



SEMICONDUCTOR BASED NANO-PHOTONIC STRUCTURES FOR INTEGRATED QUANTUM PHOTONICS

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Dedicated to
my parents & extended family

Abstract

This thesis presents experimental, theoretical and simulation based studies on nano-photonic structures for the newly emerging integrated quantum photonics technology.

Integrated quantum photonics aims to utilize the photons to implement the concepts of quantum mechanics that offer revolutionary new methods in the fields such as computation, communication, metrology and simulation on miniature scale optical devices. We begin with discussing benefits of quantum information, quantum photonics and integrated quantum photonics in more detail in the first chapter.

We demonstrate non-classical interference in a gallium nitride directional coupler which is the first demonstration in this material system, and will be discussed in the second chapter. Gallium nitride is important due to the fact that it opens a new spectral range for integrated quantum photonics, namely the blue and ultra-violet.

Another contribution of this study is the enhancement in the pair generation efficiency of silicon nano-wire based single photon sources, and this will be covered in the fourth chapter. Integrated single photon sources are an essential part of integrated quantum photonics. Advancements in this field require single photon sources to be scaled down to fit in a photonics chip. When miniaturization is of concern silicon is the first thing that comes into our minds. While its high refractive

index allows tight confinement of light and efficient guiding of light around sharp corners, its Kerr non-linearity enables generation of photon pairs through spontaneous four-wave mixing which recently attracted the attention of scientists. Here we study the pair generation in a straight waveguide and a ring resonator. We investigate the enhancement provided by the presence of the ring. Furthermore we address the well known detrimental effects of free carriers in silicon, which are generated by the high intensity pump beam, by reverse biasing the PN junction across the ring and show an enhancement in pair generation.

In the second part of the thesis we focus on photonic crystal structures which are attractive due to the size reduction they offer and can have potential benefits for integrated quantum photonics applications. In Chapter 5, we present the theory behind photonic crystals and describe methods used in simulating photonics crystals. Then we provide examples from the literature which can be found in Chapter 6. In Chapter 7 we show our simulation results where we present a systematic design of a miniature size photonic crystal directional coupler and also study its tuning capabilities.

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Author's declaration

Unless otherwise acknowledged, the content of this thesis is the original and sole work of the author. No portion of the work in this thesis has been submitted by the author in support of an application for any other degree or qualification, at this or any other university or institute of learning. The views expressed in this thesis are those of the author, and not necessarily those of the University of Bristol.

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List of Abbreviations

ABC	absorbing boundary conditions
APD	avalanche photo diode
BBO	beta barium borate
CAR	coincidence to accidental ratio
CARS	Coherent Anti-Stokes Raman Spectroscopy
CNOT	controlled-NOT
CMOS	complementary metal-oxide-semiconductor
CSRS	Coherent Stokes Raman Spectroscopy
DUT	device under test
DWDM	dense wavelength division multiplexer
EDFA	Erbium doped fiber amplifier
EFC	equifrequency contour
FBG	fiber Bragg grating
FCA	free-carrier absorption
FCD	free-carrier dispersion
FDTD	finite difference time domain
FSR	free spectral range
FWHM	full width at half maximum
FWM	four-wave mixing
HBT	Hanbury-Brown and Twiss
HOM	Hong-Ou-Mandel
ICP	inductively coupled plasma
KDP	Potassium dideuterium phosphate

KLM	Knill, Lafamme and Milburn
MEEP	MIT Electromagnetic Equation Propagation
MIT	Massachusetts Institute of Technology
MMI	multimode interference
MPB	MIT Photonic Bands
NV	nitrogen vacancy
OAM	orbital angular momentum
PCF	photonic crystal fiber
PhC	photonics crystal
PMF	polarization maintaining fiber
PML	perfectly matched layer
PPLN	periodically poled lithium niobate
QD	Quantum dots
Q-factor	quality factor
RIKES	Raman Induced Kerr Effect Spectroscopy
sFWM	spontaneous four-wave mixing
SMF	single mode fiber
SnW	Silicon nano-wires
SOI	silicon-on-insulator
SPDC	spontaneous parametric down conversion
SPM	self phase modulation
SPS	single photon source
SRGS	Stimulated Raman Gain Spectroscopy
SSPD	superconducting single photon detector
TE	transverse-electric
TIA	time interval analyzer
TIR	total internal reflection
TIRES	The Inverse Raman Effect Spectroscopy
TM	transverse-magnetic
TOE	cross-phase modulation
TPA	two-photon absorption
UV	ultra-violet
XPM	cross-phase modulation

List of Symbols

$\tilde{\alpha}^\dagger$	photon creation operator for photon in mode a
r	reflectivity
t	amplitude transmission coefficient
$ 1\rangle$	single photon state
CC_{min}	minimum coincidence count
CC_{max}	maximum coincidence count
Δ	relative delay between photons
σ^2	variance
V	visibility
L_π	the length at which the light propagating in one port of the directional coupler, couples completely to the other port
L_{3dB}	synonym for L_π
β_e	propagation constant of the even mode of a directional
β_o	propagation constant of the odd mode of a directional
C_i	count of single photons detected by channel i
W	width of detection window
$g^{(2)}$	second order correlation function
$\chi^{(n)}$	n^{th} order component of electrical susceptibility
ω	angular frequency
k	propagation constant
γ	effective non-linearity of the waveguide
A_{eff}	effective mode area
n_0	linear refractive index

LIST OF SYMBOLS

n_2	<i>second order non-linear refractive index coefficient</i>
I	<i>intensity of light</i>
Δk	<i>phase mismatch</i>
Δk_L	<i>linear phase mismatch</i>
D	<i>group velocity dispersion</i>
λ	<i>wavelength</i>
E_g	<i>band gap energy</i>
$\tilde{\beta}$	<i>two-photon absorption coefficient</i>
j	<i>imaginary number</i>
μ	<i>magnetic permeability</i>
ε	<i>electric permittivity</i>
σ	<i>electrical conductivity</i>
σ^*	<i>equivalent magnetic loss</i>
μ_0	<i>magnetic permeability of vacuum</i>
ε_0	<i>electrical permittivity of vacuum</i>
α	<i>attenuation constant</i>
τ	<i>effective free-carrier life time</i>
$\hbar$	<i>reduced Plank's constant</i>
σ_r	<i>coefficient relating the free carrier absorption to the free carrier density.</i>
σ_n	<i>coefficient relating the change of index to the free carrier density</i>
κ	<i>thermo-optic coefficient</i>
ν	<i>thermal dissipation time</i>
$\tilde{C}$	<i>thermal capacity</i>
ρ	<i>density</i>
L_{eff}	<i>effective length</i>
ϕ	<i>phase of the wave</i>
ACC	<i>accidental counts</i>
CC	<i>coincidence counts</i>
CAR	<i>coincidence to accidental ratio</i>
$errCC$	<i>error on coincidence counts</i>
$erACC$	<i>error on accidental counts</i>
$errCAR$	<i>error on coincidence to accidental ratio</i>
dc_i	<i>dark counts of detector channel i</i>
η_i	<i>collection efficiency of channel i</i>

α_{sp}	<i>coefficient of linear noise in the single photon counts with respect to power injected to the device</i>
α_{cc}	<i>coefficient of linear noise in the coincidence counts with respect to power injected to the device</i>
P_{in}	<i>power injected to the device</i>
β_p	<i>pair generation rate</i>
$\tilde{P}_{av}$	<i>average pump power</i>
P_{peak}	<i>peak pump power</i>
t_p	<i>duration of the pulse</i>
T	<i>period of the pulse train</i>
P_{mon}	<i>monitor power</i>
SNR	<i>signal to noise ratio</i>
α_t	<i>the ratio of the power inside the chip to the measured monitor power</i>
$\check{\beta}_p$	<i>the pair generation rate in the ring multiplied by the probability of the generated pairs escaping the ring</i>
P_{ring}	<i>power inside the ring</i>
δ	<i>amplitude transmission factor in one cycle of field inside the ring</i>
Γ_{ring}	<i>ratio of the power inside the ring to the injected power to the device</i>
$\vec{a}_1, \vec{a}_2$	<i>basis vectors of a two dimensional lattice</i>
$\vec{R}$	<i>lattice position vector</i>
$\vec{g}_1, \vec{g}_2$	<i>basis vectors of reciprocal lattice</i>
$\mathbf{H}(\vec{r})$	<i>magnetic field distribution in space in phasor notation</i>
$\mathbf{E}(\vec{r})$	<i>electric field distribution in space in phasor notation</i>
$\varepsilon(\vec{r})$	<i>dielectric distribution in space</i>
$u_k(\vec{r})$	<i>Bloch function</i>
k_B	<i>wave vector in irreducible Brillouin zone</i>
H_i	<i>magnetic field component in direction i</i>
E_i	<i>electric field component in direction i</i>
f	<i>frequency</i>
σ_l	<i>parameter related to the sharpness of Lorentzian curve</i>
Q	<i>quality factor</i>
f_0	<i>center frequency of resonant mode</i>
a	<i>lattice constant of photonic crystal</i>
r_{31}	<i>electro-optic coefficient</i>

LIST OF SYMBOLS

$Q_{ }$	<i>in-plane quality factor</i>
$Q_{\perp}$	<i>quality factor related to vertical confinement</i>
Q_{Total}	<i>total (observed) quality factor</i>
λ_{eff}	<i>effective wavelength</i>

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- [2]. E. Engin, J. L. O'Brien, and M. J. Cryan, "Tunable Photonic Crystal Directional Coupler in Gallium Nitride-on-Sapphire," in *10th International Symposium on Photonic and Electromagnetic Crystal Structures (PECS-X)*, Santa Fe, New Mexico, June 3-8, 2012.
- [3]. E. Engin, N. H. Hueting, J. L. O'Brien, and M. J. Cryan, "Effect of Layer Thickness in Quality Factors of GaN Photonic Crystal Cavities," in *10th International Symposium on Photonic and Electromagnetic Crystal Structures (PECS-X)*, Santa Fe, New Mexico, June 3-8, 2012.
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Chapter 1

Introduction

This chapter will initially provide an overview of quantum information and integrated quantum photonics as the motivation of the studies covered in this thesis. We will then introduce the single photon generation in silicon nano-wire waveguides and its benefits in integrated quantum photonics. And lastly we will mention about the use of photonic crystals for miniaturization of photonic circuits and introduce the work we have performed on photonic crystal structures in gallium nitride.

1.1 Quantum Information

Information is generally defined as an event affecting the state of a dynamic system. More specifically, information is an entity that contains a knowledge or intelligence. It was the advancements in electronics and optics in efficient transmission and processing of information behind the rapid developments in technology in the last few decades. Now people are in search for new methods for harnessing the laws of nature to handle information in a more beneficial way.

Quantum information involves processing and transmission of quantum states. Properties of quantum states which make them attractive for information processing are super-position, interference, entanglement and non-clonability [1] which provide unique advantages over classical information processing. Quantum information provides solutions to some problems which are practically impossible to solve with classical algorithms due the complexity of the problem growing exponentially with the size of the input. A well known example of this is the Shor's algorithm [2] which provides exponential speed up in factorizing a number as compared to the fastest classical method known so far [3]. Grover's search algorithm [4] for searching in an unsorted database, quantum simulation [5],[6] for simulation quantum systems with others in a more controlled manner are examples where quantum information provides unique advantages. Secure communication between two parties where the security is guaranteed by the laws of quantum mechanics [7], quantum metrology which beats the limits of resolution offered by classical measurement methods [8] are other fields where quantum information is superior over the classical.

There are different approaches aiming to harness the quantum mechanical properties of particles such as trapped ions systems [9], nuclear magnetic resonance based

systems [10] and quantum photonics [11]. A review and comparison of different approaches are provided in [12]. While building a “quantum computer” is one of the common ultimate goals, quantum photonics approach is what we will consider in the scope of this thesis.

1.1.1 Quantum Photonics

In quantum photonics photons are used as the carrier of information. The quantum states can be encoded in the degrees of freedom of the photon such as polarization [13], time bin [14], orbital angular momentum (OAM) [15]...etc. Photons are well isolated from the environment and they have excellent coherence time. Photons are easy to manipulate and they can carry information over long distances. These are what make quantum photonics an attractive and promising field of research. The stages of quantum information processing is the generation of single photons with single photon sources, manipulation of photons in the photonic circuit and detection of photons by single photon detectors.

Details about single photon sources are provided in chapter 2 however we are going to mention about the key concepts here. In quantum photonics applications more than a single photon is required and the photons need to be indistinguishable. Indistinguishable photons have exactly the same frequency, spatio-temporal mode and spectrum. A perfect interference only occurs between indistinguishable photons and the realization of many quantum algorithms is based on photon interference. The ideal single photon source produces a “single” photon with exactly same spatial and temporal characteristics each time upon request, with zero jitter. Various techniques are available for single photon generation such as trapped particles (neutral atoms, ions, molecules), quantum dots, defects in diamond, attenuated laser beams,

spontaneous parametric down conversion (SPDC) and four-wave mixing (FWM). The wavelengths of operation vary among different technologies. While the first three of these can be classified as deterministic sources, the latter three are classified as probabilistic sources [16]. Deterministic single photon sources work based on the excitation of a single quantum emitter, which can only emit a single photon upon returning back to its initial state. So they are physically restricted to generate a single photon at a time. On the other hand the probabilistic sources have a finite probability of generating multiple photons at a time. Indeed the probability distributions on generating one, two, three... photons at a time is a Poissonian distribution. Many applications such as the KLM scheme is based on the interference of two “single” photons. Bunched photons result in unsuccessful operations. Also in quantum communications multiple pairs makes the system open to eavesdropper attacks degrading the security [17]. Hence for reliable operation these sources have to be operated at a very low power so that the multi-photon generation probability is reasonably low. Today SPDC sources are the most common type of sources because they are simply implemented, cost effective and also enable generation of two photon entangled states (a property which is required to realize some algorithms).

The second stage of quantum information processing is the photonic circuits. Typical operations on photons are filtering, polarization rotation, splitting and phase shifting which are performed by optical components such as half-wave plates and beam splitters. An ideal quantum photonic circuit should not induce loss on the photons and performs the desired operations on the photons precisely. Consider a circuit where the transmission from the input to the output is measured as 70% ($\sim$ 1.5dB) by classical means. That means that when we send 10 single photons to this circuit on the average the circuit will work successfully only 7 times. When two of

these circuits are cascaded the overall probability of successful operation is the square of the individual success probability and becomes $\sim 49\%$. The success probability decreases exponentially as more of these circuits are cascaded.

One of the important operations is the interference of two photons which many quantum algorithm relies on. In order for two photons to interfere perfectly they need to be indistinguishable. A basic demonstration of quantum interference is the Hong-Ou-Mandel (HOM) interference experiment [18] where the photons are interfered in a beam splitter. We will explain quantum interference in directional couplers in more details in chapter 3 where we present quantum interference in a gallium nitride directional coupler [19].

A fundamental component to perform universal quantum computing is the controlled-NOT (CNOT) gate. A CNOT gate has two inputs called the control and the target, where the sign of the target bit is flipped with the control input. The sign of the photon can be encoded in one of its degrees of freedom. One of the challenges of this is the requirement of strong non-linear interactions between the photons such that for example the phase of the target photon can be changed by an amount of π with the control photon. To overcome this challenge, in 2001 Knill, Lafamme and Milburn [11] proposed a scheme, the KLM scheme, which enabled the realization of the CNOT gate by using only linear optical elements in addition to sources and detectors. The circuit is composed of beam splitters and the π phase shift on the target photon is induced by the interference of the control photon and the target photon in one of the beam splitters. However not all operations of this gate are successful hence is called a non-deterministic (or probabilistic) gate. Nevertheless the successful operations are signalled by the detector clicks. The original scheme has a probability of success of $1/16$. A non-deterministic operation of the gate is a major issue since as the non-

deterministic gates are cascaded one after the other the overall probability of successful operation of the device drops down exponentially. The KLM scheme was a mile stone in the field of quantum optics however the original implementation is not scalable (meaning that the required resources do not grow exponentially with the complexity of the task to be performed) due to the probabilistic operations. However with the modifications such as the use of “teleportation”, more deterministic operations and scalability is made possible [20], [21].

Detection of single photons is yet another challenge in quantum photonics. The availability of efficient detectors is an important factor in choosing the wavelength regime of operation of the whole application. An ideal detectors has unity efficiency which means it signals every time a photon is incident on it. However this is not the case in practice. Silicon avalanche photo diodes (APD) that are able to detect photons in the visible range, at room temperature around 70% efficiencies [22] are commonly used. In the telecom wavelengths besides APDs, superconducting single photon detectors are shown to be working around 60% [23], 95% [24] efficiencies. The superconducting detectors need to be operated at cryogenic temperatures therefore the resource overhead is high. While most of the single photon detectors flag no-photons or more-than-one-photon, detectors that can count the number of photons detected are also being investigated and promising results are shown [24],[25],[26]. Besides the detection efficiency, dead-time, jitter, dark count rate are other important performance parameters of a detector. Dead-time is the minimum required time for successive detections, jitter is the uncertainty of the delay between when the photon is incident on the detector and when the detector flags the detection and dark count rate is the count per unit time of the detector signals when there is no light incident on it. The scalability of quantum circuits imposes stringent requirements on the efficiencies of

detectors. For example 70% efficiency of a detector means, if two detectors were to be operated for detecting coincident arrival of photons (which is a common configuration in quantum photonics), then they would only be able to detect half of the coincidence events that occurred. And as the number of outputs of the circuit increase the probability of successful operation drops exponentially.

Quantum photonics faces the challenge of realizing sources, circuits and detectors as close as possible to ideal ones so to increase the probability of successful operations. The speed of processing which is not yet a great concern among the others at the current state can be identified as a long term challenge. An important challenge which we will focus on is increasing the complexity of the quantum photonic circuits which will enable to transform quantum photonics applications from proof of principle demonstrations to real life applications. To increase the complexity the components have to be scaled down and implemented on chip which is the topic of the next section. We are going to introduce integrated quantum photonics and discuss the advantages and disadvantages of it as comparison to non-integrated approach.

1.1.2 Integrated Quantum Photonics

The aim of integrated quantum photonics is to generate single photons and quantum states of single photons, manipulate and detect these states in “miniature” sized photonic chips. To put it into context we also need to introduce non-integrated (or bulk) optics approach. The bulk optical circuits are composed of components such as free-space polarizers, beam splitters, mirrors each being on the size of centimetres require a large space on the optical table, the overall circuit sizes sometimes being on the order of meters. These bulk circuits need to be well isolated from the environment to prevent vibration, particles in air or undesired illumination from the environment

which can only be satisfied inside isolated labs on vibration free optical tables. Even so the alignment of components degrades over time and they need to be realigned quite often which becomes a problem for circuits including many components. As a summary the complexity of the bulk optical circuits are limited by the space and stability requirements. Figure 1.1 shows show an example circuit to perform CNOT operation implemented using bulk optical components which cover an area of approximately 1m^2 .

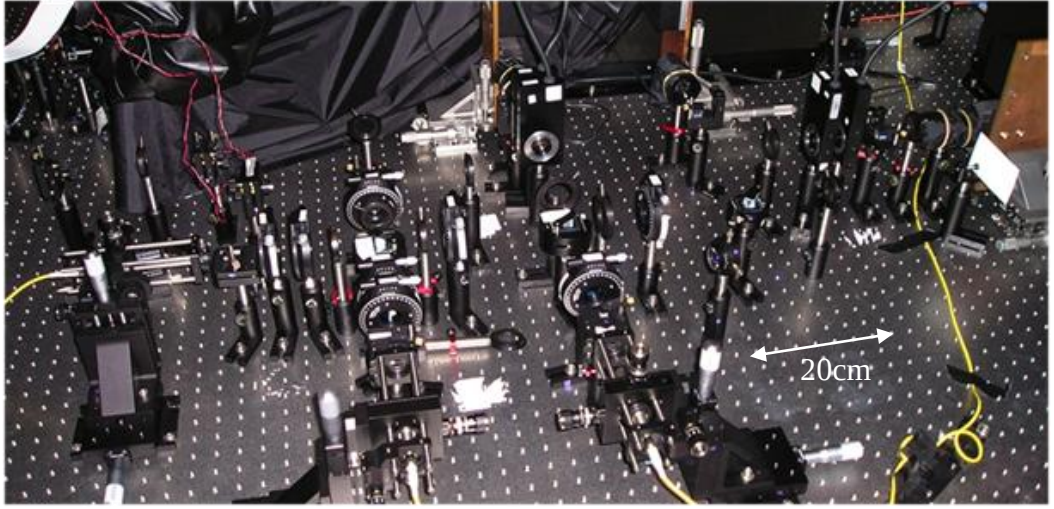


Figure 1.1: (a) Photo of a CNOT gate optical circuit implemented with bulk optical components. The circuit is fed by single photons carried by fibers generated by a separate source. The area of the circuit on the optics table is around 1m^2 . [Photo taken at University of Bristol labs]

Integrated quantum photonics aims to implement single photon sources, photonic circuits and single photon detectors all inside a single chip. In the integrated approach the photons are guided and manipulated in waveguides or structures fabricated usually in semiconductor materials. Splitting the light, filtering, polarization rotation, phase shift or modulation can be performed on-chip. As opposed to using mirrors or lenses in bulk optics the light can be directed by the bends in the waveguides. To have a better idea, pictures of quantum photonics chips realized in different material platforms are shown in Figure 1.2(a)-(c). Silica-on-silicon circuit shown in Figure 1.2

(a) implements Shor's algorithm to factorize the number 15. The waveguides are realized by direct laser writing technique and have a cross sectional area of $3.5\mu\text{m} \times 3.5\mu\text{m}$. This technique increases the refractive index of the laser written region slightly compared to the unperturbed regions which enable the guiding of light. Because of small index contrast the light is weakly confined hence the mode size is large. This is good for coupling the light from a fiber to the waveguide due to the good overlap between the two modes. However because the light is weakly confined the radii of the bends have to be large so that the light does not leak causing loss. The size of the chip is around 2 cm and the length is mainly due to these large radii bends which are required to couple the light to the chip using fiber arrays with standard spacing. Figure 1.2 (b) shows a quantum walk circuit with 21 parallel waveguides fabricated on silicon oxynitride material [27] which has an index contrast 9 times larger than that of silica allowing the bend radii to be about 25 times smaller. As a result the size of the chip is reduced down to few millimetres. Quantum walk is analogous to random walk in classical physics where a particle makes successive and random choices of whether to stay on the position it is occupying or to move to one of the adjacent positions. In case of quantum random walks of photons, the photon (or multiple photons) is injected to an array of waveguides parallel to each other. The photon has a probability of coupling to the adjacent waveguides (the probability determined by structural parameters such as the distance between the waveguides) forms quantum states. Quantum walks finds applications in simulating physical, chemical, biological phenomena and also in quantum computation [Quantum computation and decision trees]. Figure 1.2 (c) shows again a quantum walk circuit this time realized with around 300 individual multimode interference (MMI) couplers on silicon material. All these 300 MMIs are packed in a $350\mu\text{m} \times 100\mu\text{m}$ region in the

center and the remaining are fan-outs and spot size converters which transform the mode from the input fiber lenses which has $\sim 2\mu\text{m}$ in diameter, to the mode of the silicon nano wires of 450nm width ($\sim 2\text{dB}$ loss in mode conversion).

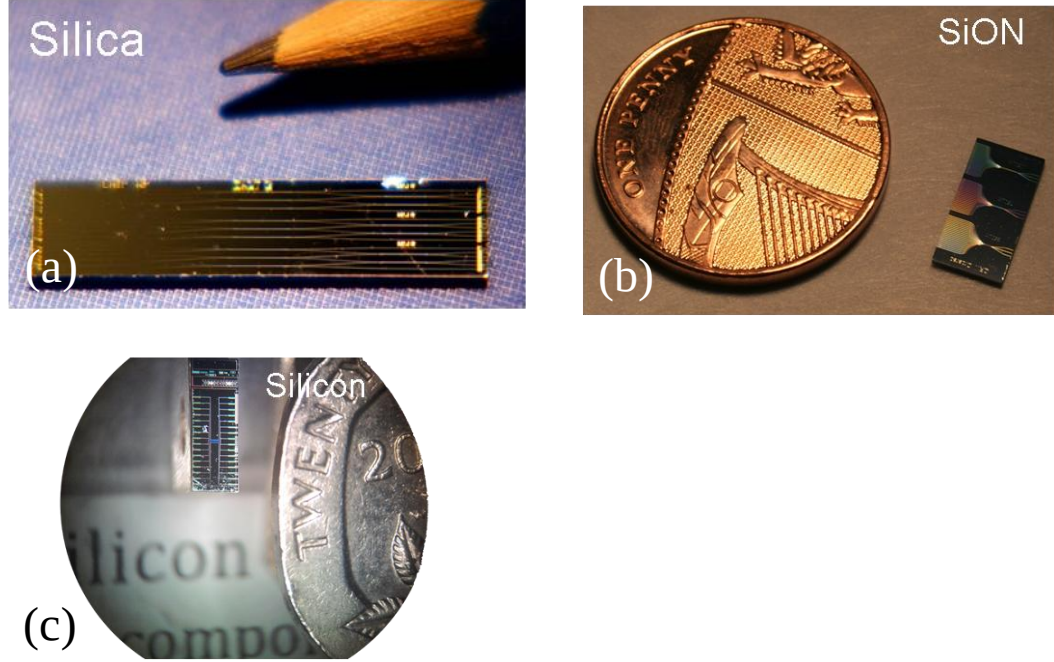


Figure 1.2: Examples of integrated quantum circuits: (a) silica-on-silicon chip for implementing Shor's algorithm [29], (b) quantum walk circuit on silicon oxynitride[26] and (c) quantum walk circuit on silicon on insulator material system realized by MMIs. (d) Illustration of a fully integrated quantum chip where sources, circuits, detectors and control electronics are implemented all in one chip.

Besides size reduction integrated circuits are more robust to the environmental effects, they do not suffer from misalignment and can be isolated with appropriate packaging technologies. This allows the ability to realize more complex circuits using integrated approach. On the other hand loss is a major obstacle in the integrated approach while the loss of bulk optical components is generally low such that mirrors, beam splitters, quarter wave plates with more than 90% reflectivities or transmissions are commercially available. Besides propagation losses of light in free space is much lower than in waveguides. The source of loss can be scattering, absorption or radiation.

Scattering is mainly due to the imperfections in the crystal structure, roughness of the surfaces due to the defects in fabrication. Absorption is due to the material properties such as the electronic band structure, the non-linearity of the material which converts the optical mode energy to other forms of energy such as heat. And radiation is when the light is not confined within the waveguide and radiates away from it such as when the waveguide is bent. While some of these losses can be reduced by proper design and improved fabrication technologies enabling better tolerances some of them are intrinsic to the material and cannot be avoided. Loss results on increased probability of error and there exists certain thresholds of error probability for realizing scalable quantum computing [28]. Therefore optical materials which are low loss but also enable size reduction is always an interest in quantum photonics as well as classical.

The first demonstration of integrated quantum photonics was performed by Politi et al. [29] in 2006 using silica-on-silicon waveguides where two photon interference with $\sim 95\%$ visibility (a measure of how well the photons interfere with each other and will be discussed in Chapter 3) and also a CNOT gate with a fidelity (a measure of how close the measured quantum states of the device is to the ones desired) of $\sim 94\%$ was demonstrated. Since then there has been a lot of progress in the field of integrated quantum photonics such as the on-chip implementation of Shor's algorithm [30], quantum walks [27], [31], gates [32], manipulation of entanglement [33],[34], quantum state preparation and tomography [35], heralded entanglement generation [36].

Early realizations of integrated quantum circuits was on silica-on-silicon material platform which offers very low loss (0.1dB/cm) and good coupling efficiency from a fiber to the waveguide. However the drawback is the small index contrast of the waveguide meaning weak confinement of light which requires the bend radii of the

waveguides to be large (on the order of centimetres) to prevent losses. Using higher index contrast material platforms such as silicon oxynitride [27] or silicon [34] the chips could be further scaled down.

Towards full integration of quantum photonic circuits it is required that the sources are also implemented on chip in a scalable manner. The output photons have to be coupled efficiently to single moded waveguides and fed to quantum circuits. Towards this aim we see examples of and photon generation by quantum dots [37], [38], NV centres in diamond placed inside waveguide coupled cavities [39] and SPDC in non-linear waveguides [40], [41]. In the scope of this thesis we will focus on FWM in silicon nano-wire waveguides [42], [43]. We will present the particular advantages and disadvantages of the silicon platform in the next section.

And finally there are studies on realizing on-chip single photon detectors that are suitable for integration with waveguide based sources and circuits fabricated on GaAs [44], silica [45] and silicon [46]. The working principle is based on the absorption of the photon propagating inside the waveguide by a nano-wire deposited on top of the waveguide. These detectors are working in cryogenic temperatures which means increased resources to perform cooling. An efficiency of 20% was reported in [44] and 7.2% in [45] and 91% in [46]. The silicon waveguide based detector in [46] is particularly interesting for us since it has a potential of being integrated with the single photon source that we study here.

1.1.3 Silicon Quantum Photonics and Four Wave Mixing in Silicon

Silicon is the material behind the revolution in digital communication and

computation that the world has experienced in the last few decades. Silicon fabrication technology has been developed over the years enabling devices having resolutions in nano meter scale to be realized. Many optical components for silicon photonics have been developed for telecommunication applications [47]. Its large refractive index enables tight confinement of light in waveguides with very small cross sections and efficient guiding around curves with very small radii. Additionally silicon photonics components can be easily integrated with CMOS electronics which has already been performed for telecommunication applications in classical photonics [48]. These make silicon an attractive material for photonics applications.

Silicon is available in the form of silicon-on-insulator (SOI) wafers. The transparency window of silicon is $\sim 1.1\mu\text{m} - 4.5\mu\text{m}$ and has a refractive index of ~ 3.45 around $1.55\mu\text{m}$ wavelength. The lower cladding being silica which has a low refractive index of 1.45 provides a high refractive index contrast between the core and the cladding. As a result the light is well confined inside silicon nano-wire waveguides. The light can be bend with a negligible loss around curvatures having a radii of on the order of microns [49]. This enables realization of very compact devices and is essential for large scale integration. Two photon interference and entanglement manipulation have already been demonstrated in silicon waveguide circuits [34]. MMI couplers having a footprint of $3 \times 27\mu\text{m}^2$ were used instead of directional couplers for splitting the light.

For changing the phase of light changes in the optical properties of the medium should be induced by external effects such as electrical, optical or thermal. Due to having a centrosymmetric crystal structure the Pockel's effect (which enables change in refractive index by applied electric field) is weak [50]. There are methods where the refractive index of silicon is modified by changing the density of free carriers

(electron-hole pairs) such as in free carrier injection modulation [51] however this not desired in quantum photonics since free carriers increase the losses. Thermo-optical tuning where the effective mode index is changed with applying heat locally to the waveguide by the resistive heaters deposited on top is left as the only alternative for quantum photonics applications.

$\chi^{(3)}$ non-linearity in silicon makes generation of photon pairs through four-wave mixing possible. In FWM two photons of the pump laser are converted into a pair of photons at different wavelengths under the energy conservation constraint. Using silicon nano-wires as single photon sources is relatively a new concept [42], [43], [52–57]. The high $\chi^{(3)}$ of the material is efficiently utilized by the small cross section of the waveguide enabling high optical intensities to be reached. The phase matching condition required for efficient operation is satisfied by the appropriate design of the waveguide cross section.

Another technique to generate single photon sources in the infrared regime is the SPDC in periodically poled lithium niobate (PPLN) waveguides [41], [58–61]. However, the sizes of PPLN waveguides are on the order of centimetres. Additionally the phase-matching is satisfied by periodic poling which causes a timing jitter which degrades the indistinguishability among independent sources [43].

The main problem in silicon when dealing with high powers (such as in FWM) is the two-photon absorption (TPA) and free-carrier related effects triggered by TPA [62] namely the free-carrier absorption (FCA) and free-carrier dispersion (FCD). These increase the loss of the medium limiting the power and also change dispersion properties of the waveguide, hence effect phase matching.

In this study besides simple waveguides we have also investigated photon pair generation in a silicon nano-wire ring resonator. The ring resonators enable the power

to build up inside the cavity; therefore the intensity of the input light is multiplied. This is beneficial since the pair generation rate in theory increases quadratically with the intensity. The resonances of a ring cavity overlap with the pump and the generated photons. Therefore the generated photons are also allowed to resonate inside the ring and they are spectrally shaped by the device. Another advantage of the ring resonator is that helps to filter out some of the pump photons although further filtering is necessary to sift the single photons from the pump at the output of the device.

1.2 Gallium Nitride Photonic Crystal Structures

Gallium nitride (GaN) is a III-V semiconductor that has found a wide range of applications in the recent years especially in light generation in the blue and green wavelength regime. Its direct band gap makes it an efficient light generator. The difficulties in fabrication was the main obstacle preventing its usage for light generation before early 90s [63]. Since then GaN is widely used in bright LEDs however its usage in integrated optics is relatively new. It has a wide range of transparency window from 365nm to 13 μ m [64].

Photonics crystals (PhC) are structures having periodic dielectric constant where the period is on the order of the wavelength of light. PhC structures exhibit band gaps for photons, analogous to the semiconductors band gaps for electrons, where light at certain wavelengths are not allowed to exist inside the PhC structure. This way the light can be trapped in PhC cavities or guided in PhC waveguides allowing various kinds of manipulation of light possible [65]. The distinguishing properties of PhCs are the tight confinement of light in small volumes [66],[67], thereby achieving high quality factors, guiding of light around sharp corners and the ability to generate so

called ‘slow light’ where the light propagating has a large group index [68]. PhCs offer the realization of optical components such as splitters, modulators, filters...etc in a much more compact way compared to conventional counterparts. This is what makes them attractive for integrated quantum photonics. Increased light material interaction can lead to utilizing the material non-linearities more efficiently to generate photon pairs or enhance absorption of photons which has potential benefits in photon detection. Slowing down the light increases the control over the phase of the photon which has key role in realization of quantum algorithms using photons while reduced component size is important in increasing of the complexity of circuits. Therefore PhCs have many potential benefits for integrated quantum photonics as well as classical photonics.

Fabrication of PhCs is a major challenge in utilizing the benefits they provide. Due to the fabrication process tolerances the side walls of the holes cannot always be perfectly perpendicular to the surface. This results in scattering of light away from the structure causing loss. In order to achieve good vertical confinement membrane type PhC structures are preferred where the holes are patterned on a slab of material suspended in air [69]. This required under etching techniques, another difficulty in fabrication, and might not be doable for every material. However it is easier to achieve good side wall verticality in suspended membranes as compared to ones having a substrate.

The variations in the diameters of the holes disturb the perfect periodicity of the structure and degrade the confinement of light by the photonic bandgap. A loss value of 24db/cm is reported as the world record in 2003 in [70] for a PhC waveguide fabricated on a silicon membrane. In 2005 a measured loss of 5dB/cm was reported [71] and in 4.1dB/cm in 2006 [72] on the same material system and around

telecommunication wavelengths near $1.55\mu\text{m}$. These values are still high when compared to traditional silicon nano wire waveguides having losses that can reach values lower than 0.3dB/cm [73] on silicon-on-insulator material system. However it is noteworthy that PhC structures are usually smaller in size therefore a direct comparison of loss per length would not be fair. Another challenge is coupling light into and out of PhC waveguides due to the mode mismatch between the mode of the PhC waveguides and of conventional ridge wires. Theoretical studies show that near unity coupling efficiencies can be reached using tapering the periodicity of the PhC at the interface [74] and also an insertion loss of down to 2.4dB has been demonstrated experimentally [75].

In this study we aimed to design a PhC directional coupler which is the integrated equivalent of a beam splitter in bulk optics and has intensive usage in integrated quantum photonics circuits. The layer structure under study is composed of GaN, aluminium nitride (AlN) and sapphire. This layer structure is considered as an asymmetric layer structure as opposed to membrane type, symmetric structures. In symmetric structures it is possible to separate the modes into transverse-electric (TE) and transverse-magnetic (TM) polarizations and they can exhibit complete band gaps (band gap for all directions of propagation). Asymmetric layer structures however are easier to fabricate since they do not require under cutting etch step in fabrication and they are mechanically more stable. Asymmetric layer PhCs can have a quasi-band gap meaning although some loss due to coupling to the modes supported by the PhC or radiation modes occurs; there is still a useful amount of confinement of light.

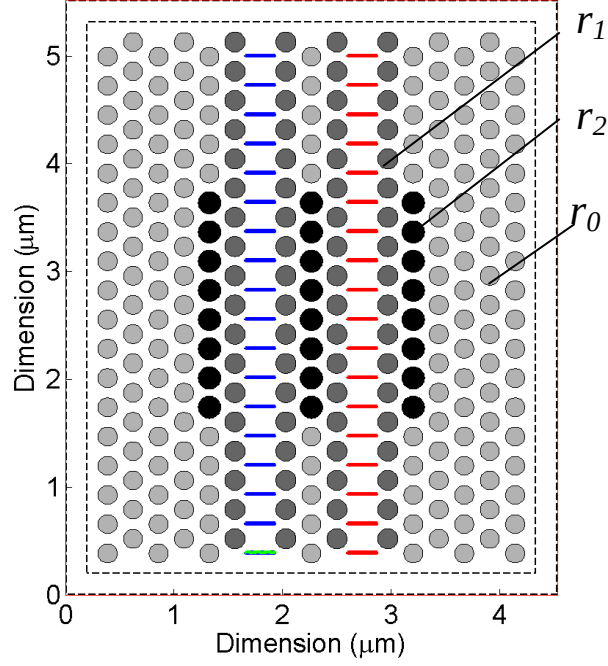


Figure 1.3: The top view of the coupler structure simulated. The radii of the holes surrounding the cavity are altered to increase/decrease coupling between the waveguides. The red and blue bars represent the positions of the flux planes used in the simulation

We have performed our simulations using three dimensional finite difference time domain (FDTD) method using an open source package, MEEP. We have designed a PhC 50-50% directional coupler for the 800nm wavelength regime. Figure 1.3 shows the top view of the simulated coupler structure having a similar topology to the one presented in [76]. The radii of the holes around the waveguides are different as shown with r_0 , r_1 and r_2 . The middle region has a slightly larger value of r_2 than the rest and this increases the coupling between the waveguides. The red and blue bars represent the position of the flux planes used in the simulation while the green one is the excitation. The device is around $5.4\mu\text{m}$ with a coupling region length of around $2.2\mu\text{m}$. We also investigated the tuning capability with a slightly longer coupler ($\sim 6\mu\text{m}$) and the simulation results show that the splitting ratio of the coupler can be altered between 27-63% to 77-23% with a 0.1% change in the refractive index.

