

MODELING OF REINFORCING BAR BUCKLING IN STRUCTURAL WALLS

by

Sura Sabah Salih Salih

B.S., Civil Engineering, University of Benghazi, 2014

Submitted to the Institute for Graduate Studies in
Science and Engineering in partial fulfillment of
the requirements for the degree of
Master of Science

Graduate Program in Civil Engineering
Boğaziçi University

2019

MODELING OF REINFORCING BAR BUCKLING IN STRUCTURAL WALLS

APPROVED BY:

Assoc. Prof. Kutay Orakçıl
(Thesis Supervisor)

Prof. Hilmi Luş

Prof. Alper İlki

DATE OF APPROVAL: DD.MM.YYYY

ACKNOWLEDGEMENTS

I would like to express my sincere gratitude and indebtedness to my supervisor, Associate Professor Kutay Orakçal, for his invaluable guidance and continuous encouragement, support and patience throughout the course of this study. Without his stimulating ideas, constructive suggestions and feedback, this work would not have been possible to achieve.

I am deeply grateful to the jury members, Professor Hilmi Luş and Professor Alper İlki, for agreeing to read the manuscript and to participate in the defense of this thesis.

I would like to extend my appreciation to Dr. M. Fethi Güllü who devoted considerable amounts of his time to help me with the FSAFE model code, and provided valuable advice throughout this study.

My heartfelt thanks to my friends who have always been a major source of support when things would get a bit discouraging.

Most of all, I am deeply indebted to my parents, who have always believed in me and whose boundless support, inspiration and love have brought me to this stage of life, so I would like to dedicate this work to them.

ABSTRACT

MODELING OF REINFORCING BAR BUCKLING IN STRUCTURAL WALLS

Buckling of longitudinal reinforcement is a critical damage state that is frequently observed in reinforced concrete structures after major seismic events that warrants the replacement of the structural component or render the entire structure unfit for rehabilitation if it is wide spread throughout the structure.

The reason for the emphasis placed on adequate design/detailing against buckling of longitudinal reinforcement is that it inhibits the energy dissipation capacity of the reinforcing steel in the plastic hinge regions and consequently limits the ductile deformation capacity of the structure as a whole, especially if it occurs in a critical component of the lateral load resisting system such as structural walls. Buckling may also cause sudden loss of axial load carrying capacity in vertical structural members, triggering possible collapse mechanisms. Therefore, it is of utmost importance to demystify the factors affecting this phenomenon and accurately model it in order to improve the abilities of existing nonlinear finite element modelling approaches, which are essential for performance-based design and assessment of structures, so that engineers can predict and avoid this failure mechanism and therefore provide safe designs that would guarantee that the structure maintains its intended performance level under different earthquake intensity scenarios.

Motivated by the aforementioned incentives, three constitutive hysteretic steel models incorporating buckling effects were proposed and investigated in this study by implementing them into the FSAFE model—an FE modeling approach to simulate the hysteretic lateral load behavior of RC walls—to assess each model’s effectiveness in capturing the buckling-induced degradation in the lateral load-displacement responses of four wall test specimens.

ÖZET

PERDELERDE DONATI BURKULMASININ MODELLENMESİ

Betonarme yapı elemanlarında boyuna donatıların burkulması, özellikle deprem etkileri altında sıkça rastlanan kritik bir hasar durumudur. Burkulma hasarı, çok zaman yapı elemanının kapsamlı bir şekilde onarımı veya yenilenmesini gerektirdiği gibi, yapı genelinde yaygın bir şekilde görüldüğü durumda yapının onarımını imkansız duruma getirebilecek ciddiyette bir hasar türüdür.

Donatı burkulmasının önlenmesi konusuna tasarım/detaylandırma yönetmeliklerinde azami önem vermesinin nedeni, özellikle perdeler veya süneklik düzeyi yüksek çerçeve elemanları gibi deprem etkileri altındaki davranışı kritik olan yapı elemanlarında, donatı burkulması etkisinin elemanın plastik mafsallardaki süneklik ve enerji sönümleme kapasiteleri üzerindeki sınırlayıcı etkisidir. Düşey taşıyıcı elemanlarında boyuna donatının burkulması, ayrıca elemanın eksenel yük taşıma kapasitesinde ani azalmaya sebep olarak yapıda göçme mekanizmalarının oluşmasını tetikleyebilir. Donatı burkulmasını etkileyen faktörleri doğru bir şekilde tespit edebilmek ve analitik olarak modelleyebilmek, özellikle betonarme yapıların performansa dayalı tasarım ve değerlendirilmesi kapsamında kullanılan doğrusal olmayan analiz yaklaşımlarının güvenilirliğinin artırılması ve olası burkulma hasarlarının güvenilir bir şekilde kestirimi açısından son derece önemlidir. Burkulma davranışını da içeren geliştirilmiş modelleme ve analiz yöntemleri ile betonarme yapıların farklı deprem seviyeleri altında beklenen yapısal performansları daha doğru bir şekilde temsil edilebilir donatı burkulmasının önlediği daha güvenli tasarımlara ulaşılabilir.

Dolayısıyla bu çalışma kapsamında donatı çeliği için burkulma etkilerini gözeterek şekilde geliştirilmiş olan üç farklı çevrimsel gerilme-birim şekil değiştirme modeli, betonarme perdeler için daha önceden geliştirilmiş bir sonlu eleman model formülasyonu içerisine eklenmiş, geliştirilen modellerin dört ayrı perde numunesinin analitik davranışına etkisi incelenmiş ve analitik kestirimler deney sonuçlarıyla karşılaştırılmıştır.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS	iii
ABSTRACT.....	iv
ÖZET	v
LIST OF FIGURES	viii
LIST OF TABLES	xiii
LIST OF SYMBOLS	xiv
LIST OF ACRONYMS/ABBREVIATIONS	xvii
1. INTRODUCTION.....	1
1.1. General	1
1.2. Overview of Classical Buckling Theories.....	2
1.3. Thesis Motivation.....	4
1.4. Thesis Objective and Scope	5
1.5. Thesis Outline	5
2. LITERATURE REVIEW.....	6
2.1. Buckling Length and Local vs. Global Buckling.....	6
2.2. Isolated vs. Embedded Bar Behavior	14
2.3. Effect of Loading History on Reinforcement Buckling.....	20
2.4. Overview	23
3. PROPOSED HYSTERETIC UNIAXIAL STRESS-STRAIN STEEL MODEL INCLUDING THE EFFECT OF BUCKLING	24
3.1. Basis for Model Selection	24
3.2. Buckling Length Calculation	25
3.3. Fixed Envelope Model	30
3.3.1. Monotonic Response.....	30

3.3.1.1. Tension Envelope.	30
3.3.1.2. Compression Envelope.	30
3.3.2. Cyclic Response.....	31
3.3.2.1. Unloading from the Fixed Compression and Tension Envelopes.	31
3.4. Shifting Envelope Model	33
3.4.1. Monotonic Response.....	33
3.4.2. Cyclic Response.....	33
3.4.2.1. Unloading from the Shifting Compression Envelope.....	33
3.4.2.2. Unloading from the Fixed Tension Envelope.....	33
3.5. Buckling-Modified Menegotto and Pinto Model	35
3.5.1. Overview of the Original Menegotto and Pinto Model	35
3.5.2. Adjustment for Buckling	37
4. IMPLEMENTATION OF PROPOSED BUCKLING MODELS AND EXPERIMENTAL VERIFICATION	41
4.1. Overview of the FSAFE Model	41
4.2. Implementation of the Proposed Steel Buckling Models in FSAFE Wall Model	43
4.3. Experimental Verification of the Proposed Steel Buckling Models	44
4.3.1. Calculation of Buckling Length and Slenderness Ratio for Boundary Vertical Steel in Wall Specimens	45
4.3.2. WSH6.....	49
4.3.3. RW-A20-P10-S63.....	54
4.3.4. TW2	60
4.3.5. RW2	65
5. SUMMARY AND CONCLUSIONS.....	71
REFERENCES	75

LIST OF FIGURES

Figure 1.1.	Typical Detailing and Observed Damage to Wall Boundary Regions: (a) Ground Floor Level of a 23-Story Building in Coronel; (b) First Subterranean Level of a 20-Story Building in Santiago (NEHRP, 2014).....	1
Figure 1.2.	Column Theories and Test Data (Shanley, 1947).....	4
Figure 2.1.	Local vs. Global Buckling (Massone and Lopez, 2013).....	7
Figure 2.2.	Mechanical System Used for Energy Analysis (Bresler and Gilbert, 1961).....	8
Figure 2.3.	Experimental Setup for Cyclic Tests on Isolated Steel Bars (Monti and Nuti, 1992).....	9
Figure 2.4.	Experimental Monotonic Compression Curves (Monti and Nuti, 1992)	9
Figure 2.5.	Experimental Cyclic Stress-Strain Curves for Different L/D Ratios (Monti and Nuti, 1992).....	10
Figure 2.6.	Unloading Steel Path with Buckling (Gomes and Appleton, 1997).....	10
Figure 2.7.	Equilibrium of a Buckled Steel Bar (Gomes and Appleton, 1997).....	11
Figure 2.8.	Simulation of Reinforcing Bars and Lateral Ties (Dhakai and Maekawa, 2002b).....	12
Figure 2.9.	Representation of Reinforcing Bar Buckling Between Two Hoops (Massone and Moroder, 2009).....	13
Figure 2.10.	Global Buckling Model (Massone and Lopez, 2014).....	14
Figure 2.11.	Flawed and Correct Analogies of Bar Buckling Models (Zong, 2010)...	15

Figure 2.12.	Proposed Phenomenological Hysteretic Model by Kashani <i>et al.</i> (2015a).....	17
Figure 2.13.	Cyclic Buckling Strength Degradation (Kashani <i>et al.</i> , 2015a).....	17
Figure 2.14.	Modelling Pinching Response: (a) Linear-Hyperbolic Curves to Model Compression Unloading Branch at Large Strain Demand; (b) Multi-Linear Curves to Model Compression Unloading Branch at Small Strain Demand (Kashani <i>et al.</i> , 2015a).....	18
Figure 2.15.	Cyclic Response of Hysteretic (OpenSees) Material Model: (a) Buckling Envelope; (b) Cyclic Response (Kashani <i>et al.</i> , 2016).....	19
Figure 2.16.	Deformation Leading to Lateral Instability in Structural Walls (Paulay and Priestley, 1993).....	21
Figure 2.17.	Tension Based Buckling Mechanism of Longitudinal Reinforcement in Columns (Feng <i>et al.</i> , 2014c).....	21
Figure 2.18.	Sectional View under Flexural Load (Feng <i>et al.</i> , 2014c).....	22
Figure 3.1.	Simulation of Reinforcing Bars and Lateral Ties (Dhakal and Maekawa, 2002b).....	27
Figure 3.2.	Flow-Chart of Buckling Length Determination (Dhakal and Maekawa, 2002b).....	27
Figure 3.3.	Values of n_b and n_l for Common Reinforcement Arrangements (Dhakal and Maekawa, 2002b).....	28
Figure 3.4.	Tension Envelope Adopted for all Buckling Models.....	30
Figure 3.5.	Compression Envelope (Kashani <i>et al.</i> , 2015a).....	31
Figure 3.6.	Cyclic Rules for the Fixed Envelope Model.....	32

Figure 3.7.	Cyclic Rules for the Shifting Envelope Model: (a) Unloading from the Shifting Compression Envelope; (b) Unloading from the Fixed Tension Envelope.....	34
Figure 3.8.	Constitutive Model for Steel (Menegotto and Pinto, 1973).....	35
Figure 3.9.	Degradation of Cyclic Curvature.....	37
Figure 3.10.	Determining the Negative Slope of the Compression Yield Asymptote	38
Figure 3.11.	Cyclic Rules for the Buckling-Modified M-P Model.....	39
Figure 3.12.	Stress-Strain Relationship Generated by the Buckling Models: (a) Fixed Envelope Model; (b) Shifting Envelope Model; (d) Buckling-Modified M-P Model.....	40
Figure 4.1.	Constitutive Fixed-Strut-Angle Model (FSAM): (a) Strain Field on Panel Element; (b) Concrete Stresses along Fixed Struts; (c) Shear Aggregate Interlock Stresses along Cracks; (d) Uniaxial Stresses in Reinforcement; (e) Dowel Stresses in Reinforcement (Gullu <i>et al.</i> , 2019).....	42
Figure 4.2.	Material Constitutive Models for: (a) Concrete (Chang And Mander, 1994); (b) Reinforcing Steel (Menegotto and Pinto 1973).....	43
Figure 4.3.	Behavioral Models for Shear Stress Transfer Mechanisms across Cracks: (a) Peak-Oriented Hysteretic Model for Shear Aggregate Interlock; (b) Origin-Oriented Hysteretic Model for Dowel Action (Gullu <i>et al.</i> , 2019).....	43
Figure 4.4.	Wall Cross-Sections and Reinforcement Details for Specimens: (a) WSH6; (b) RW-A20-P10-S63; (c) TW2; (d) RW2.....	46
Figure 4.5.	Buckling Directions Considered in the Wall Boundary Regions.....	47

Figure 4.6.	Lateral Load vs. Top Displacement Relations for Specimen WSH6: (a) Original M-P Model; (b) Fixed Envelope Model; (c) Shifting Envelope Model; (d) Buckling-Modified M-P Model.....	51
Figure 4.7.	Steel Stress-Strain Histories for Right Corner Element at Wall Base for Specimen WSH6: (a) Original M-P Model; (b) Fixed Envelope Model; (c) Shifting Envelope Model; (d) Buckling-Modified M-P Model.....	52
Figure 4.8.	Vertical Normal Strain Profiles at Wall Boundaries for Specimen WSH6: (a) Original M-P Model; (b) Fixed Envelope Model; (c) Shifting Envelope Model; (d) Buckling-Modified M-P Model.....	53
Figure 4.9.	Lateral Load vs. Top Displacement Relations for Specimen RW-A20-P10-S63: (a) Original M-P Model; (b) Fixed Envelope Model; (c) Shifting Envelope Model; (d) Buckling-Modified M-P Model.....	56
Figure 4.10.	Steel Stress-Strain Histories for the Right Corner Element at Wall Base for Specimen RW-A20-P10-S63: (a) Original M-P Model; (b) Fixed Envelope Model; (c) Shifting Envelope Model; (d) Buckling-Modified M-P Model; (e) Shifting Envelope Model (Trimmed); (f) Buckling-Modified M-P Model (Trimmed).....	57
Figure 4.11	Vertical Normal Strain Profiles at Wall Base for Specimen RW-A20-P10-S63: (a) Original M-P Model; (b) Fixed Envelope Model; (c) Shifting Envelope Model; (d) Buckling-Modified M-P Model.....	58
Figure 4.12.	Lateral Load vs. Top Displacement Relations for Specimen TW2 (Top Displacement with Pedestal Movement Contribution Subtracted): (a) Original M-P Model; (b) Fixed Envelope Model; (c) Shifting Envelope Model; (d) Buckling-Modified M-P Model.....	62

Figure 4.13. Steel Stress-Strain Histories for the First Bottom Element at the Free End of the Wall Web for Specimen TW2: (a) Original M-P Model; (b) Fixed Envelope Model; (c) Shifting Envelope Model (Trimmed); (d) Buckling-Modified M-P Model (Trimmed).....	63
Figure 4.14. Vertical Normal Strain Profiles for Selected Nominal (Unreduced) Drift Levels at the Base of the Wall Web Under Negative Displacements (Wall Flange Is Under Tension) for Specimen TW2: (a) Original M-P Model; (b) Fixed Envelope Model; (c) Shifting Envelope Model; (d) Buckling-Modified M-P Model.....	64
Figure 4.15. Lateral Load vs. Top Displacement Relations for Specimen RW2 (Top Displacement with Pedestal Movement Contribution Subtracted): (a) Original M-P Model; (b) Fixed Envelope Model; (c) Shifting Envelope Model; (d) Buckling-Modified M-P Model.....	67
Figure 4.16. Steel Stress-Strain Histories for the Right Corner Element at Wall Base for Specimen RW2: (a) Original M-P Model; (b) Fixed Envelope Model; (c) Shifting Envelope Model (Trimmed); (d) Buckling-Modified M-P Model.....	68
Figure 4.17. Vertical Normal Strain Profiles at Wall Base for Selected Nominal (Unreduced) Drift Levels for Specimen RW2: (a) Original M-P Model; (b) Fixed Envelope Model; (c) Shifting Envelope Model; (d) Buckling-Modified M-P Model.....	69

LIST OF TABLES

Table 3.1.	Required Spring Stiffness for Different Buckling Modes (Dhakai and Maekawa, 2002b).....	29
Table 4.1.	Properties of Wall Specimens.	44
Table 4.2.	Summary of Buckling Length and Slenderness Ratio Calculation for Wall Specimens.....	48
Table 4.3.	Summary of Response Quantities for Specimen WSH6.....	54
Table 4.4.	Summary of Response Quantities for Specimen RW-A20-P10-S63.....	59
Table 4.5.	Summary of Response Quantities for Specimen TW2.	65
Table 4.6.	Summary of Response Quantities for Specimen RW2.	70

LIST OF SYMBOLS

a_1, a_2	Experimentally determined parameters that represent cyclic degradation of the curvature in the Menegotto and Pinto model (1973)
a_3, a_4	Experimentally determined parameters used in calculating the isotropic strain hardening-induced stress shift for the yield asymptote in the Menegotto and Pinto model (1973) extended by Filippou <i>et al.</i> (1983)
A_{cv}	Gross area of concrete section bounded by web thickness and length of section in the direction of shear force considered
A_g	Gross area of concrete section
A_t	Cross-sectional area of ties
b	Strain hardening ratio, $b = E_1/E_0$
b_{buck}	Equivalent negative strain hardening ratio used to represent the post-yield buckling curve proposed by Kashani <i>et al.</i> (2015a)
f'_c	Compressive strength of concrete
f_{yBE}	Vertical reinforcement ratio in the boundary regions of structural walls
D, D_b	The diameter of a longitudinal reinforcing bar
E_0, E_s, E	Elastic modulus of longitudinal reinforcing steel
E_1	Strain hardening modulus or slope of yield asymptote in a bilinear hysteretic steel model
E_b	Reduced stiffness for unloading from compression
E_r	Reduced modulus (an effective value lower than E and higher than E_t)
E_t	Tangent modulus (local slope of stress-strain diagram in inelastic range)
E_t	Elastic modulus of ties
k	Normalizing stiffness or longitudinal bar flexural stiffness
k_{eq}	Non-dimensional spring stiffness parameter $k_{eq} = k_t/k$
k_t	Axial stiffness of tie legs along buckling direction
l_e	Tie leg length along buckling direction
l_w	Length of structural wall

L, L_{eff}	The length along which buckling takes place in a longitudinal reinforcing bar
L/D	Length to diameter ratio or slenderness ratio for a reinforcing bar
$L/s, n$	Buckling mode
n_l	Number of tie legs along buckling direction
n_b	Number of longitudinal bars that are prone to simultaneous buckling
R	A parameter used in the Menegotto and Pinto model (1973) for representing the Bauschinger effect
R_0	Value of the R parameter for initial or monotonic loading in the Menegotto and Pinto model (1973)
s	Spacing between lateral ties or the spiral pitch
ε	Instantaneous strain level in longitudinal steel
ε^*	Normalized strain in the Menegotto and Pinto model (1973)
ε_0	Cyclic yield strain
ε_m	Maximum or minimum strain at the previous point of strain reversal in the Menegotto and Pinto model (1973)
ε_{max}	Absolute maximum strain ever reached in the Menegotto and Pinto model (1973) extended by Filippou <i>et al.</i> (1983)
ε_p	Plastic strain, $\varepsilon_p = \varepsilon - \varepsilon_y$
ε_r	Reversal strain
ε_y	Yield strain of a longitudinal reinforcing bar
λ_p	Non-dimensional slenderness ratio proposed by Dhakal and Maekawa (2002a)
ξ	Absolute strain difference between the current asymptote intersection point and the previous maximum or minimum strain reversal point in the Menegotto and Pinto model (1973)
ρ_1	Initial tangent of the post-yield buckling curve proposed by Kashani <i>et al.</i> (2015a)
ρ_2	Rate of change of the tangent to the post-yield buckling curve proposed by Kashani <i>et al.</i> (2015a)
$\rho_{h,web}$	Horizontal reinforcement ratio in the web region of structural walls

$\rho_{v,BE}$	Vertical reinforcement ratio in the boundary regions of structural walls
$\rho_{v,web}$	Vertical reinforcement ratio in the web region of structural walls
σ	Instantaneous stress level in longitudinal steel
σ^*	Minimum stress asymptote of the post-yield buckling curve proposed by Kashani <i>et al.</i> (2015a)
σ^*	Normalized stress in the Menegotto and Pinto model (1973)
σ_0	Cyclic yield stress
σ_r	Stress level at reversal of strain
σ_{st}	Stress shift for the tensile or compression yield asymptote that represent isotropic strain hardening in the Menegotto and Pinto model (1973) extended by Filippou <i>et al.</i> (1983)
σ_y, f_y	Yield stress of a longitudinal reinforcing bar

LIST OF ACRONYMS/ABBREVIATIONS

C	Compression
Env.	Envelope
FE	Finite Element
FSAM	Fixed Strut Angle Model
FSAFE	Fixed-Strut-Angle Finite Element
M-P	Menegotto-Pinto
RC	Reinforced Concrete
T	Tension

1. INTRODUCTION

1.1. General

Buckling of longitudinal reinforcing bars is one of the most critical damage states frequently reported in reinforced concrete members at the plastic hinge regions following a major seismic event, such as in columns and boundary regions of structural walls, (Figures 1.1a,b). This phenomenon has a detrimental impact on the overall hysteretic characteristics of reinforced concrete members where it will compromise their strength, ductility and energy dissipation capacity. This is because of the degradation in the axial stiffness of the reinforcing bar under compressive stress which ultimately leads to its fracture due to the propagation of microcracks resulting from buckling-induced bending deformations in the reinforcing bars, and finally the overall loss of the axial load carrying capacity of the structural member. Furthermore, reinforcement buckling is sometimes responsible for the initiation or exacerbation of some global failure modes, like out of plane deformation (lateral instability) of structural walls, which subsequently leads to lateral force capacity degradation of the entire structure.

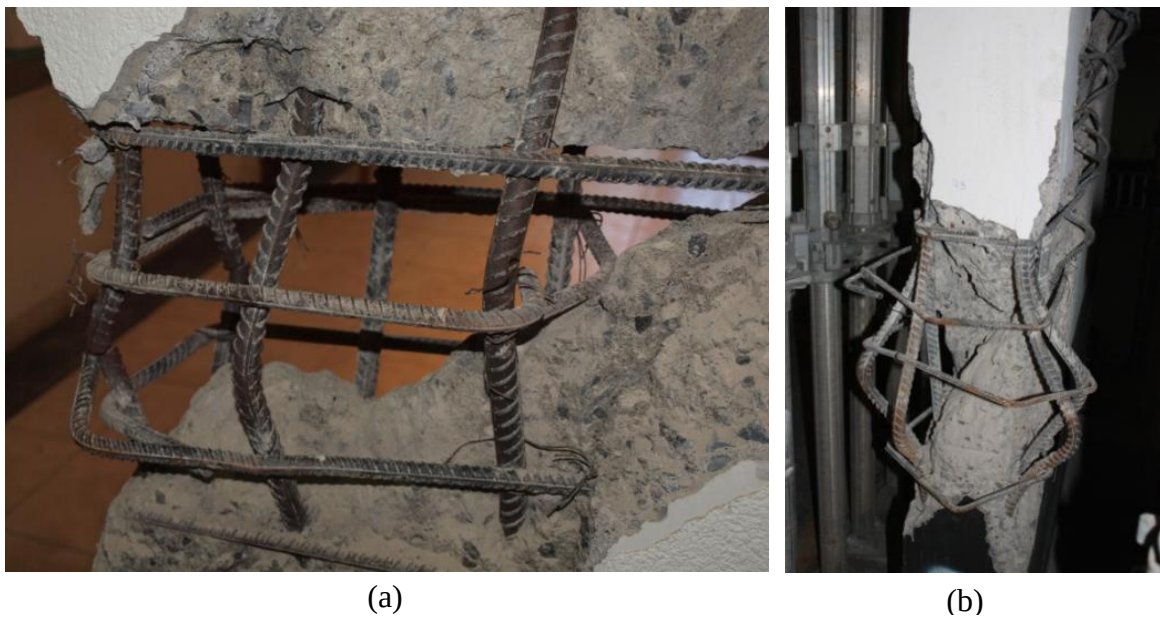


Figure 1.1. Typical Detailing and Observed Damage to Wall Boundary Regions:
(a) Ground Floor Level of a 23-Story Building in Coronel; (b) First Subterranean Level of
a 20-Story Building in Santiago (NEHRP, 2014).

Buckling is a purely geometrically nonlinear phenomenon and several finite element formulations were introduced by researchers (e.g., Mau and El-Mabsout, 1989) to simulate it for isolated bars; however, explicitly simulating localized nonlinear behavior such as buckling of reinforcing steel bars embedded in reinforced concrete members for general analysis and design purposes comes at extensive computational cost. Therefore, research efforts have been directed towards implicitly translating the degradation characteristics associated with rebar buckling into equivalent average stress vs. strain steel constitutive models that can be implemented into either concentrated or distributed plasticity (either macro- or microscopic) models that are used to analyze the response of reinforced concrete structural components under cyclic loading.

Such phenomenological material models that incorporate reinforcement buckling effects can be derived using one of the following primary approaches:

- (i) Laboratory testing of isolated reinforcing steel bars under monotonic compression (to derive the monotonic envelope) or cyclic loading conditions.
- (ii) Detailed finite element analyses capable of simulating geometric nonlinearities that either consider isolated bar behavior or fully or partially consider full structural members where the steel bars' interaction with the restraining hoops and surrounding concrete is studied (high computational cost).

The recorded average stress-strain responses from either of the two methods are then used to construct representative phenomenological material models that incorporate buckling effects into their formulation using curve fitting techniques.

1.2. Overview of Classical Buckling Theories

In engineering practice, columns or struts are generally classified according to the type of stresses developed within the column at the time of failure. Long slender columns will become unstable even the compressive stress remain in the linear elastic range. Such failure is referred to as elastic buckling. Intermediate-slenderness columns fail due to inelastic buckling, meaning that the compressive stress at failure is greater than the material's proportional limit. And short columns do not become unstable; rather the material simply yields or fractures. The Swiss mathematician Euler (1759), was the first to solve the buckling problem.

Modern design practices for reinforced concrete columns provide maximum spacing limits for hoops or spiral pitch for spiral reinforcement that precludes elastic buckling of longitudinal reinforcement and ensures they reach their yield strength first, so the problem at hand is limited to characterizing the inelastic buckling response of longitudinal reinforcing bars.

Two major theories were proposed to describe inelastic buckling and calculate the critical buckling load, which are based on replacing Young's modulus in Euler's method to calculate the critical buckling load of an elastic column by a smaller value. The first theory proposed for explaining inelastic buckling was the tangent modulus theory formulated by Engesser (1889), which is based on the presumption that there is no strain reversal when the compression member passes from a straight to a bent configuration and that the tangent modulus (E_t) applies over the whole cross-section.

However, it was pointed out by other researchers that the tangent modulus theory is not theoretically accurate. In 1889, Considère suggested that as an ideal column stressed beyond the proportional limit begins to bend, the stress on the concave side increases according to the law of the compressive stress-strain diagram, and the stress on the convex side decreases according to Hooke's law, and that therefore the buckling load would be calculated using a reduced modulus, E_r , the value of which lies between the modulus of elasticity and the tangent modulus.

