



MARMARA UNIVERSITY
INSTITUTE FOR GRADUATE STUDIES
IN PURE AND APPLIED SCIENCES



HEAVY HYBRID MESONS IN QUANTUM CHROMODYNAMICS AS THE QUANTUM FIELD THEORY OF THE STRONG INTERACTION

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Ph.D. THESIS

Department of Physics

Thesis Supervisor

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ISTANBUL, July 2025



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SCIENCES**

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TO MY PARENTS.....

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ÖZET

GÜÇLÜ ETKİLEŞİMİN KUANTUM ALAN TEORİSİ OLARAK KUANTUM KROMODİNAMİĞİNDE AĞIR HİBRİT MEZONLAR

Kuantum Renk Dinamiği (KRD) iki-nokta toplam kuralı yaklaşımı, sırasıyla $J^{PC} = 0^{++}, 0^{+-}, 0^{-+}, 0^{--}$ ve $1^{++}, 1^{+-}, 1^{-+}, 1^{--}$ spin-paritelerine sahip çarmonyum ve botomonyum hibritlerinin kütlelerini ve akım kuplajlarını belirlemek için kullanılmıştır. Hesaplamalar, $\langle g_s^3 G^3 \rangle^2 \sim$ terimlerini de içeren, operatör boyutu 12'ye kadar olan gluon yoğunlukları dikkate alınarak yürütülmüştür. $J^{PC} = 0^+, 0^-, 1^+, 1^-$ kuantum sayılarına sahip alt-tılsım hibritlerinin parametreleri de belirlenmiştir. Hesaplamalarda, toplam kuralı sonuçlarına kutup katkısının hakim olması garanti edilmiştir. Tüm çarmonyum hibritlerinin güçlü bir şekilde iki-mezonlu son durumlara bozunduğu gösterilmiştir. $0^{-+}, 1^{-+}$ botomonyum hibritlerinin yanı sıra $0^{-(+)}, 1^{-(+)}$ alt-tılsım hibrit mezonlarının da iki-mezon güçlü bozunma modlarına karşı koyabilir. Bu çalışmanın sonuçları, toplam kuralı ve alternatif metodolojiler kullanılarak elde edilenlerle karşılaştırılmıştır. Ayrıca, bu sonuçlar geleneksel mezonlarla da karşılaştırılmıştır. Gluonik etki, kütle farkı büyük olan hibrit mezon hesaplamalarında ortaya çıkmıştır. Diğer araştırmalarla yapılan karşılaştırma, özellikle daha büyük hibrit kombinasyonları için kütle tahminlerinde farklılıklar göstermektedir; bu durum, incelemede dikkate alınan kutup katkılarındaki sınırlamalardan kaynaklanıyor olabilir. Sonuçlar, yüksek enerji fiziğindeki mevcut ve gelecekteki deneysel çalışmalar için önemli olan ağır hibrit mezonların bozunma kanalları ve özellikleri hakkında değerli bilgiler sunmayı amaçlamaktadır. Ağır hibrit mezonların parametrelerine ilişkin tahminlerimiz, mevcut ve gelecekteki deneylerin yorumlanması için önemli olan çeşitli bozunma kanallarını incelemek için yararlı olabilir.

Anahtar Kelimeler: İki nokta KRD toplam kuralı, hibrit mezon, çarmonyum, bottomonyum.

ABSTRACT

HEAVY HYBRID MESONS IN QUANTUM CHROMODYNAMICS AS THE QUANTUM FIELD THEORY OF THE STRONG INTERACTION

The QCD two-point sum rule approach is used to determine the masses and current couplings of charmonium and bottomonium hybrids $\bar{c}gc$ and $\bar{b}gb$ with spin-parities $J^{PC} = 0^{++}, 0^{+-}, 0^{-+}, 0^{--}$ and $1^{++}, 1^{+-}, 1^{-+}, 1^{--}$, respectively. Computations are conducted by considering gluon condensates up to dimension 12, including terms $\sim \langle g_s^3 G^3 \rangle^2$. The parameters of bottom-charm hybrids ($\bar{b}gc$) with quantum numbers $J^{PC} = 0^+, 0^-, 1^+, 1^-$ are also determined. In computations, the pole contribution to sum rule outcomes is guaranteed. It is shown that all charmonia hybrids decay aggressively to two-meson final states. Bottomonium hybrids 0^{+-} and 1^{-+} , as well as bottom-charm hybrid mesons $0^{-(+)}$ and $1^{-(+)}$, might withstand powerful two-meson decay modes. The results of the present work are compared to those achieved using the sum rule and alternative methodologies. Another comparison had been done with these results with conventional mesons. The gluonic effect have appeared in hybrid mesons calculations of mass with large difference in value. The comparison with research shows differences in mass estimations notably for larger combinations of hybrids; this could be due to the limitations, on pole contributions considered in this examination. The results aim to provide valuable insights into the decay channels and properties of heavy hybrid mesons, which are relevant for ongoing and future experimental endeavors in high-energy physics. Our estimates for heavy hybrid meson parameters may be valuable in studying their numerous decay channels, which are critical for the interpretation of current and future research.

Keywords: Two point QCD sum rule, hybrid meson, charmonium, bottomonium.

DECLARATION OF NOVELTY

HEAVY HYBRID MESONS IN QUANTUM CHROMODYNAMICS AS THE QUANTUM FIELD THEORY OF THE STRONG INTERACTION

I hereby declare that the research work presented in this dissertation introduces novel contributions to the theoretical study of heavy hybrid mesons within the framework of Quantum Chromodynamics (QCD). While the concept of hybrid mesons has been theoretically discussed in the literature, this dissertation provides a new and comprehensive computational approach by applying the QCD sum rule technique to eight distinct heavy hybrid meson states with various quantum numbers.

To the best of my knowledge, the following aspects of this research are presented for the first time:

- The construction and analysis of advanced two-point correlation functions that include both quark-gluon interactions and higher-dimensional gluonic condensates such as $\langle g_s^3 G^3 \rangle$.
- A systematic optimization of Borel transformation windows and continuum thresholds to improve the precision and stability of extracted hadronic parameters.
- The prediction of masses, decay constants, and pole contributions for unexplored hybrid configurations, contributing original results not previously published in academic literature.

All theoretical derivations, numerical simulations, and interpretations of results were carried out independently. The findings of this dissertation are believed to represent a new contribution to the non-perturbative QCD analysis of exotic hadronic matter.

This declaration is made in good faith and with full awareness of the academic standards of novelty and scholarly contribution.

July 2025

Ashwaq Eyad Kadhim Al-aakol

SYMBOLS

q	: Four-momentum of the meson
q^2	: Momentum transfer squared
s_0	: Continuum threshold parameter
M^2	: Borel mass parameter (Borel window)
$\Pi(q^2)$	: Two-point correlation function
$\rho(s)$	: Spectral density
f_H	: Decay constant of the hybrid meson
m_H	: Mass of the hybrid meson
Γ	: Decay width
J^{PC}	: Spin-parity-charge conjugation quantum numbers
$G_{\mu\nu}^a$	: Gluon field strength tensor
$\tilde{G}_{\mu\nu}^a$	: Dual gluon field strength tensor
$\langle G^2 \rangle$	: Gluon condensate $\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{a\mu\nu} \rangle$
$\langle g_s^3 G^3 \rangle$	: Triple gluon condensate
α_s	: Strong coupling constant
$\delta(s - m_H^2)$	: Dirac delta function to isolate the ground state
$\mathcal{L}$	: Lagrangian density

ABBREVIATIONS

QFT	: Quantum Field Theory
QCD	: Quantum Chromodynamics
HQET	: Heavy Quark Effective Theory
SM	: Standard Model
QCDSR	: Quantum Chromodynamics Sum Rules
LQCD	: Lattice Quantum Chromodynamics
NRQCD	: Non-Relativistic Quantum Chromodynamics
OPE	: Operator Product Expansion
PC	: Pole Contribution
BOFET	: Born-Oppenheimer effective field theory

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1. CHAPTER ONE - INTRODUCTION

The study of hadronic matter extends beyond conventional mesons and baryons, as Quantum Chromodynamics (QCD) and the parton model predict the possible existence of more complex structures such as multiquark states, glueballs, and hybrids. Recent experimental advances have significantly enriched our understanding of exotic hadronic matter, particularly in relation to tetraquark and pentaquark configurations. For instance, the X states, some of which are believed to contain charm-anticharm quark pairs with possible tetraquark configurations [1–3], and the observed resonances $P_c(4330)$, $P_c(4440)$, and $P_{cs}(4459)$ [4–6], are prominent candidates for exotic hadrons such as tetraquarks and pentaquarks, respectively. These discoveries have broadened the scope of hadron spectroscopy beyond the traditional quark model.

Hybrid mesons—such as charmonium and bottomonium hybrids—are particularly interesting because they incorporate not only valence quark-antiquark pairs but also valence gluons. Bottom-charm hybrids fall within this category as well. In the light meson sector, several states have been proposed as potential hybrid candidates, including $\pi_1(1400)$, $\pi_1(1600)$, and $\pi_1(2105)$. A more recent example is the $\eta_1(1855)$, a resonance with quantum numbers $J^{PC} = 1^{-+}$, reported by the BESIII Collaboration [7]. This particle was identified in the decay process $J/\psi \rightarrow \gamma \eta_1(1855) \rightarrow \gamma \eta \eta'$, and is being considered a hybrid meson candidate [8–10]. However, alternative models such as hadronic molecules or diquark-antidiquark structures have also been proposed to explain the observed data [10–11].

Potential hybrid candidates have also been identified among heavier resonances. For example, the vector states $\psi(4230)$ and $\psi(4360)$ have been interpreted as possible hybrid charmonia or as states with significant hybrid components [11, 12]. A more comprehensive review of hybrid quarkonia candidates is available in Ref. [13]. Furthermore, exotic baryonic hybrids are under active investigation. The $\Lambda(1405)$, a long-known resonance [14], has been studied extensively as a potential hybrid baryon by several experimental groups [15–17].

Recent studies have prioritized light-flavor hybrids like the $1^{-+} \pi_1(1400)$ and $\pi_1(1600)$ states, observed in $\eta\pi$ and $\rho\pi$ decays at COMPASS and GlueX, though their potential mixing with tetraquarks remains unresolved [18]. In the heavy-quark sector, the $Y(4260)$ charmonium state, produced in e^+e^- collisions, exhibits hybrid-like decay patterns ($J/\psi\pi\pi$) consistent with theoretical expectations [19]. Theoretical

investigations employ a multi-framework approach: lattice QCD (LQCD) provides first-principles mass spectra and reveals significant glueball-hybrid mixing in the scalar sector [20], while systematic studies of exotic hadrons via LQCD and effective field theories (EFTs) further constrain hybrid properties [24]; complementary approaches like HQET (for charmonium hybrids) and flux-tube models (for light states) bridge predictions with observables [21]. Phenomenological models (QCD-inspired Hamiltonians, holographic QCD) additionally constrain production and decay properties [22]. Experimental progress at GlueX, LHCb, and future EIC experiments will test these predictions, particularly for the long-sought 1^{+-} exotics and charmonium hybrid candidates [18,19,23].

The theoretical investigation of hybrid hadrons has a rich history, tracing back to foundational works over four decades ago [25, 26]. Initial studies produced insightful results on spectroscopic properties, decay mechanisms, and production processes of these exotic particles through diverse methodologies [27–38]. Subsequent research [39–54] aimed to enhance the theoretical approaches and computational tools applied in these analyses. Numerous theoretical models have been utilized to explore hybrid hadrons, including QCD sum rules, lattice QCD, the constituent gluon model, the flux-tube model, and nonrelativistic field theory. These frameworks have collectively contributed to the theoretical basis of hybrid spectroscopy.

Despite progress, inconsistencies in the predicted properties of hybrid mesons make this an ongoing area of inquiry. For instance, Ref. [40] employed QCD sum rules with corrections up to the three-gluon condensate $\langle g_s^3 G^3 \rangle$ to estimate the masses of vector hybrids with $J^{PC} = 1^{--}$, denoted as $H_c = \bar{c}gc$ and $H_b = \bar{b}gb$. Their calculations predicted masses in the ranges of $m_{H_c} = 4.12 - 4.79$ GeV and $m_{H_b} = 10.24 - 11.15$ GeV, respectively. Conversely, Ref. [43] reported lower values (3.36 ± 0.15) GeV and (9.70 ± 0.12) GeV. Lattice QCD simulations provided estimates of (4.41 ± 0.02) GeV and (10.95 ± 0.02) GeV for these states [45, 51], while Ref. [50] yielded similar predictions of approximately $\simeq 4.40$ GeV and $\simeq 10.74$ GeV. In more recent work using the Born-Oppenheimer Effective Field Theory (BOEFT), the masses of the 1^{--} states were estimated as (4.011 ± 0.030) GeV and (10.6902 ± 0.003) GeV, respectively [59].

Other exotic mesons have also garnered interest. For example, bottom-charm hybrid states $\bar{b}gc$ with quantum numbers $J^P = 0^+, 0^-, 1^+, 1^-, 2^+, \text{ and } 2^-$ were studied in Ref. [40] using QCD sum rules that

incorporated dimension-six condensate terms. These hybrids were predicted to have masses ranging from 6.8 to 8.5 GeV, and their possible strong decays via two-body open- and hidden-flavor channels were also analyzed [55,58]. Ref. [49] explored hybrid mesons composed of magnetic and electric gluon excitations, applying the constituent gluon model to investigate their spectra and decay dynamics [56,57].

In this thesis, we calculate the masses and current couplings of hybrid quarkonia mesons $\bar{c}gc$ and $\bar{b}gb$, and exotic mesons $\bar{b}gc$ possessing various spin-parity quantum numbers. The analysis is conducted within the framework of the QCD two-point sum rule (SR) approach, incorporating nonperturbative contributions that are proportional to the squared three-gluon condensate, $\langle g_s^3 G^3 \rangle^2$ [54-58]. The QCD sum rule method stands out as a powerful nonperturbative technique in the study of hadronic physics. Originally developed to explore the spectroscopic properties and decay mechanisms of conventional hadrons [60, 61], this approach has proven to be equally valuable for investigating exotic states beyond the traditional quark model [62–64]. Notably, the QCD sum rule method was applied to hybrid quarkonia soon after its inception, demonstrating its relevance and effectiveness for such studies [31–33].

The thesis is structured in the following way: chapter two introduces the Standard Model of particle physics, outlining its fundamental particles and interactions. This chapter aims to provide the theoretical foundation necessary for understanding the later discussions on hybrid mesons. It also highlights the key limitations of the Standard Model that motivate further theoretical developments.

In chapter three, exotics hadrons and hybrid mesons: an exploration of exotic hadrons, we give a short introduction to exotic hadrons including the history and development of exotic hadrons and studied exotic hadrons and quantum numbers. We also discuss the definition and significance of the exotic hadrons we chose, the hybrid mesons. The classification of hybrid mesons is generally and precisely shown in different sections. As a framework, the QCD sum rules introduce an investigation of hybrid mesons also. The studies and experiments of the light and heavy hybrid mesons are an example of these investigations, which exhibit the extent of the development that has taken place through the years. As an aim for our study, we chose to predict the heavy hybrid mesons: Charmonium, Bottomonium Hybrids $\bar{c}gc$, $\bar{b}gb$, and the hybrid meson $\bar{b}gc$. The chapter presents the prior researches on these hybrids by reviewing the approaches and methods to predict the spectroscopic properties.

Chapter four (quantum chromodynamics (QCD) theory) starts with a short introduction and presents the QCD theory. We explain the fundamentals of QCD, including the gauge symmetry, SU group (3), and the Lagrangian. We also interpret the important phenomena associated with QCD theory. The applications and the implications of QCD are discussed as well. Prediction methods of QCD theory are mostly introduced briefly, but the important highlight here is the QCD sum rule.

In chapter five, theoretical analysis and computational methodologies have been explored; QCD sum rules for the heavy hybrid quarkonia is the approach that is depended on. We present the two-point correlation function as a novel framework for determining the spectroscopic properties of the selected hybrids. The main concept of both the phenomenological and theoretical aspects of the QCD sum rule are clearly demonstrated. The steps of QCD sum rules have been demonstrated by presenting the interpolating current. We calculate spectral parameters of the heavy hybrids $\bar{c}gc$ and $\bar{b}gb$ with spin-parities $J^{PC} = 0^{++}, 0^{+-}, 0^{-+}, 0^{--}$ and $1^{++}, 1^{+-}, 1^{-+}, 1^{--}$, then we devote ourselves to analysis of the hybrid mesons $\bar{b}gc$ with quantum numbers $J^P = 0^+, 0^-, 1^+, \text{ and } 1^-$. Finally, the last chapter is the conclusion.

2. CHAPTER TWO - THE STANDARD MODEL OF PARTICLE PHYSICS

2.1 Introduction to standard model

The standard model (SM) is a theoretical framework in physics that explains electromagnetic, weak and strong nuclear interactions between primary particles. This principle is an explanation of a very high level of accuracy for a wide range of phenomena of physics and is ranked in the most successful principles of the history of physics. The SM classifies all known primary particles into two groups: Fermion, or Quark and Lepton, and Bosons, which some of them mediate forces [140-146].

2.2 Historical Development of The Standard Model

The standard model development discovers its history in the early 20th century when research was originally conducted by researchers such as Max Planck, Albert Einstein and Niels Bohr. Quantum Field Theory was developed with a mathematical structure to describe particle interaction in the 1930s, Quantum Electrodynamics (QED) reached to the peak by Richard Feynman, Julian Schwinger and Sin-Itiro Tomonaga where the QED became one of the most precisely tested theories in physics [141].

The electroweak theory of the 1970s integrated electromagnetic and weak interactions in a structure and paved the way for discovery of both W and Z bosons in the experiments of CERN. Higgs Boson's recent discovery in the Large Hadron Collider in 2012 confirmed a central component of the Higgs Field and it is fundamental in the SM [65].

2.3 Fundamental Particles and Forces

The standard model describes three essential forces: electromagnetic, weak nuclear and strong nuclear forces. These forces are mediated by exchange particles known as gauge bosons. The Photon is the particle that conveys electromagnetic interactions, while W^+ , W^- and Z^0 Boson convey weak interactions [72,122]. Strong forces responsible for holding quarks together in protons and neutrons are conveyed by gluons [122].

Fermions, which form substances, are further divided into quarks and leptons. Quark is divided into three families, each consists of two types, while Lepton contains three charged leptons and their

respective neutrinos. The Higgs boson, discovered in 2012, is a scalar boson that plays a crucial role in giving mass to other particles through the Higgs mechanism Fig.(2-1) [65,72].

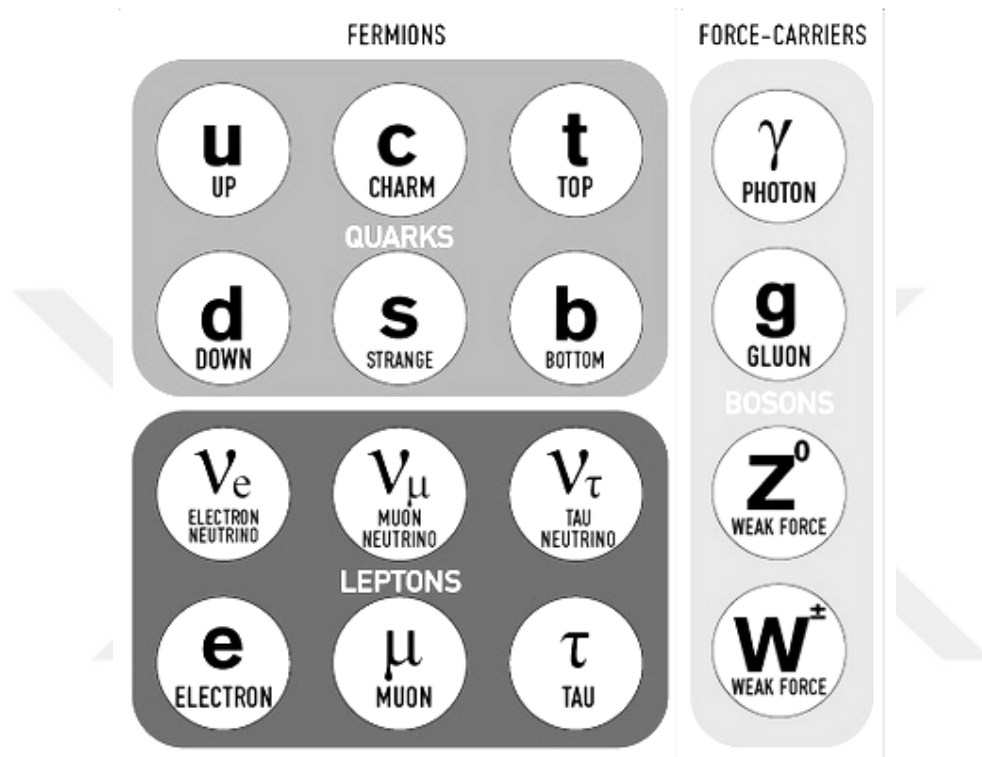


Figure 2-1: Explain the elementary particle in the stander models

2.4 Higgs Mechanism

The Higgs boson is a particle associated with the Higgs field, which gives mass to other particles through their interaction with it. In the electroweak theory, symmetry breaking occurs when the Higgs field receives a vacuum non for expectation. This breaking results in W and Z bosons gaining mass but leave the Photon with massless [140].

