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WIDEBAND MODELING OF
TRANSMISSION LINES AND CABLES

ILHAN KOCAR

DÉPARTEMENT DE GÉNIE ÉLECTRIQUE
ÉCOLE POLYTECHNIQUE DE MONTRÉAL

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présentée par : KOCAR Ilhan

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a été dûment acceptée par le jury d'examen constitué de :

M. ROY Gilles, M.Sc.A., président

M. MAHSEREDJIAN Jean, Ph.D., membre et directeur de recherche

M. OLIVIER Guy, Ph.D., membre et codirecteur de recherche

M. SIROIS Frédéric, Ph.D., membre

M. MORCHED Atef, Ph.D., membre

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RÉSUMÉ

L'objectif de cette thèse est de développer un modèle précis pour la simulation des lignes de transmission et des câbles souterrains en régime transitoire. Le modèle proposé vise à surmonter les limitations associées aux modèles existants dans les logiciels de type EMTP (Logiciel de simulation des transitoires électromagnétiques - Electromagnetic Transients Program).

Dans ce travail, la méthode des ondes mobiles est adoptée pour le calcul précis des transitoires électromagnétiques qui se produisent sur les câbles souterrains et les lignes de transmission. La solution effective des équations des ondes mobiles exige la caractérisation de deux fonctions matricielles dans le domaine fréquentiel : la fonction de propagation et l'admittance caractéristique. Ces paramètres sont identifiés directement dans le domaine de phase par des fonctions rationnelles. La fonction de propagation constitue un système à large bande d'ordre élevé avec des multiples délais de temps intrinsèques. Une technique de lissage pondéré est introduite pour l'identification de la fonction de propagation par une fonction de transfert rationnelle. Cette technique de lissage proposée est également applicable à l'admittance caractéristique. Il est démontré que cette technique produit des approximations robustes d'ordre moins élevé d'environ 10-50% pour les paramètres de lignes de transmission et de câbles par rapport aux techniques de lissage disponibles dans les logiciels de type EMTP. Des approximations d'ordre réduit augmentent la vitesse des simulations dans le domaine temporel.

L'identification de la fonction de propagation dans le domaine de phase requiert le calcul des délais de temps dans le domaine modal. On explique l'estimation initiale des délais et on propose une méthode de la recherche des délais minimisant l'erreur d'approximation.

Ce travail contribue également à l'amélioration de la stabilité numérique des modèles de ligne et de câble utilisés pour l'analyse des transitoires. Les systèmes de transmission décrits par les fonctions rationnelles peuvent être transformés dans le domaine temporel par des opérations de convolution efficaces. Cependant, la discrétisation des intégrales de convolution génère des erreurs numériques. Les erreurs numériques sont estimées en terme des paramètres des fonctions de transfert pour un modèle donné. En se basant sur cette estimation, on présente une méthodologie pour l'amélioration de la stabilité numérique qui ajuste les paramètres par une technique des moindres carrés contraints. Les avantages numériques de la nouvelle méthode sont démontrés en comparaison avec les méthodes et modèles existants.

ABSTRACT

The objective of this dissertation is to develop an accurate model to simulate the response of transmission line and underground cable systems under transient conditions. The proposed model is targeted to overcome the limitations associated with the existing models in EMTP-RV software (Electromagnetic Transients Program Restructured Version).

In this work, the method of traveling waves is adopted for the accurate computation of electromagnetic transients on underground cables and overhead transmission lines. Efficient solution of traveling wave equations requires the frequency domain characterization of two matrix functions: propagation and characteristic admittance. These parameters are identified directly in the phase domain as rational transfer functions. The propagation function constitutes a high order wideband system with inherent time delays. A weighted fitting technique is presented and advocated for the approximation of propagation function as a rational transfer function. The proposed fitting technique is also applicable to the identification problem of characteristic admittance. It is shown that this technique provides low order and robust approximations for transmission line and cable parameters compared to existing fitting techniques in the literature. Low order approximations provide faster simulations in the time domain.

Phase domain identification of propagation function requires the computation of modal time delays. The initial estimation of modal delays is explained and a time delay search algorithm is proposed. The search is done by direct experimentation with rational fitting with delay as a parameter.

This work also contributes to the improvement of numerical stability of frequency dependent transmission line and cable models used for electromagnetic transient analysis. Transmission devices described with rational functions are transformed into time domain with efficient convolution operations. However, recursive computation of

convolution integrals creates numerical errors. The numerical errors in time domain are estimated in terms of the transfer function parameters for a given line or cable model. On the basis of this estimation a methodology based on parameter scaling using constrained least squares technique is developed for the improvement of numerical stability. The numerical advantages of the new method are supported by demonstration and comparison with existing models.



CONDENSÉ EN FRANÇAIS

Modélisation des Lignes de Transport et des Câbles pour une Large Bande de Fréquences

Cette thèse présente des nouvelles méthodes mathématiques pour le calcul des transitoires électromagnétiques qui se produisent sur les lignes de transmission et les câbles souterrains en considérant les dépendances fréquentielles des paramètres. L'objectif principal de ce travail est de surmonter les limitations des modèles de lignes et câbles qui sont disponibles dans les logiciels de type EMTP [1] comme EMTP-RV (Electromagnetic Transients Program Restructured Version) [17] en développant un nouveau modèle qui est stable, précis et rapide.

L'analyse des transitoires dans les réseaux est un processus mathématiquement complexe et nécessite l'utilisation de logiciels spécialisés. Des logiciels de type EMTP, comme EMTP-RV, appliquent des méthodes numériques pour la représentation et la résolution de réseaux de différentes natures. La simulation des transitoires électromagnétiques s'applique à une large gamme de fréquences et nécessite donc une représentation très détaillée des composants. La simulation est effectuée entièrement dans le domaine du temps et l'objectif est de calculer les formes d'onde des variables d'état à un point arbitraire dans le réseau simulé.

Les modèles transitoires des lignes et câbles doivent prendre en compte la dépendance fréquentielle des paramètres pour une simulation précise. Le calcul précis des périodes transitoires est très important pour l'opération et la planification des réseaux électriques de façon fiable, sécuritaire et efficace.

Les transitoires se propagent d'un côté à l'autre des réseaux électriques à travers des lignes et des câbles. La modélisation des lignes et câbles peut devenir très complexe à cause de plusieurs facteurs tels que la dépendance fréquentielle des paramètres,

particulièrement pour le mode de retour par la terre, les dispositions asymétriques des conducteurs et les paramètres répartis continuellement.

En utilisant les paramètres répartis, on peut décrire la relation entre la tension et le courant à travers la ligne de transmission par les équations différentielles partielles (EDP) dans le temps et l'espace. Dans le cas de systèmes multi fils, les fils de lignes ou câbles sont couplés; alors la relation se transforme dans un système d'équations différentielles couplées avec les coefficients en fonction de la fréquence.

Considérons une ligne caractérisée par sa résistance et son inductance linéiques R' et L' , sa conductance et sa capacité linéiques G' et C' . En tout point, la tension et le courant sont liés par les équations suivantes [22] :

$$\frac{\partial v(x,t)}{\partial x} = -R' i(x,t) - L' \frac{\partial i(x,t)}{\partial t}$$

$$\frac{\partial i(x,t)}{\partial x} = -G' v(x,t) - C' \frac{\partial v(x,t)}{\partial t}$$

où v est la tension et i est le courant de la ligne à un point arbitraire.

Les méthodes aux différences finies utilisent une discrétisation dans le temps et l'espace des équations, prises sous la forme temporelle. L'obstacle fondamental de cette approche est la difficulté de prendre en compte la dépendance fréquentielle. Un autre désavantage est le temps de calcul excessif à cause du sectionnement de la ligne.

L'approche classique la plus répandue consiste à utiliser la méthode des ondes qui est la méthode généralement jugée la plus efficace. Elle est basée sur les équations dans le domaine fréquentiel. Pour un système de transmission à plusieurs phases, les équations des lignes dans le domaine fréquentiel sont données par :

$$\mathbf{I}_m = \mathbf{Y}_c \mathbf{V}_m - \mathbf{H}(\mathbf{Y}_c \mathbf{V}_k + \mathbf{I}_k)$$

$$\mathbf{I}_k = \mathbf{Y}_c \mathbf{V}_k - \mathbf{H}(\mathbf{Y}_c \mathbf{V}_m + \mathbf{I}_m)$$

où l'indice inférieur m dénote une extrémité arbitraire de la ligne tandis que k dénote l'autre extrémité. La matrice $\mathbf{H}$ est la fonction de propagation et la matrice $\mathbf{Y}_c$ est l'admittance caractéristique. $\mathbf{H}$ est une fonction exponentielle donnée par :

$$\mathbf{H} = \mathbf{e}^{-\sqrt{(\mathbf{G}' + j\omega\mathbf{C}')(\mathbf{R}' + j\omega\mathbf{L}')} \ell}$$

où j est l'opérateur des nombres complexes, ω est la fréquence angulaire et ℓ signifie la longueur de la ligne. L'admittance caractéristique est donné par :

$$\mathbf{Y}_c = \sqrt{\frac{(\mathbf{G}' + j\omega\mathbf{C}')}{(\mathbf{R}' + j\omega\mathbf{L}')}}}$$

On peut aussi transformer les grandeurs de phase en grandeurs modales en utilisant une transformation modale. Dans la nouvelle base, les équations des ondes mobiles sont indépendantes. La matrice de transformation varie également avec la fréquence.

L'analyse transitoire nécessite la transformation inverse des équations au domaine temporel :

$$\mathbf{i}_m = \mathbf{y}_c * \mathbf{v}_m - \mathbf{h} * (\mathbf{y}_c * \mathbf{v}_k + \mathbf{i}_k)$$

$$\mathbf{i}_k = \mathbf{y}_c * \mathbf{v}_k - \mathbf{h} * (\mathbf{y}_c * \mathbf{v}_m + \mathbf{i}_m)$$

Dans les équations précédentes on a utilisé le théorème de convolution. Il y a quatre convolutions à calculer à chaque pas de temps de la simulation. On peut efficacement calculer les intégrales de convolution par récursivité si les matrices $\mathbf{H}$ et $\mathbf{Y}_c$ sont décrites avec les approximations par des fonctions rationnelles [7]-[13].

La méthode des ondes mobiles est utilisée dans le logiciel EMTP-RV par les différents modèles de lignes/câbles, soient le modèle FD (Frequency Dependent) [7], le modèle FDQ (Full Frequency Dependent) [9] et le modèle WB (Wideband) [11].

Le modèle FD transforme les grandeurs de phase en grandeurs modales [5]-[6] pour minimiser le temps de calcul dans le domaine temporel. On identifie les grandeurs modales par une technique de lissage asymptotique de type Bode [70]. Une transformation constante est utilisée pendant les calculs pour éviter des convolutions multiples. Le FD est seulement applicable aux lignes de transmission, car dans le cas de câbles les matrices de transformation varient fortement avec la fréquence. Également, dans le cas des lignes disposées asymétriquement la précision du modèle FD diminue.

Le modèle FDQ résout le problème de la dépendance fréquentielle des matrices de transformation en les représentant avec des fractions rationnelles au prix de l'augmentation du temps de calcul. Le FDQ est seulement applicable aux câbles parce que des pôles instables deviennent nécessaires pour le lissage des matrices de transformations des lignes. De plus, le FDQ n'est pas stable numériquement à cause des convolutions multiples.

Le modèle WB identifie les paramètres des lignes/câbles dans le domaine de phase. Le modèle prend en compte les délais multiples dans la fonction de propagation. Cependant, à cause des procédures appliquées, le modèle pose plusieurs problèmes numériques. Dans ce travail, on exploite l'idée de base du modèle WB pour développer un nouveau modèle.

Les approximations rationnelles des matrices $\mathbf{H}$ et $\mathbf{Y}_c$ doivent représenter la dépendance fréquentielle des paramètres pour une large bande de fréquences. Il y a plusieurs méthodes de lissage pour obtenir une approximation rationnelle d'une réponse fréquentielle d'un système échantillonné [53]. Mais, la plupart des méthodes ne sont pas appropriées pour le lissage des fonctions de ligne/câble, étant donné la particularité des fonctions et la gamme de fréquences.

Dans ce travail une technique de lissage pondéré (Weighted Vector Fitting - WVF) [19] est introduite pour l'identification de la fonction de propagation par une fonction de transfert rationnelle. Cette nouvelle technique de lissage est également applicable au

problème d'identification de l'admittance caractéristique. Il est démontré que cette technique produit des approximations robustes d'ordre moins élevé d'environ 10-50% pour les paramètres de lignes/câbles par rapport aux techniques de lissage disponibles dans la littérature. Des approximations d'ordre réduit augmentent la vitesse des simulations dans le domaine temporel.

Lissage pondéré par les fractions rationnelles (Weighted Vector Fitting Technique - WVF)

L'objectif de l'identification d'une réponse fréquentielle est d'obtenir un système invariant linéaire dans la forme d'une fonction de transfert qui s'exprime comme une fonction rationnelle donnée par :

$$f(s) \equiv \frac{N(s)}{D(s)} = \frac{b_0 + b_1s + b_2s^2 + \dots + b_{N'}s^{N'}}{1 + a_1s + \dots + a_{N-1}s^{N-1} + a_Ns^N} \quad (\text{a.1})$$

où $N' < N$ et $s = j\omega$. On peut aussi identifier $f(s)$ par une fraction rationnelle :

$$f(s) \equiv \sum_{n=1}^N \frac{r_n}{s + p_n}$$

L'identification de $f(s)$ par une fraction rationnelle est un problème non linéaire. Par contre, on peut multiplier la fonction $f(s)$ dans l'équation (a.1) par le dénominateur à la partie droite de l'équation et en écrivant l'égalité approximative pour plusieurs échantillons de fréquence, on peut obtenir un système d'équations hyperdéterminé et linéaire. On peut résoudre le système d'équations comme un problème de minimisation en utilisant la méthode des moindres carrés. Cependant, ce système d'équations devient mal conditionné à cause de la large bande de fréquences étudiée et des termes s^n . De plus, les égalités approximatives sont biaisées dû à la multiplication par le dénominateur. Donc les polynômes ne sont pas adaptés pour l'identification des fonctions de ligne/câble.

Dans ce travail on propose une méthode qui utilise les fractions rationnelles. Cette méthode est inspirée de la méthode de Vector Fitting (VF) [39]. Premièrement divisons le numérateur et le dénominateur de l'équation (a.1) par un polynôme arbitraire $D'(s)$ de l'ordre N :

$$\frac{N(s)/D'(s)}{D(s)/D'(s)} \cong f(s) \quad (\text{a.2})$$

Quand on applique le théorème de Heaveside à l'équation (a.2) on obtient :

$$\frac{\sum_{n=1}^N \frac{\hat{r}_n}{s + \bar{p}_n}}{\sum_{n=1}^N \frac{\tilde{r}_n}{s + \bar{p}_n} + 1} \cong f(s) \quad (\text{a.3})$$

Les pôles $\bar{p}_n$ et les résidus $\hat{r}_n$ et $\tilde{r}_n$ sont les inconnues de cette équation et ils peuvent être réelles ou complexes conjugués. On peut donc écrire le problème de minimisation de l'équation (a.3) par :

$$\min \left| \frac{1}{\sum_{n=1}^N \frac{\tilde{r}_n}{s + \bar{p}_n} + 1} \left[\sum_{n=1}^N \frac{\hat{r}_n}{s + \bar{p}_n} - \left(\sum_{n=1}^N \frac{\tilde{r}_n}{s + \bar{p}_n} + 1 \right) f(s) \right] \right| \quad (\text{a.4})$$

On remarque ici que le facteur au dénominateur de l'équation (a.4) donné par

$$C(s) = \sum_{n=1}^N \frac{\tilde{r}_n}{s + \bar{p}_n} + 1$$

est le facteur de pondération. Pour la solution de l'équation (a.4) on part d'un ensemble de pôles initial $\bar{p}_n \in \mathbb{C}^{N \times 1}$ distribués logarithmiquement sur le spectre de fréquences étudié. Pour la première itération, on peut mettre le terme $C(s)$ à 1. L'équation (a.4) devient donc linéaire et hyperdéterminée par rapport à ses inconnues : $\hat{r}_n$ et $\tilde{r}_n$. On

remarque ici que les $\tilde{r}_n$ sont les résidus qui correspondent aux pôles de la fonction $f(s)$ (voir l'équation (a.3)). Pour calculer les pôles de la fonction $f(s)$ à partir des $\tilde{r}_n$ il faut résoudre le problème de valeurs propres suivant :

$$\mathbf{Q} = \mathbf{W} - \mathbf{g}_1 \tilde{\mathbf{x}}^T$$

où la matrice $\mathbf{W}$ est une matrice diagonale contenant les pôles initiaux $\bar{p}_n \in \mathbb{C}^{N \times 1}$, $\mathbf{g}_1$ est un vecteur colonne ne contenant que des 1 et finalement $\tilde{\mathbf{x}}$ est un vecteur ligne contenant les résidus $\tilde{r}_n$.

Pour calculer les résidus de $f(s)$ rapidement, on résout le problème :

$$f(s) \cong \sum_{n=1}^N \frac{r_n}{s + p_n}$$

en prenant les pôles calculés ci-dessus comme nouveaux pôles de $f(s)$.

En général pour arriver à une approximation satisfaisante, il faut résoudre l'équation (a.4) 3 à 4 fois en moyenne. À chaque itération on commence par des nouveaux pôles calculés à l'itération précédente. Pour enlever l'effet de facteur de biais, on maintient le facteur de pondération, calculé à partir des paramètres précédents :

$$C^{(j)}(s) \cong \sum_{n=1}^N \frac{\tilde{r}_n^{(j-1)}}{s + \bar{p}_n^{(j-1)}} + 1$$

où j signifie le compteur des itérations. L'utilisation du facteur de biais est nouvelle par rapport aux méthodes existantes. La nouvelle méthode est nommée WVF (Weighted Vector Fitting) [19].

Si l'approximation finale n'est pas satisfaisante on peut toujours augmenter l'ordre d'approximation. L'erreur EMQ (Erreur Moyenne Quadratique) de l'approximation est donnée par :

$$EMQ = \sqrt{\frac{1}{K} \sum_{k=1}^K |f(s_k) - \tilde{f}(s_k)|^2}$$

où K est le nombre total des échantillons de fréquence et $\tilde{f}(s)$ est l'approximation obtenue par la méthode de WVF.

Lissage de la fonction de propagation et de l'admittance caractéristique

Le lissage de la fonction de propagation est complexe en particulier dans le cas des câbles à cause des délais de temps multiples qui contribuent à chaque élément de la matrice $\mathbf{H}$ [11]. Alors, dans ce travail, on fait d'abord le lissage des modes de la fonction de propagation dans le domaine modal, ensuite on utilise les pôles et les délais distincts déjà calculés pour le lissage de $\mathbf{H}$ dans le domaine de phase. Les étapes de l'identification de la matrice $\mathbf{H}$ sont les suivantes :

1. Le calcul des modes en utilisant une transformation modale pour le spectre de fréquences donné. Les modes de $\mathbf{H}$ sont les valeurs propres de la matrice.
2. Les rotations des modes pour minimiser la partie complexe des modes.
3. L'ordre des valeurs diagonales est conservé lors du changement de fréquence. Cette condition est nécessaire pour assurer la continuité des valeurs propres pendant leur calcul.
4. L'identification des délais de temps de chaque mode.
5. Le groupement des modes ayant des délais approximativement égaux.
6. Le lissage des modes par la méthode de WVF et la recherche des délais minimisant l'erreur d'approximation.
7. Le lissage dans le domaine de phase en utilisant les pôles et délais déjà calculés.

Les étapes 1 à 5 et 7 sont inspirées directement de la littérature [11], [33], [66]. La contribution principale est dans l'utilisation de la méthode de l'étape 6 [19]. On peut formuler l'idée principale de cette approche pour chaque élément matriciel ij de $\mathbf{H}$ par :

$$\tilde{H}_{ij}(s) = \sum_{m=1}^M \left[\sum_{n=1}^{N_m} \frac{c_{ijnm}}{s + p_{nm}} \right] e^{-(s \cdot \tau_m)} \text{ alors que} \quad (a.5)$$

$$\min_c \|H_{ij}(s) - \tilde{H}_{ij}(s)\|_2^2$$

où N_m est le nombre des pôles de mode m , M est le nombre des modes et τ_m est le délai de temps du mode m . Chaque mode est donné par :

$$H_m(s) = e^{\Lambda_m} = H'_m(s) e^{-(s \cdot \tau_m)} \cong e^{-s \tau_m} \sum_{n=1}^N \frac{c_n}{s + p_{nm}}$$

où le terme e^{Λ_m} est une matrice diagonale ne contenant qu'un élément non nul diagonal du mode m . On calcule les pôles p_{mn} par la méthode de WVF et on calcule les résidus c_{ijnm} par une solution directe de type linéaire algébrique en utilisant la méthode des moindres carrés.

En général la procédure de lissage de $\mathbf{H}$ donne des résultats satisfaisants, mais, dans certains cas, les contributions des modes aux éléments de $\mathbf{H}$ sont mal représentées, bien que le lissage soit exact dans le domaine de phase. Par conséquent, on peut avoir des rapports très élevés entre les résidus et les pôles, qui augmentent les erreurs numériques des opérations de convolution dans le domaine temporel. La problématique est formulée par [20] :

$$H_{ij,m} = \left[\mathbf{T} e^{\Lambda_m} \mathbf{T}^{-1} \right]_{ij} \neq \left(\sum_{n=1}^{N_m} \frac{c_{ijnm}}{s + p_{nm}} \right) e^{-(s \tau_m)} \quad (a.6)$$

où $\mathbf{T}$ est la matrice de transformation modale de $\mathbf{H}$. La partie à gauche de l'inégalité (a.6) est la contribution du mode m à l'élément ij calculé analytiquement et la partie à droite de l'inégalité est celle calculée par la procédure de lissage. Cette thèse propose d'ajuster les rapports entre les résidus et les pôles par une technique des moindres carrés avec contraintes, sans sacrifier la précision de lissage. Cet ajustement est très important en termes de la stabilité numérique dans le domaine temporel. Le problème d'identification de $\mathbf{H}$ dans le domaine de phase devient donc :

$$\min_c \left\| H_{ij}(s) - \tilde{H}_{ij}(s) \right\|_2^2 \text{ alors que} \quad (a.7)$$

$$-K p_{mn} \leq c_{ijmn} \leq K p_{mn}$$

où $\tilde{H}_{ij}$ est l'approximation de H_{ij} par une fraction rationnelle, K est une contrainte des rapports entre les résidus et les pôles. La valeur de K est définie basée sur l'estimation de l'erreur de l'opération de convolution. Dans ce travail elle est fixée à 10.

L'identification de l'admittance caractéristique est relativement simple, car c'est une fonction lissée sans les facteurs exponentiels. Pour avoir un ensemble de pôles communs, on fait d'abord le lissage de la trace de $\mathbf{Y}_c$ et ensuite on utilise ces pôles pour l'identification de chaque élément matriciel de $\mathbf{Y}_c$ par les fractions rationnelles :

$$Y_{c_{ij}}(s) \cong \sum_{n=1}^N \frac{\tilde{c}_{ij_n}}{s + \tilde{p}_n} + d_{ij}$$

où d_{ij} est la valeur à la fréquence infinie.

Le calcul des intégrales de convolution dans le domaine temporel, les erreurs numériques associées

Les erreurs numériques associées avec le calcul des intégrales de convolution sont expliquées en [20]. Considérons une ligne monophasée. La multiplication de $\mathbf{H}$ dans les équations des ondes mobiles devient :

$$Y = HU \quad (\text{a.8})$$

où

$$U = Y_c V + I$$

Si on introduit l'approximation de H à l'équation (a.8) on obtient :

$$Y = \sum_{n=1}^{N_m} \frac{\hat{c}_n}{s + \hat{p}_n} e^{-s \cdot \tau} U \quad (\text{a.9})$$

Dans le domaine temporel l'équation (a.9) devient :

$$y(t) = h(t) * u(t) \quad (\text{a.10})$$

$$y(t_1) = \sum_{n=1}^{N_m} \hat{c}_n \int_0^{t_1} e^{-\hat{p}_n \cdot t} u(t_1 - t - \tau) dt$$

où t_1 est un point de temps. La représentation de l'équation (a.10) dans l'espace d'état est donnée par :

$$\dot{\mathbf{x}}(t) = \mathbf{a} \mathbf{x}(t) + \mathbf{b} u(t - \tau)$$

$$y(t) = \mathbf{c} \mathbf{x}(t)$$

où le vecteur $\mathbf{x}$ est le vecteur des variables d'état et y est la variable observée.

$$\mathbf{a} = \text{diag} \begin{bmatrix} -\hat{p}_1 & -\hat{p}_2 & \dots & -\hat{p}_{N_m} \end{bmatrix}$$

$$\mathbf{b} = [1 \quad 1 \quad \dots \quad 1]^T$$

$$\mathbf{c} = [\hat{c}_1 \quad \hat{c}_2 \quad \dots \quad \hat{c}_{N_m}]$$

$$\mathbf{x} = [x_1 \quad x_2 \quad \dots \quad x_{N_m}]^T$$

La solution analytique des équations d'état est donnée par :

$$\mathbf{x}_{n+1} = \mathbf{e}^{\mathbf{a}(t_{n+1}-t_n)} \mathbf{x}_n + \mathbf{e}^{\mathbf{a}t_{n+1}} \int_{t_n}^{t_{n+1}} \mathbf{e}^{-\mathbf{a}t} u(t-\tau) dt$$

où $\mathbf{x}_n = \mathbf{x}(t_n)$. La solution analytique des équations d'état dans le domaine temporel requiert l'utilisation d'une méthode d'intégration numérique. Dans la plupart des logiciels de simulation des transitoires électromagnétiques, on utilise la méthode d'intégration trapézoïdale avec un pas fixe [16] :

$$\mathbf{x}_{n+1} \cong \frac{\mathbf{I} + \mathbf{a} \frac{\Delta t}{2}}{\mathbf{I} - \mathbf{a} \frac{\Delta t}{2}} \mathbf{x}_n + \frac{\mathbf{I} \frac{\Delta t}{2}}{\mathbf{I} - \mathbf{a} \frac{\Delta t}{2}} \mathbf{b}(u_{n+1} + u_n) \quad (\text{a.11})$$

où $u_n = u(t_n - \tau)$. La variable Δt constitue le pas d'intégration et donc le pas d'échantillonnage de l'excitation u . Entre deux points de temps t et $t + \Delta t$ la solution est linéaire. On peut aussi écrire l'équation (a.11) comme :

$$\mathbf{x}_{n+1} \cong \boldsymbol{\alpha} \mathbf{x}_n + \boldsymbol{\lambda}(u_{n+1} + u_n)$$

Étant donné que τ n'est pas en général un multiple entier de $\Delta \tau$, l'interpolation linéaire de l'excitation est nécessaire. L'erreur d'interpolation locale de la variable observée est donnée par :

$$\varepsilon_1 = \mathbf{c} \int_0^{\Delta t} \mathbf{e}^{\mathbf{a}t} R(t) dt \quad (\text{a.12})$$

où le terme $R(t)$ est le reste. Si $R(t)$ est considéré constant entre deux points de temps t et $t + \Delta t$, on peut approximer l'erreur d'interpolation par :

$$\varepsilon_1 \cong \left(\sum_{n=1}^{N_m} \frac{\hat{c}_n}{\hat{p}_n} (1 - e^{-p_n \Delta t}) \right) R_m \quad (\text{a.13})$$

L'erreur de troncature locale (LTE) due à la méthode d'intégration trapézoïdale est approximée par :

$$\varepsilon_2 = \frac{1}{12} \left(\alpha(x_n - x_{n-3}) + 3\alpha(x_{n-2} - x_{n-1}) + \lambda(u_{n+1} + u_n - u_{n-2} - u_{n-1}) \right) \quad (\text{a.14})$$

Le LTE est une fonction du pas d'intégration. Le pas d'intégration constitue une donnée pour la simulation. Il s'agit d'utiliser le Δt qui permet de capter la fréquence de la transitoire la plus rapide du phénomène simulé. Plus Δt est petit, plus la précision des calculs augmente, mais le temps de calcul augmente aussi. Il est nécessaire de trouver un équilibre entre une précision suffisante et un temps de calcul acceptable sur l'ordinateur utilisé.

Le LTE peut diminuer la précision, mais elle n'est pas la raison de l'explosion numérique, car la méthode d'intégration trapézoïdale est une méthode A-stable, donc stable si le système solutionné est stable. Le problème avec les lignes/câbles est l'erreur d'interpolation. Comme on peut le voir dans l'équation (a.13) l'erreur d'interpolation est une fonction des rapports entre les résidus et les pôles correspondants. Pour une ligne monophasée ou pour un mode isolé, le rapport calculé entre un résidu et pôle est inférieur à 1. Par contre, pour une ligne à plusieurs phases, on peut trouver des rapports très élevés, en particulier dans le cas de lignes/câbles courts où les délais de temps sont rapprochés. C'est dû au fait que le problème de minimisation dans l'équation (a.5), si calculé sans contraintes, peut mal représenter les contributions des modes de propagation. Dans ce travail, on a fixé le rapport maximal entre un résidu et pôles à 10 en utilisant la méthode des moindres carrés contraints. Ceci est justifié par les cas problématiques dans le logiciel EMTP-RV.

Conclusion

Dans cette dissertation on développe un modèle précis et stable pour la simulation des lignes de transmission et des câbles souterrains en régime transitoire. Le modèle proposé est un modèle dans le domaine de phase et inspiré du modèle existant WB. Les nouvelles méthodes mathématiques de calcul surmontent les limitations des modèles de lignes et de câbles qui sont disponibles dans le logiciel EMTP-RV.

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LIST of NOTATION and ABBREVIATIONS

(1.1)	Equation 1.1
[1]	Reference 1
C'	Shunt Capacitance per line length
f	Frequency
G'	Shunt Conductance per line length
$\mathbf{H}$	Propagation Function Matrix
H_m, H	Modal Propagation Function (Scalar), Propagation Mode
$\mathbf{H}_m$	Modal contribution matrix to $\mathbf{H}$
i	Current (Scalar), Time Domain
j	Complex Variable
ℓ	Length of Line
L'	Series Inductance per line length
N_c	Number of conductors in a multiconductor transmission device
R'	Series Resistance per line length
s	Complex Angular Frequency
τ	Time Delay
v	Voltage (Scalar), Time Domain
ω	Angular Frequency
Y	Shunt Admittance per line length
$\mathbf{Y}_c$	Characteristic Admittance Matrix
Z	Series Impedance per line length
$\mathbf{Z}_c$	Characteristic Impedance Matrix

ARMA	Auto-Regressive Moving Average
CEWB	Convex Wideband Model with Euler Backward Integration Scheme
CIWB	Convex Improved Wideband Model
CP	Constant Parameter Model
CWB	Convex Wideband Model
EMTP	Electromagnetic Transients Program
EMTP-RV	Electromagnetic Transients Program Restructured Version
FD	Frequency Dependent Line Model
FDTD	Finite Difference Time Domain
FDQ	Full Frequency Dependent Cable Model
FFT	Fast Fourier Transform
LS	Least-squares
LTI	Linear Time Invariant
MIMO	Multi Input Multi Output
MPS	Minimum Phase Shift
MVF	Modal Vector Fitting
OVF	Orthonormal Vector Fitting
PDE	Partial Differential Equations
PFE	Partial Fraction Expansion
RVF	Relaxed Vector Fitting
TD	Time Domain
TSR	Two-Sided Recursions
ULM	Universal Line Model
VF	Vector Fitting
WB	Wideband Model
WVF	Weighted Vector Fitting

CHAPTER 1.

INTRODUCTION

1.1. Overview

This dissertation presents new solution procedures for the computation of electromagnetic transients on transmission lines and underground cables. Proposed solution procedures take into consideration the frequency dependence and distributed nature of the transmission line and underground cable parameters. Multi propagation delays are adequately represented and the numerical stability of the time domain solution is improved.

An electromagnetic transient is a short-lived oscillation in power systems caused by a sudden change of voltage, current or load, as when a switch opens or closes, a lightning strikes the power transmission lines or substations, or when a fault occurs on the system. Calculation of electromagnetic transients is one of the most complicated and important studies regarding power systems. Precise calculation of transient periods is extremely important for the reliable, safe and efficient operation of power systems as it is at such times that the circuit components are subjected to the maximum stresses from extreme currents and voltages.

Reliable protective strategies for transient conditions can only be properly determined if the behavior of the power system during transient periods is accurately predicted. Inaccurate calculation of transients will result into wrong estimations of excessive voltages during transient periods. In case of overestimation, ratings of the protective equipment will be unnecessarily high so will their associated financial and environmental costs. In case of underestimation, damages will occur; this may disable a power plant or black out a city in extreme cases.

Transients propagate from one part of the power system to another through transmission lines and power cables whether they originate in the lines or substations. Both transmission lines and power cables have relatively simple geometries with simple circuit parameters, however transient modeling of transmission lines and cables can be complicated due to a number of factors such as frequency dependence of the parameters particularly for the ground return mode, asymmetrical arrangement of conductors with return through ground and distributed parameters as summarized in [1].

Electromagnetic transients in power systems can be very fast and peak values may occur within microseconds. When the wavelength of the signal becomes comparable to the length of the line, lumped parameter representations are not valid and the mathematical description of the phenomena must be formulated in terms of distributed parameters. Using distributed parameters, the voltage and current descriptions along the transmission lines and cables can be formulated in terms of partial differential equations in time and space. The conductors of either transmission lines or cables are coupled to each other; hence the formulation results in a system of mutually coupled differential equations with non-constant frequency dependent coefficients. A direct numerical solution of partial differential equations (PDE) in time domain (TD) could be achieved by using a finite difference based model. The main obstacle for this approach is the difficulty to account for the frequency dependency of the equation parameters. It is also necessary to divide the line into sections, which increases the number of numerical operations.

The most common transient solution method of lines is based on traveling wave equations obtained by transforming the frequency domain equations into time domain. The frequency domain equations allows taking into account the frequency dependence; in addition to this, partial differential equations written in the time domain become ordinary type as the derivative with respect to time vanishes in the frequency domain. In the frequency domain, transmission lines and cables can be characterized with two non-constant coefficients: the propagation function $\mathbf{H}$ and the characteristic admittance $\mathbf{Y}_c$.

These two are both functions of series impedance and shunt admittance. The series impedance can be quite sensitive to variations in the frequency due to the skin effect and ground return.

Frequency domain equations of the lines can be directly employed for steady state analysis. However they should be converted into time domain for transient analysis. This could always be done by using a Fast Fourier Transform (FFT) with full numerical convolutions carried out in the time domain as done in [2] or [3]; however this approach is numerically costly in terms of CPU and memory usage. The most popular approach is to use rational function approximations for $\mathbf{H}$ and $\mathbf{Y}_c$. This approach eliminates the use of FFT and allows efficient calculation of convolution integrals with a recursive scheme [4]. The basic idea of many rational function models is first to decompose the physical system (phase domain) into a mathematically decoupled system (modal domain) in order to solve a multiphase line as if it consisted of a number of single phase lines [5], [6]. References [7]-[13] provide a list of industry grade applications based on rational function modeling of lines and cables in modal as well as phase domain. The transfer function approximation should be able to fit frequency dependent parameters of the transmission lines and cables for a specified range of frequencies defined according to the nature of transients.

1.2. Motivation

The most important advance in solving the transient simulations in power systems is probably the Electromagnetic Transients Program (EMTP). The fundamental idea of EMTP type programs is to transform differential equations describing the transient behaviour of the system into discrete models at finite time increments. Then a complete network solution is obtained at each time step [14]-[16].

As EMTP provides a complete network solution, developing a line model that would be compatible with EMTP or that could be easily integrated into EMTP have been the fundamental issues of the recent researches in this field.

In this study, EMTP studies are done in EMTP-RV software [17]. EMTP-RV presents three different frequency dependent models for transmission lines and cables, i.e. Frequency Dependent Line Model (FD) [7], Full Frequency Dependent Cable Model (FDQ) [9] and Wideband Model (WB) derived from Universal Line Model (ULM) [11]. Each model has some advantages and disadvantages compared to others considering modeling accuracy, numerical stability, generality and computation speed.

The FD line model is probably the most popular transmission line model. This model includes the frequency dependence of the line parameters and their distributed nature and assumes a real and constant transformation matrix to decouple the propagation modes. The FD line model can give accurate results for most of the transmission line cases. However, in the case of cables or long and untransposed lines (strongly asymmetrical), erroneous results could be obtained as the elements of the modal transformation matrices would depend strongly on frequency. It is also reported that [33] this model may be unable to simulate low frequency coupling effects.

The full frequency dependent FDQ cable model solves the problem of a strongly frequency dependent transformation matrix by fitting this matrix with rational functions in the frequency domain. However, in order to implement the recursive convolutions for the purpose of updating the history sources of the equivalent model circuits, all elements of the frequency dependent propagation matrix, characteristic admittance matrix and modal transformation matrix should be fitted with rational functions in the frequency domain. This results in a large number of operations in the time domain and it is not unusual that this model numerically explodes in EMTP after a certain number of time steps due to excessive number of convolutions. Although the FDQ model can be used for cable systems, it is in general not applicable to transmission lines because fitting of transformation matrices cannot be achieved without using unstable poles [10].

The WB model can be employed for both transmission lines and underground cables. The key idea is to first decompose the phase domain described propagation

function into modes, each fitted with a set of poles and a single time delay, followed by a final fitting in the phase domain with only residues as unknown quantities. The idea is quite innovative but it is likely to encounter numerically unstable models due to the modeling and fitting procedures applied especially for short transmission line or cable configurations. The WB model is also slower than the other models. In addition to these problems, cases where pole-identification procedure is unable to obtain an appropriate model were also encountered [18].

1.3. Contributions of this thesis

It is evident that whether the cable is short or not, whether the transmission line is transposed or not, a reliable and accurate transient model of a general nature is necessary; that is the model should not be restricted to particular cases or conditions, and, the mathematical complexity of the problem should not obscure the physical concepts involved.

The main objective of this thesis is to overcome the existing limitations associated with the WB model and other models available in EMTP-RV, by introducing an improved wideband model which is named as Convex Improved Wideband (CIWB) Model. CIWB model exploits the key idea of Universal Line Model [11], i.e., solves the phase domain identification problem by using modal domain parameters.

The proposed model provides a general, computationally fast and reliable transient model for transmission lines and cables in two steps. These two steps are interrelated and influence the overall performance of the model.

1. Modeling of Transmission Line and Cable Parameters as Rational Transfer Functions in the Frequency Domain: One of the complexities of electromagnetic transient analysis is the fact that transient waves are comprised of multi frequency signals from a few Hz to MHz. Therefore, transmission line and cable models should be valid for a wideband frequency range. It is unfortunate that, although many

identification techniques exist, most of them are not suitable for a wideband modeling due to mathematical limitations as will be discussed in Chapter 4. This is one of the hot research topics. In this study Weighted Vector Fitting (WVF), a robust identification technique based on least squares technique, is proposed which can be used to identify frequency responses for a wideband frequency range as a sum of first order poles with corresponding residues, that is, with “partial fraction expansions” (PFE) [19]. It is shown that this technique provides low order and robust approximations for transmission line and cable parameters in phase domain. Low order approximations provide faster simulations in the time domain.

A given modal propagation function can be identified as a rational transfer function associated with an exponential time delay term. There is a strong relation between fitting error and delay. In this work initial estimation of modal delays is explained and a time delay search algorithm is proposed. The search is done by direct experimentation with rational fitting with delay as a parameter.

2. Incorporation of the Frequency Domain Model into the EMTP Solver in the Time Domain as a Linear Time Invariant (LTI) System: Incorporating a transmission line or cable model into an EMTP type program as a rational transfer function requires computation of recursive convolution integrals in the time domain which brings numerical errors. Although much effort is given for the meticulous fitting and modeling of transmission system functions in the frequency domain, numerical stability problems of these models in the time-domain remain complex and require further research. Numerical errors due to computation of convolution integrals in the time domain are closely related to transfer function parameters and these parameters can be scaled in order to minimize numerical errors and improve stability by using a convex programming approach such as constrained least squares.

In this work a state space solution is used to compute convolution integrals in an efficient manner. State space parameters are predetermined from rational

approximations of transmission device functions then these parameters are scaled with a constrained least squares algorithm to improve numerical stability in the time domain [20].

1.4. Methodology

In this study, transmission line parameters, i.e., series impedance and shunt admittance parameters with respect to frequency are obtained from EMTP-RV software and they are assumed to be correct. These parameters are used for the numerical computation of the propagation function and characteristic admittance matrices of the lines and cables. Then these numerically available matrices are approximated as rational functions. The line and cable models are developed based on these rational function approximations and the models are integrated to the EMTP-RV solver. In EMTP-RV different circuit configurations are created based on practical cases. The simulation results of the proposed models are verified with the results of the other line and cable models available in EMTP-RV. The cross validation cannot replace the experimental validation but given that the experimental results are scarce, different models giving the same results can be considered to be an important verification. In addition to this, the models available in EMTP-RV had been verified before for similar cases with indisputable solutions obtained from completely different approaches such as FFT.

1.5. About this Thesis

The work presented in this thesis is divided into following chapters:

- Chapter I: Introduction: Brief Presentation of Electromagnetic Transient Analysis and EMTP, Transient Modeling of Transmission Lines and Cables
- Chapter II: Wave Propagation in Power Systems: Problem Description, Problem of Frequency Dependency of Transmission Line and Cable Parameters
- Chapter III: Literature Review: Important Contributions to the Transient Modeling of Transmission Lines and Cables

- Chapter IV: Frequency Domain Identification: LS technique and WVF
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CHAPTER 2.

WAVE PROPAGATION IN POWER SYSTEMS

2.1. General Line Equations

Assuming a single phase line as the reference, basic line equations applicable both to transmission lines and cables (transmission devices) are next formulated. The figures are reproduced from [21]. In this study lower case letters denote the quantities in the time domain whereas capital letters denote quantities in the frequency domain, unless stated otherwise.

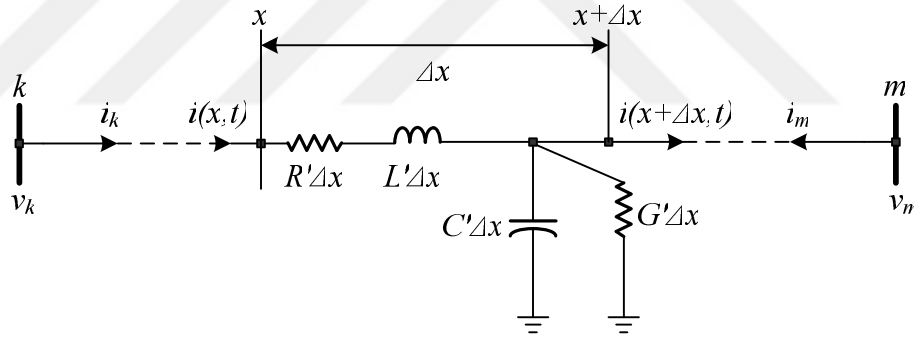


Figure 2.1 Distributed Line Parameters

The basic relationship between voltage and current in the time domain along the line shown in Figure 2.1 is given by:

$$\frac{\partial v(x,t)}{\partial x} = -R'i(x,t) - L'\frac{\partial i(x,t)}{\partial t} \quad (\text{Series Voltage Drop Equation}) \quad (2.1)$$

$$\frac{\partial i(x,t)}{\partial x} = -G'v(x,t) - C'\frac{\partial v(x,t)}{\partial t} \quad (\text{Shunt Current Drop Equation}) \quad (2.2)$$

where x is the distance from node k , and t is time. Resistance R' , capacitance C' , conductance G' and inductance L' parameters are given in per line length. Note that frequency dependence of these parameters are neglected in equations (2.1) and (2.2).

