

A NUMERICAL MODEL TO SIMULATE THE BEHAVIOR
OF REINFORCED CONCRETE MEMBERS SUBJECTED TO BIAXIAL
EARTHQUAKE EXCITATION

A Thesis

Submitted to the Faculty

of

Purdue University

by

Cemalettin Donmez

In Partial Fulfillment of the

Requirements for the Degree

of

Doctor of Philosophy

December 1998

PURDUE UNIVERSITY
GRADUATE SCHOOL
Thesis Acceptance

This is to certify that the thesis prepared

By CEMALETTIN DONMEZ

Entitled

A NUMERICAL MODEL SIMULATE THE BEHAVIOR OF REINFORCED CONCRETE
MEMBERS SUBJECTED TO BIAXIAL EARTHQUAKE EXCITATION

Complies with University regulations and meets the standards of the Graduate School for originality
and quality

For the degree of DOCTOR OF PHILOSOPHY

Signed by the final examining committee:

Mark A. Green, chair
W. J. Chen
Elisa Bruggi Litali
Rodrigo Sepedo

Approved by:

Vincent P. D'Amico
Department Head

Sept. 14, 1998
Date

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Mark A. Green

Major Professor

Format Approved by:

Chair, Final Examining Committee

or

Rita Pritchett
Thesis Format Adviser

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To all those who brought me to these days

ACKNOWLEDGEMENTS

I gratefully acknowledge the financial support given by İzmir Institute of Technology and Higher Education Council (YÖK) of Turkey.

I am deeply indebted to my supervisor Professor M. A. Sozen for all the endless stimulating discussions, encouraging enthusiasm, and the time and the effort he put not only in technical matters but also in intellectual detailing of the writer.

Special recognition is given to Professors Wai-Fah Chen, Elisa D. Sotelino, and Rodrigo Salgado for serving in the advisory committee.

The writer would also like to acknowledge the valuable discussions and contributions of his dear friends JoAnn Browning, Adolfo Matamoros, Santiago Pujol and Koray Türeyyen . The assistance given by Linda Vail is also greatly appreciated.

Finally, I would like to offer my deepest appreciation to my parents Halil and Birsen, to my sister Ferda and my friends Sibel Sayili and Behçet Açıkmeşe for their limitless support and encouragement throughout the course of the study.

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ABSTRACT

Donmez, Cemalettin Ph.D., Purdue University, December, 1998. A Numerical Model to Simulate the Behavior of Reinforced Concrete Members Subjected to Biaxial Earthquake Excitation. Major Professor: Mete A. Sozen.

Constitutive models are developed to define the dynamic response of concrete and steel reinforcement subjected to cyclic loading varying randomly in magnitude. The models are combined to determine the behavior of reinforced concrete systems subjected to biaxial excitation. The composite model is tested using data from static and dynamic tests. Comparisons with the measurements showed that the proposed algorithm is a reliable vehicle for modeling dynamic response at levels of excitation ranging from low (less than yield) to high excursions. The sensitivity of the proposed algorithm was checked. The algorithm was found to be insensitive to the changes in input parameters related to concrete and sensitive to input parameters related to reinforcing steel. A numerical study was also performed to study the nonlinear response of reinforced concrete structures to identify the differences in their maximum-displacement response subjected to biaxial and uniaxial base excitations. Single-mass systems were analyzed using the proposed algorithm. Initial stiffness, base shear strength, and earthquake frequency content were the main variables. Results indicated that maximum-displacement ratio for biaxial base excitation to uniaxial base excitation remained below 1.5, with most of the results around 1.0, provided that a base shear requirement was satisfied.

1. INTRODUCTION

1.1 Object and Scope

The primary objective of the study is the development of a numerical model for response analysis of reinforced concrete members subjected to biaxial ground excitation.

The secondary objective is to study the response of reinforced concrete structures, for identifying the differences in their response subjected to biaxial and uniaxial ground excitations.

To calculate the nonlinear dynamic response of reinforced concrete, it is necessary to have a set of rules, an algorithm, that defines the changes in incremental stiffness at all levels and directions of response. The Takeda hysteresis model (1970) is one such algorithm that has been shown to simulate successfully the response of reinforced concrete elements subjected to moment reversals about one axis. The algorithm presented in this report has been developed to determine changes in stiffness for reinforced concrete elements subjected to moment reversals about two orthogonal axes. The proposed algorithm is based on a model that represents the reinforced concrete section as a collection of nonlinear springs. The hysteretic properties of the springs are based on observed behavior of concrete and steel in uniaxial loading. The nonlinear uniaxial springs representing these two materials working together give the section its composite characteristics. The selected properties of the material hystereses effect changes in incremental stiffness and hysteretic damping.

The algorithm is inherently capable of simulating constant or variable axial load. In this study, the cases considered have involved flexure only. It has been assumed that the element is not vulnerable to shear failure. The results of the algorithm have been tested using measurements obtained in static and dynamic tests. The concrete strength in the tests varied between 3,000 and 7,500 psi and steel yield strength between 59,000 to 68,000 psi.

1.2 Organization

Chapter 1 states the object and the scope of the study. Chapter 2 includes a detailed description of the material hystereses models for concrete and reinforcing steel used in the algorithm, an explanation of numerical procedures used for calculation of the moment-deflection relation and descriptions of the member deflection components.

In Chapter 3, the performance of the proposed hysteresis algorithm is tested using experimental data from static and dynamic tests. The algorithm defines the force-displacement relationship of a reinforced concrete element by combining the effect of rules defining the behavior under load reversals of the concrete and the steel. In the nonlinear range of response, rules are also invoked defining the spread of the plastic region along the span of the beam and slip of the reinforcement. The correctness of the combined results of these rules are tested directly using data from static tests of reinforced concrete elements subjected to biaxial bending. The reliability of the use of the algorithm in dynamic modeling for nonlinear response is evaluated using data from dynamic experiments.

The sensitivity of the results of the proposed algorithm to changes in its input parameters is evaluated in Chapter 4. The study is focused on parameters defining material behavior and rules for spreading of the plastic region.

The first four chapters focus on the first objective of the study. Chapter 5 addresses the second objective. Maximum-Displacement response of the reinforced concrete subjected to biaxial base excitation is the subject of the chapter. A series of single-mass systems are analyzed using the proposed algorithm in Chapter 2. The main variables in the study are stiffness and strength of the systems as well as different earthquake intensity and frequency content.

Summary and conclusions are presented in Chapter 6.

2. A MODEL TO COMPUTE CYCLIC RESPONSE OF REINFORCED CONCRETE MEMBERS SUBJECTED TO BIAXIAL FORCES

2.1 Introduction

Three different types of models have been used by previous investigators to define response of reinforced concrete sections subjected to biaxial moment reversals. They differ in the way the biaxial response is defined, as indicated below:

1. Definition of biaxial response based on uniaxial-bending response (Colina 1992, Takizawa and Aoyama 1976).
2. Implicit definition of section behavior by rules for yielding and for loading/unloading (Chen and Powell 1982, Sfakianakis and Fardis 1991, Ricles et al. 1998).
3. Representing the section by a set of nonlinear springs distributed uniformly (Aktan et al. 1973, Zeris 1986, Kitajima et al. 1991, Taucer et al. 1991) or concentrated at specific locations over the section (Suharwardy and Pecknold 1978, Saiidi et al. 1989, Lai 1987, Jiang and Saiidi (1990).

Detailed reviews of the previous studies on cyclic biaxial response of reinforced concrete members can be found in Colina (1992) and Zeris (1986).

The model proposed in this study can be categorized to be of Type 3. The design of the model components was initiated with the conviction that its elements should be as simple as they could be. At the beginning, concrete was modeled by the Ritter parabola and a linear extension, similar to Hognestad's model (1951). The unloading path was

defined to be linear with its slope based on the observations by Karsan and Jirsa (1969). Loading used the unloading path. Pilot studies indicated that this failed to simulate observed response because crack closing was not as simple a phenomenon as reflected in using the same path in loading and unloading. To respond to that demand, the loading slope was modified to recognize the fact that the crack closed (or that compressive stress redeveloped) at a strain lower than that at which the return path crossed the strain axis. Similarly, the reinforcement was initially modeled very simply as a bilinear curve with three parameters defining the initial and secondary slopes and the yield stress. When it was observed that this failed to model energy dissipation at low amplitudes, a second model for steel based on the Menegotto-Pinto equation (Menegotto and Pinto, 1973) was used. At every step of progress, nothing was added to the basic model unless it appeared to be required by the phenomena modeled.

In the proposed model, the section is replaced by a set of nonlinear springs distributed uniformly (Fig. 2.1). Even though the chosen model is computationally more expensive than Type-1 and Type-2, it is a convenient and flexible way of modeling the nonlinear behavior of reinforced concrete under generalized loading. The variety of physical phenomena associated with flexural behavior of reinforced concrete can be easily defined through this model. Crack opening, crack closing, softening of concrete, the Bauschinger effect, permanent strains of reinforcing steel, and spreading of the plastic hinge region along the member can all be modeled conveniently. The model is capable of representing pinching of hysteresis loops, strength/stiffness interaction about orthogonal axes of the members, and the effect of axial load on strength/stiffness dependence.

Throughout the text, several special definitions are used. Section 2.2 contains a list of these definitions in alphabetical order. In sections 2.3 and 2.4, the proposed concrete and the steel hysteresis models are described. Section 2.5 presents the detailed explanation of the numerical procedure used for the calculation of incremental moments for given incremental deflections.

2.2 Definitions Used To Describe Hysteresis Curves

The hysteresis curves are used to define the “travel” of a point in the stress-strain plane. The point is located by a pair of Cartesian coordinates. Explanation of how this travel is conducted in the stress-strain plane requires several special definitions. These are listed below to facilitate the descriptions that follow in sections 2.3 for concrete and 2.4 for the reinforcement.

Asymptote: A straight line defining the limit of the stress-strain curve for steel at infinite strain. The position of this line changes depending on history.

Current Position: The position on the stress-strain plane defined by deformation and force coordinates at the beginning of the calculation for a new position.

Envelope Curve: A curve defining the stress limit for compressed concrete at different strains. It is, in effect, the stress-strain curve for concrete subjected to monotonically increasing compressive strain. The envelope curve is assumed to be independent of history.

Fiber: A material element (steel or concrete) representing a small area of the cross section.

Flag: A memory location used to contain information about the state of a fiber.

History: The stress and strain coordinates through which the fiber has traveled.

Incremental Stiffness: The secant stiffness of the fiber at the current position.

Loading: Travel away from the strain axis. For concrete, loading includes travel along the strain axis with increasing strain.

Loading History Parameters: The parameters that hold the information on past extremes of the loading path and the states of the materials, necessary for definition of the travel path for the new step.

Position of Maximum Excursion on the Envelope Curve: The extreme position reached on the envelope curve for compressed concrete.

Nonstationary: Refers to curves that change their position on the stress-strain plane depending on history.

Position: A point on the stress-strain plane.

Primary Curve: A straight line defined by E_s , Young's modulus for steel.

Refresh: Changing the stress-strain, state, and the loading history parameters to the values at the end of the calculation step.

Reversal: Change from loading to unloading or vice versa

Reversal Point: The position of the reversal.

Secondary Curve: A straight line defined by STH, strain-hardening slope for steel.

Secondary Stiffness: The tangential slope of the bounding curve.

State of Complete Contact: Complete closing of the crack in the concrete.

State of Cracking: Indicates (by a flag) whether the concrete fiber has ever been cracked.

State of Incomplete Contact: Bearing attributed to regions of the cracked surface that make contact before the fiber strain reaches the strain at which the crack formed.

Stationary: Refers to curves that do not change their positions on the stress-strain plane depending on history

Transition: Change in stiffness from the primary stiffness to that controlled by the asymptote.

Travel: Change of position on the stress-strain plane.

Unloading: Travel toward the strain axis, for concrete unloading includes travel along the strain axis with decreasing strain.

2.3 Concrete Hysteresis Model

At start, the concrete hysteresis model was adopted from Dradjat (1983). Figure 2.2 illustrates the general features of the Dradjat model. The limits of compressive-stress

response are bounded by the Ritter parabola up to the peak stress and a straight line joining the peak stress coordinates to a point on the strain axis (Hognestad, 1951).

The rules for the bounds to the stress-strain curve for concrete are:

$$1. \quad f_c = 0.0 \quad \varepsilon_c < \varepsilon_t \quad (2.1)$$

$$2. \quad f_c = E_c \varepsilon_c \quad \varepsilon_t \leq \varepsilon_c \leq 0.0 \quad (2.2)$$

$$3. \quad f_c = f_c' \left[2 \frac{\varepsilon_c}{\varepsilon_{co}} - \left(\frac{\varepsilon_c}{\varepsilon_{co}} \right)^2 \right] \quad 0.0 < \varepsilon_c \leq \varepsilon_{co} \quad (2.3)$$

$$4. \quad f_c = f_c' [1 - \Omega(\varepsilon_c - \varepsilon_{co})] \quad \varepsilon_{co} < \varepsilon_c \leq \varepsilon_{cx} \quad (2.4)$$

$$\varepsilon_{cx} = 7\varepsilon_{co} \quad (2.4a)$$

$$\Omega = \frac{1}{(6\varepsilon_{co})} \quad (2.4b)$$

$$5) \quad f_c = 0.0 \quad \varepsilon_{cx} < \varepsilon_c \quad (2.5)$$

where,

$$\varepsilon_t = \frac{f_t}{E_c}, \quad \text{Tensile strain corresponding to maximum tensile stress.}$$

$$\varepsilon_{co} = \left(\frac{C_1 f_c'}{E_c} \right), \quad \text{Compressive strain of concrete corresponding to the compressive strength } f_c'. \quad C_1 \text{ is a constant defined to be 2.0 based on experimental data.}$$

f_t = Tensile strength of the concrete, psi.

f_c = Compressive stress of the concrete, psi.

ε_c = Concrete strain.

Ω = Constant, that defines the descending slope of the concrete stress-strain curve.

f_c' = Compressive strength of the concrete, psi.

ε_{cx} = Maximum compressive strain of the concrete.

E_c = Young's Modulus of the concrete, psi.

Equation 2.3 defines the second-degree curve approximation to the concrete stress-strain curve proposed by Ritter (Hognestad, 1951). Equation 2.4 defines the variation of the compressive stress capacity of unconfined concrete beyond the strain, ϵ_{co} . The rules for cyclic loading are also illustrated in Fig. 2.2. Unloading and loading within the envelope curve follows a straight line, connecting the unloading point on the envelope (ϵ_{cen} , f_{cen}) and the loading point (ϵ_{cp} , 0.0). The strain ϵ_{cp} at loading point is computed from:

$$\epsilon_{cp} = \epsilon_{co} \left[\frac{1}{7} \left(\frac{\epsilon_{cen}}{\epsilon_{co}} \right)^2 + \frac{1}{8} \frac{\epsilon_{cen}}{\epsilon_{co}} \right] \quad (2.6)$$

where,

ϵ_{cp} = Concrete strain on the strain axis for unloading from the position of maximum excursion (Fig. 2.2).

ϵ_{cen} = Position of the maximum excursion on the envelope curve (Fig. 2.2).

Equation 2.6 is a slightly simplified version of the unloading rule proposed by Karsan and Jirsa (1969). The unloading from the envelope curve is illustrated by line AB in Fig. 2.2. If the unloading continues and the concrete has not been previously cracked, travel follows line BC with a slope equal to E_c to position C defined by the tensile strength of the concrete. Further travel will drop the stress to zero to follow path CDE. If the concrete is previously cracked, the unloading path follows BDE. Loading follows the path used in the most recent unloading. However, if the concrete is cracked, the loading path is EDBA. If loading continues beyond the last position on the envelope (ϵ_{cen} , f_{cen}), the loading follows the envelope curve. If compressive strain goes beyond ϵ_{cx} , concrete stress becomes zero.

The concrete surfaces bounding a crack, on being compressed together, may not have a perfect fit. Consequently, raised portions on each bounding surface are likely to make contact before the crack closes completely. Compression is developed before the strain, in this case at point B, is reached. The path connecting E to F in Fig. 2.2 is added to the Dradjat model for simulating the early resistance gain of the concrete during crack closing. Recognizing that there is a range of "incomplete contact" before complete contact occurs, a set of hysteresis rules to simulate this phenomenon was selected. These rules are described in reference to Fig. 2.3. The detailed plot in Fig. 2.3b shows a portion of the stress-strain curve for the concrete (Fig. 2.3a). A point E is defined at a strain $6f_t/E_c$ from point B, which is assumed to be the strain at zero stress for the most recent unloading from the envelope curve. If the calculated strain goes to a strain to the right of point E, upon reversal, the stress-strain relationship follows path EF. The point F is located on the most recent unloading from the envelope curve at a stress of $f_{cen}/5$. If the stress reverses again before reaching point F, unloading is assumed to occur along line EF.

If, after unloading to point B, there is a stress reversal before reaching point E, the loading curve makes a beeline for point F, as illustrated by line E'F. Again, if there is another reversal within line E'F, unloading occurs along line E'F. If point F is reached during loading, unloading occurs along line AB.

Details of the algorithm for concrete hysteresis are given in Appendix A.1. A representative measured stress-strain curve for concrete under cyclic loading is shown in Fig. 2.4, and compared with the curve based on the proposed concrete model.

2.4 Models for Hysteresis of Reinforcing Steel

2.4.1 Steel Hysteresis Model 1: The Bilinear Model

The bilinear steel hysteresis is the simplest hysteresis rule available for a yielding material. In order to simulate the Bauschinger effect, the kinematic hardening rule is chosen for the model. The following rules are defined with reference to Fig. 2.5.

1. If $|\epsilon| \leq |\epsilon_y|$ and the fiber has not yielded, both loading and unloading follows the primary curve, which has a slope of E , Young's modulus of the steel.
2. After first yield, loading follows the secondary curve. If fiber unloads on the secondary curve, point B_1 , it has a slope of E , for $2\epsilon_y$ strain in unloading direction, $|B_1B_3|$. Beyond B_3 , the slope changes to STH . If unloading is from an intermediate position, B_2 , it has a slope of E to point B_3 . The same rules apply for the other direction.

where:

ϵ = Current strain of the steel.

ϵ_y = Yield strain of the steel.

STH = Slope of the secondary curve (strain hardening).

The bilinear hysteresis model is not effective in modeling hysteretic energy dissipation for small load cycles. The reason is the constant stiffness used to define travel between the two secondary curves. If the loading keeps cycling between the secondary curves, no energy is dissipated. Details of the algorithm for the bilinear model are given in Appendix A.2.

2.4.2 Steel Hysteresis Model 2: A Model Based on the Menegotto-Pinto Curves

Variable-Parameter models have been used by many researchers to simulate steel hysteresis (Kent 1969, Menegotto-Pinto 1973, Aktan 1973, Thompson 1975, Stanton and McNiven 1979). Model 2, adopted for the current study, was a modification of the one developed by Stanton and McNiven. Their model was based on the Menegotto-Pinto (M-P) curves. A typical M-P curve is illustrated in Fig. 2.6. The formulation is:

$$\sigma^* = b\varepsilon^* + \frac{(1-b)\varepsilon^*}{(1+\varepsilon^{*R})^{\left(\frac{1}{R}\right)}} \quad (2.7)$$

where,

$$\varepsilon^* = \frac{\varepsilon - \varepsilon_r}{\varepsilon_o - \varepsilon_r}, \quad \sigma^* = \frac{\sigma - \sigma_r}{\sigma_o - \sigma_r}$$

σ = Steel stress.

ε_r, σ_r = Strain and stress coordinates at the reversal point (Fig. 2.6).

ε_o, σ_o = Strain and stress coordinates at the intersection of the primary curve and the asymptote (Fig. 2.6)

$b = \frac{E_2}{E_1}$, ratio of the slopes of the primary curve and the asymptote.

$E_1 = \frac{\sigma_o - \sigma_r}{\varepsilon_o - \varepsilon_r}$, Slope of the primary stiffness curve.

E_2 = Slope of the asymptote (Fig.2.6).

R = Parameter defining the transition of travel from the primary curve to the asymptote.

There are several advantages of using the M-P curves. A model based on the M-P curves is computationally economical in this application, because stress is computed from

strain. It is explicit. Distinct characteristics of the steel hysteresis can be captured with the parameters of the M-P curves. This permits the manipulation of the parameters independently. In addition, the parameters can be obtained conveniently from steel hysteresis tests (Stanton and McNiven, 1979). Figure 2.7 shows possible shapes for M-P curves.

Before the first reversal, the results of the M-P model are similar to those of the bilinear steel model. If the fiber has not yielded in either direction, travel occurs along the primary curve (Fig A.3a). After the first yield, in either direction, travel occurs along the secondary curve (Fig A.3a). At the first unloading from the secondary curve, travel follows M-P curves. Primary curve, asymptote (secondary curve in Fig. A3.a) and the transition parameter R have to be defined for constructing an M-P curve. The primary curve and the asymptote need two points for their definitions, the point of reversal (ϵ_r, σ_r , Fig. 2.8) and the point of intersection for the primary curve and the asymptote (ϵ_o, σ_o , Fig. 2.8). The point of reversal is the unloading point. Consider the reversal point (ϵ_r, σ_r) in Fig. 2.8. A tangent, with slope E_o , is drawn at point (ϵ_r, σ_r). This tangent crosses the strain axis at ϵ_z . The initial bilinear loading curve is shifted so that its origin is at point ϵ_z . The tangent drawn at point (ϵ_r, σ_r) is extended to intersect with the shifted curve. The point of intersection (ϵ_o, σ_o) becomes the new breakpoint for defining travel in that direction. A new set of bounding curves has to be defined at every reversal. Details of the algorithm used for Steel Model 2 are described in Appendix A.3.

Values of E_o and R are defined by Stanton and McNiven (1979) based on experimental data. The stiffness E_o , is modified to have an initial modulus of 29,000 ksi,

$$E_o = 29000 - 108000\epsilon_r - 1928(1.0 - \exp(-390.0\epsilon_r)), \text{ in ksi} \quad (2.8)$$

$$E_o \geq 22000, \text{ in ksi}$$

Definition of the R is modified by adding constant, C_2 to the original form,

$$R = \frac{r\sigma_y}{C_2} \quad (2.9)$$

where,

$$r = 0.074 - 1.092\varepsilon_{\max c}^p \quad \text{in tension,} \quad (2.9a)$$

$$r = 0.069 - 0.632\varepsilon_{\max p}^p \quad \text{in compression.} \quad (2.9b)$$

ε_r = Strain at the last reversal.

$\varepsilon_{\max t}^p$, $\varepsilon_{\max c}^p$ = The maximum plastic strain in a half loop, in tension and compression.

σ_y = Yield stress of the steel, in ksi.

$C_2 = 17$, based on experimental data obtained by Aktan (1973),

Details of the algorithm for the second steel model are given in App. A.3. A representative experimental result (Aktan, 1973) is plotted in Fig. 2.9 and 2.10, with the corresponding simulations using models 1 and 2. In addition, a hypothetical strain history is used to compare the results of the two models for low-strain cycles (Fig. 2.11). The effect of steel models (relative to each other) on response histories of RC members under dynamic loading will be discussed in Chapter 4.

2.5 Calculation of Incremental Moments for Given Incremental Deflections

This section describes the procedure used to calculate the moment increments in a reinforced concrete cantilever for given incremental deflections at its free end. Calculation of incremental moment for given incremental deflections is not a straightforward procedure. The nonlinear and path dependent character of the assumed material hystereses necessitated an iterative procedure.

Member deflection is considered in three components: deflections attributed to flexure, shear and slip.

For the sake of clarity, this section is organized in five subsections. In the first three subsections, calculations of the deflection components are explained. In section 2.5.4, a step-by-step procedure for calculating the incremental moment for the given incremental deflections is presented. The calculation of moments for the given curvatures are presented in Section 2.5.5.

2.5.1 Calculation of Flexural Component of the Deflection

Flexural deflection is determined using the curvature distribution along the span of the “cantilever column” shown in Fig. 2.13a. The interrelationship of moment and curvature at the section of maximum moment, m_2 and ϕ_2 , is set by the hysteresis algorithm. If m_2 is more than the yield moment, the location of the yield moment, m_1 , in the span is determined directly from statics. The curvature, ϕ_1 , is the yield curvature (Fig. 2.13c). If m_2 increases, the location of m_1 moves toward the free end of the cantilever. The curvature at the section where m_1 occurs is the yield curvature. If m_2 decreases (after having exceeded the yield moment), the “yield region,” the distance between the locations of the moments m_1 and m_2 (Fig. 2.13a), is not reduced but the curvature ϕ_1 is reduced proportionally. If the moment m_1 , is reduced to the level corresponding to the early-contact region (Fig. 2.13d) the rate of reduction depends on the level of contact. A representative summary of the assumed curvature distribution is given in Fig. 2.14 at different stages of loading.

2.5.2 Calculation of the Shear Component of the Deflection

The maximum calculated contribution of the shear deflection to the total calculated deflection was 0.35% of total. The shear deflection was based on a linear, uncracked reinforced concrete member. By this assumption, shear deflection at the free end of a cantilever for a rectangular section is:

$$\Delta_{shear} = \frac{6}{5} \frac{VL}{GA} \quad (2.22)$$

where,

Δ_{shear} : Shear deflection.

V: Shear force at the tip.

L: Length of the element.

A: Cross sectional area of the element.

G: Shear modulus.

$$G = \frac{E_c}{2(1+\nu)} \quad (2.23)$$

E_c : Young's Modulus of concrete.

ν : Poisson ratio assumed to be 1/6.

2.5.3 Calculation of Deflection Component Associated with Slip of the Reinforcing Bars

The slip of the reinforcing bar is estimated by assuming a constant development length (based on the experiments made by Dragovich (1996), 50 bar diameter for coupled and 40 bar diameter for uncoupled members) and a linear decrease of the strain in the bar from the face of the support, up to end of defined development length (Fig. 2.15). The slip is calculated for the reinforcing bar that has the highest strain in the section. Calculated slip is converted to rotation by using the distance of the bar from the neutral

axis (Fig. 2.15). Then the deflection of the cantilever at the free end is calculated by multiplying the rotation due the slip with the length of the member.

2.5.4 Procedure for Calculating the Incremental Moments for Given Incremental Deflections

In the step-by-step procedure of calculating the force-displacement history, the following parameters are obtained from the previous step and are refreshed after reaching convergence in every step:

- Curvature ϕ_Y and ϕ_Z along axes Y and Z (Fig. 2.16).
- Moments M_Y and M_Z along axes Y and Z.
- Length of the “yield region”, d_{yr} .
- Displacements d_Y and d_Z along axes Y and Z.

Step1: Make the first guess for incremental curvatures using the relation between incremental displacement and the curvature for a linearly elastic member.

$$\Delta\phi_Y = \frac{3\Delta d_Z}{L^2} \quad , \quad \Delta\phi_Z = \frac{3\Delta d_Y}{L^2}$$

where,

Δd_Y and Δd_Z = Given incremental deflections at the free end of the member along axes Y and Z.

$\Delta\phi_Y$ and $\Delta\phi_Z$ = First guess for the incremental curvatures.

Step 2: Calculate direction of curvature using the refreshed curvature values with the assumed curvature increments. The direction of deflection has a 90-degree difference with the direction of the curvature (Fig. 2.16).

Step 3: Calculate the incremental moment on the section using axial stress equilibrium.

The procedure for calculating the incremental moments is presented in Section 2.5.5.

Step 4: Calculate the new length for the “yield region” of the member using the incremental moments calculated in Step 3. If the new length is larger than the current length, refresh the length with the new value, otherwise retain the old value. The “yield region” length is calculated using the moment reached with the latest increments and the yield moment in the direction of the current moment.

$$M_{calc} = \sqrt{(M_y + \Delta M_y)^2 + (M_z + \Delta M_z)^2}$$

$$d_{yr} = \frac{(M_{calc} - M_{y0})}{\left(\frac{M_{calc}}{L}\right)}$$

where,

M_{calc} = Magnitude of calculated moment vector.

d_{yr} = “Yield region” length.

M_{y0} = Yield moment in the current direction.

L = Length of the member.

The yield moment in the current direction is calculated by assigning yield strain to the bar that is farthest to the neutral axis (Fig. 2.17). After obtaining the “yield region” length, separate it to its components along axes Y and Z, l_y and l_z .

Step 5: Calculate the moments at the end of the “yield region”, M_{y1} and M_{z1} (Fig. 2.18).

Step 6: Using the simplified moment-curvature relationships (Fig. 2.19) and the moment values obtained in the Step 5, calculate the curvatures at the end of the “yield region”, ϕ_{y1} and ϕ_{z1} .

Step 7: Using the curvature distributions obtained (Fig. 2.13c), calculate the flexural deflections.

Step 8: Calculate the deflection due to the bar slip using procedure given in Sec. 2.5.3.

Step 9: Calculate the deflection due to the shear using procedure given in Sec. 2.5.2.

Step 10: Add the deflection components calculated at Steps 7,8 and 9 to determine the total deflection. Incremental deflection is the difference between the deflection calculated and the deflection at the beginning of the increment.

$$(\Delta d_Y)_{calc} = (d_Y)_{calc} - d_Y$$

$$(\Delta d_Z)_{calc} = (d_Z)_{calc} - d_Z$$

where,

$(\Delta d_Y)_{calc}$ = Calculated deflection increment along the Y-axis.

$(\Delta d_Z)_{calc}$ = Calculated deflection increment along the Z-axis.

$(d_Y)_{calc}$ = Calculated deflection along the Y-axis.

$(d_Z)_{calc}$ = Calculated deflection along the Z-axis.

If the calculated deflection increments are within the convergence limit of the given deflection increments, assumed curvatures and the calculated moments are the values looked for. Otherwise, calculate new curvature increments using the following formulation.

$$(\Delta \phi_Y)_{new} = \frac{\Delta d_Z}{\frac{(\Delta d_Z)_{calc}}{\Delta \phi_Y}}, \quad (\Delta \phi_Z)_{new} = \frac{\Delta d_Y}{\frac{(\Delta d_Y)_{calc}}{\Delta \phi_Z}}$$

Repeat steps 2-10 until acceptable convergence is reached. This procedure requires a series of iterations of assuming a curvature increment and arriving at an incremental displacement. Convergence is assumed to have been achieved if the calculated deflection increment is within 0.001 in. of the incremental deflection. For the time increments used, the typical increment value in the nonlinear range is 0.09 in. The distribution of the curvature along the span is set by the magnitude of the assumed curvature. In each successive iteration, the curvature changes and its distribution may also change. It may be noted that the numerical procedure outlined above give results that are independent of the step size. However, the number of iterations required to predict the correct curvatures and the neutral axis depth to satisfy the axial stress equilibrium increase with larger step size.

2.5.5 Calculation of Moment-Curvature Relation

This section gives the details of the numerical procedure used to compute biaxial moment-curvature relationships for reinforced concrete sections. The cross section is discretized into sub-areas or fibers (Fig. 2.1), each with a specified centroidal coordinate and area. The procedure for calculating the strains over the cross section is adopted from Dragovich (1996).

Assuming linear variation of strain over the cross-section, the strain at the center of a given fiber is related to the position and inclination of the neutral axis (Fig. 2.20). The shortest distance from the fiber to the neutral axis that has the inclination Θ , is $|\vec{D}| + D_o$. The alpha line passes through the section centroid and parallel to the neutral axis. The unit vector $\vec{\alpha}$ is obtained by rotating the x-axis unit vector Θ degrees counterclockwise. The vector $\vec{D}$ is calculated from vector sum of $\vec{R}$,

$$\vec{D} = \vec{R} - \vec{P} \quad (2.10)$$

$\vec{R}$ is known, but $\vec{D}$ is not. $\vec{P}$ is the projection of $\vec{R}$ on the line α and defined as,

$$\{P\} = [Q]\{R\} \quad (2.11)$$

where,

$$\{P\} = \begin{Bmatrix} P_x \\ P_y \end{Bmatrix} \quad : \text{The x- and y-axis components of } \vec{P}.$$

$$\{R\} = \begin{Bmatrix} r_x \\ r_y \end{Bmatrix} \quad : \text{The x- and y-axis components of } \vec{R}.$$

$[Q]$: The projection matrix, and

$$= \begin{Bmatrix} \vec{\alpha} \\ \vec{\alpha} \end{Bmatrix} \begin{Bmatrix} \vec{\alpha} \\ \vec{\alpha} \end{Bmatrix}^T \text{ for } \vec{\alpha} = \begin{Bmatrix} a_x \\ a_y \end{Bmatrix}, \text{ a unit vector (Strang, 1986).}$$

Combining equations 2.10 and 2.11,

$$\{D\} = \begin{Bmatrix} r_x(1 - a_x^2) - r_y a_x a_y \\ r_y(1 - a_y^2) - r_x a_x a_y \end{Bmatrix} \quad (2.12)$$

The compressed portion of the section can be determined by a 90° counterclockwise rotation of the α unit vector, as shown in Fig. 2.20. The sign of $\|\vec{D}\|$ is determined from the sign of the z-axis component of the cross product, $\vec{\alpha} * \vec{D}$, which is positive for the elements on the compressive side of the line α , and negative for the elements on the tension side.

Summing up all the definitions given above, the strain ε at a point on the cross section is,

$$\varepsilon(x, y) = \phi \left(\text{sgn}(a_x d_y - a_y d_x) \left\| \vec{D} \right\| - D_o \right) \quad (2.13)$$

where,

d_x, d_y : The x and y components of $\vec{D}$.

$\text{sgn}(\)$: The signum function, defined as -1, 0, 1 for values that are negative, zero, and positive.

Having calculated the strains at the centroids of the fibers, stresses corresponding to these strains can be calculated using the strain values and the loading history parameters, which are preserved in memory for each uniaxial material hysteresis separately. Because of the path-dependent characteristics of the material hysteresis, analysis has to be done incrementally.

The unknown D_o (Fig. 2.20) can be defined, if the position of the neutral axis is known. Due to the normal stress equilibrium, there are unique neutral axis positions for the given curvature and the loading history parameters. The normal stress and moments on the section can be expressed as,

$$P = \sum_{i=1}^n \sum_{j=1}^m \sigma_{ij} A_{ij} \quad (2.14)$$

$$M_x = \sum_{i=1}^n \sum_{j=1}^m \left(-\sigma_{ij} A_{ij} y_{ij} + (E_t)_{ij} (I_x)_{ij} \phi_x \right) \quad (2.15)$$

$$M_y = \sum_{i=1}^n \sum_{j=1}^m \left(\sigma_{ij} A_{ij} x_{ij} + (E_t)_{ij} (I_y)_{ij} \phi_y \right) \quad (2.16)$$

where,

σ_{ij} : Stress on the sub-area.

x_{ij}, y_{ij} : Components of the sub-area centroid.

A_{ij} : Area of the sub-area.

$(E_t)_{ij}$: Tangential modulus of the sub-area.

ϕ_x, ϕ_y : Components of the curvature.

$(I_x)_{ij}, (I_y)_{ij}$: Second moment of areas of the sub-areas.

The moments on the section are calculated for given loading-history parameters, curvature, and the condition that the normal stresses must integrate to zero. The moment may be calculated from Eq. 2.15, 2.16 and the material hysteresis algorithm if the location of the neutral axis is known. The normal-stress constraint provides the necessary condition to determine the neutral axis depth. But, in its current form, the normal-stress equation (Eq. 2.14) is not directly related to the parameters given. A function that relates the normal stress resultant to given parameters needs to be defined. The change in normal stresses can be related to the change in the strain at section centroid according to the following equation:

$$\Delta P = \sum_{i=1}^n \sum_{j=1}^m (E_T)_{ij} d\varepsilon_o A_{ij} \quad (2.17)$$

where,

$d\varepsilon_o$: Change in the strain of the section centroid.

ΔP : Change in the normal stress for a change in centroid strain, $d\varepsilon_o$.

This equation can be rewritten as,

$$d\varepsilon_o = \frac{\Delta P}{K_{TP}} \quad (2.18)$$

where,

$K_{TP} = \sum_{i=1}^n \sum_{j=1}^m (E_T)_{ij} A_{ij}$, is the tangent stiffness of the cross section to the normal stresses.

The strain at the section centroid is related to the neutral-axis depth and curvature (Fig. 2.12),

$$\epsilon_o = D_{NA} * \phi \quad (2.19)$$

The change in strain at the section centroid is related to the change in neutral axis depth and curvature (Fig. 2.12),

$$d\epsilon_o = \Delta D_{NA} * \phi \quad (2.20)$$

where,

ϵ_o : Strain of the section centroid.

$d\epsilon_o$: Change in the strain of the section centroid.

D_{NA} : Neutral axis depth.

ΔD_{NA} : Change in the neutral axis depth.

ϕ : Curvature.

Combining Eq. 2.18 and 2.20,

$$\Delta P = (K_{TP}) (\Delta D_{NA}) \phi \quad (2.21)$$

Equation 2.21 provides the explicit formulation needed for the iterations described below.

1. Use the neutral axis depth from the previous step and the new curvature, to calculate the normal stresses on the section. The difference between the calculated normal stress resultant, P , and the given normal stress resultant is the incremental axial load, ΔP .

2. Calculate the change in the neutral axis depth using equations 2.21.
3. Calculate the new values of ΔP and P for the new neutral axis.
4. If the ΔP in Step 3 is of the same sign as in Step 1, repeat the steps 2 to 4 until the absolute value of the ΔP is less than the allowable tolerance in axial load. If ΔP in Step 3 has changed sign, use the bisection method for finding the root.

The procedure explained above for finding the neutral axis position is adopted from Suharwardy and Pecknold (1978). Its convergence is very fast. However, convergence is not certain. In order to overcome this problem, a limited number of trials are made with the algorithm given above. If convergence is not reached in these trials, another iterative procedure (Brent's Method, Press et al. 1995) is used, which is guaranteed to converge given a range that includes the desired root.

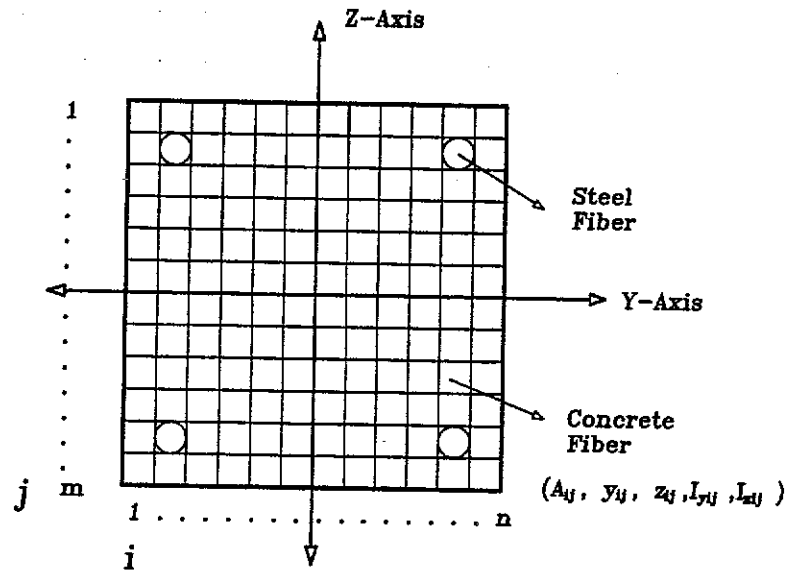


Fig. 2.1 Representation of a Rectangular Section as a Collection of Steel and Concrete Fibers

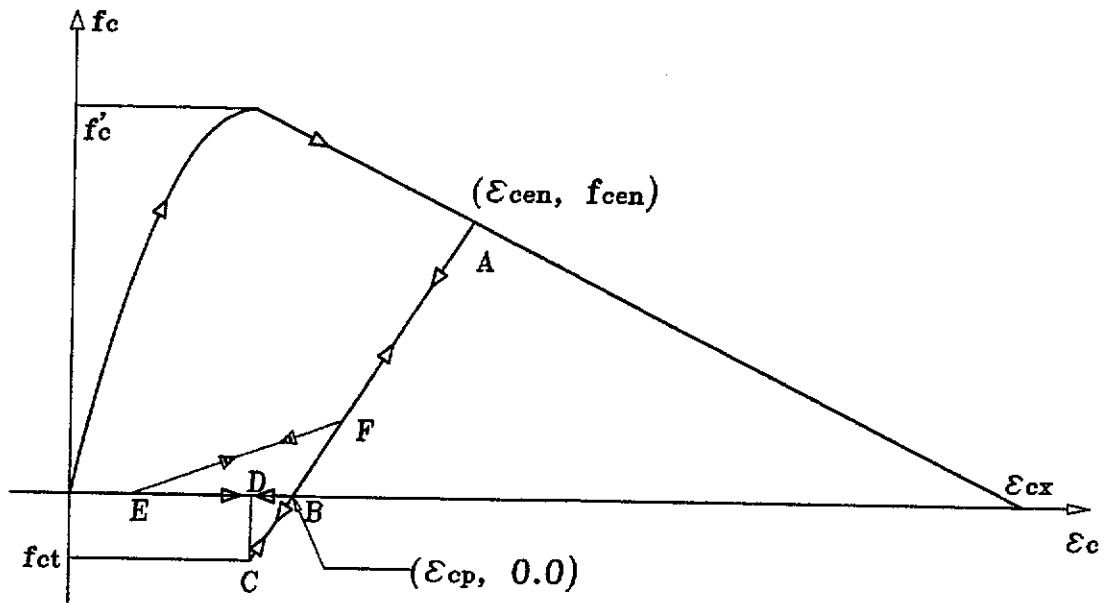


Fig. 2.2 Concrete Hysteresis

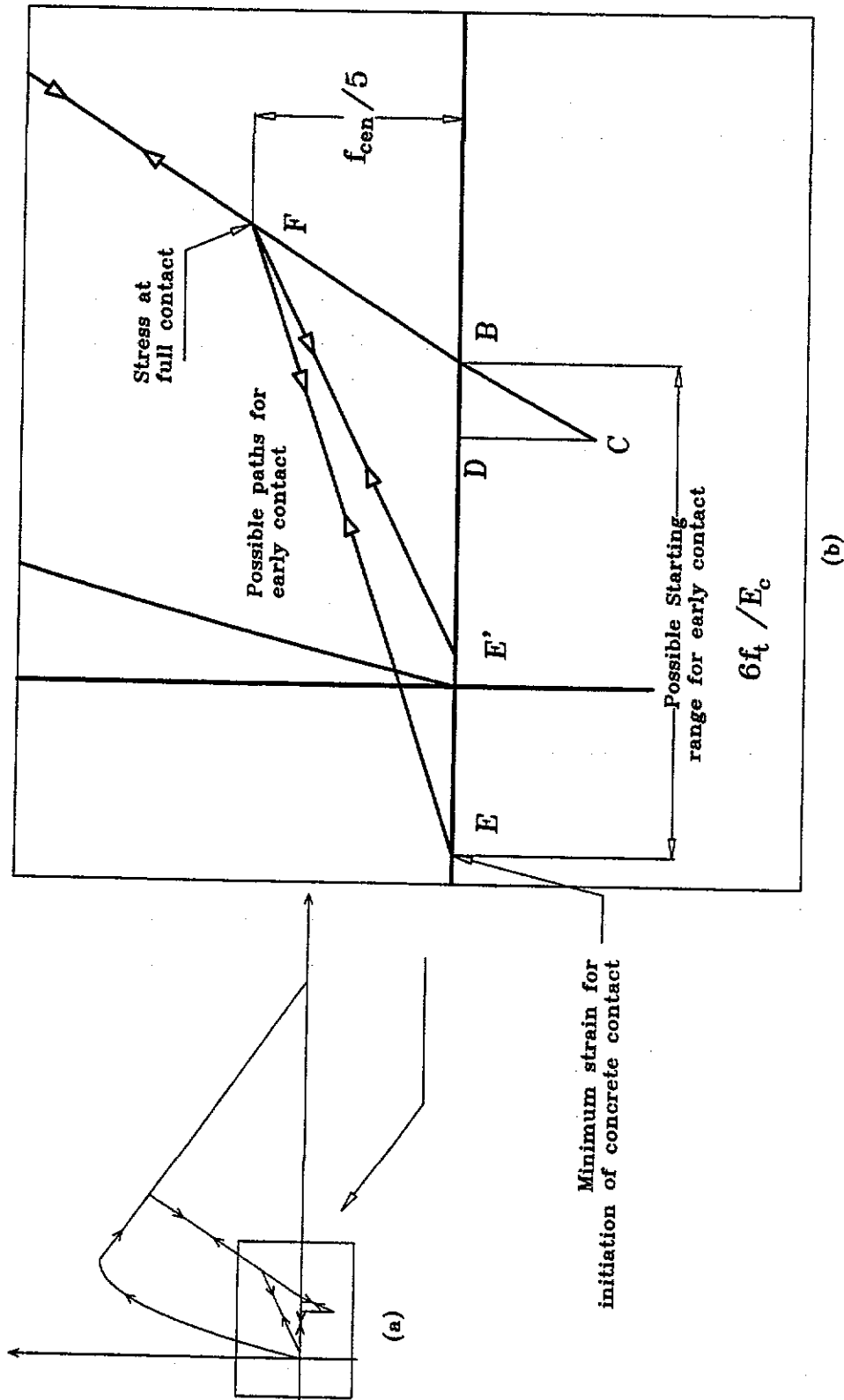


Fig. 2.3 Rules for Defining ‘Early Contact’ of Concrete at a Cracked Section

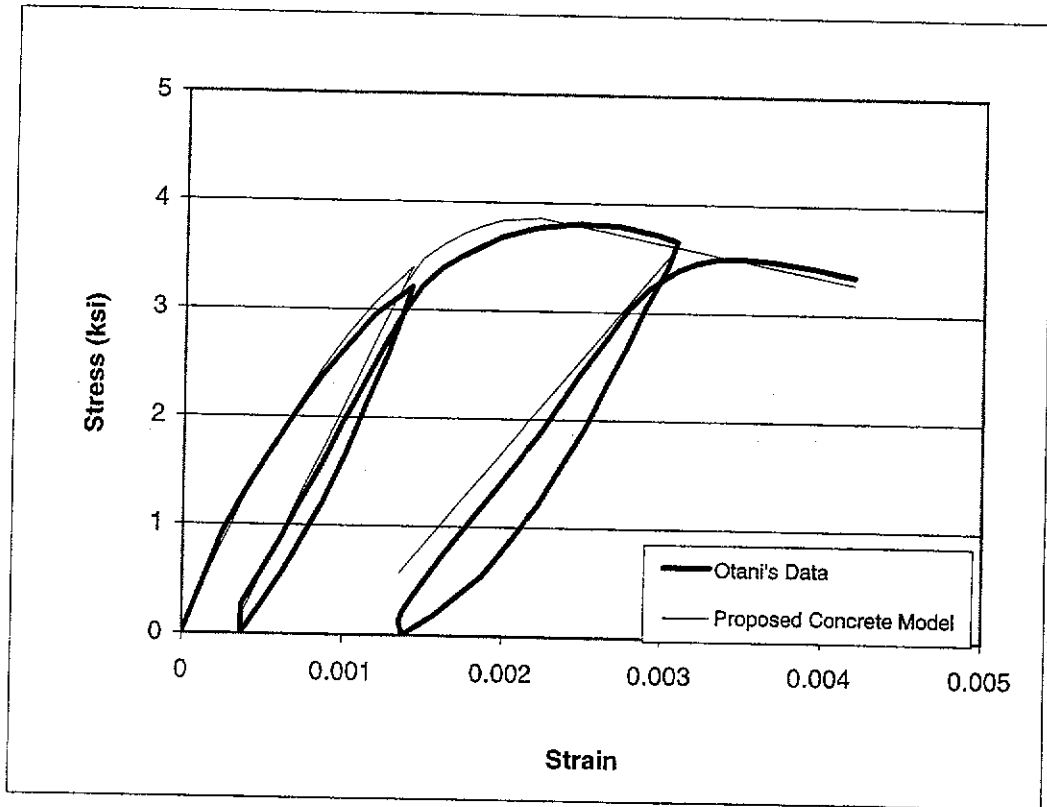


Fig. 2.4 Comparison of the Measured and Calculated Hysteresis for Compressed Concrete (Experimental Data from Otani, 1981)

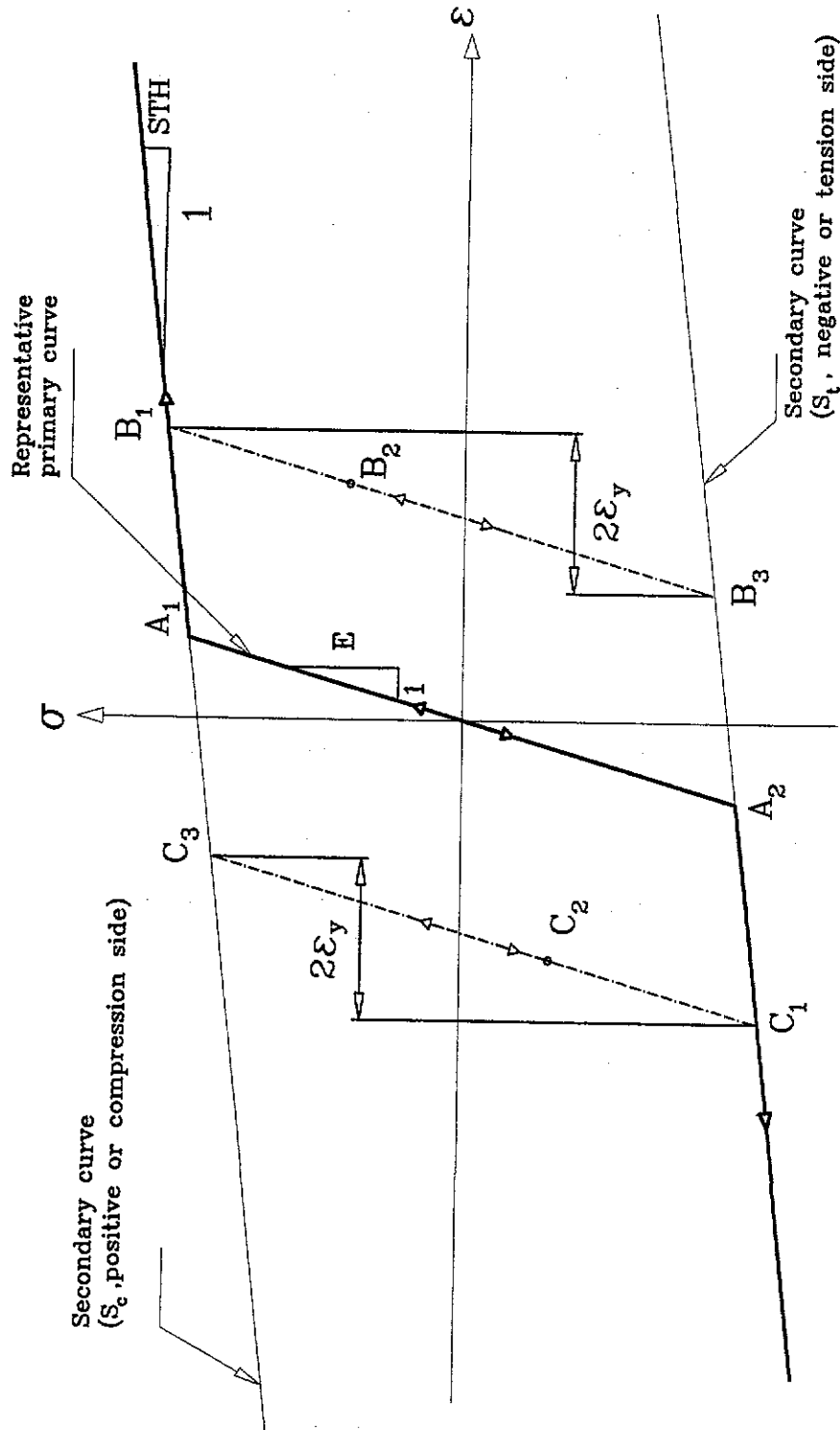


Fig. 2.5 Bilinear Hysteresis Model for Reinforcing Steel

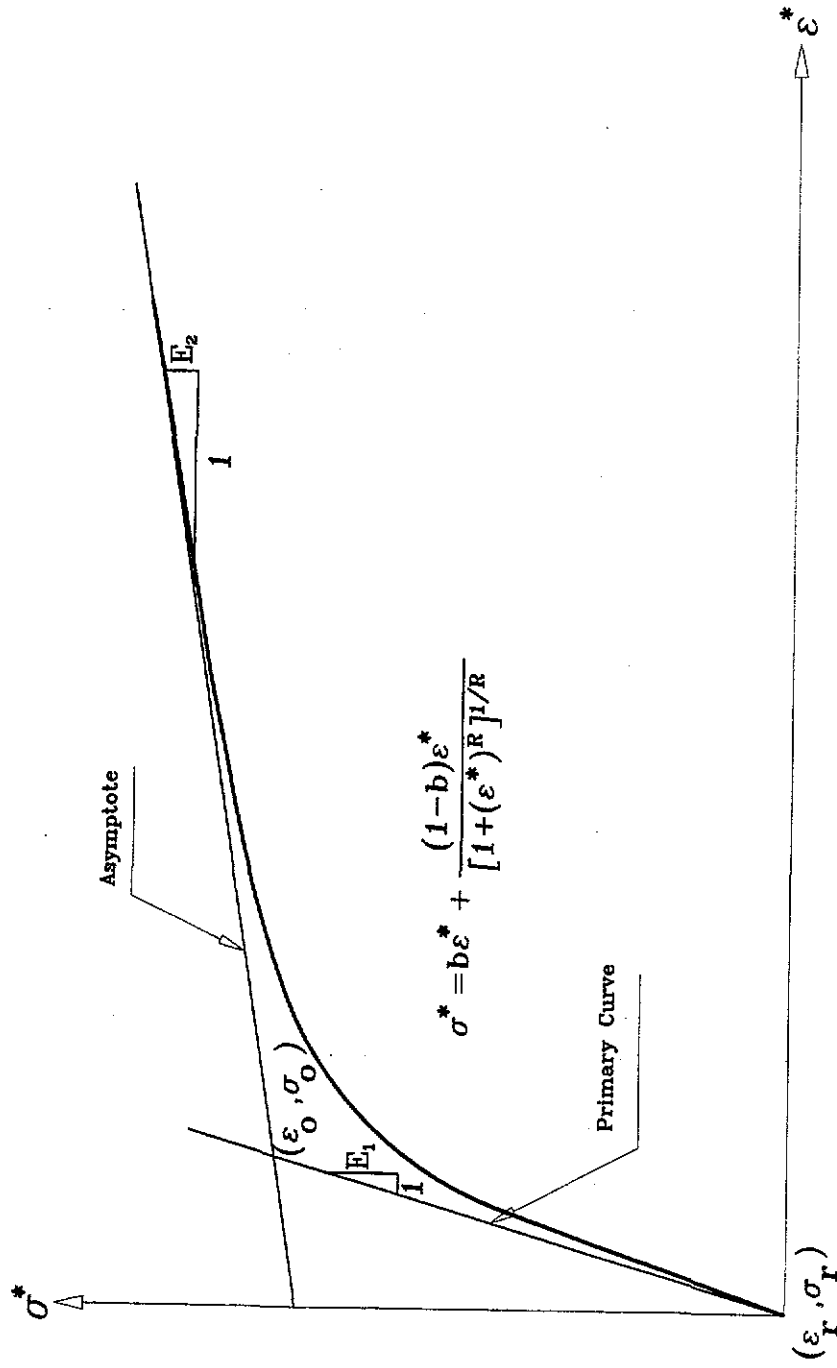


Fig 2.6 Representative Menegotto-Pinto Model and Equation

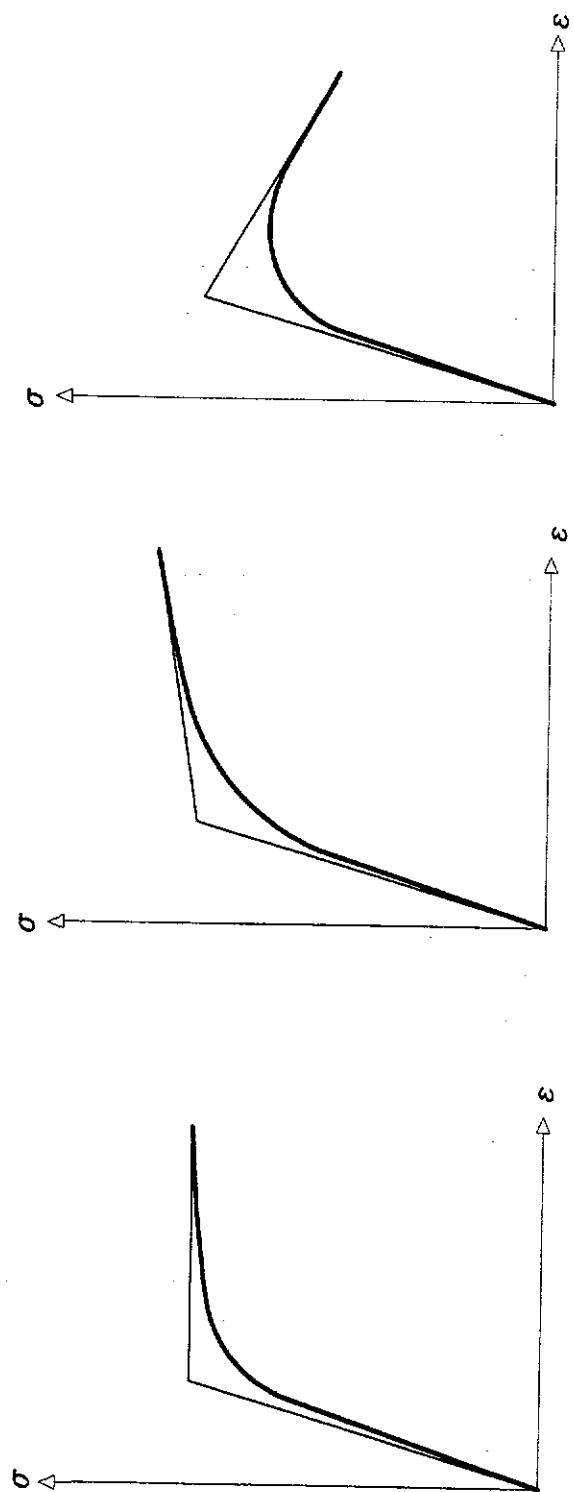


Fig. 2.7 Possible Shapes for Menegotto-Pinto Curves for Monotonically Increasing Load

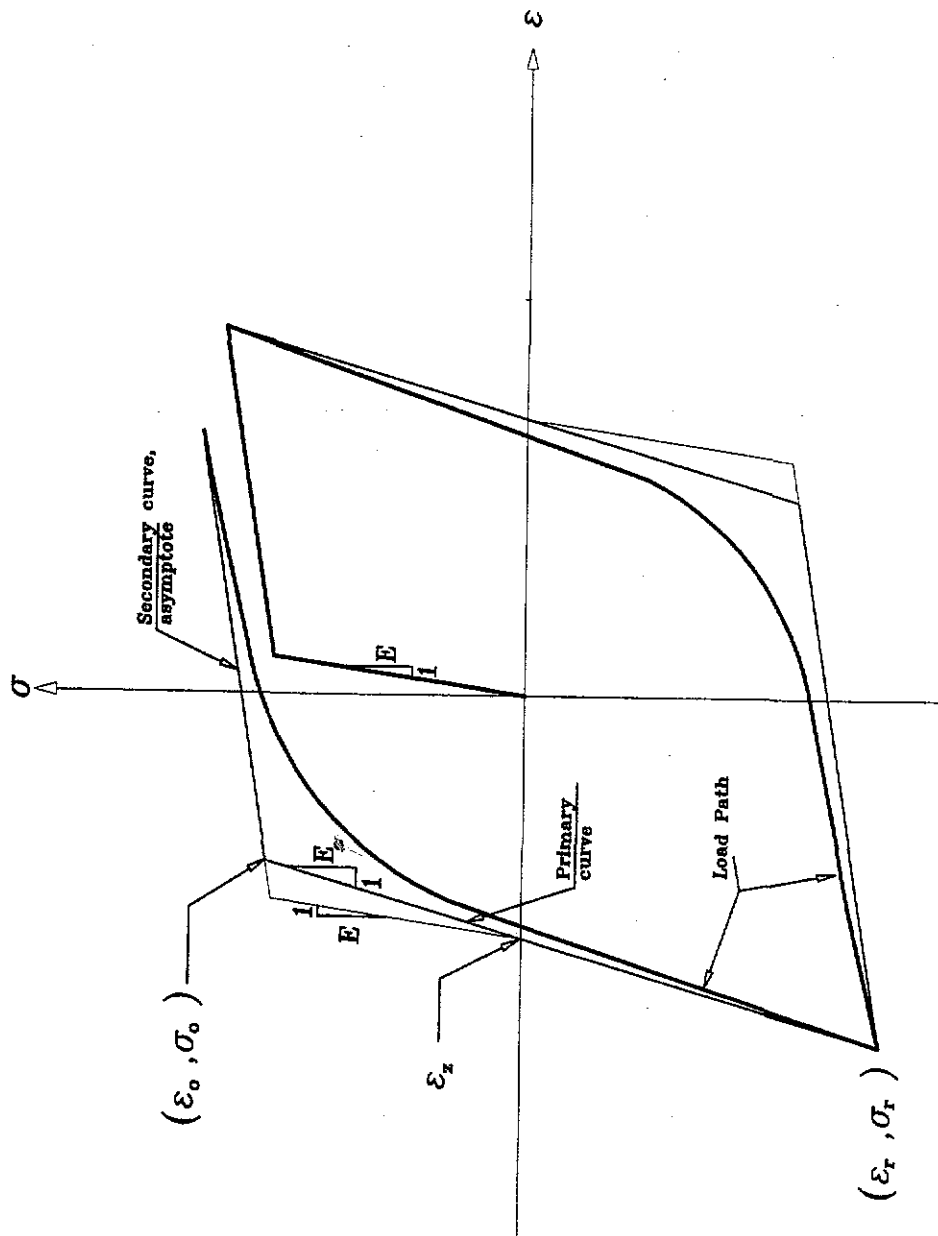


Fig. 2.8 Envelope Strain Shift

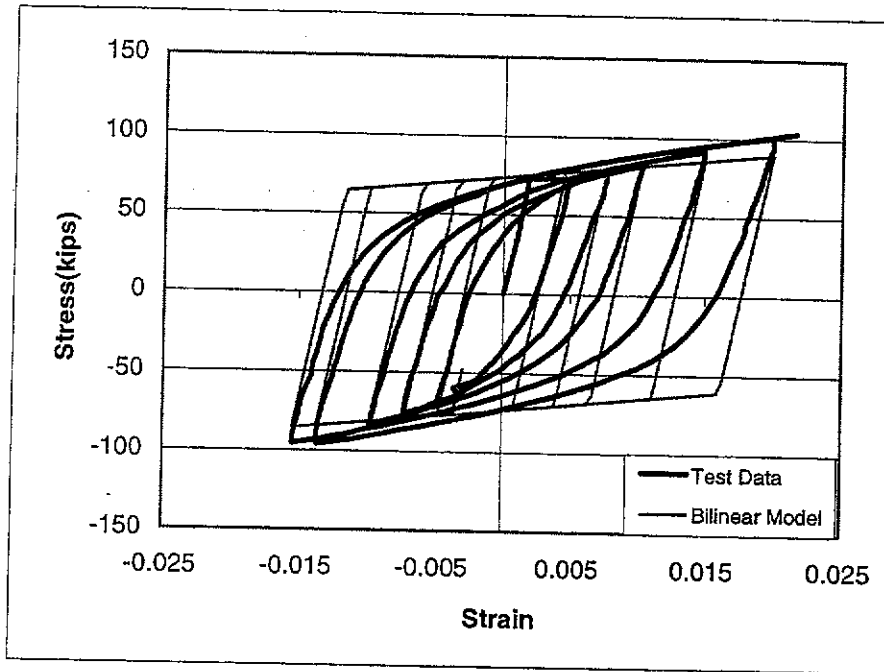


Fig. 2.9 Measured (Aktan et al. 1973, Test 9) and Calculated Hysteresis Curves for a #6 Reinforcing Bar. Based on the Bilinear Model.

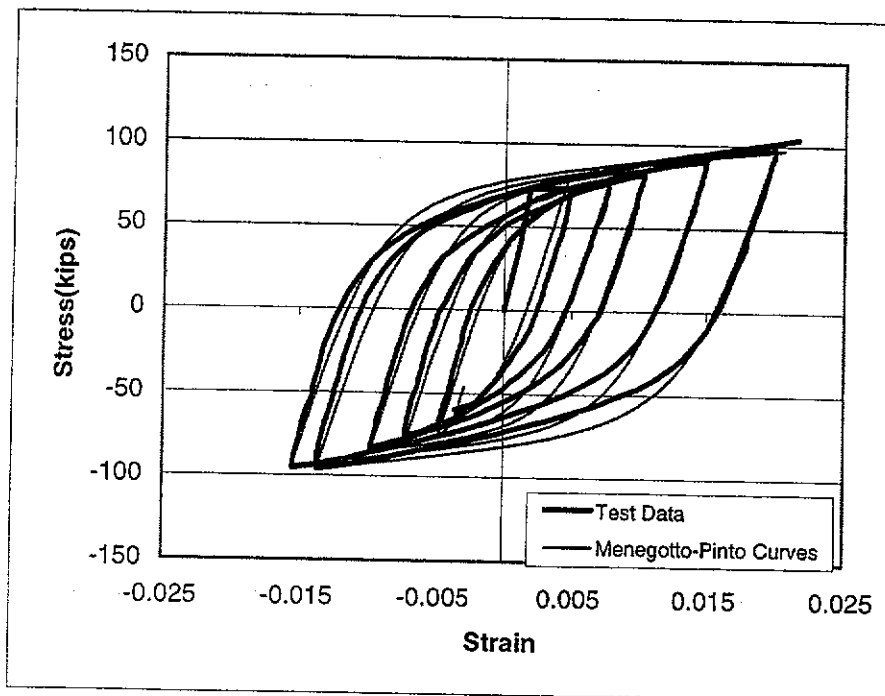


Fig. 2.10 Measured (Aktan et al. 1973, Test 9) and Calculated Hysteresis Curves for a #6 Reinforcing Bar, Based on the Menegotto-Pinto Curves.