The discovery of the Higgs boson in LHC confirmed this mechanism, strengthened the basis for the standard model. This discovery in 2012 was an important milestone and provided the experimental verification of the mechanism responsible for particle mass within the standard model [65]. Despite its remarkable success, standard model has limitations, such as the absence of quantum details on gravity and dark matter and dark energy [78-80].

2.5 Symmetry and Conservation Laws in the Standard Model

Symmetries in standard model are necessary to understand the conservation laws. According to Noether's theorem, each continuous symmetry corresponds to a conserved quantity [141]. In SM, measuring -symmetry is a basic concept that supports the interactions between particles and gauge bosons. The SM is run under local measures symmetry, such as $U(1)_Y$ for electromagnetism and hypercharge, $SU(2)_L$ for weak interactions and $SU(3)_C$ for strong interactions, which determine the interaction between particles with the same gauge boson [140].

The symmetry could provide the conservation of parity (P) under the spatial inversion, while charge conjugation (C) exchanges the particles with the antiparticles. Time reversal (T) symmetry reverses the direction of the time. CPT -symmetry, combination of these three, is considered a basic symmetry of nature [141].

2.6 Standard Model Extensions and Open Questions

Despite success, the standard model is not a complete principle of basic interaction. Several extensions have been proposed to address its limits

1. Supersymmetry (Susy):

Susy suggests that each particle has a super partner with different spinning features, possibly addressing the hierarchy problem and providing dark matter candidates [76].

2. String Theory:

This theoretical model suggests that the fundamental constituents of matter are not point-like particles, but rather one-dimensional strings. The string theory tries to unite all the forces, including gravity, but has not yet provided testable predictions [139,142].

3. Neutrino Mass:

The standard model originally predicted massless neutrinos, but experiments showed that neutrinos have mass, suggesting the need for new physics, such as the seesaw mechanism [80,81].

4. Dark Matter and Dark Energy:

Observations show that dark matter constitutes a significant portion of the universe's mass, yet it does not interact electromagnetically and is not accounted for by SM particles. Similarly, the dark energy responsible for rapid expansion of the universe is not interpret through SM [74,77].

2.7 Experimental Evidence for the Standard Model

Experimental verification of standard model is established through several particle physics experiments with high energy. Large findings are important to confirm the models' predictions:

1. Deep Inelastic Scattering:

Experiments performed at the Stanford Linear Accelerator Center (SLAC) in the late 1960s showed that protons and neutrons are not elementary particles, but are made of quarks, quantum chromodynamics (QCD) [83, 141] predictions Fig.(2-2).

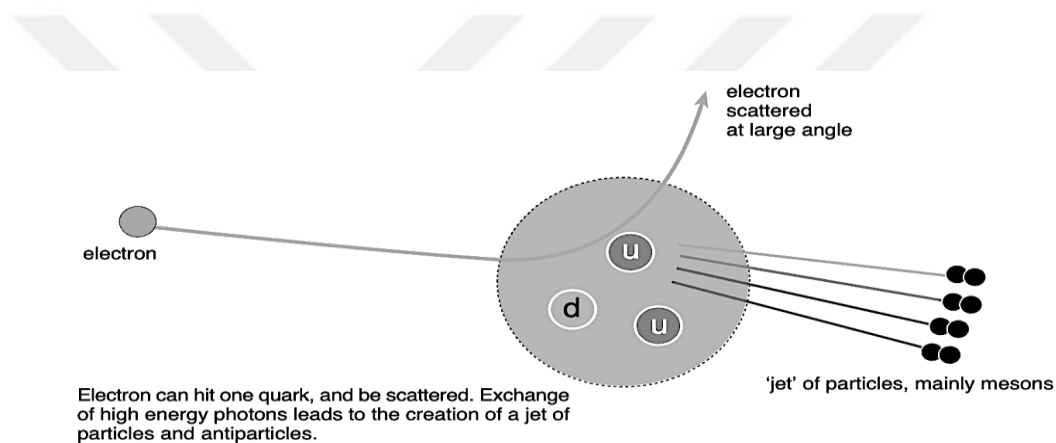


Figure 2-2: Deep inelastic scattering reveals quark structure inside nucleons

2. Discovery of the W and Z Bosons:

In 1983, the discovery of W and Z boson in CERN was an important milestone, when it confirmed the integration of electroweak theory and unification of electromagnetic and weak interactions. The measured properties of these bosons matched the predictions of the standard model, providing strong experimental validation of its framework [130].

3. The Higgs Boson:

The Higgs boson, discovered at the LHC in 2012, provided the last missing piece of the Standard Model. Its existence confirmed the mechanism by which particles acquire mass [65]. The properties of the Higgs boson, including its mass and decay channels, align well with theoretical expectations, providing strong evidence for the Standard Model [65,82,140].

2.8 The Role of Quantum Field Theory (QFT) in the Standard Model

Quantum field theory (QFT) forms the mathematical foundation of the Standard Model. It blends quantum physics and special relativity to describe particle behavior as excitations in their respective fields.

1.Fermionic Fields:

Fermions, which make up matter, are described by fields that obey the Pauli exclusion principle. Each fermionic field corresponds to a particle type, such as quarks and leptons, which are combined in QFT to understand their interactions [140].

2.Bosonic fields:

Bosons are produced as a result of the quantization of the associated force fields for the particles. For instance, the photon is the quantum of the electromagnetic field, whereas gluons are the quantized excitations of the strong force field [140,141].

3.Interactions and Perturbation Theory:

The interaction between particles in the standard model is analyzed using a disturbance theory, where interaction is considered a slight improvement of free particle behavior. This approach allows physicists to calculate resolution dimensions and cross sections, leading to the predictions that can be tested against experimental data [140].

2.9 Challenges and Future Directions of the Standard Model

While the standard model has achieved significant success, many unresolved questions challenge its perfection and purpose:

1. Hierarchy Problem:

The hierarchy problem refers to the question of why the Higgs boson's mass is so much lighter than the Planck mass, despite receiving large quantum corrections. This discrepancy raises issues of stability and fine-tuning in the Standard Model, suggesting the need for new physics beyond it [76].

2. Matter-Antimatter Asymmetry:

Assessments indicate an asymmetry between matter and antimatter in the universe, with more matter being present. The standard model may not be completely responsible for this imbalance, inspires exploration in new physics outside its structure [72].

3.Dark Matter and Dark Energy:

The existence of dark matter and dark energy, which together contains about 95% of the universe, remains unexplained by the standard model. These components require new theoretical frameworks, including potential candidates such as weak large-scale particles (Wimps) and axions [70,74,76].

4.Neutrino Masses:

The discovery of neutrino oscillations implies that neutrinos have mass, but the Standard Model originally proposed massless neutrinos. Theories like the seesaw mechanism try to explain this disparity [81].

2.10 Supersymmetry and Extensions of the Standard Model

While the standard model has been very successful, there are many theories that expand the framework. One of the most prominent is a Supersymmetry (Susy), which suggests that the standard model has a super partner with different spinning features in each particle.

For example, fermions would have bosonic superpartners, and bosons would have fermionic ones. SUSY plays a key role in addressing unresolved problems such as the hierarchy problem in the Standard Model. It can also stabilize the Higgs boson mass by introducing superpartner particles that cancel certain quantum divergences [76].

In addition to addressing the hierarchy problem, SUSY also proposes a natural candidate for dark matter: the lightest supersymmetric particle (LSP), which could be stable and weakly interacting. Supersymmetry is actively tested in high energy experiments, including the LHC. Despite extensive discoveries, no direct evidence of SUSY particles has been seen yet [2]. However, many physicists remain optimistic that future runs of the LHC or other colliders may yield signs of supersymmetry.

2.11 Grand Unified Theories and String Theory

Beyond the supersymmetry, the purpose of the other frameworks is to unify the nature's forces. Grand Unified Theories (GUTs) tries to merge electromagnetic, weak and strong forces in the same force. The fact that these forces are expected to converge at very high energies according to GUTs suggests that they were once united just after the Big Bang [72].

String theory also adds to the idea of GUTs by showing particles as one-dimensional strings instead of points. The goal is to unite all basic forces, such as gravity. String theory claims that different vibrations

of these strings correspond to different particles [142]. Although string theory still needs experimental validation and is highly theoretical, it offers a potential framework for comprehending the universe at its most basic level.

The standard model successfully unites three of the four basic forces: electromagnetism, the weak force and the strong force. However, gravity is not yet included in this framework. A quantum principle for gravity, perhaps through string theory or loop quantum gravity, is one of the greatest unresolved problems in physics [69].

2.12 The Standard Model and Cosmology

The Standard Model is essential for elucidating elements of the early cosmos and its development. One important example is Big Bang Nucleosynthesis (BBN), which explains how light elements like hydrogen, helium, and lithium were made in the first few minutes of the universe. The observed abundances of these elements correspond closely with the predictions of the Standard Model, thereby validating its correctness [80].

The Cosmic Microwave Background (CMB) offers a "snapshot" of the nascent universe, featuring minute variations that align with the Standard Model's predictions for particle interactions, thereby elucidating the formation of large-scale cosmic structures [75,78].

Certain phenomena, like baryon asymmetry—the apparent excess of matter over antimatter—remain unexplained by the Standard Model. Extensions that incorporate CP violation may provide viable explanations for this anomaly, indicating probable novel physics beyond the Standard Model [71-79]. These connections show the significance of the Standard Model in cosmology and point to potential areas for further research.

2.13 Beyond the Standard Model: Exploring New Physics

As shown in the previous sections, although the Standard Model has shown amazing success, other events point to the existence of physics outside its domain. Observations of dark matter and dark energy, which together make up approximately 95% of the universe's energy content, remain unexplained within the Standard Model framework, as evidenced by galaxy rotation curves, gravitational lensing, and the universe's accelerated expansion [74,77]. Additionally, neutrino oscillations reveal that neutrinos have mass, contradicting the Standard Model's prediction of massless neutrinos, and suggesting alternative

mechanisms, such as the see-saw model, to account for this mass [81]. GUTs propose that the electromagnetic, weak, and strong forces might unify at high energies, a concept yet to be observed but predicted by certain GUT models [72]. SUSY also proposes a symmetrical partner for each Standard Model particle, potentially addressing limitations like the hierarchy problem, although it has not been experimentally confirmed [131,141]. By means of these hypotheses, the Standard Model is meant to be reinforced by bridging the gaps between particle physics and cosmology, hence enhancing its strength.



3.Chapter Three - EXOTICS HADRONS

HYBRID MESONS: AN EXPLORATION OF EXOTIC HADRONS

3.1 Introduction to Exotic Hadrons

Usually known in the standard models, the hadrons are composite particles, divided into baryons (composed of three quarks) and mesons (composed of a quark-antiquark pair) [102]. While most hadrons fit into this traditional quark model, researchers have found evidence of “exotic” hadrons—particles that go beyond the simple quark-antiquark structure of mesons and the three-quark composition of baryons [92]. These exotic hadrons include tetraquarks, pentaquarks, hybrid mesons, and glueballs, offering unique opportunities to deepen our understanding of QCD and the strong interaction [93].

Exotic hadrons challenge the hypothesis guiding the strong interactions between quarks and gluons, QCD. Because these particles often include additional quark content or excited gluonic fields, their study is critical for probing the non-perturbative regime of QCD [94–98].



Figure 3-1: Illustrate some kinds of exotic hadrons

3.2 History and Development of Exotic Hadron Studies

Theoretical predictions of exotic hadrons began in the 1970s when the quark model suggested the possibility of multi-quark states and hybrid configurations [99]. However, experimental efforts to identify these exotic states were initially inconclusive due to the complexity of distinguishing them from ordinary hadrons [100]. A significant breakthrough occurred in 2003 with the discovery of the $X(3872)$ by the Belle collaboration, which exhibited properties inconsistent with conventional mesons, providing the first compelling evidence of exotic hadrons [97]. This discovery was soon followed by observations of other exotic states, including the $Z_c(3900)$ and $P_c(4450)$ by BESIII and LHCb, respectively, further solidifying the existence of these unconventional hadrons [85].

3.3 Exotic Hadrons and Quantum Numbers

Exotic hadrons—including hybrid mesons, tetraquarks, and pentaquarks—stand out from normal hadrons in their quantum numbers not fit for a fundamental quark-antiquark or three-quark structure. Hybrid mesons are distinguished by their unique quantum numbers, such as $J^{PC} = 1^{-+}$, which are not possible in conventional mesons. This is because the combinations of orbital angular momentum L and spin S in a quark-antiquark system cannot simultaneously produce the required parity $P = (-1)^{L+1}$ and charge conjugation $C = (-1)^{L+S}$. In these notations, J stands for total angular momentum, P for parity, and C for charge conjugation. These quantum numbers are key to identifying hybrid mesons in experimental data, as they provide clear signatures that distinguish them from ordinary mesons [84-89]. Defining the characteristics and classification of mesons depends critically on quantum numbers. In hybrid mesons, the gluonic excitation allows for combinations of J^{PC} that are otherwise forbidden. This distinction has driven research into exotic hadrons, providing a platform to explore new states of matter predicted by QCD [107-109].

3.4 Hybrid Mesons: Definition and Importance

Hybrid mesons, among the exotic hadrons, have attracted particular interest due to their distinctive quark and gluon arrangements. Unlike conventional mesons, hybrid mesons include both quarks and an active gluonic component, where the gluons themselves carry quantum numbers and contribute to the overall properties of the particle [109]. This gluonic excitation is what sets hybrid mesons apart from their ordinary counterparts and allows them to possess quantum numbers that traditional mesons cannot achieve, such as exotic J^{PC} combinations [103]. Hybrid mesons are essential for examining the dynamics of gluon fields and the non-Abelian characteristics of QCD.

3.4.1 Vector and Pseudoscalar Mesons

One important feature of hybrid mesons is their ability to show "exotic" quantum numbers—quantum states impossible with conventional quark-antiquark mesons. For instance, ordinary mesons can have quantum numbers such as $J^{PC} = 0^{-+}$ (pseudoscalar mesons) or $J^{PC} = 1^{-}$, (vector mesons). In contrast, hybrid mesons can exhibit exotic quantum numbers like $J^{PC} = 1^{-+}$, which are forbidden in conventional quark-antiquark systems [110].

In the context of meson classification, vector mesons are those with quantum numbers $J^{PC} = 1^-$, meaning they have total angular momentum $J = 1$, negative parity $P = -1$, and negative charge conjugation $C = -1$ [8]. Examples of vector mesons include the $\rho(770)$, $\omega(782)$, and $\phi(1020)$ mesons.

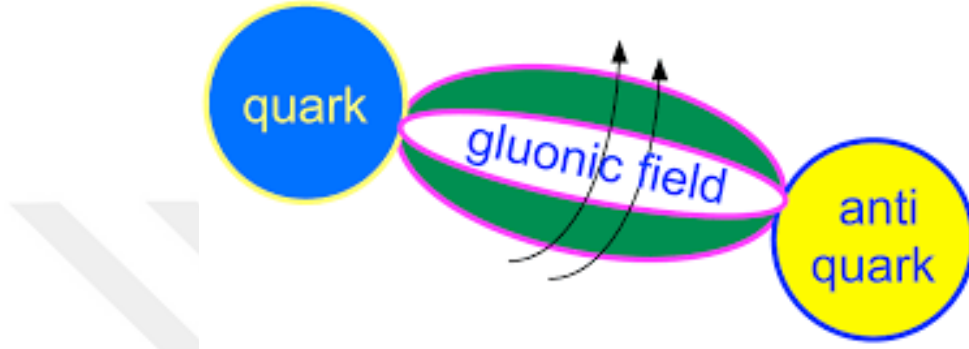


Figure 3-2: Explain the structure of hybrid meson.

In the framework of scattering processes, these particles commonly play the mediator between hadrons. On the other hand, pseudoscalar mesons are characterized by quantum numbers $J^{PC} = 0^{-+}$, which indicates they have zero spin ($J = 0$), negative parity ($P = -1$), and positive charge conjugation ($C = +1$). Examples of pseudoscalar mesons include the pion (π), kaon (K), and (η) mesons. These mesons are fundamental for chiral symmetry breaking and operate like the Goldstone bosons in QCD [111].

3.4.2 Hybrid Mesons and Exotic Quantum Numbers

The quark model faithfully captures vector and pseudoscalar mesons; hybrid mesons have more complex internal structures since gluonic excitations are explicitly involved. The gluons inside hybrid mesons can carry angular momentum, which allows for exotic quantum numbers like 1^{-+} , 0^{+-} , and 2^{+-} [110]. These quantum states are impossible for ordinary quark-antiquark mesons, making the identification of these exotic states a key indicator of hybrid meson existence [108]. For example, the exotic quantum state 1^{-+} , does not fit into the simple quark-antiquark model but can arise in hybrid mesons due to the contribution of gluonic degrees of freedom. This allows hybrid mesons to manifest themselves as resonances with quantum numbers that lie outside the traditional quark model spectrum [112].

Experiments like those at the GlueX facility at Jefferson Lab and LHCb at CERN are specifically designed to search for such states.

In addition, that the hybrid mesons possess unique quantum numbers, and they display exclusive mass spectra and decay channels. Pseudoscalar and vector mesons typically decay into lighter mesons or photon pairs, but hybrid mesons, due to their gluonic component, have more complex decay modes. For instance, they might decay into numerous mesons or mesons with different quantum numbers, offering a significant region for study by both experimentalists and theorists [99]. Understanding the behavior of gluonic fields in QCD depends on an awareness of hybrid mesons, particularly those with unusual quantum numbers. Not just as force carriers but also as active contributors to the evolution of hadronic states, hybrid mesons offer understanding of the role of gluons.

3.5 Classification of Hybrid Mesons

The internal structure and quantum numbers connected with hybrid mesons, which result from the special interaction of quarks and gluons, define their classification most essentially. Unlike conventional mesons, hybrid mesons incorporate extra gluonic excitation that helps to define their whole quantum state. This gluonic component allows hybrid mesons to be categorized in ways that reflect the nontrivial dynamics of QCD, distinguishing them from traditional hadrons [111].

3.5.1 Based on Quantum Numbers

Hybrid mesons can be classified according to their quantum numbers J^{PC} . While ordinary mesons are constrained to specific quantum numbers due to their quark-antiquark structure, hybrid mesons, because of their gluonic content, are capable of possessing exotic quantum numbers that are forbidden for standard quark-antiquark mesons [107]. These exotic quantum numbers include states like: (1^{-+}) , which is the most famous exotic quantum number combination and has been the primary target in hybrid meson searches. No conventional meson can have this J^{PC} value, making its identification a strong signature of hybrid mesons [115]. Additionally, other hybrid mesons, such as those with quantum numbers $J^{PC} = 0^{+-}$ and 2^{+-} also exhibit exotic quantum states with no counterparts in the standard quark model.

The identification of these states provides further evidence of the existence of hybrid mesons [102]. In addition to exotic quantum numbers, hybrid mesons can also manifest in non-exotic J^{PC} states such as 1, but their distinguishing feature is the presence of gluonic excitations, which affect their mass spectra and decay modes compared to ordinary mesons [110].

3.5.2 Based on Gluonic Excitations

The gluonic structure of hybrid mesons is another key aspect of their classification. In QCD, gluons, being the mediators of the strong force, can carry angular momentum and form flux tubes between quarks. These flux tubes can become excited, contributing to the overall energy of the system and generating hybrid mesons with unique properties [23]. Normally, hybrid mesons are categorized by the type and degree of gluonic excitation:

- **Low-lying gluonic excitations:** In this this spatial case, founding hybrid mesons are formed when the gluonic field between the quark and antiquark is in its lowest excited state. These states are typically lighter in mass and more likely to be observed experimentally [109].
 - **Higher-order gluonic excitations:** In contrast with other case previous case, more complex hybrid mesons can form when the gluonic field is excited to higher energy levels. These states are predicted to be heavier and more difficult to detect, requiring high-energy collisions and sensitive detectors [110].
- Lattice QCD simulations have been useful in estimating the masses and decay patterns of these distinct hybrid mesons. According to [110], the lowest-lying hybrid mesons are projected to have masses ranging from 1.8 to 2.5 GeV/c², making them accessible to existing particle physics experiments such as GlueX and the LHCb.

3.5.3 Based on Decay Patterns

Hybrid mesons also show ways of breaking down after formations occur which can be used to categorize them differently from ordinary mesons due to their structure involving gluons [112]. Hybrid mesons, with quantum properties might break down into two pseudoscalar mesons or a vector meson and a pseudoscalar meson based on their particular quantum state. Studying these decay pathways assists researchers in recognizing hybrid mesons and separating them from ambient signals detected by particle detectors [85].

Some notable decay channels include:

- **Two-meson decays:** Hybrid mesons often decay into pairs of pseudoscalar mesons (such as pions or kaons), which can be detected in collider experiments [108].
- **Radiative decays:** Some hybrid mesons may decay by emitting photons, providing another experimental signature that can be used for identification [110].

