

ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL OF SCIENCE
ENGINEERING AND TECHNOLOGY

**UNDERSTANDING SHAPE PREFERENCES IN ARCHITECTURAL DESIGN
THROUGH EVOLUTIONARY COMPUTATION**



M.Sc. THESIS

Ecenur YAVUZ

Department of Informatics

Architectural Design Computing Programme

JULY 2020

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**Ecenur YAVUZ
(523171001)**

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Thesis Advisor: Dr. Michael S. BITTERMANN

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**MİMARİ TASARIMDAKİ ŞEKİL TERCİHLERİNİ
EVİRİMSEL HESAPLAMA İLE ANLAMA**

YÜKSEK LİSANS TEZİ

**Ecenur YAVUZ
(523171001)**

Bilişim Anabilim Dalı

Mimari Tasarımda Bilişim Programı

Tez Danışmanı: Dr. Michael S. BITTERMANN

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Ecenur YAVUZ, a M.Sc. student of ITU Graduate School of Science Engineering and Technology student ID 523171001, successfully defended the thesis entitled “UNDERSTANDING SHAPE PREFERENCES IN ARCHITECTURAL DESIGN THROUGH EVOLUTIONARY COMPUTATION”, which she prepared after fulfilling the requirements specified in the associated legislations, before the jury whose signatures are below.

Thesis Advisor : **Dr. Michael S. BITTERMANN**
Istanbul Technical University

Jury Members : **Prof. Dr. M. Birgül ÇOLAKOĞLU**
Istanbul Technical University

.....
Prof. Dr. Arzu GÖNENÇ SORGUÇ
Middle East Technical University

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To my mother and father,



FOREWORD

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ABBREVIATIONS

ANN	: Artificial neural network
EA	: Evolutionary algorithm
EMO	: Evolutionary multi-objective optimization
FNT	: Fuzzy neural tree
GA	: Genetic algorithm
MSE	: Mean squared error
NURBS	: Non-uniform rational basis splines



SYMBOLS

x_0, x_1, x_2, x_3	: Independent variables of the parametric shape
x_4, x_5, x_6, x_7	: Independent variables of the parametric shape
x_8, x_9, x_{10}	: Independent variables of the parametric shape
P_1, P_2, P_3	: Curve control points
P'_1, P'_2, P'_3	: Specific control point locations
h_1, h_2, h_3	: Heights of the curves
α, π, θ	: Interior angles of the triangle
σ	: Width parameter of the Gaussian function
y	: Membership degree
y	: Trapezoidal membership degree
L_i	: FNT logic operation output
w_1, w_2, w_3, w_4	: Weights of FNT AND operation
$\bar{w}_i$	: Mean of the weights
$\mathcal{A}$	: Accuracy of the model
$\mathcal{E}$	: Expressiveness of the representation



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UNDERSTANDING SHAPE PREFERENCES IN ARCHITECTURAL DESIGN THROUGH EVOLUTIONARY COMPUTATION

SUMMARY

Architecture arouses feelings in its experiencers. Anything, colors, lights, scales, and shapes we perceive by our senses, causes a cognitive process and results in a feeling. The aesthetic experience arises from our perception, and it plays an important role in the design fields such as product design, interior design, and architecture. The general design goal in these applications is to choose the shape and material for the object in such a way that the product is desirable by its perceivers/users. Here, we can distinguish two kinds of desirable properties of a product. One kind is the fulfillment of a utilitarian purpose. Another kind is the aesthetical pleasure, which arises solely from the act of visually perceiving the object. Defining the visual pleasantness of a shape is a ‘soft’ issue. Pleasantness is a linguistic concept with associated imprecision and uncertainty. This is because it is a form of qualitative evaluation. As preferences related to shape, and in particular those that are due to aesthetical inclinations, are merely loosely related to rationality, the reasoning underlying an evaluation is problematic to pinpoint. For instance, commonly designers are having difficulty expressing the rationale both behind their own aesthetical preferences and those of the user.

The study aims to devise a computational method to get to know the rationale that underlies certain shape preferences. The method consists of three steps: abstracting the detailed shape attributes (1); aggregating several attributes to characterize the shape in more general terms (2); and tuning the representation of the shape character based on preference data. The suitability of the method is verified by applying it to a shell type of shape which has got a triangular plan and has got three support locations. This shape has been selected to have simplicity in the explanation, as it involves only a few parameters.

As a first step, the shape is analyzed with respect to its physical attributes in the vertical and horizontal sense. The basic shape elements are the boundary curves, which give the salient character to the overall shape. The exemplary shell shape is defined with six support location points, two-point pairs of which determine the support locations as well as *triangularity* of the shape; additionally, the control point locations of three NURBS curves determine the *symmetry*, *verticality*, and *height* of the shape. Based on the detailed attributes of the shape, four conditions of symmetry are distinguished. These are that all three boundary curves are symmetric (1), two of the curves are symmetric (2), two of the curves are asymmetric (3), and all three curves are asymmetric (4). Six conditions as to verticality are distinguished. These are that all three curves are oriented such that each forms an overhang (1); all three curves are oriented vertically so that there is no overhang (2); all three curves are recessed with

respect to the vertical orientation (3); two of the curves yield overhangs (4); two of the curves are oriented vertically (5); two of the curves are recessed (6). Three conditions of height relations among the boundary curves are distinguished. These are that the three curves have similar heights (1); their heights are gradually changing (2); they have heights that are different in a diverse manner (3). Four conditions as to the triangularity of the plan are distinguished. These are that the plan is an equilateral triangle (1); it is an isosceles triangle with one pointed corner (2); it is a right-angled triangle (3); it is a skewed obtuse triangle (4).

When the physical attributes of a shape are exactly matching to the prototypical example of one of the shape conditions above, then the linguistic label expressing the condition undoubtedly applies to the shape. However, when there is some discrepancy between the triangle at hand and its characteristic prototype, then the applicability of the label is dependent on the magnitude of the discrepancy. To express the association strength of linguistic shape attributes, in this thesis, fuzzy sets are used. However, the precise shape of fuzzy membership functions is problematic to specify due to the imprecision associated with the concepts the functions are to represent. In particular, the imprecision refers that people differ to some extent with regards to the meaning they associate with the concepts. As understanding implies some general validity of the knowledge, the identification of the membership functions that characterize a shape is a matter of obtaining and modeling data from multiple people. This is accomplished in this study by means of surveys. The data obtained in this way is used to identify the shape parameters of the membership functions through curve fitting. In this way an ontology of the basic shape features is established.

While the meaning of the basic shape descriptors is subject to agreement among different designers, when we come to more general shape characteristics, different designers can have different aesthetic preferences. Therefore, the rationale underlying individual aesthetical preference requires obtaining multiple manifestations instances of the individual's preference. In this study, this takes the following form. The architect selects among a set of random shapes the subset of shapes that he/she deems as preferable over the others. The next step toward understanding the architect's shape preference is to identify common characteristics among the shapes in the preferred subset.

The method to represent the range of distinct shape characters is a neural computing method known as fuzzy neural tree (FNT). Having distinguished several possible character types respectively for a shape's symmetry, verticality, height topology, and plan triangularity, and noting that the relative importance among these four characteristics is not to assume for an individual designer, it is clear that the number of possible explanations for a designer's preference is large. Each possible explanation is represented in the work by one particular FNT. The task in this modeling step is to identify among the large set of understandings of the designer's preference, the understanding that is most likely to be correct. This is accomplished by searching the space of possible FNT models using an evolutionary algorithm.

The evolutionary algorithm is run multiple times until as many as possible among the preferred designs are represented. Multiple runs are necessary because different patterns of preferences have been applied by the architect so that a single FNT model does not represent all of the preferred solutions. With different possible combinations of property conditions and with a different possible degree of memberships, there appear a variety of shape characters. The identification process reveals that there are

four patterns of preferences applied by the architect. Thus, there are four different resulting FNT models explaining the preferences. Among four different trees, the first tree can be considered as the most important one as it represents four of the ten preferred samples whereas the other three respectively represent one solution each. It can be considered as the model of the aesthetic preference of the architect. And finally, this FNT model can be used to understand the preference for any shape. When we put any shape to this model, the magnitudes at the FNT outputs predict the likelihood the architect will prefer the shape. Due to the transparency of FNT, the reason why he/she will like the shape is also given. Due to the interpretability of the reasoning, it is appropriate to refer to the result from this modeling effort as ‘understanding’ the designer’s preference, beyond merely representing it in some computational form of black-box type.

The benefit of attaining the understanding of preferences by computation is that then the preferences become subject to satisfaction in extreme form by a systematic search. A second benefit is that aesthetics related objectives become compatible with non-aesthetic related ones so that the best compromise is found taking the aesthetic preference duly into account. The subject matter dealt with in the thesis is of generic relevance in architectural design research, since the precise description of soft objectives has been a long-standing problematic issue. The study puts forward an innovative approach to tackle the problem, going beyond using computation as a mere mathematical tool for representing a non-linear relationship in a data; but yielding insight into a cognitive process. The particularity of the approach is the synergetic treatment by several computational intelligence methods in such a way that the result is subject to interpretation in the verbal form the designers are familiar with.



MİMARİ TASARIMDAKİ ŞEKİL TERCİHLERİNİ EVRİMSEL HESAPLAMA İLE ANLAMA

ÖZET

Mimari, insanların duygularını harekete geçirir. Duyularımız tarafından algıladığımız her şey, renkler, ışıklar, ölçekler ve şekiller, bilişsel bir süreçten neden olur ve bir duygu ile sonuçlanır. Estetik deneyim doğrudan algılarımızdan kaynaklanır ve ürün tasarımı, iç tasarım, mimari tasarım gibi alanlarında önemli bir rol oynar. Bu uygulamalardaki genel hedef, nesnenin şeklini ve malzemesini, kullanıcıları tarafından arzu edilen bir ürün haline getirecek şekilde seçmektir. Burada, ürün için iki tür arzu edilen özellikten bahsedebiliriz. Bunlardan ilki, ürünün belirli ve ihtiyaç duyulan bir hizmeti vermesi yani fayda sağlayan bir amacın gerçekleştirilmesidir. Diğeri ise ürünün sadece görsel olarak algılanma eyleminden kaynaklanan estetik zevktir. Bir şeklin görsel anlamda göze hoş gelmesini tanımlamak nitel bir değerlendirmedir, dille ilişkili ve çoğunlukla belirsizdir. Şekille ilgili tercihler ve özellikle estetik eğilimlere bağlı olanlar, muhakeme ile ilişkisi kopuk ve değerlendirmenin altında yatan gerekçeleri tespit etmek ise problemlidir. Örneğin, tasarımcılar çoğunlukla hem kendi estetik tercihlerinin hem de kullanıcının tercihlerinin gerekçelerini kesin olarak saptamakta zorlanırlar.

Bu çalışma, belirli şekil tercihlerinin altında yatan mantığı anlamak için bir hesaplama model geliştirmeyi amaçlamaktadır. Modelin geliştirilmesi üç adımdan oluşur: şekil niteliklerinin soyutlanması (1); şekli daha genel terimlerle karakterize etmek için niteliklerin bir araya getirilmesi (2); ve örnek tercihlerden gelen veriye göre şekil karakteri temsilinin tanımlanması. Modelin uygunluğu, üçgen bir plana ve üç destek noktasına sahip olan kabuk tipi bir şekle uygulanarak doğrulanmıştır. Bu şeklin seçilmesinin nedenlerinden biri, şekli tanımlayacak az sayıda parametre olması nedeniyle şeklin açıklanmasındaki sadeliktir. Üç destek noktasına sahip bu şekil yalnızca üç eğriden oluştuğu için bunlar arasındaki ilişkiyi takip etmek daha kolaydır. Bir diğer neden de genellikle dikdörtgen şekillerin yatay ve düşey olmak üzere iki eksenel simetriye sahip olması, üçgen şekillerin ise bir tek düşey simetriye sahip olmasıdır. Şekle belirgin bir simetrik karakter eklememek adına üçgen şekil seçilmiştir.

İlk adımda şekil, yatay ve düşey olarak fiziksel özelliklerine göre analiz edilir. Temel şekil elemanlarından sınır eğrileri şeklin göze çarpan genel karakterini oluşturur. Örnek kabuk şeklin, üç nokta çiftiyle belirlenen destek konumları şeklin üçgenliğini ve üç NURBS eğrisinin kontrol noktası konumları şeklin simetrisini, dikeyliğini ve yüksekliğini belirler. Şeklin fiziksel elemanları dört simetri koşulu oluşturur. Bunlar, üç sınır eğrisinin hepsinin simetrik (1), eğrilerden ikisinin simetrik (2), eğrilerden ikisinin asimetrik (3) ve üç eğrinin hepsinin asimetrik (4) olmasıdır. Dikeyliğe ilişkin altı koşul oluşur. Bunlar her üç eğrinin de her biri saçak oluşturacak şekilde (1); her üç eğri de dikey, böylece saçak oluşmaz (2); her üç eğri de dikey aksa göre geri çekilmiş (3); eğrilerden ikisi saçak (4); eğrilerden ikisi dikey (5); eğrilerden ikisi geri çekilmiş (6). Eğrilerin yükseklik ilişkileri üç koşul oluşturur. Bunlar üç eğrinin benzer

yüksekliklere sahip olması (1); yüksekliklerinin kademeli olması (2); yüksekliklerinin birbirinden farklı olması (3) dır. Planın üçgenliğine ilişkin dört koşul oluşur. Bunlar planın eşkenar üçgen (1); sivri uçlu ikizkenar üçgen (2); dik açılı üçgen (3); eğik ve geniş açılı üçgen (4) olmasıdır.

Bir şeklin fiziksel özellikleri yukarıda tanımlanan şekil koşullarının prototip örneğiyle tam olarak eşleştiğinde, koşulu ifade eden linguistik etiket şüpheye yer bırakmaksızın şekil için geçerlidir. Fakat, elimizdeki üçgenle karakteristik prototipi arasında farklılık varsa etiketin uygunluğu, farklılığın şiddetine bağlıdır. Bu çalışmada, şeklin linguistik niteliklerini ifade etmek için bulanık kümeler kullanılmıştır. Bununla birlikte, bulanık üyelik fonksiyonlarının tam şekliğini belirlemek, temsil ettikleri konseptlerdeki belirsizlik nedeniyle problemlidir. Belirsizlik, insanların kavramlarla ilişkilendirdikleri anlamın bir ölçüde farklılık gösterdiğini ifade eder. Anlama, bilginin genel geçerliliğini gerektirdiğinden, bir şekli karakterize eden üyelik işlevlerinin tanımlanması, birden fazla kişiden veri elde edilmesi ve modellenmesiyle sağlanabilir. Bu veri çalışmada anket yoluyla elde edilmiştir. Elde edilen veriler, eğri uydurma yoluyla üyelik fonksiyonlarının şekil parametrelerini tanımlamak için kullanılır. Böylece temel şekil özelliklerinin bir ontolojisi oluşturulur.

Temel şekil tanımlayıcılarının anlamı farklı tasarımcılar arasında anlaşmaya açıkken, daha genel şekil karakteri söz konusu olduğunda, farklı tasarımcılar farklı estetik tercihlere sahip olabilirler. Bu nedenle, bireysel estetik tercihin altında yatan mantığı anlamak, bireyin tercihi olan birden fazla örneği dışa vurmasıyla elde edilmesini gerektirir. Çalışmada bu dışa vurum, mimarın rasgele şekiller arasından, diğerlerine göre tercih ettiği şekiller kümesiyle elde edilir. Şekil tercihlerini anlamamanın bir sonraki adımı, tercih edilen şekiller arasındaki ortak özellikleri tanımlamaktır.

Bir şeklin karakterini, şeklin simetri, dikeylik, yükseklik topolojisi ve plan üçgenliği özelliklerinin her biri için birer olası koşulun bir araya gelmesiyle oluştuğu ve ayrıca karakteri oluşturan bu dört özellik arasındaki göreceli önemin her bir tasarımcı için aynı olmadığı göz önünde bulundurduğunda, tasarımcının tercihi için olası açıklamaların sayısının oldukça fazla olduğu görülür. Çalışmada farklı şekil karakterlerini temsil edebilen hesaplamalı model, fuzzy neural tree (FNT) bir sinirsel hesaplama (neural computing) yöntemidir. Her olası açıklama, belirli bir FNT tarafından temsil edilir. Modellemenin bu adımı hedef, tasarımcının tercihinin ilişkin çok sayıda anlama arasından, doğru olma olasılığı en yüksek olan anlamayı tanımlamaktır. Bu, evrimsel algoritma kullanılarak olası FNT modellerin uzayında arama yapılarak gerçekleştirilir.

Evrimsel algoritma, tercih edilen tasarımlar arasında mümkün olduğunca çok tasarım temsil edilene kadar birçok kez çalıştırılır. Algoritmanın birden fazla kez çalıştırılması gereklidir, çünkü mimar tarafından farklı tercih paternleri uygulanmıştır, bu yüzden tek bir FNT modeli tercih edilen tüm tasarımları temsil etmez. Şekil özelliklik koşullarının farklı olası kombinasyonları ve olası farklı üyelik dereceleri ile çeşitli şekil karakterleri ortaya çıkar. Tanımlama süreci, mimar tarafından uygulanan dört tercih paterni olduğunu ortaya koymaktadır. Dolayısıyla, tercihleri açıklayan dört farklı FNT modeli ortaya çıkar. Dört farklı FNT modeli arasından, ilki en önemli model olarak kabul edilebilir. Çünkü bu ilk model tercih edilen 10 tasarımdan 4'ünü temsil ederken, diğer üç FNT'nin her biri yalnızca bir tasarımı temsil etmektedir. Bu model, tanımlama için tercih örnekleri oluşturan mimarın, estetik tercihinin bir modeli olarak düşünülebilir ve herhangi bir yeni şekil için tercihi anlamak üzere kullanılabilir. Bu FNT modele herhangi bir şekil verdiğimizde, çıkan değerın büyüklüğü, şeklin

mimar tarafından tercih edilme olasılığı tahmin eder. FNT'nin şeffaf yapısı nedeniyle şeklin beğenilmesinin nedeni takip edilebilir. Ayrıca tercihteki akıl yürütmenin uyarlanabilirliği nedeniyle, bu modelleme çabasından elde edilen sonuca, tasarımcının tercihini bir 'kara kutu tipi' biçiminde temsil etmenin ötesinde 'anlama' olarak ifade etmek uygundur.

Estetik tercihlerin hesaplama yoluyla anlaşılmasının yararı, sistematik bir arama ile bu tercihlerin memnuniyetinin üst düzeyde sağlanabilecek olmasıdır. Bir diğer yarar ise estetikle ilgili hedeflerin, estetikle ilgili olmayan hedeflerle birlikte çalışabilir hale gelmesidir. Böylece estetik tercihlerin göz önünde bulundurulduğu diğer hedeflerle en iyi uzlaşma sağlanan tasarım çözümü bulunabilir. 'Yumuşak' hedeflerin kesin tanımları uzun süredir devam eden problemleri bir konu olduğundan tezde ele alınan konu, mimari tasarım araştırmalarında genel bir öneme sahiptir. Çalışma, bir problemi çözmek için hesaplamayı, sadece matematiksel bir araç olarak verideki doğrusal olmayan ilişkiyi temsil etmek için kullanmanın ötesinde yenilikçi bir yaklaşım ortaya koymaktadır. Bilişsel bir süreçle ilişkin pek çok fikir verir. Yaklaşımın özelliği elde edilen sonucun tasarımcıların aşına oldukları sözlü bir formda ve uyarlanabilir olmasıdır. Bu çalışma genel bir mimari soruna yönelik bir çok hesaplamalı zeka metodunun kullanıldığı multi disiplinler bir araştırmadır.



1. INTRODUCTION

Architecture arouses our feelings. We spend most of our time surrounded by architecture and share the intuition that the physical features of the places we live and work in influence how we feel and act. What we perceive of space by our senses, its shape, colors, textures, causes cognitive processes that refer to both intellect and emotion. One of our senses, the vision is a major source of information in our comprehension of the built environment. Visual perception involves excessive series of actions between our eyes and brain. Despite little is known about what visual perception exactly is, this complex physical process plays an important role while producing aesthetic experience of a design such as product, interior, and architecture.

The general design goal in these design applications is to choose the shape and the material for an object in such a way that the artifact has desirable properties. We can distinguish two kinds of desirable perceptual properties. One kind is the fulfillment of a utilitarian purpose, such as to make the object stand out in its environment to increase the likelihood the object is noticed. A purpose of another kind is when the designer's choice is to cause pleasure solely from the act of visually perceiving the scene. The former category we can refer to as non-aesthetical application; the latter as aesthetical application, conforming to the definition of the concept in philosophy. Designs, where pleasure is not merely to arise from the bare perception act, but where the perceptually gained information is to satisfy some conceptual objective, are a grey area. Some examples are letting an object be different from the norm or be matching with a fashion or convention. The notion of 'aesthetic experience' is an issue of traditional relevance in architecture, that has remained relevant to date. It commonly arises in response to art and design works. We all enjoy watching a movie with impressive scenes, colors and sounds in it, having a 'cool' Instagram profile, to own a sleek mobile phone, use a stylish app, and eat in a place with beautiful interior design. The enjoyment is clearly related to visual perception. Now we are in an era, in which aesthetic visual stimuli are omnipresent, referring to the diverse forms of advertisement for instance. Although the status of aesthetics in contemporary society is a complicated issue, from the fact that the pursuit of aesthetic pleasure is a human inclination since time memorial, the

subject has at least some relevance. As can be seen from the literature review below, despite the permanent relevance of the topic, the answers as to what aesthetics is precisely, and how it arises, are without consensus.

Visual pleasantness of a shape is a ‘soft’ requirement while designing a building. The softness denotes that the requirement is linguistic, imprecise, and has an uncertainty associated with it. Defining visual pleasantness is a form of qualitative evaluation. This is because it refers to events taking place in a human mind, about which still much remains to be learned. So accordingly, the reasoning underlying the evaluation of a soft requirement is problematic to pinpoint and commonly, designers are having difficulty expressing the rationale both behind their own aesthetical preferences and those of the user. The aim of this study is to devise a computational method to get to know the rationale that underlies certain shape preferences for confident decision making and appraisal of the design quality. According to the author’s best knowledge, there are no studies published in the literature that address the aesthetics issue of an architectural shape with precision. The reason behind this gap is that the subject of such a study requires having in-depth knowledge from multiple disciplines, in particular architecture and computational intelligence. The developed novel computational approach made it possible to gain deeper insight into the hidden rationale.

The method in the study consists of three steps. The first step, abstracting the physical attributes of the shape is described in section 3; the second step, aggregating several attributes to characterize the shape in more general terms is described in section 4; and tuning the representation of the shape character based on the preference data is described in section 5.

2. BACKGROUND

Understanding aesthetic appreciation is a subject of interest and research in many fields. Thus, in this review of related studies, there are considerations by philosophers, painters, physiologists, mathematicians, and engineers. In sub-section 2.1, philosophical approaches to aesthetics are outlined, in sub-section 2.2 attempts on measuring aesthetics are mentioned, in sub-section 2.3 studies seeking to establish aesthetics as dependent on the object features by experiment are mentioned, in sub-section 2.4 approaches considering the principles of perception playing role in aesthetic judgment are introduced. In sub-section 2.5, two studies that are methodologically considered as related to the thesis are described and in the last sub-section 2.6, applications of shell structures are described as examples of architectural shapes that are commonly found to be quite aesthetic.

2.1 Philosophical Approaches to Aesthetics

An aesthetic experience is defined in opposition to an experience involving reasoning. Namely, the experiences of beauty or the sublime by definition arise directly from perception. ‘Directly’ means that the information obtained from perception is not aggregated by some form of reasoning. Based on this notion, Kant introduced the concept of ‘pure aesthetics,’ and the ‘des-interested’-ness of the observer. The latter denotes that in a purely aesthetic experience the notions of purpose and utility do not play a role (Kant, 1952). Kant established the definition of *the beautiful* and *the sublime* without reference to any particular experimental experience. Beauty has been characterized as that which pleases immediately due to congruence between the interest of the senses and content in mind obtained via perception. Whereas the sublime pleases immediately due to discrepancy between the interest of the senses and the mind content obtained from perception. The interest of the senses is to classify mind content as ‘small’ in comparison with content that is deemed possible due to previous experience. This verbal description has remained the essential reference in the modern aesthetic theories. Despite the probable correctness of Kant’s philosophical assertion, precise interpretation of the assertion has remained a pending issue.

Also, Burke (1757) distinguishes *beauty* from the *sublime*. Beauty demands no assistance from our reasoning; even the will is unconcerned; the appearance of beauty

causes some degree of love in us. He believes that the properties on which beauty depends; properties that operate by nature, and are less liable to be altered by caprice, or confounded by a diversity of tastes, than any other. He defines the sensible qualities of beauty which are comparatively small, smooth, having variety in the direction of the parts, and those parts not angular, but like melted into each other, being of a delicate frame without any remarkable appearance of strength, having clear and bright colors, but not very strong and glaring or if it should have any glaring color, to have it diversified with others.

Hogarth was an English painter who searches for beauty, essentially beauty in nature, and he seeks the principles of beauty (Hogarth, 1918). And thus, also of taste, in the forms of bodies, determined by lines and their combinations. Although Hogarth is not usually considered as a writer on architecture, his treatise is not devoid of architectural ideas, as his proposals for greater variety in architecture testify. He places an abstract formal conception as the central constituent element of his aesthetic system. According to him, *beauty* is determined by multiple criteria and he defines the five principles which are “*fitness, variety, uniformity, simplicity, intricacy, and quantity.*” His six principles of beauty are also ones exemplified in earlier treatises on art, and on painting in particular. With the analysis of the beauty of line, returns to his conception of the form of objects in terms of a surface corresponding to a shell of lines. The waving line (composed of contrasting curves in a plane) and the serpentine line (waving and winding, or twisting, in space) are two lines which are the most varied in form, and they contribute most to producing beauty. Due to the vagueness of the principles and their partly conflicting nature, verification of the validity of his analysis remained a problematic issue.

2.2 Measuring Aesthetics of a Geometric Element

No matter what aesthetics arises from, there has been a long-standing interest in finding a mathematical/geometrical explanation for the admiration of certain shapes. Mathematics and geometry have traditionally been used in engineering the buildings such as to meet environmental goals like minimizing the wind speeds and slowing down the running water. However, apart from the engineering needs, architects have also used geometry to create harmonious spatial forms of a building and its surroundings to match religious imaginations and aesthetical desires. In ancient times,

buildings including pyramids, temples, mosques, palaces, and mausoleums were laid out with specific proportions, the facades, entrances, ceilings of important buildings were decorated with paintings and ornaments.

In the 1st century BC, Vitruvius believed that an architect should focus on three central themes while designing a building, *strength* (firmitas), *functionality* (utilitas), and *beauty* (venustas). In 1490, with the inspiration of Vitruvius, Da Vinci made a drawing of a human body in a circle and square (figure 1.1a). This drawing is believed to represent a model of natural proportional perfection, and the similarities between the divine creation of the human body and the human beings' production of a good building. Thereafter the proportion remained as interests of people in the world of mathematic, arts, and architecture. Although Da Vinci's belief that nature produces beauty inherently has compelling aspects, it is problematic to reconcile with the common experience of natural objects that are lacking in aesthetical quality. During the Renaissance, symmetry, proportion, geometry, and regularity were deliberately emphasized by the architects in the building designs. Orderly arrangements of columns, pilasters, and lintels, as well as the use of geometric shapes such as semicircular arches, hemispherical domes, niches, and aediculae, replaced the more complex proportional systems and irregular profiles of medieval buildings.

