

**OPTIMUM DESIGN OF GRILLAGE SYSTEMS USING HARMONY
SEARCH ALGORITHM**

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ABSTRACT

OPTIMUM DESIGN OF GRILLAGE SYSTEMS BY USING HARMONY SEARCH ALGORITHM

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Harmony search method based optimum design algorithm is presented for the grillage systems. This numerical optimization technique imitates the musical performance process that takes place when a musician searches for a better state of harmony. For instance, jazz improvisation seeks to find musically pleasing harmony similar to the optimum design process which seeks to find the optimum solution.

The design algorithm considers the displacement and strength constraints which are implemented from LRFD-AISC (Load and Resistance Factor Design-American Institute of Steel Construction). It selects the appropriate W (Wide Flange)-sections for the transverse and longitudinal beams of the grillage system among 272 discrete W-section designations given in LRFD-AISC so that the design limitations described in LRFD are satisfied and the weight of the system is confined to be minimal. Number of design examples is considered to demonstrate the efficiency of the algorithm presented.

Keywords: Optimum structural design, harmony search algorithm, minimum weight, search technique, combinatorial optimization, grillage systems.

ÖZ

IZGARA SİSTEMLERİN HARMONİ ARAMA YÖNTEMİ KULLANILARAK OPTİMUM BOYUTLANDIRILMASI

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Optimum tasarım algoritmasına dayalı harmoni arama yöntemi ızgara sistemlerin boyutlandırılması için sunulmaktadır. Bu sayısal optimizasyon tekniği, bir müzisyenin daha iyi bir müzikal uyum arayışı içinde uygulamaya çalıştığı müzikal performans sürecine benzetilmektedir. Örneğin jazz doğaçlaması, optimum çözüme ulaşmaya çalışan optimum tasarım sürecine benzer şekilde, müzikal açıdan tatmin edici uyumu bulmayı amaçlar. Tasarım algoritması LRFD-AISC (Load and Resistance Factor Design-American Institute of Steel Construction) uygulaması sonucu oluşan yer değiştirme ve dayanım sınırlamalarını göz önüne almaktadır. Harmoni arama yöntemi LRFD-AISC’de verilen 272 farklı W-profilii arasından ızgara sistemin enine ve boyuna kirişleri için uygun profili seçer böylece LRFD’de tanımlanan tasarım sınırlamaları sağlanır ve sistemin ağırlığı minimuma indirgenir. Tasarım örneklerinin sayısı sunulan algoritmanın etkinliğini göstermeyi amaçlamaktadır.

Anahtar Kelimeler: Optimum yapısal tasarım, Harmoni arama yöntemi, minimum ağırlık, arama tekniği, kombinasyonel optimizasyon, ızgara sistemler

To my family

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CHAPTER 1

OPTIMIZATION IN GENERAL

1.1 Introduction to Optimization

Optimization is concerned with achieving the best outcome of a given operation while satisfying certain restrictions. Human beings, guided and influenced by their natural surroundings, almost instinctively perform all functions in a manner that economizes energy or minimizes discomfort and pain. The motivation is to exploit the available limited resources in a manner that maximizes output or profit.

Common problems faced in the optimization field are static and dynamic response, shape optimization structural systems, reliability-based design and optimum control of systems. Any optimization problem requires proper identification of objective function, design variables and constraints on problem formulation state. Depending on the class of problems and needs, several types of design variables and objective functions can be identified. Constraints usually involve physical limitations, material failure, buckling load and other response quantities.

The main goal of the optimization process is to find the optimal solution from the variety of possible combination of variables defined in the mathematical problem. Structural optimization problems are characterized by various objectives and constraints which are generally non linear functions of the design variables. Each

objective and constraint choice defines a different optimization problem. Optimization problems can be expressed in standard mathematical terms as:

$$\text{Minimize } f(x) \quad . \quad (1.1)$$

Subject to:

$$g_i(x) \leq 0 \quad i = 1, \dots, m \quad (1.2)$$

$$h_j(x) = 0 \quad j = 1, \dots, l \quad (1.3)$$

$$x_k^l \leq x_k \leq x_k^u \quad k = 1, \dots, n_{dv} \quad (1.4)$$

in which, x is design variable vector, $f(x)$ is the objective function. In general, the constraint functions are grouped into three classes: equality constraints h_j , inequality constraints g_i , and the geometric constraints, x_k^l and x_k^u represent the lower and the upper bounds of the design variable x_k , m is the number of design variables used [1-2].

The structural optimization design is conditioned by the choice of the objective and constrained functions expressed in terms quantities. In most practical work weight of the structure is chosen as the objective function, while the maximum displacement or/and maximum stress are imposed as the constraints.

1.2 Structural Optimization

The field of structural optimization is a relatively new field undergoing rapid changes in methods and focus. Until recently there was a severe imbalance between enormous amount of literature on the subject and paucity of applications to practical design problems. This imbalance is gradually redressed. There is still no shortage of new

publications, but there are also exciting applications of the methods of structural optimizations in the civil engineering, machine design, aerospace and other engineering fields. As a result of the growing pace of applications, research into structural optimization methods is increasingly driven by real-life problems.

Most engineers who design structures employ complex general-purpose software packages for structural analysis. Often engineers do not have any access to the source program, and even more frequently they have only scant knowledge of the details of the structural analysis algorithms used in this software packages. Another major challenge is the high computational cost associated with the analysis of many complex real-life problems. In many cases the engineer who has the task of designing a structure cannot afford to analyze it more than a handful of times.

Structural optimization when first emerged has attracted a widespread attention among designers. It has provided a systematic solution to age-old structural design problems which were handled by using trial-error methods or engineering intuition or both. Application of mathematical programming methods to structural design problems has paved the way in obtaining a design procedure which was capable of producing structures with cross-sectional dimensions.

In the structural optimization problems, usually more than one objective is required to be optimized, such as, minimum weight which is related to cost, maximum stiffness, minimum displacement at specific structural points and minimum structural strain energy while all the constraints are satisfied. The constraints provide bonds on member stress, deflection, frequency, local buckling, system buckling and dynamic response. In the last four decades vast amount of research work has been conducted in structural optimization which covers the field from optimum design of individual elements to rigid frames and finite element structures. However, due to the fact that mathematical programming techniques deal with continuous design variables, the algorithms developed has provided to designer cross-sectional dimensions that were neither standart nor practical [3-5].

Consequently, the structural optimization has not enjoyed the same popularity among the practicing engineers as it has enjoyed among the researchers. As a result, efforts have been concentrated on the area of rectifying the structural optimization algorithms to be able to work with discrete set of variables.

1.2.1 Structural optimization problems

The discrete size optimization of structural system involves arriving at optimum values for discrete member design vectors x that minimizes the objective function $f(x)$, which is subjected to constraints related to the design and the behaviour of the structure. Some constraints may not be expressed explicit, but can be numerically evaluated using the structural element analysis. General optimization problems can be stated mathematically formulation as minimizing the structural weight as follows [6]:

Find a design vector $x, x^T = (x_1, x_2, \dots, x_{Ne})$ and $x_i \in T$

To minimize $f(x)$,

$$\text{For weight optimization; } f(x) = \sum_{i=1}^{Ne} \gamma_i L_i x_i \quad (1.5)$$

Subject to;

$$\begin{aligned} g'_i(x) = \sigma'_i - \sigma_i^a \leq 0 & \quad i = 1, \dots, N_e \\ & \quad l = 1, \dots, N_L \end{aligned} \quad (1.6)$$

$$f'_k(x) = u'_k - u_k^a \leq 0 \quad \begin{matrix} k = 1, \dots, N_d \\ l = 1, \dots, N_L \end{matrix} \quad (1.7)$$

$$LB \leq x \leq UB \quad (1.8)$$

Where;

$f(x)$: the objective function (usually the weight of the structure)

T : table of available discrete size

N_e : total number of design variables or elements

N_L : total number of load condition

γ_i : the specific weight of the i-th element

L_i, x_i : the length and the cross sectional area of the i-th element respectively

σ'_i, σ_i^a : the absolute value of stress under the l-th load condition and allowable stress in the i-th element respectively.

u'_k, u_k^a : the absolute value of displacement under the l-th load condition at the degree of freedom corresponding to the k-th displacement constraint and corresponding allowable value respectively.

LB : the vector of lower bounds on design variables

UB : the vector of upper bounds on designs variables

1.2.2 Structural Optimization Methods

Structural optimization methods can be divided into two categories called analytical methods and numerical methods. While analytical methods emphasize the conceptual aspect, numerical methods are concerned with the algorithmical aspect.

1.2.2.1 Analytical Methods:

Analytical methods usually employ the mathematical theory of calculus and variational methods in studies optimal layouts or geometrical form of structural elements, such as columns, beams and plates. These analytical methods are most convenient for such fundamental studies of single structural components, but they are not intended to handle larger structural systems. The structural design is represented by a number of unknown functions and the goal is to find the form of these function. The optimal design is theoretically found exactly through the solution of a system of equations expressing the conditions for optimality [7].

Applications based on analytical methods though they sometimes lack the practical aspects of realistic structures, is nonetheless basic importance. Analytical solutions provide valuable insight and theoretical lower bound optimum against which more practical designs may be judged. Problems solved by analytical methods are called continuous problems or distributed parameter optimization problems.

1.2.2.2 Numerical Methods:

Numerical methods employ a branch in the field of numerical mathematics called mathematical programming. Closed form analytical solution techniques for practical optimization problems are difficult to obtain if the number of design variables is more than two or the constraint expressions are complex. Therefore numerical methods and computer programmings is preferred to solve most optimization problems. The recent developments of the numerical methods are closely related to the rapid growth in computing capacities. In numerical methods, an initial design for the system is selected which is iteratively improved until to further improvements are possible without violating any of the constraints. The search is terminated when certain convergence criteria are satisfied, indicating that current design is sufficiently close to the optimum.

Early numerical optimization algorithms are all in the class of mathematical programming methods. The common feature of these methods is that the design variables are considered to be continuous and the objective function as well as constraints are expressed as functions of these design variables. Most of the techniques make use of the gradient vectors of the objective function and constraint which requires first derivatives of these functions with respect to the design variables.

Some of the mathematical programming methods, such as linear, quadratic, dynamic, and geometric programming algorithms, have been improved to deal with specific classes of optimization problems [8]. Although the history of mathematical programming is relatively short, there has been a large number of algorithms developed for the solution of numerical optimization problems [9].

Another approach for numerical optimization of structures is based on derivation of a set of essential conditions that must be satisfied at the optimum design and improvement of an iterative redesign procedure. These methods are called Optimality Criteria (OC) methods, which were presented in analytical form by Prager [10] and in numerical form by Venkayya [11]. Its principal attraction was that the method was easily programmed for the computer, was relatively independent of problem size, and usually provided a near-optimum design with a few structural analyses. This last feature represented a remarkable improvement over the number of analyses required in mathematical programming methods to reach a optimum solution.

In recent years, the range of applicability of structural optimization has been widened and much progress has been made in various topics associated with this area. Efficient search methods, such as genetic algorithms, simulated annealing, ant colony optimization and harmony search, for derivative calculation have been developed, and problems with complex analysis model and various types of constraints and objective function have been investigated. The important progress in these advanced topics

emphasizes the need for a deeper insight and understanding of the fundamentals of structural optimization.

1.2.2.2.1 Mathematical Programming

Mathematical programming can be subdivided into linear programming and non-linear programming. The major characteristic of linear programming is that the objective functions and the associated constraints are expressed as a linear combination of the design variables. To apply linear programming techniques to structural optimization, the relationship between the objective function and the constraints are to be expressed as linear functions of design variables. On the other hand if they are nonlinear, they have to be linearized. However, when a linear relationship is used to model a non-linear structural response, errors are inevitable.

Non-linear mathematical programming is developed for non-linear unconstrained optimization problems. Mathematical non-linear programming algorithms require either differentiability or gradient information of both the objective function and constraints with respect to the design variables. Moreover, they are unsuited for problems where the design space is discontinuous, as the derivatives of the objective function and constraints become singular across the boundary of discontinuity. The product of optimisation with these methods is mostly contingent to the starting point of optimisation process due to the locating the relative optimum closest to the initial estimate of the optimum design.

The well-known Kuhn-Tucker conditions provide the necessary conditions for optimum solutions. The calculation of gradients and the solution of the correlated non-linear equations prohibit the direct application of the Kuhn-Tucker conditions for structural optimisation problems, so direct application of the Kuhn-Tucker conditions is extremely difficult for structure problems [12-13].

1.2.2.2.2 Optimality Criteria

The Optimality Criteria methods are developed from indirectly applied the Kuhn-Tucker conditions of non-linear mathematical programming combined with Lagrangian multipliers. The Kuhn-Tucker conditions provide the necessary requirements for an optimum solution and the Lagrangian multipliers are used to include the associated constraints. After deriving the necessary condition, a recursive relationship is developed iterative use of which convergence to the near optimum solution.

Optimality Criteria methods are based on continuous design variable assumption. For the case where discrete variables are desired using Optimality Criteria methods a two-step procedure is typically used. First, the optimisation problem is solved using continuous variables. Second, a set of discrete values is estimated by matching the values obtained from the continuous solution. Optimality Criteria methods use a single cross-sectional property of a structural member as the design variable. All other cross-sectional properties are expressed as a function of the selected design variable [14-15].

1.2.3 Stochastic search methods

A class of optimization algorithms developed recently is known as stochastic search algorithms. These algorithms employ the generation of random numbers as they search for the optimum. Although they do not require the evaluation of gradients of the objective and constraint functions, they typically require many more function evaluations than do the gradient-based nonlinear programming algorithms. Unlike nonlinear algorithms, stochastic search algorithms may be applied to optimization problems involving discrete variables.

1.2.3.1 Genetic algorithms

The genetic algorithm is a search procedure inspired by principles from natural selection and genetics. Genetic algorithm is used to improve the designs after stochastic generation of initial population of designs. Genetic algorithm uses techniques derived from biology, and rely on the principle of Darwin's theory of survival of the fittest. Genetic algorithm basically consists of three parts [16]:

- (1) coding and decoding variables into strings;
- (2) evaluating the fitness of each solution string;
- (3) applying genetic operators to generate the next generation of solution strings.

Genetic algorithms are implemented with population of individuals, coded as bit strings of finite lengths, each of which represents a search point in the space of potential solutions to a given optimization problem. Using transformations analogous to biological reproduction and evolution over generations creates chromosomal strings that favorably adapt to the changing environment. The chromosomal structures, are changed through reproduction, a crossover of genetic information exchange and occasional mutation. The individuals that judged most fit are given opportunities of producing larger number of offsprings and crossed with other fit members of the population. Combination of the most suitable characteristics of the mating members results in the spreading of good characteristics throughout the population and the next generation population. If genetic algorithm is implemented properly, successive generation produces better values of design variables and the population will converge to optimal solution.

Genetic algorithms are developed by applying the principal of survival of the fittest into a numerical search method. Genetic algorithms are used as a function optimizes particularly when the variables have discrete values. They, first select an initial population where each individual is constructed by bringing together the total number

of variables in a binary coded form. This code has the most importance of two, that means each character can take either the symbol of '0' or '1'. The binary code for each design variable represents the sequence number of this variable in the discrete set. For example, consider the following simple mathematical maximization problem with x_1 , x_2 and x_3 being its design variables.

$$\begin{aligned}
\max \quad & f(x_1, x_2, x_3) = 2x_1 - x_2 + 3x_3 ; \\
0 \leq & x_1 \leq 2.0 \\
0 \leq & x_2 \leq 2.0 \\
0.5 \leq & x_3 \leq 2.0
\end{aligned} \tag{1.9}$$

If we resolve that six bits are enough to provide a desired degree of accuracy in the representation of each design variable separately, encoded variables are decoded through normalization of the corresponding binary integer by $2^6 - 1$. Our individuals would, therefore, contain three gens and consist of 18 binary digits, representing the arrangement of the codings for x_1 , x_2 and x_3 . By that way, while substring '111111' represents a value of 2.0 for all variables, substring '000000' corresponds to a real value of 0 for the variables x_1 and x_2 , 0.5 for the variable x_3 .