1.3 Organization of the Thesis

In this chapter we have discussed quantum information and its significance. In chapter 2 we will discuss about single photon sources in general and different methods of generating single photons

In chapter 3 we will present our experimental results of quantum interference of photons in a GaN directional coupler. We will explain the Hong-Ou-Mandel interference experiment and non-classical behaviour of indistinguishable photons.

In chapter 4 we will concentrate on four-wave mixing in silicon nano-wires as a source of single photons. We will present our various experiments including four-wave mixing in a ring resonator and reverse biasing.

Chapter 5 will be about modelling and simulation of photonic devices. Some methods such calculating band diagrams of photonic crystal and finite difference time domain simulation will be explained in more detail.

Chapter 6 includes literature review about photonic crystals. Some examples of photonic crystal cavities, waveguides and couplers will be presented.

Chapter 7 will be about the design of photonic crystal structures in GaN. The systematic design of a coupler with first 2D and later 3D simulations will be presented.

Finally chapter 8 provides general conclusions on this work as well as suggestions for future improvements.

Chapter 2

Single Photon Sources

This chapter includes a review of the literature on single photon sources. The ideal single photon source will be described and alternative approaches to generate single photons will be presented together with spontaneous four-wave mixing which is the method that will be the focus of this thesis.

2.1 Overview of Single Photon Sources

Single photon sources (SPS) are one of the key components for integrated quantum photonics. The performance and realizability of tasks involving quantum photonics brings certain requirements on the source of the single photons. We will first describe an ideal single photon source. Then we will discuss the properties that determine the performance of a single photon source. Then we will introduce different approaches for single photon generation as well as the approach studied in this thesis which is four-wave mixing (FWM). There are few review articles in the literature about single photon sources that present an overview of the topic and advancements in the field to date, [77] [78] and [16].

A single photon is the fundamental particle of light. Two main characteristics of a single photon are its frequency spectrum and its spatial mode which is the distribution of the electro-magnetic field in the dimension transverse to the direction of propagation. The properties of an ideal SPS can be listed as follows.

1. An ideal SPS generates a single but only a single photon upon request, with a unity probability.
2. Each generated photon by an ideal SPS has the same spectrum and spatial mode.
3. An ideal SPS outputs the generated photon by a certain, non-probabilistic delay with respect to the request. In other words it has zero jitter.

Different approaches are shown to partially satisfy the requirements of an ideal SPS. Usually the “request” sent to the SPS is of the form of an optical or electrical pulse, For example in the case of quantum dot based SPS the system jumps to the excited state upon receiving an electrical or optical pulse and releases a photon while

recovering its initial state.

The measure of purity of a single photon source is its second-order correlation, $g^{(2)}(\tau)$ which can be measured by a Hanbury-Brown and Twiss (HBT) interferometer [79]. The schematic of the HBT interferometer is given in

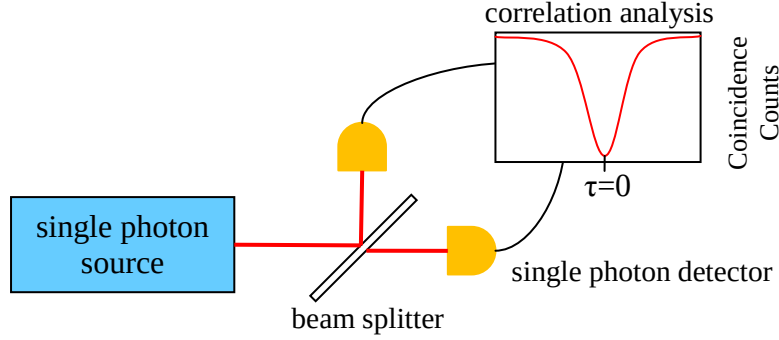


Figure 2.1: Schematic of a Hanbury-Brown and Twiss interferometer setup. The photons from the source are incident on a beam splitter. The transmitted and reflected photons are detected by detectors and correlation analysis is performed on the detector output to get a graph showing the coincidence event with respect to the relative delay between detector clicks.

This is composed of a beam splitter where the output of the single photon source is directed onto, two detectors that detect the photons transmitted and reflected by the beam splitter. The correlation between the two clicks is analyzed. The output of the analysis is the second order correlation function given by the second-order correlation is given by the formula [80]

$$g^{(2)}(\tau) = \frac{p(t:t+\tau)}{p(t)} \quad (2.1)$$

where $p(t:t+\tau)$ is the probability of detecting one photon by the first detector at time t and detecting another photon by the second detector at time $t+\tau$; while $p(t)$ is the probability of detecting only a single photon by either of the detectors. In an ideal single photon source does not emit more than one photons at a time therefore $p(t:t)$ or equivalently $g^{(2)}(0)$ is 0.

In many applications of quantum photonics the process depends on the interference of two photons. In order for the photons to interfere; the photons need to be indistinguishable, which brings another requirement on the ideal SPS. The indistinguishability of two photons is tested by interfering them on a beam splitter. Due to the bosonic behaviour of photons indistinguishable photons bunch together when they interfere and exit from the same port of the beam splitter. This is first demonstrated by Hong, Ou and Mandel in 1987 [18]. Two indistinguishable photons have the same spatial mode, polarization and frequency spectrum. The coherence time of two photons (the maximum delay between two photons which ensures that they can still interfere reasonably well) is related to their spectrum. The last property of SPS which is zero jitter ensures that the time and position of the generated photon is well known and the generated photons are well synchronized with each other.

We know that a single photon is a quantum and behaves in a non-classical manner and when large amounts of photons come together they form a classical light. So a question is what is in between these two? To answer this, consider a pulse of light from a coherent source which follows Poissonian statistics which is given by the formula

$$p(n) = \frac{\mu^n}{n!} e^{-\mu} \quad (2.2)$$

where μ is the average photon count in the pulse and $p(n)$ is the probability of finding n photons in the pulse. To maximize the probability of getting single photons, we have to attenuate the laser beam so that the average photon count is 1. So the source will some times shoot single photons, some times will give multiple photons and some times no photons at all. This type of single photon source is called a Poissonian source and has a distinguishing property such that the second order correlation,

$g^{(2)}(\tau)=1$ for all τ . A source which has $g^{(2)}(0) < 1$ is therefore called a sub-Poissonian source.

The common SPSs can be divided into two classes; deterministic and probabilistic which will be covered in the next sections.

2.1.1 Deterministic Single Photon Sources

Deterministic SPSs are by nature limited to generating only one photon at a time. These work based on the excitation of a single neutral atom, ion or molecule which emits a single photon decaying back to the ground state. The coupling of the generated single photon efficiently to a single spatial mode is an important concern.

2.1.1.1 Trapped Particles

In this type of SPSs the neutral atoms, ions or molecules are trapped in an optical cavity so that the emission is controlled and directed. Second order correlation of $g^{(2)}(0)=0.06$ has been demonstrated with neutral atom based sources [81]. However the stability of the atom inside the cavity is not sufficient for long measurement periods. Single ion based single photon sources [82] [83] overcome these difficulties providing stability hence longer lifetimes. In [83] using a single ion in cavity $g^{(2)}(0)=0.015$ is measured. Single molecules in either liquid solvent or in a solid host can also be used for generating single photons [84] [85]. This method suffers from the photo-bleaching which is an irreversible damage on the molecule typically after around a billion pair generation cycles. And additionally these methods require rather complicated setups involving the placement of a single particle inside a cavity and

ensuring its stability. Therefore they are not practical for an integrated approach which will be discussed in later sections.

2.1.1.2 Nitrogen Vacancy Centres in Diamond

Another popular method to use a nitrogen vacancy (NV) centres in diamond crystals which are formed by a nitrogen atom and a vacancy in the uniform carbon crystal. This forms a three level system with ground, excited and meta-stable states. The electron falling into the meta-stable state slows down the photon generation rate and therefore not desired. The NV centres in diamond are used for generating photon pairs at 637nm wavelength and can operate at room temperature. The emission bandwidth is around 100nm. A challenge is to couple the generated photon efficiently to free space. To address this problem, the use of nano-crystals of diamond whose size are much smaller than the wavelength are demonstrated [86], solid immersion lenses have been proposed [87]. In [86] a $g^{(2)}(0)$ value of 0.07 has been reported. It is hard to achieve indistinguishable photons from a diamond NV center due to its large emission bandwidth and also there is an upper limit in the excitation rate (over which the electrons are more likely to shelf at the meta-stable state) which is quoted as 5MHz in [88].

2.1.1.3 Quantum Dots

Quantum dots (QD) in III-V semiconductors is also a widely studied method for single photon generation. A small band gap material surrounded by a larger band gap material forms a quantum dot. The quantum dots can be pumped optically or electrically. In electrical pumping, the charge carriers are moved to the quantum dot

with so called the Coulomb blockade effect, which ensures only one carrier is moved at a time. This is referred to as single photon turnstile [89]. Quantum dots usually have to be operated at cryogenic temperatures to suppress the generation of multiple photons however relatively higher temperature operations are also reported [90]. In order to enhance spontaneous emission characteristics usually a cavity is fabricated around the dot such as micro-pillars [91] micro-disks [89], photonic crystal cavities [92]. When the wavelength of the generated photon is matched with the resonance of the cavity the emission from the dot is enhanced. The cavity also helps coupling the generated photon into a single spatial mode. Very narrow spectral line widths are typical properties of QDs. There is an extensive research in order to reduce the second order correlation, $g^{(2)}(0)$, values of QDs. A $g^{(2)}(0)$ value of 0.02 is reported in [93] where the structure is an InAs QD in a GaAs a micro-post structure.

2.1.2 Probabilistic Single Photon Sources

The methods mentioned above were using physical phenomena which enables the generation of only a single photon at a time. Alternatively there are probabilistic sources having a Poissonian statistics as described in section 2.1. These multiple photons deviates the source from being an ideal SPS. Hence the probability of generating multiple pairs has to be reduced compared to the probability of generating a single photon and the way to do this is to reduce the average photon count (typically 0.1 per pulse [16]). In other words increase the attenuation. The draw back of this is the reduction in the number of single photons output per unit time.

There are physical processes examples of which will be discussed later, that enable the generation of pairs of photons again subject to Poissonian statistics. The

generated pairs can be in the same wavelength (degenerate) or in different wavelengths (non-degenerate). The generated photons are usually referred to as “signal” and “idler” photons; in the non-degenerate case the photon with lower wavelength is the signal photon. In the degenerate case both photons can be used in a quantum circuit [19], [35], [94] whereas in the non-degenerate case the existence of a photon can be determined by the detection of the other photon [95], [96]. Hence these types of sources are also referred to as “heralded photon sources”. Heralding provides the source to be operated in sub-Poissonian statistics (by only activating the detection upon receiving the heralding signal) such as in [97] where $g^{(2)}(0)$ value of 0.002 has been reported.

In the scope of this thesis we will discuss about spontaneous parametric down conversion (SPDC) and four-wave mixing (FWM) techniques for single photon generation which are based on the optical non-linearities of the material. Optical non-linearities of the medium is the dependence of the optical properties of the medium, namely the refractive index and absorption with respect to the applied electromagnetic fields. Since light itself is an electro-magnetic field it can modify the properties of the medium it propagates. The relation between the electric displacement vector $\vec{D}$ is related to the electric field vector $\vec{E}$ and the polarization density of a medium $\vec{P}$ by

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} \quad (2.3)$$

where ϵ_0 is the permittivity of vacuum. In linear optics the polarization density is given by

$$\vec{P} = \epsilon_0 \tilde{\chi}^{(1)} \quad (2.4)$$

where $\tilde{\chi}^{(1)}$ is the first order electric susceptibility tensor. Combining equations (2.3)

& (2.4) we get

$$\vec{D} = \varepsilon_0(1 + \tilde{\chi}^{(1)})\vec{E} = \varepsilon_0\varepsilon_r\vec{E} \quad (2.5)$$

Here $\varepsilon_r = 1 + \tilde{\chi}^{(1)}$ is the relative permittivity of the material. In non-linear optical materials the polarization density is a function of the electric field $\vec{E}$ is given by [98]

$$\vec{P} = \varepsilon_0\tilde{\chi}^{(1)}\vec{E} + \varepsilon_0\tilde{\chi}^{(2)}:\vec{E} + \varepsilon_0\tilde{\chi}^{(3)}:\vec{E}^2 + \dots \quad (2.6)$$

where $\tilde{\chi}^{(n)}$ is the n th order electric susceptibility tensor of the material and the operators “.” and “:” are the scalar matrix products. equation (2.6) shows that the polarization density inside a material has dependent of the electric field. Centrosymmetric crystals such as silicon the even order susceptibilities ($\tilde{\chi}^{(2)}, \tilde{\chi}^{(4)}, \dots$) are zero so they exhibit only third order non-linearities while all crystals have non-zero odd order susceptibilities ($\tilde{\chi}^{(1)}, \tilde{\chi}^{(3)}, \tilde{\chi}^{(5)}, \dots$) [Physics of nonlinear]. $\tilde{\chi}^{(2)}$ and $\tilde{\chi}^{(3)}$ are the two important components in non-linear optics giving rise to second order and third order non-linear phenomena respectively. We will refer $\tilde{\chi}^{(2)}$ and $\tilde{\chi}^{(3)}$ as $\chi^{(2)}$ and $\chi^{(3)}$ respectively since the former are tensors and in practice when light in a single polarization along a single crystal axis these can be reduced to constants.

Two important non-linear processes for single photon generation are spontaneous parametric down conversion (SPDC) which is a second order non-linear process and four-wave mixing (FWM) which is a third order non-linear process. The former is related to $\chi^{(2)}$ non-linearity and the latter is related to $\chi^{(3)}$. In this thesis we will focus on the $\chi^{(3)}$ non-linearity which enables the generation of photon pairs through four-wave mixing (FWM). More detailed theory of $\chi^{(3)}$ non-linearity will be given in Chapter 4. In the next sub sections we will give an overview about probabilistic single photon sources utilizing SPDC and FWM.

2.1.2.1 Spontaneous Parametric Down Conversion

A well known photon generation process is the spontaneous parametric down conversion (SPDC) which was discovered in 1960s and early 1970s [99]. A crystal having $\chi^{(2)}$ non-linearity is used in this process where a photon of the pump beam (laser directed onto the crystal) is converted into a pair of photons as shown in Figure 2.2.

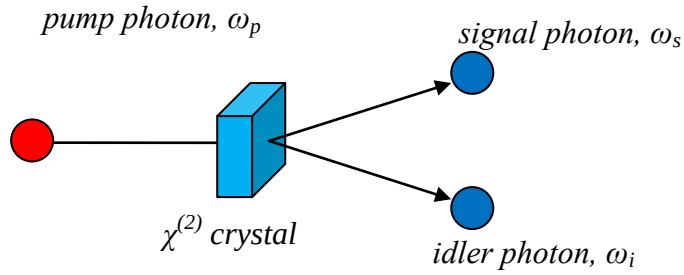


Figure 2.2: Diagram illustrating the spontaneous parametric down conversion process. A pump photon with frequency ω_p are annihilated to generate signal and idler photons at frequencies ω_s and ω_i , in a $\chi^{(2)}$ crystal.

In this process the energy and momentum has to be conserved, that is

$$\omega_p = \omega_s + \omega_i \quad (2.7a)$$

$$k_p = k_s + k_i \quad (2.7b)$$

where ω represents the frequency and k represents the wavevector. Subscripts p, s and i represent the pump, signal and idler photons respectively. In quantum mechanics perspective this process can be described by the wave equation given in [100]

$$j\hbar c \frac{\partial \psi}{\partial z} = H\psi \quad (2.8)$$

where z is the direction of propagation ψ is the wavefunction and H is the Hamiltonian given by

$$H = g\hbar c(\hat{a}_s^\dagger \hat{a}_i^\dagger \hat{a}_p + \hat{a}_s \hat{a}_i \hat{a}_p^\dagger) \quad (2.9)$$

g represents the gain per unit length, $\hat{a}_{s,i,p}$ and $\hat{a}_{s,i,p}^\dagger$ are the annihilation and creation operators for signal, idler and pump photons. The solution to the wave equation (2.8), $\hat{U}$ is

$$\hat{U} = \exp\left(\frac{-jHz}{\hbar c}\right) \quad (2.10)$$

When the phase matching condition (the pump signal and idler photons propagating at the same speed in the crystal) is satisfied and the pump beam is a strong classical beam the output wave function becomes

$$|\psi_{out}\rangle = \exp\left(-jg\frac{E_p}{E_0}\hat{a}_s^\dagger \hat{a}_i^\dagger z\right) |vac\rangle \quad (2.11)$$

where E_p is the amplitude of pump field and E_0 is the field per photon at pump wavelength.

Typical crystals that are widely used for SPDC are Potassium dideuterium phosphate (KDP) [101], beta barium borate (BBO) [102], lithium niobate (LiNbO₃) [103]. These have high $\chi^{(2)}$ non-linearity due to the asymmetry in their crystal lattices. The SPDC sources are classified as Type-I or Type-II according to the polarization of the generated photons. In Type-I the polarizations are equal whereas in Type-II the polarizations are orthogonal to each other. Type-II degenerate SPDC which is commonly used today was first proposed by Kwiat et al [104]. In this orientation the location of the ordinary polarized and extra ordinary polarized photons define different cones as depicted in Figure 2.3. The intersections of these cones define lines where the polarization entangled photon pairs are collected at locations shown with letters “A” and “B” by mirrors and lenses while the residual pump beam is

disregarded.

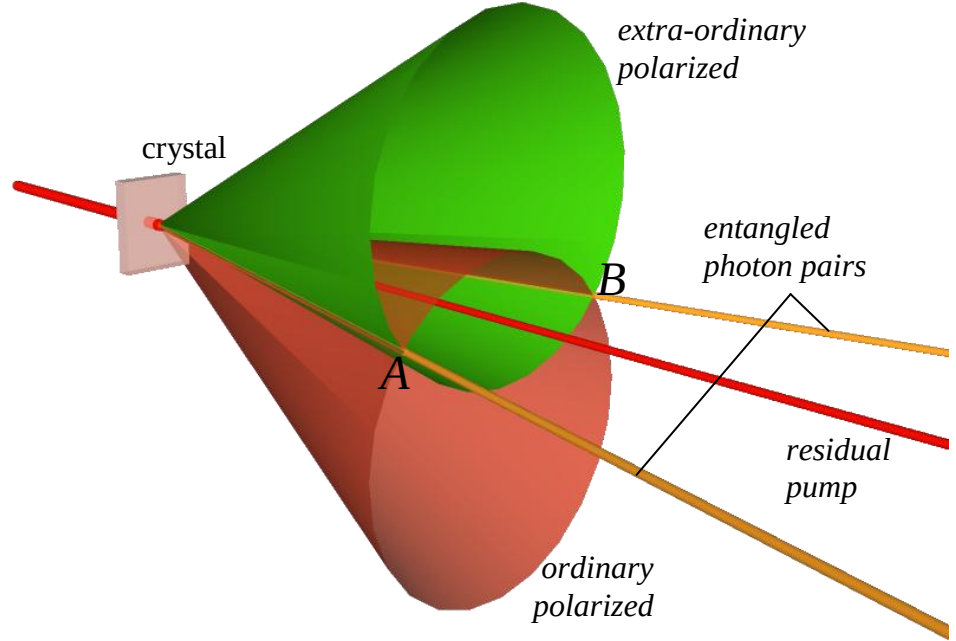


Figure 2.3: Illustration of generation of entangled photons through Type-II spontaneous parametric down conversion. In this configuration the locations of ordinary and extra-ordinary polarized photons define the red and green cones. Entangled pairs of photons are collected along the two lines formed by the intersection of these cones marked as A and B.

The polarization entangled state is defined by

$$\phi = \frac{1}{\sqrt{2}}(|H\rangle_A|V\rangle_B + e^{j\phi}|V\rangle_A|H\rangle_B) \quad (2.12)$$

where H and V are horizontal and vertical polarizations respectively. This method has been the major source of entangled photons for many quantum photonics experiments [35] [34]. The challenging part of SPDC is to satisfy the phase matching condition which affects the efficiency of the process. Since the pump and the generated photons are far apart in spectrum ensuring they all propagate at the same speed is hard to achieve. One of the most commonly employed technique to solve this problem is periodic poling in which the sign of the non-linearity is altered. The walk-off effect is also another obstacle in phase matching these birefringent materials. It is the

divergence between the pump and the generated photons in space so that their spatial overlap degrades with the length of the propagation. Also coupling the photons from free space to optical fibers is poor reducing the overall efficiency of the process. However SPDC is so far the leading method to reach very low second order correlation values and also multi-photon entangled states. In [97] a $g^{(2)}(0)$ of 0.0014 is reported from a bulk crystal SPDC source. A state-of-the-art example is the production of eight photon polarization entangled state [105]. The count rate is reported as approximately 9 eight-photon coincidence event per hour.

$\chi^{(2)}$ non-linear materials can be micro structured to guide the light which also provides more flexibility in engineering the dispersion by changing the dimensions of the waveguides besides periodic poling. Another advantage of this method is that the spatial mode of the photon can be predetermined by designing the waveguide to support only a single mode. Periodically poled lithium niobate (PPLN) waveguides are popular for integrated photon pair generation [40], [103], [61], [41]. HOM dip visibilities (discussed in Chapter 3) between the generated photons close to unity ($99\pm3\%$ after correcting for the dark counts) and also entanglement has been demonstrated in [61]. Also in [106] $g^{(2)}(0)$ of 0.17 from a PPLN waveguide is reported. Typical lengths the PPLN waveguides for single photon generation are on the order of few centimetres. This makes large scale integration of sources, circuits and detectors all in the same substrate difficult.

The other type of heralded single photon generation method is spontaneous four-wave mixing which is the one studied in this thesis. We will explain the four-wave mixing process in more details over the next section.

2.1.2.2 Four Wave Mixing

Four-wave mixing (FWM) process is associated with the third order-susceptibility $\chi^{(3)}$ of the materials. In simple words it can be defined as the interaction of three electro-magnetic fields inside the material to produce a fourth electro-magnetic field. If the wavelengths of the three interacting photons are different the process is named as non-degenerate four-wave mixing whereas in the case where two of them have the same wavelength it is called degenerate four-wave mixing. Table below is a list $\chi^{(3)}$ values of several materials [107].

Table 2-1: Third order susceptibilities of various materials

Material	$\chi^{(3)} \text{ m}^2/\text{V}^2$
Al_2O_3	3.1×10^{-22}
Diamond	2.5×10^{-21}
GaAs	1.4×10^{-18}
Si	2.8×10^{-18}
Fused Silica	2.5×10^{-22}
As_2S_3	4.1×10^{-19}

The physical process of FWM is based on the polarizations induced by the fields within the crystal. Field propagation inside the crystal creates local oscillations in the polarization. When a second field is also causes oscillations in the same medium and due to the beating between the frequencies of the two oscillations, two other harmonics at the sum and difference frequencies of the first and the second field are generated resulting in a total of four harmonic oscillations in the material. The FWM occurs when a third field is introduced to the medium interacting with these harmonic components to produce more harmonics.

Coherent Anti-Stokes Raman Spectroscopy (CARS), Coherent Stokes Raman Spectroscopy (CSRS), Stimulated Raman Gain Spectroscopy (SRGS), The Inverse Raman Effect Spectroscopy (TIRES), Raman Induced Kerr Effect Spectroscopy (RIKES) are examples of methods that use these harmonics. FWM is also used for wavelength conversion and signal regeneration in telecommunication applications [108].

In this study we are particularly interested in spontaneous FWM (sFWM) where two pump photons are converted into signal and idler photons allowing the utilization of this process as a heralded single photon source. There had been studies on utilizing the $\chi^{(3)}$ non-linearity of silica optical fibers for single photon generation [109], [110]. The dispersion of the fiber needs to be engineered in order to satisfy the phase matching condition, so these fibers are called dispersion shifted fibers. To increase the interaction length often few hundreds of meters of fiber is used. In [111] a 300m long single mode fiber (SMF) is pumped with a pulsed laser. The coincidence rate is reported as 0.04 per pulse and the CAR is around 10. The reason the CAR being so low is the Raman scattering in these silica fibers. Raman scattering results in the generation of photons which can share the same frequency spectrum with the desired photon pairs and cannot be filtered out. In an experiment these photons can result in coincident detection (if they arrive at the same time to the detector) and can be counted as photon pairs. This results in an increase of accidental counts and reduces the CAR.

Raman scattering is a source of noise in silica fibers. Raman scattering is the inelastic scattering of photons and is a process where a photon is absorbed by the material and instantaneous re-emitted at a different frequency [112]. These reemitted photons can share the same frequency range with the generated photons and cannot be

filtered out. To suppress Raman scattering cooling down the fiber using liquid nitrogen is employed in [113] where a 500m long fiber was used. Comparing the performance of the uncooled fiber and the cooled fiber, in the uncooled case a pair generation rate of 0.01/pulse and a CAR of about 9 was obtained whereas in the cooled case the pair production rate is doubled reaching 0.02/pulse and the CAR is increased to 28. In a later study the CAR in the cooled case has been increased up to around 60 and in the uncooled case up to 16 [43]. Polarization entanglement has been demonstrated with the photons generated in dispersion shifted fibers as well the preservation of entanglement at a distance of 20km of fiber [109].

Use of photonic crystal (some resources mention as micro structured) fibers (PCF) supplies an increased control over the dispersion characteristics, enabling single mode operation for a wide range of wavelengths and also tight confinement of light [114], [115]. $g^{(2)}(0)$ of 0.014 is reported in [116] using such a device. Similar to the standard fibers they also suffer from the Raman scattering reducing the quality of the source, however compared to standard fibers, photonic crystal fibers used as sources are much shorter in length. In [115] using a 3m long photonic crystal fiber, a pair generation frequency of 3MHz (collection efficiency excluded) and a CAR of around 5 is obtained at room temperature with continuous wave pumping.

FWM in silicon nano-wires is relatively a new technique. This technique is the focus of this thesis and will be discussed in more detail in the following chapter.

2.2 Conclusions

We have presented various techniques to generate single photons. Generating only a single photon at a time, indistinguishability, timing jitter are major concerns of a

single photon source. Deterministic sources are based on a single emitter embedded in a resonant cavity. These can avoid multi-photon generation by nature however room temperature operation, coupling the photons efficiently out of the cavity are some of the challenges. In probabilistic sources, multi photon generation probability is a major problem which requires them to be operated at lower pump powers, hence reduced photon rate. Table below shows second order correlation values and emission efficiencies for some of the key studies in the field [16]. The emission efficiency is the ration of the number of photons extracted from the device to the ones generated inside the device (excludes detection efficiencies).

Table 2-2: Second order correlation values for some SPSs in the literature

Method	$g^{(2)}(0)$	Emission Efficiency	Ref.
QD	0.02	0.1	[91]
Single Ion	0.015	0.08	[83]
NV Center	0.07	0.022	[117]
SPDC (Bulk)	0.0014	0.6	[97]
SPDC (PPLN)	0.17	-	[106]
FWM PCF	0.014	0.18	[116]

Currently SPDC in bulk crystals is the most commonly employed technique by many research groups around the globe due to is ease of implementation, ability to generate photons which are indistinguishable to a high degree and entanglement. Integration of sources with circuits and detectors is another concern for the emerging integrated quantum photonics technology. This way multiple sources can be implemented on the same chip and combined with on-chip circuits and detectors. This will also remove the coupling losses experienced coupling the photons from an external source to on-chip circuits. Ensuring the sources being identical to produce indistinguishable photons is a challenge related to the fabrication tolerances which

was discussed in Section 1.1.2. Quantum dots embedded in cavities, NV centres in diamond are promising methods for on-chip deterministic single photon sources. On the other hand SPDC in PPLN waveguides or FWM in silicon nano-wires are methods for implementing integrated probabilistic sources. In the next section we are going to see that FWM in silicon nanowires are also promising in terms of size and performance for near future integrated quantum photonics applications.

Chapter 3

Gallium Nitride Directional Coupler for Quantum Photonics

In this chapter we are going to present a gallium nitride directional coupler for integrated quantum photonics applications. Directional couplers are used in quantum photonics applications for interfering photons which is a fundamental concept in realizing quantum algorithms. We will initially present the theory behind the non-classical interference of photons, namely the Hong-Ou-Mandel experiment, and then move on to our experimental results.

3.1 Interference of Indistinguishable Photons in a Beam Splitter

When a beam of light is incident on a 50-50% (or 3dB) beam splitter as shown in Figure 3.1(a), the transmitted and reflected beams have equal power. Notice that the incident beam on port b undergoes a π phase shift upon reflection while the beam incident on port a does not. When a single photon is incident on a beam splitter as illustrated in Figure 3.1(b), the photon is either transmitted or reflected with equal probabilities of $1/2$. When two indistinguishable photons hit the beam splitter simultaneously from either port, one can think of four possibilities as illustrated in Figure 3.1(c). However two of these cases which are crossed out cannot exist. Therefore two indistinguishable photons interfering in a beam splitter bunch together and randomly choose to exit from one of the ports with equal probabilities.

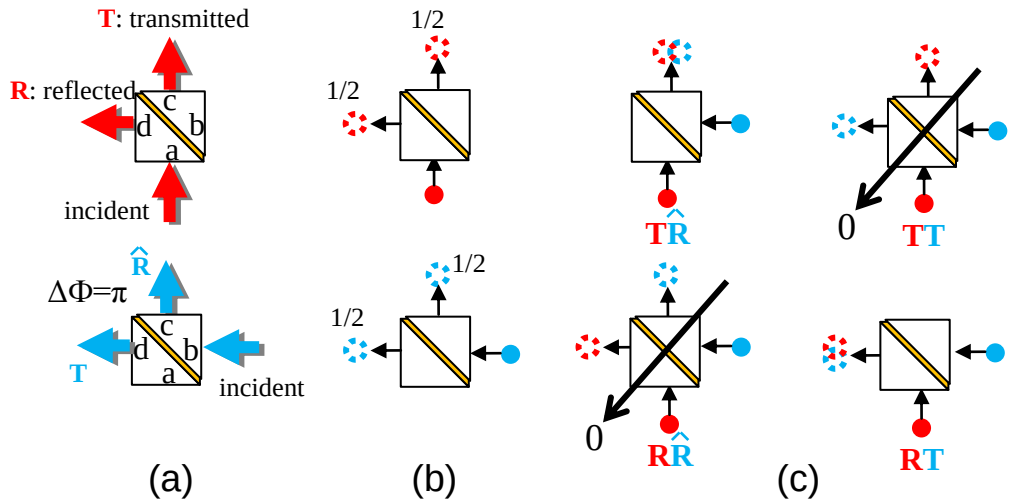


Figure 3.1: (a) When a beam of light is incident on a 50%-50% beam splitter either from port “a” (top) or from port “b” (bottom), it is split into two (the transmitted and the reflected) waves with equal power. Notice that the reflected wave in the bottom figure goes under a π phase shift (b) When a single photon hits this beam splitter it either transmits or reflects with equal probabilities of $1/2$. (c) When two indistinguishable photons hit the beam splitter simultaneously one can think of four possibilities which are permutations of each photon being either transmitted or reflected. However two of these cases crossed out have no probability of happening.

To analyze this mathematically, we represent the modes at the input and output ports of the beam splitter by corresponding creation operators $\check{a}^\dagger, \check{b}^\dagger, \check{c}^\dagger, \check{d}^\dagger$ where for instance $\check{a}^\dagger$ is the creation operator for the mode in port a of the beam splitter and so on. We will also represent the transmission constant (the power of the transmitted wave divided by the incident power) of the beam splitter with t and the reflectivity (the power of the reflected wave divided by the incident power) by r . The following relation exists between t and r for a lossless beam splitter.

$$r^2 + t^2 = 1 \quad (3.1)$$

The transformation of input modes to the output modes can be represented by

$$\check{a}^\dagger \rightarrow r\check{d}^\dagger + t\check{c}^\dagger \quad (3.2a)$$

$$\check{b}^\dagger \rightarrow t\check{d}^\dagger - r\check{c}^\dagger \quad (3.2b)$$

The Fock state at the input when two indistinguishable photons hit the beam splitter simultaneously is given by

$$|1,1\rangle_{a,b} = \check{a}^\dagger \check{b}^\dagger |0,0\rangle_{a,b} \quad (3.3)$$

When this state undergoes the transformation by the beam splitter we get the following state

$$\begin{aligned} \check{a}^\dagger \check{b}^\dagger |0,0\rangle_{a,b} &\rightarrow (r\check{d}^\dagger + t\check{c}^\dagger)(t\check{d}^\dagger - r\check{c}^\dagger) |0,0\rangle_{c,d} \\ &= \left[tr(\check{d}^{\dagger 2} - \check{c}^{\dagger 2}) - (t^2 - r^2)\check{c}^\dagger \check{d}^\dagger \right] |0,0\rangle_{c,d} \end{aligned} \quad (3.4)$$

In a 50-50% beam splitter where $t^2 = r^2 = 0.5$ equation (3.4) becomes

$$\frac{1}{2}(\check{d}^{\dagger 2} - \check{c}^{\dagger 2}) |0,0\rangle_{c,d} = \frac{1}{2}(|2,0\rangle_{c,d} - |0,2\rangle_{c,d}) \quad (3.5)$$

This shows that in a 50-50% beam splitter, at the output both photons are either in mode c or mode d . This bunching of photons is the underlying physical process of the

Hong-Ou-Mandel interference experiment which will be discussed in the next section.

3.2 Hong-Ou-Mandel Interference

Hong-Ou-Mandel (HOM) interference takes its name from the scientist who first observed the photon bunching effect experimentally [18]. The HOM interference experiment gives a measure of the indistinguishability of two photons. A schematic of experimental setup for two photon interference is shown in Figure 3.2(a). The single photons generated by a source are continuously from the two inputs of the beam splitter while the relative delay between the photons can be controlled via a delay line in one arm. At each output of the beam splitter there are single photon detectors. The outputs of the detectors are analyzed for counting coincidence events using a time interval analyzer (TIA). In Figure 3.2(b) the plot of coincidence counts versus the relative delay between the two photons is shown. This coincidence histogram is also referred to as a HOM dip. In practice, the relative delay is changed in discrete steps and the coincidence counts are accumulated over period called the integration time at each step to acquire the HOM dip curve.

When the delay between the photons is zero the coincidence count is at its lowest, in ideal case (indistinguishable photons, and unity efficiency detectors) it is zero due to the bunching effect. As the delay is increased the photons gradually lose coherence and they start independently choosing either of the ports with equal probability. When the photons are completely decoherent, the probability of each exiting the beam splitter from different ports and causing a coincidence event is $1/2$ (two outcomes out of four possibilities shown in Figure 3.1(c)). The shape of the HOM dip curve depends on the correlation characteristics of the photons. In our case

since the generated photons are filtered to have a Gaussian spectrum the resulting HOM dip curve is expected to be Gaussian (the convolution of two Gaussian signals in time is also a Gaussian) as well which is of the form [18]

$$CC(\Delta) = CC_{max} - CC_{min} \exp(-\Delta^2/\sigma^2) \quad (3.6)$$

Here CC_{max} and CC_{min} are the maximum and minimum points of the curve, Δ is the relative delay between the photons either in time or space and σ^2 is the variance related to the coherence length of the photons.

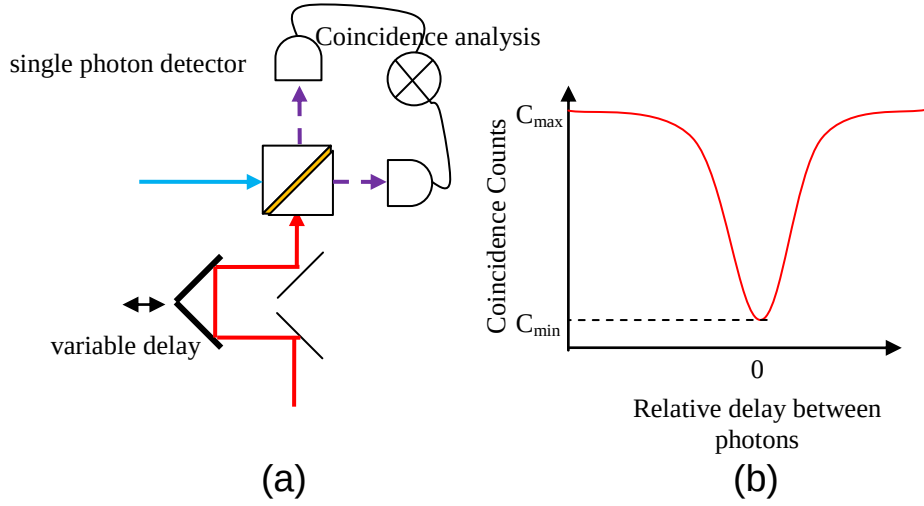


Figure 3.2: (a) The setup for measuring HOM interference. (b) A typical coincidence histogram which if the coincidence detected versus the relative delay between the photons.

The visibility of the HOM dip is a figure of merit for the indistinguishability of the photons and is calculated by the following formula

$$V = \frac{C_{max} - C_{min}}{C_{max}} \quad (3.7)$$

The visibility is determined by several factors. One of them is the indistinguishability of the photons which is determined by the single photon source as well as the device where the interference occurs. A unity visibility HOM dip can be obtained in the case where the photons generated by the source has exactly the same spectral content, the

two photons have to have the exactly same polarization and spatial mode and amplitude when they meet with each other and the splitting ratio of the beam splitter has to be exactly 1/2. Deviation from these requirements results in the degradation of the visibility. One potential reason for reduced visibility when using probabilistic sources is the multi photon events as described in Chapter 2. When the source generates more than one pair of photons are generated by the source interfered on the beam splitter, they do not exhibit the same behaviour of bunching as in the single pair case hence reduce the visibility. The detector dark counts and the accidental counts (which are false flags of the detector) adds a constant offset to the curve however their value is known and is subtracted before calculating the visibility. On the other hand the efficiency of the detectors do not have an effect on the visibility. The average accidental counts (false flags of the detector) resulting from non-unity detector efficiencies as well as the losses in the path of photons can be determined and subtracted as will be described below.

We have seen the two photon interference in a beam splitter. In integrated optics its equivalent is a directional coupler which will be described in the next section.