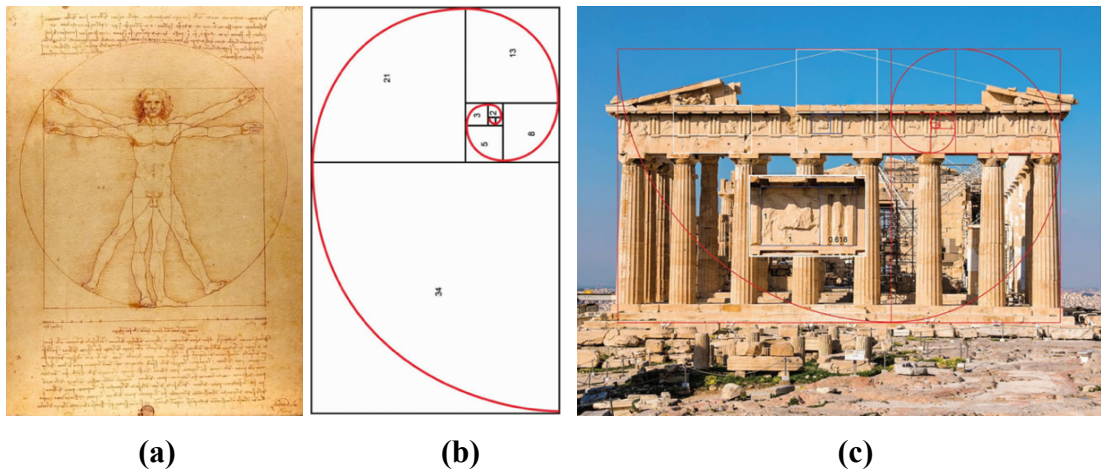


Figure 1.1: (a)Da Vinci's Vitruvian man, photo from (Luc Viatour); (b)Golden ratio and Fibonacci spiral (Meisner, 2018); (c) Golden proportions of the nearly 2,500-year-old Parthenon taken from (Meisner, 2018).

In the 1800s, it is believed that the term 'golden' to describe the Golden Ratio is used the first time by Martin Ohm. The Golden Ratio comes from the 'Fibonacci series', which is approximately equal to 1.618 and it is often denoted by ϕ phi (figure 1.1b, c). There is consensus among designers and scholars dealing with aesthetics that the

Golden Ratio plays a role in the aesthetic quality in architecture, painting, music, and industrial design. However, there is no consensus up till now as to the exact reason why Golden Ratio causes an aesthetic experience.

Friedenberg (2012) advocated that he obtained evidence for a ratio effect in the judgment of triangular geometric shape and that wasn't a golden ratio effect. He experimented on the right triangles with different length sides and different orientations, another experiment on the isosceles triangles. Participants judged low ratio triangles as being the most attractive and elongated triangles with long axis ratios were considered less beautiful. Triangles that pointed upward were judged as significantly more attractive than those that pointed down. They interpreted these results according to a shape's compactness. Compact shapes in the real world are less likely to break and may be considered more perceptually 'stable' than those with long axes. According to him, to understand the results from the experiment, we need to factor in the influence of familiarity. If we see the triangles in a way more than in other orientations, we end up liking them more. It is worth noting that how to factor 'the influence of familiarity' is a remaining question. Familiarity would be beneficial or detrimental. For instance, according to this claim, it would be expected that there isn't any aesthetic appreciation of a building that has such an appearance that it hasn't be seen so far. However, the history of architecture is full of these examples.

Birkhoff tried to quantify aesthetics based on multiple attributes of a shape. He was a mathematician, with the intention to identify formal rules of aesthetics, which would be universally transferable and applicable (Birkhoff, 1933). Birkhoff realizes that the order, or pattern, of the tones, plays an important role in the aesthetic perception of music while listening to classical music. Inspired by the observations, he puts forward a theoretical aesthetics measure and applied it to some objects such as vases, tiles, and polygons. He puts forward the ratio as a measure for aesthetics, where denotes a degree of order and complexity. The proposition is based on the view that an aesthetic experience consists of three phases: effort of attention and perception, which increases in proportion to what he calls the complexity (C) of the object; feeling of the value of the aesthetic measure (M) which rewards this effort; and a realization that the object is characterized by a certain order (O). The specific definitions of and depend on the type of the analyzed object and comparison in the values of aesthetic measures can only be applied within a group of similar objects and not across types. In the case of a

polygonal form (figure 2.2), is separated into five-element and represents the number of indefinitely extended straight lines (-), and vertical symmetry (+), equilibrium (+), rotational symmetry (+), relation to a horizontal-vertical network (+), unsatisfactory form (-). Plus, and minus signs represent each element's positive or negative effect on the aesthetic value. Birkhoff finds awkward examples with such as too small side lengths, angles too near 0-degree or 180-degree, unsupported entrant sides, diversity of niches, diversity of directions, lack of symmetry and call them as unsatisfactory forms.

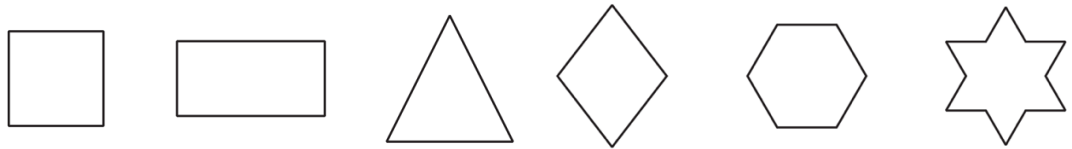


Figure 2.2: According to Birkhoff's measurement, aesthetic measure of from left to right: $M=1.5$, $M=1.25$, $M=7/6$, $M=1$, $M=1$, taken from (Douchová, 2016). Concerning Birkhoff's aim of understanding aesthetical preference quantitatively, this coincides with the aim of this study to some extent. However, besides obvious problems associated with the units in the summation yielding the quantity, it is worth noting that in nature the shapes studied by Birkhoff hardly occur in the form he presents them, namely on the same picture plane. Since aesthetics is due to visual perception an aesthetic measure involves the geometric relation between observer and object in it in one way or another. That is when the same shape is looked at from a different viewpoint it may or may not cause an aesthetic experience with the same intensity. Therefore, looking for the causes of aesthetics exclusively in the properties of the object and irrespective of the observer is problematic.

Harada et al (1999) proposed a mathematical method for quantifying the characteristics of a curve and tried to understand what characteristic makes it aesthetic. The method uses a relation between a length frequency of the curve and a Radius of the curvature in a log-log coordinate system. In the paper, the 'gradient' is defined by an equation when after transforming the coordinate system to an X-Y rectangular coordinate system, and 'C Curve' is shown in the graph of logarithmic distribution diagram of curvature. If the 'gradient' is constant, then the curve has a 'self-affine property'. As they couldn't abstract a C curve with more than three straight-lines of some constant *gradient*, they consider this type of curves as not aesthetic because the curvature change is complex. They analyzed sample curves, which are the section lines

of cars drawn by expert car designers by using this quantitative analysis method, and classified these curves into four typical types. As a result of the analyses, they clarified that there are a lot of curves that one cannot represent by only arcs of Cornu spirals and PH spirals, and the designer controls the curvature change with a self-affine property. They developed an algorithm for generating a curve having self-affine properties. By using this algorithm, not only 'decelerating' curves like spirals but also 'accelerating' curves by the same parameters can be obtained.

As Harada suggested that aesthetic curves have such a property that their logarithmic curvature histograms (LCHs) can be approximated by straight lines and there is a correlation between the slopes of the lines and the impressions of the curves. Miura et al (2005) defined the LCH analytically intending to approximate it by a straight line and formulated the curve whose LCH graph is exactly expressed by a straight line. The proposed formula of aesthetic curves describes the relationship between their radiuses of curvature and lengths. And they showed that the logarithmic spiral and the clothoid curve satisfy the general formula of aesthetic curves, and they are the two most typical aesthetic curves. Another work, which has a similar approach to this one, elucidates the properties of Generalized Log-Aesthetic Curves (GLAC) for CAD practicalities (Gobithaasan et al, 2013). GLAC's bounds and inflection points are identified to facilitate the designers' CAD application. The identification of bounds and inflection points enables the designer to form various GLAC shapes with given three control points. The GLAC shape depends on the bounds of its arc lengths and turning angle. As a result, the designer would be able to create various algorithms to design C-Shape and S-shape GLACs.

2.3 Aesthetics Due to Shape Features

Understanding preferences is relevant in many industrial applications, especially in the design professions. To be able to meet the intended goals, it is clearly necessary to learn about the preferences of the customer/user. Personalization and customization are two trend terms that refer to the use of technology to accommodate differences between individuals. The principles are designing or producing services/products to meet individual requirements. Period trends, fashion, individual differences are playing a big role in forming the preference of a product. When a new product is designed, or its success it is relevant to account for factors that influence the

customer's choice about the product. Customer choices are also influenced by the visual appeal of the products.

Here, three studies that support the idea that shape features of a product takes play an important role in preferences of them. A study that takes the qualitative attribute of a form of a car and captures preference for it in a utility function is presented (Orsborn et al., 2009). They use the utility function tool, which is an important concept in economics, and discrete choice conjoint analysis to estimate the utility weights, which is analytical modeling. They define the form of a car with seven curve parameters as they called 'atomic attributes' (figure 2.3a) and the preference for these attributes is represented in a utility function that assumes a linear relationship between the atomic attributes with quadratic utility over the levels of the attribute. A measurement of the utility of a design has been devised by analyzing the results from the vehicle survey using the Bradley–Terri–Luce method which is computing the probability of a design was chosen by a consumer. The resulting utility function, $U(\bar{x})$ was composed of a utility function u_i for each atomic attribute. The attribute weights, β_i which vary depending on the function. Here, for each attribute u_i , they assume quadratic utility.

Kelly (2011) presents a similar work that combines shape preference (form) and engineering performance (function) in the context of optimization. Two variables used for the parametric definition of the form of a bottle and the least amount of material to hold the desired amount of fluid and resisted the internal pressure without plastic deformation is used for the function of the bottle (figure 2.3b). The authors establish the utility function based on experimental data. Their findings showed that there is a dislocation between what is most appealing to users and what is most sound technically. The results of the study suggest that individuals gravitate towards a bottle shape that they are familiar with.

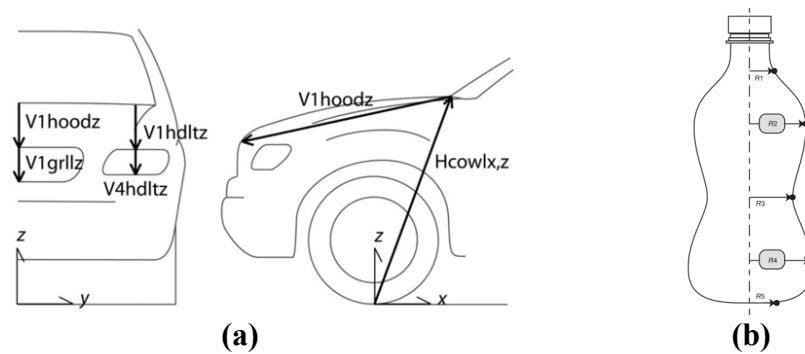


Figure 2.3: (a) Seven sports utility vehicles atomic attributes (Orsborn et al., 2009); (b) Parametrized bottle shape with variables R2 and R4 (Kelly et al., 2011).

Leder and Carbon (2005) reported two studies in which the interplay between stimulus properties and perceiver profiles in the appreciation of car interiors was investigated. They examined the role of curvature, complexity, and innovation with different design dimensions of a car example. In their study, complexity refers to the variation of the number of displays and elements in the different design versions, and innovation in design refers to unusual or indeed new aspects of design, that are unfamiliar to the perceiver. A three-way repeated measurement analysis of variance which is a collection of statistical models is used to analyze the differences samples. Based on the statistical results the authors drew three conclusions: customers prefer curved designs over non-innovative, i.e. rather classic designs, the effects of individual differences, depending on interest and expertise with design or art, were much smaller than expected and experts were more sensitive to differences in curvature and found straight models less attractive than did non-experts.

2.4 Aesthetics Due to Perceptual Aspects

In the 1920s, a group of cognitive psychologists in Germany developed a series of theories on visual perception known as the Gestalt Principles. According to Gestalt theory, the evaluation of human needs that are met by the architecture is incomplete without taking into account the psychological aspects. Arnheim undertakes to examine these psychological qualities based on a number of principles he deems to be at work in human object recognition. The principles are similarity, proximity, continuation, figure-ground, closure, unity-harmony, common fate, and many more. To the extent that 'good' Gestalts contribute to the qualities of the whole, it follows that such properties can relate to aesthetic qualities. However, it is worth noting that Gestalt principles provided insights and guidelines into understanding how the shape is perceived. The principles expressed the relation among elements of shape representation and the likelihood of recognizing the represented shape. Thus, gestalt theory posits psychophysical mechanisms related to shape recognition. In this respect, its relationship to aesthetics is not clearly defined.

Symmetry, which is one of the basic principles put forward in Gestalt theory, has traditionally been considered as a predominant structural factor in natural and manufactured objects. Zimmer (1984) focuses on symmetry in detail and names the different types of symmetry as bilateral, translator, rotational, and central symmetry.

He supported the idea that detecting the type of symmetry characterizing a form is important for the identification of this form. The measurement of the degree of symmetry is a necessary precondition for the interpretation of behavioral data about interpreting the form as an object.

Freyd and Tversky (1984) summarize two basic survival values of symmetry detection: (a) the world is full of symmetric objects they speculate that this property should make object detection easier for a human; and (b) symmetry in an image allows it to be perceived and coded abstractly and economically. In their research, they focus on systematic distortions, which means a discrepancy between the physical or objective character of a form and the mental representation of that form. They conducted three experiments and the main result from all three experiments provide converging evidence for the force of symmetry in object recognition. The representation of the form is distorted depending on their visually asserted symmetry and the degree of symmetry of that form determines the inclination in the distortion, whether towards symmetry or asymmetry. It is worth noting that gestalt theory did not conclusively address the question to what extent the two phenomena of recognizing an object and appreciating it aesthetically are interdependent.

Leyton (2013) introduces two concepts involved in aesthetic judgment, and the two principles are *maximizing transfer of structure* and *maximizing recoverability of the generative operations*. According to him, the two principles are also fundamental to memory because *recoverability* means the reconstruction of the past from what is available in the present, and the only recoverable operations are symmetry-breaking ones. And *transfer* means seeing the present in terms of the past, that is, as a transfer of the past. *Transfer* is also the basis of Gestalt, that is, *cohesion* is formed by *transfer*. (The transfer-based theory of object-oriented-ness is explaining *cohesion* in programming) “The human perceptual system forms cohesive wholes, that is, Gestalts, by transferring stimuli onto each other. Stimuli are thus bound together by transfer. Consequently, a Gestalt is an n -fold transfer hierarchy, $G_1(T) G_2(T) \dots (T) G_n$.” (A group (G) is a complete system of transformations and by complete he means satisfying the three properties: closure, identity element, inverses.)

He showed his principles with a couple of examples. In the Line drawing of Holbein’s Ann of Cleves, Holbein transfers any downward action just described onto a corresponding upward action; in the famous motif of Beethoven’s 5th Symphony

being transferred to different levels of pitch, in eight successive bars from the symphony; and the term aesthetics in science is used consistently about symmetries of a law. He considers those principles are the basis of memory storage; asserting that aesthetics implies producing memory stores of maximal size. Science achieves this by converting existing objects into maximal memory stores. Art achieves this by creating new objects as maximal memory stores.

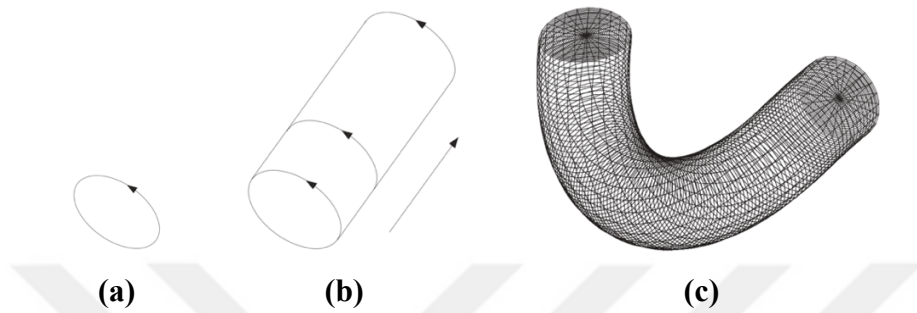


Figure 2.4: (a) A point is transferred by rotations, producing a circle; (b) The circle is then transferred by translations, producing a straight cylinder; (c) A deformed cylinder, taken from (Leyton, 2013).

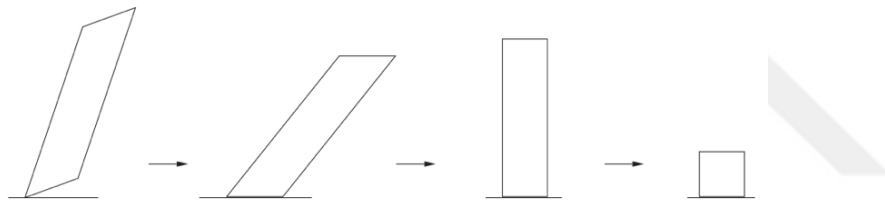


Figure 2.5: Psychological results related to recoverability, taken from (Leyton, 2013). The study of Leyton is a kind of generative theory of shape. The theory characterizes the structure of shape by a sequence of actions needed to generate it. By maximizing the number of the transfer, we increase the complexity of the shape, as he defines transfer every transformation step applied to shape. Then this results that high complexity in geometric structure brings aesthetics to the shape, which is obviously open to discussion.

2.5 Representational Aspects of Aesthetics

The main objective of the study by (Giannini & Monti, 2002) was to develop a CAD tool that preserves the aesthetic design intent for the designer (they call it the stylist) while modeling and designing the product. To explore aesthetic aspects besides the emotional reactions and also to achieve the desired character changes in the design, they identified a common vocabulary, used by designers in their daily activity, with some conducted questionnaires and interviews (figure 2.6 b). The results were

analyzed to see which elements have something in common concerning the distribution of choices. The vocabulary used when marketing and exchanging data is composed of emotional terms such as dynamic, aggressive, expressing the final product character, is termed as ‘language of the trends’.

On the other hand, the vocabulary used when creating and modifying the digital model to achieve the aesthetic intent, corresponding to shape properties, is termed as ‘language of the trade’. To link geometric properties with these stylist terms, in other words understanding the procedures followed by designers, Case-Based Reasoning techniques are used. The user specifies which curves are the most important for the characterization of the product from the emotional point of view, i.e. he specifies the characterizing shape elements and the main product characters in terms of the language of the trends. The system automatically describes each curve by vectors of the language of the trade terms with the associated property values. In the study, the example modifications applied to a sky-box by editing two characteristic planar curves (figure 2.6 a).

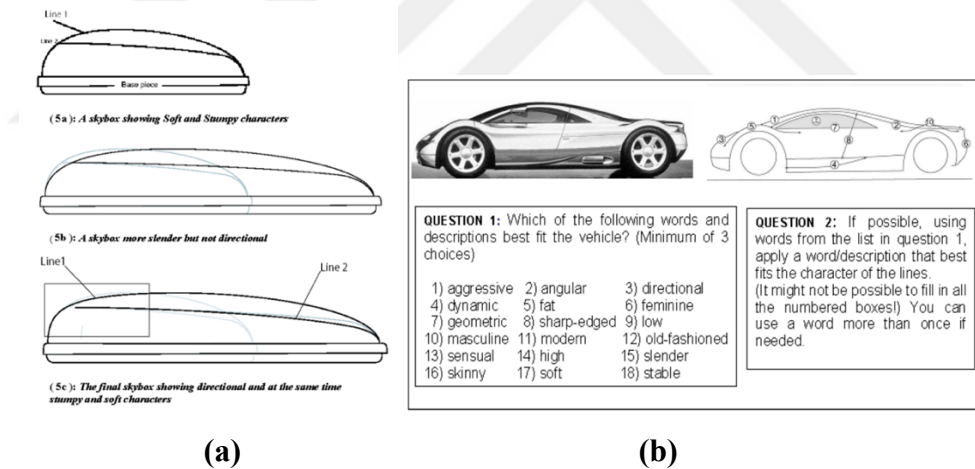


Figure 2.6: (a) An example of application of modifiers to a skybox; (b) A part of the Web questionnaire concerning, taken from (Giannini & Monti, 2002).

An existing strategy to incorporate evaluation into a computational design process without establishing a representation of the evaluation process is interactive evolution. Here the human is directing the search process by assigning the fitness values to the population members of an evolutionary algorithm. An application of interactive evolution is proposed by Machwe et al (2005). The proposed system brings together agent-based machine learning, evolutionary computing, and subjective evaluation in design space search (figure 2.7 a). The agents create the initial population based on design requirements and the evolutionary system searches, explores, and optimizes

within the space of possible designs. In this study, the system searches for the most efficient bridge designs, in terms of stability and material usage. among three kinds of bridges namely a simple unsupported span, a simple supported span, and a slanted span bridge. Slanted span bridges turned out to be most efficient and the other two design types were evolved out of the population.

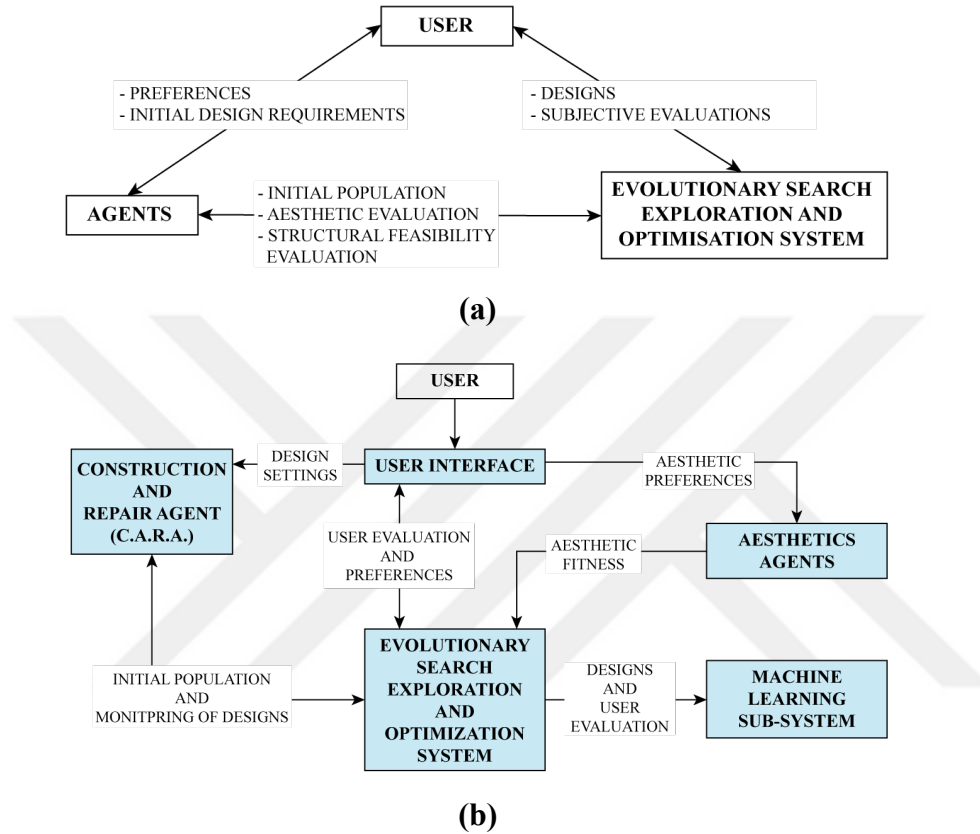


Figure 2.7: (a) The proposed system (A. T. Machwe et al., 2005), (b) Interactive evolutionary design environment (A. Machwe & Parmee, 2006).

The second study proposed by Machwe and Parmee (2006), is the inclusion of the aesthetic criteria, shown with an example of a seating arrangement design. The Case-based approach, which is a sub-system of machine learning, is used. This time, the designer ranks the solutions for the system store and the stored solutions are then used to evaluate future solutions which result in a reduction of the number of user-led evaluations to be made per generation (figure 2.7 b). With an interactive evolutionary system, the urban furniture designs, created by the agent, are assessed in terms of the designer's aesthetic preferences in addition to structural and cost criteria. The solutions ranked by the designer are stored and then used to evaluate future solutions which result in a reduction of the number of user-led evaluations to be made per generation. This study uses a fitness function with five objectives which are combined into a single

objective using a weighted sum. The five objectives are engineering fitness, rule-based aesthetic fitness, cantilever deflection analysis, materials usage, user assigned aesthetic fitness.

However, without a representation of qualitative objectives, the consistency of evaluations during the evolutionary process is problematic to ensure. Representing evaluations computationally requires making explicit the relationship between the independent, detailed properties of an object and its abstract description. The latter is generally desirable to let other people get some insight into the rationale the designer applied in his choices, for confident decision making and appraisal of the design quality in perspective.

2.6 Applications of Shell Structures in Architecture

Shell structures have found applications in architecture for aesthetical, semiotic, function-related, and structural reasons. From the structural viewpoint, their capability to resist loads efficiently, and thus span larger distances than their non-curved counterparts, has been appreciated. From the functional viewpoint, enclosing a space with a minimal need for vertical supports has been useful when visual openness or freedom of movement is to maximize. From the aesthetical viewpoint, they belong to the category of beautiful shapes rather than sublime ones. From the semiotic viewpoint, this is because their gradual shape change symbolizes processes of a gentle nature, and the delicacy of their membranes evokes notions of ephemerality (Burke, 1757). Although curved shapes have been analyzed by engineers regarding their structural and manufacturing properties, analyses from the aesthetical viewpoint have remained imprecise, because of the soft nature of aesthetical judgment.

From the construction viewpoint, Shell structures are surface like curved construction systems. The thickness of the shell is significantly smaller than the other two dimensions, the curvature can be in one or multiple directions, and the curvature may become greater or smaller at different points on the shell surface. Shell structures have been constructed in different scales, for different functions around the world throughout the history of the architecture discipline. There is a wide range of geometries that can be classified as shell structures such as vault, domes, tunnels. With the improvement of reinforced concrete and significant advances in construction methods, shells evolved enormously from the beginning of the XX century. In a shell

structure, there isn't any distinction between the roof and the walls unlike frame-like structures. They distribute the weight they are exposed to via the surface and transfers it to the supports and ground.



Figure 2.8: Shell type structures of well-known architects.

Typical applications of shells include airport terminals, stadiums, and performance spaces as these require large uninterrupted space. Pier Luigi Nervi's Palazzetto dello Sport in Rome is one of the earliest examples and Hadid's London Aquatics Center is one of the contemporary examples of the shell structures in architecture. Alternative applications of shells include galleries, acoustic shells, pavilions as well as gathering spaces for the public. Well-known architects designed shell structures with different construction methods and different visual styles (figure 2.8). It is not clear whether there is a common ground in their preferences of certain shell types. There are diverse ways to create a shell type shape as it is seen in figure 2.9, Isler's sketching on new shapes for shells. However, we can roughly deduce that if the shape is a dome type and has many support points, then its shape character is strongly centralized and

symmetrical to many axes and if it has few support points like two or three, then its shape character is generally extroversive and symmetrical to one axis on the other hand asymmetrical to other axes.