A genetic algorithm initiates the search for finding the optimum solution in a discrete space by first selecting the number of individuals randomly and collecting them together to constitute the generation of initial population. In each cycle of generation, simple genetic algorithm has the individuals passed through selection, mating, crossover and mutation operations to create next generation, which comprise more adapted individuals than the previous has. This process iterates over a fixed number of generation or until a stopping criterion. Genetic algorithm pseudo-code has shown in Figure 1.1 [17].

```

L is chromosome length
N is population size
p[i] is probability vector
1. Initialize probability vector
   For i := 1 to L do p[i] := 0.5;
2. Generate two individuals from the vector
   a:= generate (p);
   b:= generate (p);
3. Let them compete
   Winner, Loser := evaluate (a,b);
4. Update the probability vector toward the better one
   For i := 1 to L do
     if winner [i] != loser [i] then
       if winner [i] = 1 then p[i] += 1/N
       else p[i] -= 1/N
5. Check if the probability vector has converged
   for i := 1 to L do
     if p[i] > 0 and p[i] < 1 then go to step 2
6. P represents the final solution

```

Figure 1.1 Pseudo code of Genetic Algorithm

1.2.3.2 Simulated Annealing

The simulated annealing algorithm is a random-search technique which exploits an analogy between the way in which a metal cools and freezes into a minimum energy crystalline structure (the annealing process) and the search for a minimum in a more general system. It forms the basis of an optimization technique for combinatorial and other problems.

Simulated annealing (SA) was developed in 1983 to deal with highly nonlinear problems [18]. The development of the simulated annealing method was motivated by studies in statistical mechanics which deal with the equilibrium of large number of atoms in solids and liquids at a given temperature. During solidification of metals or formation of crystals, for example, a number of solid states with different internal atomic or crystalline structure that correspond to different energy levels can be achieved depending on the rate of cooling. If the system is cooled too rapidly, it is likely that the resulting solid state would have a solid margin of stability because the atoms will assume relative positions in the lattice structure to reach an energy state which is only locally minimal. In order to reach a more stable, globally minimum energy state, the process of annealing is used in which the metal is reheated to a high temperature and cooled slowly, allowing the atoms enough time to find positions that minimize a steady state is reached. It is this characteristic of the annealing process which makes it possible to achieve near global minimum energy states.

Simulated annealing algorithm's major advantage is an ability to avoid becoming trapped in local minima. The algorithm modifies the serial random search algorithm so that designs with higher objective function f (assuming a minimisation problem) are occasionally accepted.

In order to generate annealing behaviour, the algorithm process is arranged. First, a starting temperature T and a starting feasible design variable x^* are obtained. Then, a new candidate design variable x close to x^* generate randomly and candidate variable is analyzed. If it is feasible and $f(x^*) > f(x)$, let $x^* = x$. If the candidate design variable is feasible but $f(x) > f(x^*)$, then a random number r is generated. If $r < P$ where;

$$P = \left(\frac{f(x^*) - f(x)}{T} \right) \quad (1.10)$$

If x^* has not changed for several iterations, search is stopped; otherwise update $T = \alpha T$ where α is a number less than one, and a new candidate variable is generated. The procedure for SA algorithm is as follows in Figure 1.2 [19].

```

initialize temperature
for i := 1...ntemps do
    temperature := factor * temperature
for j := 1...nlimit do
    try swapping a random pair of points
    delta := current_cost - trial_cost
    if delta > 0 then
        make the swap permanent
        increment good_swaps
    else
        p := random number in range [0...1]
        m := exp( delta / temperature )
        if p < m then // Metropolis criterion
            make the swap permanent
            increment good_swaps
        end if
    end if
exit when good_swaps > glimit
end for

```

Figure 1.2 Pseudo code of Simulated Annealing Algorithm

Where;

factor - annealing temperature reduction factor;
ntemps - number of temperature steps to try;
nlimit - number of trials at each temperature;
glimit - number of succesful trials.

Note that as the temperature T decreases from iteration to iteration. Temperature decreases the probability P of accepting designs with higher f . This means that the

algorithm is likely to accept designs with higher f in the initial solutions, but it is less likely to accept worse designs in the final solutions as it converges to the global minimum. This situation simulates the annealing of metals as they cool from liquid to solid states. If the cooling is performed very slowly, the metal will solidify in a crystalline state which is the global minimum of the internal energy function. On the other hand, if cooling is performed very rapidly, the metal will solidify in a glass state which is a local minimum of the energy function. If the temperature drops rapidly, the acceptance probability for designs with higher f goes to zero.

1.2.3.3 Ant Colony Optimization

A new computational paradigm called ant colony optimization attempts to model some of the fundamental capabilities observed in the behaviour of ants as a method of stochastic combinatorial optimization. The fundamental theory in an ant colony optimization algorithm is the simulation of the autocatalytic, positive feedback process exhibited by a colony of ants. This process is modelled by utilizing a virtual substance called 'trail' that is analogous to pheromones used by ants [20]. Each ant colony optimization move behind in the same direction a basic computational structure outlined by the pseudo-code in Figure 1.3 [21]. An ant starts at a randomly selected point and must decide which of the available paths to travel. The selection criterion is based on the intensity of the paths leading to the intensity of trail present upon each path leading to the adjacent points. The path with the most trail has a higher probability of being chosen.

After each ant selects a path using a decision mechanism and travels along it to another point, a local trail update rule may be applied to the path. The local update rule reduces the intensity of trail on the selected path by the ant. After subsequent ants arrive at this point, they will have a slightly smaller probability of selecting the same path as other ants before them. This process is purposed to promote exploration of the

search space, which helps prevent early stagnation of the search and premature convergence of the solution. Each ant continues to select paths between points, until all points have been visited and it arrives back its starting point. When it turns to first point, the ant completed a tour.

The combination paths an ant selects to complete a tour is a solution to the problem, and is analyzed to determined how to solve the problem better. The intensity of trail upon each path in the tour is then adjusted through a global update process. The magnitude of the global trail adjustment reflects how well a particular solution produced by an ant's tour solves the problem. Tours that best solve the problem receive more trail than those tours that represent poor solutions. In this way, when the ants begin the next tour, there is a greater probability that an ant will select a path that was part of a tour that performed well in the past. When all of the tours have been analyzed and the trail levels on the paths have been updated, an ant colony optimization is completed. A new cycles now begins and entire process is repeated. Ultimately, all of the ants will choose same tour on every cycle, representing the convergence to a solution. Stopping criterions are typically based upon comparing to best solution from the last cycle to the best global solution found in all previous cycles. If the comparison shows that the algorithm is no longer improving the solution, then the criteria are reached.

```
Initialize Trail
Do While (Stopping Criteria Not Satisfied) – Cycle Loop
    Do Until (Each Ant Completes a Tour) – Tour Loop
        Ant Decision Mechanism
        Local Trail Update
    End Do
    Global Trail Update
End Do
```

Figure 1.3 Ant colony optimization algorithm procedure

1.2.3.4 Harmony Search Algorithm

The new optimization technique Harmony Search algorithm was conceptualized using the musical process of searching for a perfect state of harmony. Harmony search (HS) algorithm uses stochastic random search instead of gradient search to derive information. If HS algorithm is compared with other optimization techniques, It requires fewer mathematical expression for solving optimization problems. Figure 1.4 shows the design procedure that was used to apply the HS algorithm to optimization problems [22].

Step 1 Initialize Problem

The discrete optimization problem is defined

Step 2 Initialize Harmony Memory

HM matrix is randomly filled as many solution vectors as HMS

Step 3 Improvise New Harmony

A new harmony vector is improvised. For selecting one value for each variable: memory consideration, pitch adjustment and random selection

Step 4 Update Harmony Memory

If the new harmony is better than the worst harmony in the HM, the new harmony is added in the HM and the worst harmony is excluded from HM

Step 5 Check Termination Criterion

The computation is terminated when the termination criterion is satisfied.

Figure 1.4 Harmony Search Algorithm procedure

Harmony Search algorithm has been successfully applied to various optimization problems including function minimization problems, the layout of pipe networks, pipe capacity design in water supply networks, the travelling salesman problem, and truss examples.

In this thesis, HS algorithm was modified for the function minimization and several grillage system examples from the literature are presented to show its effectiveness and robustness.

1.3 Grillage systems

Rigidly jointed frames, if loaded perpendicular to their plane, are called as grillage. The steel grillage systems as shown in Figure 1, are made out of thin walled members and they are subjected to out-of-plane loading. A horizontal grid frame consists of two sets of parallel beams, with one set perpendicular to the other. Each beam may be simply supported at the ends, a fixed in rotation about the transverse axis as well or sometimes also fixed against rotation about the longitudinal axis. If one set of beams sits directly above the other set then there are only vertical interaction between two sets at the points of intersection. On the other hand, if the two sets of beams are all the same elevation and if the intersecting joints are rigid (welded steel) then each of the elements is capable of resisting torsional and bending moments, by virtue of its end connections is called a combined beam and torsion element.

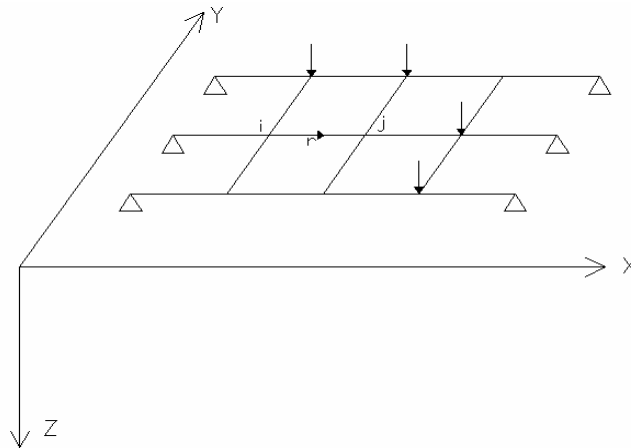


Figure 1.5 Steel Grillage System

1.3.1 Using Areas of Grillage Systems

Grillage systems are used in various design of structural buildings. Some of them are bridges, deck of the ships, deck and wings of the plane and etc.



Figure 1.6 Bridge deck

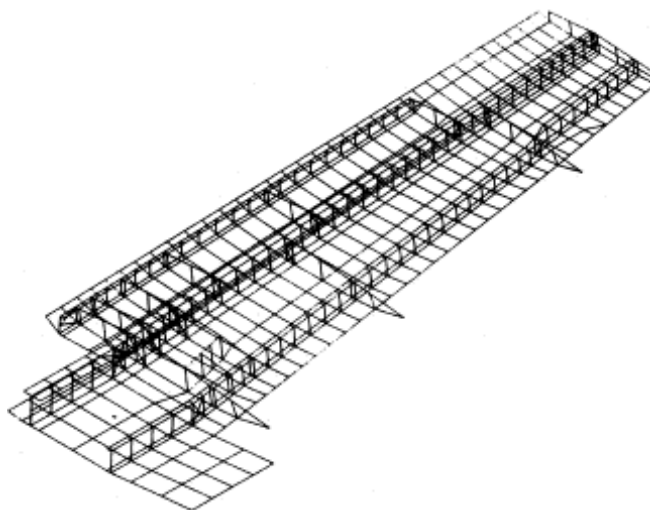


Figure 1.6 Internal structure and lower skin of the aircraft.

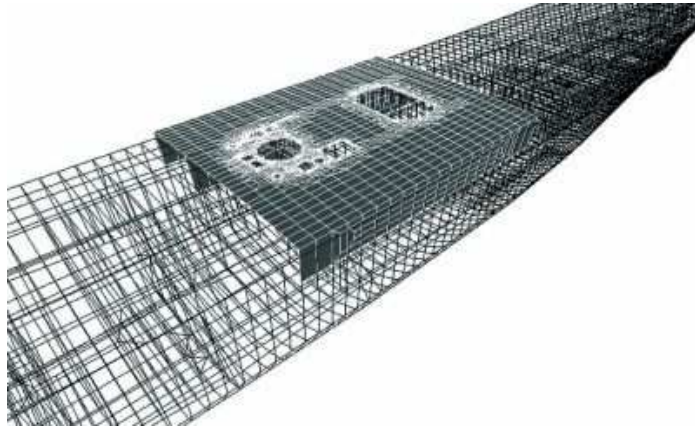


Figure 1.7 A detail model of the midship main deck

1.4 Scope of Work

This thesis is concerned with optimum design of grillage systems using harmony search method. This thesis is organized as follows. In the first chapter, a brief introduction is given to optimization, structural optimization, an overview on existing structural optimization methods, and grillage systems. Chapter 2 discusses the fundamentals of harmony search algorithm and some of the key aspects of its current theory. The last part of the chapter, current numerical test problems are solved by using harmony search algorithm and compared with the results of other optimization techniques. Chapter 3 deals with laterally supported beams, design of laterally supported beams to LRFD, structural optimization of grillage systems including the definition and selections of design variables. Chapter 4 is devoted to optimum design of several grillage system examples with harmony search algorithm. In the fifth and last chapter, some brief discussions and conclusions are presented.

CHAPTER 2

HARMONY SEARCH ALGORITHM

2.1 General Information about Harmony Search Method

The Harmony Search (HS) algorithm developed by Geem et al belongs to the group of stochastic search techniques [23-25]. It is comparatively simple method that imposes fewer mathematical requirements. Similar to the other stochastic search method, it randomly selects candidate solutions to the optimization problem from a discrete or continuous set. This selection is checked to find out whether it is feasible or not. If it is then it is inserted into what is called is a harmony search memory where each candidate solution is stored in a decending order. The method after filling the harmony search memory matrix continuous selection of the new solutions depending on two parameter either from the harmony memory considering rate and the pitch adjusting rate. Harmony Search algorithm is comparatively simple approach compared to mathematical programming techniques and it does not require neither initial starting values for the decision variables nor the derivative information of the objective function and consraints. Thus, the harmony search method provides easy programming among the combinatorial optimization algorithms.

The basic idea behind the harmony search algorithm is similar to the ideas of all meta-heuristic algorithms that are found in the paradigm of natural phenomena. Following the idea of meta-heuristic algorithms that all seek a stable state, the harmony search method drives its roots in the harmony of a musical performance which exists in the nature. Music harmony is a combination of sounds considered pleasing from an aesthetic point of view. Music harmony in nature is a kind of beat phenomenon made

by several sound waves that have different frequencies. Since the Greek philosopher and mathematician Pythagoras (BC 582-BC 497) many have researched this phenomenon. French composer Philippe Rameau (1683-1764) established the classic harmony theory.

Harmony search algorithm is based on natural musical performance processes that occur when a musician searches for a better state of harmony, such as during jazz operation. Jazz improvisation tries to reach musically pleasing a best state harmony as determined by an aesthetic standard, just as the optimization process seeks to find a best solution (global optimum-minimum cost or maximum benefit or efficiency) as determined by objective function. The pitch of each musical instrument determines the aesthetic quality, just as the objective function value is defined by the set of values assigned to each design variable. The sounds for better aesthetic quality can be improved through practice after practice, just as the values for better objective function evaluation can be improved iteration by iteration. Musical Performance and Optimization observation process are shown in Table 2.1. Harmony Search algorithm design procedure is shown in Figure 2.2

Table 2.1 Comparison between Musical Performance and Optimization Process

COMPARISON FACTOR	PERFORMANCE PROCESS	OPTIMIZATION PROCESS
Best State	Fantastic Harmony	Global Optimum
Estimated by	Aesthetic Standard	Objective Function
Estimated with	Pitches of Instruments	Values of Variables
Process Unit	Each Practice	Each Iteration

In harmony search algorithm first, initialize the optimization problem and algorithm parameters. The discrete size optimization problem is specified as objective function $f(x)$. The number of discrete design variables (x_i) and the set of available discrete values D_i .

$$D_i = \{x_i(1), x_i(2), \dots, x_i(K)\} \text{ for discrete decision variables.} \quad (2.1)$$

$$\text{Minimize } f(x) \quad (2.2)$$

$$\text{Subject to } x_i \in D_i, \quad i = 1, 2, 3, \dots, N \quad (2.3)$$

N is the number of variables; K is the number of possible values for the variables.

