

ANALYSIS OF MZI MODULATORS WITH ONE-DIMENSIONAL PHOTONIC
CRYSTAL WAVEGUIDES

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

FATİH UTKU YILMAZ

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
ELECTRICAL AND ELECTRONICS ENGINEERING

JUNE 2023

Approval of the thesis:

**ANALYSIS OF MZI MODULATORS WITH ONE-DIMENSIONAL
PHOTONIC CRYSTAL WAVEGUIDES**

submitted by **FATİH UTKU YILMAZ** in partial fulfillment of the requirements for
the degree of **Master of Science in Electrical and Electronics Engineering Department, Middle East Technical University** by,

Prof. Dr. Halil Kalıpçılar
Dean, Graduate School of **Natural and Applied Sciences**

Prof. Dr. İlkey Ulusoy
Head of Department, **Electrical and Electronics Engineering**

Assoc. Prof. Dr. Serdar Kocaman
Supervisor, **Electrical and Electronics Engineering, METU**

Examining Committee Members:

Prof. Dr. Gönül Turhan Sayan
Electrical and Electronics Engineering, METU

Assoc. Prof. Dr. Serdar Kocaman
Electrical and Electronics Engineering, METU

Prof. Dr. Haluk Külâh
Electrical and Electronics Engineering, METU

Assist. Prof. Dr. Erdiñç Tatar
Electrical and Electronics Engineering, Bilkent University

Assist. Prof. Dr. Ahmet Cemal Durgun
Electrical and Electronics Engineering, METU

Date: 23.06.2023



I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Surname: Fatih Utku Yılmaz

Signature :

ABSTRACT

ANALYSIS OF MZI MODULATORS WITH ONE-DIMENSIONAL PHOTONIC CRYSTAL WAVEGUIDES

Yılmaz, Fatih Utku

M.S., Department of Electrical and Electronics Engineering

Supervisor: Assoc. Prof. Dr. Serdar Kocaman

June 2023, 75 pages

Photonic Crystals, also known as PhCs, can control light in unique ways, which is why they are getting more and more attention in the world of integrated optics. This thesis looks into how one-dimensional PhCs can make Mach-Zehnder Interferometer modulators operate with higher performance, offering a new choice besides the usual silicon-based options.

The research focuses on how flexible PhCs are, meaning they can be tweaked to work best around a 1550 nm centered bandwidth. This special feature, along with certain measurements of width, radius, and height, are the basis of a PhC design that aims to boost the modulator's performance.

The study starts with a thorough look into how bandwidth changes with different device lengths. Then, a new MZI modulator design is suggested, which differs from the norm by using one-dimensional PhC waveguides as interferometer's arms. This setup makes the most of the PhCs' unique abilities to support high-bandwidth operations while making the modulator a lot smaller overall.

By going deep into the analysis and using numerical evaluations of the PhC's band

diagram, it is proven that this design's functionality can be supported by more than one concept. The findings show a big jump in bandwidth and efficient working, along with a notable shrink in device size, making it clear that this is a big step up from traditional silicon-based designs.

Keywords: Photonic Crystals, Mach-Zehnder Interferometer, Mach-Zehnder Interferometer Enhancement, Photonic Crystal Waveguide



ÖZ

BİR BOYUTLU FOTONİK KRİSTAL DALGA KILAVUZLARI İLE MZI MODÜLATÖRLERİNİN ANALİZİ

Yılmaz, Fatih Utku

Yüksek Lisans, Elektrik ve Elektronik Mühendisliği Bölümü

Tez Yöneticisi: Doç. Dr. Serdar Kocaman

Haziran 2023 , 75 sayfa

Fotonik Kristaller, ışığı eşsiz şekillerde kontrol edebilmeleri sebebiyle entegre optik dünyasında giderek daha fazla dikkat çekmektedirler. Bu tez, bir boyutlu Fotonik Kristal yapılarının Mach-Zehnder Interferometresinin daha yüksek performansla çalışmasını nasıl sağlayabileceğini incelemektedir ve bu sayede genellikle silikon tabanlı seçeneklerin dışında yeni bir seçenek sunmaktadır.

Bu tezdeki araştırma, Fotonik Kristallerin ölçeklenebilirlikleri sayesinde 1550 nm merkezli bir bant genişliği etrafında en iyi şekilde çalışacak şekilde ayarlanabileceği üzerine odaklanmaktadır. Bu özel özellik, genişlik, yarıçap ve yükseklik ölçümleriyle birlikte, modülatörün performansını artırmayı amaçlayan bir Fotonik Kristal tasarımının temelini oluşturur.

Çalışma, bant genişliğinin farklı cihaz uzunluklarıyla nasıl değiştiğine dair ayrıntılı bir incelemeyle başlar. Ardından, interferometrenin kolları olarak bir boyutlu kristal dalga kılavuzları kullanarak klasik kullanımdan farklı bir yeni MZI modülatör tasarımı önerilir. Bu kurulum, Fotonik Kristallerin yüksek bant genişliği operasyonlarını

destekleme ve modölatörü genel olarak çok daha küçük hale getirme gibi benzersiz yeteneklerinden en iyi şekilde yararlanır.

Bu tez, derinlemesine bir analiz yaparak ve Fotonik Kristallerin bant diyagramının sayısal deęerlendirmelerini kullanarak, bu tasarımın işlevselliğinin birden fazla kavram tarafından desteklenebileceğini gösterir. Bulgular, bant genişliğinde büyük bir artış ve verimli çalışmayı, cihaz boyutunda belirgin bir küçölme ile birlikte gösterir ve bu durum, geleneksel silikon tabanlı tasarımlardan büyük bir adım öteye gidilebileceğini ve gidildiğini açıkça ortaya koyar.

Anahtar Kelimeler: Fotonik Kristaller, Mach-Zhender Interferometre, Mach-Zehnder Interferometre Performans Arttırımı, Fotonik Kristal Dalga Kılavuzu



To My Family

ACKNOWLEDGMENTS

To begin with, I'd like to express my gratitude to my advisor Assoc. Prof. Serdar Kocaman for his unwavering support and mentorship throughout the course of my research. His keen insight, patience, and commitment to excellence have not only shaped this work but also my intellectual growth and academic journey. I am particularly thankful for his ability to balance guidance with giving me the autonomy to explore and learn independently.

I thank Prof. Dr. Gönül Turhan Sayar, Prof. Dr. Haluk Külâh, Assist. Prof. Ahmet Cemal Durgun, Assist. Prof. Erdinç Tatar for taking part in my thesis committee.

I'd like to thank my group mates Cem Kılıç, Alper Şahin, Onur Akdeniz, Ozan Berk Boyraz, and Mert Doğan, although Mert was not a member of our group, and many more for their contribution in terms of academic learning and their quality company.

I am forever grateful to my family for their unconditional love, trust and support. Their teachings and values have been my guiding light, giving me the passion to learn and overcome challenges, for which I am.

I want to say that I'm very lucky to have Nilay Göktepe in my life whose assistance, enduring patience, and ceaseless support have been invaluable. Her unwavering belief in me has continually helped me. Her influence in my life extends beyond the scope of this work and for this, I am immensely grateful.

I also thank Narin Işık for being such a great mentor and for her guidance in my professional journey. Her patience, understanding, and generosity with her time greatly contributed to my personal and professional development.

During this study, I acknowledge taking a part in TUBITAK ARDEB 116E728 project, and the financial support from it.

TABLE OF CONTENTS

ABSTRACT	v
ÖZ	vii
ACKNOWLEDGMENTS	x
TABLE OF CONTENTS	xi
LIST OF TABLES	xiv
LIST OF FIGURES	xv
LIST OF SYMBOLS AND ABBREVIATIONS	xix
CHAPTERS	
1 INTRODUCTION	1
1.1 Photonic Integrated Circuits	1
1.2 The Scope and Organization of the Thesis	3
2 THEORETICAL BACKGROUND	5
2.1 Fundamentals of Electromagnetics	5
2.1.1 Introduction	5
2.1.2 Maxwell's Equations	5
2.1.3 Wave Equation and Propagation of Electromagnetic Waves	6
2.1.4 Snell's Law and Total Internal Reflection	7
2.2 Propagation of Light in Dielectric Media	9

2.2.1	Electric Field in a Dielectric	9
2.2.2	Wave Equation in Dielectric Media	9
2.2.3	Polarization and Refraction	10
2.3	Propagation of Light in Periodic Media	10
2.3.1	Photonic Crystals	10
2.3.2	Bloch's Theorem	11
2.3.3	Dispersion Relation and Photonic Band gaps	11
2.3.4	Brillouin Zones and Negative Refraction	12
2.4	Optical Modulators and Modulation Techniques	12
2.4.1	Basics of Optical Modulation	12
2.4.2	Modulation Techniques	13
2.4.3	Underlying Physical Effects of Modulation Techniques	13
2.4.3.1	Electric Field Effects	13
2.4.3.2	Thermo-optic Effects	14
2.4.3.3	Charge-Carrier Effects	14
2.4.4	Optical Modulation Mechanisms	14
2.4.4.1	Electro-Optic Effect	14
2.4.4.2	Acousto-Optic Effect	15
2.4.4.3	Free Carrier Dispersion	15
2.4.5	Optical Modulators	16
2.4.5.1	Micro-Ring Resonators	17
2.4.5.2	Mach-Zehnder Interferometers	19
3	PHOTONIC CRYSTALS	23

3.1	Theory of Photonic Crystals	25
3.2	1D Photonic Crystals	28
3.2.1	Effective Refractive Index Calculation	30
3.3	2D Photonic Crystals	31
4	ANALYSIS AND DESIGN OF MZI MODULATOR WITH ONE DIMEN- SIONAL PHOTONIC CRYSTAL WAVEGUIDES	35
4.1	Analysis of One Dimensional Photonic Crystal	37
4.1.1	Proposed Methods and Models	37
4.1.2	Analyses Results	42
4.2	Proposed Design	54
4.3	Experimental Work	63
5	CONCLUSIONS AND FUTURE WORK	67
	REFERENCES	69

LIST OF TABLES

TABLES

Table 4.1	Bandwidth vs ext. ratio for various PS lengths, with $r = 0.285a$.	49
Table 4.2	FSR calculations of PCW-MZI designs with different PS and PCW lengths.	62

LIST OF FIGURES

FIGURES

Figure 2.1	Refraction of light, using ray model.	8
Figure 2.2	Top view of a micro-ring resonator. Structure consists of a ring-shaped cavity and a waveguide.	18
Figure 2.3	Schematic of an MZI.	19
Figure 3.1	(a) One dimensional photonic crystal structure. (b) Two dimensional photonic crystal structure. (c) Three dimensional photonic crystal structure.	24
Figure 3.2	Band diagram of an arbitrary 1D PhC.	28
Figure 3.3	Isometric view of a 1D PhC unit cell.	29
Figure 3.4	Top view of a 1D PhC unit cell.	29
Figure 3.5	A nanobeam structured with a 1D photonic crystal array.	29
Figure 3.6	1D photonic crystal structure	30
Figure 3.7	First TE bands of 1D PhCs with $n = 3.48$ and $n = 3.47$, enhanced view. Illustration of calculating Δk_x	31
Figure 3.8	(a) Square lattice of dielectric rods on air, top view, where circles are dielectric rods. (b) Isometric view of dielectric rods on air. (c) Band diagram of the structure in (b). Red bands are for TE and blue bands are for TM polarization. (d) Irreducible Brillouin Zone of square lattice configuration.	32

Figure 3.9	(a) Band diagram of a 2D PhC with triangular lattice of air holes configuration. Red bands represent TE and blue bands represent TM modes. (b) Irreducible Brillouin Zone of a triangular lattice formation.	33
Figure 3.10	SEM image of a fabricated chip's waveguide composed of triangular lattice of air holes.	34
Figure 4.1	Bandwidth vs. Phase Shifter (Silicon Waveguide) Length.	37
Figure 4.2	Schematic of an MZI with Photonic Crystal Waveguides.	37
Figure 4.3	(a) First and second bands of on and off states. Gray triangle corresponds to light cone and red shaded area shows the bandgap between first and second TE bands. (b) Enlarged view, first bands of on and off states. (c) Enlarged view, second bands of on and off states.	39
Figure 4.4	First TE Bands of Varying Air-hole Radius (r).	40
Figure 4.5	First TE Bands of Varying Slab Thickness (h).	41
Figure 4.6	First TE Bands of Varying PCW Width (w).	41
Figure 4.7	Effective index difference between on and off states for PCW with PhC parameters $r = 0.285a$, $h = 0.710a$, $w = 1.2a$	42
Figure 4.8	Cosine of phase difference of PCW with $r = 0.285a$, $h = 0.710a$, $w = 1.2a$, and $a = 0.33272 \mu\text{m}$	44
Figure 4.9	Cosine of phase difference of PCW with $r = 0.285a$, $h = 0.710a$, $w = 1.2a$, and $a = 0.33272 \mu\text{m}$, after moving average filter is applied. Filtering is done with window size = 14, 28.	44
Figure 4.10	Region chosen as bandwidth for extinction ratio ≥ 10 dB.	45
Figure 4.11	Region chosen as bandwidth for extinction ratio ≥ 15 dB.	45
Figure 4.12	Region chosen as bandwidth for extinction ratio ≥ 20 dB.	46

Figure 4.13	Cosine of phase difference of PCW with $r = 0.285a$, $h = 0.710a$, $w = 1.2a$, and $a = 0.31994 \mu\text{m}$	46
Figure 4.14	Cosine of phase difference of PCW with $r = 0.285a$, $h = 0.710a$, $w = 1.2a$, and $a = 0.33680 \mu\text{m}$	47
Figure 4.15	Cosine of phase difference of PCW with $r = 0.285a$, $h = 0.710a$, $w = 1.2a$, and $a = 0.34487 \mu\text{m}$	47
Figure 4.16	Cosine of phase difference of PCW with $r = 0.285a$, $h = 0.710a$, $w = 1.2a$, and $a = 0.35120 \mu\text{m}$	48
Figure 4.17	Cosine of phase difference of PCW with $r = 0.285a$, $h = 0.710a$, $w = 1.2a$, and $a = 0.35559 \mu\text{m}$	48
Figure 4.18	Bandwidth vs. PCW length. PCWs are with refractive indices $n = 3.48$ and $n = 3.47$, with $r = 0.285a$. Bandwidths calculated for four different extinction ratios are plotted with different colors and markers, as shown in the legends.	49
Figure 4.19	Bandwidth vs. Phase Shifter Length for Si-Waveguide and PCW.	50
Figure 4.20	Bandwidth vs. Phase Shifter Length for Si-Waveguide and PCW, enhanced view.	50
Figure 4.21	Bandwidth vs. Air-hole Radius (r) for extinction ratios 10, 15, and 20 dB for PCW length $L = 54.25 \mu\text{m}$	52
Figure 4.22	Bandwidth vs. Air-hole Radius (r) for extinction ratios 10, 15, and 20 dB for PCW length $L = 46.5 \mu\text{m}$	53
Figure 4.23	Bandwidth vs. Air-hole Radius (r) for extinction ratios 10, 15, and 20 dB for PCW length $L = 38.75 \mu\text{m}$	53
Figure 4.24	Bandwidth vs. Air-hole Radius (r) with 5 dB extinction ratio for PCW lengths $L = 54.25 - 38.75 \mu\text{m}$	55

Figure 4.25	(a) Schematics of proposed MZI, top view. (b) Photonic Crystal Waveguide used in the proposed design with length $L = 38.0664 \mu\text{m}$. Number of air-holes are representative.	56
Figure 4.26	(a) Electric field distribution at the input of phase shifters with length $L = 38.0664 \mu\text{m}$. Waves are in phase. (b) Electric field distribution at the output of phase shifters with length $L = 38.0664 \mu\text{m}$. Waves are out of phase.	57
Figure 4.27	Normalized transmission spectra of PCW-MZI with PS length L_{ps} and (a) PCW length $0.5L_{\text{ps}}$. (b) PCW length $0.75L_{\text{ps}}$. (c) PCW length L_{ps} (proposed design).	59
Figure 4.28	Normalized transmission spectra of PCW-MZI with PS length $2L_{\text{ps}}$ and (a) PCW length $1L_{\text{ps}}$. (b) PCW length $1.5L_{\text{ps}}$. (c) PCW length $2L_{\text{ps}}$	60
Figure 4.29	Number of fringes in normalized transmission spectra.	61
Figure 4.30	Theoretical vs. calculated FSR values.	61
Figure 4.31	Experimental setup.	65
Figure 4.32	(a) SEM image of fabricated chip, showing a PCW-MZI with 1D PCWs. Blue rectangle contains the p-i-n diode structure where intrinsic region is made of 1D PhCs. (b) Enhanced view of the p-i-n region. (c) Intrinsic region of the diode structure. (d) Enhanced view of the intrinsic region. Roughness in the waveguide and non-uniformity in air-holes depicted.	66

LIST OF SYMBOLS AND ABBREVIATIONS

1D	One Dimensional
2D	Two Dimensional
3D	Three Dimensional
FDTD	Finite-Difference Time-Domain
PCW	Photonic Crystal Waveguide
PS	Phase Shifter
MZI	Mach-Zehnder Interferometer
Si-MZI	MZI with Silicon Waveguide
PCW-MZI	MZI with Photonic Crystal Waveguide
MRR	Micro Ring Resonator
PhC	Photonic Crystal
MEEP	MIT Electromagnetic Equation Propagation
MPB	MIT Photonic Bands
SOI	Silicon-On-Insulator
TIR	Total Internal Reflection
TE	Transverse Electric
TM	Transverse Magnetic
FSR	Free Spectral Range
IBZ	Irreducible Brillouin Zone
n_{eff}	Effective Refractive Index
a	Lattice Constant
$\mathbf{B}$	Magnetic Flux Density Vector
$\mathbf{D}$	Electric Displacement Vector
$\mathbf{E}$	Electric Field Intensity Vector

H	Magnetic Field Intensity Vector
J	Current Density Vector
k	Wave Vector
α	Alpha
δ	Delta
ϵ	Epsilon
θ	Theta
λ	Lambda
μ	Mu
π	Pi
ϕ	Phi
ω	Omega

CHAPTER 1

INTRODUCTION

1.1 Photonic Integrated Circuits

The remarkable evolution of information technology, communication, and data processing has largely been driven by continuous advancements in microelectronics. However, as demands for faster data processing and communication continue to grow, the limitations of electronic systems have become increasingly evident. This has given rise to the field of photonic integrated circuits (PICs). PICs, which use light for functions such as signal processing, transmission, and detection, offer numerous advantages over electronic integrated circuits. These include high-speed operation, large bandwidth, reduced power consumption, and immunity to electromagnetic interference [1,2]. Furthermore, the integration of multiple optical functions on a single chip enables more compact and reliable systems. One of the most promising platforms for PICs is silicon photonics. The strength of silicon as a photonic material lies in its high refractive index contrast between silicon (Si) and silicon dioxide (SiO_2), allowing for the fabrication of highly confined waveguides and compact photonic devices. Moreover, silicon is transparent at telecommunications wavelengths, providing low propagation losses. But perhaps the most significant advantage of silicon photonics is its compatibility with the established CMOS fabrication processes, enabling large-scale and high-yield manufacturing [3–6].

Optical communication systems, one of the key applications of PICs, have revolutionized data transmission. These systems provide substantial improvements in speed and capacity compared to electronic systems. Fiber-optic communication, for example, is capable of carrying vast amounts of data over long distances, making it integral

to the infrastructure of global communication. As data traffic grows and demand for higher bandwidth increases, the need for efficient and reliable optical communication systems becomes even more urgent. In these optical communication systems, silicon photonic devices, such as waveguides, couplers, modulators, and detectors, play crucial roles. These devices can be integrated on a single chip to form a complete communication system. Among these devices, electro-optic (EO) modulators are especially central as they enable the dynamic control of the amplitude, phase, and polarization of light signals. EO modulators, which can efficiently convert electronic signals into optical ones, are integral to a variety of applications. These include switching, data transmission, fiber-optic communication, optical signal processing, and sensing. As such, the development of EO modulators, including Mach-Zehnder Interferometers (MZIs), LiNbO₃ modulators, and microring resonators (MRRs), that offer high modulation efficiency, low insertion loss, low drive voltage, and wide bandwidth is of great interest [7–17].

PICs also find significant use in biosensing applications. Optical biosensors leverage the interaction of light with biological samples and have shown potential for sensitive and selective detection of various biological entities. Photonic biosensors can provide real-time, label-free, and high-throughput analysis, which makes them attractive for a range of applications in healthcare, environmental monitoring, food safety, and biotechnology [18–24].

Photonic crystals (PhCs) are periodic nanostructures that manipulate the propagation of light, opening new opportunities in optical communication systems and biosensing. By introducing a periodic variation of refractive index, PhCs can create a photonic bandgap, a range of frequencies in which light propagation is forbidden. This unique property enables enhanced control and manipulation of light. In optical communication systems, PhCs can improve the performance of devices such as waveguides, filters, and modulators. PhC-based MZIs, for example, hold potential for high-speed operation, reduced device footprint, and enhanced modulation efficiency. In biosensing applications, PhC cavities can increase the sensitivity and selectivity of sensors by enhancing light-matter interaction. Furthermore, the resonant behavior of PhC cavities makes them ideal for label-free sensing, where a shift in resonant frequency indicates the presence of target analytes. The integration of silicon photonics and pho-

tonic crystals holds promise for the development of high-performance optical communication systems and biosensing applications. Moreover, the use of silicon photonics, which is highly compatible with complementary metal-oxide-semiconductor (CMOS) technology and allows for a high degree of integration, is advantageous in terms of cost, size, and efficiency of the resulting systems [21, 25–33].

1.2 The Scope and Organization of the Thesis

In this study, focus will be on use of 1D photonic crystals in MZI design and their analysis, aimed at optimizing their performance for use in optical communication systems and sensors. The motivation behind this work is that traditional MZIs are too long to be efficient in on-chip designs where device footprint is of essential along with wide bandwidths. To overcome this issue, we show that PhCs can solve this problem up to a point, where length can be reduced almost to half of its conventional value while other performance parameters remain high. The emphasis should be made on not only reduced lengths but also on wide bandwidth, reason to which will be clarified in the subsequent chapters.

This thesis consists of 4 chapters excluding the introduction chapter Chapter 1. In Chapter 2, background information regarding Maxwell’s equations, light propagation in dielectric medium, and how electromagnetic waves behave in general are given. Chapter 3 is dedicated to photonic crystals. There, the theory behind photonic crystals, how they are analyzed, what purpose they serve, and their possible configurations and use areas are discussed. Having the necessary knowledge in photonic crystals, Chapter 4 includes the analysis of 1D PhC in depth, further focusing on use of 1D PhC waveguides in MZI designs, parameter impact on figures of merit and optimization of PhC parameters. Then, a design is proposed with relevant transmission spectra, electric field distribution, and bandwidth calculations. Chapter 4 is finalized by a brief discussion on how the experimental procedure is followed in our lab, and images from a fabricated chip. In Chapter 5, a conclusion is made with a discussion regarding the contents of this thesis being used for further improvements in PhC usage in optical component designs, and wrapping up what is discussed in this thesis overall.



CHAPTER 2

THEORETICAL BACKGROUND

2.1 Fundamentals of Electromagnetics

2.1.1 Introduction

The fundamental laws of electromagnetic fields are encompassed within Maxwell's equations. These equations are named after the physicist James Clerk Maxwell, who derived/composed them from various laws of electricity and magnetism that were previously known. Maxwell's equations, with the Lorentz force law, form the basis of classical electrodynamics, optics, and electric circuits.