The relationship between voltage and current in the frequency domain is given by:

$$\frac{dV(x, s)}{dx} = -ZI(x, s) \quad (2.3)$$

$$\frac{dI(x, s)}{dx} = -YV(x, s) \quad (2.4)$$

where

$$\begin{aligned} \text{Series Impedance: } Z &= R' + sL' \\ \text{Shunt Admittance: } Y &= G' + sC' \end{aligned} \quad (2.5)$$

Note that frequency dependence of the line parameters can be taken into consideration in the frequency domain. Taking the derivative of (2.4) with respect to x enables us to combine first order differential equations (2.3) and (2.4) in a decoupled second order differential equation

$$\frac{d^2 I}{dx^2} = (YZ)I \quad (2.6)$$

The same operation can be also done for the voltage

$$\frac{d^2 V}{dx^2} = (ZY)V \quad (2.7)$$

The general solution of (2.6) for current is given as follows

$$I(x, s) = e^{-\gamma x} I_f(s) + e^{\gamma x} I_b(s) \quad (2.8)$$

The “constants” $I_f(s)$ and $I_b(s)$ are in general functions of s but are independent of x , and they can be evaluated from boundary conditions. The term γ is given by

$$\gamma = \sqrt{YZ} \quad (2.9)$$

Similarly the general solution of (2.7) can be given by

$$V(x, s) = e^{-\gamma x} V_f(s) + e^{\gamma x} V_b(s) \quad (2.10)$$

where $V_f(s)$ and $V_b(s)$ are constants which can be related to $I_f(s)$ and $I_b(s)$ by applying the relation given in (2.4) to (2.8)

$$V(x, s) = -Y^{-1} \frac{dI}{dx} = Y_c^{-1} (e^{-\gamma x} I_f - e^{\gamma x} I_b) \quad (2.11)$$

where Y_c stands for characteristic admittance and is given by

$$Y_c = \sqrt{\frac{Y}{Z}} \quad (2.12)$$

Emphasizing frequency dependence, γ in (2.9) can be separated into two parts

$$\gamma(s) = \alpha(s) + j\beta(s) = \alpha(s) + j\omega\tau'(s) \quad (2.13)$$

where

$\alpha(s)$: Attenuation

$\beta(s)$: Phase Displacement

An interesting frequency domain expression yielding time domain steady state solution can be obtained by assuming that I_f and I_b in (2.8) are phasors with angles of θ^+ and θ^- respectively [22]:

$$I(x, s) = |I_f| e^{j\theta^+} e^{-\alpha x} e^{-j\beta x} + |I_b| e^{j\theta^-} e^{\alpha x} e^{j\beta x} \quad (2.14)$$

$$i(x, t) = |I_f| e^{-\alpha x} \cos(\omega t - \beta x + \theta^+) + |I_b| e^{\alpha x} \cos(\omega t + \beta x + \theta^-) \quad (2.15)$$

Using (2.11), the voltage in time domain can be obtained as

$$v(x, t) = \frac{|I_f|}{|Y_c|} e^{-\alpha x} \cos(\omega t - \beta x + \theta_{Y_c} + \theta^+) + \frac{|I_b|}{|Y_c|} e^{\alpha x} \cos(\omega t + \beta x + \theta_{Y_c} + \theta^-) \quad (2.16)$$

The term $e^{\pm\alpha x}$ is the attenuation of amplitudes of the waves. These expressions are the sums of forward traveling waves (f), and backward traveling waves (b).

The concept of traveling waves and the generic time domain solutions to the second order differential equations given in (2.6) and (2.7) are first given by d'Alembert in 1747 (for the lossless and distortionless case):

$$\begin{aligned} v(x, t) &= V_f(x - at) + V_b(x + at) \\ i(x, t) &= I_f(x - at) + I_b(x + at) \end{aligned} \quad (2.17)$$

Comparing (2.17) with (2.16) speed of the traveling wave can be written as

$$a = \frac{\omega}{\beta} : \text{speed of the traveling wave} \quad (2.18)$$

The forward traveling wave concept is illustrated in Figure 2.2 for the waveform $V_f(x - at)$. The traveling wave is first shown at $t = 0$ where at $x = x_1$ it has a value of $V_f(x_1)$. At any subsequent time t_x it has the same value at $x = x_1 + at_x$ (distortion neglected) as it formerly had at $x = x_1$. It means that the voltage distribution has moved in the direction of positive x . A similar explanation holds for $V_b(x + at)$, which is traveling in the negative x direction.

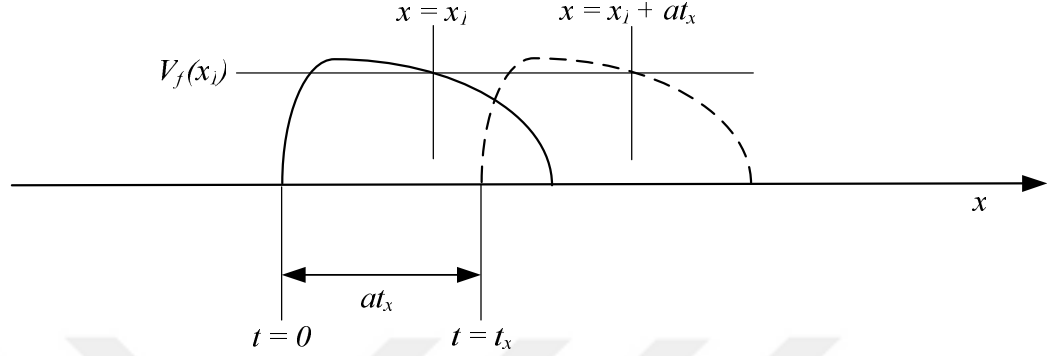


Figure 2.2 Traveling wave function

2.2. Solution of the Multiconductor Line Equations in the Frequency Domain

2.2.1. General Frequency Domain Equations

In matrix form the basic transmission line equation for the line currents can be expressed as

$$\frac{d^2 \mathbf{I}}{dx^2} = (\mathbf{YZ}) \mathbf{I} \quad (2.19)$$

$\mathbf{I}$ represents the current column vector with a length of N_c for a multiphase transmission line system with N_c conductors. $\mathbf{Y}$ and $\mathbf{Z}$ are N_c by N_c shunt admittance and series impedance matrices per unit length respectively. The general solution of (2.19) is

$$\mathbf{I}(x, s) = \mathbf{e}^{-\gamma x} \mathbf{I}_f + \mathbf{e}^{\gamma x} \mathbf{I}_b \quad (2.20)$$

where $\gamma = \sqrt{\mathbf{YZ}}$. $\mathbf{I}_f$ and $\mathbf{I}_b$ stand for vector of forward and backward current constants respectively. From the analogy with (2.4), voltage vector can be found as

$$\mathbf{V}(x, s) = -\mathbf{Y}^{-1} \frac{d\mathbf{I}}{dx} = \mathbf{Y}^{-1} \sqrt{\mathbf{YZ}} (\mathbf{e}^{-\gamma x} \mathbf{I}_f - \mathbf{e}^{\gamma x} \mathbf{I}_b) = \mathbf{Y}_c^{-1} (\mathbf{e}^{-\gamma x} \mathbf{I}_f - \mathbf{e}^{\gamma x} \mathbf{I}_b) \quad (2.21)$$

From (2.21) the characteristic admittance is given by

$$\mathbf{Y}_c = \sqrt{(\mathbf{Y}\mathbf{Z})^{-1}} \mathbf{Y} \quad (2.22)$$

Multiplying (2.21) with characteristic admittance matrix $\mathbf{Y}_c$ and summation of the resulting equation with (2.20) gives

$$\mathbf{Y}_c \mathbf{V}(x, s) + \mathbf{I}(x, s) = 2\mathbf{e}^{-\gamma x} \mathbf{I}_f \quad (2.23)$$

If we redefine the direction of the end currents as flowing into the line as shown in Figure 2.3 and apply boundary conditions to equation (2.23) at both ends

$$\begin{aligned} \mathbf{I}_f &= \mathbf{I}_{kr} \Rightarrow \\ \text{At Node } k \quad x=0 &\Rightarrow \mathbf{Y}_c \mathbf{V}_k + \mathbf{I}_k = 2\mathbf{I}_{kr} \\ x=\ell &\Rightarrow \mathbf{Y}_c \mathbf{V}_m - \mathbf{I}_m = 2\mathbf{e}^{-\gamma \ell} \mathbf{I}_{kr} \end{aligned} \quad (2.24)$$

$$\begin{aligned} \mathbf{I}_f &= \mathbf{I}_{mr} \Rightarrow \\ \text{At Node } m \quad x=0 &\Rightarrow \mathbf{Y}_c \mathbf{V}_m + \mathbf{I}_m = 2\mathbf{I}_{mr} \\ x=\ell &\Rightarrow \mathbf{Y}_c \mathbf{V}_k - \mathbf{I}_k = 2\mathbf{e}^{-\gamma \ell} \mathbf{I}_{mr} \end{aligned} \quad (2.25)$$

following important equations can be obtained:

$$\text{At node } k, \quad \mathbf{Y}_c \mathbf{V}_k - \mathbf{I}_k = 2\mathbf{I}_{ki} = 2\mathbf{e}^{-\gamma \ell} \mathbf{I}_{mr} \quad (2.26)$$

$$\mathbf{I}_k = \mathbf{I}_{kr} - \mathbf{I}_{ki} \quad (2.27)$$

$$\text{At node } m, \quad \mathbf{Y}_c \mathbf{V}_m - \mathbf{I}_m = 2\mathbf{I}_{mi} = 2\mathbf{e}^{-\gamma \ell} \mathbf{I}_{kr} \quad (2.28)$$

$$\mathbf{I}_m = \mathbf{I}_{mr} - \mathbf{I}_{mi} \quad (2.29)$$

where ℓ is the length of the line, $\mathbf{I}_{ki}$ is the incident current wave, $\mathbf{I}_{kr}$ is the reflected current wave for the k end, and

$$\mathbf{H} = \mathbf{e}^{-\gamma \ell} = \mathbf{e}^{-\sqrt{\mathbf{Y}\mathbf{Z}} \ell} \quad (2.30)$$

is the propagation function matrix.

Rearranging above equations one can also obtain

$$\mathbf{I}_m = \mathbf{Y}_c \mathbf{V}_m - \mathbf{H}(\mathbf{Y}_c \mathbf{V}_k + \mathbf{I}_k) \quad (2.31)$$

$$\mathbf{I}_k = \mathbf{Y}_c \mathbf{V}_k - \mathbf{H}(\mathbf{Y}_c \mathbf{V}_m + \mathbf{I}_m) \quad (2.32)$$

Equation pairs (2.31) and (2.32) or (2.26) and (2.28) represent the relation between voltage and current at the sending and receiving ends and they are valid both for underground cables and transmission lines. The propagation function is an exponential function and that is why its time domain counterpart corresponds to a time delay. These equations are extensively used in frequency dependent line models of EMTP. Time domain solution can be obtained by transforming the equations into time domain which result in convolution integrals.

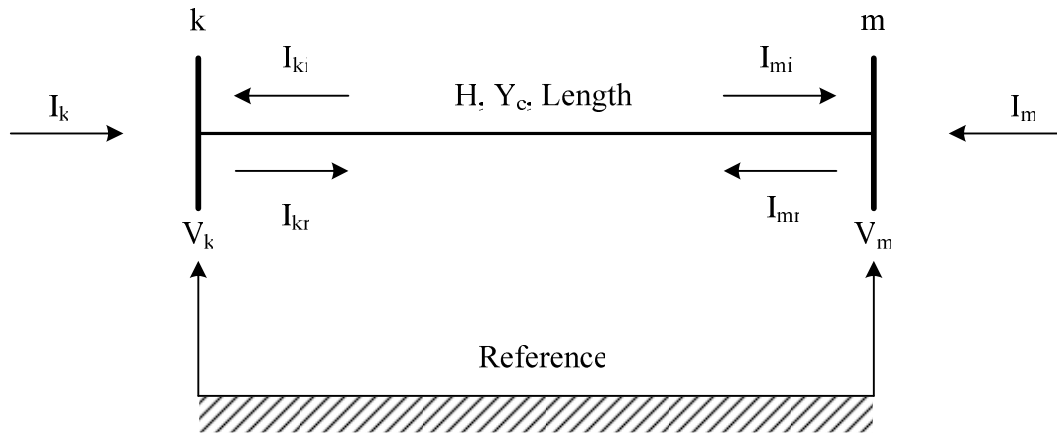


Figure 2.3 Boundary Conditions for Frequency Domain Line Equations

2.2.2. Alternative Form of Frequency Domain Equations

Instead of starting from equation (2.19) one can start from the basic transmission line equation for the line voltages

$$\frac{d^2 \mathbf{V}}{dx^2} = (\mathbf{ZY}) \mathbf{V} \quad (2.33)$$

Then an alternative form of frequency domain equations for the voltage can be obtained as follows

$$\mathbf{V}_k = \mathbf{Z}_c \mathbf{I}_k + \mathbf{H}^T (\mathbf{V}_m + \mathbf{Z}_c \mathbf{I}_m) \quad (2.34)$$

$$\mathbf{V}_m = \mathbf{Z}_c \mathbf{I}_m + \mathbf{H}^T (\mathbf{V}_k + \mathbf{Z}_c \mathbf{I}_k) \quad (2.35)$$

where superscript T stands for transpose of a matrix and the new propagation function is given as (See section 3.1.2 for verification)

$$\mathbf{H}^T = \mathbf{e}^{-\sqrt{\mathbf{ZY}}l} = (\mathbf{e}^{-\sqrt{\mathbf{YZ}}l})^T \quad (2.36)$$

and the characteristic impedance matrix is given as

$$\mathbf{Z}_c = \mathbf{Y}_c^{-1} \quad (2.37)$$

Theoretically there is no difference between two sets of equations. Both $\mathbf{Y}_c$ and $\mathbf{H}$ can be calculated from the eigenvalues of $\sqrt{\mathbf{ZY}}$. This is due to the fact that $\mathbf{Y}$ and $\mathbf{Z}$ matrices are symmetrical, therefore $\mathbf{ZY}$ and $\mathbf{YZ}$ have the same eigenvalues (See section 3.1.2 for verification).

Here, let us just remind that the WB model uses the set of equations (2.26) and (2.28) whereas FD and FDQ models in EMTP-RV use modal domain versions of (2.34) and (2.35).

2.2.3. Frequency Scan Model

For frequency scan (or steady-state) calculations in EMTP, a model similar to ABCD parameter models would be more convenient. For example applying the boundary conditions to (2.20) at the receiving end of a two terminal line as shown in

Figure 2.1 ($x = \ell$) and redefining the direction of current at node m as flowing into the line

$$-\mathbf{I}_m = \mathbf{e}^{-\gamma \ell} \mathbf{I}_f + \mathbf{e}^{\gamma \ell} \mathbf{I}_b \quad (2.38)$$

$$\mathbf{V}_m = \mathbf{Y}_c^{-1} \left(\mathbf{e}^{-\gamma \ell} \mathbf{I}_f - \mathbf{e}^{\gamma \ell} \mathbf{I}_b \right) \quad (2.39)$$

from where

$$\mathbf{I}_{f,k} = \left(\frac{\mathbf{Y}_c \mathbf{V}_m - \mathbf{I}_m}{2} \right) \mathbf{e}^{\gamma \ell} \quad (2.40)$$

$$\mathbf{I}_{b,k} = - \left(\frac{\mathbf{Y}_c \mathbf{V}_m + \mathbf{I}_m}{2} \right) \mathbf{e}^{-\gamma \ell} \quad (2.41)$$

Then as a function of the conditions at node m , the voltage and current at node k can be found by substituting (2.40) and (2.41) into (2.20) and (2.21) for $x = 0$:

$$\mathbf{V}_k = \left(\frac{\mathbf{e}^{\gamma \ell} + \mathbf{e}^{-\gamma \ell}}{2} \right) \mathbf{V}_m - \mathbf{Y}_c^{-1} \left(\frac{\mathbf{e}^{\gamma \ell} - \mathbf{e}^{-\gamma \ell}}{2} \right) \mathbf{I}_m \quad (2.42)$$

$$\mathbf{I}_k = \mathbf{Y}_c \left(\frac{\mathbf{e}^{\gamma \ell} - \mathbf{e}^{-\gamma \ell}}{2} \right) \mathbf{V}_m - \left(\frac{\mathbf{e}^{\gamma \ell} + \mathbf{e}^{-\gamma \ell}}{2} \right) \mathbf{I}_m \quad (2.43)$$

A similar pair of equations can be written for the m node this time using a matrix notation:

$$\begin{bmatrix} \mathbf{V}_m \\ \mathbf{I}_m \end{bmatrix} = \begin{bmatrix} \cosh(\gamma \ell) & -\mathbf{Y}_c^{-1} \sinh(\gamma \ell) \\ \mathbf{Y}_c \sinh(\gamma \ell) & -\cosh(\gamma \ell) \end{bmatrix} \begin{bmatrix} \mathbf{V}_k \\ \mathbf{I}_k \end{bmatrix} \quad (2.44)$$

These equations represent the classical solution to the line equations in the frequency domain. It is also possible to separate the hyperbolic functions into real and imaginary parts.

2.3. Solution of the Transmission System Equations in the Time Domain

Frequency domain described lines should be converted into time domain for transient analysis. Time domain solution of transmission systems can be obtained by taking the inverse transform of the frequency domain equations:

$$\mathbf{i}_k = \mathbf{y}_c * \mathbf{v}_k - \mathbf{h} * \mathbf{u}_m \quad (2.45)$$

$$\mathbf{i}_m = \mathbf{y}_c * \mathbf{v}_m - \mathbf{h} * \mathbf{u}_k \quad (2.46)$$

where

$$\mathbf{u}_m = \mathbf{y}_c * \mathbf{v}_m + \mathbf{i}_m = 2\mathbf{i}_{mr} \quad (2.47)$$

$$\mathbf{u}_k = \mathbf{y}_c * \mathbf{v}_k + \mathbf{i}_k = 2\mathbf{i}_{kr} \quad (2.48)$$

Note that in the above equations convolution theorem is used, i.e., point-wise multiplication of two functions in frequency domain equals to convolution in time domain:

$$L^{-1}(\mathbf{YV}) = \mathbf{y} * \mathbf{v} \quad (2.49)$$

where the convolution of two functions at time t_1 is given as

$$\mathbf{y} * \mathbf{v} = \int_0^{t_1} \mathbf{y}(t) \mathbf{v}(t_1 - t) dt \quad (2.50)$$

Equation (2.50) can be computed to obtain the time domain solution for equations (2.45) and (2.46). Computation of (2.50) using a discrete integration scheme is called numerical convolution.

Numerical convolution requires excessive number of operations and time. As an example, in order to compute the convolution integral at time $t_1 + \Delta t$, all the history

values are required from $t = 0$ to $t = t_1$ where Δt is the integration step. Under certain circumstances convolution integral at time $t_1 + \Delta t$ can be computed by using only the history values at time t_1 . This is called recursive convolution. Therefore, for a model that will be incorporated to EMTP full numerical convolutions should be avoided and recursive convolution methods should be preferred in favor of reducing computational time.

The most practical approach is to obtain rational function approximations for $\mathbf{H}$ and $\mathbf{Y}_c$. This approach allows analytical calculation of convolution integrals with a recursive scheme. The transfer function approximation should be able to represent frequency dependent $\mathbf{H}$ and $\mathbf{Y}_c$ for a specified range of frequency defined according to the nature of transients. Fitting of characteristic admittance $\mathbf{Y}_c$ is relatively easy since $\mathbf{Y}_c$ is a smooth function of frequency. Fitting of $\mathbf{H}$ is complex and requires the extraction of a suitable time delay. The basic idea of many transfer function models is first to diagonalize the exponential propagation function with a proper time delay assigned to each eigenvalue, then to use these eigenvalues for the identification of the matrix. Generally speaking all eigenvalues of $\mathbf{H}$ are present in all matrix elements of $\mathbf{H}$.

All transfer function models are extensions of the simple ideal case of an undistorted wave propagating through a single conductor, by means of some elaborate mathematical manipulations. Therefore it would be very useful to examine a case of single phase, lossless transmission system in order to facilitate the comprehension of subsequent chapters.

2.3.1. Single Phase Lossless Transmission System Model with Constant L

Neglecting the losses ($R' = G' = 0$ in Figure 2.1) and frequency dependency of L' , the frequency domain equations become:

$$I_m = Y_c V_m - H(Y_c V_k + I_k) \quad (2.51)$$

$$I_k = Y_c V_k - H(Y_c V_m + I_m) \quad (2.52)$$

where,

$$Y_c = \sqrt{\frac{C'}{L'}} : \text{surge admittance (conductance)} \quad (2.53)$$

$$H = e^{(-j\omega\sqrt{L'C'}\ell)} : \text{propagation Constant} \quad (2.54)$$

Recalling the inverse transform of

$$e^{-cs} f(s) \xrightarrow{L^{-1}} f(t-c) \quad (2.55)$$

equations (2.51) and (2.52) can now be easily transformed back into time domain as follows

$$i_m = Y_c v_m - (Y_c v_k(t-\tau) + i_k(t-\tau)) \quad (2.56)$$

$$i_k = Y_c v_k - (Y_c v_m(t-\tau) + i_m(t-\tau)) \quad (2.57)$$

These equations represent Bergeron's method [24] and show that the present value of the quantities at node k is related to the past value of the quantities at node m and vice versa. This relation can be interpreted by saying that the quantity $Y_c v + i$ travels along the line in τ units of time with a velocity " a ". The time needed by this wave to travel from one extreme of the line to the other is

$$\tau = \sqrt{L' C'} \ell \quad (2.58)$$

As τ can also be expressed as the line length divided by velocity, the velocity of propagation can be found as

$$a = \frac{1}{\sqrt{L' C'}} \quad (2.59)$$

In the case of a lossless line with non frequency dependent parameters, the propagation constant can be directly related to a constant time delay.

$$H = e^{(-j\omega\tau)} \quad (2.60)$$

With these relations, the equivalent circuits shown in Figure 2.4 can be obtained. Consequently, the relation between current and voltage at node k at a given time t is known if the past values of the corresponding quantities at node m at τ units of time earlier have been recorded. The history terms in (2.56) and (2.57) are given by:

$$i_{kh} = Y_c v_m(t - \tau) + i_m(t - \tau) \quad (2.61)$$

$$i_{mh} = Y_c v_k(t - \tau) + i_k(t - \tau) \quad (2.62)$$

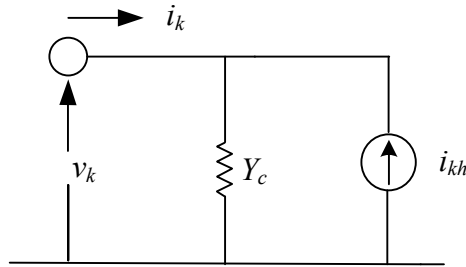


Figure 2.4 EMTP distributed parameter lossless line model for TD calculations

2.4. Frequency Dependent Parameters of Transmission Devices

We have seen that transmission systems in the frequency domain can be characterized by two frequency dependent matrix functions: propagation function $\mathbf{H}$ and characteristic admittance $\mathbf{Y}_c$ which are both functions of series impedance $\mathbf{Z}$ and shunt admittance $\mathbf{Y}$ matrices. Especially the series impedance matrix parameters, R' and L' , can be quite sensitive to variations in frequency. The resistance matrix $\mathbf{R}'$ (in Ohm/m) consists of the contributions from the conductor internal resistances and the earth path

resistance. The inductance matrix $\mathbf{L}'$ (in H/m) represents the internal inductance of the conductors, the earth path, and the external mutual inductances between phases.

In this work $\mathbf{Z}$ and $\mathbf{Y}$ matrices are assumed to be available and correct. Both matrices are obtained using EMTP-RV line and cable modules. These modules are called “line data” and “cable data” devices respectively and they generate files that contain numerical values of $\mathbf{Z}$ and $\mathbf{Y}$ matrices for a user defined set of frequency points. Readers willing to get more information about how these parameters are calculated can refer to [25]-[27] for transmission lines and [28]-[31] for underground cables.

In transmission lines for the simplest case of a conductor carrying an electrical current, the distribution of this current across the conductor is not uniform but depends on the frequency of the current. As the frequency increases the current is displaced towards the outside of the conductor (skin effect) changing the resistance and internal inductance. The analytical solutions describing the variation of these parameters are complex and involve Bessel functions. On the other hand for the simple case of a single conductor the most important component of the total impedance is the external inductance which is not affected by the frequency and studies that neglect frequency dependency can still produce reasonable results. This approximation however will not be valid for cables as external inductance has little contribution to the total impedance. In cables, resistance increases and inductance decreases with increasing frequency due to the skin effect.

Another important case for transmission lines is that of a phase conductor with ground return. In the case with the ground, which can be considered as a semi infinitely extended non-ideal conductor, the resistance and internal inductance constitute a large portion of the total impedance and the effect of the frequency is thus very important.

As seen in equation (2.5) shunt admittance has two parameters: capacitance C' and conductance G' . In transmission lines C' is not sensitive to variations in frequency and it is usually assumed to be constant for frequencies up to about 1 MHz. In underground

cables, C' depends on relative permittivity of insulation layers which can be assumed to be constant without loss of accuracy.

In transmission lines the shunt conductance G' is associated mainly with leakage at the insulators and its value is affected by a series of external factors such as humidity, pollution and maintenance. The correct value of G' is therefore difficult to assess with accuracy. Fortunately this value is very small compared to the other system parameters.

In cables the shunt conductance G' is determined by the dielectric dissipation power factor, capacitance and frequency according to the relationship

$$G' = \omega C' \tan \delta \quad (2.63)$$

where $\tan \delta$ is the dielectric dissipation factor.

2.5. Test Cases

Figure 2.5 illustrates a 230 kV single-core coaxial cable system as one of the test cases that will be used throughout the thesis (Case CAB01). Physical data of this system are given in Table 2.1. Self inductance and resistance of core and sheath conductors are given in Figure 2.6 and Figure 2.7. Mutual impedance between core and sheath is given in Figure 2.8 and Figure 2.9. Mutual impedances between phases are given in Figure 2.10 and Figure 2.11.

Figure 2.12 shows a 12 conductor single core coaxial cable system as another test case (Case CAB02). This case constitutes a short cable and physical data are given in Table 2.1. Frequency dependent parameters of this cable show a very similar characteristic to the previous example, therefore they are skipped. This case will be useful in the following chapters to show the difficulties in modeling short cables.

Figure 2.13 illustrates a 345 kV three phase overhead transmission line as another test case that will be used throughout the thesis. Physical data of this system are given in Table 2.3. As seen from the physical layout, transmission line test case has 2 conductors

per phase, 2 ground wires and ground return. However, this system can be reduced to a system with only one equivalent conductor per phase and a neutral return. The effect of the bundling of phase conductors, the presence of ground wires and the behavior of the ground return path are all included in the equivalent impedances and admittances of the reduced system. In this work line impedances and admittances are obtained from EMTP-RV in reduced equivalent form.

Self impedance parameters of the transmission line are illustrated from Figure 2.14 to Figure 2.17. Mutual impedance parameters between phases are shown in Figure 2.18 and Figure 2.19. To show the effect of ground conductors, each figure shows the parameters with ground conductors included in the system as shown in Figure 2.13 and with ground conductors eliminated from the system. Another purpose here is to provide the full details of the cases used in this thesis in order to allow researchers who may want to reproduce the cases in the future for comparison.

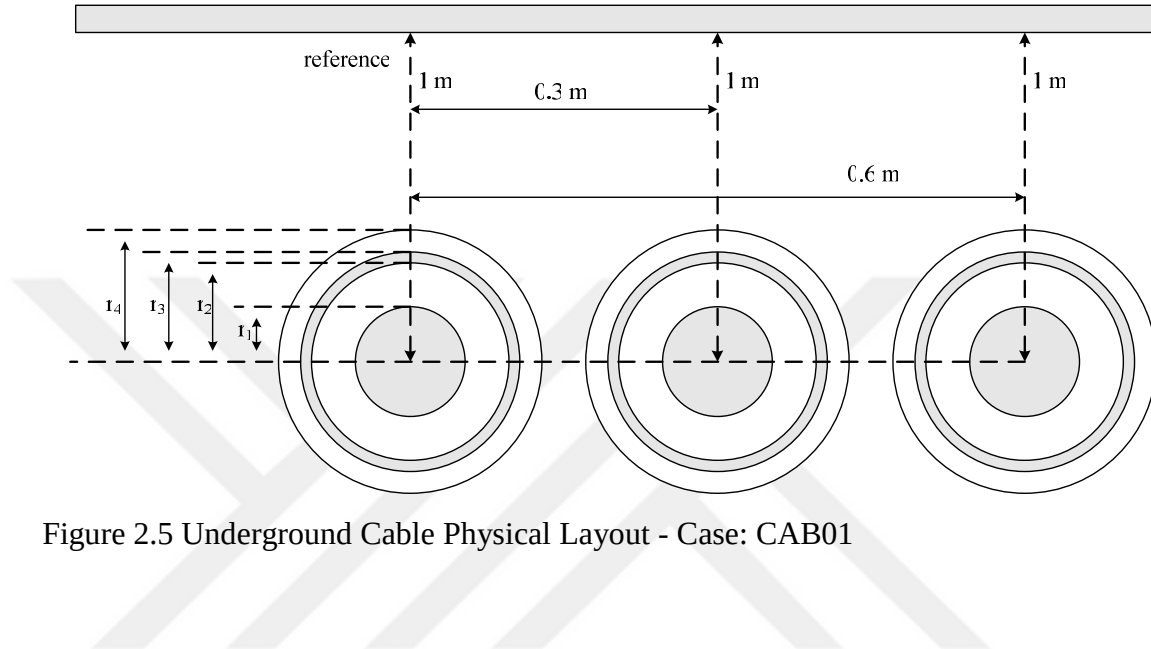


Figure 2.5 Underground Cable Physical Layout - Case: CAB01

Table 2.1 Physical Data of the 230 kV Underground Cable CAB01

Definition	Value
Inner Radius of the Core	0 mm
Outer Radius of the Core	19.50 mm
Inner Radius of the Sheath	37.75 mm
Outer Radius of the Sheath	37.97 mm
Outer Insulation Radius	42.50 mm
Resistivity of Core	1.718×10^{-8} Ohm m
Resistivity of Sheath	3.365×10^{-8} Ohm m
Relative Permeability	1
Insulator Relative Permeability	1
Core Insulator Relative Permittivity	2.85
Shield Insulator Relative Permittivity	2.51
Insulation Loss Factor	0.001
Cable Length	10 km
Earth Resistivity	100 Ohm m

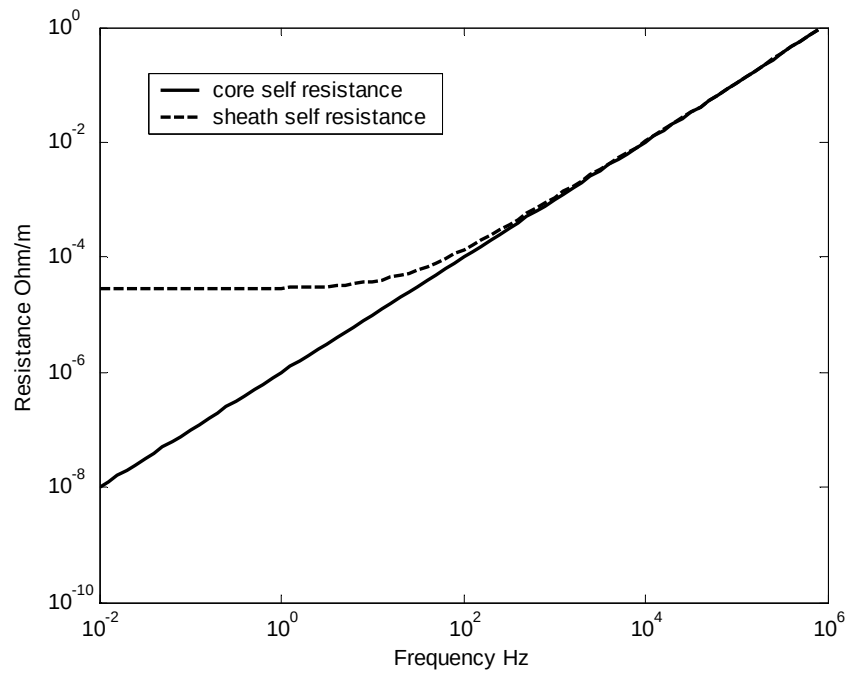


Figure 2.6 Self Resistance of Core and Sheath

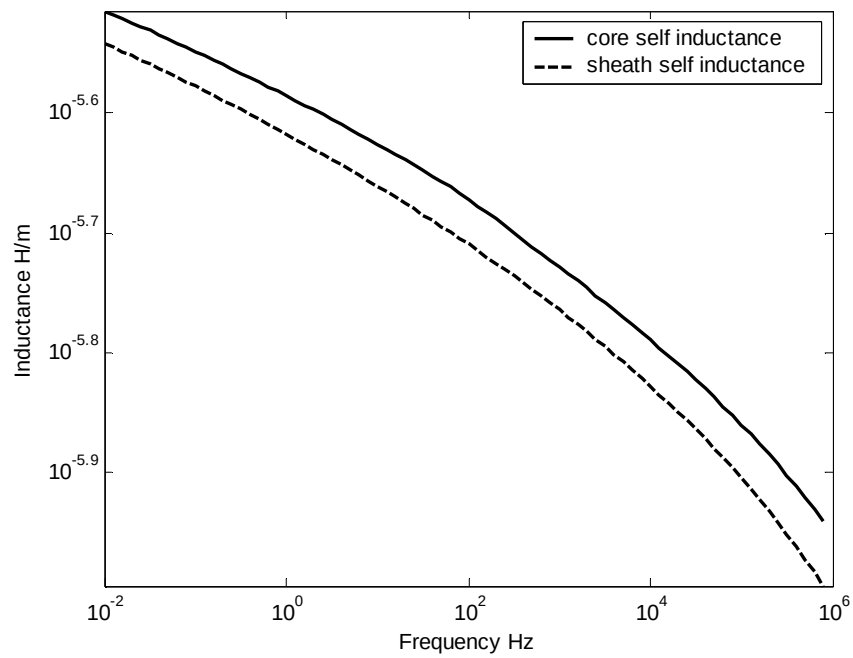


Figure 2.7 Self Inductance of Core and Sheath

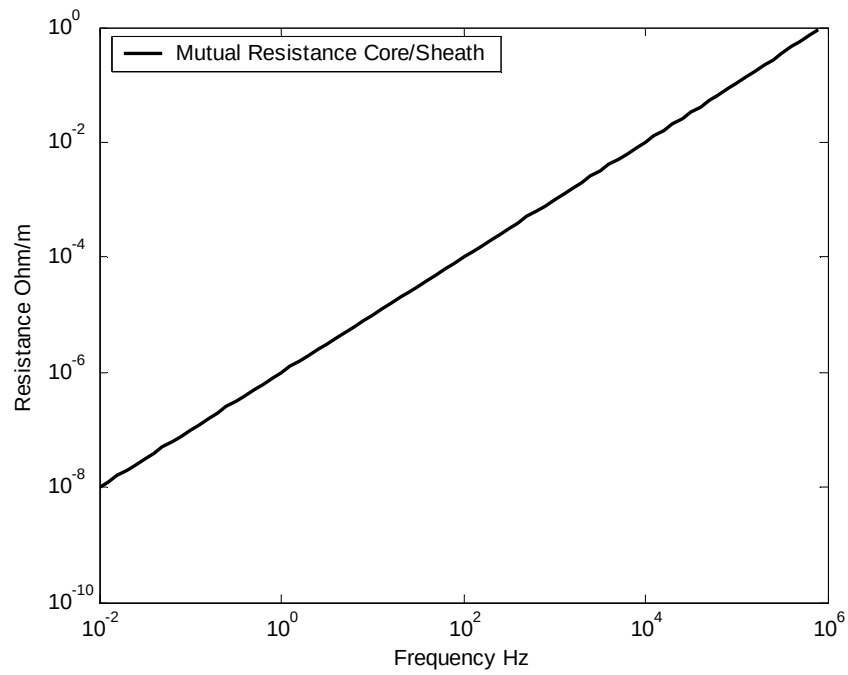


Figure 2.8 Mutual Resistance between Core and Sheath

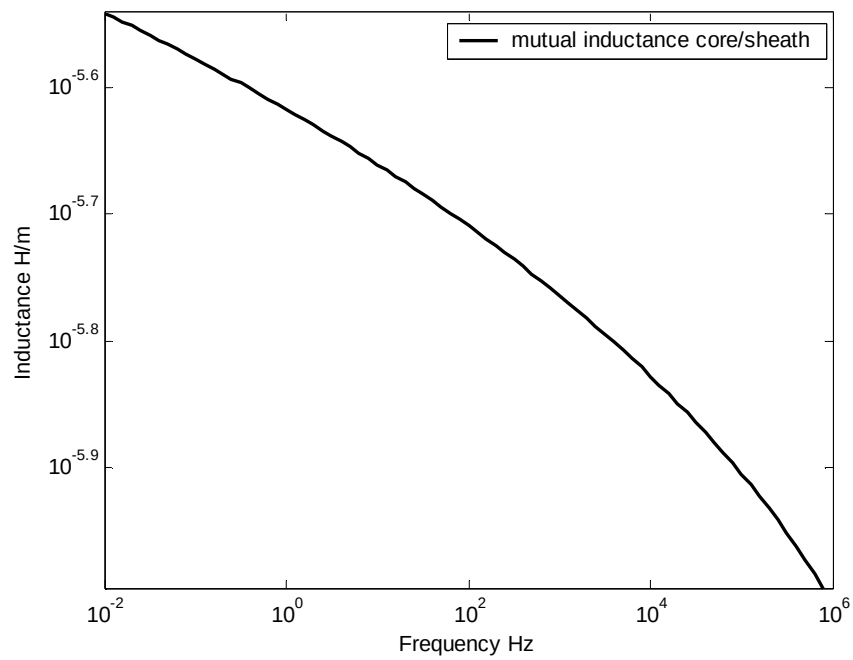


Figure 2.9 Mutual Inductance between Core and Sheath

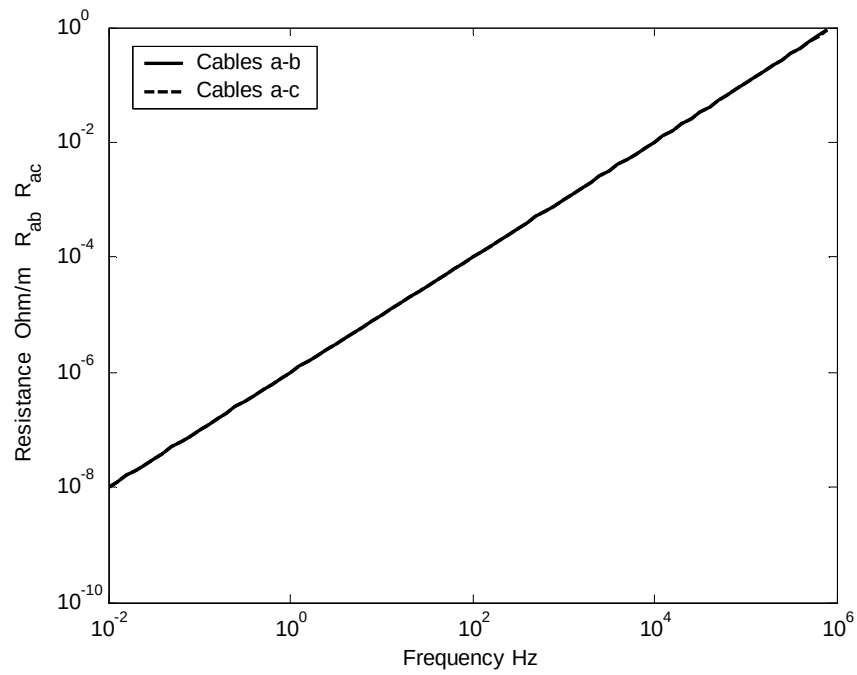


Figure 2.10 Mutual Resistances between Phases, overlapped

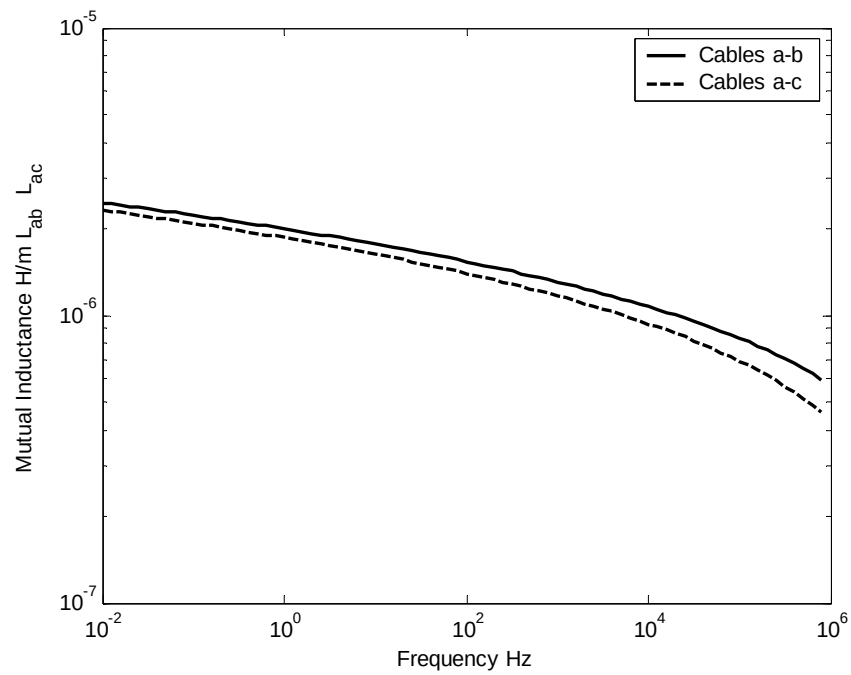


Figure 2.11 Mutual Inductances between Phases

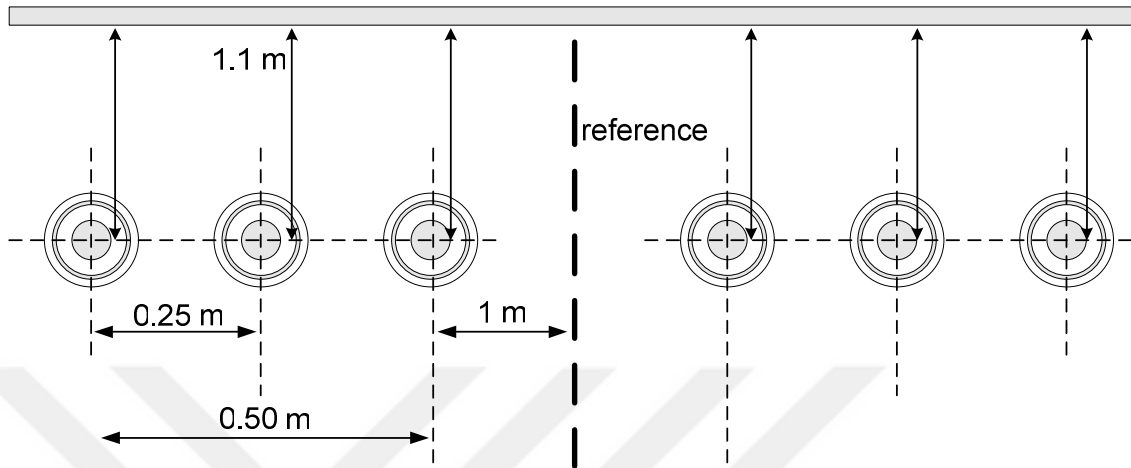


Figure 2.12 Underground Short Cable Physical Layout – Case CAB02

Table 2.2 Physical Data of the 12 conductor Short Cable CAB02

Definition	Value
Inner Radius of the Core	3.175 mm
Outer Radius of the Core	12.54 mm
Inner Radius of the Sheath	22.735 mm
Outer Radius of the Sheath	26.225 mm
Outer Insulation Radius	29.335 mm
Resistivity of Core	1.718×10^{-8} Ohm m
Resistivity of Sheath	3.365×10^{-8} Ohm m
Relative Permeability	1
Insulator Relative Permeability	1
Core Insulator Relative Permittivity	3.5
Shield Insulator Relative Permittivity	2.0
Insulation Loss Factor	0.001
Cable Length	1 km
Earth Resistivity	250 Ohm m

