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A dissertation submitted in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY IN COMPUTER SCIENCE AND INFORMATICS

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Dedicated to my parents

## ACKNOWLEDGMENTS

First of all, I would like to express my deepest gratitude to my advisor, Dr. Mohammad-Reza Siadat, for his excellent support, encouragement and guidance. I would like to thank Dr. Theophilus Ogunyemi for his great support and guiding my research for the past several years. I would also like to thank my committee members Dr. Ishwar Sethi and Dr. Guangzhi Qu who have contributed to this dissertation with numerous suggestions. Special thanks to Dr. Beth Zou who was willing to participate in my final defense.

I am also deeply thankful to my wonderful family for their faith in me, always been there to support me and encourage me with their best wishes. My research would not have been possible without their help.

Suzan Arslanturk

# ABSTRACT <br> CLINICAL DATA ANALYSIS FOR PREDICTION OF URINARY INCONTINENCE 

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It is common for clinical data in survey trials to be incomplete and inconsistent for several reasons. One objective of this study is to identify and eliminate inconsistent data as an important data mining preprocessing step. We define three types of incomplete data: missing data due to skip pattern (SPMD), undetermined missing data (UMD) and genuine missing data (GMD). Identifying the type of missing data is another important objective as all missing data types cannot be treated the same. This goal cannot be achieved manually on large data of complex surveys since each subject should be processed individually. Experiments are conducted on a longitudinal questionnaire (MESA). MESA dataset was collected between 1983-1990 to create a set of questions that can reliably predict future Urinary Incontinence (UI). The analyses are accomplished in a mathematical framework by exploiting graph theoretic structure inherent in the questionnaire. An undirected graph is built using mutually inconsistent responses as well as its complement. The responses not in the largest maximal clique of complement graph are considered inconsistent. Further, all potential paths in questionnaire's graph are considered, based on responses of subjects, to identify each type of incomplete data. Once SPMD is determined, MESA data is stratified to divide the data into stratums with
potentially different UI risk factors. Rough set imputation is applied, on the GMD portion of the incomplete data. ReliefF attribute selection technique and logistic regression is used to determine the potential predictive factors with their corresponding prediction probabilities forming the continence index on the preprocessed MESA data. The incomplete data analysis results show $15.4 \%$ GMD, $9.8 \%$ SPMD, $12.9 \%$ UMD and $0.021 \%$ inconsistent data. Proposed preprocessing methods are prerequisites for any data mining of clinical survey data. The predictive index can be applied for immediate screening and for predicting future urinary incontinence in older woman of comparable demographics.

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## LIST OF ABBREVIATIONS

| ACG | Adjusted Clinical Groups |
| :---: | :---: |
| CPS | Current Population Survey |
| DI | Deterministic Imputation |
| FEG | Failed Edit Graph |
| GMD | Genuine Missing Data |
| KNN | K-Nearest Neighbor |
| MAR | Missing At Random |
| MCAR | Missing Completely Random |
| MDS | Minimal Deletion Set |
| MESA | Medical Epidemiological and Social Aspect of Aging |
| MSLP | Multiple Sub Linear Path |
| NIA | National Institute of Aging |
| NMAR | Not Missing At Random |
| SI | Random Imputation |
| SOM | Self-organization Maps |
| SPMD | Missing Data Due to Skip Pattern |
| SSLP | Single Sub Linear Path |
| UI | Urinary Incontinence |
| UMD | Undetermined Missing Data |
| WEKA | Waikato Environment for Knowledge Analysis |

## CHAPTER ONE

## INTRODUCTION

### 1.1. Urinary Incontinence

Urinary incontinence (UI) is a condition in which involuntary urine leakage is demonstrable. UI is commonly seen on older women 60 years and older and has tremendous social and economic costs. It is one of the chronic health conditions that have the greatest effect on a woman's health related quality of life. One in three adult women in the United States suffers from UI. Estimates vary, but in general twice as many women suffer from UI than men. One meta-analysis reported ranges of UI prevalence from 4.5\% to $44 \%$ (mean $23.5 \%$ ) in women and $4.6 \%$ to $24 \%$ (mean $14.5 \%$ ) in men.

UI is not just a medical problem; it also has adverse social and psychological effects on sufferers and their families by contributing to social isolation and depression. UI also leads to dependency and is a significant factor in nursing home admissions. The economic burden on individuals, families and communities is considerable. Since average life expectancy is increasing and population estimates project that the percentage of women over 65 years old will continue to grow through 2030, the negative implications of UI are likely to increase.

There are three types of urinary incontinence: stress, urge and mixed incontinence. Stress incontinence is due essentially to insufficient strength of the pelvic floor muscles to prevent the passage of urine. It can be caused by coughing, sneezing or
movements that put pressure into bladder. Urge incontinence is involuntary loss of urine occurring from no apparent reason while suddenly feeling the need or urge to urinate. This may occur because of damage to nerves of the bladder, the nervous system, or muscles themselves. Mixed incontinence, on the other hand, is a combination of both stress incontinence and urge incontinence. Incontinence varies in degree of severity from several drops to complete bladder emptying. It may occur daily, or many times a day, or only occasionally, perhaps once a month. It may be fairly predictable (low grade stress incontinence) or totally unpredictable (urge incontinence).

A scientifically developed and tested predictive UI index would help identify women who are most likely to develop UI and permit widespread prevention or early treatment. Already identified risk factors include aging, onset during pregnancy tract symptoms, parity, higher body mass index, and functional and cognitive impairment. However, those risk factors were extracted to predict post prostatectomy continence in men after catheter removal by evaluating clinical parameters. This was the only study reported in the literature.

Studies with prospective or longitudinal designs are required to establish the temporal ordering between risk factors and the onset of UI. Furthermore, longitudinal study is needed to determine the role of natural history or medical interventions or factors in inducing UI remission.

### 1.2. MESA Dataset

The Medical Epidemiological and Social Aspects of Aging (MESA)
epidemiology study was conducted with National Institutes of Aging (NIA) funding from

1983-1990. This longitudinal population based study consisted of three detailed household interviews at 1-2 year intervals. The baseline of Medical, Epidemiological and Social Aspects of Aging (MESA) collected in 1983, is a questionnaire containing 825 questions and 1956 respondents, of which the majority, 1154 are female. The female are seniors age 60 years and older. (1,099 subjects age 60-69 years, 589 age $70-79$ years, and 268 age 80 and up; $59 \%$ women; $91 \%$ white). The respondents were interviewed for approximately two hours at home at baseline (1983-1984 interviews) and then reinterviewed at 1-2 year intervals. Re-interview response rates were $69 \%$ and $72 \%$ in those subjects that were still living. The respondents are interviewed on a variety of health related questions that may play a role in the prevalence of urinary incontinence (UI). Although, the survey focused on the epidemiology of UI, many other attributes were also assessed including medical history, mobility, cognitive function, current health, and quality of life.

One challenge of MESA data is that, it has a considerable amount of incomplete data $(37.1 \%)$. The incomplete data rates for the first, second and third follow-up are, $65.1 \%, 43.5 \%, 64.1 \%$, respectively. There is also, noise and multi-colinearity in the dataset, in which the percentages are unknown at this point.

### 1.3. Problem Statement

The first step is to preprocess MESA by identifying each type of incomplete data and treating them separately. Next, determining the risk factors that play an active role in the prevalence of UI and generating a predictive index that helps to more readily identify women who are most likely to develop UI are the major steps of this study.

### 1.4 Related Work

A common challenge of analyzing clinical survey data is to deal with incomplete and inconsistent data. A variety of methods have been developed to enable such analysis of survey data to deal with incomplete data (also referred as missing values) such as imputation, partial deletion, interpolation (Junninen et al. 2004) and maximum likelihood estimation (Beale et al. 1975). Imputation is the most commonly used among the above methods (Heijden et al. 2006; Zhang et al. 2006; Penny et al. 2006). Besides these techniques, an important approach to deal with missing data is to distinguish between different types of missing values so that each group can be treated differently. Another important challenge is to determine and eliminate the inconsistent data, which is considered as noise. Once the inconsistencies are eliminated and the incomplete data are distinguished into subcategories the data is prepared for further analysis with better representation and quality.

We define the types of inconsistent and incomplete data here starting with the definition of inconsistent data. The answer to a branching question determines which alternative set of following questions to be presented to a respondent. When more than one alternative set of questions are answered, it causes inconsistent data. Therefore, inconsistent data occur when subjects do not follow the questionnaire instructions by answering questions that they were not supposed to answer (i.e. questions that they were supposed to skip). Answers given by a non-smoker to an exclusive set of questions that are specific to smoker respondents are an example of inconsistent data.

Data can be incomplete for several reasons: A question can be left unanswered because it is not applicable to a subject. For example, a non-smoking respondent is not supposed to answer a question about the 'number of cigarettes smoked per day' since the questionnaire instructs such a person to skip this question. We refer to this type of missing data that cause data incompleteness as missing data caused by skip patterns (SPMD). Most data analysts handle $S P M D$ s by recoding the data. If we have a $\mathrm{y} / \mathrm{n}$ question on smoking and a subsequent question of number of cigarettes smoked per day for smokers, recoding would consist of imputing zero for non-smokers in the latter item. The approach described in this study alleviates the labor of manually detecting and recoding each SPMD. A question that is applicable to a subject can be left unanswered out of negligence, discomfort or other reasons. We refer to this type of missing data as genuine missing data (GMD). If a branching question is not answered along with any of the alternative set of following questions it causes undetermined missing data (UMD). For example, leaving female surgery related questions as well as all the alternative questions entirely unanswered along with their branching question that asks whether the person had a female surgery causes $U M D$.

As mentioned before, one common approach to resolve incomplete data is to use imputation techniques. There have been several imputation techniques in the literature for filling the missing information in a dataset. One way is to ignore all the entries that have missing values and only focus on a subset of the data with known entries. Another way is to replace the missing entries with a statistical model, usually distributional assumptions such as multinomial normal distribution or iterated linear regression. However, the
critical assumption here is that, there is a linear relationship among the variables. Even if there is a non-linear relationship, a different model can still be modelled, but the main point here is to decide what type of a model to choose. A badly chosen model can even show worse results than the ignorance strategy. The basic idea in imputation should be to estimate the missing values by minimizing a loss function.

Penny et al. has studied three different imputation methods to confront missing data in a trauma injury dataset (Penny et al. 2006). The data is collected at a UK hospital and the injury severity related measure that is associated with patient death is missing for $12 \%$ of the patients. Three different imputation methods (hot-deck imputation, predictive model based imputation and propensity score imputation) are used. The imputed datasets are classified by artificial neural network and logistic regression. Results show that the complete case analysis (no imputation) shows slightly more accurate results (0.89) than the imputation methods (hot-deck imputation: 0.86 , predictive model based imputation: 0.86 , propensity score imputation: 0.85 ). This paper has used different imputation techniques to compare the effectiveness of the imputation methods with complete case analysis. However, they did not consider different types of incomplete data (UMD, $S P M D, G M D$ ) in their analysis.

Another possible solution to impute the incomplete data is to automatically trace the generated graph for each subject and impute the incomplete data of a particular subject with values of another subject whose pattern shows the highest similarity with that subject's pattern (similarity based imputation). In similarity based imputation, a similarity model between subjects with incomplete data (missing data) is generated,
constructing a similarity matrix of subjects and the nearest undifferentiated subject sets of each subject to impute the missing data iteratively. This approach will improve the prediction performance on small sample sizes.

Imputation methods based on rough set theory have been proposed in the literature and shown to be effective. In rough set theory, data is stored in an information table. Let $I=(U, A)$ be an information system, where $U$ be a set of objects and $A$ be a non-empty set of attributes such that $a: U \rightarrow V_{a}$. With any $P \subseteq A$ there is an equivalence relation $I N D(P): I N D(P)=\left\{(x, y) \in U^{2} \mid \forall a \in P, a(x)=a(y)\right\}$. Let $X \subseteq U$ represent the attribute subset $P$; and an arbitrary set of objects comprising $X$, and we wish to express this subset of objects using the equivalence classes induced by attribute subset $P$. Since $X$ cannot be expressed exactly (because there may cases when some objects will be included and/or excluded) the lower bounds $\underline{P} X=\left\{x \mid[x]_{P} \subseteq X\right\}$ and upper bounds.
$\bar{P} X=\left\{x \mid[x]_{P} \cap X \neq \varnothing\right\}$ of $X$ need to be defined.
Hu et al. have used a rough set theory based approach in order to impute the missing data. The data is represented in a "condition" $\rightarrow$ "decision" format where the "condition" are the attributes and the "decision" is the class label (Hu et al. 2014). They proposed an approach where the table is rearranged in such a way where the attribute with the missing values always becomes the decision attribute, and decision rules can be deducted from the attributes excluding the object with the missing attribute. Then the decision rules deducted can be used to replace the missing values. They have introduced the concept of roughness of rearrangement. The roughness of rearrangement on concept $Y$ can be defined as the following: $\beta\left(\mathcal{T}_{Y}\right)=\frac{|\overline{A(Y)}-\underline{A}(Y)|}{\bar{A}(Y)}$. Once the roughness of
rearrangements are all calculated and sorted in the descending order they end up with a list of attributes $a_{k_{1}} \ldots . a_{k_{n}}$. The roughness of an attribute determines the relationship between the attributes and the decision. If the relationship is strong, the decision rules derived from the rough set theory can be used to determine the value of the missing items; otherwise the missing values can be imputed by using other approaches. Optimal logical flow indicates the logical relationships among the attributes in an information table under a group of selected concepts. The last attribute in the ordered list, $a_{k_{n}}$, which yields the smallest roughness value, is the optimal logic attribute, i.e. the attribute with the strongest attribute decision relation. If the optimal logic attribute's roughness is less than a predetermined threshold value, then decision rules can be derived and the missing values can be imputed based on the decision rules. Otherwise, any other traditional method can be used.

Erden et al. have used rough set theory in order to calculate the accuracy of the imputation and to determine decision rules in a real life data from the US government website including the traffic accidents that took place in the USA in 2011 according to their occurrence reason in order to discover useful knowledge from the dataset (Erden et al. 2014). The dataset contains a collection of data about people involved in car accidents with fatalities, the final injuries, and alcohol/drugs tests. The decision parameter is chosen to be the fatalities in the accidents. The condition variable, fatalities, is discretized taking either " 1 " or " 2 " as " 1 " denoting 1 person and " 2 " denoting 2 or more people. The Expectation Maximization algorithm is used to impute the data. The lower and upper approximations for decision variables $d=1$ and $d=2$ are calculated. The accuracy, $\alpha$,
of the approximations are calculated $(\alpha(d, m)=\underline{A}(X) / \bar{A}(X)$, where $m=1$ or 2$)$ after imputation. We have also used a rough set based approach in order to impute the incomplete data in our dataset. However, the imputation is not applied on the entire set of incomplete data, but only on the GMDs in order to minimize the misinformation imputation may lead.

Heijden et al. used a clinical data of 398 subjects that have missing values to evaluate different imputation techniques (missing indicator method, single imputation of unconditional and conditional mean, and multiple imputations). Their aim is to diagnose the presence or absence of pulmonary embolism (Heijden et al. 2006). By using multivariable logistic regression analysis, a diagnostic prediction model is trained. Finally, the area under the ROC curve for complete case analysis, missing-indicator method, single imputation of unconditional and conditional mean, and multiple imputations are $0.794,0.813,0.775,0.792$, and 0.787 , respectively. In conclusion, the risk factors obtained from the data based on the complete case analysis were more biased than imputation techniques studied in this paper. Single imputation methods perform well because of the low overall number of missing values.

Zhang et al. has studied a clustering based imputation technique for data preprocessing (Zhang 2006). The dataset is first divided into clusters excluding the instances with missing values. Each instance with missing values is then assigned to the most similar cluster. The missing values are then replaced with values generated using kernel based methods: Deterministic Imputation (DI) and Random Imputation (SI).

Simulation data is used to evaluate the effectiveness of the strategy under missing rates of
$5 \%, 10 \%$ and $40 \%$. The results show that in most cases filling the missing values by the proposed clustering strategy lead to lower error rate than without using it in a clustering task. The SI shows better results than DI when the dataset is divided into clusters. The missing data in this paper is MCAR. That is, if the probability of a missing observation does not depend on its measurement or on other observed or unobserved measurements then the observation is MCAR (Little and Rubin 1987). Note that there is no clear mapping between the two sets of definitions MCAR, MAR, NMAR, and GMD, UMD, SPMD. In the former the definitions are based on dependencies on observed or unobserved measurements. While in latter the definitions are based on the structure of questionnaire and in particular branching questions. This fundamental difference in the bases on which such definitions is formed prevents a clear mapping of one set to the other.

Zhong has experimented with health related individual level survey data that has incompleteness (Zhong 2009). The missing variables are imputed by concentration indices. Concentration indices quantify the degree of inequality across the distribution of a variable. Missing values on health variables are unlikely to be always MCAR. A possible solution to correct the introduced bias due to the concentration indices is proposed. The imputation results are discussed when the data is MCAR, MAR or NMAR. A case study and a simulation study are used. In addition to the type of missing data, the imputation technique being utilized determines the effectiveness of the procedure. Therefore, choosing the proper imputation technique based on the data and missing data models can result in an unbiased treatment of NMAR, MAR, MCAR. In our study, on the
other hand, SPMD missing data type is not supposed to be imputed based on the instructions of the questionnaire. Hence, performing any imputation on SPMDs will introduce misinformation. In the case of UMD, the mutually exclusive paths of questions are unanswered along with the branching question itself. So, it is ambiguous which alternative set of questions (branch) is supposed to be answered by the respondent (GMD) and which set(s) is (are) supposed to be skipped (SPMD). Therefore, imputing all UMD questions results in imputing their corresponding SPMDs as well. However, if one can determine which alternative set of questions (branch) in a UMD is supposed to be answered (GMD) and which one(s) are supposed to be skipped (SPMD), then the SPMD portion should not be imputed. However, the GMD missing data type and GMD branch of UMD can be treated where an analyst may identify the type of missing data, whether MCAR, MAR or NMAR, to determine a statistically valid imputation technique.

Yuanyuan et al. have used a nearest neighbor imputation in wireless sensor networks (Yuanyuan 2014). Ambler et al. investigated a number of methods for imputing missing data to evaluate their effect on risk model estimation and the reliability of the predictions. A large national cardiac surgery database is used in this study (Ambler 2007). Jerez et al.(Jerez 2010) have evaluated the performance of several statistical and machine learning imputation methods to predict the recurrence in a real breast cancer dataset. The imputation techniques studied are mean, hot-deck and multiple imputation, and machine learning techniques, e.g., multi-layer perceptron (MLP), self-organization maps (SOM) and k-nearest neighbor (KNN). The imputation methods based on machine learning algorithms outperformed imputation statistical methods in the prediction of patient
outcome. Ouzienko et al. has proposed an imputation technique on longitudinal social surveys. The experiments are constructed on synthetic and real life datasets with $20 \%$ to $60 \%$ of nodes missing (Ouzienko 2014). As mentioned before, SPMD type missing data are not supposed to be imputed. This important notion is not of a major concern in any of these studies.

Dillman et al. on the other hand, has considered SPMD, GMD and inconsistent data, but not $U M D$. They studied the skip pattern compliance in three test forms. They defined two terms: loop error and gap error. The loop error is defined as questions that were not skipped by the respondent when they were supposed to, and the gap error is defined as questions that were skipped by the respondent when they were not supposed to. The loop error corresponds to our inconsistent data and the gap error corresponds to our GMD. This study shows that there is some likelihood that getting perfect compliance that reduces loop error may increase the likelihood of making gap errors. Three test forms (Census Form, Arrow Form and Right Box Form) are tested and compared in terms of their error rates. The three test forms only vary with regard to how the skip pattern instructions are provided. The loop error rates for three test forms are $11.4 \%, 4.3 \%, 5.2 \%$ and the gap error rates for three test forms are $0.8 \%, 1.8 \%$ and $1.6 \%$, respectively (Dillman et al. 1999). This work is one of the very few studies about skip patterns in the literature. Unlike our study, they focus on human factor analysis in order to make it easy for people to understand how to design a questionnaire with skip patterns such that it lowers the rate of error. However, they do not perform any analysis on determining the skip patterns.

Fagan and Greenberg's interesting report has inspired and given us the basis on which we have built a comprehensive mathematical framework with all required definitions, lemmas and theorems as well as their proofs (Fagan and Greenberg 1988). They focus on skip pattern analysis on questionnaires using graph theory. The missing values are grouped into undetermined data, skip patterns and missing data, which correspond to UMD, SPMD and GMD, respectively, in our study. Their report has not provided an accurate account on how to determine GMD, UMD and SPMD as it considers the questions (vertices of the questionnaire graph) and not the answers (edges of the graph). They have not applied their proposed method on real data nor have they proposed any algorithm to implement it. As a result, they have not performed complexity analysis and they have not validated their method using simulated data.

The UMD's or GMD's determined from data do not provide valuable information for the analyses without any imputation. On the other hand, the SPMD's can be used for data stratification. It is important to stratify the data into stratums which will provide the opportunity to evaluate each stratum separately.

In clinical survey data, it is common for subjects to have their own unique set of answers to the questionnaire. However, the subjects can be grouped into populations based on common answers to specific questions. It is important to divide the dataset into sub-populations based on these questions that have common answers for each population. This will help us to investigate heterogeneous results, or to answer specific questions about particular patient groups and to see whether and how risk factors vary across sub populations. This approach leads us to extract maximum amount of information from the
data and gives the clinicians the possibility to apply different treatments for different groups of people. It is important based on what questions to divide (stratify) the population into groups, which we will refer as the stratifying factors throughout this study. This part of the study proposes a method that shows how to stratify a population based on a simulated study and a longitudinal clinical survey data. Here, we use the branching questions to stratify the population. The idea is to find the significant branching questions that divide the population into well represented groups.

There have been several studies in sub group analyses. Su et al. have focused on a comparative study where two or more treatments are compared and how the treatment effect varies across subgroups induced by covariates. Treatment effect can be defined as the amount of change in a condition or symptom because of receiving a treatment compared to not receiving the treatment. They have considered a binary treatment effect (0 or 1), a continuous output and a number of covariates where the components are of mixed types (categorical and continuous). They have used a tree-structured subgroup analyses algorithm since; the tree algorithm is a well-known tool for determining the interactions between the treatment and the covariates. Their goal in subgroup analysis is to find out whether there exist subgroups of individuals in which the treatment shows heterogeneous effects, and if so, how the treatment effect varies across them. By recursively partitioning the data into two subgroups that show the greatest heterogeneity in the treatment effect, they were able to optimize the subgroup analyses. They have used simulated studies to validate their approach. Also, they have used the Current Population Survey ( $C P S$ ) database conducted by the U.S Census Bureau for the Bureau of Labor and

Statistics, in 2004. The CPS is a survey data of 60.000 households. The investigators were interested in specific subgroups of the working population where the pay gap between sexes is dominant. The questions in the survey were related to some demographic characteristics of the respondents, the employment status, hours worked and the income earned from their work. There were different covariates in the data such as gender, age, education, race, citizenship, tax status, etc. The results show that for most of the subgroups that constitute the majority of the population, women are paid significantly less than men. Also, the wage disparity between men and women varies with the industry, occupation and age. In our study, on the other hand, instead of recursively partitioning the data into sub populations, we are using the branching questions leading the skip patterns to occur as our stratifying factor.

Subgroup analyses are a highly subjective process since the subgroups themselves as well as the number of subgroups are determined by the investigator beforehand (Assmann et al. 2000). It is important to determine which specific subgroup to use in the experiment. The incorrect selection of the subgroups may cause unreliable results. Therefore, significance testing is a common approach in subgroup analyses. That is, testing the numerous plausible possibilities to see which subgroup performs better. However, this approach cannot be considered as an efficient way of splitting the data. We are utilizing the wisdom of experts embedded in the data through the questionnaire design processes when selecting the branching questions as stratifying factors.

There have been several studies on determining variables that are for understanding the underlying phenomena of interest. It is important to reduce the
dimension of original data prior to any modeling. Different attribute selection techniques have been used in order to reduce the dimensionality and the computational complexity (Azhagusundari et al.). The attributes that are significant can be extracted, ranked and weights can be assigned to each attribute to compare the significance. Decision trees based on information gain techniques have been widely used in order to perform feature selection. Decision trees divide the population into subgroups recursively until the leaf nodes represents the class labels. However, in this study none of these techniques are used to determine the significant branching questions. The feature selection techniques cannot be directly used in order to determine the significant branching questions, since the class labels of individual subjects are irrelevant. Instead high support and confidence for prospective extracted rules from each stratum is of interest. That is, a population with mixed class labels in a stratum would be favorable as long as it lends itself to rules with high confidence factor and support. For instance, smoking could be a significant branching question if the rules applied to smokers are different than those applied to nonsmokers. However, there could be populations with mixed class labels in smokers and non-smokers groups.

Risk stratification in clinical data is used to divide patients into different acuity levels and to determine a person's risk for suffering a particular condition and the need for preventive intervention. Haas et al. have used several risk stratification techniques to evaluate the performance in predicting healthcare utilization. They have studied 83 patients empanelled in 2009 and 2010 in a primary care practice. 7 different risk stratification techniques were used: Adjusted Clinical Groups (ACGs), Hierarchical

Condition Categories (HCCs), Elder Risk Assessment, Chronic Co morbidity Count, Charlson Co morbidity Index, and Minnesota Health Care Home Tiering and a combination of Minnesota Tiering and ERA. To predict the healthcare utilization and cost, historical data (data from 2009) have been used by a logistic regression model using demographic characteristics and diagnosis such as emergency department visits, hospitalizations, 30 day readmissions. The results show that ACG model outperforms the other risk stratification methods. They have studied data stratification based on the acuity of each patient and generated different results for each stratum. However, in our study the stratifying factor is unknown beforehand and needs to be determined from the existing branching questions by using statistical methods. We have used the contingency tables to divide the population based on branching questions. The Fisher's Exact Test is used in order to compare the significance of the selected branching questions.

Once the data is preprocessed, a predictive index can be constructed by starting with the detection of important attributes (risk factors). Attribute selection is a machine learning method that selects an optimal subset of attributes by eliminating the ones which contain less predictive information. Reducing the dimensionality of an attribute space improves the performance by diminishing the curse of dimensionality effect. Attribute selection also gains advantage from efficiency in terms of storage and computational costs. Also, the execution time spent for both training and testing phases will decrease.

The attribute selection methods are categorized into three different forms: filter, wrapper or embedded (Molina 2002). It is important to evaluate the existing methods and figure out which one performs better in certain situations. Some algorithms perform well
on correlated data while others can handle noise or missing values depending on the nature of the data.

In filtering methods, attribute selection is a preprocessing step independent of the induction algorithm. Information gain algorithm determines the importance of each attribute by evaluating the uncertainty reduction, while ReliefF is sampling an instance and evaluating the difference between its nearest neighbors from both the same and the opposite class. Relevance scores are assigned to each attribute. Correlation based feature selection methods eliminate one of the less important attribute that correlates with another attribute (Hall 1998). In embedded methods, inducer has its own attribute selection algorithm embedded such as J48, a widely used decision tree algorithm. The occurrence of an attribute in a tree provides information about the importance of that particular attribute. Information gain and entropy reduction methods can be applied to each candidate attribute of the decision tree node to evaluate the importance of each attribute (Sugumaran 2006). There have been several studies on attribute selection methods. Hall and Holmes (Hall 1998) use the UCI dataset, a real world data set, which contains different data types such as categorical, continuous and multivariate. Six attribute selection methods are compared in terms of classification accuracy, reduction rate and speed. Molina et al., (Molina 2002) use a simulation based randomly created binary and nominal valued dataset. Different types of syntactic functions are applied to the dataset to generate the class labels. Since it is a fully controlled case, varying number of relevant, irrelevant and redundant attributes are placed in the dataset. However, the simulated dataset doesn't lend itself to real longitudinal dataset of clinical trials and the
functions used in this study do not care about any particular attribute when deciding about the resultant, which does not fit to what we usually see in real world biomedical datasets.

Rule extraction is a machine learning method that selects formal rules from a set of observations. Given a set of training examples where the class labels are known, the aim is to find out the classification rules that will help to predict the new instances. Agrawal et al. introduced association rules for discovering the relations between variables in large databases. Rule extraction gains advantage from efficiency in terms of computational costs and after extracting the proper rules from the dataset the time spent for classification of the new instances decrease.

The different rule extraction methods used in this study are Apriori, PART, Prism and Jrip. There have been several studies on rule induction methods. Pires and Branco use a simulation based dataset to compare the results of two multinomial classification rules in terms of their performances. One of those classification rules is the Bayesian approach while the other one is the likelihood measures.

In this research, on the other hand, several attribute selection and rule extraction algorithms are applied to a simulation based dataset with longitudinal trials in order to compare the performances of the algorithms in terms of different noise levels and different incomplete data levels and combination of both. Multicollinearity is added to the dataset to evaluate the most robust algorithms when dependencies between attributes are in question. Same attribute selection and rule extraction techniques are also applied to
the combination of the longitudinal datasets. A simulation dataset is chosen since full control over the dataset is achieved.

Besides determining the most robust attribute selection and rule extraction algorithm, this experiment also helps us to determine up to what percentage of incomplete data, noise and multicollinearity can be handled.

Logistic regression is one of the most commonly used predictive modeling techniques. Wang et al. have used multilinear sparse logistic regression in order to predict the risk on clinical data (Wang 2014). We have used logistic regression in order to determine the potential predictive factors and to determine the predictive probability of each factor. The effectiveness of the potential predictive factors is determined by odd ratios and the predictive index performance is determined based on wald scores and confidence intervals.

## CHAPTER TWO

## METHOD - DATA PRE-PROCESSING

### 2.1 Analysis of Incomplete and Inconsistent Data

The next step of the proposed method is to further preprocess the incomplete and inconsistent data by converting the entire questionnaire into a directed acyclic graph $G(N, A)$ where, each question is represented by a vertex, $v \in N$, and each answer is represented by an edge $a \in A$. The questionnaires are usually designed so they can be directly converted to an acyclic graph. That is, each question is answered at most once. If this rule is not followed, one should be able to devise an acyclic equivalent of a cyclic graph. Therefore, in this study we assume a questionnaire with a corresponding acyclic graph.

There can be vertices with more than one child (branching questions) in $G$. The children of such vertices are mutually exclusive in the sense that the respondent should answer only one of them to insure only one path is visited. This corresponds to the case where the questionnaire instructs the respondent to skip some questions. The unvisited vertices based on the questionnaire's instruction (i.e., vertices out of the path) cause skip pattern missing data (SPMD). Refusing or neglecting to visit (answer) a vertex on a path, when it is not supposed to, causes genuine missing data (GMD). Refusing or neglecting to visit any of the alternative paths going through a branching vertex along with the branching vertex itself causes undetermined missing data (UMD). This type of missing
data is called undetermined because there is no way to determine whether they are GMD or $S P M D$. When more than one child of a branching vertex is visited, it causes inconsistent data. All the answers causing data inconsistency have to be removed (leaving just one valid path) as if they were never visited. Once the answers causing data inconsistency are removed, the incompleteness caused by the removal can be considered as $S P M D$.

Figure 2.1 shows a subgraph generated from questions $v_{17}$ through $v_{26}$ in MESA. The answers (edges) are labeled as $a_{k}$. For example, $a_{12}$ in Figure 2.1 represents the first set of answers to $v_{22}$ that leads the respondent to $v_{23}$, and $a_{11}$ represents the second set of answers to $v_{22}$ that leads the respondent to $v_{24}$. Unlike this example, there could be more than two possible set of answers to a question as well as only one set. The latter means regardless of the answer to such a question, the respondent will always be led to only one following question. In this case, leaving a question unanswered causes GMD. The former means, the respondent will select one answer out of two or more possible set of answers. For $v_{22}$, one option is to answer ' $a_{12}$ ' and continue with $v_{23}$. The alternative option is to answer ' $a_{11}$ ', skip $v_{23}$ and continue with $v_{24}$. Missing value caused by skipping $v_{23}$ is referred to as $S P M D$. If none of the three vertices $\left(v_{22}, v_{23}\right.$ and $\left.v_{24}\right)$ is answered the type of the missing values corresponding to these vertices cannot be determined $(U M D)$. If a subject, on the other hand, answers both questions $v_{25 a}$ and $v_{25 b}$, those two answers become mutually inconsistent and one of the alternative answers has to be removed from that subject's answer set.


Figure 2.1- Subgraph of MESA with labeled responses

The answer set for each subject generates a different path on the graph. Thus the answer set that are incomplete or inconsistent are subject dependent. Therefore, incomplete and inconsistent data have to be extracted specific to each subject. Manual methods may show less than desired reliability especially when dealing with large amounts of data. This entails an automated method to be devised.

In this part of the study, we focus solely on the female population of the MESA baseline survey (HH1). There are 1154 women in MESA aged 60 and older. First, the inconsistent data are extracted from the dataset in order to minimize the noise. Then, our objective is to distinguish the incomplete data into $G M D, S P M D$ and $U M D$. Table 2.1. shows the definitions of the basic terms being used throughout this study.

Table 2.1- Definition of basic terms

| $\boldsymbol{Q}$ | Set of Consistent Questions Answered |
| :---: | :--- |
| $Q^{*}$ | Set of Questions Answered |
| $R^{*}$ | Set of Responses of a Subject |
| $\boldsymbol{R}$ | Set of Consistent Responses of a Subject |
| $\boldsymbol{P} P_{i}$ | An augmented set of answers |
| $\cup P P_{i}$ | Actual Path |
| $\boldsymbol{I}$ | Inconsistent Data Vertices |
| $\boldsymbol{S}$ | SPMD Vertices |
| $\boldsymbol{N}$ | All Vertices |
| $\boldsymbol{M}$ | $G M D$ Vertices |
| $\boldsymbol{U}$ | $U M D$ Vertices |
| $\boldsymbol{L P}$ | Linear Path |

The first step of the analysis is to determine the inconsistent answers. After the inconsistent answers are detected and removed from the dataset, the remaining data will be consistent. The $S P M D$, the $G M D$ and $U M D$ analyses are performed on the consistent dataset.

### 2.1.1 Inconsistent Data Analysis

In this section, we present the mathematical framework of the method that determines inconsistent answers. We define two functions that return the corresponding questions of a given answer set where one returns the previous question, $P_{b}: A \rightarrow N$, and the other returns the next question $P_{a}: A \rightarrow N . P_{a}\left(\left\{a_{k}\right\}\right)$ returns the question that follows the answer $a_{k} \cdot P_{b}\left(\left\{a_{k}\right\}\right)$ returns the question corresponding to answer $a_{k}$. One can observe the following:
$P_{a}\left(\left\{a_{k}\right\}\right)=\left\{v_{j}\right\}$ and $P_{a}^{-1}\left(\left\{v_{j}\right\}\right)=\left\{a_{k}\right\}$ where $a_{k}=\left(v_{i}, v_{j}\right) \in A$.
$P_{b}\left(\left\{a_{k}\right\}\right)=\left\{v_{i}\right\}$ and $P_{b}^{-1}\left(\left\{v_{i}\right\}\right)=\left\{a_{k}\right\}$ where $a_{k}=\left(v_{i}, v_{j}\right) \in A$.
Example I: Let's consider $R_{s}^{*}=\left\{a_{2}, a_{5}, a_{4}, a_{6}, a_{8}, a_{9}, a_{10}, a_{15}, a_{17}, a_{19}\right\}$ as the set of responses of $s^{\text {th }}$ subject to the sub-questionnaire in Figure 2.1.
$Q_{S}^{*}=\left\{v_{17}, v_{17 b}, v_{17 c}, v_{17 d}, v_{19}, v_{20}, v_{21}, v_{24}, v_{25}, v_{25 a}\right\}$ contains the corresponding vertices of each edge in $R_{s}^{*}$, i.e., $Q_{s}^{*}=P_{b}\left(R_{s}^{*}\right)$.

Answering a questionnaire by a subject is equivalent to visiting the vertices of a linear path, $L P_{i}$, in the questionnaire's corresponding graph. We can formally define a linear path as follows.

Definition I: A linear path, $L P_{i}$, is defined as a set of vertices that includes the root and the terminal vertices of the graph, $G$. Further, for any vertex, $v_{k}$ in $L P_{i}$, there exists one and only one vertex, $v_{j}$ such that $\left(v_{k}, v_{j}\right)$ is in $A$, except for the terminal vertex.

We define $L P$ as the set of all such sets shown below.

$$
L P=\bigcup_{i} L P_{i}
$$

An example of a linear path can be
$L P_{1}=\left\{v_{17}, v_{17 c}, v_{17 d}, v_{18}, v_{19}, v_{20}, v_{21}, v_{22}, v_{23}, v_{23 a}, v_{25}, v_{25 a}, v_{26}\right\}$ as can be constructed from Figure 2.1.

## Definition II:

$P P_{s_{i}}=\left\{a_{j} \mid a_{j} \in A, a_{j} \in P_{b}^{-1}\left(L P_{q}\right), \exists a_{m} \in R_{s}^{*}\right.$ s.t. $a_{m} \in P_{b}^{-1}\left(L P_{q}\right)$ and $\nexists L P_{r}$ s.t. $R_{s}^{*} \cap$ $\left.P_{b}^{-1}\left(L P_{q}\right) \subset R_{s}^{*} \cap P_{b}^{-1}\left(L P_{r}\right)\right\}$

The $i$-th potential path for subject $s, P P_{s_{i}}$, is defined as a set of edges that connect the vertices of the $j$-th linear path, i.e., $P_{b}^{-1}\left(L P_{j}\right)$. Also, each $P P_{s_{i}}$ should include all the answers in $R_{S}$ (set of consistent answers). Note that there are usually more than just one potential path for each subject.