2.1.2 Maxwell's Equations

Maxwell's Equations are a set of four differential equations that provide a comprehensive description of electromagnetic phenomena. They are universally valid, in free space and material media, as well as at both static and dynamic cases. For an arbitrary medium, these equations are presented in their differential form below [34, 35]:

$$\nabla \cdot \mathbf{D} = \rho \quad (2.1)$$

where $\mathbf{D}$ is the electric displacement field, $\nabla \cdot$ denotes the divergence of a vector field, and ρ is the free charge density. In a medium with no free charges, the divergence of the electric displacement field is zero.

$$\nabla \cdot \mathbf{B} = 0 \quad (2.2)$$

where $\mathbf{B}$ is the magnetic field. This equation represents the non-existence of magnetic monopoles in nature, implying that magnetic fields always form closed loops.

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (2.3)$$

where $\nabla \times$ represents the curl of a vector field, and $\frac{\partial}{\partial t}$ denotes the partial time derivative. This law relates a changing magnetic field to an induced electric field.

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad (2.4)$$

where μ_0 is the permeability of free space, $\mathbf{H}$ is the magnetizing field, and $\mathbf{J}$ is the current density vector. This law expresses the creation of magnetic fields due to electric currents and changes in electric fields.

It should be noted that these equations are not presented in their free space form, which assumes a vacuum with no charges or currents. Instead, they are valid for arbitrary media. However, in the scope of this thesis, it will be assumed that the medium is free space, with no charges and currents. If the assumption becomes no longer valid, they must be modified to account for the presence of electric charges and currents, as well as for dielectric and magnetic materials. However, in the scope of this chapter, things will be kept in their simple forms mostly.

2.1.3 Wave Equation and Propagation of Electromagnetic Waves

Maxwell's equations are the basis of the dynamics of electric and magnetic fields. When these fields are coupled, the result is a propagating electromagnetic wave, where the oscillating electric and magnetic fields mutually generate each other. This is a very basic behaviour of electromagnetic waves. This propagation is described by the wave equation, derived from Maxwell's equations.

The intrinsic relationship between electric and magnetic fields, governed by Maxwell's equations, results in the creation of a propagating electromagnetic wave. This fundamental electromagnetic wave behavior illustrates how oscillating magnetic and electric fields generate one another. The resulting wave propagation is guided by the wave

equation, which is also derived from Maxwell's equations.

By taking the curl of eq. (2.3) and substituting eq. (2.4), wave equation for the electric field is obtained:

$$\nabla^2 \mathbf{E} - \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \quad (2.5)$$

where ∇^2 represents the Laplacian operator, which is the divergence of the gradient of a scalar field. The wave equation for the magnetic field can be derived in a similar manner. The speed of propagation of electromagnetic waves in a vacuum, denoted as c , is given by:

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \quad (2.6)$$

This speed is approximately 3×10^8 meters per second, which is the speed of light, thus linking the theories of optics and electromagnetism. For a harmonic electromagnetic uniform plane wave propagating in the z direction, the electric and magnetic fields can be represented as:

$$\mathbf{E}(z, t) = \mathbf{E}_0 e^{j(kz - \omega t)} \quad (2.7)$$

$$\mathbf{B}(z, t) = \mathbf{B}_0 e^{j(kz - \omega t)} \quad (2.8)$$

where $\mathbf{E}_0$ and $\mathbf{B}_0$ are the amplitudes of the electric and magnetic fields, respectively, k is the wave number, and ω is the angular frequency. The electric and magnetic fields are orthogonal to each other and to the direction of propagation.

2.1.4 Snell's Law and Total Internal Reflection

Snell's Law relates the angle of incidence to the angle of refraction when a wave is transmitted through the boundary separating two media with different indices of refraction.

For the incident wave in medium 1 and the refracted wave in medium 2, Snell's Law is given as:

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2) \quad (2.9)$$

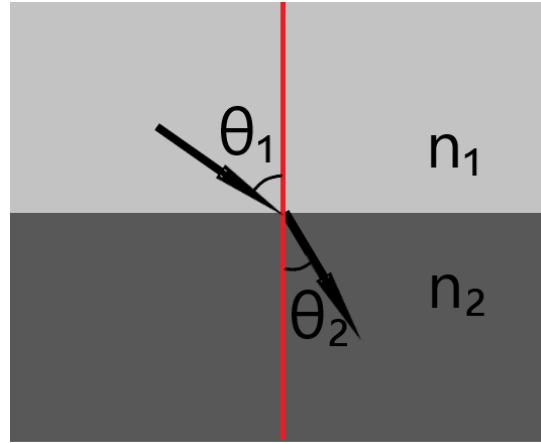


Figure 2.1: Refraction of light, using ray model.

where n_1 and n_2 are the refractive indices of medium 1 and medium 2, respectively, θ_1 is the angle of incidence measured from the normal to the boundary, and θ_2 is the angle of refraction, which is also measured from the normal.

If the angle of incidence is large enough such that $\sin(\theta_2)$ exceeds the maximum sine value (1), which is only possible when the light is moving from a medium with a higher refractive index to a medium with a lower refractive index, then there is no refraction. This phenomenon is known as total internal reflection (TIR) and is the principle behind the operation of optical fibers and other integrated optical components. TIR is what keeps a light in its path, allowing it to travel through it without too high of a loss.

The critical angle θ_c can be calculated using Snell's law:

$$\sin \theta_c = \frac{n_2}{n_1} \quad (2.10)$$

where n_1 is the refractive index of the medium the wave is coming from, and n_2 is the refractive index of the medium the wave is trying to enter. In the context of electromagnetism, TIR is typically discussed with respect to electromagnetic waves, such as light. TIR is utilized in various applications, including fiber-optic communication, where it allows light to propagate through fiber cables over long distances without significant loss.

2.2 Propagation of Light in Dielectric Media

When discussing the propagation of light, it is important to consider the medium through which it is passing. Light behaves differently in various media, depending on the properties of that medium. In this section, the propagation of light in a dielectric medium will be considered. Dielectric media are insulating materials that can be polarized by an electric field, and in this case, it is assumed a non-conducting, non-magnetic medium with no free charges or currents present, to simplify the calculations.

2.2.1 Electric Field in a Dielectric

When light enters a dielectric medium, it interacts with the atoms of the material. This interaction causes the atomic electrons to oscillate, inducing a dipole moment. This property is observed by the dielectric constant (or relative permittivity) ϵ_r of the material, and the electric displacement field $\mathbf{D}$ can be expressed as $\mathbf{D} = \epsilon_0 \epsilon_r \mathbf{E}$, where $\mathbf{E}$ is the electric field and ϵ_0 is the permittivity of free space.

2.2.2 Wave Equation in Dielectric Media

For an electromagnetic wave propagating in a dielectric, Maxwell's equations (again, no free charges or currents) yield the following wave equations for the electric field:

$$\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \quad (2.11)$$

However, in a dielectric medium, the speed of light c is replaced by the phase velocity $v = c/\sqrt{\epsilon_r}$, which takes into account the slowing of light wave due to medium's dielectric property. Therefore, the wave equation becomes:

$$\nabla^2 \mathbf{E} - \frac{1}{v^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \quad (2.12)$$

The phase velocity v of the wave is the speed at which the phase of the wave propa-

gates. However, the speed at which the information carried by the wave propagates, known as the group velocity, can differ from the phase velocity in a medium with non-zero dispersion, implying that the refractive index is dependent on the frequency.

2.2.3 Polarization and Refraction

As light propagates through a dielectric medium, it can induce a polarization in the medium due to the oscillation of the medium's atomic or molecular dipoles. This induced polarization gives rise to the refraction of the light, described by Snell's Law, and to the potential dispersion of the light, where different frequencies (or colors) of the light propagate at different speeds. Most known example of dispersion is light passing through a prism and coming out with different colors.

2.3 Propagation of Light in Periodic Media

When an electromagnetic wave, such as light, propagates through a periodic medium, the interaction can lead to phenomena such as , slow light effect, photonic band gaps, and negative refraction. The study of these effects has led to the development of photonic crystals, meta-materials, and many other areas of research, some of which are also a part of this thesis.

2.3.1 Photonic Crystals

A photonic crystal is a structure that has a periodic variation in its refractive index. When light propagates through a photonic crystal, it can encounter a wavelength range, where it cannot propagate. This range is called as the photonic bandgap. The presence of a photonic bandgap is analogous to the electronic bandgap in semiconductors, where certain energies are forbidden for electrons, like the region between conduction and valence band. This effect is derived from the general principles of wave propagation in periodic media, which is outlined in Bloch's theorem.

2.3.2 Bloch's Theorem

In a periodic medium, the properties of the medium repeat periodically, leading to a distinctive behavior of the propagating wave. The behavior of waves in periodic structures is encapsulated in Bloch's theorem, which states that the solution to a wave equation in a periodic medium can be expressed as a product of a plane wave and a periodic function with the periodicity that is same with that of the medium. For an electromagnetic wave propagating in a one-dimensional periodic medium, the electric field $\mathbf{E}(z, t)$ can be expressed as:

$$\mathbf{E}(z, t) = \mathbf{u}(z)e^{j(kz - \omega t)} \quad (2.13)$$

where $\mathbf{u}(z)$ is a periodic function of position with the same period as the periodic structure, k is the wave number of the plane wave, ω is its angular frequency, and j is the imaginary unit. Bloch's theorem ensures that the wave equation is satisfied in the periodic structure.

2.3.3 Dispersion Relation and Photonic Band gaps

The relationship between the wave number k and the angular frequency ω is known as the dispersion relation. It provides essential information about the propagation characteristics of the wave in the medium. When the dispersion relation of a photonic crystal is plotted, typically a series of allowed bands of propagation separated by forbidden bands or photonic band gaps is revealed. Within these band gaps, light with specific frequencies cannot propagate.

For instance, a simple one-dimensional photonic crystal can be represented as a periodic stack of two dielectric layers with refractive indices n_1 and n_2 , and layer thicknesses d_1 and d_2 , respectively.

The dispersion relation of such a photonic crystal can be derived from the continuity of the tangential components of the electric and magnetic fields at the interfaces between the dielectric layers. From this, it is possible to obtain allowed bands and band gaps as a function of the ratio of the layer thicknesses and the ratio of the refractive

indices.

Noting the mentioned theoretical method of obtaining the dispersion curve, it is not feasible and/or possible to use paper-pencil to get the entire curve. There exist various software and algorithms that calculate the curve and they are widely adopted.

2.3.4 Brillouin Zones and Negative Refraction

The dispersion relation typically repeats for each multiple of the wave number corresponding to the basic spatial period of the crystal. These repeated regions in wave number space are known as Brillouin zones. The first Brillouin zone contains the lowest allowed band and the first bandgap. The dispersion relation in the higher Brillouin zones can lead to unusual effects such as negative refraction, where light bends in a direction opposite to that expected from Snell's law [36].

2.4 Optical Modulators and Modulation Techniques

2.4.1 Basics of Optical Modulation

Modulation is the process of altering one or more properties of a carrier signal in accordance with an information signal. In the context of optical modulators, the carrier signal is a light wave, and its properties, such as amplitude, phase, or polarization, can be modulated. As a result of this modulation, bits to be transmitted are obtained, namely ones and zeros.

Optical modulation is especially significant for telecommunications because it enables the transmission of information over long distances with minimal loss and interference. It's also crucial for optical computing and quantum computing applications, where light is used to process and store information.

2.4.2 Modulation Techniques

Different modulation mechanisms are used to achieve the desired changes in light properties [3, 37–40]. These are:

- **Amplitude modulation (AM):** In amplitude modulation, the modulator varies the light's intensity proportional to input signal's. This modulation is achieved by using either an electro-optic or acousto-optic modulator, which changes the medium's refractive index, resulting with changes in the light's transmission or reflection.
- **Phase modulation (PM):** Phase modulation involves changing the phase of light based on the input signal. This can be done using an electro-optic modulator, which changes the refractive index of the material, altering the light's phase through the device.
- **Frequency modulation (FM):** In frequency modulation, the modulator adjusts the light's frequency according to the input signal. This can be achieved using an acousto-optic modulator, by creating an acoustic wave which the light interacts with, resulting in a shift in its frequency.
- **Polarization modulation:** Polarization modulation alters the light's polarization state in response to the input signal. This can be done using a magneto-optic or liquid crystal modulator, which changes the material's polarization properties when subjected to an external field.

2.4.3 Underlying Physical Effects of Modulation Techniques

Different modulation methods rely on various physical effects induced by external signals [3, 34, 35, 41–43]. These include:

2.4.3.1 Electric Field Effects

Electric field effects play a crucial role in electro-optic modulators. These effects arise due to the interaction of the electric field with the material's electronic structure,

leading to changes in the refractive index. The Pockels effect and Kerr effect are examples of electric field-induced phenomena that can be exploited in electro-optic modulators.

2.4.3.2 Thermo-optic Effects

Thermo-optic effects involve changes in the refractive index of a material due to temperature variations. These effects can be utilized in thermo-optic modulators, where a change in temperature is induced by an external signal, causing a corresponding change in the material's refractive index and, consequently, the modulation of light properties.

2.4.3.3 Charge-Carrier Effects

Charge-carrier effects occur when the presence of free charge carriers, such as electrons or holes, in a material affects its optical properties. These effects can be used in modulators based on semiconductor materials, where the concentration of charge carriers can be controlled by an external signal, leading to changes in the refractive index and light modulation.

2.4.4 Optical Modulation Mechanisms

There are several mechanisms used to modulate an optical signal, which are [3, 37–40]:

2.4.4.1 Electro-Optic Effect

The electro-optic effect refers to the change in the refractive index of a material as a result of an applied electric field. There are two primary types of electro-optic effects: the Pockels effect and the Kerr effect.

The Pockels effect, a linear electro-optic effect, causes a refractive index change proportional to the applied electric field. This effect is typically observed in non-

centrosymmetric materials and is exploited in electro-optic modulators to alter the phase, and thus the intensity, of the light passing through the material.

On the other hand, the Kerr effect, also known as the quadratic electro-optic effect, causes a refractive index change proportional to the square of the applied electric field. This effect is observed in all types of materials, but is typically much weaker than the Pockels effect [15, 44].

2.4.4.2 Acousto-Optic Effect

The acousto-optic effect, or the Bragg diffraction, involves the interaction of light with an acoustic wave, leading to a change in the direction and intensity of light. Acousto-optic modulators, or Bragg cells, use this effect to modulate light.

2.4.4.3 Free Carrier Dispersion

Free Carrier Dispersion (FCD), also referred as Plasma Dispersion Effect, refers to the change in the refractive index of a semiconductor material due to the presence of free charge carriers (electrons and holes). In free carrier dispersion modulators, an electric field is applied to change the carrier concentration in the semiconductor, leading to a change in the refractive index and hence the phase of light.

In case of silicon modulators, the centrosymmetric crystal structure of silicon results in the absence of a linear electro-optic (Pockels) effect. Moreover, it is shown that Kerr Effect is small in silicon. The other alternative is plasma dispersion effect. The plasma dispersion effect refers to the alteration of a semiconductor's refractive index due to changes in its free carrier density. The change in the density can be due to depletion or injection. The required charge density to obtain an index difference Δn can be calculated via

$$\Delta n = \Delta N_e + \Delta N_h = -8.8 \times 10^{-22} \Delta n_e - 8.5 \times 10^{-18} (\Delta n_h)^{0.8} \quad (2.14)$$

where Δn_e , Δn_h are index changes due to electron and hole density, and ΔN_e , ΔN_h are electron and hole densities, respectively. Based on eq. (2.14), an index change of

0.01 in bulk Si is obtainable via a carrier density between 10^{17} - 10^{18} cm^{-3} . Change in α is given by

$$\Delta\alpha = \frac{e^3\lambda^2}{4\pi^2c^3\epsilon_0n_{si}} \left(\frac{\Delta N_e}{m_{ce}^{*2}\mu_e} + \frac{\Delta N_h}{m_{ch}^{*2}\mu_h} \right) \quad (2.15)$$

where e is the electronic charge, ϵ_0 is the permittivity of free space, n is the refractive index of unperturbed Si, m_{ce}^* is the conductivity effective mass of electrons, m_{ch}^* is the conductivity effective mass of holes, μ_e is the electron mobility, and μ_h is the hole mobility [45].

2.4.5 Optical Modulators

Optical modulators are crucial in modern optical communication systems as they transform electrical signals into optical signals. This chapter discusses two common optical modulator types micro-ring resonators and MZIs, starting from the basic concept and principles.

The length of the device is important because it affects the size and footprint of the modulator. A smaller length may lead to a compact design, but it can limit the maximum possible bandwidth and extinction ratio. On the other hand, a longer length can enhance the bandwidth and extinction ratio but may cause increased insertion loss and a larger size, which will be further investigated and shown numerically in next chapters.

Bandwidth refers to the range of frequencies over which the modulator can perform modulation at the desired high/low levels. A broader bandwidth allows the device to work on a wider range of input signals, thus improving its functionality. However, achieving a broader bandwidth may require a more complex design and higher fabrication tolerances and requires compromises from other performance parameters.

Insertion loss refers to the amount of signal power lost when light travels through the device. Lower insertion loss is desirable as it allows more efficient signal transmission and reduces the need for extra amplification. To minimize insertion loss in the device, one needs to consider factors like waveguide loss, coupling loss, and bending loss in

mostly coupler regions.

Extinction ratio is the relationship between the maximum and minimum transmission levels. To put it in equation form, it is simply $10 \log \frac{P_{max}}{P_{min}}$ where P_{max} is the transmission where interference is constructive, and P_{min} is the minimum transmission where interference is destructive. A high extinction ratio ensures the device can switch and modulate signals effectively. To reach a high extinction ratio, one needs to optimize the device length, phase shifter performance, and coupling efficiency between couplers and waveguides.

Fabrication tolerances play a significant role in determining the overall performance and reliability of the design. Tighter tolerances can lead to improved device performance and lower variability between devices. However, achieving these tolerances may require advanced fabrication techniques and increase manufacturing costs. In this thesis, fabrication and fabrication related issues are not addressed.

To wrap up, designing an optical modulator involves finding the right balance and trade-off between multiple parameters to achieve the desired performance. The optimum design will depend on the specific application and requirements of the design. In optical communications and optical signal processing, wide bandwidth is a crucial factor whereas small device length/footprint is rather more important in integrated photonics, optical interconnects and quantum optics.

2.4.5.1 Micro-Ring Resonators

A micro-ring resonator (MRR) is a particular kind of optical modulator that employs the resonant attributes of a ring-shaped cavity to modulate the light signal, and is highly used in various areas [8–11, 15]. MRRs are composed of a waveguide, a ring-shaped cavity specifically designed to resonate at a certain wavelength, as depicted in Fig. 2.2. To understand the concept, one should start with examining propagation along a ring-shaped cavity. It will be assumed that TIR conditions are satisfied, that is n_r , refractive index of ring, is greater than the surrounding medium, which is assumed to be air.

As light enters the ring, it completes a round-trip by going through a length of L ,

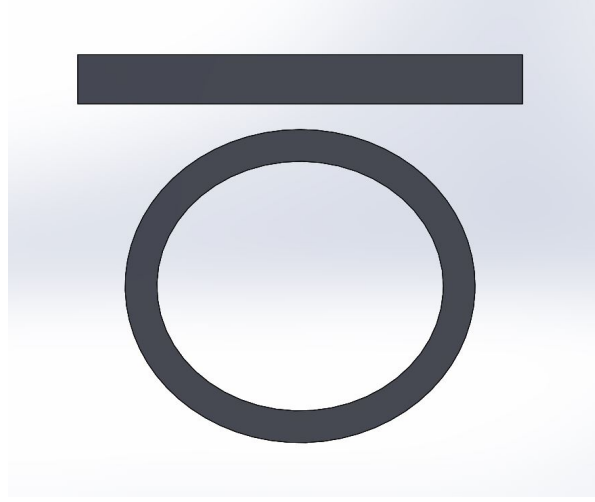


Figure 2.2: Top view of a micro-ring resonator. Structure consists of a ring-shaped cavity and a waveguide.

which is equal to $2\pi r$ where r is the radius of the ring. Then, it will be accumulating a phase that is equal to

$$\phi = \frac{2\pi}{\lambda} n_{\text{eff}} L. \quad (2.16)$$

The subsequent wave, which is assumed to arrive following the completion of the first wave's round-trip, will interfere with the first wave. This interference phenomenon is essential for the functionality of the MRR. For the interference to be constructive, leading to the amplification of the light signal, the accumulated phase should be an integer multiple of 2π . In other words, the phase condition for constructive interference and thus resonance in the MRR is discretized to integer multiples of 2π , ensuring the modulated light signal maintains its intensity. Likewise, for interference to be destructive, the two waves must be out of phase, that is having π radian phase difference.

To express interference conditions in simple equations, it is established that for constructive interference, the equation

$$\phi = m2\pi, \quad m = 0, 1, 2, 3, \dots \quad (2.17)$$

holds, and for destructive interference, the equation

$$\phi = m\pi, \quad m = 1, 2, 3, \dots \quad (2.18)$$

is applicable.

When the equations are expanded, it is found that for constructive interference, which signifies the resonance condition, the equation

$$\lambda_{\text{res}} = \frac{n_{\text{eff}}L}{m}, \quad m = 0, 1, 2, 3, \dots \quad (2.19)$$

is obtained. Similarly, for destructive interference, the equation

$$\lambda = \frac{2n_{\text{eff}}L}{m}, \quad m = 1, 2, 3, \dots \quad (2.20)$$

is derived.

By introducing an electrical signal, the refractive index of the waveguide material can be altered, leading to a shift in the resonant wavelength of the cavity. This shift results in the modulation of the output signal's intensity. MRRs are compact and easily integrated into photonic circuits with high density. Additionally, they are well-suited for applications such as optical filtering, sensing, and switching due to their resonance-based operation. However, resonance-based operation can be a limitation in systems with wide bandwidth requirements [46].

2.4.5.2 Mach-Zehnder Interferometers

Mach-Zehnder Interferometers (MZIs), another type of optical modulator, which utilize the interference of two waveguide arms to modulate the phase and amplitude of the light signal. MZIs are made up of a Y-shaped waveguide splitter that divides the input light into two arms, which are then recombined using another waveguide. Schematic of an MZI is provided in Fig. 2.3. By applying an electrical signal to one of the arms, the relative phase of the two arms can be modulated, causing a change in the interference pattern of the output signal.



Figure 2.3: Schematic of an MZI.

Consider two waves, $\mathbf{E}_1$ and $\mathbf{E}_2$, with the same frequency traveling through the two arms of an MZI. We can represent these waves in phasor form as:

$$\mathbf{E}_1 = \mathbf{E}_{10}e^{jkz} \quad (2.21)$$

$$\mathbf{E}_2 = \mathbf{E}_{20}e^{jk(z+\phi)} \quad (2.22)$$

where:

- $|E_{10}|$ and $|E_{20}|$ are the amplitudes of the two waves.
- z is the direction of propagation.
- k is the wave number.
- ϕ is the phase difference between the two waves.

The phase difference ϕ can be altered by changing the refractive index or length of one arm relative to the other. This phase difference governs the interference at the output. As the waves recombine at the second beam splitter, they may interfere constructively or destructively, depending on their phase difference.

