step 4. The IVSF Z matrices of all DMs are aggregated to obtain a single decision matrix for the calculation process using IVSWAM or IVSWGM operators that are given in Definitions 7.9 and 7.10, respectively.

Assume three DMs assign the following IVSF Z-numbers:

$$\tilde{Z}^{DM1} = (\tilde{A},\tilde{R}) = \begin{pmatrix} \left([\mu_{A,L_{ij}}^{DM1},\mu_{A,U_{ij}}^{DM1}],[\vartheta_{A,L_{ij}}^{DM1},\vartheta_{A,U_{ij}}^{DM1}],[\pi_{A,L_{ij}}^{DM1},\pi_{A,U_{ij}}^{DM1}]\right), \\ \left(\left[\mu_{R,L_{ij}}^{DM1},\mu_{R,U_{ij}}^{DM1}\right],[\vartheta_{R,L_{ij}}^{DM1},\vartheta_{R,U_{ij}}^{DM1}],[\pi_{R,L_{ij}}^{DM1},\pi_{R,U_{ij}}^{DM1}]\right) \end{pmatrix} \quad (7.33)$$

$$\tilde{Z}^{DM2} = (\tilde{A},\tilde{R}) = \begin{pmatrix} \left([\mu_{A,L_{ij}}^{DM2},\mu_{A,U_{ij}}^{DM2}],[\vartheta_{A,L_{ij}}^{DM2},\vartheta_{A,U_{ij}}^{DM2}],[\pi_{A,L_{ij}}^{DM2},\pi_{A,U_{ij}}^{DM2}]\right), \\ \left(\left[\mu_{R,L_{ij}}^{DM2},\mu_{R,U_{ij}}^{DM2}\right],[\vartheta_{R,L_{ij}}^{DM2},\vartheta_{R,U_{ij}}^{DM2}],[\pi_{R,L_{ij}}^{DM2},\pi_{R,U_{ij}}^{DM2}]\right) \end{pmatrix} \quad (7.34)$$

$$\tilde{Z}^{DM3} = (\tilde{A},\tilde{R}) = \begin{pmatrix} \left([\mu_{A,L_{ij}}^{DM3},\mu_{A,U_{ij}}^{DM3}],[\vartheta_{A,L_{ij}}^{DM3},\vartheta_{A,U_{ij}}^{DM3}],[\pi_{A,L_{ij}}^{DM3},\pi_{A,U_{ij}}^{DM3}]\right), \\ \left(\left[\mu_{R,L_{ij}}^{DM3},\mu_{R,U_{ij}}^{DM3}\right],[\vartheta_{R,L_{ij}}^{DM3},\vartheta_{R,U_{ij}}^{DM3}],[\pi_{R,L_{ij}}^{DM3},\pi_{R,U_{ij}}^{DM3}]\right) \end{pmatrix} \quad (7.35)$$

Aggregation of these evaluations is made by using IVSWAM or IVSWGM aggregation operators given in Eqs. (7.22-7.23), respectively. The aggregated IVSF Z number based on IVSWGM operator is given in Eq. (7.36).

$$
\tilde{c}_{ij} = \left(\tilde{A}, \tilde{R}\right) = \left(
\begin{array}{c}
\left(
\begin{array}{c}
\left[\left(\left(\mu_{A,L_{ij}}^{DM1}\right)^{w_j} * \left(\mu_{A,L_{ij}}^{DM2}\right)^{w_j} * \left(\mu_{A,L_{ij}}^{DM3}\right)^{w_j}\right), \left(\left(\mu_{A,U_{ij}}^{DM1}\right)^{w_j} * \left(\mu_{A,U_{ij}}^{DM2}\right)^{w_j} * \left(\mu_{A,U_{ij}}^{DM3}\right)^{w_j}\right)\right], \\
\left[\begin{array}{c}
\sqrt{\left(1-\left(1-\left(\vartheta_{A,L_{ij}}^{DM1}\right)^2\right)^{w_j} * \left(1-\left(\vartheta_{A,L_{ij}}^{DM2}\right)^2\right)^{w_j} * \left(1-\left(\vartheta_{A,L_{ij}}^{DM3}\right)^2\right)^{w_j}\right)}, \\
\sqrt{\left(1-\left(1-\left(\vartheta_{A,U_{ij}}^{DM1}\right)^2\right)^{w_j} * \left(1-\left(\vartheta_{A,U_{ij}}^{DM2}\right)^2\right)^{w_j} * \left(1-\left(\vartheta_{A,U_{ij}}^{DM3}\right)^2\right)^{w_j}\right)}
\end{array}\right], \\
\left[\begin{array}{c}
\sqrt{\begin{array}{c}
\left(1-\left(\vartheta_{A,L_{ij}}^{DM1}\right)^2\right)^{w_j} * \left(1-\left(\vartheta_{A,L_{ij}}^{DM2}\right)^2\right)^{w_j} * \left(1-\left(\vartheta_{A,L_{ij}}^{DM3}\right)^2\right)^{w_j} - \\
\left(\begin{array}{c}\left(1-\left(\vartheta_{A,L_{ij}}^{DM1}\right)^2-\left(\pi_{A,L_{ij}}^{DM1}\right)^2\right)^{w_j} * \left(1-\left(\vartheta_{A,L_{ij}}^{DM2}\right)^2-\left(\pi_{A,L_{ij}}^{DM2}\right)^2\right)^{w_j} \\ * \left(1-\left(\vartheta_{A,L_{ij}}^{DM3}\right)^2-\left(\pi_{A,L_{ij}}^{DM3}\right)^2\right)^{w_j}\end{array}\right)
\end{array}}, \\
\sqrt{\begin{array}{c}
\left(1-\left(\vartheta_{A,U_{ij}}^{DM1}\right)^2\right)^{w_j} * \left(1-\vartheta_{A,U_{ij}}^{DM2}\right)^{w_j} * \left(1-\vartheta_{A,U_{ij}}^{DM3}\right)^{w_j} - \\
\left(\begin{array}{c}\left(1-\left(\vartheta_{A,U_{ij}}^{DM1}\right)^2-\left(\pi_{A,U_{ij}}^{DM1}\right)^2\right)^{w_j} * \left(1-\left(\vartheta_{A,U_{ij}}^{DM2}\right)^2-\left(\pi_{A,U_{ij}}^{DM2}\right)^2\right)^{w_j} \\ * \left(1-\left(\vartheta_{A,U_{ij}}^{DM3}\right)^2-\left(\pi_{A,U_{ij}}^{DM3}\right)^2\right)^{w_j}\end{array}\right)
\end{array}}
\end{array}\right]
\end{array}
\right), \\
\left(
\begin{array}{c}
\left[\left(\left(\mu_{R,L_{ij}}^{DM1}\right)^{w_j} * \left(\mu_{R,L_{ij}}^{DM2}\right)^{w_j} * \left(\mu_{R,L_{ij}}^{DM3}\right)^{w_j}\right), \left(\left(\mu_{R,U_{ij}}^{DM1}\right)^{w_j} * \left(\mu_{R,U_{ij}}^{DM2}\right)^{w_j} * \left(\mu_{R,U_{ij}}^{DM3}\right)^{w_j}\right)\right], \\
\left[\begin{array}{c}
\sqrt{\left(1-\left(1-\left(\vartheta_{R,L_{ij}}^{DM1}\right)^2\right)^{w_j} * \left(1-\left(\vartheta_{R,L_{ij}}^{DM2}\right)^2\right)^{w_j} * \left(1-\left(\vartheta_{R,L_{ij}}^{DM3}\right)^2\right)^{w_j}\right)}, \\
\sqrt{\left(1-\left(1-\left(\vartheta_{R,U_{ij}}^{DM1}\right)^2\right)^{w_j} * \left(1-\left(\vartheta_{R,U_{ij}}^{DM2}\right)^2\right)^{w_j} * \left(1-\left(\vartheta_{R,U_{ij}}^{DM3}\right)^2\right)^{w_j}\right)}
\end{array}\right], \\
\left[\begin{array}{c}
\sqrt{\begin{array}{c}
\left(1-\left(\vartheta_{R,L_{ij}}^{DM1}\right)^2\right)^{w_j} * \left(1-\left(\vartheta_{R,L_{ij}}^{DM2}\right)^2\right)^{w_j} * \left(1-\left(\vartheta_{R,L_{ij}}^{DM3}\right)^2\right)^{w_j} - \\
\left(\begin{array}{c}\left(1-\left(\vartheta_{R,L_{ij}}^{DM1}\right)^2-\left(\pi_{R,L_{ij}}^{DM1}\right)^2\right)^{w_j} * \left(1-\left(\vartheta_{R,L_{ij}}^{DM2}\right)^2-\left(\pi_{R,L_{ij}}^{DM2}\right)^2\right)^{w_j} \\ * \left(1-\left(\vartheta_{R,L_{ij}}^{DM3}\right)^2-\left(\pi_{R,L_{ij}}^{DM3}\right)^2\right)^{w_j}\end{array}\right)
\end{array}}, \\
\sqrt{\begin{array}{c}
\left(1-\left(\vartheta_{R,U_{ij}}^{DM1}\right)^2\right)^{w_j} * \left(1-\vartheta_{R,U_{ij}}^{DM2}\right)^{w_j} * \left(1-\vartheta_{R,U_{ij}}^{DM3}\right)^{w_j} - \\
\left(\begin{array}{c}\left(1-\left(\vartheta_{R,U_{ij}}^{DM1}\right)^2-\left(\pi_{R,U_{ij}}^{DM1}\right)^2\right)^{w_j} * \left(1-\left(\vartheta_{R,U_{ij}}^{DM2}\right)^2-\left(\pi_{R,U_{ij}}^{DM2}\right)^2\right)^{w_j} \\ * \left(1-\left(\vartheta_{R,U_{ij}}^{DM3}\right)^2-\left(\pi_{R,U_{ij}}^{DM3}\right)^2\right)^{w_j}\end{array}\right)
\end{array}}
\end{array}\right]
\end{array}
\right)
\end{array}
\right)
$$

i=1,2,…,n; j=1,2,…,n (7.36)

These aggregated values construct the matrix given in Eq. (7.37).

$$
\tilde{Z}^{Agg} = \begin{bmatrix}
\tilde{c}_{11} & \tilde{c}_{12} & \cdots & \tilde{c}_{1n} \\
\tilde{c}_{21} & \tilde{c}_{22} & \cdots & \tilde{c}_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{c}_{i1} & \tilde{c}_{i2} & \tilde{c}_{ij} & \tilde{c}_{in} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{c}_{n1} & \tilde{c}_{n2} & \cdots & \tilde{c}_{nn}
\end{bmatrix} \quad \text{i=1,2,…n,} \quad \text{j=1,2,…,n} \qquad (7.37)
$$

Step 5. Defuzzify IVSF Z reliability functions in $\tilde{Z}^{Agg}$ using Eq. (7.32). The reliability values below the diagonal of the aggregated pairwise comparison matrix are determined by multiplicative inverses after the reliability values above the diagonal are defuzzified as in Eq. (7.38).

$$\tilde{Z}_{def\tilde{R}}^{Agg} = \begin{bmatrix} (\tilde{A}_{11}, def\tilde{R}_{11}) & (\tilde{A}_{12}, def\tilde{R}_{12}) & \dots & (\tilde{A}_{1n}, def\tilde{R}_{1n}) \\ (\tilde{A}_{21}, 1/def\tilde{R}_{12}) & (\tilde{A}_{22}, def\tilde{R}_{22}) & \dots & (\tilde{A}_{2n}, def\tilde{R}_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (\tilde{A}_{i1}, 1/def\tilde{R}_{1i}) & (\tilde{A}_{i2}, 1/def\tilde{R}_{2i}) & (\tilde{A}_{ij}, def\tilde{R}_{ij}) & (\tilde{A}_{in}, def\tilde{R}_{in}) \\ \vdots & \vdots & \ddots & \vdots \\ (\tilde{A}_{n1}, 1/def\tilde{R}_{1n}) & (\tilde{A}_{n2}, 1/def\tilde{R}_{2n}) & \dots & (\tilde{A}_{nn}, def\tilde{R}_{nn}) \end{bmatrix} \tag{7.38}$$

Step 6. Calculate the restriction function integrated by the square root of defuzzified reliability ($def\tilde{R}$) values as in Eq. (7.39). After this step, IVSF Z matrix in Eq. (7.38) is transformed to ordinary IVSF matrix ($\widetilde{IVSM}_{def\tilde{R}}^{Agg}$).

