Dynamic Pricing and Inventory Control for Perishable Products under Uncertain and Time Dependent Demand

by

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This is to certify that I have examined this copy of a master's thesis by

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Abstract

In this thesis, we consider inventory management and pricing of perishable products like sheet metals, dairy products, vegetables, pharmaceuticals, fruits, bloods and other related products considering a time-dependent demand function. The main difference that separates perishable products from durable ones is that perishable products have a decreasing demand as they age which leads to being completely obsolete after a certain amount of time. Therefore, both the amount and age of the products are the crucial variables for inventory and pricing decisions which make this decision equations harder to solve.

The inventory and pricing decisions problem is a very interesting topic when we look back in the literature; however, these decisions were generally handled separately, and studies that coordinate inventory and pricing decisions were lacking. Most of these studies focused on single product systems with a first-in-first-out structure that assumes the oldest product will be sold to the first customer. Additionally, demand is time independent and not affected by age. However, in reality, customer choices are dependent on the age of the product at hand, because it is rational to say that customers will choose fresher products that lead to a higher demand for new orders, this demand will direct customer interest to fresher products, and older ones will become obsolete. According to this system, first-in-first-out logic may not work, because when a customer has two options to choose between fresher and older products, they will always prefer fresher ones. In this study, we aim to fill gaps in literature

by considering the coordinated inventory and pricing decisions in systems where the demand is a function depending on the age of the products at hand.

We tried to find an effective way to manage inventory and pricing decisions, which also leads to decreased food waste. This proposed system is important for companies because perishability, unless managed correctly, decreases profit. By considering these facts, we build a model that reflects these real life problems for perishable products and try to find the optimal solution for such systems. When finding the optimal solution is not possible, we build heuristics and find approximate solutions. All in all, we focus on the inventory and pricing decisions by working on a continuously reviewed inventory system with a single type of product and a random lifetime decaying continuously.

List of Figures

List of Tables

Table of Contents

CHAPTER 1

Introduction

Perishable products like sheet metals, dairy products, vegetables, pharmaceuticals, fruits, bloods have been a very interesting topic since the first research in the 1950's. Today the retail market consists of $3 trillion in its entirety annually and almost one third of this market is perishable. The main problem with perishable products is profitability because of its unstable structure. As understood from their name, these products deteriorate over time, which affects demand continuously, and when we consider lifetime, they will all become obsolete after a certain amount of time. Perishable products should be effectively managed because each month billions of dollars' worth of products become obsolete due to their expired life as mentioned in Minner and Transchel [64].

Another perspective is from business owners in perishable products related sectors. Sustainability is the main issue in business and it ultimately depends on profitability. Additionally, a company may have some ups and downs on balance sheet year by year depending on the growth and investment strategy. In 21st century, profitability has more complex structure than 20th century, but when we consider main components, adding pricing effects and competitive advantage of product diversity of other suppliers, demand structure of customers is less predictable, and more uncertainty leads to less profitability.

Aging will also decrease the demand rate due to the customer preferences for freshness. Another dimension of the problem is the amount of product, which has been ordered and kept. All in all, it is a multidimensional and difficult area to work because of the diversity, complexity and low predictability of problems on perishable products.

There has been a broad history of research since 1950's but not many of the sources include coordinated inventory and pricing decisions. In our study, we focused on deciding on the ordering time and order amount for constant price model. And then ordering time, amount and pricing decisions for variable price model of perishable products that have uncertain demand and uncertain shelf time. We observed the demand function as descending by increasing time and price, and divide time into small pieces in which demand occurs or does not occur in this time unit. This demand structure requires a continuous review model, so we should continuously check inventory amount and decide on pricing or ordering decisions depending on current situation. We benefit from dynamic programming for the framework and try to find an optimal solution for the best replenishment time, best order amount, and best prices that are feasible for the problem. We conduct analytical analysis and develop heuristics to come up with a simpler solution for companies that face more complex problems.

In the literature, research were conducted in several areas that consider pricing and inventory control on deterministic and stochastic demand for perishable and non-perishable products. Benkherouf [6], Mishra and Singh [45], Broekmeulen and Donselaar [11] analyze the perishable products'

inventory decision process for a time-dependent demand, however they do not consider pricing decisions. Transchel and Minner [44] considered inventory and pricing decision for non-perishable products with deterministic demand. There are also several papers such as Rajan and Steinberg [57], Abad [1, 2, 3, 4], Burwell et al. [12], Wee [66], Mukhopadhyay et al. [47], Sana [61] and P-S You [55], which investigate coordinated pricing and inventory decisions for perishable products with deterministic demand.

In the history of coordinated pricing and inventory decision research, there have been studies that focus on pricing and inventory decisions separately for random demand and uncertain shelf life. Differing from our study, these papers mostly consider perishable products as usable after a new order arrival with indefinite shelf life. There is also a study from Minner and Transchel [64] for perishable products under consideration of service levels that satisfies required constraints, and the researchers decide on the order quantity, but again pricing decisions are not considered in this study. Nahmias [51] review literature of perishable products with exponential decay and fixed shelf life, additionally there are surveys on deteriorating items.

When we consider literature of inventory management that includes dynamic pricing where a certain amount of lifetime is assigned to perishable products to be sold, and with no replenishment opportunities, as we can see examples in airline or hotel management industries, Gallego and van Ryzin [26], Zhao and Zheng [70], Monahan et al. [46], Gayon et al. [27] are the studies on this topic and Bitran and Caldentey [10] have a broad review on dynamic pricing

literature. Details of these studies show that there is no time dependency on demand and no replenishment decision is considered.

There are many research with time dependent demand for perishable products. Chen and Sapra [15] research integrated the pricing and inventory decision problem for perishable products which have two-period lifetime with periodic review. Chen et al. [14] focus on integrated pricing and inventory decision problem for perishable products with stochastic demand and lead time. They investigate structural properties of the optimal solution. Chulung and Peng [16] have a discrete time dynamic programming model for perishable products with two-period lifetime, and observe the structural properties of the optimal solution. They also develop three heuristics to decide inventory allocation and prices of the products that extend past the two-period lifetime, which is an extension to the optimality model.

Different than the previous researches, in this study we consider perishable products for integrated inventory and pricing decisions with uncertain shelf life and a stochastic demand which is time and price dependent. We have a continuous review system and decide on optimal price momentarily, ordering time and quantity of inventory to order. We use dynamic programming to decide on these decision variables, and develop analytical approaches for the optimal solution. We do not restrict ourselves to a limited lifetime model, instead we use last-in-first-out system. Because of the structure of the perishable products, and this system requires old products to be deteriorated immediately after the new order arrives. We also prove some structural properties of our model in certain conditions, and observe changes on decision

variables with different parameters. We create four heuristics, in which the first and second one are specific applications of the third and fourth.

In the next chapter we review the literature in detail, and in chapter 3 we analyze our models and provide analytical results, in chapter 4 we inspect heuristics that we create and optimal results that we obtain depending on these models. In chapter 5, we present the numerical results and comparisons of them. And lastly, we explain the conclusion of our study.

CHAPTER 2

Literature Review

There has been different studies about perishable products since 50's, and researches evolved by including broader variables to the equation and provided better results in each step. Nahmias [51], Rafaat [56], Goyal and Giri [64], Karaesmen et al. [36] have cited detailed literature reviews.

Van Zyl [65], search the optimal inventory strategy for a perishable product that has two period life-time with a periodic review system. These researches are extended later for products with longer life-time, however, because it is hard to find optimal solutions, heuristics are established.

For the inventory system with periodic review Nahmias [51] and Fries [24] observe optimal order quantities of fixed shelf life products. Depending on the lifetime of the product, too many decision variables are demanded, so it is impossible to obtain a solution in a feasible time when the product has a longer lifetime. Therefore, in the later researches heuristics are developed.

For shrinking the number of decision variables of the inventory system, Nahmias [50] group these variables to form the equation solvable. Cohen [18] and Chazan and Gall [13] consider when and how much to order by using Markov processes for the m-period life-time products. However, for all these systems FIFO system was assumed to be valid, customer perspective was ignored, and inventory strategies like (s,S) were applied. For the inventory

systems like (s,S), inventory amount is checked and if the total inventory amount is less than s, the amount that completes inventory amount to S is ordered, but the idea that order time and amount should depend on inventory amount and depreciation, is ignored.

Chiu [17], instead of finding optimal solution, investigates effective heuristics to observe how frequent the inventory should be reviewed and optimal order amount. Ferguson and Ketzenberg [23] analyze the value of sharing maturity information of the products between a supplier and retailer.