In order to explain the last condition in Definition II $\nexists L P_{r}$ s.t. $R_{s}^{*} \cap$ $P_{b}^{-1}\left(L P_{q}\right) \subset R_{s}^{*} \cap P_{b}^{-1}\left(L P_{r}\right)$, let's consider a subgraph consisting of only $v_{22}$ to $v_{25}$ in Example I with following answer set: $R_{s}^{*}=\left\{a_{13}, a_{15}\right\}$. One may imagine that there are two potential paths: $P P_{s_{1}}=\left\{a_{12}, a_{13}, a_{15}\right\}$ and $P P_{s_{2}}=\left\{a_{11}, a_{15}\right\}$. However, since $R_{s}^{*} \cap$ $P_{b}^{-1}\left(L P_{2}\right) \subset R_{s}^{*} \cap P_{b}^{-1}\left(L P_{1}\right)$, where $L P_{1}=\left\{v_{22}, v_{23}, v_{24}, v_{25}\right\}$ and $L P_{2}=\left\{v_{22}, v_{24}, v_{25}\right\}$ Definition II does not allow $P P_{S_{2}}$. This is important because later in this section when the
inconsistent answers are formally defined based on potential paths, it does not consider $a_{13}$ and $a_{15}$ inconsistent, which is intuitively the case.

Observation I: A linear path is subject independent, whereas a potential path is subject dependent.

An acyclic questionnaire graph $G$, can contain areas with multiple sub-linear paths (MSLP) and areas with a single sub-linear path (SSLP). $M S L P, B\left(N_{B}, A_{B}\right)$, can be defined as a connected subgraph where
$N_{B}=\left\{v_{i} \mid v_{i} \in\left(P_{b}\left(A-\cap_{j} P_{b}^{-1}\left(L P_{j}\right)\right) \cup P_{a}\left(A-\cap_{j} P_{b}^{-1}\left(L P_{j}\right)\right)\right)\right\}$.
If there is $v_{i} \in N_{B}$ s.t. $v_{i} \in \bigcap_{k} L P_{k}$ and $v_{i} \neq v_{B I}, v_{i} \neq v_{B T}$ where $v_{B I}$ is the initial vertex of subgraph $B$ and $v_{B T}$ is the terminal vertex of subgraph $B$ then $B\left(N_{B}, A_{B}\right)$ should be decomposed into sub MSLP's. An example of such situation is the following:

Example II: Figure 2.1 shows a connected subgraph $B^{\prime}$
where $N_{B}^{\prime}=\left\{v_{23}, v_{23 a}, v_{24}, v_{25}, v_{25 a}, v_{25 b}, v_{26}\right\}$. The initial vertex of $N_{B}^{\prime}$ is $v_{23}\left(v_{B I^{\prime}}=\right.$ $\left.\left\{v_{23}\right\}\right)$ and the terminal vertex of $N_{B}^{\prime}$ is $v_{26}\left(v_{B T^{\prime}}=\left\{v_{26}\right\}\right)$. Since
$\bigcap_{k} L P_{k}=\left\{v_{23}, v_{25}, v_{26}\right\}$ and $v_{25} \in N_{B}^{\prime}$ and $v_{25} \neq v_{B I^{\prime}}$ and $v_{25} \neq v_{B T^{\prime}}, B^{\prime}$ should be decomposed.

To decompose such $M S L P$ 's, we define $N_{B_{j}}: N_{B_{j}}=\left(\mathrm{U}_{i} L P_{i} \left\lvert\, \begin{array}{c}v_{j-1} \\ v_{j}\end{array}\right.\right)$ where $v_{j} \in$ $\left(\cap_{i} L P_{i} \left\lvert\, \begin{array}{l}v_{B I} \\ v_{B T}\end{array}\right.\right)$ and $v_{0}$ is $v_{B I}$. Note that $\left(\left.L P_{i}\right|_{v_{l}} ^{v_{k}}\right.$. refers to all linear paths starting from $v_{k}$ and ending at $v_{l}$ in $G$. Hence, the decomposition of $B^{\prime}$ in Example II returns two
connected subgraphs $B_{1}$ and $B_{2}$ where $N_{B_{1}}=\left(U_{i} L P_{i} \left\lvert\, \begin{array}{l}v_{23} \\ v_{25}\end{array}\right.\right)=\left\{v_{23}, v_{23 a}, v_{24}, v_{25}\right\}$ and $N_{B_{2}}=\left(\mathrm{U}_{i} L P_{i} \left\lvert\, \begin{array}{c}v_{25} \\ v_{26}\end{array}\right.\right)=\left\{v_{25}, v_{25 a}, v_{25 b}, v_{26}\right\}$.

The edge set, $A_{B}$, of a $\operatorname{MSLP}, B\left(N_{B}, A_{B}\right)$, can easily be defined as, $A_{B_{i}}=P_{b}^{-1}\left(N_{B_{i}}\right) \cap$ $P_{a}^{-1}\left(N_{B_{i}}\right) . S S L P, L\left(N_{L}, A_{L}\right)$, is a connected subgraph where $N_{L} \subseteq P_{b}\left(\cap P_{b}^{-1}\left(L P_{i}\right)\right) \cup$ $P_{a}\left(\cap P_{b}^{-1}\left(L P_{i}\right)\right)$ and $A_{L} \subseteq \cap P_{b}^{-1}\left(L P_{i}\right)$.

The inconsistent data occurs when more than one alternative path of a branching question is visited. For example, subject $s$ has answered both $a_{5}$ and $a_{6}$ (See $R_{S}^{*}$ ). However, there is no linear path $\left(L P_{i}\right)$ that contains both of these answers ( $a_{5}$ and $a_{6}$ ) as can be seen in Figure 2.1. We refer to those as mutually inconsistent responses. Similarly, according to subject $s$ 's responses, the following set of answer pairs are mutually inconsistent: $\left(a_{2}, a_{5}\right),\left(a_{4}, a_{5}\right)$ and $\left(a_{6}, a_{5}\right)$. However, those responses $\left(\left\{a_{2}, a_{5}, a_{4}, a_{6}\right\}\right)$ are not mutually inconsistent with any other edge in the response set $\left(\left\{a_{8}, a_{9}, a_{10}, a_{15}, a_{17}, a_{19}\right\}\right)$, since a linear path $\left(L P_{i}\right)$ can be generated from any of those edges to the remaining edges in the response set. At this point, we formally define mutually inconsistent answers.

Definition III: $a_{i}$ is inconsistent with $a_{j}$ iff $a_{i} \neq a_{j}$ and $a_{i}, a_{j} \in A_{B_{n}}$ and $a_{i}$ or $a_{j} \notin$ $\left(P P_{s_{q}} \cap P P_{s_{r}}\right)$ where $a_{i} \in P P_{s_{q}}, a_{j} \in P P_{s_{r}}$ and $P P_{s_{q}} \neq P P_{s_{r}}$.

Two different responses $(a i \neq a j)$ of a subject are mutually inconsistent with each other when they are on exclusive paths of the same multiple sub linear paths (MSLP).

Let's consider two scenarios: $R_{s}^{*}=\left\{a_{1}, a_{3}, a_{19}, a_{20}\right\}$.

Clearly, $a_{1}$ and $a_{3}$ are consistent with each other. However, one can construct two potential paths as follows: $P P_{s_{1}}=\left\{a_{1}, a_{3}, a_{5}, a_{7}, a_{8}, a_{9}, a_{10}, \ldots, a_{17}, a_{19}\right\}$ and $P P_{s_{2}}=$ $\left\{a_{1}, a_{3}, a_{5}, a_{7}, a_{8}, a_{9}, a_{10}, \ldots, a_{18}, a_{20}\right\}$. In this example, $a_{1} \neq a_{3}, a_{1}$ and $a_{3} \in A_{B_{n}}$, $a_{1} \in P P_{s_{1}}, a_{3} \in P P_{s_{2}}, P P_{s_{1}} \neq P P_{s_{2}}$. Therefore, without condition $a_{i}$ or $a_{j} \notin$ $\left(P P_{s_{i}} \cap P P_{s_{j}}\right), a_{1}$ would be inconsistent with $a_{3}$, which is not intuitively true. The second scenario: $R_{s}^{*}=\left\{a_{11}, a_{13}, a_{15}\right\}$. Obviously, $a_{13}$ and $a_{15}$ are consistent answers. However, one can construct two potential paths as follows: $P P_{s_{1}}=\left\{\ldots, a_{10}, a_{11}, a_{15}, a_{18}, a_{20}\right\}$ and $P P_{s_{2}}=\left\{\ldots, a_{10}, a_{12}, a_{13}, a_{15}, a_{18}, a_{20}\right\}$. In this example, $a_{13} \neq a_{15}, a_{13}$ and $a_{15} \in A_{B_{n}}, a_{15} \in P P_{s_{1}}, a_{13} \in P P_{s_{2}}, P P_{s_{1}} \neq P P_{s_{2}}$. Therefore, without condition $a_{i}$ or $a_{j} \notin\left(P P_{s_{i}} \cap P P_{s_{j}}\right), a_{13}$ would be inconsistent with $a_{15}$, which is not intuitively true. Condition $a_{i}$ or $a_{j} \notin\left(P P_{s_{i}} \cap P P_{s_{j}}\right)$ resolves both of these counterintuitive situations explained in the above two scenarios.

Once the mutually inconsistent edges are detected, a failed edit graph (FEG) and its complement is generated to find a minimal deletion set (MDS). FEG, $F(N, A)$, is generated by creating a vertex for each inconsistent answer and then joining each pair of mutually inconsistent vertices with an edge. Figure 2.2 shows a FEG for the given example above.

Definition IV: A failed edit graph (FEG) is an undirected graph $F\left(N_{F}, A_{F}\right)$ s.t. $N_{F} \subseteq$ $\left(A-\bigcap_{l} P_{b}^{-1}\left(L P_{l}\right)\right)$ where for any $a_{i}, a_{j} \in A_{B_{n}}$ and $a_{i}, a_{j} \in N_{F},\left(a_{i}, a_{j}\right) \in$
$A_{F}$ iff $\exists P P_{s_{k}}$ s.t $a_{i} \in P P_{s_{k}}$ and $a_{j} \notin P P_{s_{k}}$ and $a_{i}$ or $a_{j} \notin\left(P P_{s_{i}} \cap P P_{s_{j}}\right)$ where $a_{i} \in$ $P P_{s_{i}}, a_{j} \in P P_{s_{j}}$ and $P P_{s_{i}} \neq P P_{s_{j}}$ for subject $s$.

Observation II: A different FEG is generated for each MSLP with inconsistent responses.

MDS is the minimum number of vertices that need to be removed from the answer set in order to generate a consistent answer set. If $a_{2}, a_{4}$ or $a_{6}$ is removed from the dataset, FEG will still be connected. However, if $a_{5}$ is removed, the FEG becomes disconnected, which means there are no inconsistencies. Hence, $a_{5}$ should be removed from the response set.

The new response set will be referred to by $R_{s}$ where the absence of the asterisk denotes the fact that this response set is consistent:


Figure 2.2- Failed Edit Graph
$\left(R_{S}=\left\{a_{2}, a_{4}, a_{6}, a_{8}, a_{9}, a_{10}, a_{15}, a_{17}, a_{19}\right\}\right)$. The corresponding vertices for $R_{S}$ will be: $Q_{S}=\left\{v_{17}, v_{17 c}, v_{17 d}, v_{19}, v_{20}, v_{21}, v_{24}, v_{25}, v_{25 a}\right\}$. Since $a_{5}$ is removed from $R_{S}^{*}, v_{17 b}$ which is the corresponding vertex (question) of $a_{5}$ also needs to be removed from $Q_{S}^{*}$. However, the FEG is not always as simple as it is in the given example, which means finding MDS could become fairly complex. Therefore, a method is generated by Fagan and Greenberg to find MDS (Fagan et al. 1988).

The method for detecting the MDS is the following: Given a FEG, $F$, the vertex set in $F$, i.e., $N$ and a largest maximal clique $C^{M}$ in the complement of the FEG, $F^{c}$, the minimal deletion set is $N-C^{M}$. Note that a clique is an undirected graph such that every two vertices are connected by an edge.

The complement of $F$ is $F^{c}\left(N_{F}, A_{F}^{c}\right)$ which consists of a set of connected graphs (cliques) each of which are denoted by $C\left(N_{c}, A_{c}\right)$ where $N_{c} \subseteq N_{F}, A_{c} \subseteq A_{F}^{c}$ and $F^{C}=$ $\mathrm{U}_{i} C_{i}$. The vertices on each clique are answers in a linear path, whereas, the vertices on two different cliques are not on the same linear path. We define $C^{M}\left(N_{c}^{M}, A_{c}^{M}\right)$ as the clique with a vertices set, $A_{c}^{M}$, of maximum cardinality (largest maximal clique). Now consider the same example given in Figure 2.2. The complement of the graph is shown in Figure 2.3, where the largest maximal clique is $C_{s}^{M}$. The set $N-C_{s}^{M}$ returns $\left\{\left\{a_{5}\right\}\right\}$, which is the minimal deletion set.


Figure 2.3-Minimal Deletion Set

Lemma I: $C$ is complete.
Proof: Let's assume $C$ is not complete. It means $\exists a_{i}, a_{j} \in N_{c_{s}}$, for the s-th subject, such that $\left(a_{i}, a_{j}\right) \notin A_{c_{s}}$, which means $\left(a_{i}, a_{j}\right) \in A_{F_{s}}$ for subject $s$ (Since the cliques are generated based on the complement of the FEG). However, from the definition of $C$ we know that $a_{i}, a_{j} \in P_{b}^{-1}\left(L P_{k}\right)$ and therefore $\left(a_{i}, a_{j}\right) \notin A_{F_{s}}$, which is a contradiction and our assumption that $C$ is not complete is not true.

Lemma II: Inconsistencies only happen in a MSLP.
Proof: One has to show two things to prove this. 1) It is possible to have inconsistencies in MSLP , and 2) it is impossible to have inconsistencies in SSLP. We prove (1) by constructing an answer set, $R_{S}^{*}$, where $\exists a_{i}, a_{j} \in R_{s}^{*}$ s.t. $a_{i} \in P_{b}^{-1}\left(L P_{k}\right)$ and $a_{j} \in P_{b}^{-1}\left(L P_{l}\right)$ and $L P_{k} \neq L P_{l}$ and $a_{i}, a_{j} \in A_{B_{n}}$ and $R_{S}^{*} \cap_{l} P_{b}^{-1}\left(L P_{l}\right) \not \subset R_{S}^{*} \cap_{k} P_{b}^{-1}\left(L P_{k}\right)$. Based on

Definition II $a_{i} \in P P_{s_{i}}$ and $a_{j} \in P P_{s_{j}}$ where $P P_{s_{i}} \neq P P_{S_{j}}$. Definition III constitutes inconsistency between $a_{i}$ and $a_{j}$. Proof of (2) can be directly derived from Definition III, where inconsistencies are only defined for responses in $\operatorname{MSLP}\left(a_{i}, a_{j} \in A_{B_{n}}\right)$, which means responses in $\operatorname{SSLP}\left(a_{i}, a_{j} \in A_{L}\right)$ cannot be inconsistent with each other. This proves Lemma II and further the latter leads to Observation II.

Note that, lack of inconsistency in only part of an MSLP does not guarantee consistency. Therefore in order to identify inconsistency an MSLP is supposed to be investigated in its entirety.

Observation III: There can be no inconsistencies in SSLP.
Lemma III: There can be no inconsistencies between two MSLP's.
Proof: This can be directly derived from Definition III, where inconsistencies only occur within a $\operatorname{MSLP}\left(a_{i}, a_{j} \in A_{B_{k}}\right)$. Since $a_{i} \in A_{B_{m}}$ and $a_{j} \in A_{B_{n}}$ and $A_{B_{m}} \neq A_{B_{n}}$ in two different MSLP's $a_{i}$ and $a_{j}$ cannot be inconsistent.

Theorem I: The set of all inconsistent answers for subject, $s, I_{s}$ can be computed by the following equation. ( $N_{F_{S_{n}}}$ denotes the vertices of the $n-t h$ failed edit graph for subject s.)

$$
I_{s}=\bigcup_{n}\left(N_{F_{s_{n}}}-N_{c_{s_{n}}}^{M}\right)
$$

Proof: To show this, we need to prove two statements: 1) $\forall a_{i} \in I, \exists a_{j} \in R_{s}^{*}-I$ s.t. $a_{i}$ and $a_{j}$ are inconsistent, and 2) there is no inconsistency within $R_{s}^{*}-I$. We show the first statement (1) using contradiction. Assume the subject only has one failed edit graph ( $n=1$ ). Suppose $a_{i} \in N_{c_{s}}$ where $N_{c_{s}} \subseteq N_{F_{S}}$. Since $\exists N_{c_{s}}^{M} \subset N_{F_{s}}$ and $N_{c_{s}}^{M} \cap N_{c_{s}}=\emptyset, \nexists a_{j} \in$
$N_{c_{s}}^{M}$ s.t. $\left(a_{i}, a_{j}\right) \in A_{c_{s}}^{M}$, which means $a_{i}$ is not connected to any vertex in $N_{c_{s}}^{M}$. Therefore, $a_{i}$ is inconsistent with all answers (vertices) in $C_{s}^{M}$, which means that $\exists a_{j} \in N_{F_{S}} \subseteq R_{s}^{*}-$ $I$ s.t. $a_{i}$ and $a_{j}$ are inconsistent. The second statement (2) can be shown as follows: We know that $R_{s}^{*}-I=N_{c_{s}}^{M} \cup\left(R_{s}^{*} \cap\left(\bigcap_{k} P_{b}^{-1}\left(L P_{k}\right)\right)\right)$. If $a_{i}, a_{j} \in N_{c_{s}}^{M} \Rightarrow\left(a_{i}, a_{j}\right) \in A_{c_{s}}^{M}$, which means $C_{s}^{M}$ is complete as proved in Lemma 1. Therefore, there is no inconsistency within $N_{c_{s}}^{M}$. Also, from Lemma II; we know that answers on the same linear path cannot be inconsistent. Therefore, there is no inconsistency within $\left(R_{s}^{*} \cap\left(\bigcap_{k} P_{b}^{-1}\left(L P_{k}\right)\right)\right)$. Finally, we have to show that answers in $\left(R_{s}^{*} \cap\left(\bigcap_{k} P_{b}^{-1}\left(L P_{k}\right)\right)\right)$ and $N_{c_{s}}^{M}$ are mutually consistent. Assume, $\exists a_{i}, a_{j}, a_{i} \in N_{c_{s}}^{M}, a_{j} \in\left(R_{s}^{*} \cap\left(\bigcap_{k} P_{b}^{-1}\left(L P_{k}\right)\right)\right)$ s.t. $a_{i}$ is inconsistent with $a_{j}$. This implies that $\nexists P P_{s_{i}}$ s.t. $a_{i}, a_{j} \in P P_{s_{i}}$. However, one can construct a $P P_{s_{i}}$ that includes all $N_{c_{s}}^{M}$ and $\left(R_{s}^{*} \cap\left(\bigcap_{k} P_{b}^{-1}\left(L P_{k}\right)\right)\right)$, which contradicts with the implication of the assumption. Therefore, the answers in $\left(R_{s}^{*} \cap\left(\bigcap_{k} P_{b}^{-1}\left(L P_{k}\right)\right)\right)$ and $N_{c_{s}}^{M}$ are mutually consistent.

Theorem II: $R_{S}^{*}-I$ provides one and only one possible path iff $\exists a_{i} \in R_{S}^{*}$ s.t. $a_{i} \in A_{B_{n}}$ the edge set of a $M S L P$.

Proof: First, we should prove that there is a potential path using $R_{S}^{*}-I$ (1). Then, we have to show the uniqueness of this path (2).
(1) Since $\exists a_{i} \in R_{S}^{*}$, according to Definition II, $\exists P P_{S_{i}}$.
(2) $a_{i}$ and $a_{j}$ cannot be in $A_{L}$, because of Lemma II. Let us consider $a_{i}, a_{j}$ in $N_{c_{s}}^{M}$. If $a_{i}, a_{j} \in N_{c_{s}}^{M}$ they cannot represent two different $P P_{s_{k}}$ 's, that is in contradiction
with $P P_{s_{i}} \neq P P_{S_{j}}$. If $a_{i} \in N_{c_{s}}^{M}$ and $a_{j} \in N_{C_{s}} \neq N_{c_{s}}^{M}$ then $a_{j} \notin R_{s}^{*}-I$ which means $P P_{S_{j}}$ cannot exist.

### 2.1.2. Incomplete Data Analysis

In this section, we introduce a method to detect incomplete data (SPMD, GMD and $U M D$ ). The first step of the analysis is to determine the potential paths for each response set. Example I shows that the potential paths for
$R_{S}=\left\{a_{2}, a_{4}, a_{6}, a_{8}, a_{9}, a_{10}, a_{15}, a_{17}, a_{19}\right\}$ are the following:
$P P_{s_{1}}=\left\{a_{2}, a_{4}, a_{6}, a_{7}, a_{8}, a_{9}, a_{10}, a_{11}, a_{15}, a_{17}, a_{19}\right\}$
$P P_{s_{2}}=\left\{a_{2}, a_{4}, a_{6}, a_{7}, a_{8}, a_{9}, a_{10}, a_{12}, a_{13}, a_{15}, a_{17}, a_{19}\right\}$
The determined potential paths can then be used in the analysis of SPMD, GMD and $U M D$.
2.1.2.1. SPMD Analysis. The first step of the analysis is the detection of SPMD. $S P M D$ can only occur within MSLP's. When an edge is not an element of any potential path, it will be a $S P M D$ referred to by $S . S P M D$ can be formally defined as;

Definition V: $a_{k} \in A$ is in $S P M D$ if $\nexists P P_{s_{i}}$ s.t. $a_{k} \in P P_{s_{i}}$
Therefore, $S P M D$ can be estimated by the following equation:

$$
S_{s}=A-\bigcup_{i} P P_{S_{i}}
$$

Hence; Example I shows that $S_{s}=\left\{a_{1}, a_{3}, a_{5}, a_{14}, a_{16}, a_{18}, a_{20}\right\}$.
2.1.2.2.GMD Analysis. The next step of incomplete data analysis is to determine the $G M D$ referred to by $M$. If an edge is an element of a potential path, but not a member of the consistent answer set, $R_{S}$, it can be either a $U M D$ or a $G M D$. If this condition is met ( $a_{m} \in P P_{S_{i}}$ for any i, and $a_{m} \notin R_{S}$ ) within an $\operatorname{SSLP}\left(a_{m} \in A_{L}\right)$, we can refer to this edge as a $G M D$. Otherwise, if $a_{m} \in A_{B}$, we can refer to $a_{m}$ as a $G M D$ only if there exists at least one edge , $a_{k}$, in the same $P P_{i}$ within the MSLP that ensures $a_{k} \in R_{s}$.

GMD ( $M$ ) can be formally defined as;
Definition VI: $M=\left\{a_{m} \mid \forall i, a_{m} \in \cap_{i} P P_{s_{i}}\right.$ and $\left.a_{m} \notin R_{s}\right\}$
Hence, $G M D(M)$ can be calculated by the following formula:

$$
M_{s}=\bigcap_{i} P P_{S_{i}}-R_{S}
$$

Example I shows that $M_{s}=\left\{a_{7}\right\}$ for the $s^{\text {th }}$ subject.
Observation IV: $M_{s}=\bigcap_{i} P P_{s_{i}}-R_{s} \neq P_{b}^{-1}\left(\cap_{i}\left(P_{b}\left(P P_{s_{i}}\right)\right)-Q_{s}\right)$
This observation can be shown by a counter example shown below:
The potential paths for the response set $R_{s}$ are;

$$
\begin{aligned}
& P P_{s_{1}}=\left\{a_{2}, a_{4}, a_{6}, a_{7}, a_{8}, a_{9}, a_{10}, a_{11}, a_{15}, a_{17}, a_{19}\right\} \\
& P P_{s_{2}}=\left\{a_{2}, a_{4}, a_{6}, a_{7}, a_{8}, a_{9}, a_{10}, a_{12}, a_{13}, a_{15}, a_{17}, a_{19}\right\} . \text { The intersection of all }
\end{aligned}
$$

$$
\text { potential paths is } \bigcap_{i} P P_{s_{i}}=\left\{a_{2}, a_{4}, a_{6}, a_{7}, a_{8}, a_{9}, a_{10}, a_{15}, a_{17}, a_{19}\right\} \text {. Therefore, the left }
$$ hand side of the equation becomes $\bigcap_{i} P P_{S_{i}}-R_{s}=\left\{a_{7}\right\}$. The right hand side of the equation first converts the edges within the potential paths to their corresponding vertices:

$$
\left.P_{b}\left(P P_{s_{1}}\right)=v_{17}, v_{17 c}, v_{17 d}, v_{18}, v_{19}, v_{20}, v_{21}, v_{22}, v_{24}, v_{25}, v_{25 a}\right\} \text { and } P_{b}\left(P P_{s_{2}}\right)=
$$ $\left\{v_{17}, v_{17 c}, v_{17 d}, v_{18}, v_{19}, v_{20}, v_{21}, v_{22}, v_{24}, v_{25}, v_{25 a}\right\}$. The intersection of those sets becomes $\bigcap_{i} P_{b}\left(P P_{s_{i}}\right)=\left\{v_{17}, v_{17 c}, v_{17 d}, v_{18}, v_{19}, v_{20}, v_{21}, v_{22}, v_{24}, v_{25}, v_{25 a}\right\}$. Therefore, the set of genuine missing questions of the $s^{\text {th }}$ subject is $\bigcap_{i} P_{b}\left(P P_{s_{i}}\right)-Q_{s}=\left\{v_{18}, v_{22}\right\}$. The corresponding edges of the genuine missing questions are $P_{b}^{-1}\left(\cap_{i}\left(P_{b}\left(P P_{s_{i}}\right)\right)-Q_{s}\right)=\left\{a_{7}, a_{12}, a_{11}\right\}$. The left hand side of the equation is not equal to the right hand side $\left(\left\{a_{7}\right\} \neq\left\{a_{7}, a_{12}, a_{11}\right\}\right)$. Therefore, $\cap_{i} P P_{s_{i}}-R_{s} \neq$ $P_{b}^{-1}\left(\cap_{i}\left(P_{b}\left(P P_{s_{i}}\right)\right)-Q_{s}\right)$.

2.1.2.3.UMD Analysis. The last step of our incomplete data analysis is to determine the $U M D(U) . U M D$ occurs when none of the questions are responded within a $\operatorname{MSLP}\left(A_{B} \neq \emptyset\right)$. In this case, due to the lack of responses in $A_{B}$, it is impossible to entitle an edge as $G M D$ or $S P M D$. For such edges, the status is not determined. $U M D(U)$ can be estimated by the following formula:

$$
U_{s}=\bigcup_{i} P P_{s_{i}}-\bigcap_{i} P P_{s_{i}}
$$

Clearly, $U M D$ can also be estimated by; $U_{s}=A-R_{s}-M_{s}-S_{s}$
Hence $U_{s}=\left\{a_{11}, a_{12}, a_{13}\right\}$.
Observation V: SPMD and $U M D$ edges can occur within a $M S L P$; however $G M D$ edges can occur both within both MSLP and SSLPs.

The same method (detection of $S P M D, U M D, G M D$ and inconsistent data analysis) is applied over the entire MESA data starting from the first attribute as the initial vertex and the $825^{\text {th }}$ attribute as the terminal vertex.

### 2.2. Implementation

In this section the implementation of inconsistent and incomplete data are discussed.

### 2.2.1. Implementation of Inconsistent Data Analysis

In this section, we present a technique for detecting inconsistent data. According to Lemma I, all the subgraphs in the complement of the failed edit graph are complete and therefore they are already maximal cliques. Therefore, the number of vertices of each maximal clique should be counted and the one with the maximum number of vertices should be determined (i.e. largest maximal clique), which has a polynomial time complexity. Once the largest maximal clique is determined, the vertices that are not components of the largest maximal clique are removed from $R_{S}^{*}\left(R_{s}^{*}-N_{C_{s}}^{M}\right)$, in order to prevent inconsistencies.

It is expected that the number of questions being asked in a questionnaire to be very limited, since the questionnaires are designed for human. Even in the case when the input data is large and complex, the largest maximal clique detection algorithm is only applied on areas that contain multiple sub-linear paths (MSLP) with mutually inconsistent edges (not on the entire graph, $G$, since inconsistencies can only occur on MSLP portions). Therefore, extending this work to other datasets would not cause poor run times or inefficient memory usage.

### 2.2.2. Implementation of Incomplete Data Analysis

In this section, we present an implementation for detecting the potential paths for each subject. Once the potential paths are detected, the GMD, SPMD and $U M D$ vertices can be calculated by the formulas defined in the previous section. The first step is to create two different matrices that represent the MESA graph (Node Matrix $\left(M_{1}\right)$, and Edge Immediate Successor Matrix $\left(M_{2}\right)$ ). Note that, these matrices are subjectindependent.

The Node Matrix shown in Table $2.2,\left(M_{1}\right)$, corresponds to a portion of the graph in Figure 2.1 (from $v_{17}$ to $v_{20}$ ). $M_{1}$ is generated by the following function.
$M_{1}(i, j)=\left\{\begin{array}{cc}a_{m} & \text { if } \exists a_{m} \in \text { A s.t. } a_{m} \\ 1 & \text { if } i=j \\ 0 & \text { otherwise }\end{array}\right.$
The Edge Immediate Successor Matrix $\left(M_{2}\right)$ shown in Table 2.3 is generated by the following function.
$M_{2}(i, j)=\left\{\begin{array}{cc}1 & \text { if the terminal node of } a_{i} \text { is the initial node of } a_{j} \\ 1 & i=j \\ 0 & \text { otherwise }\end{array}\right.$
Once the matrices are generated, Algorithm 1 is designed to find the linear and potential paths. After the linear and potential paths are determined, the formulas explained in the Method Section can be used to determine the $G M D, U M D$ and $S P M D$ s.

## Algorithm 1-Implementation for Determining Linear and Potential Paths /*The algorithm begins with an "initial" vertex, $v_{\text {init }}$. It then iteratively transitions from the current vertex to an adjacent, $P_{a}\left(P_{b}^{-1}\left(v_{\text {init }}\right)\right)$, until it can no longer find an

unexplored vertex to transition to from its current location. The algorithm is executed degree $\left(v_{\text {init }}\right)$ times.*/

Define global $k=1, t=0 \quad / / k$ and $t$ are arbitrary indices.
$v_{\text {current }}=v_{\text {init }} \quad / / v_{\text {init }}$ is the initial vertex and $L P$ is a linear path
Define FindLinearPath $\left(v_{\text {current }}, L P\right)$
\{
For all $P_{b}^{-1}\left(v_{\text {current }}\right)$ incident to $v_{\text {current }}$ If $P_{b}^{-1}\left(v_{\text {current }}\right)$ is unexplored then
$L P_{k}=L P_{k} \cup P_{a}\left(P_{b}^{-1}\left(v_{\text {current }}\right)\right)$
FindLinearPath $\left(P_{a}\left(P_{b}^{-1}\left(v_{\text {current }}\right)\right), L P_{k}\right)$
Increment $k$

## End If

End for
\}
$/ *$ Below for each linear path, a condition is checked to see whether that linear path's corresponding edges, the consistent response set $R_{s}$ is a subset of $P_{b}^{-1}\left(L P_{i}\right)$. Each $P_{b}^{-1}\left(L P_{i}\right)$ satisfying this condition is assigned to a potential path. */

Define FindPotentialPath $\left(R_{S}, L P\right)$
\{
For $i=1: k-1$
If $R_{S}$ is a subset of $P_{b}^{-1}\left(L P_{i}\right)$ then

$$
P P_{s_{t}}=P_{b}^{-1}\left(L P_{i}\right)
$$

## Increment $t$

## End If

## End For

\}
We analyze this algorithm for its worst case time complexity starting with FindLinearPath function. The worst case happens when each vertex is connected to as many other vertices as possible. This entails $n-1$ vertices (and the same number of edges) for the first vertex and $n-2$ vertices for the second vertex and so on. The total number of recursive calls to this function would therefore be $W(n)=(n-1)+(n-$ 2) $+\cdots+1=n(n-1) / 2 \in \Theta\left(n^{2}\right)$. The second function, FindPotentialPath, requires each element in $R_{s}$ to be compared with elements in $P_{b}^{-1}\left(L P_{i}\right)$. This should be performed for each linear path. An upper bound for the worst case complexity for the latter function would be for a case when $\operatorname{Card}\left(R_{s}\right), \operatorname{Card}\left(P_{b}^{-1}\left(L P_{i}\right)\right)$ and $\#\left(L P_{i}\right)$ are at their maximum value, which is $\operatorname{Card}(A)$. The notations $\operatorname{Card}(A)$ and $\#\left(L P_{i}\right)$ are the cardinality of the set $A$ (set of edges in $G$ ) and the number of linear paths, respectively. As mentioned this is an upper bound for the worst case complexity as \# $\left(L P_{i}\right)$ cannot be more than half of $\operatorname{Card}(A)$ since there should always be one initial and one terminal vertices in a questionnaire's graph. Moreover, when $\operatorname{Card}\left(P_{b}^{-1}\left(L P_{i}\right)\right)$ is at its maximum value, $\#\left(L P_{i}\right)$ is one. Note that the largest value of $\operatorname{Card}(A)$ in terms of $n, \operatorname{Card}(N)$, is in the order of $n^{2}$. Therefore, the upper bound would be $O\left(n^{6}\right)$. We can conclude that the upper bound of the worst case time complexity of the entire algorithm is $\left(n^{6}\right)$, which is a polynomial complexity.

Table 2.2-Node matrix ( $M_{1}$ )

|  | $v_{17}$ | $v_{17 a}$ | $v_{17 b}$ | $v_{17 c}$ | $v_{17 d}$ | $v_{18}$ | $v_{19}$ | $v_{20}$ | $\ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $v_{17}$ | 1 | $a_{1}$ | 0 | $a_{2}$ | 0 | 0 | 0 | 0 |  |
| $v_{17 a}$ | 0 | 1 | $a_{3}$ | 0 | 0 | 0 | 0 | 0 |  |
| $v_{17 b}$ | 0 | 0 | 1 | 0 | 0 | $a_{5}$ | 0 | 0 |  |
| $v_{17 c}$ | 0 | 0 | 0 | 1 | $a_{4}$ | 0 | 0 | 0 |  |
| $v_{17 d}$ | 0 | 0 | 0 | 0 | 1 | $a_{6}$ | 0 | 0 |  |
| $v_{18}$ | 0 | 0 | 0 | 0 | 0 | 1 | $a_{7}$ | 0 |  |
| $v_{19}$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | $a_{8}$ |  |
| $v_{20}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |  |
| $\ldots$ |  |  |  |  |  |  |  |  |  |

Table 2.3-Edge Immediate successor matrix ( $M_{2}$ )

|  | $a_{1}$ | $a_{2}$ | $a_{3}$ | $a_{4}$ | $a_{5}$ | $a_{6}$ | $a_{7}$ | $a_{8}$ | $a_{9}$ | $a_{10}$ | $a_{11}$ | $a_{12}$ | $\ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $a_{1}$ | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| $a_{2}$ | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| $a_{3}$ | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| $a_{4}$ | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| $a_{5}$ | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |  |
| $a_{6}$ | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |  |
| $a_{7}$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |  |
| $a_{8}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |  |
| $a_{9}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |  |
| $a_{10}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |  |
| $a_{11}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |  |
| $a_{12}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |  |
| $\ldots$ |  |  |  |  |  |  |  |  |  |  |  |  |  |

### 2.3. Stratification

Fisher's Exact Test is a statistical significance test used in the analysis of contingency tables. It is used for all sample sizes. The significance of the deviation from a null hypothesis can be calculated exactly. Therefore, it is not necessary to rely on an approximation that becomes exact in the limit as the sample size grows to infinity. The Fisher's Exact Test is used to determine if there are nonrandom associations between the two variables.

As mentioned before, the dataset is divided into sub populations $\left(\right.$ Branch $_{1}$, Branch $\left._{2}\right)$ based on each branching question. Even though, this method can be applied on each question, we limit the number of tests we are using by the branching questions for the following reason: when dividing the population into subgroups, the split is induced by a threshold which is determined by the expert knowledge for each branching question. However, for each non-branching question determining a threshold for each type of answer (categorical, binary, numeric) may lead to incorrect classifications.

Table 2.4 shows a contingency table where the p values are calculated by the following formula:

$$
P=\frac{\left(\binom{a+b}{a}\binom{c+d}{c}\right.}{\binom{n}{a+c}}=\frac{(a+b)!(c+d)!(a+c)!(b+d)!}{a!b!c!d!n!}
$$

Here, $R_{1}, R_{2}$, and $R_{3}$ are associations rules extracted from datasets Branch $_{1}$ or Branch $_{2}$. The values $a, b, c$ and $d$ are the number of subjects that support/contradict the extracted rules in those two datasets. A different contingency table needs to be generated for each branching question.

Table 2.4 - Contingency Table for Stratification


### 2.3.1. Simulation

The simulation is created by generating two independent binary datasets. The first dataset $D_{1}$ and the second dataset $D_{2}$ both contain 1000 subjects where $S_{1}=$ $\left\{S_{11}, S_{21}, S_{31}, \ldots, S_{10001}\right\}$ is the subject set of $D_{1}$ and $S_{2}=\left\{S_{12}, S_{22}, S_{32}, \ldots, S_{10002}\right\}$ is the subject set of $D_{2}$ and 15 common attributes $\left(A=\left\{A_{1}, A_{2}, A_{3}, \ldots, A_{15}\right\}\right)$. Three different rules are embedded to each dataset. The rules are generated in the sense that none of the rules contradict with another rule. There is no attribute being used in more than one rule. The class labels contain both the classes from the baseline and the first follow up. ' $C-I$ ' is an example of a response indicating that the subject was continent in the baseline and became incontinent in the first follow-up. The rules that are embedded to $D_{l}$ are as follows:

$$
\begin{aligned}
& R_{11}=A 1=0 \& A 3=1 \Rightarrow I-C \\
& R_{21}=A 5=0 \& A 7=1 \Rightarrow C-I
\end{aligned}
$$

$R_{31}=A 10=1 \& A 12=1 \Rightarrow C-I$
The rules that are embedded to $D_{2}$ are as follows:
$R_{12}=A 2=0 \& A 11=0 \Rightarrow C-I$
$R_{22}=A 4=1 \& A 9=0 \Rightarrow C-I$
$R_{32}=A 6=0 \& A 8=1 \Rightarrow I-C$
The two datasets are then combined. The combination $\left(D_{1}+D_{2}\right)$, have 2000 subjects $S_{D_{1}+D_{2}}=\left\{S_{11}, S_{21}, S_{31}, \ldots, S_{10001}, S_{12}, S_{22}, S_{32}, \ldots, S_{10002}\right\}$ and 15 attributes $\left(A=\left\{A_{1}, A_{2}, A_{3}, \ldots, A_{15}\right\}\right)$. Three attributes are then added to the combined dataset. Those attributes each represent a branching question having binary values. First branching question $B Q_{1}$, takes value ' 0 ' for each subject existing in $D_{1}$ and value ' 1 ' for each subject existing in $D_{2}$. The second and third branching questions, $B Q_{2}$ and $B Q_{3}$ take random binary values.