The harmony search algorithm uses some randomly generated parameters which are required to solve optimization problem. These parameters are the harmony memory size (number of solution vectors, HMS), harmony memory considering rate (HMCR), pitch adjusting rate (PAR), and termination criterion (maximum number of searches). HMCR and PAR parameters are used for improving the solution vector (HMS).

After initializing the optimization problem and algorithm parameters harmony memory (HM) matrix is randomly filled with as many solution vectors as harmony memory size (HMS). Harmony memory matrix has the following form

$$H = \begin{bmatrix} x_1^1 & x_2^1 & \dots & x_{N-1}^1 & x_N^1 \\ x_1^2 & x_2^2 & \dots & x_{N-1}^2 & x_N^2 \\ \vdots & \dots & \dots & \dots & \dots \\ x_1^{HMS-1} & x_2^{HMS-1} & \dots & x_{N-1}^{HMS-1} & x_N^{HMS-1} \\ x_1^{HMS} & x_2^{HMS} & \dots & x_{N-1}^{HMS} & x_N^{HMS} \end{bmatrix} \Rightarrow \begin{matrix} f(x^1) \\ f(x^2) \\ \vdots \\ f(x^{HMS-1}) \\ f(x^{HMS}) \end{matrix}$$

The feasible solutions in the harmony memory matrix are sorted in descending order according to their objective function value. Harmony search matrix is initialized by inserting zero value for each design variable. There are three rules to select a new value for a design variable. These rules are memory consideration, pitch adjustment and random selection.

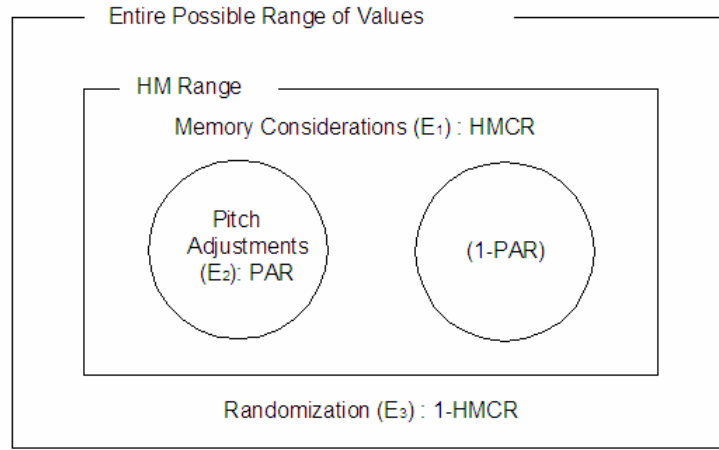


Figure 2.1 HM improvisation process

The new harmony improvisation process has shown in Figure 2.1 This process is based on memory considerations (E1), pitch adjustments (E2) and randomization (E3). In the memory consideration process, the new value of the first design variable (x'_1) for the new vector is selected any discrete value in the specified HM range $\{x_1^1, x_1^2, \dots, x_1^{HMS}\}$. The same manner is applied to all other design variables. Here, there is a possibility that the new value can be selected using the HMCR, which varies between 0 and 1.

$$x_i^{new} = \left\{ \begin{array}{ll} x'_i \in \{x_i^1, x_i^2, \dots, x_i^{HMS}\} & \text{with probability HMCR} \\ x'_i \in D_i & \text{with probability } (1 - HMCR) \end{array} \right\} \quad (2.4)$$

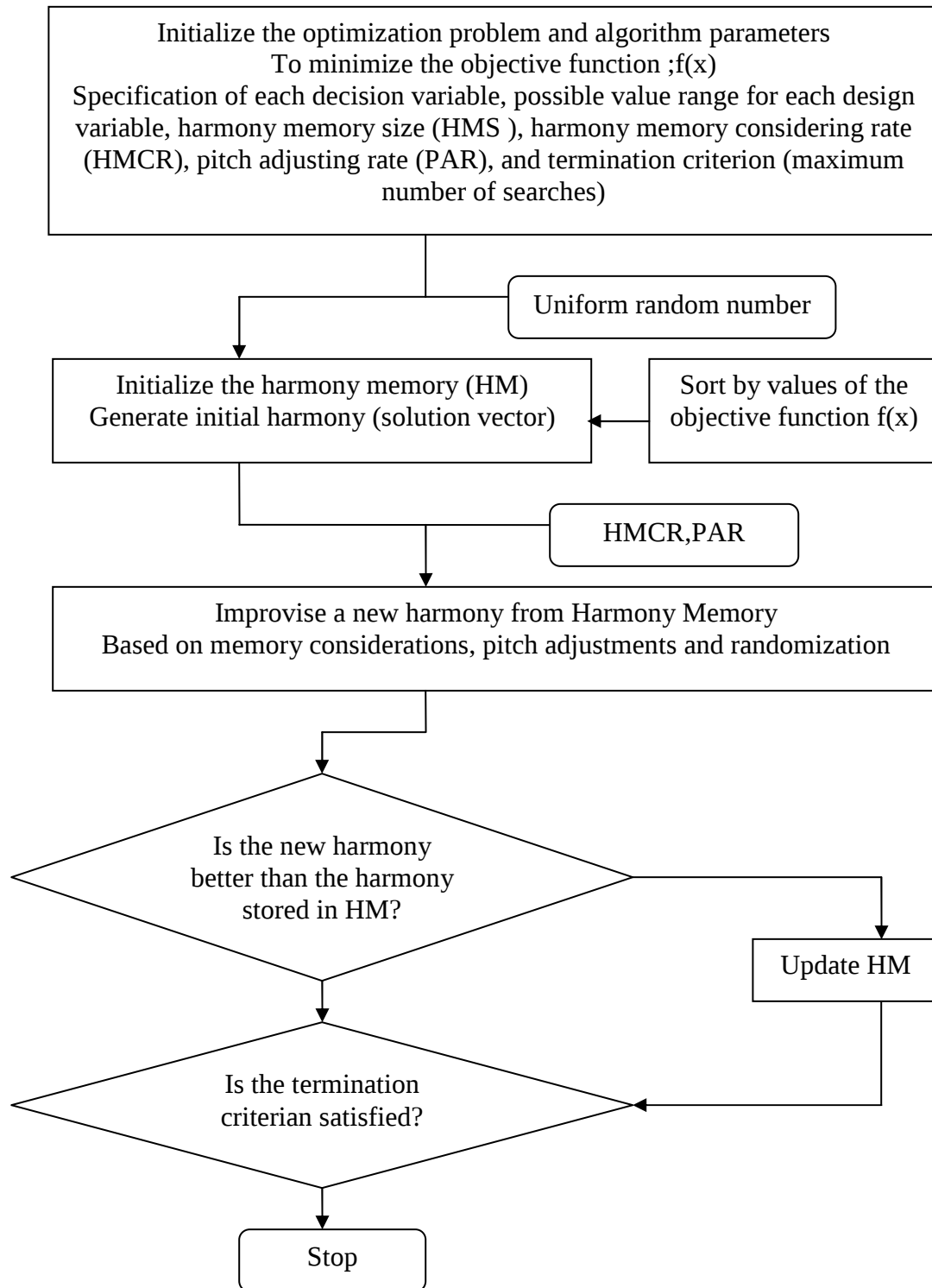


Figure 2.2 Harmony Search Algorithm Procedure

If a randomly generated value between 0 to 1 occurs the current value of the HMCR, then HS finds notes randomly within the possible playable range without considering harmony memory (HM). For example, a HMCR of 0,80 means that at the next step, HS algorithm chooses a variable value from HM with a 80% probability and undivided possible range with a 20% probability. A HMCR value of 1.0 is not advised, as there is a chance that the solution will be developed by values not stored in the HM. A new value of the design variable is chosen among the design variables in the discrete set of harmony memory matrix. This value is then checked to define whether it should be pitch-adjusted. This operation uses pitch adjusting parameter (PAR) that sets the rate of pitch-adjustment decision as follows:

$$\text{Is } x_i^{new} \text{ to be pitch-adjusted? } \left\{ \begin{array}{ll} \text{Yes} & \text{with probability of } PAR \\ \text{No} & \text{with probability of } (1 - PAR) \end{array} \right\} \quad (2.5)$$

For computation, the pitch adjustment mechanism is devised as shifting to neighboring values within a range of possible values. If there are six possible values such as (1, 2, 4, 5, 6, 8), (5) can be moved to neighboring {4} or {6} in the pitch adjusting process. A PAR of 0.20 means that the HS algorithm will select a neighboring value with 20% x HMCR probability. Assuming that the new pitch-adjustment decision for x_i^{new} came out to be yes from the test and if the value selected for x_i^{new} from the harmony memory is the k^{th} element in the general discrete set, then the neighboring value $k+1$ or $k-1$ is taken for new x_i^{new} . This process improves the harmony memory for diversity with a greater change of reaching the global optimum.

After selecting the new values for each design variable the objective function value is calculated for the newest harmony vector. If this value is better than the worst harmony vector in the harmony matrix, it is then included in the matrix while the worst one is taken out of the matrix. The harmony memory matrix is then sorted in descending order

by the objective function value. The above steps are repeated until no further improvement is possible in the objective function.

2.2 Numerical Applications

The Harmony Search method explained in the previous sections is used to determine the optimum solutions of number of optimization problems. The results obtained are compared with other heuristic algorithms.

2.2.1 Example 1.

In this example, we refer to following optimization problem. Find; $x = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$

The values of x_1 and x_2 are restricted to the set $\{0.5, 1.0, 1.5, 2.0, 2.5, 3.0, \dots, 10.0\}$

It minimizes $f(x) = 5x_1^2 - 9x_1x_2 + 5x_2^2$ which is (2.6)

Subjected to $g(x) = 25 - 16x_1x_2 \leq 0$ (2.7)

The objective function is $f(x)$, which is one of the standard test functions in optimizations problems [26]. The discrete optimum to the example problem is $\{1.5 \ 1.5\}^T$, which was verified from the Kuhn-Tucker conditions.

When applying the HS algorithm to the function, possible value bounds between 0.5 and 10.0 were used for the design variables, x_1 and x_2 shown in objective function. The total number of the solution vectors, i.e., the (*HMS*), is taken as 10. Harmony

memory considering rate (*HMCR*) is selected as 0.9 while pitch- adjusting rate is considered as 0.2 as suggested in [24]. After 10 searches the initial harmony matrix is obtained as given in Table 2.2.

Table 2.2 Initial harmony search matrix after 10 searches

Row Number	x_1	x_2	$f(x)$
1	3.0	4.5	24.75
2	2.0	4.0	28.00
3	7.5	6.0	56.25
4	4.5	7.0	62.75
5	5.5	8.0	75.25
6	8.5	5.5	91.75
7	10.0	9.0	95.00
8	10.0	10.0	100.00
9	5.5	1.0	106.75
10	1.0	9.0	329.00

As shown in Table 2.2, the HM was initially structured with randomly generated solution vectors within the bounds. The solution vectors are sorted according to the values of the objective function. 11th, 12th and 13th search can not find a better solution than the ones shown in Table 2.2. However, 14th search gives a better harmony search matrix as shown in Table 2.3. A new harmony vector $x_i' = (6.0, 9.0)$ was improvised based on three rules: memory considerations with a 72.0% probability ($0.9 \times 0.8 = 0.72$), pitch adjustments with a 0.18% probability ($0.9 \times 0.2 = 0.18$), and randomization with a 10% probability ($1 - 0.9 = 0.1$). As the objective function value of the new harmony (6.0, 9.0) is 99.00, the new harmony is included in the HM and the worst harmony (1.0, 9.0) is excluded from the HM, as shown in Table 2.3 (Subsequent HM).

Table 2.3 Harmony search matrix after 14 searches

Row Number	x_1	x_2	$f(x)$
1	3.0	4.5	24.75
2	2.0	4.0	28.00
3	7.5	6.0	56.25
4	4.5	7.0	62.75
5	5.5	8.0	75.25
6	8.5	5.5	91.75
7	10.0	9.0	95.00
8	6.0	9.0	99.00
9	10.0	10.0	100.00
10	5.5	1.0	106.75

The new combination does not affect the first row of the harmony memory matrix. However, when the harmony search algorithm continued to seek better combination, newly found combination changes the harmony matrix. For instance after 100 searches, the combinations shown in Table 2.4.

Table 2.4 Harmony search matrix after 100 searches

Row Number	x_1	x_2	$f(x)$
1	3.0	3.5	11.75
2	3.0	4.0	17.00
3	4.5	4.0	19.25
4	4.5	4.5	20.25
5	4.5	3.5	20.75
6	3.0	4.5	24.75
7	2.0	4.0	28.00
8	5.5	4.5	29.75
9	5.5	5.5	30.25
10	5.5	4.0	33.25

The probability of finding the minimum vector, $x^* = (1.5, 1.5)$ increased with the number of searches. The results obtained after 500 searches are given in Table 2.5

Table 2.5 Harmony search matrix after 500 searches

Row Number	x_1	x_2	$f(x)$
1	2.0	1.5	4.25
2	2.5	2.5	6.25
3	3.0	2.5	8.75
4	3.0	3.0	9.00
5	3.0	3.5	11.75
6	3.5	3.5	12.25
7	2.5	3.5	13.75
8	3.5	4.0	15.25
9	3.0	1.5	15.75
10	3.0	4.0	17.00

Finally, the combinations given in the first row of the harmony search matrix which is improvised the optimal harmony, $x^* = (1.5, 1.5)$, after 1000 searches, which has a minimum function. The final results obtained after 1000 searches are given in Table 2.6

Table 2.6 Harmony search matrix after 1000 searches

Row Number	x_1	x_2	$f(x)$
1	1.5	1.5	2.25
2	2.0	1.5	4.25
3	2.5	2.5	6.25
4	3.0	2.5	8.75
5	3.0	3.0	9.00
6	2.0	3.0	11.00
7	3.0	3.5	11.75
8	3.5	3.5	12.25
9	2.5	3.5	13.75
10	3.5	4.0	15.25

It is interesting to see the function $f(x) = 5x_1^2 - 9x_1x_2 + 5x_2^2$ we have optimised and we give both a contour plot in Figure 2.3 and three-dimensional plot in Figure 2.4.