The total electric field $\mathbf{E}_{\text{total}}$ at the output of the MZI can be computed as the sum of the two electric fields:

$$\mathbf{E}_{\text{total}} = \mathbf{E}_1 + \mathbf{E}_2 = \mathbf{E}_{10}e^{jkz} + \mathbf{E}_{20}e^{jk(z+\phi)} \quad (2.23)$$

Assuming that the magnitudes of the amplitudes of the two waves are equal, i.e., $|E_{10}| = |E_{20}|$, the equation can be simplified as follows:

$$\mathbf{E}_{\text{total}} = 2\mathbf{E}_{10} \cos\left(\frac{\phi}{2}\right) e^{j(kz + \frac{\phi}{2})} \quad (2.24)$$

The intensity at the output is proportional to the square of the magnitude of the total electric field, which can be written as:

$$I = |\mathbf{E}_{\text{total}}|^2 = 4|\mathbf{E}_{10}|^2 \cos^2 \left(\frac{\phi}{2} \right) \quad (2.25)$$

For an ideal Mach-Zehnder modulator, the input light is split evenly between the two paths, and so the power in each path is half of the total input power. Therefore, the total power at the output of the MZI is:

$$I_{\text{total}} = I_{\text{in}}(1 + \cos(\phi)). \quad (2.26)$$

The index change change allows for the modulation of both the amplitude and phase of the output signal, offering more versatility in optical signal processing compared to other modulator types. MZIs are frequently employed in fiber-optic communication systems for tasks such as optical switching, modulation, and demodulation. They exhibit high modulation bandwidth, low insertion loss, and low power consumption, which make them ideal for high-speed and low-power applications.

An MZI is designed in multiple steps. There are several parameters one has to take into account when carrying out design steps. These are the device length L , bandwidth, insertion loss, extinction ratio, and fabrication tolerances. Details of design procedure, including the design of phase shifters (arms) of the MZI are discussed in the subsequent chapters.



CHAPTER 3

PHOTONIC CRYSTALS

Photonic crystals are periodic structures with periodicity in their refractive index, changing spatially. The refractive indices in these structures vary periodically in one, two, or three dimensions, creating a platform to manipulate light waves. One, two and three dimensional photonic crystals can be represented as in Fig. 3.1. Photonic crystals are highly utilized in waveguide designs, also referred as photonic crystal waveguides, to enhance the overall performance [47–53].

In this thesis, two numerical simulation algorithms have been used, which are MIT Electromagnetic Equation Propagation (MEEP) and MIT Photonic Bands (MPB) [44, 54]. MEEP is an open-source algorithm based on Finite Difference Time Domain (FDTD) method that takes a user-defined structure and breaks it into grids. Then, as time is incremented, algorithm iteratively solves Maxwell's equations to obtain field values s at each point. MPB is an open-source algorithm used for analyzing dispersion relations, eigenvalue problems, etc. MPB is a frequency domain solver, unlike MEEP.

Photonic crystals, drawing a parallel to crystalline materials in solid-state physics, have a similar attribute of periodic potential. This recurring structure forms distinctive bands of allowed and disallowed energies, similar to the electronic band structure found in conventional crystals.

In photonic crystals, the periodic variation in refractive index results in photonic band structures, or ranges in which light can and cannot propagate. These frequency ranges where light can propagate are known as photonic bands, and regions where it cannot propagate are photonic band gaps [55].

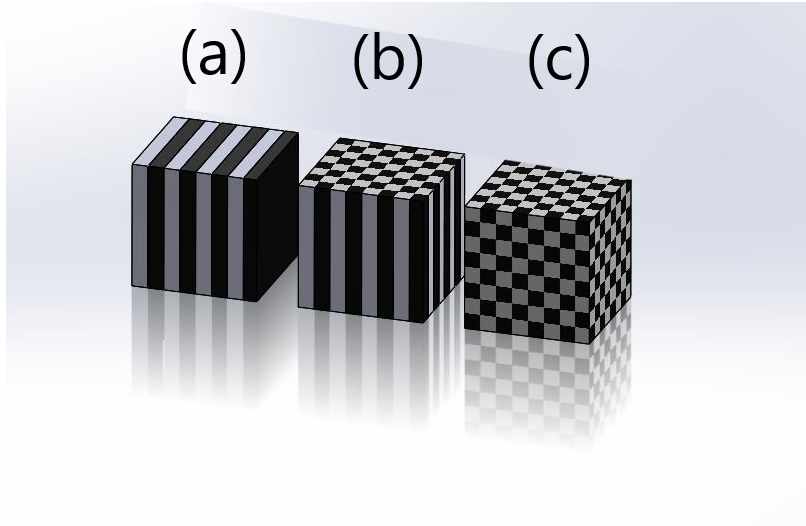


Figure 3.1: **(a)** One dimensional photonic crystal structure. **(b)** Two dimensional photonic crystal structure. **(c)** Three dimensional photonic crystal structure.

There are several topics one must get familiar with, starting with dispersion relations and curves when studying photonic crystals. Dispersion relation of a PhC constructs its photonic band diagram, which will often be mentioned as band diagram. These diagrams provide information of PhC under study regarding frequency (ω) relation with wave vector ($\mathbf{k}$). Normalized units will be used throughout this thesis with $c = 1$. These normalized units are $\omega a/2\pi c$ for frequency and $2\pi/a$ for wave number.

The reason why this diagram is essential is that it is possible to determine the bandgap location, effective refractive index, and group velocity, from the plot itself. Extracted information is necessary when designing a PhC with constraints in performance parameters.

Secondly, one can observe the electric field distribution of a waveguide consisting of PhC under investigation, to observe the effective index and phase accumulation for a certain length of propagation. In this chapter, focus will be on band diagrams rather than fields.

3.1 Theory of Photonic Crystals

By applying Maxwell's equations and deriving the boundary conditions, it is possible to establish a relationship between the electric field components across the layers of photonic crystal structure, leading to an eigenvalue problem.

To see how Maxwell's equations and boundary conditions lead to an eigenvalue problem, it is better to start with several assumptions and simplify Maxwell's equations accordingly. To do so, assume that the medium is lossless and there is no free charge density. Then, one obtains the following:

$$\nabla \cdot \mathbf{D} = 0 \quad (3.1)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (3.2)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (3.3)$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} \quad (3.4)$$

These are Maxwell's equations in differential form, where $\mathbf{D}$ is the electric displacement field, $\mathbf{B}$ is the magnetic flux density, $\mathbf{E}$ is the electric field, and $\mathbf{H}$ is the magnetic field [34].

The medium was already assumed to be free of charge and current. In the case of a typical 1D PhC, it is reasonable to assume that the medium is linear, time-invariant, and isotropic in the dimensions where the properties are uniform.

These assumptions allow us to write the relationship between displacement field and electric field as

$$\mathbf{D}(\mathbf{r}) = \varepsilon_0 \varepsilon \mathbf{E}(\mathbf{r}), \quad (3.5)$$

where ε_0 is the permittivity of the vacuum and ε is the relative permittivity. Similarly,

it is possible to express the relationship between the magnetic flux density and the magnetic field as

$$\mathbf{B}(\mathbf{r}) = \mu_0 \mu \mathbf{H}(\mathbf{r}), \quad (3.6)$$

with μ_0 being the permeability of vacuum and μ the relative permeability.

Assuming the time-harmonic behavior of fields, we can separate the temporal and spatial components of the fields, leading to the wave equation, an eigenvalue problem for $\mathbf{H}$ (or $\mathbf{E}$):

$$\nabla \times \frac{1}{\varepsilon(\mathbf{r})} \nabla \times \mathbf{H}(\mathbf{r}) = \left(\frac{\omega}{c}\right)^2 \mathbf{H}(\mathbf{r}), \quad (3.7)$$

Now, for the PhCs, the permittivity $\varepsilon(\mathbf{r})$ is periodic due to the crystal's translational symmetry. In other words,

$$\varepsilon(\mathbf{r}) = \varepsilon(\mathbf{r} + \mathbf{R}) \quad (3.8)$$

where $\mathbf{R}$ is a lattice vector. As a result, solutions to the wave equation in periodic media are products of a plane wave and a periodic function, which is known as the Bloch theorem.

For 1D PhCs, the lattice vector $\mathbf{R}$ is a scalar multiple of a basis vector. It can be expressed as

$$\mathbf{R} = m\mathbf{a}, \quad (3.9)$$

where m is an integer and $\mathbf{a}$ is the basis vector.

For 2D PhCs, the lattice vector $\mathbf{R}$ can be defined as the linear combination of two basis vectors, such as

$$\mathbf{R} = m_1 \mathbf{a}_1 + m_2 \mathbf{a}_2 \quad (3.10)$$

where m_1 and m_2 are integer numbers and $\mathbf{a}_1$ and $\mathbf{a}_2$ are basis vectors for a 2D PhC lattice.

Solutions to the wave equations for periodic media lead to photonic bands, where the eigenvalues of the product of the plane wave and a periodic function, $\omega_n(\mathbf{k})$, are functions of the wavevector $\mathbf{k}$. In this context, the function $\omega_n(\mathbf{k})$ defines the dispersion relation of the system, which details the relationship between the frequency (ω_n) and the wavevector ($\mathbf{k}$) of the wave [55].

Each band, denoted by the index n , corresponds to a different mode of the electromagnetic field. As a result, this dispersion relation forms the band structure of the crystal, which can be computed numerically for specific geometries and material properties.

Given the periodicity in the permittivity, the dispersion relation is also periodic in the wavevector space. Hence, it is enough to examine this relation within a specific range in the wavevector space. This specific range is called the Brillouin zone.

To define the Brillouin zone, we first need to introduce the concept of the reciprocal lattice. For a 1D photonic crystal, the reciprocal lattice vector G is defined as

$$G = 2\pi/a \quad (3.11)$$

where a is the lattice constant. For a 2D photonic crystal, the reciprocal lattice vectors $\mathbf{G}_1$ and $\mathbf{G}_2$ can be defined as

$$\mathbf{G}_1 = 2\pi \left(\frac{\mathbf{a}_2}{\mathbf{a}_1 \cdot \mathbf{a}_2} \right), \quad \mathbf{G}_2 = 2\pi \left(\frac{\mathbf{a}_1}{\mathbf{a}_1 \cdot \mathbf{a}_2} \right). \quad (3.12)$$

The first Brillouin zone is then the set of points in reciprocal space that are closer to the origin than to any other reciprocal lattice points.

Considering the aforementioned periodicity in the wavevector space, it is common to represent the photonic band structure within the first Brillouin zone. In first Brillouin zone, key features of the band structure, such as band gaps, can be identified.

The irreducible Brillouin Zone is a further reduction of the first Brillouin Zone taking into account the symmetry of the system. The irreducible Brillouin zone is the minimum volume in the reciprocal space that contains all the unique information about the system. It is beneficial to calculate the band structure only within the irreducible Brillouin zone, since the symmetry of the modes can be deduced from the symmetry of the zone [55].

Gaps in this band structure, where no solutions exist for $\mathbf{k}$, represent the photonic band gaps. Within these frequency ranges, light cannot propagate through the photonic crystal. The bandgap is a result of destructive interference of the waves reflected at each interface of the crystal layers. The position and width of the bandgap can be adjusted by changing the structural parameters of the PhC. Position change is done

by changing the lattice constant, and width of the bandgap depends on refractive indices and relative thicknesses. This is due to the fact that normalized frequency can be converted to wavelength by scaling the lattice constant a , such that

$$\lambda = a/\omega. \quad (3.13)$$

In Fig. 3.2, band diagram of an arbitrary 1D PhC is provided. Gray region, named as the "Light Cone", corresponds to modes that propagates in the surrounding medium, thus is not supported. There exists a bandgap region shaded with red.

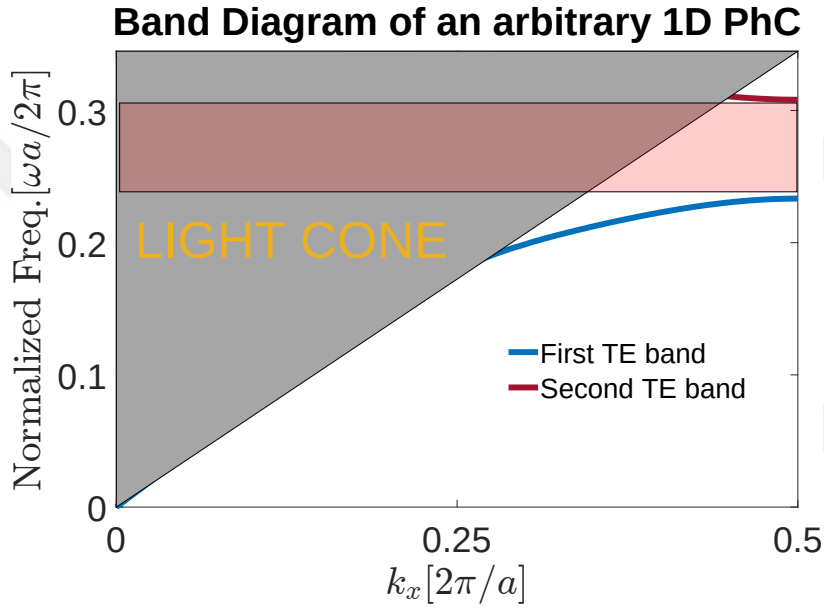


Figure 3.2: Band diagram of an arbitrary 1D PhC.

3.2 1D Photonic Crystals

One-dimensional photonic crystals constitute a particularly interesting class of photonic structures, displaying unique characteristics due to their spatially periodic refractive index modulation along one dimension. These structures can provide invaluable insight into the behavior of light-matter interaction, standing as a versatile platform for the development of devices based on the principles of optical interference and diffraction. Mostly, 1D PhCs are used for micro-cavity, nanobeam and grating design [30,31,56–58].

Schematics of one-dimensional PhC unit cell are provided in Fig. 3.3 and Fig. 3.4, and a photonic crystal nanobeam waveguide consisting of 1D PhC array made of air-holes is illustrated in Fig. 3.5.

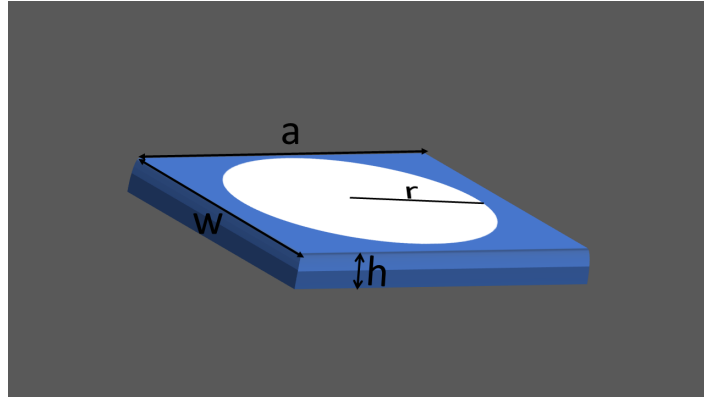


Figure 3.3: Isometric view of a 1D PhC unit cell.

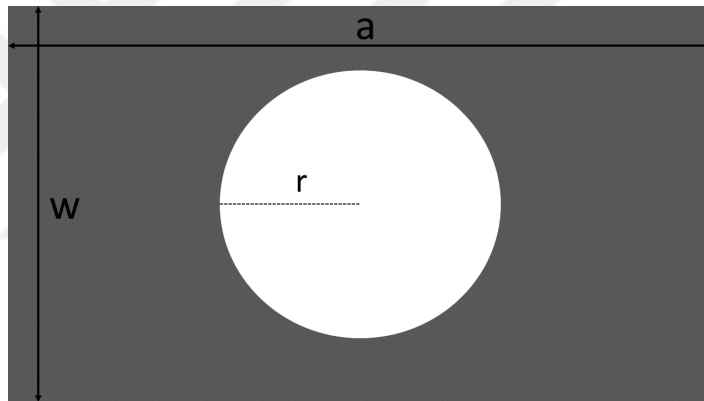


Figure 3.4: Top view of a 1D PhC unit cell.



Figure 3.5: A nanobeam structured with a 1D photonic crystal array.

The structure of 1D PhCs, as described previously, is typically formed by a periodic sequence of two different dielectric layers, having indices n_1 and n_2 , and thicknesses, d_1 and d_2 respectively. These parameters directly affect the optical properties of the 1D PhC, such as the spectral position and width of the photonic bandgap. Example of a 1D PhC structure, shown as vertical layers and not air-holes, is provided in Fig. 3.6.

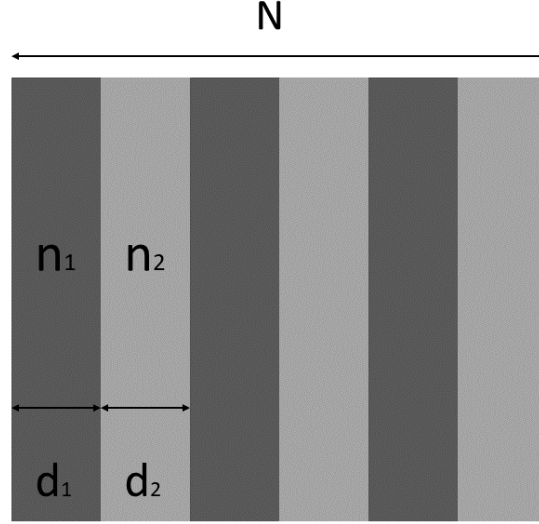


Figure 3.6: 1D photonic crystal structure

In the case where $n_1 < n_2$, light waves will travel slower in the n_2 material, and thus, the phase of the wave will change more drastically in the higher index material. If the optical path of the two layers are appropriately matched, destructive interference occurs for certain wavelengths, leading to the formation of photonic band gaps.

3.2.1 Effective Refractive Index Calculation

Effective index is a critical aspect in PhC designs, regardless of the type of PhC being designed. One controls the effective index in photonic crystal designs by carefully tailoring the geometry and refractive index contrast within the unit cell. This involves adjusting parameters such as the size and shape of the features (e.g., air holes or rods), and the relative difference in refractive index between these features and the surrounding material. From a normalized frequency perspective, the lattice constant does not impact the effective index, but the geometrical properties and material contrasts within the unit cell play a critical role. By manipulating these parameters, one can engineer the photonic band structure to achieve desired optical properties. This includes controlling the location and size of photonic band gaps, the dispersion relationship of guided modes, and the confinement of light within defect states. Thus, the effective index becomes a powerful tool for the design and optimization of photonic crystal applications. Effective refractive index of a crystal structure is $n_{eff} = k_x/\omega$,

where ω is the normalized frequency at which wave vector difference is calculated.

For a 1D PhC, the first step in obtaining the effective index difference is calculating the wave vector difference using the n_{eff} expression, such that

$$\Delta n_{eff} = \Delta k_x / \omega. \quad (3.14)$$

In Fig. 3.7, calculation of Δk_x is shown with enlarged view of first two bands, showing the difference clearly. Values of k_x in both bands ($n = 3.47$ and $n = 3.48$) at the same frequency is obtained and their difference provides the desired Δk_x value. Then, Δn_{eff} is found by using eq. (3.14).

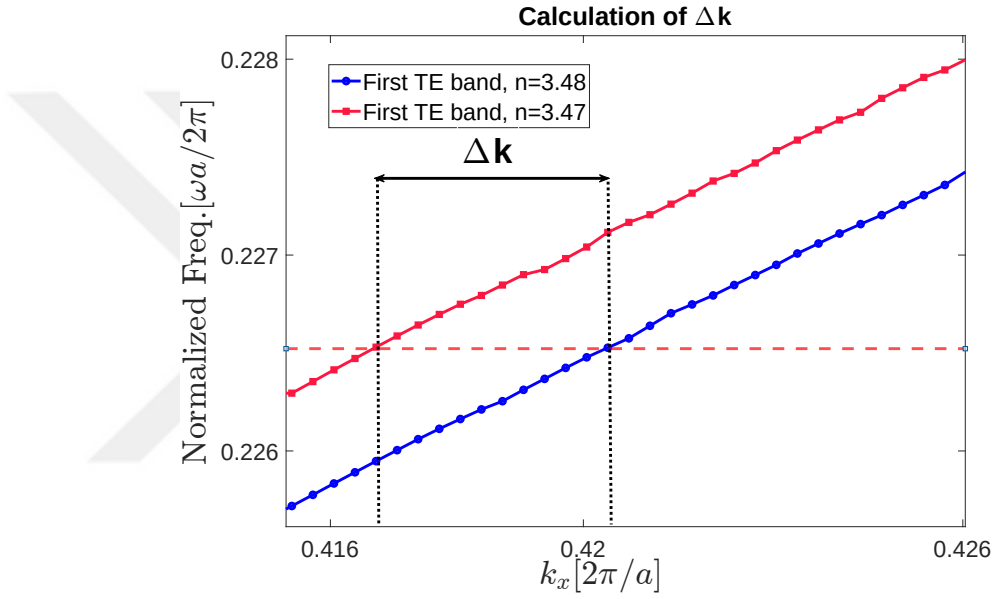


Figure 3.7: First TE bands of 1D PhCs with $n = 3.48$ and $n = 3.47$, enhanced view. Illustration of calculating Δk_x .

3.3 2D Photonic Crystals

Two-dimensional photonic crystals are structures in which the refractive index varies periodically in two spatial dimensions. A higher degree of control over light is achieved in 2D PhCs compared to 1D PhCs, due to additional degree of freedom. 2D PhCs guide light along arbitrary paths within the plane of the crystal, thanks to the additional dimension. In the third direction, which is also called out-of-plane direction, refractive index is uniform or varies slowly. Through this chapter, index in

this direction is assumed to be uniform. Some of the most common 2D configurations are square and triangular (hexagonal) lattices.

- **Square Lattice:** This is the simplest lattice configuration, where the unit cell is a square. This structure is isotropic in the plane, meaning its properties are the same in all in-plane directions. However, it typically has smaller band gaps compared to the triangular lattice. Representation of a square lattice PhC configuration, along with a band diagram of dielectric rods on air with Irreducible Brillouin Zone of square lattice is provided in Fig. 3.8.

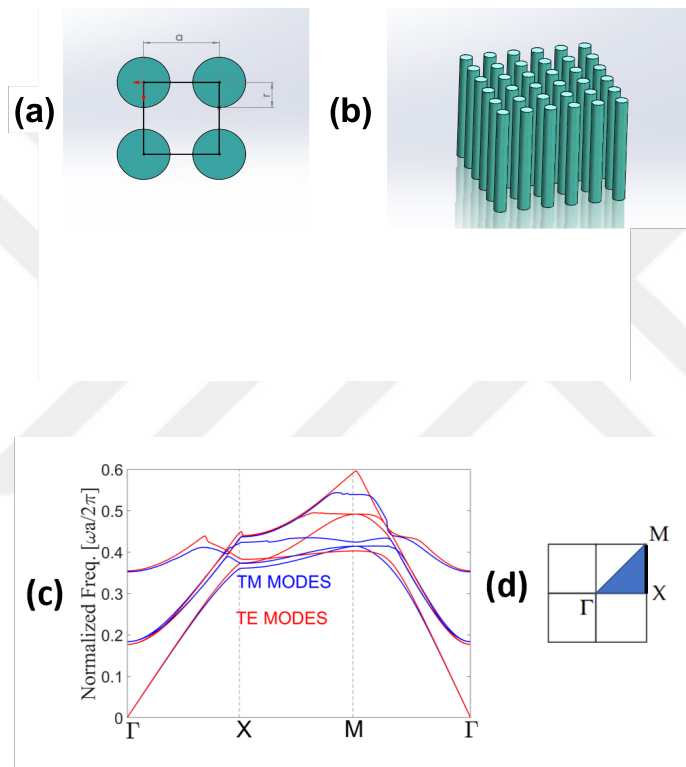


Figure 3.8: (a) Square lattice of dielectric rods on air, top view, where circles are dielectric rods. (b) Isometric view of dielectric rods on air. (c) Band diagram of the structure in (b). Red bands are for TE and blue bands are for TM polarization. (d) Irreducible Brillouin Zone of square lattice configuration.