Broekmeulen and van Donselaar [11], improve inventory strategy with (R, s, nQ) system which indicates to order nQ amount of product if inventory on hand is less than s in each R duration, and they suggest an inventory management algorithm that considers amount and age of the inventory. In the studies above, lead time is assumed to be zero.

Williams and Patuwo [68], obtain optimal order quantities for a two-period life-time product by having a positive lead time. Kapalka et al. [35] by also taking lead time into account, try to find the optimal values of (s, S) inventory strategy in a multi-period model. Minner and Transchel [44], offer strategies different than (s,S) and compare these strategies with (s, S) strategy by using a dynamic ordering strategy. In these studies, first-in-first-out system which may not be valid for perishable products is used.

For the inventory systems with continuous review, Weiss [67] shows that when inventory level is zero ordering S amount of product is the optimal strategy

assuming that fixed life products without lead time and demand with Poisson distribution.

Kalpakam and Sapna [34] and Liu and Yang [42] search optimal ordering time and amount in a continuously reviewed system by assuming a lead time with exponential distribution and using Markov renewal processes. Analyzing studies with constant lead time, different than zero is harder. Schmidt and Nahmias [62] and Perry and Posner [54], work on (S-1, S) strategy for fixed amount of order without payment. Finding optimal strategy for continuously reviewed system with lead time different than zero is really complex and hard, therefore (s, S) systems and different heuristics are developed for the solution. Nahmias and Wang [51] develop heuristics for a system with poisson demand process by taking fixed ordering price into account. Chiu [17] and Ravichandran [59], similar to Wang [51], develop different heuristics.

Berk and Gürler [7] analyze (Q, r) inventory strategy and developed heuristics by considering fixed order cost and positive lead time. Schultz [63] studies on perishable products and demonstrated that applying an inventory strategy depending on time instead of inventory amount can decrease costs significantly.

In these models one product is considered and none of them considered costumer preferences, however for perishable products, while customers are looking to the products on shelf, they prefer to buy fresher products instead of older ones. Different strategies are also developed for selling old products such as using discounts to prevent customers buying substitutes which are younger

than products we have. There are also studies that consider effects of substitutes and apply them to the customer demand equations which may lead to have different strategies for substitutable products. McGillivray and Silver [43], under a random demand distribution, observe possibility of finding optimal solution for two different scenarios about customer preferences. They show that there is an optimal solution when all of the customers prefer substitute if the product we have is out of stock, however, if they do not, it is impossible to have an analytical solution. Therefore, there are different heuristics for the second scenario.

Reyes [60] focus on product types by taking substitutes of the product into account. Kök [39] and Kök and Fisher [40] analyze how to manage different types of products using heuristics and optimality techniques under consideration of substitutes.

Because considering substitutes makes the problem harder, instead of using multi-period models, single-period models are used in random inventory models, Pasternack and Drezner [53], Gerchak and Mossman [28], Khouja et al. [38], Bassok et al. [5], Ernst and Kouvellis [19], Rajaram and Tang [58], studies on this area. Kaya [37] works on finding optimal inventory strategy for two newly produced substitutes, and necessary stimulus for them.

There are also studies that consider perishable products with their substitutes in the literature. It is harder to analyze this kind of model because of additional complexity of dependency to both order quantity and product condition. Parlar

[52] work on optimal ordering amount in each period for a two-period lifetime perishable product.

Myers [48] analyze optimal frequency of review and how much product to order in each review for perishable products that are periodically reviewed and that are substitutable, by using heuristics.

Pricing decisions in different areas were considered many times in literature (Gallego and van Ryzin [25, 26], Bitran and Mondschein [8], Feng and Gallego [20, 21], Zhao and Zheng [70], Feng and Xiao [22], Hall et al. [31]). However, these studies mostly focus on single product pricing decision in limited time and inventory, and they assume that order/production related decisions are pre-taken. Therefore, costs that are incurred as a result of this is not considered.

Rajan and Steinberg [57] and Abad [3, 4] are the examples of the studies with mixed order and pricing decisions. Jalan and Chaudhuri [32], Mukhopadhyay et al. [47], Sana [61] and Ghosh et al. [29] analyze inventory and pricing decisions for perishable products with uncertain lifetime. In these models, customer preferences are not considered and they assume a certain demand function and deterioration process and uncertainties on the system are not taken into account.

CHAPTER 3

Technical Background

We handle the problem in two main ways; first one is dynamic programming approach and second one is analytical approach. Both have similar ways to analyze problem, but the main difference is about the structure of the equations, dynamic programming approach focuses on the infinite horizon, considers full scope of the system, however in analytical approach, calculations are made by sampling repeating cycles, and calculating results for that sample with expected values.

3.1 Models

In our models, we analyze a continuous time system by dividing time into small pieces such that there is a certain probability that in that amount of time one demand arrives, which means that it is not possible to have two or more demands in this time unit. We apply a demand rate, which decreases by time and increasing price. This structure is similar to non-homogenous Poisson process and all of its properties are applicable to our models. We consider a continuously reviewed model and decide ordering amount and pricing decisions by checking the system continuously. In these models, we denote t as current time, and it increases by one when each small amount of time passed. Q denotes the amount we order, and q denotes the current inventory amount, c is cost per ordered product, and K is the ordering cost for each replenishment order.

There are some aspects of the problem for the sake of the more feasible structure. When we have a new order we assume that all of the products that we have on hand will be removed from shelves, so we can work on products with same ages which makes problem easier to solve. Unless we use this approach, there will be assumingly k different aged products that will make problem k-dimensional and much harder to solve. This approach is also rational when we take the situation from customer perspective, especially for the perishable products. Customers do not prefer to buy older products unless all new arrivals would have been sold out. However, even if it happens anyway, the demand for the older products will be very low when we consider each and every variable that affects customers' preferences. This is thought as a negligible situation that makes equations more complex and the profit gained from it is too small.

Another decision variable is the price for our problem. As a result of continuous review, we always check and change the price of the products. We denote p(q,t) as the price of the product depending on time and inventory on hand.

Mainly, problem is about deciding when and how much to order perishable products and the prices of these products with a stochastic demand function. Demand function depends on the remaining lifetime of the products, so it is a decreasing function by time. We try to use dynamic programming in infinite horizon to decide values of unknown decision variables. First we keep prices constant then change them as variables depending on time to use different prices for different remaining life times, which will affect demand too. After

that, our main consideration is trying to show results in analytical approach and compare them with the results that we find in dynamic models.

The main equation that we try to maximize is;

Definitions:

age(x): Age of batch depending on the last ordering time

Order(t)= Binary variable indicates occurrence of order at time t

$$\max_{p,Q} \quad \lim_{T\to\infty} \sum_{t=0}^{T} \lambda\Big(p\big(age(t)\big), age(t)\Big) . p\big(age(t)\big). 1_{\{q>0\}} - (A - cQ). 1_{\{Order(t)=1\}}$$

$$age(x) = \{\min(x - y) \mid order\ (y) = 1\ , y \leq x\ \}$$

$$Order(t)= \begin{cases} 1 & \text{if Order,} \\ 0 & \text{if No Order.} \end{cases}$$

3.1.1 Dynamic Programming Model

In our dynamic programming model, we use average cost model which fits into our model considering the structure of the problem. We approach the problem by dividing time slots into little pieces to apply non-homogenous Poisson

process features as we explained before. We also use the discounted dynamic programming approach as an alternative modelling of the problem.

3.1.1.1 Average Cost Dynamic Programming Model

In this model, the notations we use are as follows:

p(q,t): price depends on time and inventory

p: price independent on time

A: Fixed Ordering Cost

c: Unit Cost per Product

Q: Order size

$\lambda(p,t)$: Demand function that depends both on price and time

h(q,t): relative value function for the average cost model

μ: average profit

T: Terminal time limit

We first analyze the model in which the price is kept constant over time such that p(t)=p for all t. Then, we extend our model by allowing prices to change over time.

Because average cost dynamic models is equivalent to a minimum cycle cost problem, the objective is finding stationary policy that minimizes average cycle cost. With the lead of these insights, Bellman's equation is applicable to this infinite horizon problem.

