JANUARY 2019 M.Sc. in Mechanical Engineering İBRAHİM BAŞAR KILIÇPARLAR

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MODELING OF REDUNDANT PLANAR PARALLEL MANIPULATORS

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MECHANICAL ENGINEERING

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İBRAHİM BAŞAR KILIÇPARLAR

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Modeling of Redundant Planar Parallel Manipulators

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Supervisor

Prof. Dr. Sadettin KAPUCU

by

İbrahim Başar KILIÇPARLAR

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This is to certify that we have read this thesis and that in our consensus/majority opinion it is fully adequate, in scope and quality, as a thesis for the degree of Doctor of Philosophy.

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I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

İbrahim Başar KILIÇPARLAR

ABSTRACT

MODELING OF REDUNDANT PLANAR PARALLEL MANIPULATORS

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In this study, a method originally designed for planar open-chain serial manipulators model which is called as a “virtual joint” is extended to model planar parallel manipulators (closed-chain systems). This method describes how the configuration of a robotic chain is generated by fixing either a revolute or a prismatic joint already exist at the same joint and fixed to the ground by appropriate constraint forces which converts the open chain into a closed link loop. Two joints, a revolute and a prismatic are assumed to exist between each link pair. According to the type of the manipulator to simulate, one of the two joints existent is allowed to move while other constrained by appropriately shaped force profiles. Once the model is constructed with this model, it is possible to examine the dynamic behavior of the almost all kinds of multi link planar manipulators. By the proposed method, a set of sub open chain system can be converted into anyone of the planer parallel manipulator type mechanisms and motion equation is properly shaped up to model this particular mechanism. The motion equations are fewer and numerical integration simpler, with fewer tendencies of numerical errors to build up.

Keywords: Modeling of multi link mechanisms, Virtual joint method, Planar parallel manipulators

ÖZET

GEREĞİNDEN FAZLA SERBESTLİK DERECELİ DÜZLEMSEL PARALEL MANİPÜLATÖRLERİN MODELLENMESİ

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61 sayfa

Bu çalışmada, orijinalinde düzlemsel açık zincir seri manipülatör modeli için tasarlanmış "sanal eklem" olarak adlandırılan bir yöntem, düzlemsel paralel manipülatörler (kapalı-zincir sistemleri) modelini de kapsayacak şekilde genişletilmiştir. Bu yöntem, bir robot kinematik zincir konfigürasyonunun, aynı eklemde halihazırda var olan bulunan bir dönme veya prizmatik bir eklemin sabitlenmesiyle ve açık zinciri kapalı bir zincir bağlantı döngüsüne dönüştüren uygun kısıtlama kuvvetleri ile nasıl üretildiğini açıklamaktadır. Her bağlantı çifti arasında bir döner ve bir de kayar (prizmatik) iki eklem olduğu varsayılmaktadır. Benzetim yapılacak manipülatörün tipine göre, mevcut olan iki eklemden birinin uygun şekilde şekillendirilmiş kuvvet profilleri tarafından kısıtlanmışken diğerinin hareket etmesine izin verilir. Bu model ile hemen hemen her çeşit çoklu bağlantı manipülatörünün dinamik davranışlarını incelemek mümkündür. Önerilen yöntemle, bir dizi alt açık zincir sistemi, düzlemsel paralel manipülatör tipi mekanizmalardan herhangi birine dönüştürülebilir ve hareket denklemi, bu özel mekanizmayı modellemek için uygun şekilde biçimlendirilir. Hareket denklemleri daha az ve sayısal bütünleşme daha basittir, daha az sayısal hata eğilimi oluşur.

Anahtar Kelimeler: Çok uzuvlu mekanizmaların modellenmesi, Sanal mafsal yöntemi, Düzlemsel paralel manipülatörler

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LIST OF SYMBOLS/ABBREVIATIONS

$\boldsymbol{m^i}$	The rigid body total mass
$\boldsymbol{a^i}$	The complete acceleration of body
$\boldsymbol{F^i}$	The force vector on the mass body center
$\boldsymbol{J^i}$	The mass moment of inertia
$\boldsymbol{\ddot{\theta}^i}$	The angular acceleration of body
$\boldsymbol{M^i}$	The moment acting on the body
$\mathbf{n_b}$	The number of system coordinate
$\mathbf{n_b}$	The independent constraint equations
$\boldsymbol{l}$	The total amount of rigid bodies
$\boldsymbol{d_i}$	The amount of D.O.F. of the i'th joint.
$\boldsymbol{n}$	The total amount of joints
$\boldsymbol{K_p}$	The diagonal proportional gains
$\boldsymbol{K_v}$	The diagonal derivative gains
$\boldsymbol{\lambda}$	Lagrangian Multiplier corresponding to constraint
$\boldsymbol{C(q)}$	Closure Constraint Function
$\boldsymbol{EF_i}$	Existence Factor
$\boldsymbol{a_i}$	Length of i^{th} link
$\boldsymbol{p_i}$ ***and*** $\boldsymbol{q_i}$	Position parameter of mass at i^{th} link
$\boldsymbol{m_i}$	Mass of the i^{th} link
$\boldsymbol{R}$	The revolute joint
$\boldsymbol{P}$	The prismatic joint
$\boldsymbol{\theta_i}$	Generalized coordinate for revolute joint of i^{th} link

$\boldsymbol{B_i}$	Generalized coordinate of i'th link for prismatic joint
$\boldsymbol{F_i}$	Generalized force
$\boldsymbol{\tau_i}$	Generalized Torque
$\boldsymbol{I_i}$	Moment of inertia of i'th link
$\boldsymbol{x_{tip}}$	The x component of position of tip point
$\boldsymbol{y_{tip}}$	The y component of position of tip point
$\boldsymbol{M}$	The symmetric inertial matrix
$\bar{M}$	The symmetric inertial matrix with existence factors
$\boldsymbol{h}$	The vector of externally applied actuator forces/torques and the sum of centrifugal, Coriolis and gravity terms
$\boldsymbol{q}$	The vector of generalized coordinate
$\boldsymbol{\dot{q}}$	The vector of generalized velocity
$\boldsymbol{\ddot{q}}$	The vector of generalized acceleration
$\boldsymbol{u}$	The vector of generalized constraint forces
$\boldsymbol{I}$	The inertia matrix of the whole system
$\boldsymbol{g}$	The vector containing; forces and torques of the whole system
$\boldsymbol{A}(\boldsymbol{g})$	The jocabian of the constraint equation
$\boldsymbol{F_{Bi}}$	The applied actuator force
$\boldsymbol{T_{\theta_i}}$	The applied actuator torque
$\emptyset_{q_i}$	The sum of centrifugal, coriolis and gravity terms due to generalized coordinate
DOF	The number of system degrees of freedom
PID	Proportional, Integration and Derivative
NOC	Natural Orthogonal Complement
DAE	Differential Algebraic Equation
ODE	Ordinary Differential Equation
RPDR	Rice Planar Delta Robot

CHAPTER 1

INTRODUCTION

1.1 Introduction

In the past recent years parallel manipulators which is a closed loop mechanism consisting of the mobile platform connected base by at least two serial kinematic chains (legs) have advantages such as high structural stiffness, position accuracy and payload over serial manipulators.

Modeling of the dynamic systems has been very important in terms of their usage in design, analysis and model based control of multi-body systems. Machine designers require to see the dynamic behavior of the system and verify that the kinematics of the system are conformable and kinetics, realizable. For this purpose motion equations of the system are required. Efficient formulation exists for serial chain and three structured multi-body system, the adaptation of these methods for simulation of parallel manipulators is relatively more difficult.

Therefore, a generalized model to present anyone of the redundant parallel manipulator is necessary for design, analysis and control.

1.2 Thesis Objectives

The main aim of this thesis is to propose a generalized model to present anyone of the redundant and/or non-redundant parallel manipulator is necessary for design, analysis and control.

1.3 Thesis Scopes

This thesis describes mainly modeling, any configuration of planar parallel manipulator. The scopes are listed as follows:

- ***Modeling*****:** Introducing a detailed method to get the kinematics and dynamic equations of open chain virtual joints model
- ***Converting open-chain to close-chain mechanism algorithm*****:** Proposing appropriately shaped force profiles to obtain close chain from open sub chains.

1.4 Definition of Multi-body Systems

In general, a multi-body system is formed by bodies and components or sub-structures. Movement of the subsystem is achieved by means of different joint types, also translations and rotational or both displacements may occur in each subsection. The motion of the bodies has a field that includes mechanics of a rigid body; structural and continuum. The rigid body is assumed to be the ideal body that is not affected by the gross body motion in terms of deformation. In this manner, at all durations and arrangements of the rigid body, the length between any of its two particles stays constant under the action of all kind of forces. Conceptually, a multi-body system which has bodies and components can be depicted in Figure 1.1.

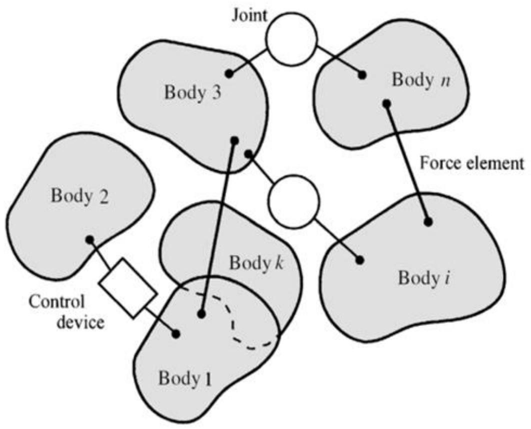

Figure 1.1 A general multi-body system [2].

1.5 Classification of Multi-body Systems

Multi-body systems are categorized into two classes which are open-chain or closed-chain systems. If any system consists of bodies without closed branches (or loops), then it is named an open-chain system. A triple pendulum can be given as an example depicted in Figure 1.2. Instead of letting the end of the system free motion if it is constraint as shown in Figure 1.3, such a system that is named a closed-chain multi-body system. Indeed, it is also well known as a four-bar mechanism.

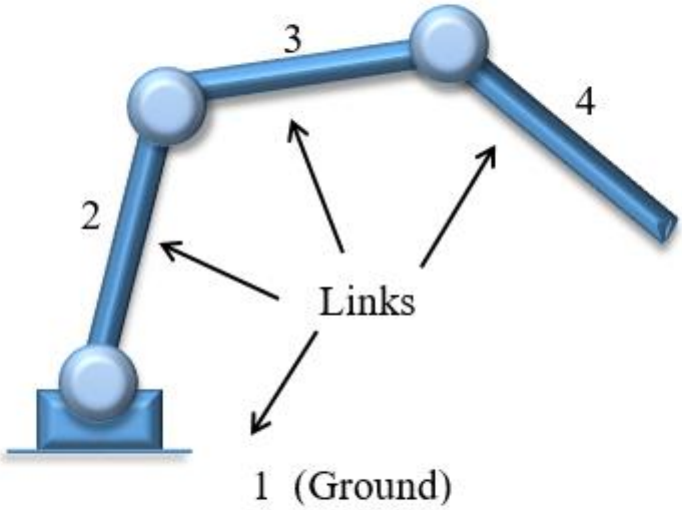

Figure 1.2 An example of an open chain system.

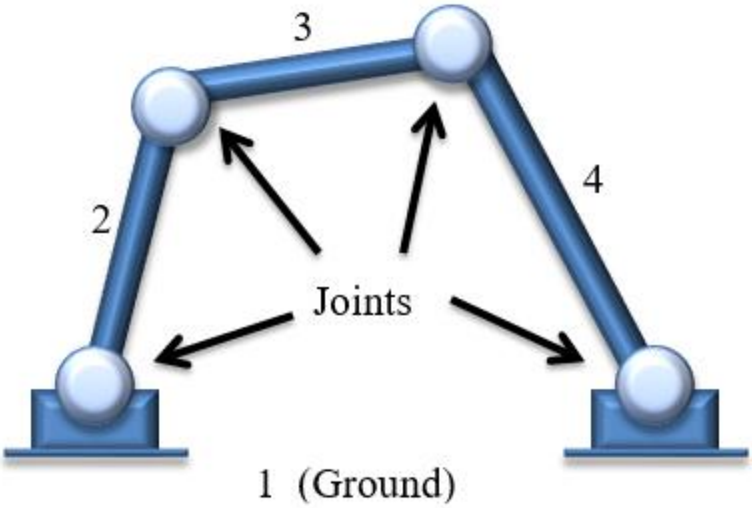

Figure 1.3 An example of a closed chain system.

1.5 Dynamics of Mechanical Systems

Dynamics is a science of particles and/or bodies motion. It can be separated into two sections as kinematics and kinetics. The kinematic analysis deals with the motion regardless of the forces which have attention on the geometric motion aspects which are positions, velocities, and accelerations. But the kinetics deals with the force-motion relationships. So the multi-body systems dynamics contains the analysis of rigid and/or deformable bodies. In the context of the thesis, analysis of rigid bodies will be in consideration.

1.5.1 Rigid Body Dynamics

The dynamic equations that concern with rigid bodies motion which can be systematically derived from the particle equations. The rigid body includes a significant number of particles. By the help of six equations it is possible to determine the unconstrained three-dimensional motion of the rigid body, and the three of these six equations are related to the rigid body and the other three are connected with the body rotation. For a centroid body coordinate system; Newton equations is the name for the translational equations, whereas Euler equations are the name for the rotational equations. Newton–Euler equations are explained by accelerations and forces which effect on the body for any arbitrary rigid body motion. For the special case of planar motion, the Newton–Euler equations are combined into to three scalar equations. For i'th body of the multi-body system it can be written as;

$$\left.\begin{matrix} m^i a^i = F^i \\ j^i \ddot{\theta}^i = M^i \end{matrix}\right\} \tag{1.1}$$

Where, m^i refers to the rigid body total mass, a^i refers to the two-dimensional vector which specifies the complete acceleration of the body about its mass center, F^i is the force vector which acts on the mass body center, J^i is the mass moment of inertia described by its relation to the mass center, $\ddot{\theta}^i$ is the angular acceleration of the body, and M^i is the moment acting on the body.

Selecting the mass center as the origin of the body coordinate system makes it easier to calculate the dynamic equations. A result of the selection of the body reference is that no inertia coupling can be seen between the translational and rotational coordinates of the rigid body while using Newton-Euler equations. When deformable bodies are considered, decoupling of the coordinates becomes more difficult.

1.5.2 Constraint Motion

In multi-body systems, the motion of the bodies is limited or restricted depending upon the mechanical joints; such as revolute, spherical, and prismatic joints or specified trajectories as depicted in Figure 1.4. Six coordinates are needed to determine the rigid body configuration in space, $6 \times n_b$ coordinates are necessary to determine n_b unconstrained body motion.

Mechanical joints or specified trajectories decrease the system mobility since the various bodies' motion is no longer independent. By using some non-linear algebraic constraint equations, it is possible to explain the mechanical joints and specified motion trajectories mathematically. Assuming that constraint equations are linearly independent, each constraint equation limits a possible system motion. Thus, degree of the freedom of a system is determined by the number of the system coordinates minus the independent constraint number equations. For a n_b rigid body system with n_c independent constraint equations, the number of system degrees of freedom (DOF) is given by

$$DOF = 6 \times n_b - n_c \tag{1.2}$$

and it is called the Kutzbach criterion.

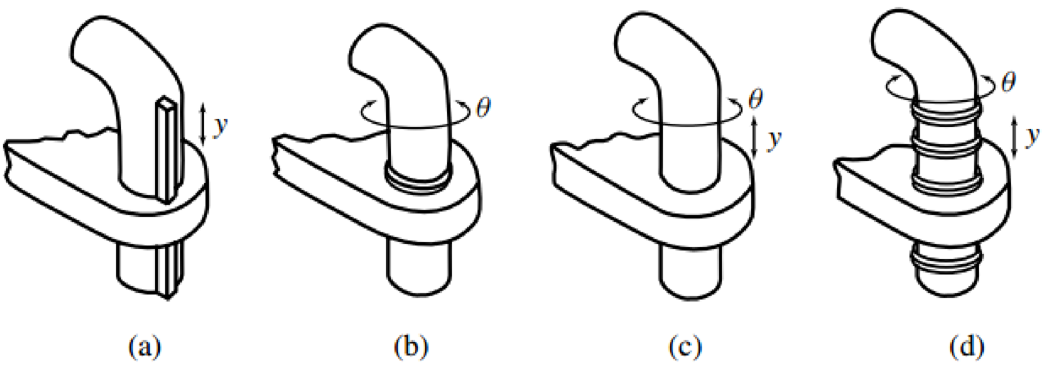

Figure 1.4 Examples of mechanical Joints: a) Prismatic joint b) Revolute joint, c) Cylindrical Joint d) Screw Joint. [2]

1.6 Introduction to Parallel Manipulator

Notwithstanding, common serial manipulators have large workplace and proficient maneuverability, parallel manipulators are recently more popular. The serial manipulators have many kinds of problems depending upon task requirement from them. One of these problems is their cantilever type construction. This structure makes them reactive to change their form at high load. Another problem of the serial manipulators is that they vibrate at higher speeds and this situation results in a less precise functioning. The parallel manipulators are better alternatives for the applications that require high load carrying capacity and accurate positioning.

When compared with their serial counterparts, parallel kinematic manipulators represent several advantages, which are higher load carrying capacities, low inertia, higher structural inflexibility, decreased sensitivity to certain errors, easy controlling and built-in redundancy. Fully parallel robots have all of their positioners in their base or near their base. This structure results in a very low inertia at the moving parts. Thereupon, it is possible to achieve a higher bandwidth with the same propelling force. That is the reason that manipulators with parallel structure provide a good use for flight simulators and robots which are used for picking and placing. Parallel manipulators are always beneficial for the great number of applications to make robot performance better when compared to serial manipulators. Parallel kinematic robots have a different kind of structure. There are arms and links which are paired in a parallel structure and attached to the Tool Centre Point. Then, for drilling, tapping, and welding purposes, all operating motors/gearboxes might be assembled on the base.

Since there are closed kinematic loops in the architecture, the kinematic equations for a parallel manipulator would be more complicated than for a serial manipulator. Closed loop manipulator is another name that is used for parallel manipulators. Models will be necessary for simulation and performance prediction in terms of developing high performance robots. Among the mechanical systems including conventional joints, traditional parallel manipulators experience errors that are the result of production faults in the joints, kick back, and hysteresis. Contrary to robots with serial structure, un-actuated or passive joints can exist. Kinematic analysis of parallel structures differs from kinematic analysis of serial manipulators because there are multi degrees of freedom joints and passive joints within parallel manipulators. Direct kinematics is much more complex and it includes the expulsion of the variable of the passive joints. The inverse kinematics is less complicated in parallel manipulators when compared to direct kinematics. Parallel manipulators are essentially more definite than the serial ones; due to their closed loop architecture and many parallel connections, their errors are averaged. These robots have many subsistent features over serial robots; therefore it will be many kinds of applications with parallel robots in near future in various fields. In this thesis only planar parallel manipulators are taken into consideration.

1.6.1 Planar Parallel Robots

A manipulator with a fully parallel structure includes an end-effector that has three D.O.F., and the two of D.O.F.s are translations and the remaining one is rotation. The shape of the end-effector is usually taken as triangle, and when the actuators are locked the mobility is seen as zero. Defining a general mobility criterion for closed-loop kinematic chains is a difficult task. Below is the Grübler's formula which is used for planar mechanism mobility.

$$m = 3(l - n - 1) + \sum_{i=1}^{n} di \quad (1.3)$$

In this formula, l refers to the total amount of rigid bodies within the structure, and n represents the total amount of joints. Lastly, d_i is the amount of D.O.F. of the i^{th} joint.

1.7 Thesis Layout

The rest of the thesis is organized as follows. Chapter 2 presents literature survey and modeling methods of the equation of motion of closed chain robots. Chapter 3 introduces material and method of Virtual Joint Method for closed-chain mechanisms. Chapter 4 presents modeling examples of single degree of freedom mechanisms and multi-degree of mechanism using Virtual Joint Method. Finally, in chapter 5, the concluding arguments of closed-chain mechanism modeling which are done using Virtual Joint Method and the further recommendations are presented.

CHAPTER 2

LITERATURE SURVEY

2.1 Introduction

It is really an important issue that machine designers to see the dynamic behavior of the mechanism that they designed before including them into their machines, and also verify that the kinematics of the mechanisms are proper and kinetics are realizable. For this aim, motion equations of the system are required to simulate to obtain the dynamic behaviors.