3.5.4 According to Spectroscopic Properties

Hybrid mesons possess characteristics due to their gluonic components and can be categorized not only based on their quantum traits and decay behaviors but also on their spectrum of masses. The mass of a meson is predominantly determined by the mass of the quarks it contains. When it comes to classifying hybrid mesons in this context; light hybrid mesons like π_1 (1600) made up of quarks (up, down and strange) and heavy hybrid mesons that include heavier quarks such as charm or bottom, like $Y(4260)$ [91].

3.6 Theoretical Predictions and Experimental Evidence for Hybrid Mesons

By the years, several theoretical frameworks have been created to investigate their characteristics and anticipate the presence of states known as hybrid mesons. The Quark Gluon Model is an approach used to explain mesons behavior by considering the involvement of gluonic excitation, in determining the mesons quantum properties resulting in unique quantum attributes that are not typically observed in conventional mesons. For instance, the 1^{-+} hybrid meson is not just formed by an antiquark pair but can exist as a hybrid because of contributions, from gluons [110]. Another idea that has been put forward is the Bag Model, which describes mesons as quarks and gluons confined within a "bag." When the gluonic field, inside the bag is excited it leads to the formation of states. The baryon $b_1(1235)$ and hadron $h_1(1170)$ are instances of states that could be identified as mesons based in this model [104].

Certainly, the phenomenological models, like the flux-tube model have effectively predicted the actions of the fields for creating these particles [105]. According to the flux tube model proposed by (Isgur and Paton, 1985), it explains how gluons create flux tubes connecting quarks and how these tubes get excited to produce mesons, with quantum properties. The particle known as $Y(4260)$ according to this models

theory is considered a meson due to its characteristics that deviate from the traditional quark model [109].

Lattice QCD, simulations have provided significant insights into the mass spectrum and decay patterns, this a numerical approach was trying to solve the equations of QCD on a discrete space-time lattice, has been instrumental in predicting the properties of hybrid mesons [100]. Both light and heavy hybrid mesons were subject to this study, for light hybrid mesons, lattice calculations predict that the lowest-lying states should have masses in the range of 1.8–2.5 GeV/c², aligning with the observed candidates like the $\pi_1(1600)$ and $\eta_1(1855)$ (Dudek et al., 2009). For heavy hybrid mesons, the mass predictions are higher, typically in the range of 4–12 GeV/c², depending on the specific quark content [108].

While the LHCb experiment at CERN has made major progress in identifying candidates for heavy hybrid mesons, the Experiments like GlueX, COMPASS, and the BESIII cooperation have been very helpful in hunting for light hybrid mesons. These experiments aim to study the mass spectra and decay channels of these mesons in greater detail to determine whether their properties match theoretical predictions [110].

3.6.1 QCD Sum Rules to Investigate the Hybrid Mesons

When it comes to analyzing exotic states, such as hybrid mesons, the QCD sum rule method is one of effective approach. This approach involves constructing a correlation function of the interpolating current for the hybrid meson, then matching it to QCD predictions using the operator product expansion (OPE). The QCD sum rules allow researchers to estimate properties such as the mass and decay widths of hybrid mesons [60]. In the context of hybrid mesons, QCD sum rules have been applied to predict the mass and structure of various states. The $\pi_1(1600)$ and the $Y(4260)$ have been explored using sum rules, which help to confirm their exotic nature and hybrid composition [104]. According to the sum rules, the approach is suggested that the gluonic field excitation contributes significantly to the mass of these particles, which sets them apart from conventional mesons. Furthermore, the QCD sum rule structure has proved rather useful in clarifying the function of gluons in hybrid meson generation. It provides insight into how gluonic excitations manifest in observable meson states and offers predictions for hybrid meson masses that are consistent with experimental data [117]. As an examples, $\pi_1(1600)$, a candidate for a hybrid meson, was investigated using QCD sum rules, which estimated its mass to be around 1.6 GeV, matching some experimental findings [81]. Another example is the $Y(4260)$, where

QCD sum rule methods suggest it has hybrid components due to its exotic decay patterns and mass, which do not align with typical quarkonium predictions [108].

3.7 Light and Heavy Hybrid Mesons

As a definition for light hybrid mesons, it is a particle which is composed of lighter quarks, such as up, down, and strange quarks, with an additional gluonic excitation contributing to the meson's overall properties. These mesons are of particular interest because their relatively lower mass makes them accessible to a wide range of experimental facilities and they are often searched for in experiments like COMPASS at CERN, GlueX at Jefferson Lab, and other hadron collider experiments [110].

One of the most well-studied light hybrid mesons is the $\pi_1(1600)$, which is a candidate for the exotic 1^{-+} quantum number state. The $\pi_1(1600)$ has been observed in several decay channels, including decays to $\eta\pi$ and $\rho\pi$, and its mass is around $1.6 \text{ GeV}/c^2$ [86,87]. This makes it one of the lightest hybrid mesons predicted by lattice QCD calculations [92].

Another example of a light hybrid meson is the $\eta_1(1855)$, which was recently observed and is another candidate for a hybrid meson with exotic quantum numbers. Its mass, around $1.85 \text{ GeV}/c^2$, fits into the range predicted for light hybrid mesons, and its discovery has provided additional evidence supporting the existence of these states [110].

Heavy hybrid mesons, in contrast, contain at least one heavy quark, such as a charm (c) or bottom (b) quark, and an antiquark, along with gluonic excitations, these heavy masses made an expected mesons to have much higher masses compared to their lighter counterparts due to the increased mass of the constituent quarks [19,24,109]. Because of their larger mass, heavy hybrid mesons are more challenging to produce and detect, requiring high-energy collisions that can only be achieved in experiments like those at the LHC [91]. One prominent candidate for a heavy hybrid meson is the $Y(4260)$, a charmonium-like state that has been observed with a mass of approximately $4.26 \text{ GeV}/c^2$, it has been discovered exotic properties that do not fit neatly into the quark-antiquark model, suggesting it may have a gluonic excitation, making it a potential hybrid meson [87]. Its decay modes, which include final states like $J/\psi\pi\pi$, further support the hypothesis that it is not a conventional meson [115].

Another heavy hybrid meson candidate is the $Z_b(10610)$, which contains a bottom quark and has been observed in the decay channels involving $Y(1S)$ and $Y(2S)$. Its mass is around $10.6 \text{ GeV}/c^2$, making it one of the heaviest hybrid meson candidates discovered so far [90].

3.8 Predictions of Heavy Hybrid Mesons: Charmonium and Bottomonium Hybrids

In the realm of heavy hybrid mesons, the focus is on systems containing heavy quarks like charm (c) and bottom (b), which form charmonium and bottomonium hybrids, respectively because the inclusion of heavy quarks allows for the study of non-perturbative QCD effects in a controlled manner, where the quark masses are much larger than the QCD energy scale Λ_{QCD} . This makes heavy hybrids a promising area for both theoretical predictions and experimental searches [91].

3.8.1 Charmonium Hybrids ($\bar{c}gc$)

Charmonium hybrids are mesons that consist of a charm quark (c) and an anti-charm quark ($\bar{c}$) bound together by a gluonic excitation (g). These states are predicted to have exotic quantum numbers not possible in ordinary quark-antiquark mesons, such as 1^{-+} , 0^{+-} , and 2^{+-} . In addition to these exotic states, charmonium hybrids can also appear with non-exotic quantum numbers like $J^{PC}=1^{--}$ and 0^{-+} , which makes distinguishing them from conventional charmonium challenging [101].

Theoretical studies, particularly those utilizing lattice QCD, predict that the lowest-lying charmonium hybrid states should have masses in the range of 4.0–4.5 GeV/c², also the studies show that these predictions are consistent with the discovery of candidates like Y(4260) and Y(4360), which are thought to be potential charmonium hybrid mesons due to their exotic properties and decay patterns [87,114].

The quantum numbers predicted for charmonium hybrids include states such as $J^{PC} = 1^{-+}$ which is an exotic quantum state that is a clear signature of hybrid mesons and $J^{PC} = 1^{--}$ a non-exotic quantum state, but with properties that suggest gluonic contributions. Lattice QCD simulations predict that these charmonium hybrids should exhibit distinctive decay patterns, often involving radiative or hadronic transitions to lower charmonium states, such as J/ψ or $\psi(2S)$, along with the emission of light mesons like pions [99].

3.8.2 Bottomonium Hybrids ($\bar{b}gb$)

Bottomonium hybrids consist of a bottom quark (b) and an anti-bottom quark ($\bar{b}$) bound by an excited gluonic field. They are considered interesting state due to their large mass and the ability to study the gluonic degrees of freedom in a regime where the bottom quark mass dominates the dynamics. Bottomonium hybrids are predicted to be heavier than their charmonium counterparts, with masses typically predicted in the range of 10–12 GeV/c² [91].

Some key quantum numbers predicted for bottomonium hybrids include: $J^{PC} = 0^{++}, 0^{+-}, 0^{-+}, 0^{--}$ and $J^{PC} = 1^{++}, 1^{+-}, 1^{-+}, 1^{--}$ while experimental evidence for bottomonium hybrids is still sparse, theoretical models like the flux-tube model predict that the lowest-lying bottomonium hybrid should have $J^{PC} = 1^{--}$, making it a prime target for searches in high-energy experiments like LHCb and Belle II [104]. According to the quantum numbers, theoretical predictions suggest that such hybrid states are expected to decay into bottomonium states like $\Upsilon(1S)$ or $\Upsilon(2S)$, along with pion emission.

The $Z_b(10610)$ and $Z_b(10650)$ are possible candidates for bottomonium-like hybrids or tetraquark states, as they exhibit unconventional properties that deviate from the traditional quark-antiquark model [90], also these states are also expected to decay into $\Upsilon(nS)$ and pions, consistent with theoretical predictions for hybrid mesons.

3.8.3 Gluonic Components in Heavy Hybrids: $\bar{c}gc$, $\bar{b}gb$, and $\bar{b}gc$

Hybrid mesons involving gluonic excitations are not limited to states with identical quarks. There are predictions for hybrid mesons with distinct gluonic components, such as $\bar{c}gc$ (charm-gluon-charm), $\bar{b}gb$ (bottom-gluon-bottom), and $\bar{b}gc$ (bottom-gluon-charm) configurations. These systems offer a larger spectrum of quantum numbers and masses due to the several quark flavors and the contributions of the gluonic field.

- $\bar{c}gc$ and $\bar{b}gb$ hybrids: These hybrids involve quarks of the same flavor bound by a gluonic excitation, leading to quantum numbers such as:

- $J^{PC} = 0^{++}, 0^{+-}, 0^{-+}, 0^{--}$

- $J^{PC} = 1^{++}, 1^{+-}, 1^{-+}, 1^{--}$

These heavy hybrid mesons are predicted to have higher masses, with bottomonium hybrids ($\bar{b}gb$) being the heaviest among them, and charmonium hybrids ($\bar{c}gc$) falling in a slightly lower mass range [91].

- $\bar{b}gc$ hybrids: These mesons contain one anti-bottom quark, one charm quark, and a gluonic excitation, leading to different quantum numbers: $J^{PC} = 0^+, 0^-, 1^+, 1^-$, these states are predicted to have masses in the range of 6–8 GeV/c². Due to the mixed heavy-quark content, they present a unique opportunity to study the interplay between charm and bottom quarks in a gluonic environment [99].

3.9 QCD Sum Rule Predictions for Heavy Hybrid Mesons

The QCD sum rules provide a structure for deducing characteristics of hadrons from the underlying QCD theory by linking the dynamics between quarks and gluons with observable properties, which proves beneficial, in situations where conducting lattice QCD calculations is demanding or when there is limited experimental data available. In particular, QCD sum rules have been employed to predict the masses and quantum numbers of both light and heavy hybrid mesons [112,145].

For hybrid mesons with a mix of quarks and gluons in them known as heavy hybrid mesons, researchers use QCD sum rules to analyze how currents connect to these unique states, combining both quark and gluonic aspects for a comprehensive understanding, making it possible to incorporate the effects of gluonic contributions accurately in the calculations, thereby enabling predictions of various properties like masses for both charmonium ($\bar{c}gc$) and bottomonium ($\bar{b}gb$) hybrid mesons through this approach [96,145].

3.9.1 Predictions for Charmonium Hybrids via QCD Sum Rules

The application of QCD sum rules to charmonium hybrids has provided mass predictions for states with exotic quantum numbers, such as 1^{-+} and 0^{-+} , which are clear signatures of hybrid mesons. For example, QCD sum rule calculations predict that the mass of the lowest-lying charmonium hybrid should be around 4.2–4.5 GeV/c², in agreement with the experimentally observed states like the Y(4260) and Y(4360) [96,115].

It was clear that QCD sum rule calculations provide predictions for the decay constants and coupling strengths of these hybrids mesons. Moreover, it has been suggest that charmonium hybrids should decay predominantly into lower charmonium states like J/ψ , along with pions or photons, which is consistent with experimental observations [95].

3.9.2 Predictions for Bottomonium Hybrids via QCD Sum Rules

For bottomonium hybrids, QCD sum rules predict that the lowest-lying hybrid states should have masses in the range of 10.0–12.0 GeV/c². These predictions align with the observed bottomonium-like exotic states, such as the Z_b(10610) and Z_b(10650), although it is still debated whether these are hybrid mesons or tetraquark states [112]. The decay channels predicted by QCD sum rules for these heavy hybrids involve transitions to lower bottomonium states like Y(1S), Y(2S) or χ_{bJ} , accompanied by pion or

photon emission [96]. These decay patterns play an essential role in guiding experimental searches at high-energy facilities such as the LHC and Belle II.

3.10 Challenges and Future Prospects in Detection

One of the main challenges in detecting both light and heavy hybrid mesons is their relatively short lifetimes and the difficulty in distinguishing them from conventional mesons. Hybrid mesons are often produced as resonances in high-energy collisions, and their experimental signatures frequently overlap with those of ordinary mesons, making it difficult to isolate their unique properties [94-97,100]. Although heavy hybrid mesons require significantly higher energy levels for production—thus limiting the number of experimental facilities capable of studying them—ongoing advancements in experimental techniques and the development of higher-energy colliders continue to provide promising opportunities for the discovery and investigation of both light and heavy hybrid mesons [99,100].

QCD sum rule predictions for heavy hybrid mesons have guided experimental searches at facilities such as LHCb, BESIII, and Belle II. As experimental techniques improve, particularly with higher energy collisions, it is expected that more hybrid mesons will be discovered, potentially confirming the masses and properties predicted by QCD sum rules [145]. Predictions from QCD sum rules extend beyond masses and quantum numbers. They also offer valuable insights into the makeup of hybrid mesons and shed light on their intricate quark gluon dynamics internally. As new data emerges the QCD sum rule framework will play a role in enhancing these predictions by further delving into our comprehension of QCD, within the non-perturbative regime [94,97,145].

4. CHAPTER FOUR - QUANTUM CHROMODYNAMICS (QCD) THEORY

4.1 Introduction

Quantum chromodynamics (QCD), a quantum field theory that describes strong interactions the fundamental forces that hold quarks and gluons within protons, neutrons, and hadrons are generated within the Standard Model of particle physics and plays an important role in explaining structure and behavior of the atomic nucleus. When compared to other basic forces the intense interactions are unique, This is because it exhibits properties such as color suppression and asymptotic independence and this is a characteristic of non-Abelian gauge structures. QCD is formulated as a gauge theory based on the SU(3) color symmetry group, where quarks carry color charges and the gluons that interact with each other acquire a color charge through the energetic exchange of gluon medium lead to creates highly non-linear dynamics. The emergence of QCD as a perfect theory of strong interactions has greatly improved our understanding of the phenomenon deeper from the binding of nucleons to the high-energy collisions studied in particle accelerators. The theoretical framework and experimental confirmation of QCD continue to shape modern physics especially at high levels energy physics and atomic [118, 122].

4.2. Theoretical basis of QCD

4.2.1 Gauge Symmetry and SU(3) group

The theory of QCD is based upon the non-Abelian gauge symmetry belonging to the SU(3) group, in fact this symmetry governs how quarks and gluons interact with each other. Unlike quantum electrodynamics (QED) with its Abelian U(1) symmetry, where neutral photons do not interact QCDs Abelian nature implies that gluons exhibit self-interactions, within the theory. This makes the theory more complicated and more colour-blocking at low power This theory explains phenomena such as strong coupling of quarks [122-124]. SU(3) symmetry is a fundamental feature of QCD and underlies strong interaction behavior at all energy levels [136]. The relationship between quarks and gluons is controlled by the force and is influenced by the coupling constant α_s . This constant varies with the systems energy level; when distances are extremely short the interaction weakens due, to asymptotic freedom. As quarks move apart the force intensifies, resulting in quark confinement—explaining why isolated quarks are never seen in nature. The behavior of α_s as energy changes is explained by the

renormalization group equations, in QCD which elucidates the behavior of the force at various energy levels [126-128].

4.2.2 The QCD Lagrangian

The dynamics of quarks and gluons are described by the QCD Lagrangian, which respects the SU(3) gauge symmetry. The QCD Lagrangian is expressed as:

$$L_{QCD} = \sum_f \bar{q}_f (i\gamma^\mu D_\mu - m_f) q_f - \frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a. \quad (1)$$

Here, q_f represents the quark field of flavor f , D_μ is the covariant derivative that includes the gluon field, and $G_a^{\mu\nu}$ is the gluon field strength tensor, which accounts for both gluon propagation and their self-interactions. The term m_f stands for the quark mass. The terms in the Lagrangian describe the kinetic energy of quarks, their interactions with gluons, and the self-interactions of gluons [150].

The renormalizability of the QCD Lagrangian marked a significant theoretical advancement, ensuring the theory's internal consistency and predictive power when quantum corrections are taken into account—particularly in high-energy regimes where perturbative techniques are applicable [132].

4.3. Phenomena in QCD

4.3.1 Confinement of Quarks and Gluons

A defining feature of QCD is confinement, which dictates that quarks and gluons cannot exist as free particles under normal conditions. Confinement arises because the force between quarks does not diminish as the distance between them increases. Instead, the interaction potential continues to grow with distance. In many phenomenological and lattice QCD models, this potential is often approximated as a linearly increasing potential ($\propto r$) or even quadratically ($\propto r^2$), depending on the theoretical framework. This behavior prevents quarks from being isolated and leads to the formation of new quark-antiquark pairs when sufficient energy is introduced [118, 119].

This phenomenon is often described through the concept of flux tubes or strings, in which gluon fields form a tight bundle between quarks [149].

The absence of free quarks and gluons has been confirmed experimentally, as only color-neutral hadrons are observed in nature. Despite its empirical success, confinement has not been analytically proven from first principles, making it one of the most profound unsolved problems in theoretical physics. The

confinement mechanism remains a focus of research, particularly through the use of lattice QCD simulations, which numerically explore the behavior of quarks and gluons on a discrete spacetime grid Fig.(4-1)[[123].

4.3.2 Asymptotic Freedom

Another key property of QCD is asymptotic freedom, which describes the weakening of the strong force at very short distances or high energies. As the energy scale increases, the strong coupling constant α_s decreases, leading quarks to behave as nearly free particles in high-energy processes. This phenomenon was a groundbreaking discovery, as it provided a theoretical explanation for the behavior of quarks in deep inelastic scattering experiments [122].

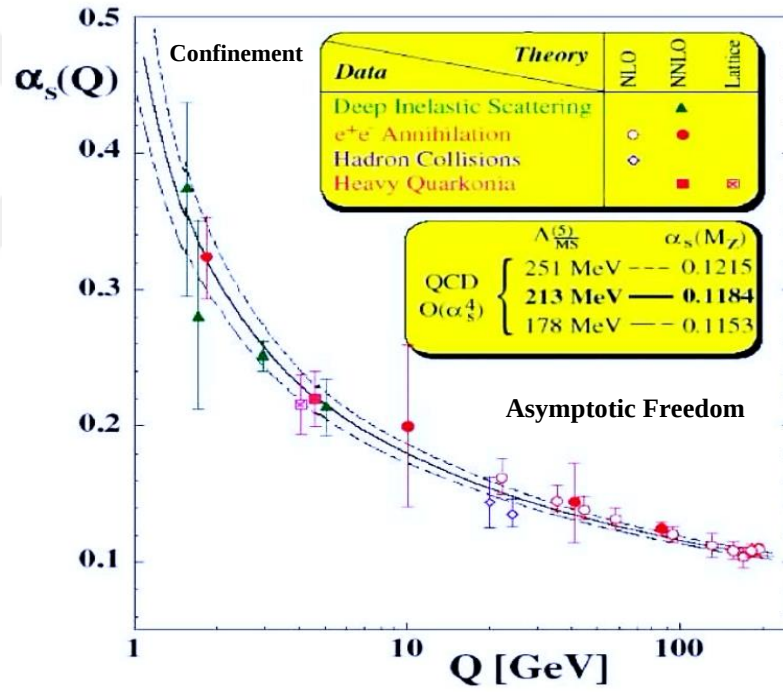


Figure 4-1: Logarithmic running of the QCD coupling constant $\alpha_s(Q)$: Illustrating confinement at low Q and asymptotic freedom at high Q .

Asymptotic freedom not only helped establish QCD as the correct theory of the strong interaction but also played a crucial role in understanding the behavior of quarks and gluons in high-energy particle collisions. At large momentum transfers, perturbative QCD can be employed to make accurate predictions about scattering cross-sections and particle production rates, as seen in experiments at particle colliders [118, 127].