$$\widetilde{IVSM}_{def\tilde{R}}^{Agg} = \begin{bmatrix} \left(\tilde{A}_{11} * \sqrt{def\tilde{R}_{11}}\right) & \left(\tilde{A}_{12} * \sqrt{def\tilde{R}_{12}}\right) & \dots & \left(\tilde{A}_{1n} * \sqrt{def\tilde{R}_{1n}}\right) \\ \left(\tilde{A}_{21} * \sqrt{1/def\tilde{R}_{12}}\right) & \left(\tilde{A}_{22} * \sqrt{def\tilde{R}_{22}}\right) & \dots & \left(\tilde{A}_{2n} * \sqrt{def\tilde{R}_{2n}}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\tilde{A}_{i1} * \sqrt{1/def\tilde{R}_{1i}}\right) & \left(\tilde{A}_{i2} * \sqrt{1/def\tilde{R}_{2i}}\right) & \left(\tilde{A}_{ij} * \sqrt{def\tilde{R}_{ij}}\right) & \left(\tilde{A}_{in} * \sqrt{def\tilde{R}_{in}}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\tilde{A}_{n1} * \sqrt{1/def\tilde{R}_{1n}}\right) & \left(\tilde{A}_{n2} * \sqrt{1/def\tilde{R}_{2n}}\right) & \dots & \left(\tilde{A}_{nn} * \sqrt{def\tilde{R}_{nn}}\right) \end{bmatrix} \tag{7.39}$$

Step 7. Calculate the interval-valued spherical mean ($\widetilde{IVSM}$) vector using IVSWAM or IVSWGM operators given in Eqs. (7.41-7.42). Let $\left(\tilde{A}_{ij} * \sqrt{def\tilde{R}_{ij}}\right)$ be represented by $\tilde{h}_{ij} = ([\mu_{\tilde{h}_{ij},L}, \mu_{\tilde{h}_{ij},U}], [\vartheta_{\tilde{h}_{ij},L}, \vartheta_{\tilde{h}_{ij},U}], [\pi_{\tilde{h}_{ij},L}, \pi_{\tilde{h}_{ij},U}])$. Thus, IVSF local weights are obtained, and $n \times n$ matrix transforms to $n \times 1$ vector as in Eq. (7.40).

$$\widetilde{IVSM} = \begin{bmatrix} \tilde{h}_{11} \\ \tilde{h}_{21} \\ \vdots \\ \tilde{h}_{n1} \end{bmatrix} \tag{7.40}$$

where

$$IVSWAM(\tilde{h}_{i1}) =$$

$$\left(\begin{array}{c} \left[\sqrt{1-\prod_{j=1}^{n}\left(1-\left(\mu_{\tilde{h}_{ij},L}\right)^2\right)^{w_j}}, \sqrt{1-\prod_{j=1}^{n}\left(1-\left(\mu_{\tilde{h}_{ij},U}\right)^2\right)^{w_j}}\right], \\ \left[\prod_{j=1}^{n}\left(\vartheta_{\tilde{h}_{ij},L}\right)^{w_j}, \prod_{j=1}^{n}\left(\vartheta_{\tilde{h}_{ij},U}\right)^{w_j}\right], \\ \left[\begin{array}{c} \sqrt{\prod_{j=1}^{n}\left(1-\left(\mu_{\tilde{h}_{ij},L}\right)^2\right)^{w_j} - \prod_{j=1}^{n}\left(1-\left(\mu_{\tilde{h}_{ij},L}\right)^2-\left(\pi_{\tilde{h}_{ij},L}\right)^2\right)^{w_j}}, \\ \sqrt{\prod_{j=1}^{n}\left(1-\left(\mu_{\tilde{h}_{ij},U}\right)^2\right)^{w_j} - \prod_{j=1}^{n}\left(1-\left(\mu_{\tilde{h}_{ij},U}\right)^2-\left(\pi_{\tilde{h}_{ij},U}\right)^2\right)^{w_j}}, \end{array}\right] \end{array}\right) \quad (7.41)$$

$$IVSWGM(\tilde{h}_{i1}) =$$

$$\left(\begin{array}{c} \left[\prod_{j=1}^{n}\left(\mu_{\tilde{h}_{ij},L}\right)^{w_j}, \prod_{j=1}^{n}\left(\mu_{\tilde{h}_{ij},U}\right)^{w_j}\right], \\ \left[\sqrt{1-\prod_{j=1}^{n}\left(1-\left(\vartheta_{\tilde{h}_{ij},L}\right)^2\right)^{w_j}}, \sqrt{1-\prod_{j=1}^{n}\left(1-\left(\vartheta_{\tilde{h}_{ij},U}\right)^2\right)^{w_j}}\right], \\ \left[\begin{array}{c} \sqrt{\prod_{j=1}^{n}\left(1-\left(\vartheta_{\tilde{h}_{ij},L}\right)^2\right)^{w_j} - \prod_{j=1}^{n}\left(1-\left(\vartheta_{\tilde{h}_{ij},L}\right)^2-\left(\pi_{\tilde{h}_{ij},L}\right)^2\right)^{w_j}}, \\ \sqrt{\prod_{j=1}^{n}\left(1-\left(\vartheta_{\tilde{h}_{ij},U}\right)^2\right)^{w_j} - \prod_{j=1}^{n}\left(1-\left(\vartheta_{\tilde{h}_{ij},U}\right)^2-\left(\pi_{\tilde{h}_{ij},U}\right)^2\right)^{w_j}}, \end{array}\right] \end{array}\right) \quad (7.42)$$

and $\tilde{h}_{n1}$ is still an IVSF number.

Step 8. Apply Steps (3-7) for all IVSF Z pairwise comparison matrices of DMs. This step includes the pairwise comparisons of the main criteria among themselves, the pairwise comparisons of all the sub-criteria under each related main criterion, and the pairwise comparisons of the alternatives according to all sub-criteria. After this step, we have IVSF local weights of main criteria $\left(\tilde{w}_{C_1}, \tilde{w}_{C_2}, \dots, \tilde{w}_{C_j}, \dots, \tilde{w}_{C_n}\right)$ and sub-criteria $\left(\tilde{w}_{C_{11}}, \tilde{w}_{C_{12}}, \dots, \tilde{w}_{C_{js}}, \dots, \tilde{w}_{C_{nt}}\right)$, s=1,2,…t, IVSF local weights of alternatives $\left(\tilde{w}_{C_{11}A_1}, \tilde{w}_{C_{12}A_1}, \dots, \tilde{w}_{C_{js}A_i}, \dots, \tilde{w}_{C_{nt}A_m}\right)$ according to sub-criteria as given in Table 7.9. In Table 7.9, $\tilde{w}_{A_i}^{G}$ represents global (overall) weight of m^{th} alternative.

Table 7.9 : IVSF local weights of main criteria, sub-criteria and alternatives.

	$\widetilde{w}_{C_1}$				$\widetilde{w}_{C_2}$				...	$\widetilde{w}_{C_n}$		
	$\widetilde{w}_{C_{11}}$	$\widetilde{w}_{C_{12}}$	...	$\widetilde{w}_{C_{1s}}$	$\widetilde{w}_{C_{21}}$	$\widetilde{w}_{C_{22}}$	...	$\widetilde{w}_{C_{2s}}$	...	$\widetilde{w}_{C_{n1}}$	$\widetilde{w}_{C_{n2}}$	...	$\widetilde{w}_{C_{ns}}$
$\widetilde{w}_{A_1}^G$	$\widetilde{w}_{C_{11}A_1}$	$\widetilde{w}_{C_{12}A_1}$	...	$\widetilde{w}_{C_{1s}A_1}$	$\widetilde{w}_{C_{21}A_1}$	$\widetilde{w}_{C_{22}A_1}$	...	$\widetilde{w}_{C_{2s}A_1}$	...	$\widetilde{w}_{C_{n1}A_1}$	$\widetilde{w}_{C_{n2}A_1}$	...	$\widetilde{w}_{C_{ns}A_1}$
$\widetilde{w}_{A_2}^G$	$\widetilde{w}_{C_{11}A_2}$	$\widetilde{w}_{C_{12}A_2}$	...	$\widetilde{w}_{C_{1s}A_2}$	$\widetilde{w}_{C_{21}A_2}$	$\widetilde{w}_{C_{22}A_2}$	...	$\widetilde{w}_{C_{2s}A_2}$	...	$\widetilde{w}_{C_{n1}A_2}$	$\widetilde{w}_{C_{n2}A_2}$	...	$\widetilde{w}_{C_{ns}A_2}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$\widetilde{w}_{A_i}^G$	$\widetilde{w}_{C_{11}A_i}$	$\widetilde{w}_{C_{12}A_i}$	...	$\widetilde{w}_{C_{1s}A_i}$	$\widetilde{w}_{C_{21}A_i}$	$\widetilde{w}_{C_{22}A_i}$	...	$\widetilde{w}_{C_{2s}A_i}$	...	$\widetilde{w}_{C_{n1}A_i}$	$\widetilde{w}_{C_{n2}A_i}$	...	$\widetilde{w}_{C_{ns}A_i}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$\widetilde{w}_{A_m}^G$	$\widetilde{w}_{C_{11}A_m}$	$\widetilde{w}_{C_{12}A_m}$	...	$\widetilde{w}_{C_{1t}A_m}$	$\widetilde{w}_{C_{21}A_m}$	$\widetilde{w}_{C_{22}A_m}$	...	$\widetilde{w}_{C_{2t}A_m}$	...	$\widetilde{w}_{C_{n1}A_m}$	$\widetilde{w}_{C_{n2}A_m}$	...	$\widetilde{w}_{C_{nt}A_m}$

**t: number of sub-criteria*

In Table 7.9, all local weights are IVSF numbers. There are two possible ways to continue to obtain global weights. Way 1 is to continue from Step 9 to Step 11.1 with defuzzification process. Way 2 is to directly continue without defuzzification process with Step 11.2 as illustrated in Figure 7.4.

Step 9. Defuzzify $\widetilde{IVSM}$ vector using Eq. (7.31) and obtain the vector of corresponding crisp values ($\widetilde{IVSM}_{def}$) by Eq. (7.43).

$$\widetilde{IVSM}_{def} = \begin{bmatrix} def\tilde{h}_{11} \\ def\tilde{h}_{21} \\ \vdots \\ def\tilde{h}_{j1} \\ \vdots \\ def\tilde{h}_{n1} \end{bmatrix} \tag{7.43}$$

For example, let $\tilde{h}_{11} = ([0.58, 0.73], [0.25, 0.42], [0.13, 0.29])$ be one of the IVSF numbers we have obtained in Step 7. For this IVSF number;

$$def\tilde{h}_{11} = 7 * 0.58 + 5 * 0{,}73 - 3 * 0.25 - 3 * 0.42 - \left({}^{0.13}/_{2}\right)^2 - \left({}^{0.29}/_{2}\right)^2 = 4.31$$

is obtained.

Step 10. Apply the normalization procedure to obtain local crisp weights by Eq. (7.44).

$$w_j = \frac{def\tilde{h}_{j1}}{\sum_{j=1}^{n} def\tilde{h}_{j1}} \quad \text{j=1,2,…,n} \tag{7.44}$$

Step 11. Combine all the obtained weights as in Eq. (7.45) if it is based on crisp local weights or Eq. (7.46) if it is based on fuzzy local weights.

Step 11.1. Eq. (7.45) can be used to obtain the overall crisp weights of alternatives ($w_{A_i}^{G}$).

$$w_{A_i}^{G} = w_{C_1} \sum_{s=1}^{t} w_{C_{1s}A_i} * w_{C_{1s}} + w_{C_2} \sum_{s=1}^{t} w_{C_{2s}A_i} * w_{C_{2s}} + \dots + w_{C_n} \sum_{s=1}^{t} w_{C_{ns}A_m} * w_{C_{ns}} \qquad i = 1,2,\dots,m \tag{7.45}$$

In this step, $w_{C_{js}}^{G} = w_{C_j} * w_{C_{js}}$ (s=1,2,…,t) can be used to obtain global crisp weights of criteria.

where w_{C_j} represents local crisp weight of j^{th} main criterion and $\widetilde{w}_{C_{js}}$ represents the local crisp weight of s^{th} sub-criterion of j^{th} main criterion.