Figure 2.9: Isler's sketch for shell type structures, taken from (Chilton, 2000).



3. MARGINAL ABSTRACTIONS OF A SHAPE

“Everything physical must occupy some region of space and time, but abstract forms can be imagined that are smaller and simpler than anything physically possible. The geometrical point and the temporal instants are such abstractions: a point is a spatial unit that does not take up any space, and an instant is a temporal unit that takes no time. Such abstractions - points, elements, instants, and transitions - are fundamental primitives that can be used to define more complex forms.” (Sowa, 1999)

The first step of the model is the abstraction. Abstraction denotes establishing a simplified representation of a physical event. Abstract entities have no physical existence and they are only subjects for our comprehension. For example, a curve is an abstract entity as its description is entirely by means of mathematical terms. It is an object similar to line and has only one dimension, namely length, without any width nor depth, which can be considered as a flow or run of a point. Abstraction is necessary for architectural design. It is used to communicate the information about its major characteristic features, while minor details and irregularities are omitted. It is used in the design process due to its offering more room for interpretation caused by perceptual ambiguities and promotes design creativity by triggering the imagination. It is also used to analyze a shape to simplify and to extract the essence of it.

Abstractions of a shape can be classified as marginal and essential abstractions; the former specify detailed qualities of the shape; the latter specify its general qualities. In the context of shape analysis, abstraction refers to extracting relevant information of a shape. Architectural shapes are 3d geometries that generally defined by means of two planes, a horizontal plane (ground plane) and a vertical plane. With a plan, we define horizontal attributes of a shape such as a width and a dept and with the façade and section, we define vertical attributes of a shape such as height. These attributes are marginal abstractions of shape as they define its geometric properties that are directly relevant for its physical realization. However, in the context of understanding shape reference the quality of a shape, such as its symmetry, is an essential abstraction. In architecture, different shapes can be relevant, depending on the particularity of the design task. Thus, the meaning of the shape quality concept can take some different

forms from task to task. In any case, however, does the essential property of ‘quality’ constitute from the marginally abstract features of the shape.

In this study, to eventually understand the shape preference of a person, as the first step shape is analyzed with respect to its marginal abstractions, attributes in a vertical and horizontal sense. In this study, a shell type of shape which has got a triangular plan and has got three support locations is treated. One of the reasons behind this shape selection is to have simplicity in the explanation due to the few numbers of parameters to define it. As it has got three support locations and is formed only with three curves, which are few in number, it is easier to trace the relationships among them. Another reason is that rectangular shapes commonly have two axial symmetries, namely vertical and horizontal, whereas triangular shapes have one axial symmetry, vertical. To avoid giving an already discernable symmetry character to the shape, a triangular one is selected. Another advantage of such a shape is that it is observed equally from multiple points viewed from.

In the following section, marginal abstractions of a geometric entity which are the geometric variables defining it are described through the exemplary shell type shape. The parametric definition of the shape will be explained. The independent variables $x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}$ related to the shape are identified.

3.1 Shape Attributes in Vertical Sense

A shell structure can show varying shapes depending on the thickness, the surface geometry, and the boundary curves. There are many ways to define the surface geometry, mathematical ways such as hyperboloids, ellipsoids, and hyperbolic or elliptic paraboloids, and many more. However, the focus of this study is the boundary curves, which give the salient character of the overall shape from the aesthetical viewpoint and the surface geometry with its thickness is not the focus.

The exemplary shell shape is defined with boundary curves digitally and the surface geometry is described by higher degree polynomials which is a patch of Non-Uniform Rational Basis Splines (NURBS). As it is seen in figure 3.1, there are different ways to define a parametric curve. Hermite and Bezier curves are two examples of cubic polynomial curves that are determined by four parameters. A Hermite curve (interpolate(t)) is determined by two endpoints and two tangent vectors at these points

like a Bezier Span. While they share similar mathematical rules, they differ characteristically as. NURBS curves (The non-uniform rational B-spline) and surfaces are the main mathematical representations used in digital modeling to represent a surface geometry. NURBS is a powerful curve representation that maintains smoother and more continuous curves. Segments share more control points to achieve even smoother curves with less storage (Issa, 2013). In general, there are four main attributes to define the NURBS curve: vertices, degree and periodic. The one used in this study, NURBS curves in figure 3.1, vertices are $A(0,0,0)$, $B(60,0,0)$ and $C(30,30,0)$ degree is equal to 2 and periodic is a Boolean equal to false.

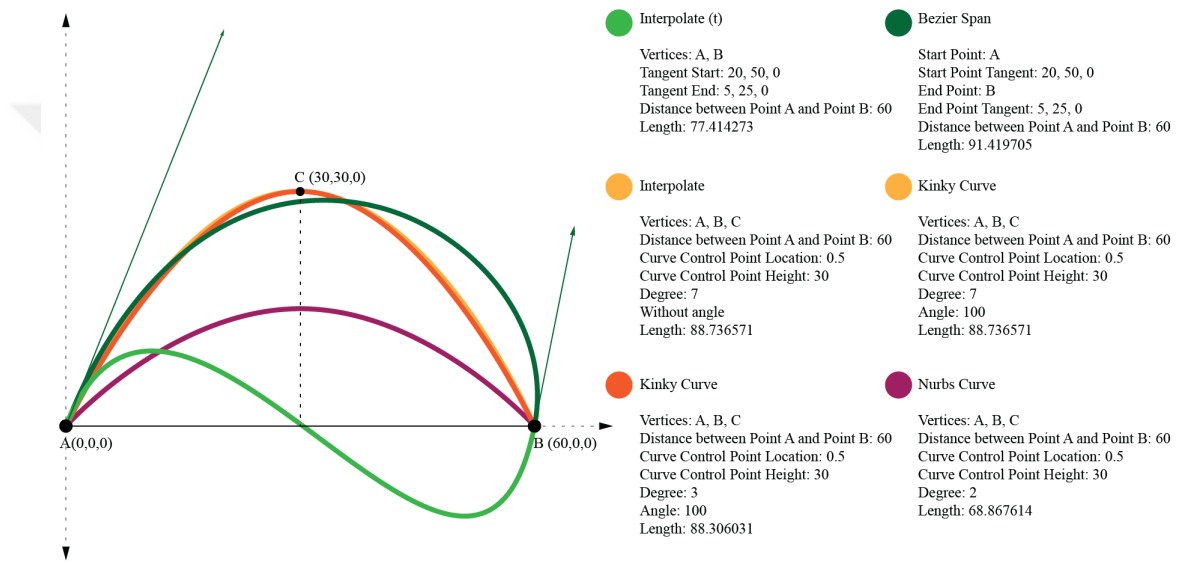


Figure 3.1: Different ways to define a parametric curve.

The base for the exemplary shape in figure 3.2 is a triangle, the vertices of which are labelled A , B and C . Points $A1$ and $A3$ are located on the vector which is obtained by rotating $\overrightarrow{AC}$ counter-clockwise with an angle $(\alpha + \pi)/2$ and connected with a line, points $B1$ and $B2$ are located on the vector which is obtained by rotating $\overrightarrow{BC}$ counter-clockwise with an angle $(\beta + \pi)/2$ and connected with a line, points $C2$ and $C3$ are located on the vector which is obtained by rotating $\overrightarrow{CB}$ counterclockwise with an angle $(\theta + \pi)/2$ and connected with a line. Line segments between each point pairs, $A1$ and $A3$, $B1$ and $B2$, $C2$ and $C3$ are the supports for the shape and the support widths are fixed to 2.00 m as a boundary condition.

Point $A1$ is connected to point $B1$ via a curve denoted by *Curve 1*; $B2$ to $C2$ via *Curve 2*; and $B3$ to $C3$ via *Curve 3*. Each curve is constructed as a NURBS curve with two construction points which are support location points and one control point. The

location of control points P'_1 , P'_2 and P'_3 are determined by the variables x_2 , x_3 and x_4 . The value of the variable x specifies a scale factor for the vector pointing from $A1$ to $B1$, fulfilling the condition $0 \leq x_2 \leq 1$. The condition is to ensure the locations of the curve points on the base plane are bound to be between the support locations, irrespective of alternative positions for the support locations in another design task. With the same rationale x_3 specifies a scale factor for the vector pointing from $B2$ to $C2$, fulfilling the condition $0 \leq x_3 \leq 1$; and x_4 specifies a scale factor for the vector pointing from $C3$ to $A3$, fulfilling the condition $0 \leq x_4 \leq 1$.

The scale factors belonging to the exemplary shape shown in figure 3.2e are $x_2 = 0.02$, $x_3 = 0.94$, and $x_4 = 0.85$. The resulting three points are shown in figure 3.2e and labelled P'_1 , P'_2 and P'_3 . The second step is to translate each of the points in vertical direction. The unit vector for the vertical direction is denoted by $\vec{z}$ and symbolically illustrated in figure 5e. The magnitude of the translation vectors are specified by three independent variables, each expressed in units of $\vec{z}$. The magnitude of the vector for P'_1 is specified by x_8 , the magnitude of the vector for P'_2 is specified by x_9 , magnitude of the vector for P'_3 is specified by x_{10} . For the exemplary shape shown in figure are $x_5 = 61.73$, $x_6 = 55.22$ and $x_7 = 48.70$.

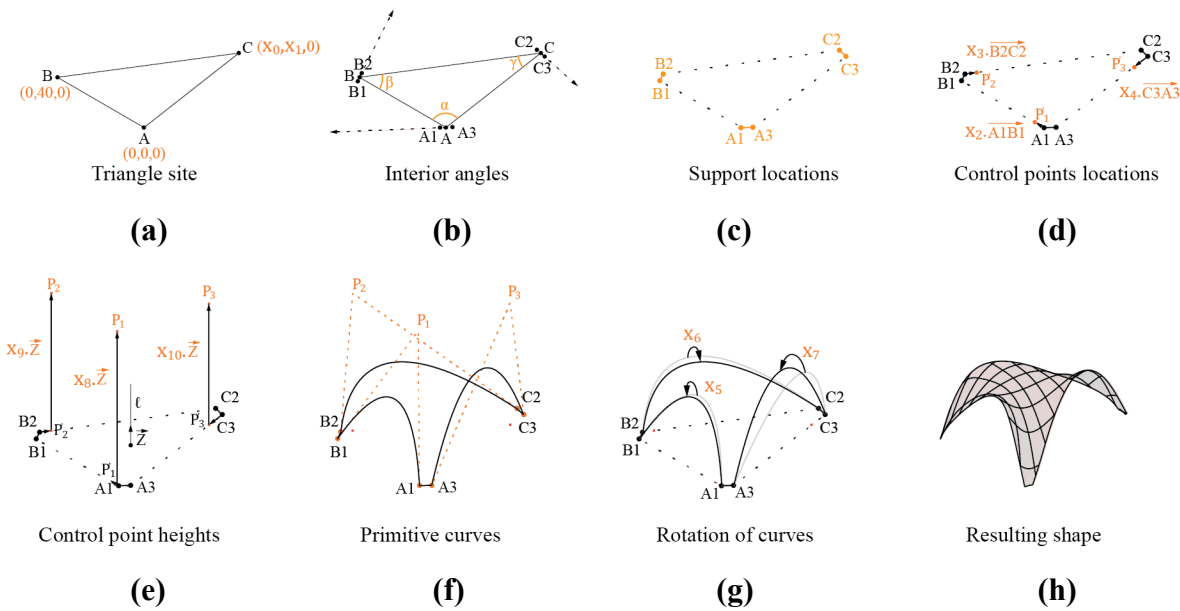


Figure 3.2: Parametric definition of the shell shape.

The resulting three points are shown in figure 3.2e and labelled P_1 , P_2 and P_3 . As the unit vector $\vec{z}$ is perpendicular to the base triangle, so that the plane defined by the set of points $\{A1, P_1, B1\}$ has a vertical orientation, and the same applies to the planes

defined by the set $\{B2, P_2, C2\}$ and the set $\{C3, P_3, A3\}$. Therefore, the three NURBS curves being carried out per set of points, yield curves that are each oriented vertically, too. The curves are referred to as primitive curves in figure 3.2f. The last step towards defining the boundary for the shape is rotating each primitive curve around an axis that is identical to the triangle side above which the curve has been constructed. The rotation angles are specified in unit degree by x_5, x_6 and x_7 . A positive angle denotes rotation away from the line ℓ in $\vec{z}$ direction at the center of the triangle in figure 3.2e; whereas negative rotation angle denotes rotation toward the vertical line in the center of the triangle. In the particular shape example shown in the figure, the angles take the values $x_8 = 3.27$, $x_9 = -6.50$, and $x_{10} = -16.28$. The resulting curves are shown in figure 3.2h.

The contour of the shape is formed by the union of the set of three-line segments between support points with the set of three NURBS curves. The final shape is a surface obtained from the patch algorithm. The patch algorithm first places a plane through a number of points sampled from the contour curves in two orthogonal directions u and v . For the exemplary surface, 10 sample points in u direction and 10 in v direction are used. Then the surface patch deforms the vertices of the spline grid in $\vec{z}$ direction to match the sampled points on the contour and trim the surface exceeding the contour with minimal error, as seen in figure 3.2h.

3.2 Shape Attributes in Horizontal Sense

In this study, triangular plots are addressed. Their occurrence in architecture is generally due to the shape of the site. The reason such shapes are dealt with is that they possess fewer shape parameters than shapes with four or more corners. This is to keep the case simple enough for studying the effectiveness of the computational design method proposed in this study. The eight floor plans, shown in figure 3.3, have different types of triangle sites and embrace the challenges of a corner site. A varied range of design resolutions are taken in these samples. The projects in figures 3.3f, g and h take the corner geometry through and resolve the building in a sharp edge, the projects in figures 3.3a, c, d and e cut the façade before it touches the corner point and adds more windows on the resulting façade segment, while the project in figure 3.3b employs gently rounded corners on façades. As a result of their structural shapes, the interiors of these spaces are shaped accordingly.

In geometry, triangles are classified according to two types of geometric features: their sides lengths and the interior angles. With respect to sides, there four types of triangles: equilateral if all sides have the same length, isosceles if two sides have the same length and scalene if all sides have different lengths. With respect to angles, there four types of triangles: acute if all angles are less 90° , equiangular, right-angled if one angle is equal to 90° and obtuse if one angle is more than 90° . If we classify the exemplary projects due to the geometric definitions of triangles. When with respect to the sides, then figures a, b, c, d, e and f should be in the same group of scalene, figure g in the group of equilateral and figure h in the group of isosceles. It could be advocated that there is a similarity in their shape as an architectural character between a, b and e but definitely there isn't any them with c, d and f and neither among c, d and f. When with respect to the interior angles, then figures c and g should be in the same group of acute, figures a, b, e, f and h in the group of right and figure d in the group of obtuse. And again, it could be advocated that there is a similarity in their shape as an architectural character between a, b, e, f and h but definitely, there isn't any c and g. As can be understood from this analysis, geometric definitions are quite deficient in defining the architectural shape characters of triangles.

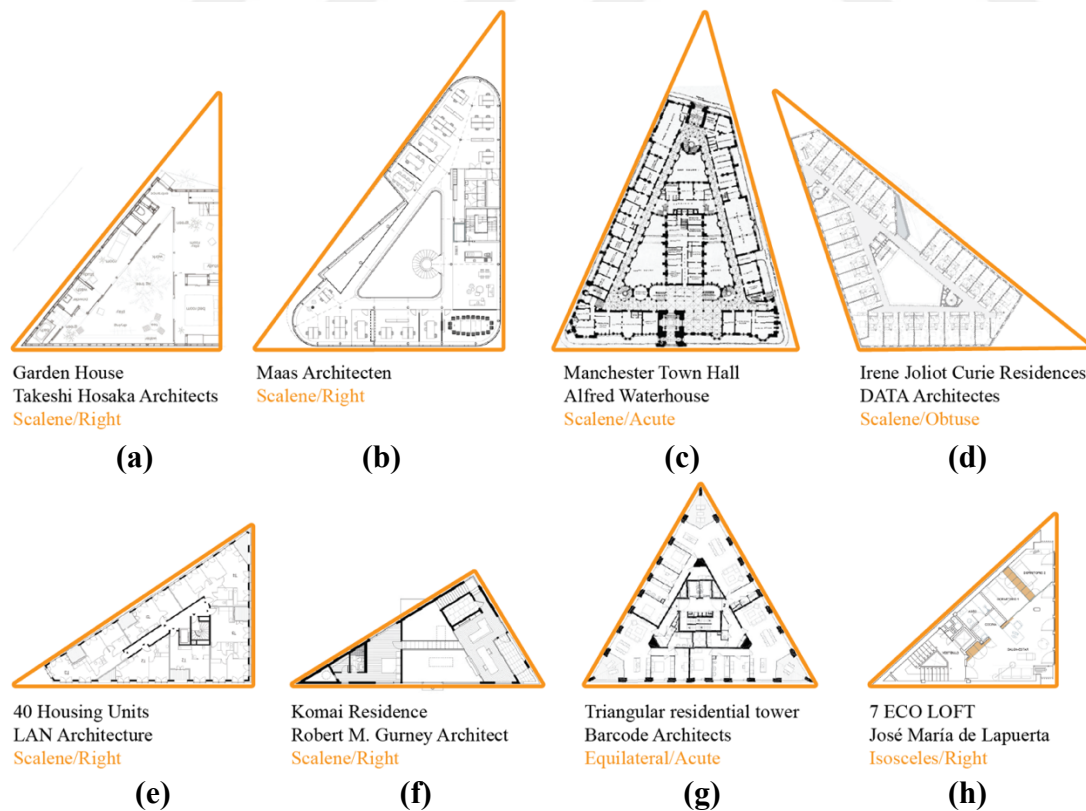


Figure 3.3: Some examples of triangle floor plans in architecture.

The base for our shape is a triangle, the vertices of which are labeled A , B and C , as it is mentioned in the previous section. Points $A1$ and $A3$, points $B1$ and $B2$, points $C2$ and $C3$ are the support location points which parametrically depend on the vertices of base triangle. A , B and C vertices are 3d points constructed with three coordinates (x, y, z) . In this study, point A and point B have constant coordinates which are $(0, 0, 0)$ and $(0, 40, 0)$. However, point C is constructed with one constant value for z and two independent variables $(x_0, x_1, 0)$ which makes C the one vertex determining the shape's triangular plan character. The shape's triangular plan is a right-angled triangle when $(x_0 \neq 0 \text{ and } x_1 = 0 \mid x_1 = 40)$, an isosceles triangle when $(x_0 \neq 0 \text{ and } x_1 = 20)$, an obtuse triangle when $(x_0 \neq 0 \text{ and } x_1 < 0 \mid x_1 > 40)$, and an equilateral triangle when $(x_0 = 20 \text{ and } x_1 = 34.64)$. For the exemplary shape shown in figure 5, $x_0 = 51.48$ and $x_1 = 7.80$. According to binary logic, the exemplary shape belongs to none of these triangular character groups.



4. ESSENTIAL ABSTRACTIONS OF A SHAPE

The second step of the model is the aggregation of marginal abstractions to represent the essential properties of the shape such as shape's being symmetric, asymmetric or vertical. In the following section, how the representation of overall shape character by fuzzy neural tree (FNT) and the representation of each shape property as FNT neuron are shown.

4.1 Fuzzy Neural Tree

An artificial neural network (ANN) represents in computational form the abstraction capability of brain to some extent. An ANN consists of interconnected neurons – that are referred to as nodes from the network organization viewpoint. A node picks up information from a network input or from other neurons, processes the information, and passes the result on to multiple other neurons. At each neuron the processing of the incoming information is carried out by means of a nonlinear function, mimicking the behavior of biological neurons to some extent. The knowledge a neural network is able to represent is stored in the connections among the neurons and their associated synaptic connection strengths. The strength parameters are referred to as the connection weights. Different arrangements of these networks with different training methodologies have been widely used to solve pattern recognition problems in modern application fields such as security, Medical diagnosis, and control of complex engineering systems. In these applications the transparency of a neural tree is a type of feed-forward neural computing structure organized not layer by layer, but node by node. This is in contrast to the conventional feed-forward neural network structures which have a hierarchical layer structure. In feed-forward ANNs every node at a lower level is connected to all nodes of the layer one level above it in the hierarchy. In a neural tree, every node has a unique set of input neurons and output neurons so that its structure can be considered a free form version of that of neural networks. This structural difference has important implications in the context of understanding architectural preferences as follows.

A neural tree structure consists of three different kinds of nodes. These are *terminal nodes*, inner nodes, and root nodes. Pairs of nodes are connected via links that have an associated weight value. The tree structure elements are schematically shown in figure

4.1a. The terminal nodes are input units. In the context of this study, they specify the marginal abstractions of the shape that will be described after the general properties of FNT have been laid down. Each inner node obtains input information from multiple other nodes, each of which may be either a terminal node or another inner node, as seen in the figure. The information reaching an inner node is aggregated by a non-linear function. This corresponds to the operation of a biological neuron to some extent. In FNT the non-linearity takes the shape of Gaussian function. The output of an inner node emulates the result of fuzzy logic operation, so that FNT can be considered as a fuzzy logic system representing human-like reasoning. However, due to the computational treatment, the output value is to interpret as a likelihood (Ciftcioglu & Bittermann, 2015).

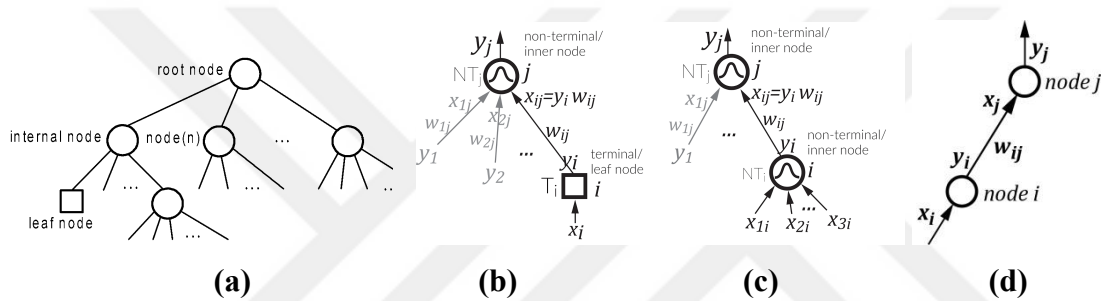


Figure 4.1: (a) Structure of a neural tree; (b) connection from a terminal to an inner node of a FNT; (c) from an inner FNT node to another inner node; (d) same as case c, yet for single node input, taken from (Ciftcioglu & Bittermann, 2015).

Figure 4.1b shows the detailed view of a connection from a terminal node i to an internal node j . Figure 4.1c shows the connection from an internal node i to another internal node j . Figure 4.1d is the simplified form of figure 1c, where each node has a single input. In figures 4.1b-4.1d a connection weight is denoted by w_{ij} , and the output of node i is denoted by y_i . The input x_{ij} of neuron j is given by $x_{ij} = y_i w_{ij}$.

4.1.1 Membership function concept in FNT

A fuzzy membership function expresses the association strength of a linguistic label and an object. It associates a number between zero and unity with the different values a variable specifying an object at hand can take. The number is referred to as a membership degree as it specifies the degree by which a specific object belongs to the set of objects that carry the same label. Several equivalently valid interpretations apply to the membership degree concept, as neatly exemplified by (Bilgiç & Türkşen, 2000).

In the context of fuzzy neural tree, the *random set* interpretation applies. The meaning is best explained with an example. Consider the vague predicate John (x) is tall (T)’ being represented by a number t in the unit interval $[0, 1]$. One possible answer to the question ‘What does it mean to say $t=0.7$?’ is referred to as the random set view. When a given population of experts is asked to individually provide an interval of height that corresponds to ‘tall,’ then 70% of the experts provided an interval that includes John’s height in it, whereas the intervals of 30% of the experts do not include John’s height in it.

4.1.2 Representing logic inference by inner FNT nodes

Based on this interpretation of fuzzy membership the input of an inner FNT neuron is considered to be a Gaussian random variable. Referring to node j in the simplified case in figure 4.1d, the input x_j of the node is a random variable, which can be modelled as a Gaussian probability density around a mean x_m . The probability density is given by

$$f_{x_j}(x_j) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma_j^2}(x_j - x_m)^2} \quad (4.1)$$

where x_m is the mean; σ_j is the width of the Gaussian probability distribution characterizing input to node j .

Definition: Assuming a statistical model parameterized by a fixed and unknown θ the likelihood $L(\theta)$ is the probability of the observed data x considered as a function of θ . In the likelihood context the unknown mean x_m is the likelihood parameter θ of our statistical model. The likelihood function of the mean value is given by

$$L(\theta) = e^{-\frac{1}{2\sigma^2}(x - \theta)^2} \quad (4.2)$$

Based on these considerations, as derived in (Ciftcioglu & Bittermann, 2015) the maximum likelihood of the output y_j is given by

$$y_j = L_j(\theta_j) = e^{-\frac{1}{2\sigma_j^2}(1 - y_i)^2} \quad (4.3)$$

Based on eq. (3), and additional possibility theoretic and information theoretic considerations, the non-trivial cases illustrated in figures 1.4b and 1.4c were treated in

the referred publication. Each input to a node is y_i assumed to be independent of the others, so that the maximum likelihood of the output of node j with n inputs is given by

$$y_j = L_j(\theta) = \exp \left[-\frac{1}{2\sigma_j^2} \sum_{i=1}^n \left[\frac{1-y_i}{\sum_{i=1}^n (1-y_i)} \right]^2 (y_i - 1)^2 \right] \quad (4.4)$$

From the fuzzy modeling perspective, the computation represents a logic operation of type AND. Applying the de Morgan's law the maximum likelihood of a neuron j with n inputs representing a logic operation of type OR is given by

$$y_j = L_j(\theta_j) = 1 - \exp \left[-\frac{1}{2\sigma_j^2} \sum_{i=1}^n \left(\frac{y_i}{\sum_{i=1}^n y_i} \right)^2 y_i^2 \right] \quad (4.5)$$

The sigma parameter characterizing the width of the Gaussians in (4) and (5) is subject to identification by optimization respectively. This is accomplished in both AND and OR cases by maximizing the same possibility theoretic condition, which is referred to as consistency condition in the publication referred to.

4.2 Marginal Abstractions of a Shape by Fuzzy Sets

Expressing the character of a shape is soft. When the marginal shape attributes are exactly matching to the prototypical example of a linguistic attribute, then the linguistic label undoubtedly applies to the shape. For instance, clearly a triangle with an angle of exactly 90° is, by all means, a right-angled triangle. However, when there is some discrepancy between the triangle at hand and its characteristic prototype, then the applicability of the label is dependent on the magnitude of the discrepancy. In the example, when the angle of a triangle deviates slightly from 90° then the triangle still to some extent has a right-angled character. The association of the linguistic attribute does not vanish abruptly with increasing discrepancy from the prototype, but gradually. This is a common experience in shape description and it applies in the interpretation of diverse shape attributes in linguistic terms. To represent this, the linguistic variable is represented in this study by a fuzzy set.