Based on the observation made by Considère, Engesser acknowledged the error in his original theory and proposed the reduced modulus (or double-modulus) theory in 1895 where he determined E_r in the general form. In 1910, Kármán presented the reduced modulus theory again and added the actual evaluation of E_r for the rectangular cross section and the idealized H-section accompanying his research with experimental work. Nonetheless, test data indicate that the actual buckling loads are usually closer to the tangent-modulus values than to the reduced-modulus values, (Figure 1.2).

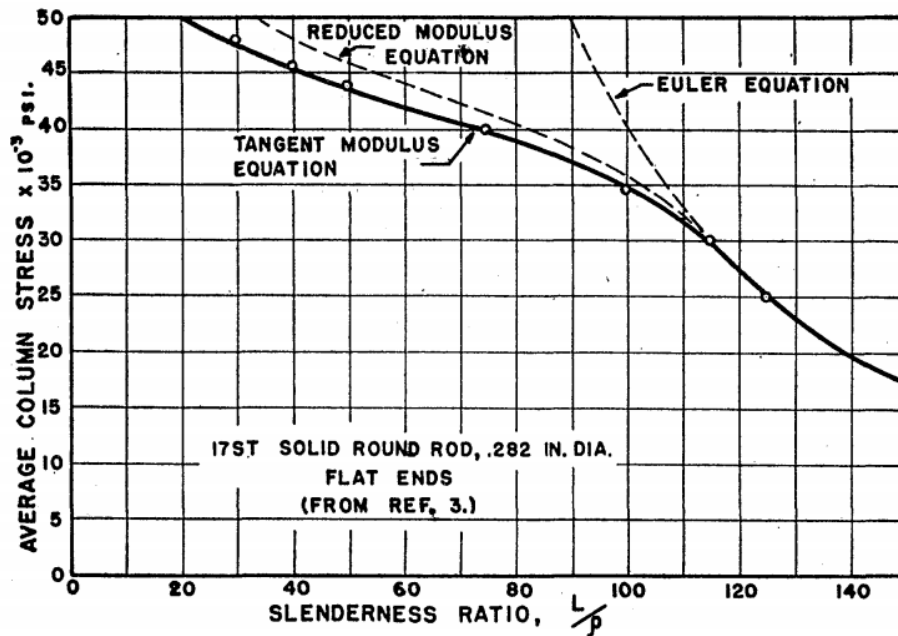


Figure 1.2. Column Theories and Test Data (Shanley, 1947).

Shanley (1947) proposed the column paradox where he called attention to the fact that the reduced modulus theory is derived by assuming that, after the column reaches the critical uniform stress, the column bends, i.e., the column remains perfectly straight prior to the onset of buckling. However, because no column is initially perfectly straight, bending will initiate before the critical buckling load is reached. Therefore, the true effective modulus lies between the tangent modulus and the reduced modulus, and evidently, the maximum column load will be reached somewhere between the loads predicted by the two theories.

1.3. Thesis Motivation

From a performance-based design and assessment point of view, it is desired to equip the existing finite element modelling approaches for reinforced concrete (e.g., the FSAFE model developed by Gullu *et al.*, 2019) with the necessary tools to accurately predict their post-peak response by accounting for the possible failure mechanisms encountered under seismic loading. This is necessary to provide accurate estimations of key performance-based design parameters such as the drift capacities of structural walls. Therefore, buckling of longitudinal reinforcement will be considered in this study as it is one the frequently encountered failure mechanisms observed in reinforced concrete structures following major seismic events.

1.4. Thesis Objective and Scope

The aim of this study is to scan the available literature on reinforcing bar buckling and select a number of suitable formulations of steel constitutive stress-strain models that capture this effect in order to implement them into the Fixed-Strut-Angle Finite Element (FSAFE) model (Gullu and Orakcal, 2019), which is a mesoscopic analysis approach for modeling the nonlinear behavior of structural walls. The effectiveness of each model is then evaluated by comparing their capabilities of capturing the experimentally observed strength degradation in a set of chosen wall test specimens for which reinforcement bar buckling was one of the reported damage modes.

1.5. Thesis Outline

Following the brief introduction in this chapter, Chapter 2 presents a comprehensive review of the literature available on reinforcing bar buckling, focusing on specific aspects related to this phenomenon such as the distinction between local and global buckling and between isolated versus embedded reinforcement behavior.

Chapter 3 provides a detailed description of the steel buckling models adopted in this thesis in terms of the monotonic compression and tension envelopes and the cyclic rules.

Chapter 4 introduces the validation methodology for the adopted steel buckling models, describing the implementation in the FSAFE (Gullu and Orakcal, 2019) wall model to simulate the behavior of wall specimens in which buckling of reinforcement was one of the observed failure modes. FSAFE model predictions for global and local responses using the proposed buckling models are compared with experimental results as well as the FSAFE model results when implementing the original Menegotto-Pinto (1973) steel model.

Finally, Chapter 5 summarizes the major observations and conclusions drawn from this study and makes recommendations for further research.

2. LITERATURE REVIEW

Buckling of longitudinal reinforcement has been the focus of numerous experimental and numerical studies which investigated various aspects pertaining to this complicated nonlinear phenomenon. The following is a summary of some of the most well-known works in this area of research.

2.1. Buckling Length and Local vs. Global Buckling

Early research efforts on different aspects of reinforcing bar buckling considered the simplifying assumption that the restraining ties are always sufficiently stiff to prevent rotations of the reinforcing bar at tie levels, and thus the unsupported length, i.e. buckling length, of the bar is solely determined by the tie spacing or spiral pitch and can be taken equal to them.

In reality, the buckled length may be restricted to two adjacent layers of transverse steel in some cases if there is a very high axial stiffness of the stirrups compared with the flexural stiffness of the longitudinal reinforcement, as is the case of very thick stirrups or very slender longitudinal reinforcement because of a large separation of stirrups (large spacing) or a very small longitudinal reinforcement diameter. Such behavior is referred to as local buckling. On the other hand, experimental and post-earthquake observations have shown that in elements with a good distribution of stirrups, the buckling length may be longer, stretching those stirrups that are within the buckling length, prevailing over the local buckling. This is called global buckling of the reinforcement (Massone and López, 2013). Thus, the buckling length is not completely determined by the spacing of stirrups, but also by the flexibility of the longitudinal and transversal reinforcement (Figure 2.1).

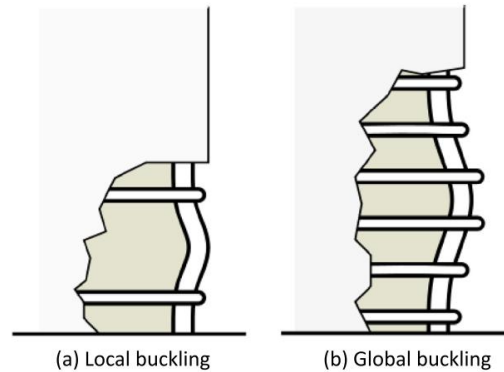


Figure 2.1. Local vs. Global Buckling (Massone and Lopez, 2013).

In terms of developing design guidelines to minimize the chances of buckling, Bresler and Gilbert (1961), focused on the effect of tie reinforcement characteristics on the behavior of RC columns and suggested a hypothesis outlining the mechanism by which RC columns subjected to monotonic compression fail. With increasing axial load, the resulting increase in strains occurring in the unconfined outer concrete cover ultimately leads to cracking and then spalling of the cover concrete when the local strains reach the crushing strain of concrete. After the outer concrete cover spalls, the reduced effective area of the column cross section subjects the concrete core and longitudinal reinforcement to additional stresses and the reinforcement ultimately yields and buckles. Stiffness reduction in the longitudinal reinforcement due to yielding and/or buckling imposes additional strains on the concrete core, which eventually cause it to crush after reaching its ultimate strength.

Bresler and Gilbert (1961) then developed an expression for minimum tie spacing that would prevent buckling of longitudinal reinforcement at a stress below yield (elastic buckling). For that purpose they assumed the existence of sufficiently stiff ties and used the tangent modulus formula to solve for the unrestrained length by equating the critical stress to the yield strength of the reinforcing bar.

Since any displacement at the level of the tie will reduce the critical buckling stress for the reinforcing bar and in effect reduce the load carrying capacity for the column, they considered an idealized system of a bar segment with an intermediate elastic spring to represent the tie. Using the Ritz method (Bleich, 1952), they solved for the smallest value of the tie stiffness that would prevent buckling over two tie spacing (Figure 2.2).

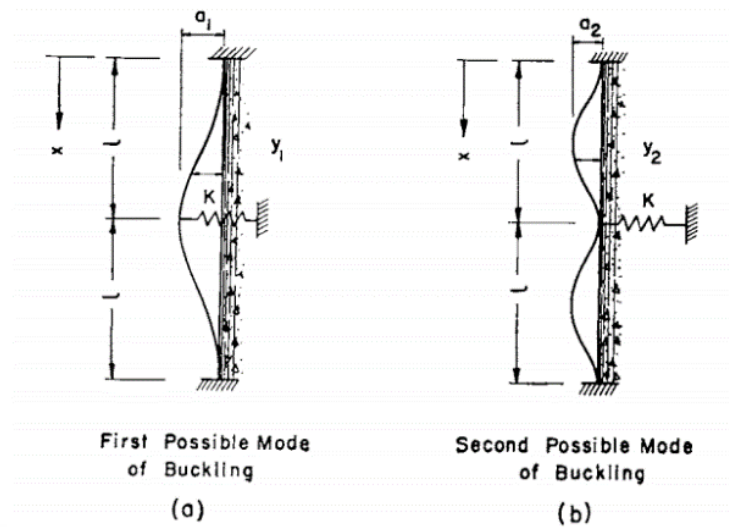


Figure 2.2. Mechanical System Used for Energy Analysis (Bresler and Gilbert, 1961).

Then they derived a formula relating the required ratio of bar to tie diameter in order to provide adequate tie axial stiffness to limit excessive tie deformation and in effect confine buckling to occur over one tie spacing. The simplified version of this formula states that the tie diameter should be at least one seventh of the longitudinal reinforcing bar diameter. Noting that this ratio is only valid for preventing the second buckling mode, higher modes will require separate analyses.

With regard to developing hysteretic steel models that captures the effect of buckling, Monti and Nuti (1992), considered the effects of inelastic buckling on the monotonic and cyclic behavior of reinforcing bars by conducting monotonic and cyclic tests on slender bars (Monti and Nuti, 1989, 1991) where three different bar length to diameter ratios were considered: $L/D = 5, 8, 11$, (Figure 2.3). They assumed that the length (L) of the tested specimens between the fixed supports is equal to the tie spacing for the case in which the tie stiffness is sufficiently high to restrain any rotation or deflection at the tie levels.

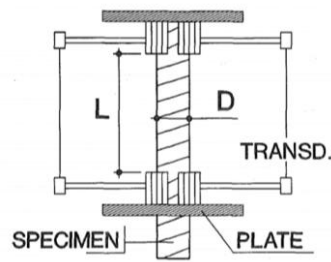


Figure 2.3. Experimental Setup for Cyclic Tests on Isolated Steel Bars (Monti and Nuti, 1992).

They observed the following phenomena:

- (i) Buckling occurs when the bar length to diameter ratio exceeds 5.
- (ii) For monotonic behavior, inelastic buckling results in a softening branch after yielding which gets more severe with larger L/D ratios (Figure 2.4).
- (iii) For cyclic behavior, hysteresis cycles are observed to be contracting, this phenomenon is referred to as isotropic softening as seen in Figure 2.5. Moreover, the reloading stiffness upon reversal from compression reduces with increasing cycle width due to buckling, which causes the stiffness of the bar to be mainly controlled by flexural stiffness while the axial stiffness is restored only when the bar is straightened again by the applied tensile strains.

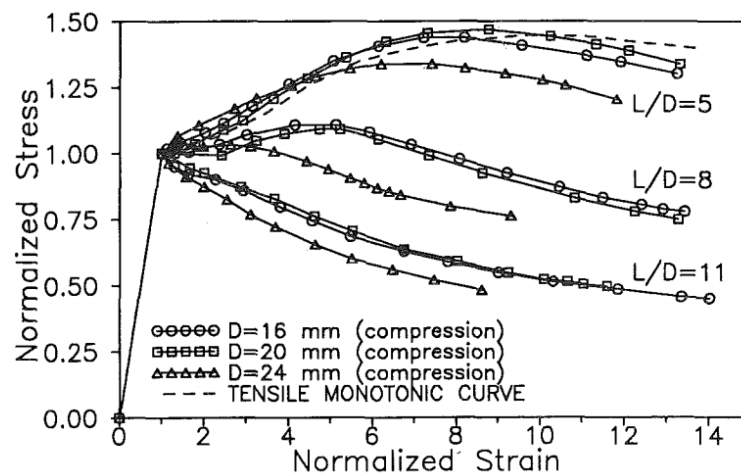


Figure 2.4. Experimental Monotonic Compression Curves (Monti and Nuti, 1992).

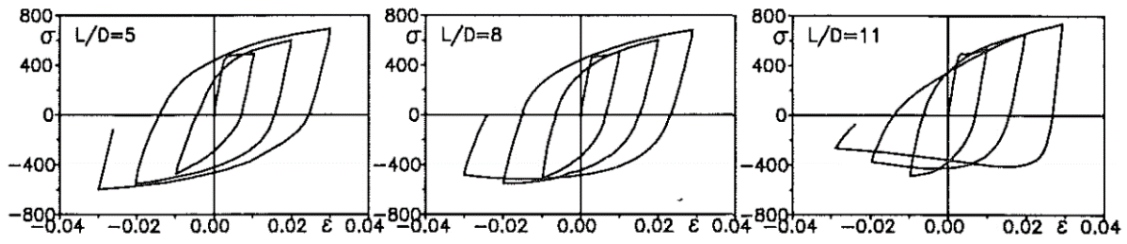


Figure 2.5. Experimental Cyclic Stress-Strain Curves for Different L/D Ratios (Monti and Nuti, 1992).

Based on these observations, they expanded the Menegotto and Pinto (1973) steel model formulation and developed a rule-based hysteretic model, using kinematic, isotropic, memory and saturation hardening rules that were modified for buckling by introducing a single governing parameter which is the ratio between the bar length between two ties and bar diameter. An empirical relationship was derived experimentally for monotonic compression which was used in the modification of the cyclic hardening rules. Considering that experimental tests show that the buckling phenomenon occurs when the bar length to diameter ratio exceeds 5, the authors defined it as the critical ratio, $(L/D)_{cr}$, and the modified hardening rules are used only beyond this critical ratio. The proposed model is reported to accurately simulate test results both in the presence and absence of buckling.

Another attempt to develop a hysteretic steel buckling model was made by Gomes and Appleton (1997), who introduced a modification of the Menegotto-Pinto cyclic stress-strain steel relationship to take into account the effect of the inelastic buckling of longitudinal bars (Figure 2.6).

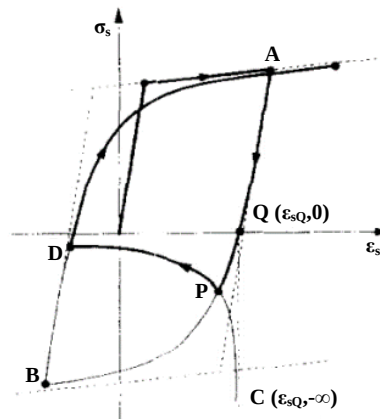


Figure 2.6. Unloading Steel Path with Buckling (Gomes and Appleton, 1997).

The buckling stress-strain path is simulated by a simplified model based on the equilibrium of a plastic mechanism of a buckled bar of length equal to the distance between two consecutive stirrups (Figure 2.7).

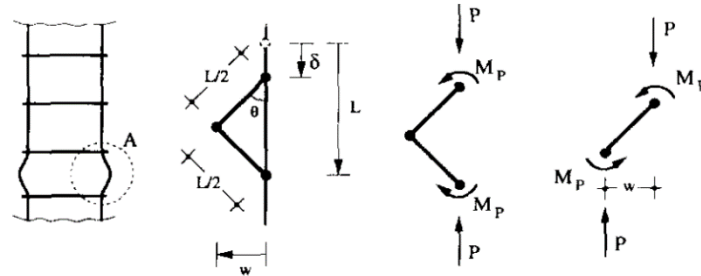


Figure 2.7. Equilibrium of a Buckled Steel Bar (Gomes and Appleton, 1997).

In the numerical fiber model developed by the two researchers to predict the moment-curvature response of RC columns, the buckling model is considered only after the rupture of the concrete cover occurs, which is detected by the failure of all concrete cover layers. It is reported that the comparison between experimental and numerical moment curvature analysis results of 9 reinforced concrete columns shows that a better agreement with experimental results was achieved by introducing the presented modification in the steel cyclic stress-strain relationship.

Dhakar and Maekawa (2002b), were probably the first to recognize that the assumption of the buckling length for reinforcement inside RC members being equal to the tie spacing does not always hold true except in some special cases that are rarely encountered in modern detailing practice:

- (i) lateral ties are very stiff;
- (ii) longitudinal reinforcing bars are very slender; and
- (iii) tie spacing is very large.

In light of that recognition, they proposed an analytical method, which is an expansion of Bresler and Gilbert's work (1961), to determine the buckling length of longitudinal reinforcing bars inside reinforced concrete members, which is equal to an integral multiple of tie spacing. Considering the stability of an idealized bar with springs model, energy principles based on the Ritz method (Bleich, 1952) are used to determine the spring constant, i.e. tie stiffness necessary to prevent buckling of longitudinal reinforcing bars in different

modes or configurations, which is then compared with the actual tie stiffness—computed based on the geometrical and mechanical properties of the lateral ties—to establish the buckling mode. Finally, multiplying the buckling mode by the tie spacing yields the buckling length, which is used in calculating the slenderness ratio L/D . This buckling length determination methodology has been validated against experimental observations from large column data set.

In their stability analysis illustrated in Figure 2.8, they considered an idealized system consisting of a flexural member fixed at both ends for simulating the reinforcing bar with an approximate average flexural rigidity, and discrete elastic springs for simulating the lateral ties with the springs around the middle of the buckling length eliminated from the system as they are more likely to yield and lose their stiffness due to larger flexural deformation at the mid-height of the longitudinal bars. A cosine shape function is chosen to approximate the deformation experienced by the bar as it buckles. By minimizing the total energy corresponding to two successive buckling modes, n and $n + 1$, with respect to their maximum amplitudes, a_n , and solving the resulting equations, the required spring stiffness is computed. Similar analysis is conducted for all possible buckling modes to estimate the corresponding spring stiffnesses.

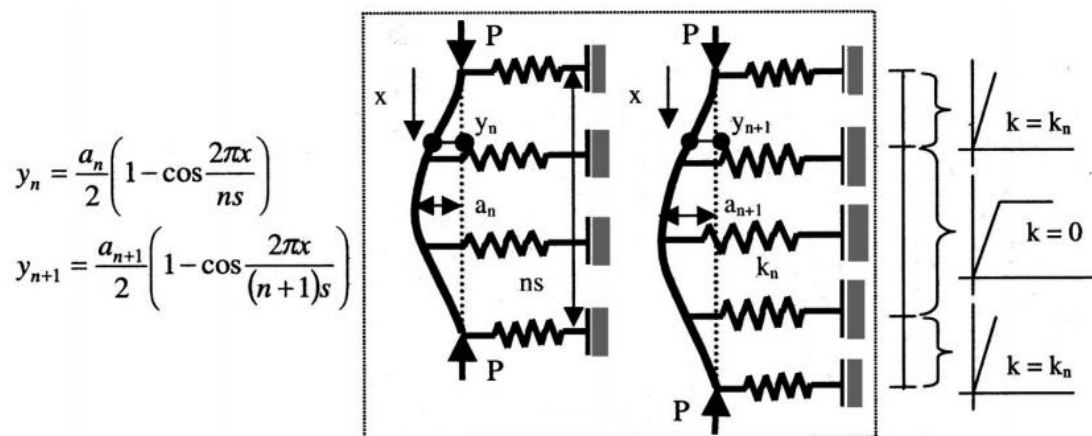


Figure 2.8. Simulation of Reinforcing Bars and Lateral Ties (Dhakal and Maekawa, 2002b).

To conclude their work, Dhakal and Maekawa presented a cyclic stress–strain model for reinforcing steel bars (Dhakal and Maekawa, 2002c), in which the compression envelope included a linear elastic range followed by a non-linear buckling model developed by the

authors using finite element simulations (Dhakal and Maekawa, 2002a). The buckling model is a function of the product of slenderness ratio (L/D) and the square root of yield strength ($\sqrt{f_y}$). The cyclic loops are defined based on the Giuffre–Menegotto–Pinto curves. A complete path-dependent cyclic constitutive model is then obtained by assembling the equations representing the monotonic tension and compression envelopes and the cyclic loops. Comparison with bare (isolated) bar test results shows that the proposed model could reasonably predict the cyclic behavior of buckling bars.

Massone and Moroder (2009), modeled the buckling behavior of isolated reinforcing bars using a lumped plasticity fiber model with an induced initial imperfection (Figure 2.9). Sectional analysis, based on uniaxial constitutive material laws for steel, is used to obtain average stress-strain buckling response of bare longitudinal reinforcement under monotonic compression. Comparison with experimental results reveals that the peak capacity is well captured, as well as the post-peak response shape with some differences in the peak capacity for specimens with high imperfection values, and in the shape of the post-peak response for specimens with low length-to-diameter ratio.

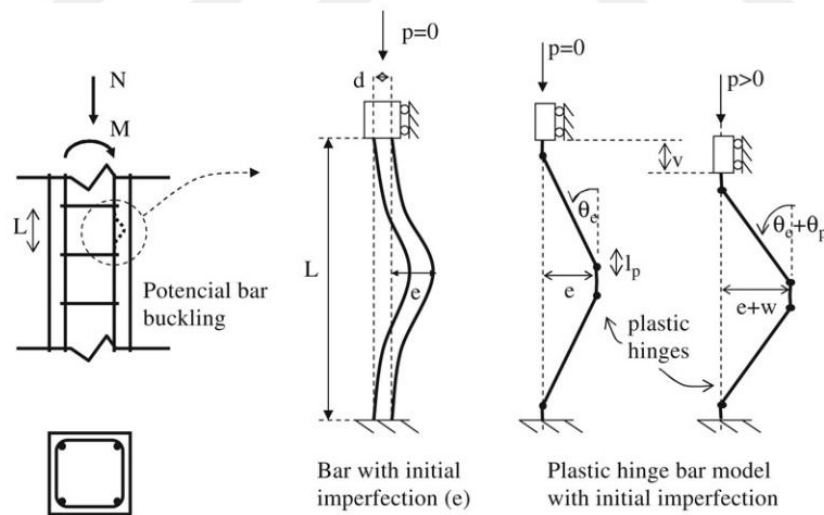


Figure 2.9. Representation of Reinforcing Bar Buckling Between Two Hoops (Massone and Moroder, 2009).

The same model was then extended by Massone and Lopez (2014) to simulate global buckling by introducing the effect of transversal reinforcement and expansion of the core concrete in the analysis (Figure 2.10).

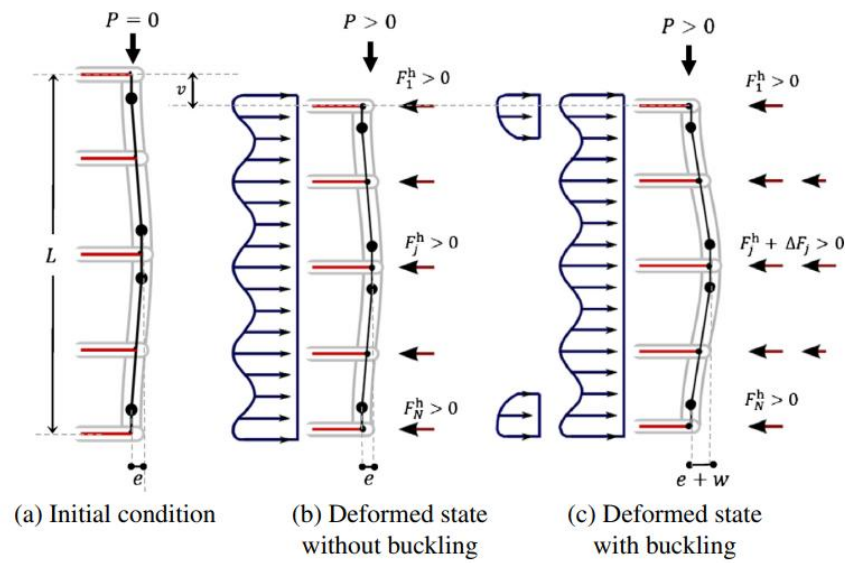


Figure 2.10. Global Buckling Model (Massone and Lopez, 2014).

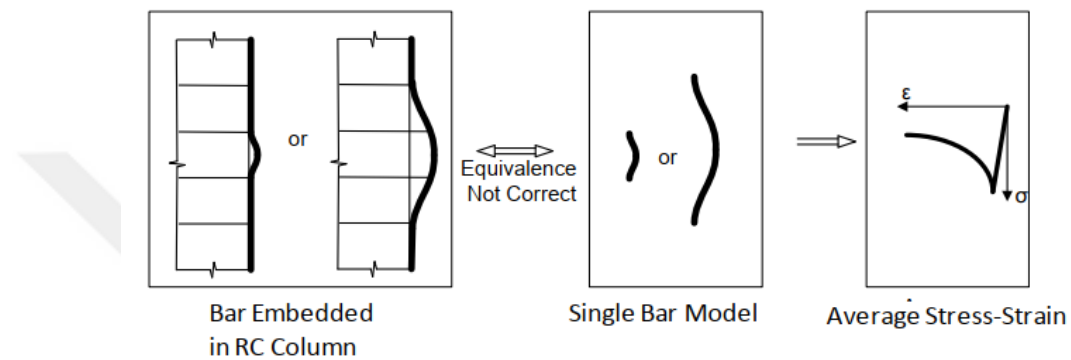
Analyses were performed for several possible buckling modes (L/s) and the peak capacities in the generated average monotonic stress-strain curves were compared. The buckling mode that yields the lowest peak capacity was considered to be the most probable mode and its corresponding monotonic stress-strain envelope can be used in sectional, element or system nonlinear analysis. The predicted buckling modes were in good agreement with both experimentally observed ones and with the modes predicted by Dhakal and Maekawa's model (2002b).

2.2. Isolated vs. Embedded Bar Behavior

The collective result of numerous studies on reinforcing bar buckling assert that the slenderness ratio of a reinforcing bar is the main parameter that affects the degree to which average compressive stress along the bar degrades once buckling has initiated. The vast majority of the proposed hysteretic steel buckling models, whether they consider local or global buckling with computed buckling lengths, use the monotonic average stress-strain curves derived from numerical or experimental studies conducted on isolated bars only. This approach is unrealistic, as it entails an implicit assumption that the buckling response of a single isolated bar matches that of longitudinal bars embedded in an RC member, as long as they have equal buckling lengths (Figure 2.11).