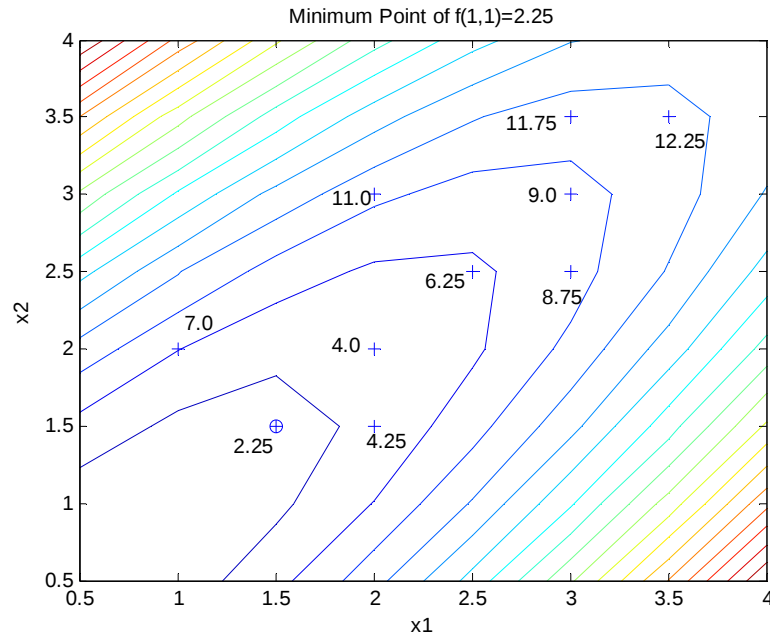


Figure 2.3 Contour plot of the function $f(x) = 5x_1^2 - 9x_1x_2 + 5x_2^2$

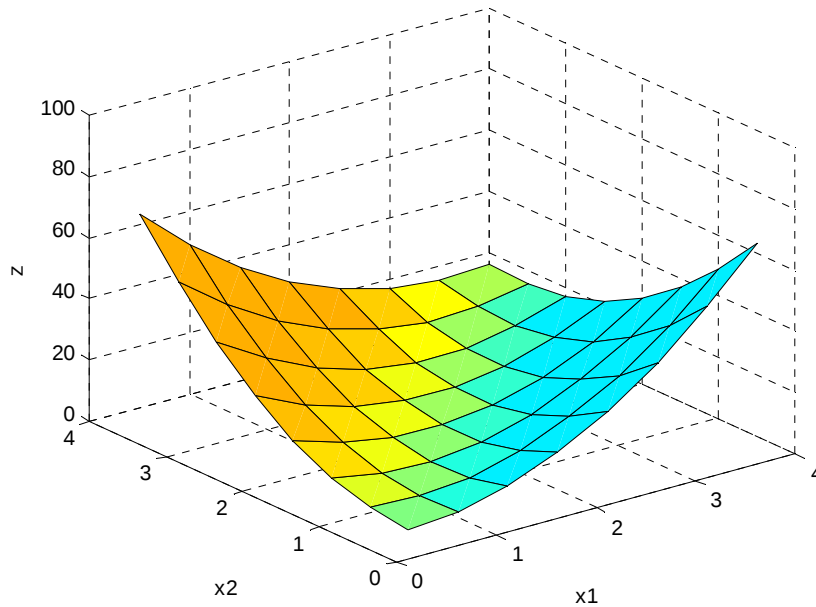


Figure 2.4 Three Dimensional plot of the function $f(x) = 5x_1^2 - 9x_1x_2 + 5x_2^2$

2.2.2 Example 2.

The second example selected for application of Harmony Search method is a common benchmark problem which has nonlinear constraints and objective function. The problem is called Himmelblau's function [27]. This problem was adopted to test Harmony search (HM) algorithm which has an improved constraint capability. The optimization problem, which has five design variables and fifteen nonlinear constraints, is as shown in the following.

$$\begin{aligned} &\text{Minimize} \\ &f(x) = 5.3578547x_3^2 + 0.8356891x_1x_5 + 37.293239x_1 - 40792.141 \end{aligned} \quad (2.8)$$

subject to:

$$0 \leq g_1(x) \leq 92 \quad (2.9)$$

$$90 \leq g_2(x) \leq 110$$

$$20 \leq g_3(x) \leq 25$$

where

$$\begin{aligned} g_1(x) &= 85.334407 + 0.0056858x_2x_5 + 0.0006262x_1x_4 - 0.0022053x_3x_5 \\ g_2(x) &= 80.51249 + 0.0071317x_2x_5 + 0.0029955x_1x_2 + 0.0021813x_3^2 \\ g_3(x) &= 9.300961 + 0.0047026x_3x_5 + 0.0012547x_1x_3 + 0.0019085x_3x_4 \end{aligned} \quad (2.10)$$

and side constraints

$$78 \leq x_1 \leq 102, \quad 33 \leq x_2 \leq 45, \quad 27 \leq x_3 \leq 45, \quad 27 \leq x_4 \leq 45, \quad 27 \leq x_5 \leq 45 \quad (2.11)$$

Himmelblau [27] used the generalized reduced gradient (GRG) method to solve this problem. The same problem was also tackled by Gen and Cheng [28] using genetic algorithm (GA) based on both local and global references. Premkumar and Wu [29] used a Particle Swarm Optimization (PSO) with stochastic ranking to solve this problem.

To apply HS algorithm to the Himmelblau's function, the five design variables x_1, x_2, x_3, x_4 and x_5 were assumed to be discrete variables, and their possible values were taken from the set $D \in \{25.00, 25.01, 25.02, \dots, 89.98, 89.99, 90.00\}$, which has 6501 discrete values. The ten cases shown in Table 2.7, each case with a different set of HS algorithm parameters (i.e. HMS, HMCR, and PAR), were tested for this example. These parameter values were arbitrarily selected on the basis of the empirical findings by Geem.

Table 2.7 HS algorithm parameters used for Himmelblau's Function

Cases	HMS	HMCR	PAR	f(x)
1	40	0,9	0,45	-30141,52
2	40	0,8	0,40	-30473,91
3	50	0,9	0,30	-30477,98
4	30	0,7	0,40	-30499,07
5	50	0,8	0,35	-30561,30
6	40	0,8	0,30	-30567,34
7	50	0,9	0,40	-30607,43
8	30	0,7	0,35	-30610,69
9	40	0,9	0,30	-30615,84
10	40	0,8	0,45	-30622,36

For Himmelblau's function, all the results obtained from these methods mentioned below are listed in Table 2.8. All the results are compared against those obtained from HS algorithm. The total number of solution vectors, i.e. the HMS, is 40, and the HMCR and PAR are 0.8 and 0.45, respectively.

Table 2.8 Optimum solution of Himmelblau's function

Optimum solutions obtained by different methods				
Design variables	HS	Runarsson[30]	GRG[27]	Gen[28]
x_1	77.9500	78.0000	78.6200	81.4900
x_2	33.1200	33.0000	33.4400	34.0900
x_3	30.3300	29.9953	31.0700	31.2400
x_4	44.9600	45.0000	44.1800	42.2000
x_5	35.8300	36.7758	35.2200	34.3700
$g_1(x)$	91.8801	92.0000	91.7927	91.7819
$g_2(x)$	98.7220	98.8405	98.8929	99.3188
$g_3(x)$	19.9811	20.0000	20.1316	20.0604
$f(x)$	-30622.36	-30665.539	-30373.949	-30183.576

The range of each design variable has been narrowed from the lower bound value to the upper bound value which has been stored in HM with number of searches. Finally, the HS heuristic algorithm improvised the optimal harmony ($f(x) = -30622.36$) after 120,000 searches. HS algorithm proves to outperform the other methods in this continuous-variable problem.

2.2.3 Example 3.

The design of welded connection shown in Figure 2.5 is taken as third example. A rectangular beam is designed as a cantilever beam to carry a certain load with minimum overall cost of fabrication. The problem involves four design variables: the thickness of the weld $h = x_1$, the length of the welded joint $l = x_2$, the width of the beam $t = x_3$ and the thickness of the beam $b = x_4$. The values of x_1 and x_2 are coded with integer multiplies of 0.0065. There are fifteen constraints, which involve shear stress (τ), bending stress in the beam (σ), buckling load on the bar (P_c), deflection of the beam (δ) and side constraints[31].

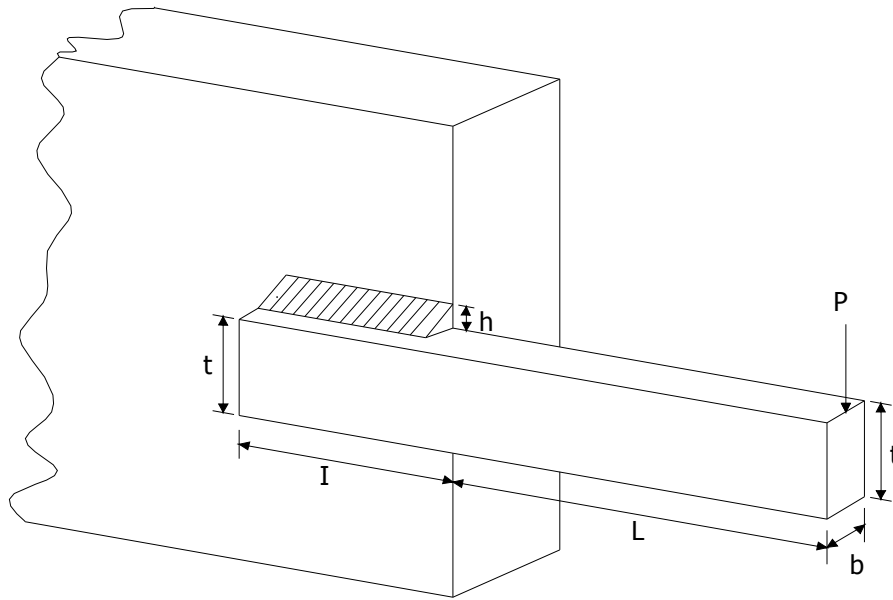


Figure 2.5 Welded beam design

Minimize

$$f(x) = 1.10471x_1^2 x_2 + 0.04811x_3 x_4 (14.0 + x_2) \quad (2.12)$$

Subject to:

$$g_1(x) = \tau(x) - \tau_{\max} \leq 0 \quad \rightarrow \quad \text{shear stress} \quad (2.13)$$

$$g_2(x) = \sigma(x) - \sigma_{\max} \leq 0 \quad \rightarrow \quad \text{bending stress in the beam} \quad (2.14)$$

$$g_3(x) = x_1 - x_4 \leq 0 \quad \rightarrow \quad \text{side constraint} \quad (2.15)$$

$$g_4(x) = 0.10471x_1^2 + 0.04811x_3x_4(14.0 + x_2) - 5 \leq 0 \quad \rightarrow \quad \text{side constraint} \quad (2.16)$$

$$g_5(x) = 0.125 - x_1 \leq 0 \quad \rightarrow \quad \text{side constraint} \quad (2.17)$$

$$g_6(x) = \delta(x) - \delta_{\max} \leq 0 \quad \rightarrow \quad \text{end deflection of the beam} \quad (2.18)$$

$$g_7(x) = P - P_c(x) \leq 0 \quad \rightarrow \quad \text{buckling load on the bar} \quad (2.19)$$

Where

$$\tau(x) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + (\tau'')^2} \quad (2.20)$$

$$\tau' = \frac{P}{\sqrt{2}x_1x_2} \quad \tau'' = \frac{MR}{J} \quad (2.21)$$

$$M = P\left(L + \frac{x_2}{2}\right) \quad R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2} \quad (2.22)$$

$$J = 2\left\{\frac{x_1x_2}{\sqrt{2}}\left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\} \quad \delta(x) = \frac{4PL^3}{Ex_3^3x_4} \quad (2.23)$$

$$\sigma(x) = \frac{6PL}{x_4x_3^2} \quad P_c(x) = \frac{4.013\sqrt{\frac{EGx_3^2x_4^6}{36}}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right) \quad (2.24)$$

$$P = 6000 \text{ lb}, \quad L = 14 \text{ in.}, \quad E = 30 \times 10^6 \text{ psi}, \quad G = 12 \times 10^6 \text{ psi} \quad (2.25)$$

$$\tau_{\max} = 13,600 \text{ psi}, \quad \sigma_{\max} = 30,000 \text{ psi}, \quad \delta_{\max} = 0.25 \text{ in.}$$

The ranges for the design variables are given as follows:

$$\begin{aligned} 0.1 \leq x_1 \leq 2.0, \quad 0.1 \leq x_2 \leq 10 \\ 0.1 \leq x_3 \leq 10, \quad 0.1 \leq x_4 \leq 2.0 \end{aligned} \quad (2.26)$$

To apply HS algorithm to the welded beam, the five design variables x_1, x_2, x_3 and x_4 were assumed to be discrete variables, and their possible values have shown above. The ten cases shown in Table 2.9, each case with a different set of HS algorithm parameters (i.e. HMS, HMCR, and PAR), were tested for this example. These parameter values were arbitrarily selected on the basis of the empirical findings by Geem and Lee [32].

Table 2.9 HS algorithm parameters used for Welded Beam

Cases	HMS	HMCR	PAR	f(x)
1	30	0,70	0,40	3.0122
2	35	0,75	0,40	3.0008
3	30	0,70	0,35	2.9749
4	40	0,80	0,40	2.7775
5	50	0,90	0,30	2.7210
6	40	0,80	0,30	2.6977
7	50	0,85	0,40	2.5821
8	40	0,90	0,45	2.4955
9	50	0,9	0,50	2.3710
10	50	0,9	0,40	2.2290

The size of HM matrix (HMS) is taken as (50x5). The total number of solution vectors, i.e. the HMCR and PAR are 0.8 and 0.40, respectively. The minimum fuction is obtained after 1000 searches.

The same problem was also solved by Ragsdell and Philips [33] using geometric programming. Deb [34] used a simple Genetic Algorithm (GA) with traditional penalty function to solve the same problem.

Table 2.10 Optimum Solution of welded beam design

Optimum solutions obtained by different methods				
Design variables	HS	PSO[29]	Ragsdell[33]	Deb[34]
x_1	0.22200	0.244369	0.245500	0.248900
x_2	3.05100	6.217519	6.196000	6.173000
x_3	9.54500	8.291471	8.273000	8.178900
x_4	0.26300	0.244369	0.245500	0.253300
$g_1(x)$	-3240.0106	-5741.176933	-5743.82652	-5758.6038
$g_2(x)$	-8965.949	-0.0000007	-4.715097	-255.5769
$g_3(x)$	-0.04100	0.000000	0.000000	-0.004400
$g_4(x)$	-2.935552	-3.022954	-3.020288	-2.982900
$g_5(x)$	-0.097000	-0.119369	-0.120500	-0.123900
$g_6(x)$	-0.158385	-0.234241	-0.234208	-0.234160
$g_7(x)$	-2212.05268	-0.000309	-74.276856	-618.818492
$f(x)$	2.229000	2.380956	2.385937	2.433116

All the results are compared against those obtained from HS algorithm. Table 2.10 shows the comparison of the optimum solutions obtained by Harmony Search algorithm and other optimization methods as shown in the last row of the Table 2.9, Harmony Search algorithm determined the lowest value for the objective function compare to other methods. It took 1000 iterations to reach to $f(x) = 2.229000$.

CHAPTER 3

OPTIMUM DESIGN OF GRILLAGE SYSTEMS

3.1 Design of Grillage Systems

The design of grillage systems is one of the common problems of steel structures that practicing engineer has to deal with. Optimum design of grillage systems aims at finding the cross sectional properties of transverse and longitudinal beams such that the response of the system under the external loading is within the allowable limits defined in code of practice while the weight or the cost of the system is the minimum.

In one of the early studies, the optimum design problem is formulated by treating the moment of inertias of the beams and joint displacements as design variables [35]. Stiffness, stress, displacement and size constraints are included in the design formulation. The effect of warping is taken into account in the computation of the stresses in the members. The nonlinear programming problem obtained is solved by the approximating programming method. The formulation of the same design problem is carried out only treating the cross-sectional of the members in the grillage system in [36] where the warping and shear effects are also considered in the computation of the response of the system under the external loading. Displacements, stress and size limitations are included in the design formulation according to ASD-AISC code. The solution of the optimum design problems achieved using optimality criteria approach. In [37], genetic algorithm is used to determine the optimum universal beam sections (UB) for the members of grillage system from set of British Standards Universal Beam sections. The deflection limitations and the allowable stress constraints are considered

in the formulation of the design problem. The algorithm developed is utilized to investigate the effect of warping in the optimum design of grillage systems. Previous study is extended to cover the determination of the optimum spacing between both transverse and longitudinal beams in addition to optimum sectional designations in the grillage system in [38]. The optimum spacing between transverse beams as well as longitudinal beams is determined both considering and not taking into account the effect of warping in the optimum design.