Step 11.2. Eq. (7.46) can be used to obtain the overall IVSF weights of alternatives ($\widetilde{w}_{A_i}^{G}$).

$$\widetilde{w}_{A_i}^{G} = \widetilde{w}_{C_1} \otimes \sum_{s=1}^{t} \widetilde{w}_{C_{1s}A_i} \otimes \widetilde{w}_{C_{1s}} \oplus \widetilde{w}_{C_2} \otimes \sum_{s=1}^{t} \widetilde{w}_{C_{2s}A_i} \otimes \widetilde{w}_{C_{2s}} \oplus \dots \oplus \widetilde{w}_{C_n} \otimes \sum_{s=1}^{t} \widetilde{w}_{C_{ns}A_m} \otimes \widetilde{w}_{C_{ns}} \qquad i = 1,2,\dots,m \tag{7.46}$$

In this step, $\widetilde{w}_{C_{js}}^{G} = \widetilde{w}_{C_j} \otimes \widetilde{w}_{C_{js}}$ (s=1,2,…,t) can be used to obtain global IVSF weights of criteria.

where $\widetilde{w}_{C_j}$ represents local IVSF weight of j^{th} main criterion and $\widetilde{w}_{C_{js}}$ represents the local IVSF weight of s^{th} sub-criterion of j^{th} main criterion.

Step 12. Go to Step 9.

7.5 Application

7.5.1 Problem definiton

A company wants to choose a green supplier among five different alternatives. For this problem, in Step 1, main criteria and sub-criteria have been determined as shown in Section 7.2, and hierarchical structure has been constructed as given in Figure 7.6.

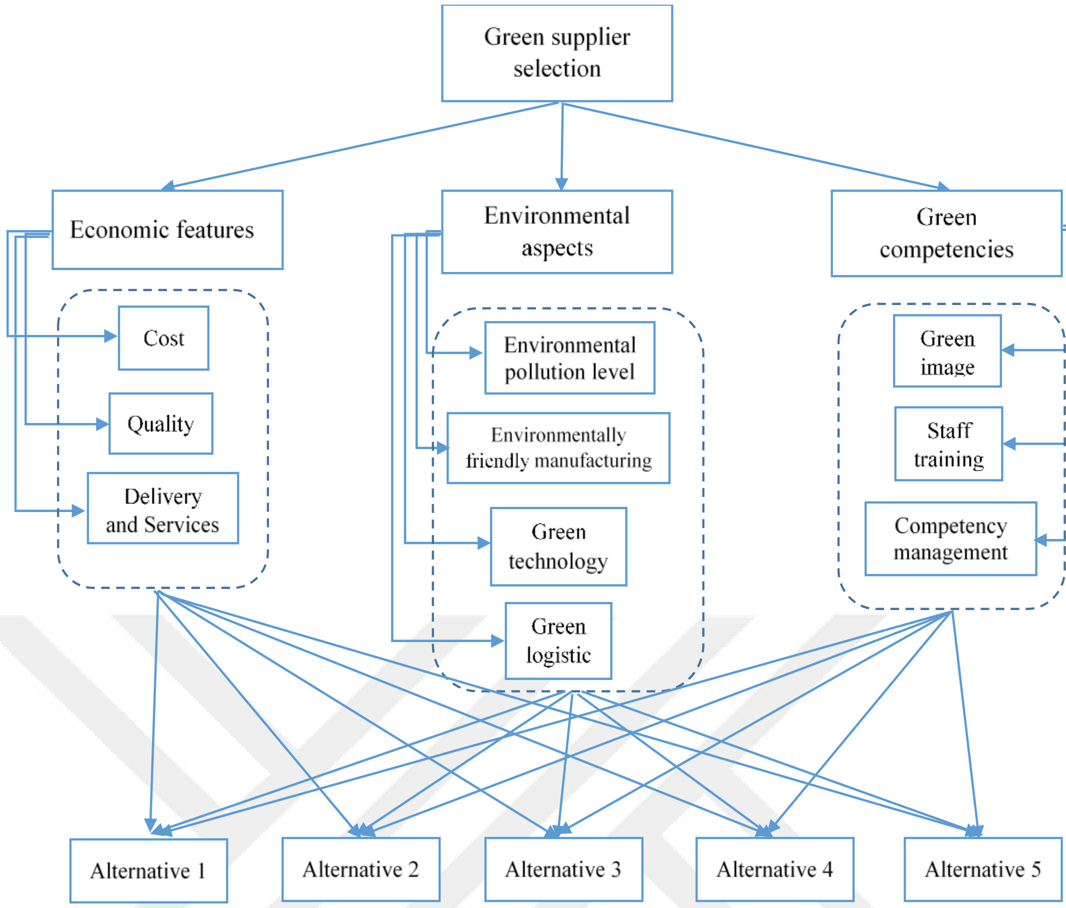


Figure 7.6 : Hierarchical structure for GSS problem.

7.5.2 Problem data

There are three DMs in the company, who know the characteristics of alternative firms, and they make necessary comparisons independently from each other. In Step 2, linguistic terms and corresponding IVSF Z numbers are defined in order to use them in pairwise comparisons. Tables 7.5 and 7.6 have been used in DMs' pairwise comparisons for main criteria, sub-criteria and alternatives.

Table 7.10 : Pairwise comparisons for main criteria.

	Goal	C1	C2	C3
DM1	C1	(EI, VSR)	(VLI, SR)	(SLI, VSR)
	C2	(VHI, SR)	(EI, VSR)	(SHI, VHR)
	C3	(SHI, VSR)	(SLI, VHR)	(EI, VSR)
	CR=0.007			
DM2	Goal	C1	C2	C3
	C1	(EI, VSR)	(ALI, VSR)	(SLI, SR)
	C2	(AHI, VSR)	(EI, VSR)	(HI, HR)
	C3	(SHI, SR)	(LI, HR)	(EI, VSR)
	CR=0.028			
DM3	Goal	C1	C2	C3
	C1	(EI, VSR)	(ALI, SR)	(LI, VSR)
	C2	(AHI, SR)	(EI, VSR)	(SHI, VHR)
	C3	(HI, VSR)	(SLI, VHR)	(EI, VSR)
	CR=0.028			

All IVSF Z-pairwise comparisons are collected from DMs using a suitable questionnaire. DMs' pairwise comparison matrices are given in Tables 7.10-7.23.

Table 7.11 : DMs' pairwise comparisons for C11, C12 and C13 with respect to economic features.

DM1's pairwise comparisons			
C1	C11	C12	C13
C11	(EI, VSR)	(SHI, HR)	(HI, SR)
C12	(SLI, HR)	(EI, VSR)	(SHI, VSR)
C13	(LI, SR)	(SLI, VSR)	(EI, VSR)
CR=0.037			
DM2's pairwise comparisons			
C1	C11	C12	C13
C11	(EI, VSR)	(HI, VHR)	(AHI, VHR)
C12	(LI, VHR)	(EI, VSR)	(SHI, SR)
C13	(ALI, VHR)	(SLI, SR)	(EI, VSR)
CR=0.028			
DM3's pairwise comparisons			
C1	C11	C12	C13
C11	(EI, VSR)	(SHI, VHR)	(VHI, HR)
C12	(SLI, VHR)	(EI, VSR)	(HI, VHR)
C13	(VLI, HR)	(LI, VHR)	(EI, VSR)
CR=0.063			

Table 7.12 : DMs' pairwise comparisons for C21, C22, C23 and C24 with respect to environmental aspects.

	C2	C21	C22	C23	C24
DM1	C21	(EI, VSR)	(LI, VHR)	(SLI, SR)	(SHI, SR)
	C22	(HI, VHR)	(EI, VSR)	(SHI, VHR)	(HI, HR)
	C23	(SHI, SR)	(SLI, VHR)	(EI, VSR)	(VHI, VHR)
	C24	(SLI, SR)	(LI, HR)	(VLI, VHR)	(EI, VSR)
	CR=0.090				
DM2	C2	C21	C22	C23	C24
	C21	(EI, VSR)	(LI, WR)	(SHI, VHR)	(HI, HR)
	C22	(HI, WR)	(EI, VSR)	(VHI, HR)	(AHI, WR)
	C23	(SLI, VHR)	(VLI, HR)	(EI, VSR)	(SHI, HR)
	C24	(LI, HR)	(ALI, WR)	(SLI, HR)	(EI, VSR)
	CR=0.065				
DM3	C2	C21	C22	C23	C24
	C21	(EI, VSR)	(EI, HR)	(HI, HR)	(AHI, VHR)
	C22	(EI, HR)	(EI, VSR)	(VHI, WR)	(HI, HR)
	C23	(LI, HR)	(VLI, WR)	(EI, VSR)	(SLI, SR)
	C24	(ALI, VHR)	(LI, HR)	(SHI, SR)	(EI, VSR)
	CR=0.098				

Table 7.13 : DMs' pairwise comparisons for C31, C32 and C33 with respect to green competencies.

DM1's pairwise comparisons			
C3	C31	C32	C33
C31	(EI, VSR)	(SHI, SR)	(SLI, HR)
C32	(SLI, SR)	(EI, VSR)	(LI, VSR)
C33	(SHI, HR)	(HI, VSR)	(EI, VSR)
CR=0.037			
DM2's pairwise comparisons			
C3	C31	C32	C33
C31	(EI, VSR)	(HI, WR)	(SLI, SR)
C32	(LI, WR)	(EI, VSR)	(ALI, VSR)
C33	(SHI, SR)	(AHI, VSR)	(EI, VSR)
CR=0.028			
DM3's pairwise comparisons			
C3	C31	C32	C33
C31	(EI, VSR)	(HI, HR)	(VHI, VSR)
C32	(LI, HR)	(EI, VSR)	(SHI, SR)
C33	(VLI, VSR)	(SLI, SR)	(EI, VSR)
CR=0.063			

Table 7.14 : DMs' pairwise comparisons for alternatives with respect to C11.

DM1's pairwise comparisons					
C11	A1	A2	A3	A4	A5
A1	(EI, VSR)	(VLI, HR)	(SLI, VHR)	(SHI, VHR)	(LI, HR)
A2	(VHI, HR)	(EI, VSR)	(SHI, SR)	(AHI, VSR)	(SHI, VHR)
A3	(SHI, VHR)	(SLI, SR)	(EI, VSR)	(HI, VHR)	(LI, HR)
A4	(SLI, VHR)	(ALI, VSR)	(LI, VHR)	(EI, VSR)	(VLI, SR)
A5	(HI, HR)	(SLI, VHR)	(HI, HR)	(VHI, SR)	(EI, VSR)
CR=0.082					
DM2's pairwise comparisons					
C11	A1	A2	A3	A4	A5
A1	(EI, VSR)	(ALI, SR)	(VLI, VSR)	(SLI, SR)	(ALI, VSR)
A2	(AHI, SR)	(EI, VSR)	(SHI, SR)	(AHI, SR)	(SHI, WR)
A3	(VHI, VSR)	(SLI, SR)	(EI, VSR)	(HI, HR)	(EI, HR)
A4	(SHI, SR)	(ALI, SR)	(LI, HR)	(EI, VSR)	(ALI, VSR)
A5	(AHI, VSR)	(SLI, WR)	(EI, HR)	(AHI, VSR)	(EI, VSR)
CR=0.066					
DM3's pairwise comparisons					
C11	A1	A2	A3	A4	A5
A1	(EI, VSR)	(SLI, FR)	(VLI, HR)	(SHI, SR)	(SLI, VHR)
A2	(SHI, FR)	(EI, VSR)	(ALI, WR)	(VHI, HR)	(SLI, HR)
A3	(VHI, HR)	(AHI, WR)	(EI, VSR)	(AHI, SR)	(SHI, VHR)
A4	(SLI, SR)	(VLI, HR)	(ALI, SR)	(EI, VSR)	(VLI, SR)
A5	(SHI, VHR)	(SHI, HR)	(SLI, VHR)	(VHI, SR)	(EI, VSR)
CR=0.096					

Table 7.15 : DMs' pairwise comparisons for alternatives with respect to C12.