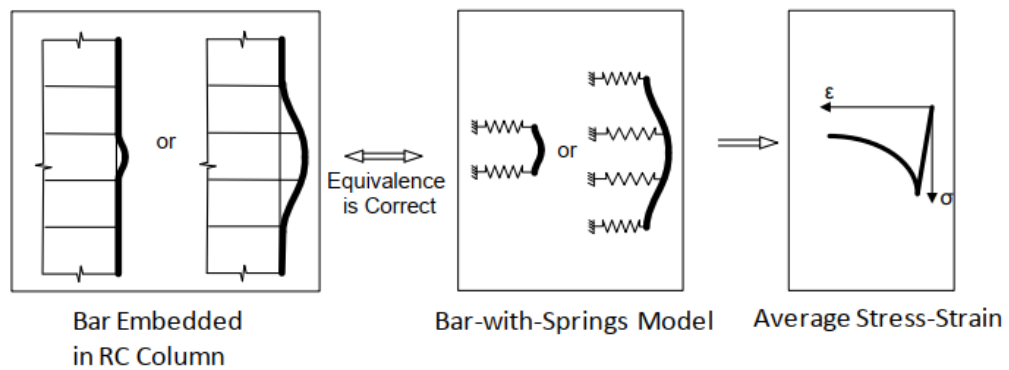
In reality, the buckling of reinforcing bars embedded in RC columns or wall boundary regions is an inherently complex phenomenon owing to the interaction of the core and cover concrete and confining lateral reinforcement with the longitudinal reinforcement.

Zong (2010) was the first to make a distinction between isolated and embedded bar behavior and verify the effect of lateral restraint on the monotonic stress-strain response of buckling bars using finite element simulations.



(a) Analogy is Correct for Local Buckling but Flawed for Global Buckling.

Figure 2.11. Flawed and Correct Analogies of Bar Buckling Models (Zong, 2010).



(b) Correct Analogy for Global Buckling

Figure 2.11. Cont. Flawed and Correct Analogies of Bar Buckling Models (Zong, 2010).

In the first part of their study, Zong simulated individual (bare) bars using both high-fidelity finite element model with 3-D solid elements and simplified macro-model with flexural beam elements. After performing a parametric study using these models it was found that the parameter $L/D\sqrt{f_y/420}$ is the only combined variable which controls individual bar buckling behavior.

Next, the behavior of bars embedded inside concrete columns was investigated using “full column” models, where their adequacy is established by comparing the rebar buckling shapes of the model with experimental results. To facilitate computational efficiency, a simplified bar with springs model is introduced and validated against experimental results in terms of buckling shape predictions and against the full column high-fidelity finite element model in terms of average stress-strain relations.

Finally, using simulations performed by the validated bar with springs model, average stress-strain curves are obtained in two steps, first, an upper boundary curve that is uniquely controlled by the combined parameter $s/D_b\sqrt{f_y/420}$ is defined and then this curve is reduced to the actual stress-strain curve for a given reinforcement configuration based on the stiffness of the horizontal ties. Comparison of the simulated force-displacement responses of RC columns obtained using the proposed buckling model showed an improved prediction of the softening of lateral force observed experimentally.

It is evident that Zong’s methodology to derive the average stress-strain envelopes is fundamentally different from previous works (e.g., Dhakal and Maekawa, 2002b) in the sense that they are generated from FE analyses using the proposed bar-with-springs model which accounts for restraining effect provided by the horizontal reinforcement on the post buckling relation, i.e., tie restraint is translated into less severe softening in the post buckling stress values.

Kashani *et al.* (2015a) developed a new hysteretic model (Figure 2.12) for reinforcing bars with and without corrosion damage that simulates a number of phenomena related to strength loss including: buckling and its effect on degrading the reinforcement response in compression and low cycle fatigue and its effects on the tension response of the reinforcement.

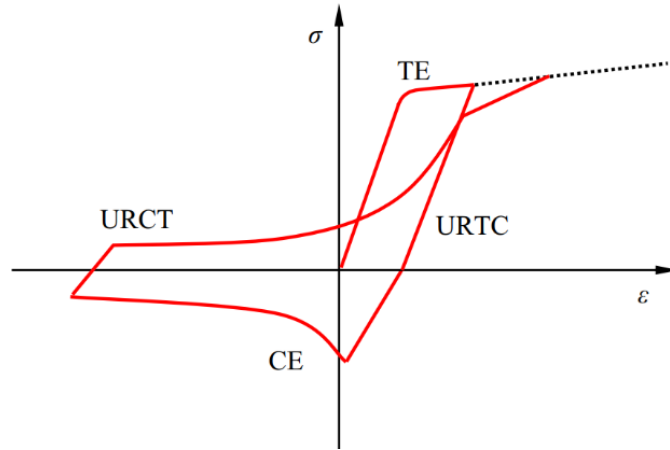


Figure 2.12. Proposed Phenomenological Hysteretic Model by Kashani *et al.* (2015a).

Numerical simulations of isolated reinforcing bars using a force-based beam-column element available on the OpenSees platform were used to calibrate the different model components for uncorroded reinforcing bars including the post-yield buckling response, degradation in cyclic buckling strength (Figure 2.13) and the unloading-reloading from compression to tension response.

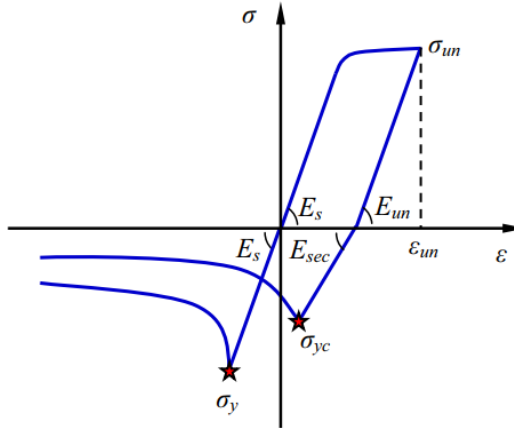


Figure 2.13. Cyclic Buckling Strength Degradation (Kashani *et al.*, 2015a).

Both experimental results and data from these simulations indicate that for commonly used reinforcing bars with a yield strength range between 400 to 500 MPa, buckling results in significant softening post buckling behavior only for slenderness ratios exceeding 8. The authors proposed an exponential function to define the post-buckling curve which is a function of a single compound variable called the non-dimensional slenderness ratio $\lambda_p = L/D\sqrt{f_y/100}$ first introduced by Dhakal and Maekawa (2002a). Moreover, they

recommended using the methodology proposed by Dhakal and Maekawa (2002b) to calculate the buckling length considering the influence of the stiffness of the horizontal ties on buckling mode of vertical reinforcement within RC elements.

Experimental and simulation data show that for bare reinforcing bars with $L/D \geq 8$, inelastic buckling results in a pinched hysteretic response when unloading from compression to tension at large strain demands, therefore a hyperbolic curve was used by the authors to capture this behavior, while a trilinear curve that captures the observed convexity at small strain demands is used in the unloading from compression to tension state (Figure 2.14).

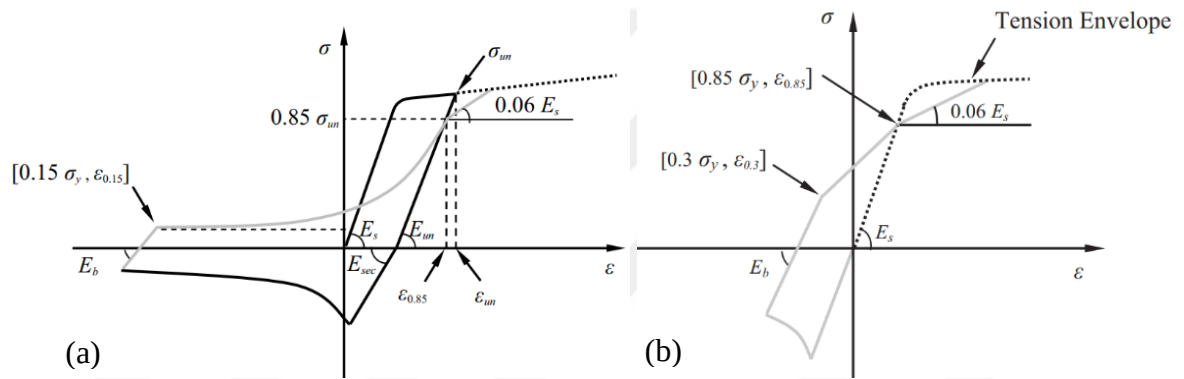


Figure 2.14. Modelling Pinching Response: (a) Linear-Hyperbolic Curves to Model Compression Unloading Branch at Large Strain Demand; (b) Multi-Linear Curves to Model Compression Unloading Branch at Small Strain Demand (Kashani *et al.*, 2015a).

It was reported that upon validating the proposed model for uncorroded reinforcing bars against laboratory test results for isolated reinforcing bars, the error in total hysteretic energy dissipation was found to be less than 1%.

Kashani *et al.* (2016) compared between the original M-P steel model and the buckling model developed by in their previous study (Kashani *et al.*, 2015a), in terms of their ability to predict the nonlinear cyclic response of an experimental column data set comprising 10 circular RC columns, where they were implemented in a finite element fiber model of a circular RC column.

The M-P model (Steel02 in OpenSees) which doesn't take inelastic buckling into account was implemented first and it was chosen to act as a benchmark to compare with the

behavior of the other considered reinforcing steel model (Kashani *et al.*, 2015a). Upon comparison of the computed responses using this model with experimental results it was observed that the prediction are in good agreement up until the point at which strength degradation starts in the experimental responses especially for columns having a large effective buckling length.

Then the column fiber model predictions, incorporating the steel model developed by the authors, which accounts for inelastic buckling and low-cycle fatigue degradation, (CorrodedReinforcingSteel in OpenSees), were compared to the experimental results and it was found that it significantly underestimated the strength of the tested columns due to the fact that this model was calibrated using simulations of isolated bar buckling that doesn't take into account the interaction between horizontal tie reinforcement and vertical reinforcement under cyclic loading.

To partially remedy this shortcoming, a third model (Hysteretic in OpenSees) (Figure 2.15) was used in a parametric study to find the optimum model pinch parameters that will minimize the error in hysteretic energy dissipation. The tension and compression envelopes of the Hysteretic model were fitted to those of the original CorrodedReinforcingSteel and different pinching parameter combinations (pinch x and pinch y in Hysteretic model) were investigated and arrived at an optimum combination. Finally, the calibrated Hysteretic model was validated against column experiments that were not included in the calibration process.

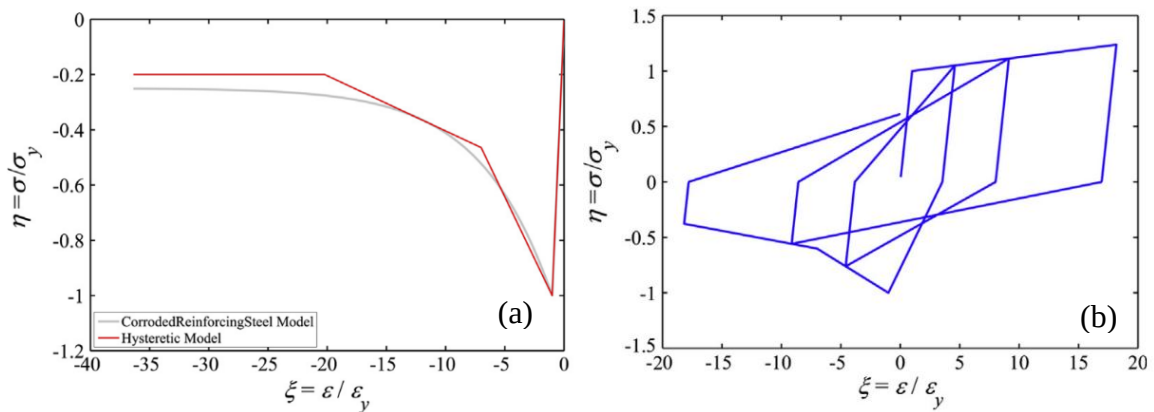


Figure 2.15. Cyclic Response of Hysteretic (OpenSees) Material Model: (a) Buckling Envelope; (b) Cyclic Response (Kashani *et al.*, 2016).

Unlike the Zong (2010) study which stressed the effect of lateral tie restraint on the post buckling softening in the monotonic stress-strain response, this study sheds light on the effect of lateral tie/spirals restraint on the cyclic rules of the stress-strain response upon unloading from buckling. It is stated that besides the buckling mode and length, the stiffness of horizontal ties are also affecting the cyclic stress-strain behavior of bars. While the buckling response of isolated bars exhibit significant pinching due to the influence of geometrical nonlinearity on stress-strain behavior, the restraint provided by lateral ties surrounding reinforcing bars in RC members acts as a mitigating mechanism that reflects as less severe pinching in the average stress-strain response of embedded bars (Kashani *et al.*, 2016).

2.3. Effect of Loading History on Reinforcement Buckling

It has been demonstrated by a great number of researchers that the reversed cyclic loading history plays an important role in the inelastic bar buckling phenomenon, where it was found to influence the onset of bar buckling and subsequent post-buckling response of the reinforcing bar.

It is well established that reinforcing bars are most prone to buckle upon reversal from tension loading (Rodriguez *et al.*, 1999; Moyer and Kowalsky, 2003; Feng *et al.*, 2014b,c). Paulay and Priestley (1993) were the first to correlate between tension and the buckling phenomenon in structural walls where they proposed a mechanism by which a boundary region of the wall initially subjected to the flexural tension force undergo out-of-plane instability after a moment reversal when the internal flexural compression force must be resisted solely by the vertical reinforcement until the flexural cracks close. However, due to unavoidable irregularities in the placement of reinforcement, small dislocated concrete particles and uneven spalling of the cover concrete on the two wall faces, eccentricities in the position of the flexural compression force with respect to the vertical reinforcement are formed that ultimately leads to out-of-plane deformations. (Figure 2.16).

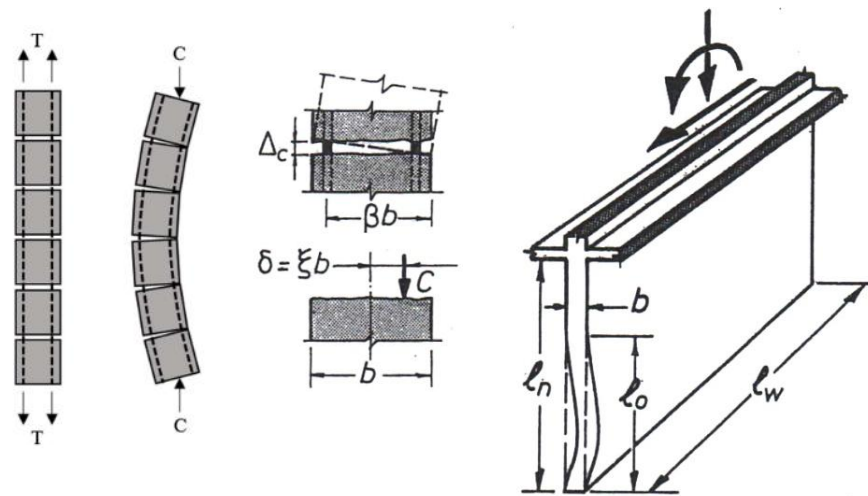


Figure 2.16. Deformation Leading to Lateral Instability in Structural Walls (Paulay and Priestley, 1993).

The basic concept involved in the wall buckling mechanism described by Paulay and Priestley is similar to the one responsible for activating the tension based reinforcing bar buckling mechanism first described by Moyer and Kowalsky (2003) and revisited later by Feng *et al.* (2014b,c) (Figure 2.17). After the occurrence of large flexural cracking on the tensile side of a column, a load reversal into compression will cause the longitudinal reinforcement to be prone to buckling as it becomes the sole source for compression zone stability until cracks are fully closed (Figure 2.17).

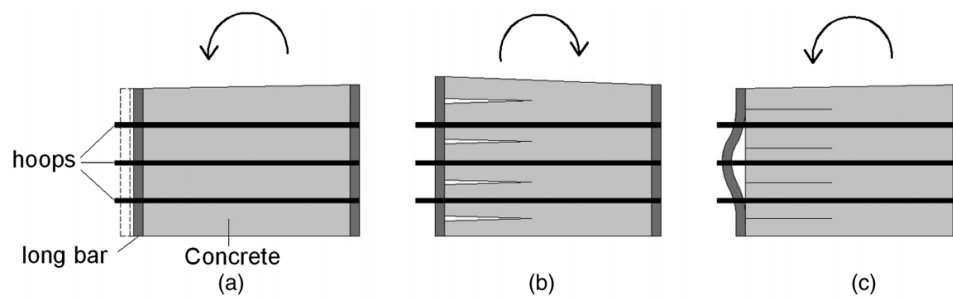


Figure 2.17. Tension Based Buckling Mechanism of Longitudinal Reinforcement in Columns (Feng *et al.*, 2014c).

Furthermore, plastic tensile strain demands' effect on inelastic buckling can be explained on the material level by the tangent modulus theory of inelastic buckling (Engesser, 1889) where reversal from a large tensile strain imposes a significant reduction in tangent modulus due to the Bauschinger effect and a large increase in nominal bar stress

from isotropic strain hardening. Both of the phenomena increase the likelihood of bar buckling, as explained in detail by Feng *et al.* (2014b).

On the other hand, compression strain was found to affect the boundary conditions of the longitudinal bar and the degree of eccentricity (Feng *et al.*, 2014b,c). Feng *et al.* postulated that concrete core dilation resulting from compressive strain may lead to yielding of the confining transverse steel and subsequent plastic elongation (Figure 2.18), as was observed in numerical studies using the finite element bar buckling model they proposed, which in turn will impair the restraining ability of horizontal hoops or spirals and will increase the unsupported length of the bars on later cycles.

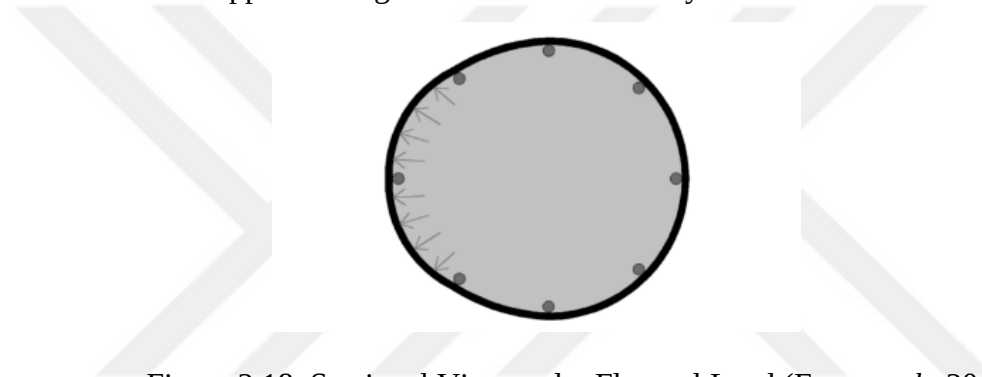


Figure 2.18. Sectional View under Flexural Load (Feng *et al.*, 2014c).

This process highlights the transient nature of the lateral restraint provided by hoops and spirals as described by Feng *et al.* (2014c). While the initial unsupported length can be determined approximately based on the section detailing and the stiffness of the longitudinal and horizontal reinforcement and hoop spacing, the load history will cause the length to “evolve” based on many interrelated factors. Moreover, dilation of the core concrete was shown to cause a non-uniform strain distribution in longitudinal bars which will accumulate on subsequent cycles and exacerbate the already present eccentricities resulting from unavoidable irregularities in the placement of reinforcement. Evolving unsupported length and accumulation of imperfections in longitudinal reinforcing ultimately facilitate buckling by reducing the critical buckling stress.

2.4. Overview

When the available literature on the subject is considered, it can be concluded that two major aspects of constitutive stress-strain steel models incorporating buckling remain largely unknown in terms of their quantitative effects on the overall cyclic behavior of reinforcing steel bars susceptible to buckling:

- (i) Determining the onset of bar buckling in the context of seismically excited RC members taking into account the effect of the loading history and core concrete expansion.
- (ii) Quantifying buckling effect on both the compression envelope and cyclic rules while accounting for the restraining effect of the horizontal reinforcement and its transient nature leading to unsupported length evolution.

With regard to accounting for the effect of tie restraint on the response of a buckled bar under monotonic compression, Massone and Lopez's effort (Massone and Lopez, 2013) to characterize the monotonic average stress-strain response of a group of bars taking into account the restraining effect of the ties and the expansion of core concrete by using an equilibrium approach was one of the rare attempts at tackling the deficiency present in Dhakal and Maekawa's approach (Dhakal and Maekawa, 2002c) and Kashani's approach (Kashani *et al.*, 2016) that was first pointed out by Zong (2010). However, Massone and Lopez (2013) didn't define any parametric stress-strain curves corresponding to different buckling modes to be readily available for implementation into FE and fiber models.

Considering the aforementioned shortcomings, this study will focus on investigating the effect of different formulations for the buckling envelope and cyclic rules on the post-peak analytical response of a group of RC wall specimens that experienced reinforcement buckling under testing.

3. PROPOSED HYSTERETIC UNIAXIAL STRESS-STRAIN STEEL MODELS INCLUDING THE EFFECT OF BUCKLING

A group of the salient features of hysteretic steel models that incorporate buckling into their formulation proposed by earlier researchers are selected to be used in this study to assemble and construct three different steel stress-strain models with buckling incorporated, in order to explore their effects on the analytical response of RC walls modeled using the FSAFE wall model (Gullu and Orakcal, 2019).

These features include:

- (i) Post-yield buckling envelope for simulating buckling-induced compressive stress softening (Kashani *et al.*, 2015a).
- (ii) Cyclic degradation of buckling stress (Kashani *et al.*, 2015a).
- (iii) Incorporating pinching in the unloading/reloading rules (Kashani *et al.*, 2016).
- (iv) Incorporating the Bauchinger effect in the unloading/reloading rules (Dhakal and Maekawa, 2002c).

3.1. Basis for Model Selection

One of the conclusions arrived at after scanning the past research on the constitutive behavior of reinforcing bars subjected to buckling was the uncertainty in the nature of the cyclic rules that will reflect the effect of inelastic buckling of a reinforcing bar embedded in concrete and restrained by lateral ties (Refer to Chapter 2), where it was first investigated by Kashani *et al.* (2016). Therefore, his work is chosen as a starting point for the selection of steel buckling models to explore in this study.

Kashani *et al.* (2016) emphasized that the pinching in his original model (Kashani *et al.*, 2015a) was too severe, in other words, he reasoned that the restraining effect of the ties will manifest itself in the cyclic rules only, namely that the pinching effect in the cyclic response of reinforcing bars inside the concrete is not as severe as the isolated bars (Kashani *et al.*, 2016) (Refer to Section 2.2 of this thesis).

To put their reasoning to the test, Kashani *et al.* (2016) implemented a generic fixed envelope model available on OpenSees (Hysteretic) in a fiber-based beam-column element to simulate column load-displacement responses. After trying out different pinching parameter combinations (pinch x and pinch y in Hysteretic model), they settled on optimum values that yielded reasonable simulated column responses.

Building on the aforementioned conclusions, it was first decided in this thesis to investigate two buckling model formulations, the first identical to the fixed envelope model (Hysteretic) with the adopted pinching parameters, whereas the second omits the pinching when unloading from tensile loads only and uses a shifting buckling envelope, which can be considered as a simplified version of the hysteretic model proposed by Kashani *et al.* (2015a). Furthermore, upon recognizing the shortcomings in neglecting the Baughinger effect in favor of pinching in the cyclic rules, a third buckling model formulation was considered, in a similar manner to Dhakal and Maekawa's model (2002c)—which is based on the original formulation proposed by Menegotto and Pinto (1973)—in which a negative strain hardening ratio in compression was used to coarsely approximate the effect of the degradation in the post-yield buckling envelope curve developed by Kashani *et al.* (2015a), while degradation in the unloading stiffness from compression was accounted for by adopting a reduced reloading stiffness expression (Eq. 3.25).

3.2. Buckling Length Calculation

The slenderness ratio (L/D) of vertical reinforcement is the sole buckling-related parameter needed as an input for the proposed buckling models.

A series of nonlinear finite element analyses on isolated reinforcing bars with different L/D ratios performed by Dhakal and Maekawa (2002a) led them to discern that the post-yield buckling behavior of reinforcing bars is controlled by the combined effect of the slenderness ratio and the yield strength of the reinforcing bars. Based on that recognition, they concluded that the average compressive stress-strain relationship for buckling reinforcement can be described completely by $L/D\sqrt{\sigma_y}$.

The post-buckling curve incorporated into the proposed buckling models in this study is a function of a single compound variable called the non-dimensional slenderness ratio λ_p (Kashani *et al.*, 2015a), which is defined by the following equation:

$$\lambda_p = \sqrt{\frac{\sigma_y}{100}} \frac{L}{D} \quad (3.1)$$

where σ_y has units of MPa.

Based on the literature review conducted in this study, it was found that one of the most convenient methods to compute the buckling length (L) is the one proposed by Dhakal and Maekawa (2002b), which is an expansion of Bresler and Gilbert's work (Bresler and Gilbert, 1961). In their approach, Dhakal and Maekawa performed an energy-based stability analysis considering the configuration of the longitudinal reinforcing bars and lateral ties and their mechanical properties.

The tie stiffness required to hold longitudinal reinforcing bars in different buckling modes is derived from energy principles, and it is compared with actual tie stiffness to determine the stable buckling mode. The buckling length is computed as the product of the stable buckling mode and the tie spacing. The proposed buckling length determination method is experimentally verified for various cases.

In their analysis illustrated in Figure 3.1, Dhakal and Maekawa (2002b) considered an idealized system consisting of a flexural member fixed at both ends for simulating the reinforcing bar with an approximate average flexural rigidity, and discrete elastic springs for simulating the lateral ties with the springs around the middle of the buckling length eliminated from the system as they are more likely to yield and lose their stiffness due to larger flexural deformation experienced by the longitudinal bars. A cosine shape function is used to define the deformational shape of the buckled bar. By minimizing the energy of two consecutive buckling modes, n and $n + 1$, with respect to their maximum amplitudes and simultaneously solving the resulting equations, the required spring stiffness is computed. Similar analysis is conducted for all possible buckling modes to estimate the corresponding spring stiffnesses.

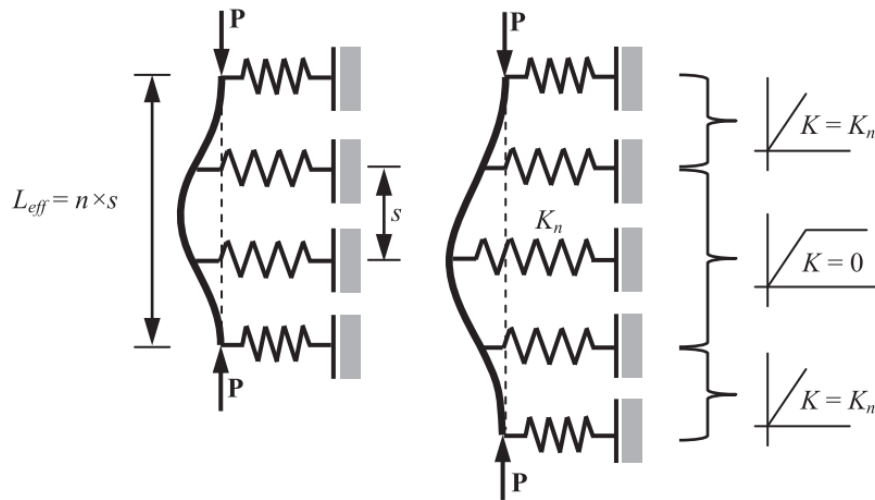


Figure 3.1. Simulation of Reinforcing Bars and Lateral Ties (Dhakar and Maekawa, 2002b).