- **Triangular (Hexagonal) Lattice:** This configuration is more complex, with a hexagonal unit cell. It is not isotropic, but it often exhibits larger and more robust band gaps, which can be beneficial for controlling light propagation.

SEM image of a fabricated chip's waveguide composed of triangular lattice of air holes is provided in Fig. 3.10. Band diagram of a 2D PhC with triangular lattice of air holes configuration, and its irreducible Brillouin Zone is provided in section 3.3.

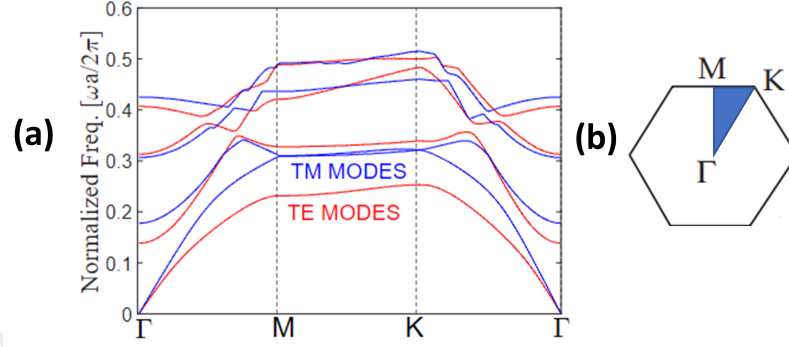


Figure 3.9: **(a)** Band diagram of a 2D PhC with triangular lattice of air holes configuration. Red bands represent TE and blue bands represent TM modes. **(b)** Irreducible Brillouin Zone of a triangular lattice formation.

One critical aspect to consider in 2D PhCs is the polarization of light. This is due to the fact that the response of the PhC to light is different, depending on whether the polarization is TE or TM.

Different configurations of 2D PhCs are used with different polarizations due to their symmetry properties. For instance, square lattice PhCs are often used with TE-polarized light, while triangular lattice PhCs are generally used with TM-polarized light.

In the case of TE polarized light, electric field is oriented parallel to the plane of layers (or rods, in some cases), and hence it is exposed to periodicity more directly. The changing refractive index from one layer (or rod) to the next effectively modulates the electric field, meaning strong interaction. This results in destructive interference for certain wavelengths, leading to the formation of band gaps, as mentioned previously.

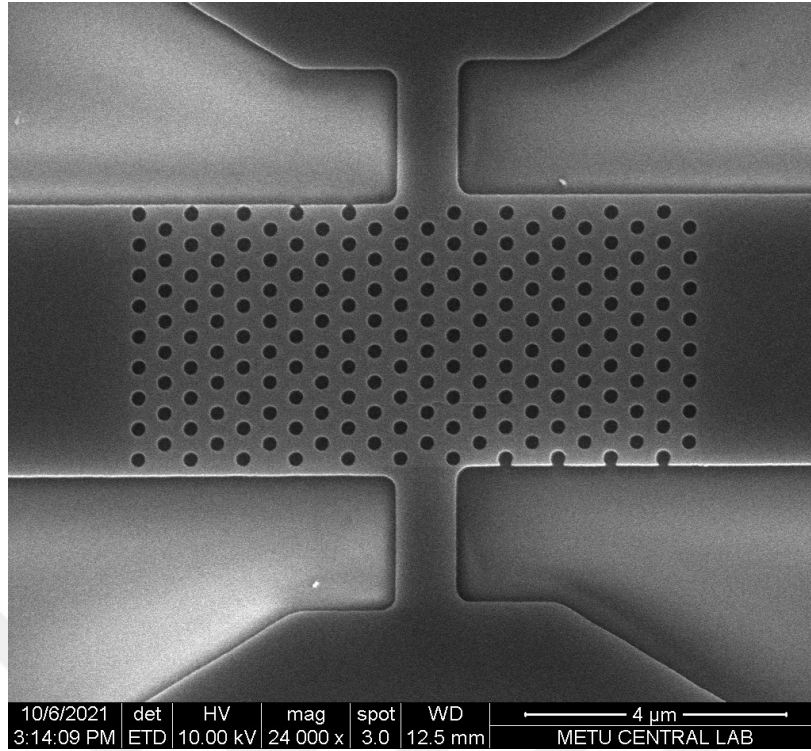


Figure 3.10: SEM image of a fabricated chip's waveguide composed of triangular lattice of air holes.

On the other hand, for TM polarized light, the electric field is perpendicular to the plane of the layers. Thus, it sees a kind of averaged refractive index. Thus, it is reasonable to say that it is not exposed to the periodicity as much as TE polarized light is. As a result, it does not interact strongly with the periodic changes. In this case, the electric field tends to be confined within regions of high refractive index, because light tends to stay where it travels slower, since higher refractive index implies slower speed of light. Because of the weaker interaction of electric field with periodicity, band gaps that are formed are typically smaller than the ones formed in the TE case [55].

CHAPTER 4

ANALYSIS AND DESIGN OF MZI MODULATOR WITH ONE DIMENSIONAL PHOTONIC CRYSTAL WAVEGUIDES

Length of an MZI is a crucial factor in on-chip designs. Traditional silicon based MZIs (Si-MZIs) are too long for integrated systems and thus the need to reduce the length arises. The motivation behind this study conducted on use of photonic crystals in MZI design is to address this issue. Moreover, theoretically showing that use of PhCs cannot only reduce device length but also provide operation bandwidth at lengths which silicon is unable to.

To reduce the loss and applied voltage to enable index shift in MZI arms, we aimed a bulk refractive index change of 0.01 for reduced loss, to minimize insertion loss while keeping device functionality [59, 60], as the loss and index change relation can be seen in eq. (2.15). Similarly, in all of the analyses, silicon bulk index change is taken as $\Delta n_{eff} = 0.01$, which is the difference between $n_{Si,on} = 3.48$ and $n_{Si,off} = 3.47$ where $n_{Si,on}$ is the refractive index when no voltage is applied to MZI arm and $n_{Si,off}$ is the index when a voltage is applied to one of the MZI arms to enable modulation by inducing 0.01 bulk index shift to create phase difference between MZI arms.

The trade-off between length (referring to phase shifter length, which is assumed equal to the device length) and bandwidth is investigated with the argument that PhCs can offer wide bandwidth, with the constraint of not operating near the band edge. Reason for this constraint is that slope of bands show dramatic shifts even at small changes in wave vector. This results in high deviations in effective index difference in small intervals, limiting the bandwidth significantly. Having said that, near band edge regions can provide high effective index differences provided that low bandwidth is not a problem for a specific application. However, operation does not have to be near

band edge to provide ultra-short devices with low bandwidths. There are numerous studies on ultra-short devices with a few nm bandwidths which do not necessarily operate near band edge, which is more preferable [61–63].

In this chapter, a detailed analysis on one-dimensional photonic crystals is conducted. This involves first obtaining the band diagram of the PhC, and through the diagrams, examining the effects of PhC parameters such as radius, width and slab height on the diagram. Then, calculating the effective index difference based on a given bulk index difference, which is also done from the information in band diagrams.

Secondly, extracted effective index difference spectrum is used to obtain the phase difference spectrum, which contains the information regarding possible operating bandwidths for a given extinction ratio requirement and a PCW length. This provides all the information one needs to know about the trade-off between extinction ratio and bandwidth, and device length and bandwidth. Note that in the calculations mentioned so far, a specific set of parameters (r , w , h , a) is assumed. The reasoning behind the choice of these parameters is explained later on.

After explaining the reasoning of choosing the PhC parameters, an MZI design is proposed, based on the analyses done up to that point. Proposed design is an MZI with 1D PCWs as phase shifters (PCW-MZI). To validate the operation of the proposed design, several investigations have been carried out. These involve changing both the overall device length and the length of the PCW independently. These length variations were then observed for their impact on the interference fringes, offering a deeper understanding of the design's behavior.

Furthermore, the Free Spectral Range (FSR) has been calculated for these variations, contributing to a comprehensive assessment of the MZI's performance characteristics. Importantly, the variations in FSR and fringes in response to changes in device and PCW lengths aligned with theoretical expectations, ensuring the effectiveness and viability of the proposed design.

4.1 Analysis of One Dimensional Photonic Crystal

4.1.1 Proposed Methods and Models

Before advancing with PhC analyses, one must first consider the bandwidth-device length plot of the conventional MZI with silicon waveguides. In Fig. 4.1, bandwidth for varying Si-MZI lengths is observed. It is seen that for lengths less than 54 μm , usage of silicon waveguides around 1550 nm is not feasible even for a low extinction ratio such as 5 dB.

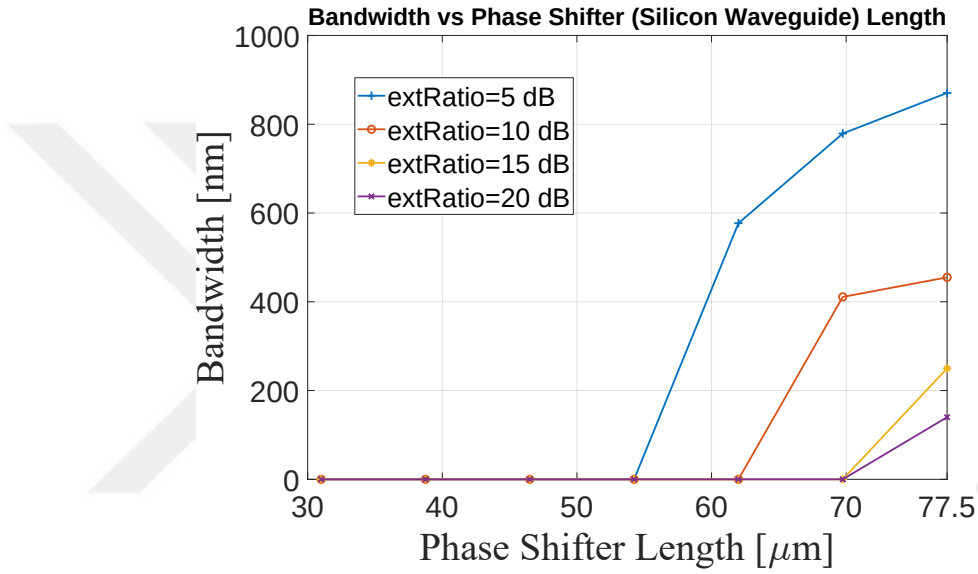


Figure 4.1: Bandwidth vs. Phase Shifter (Silicon Waveguide) Length.

To offer a solution to this problem, it will later on be shown that operation at lengths from 77.5 μm to 38.75 μm with photonic crystal waveguides is achievable. MZI consisting of PCWs, which is what is being analyzed in this study, is provided in Fig. 4.2.



Figure 4.2: Schematic of an MZI with Photonic Crystal Waveguides.

This remarkable improvement is facilitated by the incorporation of photonic crystal structures. The primary objective here, however, is not just length reduction. While designs offering up to 300 times enhancement in index difference exist, facilitating device lengths on the order of a few micrometers, they typically operate within narrow bandwidths [64]. Here, the motive for using PCWs is to ensure dependable operation across broad bandwidths. Notably, for the various lengths investigated, the lattice constant is adjusted to center the bandwidth around 1550 nm, since the wavelength is dependent on the lattice constant as seen in eq. (3.13). This reemphasizes the benefits of utilizing photonic crystal structures.

The forthcoming sections will discuss the effects of each parameter on the band diagram with the aid of numerous plots, highlighting the impact of parameter changes on the diagram. The air-hole radius's effect on the bandwidth will also be outlined. For simplicity, only the first bands will be shown in the figures, as the interest primarily and solely lies in the first TE band. However, to provide a broader understanding, several figures will represent multiple bands in the diagram.

In Fig. 4.3, band diagram of a one dimensional photonic crystal with $w = 1.2a$, $h = 0.710a$ and $r = 0.285a$ with silicon bulk refractive indices of $n = 3.47$ (on-state) and $n = 3.48$ (off-state) are provided. For simplicity, first two TE bands are depicted. Y-axis represents normalized frequency ω in units of $\omega a/2\pi$ and x-axis represents normalized wave vector with units of $2\pi/a$. In the diagram, it is seen that there exists a bandgap between the two bands, covering the frequency range $\omega = 0.2333$ to $\omega = 0.3081$. The difference between the bands of $n = 3.47$ and $n = 3.48$ is also clearly shown, which is the source of the effective index change between states.

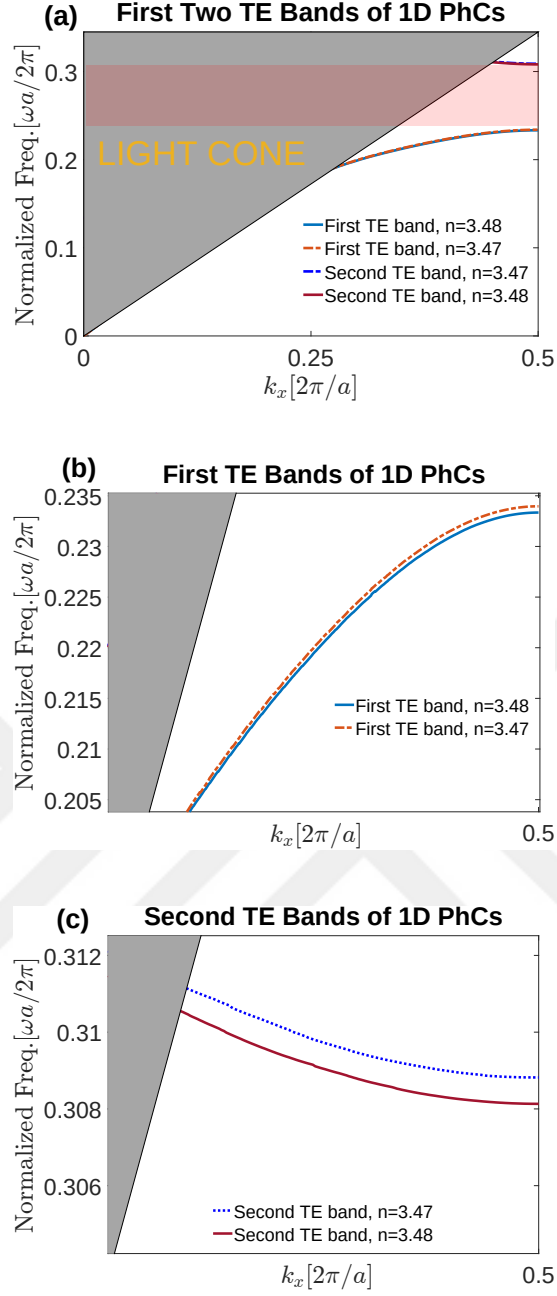


Figure 4.3: **(a)** First and second bands of on and off states. Gray triangle corresponds to light cone and red shaded area shows the bandgap between first and second TE bands. **(b)** Enlarged view, first bands of on and off states. **(c)** Enlarged view, second bands of on and off states.

First parameter of interest is the air-hole radius, denoted as r . As one can expect, from the effective index perspective, increasing the proportion of air in a air-dielectric

mix will result in shift of the effective index towards the index of air. Similarly, since PCW is made of silicon and air-holes, increasing the radius will yield a decrease in the effective index. This corresponds to an elevation in the band diagram, i.e., increase of frequency at an arbitrary k point compared to lower radius frequency, since effective index is inversely proportional to frequency at a specific k point. Impact of radius change on the first TE bands is depicted in Fig. 4.4. In these bands, w and h are kept constant at $1.2a$ and $0.710a$, respectively, and r is between $0.275a$ and $0.290a$.

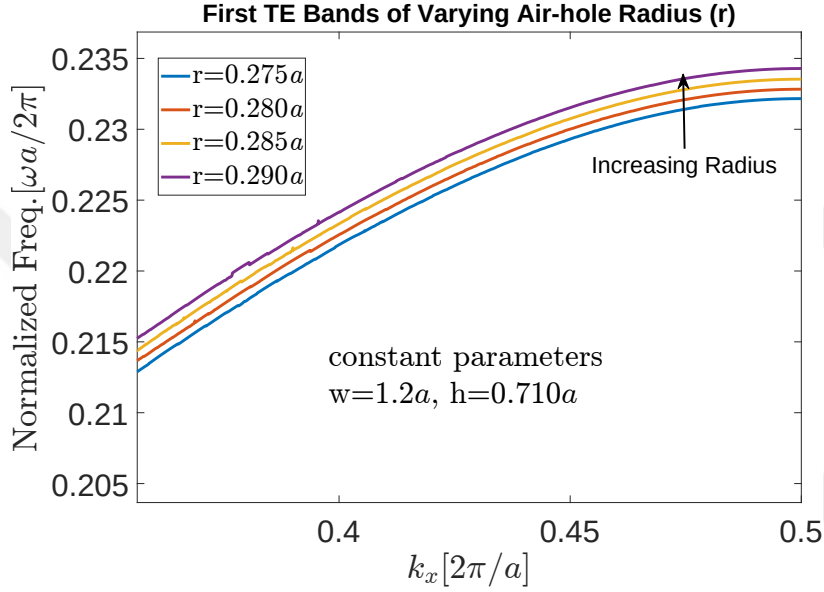


Figure 4.4: First TE Bands of Varying Air-hole Radius (r).

Second parameter of interest is the slab thickness, h . Thickness of silicon has the opposite effect of air-hole radius. Again, from the effective index perspective, there are three vertical layers, and these layers are air, silicon, and oxide, from top to bottom. Air layer is assumed infinite, oxide layer is $2\text{-}3\text{ }\mu\text{m}$, so the only variable in this situation is the silicon thickness. Increasing slab thickness will yield an increase in effective index. In this case, one can expect the frequency to decrease at an arbitrary k point compared to lower slab thickness, since effective index is inversely proportional to frequency at a specific k point. Impact of thickness change on first TE bands is depicted in Fig. 4.5. In these bands, r and w are kept constant at $0.285a$ and $1.2a$, respectively, and h is between $0.690a$ and $0.730a$.

The third parameter of interest is the photonic crystal width, w , and represents the transverse size of the photonic crystal waveguide. The effect of changing the PCW

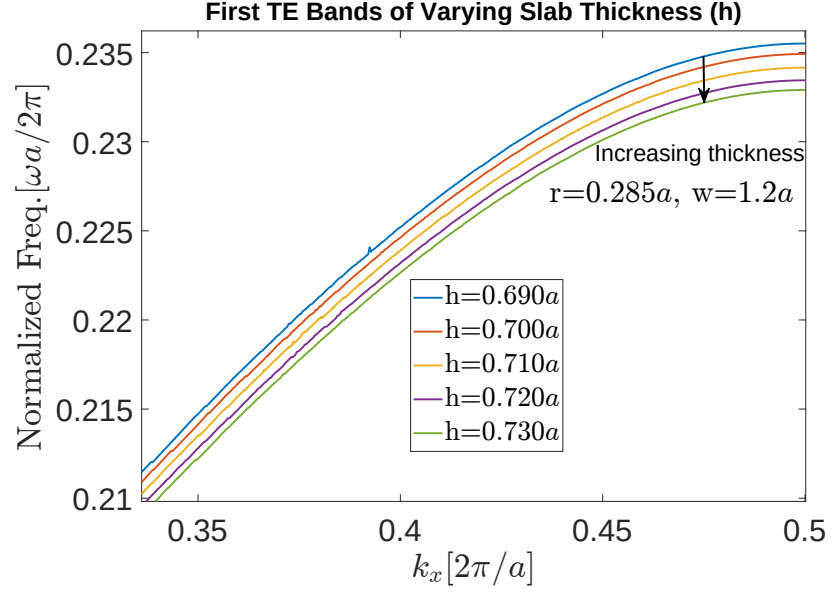


Figure 4.5: First TE Bands of Varying Slab Thickness (h).

width on the effective index and the band diagram is similar to the effect of changing the slab thickness. However, unlike the thickness which affects the vertical layering, PCW width affects the lateral structuring of the silicon and air holes. Change in width will result in similar changes in the band diagram with that of slab thickness. The impact of the width on first TE bands is depicted in Fig. 4.6. In these bands, r and h are kept constant at $0.285a$ and $0.710a$, respectively, and w is between $1.0a$ and $1.4a$.

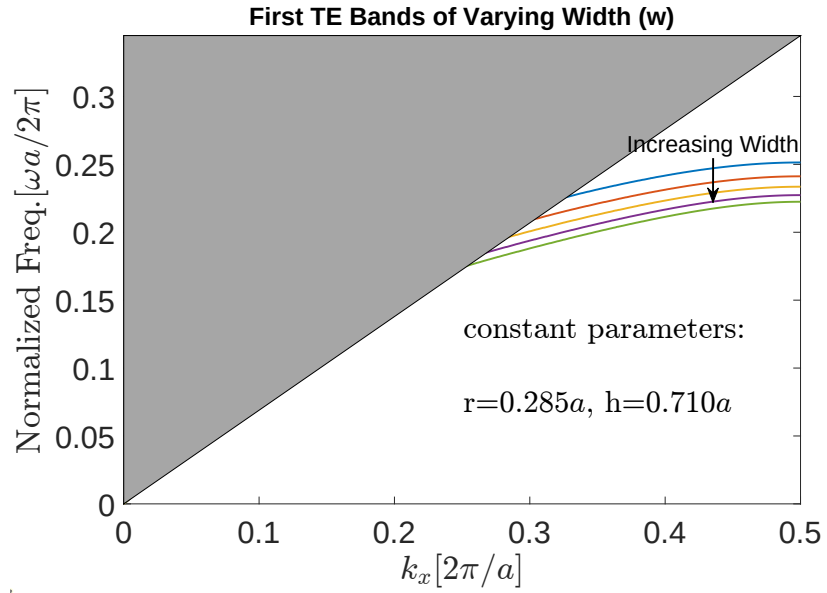


Figure 4.6: First TE Bands of Varying PCW Width (w).

From here on, until it is stated otherwise, PhC with $r = 0.285a$, $w = 1.2a$, and $h = 0.710a$ will be used in the analyses, with reasoning being explained later. For analyses, referring to interference formula eq. (2.26), effective index difference is used to derive the phase difference spectrum, which is given by

$$\phi = 2\pi \times \Delta n_{eff} \times L/\lambda \quad (4.1)$$

where ϕ is the phase difference between PCWs (thus phase shifters), λ is free space wavelength, and L is the phase shifter length along which fields propagate.

4.1.2 Analyses Results

Effective index difference spectrum between the first TE bands of 1D photonic crystals having refractive indices $n = 3.47$ and $n = 3.48$ is provided in Fig. 4.7. Spectrum is obtained by calculating the wave vector difference Δk_x and using eq. (3.14). Note that the x-axis is the normalized frequency, which will later be converted to wavelength axis using eq. (3.13).