Accordingly, the family of shell shapes that can be generated by assigning different values to the eleven independent variables $x_0 \dots x_{10}$ are subject to interpretation using

fuzzy sets. In total 17 fuzzy membership functions (mf) are used for this purpose. Each mf shape is determined in two steps as follows. First, based on a general notion about the character of the marginally abstract property the mf shape is selected by the author. Then, the mf function parameters are optimized to minimize the discrepancy between model output and expert data that has been obtained via the survey. In the selection of the shape, the guiding principle is to keep the degree of non-linearity of the function low, yet minimally compromising the probable effectiveness in the second step of the modeling. The survey outputs are in Appendix C and the mf identification procedure is explained in greater detail in section 4.3 and it is exemplified in section 5.2.

In the context of modeling of shape preferences by FNT, the marginal abstractions described in the following subsections provide details of the computations carried out at the input nodes of the tree.

4.2.1 Symmetry of a shape with respect to its elevations

A set of 6 membership functions is used for evaluating the symmetry characteristics of the shape. They are shown in figures 4.2d - 4.2i. The symmetry of the shape is dependent on the control point locations of the three curves. The symmetry evaluation takes place through multiple fuzzifications; two for *Curve 1*, two for *Curve 2*, and two for *Curve 3*.

The two graphs shown in figures 4.2d and 4.2e depend on x_2 as they concern the evaluation of *Curve 1*. The role of x_2 with respect to the shape of the curve is illustrated in figure 4.2a, which is a view perpendicular to the $\overrightarrow{A1B1}$. The shape of the curve is analyzed regarding its axial symmetry, referring to the Z axis. This is due to the two membership functions y_1 and y_4 shown in the graphs. The shape of *Curve 1* is perfectly symmetrical around the Z axis when $x_2 = 0.5$ and the function $y = f(x_i)$ is a normalized Gaussian centered at $x_1 = 0.5$, measuring the symmetry of *Curve 1* in the form of a fuzzy membership degree. It is worth noting that the location of the center is inherently given by meaning of the shape quality concept our fuzzy set expresses, namely *axial symmetry*. In figure 4.2d the value $\sigma_1 = 0.16$ is used, so that the symmetry of the exemplary shape in figure 4.2a, where $x_1 = 0.016$, is evaluated to be $y_1 = 0.01$.

The shape of *Curve 1* is perfectly asymmetrical with respect to the Z axis under two conditions. In case $x_2 = 0$, the asymmetry arises as the control point P_1 lands above the point $A1$ and in case $x_2 = 1$, then the asymmetry arises as the control point P_1

lands above the point $B1$. The limits for the locations of P_1 as the points $A1$ and $B1$ are inherent to the curve creation method described in the previous subsection. The extreme locations of the two half parabolas are at the same limits, since there the curve is the most *asymmetric*. The membership function of asymmetry is a quadratic function $y(x_i) = -a(1 - x_i)^2 + 1$ when $x_i > 1 - \sqrt{1/a}$, $y(x_i) = -ax_i^2 + 1$ when $x_i < \sqrt{1/a}$ and $y(x_i) = 0$ when $1 - \sqrt{1/a} \leq x_i \leq \sqrt{1/a}$. In figure 7d the value $a = 4.97$ is used, so that the asymmetry of the exemplary shape in figure 7a, is evaluated to be $y_4 = 1.00$.

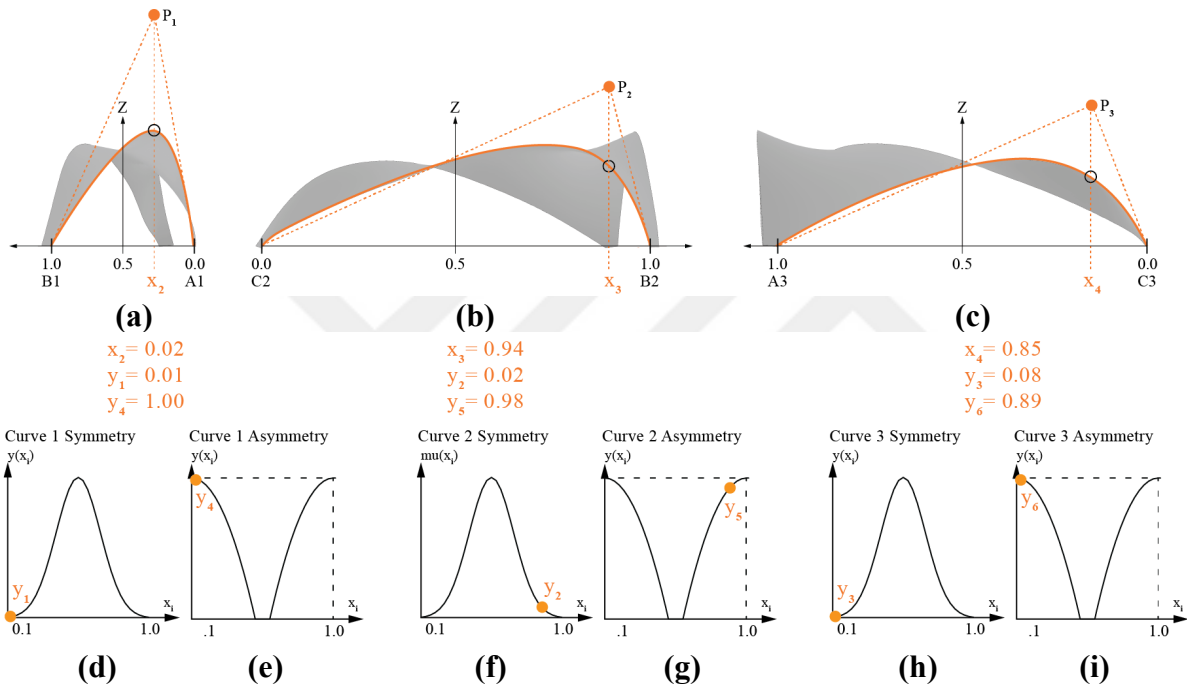


Figure 4.2: Symmetry and asymmetry membership degrees of the exemplary shape.

The two graphs shown in figures 4.2f and 4.2g depend on x_3 as they concern the evaluation of *Curve 2*; those graphs in figure 4.2h and 4.2i on x_4 as they concern the evaluation of *Curve 3*. The role of x_3 with respect to the shape of *Curve 2* is illustrated in figure 4.2b, which is a view perpendicular to the $\overrightarrow{C2B2}$. The shape of the curve is analyzed regarding its axial symmetry, referring to the Z axis. This is due to the two membership functions y_2 and y_5 shown in the graphs. The shape of *Curve 2* is perfectly symmetrical around the Z axis when $x_3 = 0.5$ and the function $y = f(x_1)$ is a normalized Gaussian centered at $x_3 = 0.5$, and for the symmetry assessment of *Curve 2*, the same value for the width of the Gaussian $\sigma_2 = \sigma_1 = 0.16$ and for the asymmetry assessment, the same value for the slope of the parabola $a = 4.97$ are used. Accordingly, the symmetry of exemplary shape in figure 4.2b is evaluated to be $y_2 =$

0.02 and the asymmetry $y_{52} = 0.98$. The symmetry and asymmetry of *Curve 3* are measured in the same way as it is done for *Curve 1* and *Curve 2*. The symmetry of the exemplary shape in figure 4.2c, where $x_4 = 0.85$, is evaluated to be $y_3 = 0.08$ and the asymmetry $y_6 = 0.89$.

4.2.2 Verticality of a shape with respect to its elevations

A set of 9 membership functions is used for evaluating the verticality of the shape. They are shown in figures 4.3d-4.3l. The verticality of the shape is due to the orientation of the three curves. Therefore, the verticality evaluation takes place through multiple fuzzifications; three for *Curve 1*, three for *Curve 2*, and three for *Curve 3*.

The three graphs shown in figures 4.3d, 4.3e, and 4.3j depend on x_5 , as they concern the evaluation of *Curve 1*. The role of the angle x_5 with respect to the verticality of the curve is illustrated in figure 4.3a, which is a view perpendicular to the $\overline{B1A1}$. The verticality of the curve is measured with reference to a vertical plane passing through the line $\overline{B1A1}$, where we distinguish three possible situations for the curve orientation. We remember that $x_5 > 0$ denotes rotation away from the line ℓ in figure 3.2e, while $x_5 < 0$ denotes rotation towards the line. In case the rotation is away from the line, the verticality of the curve is referred to as *overhang*. In case the rotation is toward the line, the verticality of the curve is referred to as *recessed*. In case the rotation is parallel to the line, in other words the rotation degree is zero, the verticality of the curve is referred to as *vertical*.

Curve 1 is perfectly vertical when $x_5 = 0$. Therefore the function $y = f(x_i)$ is a normalized Gaussian centered at $x_5 = 0$, measuring the verticality of the curve in the form of a fuzzy membership degree. It is worth noting that the location of the center is inherently given by the meaning of the shape quality concept our fuzzy set expresses, namely being *vertical*. In figure 4.3j the value $\sigma_{10} = 0.46$ is used, so that the verticality of the exemplary shape in figure 4.3a, is evaluated to be close to unity.

Unlike *vertical*, both the maximum and the minimum values for *overhang* and *recessed* situations are problematic to specify for a domain expert. This is because the concept of them are ill-defined and dependent on the context in which they are used. The extreme locations of the half parabolas in figure 4.3d and 4.3e represents the maximum and minimum overhang and recessed values which are determined with the data obtained by the survey results. Although these two are apparently similar concepts

but the two conditions are not equivalent from the geometric viewpoint. When $1.76r \geq x_5 \geq -0.24r$, $y(x_i) = -a(c - x_i)^2 + 1$ and represent being overhang, when $0.22r \geq x_5 \geq -1.84r$, $y(x_i) = -a(x_i - c)^2 + 1$ and represents being recessed. Accordingly, the verticality of *Curve 1* is evaluated to be an overhang with a fuzzy membership $y_7=0.27$ and is evaluated to be a *recessed* with a fuzzy membership $y_{13} = 0.15$.

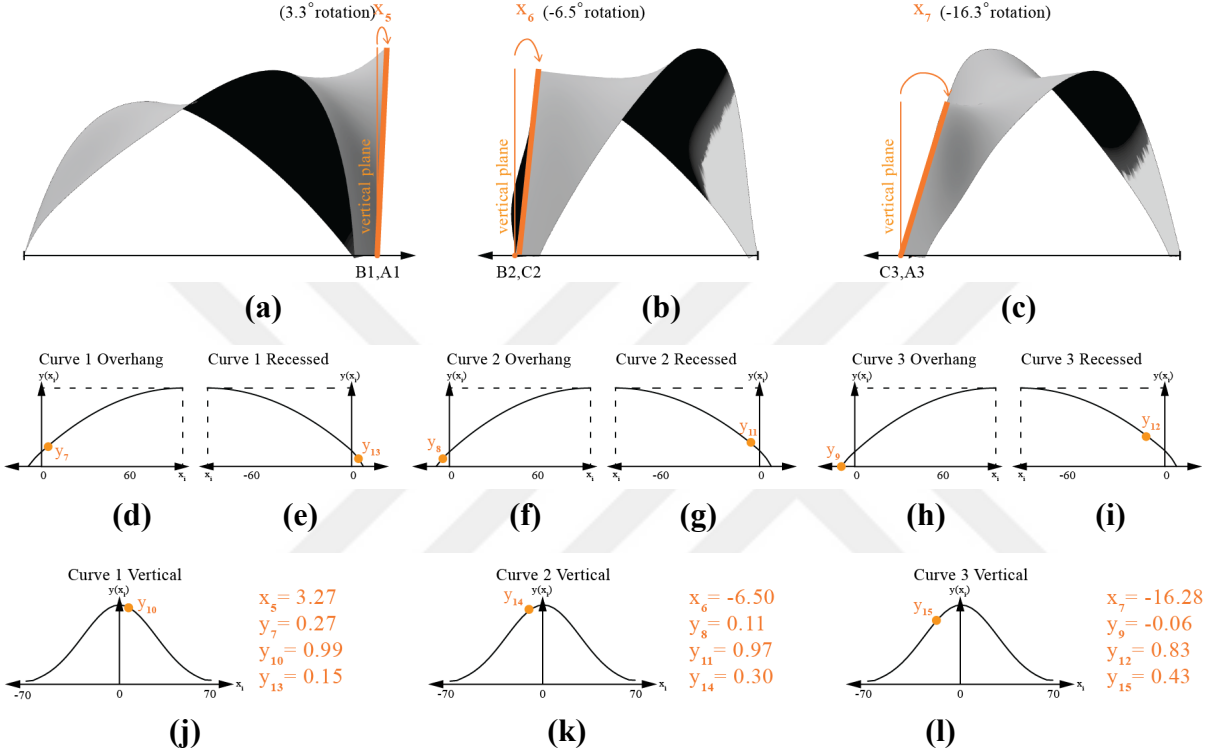


Figure 4.3: Overhang, verticality and recessed membership degrees of the exemplary shape.

The three graphs shown in figures 4.3f, 4.3g, and 4.3k depend on x_6 , as they concern the evaluation of *Curve 2*. The role of the angle x_6 with respect to the verticality of the curve is illustrated in figure 4.3b, which is a view perpendicular to the $\overline{B2C2}$. The three graphs shown in figures 4.3h, 4.3i, and 4.3l depend on x_7 , as they concern the evaluation of *Curve 3*. The role of the angle x_7 with respect to the verticality of the curve is illustrated in figure 4.3c, which is a view perpendicular to the $\overline{C3A3}$. In the same way as we did for *Curve 1*, also for *Curve 2* and *Curve 3* we distinguish the same three possible situations of vertical, overhang, and recessed, respectively. As none of the three sides of our shape has a particular geometric status compared to the other two, the membership functions $y_8 = f(x_6)$ and $y_9 = f(x_7)$ have the same a and c values as y_7 . Likewise the membership functions $y_{14} = f(x_6)$ and $y_{15} = f(x_7)$ have

the same a and c values as y_{13} ; and the membership functions $y_{11} = f(x_6)$ and $y_{12} = f(x_7)$ have the same σ value, $\sigma_{10} = \sigma_{11} = \sigma_{12} = 0.46$ as y_{10} . For *Curve 2* the higher verticality degree *vertical* is evaluated to be $y_{11} = 0.97$ and recessed is $y_{14} = 0.30$ and for *Curve 3* the higher verticality degree *vertical* is evaluated to be $y_{12} = 0.97$ and recessed is $y_{15} = 0.43$.

4.2.3 Height of a shape with respect to its elevations

A set of 4 membership functions is used for evaluating the height characteristics of the shape. They are shown in figures 4.4d-4.4g. The heights of the shape are dependent on the verticality of the three curves, as well as their respective symmetries. Therefore, the evaluation takes place through multiple fuzzifications. The heights of three curves are represented with the variables h_1, h_2 , and h_3 illustrated in figure 4.4a, 4.4b and 4.4c, which are the distances in z direction between the highest point of each curve and the base plane. All figures are parallel projections; 4.4a is to the $\overline{A1C1}$, 4.4b to the $\overline{B2C2}$ and 4.4c to the $\overline{A3C3}$.

The graph shown in figure 4.4d is to measure the similarity among the heights of *Curve 1*, *Curve 2* and *Curve 3*. To measure the similar height relation of a shape, we carry out three pairwise comparisons, each time comparing the difference between heights by considering two curves at a time, *Curve 1* and *Curve 2*, *Curve 2* and *Curve 3*, *Curve 1* and *Curve 3*. The deviation from zero implies some diminished membership in the fuzzy set. Clearly, *similar* character is unity when each difference is equal to zero. This dependence of differences is represented by logical AND operation implemented in the form of a FNT neuron. In the exemplary shape in figure 4.4, the height relation is perfectly similar when $|h_1 - h_2| = 0, |h_2 - h_3| = 0$ and $|h_1 - h_3| = 0$. The function $y(x_i) = (ax_i + 1)$ is a linear function measuring the similar height relation of the shape in the form of a fuzzy membership degree. It is worth noting that the slope of the graph is inherently given by meaning of the shape quality concept our fuzzy set expresses. In figure 4.4d the value $a = -0.04$ is used, so that for the exemplary shape shown in figure 4.4 the similarity is evaluated to be $L_{11}(x_i) = 0.82$.

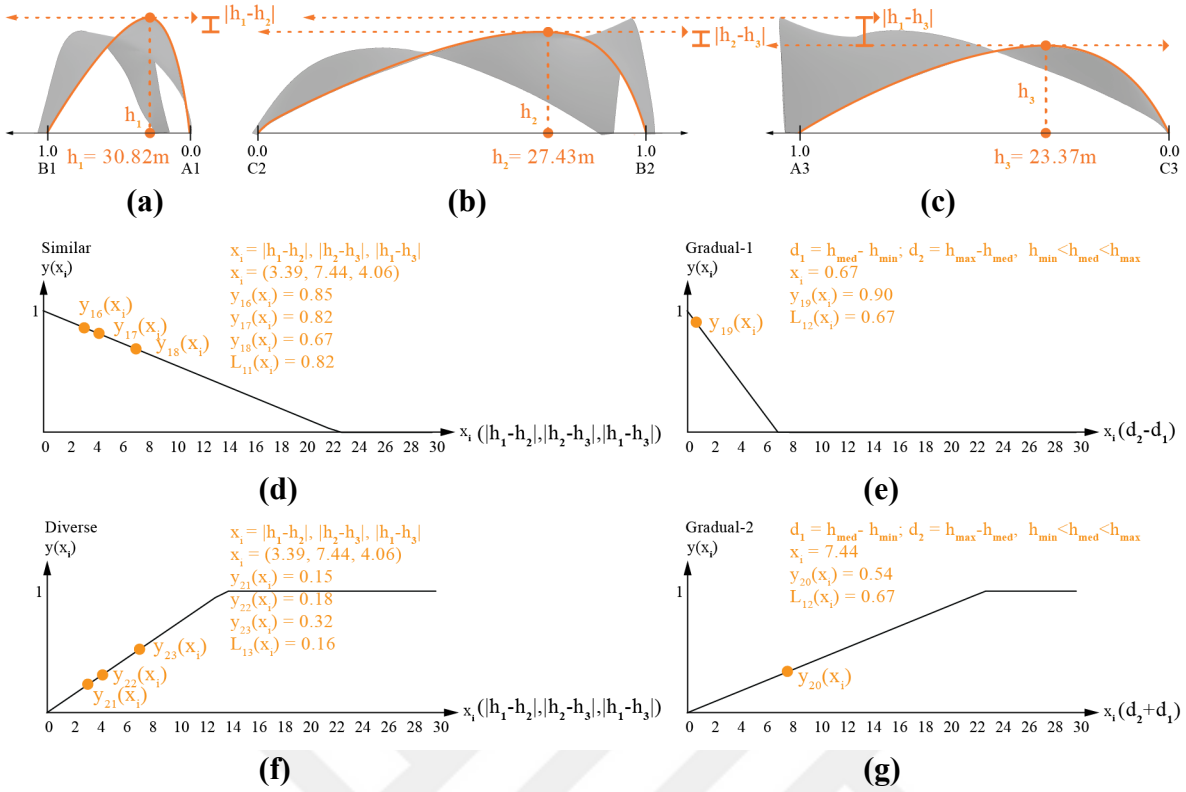


Figure 4.4: Similar, gradual, and diverse membership degrees of the exemplary shape.

The graphs shown in figures 4.4e and 4.4g are to measure the graduality among the heights of *Curve 1*, *Curve 2* and *Curve 3*. To measure the *gradual* height relation of a shape, we calculate two things. First, three possible differences that can be measured among three heights d_1 , d_2 , and d_3 and the gradualness of three quantities is maximal, when the two smallest absolute height differences are equal to each other. Second, the sum of the two smallest absolute height differences and the gradualness of three quantities is exists when the two smallest absolute height differences are big enough. This dependence of two conditions is represented by logical AND operation implemented in the form of a FNT neuron. For these calculations, we sort the absolute values of the three differences $|d_1|$, $|d_2|$, and $|d_3|$, so that d_{max} is the greatest, d_{med} is the median, and d_{min} the smallest of the absolute values. With this preparation we are ready to define the input dimension of the first membership function y_{19} as the sum of $d_{max} + d_{min}$ and the input dimension of the second membership function y_{20} as the difference of $d_{max} - d_{min}$. The two membership functions of *gradual* are $y(x_i) = 1 - (d_{max} - d_{min})/a$ and $y(x_i) = (d_{max} + d_{min})/2b$. The weights of FNT AND logic operation are equal to $w_9 = 0.41$ and $w_{10} = 0.59$. In figure 4.4e the value $a = 7.01$ and in figure 4.4g the value $b = 6.86$ are used, so that for the exemplary shape shown in figure 4.4 the similarity is evaluated to be $L_{11}(x_i) = 0.67$.

The graph shown in figure 4.4f is to measure the diversity among the heights of *Curve 1*, *Curve 2* and *Curve 3*. To measure the *diverse* height relation of a shape, we carry out three pairwise comparisons, each time comparing the difference between heights by considering two curves at a time, *Curve 1* and *Curve 2*, *Curve 2* and *Curve 3*, *Curve 1* and *Curve 3*. The deviation from zero implies raised membership in the fuzzy set. Clearly, *diverse* character is unity when each difference is equal to enough amount. This dependence of differences is represented by logical AND operation implemented in the form of a FNT neuron. In the exemplary shape in figure 4.4, the height relation is perfectly similar when $|h_1 - h_2| \neq 0$, $|h_2 - h_3| \neq 0$ and $|h_1 - h_3| \neq 0$. The function $y(x_i) = (ax_i)$ is a linear function measuring the *diverse* height relation of the shape in the form of a fuzzy membership degree. It is worth noting that the slope of the graph is inherently given by meaning of the shape quality concept our fuzzy set expresses. In figure 4.4f the value $a = 0.04$ is used, so that for the exemplary shape shown in figure 4.4 the diversity is evaluated to be $L_{13}(x_i) = 0.16$.

4.2.4 Triangular plan character of a shape

A set of 4 membership functions is used for evaluating the plan characteristics of the shape. They are shown in figure 4.5b-4.5e. The triangular plan character of the shape is dependent on the location of start vertices namely *A*, *B* and *C*. Since, the shape is considered to fit on a flat topography *z* values of vertices are always equal to zero and it is possible to obtain all kinds of triangles by changing the coordinates of only one vertice, we set *A* to (0,0,0) and *B* to (40,0,0) as constant variables in this study. Therefore, the evaluation takes place through multiple fuzzifications of the (*x*,*y*) coordinates of the vertice *C* namely x_0 and x_1 . In this study, x_0 can take any number between 30 – 150 and x_1 can take any number between 0-80. In the exemplary shape, $x_1 = 51.48$ and $x_0 = 7.80$. The variables α , β and θ illustrated in figure 4.5a, which are the internal angles of triangle, depend on the independent variables x_0 and x_1 . α represents the angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$, β represents the angle between $\overrightarrow{BA}$ and $\overrightarrow{BC}$ and θ represents the angle between $\overrightarrow{CA}$ and $\overrightarrow{CB}$. In the exemplary shape, $\alpha = 81.38deg = 1.42rad$, $\beta = 57.98deg = 1.01rad$ and $\theta = 40.64deg = 0.71rad$.

As it is seen in figure 4.5a, when point C has a constant value $x_1 = 51.48$ and moves only on the line $y = 51.48$ in other words moving horizontally, the triangular plan character of the shape changes accordingly. For example, in case $x_0 = 0$, point C lands above the point A and in case $x_0 = 40$, point C lands above the point B , then the shape is a right-angled triangle. In case $x_0 = 20$, point C lands above the midpoint of the line $\overline{AB}$ and the shape is an isosceles triangle. In case $x_0 > 40$, the shape is an obtuse triangle. When point C has a constant value of $x_0 = 20$ and moves only on the line $x = 20$ in other words moving vertically, the shape is bound to be an isosceles triangle. Only in case $x_1 = 34.64$, point C lands above the midpoint of line $\overline{AB}$ and the shape becomes an equilateral triangle.

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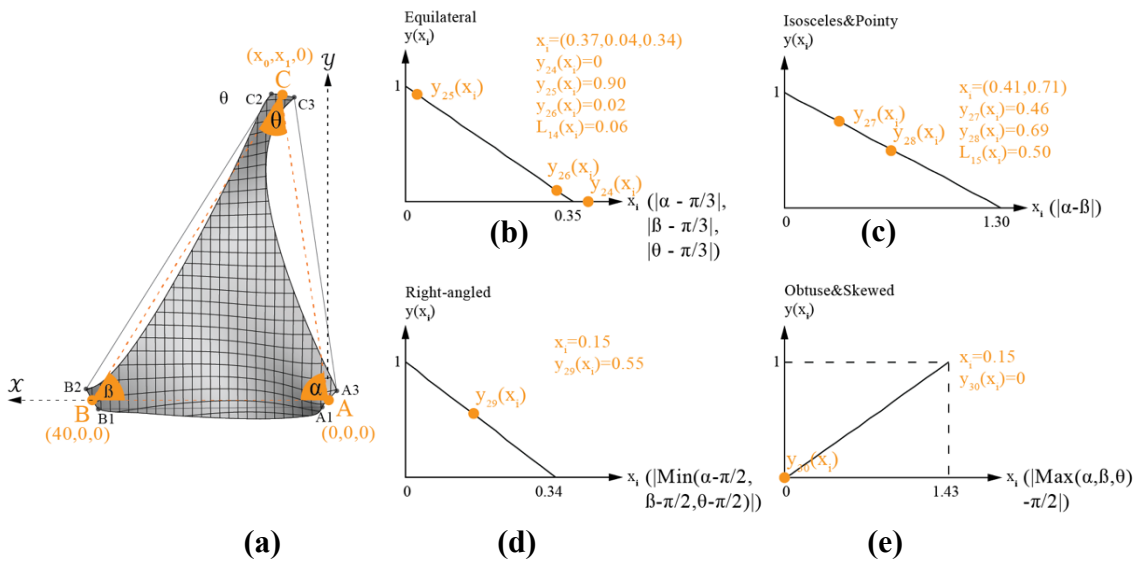


Figure 4.5: (b) Equilateral, (c) isosceles&pointy, (d) right-angled and (e) obtuse & skewed membership degrees of the exemplary shape.

As it is seen in figure 4.5a, when point C has a constant value $x_1 = 51.48$ and moves only on the line $y = 51.48$ in other words moving horizontally, the triangular plan character of the shape changes accordingly. For example, in case $x_0 = 0$, point C lands above the point A and in case $x_0 = 40$, point C lands above the point B , then the shape is a right-angled triangle. In case $x_0 = 20$, point C lands above the midpoint of the line $\overline{AB}$ and the shape is an isosceles triangle. In case $x_0 > 40$, the shape is an obtuse triangle. When point C has a constant value of $x_0 = 20$ and moves only on the line $x = 20$ in other words moving vertically, the shape is bound to be an isosceles triangle. Only in case $x_1 = 34.64$, point C lands above the midpoint of line $\overline{AB}$ and the shape becomes a perfectly equilateral triangle.

There are two alternative ways to describe one via length or angle. we take angle here, while the other manner is applicable too. Expressing the triangular character of a plan is soft. Of course, when the three angles are exactly equal the label *equilateral*, when one angle is exactly equal to $\pi/2$ the label *right-angled* and when one angle is exactly equal to $\pi/2$ the label *right-angled* undoubtedly applies to the triangle. Besides, when the two base angles are exactly equal although the top angle has any degree the top angle takes, the label *isosceles* undoubtedly applies to the triangle. However, the degree of the top angle in an isosceles triangle has a distinctive role in defining the triangular character of it. We call triangle's being a long type of isosceles in this study as *isosceles&pointy*. And similarly, when wider angle has any degree bigger than $\pi/2$, the label *obtuse* undoubtedly applies to the triangle. However, the degree of the wider angle in an obtuse triangle has a distinctive role in defining the triangular character of it. We call triangle's being skewed enough obtuse in this study as *obtuse&skewed*. When there is a small difference from the prototypical triangle, described in the previous paragraph, still to some degree the prototypical character prevails. i.e. the linguistic attribute does not vanish abruptly but gradually. To represent this, the linguistic variable equilateral is represented by a fuzzy set.