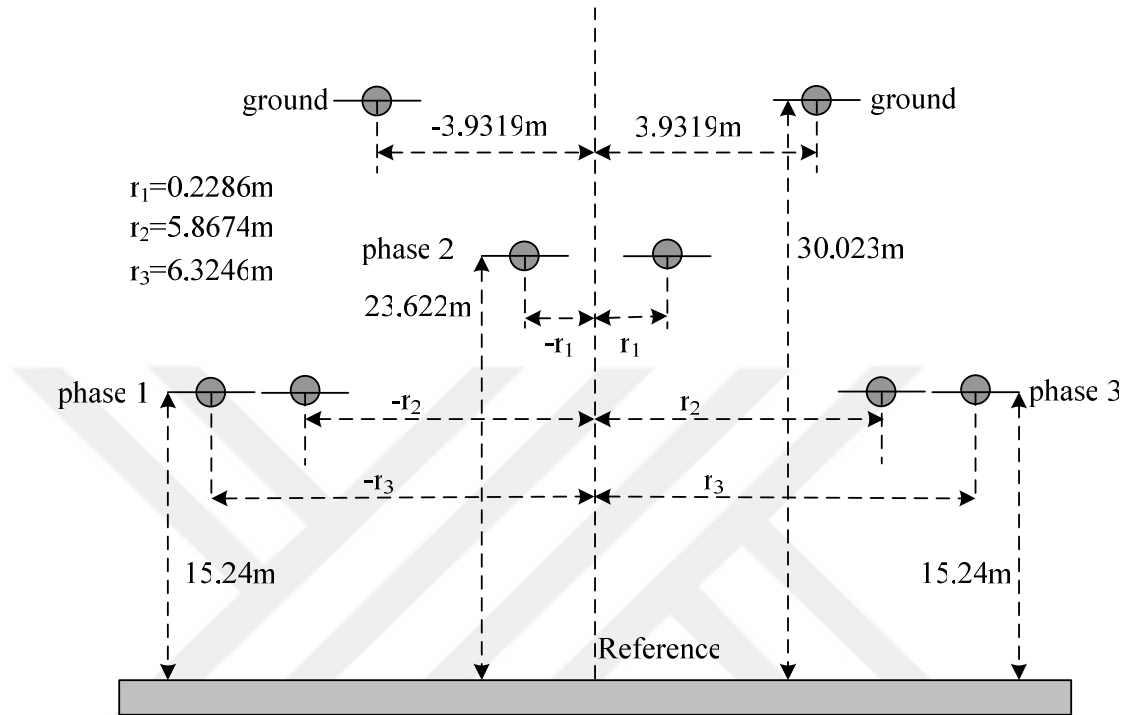


Figure 2.13 Overhead Transmission Line Physical Layout – Case TRL01

Table 2.3 Physical Data for the 345 kV Untransposed Transmission Line TRL01

Definition	Value
Outside Diameter of the Phase Conductors	4.06908 cm
Outside Diameter of the Ground Conductor	0.98044 cm
DC Resistance Phase	0.0324 Ohm/km
DC Resistance Ground	1.6216 Ohm/km
Model Frequency	60 Hz
Line Length	200 km
Earth Resistivity	100 Ohm m
Skin Effect Correction Factor - Phase	0.363 (Thick/Diam)
Skin Effect Correction Factor - Ground	0.5 (Thick/Diam)

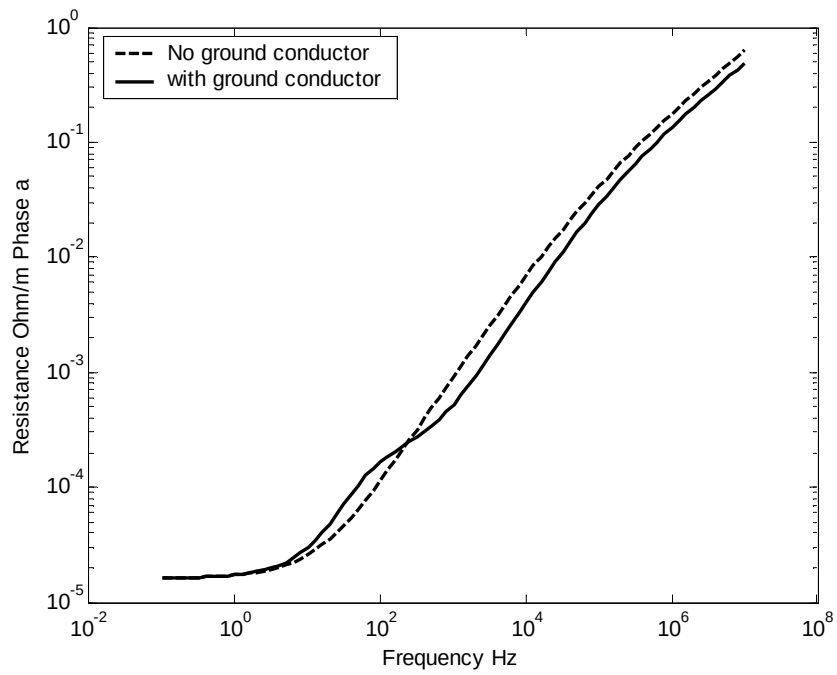


Figure 2.14 Resistance of Phase A, Transmission Line Test Case

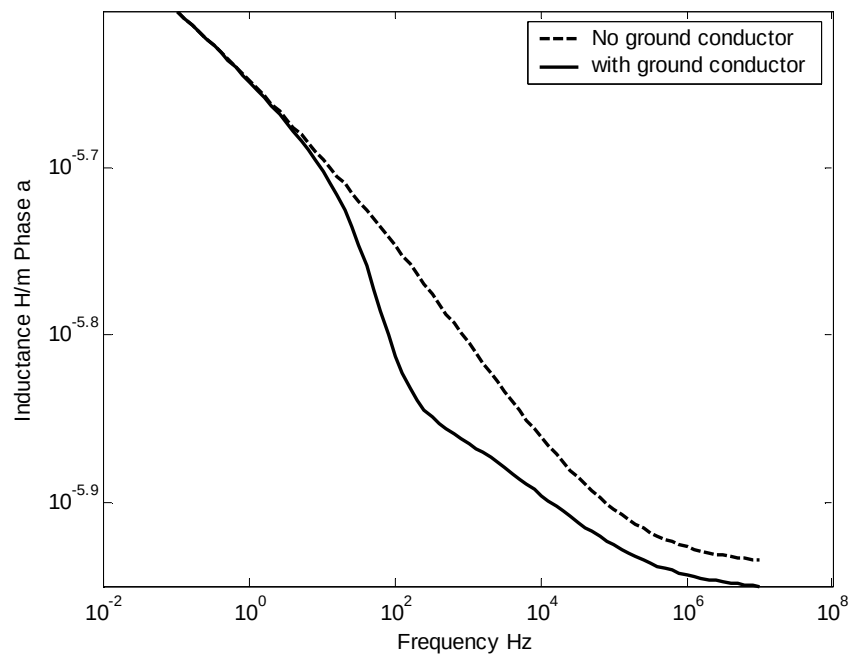


Figure 2.15 Inductance of Phase A, Transmission Line Test Case

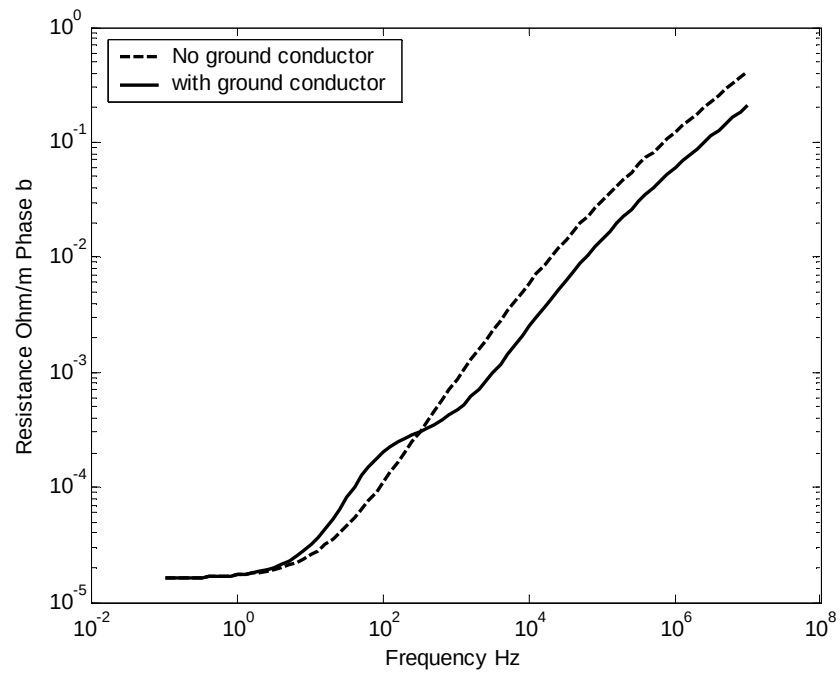


Figure 2.16 Resistance of Phase B, Transmission Line Test Case

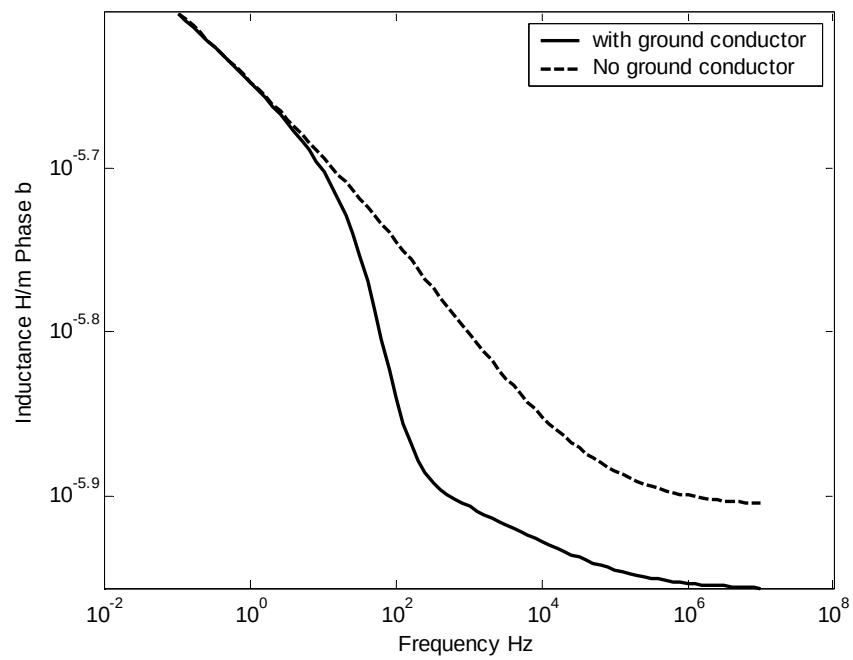


Figure 2.17 Inductance of Phase B, Transmission Line Test Case

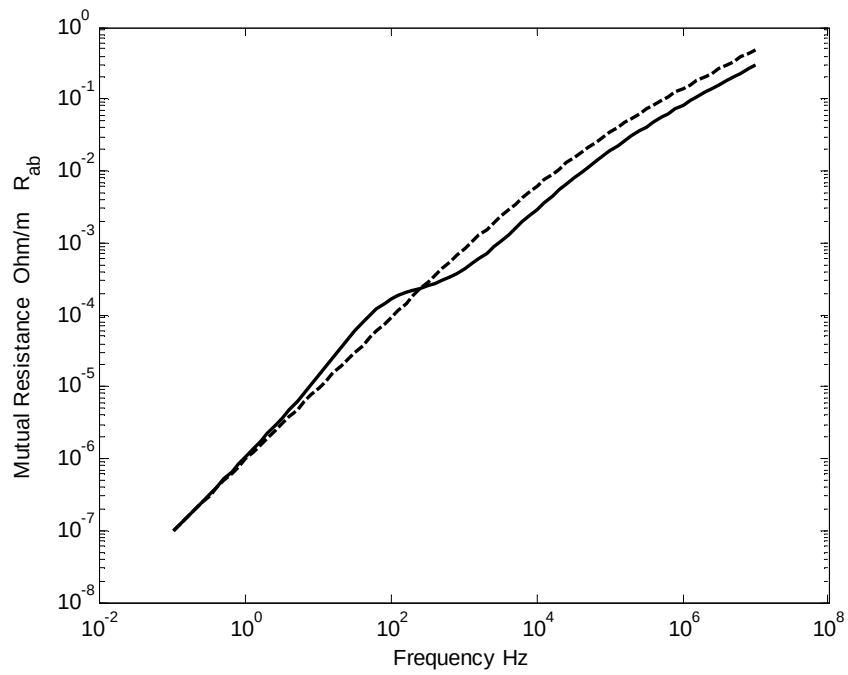


Figure 2.18 Mutual Resistance between Phase a and b, Transmission Line Test Case

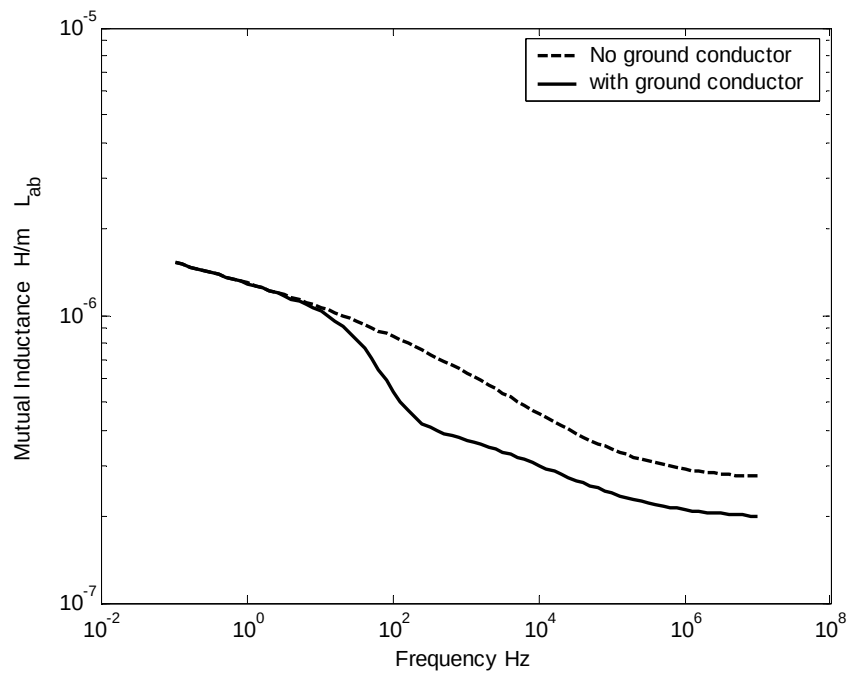


Figure 2.19 Mutual Inductance between Phase a and b, Transmission Line Test Case

CHAPTER 3.

LITERATURE REVIEW

Owing to the complexity of the frequency functions involved in the equations; these equations must be converted into the time domain by means of numerical methods. Several articles proposed numerical methods in order to obtain time domain solutions of transient phenomenon. Some important contributions are presented next.

3.1. Modal Domain Solutions

Equations (2.31) and classical equations (2.44) in the frequency domain can be decoupled through matrix transformations [5], [6] which allow transformation from phase domain into modal domain. Under certain conditions a square matrix $\mathbf{A}$ can be diagonalized by a similarity transformation of the form

$$\mathbf{A} = \mathbf{T}\mathbf{A}_M\mathbf{T}^{-1} \quad (3.1)$$

where $\mathbf{A}_M$ is a diagonal matrix of eigenvalues of $\mathbf{A}$ and $\mathbf{T}$ is a full matrix whose columns are the corresponding eigenvectors. In this work this transformation will be called modal transformation. Modal domain components are decoupled therefore each mode can be studied individually as a single phase system.

3.1.1. Early Line Models in EMTP

Early models given in [2], [23] and [32] are developed in modal domain. In [2] the line is analyzed as a two port network. From the classical line solution in the frequency domain for a single phase system

$$\begin{aligned} V_k &= \cosh(\gamma l)V_m - Z_c \sinh(\gamma l)I_m \\ I_k &= \frac{1}{Z_c} \sinh(\gamma l)V_m - \cosh(\gamma l)I_m \end{aligned} \quad (3.2)$$

If the line is seen as a two port network, the equations of this network can be written by nodal analysis:

$$\begin{aligned} I_k &= Y_{kk}V_k + Y_{km}V_m \\ I_m &= Y_{mk}V_k + Y_{mm}V_m \end{aligned} \quad (3.3)$$

where the admittances are given by;

$$\begin{aligned} Y_{kk} &= Y_{mm} = \frac{1}{Z_c} \coth(\gamma l) \\ Y_{km} &= Y_{mk} = \frac{1}{Z_c} \csc h(\gamma l) \end{aligned} \quad (3.4)$$

Taking the inverse Fourier Transform (FT), time domain equations can be written as follows:

$$\begin{aligned} i_k(t) &= y_{kk}(t) * v_k(t) + y_{km}(t) * v_m(t) \\ i_m(t) &= y_{km}(t) * v_k(t) + y_{kk}(t) * v_m(t) \end{aligned} \quad (3.5)$$

The y_{kk} and y_{km} functions are called weighting functions. When convolutions are evaluated using inverse FT i_k and i_m can be expressed in terms of history values

$$\begin{aligned} i_k(t) &= y_{kk}v_k(t) + i_{kh}(t) \\ i_m(t) &= y_{km}v_k(t) + i_{mh}(t) \end{aligned} \quad (3.6)$$

Time domain form of the admittances in (3.4) are calculated using a Fast Fourier Transform Program (FFT) during which numerical complications are encountered due to the nature of the discrete FT and due to the oscillatory form of the Y_{kk} and Y_{km} functions, moreover evaluation of the convolution integrals is itself a complex and time consuming process. An example is provided in [8].

In order to have simpler weighting functions, traveling functions are employed in [32] and later improved in [23] by analogy with the lossless, constant-parameter transmission line solution given in equations (2.56) and (2.57). In [23], characteristic impedance is chosen as a constant with a particular choice i.e. $R = \lim_{\omega \rightarrow \infty} Z_c(\omega)$. Although this method is much faster than the method in [2], some problems are encountered due to the behavior of the routines at low frequencies as the characteristic impedance is chosen a value at high frequencies. Moreover, employment of two weighting functions (analogous to propagation function) and their nature are also complex.

3.1.2. FD (Frequency Dependent) Line Model

One of the most widely used transmission line models is the FD line model [7], [8]. This model transforms the line equations into modal domain by using a constant transformation matrix. It is well worth to review the FD model in details as it constitutes a milestone for other models in this field. Let us first recall the second order differential equations in the frequency domain

$$\frac{d^2 \mathbf{I}}{dx^2} = (\mathbf{YZ}) \mathbf{I} \quad \text{and} \quad \frac{d^2 \mathbf{V}}{dx^2} = (\mathbf{ZY}) \mathbf{V}$$

with the solution

$$\mathbf{V}_k - \mathbf{Z}_c \mathbf{I}_{km} = \mathbf{H}^T (\mathbf{V}_m + \mathbf{Z}_c \mathbf{I}_{mk})$$

These equations are developed in phase domain, however, using certain mathematical operations they can be written in the modal domain. First let us assume that $\mathbf{ZY}$ matrix product can be diagonalized then it follows that $\mathbf{YZ}$ is also diagonalizable:

$$\mathbf{D}_{zy} = \mathbf{P}(\mathbf{ZY})\mathbf{P}^{-1} \tag{3.7}$$

Transposing both sides of (3.7),

$$\mathbf{D}_{zy}^t = (\mathbf{P}^{-1})^t (\mathbf{Y}^t \mathbf{Z}^t) \mathbf{P}^t \tag{3.8}$$

Since $\mathbf{Y}$ and $\mathbf{Z}$ are symmetric and $\mathbf{D}_{zy}$ is diagonal, then

$$\mathbf{D}_{zy} = (\mathbf{P}^{-1})^t (\mathbf{YZ}) \mathbf{P}^t \quad (3.9)$$

Let $\mathbf{Q}^{-1} = \mathbf{P}^t$, then (3.9) becomes

$$\mathbf{D}_{zy} = \mathbf{D}_{yz} = \mathbf{D} = \mathbf{Q}(\mathbf{YZ})\mathbf{Q}^{-1} \quad (3.10)$$

Using (3.7) and (3.10) differential equations can be easily transformed into modal domain which results in decoupled equations. When the equations are transformed to modal domain the following change of variables occurs

$$\mathbf{H}_{mode} = \mathbf{P} \mathbf{H}^T \mathbf{P}^{-1} \quad (3.11)$$

$$\mathbf{Z}_{c,mode} = \mathbf{P} \mathbf{Z}_c \mathbf{Q}^{-1} \quad (3.12)$$

$$\mathbf{V} = \mathbf{P} \mathbf{V}_{mode} \quad (3.13)$$

$$\mathbf{I} = \mathbf{Q} \mathbf{I}_{mode} \quad (3.14)$$

In FD model, approximate real and constant transformation matrices for modal transformation are used. This allows transforming modal voltage and current quantities to phase domain or vice versa easily either in the frequency domain or in the time domain. If frequency dependent transformation matrices were used, convolution of transformation matrix with voltage or current would be required for modal to phase transforms in the time domain.

Modal domain can be assumed to describe a multiphase system as a superposition of single phase systems. As the necessity of describing the variables with vectors is no longer required we can rewrite the equations for an arbitrary mode as follows:

$$V_k - Z_c I_{km} = H(V_m + Z_c I_{mk}) \quad (3.15)$$

or as follows:

$$I_{km} = Y_c V_k - H(Y_c V_m + I_{mk}). \quad (3.16)$$

This is very similar to Bergeron's equation except for the H factor, and that equation (3.16) is in the frequency domain here instead of time domain. Equation (3.15) can be transformed into time domain with convolutions

$$v_k - Z_c * i_k = h * (v_m + Z_c * i_m) = 2h * i_{mr} \quad (3.17)$$

Before proceeding further, it may be worthwhile to look at the relationship between the equations in the frequency and time domain for the simple case of a lossless line. In that case

$$Z_c = \sqrt{\frac{L'}{C'}} \quad \gamma = j\omega\sqrt{L'C'} \quad H = e^{-\gamma\ell} \approx e^{-j\omega\tau} \quad (3.18)$$

As discussed before a phase shift of $e^{-j\omega\tau}$ in the frequency domain will become a time delay τ in the time domain. Besides, Z_c is now just a constant and (3.16) therefore transforms to

$$v_k(t) - Z_c i_{km}(t) = v_m(t - \tau) + Z_c i_{mk}(t - \tau) \quad (3.19)$$

which is indeed Bergeron's equation (2.57).

For the general lossy case, propagation function

$$H = e^{-\gamma\ell} = e^{-\alpha\ell} e^{-j\beta\ell}$$

with $\gamma = \alpha + j\beta$, contains an attenuation factor $e^{-\alpha\ell}$ and a phase shift $e^{-j\beta\ell}$, which are both functions of frequency.

In [8] it is shown that it is best to approximate H and Z_c by rational functions directly in the frequency domain. Both functions can then be written explicitly as a sum of exponentials in the time domain without any need for numerical inverse Fourier

transformation. Implementation of this work in EMTP is the FD line model where propagation function and characteristic impedance are approximated as rational functions by using Bode type asymptotic fitting technique. In order to avoid possible confusions it is important to remind that propagation function is named as “weighting function” in [8].

In FD line model each element of modal propagation function, H , is approximated as a minimum phase shift function $h_{\min}$ plus a constant time delay $\tau_{\min}$ in the form of

$$H \cong h_{\min} e^{(-s\tau_{\min})} \quad (3.20)$$

A reduced order Minimum Phase Shift (MPS) equivalent of H is found only from the magnitude data by applying Bode type asymptotic fitting technique. Then time delay is calculated by comparing the given phase angle with that of the identified equivalent. Even after a time delay is removed from H , the remaining part is still a frequency dependent exponential function and it may not be a minimum phase shift function. In general advantage of an MPS function is that rational approximation with low order is achievable compared to non minimum phase functions

In the FD line model the MPS property is ensured by allowing only negative and real zeros during the asymptotic fitting process. However, this method requires much higher order of approximation compared to least squares identification methods.

The rational function which approximates H has the form

$$H(s) = e^{-s\tau} k \frac{(s+z_1)(s+z_2)....(s+z_n)}{(s+p_1)(s+p_2)....(s+p_m)} \quad (3.21)$$

All poles and zeros are negative, real and simple. Given that $n < m$, the rational function part of (3.21) can be expanded into partial fractions,

$$k \frac{(s+z_1)(s+z_2)....(s+z_n)}{(s+p_1)(s+p_2)....(s+p_m)} = \frac{k_1}{s+p_1} + \frac{k_2}{s+p_2} + ... + \frac{k_m}{s+p_m} \quad (3.22)$$

The corresponding time domain form of H becomes

$$h(t) = \begin{cases} k_1 e^{-p_1(t-\tau)} + k_2 e^{-p_2(t-\tau)} + ... + k_m e^{-p_m(t-\tau)} & \text{for } t \geq \tau \\ 0 & \text{for } t < \tau \end{cases} \quad (3.23)$$

This function is used to calculate the history terms at each time step. If we look at the contribution of one exponential term in (3.17)

$$y_i(t) = \int_{\tau}^{\infty} i(t-u) k_i e^{-p_i(t-\tau)} du \quad (3.24)$$

then we can show that $y_i(t)$ can be directly obtained from the value $y_i(t-\Delta t)$ from the previous time step with a recursive integration scheme [4] using a particular type of difference formulation such as trapezoidal or Backward Euler. The resulting equation becomes [8]

$$y_i(t) = c_1 y_i(t-\Delta t) + c_2 i(t-\tau) + c_3 i(t-\tau-\Delta t) \quad (3.25)$$

with c_1, c_2, c_3 being constants which depend on the particular type of difference formulation and interpolation used for current i .

In a similar manner the characteristic impedance is approximated by a rational function of the form

$$Z_c \cong k \frac{(s+z_1)(s+z_2)....(s+z_n)}{(s+p_1)(s+p_2).....(s+p_n)} \quad (3.26)$$

All poles and zeros are again real, negative and simple, but the number of poles is equal to the number of zeros now due to asymptotic properties of the characteristic impedance. This can be again expanded into partial fractions

$$Z_c = k_0 + \frac{k_1}{s + p_1} + \frac{k_2}{s + p_2} + \dots + \frac{k_n}{s + p_n} \quad (3.27)$$

This function is again used to evaluate $Z_c * i$ convolution integrals in a recursive manner.

Bode's procedure for approximating the magnitude of a rational function is explained in [8] (pp. 70-73). The basic principle of the procedure is to allocate poles and zeros by tracking the magnitude of the original function. A new pole or zero is allocated whenever the asymptote of the fitting function deviates from the original one by more than a predefined tolerance. Accuracy is further improved by relocating the poles and zeros about their first positions. Only real poles and zeros located in the left half plane are used therefore the rational function approximation is of minimum phase shift type.

Note that, asymptotic fitting technique is limited to smooth functions of frequency and requires higher order of approximation compared to least squares identification methods. There are also computational difficulties such as: a lot of parameters to control and determining the order of approximation.

In the FD line model the characteristic impedance and weighting function are formulated using only real and negative poles to avoid ripples and peaks in the frequency response of the functions. Although this is generally acceptable for the case of transmission lines, there may be cases where ripples and peaks occur. In this case, employment of complex conjugate poles will not only result in better fitting but also will reduce the order of the approximation. On the other hand, when all poles and zeros are real, negative and simple the resulting rational functions are guaranteed to be passive and this is an advantage in terms of numerical stability in the time domain.

The FD line model ignores the frequency dependence of the modal transformation matrices and assumes them to be real and constant. The benefit of this assumption is to obtain high computational efficiency, but it can lead to erroneous results in the cases of

strongly asymmetrical overhead lines or underground cables where the elements of the modal transformation matrices strongly depend on frequency.

Sometimes, this model is also unable to simulate low frequency coupling effects as explained in [33].

Here are some remarks about the FD Line implementation in EMTP-RV

1. FD Uses frequency dependent transformation matrices evaluated at each frequency sample in order to calculate modal H and Z_c parameters to be fitted
2. A real and constant transformation matrix is used for switching between modal and phase variables (Current and Voltage) while evaluating transients in time domain
3. Frequency dependence of propagation function H and characteristic admittance Z_c are taken into account by representing them as rational transfer functions
4. Rational transfer functions are identified using Bode type asymptotic fitting
5. Frequency domain equations are converted into time domain and can be represented as analytical functions due to rational forms of line parameters. Resulting convolution integrals are solved recursively by using trapezoidal approximation.
6. FD model automatically switch to halved time step Backward Euler when rapid changes in the input are observed such as during switching. This is done in order to avoid artificial oscillations due to trapezoidal method.

3.1.3. FDQ Cable Model

The full frequency dependent FDQ cable model given in [9] and [10] solves the problem of a strongly frequency dependent transformation matrix by fitting the elements

of this matrix with rational functions in the frequency domain using previously mentioned Bode type asymptotic fitting. Then relation between phase domain and modal domain can be written as follows:

$$\mathbf{V} = (\mathbf{Q}_{rat}^t)^{-1}(s)\mathbf{V}_{mode}$$

$$\mathbf{I} = \mathbf{Q}_{rat}(s)\mathbf{I}_{mode}$$

where $\mathbf{Q}_{rat}(s)$ is a rational approximation of the transformation matrix $\mathbf{Q}$. Accordingly in the time domain, the following convolutions becomes necessary:

$$\mathbf{i}(t) = \mathbf{Q}_{rat}(t) * \mathbf{i}_{mode}$$

$$\mathbf{Q}_{rat}^T(t) * \mathbf{v} = \mathbf{v}_{mode}.$$

This model can give good results for cable simulation in low and high frequency ranges. However, in order to implement the recursive convolutions for the purpose of updating the history sources of the equivalent model circuits, all elements of the frequency dependent propagation matrix, characteristic admittance matrix and modal transformation matrices should be fitted with rational functions in the frequency domain. This results in a large number of operations in the time domain convolutions. Although this model gives accurate results for cable systems, it is in general not applicable to overhead lines because the modal decomposition is sometimes unstable. Hence, there is not a guarantee for all cases that the numerical stability of the rational functions for the frequency dependent transformation matrices can be satisfied. It is not rare that this model numerically explodes in EMTP after a certain number of time steps.

3.2. Phase Domain Models

The problem of a frequency dependent transformation matrix can be overcome by formulating the model directly in the phase domain. With this approach it is necessary to

fit the matrix elements of the propagation function $\mathbf{H}$ and the characteristic impedance $\mathbf{Y}_c$ in the phase domain. The elements of $\mathbf{Y}_c$ are smooth functions of frequency and can be easily fitted. The fitting of $\mathbf{H}$ is more difficult as its elements contain modal contributions with different time delays.

In this part several phase domain models will be briefly presented whereas the Universal Line Model [11] which is implemented as WB model in EMTP-RV, will be investigated in full in section 3.2.2.

In [34] a traveling wave model for overhead lines and cables is proposed. In this model convolutions are performed in the phase domain, therefore frequency dependence of the modal transformation matrices is taken into account. In this model, the tail portion of the time domain step responses are fitted with piecewise linear functions while the front portion is fitted in its detailed form. The propagation and characteristic admittance matrices can be found by differentiating the front portion numerically and the tail portion analytically for convolution operations. The convolution integrals can then be evaluated using a combination of numerical and recursive convolution. This model has a problem of poor efficiency compared to other models.

In [35] a method called ‘Two-Sided Recursions (TSR)’ is proposed, which performs short convolutions with both input and output variables instead of using convolutions of the input variables only. TSR coefficients are identified directly from transfer functions over a wide range of frequency without any intermediate procedures such as rational functions. However, stability of the TSR coefficients is affected by the selection of the fitting frequencies and the weighting factors at each frequency. Phase domain TSR method is suspicious from the stability point of view and it requires much more computational effort compared to rational function fitting.

In [36] an overhead line model based on fitting all elements of the characteristic impedance and propagation matrices directly in the phase domain is introduced. As fitting of the frequency dependent modal transformation is avoided, this model requires

fewer convolutions at each time step. The main assumption of this model is that elements of the phase domain propagation matrix can be fitted using stable minimum phase transfer functions with only single time delay attributed to each element. However, each element of the propagation matrix contains modal contributions which have different time delays. Thus, uncompensated part of the time delays may result in an oscillating behaviour in the frequency domain. Although application of one time delay is possible for most practical cases of overhead lines because different modal time delays are usually close to each other, for lines with high ground resistance a large number of oscillations may occur, thus requiring a high order of fitting. For cable configurations, use of a single time delay would be a serious mistake as different modal time delays are apart from each other.

3.2.1. Modal Time Delays in Phase Domain

The problem of different time delays in $\mathbf{H}$ can be solved by including modal time delays in the phase domain. Four line models have been proposed to include modal time delays in the phase domain.

In [12] the ARMA Model (Auto-Regressive Moving Average) is introduced. This model has been implemented in ATP-EMTP and it is based on fitting the elements of $\mathbf{H}$ (and $\mathbf{Y}_c$) in the z domain, with time delays obtained from the modes. As the model uses z -domain fitting it is dependent on time step and is not applicable for an arbitrary time step. The authors have solved this restriction by introducing an IARMA (Interpolated ARMA) model [13], which allows each ARMA model to use a different time step according to its desired frequency response.

Idempotent line model [37] expands $\mathbf{H}$ into a sum, where each term is given as an idempotent matrix $\mathbf{M}$ multiplied with a corresponding mode. The rational approximation of $\mathbf{H}$ is obtained by fitting the modes and the idempotent. However, it is found that [33] in some cases of overhead line modeling, unstable poles are required in order to achieve accurate fitting of $\mathbf{M}$.

The polar decomposition model [38] has a kernel (diagonal matrix) containing poles and time delays from the modes, which is multiplied by two rectangular matrices **C** and **B**. The difficulty with this model is the identification of **C** and **B** due to nonlinear nature of the problem. Although **B-C** iterations may give accurate results, optimal solution is not guaranteed.

The Universal Line Model (ULM) is introduced in [11]. This model has been found to give highly accurate results and was selected as basis for a new standard model for EMTP-RV (Wideband Model - WB). This model identifies **H** in two steps. First poles and time delays are calculated from the modes. For modal calculations complex and frequency dependent transformation matrices are used. Then unknown residues are calculated. All poles are assumed to contribute to all elements of **H**, but without the usage of idempotent or a pole sharing kernel.

3.2.2. Universal Line Model

Modal decomposition of **H** can be expressed by extracting the time delays as follows

$$\mathbf{H}(\omega) = \mathbf{T} \mathbf{e}^{-\Lambda(\omega)\ell} \mathbf{T}^{-1} = \sum_{m=1}^M \Gamma_m(\omega) e^{-\lambda_m(\omega)\ell} = \sum_{m=1}^M \mathbf{H}'_m(\omega) e^{-j\omega\tau_m} \quad (3.28)$$

where Λ is diagonal, **T** is the eigenvector matrix, and Γ_m is the square matrix which results from multiplying the m th column of **T** with the m th row of $\mathbf{T}^{-1}$. τ_m is a constant time delay associated with mode m . Then each matrix of modal contribution to **H** can be written as

$$\mathbf{H}_m(\omega) = \Gamma_m(\omega) e^{-\lambda_m(\omega)\ell} = \Gamma_m(\omega) e^{-\lambda'_m(\omega)\ell} e^{-j\omega\tau_m} \quad (3.29)$$

One can find an approximation for $\mathbf{H}$ by fitting each modal contribution which is designated by $\mathbf{H}_m$. However, in the ULM paper [11] it is assumed that the frequency variation in $\mathbf{\Gamma}_m$ is always accompanied by a frequency variation in $e^{(-\lambda'_m \ell)}$ and therefore it is postulated that $\mathbf{H}$ can be accurately fitted in phase domain using only the poles obtained from the fitting of modal propagation function (H_m). This assumption is useful to reduce the computational effort and the order of approximation. Let us rewrite the modal propagation function using the above notation

$$H_m = e^{(-\lambda'_m \ell)} \quad (3.30)$$

$$H'_m(s) = H_m e^{(s \cdot \tau_m)} = e^{(-\lambda'_m \ell)} \quad (3.31)$$

ULM identifies every H_m as a rational function approximation using the fitting technique in [39] as follows

$$H_m \cong e^{-(s \tau_m)} \sum_{n=1}^N \frac{\hat{c}_n}{s + p_{mn}} \quad (3.32)$$

Note that modal time delays are removed to compensate the excessive phase lag of modes which constitutes an obstacle for low order identification. The extraction also impacts on the convergence of the identification process. Keeping the poles of propagation modes as known variables, the residues for phase domain fitting of each matrix element ij of $\mathbf{H}$ can be calculated by

$$H_{ij}(s) = \sum_{m=1}^M \left(\sum_{n=1}^{N_m} \frac{c_{ijnm}}{s + p_{nm}} \right) e^{-(s \cdot \tau_m)} \quad (3.33)$$

where $s = j\omega$, M is the number of propagation modes, and N_m is the order of PFE corresponding to propagation mode m , and τ_m is the time delay of mode m . The matrix element residues $c_{ij_{nm}}$ are found from a direct linear algebra solution.

Equation (3.33) assumes that the frequency dependent contribution of modes to each matrix element can be adjusted with residues. In other words the influence of the transformation matrix can be compensated with residues. Even if the presence of some modes is weak in certain elements of $\mathbf{H}$, these modes are written in (3.33). These redundant modes may improve the phase domain fitting performance.

3.2.3. Pitfalls in the Universal Line Model

As the mathematical description of $\mathbf{H}$ in (3.28) suggests, all eigenvalues are present in all elements of $\mathbf{H}$ as well as their poles given in (3.32). But how to define a reduced order of modes that is sufficient to identify $\mathbf{H}$? When fitting $\mathbf{H}$ using the ULM scheme, there is no direct relationship between the accuracy of the modal fitting and the final result provided by the phase domain fitting. This ambiguity makes programming of ULM complicated.

Another problem that is encountered is related to the linear solution of multi delay equation. Note that (3.33) is an overdetermined problem containing multiple delay groups and the problem is solved by using the least squares method. The only criterion is the minimization of the difference between fitted and calculated responses. Even though the combination of delay groups fits $\mathbf{H}$ perfectly, PFE of separate delay groups in (3.33) does not necessarily match the isolated contribution of each mode which can be calculated through (3.29). Consequently, the solution may come up with delay groups having unexpectedly high magnitude curves with high residue pole ratios. This may lead to numerically unstable models in the time domain as will be explained in Chapter 5. This situation is especially encountered in multiconductor short cables where time delays of separate modes can come very close.

The fitting technique used in ULM [39] is a two-stage linear least-squares procedure with iteration; called vector fitting (VF). VF is able to identify PFE models in (3.32) for complex frequency responses with many peaks. The VF technique has a good minimization characteristic as long as its equations are well-conditioned and the starting poles are properly chosen. When the VF method equations become ill-conditioned for rather broad frequency ranges, it creates problems for low order rational fitting in addition to needlessly increasing the number of iterations. These problems will be discussed in details in the following chapters where new techniques that cure the problems will be proposed.

Although the WB model has some disadvantages, it is the most general and accurate standard model available in EMTP. Therefore it is selected as the theoretical basis in this work.

3.2.4. Sectionalized Line Models

A sectionalized line model was proposed in [40] for three phase overhead lines with frequency dependent parameters and corona. In this model the overhead line is subdivided into a number of short segments each with a small traveling time. The model contains two different types of blocks for each segment a) a longitudinal block for the modeling of frequency dependence, and b) a transversal block for the modeling of the nonlinear effect of corona. Computations are carried out in the modal domain for the longitudinal block and in the phase domain for the transversal branches, which include corona in each phase. The traveling time delay in the longitudinal block decouples the nodes for the corona branches, and it allows the representation of the line segment by Norton equivalents at its ends. There are two drawbacks of this model: 1) the model introduces errors due to constant transformation matrix usage, 2) the line discretization and corona modeling used in this model requires approximately three times more computational time than that of usual frequency dependent line representation.

In [41], a frequency dependent distributed parameter line model is developed by combining an ideal lossless distributed parameter line and a lumped impedance circuit which takes into account the frequency dependent effects. The impedance circuit is derived from four terminal parameters theory. A lattice circuit with two or three branches is used to represent the frequency dependent effect. The distributed line can be a frequency independent lossless line of which the parameters are given at very high frequency. The lumped impedance circuit Z_n may be subdivided into: $Z_n/4$ connected to the sending end receiving ends, and $Z_n/2$ connected at the center of the ideal distributed line in order to represent the distributed nature of the losses. The distributed line can also be subdivided into more segments with more complicated analysis in order to increase accuracy. However, this model needs a better approximation of the oscillating waveform during the transient period especially for the sheath voltage.

The zLine model is introduced in [42] and [43] for frequency dependent overhead lines. The basic concept of the zLine model is to subdivide the representation of the total length line into a number of segments, each segment containing two parts a) the ideal line section with constant parameters, and b) the loss section with frequency dependent parameters. This model can be formulated directly in the phase domain thus avoiding the use of modal transformation matrices. Its limitation is that the line has to be sectionalized into a reasonable number of segments so as to simulate the actual distributed nature of losses. This model has also a disadvantage of poor computational speed.

This model is further extended to cables as zCable model [44] which separates the representation of the wave propagation into two parts: a constant ideal propagation and a frequency dependent distortion. A pi circuit correction is proposed to solve the problem of different traveling times in the ideal line propagation. This model is extremely slow so that it takes almost 20 times more time to simulate the same configuration in the TD compared to full frequency dependent models such as FDQ. Moreover, this model results in complicated equivalent circuits.

Perhaps the most important drawback of zLine and zCable models is the fact that the total cable length is subdivided into a number of short segments to simulate the distributed nature of losses. The interpolation error in each segment can accumulate and lead to a severe error for the complete line or cable solution.

As a conclusion, sectionalized line models are found to be computationally inefficient. Therefore, for the proposed research no more attention will be given to sectionalized line models.

3.2.5. Finite Difference Time Domain (FDTD) Line Models

In addition to traveling wave methods, one can use finite differences to produce a TD line formulation in time and space. Finite differences-based models which attempt to solve partial differential equations directly in the TD are based on the voltage and current distributions inside the line. These formulations are multidimensional and computationally extensive. However, they might be more convenient than the traveling wave methods in some special cases when the lines under analysis have to be subdivided such as in segmentation models [45], [46].

Although R' , L' , G' and C' are analytically complicated functions (Bessel functions etc.) they are smooth functions of frequency like the characteristic impedance so they can be easily and very accurately fitted by rational functions. Therefore in the time domain counterpart one can obtain partial differential equations that can be efficiently solved. This will be especially useful for wave propagation animation, non uniform lines, lines subjected to non uniform external electromagnetic fields, cross bonded cables and lines under corona.

One of the recent research fields is to develop animations for viewing the transient phenomena in transmission lines [47]-[49]. Direct FDTD modeling may be more useful for this purpose than traveling wave methods. By using direct TD approach transients of

non-homogeneous lines, 3 phase lines, lines subjected to external EMF, or any other line configuration can be animated.

A recent work [45] can be a basis for further research in this field. This paper presents a finite difference-based model intended for complementing the traveling wave methods for studies requiring line subdivision. The new model is based on the transmission line equations below proposed by Radulet, et al. [50], as well as on the method of characteristics of partial differential equations (PDEs) [51], [52].

$$\begin{aligned} \frac{\partial \mathbf{v}}{\partial x} + \mathbf{L}_0 \frac{\partial \mathbf{i}}{\partial t} + \frac{\partial}{\partial t} \int_0^t \mathbf{R}'(t-\tau) \mathbf{i}(\tau) d\tau &= \mathbf{v}_s \\ \frac{\partial \mathbf{i}}{\partial x} + \mathbf{C}_0 \frac{\partial \mathbf{v}}{\partial t} + \frac{\partial}{\partial t} \int_0^t \mathbf{G}'(t-\tau) \mathbf{v}(\tau) d\tau &= \mathbf{i}_s \end{aligned} \quad (3.34)$$

The adopted line equations require determining two matrix functions; namely, the transient resistance matrix $\mathbf{R}'(t)$ and the transient conductance matrix $\mathbf{G}'(t)$. These matrices are identified in the frequency domain and synthesized by using VF. The line equations are solved numerically in the TD by using a mesh prescribed by their characteristic curves. Convolution operations are incorporated into this mesh through a recursive scheme. The resulting model possesses some notable features. One feature is that matrix function synthesis is much simpler than in the traveling wave models. Whereas the latter require synthesizing two matrix functions (namely, the characteristic admittance and the exponential propagation matrices), the new model usually requires only the synthesis of $\mathbf{R}'(t)$. The rational fitting of this function is straightforward due to its smoothness in the FD and there is no need to extract time delays. A second feature is that only two matrix vector convolutions are required per line section and per time step. Traveling wave models require instead four such operations. It is obvious that when there is no need for line subdivision, this second feature ceases to be an advantage of the new model. A third feature is its ability to include distributed sources in an effective manner.