3.1.1.1.1 Average Cost Dynamic Programming Model with Constant Price

We formulate the problem as an infinite horizon average cost dynamic program with constant price. Equations of the problem considering Bellman's equation is below;

$$h(q,t)+\mu = \max\{\text{Continue},\text{Order}\} \quad (3\text{-}1)$$

$$= \max\{f(q,t), f_0\}$$

$$f(q,t) = (\lambda(t).(p+h(q-1,t+1)+(1-\lambda(t)).h(q,t+1) \quad (3\text{-}2)$$

$$f_0 = (-A-c.Q+\lambda(1).(p+h(Q-1,1))+(1-\lambda(1)).h(Q,1)) \quad (3\text{-}3)$$

$$f(0,t) = f_0 \quad (3\text{-}4)$$

$$f(q,T) = f_0 \quad (3\text{-}5)$$

Here, we try to maximize average profit value by deciding "continue" or "order" in each time interval and inventory amount. We try to maximize average profit which will show us profit per time unit for an infinite time period. At any state, if we choose to "continue", with probability λ(t) one unit of demand arrives and we multiply it with selling price p and new relative value h(q-1,t+1) when time passed by 1 and inventory decreased by 1 too. Therefore first part will be λ(t).(p+h(q-1,t+1). Second part consists of not having a demand situation, so the probability will be (1-λ(t)) and we multiply it with new relative value function h(q,t+1) in which inventory amount stays constant and time passes by

1. Because there is no selling, we will get no money. Therefore the total value equation, if we choose to continue, will be $(\lambda(t).(p+h(q-1,t+1)+(1-\lambda(t)).h(q,t+1)$.

For the second part of the equation, we consider "ordering" for that time unit and inventory amount. In that case, we pay a constant amount A for fixed ordering cost, and c for each product we ordered. Then, for these newly ordered products, age of the products is updated to "1" and with probability $(1-\lambda(1))$ no demand arrives and the new relative value function will be $h(Q,1)$ which indicates an inventory amount of Q and time will be reset to 1, beginning of the cycle. With probability $\lambda(1)$, because it is the beginning of cycle, one unit of demand arrives and the new relative value function will be $h(Q-1,1)$, which indicates an inventory amount of Q-1 at age 1. The selling price of one product, p, is also added.

Next, we show some properties related to the optimal solution of the above model:

Proposition 1: Value function h(q,t) will increase when q is increased.

Proof:

We will prove it by using induction. We should consider two scenarios for h(q,t), first one is if our decision is ordering, second one is if not. And because μ values will be same for each of them, we can deduce it from comparison. When we write down h(q,t) and h(q+1,t) in open form we have that;

$h(q,t)+\mu$ $=\max\{\text{Continue},\text{Order}\}$ (3-6)

$=\max\{f_c(q,t), f_0\}$

$f_c\ (q,t)$ $=(\lambda(t).(p+h(q-1,t+1)+(1-\lambda(t)).h(q,t+1)$ (3-7)

f_0 $=(-A-c.Q+\lambda(1).(p+h(Q-1,1))+(1-\lambda(1)).h(Q,1))$ (3-8)

$h(q+1,t)+\mu$ $=\max\{\text{Continue},\text{Order}\}$ (3-9)

$=\max\{f_c\ (q+1,t), f_0\}$

$f_c\ (q+1,t)$ $=(\lambda(t).(p+h(q,t+1))+(1-\lambda(t)).h(q+1,t+1)$ (3-10)

f_0 $=(-A-c.Q+\lambda(1).(p+h(Q-1,1))+(1-\lambda(1)).h(Q,1))$ (3-11)

Scenario 1:

For the first scenario let's assume that optimal decision for h(q,t) is "order". We can say that $h(q,t)= f_0-\mu$. Same manner is valid for h(q+1,t) and its value will be one of the components of maximization function. Therefore the minimum value of h(q+1,t) will be $f_0-\mu$, so, for first scenario we can say that $h(q+1,t)\geq h(q,t)$

$h(q,t) = f_0-\mu$ (3-12)

$h(q+1,t) = \max\{f_c\ (q+1,t), f_0\}-\mu >= f_0-\mu >= h(q,t)$ (3-13)

$h(q+1,t) >= h(q,t)$ (3-14)

Scenario 2:

For the second scenario let's assume that optimal decision for h(q,t) is "continue". Now, we will use proof by induction. First, let's write down the relative value functions as below;

$$h(q,t)+\mu = (\lambda(t).(p+h(q-1,t+1)+(1-\lambda(t)).h(q,t+1) \quad (3\text{-}15)$$

$$h(q+1,t)+\mu = \max\{(\lambda(t).(p+h(q,t+1)+(1-\lambda(t)).h(q+1,t+1), f_0\}$$

$$\geq (\lambda(t).(p+h(q,t+1)+(1-\lambda(t)).h(q+1,t+1) \quad (3\text{-}16)$$

Since there is an expiration date of these products, let's denote it with k, we should order new products at that time. Thus $h(q+1,k)+\mu = h(q,k)+\mu = f_0$ for all q. Then, if we assume that $h(q+1,t+1) \geq h(q,t+1)$ for all q, we have that $h(q+1,t) \geq h(q,t)$ due to the above relations, which completes the proof..

It can also be explained logically. Since q indicates number of inventory on hand, when we have more of them, we have more chance to sell them, thus having more leads to a higher value of relative value function.

Proposition 2: Value of function h(q,t) will decrease when t increases.

Proof:

We will inductively prove this relation. Since there is an expiration date of these products, let's denote it with k, we should order new products at that time. Thus $h(q,k)+\mu=f_0$. Then,

$$h(q,k-1)+\mu = \max\{(\lambda(t).(p+h(q-1,k)+(1-\lambda(t)).h(q,k), f_0\} \geq f_0$$

which leads to $h(q,k) \leq h(q,k-1)$ for all q. Then, we assume that $h(q,t+1) \leq h(q,t)$ for all q and analyze the DP equations below.

$$h(q,t)+\mu = \max\{(\lambda(t).(p+h(q-1,t+1)+(1-\lambda(t)).h(q,t+1), f_0\}$$

$$= \max\{(\lambda(t).[p+h(q-1,t+1) - h(q,t+1)]+h(q,t+1), f_0\} \quad (3\text{-}17)$$

$$h(q,t-1)+\mu = \max\{(\lambda(t-1).(p+h(q-1,t)+(1-\lambda(t-1)).h(q,t), f_0\}$$

$$= \max\{(\lambda(t-1).[p+h(q-1,t) - h(q,t)]+h(q,t), f_0\} \quad (3\text{-}18)$$

We note that $h(q,t+1) \leq h(q,t)$ due to the induction assumption and $\lambda(t) \leq \lambda(t-1)$. In addition, since the value of an additional inventory can at most be p, we can state that $p+h(q-1,t) - h(q,t) \geq 0$. As a results, when we analyze equations 3-12 and 3-13 above, we can conclude that $h(q,t) \leq h(q,t-1)$ due to the above relations, which completes the proof.

Proposition 3: Ordering decision will be based on only a time threshold, independent of the inventory amount at hand.

Proof:

We will inductively prove this proposition.

From propositions 1 and 2, we know that $h(q-1,t) \leq h(q,t)$ and $h(q,t) \geq h(q,t+1)$.

Let k denote the maximum lifetime of the product, so we can conclude that for each t from 1 to k-1; if we order at state (q,t) we should order in every state (i,t) where i≤.q due to proposition 1 and the fact that f_0 is a constant value that is independent of q. However the question is do we have to order at states (i,t) where i>q when we order at states (q,t). For that one we analyze the average cost dynamic program equation.

$$h(q,t)+\mu = \max\{(\lambda(t).(p+h(q-1,t+1)+(1-\lambda(t)).h(q,t+1), f_0\}$$

We know that we should order at the end of the lifetime of the product, thus, $h(q,k)= f_0-\mu$, for all q, meaning that an order of Q should be given at time k, independent of the inventory at hand, q. Then, due to induction, we assume that at some time t, we give an order of Q independent of the value of q, inventory at hand, such that $h(q,t)= f_0-\mu$ for all q. Then, at time t-1, the DP equation will be:

$$h(q,t-1)+\mu = \max(\lambda(t-1).(p+f_0-\mu)+(1-\lambda(t-1)).(f_0-\mu), f_0) \quad (3\text{-}19)$$

$$= \max(f_0-\mu+\lambda(t-1).p, f_0)$$

Therefore, the only relation that we are looking for is $(-\mu+\lambda(t-1).p)$ which does not depend on q, the amount of inventory at hand. Thus, by induction, we can state that, there will be a time threshold t which defines the ordering decision

independent of the amount of inventory at hand. We can define this relation as below:

If $\lambda(t).p > \mu$ Decision is: "No order" ;

If $\lambda(t).p < \mu$ Decision is: "Order"

We can see an example below for ordering decision as a time threshold.