3.3 Directional Couplers

The directional coupler is simply realized by having two waveguides closely spaced so that the evanescent fields of the light in one of the waveguide couples gradually to the next waveguide. After a certain distance referred to as L_π all the light in the input port couples to the adjacent waveguide and at a distance of $2L_\pi$ (or $L_{2\pi}$) all the light is back in the initial waveguide. The light periodically switches between the waveguides with a sinusoidal behaviour as illustrated in Figure 3.3 below. The distance L_π

depends on the spatial overlap of the individual modes of the waveguides and also the separation between the two waveguides.

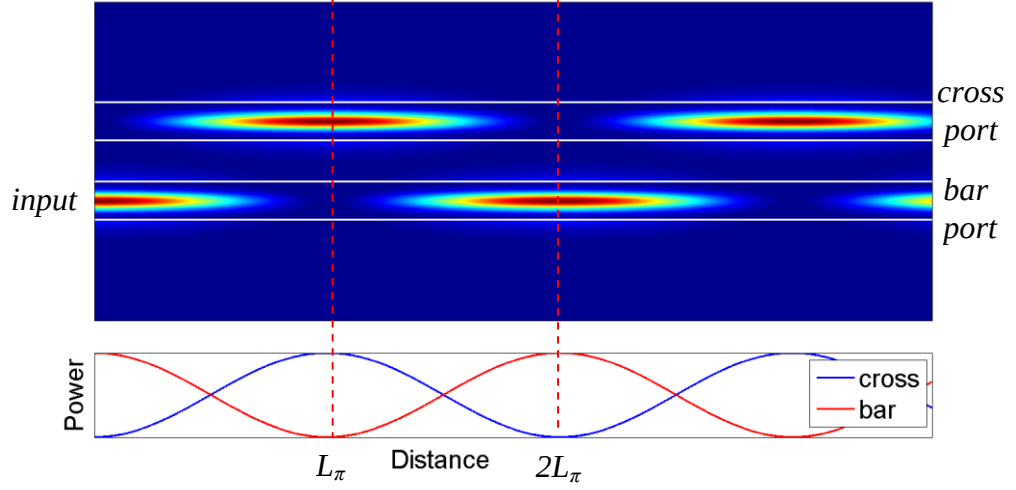


Figure 3.3: Illustration of the behaviour of the light in a directional coupler. The power periodically switches between the two waveguides with a sinusoidal behaviour. The distance at which all the input power is switched to the next waveguide is called L_π .

One way of solving L_π is by the coupled mode theory [118]. In this method the modes of the two individual waveguides are computed separately and then the modes are brought close to each other as in the coupler configuration and the overlap integral of the two modes is calculated. This gives the information about the coupling coefficient between the two waveguides from which L_π can be deduced. In most of the applications the two waveguides are desired to be identical (symmetric coupler) and another technique called the even-odd mode analysis can be applied. This is by interpreting the behaviour of the coupler by the beating between the even and the odd modes of the structure. When we think about the cross section of the structure as shown in Figure 3.4, the amplitude of the electric field in the even mode has even symmetric distribution around the symmetry line separating the two waveguides. Similarly the in the odd mode the electric field distribution is odd symmetric around the symmetry line. These even and odd modes can be solved by the help of a mode

solving software. A sample mode solution of a coupler is shown in Figure 3.4. When there is no phase difference between the even and odd modes, the fields in the right hand side waveguide destructively interfere and cancel each other so that all the light is confined in the left hand side waveguide. As the two modes co-propagate (each having a different propagation constant), when the relative phase difference between the modes becomes π , in other words one of the modes switches sign, this time the fields in the left hand side cancel each other and all the power is confined in the right hand side waveguide. So the power is switched from one waveguide to the other. This distance L_π can be calculated from the formula

$$L_\pi = \frac{\pi}{|\beta_e - \beta_o|} \quad (3.8)$$

where β_e and β_o are the propagation constants of the even and odd modes respectively. 50-50% splitting of light can be achieved at a distance $L_\pi/2$. As the waveguides are separated from each other the values of β_e and β_o become closer hence the L_π increases.

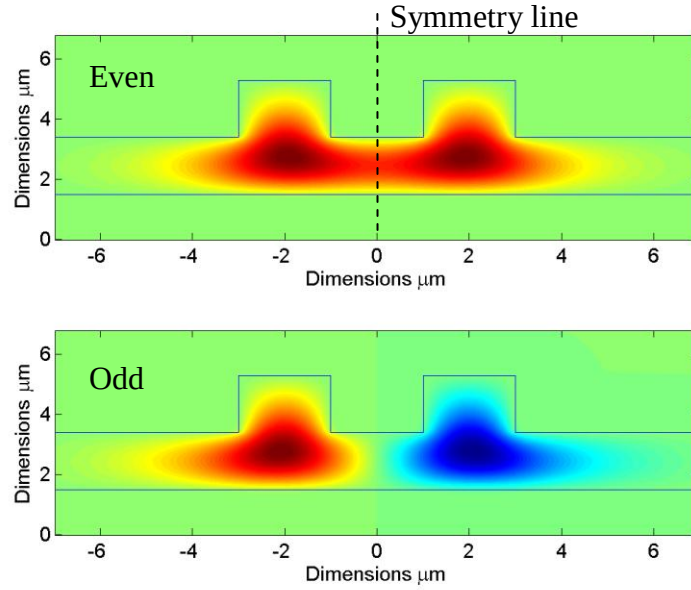


Figure 3.4: The E_x field of TE even and the odd modes of a coupler. The layer structure is $3.8\mu\text{m}$ of GaN on Sapphire. The width of the ridges is $2\mu\text{m}$ as well as the gap between the ridges. The etch depth is $1.9\mu\text{m}$. L_π is found to be $900\mu\text{m}$.

In the next section we will present our experimental results for two photon interference in a GaN directional coupler.

3.4 Two photon interference in a GaN Directional Coupler

Two photon interference has been first demonstrated in integrated optical circuits in silica-on-silicon material platform in 2006 [29]. Since then photon interference have been performed in various material platforms such as lithium niobate [119], silicon [34]. Here we present the quantum interference performed in a GaN directional coupler [19].

In order for two photons to interfere perfectly inside the coupler they need to be indistinguishable as described earlier. To satisfy this, the individual modes of the two waveguides have to have the same properties such as spatial distribution, polarization. Asymmetry in the dimensions of the waveguides, defects can degrade photon interference.

The directional coupler used in this study was fabricated by University of Strathclyde, Glasgow, UK; the test setup and experiments are performed by Loyd McKnight from University of Strathclyde and myself; software for controlling the instruments in the setup, acquiring and post processing the data are done by myself. Some details about the structure are shown in Figure 3.5. The layer structure is 3.8 μm thick GaN on top of sapphire (Al_2O_3). The device is composed of input/output taper and coupling regions and has a total length of around 5mm. The input/output separation at the facets is 50 μm and the waveguides are tapered from $\sim 5\mu\text{m}$ at the facets down to 2 μm at the coupling region. The coupler is designed to be single

moded at the wavelength of operation, 800nm. In the coupling region the width of the ridges and the gap between the two waveguides is 2 μm . The length of the coupling region is around 400 μm . Fabrication was performed by optical lithography and inductively coupled plasma (ICP) etching. The input output waveguides are separated from each other with an angle of 1.1° as shown in Figure 3.5(d). The splitting ratio device used in the HOM interference experiments is estimated to be 45:55% by classical measurements done by Loyd McKnight.

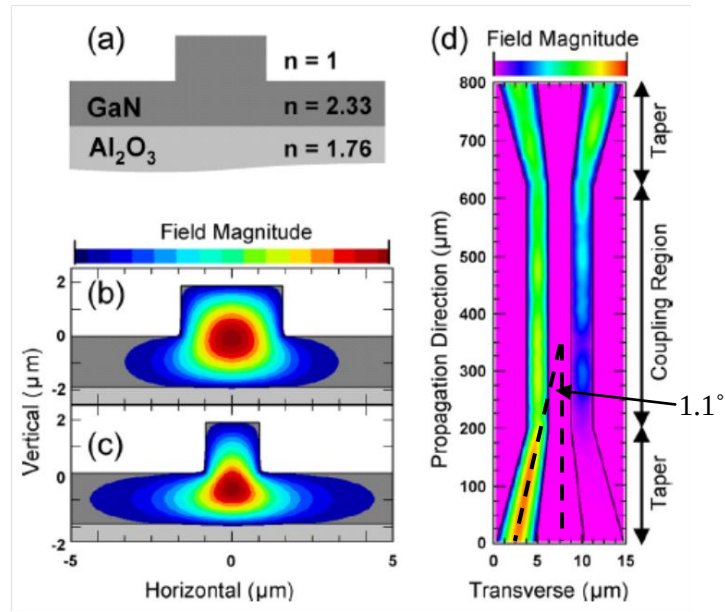


Figure 3.5: (a) The schematic of the ridge waveguide showing the layer structure. (b) The fundamental TE mode profile at 800nm wavelength of the ridge waveguide at the facet. (c) the TE mode profile of the waveguide at the coupling region. (d) Beam propagation simulation of the whole coupler structure. (Source: [17])

The experimental setup used is depicted in Figure 3.6. The single photon source is a Type-I spontaneous parametric down conversion source realized by a β -barium borate crystal pumped by 402nm wavelength light which was already built and being used by our colleagues in University of Bristol. The generated photon pairs at 804nm wavelength are coupled to polarization maintaining fibers (PMF). One arm of the source is delayed with respect to the other using a motorized translation stage. The single photons output from the fibers are focused on the device input via a collimating

lens, a mirror and a focusing lens having 20 times magnification as shown in the figure. The output of the device is coupled to PMF by a similar manner this time using a 50 times magnification lens after the chip. The fibers are then fed to avalanche photo diode (APD) single photon detectors. The electrical signals produced by the detectors are processed by a time interval analyzer (TIA) for coincidence analysis. The coincidence count rate when the source is directly connected to the detectors is measured as around 10,000 per second.

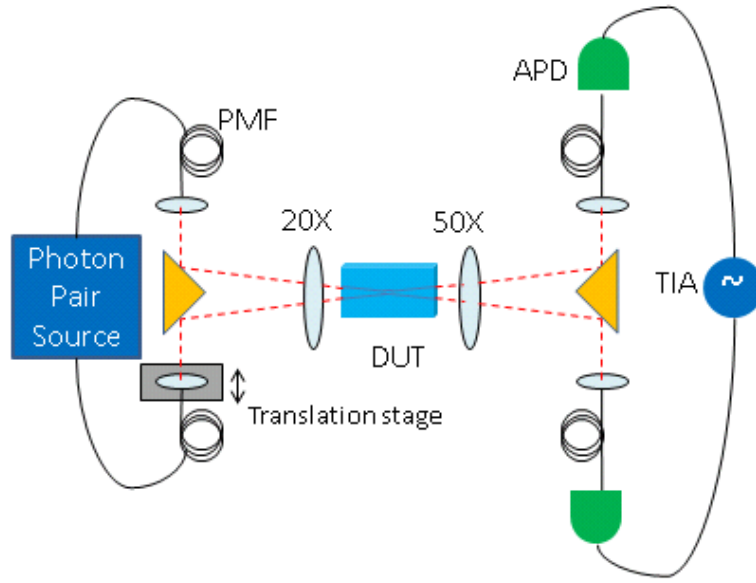


Figure 3.6: The experimental setup used for photon interference experiment. The single photon source is a Type-I SPDC source producing photon pairs at 804nm wavelength. Single photons are coupled in and out of the chip using lenses and mirrors. APDs are used for detection and the coincidence events are counted by a time interval analyzer.

The position of translation stage is changed by steps of $15\mu\text{m}$ while the coincidence counts are integrated over 35 seconds at each step. Accidental counts are computed at each step using the single photon counts of the two channels, C_1 and C_2 by the following formula

$$ACC = C_1 C_2 W \quad (3.9)$$

where W is the coincidence detection window width which is 5nsec for our case. The

mean value of the *ACC* through all data points is 3.7 counts per second. The coincidence counts (after the accidentals have been subtracted) are plotted in the HOM dip shown in Figure 3.7. The value of error bars is the square root of the coincidence count value due to the Poissonian statistics of the photon source. The red curve is the Gaussian fit to the coincidence counts as explained in Section 3.2. The visibility of the HOM dip is calculated to be 96%. This demonstrates that the modes in the waveguide are indistinguishable to a high degree. The same source was used in a HOM dip measurement on a silica-on-silicon directional coupler performed by Pete Shadbolt and the reported visibility is 0.978 ± 0.007 .

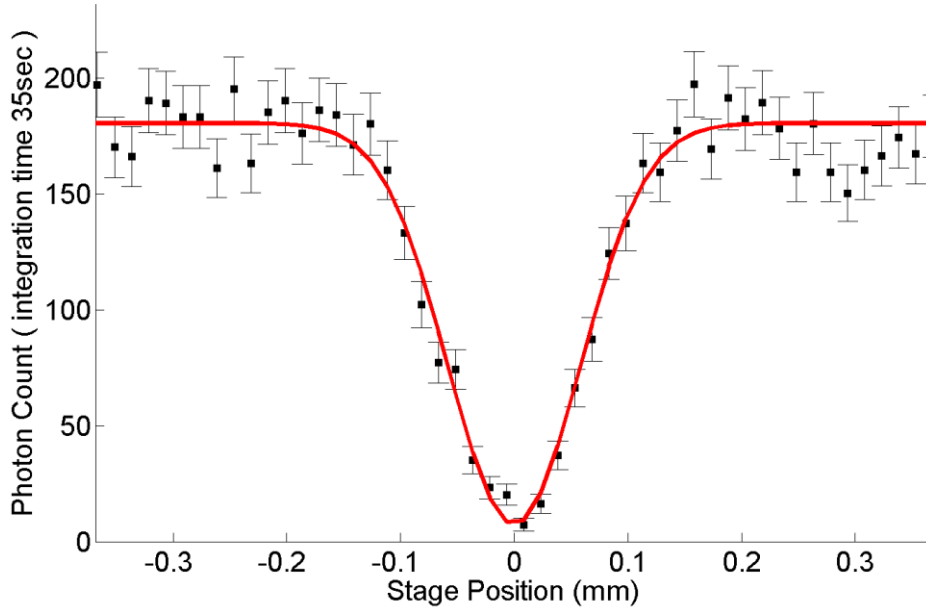


Figure 3.7: The coincidence counts with respect to the stage position are plotted in black (accidental counts are subtracted). The red curve is the fit to the coincidence data having a visibility of 96%.

3.5 Summary

In this chapter we have presented background information about non-classical interference behaviour of photons in a beam splitter. We have explained the Hong-

Ou-Mandel interference experiment which gives a measure of the indistinguishability of the photons. Then we have explained the directional coupler which is the integrated optics equivalent of a beam splitter. And finally we presented our experimental results of quantum interference in a GaN directional coupler. An interference visibility of 96% is achieved which shows the suitability of GaN platform for quantum photonics applications.

3.6 Conclusions

This experiment and related publication [19] is the first demonstration of non-classical interference in a III-V semiconductor material system. III-V semiconductors are important in integrated photonics due to their direct band-gap providing on-chip light generation [120],[121]. Their band gap properties can be engineered by doping with other materials to tailor the emission wavelength [122]. We have particularly chosen GaN material due to its wide transparency window from $\sim 365\text{nm}$ to $\sim 13.6\mu\text{m}$ [64] which makes it suitable for wide applications in this range. Single photon source [90] and single photon detector [123] has already been demonstrated on GaN both operating in the blue and ultra-violet regime. The interference measurement we demonstrate here encourages the research on GaN integrated quantum photonics circuit operating in the same wavelength regime. So far wavelengths of around 800nm [35] or around 1550nm [34] have been used in integrated quantum photonics circuits however using lower wavelengths the coherence length of the photons will be reduced and higher resolution can be achieved for example in quantum metrology applications [8]. The particular reason we have chosen 800nm wavelength is the availability of single photons source and detectors.

Here a waveguide structure having large cross sectional dimensions is used so that the mode size of the waveguides is large and we can couple light easier. However optimizations in the layer structure and the ridge dimensions can be made to achieve devices having a smaller footprint.

Chapter 4

Four-wave Mixing in Silicon nano-Wires

We have made an introduction to single photon sources and four-wave mixing in the previous chapter. In this chapter we will focus on spontaneous four-wave mixing in silicon nano-wires. Theoretical concepts of the process and non-linear model for silicon nano-wire waveguides for high intensity excitations will be presented. Experimental results obtained for a straight waveguide and ring resonator will be shown. We demonstrate the enhancement in pair generation rate and coincidence-to-accidental ratio by the use of a ring cavity. The detrimental effects of two-photon absorption in silicon are addressed by applying reverse bias across the waveguide to sweep the free carriers away. This brought an enhancement in the pair count rate by a factor of two

4.1 Four-Wave Mixing in Silicon nano-wire waveguides

Silicon nano-wires (SnW) are waveguides, typically having dimensions of few hundred nanometres in width and height, fabricated on silicon-on-insulator (SOI) material. The refractive index of silicon and silica is about 3.5 and 1.45 respectively, around telecommunication wavelengths. This produces a large index contrast which allows tight confinement of light within the guiding layer and allows lossless guiding of light around sharp corners (less than $5\mu\text{m}$ radius). The fabrication technology is being developed over many years for the electronics industry hence very small feature sizes (10nm) are already achievable. Also cost effectiveness is another advantage of silicon photonics.

Recent developments in the field of quantum photonics made the scientist to search for platforms where sources, circuits and detectors can be implemented on a monolithic platform. Silicon is one of the candidates for this. Recently silicon quantum photonic circuits [34] and detectors [46] have been demonstrated. The two photon interference experiment in [34] was performed using an bulk type-II SPDC source and super-conducting nano wire detectors. Both of these external components occupy a quite large space on the optics table and the photons need to be coupled by optical fibers in and out of the chip. The motivation of integrated silicon quantum photonics is to fabricate sources, circuits and detectors all in one substrate while we focus on developing the integrated source. Example applications of a fully integrated quantum photonics chip can be implementing Shor's algorithm [30], Grover's search algorithm [124] or quantum simulations [125].

Single photon generation in silicon nano-wires has gained interest in the first decade of 2000s. Having a high value of $\chi^{(3)}$ non-linearity compared to other materials (see Table 2-1) and small effective mode area which means high optical intensity (boosting the non-linear effect), makes silicon particularly interesting for FWM applications. A theoretical study of photon pair generation in silicon nano-wires is presented in 2006 [42], and several results on straight waveguides [43], [52–54], [56], [57], ring resonator [55] have been shown. Time-bin [56] and polarization [43] entanglements with 95% and 83% visibilities respectively, have also been demonstrated. Also a significant result is the measurement of four-fold coincidences of photons generated by independent sources [126] although it has relatively low visibility of 73%.

Two important performance parameters in FWM experiments are the photon count rate, and the coincidence to accidental ratio (CAR) which is the ratio of the number of true single photon detections to the false detections. The CAR is dependent on the performance of the detectors as well as the device.

4.2 Theory of Four-Wave Mixing in Silicon nano-wires

In this section we will provide theoretical aspects of the process of spontaneous four-wave mixing (sFWM) in silicon nano-wires. In the sFWM, two photons from the pump beam are annihilated and a pair of photons with different wavelengths called the signal and the idler is generated under energy and momentum conservation constraint. The relationship between the frequencies of the pump and the photon pairs are given by:

$$\omega_{idler} + \omega_{signal} = 2\omega_{pump} \quad (4.1)$$

From the conservation of momentum the three photons should also satisfy

$$k_{idler} + k_{signal} = 2k_{pump} \quad (4.2)$$

In the next section we are going to discuss phase matching condition and parasitic non-linear effects in silicon, namely the two-photon absorption and free-carrier effects.

4.2.1 Phase Matching

Because FWM is based on the co-propagation of the pump signal and idler photons mismatch between the phases of the waves will lead to destructive interference and reduce the efficiency of the FWM process. Propagation of the three waves with the same speed is dependent on the dispersion of the waveguide which can be satisfied by proper design of the cross sectional dimensions. When operating in high power regimes, like in sFWM, one should also consider the self-phase modulation, and cross-phase modulation effects. As a high intensity beam propagates it also modifies the optical properties of the medium such as the refractive index. This modification of the optical properties induces a change in its own phase. This is called self phase modulation (SPM) [127]. Similarly there is cross-phase modulation (XPM) [128], which is the effect of the high power beam on the phase of the generated signal and idler photons. Due to these effects under high pump powers an additional term is needed, therefore equation (4.2) has to be modified as:

$$k_{idler} + k_{signal} + 2\gamma P_{pump} = 2k_{pump} \quad (4.3)$$

Where the effective non-linearity of the waveguide γ is defined as,

$$\gamma = \omega_{pump} n_2 / c_0 A_{eff} \quad (4.4)$$

Here c_0 is the speed of light in vacuum, A_{eff} is the effective area of the guided mode in pump wavelength and n_2 is the second order non-linear refractive index coefficient. The refractive index of the medium is related to the pump intensity, I_{pump} by the following equation

$$n = n_0 + I_{pump} n_2 \quad (4.5)$$

Here n_0 is the linear refractive index of the material. n_2 is related to third order susceptibility, $\chi^{(3)}$ by the following equation

$$n_2 = \frac{12\pi^2 \chi^{(3)}}{n_0 c_0} \quad (4.6)$$

The table below shows the typical parameters for silicon around 1550nm wavelength.

Table 4-1: Parameters for the refractive index of silicon

	Value	Unit	Definition
n_0	3.477	N/A	Linear refractive index
$\chi^{(3)}$	3.5212×10^{-11}	$m^3/W.sec$	Third order susceptibility

The phase mismatch is defined by

$$\Delta k = \Delta k_L - 2\gamma P_{pump} \quad (4.7)$$

where Δk_L is the linear phase mismatch defined by

$$\Delta k_L = 2k_{pump} - k_{idler} - k_{signal} \quad (4.8)$$

The maximum efficiency of FWM occurs when phase mismatch, Δk is zero. This requires the condition $\Delta k_L > 0$ which leads to

$$k_{pump} > \frac{k_{idler} + k_{signal}}{2} \quad (4.9)$$

Knowing the relation between the propagation constants of the signal, idler and pump fields $k_{idler}, k_{pump}, k_{signal}$

$$k_{idler} < k_{pump} < k_{signal} \quad (4.10)$$

in order to satisfy equation (4.9) the waveguide has to have anomalous group velocity dispersion, D , at the pump wavelength.

$$D = -\frac{2\pi c_0}{\lambda^2} \frac{\partial^2 k_{pump}}{\partial \omega^2} > 0 \quad (4.11)$$

In SnWs it is possible to achieve anomalous dispersion by choosing the appropriate dimensions of the cross section of the waveguide. A study on designing the dispersion of SnWs for phase matching is presented in [129].

4.2.2 Two Photon Absorption and Free Carrier Effects

Two-photon absorption (TPA) is the process of two photons being absorbed in order to excite an electron in the valance band to conduction band. This excited electron is called a free carrier. This is illustrated in Figure 4.1 In this transition a phonon is also required due to the indirect nature of the band gap of silicon. The absorbed two photons should have a total energy higher than the bandgap energy ($E_g \approx 1.12eV$ for silicon) to generate a “TPA induced free carrier”. For silicon TPA occurs for wavelengths below $2.2\mu m$. The absorption is proportional to the intensity of the field propagating in the medium through so called the TPA coefficient, $\tilde{\beta}$. Several studies have been performed to investigate the TPA coefficient of silicon [130] [131]. A typical value for silicon at $1550nm$ wavelength is $\sim 1cm/GW$. The loss caused by the

TPA is negligible in most cases; however it is the free carriers generated by TPA that plays a major role in the observed non-linear effects under high field intensities.

The two major effect of the free carriers are the free carrier absorption (FCA) and free carrier dispersion (FCD). FCA as the name suggests is the absorption of a photon by a free carrier in order to change its state within the conduction band as illustrated in Figure 4.1. FCD is the modification of the refractive index of the medium by the density of free carriers which in turn affects the dispersion of the medium.

The TPA and free carrier effects are utilized in applications such as photon detection [132], auto-correlation [133] or optical modulation [134], whereas it is seen as a detrimental effect for many applications such as, Raman amplification [62], wavelength conversion [135] and photon pair generation [52].

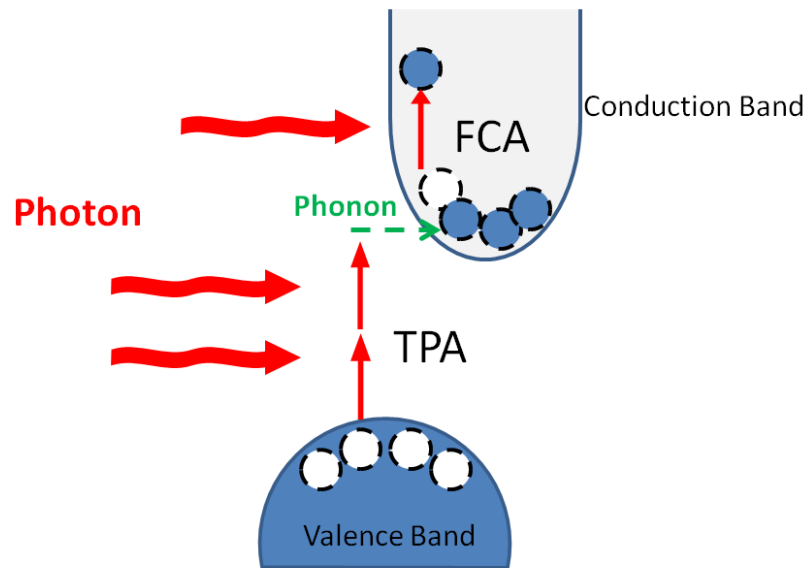


Figure 4.1: Illustration of the TPA and FCA processes. An electron in the conduction band absorbs two photons and a phonon to jump to the conduction band in the TPA process. In FCA an electron in the conduction band absorbs a photon to change its state within the conduction band.

4.2.3 Noise Mechanism in Silicon nano-Wires

Any photon detected which is not from a generated pair is considered as noise and degrade the quality of the source. In a FWM experiment the output of the device are filtered before sending to the detectors, therefore by noise photons we only consider the ones that fall in to the spectral window defined by the output filters. The noise photons can cause false coincidence counts if they arrive within the same time bin of the detector.

Raman noise is one of the non-linear noise sources in silicon waveguides however it is not as pronounced as the Raman noise in silica fiber based sources discussed in section 2.1.2.2. In [52], it has been reported that the noise measured in the silicon nano-wire is at least a order of magnitude lower than in a silica fiber. However in a typical experiment because silica fibers are used in between the elements of the setup the Raman noise originating from the setup itself becomes a source of noise.

Brillouin scattering is another noise mechanism. It is the scattering and the conversion of the frequency of the propagating wave caused by the time dependant changes in the properties of the material. These changes can be caused by acoustic, magnetic waves or temperature gradients. Raleigh scattering, FCA and TPA also introduce noise through coupling to a thermal phonon reservoir [42] and contributing to the black body radiation. However this noise is much weaker and has been shown to be negligible [52].

As a summary Raman scattering and Brillouin scattering are the main sources of noise in silicon waveguide based devices. There has not been a study to truly distinguish between these two types of sources. In a typical FWM experiment the Raman noise contribution from the silica fibers in the setup dominate the noise

resulting from the silicon waveguide itself.

4.2.4 Modelling Non-Linear Effects in a Silicon Nano-wire

This section will present a model for the propagation of the pump beam in silicon nano-wire that includes the non-linear effects, TPA, FCA, FCD and thermo-optic effect (TOE). Then in the next section we are going to apply this model to a ring resonator. The reason for using this model is to find the relationship between the power injected to the chip and the power inside the ring under these non-linear effects. The experiment gives us the coincidence count data with respect to input power. We expect the coincidence counts to have a quadratic dependence on the power inside the ring. Therefore knowing the relation between the input power and the power inside the ring we can fit our experimental data to find the pair generation rate. This model is described in [136].

Here we are analyzing a straight, single moded silicon nano-wire, excited with a monochromatic continuous wave. The electric field is defined as

$$E(z) = \bar{\omega}A(z)\exp(j\beta_0z) \quad (4.12)$$

In this equation $A(z)$ is the complex amplitude of the field propagating in z direction and j is the imaginary number. β_0 is the propagation constant given by n_0k_0 , where n_0 is the linear refractive index and k_0 is the propagation constant in free space. $\bar{\omega}$ is defined by

$$\bar{\omega} = \left(\frac{\mu_0}{\varepsilon_0}\right)^{\frac{1}{4}} (2n_0)^{1/2} \quad (4.13)$$

where μ_0 and ε_0 is the permeability and permittivity of the vacuum respectively. The propagation of the field in a silicon nano-wire is governed by the following set of

equations.

$$\frac{1}{A} \frac{dA}{dz} = -\frac{\alpha}{2} - \left(\frac{\tilde{\beta}}{2} - j\tilde{\gamma} \right) |A|^2 - \left(\frac{\xi_r}{2} - j\xi_i \right) |A|^4 \quad (4.14a)$$

$$\tilde{\gamma} = k_0 n_2 \quad (4.14b)$$

$$\xi_r = \frac{\sigma \tau \tilde{\beta}}{2 \hbar \omega_0} \quad (4.14c)$$

$$\sigma = \sigma_r \left(\frac{\omega_r}{\omega} \right)^2 \quad (4.14d)$$

$$\omega_r = \frac{2\pi c_0}{\lambda_0} \quad (4.14e)$$

$$\xi_i = \frac{\mu}{2} \xi_r \quad (4.14f)$$

$$\mu = 2k_0 \left(\frac{\sigma_n}{\sigma_r} - \frac{\tilde{\beta} \kappa \nu}{\xi_r \tilde{C} \rho} \right) \quad (4.14g)$$

The parameters used are defined as follows:

α : attenuation constant,

k_0 : free space wavevector of the propagating continuous wave field,

n_2 : Kerr coefficient,

τ : effective free-carrier life time,

$\tilde{\beta}$: TPA coefficient,

$\hbar$: reduced Plank constant,

ω : angular frequency,

σ_r : coefficient relating the free carrier absorption to the free carrier density,

ω_r : the angular frequency corresponding to 1550nm wavelength light,

c_0 : speed of light in vacuum,

λ_0 : wavelength of the propagating continuous wave field,

σ_n : coefficient relating the change of index to the free carrier density,

κ : thermo-optic coefficient,

ν : thermal dissipation time,

$\tilde{C}$: thermal capacity,

ρ : density of silicon.

The table below is typical parameters for a silicon nano-wire waveguide.

Table 4-2: Typical values for some of the material properties of silicon

Parameter	Value	Units
n_0	3.484	N/A
n_2	6×10^{-5}	cm^2/GW
$\tilde{\beta}$	0.5	cm/GW
σ_r	1.45×10^{-21}	m^2
σ_n	5.3×10^{-27}	m^3
κ	1.86×10^{-4}	K^{-1}
C	705	J/kgK
ρ	2300	kg/m^3

The value of the free-carrier life time τ and the thermal dissipation time, ν depends on the geometry of the structure. Typical values for the linear attenuation constant, α for silicon nano-wire waveguides is are between 0.5/cm and 0.8/cm (1-3dB/cm).

It is possible to have an analytical solution to equation (4.14a) when the attenuation of the field due to TPA is assumed to be negligible ($\tilde{\beta} = 0$). In this case the solution to (4.14a) is of the form

$$A(z) = \sqrt{I(z)} \exp(j\phi(z)) \quad (4.15a)$$

$$I(z) = \frac{I_0 \exp(-\alpha z)}{\sqrt{1 + I_0^2 \left(\frac{\xi_r}{a}\right) (1 - \exp(-2\alpha z))}} \quad (4.15b)$$

$$\phi(z) = \phi_0 + \beta z + \tilde{\gamma} I_0 L_{eff}(z) - \frac{\xi_i}{\xi_r} \ln\left(\frac{I_0}{I(z)} - \alpha z\right) \quad (4.15c)$$

$$L_{eff}(z) = \frac{\tan^{-1}\left(I_0 \sqrt{\frac{\xi_r}{a}}\right) - \tan^{-1}\left(I(z) \sqrt{\frac{\xi_r}{a}}\right)}{I_0 \sqrt{a \xi_r}} \quad (4.15d)$$

The assumption of $\tilde{\beta} = 0$ is valid in a structures with $\alpha \xi_r \gg \tilde{\beta}^2$. In these equations I_0 is the intensity of light at the input ($z=0$), ϕ_0 is the phase at the input and β is the propagation constant of the mode.

4.2.5 Modelling non-linearities in Silicon Ring Resonator

Here we will apply the non-linear equations for the SnW described in the previous section to a SnW ring resonator. Ring resonators enable the build up of power inside the device and provide enhancement of FWM. For consistency, initially we will present the derivation of the linear model of the ring resonator. Figure 4.2 shows a schematic of a ring coupler.

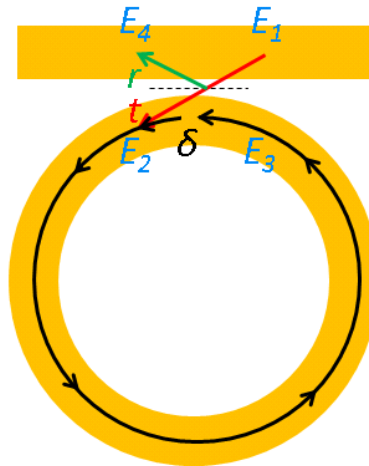


Figure 4.2: The diagram of a ring coupler showing the fields E_1 , E_2 , E_3 and E_4 in the coupling region of the ring coupler; the reflection and transmission coefficients r, t and the round trip amplitude transmission factor τ .

The relation between the fields E_1 , E_2 , E_3 , and E_4 are given by the following set of equations:

$$E_2 = jtE_1 + rE_3 \quad (4.16a)$$

$$E_3 = \delta E_2 \exp(-j\phi_{ring}) \quad (4.16b)$$

$$E_4 = jtE_3 + rE_1 \quad (4.16c)$$

where t is the transmission coefficient and $r = \sqrt{1 - t^2}$ is the reflection coefficient of the coupler, ϕ_{ring} is the total phase accumulated in one cycle inside the ring and δ is the amplitude transmission factor in one cycle. equations (4.16a) & (4.16b) lead to

$$E_2 = \frac{jtE_1}{1 - r\delta \exp(-j\phi_{ring})} \quad (4.17)$$

If we write (4.17) in terms of the intensities $I_n = |E_n|^2 / \bar{\omega}^2$ we get

$$I_2 = I_1 \frac{t^2}{(1 - r\delta \cos \phi_{ring})^2} \quad (4.18)$$

To derive the non-linear model [136], we have to redefine the relation between E_2 and E_3 taking into account the non-linear effects inside the ring. Using the non-linear relationship of (4.15a) we can express E_3 in terms of E_2 as

$$E_3 = \sqrt{\frac{|E_2|^2 \delta^2}{\sqrt{1 + |E_2|^4 \left(\frac{\xi_r}{\delta^2}\right)} (1 - \delta^4)}} \exp(-j\phi(E_2)) \quad (4.19)$$

Here $\phi(E_2)$ is the phase term obtained from (4.15c) by setting ϕ_0 to 0, and z to the perimeter of the ring. Here in order to make things simple we are going to assume $\phi(E_2) = 2\pi m$, where m is an integer and drop the term $\exp(-j\phi(E_2))$ from equation (4.19). In this case we also have the relation $|E_2| = E_2$. This physically means that we are solving the non-linear equations for the resonance dip wavelengths only. This will be the case in our experiments that we are going to present in later

sections. Substituting equation (4.19) into equation (4.16a) we get the following:

$$E_2 = jtE_1 + r \sqrt{\frac{E_2^2 \delta^2}{\sqrt{1 + E_2^4 \left(\frac{\xi_r}{\delta^2}\right)(1 - \delta^4)}}} \quad (4.20)$$

In a simpler form the equation (4.20) can be written as:

$$E_2 = jtE_1 + r\delta E_2 \left(1 + \Lambda E_2^4\right)^{-1/4} \quad (4.21)$$

where we have defined a new variable,

$$\Lambda = \left(\frac{\xi_r}{\delta^2}\right)(1 - \delta^4) \quad (4.22)$$

The relationship between the intensities $I_1 = |E_1|^2$ and $I_2 = |E_2|^2$ can be derived from equation (4.21) as:

$$I_1 = \frac{I_2}{t^2} \left(1 - r\delta \left(1 + \Lambda I_2^2\right)^{-1/4}\right)^2 \quad (4.23)$$

We will consult equation (4.23) when extracting the pair generation rate from our experimental results.

4.3 Experimental Results

4.3.1 Experimental Setup

The experimental setup is illustrated in Figure 4.3. The external-cavity diode laser (ECDL) is a Photonetics Tunics tunable laser having a tuning resolution of 1pm and a maximum output power around 7mW. In the case when continuous wave excitation is needed, the output of the tunable laser is passed through an Erbium doped fiber amplifier (EDFA) which amplifies the signal up to 100mW. The output of EDFA is first filtered by a circulator-fiber Bragg grating (FBG) circuit and then by a dense wavelength division multiplexer (DWDM). The FBG has a 3dB bandwidth of around

0.52nm and the DWDM has a channel spacing of 200GHz. By this way more than 100dB filtering is achieved.

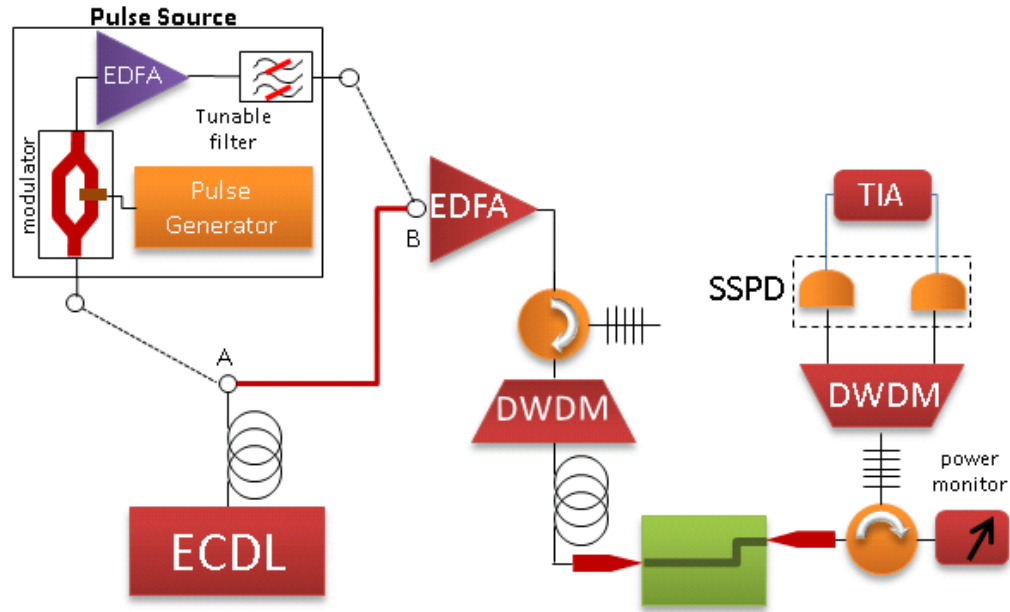


Figure 4.3: The experimental setup used in FWM in silicon nano-wire experiments. The experiments are performed for both continuous wave and pulsed excitation. In pulsed excitation case a modulator setup is inserted between the tunable laser and the EDFA.