A manipulator which is designed to do a specific job, the reliability of the dynamic behavior must be tested. Hence, the motion equations of the manipulators are derived with their actuators and then control strategies are developed. However, deriving the motion equations of the manipulator takes a long time and is also prone to make mistakes, especially when it comes to closed chain mechanisms.

2.2 Equations of Motion of Closed Chain Robots

Many books [1- 4] and studies are available on how to obtain the mathematical models of multi body and multi degree of freedom mechanical systems and some of them are given below.

Wang and Gosselin (J. Wang and Gosselin 1998) studied on virtual work principle for the dynamic parallel manipulators analysis. It could be used for any planar and spatial parallel mechanisms types and refer to faster computational and efficient algorithms by applying classical Newton–Euler method.[5]

Xu et. al. (Xu, Chung, and Choi 1999) developed a new repeatable robot dynamics algorithm by using dynamic equations created by Kane and formulations of Newton-Euler. It can be applied for robots that have open-chain design and some closed-chain robots. While eliminating the process of cutting closed-chains open, it can be used to solve the dynamic problem of robots with closed-chains. It is suitable for robots which consist of rotary joints and also for robots that have prismatic joints. It is efficient for robots with closed-chains of computational load. The algorithm also applies to robots with multiple closed-chain mechanisms. [6]

H.Cheng et. al. (H. Cheng et al. 2001) studied a 2-DOF planar parallel redundant manipulator focusing on its dynamic model. The formulation of dynamics of the system was determined by using Lagrange-D'Alembert. A new way of calculating the dynamics of such systems was proposed. For the sake of applying the dynamic modeling and control, a 2-DOF planar redundant parallel manipulator was designed as a test-bed. PID control in the junction was empirically applied on the mechanism. It was verified via these experimental results that PID control law has its effectiveness. [7]

A new and accelerated method for dynamic analysis was suggested by Lee (Lee 1993). The right Lagrange multipliers are figured by the monotone decrease of the imperative blunder vectors and the subsequent differential conditions of movement which are understood by a settled procedure of normal differential conditions that time mix is performed by utilizing program DOPRIN which depends on unequivocal Runge Kutta Nystrom technique. The solidness of the calculation is kept up on the grounds that the speed and the increasing speed requirements, and also the position imperatives, are fulfilled inside the predefined resistance at each time step. A correct arrangement is gotten without the time requesting systems, for example, the choice of autonomous directions, decay of the requirement Jacobian network, and Newton Raphson iterations. The numerical models demonstrate the sharp decreases of the requirement blunders by the recommended increasing speed strategy and show the financial calculation notwithstanding for a to a great degree little resistance. [8]

(Alıcı 2002) utilized Sylvester's dialytic end procedure to decide joint data sources required to discover any purpose of five-bar planar parallel controllers that dependent on five inflexible connections and five single degrees of opportunity joints– revolute and kaleidoscopic joints. As per the controller's topology, two scientific definitions clarify the way followed by the tip of two connections mix to one another are find and acknowledged at the same time with the end goal to clarify two ways for connections crossing point focuses (Cartesian directions). The technique is simple execution with computational effectiveness and sound to figure all circumstances. As an outcome, the strategy is a straightforward and effective proficiency for direction arranging with reverse position and control of five-bar planar parallel controllers in joint space. [9]

Zheng (Luh and Zheng 1985) developed a calculation method for robots being used in the industry with three-dimensional closed kinematic chain mechanisms. This is used to determine the input generalized forces for the virtual open-chain. A result of calculations, a recursive Newton-Euler algorithm is advised. [10]

Another way to deal with the dynamic demonstrating (blend of screw hypothesis and the lessened framework) connected on a producing controller by a complex planar 3-DOF instrument with a multi shut chain system (parallel components). The real data compared with simulation data which are in agreement in each result. The model set free the actuator forces for straightforward calculation simple. The model resembles Lagrangian formulations which can be improved for serial robots and is divided into few serial mechanisms. The sequential system is reproduced from the sub-mechanisms local coordinates. [11] As a result, forging manipulator is modeled as a sophisticated multiple-closed-loop mechanism (Ding et al. 2014).

(Sugimoto 2003) presented another autonomous calculation for the dynamic examination of closed loop systems (planar, round, spatial components e.t.c.). The twist bases areas are expressed independently to each mechanism loop with passive joints as based on vectors. Then, the wrench bases areas can be explained without any computation and these are the dual bases that belong to the twists bases. For each loop, the calculation is used individually and with this way the cost of calculation can be decreased. When this formula is implemented to a system with a different motion-space, the algorithm is verified. [12]

Attia (Attia 2005) worked on the motion equations for generalized planar linkages which contain a rigid bodies system with all generalized kinematic joints kinds, are produced by using a recursive method. The system is inserted with dynamical equivalent constrained pieces. Then, to determine motion equations the linear and angular momentums are applied. However, they are determined by the rectangular Cartesian coordinates. The groups have advantages as unknown internal constraint forces for automatic elimination and determine the rigid body motion by Cartesian coordinates without either expressing the inconvenient rotational coordinates or distributing the external forces and force couples over the particles. Moreover, the technique does not require the introduction of the coordinates of the center of mass of each link among the geometric parameters. The method is extended to rigid body systems. [13]

Nakamura and Ghodoussi (Nakamura and Ghodoussi 1989) discussed the overall and systematic computational method of the inverse dynamics of closed-link systems. The method applies to the multiple-loop or closed-link mechanisms. This is developed by d’Alembert’s principle and derived with no calculation of the Lagrange multipliers. To explain the restrictions, it is needed to use the Jacobian matrix of the passive joint angles. When a non-redundant actuator system is taken into consideration, it makes it easy to represent the limitations even for sophisticated multi-loop closed-link mechanisms. The scheme is useful and the reason is the Lagrange multipliers is not needed. [14]

Kordjazi and Akbarzadeh (Kordjazi and Akbarzadeh 2011) presented a new model for 3-PRR (prismatic–revolute–revolute) planar parallel manipulator with the help of NOC method. By using the NOC method, Lagrange multipliers and passive joint coordinates are not needed. Also, this useful method does not require using velocity and acceleration inversions for deriving the dynamics equations. Motor forces of the system are computed using NOC and a joint trajectory is put out. Moreover, a simplified model of the 3-PRR is built-up and simulated using both SimMechanics and COSMOS-Motion of Solidworks software. The results gathered from this simulation matches with results which are obtained using the NOC method. [15]

Staicu (Staicu 2009) designed a repetitive model of kinematics and 3-PRR dynamic planar parallel. The technique for virtual examination is made in the opposite dynamics problem. For the comparison of each of three actuators in types of prismatic and revolute, matrix equations and graphs are used. The model considers by the masses and inertia forces related to all elements of the parallel mechanism. All internal joint forces removed by virtual work principle and produced a time-history evolution of powers satisfied for the actuators. The method has the validity for forward and inverse mechanics of all serial or planar parallel mechanisms. Because of the power supplication on the three actuators trust on the impelling setup, anyway the aggregate power consumed by the three actuators set as the equivalent, at any moment, for both driving frameworks. The investigation of the elements of the parallel components is done basically to illuminate effectively the control of the movement of such automated frameworks. [16]

Ghorbel et. al. (Ghorbel et al. 2000) improved a method for derivation equations of general closed-chain mechanisms motion. To observe a reduced method, it is comprehensively declared as independent generalized coordinates and two special points for open-chain mechanisms models. The first one is determined as locally in the generalized coordinates. The understood capacity hypothesis in the improvement of the decreased model can just ensure the presence of the arrangement of a zero of a capacity in a little nearby area with unspecified limits. Kantorovich theorem defines model domain validity where provide constraints and is not in a singular configuration. The second one is the reduced model (implicit) that some motion equations are not explained explicitly without improvement of implicit function. The local nature suggests good local stability properties (for the reduced model). Additionally, the implicit nature can be a difficulty resource (during setting of control laws used for). The open chain mechanism suggested by control laws used for regulation problem and requirement of the implicit model expressions. The PD-based control with simple gravity compensation guarantees asymptotic stability which is local since the reduced model. And the results were valid as test-bed to perform control experiments on closed-chain mechanisms. The RPDR explicit equations of motion used for general modeling and control ideas with more general closed-chain mechanisms. Besides, the explicit equations defined the full domain model and as a result to compare and estimate the boundaries of this domain results. [17]

Modular methods have effects on developing approach efficiency in a multibody system which named as a bodies-joints composite simulation of mechanisms with closed loops. The kinematic chains appointed generalized number coordinates and equal to freedom degree for simulation. The zero degree of freedom module is determined as closely all planar mechanisms with closed loops and isolated by algebraic equations and ODEs of differential-algebraic equations. The forward dynamics simulation can be expressed by three sets of algebraic equations and one set of ODEs. The algebraic equation is inevitable. The forward dynamics method computed by algebraic closed-form loop which applied implicit coupling equations. The 5R mechanism casework shows the feasibility of the proposal by comparison with a commercial code and others. The CPU times of the casework show that the closed-form algebraic solver is more efficient than the numerical one in modular simulation (H. Wang, Eberhard, and Lin 2010). [18]

Wang et. al. (J. Wang, Gosselin, and Cheng 2002) studied on virtual spring method for robotic (mechanical) systems with closed kinematic chains which virtual springs and dampers were used to contain the kinematic constraints thus preventing the solution of the differential-algebraic equations. Its advantage is completely decoupled dynamic approach which is ideal for real-time dynamic simulation. The conclusions give a good relation with other conventional approaches results. [19]

Khan et. al. (Khan, Tang, and Krovi 2007) studied development and performance-evaluation methods for distributed forward dynamics simulations of constrained mechanical systems characterized by the four-bar linkage. The same state valid for governing equations which define ODEs coupled with algebraic constraints and DAEs combined system solution requirement. The natural spatial parallelism of closed-chain manipulators is divided for modular development and distributed numerical simulation. The numerical simulation problem has followed by two stages as an initial algorithm development and then a numerical integration. While two stages do not relate with unconstrained mechanical systems but is have an important position for the constrained simulation systems. The results were advisable for four-linked linkages. [20]

Gosselin (Gosselin 1993) created a unique paradigm for calculating the inverse dynamics that the parallel manipulators have. It is a fact that, it is easy to parallelize the inverse kinematics and the inverse dynamics for this type of a manipulator. Therefore, a closed-form efficient algorithm which depends on using n processors (n represents the number of kinematic chains that link the base and the end-effector). The calculations of dynamics are done by using Newton-Euler formulas. The parallel algorithm is based on choosing coordinate frames which are attached to each leg and this exploits directly the parallel nature of the system. Furthermore, this algorithm can be used for planar, spherical and spatial parallel manipulators that include prismatic actuators.[21]

Aghili and Piedbœuf (Aghili and Piedbœuf 2003) have found a new formula for answering the Differential Algebraic Equation (DAE) of a constrained mechanical system which determines linear operator equation concept by using the orthogonal projection notion. For a solution, no need for Lagrangian multipliers to compute acceleration. Actually, the acceleration and the generalized force related with motion equation through the named constraint mass matrix. The matrix can be computed by a mass matrix and the null-space projection matrix with singular Jacobian. The constraint mass matrix proved as always invertible. Thus the acceleration can be calculated by the presence of redundant constraints (singular configuration). The mass matrix versus to Lagrangian formulation, approach to singularities, without iteration related to the computing acceleration process. And also related to computational efficiency.Besides, the formula is only the null-space component of the generalized force accommodate motion dynamics which is useful in the optimal control of the constrained multibody system, named as inverse-dynamics. As a result, a five-bar mechanism numerical simulation shown smoothly and accurately in the region of a singular configuration. [22]

Ellis and Ricker (Ellis and Ricker 1994) studied on a new computing closed-chain dynamics, related with the systematic variables elimination which is redundant and effect on calculation. The method has a stable solutions for motion differential equations which are less stiff than traditional force-closure approach equations. Numerical solution of motion of a kinematically constrained mechanism also seems simple structures. The Lagrange applied to undefined multipliers to holonomic equations for generating kinematic constraints to determine forces which fixed the constraint. The constraint forces are greater than joint forces which hardly solved by differential equations for a mechanism. A constraint formulation has an undesirable effect and if equals that relate with a different coordinate system, then Jacobian singularity of the transformation from constraint variables to the configuration changes leads to dismissing constraint on the configuration accelerations. The result of the mass matrix is singular or ill-conditioned which are redundant constraints. The configuration reduction scheme only pertinent variables are immune to the latter effect. The numerical calculation does not effect on stiff equations of motion, according to the reduced system formulation. The reduction has no important additional on numerical stiffness.[23]

2.3 Equations of Motion of Closed Chain Robots using Open Chain Mechanism

One of the most common easiest way to model closed chain link mechanism as open link mechanism is to; cut closed chain on proper joint to form as open link chain(s) hence derivation of the equation of the motion of the open chain mechanisms are straightforward, obtain the equation of motion of the open link mechanism(s) and then convert the open chains into closed chains using the appropriate constraints. Simulation of the closed chain mechanism modeled in such a way includes the solution of coupled differential equations of motion and algebraic constrains. The equations should be solved in small time interval for numerical consistency. This situation causes computation time increase and leads to problems in real-time simulation and control of the system. To overcome this drawback as result of the open chain modeling of the closed chain links, calculation load can be distributed on a few processors [24, 25]. Modular and reconfigurable mathematical modeling of closed chain systems, as open chain systems is quite interesting. Mechanisms or manipulators having different topological properties can be obtained with the help of above mentioned modeling.

Alıcı et. al. have discussed 5 link planar mechanism modeling and control. Their closed chain linkage mathematical model is based on revolute jointed articulated open chain linkages where the last link tip is kept at specified location act as a revolute joint by calculated constraint forces. Bayseç and Jones [26] have proposed "The method of Fictititous Degrees of Freedom" for modeling all possible topologies of open chain planar robot manipulator composed of prismatic and revolute joint. The proposed model is formed three moving links in articulation with one revolute and one prismatic joint between each pair. Depending upon the linkage topology, one of the joint existent is allowed to move while the other is constraint with appropriate shaped force/torque. Nacarkahya [27] has extend this study by fixing the tip of the open chain robotic arm to the ground by appropriate force which can be generated by either a revolute or a prismatic joint and convent the open chain into a closed link loop to model anyone of the 4-link Grübler type mechanisms.

Our aim is to propose a general formulation which enables to model almost all planar parallel manipulator using "imaginary joint" model. This method is originally designed to model open chain planar robotic systems, describes how the tip of a robotic chain is fixed to the ground by appropriate constraint forces which can be generated by either a revolute or a prismatic joint and convert the open chain into a closed link loop. Two joints, a revolute and a prismatic are assumed to exist between each link pair. According to the type of the manipulator to simulate, one of the two joints existent is allowed to move while other constrained by appropriately shaped force profiles.

CHAPTER 3

OPEN CHAIN GENERALIZED PLANAR ROBOT MODEL

3.1 Introduction

This section briefly explains a method developed by Bayseç and Jones [26] which they call "The Method of Fictitious Degrees of Freedom". This method was proposed to model 4 link, planar open chain linkages composed of revolute and prismatic pairs only. These systems are assumed as the sub parts of planar parallel robots. First link is the ground and the three links connected to it in articulation, and the last link rigidly connected to platform are moving and hence comprise a 3 degree of freedom motion. The structure of the generalized sub chain manipulator model is shown in Figure3.1.

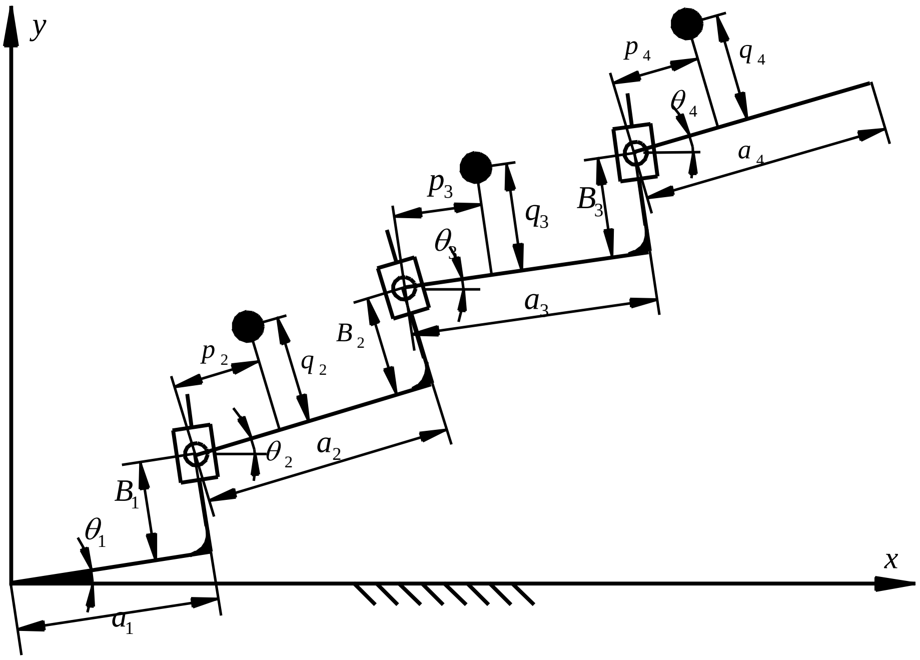

Figure 3.1 Generalized PRPRPR open chain linkage for modeling all possible topologies of open chain planar robot manipulator [26].

3.2 Equation of motion of the Generalized Open Chain

The equations of motion for the generalized chain are derived using Lagrange's formulation that results six second order ordinary differential equations. Generalized coordinates used are $B_1, \theta_2, B_2, \theta_3, B_3$ and θ_4. Generalized Joint Constraint Forces and Torques, which are activated by appropriate Existence Factors, should be added on the motion equation of each degree of freedom in the same format as the external driving forces and torques. Generalized constraint force of each degree of freedom is equal in magnitude opposite in direction to the sum of all external and velocity dependent forces. Each generalized joint constraint force is activated or kept inactive by an Existence Factor that is a binary information bit defining the actual type of joint. If degree is real its EF is 1, if not its EF is 0, EF's 1 to 6 denote the existence of coordinates $B_1, \theta_2, B_2, \theta_3, B_3, \theta_4$ respectively and

$$
\begin{aligned}
EF_1 &= \overline{EF_2}\ .OR.\ EF_2 = \overline{EF_1} \\
EF_3 &= \overline{EF_4}\ .OR.\ EF_4 = \overline{EF_3} \qquad (3.1) \\
EF_5 &= \overline{EF_6}\ .OR.\ EF_6 = \overline{EF_5}
\end{aligned}
$$

are always true. After the inclusion of Existence Factors and the necessary arithmetic operations of the Lagrange formulation produces final form of the motion equation as [66]:

$$\overline{\boldsymbol{M}}\ddot{\boldsymbol{q}} = \boldsymbol{h}(\dot{\boldsymbol{q}}, \boldsymbol{q}, \boldsymbol{u}) \qquad (3.2)$$

Where $\boldsymbol{q} = [B_1\ \theta_2\ B_2\ \theta_3\ B_3\ \theta_4]^T$ is the vector of generalized coordinates.

$\overline{\boldsymbol{M}}$ is the symmetric inertial matrix with existence factors.

$\boldsymbol{h} = (\dot{\boldsymbol{q}}, \boldsymbol{q}, \boldsymbol{u})$ is the vector containing; externally applied actuator forces/torques, F_{B1}, T_{θ_2}, F_{B2}, T_{θ_3}, F_{B3}, T_{θ_4} and the sum of centrifugal, Coriolis and gravity terms, $\emptyset_{B1}, \emptyset_{\theta_2}, \emptyset_{B2}, \emptyset_{\theta_3}, \emptyset_{B3}, \emptyset_{\theta_4}$. Matrix and vector elements are given in Appendix A.

By using the proper existence factors of Equation (3.1), the motion of any 3 degrees of freedom open loop chain is defined by Equation (3.2). The portion of the original model, which is presented here does not contain the dynamics of intermediary links such as actuators, balance springs, dashpots etc. The model in full can be found in [26].