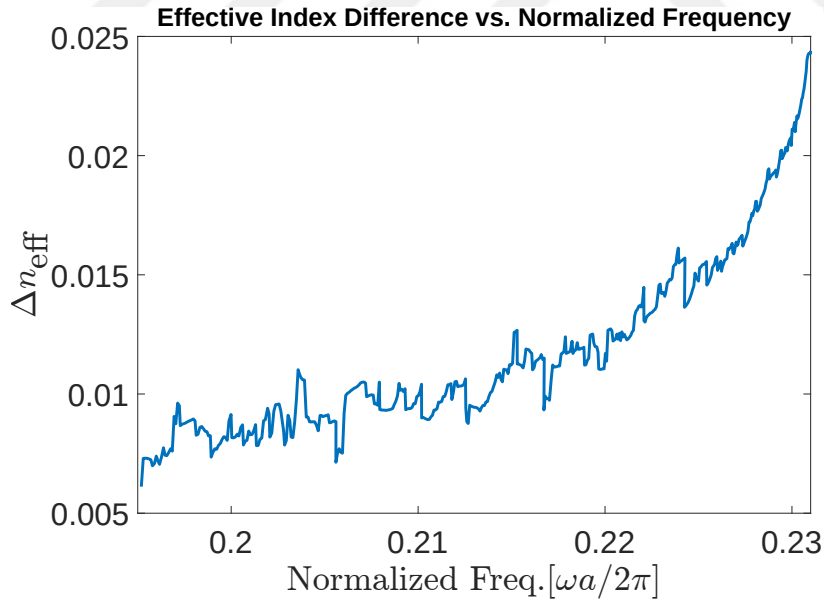


Figure 4.7: Effective index difference between on and off states for PCW with PhC parameters $r = 0.285a$, $h = 0.710a$, $w = 1.2a$.

Choosing a lattice constant a , effective index difference spectrum is obtained for varying wavelength intervals covering 1550 nm. Then, $\cos(\phi)$ is obtained for vari-

ous phase shifter lengths (L), and resulting spectrum is analyzed to determine regions where modulation is possible for specific extinction ratios, using eq. (4.1).

Modulation term ($\cos(\phi)$) values for specified extinction ratio values (dB) are obtained by simply using logarithmic conversion

$$\text{extinction ratio} = 10^{(-\text{extRatio}_{\text{dB}}/10)} - 1 \quad (4.2)$$

where extinction ratio on LHS is in decimals. For this analysis, extinction ratio values are chosen as 5, 10, 15, and 20 dB, which implies that the modulation term $\cos(\phi)$ should be -0.6837, -0.9000, -0.96837, and -0.99000, respectively, based on eq. (4.2).

As mentioned in Chapter 2, conventional silicon modulators need to be at least $77.5 \mu\text{m}$ long. Taking this value as initial ($1L$) length, possible bandwidth of $1L$ to $0.4L$ long MZI with PhC waveguides (PCW-MZI) as phase shifters is inspected. Cosine of the phase difference between the MZI arms is provided for the specified lengths.

A detailed version of this analysis is provided in Fig. 4.9, based on $0.9L$ ($69.75 \mu\text{m}$) long device with $a = 0.33272 \mu\text{m}$ showing calculated $\cos(\phi)$ spectrum and calculation steps. Four levels corresponding to four specified extinction ratios are provided as horizontal lines, with colors black, cyan, purple, and blue, respectively. Bandwidths for these extinction ratios are also provided in the same coloring, where vertical dashed lines of the same color represent bandwidth corresponding to an extinction ratio level. Note that in succeeding figures, horizontal lines are not colored or provided with dB correspondence as texts on top of them.

As one can see in Fig. 4.8, since the data is naturally noisy and has inherent numerical errors, smoothing would provide a clearer picture of the spectrum. Smoothing is done by applying a moving average filter to the data in MATLAB. It should be noted that calculations are based on both smoothed and raw data, to prevent possible errors. Moreover, when determining the bandwidth, since the expected spectrum is a cosine derivative, unlikely behaviour such as impulsive changes are considered as numeric errors where needed. This is necessary to make a correct analysis since the simulations which are run to obtain the data use numerical methods and algorithm. Shaded regions in Figs. 4.11 and 4.12 show how this assumption is applied to the spectra.

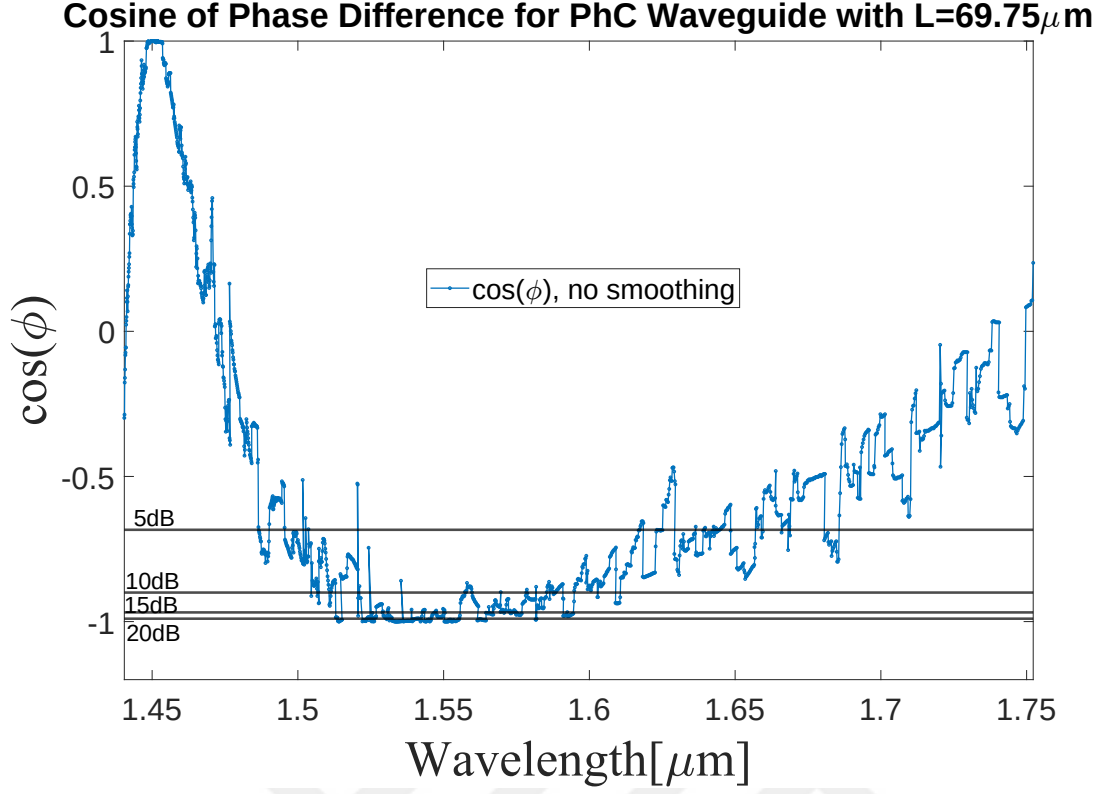


Figure 4.8: Cosine of phase difference of PCW with $r = 0.285a$, $h = 0.710a$, $w = 1.2a$, and $a = 0.33272\mu\text{m}$.

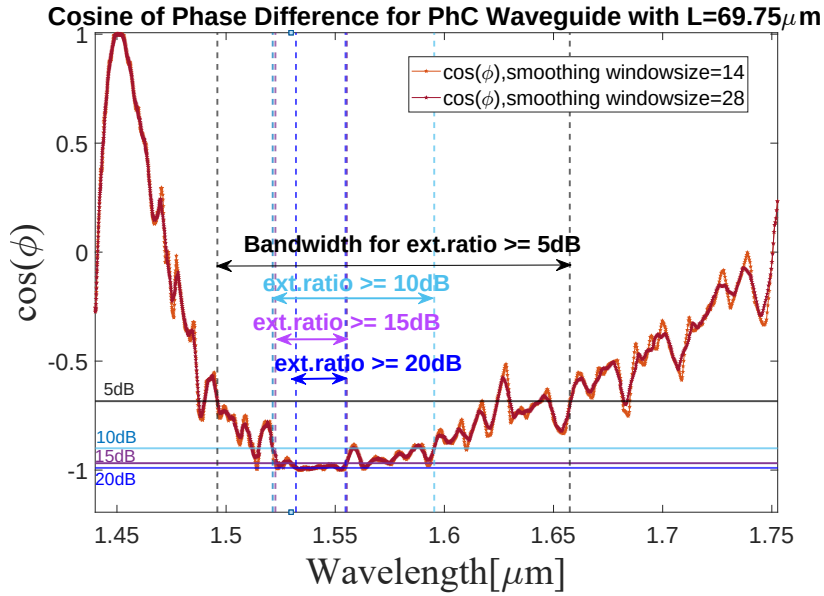


Figure 4.9: Cosine of phase difference of PCW with $r = 0.285a$, $h = 0.710a$, $w = 1.2a$, and $a = 0.33272\mu\text{m}$, after moving average filter is applied. Filtering is done with window size= 14, 28.

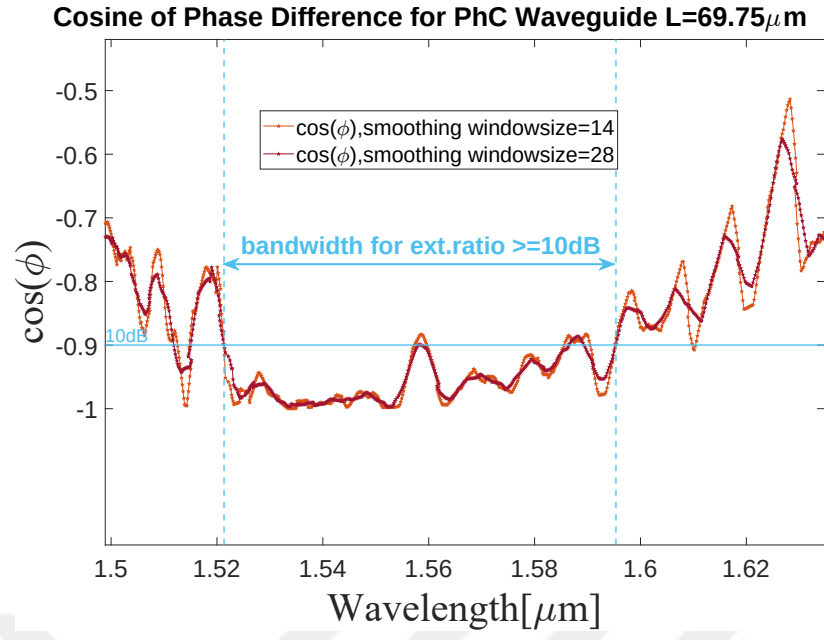


Figure 4.10: Region chosen as bandwidth for extinction ratio ≥ 10 dB.

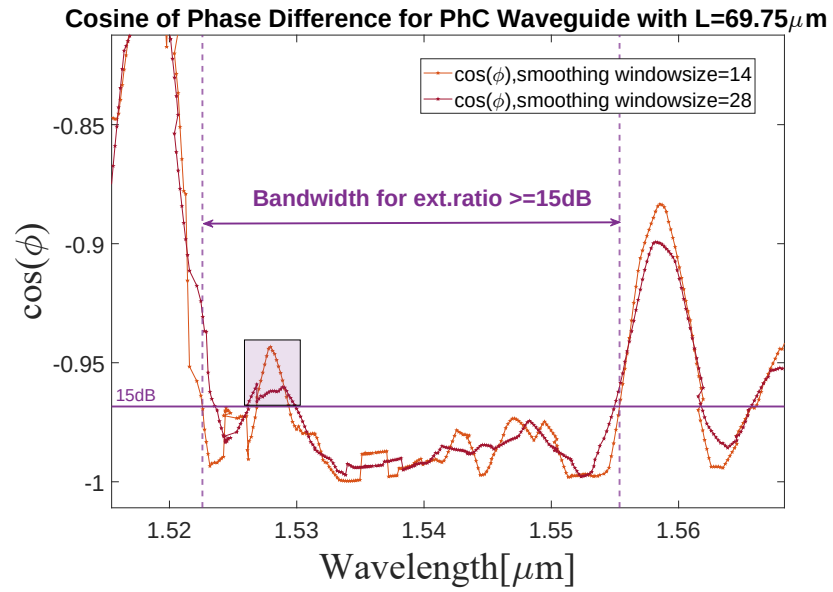


Figure 4.11: Region chosen as bandwidth for extinction ratio ≥ 15 dB.

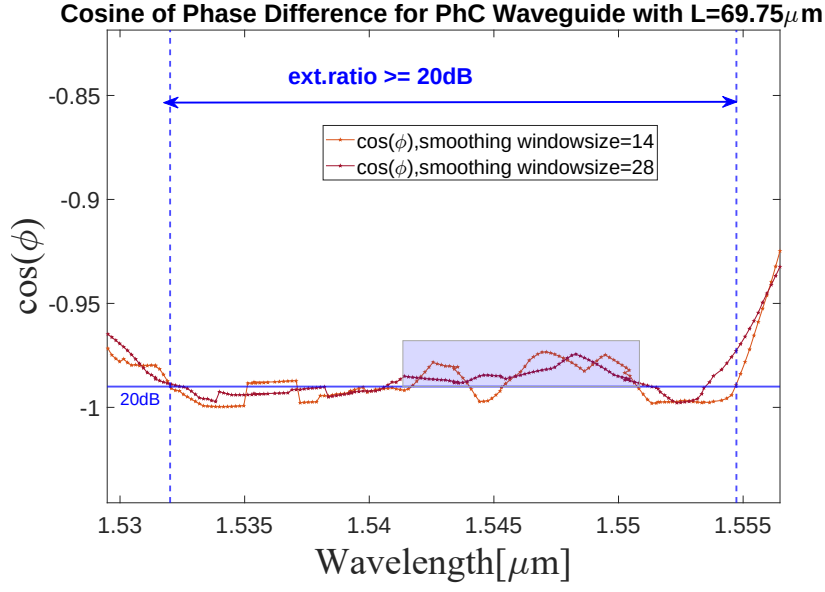


Figure 4.12: Region chosen as bandwidth for extinction ratio ≥ 20 dB.

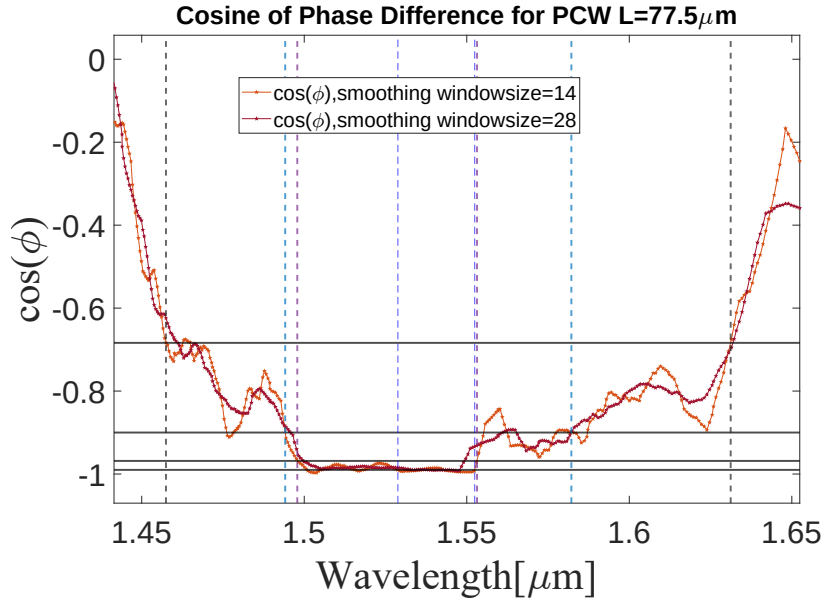


Figure 4.13: Cosine of phase difference of PCW with $r = 0.285a$, $h = 0.710a$, $w = 1.2a$, and $a = 0.31994\mu\text{m}$.

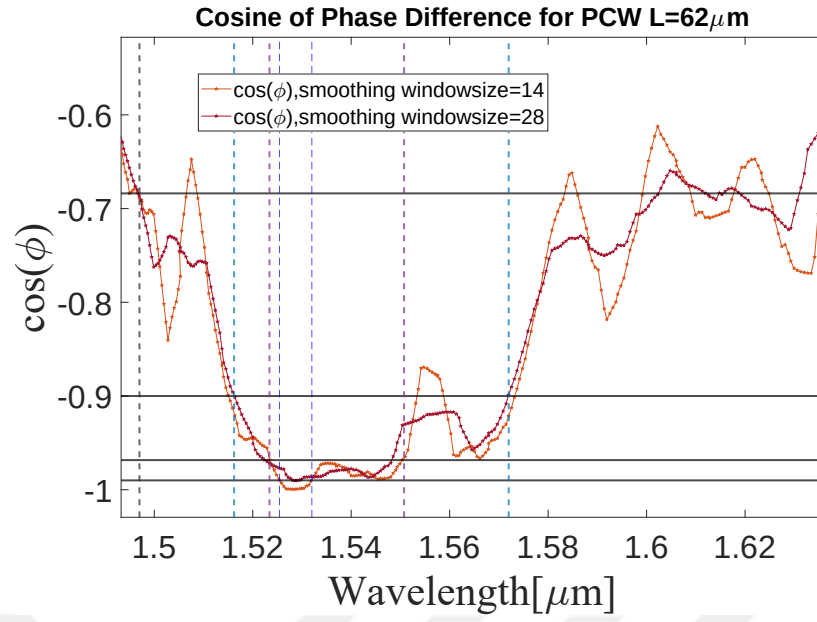


Figure 4.14: Cosine of phase difference of PCW with $r = 0.285a$, $h = 0.710a$, $w = 1.2a$, and $a = 0.33680 \mu\text{m}$.

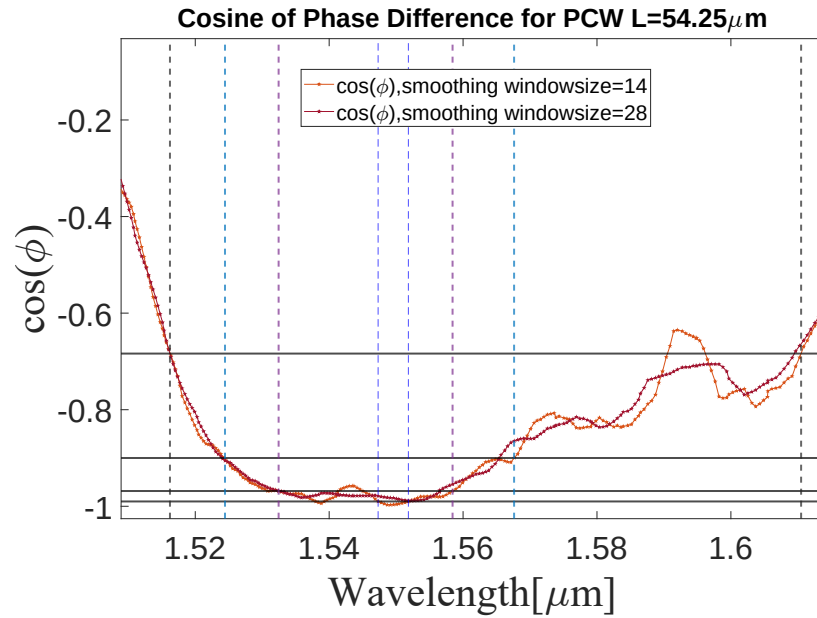


Figure 4.15: Cosine of phase difference of PCW with $r = 0.285a$, $h = 0.710a$, $w = 1.2a$, and $a = 0.34487 \mu\text{m}$.

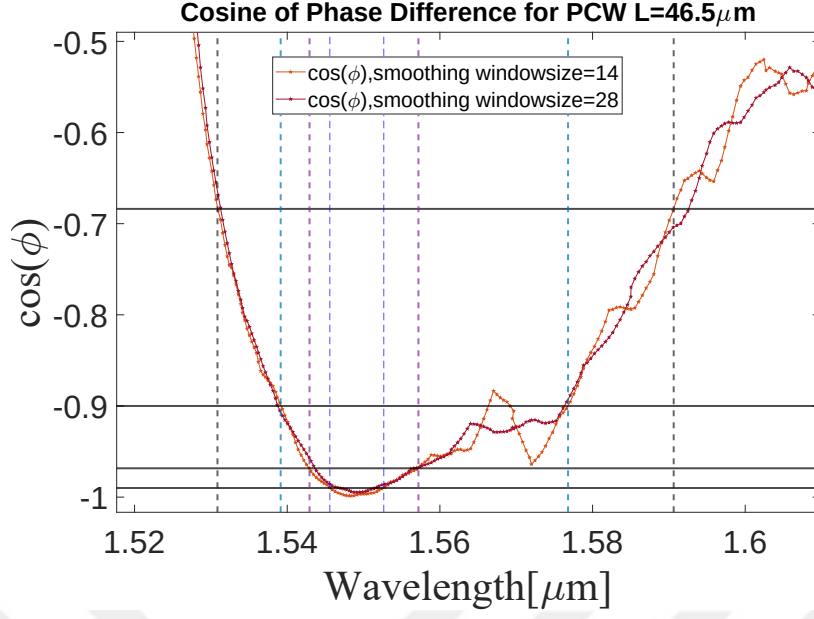


Figure 4.16: Cosine of phase difference of PCW with $r = 0.285a$, $h = 0.710a$, $w = 1.2a$, and $a = 0.35120 \mu\text{m}$.

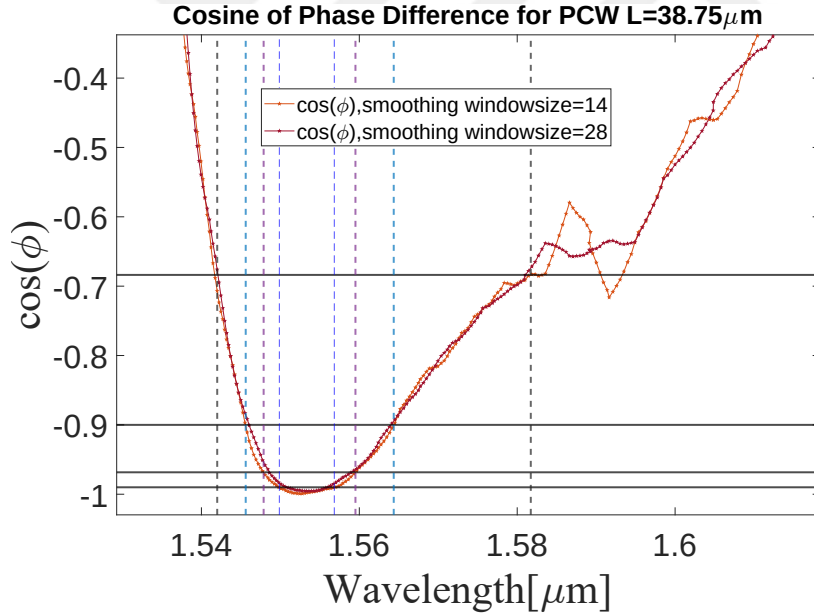


Figure 4.17: Cosine of phase difference of PCW with $r = 0.285a$, $h = 0.710a$, $w = 1.2a$, and $a = 0.35559 \mu\text{m}$.