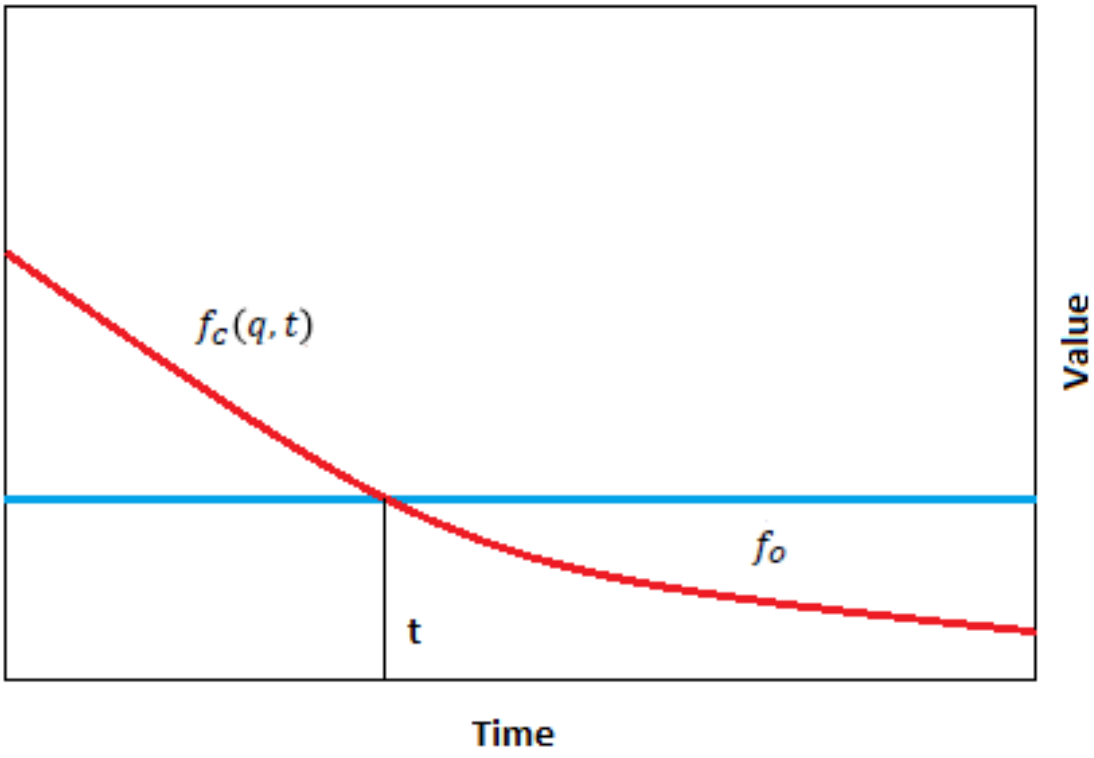

Figure 3.1: Relation between order and no-order values

Table 3.1 Ordering Decisions

	Time						
Inventory	***1***	***2***	***3***	**….**	***167***	***168***	***169***
1	No	No	No	No	No	Yes	Yes
2	No	No	No	No	No	Yes	Yes
3	No	No	No	No	No	Yes	Yes
4	No	No	No	No	No	Yes	Yes

Next, we observe results of relation between variables, and understand how function reacts to changes.

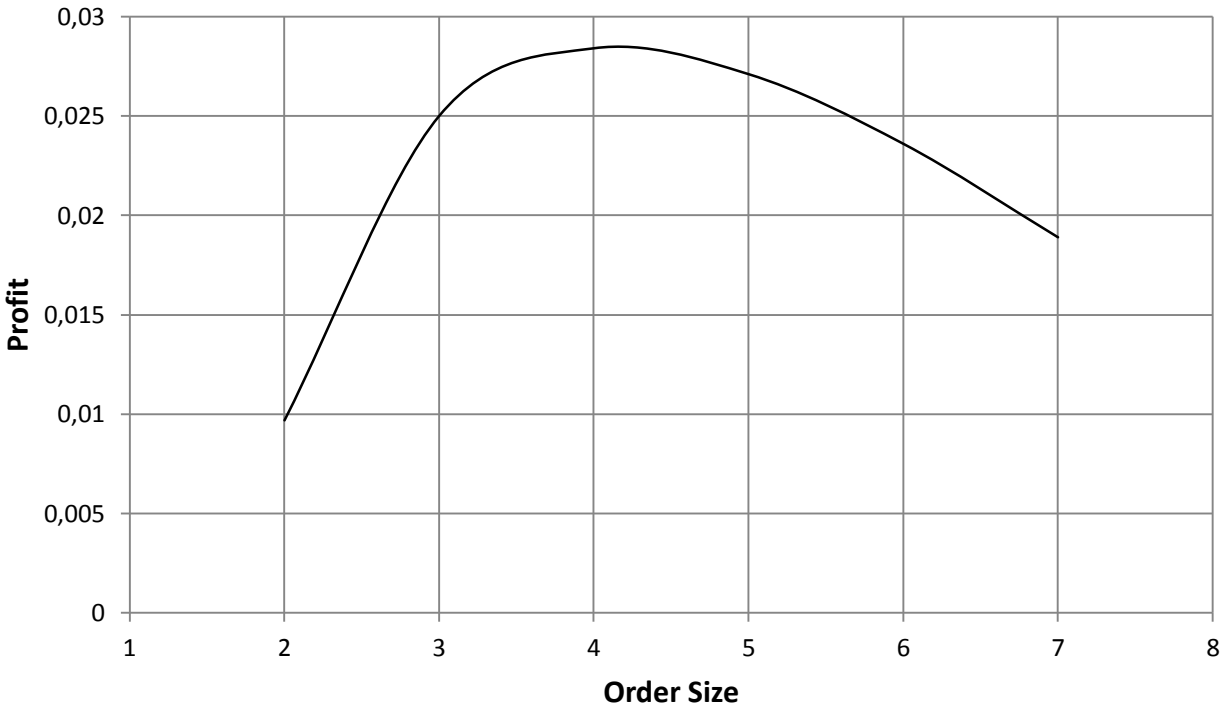

Figure 3.2: Relation between order size and profit

In figure 3.2 we observe how order size change affects the profit function and there is a peak order size.

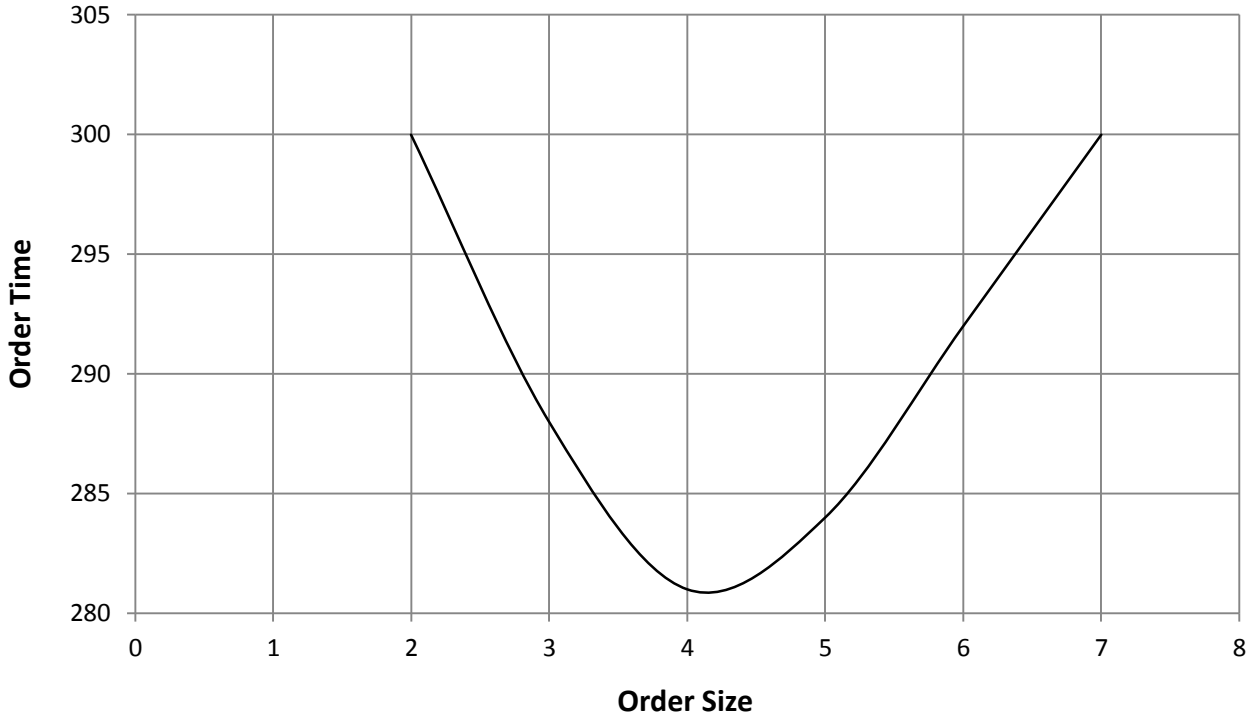

Figure 3.3: Relation between order size and order time

In figure 3.3 we observe that in the optimal inventory size, ordering threshold minimizes.