DM1's pairwise comparisons					
C12	A1	A2	A3	A4	A5
A1	(EI, VSR)	(HI, SR)	(VHI, VSR)	(SLI, SR)	(SHI, VSR)
A2	(LI, SR)	(EI, VSR)	(SHI, WR)	(VLI, FR)	(LI, VHR)
A3	(VLI, VSR)	(SLI, WR)	(EI, VSR)	(VLI, VHR)	(SLI, SR)
A4	(SHI, SR)	(VHI, FR)	(VHI, VHR)	(EI, VSR)	(SHI, VSR)
A5	(SLI, VSR)	(HI, VHR)	(SHI, SR)	(SLI, VSR)	(EI, VSR)
CR=0.089					
DM2's pairwise comparisons					
C12	A1	A2	A3	A4	A5
A1	(EI, VSR)	(SHI, HR)	(HI, VHR)	(SLI, VHR)	(HI, SR)
A2	(SLI, HR)	(EI, VSR)	(SLI, SR)	(ALI, SR)	(SHI, VHR)
A3	(LI, VHR)	(SHI, SR)	(EI, VSR)	(LI, FR)	(SHI, FR)
A4	(SHI, VHR)	(AHI, SR)	(HI, FR)	(EI, VSR)	(AHI, VSR)
A5	(LI, SR)	(SLI, VHR)	(SLI, FR)	(ALI, VSR)	(EI, VSR)
CR=0.082					
DM3's pairwise comparisons					
C12	A1	A2	A3	A4	A5
A1	(EI, VSR)	(VHI, FR)	(AHI, SR)	(SLI, VSR)	(SHI, WR)
A2	(VLI, FR)	(EI, VSR)	(SLI, HR)	(VLI, HR)	(SLI, SR)
A3	(ALI, SR)	(SHI, HR)	(EI, VSR)	(ALI, VHR)	(SLI, HR)
A4	(SHI, VSR)	(VHI, HR)	(AHI, VHR)	(EI, VSR)	(HI, FR)
A5	(SLI, WR)	(SHI, SR)	(SHI, HR)	(LI, FR)	(EI, VSR)
CR=0.085					

In Step 3, for all collected matrices, consistency check procedure is performed. For this purpose, CR is computed for restriction function of each IVSF Z matrix using the corresponding crisp value which is calculated by Eq. (7.31).

Since CR<0.10 for all matrices, we continue with Step 4. All CR values are indicated in bottom of the matrices.

Table 7.16 : DMs' pairwise comparisons for alternatives with respect to C13.

DM1's pairwise comparisons					
C13	A1	A2	A3	A4	A5
A1	(EI, VSR)	(VLI, FR)	(SLI, VHR)	(VLI, VSR)	(LI, SR)
A2	(VHI, FR)	(EI, VSR)	(HI, HR)	(SLI, VHR)	(SHI, VHR)
A3	(SHI, VHR)	(LI, HR)	(EI, VSR)	(LI, FR)	(SLI, HR)
A4	(VHI, VSR)	(SHI, VHR)	(HI, FR)	(EI, VSR)	(HI, VSR)
A5	(HI, SR)	(SLI, VHR)	(SHI, HR)	(LI, VSR)	(EI, VSR)
CR=0.075					
DM2's pairwise comparisons					
C13	A1	A2	A3	A4	A5
A1	(EI, VSR)	(LI, SR)	(SLI, FR)	(LI, SR)	(SHI, VHR)
A2	(HI, SR)	(EI, VSR)	(SHI, SR)	(SLI, HR)	(AHI, SR)
A3	(SHI, FR)	(SLI, SR)	(EI, VSR)	(LI, VHR)	(SHI, VSR)
A4	(HI, SR)	(SHI, HR)	(HI, VHR)	(EI, VSR)	(HI, SR)
A5	(SLI, VHR)	(ALI, SR)	(SLI, VSR)	(LI, SR)	(EI, VSR)
CR=0.094					
DM3's pairwise comparisons					
C13	A1	A2	A3	A4	A5
A1	(EI, VSR)	(SHI, VHR)	(LI, SR)	(VLI, VSR)	(SLI, VHR)
A2	(SLI, VHR)	(EI, VSR)	(VLI, VSR)	(ALI, SR)	(VLI, VSR)
A3	(HI, SR)	(VHI, VSR)	(EI, VSR)	(LI, SR)	(SHI, SR)
A4	(VHI, VSR)	(AHI, SR)	(HI, SR)	(EI, VSR)	(HI, SR)
A5	(SHI, VHR)	(VHI, VSR)	(SLI, SR)	(LI, SR)	(EI, VSR)
CR=0.095					

Table 7.17 : DMs' pairwise comparisons for alternatives with respect to C21.

DM1's pairwise comparisons					
C21	A1	A2	A3	A4	A5
A1	(EI, VSR)	(VHI, VSR)	(SLI, HR)	(SHI, VHR)	(LI, SR)
A2	(VLI, VSR)	(EI, VSR)	(VLI, SR)	(SLI, SR)	(ALI, VSR)
A3	(SHI, HR)	(VHI, SR)	(EI, VSR)	(AHI, SR)	(SLI, HR)
A4	(SLI, VHR)	(SHI, SR)	(ALI, SR)	(EI, VSR)	(ALI, SR)
A5	(HI, SR)	(AHI, VSR)	(SHI, HR)	(AHI, SR)	(EI, VSR)
CR=0.083					
DM2's pairwise comparisons					
C21	A1	A2	A3	A4	A5
A1	(EI, VSR)	(HI, HR)	(LI, VHR)	(SHI, VHR)	(SLI, VHR)
A2	(LI, HR)	(EI, VSR)	(ALI, VSR)	(SLI, HR)	(VLI, VSR)
A3	(HI, VHR)	(AHI, VSR)	(EI, VSR)	(HI, VSR)	(SLI, SR)
A4	(SLI, VHR)	(SHI, HR)	(LI, VSR)	(EI, VSR)	(VLI, FR)
A5	(SHI, VHR)	(VHI, VSR)	(SHI, SR)	(VHI, FR)	(EI, VSR)
CR=0.092					
DM3's pairwise comparisons					
C21	A1	A2	A3	A4	A5
A1	(EI, VSR)	(SHI, SR)	(VLI, SR)	(HI, VSR)	(LI, SR)
A2	(SLI, SR)	(EI, VSR)	(ALI, VSR)	(SHI, VHR)	(ALI, FR)
A3	(VHI, SR)	(AHI, VSR)	(EI, VSR)	(AHI, VSR)	(SHI, VHR)
A4	(LI, VSR)	(SLI, VHR)	(ALI, VSR)	(EI, VSR)	(ALI, SR)
A5	(HI, SR)	(AHI, FR)	(SLI, VHR)	(AHI, SR)	(EI, VSR)
CR=0.093					

Table 7.18 : DMs' pairwise comparisons for alternatives with respect to C22.

DM1's pairwise comparisons					
C22	A1	A2	A3	A4	A5
A1	(EI, VSR)	(SHI, VHR)	(ALI, SR)	(LI, VSR)	(SLI, SR)
A2	(SLI, VHR)	(EI, VSR)	(ALI, VHR)	(VLI, SR)	(VLI, VHR)
A3	(AHI, SR)	(AHI, VHR)	(EI, VSR)	(SHI, VHR)	(SHI, SR)
A4	(HI, VSR)	(VHI, SR)	(SLI, VHR)	(EI, VSR)	(HI, HR)
A5	(SHI, SR)	(VHI, VHR)	(SLI, SR)	(LI, HR)	(EI, VSR)
CR=0.095					
DM2's pairwise comparisons					
C22	A1	A2	A3	A4	A5
A1	(EI, VSR)	(HI, SR)	(VLI, VSR)	(SHI, SR)	(LI, FR)
A2	(LI, SR)	(EI, VSR)	(ALI, FR)	(SLI, VHR)	(VLI, VSR)
A3	(VHI, VSR)	(AHI, FR)	(EI, VSR)	(AHI, SR)	(SHI, SR)
A4	(SLI, SR)	(SHI, VHR)	(ALI, SR)	(EI, VSR)	(SLI, FR)
A5	(HI, FR)	(VHI, VSR)	(SLI, SR)	(SHI, FR)	(EI, VSR)
CR=0.094					

Table 7.18 (continued) : DMs' pairwise comparisons for alternatives with respect to C22.

DM3's pairwise comparisons					
C22	A1	A2	A3	A4	A5
A1	(EI, VSR)	(SHI, SR)	(LI, SR)	(SHI, SR)	(SLI, VHR)
A2	(SLI, SR)	(EI, VSR)	(VLI, HR)	(SLI, FR)	(ALI, VSR)
A3	(HI, SR)	(VHI, HR)	(EI, VSR)	(VHI, VSR)	(SHI, VHR)
A4	(SLI, SR)	(SHI, FR)	(VLI, VSR)	(EI, VSR)	(LI, SR)
A5	(SHI, VHR)	(AHI, VSR)	(SLI, VHR)	(HI, SR)	(EI, VSR)
CR=0.066					

Table 7.19 : DMs' pairwise comparisons for alternatives with respect to C23.

DM1's pairwise comparisons					
C23	A1	A2	A3	A4	A5
A1	(EI, VSR)	(SLI, FR)	(AHI, SR)	(HI, HR)	(SHI, VHR)
A2	(SHI, FR)	(EI, VSR)	(VHI, VSR)	(AHI, SR)	(VHI, HR)
A3	(ALI, SR)	(VLI, VSR)	(EI, VSR)	(SLI, FR)	(LI, SR)
A4	(LI, HR)	(ALI, SR)	(SHI, FR)	(EI, VSR)	(SLI, HR)
A5	(SLI, VHR)	(VLI, HR)	(HI, SR)	(SHI, HR)	(EI, VSR)
CR=0.082					
DM2's pairwise comparisons					
C23	A1	A2	A3	A4	A5
A1	(EI, VSR)	(LI, SR)	(SHI, VHR)	(AHI, SR)	(HI, VSR)
A2	(HI, SR)	(EI, VSR)	(VHI, HR)	(AHI, VSR)	(AHI, HR)
A3	(SLI, VHR)	(VLI, HR)	(EI, VSR)	(HI, HR)	(SHI, VHR)
A4	(ALI, SR)	(ALI, VSR)	(LI, HR)	(EI, VSR)	(SLI, FR)
A5	(LI, VSR)	(ALI, HR)	(SLI, VHR)	(SHI, FR)	(EI, VSR)
CR=0.088					
DM3's pairwise comparisons					
C23	A1	A2	A3	A4	A5
A1	(EI, VSR)	(LI, HR)	(HI, HR)	(SHI, SR)	(SLI, SR)
A2	(HI, HR)	(EI, VSR)	(VHI, VSR)	(VHI, VHR)	(SHI, VHR)
A3	(LI, HR)	(VLI, VSR)	(EI, VSR)	(SLI, HR)	(VLI, FR)
A4	(SLI, SR)	(VLI, VHR)	(SHI, HR)	(EI, VSR)	(LI, VSR)
A5	(SHI, SR)	(SLI, VHR)	(VHI, FR)	(HI, VSR)	(EI, VSR)
CR=0.070					

Table 7.20 : DMs' pairwise comparisons for alternatives with respect to C24.

DM1's pairwise comparisons					
C24	A1	A2	A3	A4	A5
A1	(EI, VSR)	(HI, SR)	(AHI, VSR)	(VHI, SR)	(SHI, SR)
A2	(LI, SR)	(EI, VSR)	(SHI, SR)	(HI, HR)	(SLI, HR)
A3	(ALI, VSR)	(SLI, SR)	(EI, VSR)	(SLI, HR)	(LI, VSR)
A4	(VLI, SR)	(LI, HR)	(SHI, HR)	(EI, VSR)	(LI, VWR)
A5	(SLI, SR)	(SHI, HR)	(HI, VSR)	(HI, VWR)	(EI, VSR)
CR=0.091					
DM2's pairwise comparisons					
C24	A1	A2	A3	A4	A5
A1	(EI, VSR)	(VHI, VSR)	(SHI, VHR)	(HI, VSR)	(AHI, VHR)
A2	(VLI, VSR)	(EI, VSR)	(VLI, VSR)	(SLI, SR)	(SLI, SR)
A3	(SLI, VHR)	(VHI, VSR)	(EI, VSR)	(HI, SR)	(SHI, VHR)
A4	(LI, VSR)	(SHI, SR)	(LI, SR)	(EI, VSR)	(SLI, FR)
A5	(ALI, VHR)	(SHI, SR)	(SLI, VHR)	(SHI, FR)	(EI, VSR)
CR=0.095					
DM3's pairwise comparisons					
C24	A1	A2	A3	A4	A5
A1	(EI, VSR)	(HI, VWR)	(SHI, SR)	(VHI, FR)	(HI, FR)
A2	(LI, VWR)	(EI, VSR)	(LI, VHR)	(SHI, VSR)	(SHI, VHR)
A3	(SLI, SR)	(HI, VHR)	(EI, VSR)	(AHI, SR)	(VHI, SR)
A4	(VLI, FR)	(SLI, VSR)	(ALI, SR)	(EI, VSR)	(SLI, VSR)
A5	(LI, FR)	(SLI, VHR)	(VLI, SR)	(SHI, VSR)	(EI, VSR)
CR=0.098					

Table 7.21 : DMs' pairwise comparisons for alternatives with respect to C31.