If the line does not need to be subdivided, then direct TD modeling is computationally more extensive than the traveling wave models. Further research effort is also required to efficiently apply FDTD models to cables. This thesis is not intended to further elaborate FDTD models however in order to alleviate further research efforts they are discussed here in detail.



CHAPTER 4.

FREQUENCY DOMAIN IDENTIFICATION

This chapter is focused on the development of a new technique for the identification of a given wideband frequency response as a rational transfer function, and application of this technique to the transient analysis of transmission devices.

In this chapter Weighted Vector Fitting (WVF) technique [19] is proposed for the transmission line and cable parameters. The proposed WVF method presents a low order of approximation and less computation time when compared to the existing EMTP fitting procedures.

First some theoretical background on the identification of transfer functions with least squares method is presented. Then three different algorithms are presented and discussed: VF, OVF and the new WVF. Some theoretical clarifications on existing methods are also included to alleviate further research efforts in this field and increase the level of common comprehension. Examples are given for the fitting of modal propagation function. As mentioned, accurate approximation of the modal propagation function (H) usually provides sufficient knowledge of poles to identify propagation function $\mathbf{H}$.

4.1. Background

The aim of frequency response identification is to obtain a linear time-invariant (LTI) system by approximating the frequency response of a numerically computational function with a rational transfer function expressed as a ratio of two polynomials with $N' < N$

$$f(s) \equiv \frac{N(s)}{D(s)} = \frac{b_0 + b_1s + b_2s^2 + \dots + b_{N'}s^{N'}}{1 + a_1s + \dots + a_{N-1}s^{N-1} + a_Ns^N} \quad (4.1)$$

It can be also expressed using partial fraction expansion (PFE)

$$f(s) \cong \sum_{n=1}^N \frac{r_n}{s + p_n} \quad s = j\omega \quad (4.2)$$

where ω is the angular frequency in rad/s. The identification of (4.1) can be formulated as a nonlinear least squares problem stated as the minimization of the following cost function [53]

$$\min_{a,b} \sum_{k=1}^K \left| f(s_k) - \frac{N(s_k)}{D(s_k)} \right|^2 \quad (4.3)$$

where the parameters a and b represent the coefficients in (4.1). Equation (4.3) can be also written as

$$\min \sum_{k=1}^K \frac{1}{|D(s_k)|^2} |f(s_k)D(s_k) - N(s_k)|^2 \quad (4.4)$$

The minimization problem is defined at discrete angular frequencies ω_k for $k = 1, 2, \dots, K$ and over a prescribed frequency range of interest from $\omega_{\min}$ to $\omega_{\max}$. For the case of the propagation function $\mathbf{H}$ and characteristic admittance $\mathbf{Y}_c$ numerous data samples can be calculated to obtain an overdetermined nonlinear equation. However due to its nonlinear nature it becomes difficult to estimate the system parameters in a fast and accurate way.

The denominator term $|D(s_k)|$ in (4.4) can be set to one for every given frequency point s_k . This allows obtaining a linearized cost function

$$\min_{a,b} \sum_{k=1}^K |f(s_k)D(s_k) - N(s_k)|^2 \quad (4.5)$$

which is a linear LS problem first mentioned in [54]. However, ignoring the denominator term will bias the fitted transfer function. To avoid this problem the Sanathanan-Koerner iteration can be used [55]. First an estimate of poles is obtained by

minimizing (4.5) and then the model parameters are calculated iteratively by maintaining the denominator term from the previous step (iteration step $j-1$) parameters:

$$\min_{a,b} \sum_{k=1}^K \frac{1}{|D^{(j-1)}(s_k)|^2} |f(s_k)D^{(j)}(s_k) - N^{(j)}(s_k)|^2 \quad (4.6)$$

A prominent problem for the modeling of transmission devices with (4.5) or (4.6) is that, the resulting overdetermined system of equations will become intrinsically ill-conditioned since the s^n terms will take a wide range of values considering that dynamic frequency range is relatively broad and the model requires a large number of poles. These numerical problems can be avoided by using the PFE form, however attempting to directly solve (4.2) for the estimation of residues and poles results into a nonlinear optimization problem.

Although a wide variety of methods exist for unconstrained optimization, such as Gradient methods and quasi Newton methods, there is always a possibility of divergence or convergence to local minima. Moreover these techniques are numerically complex and time consuming.

The Vector Fitting (VF) technique [39] uses PFE forms as given in (4.2) and estimates the system parameters with a two-stage linear least-squares procedure. In [11] the VF method is used to identify $\mathbf{H}$ and $\mathbf{Y}_c$.

The VF method is also interpreted as a reformulation of Sanathanan-Koerner (SK) iteration which uses a partial fraction basis instead of polynomial basis [56]. First a set of initial poles for the partial fraction basis is selected, and then the poles are relocated until the prescribed convergence is achieved. Convergence is tested by a second linear least-squares approximation where poles of the partial fractions are already calculated and residues are unknown parameters.

The VF method misrepresents the iterative weighting function of the SK method and it may also become ill-conditioned for rather broad frequency ranges. This situation creates problems for low order rational fitting in addition to needlessly increasing the number of iterations.

In order to improve the numerical stability of the VF method, Orthonormal Vector Fitting (OVF) is introduced [58]. OVF uses orthonormal rational functions obtained by applying Gram–Schmidt procedure on the partial fractions. Considering the transmission device examples that we investigated, this method can only provide better results given that a weighting term obtained from previous iterations is used as advocated in this work.

In the WVF technique [19] iterative weighting function of SK method is adequately represented. It is shown that application of proper weighting results in better conditioned equations, less iterations and lower order of approximation for a prescribed error limit. An initial weighting is also proposed which is obtained by an analysis of the asymptotic behavior of the unknown polynomials in the Laplace domain, then it is replaced by a weighting term calculated from the previous iteration parameters.

4.2. LS Formulation with PFE: Vector Fitting

The objective here is to recall the vector fitting (VF) method for setting the stage for the following sections. Additional information is also contributed to better understand the VF method theory and the following contributions.

The VF method is based on partial fractions. The fitting relation for equation (4.2) is given by

$$\sum_{n=1}^N \frac{\hat{r}_n}{s + \bar{p}_n} \cong \left(\sum_{n=1}^N \frac{\tilde{r}_n}{s + \bar{p}_n} + 1 \right) f(s) \quad (4.7)$$

The unknown parameters are $\hat{r}_n$, $\tilde{r}_n$ and $\bar{p}_n$. The order of approximation is N . In the analysis of the VF method in [56] it is mentioned that the VF formulation can approximate functions with evenly distributed initial arbitrary poles. The poles $\bar{p}_n$ are initialized by distributing them logarithmically over the frequency range of interest.

Equation (4.7) can be rewritten as

$$\sum_{n=1}^N \frac{\hat{r}_n}{s + \bar{p}_n} - \left(\sum_{n=1}^N \frac{\tilde{r}_n}{s + \bar{p}_n} \right) f(s) \cong f(s) \quad (4.8)$$

and calculated for the required number of frequency points to become an overdetermined least squares problem

$$\mathbf{A} \mathbf{x} = \mathbf{b} \quad (4.9)$$

In this equation bold characters are used to denote matrices and vectors. An arbitrary row of $\mathbf{A}$ corresponding to k th frequency sample would look as follows:

$$A_k = \left[\frac{1}{s_k + \bar{p}_1} \quad \dots \quad \frac{1}{s_k + \bar{p}_N} \quad -\frac{1}{s_k + \bar{p}_1} f(s_k) \quad \dots \quad -\frac{1}{s_k + \bar{p}_N} f(s_k) \right] \quad (4.10)$$

The vector $\mathbf{x}$ is the vector of unknown residues:

$$\mathbf{x} = [\hat{r}_1 \quad \dots \quad \hat{r}_N \quad \tilde{r}_1 \quad \dots \quad \tilde{r}_N]^T \quad (4.11)$$

and

$$b_k = f(s_k) \quad (4.12)$$

The matrix $\mathbf{A}$ and the vector $\mathbf{b}$ are actually built using real values by separating the real and imaginary parts and thus actually writing two equations for each frequency sample.

$$\begin{bmatrix} \text{Re}[\mathbf{A}'] \\ \text{Im}[\mathbf{A}'] \end{bmatrix} \mathbf{x} = \begin{bmatrix} \text{Re}[\mathbf{b}'] \\ \text{Im}[\mathbf{b}'] \end{bmatrix} \quad (4.13)$$

As explained in [39] a special care must be taken to accommodate complex conjugate pair conditions in successive partial fractions. Although the iteration counter is omitted to simplify the presentation, (4.9) is solved through an iterative process and is written as:

$$\mathbf{A}^{(j-1)} \mathbf{x}^{(j)} = \mathbf{b} \quad (4.14)$$

As explained above at the iteration $j=1$, $\mathbf{A}^{(0)}$ is found with initial poles distributed logarithmically over the frequency range of interest.

After finding the solution of (4.14) at the iteration $j=1$ for the residues, the second step is to recalculate the poles of the fitted transfer function $f(s)$. This is achieved by computing the eigenvalues of the matrix $\mathbf{Q}$:

$$\mathbf{Q} = \mathbf{W} - \mathbf{g}_1 \tilde{\mathbf{x}}^T \quad (4.15)$$

where $\mathbf{W}$ is a diagonal matrix containing the poles $\bar{p}_n$ for all sample frequencies, $\mathbf{g}_1$ is a column vector of ones and $\tilde{\mathbf{x}}$ contains only the $\tilde{r}_i$ terms of (4.11). The explanation of (4.15) is found from (4.7):

$$\frac{\sum_{n=1}^N \frac{\hat{r}_n}{s + \bar{p}_n}}{\sum_{n=1}^N \frac{\tilde{r}_n}{s + \bar{p}_n} + 1} = \frac{\prod_{n=1}^{N'} (s + \hat{z}_n)}{\prod_{n=1}^N (s + \tilde{z}_n)} \cong f(s) \quad (4.16)$$

It is apparent that the cancellation of products for poles makes the zeros $\tilde{z}_n$ become the poles of $f(s)$. The denominator on the left hand side of equation (4.16) is given by:

$$\sum_{n=1}^N \frac{\tilde{r}_n}{s + \bar{p}_n} + 1 = \frac{\prod_{n=1}^N (s + \tilde{z}_n)}{\prod_{n=1}^N (s + \bar{p}_n)} \quad (4.17)$$

To find the zeros $\tilde{z}_n$ from this equation it is necessary to find the roots of:

$$\sum_{i=1}^N \left(\tilde{r}_i \cdot \prod_{n=1, n \neq i}^N (s + \bar{p}_n) \right) + \prod_{n=1}^N (s + \bar{p}_n) = 0 \quad (4.18)$$

It can be shown that this is equivalent to finding the eigenvalues of $\mathbf{Q}$ in (4.15) as shown in Appendix I.

The new set of poles of $f(s)$ found above is replaced into equation (4.2) to determine the residues r_n . This is again an overdetermined linear system. The desired fitting error limit is tested at this stage for each sample $f(s_k)$. If the error is not acceptable, the new poles are used to restart the procedure with (4.8). If the desired error limit is not met after a pre-specified number of iterations then the order of approximation N in (4.8) is increased and the iterative procedure is restarted.

It is noticed that in most cases, even if the initial poles are not adequately chosen, the VF method is capable of finding a solution at the expense of more iterations. If in some cases an iteration yields unstable poles, then such poles are simply flipped into the left complex plane and a new solution is searched.

4.3. Weighted Vector Fitting

It is emphasized here that equation (4.7) of the VF method should be actually rewritten as [19]

$$\frac{\sum_{n=1}^N \frac{\hat{r}_n}{s + \bar{p}_n}}{\sum_{n=1}^N \frac{\tilde{r}_n}{s + \bar{p}_n} + 1} \cong f(s) \quad (4.19)$$

It must be realized that the theoretical foundation of the identification problem is actually the minimization:

$$\min \left| \frac{1}{\sum_{n=1}^N \frac{\tilde{r}_n}{s + \bar{p}_n} + 1} \left[\sum_{n=1}^N \frac{\hat{r}_n}{s + \bar{p}_n} - \left(\sum_{n=1}^N \frac{\tilde{r}_n}{s + \bar{p}_n} + 1 \right) f(s) \right] \right| \quad (4.20)$$

Equation (4.20) is analogous to equation (4.4). This is overlooked in the formulation of the VF method where the left hand side denominator outside the square brackets is simply discarded or set to 1 to obtain a linear problem. It constitutes a weighting factor. Its absence is the cause of ill-conditioning problems in VF and poor fitting due to overemphasized low frequency samples.

The formulation of (4.19) actually provides a straightforward explanation to the VF method. Both the numerator and the denominator of (4.1) are divided by prescribed poles given by $D'(s)$:

$$\frac{N(s) / D'(s)}{D(s) / D'(s)} \cong f(s) \quad (4.21)$$

Equation (4.21) can be converted to (4.19) by applying Heaviside's Theorem. At each iteration $D'(s)$ gets closer to $D(s)$ and a solution is reached when $D'(s) \cong D(s)$ which means that the prescribed poles are equal to the identified function poles and the ratio $N(s) / D'(s)$ gives the correct residues of $f(s)$.

Instead of setting the weighting factor to 1 it is proposed here to maintain it through an iterative process. For the first iteration the weighting function can be estimated by an analysis of the asymptotic behavior of the unknown partial fractions. If $C(s)$ represents the weighting factor from (4.20):

$$C(s) = \sum_{n=1}^N \frac{\tilde{r}_n}{s + \bar{p}_n} + 1 \quad (4.22)$$

then according to (4.21), at the convergence of the fitting iterations, this factor is supposed to approach unity. This means that independently of the order of approximation the calculation of $C(s)$ for the first iteration can be written as:

$$C^{(1)}(s) \cong \frac{\tilde{r}_m^{(1)}}{s + \bar{p}_m^{(1)}} + 1 \quad (4.23)$$

It is proposed to estimate $\tilde{r}_m^{(1)}$ and $\bar{p}_m^{(1)}$ by approximating the frequency domain response of $f(s)$ with a first order partial fraction expression. The first order approximation governs the magnitude behavior and shape of an N th order $C(s)$ sufficiently close to the exact value to produce acceptable starting values. This weighting term resembles to Maximum Likelihood weighting for frequency response functions [57]. At a subsequent iteration count j , the function $C(s)$ can be approximated by:

$$C^{(j)}(s) \cong \sum_{n=1}^N \frac{\tilde{r}_n^{(j-1)}}{s + \bar{p}_n^{(j-1)}} + 1 \quad (4.24)$$

The new system of equations now becomes:

$$\mathbf{A}'^{(j-1)} \mathbf{x}^{(j)} = \mathbf{b}' \quad (4.25)$$

The primed version of $\mathbf{A}$ is found by dividing each frequency row by the corresponding $C^{(j)}(s_k)$ factor. The same applies to $\mathbf{b}'$.

The proposed method is named hereafter WVF (Weighted Vector Fitting). In addition to better conditioning, the following numerical examples will demonstrate that it produces a lower order of approximation than the existing VF method and converges faster. The proposed initial estimation procedure of equation (4.23) contributes to the reduction of the number of iterations and approximation order.

4.4. Orthonormal Vector Fitting

It is asserted in [58] that instead of using simple partial fractions in (4.7), using orthonormal rational functions $\phi_n(s)$ improves the numerical conditioning of the system of equations. Equation (4.7) is now written as

$$\sum_{n=1}^N (\hat{r}_n \cdot \phi_n) \equiv \left[\sum_{n=1}^N (\tilde{r}_n \cdot \phi_n) + 1 \right] f(s) \quad (4.26)$$

The functions $\phi_n(s)$ are linear combinations of partial fractions:

$$\phi_n(s) = \frac{U_n(s)}{\prod_{i=1}^n (s + \bar{p}_i)} \quad (4.27)$$

for $n=1, \dots, N$. $U_n(s)$ is an arbitrary polynomial of order $n-1$ such that ϕ_n becomes orthonormal to ϕ_{n+1} . The authors of [58] have defined $U_n(s)$ by imposing the orthonormality conditions on the basis functions. The general polynomials are given by:

$$\phi_n(s) = \kappa_n \sqrt{2 \cdot \text{Re}(\bar{p}_n)} \left(\prod_{i=1}^{n-1} \frac{s - \bar{p}_i^*}{s + \bar{p}_i} \right) \frac{1}{s + \bar{p}_n} \quad (4.28)$$

where κ_n is an arbitrary unimodular complex number. A theoretical analysis on these functions is presented in [59]. As in the VF method, the overdetermined least squares method is given by (4.9), except that the matrix $\mathbf{A}$ is now different. The solution steps are similar, but the matrix $\mathbf{W}$ in equation (4.15) is no longer diagonal due to orthonormality coefficients.

The formulation used in this thesis for WVF is also applied here in the OVF method by writing (4.26) as equation (4.25), but the weighting coefficient is now given by:

$$C^{(j)}(s) = \sum_{n=1}^N \tilde{r}_n^{(j-1)} \phi_n^{(j-1)} + 1 \quad (4.29)$$

Dividing by the weighting factor is numerically important and its computation improves the efficiency of the OVF method. The weighting factor is set to 1 for the first iteration count $j=1$. It is observed that the calculated poles must be transformed to an orthonormal basis for the weighting term as well. This will ensure the concordance between the matrix $\mathbf{A}'^{(j-1)}$ and the weighting term. It has been found that if the weighting term is not orthonormalized then the performance of the OVF method decreases considerably and the total RMS error of the resulting approximation becomes significantly larger than with the VF method for the examples that will follow.

It was not possible to use OVF without the weighting term given in (4.29) for identifying the $\mathbf{H}$ matrices in the tested examples. The numerical examples presented in the following sections demonstrate that although the proposed WVF method provides an order of approximation similar to OVF, but it is much easier to implement WVF and the convergence characteristic of WVF is more robust.

4.5. Modal Vector Fitting

The weighting factor in WVF is not an arbitrary choice but the intrinsic factor present in the minimization problem. However, it is also possible to introduce an arbitrary weighting to the LS problem (4.9) for each $f(s)$ element and frequency sample. In [60], such an arbitrary weighting factor is introduced with a heuristic approach. The new fitting technique is called Modal Vector Fitting (MVF).

MVF is principally designated for the fitting problem of a matrix function with the same set of poles. The weighting factor is defined as the eigenvalues of the matrix function. If MVF is applied to a scalar function, then the weighting factor will be equal to the function itself. This is not the same weighting factor as proposed by the WVF technique. Another important difference is that, in WVF the weighting factor is iteratively improved while in MVF it is a constant.

4.6. Solution of Overdetermined Linear Equation $Ax=b$

It must be mentioned that the actual solution implementation for (4.14) and (4.25) uses an Euclidean scaling factor on the matrices A and A' respectively. This improves the conditioning number of the matrices [61]. Without this scaling factor the fitting of H will fail in most cases due to numerical accuracy issues. The Euclidean scaling is not needed for OVF since it deteriorates its orthonormality properties and inherent scaling.

All overdetermined linear equations in this work are solved by QR decomposition with Householder transformations [62] (Matlab's operator ' $\backslash$ ' is used for implementation).

4.7. Numerical Examples and Discussion

In this section some numerical examples are shown to better understand the theoretical contributions. Examples are concentrated in frequency domain identification of modal propagation function. Details regarding phase domain identification are kept

for next chapter. Phase domain fitting is rather complicated and it is not always possible to properly evaluate the effect of fitting technique as an independent parameter due to the existence of multi delays. Therefore, directly concentrating in phase domain would hinder the fair comparison of fitting methods. This section rather deals with the performance of the fitting techniques and this can be achieved by reducing the fitting problem into modal domain and dealing with single delay functions. Chapter V addresses phase domain fitting and calculation of modal time delays.

4.7.1. Comparison of Fitting Techniques

Figure 2.12 presents a test case with cable data given in Table 2.2. There are 7 distinct time delays as given for comparison basis. Modes with identical time delays are collapsed by taking their average. The identified time delays in μs are: 6.235, 19.834, 19.861, 23.777, 23.957, 40.391 and 60.958. Each mode is fitted by

$$H_m \cong e^{-s\tau_m} \sum_{n=1}^N \frac{c_n}{s + p_{nm}}. \quad (4.30)$$

A maximum absolute difference (between fitted and reference values) of 0.015 for each sample point is defined as the maximum error limit. Absolute error is defined as

$$\varepsilon_{abs}(k) = \left| H_m(s_k) - e^{-s_k \tau_m} \sum_{n=1}^N \frac{c_n}{s_k + p_{nm}} \right| \quad (4.31)$$

for the k th frequency sample. This allows achieving a sufficiently high quality fitting of $\mathbf{H}$ in phase domain (See Chapter V). The total RMS fitting error will become much less than 0.015 since this is the maximum error limit defined for one sample point. The total rms error (deviation) is computed from the absolute value of differences as follows:

$$RMS = \sqrt{\frac{1}{K} \sum_{k=1}^K (\varepsilon_{abs}(k))^2} \quad (4.32)$$

Table 4.1 presents the order of fitting and the number of iterations for each mode using the previously discussed methods VF, OVF and WVF. The order of fitting is the minimum order required to satisfy the maximum absolute error limit (0.015) for each sample point. All methods have been implemented using the same coding technique to provide fair comparisons. It is apparent that the methods OVF and WVF always result in lower order fitting than VF. This lower order of fitting has an important impact on the time-domain computations of the resulting model. The advantage of the new WVF method over OVF is in its formulation and implementation simplicity which results into reduced computer time (see CPU times in Table 4.1). In addition, as it will be seen below, WVF converges faster and is more robust than OVF.

Table 4.2 compares the condition numbers of $\mathbf{A}$ for 6 iterations and for all methods. It confirms the sustained conditioning advantage of the WVF method over VF and even OVF.

Table 4.1 Fitting Orders and Iteration Count for Fitted Modes – Case CAB02

Mode	Fitting Method					
	VF		OVF		WVF	
	Order	Iter.	Order	Iter.	Order	Iter.
1	15	20	10	14	10	15
2	16	14	12	8	12	8
3	16	16	12	7	12	12
4	13	8	10	9	10	8
5	13	8	10	12	10	7
6	9	7	7	11	8	7
7	4	9	4	6	4	6
Total	86	82	65	67	66	63
CPU Time	35		21		16	

Table 4.2 Condition numbers, Mode 3, order of fitting 12, Case CAB02

Iteration Step	VF	OVF	WVF
1	2.2259×10^5	1.1077×10^6	8.7824×10^4
2	1.3173×10^5	1.3172×10^5	3.5152×10^4
3	1.0810×10^5	8.4822×10^4	6.3553×10^4
4	1.0471×10^5	9.7443×10^4	2.540×10^4
5	1.0126×10^5	1.5383×10^5	1.4710×10^4
6	1.0116×10^5	4.8989×10^5	2.6389×10^4

In addition to the iterations count it is of interest to analyze the iterative trajectories of the methods. This is presented in Figure 4.1. It is apparent that the WVF method has a more robust convergence characteristic when compared to OVF and VF. In the VF method the increase in the number of iterations does not appear to improve precision. The iterative trajectories for an increased order of fitting are shown in Figure 4.2. Once again the WVF method shows its superior performance and robustness. The initial jump on iteration 2 in WVF is due to the fact that the weighting initialization given by equation (4.23) was intentionally disabled to demonstrate its consequence. When the initialization was kept, the trajectory was coherent with the following iteration towards error decrease. Similar results are obtained for all modes (not shown).

In Figure 4.3 the fitting error is compared against frequency for all methods. It is apparent that the error is very small for all methods at this order of fitting.

In Figure 4.4 the magnitude curves for all modes are compared against fitting with the WVF method. Figure 4.5 demonstrates the principle of equation (4.30) and the related compensation of excessive lag.

Table 4.3 and Table 4.4 list the identified residues and poles by using WVF for propagation modes and trace of characteristic admittance respectively. As it will be discussed in the next chapter, poles calculated from the fitting of the trace of characteristic admittance are sufficient for the identification of all elements.

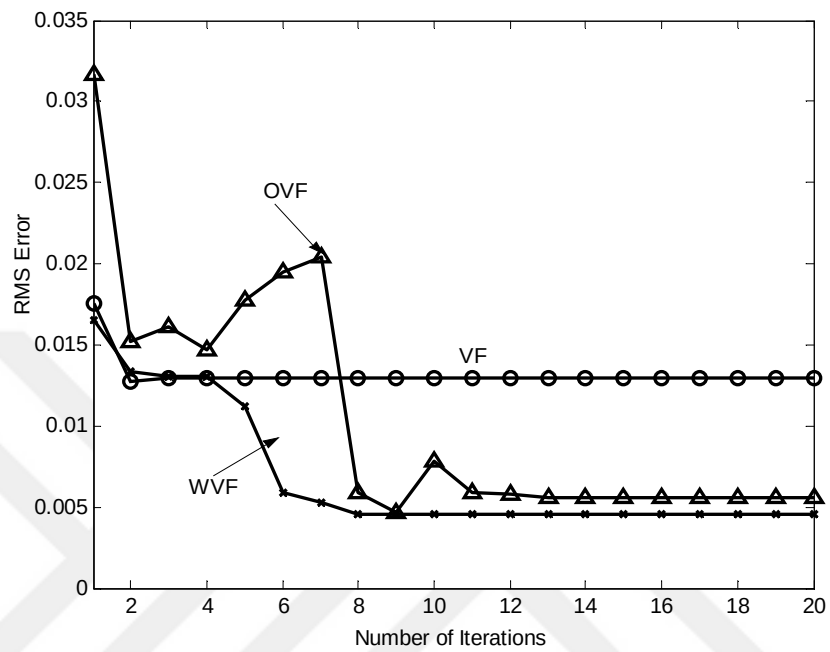


Figure 4.1 RMS error related to the number of iterations, Mode 3, order of fitting 12

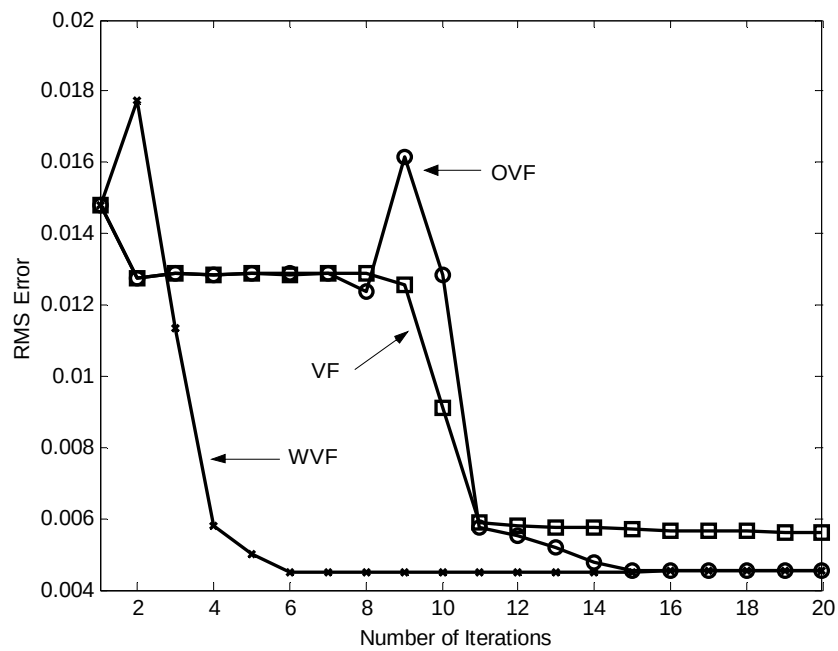


Figure 4.2 RMS error related to the number of iterations, Mode 3, order of fitting 16, No initial weighting for WVF

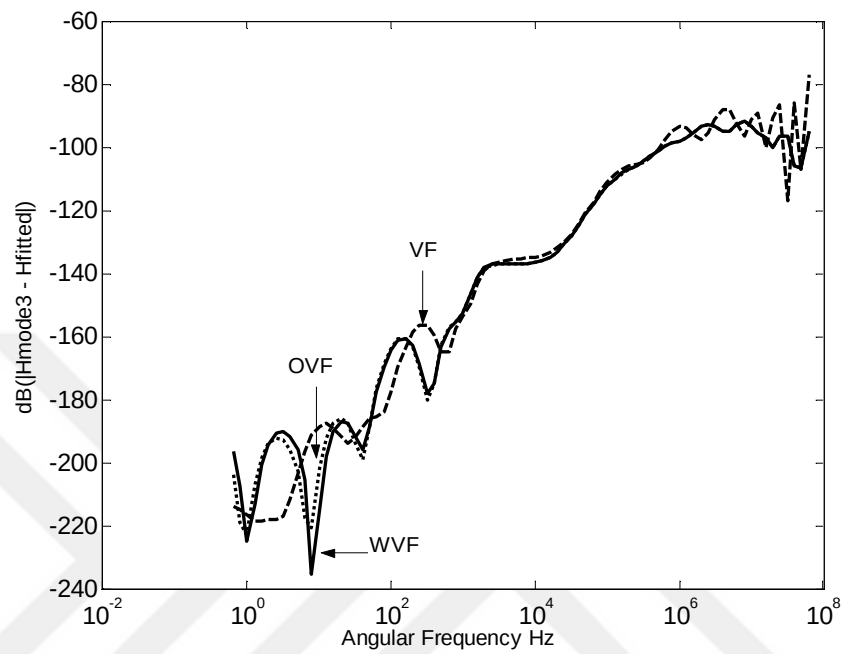


Figure 4.3 Fitting error as a function of frequency, Mode 3, order of fitting 16

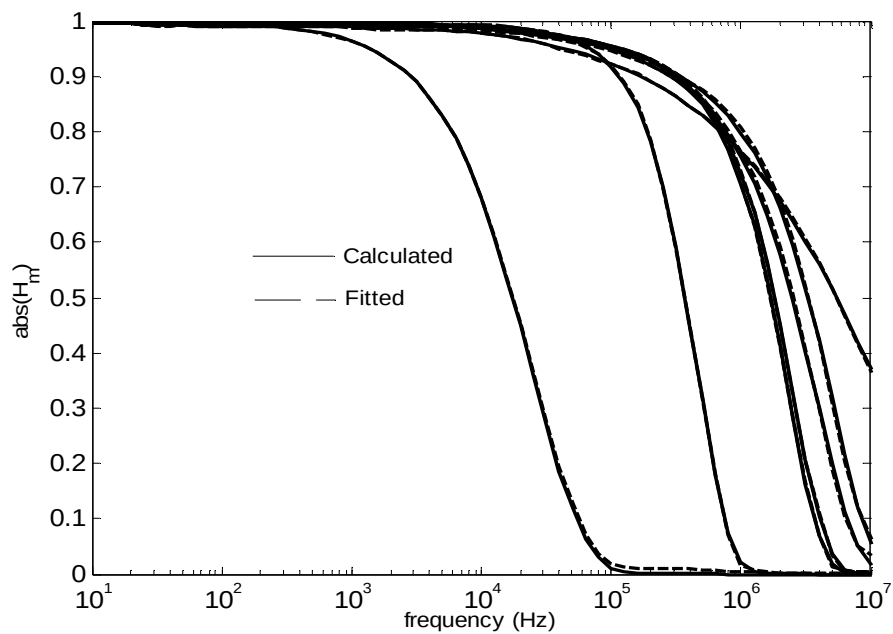


Figure 4.4 Magnitude curves of $\mathbf{H}$ matrix modes compared against WVF fitting CAB02

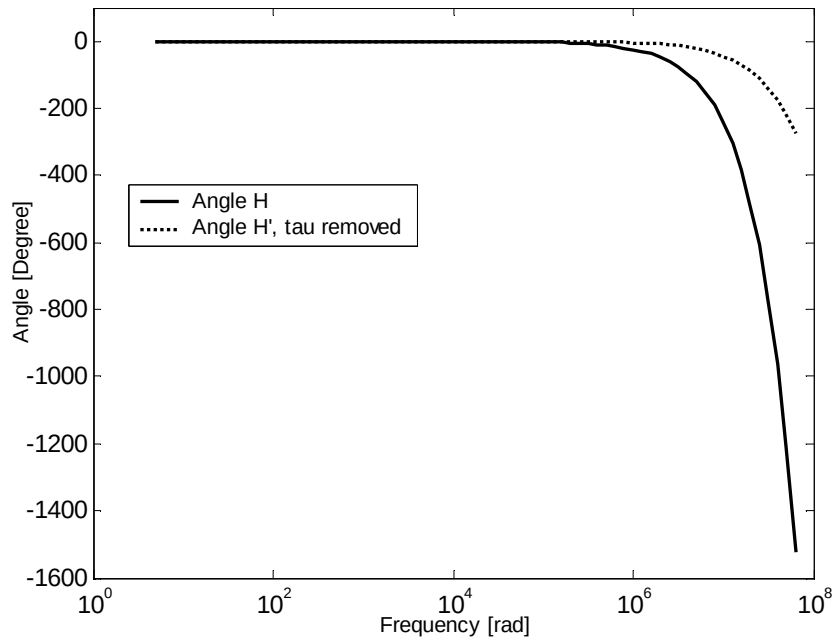


Figure 4.5 Phase angles of H'_3 and H_3 , mode 3

Table 4.3 Poles and Residues Corresponding to Modes of CAB02

Mode	Poles	Residues
Mode 1	-9.4555E+001 -1.4294E+003 -1.6442E+004 -1.6555E+005 -1.1522E+006 -6.3762E+006 -1.7319E+007 -2.9259E+007 -5.1358E+007 +5.6753E+007i -5.1358E+007 -5.6753E+007i	7.8461E-002 1.2438E+001 8.7123E+001 6.3137E+003 9.1726E+004 1.1239E+006 -7.9996E+005 1.3642E+007 1.6897E+006 -5.0822E+006i 1.6897E+006 +5.0822E+006i

Mode	Poles	Residues
Mode 2	-4.7826E+000 -7.5333E+001 -2.1244E+002 +3.0989E+002i -2.1244E+002 -3.0989E+002i -2.2489E+005 -2.4392E+006 -5.9280E+006 -1.5804E+007 -2.2647E+007 +2.4737E+007i	6.9767E-003 4.3476E-001 4.2275E-001 -5.0272E-001i 4.2275E-001 +5.0272E-001i 9.1834E+003 3.3534E+005 -6.4584E+004 4.2870E+007 -1.6233E+007 +3.2751E+007i

	-2.2647E+007 -2.4737E+007i -2.2138E+007 +5.2277E+007i -2.2138E+007 -5.2277E+007i	-1.6233E+007 -3.2751E+007i -3.6084E+006 -7.7820E+006i -3.6084E+006 7.7820E+006i
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Mode	Poles	Residues
Mode 3	-8.6163E+000 -1.7977E+001 -1.8023E+002 -3.9458E+003 -2.4668E+005 -2.3098E+006 -5.3667E+006 -1.2394E+007 -1.9785E+007 +2.0162E+007i -1.9785E+007 -2.0162E+007i -2.1114E+007 +4.4393E+007i -2.1114E+007 -4.4393E+007i	1.4894E-002 1.4094E-002 1.4306E+000 2.2540E+001 1.0101E+004 3.3539E+005 5.8790E+004 3.0459E+007 -1.2332E+007 +2.4360E+007 -1.2332E+007 -2.4360E+007 -2.2823E+006 -5.7368E+006 -2.2823E+006 +5.7368E+006

Mode	Poles	Residues
Mode 4	-1.3345E-001 -8.4380E+000 -1.1393E+002 -2.8470E+005 -3.3237E+005 -6.4326E+006 -8.0333E+006 +1.1041E+007i -8.0333E+006 -1.1041E+007i -7.1513E+006 +2.3279E+007i -7.1513E+006 -2.3279E+007i	1.6828E-004 1.6676E-002 9.4816E-001 -2.5241E+003 2.1526E+004 1.4597E+007 -7.0851E+006 +7.6171E+006i -7.0851E+006 -7.6171E+006i 3.8631E+004 -2.2389E+006i 3.8631E+004 +2.2389E+006i

Mode	Poles	Residues
Mode 5	-1.0460E+001 -9.1953E+001 -1.3107E+002 -3.9331E+005 -1.1393E+006 -6.2162E+006 -7.5878E+006 +1.0046E+007i -7.5878E+006 -1.0046E+007i -6.4408E+006 +2.1078E+007i -6.4408E+006 -2.1078E+007i	7.0251E-002 -1.9634E+000 3.2835E+000 2.4003E+004 -3.0530E+003 1.4296E+007 -7.3189E+006 +6.7205E+006i -7.3189E+006 -6.7205E+006i 3.9159E+005 -1.9946E+006i 3.9159E+005 +1.9946E+006i

Mode	Poles	Residues
Mode 6	-3.6259E+000 -4.7236E+001 -2.3590E+005 -7.7089E+005 -1.4303E+006 +1.2587E+006i -1.4303E+006 -1.2587E+006i -1.5028E+006 +3.6630E+006i -1.5028E+006 -3.6630E+006i	3.9061E-003 2.1239E-001 1.6734E+003 2.8714E+005 -4.1064E+005 -1.5823E+006i -4.1064E+005 +1.5823E+006i 2.8099E+005 +4.5612E+005i 2.8099E+005 -4.5612E+005i

Mode	Poles	Residues
Mode 7	-1.0198E+004 -7.0944E+004 -1.5948E+005 +1.9345E+005i -1.5948E+005 -1.9345E+005i	1.2255E+003 9.2867E+004 -4.5074E+004 +3.3117E+004i -4.5074E+004 -3.3117E+004i

Table 4.4 Poles Corresponding to Trace of Characteristic Admittance, CAB02

Yc	Poles
Trace	-1.7230E+000 -7.8855E+000 -3.4995E+001 -1.7813E+002 -8.8716E+002 -3.3233E+003 -2.9597E+004 -4.1352E+005 -1.4862E+007

Table 4.5 Fitting Orders and Iteration Count for Fitted Modes – Case CAB01

Mode	Fitting Method					
	VF		OVF		WVF	
	Order	Iter.	Order	Iter.	Order	Iter.
1	16	17	10	6	8	7
2	19	15	12	8	12	7
3	19	12	12	8	13	5
4	5	8	5	4	5	4
Total	59		39		38	

4.7.2. Other Modal Domain Fitting Examples

Figure 2.5 presents a test case (CAB01) with cable data given in Table 2.1. There are 4 distinct time delays as given for comparison basis. Modes with identical time delays are collapsed by taking their average. The identified time delays in μs are: 55.642, 210.726, 19.861, 258.786 and 599.023.

Similar to the previous example a maximum absolute difference (between fitted and reference values) of 0.015 for each sample point is defined as the error limit.

Table 4.5 presents the order of fitting and the number of iterations for each mode using VF, OVF and WVF. As it is observed OVF and WVF always result in lower order fitting than VF. In Figure 4.6 the magnitude curves for all modes are compared against fitting with the WVF method.

Table 4.6 and Table 4.7 list the identified residues and poles by WVF.

Figure 2.13 presents a 3 phase transmission line test case (TRL01) with line data given in Table 2.3. There are 3 distinct time delays as given for comparison basis. The identified time delays in μs are: 666.966, 667.622 and 707.934. Maximum absolute difference is again defined as 0.015 for each sample point.

Table 4.8 presents the order of fitting and the number of iterations for each mode using VF, OVF and WVF. Order of fitting calculated by Bode type fitting technique used by FD line is also given. Modal fitting of transmission lines are relatively simple compared to cables and as it can be observed OVF, WVF and VF perform similarly, and give a total order ranging from 25 to 28. However, Bode type fitting results are very apart from these techniques, i.e., a total order of 72. In Figure 4.7 the magnitude curves for all modes are compared against fitting with the WVF method.

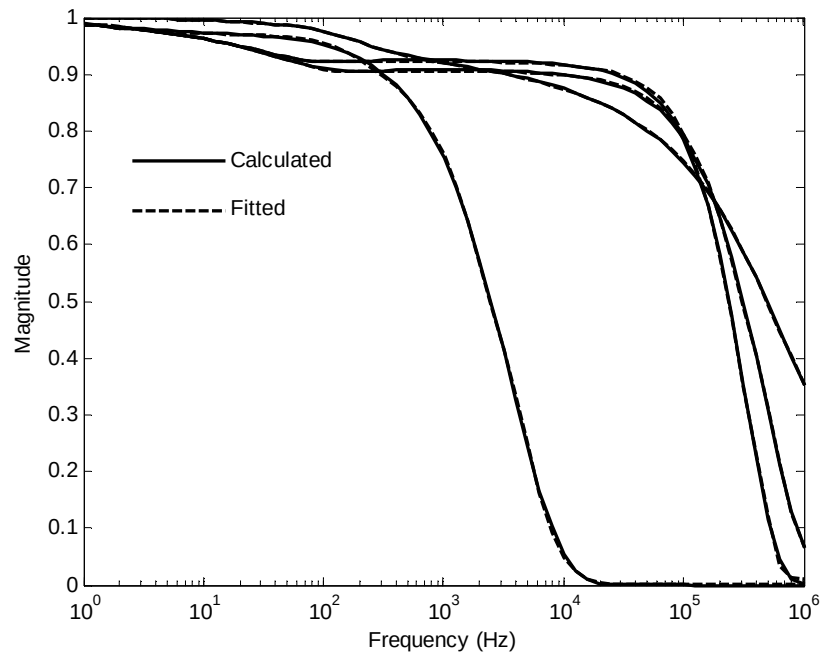


Figure 4.6 Magnitude curves of **H** matrix modes compared against WVF fitting CAB01

Table 4.6 Poles and Residues Corresponding to Modes of CAB01

Mode	Poles	Residues
Mode 1	-6.7255E+002 -5.1904E+003 -8.6724E+004 -7.0331E+005 -3.1251E+006 1.3770E+006i -3.1251E+006 -1.3770E+006i -2.6116E+006 6.2750E+006i -2.6116E+006 -6.2750E+006i	3.5721E+001 2.3082E+002 7.0141E+003 2.2475E+005 -1.5643E+006 -6.0737E+006i -1.5643E+006 6.0737E+006i 1.5362E+006 9.9254E+005i 1.5362E+006 -9.9254E+005i
Mode 2	-4.5629E+000 -4.7883E+001 -1.8064E+002 -2.7748E+002 -3.6354E+005 -4.6935E+005 -1.5633E+006 7.4588E+005i -1.5633E+006 -7.4588E+005i -1.7676E+006 2.8597E+006i -1.7676E+006 -2.8597E+006i -1.5490E+006 5.1845E+006i -1.5490E+006 -5.1845E+006i	5.4324E-002 1.3339E+000 1.9509E+000 1.1239E+001 6.1395E+004 7.5005E+002 -3.8664E+006 -6.5092E+006 -3.8664E+006 6.5092E+006 3.4870E+006 -1.4166E+006 3.4870E+006 1.4166E+006 1.7833E+005 8.4942E+005 1.7833E+005 -8.4942E+005

Mode	Poles	Residues
Mode 3	-2.4517E+000 -1.7060E+001 -1.0665E+002 -2.9644E+002 -3.5219E+002 -2.9644E+002 3.5219E+002 -1.1773E+005 -1.6498E+005 -1.1030E+006 -4.7404E+005 -1.1030E+006 4.7404E+005 -1.0177E+006 -1.7366E+006 -1.0177E+006 1.7366E+006 -7.7217E+005 -3.1887E+006 -7.7217E+005 3.1887E+006	1.4958E-002 2.7613E-001 4.3273E+000 -1.1471E-001 2.8758E+000 -1.1471E-001 -2.8758E+000 2.5402E+003 1.3885E+003 -1.1127E+006 4.2357E+006 -1.1127E+006 -4.2357E+006 1.3384E+006 -9.1583E+005 1.3384E+006 9.1583E+005 -2.2497E+005 -7.7437E+004 -2.2497E+005 7.7437E+004

Mode	Poles	Residues
Mode 4	-1.7456E+001 -1.5083E+003 -1.0177E+004 -2.1485E+004 -2.8088E+004 -2.1485E+004 2.8088E+004	4.0839E-001 1.7232E+002 1.2091E+004 -6.1781E+003 -2.6826E+003 -6.1781E+003 2.6826E+003

Table 4.7 Poles Corresponding to Trace of Characteristic Admittance, CAB01

Yc	Poles
Trace	-1.1942E+000 -3.9215E+000 -1.5572E+001 -5.2420E+001 -2.5639E+002 -1.0577E+003 -3.2994E+003 -3.5711E+004 -1.2305E+006

Table 4.8 Fitting Orders and Iteration Count for Fitted Modes – Case TRL01

Mode	Fitting Method						
	VF		OVF		WVF		FD Bode
	Order	Iter.	Order	Iter.	Order	Iter.	Order
1	17	8	15	10	16	7	30
2	6	4	5	5	5	4	22
3	5	2	5	7	5	7	20
Total	28		25		26		72

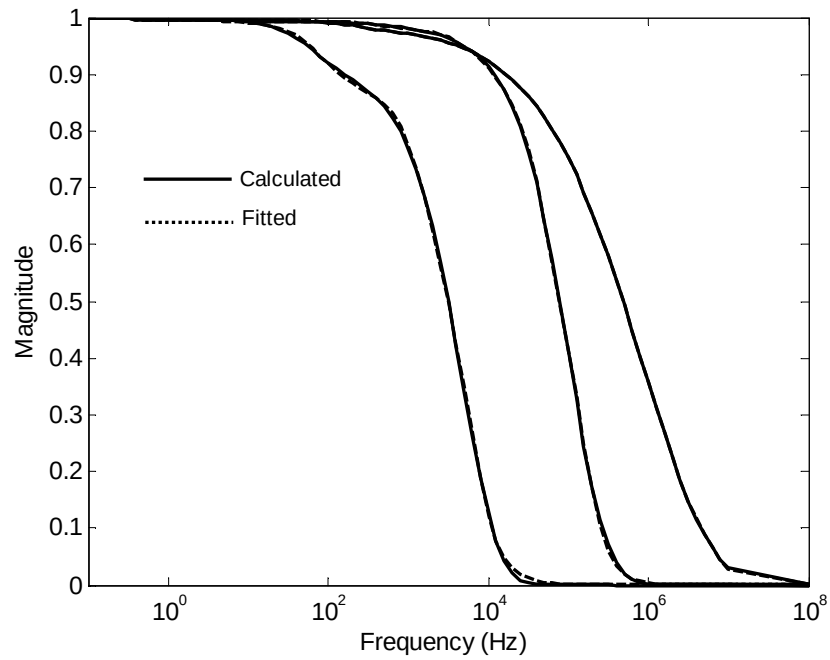


Figure 4.7 Magnitude curves of **H** matrix modes compared against WVF fitting TRL01

Table 4.9 Poles and Residues Corresponding to Modes of TRL01

Mode	Poles	Residues
Mode 1	-7.2470E-001	1.0826E-003
	-3.7803E+000	8.2171E-003
	-1.1795E+001	1.5864E-002
	-1.3268E+003	2.3172E+001
	-8.3782E+003	1.3291E+002
	-4.5958E+004	2.2811E+003
	-2.2183E+005	2.7719E+004
	-9.8267E+005	3.1466E+005
	-3.9117E+006	3.3115E+006
	-1.0897E+007 -9.1678E+006	-6.4590E+006 -4.0092E+005
	-1.0897E+007 9.1678E+006	-6.4590E+006 4.0092E+005
	-1.4700E+007 -2.8221E+007	2.6559E+006 5.2962E+006
	-1.4700E+007 2.8221E+007	2.6559E+006 -5.2962E+006
	-1.2004E+007 -5.2371E+007	1.3586E+006 -9.3212E+005
	-1.2004E+007 5.2371E+007	1.3586E+006 9.3212E+005

Mode	Poles	Residues
Mode 2	-1.6497E+001	1.0295E-001
	-2.8846E+004	2.4961E+003
	-2.9991E+005	3.7467E+005

	-7.4642E+005 -9.3382E+005 -7.4642E+005 9.3382E+005	-1.9545E+005 -1.0757E+005 -1.9545E+005 1.0757E+005
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Mode	Poles	Residues
Mode 3	-2.2177E+002 -8.2800E+002 -1.0585E+004 -2.6349E+004 -2.2190E+004 -2.6349E+004 2.2190E+004	1.1746E+001 7.1173E+001 7.4648E+003 -4.0291E+003 8.8850E+003 -4.0291E+003 -8.8850E+003

CHAPTER 5.