3.2 General formulation of optimum design problem

Structural optimization seeks the selection of design variables to achieve, within the constraints placed on the structural behaviour, geometry, or other factors, its goal of optimality defined by the objective function for specified loading or environmental conditions.

3.2.1 Design variables

The design variables of an optimum structural design problem may consist of the member sizes, parameters that describe the structural configuration and quantifiable aspects of the design. The design variables which are varied by the optimization procedure may represent the following properties of the structure:

- a. the mechanical or physical properties of the material;
- b. the topology of the structure;
- c. the geometry or configuration of the structure;
- d. the cross-sectional dimensions or the member sizes.

3.2.2 Constraints

A constraint is a restriction to be satisfied in order for the design to be acceptable. It may take the form of a limitation imposed directly on variables, or may represent a limitation on quantities whose dependence on the design variables cannot be stated directly.

From mathematical point of view, both design and behaviour constraints may usually be expressed as a set of inequalities.

$$g_j(x) \leq 0 \quad , \quad j = 1, 2, \dots, n_g \quad (3.1)$$

Where n_g is the number of inequality constraints and x is the vector of design variables.

An equality constraint, which may be either explicit or implicit, is designated as

$$h_j(x) = 0 \quad , \quad j = 1, 2, \dots, n_h \quad (3.2)$$

where n_h is the number of equalities. The constraints may be linear or nonlinear functions of the design variables. These functions may be explicit or implicit in x and may be evaluated by analytical or numerical techniques.

3.2.3 Objective function

The objective function is the function whose least value tries to reach in an optimization procedure, and constitutes a basis for the selection of one of several alternative acceptable designs. Objective function is generally a nonlinear function of

the variables x , and it may represent the weight, the cost of structure, or other criterion by which some possible designs are preferred to others.

Assuming that all equality can be eliminated, the optimal design problem can be formulated mathematically as one of choosing the vector of design variables x such that

$$Z = f(x) \rightarrow \min \quad (3.3)$$

$$g_j(x) \leq 0 \quad , \quad j = 1, 2, \dots, n_g \quad (3.4)$$

3.3 Optimum Design Problem to LRFD-AISC

The optimum design problem of a typical grillage system shown in Figure 1 where the behavioral and performance limitations are implemented from LRFD-AISC [39] can be formulated as follows.

$$\min \quad W = \sum_{k=1}^{ng} m_k \sum_{i=1}^{r_k} l_i \quad (3.5)$$

Subject to

$$\delta_j \leq \delta_{ju} \quad , \quad j = 1, 2, \dots, p \quad (3.6)$$

$$\phi_b M_{nr} \geq M_{ur} \quad , \quad r = 1, 2, \dots, nm \quad (3.7)$$

$$\phi_b V_{nr} \geq V_{ur} \quad , \quad r = 1, 2, \dots, nm \quad (3.8)$$

where m_k in Eq. 3.5 is the unit weight of grillage element belonging to group k selected from W-sections list of LRFD-AISC, r_k is the total number of members in group k , and ng is the total number of groups in the grillage system. l_i is the length of member i . δ_j in Eq. 3.6 is the displacement of joint j and δ_{ju} is its upper bound.

The joint displacements are computed using the matrix displacement method for grillage systems. Eq. 3.7 represents the strength requirement for laterally supported beam in load and resistance factor design according to LRFD-F2. In this inequality ϕ_b is the resistance factor for flexure given as 0.9, M_{nr} is the nominal moment strength and M_{ur} is the factored service load moment for member r .

Eq. 3.8 represents the shear strength requirement in load and resistance factor design according to LRFD-F2. In this inequality ϕ_v represents the resistance factor for shear given as 0.9, V_{nr} is the nominal strength in shear and V_{ur} is the factored service load shear for member r .

3.4 Matrix Stiffness Method

Matrix analysis of structures is an important subject to every structural analyst, if working in civil or mechanical engineering. Matrix analysis provides a comprehensive approach to the analysis of a wide variety of structural types, and therefore proposes a major advantage over traditional methods which often differ for each type of structure. It also provides an efficient means of describing various steps in the analysis and is easily programmed for digital computers. As matrices put up with large groups of numbers to be manipulated in a simple manner, use of matrices is natural when performing calculations with a computer.

Matrix stiffness method is a numerical technique that uses matrix algebra to analyze structural systems. It idealizes the system as an assembly of discrete elements connected to one another at points called nodes.

3.4.1 Global Coordinate System

The specification of the structure geometry is done using the Conventional Cartesian Coordinate System. This coordinate system (Figure 3.1) is a rectangular coordinate system (X, Y, Z) which follows the orthogonal right hand rule. This coordinate system may be used to define the joint locations and loading directions. The translational degrees of freedom are denoted by d_1 , d_2 , d_3 and the rotational degrees of freedom are denoted by d_4 , d_5 & d_6 .

The joint displacement vector for joint i in global coordinates is;

$$\{D\}_i = \{\theta_{xi} \quad \theta_{yi} \quad \theta_{zi}\}$$

The external load vector for joint i in global coordinates is;

$$\{P\}_i = \{P_{xi} \quad P_{yi} \quad P_{zi}\}$$

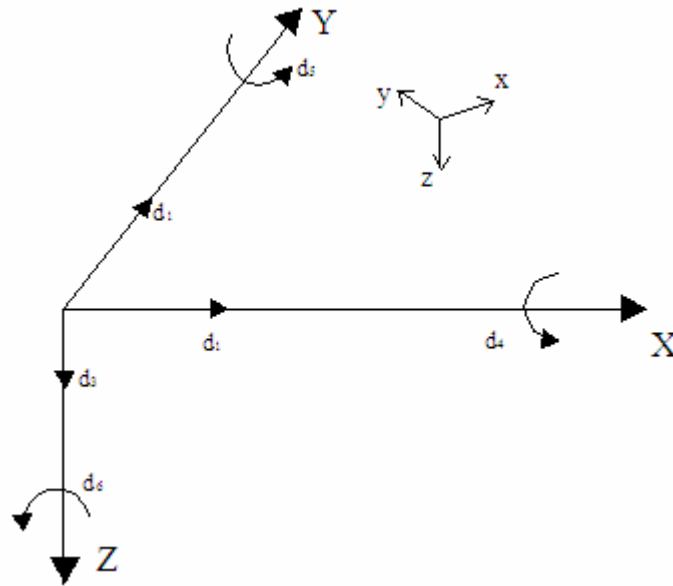


Figure 3.1 Cartesian (Rectangular) Coordinate System

3.4.2 Local Coordinate System

A local coordinate system is associated with each member. Each axis of the local orthogonal coordinate system is also based on the right hand rule. Figure 3.2 shows a beam member with start joint 'i' and end joint 'j'. The positive direction of the local x-axis is determined by joining 'i' to 'j' and projecting it in the same direction. The right hand rule may be applied to obtain the positive directions of the local y and z axes. The local y and z-axes coincide with the axes of the two principal moments of inertia. Note that the local coordinate system is always rectangular. A wide range of cross-sectional shapes may be specified for analysis. Figure 3.3 shows the local axis system(s) for these shapes.

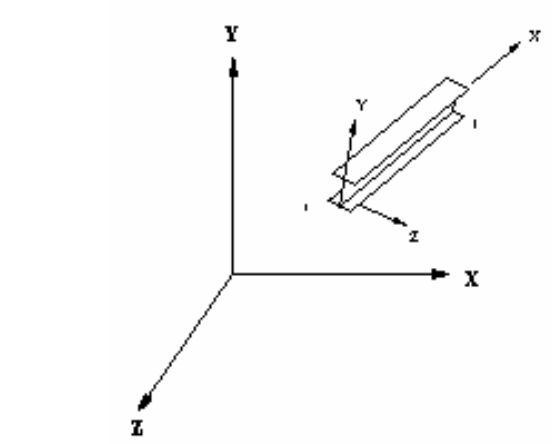


Figure 3.2. A beam member

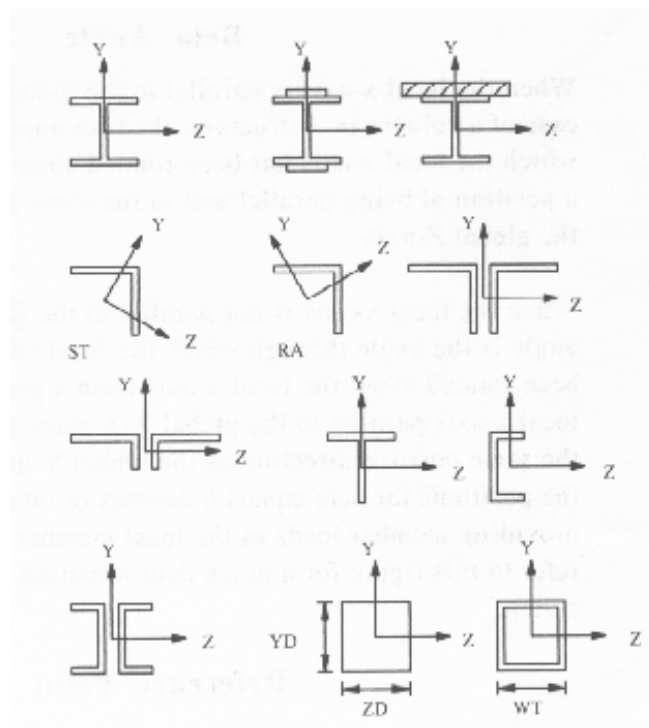


Figure 3.3 Local axis for different cross-sections

3.4.3 Relationship Between Global & Local Coordinates

Since the input for member loads can be provided in the local and global coordinate system and the output for member end forces is printed in the local coordinate system, it is important to know the relationship between the local and global coordinate systems. This relationship is defined by an angle measured in the following specified way. This angle will be defined as the beta (β) angle.

When the local x-axis is parallel to the global Y-axis, as in the case of a column in a structure, the beta angle is the angle through which the local z-axis has been rotated about the local x-axis from a position of being parallel and in the same positive direction of the global Z-axis.

When the local x-axis is not parallel to the global Y-axis, the beta angle is the angle through which the local coordinate system has been rotated about the local x-axis from a position of having the local z-axis parallel to the global X-Z plane and the local y-axis in the same positive direction as the global Y-axis. Figure 3.4 details the positions for beta equals 0 degrees or 90 degrees. When providing member loads in the local member axis, it is helpful to refer to this figure for a quick determination of the local axis system.

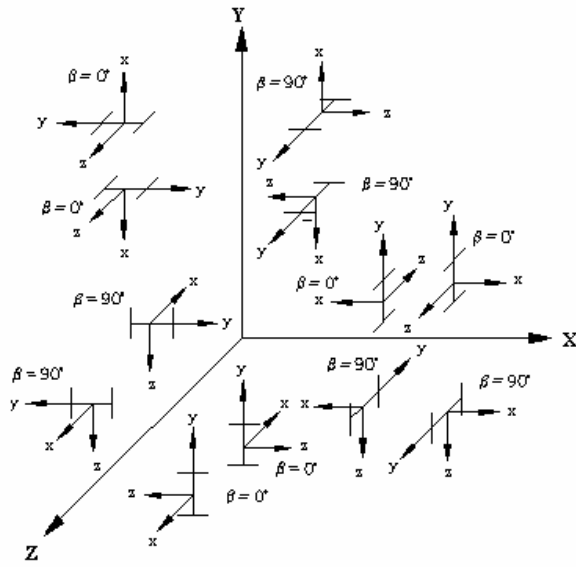


Figure 3.4 Relationship between Global and Local axes

3.4.4 Relationship between member end forces and member end deformations

To form the structure stiffness matrix of individual elements must first be constructed. Consider element r of the grillage system shown below.

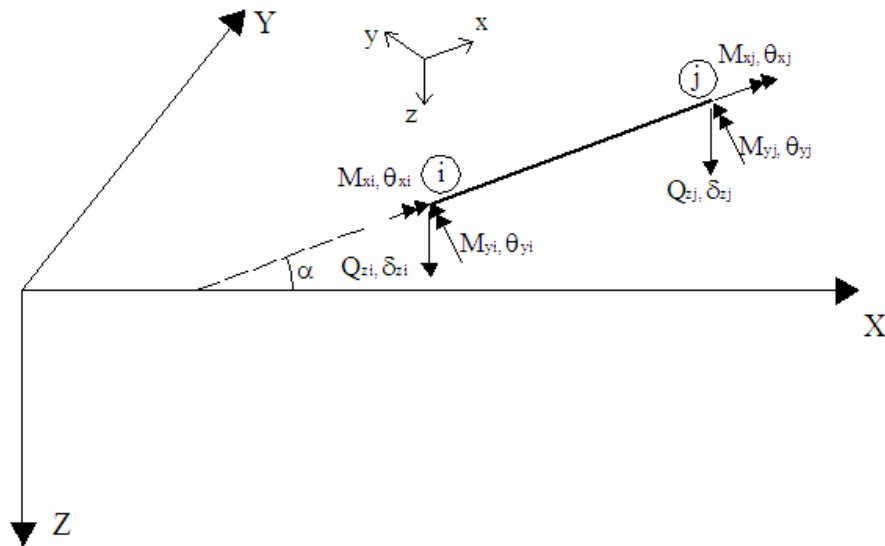


Figure 3.5 End forces of a grillage member

We derive the local stiffness matrix for a typical beam element with three degree of freedoms at each end, considering the combination of flexural and torsional effects. Subsequently, the global stiffness matrix for the grillage system is obtained from an assemblage of local stiffness matrices.

The relationship between end forces and deformations has the following form

$$\begin{Bmatrix} M_{xi} \\ M_{yi} \\ Q_{zi} \\ M_{xj} \\ M_{yj} \\ Q_{zj} \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & k_{56} \\ k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66} \end{bmatrix} \begin{Bmatrix} u_{xi} \\ u_{yi} \\ u_{zi} \\ u_{xj} \\ u_{yj} \\ u_{zj} \end{Bmatrix} \quad \text{or} \quad \{F\}_r = [k] \{u\}_r \quad (3.9)$$

where $\{F\}_r$ is vector of end forces, $\{u\}_r$ is vector of end deformations and $[k]$ is member stiffness matrix in local coordinates. This matrix can be formed by using physical definition of the concept of stiffness by applying a unit deformation in the direction of the one of the end deformation while keeping the rest equal to zero and computing the forces develop at the ends of the member.

Vector of member end forces in local coordinates ;

$$\{F\}_r = \{M_{xi} \quad M_{yi} \quad Q_{zi} \quad M_{xj} \quad M_{yj} \quad Q_{zj}\}^T \quad (3.10)$$

Vector of member end deformations in local coordinates ;

$$\{u\} = \{u_{xi} \quad u_{yi} \quad u_{zi} \quad u_{xj} \quad u_{yj} \quad u_{zj}\}^T \quad (3.11)$$

Vector of joint displacements for member r in global coordinates;

$$\{D\}_r = \{\theta_{xi} \quad \theta_{yi} \quad \delta_{zi} \quad \theta_{xj} \quad \theta_{yj} \quad \delta_{zj}\}^T \quad (3.12)$$

3.4.5 Stiffness Matrix of a both end rigidly connected grillage member

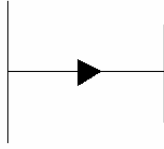


Figure 3.6 Rigidly connected member

M_{xi} and M_{xj} are the torsional moments acting at the beginning and end of the member. The relationship between the twisting of the ends and torsional moment is shown below.