DM1's pairwise comparisons					
C31	A1	A2	A3	A4	A5
A1	(EI, VSR)	(SHI, FR)	(SLI, HR)	(SHI, FR)	(LI, SR)
A2	(SLI, FR)	(EI, VSR)	(ALI, VSR)	(SLI, VWR)	(ALI, VWR)
A3	(SHI, HR)	(AHI, VSR)	(EI, VSR)	(HI, SR)	(SHI, VHR)
A4	(SLI, FR)	(SHI, VWR)	(LI, SR)	(EI, VSR)	(ALI, SR)
A5	(HI, SR)	(AHI, VWR)	(SLI, VHR)	(AHI, SR)	(EI, VSR)
CR=0.097					
DM2's pairwise comparisons					
C31	A1	A2	A3	A4	A5
A1	(EI, VSR)	(VHI, VSR)	(SLI, VHR)	(HI, SR)	(SLI, HR)
A2	(VLI, VSR)	(EI, VSR)	(VLI, SR)	(SLI, HR)	(VLI, VSR)
A3	(SHI, VHR)	(VHI, SR)	(EI, VSR)	(HI, SR)	(SHI, SR)
A4	(LI, SR)	(SHI, HR)	(LI, SR)	(EI, VSR)	(LI, VSR)
A5	(SHI, HR)	(VHI, VSR)	(SLI, SR)	(HI, VSR)	(EI, VSR)
CR=0.096					
DM3's pairwise comparisons					
C31	A1	A2	A3	A4	A5
A1	(EI, VSR)	(SLI, VHR)	(LI, SR)	(HI, SR)	(SLI, VHR)
A2	(SHI, VHR)	(EI, VSR)	(SLI, HR)	(HI, SR)	(SLI, SR)
A3	(HI, SR)	(SHI, HR)	(EI, VSR)	(AHI, VSR)	(SHI, VHR)
A4	(LI, SR)	(LI, SR)	(ALI, VSR)	(EI, VSR)	(LI, SR)
A5	(SHI, VHR)	(SHI, SR)	(SLI, VHR)	(HI, SR)	(EI, VSR)
CR=0.078					

Table 7.22 : DMs' pairwise comparisons for alternatives with respect to C32.

DM1's pairwise comparisons					
C32	A1	A2	A3	A4	A5
A1	(EI, VSR)	(HI, HR)	(SHI, SR)	(VHI, VSR)	(HI, SR)
A2	(LI, HR)	(EI, VSR)	(LI, VSR)	(SHI, SR)	(SLI, VHR)
A3	(SLI, SR)	(HI, VSR)	(EI, VSR)	(HI, HR)	(SHI, HR)
A4	(VLI, VSR)	(SLI, SR)	(LI, HR)	(EI, VSR)	(LI, VSR)
A5	(LI, SR)	(SHI, VHR)	(SLI, HR)	(HI, VSR)	(EI, VSR)
CR=0.082					
DM2's pairwise comparisons					
C32	A1	A2	A3	A4	A5
A1	(EI, VSR)	(AHI, SR)	(HI, HR)	(SHI, FR)	(SHI, VHR)
A2	(ALI, SR)	(EI, VSR)	(SLI, FR)	(SLI, HR)	(LI, VSR)
A3	(LI, HR)	(SHI, FR)	(EI, VSR)	(SHI, HR)	(SLI, VHR)
A4	(SLI, FR)	(SHI, HR)	(SLI, HR)	(EI, VSR)	(SLI, HR)
A5	(SLI, VHR)	(HI, VSR)	(SHI, VHR)	(SHI, HR)	(EI, VSR)
CR=0.075					
DM3's pairwise comparisons					
C32	A1	A2	A3	A4	A5
A1	(EI, VSR)	(VHI, VSR)	(SHI, SR)	(HI, VSR)	(HI, VHR)
A2	(VLI, VSR)	(EI, VSR)	(LI, VHR)	(SHI, SR)	(SLI, FR)
A3	(SLI, SR)	(HI, VHR)	(EI, VSR)	(HI, FR)	(SHI, SR)
A4	(LI, VSR)	(SLI, SR)	(LI, FR)	(EI, VSR)	(SLI, VSR)
A5	(LI, VHR)	(SHI, FR)	(SLI, SR)	(SHI, VSR)	(EI, VSR)
CR=0.089					

Table 7.23 : DMs' pairwise comparisons for alternatives with respect to C33.

DM1's pairwise comparisons					
C33	A1	A2	A3	A4	A5
A1	(EI, VSR)	(LI, VSR)	(HI, HR)	(SHI, VHR)	(HI, SR)
A2	(HI, VSR)	(EI, VSR)	(VHI, VSR)	(HI, SR)	(AHI, SR)
A3	(LI, HR)	(VLI, VSR)	(EI, VSR)	(SLI, HR)	(SHI, VHR)
A4	(SLI, VHR)	(LI, SR)	(SHI, HR)	(EI, VSR)	(HI, SR)
A5	(LI, SR)	(ALI, SR)	(SLI, VHR)	(LI, SR)	(EI, VSR)
CR=0.088					
DM2's pairwise comparisons					
C33	A1	A2	A3	A4	A5
A1	(EI, VSR)	(SLI, HR)	(HI, SR)	(SHI, HR)	(SHI, VHR)
A2	(SHI, HR)	(EI, VSR)	(HI, HR)	(VHI, VHR)	(VHI, VSR)
A3	(LI, SR)	(LI, HR)	(EI, VSR)	(SLI, SR)	(SLI, HR)
A4	(SLI, HR)	(VLI, VHR)	(SHI, SR)	(EI, VSR)	(SHI, VHR)
A5	(SLI, VHR)	(VLI, VSR)	(SHI, HR)	(SLI, VHR)	(EI, VSR)
CR=0.095					
DM3's pairwise comparisons					
C33	A1	A2	A3	A4	A5
A1	(EI, VSR)	(LI, SR)	(SHI, VHR)	(HI, SR)	(SHI, VHR)
A2	(HI, SR)	(EI, VSR)	(VHI, SR)	(AHI, VSR)	(HI, SR)
A3	(SLI, VHR)	(VLI, SR)	(EI, VSR)	(SHI, FR)	(SLI, FR)
A4	(LI, SR)	(ALI, VSR)	(SLI, FR)	(EI, VSR)	(SLI, HR)
A5	(SLI, VHR)	(LI, SR)	(SHI, FR)	(SHI, HR)	(EI, VSR)
CR=0.071					

7.5.3 Problem solution

In Step 4, aggregation procedure is performed for three DMs' independent pairwise comparison matrices using IVSWGM operator given in Eq. (7.23).

In Step 5, the aggregated reliability values of IVSF Z matrix are defuzzified using Eqs. (7.32) and (7.38). In Step 6, the aggregated IVSF Z matrix is converted to IVSF matrix using Eq. (7.39).

Table 7.24 : $\widetilde{IVSM}$ vectors for C1, C11, C12, C13 and for alternatives.

	$\tilde{w}_{C_1}$ = ([0, 0.33], [0.64, 0.81], [0.24, 0.34])		
	$\tilde{w}_{C_{11}}$ = ([0.48, 0.62], [0.37, 0.52], [0.28, 0.44])	$\tilde{w}_{C_{12}}$ = ([0.4, 0.56], [0.38, 0.54], [0.33, 0.49])	$\tilde{w}_{C_{13}}$ = ([0, 0.42], [0.51, 0.7], [0.29, 0.42])
$\tilde{w}^G_{A_1}$	$\tilde{w}_{C_{11}A_1}$ = ([0, 0.33], [0.68, 0.82], [0.22, 0.32])	$\tilde{w}_{C_{12}A_1}$ = ([0.43, 0.57], [0.43, 0.57], [0.26, 0.42])	$\tilde{w}_{C_{13}A_1}$ = ([0.22, 0.36], [0.64, 0.77], [0.23, 0.36])
$\tilde{w}^G_{A_2}$	$\tilde{w}_{C_{11}A_2}$ = ([0, 0.56], [0.51, 0.64], [0.24, 0.37])	$\tilde{w}_{C_{12}A_2}$ = ([0, 0.36], [0.64, 0.78], [0.23, 0.35])	$\tilde{w}_{C_{13}A_2}$ = ([0, 0.48], [0.54, 0.69], [0.24, 0.37])
$\tilde{w}^G_{A_3}$	$\tilde{w}_{C_{11}A_3}$ = ([0.47, 0.62], [0.39, 0.52], [0.28, 0.43])	$\tilde{w}_{C_{12}A_3}$ = ([0, 0.38], [0.62, 0.77], [0.24, 0.35])	$\tilde{w}_{C_{13}A_3}$ = ([0.36, 0.52], [0.49, 0.62], [0.28, 0.42])
$\tilde{w}^G_{A_4}$	$\tilde{w}_{C_{11}A_4}$ = ([0, 0.33], [0.61, 0.8], [0.22, 0.33])	$\tilde{w}_{C_{12}A_4}$ = ([0.61, 0.75], [0.26, 0.38], [0.26, 0.41])	$\tilde{w}_{C_{13}A_4}$ = ([0.6, 0.75], [0.24, 0.37], [0.27, 0.41])
$\tilde{w}^G_{A_5}$	$\tilde{w}_{C_{11}A_5}$ = ([0.57, 0.73], [0.23, 0.36], [0.33, 0.46])	$\tilde{w}_{C_{12}A_5}$ = ([0, 0.54], [0.39, 0.57], [0.32, 0.46])	$\tilde{w}_{C_{13}A_5}$ = ([0, 0.51], [0.44, 0.62], [0.29, 0.43])

In Step 7, $\widetilde{IVSM}$ vector is calculated by IVSWGM operator by utilizing Eqs. (7.40) and (7.42).

In Step 8, we obtained 14 $\widetilde{IVSM}$ vectors from pairwise comparisons. For these IVSF local weights to be understandable, Table 7.9 is divided into three parts which are given in Tables 7.24-7.26 with local weights under each main criterion.

Table 7.25 : $\widetilde{IVSM}$ vectors for C2, C21, C22, C23, C24 and for alternatives.

$\tilde{w}_{C_2}$= ([0.52, 0.67], [0.35, 0.48], [0.28, 0.44])				
	$\tilde{w}_{C_{21}}$= ([0.26, 0.37], [0.74, 0.81], [0.2, 0.32])	$\tilde{w}_{C_{22}}$= ([0.52, 0.65], [0.41, 0.53], [0.27, 0.38])	$\tilde{w}_{C_{23}}$ ([0.4, 0.59], [0.38, 0.53], [0.34, 0.45])	$\tilde{w}_{C_{24}}$= ([0, 0.44], [0.45, 0.67], [0.29, 0.42])
$\tilde{w}^G_{A_1}$	$\tilde{w}_{C_{21}A_1}$= ([0.32, 0.47], [0.53, 0.67], [0.25, 0.4])	$\tilde{w}_{C_{22}A_1}$= ([0, 0.42], [0.58, 0.73], [0.25, 0.38])	$\tilde{w}_{C_{23}A_1}$= ([0.39, 0.53], [0.49, 0.62], [0.25, 0.41])	$\tilde{w}_{C_{24}A_1}$= ([0.5, 0.65], [0.36, 0.51], [0.24, 0.4])
$\tilde{w}^G_{A_2}$	$\tilde{w}_{C_{21}A_2}$= ([0, 0.3], [0.67, 0.84], [0.2, 0.3])	$\tilde{w}_{C_{22}A_2}$= ([0, 0.29], [0.71, 0.85], [0.2, 0.29])	$\tilde{w}_{C_{23}A_2}$= ([0.59, 0.74], [0.27, 0.4], [0.24, 0.39])	$\tilde{w}_{C_{24}A_2}$= ([0.29, 0.46], [0.51, 0.66], [0.27, 0.42])
$\tilde{w}^G_{A_3}$	$\tilde{w}_{C_{21}A_3}$= ([0.54, 0.69], [0.34, 0.46], [0.26, 0.41])	$\tilde{w}_{C_{22}A_3}$= ([0.59, 0.73], [0.29, 0.41], [0.25, 0.4])	$\tilde{w}_{C_{23}A_3}$= ([0, 0.36], [0.61, 0.77], [0.23, 0.35])	$\tilde{w}_{C_{24}A_3}$= ([0, 0.53], [0.45, 0.62], [0.27, 0.41])
$\tilde{w}^G_{A_4}$	$\tilde{w}_{C_{21}A_4}$= ([0, 0.35], [0.62, 0.8], [0.24, 0.33])	$\tilde{w}_{C_{22}A_4}$= ([0, 0.49], [0.5, 0.66], [0.27, 0.4])	$\tilde{w}_{C_{23}A_4}$= ([0, 0.39], [0.57, 0.76], [0.25, 0.36])	$\tilde{w}_{C_{24}A_4}$= ([0, 0.39], [0.59, 0.74], [0.25, 0.38])
$\tilde{w}^G_{A_5}$	$\tilde{w}_{C_{21}A_5}$= ([0.62, 0.76], [0.23, 0.35], [0.27, 0.42])	$\tilde{w}_{C_{22}A_5}$= ([0.51, 0.67], [0.29, 0.43], [0.31, 0.46])	$\tilde{w}_{C_{23}A_5}$= ([0, 0.57], [0.38, 0.56], [0.3, 0.44])	$\tilde{w}_{C_{24}A_5}$= ([0, 0.57], [0.39, 0.57], [0.31, 0.43])

Table 7.26 : $\widetilde{IVSM}$ vectors for C3, C31, C32, C33 and for alternatives.