FITTING OF PROPAGATION FUNCTION AND CHARACTERISTIC ADMITTANCE

In the previous chapter WVF method is developed for the identification of a given wideband frequency response as a rational transfer function. WVF method is successfully applied to the fitting problem of modal propagation function. In this chapter fitting of propagation function $\mathbf{H}$ and characteristic admittance $\mathbf{Y}_c$ in the phase domain is examined.

For a given N_c conductor system, it is assumed that $\mathbf{Y}_c$ and $\mathbf{H}$ data given at discrete frequency samples can be represented with N_c input N_c output (MIMO) linear time invariant (LTI) systems in the form of numerical transfer function matrices. In this chapter the procedure for identifying each matrix element of $\mathbf{Y}_c$ and $\mathbf{H}$ as a transfer function in the frequency domain is explained. This is alternatively called fitting of the calculated frequency data to a rational transfer function. In order to save computations in the time-domain model, a common pole set is used for the fitting of each element of $\mathbf{H}$. A similar approach is also adopted for the fitting of $\mathbf{Y}_c$.

Since $\mathbf{H}$ and $\mathbf{Y}_c$ are assumed to be coupled LTI systems represented with rational transfer functions, according to linear system theory, any element of the matrix contains sufficient information to identify all poles. This theoretically means that all poles can be identified using the frequency response of any element of $\mathbf{H}$ and $\mathbf{Y}_c$. In practice, however, the identification of poles is carried out by a numerical procedure, and if, for instance, an element in which particular poles are dominant is used, we cannot expect good accuracy for non dominant poles [63]. With this in mind, we use the trace (sum of the diagonal elements) of $\mathbf{Y}_c$ for the pole identification. It is expected that all system

poles are dominant enough in the trace. When $\mathbf{H}$ is considered, there are modal delays present in the trace. Therefore fitting the trace would get very complicated with inherently present multiple delays. However, for a square matrix, the sum of eigenvalues is equal to the trace [64], thus instead of fitting the trace, one can instead fit the eigenvalues separately, and obtain all the poles that can automatically fit the trace as well. Since a single delay is attributed to each eigenvalue, fitting of eigenvalues is less complicated. Therefore we will start with the fitting of modal propagation function (eigenvalues).

In this chapter the fitting method of propagation function is elaborated with an emphasis on new contributions, then fitting of characteristic admittance is explained. The CIWB model of a transmission device is obtained by using the overall procedure explained in this chapter.

5.1. Fitting of Propagation Function

The identification of $\mathbf{H}$ is more complicated especially in the case of cables. There are different modal time delays contributing to each element of $\mathbf{H}$ (See Appendix IV):

$$\mathbf{H} = \mathbf{T}\mathbf{e}^{\Lambda}\mathbf{T}^{-1} \quad (5.1)$$

Thus the propagation function is first fitted in the modal domain and then resulting poles and time delays are used for fitting $\mathbf{H}$ in the phase domain. The basic idea is initially explained in [11] and improved in numerous papers such as [19], [33] and [65]. The following steps are presented here for convenience and for setting the stage to provide extra improvement into the process.

1. Calculation of modes using frequency dependent transformation matrices.
2. Rotation of modes in order to minimize the complex part

3. Application of an interchange routine to avoid the problem of artificial mode switchovers.
4. Identification of time delays for each mode.
5. Grouping (Lumping) of modes having identical or almost identical time delays.
6. Fitting of individual modes with WVF technique combined with a time delay search scheme.
7. Fitting in phase domain by using the resulting poles and time delays.

The overall idea of this approach is formulated as follows for each matrix element ij :

$$\tilde{H}_{ij}(s) = \sum_{m=1}^M \left[\sum_{n=1}^{N_m} \frac{c_{ijnm}}{s + p_{nm}} \right] e^{-(s \cdot \tau_m)} \quad \text{such that} \quad (5.2)$$

$$\min_c \|H_{ij}(s) - \tilde{H}_{ij}(s)\|_2^2$$

where $s = j\omega$, N_m is the number of poles for mode m , M is the number of modes and τ_m is the time delay of mode m . H_{ij} is calculated from (2.30). Each fitted propagation mode is actually given by:

$$H_m(s) = e^{\Lambda_m} = H'_m(s) e^{-(s \cdot \tau_m)} \quad (5.3)$$

where e^{Λ_m} is a matrix containing only one nonzero diagonal element for mode m . The poles p_{nm} are calculated using WVF.

$$H_m \cong e^{-s \tau_m} \sum_{n=1}^N \frac{c_n}{s + p_{nm}} \quad (5.4)$$

and the matrix element residues c_{ijnm} are found from a direct linear algebra solution using LS technique.

5.1.1. Calculation of Eigenvalues

For a multiphase conductor system there are as many time delays as distinct propagation modes. Propagation modes can be calculated by using a frequency dependent transformation matrix $\mathbf{T}$. The calculation of modal contributions involves diagonalization of the matrix product $\mathbf{YZ}$ at discrete frequency points.

$$\mathbf{H} = e^{(\mathbf{T}\mathbf{\Lambda}\mathbf{T}^{-1})} = \mathbf{T}e^{\mathbf{\Lambda}}\mathbf{T}^{-1} \quad (5.5)$$

where $\mathbf{\Lambda}$ is the diagonal matrix of eigenvalues of $-\sqrt{\mathbf{Y}(s)\mathbf{Z}(s)}\ell$. Each modal propagation function can be represented by

$$H = e^{\lambda} \quad \lambda \in \mathbf{\Lambda} \quad (5.6)$$

or

$$H = e^{(A+jB)} \quad (5.7)$$

where $B = -\omega\tau$ represents the phase angle of the mode and τ is the true time delay.

Usage of a constant real $\mathbf{T}$ for the calculation of propagation modes as advocated in [11] was found to give less accurate results for the phase domain fitting of $\mathbf{H}$. As explained in [18], with a constant $\mathbf{T}$, the modes are linear combinations of the elements of $\mathbf{H}$ with constant real coefficients. Since the diagonalization is not complete over the full frequency band, each mode will be composed of a dominant component plus “parasitic” components with different delays as shown below for an imaginary two conductor case.

$$H'_1 e^{-s\tau_1} + \Delta H'_2 e^{-s\tau_2} \cong H'_{rat} e^{-s\tau} \quad (5.8)$$

When fitting this impure mode with a rational function plus a single delay, the calculated poles are affected by the delay factor of the parasitic component. In other words incorrect poles result due to an incorrect model structure. As a result frequency dependent $\mathbf{T}$ is used in this work.

The rotation of modes in order to minimize the complex part of the transformation matrix is necessary for modal domain models. In these models, transformation matrix is approximated as a constant and real matrix. Therefore minimizing the complex part of the transformation matrices reduces the error of the approximation. For phase domain models rotation of the modes is obsolete, but in order to compare the fitting results of modal domain and phase domain models, this operation is applied to all cases in this work.

A general diagonalization routine such as ZGEEV LAPACK used in EMTP or Matlab might occasionally interchange the sequence of the eigenvectors when diagonalizing two subsequent frequency points. This problem is alternatively called mode switchover and has been investigated in [67] where a diagonalization routine based on the Newton Raphson approach was proposed to overcome this problem. Other non trivial methods can be also used [68].

In the wideband model, however, the switchover problem is simply overcome by tracing the direction of the eigenvectors as functions of frequency. This is done by comparing the dot product between eigenvectors. The dot product of two eigenvectors is nearly equal to 1 if no switchover occurs, and nearly equal to 0 if switchover occurs. If a switchover is detected the dot product is calculated between the concerning eigenvector and remaining vectors until the corresponding eigenvector is found. Then the two eigenvectors are interchanged.

The tracing routine works fine as long as no eigenvalues overlap. This is unlikely to happen, with the exception of coaxial modes at very high frequencies. Even this happens these eigenvalues which are close to each other can be grouped with one delay; consequently switchover problem can be covered.

Remember the test cases given in Section 2.4. For the 6 conductor cable (CAB01) and transmission line cases (TRL01) no switchover is detected. However for the 12 conductor case (CAB02) eigenvectors are observed to interchange. This can be verified

by inspecting Figure 5.1 where dashed line is the eigenvalue calculated without a switchover routine. Same problem is encountered for all modes. Mode switchover is a serious problem and should be treated before proceeding to fitting since it prevents proper identification of rational function parameters.

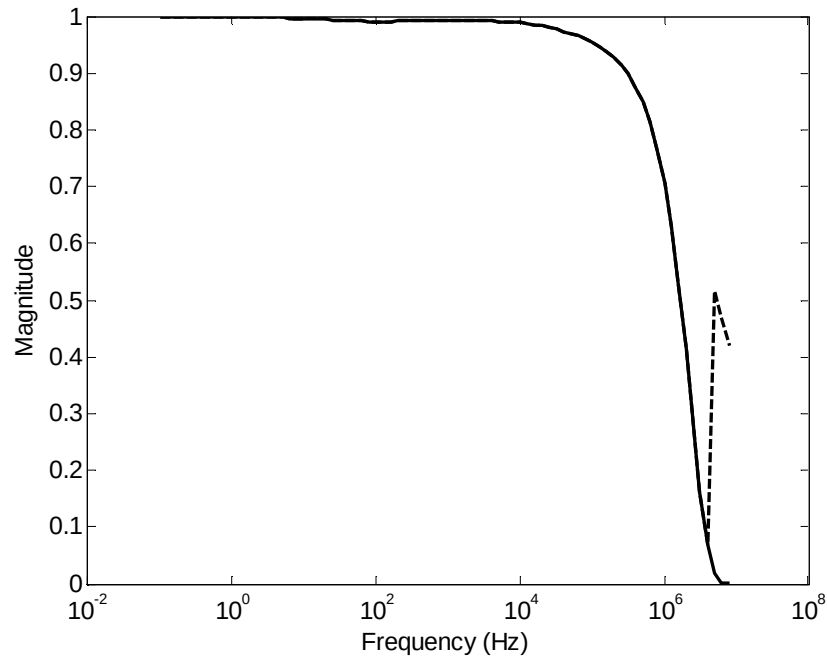


Figure 5.1 Switchover problem illustrated for mode 3, Solid line is calculated with switchover routine, Case CAB02

5.1.2. Initial Estimation of Time Delays

Another important issue is the calculation of modal time delays. The real and imaginary parts of H are fitted simultaneously when using linear LS, thus prior identification of a suitable time delay is necessary for suppressing the risk of poor fitting and improving the overall identification process.

The idea behind removing a suitable time delay from H is to compensate the excessive phase lag of each propagation mode. One can pick a time delay which produces a zero phase angle at some high frequency point where the portion of H decays to a predefined tolerance value ' tol '. This ensures compensation of the phase angle and an improved quality of fitting along the frequency range of interest.

$$\tau_0 = -\frac{B(\omega_k)}{\omega_k} \quad (5.9)$$

where ω_k is the point of frequency where the magnitude function has decayed to ‘tol’.

In [69] and EMTP-RV WB module, time delay for each mode in (5.4) is initially estimated by the following expression:

$$\tau_0 = \frac{\phi_{\min_k}}{\omega_k} - \frac{B(\omega_k)}{\omega_k} \quad (5.10)$$

$\phi_{\min_k}$ is the phase angle corresponding to the Minimum Phase Shift (MPS) equivalent of H calculated by applying Bode’s magnitude phase relation [70] at the frequency point k . Details of time delay estimation using (5.10) based on MPS formulation is given in Appendix II. In general usage of (5.10) yields less rms error of fitting compared to (5.9). This is not because of the magic formulation of MPS but rather due to the fact that having a negative phase angle at ω_k instead of a zero phase angle will prevent us from getting positive phase lag at the “toe” portion of H for the frequencies around ω_k . Positive phase angle requires usually higher order of fitting or unstable poles. Actually, trying to find an MPS equivalent is not necessary; moreover it may result in misleading estimations of time delay in some cases. Besides, a time delay obtained from MPS formulation does not guarantee minimum rms error for a given order. This observation is made in [65] too, a procedure is therefore introduced which optimizes the time delays together with the poles and residues of PFE approximation for minimizing the fitting error. There is a strong and complicated relation between the fitting error and time delay. Figure 5.2 shows the rms error versus time delay for the 3rd mode of CAB02 cable system. Minimizing rms error with respect to the time delay will be discussed later.

In this work an initial estimate of a delay is simply proposed as

$$\tau_0 = \frac{1}{N-1} \sum_{j=1}^{N-1} \frac{\angle H(\phi_{j+1}) - \angle H(\phi_j)}{\omega_{j+1} - \omega_j} \quad (5.11)$$

The initial delay proposed here (5.11) is an estimate of the sum of the true delay minus the slope of the linearized phase. The average is taken over a band at the “toe” portion of H since the compensation of the phase lag is more important at high frequencies. Calculated delay guarantees a negative phase angle at the toe portion of H . When H is multiplied by the factor $e^{(s\tau_0)}$, the phase lag becomes compensated in the frequency band of interest. Removal of τ_0 from modal propagation function suppresses the risk of poor fitting. If there is no “toe” portion for the given frequency range then τ_0 may be calculated at the highest frequency decade given.

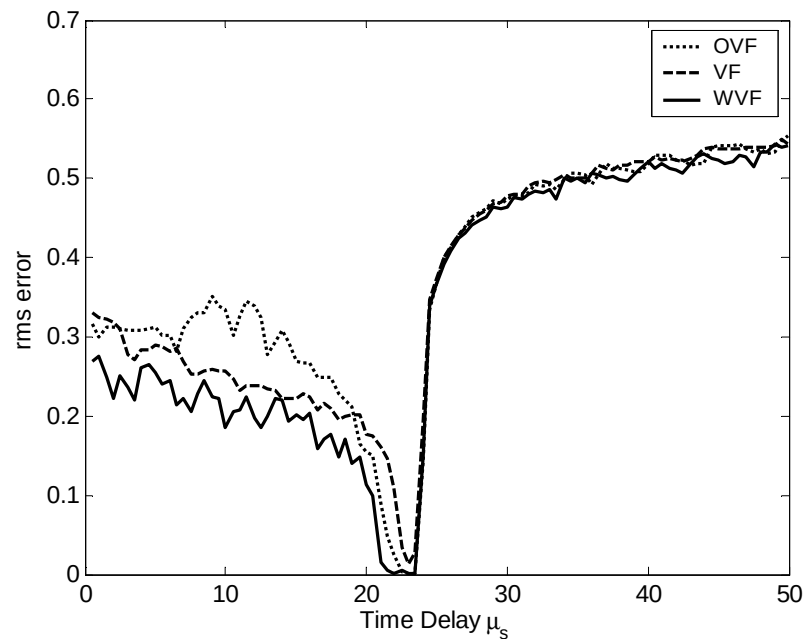


Figure 5.2 Time Delay versus rms Error calculated with OVF, VF and WVF techniques, Propagation Mode 3, Case CAB02

5.1.3. Grouping of Modes

Modes with identical or almost identical time delays are grouped. The grouping is done by adding the modes and dividing the sum with the number of modes. Each group is assigned a single time delay equal to the smallest of the time delays of the individual modes.

Concerning the delays which are not identical, the grouping criterion is defined in terms of angle difference between them which is set to 10 degrees. Angle corresponding to a delay is calculated as

$$\theta = \frac{180}{\pi} \omega_k \tau \quad (5.12)$$

5.1.4. Combined Identification of Propagation Modes and Delays

Fitting error in terms of rms-error can be further reduced by slightly altering the time delay. This operation can be called time delay search or optimization which can be achieved by direct experimentation with rational fitting with the delay as parameter. Once the interval is known (τ_a, τ_b) several classical procedures are available to refine it [71]. Golden section search in one dimension or more sophisticated methods such as Brent's method can also be applied.

In the practical implementation, following the removal of the delay term, the remaining part is approximated as a rational function by calling VF, RVF or WVF fitting routines [17]. Depending on the variation of rms error, delay is changed as a parameter in the predefined interval.

A right bracket value can be chosen by picking a time delay which produces a zero phase angle at a frequency point (10-100 kHz), where the magnitude of H has already started to decay. Note that the time delay decreases as frequency increases.

$$\tau_b = -\frac{B(\omega_b)}{\omega_b} \quad (5.13)$$

This value ensures a large enough time delay which results in a function having a positive phase angle at the “toe” portion of H , consequently an rms error significantly higher than the minimum value is guaranteed. Positive phase angle requires usually

higher order fitting or unstable poles. Unstable poles are not allowed in the present implementation.

A left value can be suggested as the smallest value of (5.14) and (5.15)

$$\tau_a = \tau_0 - \Delta\tau \quad \text{where} \quad \Delta\tau = \tau_b - \tau_0 \quad (5.14)$$

or a delay calculated at a very high frequency point where H decays to a very small value:

$$\tau_a = -\frac{B(\omega_a)}{\omega_a} \quad (5.15)$$

This value guarantees the compensation of the phase angle along the frequency range of interest. However, at medium to high frequencies compensation may not be achieved fully therefore rms-error can be expected to be larger than the minimum possible. Equation (5.14) may be useful when H does not decay to a tolerance value for a given frequency band.

A Matlab code for the initial identification of time delays and for defining the bracketing values is given in Appendix III.

After having defined the initial time delay value and bracketing values, time delays can be improved together with poles and residues for each given mode in order to minimize the fitting error. The algorithm proposed in this work is shown in Figure 5.3 which is based on sectional search method.

Practical implementation of Universal Line Model (ULM) in EMTP-RV uses a simplified version of the time delay optimization procedure described in [65]. In this application, search is initialized with a time delay obtained from (5.10) and no interval is defined in order to bracket the optimal delay. This is because time delay for minimum fitting error is expected to be close to the initial estimation. However, it is observed that minimum fitting error may correspond to an unrealistic time delay. This is rare but not

impossible. This can especially happen in short transmission devices where time delays are comparatively small. If a given time delay is small then we don't expect an excessive phase lag for a bounded frequency range (phase lag $B = -\omega\tau$, time delay small $\rightarrow$ phase lag small). Then increasing the phase with an unreal negative time delay in equation (5.4) may be better than compensating the phase with a realistic time delay in terms of fitting error; this would be of course an absurd situation. In the case of long cables it is possible that some modes decay very fast and the fitting range reduces to a very limited frequency band at low frequencies where time delays do not manifest their effects. At low frequencies the phase angle of H is practically zero and the fitting quality of H' can be improved by introducing a negative time delay which will result in a reasonable negative phase angle for H' . Consequently, bracketing values should be defined as advocated in this work in conformity with the physical characteristics of the device before proceeding to optimization to avoid unrealistic time delays including negative values.

5.1.5. Frequency Dependence of Transformation Matrices

Consider an arbitrary modal contribution to $\mathbf{H}$

$$\mathbf{T} e^{\Lambda_m} \mathbf{T}^{-1} \quad (5.16)$$

where e^{Λ_m} is a matrix containing only one nonzero diagonal element for mode m . The modal contribution matrix elements are given by (See Appendix IV)

$$\left[\mathbf{T} e^{\Lambda_m} \mathbf{T}^{-1} \right]_{ij} = T_{im} H_m T'_{mj} \quad (5.17)$$

where $T'_{mj} \in \mathbf{T}^{-1}$.

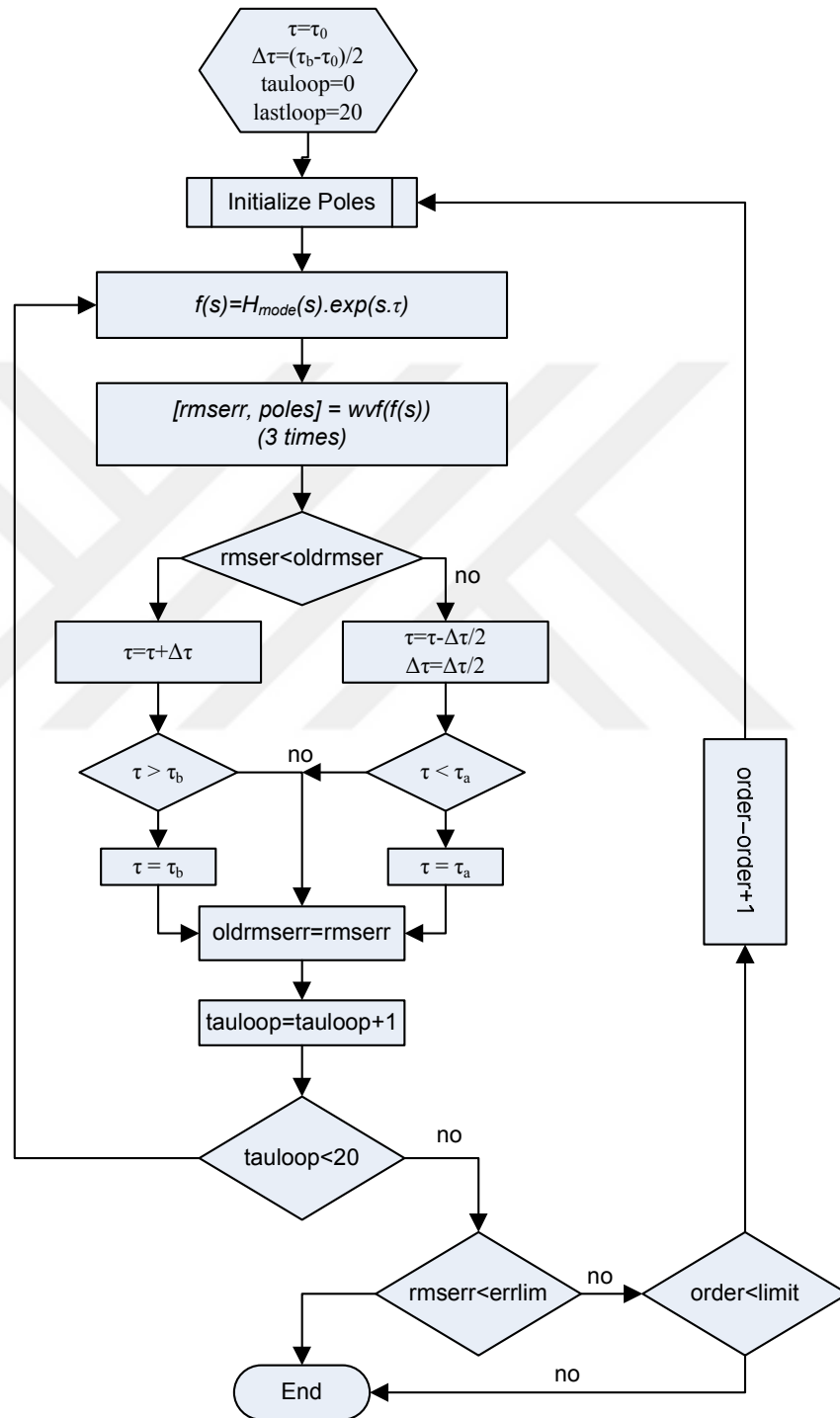


Figure 5.3 Proposed Time Delay and Pole Identification Algorithm

The effect of the frequency dependence of the transformation matrix elements to the modal contribution can be assessed by inspecting $T_{im} T'_{mj}$ multiplication. In modal domain models with constant transformation matrix assumption, this multiplication will be considered as a constant term. Figure 5.4 shows some examples for the CAB01 cable system, Figure 5.5 for TRL01 transmission line case and Figure 5.6 for transposed version of TRL01 case. As these examples suggest, frequency dependence is more important in cables than lines. If the transmission line is assumed to be continuously transposed the effect of the frequency dependence becomes negligible but if the line is not transposed significant frequency dependence is observed. Therefore for the case of lines, there is a relation between the frequency dependence of the transformation matrix and asymmetrical arrangement of the line.

5.1.6. Phase Domain Fitting

Time delay of each mode is extracted from the modal function H and the resulting function H' is finally fitted using the procedure given in Figure 5.3. The modes are fitted to an accuracy of tol , and $\mathbf{H}$ is fitted using the resulting poles and time delays using (5.2). Equation (5.2) is now an overdetermined linear problem in terms of unknowns, i.e. residues. If the required accuracy for $\mathbf{H}$ is not met, tol is reduced and the process in Figure 5.3 is repeated and an improved set of poles with a delay is obtained for a given mode. This loop is repeated until the required accuracy for $\mathbf{H}$ is obtained or until the predefined maximum order of approximation is reached for the given modes or until the number of iterations exceeds the limit.

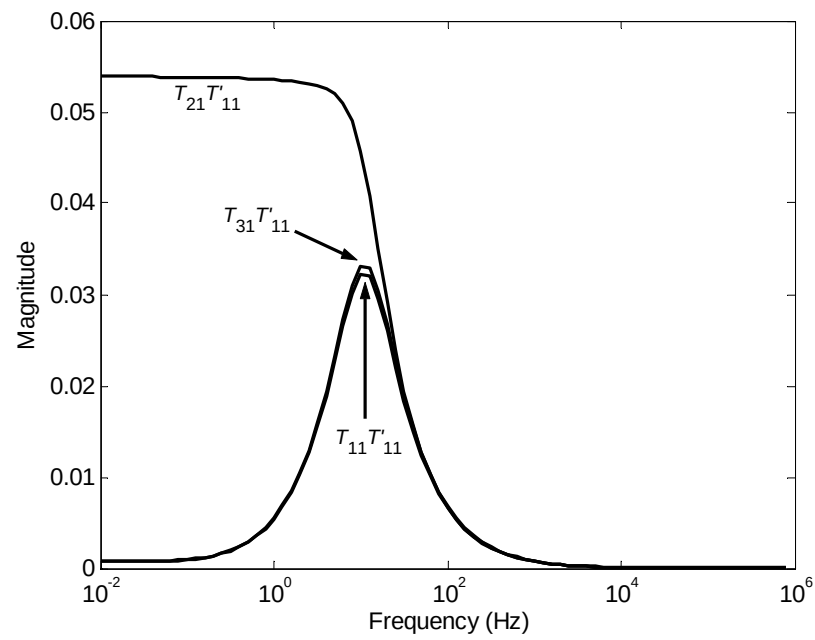


Figure 5.4 Frequency Dependence of Transformation Matrix for Cable System CAB01

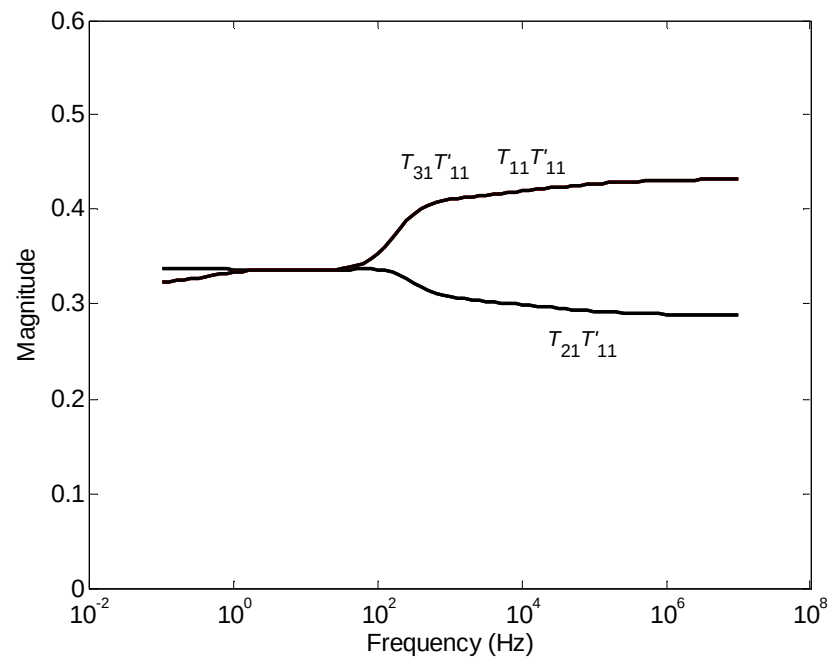


Figure 5.5 Frequency Dependence of Transformation Matrix for Transmission Line TRL01

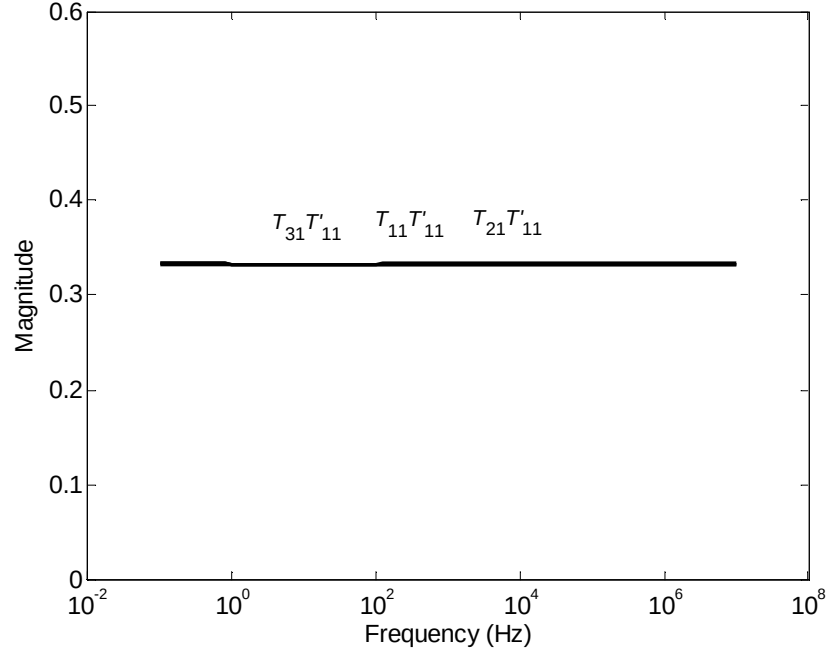


Figure 5.6 Frequency Dependence of Transformation Matrix for Transmission Line TRL01 Transposed

5.1.7. Numerical Stability Aspects of the Adopted Model

Numerical stability troubles of the resulting phase domain model should be also discussed. In the modeling approach presented here every modal time delay is present in every element of $\mathbf{H}$, therefore the interpolation error may have an important effect. Considering these issues, the frequency domain model can be improved in terms of time domain numerical stability.

Equation (5.2) assumes that the frequency dependent contribution of the eigenvalues to each matrix element can be adjusted with residues. In other words the influence of the frequency dependence of the transformation matrix can be compensated with residues. Moreover, the presence of some eigenvalues may be weak in certain elements of $\mathbf{H}$ and residue/pole pairs of these eigenvalues may also improve the phase domain fitting. In general, the above assumption gives high quality fitting results. It has

been observed however, that in some cases the WB model may encounter numerical instabilities in the time-domain solution.

Equation (5.2) is an overdetermined problem containing multiple delay groups and the problem is solved by using the least squares method. The only criterion is the minimization of the difference between fitted and calculated responses. Even though the combination of delay groups fits $\mathbf{H}$ perfectly, PFE of separate delay groups in (5.2) does not necessarily match the isolated contribution of each eigenvalue which can be calculated as follows:

$$H_{ij,m} = \left[\mathbf{T} e^{\Lambda_m} \mathbf{T}^{-1} \right]_{ij} \neq \left(\sum_{n=1}^{N_m} \frac{c_{ijn}}{s + p_{mn}} \right) e^{-(s \tau_m)} \quad (5.18)$$

Consequently, the solution may come up with delay groups having unexpectedly high magnitude curves with high residue pole ratios. This may lead to numerically unstable models in the time-domain solution. It is noticed that it is usually the case when the time delays are not significantly apart from each other, such as in short length cables.

Consider for example a 1 km cable with 6 phases for a total of 12 conductors as detailed in Table 2.2. The WB model accurately fits the $\mathbf{H}$ and $\mathbf{Y}_c$ matrices for this system, but as demonstrated later, it generates an unstable time-domain model. In this case there are 7 distinct delay groups. The magnitude curves (right hand side of the inequality in (5.18)) of these delay groups present in H_{11} are shown in Figure 5.7 together with the calculated contributions of each delay group (left hand side of the inequality in (5.18)). It is noticed that there is a magnification effect when using the WB approach. Although the magnitude curves are significantly different, the final result obtained from the summations of the groups in each case provides a very close match with an RMS error of 1.49×10^{-3} for the given frequency range.

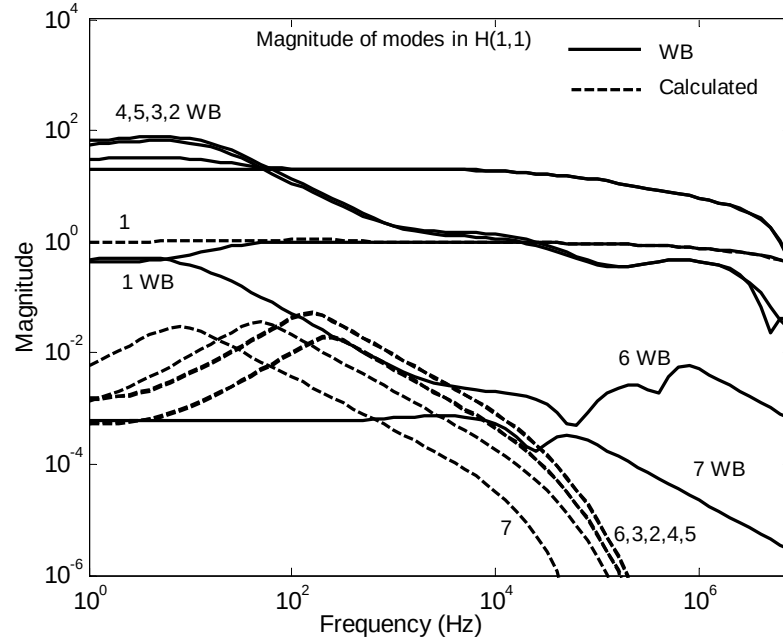


Figure 5.7 Magnitude of PFE of delay groups forming H_{11} , solid line is WB fitting, dashed line is from calculated contributions

It will be shown in the next chapter that the numerical instability problems in the time-domain solution can be avoided by scaling residues in terms of the corresponding pole values. Then equation (5.2) is solved for unknown residues as a constrained linear least squares problem by using convex programming techniques and abnormal fitting schemes such as in Figure 5.7 are avoided:

$$\min_c \|H_{ij}(s) - \tilde{H}_{ij}(s)\|_2^2 \text{ such that} \quad (5.19)$$

$$-K p_{mn} \leq c_{ijmn} \leq K p_{mn}$$

All the fitting results shown hereafter are obtained by applying the residue scaling by setting K to 10. The choice of K and its relation with numerical stability in the time domain are discussed in detail in the next chapter. Although (5.19) is sufficient for all problematic cases demonstrated in this work, other constraints can also be imposed as will be discussed in the next chapter. The model developed in this chapter will be named, hereafter, as CIWB (Convex Improved Wideband) Model.

5.2. Fitting of Characteristic Admittance

For $\mathbf{Y}_c$ common poles are calculated by fitting the sum of diagonal elements (trace). Then these poles are used to fit $\mathbf{Y}_c$ in the phase domain. Therefore each element of $\mathbf{Y}_c$ has the same set of poles

$$Y_{c_{ij}}(s) \cong \sum_{n=1}^N \frac{\tilde{c}_{ij_n}}{s + \tilde{p}_n} + d_{ij} \quad (5.20)$$

where d_{ij} is the value at infinite frequency. It is very important to set the value of d_{ij} based on the given frequency response. Although it is possible to calculate d_{ij} by defining it as an unknown parameter in the LS formulation this may lead to a wrong estimation of the sign of d_{ij} . If, for instance, d_{ij} is calculated as a negative number, this implies that characteristic impedance is negative at infinite frequency and it has a negative conductance. This is abnormal and violates the inherent passivity property of the model.

Poles that are calculated from the fitting of the traces were tabulated in the previous chapter (

Table 4.3 to **Error! Reference source not found.**) for different test cases, i.e., CAB01, CAB02 and TRL01 cases. These poles are used to identify $\mathbf{Y}_c$ in the phase domain.

5.3. Numerical Examples and Discussion

In this section fitting examples of $\mathbf{Y}_c$ and $\mathbf{H}$ are given for three different test cases given in section 2.5: CAB02 (12 Conductor Cable System), CAB01 (6 Conductor Cable System) and TRL01 (Double Circuit Transmission Line System).

From Figure 5.8 to Figure 5.15 the magnitude and phase angle curves of $\mathbf{H}$ are compared against fitting for the CAB02 case. From Figure 5.17 to Figure 5.23 the magnitude and phase angle curves of $\mathbf{H}$ and $\mathbf{Y}_c$ are compared against fitting for the CAB01 case. Note that normally $\mathbf{H}$ and $\mathbf{Y}_c$ are 12 by 12 matrices for CAB02 and 6 by 6 for CAB01, but only some of the elements, which are considered to be a good sampling for the overall illustration of the problem, are shown. For all the cases there is a perfect match between the calculated and fitted results for the defined tolerance (0.015 maximum absolute error). All the deviations in the phase angle correspond to the magnitude values below the tolerance value.

From Figure 5.24 to Figure 5.31 the magnitude and phase angle curves of $\mathbf{H}$ and $\mathbf{Y}_c$ are compared against fitting for the TRL01 case. The performance is again very good. For the given elements phase domain fitting performance of the FD line is also illustrated by multiplying modal domain fitting results with the constant transformation matrix used by the FD line. The performance of the FD line fitting is poor when the line is not transposed; if the line length is further increased the fitting quality of FD worsens. If the line is considered to be continuously transposed then the FD line generates precise results. These results are shown from Figure 5.32 to Figure 5.37. Note that, as explained in section 5.1.5, the assumption of constant transformation matrix may result in erroneous phase domain fitting in case of strongly asymmetrical transmission lines.