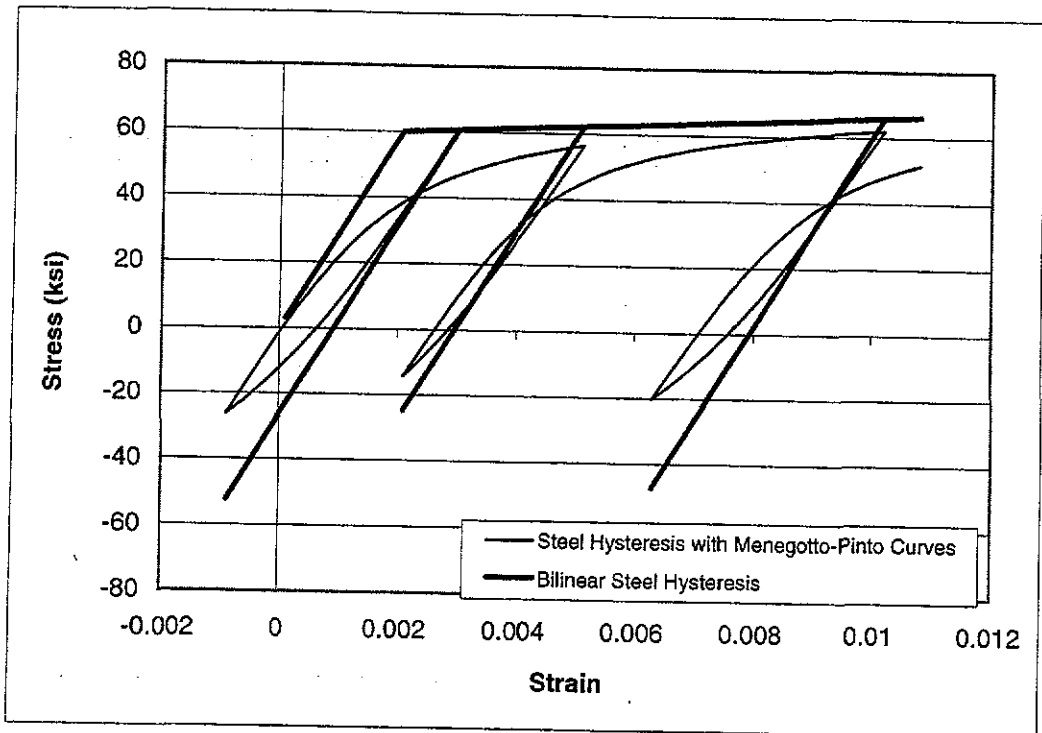


Fig. 2.11 Comparisons of the Results of the Two Models for Steel in the Range of Low Strains

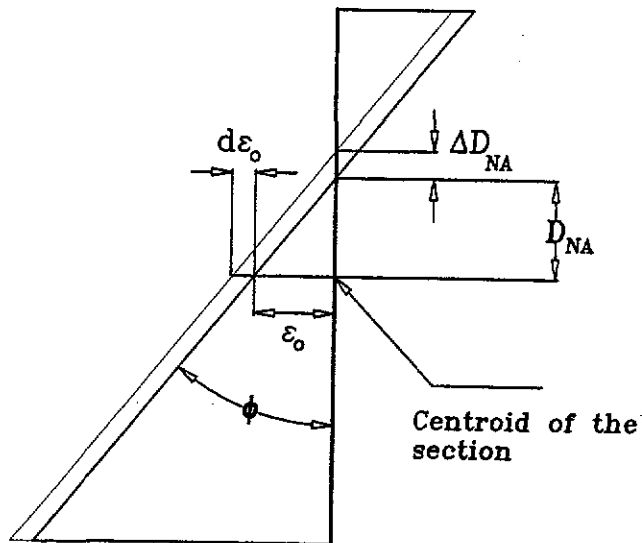
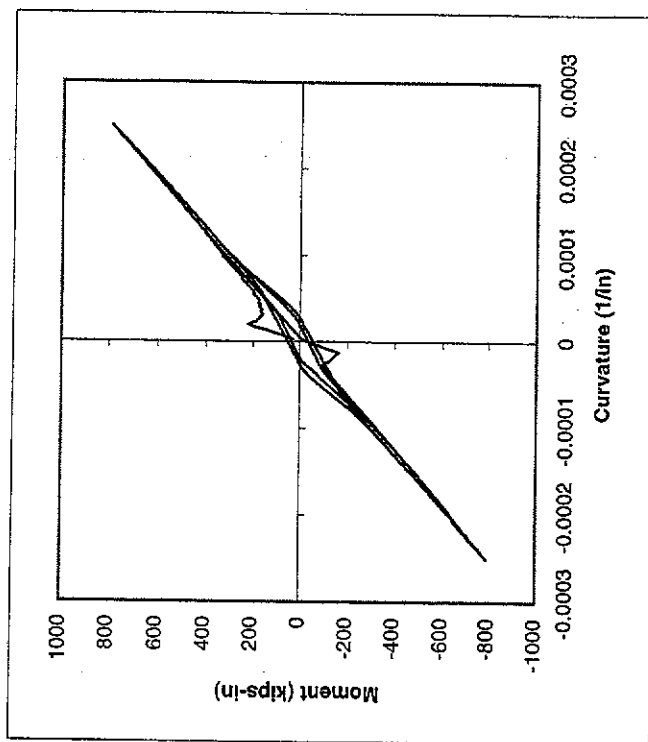
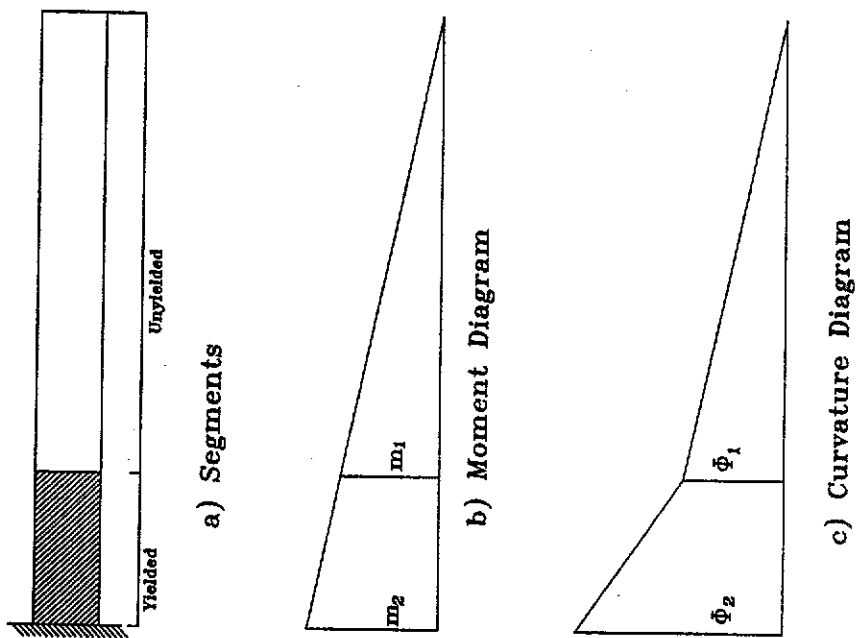


Fig. 2.12 Change in Centroid Strain Due to the Change in Neutral Axis Position



d) Representative Moment-Curvature Response of the Proposed Model for Small Moment Cycles

Fig. 2.13 Column Segments and the Assumed Curvature Distribution Along the Column

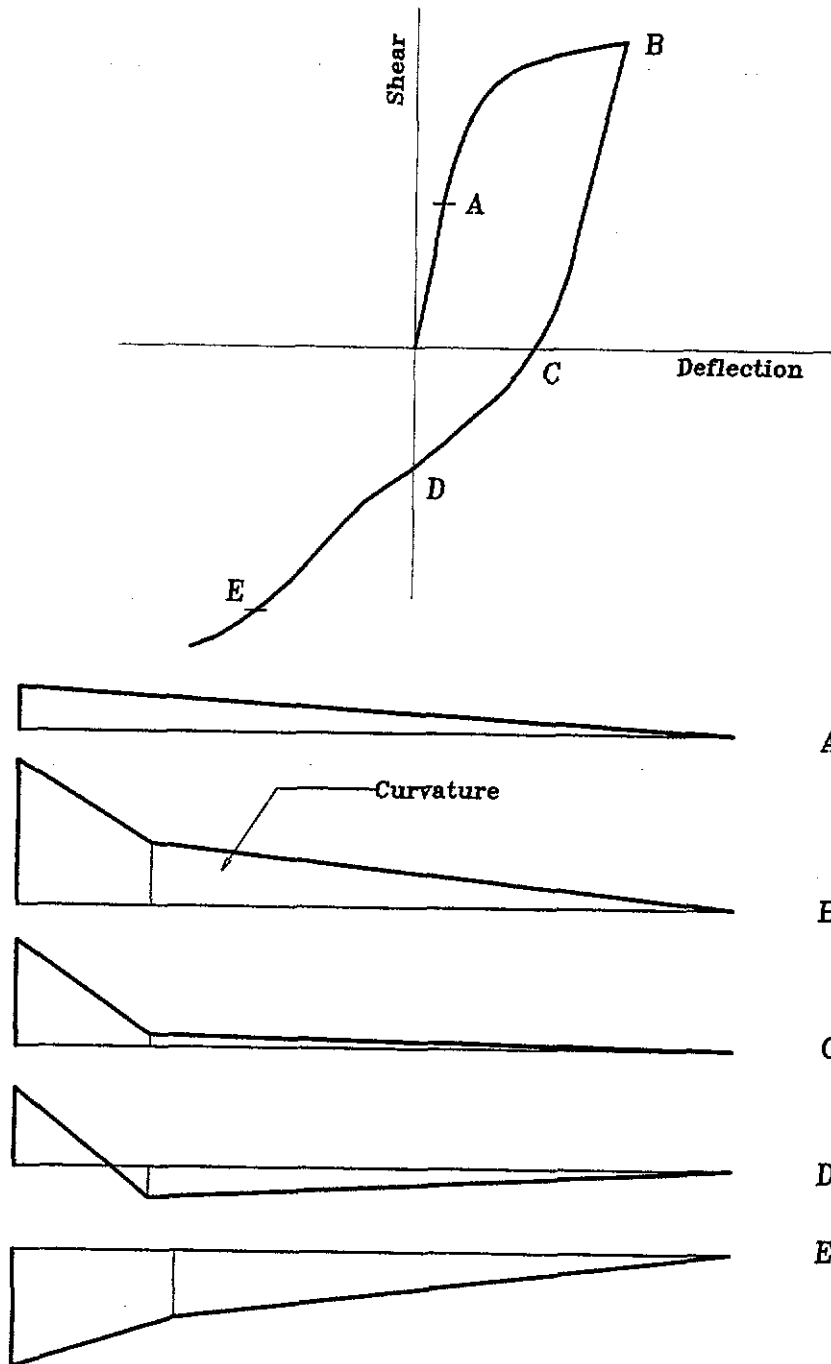


Fig. 2.14 Representative Summary of the Assumed Curvature Distribution at Different Stages of Loading

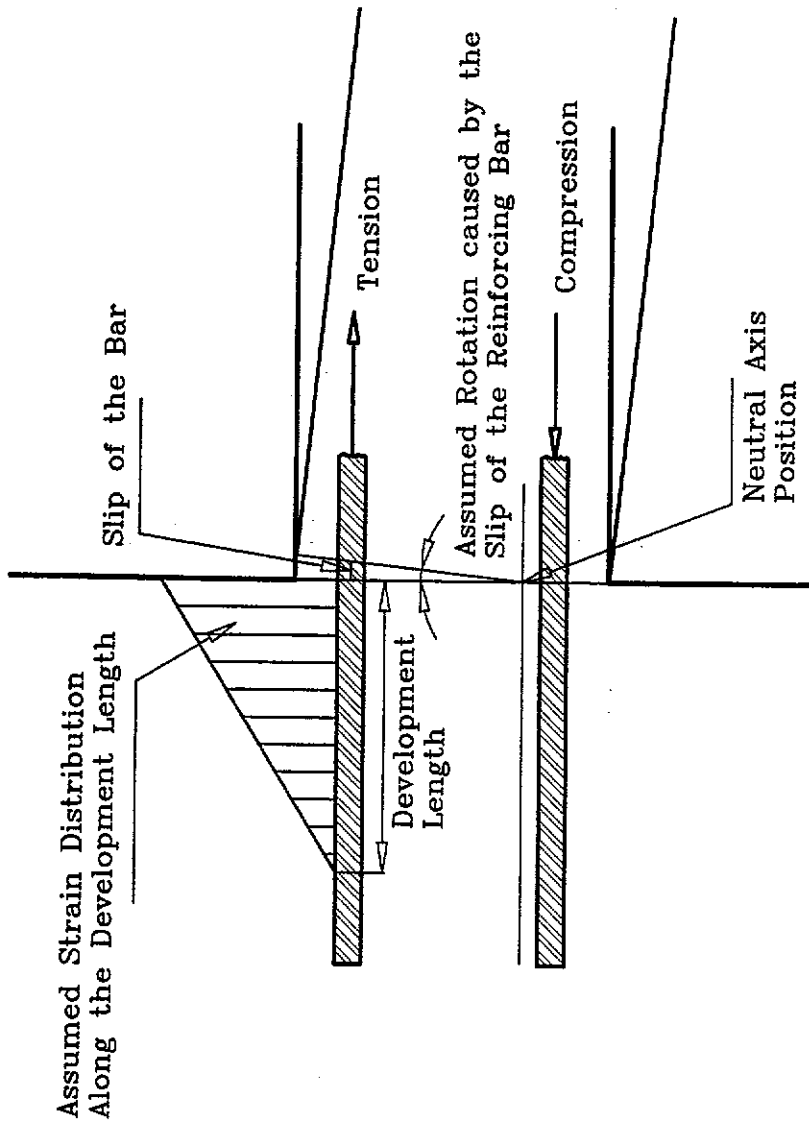


Fig. 2.15 Slip of the Bar and Related Definitions

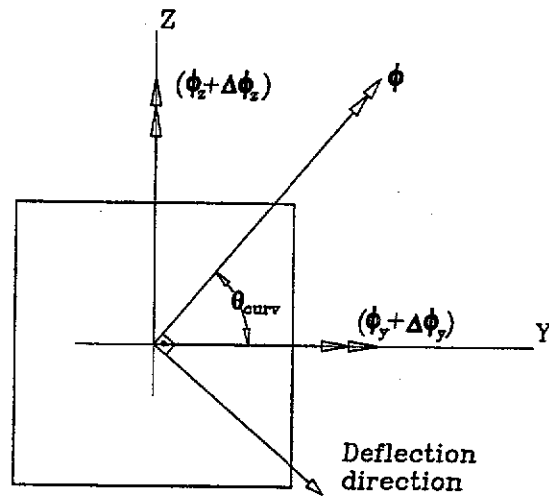


Fig. 2.16 Curvature Resultant and Its Components

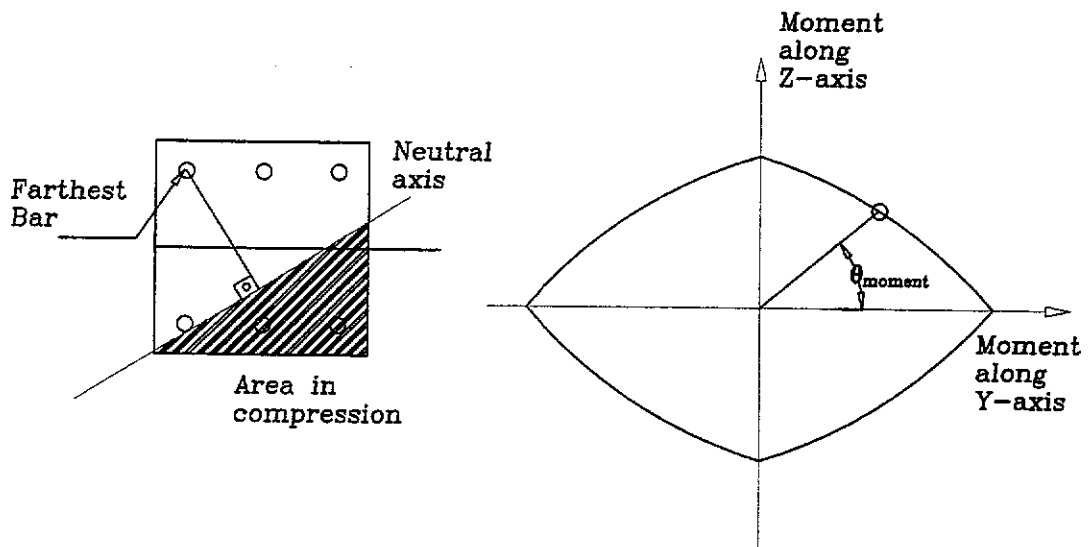


Fig. 2.17 Yield Moment Envelope for the Yield of the Farthest Bar to the Neutral Axis

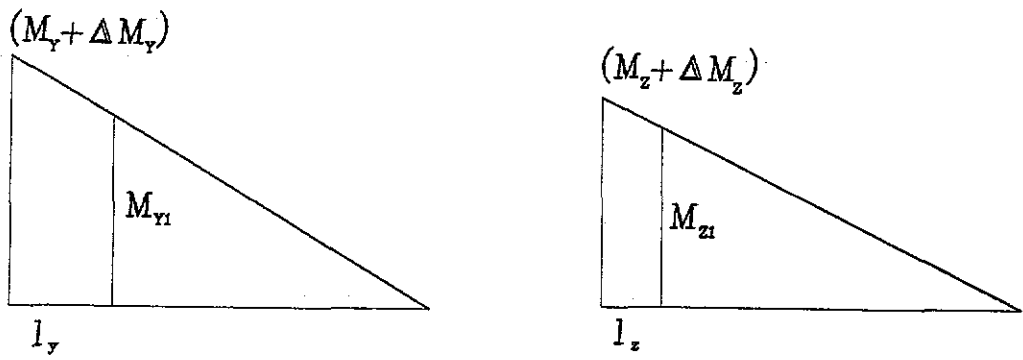


Fig. 2.18 "Yield Region" Moments

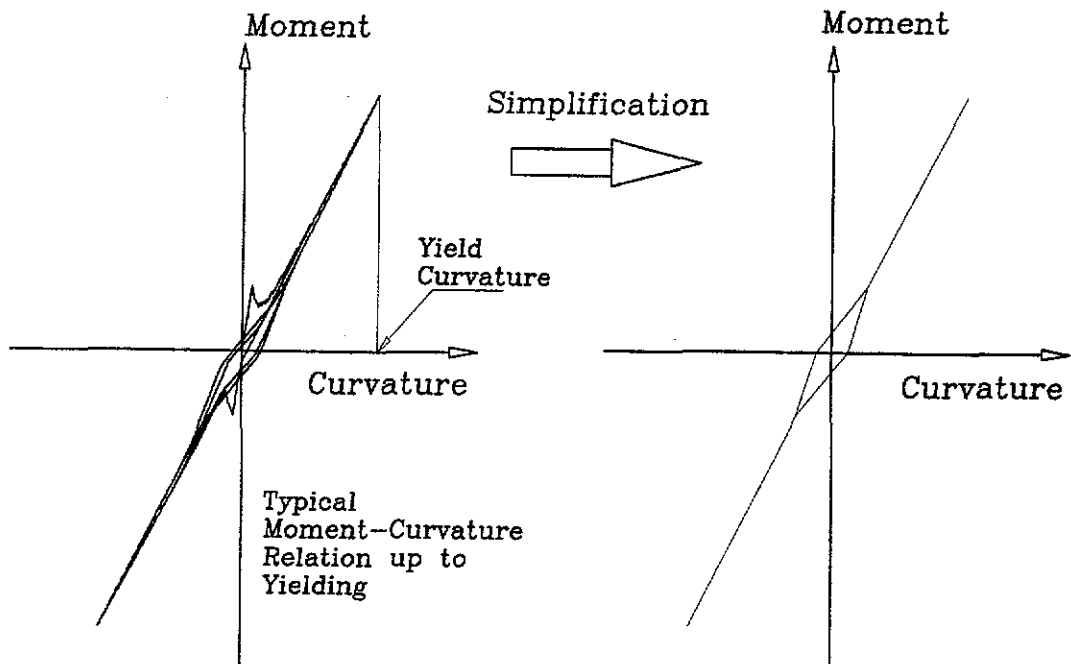


Fig. 2.19 Simplified Moment Curvature Relation

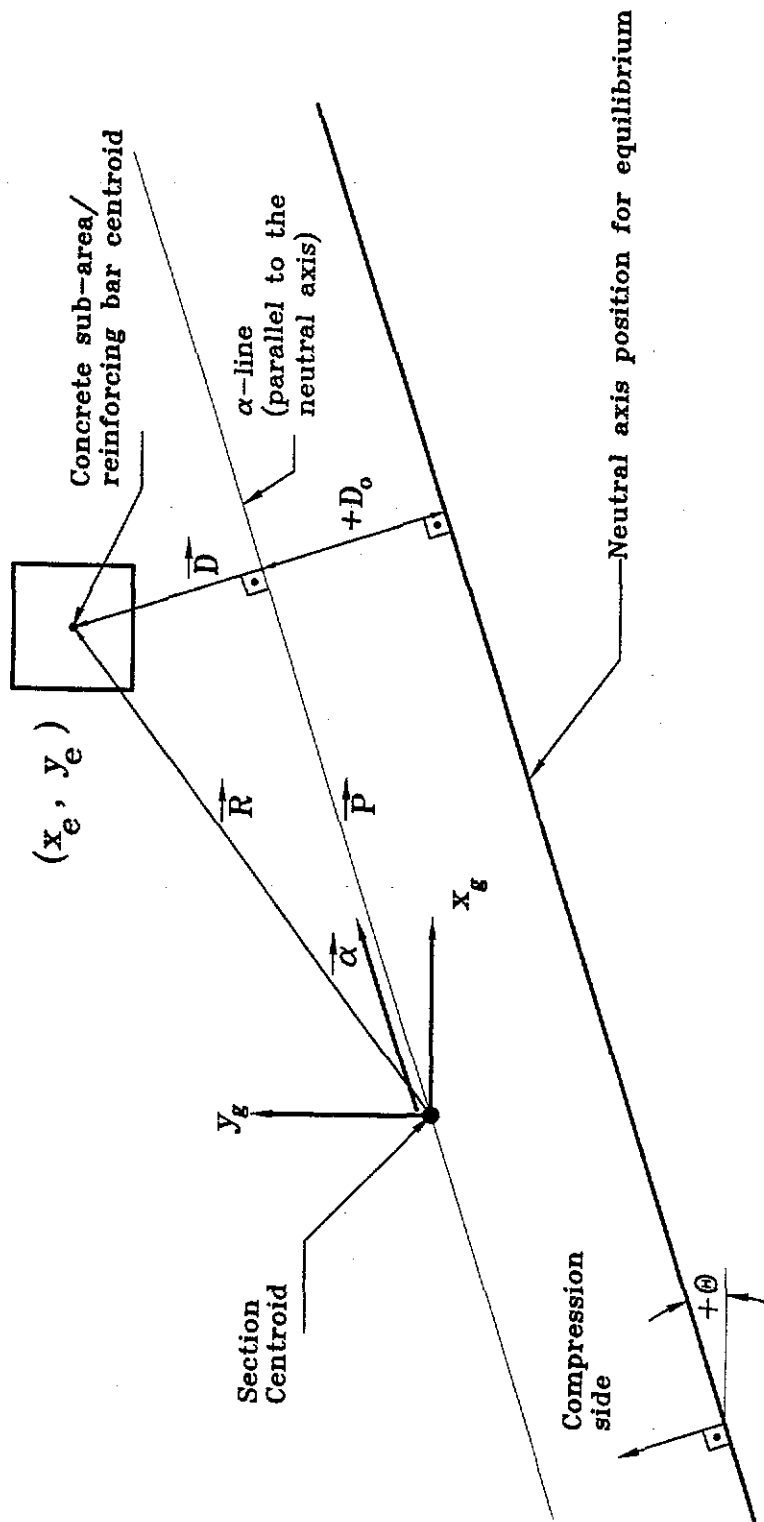


Fig. 2.20 Cross-Sectional Geometry

3. COMPARISON OF EXPERIMENTAL AND CALCULATED RESULTS

3.1 Introduction

This chapter presents comparisons of the results of the proposed model with experimental results. The comparisons are made in two groups, depending on the loading type: static and dynamic. The experimental results by Otani and Cheung (1981) were chosen for testing the proposed hysteresis algorithm in the static domain. In these experiments, reinforced concrete cantilevers were subjected to uniaxial and biaxial bending by application of static forces. Thus, the ability of the algorithm to lead to realistic estimates of hysteresis loops was tested directly. The results of the experiments by Dragovich (1996) were chosen for testing the proposed algorithm in the dynamic domain. In these experiments, reinforced concrete cantilevers were subjected to uniaxial and biaxial bending that resulted from dynamic loading (base excitation). In both loading cases, there were no axial loads on the members.

Comparisons of the calculated responses with the results of the experiments with static loading are presented in Section 3.2. This section is organized in two subsections. Section 3.2.1 presents the comparisons of a pair of specimens, which were comparable in geometry and material characteristics but had different loading histories. The first specimen was tested under cyclic-static loading about one of its axes. The second specimen was also tested under cyclic-static loading. However, the axis of application of the loading was shifted from one axis to the other orthogonal axis (load was applied along one axis at a time) throughout its loading history. Section 3.2.2 presents the comparison of the calculated responses for the specimen subjected to biaxial cyclic-static loading.

The comparisons of the calculated responses with the results of experiments with dynamic loading are presented in Section 3.3. This section is organized in two subsections. Section 3.3.1 presents the modeling of the specimens for dynamic analysis. Section 3.3.2 presents the comparisons of the specimens measured and calculated responses under dynamic-biaxial and dynamic-uniaxial loading. Finally, Section 3.4 provides a summary of this chapter.

3.2 Statically Loaded Specimens

Otani and Cheung (1981) tested reinforced concrete cantilevers to study the effect of biaxial bending on the hysteretic behavior of reinforced concrete. Three pairs of specimens were tested. For the first two pairs, one of the specimens was tested under uniaxial loading. The other specimen was also tested under uniaxial loading but, as mentioned above, loading was shifted from one orthogonal axis to the other. Both specimens were to have the same displacement history. The purpose of changing the axis, but using the same displacement history, was to investigate the effect of damage (because of prior loading) in one axis of bending about the other axis. Specimens of the last pair were subjected to biaxial bending throughout their loading histories. In all cases, loading was applied under displacement control. Data from specimens 5, 6 and 7 were chosen for evaluation of the proposed model. The dimensions and the reinforcing steel configuration of the chosen specimens are given in Fig. 3.1 and the material properties are given in Table 3.1. The longitudinal bars of the specimens were welded to a grid of steel plates at the base. The maximum possible development-length of the specimens was approximately 20-bar diameter. This value is used as the development length of the static test specimens in the numerical analysis.

3.2.1 Specimens 5 and 6

Both specimens were forced to follow the displacement paths specified in Fig. 3.2. Specimen 5 was loaded along one of its axis throughout the loading history. Specimen 6 was loaded along its north-south (NS) axis up to the 5th cycle, then along the east-west axis from the 5th to the 10th cycle. Loading was along to the NS axis after the 10th cycle. The total number of applied displacement cycles in the two specimens was equal and the amplitudes to corresponding cycles were comparable for both displacement paths. It is reported that, after the 8th cycle ($\approx 5.5\%$ drift ratio), the major source of deflection shifted from flexure to shear for both specimens. Because the proposed algorithm does not recognize "shear faulting," the comparison was stopped after the 8th load cycle.

The measured and calculated force-displacement relations of specimens 5 and 6 are presented in Fig. 3.3 and 3.4. The measured force-displacement curves for Specimen 5 are presented in three groups (because of the applied loading, Fig. 3.3). Maximum drift ratios for these three groups were approximately 1%, 3%, and 5%. The stiffness decreased and the energy dissipation (implied by the relative size of the area enclosed by the load-deformation loop) increased with increasing deflection. After yielding, reloading curves followed almost linear paths, up to about maximum force levels reached previously in that direction. The calculated force-displacement curves are also presented in three groups (Fig. 3.3). The calculated force levels are comparable with the measured values. As observed for the experimental results, the calculated stiffness decreased and the energy dissipation increased with the increasing deflection. However, the reloading curves, after yielding, did not exhibit a linear path up to the force levels reached previously. They were stiffer at the beginning and softer later, leading to a larger area enclosed by the curve. The measured to calculated ratios of the energy dissipated and the stiffness for each group are given in Fig. 3.5a (mean for each group of curves illustrated in figures 3.3 and 3.4). Energy dissipation is assumed to be proportional to the area enclosed within the force-deformation loops. It is the cumulative area of the loops in each

group. Except for the first group, the calculated dissipated energy was about 15% more than the measured. The difference between the measured and the calculated loops for the first group is attributable to the sensitivity of the response to errors in the incremental stiffness at very low amplitudes. Again except for the first group, the calculated stiffness was about 10% higher than the measured. For the first group measured stiffness and the calculated stiffness were equal.

Specimen 6 was loaded uniaxially, but loading shifted from one orthogonal axis to another through its history (Fig. 3.2b). It was observed that the force-displacement response for Specimen 6 was comparable to that for Specimen 5 (Fig. 3.4). The response was not affected significantly because of the shifting of the loading axis. Also the calculated response was not affected significantly because of shifting (Fig. 3.4) and demonstrates a match better than that for Specimen 5. The measured to calculated ratios of the energy dissipated and the stiffness for each group are given in Fig. 3.5b. Similar to Specimen 5, except the first group, the calculated dissipated energy was 15% more than the measured. For the first group, the specimen dissipated 25% more energy. The calculated stiffness was within $\pm 3\%$ of the measurements for all groups.

3.2.2 Specimen 7

The loading history imposed on Specimen 7 is shown in Fig. 3.6. The top of the specimen was forced to travel in the horizontal plane in one direction at a time so that the travel path was essentially "square" at each level of applied displacement (Fig. 3.6). The comparison of measured and calculated force-displacement relationships was stopped at a drift ratio of 4%.

The measured and the calculated force-displacement relations for Specimen 7, east-west (EW) and north-south (NS) directions, are presented in Fig. 3.7 and 3.8. In order to have a clear picture, responses of each direction were separated into three groups

on the basis of displacement demand, approximately 0.5%, 1.3% and 3% for groups 1, 2 and 3, respectively. For each group peak-force levels were in error by $\pm 10\%$. The measured to calculated ratios of the dissipated energy and the stiffness of each group are given in Fig. 3.10. It is observed (from figures 3.10a and 3.10b) that the measured to calculated values of the energy dissipated per cycle decreased with displacement. Except the first group, the energy dissipation of the proposed model was within the $\pm 15\%$ of the measurements. As it was for Specimen 5, the large difference between the measured and the calculated loops for the first group is attributable to the sensitivity of the response to errors in determining the incremental stiffness at very low amplitudes. The mean stiffness ratios also had a tendency to decrease with increasing displacements. Except for the last group, the stiffness calculated using the proposed model for the specimen was within $\pm 6\%$ of the measurements. The difference between the measured and the calculated stiffness for the last group is attributable to the insensitivity of the proposed model to the confinement of the concrete.

Several times through the loading history, the top deflection of Specimen 7 was kept at a certain level in one direction, and was increased in the orthogonal direction. Because of this loading scheme, sudden drops were observed for the moment in the direction in which the displacement was kept constant (Fig. 3.7 and 3.8). The proposed algorithm responses were sensitive to this phenomenon and indicated similar drops in moment. The disagreement in the measured and calculated reloading paths that was observed in “uniaxially” loaded specimens was not as severe for Specimen 7.

The bi-directional resistance trace for both the measured and calculated responses are presented in Fig. 3.9. The calculated response was comparable to the measured response. Otani and Cheung (1981) mentioned that the extreme traces in the north-west quadrant of Fig. 3.9a represented the limits of resistance of the column. The calculated traces exhibit a similar behavior at the same locations (Fig. 3.9b) defining the “yield surface”.

3.3 Dynamically Loaded Specimens

The purpose of Dragovich's Experiments (1996) was to study the effects of the combined torsional-translational loading in reinforced concrete. The torsional-translational response was considered in terms of dynamic interaction (coupled frequencies) and material interaction (biaxial bending). A conceptual model of the test specimens is given in Fig. 3.11a. As illustrated in Fig. 3.11a, the base motion along the X-axis generates moments about Y and Z-axes, because of the eccentricity, e , of the mass m . Configuration of the members dictates the material interaction (biaxial bending). Dragovich used two different member configurations in the experiments. The first configuration caused only dynamic interaction. As illustrated in Fig. 3.11b, the moment along the Y-axis was resisted by the lower beam, and the moment along the Z-axis was resisted by the upper beam. Since each beam was resisting moments along one axis, there was no material interaction. The second configuration caused both dynamic and material interaction. Moments were resisted along the axes of single beam (Fig. 3.11c). There was biaxial bending acting on the beam.

The specimens were designed to observe the effect of changes in three parameters: strength, mass distribution and material interaction. Specimen strength was varied by changing the amount of longitudinal reinforcement in the beam (Fig. 3.12) and the mass distribution was varied by changing the mass eccentricity along the Y-axis (Fig. 3.11a). Two values of the mass distribution and two values of the beam section strength were used both for the materially coupled and for the materially uncoupled specimens. A total of eight specimens were tested. Designation and the properties of the specimens are listed in tables 3.2 and 3.3.

The north-south component of 1940 El Centro earthquake record (California Institute of Technology, 1971) was used as base motion. The time scale was compressed by a factor of two, to shift the region of nearly constant linear velocity response closer to the frequencies of the test specimens, and to reduce the deflections of the earthquake

simulator to within its operational limits. Each test specimen was subjected to three earthquake simulations of increasing intensity to produce successively increasing levels of damage. The demand for the first set of simulations was low. The second simulations of the specimens were used for comparisons with the calculated simulations of the proposed model. The specimen responses were below yield level. During the second simulations the members of the specimens reached to drift ratios of 1% to 3%. Acceleration histories of the base motions used in the second simulations are shown in Fig. 3.14 and 3.15 for materially uncoupled and coupled specimens.

3.3.1 Modeling of the Specimens

Dragovich's Specimens were two degree-of-freedom, mass-and-stiffness-coupled systems (materially uncoupled specimens were only mass coupled). The experimental setup may be thought of as an assemblage of mass components. A mass component is any one component of the test setup for which the mass properties were computed. The definitions associated with a mass component are given in Fig. 3.13. In particular the following properties of the each mass component of the test setup were identified.

1. Mass, m_i .
2. Cartesian coordinates of the center of mass, (X_i, Y_i, Z_i) .
3. Mass moment of inertia about local Y-axis, J_{yi} .
4. Mass moment of inertia about local Z-axis, J_{zi} .

During earthquake simulations, a mass component, m_i , undergoes simultaneous linear acceleration in the X-direction, $\ddot{u}_g$, and the angular acceleration about the axes Y ($\ddot{\theta}_y$) and Z ($\ddot{\theta}_z$). Figure 3.16 shows the inertial forces acting on a component of the specimen resulting from these accelerations and the resisting moments M_y and M_z about the Y and Z-axes. These moments represent the moments provided by the resisting members.

Equilibrium of moments of the inertial forces for all mass components along the Y-axis requires (neglecting the damping forces):

$$\sum_{i=1}^n (J_{yi} \ddot{\theta}_y + \ddot{u}_g m_i z_i + m_i \ddot{\theta}_y x_i^2 + m_i \ddot{\theta}_y z_i^2 - m_i \ddot{\theta}_z y_i z_i) + M_y(\theta_y, \theta_z) = 0 \quad (3.1)$$

and along the Z-axis,

$$\sum_{i=1}^n (J_{zi} \ddot{\theta}_z - \ddot{u}_g m_i y_i + m_i \ddot{\theta}_z x_i^2 + m_i \ddot{\theta}_z y_i^2 - m_i \ddot{\theta}_y y_i z_i) + M_z(\theta_y, \theta_z) = 0 \quad (3.2)$$

Expressing equations 3.1 and 3.2 in matrix form:

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_y \\ \ddot{\theta}_z \end{Bmatrix} + \begin{Bmatrix} M_y(\theta_y, \theta_z) \\ M_z(\theta_y, \theta_z) \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} \ddot{u}_g \quad (3.3)$$

where,

$$m_{11} = \sum_{i=1}^n [m_i (x_i^2 + z_i^2) + J_{yi}] \quad (3.3a)$$

$$m_{21} = m_{12} = \sum_{i=1}^n (-m_i y_i z_i) \quad (3.3b)$$

$$m_{22} = \sum_{i=1}^n [m_i (x_i^2 + y_i^2) + J_{zi}] \quad (3.3c)$$

$$P_1 = \sum_{i=1}^n (-m_i z_i) \quad (3.3d)$$

$$P_2 = \sum_{i=1}^n (m_i y_i) \quad (3.3e)$$

The relation of the resisting moments (M_y and M_z) to the rotation of the system (θ_y and θ_z , Fig. 3.17) is nonlinear and dependent on the loading history. The proposed model, which was defined in Chapter 2, was used for calculating the moment-rotation

relation of the members. The procedure of calculation for the system rotations are presented in App. C. The nonlinear and loading-history dependent behavior of the moment-rotation relation necessitated the use of a numerical method for the integration of the differential equations. In order to write the Eq. 3.3 in incremental form, the following definitions were made:

$$\Delta\theta_i = \theta_{i+1} - \theta_i$$

$$\Delta\ddot{\theta}_i = \ddot{\theta}_{i+1} - \ddot{\theta}_i$$

$$\Delta\ddot{u}_i = \ddot{u}_{i+1} - \ddot{u}_i$$

where,

θ_i, θ_{i+1} = System rotations at the current and next time step.

$\ddot{\theta}_i, \ddot{\theta}_{i+1}$ = Rotational accelerations at the current and next time step.

$\ddot{u}_i, \ddot{u}_{i+1}$ = Base motion accelerations at the current and next time step.

By the application of these incremental forms, Eq. 3.3 can be written in the following form:

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{Bmatrix} \Delta\ddot{\theta}_{yi} \\ \Delta\ddot{\theta}_{zi} \end{Bmatrix} + \begin{Bmatrix} M_y(\theta_{y(i+1)}, \theta_{z(i+1)}) - M_y(\theta_{yi}, \theta_{zi}) \\ M_z(\theta_{y(i+1)}, \theta_{z(i+1)}) - M_z(\theta_{yi}, \theta_{zi}) \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} \Delta\ddot{u}_{gi} \quad (3.4)$$

If the moment-rotation relation is accepted linear during a step, Eq. 3.4 can be written in the form of:

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{Bmatrix} \Delta\ddot{\theta}_{yi} \\ \Delta\ddot{\theta}_{zi} \end{Bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} \Delta\theta_{yi} \\ \Delta\theta_{zi} \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} \Delta\ddot{u}_i \quad (3.5)$$

where,

K_{ij} = Secant stiffness terms.

The frequencies of the system can be found by the solution of homogenous system of Eq. 3.5. Having the frequencies of the system, mass and stiffness proportional viscous damping (Raleigh Damping) can be defined by the following:

$$[C] = a_o [M] + a_1 [K] \quad (3.6)$$

where,

$$a_o = \zeta \frac{2\omega_i \omega_j}{\omega_i + \omega_j}, \text{ and } a_1 = \zeta \frac{2}{\omega_i + \omega_j}$$

$$[M] = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}, \text{ mass matrix}$$

$$[K] = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}, \text{ secant stiffness matrix for the current step}$$

$$[C] = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}, \text{ viscous damping matrix for the current step}$$

ω_i, ω_j = system frequencies

ζ = viscous damping ratio, 2% used for the simulations

The final form of the incremental dynamic equilibrium equations is:

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{Bmatrix} \Delta \ddot{\theta}_{yi} \\ \Delta \ddot{\theta}_{zi} \end{Bmatrix} + \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{Bmatrix} \Delta \dot{\theta}_{yi} \\ \Delta \dot{\theta}_{zi} \end{Bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} \Delta \theta_{yi} \\ \Delta \theta_{zi} \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} \Delta \ddot{u}_i \quad (3.7)$$

where,

$$\Delta \dot{\theta}_i = \dot{\theta}_{i+1} - \dot{\theta}_i,$$

$\dot{\theta}_{i+1}, \dot{\theta}_i$ = Rotational velocities at the current and next time step.

The Newmark Method (1959) was used for the solution of the Eq. 3.7. The details of the solution procedure are presented in Appendix B.

3.3.2 Comparisons of the Calculated and Measured Responses

The comparisons are made in terms of maximum rotation, rotation history (waveforms) and the moment-rotation hysteresis. The measured and the calculated rotation histories of the coupled and the uncoupled specimens are presented in Fig. 3.18 and 3.19.

The measured and the calculated absolute-maximum rotations for both coupled and uncoupled specimens are listed in Table 3.4. Ratios of the measured to the calculated absolute-maximum rotations for the coupled specimens varied from 0.93 to 1.07 for the Y-axis, and 0.88 to 1.03 for the Z-axis. The mean ratios were 0.99 for the Y-axis and 0.95 for the Z-axis. Ratios of the measured to calculated absolute-maximum rotations for the uncoupled specimens varied from 0.85 to 1.27 for the Y-axis and 0.72 to 1.09 for the Z-axis. The mean ratios were 1.05 for θ_y and 0.86 for θ_z . Calculation of rotation maxima was more successful for the coupled specimens.

Measured and calculated rotation-histories are compared using the Frequency Domain Error index (FDE, Lopez 1988, Dragovich 1996), as well by visual inspection. The FDE index is a number that indicates the agreement between the waveforms. The number varies between 0 and 1. Zero means waveforms are identical and 1 means the waveforms are 180 degree out of phase. The procedure to calculate the FDE index is given in App. D. The FDE indices for the coupled and the uncoupled specimens are listed in Table 3.5. The calculated waveforms for the coupled specimens had a mean FDE index of 0.25 and the calculated waveforms of the uncoupled specimens had a mean FDE index of 0.30. The results for the coupled specimens had lower index values. Also, visual

inspection shows that the calculated waveforms for the coupled specimens matched the measured waveforms better.

In specimens UHW and CHW (both specimens with heavy masses and weak beams), the rotation response was uniform and high for the 10-sec. duration considered. For all other specimens, response-history curves exhibited phases of high and low response. Phases of high response occurred approximately in the time intervals from 1 to 3 seconds and 5.5 to 7 seconds. It is evident that the correlation of calculated and measured curves was highly satisfactory during high response phases for all specimens. The correlation was not as good for the low response phases. The reason for this is that, at low response especially after yielding, the calculated rotations are sensitive to assumed material response.

The measured and the calculated moment-rotation hystereses are given in Fig. 3.20 and 3.21. The measured and the calculated absolute-maximum moments are listed in Table 3.6. Ratios of the measured to the calculated absolute-maximum moment for the coupled specimens varied from 1.07 to 1.14 for the Y-axis and 0.96 to 1.05 for the Z-axis. The mean ratios were 1.10 for the Y-axis and 1.00 for the Z-axis. Ratios for the uncoupled specimens were varied from 1.11 to 1.18 for the Y-axis and 1.07 to 1.14 for the Z-axis. The mean ratios were 1.14 for the Y-axis and 1.09 for the Z-axis. Except for the Z-axis moments of the coupled specimens, the calculated maximum-moments were about 10% higher than the measured values. Difference between the measured and the calculated maximum-moments are attributed to strain hardening and the strain rate effects of the reinforcing bars. If the strain-rate effects observed by Dragovich (increase in yield stress of 5% for the #3 bars and 7% for #4 bars) are considered to hold at higher strains, then the mean ratios of measured to calculated maximum-moments are improved. The maximum-moment measured to calculated mean ratios for the coupled specimens became 1.04 for the Y-axis and 0.94 for the Z-axis. The mean ratios for the uncoupled specimens became 1.08 for the Y-axis and 1.02 for the Z-axis.

The biaxial-moment traces of measured and calculated responses of the coupled specimens are presented in Fig. 3.22. The loci of the measured (thin solid line) and calculated (thin broken lines) curves fall in the same regions. Both traces traveled along an axis from the 2nd quadrant to the 4th quadrant. The limiting moment-envelope curves for the coupled specimens are also shown in the figures. Two moment-envelope curves are shown, one representing the first bar yield and the second representing yielding of all bars in tension. For all specimens, the measured moments exceeded the calculated moment-envelope curves, but the difference was less than 15 %. The excess moment is attributed to the strain hardening and to the strain-rate effects of the reinforcing bars.

Returning to the data illustrated in Fig. 3.20 and 3.21, "pinching" is observed in the measured moment-rotation curves. Typically and would have been expected, the pinching occurred in regions of stress reversals with crack closing affecting the incremental stiffness. There is another reason for the pinching observed in the coupled specimens. If yielding has occurred in one direction, yielding in the orthogonal direction will occur at a lower moment creating the illusion of pinching. For example, as corroborated by the calculations shown in Fig. 3.20.2d (0-2 sec.), pinching was observed for specimen CHS (Fig3.20.2c, 0-2 sec.). The phenomenon observed there is due to material interaction rather than the effect of crack closing on stiffness.

3.4 Summary

The results from the proposed model were compared with the experimental results. Depending on the loading type, comparisons were made in two groups: static and dynamic. The experiments by Otani and Cheung (1981) were used for static loading comparisons and the experiments by Dragovich (1996) were used for dynamic loading comparisons.

The experiments by Otani and Cheung (1981) were grouped in two (depending on loading type): uniaxial and biaxial. Comparisons of the calculated and measured responses of specimens were based on the maximum force levels, the energy dissipation, and the stiffness of the responses. The calculated maximum force levels were within $\pm 10\%$ of the measured values. The maximum force levels calculated for the specimens under uniaxial loading were agreed closely with the measured maximum forces. Except for the response at rotation amplitudes lower than the yield rotation, calculated energy dissipation of the specimens was within $\pm 15\%$ of the measured values. At amplitudes below yield, the calculated energy dissipation was lower than the measured values. The stiffness of the calculated responses was within $\pm 10\%$ of the measured values.

Depending on material interaction in the specimens, the experiments by Dragovich (1996) were also grouped in two: coupled and uncoupled. Comparisons of the measured and the calculated responses of the specimens were based on maximum rotation, rotation history, and the moment-rotation hysteresis. The mean ratio of the measured to the calculated absolute-maximum rotations for the coupled specimens was 0.96 and for the uncoupled specimens, the mean ratio was 0.90.

The measured and calculated rotation histories were compared using the FDE index (App. D). The FDE index varies between 0 and 1, values closer to 0 means better agreement between the compared waveforms. The mean FDE index value for the comparison of measured and calculated rotation-history waveforms was 0.25 for the coupled specimens and 0.30 for the uncoupled specimens. Rotation-History curves of the specimens exhibited phases of high and low responses. It was evident that the agreement of measured and calculated curves was highly satisfactory for the high response phases. For the low response phases, the correlation was not as good. The mean ratio of the measured and calculated absolute-maximum moments was 1.05 for the coupled specimens and 1.12 for the uncoupled specimens. The difference between the measured and the calculated values were attributed to strain hardening and the strain-rate effects of the reinforcing bar.

Two kinds of "pinching" (changes in the slope of the moment-rotation response from steep to nearly flat, back to steep) were observed. The first kind was the type usually observed in elements with slip or crack closing and was primarily by closing of the cracks. The second kind was due to the biaxial nature of the loading. If the element had already yielded in direction-1, it yielded in direction-2 at a lower moment but could regain strength in direction-2 if the element was unloaded in direction-1. Thus, the resulting force-displacement curve would resemble the one observed for crack closing. The calculated moment-rotation hysteresis was sensitive to the pinching occurring because of the material interaction.

Table 3.1 Material Properties of the Chosen Specimens of Otani and Cheung (1981) Experiments

	Compressive Strength of Concrete (MPa)	Tensile Strength of Concrete (MPa)	Yield Stress of Longitudinal Bars (MPa)
Specimen 5 and 6	22.6	2.7	438
Specimen 7	26.7	2.2	466

Table 3.2 Designations of the Dragovich's Specimens (1996)

Weight	Uncoupled		Coupled	
	Weak	Strong	Weak	Strong
Light	ULW	ULS	CLW	CLS
Heavy	UHW	UHS	CHW	CHS

Table 3.3 Properties of Test Specimens

Specimen	UHW	ULW	UHS	ULS
Attached Weight (kips)	4.1	2.0	4.1	2.0
Total Weight (kips)	7.0	5.1	7.1	5.1
Center of Mass in Y Direction, e (in)	14.8	12.5	14.8	14.8
Center of Mass in Z Direction, h (in)	63.4	59.4	63.4	59.4
Beam Span (Y Axis, in)	30	30	30	30
Beam Span (Z Axis, in)	42	42	42	42
Longitudinal Bars	2-#3	2-#3	2-#4	2-#4
Reinforcement Ratio (Y Axis, %)	0.77	0.77	1.4	1.4
Reinforcement Ratio (Z Axis, %)	0.86	0.86	1.5	1.5
Concrete Compressive Strength, (psi)	6200	6200	6300	6500
Flexural Steel Yield Stress (Y Axis, psi)	66.3	60.0	68.3	62.5
Flexural Steel Yield Stress (Z Axis, psi)	66.7	59.7	66.3	64.5

Table 3.3(cont) Properties of Test Specimens

Specimen	CHW	CLW	CHS	CLS
Attached Weight (kips)	4.1	2.0	4.1	2.0
Total Weight (kips)	6.9	5.0	7.0	5.0
Center of Mass in Y Direction, e (in)	14.8	12.5	14.8	12.5
Center of Mass in Z Direction, h (in)	63.4	59.4	63.4	59.4
Beam Span (Y Axis, in)	42	42	42	42
Beam Span (Z Axis, in)	42	42	42	42
Longitudinal Bars	2-#3	2-#3	2-#4	2-#3
Reinforcement Ratio (Y Axis, %)	0.77	0.77	1.4	1.4
Reinforcement Ratio (Z Axis, %)	0.86	0.86	1.5	1.5
Concrete Compressive Strength, (psi)	7200	7500	6900	7000
Flexural Steel Yield Stress, (psi)	66.3	66.5	65.3	67.5

Table 3.4a Measured and Calculated Absolute-Maximum Rotations and Their Ratios for the Coupled Specimens

	Y-axis		Z-axis		Measured to Calculated Ratios	
	Measured	Calculated	Measured	Calculated	Y-axis	Z-axis
CHW	1.4	1.5	2.3	2.6	0.93	0.88
CLW	1.6	1.5	2.5	2.5	1.07	1.00
CHS	1.6	1.6	3.1	3.0	1.00	1.03
CLS	1.6	1.7	2.7	3.0	0.94	0.90

Table 3.4b Measured and Calculated Absolute-Maximum Rotations and Their Ratios for the Uncoupled Specimens

	Y-axis		Z-axis		Measured to Calculated Ratios	
	Measured	Calculated	Measured	Calculated	Y-axis	Z-axis
UHW	1.9	1.5	3.4	4.7	1.27	0.72
ULW	1.7	1.6	2.0	2.8	1.06	0.71
UHS	1.6	1.6	2.6	2.9	1.00	0.90
ULS	1.7	2.0	2.4	2.2	0.85	1.09

Table 3.5 Frequency-Domain-Error Index Values of Specimens Tested by Dragovich (1996)

	CHW	CLW	CHS	CLS	UHW	ULW	UHS	ULS
Y-axis	0.21	0.17	0.27	0.27	0.29	0.20	0.27	0.33
Z-axis	0.25	0.20	0.30	0.34	0.27	0.40	0.30	0.33

Table 3.6a Measured and Calculated Absolute-Maximum Moments and Their Ratios for the Coupled Specimens

	Y-axis		Z-axis		Measured to Calculated Ratios	
	Measured	Calculated	Measured	Calculated	Y-axis	Z-axis
CHW	150	132	64	61	1.14	1.05
CLW	145	134	55	57	1.09	0.96
CHS	244	229	96	96	1.07	1.00
CLS	262	240	91	91	1.09	1.00

Table 3.6b Measured and Calculated Absolute-Maximum Moments and Their Ratios for the Uncoupled Specimens

	Y-axis		Z-axis		Measured to Calculated Ratios	
	Measured	Calculated	Measured	Calculated	Y-axis	Z-axis
UHW	179	156	77	72	1.15	1.07
ULW	169	143	65	60	1.18	1.08
UHS	321	290	125	110	1.11	1.14
ULS	305	271	112	103	1.13	1.09

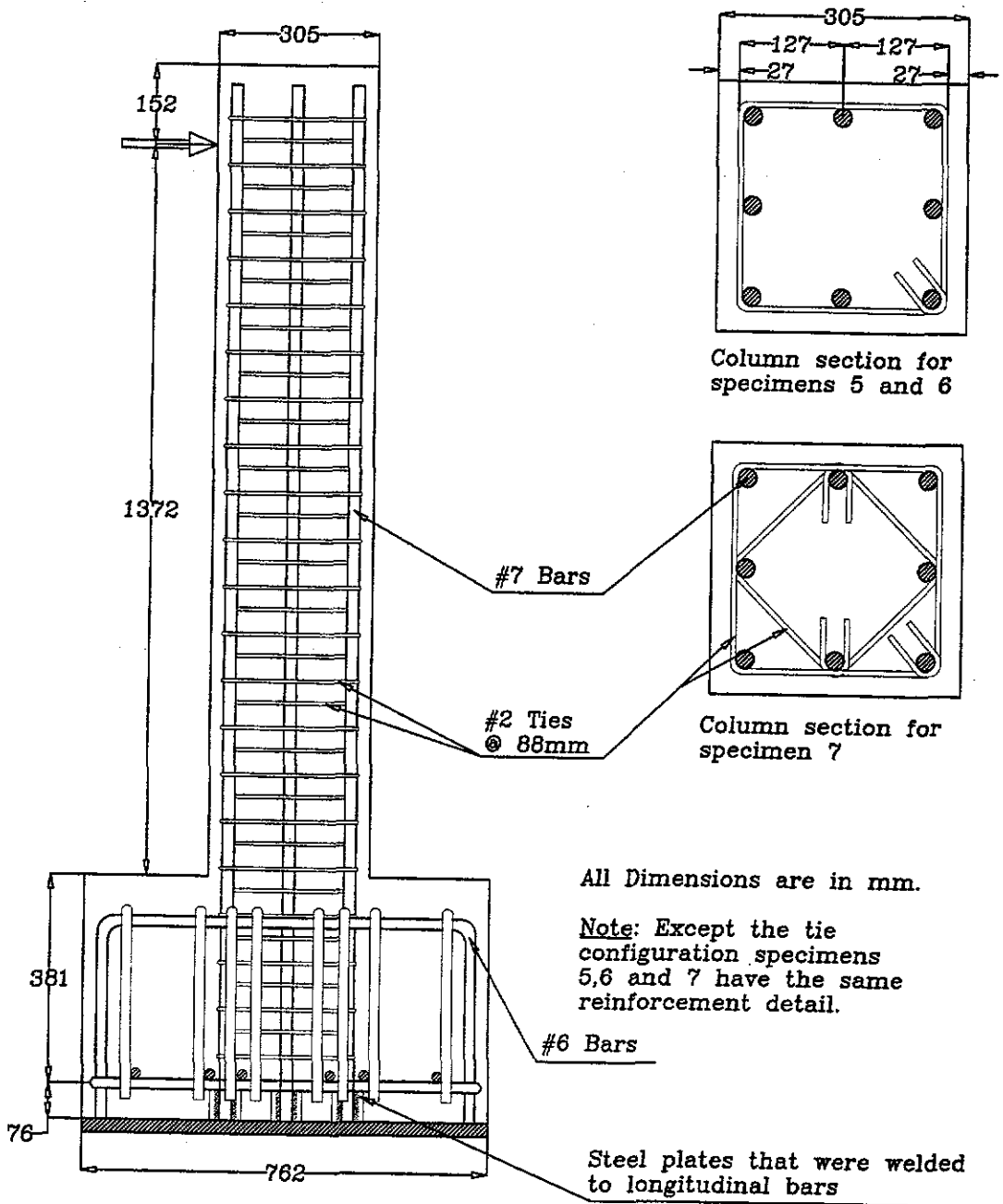


Fig. 3.1 Arrangement of Reinforcement in Specimens 5,6 and 7 (Tested by Otani and Cheung, 1981)

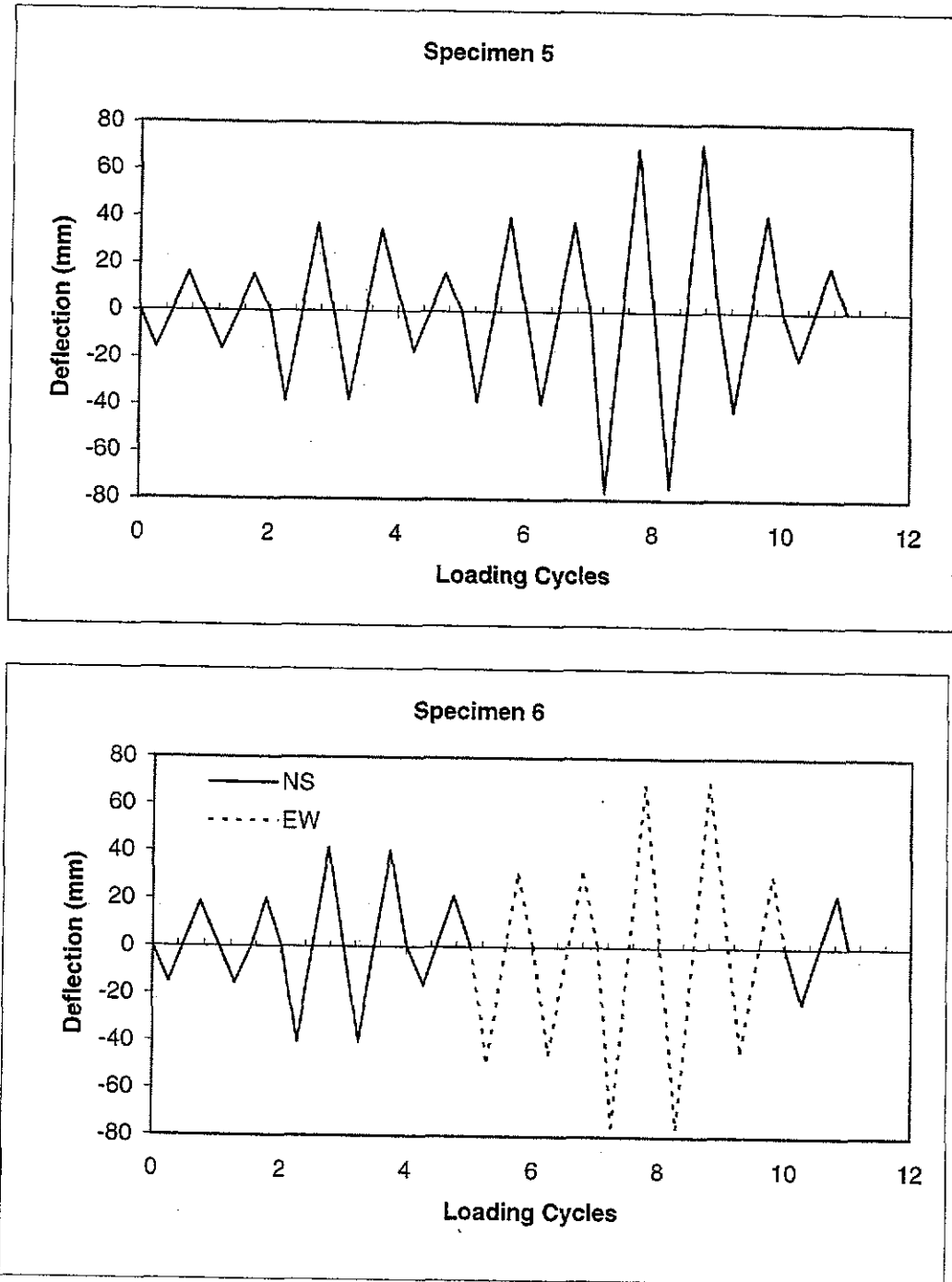


Fig. 3.2 Displacement Paths of Specimens 5 and 6

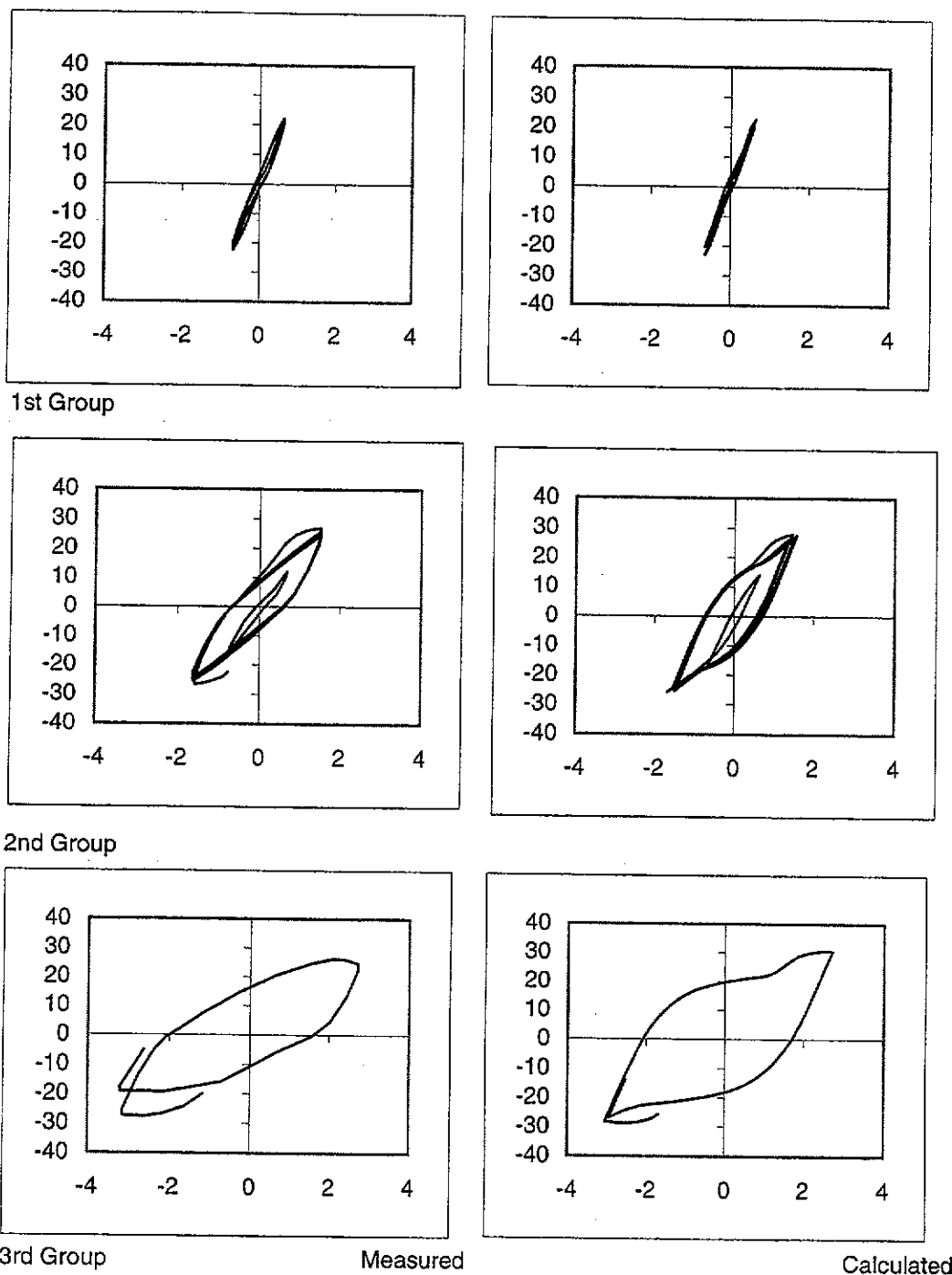
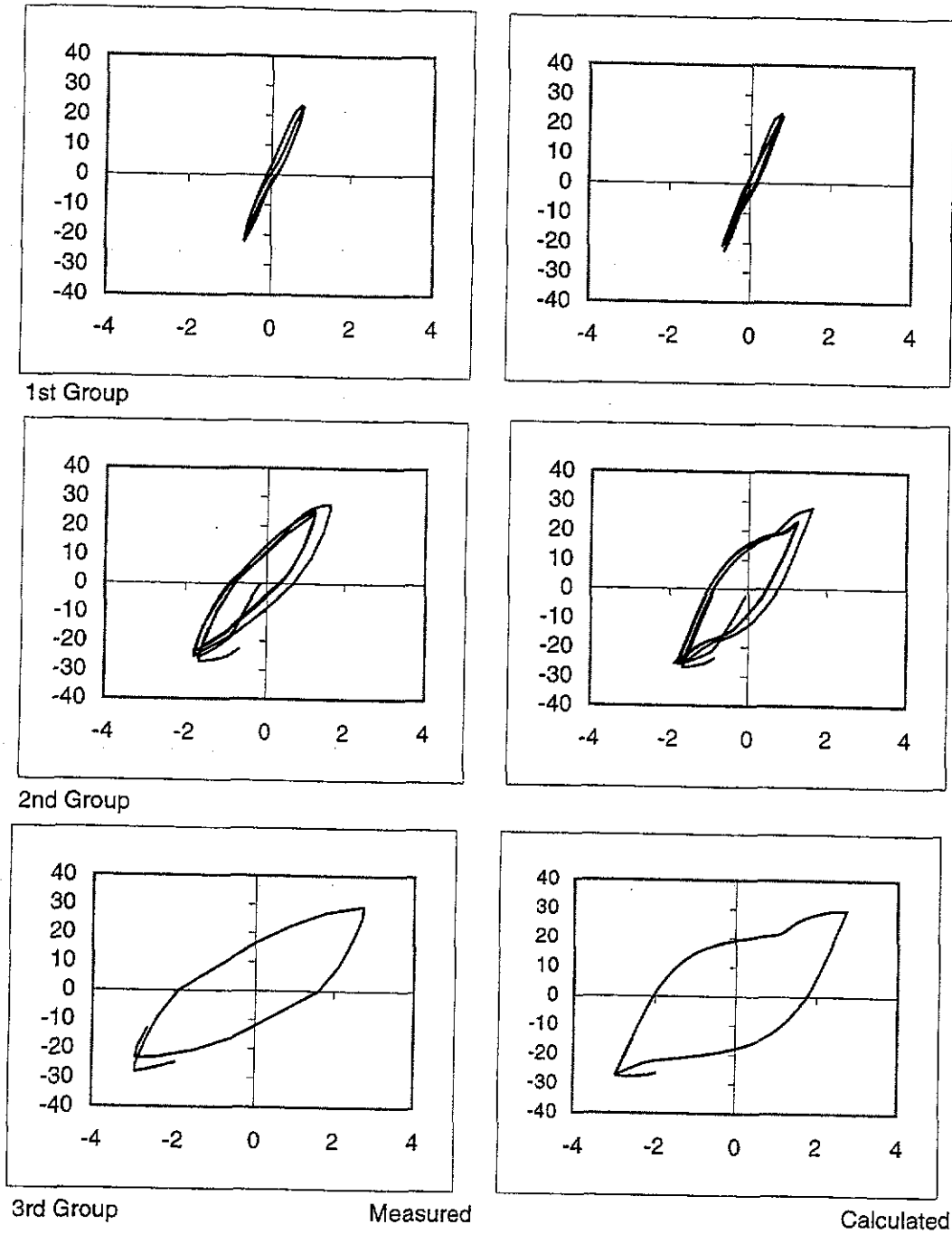


Fig. 3.3 Measured and Calculated Force-Displacement Relations of Specimen 5



Horizontal Axis: Displacement (in)
Vertical Axis: Force at the Top of the Cantilever (kips)

Fig. 3.4 Measured and Calculated Force-Displacement Relations of Specimen 6

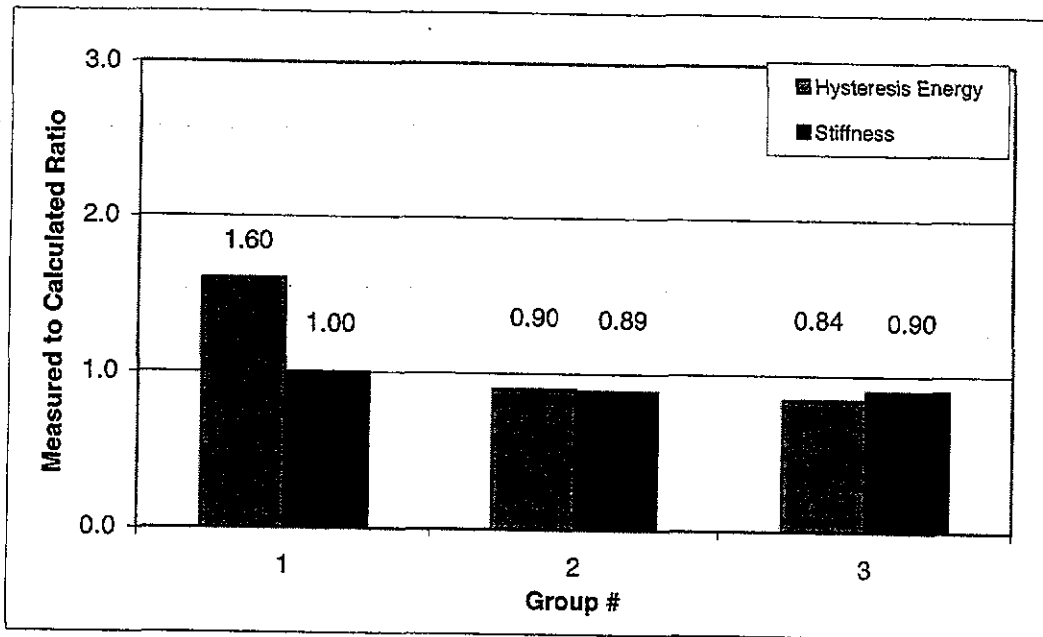


Fig3.5a Stiffness and Hysteresis Energy Comparisons of Measured and Calculated Force-Displacement Relation of Specimen 5

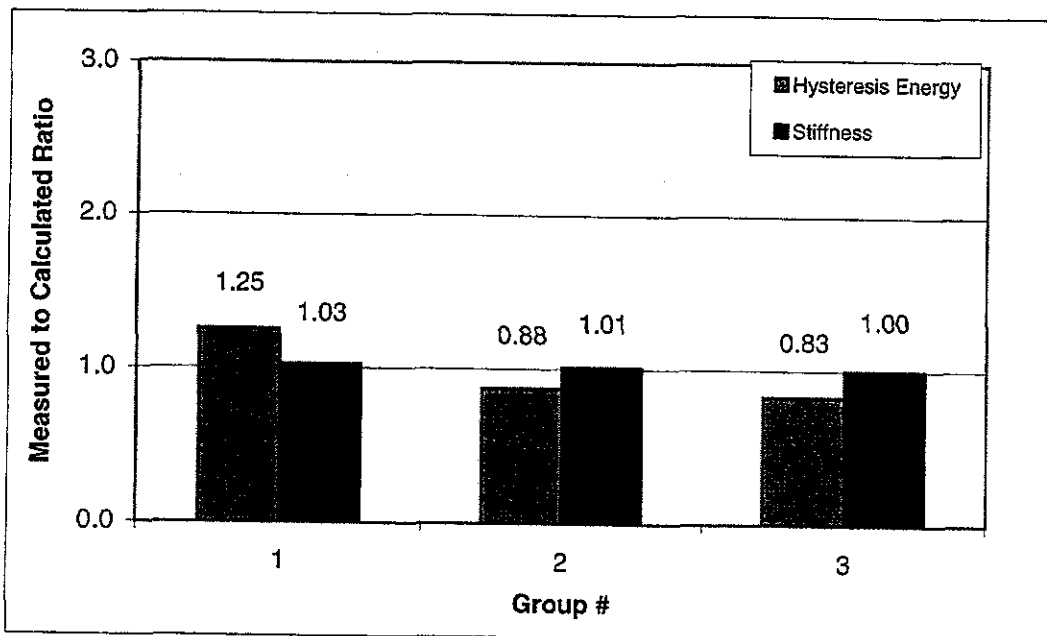


Fig3.5b Stiffness and Hysteresis Energy Comparisons of Measured and Calculated Force-Displacement Relation of Specimen 6

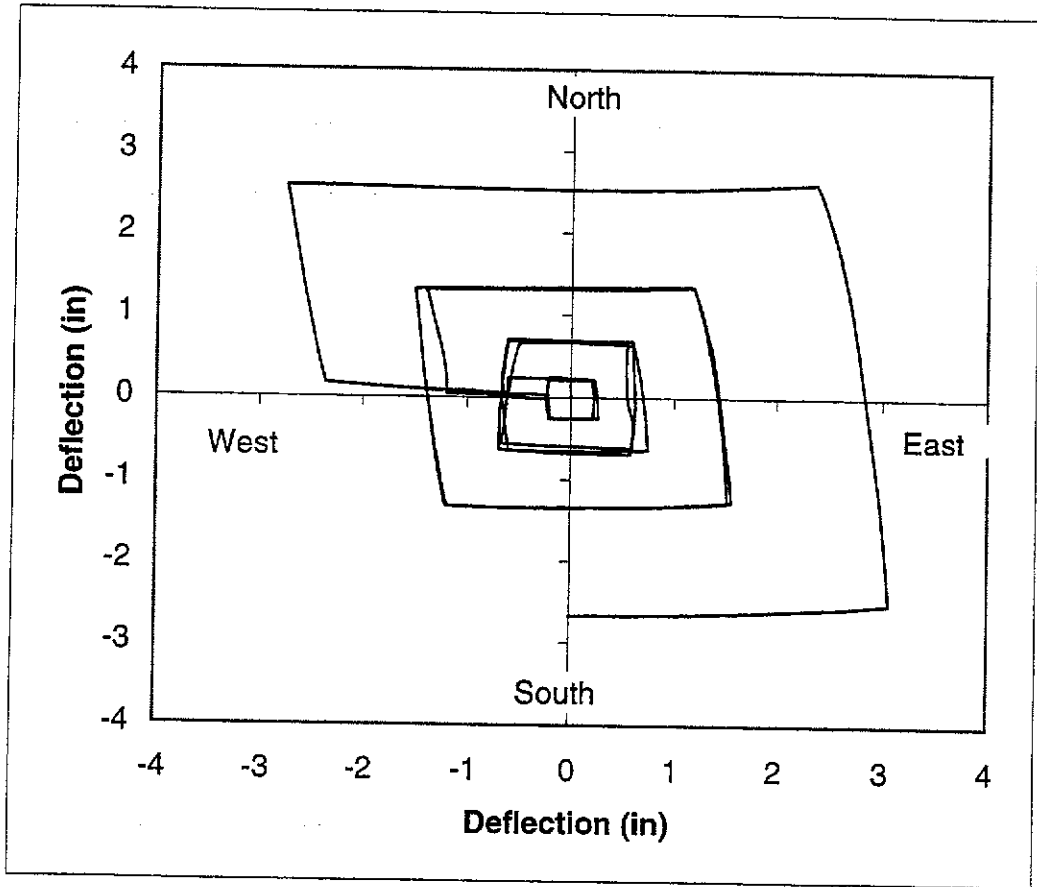


Fig. 3.6 Displacement Path of Specimen 7

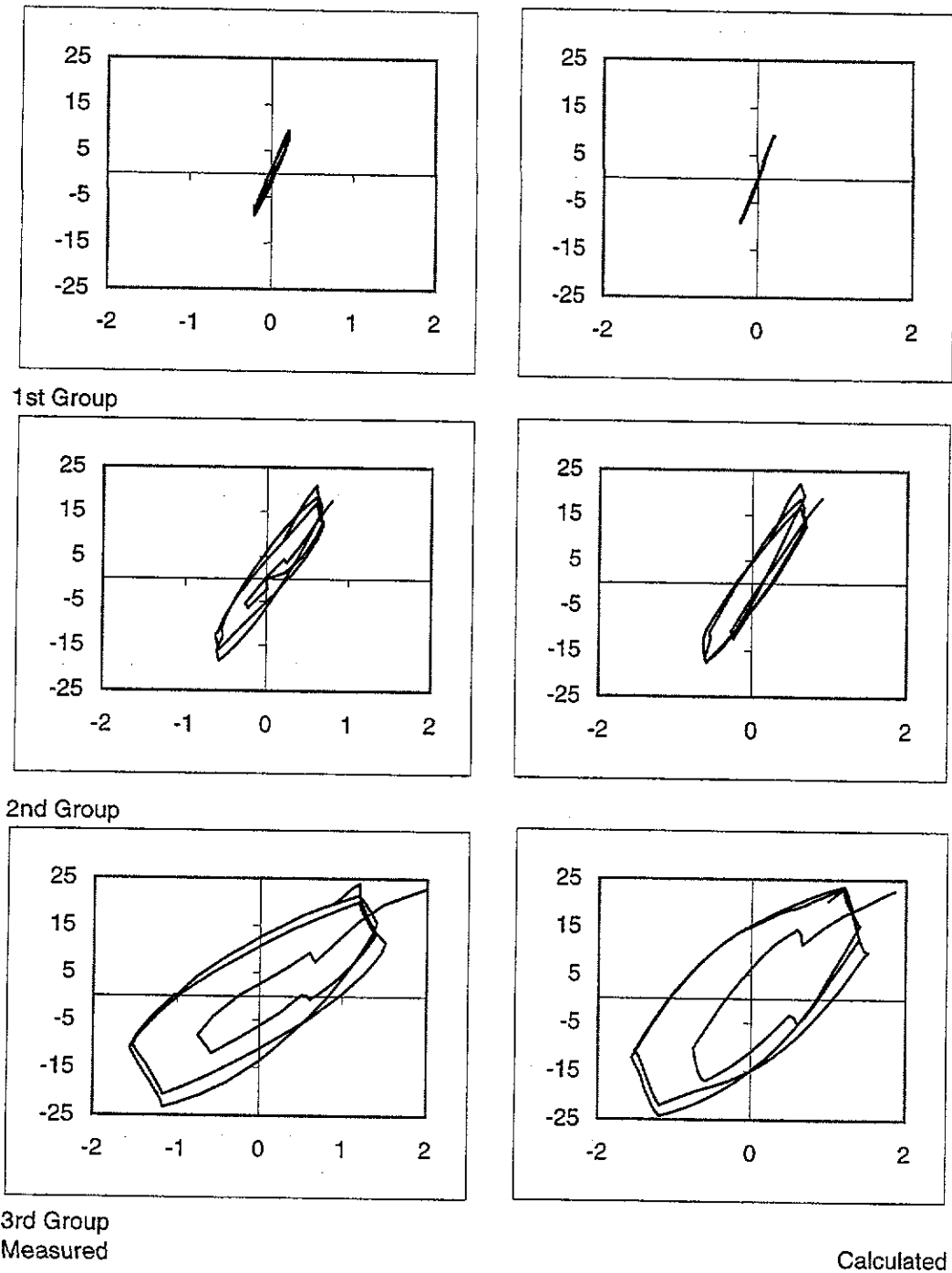


Fig. 3.7 Measured and Calculated Force-Displacement Relations of Specimen 7
Relations Of Specimen 7 EW Direction

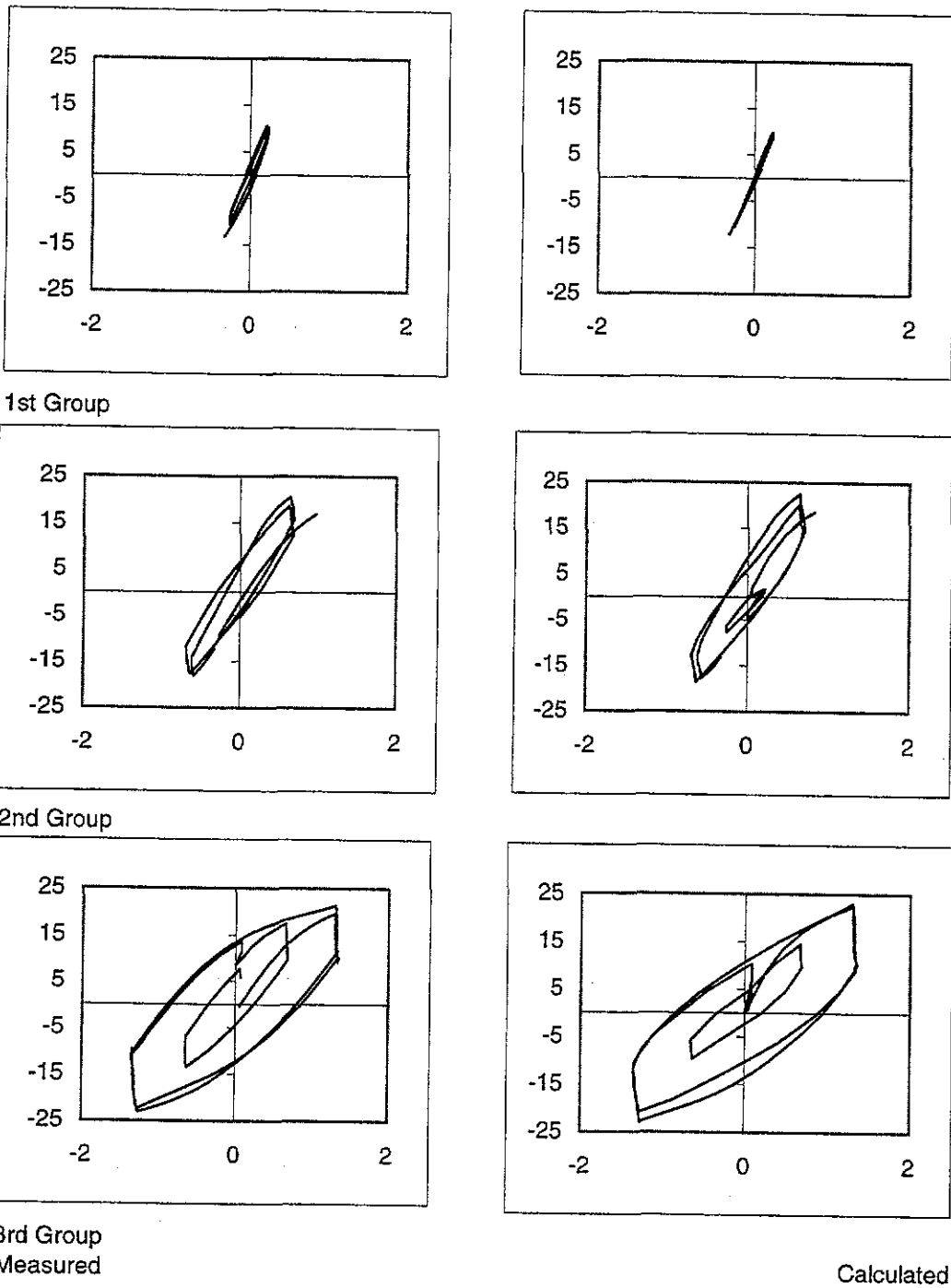


Fig. 3.8 Measured and Calculated Force-Displacement Relations of Specimen 7
NS Direction