$\tilde{w}_{C_3}$=([0.42, 0.58], [0.35, 0.51], [0.34, 0.49])			
	$\tilde{w}_{C_{31}}$ = ([0.39, 0.53], [0.48, 0.61], [0.28, 0.44])	$\tilde{w}_{C_{32}}$= ([0.00, 0.45], [0.50, 0.68], [0.30, 0.42])	$\tilde{w}_{C_{33}}$= ([0.42, 0.59], [0.37, 0.53], [0.31, 0.46])
$\tilde{w}^G_{A_1}$	$\tilde{w}_{C_{31}A_1}$= ([0.32, 0.47], [0.54, 0.67], [0.26, 0.41])	$\tilde{w}_{C_{32}A_1}$= ([0.50, 0.65], [0.35, 0.49], [0.26, 0.42])	$\tilde{w}_{C_{33}A_1}$= ([0.38, 0.53], [0.47, 0.61], [0.27, 0.42])
$\tilde{w}^G_{A_2}$	$\tilde{w}_{C_{31}A_2}$= ([0, 0.33], [0.68, 0.82], [0.22, 0.32])	$\tilde{w}_{C_{32}A_2}$= ([0.00, 0.38], [0.60, 0.75], [0.25, 0.38])	$\tilde{w}_{C_{33}A_2}$= ([0.58, 0.73], [0.25, 0.4], [0.24, 0.4])
$\tilde{w}^G_{A_3}$	$\tilde{w}_{C_{31}A_3}$= ([0.55, 0.7], [0.29, 0.43], [0.28, 0.43])	$\tilde{w}_{C_{32}A_3}$ = ([0.41, 0.56], [0.43, 0.57], [0.29, 0.44])	$\tilde{w}_{C_{33}A_3}$= ([0.26, 0.41], [0.56, 0.71], [0.26, 0.41])
$\tilde{w}^G_{A_4}$	$\tilde{w}_{C_{31}A_4}$= ([0, 0.42], [0.53, 0.71], [0.27, 0.39])	$\tilde{w}_{C_{32}A_4}$= ([0.31, 0.48], [0.46, 0.62], [0.31, 0.46])	$\tilde{w}_{C_{33}A_4}$= ([0, 0.48], [0.47, 0.65], [0.29, 0.42])
$\tilde{w}^G_{A_5}$	$\tilde{w}_{C_{31}A_5}$= ([0.55, 0.7], [0.28, 0.41], [0.3, 0.45])	$\tilde{w}_{C_{32}A_5}$= ([0.44, 0.61], [0.33, 0.49], [0.32, 0.47])	$\tilde{w}_{C_{33}A_5}$= ([0, 0.49], [0.46, 0.64], [0.3, 0.43])

We obtained the global weights of alternatives using way 1 and way 2. In way 1, we continue with Step 9 and defuzzify all $\widetilde{IVSM}$ vectors by Eq. (7.31). Then, defuzzified IVSM ($\widetilde{IVSM}_{def}$) vectors are obtained. In Step 10, we calculate normalized defuzzified IVSM vectors and then crisp local weights. In Step 11, we obtain global weights of alternatives using Eq. (7.45). Table 7.27 presents all crisp local weights and global weights of alternatives.

Table 7.27 shows that A2 has the first ranking with the weight of 0.3401 and A4 has the last ranking with the weight of 0.0524 and overall ranking of the alternatives is A2>A3>A5>A1>A4.

Table 7.27 : All crisp local weights, global weights of alternatives and their rankings (Way 1).

	w_{C_1}=0.035			w_{C_2}=0.618			
	$w_{C_{11}}$=0.584	$w_{C_{12}}$=0.347	$w_{C_{13}}$=0.069	$w_{C_{21}}$=0.034	$w_{C_{22}}$=0.573	$w_{C_{23}}$=0.323	$w_{C_{24}}$=0.071
$w_{A_1}^G$	$w_{C_{11}A_1}$=0.023	$w_{C_{12}A_1}$=0.193	$w_{C_{13}A_1}$=0.033	$w_{C_{21}A_1}$=0.033	$w_{C_{22}A_1}$=0.027	$w_{C_{23}A_1}$=0.095	$w_{C_{24}A_1}$=0.714
$w_{A_2}^G$	$w_{C_{11}A_2}$=0.048	$w_{C_{12}A_2}$=0.026	$w_{C_{13}A_2}$=0.045	$w_{C_{21}A_2}$=0.017	$w_{C_{22}A_2}$=0.018	$w_{C_{23}A_2}$=0.802	$w_{C_{24}A_2}$=0.085
$w_{A_3}^G$	$w_{C_{11}A_3}$=0.303	$w_{C_{12}A_3}$=0.027	$w_{C_{13}A_3}$=0.052	$w_{C_{21}A_3}$=0.374	$w_{C_{22}A_3}$=0.528	$w_{C_{23}A_3}$=0.035	$w_{C_{24}A_3}$=0.109
$w_{A_4}^G$	$w_{C_{11}A_4}$=0.026	$w_{C_{12}A_4}$=0.680	$w_{C_{13}A_4}$=0.803	$w_{C_{21}A_4}$=0.020	$w_{C_{22}A_4}$=0.038	$w_{C_{23}A_4}$=0.038	$w_{C_{24}A_4}$=0.058
$w_{A_5}^G$	$w_{C_{11}A_5}$=0.601	$w_{C_{12}A_5}$=0.074	$w_{C_{13}A_5}$=0.068	$w_{C_{21}A_5}$=0.555	$w_{C_{22}A_5}$=0.390	$w_{C_{23}A_5}$=0.029	$w_{C_{24}A_5}$=0.035

Table 7.27 (continued): All crisp local weights, global weights of alternatives and their rankings (Way 1).

	w_{C_3}=0.346				
	$w_{C_{31}}$=0.241	$w_{C_{32}}$=0.110	$w_{C_{33}}$=0.649	$w_{A_i}^G$	Rank
$w_{A_1}^G$	$w_{C_{31}A_1}$=0.036	$w_{C_{32}A_1}$=0.407	$w_{C_{33}A_1}$=0.104	0.1055	4
$w_{A_2}^G$	$w_{C_{31}A_2}$=0.020	$w_{C_{32}A_2}$=0.031	$w_{C_{33}A_2}$=0.736	0.3401	1
$w_{A_3}^G$	$w_{C_{31}A_3}$=0.449	$w_{C_{32}A_3}$=0.183	$w_{C_{33}A_3}$=0.043	0.2672	2
$w_{A_4}^G$	$w_{C_{31}A_4}$=0.030	$w_{C_{32}A_4}$=0.062	$w_{C_{33}A_4}$=0.057	0.0524	5
$w_{A_5}^G$	$w_{C_{31}A_5}$=0.464	$w_{C_{32}A_5}$=0.317	$w_{C_{33}A_5}$=0.060	0.2347	3

Three DMs in the company define *environmental aspects* as the most important main criterion for GSS problem as given in Table 7.27. Among the sub-criteria of *economic features*, *quality* is the most important sub-criterion. Then, *cost*, *delivery and services* follow it, respectively. In environmental aspects, the most important sub-criteria are *environmental pollution level* with the weight of 0.573, then *green technology* with the weight of 0.323. Among the *green competencies*, *competency management* is found to be the most important sub-criterion.

In way 2, after performing Step 8, we directly continue with Step 11.2 and obtain global IVSF weights of alternatives by Eq. (7.46). Then, we calculate corresponding crisp values using Eq. (7.31) and apply normalization to obtain global crisp weights and prioritization of alternatives as given in Table 7.28.

Table 7.28 : Global IVSF weights of alternatives, their corresponding values and rankings (Way 2).

Alternatives	Global IVSF weights		Corresponding values	Crisp weights	Rank
A1	$w_{A_1}^G$	([0.12, 0.45], [0.04, 0.27], [0.63, 0.86])	0.64	0.1582	4
A2	$w_{A_2}^G$	([0.16, 0.46], [0.05, 0.29], [0.61, 0.86])	0.69	0.1693	3
A3	$w_{A_3}^G$	([0.20, 0.51], [0.03, 0.23], [0.64, 0.84])	1.14	0.2797	2
A4	$w_{A_4}^G$	([0.00, 0.41], [0.04, 0.31], [0.58, 0.87])	0.29	0.0725	5
A5	$w_{A_5}^G$	([0.18, 0.54], [0.02, 0.18], [0.71, 0.83])	1.30	0.3202	1

Table 7.28 shows that A5 has the first ranking with the weight of 0.3202 and A4 has the last ranking with the weight of 0.0725 and overall ranking of the alternatives is A5>A3>A2>A1>A4.

According to these two ways to obtain global weights of the alternatives, the rankings of A1, A3 and A4 are same, but others different. The main reason for this difference is due to a comprehensive approach provided by the fuzzy set theory.

7.5.4 Comparative analysis

In this section, we compare the IVSF Z-AHP method with the IVSF AHP to demonstrate the importance and necessity of reliability information. In this analysis, the same data regarding the application of the IVSF Z-AHP method are used, and only the reliability information is ignored. We consider way 1 in the comparison of criteria since we use defuzzification process in this approach. Comparisons of the results for criteria are presented in Table 7.29.

Table 7.29 : Crisp local and global weights of criteria based on IVSF Z-AHP and IVSF AHP.

Main-criterion	Weights		Sub-criterion	Code	Local weights		Global weights	
	IVSF Z-AHP	IVSF AHP			IVSF Z-AHP	IVSF AHP	IVSF Z-AHP	IVSF AHP
Economic features	0.035	0.038	Quality	C11	0.5838	0.7479	0.0207	0.0281
			Cost	C12	0.3471	0.2065	0.0123	0.0078
			Delivery and services	C13	0.0691	0.0456	0.0024	0.0017
Environmental aspects	0.618	0.767	Environmentally friendly manufacturing	C21	0.0340	0.2425	0.0210	0.1861
			Environmental pollution	C22	0.5725	0.6608	0.3540	0.5070
			Green technology	C23	0.3225	0.0613	0.1994	0.0470
			Green logistic	C24	0.0710	0.0354	0.0439	0.0271
Green competencies	0.346	0.195	Green image	C31	0.2408	0.5872	0.0834	0.1146
			Staff training	C32	0.1099	0.0676	0.0380	0.0132
			Competency management	C33	0.6494	0.3452	0.2249	0.0674

As can be seen in Table 7.29, although the order of magnitude of the main criteria weights is the same, significant differences in the magnitudes of weights have been obtained. The *environmental pollution level* is the most important *environmental aspect* in two methods. While *competency management* is the most important criterion among *green competencies* in the IVSF Z-AHP method, the *green image* has the largest weight in the IVSF AHP method. In global weights, the ranking of all sub-criteria except *delivery and services*, *environmental pollution level* and *cost* has changed. These changes in the criteria weights are due to the ignorance of reliability information and show that there may be changes in the results when the reliability information is not used.

Comparisons of results for alternatives are given in Tables 7.30 and 7.31 for way 1 and way 2, respectively.

Table 7.30 : Comparison of results (Way 1).

	Weights			Rankings		
Alternatives	IVSF Z-AHP	IVSF AHP	Z-AHP	IVSF Z-AHP	IVSF AHP	Z-AHP
A1	0.1055	0.1003	0.1771	4	3	3
A2	0.3401	0.0980	0.1516	1	4	4
A3	0.2672	0.4700	0.3072	2	1	1
A4	0.0524	0.0410	0.0886	5	5	5
A5	0.2347	0.2907	0.2755	3	2	2

Table 7.31 : Comparison of results (Way 2).