The graph shown in figure 4.5b is to measure the “equilateral -ness” of the shape. To measure the equilateral-ness of the triangle, we carry out three pairwise comparisons, each time comparing one of the angles with the angle $\pi/3$. The deviation from $\pi/3$ implies some diminished membership in the fuzzy set. Clearly, equilateral character is unity when each angle is equal to $\pi/3$. In case this condition is not fulfilled for one of the angles the equilateral-ness vanishes. This dependence is represented by logical

AND operation implemented in the form of a FNT neuron. In our coordinate system the shape in figure 4.5a, the plan is perfectly equilateral when $x_0 = 20$ and $x_1 = 34.64$. The function $y = (ax_i + c)$, is put forward as fuzzy membership function characterizing the set of “equilateral” triangles. By the function every shape is assigned a degree of equilateralness by a fuzzy membership degree. It is worth noting that the slope of the graph specifies the change of meaning of the shape quality as the triangle deviates from perfectly equilateral. In figure 4.5b the value $a = -2.90$ and $c = 1$ is used, so that the “equilateral-ness” of the exemplary shape, is evaluated to be $L_{14}(x_i) = 0.06$.

The graph shown in figure 4.5c is to measure the “isosceles&pointy-ness” of the shape. To measure the isosceles&pointy-ness of the triangle, we carry out two measurements, one is the absolute difference between two angles α and β to evaluate how much the shape’s base angle different from each other, and second is the smallest angle among all angles to evaluate how much the shape is being lanky in other words it has one corner pointy. In case one of the two conditions is not fulfilled, the isosceles&pointy-ness vanishes. This dependence is represented by logical AND operation implemented in the form of a FNT neuron. The shape of plan is perfectly isosceles when $x_0 = 20$. Keeping $x_0 = 20$ and moving x_1 in y direction so that $x_1 > 34.64$, then the triangle changes from being an equilateral one to being an isoceles&point one. The function $y(x_i) = (ax_i + c)$ is a linear function measuring the “isosceles&pointy -ness” of the shape in the form of a fuzzy membership degree. It is worth noting that the slope of the graph is inherently given by meaning of the shape quality concept our fuzzy set expresses. In figure 4.5c, the value $a = -1.86$ and $c = 1.21$ is used, so that the “isosceles&pointy-ness” of the exemplary shape in figure 4.5a, is evaluated to be $L_{15}(x_i) = 0.50$.

The graph shown in figure 4.5d is to measure the “right-ness” of the shape. To measure the right-ness of the triangle, we carry out three measurements which are the absolute differences between $\pi/2$ with the angles α, β and θ , and then we take the minimum one among those differences to evaluate how far the shape’s closest angle to $\pi/2$ is close to be right angle. The shape of plan perfectly is right-angled when $x_0 = 0 \mid x_0 = 40$ and $x_1 > 0$. The function $y(x_i) = (ax_i + c)$ is a linear function measuring the “right-ness” of the shape in the form of a fuzzy membership degree. It is worth noting that the slope of the graph is inherently given by meaning of the shape quality concept

our fuzzy set expresses. In figure 4.5d, the value $a = -2.99$ and $c = 1$ is used, so that the “right-ness” of the exemplary shape in figure 4.5c, is evaluated to be $y_{29}(x_i) = 0.55$.

The graph shown in figure 4.5e is to measure the “obtuse&skewed-ness” of the shape. To measure the obtuse&skewed-ness of the triangle, we compare the angles α , β and θ themselves and take the maximum one among them and measure the difference to $\pi/2$ evaluate how far the shape’s widest angle from $\pi/2$. The shape of plan is perfectly obtuse&skewed when $x_0 > 40$ and $x_1 > 0$ The function $y(x_i) = (ax_i + c)$ is a linear function measuring the “obtuse&skewed-ness” of the shape in the form of a fuzzy membership degree. It is worth noting that the slope of the graph is inherently given by meaning of the shape quality concept our fuzzy set expresses. In figure 4.5e the value $a = 1.80$ and $c = -0.37$ is used, so that the “obtuse&skewed-ness” of the exemplary shape in figure 4.5a, is evaluated to be zero.

4.3 Identifying Membership Function Parameters

The precise shape of fuzzy membership functions is problematic to specify due to the imprecision associated with the concepts they are to represent. In particular, the uncertainty refers that people differ to some extent about the meaning they associate with the concepts. As understanding implies some general validity of the knowledge, the identification of the membership function shape is a matter of obtaining and modeling data from multiple people. This is accomplished in this study by means of survey. The data obtained in this way is used to identify the shape parameters of the membership functions through curve fitting. The parameters are the width of Gaussians, the scales of quadratic functions, and the slope of linear functions. In total twenty-eight people participated in the surveys, seven of them are master students in architecture, eight of them are phd students in architecture, eight of them are architects with 2-6 years’ experience in architectural firms, and the remaining seven are academicians in architecture faculty.

Although an element whether a curve or a triangle does not fulfill the condition of the concepts, defined in section 3.1, perfectly but nearly, it still has some degree of that condition according to the fuzzy point of view. The main question of the survey is to associate some degrees of that condition such as “symmetrical-ness”, “vertical-ness”,

“equilateral-ness” to that element. Although the fuzziness of the concepts, results of the judgments are highly consistent with each other (can be seen in Appendix C) and mean squared errors (MSE) of the distance to the model are relatively low. The correlation between the model of this study and the means values of judgments’ can be seen in figures 4.6, 4.7, 4.8, and 4.9 in detail.

Survey D.1 is used to specify the width of Gaussian function in symmetry evaluation and to specify the curvature degree, a value, of quadratic function in asymmetry evaluation. It is asked participants to judge six curves with different control point location values. We wanted them to grade all the curves for both symmetry and asymmetry aspects by giving any number between 0 to 10. 0 means that the curve hasn’t got even a glimpse of symmetry character and 10 means the curve has definitely got a symmetry character. It is same for the asymmetry, 0 means that the curve hasn’t got even a glimpse of asymmetry character and 10 means the curve has definitely got an asymmetry character. The 6 orange circles in figure 4.6, represent the mean values of participants’ grades for each curve asked. To find the optimum σ value and a value, we used the mean squared error (MSE), the error between model and participants’ judgement of symmetry is equal to 0.008 and asymmetry is equal to 0.003.

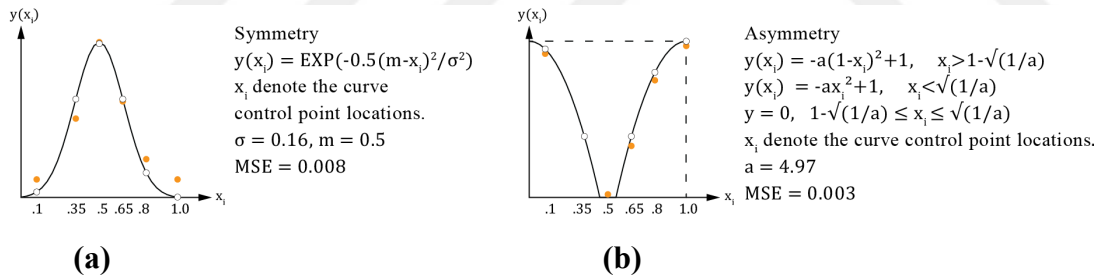


Figure 4.6: Optimized membership functions specifying the symmetry characteristics; (a) symmetry; (b) asymmetry.

Survey D.2 is used to specify the width of Gaussian function in *verticality* evaluation and to specify the curvature degree, a value and the location where it crosses the y axis, c value of quadratic function in *overhang* and *recessed* evaluation. It is asked participants to judge 12 curves with different verticality degrees. We wanted them to grade all the curves for verticality, overhang and recessed aspects by giving any number between 0 to 10. The orange circles in figure 4.7 represent the mean values of participants’ grades for each curve asked. To find the optimum σ value, a value and c value, we used MSE, the error between model and participants’ judgement of recessed is equal to 0.003, vertical is equal to 0.003 and overhang is less than 0.001.

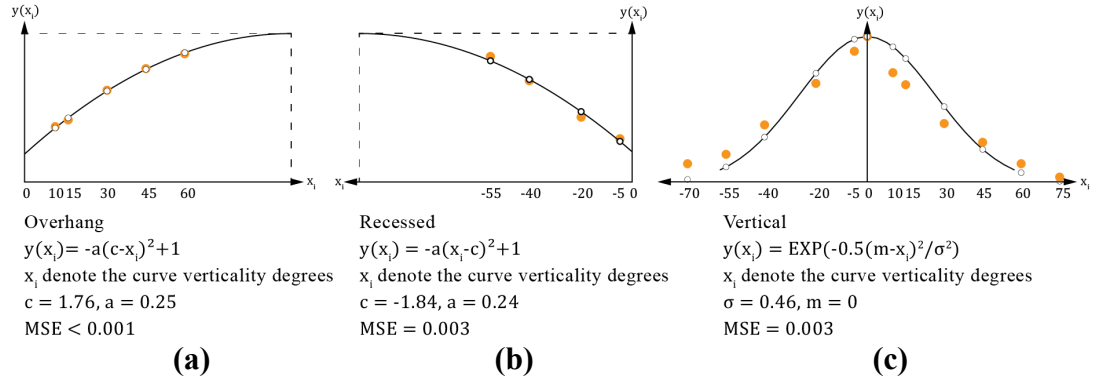


Figure 4.7: Optimized membership functions specifying the verticality characteristics; (a)overhang; (b)recessed; (c)vertical.

Survey D.3 is used to specify the slope a value, of linear function in similar and diverse height evaluations and to specify the slope a value and AND operation weight value for gradual-1 and b value and AND operation weight value for gradual-2. It is asked participants to judge different ten sets of three curves relationship. We wanted them to grade all sets for similarity, graduality and diversity aspects by giving any number between 0 to 10. 0 means that all three curves in set haven't got even a glimpse of similarity character and 10 means all three curves in set have definitely got a similarity character. It is same for graduality and diversity. The breaking points of orange line in figures 4.8a, 4.8b and 4.8c show the mean values of participants' grades for each set asked, the grey lines show the variance for mean values and the black lines show the model. To find the optimum a values, b value and weights for AND operation, we used MSE, the error between model and participants' judgement of similarity is equal to 0.013, graduality is equal to 0.018 and diversity is equal to 0.016.

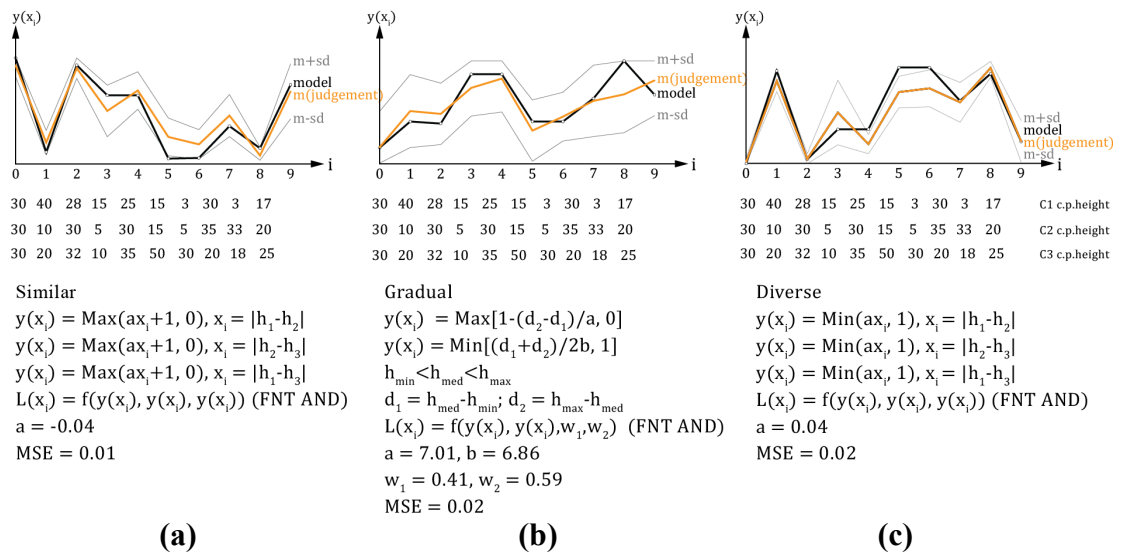


Figure 4.8: Optimized membership functions specifying the height relationship characteristics; (a)similar; (b)gradual; (c)diverse.

Surveys D.4, D.5, ..., D.9 are used to specify the slope a value of the linear function in triangular plan evaluations. This time triangle which is graded as 1.0 is given and the participants are asked to point out three triangles among 20 samples, to which they respectively assign the membership degree such as 0.1, 0.5, and 0.9. Grade 0.1 means that the triangle has got a glimpse of that specific character which is asked, 0.5 means that the triangle has to the same degree of that specific character as it hasn't got any of that specific character and 0.9 means that the triangle is nearly a perfect instance of the specific character. 4.9a shows how “equilateral-ness” changes when point C is moved to the right starting from a perfect equilateral triangle and 4.9b shows when point C is moved to up. 4.9c shows how “isosceles-ness” changes when point C is moved to the right starting from a perfect isosceles triangle and also shows how “pointy-ness” changes when point C is moved up. 4.9d shows how “right-ness” changes when point C is moved to the right starting from a perfect isosceles triangle. 4.9e shows how “skewed-ness” changes when point C is moved to the right and 4.9f shows when point C is moved to up. To find the optimum a values and c values of the linear function graph in figure 4.9, we used MSE, the errors between model and participants’ judgement of the equilateral quality is equal to 0.026, isosceles is equal to 0.001, right is equal to 0.017 and obtuse&skewed is equal to 0.011.

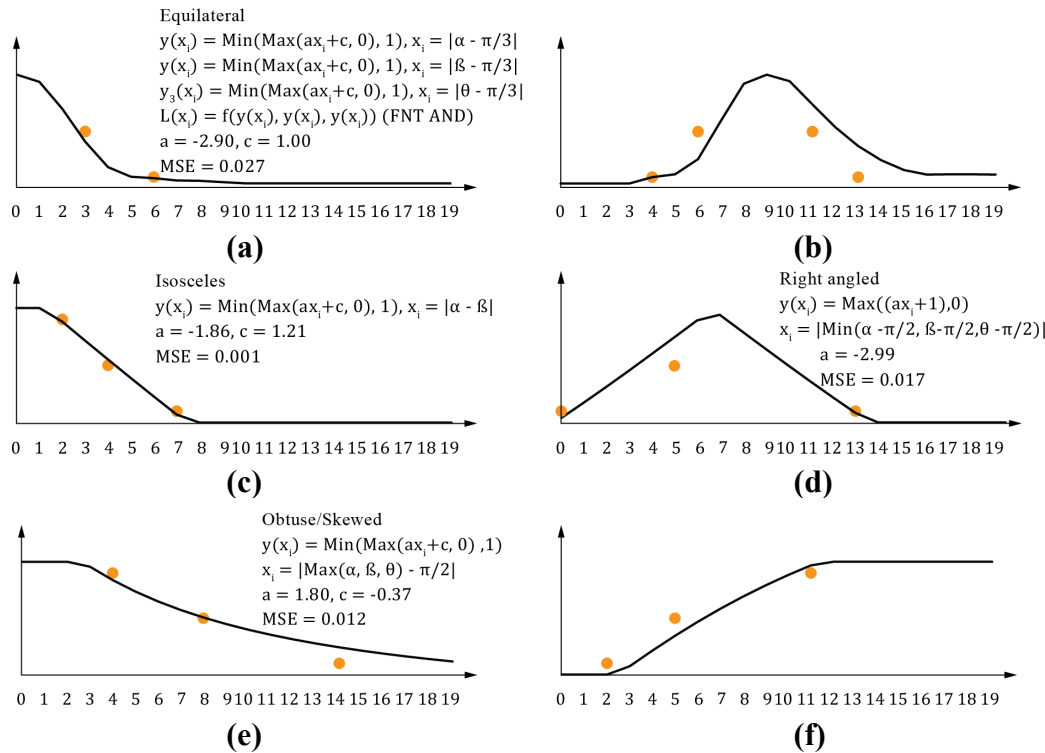


Figure 4.9: Optimized membership functions specifying the triangular characteristics; (a)(b)equilateral;(c) isosceles; (d) right-angled (d); (e)(f).obtuse&skewed.

4.4 Shape Properties by a FNT Neuron

Independent variables determining the shape subject to evaluation have been introduced in the previous section. The inputs to the tree are fuzzified by determining their degree of memberships in a fuzzy set that represents the marginally abstract property of the shape. The membership degree is used in the ensuing logic computations to obtain the essential shape properties.

The tree is consisting of the terminal nodes and inner nodes which are computed using the AND operations as described in the previous sections. The four kinds of properties of the shape namely symmetry, verticality and height are in vertical sense characters and the triangular plan is in horizontal sense and in total 17 conditions exist in those properties. Each shape satisfies each of the conditions by some degree. These are denoted by $L_1, \dots, L_{14}, L_{15}$ in figures 4.10, 4.11, 4.12 and 4.13.

With respect to the symmetry property, there are four types of symmetry conditions for characterizing the shape. They depend on control point locations (x_3, x_4, x_5) namely *all symmetry*, *two symmetry*, *two asymmetry* and *all asymmetry*. There doesn't exist a symmetry condition for three curves as it is equal to two asymmetry condition. All symmetry where symmetry of all three curves is computed by using the FNT AND operation and two symmetry where symmetry of two curves, which are the ones has similar edge lengths, is computed by using the FNT AND operation. Because only when the curves, which have similar edge lengths, are symmetric, then axial symmetry arises, otherwise plan character dominates the overall character of the shape, and symmetry cannot be observed. Exactly the same principle applies for asymmetry conditions.

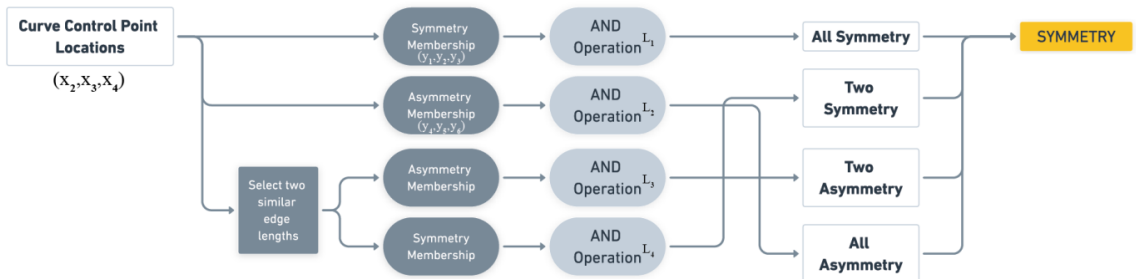


Figure 4.10: Symmetry property.

With respect to the verticality property, there are six types of verticality conditions, which depend on control point verticality degrees (x_5, x_6, x_7) , namely *all overhang*, *two overhang*, *all vertical*, *two vertical*, *two recessed*, and *all recessed*. All overhang

where overhang of all three curves is computed using the FNT AND operation and two overhang, where overhang of two curves which are the ones has similar edge lengths, is computed using the FNT AND operation. Because only when the curves, which has similar edge lengths, are overhang, then axial symmetry arouses, otherwise plan character dominates the overall character of the shape and symmetry cannot be observed. Exactly the same principle applies for vertical and recessed conditions.

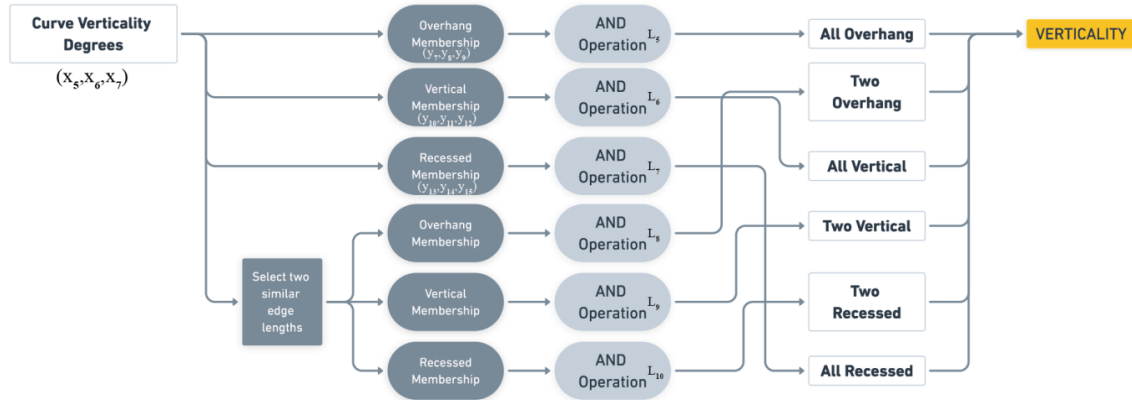


Figure 4.11: Verticality property.

With respect to the height property, there are three types of height conditions, which depend on curve heights (h_1, h_2, h_3), namely *similar*, *gradual*, and *diverse*. Height conditions represent the relationship type among three curves, so all conditions are computed using the FNT AND operation as all three curves' height memberships always should be taken into account. Only gradual condition has a FNT AND operation with weights. This is because the property gradual depends on two membership values in combination, namely *gradual-1* AND *gradual-2*. One notes that the weights associated with the two memberships in the FNT AND operation have been optimized to fit the result of the AND operation to the survey data.

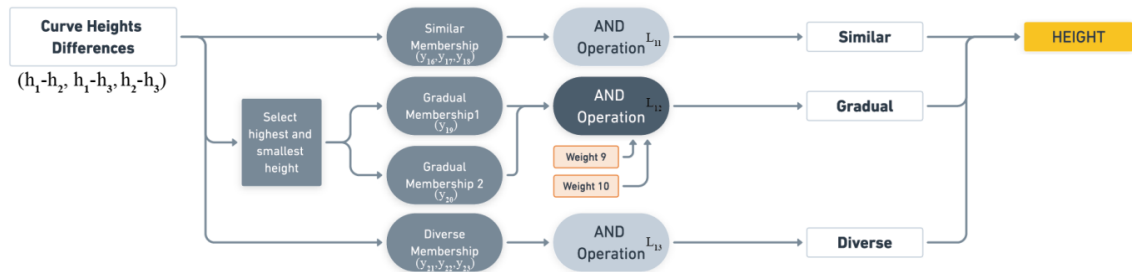


Figure 4.12: Height relation property.

With respect to the triangular plan property, there are four types of triangular character conditions, namely *equilateral*, *isosceles&pointy*, *right-angled*, and *obtuse&skewed*. They are depended on the interior angles of the triangle. *Equilateral* is computed using

the FNT AND operation as all the interior angles at the same time should close to be $\pi/2$ and also *Isosceles&pointy* is computed using the FNT AND operation as there are two different memberships to evaluate. The assessment of the right angled-ness of a triangle does not require a logic operation, since it is directly given by the membership degree of that angle among the three angles of the triangle that is closest to 90 degrees. Clearly, the values of the other two angles are dependent on the former so that their evaluations are superfluous in this context of right angled-ness. In the same way, the obtuse-ness of a triangle does not require a logic operation, since it is directly given by the membership degree of that angle among the three angles of the triangle that is most different to 90 degrees. Clearly, the values of the other two angles are dependent on the former, so that their evaluations are superfluous in the context of obtuse-ness.

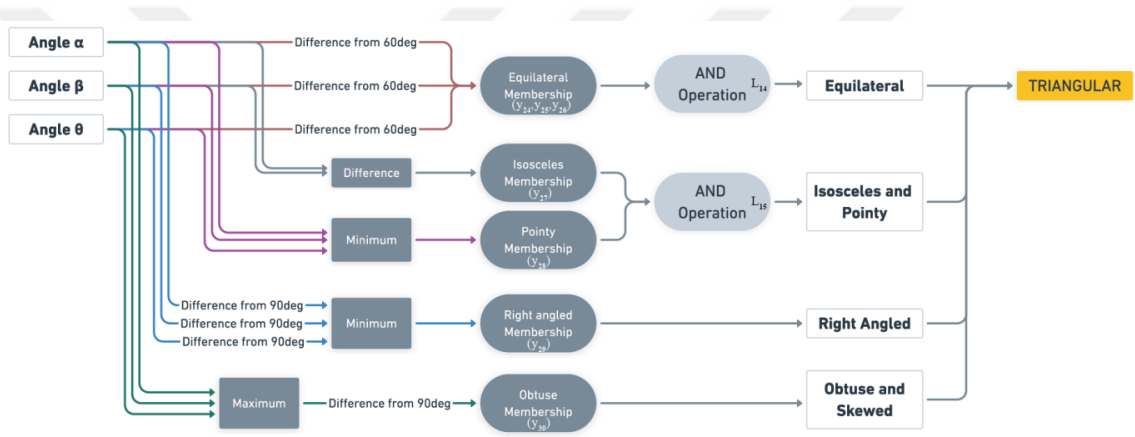


Figure 4.13: Triangular plan property.



5. FNT MODEL IDENTIFICATION

The third step of the model is tuning the representation of the shape character based on the preference data. In this section, the two criteria in the representation of the shape character and an exemplary FNT model identification of preference are described. It is to emphasize that the identification of the membership functions in the previous section has been accomplished based on survey data from multiple architects, so that the concepts represented by the functions have some general validity and that any shape preference making reference to these concepts is commonly interpretable. Till here, the basic shape descriptors, that different designers can agree with, are established. However, different designers can have different aesthetic preferences depending on the particular characteristics of a shape. Shape preference involves multiple such basic shape properties, in the sense that the simultaneous presence of a specific set of properties is more desirable than another one. Thus, understanding the shape preference of an individual or a group of people is finding that pattern of properties that best corresponds to the preference.

As preferences related to shape, and in particular those that are due to aesthetical inclinations, are merely loosely related to rationality, commonly architects are having difficulty pinpointing the rationale behind their preferences. The aim of this study is to get to know this rationale to alleviate the difficulty. Due to the elusive and tacit character of the rationale underlying aesthetical preference, the base for understanding a preference consists of manifestations of the preference; that is, actions by the architect or architects that indicate the superiority of a design over another. For an understanding of sufficient general validity, multiple manifestations of preference are necessary. The problem is thus to pinpoint that pattern of shape properties that is predominant among the multiple preferred designs.

In contrast to pattern recognition or machine learning, understanding implies that the representation of invariant relations in the data satisfies not one but two criteria. In machine learning, in general, the aim is to model data by minimizing the discrepancy between model output and the corresponding data. In representing understanding, there is a second criterion playing a role next to the aim to minimize the model error. The second criterion is that the description is of utmost meaningful. In the context of shape preference, the meaning is due to the inclusion of essential shape properties. To

let a description be of utmost meaningful thus entails letting the set of essential abstractions be present as completely as possible in the description. Explicitly this means the representation should include preferably all four qualities expressed by the neural tree branches in the previous section, which are symmetry, verticality, height relations, and triangularity of the shape. In the context of modeling, we refer to the meaningfulness of description as *expressiveness* of the model.