4.3.3 Hadronization and Jet Formation (fragmentation)

In processes with high-energy like in particle accelerators quarks and gluons are made first as particles before they come together to form hadrons through a process called hadronization. Since free quarks and gluons can't exist alone due to confinement hadronization creates color particles that can be detected in experiments Fig.(4-2).

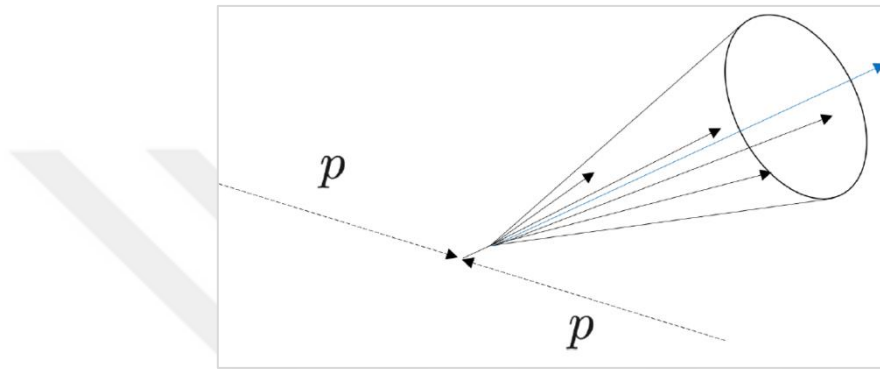


Figure 4-2: Illustrated formation of jets (fragmentation) and the hadronization

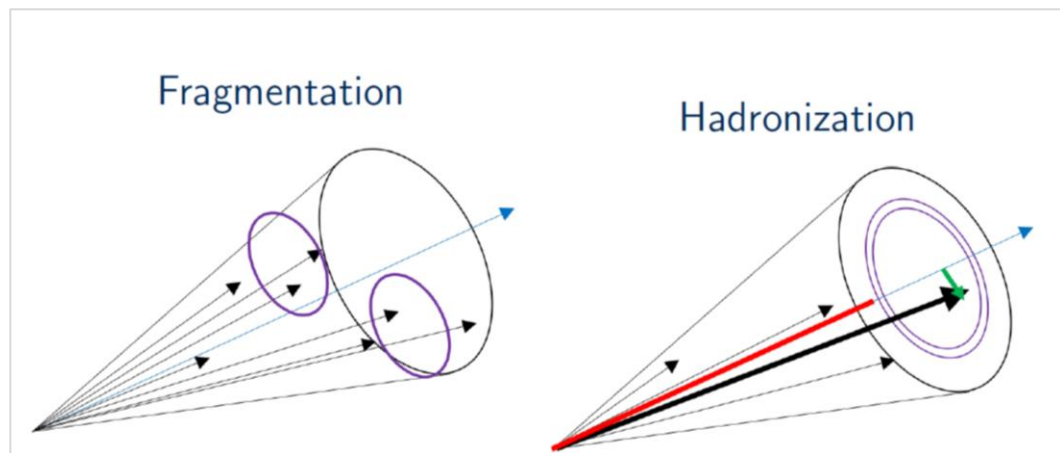


Figure 4-3 Illustrated formation of jets (fragmentation) and the hadronization.

Hadronization leads to the formation of jets, collimated streams of hadrons that result from the fragmentation of quarks and gluons. The jets provide crucial information about the underlying parton (quark or gluon) dynamics and are used to study properties like the strong coupling constant and the

internal structure of protons. Jet production is a key signature in high-energy collider experiments, such as those conducted at the LHC Fig.(4-3)[147, 132].

4.3.4 Quark-Gluon Plasma (QGP)

In situations with high heat and energy levels like those produced during heavy ion collisions in physics experiments, quarks and gluons have the ability to exist in a deconfined form referred to as the quark gluon plasma (QGP). During this stage quarks and gluons are not restricted within particles. Instead move freely within a dense and hot environment. The exploration of QGP is crucial for gaining insights into the universe, where it is theorized that matter existed in such a liberated state shortly after the commencement of the universe, in the Big Bang event.

Researchers have gathered evidence supporting the existence of QGP through high energy collisions at the Relativistic Heavy Ion Collider (RHIC) and the LHC. Studying characteristics such as thickness and movement rates of QGP in these experiments provides insights, into how strong interactions behave in situations. In the domains of theoretical and experimental nuclear physics, this continuous research on QGP still attracts attention.

At extremely high temperatures and energy densities, such as those recreated in heavy-ion collisions, quarks and gluons can exist in a deconfined state known as the QGP. Quarks and gluons in this phase flow freely in a hot, dense media, not limited inside hadrons. The study of QGP helps one to understand the early universe in which matter is supposed to have existed in such a condition quickly following the Big Bang. at collisions at the RHIC and the LHC, extremely high energy densities are attained, therefore offering experimental data for QGP. Under quite challenging conditions, the viscosity and transport coefficients of QGP provide information on the nature of the strong interaction. The study of QGP continues to be a major focus in both theoretical and experimental nuclear physics [91,128].

4.3.5 Exotic Hadrons, Hybrid Mesons, and Glueballs as an Examples

In addition to conventional hadrons, exotic hadrons, different from the traditional quark model, are predicted by QCD. While only standard mesons (quark–antiquark) and baryons (three quarks) were initially expected to exist, recent theoretical and experimental efforts have identified more complex configurations. Exotic hadrons include pentaquarks (made of four quarks and one antiquark) and tetraquarks (made of two quarks and two antiquarks), which have attracted notable experimental

attention lately. The discovery of such states in experiments like those at LHC has strengthened the case for the existence of exotic hadrons [113, 116].

Hybrid mesons are another category of exotic hadrons, the particles here consist not only of a quark and an antiquark but also an excited gluon field unlike conventional mesons. Of course, it is different from regular mesons, the excited gluon adds to the quantum numbers of the hybrid meson. Because they allow a direct window into the function of gluonic excitations in hadronic physics, hence testing the limits of QCD, hybrid mesons are especially fascinating. Experimental searches for hybrid mesons are ongoing at various facilities, including the GlueX experiment at Jefferson Lab, which aims to better understand the gluonic contributions to hadron structure [119,126].

Additionally, expected by QCD are glueballs, bound states composed solely of gluons. Because glueballs combine with ordinary mesons, their identification is tricky and requires careful detection. Still, continuous efforts in both theoretical (e.g., lattice QCD) and experimental searches probe the characteristics of these essentially gluonic particles. [125,129].

4.4 Applications and Implications of QCD

4.4.1 High-Energy Particle Physics

With high-energy physics, especially with regard to the outcomes of particle collisions at accelerators like the LHC, QCD is absolutely fundamental. The theory provides a structure for approximating the production rates of particles including jets, heavy quarks, and hadrons by via quark and gluon interactions inside protons and neutrons. The study of jet formation and deep inelastic scattering experiments has allowed physicists to extract valuable information about the strong coupling constant, α_s , and the internal structure of nucleons [118, 146].

Predictions for these processes have been much aided by perturbative QCD, a method applying QCD in high-energy environments where the strong coupling constant is modest. These forecasts are tested for accuracy of QCD and search for any novel physics beyond the Standard Model by means of experimental data comparison [143,144]. QCD calculations are vital in determining background processes in searches for new particles, such as the Higgs boson, and for exploring physics at the energy frontier [150].

4.4.2 Nuclear Physics

QCD thus offers the theoretical framework for nuclear physics since it clarifies the interactions binding protons and neutrons into atomic nuclei. Effective theories such as chiral perturbation theory assist to explain the strong force between nucleons, however these models are generated from the residual effects of QCD interactions between quarks and gluons. A deeper understanding of the nuclear force, particularly in extreme environments such as neutron stars, requires the application of QCD to describe the behavior of matter under such conditions [120,121].

4.5 Prediction Methods of QCD

Being a highly non-linear theory, QCD presents analytical difficulties notably at low energies when the strong coupling constant becomes significant. Several methods have been developed to address different aspects of the theory:

1. **Perturbative QCD:** This method applies at high energies (short distances), where the strong coupling constant becomes small. In this regime, QCD can be treated perturbatively, enabling the use of Feynman diagrams to calculate scattering amplitudes and particle production rates [122, 127,148]. The study of QCD has played a crucial role in our comprehension of high energy interactions seen at the Large Hadron Collider (LHC) [146].

2. **Lattice QCD:** In lattice QCD research methodology involves discretizing spacetime into lattice and conducting computations of QCD processes. This method is especially useful for investigating low energy occurrences, like hadron masses and quark confinement well as examining nucleon structure [132, 135]. Lattice QCD has proven to offer predictions regarding hadronic characteristics despite the substantial computational resources it demands [135].

3. **Effective Field Theories:** In low energy situations, like Chiral Perturbation Theory (ChPT) theories which belong to Effective field theories are utilized that build upon QCD symmetries to explain how pions and other light mesons interact as pseudo-Goldstone bosons formed from chiral symmetry breaking [130].

4. **QCD Sum Rules:** The method of QCD sum rule establishes a connection between QCD parameters and observable features of hadrons without causing any disturbances to the system which described as a non-perturbative method. Introduced in the 1970s by Shifman, Vainshtein and Zakharov and still

widely utilized today for investigating the spectrums of hadrons this approach effectively links the behavior of short distance QCD with data by employing operator product expansion (OPE) and associating quark gluon interactions, with measurable characteristics of hadrons [112, 138].

4.5.1 QCD Sum Rule Method

The QCD sum rule approach relies on the OPE. This method exhibits the correlation function between quark and gluon fields into non perturbative components [112]. Typically, the correlation deals with two-point currents and is broken down into segments that consider both the quick behavior of QCD and non- perturbative elements such, as quark and gluon condensates. [133, 134]. The condensates described here capture the essence of the vacuum structure in QCD. Take into account corrections that account for non-perturbative phenomena over long distances. This approach goes beyond methods outlined by [150]. By matching the sum rule with data using dispersion relations, this allows for extraction of properties like hadron masses and decay constants from calculations, in QCD. The QCD sum rule technique has proven effective in exploring types of hadronic particles such as mesons and baryons as well as unique structures like glueballs and hybrid [146,149]. This method remains essential for understanding the perturbative aspects of QCD by connecting fundamental quark gluon interactions to observable features, in experiments. The technique known as QCD sum rule has proven effective in exploring types of hadronic particles such as mesons and baryons as well as unique entities like glueballs and hybrid mesons [119,151,152]. This method remains essential for understanding the perturbative aspects of QCD by connecting fundamental quark gluon interactions with observable features, in experiments.

4.6 Correlation Function via QCD Sum Rule: Two-Point Function

Studying the properties of hadrons in quantum chromodynamics (QCD) primarily relies on correlation functions—an aspect of the QCD sum rule approach that is essential for understanding the internal structure of hadrons. The two-point correlation method stands out as the commonly employed technique for probing the makeup of hadrons.

4.6.1 Two-Point Correlation Function

The two-point correlation measure $\Pi(q^2)$ is described as the anticipated outcome of the time-based combination of two currents $J(x)$ that possess the quantum characteristics of the examined hadron. For a given hadron, the two-point correlation function is expressed as:

$$\Pi(q^2) = i \int d^4x e^{iq \cdot x} \langle 0 | T \{ J(x) J^\dagger(0) \} | 0 \rangle. \quad (2)$$

Here T the time order operator and $J(x)$ the quark current which interacts with the hadron described [60]. Eq. (2) can be formulated as follows

$$\Pi_{\mu\nu}(q) = i \int d^4x e^{iq \cdot x} \langle 0 | T \{ J_\mu(x) J_\nu^\dagger(0) \} | 0 \rangle. \quad (3)$$

We will calculate the correlation function using explicit forms of currents $J_\mu(x)$. As a result, we express $\Pi_{\mu\nu}(q)$ for each current in terms of the heavy quark propagator as well as the gluon propagator/condensate,

$$\Pi_{\mu\nu}(q) = g_s^2 i \int d^4x e^{iq \cdot x} \langle 0 | G_{\mu\theta}^n(x) G_{\nu\delta}^m(0) | 0 \rangle t^n[a, b] t^m[a', b'] Tr[S_{Q_1}^{a'a}(-x) \gamma_\theta \gamma_5 S_{Q_2}^{bb'}(x) \gamma_5 \gamma_\delta], \quad (4)$$

The current $J(x)$ which is usually associated with a bilinear like $\bar{q}\gamma_\mu q$ for vector mesons or $\bar{q}\gamma_5 q$ for pseudoscalar mesons. This section $\langle 0 | T \{ J_\mu(x) J_\nu^\dagger(0) \} | 0 \rangle$ signifies the QCD sum rules condensate for two points. The correlation function contains details about the properties of the hadron such, as its mass and decay constants [133].

By continue our steps, we explored the two-fold approach spectroscopic properties of the correlation function. First, we calculated the physical or phenomenological side, which involves hadronic degrees of freedom. Then, we computed the theoretical or QCD side, based on quantum chromodynamics (QCD) degrees of freedom. By Matching the two side together and apply Boral transformation for the high degree elimination, the new step lead us to get the spectroscopy properties like: spectroscopy density.

4.6.2 Operator Product Expansion (OPE) and QCD Sum Rules

The next step in the sum rule approach is the Operator Product Expansion (OPE) of the correlation function at large Euclidean momentum q^2 . In this expansion, the short-distance behavior is captured by perturbative QCD, while long-distance, non-perturbative effects are encoded in terms of vacuum condensates. The OPE expresses the two-point function as:

$$\Pi(q^2) = \sum_n C_n(q^2) \langle 0 | O_n | 0 \rangle, \quad (5)$$

where $C_n(q^2)$ are perturbatively calculable Wilson coefficients, and O_n are operators of increasing dimension that describe the non-perturbative vacuum structure [136].

The dominant non-perturbative terms include the quark condensate $\langle \bar{q}q \rangle$, the gluon condensate $\langle G^2 \rangle$, and mixed quark-gluon condensates. These condensates encapsulate the essential physics of the QCD vacuum [152].

4.6.3 Dispersion Relations and Physical Representation

To connect the QCD calculation with observable hadron properties, the dispersion relation is employed. The correlation function $\Pi(q^2)$ is analytically continued to the physical region, and the hadronic spectral density is related to the QCD calculation through a dispersion relation:

$$\Pi(q^2) = \int_{s_0}^{\infty} \frac{\text{Im } \Pi(s)}{s - q^2} ds, \quad (6)$$

where $\text{Im } \Pi(s)$ represents the hadronic spectral density, which includes contributions from resonances and the continuum [152]. By using quark-hadron duality, the high-energy part of the integral can be approximated by the perturbative QCD result, while the lower-energy region is dominated by the lowest-lying hadrons [133].

4.6.4 Mass Sum Rules and Hadronic Parameters

To determine quantities like the mass of hadrons and decay constants in quantum field theory – specifically in QCD sum rules – one can use the technique of matching the QCD sum rule derived from OPE with the dispersion relation method. The sum rule, for hadron mass is derived from the transformed correlation function using Borel-transformation $\mathcal{B}[\Pi(q^2)]$, which helps enhance convergence and minimize the impact of higher energy states or undesired contributions when studying hadron properties through two-point correlation functions.

An average QCD sum rule transformed by Borel is usually expressed as follows;

$$\int_{s_0}^{\infty} \rho(s) e^{-s/M^2} ds = \Pi(M^2). \quad (7)$$

In this context, s_0 the threshold value differentiates the hadronic resonance from the higher energy continuum states. The function $\rho(s)$ also known as the function or spectral density is indicative of the physical states that play into the correlation function. The M^2 stands for the Borel mass is used as an additional parameter in the transformation process to reduce higher order corrections. The part with $\Pi(M^2)$ represents the QCD aspect of the sum rule in its Borel transformed form which encompasses quark and gluon condensates along, with perturbative QCD components.

Equation (7) in essence balances the effects from the hadronic range on the left side (which includes the primary resonance and continuous spectrum) with the QCD driven OPE situated on the right side. Through this equilibrium one can deduce characteristics of hadrons like mass or decay constants of mesons or baryons as proposed by [64].

The spectral density shows the physical states that affect the correlation function and can be found by studying the aspect of the correlation function itself. By using the OPE and comparing to the representation method we simplify the equation to focus on its imaginary component. To calculate the spectral density, we divide this imaginary part by π according to the related formula

$$\rho(s) = \frac{Im [\Pi_{\mu\nu}(q)]}{\pi} . \quad (8)$$

The method has been used quite a bit to compute masses of Hadrons and their decay constants along with traits successfully in various instances. For example; researchers have used the sum rule method to analyze particles such as the rho meson and pseudoscalar mesons which has produced outcomes in line, with what's observed in experiments [137,152].

The two-point QCD sum rule proves to be a technique for studying the characteristics of hadrons linking the observable spectrum of hadrons with the fundamental theory of quarks and gluons through a combination of perturbative and non-perturbative QCD methods. It has demonstrated its effectiveness, in both theoretical and experimental exploration of QCD as evidenced by its ability to accurately determine hadron masses and decay constants.

5. CHAPTER FIVE - THEORETICAL ANALYSIS AND COMPUTATIONAL METHODOLOGIES VIA QCD SUM RULES IN THE HEAVY HYBRID QUARKONIA

5-1 QCD Sum Rules Approach

In this chapter, we try to submit the correlation function in new framework to find the spectroscopy properties of the chosen hybrids. The essential part in QCD sum rules which represented by the two-points correlation function, as we mention before the framework correlation function ($\Pi_{\mu\nu}(q)$) will be introduced by the interpolating currents, in other word, the vacuum condensates of the initial and final quarks which refer to the complete sets will be the axis of the equation

$$\Pi_{\mu\nu}(q^2) = i \int d^4x e^{iq \cdot x} \langle 0 | T \{ J_\mu(x) J_\nu^\dagger(0) \} | 0 \rangle \quad (9)$$

In this context the J_μ the interpolating current for the chosen meson (a heavy quarkonium hybrid) will be

$$J_\mu(x) = \frac{g_s}{2} \bar{Q}_1^a(x) \gamma_\theta \gamma_5 \frac{\lambda_{ab}^n}{2} \tilde{G}_{\mu\theta}^n(x) Q_2^b(x), \quad (10)$$

the strong coupling constant here refer it by g_s , a, b are color indices taken value in $\{1,2,3\}$, λ^n is the Gell-Mann matrices with $n=1,2,\dots,8$; $\tilde{G}_{\mu\theta}^n(x) = \epsilon_{\mu\theta\alpha\beta} \frac{G_{\alpha\beta}^n(x)}{2}$ is the dual field strength of and the light and heavy quarks are represented by $q = u, d, s$ and $Q = c, b$.

This part $\langle 0 | T \{ J_\mu(x) J_\nu^\dagger(0) \} | 0 \rangle$ represent to perturbative part of the QCD sum rules for two points. In our research projects methodology to determine the characteristics of the correlation function mentioned above involved two distinct approaches; one based on hadronic elements known as the phenomenological or physical side and the other utilizing QCD components referred to as the theoretical or QCD side – details of which will be elaborated in subsequent sections.

5-2 Phenomenological or physical part of the correlation function

The main concept of the phenomenological approach is a model that describes the behavior of hadronic states, such as mesons and baryons, in terms of their contribution to the correlation function. This approach leverages the fact that the correlation function can also be expressed in term of observable

hadron degrees of freedom. By doing so, the phenomenological side provides a bridge between the fundamental quark- gluon description of QCD and experimentally properties of hadrons. The physic side point out to analytic part in terms of q^2 , the amplitude $\Pi_{\mu\nu}(q^2)$ is invariant function. The q^2 could define both the positive and negative value of it. In the strong interaction, as q^2 evolves from large negative values toward the positive region, the correlation function begins to receive contributions from quark–gluon dynamics over long distances. Following this stage, hadrons start to form as a result of the confinement process.

Fundamentally, the mathematical procedure involves saturating the correlation function in Equation (9) with a complete set of hadronic states (specifically, the hybrid meson), followed by performing the space-time integration over x . As a result, Equation (9) takes the following form:

$$\Pi_{\mu\nu}(q^2) = i \int d^4x \ e^{iq \cdot x} \sum_n \langle 0 | J_\mu(x) | n \rangle \langle n | J_\nu^\dagger(0) | 0 \rangle. \quad (11)$$

Using the translation invariance made the matrix element be more simplified

$$\langle 0 | J_\mu(x) | n \rangle = \langle 0 | J_\mu(0) | n \rangle e^{-ip_n \cdot x}, \quad (12)$$

where p_n four- momentum of the state $|n\rangle$.

The correlation function will be

$$\Pi_{\mu\nu}(q^2) = \sum_n \langle 0 | J_\mu(0) | n \rangle \langle n | J_\nu^\dagger(0) | 0 \rangle \int d^4x \ e^{i(q - p_n) \cdot x}. \quad (13)$$

Eq. (13) need to the Fourier Transformation over the area d^4x which give Kronecker Delta function

$$\int d^4x \ e^{i(q - p_n) \cdot x} = (2\pi) \delta^4(q - p_n). \quad (14)$$

The last action allows to the momentum conservation $q = p_n$. So, the correlation function will be simplified to

$$\Pi_{\mu\nu}^{phys}(q) = \sum_n \frac{\langle 0 | J_\mu | n \rangle \langle n | J_\nu^\dagger | 0 \rangle}{m_n^2 - q^2 + i\epsilon}. \quad (15)$$

Here $m_n^2 = p_n$ which is the mass of the hadronic state $|n\rangle$.