This high power beam is then fed to the input of the silicon device under test (DUT) by a single mode lensed fiber which has a spot-diameter of $2\mu\text{m}$. The output light from the DUT which includes the pump beam, the generated photon pairs and noise, is then collected by another equivalent fiber lens. It is passed from another circulator-FBG setup where the filtered pump is used as a monitor power. The remaining beam is passed through another DWDM which separates the photon pairs and the remaining pump into different channels. The channels carrying single photons are fed to the two channels of the superconducting single photon detector (SSPD). The electrical pulses generated by the detector are analyzed by Picoharp time interval analyzer (TIA). When pulsed excitation is needed the modulator setup is inserted between the nodes marked with A and B as shown in Figure 4.3. In this setup a 40Gb/s LiNbO₃

modulator is used together with an Agilent 81134 pulse generator. The modulated pulse is amplified by a low noise amplifier having a 13dB maximum output power and then filtered with a tunable filter. All experiments are performed on silicon chips that are fabricated by Toshiba Corporation, Japan. The experiments were performed by me, Damien Bonneau and Mark G. Thompson from University of Bristol and Chandra M. Natarajan from Herriot-Watt University.

4.3.2 The Super-Conducting Single Photon Detectors

The Super-Conducting Single Photon Detectors (SSPD) used are designed and fabricated by Heriot-Watt University, UK and TU Delft, Netherlands [137]. They are composed of thin NbTiN nanowires of 100nm width fabricated on a silicon substrate. The substrate is cooled down to cryogenic temperatures in a cryocooler and the nanowires are biased just below the critical current density. When a photon hits on the nano-wire it generates a local spot on which the resistance converts to normal from super-conducting regime. The current in the wire tends to flow around the normal resistance region hence increasing the current density. When the current density exceeds the critical current density the wire becomes resistive. This causes an electrical glitch which is amplified and sent for electrical detection. By this way the detection of a photon is converted into an electrical pulse. This is a non-photon-number-resolving detector meaning a multiple photons simultaneously incident on the detector cannot be distinguished from a single photon. The efficiencies of our detectors are 8% and 18%. Through the experiments the SSPDs are operated at a dark count rate of 1000Hz.

4.3.3 Processing the Coincidence Histogram

The output electrical pulses from the SSPDs are input to the time interval analyzer (TIA) which counts the number of pulses received from each channel as well as the cross-correlation between them. Determining the exact time of arrival of a photon is not possible due to the jitter in both the SSPDs and the TIA. So there is an uncertainty between the time the TIA counting a photon, and the actual time of arrival of the photon at the detector. This brings the necessity of defining a window inside which if two electrical pulses are counted they are considered as coincidence counts. We will refer to this window as the coincidence window.

The accidental counts, ACC are coincidence detections caused by non-correlated photons (photons that are not of a generated single pair). The accidental counts are resultant from the noise photons generated in the device itself or in the equipments in the setup (as discussed in section 4.2.3). While processing the data the average accidental counts are calculated from the acquired histogram (see section 4.3.3.2 and 4.3.3.1) and their contribution is subtracted.

The two important parameters that we want to obtain from the experiments are the pair generation rate coefficient, which is related to the actual coincidence counts (the contribution of the accidentals subtracted), CC and the coincidence to accidental ratio, CAR (defined below). These two parameters are obtained from the processing of the coincidence histogram which will be discussed below. The pair generation rate coefficient has a unit of Hz/mW^2 and is the number of photons generated in a second per square of the input power in milliwatts.

We know that the number photons generated at a time in a FWM process is going to have a Poissonian distribution. Therefore the computed CC has an uncertainty referred as errCC given by:

$$errCC = \sqrt{CC} \quad (4.24)$$

Similarly the error on ACC, $errACC$ is:

$$errACC = \sqrt{ACC} \quad (4.25)$$

The CAR is simply given by:

$$CAR = \frac{CC}{ACC} \quad (4.26)$$

The error on the calculated CAR, $errCAR$ can be calculated from the error propagation formula given by:

$$errCAR^2 = \left(\frac{\partial CAR}{\partial CC} \cdot errCC \right)^2 + \left(\frac{\partial CAR}{\partial ACC} \cdot errACC \right)^2 \quad (4.27)$$

or equivalently

$$errCAR = \sqrt{\left(\frac{errCC}{ACC} \right)^2 + \left(\frac{errACC \cdot CC}{ACC^2} \right)^2} \quad (4.28)$$

We have performed experiments for both pulsed and continuous wave pumps. These two cases results in slightly different histograms and needs to be processed separately. We will describe the extraction of CC and CAR for both cases.

4.3.3.1 Continuous Wave Excitation

The acquired histograms have a Gaussian distribution as shown in Figure 4.4(a). The width of this Gaussian curve is dependent on the jitter of the detectors and the TIA as described in the previous section. The blue curve is the Gaussian fit to the coincidence histogram. The jitter of our detection equipment results in a FWHM of the Gaussian to be ~ 230 ps. When computing the total coincidence counts it is necessary to choose

an integration window (such as the one shown with red dashed lines) around the peak. The computed values for CC and CAR depend on the chosen window width. Figure 4.4(b) shows this dependence. The right hand side axis is for the total coincidence counts. The total coincidence counts increases with the duration of the integration window and saturates for large coincidence window durations. On the other hand the CAR values (left axis) constantly decrease as the coincidence window duration increases.

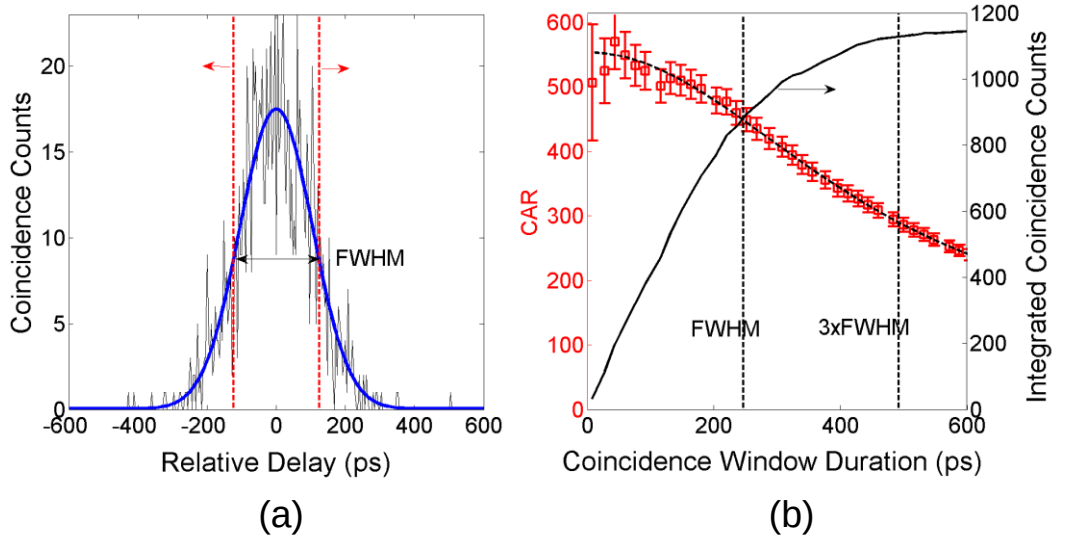


Figure 4.4: (a) the acquired coincidence histogram. (b) the dependence of the computed coincidence counts and the CAR on the coincidence window duration.

The collection efficiencies of the two detector channels (the total loss the photons undergo from after being generated to being detected including the detection efficiency of the detector) is an important parameter. The collection efficiencies help us estimate the number of pairs generated inside the chip from the coincidence counts by subtracting the effects of the losses and the detector efficiencies. In other words it is used to estimate the counts if the transmission from the chip to the detectors and the detector efficiencies were unity. The collection efficiencies are found by the single photon counts of the two channels and the coincidence counts. The calculation details

will be described in section 4.3.4.1. The coincidence counts used for this calculation has to include all the coincidence counts, therefore the coincidence window width is chosen to be 3 times the FWHM of the Gaussian fit (which covers the whole peak) in the calculation of channel efficiencies. While calculating the coincidence counts we first calculate the accidental counts, by averaging the counts far away from the peak. We normalize these counts to find the average accidental counts per the coincidence window width to find the value of ACC. Then we integrate the counts inside the main peak over the coincidence window. We subtract the found average accidental counts from this count to find the actual coincidence counts, CC.

On the other hand for calculating the CAR value there is not a standardized choice of coincidence window width. The coincidence counts and the CAR values reported in the literature are performed with arbitrary chosen various coincidence window widths. Here we would like to use the full width at half maximum (FWHM) of the fitted Gaussian (this width depends on the jitter in detection equipment) as the coincidence window width for calculating the CAR. Hence we calculate CC and ACC in this coincidence window to calculate the CAR according to equation (4.26). And we report the on-chip count rate for the same coincidence window width by normalizing this CC by the collection efficiencies found in the first step.

4.3.3.2 Pulsed Excitation

Using pulsed excitation provides reaching high amount of peak power inside the chip compared to the continuous wave excitation. Pulsed excitation help overcoming the effects of FCA. FCA related effects have a transient response depending on the free-carrier life time and by keeping the pulses shorter than the free-carrier life times [138] the FCA effects can be reduced . Figure 4.5 shows a sample histogram acquired

from the pulsed experiments showing the number of counts integrated over 60 seconds. The histogram in this case has periodic peaks with the same period of the pulses. The main peak is the coincidences obtained from the photon pairs whereas the others are the accidental counts. The inset of the figure shows a closer look to the main peak having a Gaussian like distribution. A Gaussian curve is fitted to the main peak to find its FWHM. The accidental counts, ACC, are found by averaging 10 accidental peaks over a window width of $3 \times \text{FWHM}$. Using a similar method to the one described in previous section, we find the actual coincidence counts, CC, by integrating the main peak inside a window of width $3 \times \text{FWHM}$ (the sum of red and black counts in the inset of Figure 4.5) and subtracting the accidentals. Using the single photon counts and the found value for CC, the channel collection efficiencies are extracted.

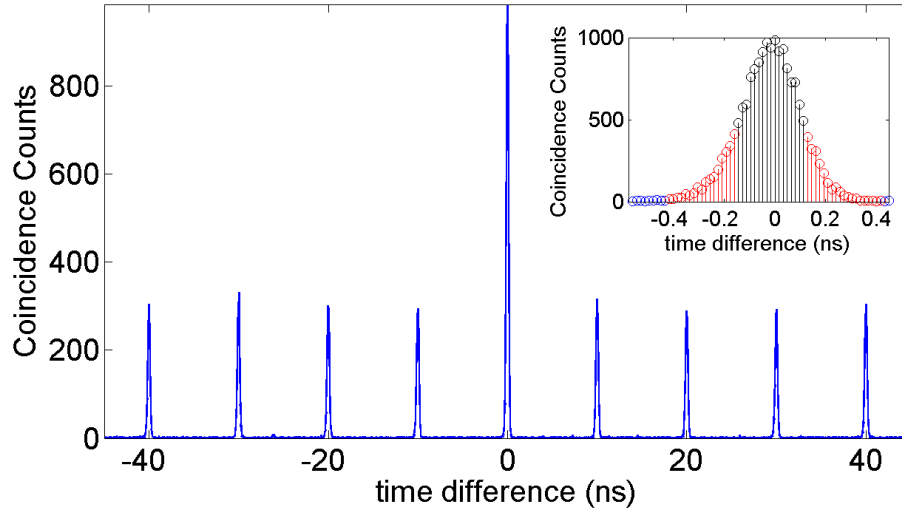


Figure 4.5: A sample coincidence histogram acquired from a pulsed source experiment. The inset shows a closer look at the middle peak. The coincidences highlighted with black cover a region of FWHM of the fitted Gaussian while the red and the black counts cover a region of $3 \times \text{FWHM}$.

For the calculation of the CAR we use a coincidence window width of FWHM of the fitted Gaussian (black counts in the inset of Figure 4.5) from the calculated values of CC and ACC over this window. And similar to the previous section to calculate the

on-chip count rate corresponding to this coincidence window we normalize the values of CC by the calculated channel collection efficiencies.

4.3.4 Photon Pair Generation in a Silicon Nano-Wire Waveguide

The cross section of the silicon waveguide used is 450nm x 220nm and the length is 3mm. It has an oxide cladding layer of 90nm on top and 30nm on the sidewalls. It has anomalous dispersion at the operation wavelength proven by optical mode simulations. In both the input and the output there are 3μm wide polymer spot-size converters in order to efficiently couple the light in and out. We have investigated photon pair generation in both continuous-wave and pulsed excitation cases.

4.3.4.1 Continuous Wave Excitation

The waveguide is pumped at 1550nm wavelength. The injected power is calculated by subtracting losses from the measured monitor power. These losses are calculated as 5dB, which includes the propagation loss of 0.6dB, the output facet coupling loss of 2dB and the loss of the circulator, 2.4dB. The single photon counts and the coincidence counts are expected to follow the equations given by:

$$C_1 = dc_1 + (\alpha_{sp}P_{in} + \beta_p P_{in}^2)\eta_1 \quad (4.29)$$

$$C_2 = dc_2 + (\alpha_{sp}P_{in} + \beta_p P_{in}^2)\eta_2 \quad (4.30)$$

$$CC = \alpha_{cc}P_{in} + \eta_1\eta_2\beta_p P_{in}^2 \quad (4.31)$$

Here P_{in} is the power injected to the device and C_n , dc_n , η_n are respectively the single photon counts, dark counts and collection efficiency of detector channel n . The

collection efficiency includes the detection efficiency and all the losses the generated photons undergo until they reach the detector. α_{sp} and α_{cc} are the linear noise coefficients for single photon and coincidence counts, and β_p is the photon pair generation rate coefficient.

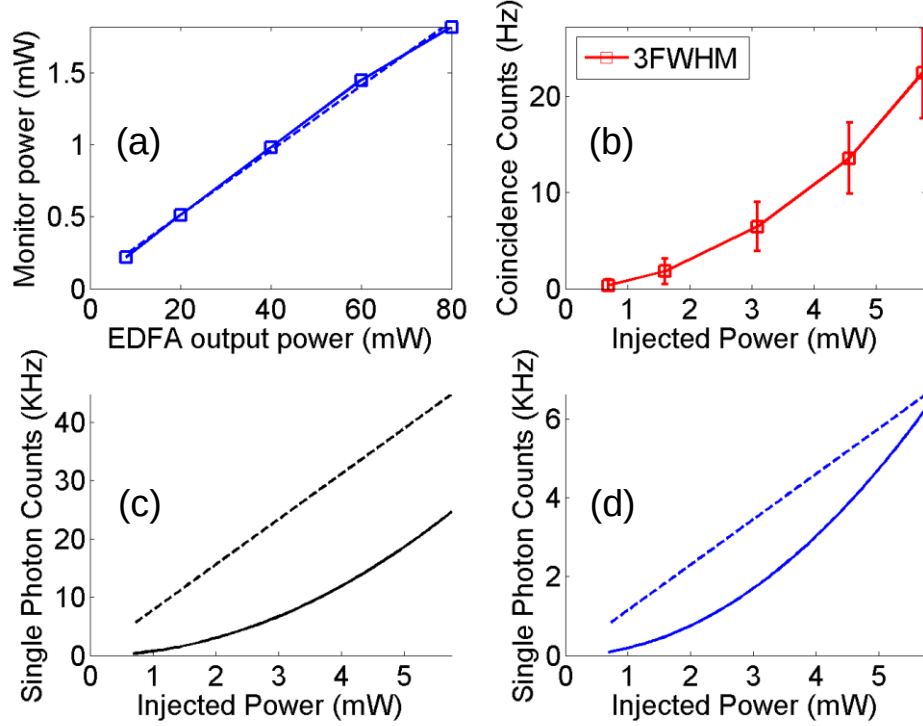


Figure 4.6: (a) EDFA power versus the monitor power. (b) Coincidence counts (3FWHM) versus the power injected to the waveguide. (c) & (d) The linear and quadratic terms of the single photon counts versus the power injected to the waveguide for the two channels of the detector.

Figure 4.6(a) shows the EDFA power versus the monitor power. The relation between the two being linear shows that the non-linear effects do not have a significant contribution to the loss of the waveguide. In Figure 4.6(b) the coincidence counts which have been integrated over a coincidence window width of 3*FWHM of the fitted Gaussian (see as section 4.3.3.1) are plotted against the power injected to the chip. Figure 4.6(c) and (d) shows the linear and quadratic part of the single photon counts plotted separately for the two channels. There is still a significant amount of

linear noise contribution to the single photon counts due to the Raman noise originating from the silica fibers in the setup.

The quadratic terms in the three equations (4.29)-(4.31), $\eta_1\beta_p P^2$, $\eta_2\beta_p P^2$ and $\eta_1\eta_2\beta_p P^2$ can be extracted from the fitted curves to the measured photon counts of Figure 4.6(b),(c) & (d) and parameters η_1 , η_2 and β_p can be calculated. The calculated values for the collection efficiencies and pair generation rate coefficient are shown in Table 4-3 below.

Table 4-3: The extracted parameters from the experiment performed on straight waveguide

	Parameter	Value
Collection efficiency of Channel 1 (3FWHM)	η_1	-24.4dB
Collection efficiency of Channel 2 (3FWHM)	η_2	-30.4dB
Pair generation rate (3xFWHM)	β_p	206KHz/mW ²
Center wavelength for idler	λ_i	1555.7nm
Center wavelength for signal	λ_s	1539.6nm

The CAR is plotted in Figure 4.7(a) using the FWHM of the Gaussian fit as the coincidence window width as described in section 4.3.3.1. Similarly the on-chip coincidence counts obtained by normalizing the calculated CC to the channel collection efficiencies ($CC/\eta_1\eta_2$) are plotted in Figure 4.7(b). In this plot again we have used the FWHM of the Gaussian fit as the coincidence window width to calculate the coincidence counts. The maximum obtained CAR is 40 ± 5 at an on-chip count rate of 88KHz.

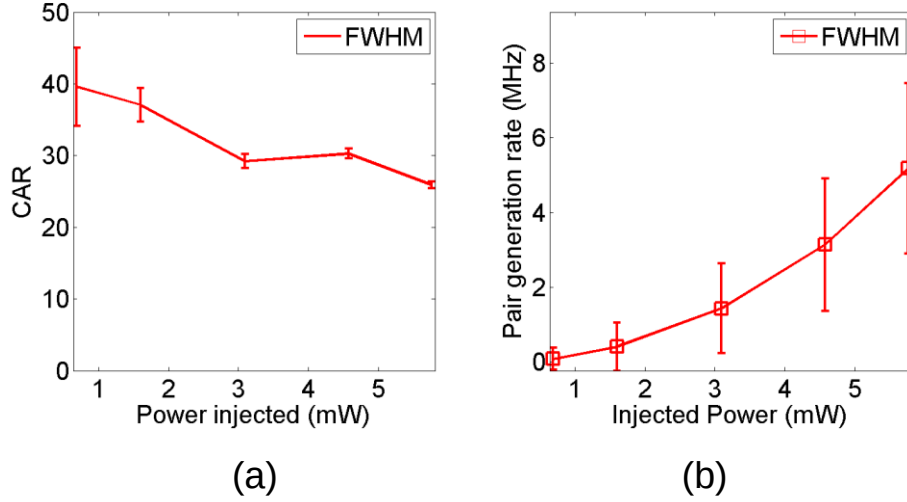


Figure 4.7: (a) The CAR and (b) the on-chip coincidence counts obtained from the straight waveguide experiments

4.3.4.2 Pulsed Excitation

With the pulsed excitation the power can be squeezed in a very short duration in time therefore reaching a very high intensity. This also reduces the free-carrier related effects provided that the duration between two pulses is longer than the life-time of the free carriers so when the next pulse arrives it will not encounter the free carriers generated by previous one. However, high peak powers generate more noise photons especially those arising from the Raman scattering in the silica fibers in the setup. The experimental setup for the pulsed case was described in Section 4.3.1.

The generated electrical pulses have an amplitude of 500mV, repetition rate of 100MHz. The pulse duration is 100ps which results in a duty cycle of 1%. As in the continuous case we have repeated measurements varying the average input power. In this case we are interested in the peak power inside the chip. To find the peak power inside the chip we have to subtract the noise since the measured monitor power is an average that includes the two. The duty cycle of the pulse is 1% which means the pulse is present in 1% of the period whereas the noise is present in the whole period.

The major contribution of the noise is the leakage while modulator is in the off state as illustrated in Figure 4.8. The duration of the pulse which is typically the FWHM, and the repetition rate are shown with t and T .

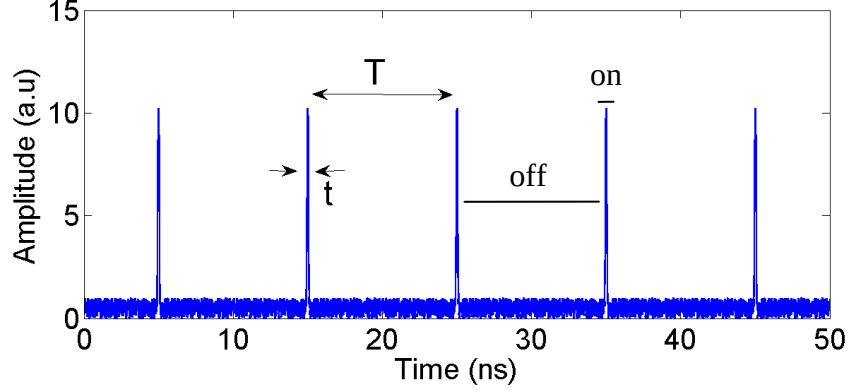


Figure 4.8: A sample pulse train with 100MHz repetition rate. When calculating the peak power inside the chip the noise has to be taken into account.

The measured monitor power, P_{mon} is the summation of the measured average power of the noise, $\tilde{P}_{noise}$ and the measured average power of the pulses.

$$P_{mon} = \tilde{P}_{pump} + \tilde{P}_{noise} \quad (4.32)$$

We experimentally found $\tilde{P}_{noise}$ by recording the monitor power when the pulse source is switched off and deduced $\tilde{P}_{pump}$. The ratio between the pump power and the noise power is called signal to noise ratio (SNR) which is found as 12dB. To project this measured power to the power injected to the chip, P_{inj} we have subtracted the losses between the chip and the power meter (5dB). The peak power inside the chip, P_{peak} is related to the injected power by the following formula

$$P_{peak} = P_{inj} \frac{T}{t} \quad (4.33)$$

Important parameters in the experiment are listed in

Table 4-4 below.

Table 4-4: Parameters for pulsed measurements

Repetition rate	100MHz
Pulse duration	100ps
Duty cycle	1%
SNR	12dB
Peak power range on chip	6mW-50mW
Integration time	60s
Coincidence window width	4ns
Histogram resolution	16ps

We have again acquired the coincidence histograms by scanning the input power by increasing the EDFA current. Figure 4.9(a) shows the single photon counts (blue and black curves, left axis) and coincidence counts (red curve, right axis). The coincidence counts are calculated by using $3 \times \text{FWHM}$ of the Gaussian fit to the main peak (see section 4.3.3.2) as the coincidence window. The thick lines are quadratic fits to the experimental data. Using equations (4.29)-(4.31), the channel collection efficiencies are found as -31.1dB and -24.2dB for the two channels. The pair generation rate coefficient, β_p is found as $760\text{Hz}/\text{mW}^2$.

The CAR is also plotted in Figure 4.9(b) (left axis) together with the calculated on-chip count rate. The maximum CAR value of 31 ± 11 corresponds to an on-chip count rate of 1.5MHz. While the on-chip count rates in this experiments can reach quite high values, usually a CAR of at least 10 is desired which limits the on-chip count rate below approximately 10MHz which is equivalent to 0.1 photon pairs per pulse in our case.

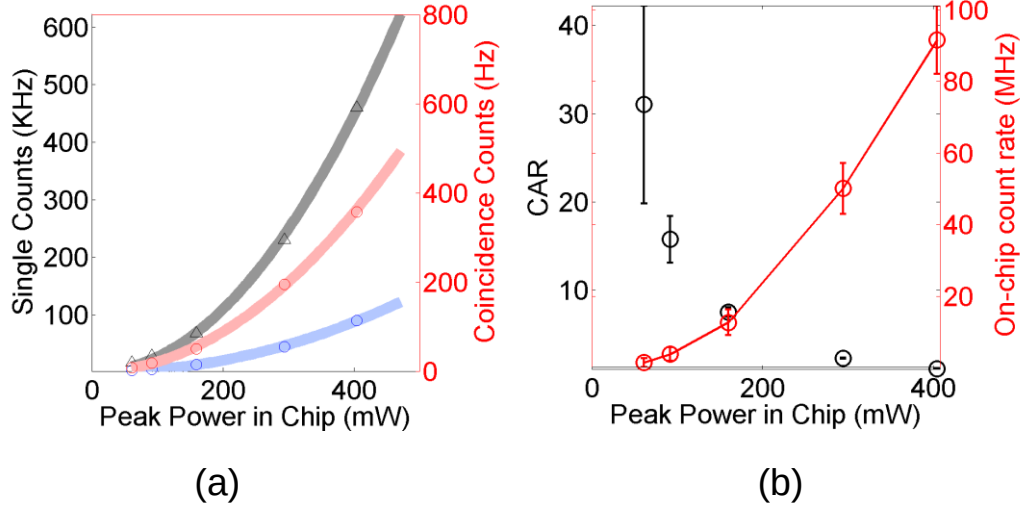


Figure 4.9: (a) The single photon counts (black&blue, left axis) and the coincidence counts (3FWHM, red, right axis) plotted against calculated peak power in chip (b) Coincidence to accidental ratio (CAR) for FWHM coincidence window width (left axis) and the corresponding on-chip count rate (right axis)

4.3.5 Photon Pair Generation in a Silicon Ring Resonator

FWM in silicon ring resonators have been previously studied theoretically in [139] and experimentally in [55]. Ring resonators are particularly interesting devices for several reasons. Firstly the field intensities are amplified in such a resonant device which enhances FWM. Secondly, the resonances of a ring resonator are almost equally spaced by nature; therefore the pump beam and the generated photon pairs are simultaneously resonant inside the ring. A typical ring resonator transmission response is plotted in Figure 4.10 where only three of the resonance dips are shown.

The photon pairs are also spectrally shaped by the ring device since only certain wavelengths are allowed to resonate inside the ring. Also by the use of a ring cavity an initial filtering is applied on the pump beam although additional filtering has to be applied externally to separate the photon pairs from the pump. However this additional filtering can also be performed in-chip by proper architectures such as cascaded rings.

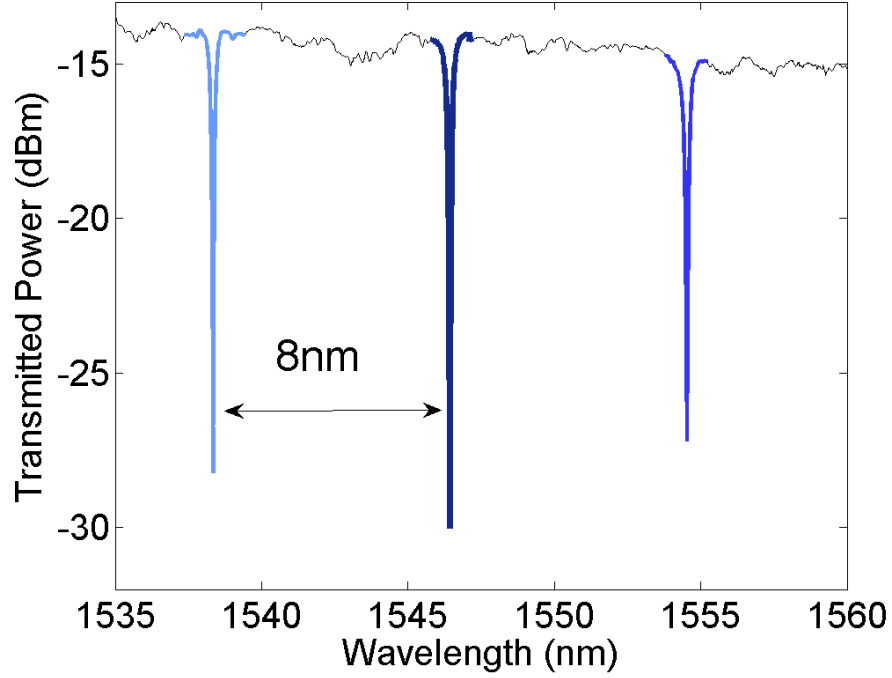


Figure 4.10: A sample transmission response of a ring resonator. In a FWM experiment the pump beam is tuned to the central resonance and the photon pairs are collected from the two adjacent resonances after applying band-pass filtering.

TPA, FCA and TOE become much more significant in a resonant device due to extensive power build up. The generated free carriers act as a loss mechanism and the free-carrier dispersion results in a shift in the resonance wavelengths. Different then previous studies we have a ring which is surrounded by a PN junction. We have also investigated the effect of applying reverse bias voltage to remove the generated free carriers.

Figure 4.11 shows an illustration of the device. The device is designed and fabricated by Toshiba Corporation, Japan on a 220nm silicon-on-silica wafer. The width of the straight bus waveguide and the ring is 450nm and the etch depth is 170nm, leaving a 50nm thick slab. The overall wafer is coated by a silica layer of thickness 90nm on top and around 30nm on the side walls. The radius of the bends is 10 μ m, the length of the coupling region is 5 μ m and the gap between the waveguides in the coupling region is 450nm.

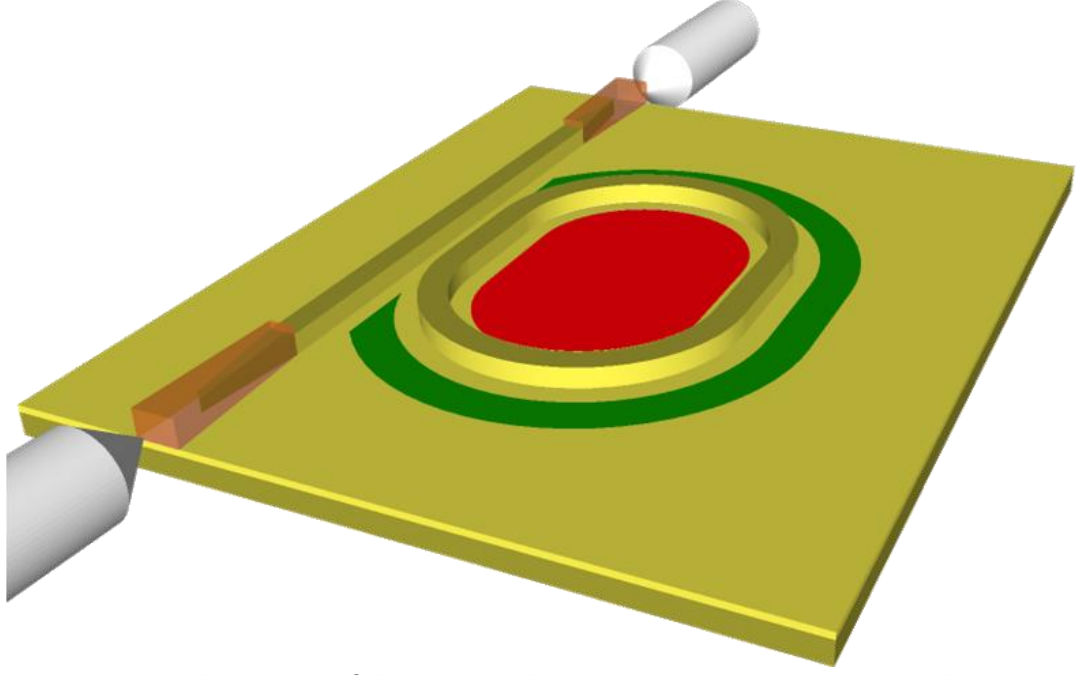


Figure 4.11: Illustration of the device. The straight bus waveguide and the ring are fabricated on a 220nm silicon on oxide by a partial etch of 170nm depth. The length of the bus waveguide is 2mm. The ring has a 10 μ m bend radius and has a race track shape with a coupling region of 5 μ m. There are polymer spot-size converters for efficient input and output coupling. There are doped regions inside and outside the ring forming a p-n junction linked with metallic pads.

This 50nm slab left after etching is necessary for the implementation of the PN junction. This brings a disadvantage in terms of phase matching. Phase matching was discussed in section 4.2.1 and for efficient FWM the dispersion has to be anomalous. Figure 4.12 shows dispersion curves for 450nm wide ridge waveguides having two different etch depths. The red curve is for the straight waveguide discussed in section 4.3.4 where the entire silicon layer is etched. The dispersion is anomalous since it is greater than zero for the wavelength regime of interest. The blue curve is for the ring resonator described in this section and it has normal dispersion, therefore phase matching condition can not be satisfied.

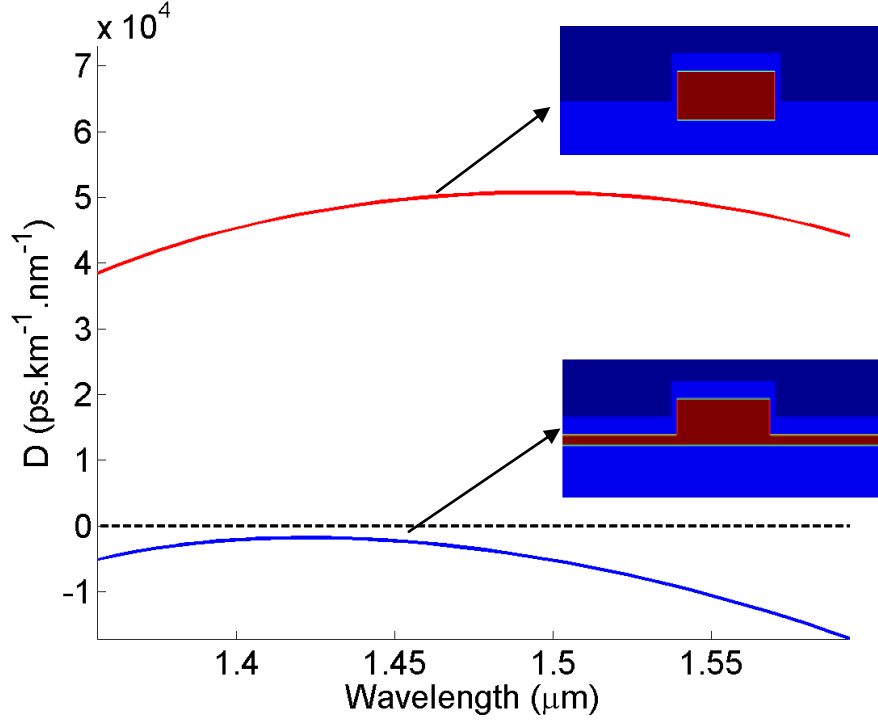


Figure 4.12: The dispersion diagrams for two waveguide structures with different etch depths. The red curve is for a 450nm wide ridge waveguide where silicon 220nm silicon layer is fully etched. The blue curve is again for a 450nm wide ridge waveguide when the etch depth is 170nm, leaving a 50nm slab. Both waveguides are coated with silica having a thickness of 90nm on top and 30nm on the side

This device has a free spectral range (FSR) of $\sim 1\text{THz}$. The chip sits on a temperature controlled stage and adjusting the temperature the refractive index of the waveguides is adjusted and the resonance wavelengths are shifted. The increase in resonance wavelengths is found as $68\text{pm}/^\circ\text{C}$. 42°C is required to shift the central TE mode (electric field parallel to plane) resonance dip to around 1547.7nm wavelength, and the adjacent resonances fall around 1539.6nm and 1555.7nm. This ensures that all three resonances are within the transmission region of FBGs and the DWDM channels. As in the straight waveguide there are polymer spot size converters on both the input and output which enables the efficient coupling of the light with a loss of 2dB per facet.

4.3.5.1 The Classical Response of the Ring Resonator

We have observed the transmission of the ring sweeping the wavelength of the input laser. The input and output light is coupled with fiber lenses as illustrated in Figure 4.11. Figure 4.13 shows the response of the ring around the central resonance. This resonance is going to be used for the pump beam in the photon generation experiments. A Lorentzian curve is fitted to the measured transmission data. The line width of the resonance is found as 43pm which corresponds to a quality factor of about 37,500.

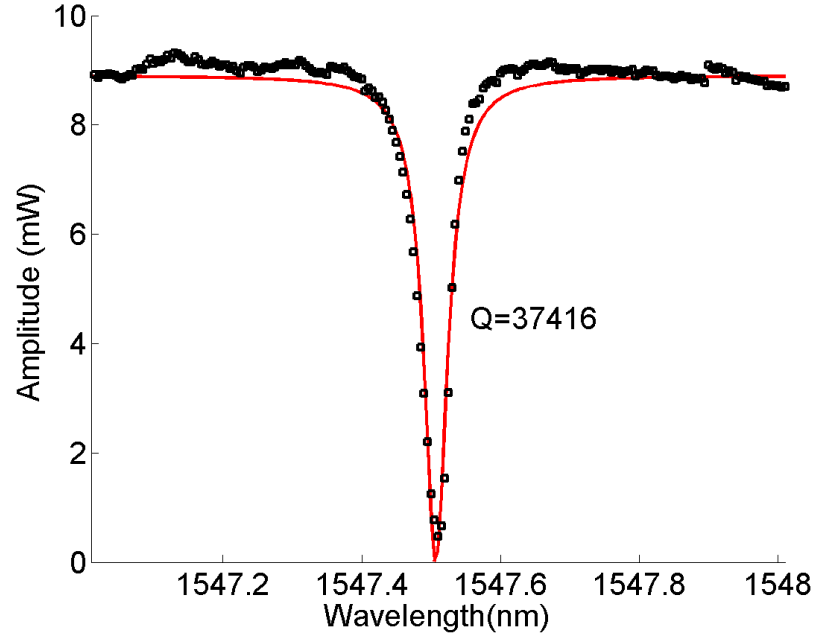


Figure 4.13: The response of the central resonance (pump wavelength) of the ring (black) and the Lorentzian fit (red).