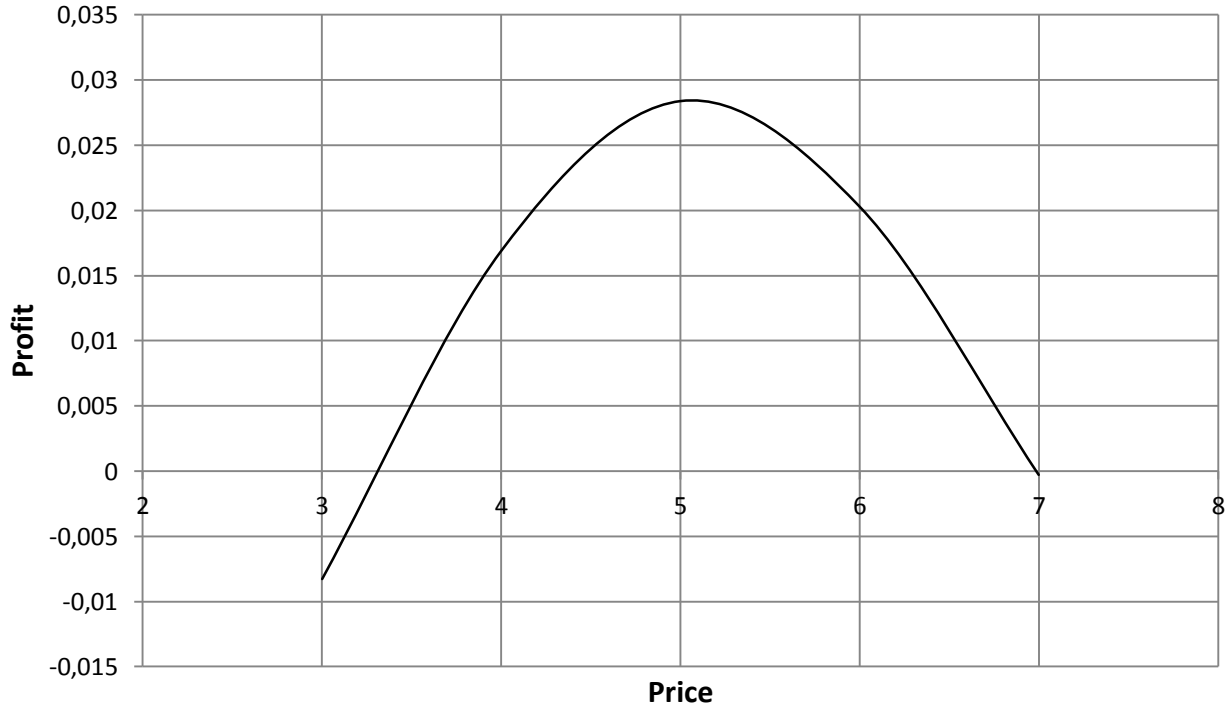

Figure 3.4: Relation between profit and price

In figure 3.4 we observe that when we keep other variables constant, there is a peak price value. When we consider optimal values below zero, price value "3" gives a negative result because it is less than unit cost, and naturally what we get is less than what we gave. For the price value "7" expected demand rate is too low and it makes barely break-even point when we take both order cost and unit cost into account.

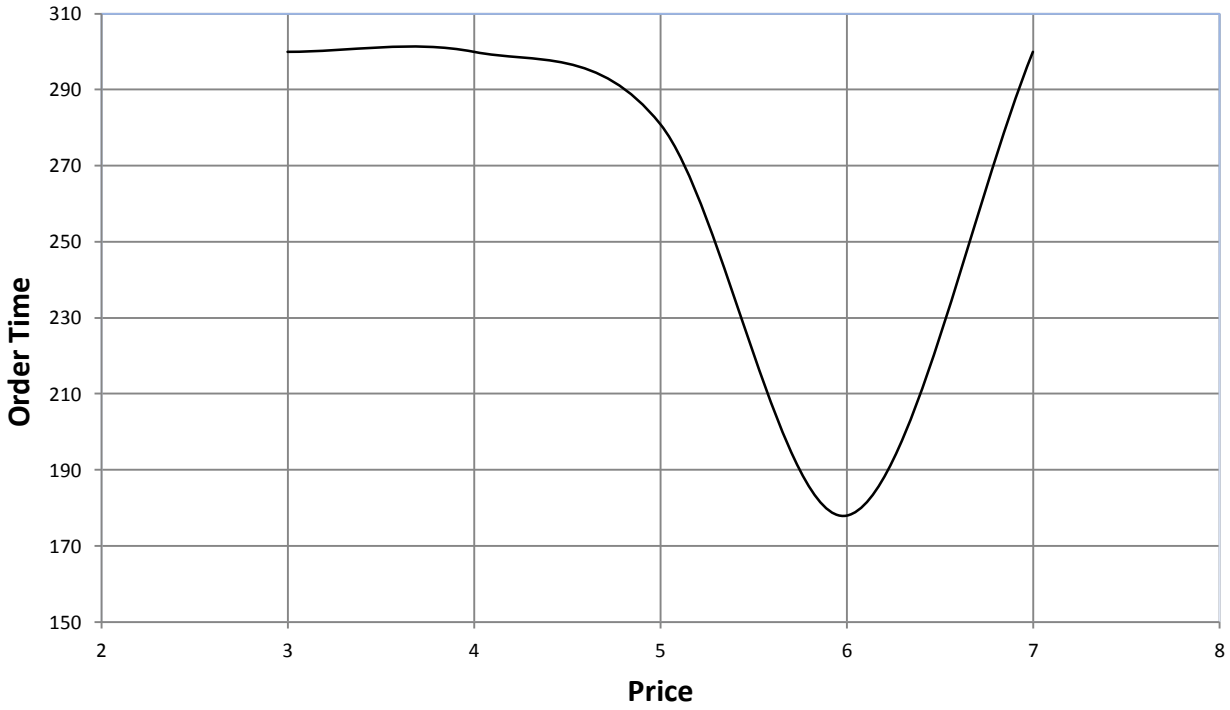

Figure 3.5: Relation between price and order time

Here we observe that there is not an observable pattern for ordering time by price changes, and it is dependent to the structure of the demand function as we display in Figure 3.5

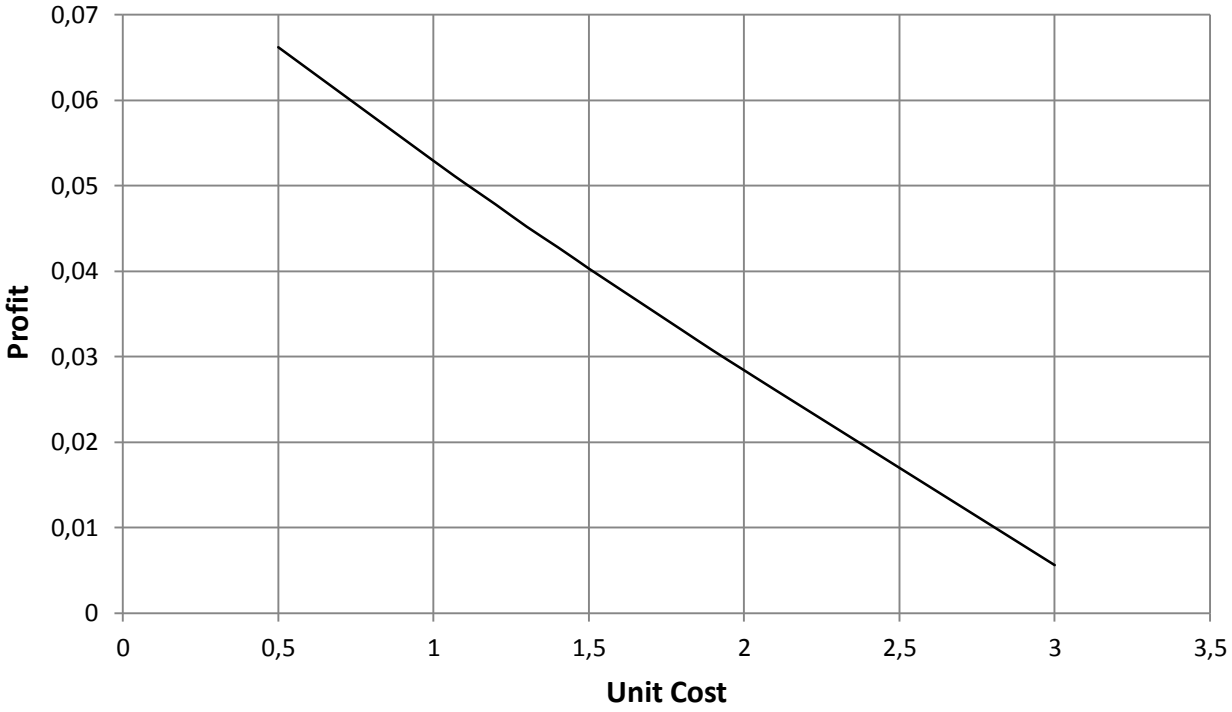

Figure 3.6: Relation between unit cost and profit

Profit value monotonically decreases by unit cost increases as seen in Figure 3.6.