	Weights			Rankings		
Alternatives	IVSF Z-AHP	IVSF AHP	Z-AHP	IVSF Z-AHP	IVSF AHP	Z-AHP
A1	0.1582	0.2007	0.1759	4	3	3
A2	0.1693	0.1383	0.1446	3	4	4
A3	0.2797	0.3307	0.3108	2	1	1
A4	0.0725	0.0559	0.0938	5	5	5
A5	0.3202	0.2744	0.2749	1	2	2

Table 7.30 shows that only alternative A4 is in the same order in IVSF Z-AHP and IVSF AHP methods with the defuzzification process (way 1). Ignoring the reliability information while applying way 2 has replaced the rankings of A1 and A2 & A3 and A5. When the results of way 1 and way 2 are compared, the rankings of A1, A3 and A4 do not change in IVSF Z-AHP method, but A2 is the best alternative in way 1 whereas A5 is the best one in way 2. In addition, the rankings obtained by way 1 and

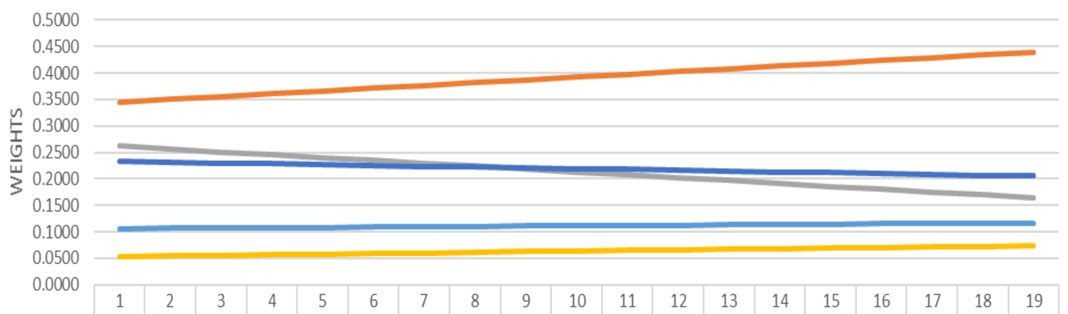
way 2 are same when reliability information is not used (IVSF AHP method). All these results show that the reliability information causes changes on the results and should be considered in decision models.

In order to prove that the IVSF Z numbers can represent the fuzziness better, the problem is also solved by the Z-AHP method. When the ordinary Z-AHP method is applied, the results of this method are the same as the IVSF AHP method in both way 1 and way 2. Alternative A4 remains in the same order when Z-AHP is compared with the IVSF Z-AHP method, while the other alternatives have different orders. This result shows that neither IVSF numbers nor ordinary Z-fuzzy numbers alone have the ability to model the fuzziness better. The integrated IVSF Z-AHP method represents the impreciseness in the linguistic evaluations in a superior way.

7.5.5 Sensitivity analysis

In the literature, various sensitivity analysis approaches are applied for MCDM problems (Alkan and Kahraman, 2022b). In addition, the sensitivity analysis approaches to be used also depend on the characteristics of the MCDM method. In this study, the proposed fuzzy approach is shaped around the AHP method. Since the AHP method usually depends on the subjective judgments of DMs, it is most logical to perform the sensitivity analysis by changing the determined criteria weights. It also should be stated that DMs or some researchers/readers usually want to know whether the obtained ranking results change according to the weights of the criteria. For these reasons, in the sensitivity analysis of this study, the changes in the ranking results with respect to the criteria weights are examined in three cases by way 1, which gives us the opportunity to change the criteria weights.

Total 19 scenarios are determined in case 1. The weight of *environmental aspects* (highest weighted criterion) has been reduced by 5% in each scenario (this decrease has been reflected as an increase in the other criteria provided that the sum of the criteria weights is equal to 1) and the changes in the weights of alternatives according to the scenarios are shown in Figure 7.7.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
A1	0.1061	0.1067	0.1073	0.1079	0.1085	0.1091	0.1097	0.1103	0.1109	0.1115	0.1121	0.1127	0.1133	0.1139	0.1145	0.1151	0.1157	0.1163	0.1169
A2	0.3453	0.3506	0.3558	0.3610	0.3662	0.3715	0.3767	0.3819	0.3871	0.3924	0.3976	0.4028	0.4080	0.4133	0.4185	0.4237	0.4289	0.4341	0.4394
A3	0.2618	0.2564	0.2510	0.2456	0.2402	0.2348	0.2294	0.2240	0.2186	0.2132	0.2078	0.2024	0.1970	0.1916	0.1862	0.1808	0.1754	0.1700	0.1645
A4	0.0536	0.0547	0.0558	0.0569	0.0580	0.0591	0.0603	0.0614	0.0625	0.0636	0.0647	0.0658	0.0669	0.0681	0.0692	0.0703	0.0714	0.0725	0.0736
A5	0.2332	0.2316	0.2301	0.2286	0.2270	0.2255	0.2240	0.2224	0.2209	0.2194	0.2178	0.2163	0.2147	0.2132	0.2117	0.2101	0.2086	0.2071	0.2055

Figure 7.7 : Changes in weights of alternatives based on 19 scenarios of case 1.

The criteria weights and ranking of alternatives obtained from case 1 are presented in Figure 7.8. The results for case 1 show that the decrease in the weight of *environmental aspects* from its current level does not change the place of the first and last alternatives. In addition, in the transition from scenario 8 to 9, the second and third alternatives are replaced.

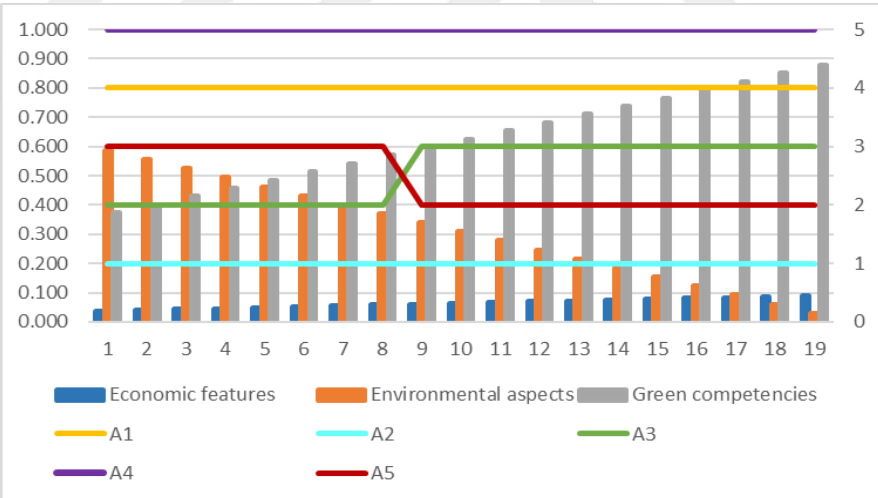
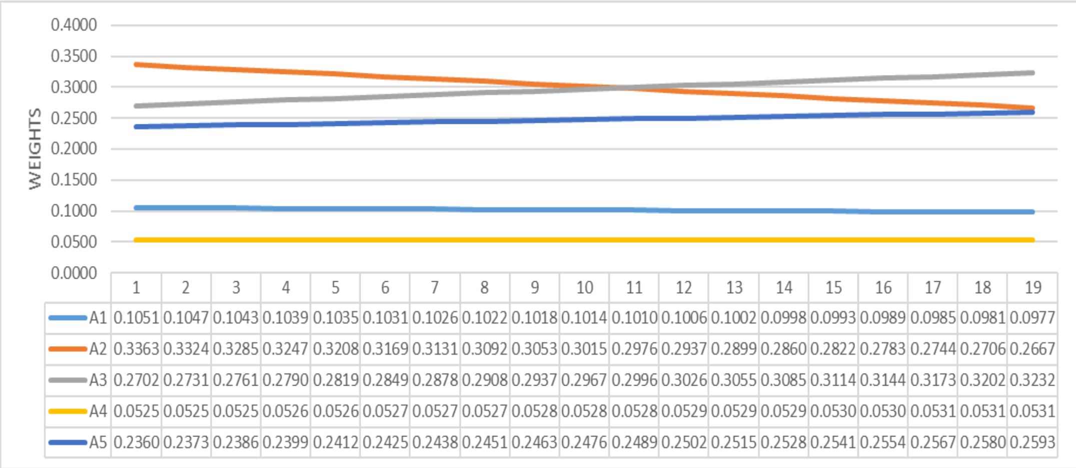


Figure 7.8 : Changes in criteria weights and ranking of alternatives based on 19 scenarios of case 1.

In case 2, the weight of *green competencies* has been reduced similar to case 1, and the changes in alternatives and criteria weights have been observed as in Figures 7.9 and 7.10. When the obtained results for case 2 are examined, it is concluded that the first and second alternatives (A2 and A3, respectively) are replaced when the eleventh scenario is applied (the weight of the *green competencies* is reduced by 55%). In addition, the weights of A1, A4 and A5 slightly change, but their rankings are not affected by case 2.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
A1	0.1051	0.1047	0.1043	0.1039	0.1035	0.1031	0.1026	0.1022	0.1018	0.1014	0.1010	0.1006	0.1002	0.0998	0.0993	0.0989	0.0985	0.0981	0.0977
A2	0.3363	0.3324	0.3285	0.3247	0.3208	0.3169	0.3131	0.3092	0.3053	0.3015	0.2976	0.2937	0.2899	0.2860	0.2822	0.2783	0.2744	0.2706	0.2667
A3	0.2702	0.2731	0.2761	0.2790	0.2819	0.2849	0.2878	0.2908	0.2937	0.2967	0.2996	0.3026	0.3055	0.3085	0.3114	0.3144	0.3173	0.3202	0.3232
A4	0.0525	0.0525	0.0525	0.0526	0.0526	0.0527	0.0527	0.0527	0.0528	0.0528	0.0528	0.0529	0.0529	0.0529	0.0530	0.0530	0.0531	0.0531	0.0531
A5	0.2360	0.2373	0.2386	0.2399	0.2412	0.2425	0.2438	0.2451	0.2463	0.2476	0.2489	0.2502	0.2515	0.2528	0.2541	0.2554	0.2567	0.2580	0.2593

Figure 7.9 : Changes in weights of alternatives based on 19 scenarios of case 2.

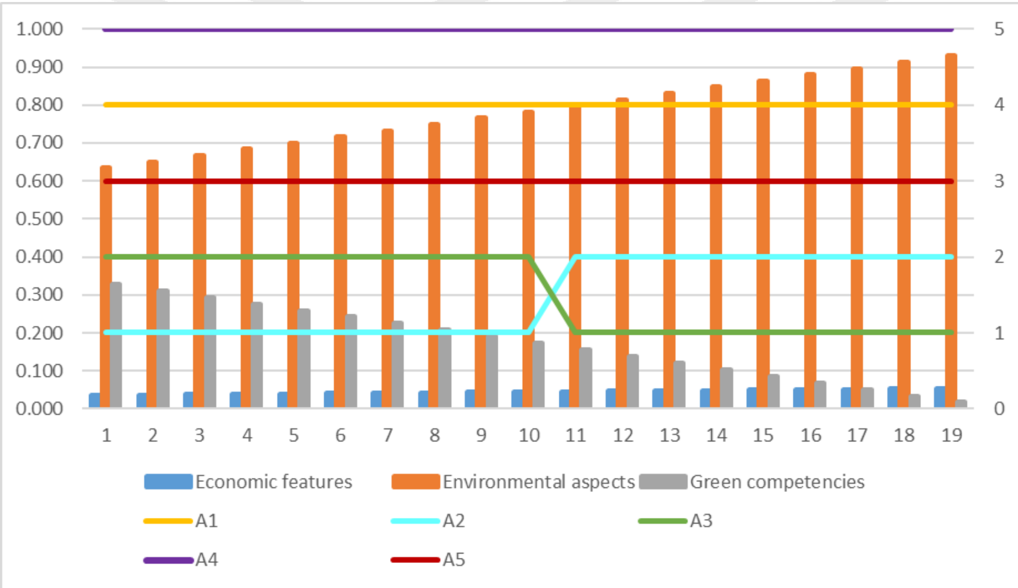
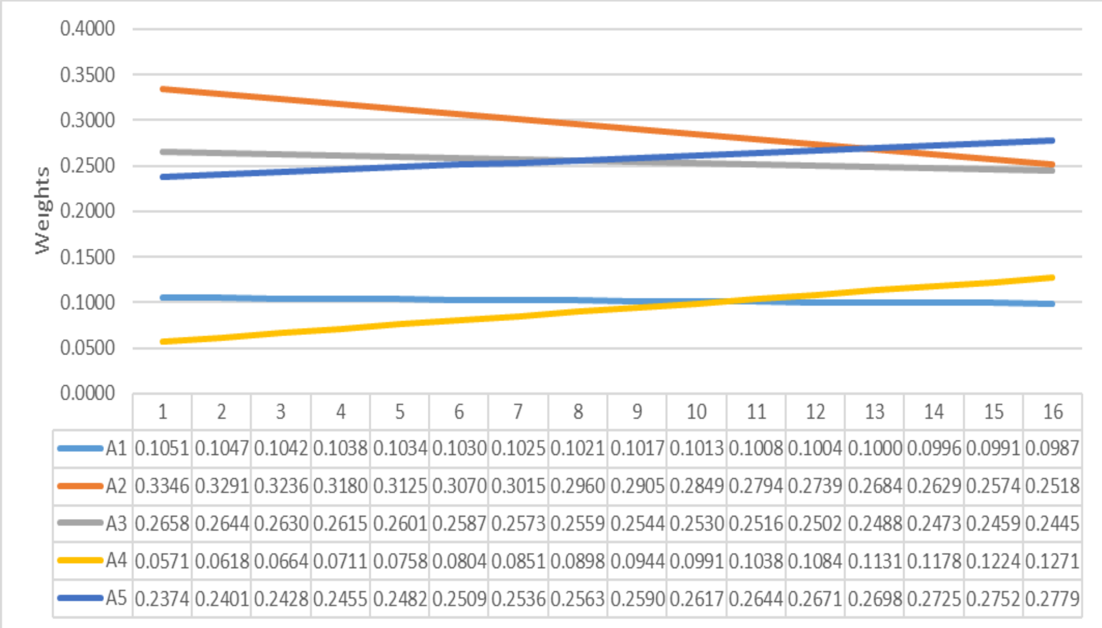


Figure 7.10 : Changes in criteria weights and ranking of alternatives based on 19 scenarios.