4.3.5.2 Optical Bi-stability

The wavelength scan shown in Figure 4.13 was performed at low input power where the non-linear effects are not significant. We understand this from the dip being somewhat symmetrical around the dip point. When the input power is large the non-

linear effects discussed in the previous section, result in so called optical bi-stability [140]. To explain the behaviour we can take a look at the black curve in Figure 4.14 which shows the output power when a high power pump beam is swept from lower wavelengths to higher. As the pump wavelength moves towards the resonance, power starts couple and build up inside the ring. This stimulates TPA, FCA processes and the loss inside the ring is increased hence the Q-factor is reduced. Due to free-carrier dispersion (FCD) and also thermo-optic effect (TOE) affect the refractive index of the ring changes and the resonant dip wavelength shifts. Figure 4.14 also shows plots of various resonance dips calculated for an arbitrary ring (using equations (4.16a-c)) where the loss and the group index of the mode inside the ring are increased simultaneously. We can see as the dip wavelength shifts to higher values the curve also widens meaning a decrease Q-factor. As the pump wavelength is increased, at each step more power starts building up inside the ring so, a different resonance dip curve represents the status of the ring. Only one point on these curves is the stable solution and is shown by the black dots. The increase of the pump wavelength carries on until the pump wavelength catches a maximum resonance point shown with “x”. At this point as the pump wavelength increases the power inside the ring decreases, so the loss and group index parameters and the solution shifts to a curve which is in the opposite direction to the direction of the sweeping pump. This triggers even less power being stored in the ring and causes a further shift. As a result of this positive feedback power inside the ring instantaneously flushes and the output power suddenly increases.

So the non-linear processes reduce the power that can build up inside the ring. The increased loss also affects the pair generation rate since the generated pairs are more likely to be absorbed inside the ring.

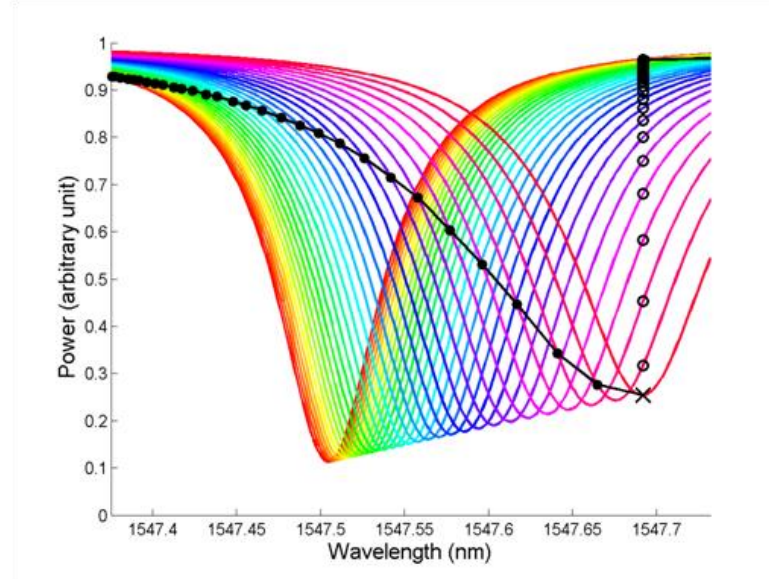


Figure 4.14: The coloured curves from left to right, illustrate the transmission response of the ring device for increased round trip loss of the cavity. The black curve illustrates the response observed by scanning the wavelength from low to higher wavelengths. As the scan beam goes into resonance the power inside the ring is increased hence the losses, and the solution is set to a different response curve.

4.3.5.3 Experimental Results

We have investigated pair generation for different input powers to the device. We have performed three sets of experiments which are named as detuned, tuned and reverse biased. We are going to present these experiments in the following sections.

4.3.5.3.1 Pair Generation in Tuned Ring

As we have described earlier the resonances of the ring can be shifted via tuning the temperature of the device. The spectral windows of the detection channels are defined by the filtering stages placed in between the chip and the detector. The spectral window for the first channel is centered at 1539.8 nm whereas for the second channel it is centered at 1557.7 nm, both having a 3dB width of 1.25 nm. For the pump beam the spectral window is defined by the filters at the input half of the setup which is

centered at 1547.8nm having a 3dB width of around 0.52nm. The input laser can be tuned within this range. In the tuned case, the device is brought to such a temperature so that the three consecutive resonances of the ring fall into these three spectral windows. The pump beam is tuned to the dip of the central resonance therefore the pump beam couples to the ring and resonates. The sFWM occurs both inside the ring and the straight bus waveguide. However the photon pairs generated inside the ring are much more intense compared to the ones in the straight bus waveguide.

We have repeated photon pair generation experiments for different input powers to the device and acquired coincidence histograms. The channel collection efficiencies are found by fitting second order polynomial curves to the single photon counts and coincidence counts (3FWHM coincidence window width). The collection efficiencies are found by the quadratic coefficients of these fits similar to in section 4.3.4.1. We have also limited the input power range below 2.5mW to calculate the collection efficiencies. This is due to the CC curve not being quadratic for high powers due to the non-linear effects described above. From this calculation the channel efficiencies are found to be -26.9dB and -34.1dB. The coincidence counts versus the injected power is plotted in Figure 4.15

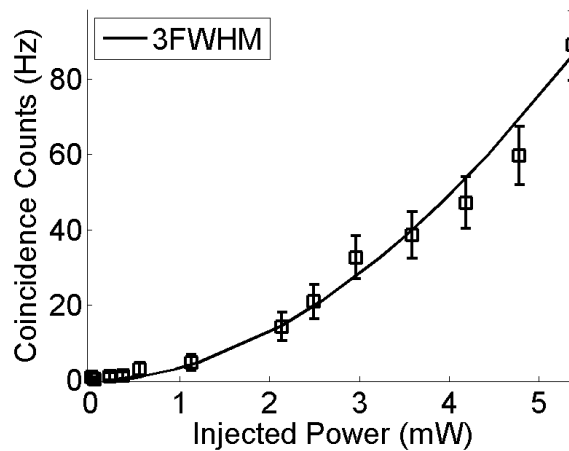


Figure 4.15: The coincidence counts versus the injected power The curve is fitted using the non-linear model described in section 4.2.5.

Here we need to be careful in extracting the pair generation rate coefficient by fitting the coincidence counts. The horizontal axis is the power injected to the straight bus waveguide, however as we have mentioned the major pair generation is taking place inside the ring. Therefore we expect the pair generation rate to be quadratic (see equation (4.31)) with the power inside the ring rather than the power injected to the chip. CC for the ring resonator is given by

$$CC = \check{\beta}_p P_{ring}^2 \quad (4.34)$$

where $\check{\beta}_p$ is the pair generation rate coefficient multiplied by the probability of the generated pairs escaping the ring. Since the power inside the ring does not have a linear relationship with the power injected to the device due to the non-linear effects of TPA and FCA, we need to use the non-linear ring model discussed in section 4.2.5 and particularly equation (4.23) combined with equation (4.34) in order to fit the CC of Figure 4.16. Remember this equation was derived assuming that the phase term in equation (4.19) is unity that is the ring operates in the resonance dip wavelength. This is indeed the case in our experiment. While collecting the data we have ensured that the pump wavelength is set to the resonance dip, the closest point where the sudden jump due to optical bi-stability occurs. We have done this by first taking a classical scan to identify the approximate wavelength where the sudden jump occurs. Then setting the pump to a wavelength that is smaller than this jumping wavelength yet still inside the resonance, we have acquired a histogram. Then we have increased the pump wavelength step by step and acquired histograms at each step until the sudden jump occurred. We have used the last acquired histogram before this jumping occurred as the valid data. For fitting the CC, we have written equation (4.23) in terms of power rather than the intensity defining a new the coefficient $\check{\Lambda} = \Lambda A_{eff}^2$

where A_{eff} is the effective mode area.

$$P_{in} = \frac{P_{ring}}{t^2} \left(1 - r\delta(1 + \tilde{\Lambda}P_{ring}^2)^{-1/4} \right)^2 \quad (4.35)$$

Incorporating the relation between the CC and P_{ring} of equation (4.34) we get:

$$P_{in} = \frac{\sqrt{CC/\check{\beta}_p}}{t^2} \left(1 - r\delta(1 + \tilde{\Lambda}CC/\check{\beta}_p)^{-1/4} \right)^2 \quad (4.36)$$

To remind, r is the amplitude reflection coefficient of the coupling region of the device, $t = \sqrt{1 - r^2}$ is the transmission coefficient, δ is the amplitude transmission coefficient in the ring in one cycle of the field and $\tilde{\Lambda}$ is a coefficient that is related to the TPA and FCA non-linear effects. We have the data points for P_{in} and CC and we are able to find the parameters r and t from the low power transmission response of the ring shown in Figure 4.13. The parameters $\tilde{\Lambda}$, δ and $\check{\beta}_p$ are found by least square fitting of the data and are as shown in Table 4-5.

Table 4-5: Parameters obtained from the least-squares fit of data points of Figure 4.16(a)

Parameter	Value
r	0.9917
t	0.128
$\tilde{\Lambda}$	1.2448E-8 mW ⁻²
δ	0.9918
$\check{\beta}_p$	3.2 Hz/mW ²

The value for $\check{\beta}_p$ that we have found here represents the pair generation rate with respect to the power inside the ring. However it would make more sense to express the pair generation rate coefficient in terms of the injected power so that we can make a fair comparison with the detuned case which will be described below. We can

achieve a value for pair generation rate coefficient when the device is operating at low powers where we can assume the non-linear effects are negligible. In the linear case (valid for low input powers) we know the following relation between the power inside the ring and the input power exists.

$$\Gamma_{ring} = \frac{P_{ring}}{P_{in}} = \left(\frac{t}{1 - r\delta} \right)^2 \quad (4.37)$$

The pair generation rate coefficient with respect to the injected power, for low input powers are therefore

$$\beta_p = \check{\beta}_p \Gamma^2 = 4.07 \text{ MHz/mW}^2 \quad (4.38)$$

When calculating the CAR again we use a coincidence window width of FWHM of the fitted Gaussian to calculate the coincidence counts needed in the CAR formula, equation (4.26). The coincidence counts found by this processing is projected to on-chip counts by normalizing to the channel collection efficiencies already calculated. The on-chip count rate and CAR resultant from the processing is shown in Figure 4.16. The count rates at the detector is also shown on the right axis of Figure 4.16(a).

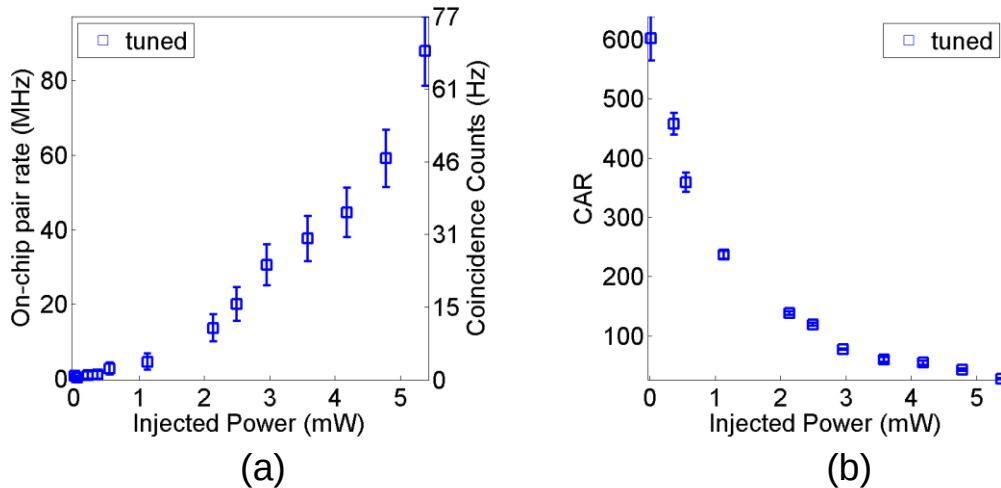


Figure 4.16: (a) The on-chip pair count rate (left axis) and coincidence counts(right axis) versus the power injected to the ring device when the ring is detuned. The right axis shows the count rate at the detector (b) The CAR versus the power injected to the ring device. The coincidence window width is FWHM of the Gaussian fit to the histograms.

The maximum value of CAR is 602 ± 37 at an on-chip count rate of 827KHz at an injected power of $19 \mu\text{W}$. The minimum value for CAR in our measurements is 27 ± 1.1 at a pair generation rate of 87.9MHz and an injected power of 5.37mW.

4.3.5.3.2 Pair Generation in Detuned Ring

When the ring resonances are shifted out of the pump wavelength and the spectral windows of the collection channels, no power couples to the ring cavity, therefore pair generation only occurs in the straight bus waveguide. This is illustrated in Figure 4.17. Figure 4.17(a) illustrates the device in this case where the ring cavity is virtually not present and Figure 4.17(b) illustrates the detuning of the ring resonances with respect to the filtering channels. The two vertical blue bars on the left and on the right show the spectral windows of the detector channels. These spectral windows are defined by the filters placed in between the chip and the detectors. The vertical bar in the middle is the spectral window for the pump wavelength, the boundaries of which is defined by the filters at the input side of the chip.

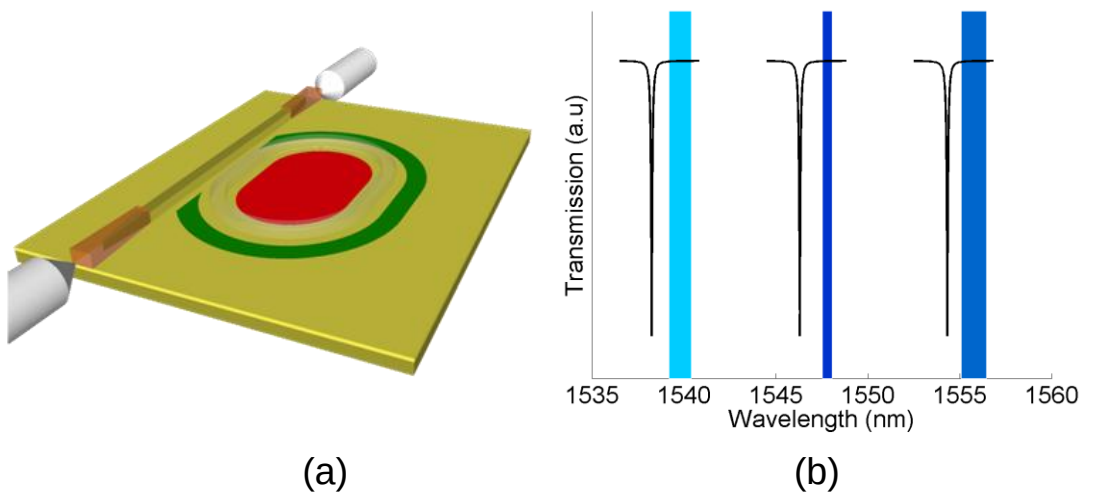


Figure 4.17: (a) Illustration of the ring device when it is detuned. No power couples to the ring cavity therefore the device is effectively a straight waveguide. (b) Illustration of the spectral response of the detuned ring. The vertical bars show the spectral windows of the pump beam (middle) and the detector channels.

Figure 4.18(a) shows the on-chip pair count rate extracted using the measured coincidence counts and the channel collection efficiencies found in section 4.3.5.3.1. Figure 4.18(b) is the plot of CAR versus the injected power. Due to the histogram acquired at low injected powers, not having a clear peak we had to discard those data points. The pair generation rate coefficient was found as 45KHz/mW^2 in the detuned case. The CAR on the other hand does not have a clear trend having a mean value around 22. Comparing the pair generation rate coefficient with that of the tuned this means that we have a 91 fold improvement due to the ring.

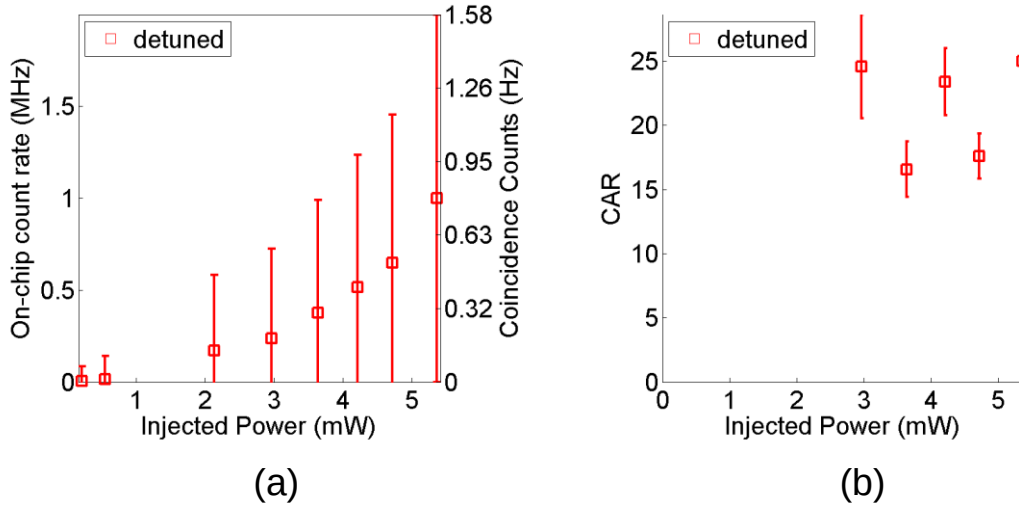


Figure 4.18: (a) The on-chip pair count rate (left axis) and coincidence counts (right axis) versus the power injected to the ring device when the ring is detuned. (b) The CAR versus the power injected to the chip.

4.3.5.3.3 Reverse Biasing

We have seen the detrimental effects of TPA and free-carriers which in summary limit the amount of power building up inside the resonator and increase the loss of the medium. Before moving to the single photon generation performance, it will be useful to present the observations of free-carrier related effects on the response of the ring resonator. We have observed the spectral response of the ring for different input pump powers and the results are shown in Figure 4.19. Here we can see the central

resonance of the ring. The high and low boundaries are due to the filtering sections in the setup. We observe a shift of the resonance dip to higher wavelengths as well as a change in the overall shape of the response with the increase of power. The shift of the resonance is due to the combined effect of free-carrier dispersion and the heat generated changing the refractive index of the waveguide.

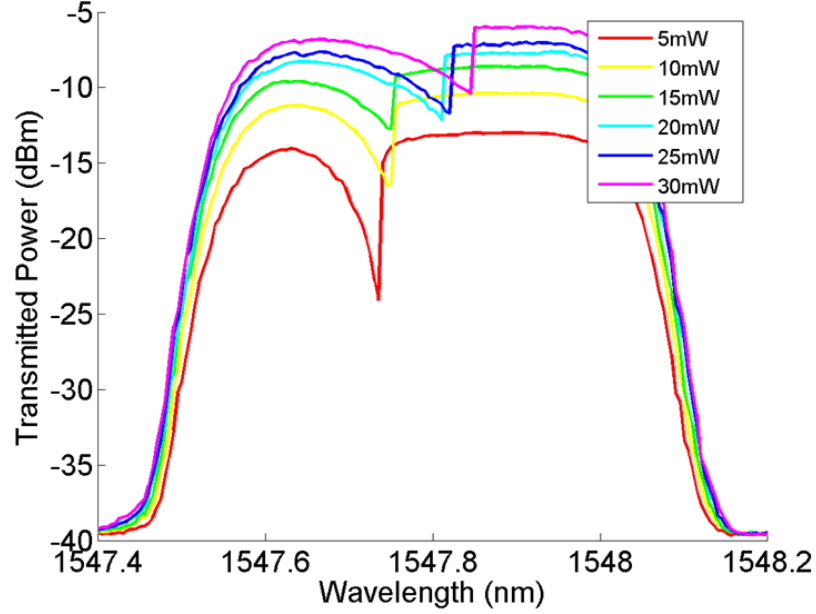


Figure 4.19: The spectral response of the ring for different EDFA output powers.

It is also worth observing the spectral response of the ring for the other resonances of interest. To do this we have pumped the central resonance of the ring and observed the adjacent resonances using a probe beam which is much reduced in power compared to the pump. The two tunable laser beams, the pump and the probe, are combined using a DWDM before injected to the device and separated at the output using another DWDM. Initially we have scanned the pump around the central resonance to get the profile shown in the left hand side axes of Figure 4.20. Then we have fixed the pump wavelength at several points on this central resonance dip and scanned the probe beam to extract the quality factor of the adjacent resonance by a Lorentzian fit. The quality factors are plotted on the right hand side axes. Both axes

share the vertical axis showing the transmission for the pump beam. As we expect as the pump beam goes deeper in to the resonance, which means the power inside the ring increases, the quality factor decreases.

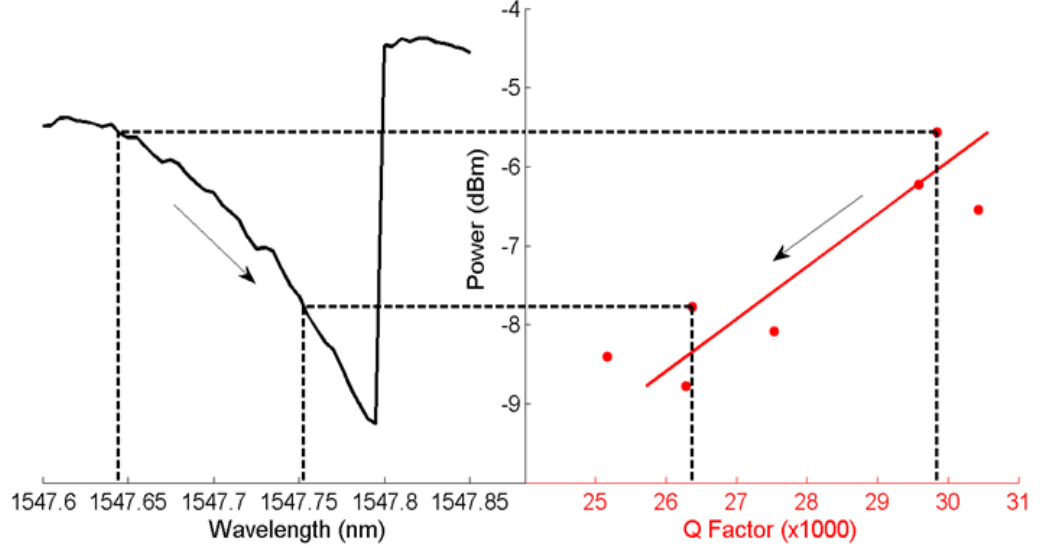


Figure 4.20: On the left, the response of central resonance of the ring is plotted. On the right the quality factor of the adjacent resonance, when the pump is fixed at different wavelengths in the resonance dip, is plotted.

In order to remove the free carriers that reduce the Q-factor we have applied reverse bias across the PN junction surrounding the ring. This is the same approach as the PN junctions used for sweeping the free-carriers to increase the gain in silicon Raman lasers [141],[142]. Figure 4.21 shows wavelength scans of the output power for different applied reverse bias voltages. As we can see as the reverse bias voltage increases the resonance wavelength shifts towards higher wavelengths. The dashed lines show the extracted current. The extracted current increases as the pump goes deeper into the resonance and also with the increased reverse bias voltage. Also shift of the resonance towards higher wavelength is observed. This shift suggests an increase in the refractive index of silicon which is caused by the thermo-optic effect. The free carriers have an opposite effect of reducing the refractive index. When more free carriers are removed applying a higher reverse bias voltage, the opposing effect

of the free carriers to the thermo-optic effect on the refractive index decreases, hence we observe this shift of resonance towards higher wavelengths. We have not tried to model this behaviour due to complex dynamics of the processes. We see a slight improvement in the resonance depth for a reverse bias voltage of 20V.

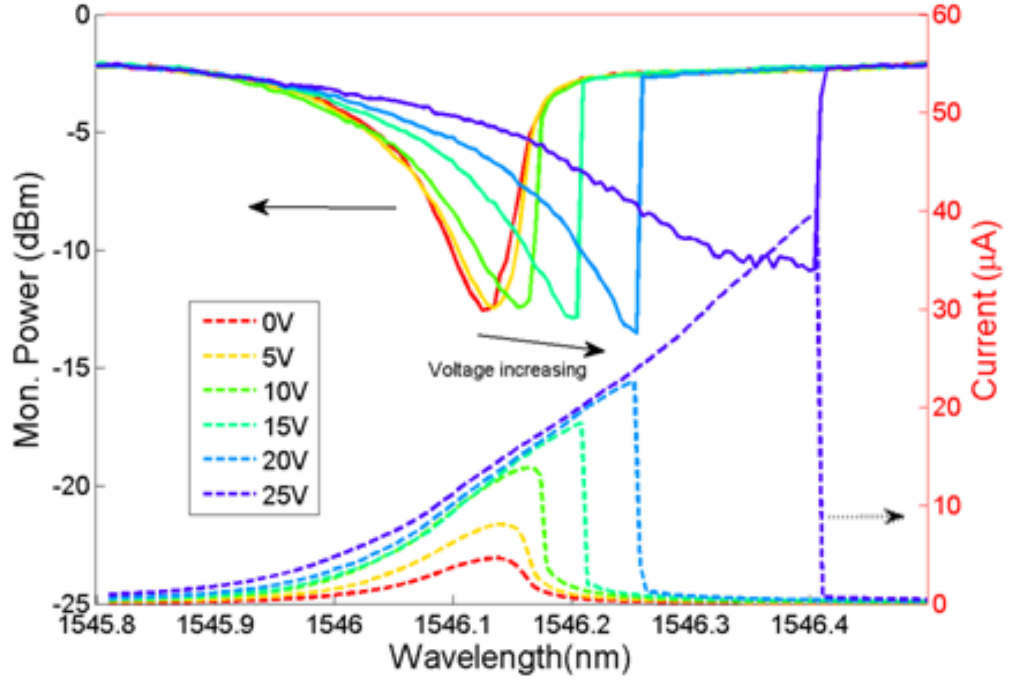


Figure 4.21: The spectral response of the ring under different reverse bias voltages (left axis) and the extracted current (right axis).

To quantify the pair generation, we have fixed the reverse bias voltage at 8V and acquired coincidence histograms at different input powers. Figure 4.22 shows the coincidence counts and the CAR versus the power injected to the device for the 8V reverse biased case. The results are plotted together with the results of the tuned and detuned experiments presented above for comparison. In the reverse bias case we have only three data points and they follow a linear trend. Therefore we cannot extract the pair generation rate coefficient as we did in the previous two experiments. If we compare the on-chip pair count rates, the maximum value in the biased case is 123MHz and in the tuned case it was 88MHz which means a 1.4 times improvement.

However the improvement occurs only after a threshold of injected power which can be estimated to be around 3.5mW. If we compare the CAR looking at the inset of Figure 4.22(b), reverse biasing improves the CAR slightly over the tuned case the maximum value of 70 ± 4 at 3.6mW injected power.

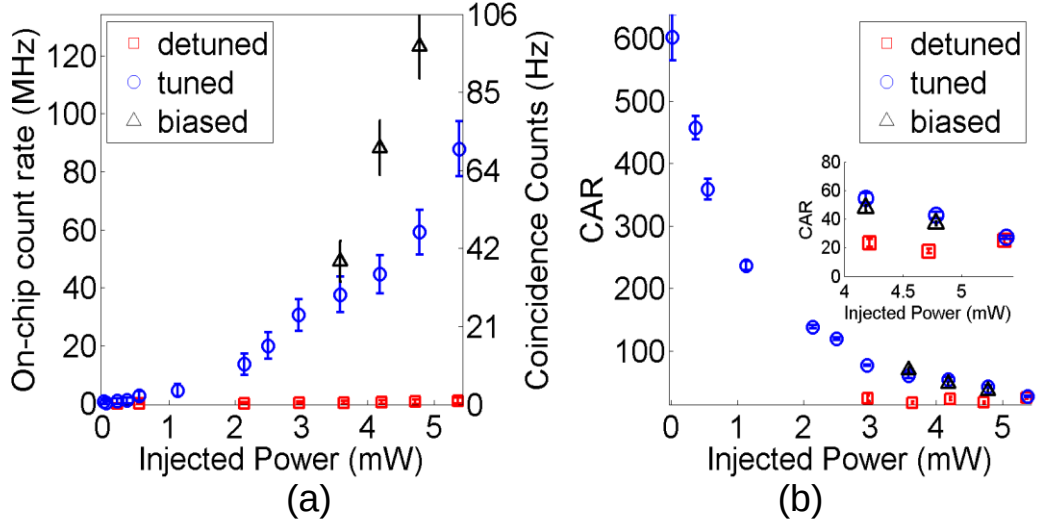


Figure 4.22: (a) The on-chip pair count rate (left axis) and coincidence counts (right axis) versus the power injected to the ring for the detuned, tuned and 8V reverse bias ring cases. (b) The CAR versus the power injected to the device for these three cases. Inset shows a closer view.

4.4 Summary

In this chapter we have presented our results in photon pair generation in silicon nano-wire devices, namely a 3mm long straight waveguide and a ring resonator. In the straight waveguide we have used both continuous wave pumping and pulsed pumping whereas in the ring resonator we have used continuous wave pumping only.

We have experimentally shown the enhancement of photon pair generation with the use of a ring resonator. Theoretically the number of pairs generated in the ideal case scales quadratic with the intensity of the pump. Therefore using a ring resonator is beneficial since the intensity inside the ring can reach to much higher values

compared to the intensity of the input light. We have investigated the enhancement provided by the ring by comparing the experimental results for the tuned and detuned cases of the ring which are explained in sections 4.3.5.3.1 and 4.3.5.3.2. We have found that the ring provides a 91 fold increase in the pair generation rate coefficient. The detrimental effects of two-photon absorption and free carrier effects are addressed by applying a reverse bias across the PN junction surrounding the ring to remove the free carriers. This further improved the pair count rate by a factor of 2.1 over the tuned case. The table below summarizes the key results.

Table 4-6: Summary of key results of continuous wave FWM experiments

	Pair generation rate coeff. (KHz/.mW⁻²)	Maximum CAR
straight waveguide (CW)	206	40±5
straight waveguide (Pulsed)	0.76	31±11
detuned ring	45	25±2
tuned ring	3120	602±37
reverse biased	N/A	70±4

In Table 4-7 below we provide a summary of results achieved by studies involving FWM in silicon nano-wire devices as well as some dispersion shifted fibers. A straight away performance comparison is not possible due to two reasons. One of them is that there are two parameters, CC and CAR, and we have to fix one of them to compare different results, which is not possible since we do not have full sets of data for other studies. Secondly the CAR does not have a fixed definition and it is very dependent on the chosen integration window as described in section 4.3.3.1. To make

a fair comparison one has to have the coincidence histograms acquired in these various experiments. However silicon nano-wires perform better than hundreds of meters long, cooled dispersion shifted fiber even at room temperature [43].

Table 4-7. Summary of result of FWM studies in the literature

	Ref	Structure	Peak CAR	Photon count rate	
Pulsed Excitation	[43]	500m cooled fiber	~60	<0.005	photons/pulse
	[113]	500m cooled fiber	~28	~0.015	
	[113]	500m un-cooled fiber	~9	~0.05	
	[143]	9.11mm silicon nano-wire	25	0.05	
	[57]	1.15cm silicon nano-wire	320	0.0013	
	[56]	1.09cm mm silicon nano-wire	50.3	0.005	
	this work	3mm long silicon nano-wire	~31±11	~0.015	
Continuous Wave Excitation	[55]	1.13cm mm silicon nano-wire	11.3	9M	photons/sec
	[55]	43μm length silicon nano-wire ring resonator	30	~0.5M	
	this work	3mm long silicon nano-wire	~40±5	88K	
	this work	73μm length silicon nano-wire ring resonator	602±37	0.827M	

4.5 Conclusions

The use of resonant structure, namely the ring resonator enhances the power inside the device and improves the pair generation rate as well as reducing the overall device footprint (comparing the 1.13cm long device of [55] with 73μm perimeter

device in this study). To our knowledge this is the first study on the effect of reverse biasing to the pair generation. Reverse biasing to remove the free-carriers help increasing the pair generation rate after a threshold of input power. (see Figure 4.22) while preserving the CAR. Figure 4.23 show the CAR values versus the on-chip count rates found by measurements. The CAR is drawn on a log scale axis for easier visualization. The inset figure shows a closer look to the straight waveguide measurement results. The ring device performs much better compared the straight waveguides. The CAR in the pulsed pump case drops faster then continuous pump case having a large value of uncertainty due to the large values of both coincidence and accidental counts. This might be caused by the noise photons originated from the Raman scattering in the silica fibers in the setup. On the other hand the tuned ring gives the best performance in terms of CAR in the low pump regime. For the higher on-chip pair generation rates (higher input power) reverse biasing provides enhancement of the CAR:

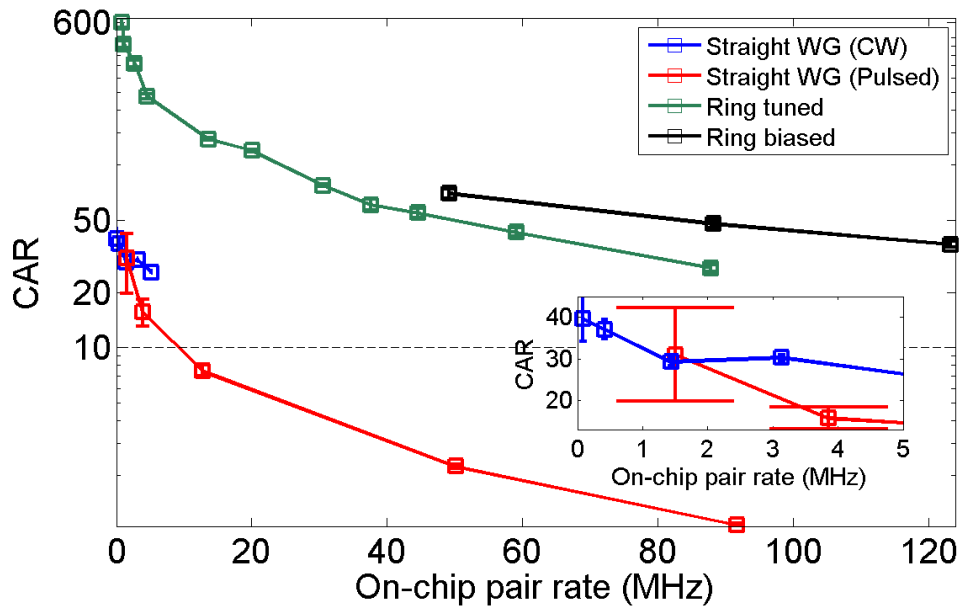


Figure 4.23: The on-chip count rates versus the CAR for the performed experiments.

A major source of the accidental counts is the Raman scattering in the silica fibers used in amplification, filtering stages and inter-connections. To reduce it, the inter-connection fibers should be kept as short as possible. Also the filtering which is currently done by external bulk components can be implemented on chip by the use of multiple ring cavities. The implementation of the PN junction requires a slab of silicon layer left on either side of the ridge which results in the waveguide to operate in the normal dispersion regime. In the normal dispersion regime the efficiency of FWM is reduced due to phase matching condition not being satisfied. We believe that better performance can be achieved using a phase-matched device.

This study is an intermediate step to develop practical on-chip single photon sources where either signal or idler photons are used. In real applications more than a single photon is required. Additionally indistinguishability among the photons generated by different sources and the synchronization between them is needed. Therefore sources have to be identical which is a challenge related to fabrication tolerances. To achieve synchronization among the sources, the multiple sources have to be excited by pulses. However since each source has a certain probability of generating a single photon, the probability of all the sources generation single photons at the same time will be much lower. The source can also be used as heralded source so that the successful operations of the sources can be flagged with the heralding signals and the overall success rate of the source is increased.

We have also analyzed the dependence of the source parameters namely the pair generation rate and the CAR on the chosen coincidence window. The optimum operating regime is can not be clearly identified. There is a trade off between the CAR and the pair rate and the optimum operating regime is dependent on the loss of the circuits and the efficiency of the detectors that are going to be used together with the

source. For example if the losses are high one would chose to operate in a higher photon generation rate to overcome the losses sacrificing from the CAR. However it is possible to make some estimation. The CAR also provides an estimation of the bit error rate contribution from the accidental noise, ε , in quantum communications systems using entangled pairs of photons. ε is related to CAR by the formula [113]

$$\varepsilon \approx \frac{0.5}{CAR - 1} \quad (4.39)$$

For quantum photonics applications the interference visibility, V of the source photons is an important figure of merit and is approximated by the formula [144].

$$V \approx 1 - \frac{1}{CAR} \quad (4.40)$$

When our ring device is operated at an injected power of 4.8mW and reverse biased with 8V, the bit error rate contribution from the accidentals in a quantum communication application that uses entangled pairs of photons, according to equation (4.39) would be 1.4% (CAR=36.8). This operating regime corresponds to an on-chip pair generation rate of ~123 MHz. On the other hand according to equation (4.40), when the chip is operated at an injected power of 2.5mW without reverse bias which corresponds to an on-chip pair generation rate of 20MHz the obtained visibility would be above ~99% (CAR = 120).

Silicon provides reduction in size compared to any other material commonly used in photonics such as GaAs, InP, GaN or LiNbO₃ and it has a mature fabrication technology inherited from digital electronics. Also it can be integrated with CMOS control electronics on the same substrate. Recent demonstrations of silicon quantum photonic circuits [34] and detectors [46] makes it an important candidate for integrated quantum photonics applications.

Chapter 5

Photonic Crystals and Modelling

In this chapter we will first introduce photonic crystals and the theory governing the propagation of light in photonic crystals. Then we are going to present techniques used in simulating photonic crystal structures. Two methods presented here are the plane wave expansion method for solving band diagrams of photonic crystals and finite difference time-domain (FDTD) method for simulating electromagnetic wave propagation in arbitrary medium

5.1 Photonic Crystals

A photonic crystal (PhC) is a material or structure having a periodic dielectric distribution in space, whose period is on the order of the wavelength of light [65]. The periodicity can be in one, two or three dimensions as illustrated in Figure 5.1. There is a strong analogy between the photons in photonic crystals and the electrons in semiconductor crystals. Electrons in a semiconductor experience periodic potentials which results in a bandgap between the valence and conduction bands. No electrons with energy in between these two bands are allowed to exist in the crystal. Similarly the light inside the photonic crystal is scattered by the dielectric interfaces and if the scattering is strong enough it can give rise to the formation of a photonic band gap, where the light at a range wavelengths (energies) is prevented from existence within the structure.