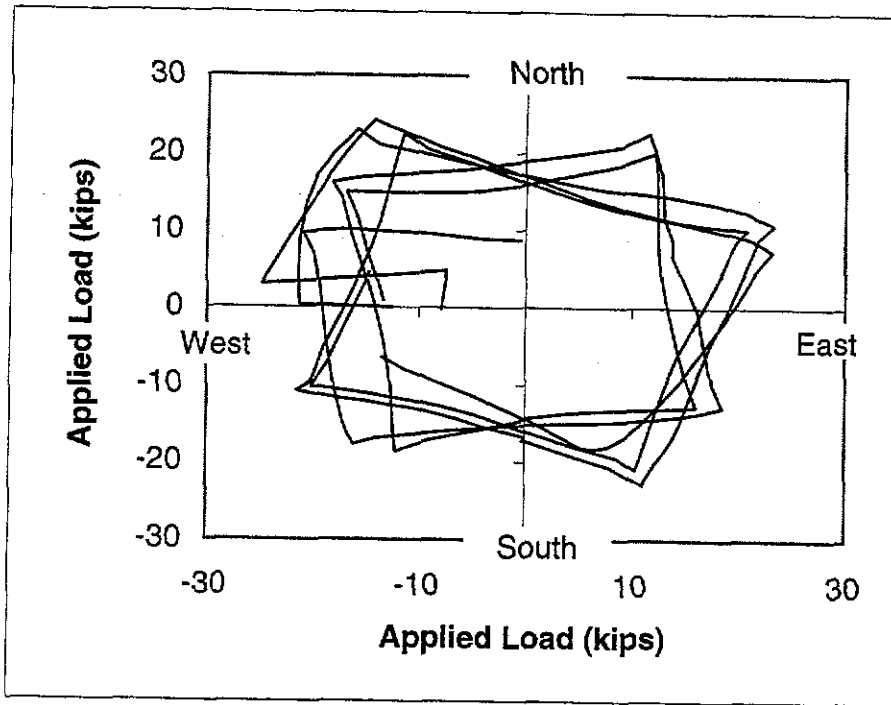


Fig. 3.9a Measured Biaxial Resistance Trace of Specimen 7

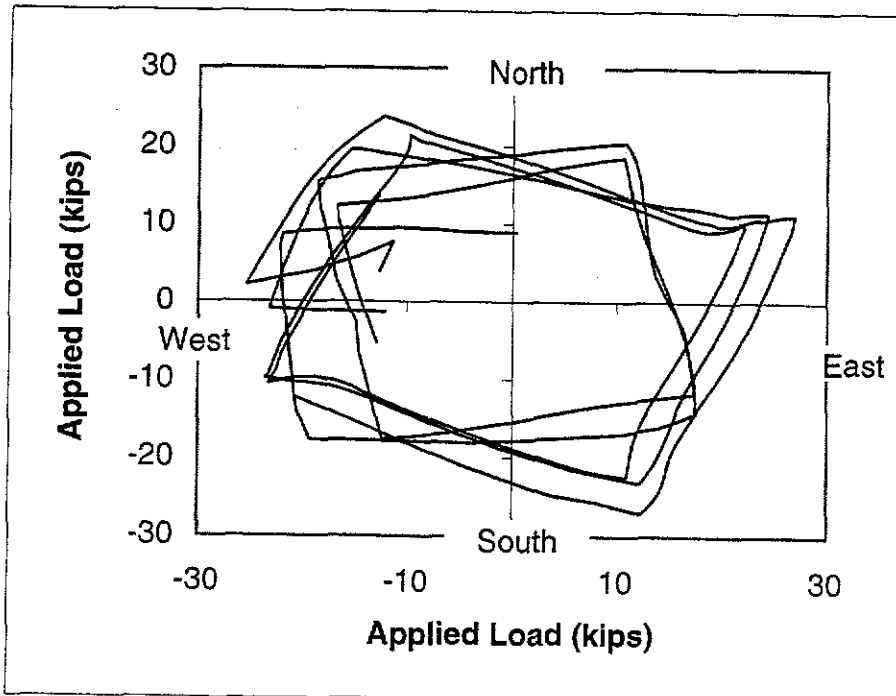


Fig. 3.9b Calculated Biaxial Resistance Trace of Specimen 7

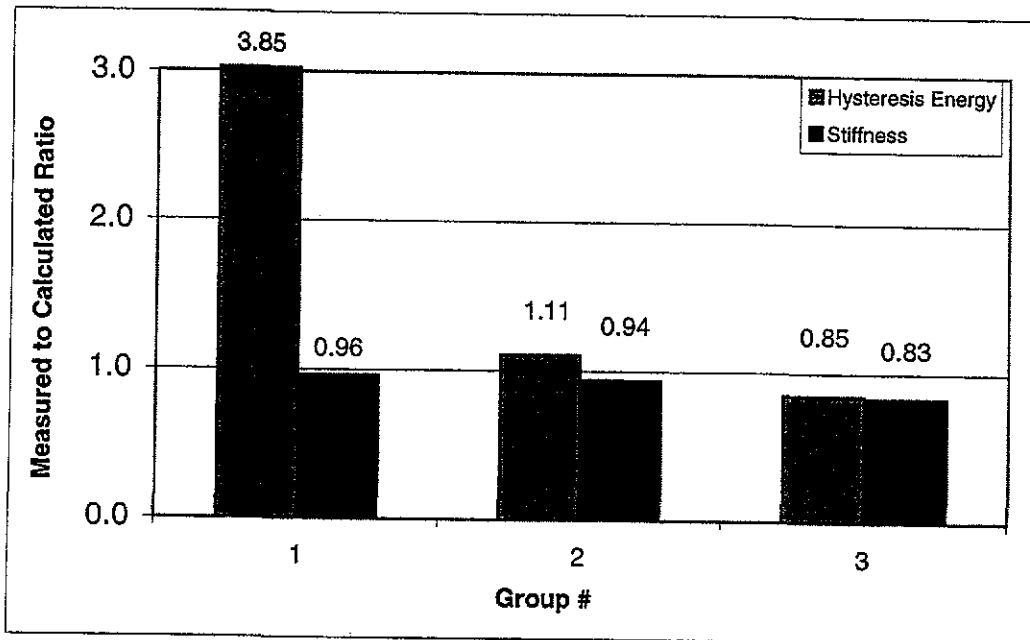


Fig3.10a Stiffness and Hysteresis Energy Comparisons of Measured and Calculated Force-Displacement Relation of Specimen 7 EW Direction

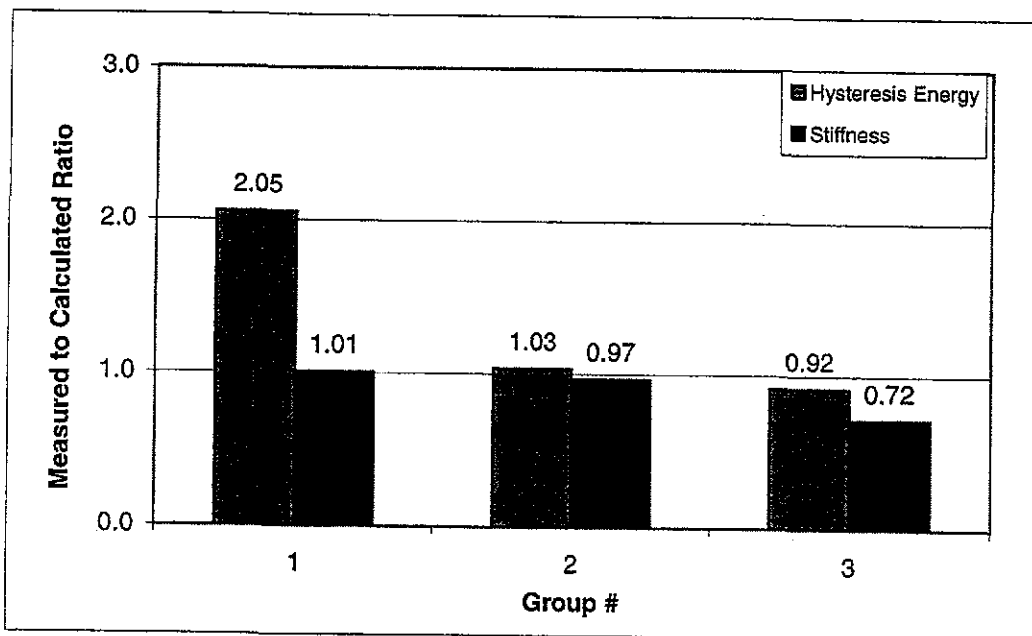


Fig3.10b Stiffness and Hysteresis Energy Comparisons of Measured and Calculated Force-Displacement Relation of Specimen 7 NS Direction

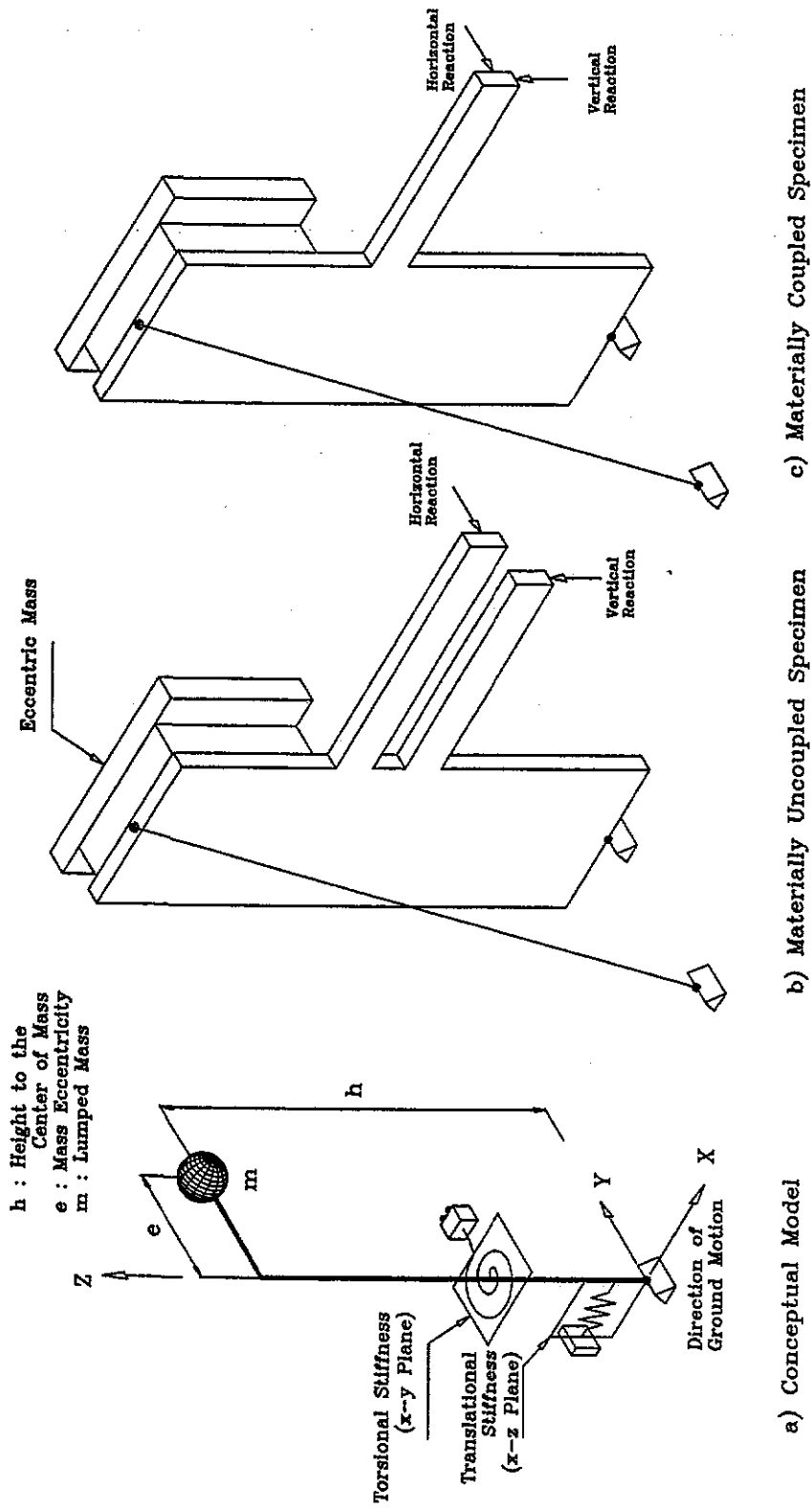


Fig. 3.11 Dragovich's Specimen

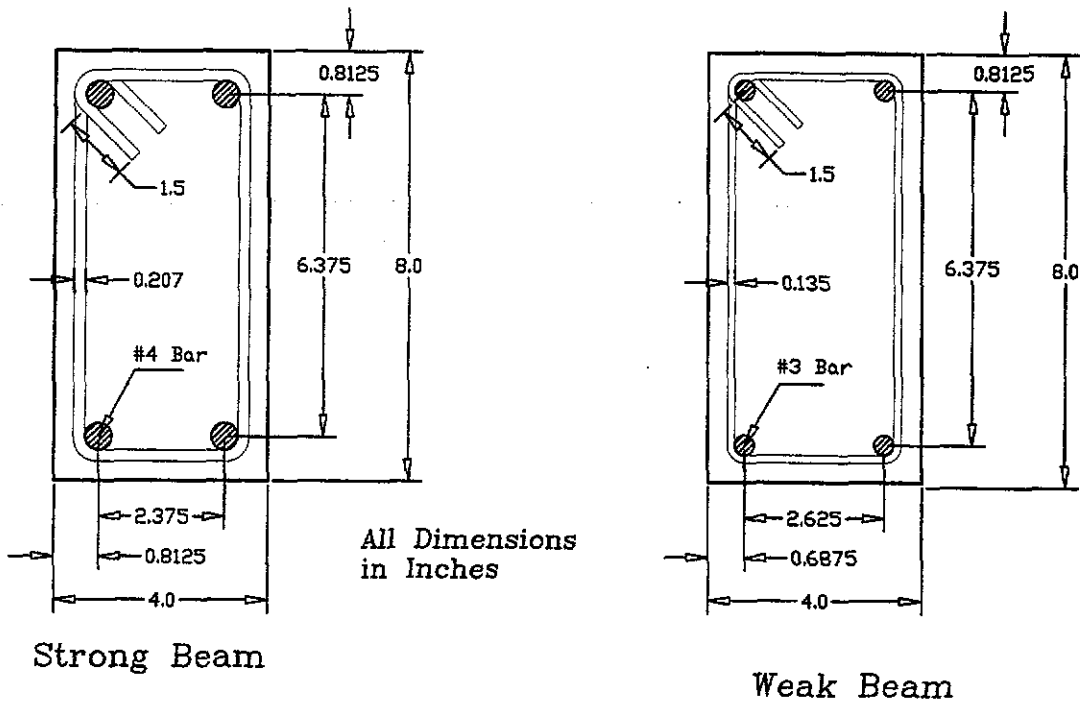


Fig. 3.12 Dragovich's Specimen Beam Sections

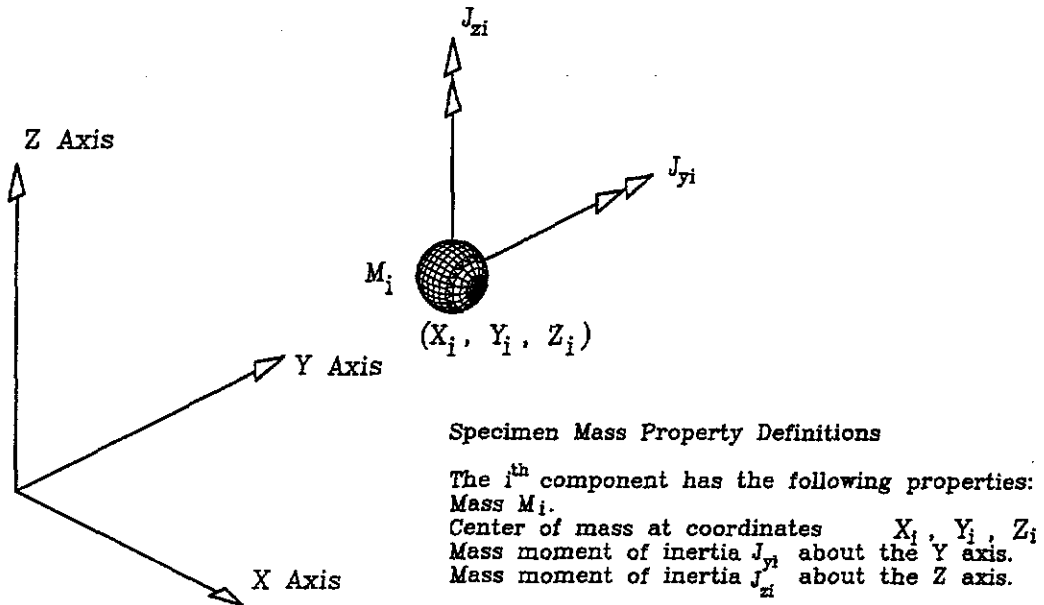
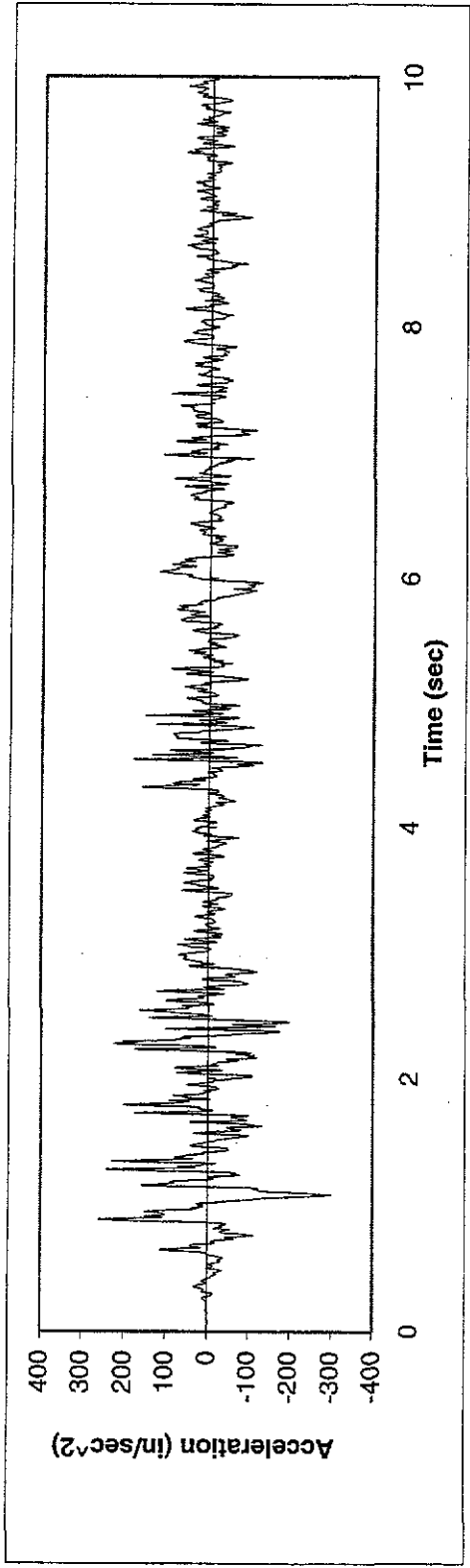
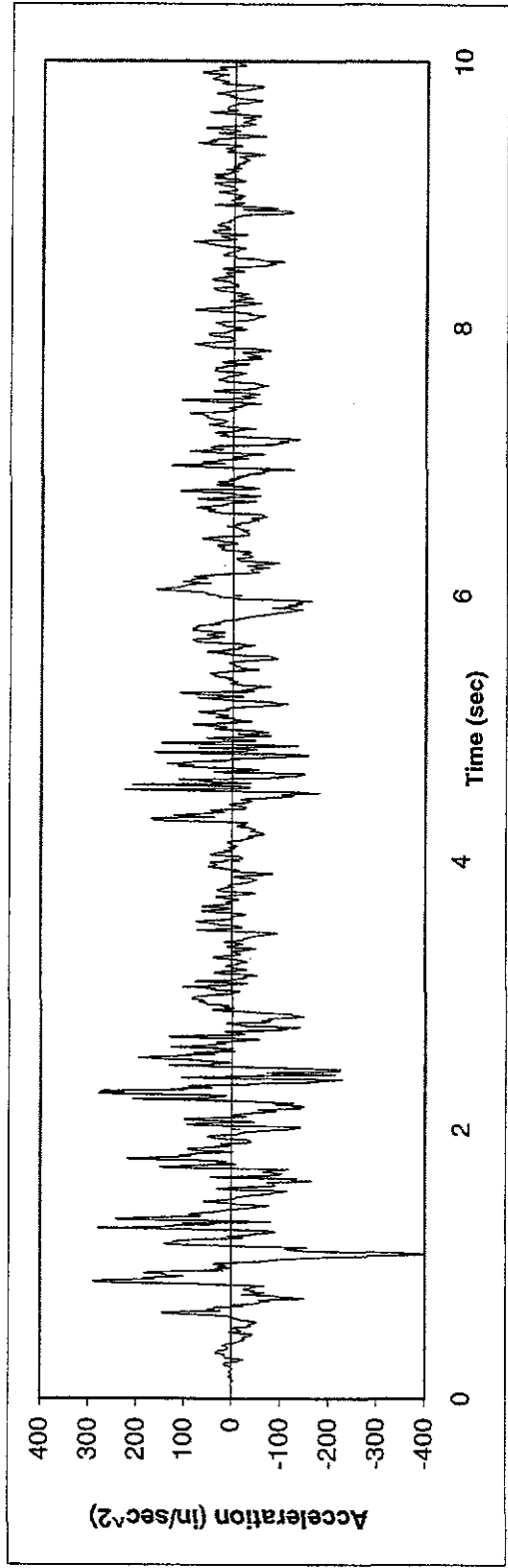


Fig. 3.13 Property Definitions of Mass Components

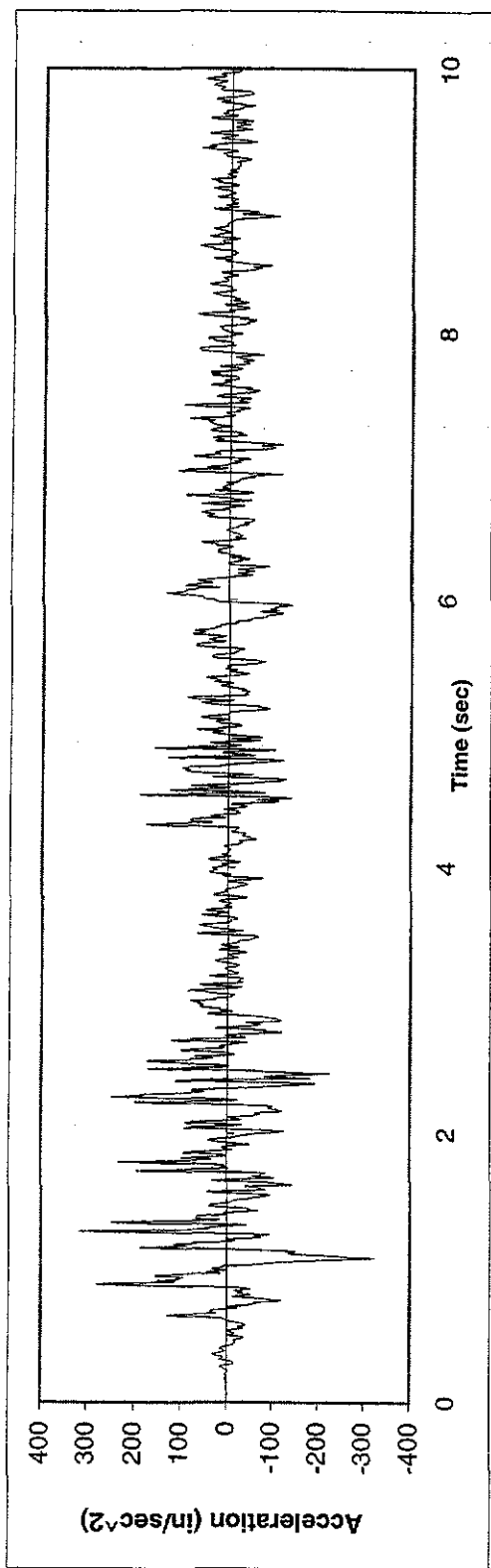


UHS

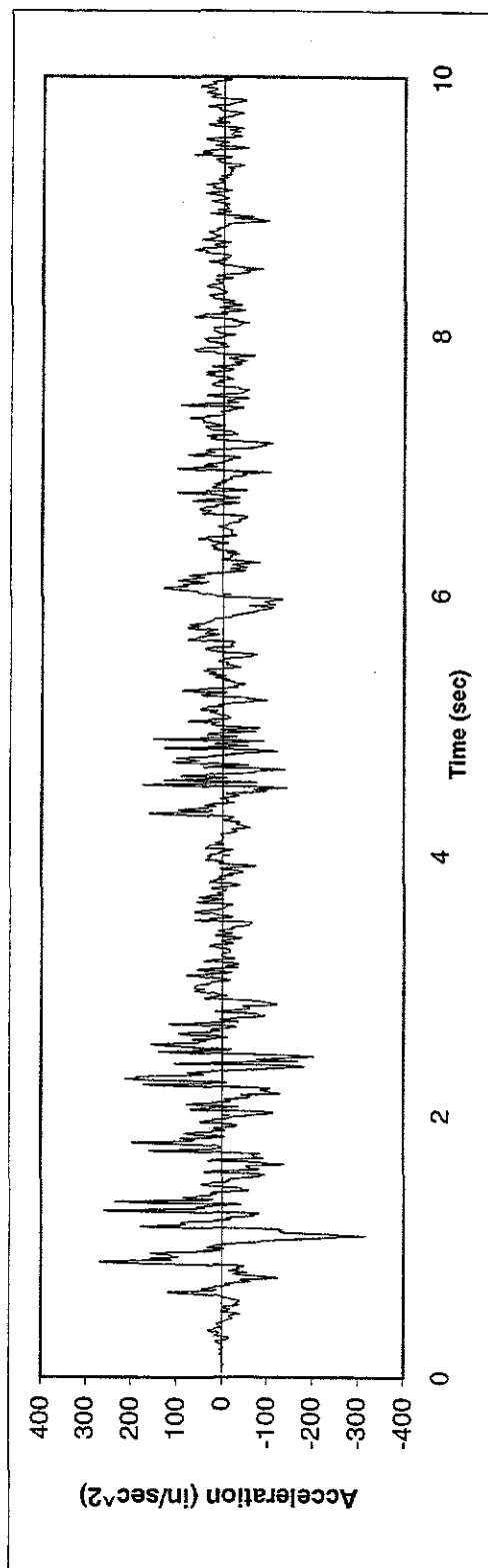


UHW

Fig. 3.14a Base Motions Used for the Second Run of the Materially Uncoupled Specimens

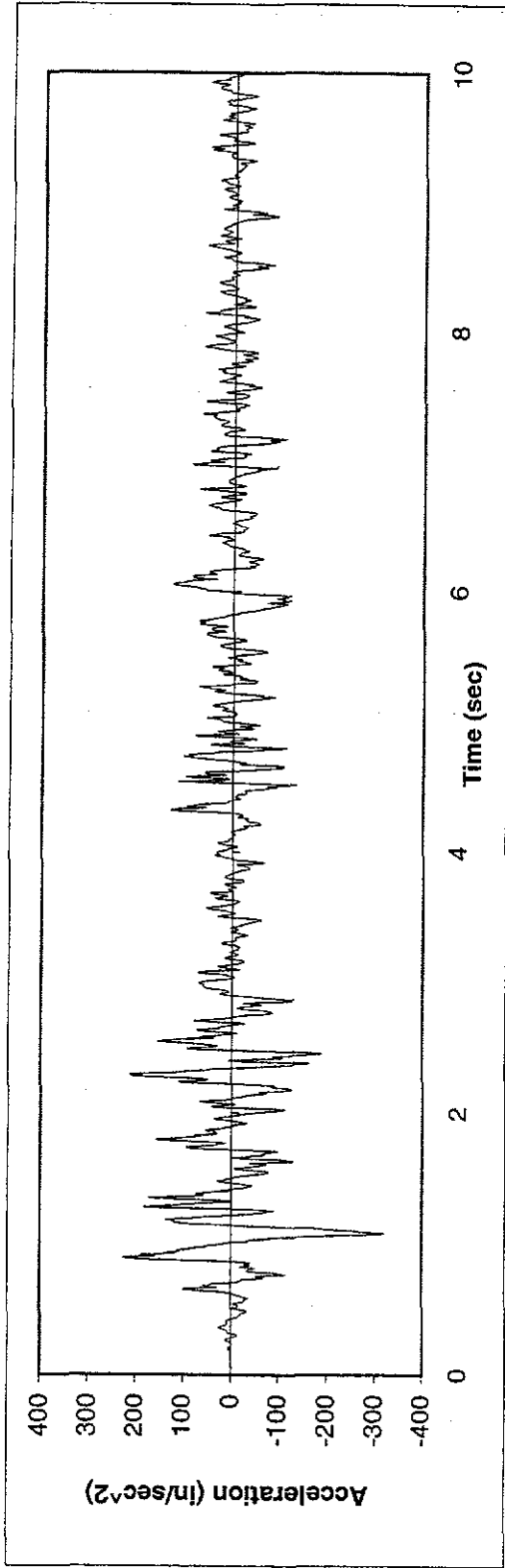


ULS

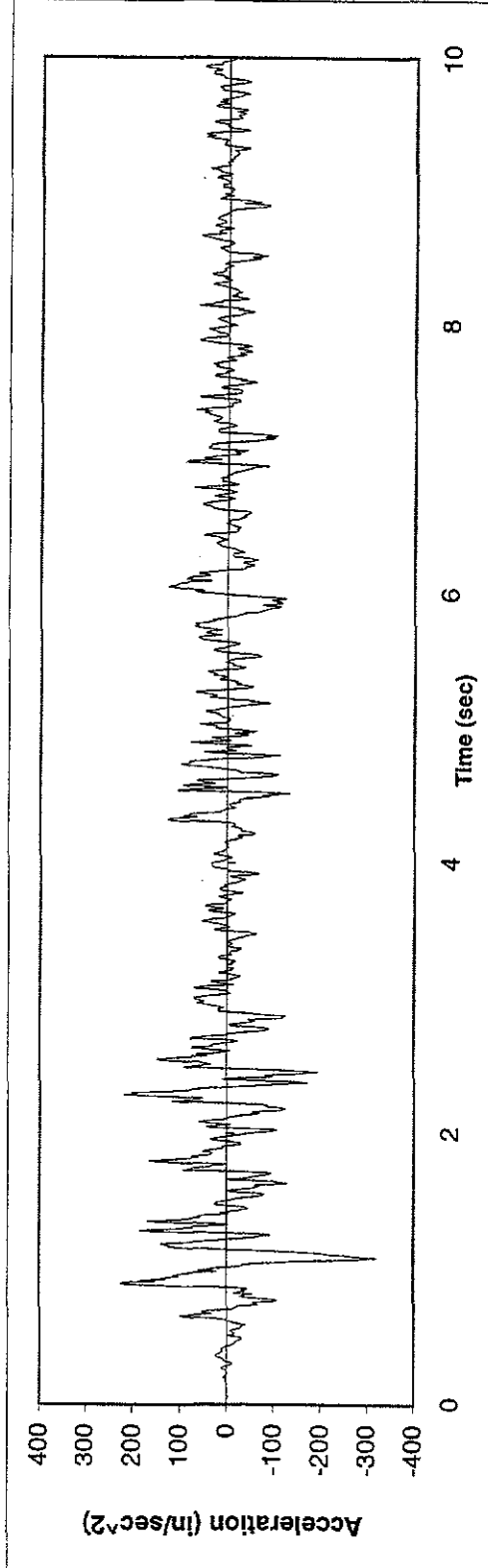


ULW

Fig. 3.14b Base Motions Used for the Second Run of the Materially Uncoupled Specimens

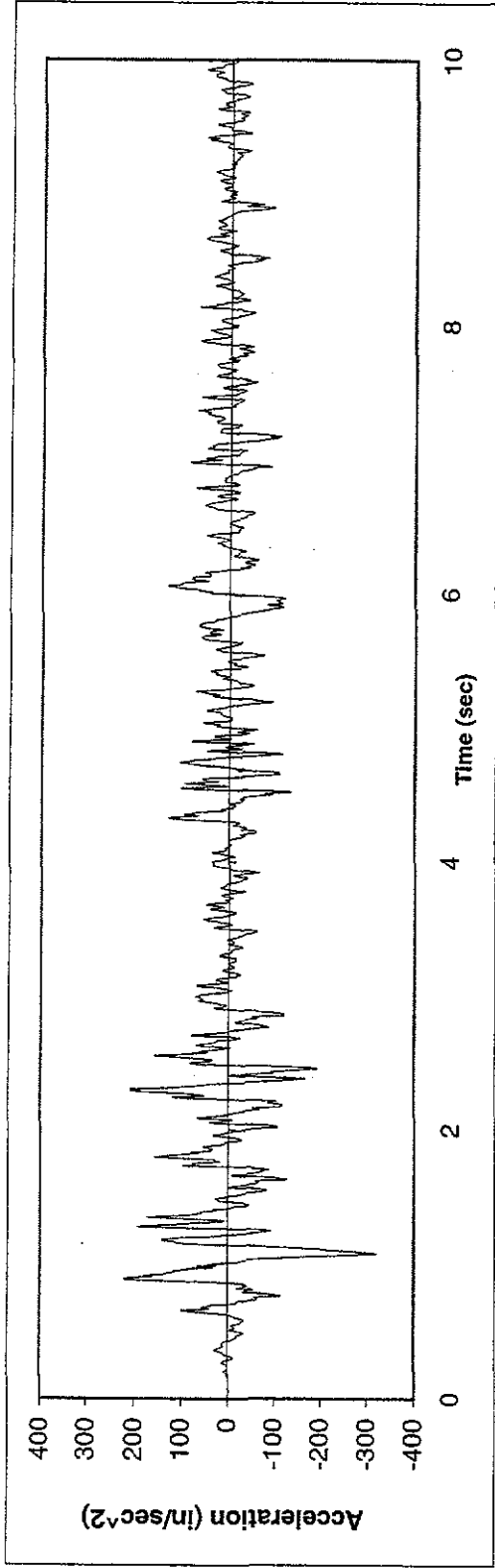


CHS

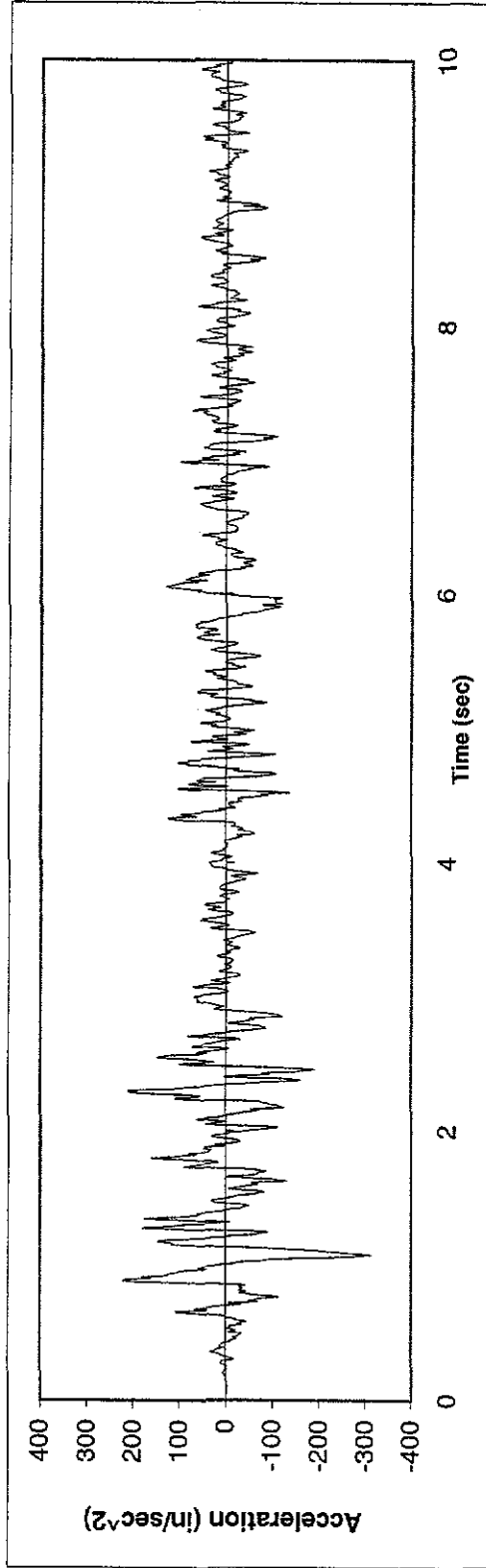


CHW

Fig. 3.15a Base Motions Used for the Second Run of the Materially Coupled Specimens



CLS



CLW

Fig. 3.15b Base Motions Used for the Second Run of the Materially Coupled Specimens

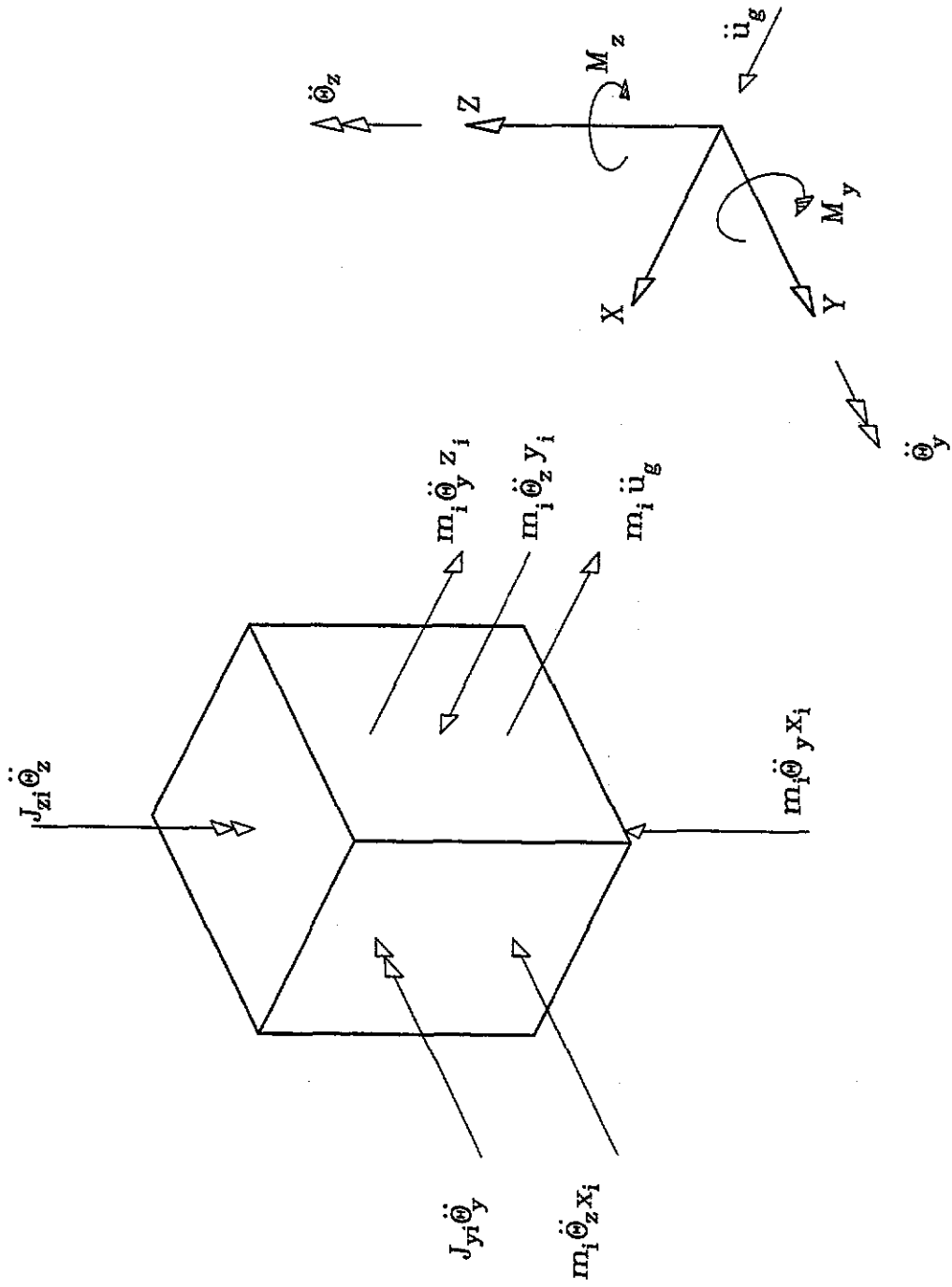


Fig. 3.16 Dynamic Equilibrium of a Mass Component

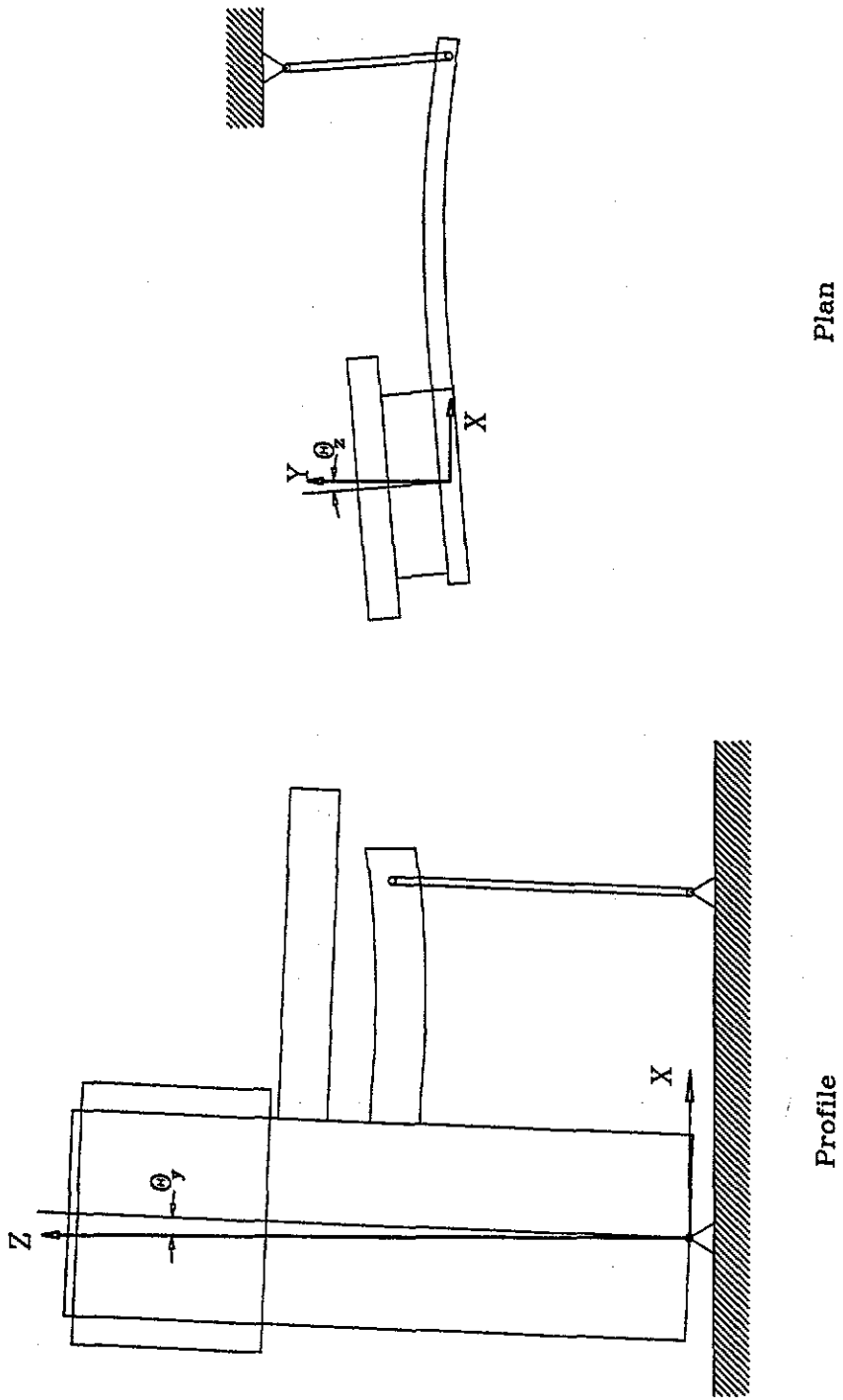


Fig. 3.17 System Rotation Definitions

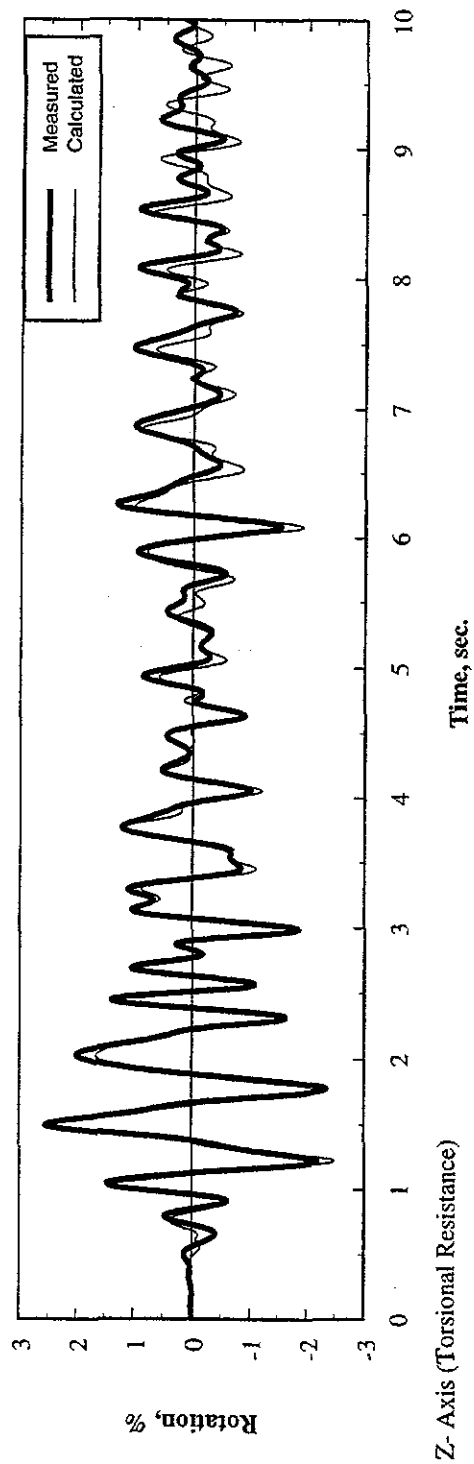
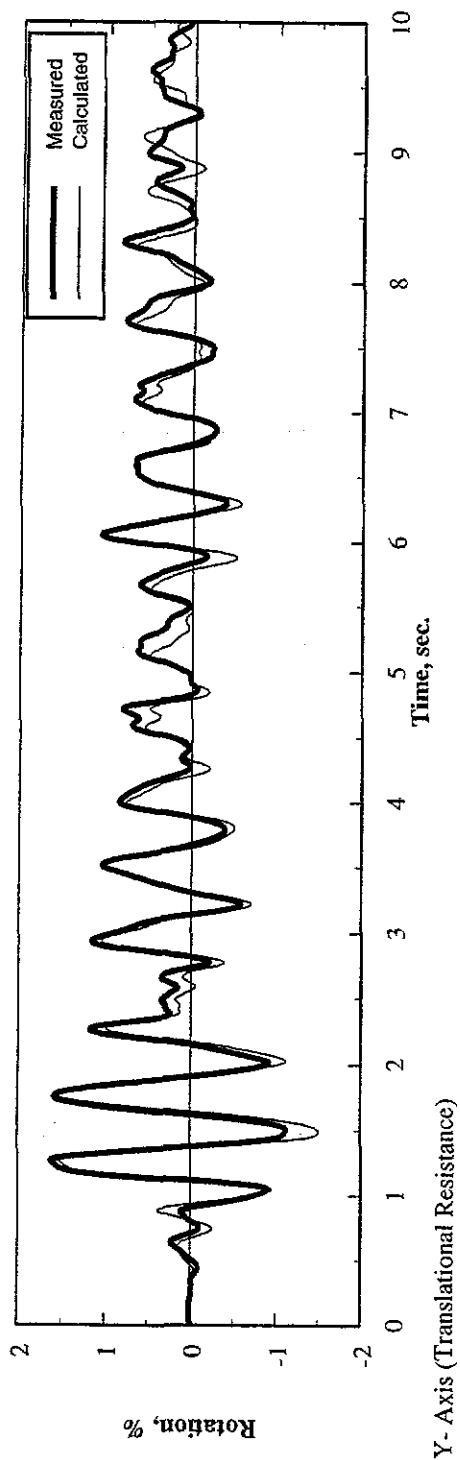


Fig. 3.18a Displacement Histories for Rotation About Axes Y and Z, CLW Run2

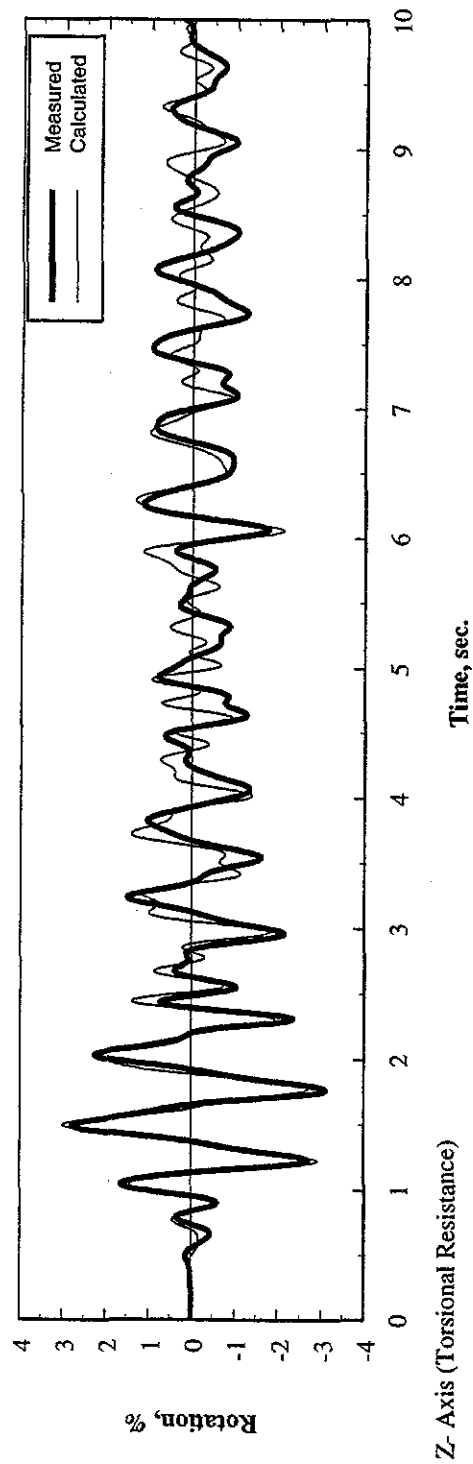
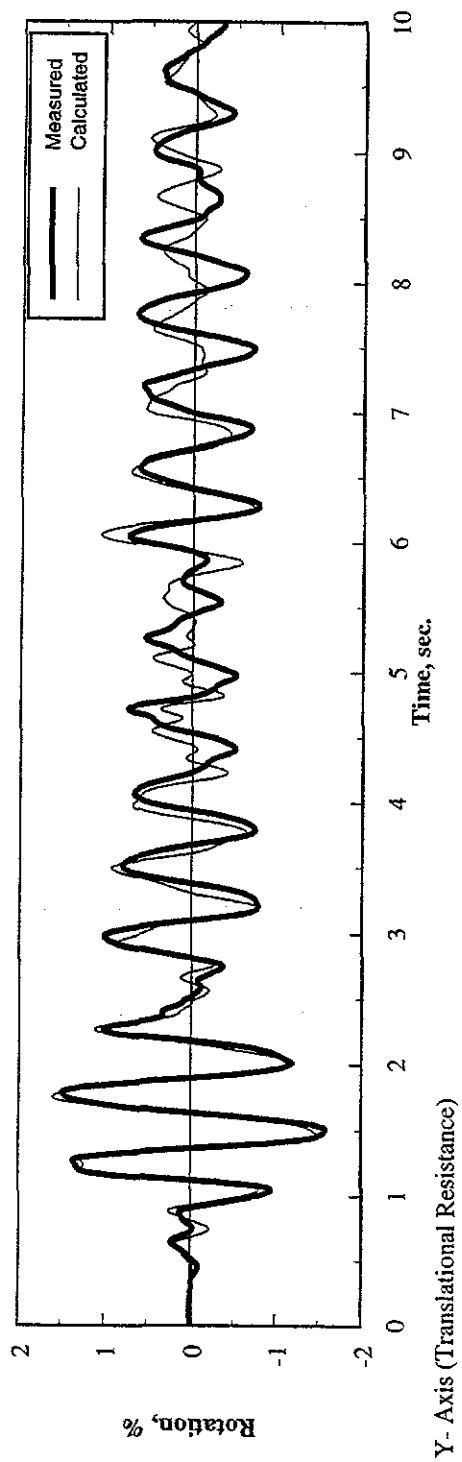


Fig. 3.18b Displacement Histories for Rotation About Axes Y and Z, CHS Run2

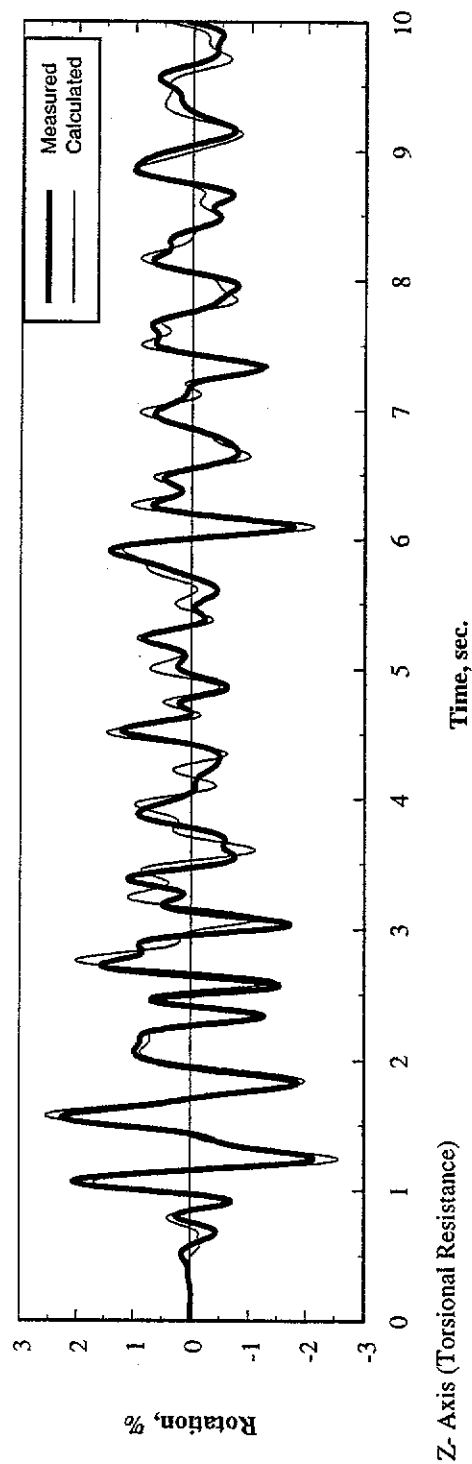
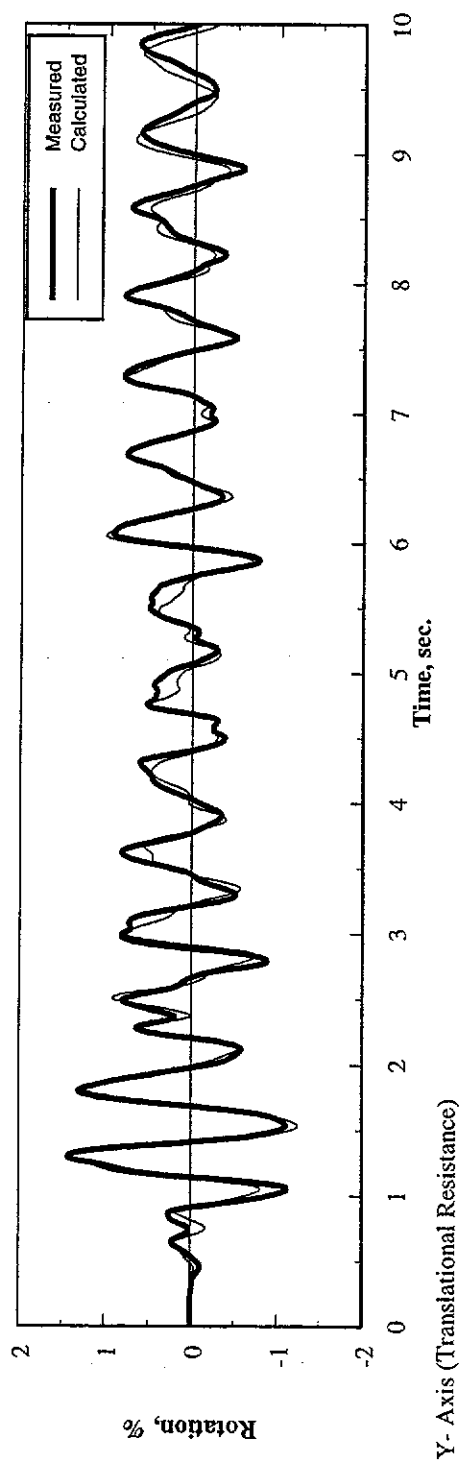


Fig. 3.18c Displacement Histories for Rotation About Axes Y and Z, CHW Run2

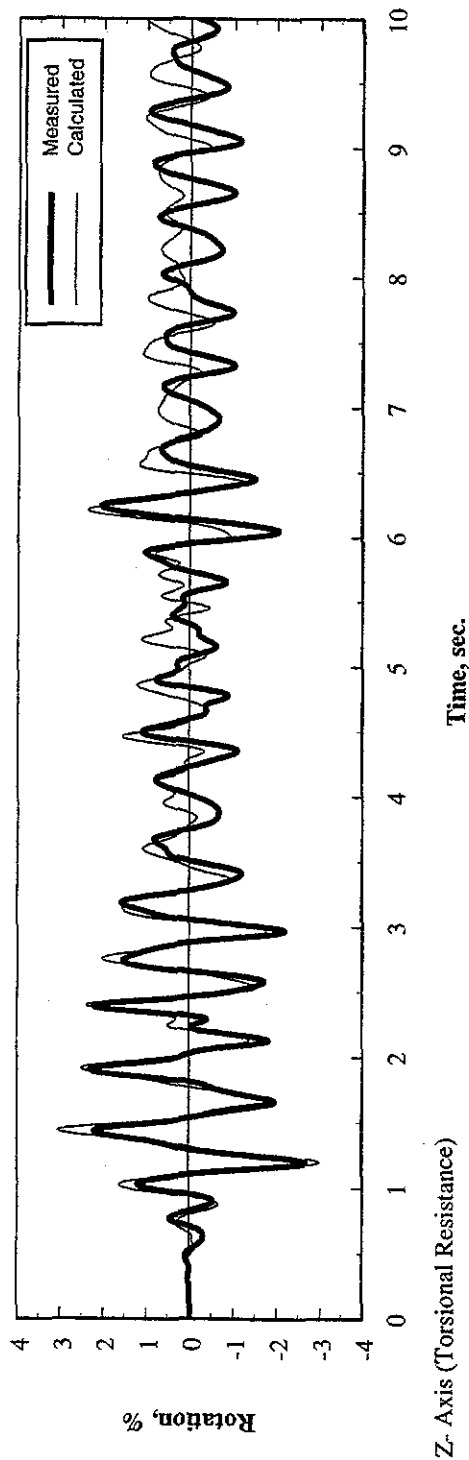
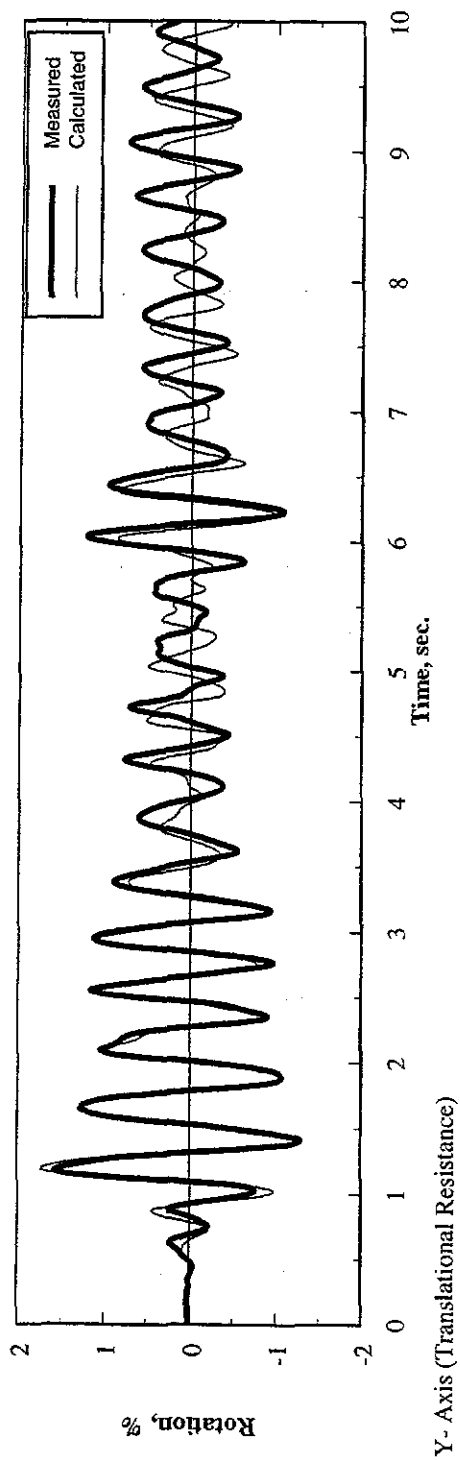


Fig. 3.18d Displacement Histories for Rotation About Axes Y and Z, CLS Run2

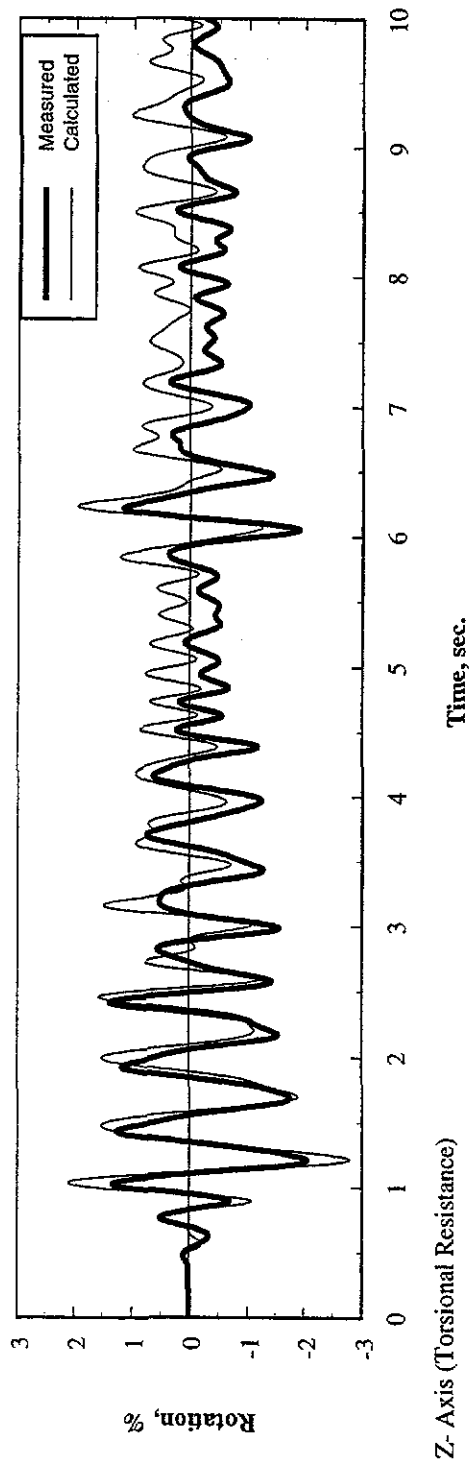
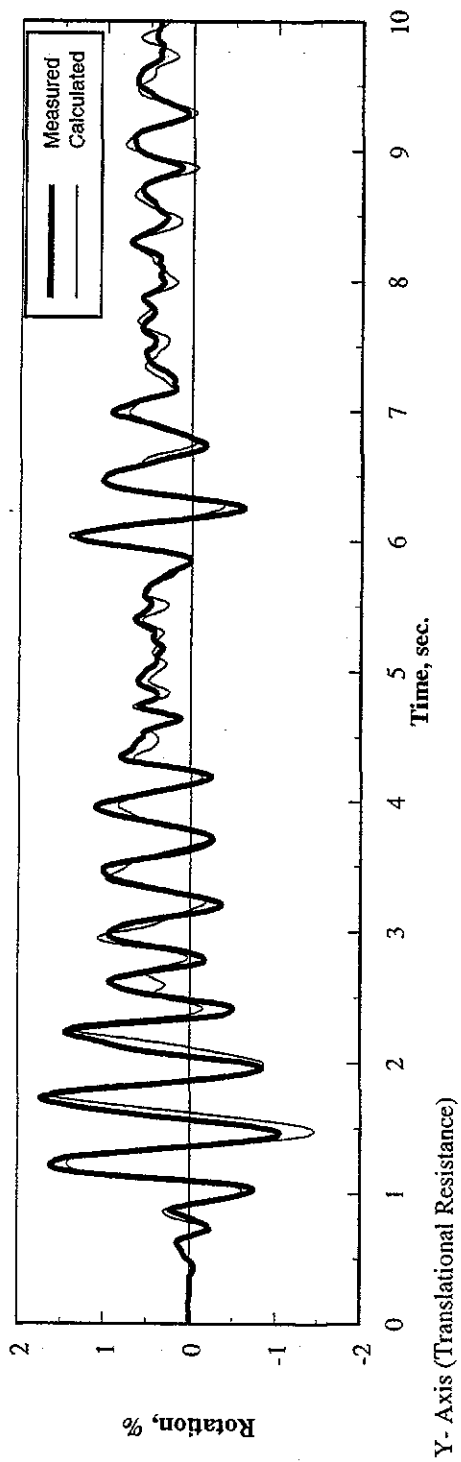


Fig. 3.19a Displacement Histories for Rotation About Axes Y and Z, ULW Run2

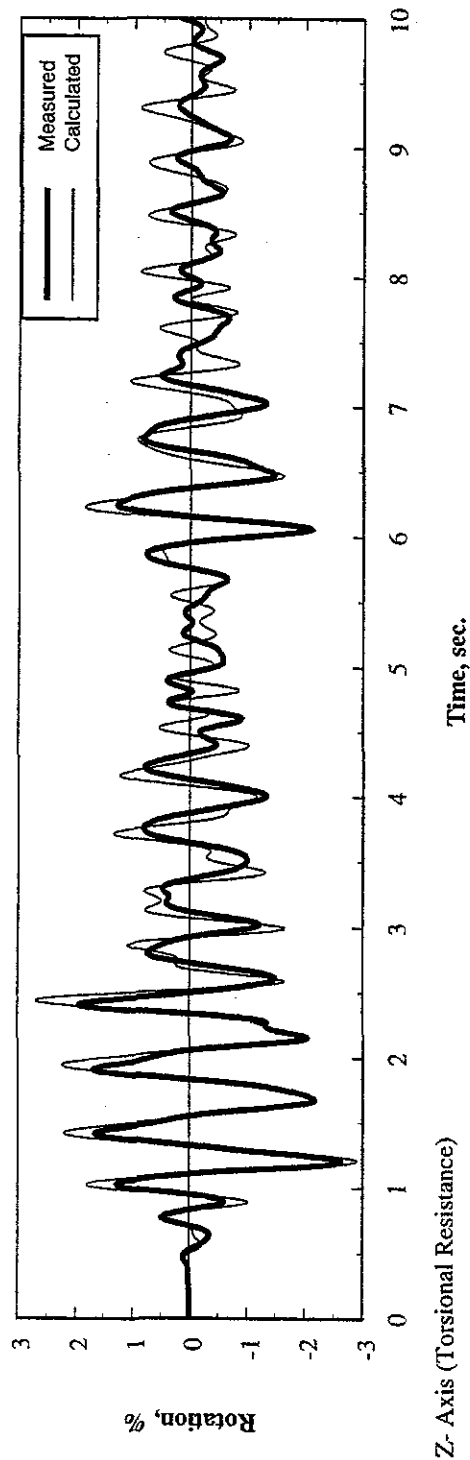
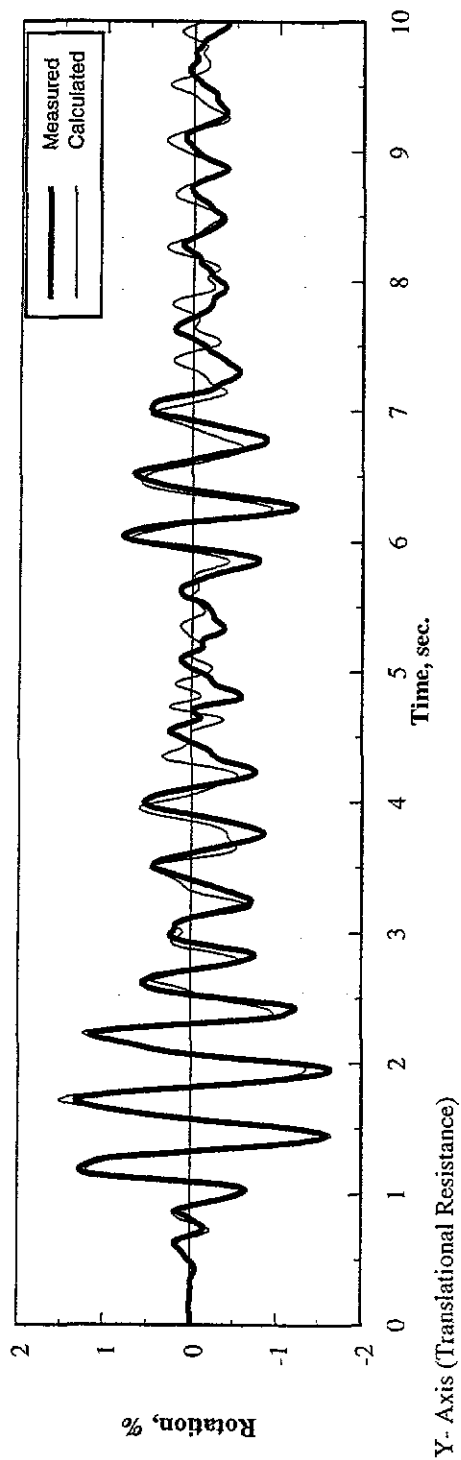


Fig. 3.19b Displacement Histories for Rotation About Axes Y and Z, UHS Run2

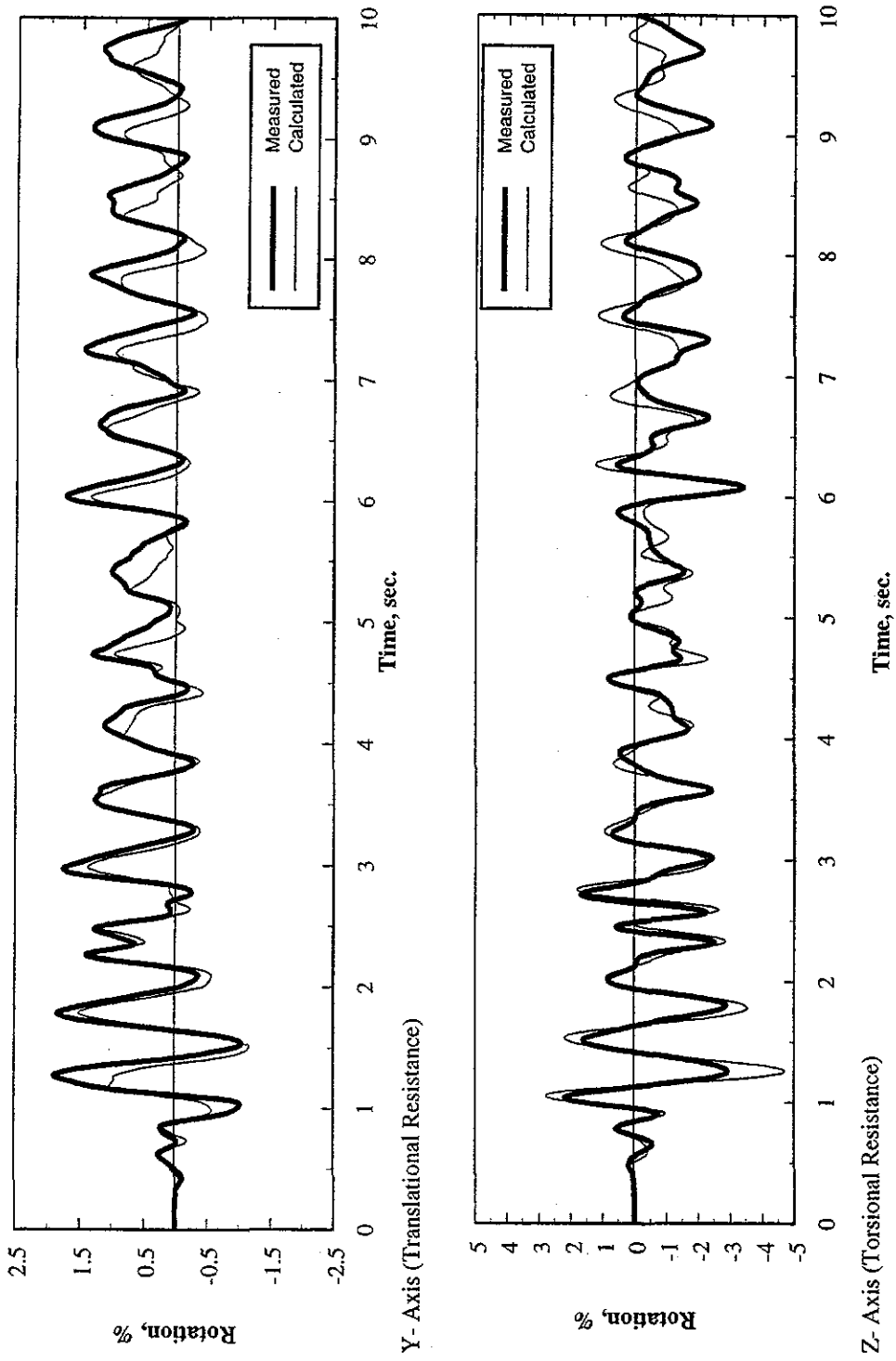


Fig. 3.19c Displacement Histories for Rotation About Axes Y and Z, UHW Run2

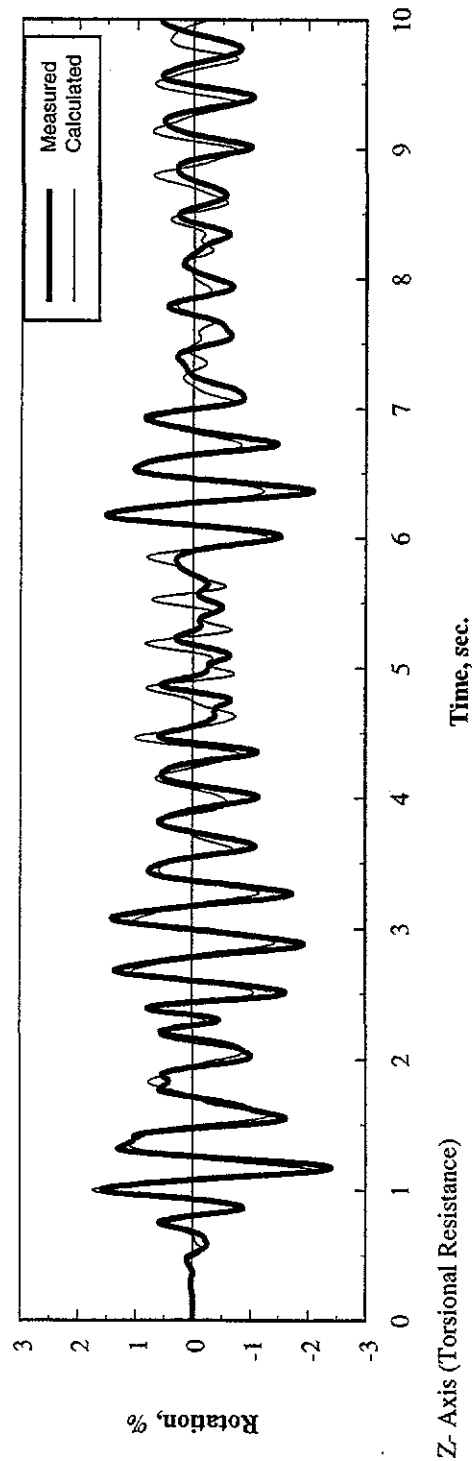
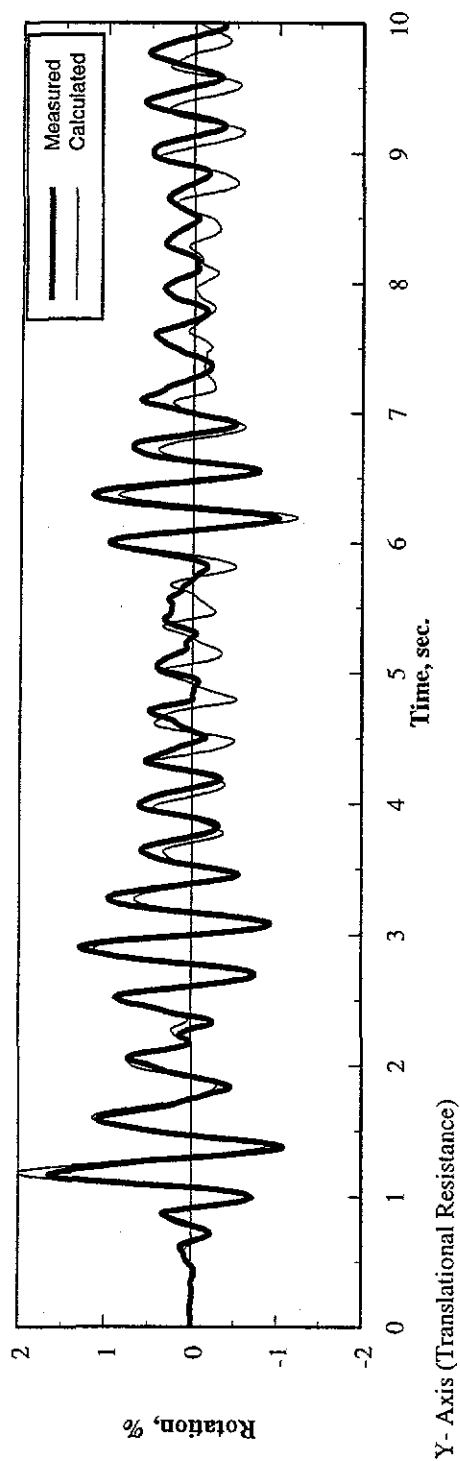