The combined dataset is then separated into two subsets based on the values of each branching question. The subset for $B Q_{1}$ taking value ' 0 ' is $D_{B Q_{1}^{0}}$ and the subset for $B Q_{1}$ taking value ' 1 ' is $D_{B Q_{1}^{1}}$, similarly the subsets for $B Q_{2}$ are $D_{B Q_{2}^{0}}$ and $D_{B Q_{2}^{1}}$ and the subsets for $B Q_{3}$ are $D_{B Q_{3}^{0}}$ and $D_{B Q_{3}^{1}}$. Figure 2.4 shows an example of the combined dataset with branching questions that are separated.

The association rule mining algorithm, Apriori, is used to extract the rules of each subset, since it was proofed to outperform other rule extraction techniques. (Arslanturk et. al.). This is explained in detail in Section 3.3. Apriori is an association rule that iteratively reduces the minimum support until it finds the required number of rules with the given minimum confidence (W. Cohen).


Figure 2.4- Simulation of Stratification

Once the rules are extracted for each subset, a contingency table is created for each branching question. A contingency table is a matrix format that displays the frequency distribution of the variables. The rows of the contingency table denote the rules associated with that branching question. The columns are the subsets. For example, for $B Q_{2}$, the rows of the contingency table are the rules extracted from subset $\boldsymbol{D}_{B Q_{2}^{0}}$ and $\boldsymbol{D}_{\boldsymbol{B} \boldsymbol{Q}_{2}^{1}}$, respectively. The columns are the datasets $\boldsymbol{D}_{\boldsymbol{B} \boldsymbol{Q}_{2}^{0}}$ and $\boldsymbol{D}_{\boldsymbol{B} Q_{2}^{1}}$. The contingency table is created to display the relative frequencies, i.e. the support/no support of each rule for each subset.

Once the contingency table is created, the p-values are calculated using the Fisher's Exact Test explained in the Methods Section. Based on the p-values that are calculated, we can determine the branching questions that are statistically significant.

A branching question is a good stratifying factor when it is statistically significant, since different rules (hence different risk factors and predictive factors) are extracted from its sub populations. Therefore, those two sub populations cannot be treated the same. If data is not stratified into sub populations when the branching question is determined to be significant, one may skip some important risk factors and predictive factors.

### 2.4. Imputation Using Rough Set Theory

In rough set theory, data is stored in an information table. Let $I=(U, A)$ be an information system, where $U$ be a set of objects and $A$ be a non-empty set of attributes such that $a: U \rightarrow V_{a} . V_{a}$ be the set of values attribute $a$ may take. The information table assigns a value $a(x)$ from $V_{a}$ to each attribute $a$ and object $x$ in the universe $U$.

With any $P \subseteq A$ there is an equivalence relation $\operatorname{IND}(P): \operatorname{IND}(P)=$ $\left\{(x, y) \in U^{2} \mid \forall a \in P, a(x)=a(y)\right\}$. The objects that are the elements of an equivalence class are indistinguishable. Let $X \subseteq U$ represent the attribute subset $P$; and an arbitrary set of objects comprising $X$, and we wish the express this subset of objects using the equivalence classes induced by attribute subset $P$. Since $X$ cannot be expressed exactly (because there may cases when some objects will be included and/or excluded due to the indistinguishable relations) the lower bounds $\underline{P} X=\left\{x \mid[x]_{P} \subseteq X\right\}$ and upper bounds $\bar{P} X=$ $\left\{x \mid[x]_{P} \cap X \neq \emptyset\right\}$ of $X$ can be defined. In other words, either all the objects being part of the equivalence class need to be included if at least one of them is an object comprising $X$ (upper bound), or all of the objects being part of the equivalence class need to be excluded (lower bound).

As you may recall, the GMDs were determined in Section 2.1.2. They are the only incomplete data type that is imputed in this study. The first step of imputation starts with dichotomizing the MESA data. 0 denotes a subject towards continence and 1 denotes a subject towards incontinence. The equivalence classes are determined from the dichotomized data. The subjects containing one or more GMD are assigned to the appropriate equivalence classes, if any. Note that one subject with one or more GMD can be assigned to more than one equivalence class. The algorithm implemented below (Algorithm 2) imputes the GMD by presenting a list of possible values, based on the observed data within the same equivalence class(es). The hypothesis here is that in most finite databases, a case similar to the missing data case could have been observed before.

Algorithm 2. Rough set based missing data imputation algorithm
Input: $\quad$ Incomplete data set $\Lambda$ with $a$ attributes and $i$ instances.

All these instances should belong to a decision $D$.
Output: Imputed dataset.
Assumption: $D$ and some attributes are always known.
For all $\boldsymbol{i}$ do $\rightarrow$ Partition the input space according to $D$ End

## For each attribute do $\rightarrow$

The family of equivalent classes $\varepsilon(a)$ containing each object $o_{i}$ for all input attributes is computed.

IF $i$ has the same attribute values with $a_{j}$ everywhere except for the missing value, replace the missing value, $a_{\text {missing }}$, with the value $v_{j}$, from $a_{j}$, where $j$ is an index to another instance within the same equivalence class.

IF more than one $v_{j}$ values are suitable for the estimation, postpone the replacement for later when it will be clear which value is appropriate.

## End

### 2.4.1 Imputation Validation

In MESA, 44\% of the subjects (instances) contain at least one missing attribute. In order to validate the imputation technique, all the subjects containing at least one missing value are excluded from the dataset forming a subset of MESA without any missing values (referred to as $D_{\text {complete }}$ ). Next, the values of several attributes are randomly removed from $D_{\text {complete }}$ in order to build a new dataset that has $44 \%$ of the
subjects containing at least one missing value ( $44 \%$ is specifically chosen in order to reflect the actual case). This dataset is referred to as $D_{\text {simulation }}$.

The rough set imputation is applied on the $D_{\text {simulation }}$ data. All the instances are partitioned into equivalence classes in the $D_{\text {simulation }}$ data. The missing attribute , $a_{\text {missing }}$, of instance $i$ is imputed with the value $v_{j}$, from $a_{j}$, where $j$ is an index to another instance (within the same equivalence class) that has the same attribute values with $a_{j}$ everywhere except for the missing value. This process is repeated for each instance with missing attributes. If there exist more than one $v_{j}$ value suitable for the missing attribute, the imputation process is postponed until only one value is appropriate. Note that, this method does not guarantee each missing attribute to be imputed. Some attributes may still remain missing at the end of imputation.

As mentioned above, the missing attributes are imputed when there exists at least one instance identical to the missing data case except for the missing attribute itself. In order to compare the performance of rough set imputation, we have extended this process to imputing the missing case only when there exists at least a predetermined number of identical instances to the instance with the missing case. The imputed values are then compared with the actual values in the $D_{\text {complete }}$ data. This process is repeated several times, by selecting a different random set of attributes to remove each time. After the removed attributes are imputed and the imputed values are compared with the actuals, the average imputation accuracy is then reported.

## CHAPTER THREE

## METHOD - PREDICTIVE INDEX ESTIMATION

### 3.1. Comparison of Attribute Selection Methods

Attribute selection techniques can be divided into three different categories: filter, wrapper and embedded methods. In filtering methods, the attribute selection method takes place before any learning algorithm. The undesirable attributes are filtered out before the classification step. All the training data is used in filtering methods (Hall 2003). In embedded methods, the learning algorithm has its own attribute selection algorithm embedded in it (Molina 2002). J48 decision tree classification algorithm is a common example of an embedded method. In wrapper mode, on the other hand, the attribute selection algorithm uses the learning algorithm as a sub-routine (John 1994).

Five different attribute selection methods are applied to the MESA dataset.
Wrapper methods which are Correlation based feature selection and ReliefF, a filtering method, information gain and an embedded method J48 decision tree based attribute selection are applied to the dataset and the results are compared in terms of sensitivity and specificities.

Correlation based feature selection. Correlation based feature selection evaluates the dependencies between attributes and eliminate the ones which are correlated to each other. The irrelevant and redundant data has to be removed. After the attribute selection,
the remaining data has to be highly correlated with the class and uncorrelated with each other.

As equation 1 (Ghiselli, 1964) formalizes:

$$
\text { Merit }_{s_{k}}=\frac{k \overline{r_{c f}}}{\sqrt{k+k(k-1) \bar{r}_{f f}}}
$$

The attribute subset $S$ contains $k$ different attributes where $r_{c f}$ is the attribute to class correlation and $r_{f f}$ is the attribute to attribute correlation. In order to have a good attribute selection algorithm the merit has to be maximized. Symmetrical uncertainty can be evaluated as follows where $H(X)$ and $H(Y)$ are marginal entropies.

$$
\text { Symmetrical uncertainty }=2.0 x\left[\frac{\text { gain }}{H(Y)+H(X)}\right]
$$

Consistency based feature selection. The consistency of the class is evaluated by first figuring out all different combinations of the attributes. For each different combination the consistency is calculated by differentiating the number of occurrences of a particular attribute from the cardinality of the majority class (Hall 1998).

Information gain. The uncertainty of the class is evaluated with and without the attribute observation (Hall 1998).
$H($ Class $)=-\sum_{c \in C} p(c) \log p(c)$
$H($ Class $\mid$ Attribute $)=-\sum_{a \in A} p(a) \sum_{c \in C} p(c \mid a) \log p(c \mid a)$
InfoGain(Class, Attribute) $=$ H(Class) - (Class $\mid$ Attribute $)$
ReliefF. ReliefF algorithm assigns a relevancy score to all the attributes in descending order. The algorithm selects an instance in each iteration and finds the nearest
neighbor (by Euclidean distance) from the same and opposite class. The algorithm starts with a $p$-long weight vector $(W)$ of zeros. The closest same-class instance is called 'nearhit', and the closest different-class instance is called 'near-miss'. The weight vector is updated in each iteration as shown in the following formula:
$W_{i}=W_{i-1}-\left(x_{i}-\text { nearHit }_{i}\right)^{2}+\left(x_{i}-\text { nearMiss }_{i}\right)^{2}$
Therefore, the weight of a given feature decreases if it differs from that feature in nearby instances of the same class more than nearby instances of the other class, and increases in the reverse case. After $m$ iterations, each element of the weight vector is divided by $m$ which will give the relevance vector. The algorithm can handle noise if the neighbor number, k , is increased.

J48 decision tree based attribute selection. J48 decision tree is a classification method that can also be used for attribute selection. Each node of a tree involves an attribute and the occurrence of each attribute provides information about the importance of that particular attribute (Sugumaran 2006). The importance of each attribute can be evaluated by applying the information gain formula to each node of the tree.

### 3.2. Selection of Potential Predictive Factors

The construction of urinary incontinence predictive index is based on subjects who were classified as continent at the baseline. The idea behind the construction of predictive index is to model what combination of factors makes a continent subject change from a continent condition to an incontinent condition. 91 subjects are classified as incontinent in the first follow up (HH2) out of 424 subjects in the baseline (HH1). As mentioned before, the factors are all dichotomized where 0 denoting a subject towards
continent and 1 denoting a subject towards incontinent. A logistic regression model is used where the independent variables are the factors of baseline and the dependent variable is the class label (outcome) of HH 2 . The same factors from HH 1 , along with the outcome from HH 4 are then used to determine the p-values and odd ratios. The reason of this attempt is to see if the same factors are still significant when a different outcome (response variable) is used.

Relieff attribute selection technique is also used to see whether the potential predictive factors extracted are similar with the regression method. Relieff assigns a weight to each attribute and orders the risk factors in descending order. The attributes are selected with the highest weight until a predetermined threshold value.

Interaction effects represent the combined effects of variables on the criterion or dependent measure. An interaction occurs when the magnitude of the effect of one independent variable $(X)$ on a dependent variable $(Y)$ varies as a function of a second independent variable ( $Z$ ). Adding an interaction term to a model drastically changes the interpretation of all of the coefficients. In this study, 2-way interactions of the regression model are determined. The new $p$-values of the potential predictive factors along with the most significant 2-way interaction factors for HH 1 HH 2 data (the baseline factors and the first follow-up response) are determined.

Once the potential predictive factors are extracted from the Relieff attribute selection technique and the logistic regression method, the data containing only the predictive factors along with the class labels are used for association rules generation. The class labels contain both the class from the baseline and the first follow up. The four
different class labels are denoted by, 'C-C', 'C-I', 'I-C', 'I-I' where 'C-I' showing the subject being continent in the baseline and incontinent in the first follow-up. The dimensionality is than further reduced by removing the subjects that have the following class labels, 'I-I', 'I-C'. The reason of removing those class labels is that, we are only interested in the subjects that became incontinent over time and the ones that remain continent.

### 3.3. Comparison of Rule Extraction Methods

Rule extraction techniques can be categorized into several different categories. In this research we will focus on the association rule learning and classification rules. Four different rule extraction methods are studied. Apriori an association rule and PART, Prism and Jrip the classification rules are examined and the results are compared in terms of sensitivity and specificities.

Apriori is an association rule available on the data mining tool Weka (Waikato Environment for Knowledge Analysis) that iteratively reduces the minimum support until it finds the required number of rules with the given minimum confidence. PART algorithm uses a divide-and-conquer approach and builds a decision tree. The "best" leafs of the tree become part of the rules. For the Jrip, a rule learner is implemented to construct the classification rules. Prism can only deal with nominal attributes. It doesn't do any pruning. Prism algorithm cannot deal with missing values. Predictive Apriori and Tertius were also two rule extraction methods available in Weka. But those two methods were not as effective in terms of the run times compared to others. This may cause inefficient results since MESA data has a high dimensional attribute space.

### 3.3.1. Simulation

10 different simulation datasets are created each containing 20 attributes and 1500 subjects. In order to gain full control over the dataset the values in the dataset are formed to create rules (Agraval 1994). The last column of the dataset specifies the resultants i.e. the class labels. Resultant is defined as continence, expressed as 0 , or incontinence, expressed as 1, of the subject. Resultant values are stored in the column space of the matrix.

Subjects are defined as an individual and are represented in the row space of the matrix. The number of subjects is a function of the number of rules and the partition size for each rule. For example with ten rules and two hundred subject partition size there would be two thousand subjects represented in the matrix, with subjects one to two hundred being in the first partition, subjects two hundred and one to four hundred in the second, etc.

The rules that are embedded to the simulation data are as follows:
$\left(A_{1}=0\right) A N D\left(A_{4}=0\right) \Rightarrow Y=0$
$\left(A_{2}=0\right) A N D\left(A_{8}=1\right) \Rightarrow Y=1$
$\left(A_{10}=0\right) A N D\left(A_{15}=1\right) \Rightarrow Y=0$
$\left(A_{6}=1\right) A N D\left(A_{13}=1\right) \Rightarrow Y=0$
$\left(A_{9}=0\right) \operatorname{AND}\left(A_{14}=1\right) \Rightarrow Y=1$
Having defined the base dataset, it is now possible to manipulate the dataset for further methodology testing. There are two types of modifications that can be performed to the data set for this testing. First is the inclusion of noise to the data set. The second
type of modification is to incorporate missing values. The expected results of the attribute selection algorithms are the attributes that are the entities of the rules embedded to the datasets.

A simulation dataset is used to allow maximum flexibility in creating and manipulating the data sets. The simulation allows for one base dataset to be processed with multiple noise levels, multiple incomplete data levels and a mixture of both noise and incomplete data. The advantage of this methodology allows the researcher to understand the impact of varying levels of these factors on attribute selection or any other metric of interest. Further, we can determine up to what percentage of incomplete data and noise levels a given data mining tool can handle. This is extremely important when one needs to use the tool on a real application. The current version of the matrix creation algorithm was intentionally restricted to binary data only. This was chosen primarily to validate the attribute selection methods that were the driving force behind creating this simulated data. With the methodology described above, modifications to alternative data types, such as categorical and/or continuous, will be possible.

The simulation shows up to what percentage of incomplete data, noise and multicollinearity can be handled by the attribute selection and rule extraction methods. If the level of incomplete data is more than the percentage that can be handled, further preprocessing is required.

### 3.4. Estimating the Predictive Index

A scientifically developed and tested predictive UI index would help to identify women who are most likely to develop UI and permit widespread prevention or early
treatment. Therefore a method is generated that calculates the probability of a new patient being incontinent in the future.

The HH1HH2 data is prepared for association rule mining. Apriori algorithm explained in Section 3.3 is used to determine the predictive index. However, the confidence and support of each rule were less than desired for this method. Also, not all combinations of the attribute values are listed, therefore; we cannot determine the status of a patient having a combination that was not listed in any of the rules. Since, the results of Apriori algorithm are not reliable and also limited to the listed rules; we listed the risk factors with all different combination of values they can take. This experiment will return all the rules that are available. For each rule the support is calculated, that is, the number of subjects that satisfy the rule. The probability can then simply be calculated by dividing the number of subjects that satisfy the antecedent of the rule by the number of subjects that satisfies the entire rule. This way when a new subject comes into the clinic, the probability of that subject going towards incontinence can be calculated by comparing her/his rules with the model and reporting that rule's probability measure.

### 3.4.1. Reliability of the Rules

Once each combination of the rules are determined, it is important whether or not these rules can be referred as reliable. For a small sample size, there may be very few or even no subjects that satisfy the rule. For example if there is a rule that is only satisfied by one subject, and contradicts with no subjects, the confidence of that rule will be $100 \%$. An example of this rule can be the following: A10T1='(0.75-inf)' A12T1='(0.75- inf)' A3T1='(-inf-0.25]' A4T1='(0.75-inf)' support: $1==>$ class=C-I support: 1 . The antecedent
of this rule is supported by 1 subject, and the rule including the consequent is supported by 1 subject. However, it will be a misleading assumption to conclude that there is a $100 \%$ probability that a new subject satisfying this rule will go towards incontinence over time. Therefore, it is important to determine a support threshold. The margin of errors is used to determine a support threshold for each rule. The rules that have less support than the threshold are considered as unreliable.

In a confidence interval, the range of values above and below the sample statistic is called the margin of error. If $n p q>5$, then the normal approximation can be used to develop a confidence interval for a binomial variable. The formula is the following:

$$
|p-\hat{p}| \leq Z \alpha / 2 \sqrt{\frac{p_{0}\left(1-p_{0}\right)}{n}}
$$

$p_{0}$ is the probability calculated from the rule by dividing the number of the number of subjects that satisfy the entire rule by the number of subjects that satisfies the antecedent of the rule. $n$ is the number of subjects that support the rule. $|p-\hat{p}|$ is the margin of error. The following are critical values for common levels of confidence. A $90 \%$ level of confidence has $\alpha=0.10$ and critical value of $z_{\alpha / 2}=1.64$. A $95 \%$ level of confidence has $\alpha=0.05$ and critical value of $z_{\alpha / 2}=1.96$. A $99 \%$ level of confidence has $\alpha=0.01$ and critical value of $z_{\alpha / 2}=2.58$. A $99.5 \%$ level of confidence has $\alpha=0.005$ and critical value of $z_{\alpha / 2}=2.81$.

### 3.4.2. Model Based Approach

This section deals with the construction of a predictive index for urinary incontinence based on predicted probabilities of all possible combinations of the selected predictive factors. Between the two regression models, with and without interaction terms, the one with higher performance will be used for the next steps. The selected predictive factors, and if needed their 2-way interaction terms, from the baseline are regressed against the class label of the follow-up. This is achieved by constructing contrasts and their corresponding 95\% confidence intervals for probabilities of incontinence.

## CHAPTER FOUR

## RESULTS AND DISCUSSION

### 4.1. Incomplete and Inconsistent Data Analysis Results

The first step of MESA preprocessing is to categorize the missing values into GMD, UMD and SPMD. Table 4.1 shows the number of subjects in the MESA baseline. Table 4.2 shows the number of cells that were incomplete before the experiment, along with their percentages. Once the experiments are conducted over the entire MESA dataset, 200 responses are determined to be mutually inconsistent, $15.4 \%$ of the responses are $G M D, 12.9 \%$ of the responses are $U M D$, and $9.8 \%$ of the responses are $S P M D$. Table 4.3 shows the results after the incomplete data analysis.

The summation of the percentages of $G M D, U M D$ and $S P M D$ for the female population shown in Table 4.3 must be equal the total percentage of incomplete data (37.1\%). Since inconsistent data is not a type of incomplete data, the percentage of inconsistent data $(0.021 \%)$ is not included in the summation.

Table 4.4 shows the percentage of data with the range of GMD/UMD and SPMD's. We know from our previous experiments that our methods can handle up to $12 \%$ of missing values and $15 \%$ of noise (Arslanturk et al. 2011). When the percentage of missing values and/or noise exceeds this threshold, the method shows less than desired reliability. Therefore, the dimensionality of the data is further reduced by removing the attributes and subjects that contain more than $15 \%$ of $G M D$ and/or $U M D$. The attributes
and subjects that contain more than $15 \%$ of $S P M D$ are not removed, since those will later be used for stratification. The size of the reduced dataset has 773 attributes and 1059 subjects.

Note that, the skip patterns cover $9.8 \%$ of the missing values. It is important to distinguish them from the GMD's. The branching questions which lead the skip patterns to occur can now be used for data stratification. This approach will help us to analyze each population with different characteristics separately.

### 4.2. Stratification Results

The stratification results of the simulated data and MESA data are explained in this section.

### 4.2.1. Simulation Results of Stratification

Table 4.5 shows the simulation results of the stratification with $0 \%$ noise. The first three rules $\left(R_{1}, R_{2}, R_{3}\right)$ belong to the dataset that contains the subjects that have $B Q_{1}^{1}=$ 0 , which can be denoted by $D_{B Q_{1}^{1}}$, and the last three rules $\left(R_{4}, R_{5}, R_{6}\right)$ belong to the dataset that contains the subjects that have $B Q_{1}^{2}=1$, which can be denoted by $D_{B Q_{1}^{2}}$. Since the first branching question, $B Q_{1}$, was designed to separate $D_{1}$ and $D_{2}$, it is expected to extract the same rules listed in Section 2.3 .1 for the subsets, $D_{B Q_{1}^{1}}$ and $D_{B Q_{1}^{2}}$. The rules extracted from $D_{B Q_{1}^{1}}$ are the same as the rules extracted from $D_{1}$, and the rules extracted from $D_{B Q_{1}^{2}}$ are the same as the rules extracted from $D_{2}$. Note that, the support of the first three rules $\left(R_{1}, R_{2}, R_{3}\right)$ of dataset $D_{B Q_{1}^{1}}$, is much higher than the support of the first three rules $\left(R_{1}, R_{2}, R_{3}\right)$ of dataset $D_{B Q_{1}^{2}}$ and the non-support of the first three

Table 4.1- The size of HH1

|  | \# of Subjects | \# of Attributes |
| :---: | :---: | :---: |
| HH1 | 1956 | 826 |
| HH1 Female | 1154 |  |
| HH1 Male | 802 |  |

Table 4.2- Missing Data in HH1, gender specified

| Missing Data | \# of Cells | Percentages |
| :---: | :---: | :---: |
| HH1 | 751132 | $46.4 \%$ |
| HH1 Female | 353637 | $37.1 \%$ |
| HH1 Male | 321637 | $48.5 \%$ |

Table 4.3-GMD, UMD, SPMD and Inconsistent Data for Female

| HH1 Female | \#of Cells | Percentages |
| :---: | :---: | :---: |
| GMD | 157278 | $15.4 \%$ |
| UMD | 114384 | $12.9 \%$ |
| SPMD | 81975 | $9.8 \%$ |
| Inconsistent | 200 | $0.021 \%$ |

Table 4.4- Average number of missing values per attribute and subject

| Avg. Missing Data | By Attribute | By Subject |
| :---: | :---: | :---: |
| GMD UMD 0-15 | $93.5 \%$ | $91.7 \%$ |
| GMD UMD $>15$ | $6.4 \%$ | $8.2 \%$ |
| SPMD 0-15 | $65.4 \%$ | $0.33 \%$ |
| SPMD $>\mathbf{1 5}$ | $34.5 \%$ | $99.6 \%$ |

rules $\left(R_{1}, R_{2}, R_{3}\right)$ of dataset $D_{B Q_{1}^{1}}$ is equal to 0 . The reason of the non-support being 0 is that, the rules were generated without any conflict. Likewise, for the second branch, $B Q_{1}^{2}$, we expect to see a lower support and a higher non-support compared to $B Q_{1}^{1}$, for the first three rules. Notice that, for the last three rules the support of $B Q_{1}^{2}$ is higher than $B Q_{1}^{1}$ and the non-support of $B Q_{1}^{2}$ is lower than $B Q_{1}^{1}$, since the last three rules were extracted from $D_{2}$.

The p-values are calculated and the results show that, for each rule, the association between the rules and two populations are considered to be extremely statistically significant. Therefore, we can define, $B Q_{1}$, as a significant branching question. That means, the two populations, $B Q_{1}^{1}$ and $B Q_{1}^{2}$ has different association rules hence; they have to be analyzed separately.

Table 4.6, 4.7, and 4.8 show the same experiment with $10 \%, 20 \%$ and $30 \%$ noise, respectively. Note that, even if there is $30 \%$ noise in the data, the p-values are still considered to be statistically significant. However, the comparison of the p-values with different noise levels also show that, the less the noise there is in the data, the smaller the p -value becomes.

Table 4.9 shows the contingency table for the second branching question, $B Q_{2}$. As mentioned in the Methods section, the binary values of the second branching question were assigned randomly, and based on the binary values $B Q_{2}$ have, $D_{1}+D_{2}$ dataset was separated into $D_{B Q_{2}^{0}}$ and $D_{B Q_{2}^{1}}$.

Table 4.5- Optimal Branching (OB) p-values for 0\% Noise

| Association Rules | Noise: 0\% | $D_{B Q_{1}^{0}}$ | $D_{B Q_{1}^{1}}$ | $\begin{aligned} & \text { Row } \\ & \text { T. } \end{aligned}$ | p-value |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \hline \text { A1T1='(-inf -0.25]' A3T1='(0.75-inf)' } 200 \\ ==>\text { class=I-C } 200 \end{gathered}$ | Support | 200 | 17 | 217 | $3.224 \mathrm{e}^{-46}$ |
|  | No Support | 0 | 61 | 61 |  |
|  | Column | 200 | 78 | 556 |  |
| $\begin{gathered} \text { A5T1='(-inf-0.25]' A7T1='(0.75-inf)' } 200 \\ ==>\text { class }=C-I \end{gathered}$ | Total Support | 200 | 31 | 231 | $3.730 \mathrm{e}^{-25}$ |
|  | No Support | 0 | 34 | 34 |  |
|  | Column Total | 200 | 65 | 530 |  |
| $\begin{gathered} \text { A10T1='(0.75-inf)' A12T1='(0.75-inf)' } 200 \\ ==>\text { class=C-I } 200 \end{gathered}$ | Support | 200 | 33 | 233 | $7.54 \mathrm{e}^{-30}$ |
|  | No Support | 0 | 43 | 43 |  |
|  | Column Total | 200 | 76 | 552 |  |
| $\begin{gathered} \text { A2T1='(-inf-0.25]' A11T1='(-inf-0.25]' } 200 \\ ==>\text { class=C-I } 200 \end{gathered}$ | Support | 26 | 200 | 226 | $4.92 \mathrm{e}^{-29}$ |
|  | No Support | 38 | 0 | 38 |  |
|  | Column Total | 64 | 200 | 528 |  |
| $\begin{gathered} \text { A4T1='(0.75-inf)' A9T1='(-inf- } 0.25]^{\prime} 200 \\ ==>\text { class=C-I } 200 \end{gathered}$ | Support | 28 | 200 | 228 | $1.30 \mathrm{e}^{-29}$ |
|  | No Support | 40 | 0 | 40 |  |
|  | Column Total | 68 | 200 | 536 |  |
| $\begin{gathered} \text { A6T1='(-inf-0.25]' A8T1='(0.75-inf)' } 200 \\ ==>\text { class=I-C } 200 \end{gathered}$ | Support | 10 | 200 | 210 | $2.74 \mathrm{e}^{-37}$ |
|  | No Support | 40 | 0 | 40 |  |
|  | Column <br> Total | 50 | 200 | 500 |  |

Table 4.6- Optimal Branching (OB) p-values for $10 \%$ Noise

| Association Rules | Noise: $10 \%$ | $D_{B Q_{1}^{0}}$ | $D_{B Q_{1}^{1}}$ | $\begin{aligned} & \text { Row } \\ & \text { T. } \\ & \hline \end{aligned}$ | p-value |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { A10T1='(0.75-inf)' A12T1='(0.75-inf)' } 137 \\ ==>\text { class=C-I } 129 \end{gathered}$ | Support | 129 | 16 | 145 | $3.02 \mathrm{e}^{-17}$ |
|  | NoSupport | 8 | 33 | 41 |  |
|  | Column | 137 | 49 | 372 |  |
|  | Total |  |  |  |  |
| $\begin{gathered} \text { A5T1='(-inf-0.25]' A7T1='(0.75-inf)' } 146 \\ ==>\text { class=C-I } \end{gathered}$ | Support | 135 | 24 | 159 | $3.94 \mathrm{e}^{-14}$ |
|  | NoSupport | 11 | 34 | 45 |  |
|  | Column | 146 | 58 | 408 |  |
|  | Total |  |  |  |  |
| $\begin{gathered} \text { A1T1='(-inf- } 0.25]^{\prime} \text { A3T1='(0.75-inf)' } 143 \\ ==>\text { class }=\text { I-C } 129 \end{gathered}$ | Support | 129 | 7 | 136 | $1.57 \mathrm{e}^{-33}$ |
|  | NoSupport | 14 | 67 | 81 |  |
|  | Column | 143 | 74 | 434 |  |
|  | Total |  |  |  |  |
| $\begin{gathered} \text { A4T1='(0.75-inf)' A9T1='(-inf- } 0.25]^{\prime} 150 \\ ==>\text { class=C-I } 140 \end{gathered}$ | Support | 26 | 140 | 166 | $2.70 \mathrm{e}^{-15}$ |
|  | NoSupport | 36 | 10 | 46 |  |
|  | Column | 62 | 150 | 424 |  |
|  | Total |  |  |  |  |
| $\begin{gathered} \text { A6T1='(-inf-0.25]' A8T1='(0.75-inf)' } 147 \\ ==>\text { class=I-C } 135 \end{gathered}$ | Support | 12 | 135 | 147 | $5.12 \mathrm{e}^{-23}$ |
|  | NoSupport | 45 | 12 | 57 |  |
|  | Column | 57 | 147 | 408 |  |
|  | Total |  |  |  |  |
| $\begin{gathered} \text { A2T1='(-inf-0.25]' A11T1='(-inf-0.25]' } 155 \\ ==>\text { class }=C-I 141 \end{gathered}$ | Support | 18 | 141 | 159 |  |
|  | NoSupport | 35 | 14 | 49 |  |
|  | Column <br> Total | 53 | 155 | 416 |  |

Table 4.7- Optimal Branching (OB) p-values for $20 \%$ Noise
$\left.\begin{array}{llllll}\hline \text { Association Rules } & \begin{array}{l}\text { Noise: } \\ \text { 20\% }\end{array} & D_{B Q_{1}^{0}} & D_{B Q_{1}^{1}} & \begin{array}{l}\text { Row } \\ \text { T. }\end{array} & \text { p-value } \\ \hline \text { A5T1='(-inf-0.25]' 169 ==> class=C-I 126 } & \text { Support } & 126 & 54 & 180 & 4.47 \mathrm{e}^{-9} \\ & \begin{array}{l}\text { No Support } \\ \text { Column }\end{array} & 43 & 78 & 121 & 132\end{array} \mathbf{6 0 2}\right)$

Table 4.8- Optimal Branching (OB) p-values for 30\% Noise

| Association Rules | Noise:30\% | $D_{B Q_{1}^{0}}$ | $D_{B Q_{1}^{1}}$ | Row $T$ | p-value |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A12T1='(0.75-inf)' 263 ==> class=C-I 150 | Support | 150 | 105 | 255 | 0.0018 |
|  | No Support | 113 | 139 | 252 |  |
|  | Column | 263 | 244 | 1014 |  |
|  | Total |  |  |  |  |
| $\begin{aligned} & \text { A10T1='(0.75-inf)' A12T1='(0.75-inf)' } 257 \\ & ==>\text { class=C-I } 143 \end{aligned}$ | Support | 143 | 90 | 233 | $7.24 e^{-005}$ |
|  | No Support | 114 | 149 | 263 |  |
|  | Column | 257 | 239 | 992 |  |
|  | Total |  |  |  |  |
| A4T1='(0.75-inf)' 249 ==> class=C-I 145 | Support | 96 | 145 | 241 | $4.69 \mathrm{e}^{-005}$ |
|  | No Support | 146 | 104 | 250 |  |
|  | Column | 242 | 249 | 982 |  |
|  | Total |  |  |  |  |
| A9T1='(-inf-0.25]' 269 ==> class=C-I 152 | Support | 81 | 152 | 233 | $7.62 e^{-005}$ |
|  | No Support | 131 | 117 | 248 |  |
|  | Column Total | 212 | 269 | 962 |  |

Since the $B Q_{2}$ values are assigned randomly, once separated, $D_{B Q_{2}^{0}}$ and $D_{B Q_{2}^{1}}$
datasets each may contain subjects from both $D_{1}$ and $D_{2}$. Therefore, the rule extraction technique will extract rules belonging to both $D_{1}$ and $D_{2}$. The rules embedded to $D_{1}$ can lead conflicts, for subjects belonging to $D_{2}$, and vice versa. These conflicts will effect the support of each rule, and may also corrupt the rules.

Therefore, Table 4.9 shows that the association between the rules and two populations are considered to be not statistically significant. Table 4.10 shows the results with $10 \%$ noise where the p -value shows an increase. Therefore, $B Q_{2}$ is not being considered as a significant branching question.

Table 4.9- Bad Branching (BB) p-values for 0\% Noise

| Association Rules | Noise: 0\% | $D_{B Q_{2}^{0}}$ | $D_{B Q_{2}^{1}}$ | $\begin{aligned} & \text { Row } \\ & \text { T. } \\ & \hline \end{aligned}$ | p-value |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { A4T1='(0.75-inf)' A9T1='(-inf- } 0.25]^{\prime} 136 \\ ==>\text { class=C-I } 117 \end{gathered}$ | Support | 117 | 111 | 128 | 0.7326 |
|  | No Support | 19 | 21 | 40 |  |
|  | Column T. | 136 | 132 | 436 |  |
| $\begin{aligned} &\text { A2T1='(-inf-0.25]' A11T1 }=\text { '(-inf- } 0.25]^{\prime} \\ & 132==>\text { class }=C-I ~ \\ & 112 \end{aligned}$ | Support | 112 | 114 | 226 | 0.8610 |
|  | No Support | 20 | 18 | 38 |  |
|  | Column | 132 | 132 | 528 |  |
|  | Total |  |  |  |  |
| $\begin{gathered} \text { A5T1='(-inf-0.25]' A7T1='(0.75-inf)' } 124 \\ ==>\text { class=C-I } 107 \end{gathered}$ | Support | 124 | 107 | 231 | 0.7160 |
|  | No Support | 17 | 17 | 34 |  |
|  | Column | 141 | 124 | 530 |  |
|  | Total |  |  |  |  |
| $\begin{gathered} \text { A10T1='(0.75-inf)' A12T1='(0.75-inf)' } 123 \\ ==>\text { class=C-I } 109 \end{gathered}$ | Support | 124 | 109 | 233 | 0.0962 |
|  | No Support | 29 | 14 | 43 |  |
|  | Column <br> Total | 153 | 123 | 552 |  |

Table 4.10- Bad Branching (BB) p-values for $10 \%$ Noise

| Association Rules | Noise: 10\% | $D_{B Q_{2}^{0}}$ | $D_{B Q_{2}^{1}}$ | Row T. | p-value |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { A4T1='(0.75-inf)' A9T1='(-inf- } 0.25] \text { ' } 152 \\ ==>\text { class=C-I } 109 \end{gathered}$ | Support | 109 | 109 | 218 | 0.3932 |
|  | No Support | 43 | 54 | 97 |  |
|  | Column | 152 | 163 | 630 | 0.4590 |
|  | Total |  |  |  |  |
| $\begin{gathered} \text { A10T1='(0.75-inf)' A12T1='(0.75-inf)' } \\ 158==>\text { class=C-I } 108 \end{gathered}$ | Support | 108 | 87 | 315 |  |
|  | No Support | 50 | 49 | 99 |  |
|  | Column | 158 | 136 | 708 | 0.4721 |
|  | Total |  |  |  |  |
| A4T1='(0.75-inf)' 156 ==> class=C-I 106 | Support | 106 | 97 | 203 |  |
|  | No Support | 50 | 55 | $\begin{aligned} & 105 \\ & 616 \end{aligned}$ |  |
|  | Column | 156 | 152 |  | 0.3361 |
|  | Total |  |  |  |  |
| $\begin{gathered} \text { A5T1='(-inf-0.25]' A7T1='(0.75-inf)' } 159 \\ ==>\text { class=C-I } 110 \end{gathered}$ | Support | 96 | 110 | 206 |  |
|  | No Support | 55 | 49 | 104 |  |
|  | Column | 151 | 159 | 620 | 1.0000 |
|  | Total |  |  |  |  |
|  | Support | 104 | 102 | 206 |  |
| A2T1='(-inf-0.25]' A11T1='(-inf-0.25]' | No Support | 52 | 50 | 102 |  |
| $152==>$ class=C-I 102 | Column T. | 156 | 152 | 616 |  |

As a result, $B Q_{1}$ was defined as a factor that perfectly splits the dataset into two populations with different characteristics, $B Q_{2}$ and $B Q_{3}$ were defined as factors that randomly split the population into two groups. Our simulation results showed that $B Q_{1}$ is a significant factor, whereas $B Q_{2}$ and $B Q_{3}$ are not. Hence, this simulation shows that the Fisher's Exact Test can be used to determine the significant branching questions.