Light can be confined, guided and manipulated inside defects created in uniform photonic crystals as it is not allowed to propagate inside the crystal. Quarter-wave stacks used as reflectors, Bragg gratings are examples of photonic crystals. Despite its advantages, fabrication of photonic crystals is a challenge since the periodicity has to be on the order of the wavelength of the light. Imperfections in the fabrication result in the scattering of light and deviates its behaviour from the expected. We will see practical applications of photonic crystals in more detail in Chapter 6.

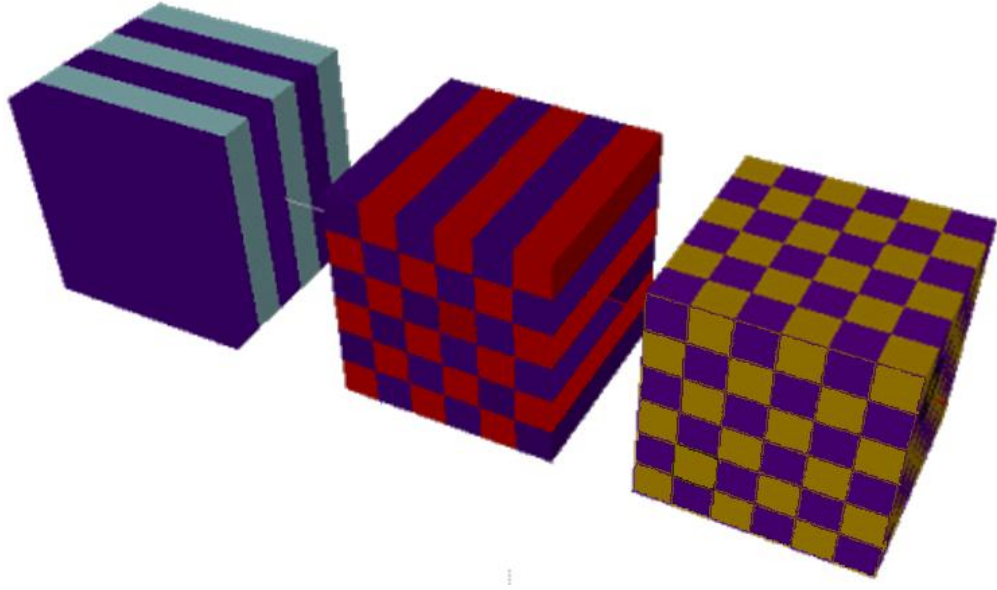


Figure 5.1: From left to right the illustrations of 1D, 2D and 3D photonic crystals. Different colours represent different dielectric constants

5.1.1 Photonic Bands

Photonic bands are a continuum of allowed modes in the dispersion diagram of Photonic Crystals (PhC). To visualize this, Figure 5.2(a) shows the band diagram of the light that propagates in a bulk GaAs. The bulk material is illustrated in the inset of the figure. The bulk medium can be assumed to be formed of stacks of GaAs layers, having the same dielectric constant, and the stacks are periodic with period a as shown in the figure. In this dispersion diagram, the frequency and propagation constant axes are normalized to $(2\pi c/a)$ and $(2\pi/a)$ respectively. The band is folded back at normalized propagation constant $k=0.5$ and a new band starts at each folding point. It seems as if there are multiple frequencies corresponding to a single value of k , however this is not the correct interpretation of the graph. The actual value for k in the n^{th} band is $k+(n-1)\pi/a$. We are going to see that this way of plotting is useful in visualizing the band gap of PhCs. When the index contrast is slightly increased by using GaAs and

GaAlAs stacks, there occurs a narrow range of frequencies, shaded with yellow in Figure 5.2(b), which are not allowed to propagate in the medium. This yellow shaded region is called the band gap. In Figure 5.2(c) when index contrast is further increased by using GaAs/air stacks we see that the band gap widens splitting the first and second photonic bands even more.

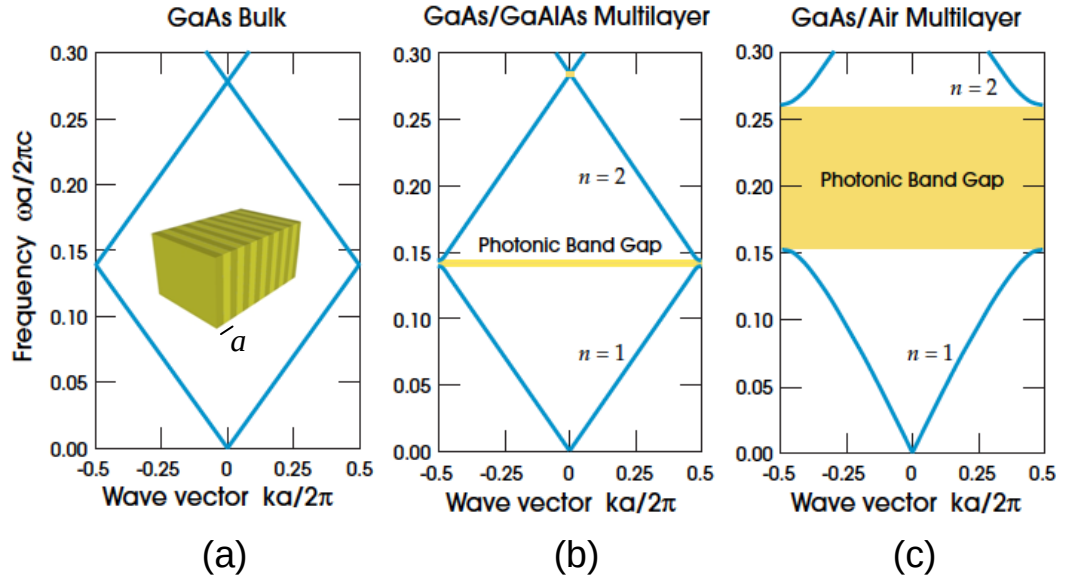


Figure 5.2: (a) Band diagram of a bulk GaAs material. Inset is the illustration of stacked layers with period a . (b) The band diagram when the index contrast is slightly increased by using GaAs/GaAlAs layers. (c) The band diagram when GaAs/air stacks are used. The band gap widens as the contrast is increased. Figures are taken from [129].

5.1.2 The Lattice and the Reciprocal Lattice

Lattices are formed by periodic patterns in space, in our case, patterns of dielectric constant. The top view of a triangular lattice structure, which is a common pattern for PhCs, is shown in Figure 5.3(a). A lattice vector is any vector from one lattice point to another such as the one shown with $\vec{R}$ in the figure. The vectors shown as $\vec{a}_1$ and $\vec{a}_2$ in the figure are the lattice basis vectors which are the set of vectors that form a basis for

all possible lattice vectors. In other words all lattice vectors are integral number of combinations of lattice basis vectors. Representation of the lattice in Fourier space (k-space) is called the reciprocal lattice. The lattice points of the reciprocal lattice are all the points k in the k-space that satisfy the following equation

$$e^{jk \cdot \vec{R}} = 1 \quad (5.1)$$

for all lattice points represented with the position vector $\vec{R}$ (linear combinations of the lattice basis vectors $\vec{a}_1$ and $\vec{a}_2$). Figure 5.3(b) shows the reciprocal lattice of the triangular lattice which is also a triangular lattice with basis vectors $\vec{g}_1$ and $\vec{g}_2$. The reciprocal lattice of the triangular lattice is also triangular as shown in Figure 5.3(b). The grey shaded area in the reciprocal lattice is called the Brillouin zone and is the primitive cell of the reciprocal lattice. The boundaries of the Brillouin zone are defined by the bisectors of lines drawn from the central point to the nearest reciprocal lattice points (the blue dashed line). The Brillouin zone can further be reduced exploiting the symmetries of the crystal such as mirror symmetry and rotational symmetry. In the triangular lattice there exists mirror symmetry and 60° rotation symmetry. The red shaded area in the Brillouin zone is referred to as the irreducible Brillouin zone. Any arbitrary vector inside the Brillouin zone such as the red one in Figure 5.3(b) corresponds to a vector inside the irreducible Brillouin zone (green vector) by applying the lattice symmetry operations (in this example a mirror around the vertical axis and then a 60° rotation counter clock wise). The irreducible Brillouin zone is important because it restricts the solution for the propagation vector components of a wave propagating inside the crystal to the ones inside the irreducible Brillouin zone. The use of the reciprocal lattice will be more clear when we introduce the solution of Maxwell's equations in periodic media.

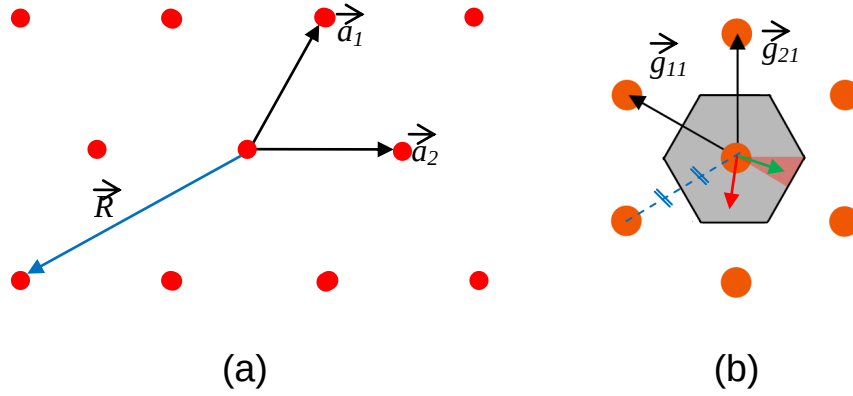


Figure 5.3: (a) The triangular lattice of holes and the lattice basis vectors. (b) The reciprocal lattice of a triangular lattice, the reciprocal lattice basis vectors. The grey shaded area is the Brillouin zone and the red is the irreducible Brillouin zone.

5.1.3 Maxwell's Equations in Periodic Media

In this section we will explain the theory governing wave propagation in periodic structures. This is the basis of the plane wave expansion (PWE) method. The PWE method is used to solve the band diagrams of PhCs. The band diagram is the plot of frequency, ω with respect to the propagation vector, k . The band diagram proves information about the band gap of the PhC (the frequency band which is not allowed inside the PhC) and also the dispersion of the propagating waves inside the PhC. We will assume the medium is lossless, has a linear dielectric constant and uniform relative magnetic permeability of 1, there are no free charges or currents in the medium. In this case the Maxwell's equations can be written in phasor form as:

$$\nabla \cdot \mathbf{H}(\vec{r}) = 0 \quad (5.2)$$

$$\nabla \times \mathbf{E}(\vec{r}) - j\mu_0\omega\mathbf{H}(\vec{r}) = 0 \quad (5.3)$$

$$\nabla \cdot [\epsilon(\vec{r})\mathbf{E}(\vec{r})] = 0 \quad (5.4)$$

$$\nabla \times \mathbf{H}(\vec{r}) + j\omega\epsilon(\vec{r})\mathbf{E}(\vec{r}) = 0 \quad (5.5)$$

Here $\mathbf{H}(\vec{r})$ and $\mathbf{E}(\vec{r})$ are the magnetic and electric field phasors with respect to the position vector $\vec{r}$, $\varepsilon(\vec{r})$ is the dielectric vector, μ_0 is the magnetic permeability of free space and ω is the angular frequency of the wave. $\varepsilon(\vec{r})$ is a periodic function satisfying the following relation for all lattice basis vectors $\vec{R}$.

$$\varepsilon(\vec{r}) = \varepsilon(\vec{r} + \vec{R}) \quad (5.6)$$

From equation (5.2) to (5.5) we can derive the following equation for $\mathbf{H}(\vec{r})$:

$$\nabla \times \left(\frac{1}{\varepsilon(\vec{r})} \nabla \times \mathbf{H}(\vec{r}) \right) = \left(\frac{\omega}{c_0} \right)^2 \mathbf{H}(\vec{r}) \quad (5.7)$$

where c_0 is the speed of light in vacuum. Defining a linear and Hermitian operator $\breve{\Theta}$:

$$\breve{\Theta} \mathbf{H}(\vec{r}) = \nabla \times \left(\frac{1}{\varepsilon(\vec{r})} \nabla \times \mathbf{H}(\vec{r}) \right) \quad (5.8)$$

we can rewrite equation (5.7) as

$$\breve{\Theta} \mathbf{H}(\vec{r}) = \left(\frac{\omega}{c_0} \right)^2 \mathbf{H}(\vec{r}) \quad (5.9)$$

This is in fact an eigenvalue equation with eigenvalues $(\omega/c_0)^2$. To solve equation (5.9) Bloch-Floquet's theorem is employed. This theorem is also used for solving the wave function of an electron moving in a medium which has periodic potential and is directly applicable to photons moving in periodic dielectric media. The theorem states that the solutions for the wave propagating in the periodic dielectric media can be expressed as a periodic envelope function, which has the same periodicity as the dielectric, multiplied by a plane wave. That is

$$\psi_k = e^{ik \cdot \vec{r}} u_k(\vec{r}) \quad (5.10)$$

Here $u_k(\vec{r})$ is the periodic envelope function or the Bloch function corresponding to a wave vector k . The periodicity of the Bloch function can be expressed by

$$u_k(\vec{r}) = u_k(\vec{r} + \vec{R}) \quad (5.11)$$

If we substitute ψ_k instead of $\mathbf{H}(\vec{r})$ in the master equation (5.9), we will get the following:

$$\nabla \times \frac{1}{\varepsilon(\vec{r})} \nabla \times e^{ik\vec{r}} u_k(\vec{r}) = \left(\frac{\omega(k)}{c_0} \right)^2 e^{ik\vec{r}} u_k(\vec{r}) \quad (5.12)$$

$$(ik + \nabla) \times \frac{1}{\varepsilon(\vec{r})} (ik + \nabla) \times u_k(\vec{r}) = \left(\frac{\omega(k)}{c_0} \right)^2 u_k(\vec{r}) \quad (5.13)$$

Defining an operator:

$$\hat{\Theta}_k \triangleq (ik + \nabla) \times \frac{1}{\varepsilon(\vec{r})} (ik + \nabla) \times \quad (5.14)$$

equation (5.13) can be written in short as:

$$\hat{\Theta}_k u_k(\vec{r}) = \left(\frac{\omega(k)}{c_0} \right)^2 u_k(\vec{r}) \quad (5.15)$$

equation (5.15) is also an eigenvalue problem. It is sufficient to solve this eigenvalue problem for k vectors inside the irreducible Brillouin zone, k_B . This is due to the fact that the solutions for the problem for $k_B + \vec{G}$, where $\vec{G}$ is a position vector in reciprocal lattice, is a higher order eigenvalue of for the problem for k_B . The band diagram which is a plot of frequency, ω versus propagation constant is obtained by choosing a k_B and finding the multiple eigenvalues $\omega(k_B)$. The first eigenvalue of the problem is for the frequency of the first band, the second eigenvalue of the problem is for the frequency of the second band and so on.

Plane-wave expansion (PWE) method is a common method to solve this eigenvalue problem in which the dielectric function in space is written in terms of its Fourier components. A matrix is constructed and its eigenvalues are solved. This often requires high computational resources. There is software that implement plane-wave expansion method to solve band diagrams such as BandSolve by RSoft Inc., CrystalWave by Photon Design and open source MIT Photonic Bands (MPB) [145]. We are not going to describe the details of plane-wave expansion method. Readers who are interested can consult references [146] or [147]. In the next section we are

going to present a sample PhC band diagram.

5.1.4 A Sample Band Diagram

In this section we will present a band diagram of a 2D triangular lattice of holes in a bulk dielectric medium. The holes are assumed to be infinitely deep in an infinitely extending bulk media. This helps us reduce the three dimensional problem to two dimensions.

The band diagram of a PhC is equivalent to dispersion (or ω - k) plots of waveguides. As we have seen in the previous section, the solution for a k -vector outside the irreducible Brillouin zone is just another eigenvalue, of the eigenvalue problem for its corresponding k -vector inside the irreducible Brillouin zone. Therefore in a PhC band diagram multiple ω values are found for one k -vector and these multiple solutions construct the multiple bands.

The irreducible Brillouin zone, shown as the red region in Figure 5.4(a) includes the k -vectors of interest. There are many k -vectors such as the one shown with a white arrow inside the irreducible Brillouin zone that we can solve bands for, however it will be hard to visualize the results on a two dimensional graph. It is a common practice to solve only for k -vectors that span the perimeter of the irreducible Brillouin zone as shown with black arrows and this is sufficient to give exclusive information about the band-gap of the crystal. The three corners of the irreducible Brillouin zone are named as Γ , K and M as shown.

The band diagram of the 2D triangular lattice of holes in a bulk medium with refractive index of 2.279 and hole radius to lattice constant ratio (r/a ratio) of 0.32, is shown in Figure 5.4(b). The bands coloured black are for TE modes (electric field orthogonal to the hole axes) and the red bands are for TM modes (magnetic field

orthogonal to the hole axes). The frequency axis is normalized by $2\pi c/a$ and the k-vector axis is normalized by $2\pi/a$. This is due to the scalability of the Maxwell's equations. The band diagram of a PhC of any lattice constant, a , having the same bulk material index and r/a ratio can be extracted from this band diagram by simply scaling the normalized frequency axis by a factor of $2\pi c/a$ and the k-vector axis by $2\pi/a$. The band gap in this example lies between frequencies $0.31 \times 2\pi c/a$ and $0.39 \times 2\pi c/a$. So if we were to choose a lattice constant of 271nm, the band gap would be between 694 nm and 874nm.

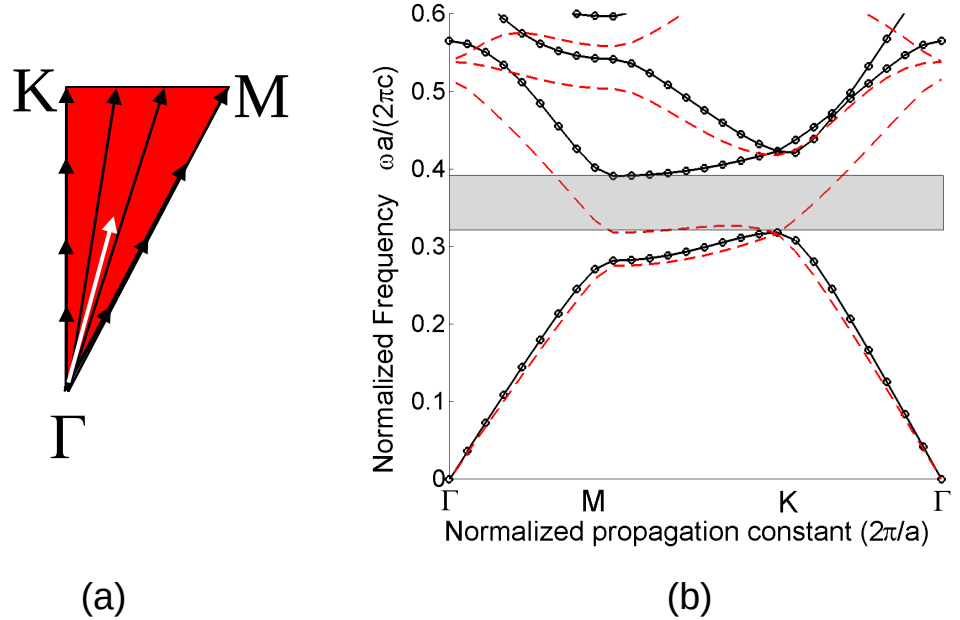


Figure 5.4: (a) The irreducible Brillouin zone of the triangular lattice and the k-vectors that are used in the band diagram plot. The corners of the irreducible Brillouin zone has special names Γ, M, K as labelled (b) The band diagram of a 2D triangular lattice of holes in a infinitely high bulk dielectric with refractive index 2.279 with hole radius to lattice constant ratio of 0.32.

5.1.5 The Super-Cell Method and the Projected Band Diagram

The plane-wave expansion method is based on periodic boundary conditions. That is, the structure is assumed to be infinitely periodic. However when it comes to defects where the periodicity of the structure is broken, the plane-wave expansion method

cannot be applied directly. Below is an illustration of point and line defects in a triangular lattice PhC.

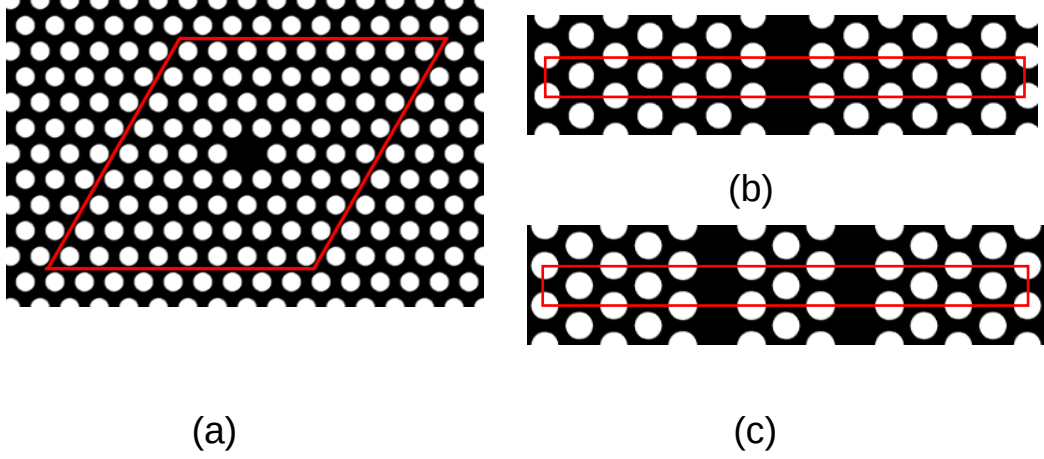


Figure 5.5: (a) A point defect in triangular lattice photonic crystal. (b) A line defect. (c) Two close line defects used as a directional coupler.

There is still a way to solve the band diagrams using plane wave expansion method which is by defining a so called “super-cell”, such as the region surrounded by the red lines in Figure 5.5(a-c). Then assuming the super-cell is the unit cell of the new virtual lattice, we can apply the plane wave expansion method. The virtual lattice for the point defect case of Figure 5.5(a) is shown in Figure 5.6. To obtain accurate results, the mutual perturbation of the adjacent defects has to be negligible and this is performed by choosing the super-cell size sufficiently large. In the Figure 5.6 super-cell size is chosen as 9×9 cells.

The super cell technique is also used to solve the band diagrams of line defects. The band diagrams of line defect PhCs is slightly different than that of uniform or point defect PhC structures since in this case we are not interested in field travelling in other directions than the one along the defect. Therefore the horizontal axis of the band diagram only includes the k -vectors along the defect and this band diagram is called the projected band diagram.

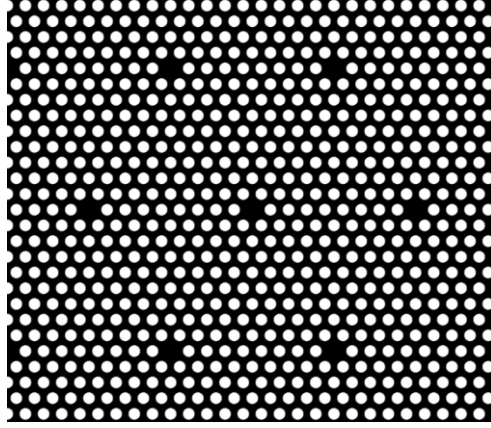


Figure 5.6: The virtual lattice constructed by a super-cell of size 9x9 cells in order to solve the bands of a point defect.

An example projected band diagram for a coupler which is composed of two parallel line defects is presented in Figure 5.7(a). The r/a ratio is again 0.32. The shaded area is the band gap and the bands inside the band gap are the defect bands separated as even and odd modes according shape of the field.

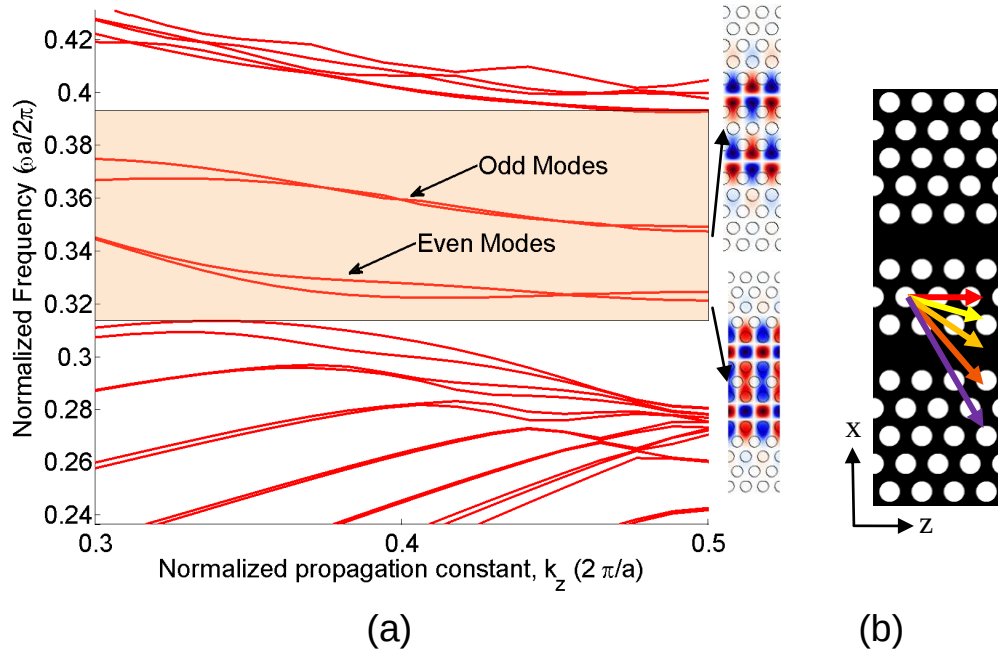


Figure 5.7: (a) The projected band diagram of a photonic crystal coupler with $r/a=0.32$ and material index 2.279. The field distributions of two of the defect bands at $k=0.5$ are shown. (b) The dielectric distribution of the simulated structure. Possible k -vectors having the same k_z component are shown with arrows.

The horizontal axis is the k -vectors in the defect direction z , k_z . For a given value of k_z , there exist multiple k -vectors inside the irreducible Brillouin zone that has the

same k_z components as shown in Figure 5.7(b). Therefore when solving for the projected band diagram, a k_z is chosen, then eigenvalues, ω , for all k-vectors inside the Brillouin zone that has a z-component of value k_z is solved. The group velocity of the modes can be extracted from the slope of the defect bands. As it is seen the bands can be close to flat meaning a very low group velocities can be achieved using PhCs.

In this section we have described basics of the theory behind photonic crystal structures. More detailed theory about PhCs can be found in references [148], [146]

5.2 Finite Difference Time Domain Method

Finite Difference Time Domain (FDTD) method is a well known method for simulating wave propagation, and was first proposed by Yee [149]. The name FDTD was given by A. Taflové [150]. It is widely used for modelling antennas, radar systems, wireless communications, waveguides, micro and nano-scale optical devices as well as acoustics. The method is based on computing the field components of an electro magnetic wave in a discretized computational space in discrete steps of time. An exclusive reference for the FDTD method is [151]. FDTD is also a common method to simulate PhCs.

As compared to PWE method FDTD method has advantages in some aspects. The PWE method solves modes that are allowed to exist inside the structure. However it does not exactly solve the overall propagating mode for a particular excitation. The response of the structure to a particular excitation can not be determined from the band diagram. In FDTD on the other hand the excitation spectrum and the spatial distribution is defined and the modes of the structure this excitation is going to excite is intrinsically computed. Additionally the propagation of these excited modes in the

structure namely their dispersion is also intrinsically computed. Additionally the PWE can be computationally much more expensive in 3D compared to FDTD simulations therefore is not practical. So generally when designing PhC structures the structure is initially reduced to a 2D one and a PWE simulation is performed to roughly determine the operating regime. And later the actual simulations are performed by FDTD method to get a more accurate result. We will describe a PhC simulation process later in Chapter 7. We will continue with describing discretization of differential equations which is the basis of FDTD method.

5.2.1 Discretizing Differential Equations

The FDTD method is a subset of finite difference methods in mathematics which solves a differential equation by an equivalent but approximate “difference equation”.

A simple differential equation is

$$y(x) = \frac{\partial f(x)}{\partial x} \quad (5.16)$$

can be discretized as

$$\tilde{y}(x_n) = \frac{f(x_{n+1}) - f(x_n)}{x_{n+1} - x_n} \quad (5.17)$$

where $\tilde{y}(x_n)$ is the approximation to $y(x = x_n)$. The Maxwell’s equations are of the form where the time derivative is on the left hand side and the space derivatives are in the right hand side of the equation similar to:

$$\frac{\partial y(x, t)}{\partial x} = \frac{\partial g(x, t)}{\partial t} \quad (5.18)$$

This equation can be discretized as

$$\frac{\tilde{y}(x_{n+1}, t_m) - \tilde{y}(x_n, t_m)}{x_{n+1} - x_n} = \frac{\tilde{g}(x_n, t_{m+1}) - \tilde{g}(x_n, t_m)}{t_{m+1} - t_m} \quad (5.19)$$

After re-arranging equation (5.19) the equation we get

$$\tilde{g}(x_n, t_{m+1}) = \frac{\Delta t_m}{\Delta x_n} (\tilde{y}(x_{n+1}, t_m) - \tilde{y}(x_n, t_m)) + \tilde{g}(x_n, t_m) \quad (5.20)$$

As it is seen from this equation the value of the function $\tilde{g}(x_n)$ at the discrete time value t_{m+1} is dependent on its own value and the value of function $\tilde{y}$ at time previous time step t_m .

5.2.2 The Yee Cell and FDTD Grid

In the FDTD method the field components of the electromagnetic waves are discretized in both space and time. The Yee cell is the fundamental building block of the whole computational grid and represents the discretization of the fields in space. In the Yee cell the electric fields and the magnetic fields are placed on the surface of a cube as shown in Figure 5.8.

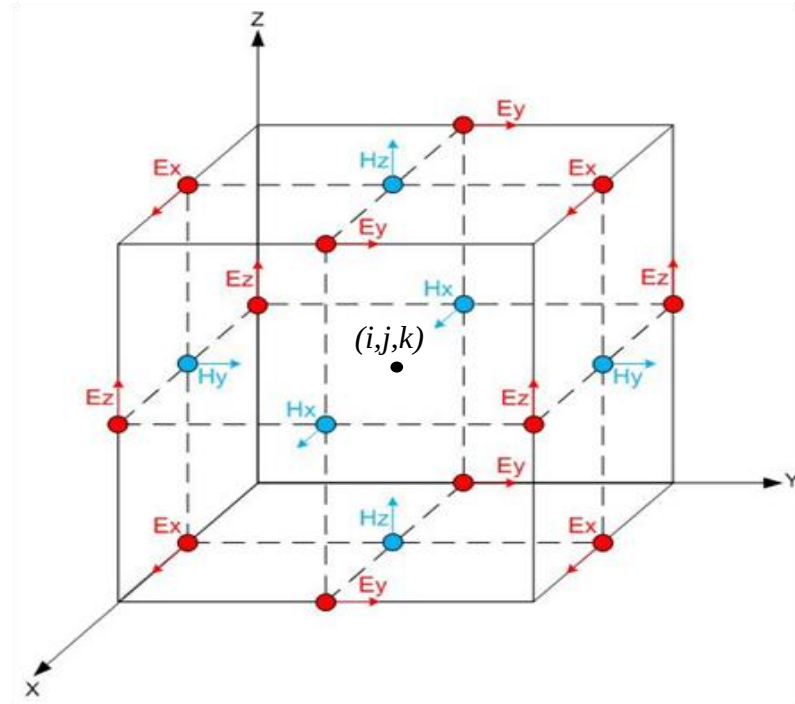


Figure 5.8: The Yee cell showing the discrete locations of the E and H fields in space.

The whole simulation can be considered as a lattice of Yee cells. In a programmers point of view the six fields are stored in three dimensional matrices. Other three dimensional matrices are needed to hold each of the material properties such as the dielectric constant and if necessary, the permeability and conductivity. The fields are updated in time steps as the simulation runs in a “leapfrog” manner in which the electric and the magnetic fields are updated one after the other.

The choice of the size of the Yee cell depends on the simulation. The wavelength of the propagating field is an important parameter affecting the choice. Usually it is chosen so that the field is sampled at least ten times in its wavelength. The value for Δt is chosen according the spatial resolution and has to be smaller than a certain value to avoid numerical instability. This is referred to as Courant stability condition and given by

$$\Delta t \leq \min(\Delta x, \Delta y, \Delta z)/c \quad (5.21)$$

where c is the speed of the light in the medium and Δx , Δy and Δz are the size of the Yee cell in x , y and z dimensions respectively.

5.2.3 Discretizing Maxwell's Equations

For simplicity we will carry on with the discretization of FDTD method in two dimensions. The problem can be reduced to two dimensions if the material is uniform and extending to infinity in the third dimension, in our case we choose it as z direction. The solution can be separated into two independent modes, the TE mode composed of E_x, E_y, H_z fields and TM mode composed of H_x, H_y, E_z fields. We will consider the TM and in this case the reduced Maxwell's equations become:

$$\frac{\partial H_x}{\partial t} = -\frac{1}{\mu} \left[\frac{\partial E_z}{\partial z} + \sigma^* H_x \right] \quad (5.22a)$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu} \left[\frac{\partial E_z}{\partial x} - \sigma^* H_y \right] \quad (5.22b)$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon} \left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \sigma E_z \right] \quad (5.22c)$$

Here μ is the magnetic permeability, ε is the electrical permittivity, σ is the electrical conductivity and σ^* is equivalent magnetic loss. The parameters σ and σ^* are zero for non lossy media. One important thing to notice is that, the center of the Yee cell in Figure 5.8 is given the coordinates (i,j,k) . The H_x component is therefore sampled in coordinates $(i+1/2, j,k)$ and $(i-1/2, j,k)$. Similarly the H_y fields are sampled at $(i, j-1/2,k)$ and $(i, j+1/2,k)$ and so on. equation (5.22a) can be discretized as

$$H_x|_{i-1/2,j}^{n+1} = D_1(i,j)H_x|_{i-1/2,j}^n + D_2(i,j)(E_z|_{i-1/2,j-1/2}^n - E_z|_{i-1/2,j+1/2}^n) \quad (5.23)$$

The superscript $n + 1$ is the discrete time step and the subscripts containing i and j are the Yee lattice coordinates. The coefficients $D_1(i,j)$ and $D_2(i,j)$ are defined as

$$D_1(i,j) = \left(1 - \frac{\sigma_{i,j}^* \Delta t}{2\mu_{i,j}} \right) / \left(1 + \frac{\sigma_{i,j}^* \Delta t}{2\mu_{i,j}} \right) \quad (5.24a)$$

$$D_2(i,j) = \left(\frac{\Delta t}{\mu_{i,j} \Delta d} \right) / \left(1 + \frac{\sigma_{i,j}^* \Delta t}{2\mu_{i,j}} \right) \quad (5.24b)$$

Here the Yee cell is assumed to be a square with side length of Δd . The rest of the equations can be similarly discretized. The sources are realized by forcing the desired field components in the desired location to a value at each time step. The source can be a continuous wave signal or a Gaussian pulse or any arbitrary source sampled with the time interval, Δt .

5.2.4 Boundary Conditions

The field of one cell in the next time step is dependent on its neighbouring cells. If a field is launched from the center of the computational domain, the excitation propagates and reaches the boundaries. Special attention is needed when the field reaches to the boundaries of the computational domain. An outer most cell is not surrounded by all the fields that are needed to compute its value in the next time step because the computational domain is truncated. We can not simply force all the fields at the boundaries to zero since this will result in reflection of the field.

There are several methods to avoid this reflection which in general are called absorbing boundary conditions (ABC), among which the widely used one is the perfectly matched layer (PML) proposed by Berenger [152]. This method defines only a few (typically less than 10) extra Yee cells at the boundaries of the original simulation domain which are lossy. The fields in these regions are attenuated as they propagate towards the outer boundaries which are perfectly reflecting mirrors. However since the amplitude of the field is very small as it reaches the boundary the reflection becomes negligible. PML works quite well at the expense of increased computational load. Considering the 2D TM case described above, to implement PMLs, the E_z field is split into two virtual components E_{zx} and E_{zy} . As a result at each time step now we have to compute the value of four fields instead of three. The PML layers are lossy having non-zero values for the conductivity σ , and the equivalent magnetic loss σ^* in equations (5.22). The loss of the PML layers is increased gradually towards the boundary. Details of implementation of PML boundary conditions can be found in [151]. There are other types of ABC such as Mur's absorbing boundary conditions [153]. UPML [154] and CPML [155] are alternative PML implementations. A comparison of Mur's ABC and PML is provided in [156].

5.2.5 Calculating Quality factor with Harmonic Inversion Technique

For analyzing the field in a particular location of the simulation domain, usually point probes are used. The probes record the fields over time. In simulations such as resonant cavities, the probes are placed inside the cavity and the decay rates of the field intensity at different frequencies are analyzed. This analysis gives the resonance frequencies of the cavity and the corresponding quality factors (Q-factor). As an example consider the time domain plot in Figure 5.9(a) which shows the E_x field recorded by a probe (light blue) inside a resonant PhC cavity. The time domain signal decays with time due to the loss of the cavity. Figure 5.9(b) shows the power spectrum of this probe data. The single peak in the spectrum shows us that there is only one resonant mode of the structure which is around 832nm wavelength. The Q-factor is calculated using two different methods. One of them is the harmonic inversion method [157] using the open source software, harminv, and the other is fitting a Lorentzian curve to the peak in the power spectrum and extract the Q-factor. The Lorentzian curve is given by the following formula

$$P(f) = I_0 \frac{\sigma_l^2}{(f - f_0)^2 + \sigma_l^2} \quad (5.25)$$

where P is the power, f is the frequency variable, f_0 is the resonant mode center frequency, I_0 is the peak amplitude and σ_l is the parameter determining the sharpness of the peak. The Q-factor is simply given by

$$Q = \frac{f_0}{2\sigma_l} \quad (5.26)$$

The Q-factors shown in Figure 5.9(a) (right axis) are found by feeding different durations of the data to both methods. The Q-factors computed with harminv (black, circles) give a stable output for data durations longer than 0.54ps. On the other hand

the Q-factor computed with Lorentzian fit is noisy even for much longer durations. This shows that harminv method greatly reduces the amount of time the simulation is required to run for to get accurate estimation of the Q-factor.