Fig. 3.19d Displacement Histories for Rotation About Axes Y and Z, ULS Run2

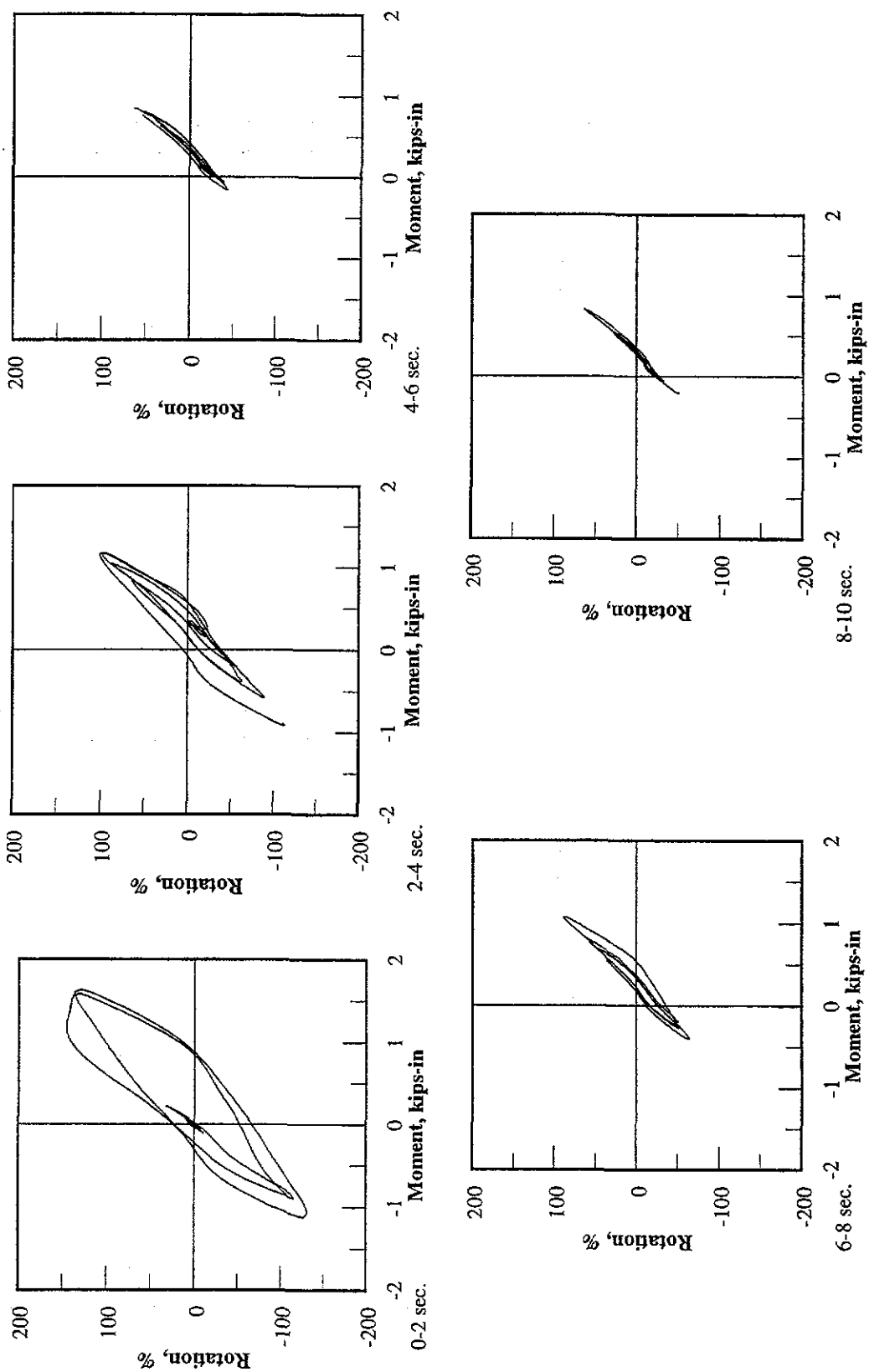


Fig. 3.20.1a Measured Moment-Rotation Hysteresis for CLW Run2, Y-Axis (Translational Resistance)

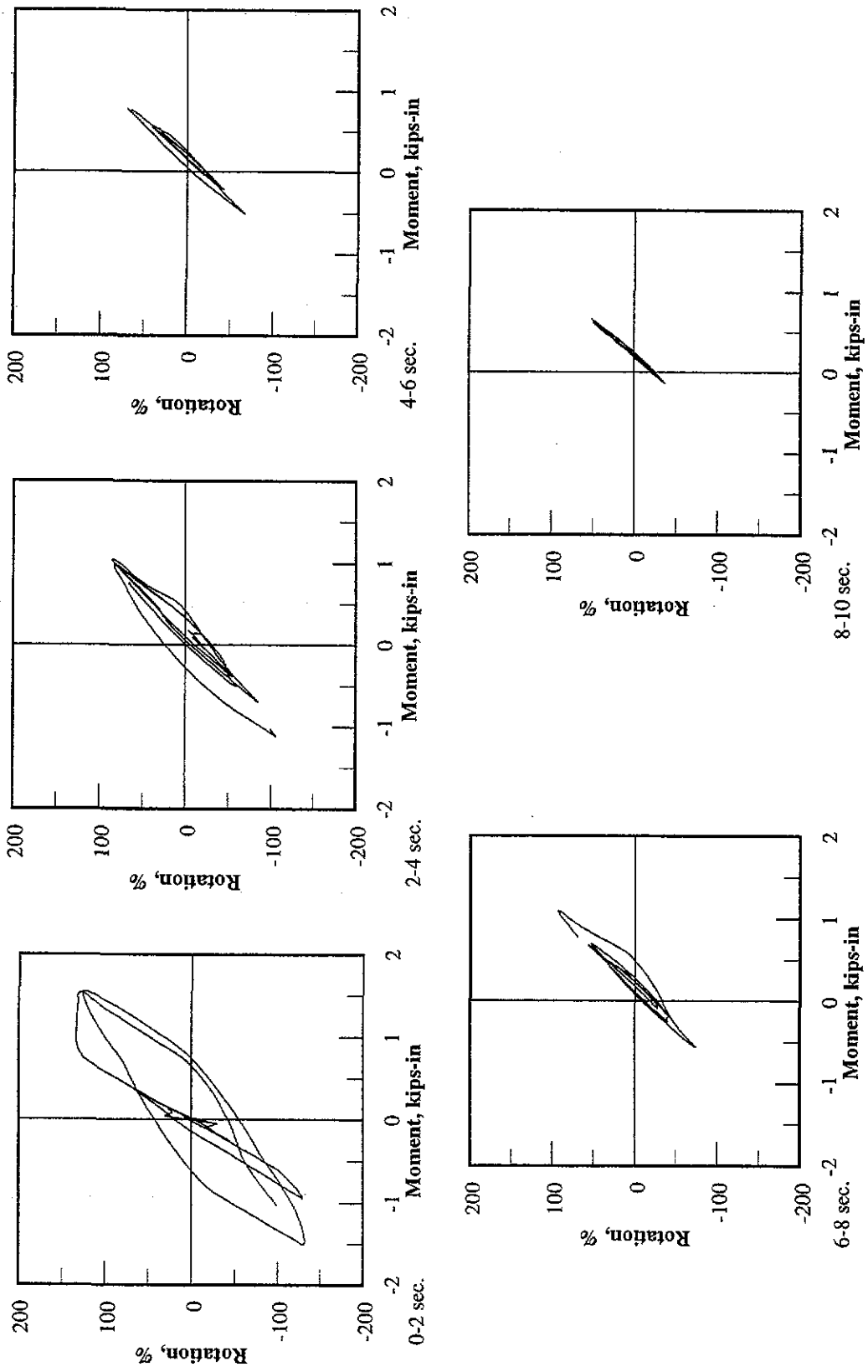


Fig. 3.20.1b Calculated Moment-Rotation Hysteresis for CLW Run2, Y-Axis (Translational Resistance)

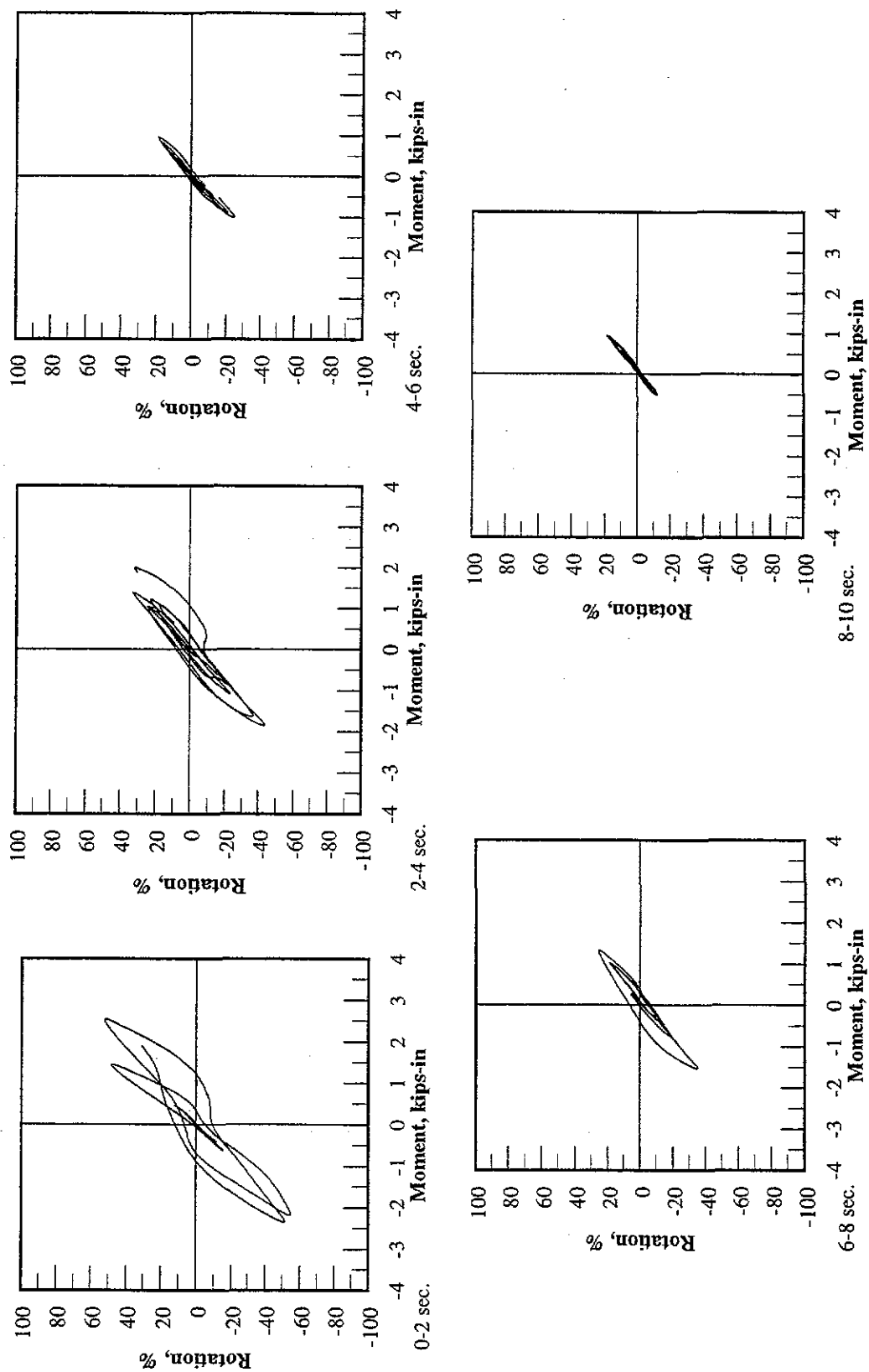


Fig. 3.20.1c Measured Moment-Rotation Hysteresis for CLW Run2, Z-Axis (Torsional Resistance)

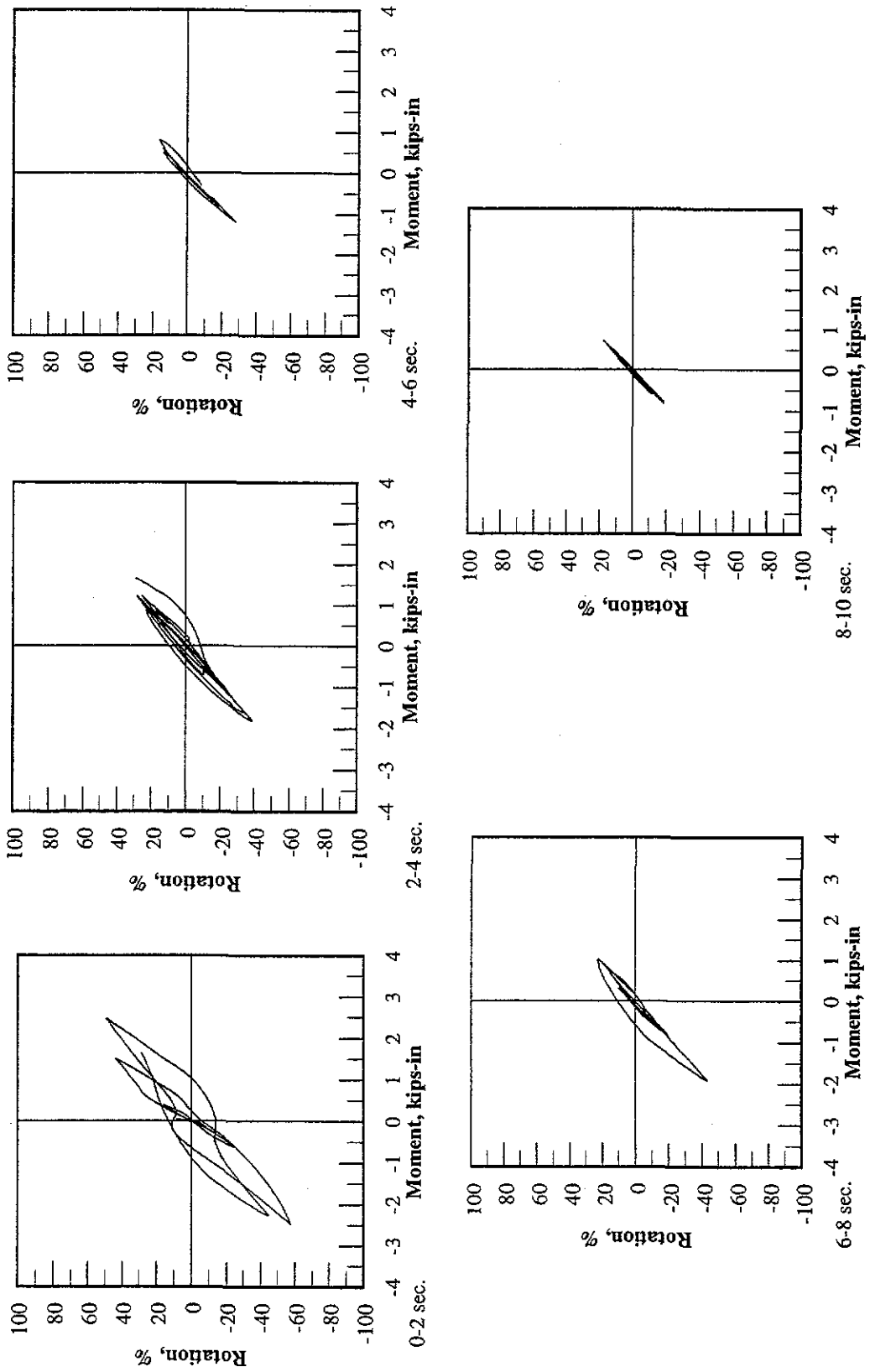


Fig. 3.20.1d Calculated Moment-Rotation Hysteresis for CLW Run2, Z-Axis (Torsional Resistance)

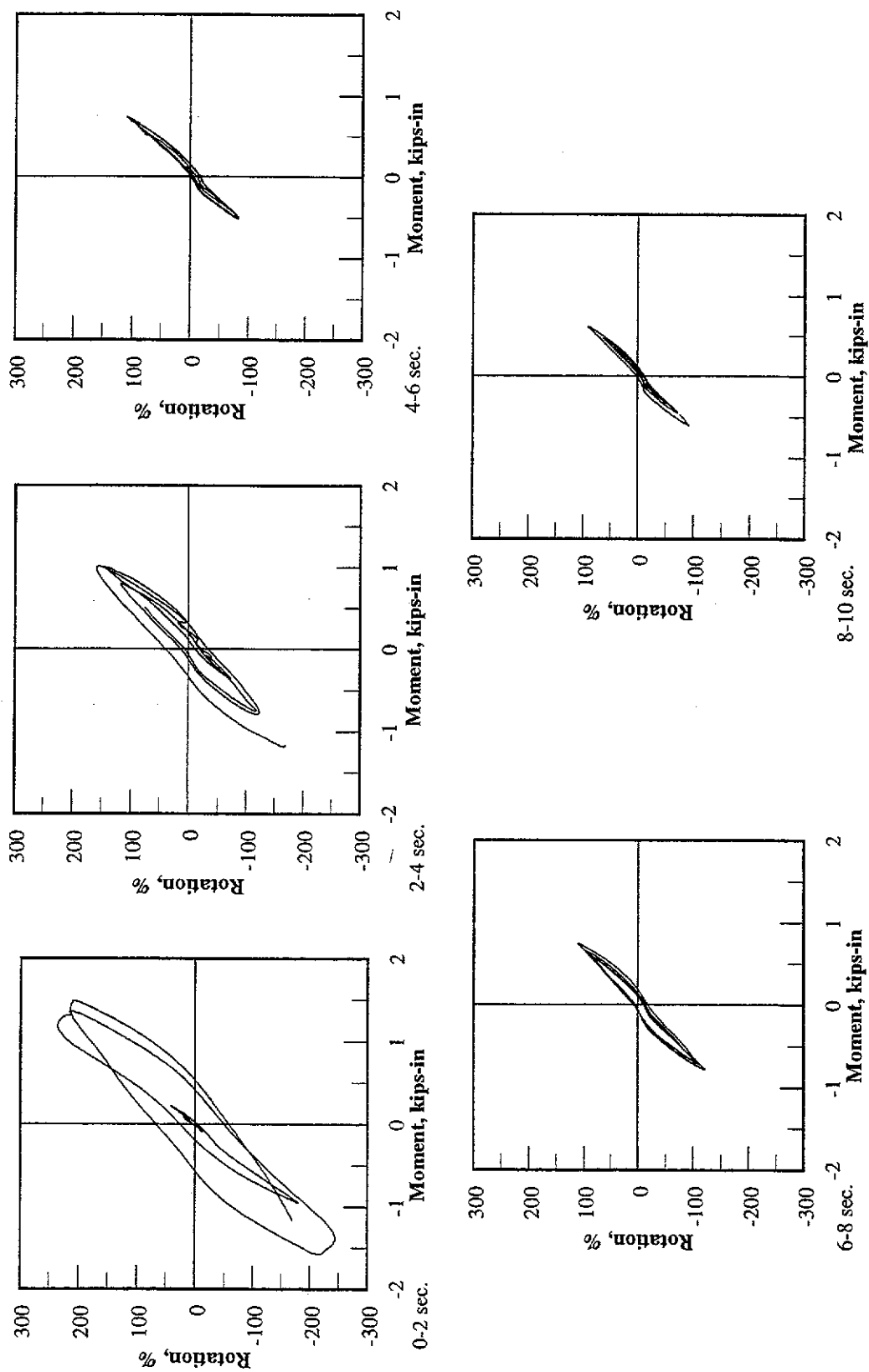


Fig. 3.20.2a Measured Moment-Rotation Hysteresis for CHS Run2, Y-Axis (Translational Resistance)

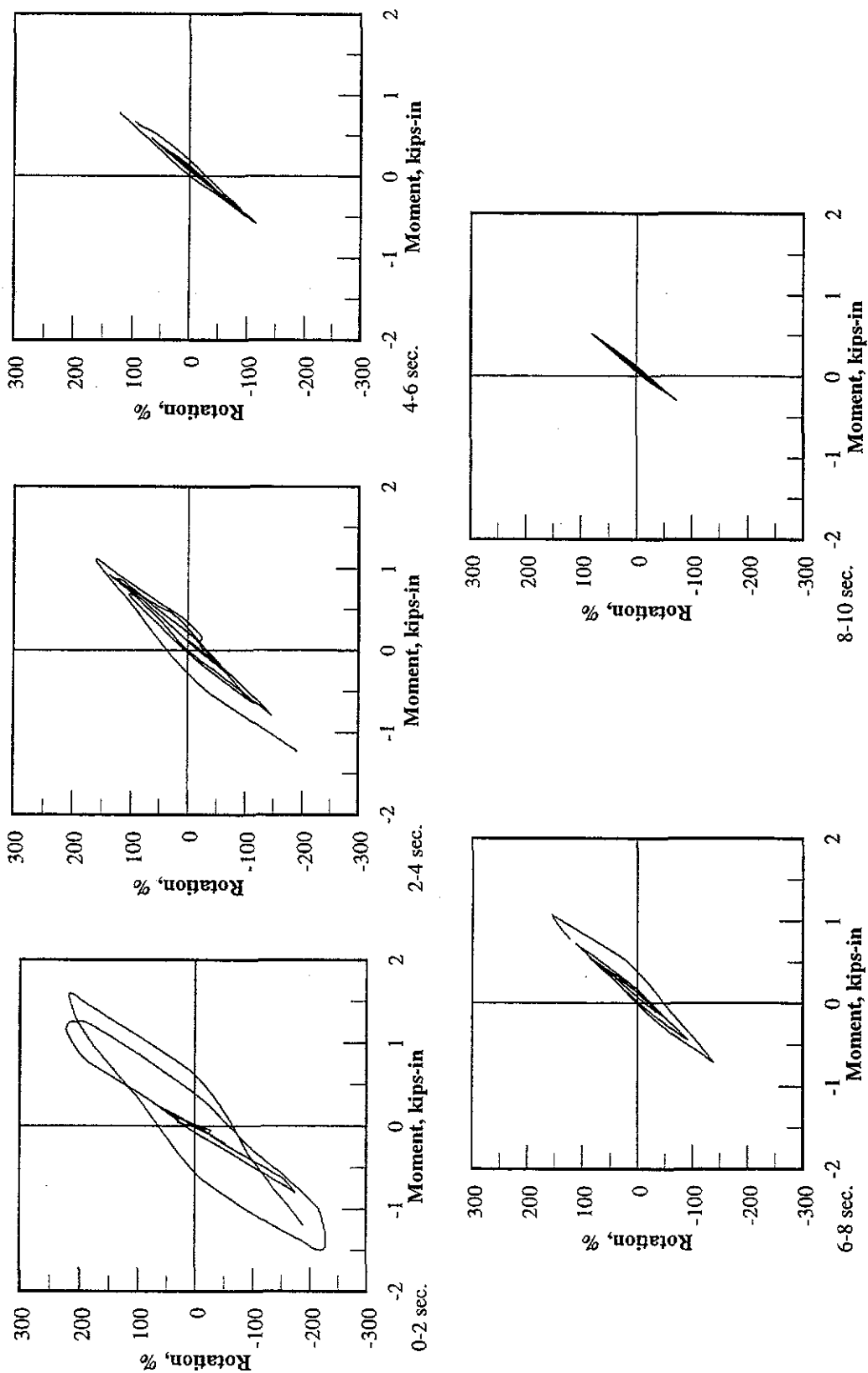


Fig. 3.20.2b Calculated Moment-Rotation Hysteresis for CHS Run2, Y-Axis (Translational Resistance)

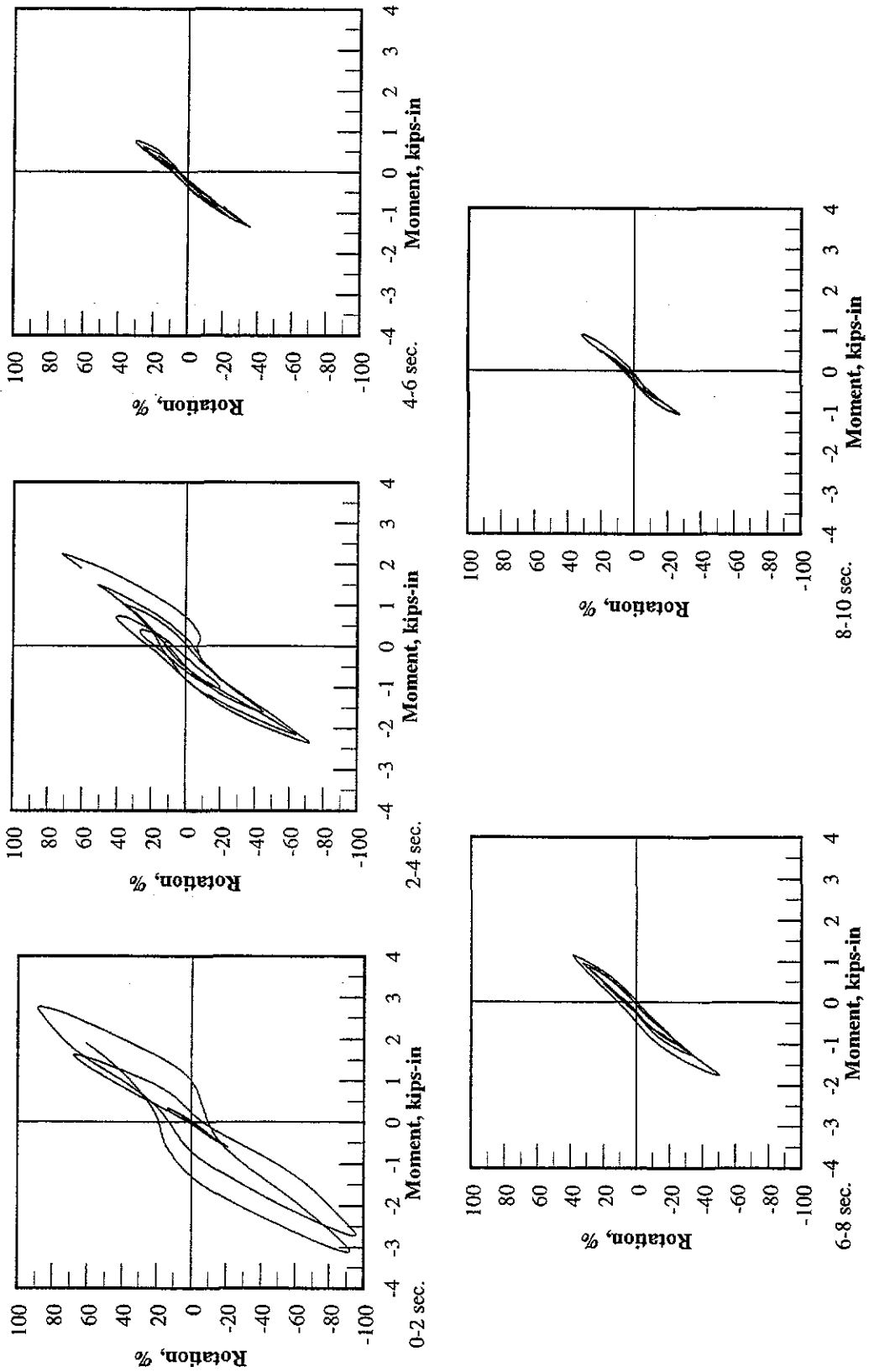


Fig. 3.20.2c Measured Moment-Rotation Hysteresis for CHS Run2, Z-Axis (Torsional Resistance)

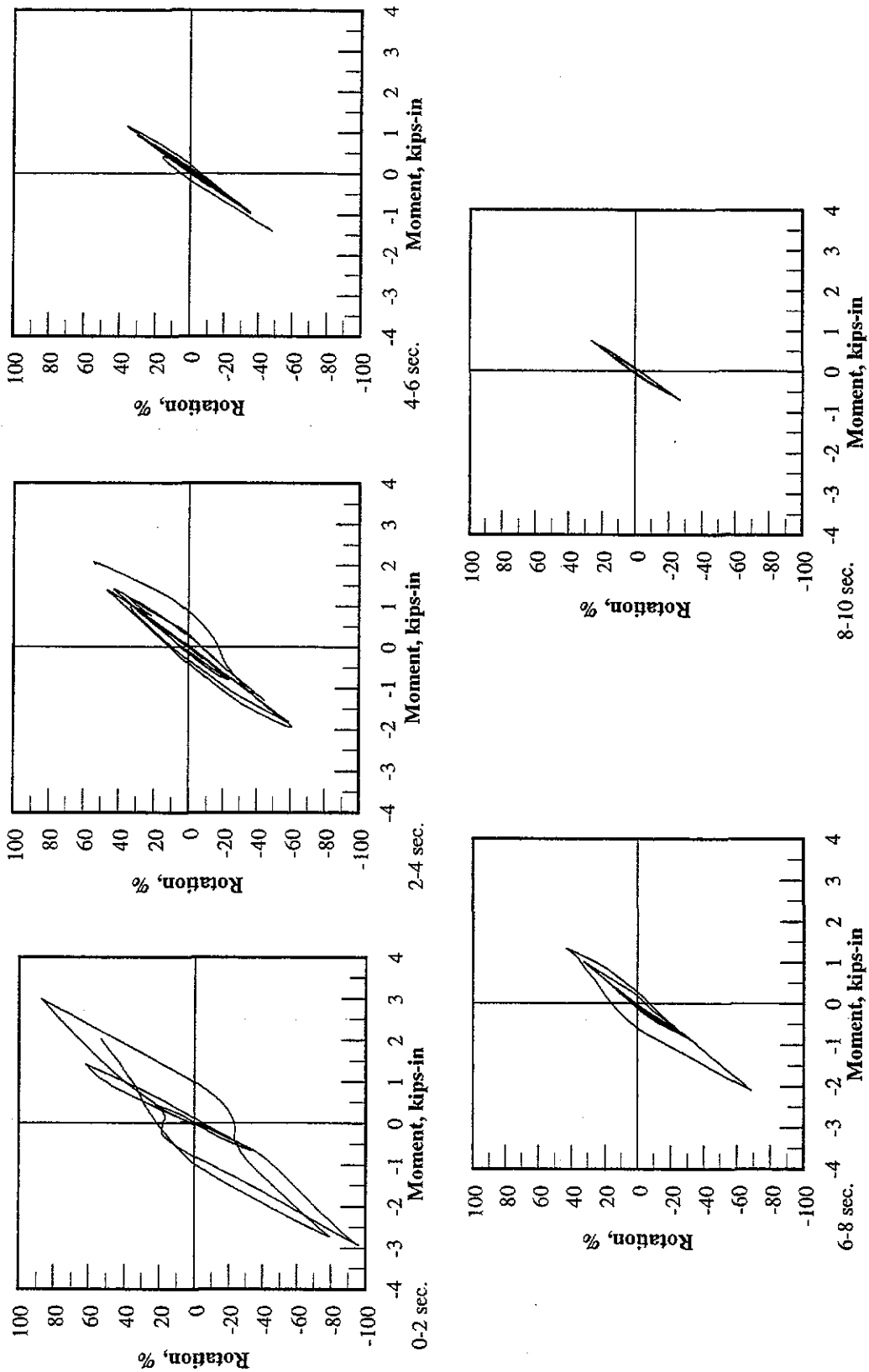


Fig. 3.20.2d Calculated Moment-Rotation Hysteresis for CHS Run2, Z-Axis (Torsional Resistance)

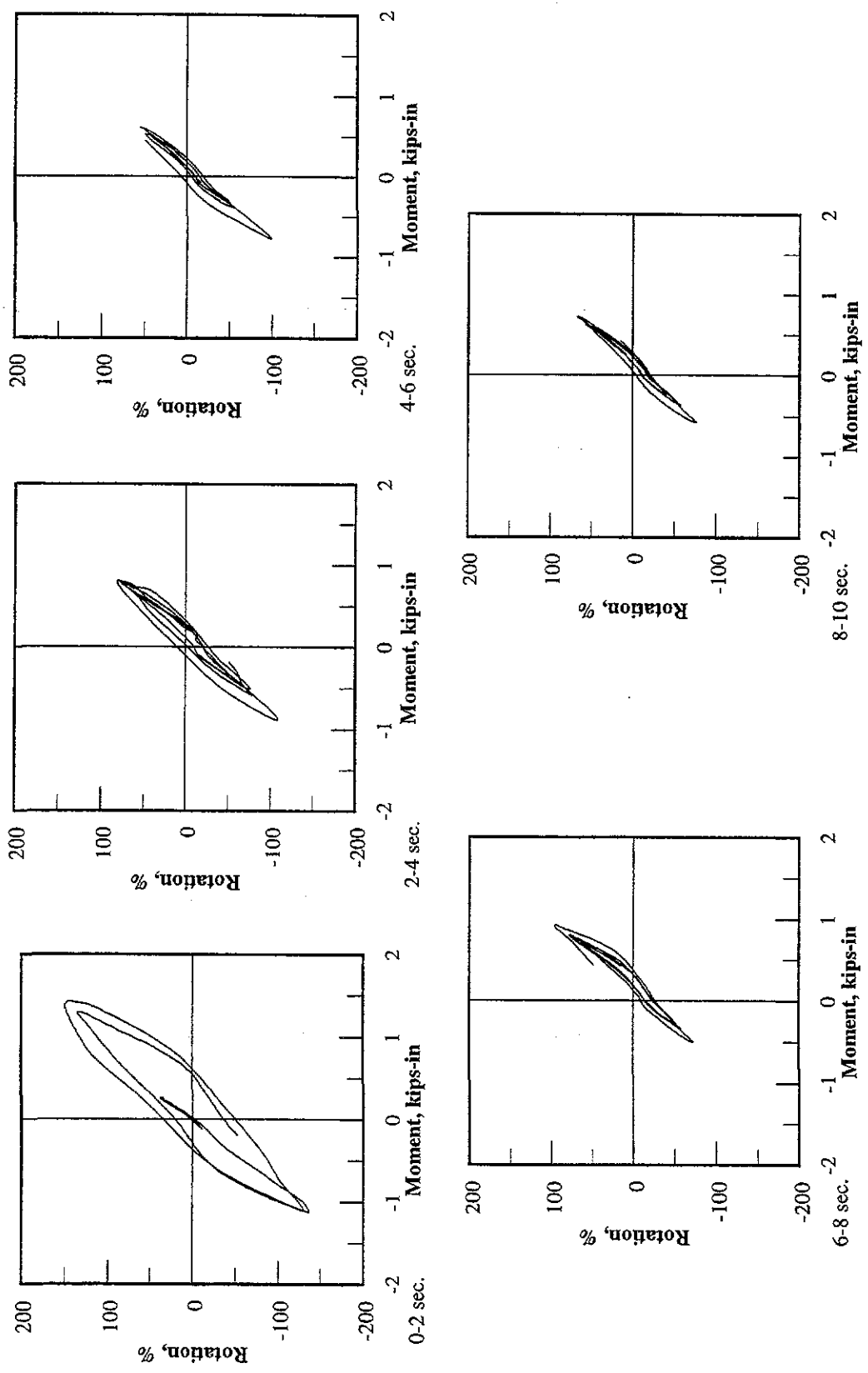


Fig. 3.20.3a Measured Moment-Rotation Hysteresis for CHW Run2, Y-Axis (Translational Resistance)

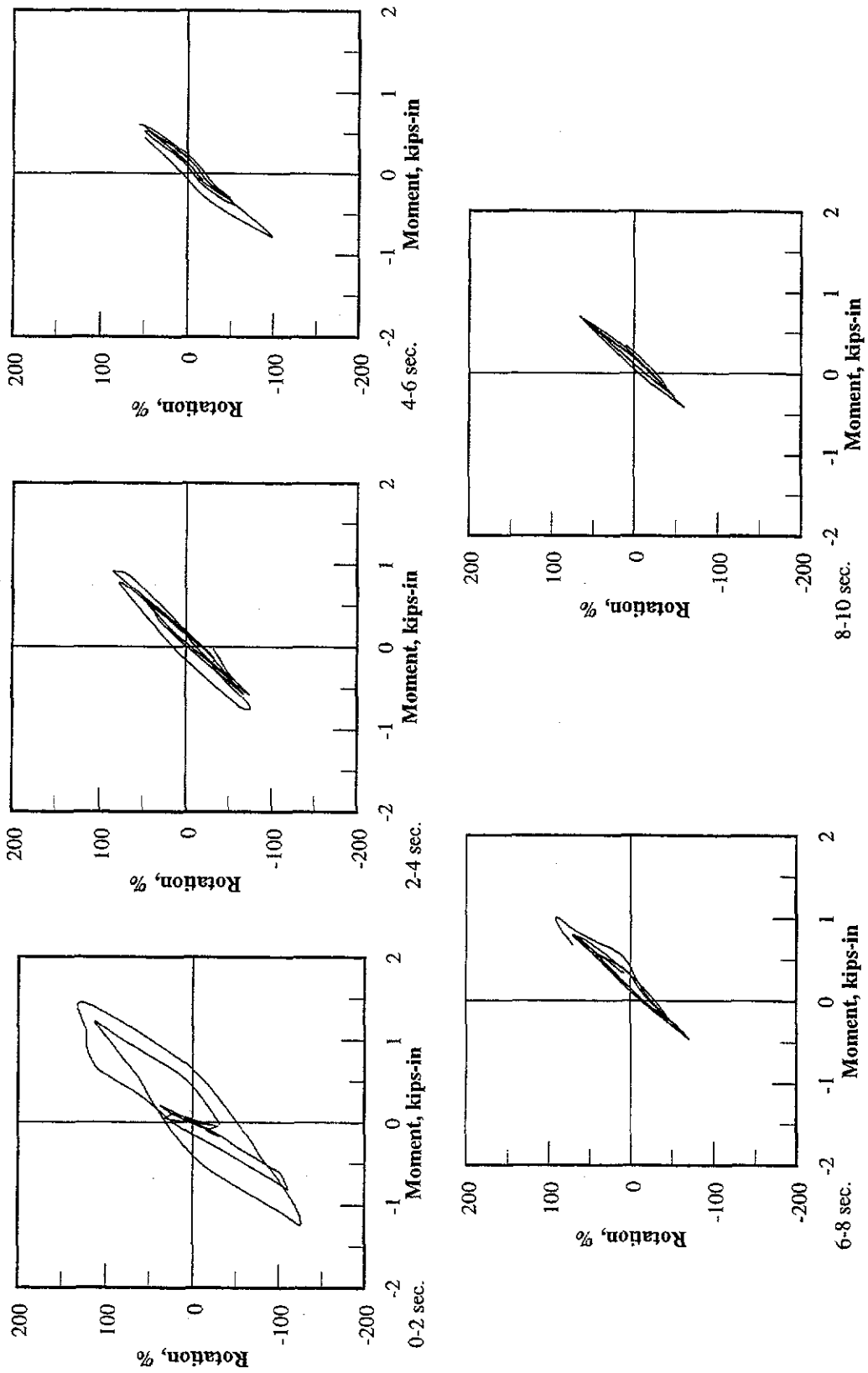


Fig. 3.20.3b Calculated Moment-Rotation Hysteresis for CHW Run2, Y-Axis (Translational Resistance)

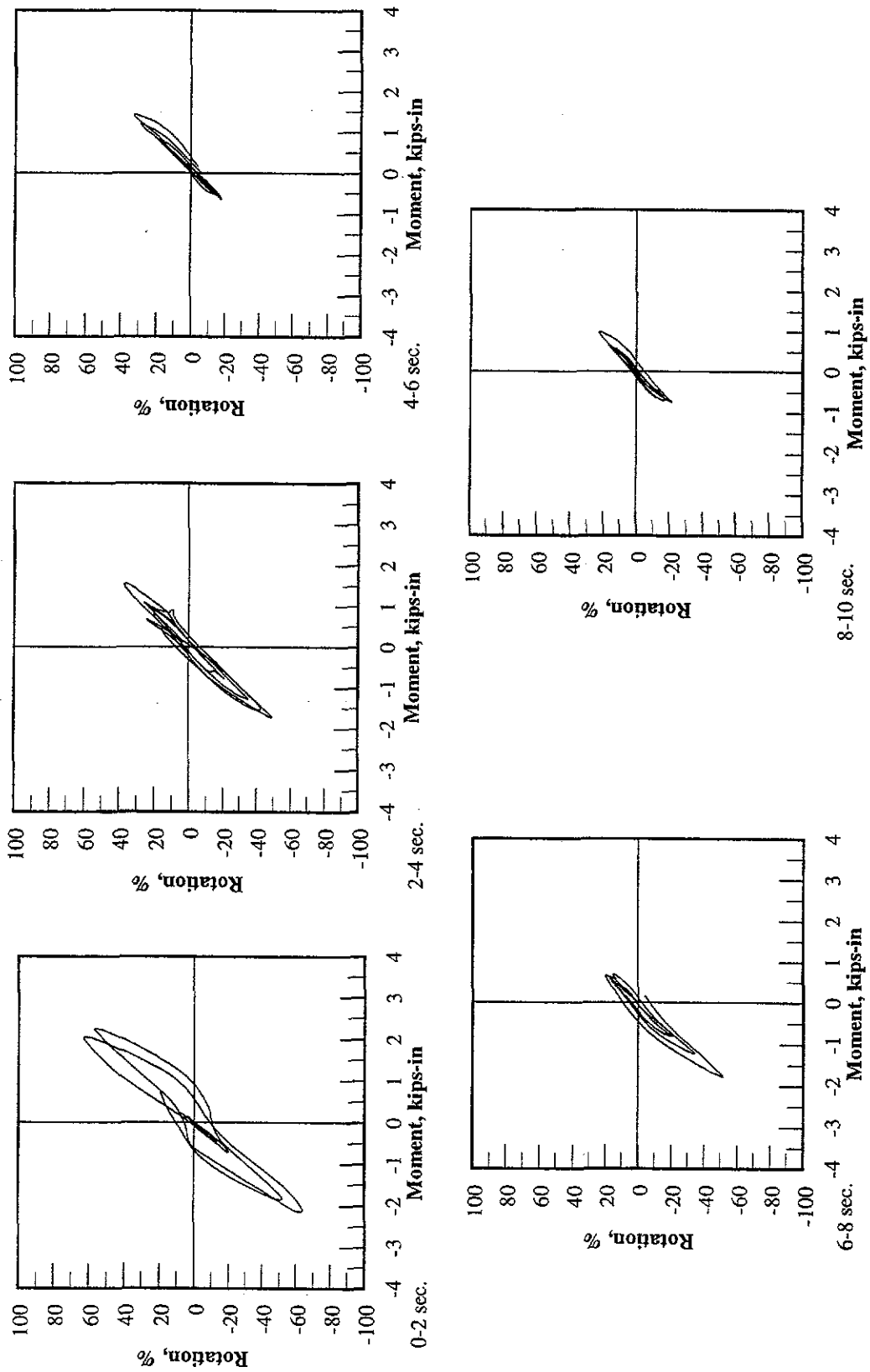


Fig. 3.20.3c Measured Moment-Rotation Hysteresis for CHW Run2, Z-Axis (Torsional Resistance)

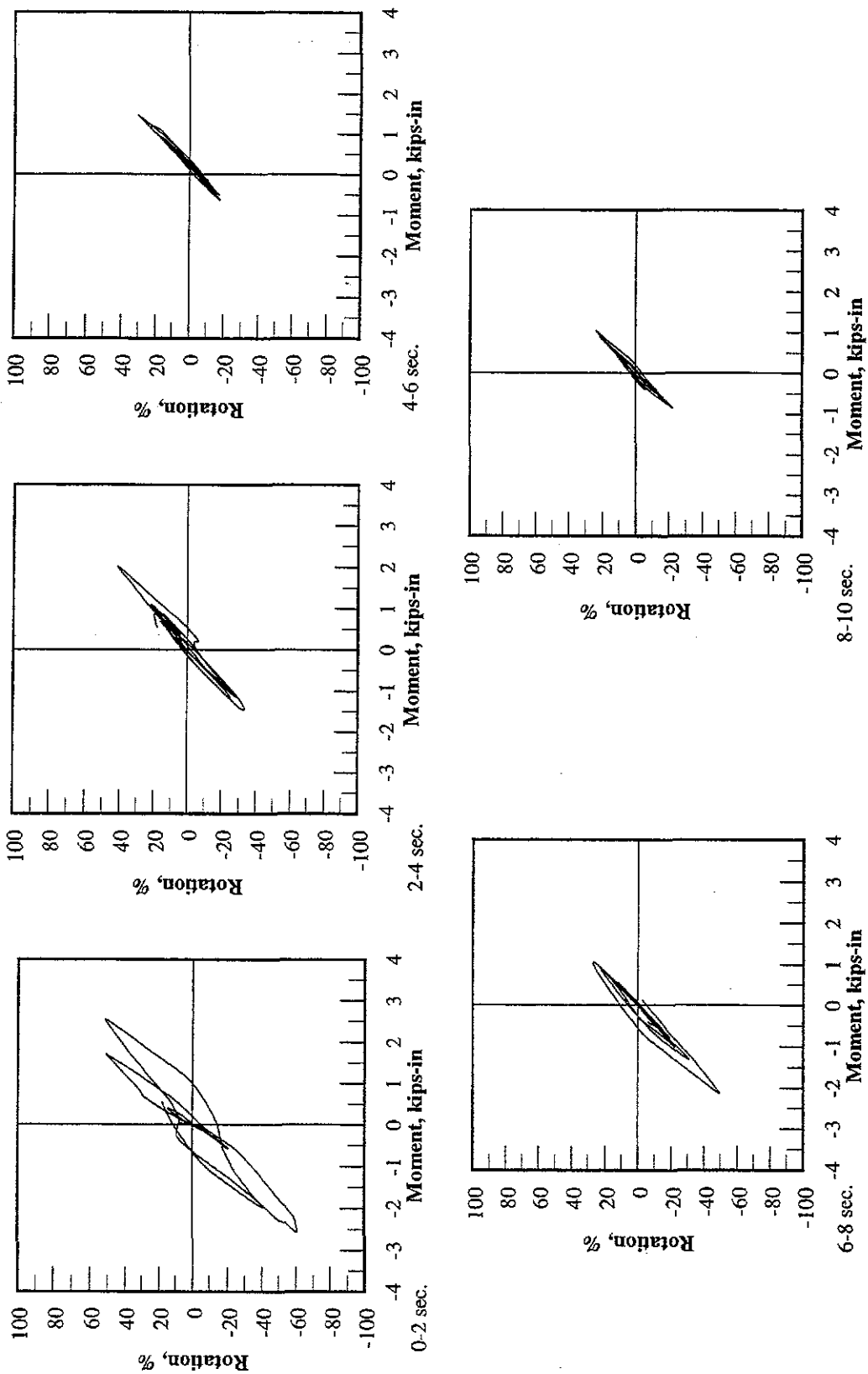


Fig. 3.20.3d Calculated Moment-Rotation Hysteresis for CHW Run2, Z-Axis (Torsional Resistance)

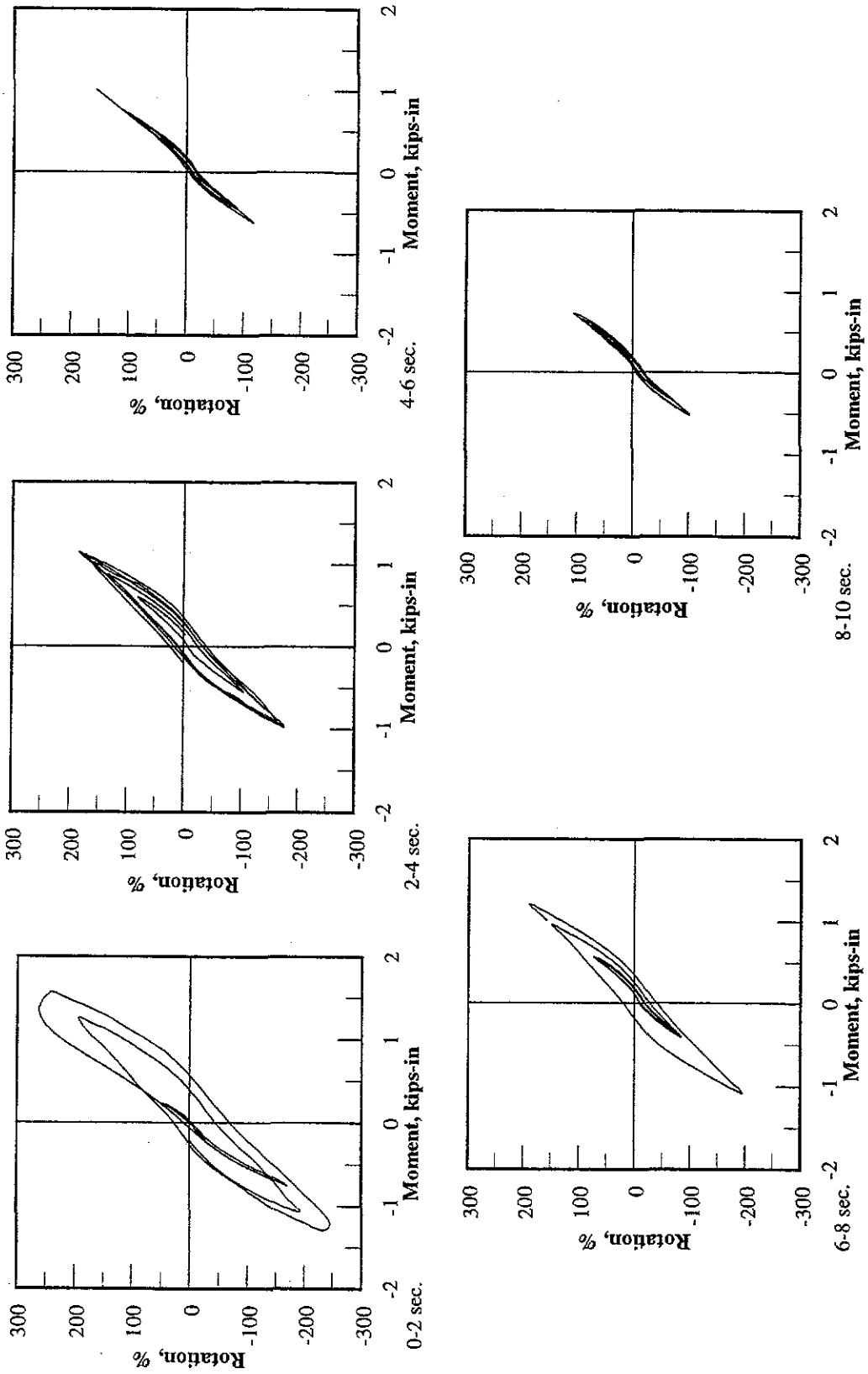


Fig. 3.20.4a Measured Moment-Rotation Hysteresis for CLS Run2, Y-Axis (Translational Resistance)

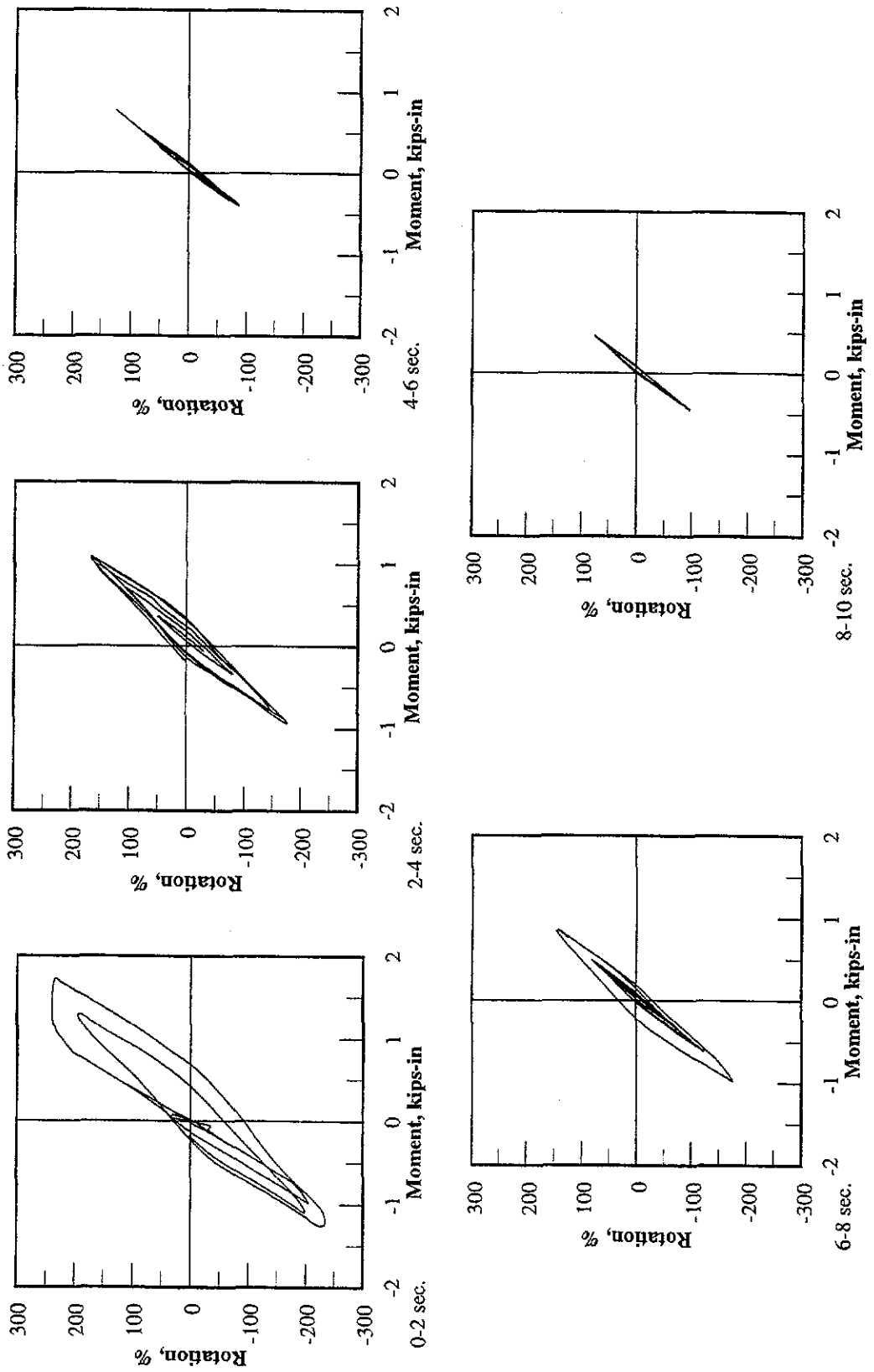


Fig. 3.20.4b Calculated Moment-Rotation Hysteresis for CLS Run2, Y-Axis (Translational Resistance)

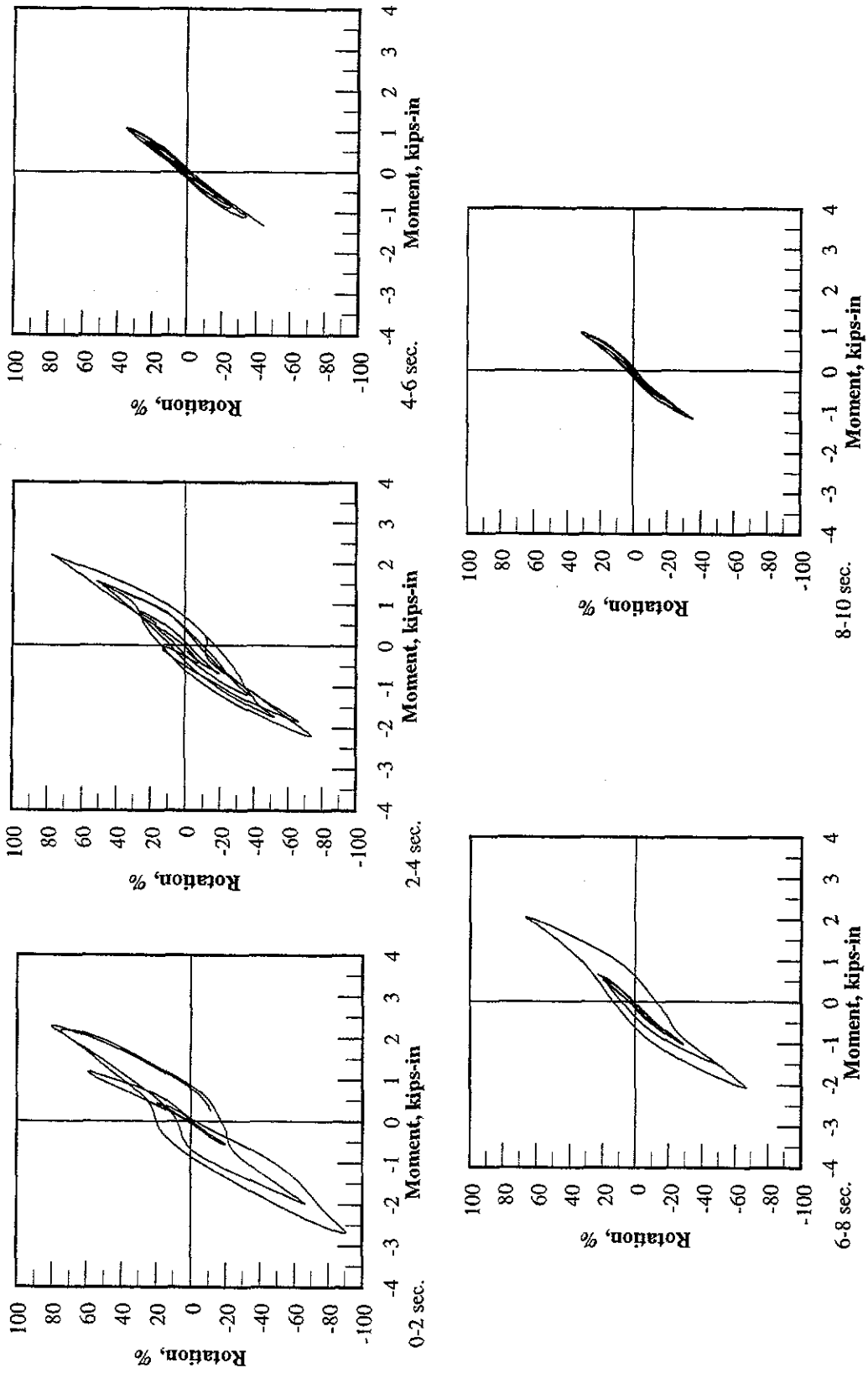


Fig. 3.20.4c Measured Moment-Rotation Hysteresis for CLS Run2, Z-Axis (Torsional Resistance)

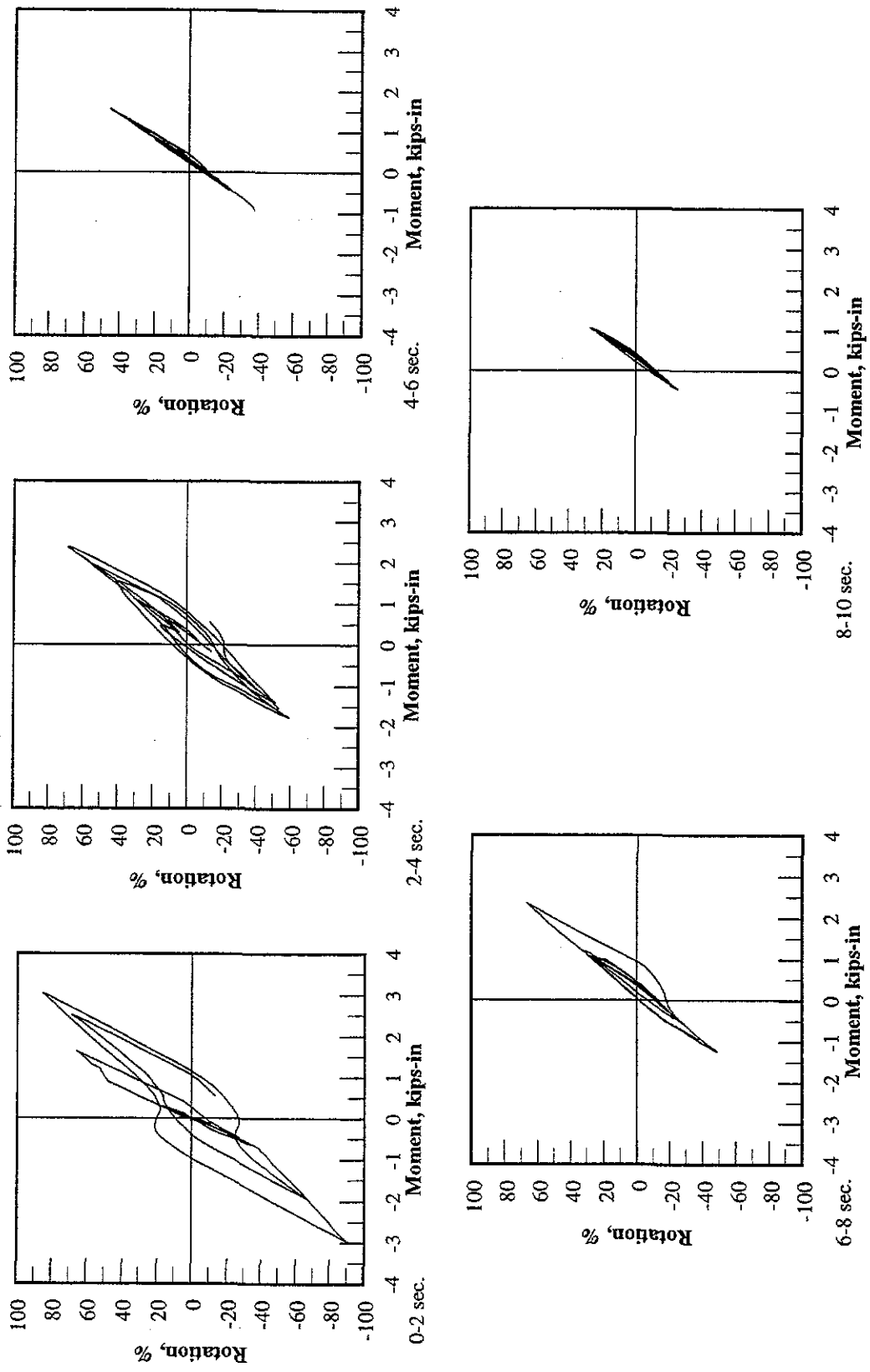


Fig. 3.20.4d Calculated Moment-Rotation Hysteresis for CLS Run2, Z-Axis (Torsional Resistance)

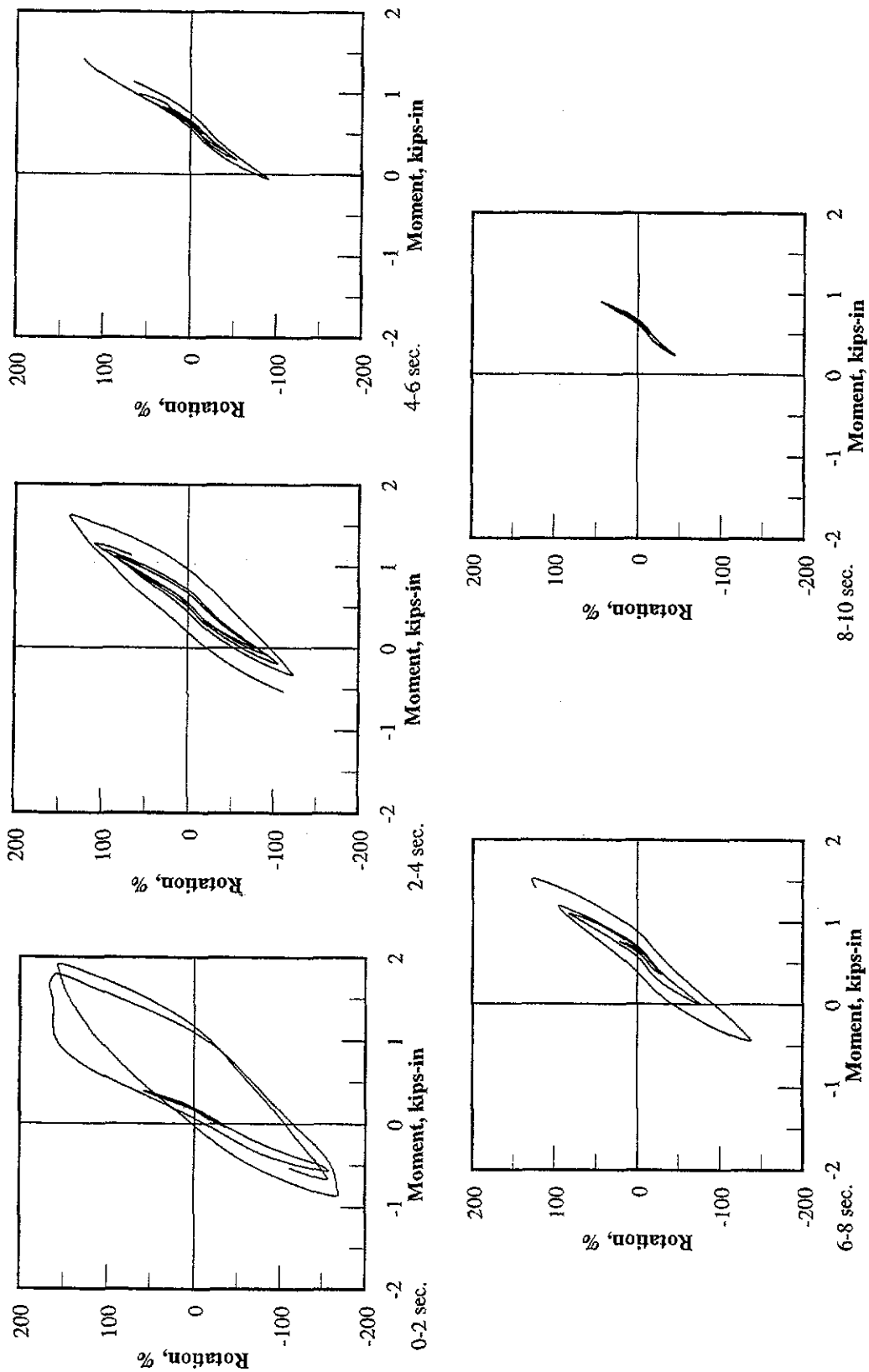


Fig. 3.21.1a Measured Moment-Rotation Hysteresis for ULW Run2, Y-Axis (Translational Resistance)

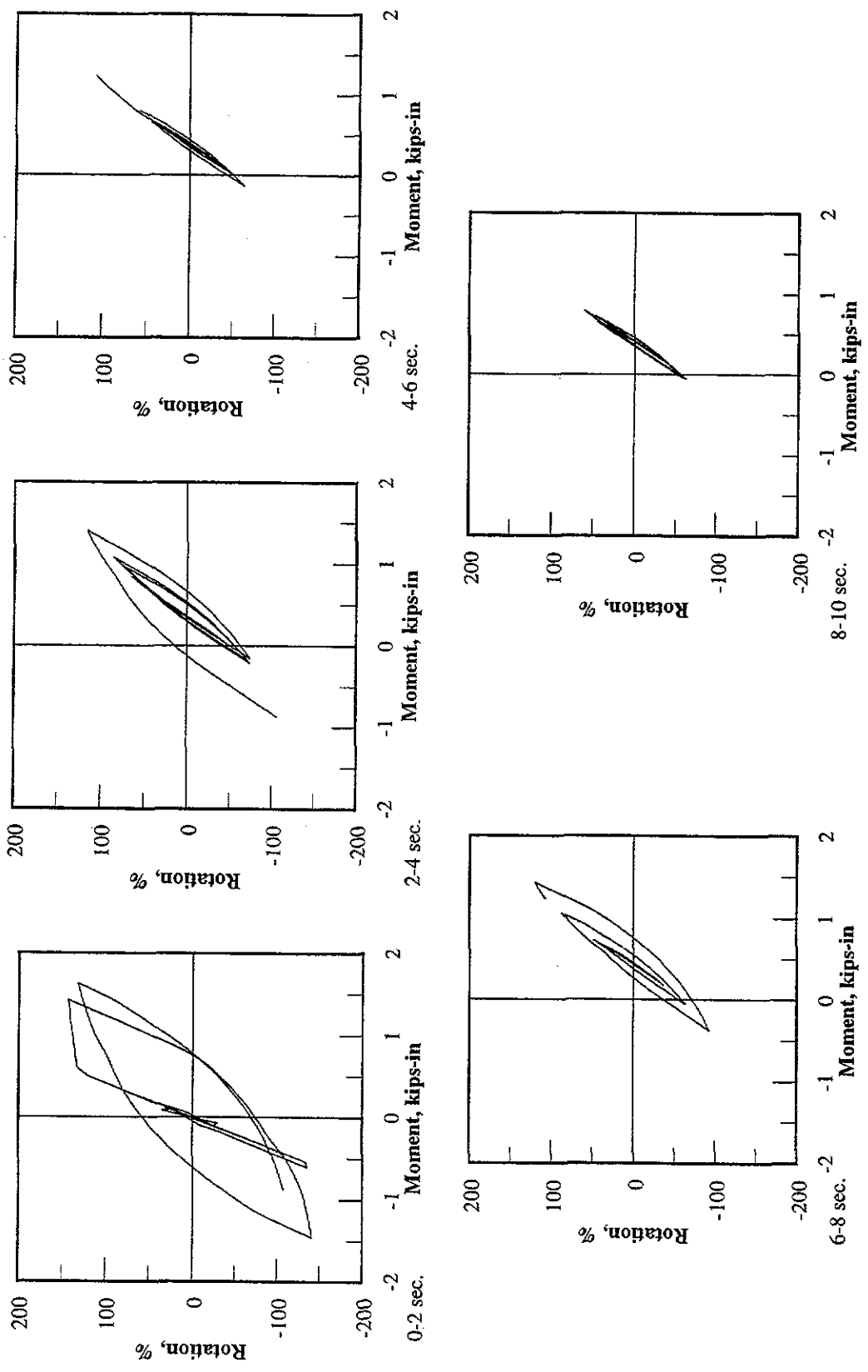


Fig. 3.2.1.1b Calculated Moment-Rotation Hysteresis for ULW Run2, Y-Axis (Translational Resistance)