In case 3, the weight of the *economic features* having the least criterion weight is increased by 50% each time, and the results are observed under 16 scenarios as in Figures 7.11 and 7.12. Since the weight of *economic features* is quite less than the others, the effect of the proportional increase will be less. For this reason, it is decided to increase by 50% in each scenario.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A1	0.1051	0.1047	0.1042	0.1038	0.1034	0.1030	0.1025	0.1021	0.1017	0.1013	0.1008	0.1004	0.1000	0.0996	0.0991	0.0987
A2	0.3346	0.3291	0.3236	0.3180	0.3125	0.3070	0.3015	0.2960	0.2905	0.2849	0.2794	0.2739	0.2684	0.2629	0.2574	0.2518
A3	0.2658	0.2644	0.2630	0.2615	0.2601	0.2587	0.2573	0.2559	0.2544	0.2530	0.2516	0.2502	0.2488	0.2473	0.2459	0.2445
A4	0.0571	0.0618	0.0664	0.0711	0.0758	0.0804	0.0851	0.0898	0.0944	0.0991	0.1038	0.1084	0.1131	0.1178	0.1224	0.1271
A5	0.2374	0.2401	0.2428	0.2455	0.2482	0.2509	0.2536	0.2563	0.2590	0.2617	0.2644	0.2671	0.2698	0.2725	0.2752	0.2779

Figure 7.11 : Changes in weights of alternatives based on 16 scenarios of case 3.

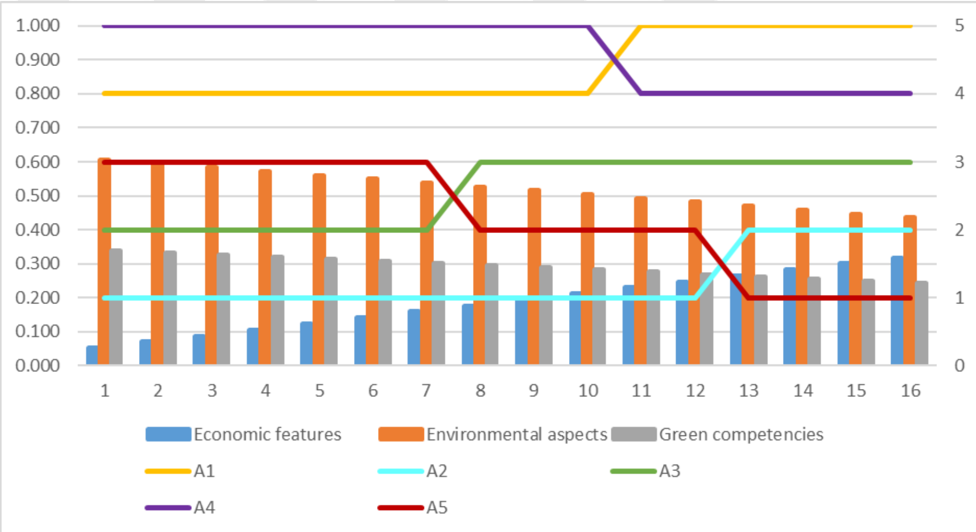


Figure 7.12 : Changes in criteria weights and ranking of alternatives based on 16 scenarios.

Increasing the weight of the *economic features*, which has the lowest weight according to the DMs' evaluations, causes some changes in the results as the scenarios progress. For example, the rankings of A3 and A5 replace when it is passed to the 8th scenario (that is, when the weight of the *economic features* is 5 times). This result actually means that there is no change in the results until the criterion weight of the *economic features* is 5 times. The ranking of the alternatives is observed until the weight of *economic features* is 9 times of the beginning value. As it can be seen from Figure 7.12, the ranking of the first alternative changes quite later. The robustness of the results is clearly seen considering such high changes cannot be in the criteria weights.

When the three cases are assessed together, the changes in the results according to the criteria weights indicate the robustness of the proposed fuzzy approach. Although some changes may cause little differences in the results, it can be interpreted that these differences do not impair the effectiveness and consistency of the developed method.

7.6 Conclusion

Since the results obtained in MCDM problems can vary depending on the used data and human judgments, the main point is the presence of reliability information about them. The fuzzy set theory and its various versions in the literature attempt to better model subjective judgments. However, they cannot incorporate reliability information alone into solution processes. Furthermore, in order to calculate the correct numerical equivalents of linguistic expressions and obtain more accurate results, not only the judgments but also the reliability of them should be included in the decision structure using the broadest framework. Although the Z-fuzzy numbers introduced by Zadeh (2011) have been used and extended to many versions to reflect reliability information, the literature is not rich in terms of these studies. For these reasons, the main purpose of this study is to integrate the IVSFSs, which have been put forward in recent years and have become very popular since then, with the Z-fuzzy numbers. Another aim is to propose a novel IVSF Z-AHP method to deal with MCDM problems involving uncertain expressions. The integration of IVSF Z-numbers with the AHP method and their presentation with a multi-expert model can be useful for problems in uncertain decision environments.

Due to the nature of speaking language, DMs use ambiguous judgments in pairwise comparisons, and these judgments are transformed to numerical equivalents with linguistic expressions defined by the help of fuzzy set theory. In this study, DMs' judgments and their reliability degrees are modeled by a novel IVSF Z-AHP method. The introduction of IVSF Z-numbers for the first time makes this study quite important. In the proposed method, a new IVSF Z linguistic scale that DMs can employ in their evaluations has been defined. A new formula is introduced to the literature to calculate the defuzzified values of the IVSF Z-numbers used in this scale. The proposed defuzzification formula produces more logical, consistent and effective responses than the existing formulas in the literature. It provides more realistic and quantitative data after the DMs' assessments have been processed during the computational stages.

In fuzzy AHP methods, the reciprocal values should be assigned to be multiplicative inverse to each other. It has been successfully achieved for IVSF Z-numbers in the proposed AHP method. This description is important in terms of accuracy and consistency in calculations, getting correct results and compliance with the fundamentals of the AHP method.

The proposed method has been successfully performed for a GSS problem to demonstrate its applicability. The main problem in the GSS is to estimate the potential environmental sensitivity of suppliers and to calculate their real values requiring the use of DMs' subjective judgments. The GSS problem is just one of the real-life MCDM problems that are based on subjective judgments in the solution process. The proposed method is applied for evaluating the five green suppliers according to the determined criteria set based on the DMs' pairwise comparisons. DMs have used IVSF Z restriction and reliability linguistic terms to evaluate five alternatives under three main and ten sub-criteria. The obtained results have been compared with the results of IVSF AHP and ordinary fuzzy Z-AHP methods. The findings of the proposed method are different according to both methods and show the importance of reliability information on the results. Finally, a sensitivity analysis is performed through the change of criteria weights under three cases consisting of different scenarios. Only in case 2, the first-ranked and the second-ranked alternatives have replaced. In this case, the sensitivity of the alternatives can be examined by changing the criteria weights. In case 3, it is aimed to show that rankings will naturally change in case of major changes in criteria weights.

The criteria used in the AHP method are grouped under three categories consisting of 10 sub-criteria totally. The most important main criterion that companies should pay attention in order to select a green supplier is *environmental aspects*, and they should focus on *environmental pollution level* among the sub-criteria. In addition, *environmental friendly manufacturing* and *green image*, which have weights greater than 0.10 among the 10 sub-criteria, can be interpreted as the most important sub-criteria to be considered in managerial applications about GSS problem. To measure the consistency of the proposed IVSF Z-AHP method and to observe the results

according to the change in criteria weights, a sensitivity analysis is carried out. The small changes that have occurred do not require a change in managerial decisions.
There are some limitations to the proposed method as well as advantages. One of them is that the cost and benefit criteria have same operations in the solution process as in the IVSF AHP method in the literature. The computational complexity is another limitation that makes the method difficult to extend. Another limitation is that only way 1 can be used rather than way 2 in sensitivity analysis because it allows us to change the criteria weights.

For future research, IVSF Z-numbers can be used with other MCDM methods such as TOPSIS, VIKOR, TODIM, MACBETH (Tolga and Basar, 2022) or EDAS (Deveci et al., 2022) to develop new approaches such as IVSF Z-AHP&TOPSIS or IVSF Z-AHP&VIKOR, and they can be applied to other types of problems such as location selection, personnel selection, or software selection. In addition, Z-fuzzy numbers can be modeled with recently emerged fuzzy sets such as fermatean fuzzy sets or circular intuitionistic fuzzy sets to see their abilities in representing fuzziness. Then, AHP method or other MCDM methods can be integrated with these extensions, and the results of this paper can be compared with them.

8. CONCLUSIONS AND RECOMMENDATIONS

Decision making in real-life is generally made based on the experiences and knowledge of experts and practitioners. Therefore, solutions of real-life problems involve subjective intense evaluations, which represent decision makers' complex thinking structure. Although fuzzy sets are successful in reflecting these knowledge and experiences into the decision process, they lack the ability to represent reliability information. For this purpose, in this thesis, new types of Z-numbers have been developed by integrating fuzzy set extensions and Z-numbers. Picture fuzzy Z-numbers, interval-valued spherical Z-numbers, decomposed fuzzy Z-numbers have been introduced to the literature, which is the main contribution of this thesis. In addition, ordinary fuzzy Z-numbers have been integrated with AHP, CODAS and EDAS methods to increase their representation capability of uncertain linguistic judgments. Then, new fuzzy MCDM method and methodologies such as interval-valued spherical fuzzy Z-AHP, picture fuzzy Z-AHP&TOPSIS methodology, decomposed fuzzy Z-TOPSIS method have been developed by integrating these proposed Z-numbers. In order to show the practicality of them, proposed method and methodologies have been applied for different type of decision making problems. The necessity of reliability information and effects on the given decisions are demonstrated with sensitivity and comparative analysis. These analyzes show that Z-fuzzy numbers may change the results and effect the managerial decisions. Therefore, it is recommended that reliability information should be considered in decision making processes.

Although this thesis focuses on problems such as wind turbine selection, green supplier selection, supplier selection, location selection, solar panel selection, etc., the proposed method and methodologies can be applied to many different real-life problems. Thus, managers or practitioners can use the proposed method and methodologies for other MCDM problems by adapting criteria and alternative sets.

Although the proposed method and methodologies present reliable decision making tools for practitioners, they have mathematical complexities due to the fuzzy operations of the Z-numbers. In addition, when the number of criteria and alternatives increases, the solution process becomes difficult as in most MCDM methods under uncertainty. It is thought that the complex evaluation systems of experts are made more reliable with high mathematical concepts. Therefore, the advantages of the proposed methods and methodologies compensate disadvantages arising from computational complexity.

For further research, other fuzzy set extensions, such as decomposed fermatean fuzzy sets, decomposed pythagorean fuzzy sets, can be used to extend Z-numbers. Then, integrated fuzzy Z-MCDM methods such as decomposed fermatean fuzzy Z-AHP&CODAS or decomposed pythagorean fuzzy Z-AHP&EDAS can be proposed to the literature and compared with the results of this study.

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