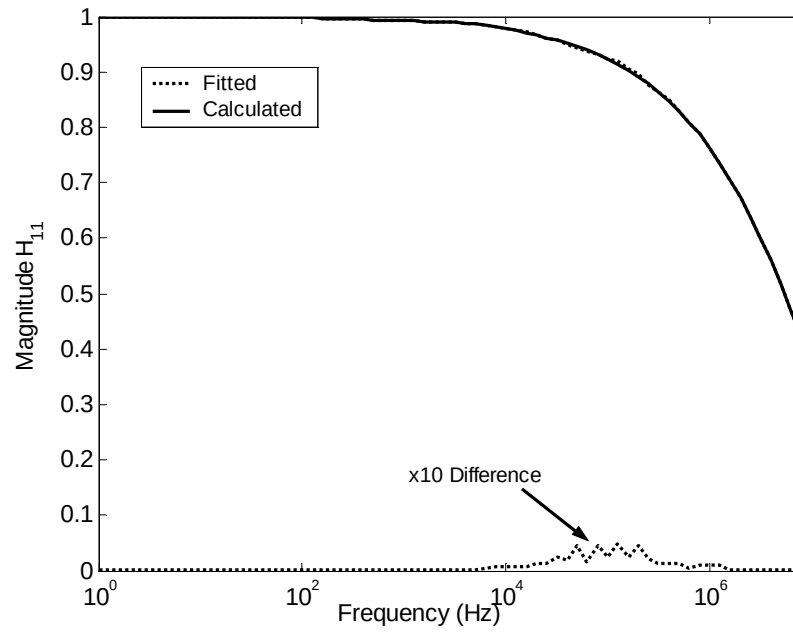


Figure 5.8 Magnitude of H_{11} solid line is Calculated, dashed line is fitted solution (overlapped), the difference (error) is magnified by 10, Case CAB02

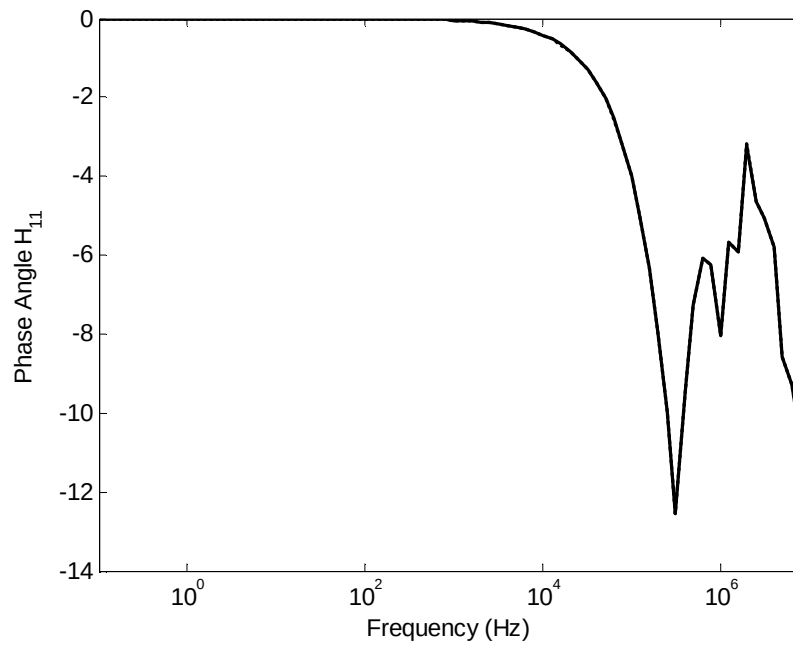


Figure 5.9 Phase Angle of H_{11} solid line is calculated, dashed line is fitted solution (overlapped), Case CAB02

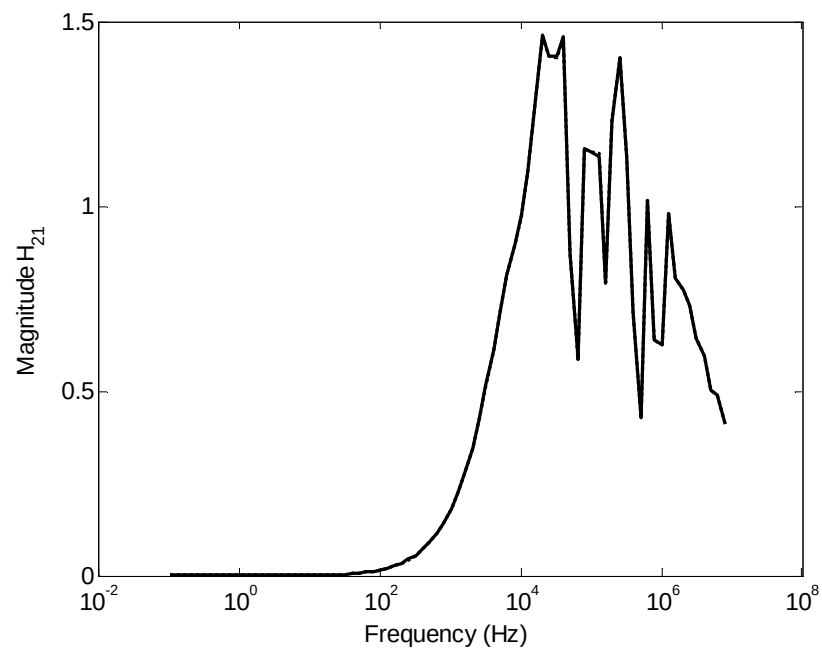


Figure 5.10 Magnitude of H_{21} solid line is calculated, dashed line is fitted solution (overlapped), Case CAB02

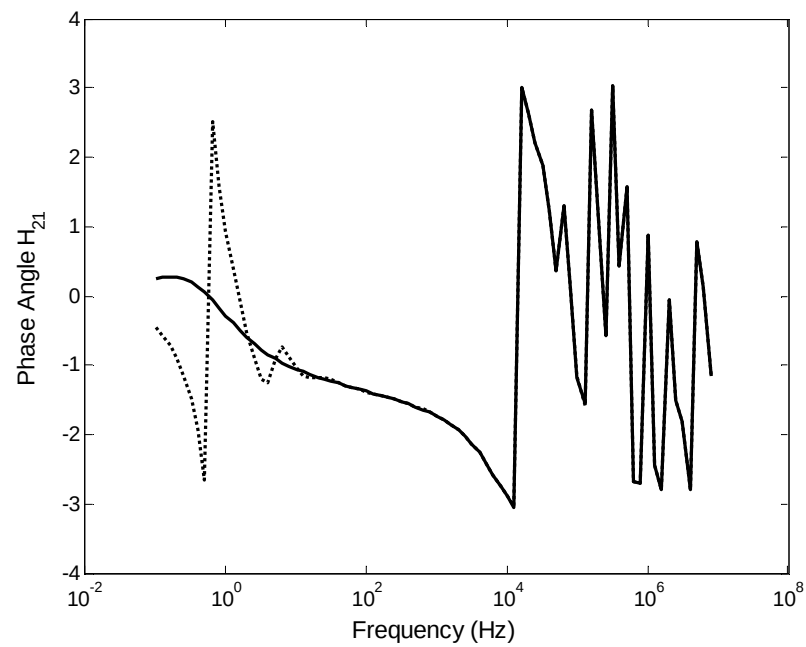


Figure 5.11 Phase Angle of H_{21} solid line is calculated, dashed line is fitted solution (overlapped), Case CAB02

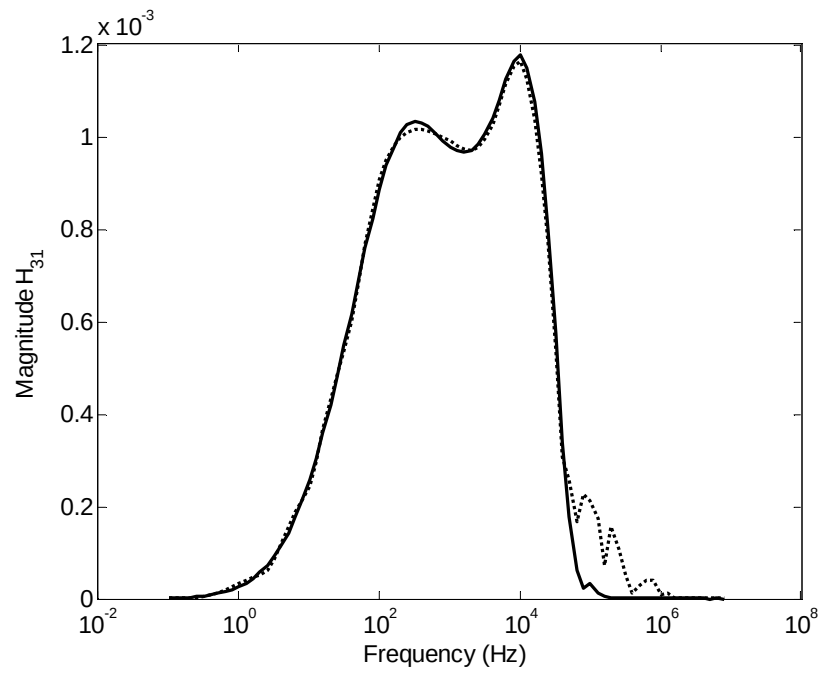


Figure 5.12 Magnitude of H_{31} solid line is calculated, dashed line is fitted solution, Case CAB02

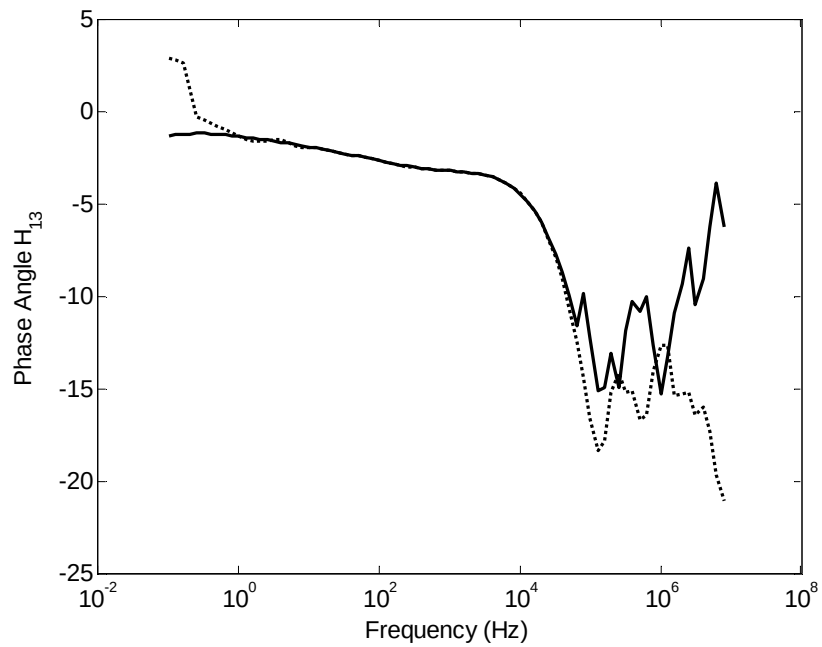


Figure 5.13 Phase Angle of H_{31} solid line is calculated, dashed line is fitted solution, Case CAB02

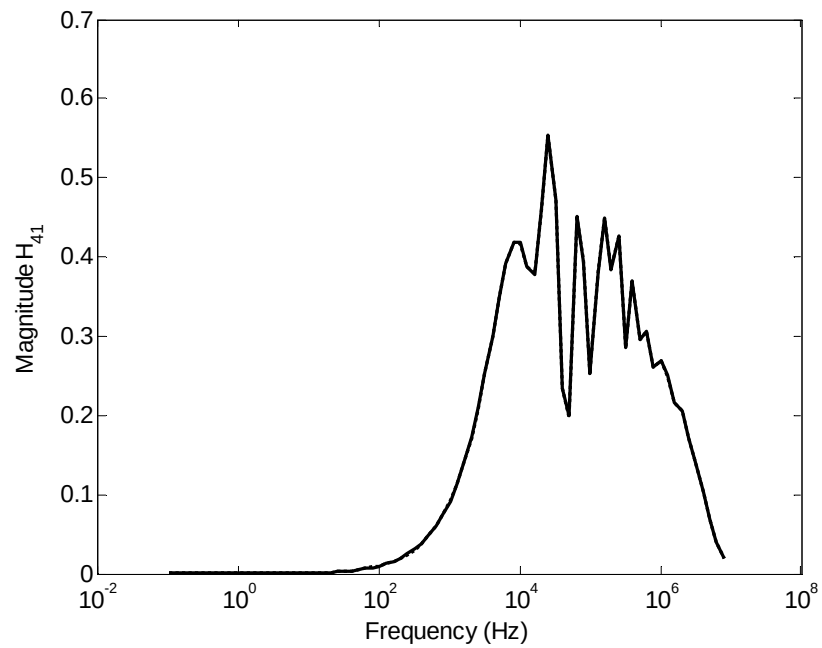


Figure 5.14 Magnitude of H_{41} solid line is calculated, dashed line is fitted solution (overlapped), Case CAB02

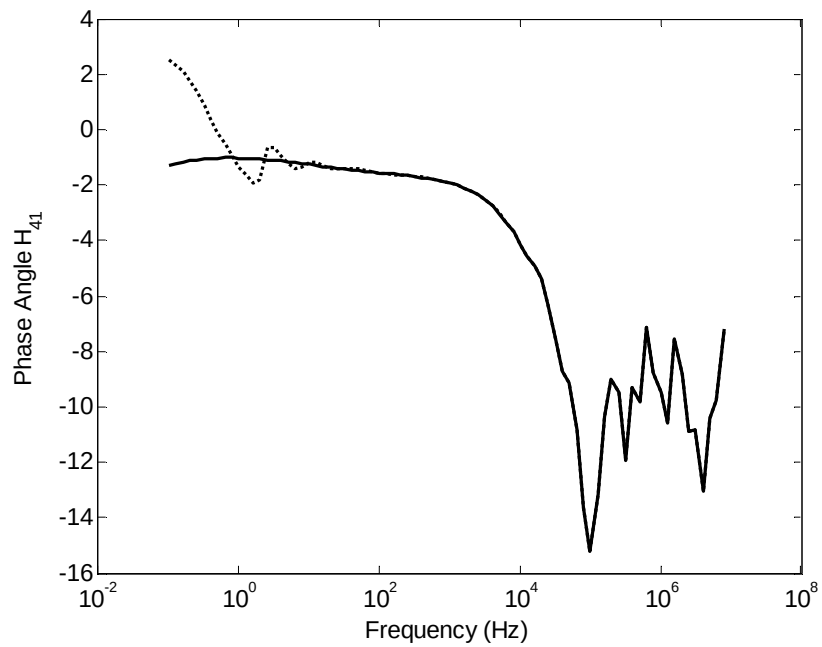


Figure 5.15 Phase Angle of H_{41} solid line is calculated, dashed line is fitted solution, Case CAB02

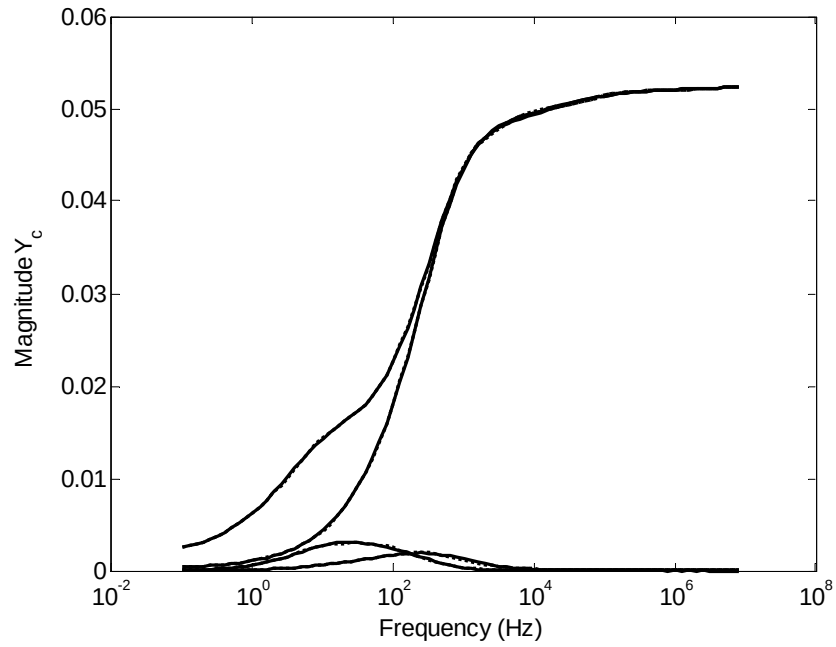


Figure 5.16 Magnitude of Y_{c11} Y_{c21} Y_{c31} Y_{c41} solid line is calculated, dashed line is fitted solution (overlapped), Case CAB02

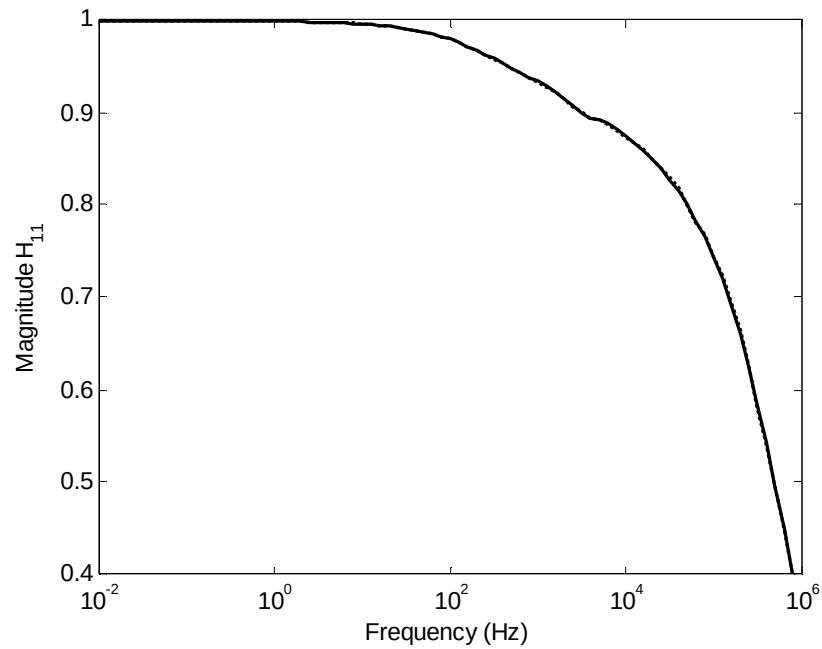


Figure 5.17 Magnitude of H_{11} solid line is calculated, dashed line is fitted solution (overlapped), Case CAB01

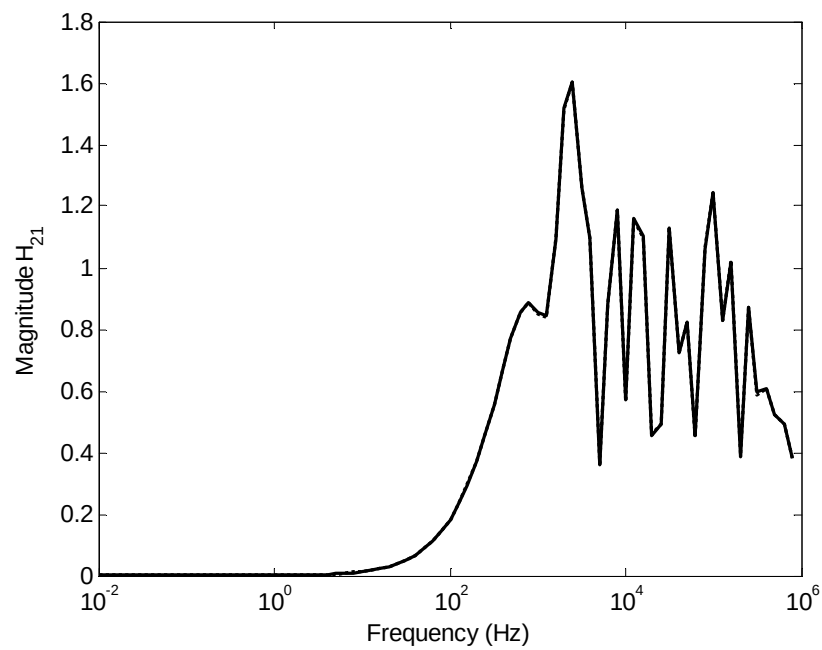


Figure 5.18 Magnitude of H_{21} solid line is calculated, dashed line is fitted solution (overlapped), Case CAB01

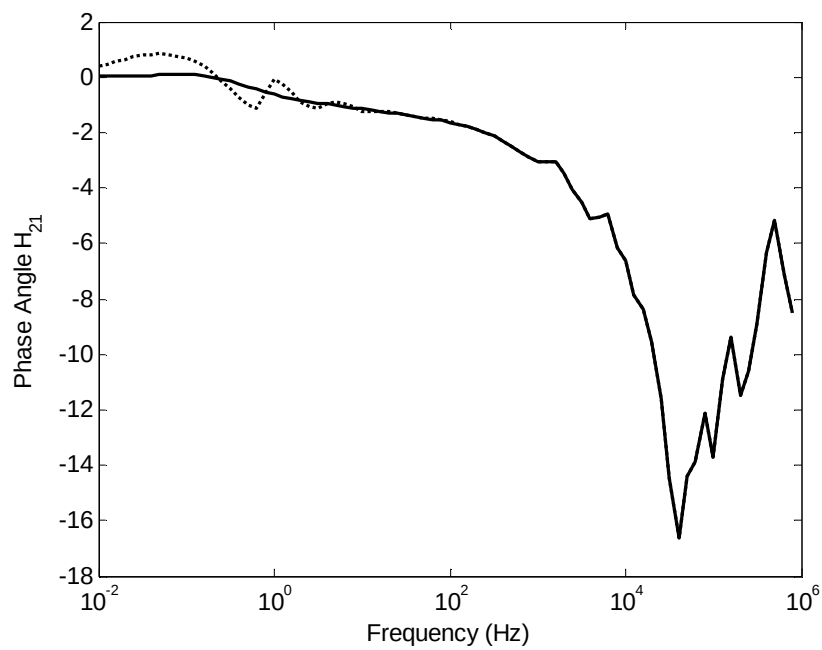


Figure 5.19 Phase Angle of H_{21} solid line is calculated, dashed line is fitted solution, Case CAB01

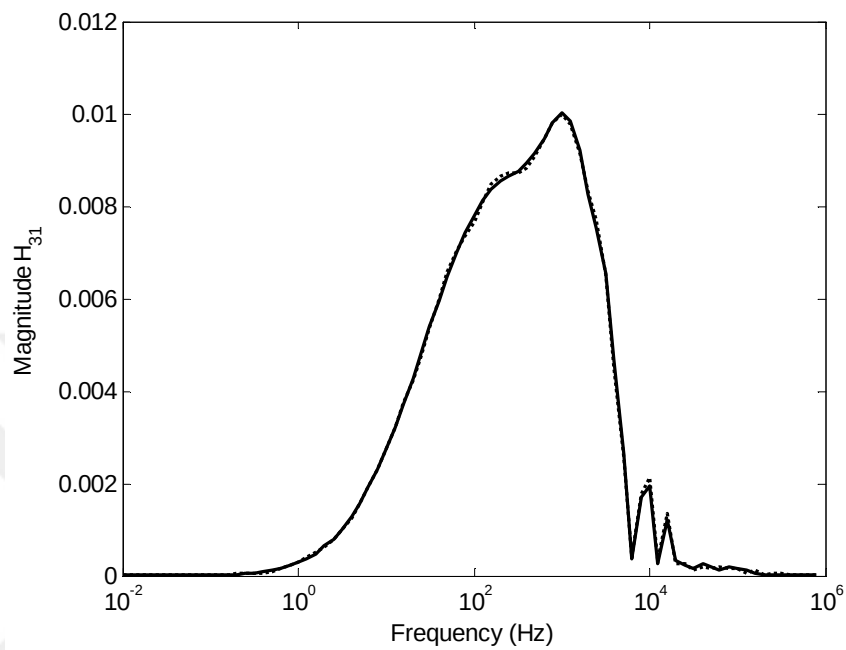


Figure 5.20 Magnitude of H_{31} solid line is calculated, dashed line is fitted solution, Case CAB01

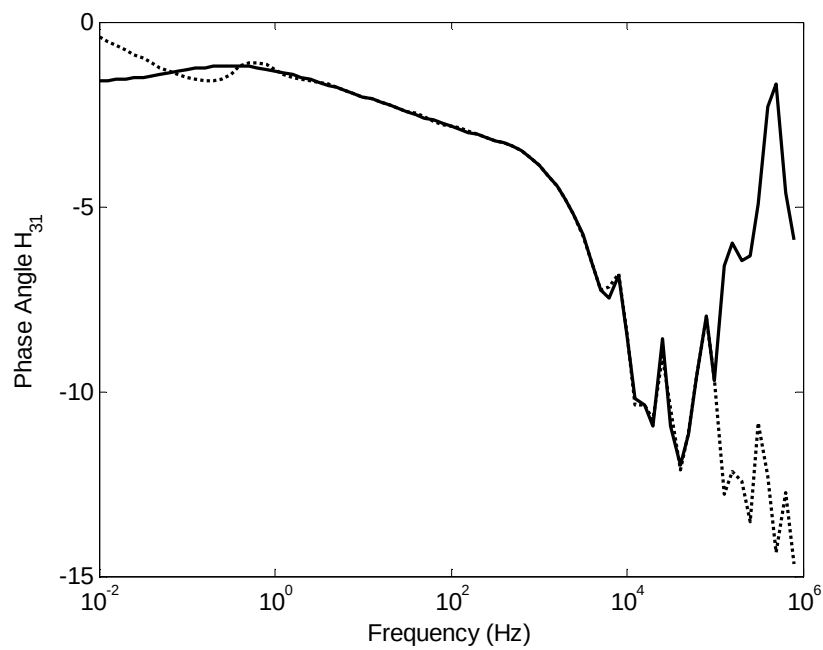


Figure 5.21 Phase Angle of H_{31} solid line is calculated, dashed line is fitted solution, Case CAB01

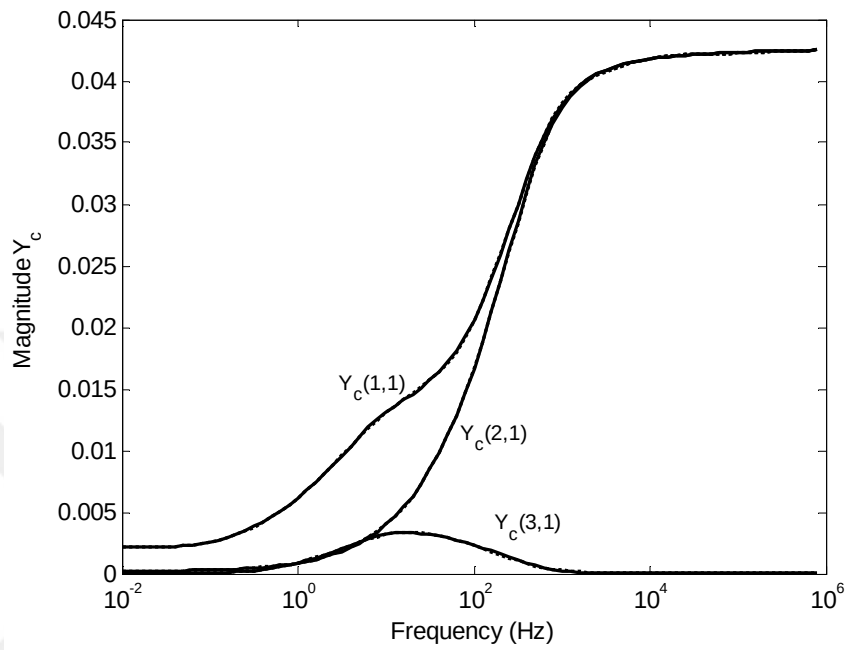


Figure 5.22 Magnitude of Y_c solid line is calculated, dashed line is fitted solution (overlapped), Case CAB01

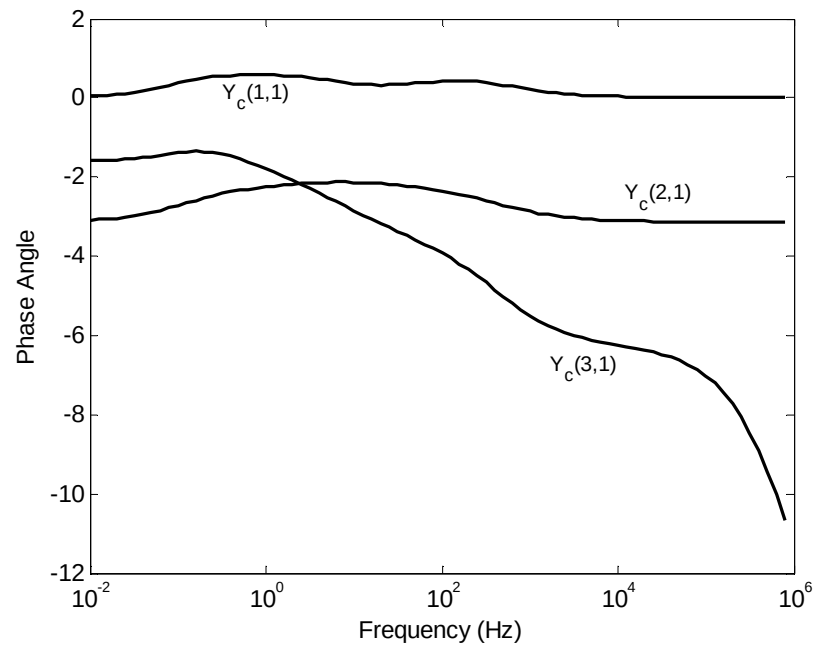


Figure 5.23 Phase Angle of Y_c solid line is calculated, dashed line is fitted solution (overlapped), Case CAB01

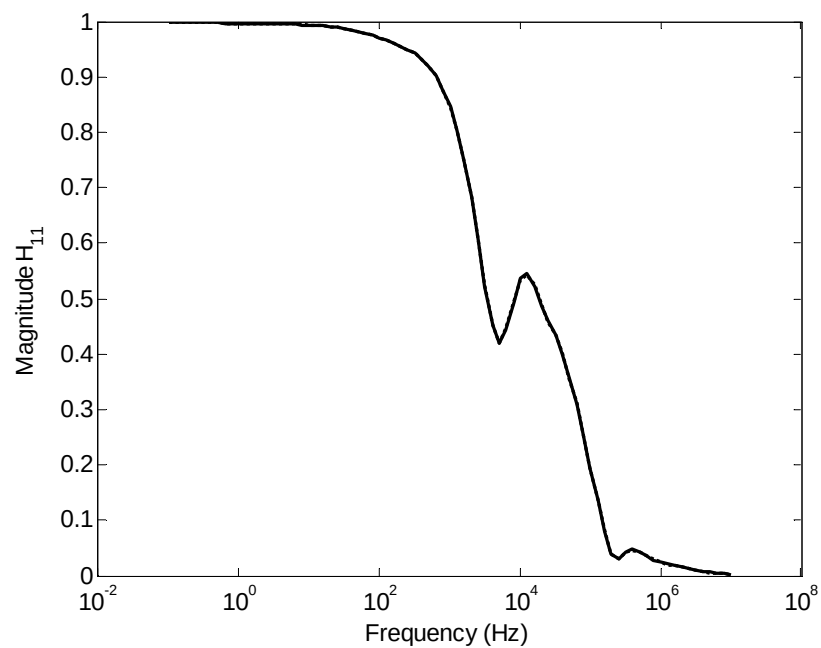


Figure 5.24 Magnitude of H_{11} solid line is calculated, dashed line is fitted solution (overlapped), Case TRL01

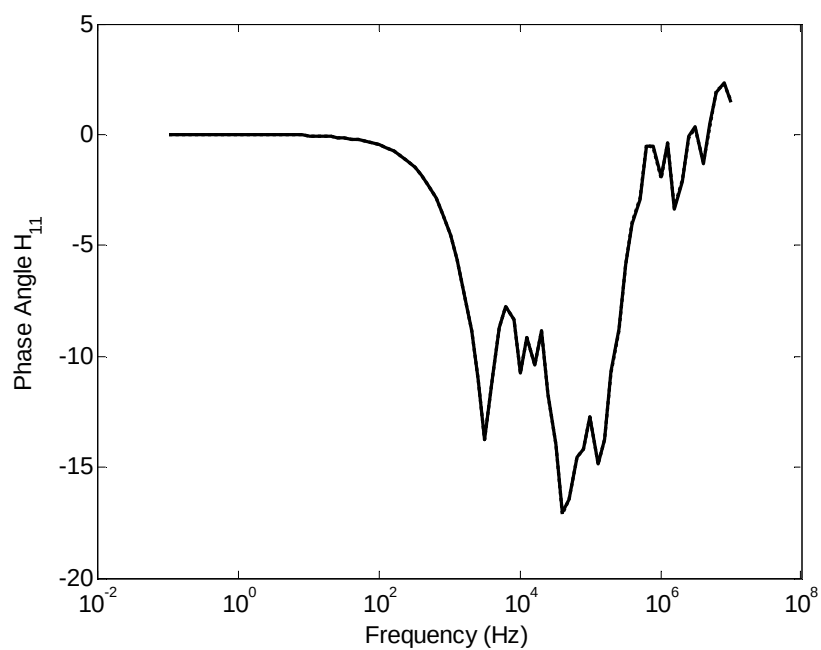


Figure 5.25 Phase Angle of H_{11} solid line is calculated, dashed line is fitted solution (overlapped), Case TRL01

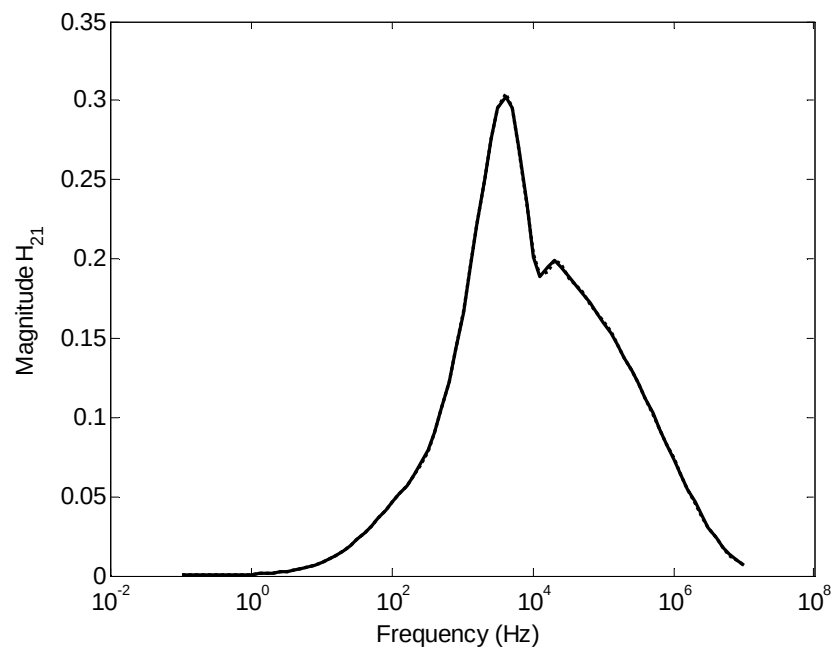


Figure 5.26 Magnitude of H_{21} solid line is calculated, dashed line is fitted solution (overlapped), Case TRL01

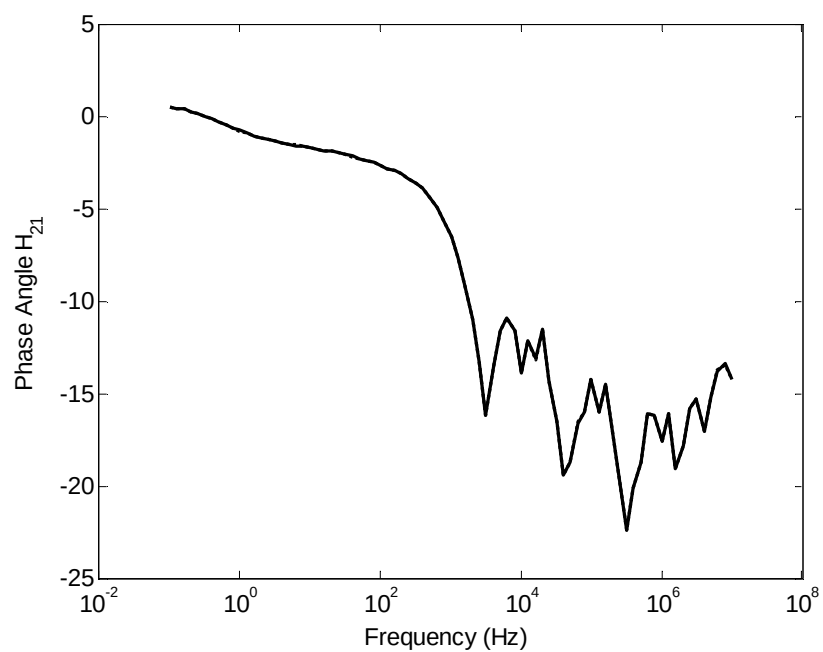


Figure 5.27 Phase Angle of H_{21} solid line is calculated, dashed line is fitted solution (overlapped), Case TRL01

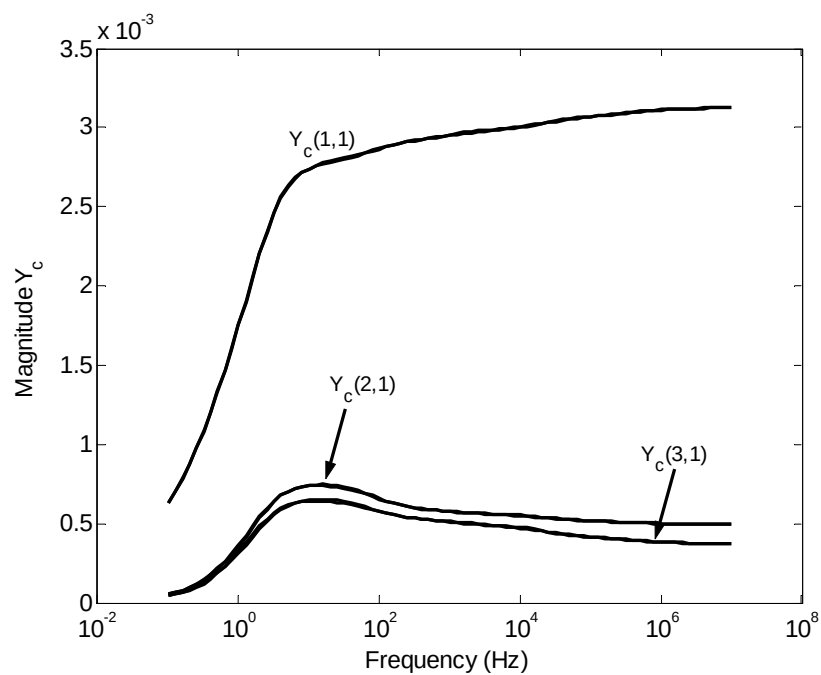


Figure 5.28 Magnitude of Y_c solid line is calculated, dashed line is fitted solution (overlapped), Case TRL01

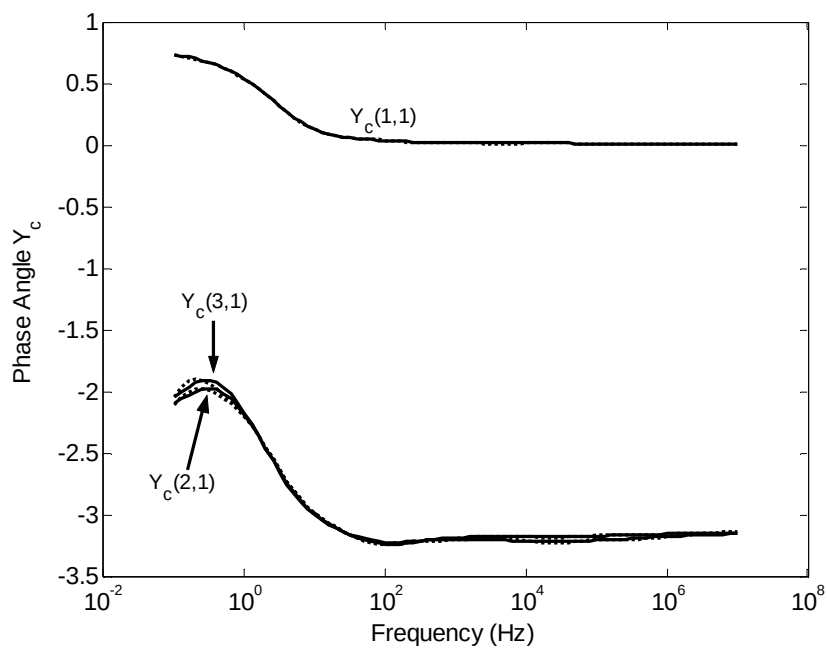


Figure 5.29 Phase Angle of Y_c solid line is calculated, dashed line is fitted solution, Case TRL01

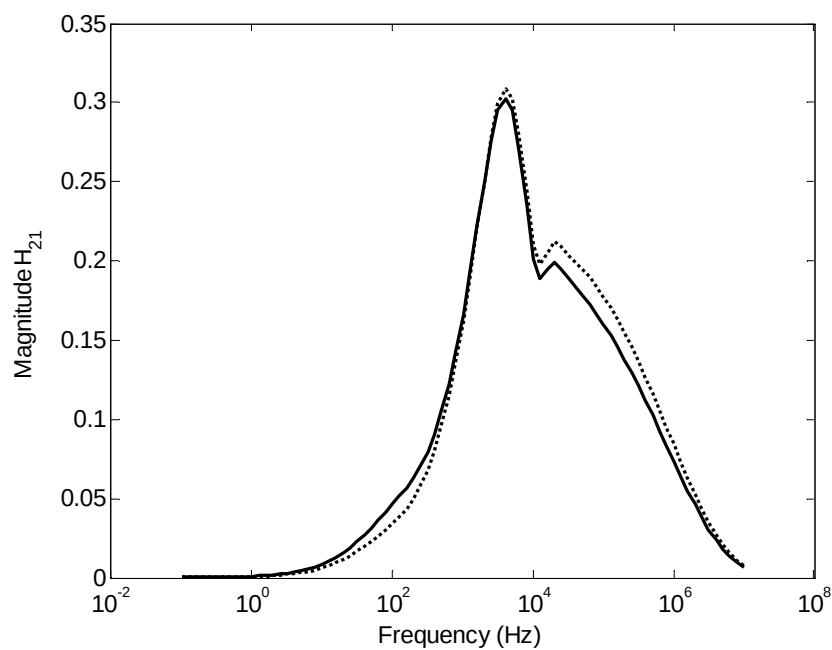


Figure 5.30 Magnitude of H_{21} solid line is calculated, dashed line is FD line solution, Case TRL01

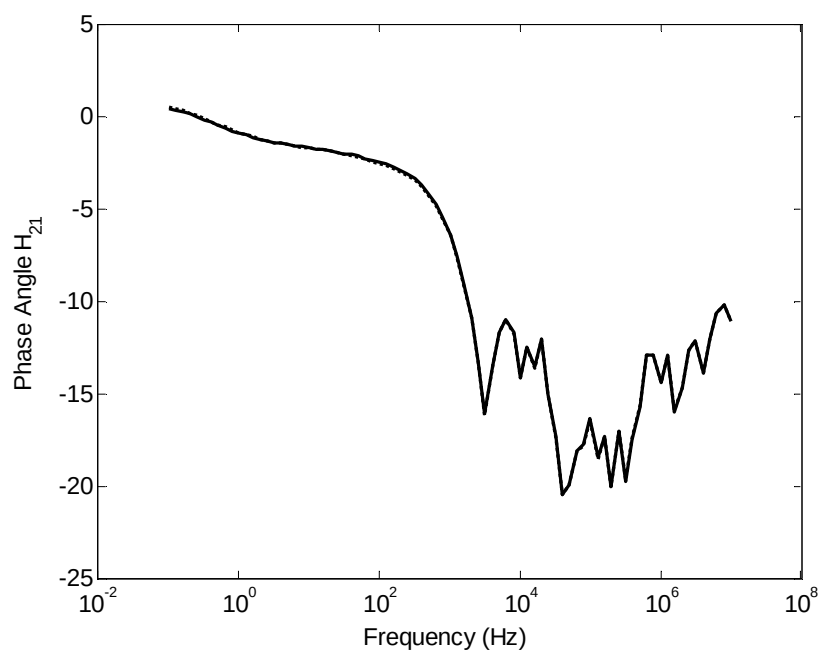


Figure 5.31 Phase Angle of H_{21} solid line is calculated, dashed line is FD line solution (overlapped), Case TRL01

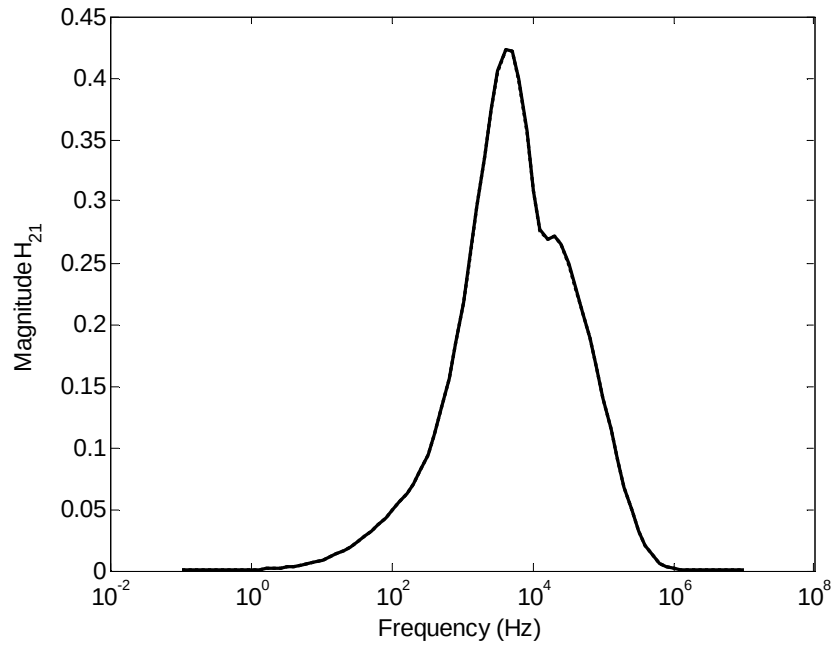


Figure 5.32 Magnitude of H_{21} solid line is calculated, dashed line is FD line solution, Case TRL01 Transposed

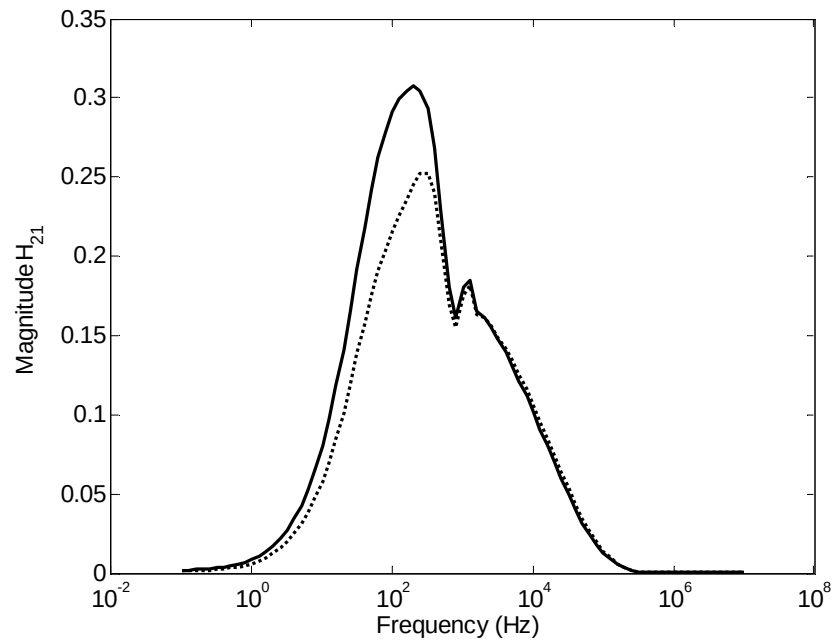


Figure 5.33 Magnitude of H_{21} solid line is calculated, dashed line is FD line solution, Case TRL01 2000 km