### 4.2.2. MESA Results of Stratification

The same experiment is applied on the MESA dataset. The branching questions that are extracted from the MESA data are listed in Table 4.11.

We have split the data into two subsets based on the following branching questions: v29, v92, v171, v180, v193, v418, v737 and v745. For most of these factors except v180 and v737, there were not enough subjects for a rule to be generated. Therefore, a contingency table could not be generated. Table 4.12 shows the contingency table for the subjects that have undergone female surgery and the ones that have not. The p -values show that the association between the rules and the data is not considered to be statistically significant. Therefore, female surgery cannot be considered as a significant branching question.

Table 4.13 shows the contingency table for the smokers and non-smokers. Here, the association between the rules and the data is considered to be statistically significant for the rules whose p-values are underlined. Therefore, smoking more than 100 cigarettes in the entire life is considered to be a significant branching question.

## Table 4.11- List of Branching Questions

## Label Branching Questions

v32 Are you married, widowed, divorced, separated, or have you never married?
v41 Do you sneeze often, sometimes, rarely or never?
v53 Do you usually need to use a wheelchair, cane, crutches or walker to help you get around?
v59 Do you have any health problems which make it difficult for you to leave your home and go visiting, shopping, or to the doctor's?
v87 Have you ever been told by a doctor that you had high blood pressure?
v90 Have you ever been told by a doctor that you had a hernia in the groin or stomach area?
v92 Have you ever had a stroke or cerebral brain hemorrhage?
v95 Have you ever been told by a doctor that you occasionally have had transient ischemic attacks or poor blood flow to the brain, where you seem to lose track of things that are happening around you for up to a few minutes?
v97 Have you ever had problems with any paralysis?
v107 Have you ever had a heart attack?
v111 Has any doctor ever told you that you have or have had arthritis or rheumatism?
v125 Have you had any other disease of the nerves or muscles?
v128 Have you ever been told by a doctor that you had cancer of any kind?
v133 Have you lost any inches in height as you have gotten older?
v135 In the last 12 months how many times have you become so dizzy that you fainted or nearly fainted?
v138 Have you broken any bones in the last 12 months?
v171 How many pregnancies have you had?
v180 Have you ever had female surgery such as on your ovaries, vagina, fallopian tubes, uterus, rectum, or urethra?
v193 Are you currently taking any female hormones?
v207 Have you ever had any other operations on your bladder, kidneys, or any other organs in your pelvic area or area normally covered by underpants or undershorts?
v219 Do you usually need help in getting into the bathroom or on or off of the toilet?
v220 Do you use any aids like a grab bar, or special toilet or anything else to help you with using the toilet in your home?
v224 Do you have your regular schedule that you usually use to get you to the toilet to urinate, for example every hour or so?
v415 Have you ever had or have you been told by a doctor that you had any kidney or bladder problems we haven't talked about already?
v418 Do you usually drink any liquids of any kind before you go to bed at night?
v422 Did your mother of father have a urine loss condition as an adult?
v458 Do you have any health problems that require medical attention that you have not been able to get treated?

Table 4.11- List of Branching Questions- Continued

## Label Branching Questions

v496 As you know some people experience memory problems as they get older. How about your memory? Has it become worse within the last five years?
v540 Have you ever had to stay in a nursing home overnight or longer because of a health problem you had?
$v 543$ Have you ever had to stay in a mental health facility overnight or longer, because of a mental or emotional problem that you had?
v725 Do you usually take one or more naps during the day?
v737 Have you smoked at least 100 cigarettes in your entire life?
v745 Do you drink wine, beer, or liquor?
v776 Is this the same occupation that you had for most of your life?
4.2.2.1 Analysis of the Stratums. The extracted rules of the significant branching questions can be used for prediction. For example when a new patient comes into the clinic who meets the following rule v69='(-inf-0.5]' v80='(-inf-0.5]' v124='(-inf-0.5]' v152='(-inf-0.5]' v230='(-inf-0.5]' $50==>~ c l a s s=C-C ~ 37 ~ c o n f:(0.74), ~ t h e ~ p a t i e n t ~ w i l l ~$ remain continent with $74 \%$ of probability.

Next, the risk factors of different stratums (smokers/non-smokers for our case) are determined separately. Relieff attribute selection is used to determine the important attributes of each population, since our previous studies have shown that Relieff outperformed other attribute selection techniques on the analysis of MESA dataset (Arslanturk et al.). The extracted attributes are defined as the risk factors. Table 4.14 shows the risk factors for smokers and non-smokers.

Table 4.12- Contingency Table of Female Surgery

| Female Surgery- Association Rules |  | $D_{F S}$ | $D_{\text {NFS }}$ | $\begin{aligned} & \hline \text { Row } \\ & \text { T. } \end{aligned}$ | p-value |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. v124='(-inf-0.5]' v152='(-inf-0.5]' | Support | 49 | 38 | 87 | 0.4988 |
| v178='(-inf-0.5]' v195='(-inf-0.5]' v230='(- | No Support | 29 | 29 | 58 |  |
| $\begin{aligned} & \text { inf } \left.-0.5]^{\prime} \text { v231='(-inf- } 0.5\right]^{\prime} 78==\text { class=C- } \\ & \text { C } 49 \text { conf:(0.63) } \end{aligned}$ | Column T. | 78 | 67 | 145 |  |
| 2. v120='All' v124='(-inf-0.5]' v152=' (-inf- | Support | 49 | 38 | 87 | 0.4988 |
| 0.5]' v178='(-inf-0.5]' v195='(-inf-0.5]' | No Support | 29 | 29 | 58 |  |
| $\begin{aligned} & \text { v230='(-inf- } \left.0.5]^{\prime} \text { v231='(-inf- }-.5\right]^{\prime} 78==> \\ & \text { class=C-C } 49 \quad \text { conf:(0.63) } \end{aligned}$ | Column T. | 78 | 67 | 145 |  |
| 3. v121='All' v124='(-inf-0.5]' v152='(-inf- | Support | 49 | 38 | 87 | 0.4988 |
| 0.5]' v178='(-inf-0.5]' v195='(-inf-0.5]' | No Support | 29 | 29 | 58 |  |
| $\begin{aligned} & \text { v230 } \left.=\text { '(-inf- } 0.5]^{\prime} \text { v231='(-inf- }-.5\right]^{\prime} 78== \\ & \text { class=C-C } 49 \text { conf:(0.63) } \end{aligned}$ | Column T. | 78 | 67 | 145 |  |
| 1. v107='(-inf-0.5]' v125='(-inf-0.5]' | Support | 32 | 43 | 75 | 0.1135 |
| v199='(-inf-0.5]' v432='(-inf-0.5]' 67 ==> | No Support | 33 | 24 | 57 |  |
| class=C-I 43 conf:(0.64) | Column T. | 65 | 67 | 132 |  |
| $\begin{aligned} & \text { 2. v78='(-inf-0.5]' v107='(-inf-0.5]' } \\ & \text { v125='(-inf-0.5]' v199='(-inf-0.5]' v432='(- } \end{aligned}$ | Support | 30 | 43 | 73 | 0.0782 |
| inf-0.5]' 67 ==> class=C-I 43 conf:(0.64) | No Support | 32 | 24 | 56 |  |
|  | Column T. | 62 | 67 | 129 |  |
| 3. v107='(-inf-0.5]' v120='All' v125='(-inf- | Support | 32 | 43 | 75 | 0.1135 |
| 0.5]' v199='(-inf-0.5]' v432='(-inf-0.5]' 67 | No Support | 33 | 24 | 57 |  |
| $=$ => class=C-I 43 conf:(0.64) | Column T. | 65 | 67 | 132 |  |

Table 4.13- Contingency Table of Smoking

| Smoke- Association Rules |  | $D_{S}$ | $D_{N S}$ | Row <br> T. | p-value |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. v69='(-inf-0.5]' v80='(-inf-0.5]' | Support | 37 | 41 | 78 | 0.0064 |
| v124='(-inf-0.5]' v152='(-inf-0.5]' | No Support | 13 | 42 | 55 |  |
| $\begin{aligned} & \text { v230='(-inf- } 0.5]^{\prime} 50==>\text { class=C-C } 37 \\ & \text { conf:(0.74) } \end{aligned}$ | Column T. | 50 | 83 | 133 |  |
| 2. v69='(-inf-0.5]' v78='(-inf-0.5]' v80='(- | Support | 37 | 41 | 78 | 0.0064 |
| inf-0.5]' v124='(-inf-0.5]' v152='(-inf- | No Support | 13 | 42 | 55 |  |
| $0.5]^{\prime}$ v230='(-inf-0.5]' $50=$ => class=C-C <br> 37 conf:(0.74) | Column T. | 50 | 83 | 133 |  |
| 3. v69='(-inf-0.5]' v79='(-inf -0.5$]^{\prime}$ v80 ${ }^{\prime}(-$ | Support | 37 | 41 | 78 | 0.0064 |
| inf-0.5]' v124='(-inf-0.5]' v152='(-inf- | No Support | 13 | 42 | 55 |  |
| $0.5]^{\prime}$ v230='(-inf-0.5]' 50 ==> class=C-C <br> 37 conf:(0.74) | Column T. | 50 | 83 | 133 |  |
| 1. v78='(-inf-0.5]' v107='(-inf-0.5]' | Support | 24 | 58 | 82 | 0.1241 |
| v138='(-inf-0.5]' v432='(-inf-0.5]' $98==>$ | No Support | 29 | 40 | 69 |  |
| class=C-I 58 conf:(0.59) | Column T. | 53 | 98 | 151 |  |
| 2. v28='(-inf-0.5]' v78='(-inf-0.5]' v79='(-inf-0.5]' v138='(-inf-0.5]' v432='(-inf- | Support | 24 | 58 | 82 | 0.0461 |
| $0.5]^{\prime} 98=$ class $=$ C-I 58 conf:(0.59) | No Support Column T. | 34 58 | 40 98 | 74 |  |
| 3. v78='(-inf-0.5]' v107='(-inf-0.5]' | Support | 24 | 58 | 82 | 0.1241 |
| v120='All' v138='(-inf-0.5]' v432='(-inf- | No Support | 29 | 40 | 69 |  |
| $0.5]^{\prime} 98==>$ class=C-I 58 conf:(0.59) | Column T. | 53 | 98 | 151 |  |

Table 4.14- Risk Factors of Stratums

|  | SMOKERS | NON-SMOKERS |
| :--- | :--- | :--- |
|  | v211- Getting yourself wet | v68- Being proud of yourself |
| n | v128- Having Cancer | v128- Having Cancer |
| v229- Difficulties going to the | v719- Having an active hobby |  |
| bathroom on time | v180- Undergone Female Surgery |  |
|  | v89- Being Diabetes | v69- Feeling lonely |

### 4.3. Imputation Results

The simulation results of imputation are explained in this section.

### 4.3.1. Simulation Results

The rough set imputation is applied on the simulated data with $44 \%$ of embedded missing values. The missing values are imputed when there exists at least $12,3,4$ and 7 , (Threshold 1, Threshold 2, Threshold 3, Threshold 4, and Threshold 7, respectively) identical instances to the instance with the missing case except for the missing attribute itself. Table 4.15 shows the accuracies of the imputation process with all different thresholds. The first column (Correct) shows the percentage of accurately imputed values while the second column (Incorrect) shows the inaccurately imputed values. At the end of the imputation process, the percentages of attributes that remain missing are shown in the third column (Not imputed). Therefore, one can see that, when the number of identical instances necessary to perform imputation increases (i.e., when the threshold increases),
inaccurately imputed values decrease. At the same time, more missing values will remain to be missing at the end of the imputation process.

The 7\% of incorrectly imputed values of the validation data is corrected (Threshold 1-Errors Corrected in Table 4.16). The wald score of this data (imputed data with errors corrected) is then compared both with the Threshold 1 (imputed data with 7\% of error rate) data and the non-imputed data. The results in Table 4.16 show that the imputed data with the corrected errors outperformed the two other datasets.

Also, a predictive index is constructed, based on the predicted probabilities of all possible combinations of the potential predictive factors (This is explained in detail in Section 4.7). This is achieved by constructing contrasts and their 95\% confidence intervals for probabilities of incontinence. For each possible combination ( $2^{n}$ combinations where $n$ is the number of potential predictive factors identified) the difference between the upper and lower bounds of the confidence intervals are identified. The average difference of the $n$ combinations is then calculated and referred as the $C I_{\text {Diff }}$.

Table 4.15- The average imputation results for different threshold levels for several runs

|  | Correct | Incorrect | Not Imputed |
| :--- | :--- | :--- | :--- |
| Threshold 1 | $36 \%$ | $7 \%$ | $56 \%$ |
| Threshold 2 | $25 \%$ | $5 \%$ | $70 \%$ |
| Threshold 3 | $22 \%$ | $3 \%$ | $74 \%$ |
| Threshold 4 | $21 \%$ | $3 \%$ | $75 \%$ |
| Threshold 7 | $11 \%$ | $2 \%$ | $88 \%$ |

Table 4.16- Testing Global Null Hypothesis

|  | Test | Chi-Square | Pr $>$ ChiSq |
| :--- | :--- | :--- | :--- |
| Threshold 1-Errors Corrected | Wald | 29.3546 | 0.0003 |
| Threshold 1-Errors Retained | Wald | 28.6637 | 0.0004 |
| Non-imputed Data | Wald | 23.4406 | 0.0028 |

$$
C I_{\text {Diff }}=\frac{\sum_{i=1}^{n}\left(\text { Upper } \text { Bound }_{i}-{\text { Lower } \left.\text { Bound }_{i}\right)}_{n}^{n}\right.}{\text { ( }}
$$

The results of the three different datasets, Table A.1, Table A. 2 and Table A. 3 (NonImputed Data, Threshold 1- Errors Retained and Threshold 1-Errors Corrected,) are then compared based on their $C I_{D i f f}$ values. Note that, a tighter confidence interval is preferred. The results show that, the improvement of correcting the $7 \%$ of errors with respect to retaining them is 0.009834 , whereas the improvement of imputing the data with respect to no imputation is 0.02855 .

These results show that, although the rough set imputation causes 7\% the data to be incorrectly imputed, the improvement of imputation with respect to leaving the data as it is (i.e. no imputation) is higher. Therefore, in order to generate a more reliable predictive index, imputation needs to be preferred.

### 4.4. Attribute Selection Results

In this experiment we have applied five different attribute selection algorithms to the simulation dataset and the results are compared when noise, incomplete data and multicollinearity are added to the data. The first set of experiments was designed to
evaluate the following attribute selection methods to see which one handles additive noise better: J48, ReliefF, information gain, Consistency based feature selection and Correlation based feature selection.

In order to refer to an attribute selection algorithm as robust, it has to both have a high sensitivity and a high specificity. Figure 4.1 shows that J 48 algorithm performs well in terms of sensitivity but there is a huge decrease in the specificity curve (Figure 4.2) which makes the algorithm less desirable than the others when there is noise.

Without any noise, Information gain and ReliefF algorithms both perform well. Consistency based attribute evaluation has a low specificity and J48 decision tree classification algorithm has a low sensitivity. The results do not change in Cfs, Information Gain and ReliefF algorithms when the noise level is $2 \%, 5 \%, 10 \%, 15 \%$, respectively. In spite, in J48 decision tree the specificity decreases and the sensitivity increases rapidly when the noise level increase. The sensitivity of Consistency based feature selection was $11 \%$ higher than the average sensitivity of other methods. However, its specificity was $37 \%$ lower than that of the average. It is important to note that, the method that maximizes both the sensitivity and specificity is of interest. The best method, when both sensitivity and specificity are considered, is information gain. This method outperformed the average performance of the other methods by $1 \%$ and $20.5 \%$ when we considered its sensitivity and specificity, respectively. Consistency based subset evaluation and J48 algorithms cannot handle noise.


Figure 4.1- Sensitivities of Attribute Selection Methods with Noise Levels


Figure 4.2- Specificities of Attribute Selection Methods with Noise Levels

When multicollinearity is embedded into the dataset without any noise and missing values, the Correlation based feature selection method outperformed other methods.

The next set of experiments is designed to evaluate the attribute selection methods to see which one handles missing values better. Figure 4.3 and Figure 4.4 show the results of different missing value levels. In this case, when considering sensitivity and specificity, ReliefF and information gain are proved to perform better compared to the other methods of our study by $7.2 \%$ and $12.4 \%$, respectively. Despite, the Consistency based feature selection and J48 algorithms cannot handle missing values effectively.

In summary, ReliefF and information gain are the best in all three situations (noise, missing value, multicollinearity) when both sensitivity and specificity measures are considered.

It is also important to note that, the reliability of these methods decrease when there is more than $15 \%$ of noise or $12 \%$ of missing values. Therefore, $15 \%$ of noise and $12 \%$ of missing values have to be defined as thresholds and data that has more than $15 \%$ of noise or $12 \%$ of missing values need to be further pre-processed.

### 4.5. Potential Predictive Factors

The UI risk factors are extracted as body mass index (bmi), sneezing frequency, urine loss problems started after deliveries, trouble getting to the bathroom on time, frequency of wetting/soiling, shutting off the flow of urine in the middle of stream by using muscles and memory problems.


Figure 4.3- Sensitivities of Attribute Selection Methods with Missing Value Levels


Figure 4.4- Specificities of Attribute Selection Methods with Missing Value Levels

The results of the logistic regression consisting eight most promising factors for prediction are given in Table 4.17 along with their corresponding p-values and odd ratios. The significant factors from the regression method and the Relieff algorithm were identical except a new factor (F6) which wasn't extracted as significant from the regression method. The analysis has shown that the factor (F6) is promising for both the datasets constructed having HH2 and HH4 as outcome variables (and HH1 as baseline input factors). Therefore, it is added to the list of potential predictive factors. Table 4.18 shows that not all of the eight risk factors identified from HH1HH2 are performing well when we regressed the third follow up response on the same eight baseline factors. F3, F4, F5 and F6 remain to be significant.

However, when the outliers are removed from the dataset, all eight factors are determined as significant both for the HH 1 HH 2 (predictors from HH 1 and response from HH2) and HH1HH4 (predictors from HH1 and response from HH 4 ) datasets. A case is described as an outlier if all or at least one half of the binary factors are 0 but the outcome is 1 , or the other way round. Table 4.19 shows the results of the p-values for HH 1 HH 2 and HH 1 HH 4 datasets with the removed outliers.

Table 4.20 shows the p-values of HH1HH2 with 2-way interactions of some factors. Possibility of future incontinence and sneezing frequency, memory problems and bmi are the most significant interactions based on their p-values. Once the most significant interactions are determined from HH 1 HH 2 data, HH 1 HH 4 dataset is used to validate the results. The table shows that the same factors are still significant on HH1HH4 data.

Table 4.17- Baseline Factors and First Follow-up Outcome (HH1HH2)

| Factor | Description | P-value | OR <br> 95\% $\boldsymbol{C} \boldsymbol{I}$ |
| :--- | :--- | :--- | :--- | :--- |
| F1 | Body Mass Index (BMI) <=24; >24 | 0.1698 | $(0.827,2.944)$ |
| F2 | Do you sneeze often, rarely or never? | 0.0274 | $(1.088,4.152)$ |
| F3 | Any urine loss problems that started after <br> deliveries: Yes;No | 0.0423 | $(1.05,15.313)$ |
| F4 | Trouble getting to the bathroom on time: Yes;No | 0.0769 | $(0.916,5.539)$ |
| F5 | Frequency of wetting/soiling yourself day/night: <br> Never; 1/week; 1 or 2/week; >3/week | 0.0064 | $(1.573,16.021)$ |
| F6* | When you are urinating into a toilet, can you shut <br> off the flow of urine in the middle of your stream <br> by using your muscles if you want to? | 0.0419 | $(1.031,5.049)$ |
| F7 | As you know people experience memory problems <br> as they get older. What about remembering <br> names? | 0.0672 | $(0.958,3.552)$ |
| F8 | Possibility of future incontinence | 0.0224 | $(1.112,4.014)$ |
| *:Factor Extracted From Relieff Algorithm |  |  |  |

Table 4.18- Baseline Factors and Third Follow-up Outcome (HH1HH4)

| Factor | Description | P-value | OR <br> 95\% $\boldsymbol{C} \boldsymbol{I}$ |
| :--- | :--- | :--- | :--- | :--- |
| F1 | Body Mass Index (BMI) <=24; >24 | 0.6109 | $(0.668,1.987)$ |
| F2 | Do you sneeze often, rarely or never? | 0.6269 | $(0.527,2.894)$ |
| F3 | Any urine loss problems that started after <br> deliveries: Yes;No | 0.0042 | $(2.097,51.848)$ |
| F4 | Trouble getting to the bathroom on time: Yes;No | 0.0116 | $(1.328,9.495)$ |
| F5 | Frequency of wetting/soiling yourself day/night: <br> Never; 1/week; 1 or 2/week; >3/week | 0.0336 | $(1.112,13.915)$ |
| F6* | When you are urinating into a toilet, can you shut <br> off the flow of urine in the middle of your stream <br> by using your muscles if you want to? | 0.0478 | $(1.007,4.639)$ |
| F7 | As you know people experience memory <br> problems as they get older. What about <br> remembering names? | 0.2564 | $(0.792,2.401)$ |
| F8 | Possibility of future incontinence |  |  |
| *:Factor Extracted From Relieff Algorithm | 0.1516 | $(0.858,2.69)$ |  |

Table 4.19- Baseline Factors- First and Third Follow-up Outcome Outliers Removed

| Factor | Description | P-value <br> HH1HH2 | P-value <br> HH1HH4 |
| :--- | :--- | :--- | :--- | :---: |
| F1 | Body Mass Index (BMI) <=24; >24 | 0.0330 | 0.0028 |
| F2 | Do you sneeze often, rarely or never? | 0.0058 | 0.0249 |
| F3 | Any urine loss problems that started after <br> deliveries: Yes;No | 0.0234 | 0.0076 |
| F4 | Trouble getting to the bathroom on time: Yes;No | 0.0354 | 0.0109 |
| F5 | Frequency of wetting/soiling yourself day/night: <br> Never; 1/week; 1 or 2/week; >3/week | 0.0021 | 0.0086 |
| F6* | When you are urinating into a toilet, can you shut <br> off the flow of urine in the middle of your stream <br> by using your muscles if you want to? | 0.0149 | 0.0032 |
| F7 | As you know people experience memory <br> problems as they get older. What about <br> remembering names? | 0.0204 | 0.0093 |
| F8 | Possibility of future incontinence | 0.0062 | 0.0077 |

Table 4.20- Regression Results with 2-way Interactions

| Factor | Description | P-value <br> HH1HH2 | P-value <br> HH1HH4 |
| :--- | :--- | :--- | :--- | :---: |
| F1 | Body Mass Index (BMI) <=24; >24 | 0.0132 | 0.0008 |
| F2 | Do you sneeze often, rarely or never? | 0.0065 | 0.0130 |
| F3 | Any urine loss problems that started after <br> deliveries: Yes;No | 0.0311 | 0.0086 |
| F4 | Trouble getting to the bathroom on time: Yes;No | 0.0429 | 0.0130 |
| F5 | Frequency of wetting/soiling yourself day/night: <br> Never; 1/week; 1 or 2/week; >3/week | 0.0008 | 0.0071 |
| F6* | When you are urinating into a toilet, can you shut <br> off the flow of urine in the middle of your stream <br> by using your muscles if you want to? | 0.0069 | 0.0013 |
| F7 | As you know people experience memory <br> problems as they get older. What about <br> remembering names? | 0.0073 | 0.0020 |
| F8 | Possibility of future incontinence | 0.0015 | 0.0109 |
| F8*F2 | Future Incontinence * Sneezing | 0.0042 | 0.0892 |
| F7*F1 | Memory Problems * BMI | 0.0245 | 0.0282 |

Table 4.21 shows the comparison between the Wald scores of HH1 and HH4 datasets with and without the interactions. The results show that both HH 1 and HH 4 with the retained outliers outperformed the linear model. When the outliers are removed from the data, the linear model outperforms the model with interactions.

### 4.6. Rule Extraction Results

In our experiment, we have also applied four different rule extraction methods to the simulation dataset and the results are compared when noise, incomplete data and multicollinearity are added to the data.

The first set of experiments was designed to evaluate the following rule extraction methods to see which one handles additive noise better: Apriori, JRip, PART and Prism. Table 4.22 and Table 4.23 show the results of those rule extraction methods when there are 5,8 and $10 \%$ noise. Apriori algorithm outperforms the other rule extraction methods if the noise level is 5 or $10 \%$. But if the noise level is beyond $10 \%$ Apriori algorithm lose its effectiveness.

Table 4.21- Comparison of Wald Scores with/without Interactions

|  |  | With Interaction | Without Interaction |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | Chi-Square | Pr $>$ ChiSq | Chi-Square | Pr > ChiSq |
| HH1 with outliers | Wald | 34.89 | 0.0001 | 33.00 | $<.0001$ |
| HH1 without outliers | Wald | 37.05 | $<.0001$ | 40.50 | $<.0001$ |
| HH4 with outliers | Wald | 29.17 | 0.0012 | 28.79 | .0003 |
| HH4 without outliers | Wald | 42.15 | $<.0001$ | 44.94 | $<.0001$ |

Table 4.22- Sensitivity of Rule Extraction Methods at Several Noise Levels

|  | $\mathbf{0 \%}$ | $\mathbf{5 \%}$ | $\mathbf{8 \%}$ | $\mathbf{1 0 \%}$ |
| :--- | :---: | :---: | :---: | :---: |
| Apriori | $100 \pm 0$ | $100 \pm 0$ | $32 \pm 10$ | $0 \pm 0$ |
| PART | $55 \pm 10$ | $0 \pm 0$ | $0 \pm 0$ | $0 \pm 0$ |
| Prism | $90 \pm 20$ | $0 \pm 0$ | $0 \pm 0$ | $0 \pm 0$ |
| JRip | $40 \pm 0$ | $32 \pm 10$ | $36 \pm 8$ | $40 \pm 0$ |

Table 4.23- Specificity of Rule Extraction Methods at Several Noise Levels

|  | $\mathbf{0 \%}$ | $\mathbf{5 \%}$ | $\mathbf{8 \%}$ | $\mathbf{1 0 \%}$ |
| :--- | :---: | :---: | :---: | :---: |
| Apriori | $100 \pm 0$ | $100 \pm 0$ | $96 \pm 2$ | $90 \pm 0$ |
| PART | $75 \pm 10$ | $28 \pm 3$ | $0 \pm 0$ | $0 \pm 0$ |
| Prism | $98 \pm 5$ | $3 \pm 10$ | $0 \pm 0$ | $0 \pm 0$ |
| JRip | $100 \pm 0$ | $96 \pm 0$ | $96 \pm 1$ | $96 \pm 0$ |

Table 4.24- Sensitivity of Rule Extraction Methods at Missing Value Levels

|  | $\mathbf{0 \%}$ | $\mathbf{5 \%}$ | $\mathbf{8 \%}$ | $\mathbf{1 0 \%}$ |
| :--- | :---: | :---: | :---: | :---: |
| Apriori | $100 \pm 0$ | $100 \pm 0$ | $50 \pm 17$ | $0 \pm 0$ |
| PART | $55 \pm 10$ | $58 \pm 26$ | $48 \pm 27$ | $26 \pm 20$ |
| Prism | $90 \pm 20$ | N/A | N/A | N/A |
| JRip | $40 \pm 0$ | $40 \pm 0$ | $40 \pm 0$ | $40 \pm 0$ |

Table 4.25-Specificity of Rule Extraction Methods at Missing Value Levels

|  | $\mathbf{0 \%}$ | $\mathbf{5 \%}$ | $\mathbf{8 \%}$ | $\mathbf{1 0 \%}$ |
| :--- | :---: | :---: | :---: | :---: |
| Apriori | $100 \pm 0$ | $100 \pm 0$ | $100 \pm 17$ | $100 \pm 0$ |
| PART | $75 \pm 10$ | $81 \pm 5$ | $79 \pm 6$ | $80 \pm 5$ |
| Prism | $98 \pm 5$ | N/A | N/A | N/A |
| JRip | $100 \pm 0$ | $99 \pm 2$ | $97 \pm 2$ | $96 \pm 2$ |

At that point the sensitivity and specificities of JRip is better than any other rule extraction method.

Table 4.24 and Table 4.25 show the results of the rule extraction methods when there are 5, 8 and $10 \%$ missing values. Apriori algorithm outperforms other rule extraction methods when there are 5 and $8 \%$ missing values. When the missing values exceeded $8 \%$ the reliability of Apriori decreases and PART gives better results.

### 4.7. Prediction Results

### 4.7.1. Constructed Predictive Index

In this section the most promising risk factors both for the first (HH2) and for the third follow up (HH4) outcome are extracted. A table is generated (Table 4.26) that determines the support and confidence for each combination of values the factors can take. The support threshold is calculated based on the margins of error formula. Note that, we are interested in the subjects that have become incontinent over time (C-I). However, the limited number of subjects that belong to the class 'C-I' prevent the technique from returning a support value that is greater than the support threshold as can be seen in Table 4.26 under the assumption of $90 \%$ confidence and $10 \%$ error rate. Table 4.26 shows that only $15 \%$ of the rules ( 10 out of 64 ) are having a support threshold that is less than or equal to the support of the rule. However, eight of those rules have a confidence of 0 . One has $9 \%$ and the other one has $60 \%$.

Table 4.26- All Possible Decision Rules For Five Promising Factors

| F2 | F3 | F6 | F4 | F5 | HH12 | Antecedent | Support | Confidence | Support Threshold | $\begin{aligned} & \text { Reliable } \\ & (\mathbf{Y} / \mathbf{N}) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | C-C | 278 | 167 | 60.0719424 | 64.51156151 | Y |
| 0 | 0 | 0 | 0 | 1 | C-C | 7 | 3 | 42.8571429 | 65.8677551 | N |
| 0 | 0 | 0 | 1 | 0 | C-C | 20 | 12 | 60 | 64.5504 | N |
| 0 | 0 | 0 | 1 | 1 | C-C | 8 | 3 | 37.5 | 63.0375 | N |
| 0 | 0 | 1 | 0 | 0 | C-C | 48 | 23 | 47.9166667 | 67.12326389 | N |
| 0 | 0 | 1 | 0 | 1 | C-C | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 0 | 0 | 1 | 1 | 0 | C-C | 6 | 2 | 33.3333333 | 59.76888889 | N |
| 0 | 0 | 1 | 1 | 1 | C-C | 1 | 0 | 0 | 0 | Y |
| 0 | 1 | 0 | 0 | 0 | C-C | 8 | 5 | 62.5 | 63.0375 | N |
| 0 | 1 | 0 | 0 | 1 | C-C | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 0 | 1 | 0 | 1 | 0 | C-C | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 0 | 1 | 0 | 1 | 1 | C-C | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 0 | 1 | 1 | 0 | 0 | C-C | 3 | 1 | 33.3333333 | 59.76888889 | N |
| 0 | 1 | 1 | 0 | 1 | C-C | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 0 | 1 | 1 | 1 | 0 | C-C | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 0 | 1 | 1 | 1 | 1 | C-C | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 1 | 0 | 0 | 0 | 0 | C-C | 26 | 11 | 42.3076923 | 65.64852071 | N |
| 1 | 0 | 0 | 0 | 1 | C-C | 4 | 0 | 0 | 0 | Y |
| 1 | 0 | 0 | 1 | 0 | C-C | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 1 | 0 | 0 | 1 | 1 | C-C | 1 | 1 | 100 | 0 | NA |
| 1 | 0 | 1 | 0 | 0 | C-C | 4 | 2 | 50 | 67.24 | NA |
| 1 | 0 | 1 | 0 | 1 | C-C | 1 | 0 | 0 | 0 | Y |
| 1 | 0 | 1 | 1 | 0 | C-C | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 1 | 0 | 1 | 1 | 1 | C-C | 2 | 0 | 0 | 0 | Y |
| 1 | 1 | 0 | 0 | 0 | C-C | 1 | 0 | 0 | 0 | Y |
| 1 | 1 | 0 | 0 | 1 | C-C | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 1 | 1 | 0 | 1 | 0 | C-C | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 1 | 1 | 0 | 1 | 1 | C-C | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 1 | 1 | 1 | 0 | 0 | C-C | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 1 | 1 | 1 | 0 | 1 | C-C | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 1 | 1 | 1 | 1 | 0 | C-C | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 1 | 1 | 1 | 1 | 1 | C-C | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 0 | 0 | 0 | 0 | 0 | C-I | 278 | 26 | 9.35251799 | 22.80195021 | Y |
| 0 | 0 | 0 | 0 | 1 | C-I | 7 | 4 | 57.1428571 | 65.8677551 | N |
| 0 | 0 | 0 | 1 | 0 | C-I | 20 | 7 | 35 | 61.1884 | N |
| 0 | 0 | 0 | 1 | 1 | C-I | 8 | 2 | 25 | 50.43 | N |

Table 4.26- All Possible Decision Rules For Five Promising Factors- Continued

| F2 | F3 | F6 | F4 | F5 | HH12 | Antecedent | Support | Confidence | Support <br> Threshold | Reliable (Y/N) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 1 | 0 | 0 | C-I | 48 | 11 | 22.9166667 | 47.51159722 | N |
| 0 | 0 | 1 | 0 | 1 | C-I | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 0 | 0 | 1 | 1 | 0 | C-I | 6 | 2 | 33.3333333 | 59.76888889 | N |
| 0 | 0 | 1 | 1 | 1 | C-I | 1 | 0 | 0 | 0 | Y |
| 0 | 1 | 0 | 0 | 0 | C-I | 8 | 3 | 37.5 | 63.0375 | N |
| 0 | 1 | 0 | 0 | 1 | C-I | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 0 | 1 | 0 | 1 | 0 | C-I | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 0 | 1 | 0 | 1 | 1 | C-I | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 0 | 1 | 1 | 0 | 0 | C-I | 3 | 1 | 33.3333333 | 59.76888889 | N |
| 0 | 1 | 1 | 0 | 1 | C-I | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 0 | 1 | 1 | 1 | 0 | C-I | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 0 | 1 | 1 | 1 | 1 | C-I | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 1 | 0 | 0 | 0 | 0 | C-I | 26 | 4 | 15.3846154 | 35.01254438 | N |
| 1 | 0 | 0 | 0 | 1 | C-I | 4 | 1 | 25 | 50.43 | N |
| 1 | 0 | 0 | 1 | 0 | C-I | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 1 | 0 | 0 | 1 | 1 | C-I | 1 | 0 | 0 | 0 | Y |
| 1 | 0 | 1 | 0 | 0 | C-I | 4 | 2 | 50 | 67.24 | N |
| 1 | 0 | 1 | 0 | 1 | C-I | 1 | 1 | 100 | 0 | N |
| 1 | 0 | 1 | 1 | 0 | C-I | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 1 | 0 | 1 | 1 | 1 | C-I | 2 | 1 | 50 | 67.24 | N |
| 1 | 1 | 0 | 0 | 0 | C-I | 1 | 1 | 100 | 0 | Y |
| 1 | 1 | 0 | 0 | 1 | C-I | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 1 | 1 | 0 | 1 | 0 | C-I | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 1 | 1 | 0 | 1 | 1 | C-I | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 1 | 1 | 1 | 0 | 0 | C-I | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 1 | 1 | 1 | 0 | 1 | C-I | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 1 | 1 | 1 | 1 | 0 | C-I | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |
| 1 | 1 | 1 | 1 | 1 | C-I | 0 | 0 | \#DIV/0! | \#DIV/0! | NA |

This shows us that the only reliable rule is the following:
IF v41 $=0 \& v 178=0 \& v 215=0 \& v 229=0 \& v 230=0$ THEN HH12 $=$ C-C

This table is generated based on only five potential predictive factors.
4.7.1.1. MESA Data Predictive Index. The MESA Data is imputed and the outliers are removed in the data preprocessing section. The predictive index is determined based on the imputed data. Table 4.27 shows all the potential risk factors along with their odd ratios and confidence limits of the imputed data.

The predicted confidence interval for incontinence for a case with any of the 256 combinations of predictive factors, based on reduced HH1HH2 data set is listed in Table A. 4 (non-imputed data) and A. 5 (imputed data). For instance, a case classified as having a combination of factors F7 and F8 denoted as F78 is coded as 00000011 . This case has a predicted probability of $58-77 \%$ as shown in Table A. 5 (where $58 \%$ is the lower limit and $77 \%$ is the upper limit) of developing incontinence in the next year.

Table 4.27- Odd Ratio Estimates of Potential Predictive Factors of the Imputed Dataset

|  | Odds Ratio Estimates |  |  |
| :--- | :--- | :---: | :---: |
| Effect | Point Estimate | $\mathbf{9 5 \%}$ Wald <br> Confidence Limits |  |
| F3 | 4.355 | 1.096 | 17.299 |
| F4 | 2.694 | 1.139 | 6.373 |
| F5 | 5.256 | 1.500 | 18.411 |
| F6 | 2.456 | 1.134 | 5.319 |
| F7 | 2.113 | 1.099 | 4.065 |
| F8 | 2.408 | 1.283 | 4.519 |
| F1 | 2.202 | 1.163 | 4.168 |
| F2 | 2.008 | 1.057 | 3.813 |

## CHAPTER FIVE

## SUMMARY AND CONCLUSIONS

We have successfully proposed, implemented and validated an automated method for differentiating between different types of missing values and determining inconsistent data. An automated method is particularly important when dealing with large data of complex surveys since each subject should be processed individually. Incomplete data analyses are important because different types of incomplete data (SPMD, UMD, and GMD) cannot be treated the same, e.g. SPMD should not be treated by imputation techniques. Determining and eliminating the inconsistent data, on the other hand, partially eliminates noise.