Figure 3.2 presents a simple flowchart for the iterative procedure to follow when calculating the buckling length, while the following steps summarize the buckling length calculation procedure for rectangular sections:

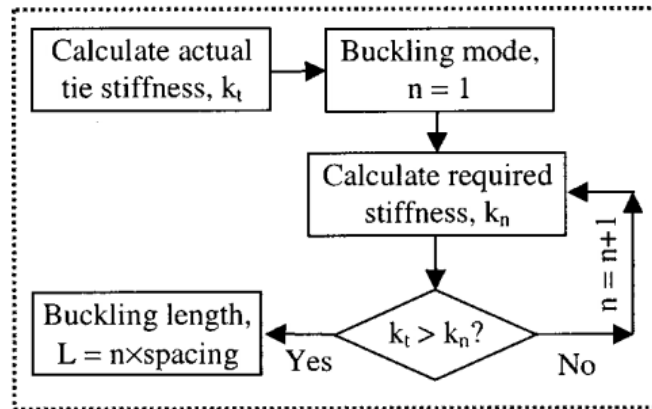


Figure 3.2. Flow-Chart of Buckling Length Determination (Dhakar and Maekawa, 2002b).

- (i) Calculate vertical reinforcement average flexural rigidity.

Dhakar and Maekawa (2002b) suggested an average flexural rigidity EI as an approximation to account for the bilinear strain distribution on the buckled bar section,

$$EI = \frac{E_s I}{2} \sqrt{\frac{\sigma_y}{400}} \quad (3.2)$$

$$I = \frac{\pi D^4}{64} \quad (3.3)$$

- (ii) Calculate the average actual stiffness of lateral ties available for each longitudinal bar at the buckling prone face.

The buckling induced lateral deformations of longitudinal bars generate axial tension in the tie legs along the buckling direction. Therefore, the axial stiffness of the ties will mainly resist longitudinal bar buckling. The axial stiffness of each tie leg is given by

$$\frac{E_t A_t}{l_e} \quad (3.4)$$

where E_t , A_t and l_e are the elastic modulus, cross-sectional area, and the tie leg-length, respectively.

It is assumed that total stiffness of n_l tie legs along the buckling direction contribute equally to n_b longitudinal bars that are prone to simultaneous buckling, therefore the restraining stiffness of the tie system that can be assigned to each individual bar is calculated by

$$k_t = \frac{E_t A_t}{l_e} * \frac{n_l}{n_b} \quad (3.5)$$

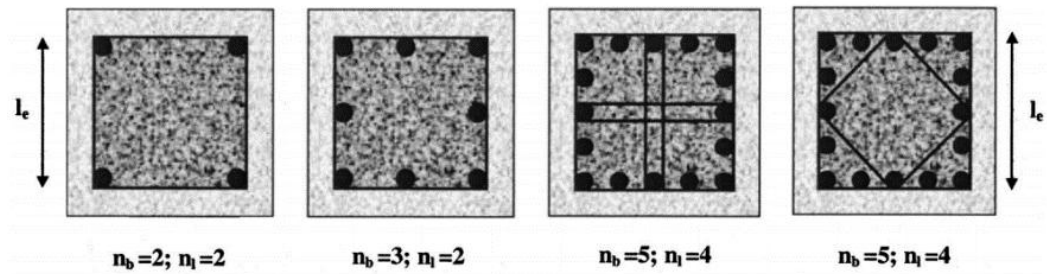


Figure 3.3. Values of n_b and n_l for Common Reinforcement Arrangements (Dhakar and Maekawa, 2002b).

Figure 3.3 shows values for n_b and n_l for common reinforcement arrangements in rectangular sections subjected to flexural loading where only the reinforcing bars in the section region below the neutral axis, i.e. under compressive strain, are prone to buckle simultaneously. Reinforcing bars located in all sides of columns under concentric compressive loads are considered to buckle together since they have equal strains. Therefore, n_b and n_l along each axis should be determined considering the tie legs parallel to the axis and longitudinal bars in both sides of the tie legs.

- (iii) Convert the average stiffness of lateral ties into non-dimensional spring stiffness parameter k_{eq} by dividing it by the normalizing stiffness k (longitudinal bar flexural stiffness)

$$k_{eq} = \frac{k_t}{k} = k_t * \frac{s^3}{\pi^4 EI} \quad (3.6)$$

- (iv) Choose a suitable range of eliminated springs from Table 3.1 and compare the non-dimensional spring stiffness parameter with the values in the table to determine the stable buckling mode, n .

- (v) Multiply the stable buckling mode, n with the tie spacing to obtain the effective buckling length L_{eff}

$$L_{eff} = n * s \quad (3.7)$$

Table 3.1. Required Spring Stiffness for Different Buckling Modes (Dhakal and Maekawa, 2002b).

Stable buckling mode, n	Equivalent Required Stiffness k_{eq} (Lateral Ties Eliminated in)			Average of centrals $L/2$ and $L/3$, k_{eq}
	Central $L/2$	Central $L/3$	Central $L/4$	
1	0.75	0.75	0.75	0.75
2	0.1649	0.1649	0.1649	0.1649
3	0.0976	0.0976	0.0371	0.0976
4	0.0758	0.0137	0.0137	0.0448
5	0.0084	0.0084	0.0084	0.0084
6	0.0063	0.0063	0.0032	0.0063
7	0.0052	0.0022	0.0022	0.0037
8	0.0046	0.0016	0.0016	0.0031
9	0.0013	0.0013	0.0008	0.0013
10	0.0011	0.0006	0.0006	0.0009

3.3. Fixed Envelope Model

3.3.1. Monotonic Response

3.3.1.1. Tension Envelope. A bilinear tension envelope (Figure 3.4) is used that has two segments for the linear elastic and the strain hardening regions as follows:

$$\sigma = \varepsilon E_0 \quad \varepsilon < \varepsilon_y \quad (3.8)$$

$$\sigma = \varepsilon E_1 = \varepsilon b E_0 \quad \varepsilon \geq \varepsilon_y \quad (3.9)$$

Where b is the strain hardening ratio defined as the ratio between the slopes of the post-yield strain hardening branch and the elastic loading/unloading branch.

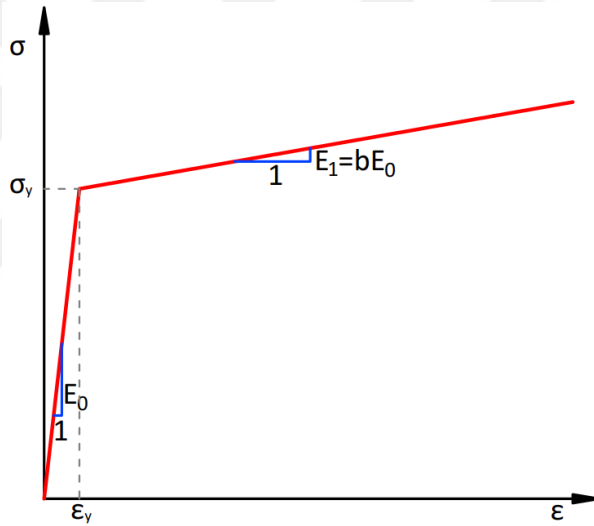


Figure 3.4. Tension Envelope Adopted for all Buckling Models.

3.3.1.2. Compression Envelope. The response in compression is defined by a straight line in the elastic range while the post-yield buckling curve suggested by Kashani *et al.* (2015a) was used to simulate the compression response of steel reinforcement with buckling length to bar diameter ratio between $8 \leq L/D \leq 30$ (Figure 3.5).

$$\sigma = \varepsilon E_0 \quad \varepsilon < \varepsilon_y \quad (3.10)$$

$$\sigma = \sigma^* + (\sigma_y - \sigma^*) \exp(-(\rho_1 + \rho_2 \sqrt{\varepsilon_p}) \varepsilon_p) \quad \varepsilon \geq \varepsilon_y \quad (3.11)$$

where;

ρ_1 is the initial tangent of the post-yield curve:

$$\rho_1 = 4.572 \lambda_p - 74.43 \quad (3.12)$$

ρ_2 is the rate of change of the tangent:

$$\rho_2 = 318.40 \exp(-0.071\lambda_p) \quad (3.13)$$

λ_p is the non-dimensional slenderness ratio proposed by Dhakal and Maekawa (2002a)

$$\lambda_p = \sqrt{\frac{\sigma_y}{100}} \frac{L}{D} \quad (3.14)$$

ε_p is the plastic strain:

$$\varepsilon_p = \varepsilon - \varepsilon_y \quad (3.15)$$

σ^* is the minimum stress asymptote of the curve:

$$\sigma^* = 3.75 \frac{\sigma_y}{\frac{L}{D}} \quad (3.16)$$

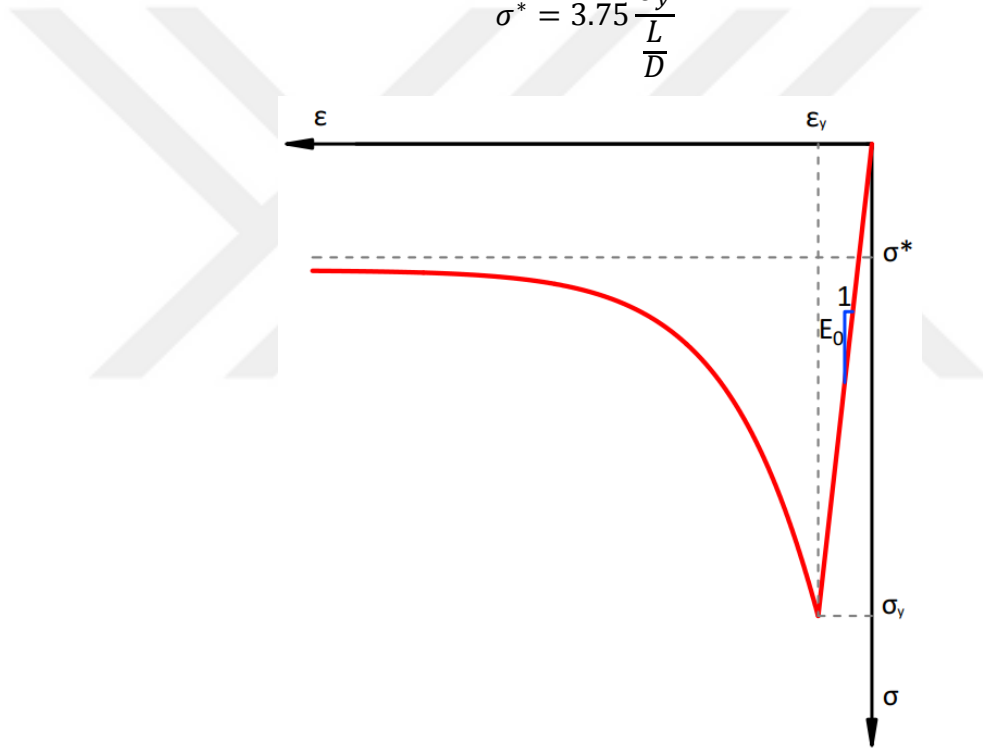


Figure 3.5. Compression Envelope (Kashani *et al.*, 2015a).

3.3.2. Cyclic Response

3.3.2.1. Unloading from the Fixed Compression and Tension Envelopes. During the first cycle when unloading from the monotonic compression or tension envelope, the path followed is defined by two straight lines (Figure 3.6). The first line continues with the elastic stiffness (E_0) until reaching the strain value corresponding to zero stress ($\varepsilon_{\sigma=0}$) while the second continues until it reaches the monotonic tensile or compression yield point ($\varepsilon_0 = \varepsilon_y$,

$\sigma_0 = \sigma_y$) with an intermediate reduced stiffness (E_{int}) that is computed as the slope of the line connecting the zero stress point and the monotonic yield point.

$$E_{int} = \frac{\sigma_0}{\varepsilon_0 - \varepsilon_{\sigma=0}} \quad (3.17)$$

A peak oriented approach is used to define a new cyclic yield point for the next reversal from the envelope. The last strain reversal point $(\varepsilon_r^{i-1}, \sigma_r^{i-1})$ from the envelope is considered to be the new yield point $(\varepsilon_0^i, \sigma_0^i)$, provided that the absolute strain value is the peak value ever reached. After reaching the new yield point, loading continues on the post-yield branches of the tension or compression envelope.

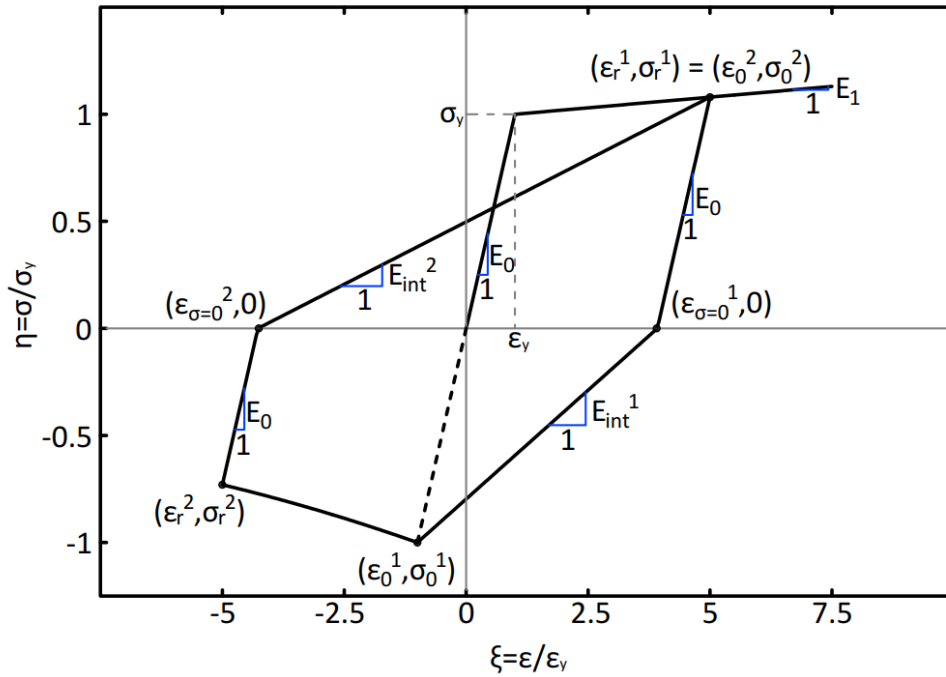


Figure 3.6. Cyclic Rules for the Fixed Envelope Model.

3.4. Shifting Envelope Model

3.4.1. Monotonic Response

The monotonic response in tension and compression follows the same envelopes defined for the fixed envelope model (Figures 3.4 and 3.5).

3.4.2. Cyclic Response

3.4.2.1. Unloading from the Shifting Compression Envelope. The path followed when unloading from monotonic compression envelope (Figure 3.7a) is defined by one straight line that continues until reaching the monotonic tensile yield point with an intermediate reduced stiffness (E_{int}) that is computed as the slope of the line connecting the strain reversal point (ε_r, σ_r) and the monotonic tensile yield point ($\varepsilon_0 = \varepsilon_y, \sigma_0 = \sigma_y$).

$$E_{int} = \frac{\sigma_0 - \sigma_r}{\varepsilon_0 - \varepsilon_r} \quad (3.18)$$

Similarly to the fixed envelope model mentioned previously, a peak oriented approach is used to define a new cyclic tensile yield point at each reversal from the compression envelope where the last strain reversal point ($\varepsilon_r^{i-1}, \sigma_r^{i-1}$) from the tension envelope is considered to be the new tensile yield point ($\varepsilon_0^i, \sigma_0^i$), provided that the tensile strain value is the maximum value ever reached. After reaching the new tensile yield point, loading continues on the post-yield branch of the tension envelope.

3.4.2.2. Unloading from the Fixed Tension Envelope. Unloading from the post-yield region of the tension envelope as illustrated in Figure 3.7b occurs with the elastic stiffness (E_0) until intersecting with the monotonic compression yield stress asymptote after which loading continues on the monotonic post-yield buckling curve which shifts with each cycle according to the intersection strain value with the cyclic compression yield stress asymptote.

A peak oriented approach is used to define a new cyclic buckling yield stress where the point corresponding to the last minimum compressive strain ever reached ($\varepsilon_r^{i-1}, \sigma_r^{i-1}$) is used to define a new buckling yield stress asymptote for the next reversal from the post-yield

part of the tension envelope. After reaching the buckling yield stress asymptote, loading continues on the shifted post-yield buckling curve.

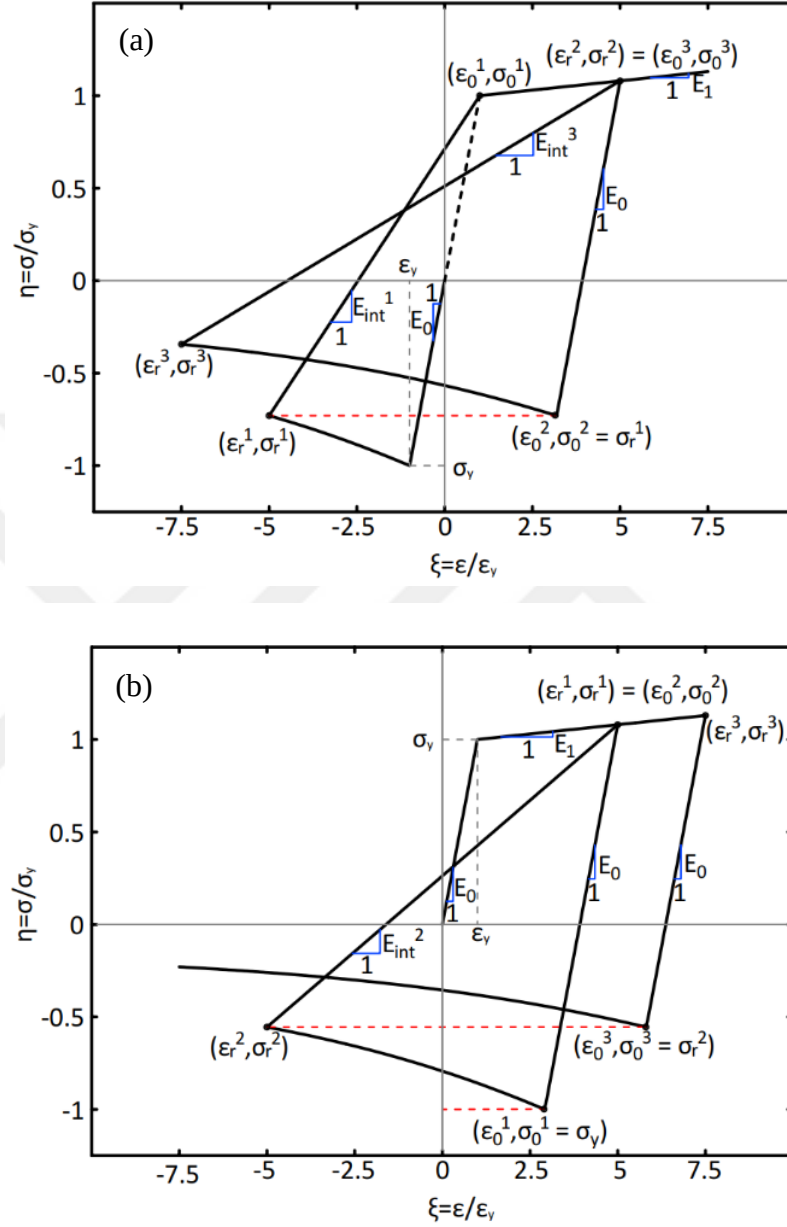


Figure 3.7. Cyclic Rules for the Shifting Envelope Model: (a) Unloading from the Shifting Compression Envelope; (b) Unloading from the Fixed Tension Envelope.

3.5. Buckling-Modified Menegotto and Pinto Model

The uniaxial constitutive stress-strain relationship implemented in the wall model for horizontal reinforcing steel as well as the vertical reinforcement in the web region of the walls is the well-known nonlinear hysteretic model of Menegotto and Pinto(1973), as extended by Filippou *et al.* (1983) to include isotropic strain hardening effects.

A brief description of the model is presented next followed by the proposed adjustment to account for buckling effects.

3.5.1. Overview of the Original Menegotto and Pinto Model

The original model of Menegotto and Pinto (1973) shown in Figure 3.8 can be expressed using the normalized stress-strain relationship as follows

$$\sigma^* = b\varepsilon^* + \frac{(1-b)\varepsilon^*}{(1+\varepsilon^{*R})^{1/R}} \quad (3.19)$$

where;

$$\varepsilon^* = \frac{\varepsilon - \varepsilon_r}{\varepsilon_0 - \varepsilon_r} \quad (3.20)$$

$$\sigma^* = \frac{\sigma - \sigma_r}{\sigma_0 - \sigma_r} \quad (3.21)$$

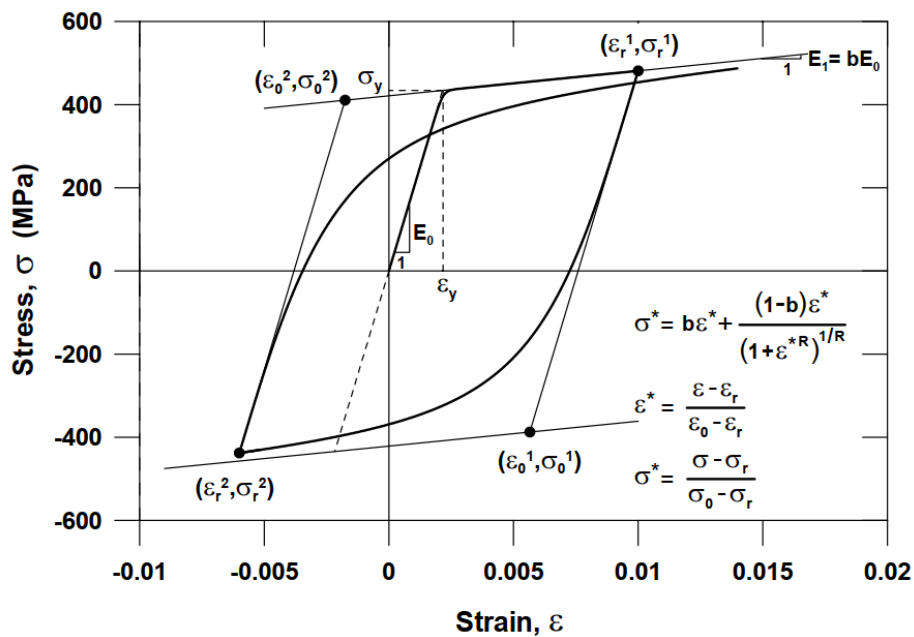


Figure 3.8. Constitutive Model for Steel (Menegotto and Pinto, 1973).

Equation (3.19) defines a curved transition from an elastic straight line asymptote with slope E_0 to the yield asymptote with slope $E_1 = bE_0$ where σ_0 and ε_0 are the stress and strain at the intersection point of these two asymptotes. σ_r and ε_r are the stress and strain at the point where the last strain reversal occurred (Figure 3.8). R is a parameter which controls the curvature of the curve at the intersection point of the elastic and yield asymptotes and allows a good representation of the Bauschinger effect. It depends on the absolute strain difference between the current asymptote intersection point and the previous maximum or minimum strain reversal point (Figure 3.9) depending on whether the corresponding steel stress is positive or negative. The expression for takes the form suggested by Menegotto and Pinto:

$$R = R_0 - \frac{a_1 \xi}{a_2 + \xi} \quad (3.22)$$

Where R_0 is the value assigned to the parameter for initial (or monotonic) loading, and a_1 and a_2 are experimentally determined parameters that represent the degradation of the curvature within subsequent cycles. The absolute strain difference between the current asymptote intersection point and the previous maximum or minimum strain reversal point is represented by the parameter ξ (Figure 3.9), which can be expressed as:

$$\xi = \left| \frac{(\varepsilon_m - \varepsilon_0)}{\varepsilon_y} \right| \quad (3.23)$$

Where ε_m is the maximum or minimum strain, at the previous point of strain reversal, depending on whether the current strain is increasing or decreasing, respectively. Parameter ε_0 is the strain at the current intersection point of the two asymptotes, and parameter ε_y is the strain at monotonic yield point (Figure 3.9).

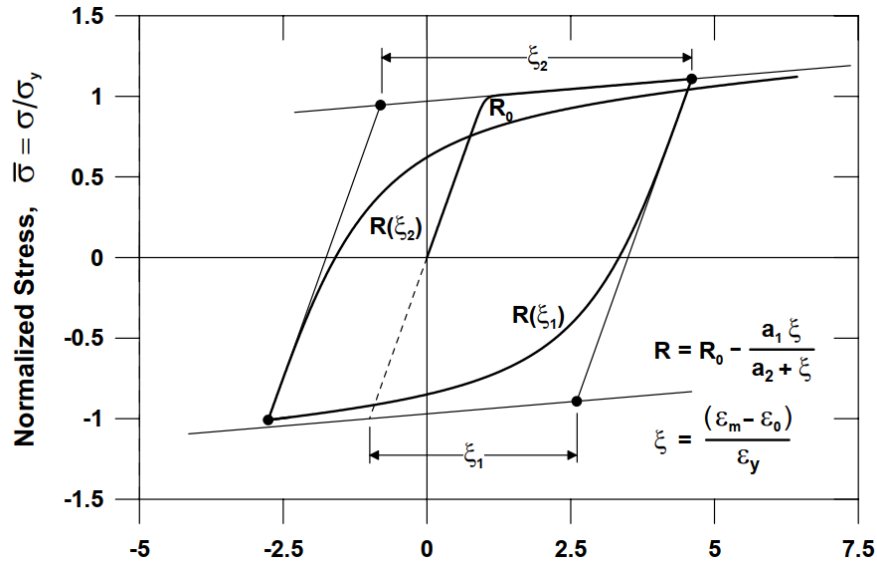


Figure 3.9. Degradation of Cyclic Curvature

The extension suggested by Filippou *et al.* (1983) to incorporate isotropic strain hardening into the Menegotto and Pinto model is considered for tensile stresses only in the buckling-modified M-P model. The stress shift for the tensile yield asymptote is given as follows

$$\frac{\sigma_{st}}{\sigma_y} = a_3 \left(\frac{\epsilon_{max}}{\epsilon_y} - a_4 \right) \quad (3.24)$$

Where ϵ_{max} is the absolute maximum strain at the instant of strain reversal and a_3 and a_4 are experimentally determined parameters.

3.5.2. Adjustment for Buckling

The yield asymptote in compression shall have a slope determined based on the buckling envelope curve developed by Kashani *et al.* (2015a), where there is a unique slope for each monotonic yield stress and slenderness ratio combination. Figure 3.10 illustrates the procedure for determining the slope of the yield asymptote in compression.

The slope of the secant line that bisects the angle between the initial tangent to the buckling curve and the line extending between the monotonic yield point and the point at which the compressive stress reaches 99.9% of the residual stress asymptote is taken as the

compression yield asymptote slope which will then be divided by elastic modulus (E_0) and inputted as a constant (b_{buck}) ratio to the buckling-modified M-P model.