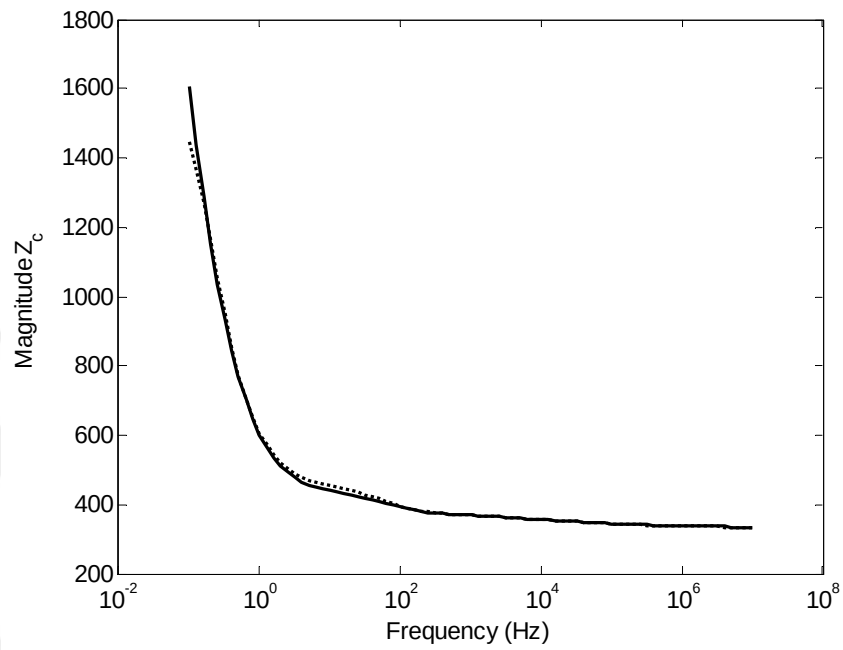


Figure 5.34 Magnitude of Z_{c11} solid line is calculated, dashed line is FD line solution, Case TRL01

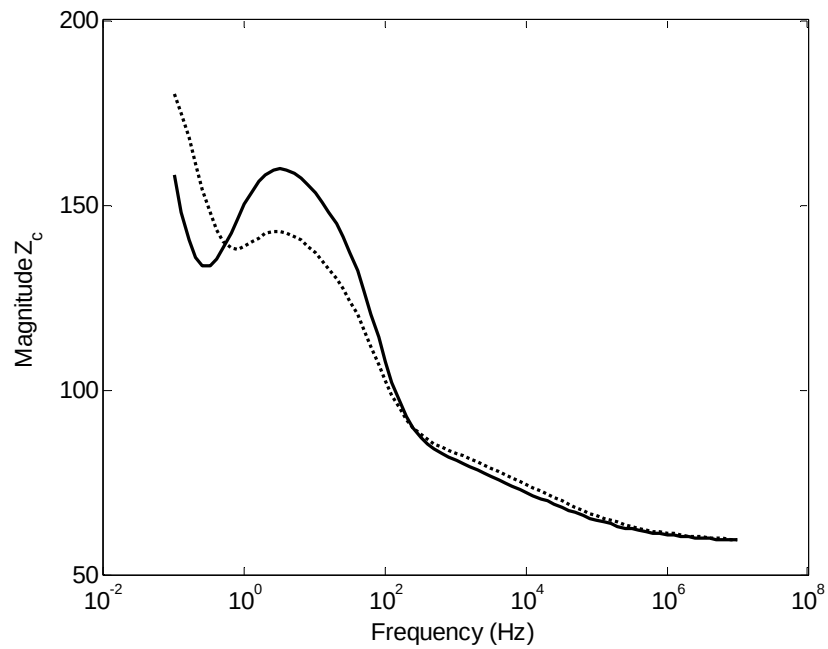


Figure 5.35 Magnitude of Z_{c21} solid line is calculated, dashed line is FD line solution, Case TRL01

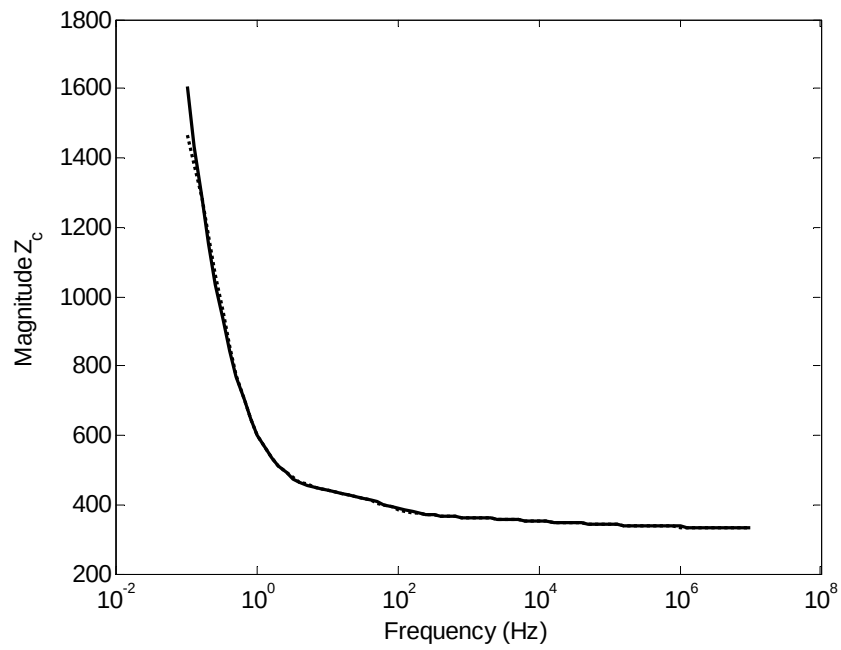


Figure 5.36 Magnitude of Z_{c11} solid line is calculated, dashed line is FD line solution, Case TRL01 Transposed

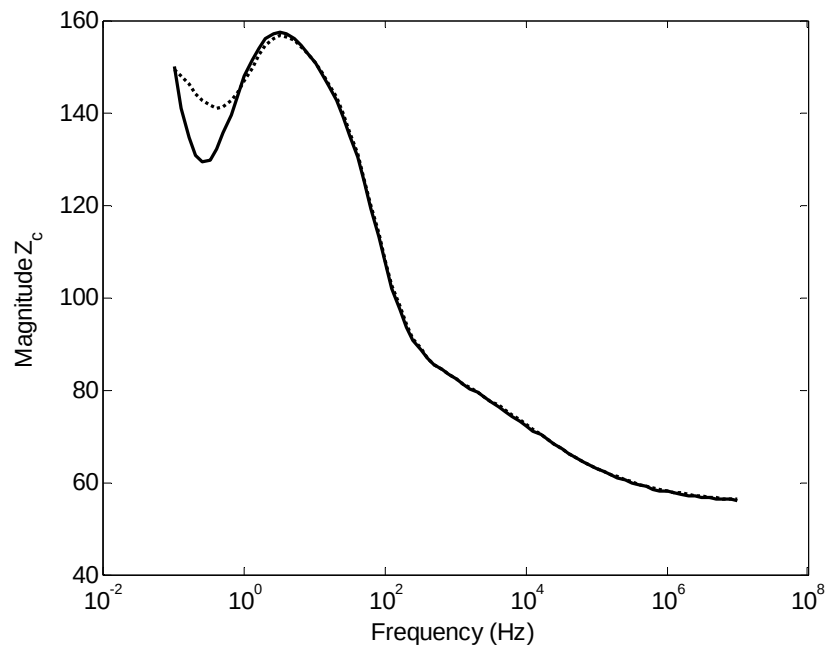


Figure 5.37 Magnitude of Z_{c21} solid line is calculated, dashed line is FD line solution, Case TRL01 Transposed

CHAPTER 6.

TRANSIENT MODEL AND ERROR ANALYSIS

In the previous chapters, the fitting of $\mathbf{H}$ and $\mathbf{Y}_c$ by using rational transfer functions in the form of PFE was discussed. Rational function approximations allow an efficient computation of convolution integrals in the time-domain with a recursive scheme. Convolution integrals are in general computed via numerical integration techniques in the time-domain in accordance with the discrete time-step of the EMTP simulation.

In this chapter convolution integrals are solved with the state-space representation which is the most reliable LTI model to use for computer analysis. This is one of the reasons for the popularity of "modern" state-space control theory.

Although application of state space theory to the transient solution of transmission devices is well known, numerical stability problems in the time-domain remain complex and require further research. Therefore this work is rather focused on the numerical stability problems.

As discussed in the previous chapter, direct solution of (5.2) can give an accurate fitting result of $\mathbf{H}$ but may come up with magnified modal contributions. This may deteriorate the time domain performance and create significant numerical errors. In this work, given a transmission device model, associated local numerical errors for time-domain analysis are estimated in terms of PFE variables. Based on this relation between errors and PFE variables, constraints are determined for the residue/pole ratios such that the time-domain errors are confined within a safe boundary. In the proposed approach the identification in phase domain becomes a constrained linear least squares problem. Therefore, the frequency domain model is improved considering the time domain error performance, a process which has not been considered before in transmission device modeling.

6.1. General Description of the Transient Line Equations

The voltage and current relations of an arbitrary transmission device connected between arbitrary ends k and m , can be written in the frequency domain as

$$\mathbf{I}_k = \mathbf{Y}_c \mathbf{V}_k - \mathbf{H}(\mathbf{Y}_c \mathbf{V}_m + \mathbf{I}_m) \quad (6.1)$$

where $\mathbf{I}$ and $\mathbf{V}$ represent current and voltage column vectors of dimension N_c standing for the number of conductors.

The time-domain solution of (6.1) is obtained by applying convolution

$$\mathbf{i}_k = \mathbf{y}_c * \mathbf{v}_k - \mathbf{h} * \mathbf{u}_m \quad (6.2)$$

where

$$\mathbf{u}_m = \mathbf{y}_c * \mathbf{v}_m + \mathbf{i}_m \quad (6.3)$$

6.2. State Space Representation of Convolution Integral

The frequency domain multiplication of $\mathbf{H}$ in (6.1) with a generic excitation $\mathbf{U}$ for any end k or m can be written as

$$\mathbf{Y} = \mathbf{H}\mathbf{U} \quad (6.4)$$

where

$$\mathbf{U} = \mathbf{Y}_c \mathbf{V} + \mathbf{I} \quad (6.5)$$

The phase domain approximation of $\mathbf{H}$ in (5.2) can be introduced into (6.4), then the i th element of $\mathbf{Y}$ can be written as

$$Y_i = \sum_{j=1}^{N_c} \left[\sum_{m=1}^M \left(\sum_{n=1}^{N_m} \frac{c_{ijmn}}{s + p_{mn}} \right) e^{(-s \cdot \tau_m)} U_j \right] \quad (6.6)$$

For the sake of error analysis it is useful to show the state-space representation for each group of residues and poles corresponding to a distinct delay group. In the first inner sum of (6.6)

$$Y_{i,jm} = \left[\sum_{n=1}^{N_m} \frac{C_{ijmn}}{s + p_{mn}} \right] e^{(-s \cdot \tau_m)} U_j \quad (6.7)$$

Equation (6.7) can be interpreted as the contribution of modal delay group m , present in the element ij of $\mathbf{H}$ to Y_i . The following version of (6.7) is written to simplify the notation

$$Y = \sum_{n=1}^{N_m} \frac{\hat{c}_n}{s + \hat{p}_n} e^{-s \cdot \tau} U \quad (6.8)$$

The state-space representation of (6.8) is now given as

$$\dot{\mathbf{x}}(t) = \mathbf{a} \mathbf{x}(t) + \mathbf{b} u(t - \tau) \quad (6.9)$$

$$y(t) = \mathbf{c} \mathbf{x}(t) \quad (6.10)$$

where $y(t)$ is the scalar output variable and

$$\begin{aligned} \mathbf{a} &= \text{diag} \left[-\hat{p}_1 \quad -\hat{p}_2 \quad \dots \quad -\hat{p}_{N_m} \right] \\ \mathbf{b} &= [1 \quad 1 \quad \dots \quad 1]^T \\ \mathbf{c} &= [\hat{c}_1 \quad \hat{c}_2 \quad \dots \quad \hat{c}_{N_m}] \\ \mathbf{x} &= [x_1 \quad x_2 \quad \dots \quad x_{N_m}]^T \end{aligned} \quad (6.11)$$

Vector $\mathbf{b}$ is of dimension N_m . The analytical solution of (6.9) at the time-point t_{n+1} is given by

$$\mathbf{x}_{n+1} = \mathbf{e}^{\mathbf{a}(t_{n+1}-t_n)} \mathbf{x}_n + \mathbf{e}^{\mathbf{a}t_{n+1}} \int_{t_n}^{t_{n+1}} \mathbf{e}^{-\mathbf{a}t} u(t - \tau) dt \quad (6.12)$$

where $\mathbf{x}_n = \mathbf{x}(t_n)$. It is noticed that in EMTP computations with fixed time-step Δt , the solution samples are separated by $\Delta t = t_{n+1} - t_n$. The input is only available at some discrete values of time. As the time delay τ is not usually an integer multiple of time-step, it is necessary to apply interpolation. The error term present in $y(t)$ due to the interpolation on the input can be written as

$$\varepsilon_1 = \mathbf{c} \int_0^{\Delta t} \mathbf{e}^{at} R(t) dt \quad (6.13)$$

where $R(t)$ is the remainder term. Although the term interpolation is used here, this is actually a generic error estimate from the computation of $\mathbf{u}$ based on the assumption that the previous solution $\mathbf{x}_n$ was exact.

In EMTP, the state-space equations are solved using the trapezoidal integration method

$$\mathbf{x}_{n+1} \cong \frac{\mathbf{I} + \mathbf{a} \frac{\Delta t}{2}}{\mathbf{I} - \mathbf{a} \frac{\Delta t}{2}} \mathbf{x}_n + \frac{\mathbf{I} \frac{\Delta t}{2}}{\mathbf{I} - \mathbf{a} \frac{\Delta t}{2}} \mathbf{b} (u_{n+1} + u_n) \quad (6.14)$$

where $u_n = u(t_n - \tau)$. The notation in (6.14) can be simplified to give

$$\mathbf{x}_{n+1} \cong \boldsymbol{\alpha} \mathbf{x}_n + \boldsymbol{\lambda} (u_{n+1} + u_n) \quad (6.15)$$

The numerical solution given by equation (6.15) has a Local Truncation Error (LTE). The LTE generated by a state variable due to the trapezoidal integration method is given by [72]

$$\varepsilon_2 = \frac{\Delta t^3}{12} x^{(3)}(\xi) \quad (6.16)$$

where $t_n \leq \xi \leq t_{n+1}$. The third derivative of $x(\xi)$ can be approximated using [72]

$$x^{(3)}(\xi) = 3! \frac{1}{6\Delta t^3} (x_{n+1} - x_{n-2} + 3x_{n-1} - 3x_n) \quad (6.17)$$

If previous time-point state variables are considered to be exact then the local truncation error for the calculation of x_{n+1} will be

$$\varepsilon_2 = \frac{1}{12} (\alpha(x_n - x_{n-3}) + 3\alpha(x_{n-2} - x_{n-1}) + \lambda(u_{n+1} + u_n - u_{n-2} - u_{n-1})) \quad (6.18)$$

The LTE is time-step dependent. It is also related to the input variable and implicitly to the interpolation error estimate given in equation (6.13). It is complicated to derive a method for estimating and controlling the error from the LTE equation, but as it will be shown in the following section, it is possible to use equation (6.13) in a limiting criterion.

6.3. Scaling Residues with Convex Programming

This section shows that it is possible to calculate an estimate of the local interpolation error for the given model (5.2). If the interpolation error is considered to be problematic the residues are recalculated such that a certain limit of residue/pole ratio is satisfied and the error between the calculated frequency response and model's frequency response is minimized. This is accomplished by solving a constrained linear least-squares problem. Since the objective function and constraints are convex functions of the optimization variables, i.e. residues, efficient programming algorithms can be applied and failure due to convergence is not a concern. In this paper the Matlab function “lsqlin” is used [73]-[75] for the solution of the constrained linear least-squares problem.

If it is assumed that the remainder term in equation (6.13) is piecewise constant for a given delay group m then the local interpolation error present in (6.10) is given by

$$\varepsilon_1 \cong \mathbf{c} \int_0^{\Delta t} \mathbf{e}^{at} R_m dt = \mathbf{c} \frac{(\mathbf{e}^{a\Delta t} - 1)}{\mathbf{a}} R_m \quad (6.19)$$

or more explicitly

$$\varepsilon_{1,ij,m} \cong \left(\sum_{n=1}^{N_m} \frac{C_{ijmn}}{p_{mn}} (1 - e^{-p_{mn} \Delta t}) \right) R_m \quad (6.20)$$

The term between the parenthesis is a weighting term W_{ijm} of the interpolation error due to mode m in H_{ij} . Therefore minimizing the weighting terms will reduce the total interpolation error due to the convolution of H_{ij} .

The combination of equation (6.20) with (6.12) and (6.10) for consecutive time-steps allows estimating the global interpolation error at a given time-point n_k (number of time-steps).

$$\varepsilon_{1,ij,m}(n_k) \cong \sum_{k=0}^{n_k-1} \left[\sum_{n=1}^{N_m} \frac{C_{ijmn}}{p_{mn}} \left(e^{-p_{mn} k \Delta t} - e^{-p_{mn} (k+1) \Delta t} \right) R_m (n_k - k) \right] \quad (6.21)$$

It is apparent that previous time-point R_m effects are minimized and the local error is the most important.

Since estimating R_m is too complicated it is preferable to limit the weighting term

$$\text{abs}(W_{ijm}) \leq K_w \quad (6.22)$$

where K_w is a positive real constraint. While generating a transmission line or cable model the simulation time-step is not determined, therefore it is preferable to choose the worst case condition in equation (6.20)

$$e^{-p_{mn} \Delta t} \triangleq 0 \quad (6.23)$$

Equation (6.22) becomes

$$\left| \sum_{n=1}^{N_m} \frac{c_{ij\ mn}}{p_{mn}} \right| \leq K_w \quad (6.24)$$

In order to find a reasonable estimate of K_w it is proposed to apply the asymptotic behavior ($s \rightarrow 0$) in the right hand side of the inequality in equation (5.18)

$$H_{ij,m}(s \rightarrow 0) \equiv \sum_{n=1}^{N_m} \frac{c_{ij\ mn}}{p_{mn}} \quad (6.25)$$

If the contribution of this approximated mode should match asymptotically the calculated contribution, then

$$\sum_{n=1}^{N_m} \frac{c_{ij\ mn}}{p_{mn}} \equiv \left[\mathbf{T} e^{\Lambda_m} \mathbf{T}^{-1} \right]_{ij} \text{ for } s \rightarrow 0 \quad (6.26)$$

As the mathematical definition of $\mathbf{H}$ suggests:

$$\left| \left[\mathbf{T} e^{\Lambda_m} \mathbf{T}^{-1} \right]_{ij} \right| \leq 1 \quad (6.27)$$

which provides an upper bound of 1 for K_w .

The purpose here is to fit the elements of $\mathbf{H}$ using suitable residue/pole pairs rather than separately fitting the contributions of modes to $\mathbf{H}$ elements. Asymptotic matching of the contribution of modes can ensure the numerical stability since high weighting terms will be automatically avoided, which leads to the reduction of numerical errors.

Additionally, it is also possible to define a lower and upper boundary for each residue in terms of corresponding poles. This is to avoid high and negative weighting terms. Finally equation (5.2) becomes a constrained linear least-squares problem

$$\begin{aligned}
& \min_c \|H_{ij}(s) - \tilde{H}_{ij}(s)\|_2^2 \text{ such that} \\
& \left[\sum_{n=1}^{N_m} \frac{c_{ij\ mn}}{p_{mn}} \right] \leq 1 \\
& -K p_{mn} \leq c_{ij\ mn} \leq K p_{mn}
\end{aligned} \tag{6.28}$$

A straightforward method can be used to define K for each residue. In equation (5.4) each propagation mode is identified as a rational transfer function and equation (5.18) gives the contribution of a mode to an element of $\mathbf{H}$. By combining these equations the K parameters can be estimated for the phase domain residues by

$$K(c_{ij\ mn}) \leq \max \left| T_{im} \frac{\hat{c}_n}{p_{mn}} T'_{mj} \right| \tag{6.29}$$

where $T'_{mj} \in \mathbf{T}^{-1}$. Choosing the maximum value provides some flexibility for the constrained least squares solution. In the cases presented below setting K to 10 provided correct results.

The fitting error may slightly increase due to the imposition of the above constraints, but this is a reasonable compromise as long as the transient response stability of the model is of concern. Optionally the K parameter can be increased within a predefined tolerance in order to reduce fitting error. In this work it was decided to use a single constraint for each residue pole.

For complex conjugate poles, the boundary condition is applied separately for real and imaginary parts.

6.4. Case Studies on the Residue Scaling

In this section, five different models will be compared in terms of numerical stability in the time-domain simulations. Three models are currently available in EMTP-RV: Frequency Dependent transmission line model (FD), Full Frequency Dependent

(FDQ) cable model, Wideband Model (WB) which is the implementation of ULM model. The other two models which are both applicable to transmission lines and cables are based on the methodology developed in this thesis. First is the Convex Wideband Model (CWB). In this model, residue, pole and time delay estimates of the WB model for a given test case are taken from EMTP-RV and residues are recalculated with convex programming using (6.28). The improvements of the previous chapter are not applied for the time being in order to better understand the contributions of this chapter. The second model is the Convex Wideband Model with Backward Euler (CEWB), where in addition to CWB the Backward Euler integration scheme is used for the solution of equation (6.12) during the entire time-domain simulation interval. Both CWB and CEWB models use the same frequency domain modeling but CEWB has a different time-domain computation algorithm.

6.4.1. Transient Response of the 12 Conductor Cable System CAB02

The transient response of the 12 conductor CAB02 cable system is analyzed using the circuit configuration shown in Figure 6.1.

Table I shows the interpolation error weighting terms of H_{11} for WB and CWB/CEWB models. There are 7 distinct propagation modes identified for this cable and it is apparent that the least squares technique used in the WB model without boundary conditions, results in high error weighting terms for some of the modes. The opposite signs of the terms do not imply cancellation of interpolation errors. Different modes have different delays and result into different R_m terms. Consequently, either cancellation or catastrophic accumulation of errors can happen at each time-step for different modes. This can be observed in Figure 6.2 where modes 2 and 3 cause explosion before $20\mu s$ in the WB model. These modes are supposed to have little effect on the core voltages of the cable. On the other hand these modes cannot be discarded as they are very important for the precise calculation of the sheath voltages.

In this case the maximum rms error between the calculated frequency samples and the approximated rational function is 1.5×10^{-3} for WB and 3.5×10^{-3} for CWB/CEWB. The difference is not significant and the CWB/CEWB model is capable to preserve its numerical stability.

Figure 6.2 also presents the transient voltage at the receiving end of the first cable core conductor using the FDQ model. Both the FDQ and CWB models give almost the same answer. It is observed however, that both FDQ and CWB produce ringing (numerical oscillations) when the step voltage reaches the receiving end. This ringing (see zoomed version in Figure 6.3) is observed despite the fact that EMTP-RV uses two halved time-step Backward Euler steps to eliminate the discontinuity triggered by the step voltage application. The reason is the numerical sensitivity of equation (6.14) where the initial discontinuity causes numerical oscillations which decay only after a few time-points. In the CEWB model it was chosen to replace equation (6.14) by its full time-step Backward Euler version for the entire simulation interval. The halved time-step version for standard discontinuity treatment was also maintained. It was observed that it does not have a significant effect on the simulation results especially for sufficiently small time-steps.

In Figure 6.2 the simulation is $1 \mu\text{s}$ and the minimum time delay of the cable system is $6.23 \mu\text{s}$. Reducing the time-step to $0.1 \mu\text{s}$ prevents the WB model from explosion, but the result remains extremely noisy as shown in Figure 6.4. Now the CWB version does not have a ringing problem. Reducing the time-step even further to $0.01 \mu\text{s}$ does not cure the numerical problems of the WB model, but CWB continues performing as required.

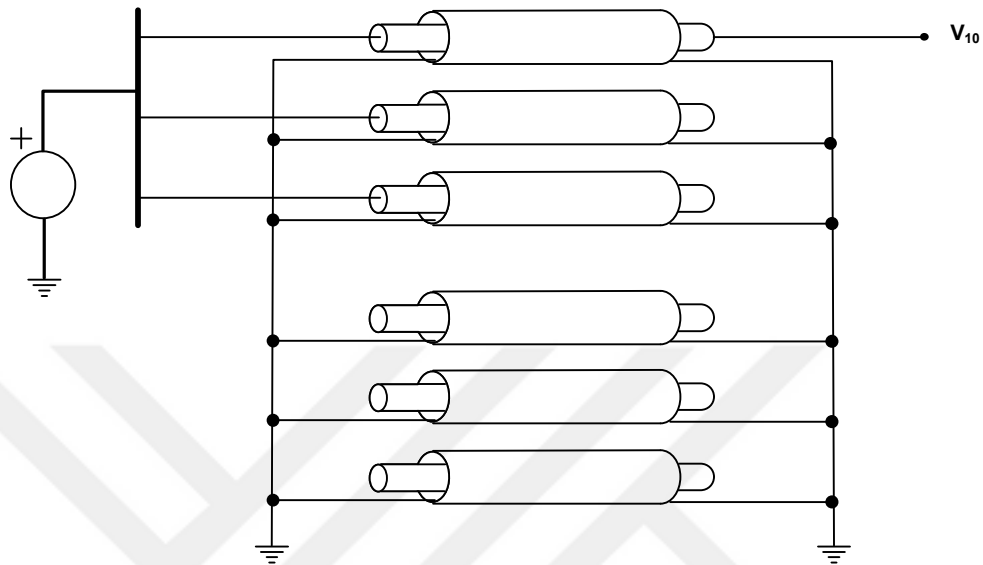


Figure 6.1 Step voltage excitation on cores of cable 1

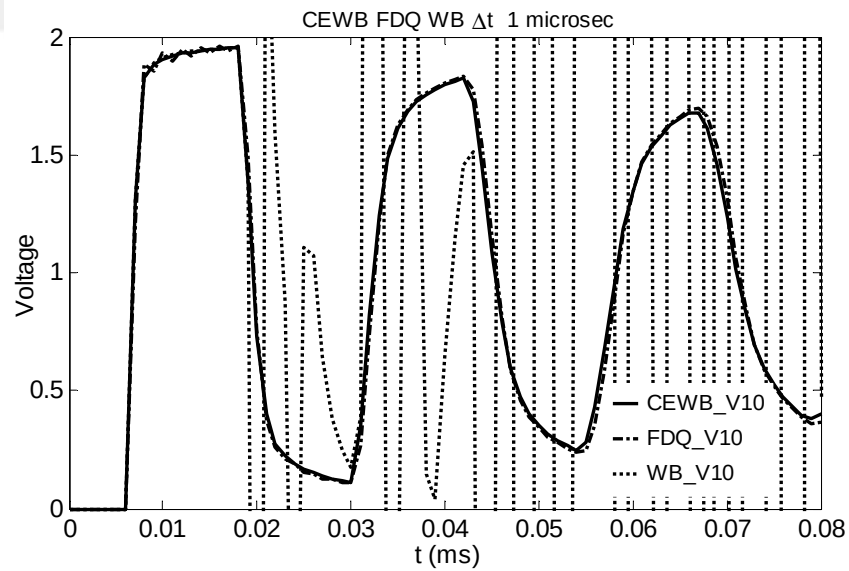


Figure 6.2 Voltage V_{10} for CEWB, FDQ and WB for $\Delta t = 1 \mu\text{s}$

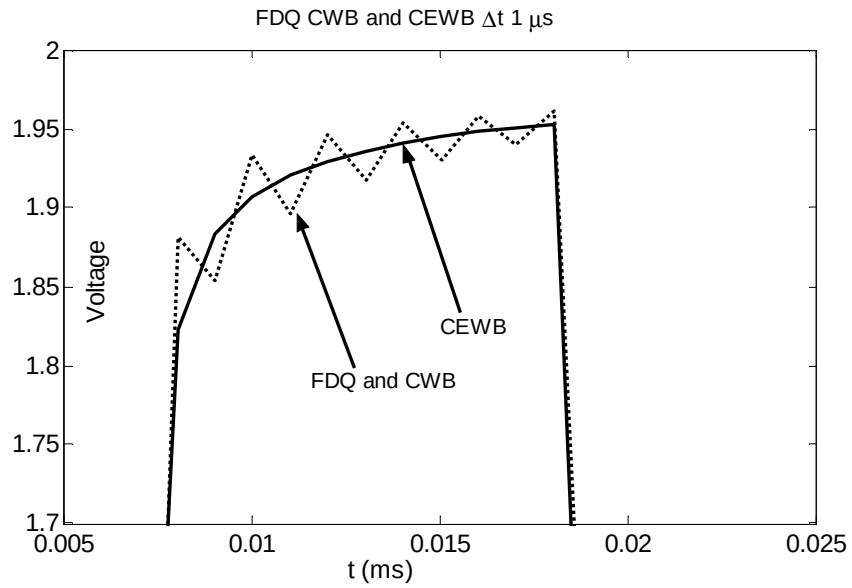


Figure 6.3 Ringing problem for FDQ and CWB, $\Delta t = 1 \mu s$

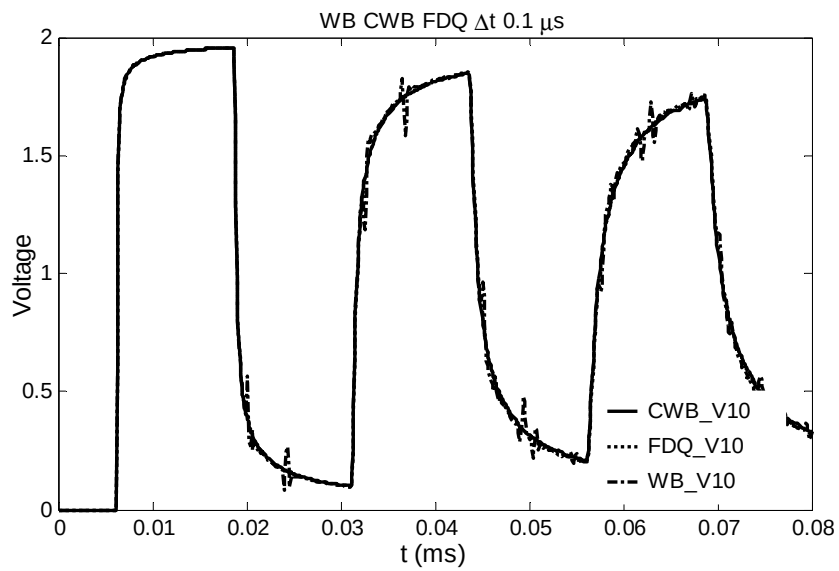


Figure 6.4 Voltage V_{10} for CEWB, FDQ and WB for $\Delta t = 0.1 \mu s$, WB still noisy

Table 6.1 Interpolation Error Weighting Terms versus Modes for H_{11} , Case CAB02

Mode	VF	CWB/CEWB Models	Time Delay μs	Order
1	0.9198	0.9177	6.235	8
2	-144.4404	-0.0188	19.835	14
3	146.4668	0.0192	19.861	13
4	-9.4665	-0.0080	23.778	13
5	10.9791	0.0080	23.958	13
6	-4.7895×10^{-4}	2.4219×10^{-4}	40.391	8
7	2.4537×10^{-4}	1.3113×10^{-4}	60.959	4

6.4.2. Double Circuit Line with Lightning Case

This case uses a double circuit transmission line as shown in Figure 6.5 with the simulation parameters given in Table 6.2. Phase 1 is submitted to a direct lightning stroke as shown in Figure 6.6. The lightning current is the CIGRE concave lightning current source as defined in [76]. The tower at each span is modeled using 6 insulator chains connected from phase wires to the constant parameter (CP) transmission line model of the tower. The CP model has a characteristic impedance of 90Ω with a length of 15.5 m. The insulator chains are represented with the leader propagation model [76].

Figure 6.7 shows the transient voltages calculated with the CWB and FD models for phase 1 at the location of lightning. The integration time-step is $0.02\mu\text{s}$. Once again the existing WB model (not shown) becomes numerically unstable even with a reduced time-step. There is a sufficiently good match between FD and CWB results. It is noticed that for the FD model the transformation matrix $\mathbf{T}$ is approximated by a constant and real matrix at 100 kHz. This is a manual selection for analyzing lightning cases. Selecting 50 Hz increases the deviation between CWB and FD results. A visual appreciation of absolute difference can be seen in Figure 6.8 and Figure 6.9.

The objective here was to validate CWB results as compared to FD. It remains that the CWB model is the most precise and automatically applicable for a wide band of frequencies.

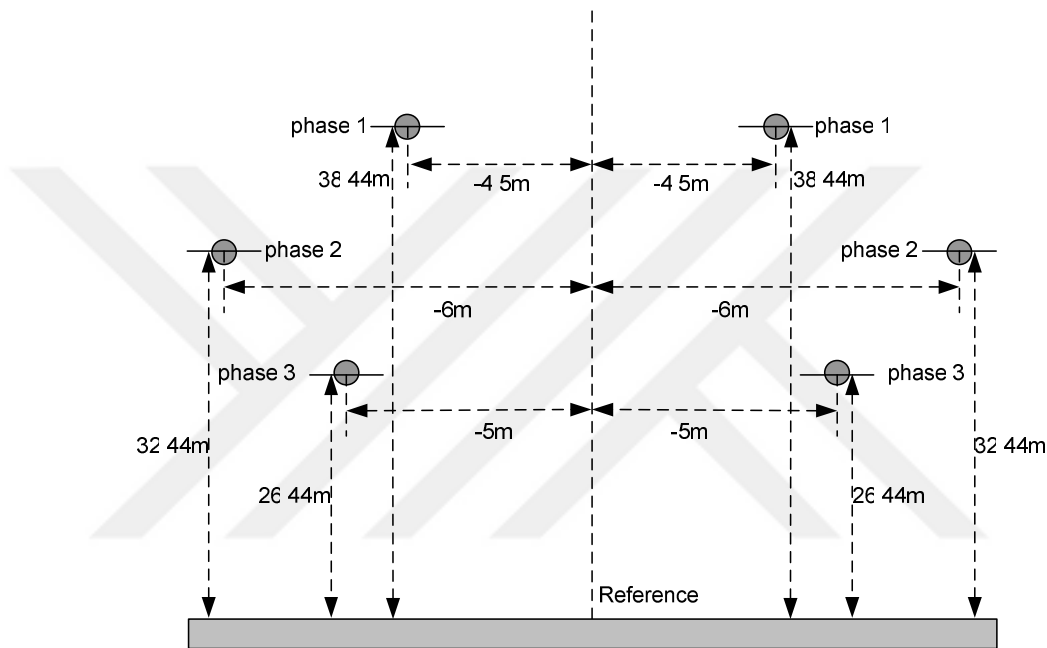


Figure 6.5 Double Circuit Transmission Line Physical Layout TRL02

Table 6.2 Line and Lightning Stroke Data for the Double Circuit Case

Outside Diameter of the Conductors	2.86 cm
DC Resistance	$0.068 \Omega / \text{km}$
Fundamental frequency	50 Hz
Tower footing resistance	300Ω
Insulator chain length	2.16 m
Lightning current source (negative polarity)	
Start Time	$10 \mu\text{s}$
Maximum current	30 kA
Rise time t_f	$3 \mu\text{s}$
Maximum slope in the rising part S_m	$40 \text{ kA} / \mu\text{s}$
Tail time (half value of maximum current) t_h	$50 \mu\text{s}$

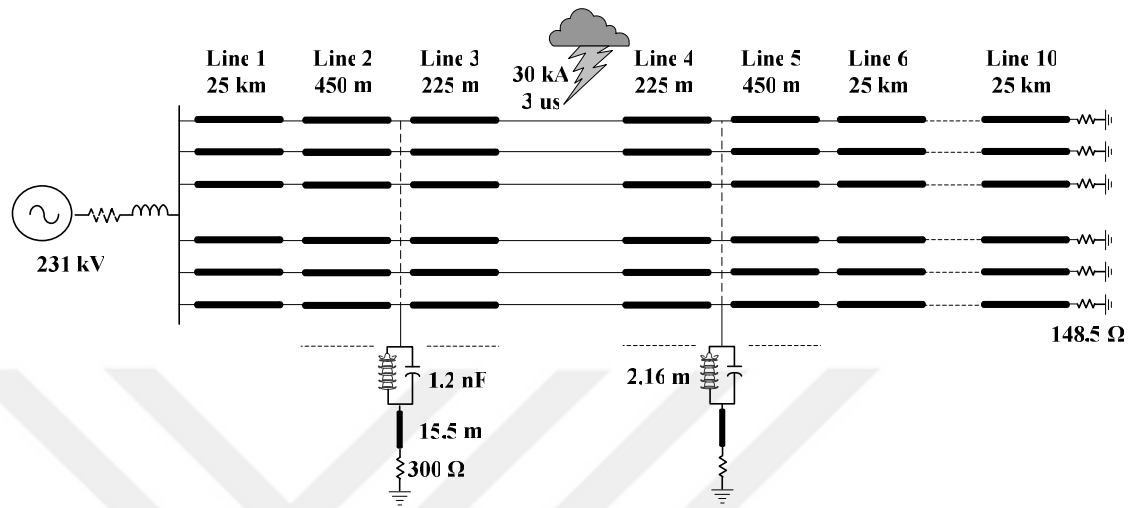


Figure 6.6 Modeling circuit for a double circuit transmission line subjected to a lightning stroke

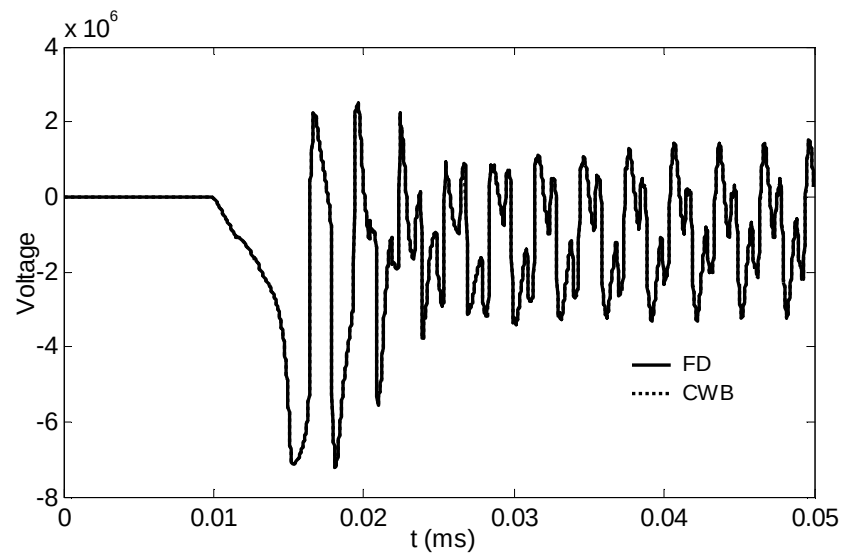


Figure 6.7 Transient response comparison with FD (100 kHz) and CWB (overlapped)

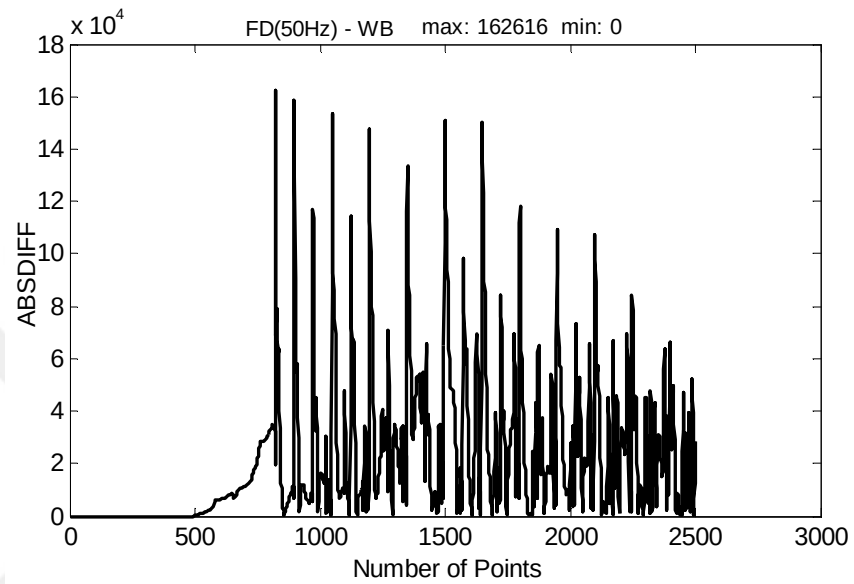


Figure 6.8 Transient Response Difference between FD (50 Hz) and CWB

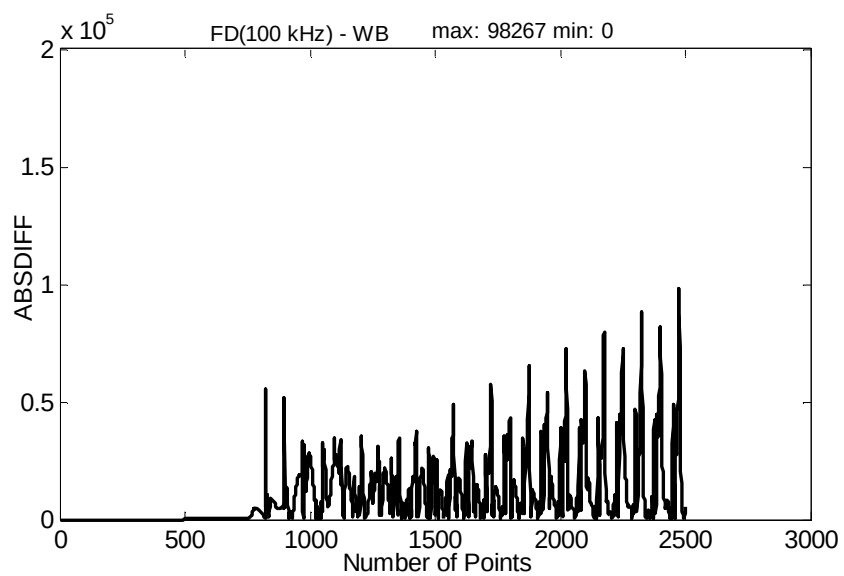


Figure 6.9 Transient Response Difference between FD (100 kHz) and CWB

6.5. Case Studies on the Transient Modeling of CIWB Model

In this section the performance of CIWB (Convex Improved WB) model will be analyzed through comparisons. The objective is to verify the validity of the CIWB model and to demonstrate the shortcomings of other models available in EMTP-RV if applicable. CIWB model is obtained by applying all the techniques mentioned in Chapter V. Therefore it fully includes all the improvements and techniques proposed in this dissertation. For convenience, fundamental differences between the CIWB model and the WB model of EMTP-RV are summarized as follows:

- Rational fitting is accomplished by newly developed WVF technique which results in low order approximations
- Time delay identification is simplified and time delay search is improved
- Phase domain fitting is reformulated as a constrained LS problem so that residues are scaled with respect to poles. This ensures numerical stability for complex cases.

The CIWB model is obtained using Matlab codes. Then model data is transferred into EMTP-RV through the WB model setup. Transient analysis is done under EMTP-RV software. In addition to the CIWB model; FDQ, WB and FD models already available in EMTP-RV are also considered.

6.5.1. Transient Response of the TRL01 Transmission Line

The transient response of the TRL01 transmission line is analyzed using the circuit configuration given in Figure 6.10. In this case, an already energized line is subjected to an unbalanced short circuit.