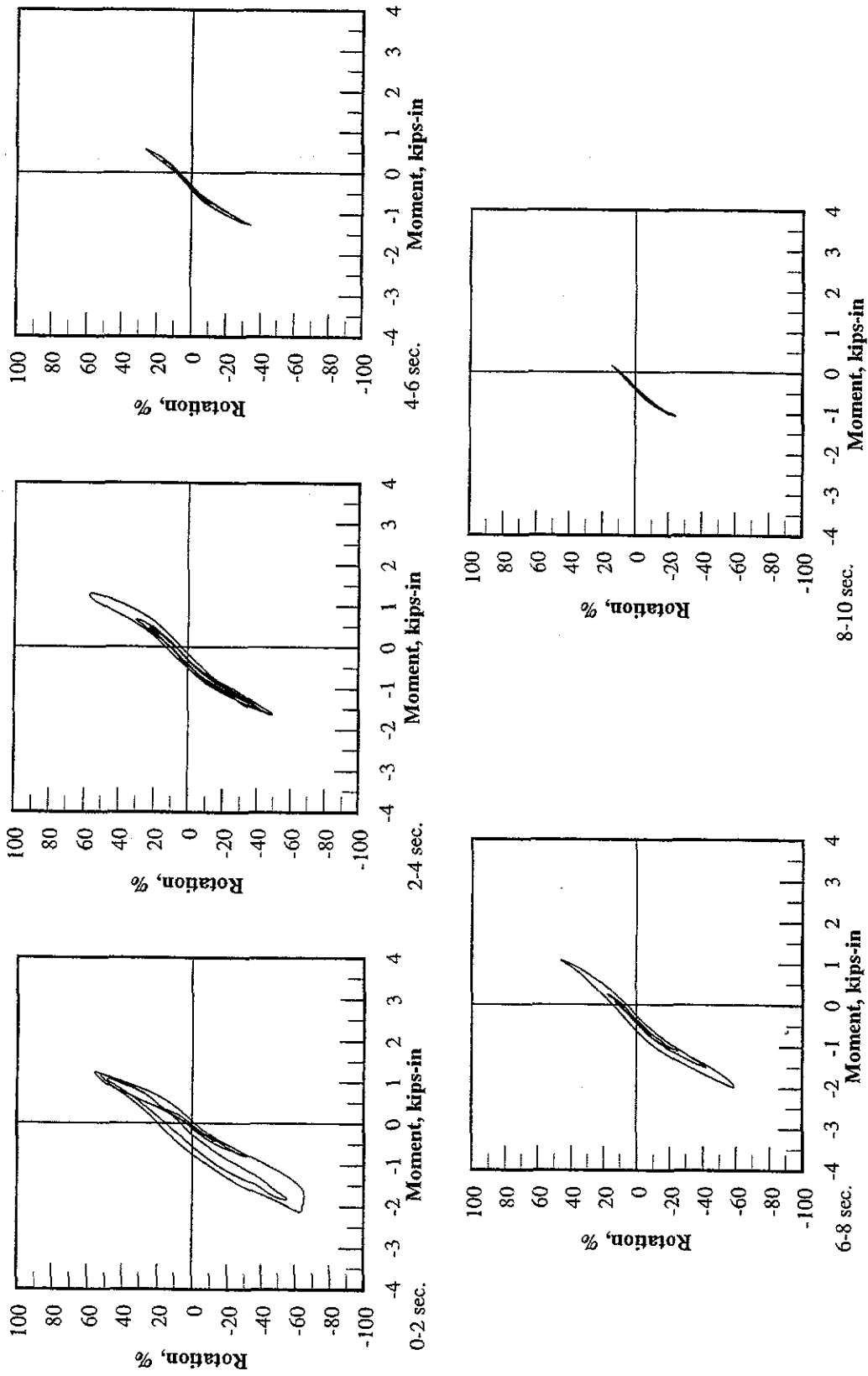


Fig. 3.21.1c Measured Moment-Rotation Hysteresis for ULW Run2, Z-Axis (Torsional Resistance)

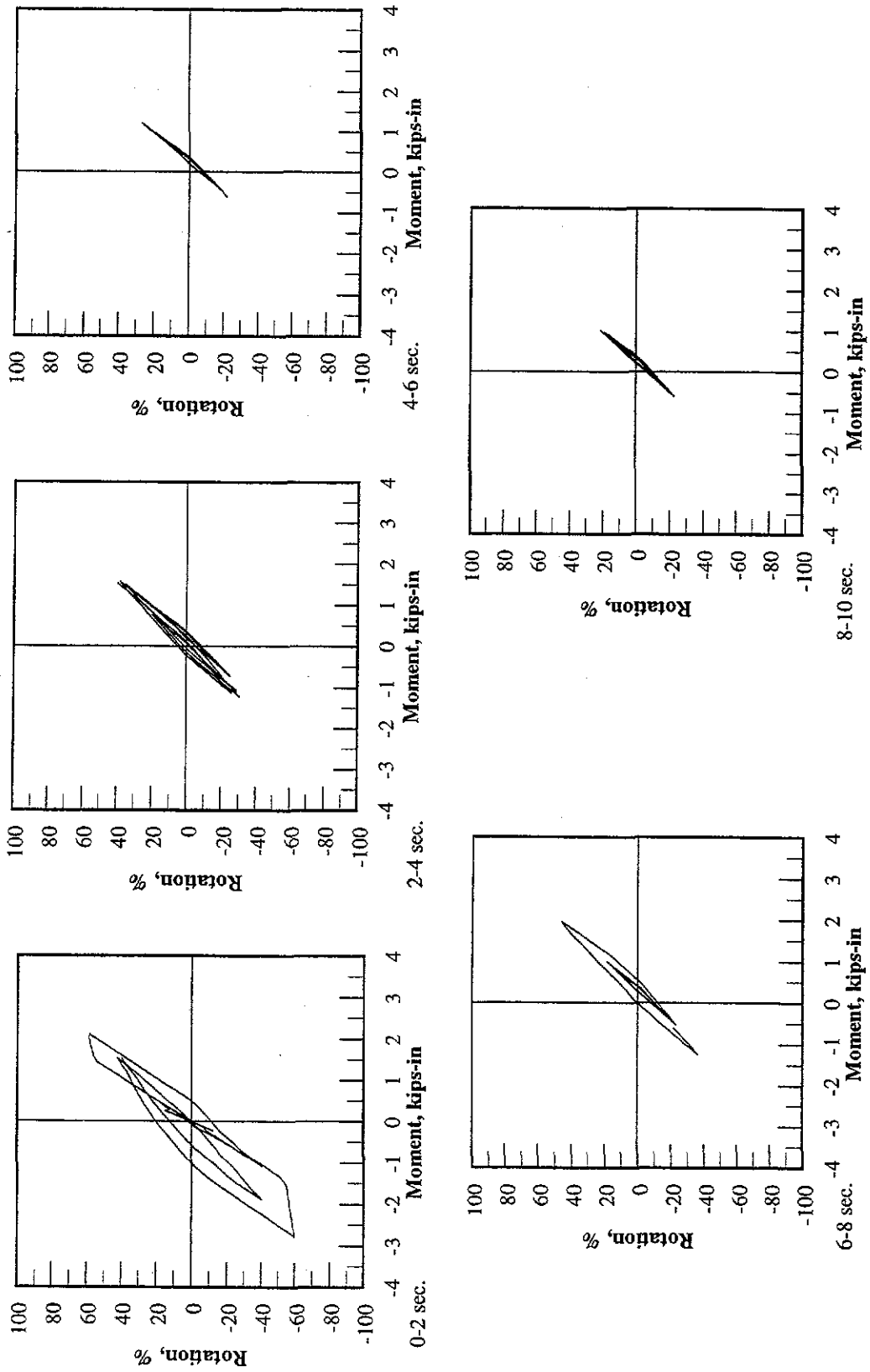


Fig. 3.21.1d Calculated Moment-Rotation Hysteresis for ULW Run2, Z-Axis (Torsional Resistance)

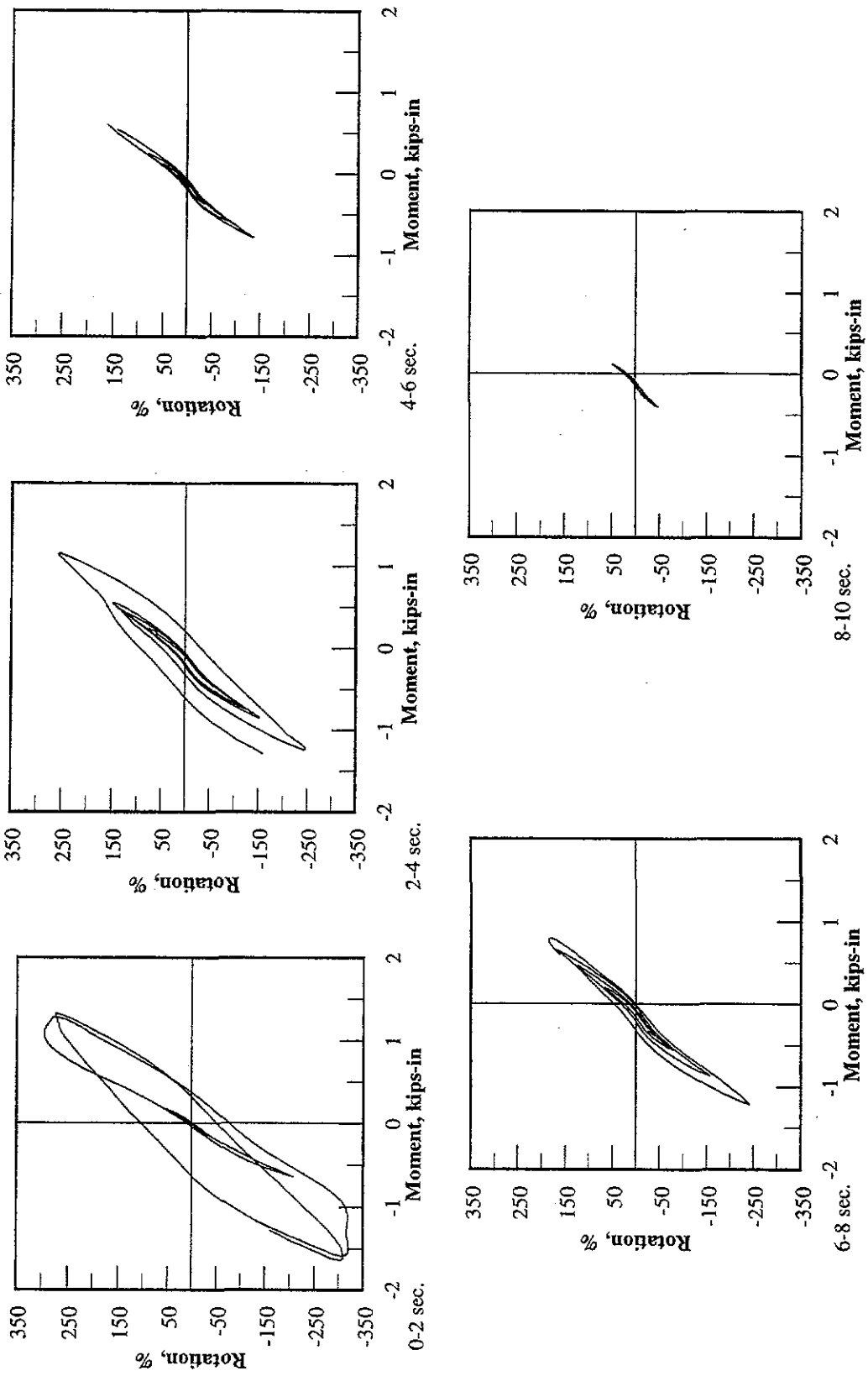


Fig. 3.21.2a Measured Moment-Rotation Hysteresis for UHS Run2, Y-Axis (Translational Resistance)

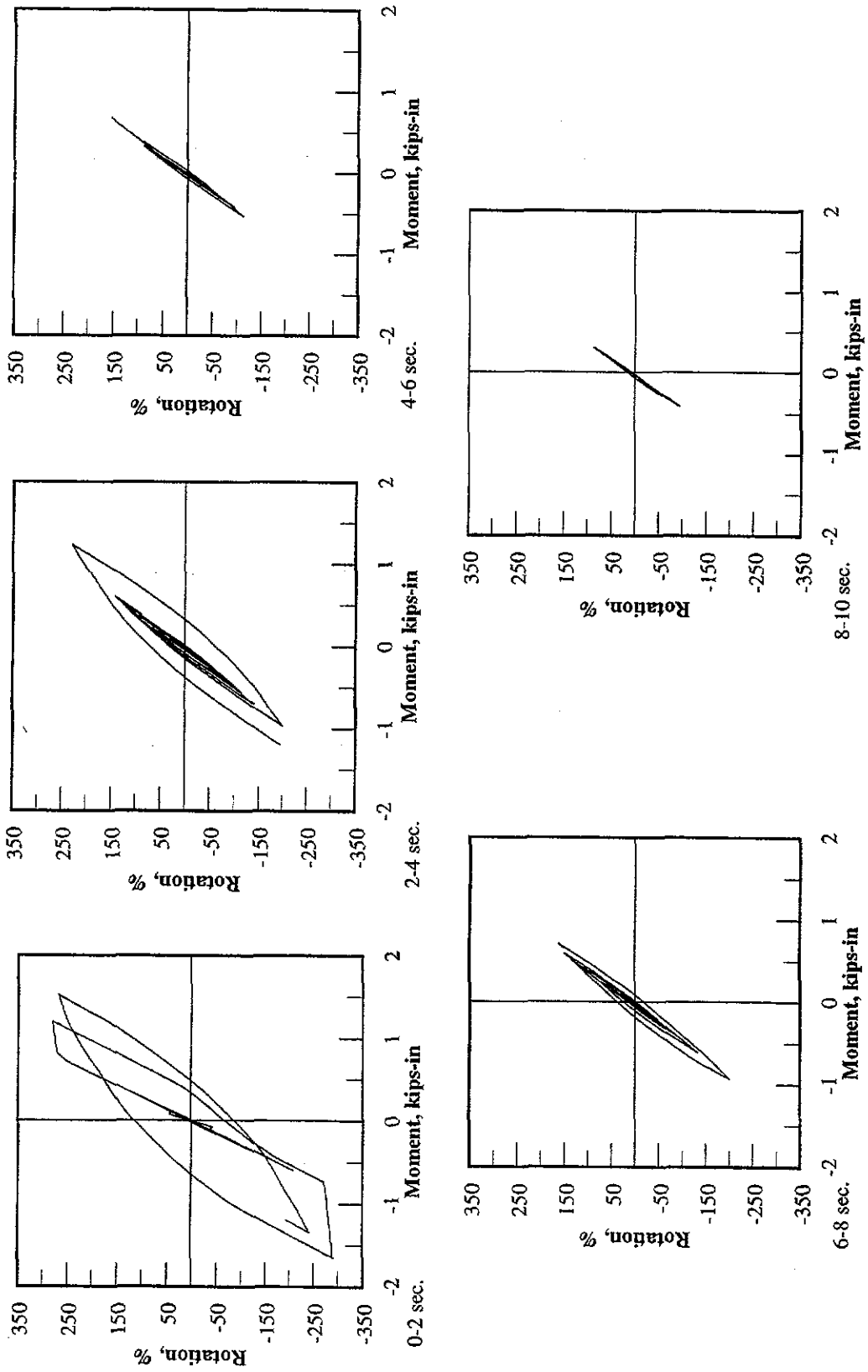


Fig. 3.21.2b Calculated Moment-Rotation Hysteresis for UHS Run2, Y-Axis (Translational Resistance)

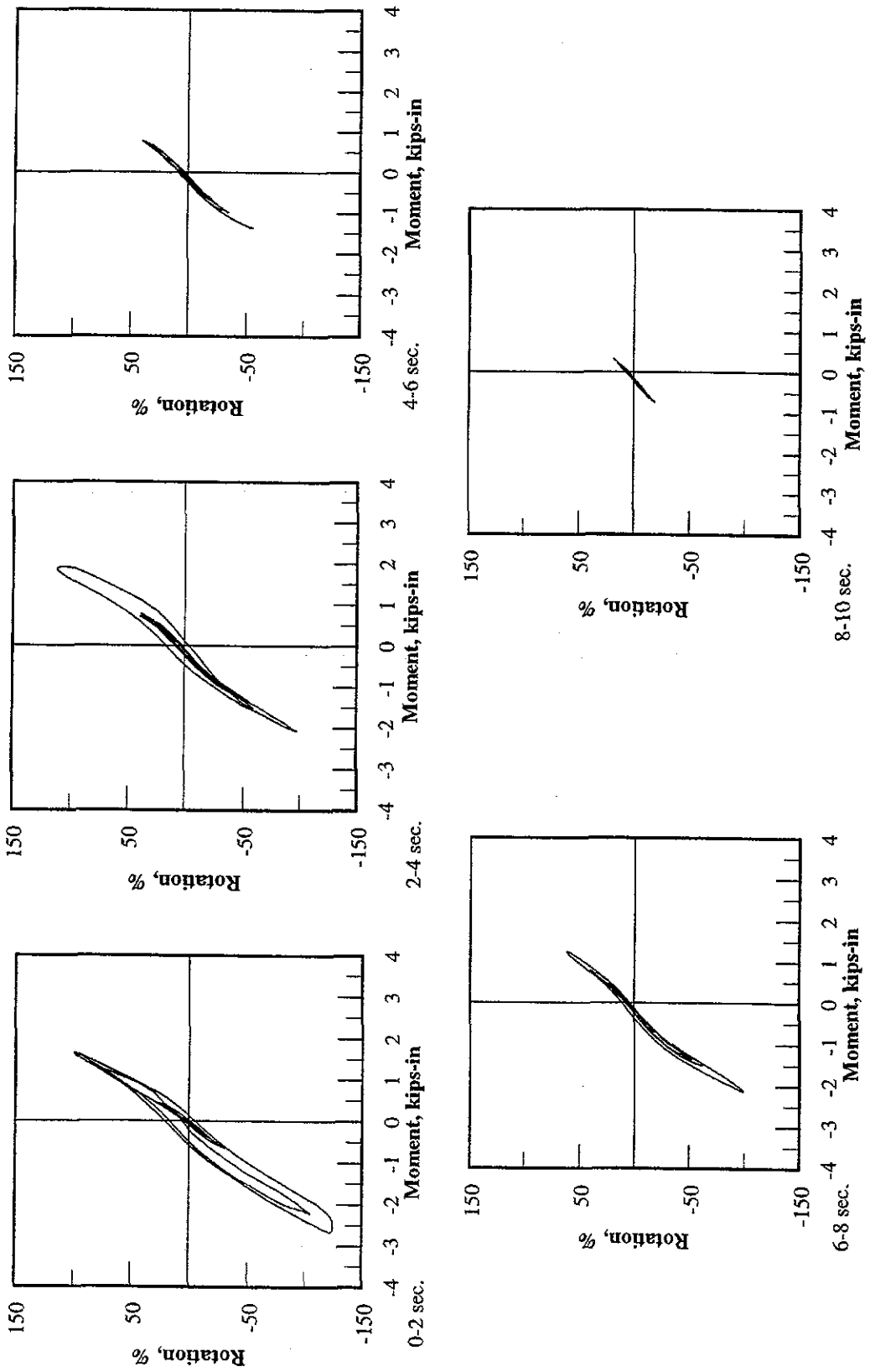


Fig. 3.21.2c Measured Moment-Rotation Hysteresis for UHS Run2, Z-Axis (Torsional Resistance)

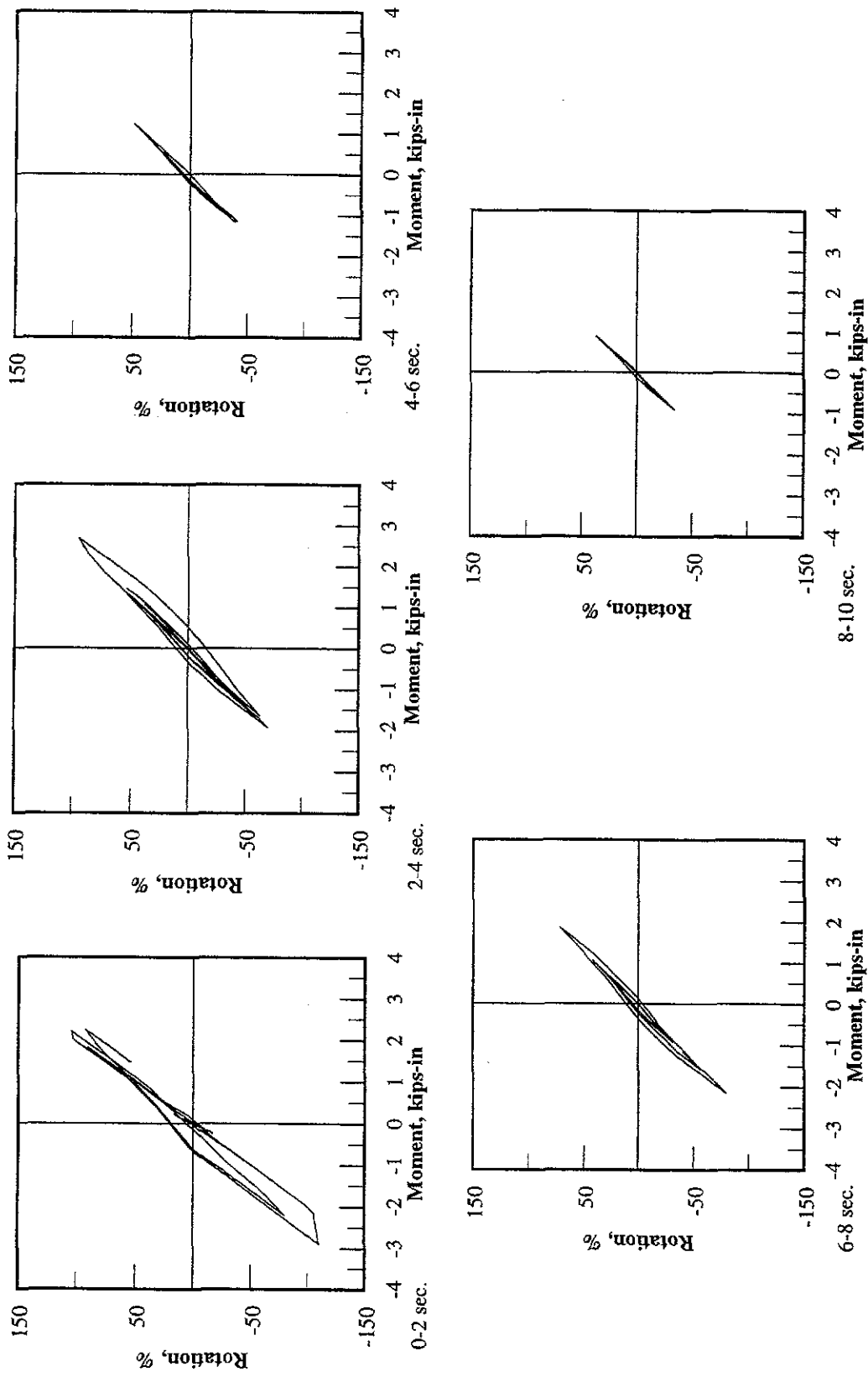


Fig. 3.21.2d Calculated Moment-Rotation Hysteresis for UHS Run2, Z-Axis (Torsional Resistance)

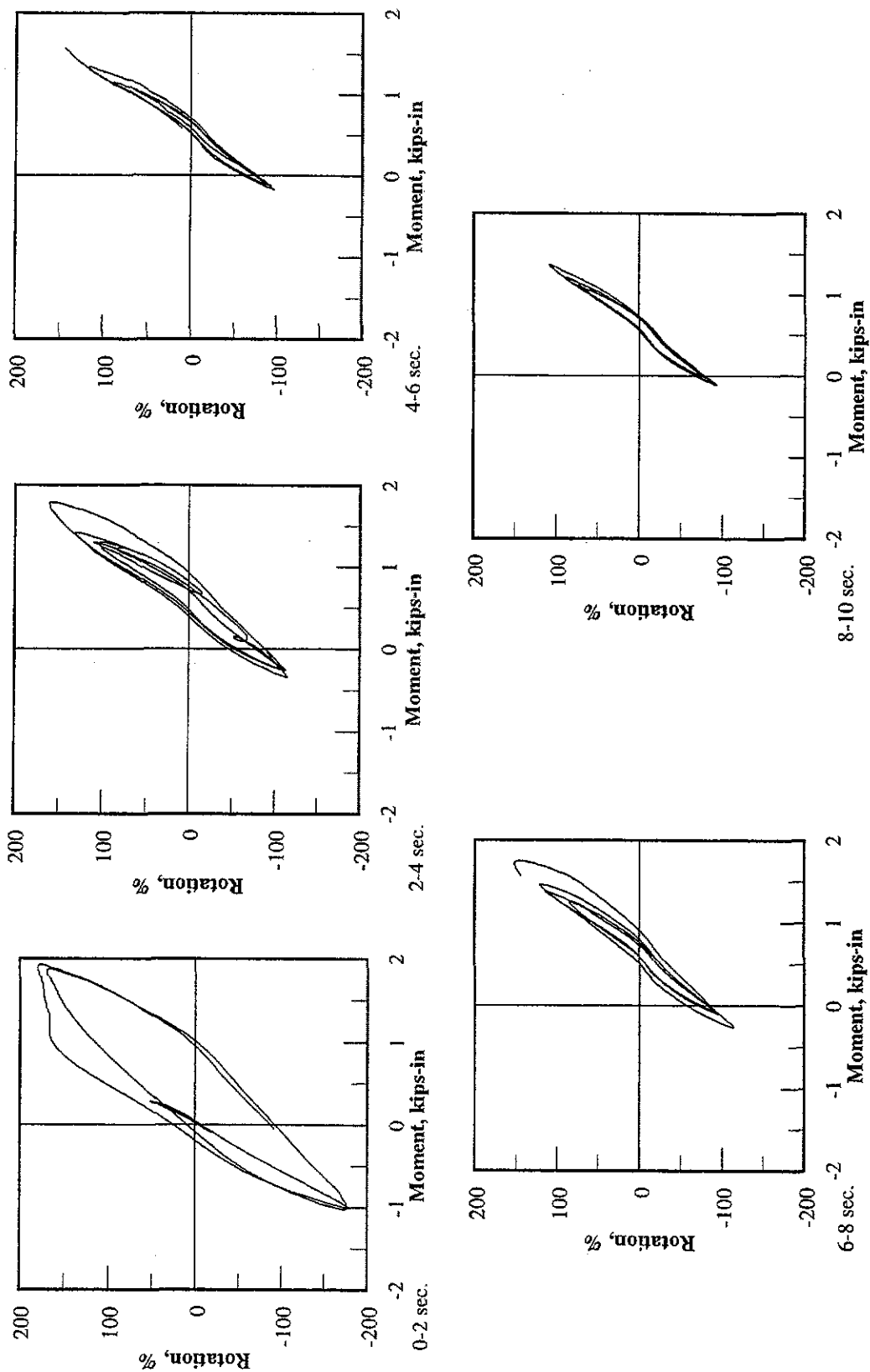


Fig. 3.21.3a Measured Moment-Rotation Hysteresis for UHW Run2, Y-Axis (Translational Resistance)

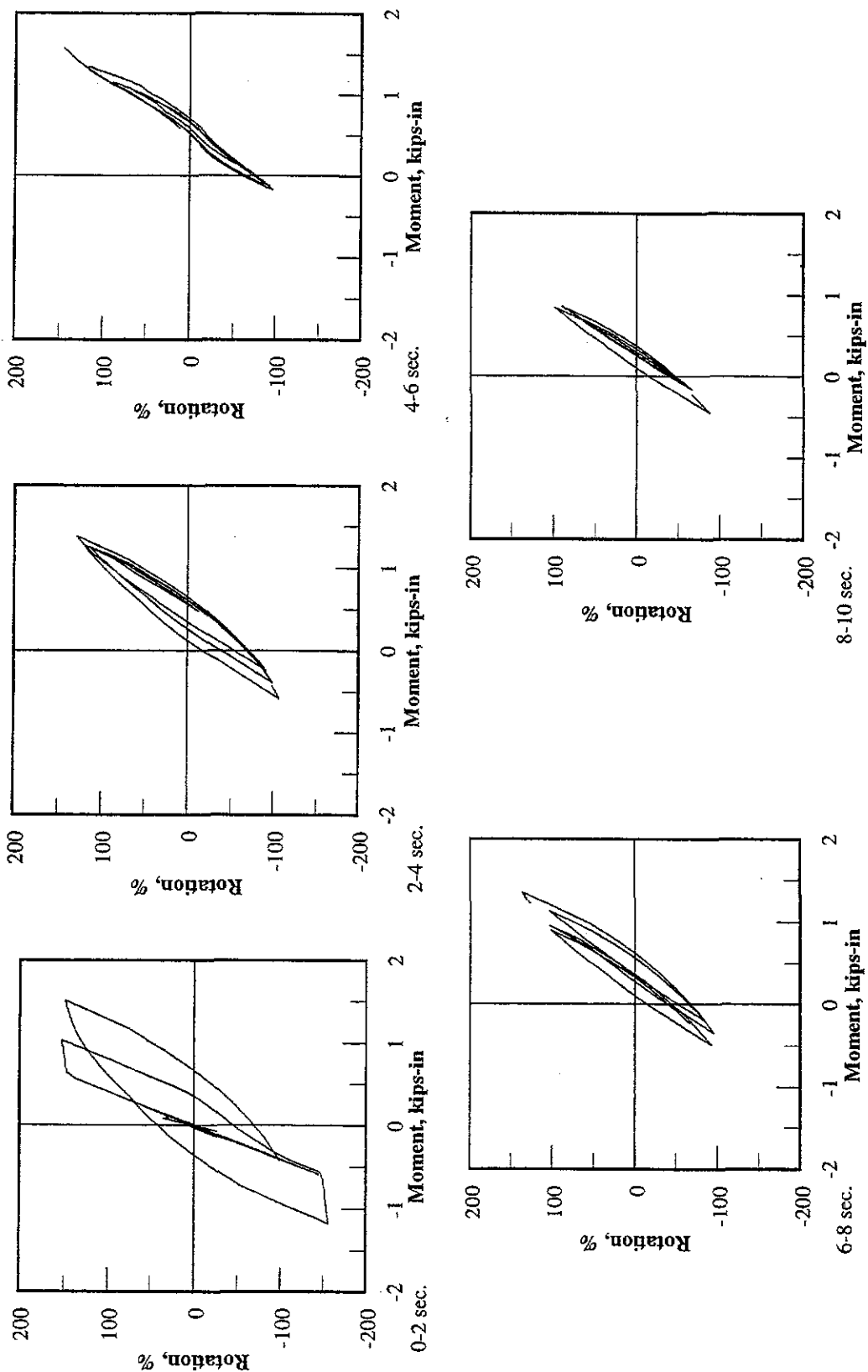


Fig. 3.21.3b Calculated Moment-Rotation Hysteresis for UHW Run2, Y-Axis (Translational Resistance)

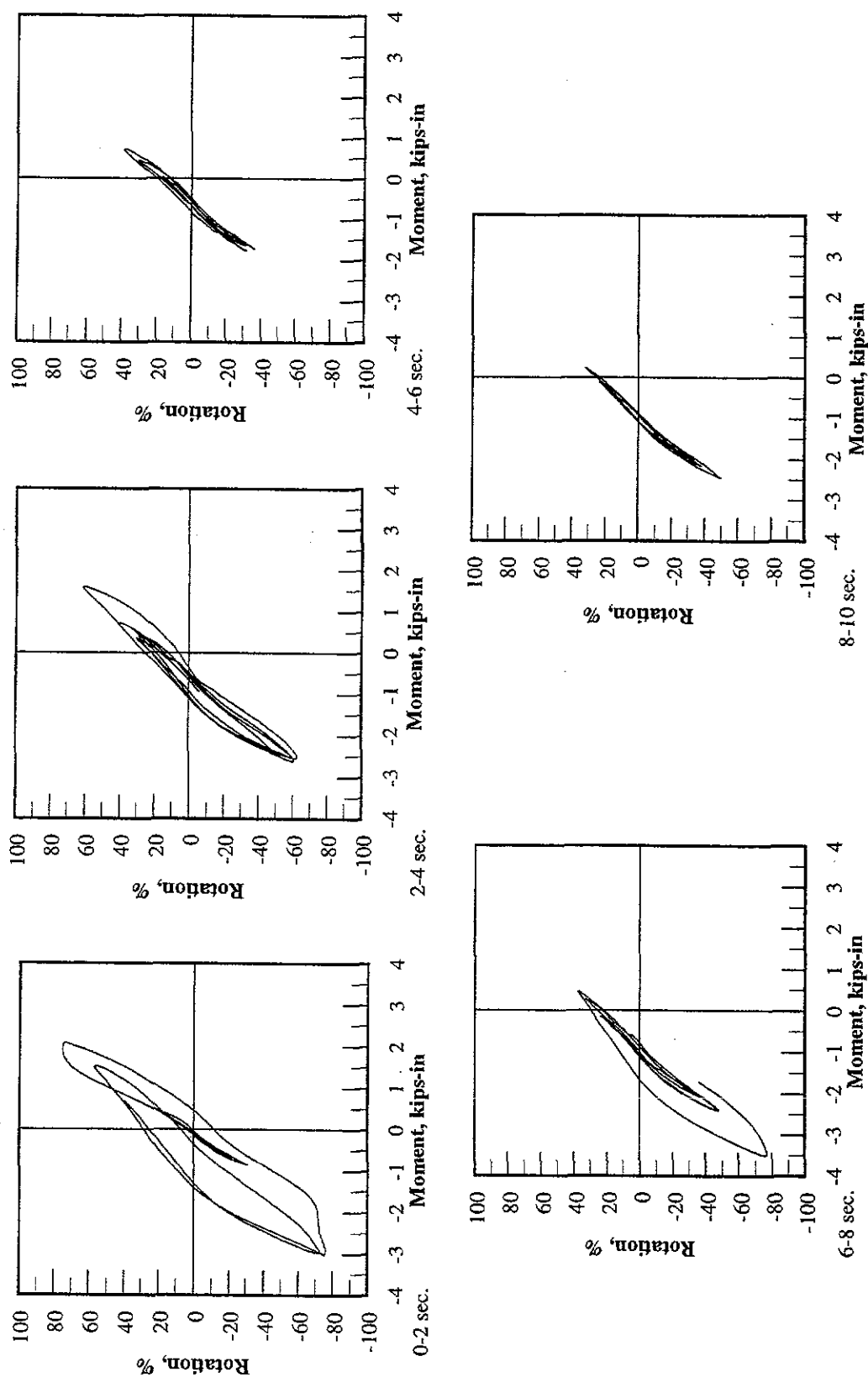


Fig. 3.21.3c Measured Moment-Rotation Hysteresis for UHW Run2, Z-Axis (Torsional Resistance)

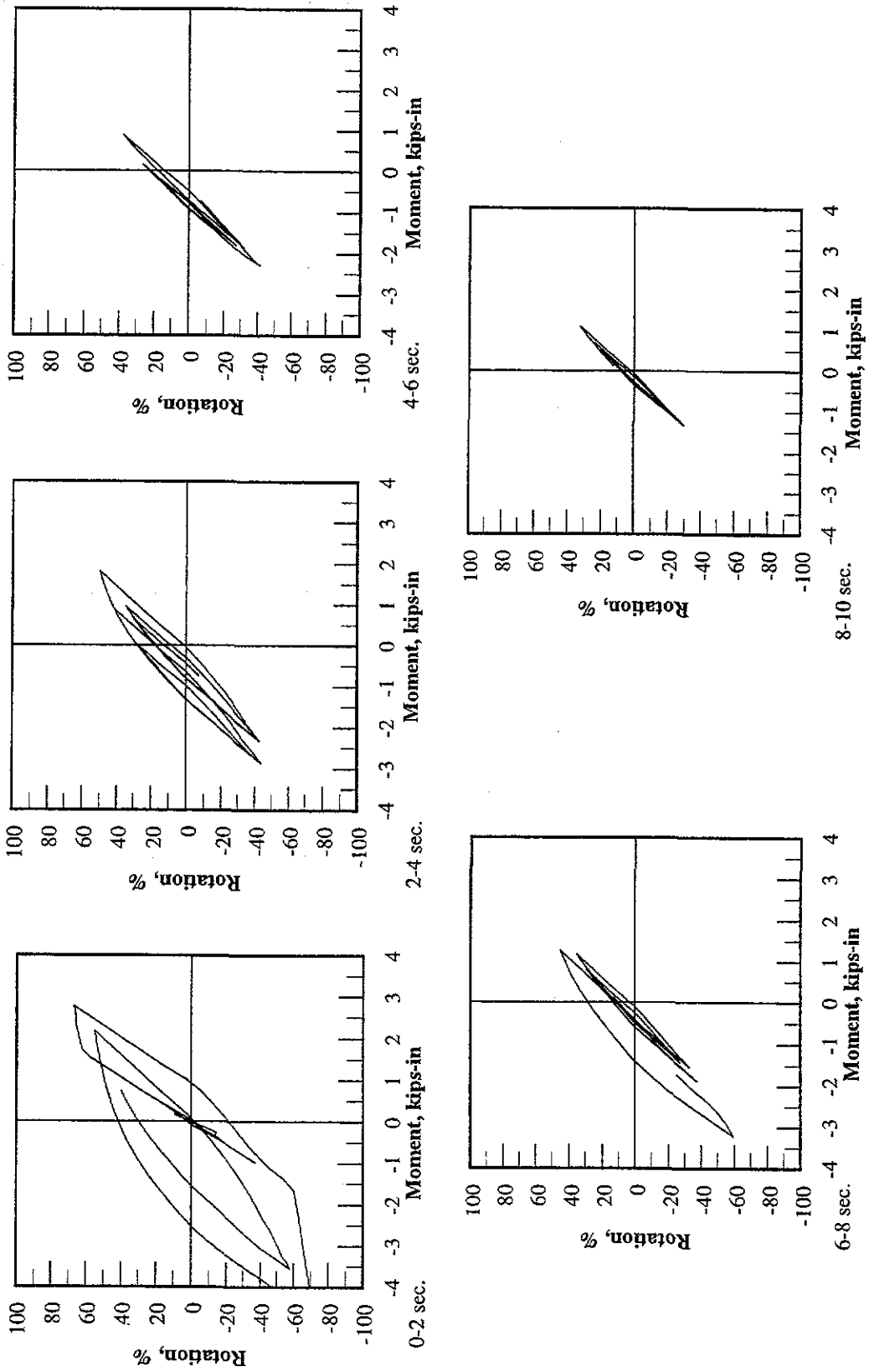


Fig. 3.21.3d Calculated Moment-Rotation Hysteresis for UHW Run2, Z-Axis (Torsional Resistance)

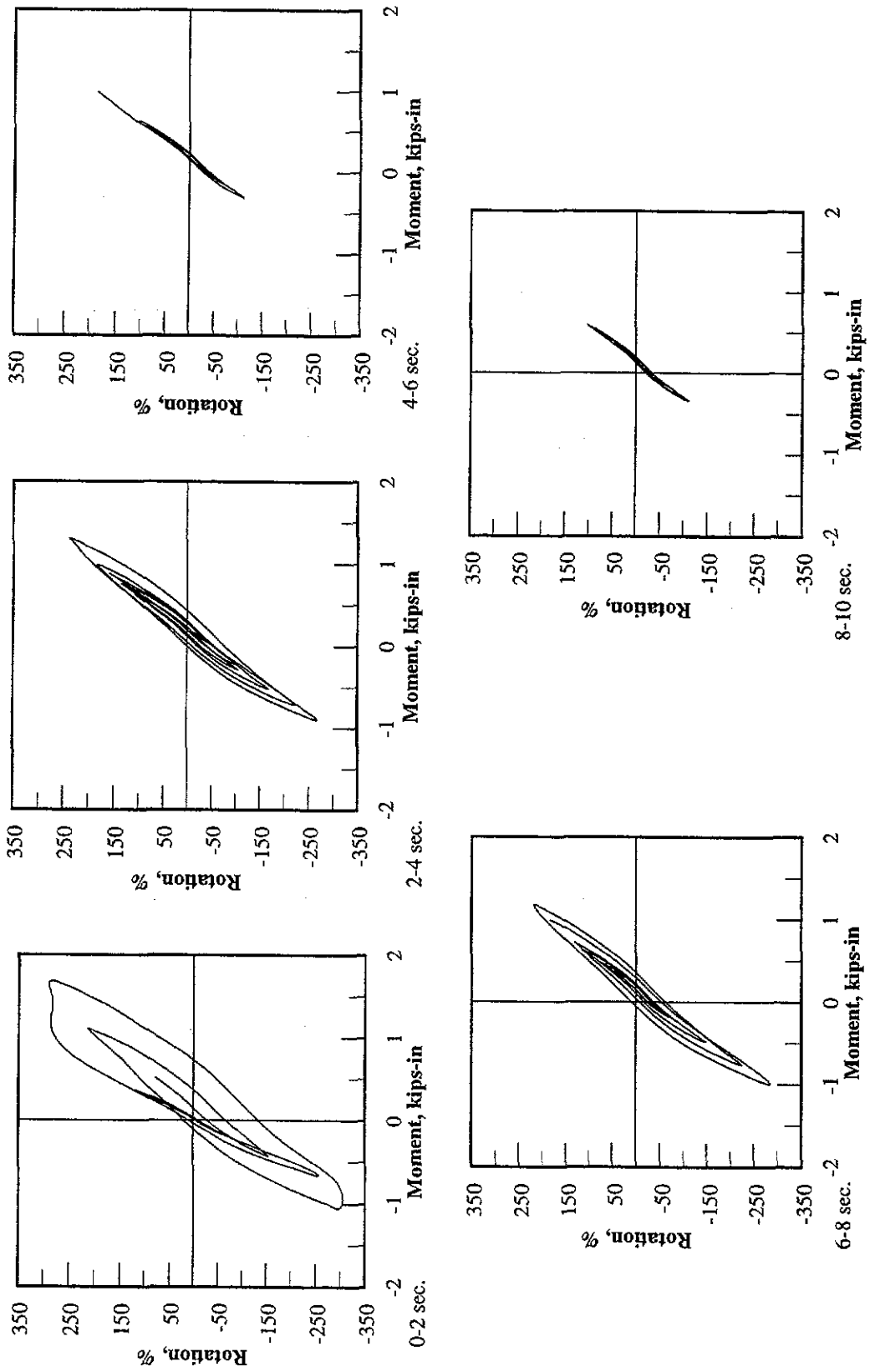


Fig. 3.21.4a Measured Moment-Rotation Hysteresis for ULS Run2, Y-Axis (Translational Resistance)

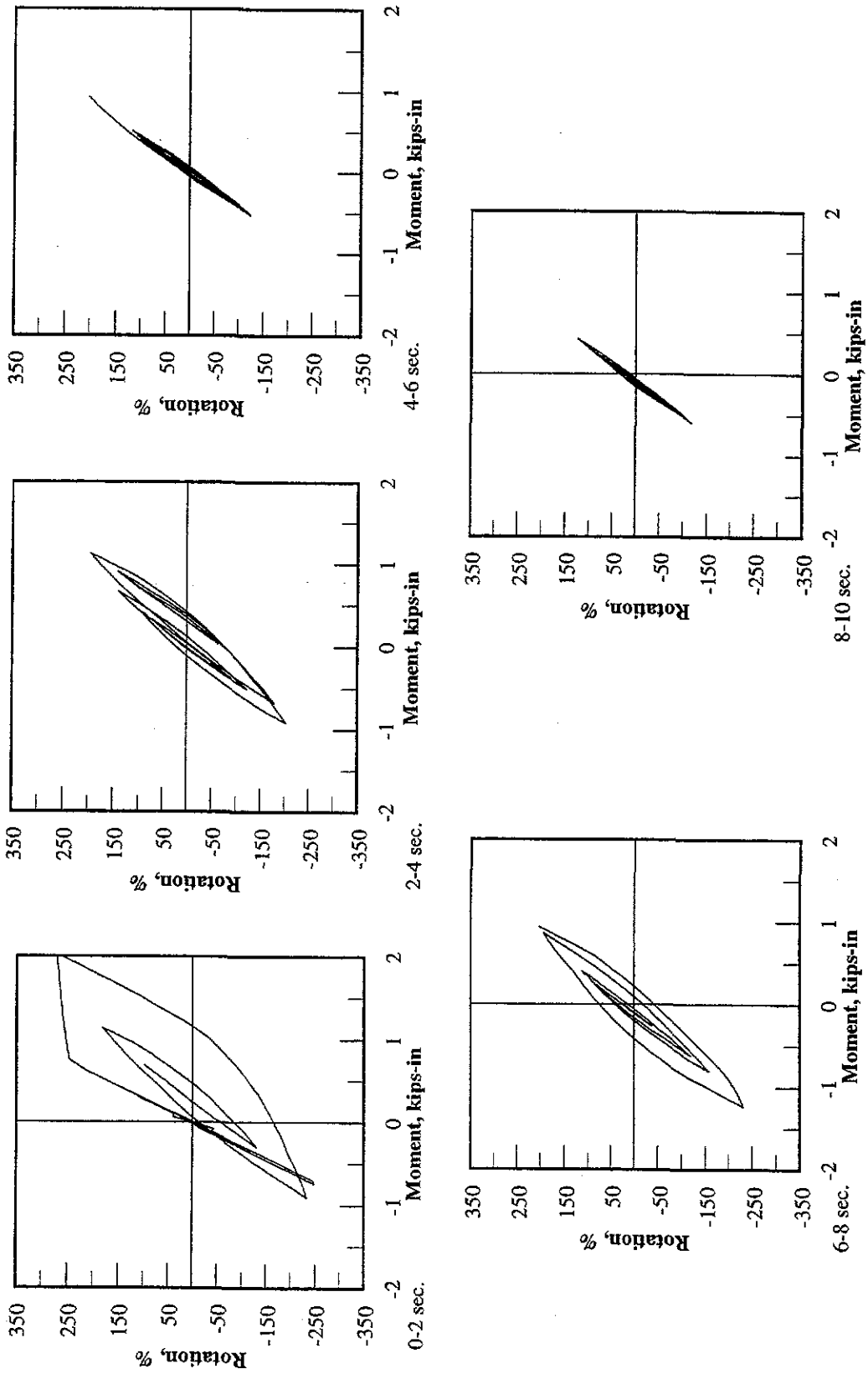


Fig. 3.21.4b Calculated Moment-Rotation Hysteresis for ULS Run2, Y-Axis (Translational Resistance)

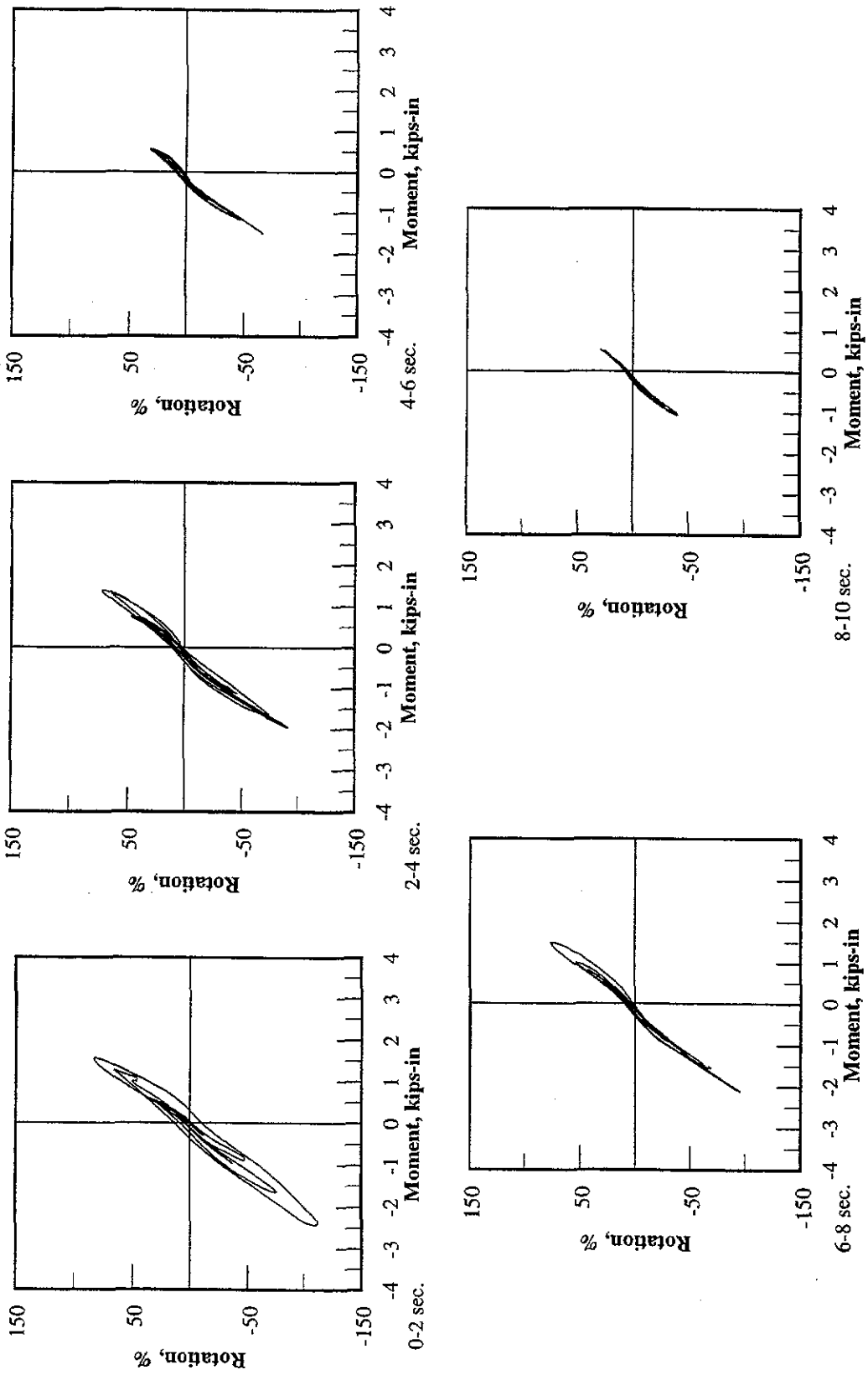


Fig. 3.21.4c Measured Moment-Rotation Hysteresis for ULS Run2, Z-Axis (Torsional Resistance)

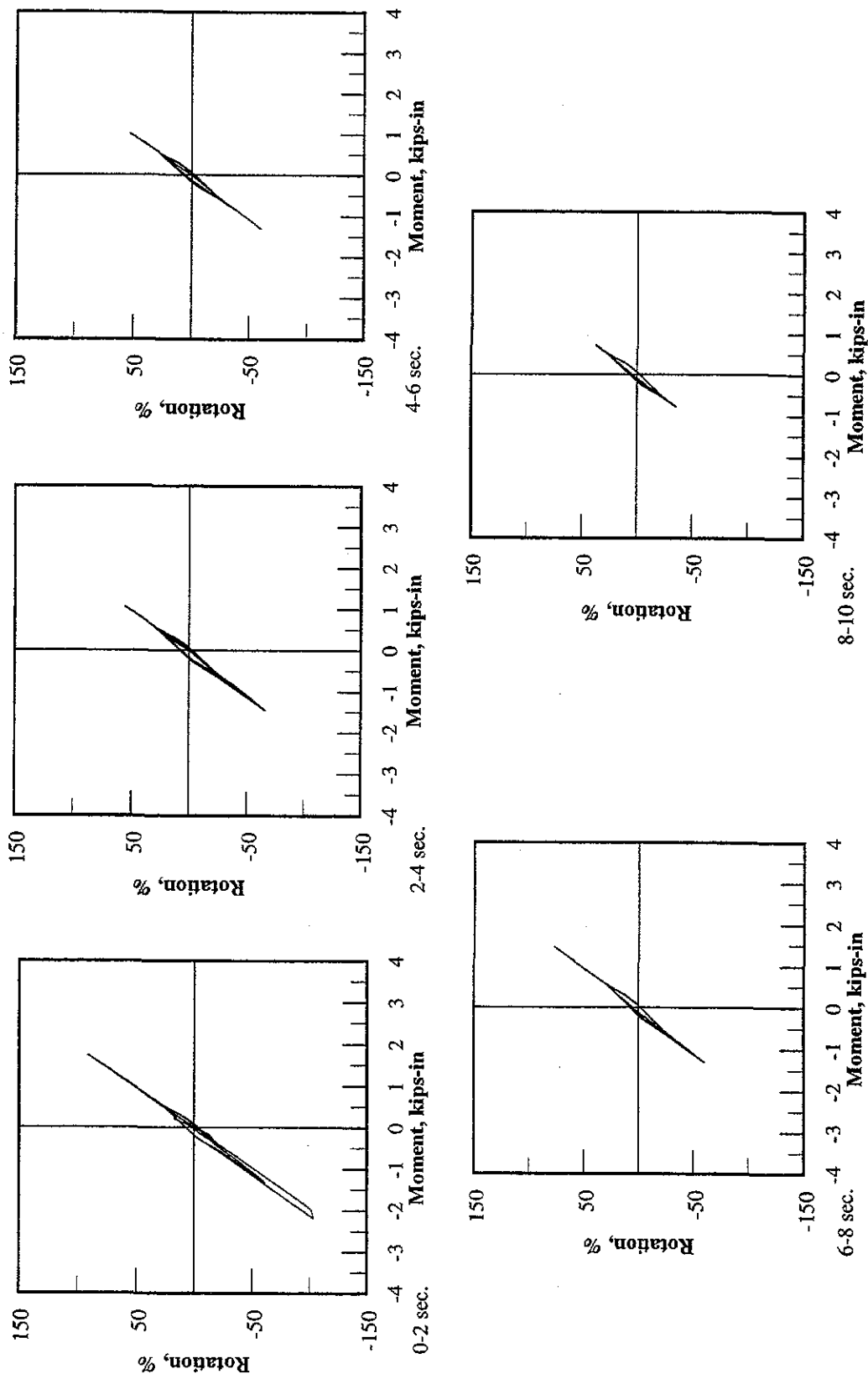


Fig. 3.21.4d Calculated Moment-Rotation Hysteresis for ULS Run2, Z-Axis (Torsional Resistance)

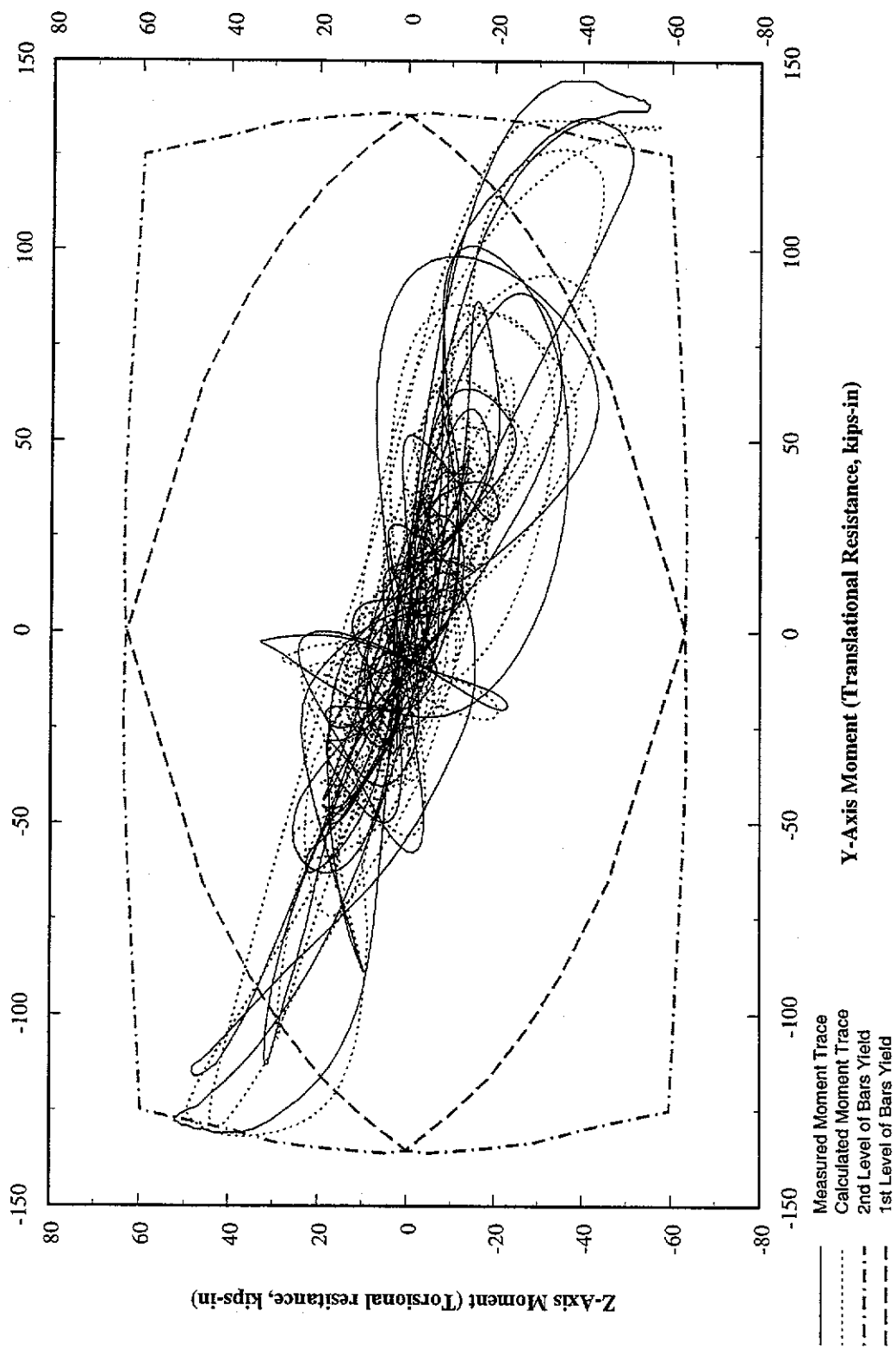


Fig. 3.22a The Measured Moment Trace, the Calculated Moment Trace, and the Calculated Moment-Interaction Curves for CLW ,Run2

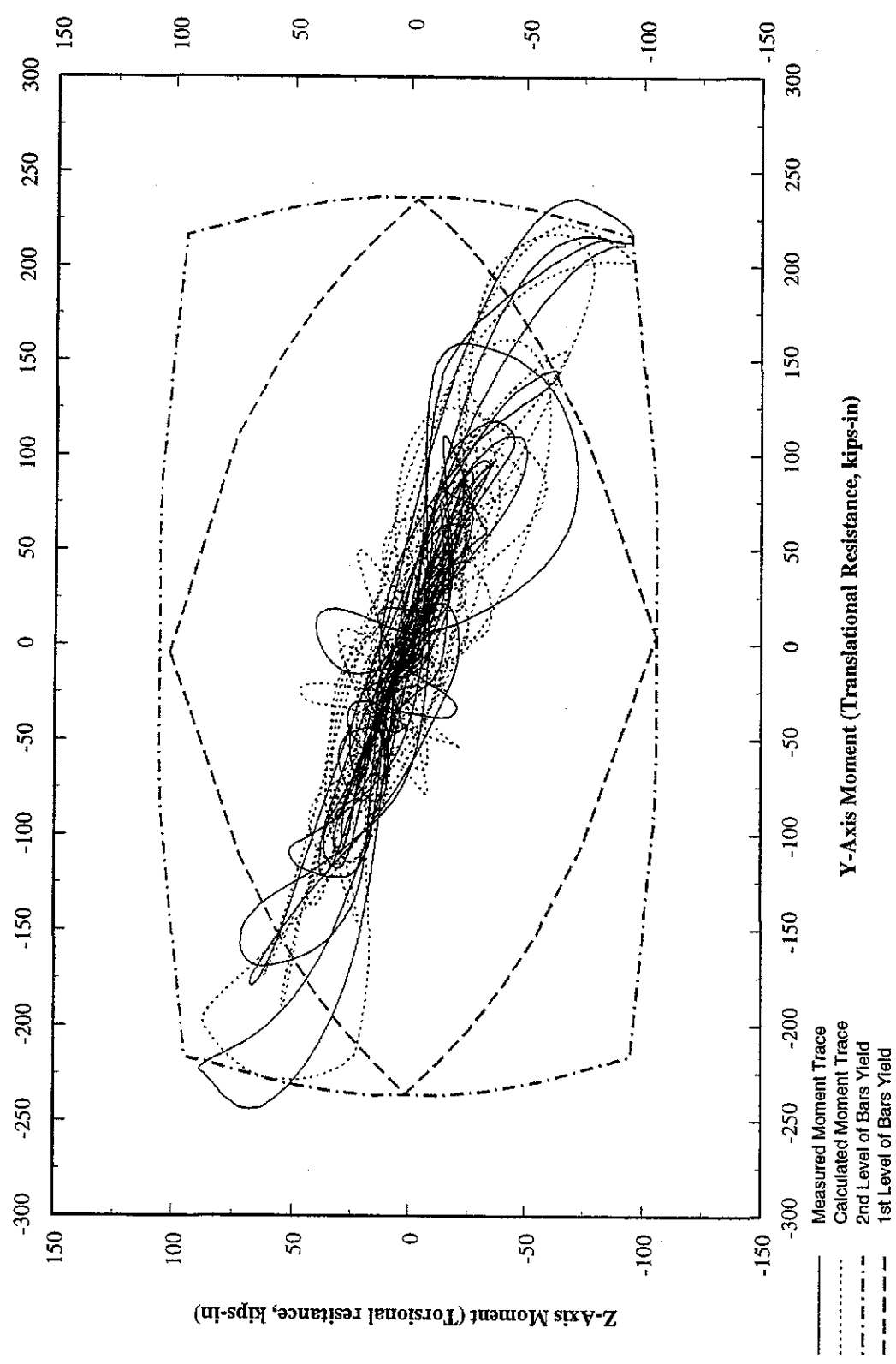


Fig. 3.22b The Measured Moment Trace, the Calculated Moment Trace, and the Calculated Moment-Interaction Curves for CHS ,Run2

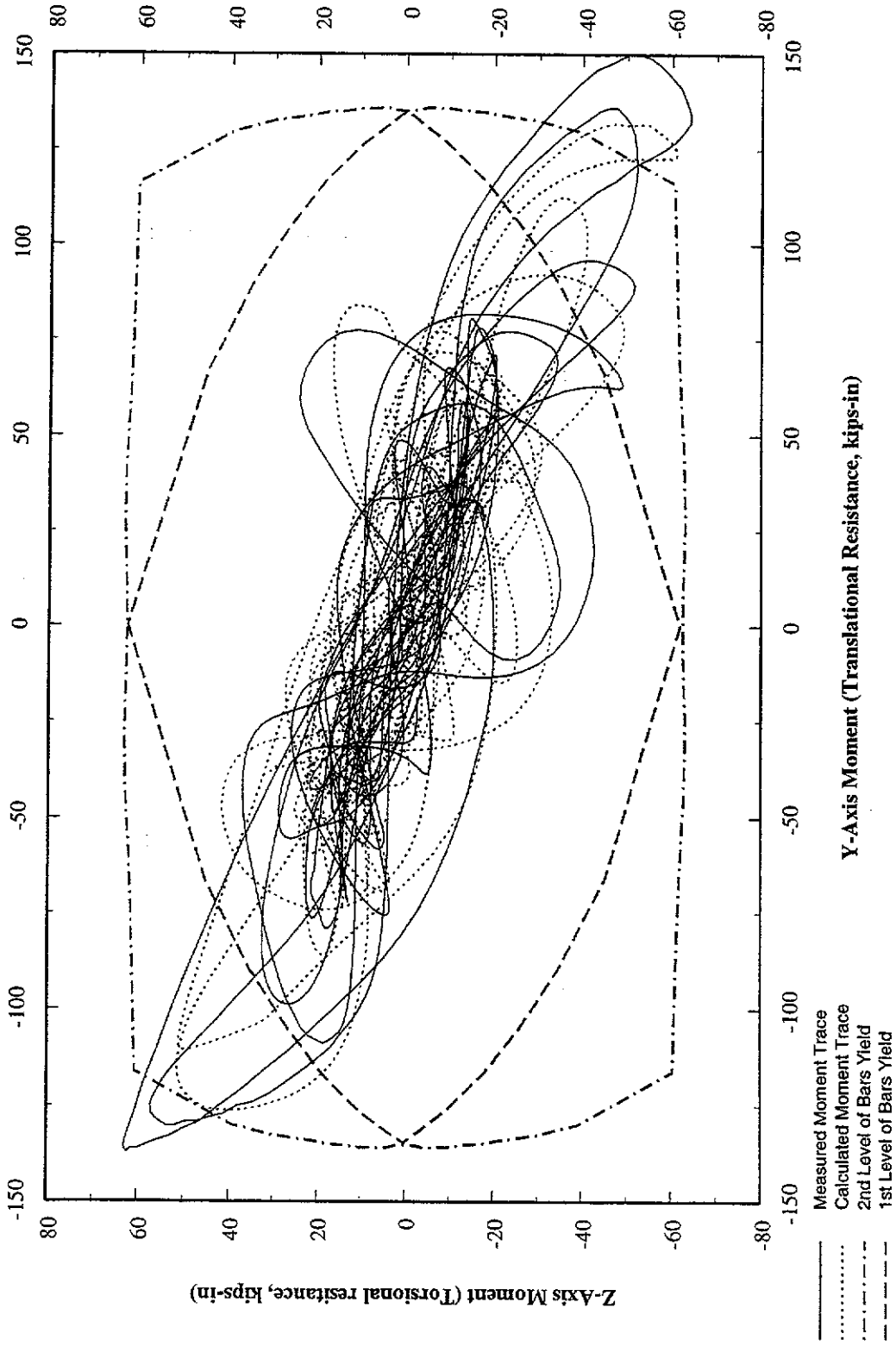


Fig. 3.22c The Measured Moment Trace, the Calculated Moment Trace, and the Calculated Moment-Interaction Curves for CHW ,Run2

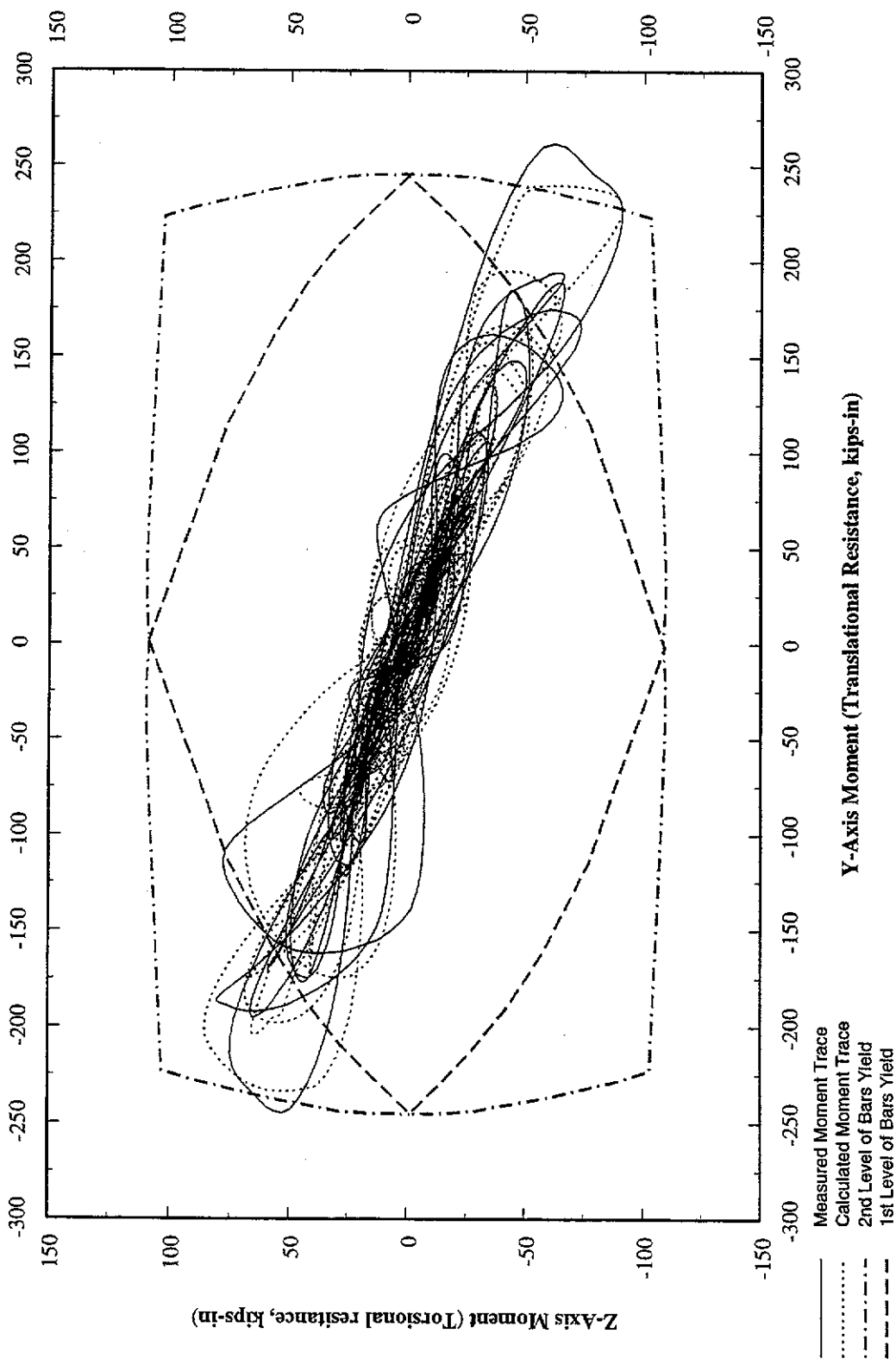


Fig. 3.22d The Measured Moment Trace, the Calculated Moment Trace, and the Calculated Moment-Interaction Curves for CLS ,Run2

4. SENSITIVITY ANALYSIS OF THE PROPOSED MODEL

4.1 Introduction

This chapter documents the studies made to check the sensitivity of the proposed model to changes in the parameters controlling output. The sensitivity study was made to define the needed accuracy for the input for applications of the model to determine response. The effect of changes in the following parameters on computed displacement history was considered:

1. Concrete strength (Sec. 2.3).
2. Parameters defining “early contact” of concrete (Sec. 2.3).
3. Initial modulus of elasticity of the concrete (Sec. 2.3).
4. Yield stress of the reinforcing steel (Sec. 2.4).
5. Hysteresis model for the reinforcement (Sec. 2.4).
6. Transition parameter for the Menegotto-Pinto model for defining reinforcement response (Sec. 2.4.2).
7. Length of the “yield region” (Sec. 2.5.1).
8. Development length of the reinforcing steel (Sec. 2.5.3).

The selected ranges of variations in the parameters are presented in Table 4.1. Parameters that were used to get the reference response of the model are given in the second column of Table 4.1. The term “consistent definition” in Table 4.1 refers to determination of the “yield region” by defining it to extend from the section where the yield moment is encountered to the section of maximum moment.

Another objective of Chapter 4 is to investigate the effect of the base excitation with two components (having different frequency contents) on the sensitivity of the proposed model response. The reason for the second objective was to probe the effect of having two different components of the ground motion in the two orthogonal directions. For this purpose, two different structural configurations were used.

The first structural configuration (System A) was one of the specimens tested by Dragovich (1996), specimen CLW. Specimen CLW was subjected to the north-south component of the El Centro earthquake record (California Institute of Technology, 1971). Motion was scaled to have a maximum acceleration of $0.81g$. Specimen CLW reached maximum rotations of approximately 2.5% during its rotation-history (Fig. 3.18a) for the reference response. Material properties of the specimen CLW are given in Table 3.3. Dimensions and the longitudinal reinforcement configuration are given in Fig. 3.11c, Table 3.3 and Fig. 3.12.

The second structural configuration (System B) was a cantilever beam that was subjected to horizontal components of the El Centro earthquake record (California Institute of Technology, 1971). The record was scaled to have a maximum acceleration of $0.5g$ (north-south component), keeping the original ratio of the two components. The scaled records are presented in Fig. 4.1. The dimensions and the longitudinal reinforcement configuration of the cantilever were given in Fig. 4.2. The strength of the concrete and the yield stress of the steel used for System B were 6,000 psi and 60,000 psi. The system reached to a maximum deflection of approximately 4.5% of its height for the reference response.

Waveforms calculated for different sets of parameters were compared by using Frequency Domain Error index (FDE, Appendix D), as well as by visual inspection. The FDE indices resulting from comparisons of the displacement history of the reference response against the displacement histories of the responses for the listed parameters are presented in Table 4.2. The displacement histories of the systems are presented in Fig. 4.3

to Fig. 4.10. The sensitivity of the proposed model response to given set of parameters is discussed in Section 4.2. A summary of the chapter is presented in Section 4.3.

4.2 Results of The Sensitivity Study

Sensitivity of the proposed model was considered in terms of maximum displacements and the waveform of the displacement histories. The observed sensitivities are presented for each input parameter below:

1. *Concrete Strength*: Changes in concrete strength (Table 4.1) caused a barely perceptible change in the calculated displacement in the phase of high response (Fig. 4.3). High-response phase refers to the duration of the test when the moment response was approximately half of the yield moment or more. The change in maximum displacement caused by increasing or decreasing the concrete strength by 20% was less than 3% (Table 4.3). In the low-response phase, there were small shifts but, in general, the shape of the waveform was preserved. The FDE indices were less than 0.07 (Table 4.2) for the entire duration. There was no clear difference in sensitivity to changes in concrete strength for systems A and B.
2. *Parameters defining "early contact" of concrete*: Change in the "early contact" parameters (Table 4.1) caused very little difference for the displacement response in high-response phases (Fig. 4.4). The change in maximum displacement was less than 3%. During low-response phases, there were small shifts but the shape of the waveform was preserved. The FDE indices were less than 0.12 (Table 4.2) for the entire duration. There was no clear difference in sensitivity for responses of systems A and B.
3. *Initial modulus of elasticity of the concrete*: Change in the initial modulus of elasticity of concrete (Table 4.1) also caused very little difference for the displacement response in high-response phases (Fig. 4.5). The change in maximum displacement was less than 3%. During low-response phases, there were notable

- shifts but the shape of the waveform was preserved. The FDE indices were less than 0.11 (Table 4.2) for the entire duration. There was no clear difference in sensitivity for responses of systems A and B.
4. *Yield Stress of Reinforcing Steel*: Change in the yield stress of the reinforcing steel caused notable changes in periodicity, waveform and the maximum displacement for both low- and high-response phases (Fig. 4.6). The change in maximum displacement caused by increasing or decreasing the steel yield stress by 20% was in the interval of $\pm 13\%$ (Table 4.3). The FDE indices were less than 0.31 (Table 4.2). There was no clear difference in sensitivity for responses of systems A and B.
 5. *Steel Hysteresis Model*: Change in the steel hysteresis model caused perceptible changes in periodicity, waveform, and the maximum displacement for both low- and high-response phases (Fig. 4.7). The inter-relationship of the histories calculated for the bilinear and the Menegotto-Pinto curves suggests that the M-P curve provided higher damping during phases of low response (Fig. 4.7b, east-west axis, 5 to 9-sec). The change in maximum displacement was less than 19% (Table 4.3). The FDE indices were less than 0.31 (Table 4.2). There was no clear difference in sensitivity for responses of systems A and B.
 6. *Transition parameter for the Menegotto-Pinto model*: Change in the transition parameter from 17 to 23 (See Sec. 2.4.2) caused notable changes in periodicity, waveform, and the maximum displacement for both low- and high-response phases (Fig. 4.8). The inter-relationship of the histories calculated for increasing values of transition parameter suggests that larger values provided higher damping during phases of low response (Fig. 4.8b, both axis, 6 to 12-sec.). The change in maximum displacement was less than 11% (Table 4.3). The FDE indices were less than 0.17 (Table 4.2). There was no clear difference in sensitivity for responses of systems A and B.
 7. *Length of "yield region"*: Change in the "yield region" length (Table 4.1) caused very little difference for the displacement response in high-response phases (Fig. 4.9). The change in maximum displacement was less than 2% (Table 4.3). On low-response phases, there were small shifts but the shape of the waveform was

preserved. The FDE indices were less than 0.02 (Table 4.2) for the entire duration. There was no clear difference in sensitivity for responses of systems A and B.