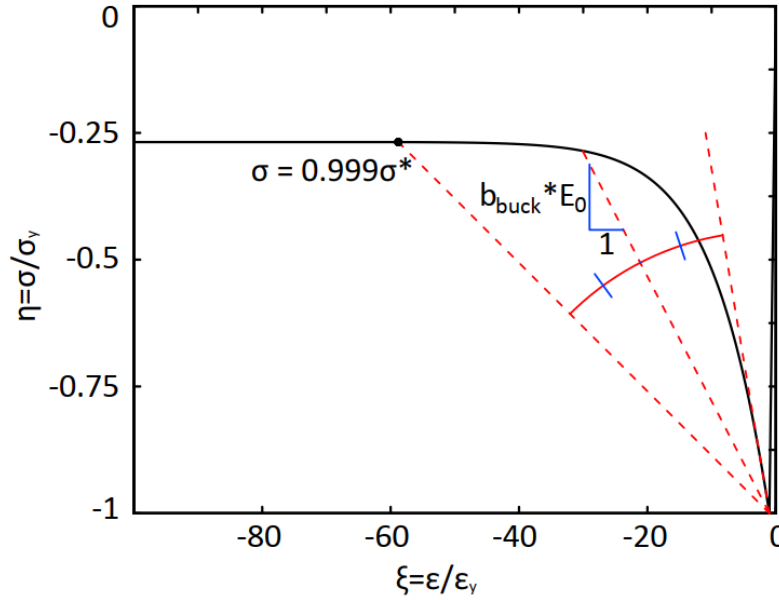


Figure 3.10. Determining the Negative Slope of the Compression Yield Asymptote

To account for degradation in the intersection point which can be considered as a cyclic buckling stress, which was observed by previous tests on bare and embedded bars (e.g., Kashani *et al.*, 2015a, Suda *et al.*, 1996), the position of yield asymptote in compression will be updated at each new cycle (Figure 3.11), by shifting the monotonic yield stress level to the stress level corresponding to the point of the last strain reversal from compression ($\varepsilon_r^{i-1}, \sigma_r^{i-1}$), provided that the reversal strain is the minimum compressive strain ever reached. The new intersection point ($\varepsilon_0^i, \sigma_0^i$) can then be computed.

It is reported in previous experimental programs (e.g., Monti and Nuti, 1992) on bare reinforcing bars that the reloading stiffness from the post-buckling stage is considerably smaller than the elastic modulus (E_0) as a result of the buckled bar's stiffness being mostly flexural and the axial stiffness is not fully restored until the bar is straightened again. Therefore, when unloading from compression, a reduced stiffness (E_b) is used (Figure 3.11) that depends on the buckling-induced compressive stress reduction at the unloading point,

$$\frac{E_b}{E_0} = \left(\frac{\sigma_r}{\sigma_y} \right)^{1.5} \quad (3.25)$$

Finally, once the compressive stress degrades to zero, the steel is considered incapable of supporting any further compressive stresses.

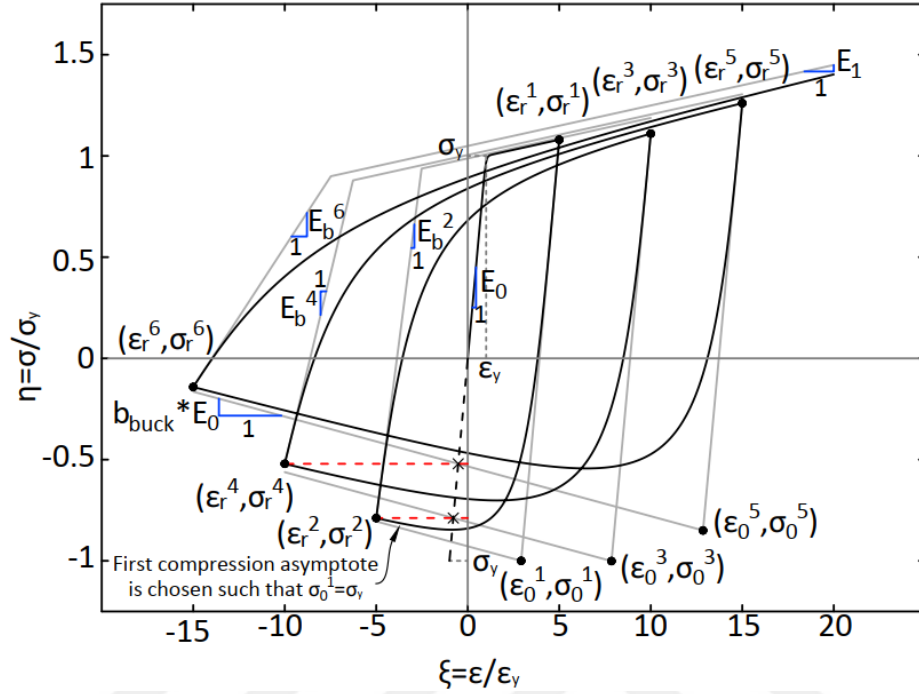


Figure 3.11. Cyclic Rules for the Buckling-Modified M-P Model.

Figure 3.12 shows an illustrative hysteretic stress-strain relationship, generated by the aforementioned buckling models for a representative strain history experienced by reinforcing bars located within the plastic hinge regions of RC wall after concrete strength has started to deteriorate, i.e., post peak loading stage.

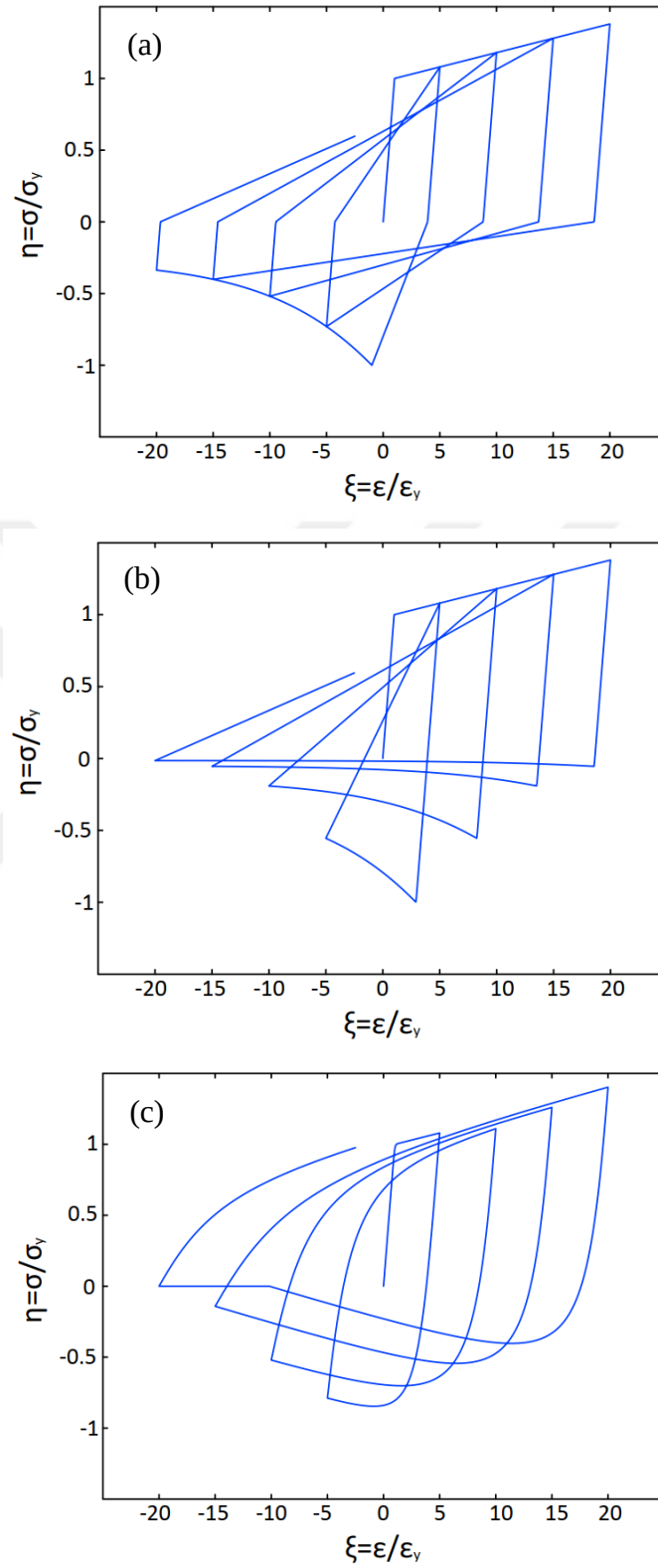


Figure 3.12. Stress-Strain Relationship Generated by the Buckling Models: (a) Fixed Envelope Model; (b) Shifting Envelope Model; (d) Buckling-Modified M-P Model.

4. IMPLEMENTATION OF PROPOSED BUCKLING MODELS AND EXPERIMENTAL VERIFICATION

In this chapter, a group of four wall test specimens previously modeled using the FSAFE mesoscopic finite element modeling approach (Gullu and Orakcal, 2019, Gullu *et al.*, 2019) in which the buckling effects were not accounted for, will be used to validate the steel buckling models proposed in this study, where they will be implemented to represent the constitutive behavior of the vertical reinforcing steel present in the boundary regions of these walls.

4.1. Overview of the FSAFE Model

The Fixed-Strut-Angle Finite Element (Gullu and Orakcal, 2019) model uses rectangular four-node panel (membrane) elements with only in-plane translational degrees of freedom defined at the nodes.

Linear interpolation functions and a single Gauss integration point are used to compute a single average strain field ($\varepsilon_x, \varepsilon_y, \gamma_{xy}$) corresponding to a single average stress field ($\sigma_x, \sigma_y, \tau_{xy}$) for each model element.

A constitutive model with a relatively simple formulation—named the Fixed Strut Angle Model (FSAM) (Orakcal *et al.*, 2012)—is used to describe the nonlinear behavior of the panel elements used in the finite element model assembly (Figure 4.1). Before formation of the first concrete crack, the FSAM assumes that the principal strain directions on the panel element coincide with the principal stress directions in concrete. The first concrete crack develops in perpendicular direction to the principal tensile strain, implying formation of the first fixed strut, when the principal tensile strain first exceeds the cracking strain value of concrete. The second crack is assumed to develop in perpendicular direction to the first crack when the tensile strain in the first strut direction exceeds the concrete cracking strain, resulting in formation of the second fixed strut. During further loading stages the two fixed

concrete struts work under interchanging compression and tension based on the applied strain field (Figure 4.1b).

A uniaxial hysteretic stress-strain relationship for concrete (Chang and Mander, 1994) (Figure 4.2a) is applied along the two fixed crack (strut) directions of the panel element, whereas a uniaxial hysteretic reinforcing steel model (Menegotto and Pinto, 1973) (Figure 4.2b) is applied along the orthogonal reinforcement directions (Figure 4.1d).

Behavioral response characteristics including tension stiffening (Belarbi and Hsu, 1994), biaxial compression softening in concrete (Vecchio and Collins, 1993), and hysteretic biaxial damage on concrete (Mansour *et al.*, 2002) are incorporated in the material model formulations. Additionally, simple behavioral models are incorporated in the FSAM to represent shear aggregate interlock effects along cracks (Figures 4.1c, 4.3a) and dowel action on the reinforcing steel bars (Figures 4.1e, 4.3b), constituting the shear stress transfer mechanisms across the cracks.

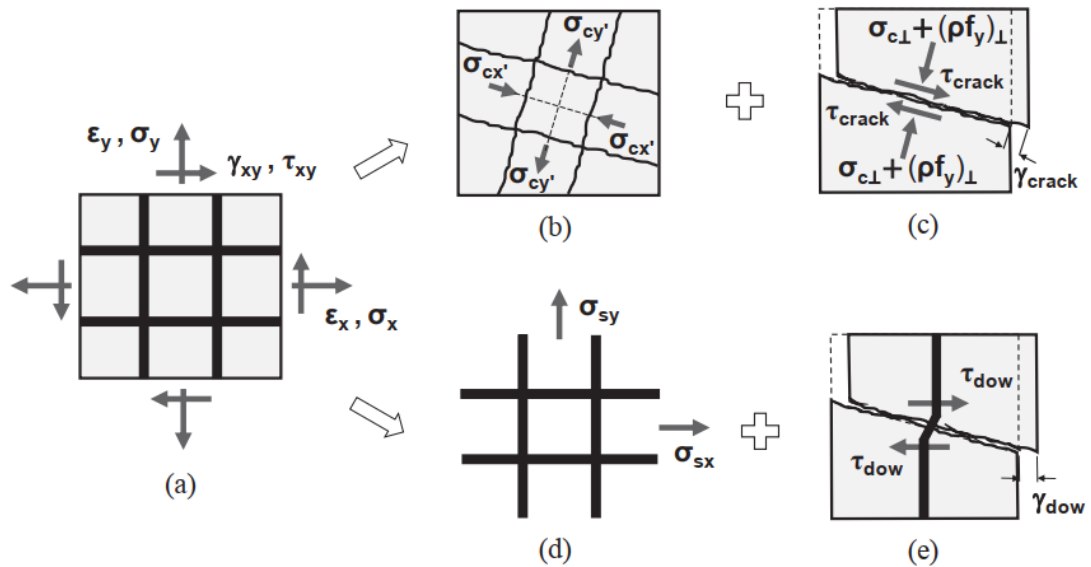


Figure 4.1. Constitutive Fixed-Strut-Angle Model (FSAM): (a) Strain Field on Panel Element; (b) Concrete Stresses along Fixed Struts; (c) Shear Aggregate Interlock Stresses along Cracks; (d) Uniaxial Stresses in Reinforcement; (e) Dowel Stresses in Reinforcement (Gullu *et al.*, 2019).

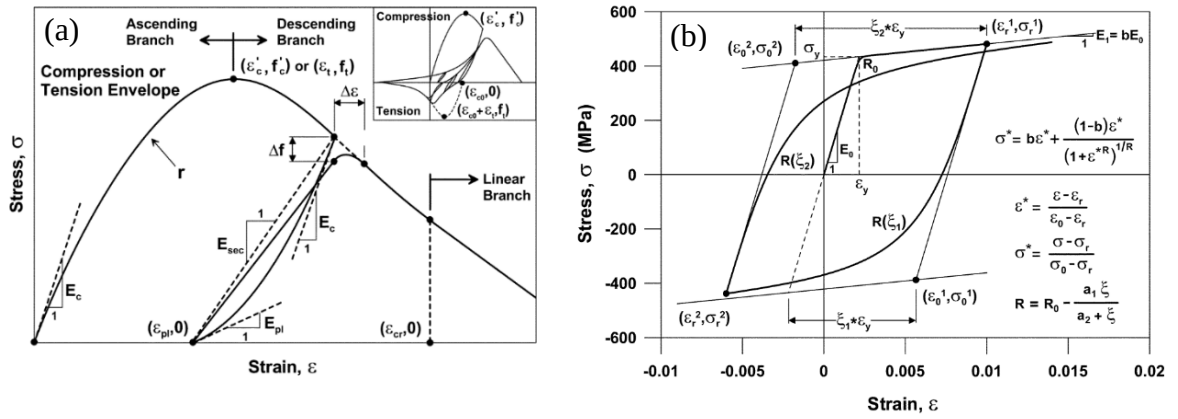


Figure 4.2. Material Constitutive Models for: (a) Concrete (Chang And Mander, 1994);
(b) Reinforcing Steel (Menegotto and Pinto 1973).

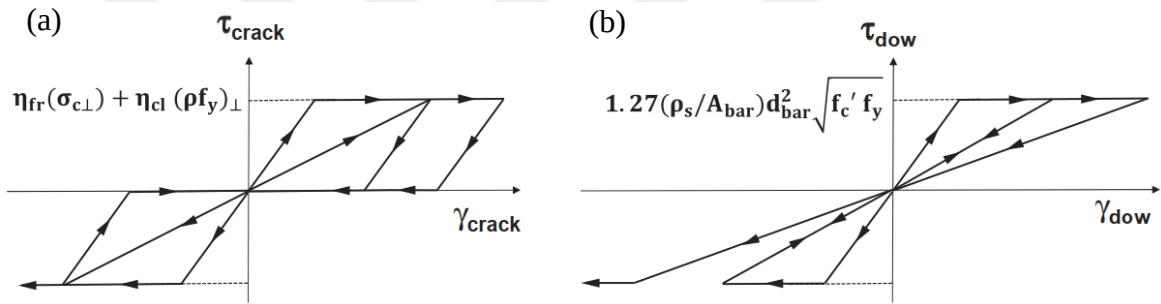


Figure 4.3. Behavioral Models for Shear Stress Transfer Mechanisms across Cracks:
(a) Peak-Oriented Hysteretic Model for Shear Aggregate Interlock; (b) Origin-Oriented
Hysteretic Model for Dowel Action (Gullu *et al.*, 2019).

4.2. Implementation of the Proposed Steel Buckling Models in FSAFE Wall Model

As buckling of longitudinal reinforcement in RC walls is frequently reported to take place at and near the boundary regions, the three proposed steel buckling models presented in Chapter 3 of this thesis are implemented as alternative constitutive models to simulate the nonlinear cyclic response of the vertical reinforcement present in the boundary regions of the FSAFE wall models.

For the rectangular wall specimens selected from the literature (WSH6, RW-A20-P10-S63 and RW2), each boundary region is discretized into two adjacent meshes, while for a selected wall specimen with T-shaped cross-section (TW2), the boundary regions are

considered at the free end of the web, the two free ends of the flange and at the flange-web junction for which four, two and three meshes are chosen as the discretization scheme, respectively.

The original Menegotto-Pinto steel model is used to simulate the cyclic behavior of all the remaining steel reinforcement in the FSAFE wall models, namely the horizontal steel in the boundary regions and the horizontal and vertical steel in the web region.

4.3. Experimental Verification of the Proposed Steel Buckling Models

Four test specimens from three different experimental programs (Figure 4.4) will be used for validation of the proposed steel buckling models. Table 4.1 provides the overall material/geometric/detailing properties as well as the experimentally observed failure mechanisms. A thorough comparison of the numerical versus experimental behavior between the proposed buckling steel model implementations in the FSAFE wall model is presented in terms of global responses (lateral load–displacement) and local responses (vertical normal strain profiles along the base and boundary regions of the walls).

Table 4.1. Properties of Wall Specimens.

Specimen ID	WSH6	RW-A20-P10-S63	TW2		RW2
			Web	Flange	
f'_c (MPa)	45.6	48.6	42.8	42.8	42.8
f_{yBE} (MPa)	576	477	434	434	434
$\rho_{v,BE}$ (%)	1.54	7.11	2.93	2.93	2.93
$\rho_{h,web}/\rho_{v,web}$ (%)	0.25/0.46	0.61/0.61	0.44/0.44	0.33/0.33	0.33/0.33
M/Vl_w	2.26	2	3		3
$P/A_g f'_c$	0.108	0.1	0.08		0.07
$V_{max,test}/A_{cv}\sqrt{f'_c}$	0.29	0.57	0.34		0.19
L_{eff}/D_b	9	12	9		8
Failure mode ^a	CB	CB/LI	CB/LI		CB

Where;

$\rho_{v,BE}$: Vertical reinforcement ratio in the boundary regions

$\rho_{h,web}$: Horizontal reinforcement ratio in the web region

$\rho_{v,web}$: Vertical reinforcement ratio in the web region

M/Vl_w : Shear span-to-depth ratio

$P/A_g f'_c$: Axial load ratio

$V_{max,test}/A_{cv}\sqrt{f'_c}$: Average shear stress over $\sqrt{f'_c}$ ratio at peak lateral load

L_{eff}/D_b : Computed slenderness ratio of vertical reinforcement in boundary regions which is the ratio of the effective buckling length, calculated as per Dhakal–Maekawa buckling model (Dhakal and Maekawa, 2002b), over the vertical reinforcement diameter

^aCB: concrete crushing/rebar buckling, LI: lateral instability of wall boundary

4.3.1. Calculation of Buckling Length and Slenderness Ratio for Boundary Vertical Steel in Wall Specimens

Buckling lengths are computed in accordance with the method introduced by Dhakal and Maekawa (2002b) described in detail in Chapter 3 of this thesis. Since the hysteretic buckling models are implemented into the boundary regions of the FSAFE wall models, buckling lengths for vertical reinforcing steel located in the wall boundary regions must be calculated by treating each boundary region cross section as a separate entity.

In order to account for the fact that the load action affecting boundary region sections in structural walls is closer to axial rather than flexural action, bars along all faces of the boundary region are assumed to be under compressive strain. Therefore, the bars along the short boundary face are considered to buckle in the direction of the applied lateral load while the bars along the other two faces are considered to buckle in the out-of-plane direction to the applied loads, see Figure 4.5.

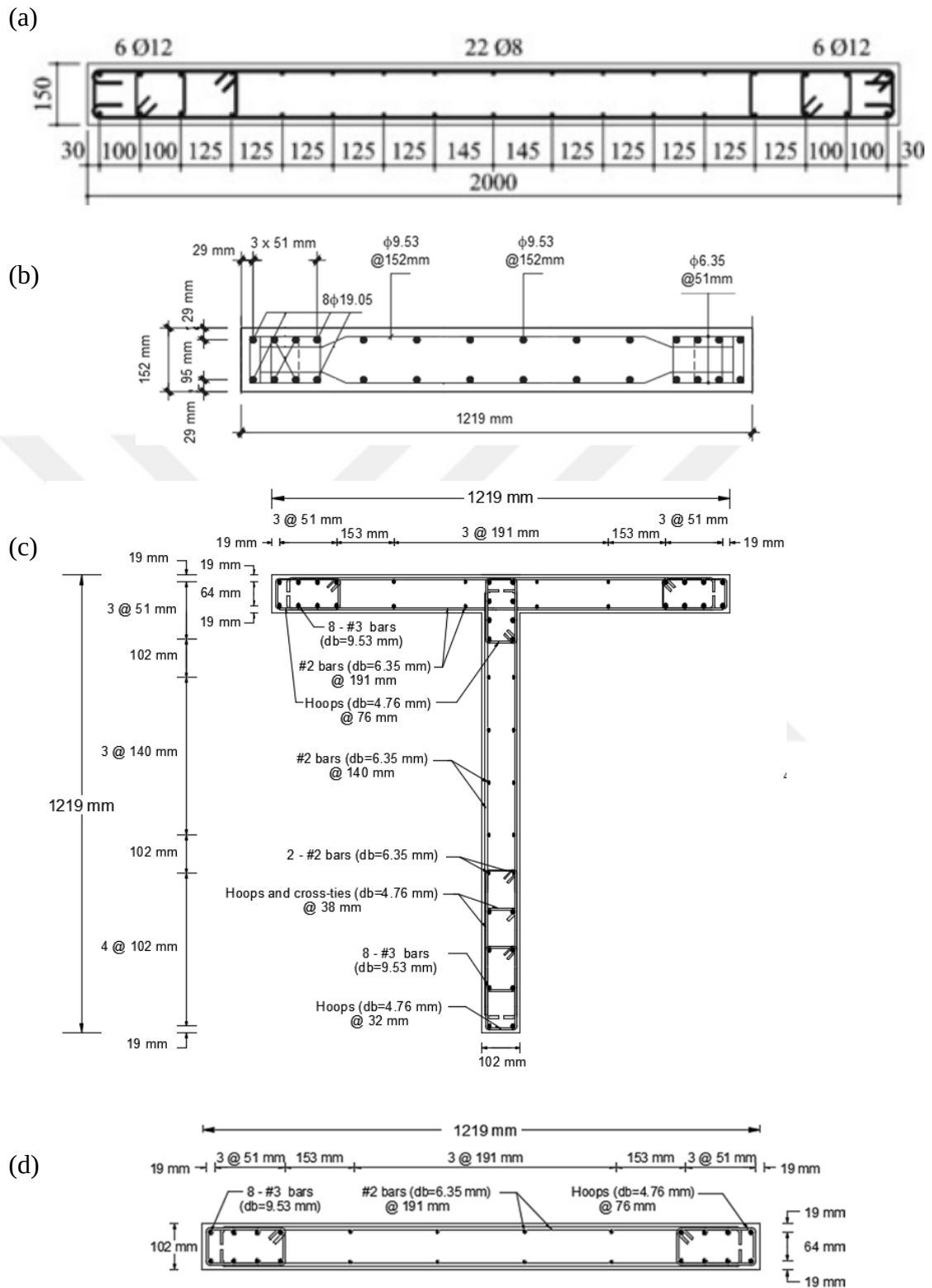


Figure 4.4. Wall Cross-Sections and Reinforcement Details for Specimens: (a) WSH6; (b) RW-A20-P10-S63; (c) TW2; (d) RW2.

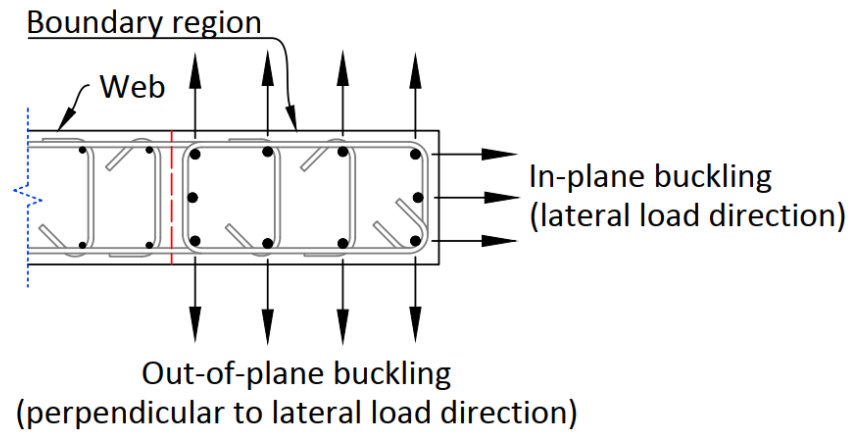


Figure 4.5. Buckling Directions Considered in the Wall Boundary Regions

Thus, for each wall, two values for the buckling length are computed based on the reinforcement configuration in the boundary region. One is for the bars on the short face of the boundary buckling in the in-plane direction, and the other is for the bars along the long faces of the boundary buckling in the out-of-plane direction.

The average value of the buckling lengths along the two principal directions is then used to compute an average slenderness ratio which is needed as an input for the proposed steel buckling models implemented in the FSAFE wall model. Since the mesh discretization of the wall model divides each boundary element into two meshes where four vertical bars are grouped into an equivalent bar for each mesh, inputting a unified average slenderness ratio for both meshes is a justified simplification, a more accurate approach being to use two slenderness ratios, the first is the averaged value for the exterior mesh while using the slenderness ratio for the bars that will buckle in the out-of-plane direction for the interior mesh.

Table 4.2 provides a summary of buckling length calculations for the wall specimens chosen in this study to validate the proposed steel buckling models.

Table 4.2. Summary of Buckling Length and Slenderness Ratio Calculation for Wall Specimens.