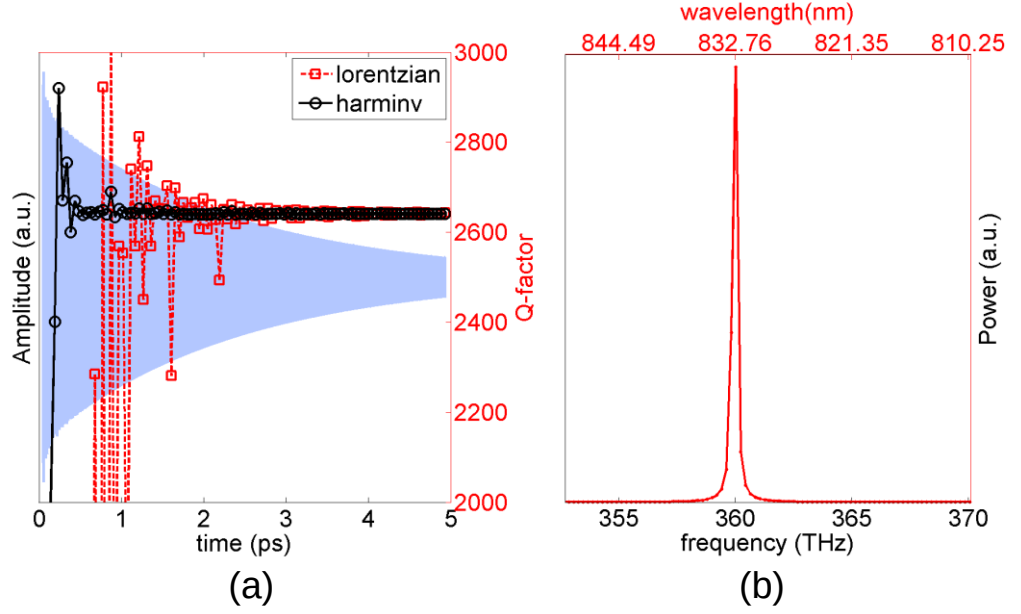


Figure 5.9: (a) The time domain signal recorded by a point probe (light blue plot) in an FDTD simulation. The right hand side axis is the Q-factors computed by harmonic inversion method (black circles) and Lorentzian fitting method (red squares) for different durations of the probe data. (b) The power spectrum of the probe data showing the peak for the resonant mode.

Chapter 6

Review of Photonic Crystals

This chapter discusses about applications of photonic crystals providing examples from the literature. The application examples include photonic crystal waveguides, couplers and cavities. Slow light which is one of the most distinguishing properties of photonic crystal is discussed in detail.

6.1 Applications of Photonic Crystals

PhCs are artificially fabricated structures although there are some examples of PhCs in nature such as the wings of the Morpho butterfly [158]. The developments in fabrication technology and the availability of fabrication of sub-micron feature sized devices enabled the realization of PhCs working in a broad range of wavelengths including the visible [159], [160]. The band gap is a range of frequencies in which light is not allowed to propagate inside the PhC. Therefore by the use of PhCs the light can be confined and manipulated. Photonic crystal devices such as directional couplers, filters, modulators can be made much smaller than their conventional waveguide based counter parts [161].

Despite their advantages the challenges in fabrication is the major reason why PhCs are not widely available in integrated photonics. The patterns forming the PhC have be on the order of the wavelength of the light. Triangular lattice of holes in a dielectric slab and the woodpile structure are two commonly employed geometries for 2D and 3D PhCs shown in Figure 6.1(a) and (b). The 2D PhC is a waveguide where a row of holes have been removed from the uniform crystal. The side walls of the holes has to be vertical to the surface in the ideal case. The angle in the side walls scatter the light out of the structure and cause loss. Also the smoothness of the surface and the side walls and the uniformity of the hole radii are other important things to prevent unwanted scattering of light. Using membrane type structures as the one in the figure makes it easier to etch smoother and more vertical holes as compared to ones which have a substrate underneath, since the etched material has an additional path to escape, however this requires additional steps of fabrication for under cutting. For the purpose of comparison of PhC waveguide losses with the conventional one we will

quote propagation losses of PhC waveguides in the literature. In [70] a propagation loss of 24dB/cm was reported as the world record of that time in a silicon membrane which was beaten by 5dB/cm reported [71] and in 4.1dB/cm in 2006 [72] on the same material and similar wavelengths ($\sim 1.55\mu\text{m}$). In conventional silicon nano wires the losses can be as low as 0.3dB/cm [73]. However since the PhC structures provides reduction in size a direct comparison might not be fair.

Coupling in and out of the PhC from conventional ridge waveguides is also a challenge due to the mode mismatch between the two modes. Although in theory unity coupling efficiency is possible [74] an coupling loss of down to 2.4dB has been demonstrated experimentally [75].

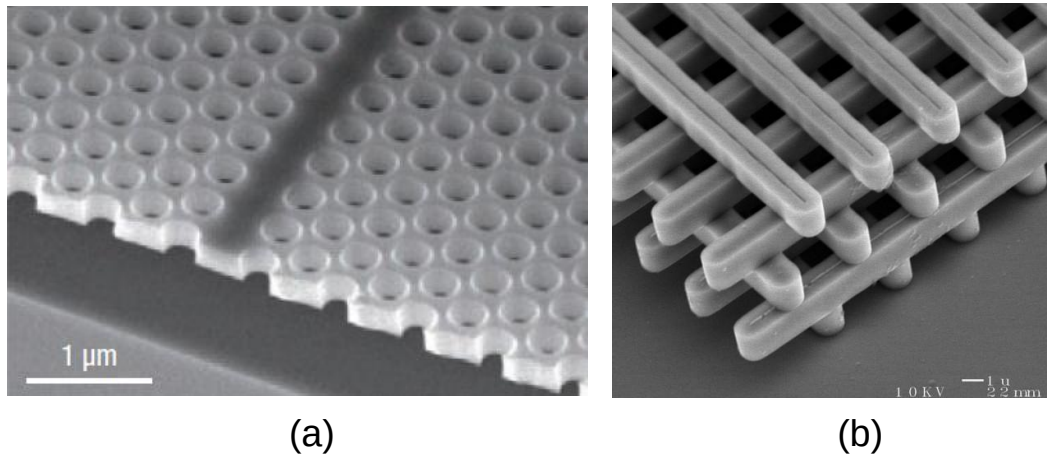


Figure 6.1: (a) A 2D photonic crystal of triangular lattice holes in dielectric. Image taken from [174]. (b) 3D woodpile structure which exhibits a band gap in all directions [197].

PhCs has been a research topic since late 19th century although not under the name of PhCs. Stacked layers of dielectrics which are 1D PhCs were being used commonly in optics. In 1987 Yablonovitch [162] and John [163] proposed the use of photonic band gaps to manipulate the light. Due to the lack of sub-micron size fabrication technologies, early PhCs were fabricated for microwave frequencies and hence sizes were on the order of centimetres. In 1991 Yablonovitch demonstrated a

3D PhC structure known as “Yablonovite” today, which works at microwave frequencies [164]. In 1996 Thomas Krauss et al. demonstrated the first artificial 2D PhC that works at wavelengths around 800-900nm [165]. Since then similar structures which can be called in general “2D PhC slabs” have been investigated one of which is shown in Figure 6.1 (a).

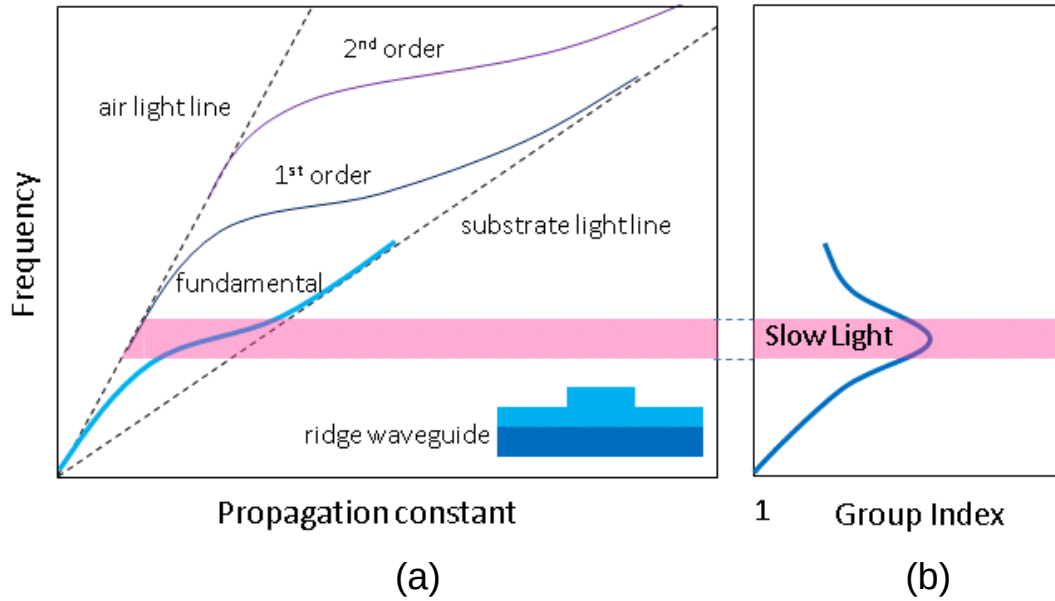


Figure 6.2: (a) A typical dispersion curve for a ridge waveguide. (b) The group index corresponding to the fundamental mode of the ridge

Photonic crystals provide the flexibility of engineering the dispersion of the medium. So it will be useful to explain the dispersion and its relation with the group velocity of light. Figure 6.2(a) shows a typical dispersion diagram corresponding to a ridge waveguide as illustrated in the inset. The bands shown correspond to fundamental, 1st order and 2nd order modes bounded by the air light line and the substrate light line. The region in between the light lines is where the guided modes exist. The modes out of this region are radiation modes. The slope of the band is inversely proportional to the group index. The group index is the ratio of the speed of a wave packet of light in free space to the speed at which it travels inside the waveguide. Figure 6.2(b) illustrates the group index for the fundamental mode. The

region shaded with pink is where the group index gets larger.

While in semiconductor ridge waveguides such as silicon nano-wires, the group index is typically below 5 [166], photonic crystal waveguides in silicon are demonstrated to have a group index above 50 in [167] and around 300 in [168]. The following subsections will show some examples of PhC devices: PhC waveguides, couplers and cavities.

6.1.1 Photonic Crystal Waveguides

PhC waveguides are formed by removing the periodicity of dielectric along a particular direction. In a triangular lattice slab PhC structure it is usually realized by removing a row of holes from along the so called ΓK direction of the lattice, as shown in Figure 6.1(a). These are called W1 defect waveguides (if n rows are removed the resultant is called a W_n defect waveguide [169]). One distinguishing property of this structure is the ability to slow down the light. A performance parameter for these devices is the group index, n_g which is related to the group velocity, v_g the velocity of the envelope of a wave packet. The group velocity is given by the following formula

$$v_g = \frac{\partial \omega}{\partial k} \quad (6.1)$$

where ω is the angular frequency of the wave and k is the propagation constant. Equivalently it is the slope of the dispersion curve. The group index is related to the group velocity by

$$n_g = \frac{c_0}{v_g} \quad (6.2)$$

where c_0 is the speed of light in free space. So the group index is the factor of slow down of the wave packet.

Many different W1 defect geometries were shown where the position and the size of holes were modified [170], [171], [172], [173]. A different geometry is demonstrated in [167] where the input light propagates in the structure in the form of even and odd modes by the use of two parallel W1 defect waveguides as shown in Figure 6.3.

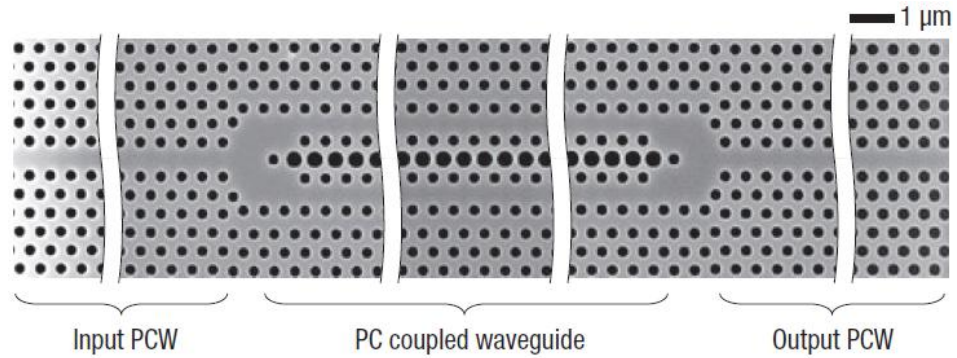


Figure 6.3: The SEM image of the slow light device presented in [148]. The radii of the holes in the center of the coupled waveguide region are gradually increased to form a chirped waveguide. The device is fabricated on silicon-on-insulator substrate.

The device is composed of three regions, the input region, the coupled waveguide region and the output region. The aim is to achieve a flat band (hence constant group index) over a range of frequencies. The flat band is illustrated in the band diagram of Figure 6.4 (a). The linear region highlighted with red is the flat region. Figure 6.4 (b) shows the corresponding group index versus frequency. The linear region of the dispersion diagram translated to a constant region in this group index versus frequency curve also highlighted with red. The band of frequencies corresponding to this flat region determines the bandwidth of the slow light operation regime. The radii of the holes at the center of the coupled waveguide region in Figure 6.3 are gradually increased which is like cascading many devices each operating at slightly different wavelengths to achieve the overall desired response. A group index of 40-60 over a 12nm bandwidth was achieved. A total delay of 57ps which translates

to a storage capacity of 12bit was also shown. Tuning of the delay using thermo-optic effect was demonstrated where the device was heated by focusing a laser on the surface.

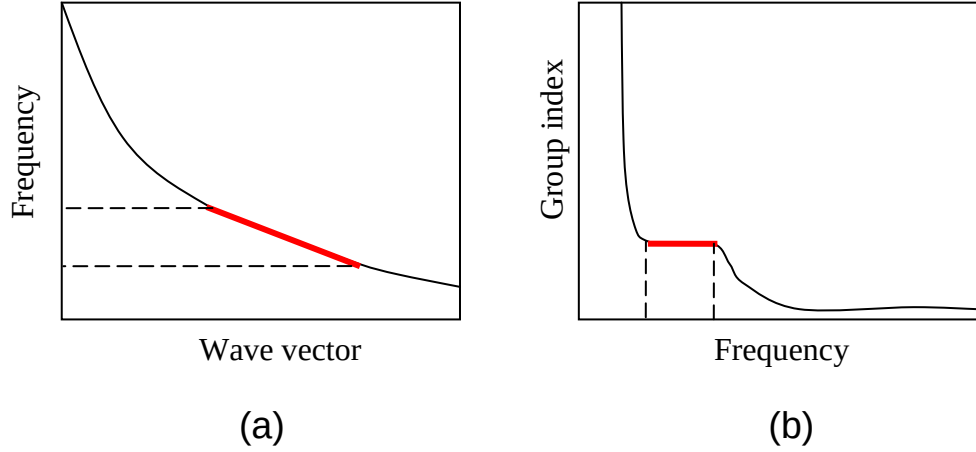


Figure 6.4: (a) Illustration of dispersion curve (band) of a waveguide. It has a linear region highlighted with red. (b) The corresponding group index versus frequency curve. The flat region of the dispersion curve corresponds to a constant group index.

6.1.2 Photonic Crystal Couplers

PhC couplers are made by bringing together two PhC waveguides. The coupling occurs as a result of the beating between the even and the odd modes of the overall structure. There are a number of simulation based studies based on PhC couplers [174], [175], [176] which involve the optimization of the performance by tweaking the geometry of the structure.

A PhC coupler by Krauss et al. is presented in [76]. This device is fabricated on silicon-on-insulator (SOI) material having 225nm thick silicon layer. The material under the PhC region is etched leaving the coupler a free standing silicon membrane as shown in Figure 6.5(a). The coupler is a cascade of three sections. The input and output sections serve as tapers to efficiently couple the light in and out. The first row

of holes neighbouring the defect waveguides have a hole radius of r_1 and the second row of holes have a radius of r_2 . The remaining holes have a radius of r_0 . In the middle section r_1 and r_2 are chosen as 0.34 and 0.39 respectively. These parameters are chosen to achieve sensitivity in switching and is explained in [176]. The working principle is based on the slow light inside the PhC structure. Figure 6.5(b) shows the band diagram for the middle coupling section of the device calculated by PWE method. The switching length which is the length of device required to couple light from one arm to the other is dependent on the difference between the propagation constants of the two modes (equation (3.8)). The choice of hole radii allows the upper band to be flat as indicated in the figure. A change in the refractive index is going to shift the two bands towards lower frequencies hence decreasing the difference between the propagation constants (increasing the switching length). Therefore the flatness of the band enables the device to be more sensitive to variations. This flat band is not possible to achieve using a conventional ridge waveguide. The switching length is also plotted in the figure in terms of lattice constant (top axis).

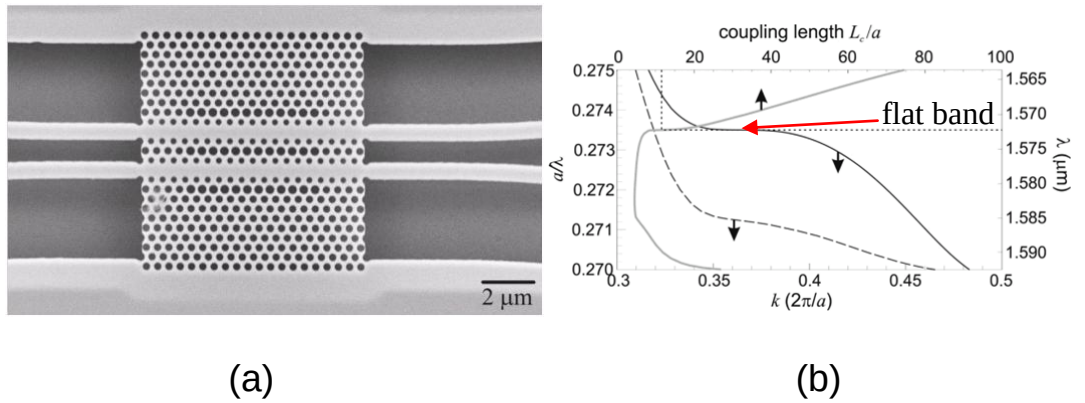


Figure 6.5: (a) The SEM image of the PhC coupler presented in [74]. (b) The band diagram for the middle coupling region of the structure calculated by PWE method showing the two modes of the coupler (bottom axis). The upper axis is for the switching length.

The length of the middle section is 12 lattice constants ($5.2\mu\text{m}$) and the input output regions are 4 lattice constants each. The light input from one arm of the coupler splits among the two output ports.

The splitting ratio was controlled by changing the temperature of the chip. By changing the temperature from 23°C to 45°C , the light can be switched from the bar port to the cross port. The length of the device is only $9\mu\text{m}$ which shows that by utilizing the slow light property the switching of light can be performed in a very small distance.

A continuation of this work is presented in [177]. In this study a similar structure is used. The switching between the ports is controlled using an external femto-second pulsed laser focused on to the device. Two consecutive pulses having a 3ps separation were successfully guided to different ports of the device using this method.

6.2 Slow Light in Photonic Crystals

Slowing down the light is important for two main reasons. One of them is to allow more time for the processing light by delaying it, and secondly to increase the light-matter interaction which is important especially in non-linear optical applications.

Converting a stream of information into pulses of light and transmitting them is a well established method for communication. Modulation of light is often performed by modifying the optical properties of the medium that the light propagates using physical phenomena such as thermo-optic effect, electro-optic effect or using free-carrier injection. The amount of change in the refractive index of the medium that can be induced via these methods does not exceed one per cent in most cases. This limits the modulation capability and requires a longer light-material interaction length to

achieve the desired modulation. This is what makes PhCs attractive in a variety of applications. As a comparison while the group index in typical silicon nano-wire waveguides is below 5 [178] using a photonic crystal values ~ 100 [179] and 300 [168] has been achieved. The delay-bandwidth product is an important parameter for evaluating the performance of the slow-light devices [180]. While the index is desired to be high it is also desired that it is constant over a wide bandwidth so that a shorter pulse can be slowed down preserving its shape (see Figure 6.4). To achieve this, the dispersion curve needs to be linear in the wavelength range of interest.

The dispersion characteristics of PhC waveguides can be tailored by adjusting the geometry of the lattice cells which are close to the defect [173], [181] or by cascading structures of different dispersion characteristics to reach the desired overall dispersion [172]. In [173] a constant group index of 30 was achieved over a bandwidth of 11nm.

Another utilization of slow light in PhC waveguides is the boosting of non-linearities of the medium resulting from the increased interaction time. This can be visualized by considering a pulse of light entering the slow light regime. The pulse is going to be compressed in space hence the intensity is going to increase with the slow down factor. As a result non-linear processes such as the ones originating from $\chi^{(2)}$ and $\chi^{(3)}$ non-linearities are expected to scale with the square of the slow down factor [182]. In practice this quadratic enhancement can not be achieved due to the change of dispersion induced by these non-linear processes. Careful design is required to reduce the negative effect of these non-linear processes on the dispersion.

6.3 Summary and Conclusions

Photonic crystals are artificially made structures having periodic dielectric, which provide flexibilities in manipulating the light. PhCs offer very small scale devices such as cavities, waveguides and couplers compared to their conventional counterparts. This makes them attractive for integrated photonics applications as well as integrated quantum photonics applications. PhC cavities are already being used in single photon source studies such as in quantum dots [92] or NV centres in diamond [183]. Two dimensional PhCs on semiconductor slab are commonly used due to their relatively easier fabrication and integration with other conventional components compared to the three dimensional ones. We have provided some examples of PhC cavities, waveguides and couplers in the literature and also explained slow light in more detail.

Chapter 7

Photonic Crystal Structures on GaN

In this chapter the systematic design of a photonic crystal directional coupler in gallium nitride material platform which has potential applications in integrated quantum photonics, is covered. Two dimensional band diagram calculations and two and three dimensional finite-difference-time-domain simulations are presented. A miniature sized directional coupler is designed and also tuning of the coupling ratio is investigated. The device designed has a much reduced footprint compared to the device discussed in chapter 3.

7.1 Design and Modelling of GaN Photonic Crystal Devices

7.1.1 Gallium Nitride

Gallium Nitride is in the class of III-V semiconductors having a direct band gap of $\sim 3.4\text{eV}$ which makes it an efficient material for light generation in the blue and green wavelength regime. The technique developed by Shuji Nakamura on p-type GaN doping enabled mass production of GaN p-n junctions which was a turning point in the commercialization of optical devices in these wavelengths [63]. Since then the GaN optoelectronic market volume has reached to multi-billion dollars.

It has a refractive index ranging between 2.3 and 2.7 in the visible and near infrared regime. GaN is a relatively new material for integrated optics [184] and its applications include second harmonic generation [185] and biosensing [186]. GaN is transparent in a wide range of wavelengths from $\sim 365\text{nm}$ to $\sim 13.6\mu\text{m}$ [64] which makes it attractive for classical photonics applications as well as quantum photonics. It enables generation of single photons with embedded quantum dots [19] and GaN single photon detectors have also been demonstrated [123]. Therefore it is a potential platform for integrated quantum photonics. Here we aim to utilize these benefits together with the size reduction provided by the photonic crystal structures.

Growth of high quality GaN layers still a challenge. Most common substrate for GaN is sapphire however the two crystals have large lattice constant and thermal expansion coefficient mismatches [187]. Usually an AlN buffer layer is used in between sapphire and GaN to prevent crystal defects resulting from this mismatch. Silicon carbide and silicon are other substrate alternatives however they result in

lower quality GaN crystal compared to sapphire.

Several materials for integrated photonics as alternative to GaN are gallium arsenide (GaAs), gallium phosphate (GaP), titania (TiO₂), lithium niobate (LiNbO₃), diamond, silica. The table below shows a comparison of these different material systems [188]. GaN is popular among the others mainly due to its ability to generate light in blue wavelength and it is important when integration of circuits and sources is required.

Table 7-1 A comparison of different material systems for photonics applications

Material	Refractive index (800nm)	Nano-photonics Fabrication	Transparency	Light Emission
Gallium Nitride	2.35	Good	All visible	Blue
Diamond		Difficult	All visible & Infrared	No
Gallium Phosphide	3.18	Very Good	Not blue	Green
Titania	2.49	Fair	All visible	No
Lithium Niobate	2.32	Difficult	All visible	No
Silica (Glass)	1.54	Difficult	All visible	No

7.1.2 Modelling and Simulations

Our layer structure is composed of a sapphire substrate, an AlN buffer layer and GaN core layer. In a photonic crystal design perspective, this layer structure is considered as asymmetric due to the layers above and below GaN having different refractive indices. There are many studies which use suspended membrane type structures [70], [76]. These are classified as symmetric PhC structures due to the existence of a

symmetry plane bisecting the slab layer. In these structures the modes can be separated into even-symmetric and odd-symmetric depending on their symmetry around this plane. The even symmetric modes are TE (Electric field in plane) modes and the odd modes are TM modes [189]. These two types of modes do not couple to each other and with proper design the structure can exhibit band gap for even-symmetric modes or a band gap for odd-symmetric modes. The existence of a band gap means in theory a lossless propagation can be achieved. On the other hand there are asymmetric type PhC structures where the structure does not have a vertical symmetry, that is the top and bottom claddings have different refractive indices. An example for such structure is PhC structures etched on SOI material [190], [191]. In these PhC structures the modes cannot be separated into even and odd. Therefore these structures do not exhibit a band gap. In these structures although there is not a pure TE or pure TM modes, there are TE-like or TM-like modes [192]. In these structures the band gap effect (suppression of a range of frequencies) can still exist although not as strong as in the symmetric structures. The advantage of using asymmetric structures is that they do not require the complex undercutting fabrication step [193],[194],[195] and they are mechanically more stable. In this study we also consider an asymmetric layer structure composed of sapphire, AlN and GaN layers. We have used this layer structure due to its availability and ease of fabrication.

Our aim is to design a photonic crystal directional coupler harnessing the slow light effect of the PhCs which enables us to perform splitting of the input light in a very short distance. Increased interaction of the light with the matter also enables the use of the electro optic effect more efficiently for tuning the operation of the coupler. We have achieved a 50-50% coupler having a length around $5\mu\text{m}$ operating around 800nm wavelength. Also we have designed a coupler which is slightly longer in

length that can be tuned from 63-37% (the first number is the percentage of power output from the bar port, the port where the light was injected, and the second number is for the power in the cross port) to 15-85% for wavelengths around 800nm by a 0.1% change of the refractive index of GaN layer.

We are going to follow a step by step design procedure starting from uniform photonic crystals followed by W1 defect waveguides and then directional couplers. The design procedure of PhCs usually starts with 2D design since it is faster and computationally less expensive compared to 3D. The results from the 2D design provide starting parameters for the actual 3D design such as the lattice constant and radii of the holes. Throughout the designs we will use TE excitation which has E-field parallel to layer interfaces since the triangular lattice PhC structure has quasi-bandgaps for this polarization. Figure 7.1 shows the layer structure used in the simulations. It is composed of a 100nm thick AlN buffer layer and a 500nm thick GaN layer grown on top of sapphire.

GaN	500nm	n=2.353
AlN	100nm	n=2.138
Sapphire		n=1.76

Figure 7.1: The layer structure used in the simulations. It is composed of a Sapphire substrate, 100nm AlN buffer layer and 500nm GaN layer.

7.1.3 Two Dimensional Design

To reduce the problem into a two dimensions, the effective index of the fundamental TE mode of slab at 800nm wavelength, 2.2787 is used.

7.1.3.1 Uniform Photonic Crystal

The simulated structure is a triangular lattice of holes in a dielectric background with hole radius to lattice constant ratio (r/a) of 0.32. The value 0.32 is chosen considering the fabrication tolerances. A 2D FDTD simulation is performed in MEEP due to its parallel computing capability. Simulations are run in the BlueCrystal, a high performance computer in University of Bristol. The simulation environment is depicted in Figure 7.2(a). The excitation is placed on one side of the PhC structure. It is a line source producing a Gaussian pulse centered at 800nm (840nm full-width-at-half-maximum (FWHM) bandwidth) covering the band gap of the structure. On the other side of the PhC there is a flux plane for computing the transmitted flux through the PhC for a range of wavelengths. Figure 7.2(b) shows the result of the 2D FDTD simulations for two different orientations of the crystal. The solid curve is for the PhC is oriented in ΓK direction (as in the Figure) and the dashed is for the PhC oriented in ΓM direction (the PhC in the figure is rotated by 90°). These are the two extreme cases so the observed band gap is expected to be valid in intermediate directions. The graph shows the normalized transmission through the structure. The normalization is performed by dividing the flux data in the case when the holes are present to the flux data obtained when the holes are removed. The band gap is the region of wavelengths where the transmission is significantly lower highlighted by the horizontal dashed lines. The ratio between the transmitted power when operating outside the band gap and when operating inside the band gap depends on the number of periods of the PhC structure. Typically a 10dB extinction ratio can be considered as a band gap. The band diagram of the structure shown in Figure 7.2(c) is calculated by MPB software. The TE (E-field parallel to surface) bands are plotted with black dots and the TM (E-field perpendicular to surface) bands are also plotted in dashed lines. The TE band gap is

between normalized frequencies of 0.318 and 0.391. For 271nm of lattice constant this means a bandgap from 693nm to 852nm and agrees with the FDTD simulation. For TM modes there is no band gap in the frequency range of interest.

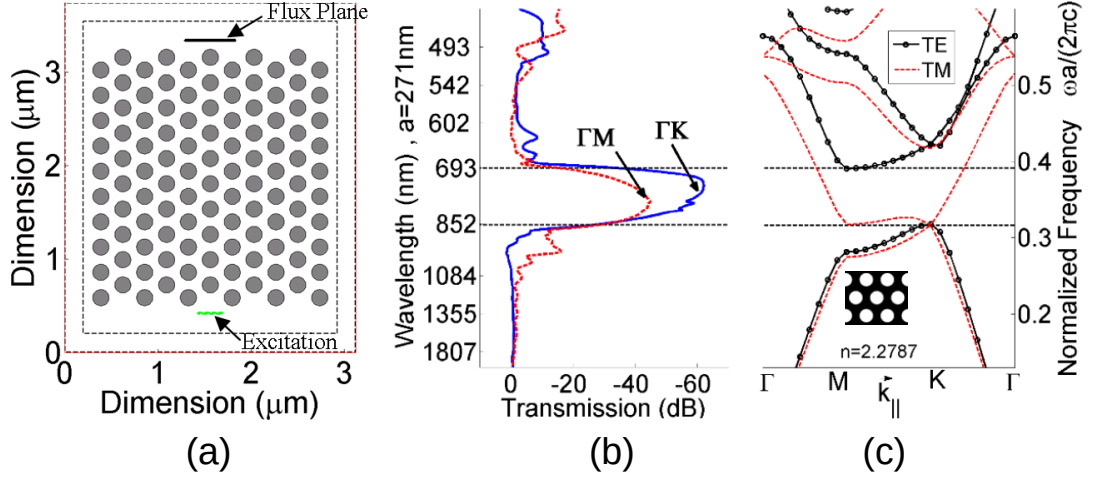


Figure 7.2: (a) The 2D FDTD environment for simulating the triangular lattice PhC with $r/a=0.32$ oriented in ΓK direction. (b) The transmission vs. wavelength graph for ΓK and ΓM orientated PhCs calculated by using structures with no holes as reference simulated by 2D FDTD. (c) Band diagram of the photonic crystal structure for TE (E-field in plane) and TM (E-field out of plane) modes.

7.1.3.2 W1 Defect Waveguide

W1 defect waveguides are obtained by removing a row of holes from the uniform lattice. In a triangular lattice, a row of holes in ΓK direction is removed. The resultant structure guides the light through the defect. One of the powerful characteristics of PhC waveguides is the increased flexibility in controlling the dispersion [170], [196], [197].

Figure 7.3 shows the simulation of a triangular lattice W1 defect waveguide PhC structure with a background index of 2.2787. Here the radius of the nearest row of holes around the defect is r_1 , the radius of the second nearest row of holes is r_2 and the radius of the holes in the background is r_0 as shown in Figure 7.3(a). These holes because they are closer to the propagating mode they have more effect on the

dispersion of the structure. We are going to use the same parameters for the coupler structure and we will see that these hole radii has a significant influence on the switching length.

Figure 7.3(b) shows the projected band diagram of structures with three different parameters for r_0 , r_1 and r_2 . Notice that the dispersion properties are changed significantly by modifying the hole radius. The bands are named as even and odd according to their symmetry around the line that bisects the defect. A sample H_z field distribution for the structure having parameters $[r_0, r_1, r_2]=[0.32a, 0.34a, 0.32a]$ at normalized wavevector 0.5 is shown in Figure 7.3(c).

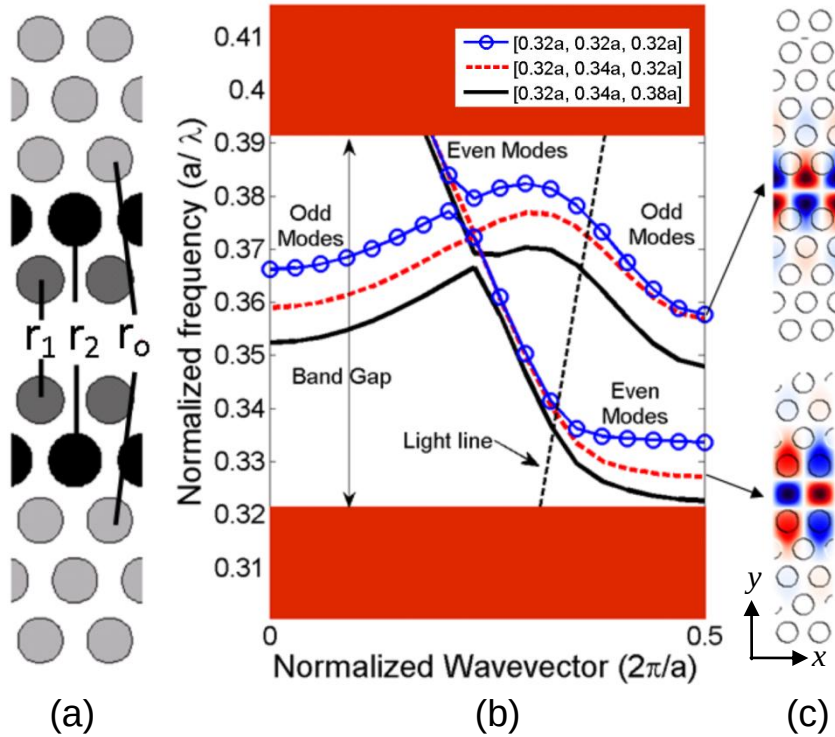


Figure 7.3: (a) Geometry of W1 defect PhC structure, (b) Band diagram for 2D W1 defect PhC, (c) Real part of H_z at normalized wavevector $k=0.5$ for the dashed curve. The solid line in (b) corresponds to the hole radii $[r_0, r_1, r_2]=[0.32a, 0.32a, 0.32a]$, the dashed line is for $[0.32a, 0.34a, 0.32a]$ and the circled line is for $[0.32a, 0.34a, 0.38a]$. The air light line is also shown.

7.1.3.3 Photonic Crystal Directional Coupler

Directional couplers are realized by bringing two W1 defect waveguides close to each other. The light propagating in one of the waveguides evanescently couples to the other. The light switches periodically between the waveguides, the period being inversely proportional to the coupling coefficient between the waveguides. As in the waveguide based directional coupler discussed in section 3.3. At this distance, L_π the light at one arm is switched to the other waveguide. L_π is given by the equation:

$$L_\pi = \frac{\pi}{|\Delta k|} \quad (7.1)$$

where Δk is the difference in the propagation constants (wavevectors) of the even and odd modes of the coupler. At half of this distance the input light is split equally across the two ports of the coupler and is called L_{3dB} .

We tune L_{3dB} by altering the radius of the holes around the waveguides. As in the W1 defect waveguide, we name the radius of the closest row of hole to the defects as r_1 , and the second nearest ones as r_2 and the remaining as r_0 as shown in Figure 7.4(a). The band diagram is calculated by MPB software for PhC couplers with two different parameters; $[r_0, r_1, r_2]=[0.32a, 0.34a, 0.32a]$, and $[r_0, r_1, r_2]=[0.32a, 0.34a, 0.38a]$. Figure 7.4(b) shows a portion of the complete band diagram where we see the even-even and even-odd defect bands. L_{3dB} is calculated from the difference between the normalized wavevectors of the two modes and is plotted in Figure 7.4(c) for the two different coupler structures. Here the 3dB coupling lengths are given in terms of the lattice constant, a .

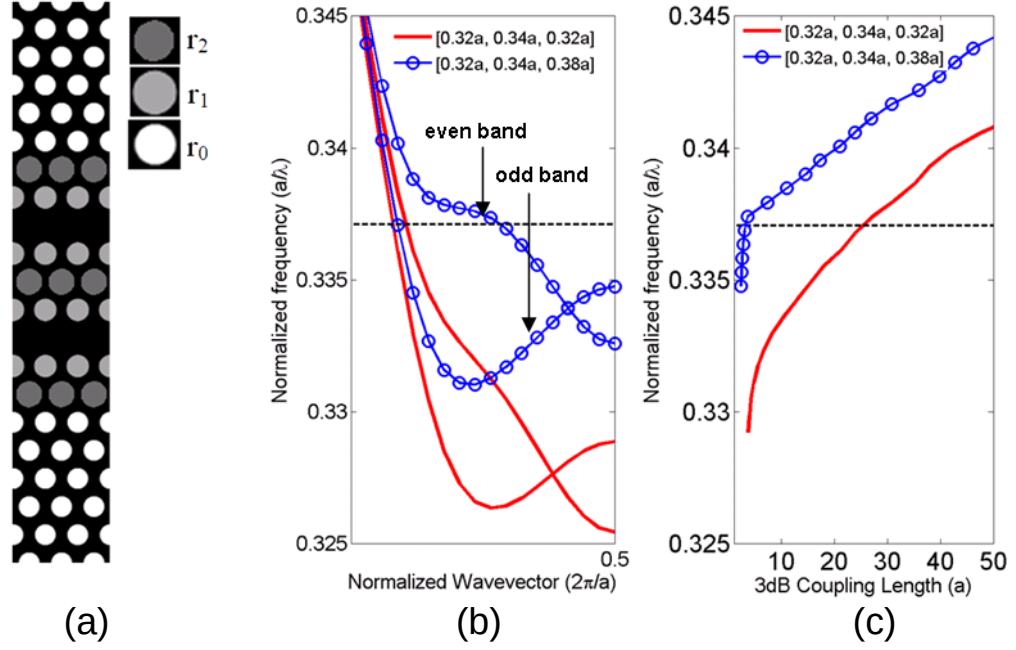


Figure 7.4: (a) The coupler geometry simulated in MPB. (b) The resultant band diagram for PhC couplers with two different parameters zoomed at the region of interest. (c) The coupling lengths versus normalized frequency obtained from the bands for the two structures.