8. *Development Length of the Reinforcing Steel*: Change in the development length also caused notable changes in periodicity, waveform, and the maximum displacement for both low and high response phases (Fig. 4.10). The change in maximum displacement was less than 11% (Table 4.3). The FDE indices were less than 0.19 (Table 4.2). There was no clear difference in sensitivity for responses of systems A and B.

4.3 Summary

The sensitivity of the proposed model was checked for changes in the input parameters, under dynamic loading. Based on sensitivity of responses, parameters can be divided into two groups:

- 1) The output of the model was found to be sensitive to changes in (i) The yield stress of the reinforcing steel, (ii) The steel hysteresis model, (iii) The transition parameter of the Menegotto-Pinto model, (iv) Development length of the reinforcing bars. Variation of transition parameter of the Menegotto-Pinto model and change of steel hysteresis model also affected the damping characteristics of the response. The FDE indices of this group for the comparisons with the reference displacement history were less than 0.31. The changes of the absolute-maximum displacements were less than 19%.
- 2) The output of the model in phases of low response was found to be sensitive to: (i) Concrete strength, (ii) Parameters defining crack closing, (iii) Initial modulus of elasticity for the concrete and, (iv) The definition of the "yield region" along the column. The FDE indices of this group for the comparisons with the reference displacement history were less than 0.12. Changes of the absolute-maximum displacements were less than 3%.

The sensitivity of the proposed model to changes in the first group of input parameters was strong for maximum-displacement, waveform, and the periodicity. The effect of the second group of input parameters was weak and limited to the low response phases. The choices for the first group of parameters have to be considered carefully in assembling a model for calculating response. Such as, it is clear that accurate knowledge of concrete strength is not important for getting satisfactory estimates of the displacement response. On the basis of the studies made for this chapter, if the actual concrete strength is 5,000 psi, an incorrect assumption of 4,000 psi (20% less) for concrete strength would affect the calculated maximum displacement by less than 3%, with negligible changes in waveform and periodicity. This was found to hold for both systems considered. On the other hand, an error of 20% in the estimate of the yield stress of reinforcing steel would change the displacement response by 15%, with notable changes in the waveform and periodicity.

No clear difference was observed between the sensitivities of the materially coupled responses caused by a single component base excitation and by a double component base excitation (two different frequency contents).

Table 4.1 Parameters Chosen for the Sensitivity Analysis and Their Variations

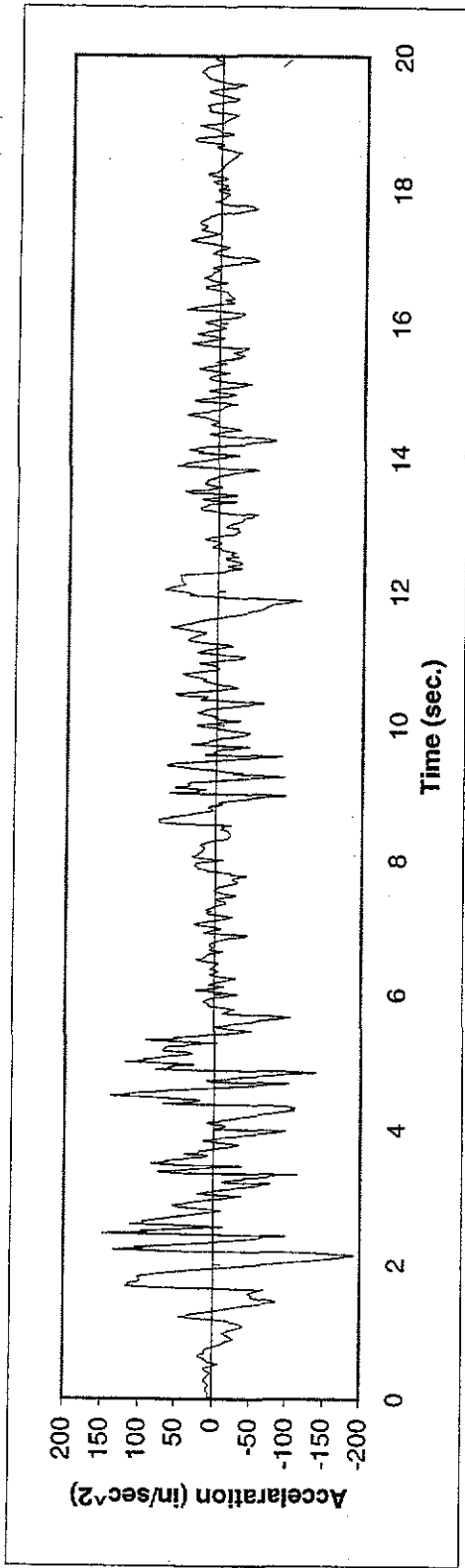
Parameters	Reference Response Parameters	Variations
Concrete Strength	f'_c	$0.8f'_c$ and $1.2f'_c$
Parameters Defining "Early Contact" of Concrete	6, 0.2	No "Early Contact" and 9, 0.4
Initial Modulus of Elasticity of the Concrete	$57000(f'_c)^{0.5}$	$50000(f'_c)^{0.5}$ and $64000(f'_c)^{0.5}$
Yield Stress of the Reinforcing Steel	f_y	$0.8f_y$ and $1.2f_y$
Transition Parameter of Menegotto-Pinto Model	17	12 and 25
Steel Hysteresis Model	Menegotto-Pinto Model	Bilinear Model
Length of "Yield Region"	Consistent Definition	0.5*(Consistent Definition)
Bar Development Length	50 bar diameter	40 bar diameter and 60 bar diameter

Table 4.2 FDE Indices of Systems for Comparisons with the Reference Response

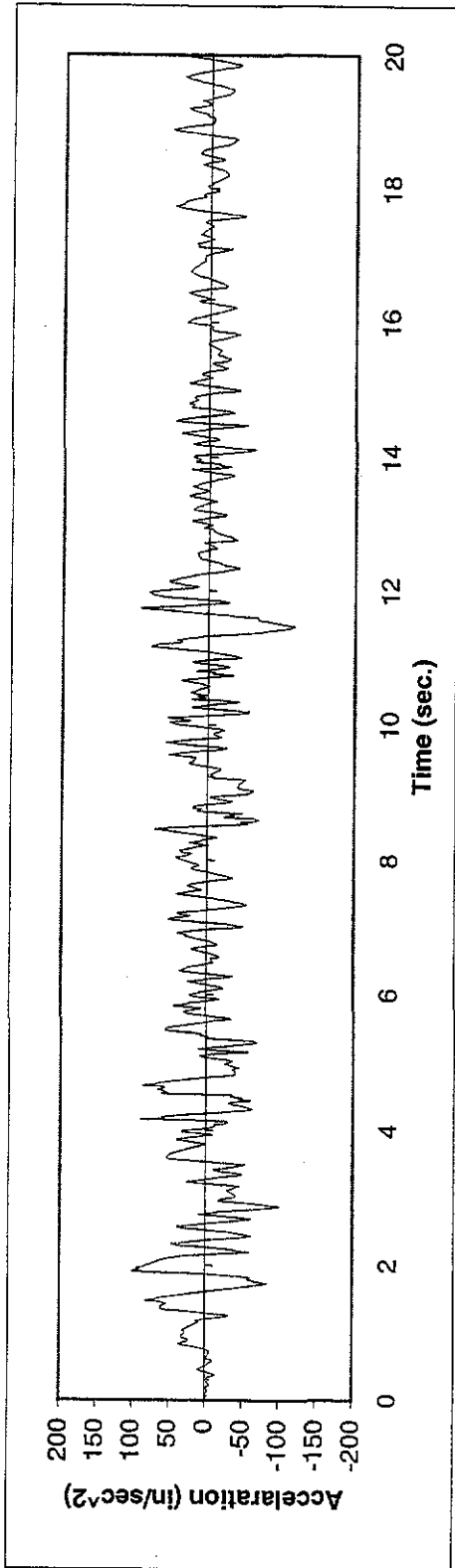
Parameters	Reference Parameters	Variations	First System (CLW)		Second System	
			Y-axis	Z-axis	Y-axis (NS)	Z-axis (EW)
Concrete Strength	f'_c	0.8 f'_c	0.04	0.05	0.05	0.06
		1.2 f'_c	0.02	0.02	0.07	0.05
Parameters Defining "Early Contact" Concrete	6, 0.2	No "Early Contact"	0.08	0.08	0.03	0.04
		9, 0.2	0.05	0.12	0.08	0.06
Yield Stress of the Reinforcing Steel	f_y	0.8 f_y	0.20	0.22	0.30	0.30
		1.2 f_y	0.30	0.26	0.18	0.23
Initial Modulus of Elasticity of the Concrete	$57000(f'_c)^{0.5}$	$50000(f'_c)^{0.5}$	0.08	0.08	0.06	0.06
		$64000(f'_c)^{0.5}$	0.09	0.11	0.04	0.04
Steel Hysteresis Model	Menegotto-Pinto	Bilinear	0.26	0.31	0.28	0.18
Transition Parameter of Menegotto-Pinto Model	17	12	0.08	0.09	0.15	0.09
		23	0.12	0.17	0.12	0.15
Length of "Yield Region"	"Consistent"	0.5* "Consistent"	0.06	0.07	0.03	0.04
Development Length of the Reinforcing Steel	50 Bar Dia.	40 Bar Dia.	0.16	0.17	0.15	0.08
		60 Bar Dia.	0.15	0.19	0.13	0.12

Table 4.3 Maximum Absolute-Displacement Ratios with the Reference Absolute-Displacement

Parameters	Reference Parameters	Variations	First System (CLW)		Second System	
			Y-axis	Z-axis	Y-axis (NS)	Z-axis (EW)
Concrete Strength	f'_c	0.8 f'_c	0.99	1.03	1.00	0.97
		1.2 f'_c	1.01	1.01	0.98	1.02
Parameters Defining "Early Contact" Concrete	6, 0.2	No "Early Contact"	1.01	1.00	0.99	1.02
		9, 0.2	1.03	1.00	1.00	0.97
Initial Modulus of Elasticity of the Concrete	57000(f'_c) ^{0.5}	50000(f'_c) ^{0.5}	1.03	1.02	0.99	1.00
		64000(f'_c) ^{0.5}	1.01	1.00	1.01	0.99
Yield Stress of Reinforcing Steel	f_y	0.8 f_y	0.91	0.87	0.92	1.03
		1.2 f_y	0.96	0.99	1.11	1.07
Steel Hysteresis Model	Menegotto-Pinto	Bilinear	0.95	1.08	0.95	0.81
Transition Parameter of Menegotto-Pinto Model	17	12	0.99	0.99	0.98	0.97
		23	1.11	1.03	1.04	0.99
Length of "Yield Region"	"Consistent"	0.5* "Consistent"	0.99	1.02	1.00	0.99
		40 Bar Dia.	1.01	1.04	1.00	0.89
Development Length of the Reinforcing Steel	50 Bar Dia.	60 Bar Dia.	1.07	1.03	1.00	1.03



North-South Component



East-West Component

Fig. 4.1 El Centro Earthquake Recorded Horizontal Components Scaled Acceleration-Histories.

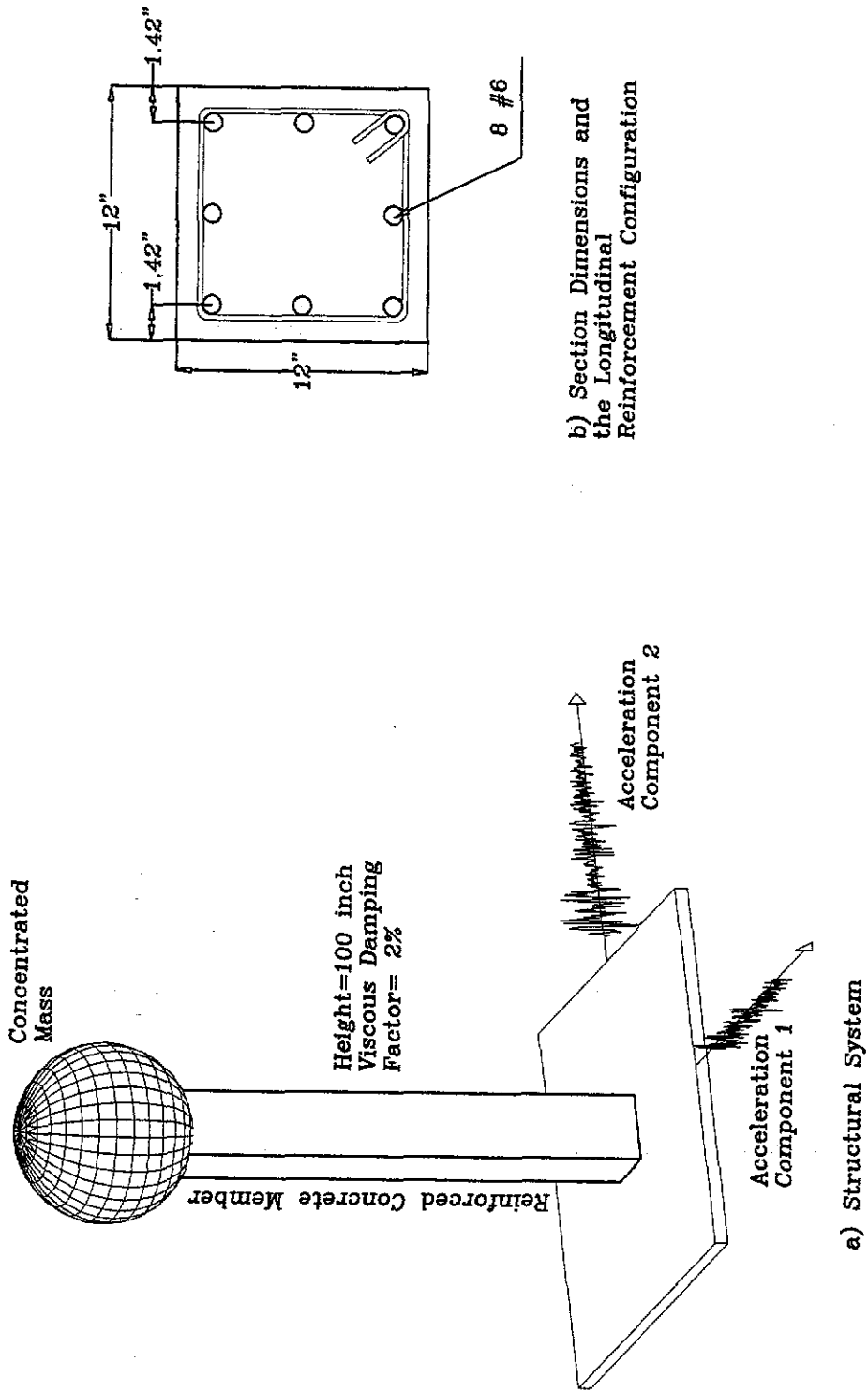


Fig. 4.2 Structural System and the Longitudinal Reinforcement Configuration of System B

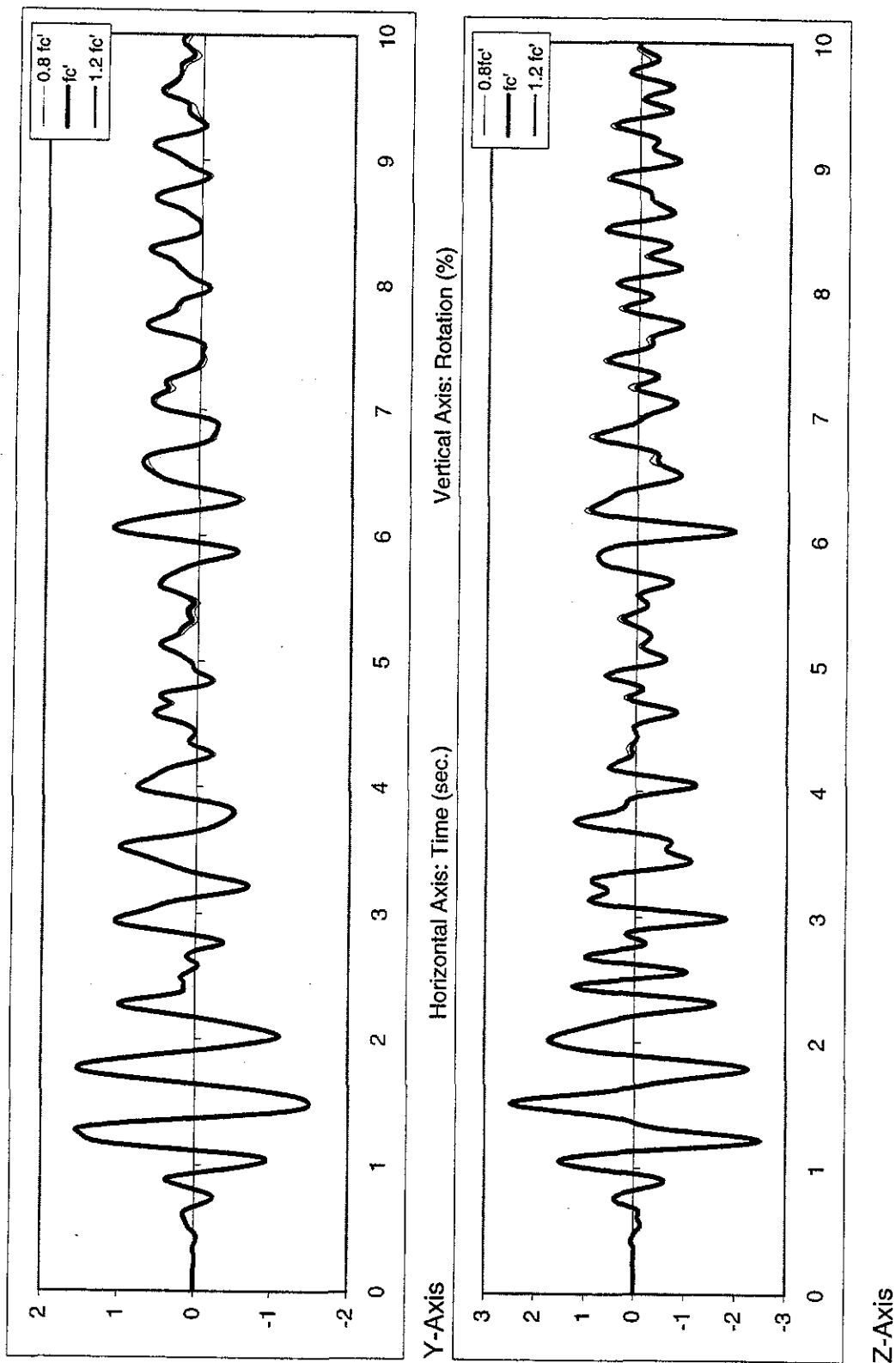


Fig. 4.3a Displacement-Histories of System A for Variations in Concrete Strength

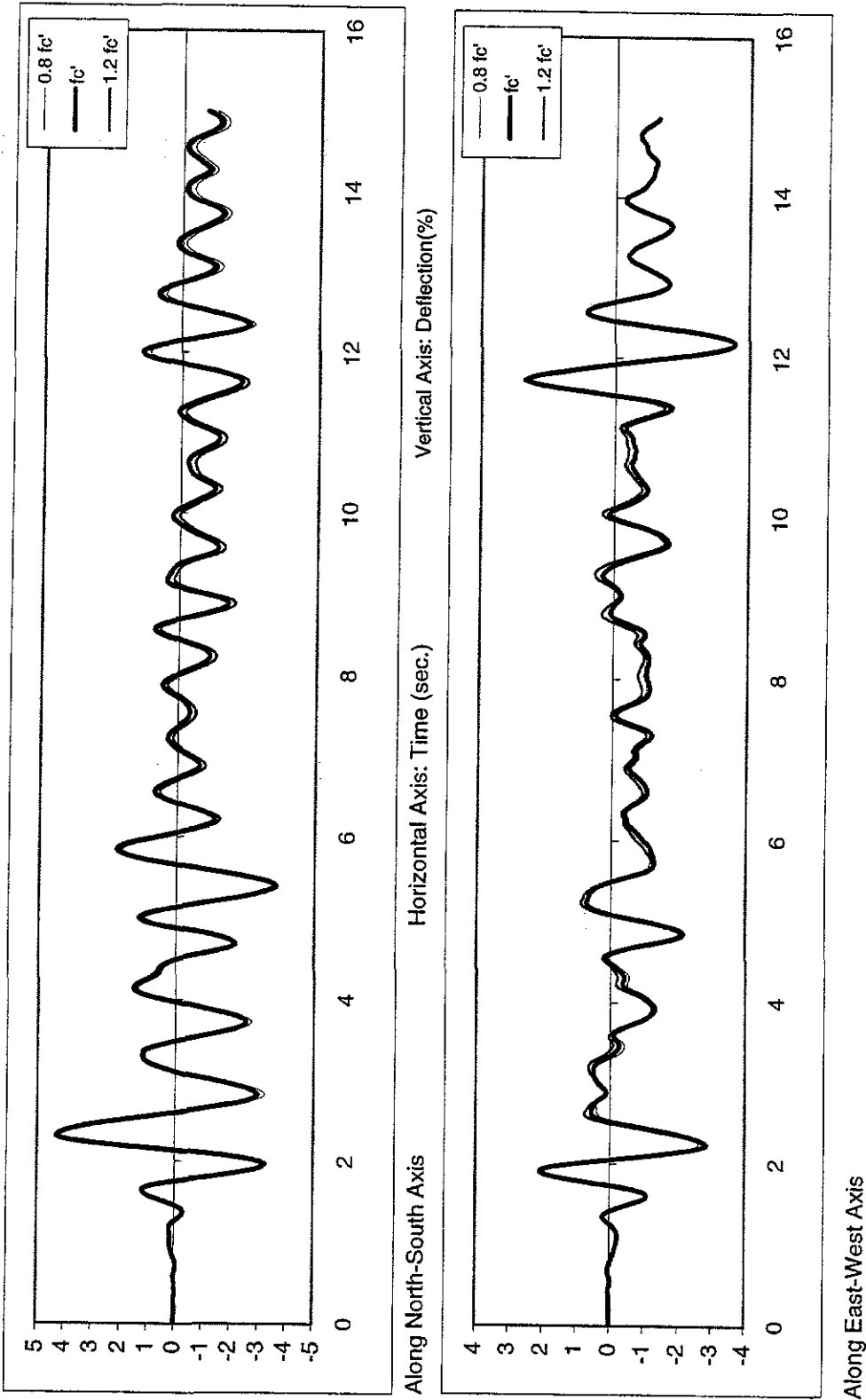


Fig. 4.3b Displacement-Histories of System B for Variations in Concrete Strength

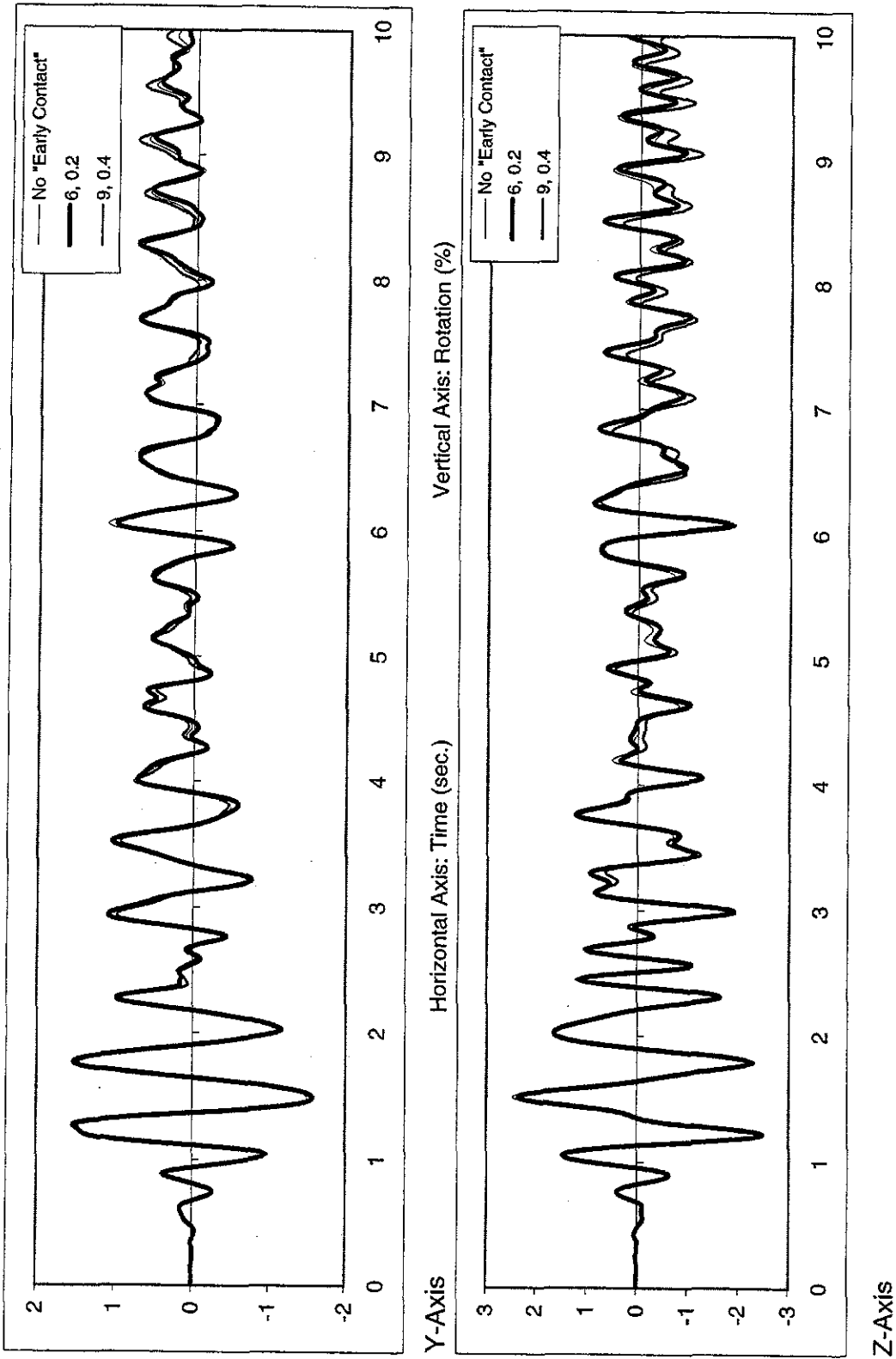


Fig. 4.4a Displacement-Histories of System A for Variations in Concrete "Early Contact" Parameters

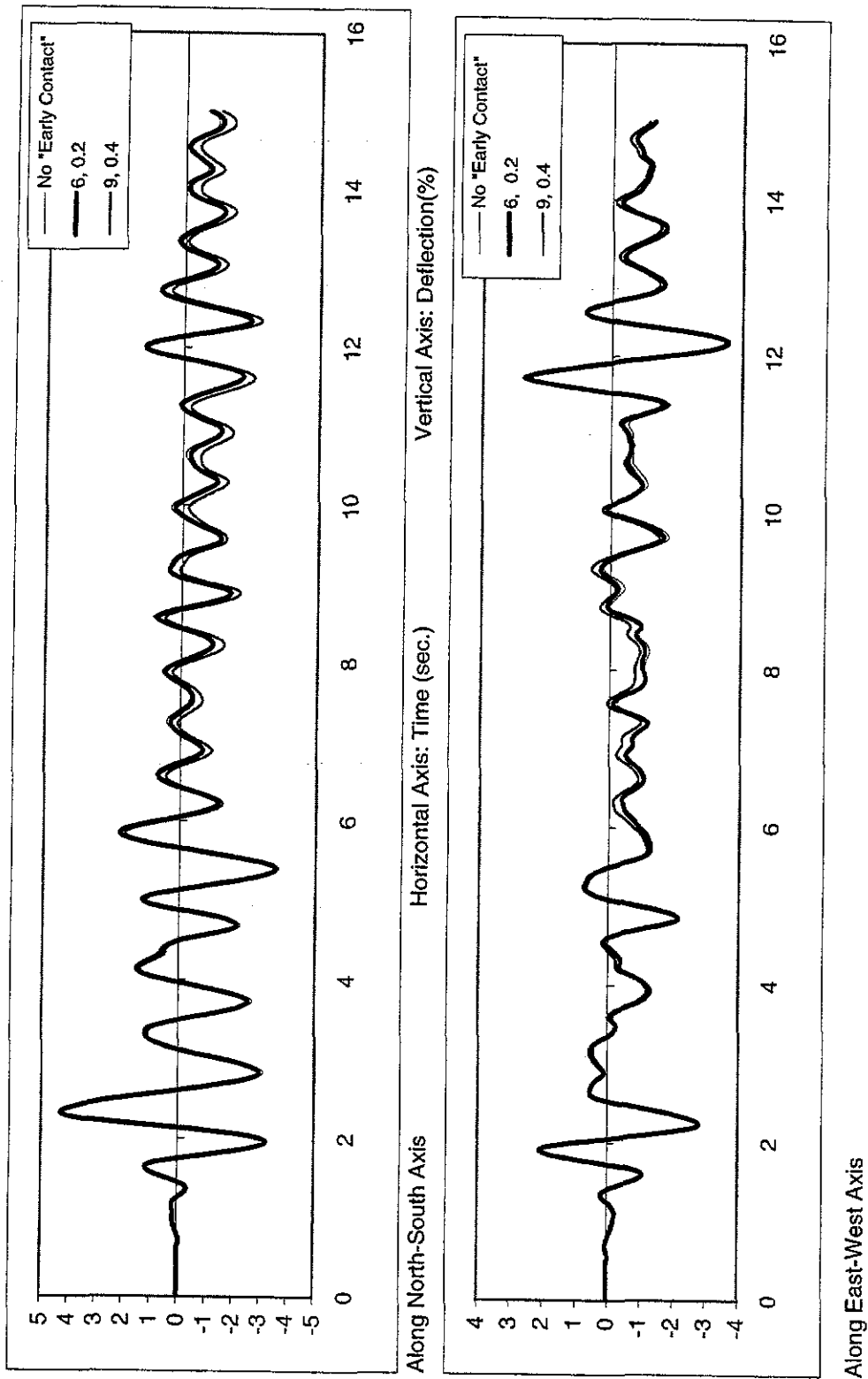


Fig. 4.4b Displacement-Histories of System B for Variations in Concrete "Early Contact" Parameters

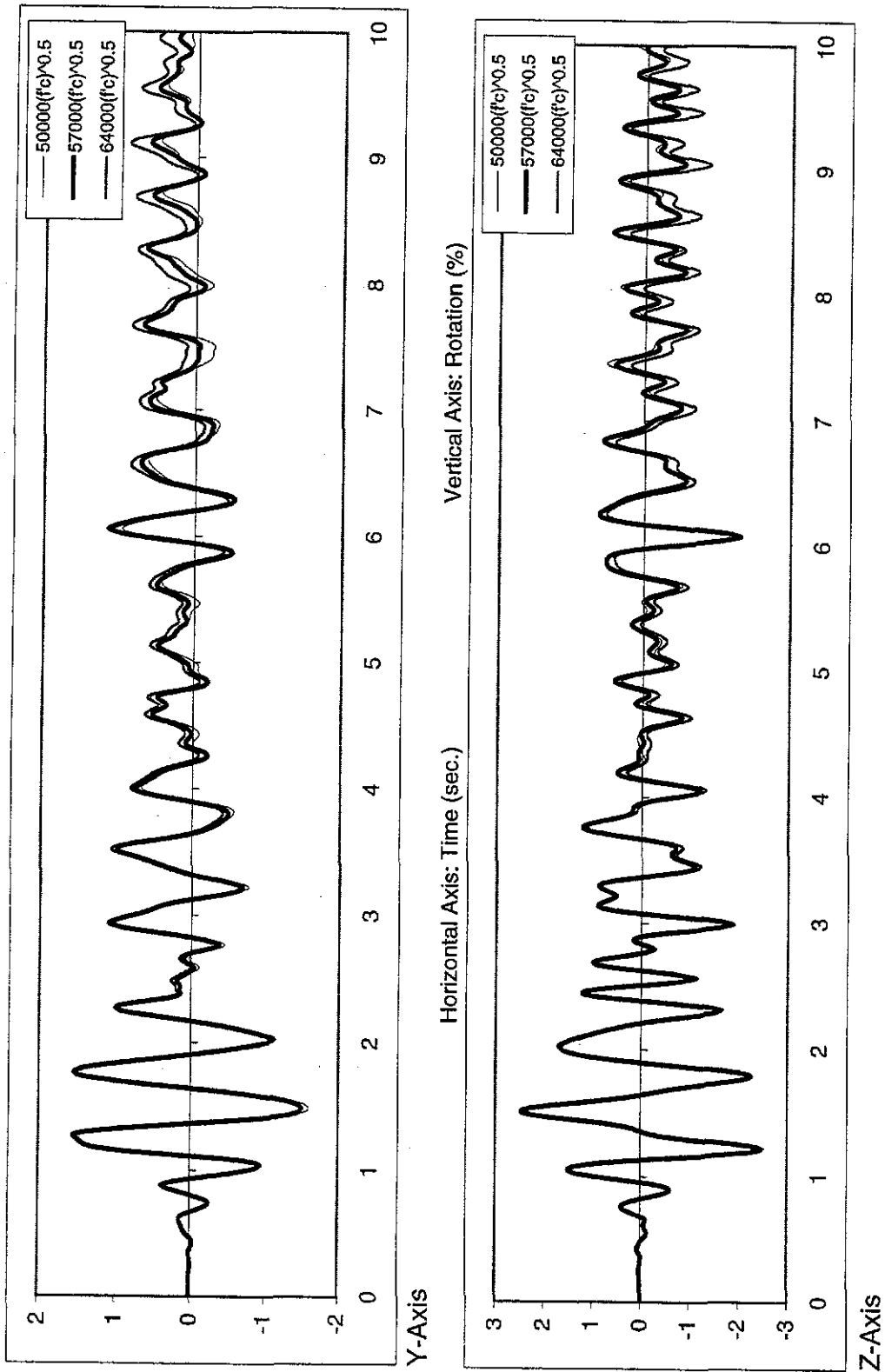
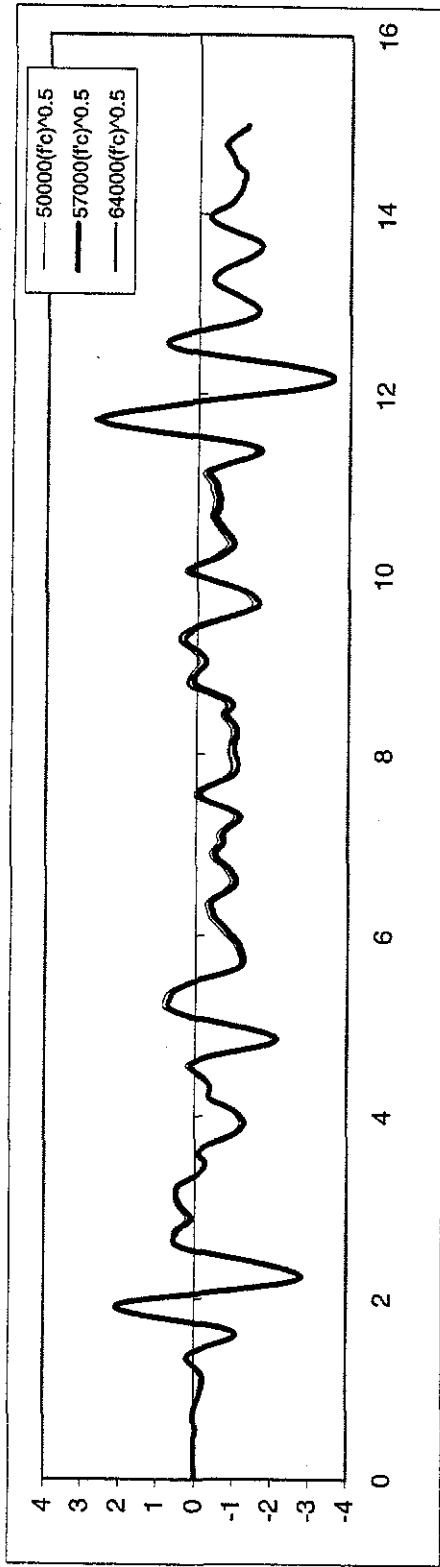
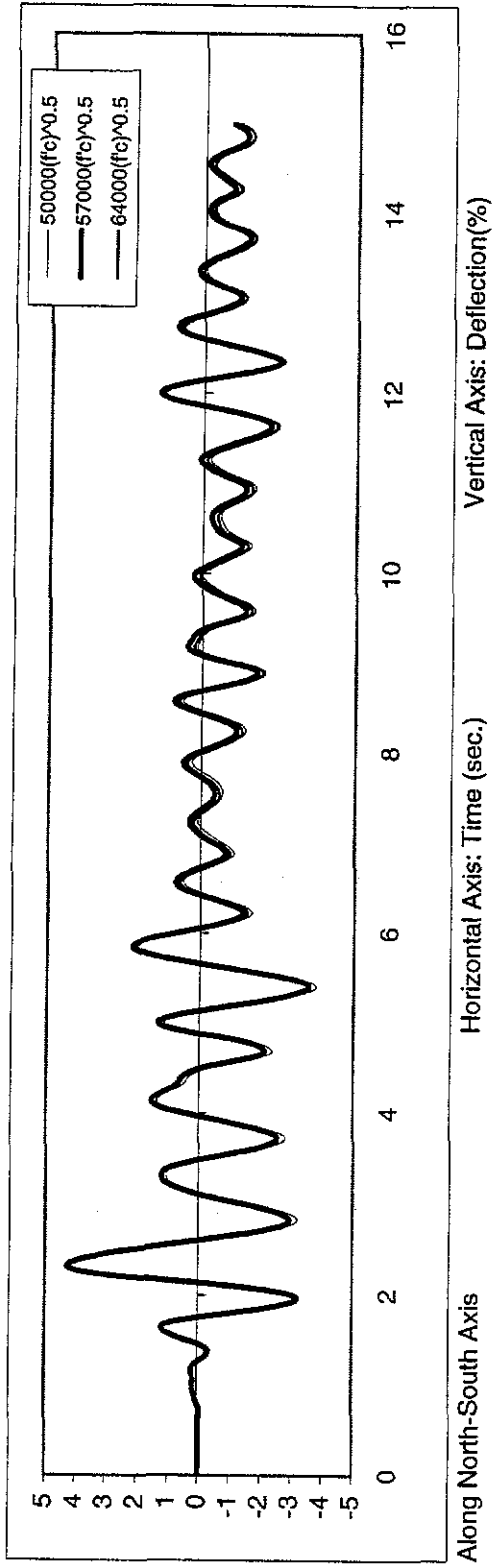


Fig. 4.5a Displacement-Histories of System A for Variations in Concrete Initial Modulus of Elasticity



Along East-West Axis

Fig. 4.5b Displacement-Histories of System B for Variations in Concrete Initial Modulus of Elasticity

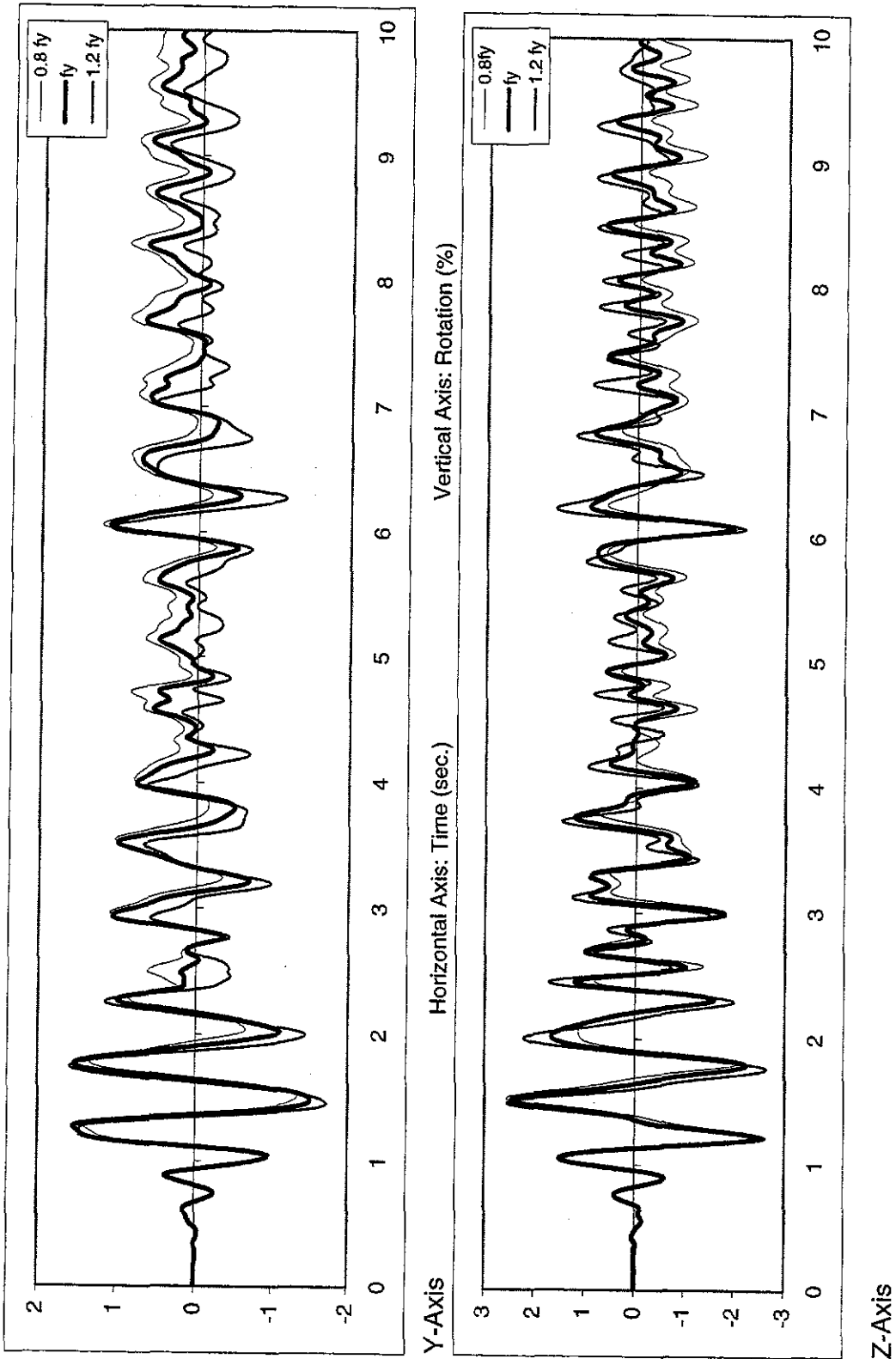


Fig. 4.6a Displacement-Histories of System A for Variations in Steel Yield Stress

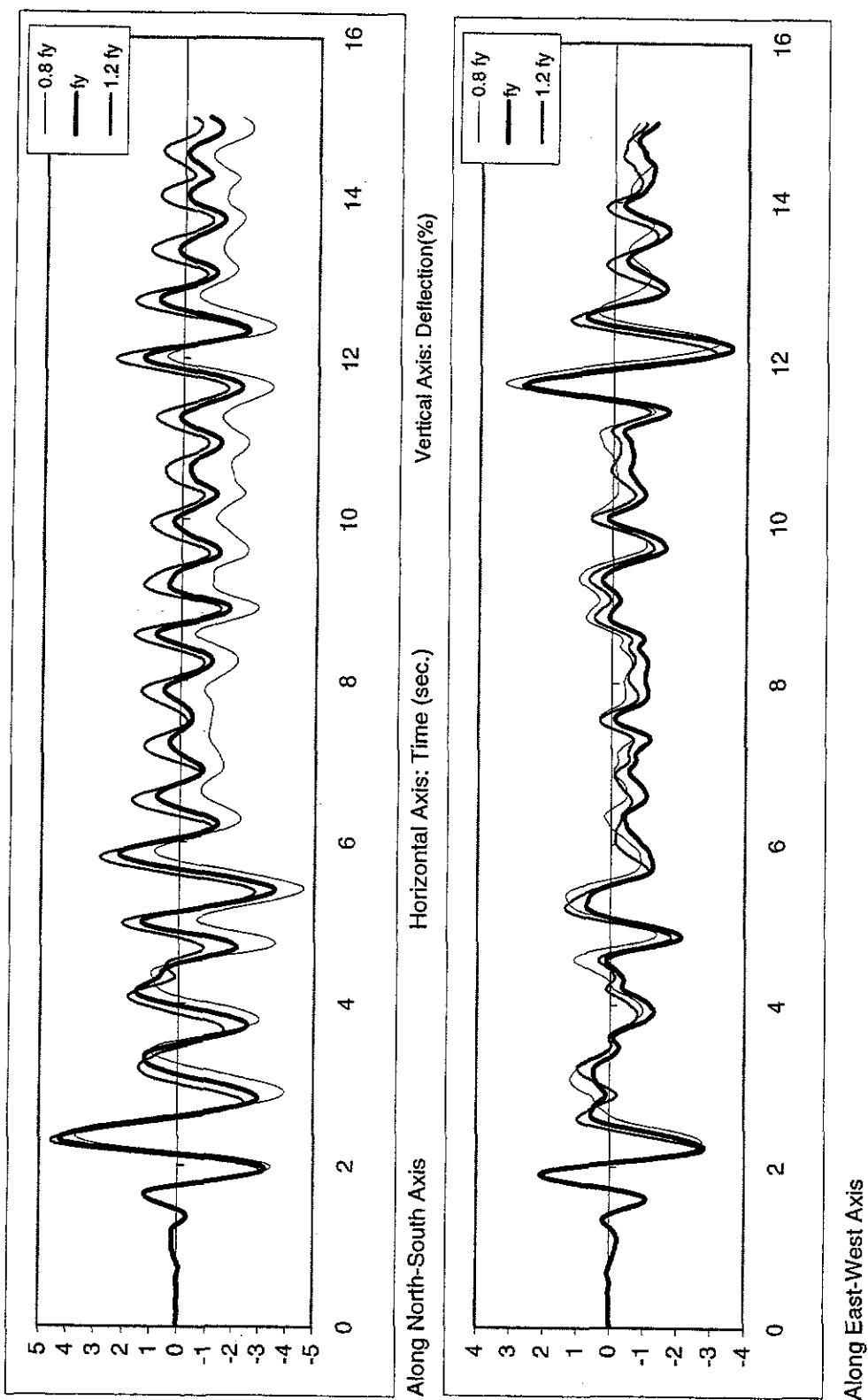
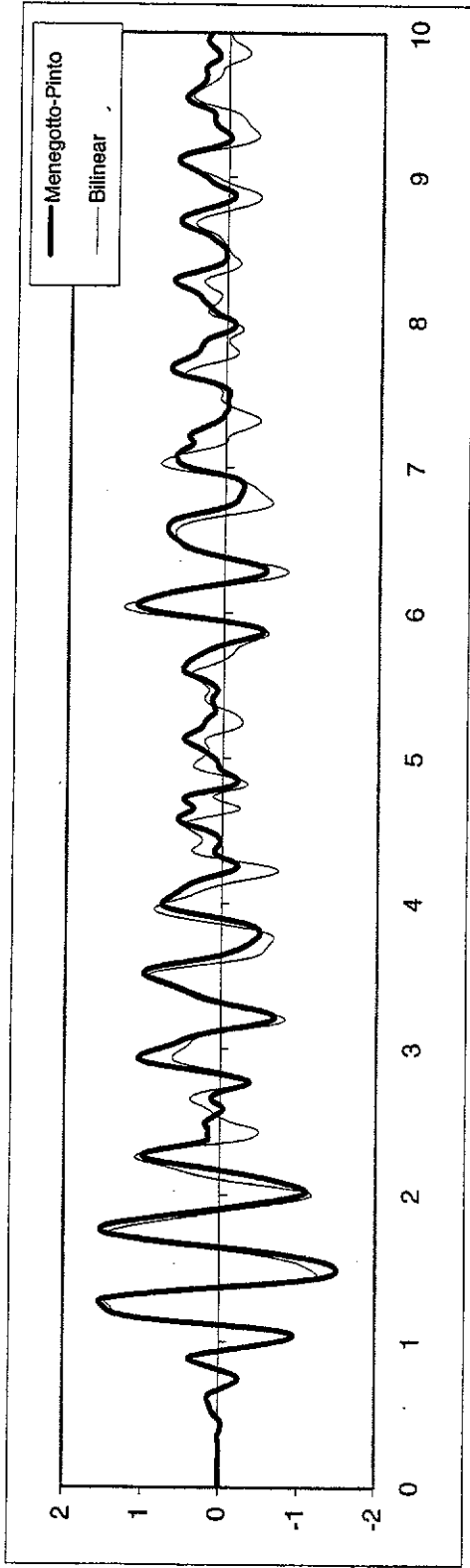
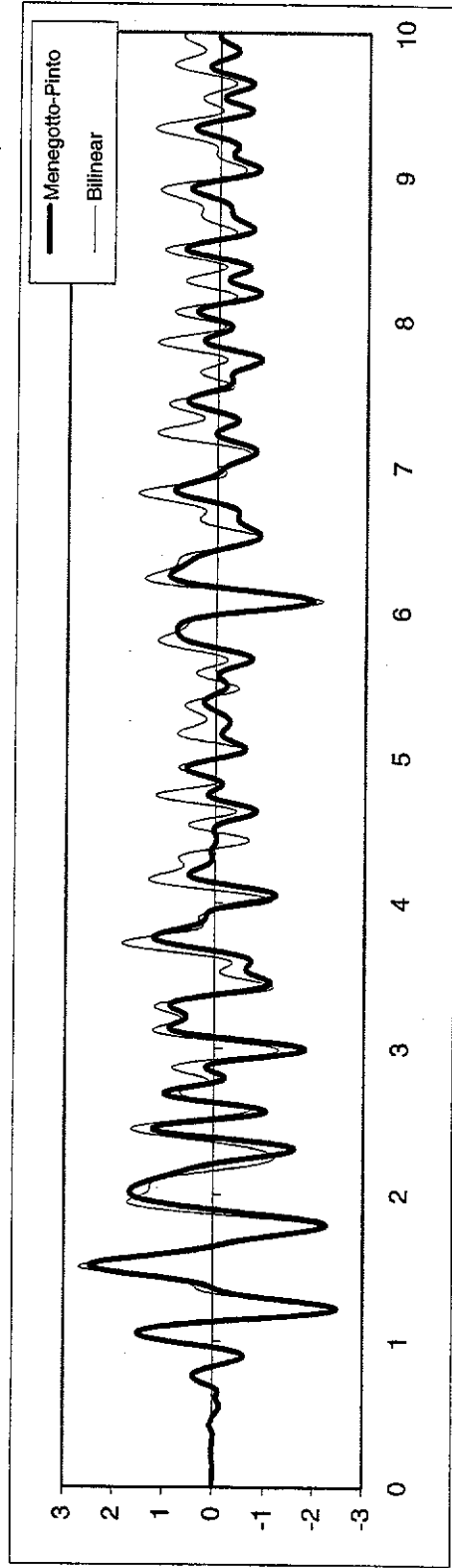


Fig. 4.6b Displacement-Histories of System B for Variations in Steel Yield Stress



Y-Axis



Z-Axis

Fig. 4.7a Displacement-Histories of System A for Variation in Steel Hysteresis Model

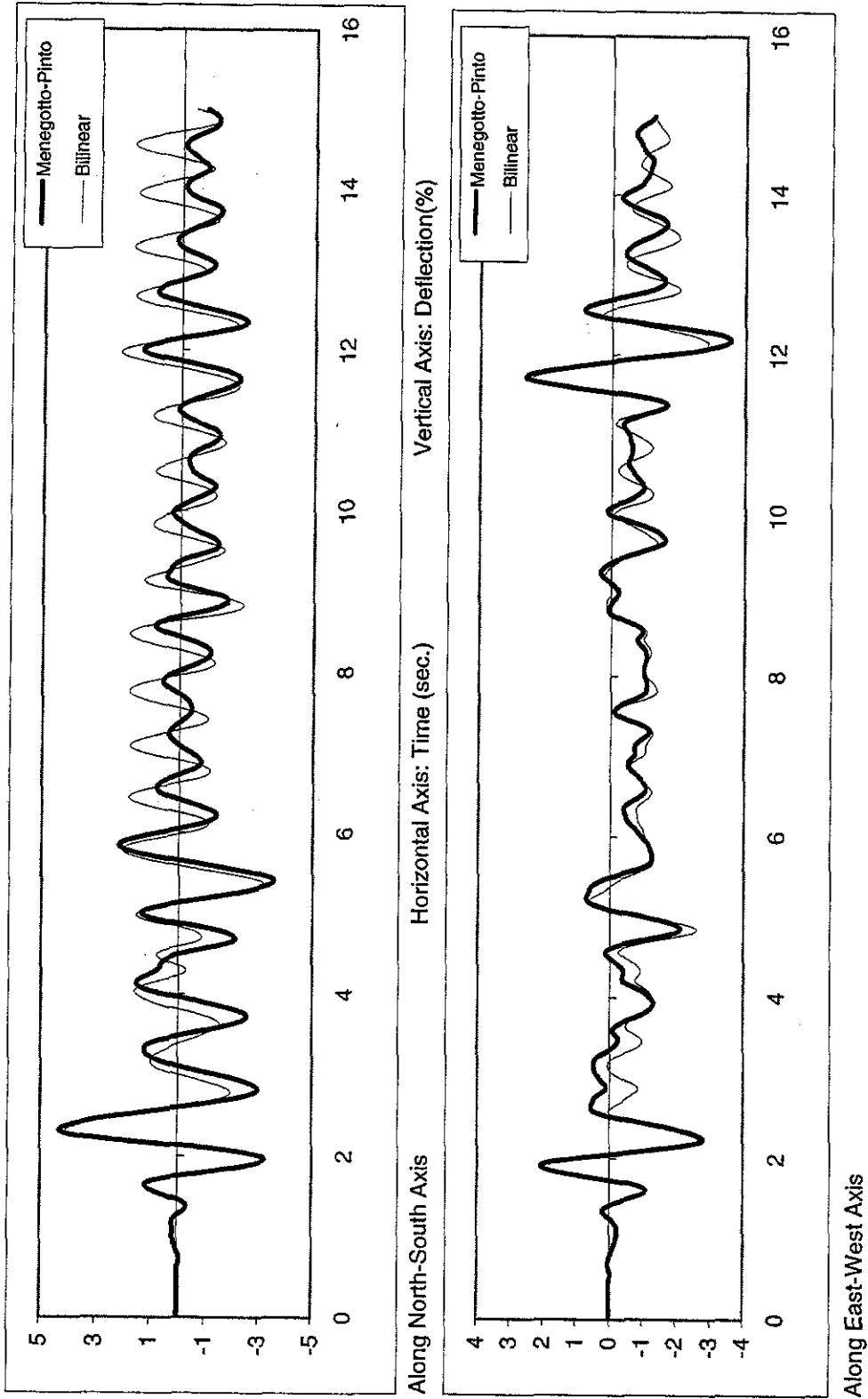


Fig. 4.7b Displacement-Histories of System B for Different Steel Hysteresis Models

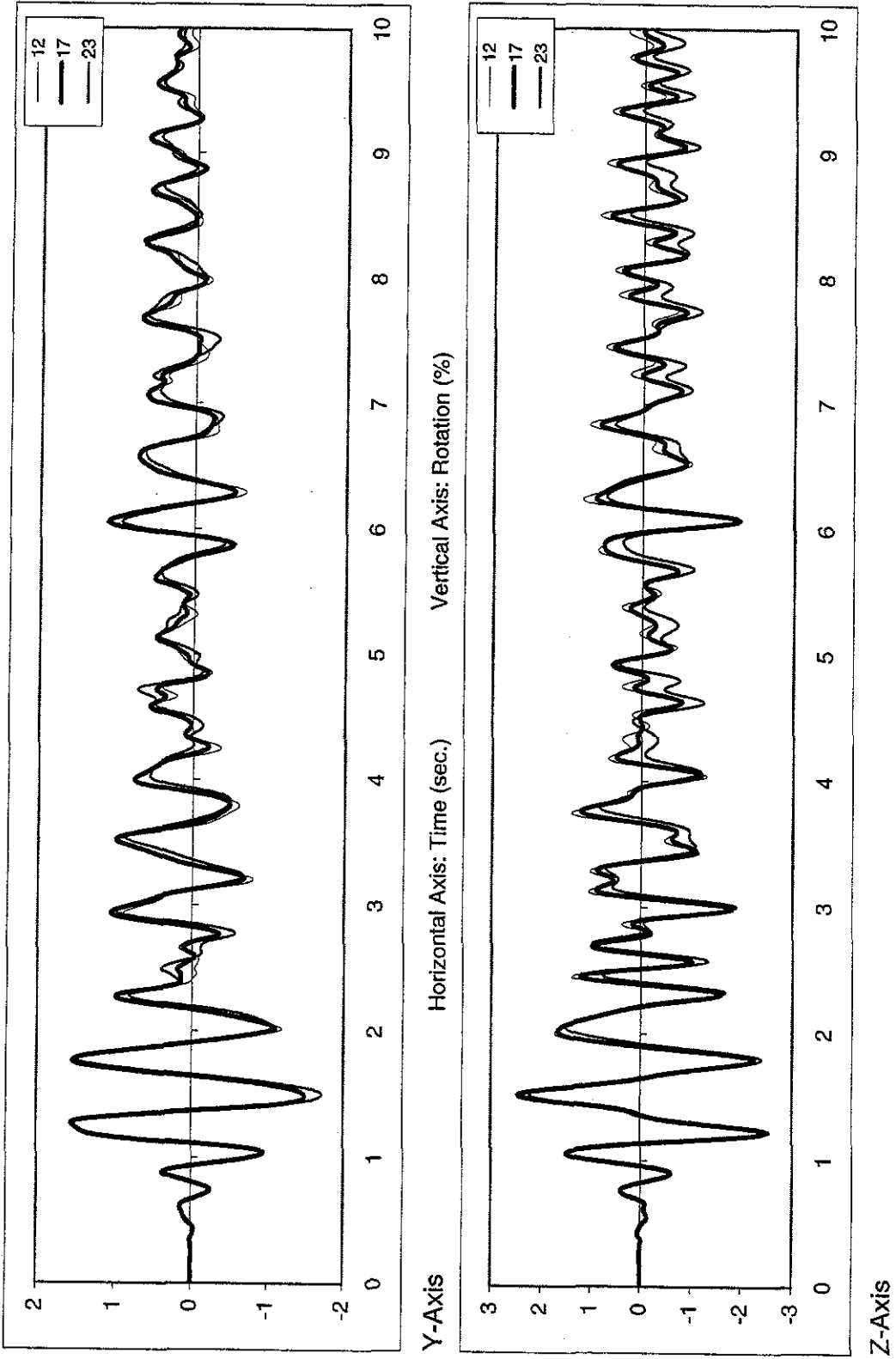


Fig. 4.8a Displacement-Histories of System A for Variations in M-P Model Transition Parameter

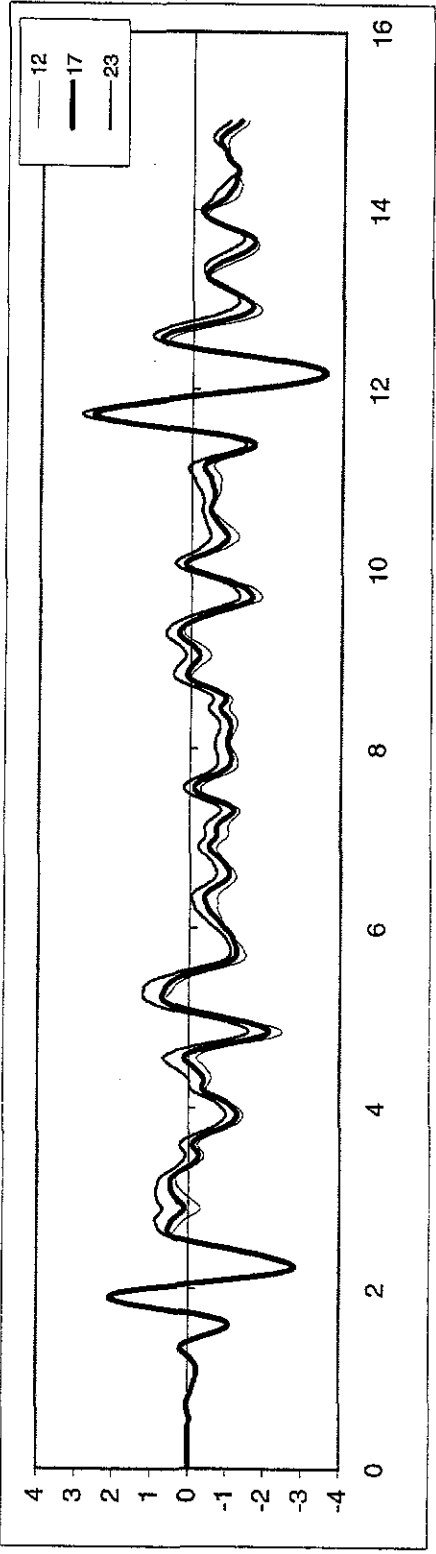
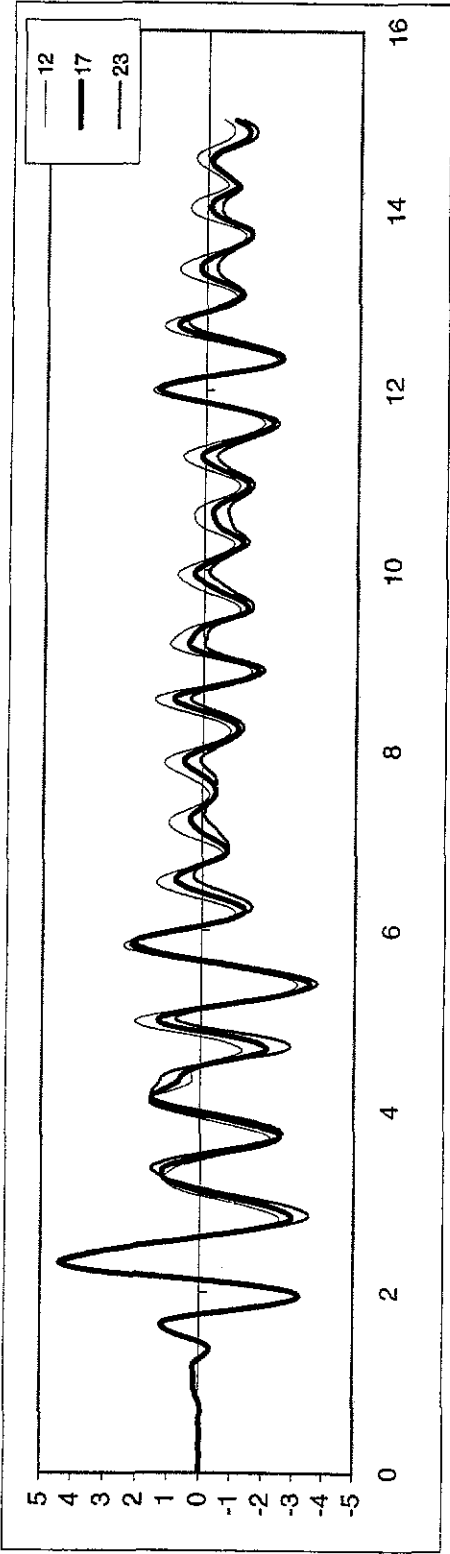
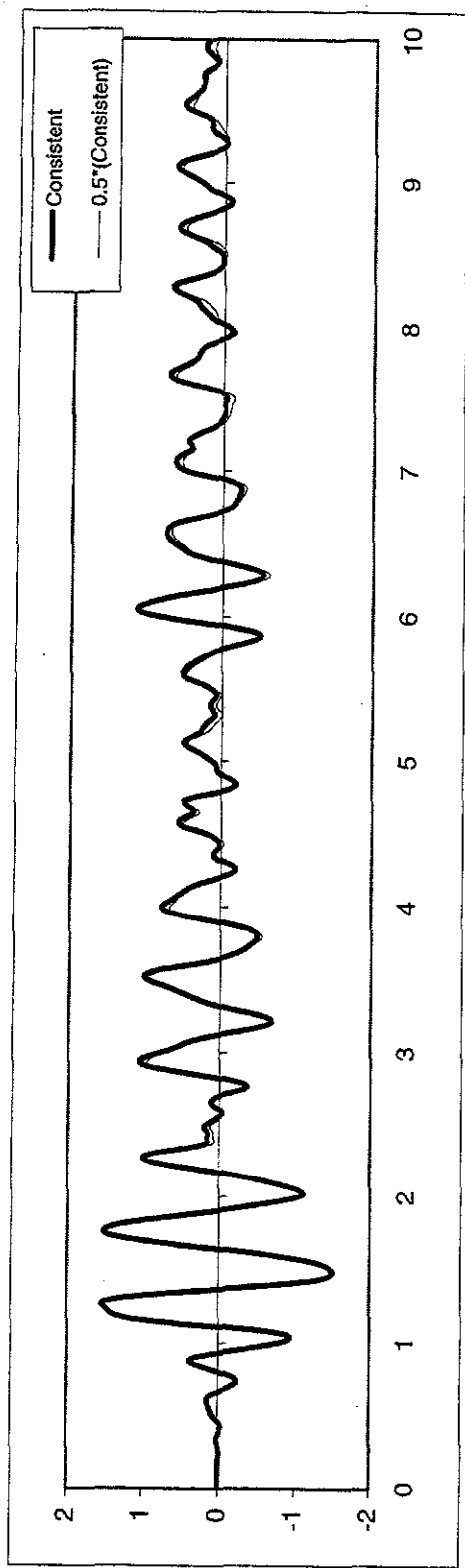
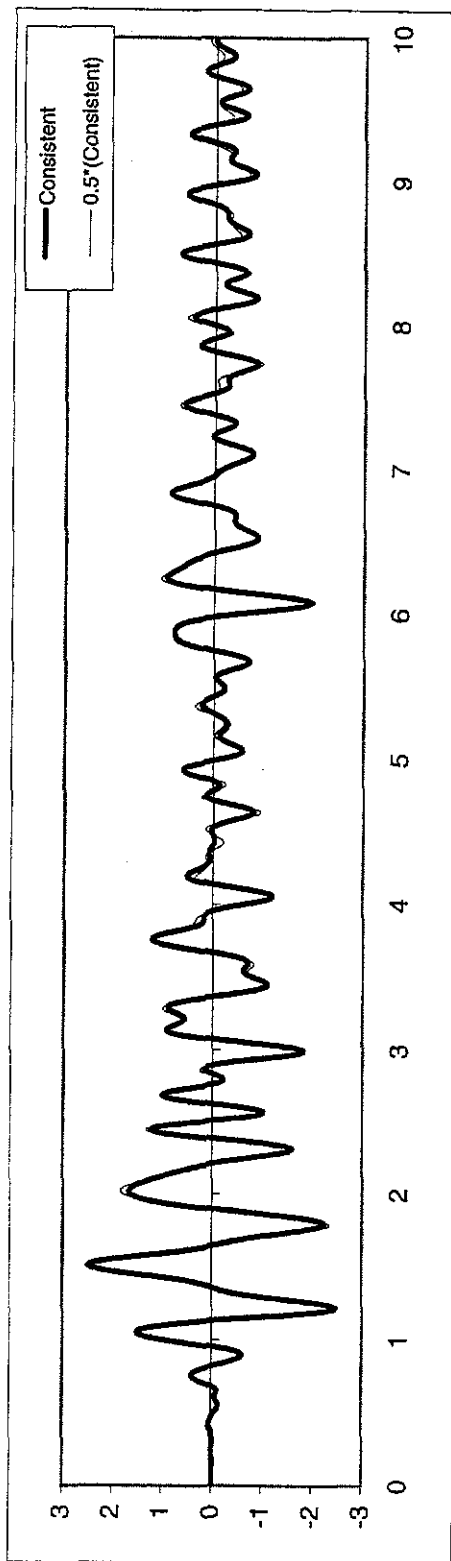


Fig. 4.8b Displacement-Histories of System B for Variations in M-P Model Transition Parameter



Y-Axis



Z-Axis

Fig. 4.9a Displacement-Histories of System A for Variations in Length of "Yield Region"

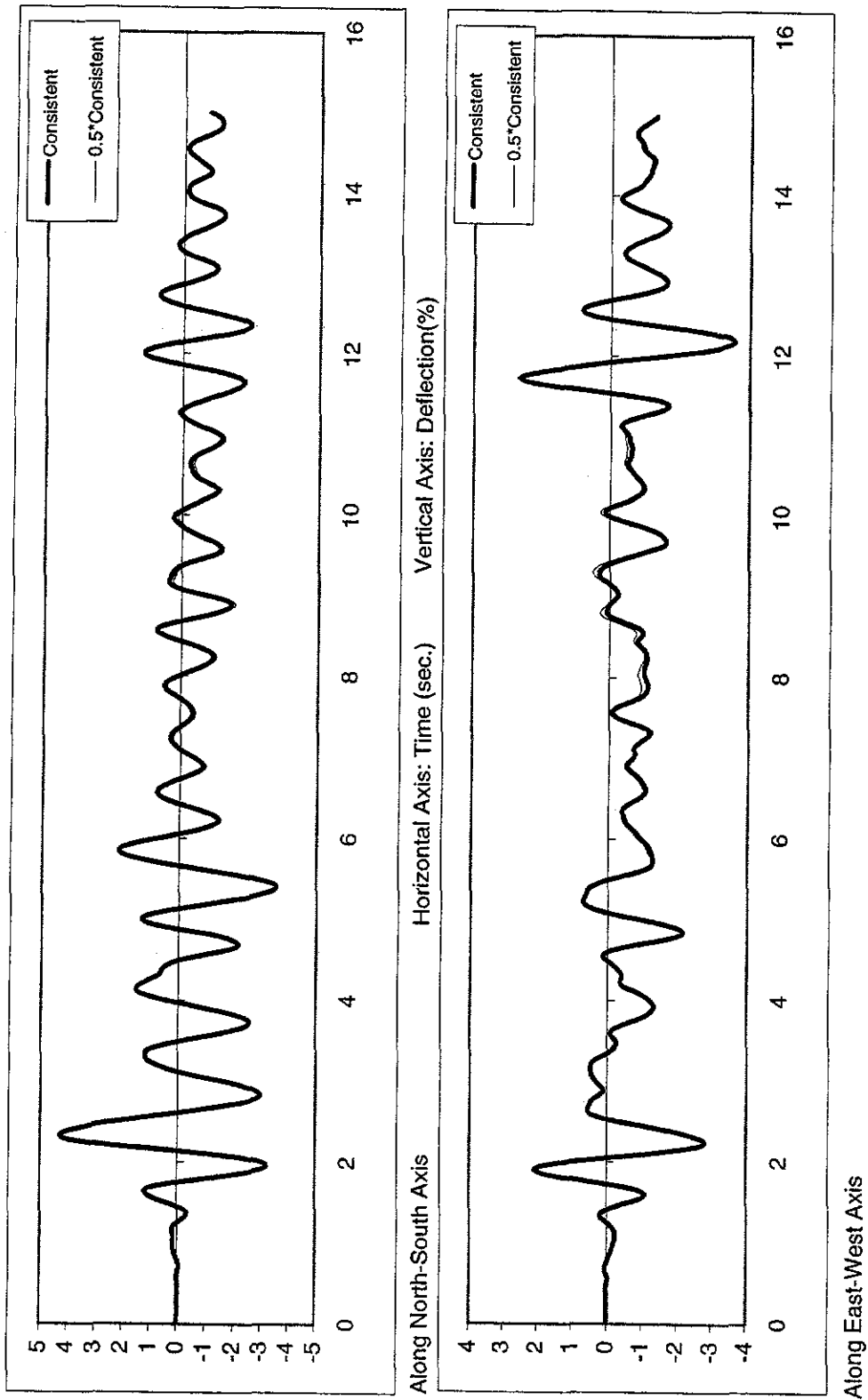


Fig. 4.9b Displacement-Histories of System B for Variation in Length of Yield Region

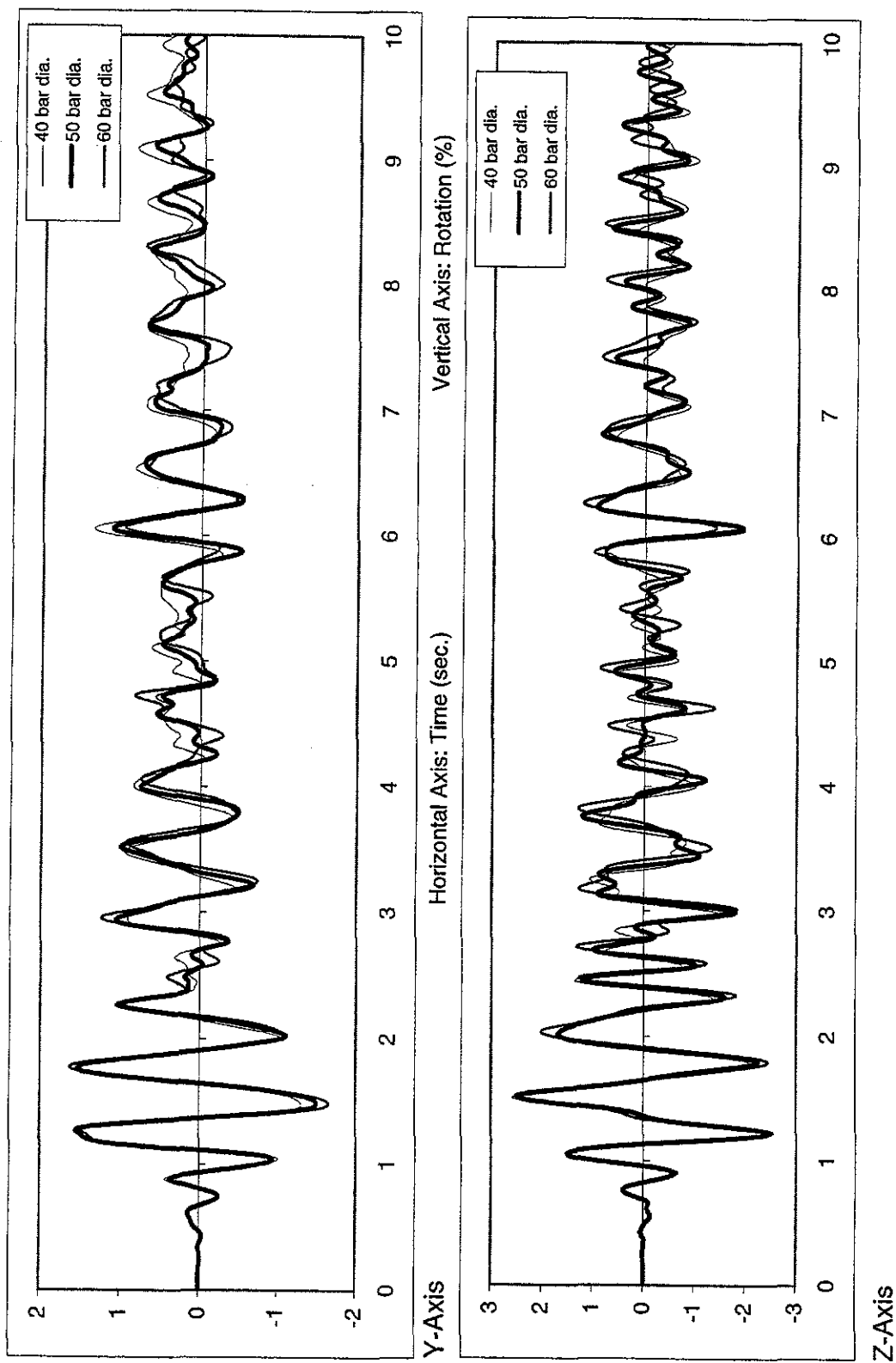


Fig. 4.10a Displacement-Histories of System A for Variations in Development Length of the Reinforcing Steel

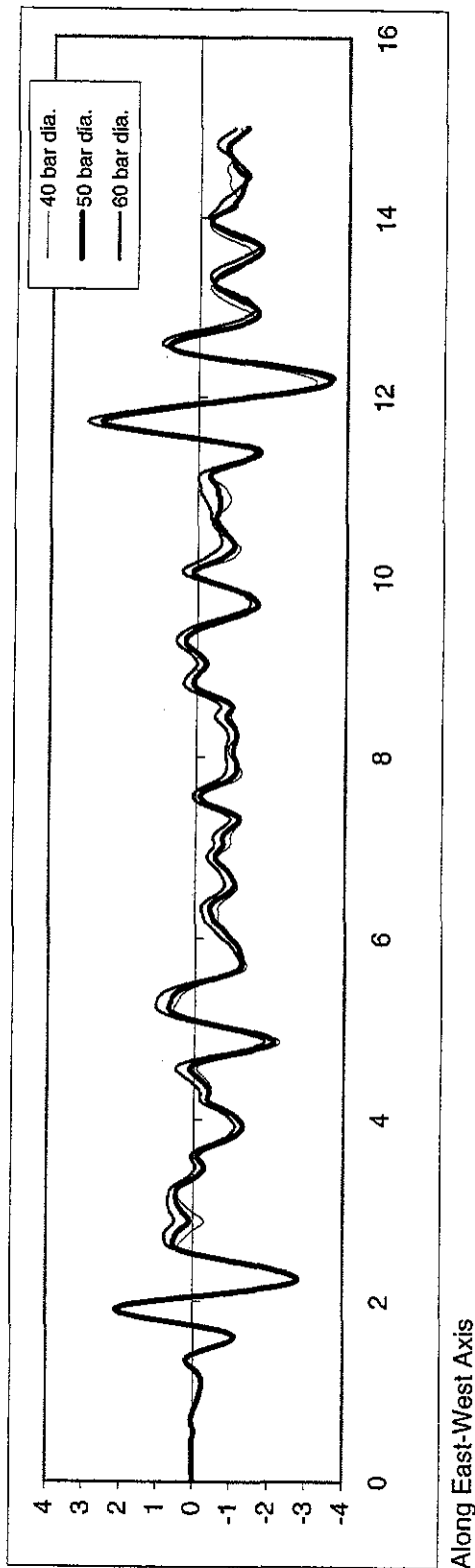
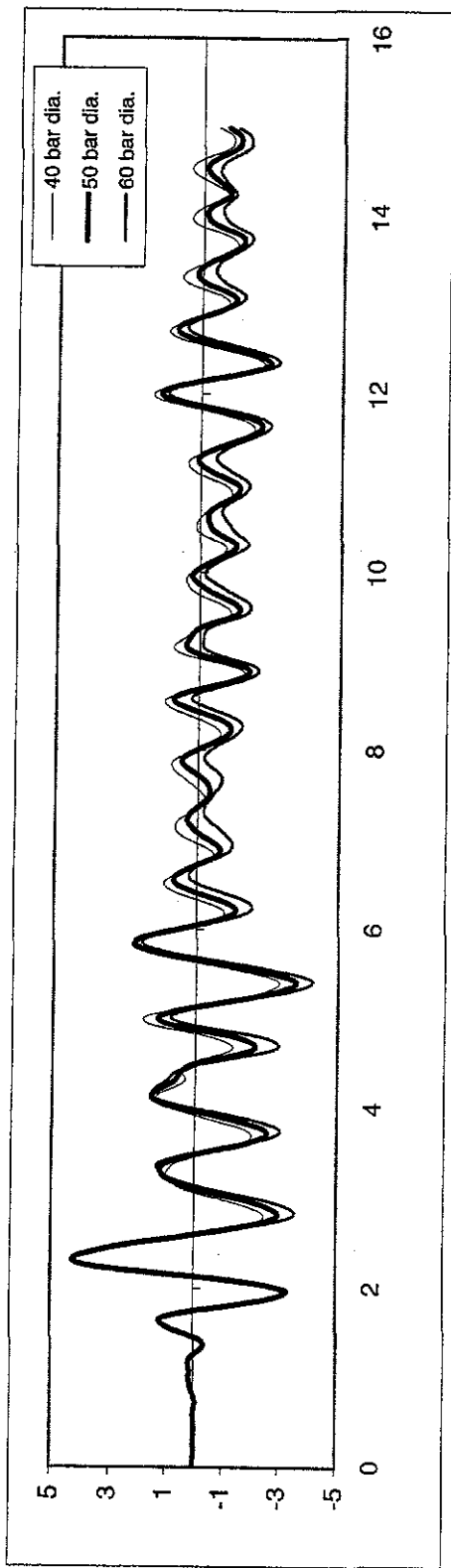


Fig. 4.10b Displacement-Histories of System B for Variations in Development Length of the Reinforcing Steel

5. LATERAL DRIFT OF SIMPLE STRUCTURAL SYSTEMS SUBJECTED TO BIAXIAL BASE EXCITATION

5.1 Introduction

The objective of this chapter is to study the nonlinear response of reinforced concrete structures to identify the differences in their response subjected to biaxial and uniaxial ground excitations. The item of interest is specifically the increase in maximum displacement caused by biaxial excitation with respect to that for uniaxial excitation.