This sum is often simplified by isolating the lowest lying hadronic state (e.g. the ground state) and approximating the contributions of higher states by a continuum starting from a threshold s_0

$$\Pi^{Phys}(q^2) = \frac{|\langle 0|J|H\rangle|^2}{m_H^2 - q^2} + \int_{s_0}^{\infty} ds \frac{\rho_{cont}(s)}{s - q^2}, \quad (16)$$

The square absolute value of set $|\langle 0|J|H\rangle|^2$ refer to the decay constant squared for mesons (or residue for baryons) of hadron with mass m_H , also $\rho_{cont}(s)$ is the spectral density function representing the continuum contribution over the threshold s_0 .

Getting of eq (16) with a higher state need to suppress, so to improve the sum rules, we will apply a Boral transformation with respect q^2

$$\mathcal{B}[\Pi^{Phys}(q^2)] = \frac{|\langle 0|J|H\rangle|^2}{M^2} e^{-\frac{m_H^2}{M^2}} + \int_{s_0}^{\infty} ds \rho_{cont}(s) e^{-\frac{s}{M^2}}, \quad (17)$$

M^2 refer to Borel parameter.

5-3 The Operator Product Expansion (OPE) to Calculate the Theoretical QCD Sum Rule Side

The important step that we should take after we got the physics part, it is focus to calculate theoretical side, by using (OPE), which is the framework that separates the perturbative (short-distance) effect from the non-perturbative (long-distance) effect. The OPE action represented by multiplying two currents, which probe to hadronic structure in terms of series of increased with dimensional operators. Each term in this series is associated with coefficient that encapsulates the perturbative effects, while the operates themselves encapsulates the non-perturbative aspects of QCD sum rules, such quark condensates and gluon field strengths.

Expanding the right-hand side of Eq. (9) leads us to the theoretical representation of the correlation function. This side is developed using the Operator Product Expansion (OPE), which enables the systematic inclusion of both perturbative and non-perturbative QCD contributions.

In order to determine the QCD side of the sum rules accurately within the OPE it is necessary to compute $\Pi_{\mu\nu}^{QCD}(q)$. To achieve this calculation of $\Pi_{\mu\nu}^{QCD}(q)$, we evaluate the correlation function by employing formulations of currents denoted as $J_\mu(x)$

$$J_\mu(x) = \frac{g_s}{2} \bar{Q}_1^a(x) \gamma_\theta \gamma_5 \frac{\lambda_{ab}^n}{2} \tilde{G}_{\mu\theta}^n(x) Q_2^b(x), \quad (18)$$

Represent $\Pi_{\mu\nu}^{QCD}(q)$, for each current by using the quark propagator and the gluon propagator or condensate, so

$$\Pi_{\mu\nu}^{QCD}(q) = g_s^2 i \int d^4x e^{iqx} \langle 0 | G_{\mu\theta}^n(x) G_{\nu\delta}^m(0) | 0 \rangle t^n[a, b] t^m[a', b'] \text{Tr}[S_{Q_1}^{a'a}(-x) \gamma_\theta \gamma_5 S_{Q_2}^{bb'}(x) \gamma_5 \gamma_\delta], \quad (19)$$

Where $t^n = \lambda^n/2$. In the above formula, $S_Q^{ab}(x)$ is propagator of c, b -quarks.

As standard framework, the heavy quark Q propagator $S_Q^{ab}(x)$ formulate as

$$S_Q^{ab}(x) = i \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \left\{ \frac{\delta_{ab}(k+m_Q)}{k^2 - m_Q^2} - \frac{g_s G_{ab}^{\alpha\beta}}{4} \frac{\sigma_{\alpha\beta}(k+m_Q) + (k+m_Q)\sigma_{\alpha\beta}}{(k^2 - m_Q^2)^2} + \frac{\langle g_s^2 G^2 \rangle}{12} \delta_{ab} m_Q \frac{k^2 + m_Q k}{(k^2 - m_Q^2)^4} + \frac{\langle g_s^3 G^3 \rangle}{48} \delta_{ab} \frac{(k+m_Q)}{(k^2 - m_Q^2)^6} [k(k^2 - 3m_Q^2) + 2m_Q(2k^2 - m_Q^2)](k + m_Q) + \dots \right\}, \quad (20)$$

Here, we use the short-hand notations

$$G_{ab}^{\alpha\beta} = G_A^{\alpha\beta} \lambda_{\alpha\beta}^A, \quad G^2 = G_A^{\alpha\beta} G_A^{\alpha\beta}, \quad G^3 = f^{ABC} G_A^{\mu\nu} G_B^{\nu\delta} G_C^{\delta\mu},$$

Here the f^{ABC} refer to the structure constants of the color group $SU(3)_c$.

This part $\langle 0 | G_{\mu\theta}^n(x) G_{\nu\delta}^m(x) | 0 \rangle$ in Eq. (19) will be as two different forms. We start by moving it using the gluon propagator in coordinate space

$$\begin{aligned} & \langle 0 | G_{\alpha\beta}^n(x) G_{\alpha'\beta'}^m(0) | 0 \rangle \\ &= \frac{\delta^{mn}}{2\pi^2 x^2} \left[g_{\beta\beta'} \left(g_{\alpha\alpha'} - \frac{4x_\alpha x_{\alpha'}}{x^2} \right) + (\beta, \beta') \leftrightarrow (\alpha, \alpha') - \beta \leftrightarrow \alpha - \beta' \leftrightarrow \alpha' \right]. \end{aligned} \quad (21)$$

Perform all the calculations needed for this task akin to diagrams where the valence gluon acts as a propagator in physics equations. Moving forward to the stage involves examining $\langle 0 | G_{\mu\theta}^n(x) G_{\nu\delta}^m(x) | 0 \rangle$, which represents the gluon condensate and focusing on the initial term of the Taylor series when x is equal to zero, that mean

$$\langle 0 | G_{\alpha\beta}^n(x) G_{\alpha'\beta'}^m(0) | 0 \rangle = \frac{\langle G^2 \rangle}{96} \delta^{mn} [g_{\alpha\alpha'} g_{\beta\beta'} - g_{\alpha\beta'} g_{\alpha'\beta}]. \quad (22)$$

Which is an abbreviation for the diagrams that depict the gluon interacting with the QCD vacuum. In the calculation, we need to use

$$t^n[a, b] t^n[a', b'] = \frac{1}{2} \left(\delta^{ab'} \delta^{a'b} - \frac{1}{3} \delta^{ab} \delta^{a'b'} \right) \quad (23)$$

To establish sum rules calculation for these structures in question we set equal the unchanging amplitudes $\Pi^{Phys}(q^2)$ and $\Pi^{QCD}(q^2)$ associated with them and employ the Boral transformation on both sides of the derived sum rules. This final step becomes crucial in minimizing the influence coming from the complex resonances and continuous states. Moving forward with our manipulations we rely on an assumption regarding the interaction for the quark-hadron duality while also removing contributions, from resonances and continuous terms on the physical side of the equation. Finally, the QCD sum rules for mass and current coupling is obtained as:

$$m_H^2 = \frac{\frac{d}{d\left(-\frac{1}{M^2}\right)} \Pi[M^2, s_0]}{\Pi[M^2, s_0]}, \quad (24)$$

$$f_H^2 = \frac{e^{m_H^2/M^2}}{m_H^2} \Pi[M^2, s_0], \quad (25)$$

The calculation of $\Pi[M^2, s_0]$ will incorporate contributions from quark, gluon, and mixed vacuum condensates, including terms up to dimension 12, as illustrated below

$$\Pi[M^2, s_0] = \int_{4m_Q^2}^{s_0} ds \rho^{QCD}(s) e^{-\frac{s}{M^2}} + \Pi(M^2). \quad (26)$$

Here the $\rho^{QCD}(s)$ is represent to the two-point spectral density, the analytical form is given in Appendix. The analytical expression of $\Pi(M^2)$ was not written in the thesis because it was too long.

5 – 4 Determination of Mass and Current Coupling for Heavy Hybrid Quarkonia (Charmonium and Bottomonium)

For scalar and vector hybrid states possessing negative charge conjugation parity C-parity $J^{PC} = 0^{+-}$ and 1^{--} , the corresponding interpolating current is given by the following expression:

$$J_{\mu}^1(x) = g_s \bar{Q}_a(x) \gamma^{\alpha} \gamma_5 \frac{\lambda_{ab}^n}{2} \tilde{G}_{\mu\alpha}^n(x) Q_b(x), \quad (27)$$

In contrast, for states with positive charge conjugation parity C-parity states with $J^{PC} = 0^{++}$ and 1^{-+} , the interpolating current takes the following form

$$J_{\mu}^2(x) = g_s \bar{Q}_a(x) \gamma^{\alpha} \frac{\lambda_{ab}^n}{2} G_{\mu\alpha}^n(x) Q_b(x), \quad (28)$$

In Eq. (27) and (28), the heavy quark field $Q(x)$ represents either the charm c or bottom (b) quarks. The symbol g_s denotes the QCD strong coupling constant, while a and b correspond to color indices. The matrices λ^n , $n = 1, 2, \dots, 8$ refer to the Gell-Mann matrices. The gluon field strength tensor and its dual are represented by $G_{\mu\nu}^n(x)$ and $\tilde{G}_{\mu\nu}^n(x) = \epsilon_{\mu\nu\alpha\beta} G^{\alpha\beta n}(x)$, respectively

The interpolating currents corresponding to the pseudoscalar and axial-vector hybrid states are defined by the following expressions:

$$J_{\mu}^3(x) = g_s \bar{Q}_a(x) \gamma^{\alpha} \gamma_5 \frac{\lambda_{ab}^n}{2} G_{\mu\alpha}^n(x) Q_b(x), \quad (29)$$

for the states $J^{PC} = 0^{--}, 1^{+-}$ quantum numbers, and

$$J_{\mu}^4(x) = g_s \bar{Q}_a(x) \gamma^{\alpha} \frac{\lambda_{ab}^n}{2} \tilde{G}_{\mu\alpha}^n(x) Q_b(x), \quad (30)$$

for the $0^{-+}, 1^{++}$ configurations.

Let us examine the current $J_\mu^1(x)$ and the charmonium hybrids H_S and H_V with quantum numbers $J^{PC} = 0^{+-}$ and 1^{--} , as a representative example.

Extending this analysis to other currents is straightforward. Following the standard approach of sum rule analysis, we begin by expressing the correlation function $\Pi_{\mu\nu}(p)$ in terms of the physical parameters of the particles, considering $q = p$.

$$\Pi_{\mu\nu}^{Phys}(p) = \frac{\langle 0|J_\mu|H_S(p)\rangle\langle H_S(p)|J_\nu^\dagger|0\rangle}{m_S^2 - p^2} + \frac{\langle 0|J_\mu|H_V(p, \varepsilon)\rangle\langle H_V(p, \varepsilon)|J_\nu^\dagger|0\rangle}{m_V^2 - p^2} + \dots \quad (31)$$

In this expression, m_S and m_V represent the masses of the respective hybrid states. The contributions from the ground-state hybrids H_S and H_V are explicitly shown, while the effects from higher resonances and continuum states are indicated by ellipses. For the sake of simplicity in the notation, we also replace $J_\mu^1 \rightarrow J_\mu$.

Eq. (31) is obtained by inserting a complete set of physical states, matching the quantum numbers of the hybrids, into Eq. (9) and performing the integration over x . The resulting expression $\Pi_{\mu\nu}^{Phys}(p)$ can be further simplified by rewriting the matrix elements in terms of the masses and current couplings of the scalar hybrid (H_S) and vector hybrid (H_V) states.

$$\langle 0|J_\mu|H_S(p)\rangle = f_S p_\mu, \quad \langle 0|J_\mu|H_V(p, \varepsilon)\rangle = m_V f_V \varepsilon_\mu. \quad (32)$$

Here, f_S and f_V represent the current couplings of the hybrid states, while ε_μ denotes the polarization vector of the hybrid vector meson (H_V). By substituting these matrix elements into Eq. (31), one can straightforwardly obtain

$$\Pi_{\mu\nu}^{Phys}(p) = \frac{f_S^2}{m_S^2 - p^2} p_\mu p_\nu + \frac{m_V^2 f_V^2}{m_V^2 - p^2} \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \right) + \dots \quad (33)$$

As observed, the correlator $\Pi_{\mu\nu}^{Phys}(p)$ includes two Lorentz structures, $g_{\mu\nu}$ and $p_\mu p_\nu$, which can be utilized to derive the sum rules of interest. The component proportional to $g_{\mu\nu}$ receives contributions

exclusively from the vector particle. Consequently, the corresponding amplitude $\Pi_{\mu\nu}^{Phys}(p^2)$ can be reliably employed to obtain the sum rules for m_V and f_V . To separate the scalar state's contribution, it is useful to multiply $\Pi_{\mu\nu}^{Phys}(p)$ by $p^\mu p^\nu / p^2$ resulting in

$$\frac{p^\mu p^\nu}{p^2} \Pi^{Phys}(p) = \tilde{\Pi}^{Phys}(p) = -f_S^2 + \frac{m_S^2 f_S^2}{m_S^2 - p^2} + \dots \quad (34)$$

The correlation function after this operation contains only a trivial Lorentz structure proportional to I . We denote corresponding invariant amplitude by $\Pi_S^{Phys}(p^2)$ and use it to obtain the SRs for parameters m_S and f_S .

The QCD side of the sum rule is determined by Eq. (9) calculated by employing the explicit expressions of the interpolating currents and replacing contractions of heavy quark fields with relevant propagators. The correlator obtained by this manner should be computed in the OPE with some accuracy. After these operations, the correlation function $\Pi_{\mu\nu}(p)$ acquires the form

$$\Pi_{\mu\nu}^{OPE}(p) = \frac{i\varepsilon_{\mu\theta\alpha\beta} \varepsilon_{\nu\delta\alpha'\beta'}}{4} \int d^4x e^{ipx} \frac{\lambda_{ab}^n \lambda_{a'b'}^m}{4} Tr[S_Q^{a'a}(-x) \gamma^\theta \gamma_5 S_{Q_2}^{bb'}(x) \gamma^\delta \gamma_5] \times \langle 0 | g_s^2 G^{n\alpha\beta}(x) G^{m\alpha'\beta'}(0) | 0 \rangle, \quad (35)$$

where $S_Q^{ab}(x)$ is the propagator of the $Q = c(b)$ quark. In present work, we use the following expression for the propagators $S_Q^{ab}(x)$

$$S_Q^{ab}(x) = i \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \left\{ \frac{\delta_{ab}(\mathbf{K} + m_Q)}{k^2 - m_Q^2} - \frac{g_s G_{ab}^{\alpha\beta}}{4} \frac{\sigma_{\alpha\beta}(\mathbf{K} + m_Q) + (\mathbf{K} + m_Q)\sigma_{\alpha\beta}}{(k^2 - m_Q^2)^2} + \frac{\langle g_s^2 G^2 \rangle}{12} \delta_{ab} m_Q \frac{k^2 + m_Q \mathbf{K}}{(k^2 - m_Q^2)^4} \right. \\ \left. + \frac{\langle g_s^3 G^3 \rangle}{48} \delta_{ab} \frac{(\mathbf{K} + m_Q)}{(k^2 - m_Q^2)^6} [\mathbf{K}(k^2 - 3m_Q^2) + 2m_Q(2k^2 - m_Q^2)](\mathbf{K} + m_Q) + \dots \right\}. \quad (36)$$

Here, we have introduced the shorthand notations

$$G_{ab}^{\alpha\beta} = \frac{G_A^{\alpha\beta} \lambda_{ab}^A}{2}, \quad G^2 = G_{\alpha\beta}^A G_A^{\alpha\beta}, \quad G^3 = f^{ABC} G_A^{\mu\nu} G_B^{\nu\delta} G_C^{\delta\mu}, \quad (37)$$

where f^{ABC} are structure constants of the color group $SU(3)_c$.

The $\Pi_{\mu\nu}^{\text{OPE}}(p)$ depends on two important factors: One of them is the trace term with the heavy quark propagators. The propagator $S_Q^{ab}(x)$ contains the perturbative component and nonperturbative terms proportional to $g_s^2 G^2$ and $g_s^3 G^3$ that after sandwiched between vacuum states give rise to well-known gluon condensates. But there also is a term $\sim g^2 G_{ab}^{\alpha\beta}$ in the propagator which multiplied with a similar component of the second propagator generates additional two-gluon condensate: All such terms are taken into account.

Another question to be clarified here is connected with the matrix element $\langle 0 | g_s^2 G^{n\alpha\beta}(x) G^{m\alpha'\beta'}(0) | 0 \rangle$. Our treatment of this matrix element is twofold. First, we replace it by the vacuum condensate $\langle g_s^2 G^2 \rangle$ keeping the first term in the Taylor expansion at $x = 0$

$$\langle 0 | g_s^2 G^{n\alpha\beta}(x) G^{m\alpha'\beta'}(0) | 0 \rangle = \frac{\langle g_s^2 G^2 \rangle}{96} \delta^{mn} [g^{\alpha\alpha'} g^{\beta\beta'} - g^{\alpha\beta'} g^{\alpha'\beta}]. \quad (38)$$

Terms generated by this way correspond to diagrams in which the gluon interacts with the QCD vacuum. Alternatively, another important aspect to clarify here involves the matrix element $\langle 0 | G^{n\alpha\beta}(x) G^{m\alpha'\beta'}(0) | 0 \rangle$. We use the full gluon propagator in the x-space

$$\langle 0 | G^{n\alpha\beta}(x) G^{m\alpha'\beta'}(0) | 0 \rangle = \frac{\delta^{nm}}{2\pi^2 x^4} \left[g^{\beta\beta'} \left(g^{\alpha\alpha'} - \frac{4x_\alpha x_{\alpha'}}{x^2} \right) + (\beta, \beta') \leftrightarrow (\alpha, \alpha') - \beta \leftrightarrow \alpha - \beta' \leftrightarrow \alpha' \right]. \quad (39)$$

The contributions derived in this approach represent diagrams involving a full valence gluon propagator.

The correlator $\Pi_{\mu\nu}^{\text{OPE}}(p)$ can be expressed as a sum of terms proportional to $\sim g_{\mu\nu}$ and $\sim p_\mu p_\nu$. The amplitude $\Pi_V^{\text{OPE}}(p^2)$ associated with the structure $g_{\mu\nu}$ is used to extract the parameters m_V and f_V . Similarly, the amplitude $\Pi_S^{\text{OPE}}(p^2)$ obtained from the correlator $\tilde{\Pi}^{\text{OPE}}(p) = p^\mu p^\nu \Pi_{\mu\nu}^{\text{OPE}}(p) / p^2$ is convenient for establishing sum rules to m_S and f_S .

By equating the relevant amplitudes $\Pi_S^{\text{OPE}}(p^2)$ and applying both the Borel transformation and continuum subtraction procedures, we derive the following sum rules:

$$m_S^2 = \frac{\Pi'_S[M^2, s_0]}{\Pi_S[M^2, s_0]}, \quad (40)$$

and

$$f_S^2 = \frac{e^{m_S^2/M^2}}{m_S^2} \Pi_S[M^2, s_0]. \quad (41)$$

Here the $\Pi_S(M^2, s_0)$ denotes the amplitude obtained following the application of the Borel transformation and the subtraction of the continuum contribution. The parameters M^2 and s_0 correspond to the Borel mass and the continuum threshold, respectively. For simplicity, we introduce the notation $\Pi'_S(M^2, s_0) = d/d(-1/M^2) \Pi_S(M^2, s_0)$ in Eq. (40). The amplitude $\Pi_S(M^2, s_0)$ is expressed using the equation below:

$$\Pi_S[M^2, s_0] = \int_{4m_Q^2}^{s_0} ds \rho_S^{OPE}(s) + \Pi_S(M^2), \quad (42)$$

where $\rho_S^{OPE}(s)$ is the two-point spectral density. The quantity $\Pi_S(M^2)$ represents the nonperturbative contributions, which are evaluated directly from the corresponding correlation function $\Pi_S^{OPE}(p^2)$. Due to the complexity and length of the explicit expressions for both the spectral density and $\Pi_S(M^2)$, we refrain from presenting them here.

In this context, $\Pi_S(M^2, s_0)$ is evaluated by including contributions up to dimension 12. The propagator defined in Eq. (36) incorporates gluon condensates of various dimensions. Terms of dimension 4 and 6, which are proportional to the $\langle g_s^2 G^2 \rangle$ and $\langle g_s^3 G^3 \rangle$, arise in the final expressions due to corresponding components present in the heavy quark propagator.