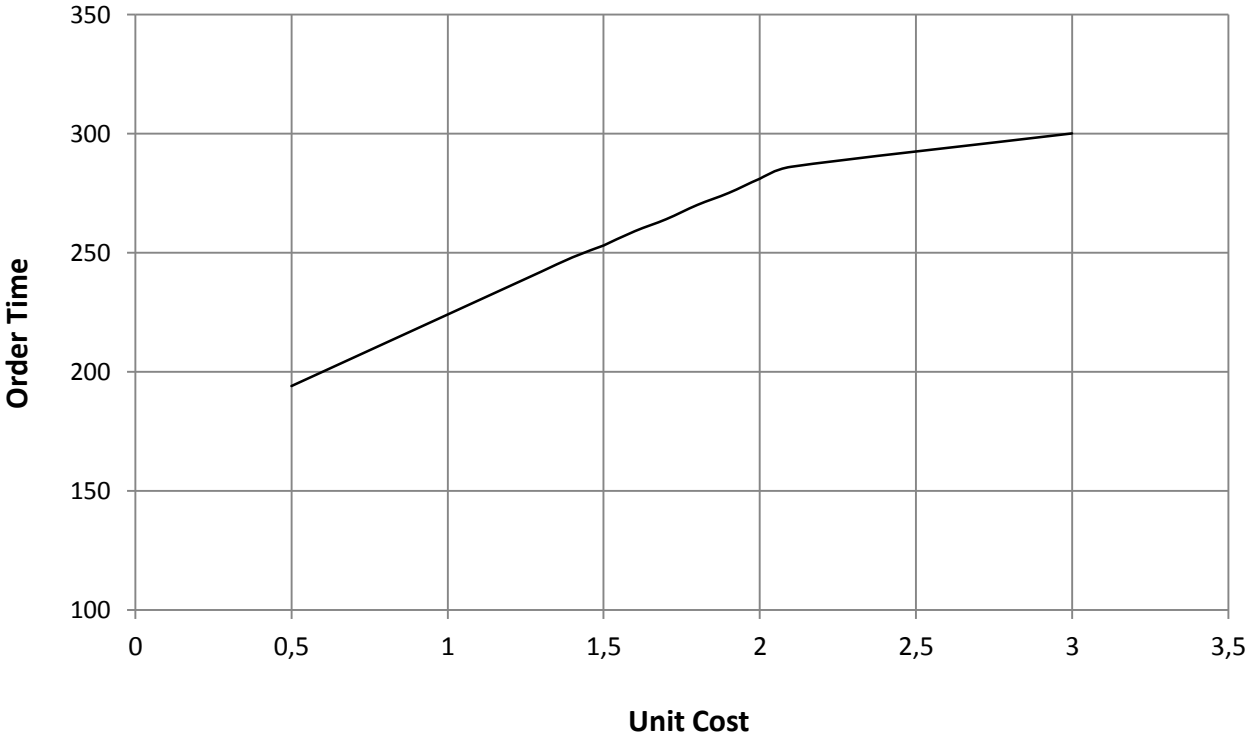

Figure 3.7: Relation between unit cost and order time

Order time value monotonically increases by unit cost as seen in Figure 3.7.

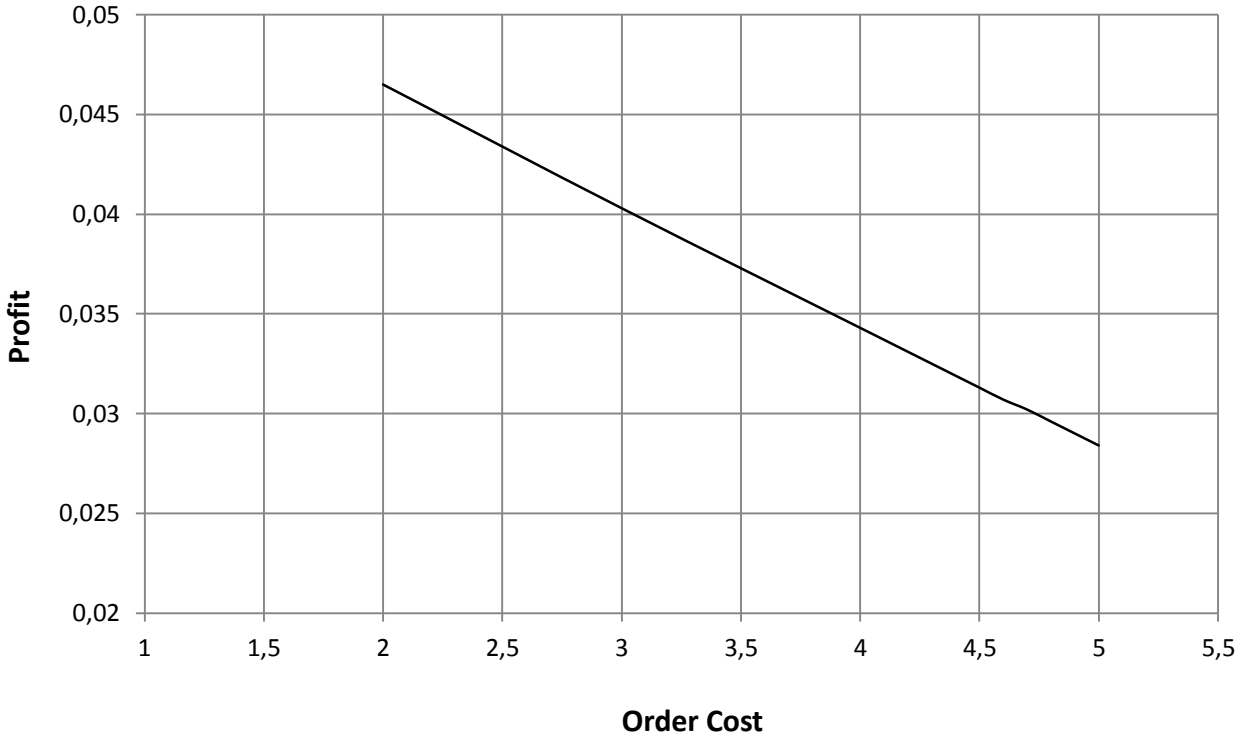

Figure 3.8: Relation between order cost and profit

Profit value monotonically decreases as order cost increases as seen in Figure 3.8.

Figure 3.9: Relation between order cost and profit

Profit value monotonically decreases as order cost increases as seen in Figure 3.9.

3.1.1.1.2 Average Cost Dynamic Programming Model with Variable Price

We formulate the problem similar to the constant price model, however the only difference in here is taking price as a variable, dependent to the inventory amount on hand and time. Equations of the problem considering Bellman's equation is below;

$$h(q,t)+\mu = \max\{\text{Continue},\text{Order}\} \quad (3\text{-}20)$$

$$= \max\{f_c(q,t), f_0\}$$

$$f_c(q,t) = (\lambda(p(q,t),t).(p(q,t)+h(q-1,t+1)+(1-\lambda(p(q,t),t)).h(q,t+1) \quad (3\text{-}21)$$

$$f_0 = (-A-c.Q+\lambda(p(Q,1),1).(p(Q,1)+h(Q-1,1)) +(1-\lambda(p(Q,1),1)).h(Q,1)) \quad (3\text{-}22)$$

Here, we try to maximize average profit value by deciding "continue" or "order" decisions for each time and inventory amount, but now we insert p(q,t) instead of p to all equations that we observe in constant price model.

Next, we show some properties related to the optimal solution of the above model:

Proposition 4: Value function h(q,t) will increase when q is increased.