3.3 Generalized Model of Planar Parallel Manipulator

Generalized model of parallel planar manipulator consists of three identical sub open chains; numbered I, II and III and a coupling platform P, as depicted in Figure 3.2. Last links of the sub chains form the equilateral platform which has a total mass of last links mass. Its mass center is located at the centroid of the equilateral triangle. Each sub parts can be modeled as an open chain with three degrees of freedom. The dynamic equations describing the each sub system can be written by using the proper existence factors of Equation (3.1), for i=I, II and III as follows:

$$\boldsymbol{\bar{M}^i}\boldsymbol{\ddot{q}^i} = \boldsymbol{h^i}(\boldsymbol{\dot{q}}, \boldsymbol{q}, \boldsymbol{u}) \tag{3.3}$$

The Newton –Euler equations of motion of the platform becomes:

$$\boldsymbol{\bar{M}^P}\boldsymbol{\ddot{q}^P} = \boldsymbol{h^P}(\boldsymbol{\dot{q}}, \boldsymbol{q}, \boldsymbol{u}) \tag{3.4}$$

where $\boldsymbol{\bar{M}^P}$ is the 3x3 inertia matrix of the platform

$\boldsymbol{h^P}$ is the 3x1 vector containing; forces and torque

$\boldsymbol{\ddot{q}^P}$ is the 3x1 vector containing; acceleration and angular acceleration

The dynamics of whole system can be formulated as Differential Algebraic Equations (DAE) whose solution is to satisfy the holonomic constrains resulting from cutting the loops at joints on the platform.

$$\boldsymbol{I}\boldsymbol{\ddot{q}} = \boldsymbol{g} - \boldsymbol{A}(\boldsymbol{q})^{\boldsymbol{T}}\boldsymbol{\lambda} \tag{3.5}$$

$$\boldsymbol{C}(\boldsymbol{q}) = 0 \tag{3.6}$$

Where

$$\boldsymbol{I} = \begin{bmatrix} \boldsymbol{\bar{M}}^{I}_{6X6} & 0 & 0 & 0 \\ 0 & \boldsymbol{\bar{M}}^{II}_{6X6} & 0 & 0 \\ 0 & 0 & \boldsymbol{\bar{M}}^{III}_{6X6} & 0 \\ 0 & 0 & 0 & \boldsymbol{\bar{M}}^{P}_{3X3} \end{bmatrix} \tag{3.7}$$

$$\boldsymbol{g} = [\boldsymbol{h^I}(\boldsymbol{\dot{q}}, \boldsymbol{q}, \boldsymbol{u})_{\boldsymbol{6x1}} \quad \boldsymbol{h^{II}}(\boldsymbol{\dot{q}}, \boldsymbol{q}, \boldsymbol{u})_{\boldsymbol{6x1}} \quad \boldsymbol{h^{III}}(\boldsymbol{\dot{q}}, \boldsymbol{q}, \boldsymbol{u})_{\boldsymbol{6x1}} \quad \boldsymbol{h^P}(\boldsymbol{\dot{q}}, \boldsymbol{q}, \boldsymbol{u})_{\boldsymbol{3x1}}]^{\boldsymbol{T}} \tag{3.8}$$

$\boldsymbol{C}(\boldsymbol{q}) = [\boldsymbol{c}(\boldsymbol{q})^{\boldsymbol{I}} \quad \boldsymbol{c}(\boldsymbol{q})^{\boldsymbol{II}} \quad \boldsymbol{c}(\boldsymbol{q})^{\boldsymbol{III}}]^{\boldsymbol{T}}$, is the loop closure constraints,

$\boldsymbol{A}(\boldsymbol{q}) = \frac{\partial \boldsymbol{C}(\boldsymbol{q})}{\partial \boldsymbol{q}}$, is the Jacobian of the constraint equation.

Here, the resulting formulation yields systems of index-3 DAEs. The numerical solution of the equations becomes problematic [28] [29]. Mainly, two approaches adopted for the dynamic simulation of such systems are: a) embedding techniques (elimination of dependent coordinates) wherein the system dynamic equations are formulated in terms of the generalized coordinates. This technique leads to smaller set of ordinary differential equations, but these may become much more complicated. b) Converting techniques wherein the constraints in position and/or velocity are differentiated and represented as second order differential equations. The resulting equations become an augmented formulation of index-1 DAE. One of the methods to solve such kind of equations is “compliance-based approaches [30] which uses the virtual springs and dampers to approximate the constraint forces [31]. Virtual springs can be considered as a form of penalty formulation of Jalon and Bayo [1] and/or stiffness of the spring and damping coefficient of damper can be considered as a form of gains of proportional and derivative gains of Alıcı at all [32].

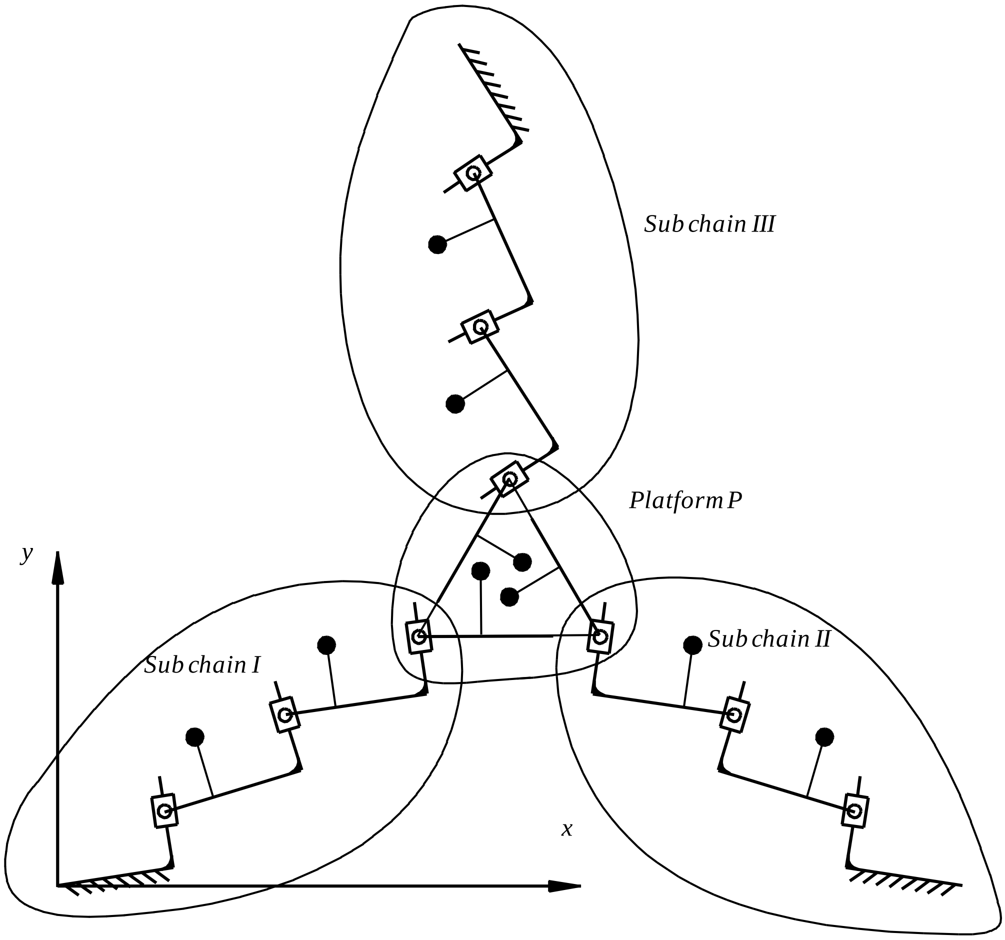

Figure 3.2 A generalized model of planar parallel manipulator with its sub open chains.

In order to convert, obtained general equations of open chain mechanisms to closed chain, end point of the open chain mechanism must be constraint on fixed or moving link via constraint forces or torques. Consequently, by helps of constraint forces and torques end points behaves like a joint. (PD control) compliance-based method will be applied for the calculation of this joint force and torque values. The Lagrangian multiplier corresponding to the constraint force is expressed as;

$$\lambda = K_p C(q) + K_v \dot{C}(q) \tag{3.9}$$

Where K_p is the diagonal proportional gains, K_v is the diagonal derivative gains.

One way to obtain the desired performance in the first term of the constraint forces and or torque can be achieved by selecting proportionality constant K_p as a high number. In the limiting case, when K_p goes to infinity, then the constraint error will tend to zero. One difficulty likely occur is that the solution under the control of infinitively large corrective constraint forces tend to go out of control, displaying high frequency and amplitude oscillations on error functions. To prevent any such oscillations K_p must be kept at moderate values. This on the other hand increases the magnitudes of errors, which may go beyond tolerable limits. To keep K_p high and eliminate the oscillation, a damping force component which is the second term of the constraint forces and or torque equation, is to be included with appropriate proportionality constant K_v. The damping term does not affect the dynamics of the system, but dampens the constraint forces only. Those proportionality constrains are analogous to the proportional gain and derivative gain in a closed loop control systems.

CHAPTER 4

SIMULATIONS AND RESULTS

4.1 Introduction

To demonstrate the versatility of the method, several examples are presented in this section.

4.2 Single Degree of Freedom Planar Mechanism

The equations of motion of the single freedom planar mechanism can be easily formed by using the method described in Chapter 3.

4.2.1 The Four Bar Mechanism

When the tip of the generalized PRPRPR open chain linkage is constrained not to move by two constraint forces, the system reduces into a 4-bar mechanism as depicted in Figure 4.1. Relevant linkage parameters (masses, mass moments of inertias), initial conditions (orientations, velocities), generalized forces and torques, location of the tip point of the open chain, existence factors to determine the type of joints and constraint force and/or torque gains to keep the given mechanism tip point at the required ground position are listed in Table 4.1. As it is seen from the Table 4,1 the crank of the four bar is brought to zero degrees position and released from rest. Chrono-cyclograph of the generated motion under the action of gravity only is shown in Figure 4.2. As there are no external motor action or friction, system is conservative and displays non-ending oscillations as shown in Figure 4.3. Displacements, velocities and acceleration of system is shown in Figure 4.3, Figure 4,4 and Figure 4.5 respectively and constraint forces are shown in Figure 4.6. The easiest and simplest way to check the validity of the model and equation is to draw profiles of the total kinetic, potential and sum of the energy curves. Since the system is conservative, total energy stays constant as shown in Figure 4.7.

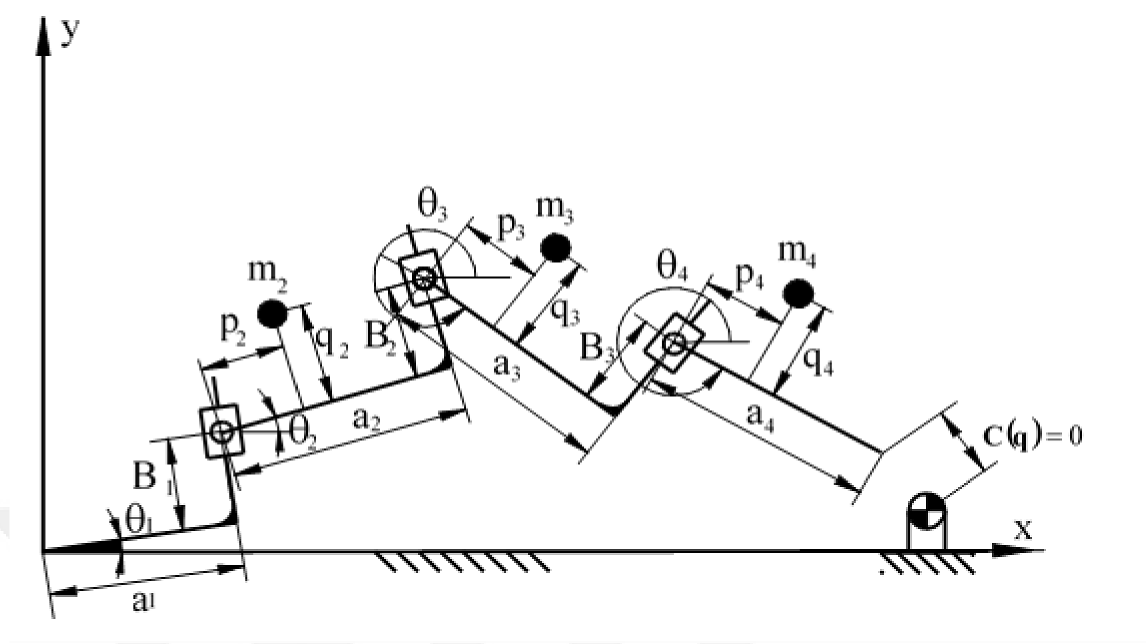

Figure 4.1 Generalized PRPRPR open chain linkage for modeling a four bar.

Table 4.1 Four bar mechanism simulation parameters

Linkage parameters		**Initial conditions**	**Existence factor**
$a_1 = 0.0\,m$	$m_2 = 0.5\,kg$	$B_1 = 0.0\,m$	$EF_1 = 0$ $EF_3 = 0$ $EF_5 = 0$
$a_2 = 0.2\,m$	$m_3 = 0.5\,kg$	$B_2 = 0.0\,m$	$EF_2 = 1$ $EF_4 = 1$ $EF_6 = 1$
$a_3 = 0.8\,m$	$m_4 = 0.5\,m$	$B_3 = 0.0\,m$	**Tip point of the open chain**
$a_4 = 0.8\,m$	$I_2 = 0.0\,kgm^2$	$\theta_2 = 0.0\,rad.$	$x_{tip} = 1.0\,m$
$p_2 = 0.2\,m$	$I_3 = 0.0\,kgm^2$	$\theta_3 = \frac{\pi}{3}\,rad.$	$y_{tip} = 0.0\,m$
$p_3 = 0.8\,m$	$I_4 = 0.0\,kgm^2$	$\theta_4 = -\frac{\pi}{3}\,rad$	**Constraint force or torque gains**
	Generalized Forces		
$p_4 = 0.8\,m$		$\dot{B}_1 = 0.0\,m/s$	$K_{p_{tip}} = 100000\,Ns/m$
$q_2 = 0.0\,m$	$F_1 = 0.0\,N$	$\dot{B}_2 = 0.0\,m/s$	$K_{v_{tip}} = 100\,Ns/m$
$q_3 = 0.0\,m$	$F_2 = 0.0\,N$	$\dot{B}_3 = 0.0\,m/s$	**Generalized Torques**
$q_4 = 0.0\,m$	$F_3 = 0.0\,N$	$\dot{\theta}_2 = 0.0\,m/s$	$\tau_2 = 0.0\,Nm$
$\theta_1 = 0.0\,rad$		$\dot{\theta}_3 = 0.0\,m/s$	$\tau_3 = 0.0\,Nm$
		$\dot{\theta}_4 = 0.0\,m/s$	$\tau_4 = 0.0\,Nm$

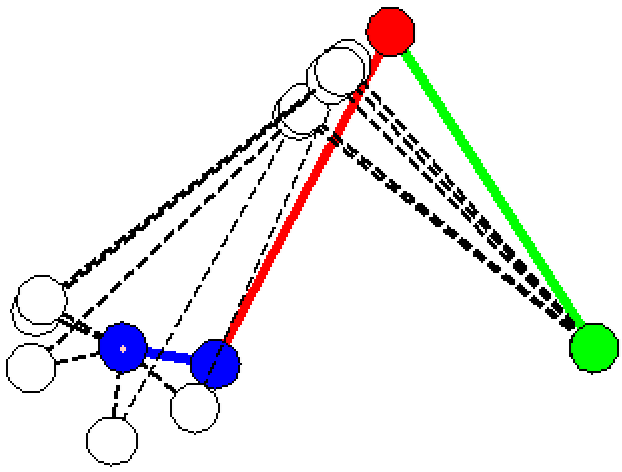

Figure 4.2 Chrono-cyclograph of the four bar.

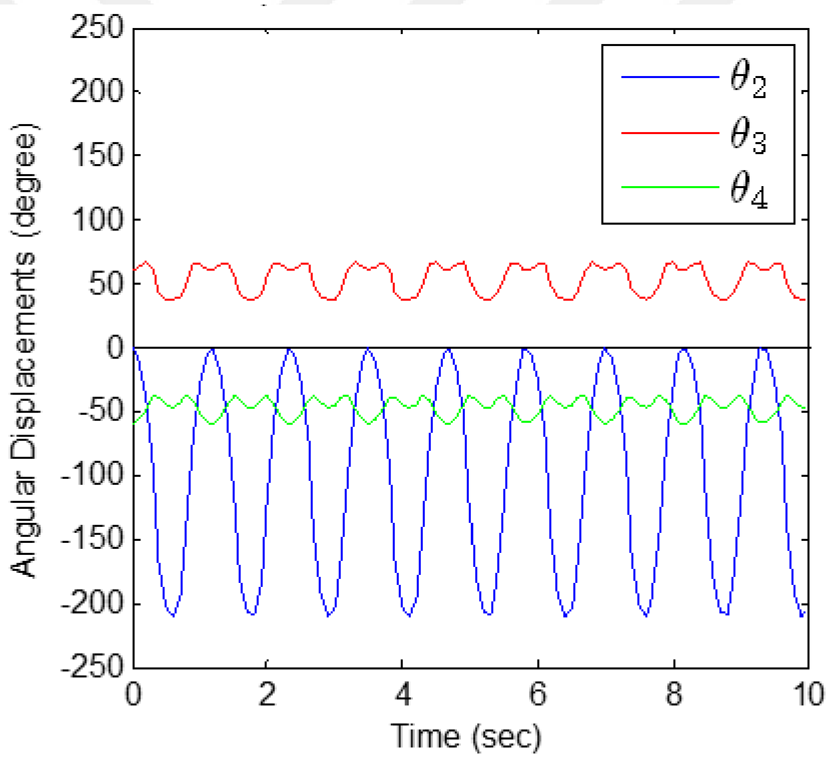

Figure 4.3 Angular displacements of the four bar links.

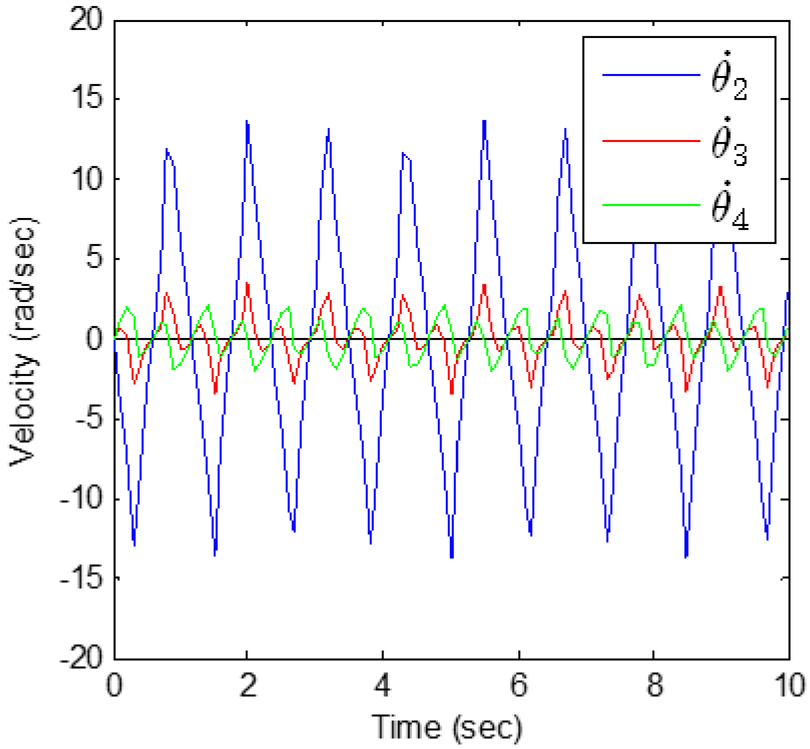

Figure 4.4 Angular velocities of the four bar links.