The proposed method was validated using a simulation study. It was applied on MESA data as a preprocessing step to prepare the data for further analysis with better representation and quality. In this step, the baseline of this longitudinal survey data (MESA) was analyzed focusing solely on the female population. Proposed method is a preprocessing prerequisite for any data mining of clinical survey data.

The method can be applied on any questionnaire which would be convertible to a directed acyclic graph (DAG). The method scales well to high dimensional data as the time complexity is polynomial for the proposed algorithms. Future work include: a) partially estimating noise or respondents' reliability score using inconsistent data, and b) automating the conversion of the questionnaire to its corresponding directed acyclic graph.

Once SPMD missing data is determined, the data is stratified based on the most significant branching questions. This stratification process leads us to determine the different risk factors of each stratum and shows the diverse outcomes on different populations. We achieve two goals by taking advantage of incomplete data in the stratification process: (1) we utilize the wisdom of experts embedded in the data through the questionnaire design processes; (2) We treat a number of null answers without any estimation or imputation technique, preventing any unavoidable misinformation introduced by such methods.

The GMD portion of the data is then imputed using the rough set theory. A simulated data is used to validate the results. One possible future work of this approach is to decompose the mutually exclusive paths of UMD into GMD and SPMD, and using the rough set imputation on the GMD portion of UMD as well.

We have presented a comparison between different attribute selection and rule extraction techniques. The results show that, there is no single attribute selection or rule extraction technique that gives the best results. The advantage of each technique differs in different situations. The results of the attribute selection and rule extraction techniques show that, both are only reliable up to a certain percent of noise and incomplete data level.

The most promising potential predictive factors of the imputed data are then determined using logistic regression based on the original baseline and first follow-up data sets. The results are validated based on the original baseline and third follow-up data. Then a predictive index for urinary incontinence is constructed based on the
predictive probabilities of all possible combinations of the predictive factors. The predictive index can be applied for immediate screening and for predicting future urinary incontinence in older woman of comparable demographics.

APPENDIX A
PREDICTIVE INDEX

Table A.1- Predictive Index of Non-Imputed Simulation Data

| Contrast Estimation and Testing Results by Row |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Contrast | Estimate | Standard <br> Error | Confidence | Limits | Wald Chi- <br> Square | Pr $>$ <br> ChiSq |
| 0 | 0.5 | 0 | . | . | . | . |
| F8 | 0.63 | 0.0532 | 0.5213 | 0.727 | 5.4446 | 0.0196 |
| F7 | 0.5961 | 0.0583 | 0.4788 | 0.7034 | 2.5871 | 0.1077 |
| F78 | 0.7154 | 0.0663 | 0.5703 | 0.8264 | 8.0038 | 0.0047 |
| F6 | 0.6296 | 0.0598 | 0.507 | 0.7374 | 4.2834 | 0.0385 |
| F68 | 0.7432 | 0.0663 | 0.5944 | 0.8511 | 9.3651 | 0.0022 |
| F67 | 0.7149 | 0.0725 | 0.5553 | 0.8344 | 6.6753 | 0.0098 |
| F678 | 0.8103 | 0.0647 | 0.6518 | 0.9069 | 11.9028 | 0.0006 |
| F5 | 0.7386 | 0.0948 | 0.5191 | 0.8809 | 4.4751 | 0.0344 |
| F58 | 0.8279 | 0.0795 | 0.6173 | 0.9349 | 7.9353 | 0.0048 |
| F57 | 0.8065 | 0.0876 | 0.5812 | 0.9261 | 6.4717 | 0.011 |
| F578 | 0.8765 | 0.0667 | 0.6794 | 0.9596 | 10.0986 | 0.0015 |
| F56 | 0.8276 | 0.0816 | 0.6101 | 0.9364 | 7.5236 | 0.0061 |
| F568 | 0.891 | 0.0614 | 0.703 | 0.9658 | 11.0381 | 0.0009 |
| F567 | 0.8763 | 0.0689 | 0.671 | 0.9609 | 9.5004 | 0.0021 |
| F5678 | 0.9235 | 0.0486 | 0.7584 | 0.9789 | 13.1427 | 0.0003 |
| F4 | 0.5837 | 0.0766 | 0.4305 | 0.7223 | 1.1492 | 0.2837 |
| F48 | 0.7048 | 0.0807 | 0.5275 | 0.8362 | 5.0341 | 0.0249 |
| F47 | 0.6742 | 0.0877 | 0.486 | 0.8191 | 3.3131 | 0.0687 |
| F478 | 0.7789 | 0.0782 | 0.5913 | 0.8956 | 7.6942 | 0.0055 |
| F46 | 0.7044 | 0.0868 | 0.5129 | 0.8435 | 4.3432 | 0.0372 |
| F468 | 0.8023 | 0.0757 | 0.6144 | 0.9118 | 8.6217 | 0.0033 |
| F467 | 0.7786 | 0.0838 | 0.5756 | 0.9011 | 6.6932 | 0.0097 |
| F4678 | 0.8569 | 0.0655 | 0.6775 | 0.9447 | 11.2124 | 0.0008 |
| F45 | 0.7984 | 0.0853 | 0.5835 | 0.918 | 6.7392 | 0.0094 |
| F458 | 0.8709 | 0.0665 | 0.679 | 0.9556 | 10.406 | 0.0013 |
| F457 | 0.8539 | 0.0745 | 0.6445 | 0.9496 | 8.7388 | 0.0031 |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |


| F4578 | 0.9087 | 0.0538 | 0.7361 | 0.9726 | 12.5359 | 0.0004 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F456 | 0.8707 | 0.069 | 0.6693 | 0.9572 | 9.6745 | 0.0019 |
| F4568 | 0.9198 | 0.0494 | 0.7554 | 0.977 | 13.289 | 0.0003 |
| F4567 | 0.9085 | 0.056 | 0.7262 | 0.9738 | 11.6144 | 0.0007 |
| F45678 | 0.9442 | 0.038 | 0.8043 | 0.9858 | 15.3462 | <. 0001 |
| F3 | 0.5218 | 0.1304 | 0.2815 | 0.7524 | 0.0279 | 0.8674 |
| F38 | 0.6501 | 0.1228 | 0.3922 | 0.8426 | 1.318 | 0.251 |
| F37 | 0.6169 | 0.1362 | 0.3422 | 0.8329 | 0.6828 | 0.4086 |
| F378 | 0.7328 | 0.1152 | 0.464 | 0.8968 | 2.9412 | 0.0863 |
| F36 | 0.6497 | 0.1266 | 0.3839 | 0.8466 | 1.2322 | 0.267 |
| F368 | 0.7595 | 0.105 | 0.5058 | 0.9069 | 4.0009 | 0.0455 |
| F367 | 0.7324 | 0.1194 | 0.4533 | 0.9003 | 2.7306 | 0.0984 |
| F3678 | 0.8233 | 0.0906 | 0.579 | 0.9404 | 6.1106 | 0.0134 |
| F35 | 0.7551 | 0.133 | 0.4296 | 0.9266 | 2.4508 | 0.1175 |
| F358 | 0.84 | 0.1 | 0.5499 | 0.9575 | 4.9704 | 0.0258 |
| F357 | 0.8198 | 0.1136 | 0.5018 | 0.9536 | 3.8786 | 0.0489 |
| F3578 | 0.8857 | 0.0799 | 0.6225 | 0.9733 | 6.7251 | 0.0095 |
| F356 | 0.8397 | 0.102 | 0.5427 | 0.9585 | 4.7787 | 0.0288 |
| F3568 | 0.8992 | 0.071 | 0.6579 | 0.9764 | 7.8149 | 0.0052 |
| F3567 | 0.8855 | 0.0818 | 0.6139 | 0.9741 | 6.4259 | 0.0112 |
| F35678 | 0.9294 | 0.0543 | 0.7221 | 0.9852 | 9.6938 | 0.0018 |
| F34 | 0.6047 | 0.1505 | 0.3081 | 0.8402 | 0.4558 | 0.4996 |
| F348 | 0.7226 | 0.1289 | 0.4248 | 0.9019 | 2.2163 | 0.1366 |
| F347 | 0.693 | 0.1439 | 0.3749 | 0.8947 | 1.4498 | 0.2286 |
| F3478 | 0.7936 | 0.1123 | 0.5007 | 0.9365 | 3.8583 | 0.0495 |
| F346 | 0.7222 | 0.1333 | 0.4142 | 0.9053 | 2.0682 | 0.1504 |
| F3468 | 0.8158 | 0.1021 | 0.539 | 0.9437 | 4.7972 | 0.0285 |
| F3467 | 0.7932 | 0.1165 | 0.4881 | 0.9392 | 3.5823 | 0.0584 |
| F34678 | 0.8673 | 0.083 | 0.6138 | 0.9641 | 6.772 | 0.0093 |
| F345 | 0.8121 | 0.1163 | 0.4924 | 0.9506 | 3.6859 | 0.0549 |
| F3458 | 0.8804 | 0.0827 | 0.6124 | 0.9717 | 6.4646 | 0.011 |
| F3457 | 0.8645 | 0.095 | 0.5655 | 0.969 | 5.2215 | 0.0223 |


| F34578 | 0.9157 | 0.064 | 0.6812 | 0.9822 | 8.2685 | 0.004 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F3456 | 0.8802 | 0.0848 | 0.603 | 0.9726 | 6.1492 | 0.0131 |
| F34568 | 0.926 | 0.0567 | 0.7119 | 0.9844 | 9.3237 | 0.0023 |
| F34567 | 0.9155 | 0.0659 | 0.6713 | 0.9829 | 7.8307 | 0.0051 |
| F345678 | 0.9486 | 0.0425 | 0.77 | 0.9903 | 11.2031 | 0.0008 |
| F2 | 0.6226 | 0.0571 | 0.5061 | 0.7264 | 4.243 | 0.0394 |
| F28 | 0.7375 | 0.0667 | 0.5884 | 0.8466 | 8.9853 | 0.0027 |
| F27 | 0.7088 | 0.0647 | 0.5684 | 0.8181 | 8.0546 | 0.0045 |
| F278 | 0.8056 | 0.0613 | 0.658 | 0.8993 | 13.1766 | 0.0003 |
| F26 | 0.7371 | 0.0664 | 0.5887 | 0.8459 | 9.0388 | 0.0026 |
| F268 | 0.8268 | 0.0608 | 0.6751 | 0.9164 | 13.5658 | 0.0002 |
| F267 | 0.8053 | 0.0626 | 0.6543 | 0.9004 | 12.662 | 0.0004 |
| F2678 | 0.8757 | 0.0507 | 0.7386 | 0.9461 | 17.5394 | <. 0001 |
| F25 | 0.8233 | 0.0778 | 0.6204 | 0.93 | 8.2858 | 0.004 |
| F258 | 0.8881 | 0.0599 | 0.7088 | 0.9628 | 11.8047 | 0.0006 |
| F257 | 0.873 | 0.0647 | 0.6866 | 0.9557 | 10.9192 | 0.001 |
| F2578 | 0.9213 | 0.0466 | 0.7685 | 0.9764 | 14.6377 | 0.0001 |
| F256 | 0.8879 | 0.0601 | 0.7079 | 0.9628 | 11.7366 | 0.0006 |
| F2568 | 0.931 | 0.0429 | 0.7848 | 0.9803 | 15.1994 | <. 0001 |
| F2567 | 0.9212 | 0.0472 | 0.7659 | 0.9766 | 14.3247 | 0.0002 |
| F25678 | 0.9522 | 0.0321 | 0.8331 | 0.9876 | 17.9647 | <. 0001 |
| F24 | 0.6981 | 0.0842 | 0.5138 | 0.835 | 4.4007 | 0.0359 |
| F248 | 0.7975 | 0.0755 | 0.6118 | 0.9078 | 8.6015 | 0.0034 |
| F247 | 0.7734 | 0.0785 | 0.5865 | 0.8914 | 7.5117 | 0.0061 |
| F2478 | 0.8532 | 0.0632 | 0.6836 | 0.9399 | 12.154 | 0.0005 |
| F246 | 0.7972 | 0.077 | 0.6071 | 0.909 | 8.2605 | 0.0041 |
| F2468 | 0.87 | 0.0607 | 0.7003 | 0.9504 | 12.5321 | 0.0004 |
| F2467 | 0.8529 | 0.0652 | 0.6768 | 0.9414 | 11.4396 | 0.0007 |
| F24678 | 0.9081 | 0.0478 | 0.763 | 0.9681 | 16.0288 | <. 0001 |
| F245 | 0.8672 | 0.0659 | 0.6804 | 0.9525 | 10.7636 | 0.001 |
| F2458 | 0.9175 | 0.0481 | 0.762 | 0.9748 | 14.3702 | 0.0002 |
| F2457 | 0.906 | 0.0527 | 0.7412 | 0.9701 | 13.3876 | 0.0003 |


| F24578 | 0.9426 | 0.0365 | 0.8139 | 0.984 | 17.184 | <. 0001 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F2456 | 0.9174 | 0.0488 | 0.7586 | 0.9751 | 13.9691 | 0.0002 |
| F24568 | 0.9498 | 0.0335 | 0.8266 | 0.9868 | 17.4799 | <. 0001 |
| F24567 | 0.9425 | 0.0373 | 0.8096 | 0.9844 | 16.5125 | $<.0001$ |
| F245678 | 0.9654 | 0.0247 | 0.8673 | 0.9917 | 20.198 | <. 0001 |
| F23 | 0.6428 | 0.1315 | 0.3693 | 0.8469 | 1.0521 | 0.305 |
| F238 | 0.754 | 0.1104 | 0.4885 | 0.9077 | 3.5439 | 0.0598 |
| F237 | 0.7265 | 0.1206 | 0.4471 | 0.8971 | 2.5922 | 0.1074 |
| F2378 | 0.8189 | 0.0925 | 0.5711 | 0.9389 | 5.8498 | 0.0156 |
| F236 | 0.7536 | 0.111 | 0.4866 | 0.908 | 3.4969 | 0.0615 |
| F2368 | 0.8389 | 0.084 | 0.6065 | 0.9462 | 7.0551 | 0.0079 |
| F2367 | 0.8186 | 0.0939 | 0.5666 | 0.9397 | 5.6823 | 0.0171 |
| F23678 | 0.8849 | 0.0663 | 0.6821 | 0.965 | 9.813 | 0.0017 |
| F235 | 0.8356 | 0.1026 | 0.5404 | 0.9565 | 4.7395 | 0.0295 |
| F2358 | 0.8965 | 0.072 | 0.6544 | 0.9754 | 7.7444 | 0.0054 |
| F2357 | 0.8824 | 0.0813 | 0.6179 | 0.9721 | 6.6233 | 0.0101 |
| F23578 | 0.9274 | 0.0544 | 0.724 | 0.9842 | 9.9489 | 0.0016 |
| F2356 | 0.8963 | 0.0725 | 0.6522 | 0.9755 | 7.6535 | 0.0057 |
| F23568 | 0.9364 | 0.0482 | 0.7509 | 0.9863 | 11.0519 | 0.0009 |
| F23567 | 0.9273 | 0.055 | 0.7205 | 0.9844 | 9.7397 | 0.0018 |
| F235678 | 0.956 | 0.0354 | 0.8068 | 0.9912 | 13.3926 | 0.0003 |
| F234 | 0.7162 | 0.1368 | 0.4029 | 0.9042 | 1.8915 | 0.169 |
| F2348 | 0.8112 | 0.1058 | 0.5258 | 0.9434 | 4.4497 | 0.0349 |
| F2347 | 0.7883 | 0.1173 | 0.4842 | 0.9366 | 3.496 | 0.0615 |
| F23478 | 0.8638 | 0.0844 | 0.6085 | 0.9628 | 6.6284 | 0.01 |
| F2346 | 0.8109 | 0.1074 | 0.5207 | 0.9442 | 4.3201 | 0.0377 |
| F23468 | 0.8796 | 0.0763 | 0.6404 | 0.9677 | 7.627 | 0.0057 |
| F23467 | 0.8635 | 0.0862 | 0.6016 | 0.9637 | 6.3676 | 0.0116 |
| F234678 | 0.9151 | 0.058 | 0.7137 | 0.979 | 10.1329 | 0.0015 |
| F2345 | 0.877 | 0.0852 | 0.6027 | 0.971 | 6.1882 | 0.0129 |
| F23458 | 0.9239 | 0.0574 | 0.7103 | 0.9836 | 9.3563 | 0.0022 |
| F23457 | 0.9132 | 0.0654 | 0.6763 | 0.9815 | 8.1401 | 0.0043 |


| F234578 | 0.9471 | 0.0424 | 0.7728 | 0.9895 | 11.5885 | 0.0007 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F23456 | 0.9237 | 0.0582 | 0.706 | 0.9839 | 9.1286 | 0.0025 |
| F234568 | 0.9538 | 0.0376 | 0.795 | 0.991 | 12.5981 | 0.0004 |
| F234567 | 0.947 | 0.0432 | 0.7679 | 0.9898 | 11.2215 | 0.0008 |
| F2345678 | 0.9682 | 0.0272 | 0.8432 | 0.9942 | 14.9198 | 0.0001 |
| F1 | 0.5635 | 0.0556 | 0.4533 | 0.6678 | 1.2784 | 0.2582 |
| F18 | 0.6874 | 0.0686 | 0.5405 | 0.8043 | 6.0967 | 0.0135 |
| F17 | 0.6558 | 0.0774 | 0.4931 | 0.7886 | 3.5344 | 0.0601 |
| F178 | 0.7644 | 0.0729 | 0.5948 | 0.8776 | 8.4556 | 0.0036 |
| F16 | 0.6869 | 0.0721 | 0.5322 | 0.8089 | 5.495 | 0.0191 |
| F168 | 0.7889 | 0.0679 | 0.6271 | 0.8925 | 10.4671 | 0.0012 |
| F167 | 0.764 | 0.0768 | 0.5843 | 0.8818 | 7.6127 | 0.0058 |
| F1678 | 0.8465 | 0.0624 | 0.6827 | 0.9339 | 12.6471 | 0.0004 |
| F15 | 0.7848 | 0.0963 | 0.5438 | 0.9177 | 5.1445 | 0.0233 |
| F158 | 0.8613 | 0.075 | 0.6447 | 0.9551 | 8.4628 | 0.0036 |
| F157 | 0.8433 | 0.0843 | 0.6064 | 0.9495 | 6.9539 | 0.0084 |
| F1578 | 0.9016 | 0.0609 | 0.7047 | 0.9724 | 10.4087 | 0.0013 |
| F156 | 0.8611 | 0.0763 | 0.6396 | 0.9559 | 8.1718 | 0.0043 |
| F1568 | 0.9135 | 0.0547 | 0.7311 | 0.9762 | 11.5993 | 0.0007 |
| F1567 | 0.9014 | 0.0623 | 0.6983 | 0.9731 | 9.9654 | 0.0016 |
| F15678 | 0.9397 | 0.0424 | 0.7826 | 0.9854 | 13.4949 | 0.0002 |
| F14 | 0.6441 | 0.0916 | 0.4527 | 0.7984 | 2.2059 | 0.1375 |
| F148 | 0.7551 | 0.0846 | 0.557 | 0.8832 | 6.0518 | 0.0139 |
| F147 | 0.7276 | 0.0946 | 0.5118 | 0.8719 | 4.238 | 0.0395 |
| F1478 | 0.8198 | 0.0772 | 0.6203 | 0.9268 | 8.4036 | 0.0037 |
| F146 | 0.7547 | 0.0887 | 0.5461 | 0.8872 | 5.5025 | 0.019 |
| F1468 | 0.8397 | 0.0715 | 0.6491 | 0.9369 | 9.7203 | 0.0018 |
| F1467 | 0.8195 | 0.081 | 0.6081 | 0.93 | 7.6264 | 0.0058 |
| F14678 | 0.8855 | 0.0599 | 0.7086 | 0.9609 | 12.0102 | 0.0005 |
| F145 | 0.8364 | 0.0838 | 0.6064 | 0.9444 | 7.1055 | 0.0077 |
| F1458 | 0.897 | 0.0615 | 0.7028 | 0.9698 | 10.5852 | 0.0011 |
| F1457 | 0.883 | 0.07 | 0.6669 | 0.966 | 8.9113 | 0.0028 |


| F14578 | 0.9278 | 0.0484 | 0.7573 | 0.9815 | 12.5002 | 0.0004 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F1456 | 0.8968 | 0.0631 | 0.6954 | 0.9707 | 10.0513 | 0.0015 |
| F14568 | 0.9367 | 0.0434 | 0.779 | 0.9842 | 13.5533 | 0.0002 |
| F14567 | 0.9277 | 0.0498 | 0.7496 | 0.9821 | 11.8106 | 0.0006 |
| F145678 | 0.9562 | 0.0329 | 0.824 | 0.9903 | 15.406 | <. 0001 |
| F13 | 0.5848 | 0.1353 | 0.3209 | 0.8077 | 0.3781 | 0.5386 |
| F138 | 0.7058 | 0.1188 | 0.4387 | 0.8805 | 2.3385 | 0.1262 |
| F137 | 0.6752 | 0.1348 | 0.3839 | 0.874 | 1.4177 | 0.2338 |
| F1378 | 0.7798 | 0.1073 | 0.51 | 0.9233 | 4.0968 | 0.043 |
| F136 | 0.7054 | 0.1216 | 0.4319 | 0.8829 | 2.2244 | 0.1358 |
| F1368 | 0.803 | 0.0952 | 0.5562 | 0.9299 | 5.4511 | 0.0196 |
| F1367 | 0.7794 | 0.1104 | 0.5009 | 0.9256 | 3.8644 | 0.0493 |
| F13678 | 0.8575 | 0.0799 | 0.6255 | 0.9559 | 7.5338 | 0.0061 |
| F135 | 0.7992 | 0.1231 | 0.4696 | 0.9471 | 3.2446 | 0.0717 |
| F1358 | 0.8714 | 0.0884 | 0.5907 | 0.9695 | 5.8802 | 0.0153 |
| F1357 | 0.8545 | 0.1018 | 0.5413 | 0.9669 | 4.6741 | 0.0306 |
| F13578 | 0.9091 | 0.0692 | 0.6597 | 0.981 | 7.5665 | 0.0059 |
| F1356 | 0.8712 | 0.0898 | 0.5849 | 0.9701 | 5.7052 | 0.0169 |
| F13568 | 0.9201 | 0.0605 | 0.6963 | 0.983 | 8.8082 | 0.003 |
| F13567 | 0.9089 | 0.0705 | 0.6528 | 0.9815 | 7.2962 | 0.0069 |
| F135678 | 0.9444 | 0.0457 | 0.7551 | 0.9894 | 10.5802 | 0.0011 |
| F134 | 0.6639 | 0.1486 | 0.3488 | 0.8793 | 1.0452 | 0.3066 |
| F1348 | 0.7708 | 0.1197 | 0.4713 | 0.927 | 3.2051 | 0.0734 |
| F1347 | 0.7445 | 0.1361 | 0.4176 | 0.9222 | 2.2351 | 0.1349 |
| F13478 | 0.8323 | 0.101 | 0.5459 | 0.9535 | 4.9044 | 0.0268 |
| F1346 | 0.7705 | 0.123 | 0.462 | 0.9292 | 3.0313 | 0.0817 |
| F13468 | 0.8511 | 0.0898 | 0.5876 | 0.9582 | 6.0482 | 0.0139 |
| F13467 | 0.832 | 0.1041 | 0.5349 | 0.9552 | 4.6131 | 0.0317 |
| F134678 | 0.894 | 0.0715 | 0.6578 | 0.9737 | 7.9883 | 0.0047 |
| F1345 | 0.848 | 0.1048 | 0.5313 | 0.9649 | 4.469 | 0.0345 |
| F13458 | 0.9048 | 0.0718 | 0.6496 | 0.9799 | 7.2929 | 0.0069 |
| F13457 | 0.8917 | 0.0834 | 0.6025 | 0.9781 | 5.9621 | 0.0146 |


| F134578 | 0.9334 | 0.0547 | 0.7141 | 0.9875 | 8.9996 | 0.0027 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F13456 | 0.9046 | 0.0733 | 0.6419 | 0.9805 | 7.0052 | 0.0081 |
| F134568 | 0.9417 | 0.0478 | 0.7454 | 0.9889 | 10.1959 | 0.0014 |
| F134567 | 0.9333 | 0.056 | 0.7059 | 0.9879 | 8.6061 | 0.0034 |
| F1345678 | 0.9597 | 0.0354 | 0.7979 | 0.9931 | 11.9538 | 0.0005 |
| F12 | 0.6805 | 0.073 | 0.5244 | 0.8044 | 5.0639 | 0.0244 |
| F128 | 0.7839 | 0.0701 | 0.6171 | 0.8908 | 9.6926 | 0.0019 |
| F127 | 0.7586 | 0.0733 | 0.5893 | 0.8731 | 8.1926 | 0.0042 |
| F1278 | 0.8426 | 0.0614 | 0.6837 | 0.9298 | 13.1519 | 0.0003 |
| F126 | 0.7835 | 0.0693 | 0.619 | 0.8896 | 9.9056 | 0.0016 |
| F1268 | 0.8604 | 0.0574 | 0.7073 | 0.9402 | 14.4876 | 0.0001 |
| F1267 | 0.8423 | 0.0618 | 0.682 | 0.9301 | 12.9543 | 0.0003 |
| F12678 | 0.9009 | 0.0467 | 0.7655 | 0.962 | 17.8243 | <. 0001 |
| F125 | 0.8575 | 0.0746 | 0.645 | 0.9522 | 8.6311 | 0.0033 |
| F1258 | 0.9111 | 0.0543 | 0.7336 | 0.9744 | 12.0477 | 0.0005 |
| F1257 | 0.8988 | 0.0601 | 0.7089 | 0.97 | 10.944 | 0.0009 |
| F12578 | 0.938 | 0.0415 | 0.7891 | 0.9839 | 14.5349 | 0.0001 |
| F1256 | 0.9109 | 0.0543 | 0.7337 | 0.9743 | 12.071 | 0.0005 |
| F12568 | 0.9457 | 0.0373 | 0.8076 | 0.9863 | 15.4925 | <. 0001 |
| F12567 | 0.9378 | 0.0417 | 0.7876 | 0.984 | 14.3633 | 0.0002 |
| F125678 | 0.9625 | 0.0276 | 0.8512 | 0.9914 | 17.938 | <. 0001 |
| F124 | 0.7491 | 0.0887 | 0.5421 | 0.8827 | 5.3718 | 0.0205 |
| F1248 | 0.8356 | 0.0727 | 0.6431 | 0.9348 | 9.4394 | 0.0021 |
| F1247 | 0.815 | 0.0786 | 0.6134 | 0.9244 | 8.0977 | 0.0044 |
| F12478 | 0.8824 | 0.0591 | 0.7107 | 0.9582 | 12.5156 | 0.0004 |
| F1246 | 0.8354 | 0.0735 | 0.6404 | 0.9353 | 9.241 | 0.0024 |
| F12468 | 0.8963 | 0.0547 | 0.7318 | 0.9647 | 13.4504 | 0.0002 |
| F12467 | 0.8822 | 0.0603 | 0.706 | 0.9589 | 12.0362 | 0.0005 |
| F124678 | 0.9273 | 0.0423 | 0.7888 | 0.9775 | 16.5073 | <. 0001 |
| F1245 | 0.894 | 0.0617 | 0.7019 | 0.968 | 10.7312 | 0.0011 |
| F12458 | 0.9349 | 0.043 | 0.7825 | 0.9829 | 14.2282 | 0.0002 |
| F12457 | 0.9256 | 0.0481 | 0.7599 | 0.98 | 13.0295 | 0.0003 |


| F124578 | 0.9549 | 0.0322 | 0.8304 | 0.9892 | 16.6887 | <. 0001 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F12456 | 0.9348 | 0.0434 | 0.7803 | 0.983 | 13.9864 | 0.0002 |
| F124568 | 0.9606 | 0.0289 | 0.845 | 0.9909 | 17.446 | <. 0001 |
| F124567 | 0.9549 | 0.0326 | 0.8274 | 0.9894 | 16.2365 | <. 0001 |
| F1245678 | 0.973 | 0.0211 | 0.8816 | 0.9943 | 19.8443 | <. 0001 |
| F123 | 0.6991 | 0.1277 | 0.4142 | 0.8842 | 1.9293 | 0.1648 |
| F1238 | 0.7983 | 0.1009 | 0.5367 | 0.9311 | 4.8154 | 0.0282 |
| F1237 | 0.7742 | 0.1128 | 0.4919 | 0.9239 | 3.6477 | 0.0561 |
| F12378 | 0.8538 | 0.0825 | 0.6154 | 0.9552 | 7.138 | 0.0075 |
| F1236 | 0.7979 | 0.1011 | 0.536 | 0.931 | 4.7962 | 0.0285 |
| F12368 | 0.8705 | 0.0731 | 0.6535 | 0.96 | 8.6349 | 0.0033 |
| F12367 | 0.8535 | 0.0833 | 0.6123 | 0.9555 | 7.0036 | 0.0081 |
| F123678 | 0.9085 | 0.0568 | 0.7224 | 0.9743 | 11.2915 | 0.0008 |
| F1235 | 0.8678 | 0.0912 | 0.5803 | 0.9689 | 5.6058 | 0.0179 |
| F12358 | 0.9179 | 0.0619 | 0.691 | 0.9824 | 8.6464 | 0.0033 |
| F12357 | 0.9064 | 0.0707 | 0.6541 | 0.9802 | 7.4212 | 0.0064 |
| F123578 | 0.9428 | 0.0461 | 0.755 | 0.9888 | 10.7239 | 0.0011 |
| F12356 | 0.9177 | 0.0621 | 0.6898 | 0.9824 | 8.5941 | 0.0034 |
| F123568 | 0.95 | 0.0404 | 0.7822 | 0.9901 | 12.0039 | 0.0005 |
| F123567 | 0.9427 | 0.0465 | 0.7525 | 0.9889 | 10.5655 | 0.0012 |
| F1235678 | 0.9656 | 0.0294 | 0.8318 | 0.9937 | 14.1845 | 0.0002 |
| F1234 | 0.7651 | 0.1274 | 0.4482 | 0.9289 | 2.7768 | 0.0956 |
| F12348 | 0.8473 | 0.0938 | 0.5725 | 0.9583 | 5.5832 | 0.0181 |
| F12347 | 0.8278 | 0.1059 | 0.5285 | 0.9537 | 4.4679 | 0.0345 |
| F123478 | 0.8912 | 0.0733 | 0.6506 | 0.973 | 7.7445 | 0.0054 |
| F12346 | 0.847 | 0.0948 | 0.5688 | 0.9587 | 5.4669 | 0.0194 |
| F123468 | 0.9041 | 0.065 | 0.6844 | 0.9762 | 8.9563 | 0.0028 |
| F123467 | 0.8909 | 0.0745 | 0.6452 | 0.9735 | 7.5078 | 0.0061 |
| F1234678 | 0.9329 | 0.0488 | 0.7508 | 0.9847 | 11.3722 | 0.0007 |
| F12345 | 0.902 | 0.0743 | 0.6393 | 0.9795 | 6.9757 | 0.0083 |
| F123458 | 0.94 | 0.0488 | 0.7422 | 0.9884 | 10.1291 | 0.0015 |
| F123457 | 0.9314 | 0.0561 | 0.7084 | 0.987 | 8.8252 | 0.003 |


| F1234578 | 0.9586 | 0.0357 | 0.7989 | 0.9926 | 12.2099 | 0.0005 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| F123456 | 0.9399 | 0.0492 | 0.7391 | 0.9886 | 9.9484 | 0.0016 |
| F1234568 | 0.9638 | 0.0313 | 0.8212 | 0.9936 | 13.3926 | 0.0003 |
| F1234567 | 0.9585 | 0.0362 | 0.7951 | 0.9928 | 11.9082 | 0.0006 |
| F12345678 | 0.9752 | 0.0225 | 0.8637 | 0.9959 | 15.542 | $<.0001$ |

Table A.2- Predictive Index of Imputed-Errors Retained Simulation Data

| Contrast Estimation and Testing Results by Row |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Contrast | Estimate | Standard Error | Confidence | Limits | Wald Chi- <br> Square | $\begin{aligned} & \text { Pr }> \\ & \text { ChiSq } \end{aligned}$ |
| 0 | 0.5 | 0 | . | . | . | . |
| F8 | 0.6047 | 0.0488 | 0.5062 | 0.6954 | 4.3294 | 0.0375 |
| F7 | 0.602 | 0.051 | 0.4992 | 0.6965 | 3.7846 | 0.0517 |
| F78 | 0.6982 | 0.0596 | 0.5706 | 0.8011 | 8.7909 | 0.003 |
| F6 | 0.6517 | 0.0537 | 0.5407 | 0.7483 | 7.0228 | 0.008 |
| F68 | 0.7411 | 0.0602 | 0.6075 | 0.8411 | 11.2337 | 0.0008 |
| F67 | 0.7389 | 0.0601 | 0.6059 | 0.839 | 11.1653 | 0.0008 |
| F678 | 0.8124 | 0.0555 | 0.6796 | 0.8983 | 16.201 | <. 0001 |
| F5 | 0.7386 | 0.0828 | 0.5494 | 0.8675 | 5.8654 | 0.0154 |
| F58 | 0.8121 | 0.0744 | 0.6243 | 0.9183 | 9.0069 | 0.0027 |
| F57 | 0.8104 | 0.0757 | 0.6194 | 0.9182 | 8.6934 | 0.0032 |
| F578 | 0.8673 | 0.0619 | 0.6948 | 0.9494 | 12.1688 | 0.0005 |
| F56 | 0.8409 | 0.0668 | 0.6652 | 0.9336 | 11.1204 | 0.0009 |
| F568 | 0.8899 | 0.054 | 0.7329 | 0.9597 | 14.3675 | 0.0002 |
| F567 | 0.8888 | 0.0545 | 0.7308 | 0.9593 | 14.2232 | 0.0002 |
| F5678 | 0.9244 | 0.0414 | 0.7928 | 0.9751 | 17.8283 | <. 0001 |
| F4 | 0.5707 | 0.0713 | 0.4291 | 0.7015 | 0.9569 | 0.328 |
| F48 | 0.6703 | 0.0809 | 0.4981 | 0.8064 | 3.7606 | 0.0525 |
| F47 | 0.6678 | 0.0793 | 0.4994 | 0.8021 | 3.8142 | 0.0508 |
| F478 | 0.7546 | 0.0764 | 0.5781 | 0.8735 | 7.4143 | 0.0065 |
| F46 | 0.7132 | 0.0778 | 0.5412 | 0.8398 | 5.7322 | 0.0167 |
| F468 | 0.7918 | 0.0728 | 0.6156 | 0.9004 | 9.1598 | 0.0025 |
| F467 | 0.79 | 0.0711 | 0.6188 | 0.8971 | 9.5475 | 0.002 |
| F4678 | 0.8519 | 0.0601 | 0.6934 | 0.9361 | 13.4895 | 0.0002 |
| F45 | 0.7897 | 0.075 | 0.6079 | 0.901 | 8.59 | 0.0034 |
| F458 | 0.8517 | 0.065 | 0.6767 | 0.9403 | 11.52 | 0.0007 |
| F457 | 0.8503 | 0.065 | 0.6762 | 0.9392 | 11.5785 | 0.0007 |
| F4578 | 0.8968 | 0.052 | 0.743 | 0.9631 | 14.8331 | 0.0001 |


| F456 | 0.8754 | 0.057 | 0.716 | 0.9514 | 13.8975 | 0.0002 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F4568 | 0.9149 | 0.0451 | 0.7754 | 0.971 | 16.7928 | <. 0001 |
| F4567 | 0.914 | 0.0449 | 0.7763 | 0.9702 | 17.1314 | <. 0001 |
| F45678 | 0.9421 | 0.0337 | 0.8289 | 0.982 | 20.3858 | <. 0001 |
| F3 | 0.5522 | 0.1285 | 0.3082 | 0.7735 | 0.1629 | 0.6865 |
| F38 | 0.6536 | 0.1218 | 0.3967 | 0.8441 | 1.3931 | 0.2379 |
| F37 | 0.651 | 0.1278 | 0.3826 | 0.8489 | 1.2294 | 0.2675 |
| F378 | 0.7405 | 0.1102 | 0.4813 | 0.8977 | 3.3461 | 0.0674 |
| F36 | 0.6977 | 0.1154 | 0.4413 | 0.8708 | 2.3375 | 0.1263 |
| F368 | 0.7792 | 0.0972 | 0.5384 | 0.9144 | 4.9832 | 0.0256 |
| F367 | 0.7773 | 0.1011 | 0.5263 | 0.9164 | 4.5791 | 0.0324 |
| F3678 | 0.8423 | 0.0791 | 0.6244 | 0.9449 | 7.9175 | 0.0049 |
| F35 | 0.777 | 0.1171 | 0.481 | 0.9291 | 3.4138 | 0.0647 |
| F358 | 0.842 | 0.0929 | 0.5755 | 0.9545 | 5.7387 | 0.0166 |
| F357 | 0.8405 | 0.0963 | 0.5631 | 0.9557 | 5.3512 | 0.0207 |
| F3578 | 0.8897 | 0.0722 | 0.6561 | 0.9715 | 8.055 | 0.0045 |
| F356 | 0.867 | 0.0811 | 0.6214 | 0.9628 | 7.0983 | 0.0077 |
| F3568 | 0.9089 | 0.0602 | 0.7061 | 0.9764 | 10.0279 | 0.0015 |
| F3567 | 0.9079 | 0.062 | 0.6972 | 0.9769 | 9.5098 | 0.002 |
| F35678 | 0.9378 | 0.0443 | 0.7731 | 0.9852 | 12.7817 | 0.0004 |
| F34 | 0.6211 | 0.1434 | 0.3318 | 0.844 | 0.658 | 0.4173 |
| F348 | 0.7149 | 0.1286 | 0.4213 | 0.8962 | 2.1229 | 0.1451 |
| F347 | 0.7126 | 0.132 | 0.4121 | 0.8977 | 1.9843 | 0.1589 |
| F3478 | 0.7914 | 0.109 | 0.5099 | 0.9326 | 4.0812 | 0.0434 |
| F346 | 0.7541 | 0.118 | 0.4685 | 0.9143 | 3.1034 | 0.0781 |
| F3468 | 0.8243 | 0.0952 | 0.5639 | 0.9445 | 5.5264 | 0.0187 |
| F3467 | 0.8227 | 0.0973 | 0.5566 | 0.9449 | 5.2937 | 0.0214 |
| F34678 | 0.8765 | 0.0739 | 0.6507 | 0.9643 | 8.2484 | 0.0041 |
| F345 | 0.8224 | 0.1025 | 0.5391 | 0.9483 | 4.7654 | 0.029 |
| F3458 | 0.8763 | 0.0791 | 0.6291 | 0.9673 | 7.2044 | 0.0073 |
| F3457 | 0.8751 | 0.0811 | 0.6206 | 0.9678 | 6.879 | 0.0087 |
| F34578 | 0.9146 | 0.0596 | 0.7058 | 0.9795 | 9.6487 | 0.0019 |