Proof:

We will prove it by using induction. The most important consideration here is that for each relative function, sale price is changed for the sake of increasing value of the function. We should consider two scenarios for h(q,t), first one is if our decision is ordering, second one is if not. And because μ values will be same for each of them, we can deduce it from comparison. When we write down h(q,t) and h(q+1,t) in open form we have that;

$$h(q,t)+\mu = \max\{\text{Continue},\text{Order}\} \quad (3\text{-}23)$$

$$= \max\{f_c(q,t), f_0\}$$

$$f_c(q,t) = (\lambda(p,t).(p(q,t)+h(q-1,t+1)+(1-\lambda(t)).h(q,t+1) \quad (3\text{-}24)$$

$$f_0 = (-A-c.Q+\lambda(p,1).(p(Q,1)+h(Q-1,1))+(1-\lambda(1)).h(Q,1)) \quad (3\text{-}25)$$

$$h(q+1,t)+\mu = \max\{\text{Continue},\text{Order}\} \quad (3\text{-}26)$$

$$= \max\{f_c(q+1,t), f_0\}$$

$$f_c(q+1,t) = (\lambda(p,t).(p(q,t)+h(q,t+1))+(1-\lambda(p,t)).h(q+1,t+1) \quad (3\text{-}27)$$

$$f_0 = (-A-c.Q+\lambda(p,1).(p(Q,1)+h(Q-1,1))+(1-\lambda(p,1)).h(Q,1)) \quad (3\text{-}28)$$

Scenario 1:

For the first scenario let's assume that optimal decision for h(q,t) is "order". We can say that $h(q,t)= f_0-\mu$. Same manner is valid for h(q+1,t) and its value will be one of the components of maximization function. Therefore the minimum

value of h(q+1,t) will be f_0-μ, so, for first scenario we can say that h(q+1,t)≥h(q,t)

$$h(q,t) = f_0-\mu \quad (3\text{-}29)$$

$$h(q+1,t) = \max\{f_c\ (q+1,t), f_0\}-\mu >= f_0-\mu >= h(q,t) \quad (3\text{-}30)$$

$$h(q+1,t) >= h(q,t) \quad (3\text{-}31)$$

Scenario 2:

For the second scenario let's assume that optimal decision for h(q,t) is "continue". Now, we will use proof by induction. First, let's write down the relative value functions as below;

$$h(q,t)+\mu =(\lambda(p(q,t),t).(p(q,t)+h(q-1,t+1)+(1-\lambda(t)).h(q,t+1) \quad (3\text{-}32)$$

$$h(q+1,t)+\mu=\max\{(\lambda(p(q+1,t),t).(p(q+1,t)+h(q,t+1) +(1-\lambda(p(q+1,t),t)).h(q+1,t+1),f_0\}$$

$$\geq(\lambda(p(q+1,t),t).(p(q+1,t)+h(q,t+1)+(1-\lambda(p(q+1,t),t)).h(q+1,t+1) \quad (3\text{-}33)$$

Since there is an expiration date of these products, let's denote it with k, we should order new products at that time. Thus $h(q+1,k)+\mu = h(q,k)+\mu = f_0$ for all q. Then, if we assume that $h(q+1,t+1) \geq h(q,t+1)$ for all q, we have that $h(q+1,t) \geq h(q,t)$ due to the above relations, which completes the proof.

It can also be explained logically. Since q indicates number of inventory on hand, when we have more of them, we have more chance to sell them, thus having more leads to a higher value of relative value function.

Proposition 5: Value of function h(q,t) will decrease when t increases.

Proof:

We will inductively prove this relation. Since there is an expiration date of these products, let's denote it with k, we should order new products at that time. Thus $h(q,k)+\mu = f_0$. Then,

$$h(q,k-1)+\mu = \max\{(\lambda(p(q,k-1),t).(p(q,k-1)+h(q-1,k)+(1-\lambda(p(q,k-1),t)).h(q,k), f_0\} \geq f_0$$

which leads to $h(q,k) \leq h(q,k-1)$ for all q. Then, we assume that $h(q,t+1) \leq h(q,t)$ for all q and analyze the DP equations below.

$$h(q,t)+\mu = \max\{(\lambda(p(q,t),t).(p(q,t)+h(q-1,t+1)+(1-\lambda(p(q,t),t)).h(q,t+1), f_0\}$$

$$= \max\{(\lambda(p(q,t),t).[p(q,t)+h(q-1,t+1)-h(q,t+1)]+h(q,t+1), f_0\} \quad (3\text{-}34)$$

$$h(q,t-1)+\mu = \max\{(\lambda(p(q,t-1),t-1).(p(q,t-1)+h(q-1,t)+(1-\lambda(p(q,t-1),t-1)).h(q,t), f_0\}$$

$$= \max\{(\lambda(p(q,t-1),t-1).[p(q,t-1)+h(q-1,t) - h(q,t)]+h(q,t),\ f_0\} \qquad (3\text{-}35)$$

We note that $h(q,t+1) \leq h(q,t)$ due to the induction assumption and $\lambda(p(q,t),t) \leq \lambda(p(q,t-1),t-1)$. In addition, since the value of an additional inventory can at most be p, we can state that $p(q,t-1)+h(q-1,t) - h(q,t) \geq 0$. As a results, when we analyze equations 3-12 and 3-13 above, we can conclude that $h(q,t) \leq h(q,t-1)$ due to the above relations, which completes the proof.

Proposition 6: Ordering decision will be based on only a time threshold, independent of the inventory amount at hand.

Proof:

We will inductively prove this proposition.

From propositions 4 and 5, we know that $h(q-1,t) \leq h(q,t)$ and $h(q,t) \geq h(q,t+1)$.

Let k denote the maximum lifetime of the product, so we can conclude that for each t from 1 to k-1; if we order at state (q,t) we should order in every state (i,t) where $i \leq .q$ due to proposition 1 and the fact that f_0 is a constant value that is independent of q. However the question is do we have to order at states (i,t) where i>q when we order at states (q,t). For that one we analyze the average cost dynamic program equation;

$$h(q,t)+\mu = \max\{(\lambda(p(q,t),t).(p(q,t)+h(q-1,t+1)+(1-\lambda(p(q,t),t)).h(q,t+1),\ f_0\}$$

We know that we should order at the end of the lifetime of the product, thus, $h(q,k)= f_0-\mu$, for all q, meaning that an order of Q should be given at time k, independent of the inventory at hand, q. Then, due to induction, we assume

that at some time t, we give an order of Q independent of the value of q, inventory at hand, such that h(q,t)= f_0-μ for all q. Then, at time t-1, the DP equation will be:

$$h(q,t-1)+\mu = \max(\lambda(p(q,t-1),t-1).(p(q,t-1)+f_0-\mu)+(1-\lambda(p(q,t-1,t-1)).(f_0-\mu), f_0) \quad (3\text{-}36)$$

$$= \max(f_0-\mu+\lambda(p(q,t-1),t-1).p(q,t-1), f_0)$$

Therefore, the only relation that we are looking for is (-μ+λ(t-1).p(q,t-1)), and we will show that at time t-1, price will be constant for each i than proof will be similar to proposition 3 in which, (-μ+λ(t-1).p(t-1)) does not depend on q, the amount of inventory at hand. For h(i,t-1), decision equation is (-μ+λ(t-1).p(q,t-1)), so optimal value of p is the same for each other which completes our proof and, by induction, we can state that, there will be a time threshold t which defines the ordering decision independent of the amount of inventory at hand. We can define this relation as below:

If λ(p,t).p(q,t)> μ Decision is: "No order" ;

If λ(p,t).p(q,t)< μ Decision is: "Order"

We can see an example below for ordering decision as a time threshold.

Table 3.2 Pricing Decisions

	Time															
Inventory	***1***	***2***	***3***	**….**	***51***	***52***	***53***	***54***	***55***	***56***	**..**	***165***	***166***	***167***	***168***	**..**
1	6	6	6	6	6	6	6	6	5	5	5	5	5	4	6	6
2	6	6	6	6	6	6	6	5	5	5	5	4	4	4	6	6
3	6	6	6	6	6	5	5	5	5	5	5	4	4	4	6	6
4	6	6	6	6	5	5	5	5	5	5	5	4	4	4	6	6

Next, we observe results of relation between variables, and understand how function reacts to changes. For the profit changes, we observe current profit change with given variables, not optimal one unless otherwise specified.

Figure 3.10: Relation between order size and optimal

In figure 3.10 we observe how order size changes affect the optimal profit function and there is a non-concavity as seen in Figure 3.14.