Based on the bandwidth calculations provided so far, below are the plot and table of bandwidth for a PCW-MZI in Fig. 4.18 and Table. 4.1 for different PCW lengths with PhC parameters $r = 0.285a$, $h = 0.710a$, $w = 1.2a$, and $a = 351.2 \text{ nm}$. It is seen that

from 77.5 to 38.75 μm , the bandwidth is never zero with PCW-MZI, unlike Si-MZI.

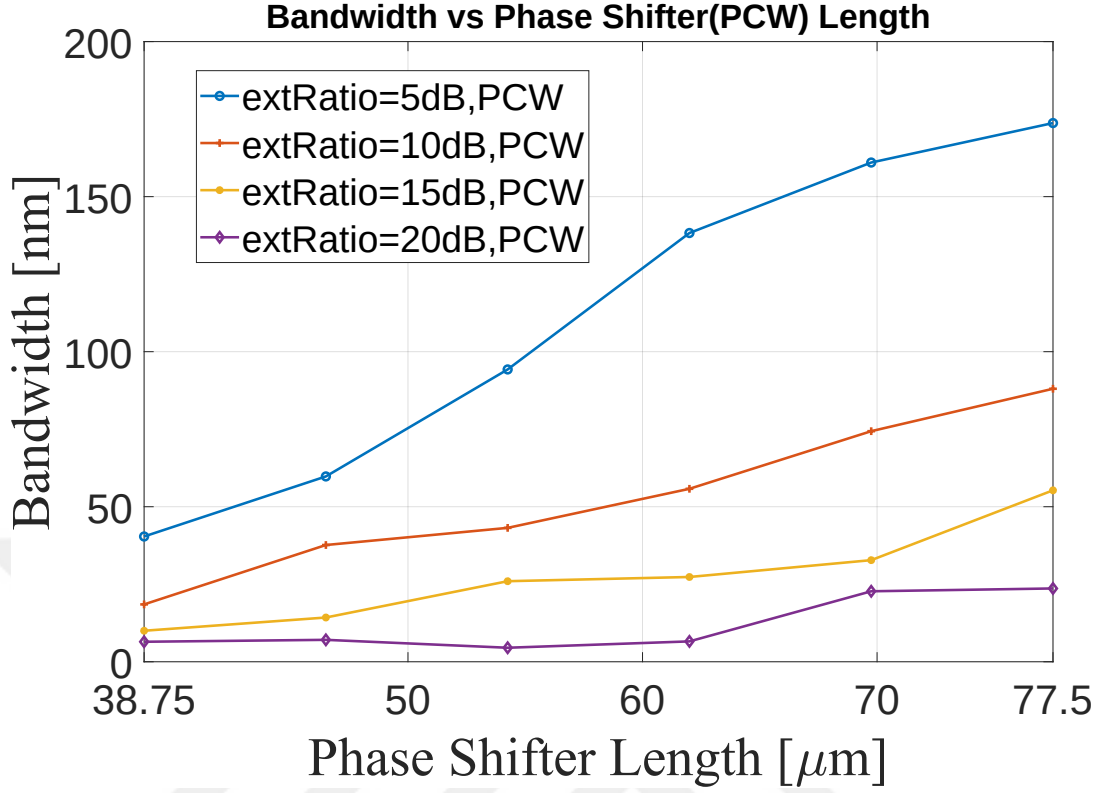


Figure 4.18: Bandwidth vs. PCW length. PCWs are with refractive indices $n = 3.48$ and $n = 3.47$, with $r = 0.285a$. Bandwidths calculated for four different extinction ratios are plotted with different colors and markers, as shown in the legends.

Table 4.1: Bandwidth vs ext. ratio for various PS lengths, with $r = 0.285a$.

Ext. Ratio / L_{ps}	77.5 μm	69.75 μm	62.0 μm	54.25 μm	46.5 μm	38.75 μm
5 dB	173.73 nm	161.00 nm	138.27 nm	94.27 nm	59.77 nm	40.4 nm
10 dB	88.04 nm	74.36 nm	55.78 nm	43.18 nm	37.65 nm	18 nm
15 dB	55.29 nm	32.77 nm	27.34 nm	25.98 nm	14.27 nm	10.0 nm
20 dB	23.64 nm	22.73 nm	6.57 nm	4.52 nm	7.07 nm	6.45 nm

Now that bandwidth comparison is possible between Si-MZI and PCW-MZI, let us see the bandwidths of both types on top of each other. In Fig. 4.19, comparison of Fig. 4.18 with Fig. 4.1, for 10 dB extinction ratio is provided, clearly showing the superiority of PCW-MZI design for shorter device lengths. In Fig. 4.20, an enhanced view of the same comparison is provided, with higher scaling in the y-axis.

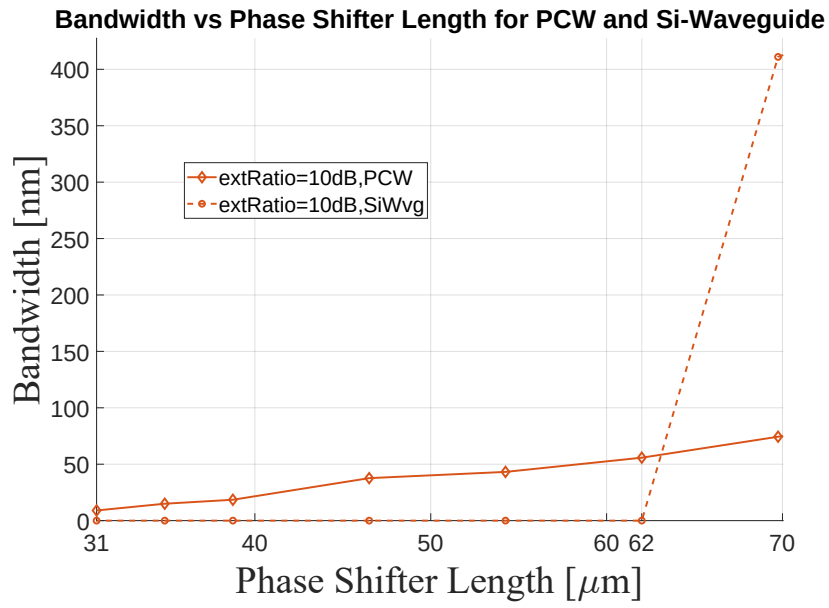


Figure 4.19: Bandwidth vs. Phase Shifter Length for Si-Waveguide and PCW.

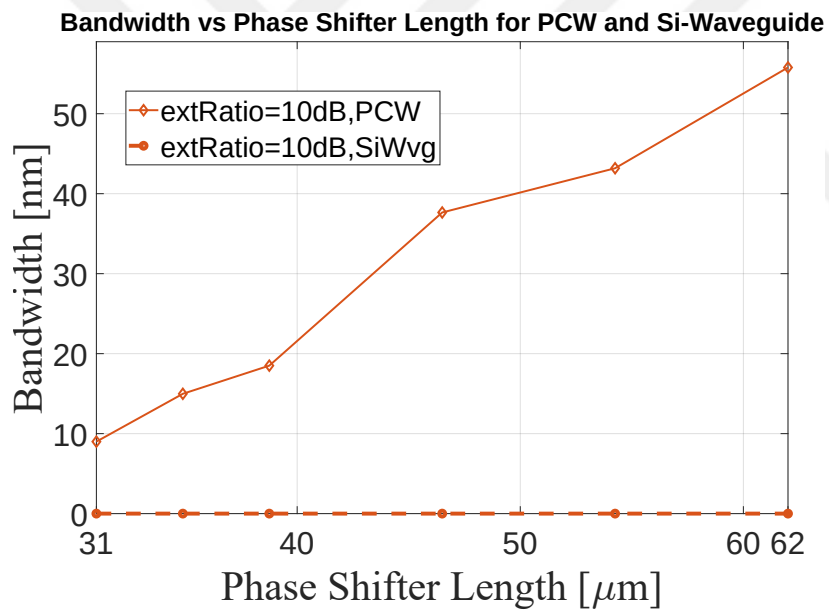


Figure 4.20: Bandwidth vs. Phase Shifter Length for Si-Waveguide and PCW, enhanced view.

The advantages of photonic crystal waveguides over silicon waveguides have been thoroughly discussed and demonstrated until now. The subsequent focus lies on detailing the rationale behind choosing the parameters of the air hole radius, waveguide width, and slab thickness as $0.285a$, $1.2a$, and $0.710a$, respectively. This selection process is iterative, commencing from a starting point of $r = 0.3a$, $h = 0.6a$, and $w = 1.0a$. The starting choice of a slab thickness around $0.6a$ is inspired by its frequent occurrence in simulations and designs. The radius, $0.3a$, aims to enhance the precision of numerical calculations due to a larger air region containing more pixels, thereby enabling moderate resolution values to yield accurate results. This stems from the working principle of numerical algorithms that divide the simulation environment into grids, the number of which is proportional to the resolution value chosen by the user. A higher resolution usually implies higher accuracy, reaching a saturation point beyond which the results will not change significantly, if it does at all. The choice of width is influenced by the radius size. The minimum unit-cell width should be large enough to encompass the air hole, which has a diameter of $0.6a$, while the maximum value is determined by the single-mode condition. An excessively large width would result in multi-mode operation, which is undesirable. Hence, a value greater than $0.6a$ and moderate enough was determined as $1.0a$, defining the starting point for the width.

To find the optimized parameters for PhC, bandwidth calculations were conducted on PCWs with varying air-hole radii while maintaining constant w and h values at $1.2a$ and $0.710a$, respectively. Bandwidth calculations were done based on 5, 10, 15, and 20 dB extinction ratios, and these calculations were done at different PCW lengths, to fully depict a picture to show that different radii may be optimal for different device length requirements. For a constant PCW length, radius-bandwidth relation is acquired, and the same procedure is done with a different length, and so on.

The radius-bandwidth relation where the PCW length is $L = 54.25 \mu\text{m}$ (corresponding to a %30 reduction in a conventional MZI), is demonstrated in Fig. 4.21. From these findings, it is evident that a radius of $r = 0.285a$ delivers the maximum bandwidth at 10 and 15 dB extinction ratios, whereas $r = 0.290a$ leads to the highest bandwidth at a 20 dB extinction ratio.

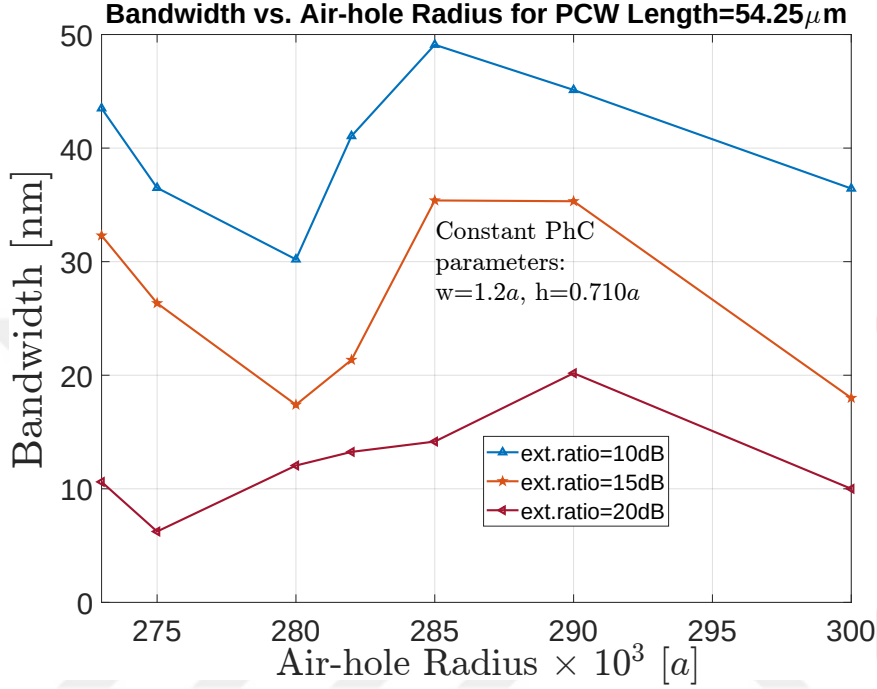


Figure 4.21: Bandwidth vs. Air-hole Radius (r) for extinction ratios 10, 15, and 20 dB for PCW length $L = 54.25 \mu\text{m}$.

Following the same process for a PCW length of $L = 46.5 \mu\text{m}$ (equivalent to a 40% reduction in a conventional MZI) as illustrated in Fig. 4.22, it becomes clear that $r = 0.273a$ provides the maximum bandwidth at a 10 dB extinction ratio. Similarly, $r = 0.282a$ and $r = 0.285a$ result in the highest bandwidth at 15 and 20 dB respectively. An 8.71% difference is observed between $r = 0.282a$ and $r = 0.285a$ at the 15 dB extinction ratio. Therefore, we can infer that $r = 0.285a$ dominates once again, mirroring its role with $L = 54.25 \mu\text{m}$.

Proceeding to a PCW length of $L = 38.75 \mu\text{m}$, (a 50% reduction in conventional MZI) as shown in Fig. 4.23, it is determined that $r = 0.282a$ provides the greatest bandwidth at a 10 dB extinction ratio. However, at 15 and 20 dB extinction ratios, $r = 0.280a$ and $r = 0.285a$ respectively deliver the highest bandwidth.

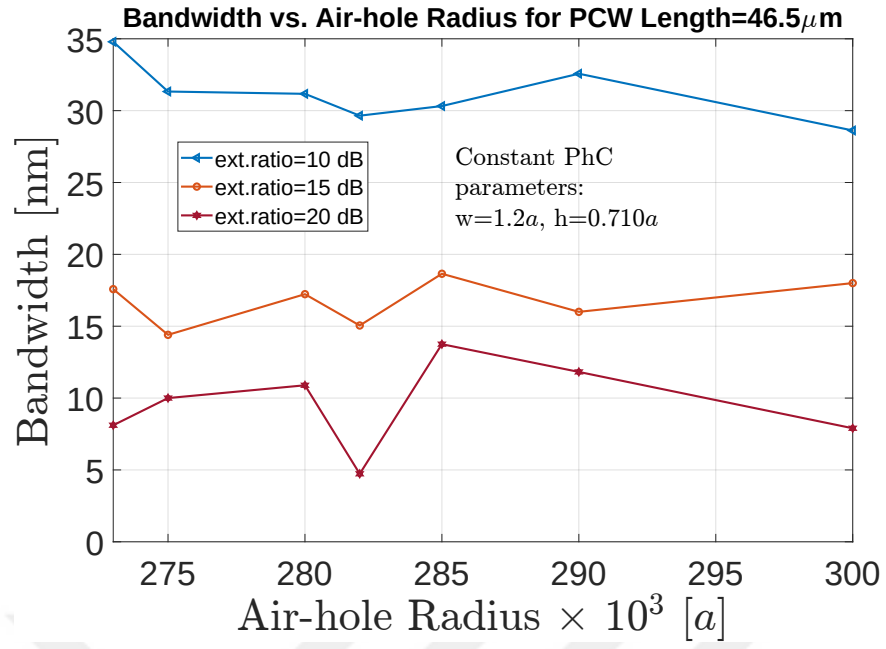


Figure 4.22: Bandwidth vs. Air-hole Radius (r) for extinction ratios 10, 15, and 20 dB for PCW length $L = 46.5 \mu\text{m}$.

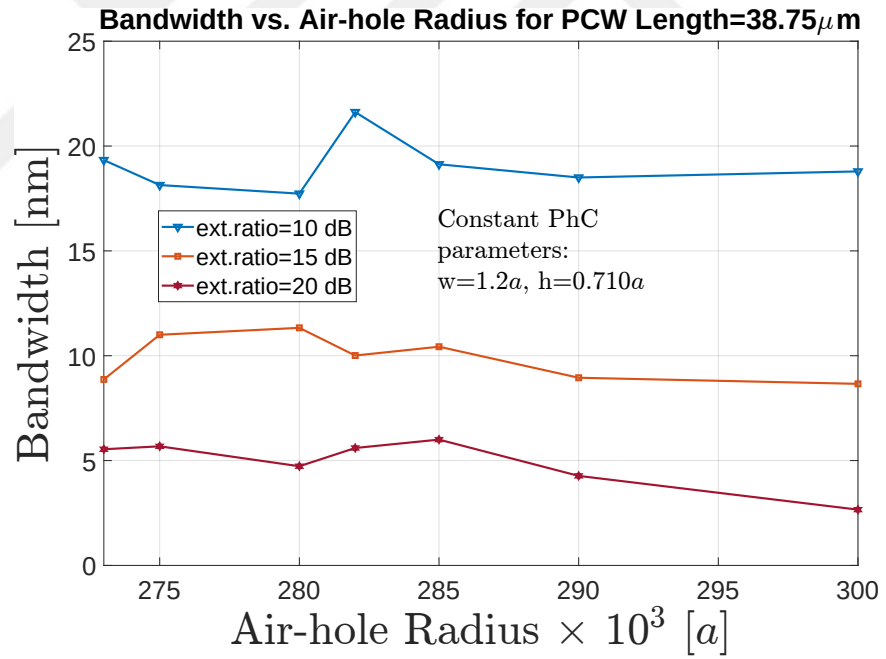


Figure 4.23: Bandwidth vs. Air-hole Radius (r) for extinction ratios 10, 15, and 20 dB for PCW length $L = 38.75 \mu\text{m}$.

Observing the radius impact on bandwidth for different lengths and extinction ratios, it is not possible to declare an "optimum" radius value for all lengths and extinction ratios. It is seen that the optimum radius is subject to change, depending on other constraints. Hence, one needs to specify remaining device parameters before moving on with optimizing the radius.

For slab thickness and waveguide width, these values will more likely not be subject to too precise optimization, i.e., 0.001 resolution is unlikely when one optimizes these values. Moreover, in fabrication, slab thickness will be usually either 220 nm or 260 nm, not applicable for too precise optimization. Thus, one will need to set a thickness before optimizing the radius anyway. As for the waveguide width, to have a tighter confinement, it is better to keep the width rather not too large, and of course to avoid multi-mode operation also. A moderate optimization can be performed on waveguide width, but determining the parameter still comes down to optimization of radius value.

4.2 Proposed Design

Analyzing the impact of radius on bandwidth, we decided that $r = 0.285a$ would yield more accurate results since its' dominance is more frequent for different device lengths and extinction ratios, compared to other radii values. To support this argument, bandwidth vs. air-hole radius for at 5 dB extinction ratio is provided in Fig. 4.24. This figure was separated from others with higher extinction ratios because it would decrease the resolution in y-axis due to having significantly higher bandwidth values. Then, keeping h and w as their previous values used in analyses ($0.710a$ and $1.2a$, respectively) inspecting the effective index difference spectrum of 1D PhC with $r = 0.285a$, $w = 1.2a$, $h = 0.710a$, and $a = 351.2$ nm, the Δn_{eff} at 1550 nm is calculated to be 0.01611. Consequently, the target device length was set to $L = 46.5$ μm , calculated via eq. (3.14). After the band diagram analyses through MPB, the initial goal involved using MEEP to locate the length which yielded a π radian phase difference between MZI arms. Therefore, electric field distributions for the E_y component were obtained at various lengths ranging from 30 to 60 μm , focusing on the targeted length. At this juncture, 15 μm offset was tolerated due to potential numerical errors possibly resulting from shifting simulation algorithms, from MPB

to MEEP.

In these simulations, it was discovered that the MZI with a phase shifter length of $38.0664 \mu\text{m}$ made of photonic crystal waveguide with PhC parameters $r = 0.285a$, $h = 0.710a$, $w = 1.2a$, $a = 351.2 \text{ nm}$ met the requisite phase difference of π radian. Electric field distribution at the input of phase shifters and at the output can be seen in Fig. 4.26a and Fig. 4.26b, respectively. Based on what was analyzed so far, in this study, an MZI design that incorporates 1D PCW with PhC parameters $r = 0.285a$, $h = 0.710a$, $w = 1.2a$, $a = 351.2 \text{ nm}$ as phase shifters is proposed, and the basic schematic of the device is provided in Fig. 4.25. Parameter choice is based on the impact of parameters on band diagrams and thus bandwidth and effective index difference, which has been discussed in detail so far, with clear depiction on radius choice especially.

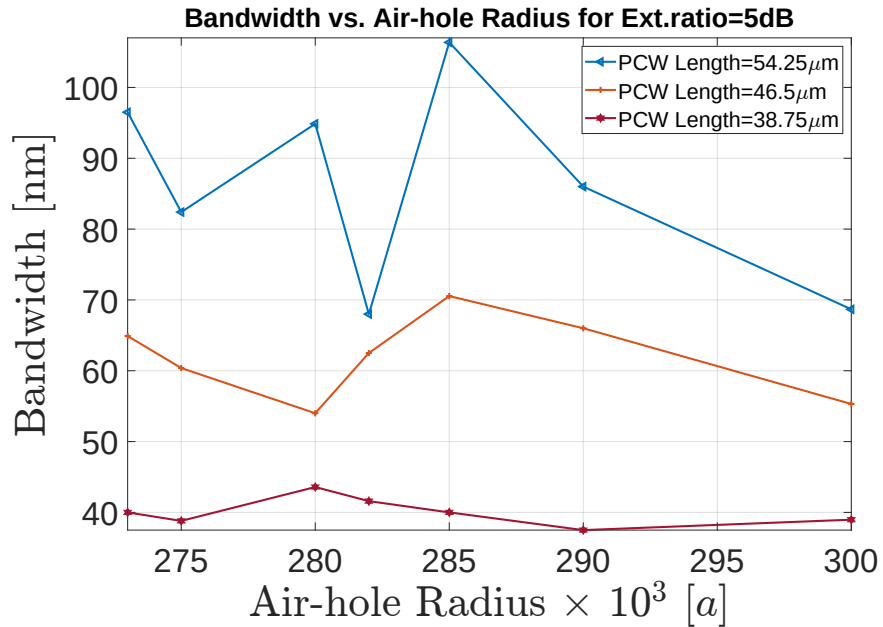


Figure 4.24: Bandwidth vs. Air-hole Radius (r) with 5 dB extinction ratio for PCW lengths $L = 54.25 - 38.75 \mu\text{m}$.

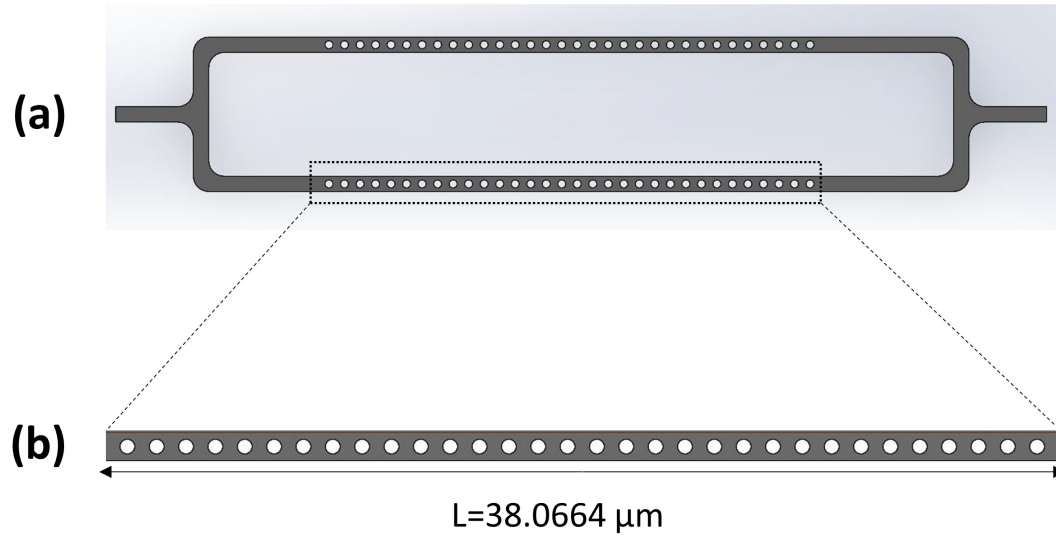


Figure 4.25: **(a)** Schematics of proposed MZI, top view. **(b)** Photonic Crystal Waveguide used in the proposed design with length $L = 38.0664 \mu\text{m}$. Number of air-holes are representative.