| F3456 | 0.8965 | 0.0679 | 0.6736 | 0.9733 | 8.6982 | 0.0032 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F34568 | 0.9298 | 0.0495 | 0.7497 | 0.9832 | 11.5977 | 0.0007 |
| F34567 | 0.9291 | 0.0506 | 0.7442 | 0.9833 | 11.2284 | 0.0008 |
| F345678 | 0.9525 | 0.0357 | 0.8103 | 0.9895 | 14.4386 | 0.0001 |
| F2 | 0.6023 | 0.0528 | 0.4958 | 0.6999 | 3.5493 | 0.0596 |
| F28 | 0.6985 | 0.0649 | 0.5587 | 0.8091 | 7.4279 | 0.0064 |
| F27 | 0.6961 | 0.0623 | 0.5625 | 0.8032 | 7.9105 | 0.0049 |
| F278 | 0.778 | 0.0614 | 0.6358 | 0.8755 | 12.4392 | 0.0004 |
| F26 | 0.7392 | 0.0609 | 0.604 | 0.8404 | 10.866 | 0.001 |
| F268 | 0.8125 | 0.0584 | 0.6715 | 0.9019 | 14.6192 | 0.0001 |
| F267 | 0.8108 | 0.0561 | 0.6766 | 0.8978 | 15.8136 | <. 0001 |
| F2678 | 0.8677 | 0.0479 | 0.7431 | 0.937 | 20.2864 | <. 0001 |
| F25 | 0.8106 | 0.0712 | 0.633 | 0.9139 | 9.8355 | 0.0017 |
| F258 | 0.8675 | 0.0601 | 0.7013 | 0.948 | 12.9034 | 0.0003 |
| F257 | 0.8662 | 0.0599 | 0.7017 | 0.9468 | 13.0782 | 0.0003 |
| F2578 | 0.9083 | 0.047 | 0.7661 | 0.9677 | 16.5055 | <. 0001 |
| F256 | 0.889 | 0.0518 | 0.741 | 0.9573 | 15.6966 | <. 0001 |
| F2568 | 0.9245 | 0.0404 | 0.7975 | 0.9744 | 18.7251 | <. 0001 |
| F2567 | 0.9237 | 0.0401 | 0.7989 | 0.9736 | 19.2403 | <. 0001 |
| F25678 | 0.9488 | 0.0298 | 0.8478 | 0.984 | 22.6708 | <. 0001 |
| F24 | 0.6681 | 0.0815 | 0.4949 | 0.8053 | 3.6269 | 0.0569 |
| F248 | 0.7549 | 0.0804 | 0.5677 | 0.8783 | 6.693 | 0.0097 |
| F247 | 0.7528 | 0.077 | 0.575 | 0.8727 | 7.2382 | 0.0071 |
| F2478 | 0.8233 | 0.068 | 0.6508 | 0.9209 | 10.8387 | 0.001 |
| F246 | 0.7902 | 0.0724 | 0.6155 | 0.8986 | 9.2291 | 0.0024 |
| F2468 | 0.8521 | 0.0624 | 0.6857 | 0.9383 | 12.4967 | 0.0004 |
| F2467 | 0.8507 | 0.0599 | 0.6932 | 0.9349 | 13.5963 | 0.0002 |
| F24678 | 0.8971 | 0.048 | 0.7587 | 0.9602 | 17.323 | <. 0001 |
| F245 | 0.8505 | 0.0619 | 0.6868 | 0.9365 | 12.7724 | 0.0004 |
| F2458 | 0.8969 | 0.0509 | 0.7474 | 0.9624 | 15.4552 | <. 0001 |
| F2457 | 0.8959 | 0.0499 | 0.7509 | 0.9609 | 16.1714 | <. 0001 |
| F24578 | 0.9294 | 0.0386 | 0.8061 | 0.9766 | 19.2119 | <. 0001 |


| F2456 | 0.9141 | 0.0431 | 0.7838 | 0.969 | 18.5366 | <. 0001 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F24568 | 0.9421 | 0.0331 | 0.832 | 0.9817 | 21.1078 | $<.0001$ |
| F24567 | 0.9415 | 0.0325 | 0.8353 | 0.9808 | 22.2301 | <. 0001 |
| F245678 | 0.961 | 0.0239 | 0.8757 | 0.9885 | 25.191 | $<.0001$ |
| F23 | 0.6513 | 0.1282 | 0.3819 | 0.8496 | 1.2252 | 0.2683 |
| F238 | 0.7407 | 0.1124 | 0.4757 | 0.9 | 3.2166 | 0.0729 |
| F237 | 0.7386 | 0.1155 | 0.4665 | 0.9013 | 3.0132 | 0.0826 |
| F2378 | 0.8121 | 0.0935 | 0.5654 | 0.9349 | 5.7114 | 0.0169 |
| F236 | 0.7775 | 0.1013 | 0.5257 | 0.9168 | 4.5617 | 0.0327 |
| F2368 | 0.8424 | 0.0805 | 0.6195 | 0.9461 | 7.6348 | 0.0057 |
| F2367 | 0.8409 | 0.0823 | 0.6129 | 0.9464 | 7.3272 | 0.0068 |
| F23678 | 0.8899 | 0.0617 | 0.7019 | 0.9652 | 11.0238 | 0.0009 |
| F235 | 0.8407 | 0.0935 | 0.5732 | 0.954 | 5.6751 | 0.0172 |
| F2358 | 0.8898 | 0.071 | 0.6615 | 0.9709 | 8.3272 | 0.0039 |
| F2357 | 0.8887 | 0.0727 | 0.6539 | 0.9712 | 7.984 | 0.0047 |
| F23578 | 0.9243 | 0.0528 | 0.7355 | 0.9817 | 10.99 | 0.0009 |
| F2356 | 0.908 | 0.0603 | 0.7056 | 0.976 | 10.0515 | 0.0015 |
| F23568 | 0.9379 | 0.0435 | 0.7773 | 0.9849 | 13.1927 | 0.0003 |
| F23567 | 0.9372 | 0.0444 | 0.7727 | 0.985 | 12.8168 | 0.0003 |
| F235678 | 0.9581 | 0.0311 | 0.8334 | 0.9905 | 16.3028 | <. 0001 |
| F234 | 0.7129 | 0.1329 | 0.4101 | 0.8986 | 1.9607 | 0.1614 |
| F2348 | 0.7916 | 0.1111 | 0.5037 | 0.9343 | 3.9279 | 0.0475 |
| F2347 | 0.7897 | 0.1125 | 0.4988 | 0.9341 | 3.8133 | 0.0508 |
| F23478 | 0.8517 | 0.0878 | 0.5951 | 0.9574 | 6.3175 | 0.012 |
| F2346 | 0.8229 | 0.0979 | 0.5548 | 0.9454 | 5.2343 | 0.0221 |
| F23468 | 0.8766 | 0.0752 | 0.6454 | 0.9652 | 7.9604 | 0.0048 |
| F23467 | 0.8754 | 0.0759 | 0.6425 | 0.9649 | 7.8566 | 0.0051 |
| F234678 | 0.9149 | 0.0556 | 0.7263 | 0.9775 | 11.0731 | 0.0009 |
| F2345 | 0.8752 | 0.0792 | 0.6287 | 0.9667 | 7.2145 | 0.0072 |
| F23458 | 0.9147 | 0.0589 | 0.7096 | 0.9792 | 9.8837 | 0.0017 |
| F23457 | 0.9139 | 0.0598 | 0.7054 | 0.9792 | 9.6696 | 0.0019 |
| F234578 | 0.942 | 0.0428 | 0.7775 | 0.9869 | 12.6549 | 0.0004 |


| F23456 | 0.9292 | 0.0494 | 0.7506 | 0.9828 | 11.7436 | 0.0006 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F234568 | 0.9526 | 0.0353 | 0.8131 | 0.9893 | 14.7744 | 0.0001 |
| F234567 | 0.952 | 0.0357 | 0.8109 | 0.9892 | 14.6135 | 0.0001 |
| F2345678 | 0.9681 | 0.0249 | 0.8623 | 0.9933 | 17.9596 | <. 0001 |
| F1 | 0.5702 | 0.0505 | 0.4698 | 0.6652 | 1.8844 | 0.1698 |
| F18 | 0.6699 | 0.0629 | 0.5375 | 0.7799 | 6.1925 | 0.0128 |
| F17 | 0.6674 | 0.0687 | 0.5224 | 0.7864 | 5.0614 | 0.0245 |
| F178 | 0.7543 | 0.0662 | 0.604 | 0.8607 | 9.8738 | 0.0017 |
| F16 | 0.7128 | 0.0628 | 0.5764 | 0.8191 | 8.7903 | 0.003 |
| F168 | 0.7915 | 0.0602 | 0.6499 | 0.8859 | 13.3521 | 0.0003 |
| F167 | 0.7897 | 0.0629 | 0.6413 | 0.8875 | 12.217 | 0.0005 |
| F1678 | 0.8517 | 0.0529 | 0.7164 | 0.9289 | 17.3987 | <. 0001 |
| F15 | 0.7894 | 0.0842 | 0.5814 | 0.91 | 6.8044 | 0.0091 |
| F158 | 0.8515 | 0.0701 | 0.6592 | 0.9444 | 9.9255 | 0.0016 |
| F157 | 0.8501 | 0.0725 | 0.6503 | 0.9453 | 9.3059 | 0.0023 |
| F1578 | 0.8966 | 0.0562 | 0.7256 | 0.966 | 12.7053 | 0.0004 |
| F156 | 0.8752 | 0.0616 | 0.6991 | 0.9549 | 11.9408 | 0.0005 |
| F1568 | 0.9147 | 0.0474 | 0.7653 | 0.9725 | 15.2493 | <. 0001 |
| F1567 | 0.9139 | 0.0486 | 0.76 | 0.9726 | 14.6591 | 0.0001 |
| F15678 | 0.942 | 0.0356 | 0.8189 | 0.9831 | 18.2707 | <. 0001 |
| F14 | 0.6381 | 0.0854 | 0.4607 | 0.7845 | 2.3525 | 0.1251 |
| F148 | 0.7295 | 0.0844 | 0.5385 | 0.8618 | 5.3862 | 0.0203 |
| F147 | 0.7273 | 0.086 | 0.5326 | 0.8619 | 5.1134 | 0.0237 |
| F1478 | 0.8032 | 0.0754 | 0.6158 | 0.9122 | 8.6974 | 0.0032 |
| F146 | 0.7674 | 0.0784 | 0.5826 | 0.8864 | 7.3949 | 0.0065 |
| F1468 | 0.8346 | 0.0676 | 0.6592 | 0.9294 | 10.9396 | 0.0009 |
| F1467 | 0.8331 | 0.0681 | 0.6565 | 0.9288 | 10.7702 | 0.001 |
| F14678 | 0.8842 | 0.0542 | 0.7302 | 0.9556 | 14.7632 | 0.0001 |
| F145 | 0.8328 | 0.0745 | 0.6358 | 0.9343 | 9.0091 | 0.0027 |
| F1458 | 0.884 | 0.0602 | 0.7069 | 0.9601 | 11.9735 | 0.0005 |
| F1457 | 0.8829 | 0.0613 | 0.7023 | 0.9601 | 11.6148 | 0.0007 |
| F14578 | 0.9202 | 0.0466 | 0.7688 | 0.9756 | 14.8489 | 0.0001 |


| F1456 | 0.9031 | 0.0519 | 0.7446 | 0.9675 | 14.1702 | 0.0002 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F14568 | 0.9345 | 0.0392 | 0.8024 | 0.9804 | 17.1934 | <. 0001 |
| F14567 | 0.9338 | 0.0397 | 0.8002 | 0.9803 | 16.978 | <. 0001 |
| F145678 | 0.9557 | 0.0289 | 0.8501 | 0.988 | 20.3047 | $<.0001$ |
| F13 | 0.6207 | 0.1294 | 0.3578 | 0.8277 | 0.8027 | 0.3703 |
| F138 | 0.7145 | 0.115 | 0.4531 | 0.8832 | 2.6461 | 0.1038 |
| F137 | 0.7122 | 0.1224 | 0.4343 | 0.8886 | 2.3032 | 0.1291 |
| F1378 | 0.7911 | 0.1 | 0.5365 | 0.9253 | 4.8467 | 0.0277 |
| F136 | 0.7538 | 0.1061 | 0.4995 | 0.9038 | 3.8284 | 0.0504 |
| F1368 | 0.824 | 0.085 | 0.5974 | 0.9366 | 6.9327 | 0.0085 |
| F1367 | 0.8224 | 0.0897 | 0.5815 | 0.9392 | 6.2262 | 0.0126 |
| F13678 | 0.8763 | 0.0675 | 0.6766 | 0.96 | 9.8986 | 0.0017 |
| F135 | 0.8222 | 0.1053 | 0.5298 | 0.9499 | 4.5176 | 0.0335 |
| F1358 | 0.8761 | 0.0803 | 0.624 | 0.9679 | 6.9937 | 0.0082 |
| F1357 | 0.8749 | 0.0839 | 0.6091 | 0.9691 | 6.445 | 0.0111 |
| F13578 | 0.9145 | 0.061 | 0.6988 | 0.9801 | 9.2348 | 0.0024 |
| F1356 | 0.8964 | 0.0691 | 0.6681 | 0.9738 | 8.4126 | 0.0037 |
| F13568 | 0.9297 | 0.0499 | 0.7478 | 0.9833 | 11.4529 | 0.0007 |
| F13567 | 0.929 | 0.0518 | 0.7372 | 0.9839 | 10.7139 | 0.0011 |
| F135678 | 0.9524 | 0.0362 | 0.8068 | 0.9897 | 14.0456 | 0.0002 |
| F134 | 0.685 | 0.1387 | 0.3816 | 0.8846 | 1.4611 | 0.2268 |
| F1348 | 0.7689 | 0.1174 | 0.4767 | 0.924 | 3.3083 | 0.0689 |
| F1347 | 0.7669 | 0.122 | 0.4634 | 0.9261 | 3.045 | 0.081 |
| F13478 | 0.8342 | 0.096 | 0.5633 | 0.9515 | 5.4133 | 0.02 |
| F1346 | 0.8027 | 0.1053 | 0.5249 | 0.9374 | 4.4524 | 0.0349 |
| F13468 | 0.8616 | 0.0815 | 0.62 | 0.9596 | 7.163 | 0.0074 |
| F13467 | 0.8602 | 0.0842 | 0.6093 | 0.9605 | 6.7303 | 0.0095 |
| F134678 | 0.904 | 0.0619 | 0.6996 | 0.9744 | 9.8949 | 0.0017 |
| F1345 | 0.86 | 0.0905 | 0.5847 | 0.9641 | 5.8318 | 0.0157 |
| F13458 | 0.9038 | 0.0674 | 0.6729 | 0.9772 | 8.3533 | 0.0038 |
| F13457 | 0.9029 | 0.0697 | 0.662 | 0.9778 | 7.8735 | 0.005 |
| F134578 | 0.9343 | 0.0499 | 0.7431 | 0.9859 | 10.6742 | 0.0011 |


| F13456 | 0.92 | 0.0572 | 0.7148 | 0.9814 | 9.8741 | 0.0017 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F134568 | 0.9462 | 0.0407 | 0.7856 | 0.9883 | 12.8331 | 0.0003 |
| F134567 | 0.9456 | 0.042 | 0.7785 | 0.9885 | 12.2502 | 0.0005 |
| F1345678 | 0.9638 | 0.0291 | 0.8384 | 0.9927 | 15.4787 | <. 0001 |
| F12 | 0.6677 | 0.0663 | 0.5281 | 0.783 | 5.4608 | 0.0194 |
| F128 | 0.7545 | 0.0674 | 0.601 | 0.8625 | 9.523 | 0.002 |
| F127 | 0.7525 | 0.0686 | 0.5962 | 0.8622 | 9.1079 | 0.0025 |
| F1278 | 0.823 | 0.0604 | 0.6735 | 0.9129 | 13.7312 | 0.0002 |
| F126 | 0.7899 | 0.0612 | 0.6459 | 0.8857 | 12.876 | 0.0003 |
| F1268 | 0.8519 | 0.0536 | 0.7145 | 0.9297 | 16.9783 | <. 0001 |
| F1267 | 0.8505 | 0.0537 | 0.713 | 0.9287 | 16.9244 | $<.0001$ |
| F12678 | 0.8969 | 0.0429 | 0.7778 | 0.9558 | 21.7026 | <. 0001 |
| F125 | 0.8502 | 0.0681 | 0.6657 | 0.9418 | 10.5519 | 0.0012 |
| F1258 | 0.8967 | 0.0541 | 0.7343 | 0.9646 | 13.6936 | 0.0002 |
| F1257 | 0.8957 | 0.0549 | 0.7306 | 0.9645 | 13.3724 | 0.0003 |
| F12578 | 0.9293 | 0.0413 | 0.7933 | 0.9782 | 16.8218 | <. 0001 |
| F1256 | 0.914 | 0.046 | 0.7714 | 0.971 | 16.3113 | <. 0001 |
| F12568 | 0.942 | 0.0345 | 0.825 | 0.9825 | 19.5027 | <. 0001 |
| F12567 | 0.9414 | 0.0348 | 0.8235 | 0.9823 | 19.3676 | <. 0001 |
| F125678 | 0.9609 | 0.0251 | 0.8688 | 0.9892 | 22.9125 | <. 0001 |
| F124 | 0.7276 | 0.0853 | 0.5346 | 0.8613 | 5.2067 | 0.0225 |
| F1248 | 0.8034 | 0.0768 | 0.6118 | 0.9137 | 8.3888 | 0.0038 |
| F1247 | 0.8016 | 0.0763 | 0.6122 | 0.9118 | 8.4804 | 0.0036 |
| F12478 | 0.8607 | 0.0627 | 0.6891 | 0.9452 | 12.1183 | 0.0005 |
| F1246 | 0.8333 | 0.0676 | 0.6582 | 0.9284 | 10.9366 | 0.0009 |
| F12468 | 0.8843 | 0.0549 | 0.7274 | 0.9563 | 14.3506 | 0.0002 |
| F12467 | 0.8832 | 0.0543 | 0.7293 | 0.955 | 14.7707 | 0.0001 |
| F124678 | 0.9204 | 0.0416 | 0.7919 | 0.9723 | 18.6275 | <. 0001 |
| F1245 | 0.883 | 0.0582 | 0.7145 | 0.9579 | 12.8757 | 0.0003 |
| F12458 | 0.9203 | 0.0452 | 0.7751 | 0.9748 | 15.7367 | <. 0001 |
| F12457 | 0.9194 | 0.0454 | 0.7746 | 0.9743 | 15.7987 | <. 0001 |
| F124578 | 0.9458 | 0.0336 | 0.8282 | 0.9844 | 18.9635 | <. 0001 |


| F12456 | 0.9339 | 0.038 | 0.8088 | 0.9792 | 18.5397 | <. 0001 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F124568 | 0.9557 | 0.0281 | 0.8544 | 0.9876 | 21.3654 | <. 0001 |
| F124567 | 0.9553 | 0.0281 | 0.8549 | 0.9873 | 21.6997 | <. 0001 |
| F1245678 | 0.9703 | 0.0201 | 0.8925 | 0.9923 | 24.8694 | <. 0001 |
| F123 | 0.7125 | 0.121 | 0.4377 | 0.8875 | 2.3599 | 0.1245 |
| F1238 | 0.7913 | 0.1004 | 0.5351 | 0.9258 | 4.8024 | 0.0284 |
| F1237 | 0.7894 | 0.1047 | 0.5218 | 0.928 | 4.4028 | 0.0359 |
| F12378 | 0.8515 | 0.081 | 0.6203 | 0.9527 | 7.4314 | 0.0064 |
| F1236 | 0.8226 | 0.0887 | 0.5849 | 0.9385 | 6.3715 | 0.0116 |
| F12368 | 0.8764 | 0.0677 | 0.6756 | 0.9602 | 9.8179 | 0.0017 |
| F12367 | 0.8752 | 0.0702 | 0.6657 | 0.9611 | 9.1933 | 0.0024 |
| F123678 | 0.9147 | 0.051 | 0.7487 | 0.9748 | 13.1757 | 0.0003 |
| F1235 | 0.875 | 0.081 | 0.6213 | 0.9676 | 6.9091 | 0.0086 |
| F12358 | 0.9146 | 0.0595 | 0.7062 | 0.9795 | 9.6747 | 0.0019 |
| F12357 | 0.9137 | 0.0615 | 0.6964 | 0.98 | 9.1412 | 0.0025 |
| F123578 | 0.9419 | 0.0436 | 0.7726 | 0.9872 | 12.2087 | 0.0005 |
| F12356 | 0.9291 | 0.0501 | 0.747 | 0.9831 | 11.4497 | 0.0007 |
| F123568 | 0.9525 | 0.0354 | 0.8122 | 0.9893 | 14.6873 | 0.0001 |
| F123567 | 0.952 | 0.0364 | 0.8061 | 0.9895 | 14.0474 | 0.0002 |
| F1235678 | 0.9681 | 0.0252 | 0.8602 | 0.9933 | 17.5868 | <. 0001 |
| F1234 | 0.7671 | 0.1214 | 0.4651 | 0.9258 | 3.0767 | 0.0794 |
| F12348 | 0.8344 | 0.0967 | 0.5609 | 0.9521 | 5.3348 | 0.0209 |
| F12347 | 0.8329 | 0.0992 | 0.5522 | 0.9527 | 5.0821 | 0.0242 |
| F123478 | 0.884 | 0.0746 | 0.6469 | 0.9694 | 7.7995 | 0.0052 |
| F12346 | 0.8604 | 0.0838 | 0.611 | 0.9603 | 6.7978 | 0.0091 |
| F123468 | 0.9041 | 0.0623 | 0.6977 | 0.9747 | 9.7624 | 0.0018 |
| F123467 | 0.9031 | 0.0636 | 0.6918 | 0.9748 | 9.4407 | 0.0021 |
| F1234678 | 0.9345 | 0.0454 | 0.7692 | 0.9839 | 12.8381 | 0.0003 |
| F12345 | 0.903 | 0.0677 | 0.672 | 0.9769 | 8.344 | 0.0039 |
| F123458 | 0.9344 | 0.0489 | 0.7487 | 0.9855 | 11.0743 | 0.0009 |
| F123457 | 0.9337 | 0.0501 | 0.7423 | 0.9857 | 10.6698 | 0.0011 |
| F1234578 | 0.9556 | 0.0352 | 0.8089 | 0.991 | 13.6745 | 0.0002 |


| F123456 | 0.9457 | 0.0408 | 0.786 | 0.988 | 12.9476 | 0.0003 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| F1234568 | 0.9638 | 0.0286 | 0.8423 | 0.9925 | 16.0353 | $<.0001$ |
| F1234567 | 0.9634 | 0.0292 | 0.8387 | 0.9926 | 15.6109 | $<.0001$ |
| F12345678 | 0.9758 | 0.0201 | 0.8842 | 0.9953 | 18.979 | $<.0001$ |

Table A.3- Predictive Index of Imputed-Errors Corrected Simulation Data
Contrast Estimation and Testing Results by Row

| Contrast | Estimate | Standard <br> Error | Confidence | Limits | Wald Chi- <br> Square | Pr $>$ <br> ChiSq |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0.5 | 0 | . | . | . | . |
| F8 | 0.6208 | 0.048 | 0.5233 | 0.7093 | 5.8494 | 0.0156 |
| F7 | 0.6109 | 0.0506 | 0.5085 | 0.7043 | 4.4962 | 0.034 |
| F78 | 0.7199 | 0.058 | 0.594 | 0.8186 | 10.7869 | 0.001 |
| F6 | 0.648 | 0.0552 | 0.534 | 0.7473 | 6.3689 | 0.0116 |
| F68 | 0.7509 | 0.0596 | 0.6175 | 0.8491 | 11.992 | 0.0005 |
| F67 | 0.743 | 0.0601 | 0.6092 | 0.8427 | 11.3565 | 0.0008 |
| F678 | 0.8255 | 0.0535 | 0.6955 | 0.9074 | 17.5026 | $<.0001$ |
| F5 | 0.7325 | 0.0785 | 0.5552 | 0.8573 | 6.3161 | 0.012 |
| F58 | 0.8176 | 0.0702 | 0.6406 | 0.9185 | 10.1653 | 0.0014 |
| F57 | 0.8113 | 0.0723 | 0.6301 | 0.9156 | 9.5316 | 0.002 |
| F578 | 0.8756 | 0.0576 | 0.714 | 0.952 | 13.6121 | 0.0002 |
| F56 | 0.8345 | 0.067 | 0.6609 | 0.9288 | 11.1279 | 0.0009 |
| F568 | 0.8919 | 0.0526 | 0.7392 | 0.9601 | 14.9789 | 0.0001 |
| F567 | 0.8878 | 0.0539 | 0.7326 | 0.9581 | 14.6008 | 0.0001 |
| F5678 | 0.9284 | 0.0394 | 0.8022 | 0.9764 | 18.6868 | $<.0001$ |
| F4 | 0.5824 | 0.0697 | 0.443 | 0.7098 | 1.3481 | 0.2456 |
| F48 | 0.6954 | 0.0759 | 0.5308 | 0.8217 | 5.3115 | 0.0212 |
| F47 | 0.6865 | 0.0755 | 0.524 | 0.8133 | 4.9871 | 0.0255 |
| F478 | 0.7819 | 0.0693 | 0.6178 | 0.8883 | 9.8662 | 0.0017 |
| F46 | 0.7197 | 0.0772 | 0.5482 | 0.8446 | 6.0796 | 0.0137 |
| F468 | 0.8078 | 0.0684 | 0.6394 | 0.9088 | 10.631 | 0.0011 |
| F467 | 0.8013 | 0.0681 | 0.6356 | 0.9031 | 10.6323 | 0.0011 |
| F4678 | 0.8684 | 0.0544 | 0.7221 | 0.9437 | 15.7359 | $<.0001$ |
| F45 | 0.7925 | 0.0714 | 0.6199 | 0.8994 | 9.5294 | 0.002 |
| F458 | 0.8621 | 0.0599 | 0.6996 | 0.9438 | 13.2349 | 0.0003 |
| F457 | 0.8571 | 0.0608 | 0.6937 | 0.9407 | 13.0027 | 0.0003 |
| F4578 | 0.9075 | 0.0466 | 0.7679 | 0.9668 | 16.9399 | $<.0001$ |
| F456 | 0.8755 | 0.0565 | 0.718 | 0.951 | 14.1616 | 0.0002 |
| F4568 | 0.9201 | 0.0426 | 0.7872 | 0.9728 | 17.7937 | $<.0001$ |
| F4567 | 0.9169 | 0.0432 | 0.7839 | 0.9711 | 17.8899 | $<.0001$ |
| F45678 | 0.9476 | 0.0308 | 0.8427 | 0.9839 | 21.7558 | $<.0001$ |
| F3 | 0.5327 | 0.1236 | 0.3011 | 0.7511 | 0.0696 | 0.7919 |
| F38 | 0.6511 | 0.1184 | 0.4019 | 0.8382 | 1.4333 | 0.2312 |
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| F37 | 0.6415 | 0.1256 | 0.3803 | 0.8392 | 1.136 | 0.2865 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| F378 | 0.7455 | 0.1071 | 0.492 | 0.8986 | 3.6222 | 0.057 |
| F36 | 0.6773 | 0.1156 | 0.4266 | 0.8555 | 1.9632 | 0.1612 |
| F368 | 0.7745 | 0.0967 | 0.5373 | 0.9104 | 4.9719 | 0.0258 |
| F367 | 0.7672 | 0.1021 | 0.5179 | 0.91 | 4.3474 | 0.0371 |
| F3678 | 0.8436 | 0.078 | 0.6288 | 0.945 | 8.1312 | 0.0044 |
| F35 | 0.7574 | 0.1181 | 0.4697 | 0.9167 | 3.1373 | 0.0765 |
| F358 | 0.8363 | 0.0926 | 0.5759 | 0.9506 | 5.8198 | 0.0158 |
| F357 | 0.8305 | 0.0977 | 0.557 | 0.9503 | 5.242 | 0.022 |
| F3578 | 0.8892 | 0.0712 | 0.6607 | 0.9706 | 8.3078 | 0.0039 |
| F356 | 0.8518 | 0.0858 | 0.6025 | 0.9561 | 6.6126 | 0.0101 |
| F3568 | 0.9039 | 0.0619 | 0.6994 | 0.9744 | 9.8895 | 0.0017 |
| F3567 | 0.9002 | 0.0652 | 0.6851 | 0.974 | 9.1859 | 0.0024 |
| F35678 | 0.9366 | 0.0448 | 0.7712 | 0.9848 | 12.7593 | 0.0004 |
| F34 | 0.6139 | 0.1395 | 0.334 | 0.8345 | 0.6206 | 0.4308 |
| F348 | 0.7224 | 0.123 | 0.4389 | 0.8965 | 2.4327 | 0.1188 |
| F347 | 0.714 | 0.1282 | 0.4218 | 0.8952 | 2.1241 | 0.145 |
| F3478 | 0.8034 | 0.1023 | 0.5345 | 0.9357 | 4.7244 | 0.0297 |
| F346 | 0.7454 | 0.1179 | 0.4643 | 0.9082 | 2.992 | 0.0837 |
| F3468 | 0.8274 | 0.0922 | 0.5749 | 0.9444 | 5.8927 | 0.0152 |
| F3467 | 0.8213 | 0.0961 | 0.5602 | 0.9431 | 5.4285 | 0.0198 |
| F34678 | 0.8827 | 0.0699 | 0.6672 | 0.9658 | 8.9426 | 0.0028 |
| F345 | 0.8132 | 0.1029 | 0.5359 | 0.9426 | 4.7182 | 0.0298 |
| F3458 | 0.877 | 0.0769 | 0.6382 | 0.9664 | 7.6003 | 0.0058 |
| F3457 | 0.8724 | 0.0805 | 0.6235 | 0.9658 | 7.0597 | 0.0079 |
| F34578 | 0.918 | 0.0568 | 0.7187 | 0.98 | 10.2684 | 0.0014 |
| F3456 | 0.8891 | 0.0707 | 0.663 | 0.9703 | 8.433 | 0.0037 |
| F34568 | 0.9292 | 0.0494 | 0.7507 | 0.9828 | 11.7494 | 0.0006 |
| F34567 | 0.9264 | 0.0517 | 0.7403 | 0.9823 | 11.1759 | 0.0008 |
| F345678 | 0.9537 | 0.0348 | 0.8149 | 0.9897 | 14.7631 | 0.0001 |
| F2 | 0.6073 | 0.053 | 0.5 | 0.705 | 3.8435 | 0.0499 |
| F28 | 0.7168 | 0.0636 | 0.5779 | 0.8239 | 8.7743 | 0.0031 |
| F27 | 0.7082 | 0.0619 | 0.5744 | 0.8137 | 8.7626 | 0.0031 |
| F278 | 0.7989 | 0.0589 | 0.6595 | 0.8907 | 14.1582 | 0.0002 |
| F26 | 0.74 | 0.0616 | 0.6031 | 0.8421 | 10.6703 | 0.0011 |
| F268 | 0.8233 | 0.0569 | 0.6841 | 0.9093 | 15.4895 | $<.0001$ |
| F267 | 0.8172 | 0.0556 | 0.6831 | 0.9026 | 16.1998 | $<.0001$ |
| F2678 | 0.8797 | 0.0454 | 0.7592 | 0.9444 | 21.4607 | $<.0001$ |
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| F25 | 0.8089 | 0.0673 | 0.6432 | 0.9086 | 10.9722 | 0.0009 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| F258 | 0.8739 | 0.056 | 0.7192 | 0.9494 | 14.5333 | 0.0001 |
| F257 | 0.8692 | 0.0565 | 0.7149 | 0.9463 | 14.5033 | 0.0001 |
| F2578 | 0.9158 | 0.043 | 0.7847 | 0.9701 | 18.3042 | $<.0001$ |
| F256 | 0.8863 | 0.0513 | 0.7419 | 0.9548 | 16.2822 | $<.0001$ |
| F2568 | 0.9273 | 0.0386 | 0.8058 | 0.9752 | 19.7355 | $<.0001$ |
| F2567 | 0.9245 | 0.039 | 0.8037 | 0.9734 | 20.1034 | $<.0001$ |
| F25678 | 0.9525 | 0.0278 | 0.8573 | 0.9852 | 23.806 | $<.0001$ |
| F24 | 0.6832 | 0.0799 | 0.5113 | 0.8164 | 4.338 | 0.0373 |
| F248 | 0.7793 | 0.0749 | 0.6005 | 0.8924 | 8.3879 | 0.0038 |
| F247 | 0.772 | 0.0731 | 0.6001 | 0.8842 | 8.6327 | 0.0033 |
| F2478 | 0.8472 | 0.061 | 0.6878 | 0.9331 | 13.23 | 0.0003 |
| F246 | 0.7988 | 0.071 | 0.6256 | 0.9042 | 9.7515 | 0.0018 |
| F2468 | 0.8667 | 0.0578 | 0.7093 | 0.9454 | 14.0146 | 0.0002 |
| F2467 | 0.8618 | 0.0567 | 0.7103 | 0.9406 | 14.7792 | 0.0001 |
| F24678 | 0.9107 | 0.0428 | 0.7842 | 0.9663 | 19.4495 | $<.0001$ |
| F245 | 0.8552 | 0.0584 | 0.701 | 0.937 | 14.1928 | 0.0002 |
| F2458 | 0.9063 | 0.0461 | 0.7693 | 0.9656 | 17.4511 | $<.0001$ |
| F2457 | 0.9026 | 0.0461 | 0.7682 | 0.9629 | 17.9955 | $<.0001$ |
| F24578 | 0.9382 | 0.034 | 0.8278 | 0.9796 | 21.4947 | $<.0001$ |
| F2456 | 0.9158 | 0.042 | 0.7889 | 0.9694 | 19.169 | $<.0001$ |
| F24568 | 0.9468 | 0.0307 | 0.8436 | 0.9833 | 22.3272 | $<.0001$ |
| F24567 | 0.9447 | 0.0307 | 0.8435 | 0.9818 | 23.2743 | $<.0001$ |
| F245678 | 0.9655 | 0.0215 | 0.8876 | 0.99 | 26.6637 | $<.0001$ |
| F23 | 0.638 | 0.1253 | 0.3783 | 0.8362 | 1.0907 | 0.2963 |
| F238 | 0.7426 | 0.1093 | 0.4848 | 0.8984 | 3.4357 | 0.0638 |
| F237 | 0.7345 | 0.1139 | 0.4683 | 0.8969 | 3.0359 | 0.0814 |
| F2378 | 0.8192 | 0.09 | 0.5792 | 0.9371 | 6.1788 | 0.0129 |
| F236 | 0.7644 | 0.1023 | 0.516 | 0.9081 | 4.2958 | 0.0382 |
| F2368 | 0.8416 | 0.0796 | 0.6225 | 0.9448 | 7.8269 | 0.0051 |
| F2367 | 0.8359 | 0.0828 | 0.6093 | 0.9433 | 7.268 | 0.007 |
| F23678 | 0.8929 | 0.06 | 0.7089 | 0.9662 | 11.4089 | 0.0007 |
| F235 | 0.8284 | 0.0944 | 0.5678 | 0.9466 | 5.6223 | 0.0177 |
| F2358 | 0.8877 | 0.07 | 0.6664 | 0.969 | 8.6799 | 0.0032 |
| F2357 | 0.8834 | 0.0732 | 0.6531 | 0.9683 | 8.1266 | 0.0044 |
| F23578 | 0.9254 | 0.0513 | 0.743 | 0.9816 | 11.4863 | 0.0007 |
| F2356 | 0.8989 | 0.0633 | 0.6942 | 0.9721 | 9.8434 | 0.0017 |
| F23568 | 0.9357 | 0.0441 | 0.7756 | 0.9839 | 13.3297 | 0.0003 |
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| F23567 | 0.9331 | 0.046 | 0.7667 | 0.9834 | 12.7635 | 0.0004 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F235678 | 0.958 | 0.0309 | 0.8347 | 0.9904 | 16.504 | <. 0001 |
| F234 | 0.7109 | 0.1299 | 0.4161 | 0.8945 | 2.0275 | 0.1545 |
| F2348 | 0.801 | 0.1053 | 0.5245 | 0.9362 | 4.4461 | 0.035 |
| F2347 | 0.7942 | 0.1086 | 0.512 | 0.9342 | 4.1304 | 0.0421 |
| F23478 | 0.8634 | 0.0813 | 0.6209 | 0.9606 | 7.1634 | 0.0074 |
| F2346 | 0.819 | 0.0975 | 0.5549 | 0.9426 | 5.2673 | 0.0217 |
| F23468 | 0.8811 | 0.0719 | 0.6587 | 0.966 | 8.5117 | 0.0035 |
| F23467 | 0.8766 | 0.0742 | 0.6493 | 0.9646 | 8.1676 | 0.0043 |
| F234678 | 0.9208 | 0.0518 | 0.7429 | 0.9791 | 11.9221 | 0.0006 |
| F2345 | 0.8707 | 0.0789 | 0.6303 | 0.9637 | 7.4063 | 0.0065 |
| F23458 | 0.9168 | 0.0564 | 0.7211 | 0.9792 | 10.5255 | 0.0012 |
| F23457 | 0.9136 | 0.0586 | 0.7117 | 0.9784 | 10.0956 | 0.0015 |
| F234578 | 0.9454 | 0.0401 | 0.7907 | 0.9875 | 13.4843 | 0.0002 |
| F23456 | 0.9253 | 0.0508 | 0.7458 | 0.9813 | 11.7191 | 0.0006 |
| F234568 | 0.953 | 0.0346 | 0.8167 | 0.9893 | 15.1434 | <. 0001 |
| F234567 | 0.9511 | 0.0359 | 0.8108 | 0.9888 | 14.7852 | 0.0001 |
| F2345678 | 0.9696 | 0.0238 | 0.8678 | 0.9936 | 18.4482 | <. 0001 |
| F1 | 0.5826 | 0.0505 | 0.4816 | 0.6771 | 2.5746 | 0.1086 |
| F18 | 0.6955 | 0.0615 | 0.5638 | 0.8015 | 8.0862 | 0.0045 |
| F17 | 0.6866 | 0.0663 | 0.545 | 0.8003 | 6.4838 | 0.0109 |
| F178 | 0.782 | 0.0619 | 0.6377 | 0.8796 | 12.3615 | 0.0004 |
| F16 | 0.7198 | 0.0622 | 0.584 | 0.8246 | 9.3692 | 0.0022 |
| F168 | 0.8079 | 0.0576 | 0.6702 | 0.897 | 14.9779 | 0.0001 |
| F167 | 0.8013 | 0.06 | 0.6582 | 0.8942 | 13.6756 | 0.0002 |
| F1678 | 0.8685 | 0.0486 | 0.7416 | 0.9382 | 19.7073 | <. 0001 |
| F15 | 0.7926 | 0.0798 | 0.5962 | 0.9082 | 7.6317 | 0.0057 |
| F158 | 0.8622 | 0.0646 | 0.6829 | 0.9478 | 11.3578 | 0.0008 |
| F157 | 0.8571 | 0.0676 | 0.6704 | 0.9465 | 10.5432 | 0.0012 |
| F1578 | 0.9076 | 0.0504 | 0.7516 | 0.9696 | 14.4666 | 0.0001 |
| F156 | 0.8756 | 0.06 | 0.705 | 0.9539 | 12.5448 | 0.0004 |
| F1568 | 0.9201 | 0.0444 | 0.7789 | 0.9741 | 16.3463 | <. 0001 |
| F1567 | 0.917 | 0.0462 | 0.7709 | 0.9732 | 15.6851 | <. 0001 |
| F15678 | 0.9476 | 0.0324 | 0.8343 | 0.9848 | 19.6901 | <. 0001 |
| F14 | 0.6606 | 0.084 | 0.4829 | 0.8023 | 3.1597 | 0.0755 |
| F148 | 0.7611 | 0.0785 | 0.5776 | 0.8813 | 7.2083 | 0.0073 |
| F147 | 0.7535 | 0.0805 | 0.5665 | 0.8773 | 6.6402 | 0.01 |
| F1478 | 0.8334 | 0.0665 | 0.6616 | 0.9275 | 11.2854 | 0.0008 |