As we can observe there is a particular frequency (marked with the horizontal dashed line) at which the slope of the blue-circled curve suddenly changes, in this case it is around 0.3374. For frequencies above this point the coupling length is more sensitive to the variations in frequency and for frequencies below, the 3dB coupling length is shorter. Therefore at this point we can achieve both short device length and sensitivity. Sensitivity is required for tuning the operating regime as we will see in Section 7.1.4.4 and for switching applications (see Section 6.1.2). This frequency is also of interest in switching applications where the band diagram is shifted down by a change in the refractive index. This results in a sudden increase in the 3dB coupling length hence blocking light to couple between waveguides. We are going to cascade the two PhC structures simulated here in order to control the coupling between the waveguides by the change of the parameter r_2 .

7.1.4 Three Dimensional Design

Now we proceed with 3D designs which are computationally more expensive yet more accurate. In this case while the in-plane confinement is provided by the band gap of the PhC, confinement in the vertical direction is provided by total internal reflections at the layer interfaces. In structures with low index contrast it is not possible to obtain a frequency range where both confinement mechanisms simultaneously exist. Therefore we have to operate in so called above the light line regime where some amount of light radiates out of plane causing loss. We have used 3D FDTD method to calculate the response of the PhC devices designed. The layer structure is used in the 3D simulations was already shown in Figure 7.1.

7.1.4.1 Uniform Photonic Crystal

The band diagram of uniform photonic crystals can be generated using MPB software, however it is not practical besides in such an asymmetric structure we do not expect to see complete band gap. In practice we are only interested in the case when TE polarized light is launched into the device, hence the propagating mode although will not be a pure TE mode, will have a dominant TE-like component. In other words the TM-like modes will be excited very weakly. Hence although there is no full band gap for the structure we can still observe quasi band gaps.

The geometry of the simulated structure is depicted in Figure 7.5. As in the 2D case, the lattice constant is 271nm and the radii of the holes are 86.7nm. Two separate simulations are performed. In the first one a 10x11 size PhC structure is placed in the Γ -K direction and in the second one a 12x9 size structure is placed in the Γ -M orientation. The excitation is a Gaussian pulse plane source, centred at 770nm

and has a FWHM of 380nm. On the other side of the crystal there is a flux plane to measure the flux for a range of frequencies. The position and size of the excitation and the flux planes are shown in Figure 7.5.

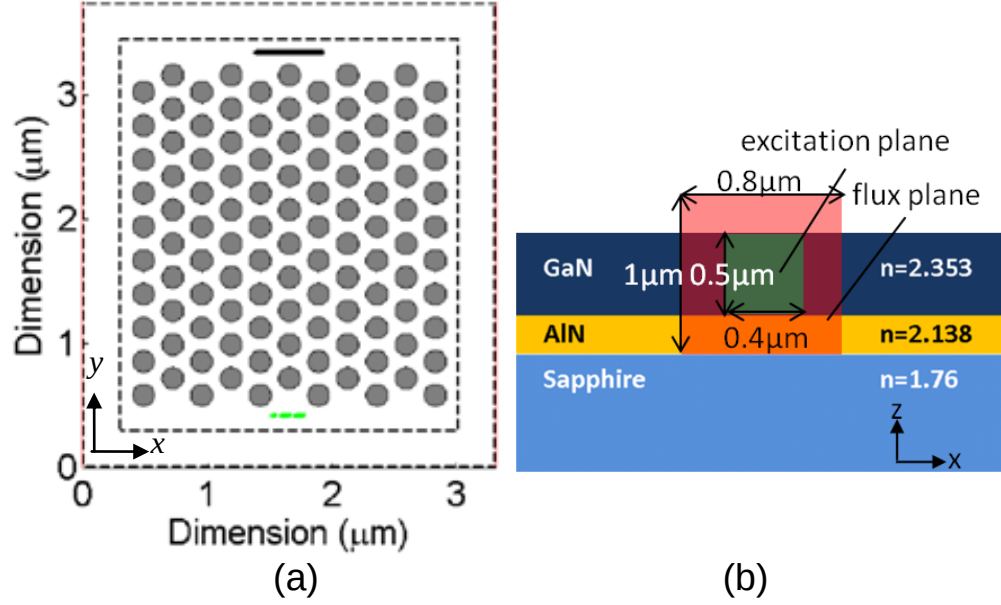


Figure 7.5: (a) The top view of the simulated PhC with $a=271\text{nm}$ and $r/a=0.32$. (b) The orientation of the excitation and the flux plane.

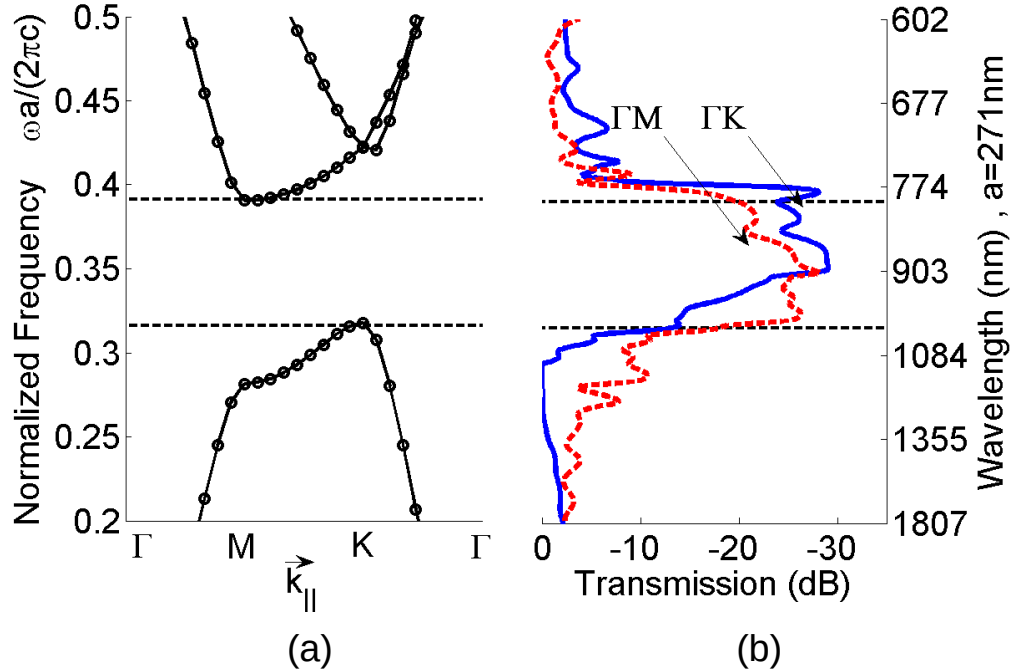


Figure 7.6: (a) The band diagram calculated by MPB of the 2D approximation of the actual PhC structure as in Figure 7.2(b). (b) The transmission for the uniform PhC simulated by 3D FDTD. The parameters of the PhC are $a=271\text{nm}$ and $r/a=0.32$.

Figure 7.6 shows the results of the FDTD simulation plotted together with the 2D band diagram calculations same as the one shown in Figure 7.2(c). We see a strong correlation between the bandgap of the MPB band diagram found using the effective index approximation and the bandgap found by the 3D FDTD transmission simulations.

In this simulation the depth of the holes was assumed to be infinite however in practice the holes will be finite. To determine the effect of the depth of the holes we have repeated the simulation for different hole depths, first when the holes span GaN and AlN layers and when the holes only span the GaN layer. The results are shown in Figure 7.7 together with the infinite hole depth case. When the holes are etched only in GaN we can barely observe a band gap since there is a non-negligible portion of the propagating mode in the AlN layer. When the holes go through AlN layer as well, the band gap can be clearly seen and is close to the ideal case when holes are infinitely deep. In fabrication of PhC etching deep (high aspect ratio) holes is a challenge. This simulations shows that etching through GaN and AlN layers is sufficient to imitate the ideal case, infinite deep holes.

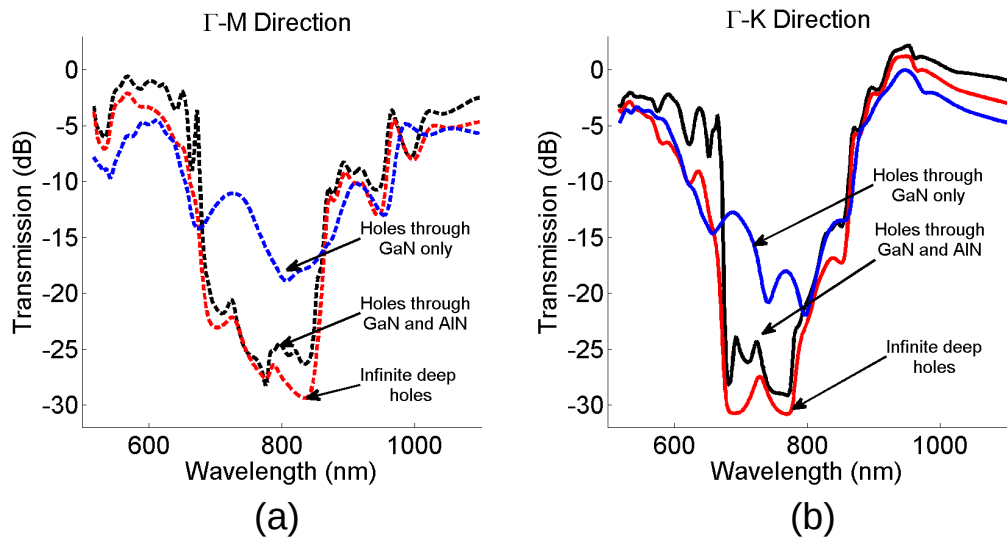


Figure 7.7: (a) The transmission through a triangular lattice PhC with $a=271\text{nm}$ and $r/a=0.32$, in Γ -M orientation for three different hole depths. (b) Transmission of the same structure in Γ -K direction.

7.1.4.2 W1 Defect Waveguide

We have seen the quasi band-gap of the structure in Section 7.1.4.1. We are now going to investigate W1 defect PhC structure on the same layer structure using 3D FDTD method. W1 defect waveguides are especially used for slow light applications as the dispersion properties of the waveguide can easily be changed by modifying the radii of the holes or shifting them. The two guiding mechanism in PhC waveguides are known as index guiding and bandgap guiding. Index guiding occurs for wavelengths which are much larger than then the period of the holes in which case the PhC is virtually a uniform dielectric medium having a refractive index average of the material and the holes. Therefore the region where there are no holes has a higher average refractive index compared to the surrounding PhC media and hence the light is guided. In the bandgap guiding case the light is not allowed to propagate through the PhC regions due to the bandgap and forced to stay in the region where the holes are removed.

Figure 7.8 shows the geometry of the simulated W1 defect PhC structure. Here first row of holes around the cavity have a radius of r_1 , and the second row of holes have a radius of r_2 . The lattice constant is 271nm and the holes in the background are chosen to be $0.32a$. We have placed the excitation at the beginning of the waveguide and flux planes through the waveguide separated by a lattice constant. The excitation and the flux planes have a cross sectional dimensions of 271nm x 500nm in xz-plane.

We have plotted the simulated flux across the waveguide at a wavelength of 799.3nm as shown in Figure 7.9 for different parameters for r_0 , r_1 and r_2 . The blue curves are for $[r_0, r_1, r_2] = [0.32a, 0.34a, 0.32a]$ and the red curve is for $[r_0, r_1, r_2] = [0.32a, 0.34a, 0.38a]$. The blue curve which is steeper is when the holes are in the GaN and the AlN layers only whereas the other two curves are for infinitely deep holes.

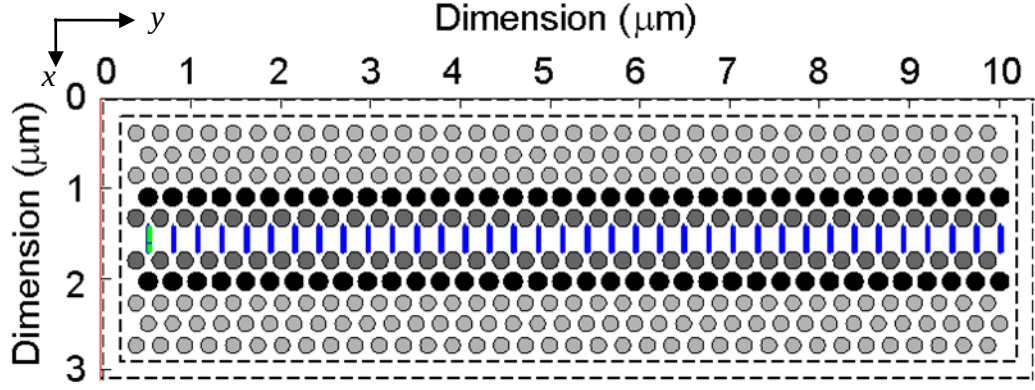


Figure 7.8: Geometry of W1 defect waveguide. The first row of holes surrounding the defect has a radius of r_1 and the second row has a radius of r_2 . The excitation is placed at the beginning of the waveguide and flux planes are put through the defect separated by a lattice constant.

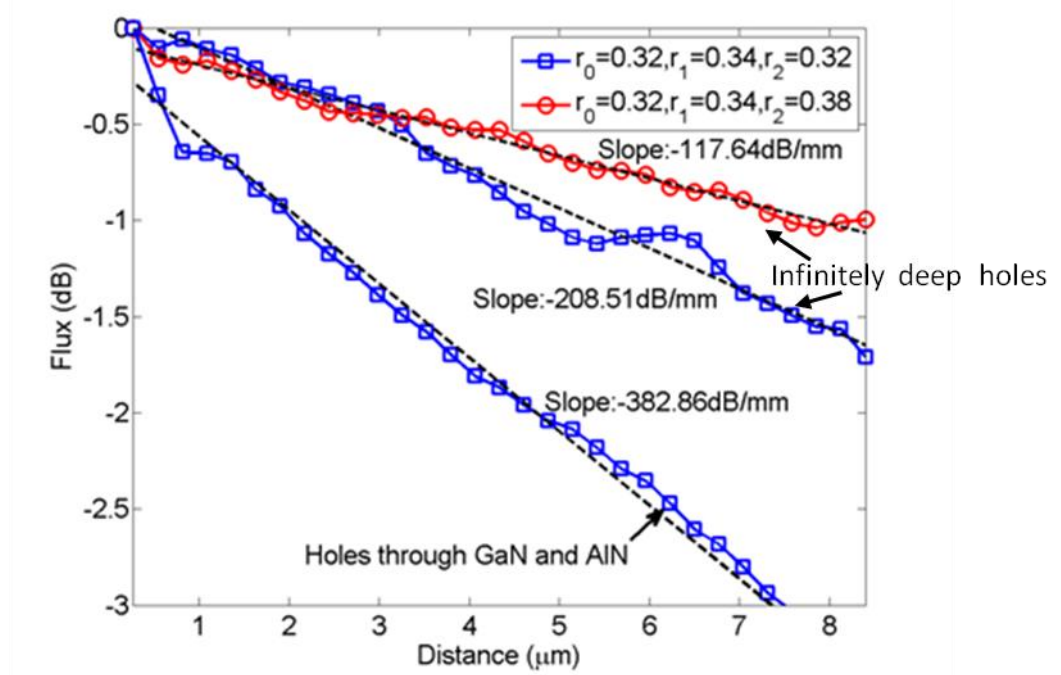


Figure 7.9: The fluxes at 799.3nm along a W1 defect PhC for PhCs with different parameters, circles: $[r_0, r_1, r_2] = [0.32a, 0.34a, 0.38a]$ and square: $[r_0, r_1, r_2] = [0.32a, 0.34a, 0.32a]$. The losses computed by linear fitting are 0.12dB/ μm and 0.2 dB/ μm respectively for infinite depth of holes. Reducing the hole depth to AlN-sapphire interface increases the loss per μm by a factor of two.

The values for loss are high due to the above light line operation. However in a device which is 5 μm in length, the losses will be around 1.5-2dB and these could be tolerable. It may also be possible to optimize the layer structure to minimize these losses.

7.1.4.3 Photonic Crystal Coupler

The band diagram of the 2D effective index approximation of the PhC coupler of Section 7.1.3.3 has given an initial idea for the parameters of the 3D PhC coupler design. Here we are going to analyze PhC coupler structures with different hole radius parameters and then cascade these two to reach the final design.

Figure 7.10 shows the geometry and simulation results of a PhC coupler with parameters $[r_0, r_1, r_2] = [0.32a, 0.34a, 0.32a]$. The simulation domain is about $4.5 \times 2 \times 5.8 \mu\text{m}^3$ including the 200nm thick perfectly matched layers. The excitation is placed at the input of the device as shown with the green dashed line. It is a Gaussian pulse centered at 804nm wavelength and the spectral FWHM width is around 200nm. Flux planes are placed along both defect waveguides to observe the coupling. The measured fluxes are normalized to the value of the first flux plane in the bar port.

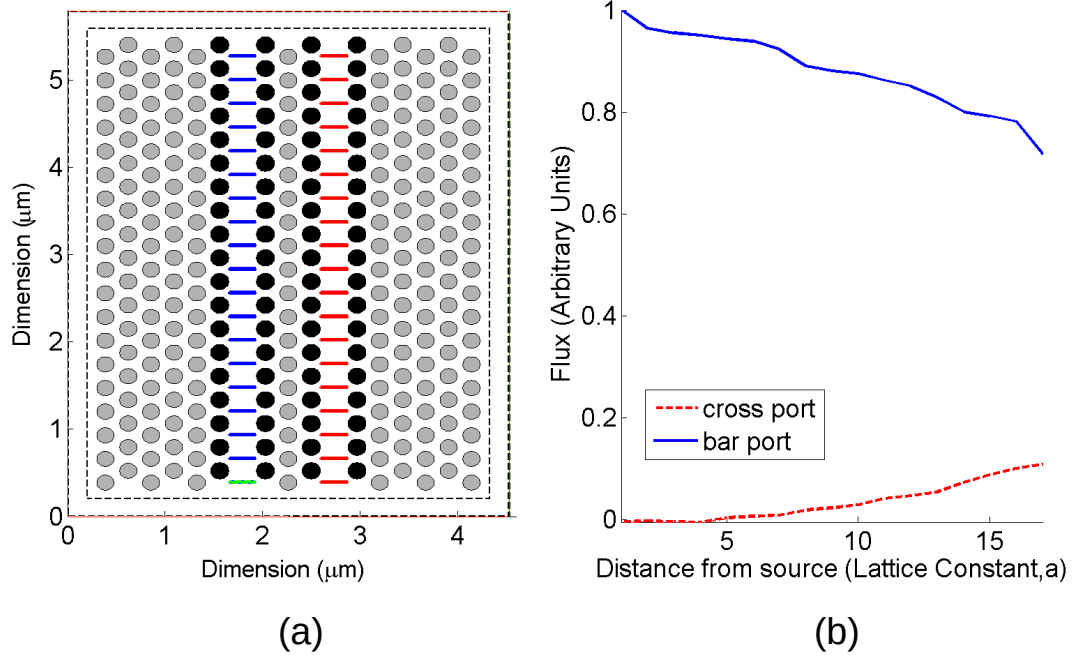


Figure 7.10: (a) The top view of the simulated 3D structure where holes sizes are: $r_0=0.32a$, $r_1=0.34a$, $r_2=0.32a$ and they are infinitely deep. (b) The flux at 798.4nm along the waveguides, solid showing the bar port (where the excitation is) and dashed is the cross port. The coupling is very small therefore the coupling length is large.

The last flux planes on both ports at the end of the device are discarded due to the undesired effects as a result of truncation of the PhC waveguides. Here we observe a small amount of coupling such that the flux at the bar port decreases and the flux at the cross port increases gradually towards the output of the device. Also the black and circled curve shows the summation of the normalized fluxes in both ports.

Figure 7.11 shows the case when the parameter r_2 is increased from 0.32 to 0.38. In this case we see a clear reduction in the coupling length.

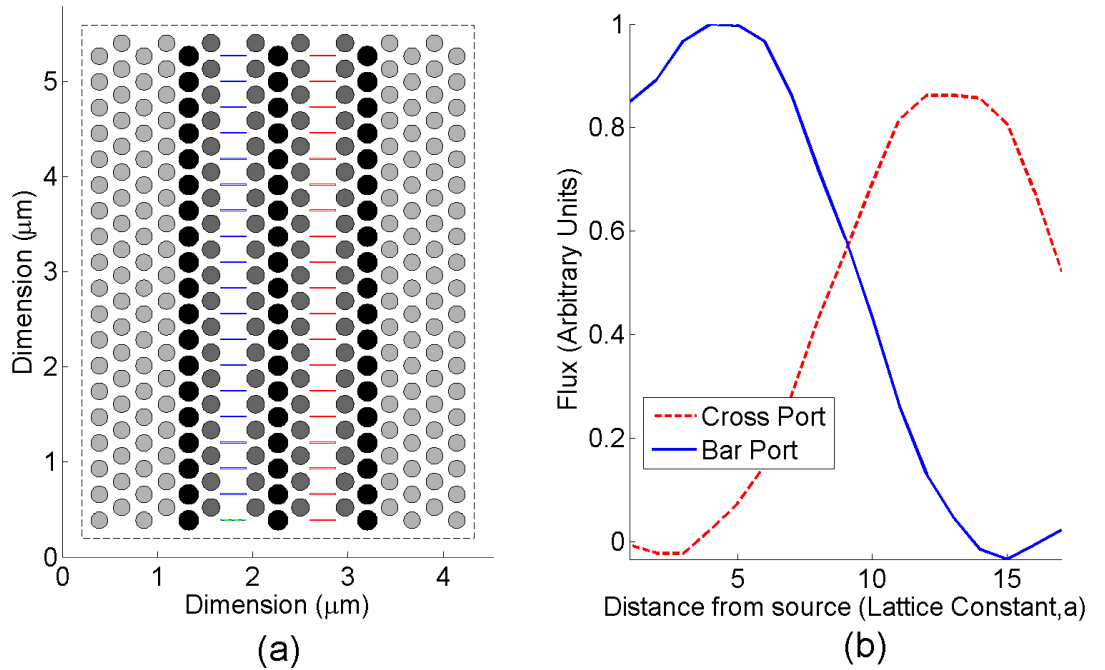


Figure 7.11: (a) The top view of the simulated PhC coupler with parameters $a=271\text{nm}$ and $r_0=0.32a$, $r_1=0.34a$, $r_2=0.38a$. The holes are infinitely deep. (b) The fluxes measured by flux planes placed along both cross and bar port of the device and the total flux at 798.4nm wavelength.

At about 9 lattice constants away from the source we observe that the power at both ports are equal. The flux in the bar makes a peak at around 2 lattice constants from the source. This is attributed to the Fabry-Perot effect occurring due to the termination of the PhC device at the input and the output.

When we cascade the two PhC structures described above, the first having $r_2 = 0.32$ and the second having $r_2 = 0.38$ we have the structure shown in Figure 7.12.

Here the input and output sections are 5 lattice constant long and have smaller r_2 and the middle 8 lattice constant long section has larger r_2 . The input output sections slow down the coupling to provide a more smooth transition to input output ridge waveguides. The input and output ridge waveguides are not included in our simulations.

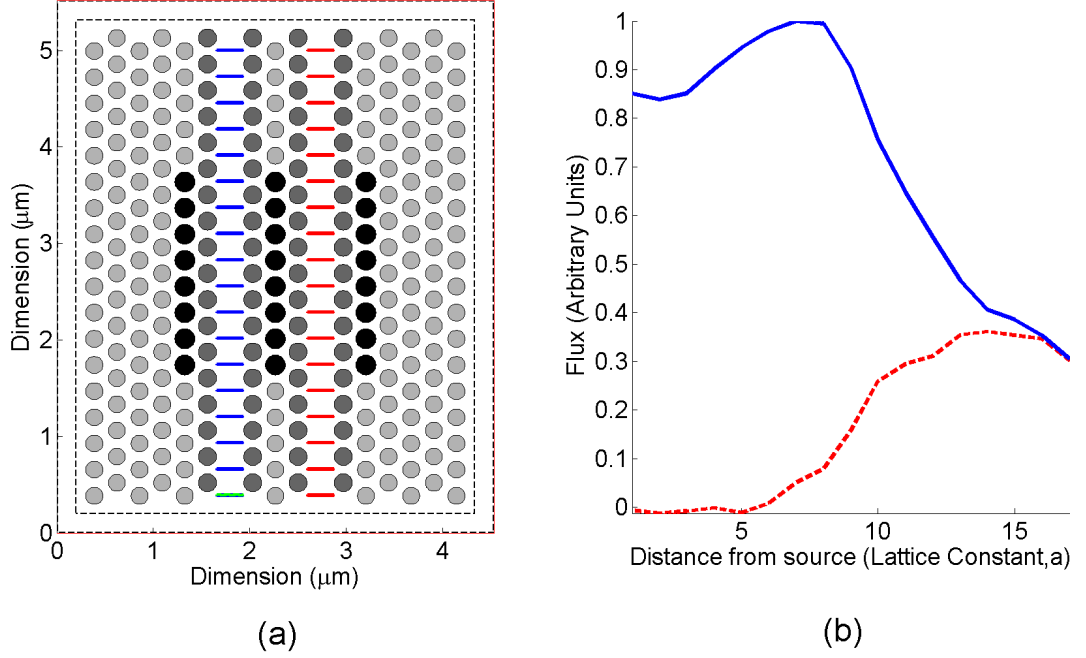


Figure 7.12: (a) The top-view of the simulated PhC coupler structure. (b) The flux along the bar and cross ports at 798.4nm wavelength.

Looking at Figure 7.12(b) the light is split equally across the two output ports at the output of the device. The device is sensitive to wavelength and to illustrate this we have plotted the fluxes in both arms and the total flux versus wavelength at the output of the device. Here the bar and cross port fluxes are normalized to the total flux at each wavelength. The vertical dashed line shows the operating wavelength, 798.4nm where the fluxes of the bar and cross ports are equal.

The fluxes at the cross port at wavelengths 797.5nm, 798.4nm and 799.3nm are 37%, 49.7% and 67.3% respectively. Since the device is so sensitive to wavelength we need to be able to tune to the wavelength of operation to compensate for the

mismatch between the model and the fabricated device. In the next section we will discuss the tuning of the device using the electro-optic effect of GaN.

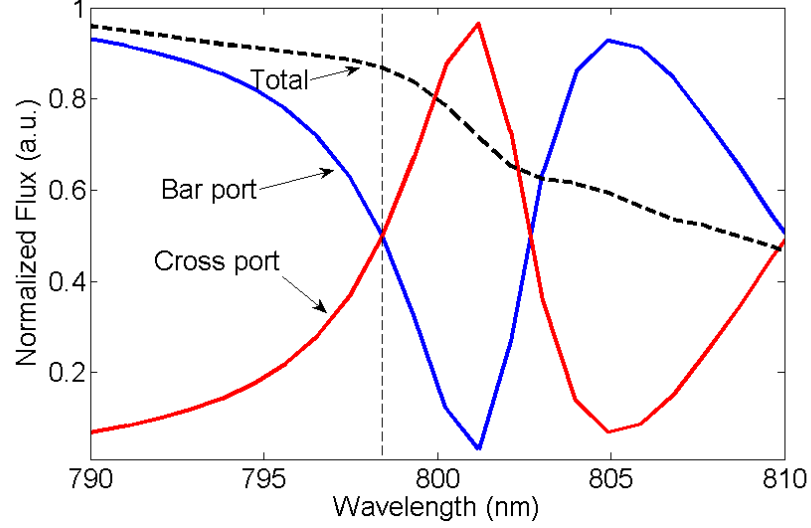


Figure 7.13: The normalized fluxes in the bar and cross ports versus the wavelength.

7.1.4.4 Tuning of Photonic Crystal Coupler

Here we investigate the effect of 0.1% change of the refractive index of the GaN layer on the 3dB coupling length. We have used 2.3535 as the refractive index of GaN. Possible methods for tuning are thermo-optic or electro-optic. The thermo-optic effect is much slower compared to electro-optic effect. For the thermo-optic effect a 10 °C change in the temperature is going to induce a refractive index change in the GaN layer by 0.027% according to the data given in [198]. That means, for 0.1% change the temperature needs to be changed by around 40 °C which is achievable by metal resistive heaters. On the other hand for electro-optic tuning an electric field has to be applied across the GaN material. Doped AlN can serve as a bottom electrode and for the upper electrode a transparent layer of Indium Tin Oxide [199], [200] can be used. The general formula for the Pockels effect is

$$n'_x = n'_y = n' = n_0 - \frac{n_0^3 r_{31} V}{2d} \quad (7.2)$$

where n_0 is the unperturbed refractive index, r_{31} is the electro-optic coefficient, V is the applied potential difference and d is the distance between the electrodes. The electro-optic coefficient for GaN at 635nm is $r_{31} = 0.57 \pm 0.11 \text{ pm/V}$ [201]. We will use these parameters since there is no report for the optical parameters of GaN in the wavelength regime we are working at. We will use the upper bound for $r_{31}=0.68 \text{ pm/V}$ to reach the following equation for the amount of index difference with applied field.

$$\Delta n = 4.4125 \times 10^{-02} \frac{V}{d} \quad (7.3)$$

In order to obtain an index difference of 0.1% we need to apply around 266V. While the required voltage is high it is achievable. Figure 7.14(a) shows the geometry of the simulated coupler. The length of the strong coupling region is longer than the previous coupler simulations in order to increase the sensitivity of the device. The observation point is marked in the figure. Figure 7.14 (b) shows the fluxes at the cross port of the device at the observation point (normalized to the total flux at that point) for two different cases; where the GaN index is unperturbed and when there is a 0.1% change in the GaN refractive index. At 800nm wavelength the cross port flux is originally 63.5% of the total and with the change of the index it becomes 15.4% of the total power at the observation point. The overall loss at the observation point at 800nm is roughly 1.7dB. The coupler is most sensitive at 799.8nm and at this wavelength it is possible to steer the power at the cross port from 26.7% to 76.8%

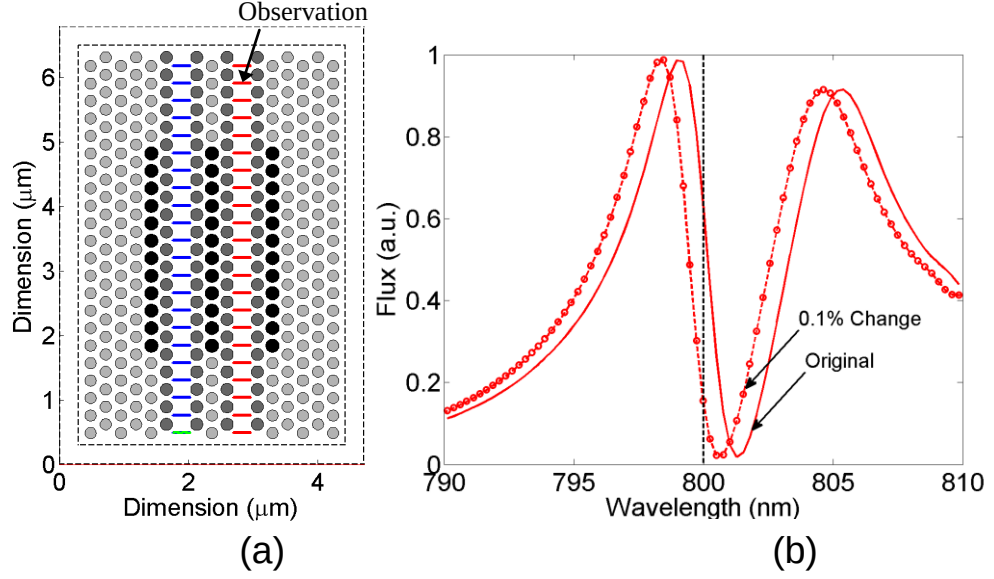


Figure 7.14: (a) The top view of the simulated PhC coupler geometry. (b) The normalized fluxes measured by the flux planes at the observation point versus wavelength.

7.2 Summary and Conclusions

In this chapter we have presented photonic crystal devices designed on GaN. We have designed our devices in an asymmetric layer structure composed of sapphire substrate, a 100nm of AlN buffer layer and GaN. This layer structure provided advantages in fabrication compared to membrane (symmetric) type structures which require complex fabrication techniques and mechanically less stable. This layer structure is considered as asymmetric where it is not possible to split the modes into TE and TM components. Although no real band gap exists there still is a quasi bandgap for the TE mode as we have shown with the simulations. Due to operating above the light line the modes inside the PhC devices can couple to the radiation modes, therefore the losses are high. However since the PhC devices are relatively shorter than their conventional counterparts the total device losses can be still comparable. We have shown 3D FDTD simulation results for W1 defect waveguides and couplers. The

results show that the losses of the W1 defect waveguides are more than 100dB/mm which is similar to what other studies (above the light line operation) show [202]. The designed 50-50% coupler operating around 800nm wavelength is only 5.4 μ m long. This is a great reduction in the length when compared to the GaN directional coupler described in Section 3.4 which had a coupling region length of 400 μ m. We have further tried to utilize the slow light effect of PhCs which increases the light material interaction, in order to tune the splitting ratio. In a $\sim$ 6 μ m long device we have seen that the coupling can be tuned from 63-37% to 15-85% around 800nm wavelength. The device loss is calculated to be around 1.7dB.

In conclusion photonics crystals provide a reduction in size compared to conventional ridge waveguide based directional couplers. The losses of this structure can not be avoided due to the lack of true band gap. The losses are expected to increase when the devices are fabricated due to the fabrication tolerances as discussed. For the devices to be used in integrated quantum photonics applications which require low loss circuits as discussed in Section 1.1.1 the fabrication techniques have to be improved. However due to a wide range of transparency GaN is an attractive material for integrated quantum photonics as well as classical.

Chapter 8

Conclusions

This chapter provides conclusions derived from the studies included in this thesis.

Quantum information is a motivating field of research for the scientist due to the unique advantages it provides in communication, metrology, simulation and computation. The current state of integrated quantum photonics can still be considered as in a “pre-mature” state. To make an analogy with the digital electronics, quantum photonics is at the stage of combining few transistors together to make simple proof of principle computations. However the challenges quantum photonics is facing are much tougher since there are strict requirements on sources, circuits and detectors. Deterministic sources, very low loss circuits and unity efficiency detectors are the main goals to achieve in the field. Additionally integration of these components are required to achieve stability, increase of complexity and reduction of losses arising from coupling photons from the sources to the circuits or from circuits to detectors. Towards this aim it is important to realize all components in a single material system.

GaN is a candidate having a large transparency window covering the visible and near & middle infrared region. We have demonstrated a high visibility (96%) two photon interference measurement on a gallium nitride (GaN) directional coupler [19] and it was the first demonstration of two photon interference in GaN material. Our wavelength regime was infrared due to the readily available single photon sources and detectors however it is possible to operate down in the visible regime where GaN single photon sources [90] and detectors [123] are also demonstrated. Towards reducing the size of the circuits photonic crystal structures are attractive since devices can be made smaller utilizing the band gap and slow light effect of the photonic crystals. We have aimed to design a photonic crystal directional coupler in GaN. Our simulation based study shows that while it provides a reduction in size the high losses might prevent the use of these circuits in integrated quantum photonics applications.

Another important material system for integration is silicon where integrated

quantum circuits [34] and detectors have been demonstrated [46]. Its high $\chi^{(3)}$ non-linearity and refractive index also makes it suitable for photon pair generation through four wave mixing (FWM). Resonant structures such as the one used in this study improves the efficiency of FWM. According to our analysis the ring resonator provides a 91 fold improvement in the pair generation rate coefficient compared to without using a ring resonator. We have also addressed the detrimental effects of free-carriers and demonstrated an increase in the pair generation rate by removing the free-carriers by reverse biasing. This is the first study on the effect of reverse biasing to the performance of the source. Since it is a waveguide based device it can easily be integrated to silicon quantum photonic circuits [34] and detectors [46] however more needs to be done for full integration. Filtering of the pump beam and separating the photon pairs inside the chip is one issue which can be done by additional ring cavities or arrayed waveguide grating (AWG) structures in silicon [203]. Another issue is the requirement of multiple indistinguishable photons. This requires the implementation of multiple sources on the same chip. In order to achieve synchronization among these sources the pumping needs to be pulsed so that photons are generated in certain time bins. Each source has a finite probability of generating photons upon receiving a pulse and when multiple sources are used the probability of the source generating a photon exponentially decreases [126]. Therefore high performance from an individual source is crucial. Another idea for steering the non-deterministic operation of the single photon sources towards more deterministic is multiplexing [204]. In this concept, single photons generated by multiple non-deterministic sources are heralded and by the feedback provided by heralding they are placed into desired time bins by configurable delay circuits.

It is hard to claim which material system is the best for realizing the ultimate

aim, a quantum computer. As we have discussed earlier loss is an issue. Silica-on-silicon platform has the lowest loss among the quantum photonics circuits realized so far but the sizes of the circuits are larger compared to other materials such as silicon or GaN. Additionally photon pair generation in silica waveguides is not practical and single photon detectors are not available. In silicon on the other hand the devices sizes can be reduced however its material loss is higher than silica due to fabrication tolerances. One can think of hybrid integrated structures where different parts are fabricated on different material platforms and then integrated, however in this case there will be losses at the interfaces namely coupling from one material system to the other. At the current state there are still things to do before these grey areas regarding the route to take towards realizing a quantum computer become more clear. Generating and manipulating and detecting quantum states involving more than one photon all inside a chip is a closer goal and silicon is a promising platform for this.

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