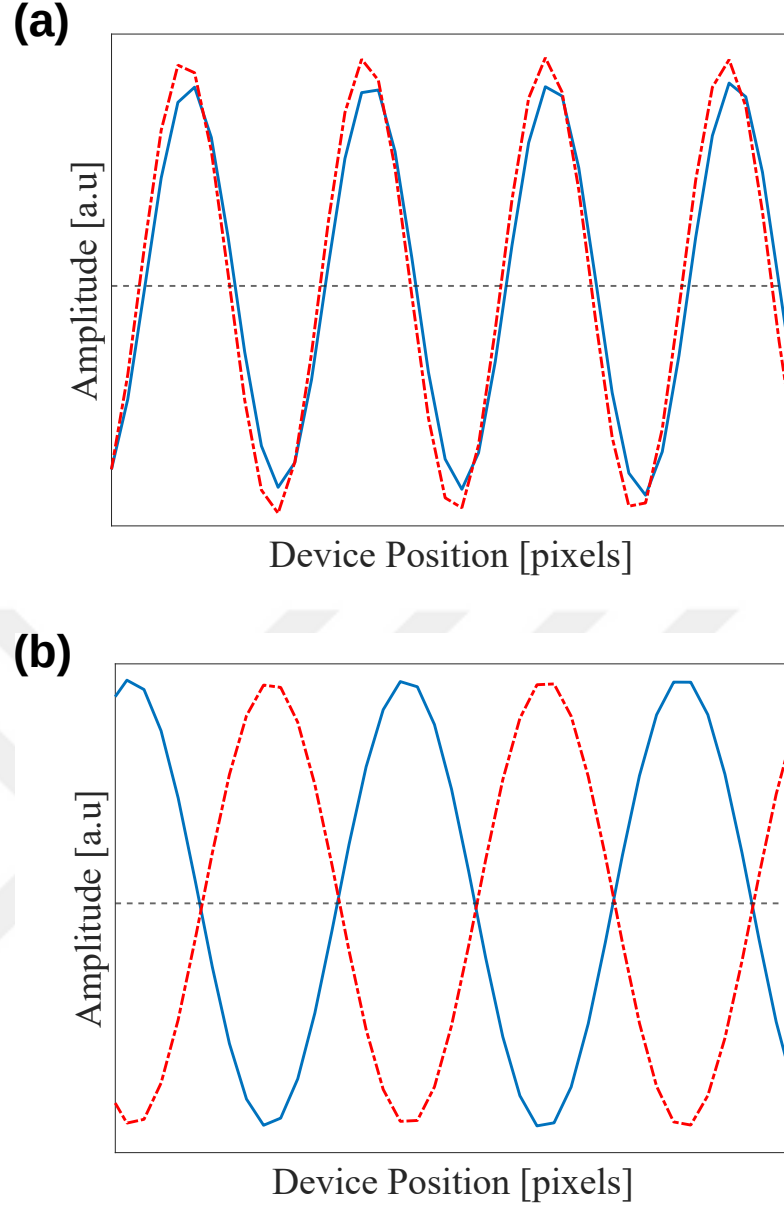


Figure 4.26: **(a)** Electric field distribution at the input of phase shifters with length $L = 38.0664 \mu\text{m}$. Waves are in phase. **(b)** Electric field distribution at the output of phase shifters with length $L = 38.0664 \mu\text{m}$. Waves are out of phase.

Next step was to obtain transmission spectra for different device lengths, to observe the number of fringes and calculate FSR values. To do so, let us first start with length at which π radian phase shift was observed between phase shifter lengths, which is $38.0664 \mu\text{m}$, which will be denoted as L_{ps} from now on. When running simulations, there are two variables to consider, phase shifter length and PCW length. Phase shifter

length does not have to be equal to PCW length, and change in PS length for constant PCW length should not make a difference in transmission spectrum, ideally.

Secondly, FSR values of different PCW lengths will be compared to proposed design's FSR value, which is provided after transmission spectra. Theoretical FSR value is calculated for a device with PCW length of L_x is calculated as

$$FSR_{L_x} = (L_{ps} \times FSR_{L_{ps}}) / L_x \quad (4.3)$$

where L_{ps} is $38.0664\mu\text{m}$. For instance, FSR of MZI with PCW length $0.5L$ and PS length $1L$ should be double of FSR of MZI with PCW length L and PS length L since

$$FSR_{0.5L} = (1L \times FSR_{1L}) / 0.5L = 2FSR_{1L}. \quad (4.4)$$

In the transmission spectra, we expect to see an increase in the frequency of oscillations as the photonic crystal waveguide length increases. This expectation is essentially a compression of the spectrum along the wavelength or frequency axis, often described as the spectrum being "squeezed" in the x-axis.

This trend holds true irrespective of the phase shifter length, and it further implies a decrease in the FSR as the PCW length increases. This expectation is logical as more tightly spaced oscillations suggest that there are more fringes within a given wavelength interval, and FSR is inversely proportional to the number of fringes.

To make these verifications, normalized transmission spectra of PCW-MZI designs with various PCW lengths and constant PS length in the interval 1510-1570 nm are provided in Fig. 4.27 and Fig. 4.28. In Fig. 4.27, PS length is kept constant at L_{ps} , and PCW length varies between $0.5L_{ps}$ and $1L_{ps}$. Next, in Fig. 4.28, PS length is kept constant at $2L_{ps}$ and PCW length varies between $1L_{ps}$ and $2L_{ps}$. With increasing PCW length, the spectra are tightening horizontally, in both figures.

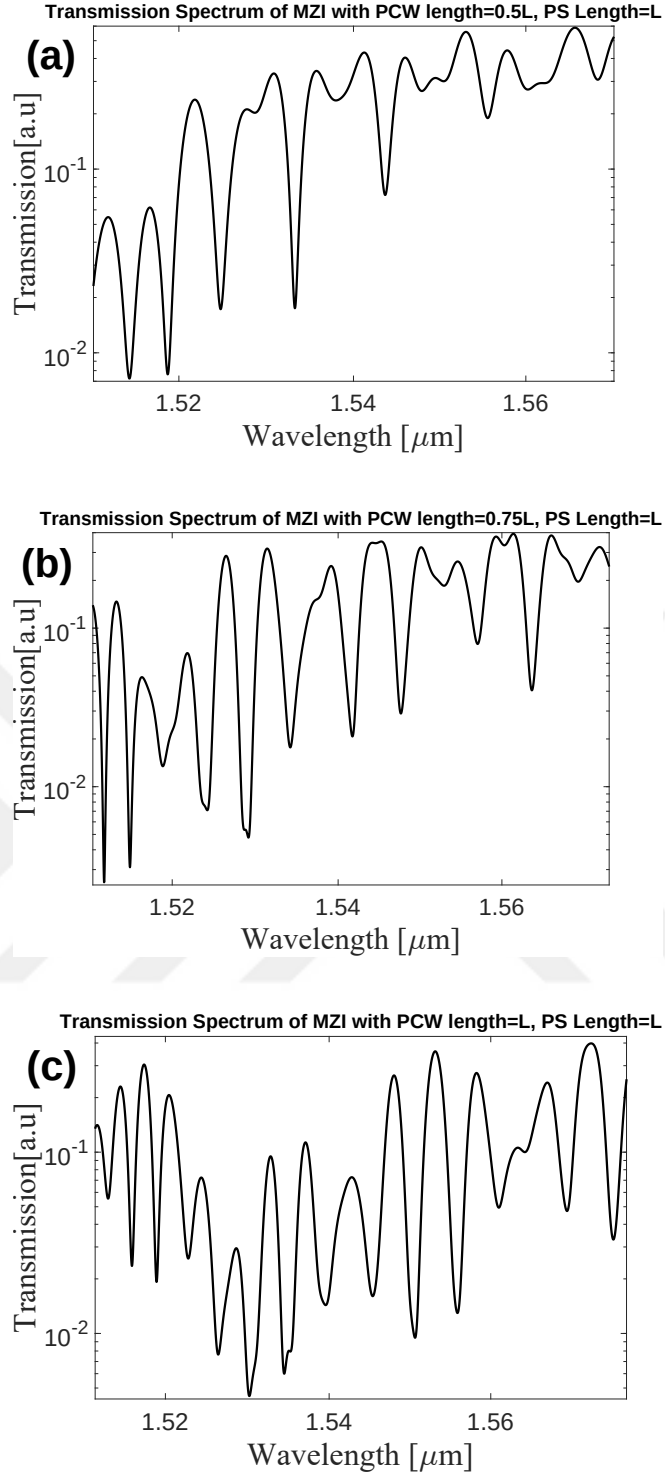


Figure 4.27: Normalized transmission spectra of PCW-MZI with PS length L_{ps} and (a) PCW length $0.5L_{\text{ps}}$. (b) PCW length $0.75L_{\text{ps}}$. (c) PCW length L_{ps} (proposed design).

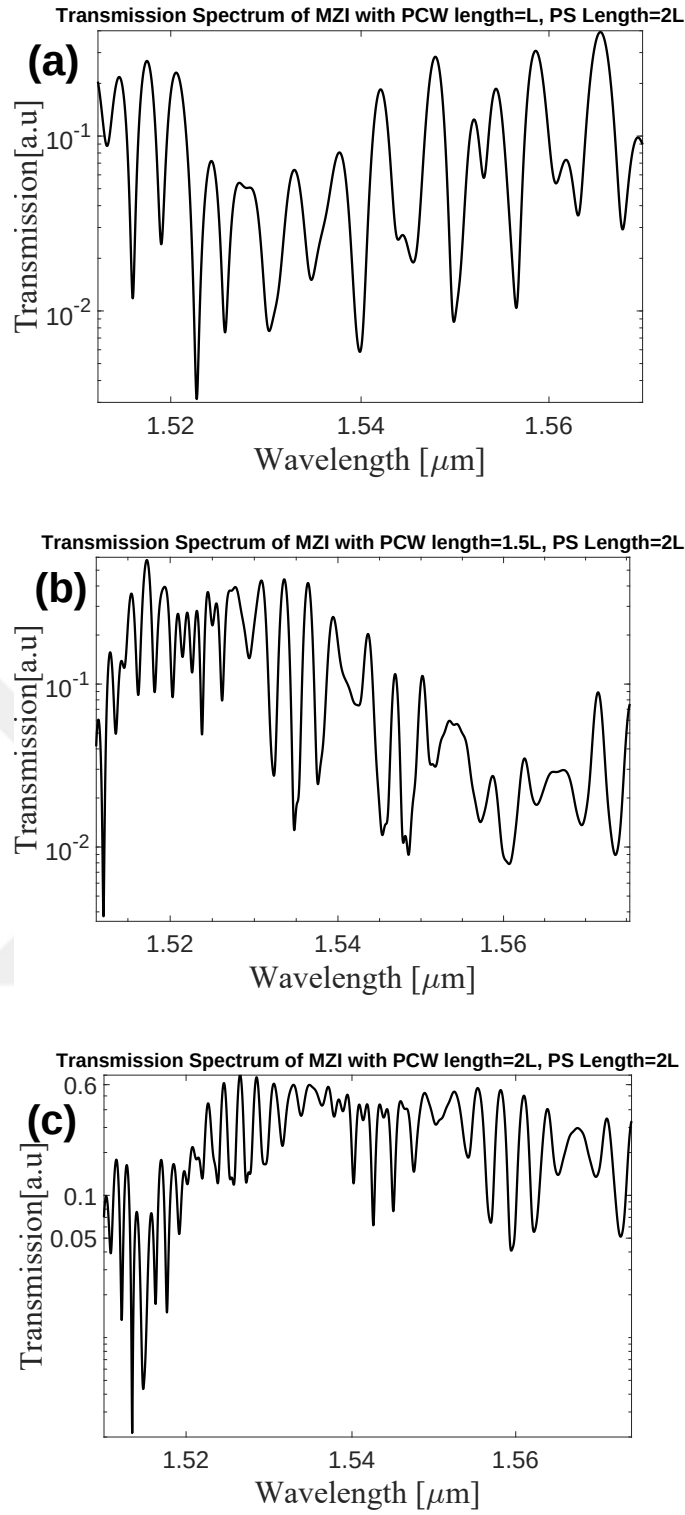


Figure 4.28: Normalized transmission spectra of PCW-MZI with PS length $2L_{ps}$ and
(a) PCW length $1L_{ps}$. **(b)** PCW length $1.5L_{ps}$. **(c)** PCW length $2L_{ps}$.

In Fig. 4.29, fringes in Fig. 4.27 and Fig. 4.28 are plotted against PCW length, with units of L_{ps} .

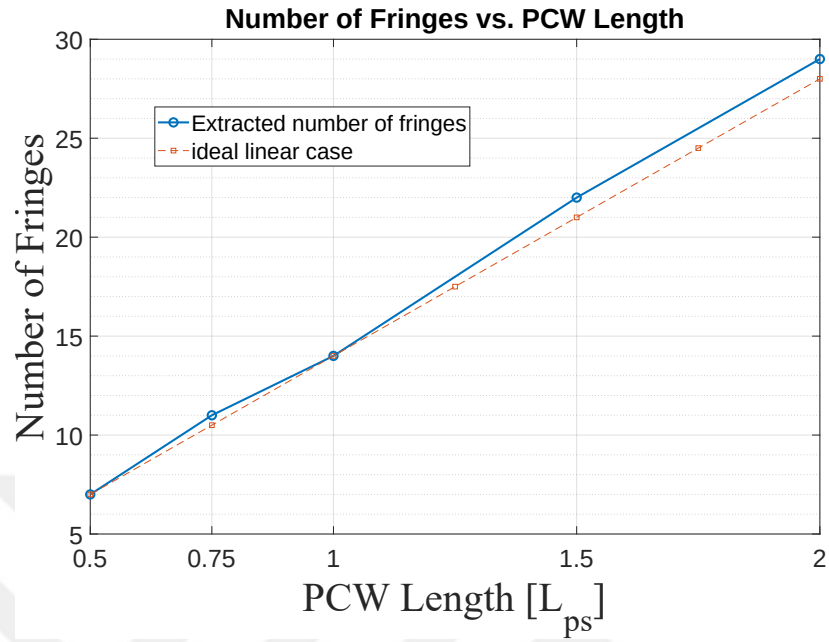


Figure 4.29: Number of fringes in normalized transmission spectra.

In Fig. 4.30, FSR values extracted from transmission spectra and ideal FSR values are plotted against PCW length, with units of L_{ps} .

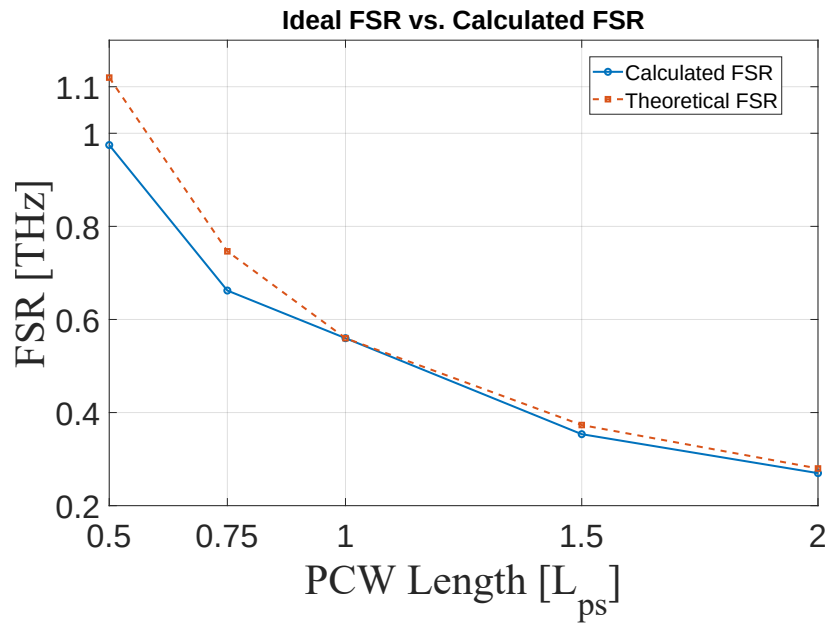


Figure 4.30: Theoretical vs. calculated FSR values.

Values plotted in Fig. 4.30 are provided as a table in Table. 4.2. Deviation values being low is an evidence showing the functionality of the proposed design. Note that deviation is higher for shorter device lengths, and lower for longer device lengths. This is most likely due to numerical errors increasing in lower device lengths, since fields are not allowed to propagate along a rather long path compared to longer device lengths, causing problems in intensity calculations in the software.

Table 4.2: FSR calculations of PCW-MZI designs with different PS and PCW lengths.

PS length (L_{ps})	PCW length (L_{ps})	Number of Fringes	Theoretical FSR (THz)	Calculated FSR (THz)	Deviation in FSR (%)
1	0.5	7	1.0764	0.9747	12.9414
1	0.75	11	0.7176	0.6621	11.2899
1	1	14	-	0.5598	-
2	1	14	0.5598	0.5142	8.18
2	1.5	22	0.3732	0.3535	5.2684
2	2	29	0.2799	0.2698	3.5920

The relation between FSR and number of fringes with PCW length in Table. 4.2 matching with theoretical expectations serve as a validation of our MZI design. They demonstrate that our MZI is operating according to our expectations and provide strong evidence of the successful implementation of our design principles. Through careful control of the PCW length in our MZI design, we can precisely control the FSR, confirming the robustness and reliability of our device.

Initially, we had set the targeted length to be $L = 46.5 \mu\text{m}$, and tried to find the length at which we could observe out of phase waves in MZI arms. Through that, at $L = 38.0664 \mu\text{m}$, required phase difference was observed. As previously mentioned, it was expected to have numerical errors and shifts due to switching between two simulation algorithms. To elaborate, MPB assumes infinite periodicity in dispersion calculations, making an ideal assumption. Hence, when we calculate the initial targeted length, we make the same assumption. However, MEEP has no such assump-

tion, the device under investigation is finite and well-defined. There exist termination layers at the beginning and at the end of the simulation region, which absorbs the fields, contributing to numerical error. It was not unexpected to have mismatches between MPB and MEEP, as it can be overcome by choosing parameters at not exact values but having a $\pm$ offset of some sort, depending on simulation parameters.

4.3 Experimental Work

We fabricated chips consisting of MZIs with 1D PCW in Sabancı University and tried to get measurements in our lab. There were several components on the chip, MZIs with PCWs, straight waveguides without PhC, waveguides with PhC such that air-hole radius was $\pm 5\text{-}10\%$ for different waveguides, to make up for shifts in values due to fabrication and observe behavior for different radii. This observation was possible after obtaining the radii values after fabrication, by using Scanned Electron Microscopy (SEM) on METU Central Lab.

In the experiments, we first started by coupling the fibers into the input and output of straight waveguides without PhC. Then, we connected the output fiber's other end to a power-meter and make further optimizations in terms of fibers' alignment, by observing the output power change when we manually changed fibers' positions. After the optimization, without changing fiber alignments, we disconnected the output fiber from the power-meter and connected it to Optical Spectrum Analyzer (OSA), and got the transmission spectrum. Power measurement of straight waveguide without PhC constructed a basis line for our later measurements, such that we knew we had a strong coupling when we measured close values to that of straight waveguide's power. In addition to power basis line, transmission spectrum of straight waveguide without PhC was used in normalizing the other components' transmission spectra.

Second, we coupled fibers to PhC waveguides, and measured the output power. Then, we optimized the fiber alignment, obtained the maximum output power. Then, we disconnected the output fiber from the power-meter and connected it to OSA, and got the transmission spectrum. After this step, we knew what spectrum and output power we had for PCWs with different air-hole radii.

Third step was obtaining the MZI transmission spectra and output power levels. On the chip, there were multiple MZIs made with PCWs of different air-hole radii. Like previous steps, we optimized the fiber alignment, measured the output power, and at optimized state, we obtained the transmission spectra.

At the end of all three steps, we exported the OSA data to MATLAB, normalized the transmission spectra of PCWs and MZIs, and inspected the output. It was possible to obtain bandgap locations, FSR values, PhC induced loss for different radii. These crucial data allowed us to see the impact of radius on bandgap location, FSR, loss, cut-off frequency, and possible fabrication limits.

1. Setup and Baseline Measurement

- Start by coupling fibers into the input and output of straight waveguides without PhC.
- Connect the output fiber's other end to a power-meter.
- Perform further optimizations by manually changing the positions of fibers while monitoring the output power.
- Once optimized, disconnect the output fiber from the power-meter and connect it to an Optical Spectrum Analyzer (OSA) to get the transmission spectrum.
- The power measurement for the straight waveguide without PhC forms the baseline for later measurements.
- The transmission spectrum of the straight waveguide without PhC is used to normalize other components' transmission spectra.

2. Measurements for PhC Waveguides

- Couple fibers to the PhC waveguides and measure the output power.
- Optimize the fiber alignment to obtain the maximum output power.
- At the optimized state, disconnect the output fiber from the power-meter and connect it to the OSA to get the transmission spectrum.
- At the end of this step, you will have the spectrum and output power data for PCWs with different air-hole radii.

3. Measurements for MZIs

- Next, obtain the MZI transmission spectra and output power levels. Multiple MZIs made with PCWs of different air-hole radii are available on the chip.
- Like the previous steps, optimize the fiber alignment, measure the output power, and at the optimized state obtain the transmission spectra.

4. Data Analysis

- Export the OSA data to MatLab, and normalize the transmission spectra of PCWs and MZIs.
- Inspect the output, from which it is possible to obtain bandgap locations, FSR values, and PhC induced loss for different radii.

Experimental setup in our lab which we conducted the above steps is provided in Fig. 4.31 during an optical excitation of a p-i-n diode region of an MZI consisting of 2D triangular lattice of air-holes.