| F146 | 0.7818 | 0.0759 | 0.5997 | 0.8955 | 8.2298 | 0.0041 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| F1468 | 0.8544 | 0.0616 | 0.6896 | 0.9394 | 12.7582 | 0.0004 |
| F1467 | 0.8491 | 0.0629 | 0.6825 | 0.9364 | 12.3844 | 0.0004 |
| F14678 | 0.9021 | 0.0471 | 0.7641 | 0.9632 | 17.3482 | $<.0001$ |
| F145 | 0.842 | 0.0702 | 0.6546 | 0.9375 | 10.0579 | 0.0015 |
| F1458 | 0.8972 | 0.0541 | 0.7345 | 0.9649 | 13.6595 | 0.0002 |
| F1457 | 0.8933 | 0.0559 | 0.7264 | 0.9635 | 13.1514 | 0.0003 |
| F14578 | 0.932 | 0.0403 | 0.7976 | 0.9794 | 16.9507 | $<.0001$ |
| F1456 | 0.9075 | 0.0498 | 0.7542 | 0.9691 | 14.817 | 0.0001 |
| F14568 | 0.9414 | 0.0357 | 0.8188 | 0.9828 | 18.4177 | $<.0001$ |
| F14567 | 0.9391 | 0.0367 | 0.814 | 0.9819 | 18.1435 | $<.0001$ |
| F145678 | 0.9619 | 0.0253 | 0.8673 | 0.9898 | 21.9487 | $<.0001$ |
| F13 | 0.614 | 0.1267 | 0.3582 | 0.8193 | 0.7546 | 0.385 |
| F138 | 0.7225 | 0.1117 | 0.4664 | 0.8858 | 2.9521 | 0.0858 |
| F137 | 0.7141 | 0.1197 | 0.4418 | 0.8874 | 2.4372 | 0.1185 |
| F1378 | 0.8035 | 0.0953 | 0.5561 | 0.9303 | 5.4439 | 0.0196 |
| F136 | 0.7455 | 0.1061 | 0.4948 | 0.8975 | 3.6963 | 0.0545 |
| F1368 | 0.8274 | 0.0831 | 0.6052 | 0.9375 | 7.257 | 0.0071 |
| F1367 | 0.8214 | 0.0888 | 0.5842 | 0.9377 | 6.3597 | 0.0117 |
| F13678 | 0.8827 | 0.0645 | 0.6895 | 0.9623 | 10.5021 | 0.0012 |
| F135 | 0.8133 | 0.1056 | 0.5271 | 0.9445 | 4.4763 | 0.0344 |
| F1358 | 0.877 | 0.0784 | 0.632 | 0.9673 | 7.3132 | 0.0068 |
| F1357 | 0.8724 | 0.0832 | 0.6123 | 0.9673 | 6.6111 | 0.0101 |
| F13578 | 0.918 | 0.0583 | 0.7107 | 0.9808 | 9.7429 | 0.0018 |
| F1356 | 0.8891 | 0.0715 | 0.6592 | 0.9708 | 8.2311 | 0.0041 |
| F13568 | 0.9292 | 0.0497 | 0.7489 | 0.983 | 11.591 | 0.0007 |
| F13567 | 0.9264 | 0.0527 | 0.7346 | 0.9828 | 10.7397 | 0.001 |
| F135678 | 0.9537 | 0.0353 | 0.8115 | 0.99 | 14.3374 | 0.0002 |
| F134 | 0.6893 | 0.1356 | 0.3909 | 0.8847 | 1.5853 | 0.208 |
| F1348 | 0.7841 | 0.111 | 0.5011 | 0.9293 | 3.8687 | 0.0492 |
| F1347 | 0.777 | 0.1168 | 0.4816 | 0.9289 | 3.4262 | 0.0642 |
| F13478 | 0.8508 | 0.0879 | 0.5947 | 0.9568 | 6.3168 | 0.012 |
| F1346 | 0.8034 | 0.104 | 0.5293 | 0.9369 | 4.5725 | 0.0325 |
| F13468 | 0.8699 | 0.0771 | 0.6376 | 0.9621 | 7.7823 | 0.0053 |
| F13467 | 0.8651 | 0.0811 | 0.6217 | 0.9616 | 7.1572 | 0.0075 |
| F134678 | 0.913 | 0.0567 | 0.7217 | 0.977 | 10.8576 | 0.001 |
| F1345 | 0.8587 | 0.0895 | 0.5886 | 0.9627 | 5.9788 | 0.0145 |
| F13458 | 0.9087 | 0.0639 | 0.6874 | 0.9783 | 8.8977 | 0.0029 |
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| F13457 | 0.9051 | 0.0674 | 0.6721 | 0.978 | 8.262 | 0.004 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F134578 | 0.9398 | 0.046 | 0.7606 | 0.9871 | 11.4427 | 0.0007 |
| F13456 | 0.918 | 0.058 | 0.7121 | 0.9806 | 9.8324 | 0.0017 |
| F134568 | 0.9482 | 0.0394 | 0.7918 | 0.9888 | 13.1469 | 0.0003 |
| F134567 | 0.9461 | 0.0414 | 0.7814 | 0.9885 | 12.4502 | 0.0004 |
| F1345678 | 0.9664 | 0.0273 | 0.8472 | 0.9933 | 15.9898 | <. 0001 |
| F12 | 0.6833 | 0.066 | 0.5427 | 0.7969 | 6.3568 | 0.0117 |
| F128 | 0.7794 | 0.0647 | 0.6281 | 0.8807 | 11.2421 | 0.0008 |
| F127 | 0.7721 | 0.0658 | 0.6194 | 0.8758 | 10.643 | 0.0011 |
| F1278 | 0.8472 | 0.0556 | 0.7051 | 0.9279 | 15.9212 | <. 0001 |
| F126 | 0.7989 | 0.06 | 0.6564 | 0.892 | 13.63 | 0.0002 |
| F1268 | 0.8667 | 0.0503 | 0.7347 | 0.9385 | 18.4806 | $<.0001$ |
| F1267 | 0.8618 | 0.0508 | 0.73 | 0.935 | 18.4207 | <. 0001 |
| F12678 | 0.9108 | 0.0388 | 0.8003 | 0.963 | 23.6964 | <. 0001 |
| F125 | 0.8553 | 0.0637 | 0.6829 | 0.9419 | 11.9016 | 0.0006 |
| F1258 | 0.9063 | 0.049 | 0.7574 | 0.9677 | 15.466 | <. 0001 |
| F1257 | 0.9027 | 0.0504 | 0.7508 | 0.9662 | 15.0713 | 0.0001 |
| F12578 | 0.9382 | 0.0363 | 0.8164 | 0.9811 | 18.8372 | <. 0001 |
| F1256 | 0.9158 | 0.0441 | 0.7798 | 0.9709 | 17.3826 | <. 0001 |
| F12568 | 0.9468 | 0.0317 | 0.8382 | 0.9839 | 20.8867 | <. 0001 |
| F12567 | 0.9447 | 0.0325 | 0.8345 | 0.983 | 20.796 | <. 0001 |
| F125678 | 0.9655 | 0.0224 | 0.8821 | 0.9905 | 24.5235 | <. 0001 |
| F124 | 0.7506 | 0.0826 | 0.5589 | 0.8773 | 6.2288 | 0.0126 |
| F1248 | 0.8313 | 0.0699 | 0.6498 | 0.929 | 10.247 | 0.0014 |
| F1247 | 0.8253 | 0.0703 | 0.6451 | 0.9247 | 10.1521 | 0.0014 |
| F12478 | 0.8855 | 0.0543 | 0.7304 | 0.9567 | 14.6015 | 0.0001 |
| F1246 | 0.8471 | 0.0644 | 0.6765 | 0.9362 | 11.8624 | 0.0006 |
| F12468 | 0.9007 | 0.0491 | 0.7555 | 0.9638 | 16.1071 | <. 0001 |
| F12467 | 0.8969 | 0.0494 | 0.7533 | 0.9612 | 16.407 | <. 0001 |
| F124678 | 0.9344 | 0.0355 | 0.8206 | 0.9779 | 21.0106 | $<.0001$ |
| F1245 | 0.8918 | 0.054 | 0.7337 | 0.961 | 14.2285 | 0.0002 |
| F12458 | 0.931 | 0.0399 | 0.7999 | 0.9785 | 17.58 | <. 0001 |
| F12457 | 0.9283 | 0.0406 | 0.7964 | 0.9772 | 17.5959 | <. 0001 |
| F124578 | 0.9549 | 0.0286 | 0.8522 | 0.9873 | 21.1459 | <. 0001 |
| F12456 | 0.9382 | 0.0358 | 0.8191 | 0.9807 | 19.4167 | <. 0001 |
| F124568 | 0.9613 | 0.0251 | 0.8688 | 0.9894 | 22.6878 | <. 0001 |
| F124567 | 0.9597 | 0.0255 | 0.8672 | 0.9886 | 23.0447 | <.0001 |
| F1245678 | 0.975 | 0.0173 | 0.9063 | 0.9937 | 26.524 | <. 0001 |


| F123 | 0.711 | 0.1188 | 0.4421 | 0.8842 | 2.425 | 0.1194 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| F1238 | 0.8011 | 0.0964 | 0.5516 | 0.9295 | 5.2998 | 0.0213 |
| F1237 | 0.7943 | 0.1017 | 0.5328 | 0.929 | 4.7136 | 0.0299 |
| F12378 | 0.8634 | 0.076 | 0.6413 | 0.9572 | 8.1908 | 0.0042 |
| F1236 | 0.8191 | 0.0883 | 0.5846 | 0.9358 | 6.417 | 0.0113 |
| F12368 | 0.8811 | 0.0653 | 0.6859 | 0.9618 | 10.3163 | 0.0013 |
| F12367 | 0.8767 | 0.0687 | 0.6716 | 0.9611 | 9.5186 | 0.002 |
| F123678 | 0.9209 | 0.048 | 0.762 | 0.9769 | 13.8955 | 0.0002 |
| F1235 | 0.8707 | 0.0805 | 0.6236 | 0.9648 | 7.1062 | 0.0077 |
| F12358 | 0.9168 | 0.0572 | 0.7169 | 0.9796 | 10.2293 | 0.0014 |
| F12357 | 0.9136 | 0.0602 | 0.7032 | 0.9793 | 9.5502 | 0.002 |
| F123578 | 0.9454 | 0.041 | 0.7852 | 0.9879 | 12.9154 | 0.0003 |
| F12356 | 0.9254 | 0.0512 | 0.7436 | 0.9815 | 11.5372 | 0.0007 |
| F123568 | 0.9531 | 0.0347 | 0.816 | 0.9894 | 15.0525 | 0.0001 |
| F123567 | 0.9511 | 0.0365 | 0.8071 | 0.9891 | 14.3231 | 0.0002 |
| F1235678 | 0.9696 | 0.024 | 0.8658 | 0.9937 | 18.0502 | $<.0001$ |
| F1234 | 0.7743 | 0.1177 | 0.4781 | 0.9278 | 3.3494 | 0.0672 |
| F12348 | 0.8489 | 0.0899 | 0.5874 | 0.9568 | 6.0718 | 0.0137 |
| F12347 | 0.8434 | 0.0936 | 0.573 | 0.9558 | 5.6395 | 0.0176 |
| F123478 | 0.8981 | 0.0669 | 0.6776 | 0.9737 | 8.8517 | 0.0029 |
| F12346 | 0.8633 | 0.0818 | 0.6187 | 0.9609 | 7.0646 | 0.0079 |
| F123468 | 0.9118 | 0.0579 | 0.7159 | 0.977 | 10.5138 | 0.0012 |
| F123467 | 0.9084 | 0.0603 | 0.7055 | 0.9762 | 10.0188 | 0.0015 |
| F1234678 | 0.942 | 0.0409 | 0.7894 | 0.986 | 13.8878 | 0.0002 |
| F12345 | 0.9038 | 0.0661 | 0.6794 | 0.9766 | 8.6923 | 0.0032 |
| F123458 | 0.939 | 0.0456 | 0.7638 | 0.9865 | 11.7959 | 0.0006 |
| F123457 | 0.9365 | 0.0477 | 0.7538 | 0.9861 | 11.2537 | 0.0008 |
| F1234578 | 0.9602 | 0.0318 | 0.8249 | 0.992 | 14.5815 | 0.0001 |
| F123456 | 0.9454 | 0.0407 | 0.787 | 0.9878 | 13.0956 | 0.0003 |
| F1234568 | 0.9659 | 0.0271 | 0.8494 | 0.993 | 16.4912 | $<.0001$ |
| F1234567 | 0.9645 | 0.0283 | 0.8433 | 0.9928 | 15.9788 | $<.0001$ |
| F12345678 | 0.978 | 0.0185 | 0.8922 | 0.9958 | 19.5793 | $<.0001$ |
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|  |  |  |  |  |  |  |

Table A.4- Predictive Index of Non-Imputed MESA Data

| Contrast Estimation and Testing Results by Row |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Contrast | Estimate | Standard Error | Confidence | Limits | Wald ChiSquare | $\begin{aligned} & \text { Pr }> \\ & \text { ChiSq } \end{aligned}$ |
| 0 | 0.5 | 0 | . | . | . | . |
| F8 | 0.6193 | 0.0419 | 0.5344 | 0.6974 | 7.4874 | 0.0062 |
| F7 | 0.6065 | 0.0445 | 0.5168 | 0.6896 | 5.3799 | 0.0204 |
| F78 | 0.7148 | 0.0526 | 0.602 | 0.806 | 12.706 | 0.0004 |
| F6 | 0.6294 | 0.0508 | 0.5258 | 0.7224 | 5.9239 | 0.0149 |
| F68 | 0.7342 | 0.0539 | 0.6166 | 0.826 | 13.5457 | 0.0002 |
| F67 | 0.7236 | 0.0559 | 0.6022 | 0.8191 | 11.8594 | 0.0006 |
| F678 | 0.8098 | 0.0504 | 0.6916 | 0.8899 | 19.61 | <. 0001 |
| F5 | 0.7248 | 0.0628 | 0.587 | 0.83 | 9.4715 | 0.0021 |
| F58 | 0.8108 | 0.057 | 0.674 | 0.8987 | 15.3244 | <. 0001 |
| F57 | 0.8024 | 0.0613 | 0.6556 | 0.8965 | 13.1413 | 0.0003 |
| F578 | 0.8685 | 0.0496 | 0.7382 | 0.9393 | 18.8936 | <. 0001 |
| F56 | 0.8173 | 0.0597 | 0.6714 | 0.9074 | 14.035 | 0.0002 |
| F568 | 0.8792 | 0.0471 | 0.7533 | 0.9455 | 20.0663 | <. 0001 |
| F567 | 0.8734 | 0.0502 | 0.739 | 0.9438 | 18.0786 | <. 0001 |
| F5678 | 0.9181 | 0.037 | 0.8103 | 0.9672 | 24.0795 | <. 0001 |
| F4 | 0.6242 | 0.0566 | 0.5087 | 0.7272 | 4.4249 | 0.0354 |
| F48 | 0.7299 | 0.062 | 0.5933 | 0.8335 | 9.9903 | 0.0016 |
| F47 | 0.7191 | 0.0606 | 0.5872 | 0.8217 | 9.8232 | 0.0017 |
| F478 | 0.8064 | 0.0565 | 0.6721 | 0.8943 | 15.5662 | <. 0001 |
| F46 | 0.7383 | 0.063 | 0.5982 | 0.8425 | 10.1079 | 0.0015 |
| F468 | 0.8211 | 0.0558 | 0.6855 | 0.9062 | 16.0805 | <. 0001 |
| F467 | 0.8131 | 0.0557 | 0.6796 | 0.8992 | 16.1067 | <. 0001 |
| F4678 | 0.8761 | 0.045 | 0.7582 | 0.941 | 22.2306 | <. 0001 |
| F45 | 0.814 | 0.0534 | 0.6869 | 0.8972 | 17.5509 | <. 0001 |
| F458 | 0.8768 | 0.0448 | 0.7593 | 0.9414 | 22.3516 | <. 0001 |
| F457 | 0.8709 | 0.0466 | 0.7497 | 0.9382 | 21.2466 | <. 0001 |
| F4578 | 0.9165 | 0.0359 | 0.814 | 0.9649 | 26.1082 | <. 0001 |
| F456 | 0.8814 | 0.0451 | 0.7615 | 0.9454 | 21.6417 | <. 0001 |
| F4568 | 0.9236 | 0.0339 | 0.8249 | 0.9688 | 26.8502 | <. 0001 |
| F4567 | 0.9197 | 0.0353 | 0.8176 | 0.967 | 25.9463 | <. 0001 |
| F45678 | 0.9491 | 0.0253 | 0.8696 | 0.9811 | 31.1451 | <. 0001 |
| F3 | 0.6956 | 0.0772 | 0.5279 | 0.8236 | 5.1372 | 0.0234 |
| F38 | 0.788 | 0.0696 | 0.6216 | 0.8937 | 9.9367 | 0.0016 |
| F37 | 0.7788 | 0.073 | 0.6055 | 0.8899 | 8.8264 | 0.003 |


| F378 | 0.8514 | 0.0594 | 0.6954 | 0.9349 | 13.8364 | 0.0002 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F36 | 0.7951 | 0.0703 | 0.6248 | 0.9005 | 9.8652 | 0.0017 |
| F368 | 0.8632 | 0.0559 | 0.7139 | 0.9411 | 15.1436 | <. 0001 |
| F367 | 0.8568 | 0.0587 | 0.7007 | 0.9386 | 13.9719 | 0.0002 |
| F3678 | 0.9068 | 0.0437 | 0.7795 | 0.964 | 19.3977 | <. 0001 |
| F35 | 0.8575 | 0.0612 | 0.6928 | 0.9414 | 12.8489 | 0.0003 |
| F358 | 0.9073 | 0.046 | 0.7703 | 0.9662 | 17.4212 | <. 0001 |
| F357 | 0.9027 | 0.0491 | 0.756 | 0.9652 | 15.855 | <. 0001 |
| F3578 | 0.9378 | 0.035 | 0.8229 | 0.98 | 20.4005 | <. 0001 |
| F356 | 0.9109 | 0.0458 | 0.772 | 0.9686 | 17.0066 | <. 0001 |
| F3568 | 0.9433 | 0.0322 | 0.8362 | 0.9819 | 21.7676 | <. 0001 |
| F3567 | 0.9403 | 0.0344 | 0.8257 | 0.9813 | 20.2245 | <. 0001 |
| F35678 | 0.9624 | 0.0235 | 0.8778 | 0.9892 | 24.9766 | <. 0001 |
| F34 | 0.7915 | 0.0754 | 0.6079 | 0.9029 | 8.5227 | 0.0035 |
| F348 | 0.8606 | 0.061 | 0.6951 | 0.9436 | 12.8256 | 0.0003 |
| F347 | 0.854 | 0.0626 | 0.6861 | 0.94 | 12.3638 | 0.0004 |
| F3478 | 0.9049 | 0.0473 | 0.7642 | 0.9654 | 16.8033 | <. 0001 |
| F346 | 0.8657 | 0.0596 | 0.7023 | 0.9463 | 13.2 | 0.0003 |
| F3468 | 0.9129 | 0.0443 | 0.7788 | 0.969 | 17.8179 | <. 0001 |
| F3467 | 0.9086 | 0.0457 | 0.7717 | 0.9669 | 17.4206 | <. 0001 |
| F34678 | 0.9417 | 0.0324 | 0.8353 | 0.981 | 22.149 | <. 0001 |
| F345 | 0.9091 | 0.0448 | 0.7757 | 0.9666 | 18.0644 | <. 0001 |
| F3458 | 0.9421 | 0.0323 | 0.836 | 0.9811 | 22.1879 | <. 0001 |
| F3457 | 0.9391 | 0.034 | 0.8279 | 0.9801 | 21.2122 | <. 0001 |
| F34578 | 0.9616 | 0.0236 | 0.8773 | 0.9888 | 25.3233 | <. 0001 |
| F3456 | 0.9444 | 0.0316 | 0.8394 | 0.9822 | 22.1983 | <. 0001 |
| F34568 | 0.9651 | 0.0217 | 0.8864 | 0.9899 | 26.4944 | <. 0001 |
| F34567 | 0.9632 | 0.0229 | 0.8808 | 0.9893 | 25.5937 | <. 0001 |
| F345678 | 0.977 | 0.0154 | 0.9173 | 0.9939 | 29.8834 | <. 0001 |
| F2 | 0.6302 | 0.045 | 0.5385 | 0.7134 | 7.6086 | 0.0058 |
| F28 | 0.7348 | 0.0538 | 0.6172 | 0.8265 | 13.6043 | 0.0002 |
| F27 | 0.7242 | 0.0535 | 0.6084 | 0.8162 | 12.9848 | 0.0003 |
| F278 | 0.8103 | 0.0512 | 0.6898 | 0.8914 | 19.0041 | <. 0001 |
| F26 | 0.7432 | 0.055 | 0.622 | 0.8358 | 13.6001 | 0.0002 |
| F268 | 0.8248 | 0.0499 | 0.7052 | 0.9026 | 20.1078 | <. 0001 |
| F267 | 0.8169 | 0.0504 | 0.6975 | 0.8962 | 19.7105 | <. 0001 |
| F2678 | 0.8789 | 0.0412 | 0.7726 | 0.9394 | 26.211 | <. 0001 |
| F25 | 0.8178 | 0.0553 | 0.6845 | 0.9028 | 16.3938 | <. 0001 |
| F258 | 0.8795 | 0.0455 | 0.759 | 0.9442 | 21.4692 | <. 0001 |


| F257 | 0.8737 | 0.0478 | 0.7475 | 0.9417 | 19.945 | $<.0001$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| F2578 | 0.9184 | 0.0363 | 0.8134 | 0.9667 | 25.018 | $<.0001$ |
| F256 | 0.884 | 0.0455 | 0.7617 | 0.9479 | 20.9693 | $<.0001$ |
| F2568 | 0.9254 | 0.0339 | 0.8258 | 0.9701 | 26.3488 | $<.0001$ |
| F2567 | 0.9216 | 0.0356 | 0.8173 | 0.9686 | 25.0098 | $<.0001$ |
| F25678 | 0.9503 | 0.0253 | 0.87 | 0.982 | 30.3684 | $<.0001$ |
| F24 | 0.739 | 0.0611 | 0.6033 | 0.8405 | 10.7794 | 0.001 |
| F248 | 0.8216 | 0.0565 | 0.6838 | 0.9074 | 15.6939 | $<.0001$ |
| F247 | 0.8135 | 0.0551 | 0.6817 | 0.8989 | 16.4546 | $<.0001$ |
| F2478 | 0.8765 | 0.0459 | 0.7554 | 0.9422 | 21.3214 | $<.0001$ |
| F246 | 0.8278 | 0.0546 | 0.694 | 0.9107 | 16.773 | $<.0001$ |
| F2468 | 0.8866 | 0.0441 | 0.768 | 0.9487 | 21.9869 | $<.0001$ |
| F2467 | 0.8811 | 0.0437 | 0.7658 | 0.9438 | 23.004 | $<.0001$ |
| F24678 | 0.9234 | 0.0332 | 0.8278 | 0.968 | 28.1859 | $<.0001$ |
| F245 | 0.8818 | 0.0427 | 0.7698 | 0.9433 | 24.0953 | $<.0001$ |
| F2458 | 0.9238 | 0.0332 | 0.8277 | 0.9684 | 27.9091 | $<.0001$ |
| F2457 | 0.92 | 0.0341 | 0.8226 | 0.9661 | 27.7908 | $<.0001$ |
| F24578 | 0.9492 | 0.0251 | 0.8709 | 0.9811 | 31.7279 | $<.0001$ |
| F2456 | 0.9268 | 0.0324 | 0.8325 | 0.9699 | 28.3178 | $<.0001$ |
| F24568 | 0.9537 | 0.0234 | 0.8795 | 0.9831 | 32.6288 | $<.0001$ |
| F24567 | 0.9513 | 0.0241 | 0.8757 | 0.9819 | 32.6342 | $<.0001$ |
| F245678 | 0.9695 | 0.0168 | 0.9124 | 0.9898 | 36.9586 | $<.0001$ |
| F23 | 0.7956 | 0.0667 | 0.6353 | 0.8969 | 10.9729 | 0.0009 |
| F238 | 0.8636 | 0.0548 | 0.7179 | 0.9403 | 15.7464 | $<.0001$ |
| F237 | 0.8572 | 0.0567 | 0.7076 | 0.937 | 14.9542 | 0.0001 |
| F2378 | 0.9071 | 0.0432 | 0.7814 | 0.9638 | 19.7545 | $<.0001$ |
| F236 | 0.8687 | 0.0535 | 0.725 | 0.9432 | 16.2048 | $<.0001$ |
| F2368 | 0.9149 | 0.0401 | 0.7967 | 0.9672 | 21.2595 | $<.0001$ |
| F2367 | 0.9107 | 0.0417 | 0.7888 | 0.9653 | 20.5356 | $<.0001$ |
| F23678 | 0.9431 | 0.0298 | 0.8482 | 0.9801 | 25.6202 | $<.0001$ |
| F235 | 0.9112 | 0.0434 | 0.782 | 0.967 | 18.8583 | $<.0001$ |
| F2358 | 0.9434 | 0.0312 | 0.8411 | 0.9813 | 23.104 | $<.0001$ |
| F2357 | 0.9405 | 0.0331 | 0.8323 | 0.9805 | 21.819 | $<.0001$ |
| F23578 | 0.9626 | 0.0229 | 0.8808 | 0.9889 | 26.044 | $<.0001$ |
| F2356 | 0.9457 | 0.0305 | 0.8447 | 0.9824 | 23.152 | $<.0001$ |
| F23568 | 0.9659 | 0.021 | 0.8905 | 0.99 | 27.5825 | $<.0001$ |
| F23567 | 0.9641 | 0.0222 | 0.8844 | 0.9895 | 26.3692 | $<.0001$ |
| F235678 | 0.9776 | 0.0149 | 0.92 | 0.994 | 30.7834 | $<.0001$ |
| F234 | 0.8661 | 0.0579 | 0.7086 | 0.9451 | 13.9895 | 0.0002 |
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|  |  |  |  |  |  |  |
|  |  |  | 0.0 |  |  |  |


| F2348 | 0.9132 | 0.0439 | 0.7805 | 0.9689 | 18.0884 | $<.0001$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| F2347 | 0.9088 | 0.0448 | 0.7755 | 0.9664 | 18.0812 | $<.0001$ |
| F23478 | 0.9419 | 0.0324 | 0.8358 | 0.981 | 22.2033 | $<.0001$ |
| F2346 | 0.9166 | 0.042 | 0.7892 | 0.9699 | 19.0308 | $<.0001$ |
| F23468 | 0.947 | 0.0299 | 0.8473 | 0.9829 | 23.3414 | $<.0001$ |
| F23467 | 0.9442 | 0.0307 | 0.8435 | 0.9815 | 23.4701 | $<.0001$ |
| F234678 | 0.965 | 0.0213 | 0.8893 | 0.9895 | 27.8018 | $<.0001$ |
| F2345 | 0.9446 | 0.0303 | 0.8457 | 0.9815 | 24.0007 | $<.0001$ |
| F23458 | 0.9652 | 0.0212 | 0.8892 | 0.9897 | 27.6211 | $<.0001$ |
| F23457 | 0.9633 | 0.0222 | 0.8847 | 0.989 | 27.1174 | $<.0001$ |
| F234578 | 0.9771 | 0.0151 | 0.9189 | 0.9938 | 30.7464 | $<.0001$ |
| F23456 | 0.9666 | 0.0204 | 0.8932 | 0.9901 | 28.2268 | $<.0001$ |
| F234568 | 0.9792 | 0.0139 | 0.9254 | 0.9944 | 32.0423 | $<.0001$ |
| F234567 | 0.9781 | 0.0145 | 0.9223 | 0.9941 | 31.6301 | $<.0001$ |
| F2345678 | 0.9864 | 0.00965 | 0.9465 | 0.9966 | 35.4426 | $<.0001$ |
| F1 | 0.5946 | 0.0433 | 0.5077 | 0.676 | 4.5454 | 0.033 |
| F18 | 0.7047 | 0.0538 | 0.5897 | 0.7984 | 11.3081 | 0.0008 |
| F17 | 0.6933 | 0.0563 | 0.5736 | 0.7916 | 9.4892 | 0.0021 |
| F178 | 0.7862 | 0.0544 | 0.661 | 0.874 | 16.1823 | $<.0001$ |
| F16 | 0.7136 | 0.057 | 0.5906 | 0.8115 | 10.7187 | 0.0011 |
| F168 | 0.8021 | 0.0526 | 0.6793 | 0.8858 | 17.8695 | $<.0001$ |
| F167 | 0.7934 | 0.0547 | 0.6662 | 0.8808 | 16.2347 | $<.0001$ |
| F1678 | 0.862 | 0.0451 | 0.7483 | 0.9292 | 23.3985 | $<.0001$ |
| F15 | 0.7944 | 0.0648 | 0.6398 | 0.8937 | 11.6185 | 0.0007 |
| F158 | 0.8627 | 0.0529 | 0.7237 | 0.9378 | 16.9504 | $<.0001$ |
| F157 | 0.8562 | 0.0565 | 0.7078 | 0.9361 | 15.1137 | 0.0001 |
| F1578 | 0.9064 | 0.0426 | 0.7833 | 0.9629 | 20.3956 | $<.0001$ |
| F156 | 0.8678 | 0.0534 | 0.7251 | 0.9423 | 16.3697 | $<.0001$ |
| F1568 | 0.9143 | 0.0396 | 0.7985 | 0.9664 | 21.9385 | $<.0001$ |
| F1567 | 0.91 | 0.0422 | 0.7865 | 0.9652 | 20.1635 | $<.0001$ |
| F15678 | 0.9427 | 0.0298 | 0.8479 | 0.9798 | 25.7151 | $<.0001$ |
| F14 | 0.7091 | 0.0641 | 0.5699 | 0.8176 | 8.2084 | 0.0042 |
| F148 | 0.7985 | 0.0603 | 0.6554 | 0.892 | 13.5176 | 0.0002 |
| F147 | 0.7897 | 0.0602 | 0.6486 | 0.8843 | 13.321 | 0.0003 |
| F1478 | 0.8593 | 0.0506 | 0.729 | 0.9328 | 18.695 | $<.0001$ |
| F146 | 0.8054 | 0.0593 | 0.6636 | 0.8968 | 14.1132 | 0.0002 |
| F1468 | 0.8707 | 0.0483 | 0.7439 | 0.9398 | 19.7764 | $<.0001$ |
| F1467 | 0.8645 | 0.0488 | 0.7381 | 0.9352 | 19.7685 | $<.0001$ |
| F14678 | 0.9121 | 0.0371 | 0.8071 | 0.9626 | 25.4937 | $<.0001$ |
|  |  |  |  |  |  |  |


| F145 | 0.8652 | 0.0506 | 0.7327 | 0.9376 | 18.339 | <. 0001 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F1458 | 0.9126 | 0.039 | 0.8 | 0.9646 | 22.9606 | <. 0001 |
| F1457 | 0.9082 | 0.0407 | 0.7915 | 0.9627 | 21.9899 | <. 0001 |
| F14578 | 0.9415 | 0.0297 | 0.8485 | 0.9788 | 26.6084 | <. 0001 |
| F1456 | 0.916 | 0.0383 | 0.8041 | 0.9666 | 22.9818 | <. 0001 |
| F14568 | 0.9466 | 0.0275 | 0.8591 | 0.981 | 27.8633 | <. 0001 |
| F14567 | 0.9438 | 0.0288 | 0.8529 | 0.9799 | 27.0208 | <. 0001 |
| F145678 | 0.9647 | 0.0199 | 0.8966 | 0.9885 | 31.8924 | <. 0001 |
| F13 | 0.7702 | 0.0731 | 0.5987 | 0.8828 | 8.5773 | 0.0034 |
| F138 | 0.845 | 0.0606 | 0.6876 | 0.931 | 13.4378 | 0.0002 |
| F137 | 0.8378 | 0.0638 | 0.6731 | 0.9284 | 12.2431 | 0.0005 |
| F1378 | 0.8936 | 0.0488 | 0.7543 | 0.9583 | 17.1736 | <. 0001 |
| F136 | 0.8506 | 0.0599 | 0.6933 | 0.9348 | 13.6185 | 0.0002 |
| F1368 | 0.9025 | 0.0451 | 0.7722 | 0.962 | 18.845 | <. 0001 |
| F1367 | 0.8977 | 0.0475 | 0.7609 | 0.9603 | 17.6171 | <. 0001 |
| F13678 | 0.9345 | 0.034 | 0.8277 | 0.977 | 22.9005 | <. 0001 |
| F135 | 0.8983 | 0.0512 | 0.7464 | 0.9636 | 15.0943 | 0.0001 |
| F1358 | 0.9349 | 0.0368 | 0.8146 | 0.9791 | 19.4365 | <. 0001 |
| F1357 | 0.9315 | 0.0393 | 0.8027 | 0.9785 | 17.9573 | <. 0001 |
| F13578 | 0.9568 | 0.0271 | 0.8595 | 0.9877 | 22.2754 | <. 0001 |
| F1356 | 0.9375 | 0.0361 | 0.8177 | 0.9805 | 19.3263 | <. 0001 |
| F13568 | 0.9606 | 0.0247 | 0.8713 | 0.9887 | 23.86 | <. 0001 |
| F13567 | 0.9585 | 0.0264 | 0.8629 | 0.9884 | 22.3846 | <. 0001 |
| F135678 | 0.9741 | 0.0176 | 0.9052 | 0.9933 | 26.9052 | <. 0001 |
| F134 | 0.8478 | 0.0649 | 0.6753 | 0.9371 | 11.6745 | 0.0006 |
| F1348 | 0.9006 | 0.0495 | 0.7542 | 0.9639 | 15.9142 | <. 0001 |
| F1347 | 0.8956 | 0.0511 | 0.746 | 0.9616 | 15.4443 | <. 0001 |
| F13478 | 0.9331 | 0.037 | 0.8136 | 0.9781 | 19.7499 | <. 0001 |
| F1346 | 0.9044 | 0.0478 | 0.7621 | 0.9654 | 16.5529 | <. 0001 |
| F13468 | 0.939 | 0.0341 | 0.8272 | 0.9802 | 21.0569 | <. 0001 |
| F13467 | 0.9358 | 0.0354 | 0.8211 | 0.9789 | 20.6556 | <. 0001 |
| F134678 | 0.9595 | 0.0245 | 0.8733 | 0.9879 | 25.2162 | <. 0001 |
| F1345 | 0.9362 | 0.036 | 0.8182 | 0.9795 | 19.8607 | <. 0001 |
| F13458 | 0.9598 | 0.0251 | 0.8696 | 0.9884 | 23.7754 | <. 0001 |
| F13457 | 0.9576 | 0.0265 | 0.8629 | 0.9878 | 22.8575 | <. 0001 |
| F134578 | 0.9735 | 0.018 | 0.9037 | 0.9931 | 26.7619 | <. 0001 |
| F13456 | 0.9614 | 0.0243 | 0.8734 | 0.989 | 24.0925 | <. 0001 |
| F134568 | 0.9759 | 0.0164 | 0.9117 | 0.9937 | 28.1714 | <. 0001 |
| F134567 | 0.9746 | 0.0173 | 0.9072 | 0.9934 | 27.31 | <. 0001 |