ID	Buckling Direction	f_y , MPa	Bar Diameter (D_b), mm	Flexural Rigidity (EI), N.mm ²	Tie Spacing (s), mm	Tie Area (A_t), mm ²	Tie Length (l_e), mm	Number of Bars n_t/n_b	Tie Stiffness (k_t), N/mm	Normalizing Stiffness (k), N/mm	Equiv. Stiffness (k_{eq})	Buckling Mode (n)	Buckling Length (L_{eff}), mm	Average Buckling Length, mm	Slenderness (L_{eff}/D_b)
RW2	In-plane	434	9.53	42.2*10 ⁶	76.2	17.80	167.26	1.00	21.3*10 ³	9.3*10 ³	2.2917	1	76.2	76.2	8.00
	Out-of-plane	434	9.53	42.2*10 ⁶	76.2	17.80	78.3	0.25	11.4*10 ³	9.3*10 ³	1.2238	1	76.2		
RW-A20-P10-S63	In-plane	477	19.05	706.0*10 ⁶	51	31.67	178.4	1.00	35.5*10 ³	518.4*10 ³	0.0685	5	255	229.1	12.02
	Out-of-plane	477	19.05	706.0*10 ⁶	67.7	31.67	100	0.38	23.8*10 ³	221.6*10 ³	0.1072	3	203.1		
WSH6	In-plane	576	12	122.1*10 ⁶	50	28.27	341	1.00	16.6*10 ³	95.2*10 ³	0.1742	2	100	100	8.33
	Out-of-plane	576	12	122.1*10 ⁶	50	20.43	102	0.50	20.0*10 ³	95.2*10 ³	0.2104	2	100		
TW2	In-plane (Web)	434	9.53	42.2*10 ⁶	32	17.80	204	1.00	17.4*10 ³	125.4*10 ³	0.1392	3	96	83.6	8.77
	Out-of-plane (Web)	434	8.89	31.9*10 ⁶	35.6	17.80	78.3	0.50	22.7*10 ³	69.0*10 ³	0.3296	2	71.2		

Notes:

- (i) Elastic modulus of steel reinforcement $E_s = 200000 \text{ MPa}$
- (ii) Some longitudinal bar and tie diameter values mentioned in the table are average values since some specimens use different values in their boundary regions.
- (iii) Some tie spacing values mentioned in the table are average values since some specimens use different spacings for hoops and ties respectively.
- (iv) Buckling modes are calculated based on k_{eq} values for lateral ties being eliminated in central L/2 (See Table 3.1).

4.3.2. WSH6

Specimen WSH6 (Dazio *et al.*, 2009) is a medium rise RC wall with an aspect ratio of 2.26, axial load ratio of $0.108A_g f_c'$, and low shear stress demand of $0.29\sqrt{f_c'}$.

The behavior of the wall during testing was dominated by flexure. Onset of buckling of corner bars at both wall ends was first observed at drift ratio of 1.7%. Shortly after that, a sudden drop in the lateral force was observed at 2% drift in the negative loading direction caused by an abrupt failure due to crushing of the confined concrete within the boundary region precipitated by fracture of confining hoops.

Numerical versus experimentally-measured lateral load-displacement responses are compared in Figure 4.6. Figure 4.6b shows that the fixed envelope model displays slightly better lateral load capacity predictions especially in the negative loading direction with respect to the original Menegotto-Pinto model results (Figure 4.6a). It is also able to reasonably capture the experimentally observed sudden drop in lateral load capacity at 2% negative drift where soft in-cycle degradation in the analysis results starts roughly at 1.75% drift followed by more abrupt degradation at 2% negative drift.

Likewise, the shifting envelope steel model succeeds in capturing the experimentally observed drop in lateral load capacity at negative 2% drift as seen in Figure 4.6c, therefore, the drift capacity prediction by both the fixed and shifting envelope models agrees reasonably well with experimental observation.

The response obtained when using the buckling-modified Menagatto-Pinto model as depicted in Figure 4.6d reveals that the lateral load capacity is slightly underestimated for most of the drift levels. The reason why this underestimation was not observed for the fixed and shifting envelope models is that they don't incorporate the Bauschinger effect which is normally responsible for cyclic strength and stiffness degradation, so their good estimation of the lateral load capacity was only a by-product of the models' inability to account for this effect. In a similar manner to the first two models, the buckling-modified Menegotto-Pinto model provides a good estimate of the drift capacity.

By comparison with the original M-P model (Figure 4.6a), the lateral load vs. top displacement responses obtained from analysis experience higher pinching (narrow width of lateral load-displacement loops) when using any of the three buckling models as opposed to the original M-P model, with the most severe pinching occurring in the fixed and shifting envelope models (Figure 4.6b,c,d). This behavior can be attributed in part to the relatively smaller area inside the hysteretic loops of the implemented steel stress-strain relations for the different buckling models as can be seen in Figures 4.7a-d, where a representative example of the stress-strain response of the vertical steel for one of the FSAFE wall model elements (the right corner base element) is presented. Moreover, the absence of the Bauschinger effect in the fixed and shifting envelope models makes the pinching response more pronounced.

The vertical tensile and compressive strain profiles along the wall boundaries corresponding to selected drift levels are presented in Figures 4.8a-d for different steel stress-strain model implementations.

It can be observed that all models overestimate the compressive strain values at the final drift levels where the overestimation is highest when using models that incorporate buckling induced compressive stress degradation. The buckling induced cyclic compressive strength degradation in the vertical steel imposes additional stress on the confined concrete, which triggers its early crushing and as a consequence, the vertical steel becomes the sole source for compression resistance, and experiences substantial compressive strains due to it reaching residual stress level.

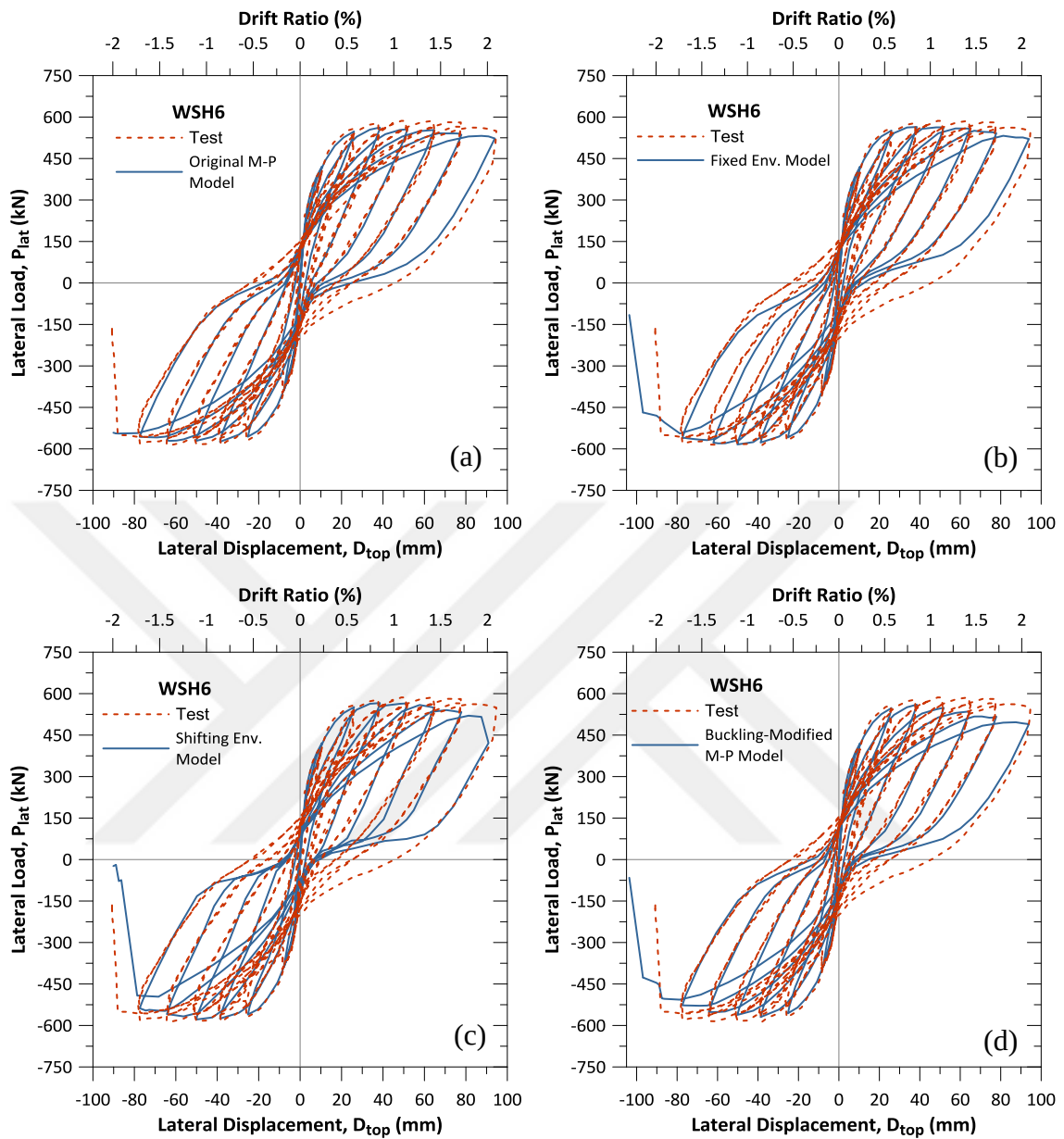


Figure 4.6. Lateral Load vs. Top Displacement Relations for Specimen WSH6: (a) Original M-P Model; (b) Fixed Envelope Model; (c) Shifting Envelope Model; (d) Buckling-Modified M-P Model.

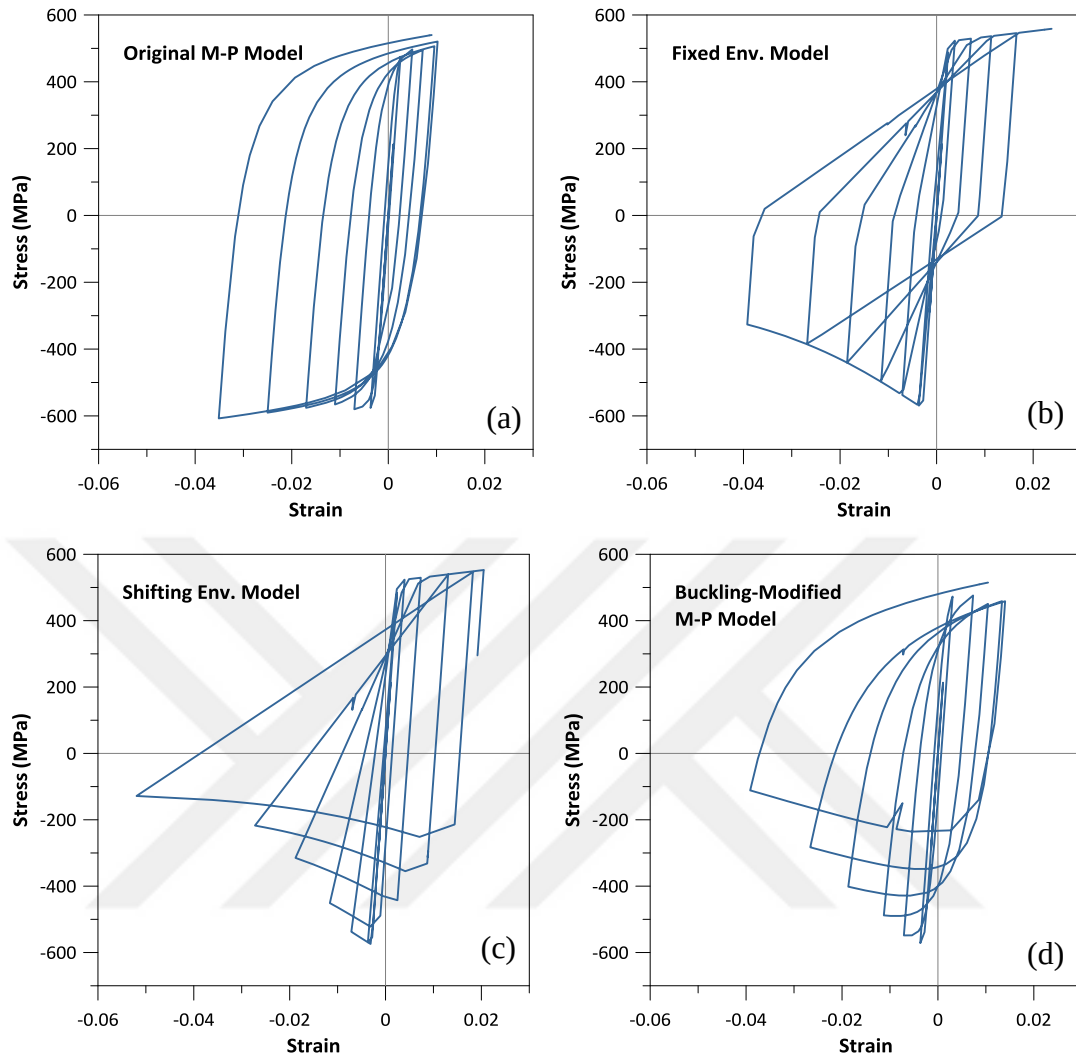


Figure 4.7. Steel Stress-Strain Histories for Right Corner Element at Wall Base for Specimen WSH6: (a) Original M-P Model; (b) Fixed Envelope Model; (c) Shifting Envelope Model; (d) Buckling-Modified M-P Model.

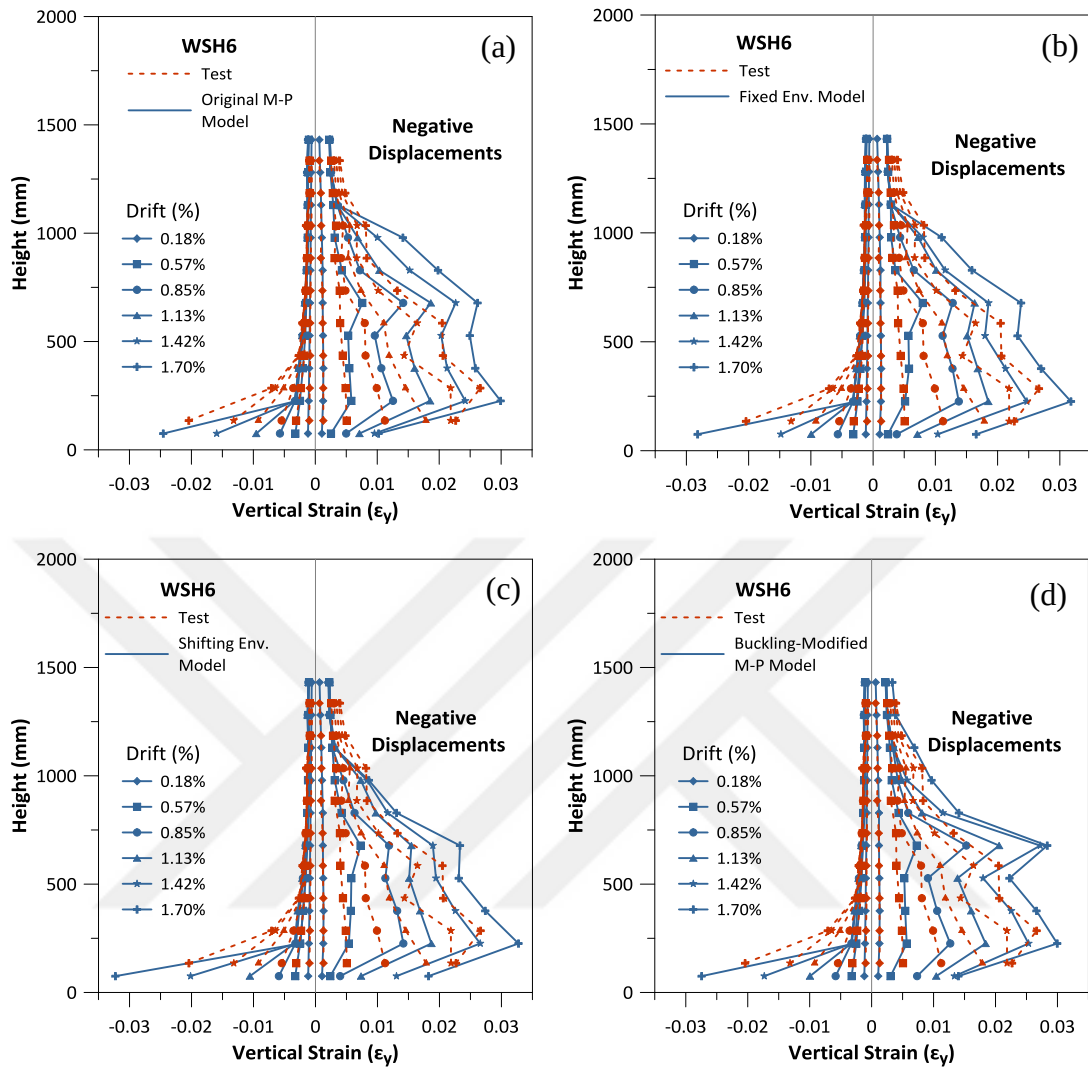


Figure 4.8. Vertical Normal Strain Profiles at Wall Boundaries for Specimen WSH6:
 (a) Original M-P Model; (b) Fixed Envelope Model; (c) Shifting Envelope Model;
 (d) Buckling-Modified M-P Model.

Table 4.3 provides a quantitative comparison of the peak lateral loads and drift capacities attained upon implementation of the proposed buckling steel models in the FSAFE wall model for specimen WSH6.

Table 4.3. Summary of Response Quantities for Specimen WSH6.

Type of Response	Peak Lateral Load (kN)		Drift Capacity (%)*	
	Positive Load	Negative Load	Positive Load	Negative Load
Experimental	585 kN @ 1.1% drift**	587 kN @ -1.4% drift	Undefined ***	-1.95%
Analytical – Original M-P Steel Model	560 kN @ 0.85% drift	-572 kN @ -0.85% drift	Undefined	Undefined
Analytical – Fixed Envelope Buckling Steel Model	565 kN @ 0.85% drift	-583 kN @ -1.1% drift	Undefined	-1.94%
Analytical – Shifting Envelope Buckling Steel Model	565 kN @ 0.85% drift	-583 kN @ -1.1% drift	1.97%	-1.55%
Analytical – Buckling-Modified M-P Steel Model	560 kN @ 0.85% drift	-570 kN @ -0.85% drift	Undefined	-1.94%

*The drift ratio at which the loss in lateral load first reached 15% or higher.

**The drift ratio at which peak lateral load was first reached.

*** The lateral load didn't drop below 15% of the peak load.

4.3.3. RW-A20-P10-S63

Specimen RW-A20-P10-S63 (Tran, 2012) is a medium rise RC wall specimen with an aspect ratio of 2, axial load ratio of $0.1A_g f_c'$, and shear demand of $0.57\sqrt{f_c'} MPa$.

The wall response during testing exhibited considerable shear-flexure interaction behavior. Concrete cover spalling at the wall boundaries was first observed at 1.5% and 2% lateral drift. During the second cycle to 3% drift in the negative direction, crushing of concrete and buckling of vertical reinforcement in the north boundary that extended to the web led to a significant drop in the lateral strength of the wall. It was also observed that this strength loss was accompanied by lateral instability of the boundary region.

Figure 4.9 compares the analytical responses of FSAFE wall model with different implemented steel models with the experimentally measured one. In comparison with the

finite element analysis results using Menegotto-Pinto model illustrated in Figure 4.9a, Figure 4.9b shows that the fixed envelope model overestimates the lateral load capacity for most of the drift levels prior to strength loss. The lateral load overestimation is greatest in the initial loading cycles, whereas the M-P model displays only a slight overestimation. The fixed envelope model's prediction of drift capacity in the negative loading direction is nearly the same as that of the M-P model (around 2%) which is in agreement with experimentally measured value. The fixed envelope model fails in capturing any abrupt strength degradation, thus providing no advantage over the original M-P model.

The shifting envelope steel model results displayed drastic strength degradation, which was an absent feature in the fixed envelope model and therefore succeeded in replicating the experimentally-observed abrupt lateral strength loss encountered during the second cycle to 3% drift, albeit at an earlier drift level. This premature strength loss leads to an even greater underestimation of the drift capacity with respect to the original M-P and fixed envelope models.

Implementing the buckling-modified Menegotto-Pinto model leads to underestimation of the lateral load capacity for most of the drift levels (Figure 4.9d), in contrast to the fixed and shifting envelope models. This discrepancy stems from the absence of the Bauschinger effect in the formulation of the first two models as was explained previously in the discussion of specimen WSH6. The lateral drift capacity estimation is comparable to the one estimated by the original M-P model around 2% lateral drift. The substantial loss in lateral strength attributed to lateral instability/longitudinal reinforcement buckling was captured reasonably well using the buckling-modified M-P model, albeit at a slightly smaller drift level than the experimentally observed one (2%).

By comparison with the original M-P model (Figure 4.9a), the lateral load versus top displacement responses obtained from analysis experience higher overall pinching when using any of the three buckling models, with the most severe pinching occurring in the shifting envelope model (Figures 4.9b,c,d). This behavior can be attributed in part to the relatively smaller area inside the hysteretic loops of the steel stress-strain relations for the different buckling models. See Figures 4.10a-d.

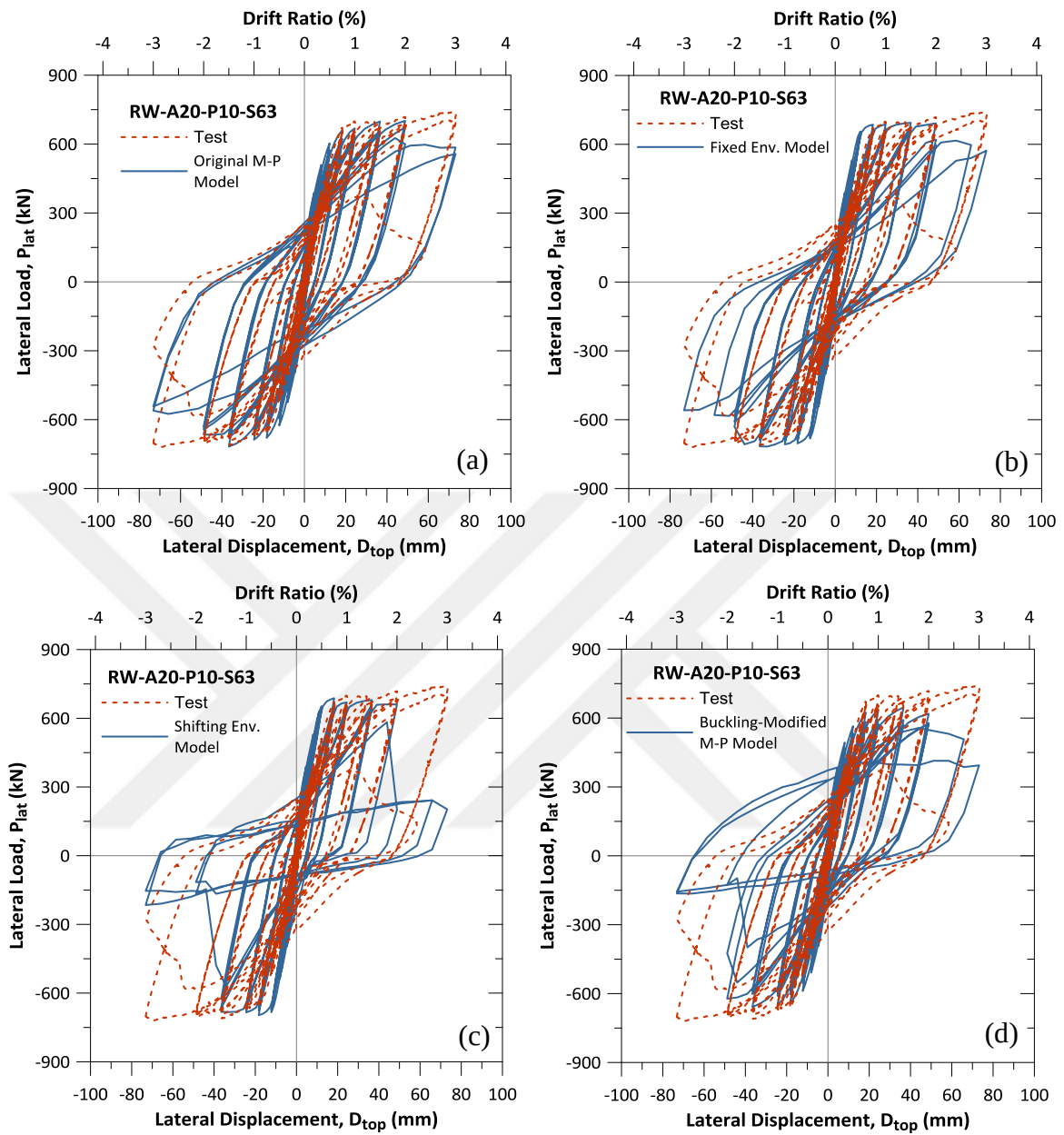


Figure 4.9. Lateral Load vs. Top Displacement Relations for Specimen RW-A20-P10-S63:
 (a) Original M-P Model; (b) Fixed Envelope Model; (c) Shifting Envelope Model;
 (d) Buckling-Modified M-P Model.

Figures 4.10a-f illustrate a representative example of the stress-strain response of the vertical steel for one of the FSAFE wall model elements (the right corner base element) obtained when using the three steel model formulations investigated in this study. To facilitate visual clarity, the stress-strain responses for the shifting and buckling-modified buckling models have been trimmed before the end of the loading history and represented in

Figures 4.10e,f due to the unrealistically large strain values reached by the buckling steel, so that only the physically attainable response is shown for these two steel models.

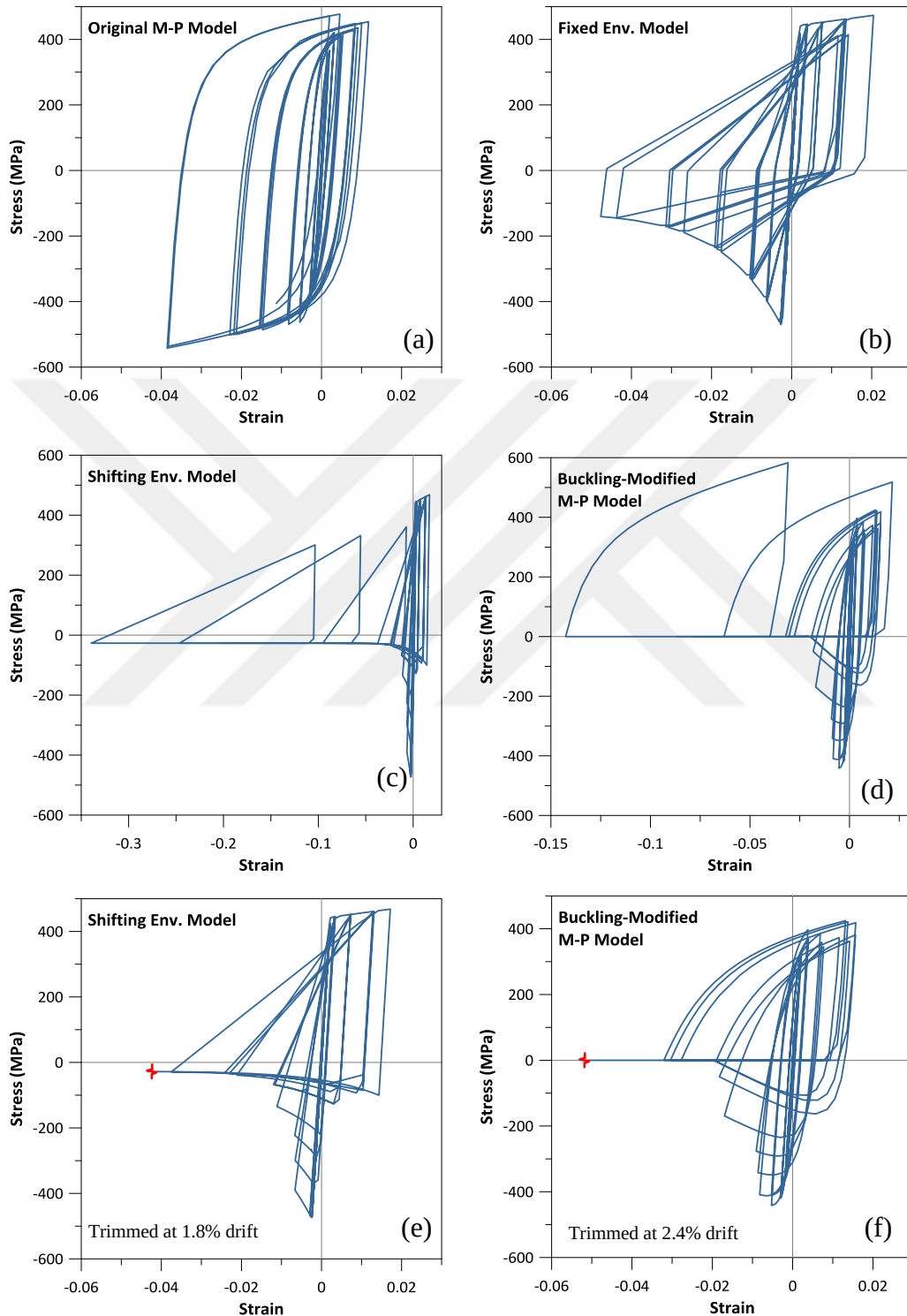


Figure 4.10. Steel Stress-Strain Histories for the Right Corner Element at Wall Base for Specimen RW-A20-P10-S63: (a) Original M-P Model; (b) Fixed Envelope Model; (c) Shifting Envelope Model; (d) Buckling-Modified M-P Model; (e) Shifting Envelope Model (Trimmed); (f) Buckling-Modified M-P Model (Trimmed).