Contributions of dimensions 8, 10, and 12 are computed under the factorization hypothesis for higher-dimensional condensates. Although this approximation is known to break down at higher dimensions [64], we proceed by neglecting the associated uncertainties, as these contributions are numerically suppressed and therefore have a minimal effect on the overall results. The sum rules for m_S and f_S involve, as input parameters, the gluon condensates and the mass of the charm or bottom quark. Below, we present the numerical values adopted for these quantities

$$\begin{aligned} \langle \alpha_s G^2 / \pi \rangle &= (0.012 \pm 0.004) \text{ GeV}^4, & \langle g_s^3 G^3 \rangle &= (0.57 \pm 0.29) \text{ GeV}^4, \\ m_c(\mu = m_c) &= (1.27 \pm 0.02) \text{ GeV}, & m_b(\mu = m_b) &= 4.18_{-0.02}^{+0.03} \text{ GeV}. \end{aligned} \quad (43)$$

Above we also present b quark's mass necessary for analysis of the hybrids $\bar{b}gb$ and $\bar{b}gc$. The m_c and m_b correspond to the running masses in the $\overline{MS}$ scheme at the scales $\mu = m_c$ and $\mu = m_b$ [61], respectively. The condensates $\langle \alpha_s G^2 / \pi \rangle$ and $\langle g_s^3 G^3 \rangle$ were extracted from analysis of hadronic processes [60, 61, 67].

Eqs. (40) and (41) are functions of the parameters M^2 and s_0 , which must be selected in accordance with the constraints imposed by the SR analysis. Specifically, these parameters should guarantee the dominance of the pole contribution (PC) in the physical observables extracted from the calculations. Furthermore, ensuring the convergence of the OPE and the stability of the results with respect to variations in M^2 are also critical criteria. To manage these aspects effectively within the SR framework, we adopt the following strategy:

$$PC = \frac{\Pi[M^2, s_0]}{\Pi[M^2, \infty]}, \quad (44)$$

and

$$R(M^2) = \frac{\Pi^{DimN}[M^2, s_0]}{\Pi[M^2, s_0]}, \quad (45)$$

where $\Pi^{DimN}[M^2, s_0] = \sum_{N=8,10,12} \Pi^{DimN}$ is a sum of last three terms in OPE which are proportional to $\langle g_s^2 G^2 \rangle^2$, $\langle g_s^2 G^2 \rangle \langle g_s^3 G^3 \rangle$ and $\langle g_s^3 G^3 \rangle^2$, respectively.

We now focus on the charmonium hybrids H_S and H_V . Fig. (5-1) displays the mass of hybrid H_S as a function of the Borel parameter M^2 in the range $3 - 10 \text{ GeV}^2$ at fixed s_0 . Our numerical calculations demonstrate that for H_S , the regions

$$M^2 \in [3.8, 4.8] \text{ GeV}^2, \quad s_0 \in [23, 25] \text{ GeV}^2, \quad (46)$$

are consistent with sum rule (SR) calculation limits. Specifically, at $M^2 = 4.8 \text{ GeV}^2$ and $M^2 = 3.8 \text{ GeV}^2$, on the average in s_0 the pole contribution is $PC_s \approx 0.5$ and $PC_s \approx 0.71$, respectively. Figure (5-1) shows the dependence of PCs on the Borel parameter M^2 . At $M^2 = 3.8 \text{ GeV}^2$, the nonperturbative contribution is positive and accounts for less than 1% of $\Pi_s(M^2, s_0)$. It should be noted that the narrow range for the Borel parameter results from the stringent requirement $PC \geq 0.5$ imposed on the pole contribution. The dependence of PC_s on M^2 is also illustrated in Fig. (5-1). The mass m_S and current coupling f_S are determined as the average values of these parameters across the regions specified in Eq. (46). Their values are:

$$m_S = (4.06 \pm 0.12 \pm 0.05 \pm 0.01) \text{ GeV}, \quad f_S = (2.80 \pm 0.30 \pm 0.18 \pm 0.09) \times 10^{-2} \text{ GeV}^3, \quad (47)$$

respectively. The results in Eq. (47) correspond to the sum rule (SR) predictions at $M^2 = 4.3 \text{ GeV}^2$ and $s_0 = 24 \text{ GeV}^2$. At these values, the pole contribution is $\text{PC}_S \approx 0.60$, ensuring its dominance in the results and confirming the ground-state nature of the hybrid state (H_S) in this class of hybrid quarkonia.

The errors in the SR computations of Eq. (47) arise from the choices of M^2 and s_0 , uncertainties in the gluon condensate $\langle \alpha_s G^2/\pi \rangle$, and the charm quark mass m_c . The primary sources of theoretical errors in this analysis are M^2 , s_0 and the gluon condensate $\langle \alpha_s G^2/\pi \rangle$. Uncertainties in the condensate $\langle g_s^3 G^3 \rangle$ result in corrections, such as to $\pm 4 \times 10^{-6} \text{ GeV}$ and $< |10^{-6}| \text{ GeV}$ for m_s and $\tilde{m}_s$, which are negligible and can be safely ignored. Throughout this work, the central value of $\langle g_s^3 G^3 \rangle$ is used in calculations.

Additional potential errors stem from the scale dependence of the gluon condensates and the charm quark mass. However, $\langle \alpha_s G^2/\pi \rangle$ is independent of the μ -scale, and the effects of $m_c(\mu)$ and $\langle g_s^3 G^3 \rangle$ rescaling are minor and can also be neglected. The mass m_s is depicted in Fig. (5-2) as a function of the Borel and continuum subtraction parameters.

The mass and current coupling of the vector hybrid $J^{\text{PC}} = 1^{--}$ can be derived from the sum rules in Eqs. (40) and (41) by substituting substitutions $\Pi_S(M^2, s_0) \rightarrow \Pi_V(M^2, s_0)$ and $(m_s, f_s) \rightarrow (m_V, f_V)$. Their values are as follows:

$$m_V = (4.12 \pm 0.11 \pm 0.06 \pm 0.004) \text{ GeV}, f_V = (4.0 \pm 0.4 \pm 0.2 \pm 0.02) \times 10^{-2} \text{ GeV}^3, \quad (48)$$

and have been extracted using the parameters

$$M^2 \in [4, 4.6] \text{ GeV}^2, \quad s_0 \in [24, 26] \text{ GeV}^2. \quad (49)$$

The quantities m_V and f_V were evaluated at the fixed values $M^2 = 4.3 \text{ GeV}^2$ and $s_0 = 25 \text{ GeV}^2$, where the perturbative contribution $\text{PC}_V \approx 0.58$. Fig. (5-3) and Fig. (5-4) illustrate the dependence of the mass m_V on these parameters M^2 and s_0 .

The parameters of charmonium hybrids with various spin-parity assignments J^{PC} are summarized in Table I. These particles are organized according to the light and heavy hybrid supermultiplet structures, as predicted by the MIT bag model [21, 22] and supported by QCD lattice simulations [42].

• **Light Supermultiplet:**

Composed of hybrids with $J^{PC} = \{(0, 1, 2)^{-+}; 1^{--}\}$, these states arise from an S-wave color-octet $c\bar{c}$ diquark and an excited gluon with $J^{PC} = 1^{+-}$. In this study, we focus on particles with $J=0,1$, limiting the multiplet to three states: $(0,1)^{-+}; 1^{--}$.

• **Heavy Supermultiplet:**

Formed from a P-wave $c\bar{c}$ diquark and a gluon with $J^{PC} = 1^{+-}$, this multiplet includes hybrids with $J^{PC} = \{(0, 1^3, 2^2, 3)^{+-}; (0, 1, 2)^{++}\}$. For our analysis, this reduces to four states: $J^{PC} = \{(0, 1)^{+-}; (0, 1)^{++}\}$.

The masses of charmonium hybrids with spins 0 and 1 range between 3.56 and 4.64 GeV. The lightest hybrid is the pseudoscalar state 0^{-+} , with a mass of 3.56 ± 0.01 GeV. These results, along with predictions for the vector particle 1^{--} are consistent with Ref. [43] within uncertainties. However, the mass of the 1^- state remains controversial. Our estimate, $m_V = (4.12 \pm 0.11)$ GeV, agrees with the range 4.12–4.79 GeV from Ref. [40] but is significantly higher than the 3.36 ± 0.15 GeV reported in Ref. [43]. The heavy supermultiplet consists of four hybrids, with masses ranging from 4.06 ± 0.12 GeV for the scalar 0^{+-} to (4.53 ± 0.17) GeV for the 0^{++} hybrid. The masses of our work the 0^{++} and 1^{++} states are notably lower than those reported in Ref. [43]. The heaviest hybrid, with exotic quantum numbers 0^{--} , has a mass of 4.64 ± 0.14 GeV, which is 0.87 GeV smaller than the prediction in Ref. [43].

Table I includes data on conventional $\bar{c}c$ mesons to compare mass hierarchies between normal and hybrid charmonia. The hybrids 0^{-+} and 1^{--} are approximately 0.58 GeV and $\delta_C(1^{--}) \approx 1$ GeV heavier than their normal counterparts, η_c and J/ψ , respectively. Similarly, the hybrids 0^{++} , 1^{+-} and 1^{++} exceed the masses of $\chi_{c0}(1P)$, $h_c(1P)$, and $\chi_{c1}(1P)$ by about $\delta_C(0^{++}) \approx 1.12$, $\delta_C(1^{+-}) \approx 0.69$, and $\delta_C(1^{++}) \approx 0.64$ GeV, respectively.

Table II presents charmonium hybrid parameters derived from different methods, including sum rules, (BOEFT), and lattice QCD. Our results show good agreement with BOEFT predictions from Ref. [57]. However, lattice simulations, such as those in Ref. [45], yield significantly higher masses. For example, the mass splitting between hybrids in the light multiplet and normal charmonia was estimated at 1.2–1.4 GeV in Ref. [45], while the heavy supermultiplet showed differences of 1.1–1.2 GeV—higher than our estimates in many cases.

The mass data for charmonium hybrids allow us to identify their kinematically allowed two-body strong decay channels. This analysis is qualitative and does not include calculations of partial widths for specific decay modes.

Hybrid mesons can decay through both open-charm and hidden-charm exclusive channels. Their decay patterns, including suppressed or preferred modes, as well as model-dependent selection rules, have been extensively studied in multiple publications, such as Refs. [11, 23, 34, 36, 37, 54, 58].

All iso-scalar hybrid charmonia discussed in this study can undergo two-body decays into standard mesons. For instance, even the lightest state, the 0^{-+} hybrid, decays via S-wave into $\eta_c f_0(500)$. Similarly, the mass of the vector hybrid 1^{-} allows it to decay through S-wave hidden-charm channels, such as J/ψ_n or $J/\psi f_0(500)$ depending on phase space and selection rules.

$$H_V \rightarrow J/\psi f_0(500), \quad (50)$$

a possible process for this particle. The open-charm P-wave decays

$$H_V \rightarrow D\bar{D}, D_0\bar{D}_0, D_s\bar{D}_s, \quad (51)$$

are among kinematically allowed modes of H_V as well.

We began by comparing our results to those presented in Refs. [43,44], which also employed the QCD sum rule approach. This framework, rooted in the principles of field theory, offers a unified treatment of both heavy and light quark flavors alongside the gluonic field. It relies on non-perturbative parameters encoded in the vacuum expectation values of gluon-based operators. These parameters, often referred to as condensates, are typically extracted from independent physical phenomena, despite being interpreted through the same theoretical lens.

Although the methodology is consistent across studies, we observe significant differences, particularly among hybrid states belonging to the heavy-mass multiplets. In several instances, discrepancies emerge even within overlapping mass regions. These variations are likely due to contributions accounted for in our present analysis but omitted in Refs. [43,44].

As previously discussed, enforcing the condition $PC \geq 0.5$ ensures the dominance of the pole contribution in our sum rule framework, enhancing the reliability of the extracted observables. Various other theoretical models have also been utilized to estimate the masses of hybrid states, employing distinct assumptions. For instance, the work in Ref. [106] uses a nonrelativistic effective field theory framework in which hybrid mass spectra are computed by solving the Schrödinger equation under different potential configurations, such as $V^{(0.5)}$ and $V^{(0.25)}$.

Another relevant study is presented in Ref. [58], where the (BOEFT) is applied in combination with lattice QCD techniques. Within this formulation, the gauge theory is discretized across space and time, allowing the dynamics of the valence gluon and the quark-antiquark pair to be analyzed in a structured computational setting. However, such lattice-based methods are sensitive not only to the underlying interaction dynamics but also to the specific lattice configurations used. This sensitivity introduces an added layer of complexity when comparing results derived from different theoretical frameworks, especially when these approaches emphasize different aspects of hybrid meson internal structure.

The analysis of hybrid bottomonium $\bar{b}gb$ states is performed using the same framework described previously. However, some differences arise due to the mass of the bottom (b) quark, necessitating appropriate choices for the Borel mass parameter M^2 and the continuum threshold s_0 in accordance with the requirements of sum rule (SR) computations.

Our findings indicate that for the bottomonium counterparts of scalar and vector charmonium hybrids, specifically the 0^{+-} and 1^{--} states, the parameters M^2 and s_0 must be adjusted within bounds to ensure the reliability of the SR predictions, so for scalar particle

$$M^2 \in [12,13.5] \text{ GeV}^2, s_0 \in [124,126] \quad (52)$$

and for the vector state

$$M^2 \in [12,14] \text{ GeV}^2, s_0 \in [120,125] \text{ GeV}^2. \quad (53)$$

The mass and current coupling of the structures 0^{+-} and 1^{--} are equal to

$$\tilde{m}_s = (10.12 \pm 0.6) \text{ GeV}^3, \quad \tilde{f}_s = (4.6 \pm 0.3) \times 10^{-2} \text{ GeV}^3, \quad (54)$$

and

$$\tilde{m}_v = (10.41 \pm 0.28) \text{ GeV}^3, \quad \tilde{f}_v = (1.2 \pm 0.3) \times 10^{-2} \text{ GeV}^3, \quad (55)$$

respectively.

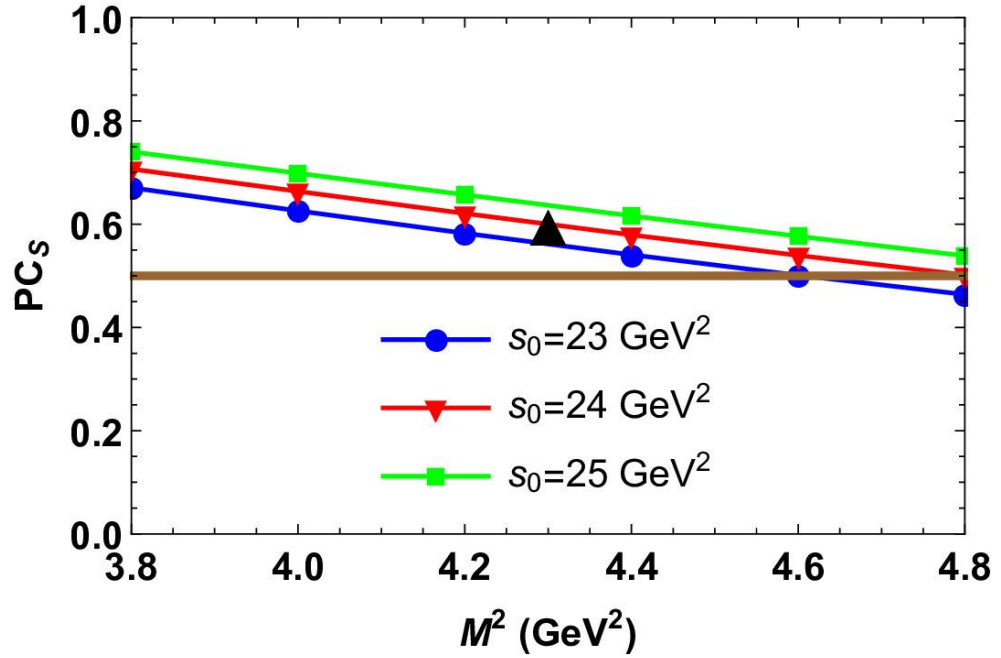
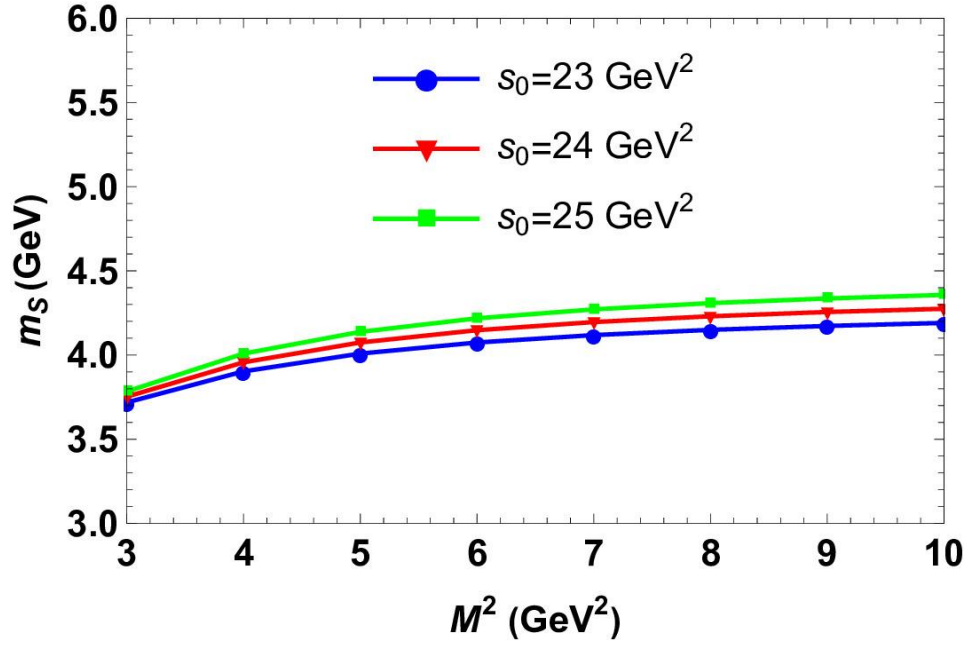


Figure 5-1: Up – Mass m_s of the scalar charmonium hybrid $J^{PC} = 0^{+-}$ vs. Borel parameter. Down – Pole contribution PC_s vs. M^2 at fixed s_0 . The horizontal line marks $PC = 0.5$; the triangle indicates the point where m_s is extracted.

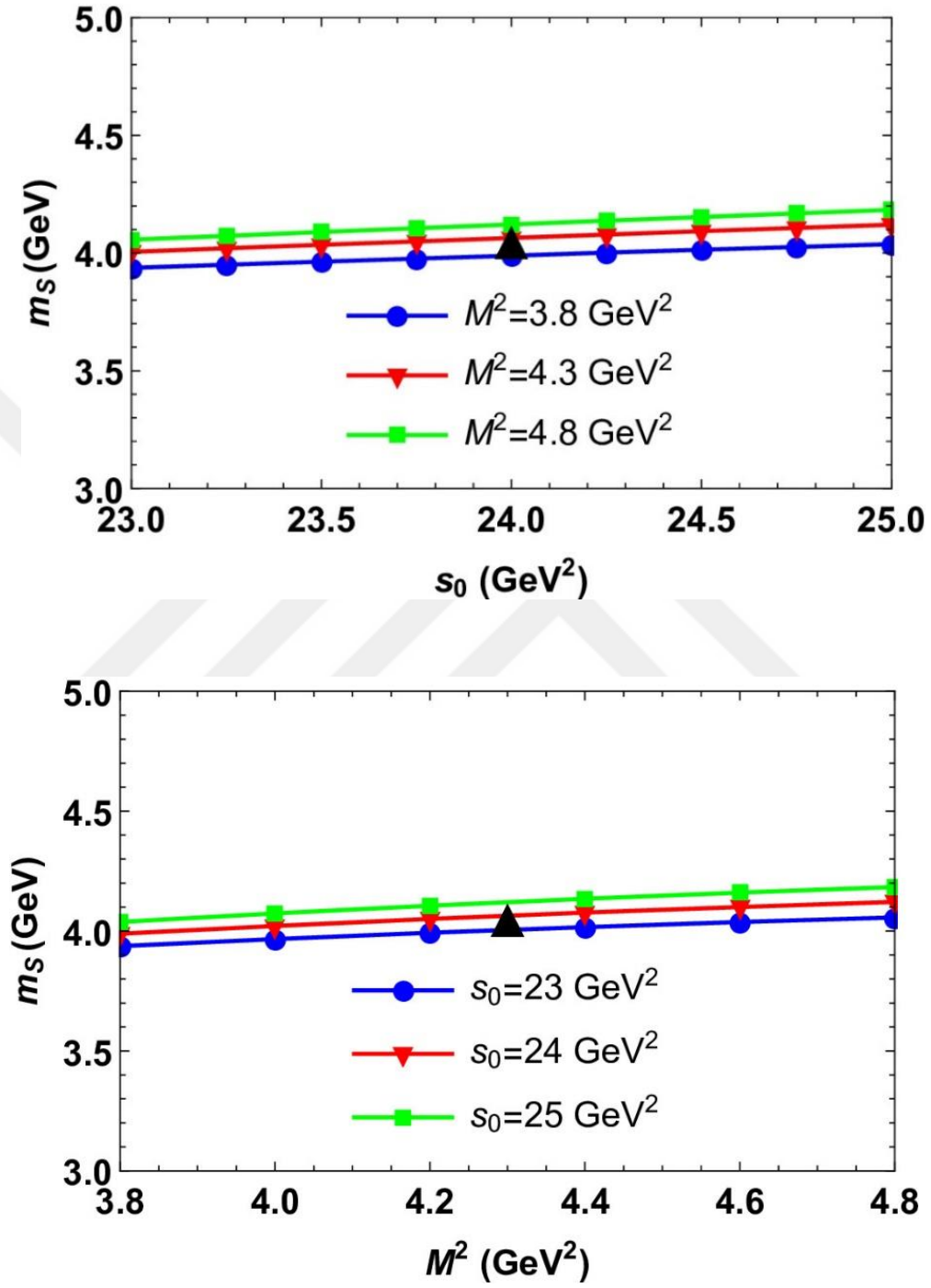


Figure 5-2: Mass m_S of hybrid H_S as a function of the Borel parameter M^2 (up) and the continuum threshold s_0 (Down). Triangles indicate the position of H_S