The two modeling criteria, minimizing the model error, and maximizing the *expressiveness* of the representation are conflicting. The conflict arises because abstraction inherently implies some model error so that stipulating the inclusion of as many as possible among the essential abstractions inherently increases the model error.

5.1 Evolutionary Computation

In the last decades, evolutionary computation turned out to be one of the main pillars of computational intelligence. Evolutionary computation approaches have been used to exploit the similarities between biological evolution searching through a space of genes and computer evolution searching through a space of computable data. There are several methods forming the realm of evolutionary computation. The methods generally are implemented in the form of an evolutionary algorithm (EA). The purpose of the algorithms is to search optimality in a voluminous search space involving nonlinear relations among multiple dimensions of this space. Therefore, local optima arise. The existence of multiple locally optimal solutions in the possible solution space makes finding the global optimum problematic when non-evolutionary methods are used. Another appealing property is that EAs do not require derivative information about the objective function to be effective. For this reason, the method found applications in a variety of domains such as engineering, and decision making.

5.1.1 Genetic algorithm

A widely used form of evolutionary algorithms is genetic algorithms (GAs). Genetic algorithm is a type of evolutionary optimization algorithm based on the mechanisms of genetics and natural selection. In the terminology of GA, each candidate solution to a given problem is referred to as a chromosome. The reason is that the decision variable values that constitute a solution represent the material from which new solutions are formed, where the formation mechanism resembles processes observed in biological

reproduction. GA carries out its search for optimality using multiple solutions at each iteration. The set of solutions is termed as *population*. GA simulates the genetic evolution of a population of individuals using a recombination operator known as *crossover* and a perturbation operator known as *mutation*. An initial population of solutions generated by a genetic algorithm is a random combination of the variables of the solutions within their respective boundaries. New solutions are created by the recombination operators for the next generation. The chromosomes of a population are graded using a scalar number termed as *fitness* that quantifies the performance of a chromosome as a solution to the problem. The GA is designed such that a higher fitness implies a greater likelihood the chromosome exerts influence on the composition of the next generation by contributing to the ‘genetic material’ it represents. The robustness of the method has been well established by ample experiments.

5.1.2 Evolutionary multi-objective optimization

Solving a multi-objective optimization problem by evolutionary computation is called Evolutionary Multi-objective Optimization (EMO). The concept of multi-objective optimization comes from the need to achieve compromise decision-making in a problem consisting of conflicting objectives. Such a problem gives rise to not a single optimal solution in the search space, but multiple equivalently valid solutions. Collectively these are referred to as the Pareto frontier. EMO effectively and efficiently identifies the frontier. This has practical importance since real-world problems can involve conflicting objectives (Deb, 2001).

In the present study, an EMO is used to identify the model representing in the best way the preferences of an architect. In this modeling context, the attribute ‘best’ refers to the two conflicting properties the model should possess namely *accuracy* of the model as well as its *expressiveness*. The meaning of these concepts is explained in section 5.2.3.

5.2 Computer Experiment in an Architectural Design

As a result of section 4, for any shape generated based on the independent attributes we have a qualified characterization of it. That is, we know which characteristics apply to shape at hand most. This is a base to understand the shape preferences of a person. In this section, FNT model developed for the representation of the shape character is

explained in detail and an exemplary identification for the shell shapes preferences of the architect, who is the author of this thesis, is shown.

5.2.1 FNT for understanding shape preferences

To understand the preferences, the principle is to obtain multiple manifestation instances of the person's preference. In this study, this takes the form of selecting among a set of random designs the subset of the designs that are deemed as preferable over the others by the architect. Based on this subset we want to understand what pattern of *shape characters* is most likely explaining the manifested preference. As the computation of the shape characters is non-linear, which is due to the Gaussianity of the likelihood computation at the inner nodes of the FNT, and the number of possible models is excessive, analytical model identification is a problematic issue. For this reason, an evolutionary algorithm is used.

This type of computation is suitable for the problem at hand because the evolutionary algorithm does not require derivative information about the fitting function. The evolutionary algorithm searches for the pattern of qualitative shape features that best explains the shape preferences of the architect. That is, the algorithm searches in a space of possible neural tree models for that one yielding maximal likelihood as to the preferred set of shapes. In this way, the preference is understood using computation. An exemplary identification is described below.

The outputs of the tree branches shown in figures 4.10, 4.11, 4.12 and 4.13 are ultimately combined via the FNT AND operation which is the output of the neural tree and represents the expressiveness of the representation of the shape character. The complete tree structure is shown in figure 5.1. The output of each tree branch has values with the number of possible property conditions in it. And the Select component is used to select one of the conditions with a number slider termed also as *weight* (w_i) in figure 5.1 (w_5, w_6, w_8 and w_9). As we don't know which property condition describes the shape character selected by the architect. To describe the shape, we search one of the conditions in each property. The condition selected is the one that has the highest numeric value among all conditions in each property branch. As any shape is described with one condition in each property and as we have four types of properties, the description of a shape character is accomplished with four values. If the representation is accomplished with the higher numerical values, then this means that

expressiveness of the representation is high and the shape can be defined with that character. For example, for the exemplary shape, described in section 3, the highest value 0.989 in symmetry property condition is all asymmetry, the highest value 0.972 in vertical property condition is all vertical, the highest value 0.817 in height property condition is similar and the highest value 0.551 in triangular property condition is isosceles&pointy.

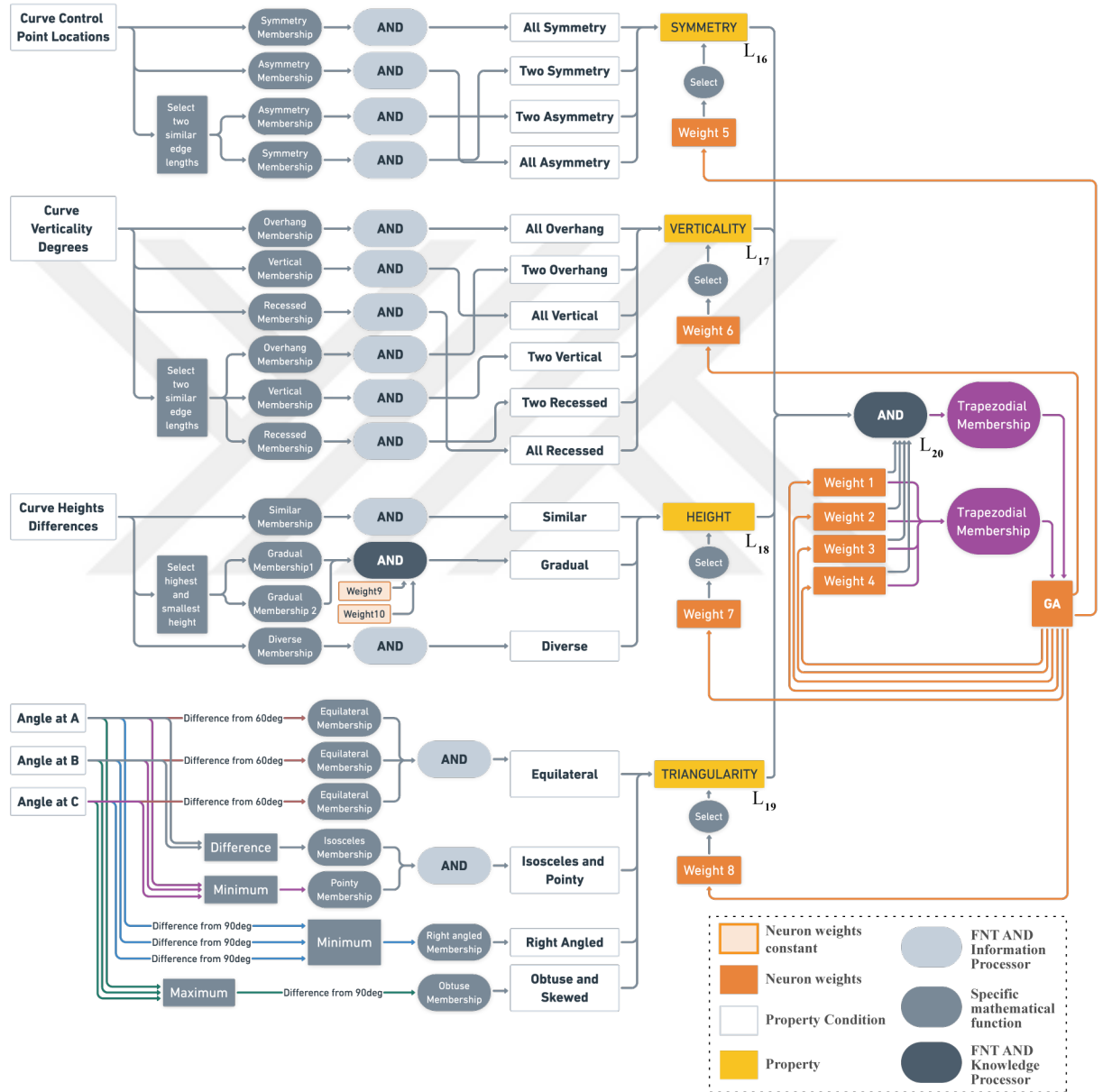


Figure 5.1: Fuzzy neural tree model developed in this study for an essential abstraction of the shape.

5.2.2 Preferred samples used for the model identification

For the identification of preferences, the architect selects a set of preferred shapes among a number of randomly generated ones. That is, multiple shapes are created by assigning random values between the minimum and maximum values of the decision variables. In this study, for the exemplary shape identification in figure 5.3, the architect preferred 10 shapes, shown in figure 5.2, among random 50 samples is shown in appendix A.1. The model identification procedure aims to find those parameter values specifying the FNT abstraction, such that the 10 preferred shapes are best represented.

When identifying preferences, the distribution of the shape characters in a randomly generated set of samples is as important as the selection of preferred ones. To ensure the variety of the characters in the set of samples, three different generating schemes are explored and the distribution of the shape characters in those three generating schemes can be seen in appendix A.2. The first generating scheme was generating a list of random numbers for 11 independent variables. 50 sample shapes are obtained.

The second generating scheme was generating 200 random numbers for every 11 independent variables with the odd numbers (starting from 1 to 21) as seed values. Then, those 200 samples are ordered to find the '*representative*' 50 samples by forcing the ordering process to include the ones which get higher values in each property. Here representative means including some samples with higher values per shape property. In the appendix, A.2 can be seen that although equilateral and obtuse characters do not appear with high probability when random values are assigned to the variables, there is a noticeable improvement as to the uniformity of the probability distribution.

Besides, those two generating schemes, we applied a third generating scheme which is generating a list of 400 random numbers for every 11 independent variables with the odd numbers (starting from 1 to 21) as seed values. Then again, those 400 samples are ordered to find the *representative* 50 samples by forcing the ordering process to include the ones which get higher values in each property. By doing this, we ensure that at least some samples show that character strongly and randomness in the distribution of properties is fair enough. For the exemplary identification in this study, we use the third generating scheme.

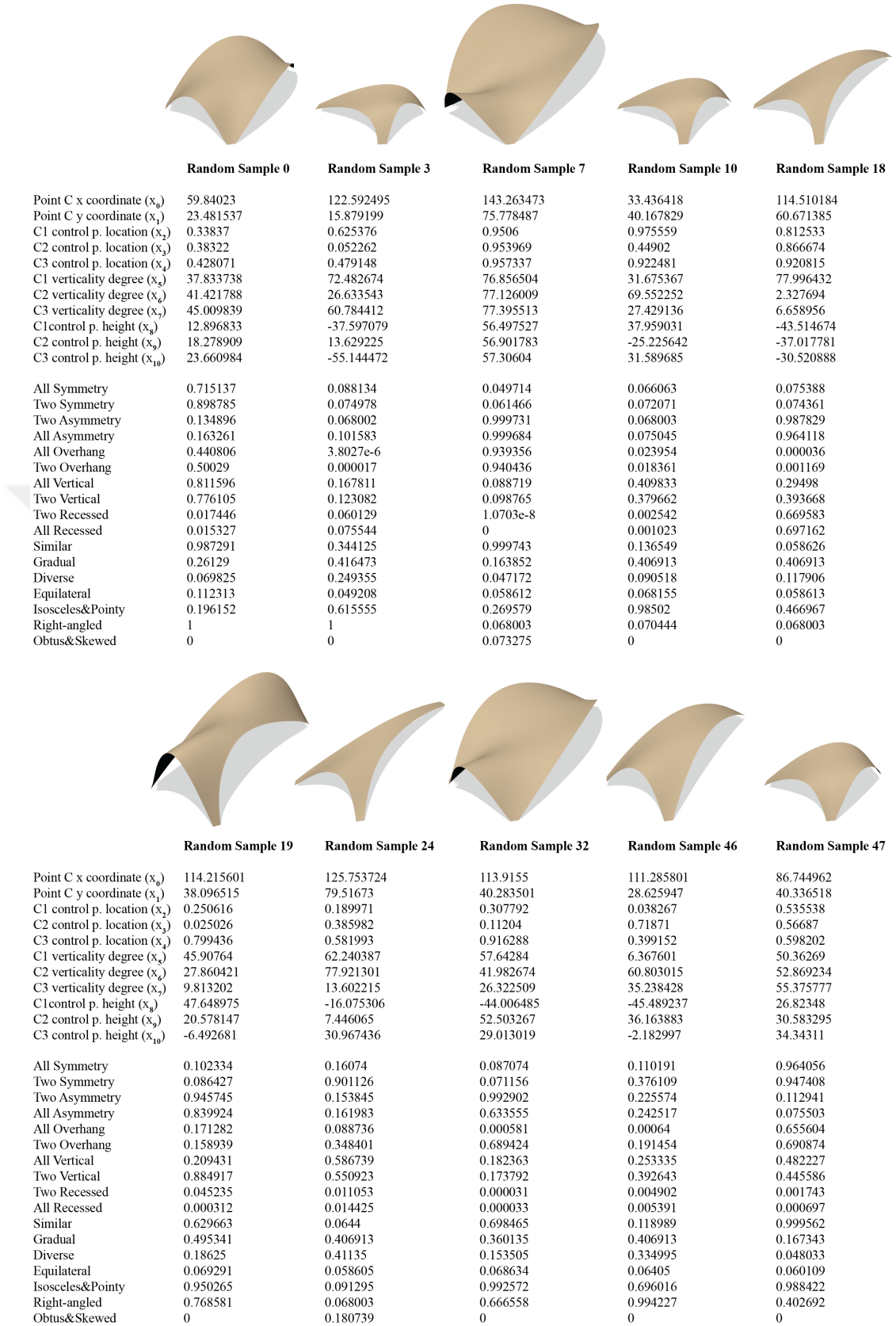


Figure 5.2: Preferred 10 samples

5.2.3 Preference identification

In this study, model identification refers to selecting the neural tree models among the possible ones that best represents the dataset in figure 5.2. Due to the four switches at the *property* neurons given in figure 5.1, the number of topologically unique tree structures is $4 \cdot 7 \cdot 3 \cdot 4 = 336$. The number of FNT models becomes excessively large, considering that next to the topological selection, four real numbers from the unit interval represent the weights associated with the AND operation neuron, which is the output of the neural tree and represents the *expressiveness* of the representation of the *shape character*. For one of the 336 topologies, each weight tuple corresponds to a different FNT model. The role of EC is to identify which topology and associated weight vector, underlie the preference of the architect. This FNT yields insight into the architect's preference, as each of the model constituents is interpretable without restriction. The transparency warrants that the resulting model allows 'understanding' of the architect's preference, beyond merely representing them.

The presence of multiple objectives in a problem generally implies multiple optimal solutions. Solutions are equivalently valid in the sense that they are all non-dominated in the space of the objectives; in our case, this refers to the *accuracy* and *expressiveness* of the model. Among the non-dominated FNT models, we look for that model yielding minimal size model error, while still being as expressive as possible in Pareto sense.

As described qualitatively in the previous section, in our context *expressiveness* of the representation, weights of the properties tend to be low, as this generally implies better model *accuracy*. Yet, we would like to express *shape character* ignoring as few characteristics in the expression as possible. The decisive quantities for our purpose are the mean value $\bar{w}_i$ of weights w_1, w_2, w_3, w_4 and the output value of FNT AND operation L_{20} . We calculate the trapezoidal membership degrees of both the output value L_{20} and the mean $\bar{w}_i$ as shown in figure 5.3. The criteria in our model identification are twofold;

maximizing *accuracy* $\mathcal{A}$

$$\max \mathcal{A} = 1 - \mathcal{Y}(L_{20}) \quad (5.1)$$

and maximizing *expressiveness* $\mathcal{E}$

$$\max \mathcal{E} = \bar{w}_i = \frac{1}{4} \sum_{i=1}^4 \mathcal{Y}(w_i) \quad (5.2)$$

simultaneously. In the equations 5.1 and 5.2, the function $\mathcal{Y}$ is a trapezoidal membership function given by

$$\begin{aligned} \mathcal{Y} &= 0 \text{ for } x < 0.5 \\ \mathcal{Y} &= 3.33x - 1.67 \text{ for } 0.5 \leq x < 0.8 \\ \mathcal{Y} &= 1 \text{ for } x \geq 0.8 \end{aligned} \quad (5.3)$$

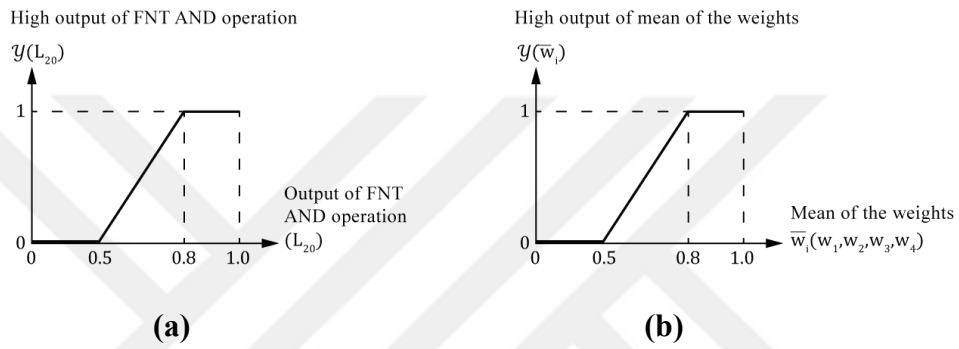


Figure 5.3: Trapezoidal membership functions of; (a) AND operation; (b) weights.

There are two extreme points due to the two conflicting objectives, one is where the weights w_1, w_2, w_3 , and w_4 are high and the other is where the output L_{20} is high. The attribute ‘high’ is a fuzzy concept expressed by the membership functions in figures 5.3a and 5.3b. In the case of a certain design, when the output value is lower than 0.5, then the design is not represented by the tree model at hand. In the same way, when one of the weights is lower than 0.5, then the associated property is excluded as a component of the expression. The decisive quantities for our purpose are the mean values of weights and output of FNT AND operation. Maximizing both of them means that as many as possible weights are high, and as many as possible designs are represented by the candidate FNT model being considered.

The genetic algorithm is run multiple times until as many as possible among the 10 preferred designs are represented. Multiple runs are necessary because multiple patterns of preferences are applied by the architect so that a single FNT model does not represent all solutions. With different possible combinations of property conditions and with a different possible degree of memberships, there appear a variety of shape characters. For example, when the plan has an *isosceles&pointy* character, the

preference of verticality might tend to be *two symmetry*, whereas when it has an *equilateral* character, the preference of verticality might tend to be *two asymmetry*, and vice versa.

Pareto fronts seen in figure 5.4 are reached after approximately 10 generations; the population size 200 and each time we remove the samples described by the model, and then the search is run for the remaining ones. With the first run of the algorithm, four shapes, with id numbers 7, 19, 32, and 47, are found to represent the pattern of the properties *two asymmetry*, *two overhang*, *similar height* in the vertical sense, and *isosceles&pointy* in plan sense character. As these designs are already explained, they are removed from the population of 10, and the identification procedure is repeated for the remaining 6 designs to understand the preference for them. With the second run of the algorithm, one shape, with id number 0 is defined as having *two symmetry*, *all vertical*, *similar height* in vertical sense, and *right-angled* in plan sense character.

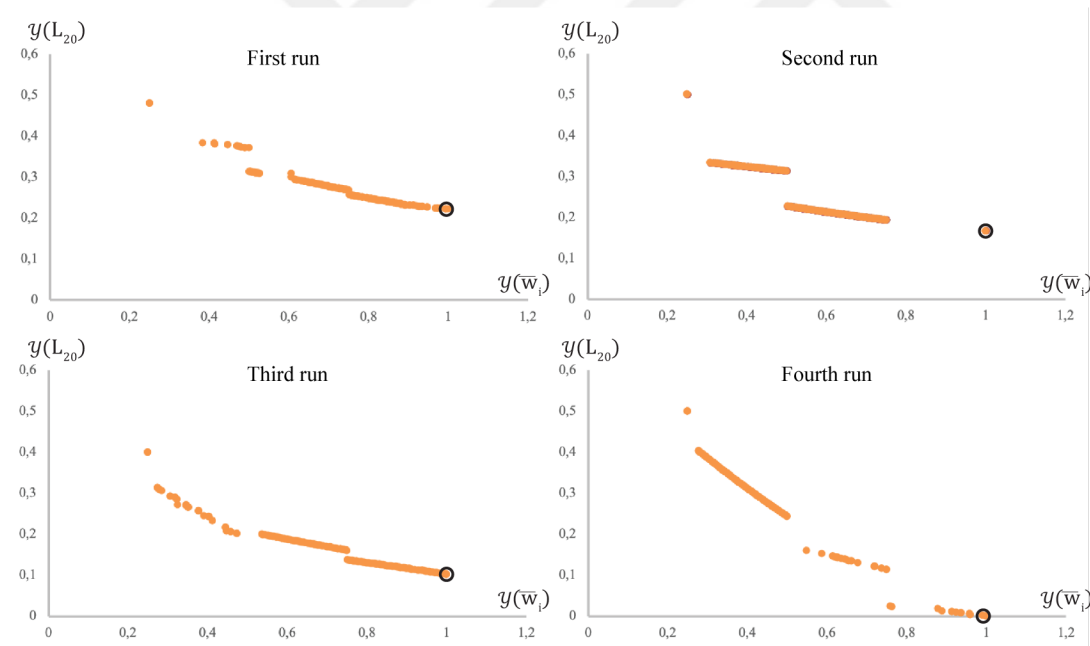


Figure 5.4: First, second, third and fourth preference identifications' pareto fronts.

The number of designs that are explained by now amounts to five; these designs are removed from the population of 10, and the identification procedure is repeated for the remaining 5 designs to understand the preference for them. With the third run of the algorithm, one shape, with id number 18 is defined as having *two asymmetry*, *all recessed*, *gradual height* in vertical sense and *isosceles&pointy* in plan sense character. The number of designs that are explained by now amounts to six; these

designs are removed from the population of 10, and the identification procedure is repeated for the remaining 4 designs to understand the preference for them. With the last run of the algorithm, one shape, with id number 46 is defined as having *two symmetry*, *two vertical*, *gradual* height in vertical sense and *isosceles&pointy* in plan sense character.

In pareto fronts seen in figure 5.4, the horizontal axis represents the trapezoidal membership degree of the mean value $\mathcal{Y}(\bar{w}_i)$, and the vertical axis represents the trapezoidal membership degree of the output value $\mathcal{Y}(L_{20})$. The solutions marked by black circles in figure 5.4 are the ones which describe the architect's preference with utmost *expressiveness*, while the model *accuracy* is still as high as possible.

5.2.4 Results and interpretation as to understanding preference

As mentioned before, to find the FNT model which explains the architect's preference, the algorithm searches in a space of possible fuzzy neural tree models. The structure of FNT model before the evolutionary search is shown in figure 5.5, where the weights of FNT AND and the selected conditions are unknown. And, the learning process of FNT model is shown in figure 5.6, in which evolutionary search identifies the weights and the condition with the data obtained from the user by the preferred shapes.

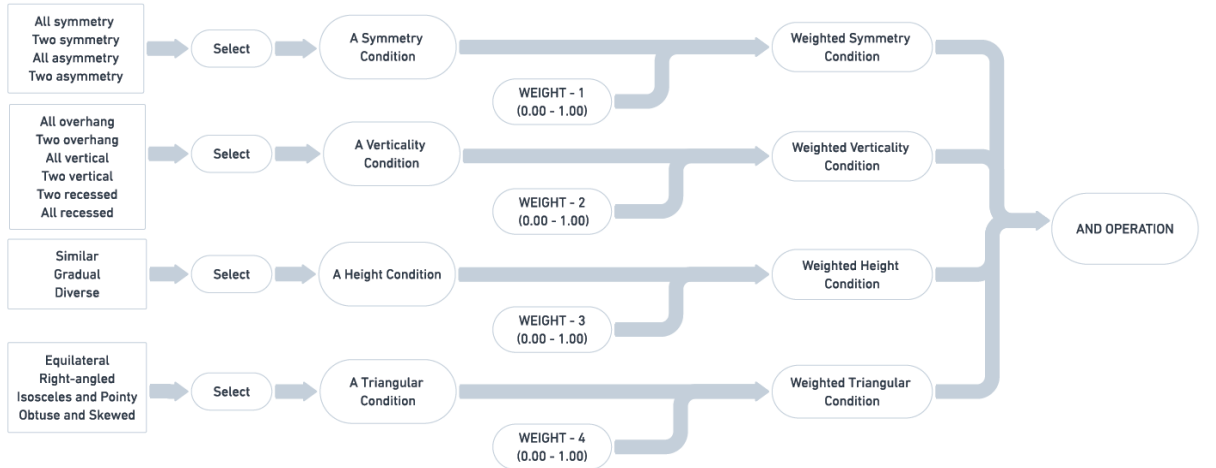


Figure 5.5: FNT models before learning.

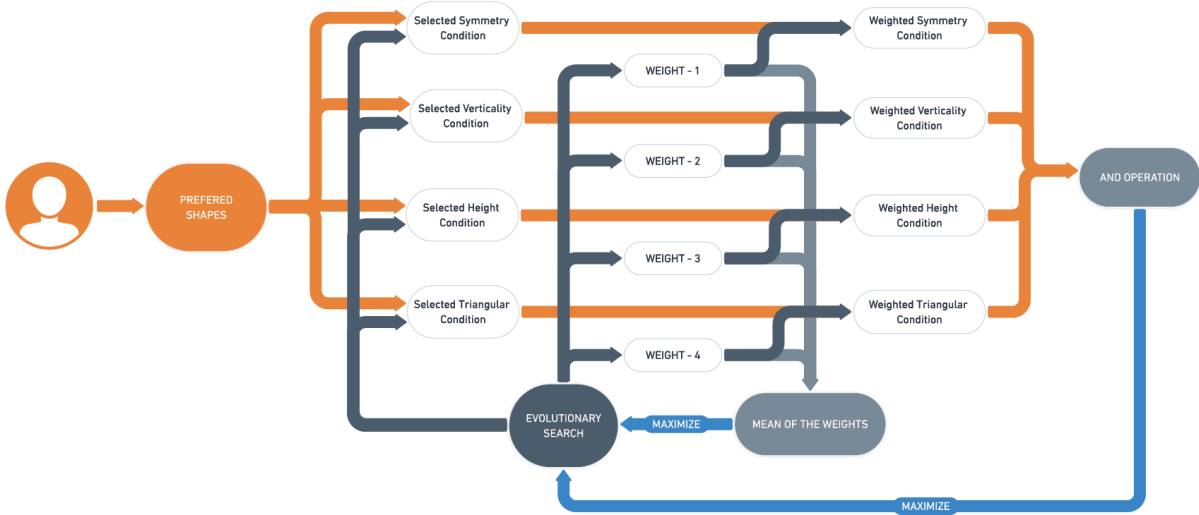


Figure 5.6: FNT models learning.