Maximum displacement is an important parameter for the design of structural systems under earthquake loading. Observations (Algan 1982, ATC Report No. SSC 96-01 1996) have confirmed that there is a good correlation between damage and the drift ratio (difference in drift response between two consecutive levels divided by story height): the lower is the drift ratio the more tolerable is the damage. Previous studies have shown that the nonlinear-displacement response of structural systems subjected to uniaxial-base excitation can be approximated using linear estimates (Veletsos and Newmark 1960, Shibata and Sozen 1976, Riddell and Newmark 1979, Shimazaki and Sozen 1984, Lepage 1997). Lepage (1997) proposed a generalized displacement spectrum for the nonlinear displacement of reinforced concrete structures, subjected to uniaxial base excitation, with the restriction of a minimum base-shear strength requirement.

In order to compare the maximum displacements for biaxial base excitation and for uniaxial base excitation, a series of single-mass systems were analyzed by using the numerical model presented in Chapter 2. A total of ten earthquake records (same set used by Lepage, 1997) were used. The record stations include locations where the geology is

best described as rock, stiff, or soft soil sites. The main variables of the study are the initial stiffness and base shear strength of the systems, as well as different earthquake intensities and frequency contents. The evaluations of the results are based on comparisons of the maximum displacements, obtained from the analysis of systems subjected to biaxial and uniaxial base excitations.

Section 5.2 presents the properties of the single-mass systems analyzed. Section 5.3 presents the descriptions of the ground motions and scaling procedure. Section 5.4 presents the procedure for calculating the base shear requirements proposed by Lepage (1997). Section 5.5 presents the results of the nonlinear dynamic analysis and their evaluation. A summary of the chapter is given in Sec. 5.6.

5.2 Properties of the Single-Mass Systems Analyzed

A series of cantilevers, with the total mass concentrated at one end, were selected to study the displacement response of reinforced-concrete systems subjected to biaxial ground excitation. Each cantilever had the same material properties, sectional dimensions and height. The compressive strength of the concrete was 6,000 psi and the yield stress of the steel was 70,000 psi. Dimensions and reinforcing bar configuration of the cantilevers are presented in Fig. 5.1. The amount of longitudinal reinforcement and the concentrated mass at the free end of the cantilevers were varied to have the required base shear strengths and the initial periods. Initial periods of the systems varied from 0.1 to 1.0-sec. The dependency of the proposed hysteresis model on material properties and the physical dimensions forced the 1.0-sec. upper limit. Periods higher than 1.0-sec. were not physically feasible for single-mass systems analyzed with the proposed model. Base-shear strength coefficient (ratio of the shear strength of the cantilevers to the weight at the free end) were obtained using Lepage's formulation for the minimum base shear strength. Lepage's formulation considers the peak acceleration, acceleration amplification, and the characteristic ground period of the motions.

5.3 Description of Ground Motions and Scaling Procedure

Ten different earthquake records were used for the analytical study. The descriptions of each earthquake record including the station name, the recorded peak ground accelerations of both components, the characteristic period, the duration of the record used for the analysis, and the information source are presented in Table 5.1.

The stations for the set selected by Lepage (1997) include locations where the geology is best described as rock, stiff and soft soil sites. The selected ground motions have characteristic periods that range from 0.35-sec., for the Castaic Record of the February 9th, 1971 San Fernando Earthquake, to 1.1-sec., for the Hachinohe Harbor record of the May 16th, 1968 Takachi-Oki Earthquake.

Components of the motions were scaled by Lepage(1997). The purpose of the scaling was to have the same idealized nonlinear-displacement spectra for all motions. Scaling was defined by the following formulation:

$$D_{\max} = \frac{F_a \alpha g T_g}{(2\pi)^2} T = 10 T \quad (5.1)$$

where,

$D_{\max}$ = Idealized nonlinear-displacement maxima, in inches.

F_a = Acceleration amplification factor (accepted as 3.75, for systems with 2% viscous damping coefficient).

α = Peak ground acceleration expressed as a coefficient of the acceleration of gravity.

g = Acceleration of gravity (in/sec²).

- T_g = Characteristic period of ground motion, it may be defined as the period at which the idealized energy-response spectrum ceases to increase with increase in period (in seconds).
- T = Effective initial period of the oscillator, for reinforced concrete structures assumed to be approximately $\sqrt{2}$ times the period that corresponds to uncracked member section properties (in seconds).

In this study, one component of the motion was scaled using Eq. 5.1. The orthogonal component was scaled in the same ratio. The components that controlled the scaling are the ones listed first for each event in Table 5.1.

A representative example of the idealized response spectra is given in Fig. 5.2 with the linear-displacement spectra for the scaled El Centro earthquake record. The selected variation of D_{max} with period corresponds to values of $\alpha=0.5$ and $T_g=0.55$ -sec. It defines the idealized nonlinear-displacement maxima for systems subjected to north-south component of El Centro 1940 record, scaled to peak acceleration of $0.5g$.

5.4 Required Base Shear Strength Proposed by Lepage (1997)

Base shear strength is the maximum base-shear force that a structure can develop for a given distribution of lateral forces. Lepage (1997) indicated that a minimum base-shear coefficient (ratio of the base shear strength to the total weight of the structure) defined by Eq. 5.2 must be sustained, in order to keep the maximum-nonlinear displacement of the reinforced concrete structures (subjected to uniaxial base excitation) under the envelope defined by Eq. 5.1.

$$C_y = \begin{cases} \frac{F_a \alpha}{(R_t - 1) TR + 1} \leq \alpha & TR < 1 \\ \frac{F_a \alpha}{R_t TR} & TR \geq 1 \end{cases} \quad (5.2)$$

where,

$TR = \frac{T_{eff}}{T_g}$, Period ratio, normalizes the effective initial period of the system, T_{eff} , in relation to the characteristic period of the ground motion T_g .

$R_t =$ Force reduction factor.

5.5 Results of the Nonlinear Dynamic Analysis

The analysis of each base motion was performed for base-shear strengths satisfying Eq. 5.2 with $R_t=8$. For every motion, nonlinear responses of 19 oscillators, initial periods ranging from 0.1 to 1.0-sec. at 0.05-second intervals, were obtained.

Results of the nonlinear analyses are presented in Fig. 5.3 to 5.12. Figures 5.3a through 5.12a present the ratio of the maximum displacement response for biaxial base excitation to the maximum displacement response for uniaxial base excitation. The y-axes of the figures represent the ratios and the x-axes represent the initial periods of the systems, in seconds. Figures 5.3b through 5.12b present the base shear coefficients used in the analyses. The y-axes of the figures represent the base shear coefficients and the x-axes represent the initial periods of the systems.

Analysis results showed that maximum-displacement ratios for biaxial base excitation to uniaxial base excitation remained below 1.5 (98% of the cases). If the distribution of the ratios considered, as it can be observed from Fig. 5.14a and b, the bulk

of the results remained around 1.0. The mean of the maximum-displacement ratios is 1.0, with a standard deviation of 0.21. All the calculated ratios are presented in Fig. 5.13. No specific tendency was observed for any of the parameters: Initial stiffness, frequency content, and the characteristic period of the ground motion.

5.6 Summary and Conclusions

A numerical study was made to investigate the maximum-displacement of reinforced concrete structures subjected to biaxial base excitation. The main variables of the study were initial stiffness, base shear strength, as well as ground-motion records with different frequency contents and sequences. A series of single-mass systems, that had initial periods changing from 0.1 to 1.0-sec., were analyzed using the model proposed in Chapter 2. A total of ten earthquake records were used. The record stations include locations where geology is best described as rock, stiff or soft soil sites. Base-shear strengths were obtained using Eq. 5.2 for the minimum base shear strength, for $R_t=8$.

The evaluation of the results was made by comparing the maximum displacements for biaxial base excitation to maximum displacements for uniaxial base excitations. The ratio of maximum-displacement for biaxial base excitation to maximum-displacement for uniaxial base excitation remained below 1.5 and most of the results were close to 1.0, provided that base shear strength satisfied Eq. 5.2.

Table 5.1 Earthquake Records Used and Related Information.

Earthquake Record	Station Name	Components	Original Acc. (g)	Scaled Acc. (g)	Duration Used	T_p
San Fernando 02-19-1971	Castaic (CALTECH 1973b) Old Ridge Route, California	N21E S69E	0.31 0.27	0.78 0.68	0-15 sec.	0.35 sec.
Northridge 01-17-1994	Tarzana (CSMIP 1994) Cedar Hill Nursey, California	NS EW	0.99 1.78	0.62 1.11	0-15 sec	0.44 sec.
Chile 03-03-1985	Llolleo (Saragoni et al. 1985) D.I.C., Chile	N10E S80E	0.71 0.44	0.55 0.34	10-60 sec.	0.50 sec.
Imperial Valley 05-18-1940	El Centro (CALTECH 1971) Irrigation District, California	NS EW	0.35 0.21	0.50 0.30	0-15 sec.	0.55 sec.
Hyogo-Ken-Nanbu 01-17-1995	Kobe (JMA 1995) KMMO, Japan	NS EW	0.83 0.63	0.39 0.30	0-15 sec.	0.70 sec.
Kern County 07-21-1952	Taft (CALTECH 1971) Lincoln School Tunnel, California	N21E S69E	0.15 0.18	0.38 0.46	0-15 sec.	0.72 sec.
Western Washington 04-13-1949	Seattle (CALTECH 1973a) Army Base, Washington	S02W N88W	0.07 0.07	0.31 0.31	0-35 sec.	0.89 sec.
Miyagi-Ken Oki 06-12-1978	Sendai (Mori and Crouse 1981) Tohoku University, Japan	NS EW	0.26 0.24	0.29 0.27	0-20 sec.	0.95 sec.
Kern County 07-21-1952	Santa Barbara (CALTECH 1971) Courthouse, California	S48E N42E	0.13 0.09	0.27 0.19	0-35 sec.	1.03 sec.
Tokachi-Oki 05-16-1968	Hachinohe (Mori and Crouse 1981) Harbor, Japan	EW NS	0.18 0.23	0.24 0.30	0-15 sec.	1.14 sec.

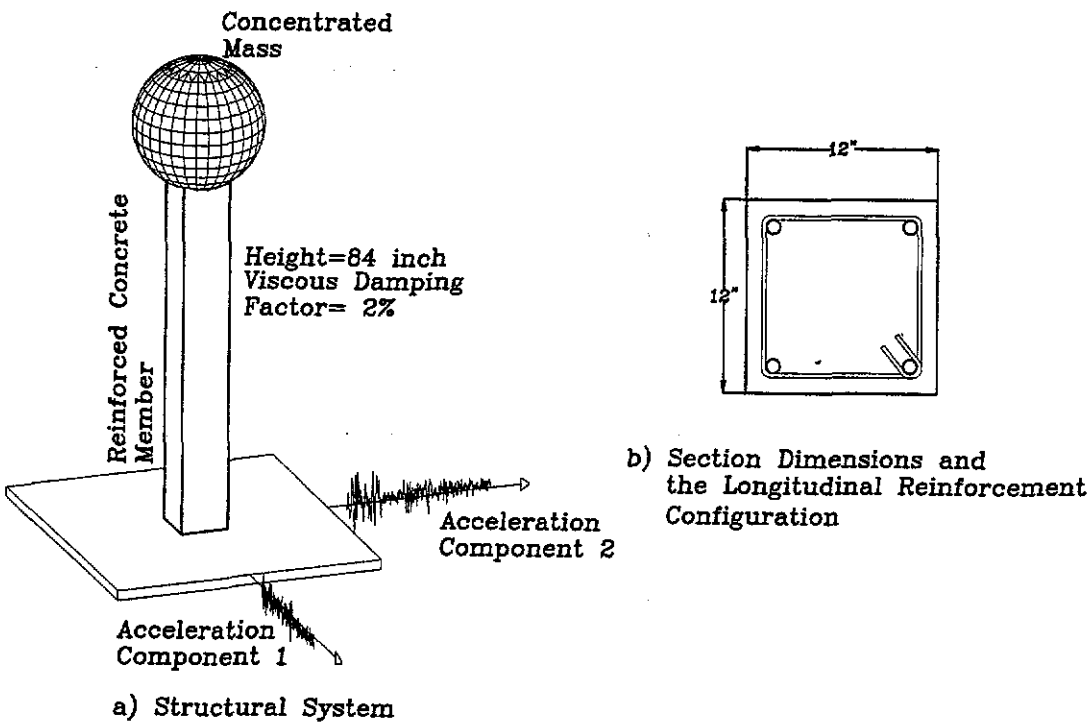


Figure 5.1 Structural System and the Longitudinal Reinforcement Configuration of the Cantilevers

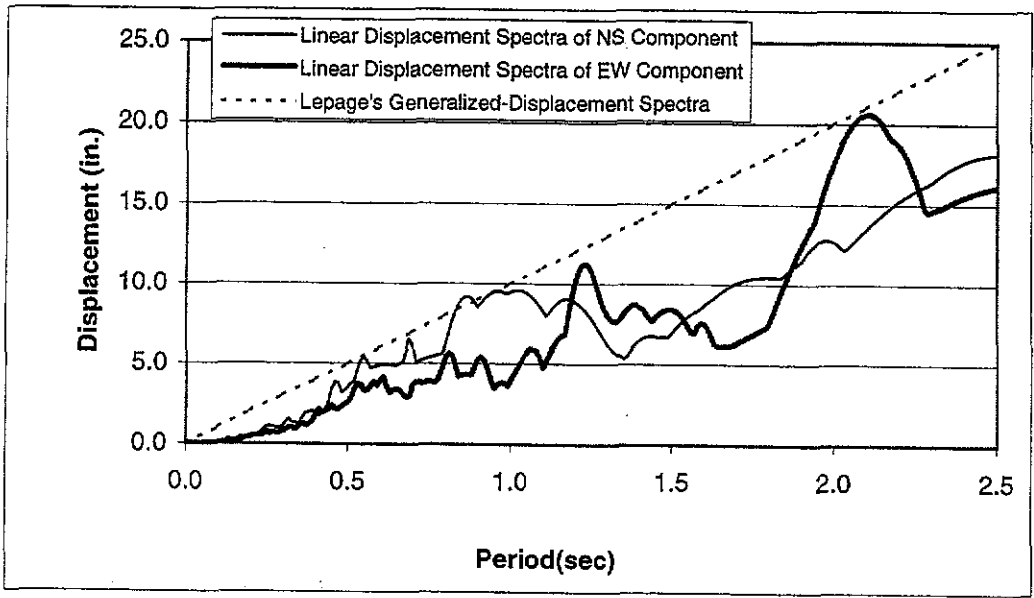
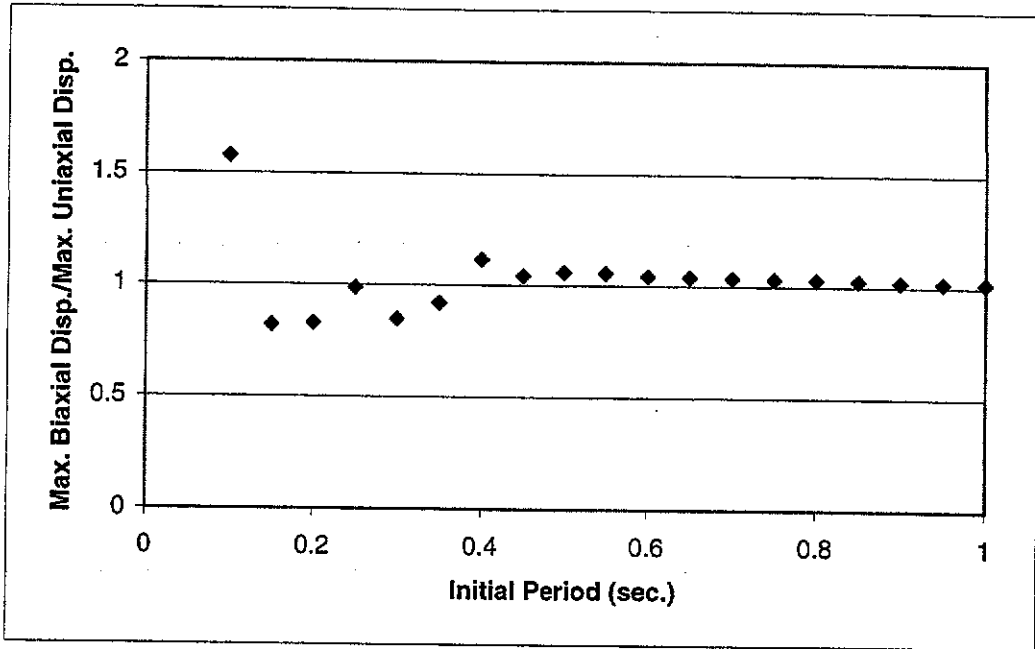
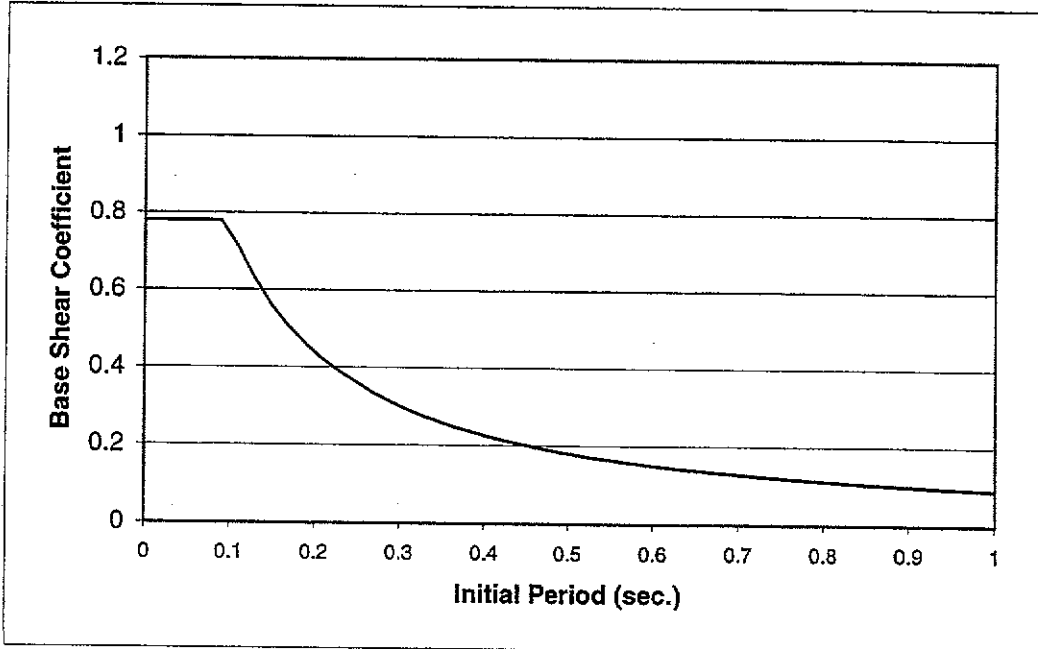


Fig. 5.2 Example of the Nonlinear-Displacement Spectra Proposed by Lepage (1997) and the Linear Displacement Spectra for El Centro Record

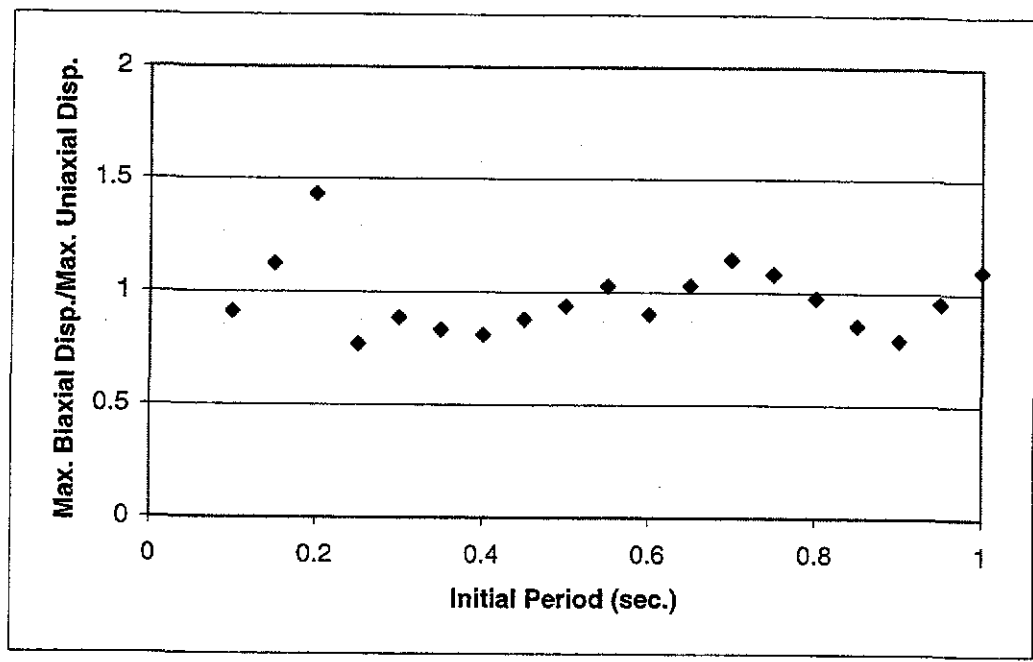


a) Ratio of the Maximum Displacement for Biaxial Base Excitation to the Maximum Displacement for Uniaxial base Excitation

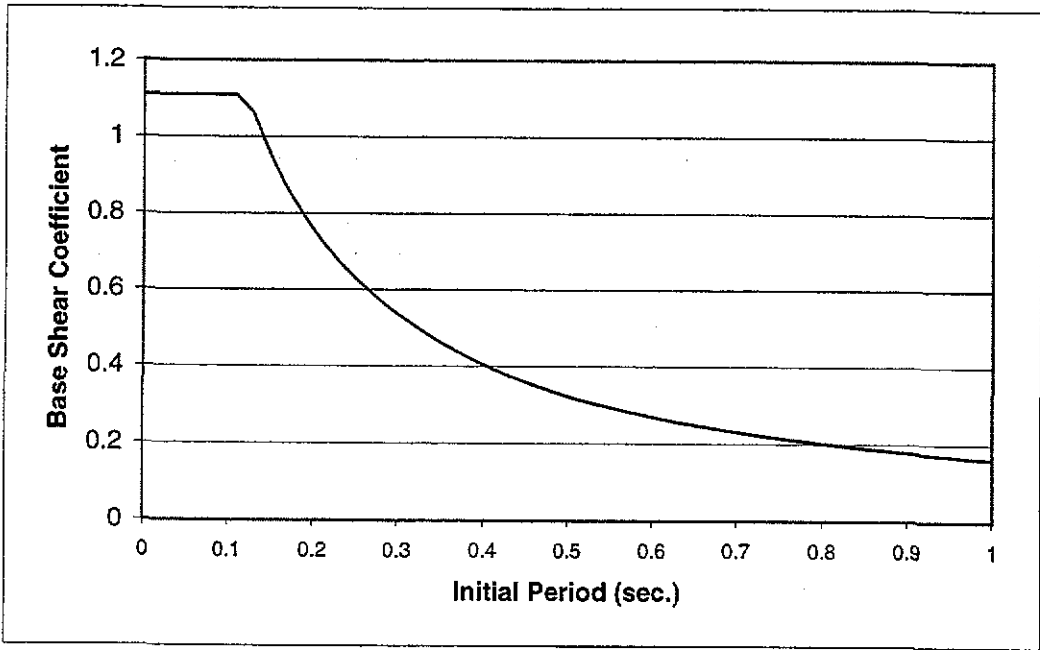


b) Base Shear Coefficient

Fig. 5.3 Analyses Results and the Base Shear Coefficients Used for Castaic Earthquake Record.

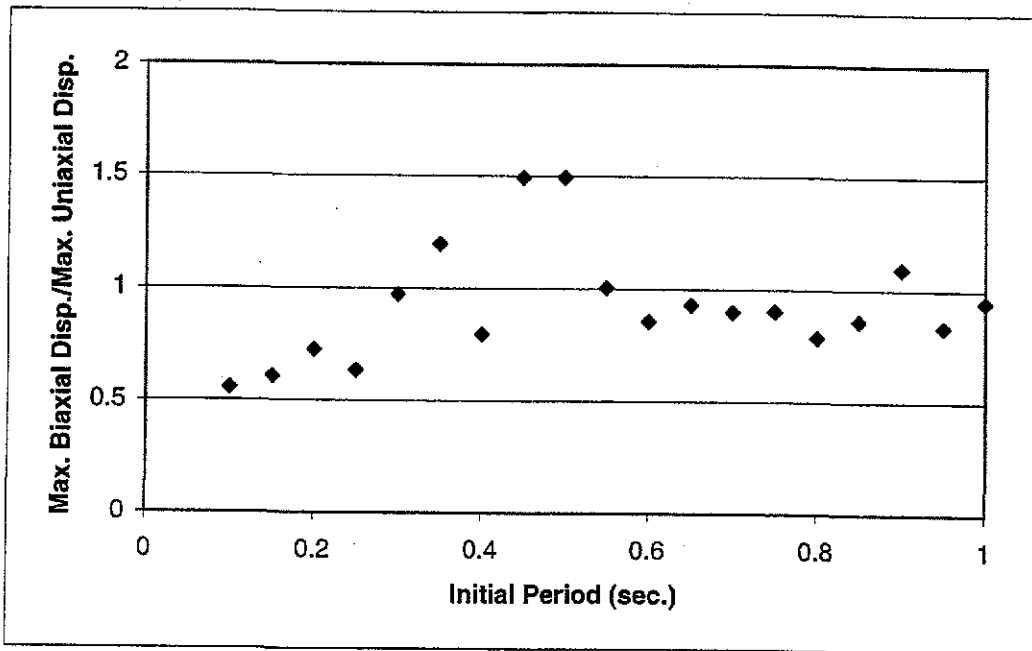


a) Ratio of the Maximum Displacement for Biaxial Base Excitation to the Maximum Displacement for Uniaxial base Excitation

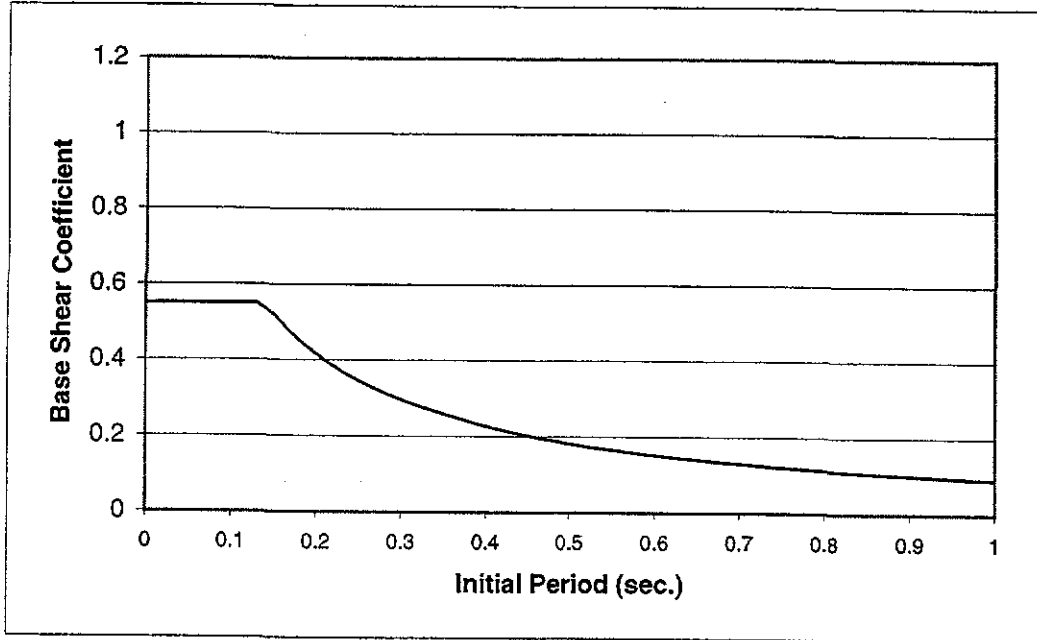


b) Base Shear Coefficient

Fig. 5.4 Analyses Results and the Base Shear Coefficients Used for Tarzana Earthquake Record.

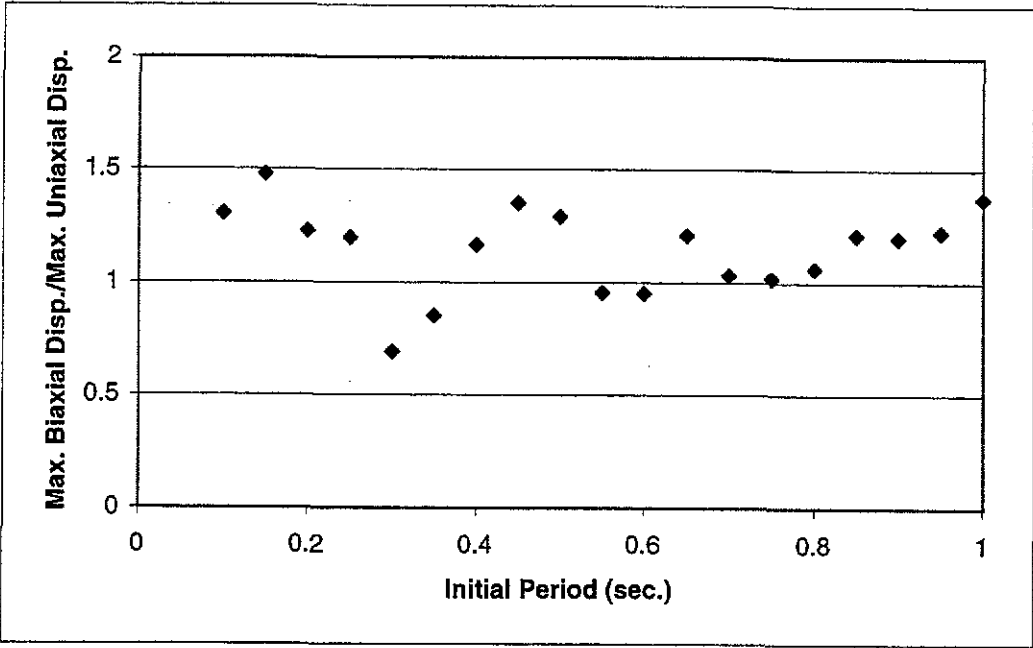


a) Ratio of the Maximum Displacement for Biaxial Base Excitation to the Maximum Displacement for Uniaxial base Excitation

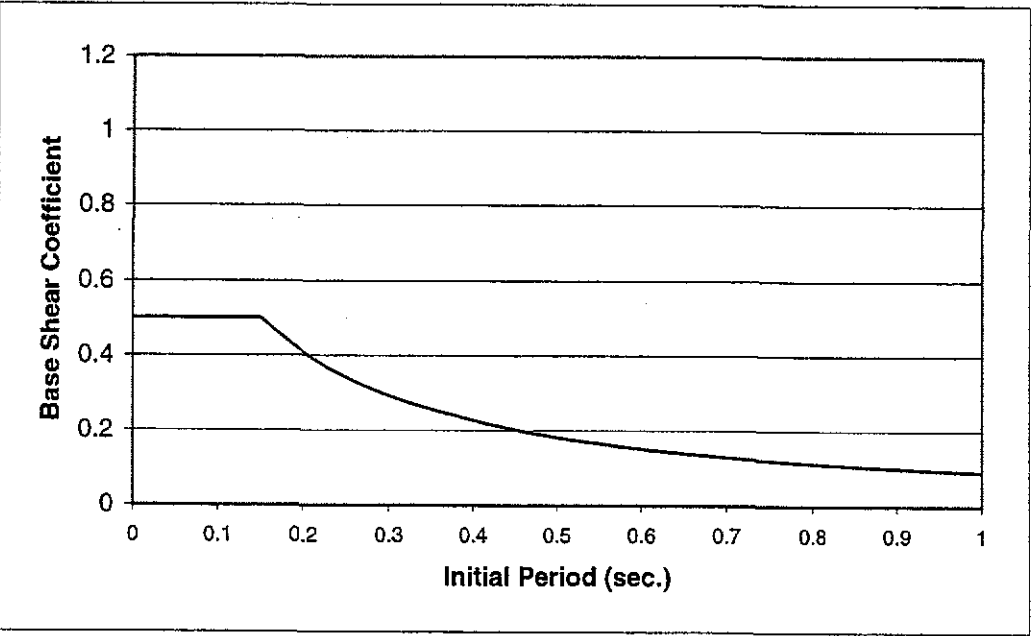


b) Base Shear Coefficient

Fig. 5.5 Analyses Results and the Base Shear Coefficients Used for Llolleo Earthquake Record.

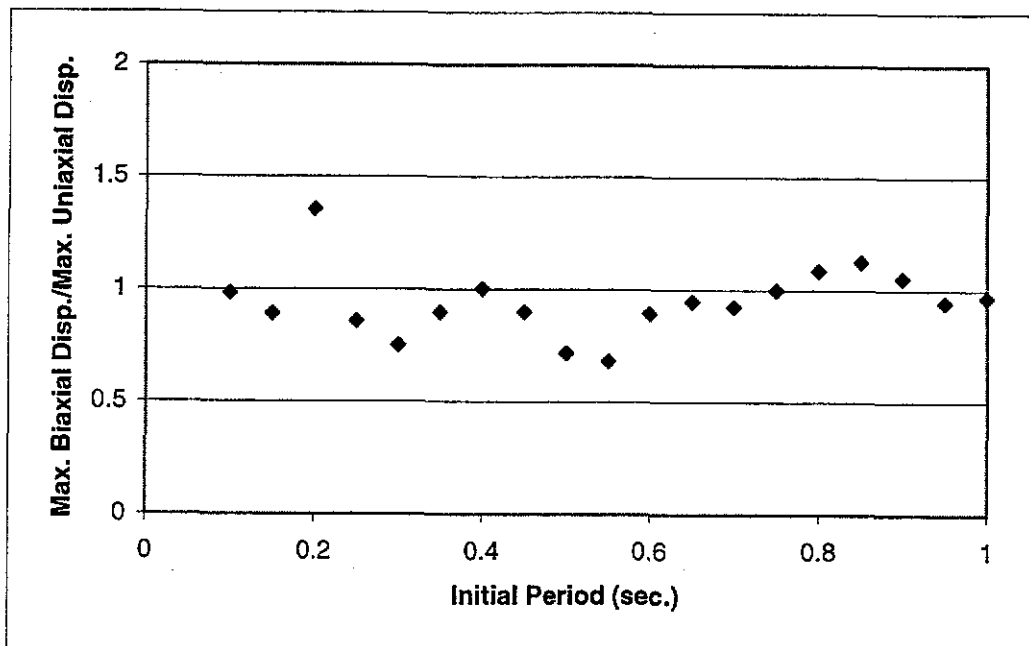


a) Ratio of the Maximum Displacement for Biaxial Base Excitation to the Maximum Displacement for Uniaxial base Excitation

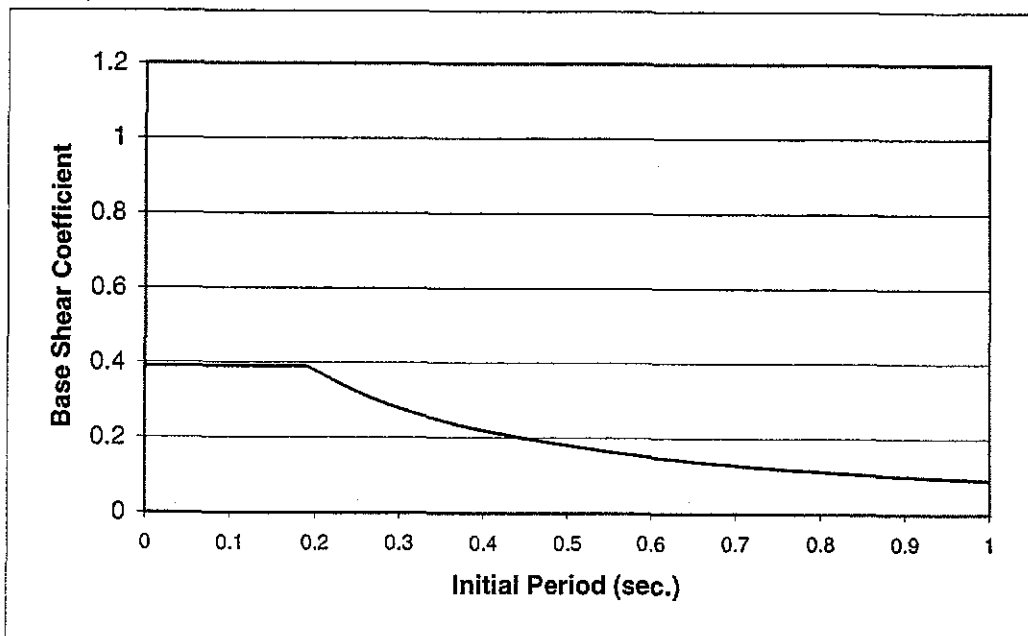


b) Base Shear Coefficient

Fig. 5.6 Analyses Results and the Base Shear Coefficients Used for El Centro Earthquake Record.

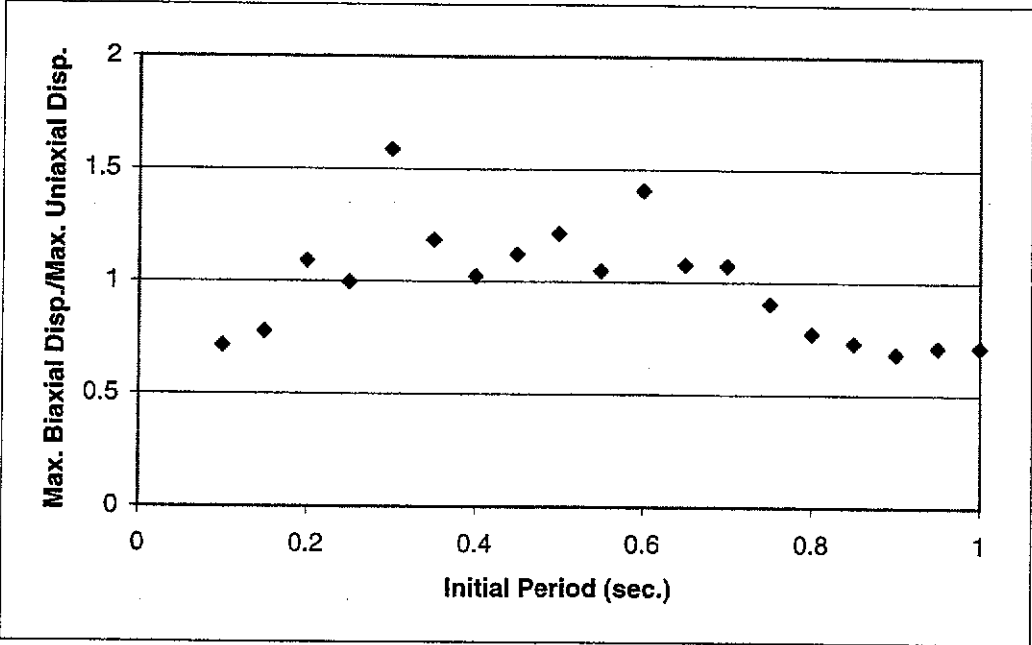


a) Ratio of the Maximum Displacement for Biaxial Base Excitation to the Maximum Displacement for Uniaxial base Excitation

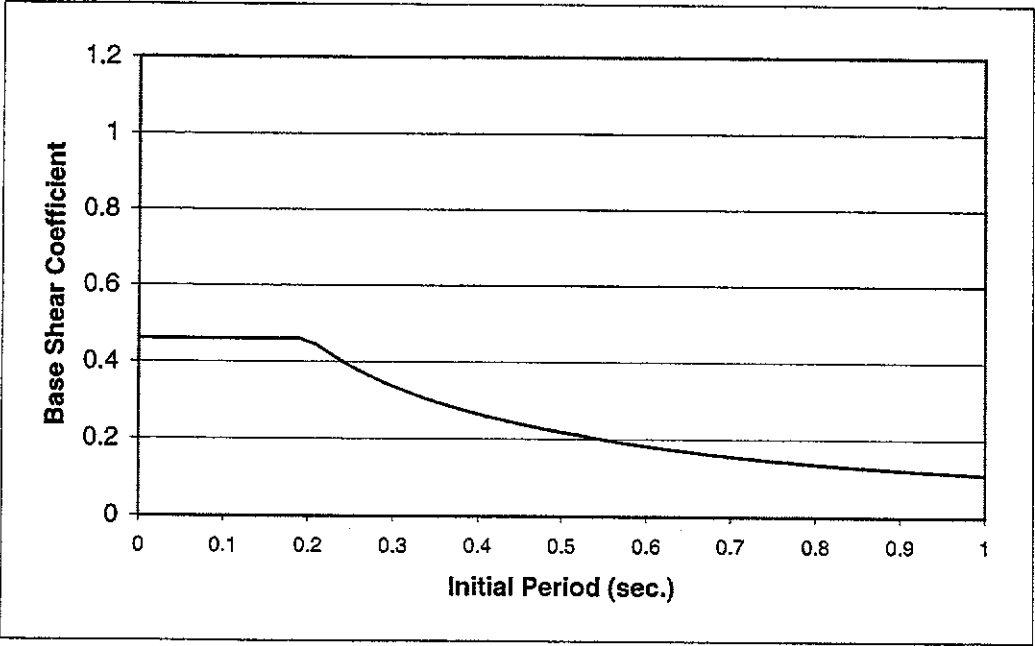


b) Base Shear Coefficient

Fig. 5.7 Analyses Results and the Base Shear Coefficients Used for Kobe Earthquake Record.

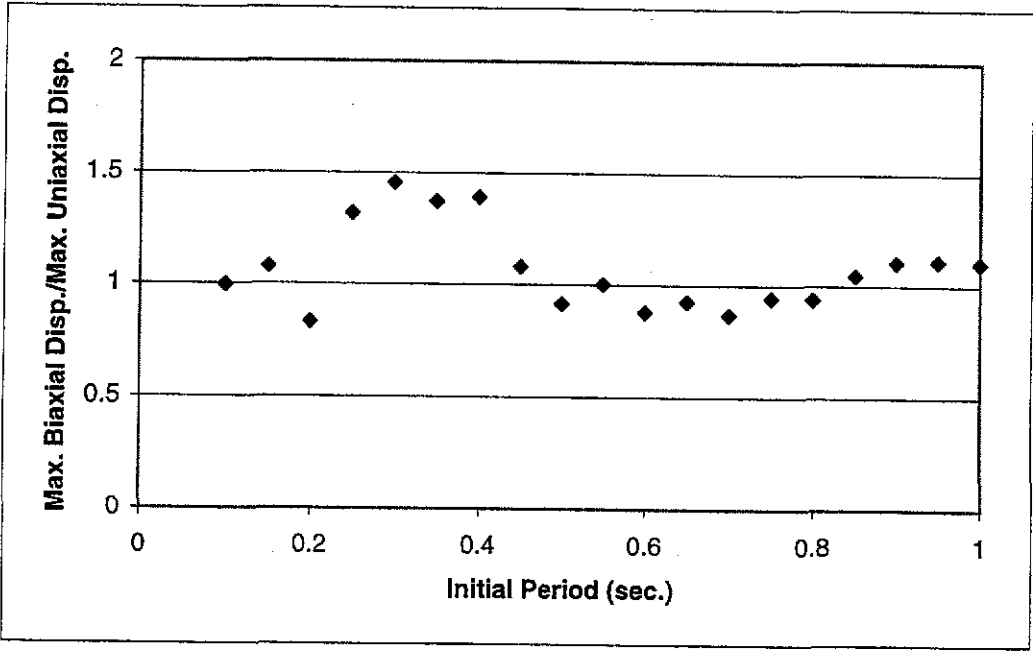


a) Ratio of the Maximum Displacement for Biaxial Base Excitation to the Maximum Displacement for Uniaxial base Excitation

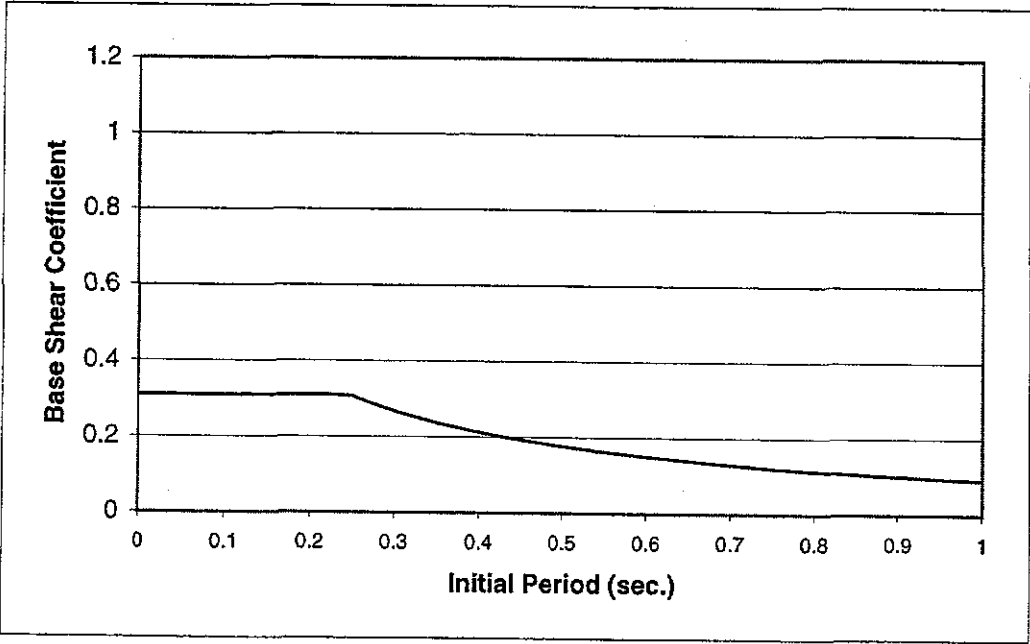


b) Base Shear Coefficient

Fig. 5.8 Analyses Results and the Base Shear Coefficients Used for Taft Earthquake Record.

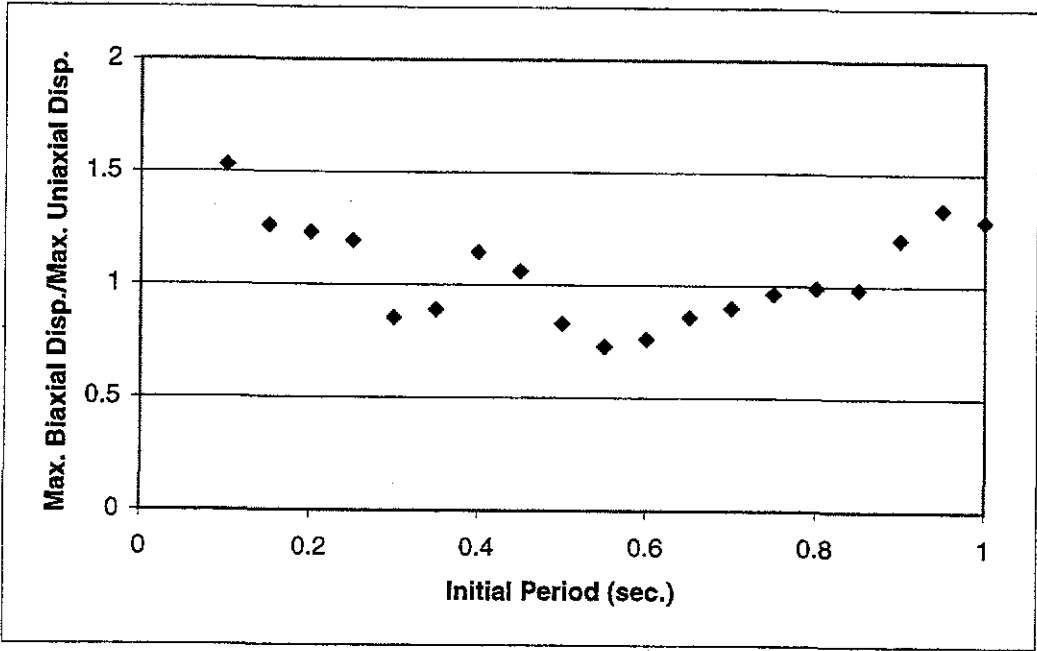


a) Ratio of the Maximum Displacement for Biaxial Base Excitation to the Maximum Displacement for Uniaxial base Excitation

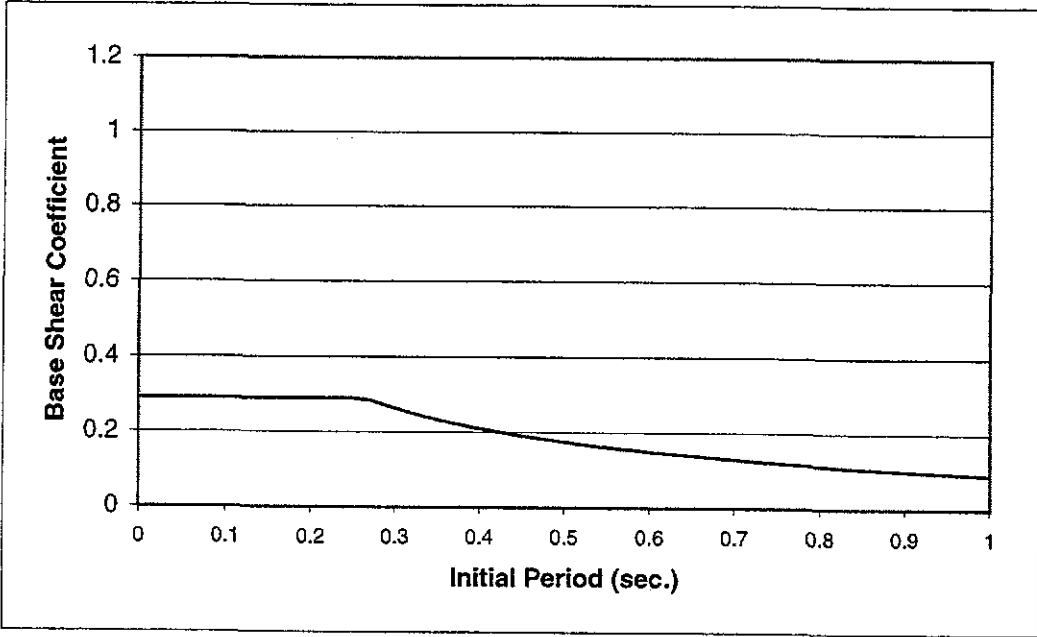


b) Base Shear Coefficient

Fig. 5.9 Analyses Results and the Base Shear Coefficients Used for Seattle Earthquake Record.

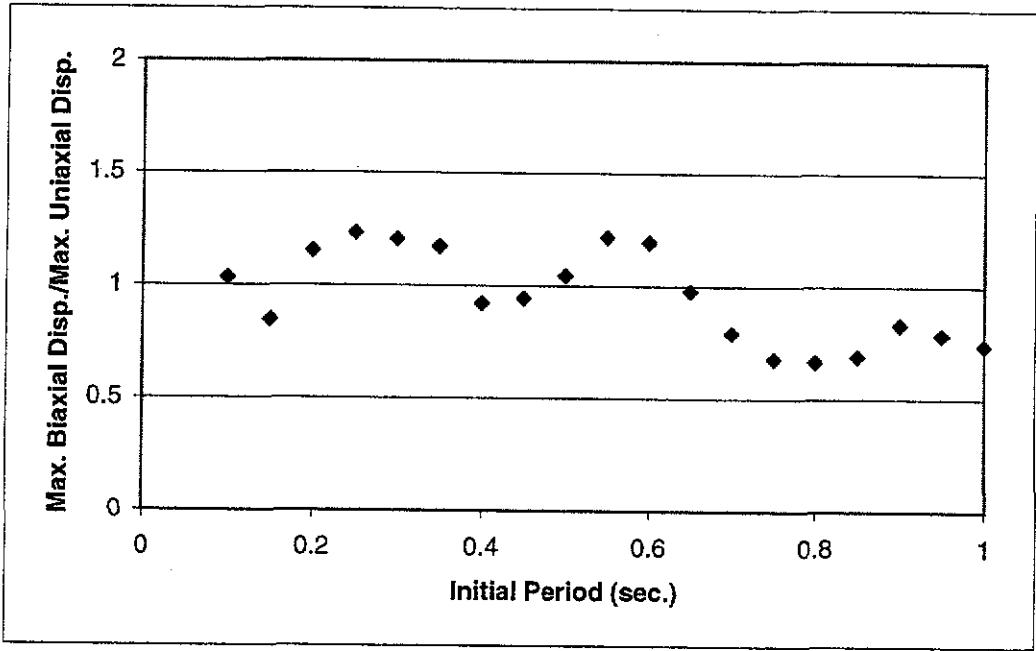


a) Ratio of the Maximum Displacement for Biaxial Base Excitation to the Maximum Displacement for Uniaxial base Excitation

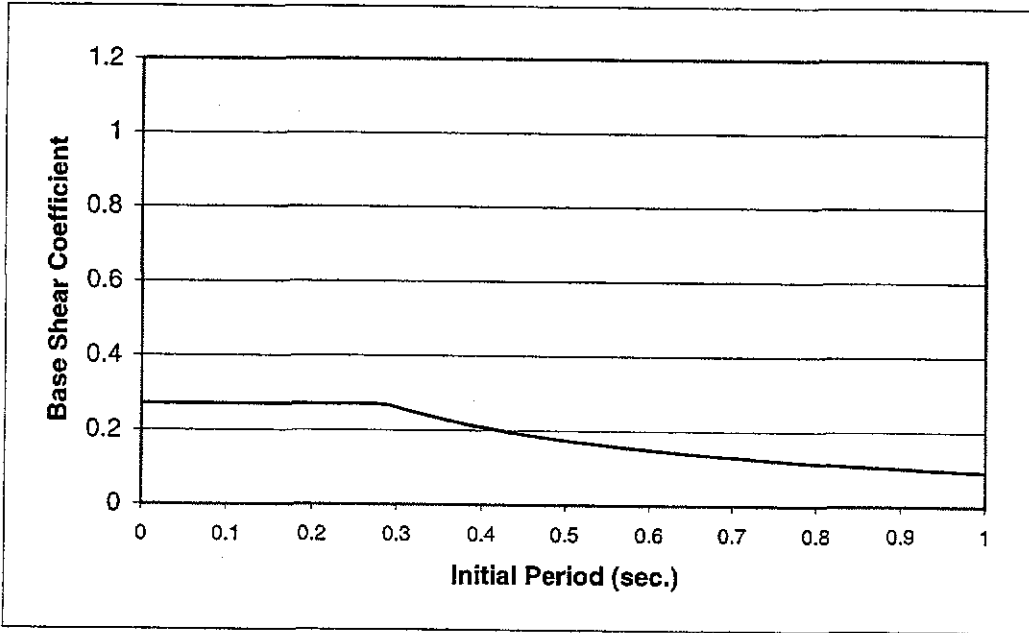


b) Base Shear Coefficient

Fig. 5.10 Analyses Results and the Base Shear Coefficients Used for Sendai Earthquake Record.

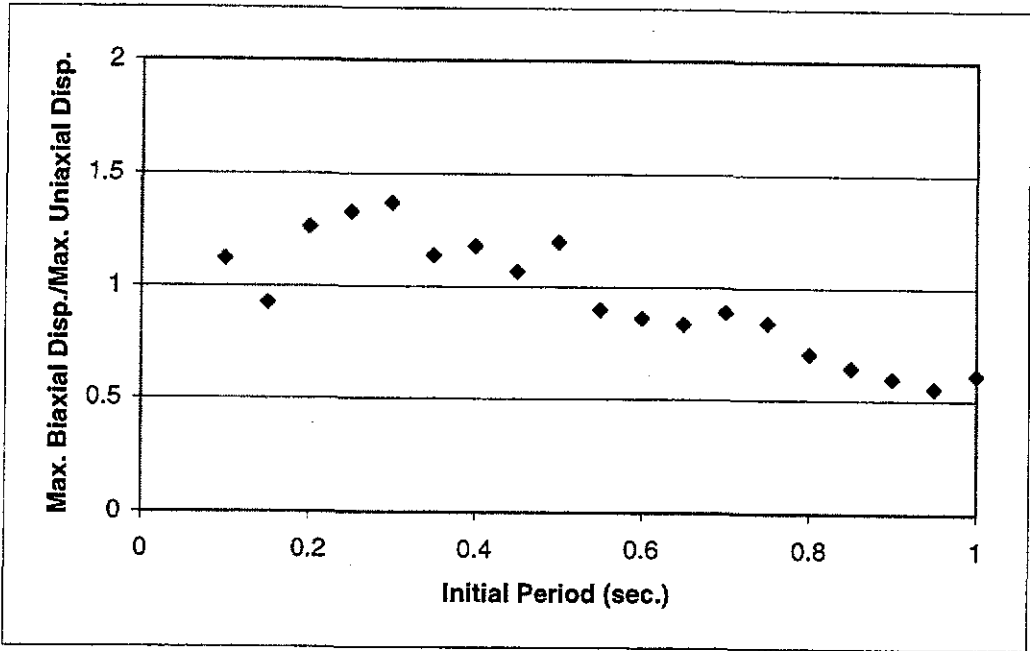


a) Ratio of the Maximum Displacement for Biaxial Base Excitation to the Maximum Displacement for Uniaxial base Excitation

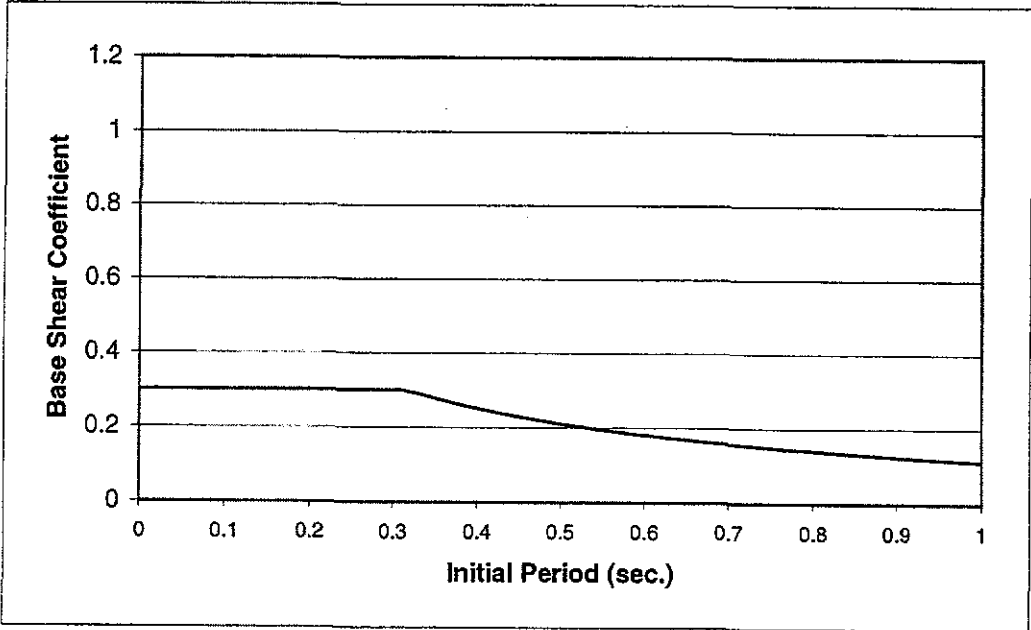


b) Base Shear Coefficient

Fig. 5.11 Analyses Results and the Base Shear Coefficients Used for Santa Barbara Earthquake Record.



a) Ratio of the Maximum Displacement for Biaxial Base Excitation to the Maximum Displacement for Uniaxial base Excitation



b) Base Shear Coefficient

Fig. 5.12 Analyses Results and the Base Shear Coefficients Used for Hachinohe Earthquake Record.

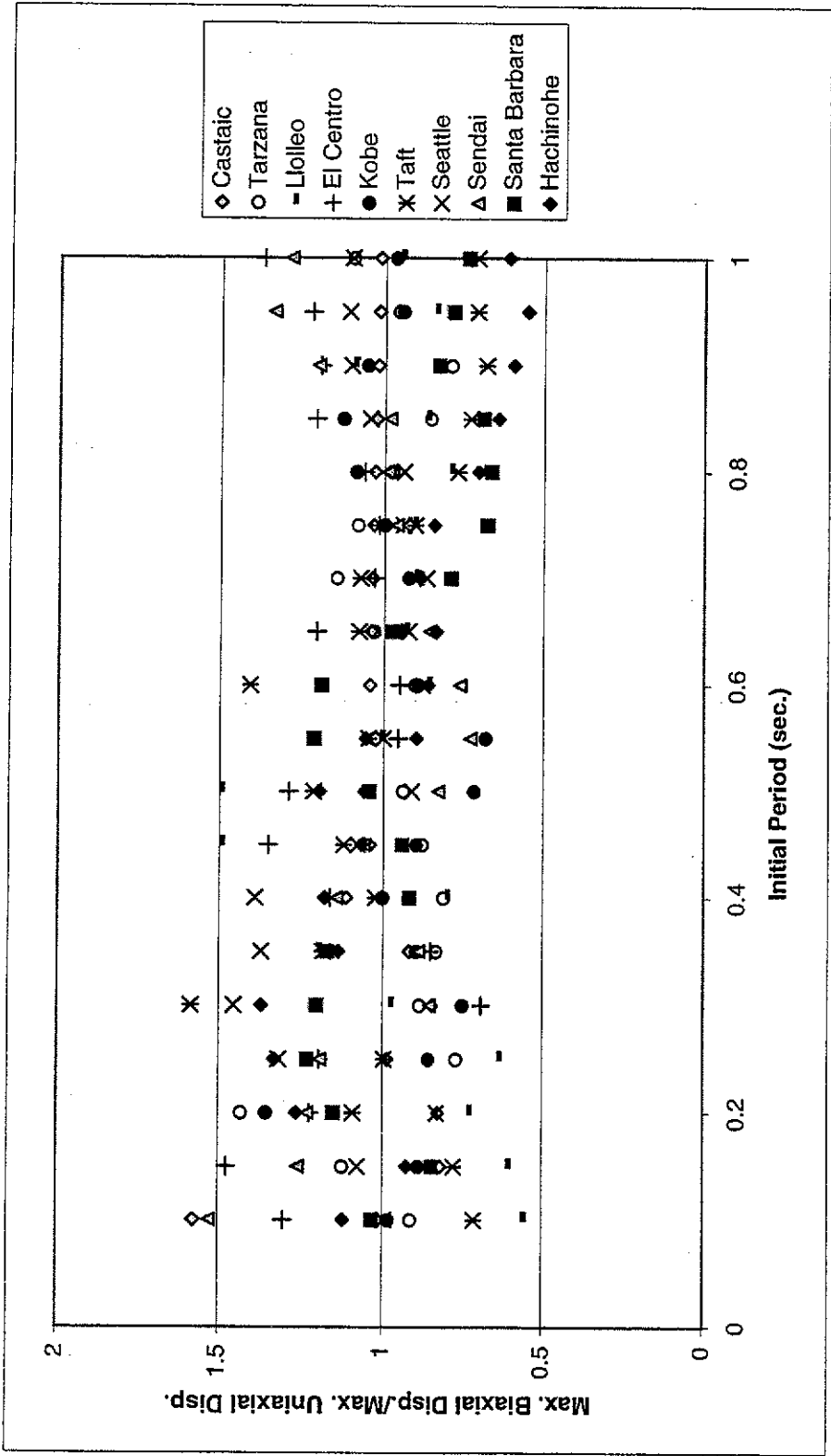
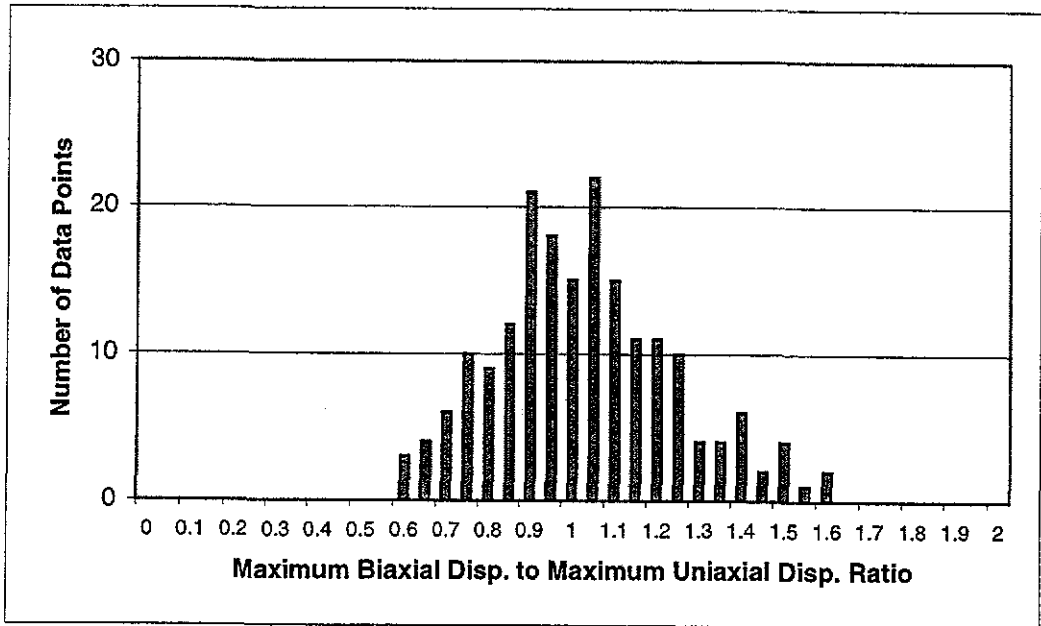
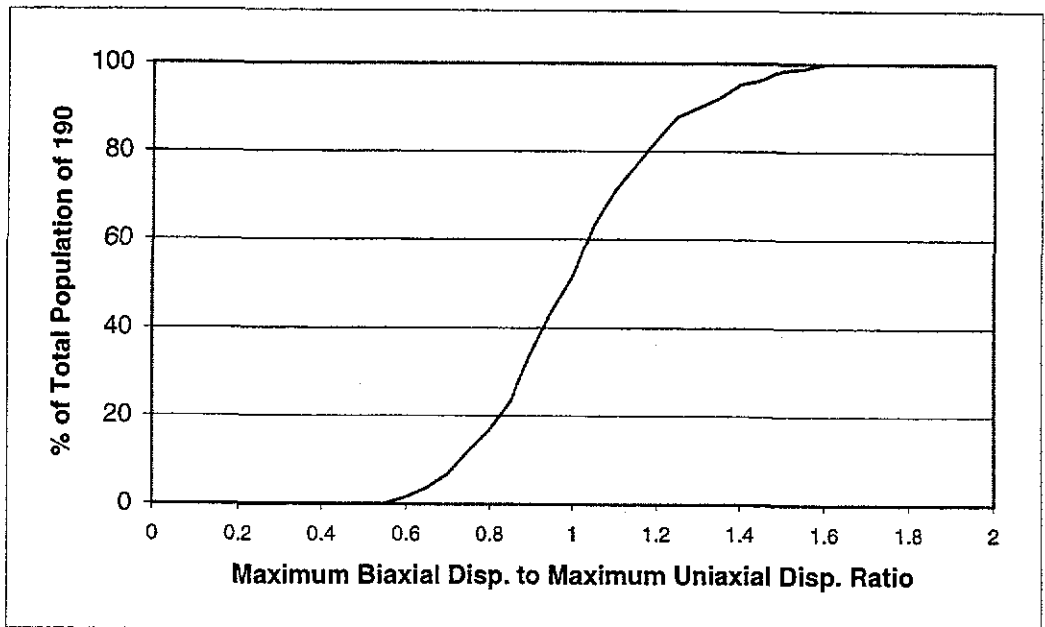


Fig. 5.13 Ratio of the Maximum Displacement for Biaxial Base Excitation to the Maximum Displacement for Uniaxial Base Excitation, All the Records Together



a) Frequency Distribution



b) Cumulative Frequency Distribution

Fig. 5.14 Distributions of the Maximum-Displacement Ratios for Biaxial Base Excitation to Uniaxial Base Excitation

6. SUMMARY AND CONCLUSIONS

The primary objective of this study was the development of a numerical model for nonlinear-response analysis of reinforced concrete members subjected to biaxial ground excitation. A new algorithm was developed to determine the moment-rotation response of reinforced concrete based on nonlinear hysteretic response of concrete and steel in terms of their unit stress-strain properties. The algorithm was tested using data from static and dynamic tests with biaxial loading. Criteria for testing the success of the algorithm included not only the maximum response but the entire behavior during an earthquake event of a reinforced concrete system through ranges of low and high response.