Vertical normal strain profiles along the base of the wall for different steel models are compared with the experimentally-measured values and presented in Figures 4.11a-d. The analytical predictions of the compressive strains provided by both the original M-P model and the buckling models are in good agreement with experimental values except for the shifting envelope model, which overestimates the compressive strains at 2% and 1.5% positive drifts.

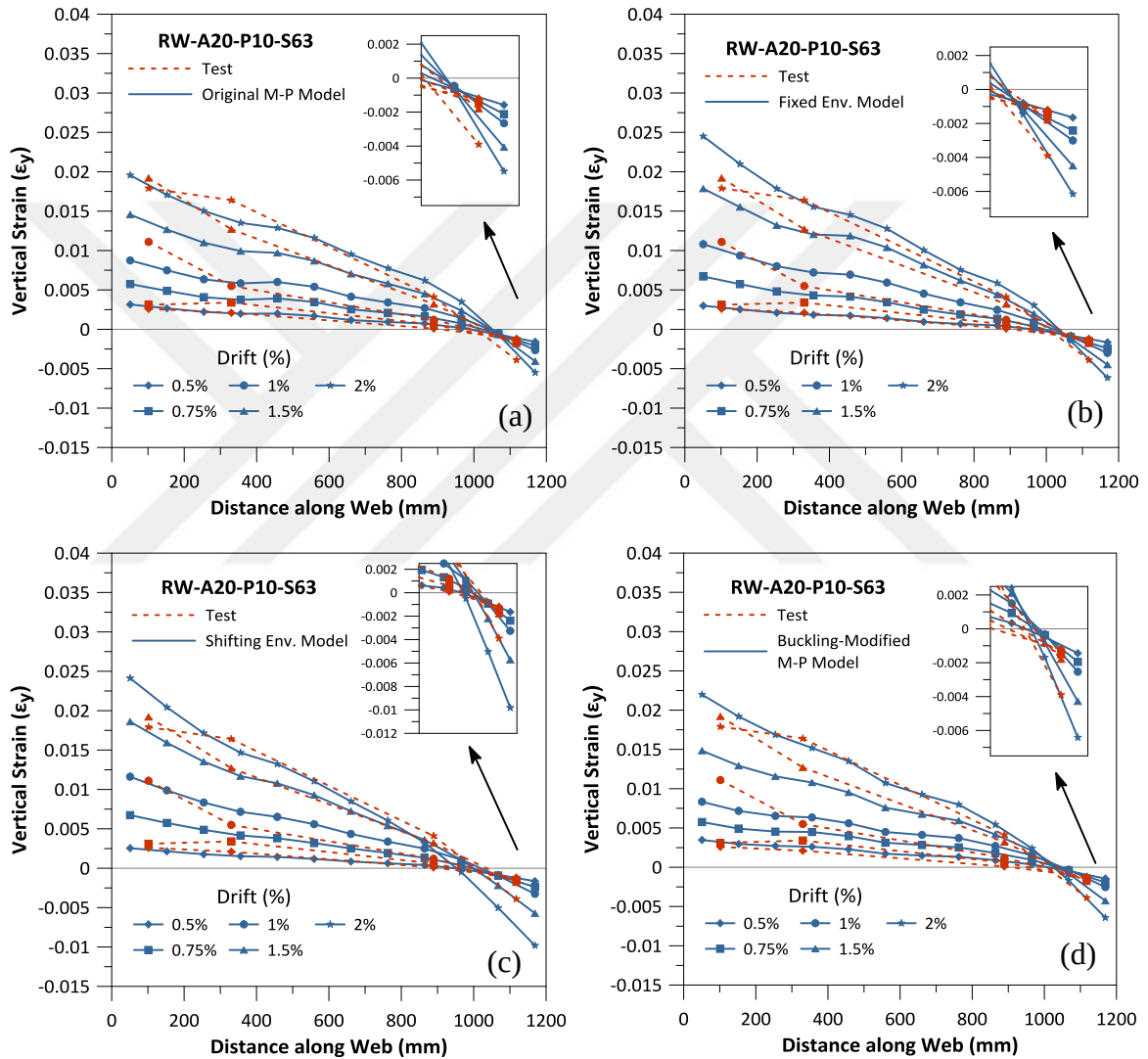


Figure 4.11. Vertical Normal Strain Profiles at Wall Base for Specimen RW-A20-P10-S63:
 (a) Original M-P Model; (b) Fixed Envelope Model; (c) Shifting Envelope Model;
 (d) Buckling-Modified M-P Model.

The extreme compressive strain values reached by the vertical steel when implementing any of the buckling steel models can be explained by the early deterioration of the confined concrete due to being overstressed as the buckling steel takes on smaller and

smaller compressive stresses as the loading progresses. Eventually, the concrete portion of the finite elements located at the bottom of the boundary regions crushes and is discarded from the analysis. As a consequence, steel is left exposed to excessive compressive strains which corresponds to negligible stress values (residual stress). Moreover, an abrupt drop in lateral load capacity in the analytical lateral force-displacement response usually occurs as a result to the loss of the concrete contribution in load resistance as can be observed in Figures 4.9c,d. It should be noted that in the case of using either shifting envelope model or buckling modified M-P model, the compressive strains reach as high as 30% (Figures 4.10c,d), which is physically nonsensical and is only due to limitations inherent in the buckling steel models.

Table 4.4 provides a quantitative comparison of the peak lateral loads and drift capacities attained upon implementation of the proposed buckling steel models in the FSAFE wall model for specimen RW-A20-P10-S63.

Table 4.4. Summary of Response Quantities for Specimen RW-A20-P10-S63.

Type of Response	Peak Lateral Strength (kN)		Drift Capacity (%)*	
	Positive Load	Negative Load	Positive Load	Negative Load
Experimental	742 kN @ 2.8% drift**	-717 kN @ -2.8% drift	Undefined***	-2.2%
Analytical – Original M-P Steel Model	700 kN @ 2% drift	-718 kN @ -1.5% drift	2%	-2%
Analytical – Fixed Envelope Buckling Steel Model	693 kN @ 1.5% drift	-784 kN @ -1.5%	2.7%	-1.9%
Analytical – Shifting Envelope Buckling Steel Model	688 kN @ 0.75% drift	-700 kN @ -0.75% drift	1.8%	-1.4%
Analytical – Buckling-Modified M-P Steel Model	645 kN @ 1.5% drift	-660 kN @ -1.5% drift	2.05%	-1.8%

*The drift ratio at which the loss in lateral load first reached 15% or higher.

**The drift ratio at which peak lateral load was first reached.

*** The lateral load didn't drop below 15% of the peak load.

4.3.4. TW2

Specimen TW2 (Thomsen and Wallace 1995) is a relatively slender RC wall with a T-shaped cross section, aspect ratio of 3, axial load ratio of $0.075A_g f_c'$ and a shear stress demand of $0.34\sqrt{f_c'} \text{ MPa}$ at peak lateral load. The lateral load is applied parallel to the web.

Failure was initiated by spalling of cover concrete in the web boundary element at progressive drift levels, which ultimately resulted in prominent crushing of the confined core in the web boundary element during the third cycle to 2.5% nominal drift followed by out of plane instability during the first cycle to 3% nominal drift. Global buckling of the longitudinal steel in the web boundary over a large number of hoop spacings was observed to have taken place at the end of the test.

In comparison with the Menegotto-Pinto model implementation results presented in Figure 4.12a, Figures 4.12b,c show that both the fixed and shifting envelope models display better lateral load capacity predictions in the positive loading direction (when the wall flange is in compression) while considerably overestimating it in the negative loading direction (when the wall flange is in tension), giving worse predictions than the original M-P model. However, using any of the two fixed and shifting envelope steel models result in noticeable degradation in the analytical response, which manifests itself as in-cycle degradation during the first cycle to 2% negative nominal drift after which the lateral load capacity drops sharply approaching zero during the first cycle to 2.5% negative nominal drift.

Using the buckling-modified M-P model (Figure 4.12d) improves on the lateral load predictions when the wall flange is in tension and under-predicts the lateral load capacity when the wall flange is in compression. By comparison with the fixed and shifting envelope steel models, this reduction in lateral load predictions is due to the presence of the Bauschinger effect in its formulation, which leads to cyclic strength and stiffness degradation in the overall analytical response. Therefore, including the Bauschinger effect mitigates the capacity overestimation issue in the negative loading direction encountered with the fixed and shifting envelope models.

The buckling-modified M-P model seems to be able to roughly capture the loss in strength observed in the test during loading to 2.5% negative nominal drift when the web began to experience out-of-plane deformation (Figure 4.12d), even though in reality the strength loss associated with out-of-plane deformations is influenced by the interaction of complex phenomena affecting both the concrete and reinforcing steel. Non-uniform crack closure caused by large residual crack widths, unsymmetrical concrete cover spalling and small dislocated concrete particles among other factors, leads to distortions in longitudinal bars which further weakens the already-compromised in-plane flexural strength of the wall section region under compression (Paullay and Priestley, 1993).

The lateral load vs. top displacement responses obtained analytically experience higher pinching in the positive loading direction (when wall flange is in compression) when using any of the three buckling models (Figures 4.12b,c,d) in contrast to the original M-P model, which agrees well with the experimentally measured response (Figure 4.12a). This is due to the inherent limitations of the three buckling models implemented in the flange region which cannot predict the onset of reinforcement buckling and will always result in compressive stress degradation as soon as the steel yields in compression even though the flange reinforcement didn't experience any visible buckling during the experiment. This also partially explain the under-predation of the lateral load capacity when the wall flange is under compression for the buckling-modified M-P model. However, this shortcoming can be deemed acceptable for rectangular walls where the chances of rebar buckling are equal for their two confined boundary regions.

Figures 4.13a-d illustrate a representative example of the stress-strain response of the vertical steel for one of the FSAFE wall model elements (the first element from the free end at the base of the wall web) obtained when implementing the three steel model formulations investigated in this study. To facilitate visual clarity, the stress-strain responses for all three buckling models have been trimmed before the end of the loading history due to the unrealistically large strain values reached by the buckling steel, so that only the physically attainable response is shown for the three steel models.

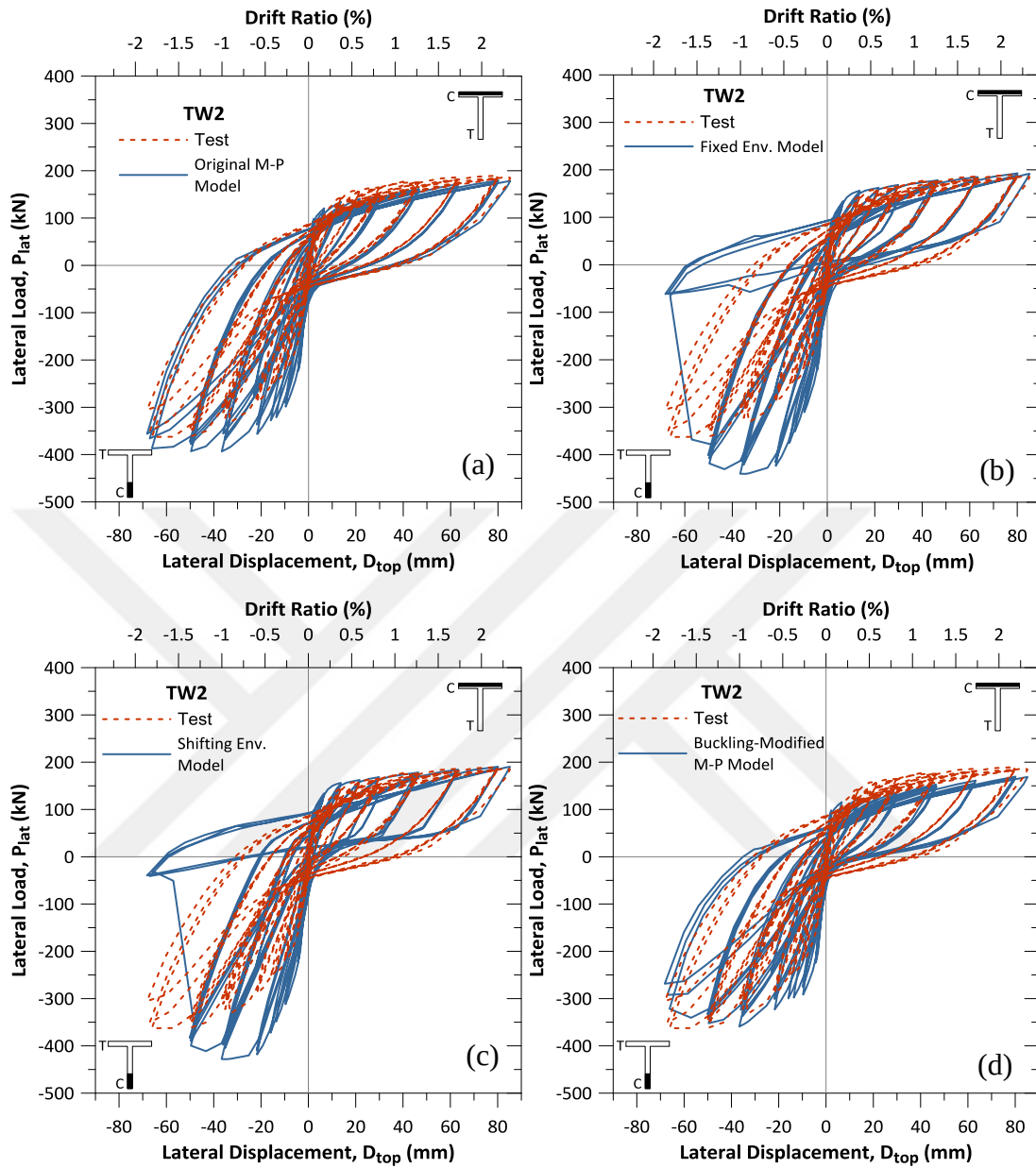


Figure 4.12. Lateral Load vs. Top Displacement Relations for Specimen TW2 (Top Displacement with Pedestal Movement Contribution Subtracted): (a) Original M-P Model; (b) Fixed Envelope Model; (c) Shifting Envelope Model; (d) Buckling-Modified M-P Model.

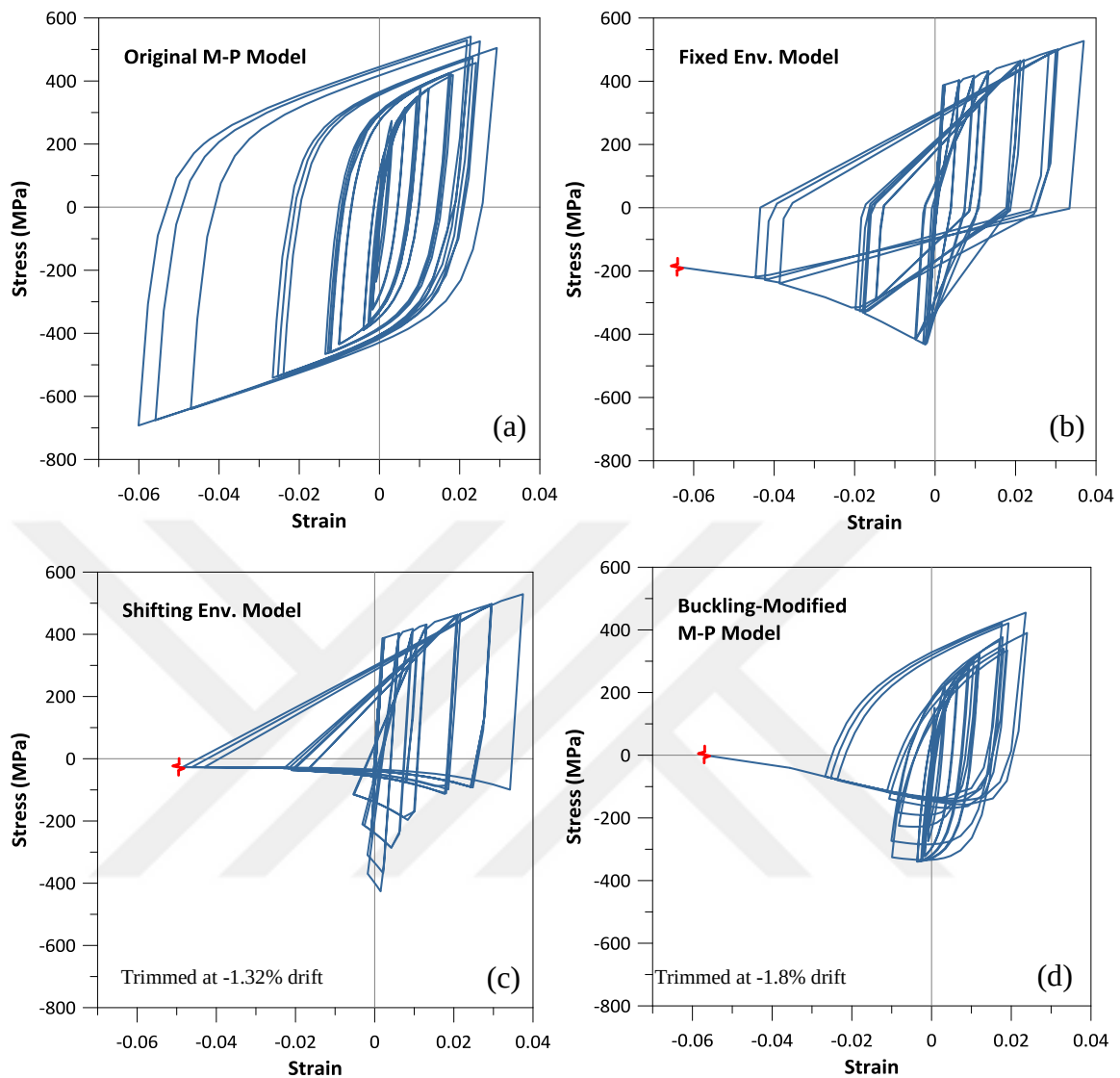


Figure 4.13. Steel Stress-Strain Histories for the First Bottom Element at the Free End of the Wall Web for Specimen TW2: (a) Original M-P Model; (b) Fixed Envelope Model; (c) Shifting Envelope Model (Trimmed); (d) Buckling-Modified M-P Model (Trimmed).

Vertical strain profiles along the base of the wall web at negative nominal drifts (wall web is under compression) for different implemented steel models are compared with the experimentally-measured values and presented in Figures 4.14a-d. The compressive strain estimation offered by the fixed and shifting envelope models at 1.5% drift agree well with the experimental values. In a similar way to the original M-P model, the buckling modified M-P model greatly underestimates the experimentally measured compressive strain values at all drift levels.

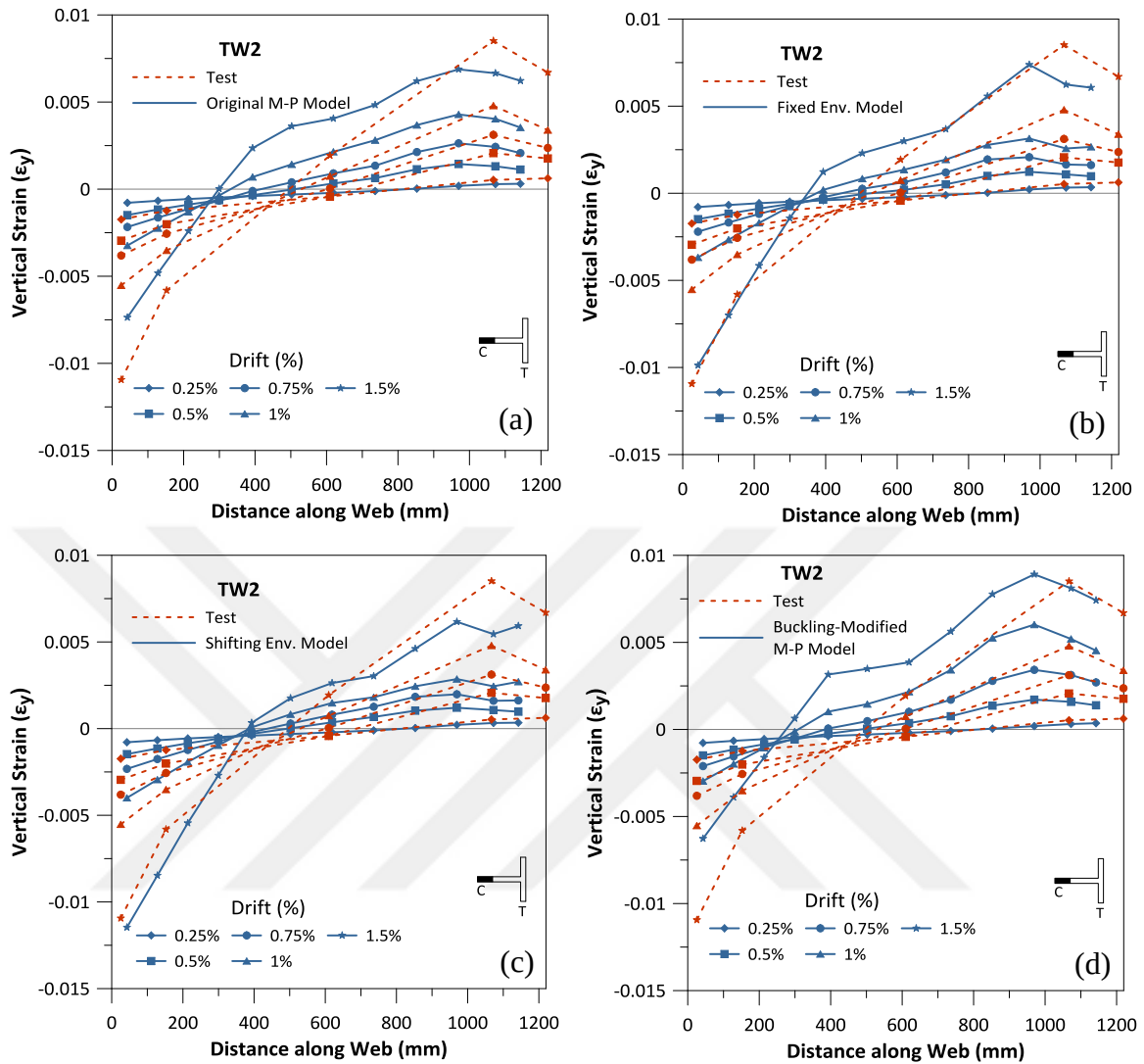


Figure 4.14. Vertical Normal Strain Profiles for Selected Nominal (Unreduced) Drift Levels at the Base of the Wall Web Under Negative Displacements (Wall Flange Is Under Tension) for Specimen TW2: (a) Original M-P Model; (b) Fixed Envelope Model; (c) Shifting Envelope Model; (d) Buckling-Modified M-P Model.

The reason behind the unrealistically large compressive strains attained by the steel when using the buckling models is the occurrence of early deterioration of the confined concrete due to its increased share of stress as the buckling steel takes on smaller and smaller compressive stresses with continuing load as was explained in the previous specimens.

Table 4.5 provides a quantitative comparison of the peak lateral loads and drift capacities attained upon implementation of the proposed buckling steel models in the FSAFE wall model for specimen TW2.

Table 4.5. Summary of Response Quantities for Specimen TW2.

Type of Response	Peak Lateral Load (kN)		Drift Capacity (%) *	
	Positive Load	Negative Load	Positive Load	Negative Load
Experimental	190 kN @ 2.2% drift**	-363 kN @ -1.56%	Undefined***	-1.87%
Analytical – Original M-P Steel Model	180 kN @ 2.2% drift	-393 kN @ -1% drift	Undefined	Undefined
Analytical – Fixed Envelope Buckling Steel Model	192 kN @ 2.2% drift	-440 kN @ -1% drift	Undefined	-1.34%
Analytical – Shifting Envelope Buckling Steel Model	190 kN @ 2.2% drift	-428 kN @ -1% drift	Undefined	-1.33%
Analytical – Buckling-Modified M-P Steel Model	170 kN @ 2.2% drift	-360 kN @ -1% drift	Undefined	-1.8%

*The drift ratio at which the loss in lateral load first reached 15% or higher.

**The drift ratio at which peak lateral load was first reached.

*** The lateral load didn't drop below 15% of the peak load.

4.3.5. RW2

Specimen RW2 (Thomsen and Wallace 1995) is a relatively slender RC wall with an aspect ratio of 3 similar to TW2, and a shear stress demand of $0.19\sqrt{f'_c} MPa$, which is the least among the selected wall specimens.

Reported test observations indicate that the wall was on the verge of failure when the test was stopped after two cycles at 2.5% lateral drift due to crushing of the concrete within the confined core and buckling of longitudinal bars in the boundaries, which initiated on the first cycle to 2.5% drift. Overall, the specimen did not experience a significant loss of

strength up until the stopping of the test due to the late onset of buckling of longitudinal reinforcement attributed to the tight spacing of the transverse boundary reinforcement.

Figure 4.15 displays the lateral load versus top displacement response predictions using different steel constitutive relations and compares them with experimentally measured responses. Implementing either the fixed or shifting envelope models significantly overestimates the yield capacity (Figures 4.15a,b) while the shifting envelope model leads to significant degradation during the cycles to 2.5% nominal drift in both the positive and negative loading directions.

The buckling-modified Menegotto-Pinto model (Figure 4.15d) does not significantly improve the response prediction over the original Menegotto-Pinto model, as it underestimates the load capacity across all drift levels and fails to capture the experimentally observed degradation in lateral load during the second loading cycle to 2.5% drift in the positive loading direction, which occurred due to initiation of buckling of longitudinal reinforcement in the wall boundary region.

Figure 4.16 presents an illustrative example of the stress-strain responses exhibited by the bottommost mesh element of the south boundary region. As can be seen in Figure 4.16b,c, the absence of the Bauschinger effect in the fixed and shifting envelope models accounts for the lateral load overestimation encountered in the corresponding global lateral load–displacement responses (Figures 4.15b,c).