Figure 6.11 shows the voltages measured at the end of Phase A calculated with CIWB, WB and FD models. In this case the transposed version of TRL01 transmission

line is used. All the results are identical. Since CIWB requires less poles it allows saving computation time over WB model. Similar gains have been observed for other cases.

In Figure 6.12 the line is not transposed. Deviation of FD from CIWB and WB models can be easily recognized. Maximum deviation is calculated as 4%. This difference can be related to the poor fitting performance of FD compared to CIWB and WB.

6.5.2. Transient Response of the 6 Conductor CAB01 Cable System

The transient response of the 6 conductor CAB01 cable system is analyzed using the circuit configuration given in Figure 6.13.

In this case the cores of the cable I are energized with 1 pu of voltage and the measurements represent the induced voltages on the core and on the sheath of cable II.

Figure 6.14 shows the voltage on the core calculated with FDQ, WB and CIWB. WB and CIWB results are identical where FDQ result is slightly different from them. Figure 6.15 shows the voltage on the shield conductor of Cable II. Again WB and CIWB results are overlapped but FDQ is slightly different in this case.

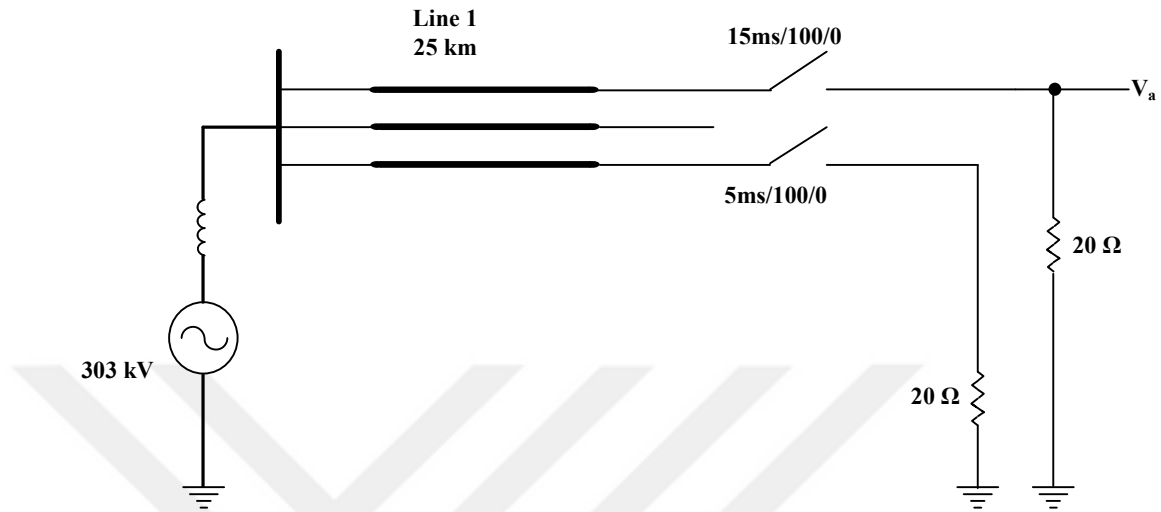


Figure 6.10 Case TRL01 Closing to Unbalanced Short Circuit,

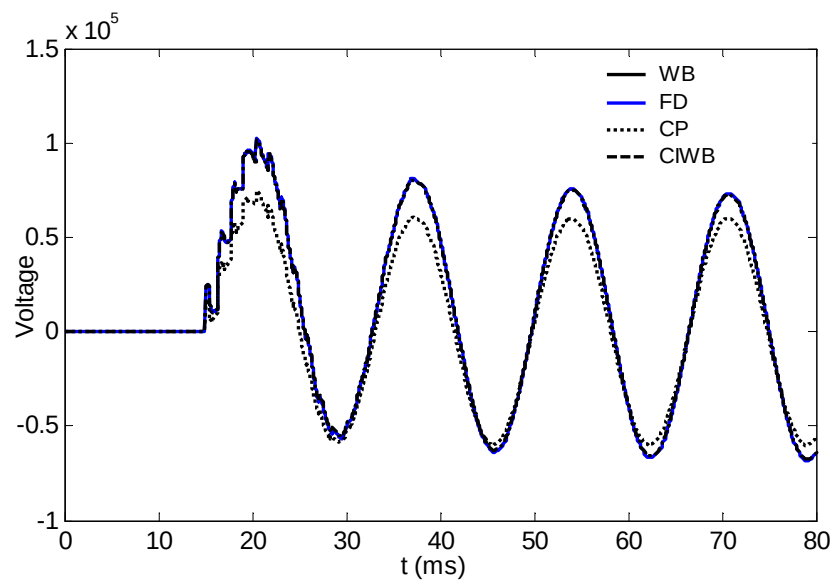


Figure 6.11 Case TRL01 Voltage Phase A, Transposed Version, CIWB, WB, FD (overlapped) and CP Models

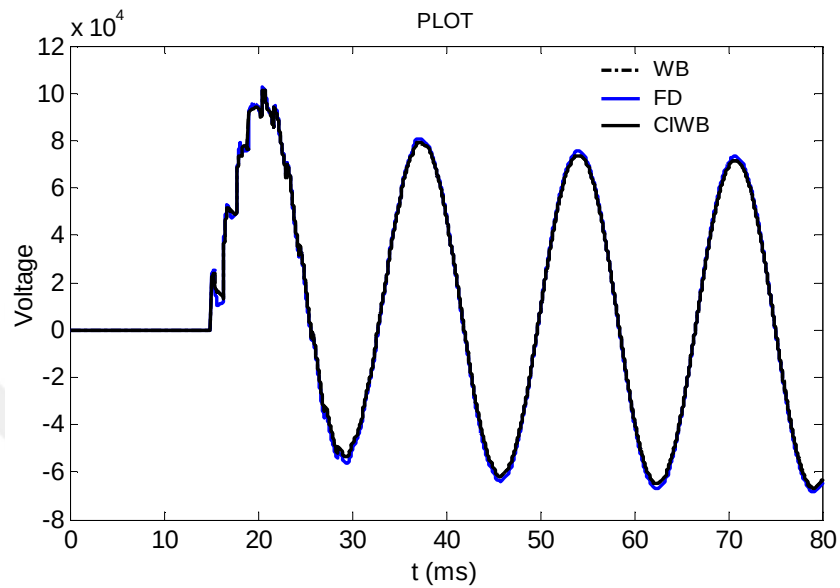


Figure 6.12 Case TRL01 Voltage Phase A, CIWB, WB (overlapped) and FD Models, No transposition

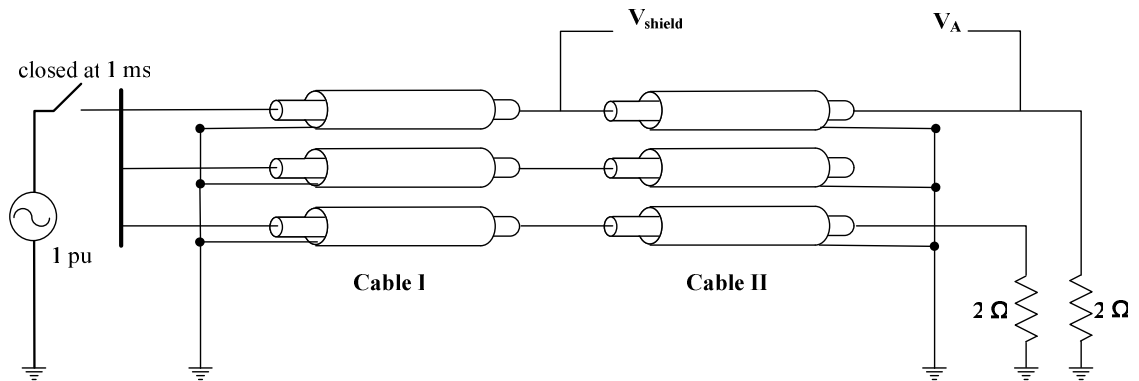


Figure 6.13 Case CAB01 Cable Energizing, Measurement of Shield and Core Voltages

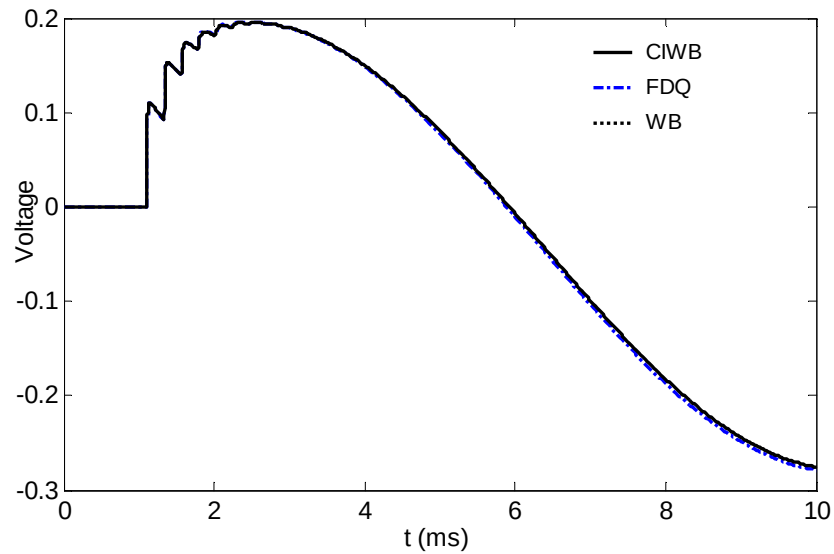


Figure 6.14 Case CAB01 Voltage Phase A, CIWB, WB (overlapped) and FDQ Models

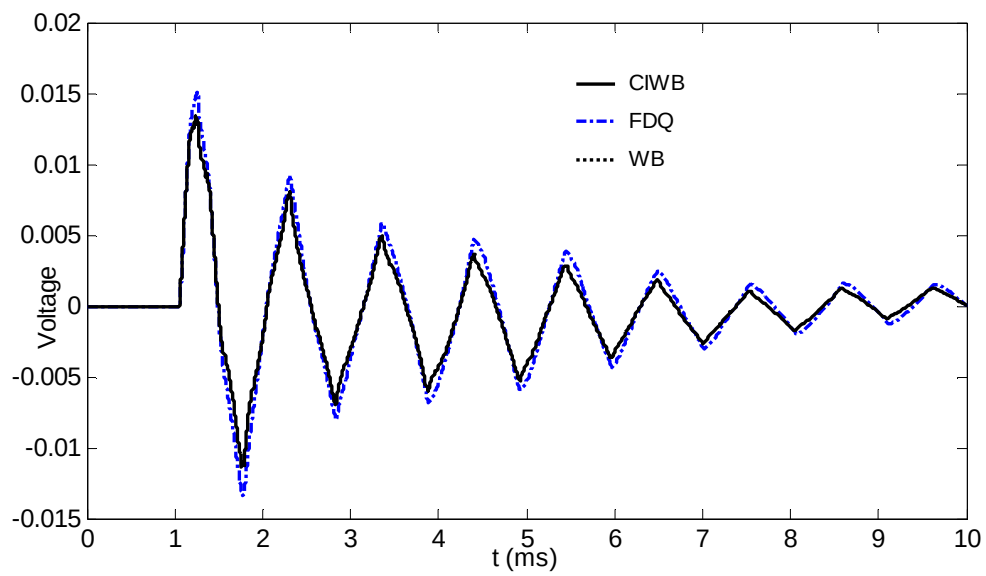


Figure 6.15 Case CAB01 Voltage, Middle Point of Shield A, CIWB, WB (overlapped) and FDQ Models

CHAPTER 7.

CONCLUSIONS

In this thesis, new solution procedures are developed for the simulation of electromagnetic transients on transmission lines and underground cables with the representation of frequency dependence. The objective of the thesis is to overcome the existing modeling limitations experienced with the WB model in EMTP-RV by means of introducing an improved model which is called CIWB. CIWB model exploits the key idea of the ULM model proposed in [11].

By using traveling wave equations, transmission lines and cables can be characterized in the frequency domain with the propagation function $\mathbf{H}$ and the characteristic admittance $\mathbf{Y}_c$ which are both frequency dependent functions. These functions should be identified for a wideband frequency range in order to properly analyze the propagation of electromagnetic transient signals which are comprised of multi frequency signals. Identification of $\mathbf{H}$ and $\mathbf{Y}_c$ as rational transfer functions allows the representation of the frequency dependence.

In Chapter IV, the WVF technique is proposed for the identification of a given wideband frequency response as a rational transfer function. The proposed WVF method presents a low order of approximation, less computation time and better conditioned equations when compared to the existing EMTP fitting procedures.

In Chapter V, the procedures for identifying each matrix element of $\mathbf{Y}_c$ and $\mathbf{H}$ as transfer functions in the frequency domain are developed. It is assumed that, for a given N_c conductor system, $\mathbf{Y}_c$ and $\mathbf{H}$ data given at discrete frequency samples can be represented with N_c input N_c output (MIMO) linear time invariant (LTI) systems in the form of numerical transfer function matrices.

For the identification of $\mathbf{H}$ the model proposed in [11] is elaborated with some theoretical clarifications and experimental experience. The mathematical formulation of $\mathbf{H}$ suggests that every propagation mode is present in every element of $\mathbf{H}$. Therefore propagation modes can be used to identify $\mathbf{H}$ in the phase domain. The key idea is to first decompose the propagation function into modes, each fitted with a set of poles and a single time delay, then to solve the final fitting problem in the phase domain with only residues as unknown quantities. After having identified the modes of $\mathbf{H}$ by using a frequency dependent transformation matrix, time delays are initially calculated with a simplified routine. It is shown that there is a strong relation between the rms error and time delay in the modal domain. Therefore initial time delay estimations are improved by direct experimentation with rational fitting with the delay as parameter. This is achieved by implementing an algorithm based on sectional search method. Bracketing intervals for time delays are defined in accordance with the physical properties of the device. Unrealistic time delays generated by the existing WB model are avoided.

It has been observed that direct LS solution of $\mathbf{H}$ can result in a model in which modal contributions are magnified. This kind of solution is likely to happen in multiconductor short cables in which time delays of two consecutive modes can come very close and the modal contributions corresponding to these modes can have opposite signs. In this case direct LS solution may end up with high residue pole ratios which may increase the integration errors and cause numerical instabilities in the time domain. Therefore phase domain identification of $\mathbf{H}$ is done by applying a constrained least squares solution which controls the modal contributions. The constraints can be either determined experimentally or by estimating the truncation and interpolation errors of the time domain model.

Frequency domain described lines should be converted into time domain for transient analysis. Transformation of the frequency domain equations into time domain result in convolution integrals. Convolution integrals are in general computed via numerical integration techniques in the time-domain in accordance with the discrete

time-step of the EMTP simulation. In Chapter VI, the convolution integrals are represented with the state-space equations which are solved with trapezoidal or Euler Backward discretization schemes. Given a transmission device model, associated local numerical errors of time-domain analysis are estimated in terms of PFE variables. Based on this relation between errors and PFE variables, constraints are determined for the residue/pole ratios such that time-domain errors are confined within a safe boundary. In the proposed approach the identification in phase domain becomes a constrained linear least squares problem. As a result, the frequency domain model is improved considering the time domain error performance.

As a conclusion, this thesis contributes to the development of generic and precise models for transmission lines and cables. It overcomes many limitations and allows solving/simulating cases that are failing with currently available EMTP type solution methods.

Although several improvements are realized in this work, the model that is generated may still fail in the time domain. The root cause for this difficulty can be traced back to the lack of passivity. Merely using stable poles may not entail a stable time-domain simulation. Passivity should be also preserved since passivity violations may induce unreal numerical oscillations. Transmission devices are passive by definition. However, it is the time-domain model that may introduce passivity violations due to improper selection of frequency band for fitting and/or poor fitting. It is shown in [77] that the ULM formulation can lead to large out-of-band passivity violations. The studies can be extended in order to consider the algebraic passivity verification of the model [78].

Future research can also target the improvement of fitting methods by further reducing the fitting order.

REFERENCES

- [1] H. W. Dommel, EMTP Theory Book, Microtran Power System Analysis Corporation, Vancouver, British Columbia, April 1996.
- [2] A. Budner, "Introduction of frequency dependent line parameters into an electromagnetic transients program", IEEE Trans., Pas-89, pp. 88-97, January 1970.
- [3] H. Nakanishi and A. Ametani, "Transient calculation of a transmission line using superposition law", IEE Proc., vol. 133, Pt. C, no. 5, July 1986, pp. 263-269
- [4] A. Semlyen and A. Dabuleanu, "Fast and accurate switching transient calculations on transmission lines with ground return using recursive convolutions", IEEE Trans., PAS-94, pp. 561-571, March/April 1975.
- [5] L. M. Wedepohl, "Application of matrix methods to the solution of traveling wave phenomena in polyphase systems", IEE Proc., Vol. 110 (12), pp. 2200-2212, December 1963.
- [6] D. E. Hedman, "Propagation on overhead transmission lines: I. Theory of modal analysis; II. Earth conduction effects and practical results", IEEE Trans., PAS-84, pp. 200-211, March 1965
- [7] J. R. Marti, "Accurate modeling of frequency dependent transmission lines in electromagnetic transient simulations", IEEE Trans. On Power App. and Syst., vol. PAS-101, no. 1, pp. 147-155, Jan. 1982
- [8] J. R. Marti, "The problem of frequency dependence in transmission line modeling", Ph. D. Thesis, Department of Electrical Engineering, The University of British Columbia, 1981
- [9] L. Marti, "Simulation of transients in underground cables with frequency dependent modal transformation matrices", IEEE Trans. On Power Delivery, vol. 3, no. 3, pp. 1099-1110, July 1988

- [10] L. Marti, "Simulation of electromagnetic transients in underground cables with frequency dependent modal transformation matrices", Ph. D. Thesis, Department of Electrical Engineering, The University of British Columbia, November 1986.
- [11] A. Morched, B. Gustavsen, M. Tartibi, "A universal model for accurate calculation of electromagnetic transients on overhead lines and underground cables", IEEE Trans. on Power Delivery, Vol. 14, No. 3, pp. 1032-1038, July 1999
- [12] T. Noda, N. Nagaoka, and A. Ametani, "Phase domain modeling of frequency-dependent transmission lines by means of an ARMA model," IEEE Trans. on Power Del., vol. 11, no. 1, pp. 401–411, Jan. 1996.
- [13] T. Noda, N. Nagaoka, and A. Ametani, "Further improvements to a phase domain ARMA line model in terms of convolution, steady state initialization and stability," IEEE Trans. on Power Del., vol. 12, no. 3, pp. 1327–1334, July 1997.
- [14] H. W. Dommel, "Digital computer solution of electromagnetic transients in single and multiphase networks", IEEE Trans., Pas-88, pp. 388-399, April 1969.
- [15] H. W. Dommel and W. S. Meyer, "Computation of electromagnetic transients", IEEE Proc., vol. 62 (7), pp. 983-993, July 1974.
- [16] J. Mahseredjian, "Simulation des transitoires électromagnétiques dans les réseaux électriques", Éditions Les Techniques de l'Ingénieur, Dossier No.D4130, Réseaux électriques et applications.
- [17] J. Mahseredjian, S. Dennerrière, L. Dubé, B. Khodabakhchian and L. Gérin-Lajoie, "On a new approach for the simulation of transients in power systems", Proceedings of International Conference on Power Systems Transients, IPST 2005 in Montréal, June 19th-23rd, 2005.
- [18] B. Gustavsen and J. Nordstrom, "Pole Identification for the universal line model based on trace fitting", IEEE Transactions on Power Delivery, Vol. 23, Issue 1, pp. 472 – 479, Jan. 2008.
- [19] I. Kocar, J. Mahseredjian, G. Olivier, "Weighting Method for Transient Analysis of Underground Cables" IEEE Trans. Power Delivery, Volume: 23, Issue: 3, pp 1629-1635, July 2008.

- [20] I. Kocar, J. Mahseredjian, G. Olivier, “Improvement of Numerical Stability for the Computation of Transients in Lines and Cables”, Submitted to IEEE Transactions on Power Delivery.
- [21] EMTP-RV Software Tutorial and Reference Documents, EMTPWorks Version 2.0.
- [22] C. R. Paul, Analysis of Multiconductor Transmission Lines, New York, Wiley Interscience, 1994.
- [23] W. S. Meyer and H. W. Dommel, “Numerical modeling of frequency dependent transmission line parameters in an electromagnetic transients program”, IEEE Trans., PAS-93, pp. 1401-1409, September/October 1974.
- [24] L. Bergeron, “Du Coup de Bélier en Hydraulique au coup de Foudre en Électricité”, Dunos, Paris, 1949 (English Translation : Water Hammer in Hydraulics and Wave Surges in Electricity, ASME Comity, Wiley, New York, 1961).
- [25] M. H. Hesse, “Electromagnetic and electrostatic transmission line parameters by digital computer” IEEE Trans. on Power App. and Syst., vol. 82, pp. 282-290, 1963
- [26] J. R. Carson, “Wave propagation in overhead wires with ground return”, Bell Syst. Tech. Journal, vol. 5, pp. 539-554, 1926.
- [27] C. Gary, “Approche complète de la propagation multifilaire en haute fréquence par utilisation des matrices complexes”, EDF Bulletin de la Direction des Études et Recherches, Série B, no. 3/4, pp. 5-20, 1976.
- [28] G. F. Moore, Electric Cables Handbook, 3rd Edition, Blackwell Science Ltd., 1997
- [29] A. Ametani, “A general formulation of impedance and admittance of cables”, IEEE Trans. on Power App. and Syst., vol. PAS-99, no.3, pp. 902-910, May/June 1980.
- [30] S. A. Schelkunoff, “The electromagnetic theory of coaxial transmission lines and cylindrical shields”, Bell System Technical Journal, pp. 532-579, 1934.

- [31] L. M. Wedepohl, D. J. Wilcox, "Transient analysis of underground power transmission systems", Proc. IEE, vol. 120, no. 2, pp. 253-260, Feb. 1973.
- [32] J. K. Snelson, "Propagation of traveling waves on transmission lines – frequency dependent parameters", IEEE Trans., PAS-91, pp. 85-91, January/February 1972.
- [33] B. Gustavsen et al., "Transmission line models for the simulation of interaction phenomena between parallel AC and DC overhead lines", IPST 99, pp. 61-67, June 1999
- [34] B. Gustavsen, J. Sletbak, T. Henriksen, "Calculation of electromagnetic transients in transmission cables and lines taking frequency dependent effects accurately into account", IEEE Trans. On Power Delivery, vol. 10, no. 2, pp. 1076-1084, April 1995.
- [35] G. Angelidis, A. Semlyen, "Direct phase domain calculation of transmission line transients using two sided recursions", IEEE Trans. On Power Delivery, vol. 10, no. 2, pp. 941-949, April 1995.
- [36] H. V. Nguyen, H. W. Dommel, J. R. Marti, "Direct phase domain modeling of frequency dependent overhead transmission lines", IEEE Trans. on Power Del., vol. 12, no. 3, pp. 1335-1342, July 1997.
- [37] F. Castellanos, J. R. Marti, F. Marcano, "Phase domain multiphase transmission line models", International Journal of Electrical Power and Energy Systems, vol. 19, no. 4, pp. 241-248, May 1997.
- [38] B. Gustavsen and A. Semlyen, "Calculation of transmission line transients using polar decomposition," IEEE Trans. Power Del., vol. 13, no. 3, pp. 855–862, Jul. 1998.
- [39] B. Gustavsen and A. Semlyen, "Rational approximation of frequency domain responses by vector fitting," IEEE Trans. Power Del., vol. 14, no. 3, pp. 1052–1061, July 1999.
- [40] Huang Wei-Gang, A. Semlyen, "Computation of electromagnetic transients on three phase transmission lines with corona and frequency dependent parameters", IEEE Trans. on Power Del., vol. PWRD-2, no. 3, pp. 887-898, July 1987.

- [41] A. Ametani, N. Nagaoka, T. Noda, T. Matsuura, "A simple and efficient method for including a frequency dependent effect in a transmission line transient analysis", IPST'95, pp. 11-16, Lisbon, Portugal, 1995.
- [42] F. Castellanos, J. R. Marti, F. Marcano, "Full frequency dependent phase domain transmission line model", IEEE Transactions on Power System, vol. 12, no. 3, pp. 1331-1339, August 1997.
- [43] F. Castellanos, "Full frequency dependent phase domain modeling of transmission lines and corona phenomena", Ph. D. Thesis, Department of Electrical Engineering, the University of British Columbia, April 1997
- [44] T. C. Yu, "Full frequency dependent modeling of underground cables for electromagnetic transient analysis", Department of Electrical and Computer Engineering, The University of British Columbia, 2001.
- [45] A. Ramirez, J. L. Naredo, P. Naredo, "Full frequency-dependent line model for electromagnetic transient simulation including lumped and distributed sources", IEEE Trans. on Power Delivery, Vol. 20, No. 1, pp. 292-299, January 2005.
- [46] C. A. Nucci, S. Guerrieri, M. T. Correia de Barros, and F. Rachidi, "Influence of corona on the voltages induced by nearby lightning on overhead distribution lines," IEEE Trans. Power Del., vol. 15, no. 4, pp. 1265–1273, Oct. 2000.
- [47] C. Y. Evrenesoglu, E. Akleman, A. Abur, "Visualisation of electromagnetic transients along transmission lines", Power Systems Conference and Exposition, 2004 IEEE PES, pp. 848 – 852, vol.2, 10-13 Oct. 2004
- [48] L. Marti and H. W. Dommel, "Calculation of voltage profiles along transmission lines", IEEE Transactions on Power Delivery, Vol. 12, No. 2, pp. 993-998, April 1997.
- [49] B. Gustavsen, J. Mahseredjian, "Simulation of Internal Overvoltages on Transmission Lines by an Extended Method of Characteristics Approach", IEEE Transactions on Power Delivery, Vol. 22, no.3, pp.1736-1742, July 2007.

- [50] R. Radulet, A. Timotin, A. Tugulea, and A. Nica, "The transient response of the electric lines based on the equations with transient line-parameters," *Rev. Roum. Sci. Techn.*, vol. 23, no. 1, pp. 3–19, 1978.
- [51] J. L. Naredo, P. Moreno, A. Soudack, and J. R. Marti, "Frequency independent representation of transmission lines for transient analysis through the method of characteristics," in *Proc. Athens Power Tech. Nat. Tech. Univ. Athens IEEE/Power Eng. Soc. Joint Int. Power Conf.*, vol. 1, Athens, Greece, Sep. 5–8, 1993, pp. 28–32
- [52] A. Jeffrey, *Quasilinear Hyperbolic Systems and Waves*, New York, Pitman, 1976.
- [53] R. Pintelon, "System Identification a frequency domain aproach", IEEE Press, 2001
- [54] E. C. Levi, "Complex curve fitting," *IEEE Trans. Autom. Control*, vol. AC-4, no. 1, pp. 37–43, Jan. 1959.
- [55] C. K. Sanathanan and J. Koerner, "Transfer function synthesis as a ratio of two complex polynomials," *IEEE Trans. Autom. Control*, vol. AC-8, no. 1, pp. 56–58, Jan. 1963.
- [56] W. Hendrickx and T. Dhaene, "A discussion of rational approximation of frequency domain responses by vector fitting," *IEEE Trans. Power Syst.*, vol. 21, no. 1, pp. 441–443, Feb. 2006
- [57] Y. Rolaine and R. Pintelon, "Generating robust starting values for frequency domain transfer function estimation", *Automatica* vol 35, pp. 965-972, 1999
- [58] D. Deschrijver and T. Dhaene, "Broadband macromodelling of passive components using orthonormal vector fitting," *Electronic letters of IEE*, vol. 41, no. 21, pp. 1160–1161, 2005.
- [59] P. S. C. Heuberger, P. M. J. Van Den Hof, and B. Wahlberg, *Modelling and Identification With Rational Orthogonal Basis Functions*. London: Springer-Verlag, 2005.

- [60] B.Gustavsen & C.Heitz, “Modal Vector Fitting: A Tool for Generating Rational Models of High Accuracy with Arbitrary Terminal Conditions”, IEEE Trans. Advanced Pack., vol.31, no.4, Nov 2008, pp 664-672.
- [61] A. O. Soysal and A. Semlyen, “Practical transfer function estimation and its application to wide frequency range representation of transformers,” IEEE Trans. Power Del., vol. 8, no. 3, pp. 1627–1637, Jul. 1993.
- [62] MATLAB Function Reference Volume 1–3, Version 6, The MathWorks, Inc., Natick, MA, 2002
- [63] T. Noda, “Identification of a multiphase network equivalent for electromagnetic transient calculations using partitioned frequency response”, IEEE Trans. Power Del., Vol. 20, No. 2, pp 1134-1142, April 2005
- [64] L. N. Trefethen and D. Bau, Numerical Linear Algebra, SIAM, Philadelphia, 1997, pp. 186
- [65] B. Gustavsen, “Time delay identification for transmission line modeling”, Proceeding of IEEE Workshop on Signal Propagation on Interconnects, May 9-12, 2004, pp. 103-106
- [66] V. Brandwajn, Modification of User’s Instructions for “Marti Setup”, EMTP Newsletter, vol. 3, number 1, pp. 76-80, August 1982
- [67] L. M. Wedepohl, H. V. Nguyen, G. D. Irwin, “Frequency dependent transformation matrices for untransposed transmission line using Newton-Raphson method”, IEEE Trans. on Power Syst., Vol. 11, No. 3, August 1996.
- [68] Alécio B. Fernandes and Washington. L. A. Neves, “Phase domain transmission line models considering frequency dependent transformation matrices”, IEEE Trans. on Power Delivery, vol. 19, no. 2, April 2004
- [69] B. Gustavsen and A. Semlyen, “Combined phase and modal domain calculation of transmission line transients based on vector fitting,” IEEE Trans. Power Del., vol. 13, no. 2, pp. 596–604, Apr. 1998
- [70] H. W. Bode, Network Analysis and Feedback Amplifier Design, Van Nostrand, New York, 1945

- [71] S. A Teukolsky, W.T. Vetterling and B.F. Flannery. "Numerical Recipes in Fortran 77". vol. 1. Second Edition, Cambridge University Press, pp. 390-392. 1996
- [72] Mariesa Crow, Computational Methods for Electric Power Systems, p. 148, Taylor & Francis Ltd., 2002
- [73] [Online]. Available at <http://www.mathworks.com>
- [74] T.F. Coleman and Y. Li, "A Reflective Newton Method for Minimizing a Quadratic Function Subject to Bounds on Some of the Variables", SIAM Journal on Optimization, Vol. 6, Number 4, pp. 1040-1058, 1996
- [75] P.E. Gill, W. Murray, and M.H. Wright, Practical Optimization, Academic Press, London, UK, 1981
- [76] "Guide to procedures for estimating the lightning performance of transmission lines", Working Group 01 (Lightning) of Study Committee 33 (Overvoltages and Insulation Co-ordination), October 1991, CIGRÉ[10]
- [77] B. Gustavsen, "Passivity enforcement for transmission line models based on the method of characteristics", IEEE Trans. Power Delivery, vol. 23, no. 4, pp. 2286-2293, October 2008.
- [78] C. Changzhong, E. Gad, M. Nakhla, R. Achar, "Passivity verification in delay-based macromodels of multiconductor electrical interconnects", IEEE Transactions on Advanced Packaging, vol. 30, no. 2. May 2007

Appendix I

Calculation of Zeros

In this appendix the objective is to show that transformation of a transfer function from residue-pole $(r_n - p_n)$ to zero-pole $(z_n - p_n)$ configuration can be accomplished by finding the roots of the characteristic equation (eigenvalues) of matrix $\mathbf{Q}$

$$\det[\lambda \mathbf{I} - \mathbf{Q}] \quad (\text{I.1})$$

such that

$$\mathbf{Q} = \mathbf{W} - \mathbf{g}_1 \tilde{\mathbf{x}}^T \quad (\text{I.2})$$

where $\mathbf{W}$ is a diagonal matrix containing the poles a_n , $\mathbf{g}_1$ is a column vector of ones and $\tilde{\mathbf{x}}$ contains only the r_n terms. In other words it is postulated that roots of (I.1) (eigenvalues of $\mathbf{Q}$) give z_n .

Heaviside theorem states the following relation

$$\sum_{n=1}^N \frac{r_n}{s + p_n} + 1 = \frac{\prod_{n=1}^N (s + z_n)}{\prod_{n=1}^N (s + p_n)} \quad (\text{I.3})$$

After some simple manipulations on the left hand side of (I.3) we obtain

$$\frac{\prod_{n=1}^N (s + z_n)}{\prod_{n=1}^N (s + p_n)} = \frac{\prod_{n=1}^N (s + p_n) - \sum_{i=1}^N r_i \cdot \prod_{n=1, n \neq i}^N (s + p_n)}{\prod_{n=1}^N (s + p_n)} \quad (\text{I.4})$$

Equation (I.4) suggests

$$\prod_{n=1}^N (s + p_n) - \sum_{i=1}^N r_i \cdot \prod_{n=1, n \neq i}^N (s + p_n) = \prod_{n=1}^N (s + z_n) \quad (\text{I.5})$$

Therefore zeros are equivalently the eigenvalues of the matrix **Q** (left hand side of equation (I.5)).



Appendix II

Time Delay Estimation Based on MPS Formulation

The goal here is to remove such a time delay from H that the phase angle of the remaining part ($\phi_{\min}$) is conformal to MPS property. This can be formulated as follows (subscript “min” is used to indicate MPS property).

$$B = -\omega\tau_{\min} + \phi_{\min}(\omega) \quad (\text{II.1})$$

$$H_{\min} = He^{(j\omega\tau_{\min})} = \sum_{n=1}^N \frac{c_n}{s + a_n} \quad (\text{II.2})$$

Now we will show the detailed derivation of $\tau_{\min}$ formulation used in [69] and EMTP-RV for the sake of clarity. If $H_{\min}$ is an MPS function then there is a relation [70] between its magnitude and phase at any frequency sample point k (Bode's gain/magnitude phase relation)

$$\phi_{\min_k} = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{d \log |H_{\min}|}{du} \log \coth \frac{|u|}{2} du \quad (\text{II.3})$$

where $u = \log(\omega / \omega_k)$. Following relation also holds for $H_{\min}$

$$\log |H_{\min}| = \log |H| \quad (\text{II.4})$$

If magnitude has a constant slope on logarithmic scale such that

$$d \log |H| / du \cong K \quad (\text{II.5})$$

$$K \cong \frac{\log |H_{k+1}| - \log |H_{k-1}|}{\log(\omega_{k+1} / \omega_{k-1})} \quad (\text{II.6})$$

then equation (II.3) becomes

$$\phi_{\min_k} \cong \frac{K}{\pi} \int_{-\infty}^{\infty} \log \coth \frac{|u|}{2} du \quad (\text{II.7})$$

Since the integral part is defined now

$$\int_{-\infty}^{\infty} \log \coth \frac{|u|}{2} du = \frac{\pi^2}{2} \quad (\text{II.8})$$

Equation (II.7) further simplifies to

$$\phi_{\min_k} \cong K \frac{\pi}{2}. \quad (\text{II.9})$$

Thus with the assumption that magnitude has a constant slope at all frequencies, phase becomes simply the constant slope of magnitude multiplied by $\pi / 2$. This is not a reliable result as magnitude of H does not always have a constant slope. Therefore approximation error of (II.9) is examined next. Second logarithmic term in (II.3) can be rewritten by replacing u :

$$\log \coth \frac{|u|}{2} = \log \left| \frac{(\omega + \omega_k)}{(\omega - \omega_k)} \right|. \quad (\text{II.10})$$

This term can be interpreted as weighting term of the integral (II.3) and it is illustrated in Figure 7.1. It is observed that this term vanishes at remote frequencies and thus the variation of the slope from the constant value K has little importance at remote frequencies for precise calculation of (II.3), i.e. after two decades from the frequency point of interest. The total error associated with calculation of (II.3) by using (II.9) can then be approximated as

$$\varepsilon_k = \frac{1}{\pi} \int_{-\infty}^{\infty} \left(\frac{d \log |H|}{du} - K \right) \log \coth \frac{|u|}{2} du \quad (\text{II.11})$$

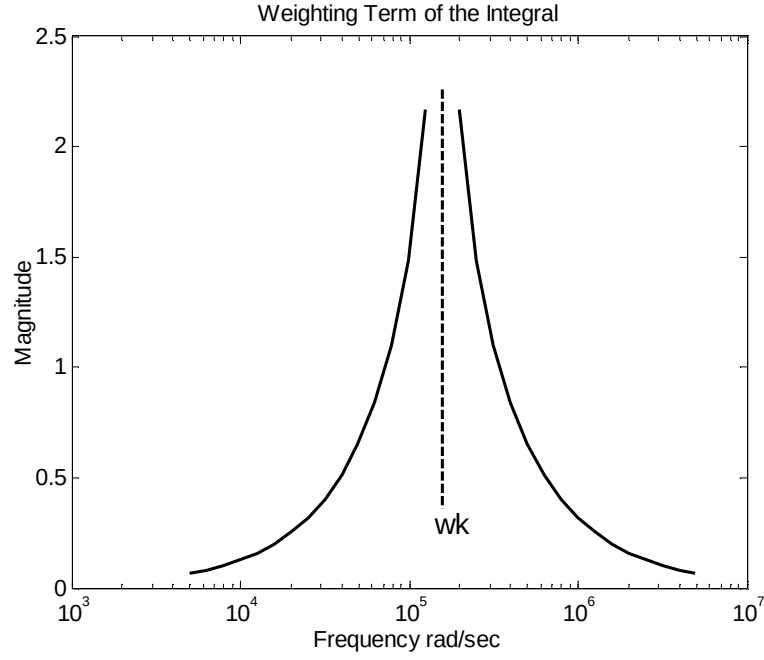


Figure 7.1 Weighting term of the integral (II.10)

If the weighting term (II.10) is inspected, it is seen that integration (II.11) can be limited to four decades apart by placing the frequency point of interest at the center of the range, i.e.

$$\varepsilon_k \cong \frac{1}{\pi} \sum_{j=1}^{N\omega-1} \left(\frac{\log |H_{j+1}| - \log |H_j|}{\Delta u} - K \right) \log \coth \frac{|u|}{2} \Delta u \quad (\text{II.12})$$

where

$$\Delta u = \log \left(\frac{\omega_{j+1}}{\omega_j} \right) \quad (\text{II.13})$$

$$\omega_j \in [0.01\omega_k \dots 100\omega_k] \quad (\text{II.14})$$

$$|u| = \left| \frac{\omega_j + \omega_{j+1}}{2\omega_k} \right| \quad (\text{II.15})$$

Finally it becomes possible to obtain an improved approximation for (II.3):

$$\phi_{\min_k} \cong K \frac{\pi}{2} + \varepsilon_k. \quad (\text{II.16})$$

$\phi_{\min_k}$ corresponds to the phase angle of $H_{\min}$, therefore $\tau_{\min}$ can now be easily calculated using (II.2):

$$\tau_{\min} = \frac{\phi_{\min_k}}{\omega_k} - \frac{B_k}{\omega_k} \quad (\text{II.17})$$

Appendix III

Matlab Code for the Initial Estimation of Time Delay and Bracketing Interval

Following is the Matlab Code for the initial identification of time delays and for defining the bracketing values for time delay optimization

```
function [tau, tau12, tau1, tau2]...
    =taucalcP2(veloc,H,Nw,w,length,errlim,tau,tau12)

% Finding the frequency point (j) to be used for the identification of
% time delay:
% Find the first frequency at which the magnitude of Hmode <= errlim
% errlim is the dumerr initially attained in the main program (0.25),
% not the errlim defined by the user e.g. 0.01
% Nw: Number of Frequency Samples
% H(Nw,1): Propagation Mode
% w(Nw): Frequency points in rad
% length: length of device
% veloc: velocity of phase displacement w/phase-angle
% tau: initial time delay
% tau12= deltau, tau-tau1 = tau2-tau
% tau1: left bracket value for time delay optimization
% tau2: right bracket value

flag=0;
j=Nw;
for k=1:Nw
    absH(k)=abs(H(k));
    if ( (absH(k)<errlim)&(flag==0) )
        j=k;
        flag=1;
    end
end

if (j==Nw)
    j=Nw-1;
end

% j is now the frequency for the identification of time-delay
% Calculate the low and high frequency point (jlow, jhigh). They will
% be used for initial tau and bracketing values
% 1 decade of frequency to each side of j
% w(j) is the errlim frequency
jlow=1;
```

```

jhigh=1;
for k=1:Nw
    if (w(k)/w(j) < 0.1d0)
        jlow=k;
    end
    if (w(k)/w(j) < 10.0d0)
        jhigh=k;
    end
end

% Find tau0 and tau2 first
% length/veloc(k) gives a tau that makes the phase angle zero at w(k)
% w(k)*length/veloc(k) gives the phase angle of H at w(k)
tau0=0;
for k=jlow:jhigh-1
    dum=w(k+1)*length/veloc(k+1) - w(k)*length/veloc(k);
    tau0=tau0 + dum/(w(k+1)-w(k));
end
tau0=tau0/(jhigh-jlow);
tau2=length/veloc(jlow);
tau12=tau2-tau0;

%find tau1,
tau1=length/veloc(jhigh);
if (tau0-tau1)<tau12
    tau1=tau0-tau12;
elseif (tau0-tau1)>tau12
    tau12=tau0-tau1;
    tau2=tau0+tau12;
end
%sanity check
if (tau1<tau0)&(tau0<tau2)&(tau12>0)&(tau12>0.01*tau0)
%3rd test is redundant
    'everything is ok';
else
    tau1=0.95*tau0;
    tau2=1.05*tau0;
    tau12=0.05*tau0;
end
tau=tau0;

```

Appendix IV

Modal Contributions to Propagation Matrix

In this appendix, modal contributions to the propagation function for a three conductor transmission device are symbolically shown. The purpose is to improve the comprehension and allow visualization while reading the thesis.

$$\mathbf{H} = \mathbf{T} \mathbf{e}^{\Lambda} \mathbf{T}^{-1}$$

$$\mathbf{H} = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} \begin{bmatrix} t'_{11} & t'_{12} & t'_{13} \\ t'_{21} & t'_{22} & t'_{23} \\ t'_{31} & t'_{32} & t'_{33} \end{bmatrix}$$

$$\mathbf{H} = \begin{bmatrix} t_{11}h_1 & t_{12}h_2 & t_{13}h_3 \\ t_{21}h_1 & t_{22}h_2 & t_{23}h_3 \\ t_{31}h_1 & t_{32}h_2 & t_{33}h_3 \end{bmatrix} \begin{bmatrix} t'_{11} & t'_{12} & t'_{13} \\ t'_{21} & t'_{22} & t'_{23} \\ t'_{31} & t'_{32} & t'_{33} \end{bmatrix}$$

$$\mathbf{H} = \begin{bmatrix} t_{11}h_1t'_{11} + t_{12}h_2t'_{21} + t_{13}h_3t'_{31} & t_{11}h_1t'_{12} + t_{12}h_2t'_{22} + t_{13}h_3t'_{32} & t_{11}h_1t'_{13} + t_{12}h_2t'_{23} + t_{13}h_3t'_{33} \\ t_{21}h_1t'_{11} + t_{22}h_2t'_{21} + t_{23}h_3t'_{31} & t_{21}h_1t'_{12} + t_{22}h_2t'_{22} + t_{23}h_3t'_{32} & t_{21}h_1t'_{13} + t_{22}h_2t'_{23} + t_{23}h_3t'_{33} \\ t_{31}h_1t'_{11} + t_{32}h_2t'_{21} + t_{33}h_3t'_{31} & t_{31}h_1t'_{12} + t_{32}h_2t'_{22} + t_{33}h_3t'_{32} & t_{31}h_1t'_{13} + t_{32}h_2t'_{23} + t_{33}h_3t'_{33} \end{bmatrix}$$

Modal Contribution Matrix for first mode

$$\mathbf{H}_1 = \begin{bmatrix} t_{11}h_1t'_{11} & t_{11}h_1t'_{12} & t_{11}h_1t'_{13} \\ t_{21}h_1t'_{11} & t_{21}h_1t'_{12} & t_{21}h_1t'_{13} \\ t_{31}h_1t'_{11} & t_{31}h_1t'_{12} & t_{31}h_1t'_{13} \end{bmatrix}$$

$$\mathbf{H} = \mathbf{H}_1 + \mathbf{H}_2 + \mathbf{H}_3$$