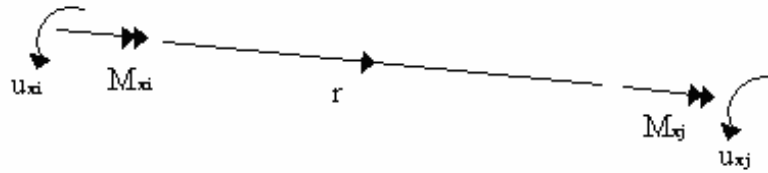


Figure 3.7 Relationship between twisting of the ends and torsional moment

The axial stiffness of the member is given as;

$$M_{xi} = \frac{GJ}{L}(u_{xi} - u_{xj}) \quad M_{xj} = -\frac{GJ}{L}(u_{xi} - u_{xj}) \quad (3.13)$$

where G is shear modulus, J is torsional constant. In Figure 3.7 and Figure 3.8 a member of a rigidly jointed plane frame is shown subject to member forces.

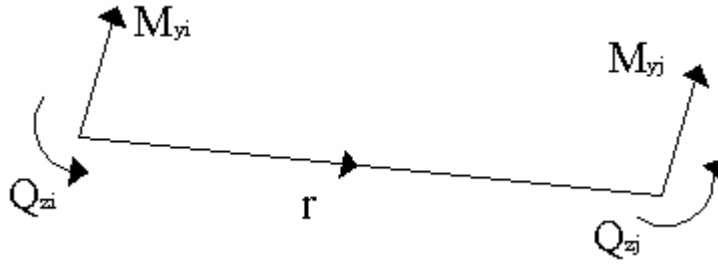


Figure 3.8 Shear forces and bending moments

The general slope-deflection equations between the end moments, shear and end deformations where obtained as;

$$M_{yi} = \frac{4EI}{L}u_{yi} + \frac{6EI}{L^2}u_{zi} + \frac{2EI}{L}u_{yj} - \frac{6EI}{L^2}u_{zj} \quad (3.14)$$

$$M_{yj} = \frac{2EI}{L}u_{yi} + \frac{6EI}{L^2}u_{zi} + \frac{4EI}{L}u_{yj} - \frac{6EI}{L^2}u_{zj} \quad (3.15)$$

$$Q_{zi} = \frac{6EI}{L^2}u_{yi} + \frac{12EI}{L^3}u_{zi} + \frac{6EI}{L^2}u_{yj} - \frac{12EI}{L^3}u_{zj} \quad (3.16)$$

$$Q_{zj} = -\frac{6EI}{L^2}u_{yi} - \frac{12EI}{L^3}u_{zi} - \frac{6EI}{L^2}u_{yj} + \frac{12EI}{L^3}u_{zj} \quad (3.17)$$

Collecting these equations in a matrix form;

$$[k]_r = \begin{bmatrix} \frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\ 0 & \frac{4EI}{L} & \frac{6EI}{L^2} & 0 & \frac{2EI}{L} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} & -\frac{12EI}{L^3} \\ -\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & \frac{2EI}{L} & \frac{6EI}{L^2} & 0 & \frac{4EI}{L} & -\frac{6EI}{L^2} \\ 0 & -\frac{6EI}{L^2} & -\frac{12EI}{L^3} & 0 & -\frac{6EI}{L^2} & \frac{12EI}{L^3} \end{bmatrix} \quad (3.18)$$

3.4.6 Relationship between the joint displacements and member end deformations

If joint forces and displacements are also defined in a manner in the global coordinates system, then the displacement transformation matrix can be obtained.

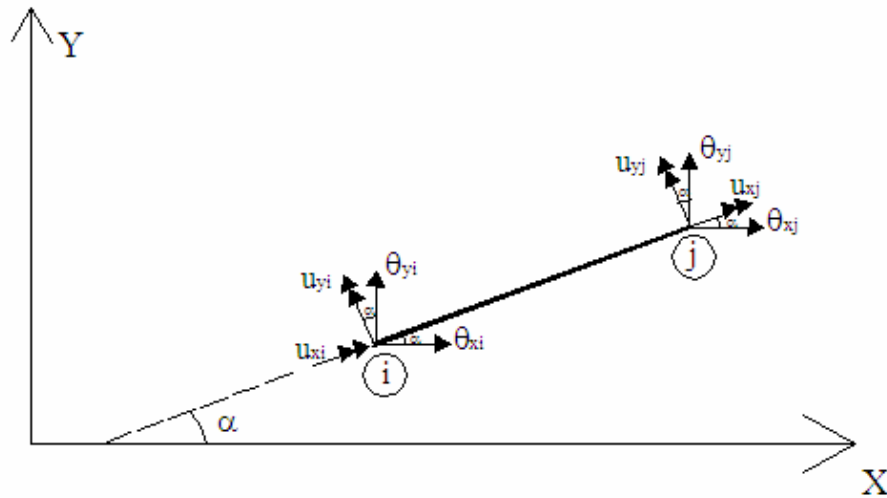


Figure 3.9 Displacements of a grillage member

At joint i;

$$\begin{aligned} u_{xi} &= \theta_{xi} \cos \alpha + \theta_{yi} \sin \alpha \\ u_{yi} &= -\theta_{xi} \sin \alpha + \theta_{yi} \cos \alpha \\ u_{zi} &= \delta_{zi} \end{aligned}$$

At joint j;

$$\begin{aligned} u_{xj} &= \theta_{xj} \cos \alpha + \theta_{yj} \sin \alpha \\ u_{yj} &= -\theta_{xj} \sin \alpha + \theta_{yj} \cos \alpha \\ u_{zj} &= \delta_{zj} \end{aligned} \quad (3.19)$$

Collecting these into matrix form;

$$\begin{bmatrix} u_{xi} \\ u_{yi} \\ u_{zi} \\ u_{xj} \\ u_{yj} \\ u_{zj} \end{bmatrix} = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 & 0 & 0 & 0 \\ -\sin \alpha & \cos \alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos \alpha & \sin \alpha & 0 \\ 0 & 0 & 0 & -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_{xi} \\ \theta_{yi} \\ \delta_{zi} \\ \theta_{xj} \\ \theta_{yj} \\ \delta_{zj} \end{bmatrix} \quad (3.20)$$

$$\{u\}_r = [B]_r \{D\}_r \quad (3.21)$$

3.4.7 Relationship between external loads and member forces

When an elastic structure is subjected to external loads, it deforms and joint displacements and member end displacements occur. In this case, the work done by the external loads is equal to the work done by the internal forces due to the principle of conservation of the energy. Hence;

$$\frac{1}{2} \{P\}_r^T \{D\}_r = \frac{1}{2} \{F\}_r^T \{u\}_r \quad (3.22)$$

Where $\{F\}_r$ is the vector of member forces, $\{u\}_r$ is the vector of member end deformations, $\{P\}_r$ is vector of external loads and $\{D\}_r$ is the joint displacement vector in the structure.

Remembering that $\{u\}_r = [B]_r \{D\}_r$

$$\frac{1}{2} \{P\}^T \{D\}_r = \frac{1}{2} \{F\}^T [B]_r \{D\}_r \quad (3.23)$$

$$\{P\}^T = \{F\}^T [B]_r \quad (3.24)$$

Taking transpose of both sides

$$\{P\} = [B]_r^T \{F\} \quad (3.25)$$

Overall stiffness matrix is obtained by collecting the equations (3.9), (3.21), and (3.25), together.

$$\{F\}_r = [k] \{u\}_r \quad (3.9)$$

$$\{u\}_r = [B]_r \{D\}_r \quad (3.21)$$

$$\{P\} = [B]_r^T \{F\} \quad (3.25)$$

Substituting (3.21) into (3.9)

$$\{F\}_r = [k] [B]_r \{D\}_r \quad (3.26)$$

Substituting (3.26) into (3.25)

$$\{P\} = \underbrace{[B]_r^T [k] [B]_r}_{[K]} \{D\}_r \quad (3.27)$$

Where $[K] = [B]_r^T [k] [B]_r$ is called overall stiffness of the structure.

$$[K] = \begin{matrix} & \begin{matrix} \text{First end} & & & \text{Second end} \end{matrix} \\ \begin{matrix} \text{First end} \\ \text{Second end} \end{matrix} & \begin{bmatrix} a & b & c & . & d & e & -c \\ b & f & g & . & e & h & -g \\ c & g & p & . & c & g & -p \\ . & . & . & . & . & . & . \\ d & e & c & . & a & b & -c \\ e & h & g & . & b & f & -g \\ -c & -g & -p & . & -c & -g & p \end{bmatrix} \end{matrix} \quad (3.28)$$

$$a = \frac{GJ}{L} \cos^2 \alpha + \frac{4EI}{L} \sin^2 \alpha$$

$$b = \left(\frac{GJ}{L} - \frac{4EI}{L} \right) \cos \alpha \sin \alpha$$

$$c = -\frac{6EI}{L^2} \sin \alpha$$

$$d = -\frac{GJ}{L} \cos^2 \alpha + \frac{2EI}{L} \sin^2 \alpha$$

$$e = \left(-\frac{GJ}{L} - \frac{2EI}{L} \right) \cos \alpha \sin \alpha$$

$$f = \frac{GJ}{L} \sin^2 \alpha + \frac{4EI}{L} \cos^2 \alpha$$

$$g = \frac{6EI}{L^2} \cos \alpha$$

$$h = -\frac{GJ}{L} \sin^2 \alpha + \frac{2EI}{L} \cos^2 \alpha$$

$$p = \frac{12EI}{L^3}$$

3.4.8 Stiffness Matrix of a grillage member with a hinge at the first end in local coordinate system

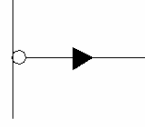


Figure 3.10 Hinge at the first end

$$M_{xi} = \frac{GJ}{L} (u_{xi} - u_{xj}) \quad M_{xj} = -\frac{GJ}{L} (u_{xi} - u_{xj}) \quad (3.29)$$

Since in such members, there will be no transmission of moments from the first end to the joint, it is apparent that $M_{yi} = 0$. The end deformation u_{yi} represents the end rotation at hinge. Since this has no relation with the joint rotation.

$$M_{yi} = 0 \rightarrow u_{yi} = -\frac{3}{2L} u_{zi} - \frac{1}{2} u_{yj} + \frac{3}{2L} u_{zj} \quad (3.30)$$

$$M_{yj} = \frac{3EI}{L^3} u_{zi} + \frac{3EI}{L} u_{yj} - \frac{3EI}{L^2} u_{zj} \quad (3.31)$$

$$Q_{zi} = \frac{3EI}{L^3} u_{zi} + \frac{3EI}{L^2} u_{yj} - \frac{3EI}{L^3} u_{zj} \quad (3.32)$$

$$Q_{zj} = \frac{3EI}{L^2} u_{zi} + \frac{3EI}{L} u_{yj} - \frac{3EI}{L^2} u_{zj} \quad (3.33)$$

Collecting these equations in a matrix form together with axial stiffness.

$$[k]_r = \begin{bmatrix} \frac{GJ}{L} & 0 & -\frac{GJ}{L} & 0 & 0 \\ 0 & \frac{3EI}{L^3} & 0 & \frac{3EI}{L^2} & -\frac{3EI}{L^3} \\ -\frac{GJ}{L} & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & \frac{3EI}{L^2} & 0 & \frac{3EI}{L} & -\frac{3EI}{L^2} \\ 0 & -\frac{3EI}{L^3} & 0 & -\frac{3EI}{L^2} & \frac{3EI}{L^3} \end{bmatrix} \quad (3.34)$$

The compability matrix for the member becomes

$$\begin{bmatrix} u_{xi} \\ u_{zi} \\ u_{xj} \\ u_{yj} \\ u_{zj} \end{bmatrix} = \begin{bmatrix} \cos \alpha & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \cos \alpha & \sin \alpha & 0 \\ 0 & 0 & -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_{xi} \\ \delta_{zi} \\ \theta_{xj} \\ \theta_{yj} \\ \delta_{zj} \end{bmatrix} \quad (3.35)$$

$$\{u\}_r = [B]_r \{D\}_r$$

After obtaining member stiffness matrix and member compability matrix, stiffness method can be applied either by following direct or indirect way. The contribution of a member with a hinge at its first end will be obtained by carrying out triple matrix multiplication.

$$[K] = \begin{bmatrix} a & 0 & -a & -b & 0 \\ 0 & c & -d & e & -c \\ -a & -d & f & g & d \\ -b & e & g & h & -e \\ 0 & -c & d & -e & c \end{bmatrix} \quad (3.36)$$

$$a = \frac{GJ}{L} \cos^2 \alpha$$

$$b = \frac{GJ}{L} \cos \alpha \sin \alpha$$

$$c = \frac{3EI}{L^3}$$

$$d = \frac{3EI}{L^2} \sin \alpha$$

$$e = \frac{3EI}{L^2} \cos \alpha$$

$$f = \frac{GJ}{L} \cos^2 \alpha + \frac{3EI}{L} \sin^2 \alpha$$

$$g = \left(\frac{GJ}{L} - \frac{3EI}{L} \right) \cos \alpha \sin \alpha$$

$$h = \frac{GJ}{L} \sin^2 \alpha + \frac{3EI}{L} \cos^2 \alpha$$

3.4.9 Stiffness Matrix of a grillage member with a hinge at second end

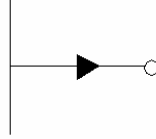


Figure 3.11 Hinge at the second end

$$M_{xi} = \frac{GJ}{L} (u_{xi} - u_{xj}) \quad M_{xj} = -\frac{GJ}{L} (u_{xi} - u_{xj}) \quad (3.37)$$

In this case M_{yj} will be equal to zero. The end deformation u_{yj} represents the hinge rotation at the second end. Since it is not relevant to the formulation, it is eliminated from the below equations.

$$M_{yj} = 0 \rightarrow u_{yj} = -\frac{1}{2}u_{yi} - \frac{3}{2L}u_{zi} + \frac{3}{2L}u_{zj} \quad (3.38)$$

$$M_{yi} = \frac{3EI}{L}u_{yi} + \frac{3EI}{L^2}u_{zi} - \frac{3EI}{L^2}u_{zj} \quad (3.39)$$

$$Q_{zi} = \frac{3EI}{L^2}u_{yi} + \frac{3EI}{L^3}u_{zi} - \frac{3EI}{L^3}u_{zj} \quad (3.40)$$

$$Q_{zj} = -\frac{3EI}{L^2}u_{yi} - \frac{3EI}{L^3}u_{zi} + \frac{3EI}{L^3}u_{zj} \quad (3.41)$$

Collecting these equations in a matrix form together with axial stiffness;

$$[k]_r = \begin{bmatrix} \frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 \\ 0 & \frac{3EI}{L} & \frac{3EI}{L^2} & 0 & -\frac{3EI}{L^2} \\ 0 & \frac{3EI}{L^2} & \frac{3EI}{L^3} & 0 & -\frac{3EI}{L^3} \\ -\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 \\ 0 & -\frac{3EI}{L^2} & -\frac{3EI}{L^3} & 0 & \frac{3EI}{L^3} \end{bmatrix} \quad (3.42)$$

The compability matrix;

$$\begin{bmatrix} u_{xi} \\ u_{yi} \\ u_{zi} \\ u_{xj} \\ u_{zj} \end{bmatrix} = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 & 0 & 0 \\ -\sin \alpha & \cos \alpha & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \cos \alpha & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_{xi} \\ \delta_{yi} \\ \theta_{zi} \\ \theta_{xj} \\ \delta_{zj} \end{bmatrix} \quad (3.43)$$

Similar to the previous case, after obtaining member stiffness matrix and member compability matrix, stiffness method can be applied either by following direct or indirect way. The contribution of the member with a hinge at its second end to the overall stiffness matrix is obtained by carrying out the triple matrix multiplication.

$$\begin{aligned}
 a &= \frac{GJ}{L} \cos^2 \alpha & b &= \frac{GJ}{L} \cos \alpha \sin \alpha & c &= \frac{3EI}{L^3} \\
 d &= \frac{3EI}{L^2} \sin \alpha & e &= \frac{3EI}{L^2} \cos \alpha & f &= \frac{GJ}{L} \cos^2 \alpha + \frac{3EI}{L} \sin^2 \alpha \\
 g &= \left(\frac{GJ}{L} - \frac{3EI}{L} \right) \cos \alpha \sin \alpha & h &= \frac{GJ}{L} \sin^2 \alpha + \frac{3EI}{L} \cos^2 \alpha
 \end{aligned}$$

3.5 Beams: Laterally Supported

A beam is generally considered to be any member subjected principally to transverse gravity loading. A simple beam [Figure 3.12] is supported vertically at each end with little or no rotational restraint, and downward loads cause positive bending moment throughout the span.