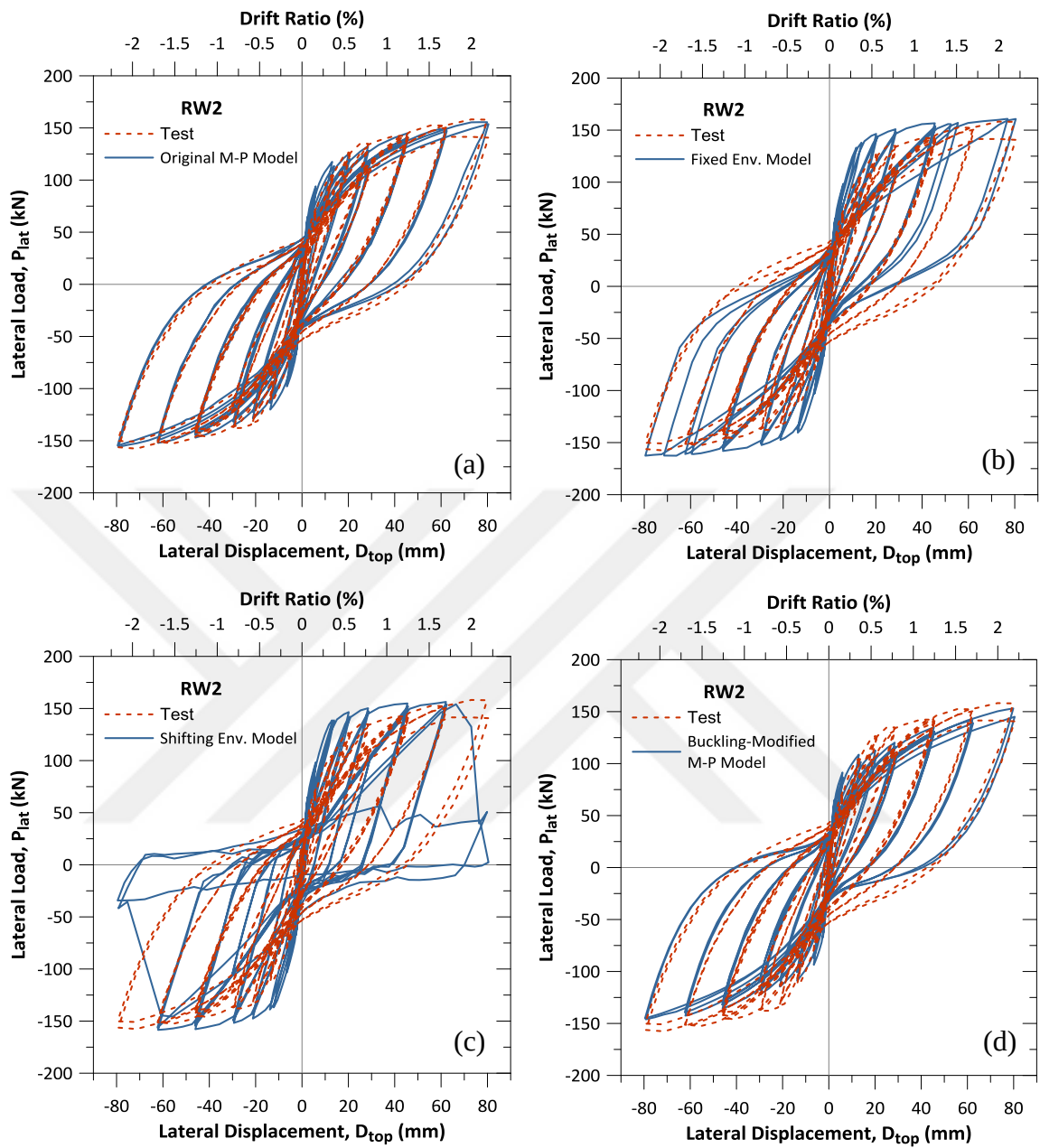


Figure 4.15. Lateral Load vs. Top Displacement Relations for Specimen RW2 (Top Displacement with Pedestal Movement Contribution Subtracted): (a) Original M-P Model; (b) Fixed Envelope Model; (c) Shifting Envelope Model; (d) Buckling-Modified M-P Model.

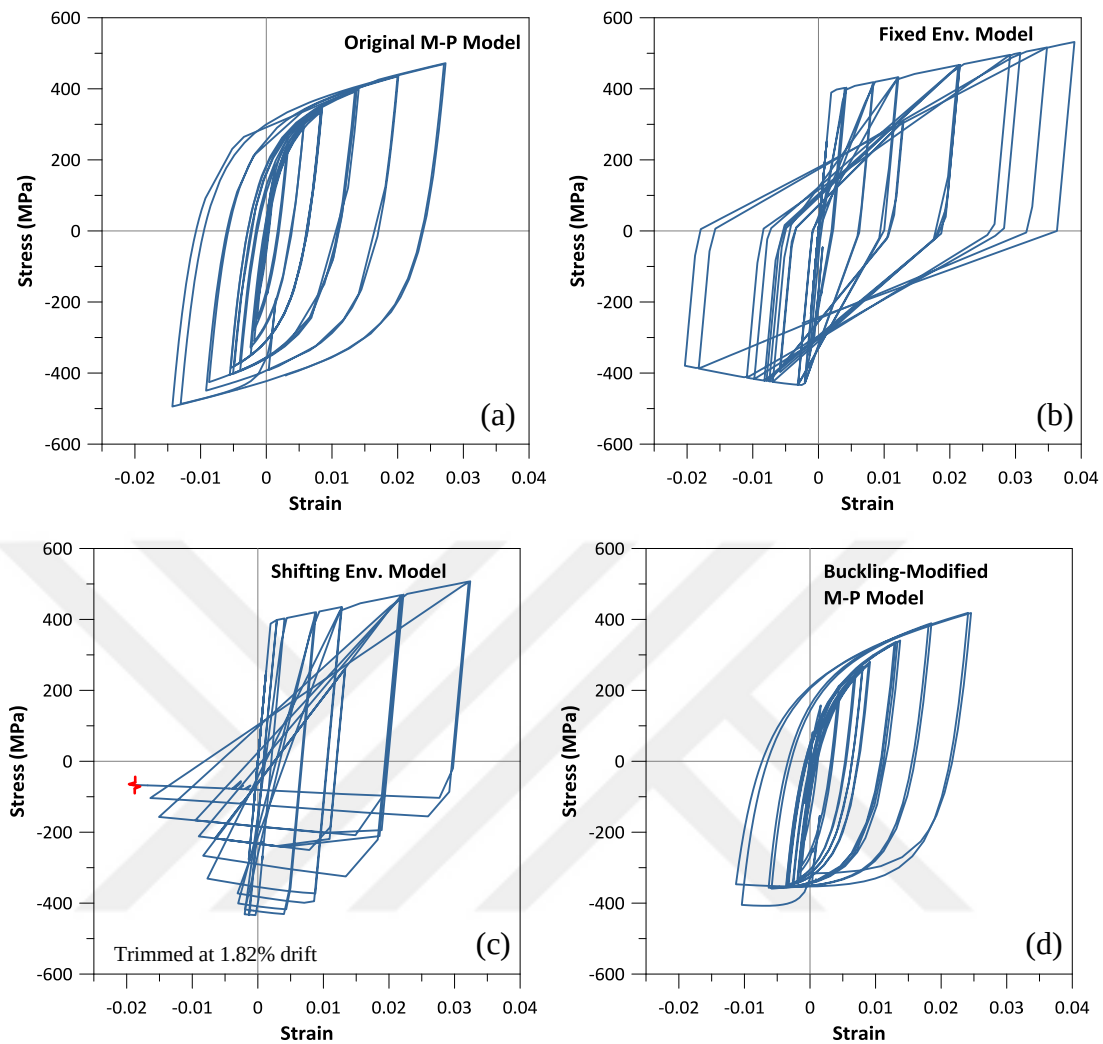


Figure 4.16. Steel Stress-Strain Histories for the Right Corner Element at Wall Base for Specimen RW2: (a) Original M-P Model; (b) Fixed Envelope Model; (c) Shifting Envelope Model (Trimmed); (d) Buckling-Modified M-P Model.

Figure 4.17 displays the concrete strain measurements of Specimen RW2 at the base in comparison with the model predictions at the positive peak of some selected drift cycles applied during the test. The analysis results using the fixed and shifting envelope models are in better agreement with the test measurements than the original M-P model predictions in terms of compressive strains at the final drift levels. However, due to the premature crushing of confined concrete when implementing the shifting envelope model, the compressive strain estimation at 2.5% nominal drift is unrealistically large (Figure 4.17c). In contrast, the buckling-modified M-P model yields an almost identical vertical stain profile with the original M-P model (Figure 4.17d).

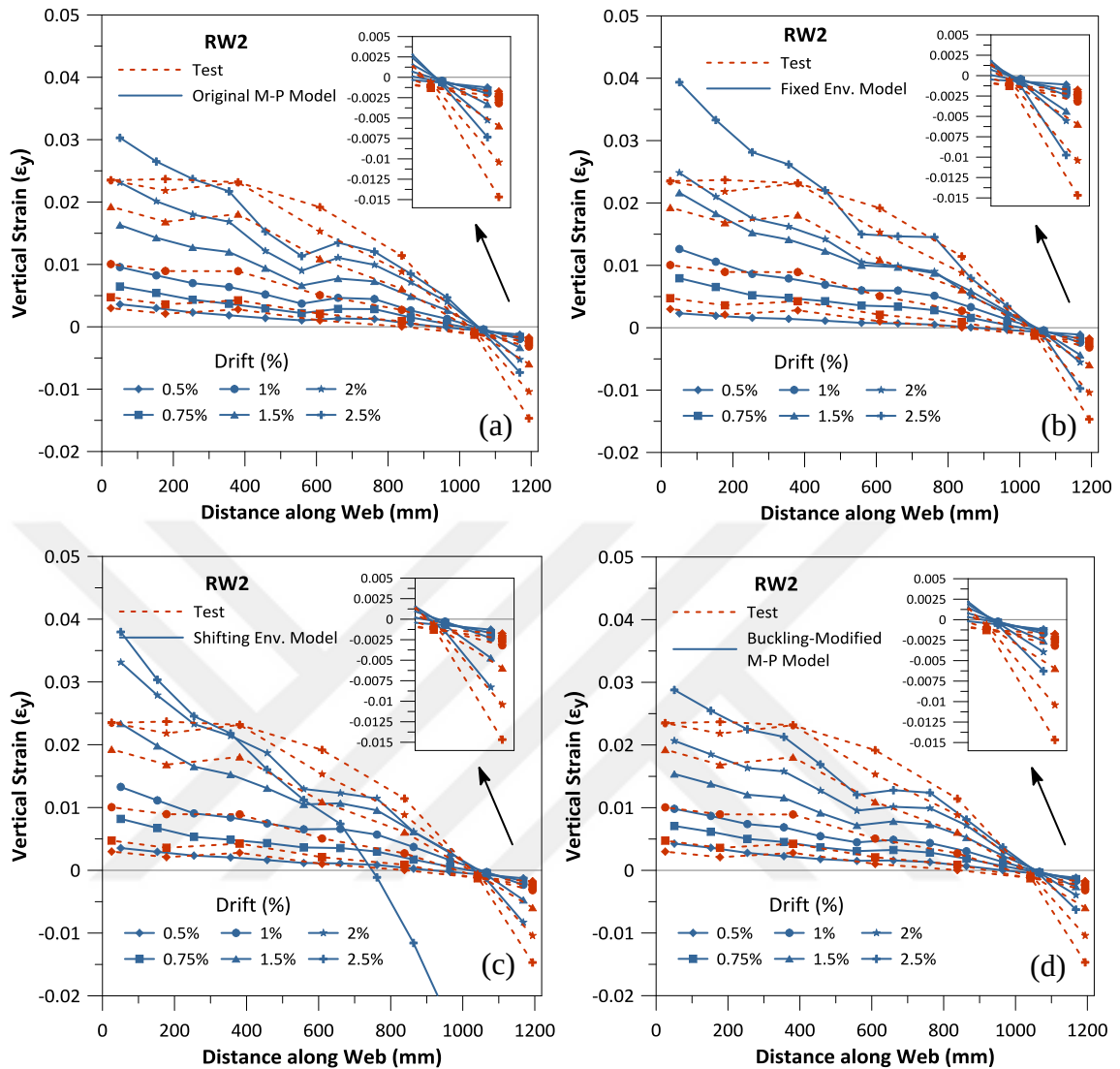


Figure 4.17. Vertical Normal Strain Profiles at Wall Base for Selected Nominal (Unreduced) Drift Levels for Specimen RW2: (a) Original M-P Model; (b) Fixed Envelope Model; (c) Shifting Envelope Model; (d) Buckling-Modified M-P Model.

Table 4.6 provides a quantitative comparison of the peak lateral loads and drift capacities attained upon implementation of the proposed buckling steel models in the FSAFE wall model for specimen RW2.

Table 4.6. Summary of Response Quantities for Specimen RW2.

Type of Response	Peak Lateral Load (kN)		Drift Capacity (%)*	
	Positive Load	Negative Load	Positive Load	Negative Load
Experimental	159 kN @ 2% drift**	-158 kN @ -1.95% drift	Undefined***	Undefined
Analytical – Original M-P Steel Model	166 kN @ 2% drift	-156 kN @ -2.17% drift	Undefined	Undefined
Analytical – Fixed Envelope Buckling Steel Model	160 kN @ 2.1% drift	-163 kN @ -1.8% drift	Undefined	Undefined
Analytical – Shifting Envelope Buckling Steel Model	156 kN @ 1.7% drift	-158 kN @ -1.7% drift	2%	-1.7%
Analytical – Buckling-Modified M-P Steel Model	153 kN @ 2.2% drift	-146 kN @ -2.17% drift	Undefined	Undefined

*The drift ratio at which the loss in lateral load first reached 15% or higher.

**The drift ratio at which peak lateral load was first reached.

*** The lateral load didn't drop below 15% of the peak load.

5. SUMMARY AND CONCLUSIONS

This study made use of the hysteretic reinforcing steel models available in the literature (e.g., Kashani *et al.*, 2015a, 2016, Menegotto and Pinto, 1973) to construct and propose three different buckling models: the fixed envelope model, the shifting envelope model and the buckling-modified Menegotto-Pinto model. These models were then implemented into the FSAFE finite element wall model (Gullu and Orakcal, 2019) in order to simulate the reinforcement buckling reported in four wall specimens tested under reversed cyclic loading. The effect of the proposed steel buckling models on the global and local responses for the selected wall specimens was examined in detail.

It was found that the fixed envelope and buckling-modified M-P models tend to provide better estimates of the wall drift capacities in comparison to the shifting envelope model, which almost always underestimates drift capacities. Moreover, all three buckling models were able to capture the abrupt losses in lateral load capacities encountered in the experimental lateral load-displacement responses, albeit occasionally at earlier drift levels compared to the experimentally-measured responses. The abrupt drop in lateral load encountered in the experimental response for specimen WSH6 was reasonably captured by all three buckling models.

It was also observed that the yield capacity and initial stiffness in the lateral load-displacement responses were being overestimated when implementing either the fixed or shifting envelope models, which is evident in specimens RW-A20-P10-S63 and TW2. This shortcoming is attributed to the absence of the Bauschinger effect in these two models' formulations. On the other hand, the buckling-modified M-P model, which incorporate the Bauschinger effect, underestimates the yield capacity and initial stiffness in addition to underestimating the lateral load capacities for the drift levels prior to lateral strength loss, as can be clearly seen in the analytical response of specimen RW-A20-P10-S63.

As a general trend, it has been observed during the analyses conducted in this study that the three buckling models tend to underestimate wall drift capacities and provide predictions of lateral strength degradation at smaller drift levels than the experimentally

measured values. This shortcoming is encountered whether or not pinching was incorporated into the cyclic rules of the buckling steel models; where the fixed and shifting envelope models considered pinching in the unloading/reloading rules while the buckling-modified M-P model didn't consider it. Since the buckling envelope implemented into all three buckling models is based on bare bar simulations (Kashani *et al.*, 2015a), the aforementioned observation confirms the fact that the lateral restraint of the horizontal steel influences both the cyclic rules and the monotonic and cyclic compression envelopes. This comes in contrast to Kashani *et al.*'s hypothesis (2016), in which they assumed that the lateral tie restraint will only lessen the severity of pinching encountered in the hysteretic response of bare bars.

Overall, the observations made when implementing the steel buckling models proposed in this study can be summarized into the following key points:

- (i) In a general sense, using any steel buckling model formulation that accounts for the Bauschinger effect would most likely result in underestimation of the lateral load predictions. So there's some uncertainty in how the Bauschinger effect interacts with buckling of steel embedded in concrete and restrained by lateral ties, where using it in its current formulation underestimates the capacity and discarding it overestimate it.
- (ii) For the steel buckling models considered, their implementation almost always results in over-pronounced pinching behavior in the global lateral load-displacement responses of the wall specimens, characterized by narrow width of the hysteretic loops and early degradation of the unloading/reloading stiffness, which is a direct consequence of the small area inside the hysteretic loops of the implemented steel buckling models. Therefore, the energy dissipation capacity for the walls will be underestimated upon implementation of the proposed hysteretic models.
- (iii) Using the buckling models leads to early crushing of the concrete portion of the FSAFE model elements, which manifests itself as degradation in the global load-displacement response. This is the sought-after analytical response that leads to improving the FSAFE wall model predictions for the drift capacity of walls. Crushing of the concrete portion of the FSAFE model elements also manifests as localized spikes in compressive strains at the wall base, where maximum moment occurs.

The limitations inherent in the proposed steel buckling models are responsible for the observed shortcomings in the analytical responses for the investigated wall specimens, and they include the following:

- (i) The compression envelope used in the three models is derived from FE simulations of isolated reinforcing bars, which consequently contributes to the overestimation of the buckling-induced lateral load degradation in the overall wall responses due to overstressing the concrete portion of the FSAFE model and triggering its failure.
- (ii) The nature of the cyclic characteristics of the average stress-strain response of buckling reinforcing bars embedded in concrete and restrained by lateral ties remains largely unknown in terms of the degree of pinching involved and the degree of cyclic curvature degradation, i.e., the Bauschinger effect, which consequently contributes to overestimation of the pinching behavior encountered in the analytical lateral load-displacement responses for the walls.
- (iii) The least obvious and most critical reason behind the aforementioned shortcomings, i.e., lateral load underestimation/overestimation, pinching behavior and premature concrete crushing has to do with the inability of the proposed steel buckling models to predict the onset of bar buckling, in order to facilitate a typical buckling-free response prior to the buckling onset. This drawback will always lead to the amplification of any misrepresentations present in the envelopes/cyclic rules of the buckling models because buckling is set to always take place at a fixed trigger, i.e., yield in compression for the proposed buckling models.
- (iv) Finally, limitations inherent in the implementation scheme of the buckling models into the FSAFE wall model, namely the grouping of bars at the center of a mesh element resulting from coarse mesh discretization in the FSAFE wall models, will more or less affect the overall accuracy of the response predictions to some extent.

Overall, despite the aforementioned complexities; implementation of the reinforcing steel buckling models developed in this study has been observed to improve the response predictions of the FSAFE model in terms of capturing wall drift capacity and post-peak

lateral strength degradation. Furthermore, based on the model verifications conducted in this study and taking into account all the positive and negative aspects of the investigated buckling models, the so-called buckling-modified M-P model is found to yield the most consistent results among them, which makes it the most reliable candidate for implementation into nonlinear fiber-section and FE analyses for the performance-based assessment and design of reinforced concrete structural members.

Finally, in order to improve the accuracy in modeling of reinforcement buckling on the average stress-strain response level, it is recommended to conduct numerical simulations to study the interaction between longitudinal reinforcement undergoing buckling and the lateral restraint provided by the ties with properly defined boundary conditions using finite element modelling techniques in order to quantify this effect and convert it into a well-defined formulation for the compression envelope and the cyclic rules to introduce a sophisticated average stress-strain hysteretic model. At the same time, a sufficiently accurate prediction scheme needs to be incorporated into such a model formulation in order to predict the onset of reinforcing rebar buckling, taking into consideration the characteristics of the loading history and its impact on the strain levels experienced by the longitudinal reinforcing steel, transverse steel and core and cover concrete.

REFERENCES

- Belarbi, A., & Hsu, T. T. C. (1994). “Constitutive laws of concrete in tension and reinforcing bars stiffened by concrete”, *ACI Structural Journal*, 91(4), pp. 465–474.
- Bleich, F. (1952). *Buckling strength of metal structures*, McGraw-Hill, New York.
- Bresler B., & Gilbert P. H. (1961). “Tie Requirements for Reinforced Concrete Columns”, *ACI Journal Proceedings*, 58(11), pp. 555-570.
- Chang, G. A., & Mander, J. B. (1994). *Seismic energy based fatigue damage analysis of bridge columns : part 1 - evaluation of seismic capacity*, NCEER Technical Report No. NCEER-94-0006, National Center for Earthquake Engineering Research, State University of New York at Buffalo, NY, USA.
- Dazio, A., Beyer, K., & Bachmann, H. (2009). “Quasi-static cyclic tests and plastic hinge analysis of RC structural walls”, *Engineering Structures*, 31(7), pp. 1556–1571.
- Dhakal, R. P., & Maekawa, K. (2002b). “Reinforcement Stability and Fracture of Cover Concrete in Reinforced Concrete Members”, *Journal of Structural Engineering*, 128(10), pp. 1253–1262.
- Dhakal, R. P., & Maekawa, K. (2002c). “Path-dependent cyclic stress-strain relationship of reinforcing bar including buckling”, *Engineering Structures*, 24(11), pp. 1383–1396.
- Engesser, F. (1889). “Ueber die knickfestigkeit gerader stabe.” *Zeitschrift des Architekten- und Ingenieur—Vereins zu Hannover*, 35, pp. 445–462.
- Euler, L. (1759). *Sur le forces des colonnes*, Memoires de l’Academie Royale des Sciences et Belles Lettres, 13, Berlin.

- Feng, Y., Kowalsky, M. J., & Nau, J. M. (2014b). "Finite-Element Method to Predict Reinforcing Bar Buckling in RC Structures", *Journal of Structural Engineering*, 141(5), 04014147.
- Feng, Y., Kowalsky, M. J., & Nau, J. M. (2014c). "Effect of Seismic Load History on Deformation Limit States for Longitudinal Bar Buckling in RC Circular Columns", *Journal of Structural Engineering*, 141(8), 04014187.
- Filippou F., Popov E., B. V. (1983). *Effects of Bond Deterioration on Hysteretic Behavior of Reinforced Concrete Joints*, EERC Report No. UCB/EERC-83/19, University of California, Berkeley.
- Gomes, A., & Appleton, J. (1997). "Nonlinear cyclic stress-strain relationship of reinforcing bars including buckling", *Engineering Structures*, 19(10), pp. 822–826.
- Gullu, M.F., Orakcal, K. & Kolozvari, K. (2019). "The fixed-strut-angle finite element (FSAFE) model for reinforced concrete structural walls". *Bulletin of Earthquake Engineering*, <https://doi.org/10.1007/s10518-019-00641-0>, accessed in June 2019.
- Gullu, M.F., & Orakcal, K. (2019). "Nonlinear finite element modeling of reinforced concrete walls with varying aspect ratios", *Journal of Earthquake Engineering*, <https://doi.org/10.1080/13632469.2019.1614498>, accessed in June 2019.
- Kashani, M. M., Lowes, L. N., Crewe, A. J., & Alexander, N. A. (2015a). "Phenomenological hysteretic model for corroded reinforcing bars including inelastic buckling and low-cycle fatigue degradation." *Computers and Structures*, 156, pp. 58–71.
- Kashani, M. M., Lowes, L. N., Crewe, A. J., & Alexander, N. A. (2016). "Nonlinear fibre element modelling of RC bridge piers considering inelastic buckling of reinforcement." *Engineering Structures*, 116, pp. 163–177.

- Mansour, M. Y., Lee, J. Y., & Hsu, T. T. C. (2002). "Pinching Effect in Hysteretic Loops of R/C Shear Elements", *ACI Special Publication*, Vol. 205, pp. 293-322.
- Massone, L. M., & López, E. E. (2014). "Modeling of reinforcement global buckling in RC elements", *Engineering Structures*, 59, pp. 484-494.
- Massone, L. M., & Moroder, D. (2009). "Buckling modeling of reinforcing bars with imperfections", *Engineering Structures*, 31(3), pp. 758-767.
- Mau, S. T., & El-Mabsout, M. (2008). "Inelastic Buckling of Reinforcing Bars", *Journal of Engineering Mechanics*, 115(1), pp. 1-17.
- Menegotto, M., and Pinto, E. (1973). "Method of analysis for cyclically loaded reinforced concrete plane frames including changes in geometry and non-elastic behavior of elements under combined normal force and bending", *Proceedings, IABSE Symposium on Resistance and Ultimate Deformability of Structures Acted on by Well-Defined Repeated Loads*, Lisbon, Portugal.
- Monti, G., and Nuti, C. (1989). *Modellazione del comportamento ciclico di barre in acciaio per armatura di elementi in cemento armato*, Report No. 1/90, Dept. of Struct. Eng., Rome, Italy.
- Monti, G., and Nuti, C. (1991). *Analytical model of cyclic behavior of reinforcing bars with inelastic buckling*, Report No. 6191, Dept. of Struct. Eng., Rome, Italy.
- Monti, G., & Nuti, C. (1992). "Nonlinear Cyclic Behavior of Reinforcing Bars Including Buckling", *Journal of Structural Engineering*, 118(12), pp. 3268-3284.
- Moyer, M. J., & Kowalsky, M. J. (2003). "Influence of tension strain on buckling of reinforcement in concrete columns", *ACI Structural Journal*, 100(1), pp. 75-85.

- NEHRP Consultants Joint Venture (2014). *Recommendations for seismic design of reinforced concrete wall buildings based on studies of the 2010 Maule, Chile earthquake*, NIST GCR 14-917-25, National Institute of Standards and Technology, Gaithersburg, MD. <https://www.nist.gov/publications/recommendations-seismic-design-reinforced-concrete-wall-buildings-based-studies-2010>, accessed in June 2019.
- OpenSees, the open system for earthquake engineering simulation. Pacific Earthquake Engineering Research Centre. Berkeley: University of California. <http://opensees.berkeley.edu/>, accessed in June 2019.
- Orakcal, K., Ulugtekin, D., & Massone, L. M. (2012). “Constitutive modeling of reinforced concrete panel behavior under cyclic loading”, *Proceedings, 15th World Conference on Earthquake Engineering*, Lisbon, Portugal, No. 3573.
- Paulay, T., & Priestley, M. J. N. (1993). “Stability of ductile structural walls”, *ACI Structural Journal*, 90(4), pp. 385–392.
- Rodriguez, M. E., Botero, J. C., & Villa, J. (1999). “Cyclic Stress-Strain Behavior of Reinforcing Steel Including Effect of Buckling”, *Journal of Structural Engineering*, 125(6), pp. 605–612.
- Shanley, F. R. (1947). “Inelastic column theory”, *J. Aeronautical Sciences*, 14(5), pp. 261–264.
- Thomsen, J.H. & Wallace, J.W. (1995). *Displacement-Based Design of Reinforced Concrete Structural Walls: An Experimental Investigation of Walls with Rectangular and T-Shaped Cross-Sections*, Report No. CU/CEE-95/06, Department of Civil Engineering, Clarkson University, Postdam, New York.
- Tran, T.A., 2012, *Experimental and Analytical Studies of Moderate Aspect Ratio Reinforced Concrete Structural Walls*, Ph.D. Thesis, Department of Civil Engineering, University of California, Los Angeles.

Vecchio, F. J., & Collins, M. P. (2006). "Compression Response of Cracked Reinforced Concrete." *Journal of Structural Engineering*, 119(12), pp. 3590–3610.

Zong, Z. (2010). *Uniaxial material model incorporating buckling for reinforcing bars in concrete structures subjected to seismic loads*, Ph.D. Thesis, Department of Civil Engineering, University of California, Los Angeles.