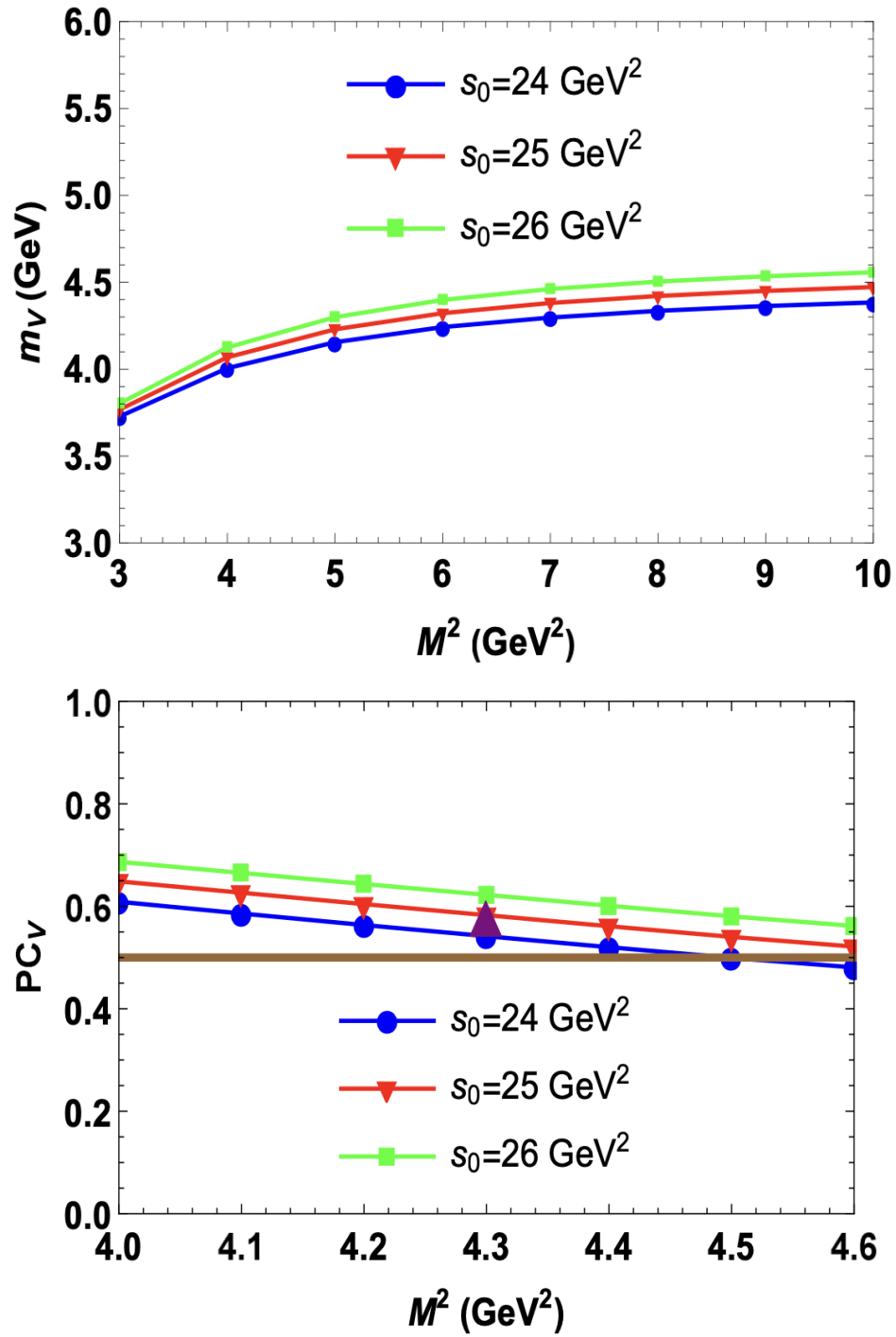


Figure 5-3: Repeats the analysis of Figure 5-1, applied specifically to the vector Charmonium hybrid ($J^{PC} = 1^{+-}$).

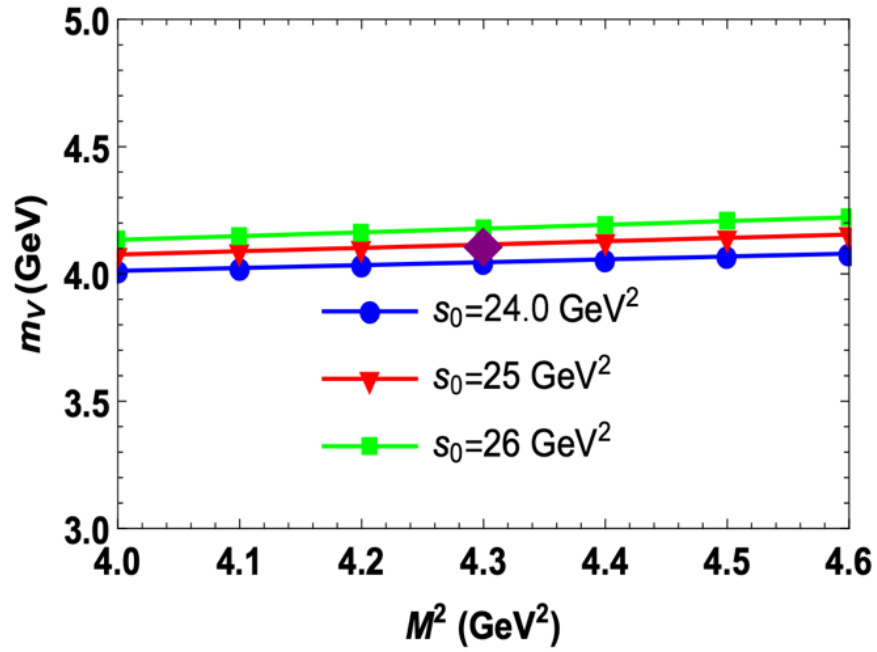
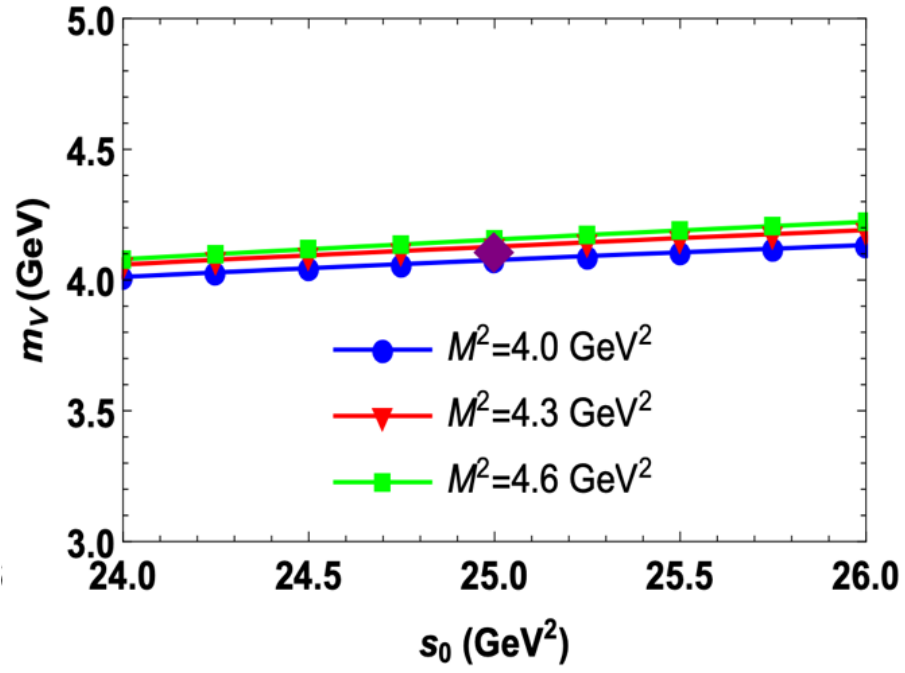


Figure 5-4: Vector hybrid H_v mass m_v vs. Borel parameter M^2 (up) and continuum threshold s_0 (down). Diamonds indicate evaluated mass points

Table I: Masses, current couplings, and quantum numbers of hybrid charmonia, along with input parameters. Conventional $\bar{c}c$ meson masses are from [59]. Errors are shown as: $3.56(09)(04)(01) = 3.56 \pm 0.09 \pm 0.04 \pm 0.01$

J^{PC}	M^2 (GeV ²)	s_0 (GeV ²)	PC(%)	Mass (GeV)	$f \times 10^2$ (GeV ³)	Mass of $\bar{c}c$ meson (MeV)
0^{-+}	3.6 – 4.7	19 – 20	72–50	3.56(09)(04)(01)	3.8(3)(4)(07)	$\eta_c : 2983.9(4)$
1^{-+}	4–5	23 – 24	69–50	3.93(10)(07)(007)	4.60(20)(17)(11)	–
1^{--}	4–4.6	24 – 26	63–50	4.12(11)(06)(004)	4.0(4)(2)(02)	$J/\psi : 3096.900(6)$
0^{+-}	3.8 – 4.8	23 – 25	71–50	4.06(12)(05)(01)	2.80(30)(18)(09)	–
1^{+-}	4–5	25 – 27	69–50	4.21(15)(07)(004)	4.50(60)(04)(14)	$h_c(1P) : 3525.37(14)$
0^{++}	4–5	26 – 28	75–50	4.53(16)(07)(007)	2.50(90)(37)(05)	$\chi_{c0}(1P) : 3414.71(30)$
1^{++}	4.5 – 5.1	26 – 28	70–59	4.15(09)(07)(007)	5.80(50)(10)(14)	$\chi_{c1}(1P) : 3510.67(05)$
0^{--}	5–6.5	30 – 32	70–50	4.64(14)(06)(01)	4.90(50)(14)(13)	–

Table II: Predicted masses (in GeV) of hybrid charmonium states $\bar{c}gc$ as reported in various studies

J^{PC}	This work	SR [43]	BOEFT [58]	Lattice [45]
0^{-+}	3.56(10)	3.61(21)	3.911(54)	4.279(18)
1^{-+}	3.93(12)	3.70(21)	3.963(38)	4.310(23)
1^{--}	4.12(13)	3.36(15)	4.011(30)	4.411(17)
0^{+-}	4.06(13)	4.09(23)	4.087(61)	4.437(27)
1^{+-}	4.21(17)	4.53(23)	4.235(35)	4.665(53)
0^{++}	4.53(17)	5.34(45)	4.486(30)	4.591(46)
1^{++}	4.15(11)	5.06(44)	4.145(30)	4.518(35)
0^{--}	4.64(15)	5.51(50)	–	–

The remaining hybrid bottomonium states are studied using a similar approach. The results are summarized in Table III, revealing a mass spectrum in the range of 9.68 to 10.57 GeV. The states 0^{-+}

and 1^{+-} , assigned to the light supermultiplet, have predicted masses of (9.68 ± 0.20) GeV and (9.79 ± 0.21) GeV, respectively. These values are consistent with those reported in Ref. [43], though they are significantly lower than the predictions obtained through QCD lattice simulations and the (BOEFT). The Figs. (5-5) and (5-6) illustrate the dependence of the bottomonium hybrid masses m_S and m_V on the Borel parameter M^2 and the continuum threshold s_0 . The observed stability across these ranges confirms the robustness of the QCD sum rule results.

For comparison, the lattice and BOEFT approaches yield masses for the 0^{+-} and 1^{+-} hybrids of (10.926 ± 0.018) GeV and (10.935 ± 0.018) GeV [43], and (10.682 ± 0.005) GeV and (10.686 ± 0.004) GeV [57], respectively. In our study, the vector hybrid 1^{--} has a mass of (10.41 ± 0.18) GeV, which is in good agreement with the result from Ref. [40]. However, this value is notably higher than the prediction of (9.70 ± 0.12) GeV provided in Ref. [43], and remains lower than the corresponding lattice and BOEFT predictions.

For bottomonium hybrids belonging to the light supermultiplet, lattice simulations predict a spin-averaged mass of approximately 10.938 GeV [50]. For hybrids in the heavy supermultiplet, this value is around 10.872 GeV. According to the Born–Oppenheimer approximation, the average masses are 10.686 GeV and 10.822 GeV for the light and heavy supermultiplets, respectively [57]. A notable feature in both lattice and BOEFT spectra is the near-degeneracy of masses within each multiplet.

Table III: Same as Table I, but for bottomonium hybrids. The $\bar{b}gb$ meson masses are taken from Ref. [66]

J^{PC}	M^2 (GeV ²)	s_0 (GeV ²)	PC(%)	Mass (GeV)	$f \times 10^2$ (GeV ³)	Mass of bb meson (MeV)
0^{--}	11 – 13	110 – 120	67 – 50	9.68(20)(07)(012)	9.2(1.9)(0.28)(0.27)	η_b : 9398.7(2.0)
1^{--}	12 – 13	115 – 120	60 – 51	9.85(11)(08)(01)	8.9(1.0)(0.13)(0.34)	–
1^{--}	12 – 14	120 – 125	72 – 51	10.41(18)(08)(012)	12.0(3.0)(0.6)(0.4)	Y(1S) : 9460.40(09)(04)
0^{+-}	12 – 13.5	124 – 126	66 – 50	10.12(06)(06)(01)	4.6(3)(7)(2)	–
1^{+-}	13.5 – 14.5	130 – 132	56 – 50	10.46(06)(07)(01)	11.60(60)(13)(42)	$h_b(1P)$: 9899.3(8)
0^{++}	13.5 – 15	125 – 130	60 – 50	10.57(08)(08)(02)	19.3(1.6)(0.2)(0.5)	$\chi_{b0}(1P)$: 9859.44(42)(31)
1^{++}	12.5 – 14.5	128 – 130	65 – 51	10.55(10)(08)(02)	13.4(1.0)(0.07)(0.44)	$\chi_{b1}(1P)$: 9892.78(26)(31)
0^{--}	12 – 14	130 – 135	76 – 50	10.51(11)(06)(02)	3.90(30)(09)(18)	–

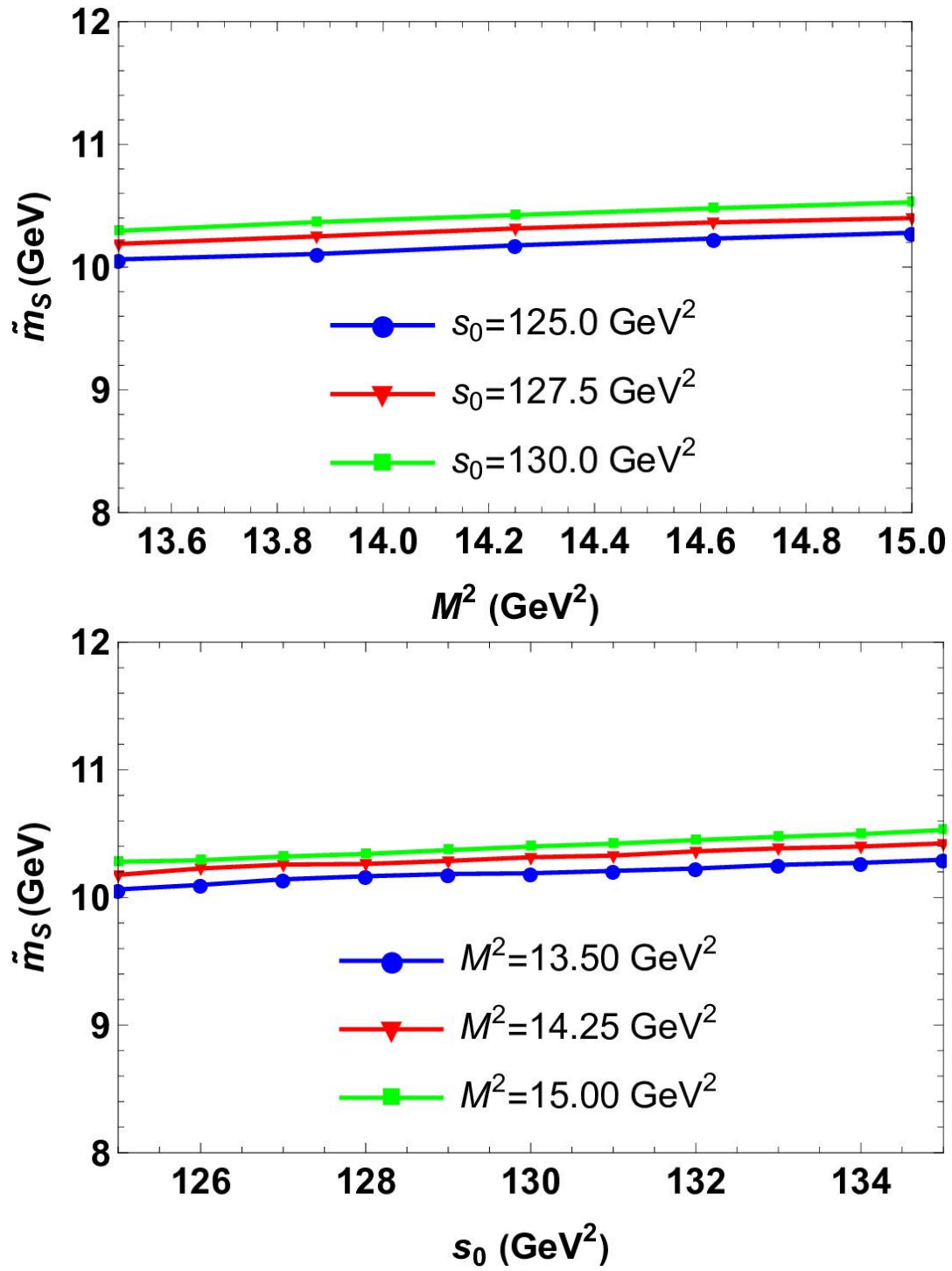


Figure 5-5: Bottomonium mass $\tilde{m}_S$ as a function of the Borel parameter M^2 (up) and the continuum threshold s_0 (down).

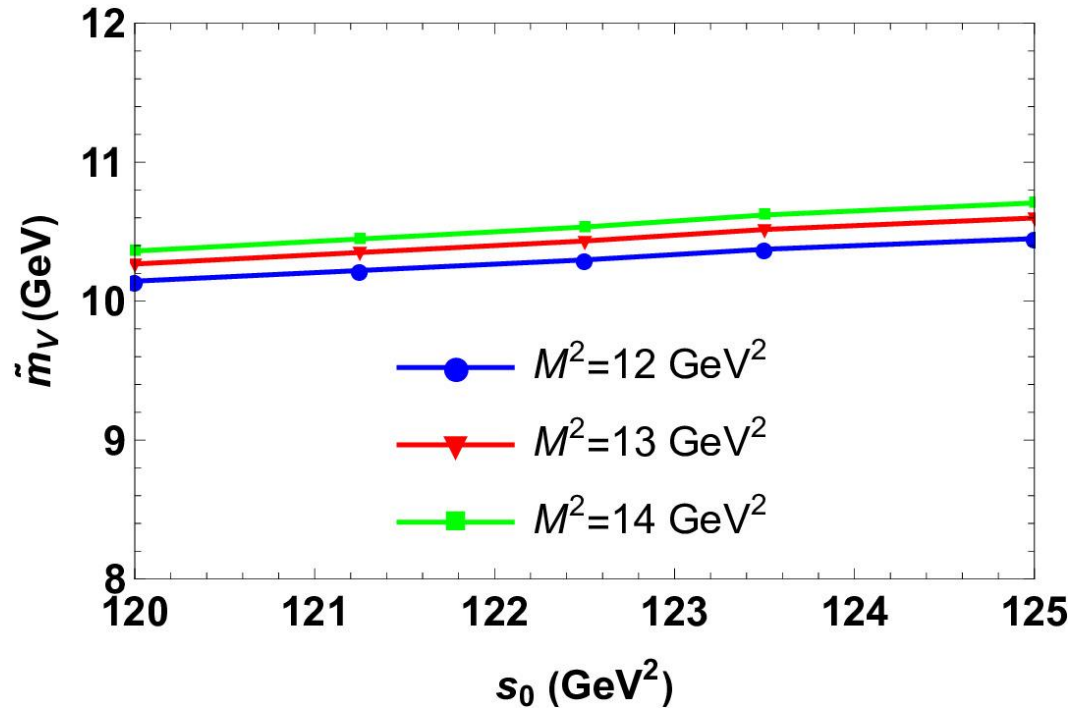
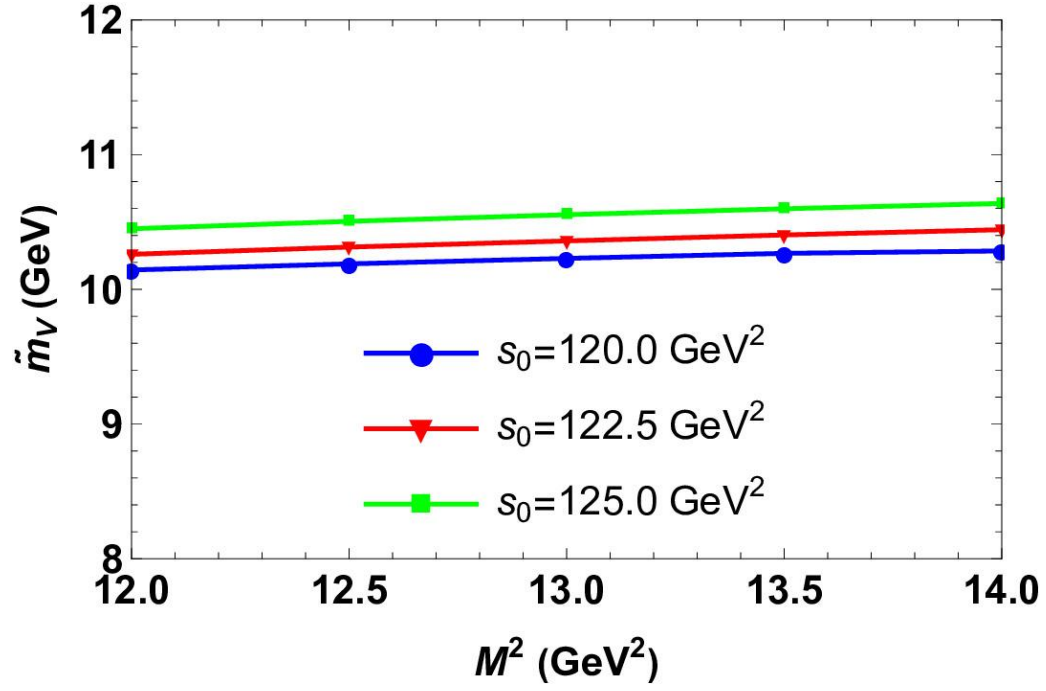


Figure (5-6): Vector hybrid H_V mass $\tilde{m}_V$ of bottomonium variation with Borel parameter M^2 (up) and continuum threshold s_0 (down).