As the identification process reveals that there are four patterns of preferences applied by the architect. The resulting FNT models are four different trees shown in figure 5.7. That is different in the sense that the output value of FNT AND, the fuzzified value of FNT AND, the weights associated with the FNT AND are different. Table 5.1 shows the output values of the exemplary identification process described in the previous section.

First preference identification			Second preference identification		Third preference identification		Fourth preference identification	
ID	AND output	M.Function	AND output	M.Function	AND output	M.Function	AND output	M.Function
0	0,36268	0	0,973116	1				
3	0,191438	0	0,244842	0	0,177303	0	0,28943	0
7	0,722069	0,740229						
10	0,158773	0	0,130271	0	0,169189	0	0,162015	0
18	0,197642	0	0,107283	0	0,653235	0,510782		
19	0,568144	0,227146						
24	0,130202	0	0,253792	0	0,125332	0	0,318592	0
32	0,864917	1						
46	0,208644	0	0,287783	0	0,203644	0	0,50172	0,005733
47	0,575581	0,251936						
AND Node Weights	Highest properties		AND Node Weights	Highest properties	AND Node Weights	Highest properties	AND Node Weights	Highest properties
0,814	Two asymmetry		0,988	Two symmetry	1	Two asymmetry	0,846	Two symmetry
0,803	Two overhang		0,806	All vertical	0,999	All recessed	0,811	Two vertical
1	Similar		1	Similar	0,802	Gradual	0,788	Gradual
0,8	Isosceles/pointy		0,882	Right angled	0,803	Isosceles/pointy	1	Isosceles/pointy

Table 5.1: Preference identification weights and outputs in FNT.

Among four different trees, the first one can be considered as the most important one as it represents four of the preferred samples whereas the other three respectively represent one solution each. Based on, we find out that the character of the shape which is *two asymmetry*, *two overhang*, *similar* and *isosceles&pointy* is most likely explaining the manifested preference. That can be considered as the model of the aesthetic preference of the architect.

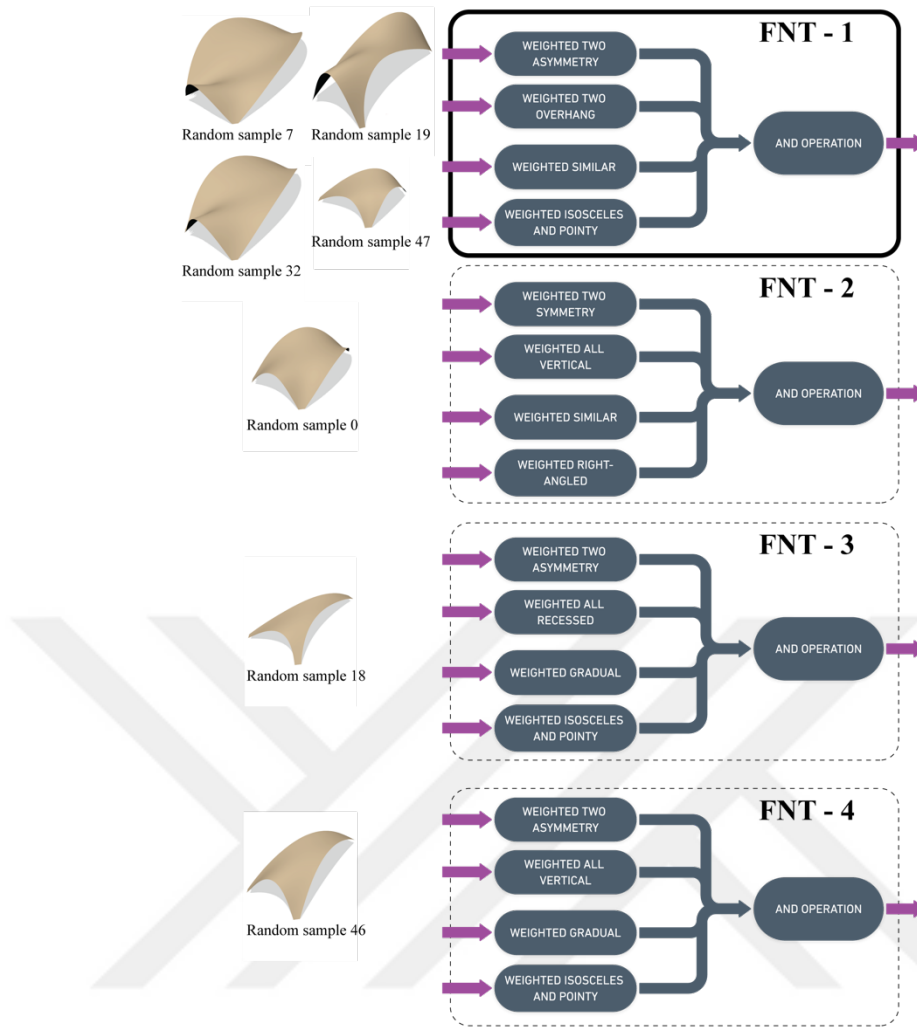


Figure 5.7: FNT models determined by the preference identification.

Finally, to understand the preference for any shell shape, the FNT model which is the model of the aesthetic preference of the architect can be used. When we put any shell shape, generated with the defined parameters, to this model, the greatest magnitude at the four FNT outputs predicts the likelihood that the architect will prefer the shape (in figure 5.8). When the output of the model is a high value, then this means that the shape will be most likely appreciated by the architect. The output tells us how much it will be liked and due to the transparency of the model, why it is liked or not. The discovered model of aesthetic preferences of the designer/user is in interpretable form, so that appreciation as a soft objective in design processes can be accomplished with great consciousness and precision.

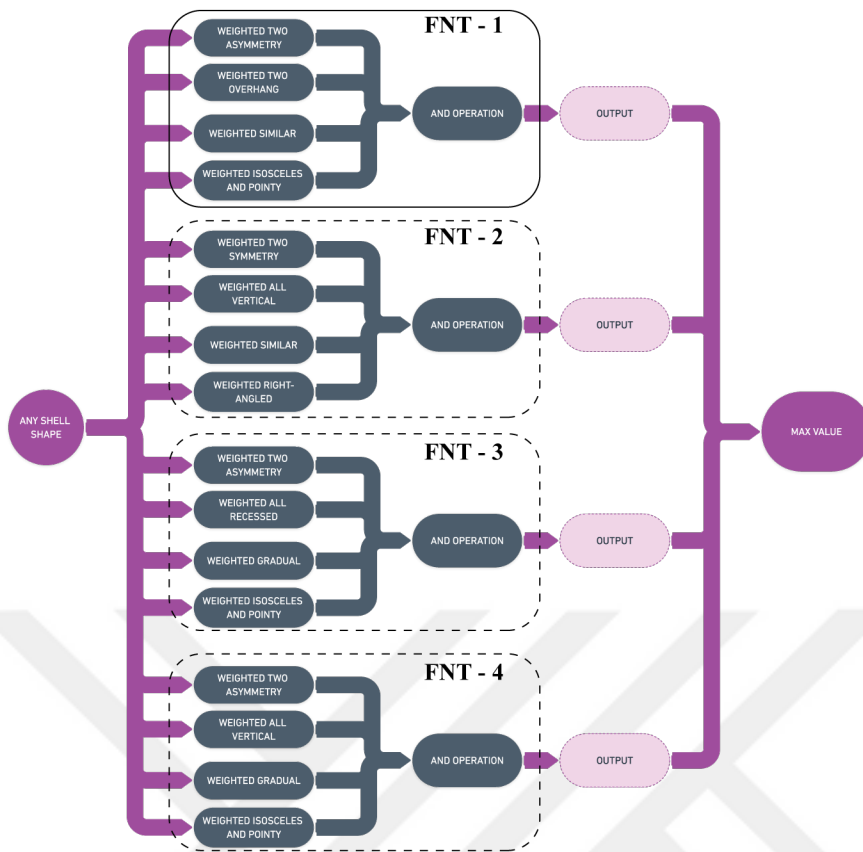


Figure 5.8: The greatest magnitude at the four FNT outputs predicts the likelihood of the preference.

6. CONCLUSION

A method for computational design is described. The purpose is to represent the aesthetic preferences of an architect as to the shape of a building. The preferences are understood using an innovative form of machine learning. It identifies the pattern of properties that best explains selections the architect has made among random designs. The properties of a shape are represented in interpretable form by the method of fuzzy neural tree. The machine learning is to identify the optimal neural tree models that explain the architect's selection, and it is carried out by a multi-objective evolutionary algorithm, which is imperative due to the non-linearity of the neural tree model computations. The algorithm searches in a space of possible neural tree models for the one that best explains the architect's preferred shapes. The search concerns the appropriate tree topology and weight values associated with certain node connections. The notion of 'best explanation' applied in this study has two aspects, which are in contrast to other forms of neural computing-based machine learning. While conventionally the sole objective in machine learning is minimizing the model error, in this study a second criterion is involved. It is that the description of a shape's character should be of utmost expressive. The latter implies that description should include as many as possible among the four essential shape properties.

The benefit of attaining the understanding of preferences by computation is that then the preferences become subject to satisfaction in extreme form by a systematic search. A second benefit is that aesthetics related objectives become compatible with non-aesthetic related ones so that a best compromise is found taking the aesthetic preference duly into account. The suitability of the method is verified by applying it to understanding an architect's preferences in the context of a shell type shape.

The practical value of the study is that the elusive preferences of a customer/user are understood by the aid of computational methods with established effectiveness. In this way, the precision of the understanding is increased compared to the conventional cases. Understanding the customer's preferences is an issue of generic relevance in architectural design. The representational bottlenecks arising from the tacit nature of such preferences are duly handled. In particular, the rationale behind a preference is problematic to pinpoint at large, and this issue is alleviated by this study.

The academic value of the study is that a methodologically challenging issue has been tackled. The problem is representing a highly non-linear relationship based on a small data set while maintaining the model interpretability. The reason why a certain shape has a greater shape quality than another one is to trace due to the transparency characterizing fuzzy neural tree modeling. The discovered preferences of the designer/user are given in explicit form without interpretability restrictions and can be applied as an objective function in ensuing computational design. The study exemplifies multi-disciplinary research in architecture, applying intelligent computing to a generic architectural issue.

For architects who are seek to benefit from the study in their design works, it is emphasised that with the method shown they can get to know customer preferences based on systematic method, rather than exclusively relying on intuition, experience etc. Alternatively, or additionally the same method can be used by architects to get insight into their own tacit design preferences which is a problematic issue at large when the topic concerns aesthetics.

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APPENDICES

APPENDIX A: 50 random samples and distribution of the properties

APPENDIX B: Digital model screenshots

APPENDIX C: Survey results

APPENDIX D: Surveys



APPENDIX A

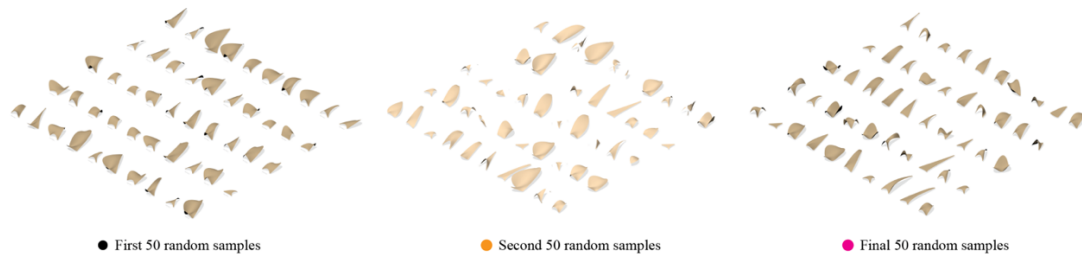


Figure A.1: 50 random samples.

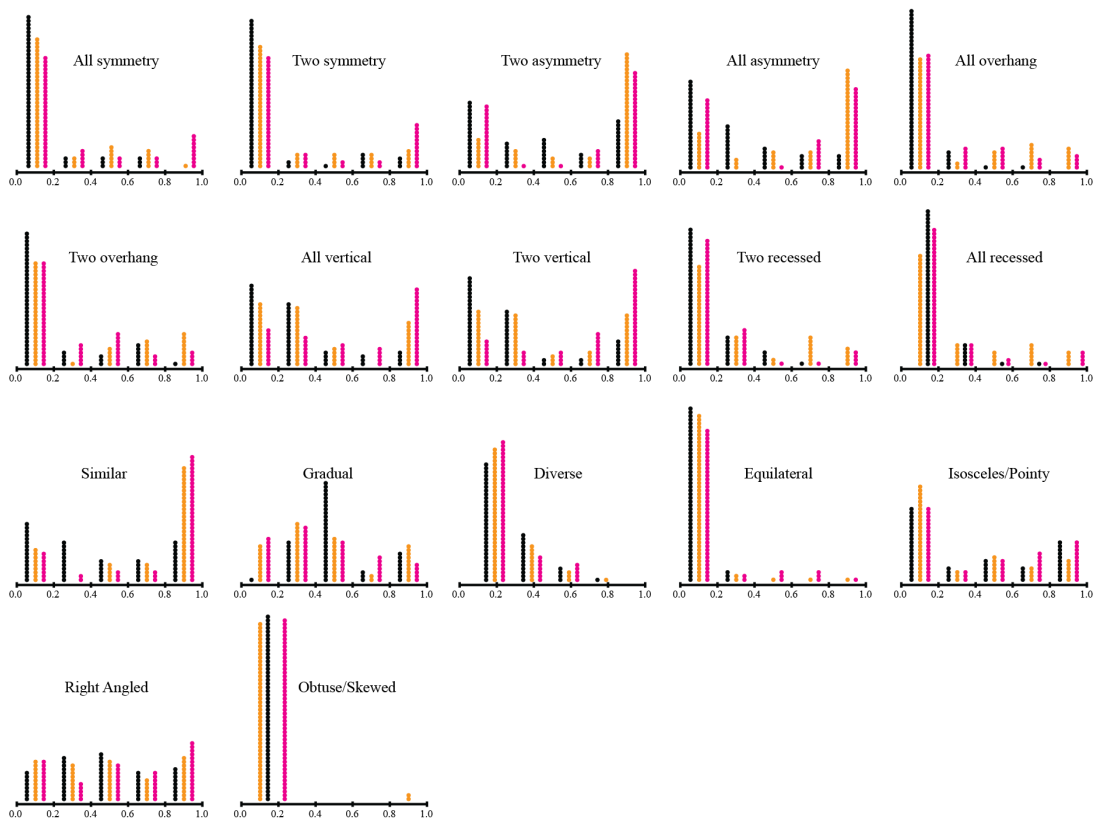


Figure A.2: Distribution of the properties in 50 random samples, randomness analysis.

APPENDIX B

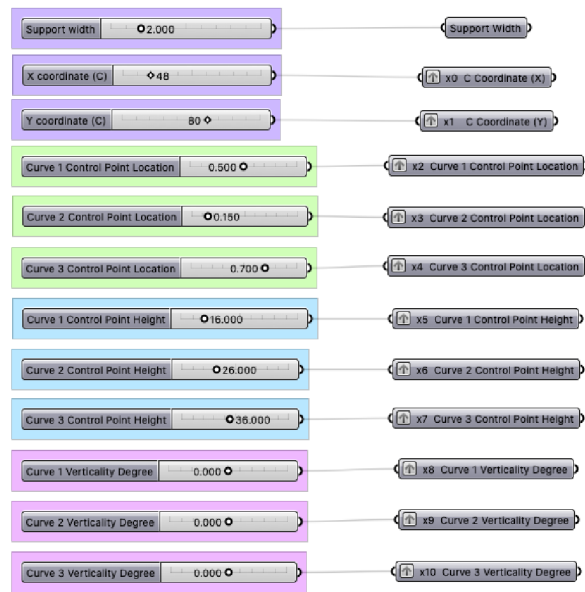


Figure B.1: Digital model screenshots – Parameter values.

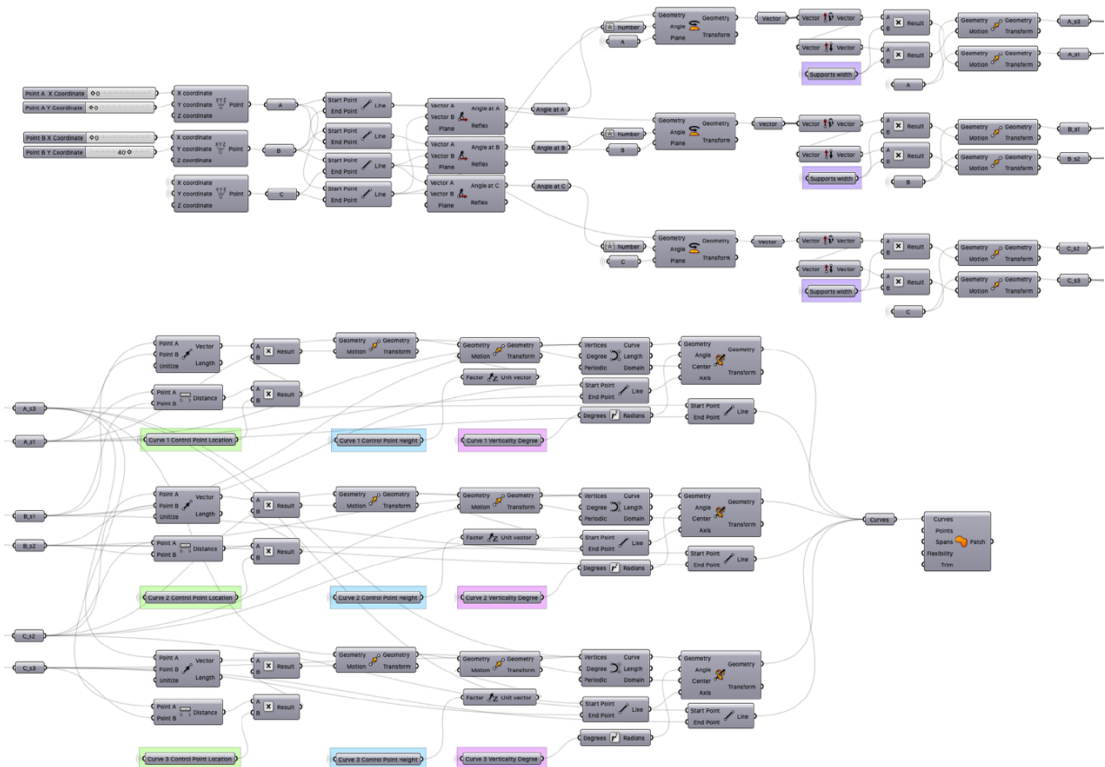


Figure B.2: Digital model screenshots – Parametric definition of the shell shape.

APPENDIX B

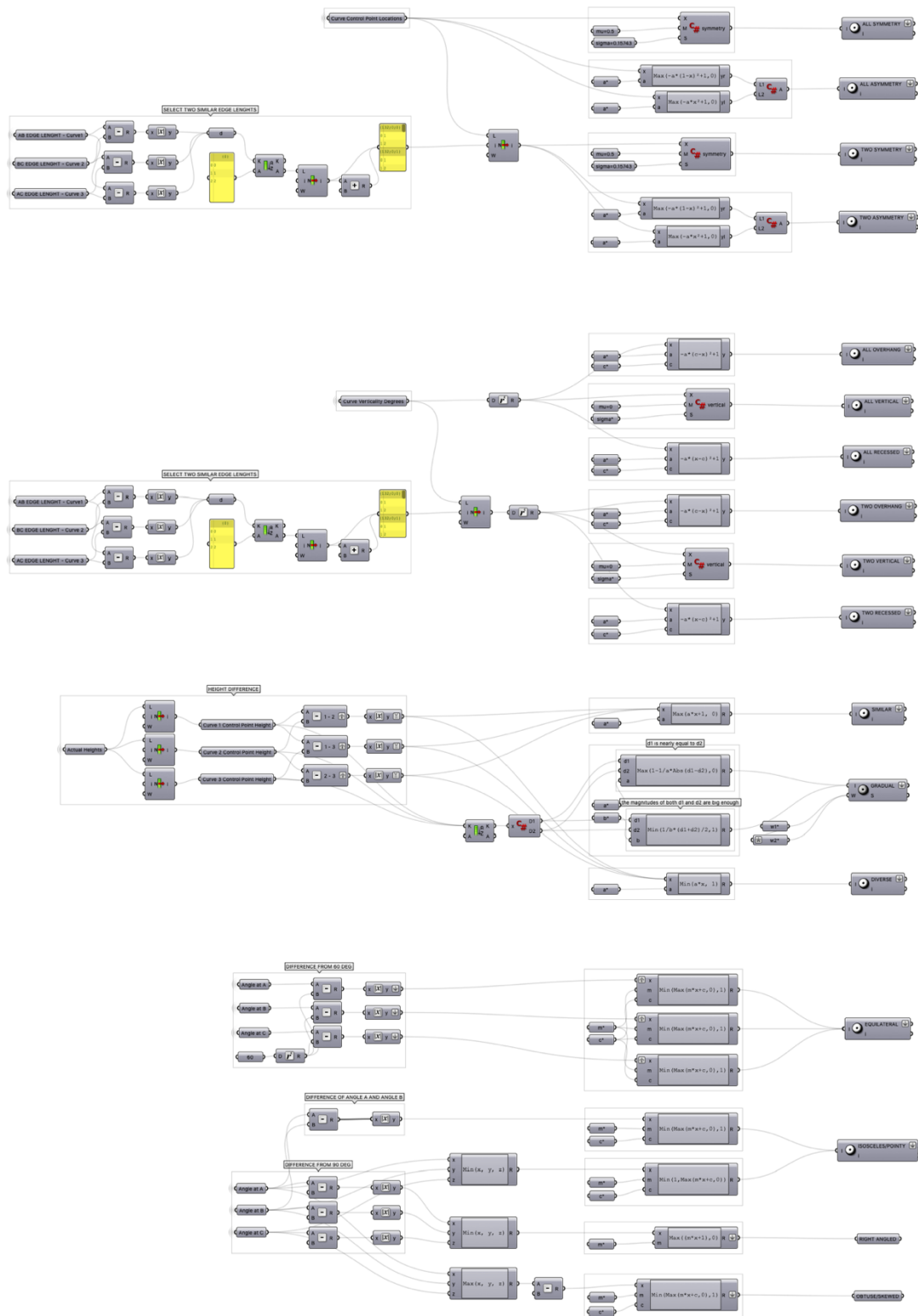


Figure B.3: Digital model screenshots – Definition of the properties.

APPENDIX B

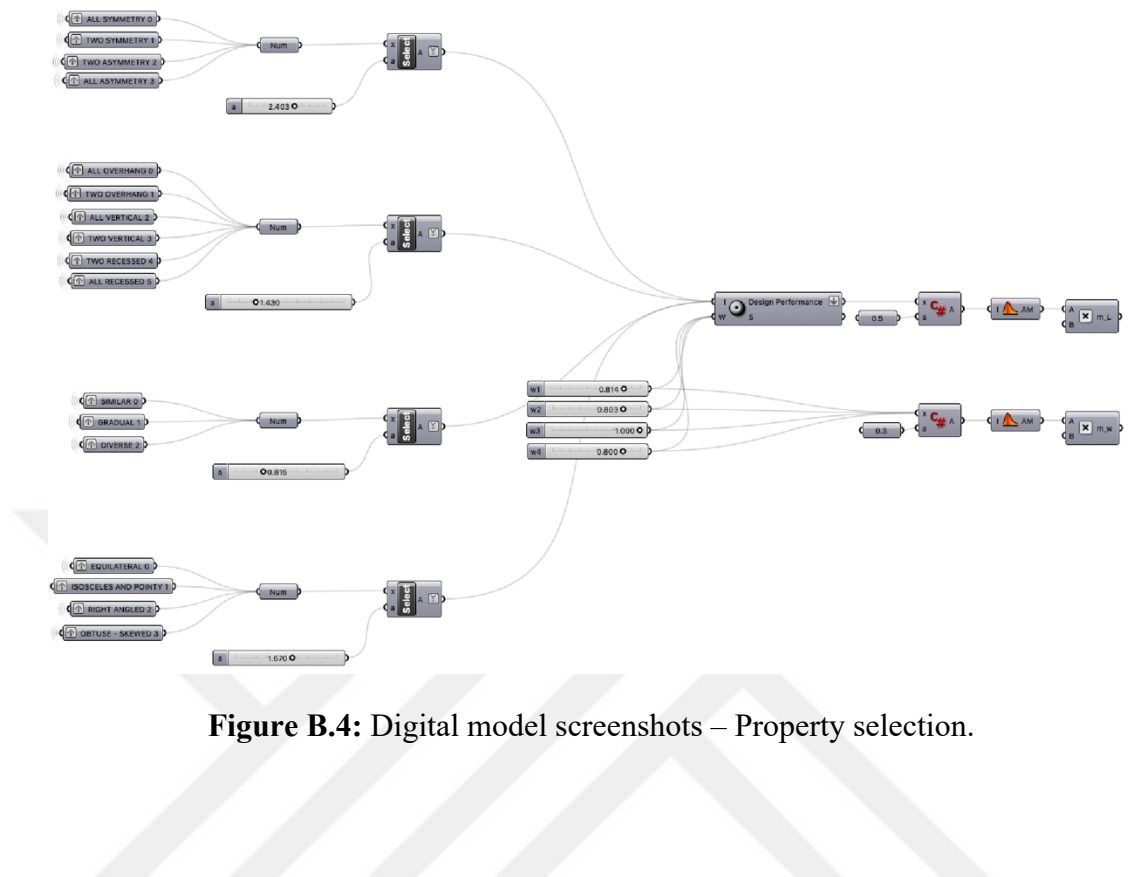


Figure B.4: Digital model screenshots – Property selection.