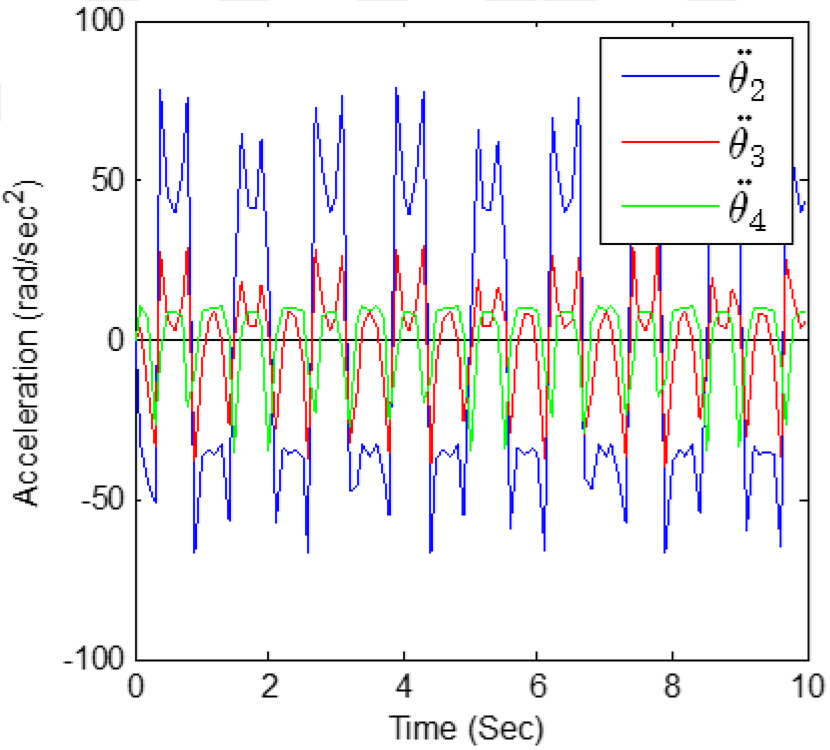

Figure 4.5 Angular accelerations of the four bar links.

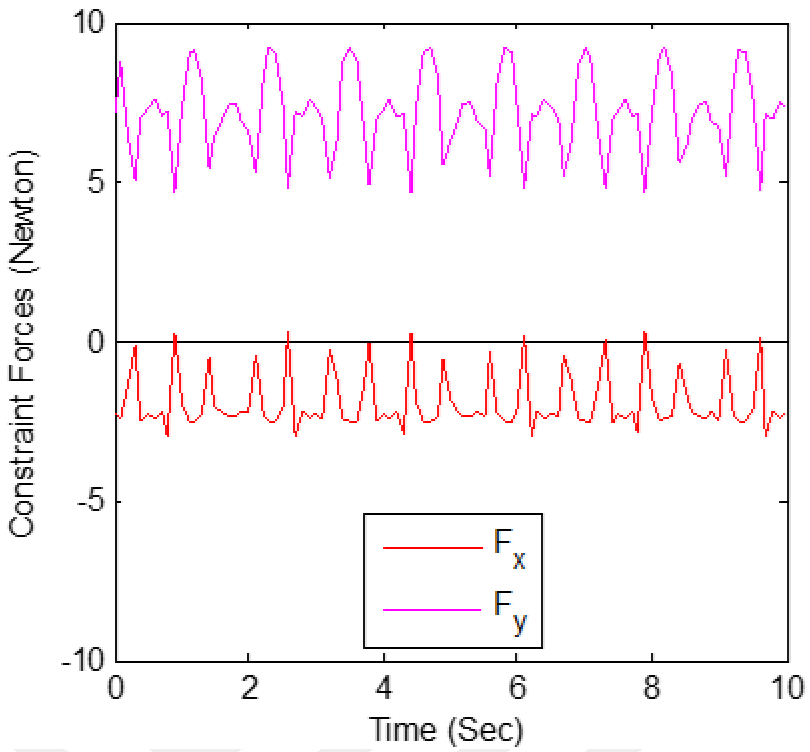

Figure 4.6 Constraint forces of the four bar links at tip points

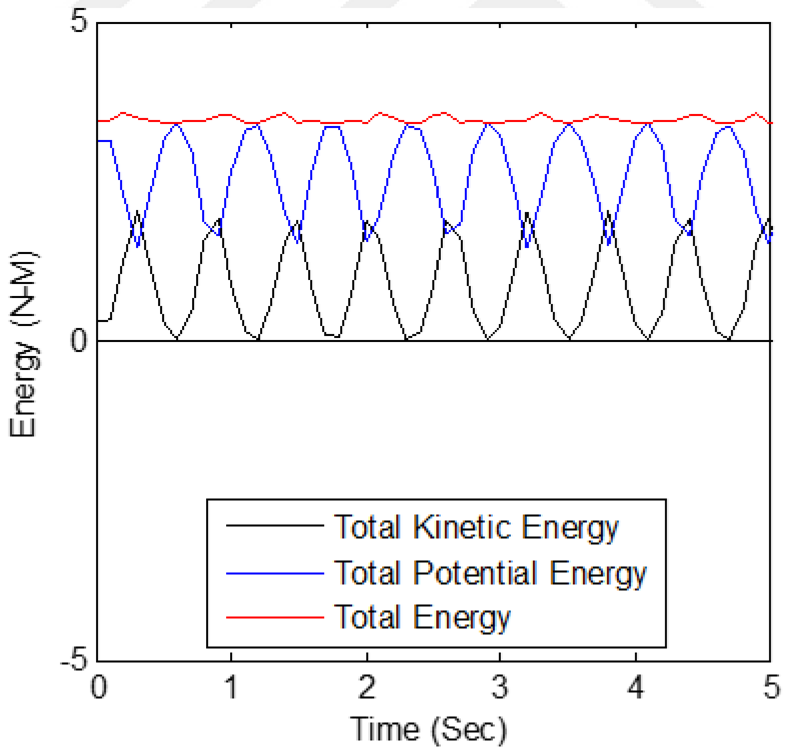

Figure 4.7 Kinetic, potential and total energy of the four bar.

4.2.2 The Six Bar Mechanism

When the tip of the generalized PRPRPR (first sub chain) open chain linkage is constrained not to move by two constraint forces, the system reduces into a 4-bar mechanism and another generalized PRPR (second sub chain) open chain linkage (to form a dyad, the generalized PRPRPR open chain linkage is reduced simply discarding last PR joints and link) is constrained to move by the rocker link of the previous four bar as shown in the Figure 4.8. Parameters of the PRPRPR and PRPR open chain mechanism are listed in Table 4.2. As in case of the four bar, the six bar mechanism is on the vertical plane. Its crank is brought to zero degrees position and released from rest. It is assumed that the mechanism has no external and dissipative force-torque. Generated motion under the action of gravity only is shown in Figure 4.9. Displacements, velocities and acceleration of system is shown in Figure 4.10, Figure 4,11 and Figure 4.12 respectively and constraint forces are shown in Figure 4.13. As expected, since there are no external motor actions or frictions, system is conservative and displays continuous oscillations and also total energy stays constant as shown in Figure 4.14.

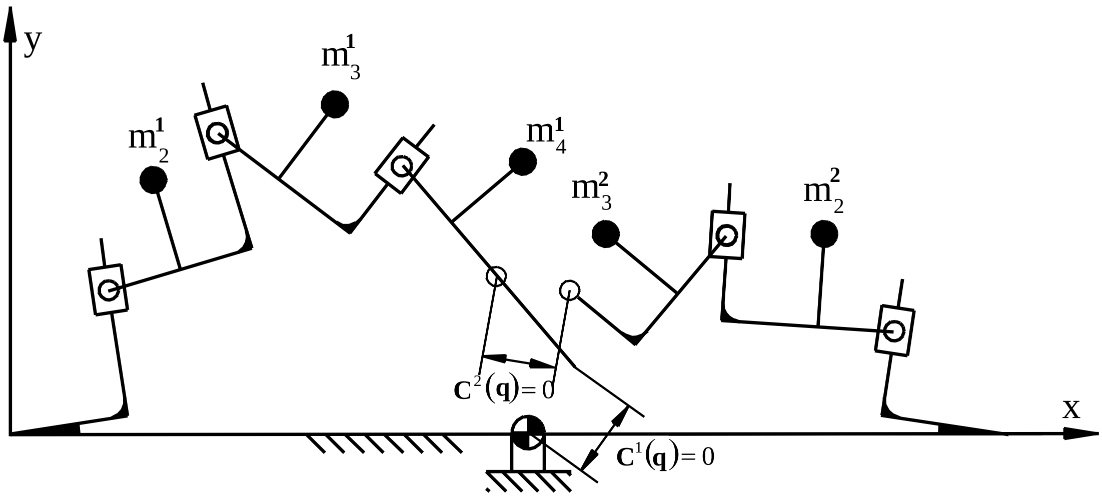

Figure 4.8 Six bar mechanism formed by a generalized PRPRPR (first sub chain) and a PRPR (second chain) open chain linkage.

Table 4.2 Six bar mechanism simulation parameters given in Figure 4.9.

First Sub Chain PRPRPR			
Linkage parameters		**Initial conditions**	**Existence factor**
$a_1 = 0.0\,m$	$m_2 = 1\,kg$	$B_1 = 0.0\,m$	$EF_1 = 0$ $EF_3 = 0$ $EF_5 = 0$
$a_2 = 0.2\,m$	$m_3 = 1\,kg$	$B_2 = 0.0\,m$	$EF_2 = 1$ $EF_4 = 1$ $EF_6 = 1$
$a_3 = 0.8\,m$	$m_4 = 1\,kg$	$B_3 = 0.0\,m$	**Tip point of the open chain**
$a_4 = 0.8\,m$	$I_2 = 0.0\,kgm^2$	$\theta_2 = \frac{\pi}{2}\,rad.$	$x_{tip} = 1.0\,m$
$p_2 = 0.2\,m$	$I_3 = 0.0\,kgm^2$	$\theta_3 = 39.09\frac{\pi}{180}\,r.$	$y_{tip} = 0.0\,m$
$p_3 = 0.8\,m$	$I_4 = 0.0\,kgm^2$	$\theta_4 = 298.29\frac{\pi}{180}\,r.$	**Constraint force or torque gains**
$p_4 = 0.8\,m$	**Generalized Forces**	$\dot{B}_1 = 0.0\,m/s$	$K_{p_{tip}} = 100000\,Ns/m$ $K_{v_{tip}} = 100\,Ns/m$
$q_2 = 0.0\,m$	$F_1 = 0.0\,N$	$\dot{B}_2 = 0.0\,m/s$	
$q_3 = 0.0\,m$	$F_2 = 0.0\,N$	$\dot{B}_3 = 0.0\,m/s$	**Generalized Torques**
$q_4 = 0.0\,m$	$F_3 = 0.0\,N$	$\dot{\theta}_2 = 0.0\,m/s$	$\tau_2 = 0.0\,Nm$
$\theta_1 = 0.0\,rad$		$\dot{\theta}_3 = 0.0\,m/s$	$\tau_3 = 0.0\,Nm$
		$\dot{\theta}_4 = 0.0\,m/s$	$\tau_4 = 0.0\,Nm$
Second Sub Chain PRPR			
Linkage parameters, PRPR		**Initial conditions**	**Existence factor**
$a_5 = 2.0\,m$	$m_5 = 0.5\,kg$	$B_5 = 0.0\,m$	$EF_1 = 0$ $EF_3 = 0$
$a_6 = 0.75\,m$	$m_6 = 1\,kg$	$B_6 = 0.0\,m$	$EF_2 = 1$ $EF_4 = 1$
$a_7 = 0.75\,m$	$I_2 = 0.0\,kgm^2$	$\theta_5 = 129.30\frac{\pi}{180}\,r.$	**Tip point of the open chain**
$p_2 = 0.75\,m$	$I_3 = 0.0\,kgm^2$	$\theta_6 = 197.71\frac{\pi}{180}\,r.$	$x_{tip} = 0.81\,m$
$p_3 = 0.75\,m$			$y_{tip} = 0.35\,m$
$q_2 = 0.0\,m$	**Generalized Forces**	$\dot{B}_5 = 0.0\,m/s$	**Constraint force or torque gains**
$q_3 = 0.0\,m$	$F_1 = 0.0\,N$	$\dot{B}_6 = 0.0\,m/s$	$K_{p_{tip}} = 100000\,Ns/m$ $K_{v_{tip}} = 100\,Ns/m$
$\theta_{21} = 0.0\,rad$	$F_2 = 0.0\,N$	$\dot{\theta}_5 = 0.0\,m/s$	
		$\dot{\theta}_6 = 0.0\,m/s$	**Generalized Torques**
			$\tau_5 = 0.0\,Nm$
			$\tau_6 = 0.0\,Nm$

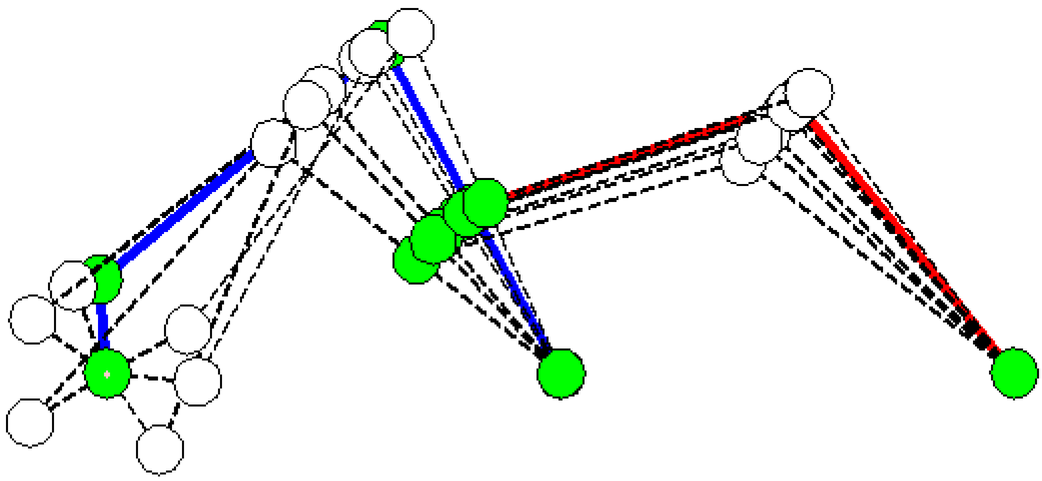

Figure 4.9 Chrono-cyclograph of the six bar mechanism formed by a generalized PRPRPR (first sub chain, drawn in solid line) and a PRPR (second sub chain, drawn in dotted line) open chain linkage.

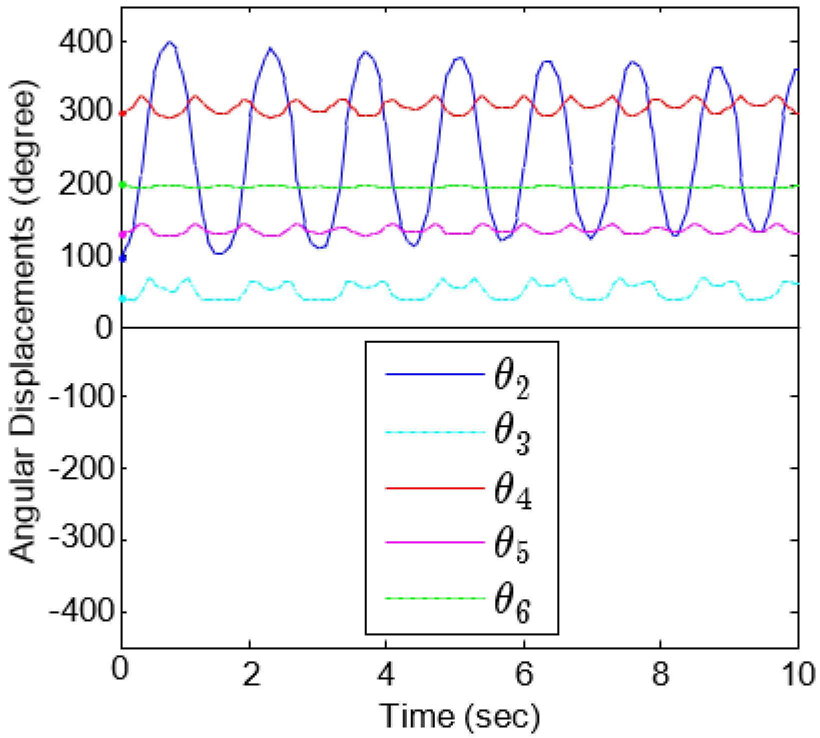

Figure 4.10 Angular displacements of the six bar mechanism.

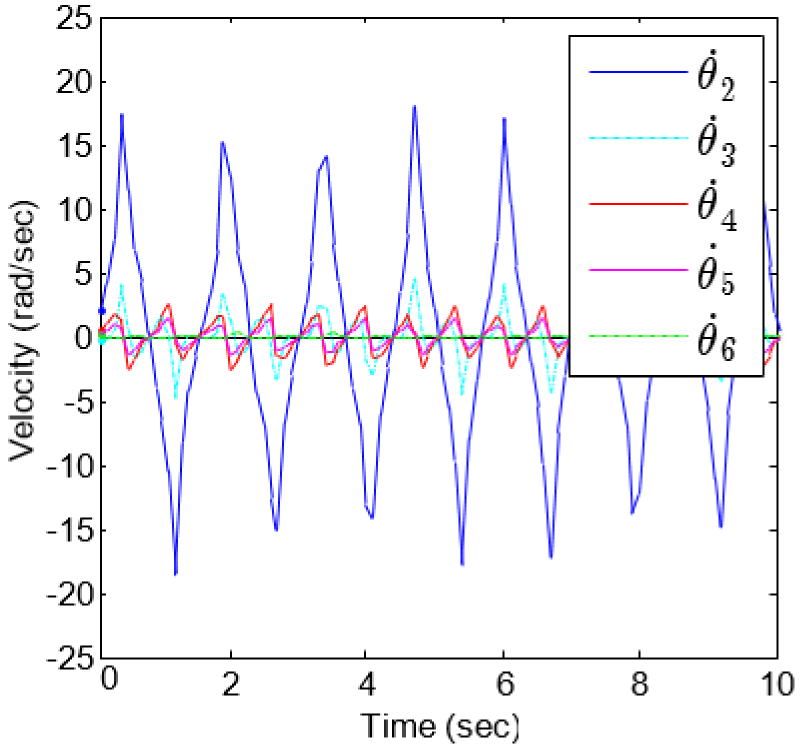

Figure 4.11 Angular velocities of the six bar mechanism

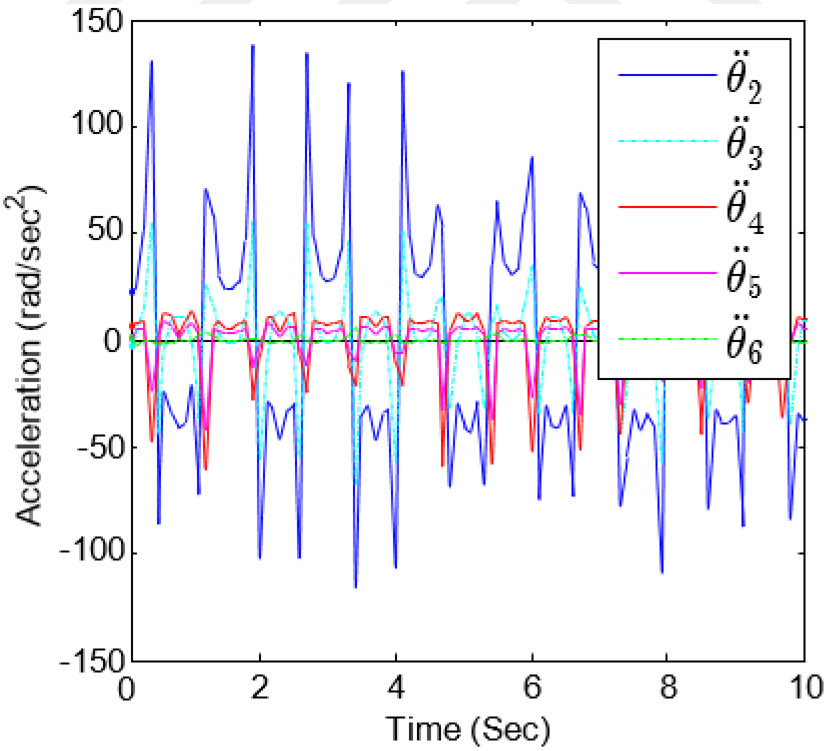

Figure 4.12 Angular accelerations of the six bar mechanism.