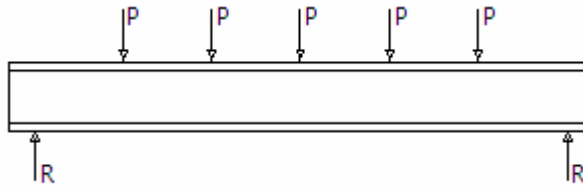


Figure 3.12 A simple beam

A beam is combination of a tension element and a compression element. Probably the large majority of steel beams are used in such a manner that their compression flanges are restrained against lateral buckling. Should the compression flange of a beam be without lateral support for some distance it will have a stress situation similar to that existing in columns. The longer and slenderer column becomes the greater the danger of its buckling for the same loading condition. When the compression flange of a beam is long and slender enough it may quite possibly buckle unless lateral support is provided [40].

The most common rolled steel beam cross section, shown in Figure 3.13, is called the W (wide-flange) shape, with much of the material in the top and bottom flange, where it is most effective in resisting bending moment. The concepts of tension members and compression members are combined in the treatment as a beam. The compression element (a flange) that is integrally braced perpendicular to its plane through its attachment to the stable tension flange by means of the web, is assumed also to be braced laterally in the direction to the plane of the web.

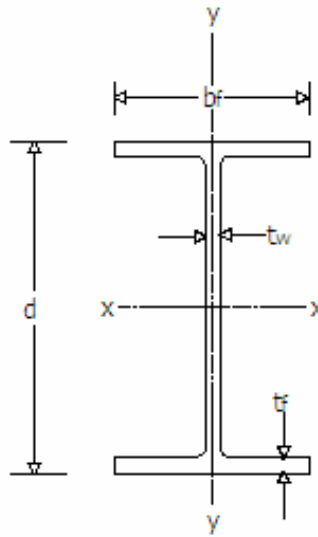


Figure 3.13 W (wide-flange) shape steel beam

b_f = the width of flange

t_f = the thickness of flange

t_w = the thickness of web

d = overall depth of steel section

3.5.1 Load and Resistance Factor Design for Laterally Supported Rolled Beams

The strength requirement for beams in load and resistance factor design according to LRFD-F2 may be stated;

$$\phi_b M_n \geq M_u \quad (3.44)$$

Where; ϕ_b = resistance (i.e., strength reduction) factor for flexure = 0.90

M_n = nominal moment strength

M_u = factored service load moment

For the computation of the nominal moment strength (M_n) of a laterally supported beam, it is necessary first to determine whether the beam is compact, non-compact or slender.

3.5.1.1 Compact Sections

In compact sections, local buckling of the compression flange and the web does not occur before the plastic hinge develops in the cross section. On the other hand in practically compact sections, the local buckling of compression flange or web may occur after the first yield is reacted at the outer fibre of the flanges.

If $\lambda \leq \lambda_p$ for both the compression flange and the web, the capacity is equal to M_p and shape is compact and nominal moment strength M_n for laterally stable compact sections according to LRFD-F1 may be stated;

$$M_n = M_p \quad (3.45)$$

where;

$$M_p = \text{plastic moment strength} = Z F_y \quad (3.46)$$

Z = plastic modulus

F_y = yield stress

3.5.1.2 Noncompact Sections

The nominal strength M_n for laterally stable noncompact sections whose width/thickness ratios λ exactly equal the limits λ_r of LRFD-B5.1 is the moment strength available when the extreme fiber is at yield stress F_y . Because of the residual stress the strength is expressed as

$$M_n = M_r = S(F_y - F_r) \quad (3.47)$$

Where; M_r is the residual moment that will cause the extreme fiber stress to rise from its residual stress F_r value when there is no applied load acting to the yield stress F_y . The elastic section modulus S equals the moment of inertia I divided by the distance from the neutral axis to the extreme fiber.

3.5.1.3 Partially Compact Sections

The nominal strength M_n for laterally stable noncompact sections whose width or thickness ratios λ are less than λ_r but not as low as λ_p must be linearly interpolated between M_r and M_p , as follows according to LRFD-F1.7

$$M_n = M_p - (M_p - M_r) \frac{\lambda - \lambda_r}{\lambda_r - \lambda_p} \quad (3.48)$$

3.5.1.4 Slender Sections

When the width or thickness ratios λ exceed the limits λ_r of LRFD-B5.3, the sections are referred to as slender. In this situation, nominal moment strength is expressed as

$$M_n = M_{cr} = S_x F_{cr} \quad (3.49)$$

where $\lambda = b_f / (2t_f)$ for I-shaped member flanges and the thickness in which b_f and t_f are the width and the thickness of the flange, and $\lambda = h / t_w$ for beam web, in which $h = d - 2k$ plus allowance for undersize inside fillet at compression flange for rolled I-shaped sections. d is the depth of the section and k is the distance from outer face of flange to web toe of fillet. t_w is the web thickness. h / t_w values are readily available in W-section properties table. λ_p and λ_r are given in table LRFD-B5.1 of the code as

$$\left. \begin{aligned} \lambda_p &= 0.38 \sqrt{\frac{E}{F_y}} \\ \lambda_r &= 0.83 \sqrt{\frac{E}{F_y - F_r}} \end{aligned} \right\} \text{for compression flange} \quad (3.50)$$

$$\left. \begin{aligned} \lambda_p &= 3.76 \sqrt{\frac{E}{F_y}} \\ \lambda_r &= 5.70 \sqrt{\frac{E}{F_y}} \end{aligned} \right\} \text{for the web} \quad (3.51)$$

in which E is the modulus of elasticity and F_y is the yield stress of steel. F_r is the compressive residual stress in flange which is given as 69 MPa for rolled shapes in the code. It is apparent that M_n is computed for the flange and for the web separately by using corresponding λ values. The smallest among all is taken as the nominal moment strength of the W section under consideration.

3.5.2 Load and Resistance Factor Design for Shear in Rolled Beams

Beams are usually selected on the basis of their bending capacity and then checked for the shear capacity.

The shear strength requirement in load and resistance factor design according to LRFD-F2 may be stated;

$$\phi_v V_n = V_u \quad (3.52)$$

Where;

ϕ_b = Resistance factor for = 0.90

V_n = Nominal strength in shear

V_u = Required shear strength

Nominal shear strength of a rolled compact and non-compact W section is computed as follows as given in LRFD-AISC-F2.2

a) When $\frac{h}{t_w} \leq 2.45 \sqrt{\frac{E}{F_{yw}}}$, shear yielding of the web is the mode of failure, and the nominal shear strength definition is expressed as;

$$V_n = 0.6F_{yw}A_w \quad (3.53)$$

b) When $2.45 \sqrt{\frac{E}{F_{yw}}} < \frac{h}{t_w} \leq 3.07 \sqrt{\frac{E}{F_{yw}}}$, inelastic shear buckling of the web is the mode of failure, and the nominal shear strength definition is

$$V_n = 0.6F_{yw}A_w \left(\frac{2.45 \sqrt{\frac{E}{F_{yw}}}}{\frac{h}{t_w}} \right) \quad (3.54)$$

c) When $3.07 \sqrt{\frac{E}{F_{yw}}} < \frac{h}{t_w} \leq 260$, elastic shear buckling of the web is the mode of failure, and the nominal shear strength definition is

$$V_n = A_w \frac{4.52Et_w^2}{h^2} \quad (3.55)$$

Where E is the modulus of elasticity and F_{yw} is the yield stress of web steel. V_n is computed from one of the expressions of (3.53)-(3.55) depending upon the value of h/t_w of the W section under consideration.

CHAPTER 4

DESIGN EXAMPLES

4.1 Introduction

Harmony search based optimum design algorithm presented in the previous sections is used to design three grillage systems. The discrete set from which the design algorithm selects the sectional designations for grillage members is considered to be the complete set of 272 W-sections starting from W100x19.3 to W1100x499mm as given in LRFD-AISC [39]. The sequence number of each section in the set is used as the design variable. Hence the terms of the harmony memory matrix represents the sequence number of W-sections in the discrete set.

4.1.1 3-member Simple Grillage System

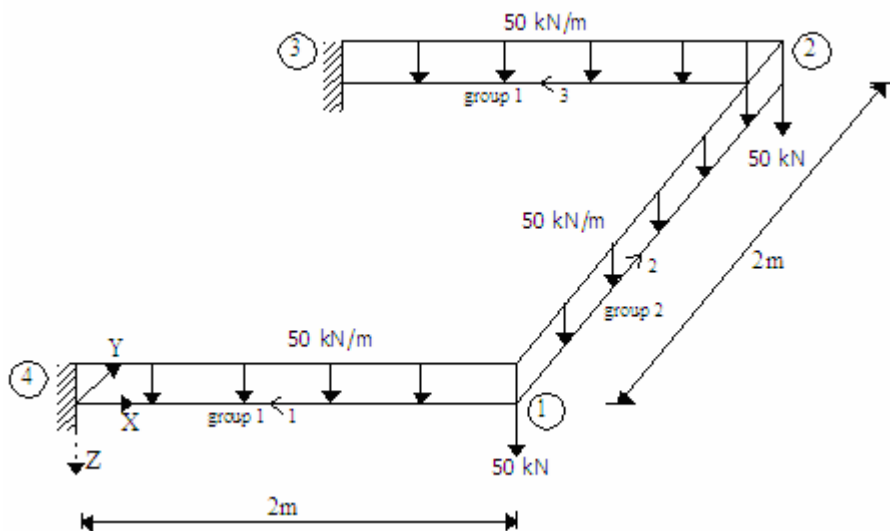


Figure 4.1 Simple Grillage System

The simple grillage system shown in Figure 4.1 is selected as first design example to demonstrate the steps of harmony search based optimum design algorithm developed. The dimensions and the loading of the system are shown in the figure. A36 mild steel is selected for the design which has the yield stress of 250MPa. The modulus of elasticity and the shear modulus are selected as 205kN/mm² and 81kN/mm² respectively. The deflections of joints 1 and 2 are restricted to 10 mm while the other design constraints are implemented from LRFD-AISC as explained in previous chapter. The members 1 and 3 are considered to be made out of the same W section while the member 2 can be made from another. Hence there are two variables in the design problem.

Harmony memory matrix size is taken as 10. Harmony memory considering rate (*HMCR*) is selected as 0.9 while pitch-adjusting rate is considered as 0.45 as suggested in [24]. After 11 searches the initial harmony memory matrix is obtained as given in Table 4.1. The first row of this matrix has the least weight and corresponds to W530x66 and W410x85 within the W-sections list. With these sections, the strength ratio is 0.83 for group 1 and 0.06 for group 2 while the vertical displacements of joints 1 and 2 are 5.1mm which are smaller than their upper bounds. 12th and 13th searches can not find better sections than the ones shown in Table 4.1. However 14th search gives a better harmony search matrix as shown in Table 4.2. Comparing to Table 4.1, it is apparent that harmony search has found a better combination of 52th and 215th W-sections in the list which gives 729.71 kg of weight. This combination yields a lighter grillage system than the last combination of 271st and 263th W-sections in the list.

Table 4.1 Initial harmony search matrix after 11 searches

Row Number	Group 1	Group 2	Weight (kg)
1	141	113	433.98
2	52	109	450.29
3	80	113	610.61
4	141	215	649.36
5	149	26	656.49
6	116	78	777.02
7	203	153	1694.77
8	221	140	2367.71
9	262	234	2667.59
10	271	263	2840.94

Naturally the last combination is discarded from the harmony search matrix and the new combination is included in the 6th row of the harmony search matrix as shown in Table 4.2. It should be noticed that newly found combination does not affect the first

Table 4.2 Harmony search matrix after 14 searches

Row Number	Group 1	Group 2	Weight (kg)
1	141	113	433.98
2	52	109	450.29
3	80	113	610.61
4	141	215	649.36
5	149	26	656.49
6	52	215	729.71
7	116	178	777.02
8	203	153	1694.77
9	221	140	2367.71
10	262	234	2667.59

row of the harmony matrix. When the harmony search algorithm is continued a better combination than the one in the first row of the harmony memory matrix of Table 4.2 is found. For example, after 1000 searches, the combinations shown in Table 4.3 are obtained. The sectional designations correspond to the sequence numbers

Table 4.3 Harmony search matrix after 1000 searches

Row Number	Group 1	Group 2	Weight (kg)
1	141	13	317.15
2	141	26	320.19
3	141	8	323.46
4	141	27	329.37
5	141	70	329.78
6	142	13	349.36
7	142	26	353.02
8	142	8	355.67
9	141	17	356.09
10	142	27	361.59

given in the first row are W530x66 for group 1 and W200x26.6 for group 2 which yield to a grillage system with a weight of 317.15 kg. The analysis of the system with these sections result in 5.1 mm vertical displacements at joints 1 and 2. The strength ratios computed for these sections are 0.83 for the members in group 1 and 0.40 for the member in group 2. These values clearly indicate that harmony search should be continued to determine even a better combination.

The harmony search matrix shown in Table 4.4 which is obtained after 2000 searches verifies this fact. The sections corresponding to the least weight of 305.91 kg in this matrix are W530x66 and W310x21. With these sections, the strength limitation for group 1 searches to their upper bound of 1 and the strength constraint ratio has the value of 0.39 in member 2. The vertical displacements at joints 1 and 2 have the values of 7mm which is less than the upper bound of 10mm. It is clear that strength constraints are dominant in the design problem.

Table 4.4 Harmony search matrix after 2000 searches

Row Number	Group 1	Group 2	Weight (kg)
1	141	41	305.91
2	141	24	308.56
3	141	2	311.42
4	141	13	317.13
5	141	43	320.60
6	141	26	320.80
7	141	8	323.45
8	141	27	329.36
9	141	70	329.77
10	142	41	338.13

The harmony search algorithm is continued to determine even better combinations. The results obtained after 7000 and 8000 searches are given in Tables 4.6 and 4.7.

Table 4.5 Harmony search matrix after 7000 searches

Row Number	Group 1	Group 2	Weight (kg)
1	141	23	299.79
2	141	5	300.01
3	141	11	302.44
4	111	23	303.87
5	111	5	304.08
6	141	41	305.91
7	111	11	306.52
8	141	24	308.56
9	111	41	309.99
10	141	2	311.42

Table 4.6 Harmony search matrix after 8000 searches

Row Number	Group 1	Group 2	Weight (kg)
1	141	10	293.88
2	111	10	297.96
3	141	23	299.79
4	141	5	300.01
5	120	10	302.04
6	141	11	302.44
7	111	23	303.87
8	111	5	304.08
9	141	41	305.91
10	111	11	306.52

The optimum sectional designations obtained by the HS based method for external loading is shown in Table 4.7.