Table IV: Masses of bottomonium hybrid states (in GeV), obtained using QCD sum rules (SR), (BOEFT), and lattice QCD methods

J^{PC}	This work	SR [43]	BOEFT [58]	Lattice [51]
0^{-+}	9.68(21)	9.68(29)	10.682(5)	10.926(18)
1^{-+}	9.85(14)	9.79(22)	10.686(4)	10.935(18)
1^{--}	10.41(20)	9.70(12)	10.6902(30)	10.952(24)
0^{+-}	10.12(09)	10.17(22)	10.756(5)	10.935(41)
1^{+-}	10.46(09)	10.70(53)	10.759(4)	11.062(35)
0^{++}	10.57(12)	11.20(48)	11.012(3)	—
1^{++}	10.55(13)	11.09(60)	10.761(3)	10.921(55)
0^{--}	10.51(13)	11.48(75)	—	—

When compared with our results presented in Table III, where the corresponding spin-averaged masses are 9.98 GeV for the light multiplet and 10.425 GeV for the heavy one, a mass gap of roughly 1 GeV and 0.4 GeV is observed between the sum rule (SR) predictions and the lattice results. This indicates that for bottomonium hybrids, the SR method also predicts an upward-trending mass spectrum. The two-body strong decay modes of these hybrid mesons were assessed based on our mass predictions. The hybrids with quantum numbers $J^P = 0^+$ and 1^+ are found to be below the thresholds for the production of open- and hidden-bottom meson pairs, even when considering the upper bounds of their mass estimates.

The vector hybrid 1^- can decay through an S-wave into conventional mesons. The scalar hybrid 0^{+-} , with a mass of (10.12 ± 0.06) GeV, undergoes a P-wave decay into a meson pair. The remaining kinematically allowed two-body strong decay channels for both charmonium and bottomonium heavy hybrid mesons can be obtained by considering the mass differences and referring to the decay schemes listed in Ref. [44].

5.5 The Heavy Hybrid Mesons $\bar{b}gc$

In this part of the study, we focus on evaluating the mass spectra of heavy hybrid mesons with spin-parity quantum numbers $J^P = 0^+, 0^-, 1^+, \text{ and } 1^-$. The interpolating currents used for these calculations are derived directly from Eqs. (27) – (30). Since charged hybrid states $\bar{b}gc$ do not possess well-defined charge conjugation parity (C-parity), they cannot be classified accordingly. However, to distinguish among the meson states generated using different interpolating currents, we retain the C-parity label of the original current in parentheses as a formal identifier. Specifically, the interpolating current corresponding to the hybrid mesons with $J^P = 0^{+(-)}$ and $1^{-(-)}$ is used in the following analysis

$$\tilde{J}_\mu^1(x) = g_s \bar{b}_a(x) \gamma^\alpha \gamma_5 \frac{\lambda_{ab}^n}{2} \tilde{G}_{\mu\alpha}^n(x) c_b(x). \quad (56)$$

The scalar and vector particles with $J^P = 0^{+(+)}$ and $1^{-(+)}$ can be investigated using the current

$$\tilde{J}_\mu^2(x) = g_s \bar{b}_a(x) \gamma^\alpha \frac{\lambda_{ab}^n}{2} G_{\mu\alpha}^n(x) c_b(x). \quad (57)$$

The particles $0^{-(-)}$ and $1^{+(-)}$ are described by the current

$$\tilde{J}_\mu^3(x) = g_s \bar{b}_a(x) \gamma^\alpha \gamma_5 \frac{\lambda_{ab}^n}{2} G_{\mu\alpha}^n(x) c_b(x). \quad (58)$$

and

$$\tilde{J}_\mu^4(x) = g_s \bar{b}_a(x) \gamma^\alpha \frac{\lambda_{ab}^n}{2} \tilde{G}_{\mu\alpha}^n(x) c_b(x). \quad (59)$$

corresponds to the pseudoscalar and axial-vector hybrids $J^P = 0^{-(+)}$ and $1^{+(+)}$.

The correlation functions $\tilde{\Pi}_{\mu\nu}(p)$ to be studied in these cases are given by Eq. (9) after replacements $J_\mu(x) \rightarrow \tilde{J}_\mu(x)$.

The methodology applied to analyze the corresponding correlation functions remains largely consistent with the approach outlined in the preceding section. The primary differences arise on the QCD side of

the sum rules. For example, when considering the interpolating current $\tilde{J}_\mu^1(x)$, the associated correlation function $\tilde{\Pi}_{\mu\nu}^{OPE}(p)$ is expressed as follows:

$$\tilde{\Pi}_{\mu\nu}^{OPE}(p) = \frac{i\varepsilon_{\mu\theta\alpha\beta} \varepsilon_{\nu\delta\alpha'\beta'}}{4} \int d^4x e^{ipx} \frac{\lambda_{ab}^n \lambda_{a'b'}^m}{4} \text{Tr}[S_Q^{a'a}(-x) \gamma^\theta \gamma_5 S_{Q_2}^{bb'}(x) \gamma^\delta \gamma_5] \times \langle 0 | g_s^2 G^{n\alpha\beta}(x) G^{m\alpha'\beta'}(0) | 0 \rangle. \quad (60)$$

Parameters of the bottom-charm hybrids obtained using these interpolating currents are presented in Table V. As samples, in Fig. (5-7) we show masses of the pseudoscalar and vector particles $0^{-(+)}$ and $1^{-(+)}$.

The predicted masses of the hybrid mesons $\bar{b}gc$ studied in this work fall within the range of 6.55 to 7.19 GeV. The average mass of the hybrids tentatively categorized as members of the light multiplet is approximately 6.73 GeV, which aligns reasonably well with the value of 6.9 GeV reported in Ref. [44]. This value is also only about 0.4 GeV higher than the mean mass of the B_c and B_c^* mesons. In contrast, our mass predictions for the hybrid states assigned to the heavy multiplet are significantly lower than those presented in the same reference. Moreover, the spectrum obtained in this analysis appears more uniform, with smoother mass variations, compared to the broader and more fluctuating range of 6.83 to 8.48 GeV observed in Ref. [44].

The possible two-body strong decay modes of the hybrid mesons are determined by the kinematic thresholds for the production of meson pairs with the appropriate quantum numbers. The hybrids with quantum numbers $J^P = 0^{-(+)}$ and $1^{-(+)}$ are relatively light states and appear to be stable against strong two-body decays into conventional heavy mesons. The next state listed, $1^{(-)}$, has energetically allowed decay channels: $1^{(-)} \rightarrow B_c^* + f_0(500)$ and $B_c + \eta$, which correspond to S-wave and P-wave decay modes, respectively. The hybrid state $0^{+(-)}$ can decay into the meson pairs $B_c + \eta$ and $B_c^* + f_0(500)$. For the states $1^{+(-)}$ and $1^{+(+)}$, the allowed decay processes are:

- $1^{(-)} \rightarrow B_c^* + \eta, B_c + f_0(500), B_c + \omega(782)$
- $1^{+(+)} \rightarrow B_c + f_0(500)$

In addition, the hybrids $0^{+(+)}$ and $0^{(-)}$ may decay through the channels:

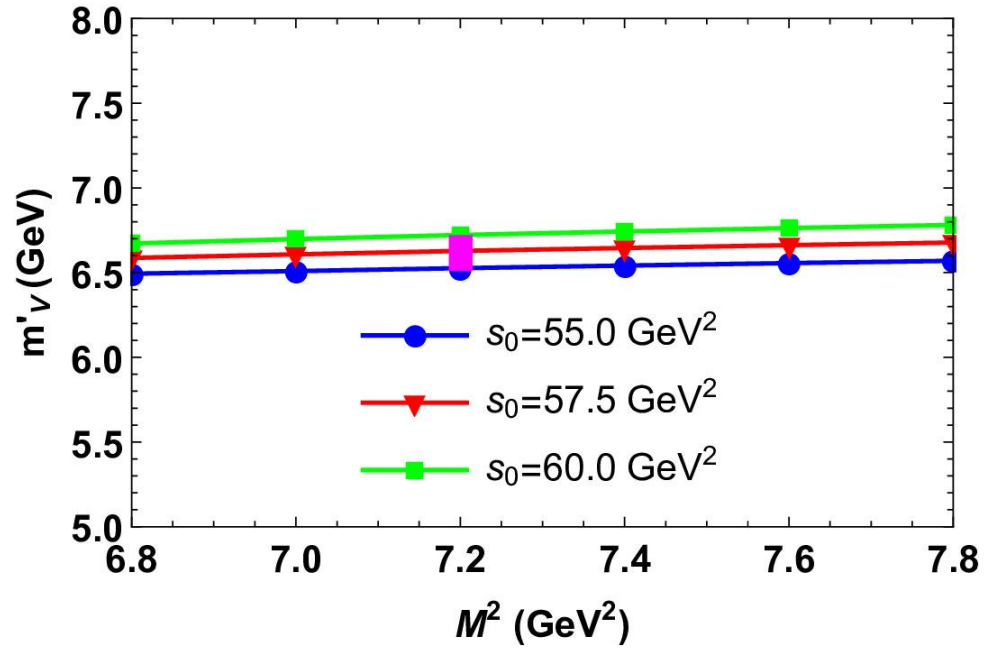
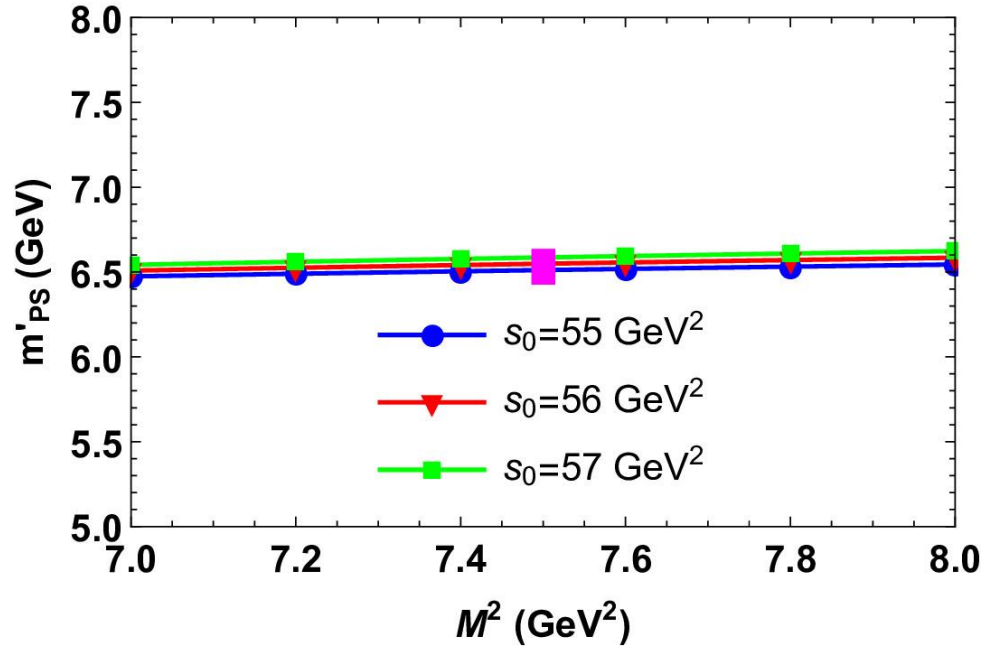


Figure 5-7. Masses of pseudoscalar (0^{-+} , up) and vector (1^{-+} , down) hybrids as functions of the Borel

- $0^{+(+) } \rightarrow B_c + \eta, B + D^0$
- $0^{(-) } \rightarrow B_c + f_0(500), B_c^* + \eta$

These decay channels reflect the underlying quantum numbers of the hybrid states and provide theoretical predictions that can be tested in future experimental searches.

Table V: Bottom-charm hybrid meson parameters with uncertainties from (M^2, s_0) , $\langle \alpha_s G^2/\pi \rangle$, condensates, and quark masses. The $B_c^+(1S)$ mass is experimental [66]; others are model-based [68].

J^{PC}	M^2 (GeV ²)	s_0 (GeV ²)	PC (%)	Mass (GeV)	$f \times 10^2$ (GeV ³)	Mass of $\bar{b}c$ meson (MeV)
$0^{-(+)}$	7–8	55 – 57	69 – 51	6.55(08)(06)(009)(007)	4.90(4)(3)(09)(04)	$B_c(1S) : 6274.47(27)(17)$
$1^{-(+)}$	6.8 – 7.8	55 – 60	62 – 50	6.63(14)(07)(007)(007)	4.50(70)(18)(09)(04)	–
$1^{(-)}$	7 – 8.3	62 – 65	71 – 55	7.01(13)(08)(006)(002)	4.80(60)(05)(11)(07)	$B_c^*[B_c(1^3S_1)] : 6338$
$0^{+(-)}$	8.5 – 9.4	58 – 60	61 – 53	6.93(07)(05)(006)(006)	2.80(20)(19)(06)(05)	–
$1^{+(-)}$	7.6 – 8.6	65 – 67	61 – 50	7.17(09)(07)(005)(004)	5.70(50)(08)(13)(07)	$B_c(1^1P_1) : 6750$
$0^{+(+)}$	8–10	65 – 67	70 – 50	7.03(12)(08)(008)(005)	8.10(8)(2)(1)(07)	$B(1^3P_0) : 6706$
$1^{+(+)}$	8 – 9.5	66 – 67	65 – 50	7.12(09)(08)(007)(004)	7.40(50)(06)(14)(08)	$B(1^3P_1) : 6741$
$0^{(-)}$	7.8 – 8.8	66 – 67	62 – 50	7.19(06)(06)(007)(006)	4.00(20)(13)(08)(06)	–

6.CHAPTER SIX - CONCLUSION

In this study, we have conducted a detailed analysis of scalar, pseudoscalar, vector, and axial-vector heavy hybrid mesons within both the charmonium and bottomonium sectors, considering states with positive and negative charge conjugation (C) parities. The work employed the QCD two-point sum rule method to calculate their masses and current couplings. The investigated states include those with spin-parity quantum numbers $J^P = 0^+, 0^-, 1^+, \text{ and } 1^-$.

The operator product expansion (OPE) was extended to include nonperturbative contributions up to dimension 12. While higher-dimensional operators tend to contribute minimally from a numerical perspective, their inclusion is essential for enhancing the stability and reliability of the sum rule (SR) calculations. To further ensure the validity of the derived predictions, a stringent constraint was applied by requiring the pole contribution (PC) to satisfy $PC \geq 0.5$.

The computed mass values of the hybrid mesons were then utilized to identify possible two-body strong decay channels. The results indicate that all charmonium hybrids are unstable under strong interactions. Notably, the vector hybrid state $\bar{c}gc$ with 1^{--} , previously hypothesized to be stable in [36], is found in our analysis to have a mass of $m_V = (4.12 \pm 0.11) \text{ GeV}$, and is capable of decaying via both open- and hidden-charm channels into various meson pairs.

In the bottomonium sector, only the pseudoscalar and exotic axial-vector hybrids with $J^{PC} = 0^{-+}$ and 1^{-+} and corresponding mass values of $(9.68 \pm 0.21) \text{ GeV}$ and $(9.85 \pm 0.14) \text{ GeV}$, appear to be stable against strong two-body decays. Their stability arises from the fact that their masses lie below the thresholds required for the production of both open- and hidden-bottom meson pairs. Furthermore, exotic hybrid mesons with quantum $0^{-(+)}$ and $1^{-(+)}$ are also predicted to be stable under strong interactions.

When compared to earlier studies, such as those presented in Refs. [36, 37], our results show notable deviations, particularly for hybrid states within the heavy multiplet sector. However, due to the relatively large uncertainties associated with the extracted mass values, some overlap with previous predictions still exists. These differences may stem from the application of the stricter condition $PC \geq 0.5$, which has been implemented in our work to enhance the fidelity of the analysis.

A good level of agreement is observed between our findings and those obtained via BOEFT for charmonium hybrids. Nevertheless, for certain mesons $\bar{b}gb$, especially those belonging to the light multiplet, BOEFT predicts higher mass values than those found in our calculations. Lattice QCD simulations, on the other hand, tend to provide the largest mass predictions for heavy hybrid mesons.

The QCD sum rule analysis conducted in this research, together with the comparative study involving alternative approaches, contributes meaningfully to the ongoing efforts to understand the properties of exotic hybrid mesons. The spectroscopic data presented herein can serve as foundational input for future investigations into the strong and electroweak decay channels of hybrids and may also be useful in analyzing their interactions with conventional hadrons.

Dedicated discoveries at facilities like LHCb, Belle II and Panda can provide important tests of our findings. Strong-interaction-stable hybrid meson's identification will provide direct evidence of gluonic degree of freedom in the hadronic bound States and deepen our understanding of QCD in non-perturbative system.

This approach can be further expanded in the future to analyze the effects of alternative non-perturbative framework such as high spin hybrids, mixed conditions and other non - perturbative framework. In addition, its role in hybrid mesons production and heavy ion conflicts and foreign multiquarks structures can provide deep insight into QCD dynamics beyond traditional Quark-antiquark mesons. The major results of our study have shown that the QCD-sum rules are actually a powerful tool in Hadrons spectroscopy. This research should be conducted to explore theoretical and experimental investigations into the fundamental processes governing nature, particularly in relation to stable hybrid mesons in bottomonium fields.

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Appendix

$$\begin{aligned} \rho^{QCD}(s) = & \int_0^1 dz \int_0^{1-z} dw \frac{g_s^2}{192 \pi^4 A^7} [swz(1-w-z) \\ & + Am_Q^2(w+z) \{A^2 m_Q^4 (w+z)^2 [w^2 + z(-1+z) + w(1+2z)] \\ & - 2s^2 w^2 z^2 (-1+z+w)^2 [w^2 + z(-1+z) + w(1+13z)] \\ & + Am_Q^2 swz(-1+z+w)(w+z)[w^2 + z(-1+z) + w(1+22z)]\}] \theta[N] \end{aligned}$$

$$\begin{aligned} \rho^{DimN}(s) = & \int_0^1 dz \int_0^{1-z} dw \frac{g_s^2}{9216 \pi^2 A^5} \left\langle \alpha_s \frac{G}{\pi} \right\rangle \\ & \times \{-8 s w^2 z^2 (-1+w+z) [w^2 + z(-1+z) + w(-1+4z)] \\ & + Am_Q^2 (w+z) [8 w^4 + 8 z^3 (-1+z) - w z^2 (5+43z) - w^3 (8+43z) \\ & + w^2 z (-5+49z)]\} \theta[N] + \int_0^1 dz \left\langle \alpha_s \frac{G}{\pi} \right\rangle \left[\frac{3m_Q^2 + s z (-1+z)}{96} \right] \theta[N_1] \end{aligned}$$

where:

$$A = w^2 + (-1+z)(w+z)$$

$$N = \frac{(-1+w)[s w z (-1+w+z) - A m_Q^2 (w+z)]}{A^2}$$

$$N_1 = s(1-z)z - m_Q^2$$

w, z, and s: are parameters

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