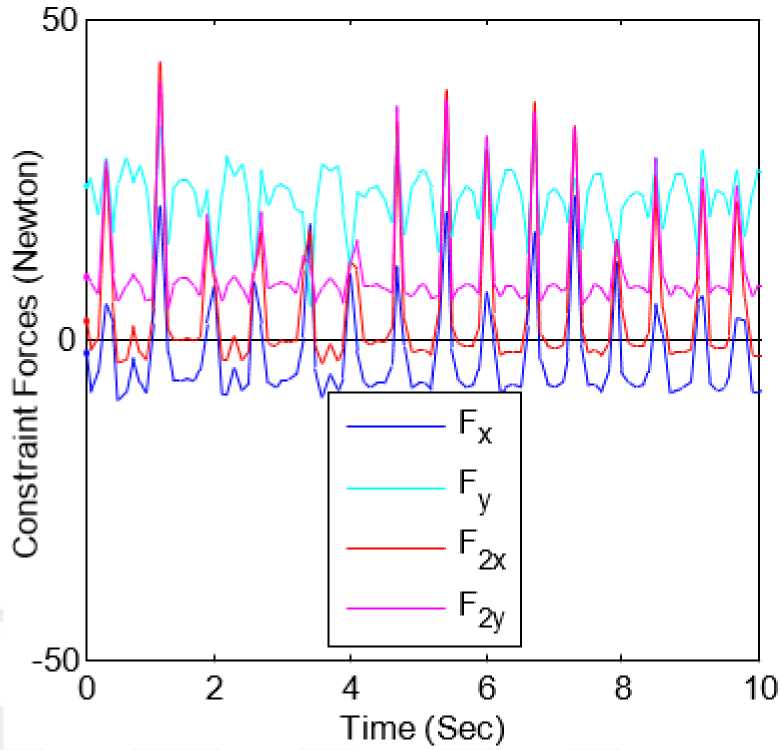

Figure 4.13 Constraint forces of the six bar mechanism links at tip points

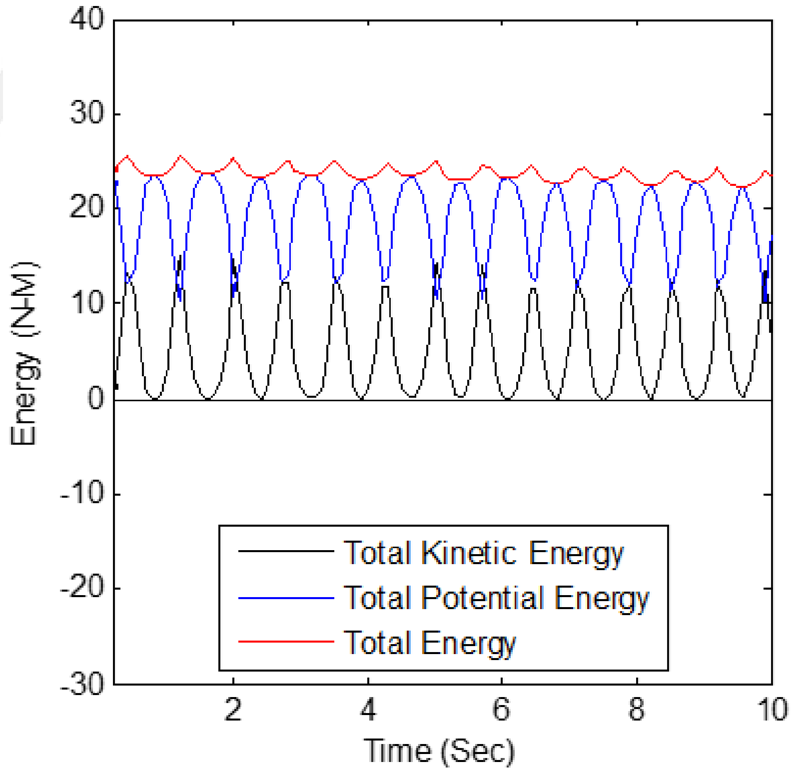

Figure 4.14 Kinetic, potential and total energy of the six bar

The total energy in the energy graph appears to be inclined downward in the profile. This is due to both the accumulation of errors during the solution and the gain coefficients in the constraint force calculation. This problem will be eliminated when optimum gain coefficients are used..

4.2.3 The Double Slider Mechanism

When the tip of the generalized PRPR open chain linkage is constrained not to move by one constraint forces at the x-direction and existence factor parameters for prismatic joint are one at the tips of mechanism, the system reduces into a double slider mechanism as depicted in Figure 4.15. Relevant linkage parameters (masses, mass moments of inertias), initial conditions (orientations, velocities), generalized forces and torques, location of the tip point of the open chain, existence factors to determine the type of joints and constraint force and/or torque gains to keep the given mechanism tip point at the required ground position are listed in Table 4.3. As it is seen from the Table 4.3 the first sliding joint of the double slider mechanism is moving at x-direction and the other slider double slider mechanism which is at end point of mechanism is moving at y-direction under the action of gravity. Chrono-cyclograph of the generated motion under the action of gravity only is shown in Figure 4.16. As there are no external motor action or friction, system is conservative and displays non-ending oscillations as shown in Figure 4.17a and 4.17b. Displacements, velocities and acceleration of system is shown in Figure 4.17a and 4.17b, Figure 4,18a and 4.18b, Figure 4.19a and 4.19b respectively and constraint forces are shown in Figure 4.20. The easiest and simplest way to check the validity of the model and equation is to draw profiles of the total kinetic, potential and sum of the energy curves. Since the system is conservative, total energy stays constant as shown in Figure 4.21.

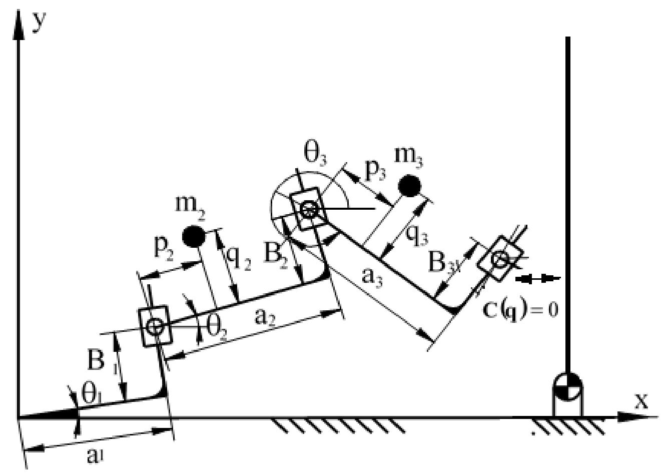

Figure 4.15 Generalized PRPR open chain linkage for modelling a double slider mechanism

Table 4.3 Double slider mechanism simulation parameters given in Figure 4.16.

Linkage parameters		**Initial conditions**	**Existence factor**
$a_1 = 0.0\ m$	$m_2 = 2\ kg$	$B_1 = 0.0\ m$	$EF_1 = 1$ $EF_3 = 0$ $EF_5 = 0$
$a_2 = 0.2\ m$	$m_3 = 1\ kg$	$B_2 = 0.0\ m$	$EF_2 = 0$ $EF_4 = 1$ $EF_6 = 1$
$a_3 = 0.8\ m$	$m_4 = 1\ kg$	$B_3 = 0.0\ m$	**Tip point of the open chain**
	$I_2 = 0.0\ kgm^2$	$\theta_2 = 0.0\ rad.$	$x_{tip} = 0.8\ m$
$p_2 = 0.0001\ m$	$I_3 = 0.0\ kgm^2$	$\theta_3 = 0.0\ rad.$	$y_{tip} = 0.0\ m$
$p_3 = 0.8\ m$	$I_4 = 0.0\ kgm^2$	$\theta_4 = 0.0\ rad.$	**Constraint force or torque gains**
$p_4 = 0.8\ m$	**Generalized Forces**	$\dot{B}_1 = 0.0\ m/s$	$K_{p_{tip}} = 100000\ Ns/m$
$q_2 = 0.0\ m$	$F_1 = 0.0\ N$	$\dot{B}_2 = 0.0\ m/s$	$K_{v_{tip}} = 100\ Ns/m$
$q_3 = 0.0\ m$	$F_2 = 0.0\ N$	$\dot{B}_3 = 0.0\ m/s$	**Generalized Torques**
$q_4 = 0.0\ m$	$F_3 = 0.0\ N$	$\dot{\theta}_2 = 0.0\ m/s$	$\tau_2 = 0.0\ Nm$
$\theta_1 = \frac{\pi}{2}\ rad$		$\dot{\theta}_3 = 0.0\ m/s$	$\tau_3 = 0.0\ Nm$
		$\dot{\theta}_4 = 0.0\ m/s$	$\tau_4 = 0.0\ Nm$

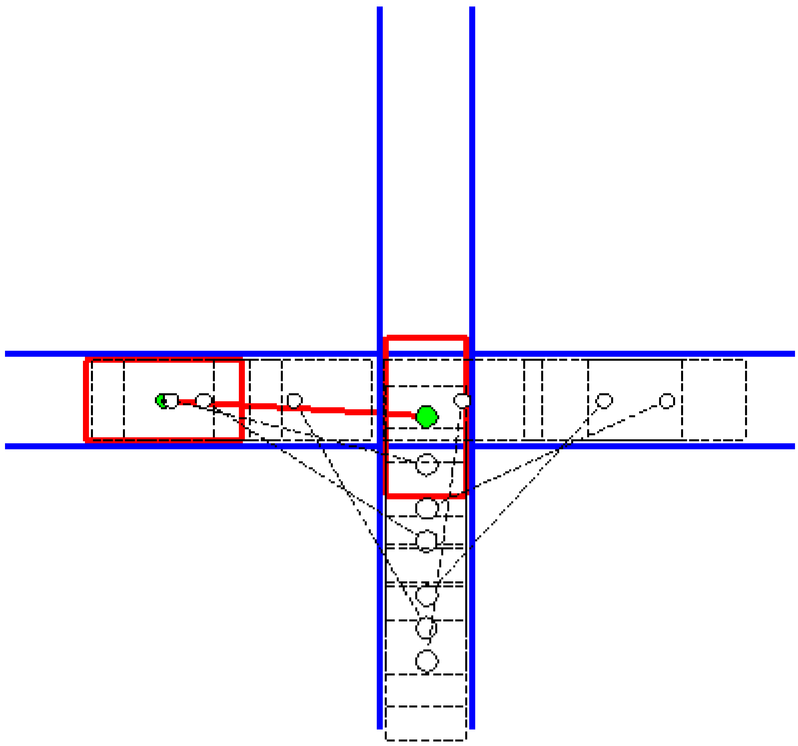

Figure 4.16 Chrono-cyclograph of the double slider mechanism.