Table 4.7 Optimum Design for 3-member grillage system with two groups

Optimum Sectional Designations		$\delta_{\max}$ (mm)	Maximum Strength Ratio	Minimum Weight (kg)
Group 1	Group 2			
W530x60	W250x17.9	7.0	0.54	293.88

It can be noticed in Table 4.7 that sections W530x60 and W250x17.9 produces a lighter grillage with a weight of 299.79 kg obtained after 7000 searches. The combinations given in the first row of the harmony search matrix which is obtained after 8000 searches even gives a lighter system. The sectional designations for this combination are W530x60 and W200x15. With these sections the vertical displacements of joints 1 and 2 are 7 mm and strength limitation ratios are 1 and 0.54 for members in groups 1 and 2. Further use of harmony search method with more than 8000 search produces the same combination. Consequently the solution found in Table 4.5 represents the

optimum solution which corresponds to the grillage system with sections W530x60 selected for members 1 and 3 and W200x15 adopted member 2.

4.1.2 23-member Grillage System

The grillage system shown in Figure 4.2 has 23 members which are collected in three groups. It is subjected to unsymmetrical loading which is also shown in the figure. The vertical displacements of joints 4, 5, 6 and 8 are restricted to 25 mm while the yield stress is taken as 250MPa which is the value for A36 mild steel.

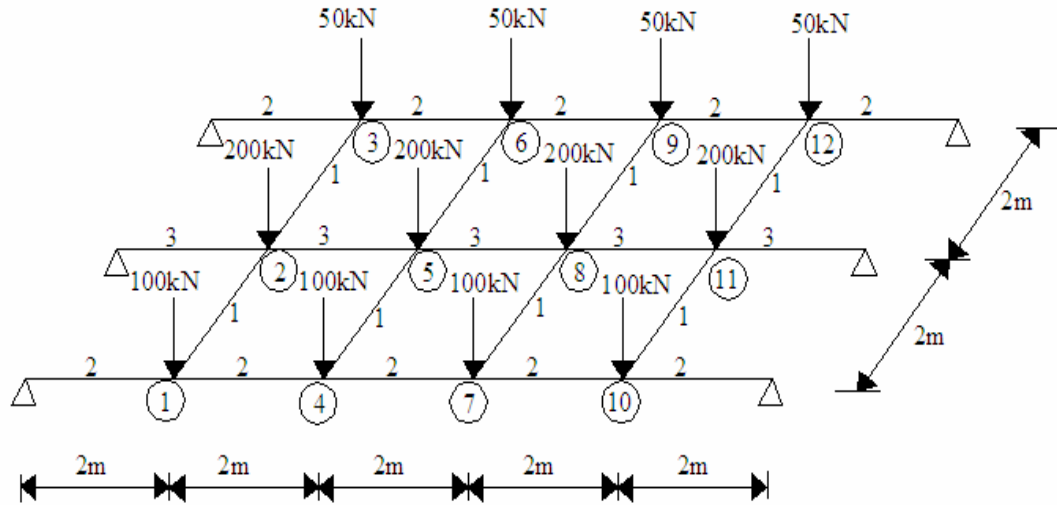


Figure 4.2 23-member grillage system

For this grillage, the ten cases shown in table 4.8, each with a different set of HS algorithm parameters were tested. These parameters were arbitrarily selected on the basis of the empirical findings by Geem and Lee [32]. The maximum number of searches was set to 2000 for this example.

Table 4.8 HS algorithm parameters used for 23-member grillage system

Cases	HMS	HMCR	PAR	Weight(kg)
1	35	0.85	0.40	5184.3
2	25	0.85	0.35	5121.0
3	35	0.9	0.45	5091.5
4	40	0.95	0.30	5081.7
5	50	0.95	0.25	5065.4
6	40	0.70	0.50	4984.4
7	30	0.8	0.45	4974.6
8	45	0.75	0.20	4892.2
9	30	0.80	0.40	4862.6
10	50	0.9	0.45	4718.4

The size of HM matrix (HMS) is taken as (50x5). The total number of solution vectors, i.e. the HMCR and PAR are 0.9 and 0.45, respectively. The optimum result presented in Table 4.8 is obtained after 1500 searches of the harmony search method. However, it was noticed that the optimum sectional designations remained the same after 2000 searches. Variation of the minimum weight from 10 iterations to 2000 iterations is shown in Figure 4.3.

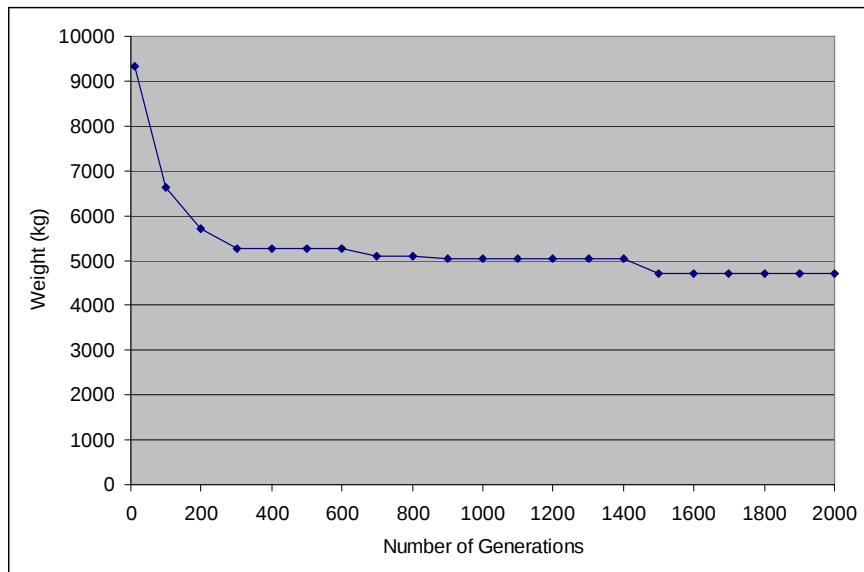


Figure 4.3 Variation of minimum weights during the generations for 23-member

The discrete set considered in the design is extended to the complete list of universal beam sections which has 272 W-sections. After 2000 iterations HS algorithm has found the optimum design as W530x66 for the first group, W840x176 for the second and W150x13.5 for the third group. This combination has resulted in the minimum weight of 4718.4 kg. Variation of the minimum weight from 10 iterations to 2000 iterations is shown in Figure 4.3.

Table 4.9 Optimum Design for 23-member grillage system with three groups

Optimum Sectional Designations			$\delta_{\max}$ (mm)	Maximum Strength Ratio	Minimum Weight (kg)
Group 1	Group 2	Group 3			
W530x66	W840x176	W150x13.5	24.5	0.79	4718.4

The optimum sectional designations obtained by the harmony search based method for the external loading shown in Figure 4.2 is given in Table 4.7. It is apparent from the table that displacement constraints are dominant in the design problem.

4.1.3 40-member Grillage System

The grillage system shown in Figure 4.4 has 40 members which are collected in two groups. The external loading is also shown in the figure. The vertical displacements of joints 6, 7, 10 and 11 are restricted to 25 mm while the yield stress is taken as the yield stress of mild steel which is 250Mpa [40].

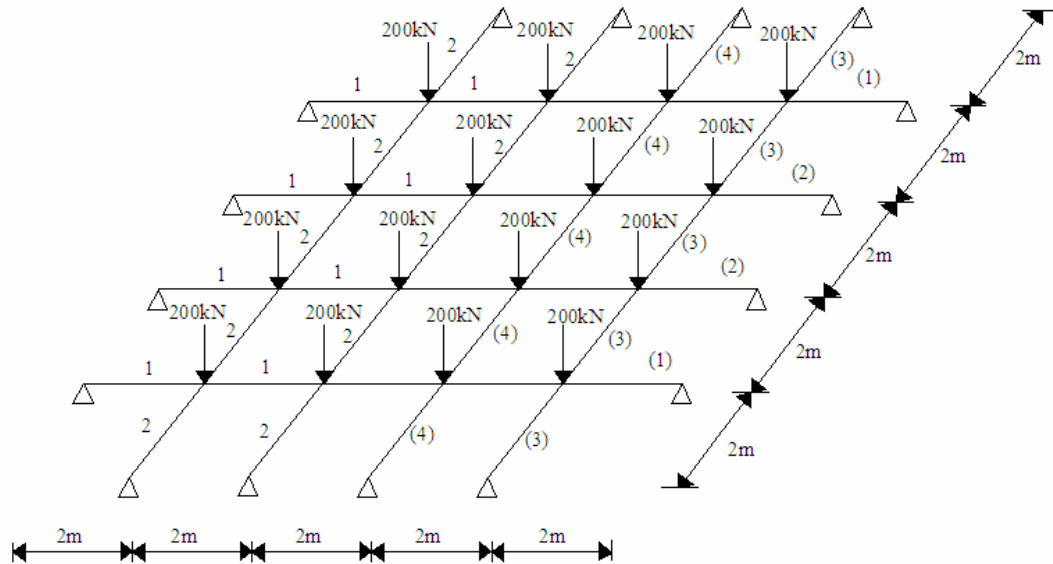


Figure 4.4 40-member grillage system

While the maximum vertical displacement was less than its upper bound, the strength ratio for members in the middle portion of the system was equal to its upper bound of 1. It is interesting to notice that harmony search method came up with the same section for the both groups due to the symmetry of the system and its loading.

40-member grillage system were tested to demonstrate the discrete search efficiency of the HS algorithm. The ten cases shown in table 4.10, each with a different set of HS algorithm parameters were tested. These parameters were arbitrarily selected on the

basis of the empirical findings by Geem and Lee [32]. The maximum number of searches was set to 2000 for this example.

Table 4.10 HS algorithm parameters used for 40-member grillage system

Cases	HMS	HMCR	PAR	Weight(kg)
1	30	0.80	0.40	8759.4
2	45	0.75	0.20	8458.6
3	25	0.85	0.35	8382.1
4	35	0.90	0.45	8317.4
5	35	0.85	0.40	8244.1
6	40	0.95	0.30	8020.0
7	30	0.8	0.45	7806.0
8	50	0.95	0.25	7795.9
9	50	0.90	0.45	7738.3
10	40	0.70	0.50	7729.5

The size of HM matrix (HMS) is taken as (40x5). The total number of solution vectors, i.e. the HMCR and PAR are 0.7 and 0.50, respectively. The optimum result presented in Table 4.10 is obtained after 1650 searches of the harmony search method. However, it was noticed that the optimum sectional designations remained the same after 2000 searches. Variation of the minimum weight from 10 iterations to 2000 iterations is shown in Figure 4.5.

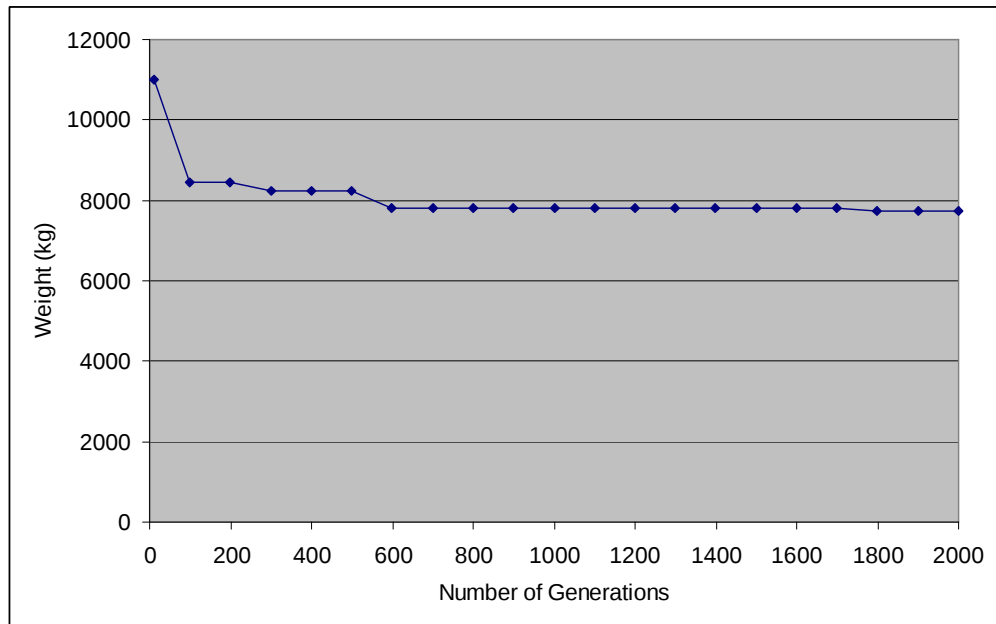


Figure 4.5 Variation of minimum weights during the generations for 40-member

The optimum sectional designations obtained by the harmony search based method for the external loading shown in Figure 4.4 is given in Table 4.9. It is apparent from the table that displacement constraints are dominant in the design problem.

Table 4.11 Optimum Design for 40-member grillage system with two groups

Optimum Sectional Designations		$\delta_{\max}$ (mm)	Maximum Strength Ratio	Minimum Weight (kg)
Group 1	Group 2			
W150x13.5	W840x176	24.2	0.79	7729.5

The same system was designed once more by collecting the members of the system in four different groups. The outer and inner longitudinal beams are considered to be group 1 and 2 respectively while the outer and inner transverse beams are taken as group 3 and 4. The optimum solution obtained for this case is given in Table 4.11. This

system is slightly lighter than the one with two groups. It is apparent from the table that strength constraints are dominant in the design problem. In this case the harmony search method selects different W-sections for groups 3 and 4.

Table 4.12 Optimum Design for 40-member grillage system with four groups

Optimum Sectional Designations				$\delta_{\max}$ (mm)	Maximum Strength Ratio	Minimum Weight (kg)
Group 1	Group 2	Group 3	Group 4			
W410x46.1	W410x53	W200x15	W1000x222	22.3	0.99	7223.7

CHAPTER 5

CONCLUSION

The structural optimization field has been continuing to gain wide acceptance amongst the design community because of its applicability in a large range of practical numerical and structural design problems. Recently, some of researchers have been concentrating on optimization area, realizing the important advantage of an optimized structure over a non-optimized structure in regard of cost- saving gained with a cost-optimal design.

In the present study the harmony search algorithm-based optimization method have been researched for engineering optimization problems. In chapter 2, several standard test examples, including two constrained function minimization problems, and one constrained structural optimization problem (welded beam design) have been solved with HS algorithm method. These examples showed that HS algorithm can easily obtain minimum value of function with fewer iterations at simple numerical problems. If number of design variables and constraints increases, Harmony search method requires more iterations for finding the minimum value of function. HS parameters, HMS, HMCR, and PAR, affect the value of objective function. For the Harmony search algorithm, high HMCR, especially from 0.7 to 0.95, contributes excellent Fortran programming outputs, while PAR and HMS demonstrate little correlation with improvement in performance. Geem's sensitivity analysis of HMCR and PAR is confirmed in this study.

When numerical results compared with other studies, especially Genetic Algorithm based methods, to show accuracy and performance of HS method. The results obtained using the harmony search algorithm may yield better solutions than those obtained using current algorithms, such as generalized reduced gradient method or genetic

algorithm based approaches. The results also showed that harmony search method needs less number of numerical analysis compare to the simple genetic algorithm based methods.

The recently developed harmony search method is applied to the optimum design problem of grillage systems where the design constraints are implemented from LRFD-AISC. The harmony search method is a new stochastic random search based numerical technique which simulates the musical process of searching for a perfect state of harmony. This mathematically simple algorithm sets up harmony search matrix each row of which consists of randomly selected feasible solutions to the design problem. In every search step, it searches the entire set rather than a local neighborhood of a current solution vector. It neither needs initial starting values for the design variables nor a population of candidate solutions to the design problem. The results obtained showed that the harmony search method is powerful and efficient in finding the optimum solution of combinatorial structural optimization problems. Harmony Search algorithm have appeared to be a promising method for optimization problems.

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