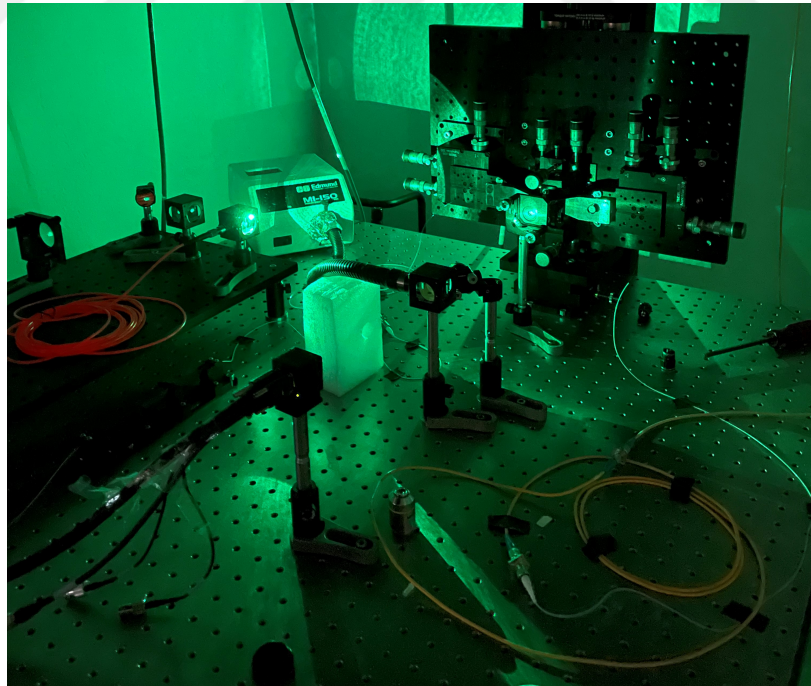


Figure 4.31: Experimental setup.

Having discussed all the experimental procedure, unfortunately, we were not able to get a meaningful data from fabricated chips with 1D PCWs, due to fabrication errors. There was too much roughness and radius mismatch between the fabricated and requested samples was over tolerable values. Fabricated chip containing MZIs with 1D PCWs, and the mentioned roughness is seen in Fig. 4.32.

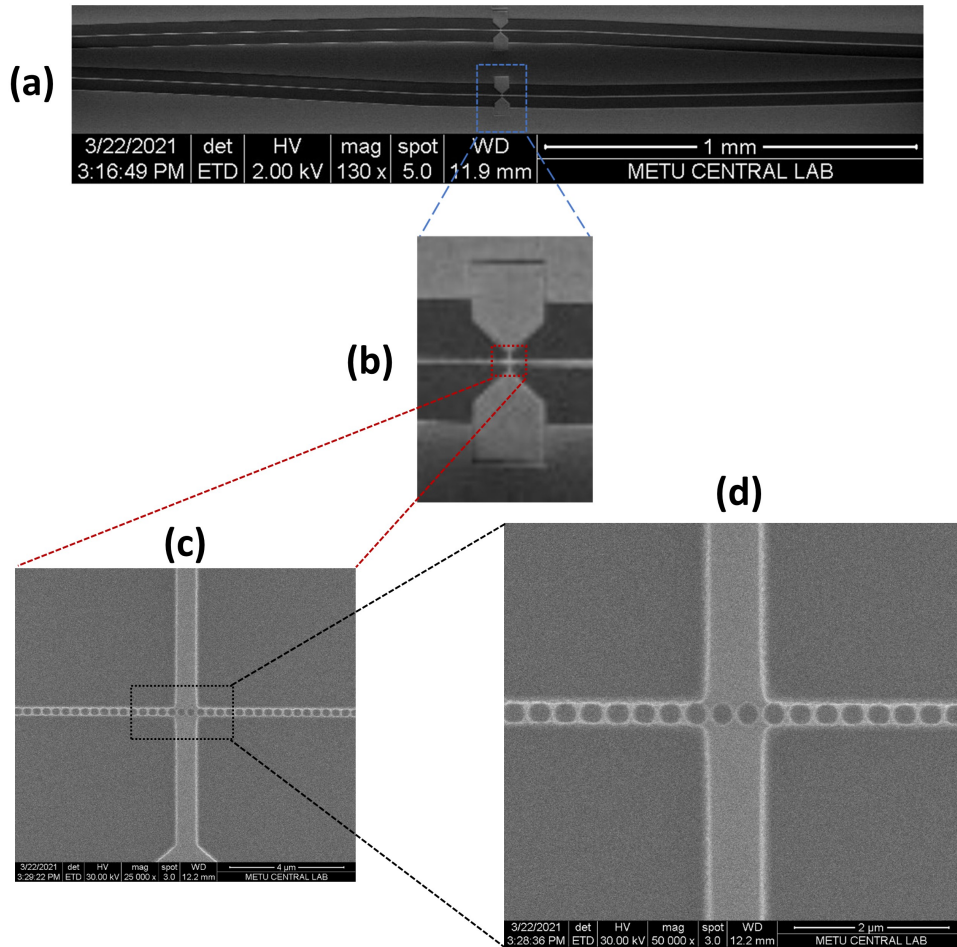


Figure 4.32: **(a)** SEM image of fabricated chip, showing a PCW-MZI with 1D PCWs. Blue rectangle contains the p-i-n diode structure where intrinsic region is made of 1D PhCs. **(b)** Enhanced view of the p-i-n region. **(c)** Intrinsic region of the diode structure. **(d)** Enhanced view of the intrinsic region. Roughness in the waveguide and non-uniformity in air-holes depicted.

CHAPTER 5

CONCLUSIONS AND FUTURE WORK

This thesis began with an investigation into the impact of one-dimensional photonic crystals on the design of Mach-Zehnder Interferometers. A deep understanding of the performance characteristics of these designed MZIs was established, yielding valuable insights for the study.

The first step involved understanding the parameters affecting the photonic band gap and, consequently, the effective index in a 1D PhC structure. Systematic evaluations were made on how the air-hole radius and slab thickness impacted these parameters. An important observation was made regarding how the effective index responded to variations in these parameters, thus affecting the frequency at a particular k-point in the band diagram.

Further exploration led to the examination of the design elements of an MZI. Calculating the effective index difference based on a given bulk index change and subsequent introduction of the bandwidth concept made it clear that the incorporation of a PhC structure in the design of phase shifters can potentially create shorter shifters with high bandwidth. The efficiency of this innovative use of PhC structures was convincingly demonstrated by comparing the bandwidth-device length plots of a conventional MZI with silicon waveguides and an MZI that utilized PhC.

A crucial part of the study focused on optimizing the air-hole radius. This was accomplished by varying the radius and PCW length values, thereby determining the optimal radius for a specific length and extinction ratio. This significant finding has profound implications on the design considerations of an MZI that utilizes PhC structures.

Drawing from the insights gained so far, an MZI design was proposed, employing 1D PhCs as phase shifters. The reliability of the operation of the proposed design was tested by altering the device length and the PCW length separately and observing the effect on the fringes and the FSR. The matching of the observed changes in FSR and fringe patterns with the theoretical expectations affirmed the functionality of the proposed design.

The proposed MZI design carries implications for the development of next-generation electro-optic switches in optical communication systems, where they play an important role in controlling light signal transmission. The design's high index contrast offers potential solutions to existing challenges such as long switching times and operational wavelength limitations.

Similarly, the proposed MZI design can be utilized in bio-sensing applications. As we have shown, design can be adjusted for different extinction ratios and bandwidths. Depending on the requirement, multiple MZIs with different parameters can be utilized to achieve desired performance. For instance, to obtain 90 nm bandwidth around 1550 nm, three MZIs with proposed design geometry can be used by changing the lattice constant while keeping dimensions the same in terms of normalized units, where normalization is done with lattice constant. Hence, bandwidths which are not possible to get with using traditional MZIs can be achieved, let alone increasing the bandwidth.

Overall, this study has provided a comprehensive analysis of the role of 1D PhC structures in the design of MZIs and proposed a novel MZI design employing these structures. The promising performance characteristics of the proposed MZI validate the efficacy of this research.

It is hoped that this research serves as a step for further explorations in this area, leading to the design of more compact and efficient photonic devices. However, these simulation based derivations should be verified experimentally, and any mismatches should be calculated and accounted for further fabrications. In the scope of this thesis, it was not possible to do the experimental work due to fabrication issues. There was too much roughness and shifts in radius, thus blocking the way of a healthy experiment, as discussed in previous chapter.

REFERENCES

- [1] T. Tsuchizawa, K. Yamada, H. Fukuda, T. Watanabe, J. ichi Takahashi, M. Takahashi, T. Shoji, E. Tamechika, S. Itabashi, and H. Morita, “Microphotronics devices based on silicon microfabrication technology,” *IEEE Journal of Selected Topics in Quantum Electronics*, vol. 11, no. 1, pp. 232–240, 2005.
- [2] M. Lipson, “Guiding, modulating, and emitting light on silicon-challenges and opportunities,” *Journal of Lightwave Technology*, vol. 23, no. 12, pp. 4222–4238, 2005.
- [3] B. Jalali and S. Fathpour, “Silicon photonics,” *J. Lightwave Technol.*, vol. 24, no. 12, pp. 4600–4615, Dec 2006.
- [4] M. Lipson, “The revolution of silicon photonics,” *Nature Materials*, vol. 21, 08 2022.
- [5] T. Baba, H. Nguyen, N. Ishikura, K. Suzuki, M. Shinkawa, R. Hayakawa, and K. Kondo, “Photonic crystal slow light devices fabricated by cmos-compatible process,” *IEICE Electronics Express*, vol. 10, pp. 20 132 002–20 132 002, 05 2013.
- [6] G. de Valicourt, C.-M. Chang, M. S. Eggleston, A. Melikyan, C. Zhu, J. Lee, J. E. Simsarian, S. Chandrasekhar, J. H. Sinsky, K. W. Kim, P. Dong, A. Maho, A. Verdier, R. Brenot, and Y. K. Chen, “Photonic integrated circuit based on hybrid iii–v silicon integration,” *Journal of Lightwave Technology*, vol. 36, no. 2, pp. 265–273, 2018.
- [7] A. Liu, L. Liao, Y. Chetrit, J. Basak, H. Nguyen, D. Rubin, and M. Paniccia, “Wavelength division multiplexing based photonic integrated circuits on silicon-on-insulator platform,” *IEEE Journal of Selected Topics in Quantum Electronics*, vol. 16, no. 1, pp. 23–32, 2010.
- [8] L. Lu, L. Zhou, Z. Li, D. Li, S. Zhao, X. Li, and J. Chen, “ 4×4 silicon optical

switches based on double-ring-assisted mach–zehnder interferometers,” *IEEE Photonics Technology Letters*, vol. 27, no. 23, pp. 2457–2460, 2015.

- [9] Q. Xu, S. Manipatruni, B. Schmidt, J. Shakya, and M. Lipson, “12.5 gbits carrier-injection-based silicon micro-ring silicon modulators,” *Opt. Express*, vol. 15, no. 2, pp. 430–436, Jan 2007.
- [10] P. Dong, S. Liao, D. Feng, H. Liang, D. Zheng, R. Shafiiha, C.-C. Kung, W. Qian, G. Li, X. Zheng, A. V. Krishnamoorthy, and M. Asghari, “Low vpp, ultralow-energy, compact, high-speed silicon electro-optic modulator,” *Opt. Express*, vol. 17, no. 25, pp. 22 484–22 490, Dec 2009.
- [11] G. Lanlan, W. Jiang, X. Chen, L. Wang, and R. Chen, “High speed silicon photonic crystal waveguide modulator for low voltage operation,” *Applied Physics Letters*, vol. 90, p. 071105, 02 2007.
- [12] Y. Hinakura, D. Akiyama, H. Ito, and T. Baba, “Silicon photonic crystal modulators for high-speed transmission and wavelength division multiplexing,” *IEEE Journal of Selected Topics in Quantum Electronics*, vol. 27, no. 3, pp. 1–8, 2021.
- [13] L. Lu, S. Zhao, L. Zhou, D. Li, Z. Li, M. Wang, X. Li, and J. Chen, “16 × 16 non-blocking silicon optical switch based on electro-optic mach-zehnder interferometers,” *Opt. Express*, vol. 24, no. 9, pp. 9295–9307, May 2016.
- [14] R. Soref, “Tutorial: Integrated-photonic switching structures,” *APL Photonics*, vol. 3, p. 021101, 02 2018.
- [15] P. Dong, R. Shafiiha, S. Liao, H. Liang, N.-N. Feng, D. Feng, G. Li, X. Zheng, A. V. Krishnamoorthy, and M. Asghari, “Wavelength-tunable silicon microring modulator,” *Opt. Express*, vol. 18, no. 11, pp. 10 941–10 946, May 2010.
- [16] T.-H. Yen, T.-C. Wang, R. Kou, and Y. Hung, “Fabrication-tolerant wavelength (de)multiplexer based on cascaded mach-zehnder interferometers on silicon-on-insulator,” in *2020 Opto-Electronics and Communications Conference (OECC)*, 2020, pp. 1–3.

- [17] A. Goban, C.-L. Hung, S.-P. Yu, J. Hood, J. Muniz, J. Lee, M. Martin, A. McClung, K. Choi, D. Chang, O. Painter, and H. Kimble, “Atom-light interactions in photonic crystals,” *Nature communications*, vol. 5, p. 3808, 05 2014.
- [18] J. Hu, L. Shao, G. Gu, X. Zhang, Y. Liu, X. Song, Z. Song, J. Feng, R. Buczyński, M. Śmietana, T. Wang, and T. Lang, “Dual mach–zehnder interferometer based on side-hole fiber for high-sensitivity refractive index sensing,” *IEEE Photonics Journal*, vol. 11, no. 6, pp. 1–13, 2019.
- [19] P. Hua, B. Jonathan Luff, G. R. Quigley, J. S. Wilkinson, and K. Kawaguchi, “Integrated optical dual mach–zehnder interferometer sensor,” *Sensors and Actuators B: Chemical*, vol. 87, no. 2, pp. 250–257, 2002.
- [20] C. Peng, C. Yang, H. Zhao, L. Liang, C. Zheng, C. Chen, L. Qin, and H. Tang, “Optical waveguide refractive index sensor for biochemical sensing,” *Applied Sciences*, vol. 13, no. 6, 2023.
- [21] R. Heideman and P. Lambeck, “Remote opto-chemical sensing with extreme sensitivity: design, fabrication and performance of a pigtailed integrated optical phase-modulated mach–zehnder interferometer system,” *Sensors and Actuators B: Chemical*, vol. 61, no. 1, pp. 100–127, 1999.
- [22] M. S. Mohamed, M. F. O. Hameed, N. F. F. Areed, M. M. El-Okr, and S. S. A. Obayya, “Analysis of highly sensitive photonic crystal biosensor for glucose monitoring,” *Appl. Comput. Electromagn. Soc. J.*, 2016.
- [23] C. Wang *et al.*, “High figure of merit ultra-compact 3-channel parallel-connected photonic crystal mini-hexagonal-h1 defect microcavity sensor array,” *Opt. Commun.*, 2017.
- [24] D. Mohammad and K. Behnam, “Design of a label-free photonic crystal refractive index sensor for biomedical applications,” *Photonics Nanostructures - Fundam. Appl.*, 2018.
- [25] E. Yablonovitch, “Inhibited spontaneous emission in solid-state physics and electronics,” *Physical Review Letters*, vol. 58, pp. 2059–2062, 1987.
- [26] J. Joannopoulos, P. Villeneuve, and S. Fan, “Photonic crystals: putting a new twist on light,” *Nature*, vol. 386, pp. 143–149, 1997.

- [27] A. Chutinan, S. Noda, and M. Imada, “Trapping and emission of photons by a single defect in a photonic bandgap structure,” *Nature*, vol. 407, pp. 608–610, 1999.
- [28] D. Dai and S. He, “A silicon-based hybrid plasmonic waveguide with a metal cap for a nano-scale light confinement,” *Optics Express*, vol. 18, pp. 23 802–23 817, 2010.
- [29] J. Li, I. M. White, and H. Oveys, “Analysis of photonic crystal waveguide-based resonant cavities for label-free biosensing,” *Optics Express*, vol. 16, pp. 21 918–21 924, 2008.
- [30] M. Erdil, Y. Ozer, and S. Kocaman, “High-q slot-mode photonic crystal nanobeam cavity biosensor with optomechanically enhanced sensitivity,” *IEEE Journal of Selected Topics in Quantum Electronics*, vol. 25, no. 2, pp. 1–6, 2019.
- [31] M. Erdil and S. Kocaman, “Optomechanically enhanced high-q slot mode photonic crystal nanobeam cavity,” in *2018 IEEE Photonics Conference (IPC)*, 2018, pp. 1–2.
- [32] T. Sünnner *et al.*, “Photonic crystal cavity based gas sensor,” *Appl. Phys. Lett.*, 2008.
- [33] C. Feng, G. Y. Feng, N. J. Chen, and S. H. Zhou, “Ultrahigh-q small-v photonic crystal nanobeam cavities based on parabolic-shaped width and taper holes,” *Wuli Xuebao Acta Phys. Sin.*, 2012.
- [34] B. E. A. Saleh and M. C. Teich, *Fundamentals of photonics; 2nd ed.*, ser. Wiley series in pure and applied optics. New York, NY: Wiley, 2007.
- [35] G. T. Reed and A. P. Knights, “Silicon photonics: An introduction,” 2004.
- [36] S. Kocaman, M. Aras, P. Hsieh, J. McMillan, C. Biris, N. Panoiu, Y. Yu, D. Kwong, A. Stein, and C. Wong, “Zero phase delay in negative-refractive-index photonic crystal superlattices,” *Nature Photonics*, vol. 5, pp. 499–505, 07 2011.
- [37] R. W. Boyd, *Nonlinear Optics*. Academic Press, 2008.

- [38] A. Liu, R. Jones, L. Liao, D. Samara-Rubio, D. Rubin, O. Cohen, R. Nicolaescu, and M. Paniccia, “A high-speed silicon optical modulator based on a metal–oxide–semiconductor capacitor,” *Nature*, vol. 427, pp. 615–618, 2004.
- [39] R. Soref, “Silicon-based optoelectronics,” *Proceedings of the IEEE*, vol. 81, pp. 1687–1706, 2006.
- [40] J. F. Nye, *Physical properties of crystals: their representation by tensors and matrices*. Oxford University Press, 1983.
- [41] R. Soref, “The past, present, and future of silicon photonics,” *IEEE Journal of Selected Topics in Quantum Electronics*, vol. 12, no. 6, pp. 1678–1687, 2006.
- [42] J.-M. Liu, *Optical Modulation*. Cambridge University Press, 2016, p. 297–361.
- [43] D. Nandi, S. Nandi, A. Sarkar, and C. Sarkar, *Optical Switching: Device Technology and Applications in Networks*. Wiley, 2022.
- [44] S. G. Johnson and J. D. Joannopoulos, “Block-iterative frequency-domain methods for maxwell’s equations in a planewave basis,” *Opt. Express*, vol. 8, no. 3, pp. 173–190, Jan 2001.
- [45] R. Soref and B. Bennett, “Electrooptical effects in silicon,” *IEEE Journal of Quantum Electronics*, vol. 23, no. 1, pp. 123–129, 1987.
- [46] W. Bogaerts, P. Heyn, T. Van Vaerenbergh, K. Vos, S. K. Selvaraja, T. Claes, P. Dumon, P. Bienstman, D. Thourhout, and R. Baets, “Silicon microring resonators,” *Laser & Photonics Reviews*, vol. 6, pp. 47 – 73, 01 2012.
- [47] K. Hirschman, N. Wescott, T. Brown, and J. Neiser, “Silicon-on-insulator based modulators and switches for high speed optical telecommunications,” in *Proceedings of the Fourteenth Biennial University Government Industry Microelectronics Symposium (Cat. No.01CH37197)*, 2001, pp. 64–69.
- [48] H. C. Nguyen, Y. Sakai, M. Shinkawa, N. Ishikura, and T. Baba, “10 gbs operation of photonic crystal silicon optical modulators,” *Opt. Express*, vol. 19, no. 14, pp. 13 000–13 007, Jul 2011.

- [49] G. Lanlan, Y. Jiang, W. Jiang, X. Chen, J. Choi, and R. Chen, “Silicon-on-insulator-based photonic-crystal mach-zehnder interferometers,” *Proc SPIE*, vol. 6128, pp. 261–268, 02 2006.
- [50] T. Baba, S. Akiyama, M. Imai, N. Hirayama, H. Takahashi, Y. Noguchi, T. Horikawa, and T. Usuki, “50-gbs ring-resonator-based silicon modulator,” *Opt. Express*, vol. 21, no. 10, pp. 11 869–11 876, May 2013.
- [51] T. Han, H.-S. Lee, and E.-H. Lee, “Design of compact silicon optical modulator using photonic crystal mzi structure,” *2008 5th International Conference on Group IV Photonics, GFP*, pp. 308–310, 01 2008.
- [52] A. Gövdeli, M. Sarıhan, U. Karaca, and S. Kocaman, “Integrated optical modulator based on transition between photonic bands,” *Scientific Reports*, vol. 8, 01 2018.
- [53] L. Gu, W. Jiang, X. Chen, and R. T. Chen, “Physical mechanism of p-i-n-diode-based photonic crystal silicon electrooptic modulators for gigahertz operation,” *IEEE Journal of Selected Topics in Quantum Electronics*, vol. 14, no. 4, pp. 1132–1139, 2008.
- [54] A. F. Oskooi, D. Roundy, M. Ibanescu, P. Bermel, J. Joannopoulos, and S. G. Johnson, “Meep: A flexible free-software package for electromagnetic simulations by the fdtd method,” *Computer Physics Communications*, vol. 181, no. 3, pp. 687–702, 2010.
- [55] J. D. Joannopoulos, S. G. Johnson, J. N. Winn, and R. D. Meade, “Photonic crystals: Molding the flow of light - second edition,” 2008.
- [56] Y. Chen, J. Li, Z. Li, and Y. Zhang, “Coupled mode theory analysis of mode-splitting in coupled cavity system,” *Optics Express*, vol. 25, no. 2, pp. 1594–1603, 2017.
- [57] P. Lalanne and D. Lemercier-Lalanne, “On the effective medium theory of sub-wavelength periodic structures,” *Journal of Modern Optics*, vol. 43, no. 10, pp. 2063–2085, 1996.

- [58] W. Zhou and P. Dastmalchi, “A review of subwavelength resonant grating technology for biosensors and optical devices,” *Journal of Micro Nanolithography, MEMS, and MOEMS*, vol. 15, no. 2, p. 020301, 2016.
- [59] V. Almeida, C. Barrios, R. Panepucci, and M. Lipson, “All-optical control of light on a silicon chip,” *Nature*, vol. 431, no. 7012, pp. 1081–1084, 2004.
- [60] L. Chen, X. Chen, I. Shieh, and H. Hsieh, “Ultrahigh-speed semiconductor-optical-modulator-based photonic mixer for high-linearity microwave photonic links,” *Journal of Lightwave Technology*, vol. 25, no. 9, pp. 2751–2767, 2007.
- [61] Y. Ding, H. Ou, and C. Peucheret, “Ultrahigh-speed silicon-organic hybrid modulator with broad optical bandwidth for 16qam coherent communications,” *Journal of Lightwave Technology*, vol. 30, no. 7, pp. 967–972, 2010.
- [62] C. Xiong, W. H. Pernice, and H. X. Tang, “Low-loss, silicon integrated, aluminum nitride photonic circuits and their use for electro-optic signal processing,” *Nano Letters*, vol. 12, no. 7, pp. 3562–3568, 2014.
- [63] Z. Wang, X. Guan, N. Chen, and D. Dai, “Ultracompact low-loss coupler between strip and slot waveguides,” *Optics Letters*, vol. 43, no. 2, pp. 310–313, 2015.
- [64] Y. Vlasov, M. O’Boyle, H. Hamann, and S. McNab, “Vlasov, y. a., o’boyle, m., hamann, h. f., mc nab, s. j. active control of slow light on a chip with photonic crystal waveguides. nature 438, 65-69,” *Nature*, vol. 438, pp. 65–9, 12 2005.