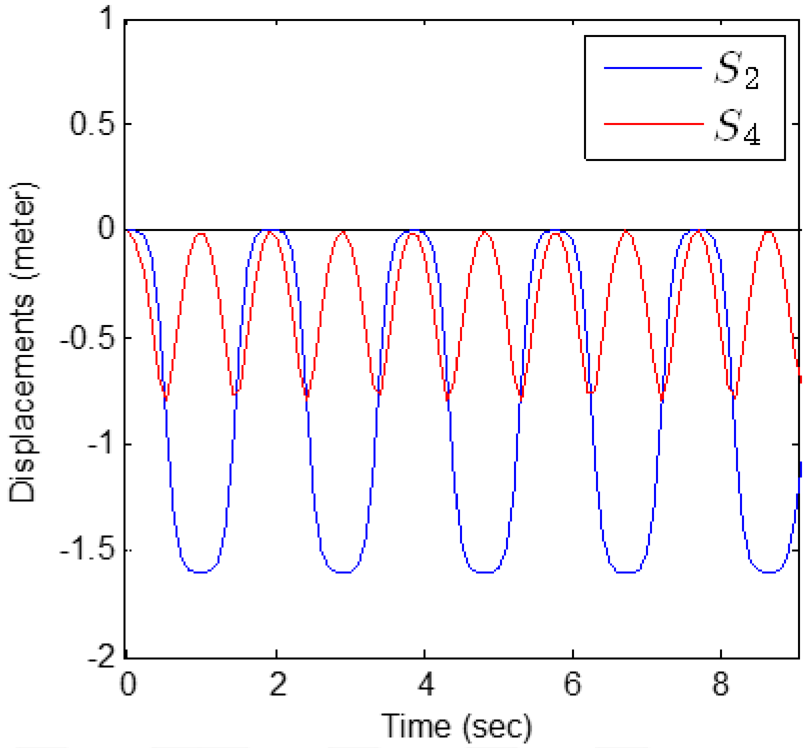

Figure 4.17a Linear displacements of the double slider mechanism

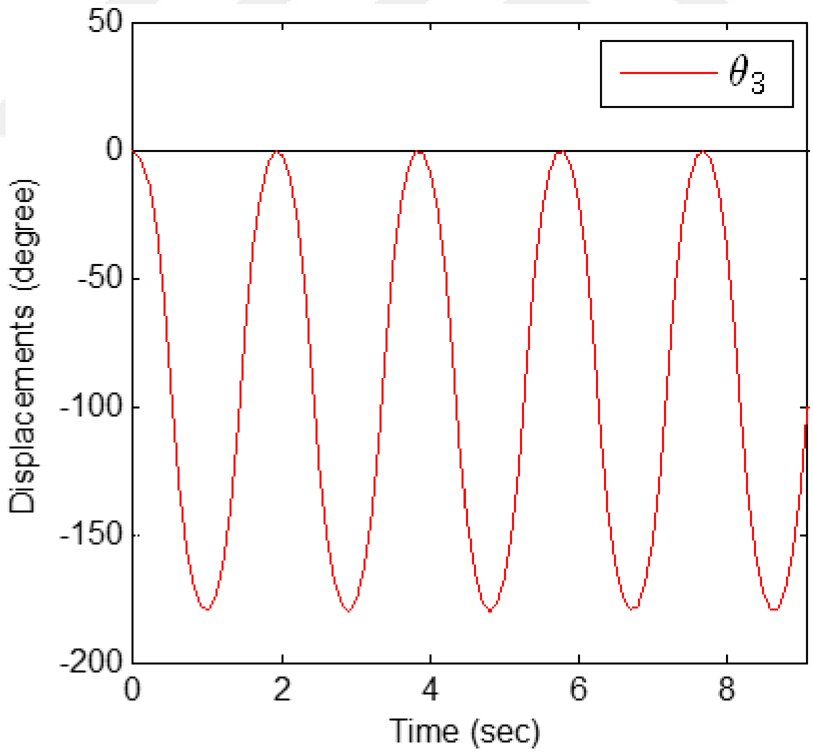

Figure 4.17b Angular displacements of the double slider mechanism

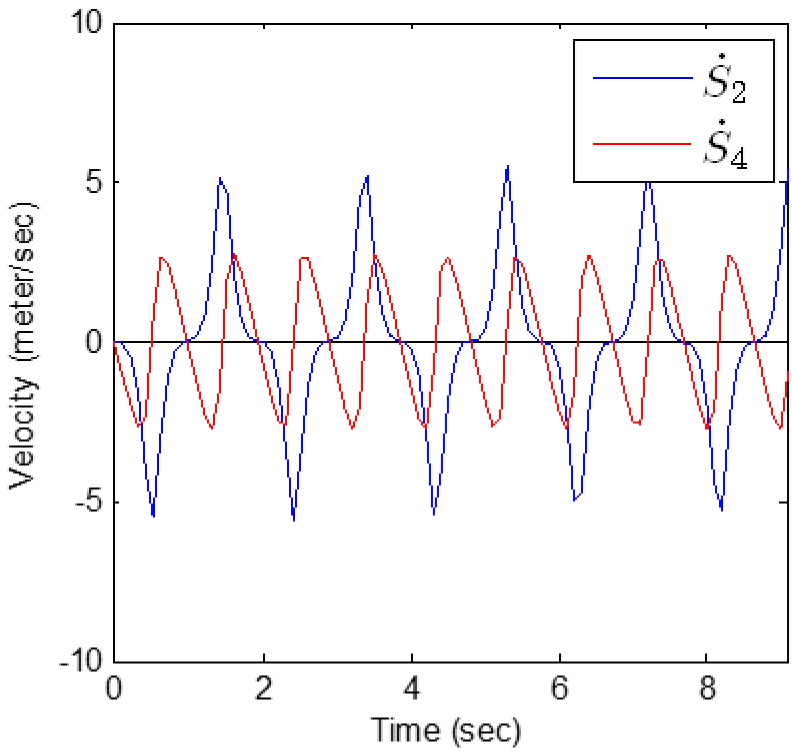

Figure 4.18a Linear velocities of the double slider mechanism

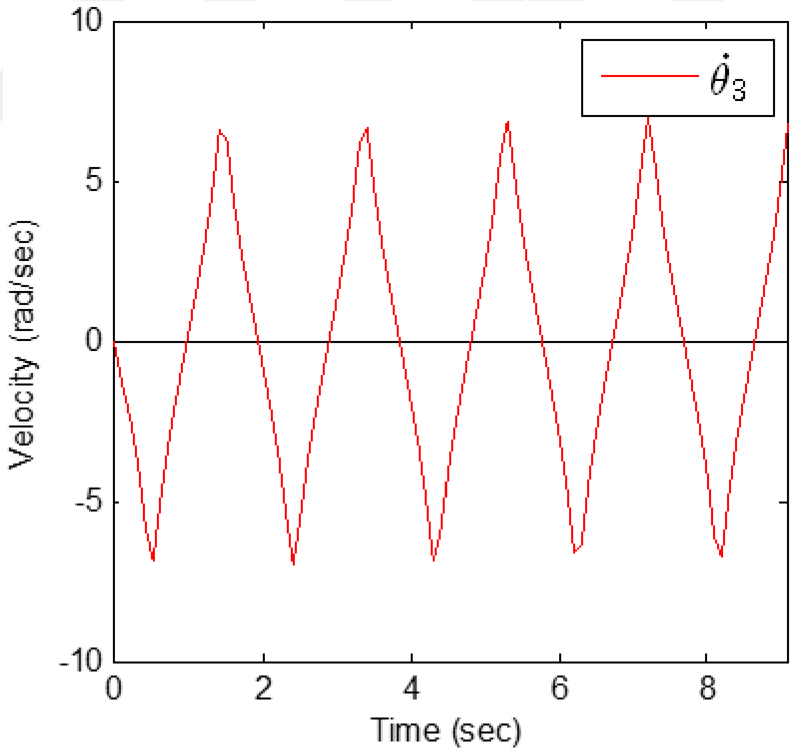

Figure 4.18b Angular velocities of the double slider mechanism

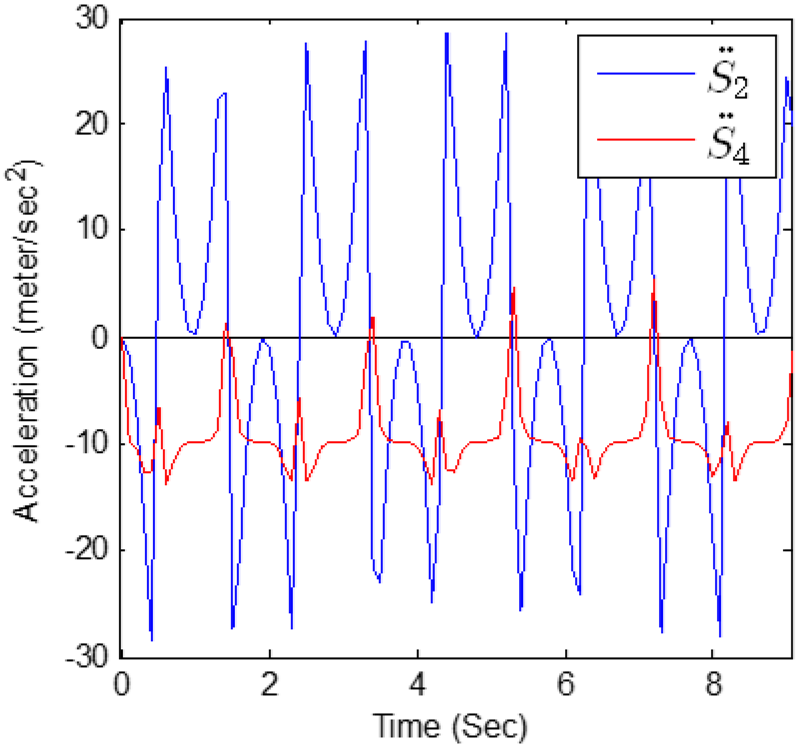

Figure 4.19a Linear accelerations of the double slider mechanism

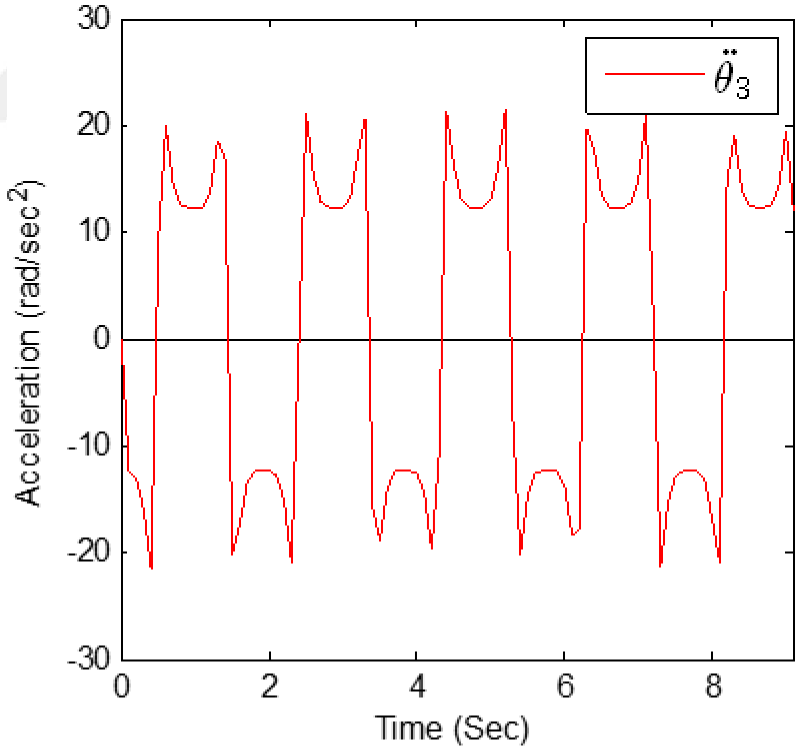

Figure 4.19b Angular accelerations of the double slider mechanism

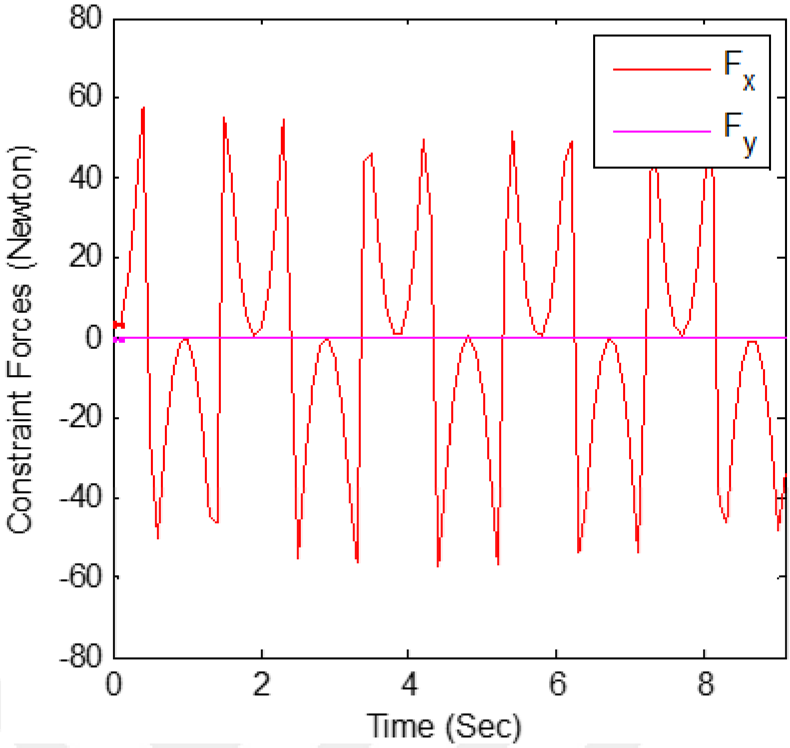

Figure 4.20 Constraint forces of the double slider mechanism at tip points

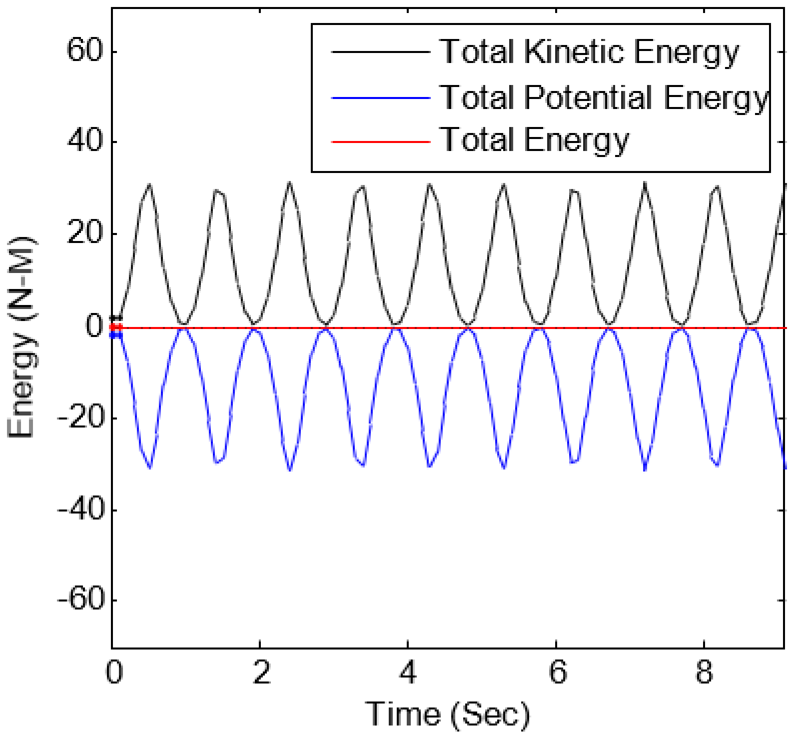

Figure 4.21 Kinetic, potential and total energy of the double slider mechanism

4.3 Multi-Degree Freedom Systems

The equations of motion for the multi-degree freedom parallel robots can also be easily constructed as discussed in Chapter 3. Let's start with the two degree freedom 5 bar planar parallel manipulator.

4.3.1 The Five Bar Planar Parallel Manipulator

Two identical generalized sub chain PRPR tip points are constrained to have the same position ($P(x, y)$) as shown in Figure 4.22. Arbitrarily choosing the appropriate joint type, any king of five bar topology can be obtained. One of the configurations consisting of all revolute joints is given in Figure 4.23 and corresponding parameters of the first and second sub open chain mechanism are listed in Table 4.4. It should also be noted here, dimensions, initial position and velocity values, and the other values can be selected arbitrarily. Here, these values are chosen for the purpose of clearly understand the simulations and the validity of the modeling. It assumed that mechanism is on the vertical plane, tip points of the sub chains are above x axis, and then released without any initial velocities. The motion of the tip points of the sub chains is expected as to move in a vertical line below and above the x at the specified location. As seen in the Figure 4.23, the same motion as anticipated are obtained. Displacements, velocities and acceleration of system is shown in Figure 4.24, Figure 4,25 and Figure 4.26 respectively and constraint forces are shown in Figure 4.27. At the same time, system is conservative, total energy stays constant as shown in Figure 4.28.

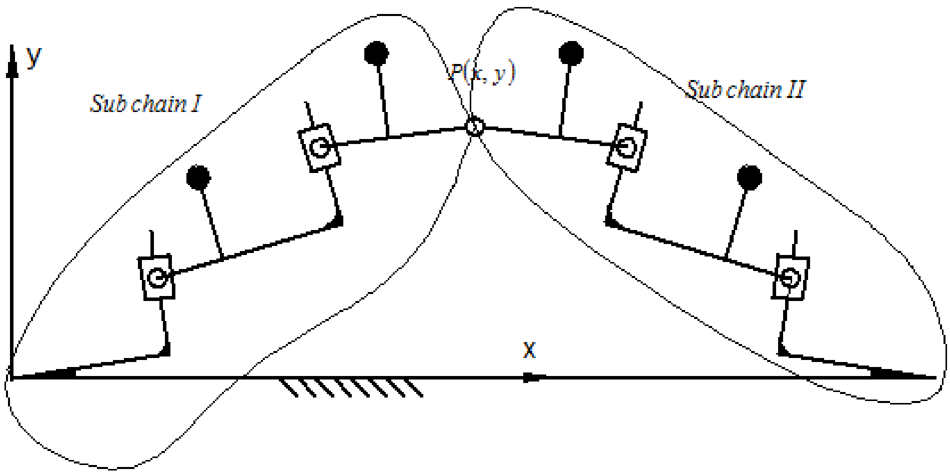

Figure 4.22 Five bar mechanism formed by a generalized PRPR (first sub chain) and a PRPR (second chain) open chain linkage.

Table 4.4 Five bar mechanism simulation parameters given in Figure 4.15.

First Sub Chain PRPRPR			
Linkage parameters		**Initial conditions**	**Existence factor**
$a_1 = 0.0\,m$ $a_2 = 0.25\,m$ $a_3 = 0.75\,m$ $p_2 = 0.125\,m$ $p_3 = 0.375\,m$ $q_2 = 0.0\,m$ $q_3 = 0.0\,m$ $\theta_1 = \frac{\pi}{2}\,rad$	$m_2 = 1\,kg$ $m_3 = 1\,kg$ $I_2 = 0.0\,kgm^2$ $I_3 = 0.0\,kgm^2$ **Generalized Forces** $F_1 = 0.0\,N$ $F_2 = 0.0\,N$	$B_1 = 0.0\,m$ $B_2 = 0.0\,m$ $\theta_2 = \frac{\pi}{4}\,rad.$ $\theta_3 = \frac{\pi}{4}\,rad.$ $\dot{B}_1 = 0.0\,m/s$ $\dot{B}_2 = 0.0\,m/s$ $\dot{\theta}_2 = 0.0\,m/s$ $\dot{\theta}_3 = 0.0\,m/s$	$EF_1 = 0 \quad EF_3 = 0$ $EF_2 = 1 \quad EF_4 = 1$ **Tip point of the open chain** $x_{tip} = 0.707\,m$ $y_{tip} = 0.707\,m$ **Constraint force or torque gains** $K_{p_{tip}} = 100000\,Ns/m$ $K_{v_{tip}} = 100\,Ns/m$ **Generalized Torques** $\tau_2 = 0.0\,Nm$ $\tau_3 = 0.0\,Nm$
Second Sub Chain PRPR			
Linkage parameters, PRPR		**Initial conditions**	**Existence factor**
$a_5 = \sqrt{2}\,m$ $a_6 = 0.25\,m$ $a_7 = 0.75\,m$ $p_2 = 0.125\,m$ $p_3 = 0.375\,m$ $q_2 = 0.0\,m$ $q_3 = 0.0\,m$ $\theta_{21} = 0.0\,rad$	$m_5 = 1\,kg$ $m_6 = 1\,kg$ $I_2 = 0.0\,kgm^2$ $I_3 = 0.0\,kgm^2$ **Generalized Forces** $F_1 = 0.0\,N$ $F_2 = 0.0\,N$	$B_5 = 0.0\,m$ $B_6 = 0.0\,m$ $\theta_5 = 3x\frac{\pi}{4}\,rad$ $\theta_6 = 3x\frac{\pi}{4}\,rad.$ $\dot{B}_5 = 0.0\,m/s$ $\dot{B}_6 = 0.0\,m/s$ $\dot{\theta}_5 = 0.0\,m/s$ $\dot{\theta}_6 = 0.0\,m/s$	$EF_1 = 0 \quad EF_3 = 0$ $EF_2 = 1 \quad EF_4 = 1$ **Tip point of the open chain** $x_{tip} = 0.707\,m$ $y_{tip} = 0.707\,m$ **Constraint force or torque gains** $K_{p_{tip}} = 100000\,Ns/m$ $K_{v_{tip}} = 100\,Ns/m$ Generalized Torques $\tau_5 = 0.0\,Nm$ $\tau_6 = 0.0\,Nm$

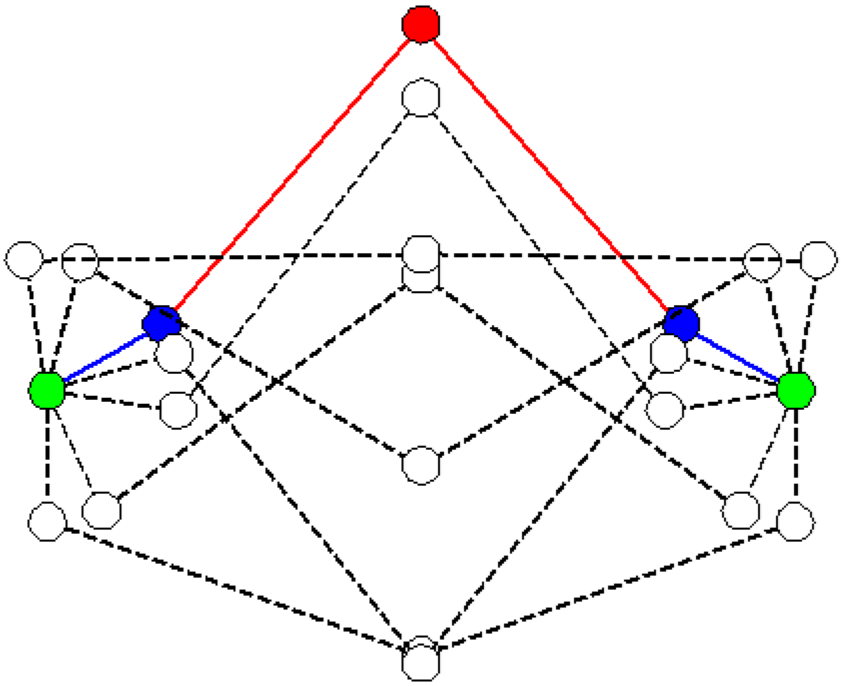

Figure 4.23 Five bar mechanism formed by a generalized PRPR (first sub chain) and a PRPR (second chain) open chain linkage.

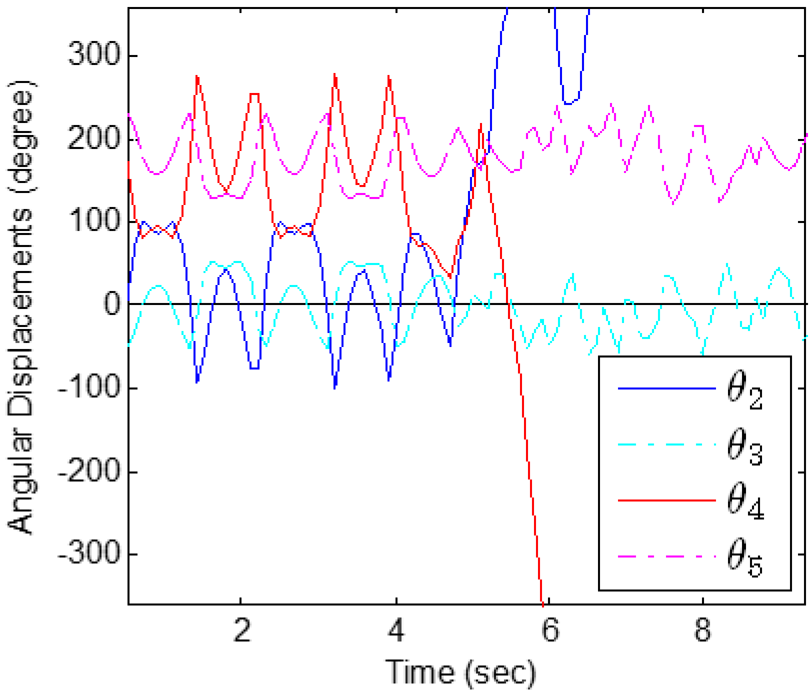

Figure 4.24 Angular displacements of the five bar mechanism

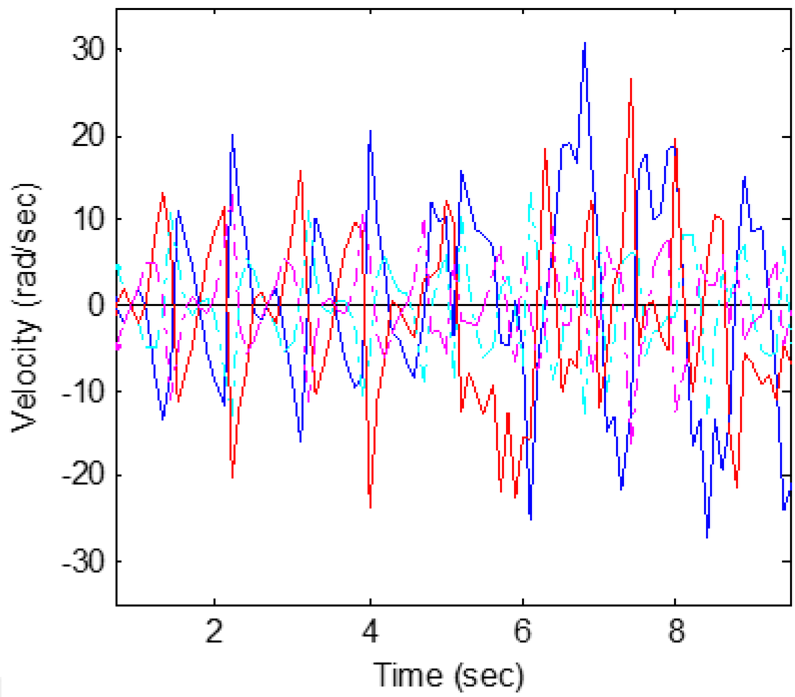

Figure 4.25 Angular velocities of the five bar mechanism

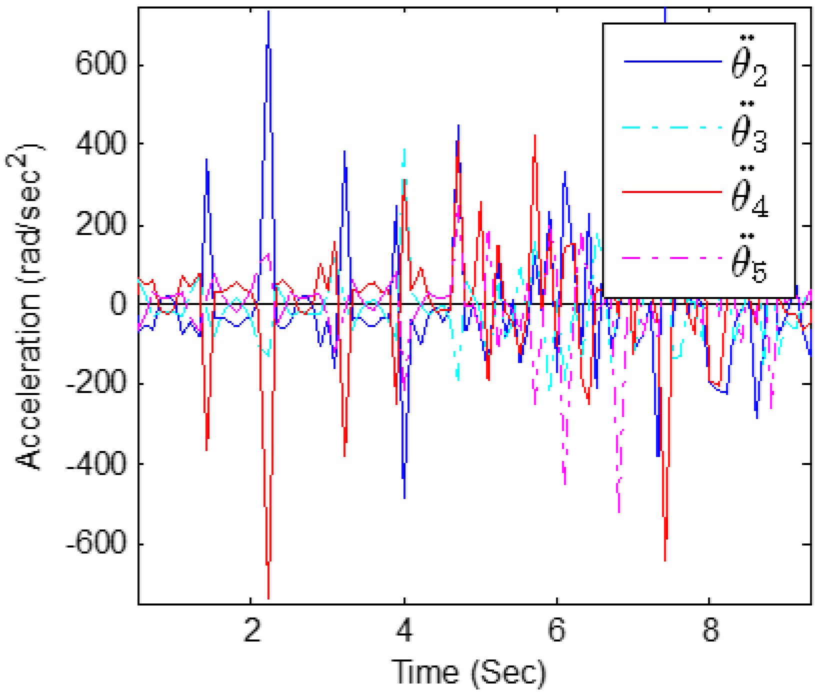

Figure 4.26 Angular accelerations of the five mechanism

After approximately five seconds, the five-bar mechanism failed to maintain non-ending oscillation in the angular displacement of joint 2 and joint 4 due to the accumulation of errors during the solution.