APPENDIX C

SAMPLE ID	0	1	2	3	4	5						
Curve control point locations	1	0.5	0.8	0.65	0.35	0.1						
SYMMETRY												
MICHAEL	0	10	2	4	4	0						
ECENUR	0	10	0	7	3	0						
BEGUM	1	10	2	9	7	2						
ELIF	2	10	3	4	4	2						
ASLI	3	10	3	4	4	2						
BURAK	4	10	5	8	8	5						
ÇİSEM	0	10	0	7	7	0						
BEGUM ESER	7	9	5	9	5	7						
ŞEBNEM	0	10	1	3	1	0						
KORAY	3	10	6	8	8	2						
MUGE	0	9	4	9	6	0						
YIGIT	3	10	4	7	7	4						
İNANC	6	10	7	8	8	6						
mean	0.94	0.054545455	0.690909091	0.327272727	0.409090909	0.836363636						
standart deviation	0.916515139	1.293339581	2.022599587	2.284333999	2.30019762	1.858640755						
ASYMMETRY												
MICHAEL	10	0	8	4	6	9						
ECENUR	9	0	8	5	6	10						
BEGUM	9	0	6	1	3	8						
ELIF	9	2	3	4	4	9						
ASLI	10	0	9	3	3	9						
BURAK	7	0	6	3	3	6						
ÇİSEM	10	0	7	3	3	10						
BEGUM ESER	9	4	7	7	5	9						
ŞEBNEM	10	0	9	8	9	10						
KORAY	10	0	8	1	1	8						
MUGE	10	0	9	2	7	10						
YIGIT	10	0	8	2	5	9						
İNANC	9.4	0	4	2	2	4						
mean	0.42	0.38	0.86	0.76	0.61	0.96						
standart deviation	1.932183566	1.988857852	0.966091783	0.966091783	1.852925615	0.699205899						
Curve verticality degrees												
SAMPLE ID	0	1	2	3	4	5	6	7	8	9	10	11
Curve verticality degrees	15	10	60	45	30	75	-20	-40	-55	-70	-5	0
OVERHANG												
MICHAEL	3	2	9	8	7	10	0	0	0	0	0	0
ECENUR	3	1	10	9	8	10	0	0	0	0	0	0
ETHEM	6	6	9	8	7	10	3	2	1	0	4	5
ILGİM	6	5	9	8	7	10	2	1	0	0	1	1
CUNEYT	2	2	7	6	5	8	0	0	0	0	1	2
SELDA	6	6	9	8	7	10	3	2	1	0	4	5
DILAN	6	6	9	8	7	10	3	2	1	0	4	5
ERDAL	6	5	9	8	7	10	0	0	0	0	0	0
SEZGİ	2	3	8	7	2	9	0	0	0	0	0	0
ELIF	2	2	7	6	4	9	0	0	0	0	0	0
BAHAR	4	3	9	8	7	10	0	0	0	0	0	0
mean	0.418181818	0.372727273	0.863636364	0.763636364	0.618181818	0.963636364	0.1	0.145454545	0.027272727	0	0.127272727	0.163636364
standart deviation	1.834021909	1.902151891	0.924416278	0.924416278	1.778661397	0.674199862	1.414213562	2.659459959	0.467099366	0	1.793929156	2.248231628
VERTICALITY												
MICHAEL	3	6	0	0	1	0	5	1	0	0	9	10
ECENUR	5	8	0	0	0	0	4	0	0	0	9	10
ETHEM	8	8	3	5	6	1	7	6	4	2	9	10
ILGİM	6	6	1	2	4	0	8	6	2	2	9	10
CUNEYT	7	8	3	4	5	2	7	6	4	3	9	10
SELDA	8	8	1	3	5	0	7	4	2	1	9	10
DILAN	8	9	2	3	5	1	8	7	2	2	9	10
ERDAL	7	8	1	5	6	0	8	5	3	2	9	10
SEZGİ	3	7	0	3	6	0	7	1	0	0	9	10
ELIF	6	6	2	3	5	1	6	4	3	2	9	10
BAHAR	8	9	2	3	2	0	8	4	2	1	9	10
mean	0.672727273	0.754545455	0.136363636	0.281818182	0.409090909	0.045454545	0.681818182	0.4	0.2	0.136363636	0.9	1
standart deviation	1.61807967	1.12815215	1.120064933	1.662418829	2.119176512	0.687551651	1.328019715	2.366431913	1.483239697	1.026910636	0	0
RECESSED												
MICHAEL	0	0	0	0	0	0	5	9	10	10	2	0
ECENUR	0	0	0	0	0	0	6	8	9	10	1	0
ETHEM	4	4	1	2	3	0	7	8	9	10	6	5
ILGİM	2	2	1	1	1	0	6	7	8	9	5	5
CUNEYT	0	0	0	0	0	0	2	4	7	8	1	0
SELDA	5	5	11	2	3	0	6	8	9	10	5	4
DILAN	4	4	1	2	3	0	7	8	9	10	6	5
ERDAL	0	0	0	0	0	0	3	8	9	10	2	0
SEZGİ	0	0	0	0	0	0	1	6	10	10	1	0
ELIF	0	0	0	0	0	0	2	4	5	6	1	0
BAHAR	0	0	0	0	0	0	3	5	8	10	2	1
mean	0.136363636	0.136363636	0.127272727	0.063636364	0.090909091	0	0.436363636	0.681818182	0.845454545	0.936363636	0.290909091	0.181818182
standart deviation	2.01359019	2.01359019	3.258555173	0.924416278	1.375103302	0	2.203303305	1.778661397	1.439696938	1.286291357	2.119176512	2.358736179
SIMILAR												
SAMPLE ID	0	1	2	3	4	5	6	7	8	9		
Curve 1 control point height	30	40	28	15	25	15	3	30	3	17		
Curve 2 control point height	30	10	30	5	30	15	5	35	33	20		
Curve 3 control point height	30	20	32	10	35	50	30	20	18	25		
SIMILAR												
ECENUR	10	1	9	5	6	1	2	4	0	7		
MICHAEL	10	2	10	4	8	0	1	2	0	8		
ETHEM	10	3	9	5	8	5	4	6	0	7		
ILGİM	10	1	9	4	8	2	0	5	1	9		
CUNEYT	8	3	8	6	7	4	1	6	2	7		
SELDA	10	3	10	6	9	2	1	6	0	9		
DILAN	10	1	9	6	9	4	4	7	2	9		
ERDAL	7	3	10	5	7	4	2	5	0	6		
SEZGİ	10	2	10	10	7	3	2	3	0	0		
ELIF	9	0	7	0	2	0	0	0	0	4		
BAHAR	8	0	8	2	7	0	0	4	0	8		
mean	9.272727273	1.727272727	9	4.818181818	6.818181818	2.272727273	1.545454545	4.363636364	0.454545455	6.727272727		
standart deviation	1.103712743	1.190874392	1	2.522624896	1.834021909	1.848832555	1.439696938	2.062654953	0.820199532	2.686667419		

Figure C.1: People's judgements on surveys I.

APPENDIX C

GRADUAL											
ECENUR	0	2	7	8	9	1	1	7	10	9	
MICHAEL	2	4	8	10	10	0	3	9	7	9	
ETHEM	0	6	10	7	10	3	3	8	5	10	
ILGIM	2	5	7	8	9	1	0	9	6	9	
CUNEYT	0	3	10	5	10	6	5	7	6	9	
SELDÄ	0	9	0	6	2	8	7	5	10	2	
DILAN	10	2	8	5	9	6	3	3	0	9	
ERDAL	1	6	0	8	10	3	4	3	5	10	
SEZGI	0	10	0	10	10	0	10	10	10	10	
ELIF	0	1	1	7	9	0	7	0	8	10	
BAHAR	2	8	2	7	3	7	7	6	7	2	
mean	1,545454545	5,090909091	4,818181818	7,363636364	8,272727273	3,181818182	4,545454545	6,090909091	6,727272727	8,090909091	
standart deviation	2,944949452	3,015113446	4,190899231	1,689540013	2,901410315	3,060005942	2,978712351	3,080731908	2,935673997	3,048099259	
DIVERSE											
ECENUR	0	9	0	4	2	8	5	7	10	2	
MICHAEL	0	9	0	4	3	8	6	7	10	0	
ETHEM	0	7	1	4	2	5	6	3	10	2	
ILGIM	0	7	1	3	2	7	8	3	8	2	
CUNEYT	1	7	1	8	3	6	9	4	10	2	
SELDÄ	0	10	1	10	2	6	7	6	10	2	
DILAN	0	8	1	2	2	4	5	5	8	2	
ERDAL	1	9	0	10	1	8	7	8	10	1	
SEZGI	0	8	0	0	0	8	8	8	10	8	
ELIF	0	8	0	5	3	8	10	7	9	2	
BAHAR	0	7	0	5	1	9	10	8	8	1	
mean	0,181818182	8,090909091	0,454545455	5	1,909090909	7	7,363636364	6	9,363636364	2,181818182	
standart deviation	0,404519917	1,044465936	0,522232968	3,16227766	0,943879807	1,549193338	1,80403588	1,949358869	0,924416278	2,040499048	
EQUILATERAL-1											
MICHAEL	0	4	8	0	3	5	9	12	14	19	
ECE	0	5	5	0	5	7	9	11	13	19	
ILGIM	0	2	5	0	5	6	9	10	11	19	
CUNEYT	0	3	6	0	2	4	9	12	14	19	
BIRGUL	0	2	3	0	6	7	9	10	11	19	
SAFA	0	1	5	0	2	4	9	11	18	19	
RUSEN	0	2	3	0	6	7	9	11	12	19	
ERDAL	0	4	7	0	5	7	9	10	12	19	
OZAN	0	2	6	0	4	6	9	12	14	19	
UYGAR	0	6	12	0	3	7	9	13	16	19	
NIDA	0	2	4	0	4	6	9	12	14	19	
FATMA	0	4	13	0	5	7	9	11	12	19	
mean		2,916666667	6,416666667		4,166666667	6,083333333		11,25	13,41666667		
standart deviation		1,378954369	3,203927514		1,403458931	1,164500153		0,965307299	2,065224326		
ISOSCELES-1											
MICHAEL	0	2	5	0	3	5	10	19			
ECE	0	2	5	7	6	9	14	19			
ILGIM	0	2	3	4	2	3	4	19			
CUNEYT	0	2	4	6	4	10	13	19			
BIRGUL	0	1	2	3	2	3	4	19			
SAFA	0	1	2	5	3	6	10	19			
RUSEN	0	1	2	3	2	5	15	19			
ERDAL	0	2	3	4	5	7	10	19			
OZAN	0	1	3	6	4	7	12	19			
UYGAR	0	4	7	13	7	11	14	19			
NIDA	0	2	3	10	2	3	9	19			
FATMA	0	2	5	10	2	7	14	19			
mean		1,833333333	3,666666667	6,583333333	3,545454545	6,454545455	10,81818182				
standart deviation		0,83484711	1,556997888	3,146667309	1,809068067	2,80583807	3,894051407				
OBTUSE-1											
MICHAEL	3	5	12	19	0	3	8	13			
ECE	5	7	13	19	0	3	5	9			
ILGIM	2	4	9	19	0	6	8	12			
CUNEYT	4	6	9	19	0	6	10	18			
BIRGUL	0	1	2	19	0	2	5	19			
SAFA	0	2	12	19	0	1	3	5			
RUSEN	1	7	13	19	0	5	10	19			
ERDAL	1	4	8	19	0	8	10	15			
OZAN	4	9	17	19	0	2	6	12			
UYGAR	4	8	11	19	0	6	10	15			
NIDA	5	7	11	19	0	4	7	14			
FATMA	0	5	18	19	0	1	8	18			
mean	2,416666667	5,416666667	11,25			3,916666667	7,5	14,08333333			
standart deviation	1,975225342	2,391588796	4,180582821			2,274696117	2,354878881	4,252450274			
RIGHT ANGLED											
MICHAEL	0,25	5	7	13	17						
ECE	0	4	7	10	14						
ILGIM	0	6	7	9	11						
CUNEYT	0,3	4	7	9	12						
BIRGUL	0	5	7	8	9						
SAFA	0	6	7	8	9						
RUSEN	0	5	7	8	11						
ERDAL	0	4	7	10	13						
OZAN	0,2	4	7	10	13						
UYGAR	0,2	4	7	12	19						
NIDA	0	5	7	9	12						
FATMA	0,1	5	7	11	16						
mean	0,0875	4,75	9,75	13							
standart deviation	0,11701787	0,753778361	1,602554779	3,074824459							

Figure C.2: People's judgements on surveys II.

APPENDIX D

EVALUATING SYMMETRY & ASYMMETRY

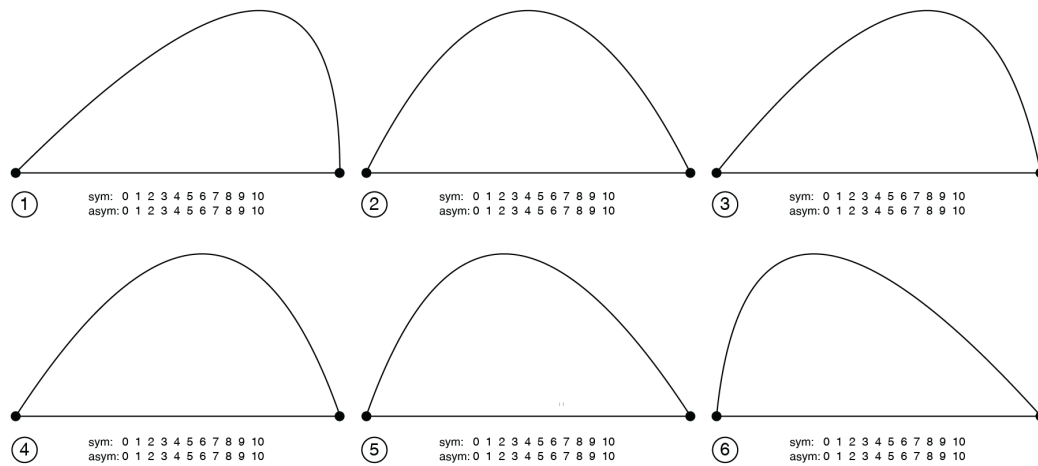


Figure D.1: Survey on the symmetry evaluation.

EVALUATING VERTICALITY

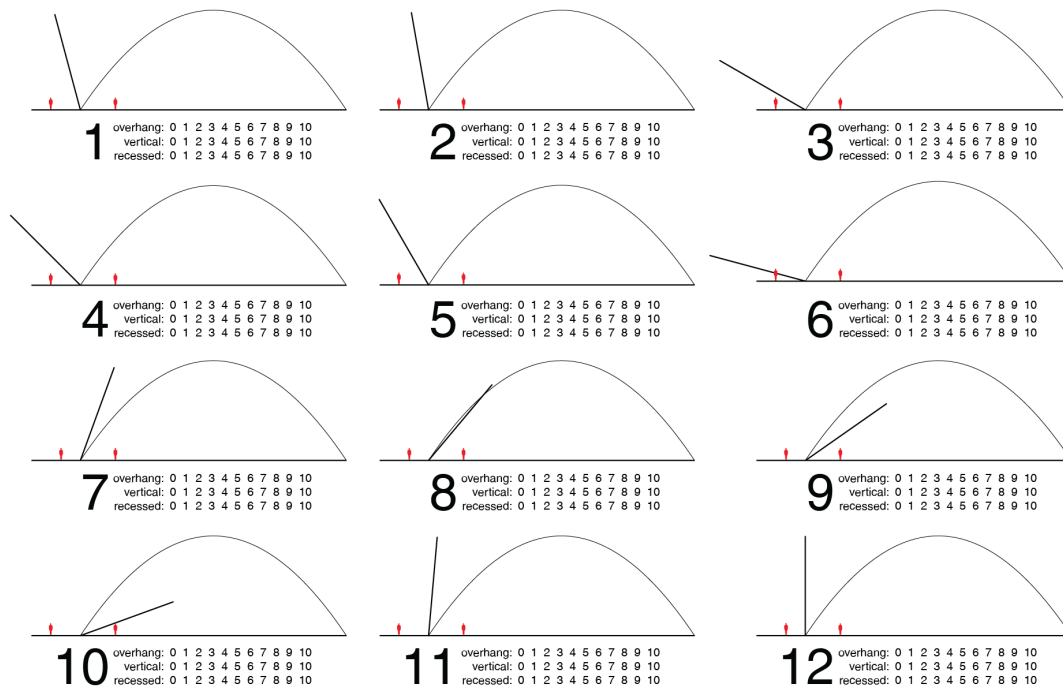


Figure D.2: Survey on the verticality evaluation.

APPENDIX D

EVALUATING HEIGHT

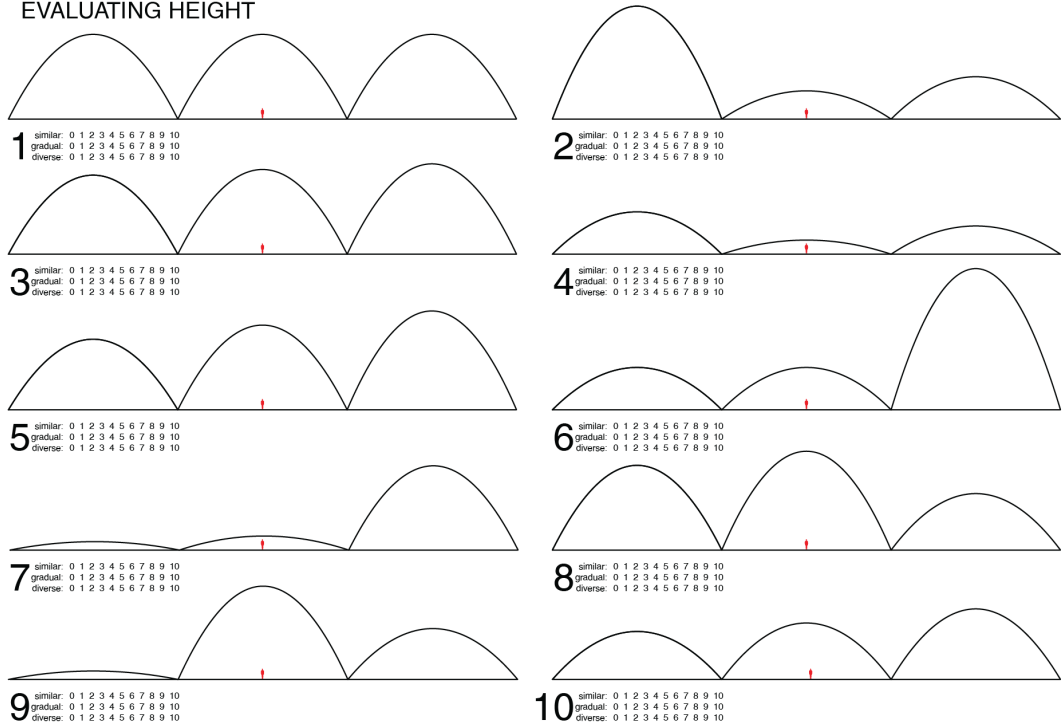


Figure D.3: Survey on the height evaluation.

Definitions: All sides of an equilateral triangle have equal length. Two examples are those, underneath which the number 1 is written in figures 1 and 2. When a triangle does not fulfill the condition perfectly but nearly, it still has some degree of "equilateral-ness".

The aim of this survey is to associate some "equilateral-ness" degrees to some triangles.

- Starting from the lower right, which triangle is the first that has a glimpse of "equilateral character"? Write the number 0.1 under this triangle.
- Which triangle has to the same degree an "equilateral character" as it has a "non-equilateral character"? Write the number 0.5 under this triangle. (şöyle böyle)

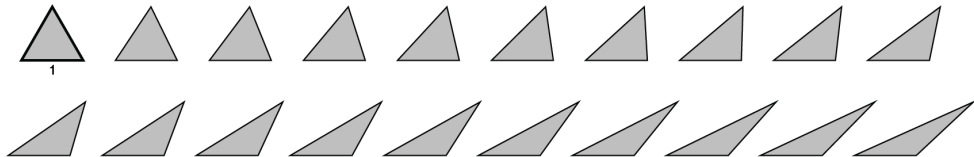


Figure 1

- Starting from the upper left, which triangle is the first that has a glimpse of "equilateral character"? Write the number 0.1 under this triangle.
- Starting from the lower right, which triangle is the first that has a glimpse of "equilateral character"? Write the number 0.1 under this triangle.
- Starting from the upper left, which triangle has to the same degree an "equilateral character" as it has a "non-equilateral character"? Write the number 0.5 under this triangle.
- Starting from the lower right, which triangle has to the same degree an "equilateral character" as it has a "non-equilateral character"? Write the number 0.5 under this triangle.

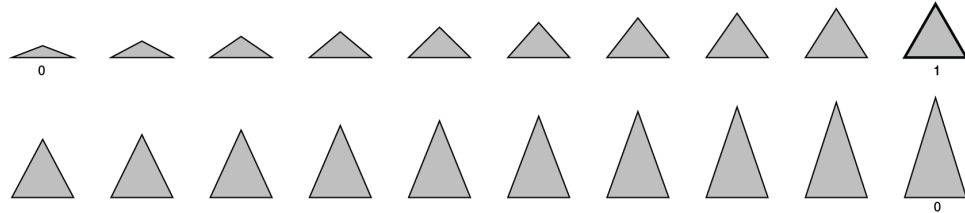


Figure 2

Figure D.4: Survey on the "equilateral-ness" evaluation.

APPENDIX D

Definitions: One angle of a right triangle is equal to 90° . Two examples are those, underneath which the number 1 is written in figure 1.
When a triangle does not fulfill the condition perfectly but nearly, it still has some degree of "right-ness". The aim of this survey is to associate some "right-ness" degrees to some triangles.

- Starting from the upper left, which triangle has to the same degree a "right angle" character as it has a "non-right angle" character? Write the number 0.5 under this triangle.
- The triangle which has a question mark underneath it: which number should it take? Write the number under the triangle.
- Starting from the triangle which the number 1 is written underneath, moving to the right, which triangle has to the same degree a "right angle" character as it has a "non-right angle" character? Write the number 0.5 under this triangle.
- Starting from the lower right, which triangle is the first that has a glimpse of "right angle character"? Write the number 0.1 under this triangle.

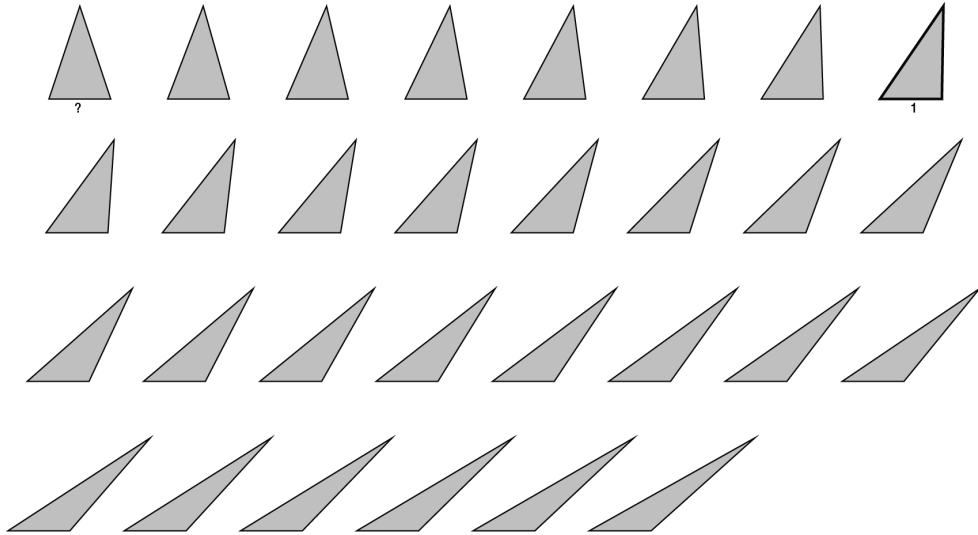


Figure 1

Figure D.5: Survey on the "right-ness" evaluation

Definitions: Exactly two sides of an isosceles triangle have equal length. Two examples are those, underneath which the number 1 is written in figures 1 and 2.
When a triangle does not fulfill the condition perfectly but nearly, it still has some degree of "isosceles-ness". The aim of this survey is to associate some "isosceles-ness" degrees to some triangles.

- Starting from the upper left, which triangle is the last that has no longer a perfectly "isosceles character"? Write the number 0.9 under this triangle.
- Starting from the lower right, which triangle is the first that has a glimpse of "isosceles character"? Write the number 0.1 under this triangle.
- Which triangle has to the same degree an "isosceles character" as it has a "non-isosceles character"? Write the number 0.5 under this triangle.

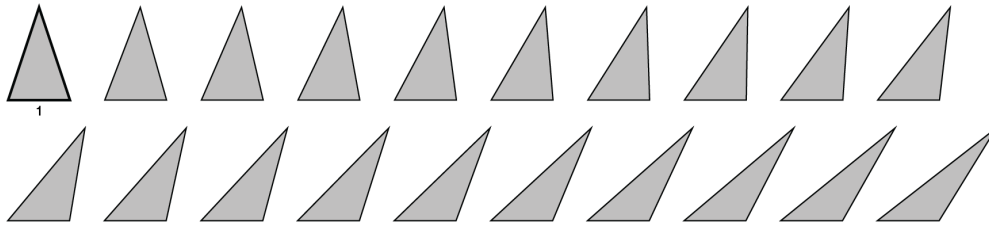


Figure 1

Two sides of a "pointy triangle" are both much longer than the third side.

- Starting from the upper left, which triangle is the first that has a glimpse of "pointy character"? Write the number 0.1 under this triangle.
- Starting from the lower right, which triangle is the last that has no longer a perfectly "pointy character"? Write the number 0.9 under this triangle.
- Which triangle has to the same degree a "pointy character" as it has a "non-pointy character"? Write the number 0.5 under this triangle.

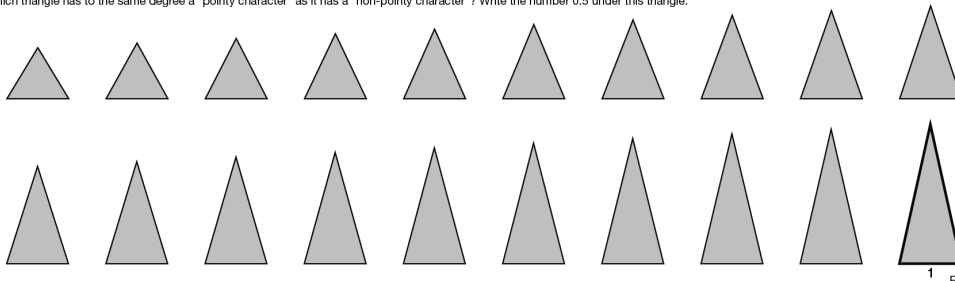


Figure 2

Figure D.6: Survey on the "isosceles&pointy-ness" evaluation.

CURRICULUM VITAE



Name Surname : Ecenur Yavuz
Place and Date of Birth : Samsun, 15.01.1992
E-Mail : ecenuryavuz@gmail.com

EDUCATION :

- **B.Sc.** : 2015, Izmir Institute of Technology, Faculty of Architecture, Department of Architecture

PROFESSIONAL EXPERIENCE AND REWARDS:

- 2017 - 2018 BKA (Bahadır Kul Architects)
- 2015 - 2016 BINAA (Building, Innovation, Arts, Architecture)
- 2017 6. Zeki Yurtbay Tasarım Yarışması (3rd prize)
- 2015 Atölye Muğla Bir - 13 İlçe 13 Desen Yarışması (jury special prize)
- 2014 5. Koçtaş İç Mekan Tasarım Yarışması (encouragement award)

PUBLICATIONS, PRESENTATIONS AND PATENTS:

- **Bittermann M.S., Yavuz E., Ciftcioglu O.** (2020) A Fuzzy Neural Tree for the Evaluation of Shape in an Architectural Design. In: Kahraman C., Cebi S., Cevik Onar S., Oztaysi B., Tolga A., Sari I. (eds) *Intelligent and Fuzzy Techniques in Big Data Analytics and Decision Making. INFUS 2019. Advances in Intelligent Systems and Computing*, vol 1029. Springer, Cham
- **Yavuz, E., Çolakoğlu, B., & Aktaş, B.** (2018). From Pattern Making to Acoustic Panel Making Utilizing. In A. Kępczyńska-Walczak & S. Białkowski (Eds.), *Proceedings of the 36th International Conference on Education and Research in Computer Aided Architectural Design in Europe* (Vol. 1). eCAADe.