| F1345678 | 0.9842 | 0.0114 | 0.9363 | 0.9962 | 31.3826 | <. 0001 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F12 | 0.7143 | 0.0548 | 0.5963 | 0.8088 | 11.6438 | 0.0006 |
| F128 | 0.8026 | 0.0535 | 0.6771 | 0.8874 | 17.2403 | <. 0001 |
| F127 | 0.7939 | 0.0542 | 0.6682 | 0.8805 | 16.5985 | <. 0001 |
| F1278 | 0.8624 | 0.0462 | 0.7449 | 0.9308 | 22.2003 | <. 0001 |
| F126 | 0.8094 | 0.0525 | 0.6854 | 0.8922 | 18.0426 | <. 0001 |
| F1268 | 0.8735 | 0.0435 | 0.7614 | 0.9373 | 24.0664 | <. 0001 |
| F1267 | 0.8674 | 0.0444 | 0.7542 | 0.9331 | 23.6329 | <. 0001 |
| F12678 | 0.9141 | 0.0341 | 0.8196 | 0.9614 | 29.6423 | <. 0001 |
| F125 | 0.8682 | 0.051 | 0.7334 | 0.9403 | 17.9223 | <. 0001 |
| F1258 | 0.9146 | 0.0389 | 0.8015 | 0.966 | 22.6965 | <. 0001 |
| F1257 | 0.9103 | 0.0409 | 0.7916 | 0.9644 | 21.3709 | <. 0001 |
| F12578 | 0.9429 | 0.0295 | 0.8492 | 0.9797 | 26.1266 | <. 0001 |
| F1256 | 0.9179 | 0.0381 | 0.8059 | 0.9679 | 22.8149 | <. 0001 |
| F12568 | 0.9479 | 0.0272 | 0.861 | 0.9816 | 27.8377 | <. 0001 |
| F12567 | 0.9452 | 0.0286 | 0.8539 | 0.9807 | 26.6176 | <. 0001 |
| F125678 | 0.9656 | 0.0197 | 0.8977 | 0.989 | 31.6236 | <. 0001 |
| F124 | 0.8059 | 0.0588 | 0.6653 | 0.8967 | 14.342 | 0.0002 |
| F1248 | 0.871 | 0.0493 | 0.7408 | 0.941 | 18.9614 | <. 0001 |
| F1247 | 0.8649 | 0.049 | 0.7378 | 0.9357 | 19.5994 | <. 0001 |
| F12478 | 0.9124 | 0.0381 | 0.8036 | 0.9636 | 24.1785 | <. 0001 |
| F1246 | 0.8758 | 0.0469 | 0.7518 | 0.9426 | 20.5188 | <. 0001 |
| F12468 | 0.9198 | 0.0357 | 0.8162 | 0.9673 | 25.405 | <. 0001 |
| F12467 | 0.9158 | 0.0359 | 0.8137 | 0.9644 | 26.3097 | <. 0001 |
| F124678 | 0.9465 | 0.0261 | 0.8657 | 0.9798 | 31.1566 | <. 0001 |
| F1245 | 0.9162 | 0.0372 | 0.809 | 0.9658 | 24.4232 | <. 0001 |
| F12458 | 0.9468 | 0.0273 | 0.8603 | 0.9809 | 28.2911 | <. 0001 |
| F12457 | 0.944 | 0.0282 | 0.8557 | 0.9796 | 28.0611 | <. 0001 |
| F124578 | 0.9648 | 0.0199 | 0.8969 | 0.9886 | 31.9622 | <. 0001 |
| F12456 | 0.9489 | 0.0262 | 0.8655 | 0.9817 | 29.1899 | <. 0001 |
| F124568 | 0.968 | 0.0183 | 0.9048 | 0.9897 | 33.35 | <. 0001 |
| F124567 | 0.9663 | 0.019 | 0.9015 | 0.989 | 33.2464 | <. 0001 |
| F1245678 | 0.979 | 0.0129 | 0.9315 | 0.9938 | 37.406 | <. 0001 |
| F123 | 0.851 | 0.0579 | 0.7001 | 0.9332 | 14.5641 | 0.0001 |
| F1238 | 0.9028 | 0.0447 | 0.7739 | 0.9618 | 19.1516 | <. 0001 |
| F1237 | 0.898 | 0.0465 | 0.7649 | 0.9597 | 18.3479 | <. 0001 |
| F12378 | 0.9347 | 0.0339 | 0.8281 | 0.977 | 22.9344 | <. 0001 |
| F1236 | 0.9066 | 0.0431 | 0.7818 | 0.9633 | 19.9823 | <. 0001 |
| F12368 | 0.9404 | 0.031 | 0.842 | 0.979 | 24.824 | <. 0001 |


| F12367 | 0.9373 | 0.0324 | 0.8355 | 0.9778 | 24.091 | $<.0001$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| F123678 | 0.9605 | 0.0225 | 0.8838 | 0.9873 | 28.9357 | $<.0001$ |
| F1235 | 0.9377 | 0.0347 | 0.8243 | 0.9797 | 20.7998 | $<.0001$ |
| F12358 | 0.9607 | 0.0242 | 0.8743 | 0.9885 | 24.8281 | $<.0001$ |
| F12357 | 0.9587 | 0.0256 | 0.8672 | 0.988 | 23.6256 | $<.0001$ |
| F123578 | 0.9742 | 0.0174 | 0.9069 | 0.9932 | 27.6366 | $<.0001$ |
| F12356 | 0.9623 | 0.0234 | 0.8782 | 0.9891 | 25.1988 | $<.0001$ |
| F123568 | 0.9765 | 0.0158 | 0.9153 | 0.9938 | 29.4027 | $<.0001$ |
| F123567 | 0.9752 | 0.0167 | 0.9104 | 0.9935 | 28.2495 | $<.0001$ |
| F1235678 | 0.9846 | 0.0111 | 0.9387 | 0.9963 | 32.439 | $<.0001$ |
| F1234 | 0.9047 | 0.0469 | 0.7658 | 0.965 | 17.1412 | $<.0001$ |
| F12348 | 0.9391 | 0.0341 | 0.8275 | 0.9803 | 21.0726 | $<.0001$ |
| F12347 | 0.936 | 0.035 | 0.823 | 0.9787 | 21.0611 | $<.0001$ |
| F123478 | 0.9597 | 0.0245 | 0.8729 | 0.988 | 24.9957 | $<.0001$ |
| F12346 | 0.9416 | 0.0324 | 0.8357 | 0.9808 | 22.3255 | $<.0001$ |
| F123468 | 0.9633 | 0.0225 | 0.883 | 0.9891 | 26.4473 | $<.0001$ |
| F123467 | 0.9613 | 0.0232 | 0.8798 | 0.9883 | 26.5694 | $<.0001$ |
| F1234678 | 0.9758 | 0.0157 | 0.9161 | 0.9934 | 30.6933 | $<.0001$ |
| F12345 | 0.9615 | 0.0236 | 0.8774 | 0.9887 | 25.4458 | $<.0001$ |
| F123458 | 0.976 | 0.0161 | 0.9133 | 0.9937 | 28.9229 | $<.0001$ |
| F123457 | 0.9747 | 0.0169 | 0.9096 | 0.9933 | 28.4232 | $<.0001$ |
| F1234578 | 0.9843 | 0.0113 | 0.9371 | 0.9962 | 31.9038 | $<.0001$ |
| F123456 | 0.977 | 0.0154 | 0.917 | 0.9939 | 29.8042 | $<.0001$ |
| F1234568 | 0.9857 | 0.0103 | 0.9427 | 0.9966 | 33.4472 | $<.0001$ |
| F1234567 | 0.9849 | 0.0108 | 0.9402 | 0.9963 | 33.028 | $<.0001$ |
| F12345678 | 0.9907 | 0.00711 | 0.9592 | 0.9979 | 36.668 | $<.0001$ |
|  |  |  |  |  |  |  |

Table A.5- Predictive Index of Imputed MESA Data
Contrast Estimation and Testing Results by Row

| Contrast | Estimate | Standard <br> Error | Confidence | Limits | Wald Chi- <br> Square | Pr $>$ <br> ChiSq |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0.5 | 0 | . | . | . | . |
| F8 | 0.6081 | 0.0383 | 0.5311 | 0.6801 | 7.479 | 0.0062 |
| F7 | 0.5925 | 0.0403 | 0.5117 | 0.6685 | 5.0246 | 0.025 |
| F78 | 0.6928 | 0.0489 | 0.5898 | 0.7797 | 12.5356 | 0.0004 |
| F6 | 0.6105 | 0.0469 | 0.5157 | 0.6975 | 5.1955 | 0.0226 |
| F68 | 0.7086 | 0.0514 | 0.5988 | 0.7985 | 12.7337 | 0.0004 |
| F67 | 0.695 | 0.0523 | 0.5842 | 0.787 | 11.1503 | 0.0008 |
| F678 | 0.7795 | 0.0495 | 0.6677 | 0.8615 | 19.1909 | $<.0001$ |
| F5 | 0.6963 | 0.0676 | 0.5506 | 0.811 | 6.731 | 0.0095 |
| F58 | 0.7806 | 0.0628 | 0.6342 | 0.8795 | 11.9717 | 0.0005 |
| F57 | 0.7692 | 0.0654 | 0.6181 | 0.8728 | 10.6669 | 0.0011 |
| F578 | 0.838 | 0.0555 | 0.6988 | 0.9202 | 16.1454 | $<.0001$ |
| F56 | 0.7823 | 0.0656 | 0.628 | 0.8843 | 11.0152 | 0.0009 |
| F568 | 0.8479 | 0.0544 | 0.7091 | 0.9273 | 16.578 | $<.0001$ |
| F567 | 0.8393 | 0.0566 | 0.6964 | 0.9224 | 15.5106 | $<.0001$ |
| F5678 | 0.8902 | 0.0443 | 0.7695 | 0.9516 | 21.3672 | $<.0001$ |
| F4 | 0.6214 | 0.0517 | 0.5162 | 0.7163 | 5.0858 | 0.0241 |
| F48 | 0.718 | 0.0569 | 0.5949 | 0.8154 | 11.079 | 0.0009 |
| F47 | 0.7047 | 0.0565 | 0.5836 | 0.8024 | 10.2667 | 0.0014 |
| F478 | 0.7873 | 0.0538 | 0.6635 | 0.8742 | 16.5724 | $<.0001$ |
| F46 | 0.7201 | 0.059 | 0.5917 | 0.8203 | 10.4101 | 0.0013 |
| F468 | 0.7996 | 0.054 | 0.6733 | 0.8854 | 16.8491 | $<.0001$ |
| F467 | 0.789 | 0.054 | 0.6644 | 0.876 | 16.5311 | $<.0001$ |
| F4678 | 0.853 | 0.0456 | 0.7399 | 0.9221 | 23.3658 | $<.0001$ |
| F45 | 0.79 | 0.0565 | 0.6588 | 0.88 | 15.1545 | $<.0001$ |
| F458 | 0.8538 | 0.0488 | 0.7307 | 0.9263 | 20.3598 | $<.0001$ |
| F457 | 0.8454 | 0.0501 | 0.7207 | 0.9206 | 19.6444 | $<.0001$ |
| F4578 | 0.8946 | 0.0404 | 0.7858 | 0.9515 | 24.9714 | $<.0001$ |
| F456 | 0.855 | 0.0497 | 0.7288 | 0.9283 | 19.59 | $<.0001$ |
| F4568 | 0.9015 | 0.0392 | 0.7938 | 0.956 | 25.1165 | $<.0001$ |
| F4567 | 0.8955 | 0.0403 | 0.7864 | 0.9523 | 24.8272 | $<.0001$ |
| F45678 | 0.9301 | 0.0305 | 0.8415 | 0.9709 | 30.5174 | $<.0001$ |
| F3 | 0.676 | 0.0771 | 0.5115 | 0.8062 | 4.3697 | 0.0366 |
| F38 | 0.764 | 0.071 | 0.5994 | 0.8751 | 8.8976 | 0.0029 |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |


| F37 | 0.7521 | 0.0748 | 0.5801 | 0.8695 | 7.6477 | 0.0057 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F378 | 0.8248 | 0.0632 | 0.6663 | 0.9173 | 12.5326 | 0.0004 |
| F36 | 0.7658 | 0.0729 | 0.5959 | 0.8788 | 8.5023 | 0.0035 |
| F368 | 0.8354 | 0.0605 | 0.6817 | 0.9232 | 13.6127 | 0.0002 |
| F367 | 0.8262 | 0.0636 | 0.6661 | 0.9189 | 12.3742 | 0.0004 |
| F3678 | 0.8806 | 0.0498 | 0.7447 | 0.9491 | 17.8109 | <. 0001 |
| F35 | 0.8271 | 0.0697 | 0.6478 | 0.9256 | 10.3004 | 0.0013 |
| F358 | 0.8813 | 0.0549 | 0.7262 | 0.9541 | 14.5797 | 0.0001 |
| F357 | 0.8743 | 0.0582 | 0.7111 | 0.9516 | 13.3954 | 0.0003 |
| F3578 | 0.9152 | 0.0437 | 0.7814 | 0.9702 | 17.8123 | <. 0001 |
| F356 | 0.8823 | 0.0556 | 0.7242 | 0.9554 | 14.1638 | 0.0002 |
| F3568 | 0.9208 | 0.0413 | 0.7929 | 0.9725 | 18.7212 | <. 0001 |
| F3567 | 0.916 | 0.0438 | 0.7814 | 0.9708 | 17.6346 | <. 0001 |
| F35678 | 0.9442 | 0.0315 | 0.8397 | 0.982 | 22.3531 | <. 0001 |
| F34 | 0.774 | 0.0755 | 0.5951 | 0.8887 | 8.1316 | 0.0044 |
| F348 | 0.8416 | 0.0629 | 0.6783 | 0.9305 | 12.541 | 0.0004 |
| F347 | 0.8327 | 0.0655 | 0.6645 | 0.926 | 11.6538 | 0.0006 |
| F3478 | 0.8854 | 0.0514 | 0.7412 | 0.9542 | 16.3071 | <. 0001 |
| F346 | 0.843 | 0.063 | 0.6787 | 0.9317 | 12.4779 | 0.0004 |
| F3468 | 0.8928 | 0.0488 | 0.7541 | 0.9577 | 17.2974 | <. 0001 |
| F3467 | 0.8864 | 0.0509 | 0.7436 | 0.9545 | 16.5455 | <. 0001 |
| F34678 | 0.9237 | 0.0378 | 0.8088 | 0.9719 | 21.6169 | <. 0001 |
| F345 | 0.887 | 0.0517 | 0.7408 | 0.9557 | 15.9804 | <. 0001 |
| F3458 | 0.9241 | 0.0389 | 0.8041 | 0.9731 | 20.274 | <. 0001 |
| F3457 | 0.9194 | 0.041 | 0.7942 | 0.9712 | 19.362 | <. 0001 |
| F34578 | 0.9466 | 0.0298 | 0.8479 | 0.9825 | 23.7474 | <. 0001 |
| F3456 | 0.9248 | 0.0389 | 0.8043 | 0.9736 | 20.1263 | <. 0001 |
| F34568 | 0.9502 | 0.0281 | 0.8563 | 0.9839 | 24.6465 | <. 0001 |
| F34567 | 0.9471 | 0.0296 | 0.8492 | 0.9827 | 23.9318 | <. 0001 |
| F345678 | 0.9652 | 0.0209 | 0.8914 | 0.9895 | 28.5702 | <. 0001 |
| F2 | 0.5862 | 0.0397 | 0.5069 | 0.6613 | 4.5336 | 0.0332 |
| F28 | 0.6874 | 0.0511 | 0.5798 | 0.7779 | 10.9838 | 0.0009 |
| F27 | 0.6732 | 0.0512 | 0.5661 | 0.7648 | 9.624 | 0.0019 |
| F278 | 0.7617 | 0.0523 | 0.645 | 0.849 | 16.2472 | <. 0001 |
| F26 | 0.6895 | 0.0539 | 0.5756 | 0.7843 | 10.0522 | 0.0015 |
| F268 | 0.7751 | 0.0524 | 0.6565 | 0.8613 | 16.9324 | <. 0001 |
| F267 | 0.7635 | 0.0526 | 0.6459 | 0.8511 | 16.1799 | <. 0001 |
| F2678 | 0.8336 | 0.0462 | 0.7227 | 0.9059 | 23.3728 | <. 0001 |


| F25 | 0.7646 | 0.0648 | 0.6159 | 0.8681 | 10.7019 | 0.0011 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F258 | 0.8344 | 0.0563 | 0.6941 | 0.918 | 15.7738 | <. 0001 |
| F257 | 0.8252 | 0.0582 | 0.6816 | 0.9124 | 14.7917 | 0.0001 |
| F2578 | 0.8799 | 0.047 | 0.7538 | 0.946 | 20.0146 | <. 0001 |
| F256 | 0.8358 | 0.0572 | 0.6923 | 0.9201 | 15.256 | <. 0001 |
| F2568 | 0.8876 | 0.0454 | 0.764 | 0.9507 | 20.6237 | <. 0001 |
| F2567 | 0.881 | 0.047 | 0.7546 | 0.9469 | 19.9413 | <. 0001 |
| F25678 | 0.9199 | 0.0356 | 0.8166 | 0.9673 | 25.5121 | <. 0001 |
| F24 | 0.6993 | 0.0572 | 0.5771 | 0.7985 | 9.6245 | 0.0019 |
| F248 | 0.783 | 0.056 | 0.6542 | 0.8731 | 15.1751 | <. 0001 |
| F247 | 0.7717 | 0.0554 | 0.646 | 0.8623 | 14.9867 | 0.0001 |
| F2478 | 0.8399 | 0.049 | 0.7198 | 0.9146 | 20.7102 | <. 0001 |
| F246 | 0.7847 | 0.0558 | 0.656 | 0.8744 | 15.3228 | <. 0001 |
| F2468 | 0.8497 | 0.0479 | 0.7304 | 0.9219 | 21.2868 | <. 0001 |
| F2467 | 0.8412 | 0.0478 | 0.7242 | 0.9145 | 21.6784 | <. 0001 |
| F24678 | 0.8915 | 0.0386 | 0.7899 | 0.9473 | 27.8722 | <. 0001 |
| F245 | 0.8421 | 0.0502 | 0.718 | 0.9178 | 19.69 | <. 0001 |
| F2458 | 0.8922 | 0.0412 | 0.7814 | 0.9504 | 24.3685 | <. 0001 |
| F2457 | 0.8857 | 0.0421 | 0.7743 | 0.946 | 24.2409 | <. 0001 |
| F24578 | 0.9232 | 0.0327 | 0.8295 | 0.9674 | 29.032 | <. 0001 |
| F2456 | 0.8931 | 0.041 | 0.7825 | 0.951 | 24.3898 | <. 0001 |
| F24568 | 0.9284 | 0.0314 | 0.8371 | 0.9703 | 29.4283 | <. 0001 |
| F24567 | 0.9239 | 0.0322 | 0.832 | 0.9675 | 29.7474 | <. 0001 |
| F245678 | 0.9496 | 0.0238 | 0.8768 | 0.9804 | 34.9032 | <. 0001 |
| F23 | 0.7473 | 0.072 | 0.5835 | 0.8619 | 8.0877 | 0.0045 |
| F238 | 0.821 | 0.0624 | 0.6661 | 0.9134 | 12.8548 | 0.0003 |
| F237 | 0.8113 | 0.0653 | 0.6506 | 0.9084 | 11.6758 | 0.0006 |
| F2378 | 0.8696 | 0.0528 | 0.7282 | 0.9432 | 16.6336 | <. 0001 |
| F236 | 0.8225 | 0.0627 | 0.6664 | 0.9149 | 12.759 | 0.0004 |
| F2368 | 0.8779 | 0.0499 | 0.7427 | 0.9471 | 17.9451 | <. 0001 |
| F2367 | 0.8707 | 0.0522 | 0.7307 | 0.9436 | 16.909 | <. 0001 |
| F23678 | 0.9127 | 0.0396 | 0.7979 | 0.9651 | 22.3083 | <. 0001 |
| F235 | 0.8714 | 0.0571 | 0.7139 | 0.9485 | 14.087 | 0.0002 |
| F2358 | 0.9132 | 0.0435 | 0.7819 | 0.9686 | 18.3554 | <. 0001 |
| F2357 | 0.9079 | 0.046 | 0.7703 | 0.9666 | 17.3023 | <. 0001 |
| F23578 | 0.9386 | 0.0338 | 0.829 | 0.9797 | 21.6643 | <. 0001 |
| F2356 | 0.914 | 0.0435 | 0.7822 | 0.9692 | 18.2282 | <. 0001 |
| F23568 | 0.9428 | 0.0317 | 0.839 | 0.9812 | 22.7397 | <. 0001 |


| F23567 | 0.9392 | 0.0334 | 0.8305 | 0.9799 | 21.8447 | <. 0001 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F235678 | 0.9599 | 0.0237 | 0.8772 | 0.9877 | 26.4756 | <. 0001 |
| F234 | 0.8291 | 0.0644 | 0.6656 | 0.9221 | 12.0646 | 0.0005 |
| F2348 | 0.8828 | 0.0514 | 0.7398 | 0.9522 | 16.5131 | <. 0001 |
| F2347 | 0.8758 | 0.0534 | 0.7293 | 0.9486 | 15.8285 | <. 0001 |
| F23478 | 0.9163 | 0.0406 | 0.795 | 0.9686 | 20.431 | <. 0001 |
| F2346 | 0.8838 | 0.0508 | 0.7427 | 0.9525 | 16.8428 | <. 0001 |
| F23468 | 0.9219 | 0.0382 | 0.8066 | 0.9709 | 21.6204 | <. 0001 |
| F23467 | 0.9171 | 0.0397 | 0.7989 | 0.9685 | 21.1649 | <. 0001 |
| F234678 | 0.9449 | 0.0289 | 0.8522 | 0.9808 | 26.1175 | <. 0001 |
| F2345 | 0.9175 | 0.0405 | 0.7957 | 0.9695 | 20.2372 | <. 0001 |
| F23458 | 0.9452 | 0.0299 | 0.8477 | 0.9817 | 24.3467 | <. 0001 |
| F23457 | 0.9418 | 0.0314 | 0.8406 | 0.9802 | 23.6998 | <. 0001 |
| F234578 | 0.9617 | 0.0225 | 0.8836 | 0.9881 | 27.8834 | <. 0001 |
| F23456 | 0.9458 | 0.0295 | 0.8494 | 0.9818 | 24.6278 | <. 0001 |
| F234568 | 0.9644 | 0.0211 | 0.8906 | 0.989 | 28.9558 | <. 0001 |
| F234567 | 0.962 | 0.0221 | 0.8856 | 0.9881 | 28.556 | <. 0001 |
| F2345678 | 0.9752 | 0.0155 | 0.9182 | 0.9928 | 32.9761 | <. 0001 |
| F1 | 0.5974 | 0.0392 | 0.5189 | 0.6712 | 5.8734 | 0.0154 |
| F18 | 0.6972 | 0.0494 | 0.5928 | 0.7845 | 12.7203 | 0.0004 |
| F17 | 0.6833 | 0.0515 | 0.5751 | 0.7747 | 10.4465 | 0.0012 |
| F178 | 0.77 | 0.0513 | 0.6549 | 0.8552 | 17.4048 | <. 0001 |
| F16 | 0.6993 | 0.0508 | 0.5915 | 0.7888 | 12.1898 | 0.0005 |
| F168 | 0.783 | 0.0492 | 0.6716 | 0.8643 | 19.6185 | <. 0001 |
| F167 | 0.7717 | 0.0506 | 0.6581 | 0.8559 | 17.9661 | <. 0001 |
| F1678 | 0.8399 | 0.044 | 0.7341 | 0.9088 | 25.6208 | <. 0001 |
| F15 | 0.7728 | 0.0672 | 0.6164 | 0.8781 | 10.2341 | 0.0014 |
| F158 | 0.8407 | 0.057 | 0.6963 | 0.924 | 15.2856 | <. 0001 |
| F157 | 0.8318 | 0.0597 | 0.6817 | 0.9195 | 14.0191 | 0.0002 |
| F1578 | 0.8847 | 0.0474 | 0.7553 | 0.9502 | 19.2269 | <. 0001 |
| F156 | 0.8421 | 0.0573 | 0.6961 | 0.9254 | 15.0787 | 0.0001 |
| F1568 | 0.8922 | 0.0449 | 0.7682 | 0.9538 | 20.5068 | <. 0001 |
| F1567 | 0.8857 | 0.047 | 0.7572 | 0.9506 | 19.4454 | <. 0001 |
| F15678 | 0.9232 | 0.0352 | 0.8196 | 0.9695 | 25.0716 | <. 0001 |
| F14 | 0.7089 | 0.0588 | 0.5822 | 0.8097 | 9.7686 | 0.0018 |
| F148 | 0.7907 | 0.0559 | 0.6609 | 0.8799 | 15.4949 | <. 0001 |
| F147 | 0.7798 | 0.0566 | 0.6498 | 0.8711 | 14.7048 | 0.0001 |
| F1478 | 0.846 | 0.0489 | 0.7248 | 0.9197 | 20.616 | <. 0001 |


| F146 | 0.7924 | 0.055 | 0.6647 | 0.8802 | 16.0702 | <. 0001 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F1468 | 0.8555 | 0.0466 | 0.7389 | 0.9253 | 22.2897 | <. 0001 |
| F1467 | 0.8473 | 0.0472 | 0.7306 | 0.919 | 22.024 | <. 0001 |
| F14678 | 0.8959 | 0.0376 | 0.7961 | 0.95 | 28.4784 | <. 0001 |
| F145 | 0.8481 | 0.0525 | 0.7151 | 0.9255 | 17.7753 | <. 0001 |
| F1458 | 0.8965 | 0.0421 | 0.7807 | 0.9547 | 22.6533 | <. 0001 |
| F1457 | 0.8903 | 0.0436 | 0.7717 | 0.9512 | 21.9545 | <. 0001 |
| F14578 | 0.9264 | 0.0333 | 0.8287 | 0.9704 | 26.938 | <. 0001 |
| F1456 | 0.8974 | 0.0416 | 0.783 | 0.955 | 23.0293 | <. 0001 |
| F14568 | 0.9314 | 0.0314 | 0.8384 | 0.9726 | 28.2258 | <. 0001 |
| F14567 | 0.9271 | 0.0325 | 0.832 | 0.9703 | 27.9134 | <. 0001 |
| F145678 | 0.9518 | 0.0237 | 0.8775 | 0.9819 | 33.244 | <. 0001 |
| F13 | 0.7559 | 0.0726 | 0.5889 | 0.87 | 8.262 | 0.004 |
| F138 | 0.8277 | 0.0619 | 0.6724 | 0.9183 | 13.0841 | 0.0003 |
| F137 | 0.8182 | 0.0655 | 0.655 | 0.9143 | 11.6648 | 0.0006 |
| F1378 | 0.8748 | 0.0522 | 0.7331 | 0.9467 | 16.6574 | <. 0001 |
| F136 | 0.8291 | 0.0616 | 0.6743 | 0.9192 | 13.2028 | 0.0003 |
| F1368 | 0.8828 | 0.0485 | 0.7502 | 0.9497 | 18.5272 | <. 0001 |
| F1367 | 0.8758 | 0.0513 | 0.7368 | 0.9468 | 17.157 | <. 0001 |
| F13678 | 0.9163 | 0.0386 | 0.8034 | 0.967 | 22.6694 | <. 0001 |
| F135 | 0.8765 | 0.0579 | 0.7132 | 0.953 | 13.4156 | 0.0002 |
| F1358 | 0.9168 | 0.0436 | 0.7824 | 0.9712 | 17.634 | <. 0001 |
| F1357 | 0.9117 | 0.0464 | 0.7695 | 0.9696 | 16.436 | <. 0001 |
| F13578 | 0.9412 | 0.0337 | 0.8292 | 0.9814 | 20.7497 | <. 0001 |
| F1356 | 0.9175 | 0.0433 | 0.7836 | 0.9716 | 17.6926 | <. 0001 |
| F13568 | 0.9452 | 0.0313 | 0.8407 | 0.9826 | 22.1988 | <. 0001 |
| F13567 | 0.9418 | 0.0332 | 0.8314 | 0.9815 | 21.0925 | <. 0001 |
| F135678 | 0.9617 | 0.0234 | 0.8784 | 0.9887 | 25.7154 | <. 0001 |
| F134 | 0.8356 | 0.0649 | 0.668 | 0.9277 | 11.8263 | 0.0006 |
| F1348 | 0.8875 | 0.0511 | 0.7431 | 0.9556 | 16.2797 | <. 0001 |
| F1347 | 0.8808 | 0.0536 | 0.731 | 0.9526 | 15.3583 | <. 0001 |
| F13478 | 0.9198 | 0.0403 | 0.7973 | 0.9709 | 19.9655 | <. 0001 |
| F1346 | 0.8884 | 0.0502 | 0.747 | 0.9555 | 16.7893 | <. 0001 |
| F13468 | 0.9251 | 0.0374 | 0.8107 | 0.9727 | 21.6303 | <. 0001 |
| F13467 | 0.9205 | 0.0392 | 0.8019 | 0.9707 | 20.8655 | <. 0001 |
| F134678 | 0.9473 | 0.0284 | 0.8551 | 0.982 | 25.8775 | <. 0001 |
| F1345 | 0.921 | 0.0412 | 0.7934 | 0.9725 | 18.8094 | <. 0001 |
| F13458 | 0.9476 | 0.03 | 0.8468 | 0.9834 | 22.93 | <. 0001 |


| F13457 | 0.9442 | 0.0317 | 0.8387 | 0.9822 | 22.0445 | <. 0001 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F134578 | 0.9633 | 0.0225 | 0.8827 | 0.9892 | 26.2405 | <.0001 |
| F13456 | 0.9481 | 0.0296 | 0.8491 | 0.9834 | 23.3794 | <. 0001 |
| F134568 | 0.9659 | 0.0209 | 0.8909 | 0.9899 | 27.7385 | <. 0001 |
| F134567 | 0.9637 | 0.0221 | 0.8852 | 0.9892 | 27.0366 | <. 0001 |
| F1345678 | 0.9763 | 0.0153 | 0.9183 | 0.9934 | 31.4904 | <. 0001 |
| F12 | 0.6777 | 0.0518 | 0.5691 | 0.7699 | 9.8161 | 0.0017 |
| F128 | 0.7654 | 0.0534 | 0.6456 | 0.8538 | 15.8293 | <.0001 |
| F127 | 0.7535 | 0.0544 | 0.6324 | 0.8445 | 14.5268 | 0.0001 |
| F1278 | 0.8259 | 0.0492 | 0.708 | 0.9027 | 20.6874 | <.0001 |
| F126 | 0.7672 | 0.0523 | 0.6498 | 0.854 | 16.5636 | <. 0001 |
| F1268 | 0.8364 | 0.0465 | 0.7244 | 0.9086 | 23.0996 | <. 0001 |
| F1267 | 0.8273 | 0.0473 | 0.7146 | 0.9016 | 22.3893 | <.0001 |
| F12678 | 0.8814 | 0.0389 | 0.782 | 0.939 | 29.1236 | <. 0001 |
| F125 | 0.8282 | 0.0598 | 0.6789 | 0.9166 | 13.9871 | 0.0002 |
| F1258 | 0.8821 | 0.0483 | 0.7507 | 0.9489 | 18.7989 | <.0001 |
| F1257 | 0.8751 | 0.0504 | 0.7396 | 0.9453 | 17.8584 | <.0001 |
| F12578 | 0.9158 | 0.0386 | 0.8032 | 0.9666 | 22.7871 | <.0001 |
| F1256 | 0.8831 | 0.0477 | 0.7532 | 0.9492 | 19.1229 | <. 0001 |
| F12568 | 0.9214 | 0.0362 | 0.8149 | 0.969 | 24.2652 | <. 0001 |
| F12567 | 0.9165 | 0.0377 | 0.8068 | 0.9665 | 23.5984 | <. 0001 |
| F125678 | 0.9446 | 0.0276 | 0.8583 | 0.9796 | 28.8909 | <. 0001 |
| F124 | 0.7753 | 0.0577 | 0.6432 | 0.8685 | 13.9726 | 0.0002 |
| F1248 | 0.8426 | 0.0508 | 0.7165 | 0.919 | 19.1964 | <. 0001 |
| F1247 | 0.8338 | 0.0513 | 0.7083 | 0.912 | 18.9854 | <.0001 |
| F12478 | 0.8861 | 0.042 | 0.775 | 0.9462 | 24.3427 | $<.0001$ |
| F1246 | 0.8439 | 0.0489 | 0.7231 | 0.918 | 20.645 | <.0001 |
| F12468 | 0.8935 | 0.0395 | 0.7882 | 0.9498 | 26.2751 | <.0001 |
| F12467 | 0.8872 | 0.04 | 0.7824 | 0.945 | 26.6924 | <.0001 |
| F124678 | 0.9242 | 0.0307 | 0.8377 | 0.9665 | 32.4797 | <.0001 |
| F1245 | 0.8878 | 0.0441 | 0.7687 | 0.9496 | 21.8555 | <.0001 |
| F12458 | 0.9247 | 0.0341 | 0.8248 | 0.9697 | 26.3058 | <.0001 |
| F12457 | 0.92 | 0.0352 | 0.8183 | 0.9671 | 26.0885 | <.0001 |
| F124578 | 0.9469 | 0.0262 | 0.8654 | 0.9802 | 30.6336 | <.0001 |
| F12456 | 0.9254 | 0.0332 | 0.8285 | 0.9695 | 27.4008 | <.0001 |
| F124568 | 0.9506 | 0.0245 | 0.8738 | 0.9816 | 32.1476 | <. 0001 |
| F124567 | 0.9474 | 0.0253 | 0.8694 | 0.9799 | 32.3682 | <.0001 |
| F1245678 | 0.9655 | 0.0182 | 0.9056 | 0.9879 | 37.2192 | <.0001 |


| F123 | 0.8144 | 0.064 | 0.6569 | 0.9095 | 12.2124 | 0.0005 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| F1238 | 0.8719 | 0.052 | 0.7322 | 0.9443 | 16.9826 | $<.0001$ |
| F1237 | 0.8645 | 0.0547 | 0.7186 | 0.9409 | 15.7438 | $<.0001$ |
| F12378 | 0.9082 | 0.0421 | 0.7863 | 0.9638 | 20.6237 | $<.0001$ |
| F1236 | 0.873 | 0.0509 | 0.7364 | 0.9442 | 17.6109 | $<.0001$ |
| F12368 | 0.9143 | 0.0389 | 0.8014 | 0.9658 | 22.7811 | $<.0001$ |
| F12367 | 0.9091 | 0.0409 | 0.7914 | 0.9634 | 21.7047 | $<.0001$ |
| F123678 | 0.9394 | 0.03 | 0.8465 | 0.9776 | 27.012 | $<.0001$ |
| F1235 | 0.9096 | 0.0459 | 0.7711 | 0.9678 | 17.1116 | $<.0001$ |
| F12358 | 0.9398 | 0.0337 | 0.8291 | 0.9805 | 21.242 | $<.0001$ |
| F12357 | 0.936 | 0.0358 | 0.8195 | 0.9792 | 20.205 | $<.0001$ |
| F123578 | 0.9578 | 0.0255 | 0.868 | 0.9874 | 24.4084 | $<.0001$ |
| F12356 | 0.9403 | 0.0332 | 0.8316 | 0.9805 | 21.6776 | $<.0001$ |
| F123568 | 0.9607 | 0.0236 | 0.8776 | 0.9882 | 26.0614 | $<.0001$ |
| F123567 | 0.9582 | 0.025 | 0.8708 | 0.9873 | 25.172 | $<.0001$ |
| F1235678 | 0.9726 | 0.0174 | 0.9077 | 0.9923 | 29.6485 | $<.0001$ |
| F1234 | 0.8781 | 0.0532 | 0.7311 | 0.9502 | 15.7881 | $<.0001$ |
| F12348 | 0.9178 | 0.0406 | 0.7957 | 0.9698 | 20.1395 | $<.0001$ |
| F12347 | 0.9128 | 0.0424 | 0.7867 | 0.9674 | 19.459 | $<.0001$ |
| F123478 | 0.942 | 0.0311 | 0.8416 | 0.9803 | 23.9185 | $<.0001$ |
| F12346 | 0.9186 | 0.0394 | 0.8008 | 0.9694 | 21.1647 | $<.0001$ |
| F123468 | 0.946 | 0.0288 | 0.8531 | 0.9814 | 25.835 | $<.0001$ |
| F123467 | 0.9425 | 0.0301 | 0.8468 | 0.9799 | 25.3998 | $<.0001$ |
| F1234678 | 0.9622 | 0.0214 | 0.8892 | 0.9878 | 30.1964 | $<.0001$ |
| F12345 | 0.9429 | 0.0316 | 0.8393 | 0.9812 | 22.8169 | $<.0001$ |
| F123458 | 0.9624 | 0.0227 | 0.8822 | 0.9887 | 26.7389 | $<.0001$ |
| F123457 | 0.96 | 0.0239 | 0.8764 | 0.9878 | 26.1093 | $<.0001$ |
| F1234578 | 0.9738 | 0.0168 | 0.9109 | 0.9927 | 30.0972 | $<.0001$ |
| F123456 | 0.9628 | 0.0222 | 0.885 | 0.9886 | 27.6623 | $<.0001$ |
| F1234568 | 0.9757 | 0.0155 | 0.9175 | 0.9931 | 31.8006 | $<.0001$ |
| F1234567 | 0.9741 | 0.0163 | 0.9136 | 0.9926 | 31.4057 | $<.0001$ |
| F12345678 | 0.9832 | 0.0113 | 0.9388 | 0.9955 | 35.623 | $<.0001$ |
|  |  |  |  |  |  |  |

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