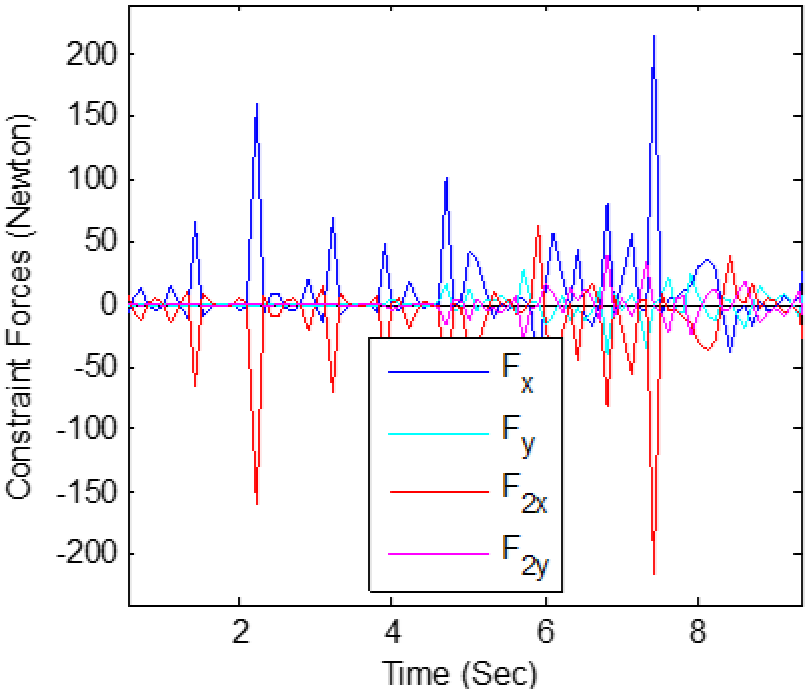

Figure 4.27 Constraint forces of the five bar mechanism at tip points

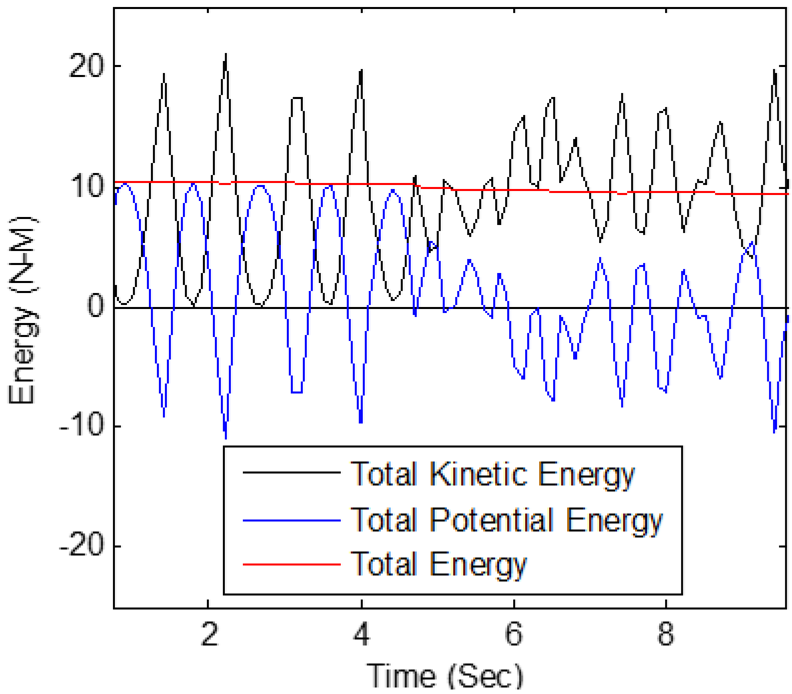

Figure 4.28 Kinetic, potential and total energy of the five bar mechanism

4.3.1 Generalized Model of Planar Parallel Manipulator

The generalized model of parallel planar manipulator consists of three sub open chains; numbered I, II and III and tip points of these are constrained to have the same position as shown in Figure 4.29. Again here, arbitrarily choosing the appropriate joint type, any king of planar parallel manipulators topology can be obtained. One of the configurations consisting of all revolute joints is given in Figure 4.30 and corresponding parameters of the sub open chain mechanisms are listed in Table 4.5. It should also be noted here, dimensions, initial position and velocity values, and the other values can be selected arbitrarily. Here, these values are chosen for the purpose of clearly understand the simulations and the validity of the modeling. It assumed that mechanism is on the vertical plane, tip points of the sub chains are above x axis, and then released without any initial velocities. The motion of the tip points of the sub chains is expected as to move in a vertical line below and above the x at the specified location. As seen in the Figure 4.30, the same motion as anticipated are obtained. Displacements, velocities and acceleration of system is shown in Figure 4.31, Figure 4.32 and Figure 4.33 respectively and constraint forces are shown in Figure 4.34. At the same time, system is conservative, total energy stays constant as shown in Figure 4.35.

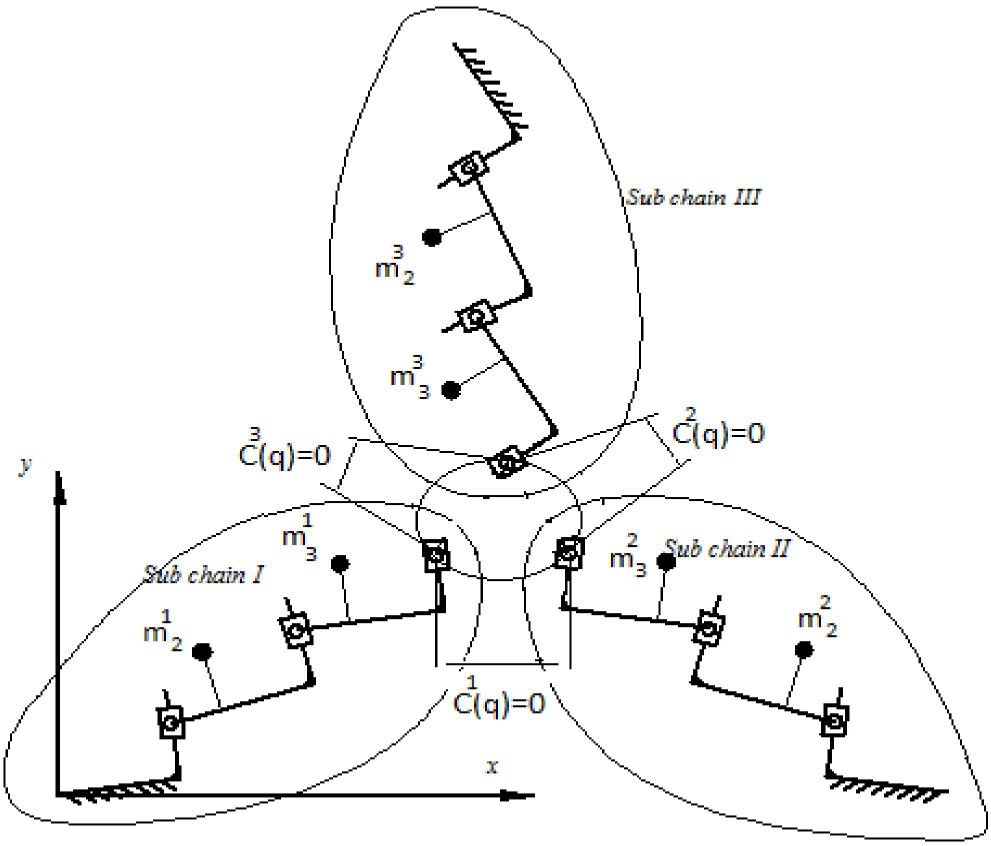

Figure 4.29 Parallel planar manipulator formed by a generalized three PRPR open chain linkage.

Table 4.5a Generalized model of parallel planar manipulator simulation parameters for first sub chain given in Figure 4.30.

First Sub Chain PRPR			
Linkage parameters		Initial conditions	Existence factor
$a_1 = 0.0\,m$	$m_2 = 1\,kg$	$B_1 = 0.0\,m$	$EF_1 = 0$ $EF_3 = 0$
$a_2 = 2\,m$	$m_3 = 1\,kg$	$B_2 = 0.0\,m$	$EF_2 = 1$ $EF_4 = 1$
$a_3 = 3\,m$	$I_2 = 0.33\,kgm^2$	$\theta_2 = 75x\frac{\pi}{180}\ rad.$	Tip point of the open chain
$p_2 = 1\,m$	$I_3 = 0.75\,kgm^2$	$\theta_3 = 345.70x\frac{\pi}{180}\ rad.$	$x_{tip} = 3.42\,m$
$p_3 = 1.5\,m$			$y_{tip} = 1.19\,m$
$q_2 = 0.0\,m$			Constraint force or torque gains
$q_3 = 0.0\,m$	Generalized Forces	$\dot{B}_1 = 0.0\,m/s$	$K_{p_{tip}} = 100000\,Ns/m$
$\theta_{11} = 0.0\,rad$	$F_1 = 0.0\,N$	$\dot{B}_2 = 0.0\,m/s$	$K_{v_{tip}} = 100\,Ns/m$
	$F_2 = 0.0\,N$	$\dot{\theta}_2 = 0.0\,m/s$	Generalized Torques
		$\dot{\theta}_3 = 0.0\,m/s$	$\tau_2 = 0.0\,Nm$
			$\tau_3 = 0.0\,Nm$

Table 4.5b Generalized model of parallel planar manipulator simulation parameters for second sub chain given in Figure 4.30.

Second Sub Chain PRPR			
Linkage parameters, PRPR		Initial conditions	Existence factor
$a_5 = 5\,m$	$m_5 = 1\,kg$	$B_5 = 0.0\,m$	$EF_1 = 0$ $EF_3 = 0$
$a_6 = 2\,m$	$m_6 = 1\,kg$	$B_6 = 0.0\,m$	$EF_2 = 1$ $EF_4 = 1$
$a_7 = 3\,m$	$I_2 = 0.33\,kgm^2$	$\theta_5 = 44.91x\frac{\pi}{180}\ rad.$	Tip point of the open chain
$p_6 = 1\,m$	$I_3 = 0.75\,kgm^2$	$\theta_6 = 184.23x\frac{\pi}{180}\ rad.$	$x_{tip} = 3.42\,m$
			$y_{tip} = 1.19\,m$
$p_7 = 1.5\,m$	Generalized Forces	$\dot{B}_5 = 0.0\,m/s$	Constraint force or torque gains
$q_6 = 1.5\,m$	$F_1 = 0.0\,N$	$\dot{B}_6 = 0.0\,m/s$	$K_{p_{tip}} = 100000\,Ns/m$
$q_7 = 0.0\,m$	$F_2 = 0.0\,N$	$\dot{\theta}_5 = 0.0\,m/s$	$K_{v_{tip}} = 100\,Ns/m$
$\theta_{21} = 0.0\,rad$		$\dot{\theta}_6 = 0.0\,m/s$	Generalized Torques
			$\tau_5 = 0.0\,Nm$
			$\tau_6 = 0.0\,Nm$

Table 4.5c Generalized model of parallel planar manipulator simulation parameters for third sub chain given in Figure 4.30.

Third Sub Chain PRPR			
Linkage parameters, PRPR		Initial conditions	Existence factor
$a_8 = 5\,m$ $a_9 = 2\,m$	$m_7 = 1\,kg$ $m_8 = 1\,kg$	$B_7 = 0.0\,m$ $B_8 = 0.0\,m$	$EF_1 = 0 \quad EF_3 = 0$ $EF_2 = 1 \quad EF_4 = 1$
$a_{10} = 4.82\,m$	$I_2 = 0.33\,kgm^2$	$\theta_7 = 155.53x\frac{\pi}{180}\ rad.$	Tip point of the open chain
$p_9 = 1\,m$	$I_3 = 1.94\,kgm^2$	$\theta_8 = 304.67x\frac{\pi}{180}\ rad.$	$x_{tip} = 3.42\,m$ $y_{tip} = 1.19\,m$
$p_{10} = 1.5\,m$	Generalized Forces	$\dot{B}_7 = 0.0\,m/s$	Constraint force or torque gains
$q_9 = 0.0\,m$ $q_{10} = 0.0\,m$ $\theta_{31} = 0.0\,rad$	$F_1 = 0.0\,N$ $F_2 = 0.0\,N$	$\dot{B}_8 = 0.0\,m/s$ $\dot{\theta}_7 = 0.0\,m/s$ $\dot{\theta}_8 = 0.0\,m/s$	$K_{p_{tip}} = 100000\,Ns/m$ $K_{v_{tip}} = 100\,Ns/m$
			Generalized Torques
			$\tau_7 = 0.0\,Nm$ $\tau_8 = 0.0\,Nm$

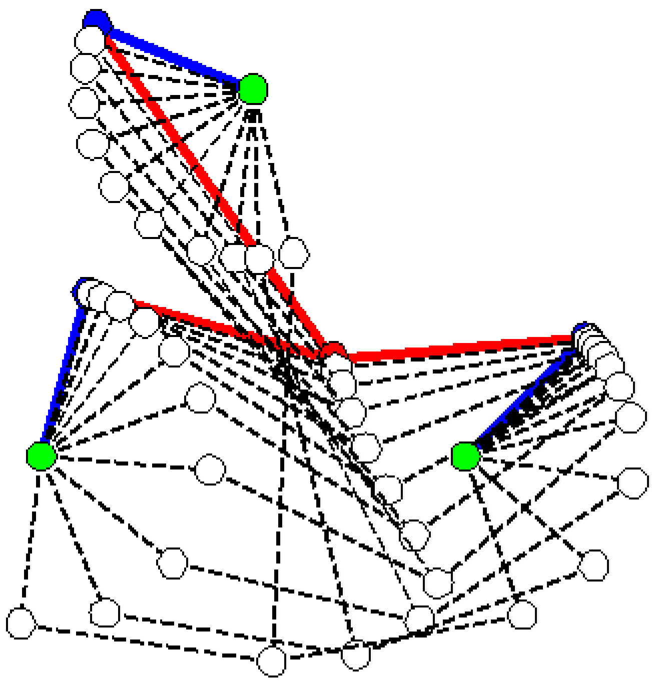

Figure 4.30 Chrono-cyclograph of parallel planar manipulator

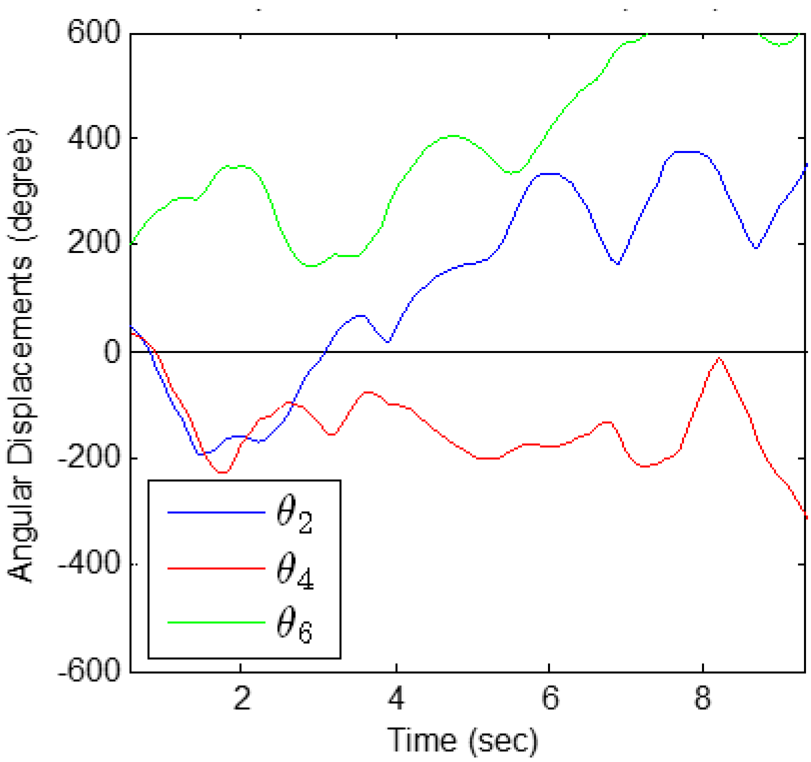

Figure 4.31 Angular displacements of parallel planar manipulator

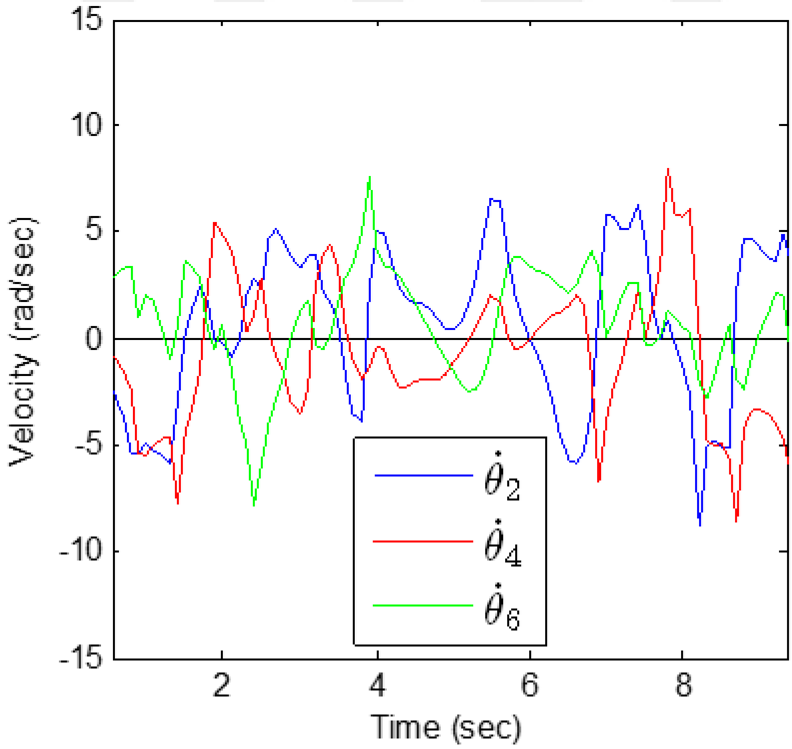

Figure 4.32 Angular velocities of parallel planar manipulator

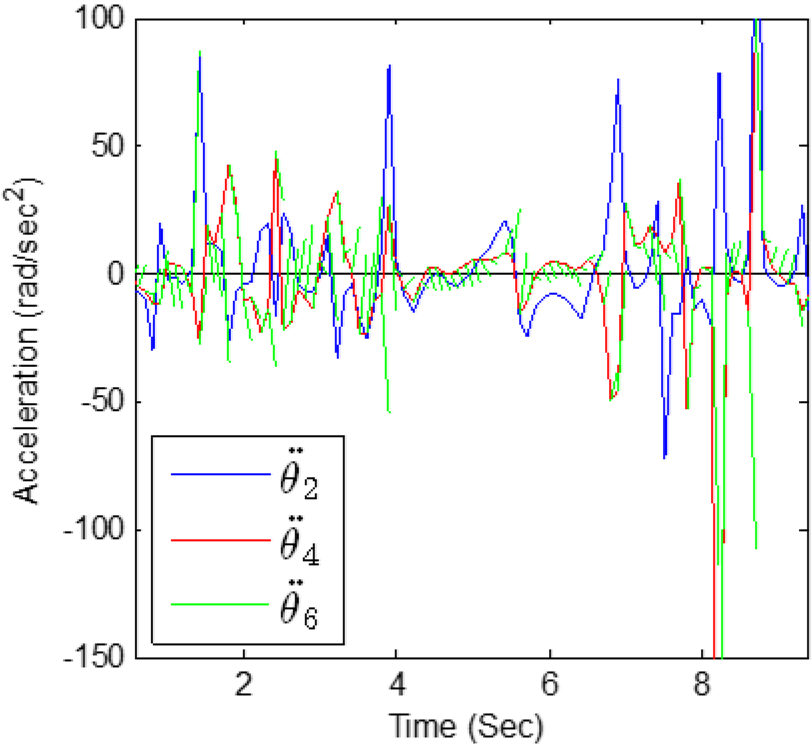

Figure 4.33 Angular accelerations of parallel planar manipulator

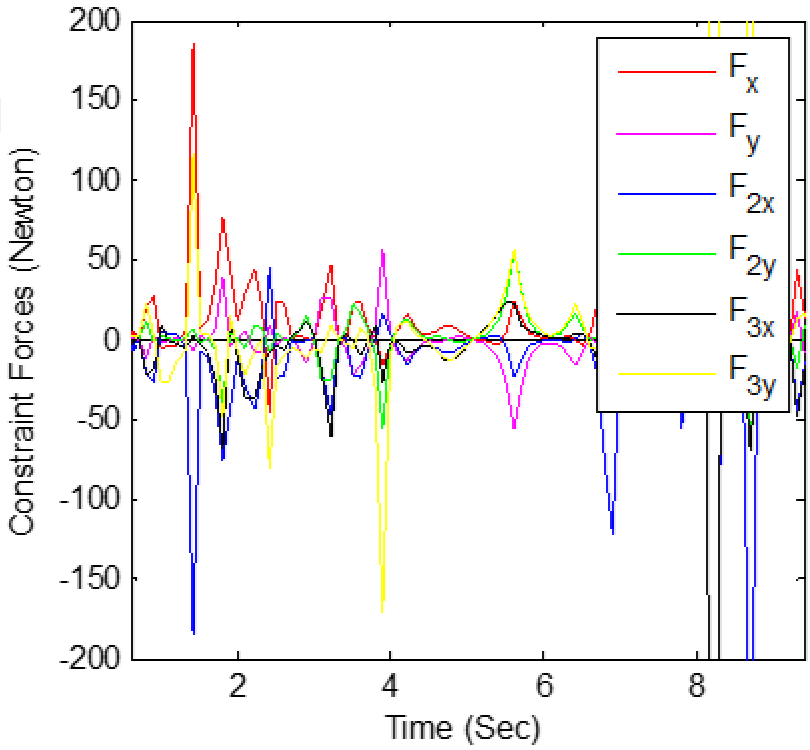

Figure 4.34 Constraint forces of parallel planar manipulator

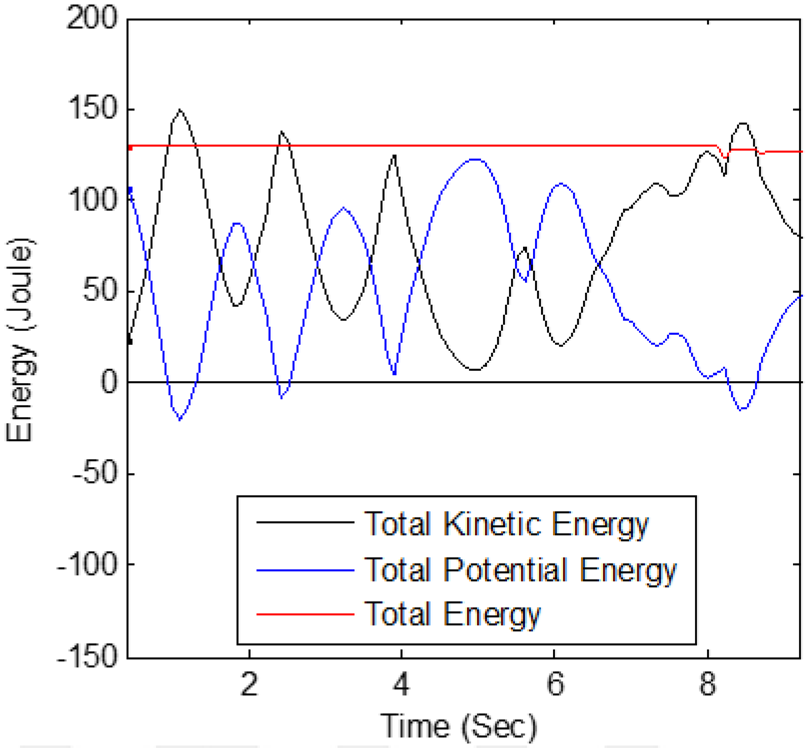

Figure 4.35 Kinetic, potential and total energy of parallel planar manipulator

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions and Recommendations in generalized planar manipulators

Within the context of this thesis, mathematical models of the appropriate constraint forces produced by the revolute and prismatic joints transforming open-chain mechanism into a closed chain mechanism by converting single and multi-degree of freedom mechanisms using the "Virtual Joint Method", which is designed for modeling the general-purpose-open-chain planar robotic systems and the movements are examined in the simulation environment. In the general model, it is assumed that there are two joints between each limb pair; revolute joint and prismatic joint. While one of the two existing joints is in operation, the other is restricted by appropriately shaped force profiles. Each of the joints is activated by the existence factor.