The proposed algorithm is based on a model that represents the reinforced concrete section as a collection of nonlinear springs. The hysteretic properties of the springs reflect observed behavior of concrete and steel in uniaxial loading. To use as simple a definition of hysteretic properties as possible, an approach evolving from simple to complex was followed. At the beginning, concrete was modeled by the Ritter parabola and a linear extension, similar to the Hognestad model (1951). The unloading path was defined to be linear with its slope based on the observations by Karsan and Jirsa (1969). Reloading followed the unloading path in the force-deformation plane. Initial studies indicated that this scheme failed to simulate observed response because crack closing was a more complex phenomenon than could be represented with these assumptions. The reloading path was modified to recognize the fact that the crack closed at a strain lower than that at which the return or unloading path crossed the strain axis. Similarly, the reinforcement was initially modeled simply by three parameters defining the initial slope, the secondary slope, and the yield stress. It was observed that this approach failed to model the proper energy dissipation at low amplitudes and a second model for steel

based on the Menegotto-Pinto equation (Menegotto and Pinto, 1973) was adopted. At every step of progress, no additions were made to the basic model unless it was required by the phenomena modeled.

The variety of physical phenomena associated with flexural behavior of reinforced concrete can be easily defined through the proposed model. Crack opening, crack closing, softening of concrete, the Bauschinger effect, permanent strains of the reinforcing steel, and the spreading of the plastic hinge region along the member can all be modeled conveniently. The model is capable of representing the pinching of hysteresis loops, strength/stiffness interaction about orthogonal axes of the members, and the effect of the axial load on strength/stiffness dependence. The focus of this study has been on flexural behavior of reinforced concrete elements. It has been assumed that the elements considered are not vulnerable to brittle modes of failure related to shear or bond stresses.

In order to check the sensitivity of the proposed model for small changes in input parameters, under dynamic loading, a sensitivity study was performed. The parameters considered were divided into two groups:

1. The steel hysteresis model, the transition parameter of the Menegotto-Pinto equation, the development length of the reinforcing bars, and the yielding stress of reinforcing bars.
2. Parameters defining crack closing, initial modulus of elasticity for the concrete, concrete strength and the definition of the "yield region" along the column.

The sensitivity of the calculated results to changes in the first group of input parameters was strong for maximum-displacement, waveforms and periodicity. The effect of the second group of parameters was weak and limited to low-response phases (low-response phase refers to the duration of the test when the moment response did not

exceed approximately half the yield moment). The choices for the first group of parameters have to be considered carefully in assembling a model for calculating response.

The sensitivity of the proposed model in the case of biaxial base excitation with components having different frequency contents was also investigated. No clear difference was observed between the sensitivities of the materially coupled responses caused by a single component base excitation and by a biaxial base excitation.

The secondary objective of the thesis was to study the nonlinear response of reinforced concrete structures to identify the differences in their maximum-displacement response subjected to biaxial and uniaxial ground excitations. A numerical study was made to reach this objective. The main variables of the study were stiffness, base shear strength, as well as different earthquake intensities and frequency contents. A series of single-mass systems, with initial periods changing from 0.1 to 1.0-sec., were analyzed using the model proposed in Chapter 2. A total of ten different earthquake records were used (Table 5.1). The record stations include locations for which the soil conditions were described as rock, stiff or soft soil.

The effect of biaxial excitation was evaluated by comparing the magnitudes of displacement calculated for biaxial and uniaxial excitation. The maximum-displacement ratio for biaxial base excitation to that for uniaxial base excitation was found not to exceed 1.5, with most of the results around 1.0, provided that the base shear strength for the system satisfied Eq. 5.2.

The results of the proposed algorithm were tested using measurements obtained in static and dynamic tests. The experiments by Otani and Cheung (1981) were used for static loading comparisons. In these experiments, reinforced concrete cantilevers were subjected to uniaxial and biaxial bending. The experiments by Dragovich (1996) were used for the dynamic loading comparisons. In these experiments, reinforced concrete

cantilevers were subjected to uniaxial and biaxial bending resulted from base excitation. In both loading cases, there were no axial loads. The range of concrete strengths in the tests varied between 3,000 psi and 7,500 psi and steel stresses between 59,000 to 68,000 psi.

Comparisons of the calculated and the measured results for the specimens tested by Otani and Cheung (1981) were based on the maximum force levels, the energy dissipation, and the stiffness. The calculated maximum force levels were within $\pm 10\%$ of the measured values. Except for the responses at rotation amplitudes lower than the yield rotation, calculated energy dissipation (area of the loops) for each specimen was within $\pm 15\%$ of the measured value in each case. The calculated stiffness (defined by the slope of a line drawn to extreme points on the hysteresis loop) was within $\pm 10\%$ of the measured values in each case.

Comparisons of the calculated and the measured results for the specimens tested by Dragovich (1996) were based on maximum rotation, rotation history and moment rotation histories (Table 3.4, Fig. 3.18 and 3.19, Fig. 3.20 to 3.22). The mean ratio of the measured to calculated absolute-maximum rotations for the materially coupled specimens was 0.96 and for the materially uncoupled specimens, the mean ratio was 0.90. Rotation-History curves of the specimens exhibited phases of low and high responses. Correlation of the measured and the calculated curves was highly satisfactory for the high-response phases. For the low response phases, the correlation was satisfactory. The mean ratio of the measured to calculated absolute-maximum moments was 1.05 for the coupled and 1.12 for the uncoupled specimens. The difference between the measured and calculated values was attributed to strain hardening and the strain-rate effects of the reinforcing bar. In the measured and calculated responses of biaxially excited test specimens two kinds of "pinching" were observed. The first kind was the type usually observed in uniaxially loaded elements with slip or crack closing. The second kind was due to the biaxial nature of the loading. If the element had already yielded in direction-1, it yielded in direction-2 at a lower moment but could regain strength in direction-2, if the element was unloaded

in direction-1. As a result, slope of the moment-rotation response could change from steep to nearly flat, back to steep (Fig. 3.20c and d, 0 to 2-sec.), creating the effect which has been described as "pinching". The proposed algorithm provided a rationale for understanding the different sources of "pinching" in hysteresis loops.

Comparisons of the calculated and measured results confirmed that the proposed algorithm was successful not only in providing excellent estimates of the maximum force and displacement responses, a result which has been achieved before, but it also resulted in very good estimates of the behavior of the biaxially loaded reinforced concrete systems in ranges of low response. Considering that the elements of the model are explicit and based directly on observed behavior of the materials, it is concluded that the proposed algorithm provides a reliable basis for analytical assessment of the biaxial response of complex systems in reinforced concrete.

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APPENDICES

APPENDIX A
MATERIAL HYSTERESIS ALGORITHMS

A.1 Concrete Model Algorithm

The stiffness to be used in each increment is calculated from four parameters:

1. The stress and strain coordinates defining the current position (ϵ_{cur} , f_{cur}).
2. The stress and strain coordinates of the maximum excursion on the envelope curve (ϵ_{cen} , f_{cen}).
3. The state of cracking (true or false).
4. Information indicating whether (ϵ_{cur} , f_{cur}) is in the state of incomplete contact (true or false).

The algorithm outputs the refreshed values of the four parameters above. In order to keep track of the travel, the range of behavior is considered in a stationary stress-strain surface (envelope curve Fig. A.1a), and five nonstationary strain intervals (Fig. A.1b-e). The notation used in the following text and in Table A.1 is defined below.

ϵ_{cx} = The compressive strain at which concrete resistance vanishes.

ϵ_{cen} = The strain of the maximum excursion on the envelope curve.

ϵ_{ec} = Strain at complete contact.

ϵ_{cp} = The permanent-set strain of the concrete defined by Eq. 2.8.

ϵ_{ct} = Concrete strain at reaching tensile strength.

ϵ_{new} = The new strain.

ϵ_{cur} = Strain coordinate of the current position.

f_{cen} = Stress of the maximum excursion on the envelope curve.

- f_{ec} = Stress at attainment of complete contact.
- f = The new stress calculated for the new strain ϵ_{new} .
- f_{cur} = Stress coordinate of the current position.
- E_c = Initial modulus of elasticity for the concrete fiber.
- $crack_flag$ = Flag indicating the cracking state. If the fiber is cracked, the flag is set at -1. Otherwise, it is set at 1.
- ec_flag = Flag indicating the incomplete contact state. If the fiber is in a state of incomplete contact, the flag is set to the strain value at the left boundary of the incomplete-interval. Otherwise, it is set to 101.

The following definitions are listed below to avoid repetitions in Table A.1:

$f_1(\epsilon_{new})$ Defines the stress on the envelope curve, using Sec. 2.2. (A.1)

$$f_2(\epsilon_{new}, \epsilon_{cen}, f_{cen}, \epsilon_{cp}) = f_{cen} \left(\frac{\epsilon_{new} - \epsilon_{cp}}{\epsilon_{cen} - \epsilon_{cp}} \right) \quad (A.2)$$

$$f_3(\epsilon_{new}, E_c, \epsilon_{cur}, \epsilon_{cp}) = -E_c \left(\frac{\epsilon_{cur} - \epsilon_{new}}{\epsilon_{cur} - \epsilon_{cp}} \right) \quad (A.3)$$

$$f_3^*(\epsilon_{new}, E_c, \epsilon_{cp}) = -E_c (\epsilon_{cp} - \epsilon_{new}) \quad (A.4)$$

$$f_4(\epsilon_{new}, f_{ec}, \epsilon_{ec}, ec_flag) = f_{ec} \left(\frac{\epsilon_{new} - ec_flag}{\epsilon_{ec} - ec_flag} \right) \quad (A.5)$$

The specific rules used for the algorithm defining concrete response are described in Table A.1 and in the accompanying Figures A.1 and A.2. The solid lines in Fig. A.1a-f indicate the assumed stress-strain relationship for that interval. The range of each expression used in the algorithm is indicated by the solid lines in the figures identified in Table A.1. It should be noted that the ranges of intervals 1 through 5 vary if the

maximum strain in a cycle exceeds the previous maximum. To help explain the process used in assembling Table A.1, the items in the first row are explained below.

The first column of the table identifies the strain interval. For example, the first row refers to a condition with the current stress-strain coordinates located on the envelope. The second column refers to loading. For the first row, it defines that, in case of loading on the envelope curve, stress is governed by the expressions defined in Section 2.4 (Eqn. A.1, the solid line in Fig. A.1a). The third column refers to unloading. For the first row, it defines that if the strain is more than that corresponding to the permanent set, ϵ_{cp} , Eq. A.2 governs (Fig. A.2a). If the unloading reaches the strain axis, there are two options for further unloading. If it is previously cracked, the stress is zero (Fig. A.2e). If it is not, Eq. A.3 governs to a strain of $(\epsilon_{cp}-\epsilon_{ct})$ (Fig. A.2b). If the strain becomes less than $(\epsilon_{cp}-\epsilon_{ct})$, the stress is set to zero (Fig. A.2g and h) and the flag indicating the state of cracking is set to -1.

Some of the parameters have to be refreshed through the steps. The refreshed parameters are identified by the term “refresh” sign in parentheses in Table A.1.

A.2 Steel Model 1

The bilinear model is defined by a primary curve (see curve P in Fig. 2.5) and two bounding curves (see line S_c and S_t in Fig. 2.5). Loading and unloading occurs with the slope defined by the primary curve until the bounding curves are reached. Then, loading occur along the bounding curves and unloading occurs along a new primary curve that has the same slope with the initial primary curve but starts at the reversal point. Unloading stays on the primary curve until the other bounding curve is reached. Upon reaching the bounding curve, travel occurs along bounding curve. In reference to Fig. 2.5, the boundary curves are defined as:

The bounding curve on the positive (compression) side:

$$\sigma = (STH) \varepsilon_{new} + (\sigma_y - (STH) \varepsilon_y) \quad (A.6)$$

The bounding curve on the negative (tension) side:

$$\sigma = (STH) \varepsilon_{new} + (-\sigma_y + (STH) \varepsilon_y) \quad (A.7)$$

where,

ε_{new} = The new strain value.

σ = The stress corresponding to ε_{new} .

σ_y, ε_y = Yield stress and strain (both defined to be positive).

STH= Slope of the bounding curves.

If the current stress-strain point is within the bounding curves, stress-strain relation of the primary curve is:

$$\sigma = E \varepsilon_{new} - (\sigma_{cur} - E \varepsilon_{cur}) \quad (A.8)$$

where,

$\sigma_{cur}, \varepsilon_{cur}$ = Stress and strain values of the at the current position.

E= Slope of the primary curve.

The algorithm given in Table A.2 separates the model into three regions, based on the position of the current values of stress and strain for the fiber.

1. Between the boundaries.
2. On the tension side boundary.
3. On the compression side boundary.

The hysteresis rules are specified in Table A.2. The strain at which the primary line meets the bounding line are defined below and used in Table A.2 (the points A₁ and A₂ for the representative primary curve in Fig. 2.5).

$$\varepsilon_{\text{comp}} = \frac{(D - B)}{((\text{STH}) - E)} \quad (\text{A.9})$$

$$\varepsilon_{\text{tens}} = \frac{(D + B)}{((\text{STH}) - E)} \quad (\text{A.10})$$

where,

$\varepsilon_{\text{comp}}$: The intersection point of transition curve with the compression boundary.

$\varepsilon_{\text{tens}}$: The intersection point of transition curve with the tension boundary.

$$B = \sigma_y - ((\text{STH})\varepsilon_y)$$

$$D = \sigma_{\text{cur}} - \varepsilon_{\text{cur}}E$$

The first row of the Table A.2 is explained below to provide an example of the Table A.2. The first column identifies the current state of the fiber. If the fiber is between the boundaries, the first row has to be used. The second column refers to loading. If the loading does not reach the compression boundary curve ($\varepsilon_{\text{new}} < \varepsilon_{\text{comp}}$), Eq. A.8 governs. If the bounding curve is reached, Eq. A.6 governs. The third column refers to unloading. If unloading does not reach to tension side boundary curve ($\varepsilon_{\text{new}} > \varepsilon_{\text{tens}}$), again Eq. A.8 governs. If the boundary is reached Eq. A.7 governs. Current position has to be refreshed for every step. The refreshed parameters are identified in the Table A.2 by the term “refresh” in parentheses.

A.3 Steel Model 2

The algorithm is defined in three intervals throughout the stress-strain history:

1. Before reaching the yield stress in any direction (Fig. A.3b).
2. After first yield, up to first reversal (Fig. A.3c).
3. Reversals after first yield (Fig. A.3d).

The following definitions are used in Table A.3.

ε_{new}	The new strain value.
σ	Stress corresponding to the new strain value.
$\varepsilon_{\text{cur}}, \sigma_{\text{cur}}$	Strain and stress values the current position.
ε_r, σ_r	Position of the maximum excursion point on the envelope .
ε_o, σ_o	Strain and stress values of the bounding curves intersection.
E_o	Initial slope of unloading curve (Eqn. 2.8).
R	Parameter that defines the curvature for transition from the first to the second asymptote(Eqn. 2.9).
yield_flag	Parameter that defines the yield state of the fiber. If it is 1, the fiber has not yield in any direction. If it is 0, the fiber has yielded once in any direction. If it is -1, the fiber has yielded more than once.
$\varepsilon^*, \sigma^*, b$	As defined in Eqn. 2.7.
ε_z	As defined in Section 2.4.2.
$\varepsilon_{\text{max}}^t, \varepsilon_{\text{max}}^c$	As defined in Eqn. 2.9.

The calculations for the strains ε_z , ε_o and the stress σ_o are given below:

1. ε_z is a function of the initial slope of the unloading curve and stress-strain coordinates, $(\varepsilon_r, \sigma_r)$, of the most recent reversal (Fig. 2.8).

$$\varepsilon_z = \varepsilon_r - \frac{\sigma_r}{E_o} \quad (\text{A.11})$$

2. To calculate the intersection of asymptotes, the direction of the reversal has to be known. If the reversal is toward the tension (negative) side:

$$\varepsilon_o = \frac{\sigma_y - (\varepsilon_z + \varepsilon_y)(STH) + \varepsilon_z E_o}{(E_o - (STH))}, \quad \sigma_o = (\varepsilon_o - \varepsilon_z)E_o \quad (\text{A.12})$$

If the reversal is toward the compression (positive) side:

$$\varepsilon_o = \frac{\sigma_y - \varepsilon_z E_o - (\varepsilon_y - \varepsilon_z)(STH)}{((STH) - E_o)}, \quad \sigma_o = (\varepsilon_o - \varepsilon_z)E_o \quad (A.13)$$

The rules for the M-P curve are specified in Table A.3. The first column of Table A.3 defines the range in which the coordinates of the current stress-strain point are. If the fiber has not yielded in either direction, travel occurs along the primary curve (Fig. A.3a) in either direction. If the fiber has yielded for the first time but has not experienced a reversal (the solid curve in Fig. A.3c) and the loading is continuing, the rule in the second column of the second row applies. If a reversal occurs, the rules in the third row of Table A.3 apply.

Table A.1 Concrete Model Algorithm

	LOADING	UNLOADING
Fiber strain on the envelope curve	$f = f_1(\epsilon_{new})$ $\epsilon_{cen} = \epsilon_{new}$ $f_{cen} = f$ $\epsilon_{cur} = \epsilon_{new}$	Fig. A1.a (refresh) (refresh) (refresh) <pre> if ($\epsilon_{new} \geq \epsilon_{cp}$) $f = f_2(\epsilon_{new}, \epsilon_{cen}, f_{cen}, \epsilon_{cp})$ else if ($(\epsilon_{cp} - \epsilon_{ct}) < \epsilon_{new} < \epsilon_{cp}$) { if (crack_flag = -1) $f = 0$ else $f = f_3(E_c, \epsilon_{cur}, \epsilon_{cp}, \epsilon_{new})$ } else if ($\epsilon_{new} < (\epsilon_{cp} - \epsilon_{ct})$) { if (crack_flag = 1) crack_flag = -1 (refresh) $f = 0$ } $\epsilon_{cur} = \epsilon_{new}$ (refresh) </pre> Fig. A2.a Fig. A2.e Fig. A2.b
Fiber strain in interval 1	<pre> if ($\epsilon_{new} \leq \epsilon_{cen}$) $f = f_2(\epsilon_{new}, \epsilon_{cen}, f_{cen}, \epsilon_{cp})$ else if ($\epsilon_{new} > \epsilon_{cen}$) { $f = f_1(\epsilon_{new})$ $\epsilon_{cen} = \epsilon_{new}$ $f_{cen} = f$ } $\epsilon_{cur} = \epsilon_{new}$ </pre>	Fig. A1.b Fig. A1.a (refresh) (refresh) (refresh) <pre> if ($\epsilon_{new} > \epsilon_{cp}$) $f = f_2(\epsilon_{new}, \epsilon_{cen}, f_{cen}, \epsilon_{cp})$ if ($(\epsilon_{cp} - \epsilon_{ct}) \leq \epsilon_{new} < \epsilon_{cp}$) { if (crack_flag = -1) $f = 0$ else $f = f_3(E_c, \epsilon_{cur}, \epsilon_{cp}, \epsilon_{new})$ } if ($\epsilon_{new} < (\epsilon_{cp} - \epsilon_{ct})$) { if (crack_flag = 1) crack_flag = -1 (refresh) $f = 0$ } $\epsilon_{cur} = \epsilon_{new}$ (refresh) </pre> Fig. A2.a Fig. A2.e Fig. A2.b

Table A.1 Concrete Model Algorithm (cont.)

	LOADING	UNLOADING
Fiber strain in interval 2	<pre> if ($\epsilon_{new} < \epsilon_{ec}$) { if ($ec_flag > 100$) $f = f_2(\epsilon_{new}, \epsilon_{cen}, f_{cen}, \epsilon_{cp})$ else $f = f_4(\epsilon_{new}, \epsilon_{ec}, f_{ec}, ec_flag)$ } if ($\epsilon_{ec} < \epsilon_{new} \leq \epsilon_{cen}$) $f = f_2(\epsilon_{new}, \epsilon_{cen}, f_{cen}, \epsilon_{cp})$ if ($\epsilon_{cen} < \epsilon_{new}$) { $f = f_1(\epsilon_{new})$ $\epsilon_{cen} = \epsilon_{new}$ $f_{cen} = f$ } $\epsilon_{cur} = \epsilon_{new}$ </pre> Fig. A2.c Fig. A2.d Fig. A1.b Fig. A1.a (refresh) (refresh) (refresh)	<pre> if ($\epsilon_{new} \geq \epsilon_{cp}$) { if ($ec_flag > 100$) $f = f_2(\epsilon_{new}, \epsilon_{cen}, f_{cen}, \epsilon_{cp})$ else { $f = f_4(\epsilon_{new}, \epsilon_{ec}, f_{ec}, ec_flag)$ if ($f < 0$) { (See note 1) $f = 0$; $ec_flag = 101$ } } } if ($((\epsilon_{cp} - \epsilon_{ci}) \leq \epsilon_{new} < \epsilon_{cp})$ { if ($ec_flag > 100$) { if ($crack_flag = 1$) $f = f_3(\epsilon_{cp}, \epsilon_{cur}, \epsilon_{cp}, \epsilon_{new})$ if ($crack_flag = -1$) $f = 0$ } if ($ec_flag < 100$) { $f = f_4(\epsilon_{new}, \epsilon_{ec}, f_{ec}, ec_flag)$ if ($f < 0$) { (See note 1) $f = 0$; $ec_flag = 101$ } } } if ($end_ec \leq \epsilon_{new} < \epsilon_{cp}$) { if ($ec_flag > 100$) { $f = 0$ $crack_flag = -1$ } } </pre> Fig. A2.c Fig. A2.d Fig. A2.b Fig. A2.c Fig. A2.d Fig. A2.g (refresh)

Table A.1 Concrete Algorithm (cont.)

LOADING	UNLOADING
Fiber strain in interval 2 (cont.)	<pre> if(ec_flag<100){ f=f₄(ϵ_{new}, ϵ_{ec}, f_{ec}, ec_flag) if(f<0){ f=0 (See note 1) ec_flag=101 } } if(ϵ_{new}<end_ec){ f=0 crack_flag=-1 ec_flag=101 } $\epsilon_{cur} = \epsilon_{new}$ </pre> Fig. A2.f Fig. A2.h (refresh) (refresh) (refresh)

Table A.1 Concrete Model Algorithm (cont.)

	LOADING	UNLOADING
Fiber strain in interval 3	<pre> if($\epsilon_{new} \leq \epsilon_{cp}$) { if($\text{crack_flag}=1$) $f=f_3^*(E_c, \epsilon_{cp}, \epsilon_{new})$ if($\text{ec_flag}>100$) { if($\epsilon_{cur}>\text{end_ec}$) $\text{ec_flag}=\epsilon_{cur}$ else $\text{ec_flag}=\text{end_ec}$ $f=f_4(\epsilon_{new}, \epsilon_{ec}, f_{ec}, \text{ec_flag})$ } if($\text{ec_flag}<100$) $f=f_4(\epsilon_{new}, \epsilon_{ec}, f_{ec}, \text{ec_flag})$ Fig. A2.f } if($\epsilon_{cp}<\epsilon_{new} \leq \epsilon_{ec}$) { if($\text{ec_flag}>100$) $f=f_2(\epsilon_{new}, \epsilon_{cen}, f_{cen}, \epsilon_{cp})$ Fig. A2.c else $f=f_4(\epsilon_{new}, \epsilon_{ec}, f_{ec}, \text{ec_flag})$ Fig. A2.d } if($\epsilon_{ec}<\epsilon_{new} \leq \epsilon_{cen}$) { $f=f_2(\epsilon_{new}, \epsilon_{cen}, f_{cen}, \epsilon_{cp})$ Fig. A2.b $\text{ec_flag}=101$ } else { $f=f_1(\epsilon_{new})$ Fig. A1.a $\epsilon_{cen}=\epsilon_{new}$ (refresh) $f_{cen}=f$ (refresh) } $\epsilon_{cur}=\epsilon_{new}$ (refresh) </pre>	<pre> if($((\epsilon_{cp}-\epsilon_{new}) \leq \epsilon_{ct})$) { if($\text{ec_flag}<100$) { $f=f_4(\epsilon_{new}, \epsilon_{ec}, f_{ec}, \text{ec_flag})$ Fig. A2.d if($f<0$) { (See note 1) $f=0$ $\text{ec_flag}=101$ } } if($\text{ec_flag}>100$) $f=f_3^*(E_c, \epsilon_{cp}, \epsilon_{new})$ Fig. A2.b else $f=0$ Fig. A2.e } if($(\text{end_ec} \leq \epsilon_{new} < (\epsilon_{cp}-\epsilon_{ct}))$ and ($\text{crack_flag}=1$)) { if($\text{ec_flag}>100$) { $\text{crack_flag}=-1$ $f=0$ } if($\text{ec_flag}<100$) { $f=f_4(\epsilon_{new}, \epsilon_{ec}, f_{ec}, \text{ec_flag})$ Fig. A2.f if($f<0$) { (See note 1) $f=0$ $\text{ec_flag}=101$ } } } if($\epsilon_{new} < \text{end_ec}$) { $\text{crack_flag}=-1$ $f=0$ $\text{ec_flag}=101$ } } $\epsilon_{cur}=\epsilon_{new}$ </pre>
		(refresh)
		(refresh)
		(refresh)
		(refresh)

Table A.1 Concrete Model Algorithm (cont.)

	LOADING	UNLOADING	
Fiber strain in interval 4	<pre> if($\epsilon_{new} \leq \epsilon_{ec}$) { if($ec_flag < 100$) $f = f_4(\epsilon_{new}, \epsilon_{ec}, f_{ec}, ec_flag)$ Fig. A2.f&d if($ec_flag > 100$) { (See note 2) if($\epsilon_{cur} > end_ec$) $ec_flag = \epsilon_{cur}$ else $ec_flag = end_ec$ $f = f_4(\epsilon_{new}, \epsilon_{ec}, f_{ec}, ec_flag)$ Fig. A2.f and d } } </pre>	<pre> if($end_ec < \epsilon_{new}$) { if($ec_flag < 100$) { $f = f_4(\epsilon_{new}, \epsilon_{ec}, f_{ec}, ec_flag)$ Fig. A2.f if($f < 0$) { (see note 1) $f = 0$; $ec_flag = 101$ } } if($ec_flag > 100$) $f = 0$ Fig. A2.g } </pre>	
	<pre> if($\epsilon_{ec} < \epsilon_{new} \leq \epsilon_{cen}$) { $f = f_2(\epsilon_{new}, \epsilon_{cen}, f_{cen}, \epsilon_{op})$ Fig. A2.b $ec_flag = 101$ (refresh) } </pre>	<pre> if($\epsilon_{new} < end_ec$) { $f = 0$ $ec_flag = 101$ Fig. A2.h } </pre>	(refresh)
	<pre> if($\epsilon_{new} < \epsilon_{cen}$) { $f = f_1(\epsilon_{new})$ Fig. A1.a $\epsilon_{cen} = \epsilon_{new}$ (refresh) $f_{cen} = f$ (refresh) $ec_flag = 101$ (refresh) } </pre>	<pre> $\epsilon_{cur} = \epsilon_{new}$ } </pre>	
	<pre> $\epsilon_{cur} = \epsilon_{new}$ </pre>		

Table A.1 Concrete Model Algorithm (cont.)

	LOADING	UNLOADING
	$\epsilon_{cur} < \epsilon_{end_ec}$ if($\epsilon_{new} \leq \epsilon_{end_ec}$) $f=0$ if($\epsilon_{end_ec} < \epsilon_{new} \leq \epsilon_{ec}$) { if($\epsilon_{cur} > \epsilon_{end_ec}$) $ec_flag = e_cur$ (See note 2) else $ec_flag = end_ec$ $f = f_4(\epsilon_{new}, \epsilon_{ec}, f_{ec}, ec_flag)$ } if($\epsilon_{ec} < \epsilon_{new} \leq \epsilon_{cen}$) $f = f_2(\epsilon_{new}, \epsilon_{cen}, f_{cen}, \epsilon_{cp})$ if($\epsilon_{new} > \epsilon_{cen}$) { $f = f_1(\epsilon_{new})$ $\epsilon_{cen} = \epsilon_{new}$ $f_{cen} = f$ } $\epsilon_{cur} = \epsilon_{new}$	$F=0$ $\epsilon_{cur} = \epsilon_{new}$ Fig. A2.h (refresh)
Fiber strain in interval 5	Fig. A2.h Fig. A2f and d Fig. A2.b Fig. A1.a (refresh) (refresh) (refresh)	

Note 1: This check is made to control the end of incomplete contact interval. If negative stresses are calculated, stress in the fiber set to zero and incomplete contact interval concluded. For transferring this information to next step, flag indicating the incomplete contact refreshed to 101.

Note 2: This block defines the beginning of the incomplete contact interval and sets the incomplete-contact flag to the strain value at the left boundary of the incomplete contact interval.

Table A.2 The Algorithm Defining the Bilinear Model for Reinforcement

LOADING		UNLOADING
Fiber strain between the boundaries.	if($\epsilon_{\text{new}} < \epsilon_{\text{comp}}$) $\sigma = \sigma_{\text{cur}} + E(\epsilon_{\text{new}} - \epsilon_{\text{cur}})$ (P-curve, Fig. 2.5) else $\sigma = \sigma_{\text{cur}} + E(\epsilon_{\text{comp}} - \epsilon_{\text{cur}}) + A(\epsilon_{\text{new}} - \epsilon_{\text{comp}})$ (S_c-curve, Fig. 2.5) $\epsilon_{\text{cur}} = \epsilon_{\text{new}}$ (refresh) $\sigma_{\text{cur}} = \sigma$ (refresh)	if($\epsilon_{\text{new}} > \epsilon_{\text{tens}}$) $\sigma = \sigma_{\text{cur}} + E(\epsilon_{\text{new}} - \epsilon_{\text{cur}})$ (P-curve, Fig. 2.5) else $\sigma = \sigma_{\text{cur}} + E(\epsilon_{\text{tens}} - \epsilon_{\text{cur}}) + A(\epsilon_{\text{new}} - \epsilon_{\text{tens}})$ (S_t-curve, Fig. 2.5) $\epsilon_{\text{cur}} = \epsilon_{\text{new}}$ (refresh) $\sigma_{\text{cur}} = \sigma$ (refresh)
Fiber strain on the tension side boundary.	if($\epsilon_{\text{new}} < \epsilon_{\text{tens}}$) $\sigma = \sigma_{\text{cur}} + A(\epsilon_{\text{new}} - \epsilon_{\text{cur}})$ (S_t-curve, Fig. 2.5) if($\epsilon_{\text{tens}} < \epsilon_{\text{new}} < \epsilon_{\text{comp}}$) $\sigma = \sigma_{\text{cur}} + A(\epsilon_{\text{tens}} - \epsilon_{\text{cur}}) + E(\epsilon_{\text{new}} - \epsilon_{\text{tens}})$ (P-curve, Fig. 2.5) if($\epsilon_{\text{new}} > \epsilon_{\text{comp}}$) $\sigma = \sigma_{\text{cur}} + A(\epsilon_{\text{tens}} + \epsilon_{\text{new}} - \epsilon_{\text{comp}} - \epsilon_{\text{cur}}) + E(\epsilon_{\text{comp}} - \epsilon_{\text{tens}})$ (S_c-curve, Fig. 2.5) $\epsilon_{\text{cur}} = \epsilon_{\text{new}}$ (refresh) $\sigma_{\text{cur}} = \sigma$ (refresh)	$\sigma = \sigma_{\text{cur}} + A(\epsilon_{\text{new}} - \epsilon_{\text{cur}})$ (S-line, Fig. 2.5) $\epsilon_{\text{cur}} = \epsilon_{\text{new}}$ (refresh) $\sigma_{\text{cur}} = \sigma$ (refresh)
Fiber strain on the compression side boundary.	$\sigma = \sigma_{\text{cur}} + A(\epsilon_{\text{new}} - \epsilon_{\text{cur}})$ (S_c-curve, Fig. 2.5) $\epsilon_{\text{cur}} = \epsilon_{\text{new}}$ (refresh) $\sigma_{\text{cur}} = \sigma$ (refresh)	if($\epsilon_{\text{new}} > \epsilon_{\text{comp}}$) $\sigma = \sigma_{\text{cur}} + A(\epsilon_{\text{new}} - \epsilon_{\text{cur}})$ (S_c-curve, Fig. 2.5) if($\epsilon_{\text{comp}} > \epsilon_{\text{new}} > \epsilon_{\text{tens}}$) $\sigma = \sigma_{\text{cur}} + A(\epsilon_{\text{comp}} - \epsilon_{\text{cur}}) + E(\epsilon_{\text{new}} - \epsilon_{\text{comp}})$ (P-curve, Fig. 2.5) if($\epsilon_{\text{new}} < \epsilon_{\text{tens}}$) $\sigma = \sigma_{\text{cur}} + A(\epsilon_{\text{comp}} + \epsilon_{\text{new}} - \epsilon_{\text{tens}} - \epsilon_{\text{cur}}) + E(\epsilon_{\text{comp}} - \epsilon_{\text{tens}})$ (S_t-curve, Fig. 2.5) $\epsilon_{\text{cur}} = \epsilon_{\text{new}}$ (refresh) $\sigma_{\text{cur}} = \sigma$ (refresh)

Table A.3 The Algorithm Defining the Menegotto-Pinto Curves

Fiber has not yielded in either direction.	$\sigma = \sigma_{cur} + E(\epsilon_{new} - \epsilon_{cur})$ $\sigma_{cur} = \sigma$ $\epsilon_{cur} = \epsilon_{new}$	On the primary curve (Fig. A.3a) (refresh) (refresh)	UNLOADING
Fiber has yielded for the first time and has not yet experienced a reversal.		LOADING	Go to the third row, reversal point.
	$\sigma = \sigma_{cur} + (STH)(\epsilon_{new} - \epsilon_{cur})$ $\sigma_{cur} = \sigma$ $\epsilon_{cur} = \epsilon_{new}$	On secondary curve (Fig. A.3) (refresh) (refresh)	

Table A.3 The Algorithm Defining the Menegotto-Pinto Curves (cont.)

REVERSAL	NOT A REVERSAL
Calculate the parameters for the new M-P curve: $\sigma_t = \sigma_{cur}$ (refresh) $\epsilon_t = \epsilon_{cur}$ (refresh) Calculate E_o using Eq. 2.8 Calculate ϵ_z using Eq. A.11 If $(\epsilon_z < \epsilon_{max}^t)$ $\epsilon_{max}^t = \epsilon_z$ If $(\epsilon_z > \epsilon_{max}^c)$ $\epsilon_{max}^c = \epsilon_z$ If the reversal is in tension direction calculate (ϵ_o, σ_o) using Eq. A.11 If the reversal is in compression direction calculate (ϵ_o, σ_o) using Eq. A.12 Calculate R using Eq. 2.9 Calculate b using Eq. 2.7 $\epsilon^* = (\epsilon_{new} - \epsilon_t) / (\epsilon_o - \epsilon_t)$ $\sigma^* = b\epsilon^* + (1-b)\epsilon^* / (1 + \epsilon^{*R_1(1/R)})$ $\sigma = \sigma^* (\sigma_o - \sigma_t) + \sigma_t$ $\sigma_{cur} = \sigma$ (refresh) $\epsilon_{cur} = \epsilon_{new}$ (refresh)	Using the set of parameters $(\sigma_t, \epsilon_t, E_o, \epsilon_z, \epsilon_{max}^t, \epsilon_{max}^c, \epsilon_o, \sigma_o, b, R)$ calculated in the most recent reversal point calculate: $\epsilon^* = (\epsilon_{new} - \epsilon_t) / (\epsilon_o - \epsilon_t)$ $\sigma^* = b\epsilon^* + (1-b)\epsilon^* / (1 + \epsilon^{*R_1(1/R)})$ $\sigma = \sigma^* (\sigma_o - \sigma_t) + \sigma_t$ $\sigma_{cur} = \sigma$ (refresh) $\epsilon_{cur} = \epsilon_{new}$ (refresh)

Reversal
has
occurred
after first
yield.

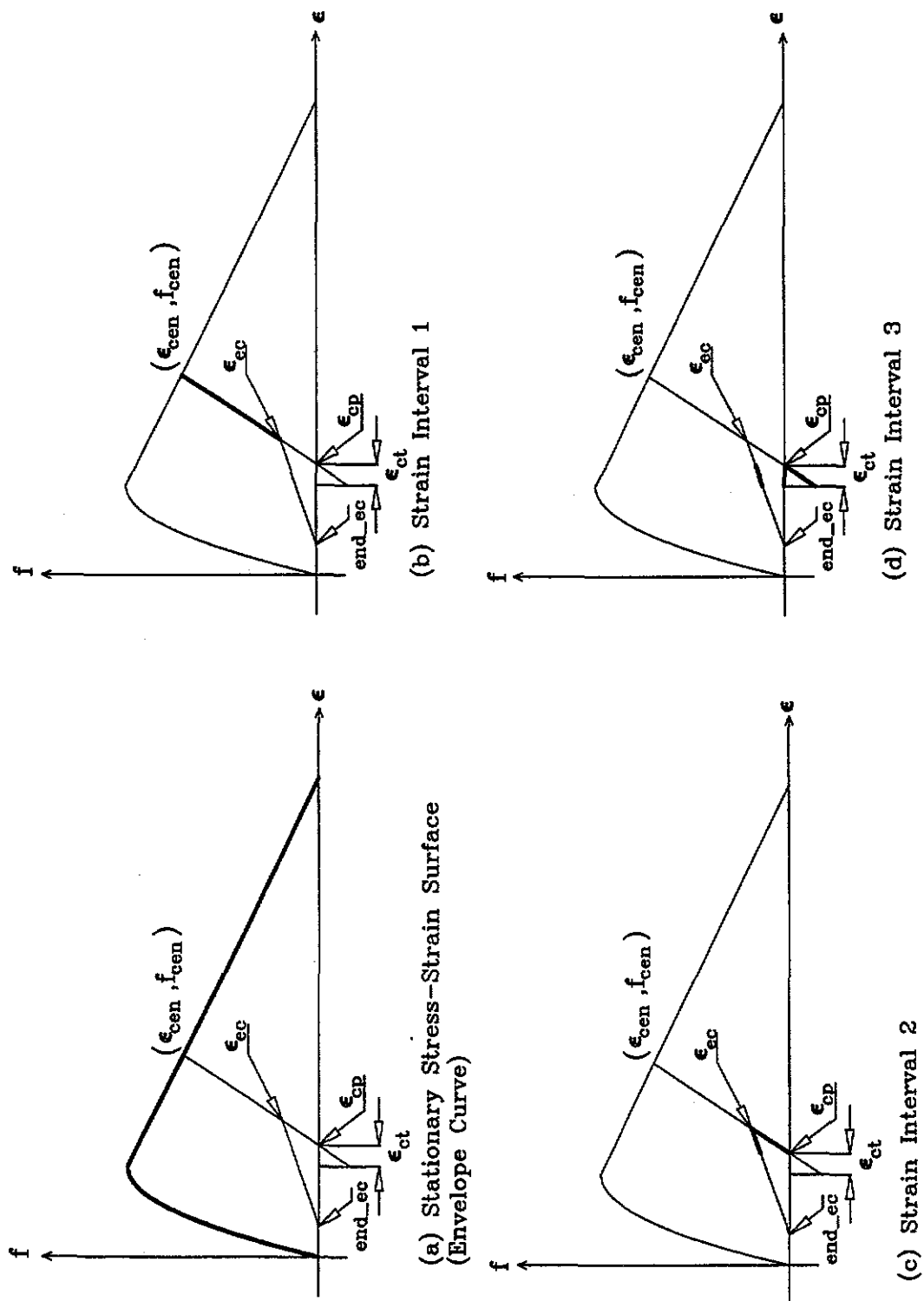


Fig. A.1 Intervals Used in the Concrete Algorithm

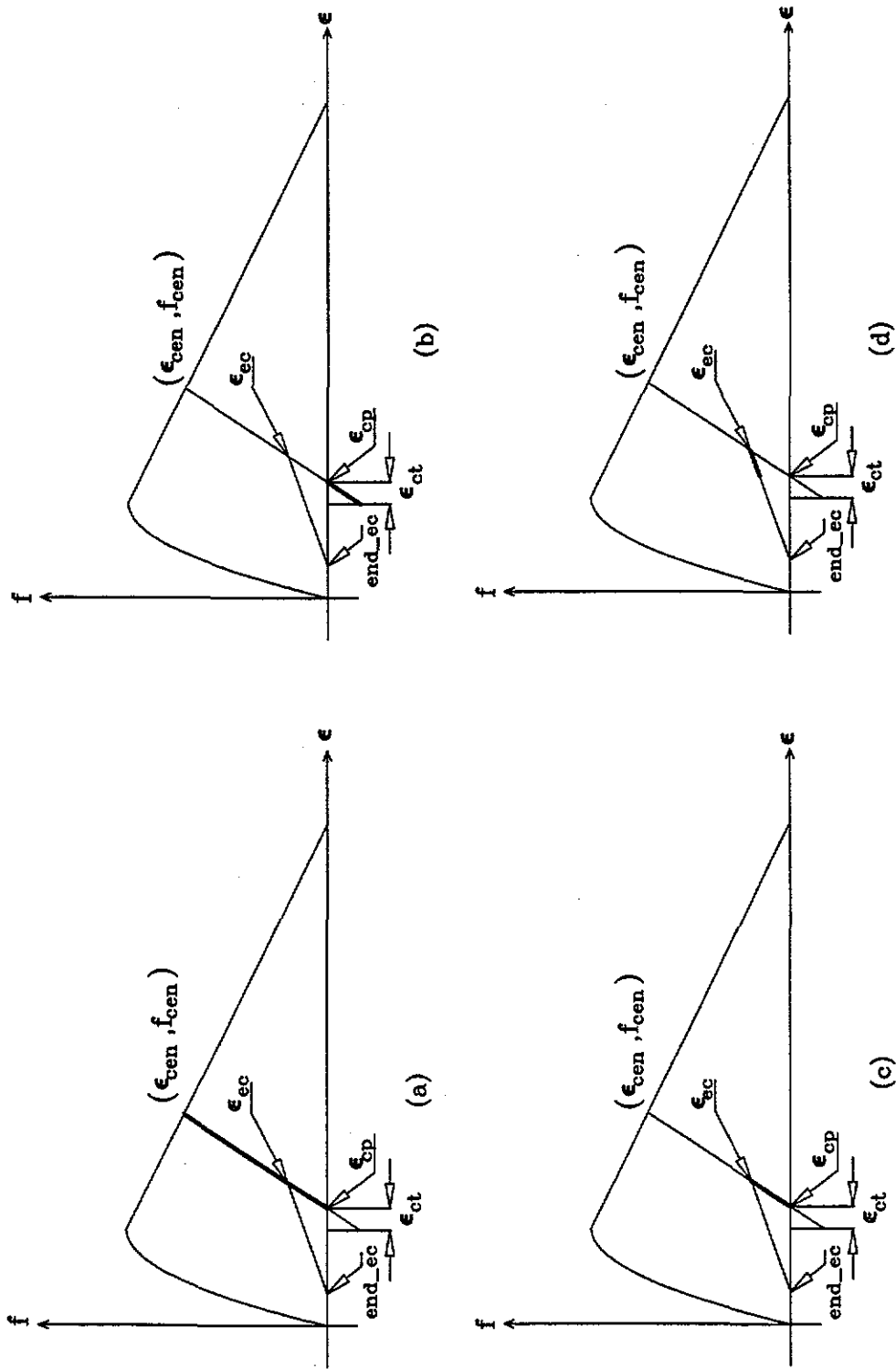


Fig. A.2 Ranges of the Expressions Given in Table A.1

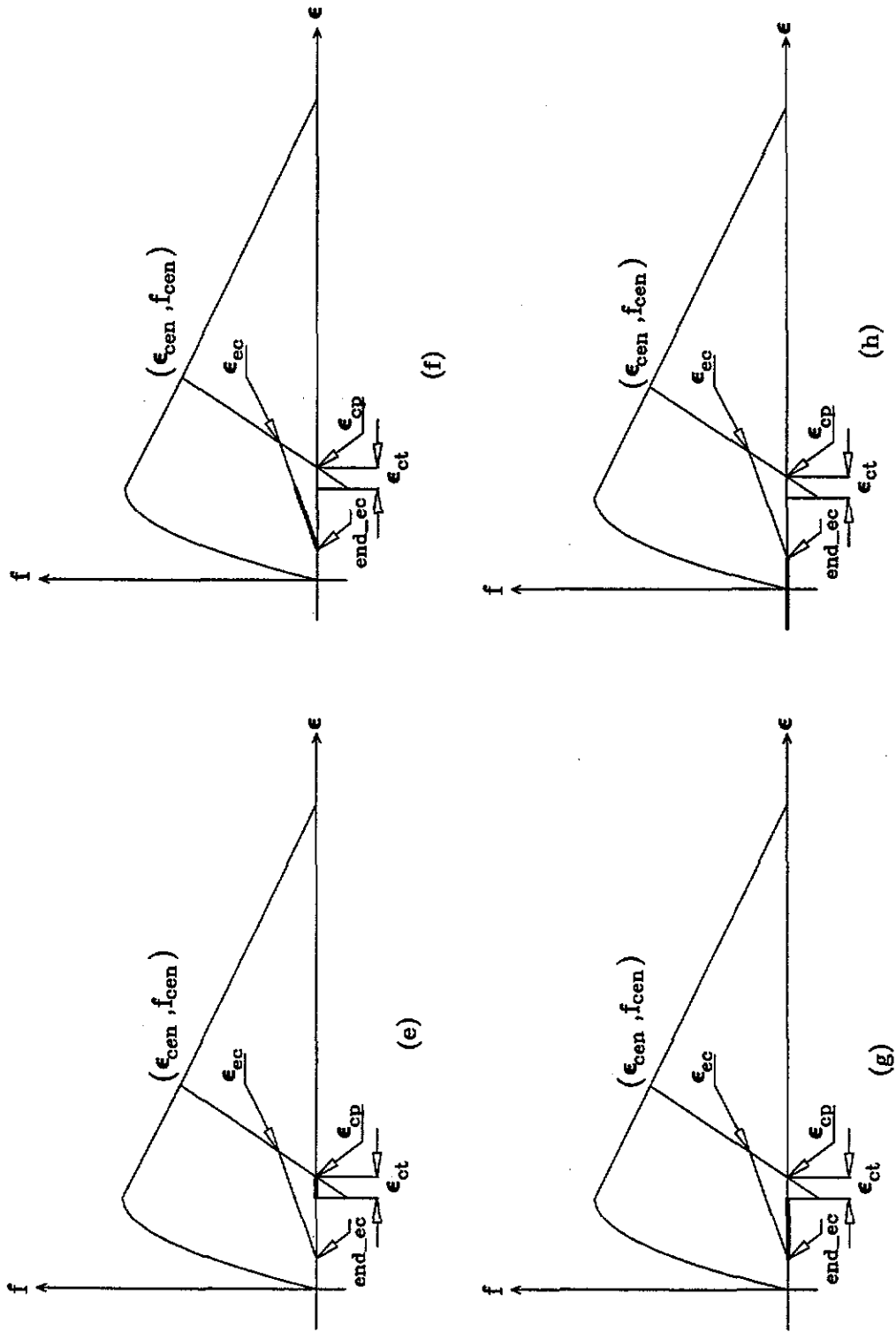


Fig. A.2 Ranges of the Expressions Given in Table A.1 (cont.)

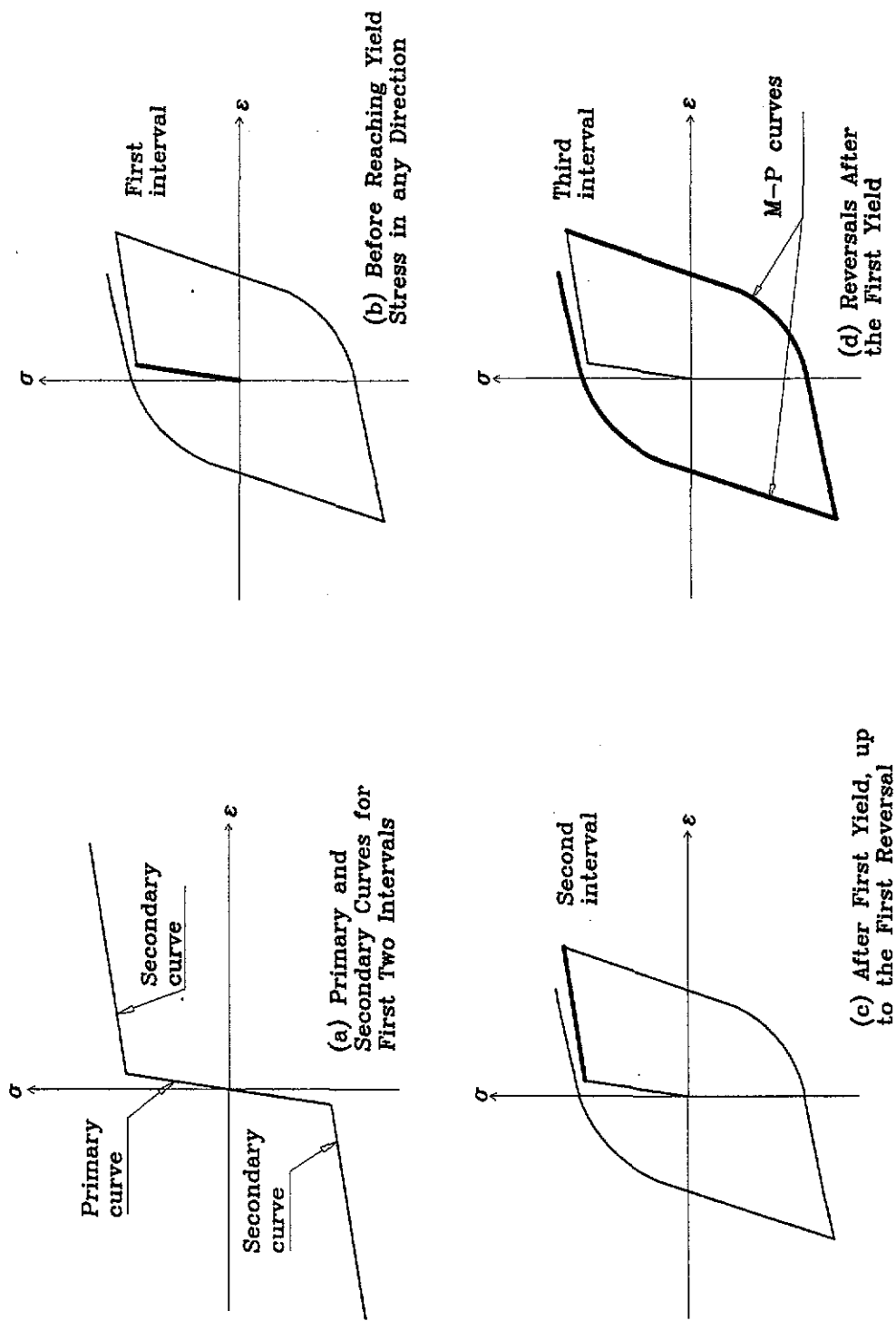


Fig. A.3 Representative Hysteresis Curves Indicating Intervals Used in the Algorithm of Steel Model for Menegotto-Pinto Curves

APPENDIX B
DYNAMIC ANALYSIS

The procedure used to obtain the response of a nonlinear system subjected to two dimensional earthquake motions is presented in this section. The equation of motion to be solved for a nonlinear system is:

$$[m]\{\ddot{u}\} + [c]\{\dot{u}\} + \{f(u)\} = \{P(t)\} \quad (\text{B.1})$$

where,

$\{\ddot{u}\}$, $\{\dot{u}\}$, $\{u\}$: Acceleration, velocity and displacement matrices.

$[m]$: Mass matrix.

$[c]$: Damping matrix.

$\{f(u)\}$: Resisting force matrix.

The system is assumed to have viscous damping. The applied force $P(t)$ is given by a set of discrete values $P_i = P(t_i)$, $i=0$ to N . The time interval, $\Delta t = t_{i+1} - t_i$, is kept constant. The response is determined at the discrete time instants t_i . The acceleration, velocity and displacement of the system at the time instant t_i are $\{\ddot{u}_i\}$, $\{\dot{u}_i\}$ and $\{u_i\}$. These values satisfy the Eq. B.1 at time i ,

$$[m]\{\ddot{u}_i\} + [c]\{\dot{u}_i\} + \{f(u_i)\} = \{P_i(t)\} \quad (\text{B.2})$$

where,

$\{f(u_i)\}$ = Resisting force at time i .

The response quantities at time instant (i+1) are $\{\ddot{u}_{i+1}\}$, $\{\dot{u}_{i+1}\}$ and $\{u_{i+1}\}$ and they also satisfy Eq. B.1.

$$[m]\{\ddot{u}_{i+1}\} + [c]\{\dot{u}_{i+1}\} + \{f(u_{i+1})\} = \{P(t_{i+1})\} \quad (\text{B.3})$$

The difference between equations B.2 and B.1 gives the incremental form of the equilibrium equation,

$$[m]\{\Delta\ddot{u}_i\} + [c]\{\Delta\dot{u}_i\} + \{\Delta f_i(u)\} = \{\Delta P_i(t)\} \quad (\text{B.4})$$

where,

$$\begin{aligned} \Delta\ddot{u}_i &= \ddot{u}_{i+1} - \ddot{u}_i, & \Delta\dot{u}_i &= \dot{u}_{i+1} - \dot{u}_i, & \Delta u_i &= u_{i+1} - u_i, \\ \Delta f_i(u) &= f(u_{i+1}) - f(u_i), & \Delta P_i &= P_{i+1} - P_i, \end{aligned}$$

The incremental resisting force can be defined as:

$$\Delta f_i(u) = (k_i)_{\text{sec}} \Delta u_i \quad (\text{B.5})$$

where,

$$(k_i)_{\text{sec}} = \text{Secant stiffness, as shown in Fig. B.1}$$

If Eq. B.5 is introduced to Eq. B.4,

$$[m]\{\Delta\ddot{u}_i\} + [c]\{\Delta\dot{u}_i\} + [(k_i)_{\text{sec}}]\{\Delta u_i\} = \{\Delta P_i(t)\} \quad (\text{B.6})$$

Newmark's Method (1959) is used to solve the Eq. B.6. This method is based on the following formulations:

$$\dot{u}_{i+1} = \dot{u}_i + [(1-\gamma)\Delta t]\ddot{u}_i + (\gamma\Delta t)\ddot{u}_{i+1} \quad (\text{B.7})$$

$$u_{i+1} = u_i + (\Delta t)\dot{u}_i + \left[(0.5 - \beta)(\Delta t)^2 \right] \ddot{u}_i + \left[\beta(\Delta t)^2 \right] \ddot{u}_{i+1} \quad (\text{B.8})$$

The parameters β and α define the variation of acceleration over a time step and determine the stability and accuracy characteristics of the method. Incremental forms of the velocity and displacement can be written from equations B.7 and B.8 as,

$$\Delta \dot{u}_i = (\Delta t)\ddot{u}_i + (\gamma\Delta t)\Delta \ddot{u}_i \quad (\text{B.9})$$

$$\Delta u_i = (\Delta t)\dot{u}_i + \frac{(\Delta t)^2}{2}\ddot{u}_i + \beta(\Delta t)^2\Delta \ddot{u}_i \quad (\text{B.10})$$

And Eq. B.10 can be solved for $\Delta \ddot{u}_i$,

$$\Delta \ddot{u}_i = \frac{1}{\beta(\Delta t)^2}\Delta u_i - \frac{1}{\beta\Delta t}\dot{u}_i - \frac{1}{2\beta}\ddot{u}_i \quad (\text{B.11})$$

And if Eq. B.11 is introduced into Eq. B.9

$$\Delta \dot{u}_i = \frac{\gamma}{\beta\Delta t}\Delta u_i - \frac{\gamma}{\beta}\dot{u}_i + \Delta t\left(1 - \frac{\gamma}{2\beta}\right)\ddot{u}_i \quad (\text{B.12})$$

If equations B.11 and B.12 are introduced to the incremental form of the equilibrium equation then,

$$\left[\hat{k} \right] \{ \Delta u_i \} = \left\{ \Delta \hat{P}_i \right\} \quad (\text{B.13})$$

where,

$$\left[\hat{k} \right] = [(k_i)_{\text{sec}}] + \frac{\gamma}{\beta \Delta t} [c] + \frac{1}{\beta (\Delta t)^2} [m] \quad (\text{B.14})$$

and

$$\left\{ \Delta \hat{P}_i \right\} = [\Delta P_i] + \left(\frac{1}{\beta \Delta t} [m] + \frac{\gamma}{\beta} [c] \right) \dot{u}_i + \left(\frac{1}{2\beta} [m] + \Delta t \left(\frac{\gamma}{2\beta} - 1 \right) [c] \right) \ddot{u}_i \quad (\text{B.15})$$

If the system is linear $\left[\hat{k} \right]$ and $\left\{ \Delta \hat{P}_i \right\}$ can be computed from the system properties $[m]$, $[k]$, $[c]$; algorithm parameters γ and β ; and the $\{\dot{u}_i\}$, $\{\ddot{u}_i\}$ at the beginning of the time step. Having the values of $\left[\hat{k} \right]$ and $\left\{ \Delta \hat{P}_i \right\}$ incremental displacement can be calculated from B.13. Once Δu_i is known, $\Delta \dot{u}_i$ and $\Delta \ddot{u}_i$ is computed from B.11 and B.12.

If the system is nonlinear, the secant stiffness $[(k_i)_{\text{sec}}]$ cannot be obtained before obtaining the displacement increment. This necessitates the use of an iterative method. Modified Newton-Raphson method is used for the solution of the nonlinear system for this study. Figure B.2 shows a schematic plot of Eq. B.13. In the original form of the method tangent stiffness at the current time instant is used as initial stiffness value. This is not possible for the solution of Eq. B.13, because continuous form of the member hysteresis function is not available. To overcome this problem a secant stiffness, for a predefined small displacement increment is calculated (Fig. B.2), and the method applied by using this value.

The solution procedure for a step is:

1. Initialize data

$$u_{i+1}^{(0)} = u_i, \quad f^{(0)}(u) = f_i(u), \quad \Delta R^{(1)} = \Delta \hat{P}_i, \quad (k_{\text{sec}})_i = (k_{\text{pre_sec}})_i$$

2. Calculations for each iteration, $j=1,2,3\ldots$

$$2.1. \text{ Solve } \hat{k} \Delta u^{(j)} = \Delta R^{(j)} \Rightarrow \Delta u^{(j)}$$

$$2.2. u_{i+1}^{(j)} = u_{i+1}^{(j-1)} + \Delta u^{(j)}$$

$$2.3. \Delta f^{(j)} = f^{(j)} - f^{(j-1)} + \left(\hat{k} - k_{\text{pre_sec}} \right) \Delta u^{(j)}$$

$$2.4. \Delta R^{(j+1)} = \Delta R^{(j)} - \Delta f^{(j)}$$

3. Repeat Step 2 till ΔR is less than the convergence criteria.

The parameters of the Newmark method are chosen to be $\gamma = \frac{1}{2}$ and $\beta = \frac{1}{4}$. Chosen values of the parameters resulted average acceleration between the steps and ensures unconditional stability.

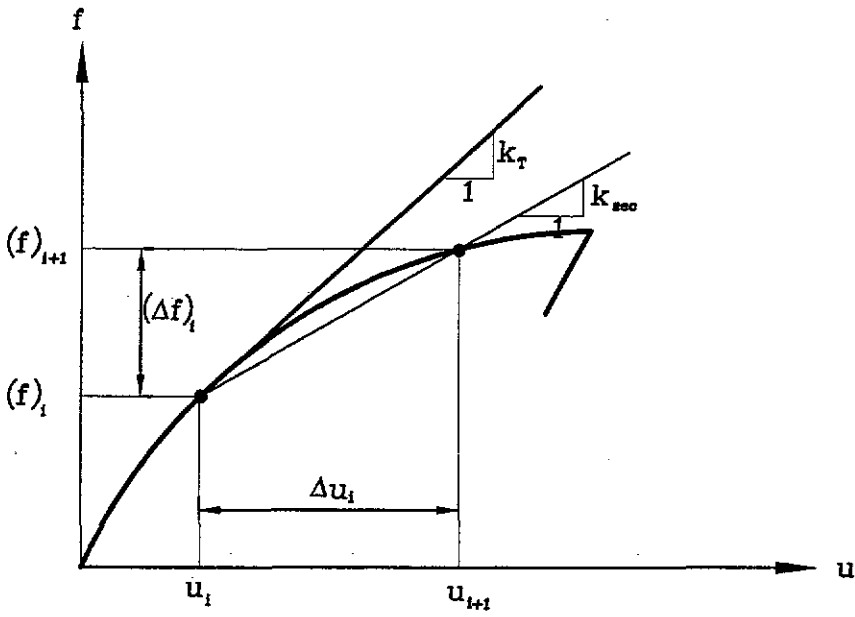


Fig. B.1 Secant Stiffness

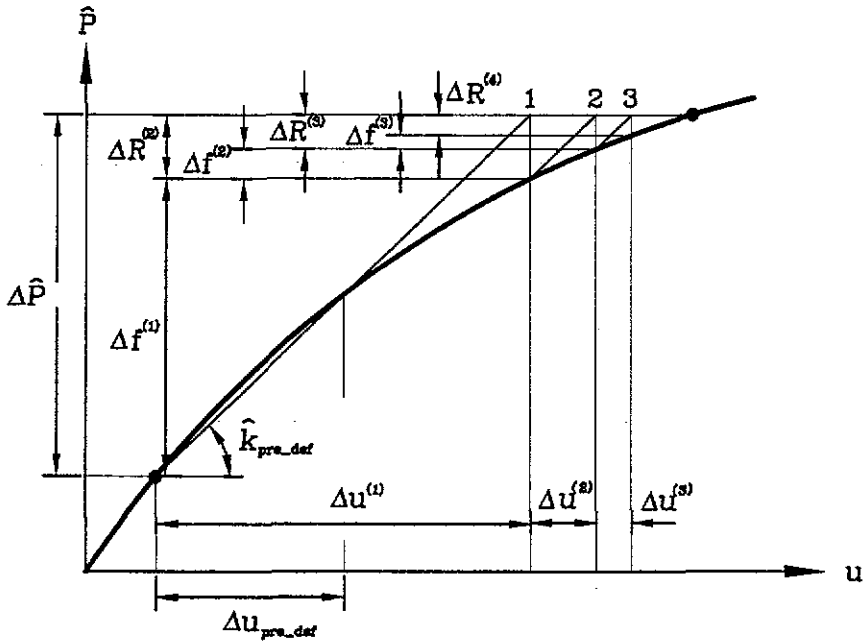


Fig. B.2 The Modified Newton-Raphson Iterations

APPENDIX C
CALCULATIONS OF ROTATIONS FOR THE SPECIMENS TESTED BY
DRAGOVICH (1996)

The specimen rotation at the panel centerline related to the flexural deformation of the beams is calculated by integrating the curvature along the beam with the appropriate boundary conditions (Fig. C.1). The expression for the rotation before yielding is:

$$\theta_f = \frac{\phi L_n^2}{3L_T} \quad (\text{A3.1})$$

and after yielding:

$$\theta_f = \phi_1 \left[\frac{L_d^2 + 2L_u^2}{6L_T} \right] + \phi_2 \left[\frac{3L_d L_u + 2L_d^2}{3L_T} \right] \quad (\text{A3.2})$$

where,

ϕ_1, ϕ_2 = Curvature values as defined in Section 2.6 and Fig. 2.4.

θ_f = Flexural rotation at the panel centerline.

L_n = Clear span length of the beam.

L_T = Total span length of the beam.

L_d = "Yield region" length.

L_u = Not yielded beam length.

Slip of the longitudinal beam reinforcement along its development length in the specimen wall causes a concentration of rotation at the beam wall intersection. The slip

of the reinforcing bar is estimated by assuming a constant development length and a linear decrease of the strain in the bar from the face of the wall, up to end of the defined development length. Rotation because of the bar slip is calculated by using the distance of the bar from the neutral axis (Fig. 2.16), and given by the expression:

$$\Delta_{slip} = \frac{\varepsilon_{bar} * L_b}{2} \quad (A3.3)$$

$$\theta_{slip} = \frac{\Delta_{slip}}{NA} \quad (A3.4)$$

where,

Δ_{slip} = Reinforcing bar slip at the wall face.

ε_{bar} = Strain in the bar.

L_b = Development length of the bar.

= 40 bar diameter for materially uncoupled specimens.

= 50 bar diameter for materially coupled specimens.

NA = Neutral axis depth.

θ_{slip} = Rotation because of the reinforcing bar slip.

Rotation at the panel centerline (Fig. 3.1) related to the shear deformation is calculated, based on a nonlinear, uncracked reinforced concrete member. The corresponding rotation because of the shear deformation, θ_v , is given by:

$$\theta_v = \frac{6 L_n M_o}{5 G A L_T^2} \quad (A3.5)$$

where,

G = Shear modulus of concrete,

$$= \frac{E_c}{2(1+\nu)}$$

ν = Poisson's ratio for concrete.

$$=1/6 \text{ (assumed)}$$

The total rotation at the panel centerline is the sum of the rotations caused by flexure, reinforcing bar slip, and shear.

$$\theta_{tot} = \theta_f + \theta_{slip} + \theta_v \quad (A3.6)$$

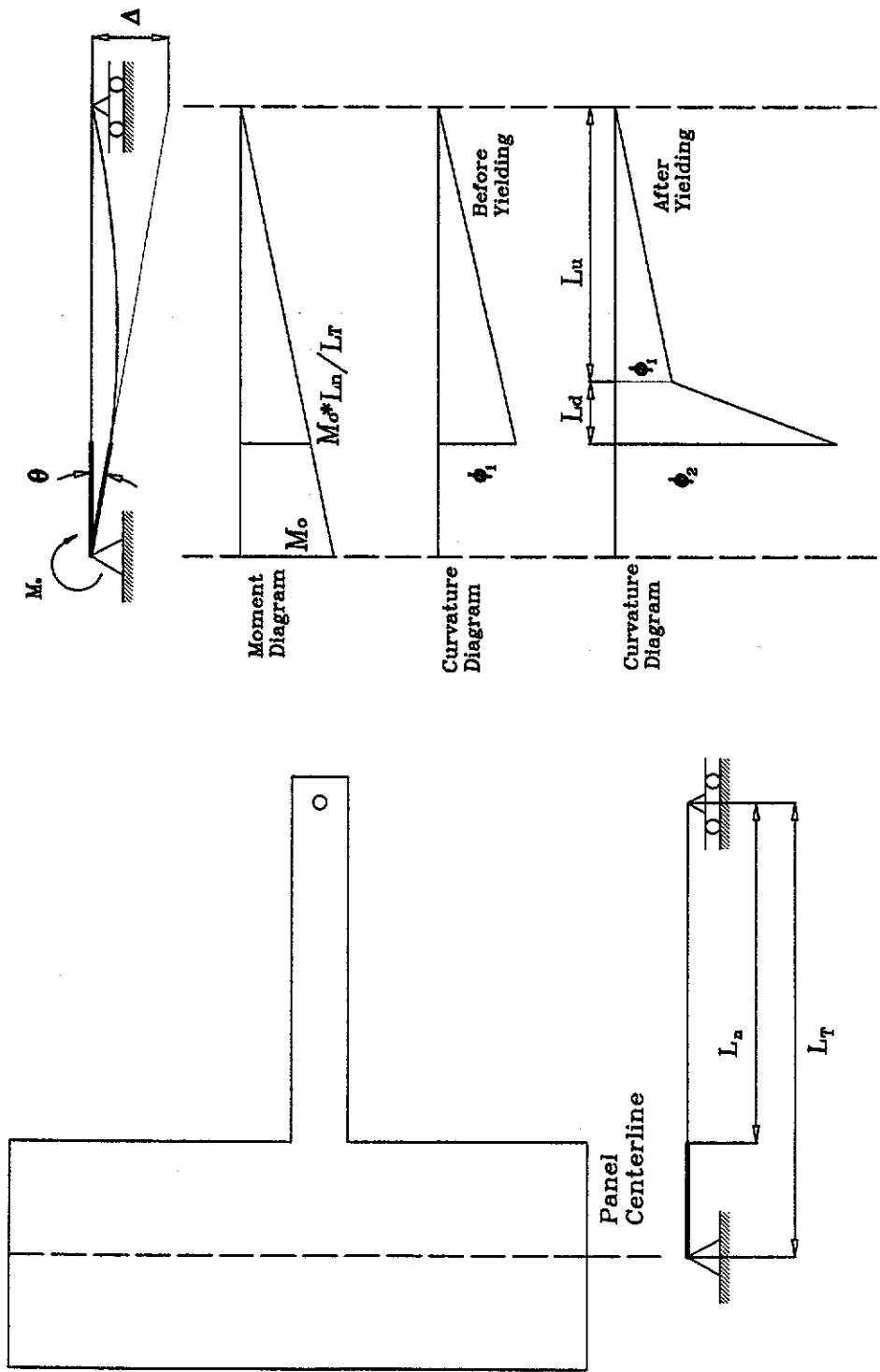


Figure C.1 Definitions for the Calculation of Rotation for the Specimens Tested by Dragovich (1996)

APPENDIX D
FREQUENCY DOMAIN ERROR INDEX

This computational method was developed to produce a number indicating the degree of agreement between two waveforms. The method is based on the frequency characteristics of the signals, and has been proven a consistent method for matching both amplitudes and periodicity between signals (Wood 1988, Dragovich 1996). The idea was originally stated by Lopez (1998). In its initial form method was producing a real number without any bound on it. And did not reflect the phase differences between the two signals. Dragovich (1996) improved the model to have an upper and lower bound and to reflect the phase differences. In its final form, the first step in the method is to compute the Fourier transforms of the both waveforms. The signals are represented as complex numbers in the frequency domain. At each frequency, the real and imaginary components of the complex frequency number can be thought of as coordinates on an x-y graph, where the x-axis represents the real component, and the y-axis imaginary component. Figure D.1 shows this x-y plane representation of real and imaginary components at a frequency f_i , for the signals compared. The method simply involves computing the frequency domain error vector between the two signals, shown as vector "error" in Fig. D.1, and normalizing this vector by the magnitudes of the both signals. So that the "error" varies from 0, for the perfect correlation between signals, to 1 for two signals 180 degrees out of phase. Symbolically,

$$E_f = \frac{\sum_{i=f_1}^{f_2} \sqrt{(R_{Mi} - R_{Ci})^2 + (I_{Mi} - I_{Ci})^2}}{\sum_{i=f_1}^{f_2} [\sqrt{R_{Mi}^2 + I_{Mi}^2} + \sqrt{R_{Ci}^2 + I_{Ci}^2}]} \quad (D.1)$$

where,

E_f = Frequency domain error, ($0 \leq E_f \leq 1$)

R_{Mi} = Real component of the first signal at frequency i .

I_{Mi} = Imaginary component of the first signal at frequency i .

R_{Ci} = Real component of the second signal at frequency i .

I_{Ci} = Imaginary component of the second signal at frequency i .

f_1 = Starting frequency for summation.

f_2 = Ending frequency for summation.

The starting and ending frequencies for the summation depends on the frequency content of the waveforms. The user has to choose the frequency interval that contains most of the energy of the waves. For this study, $f_1=0$ Hz, and $f_2=25$ Hz, based on the frequency characteristic of the measured and the calculated rotation signals.

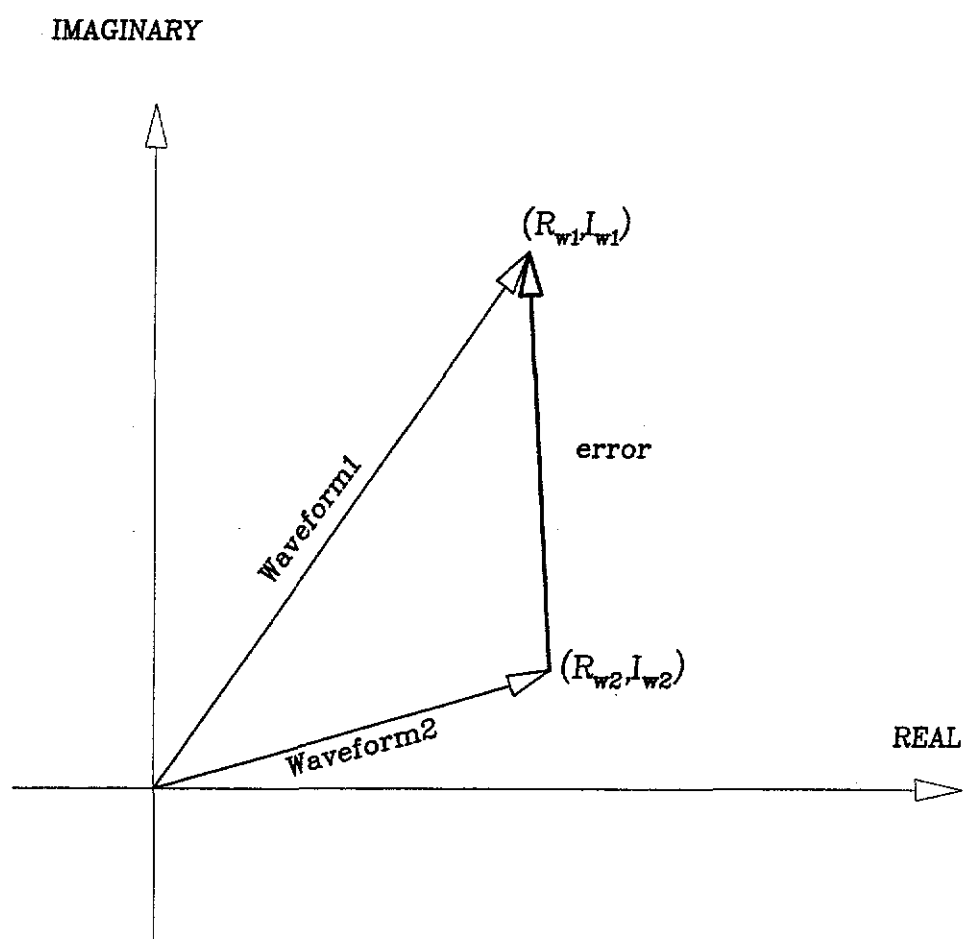


Fig. D.1 Frequency Domain Error Representation

VITA

VITA

Cemalettin Dönmez was born 27 February 1971 in Ankara, Turkey. He received a B.S. in Civil Engineering from Middle East Technical University (METU, Ankara, Turkey) in June 1992.

He started his M.S. degree in October 1992 in METU and received his degree in September 1994. His research included an experimental study of precast concrete beam-column connections under cyclic loading. During this period, he also worked as a Research Assistant in METU.

He received the scholarship of the Higher Education Council (YÖK) of Turkey in 1994. He started his Ph.D in Civil Engineering in September 1994.