Using the method described in this study, it is highly facilitated to model any mechanism and derive equations of motion. Particularly in order to derive the equations of motion of the closed chain mechanisms, it is needed to use both differential equations and algebraic equations (DAE) (Differential Algebraic Equation). Moreover, for the sake of numerical consistency, the equations should be solved under small intervals. Solving these two different types of equations at the same time and applying motion analysis is a very complex and error-prone task. Especially for multi-link mechanisms, problems arise in real-time simulation and control of the system due to the increase in calculation time. When designing the control algorithm of the system, it is easier for the control designer to derive the equations of motion modeled easily with this method instead of the equations that cause the process complexity.

Within the scope of this thesis, single-degree of freedom and multi-degree of freedom mechanisms are modeled by virtual joint method and the motion equations are simulated in real time. Mechanisms are divided into open-ended sub-systems to model the endpoints of one or more of the generalized models developed for open-chain mechanisms by combining the constraint forces of a fixed or movable limb or torque to form a joint at that point. Since any constraint force(s) and torque(s) used to convert the open-chain system into a closed-chain system will also affect other joints, these components must be added to the generalized force functions. The PD Control Method was used to calculate the joint forces and / or torque values.

Within the scope of this thesis, the simulated systems move only in the vertical direction depending on the gravitational effect without any external force, torque and friction. The easiest and simplest way to prove the accuracy of the models and equations of the mechanisms that move only with gravity is to observe the total kinetic and potential energy profiles at a given time interval. In this study, when the total energy profiles of the simulated mechanisms were examined, the system was preserved and the total energy remained constant. The fact that the total energy remains constant proves the accuracy of the motion equations and modeling method created by the Virtual Joint Method. In addition, when the movement of the mechanisms with constant energy profiles at certain time intervals is examined, observing a movement behavior as expected verifies the accuracy of the models.

Leaps and oscillations are observed especially in the total energy profiles of some mechanisms. Errors accumulate and the system loses conservation during the process of solving equations of motion after a certain period of time. Similarly, while the system is attempting to maintain its conservation, leaps or oscillations occur in the total energy profiles. At the same time, the selection of the PD control coefficients used in the calculation of the constraint forces at the end points with a certain prediction leads to irregularities in these profiles. Optimal determination of these coefficients will eliminate the total energy profile irregularity in the energy graphs. In order to prevent the accumulation of errors, the generalized model used when creating the system model should be chosen in the simplest way. For example, it is possible to construct the double slider mechanism with both the PRPRPR generalized open-chain model and the PRPR generalized sub-chain model. However, when the mechanism is constructed with the PRPRPR generalized model, it is understood from the system behavior that the error accumulation increases during the solution as the transaction volume will increase.

Finally, a motion profile, in form of an animated graphics of the mechanism is very meaningful to experienced eyes. An experienced person can judge whether an animated motion is correct or not, whether the developing motions look, natural or not. In this final point, there is no mathematics, no solid rules. No such a thing is taught as part of a formal engineering education. Such a judgment, feeling of the motion is very important and equally valuable tool for a designer.

5.2 Further Study

Within the scope of this thesis, closed-chain mechanisms are modeled by using "Virtual Joint Method" and PD control with constraint force(s) and torque(s) from end points of open-chain generalized model. For further studies;

i) When converting generalized open-chain mechanisms to closed-chain planar mechanisms, the constraint force(s) or torque values are calculated by the PD Control Method in order that the endpoints of the generalized model behave as if the hinge is present. Optimization studies can be performed in proportional and derivative gain coefficients used for force / torque values calculated according to PD control law.

ii) In order to convert the open-chain generalized model into closed-chain mechanisms, a different control method may be used instead of the PD Control Method. For the sake of conducting realistic simulations of the movement behavior of the systems, studies can be made on different control methods.

iii) A generalized model can be developed according to the three-dimensional space coordinate system of the open-chain generalized model, which is developed according to the planar two-dimensional space coordinate system. Using the generalized model depending on 3-D space coordinate system 3-D robot manipulators can be modeled and their movement behaviors can be examined in the simulation environment.

iv) Using the generalized model, a program which can run system modeling in a modular way can be developed. Modular mechanism can be created with this program and the movement behavior of the mechanism can be examined in simulation environment according to desired parameters. Control algorithms can be developed according to the movement behavior of the mechanism.

v) With the proposed method, it would be possible to carry out a study in which the manipulator should have active and passive joints and also should have a degree of freedom, in order to monitor the desired or predetermined trajectory by deriving the equations of the general parallel planar manipulators.

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APPENDIX A

$\bar{\boldsymbol{M}}$ is the symmetric 6x6 inertial matrix with existence factors. Its elements are:

$\bar{M}_{11} = M_{11}$

$\bar{M}_{12} = (M_{12}EF_1)$

$\bar{M}_{13} = (M_{13}EF_1)$

$\bar{M}_{14} = (M_{14}EF_1)$

$\bar{M}_{15} = (M_{15}EF_1)$

$\bar{M}_{16} = (M_{16}EF_1)$

$\bar{M}_{21} = ((M_{21} + M_{41} + M_{61}EF_5EF_3)EF_2)$

$\bar{M}_{22} = ((M_{22} + (M_{41} + M_{44} + (M_{62} + M_{64} + M_{66})EF_5)EF_3))$

$\bar{M}_{23} = ((M_{23} + (M_{43} + M_{63}EF_5)EF_3)EF_{2)}$

$\bar{M}_{24} = M_{24}EF_2$

$\bar{M}_{25} = ((M_{25} + (M_{45} + M_{65}EF_5)EF_3)EF_{2)}$

$\bar{M}_{26} = ((M_{26} + M_{46}EF_3)EF_2)$

$\bar{M}_{31} = M_{31}EF_3$

$\bar{M}_{32} = M_{32}EF_3$

$\bar{M}_{33} = M_{33}$

$\bar{M}_{34} = M_{34}EF_3$

$\bar{M}_{35} = (M_{35}EF_3)$

$\bar{M}_{36} = (M_{36}EF_3\,)$

$\bar{M}_{41} = ((M_{41} + M_{61}EF_5)EF_4)$

$\bar{M}_{42} = ((M_{42} + M_{62}EF_5)EF_4 - M_{44}EF_3 - (M_{42} + M_{62})EF_5EF_3)$

$\bar{M}_{43} = M_{63}EF_6$

$\bar{M}_{44} = (M_{64}EF_6 - M_{66}EF_5)$

$\bar{M}_{45} = ((M_{45} + M_{65}EF_5)EF_4)$

$\bar{M}_{46} = (M_{46}EF_4)$

$\bar{M}_{51} = M_{51}EF_5$

$\bar{M}_{52} = (M_{52}EF_5)$

$\bar{M}_{53} = M_{53}EF_5$

$\bar{M}_{54} = M_{54}EF_5$

$\bar{M}_{55} = M_{55}$

$\bar{M}_{36} = (M_{56}EF_5\,)$

$\bar{M}_{61} = M_{61}EF_6$

$\bar{M}_{62} = (M_{62}EF_6)$

$\bar{M}_{63} = M_{63}EF_6$

$\bar{M}_{64} = (M_{64}EF_5 - M_{66}EF_5)$

$\bar{M}_{55} = M_{65}\,EF_6$

$\bar{M}_{66} = M_{66}$

$\boldsymbol{h} = (\dot{\boldsymbol{q}}, \boldsymbol{q}, \boldsymbol{u})$ is the 1×6 vector containing; externally applied actuator forces/torques, F_{B1}, T_{θ_2}, F_{B2}, T_{θ_3}, F_{B3}, T_{θ_4} and the sum of centrifugal, Coriolis and gravity terms, $\emptyset_{\mathrm{B1}}, \emptyset_{\theta_2}, \emptyset_{\mathrm{B2}}, \emptyset_{\theta_3}, \emptyset_{\mathrm{B3}}, \emptyset_{\theta_4}$. Its elements are:

$h_1 = (F_{B1} + \emptyset_{\mathrm{B1}})EF_1$

$h_2 = \left(T_{\theta_2} + T_{\theta_3}EF_4 - T_{\theta_4}EF_3EF_6 + \emptyset_{\theta_2} + \emptyset_{\theta_3}EF_3 + \emptyset_{\theta_3}EF_3EF_5\right)EF_2$

$h_3 = (F_{B2} + \emptyset_{\mathrm{B2}})EF_3$

$h_4 = \left(T_{\theta_3} + \emptyset_{\theta_3} + T_{\theta_3}EF_6 + \emptyset_{\theta_4}EF_5\right)EF_4$

$h_5 = (F_{B3} + \emptyset_{\mathrm{B3}})EF_5$

$h_6 = (F_{B4} + \emptyset_{\mathrm{B4}})EF_6$

Where

$M_{11} = m_2 + m_3 + m_4,$

$$M_{12} = M_{21} = \left(m_2q_2 + b_2(m_3 + m_4)\right)\sin(\theta_1 - \theta_2) + \left(m_2p_2 + a_2(m_3 + m_4)\right)\cos(\theta_1 - \theta_2)\,,$$

$M_{13} = M_{31} = (m_3 + m_4)\cos(\theta_1 - \theta_2),$

$M_{14} = M_{41} = (m_3q_3 + m_4b_3)\sin(\theta_1 - \theta_3) + (m_3p_3 + m_4a_3)\cos(\theta_1 - \theta_3),$

$M_{15} = M_{51} = m_4\cos(\theta_1 - \theta_3),\qquad M_{16} = m_4(q_4\sin(\theta_1 - \theta_4) + p_4\cos(\theta_1 - \theta_4)),$

$M_{22} = I_2 + m_2(p_2^2 + q_2^2) + (m_3 + m_4)(a_2^2 + b_2^2),$

$M_{23} = M_{32} = a_2(m_3 + m_4),$

$M_{24} = M_{24} = \left(m_3(a_2q_3 - b_2p_3) + m_4(a_2b_3 - b_2a_3)\right)\sin(\theta_2 - \theta_3) + \left(m_3(a_2p_3 + b_2q_3) + m_4(a_2a_3 + b_2b_3)\right)\cos(\theta_2 - \theta_3)$

$M_{25} = M_{52} = m_4(a_2\cos(\theta_2 - \theta_3) - b_2\sin(\theta_2 - \theta_3)),$

$M_{26} = M_{62} = m_4((a_2q_4 - b_2p_4)\sin(\theta_2 - \theta_4) + (a_2p_4 + b_2q_4)\cos(\theta_2 - \theta_4)),$

$M_{33} = m_3 + m_4,$

$M_{34} = M_{43} = (m_3q_3 + m_4b_3)\sin(\theta_2 - \theta_3) + (m_3p_3 + m_4a_3)\cos(\theta_2 - \theta_3),$

$M_{35} = m_4\cos(\theta_2 - \theta_3),$

$M_{36} = m_4(q_4\sin(\theta_2 - \theta_4) + p_4\cos(\theta_2 - \theta_4)),$

$M_{44} = I_3 + m_3(p_3^2 + q_3^2) + m_4(a_3^2 + b_3^2),$

$M_{45} = m_4 a_3,$

$M_{46} = m_4[(a_3 q_4 - b_3 p_4)\sin(\theta_3 - \theta_4) + (a_3 p_4 + b_3 q_4)\cos(\theta_3 - \theta_4)],$

$M_{55} = m_4$

$M_{56} = m_4(q_4\sin(\theta_3 - \theta_4) + p_4\cos(\theta_3 - \theta_4)), \qquad M_{66} = I_4 + m_4(p_4^2 + q_4^2),$

$M_{21} = M_{12},\ M_{31} = M_{13},\ M_{41} = M_{14},\ M_{51} = M_{15}, M_{61} = M_{16}, M_{32} = M_{23}, M_{42} = M_{24}, M_{52} = M_{25}, M_{62} = M_{26}, M_{43} = M_{34}, M_{53} = M_{35}, M_{63} = M_{36}, M_{54} = M_{45}, M_{64} = M_{46}, M_{65} = M_{56}$

$\emptyset_{B1} = -g\cos\theta_1(m_2 + m_3 + m_4) - \left[m_2 p_2\dot{\theta}_2^2 + \dot{\theta}_2\left(a_2\dot{\theta}_2 + 2\dot{B}_2\right)(m_3 + m_4)\right]\sin(\theta_1 - \theta_2) + \left[m_2 q_2\dot{\theta}_2^2 + B_2\dot{\theta}_2(m_3 + m_4)\right]\cos(\theta_1 - \theta_2) - \left[m_3 p_3\dot{\theta}_3^2 + m_4\dot{\theta}_3\left(a_2\dot{\theta}_3 + 2\dot{B}_2\right)\right]\sin(\theta_1 - \theta_3) + \left[(m_3 q_3 + m_4 B_3)\dot{\theta}_3^2\right]\cos(\theta_1 - \theta_3) - \left[m_4 p_4\dot{\theta}_4^2\right]\sin(\theta_1 - \theta_4) + \left[m_4 q_4\dot{\theta}_4^2\right]\cos(\theta_1 - \theta_4)$

$\emptyset_{\theta_2} = -\left[2B_2\dot{B}_2\dot{\theta}_2 + g(a_2\cos\theta_2 - B_2\sin\theta_2)\right](m_3 + m_4) - m_2[g(p_2\cos\theta_2 - q_2\sin\theta_2)] - \left[m_3\dot{\theta}_3^2(a_2 p_3 + B_2 q_3) + m_4\left(2a_2\dot{B}_3\dot{\theta}_3 + \dot{\theta}_3^2(a_2 a_3 + B_2 B_3)\right)\right]\sin(\theta_2 - \theta_3)] - [-m_3\dot{\theta}_3^2(a_2 q_3 - B_2 p_3) + m_4(2a_2\dot{B}_3\dot{\theta}_3 - \dot{\theta}_3^2(a_2 B_3 - B_2 a_3))]\cos(\theta_2 - \theta_3) - (m_4\dot{\theta}_4^2(a_2 p_4 - B_2 q_4))\sin(\theta_2 - \theta_4) + m_4\dot{\theta}_4^2(a_2 q_4 - B_2 p_4))\cos(\theta_2 - \theta_4)$

$$\begin{aligned}\emptyset_{B2} = &\left(B_2\dot{\theta}_2^2 - g\cos\theta_2\right)(m_3 + m_4) \\ &- \left[m_3 p_3\dot{\theta}_3^2 + m_4\dot{\theta}_3\left(a_3\dot{\theta}_3 + 2\dot{B}_3\right)\right]\sin(\theta_2 - \theta_3) \\ &+ \left[\dot{\theta}_3^2(m_3 q_3 + m_4 B_3)\right]\cos(\theta_2 - \theta_3) - \left[m_4 p_4\dot{\theta}_4^2\right]\sin(\theta_2 - \theta_4) \\ &+ \left[m_4 p_4\dot{\theta}_4^2\right]\cos(\theta_2 - \theta_4)\end{aligned}$$

$\emptyset_{\theta_3} = -m_3 g(p_3\cos\theta_3 - q_3\sin\theta_3) - m_4[2B_3\dot{B}_3\dot{\theta}_3 + g(a_3\cos\theta_3 - B_3\sin\theta_3)] - \left[m_3\left(2p_3\dot{B}_2\dot{\theta}_2 + \dot{\theta}_2^2(a_2 p_3 + B_2 q_3)\right) + m_4\left(2a_3\dot{B}_2\dot{\theta}_2 + \dot{\theta}_2^2(a_2 a_3 + B_2 B_3)\right)\right]\sin(\theta_2 - \theta_3) - [m_3(2q_3\dot{B}_2\dot{\theta}_2 - \dot{\theta}_2^2(a_2 q_3 - B_2 p_3)) + m_4(2B_3\dot{B}_2\dot{\theta}_2 - \dot{\theta}_2^2(a_2 B_3 - B_2 a_3))]\cos(\theta_2 - \theta_3) - (m_4\dot{\theta}_4^2(a_3 p_4 - B_3 q_4))\sin(\theta_3 - \theta_4) + m_4\dot{\theta}_4^2(a_3 q_4 - B_3 p_4))\cos(\theta_2 - \theta_4)$

$\emptyset_{B3} = m_4\left(B_3\dot{\theta}_3^2 - g\cos\theta_3\right) + \left[m_4\dot{\theta}_2\left(a_3\dot{\theta}_2 + 2\dot{B}_2\right)\right]\sin(\theta_2 - \theta_3) + \left[m_4 B_2\dot{\theta}_2^2\right]\cos(\theta_2 - \theta_3) - \left[m_4 p_4\dot{\theta}_4^2\right]\sin(\theta_3 - \theta_4) + \left[m_4 q_4\dot{\theta}_4^2\right]\cos(\theta_3 - \theta_4)$

$$\emptyset_{\theta_4} = -m_4 g(p_4 cos\theta_4 - q_4 sin\theta_4) - m_4[2p_4\dot{B}_2\dot{\theta}_2 + \dot{\theta}_2^2(a_2p_4 + B_2q_4)]\sin(\theta_2 - \theta_4) - m_4\left[2q_4\dot{B}_2\dot{\theta}_2 + \dot{\theta}_2^2(a_2q_4 - B_2p_4)\right]\cos(\theta_2 - \theta_4) - m_4[2p_4\dot{B}_3\dot{\theta}_3 - \dot{\theta}_3^2(a_3p_4 + B_3q_4)]\sin(\theta_3 - \theta_4) - m_4[2q_4\dot{B}_3\dot{\theta}_3 + \dot{\theta}_3^2(a_3q_4 - B_3p_4))\cos(\theta_3 - \theta_4)$$