Figure 3.11: Relation between order size and order time

Figure 3.12: Relation between order size and profit

In figure 3.11 and 3.12 we observe that increase in order size decreases the value of profit and increases order time

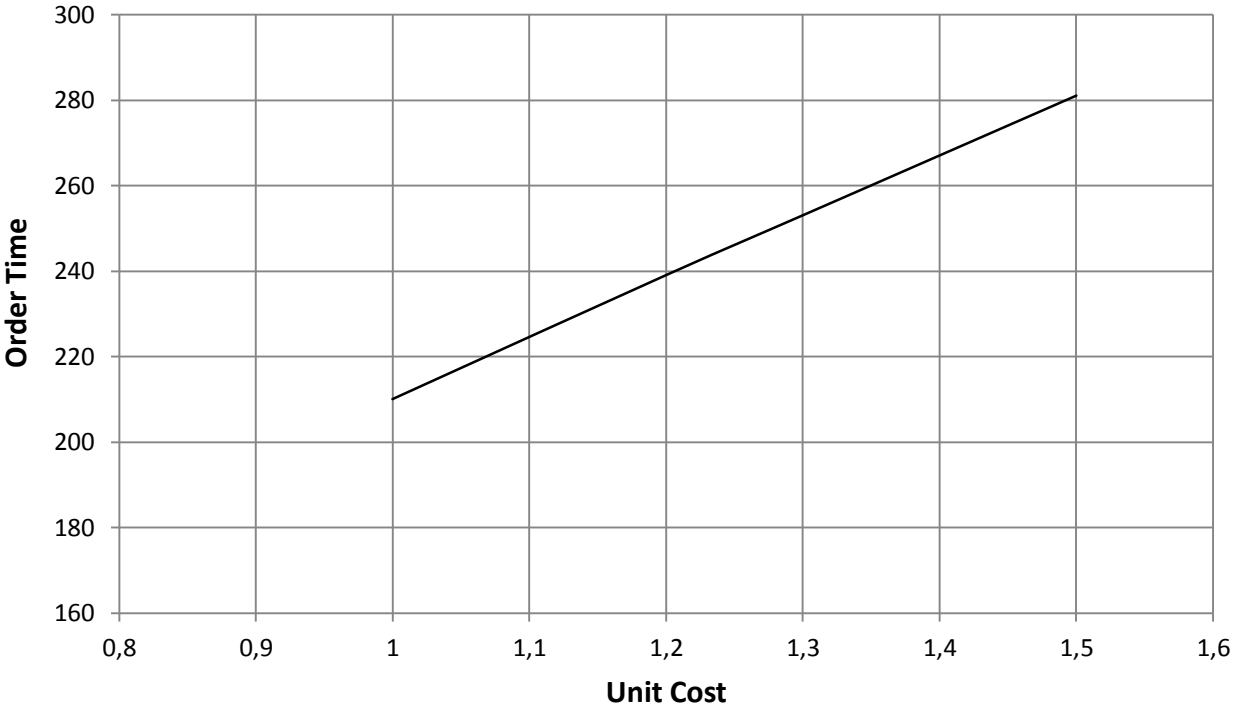

Figure 3.13: Relation between unit cost and order time

In figure 3.13 we observe that increase in unit cost decreases the value of profit and increases order time

Figure 3.14: Relation between order cost and profit

In figure 3.14 we observe that increase in unit cost decreases the value of profit and increases order time.

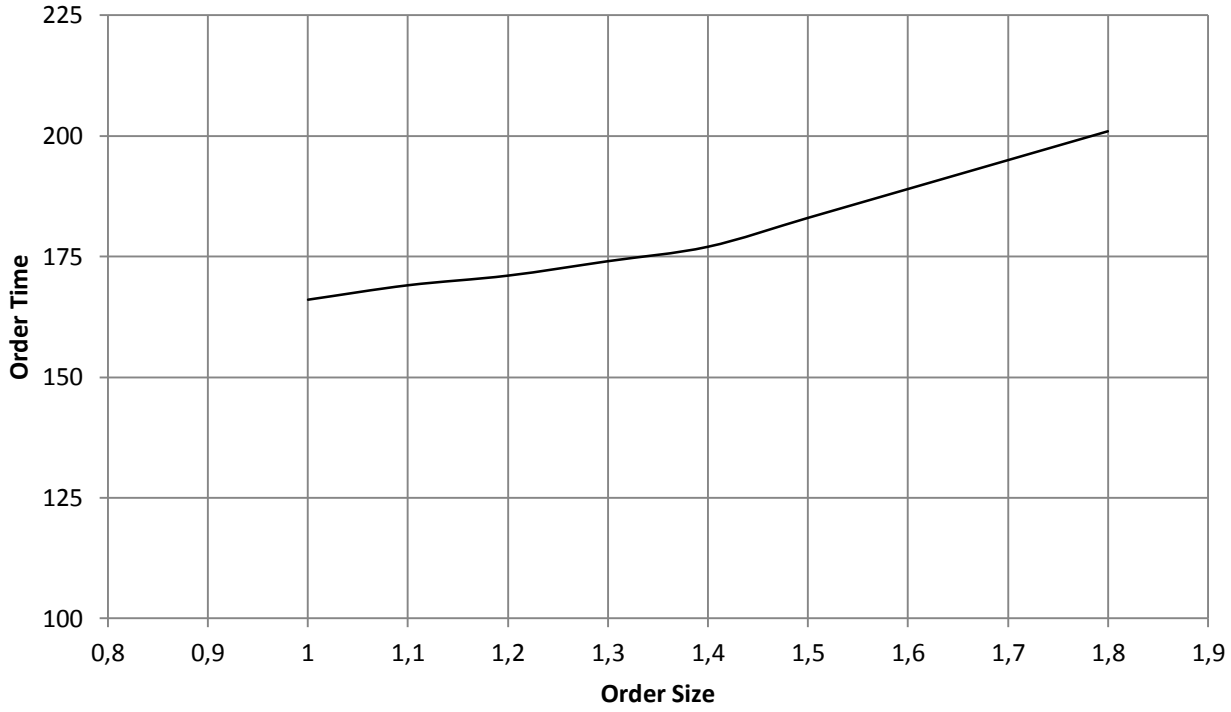

Figure 3.15: Relation between order cost and profit

In figure 3.15 we observe that increase in order size increases order time.

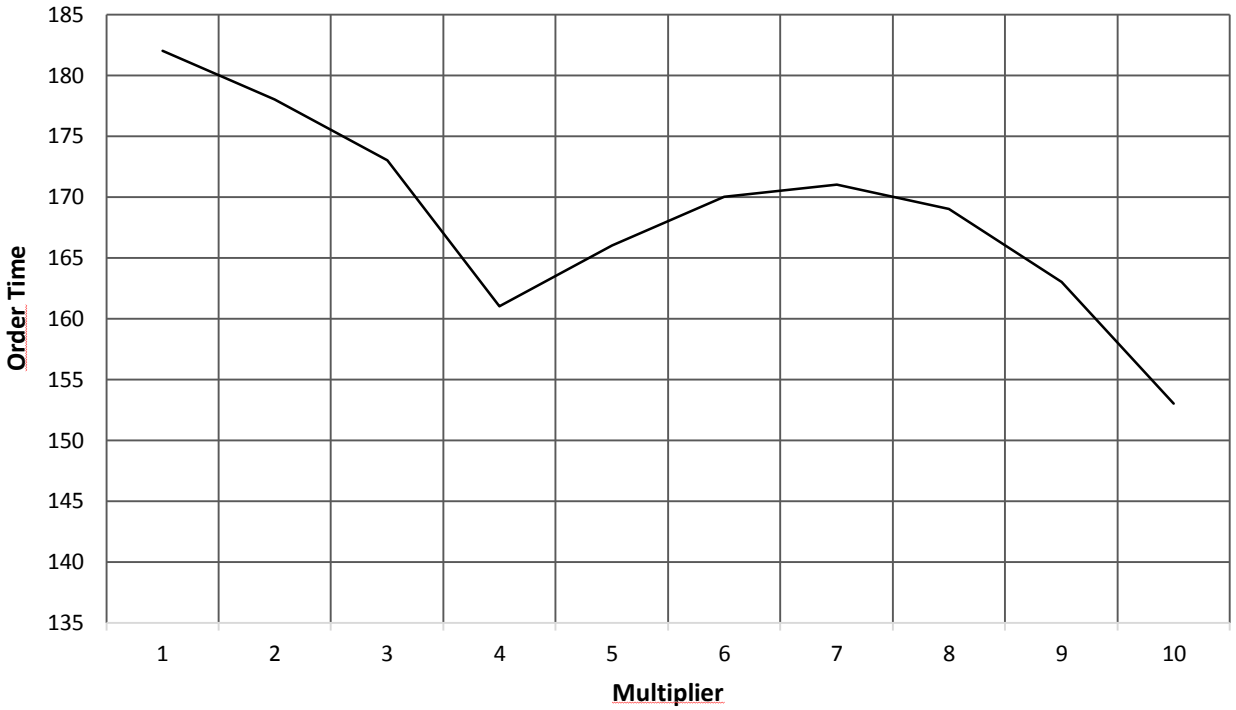

Figure 3.16: Relation between multiplier and order time

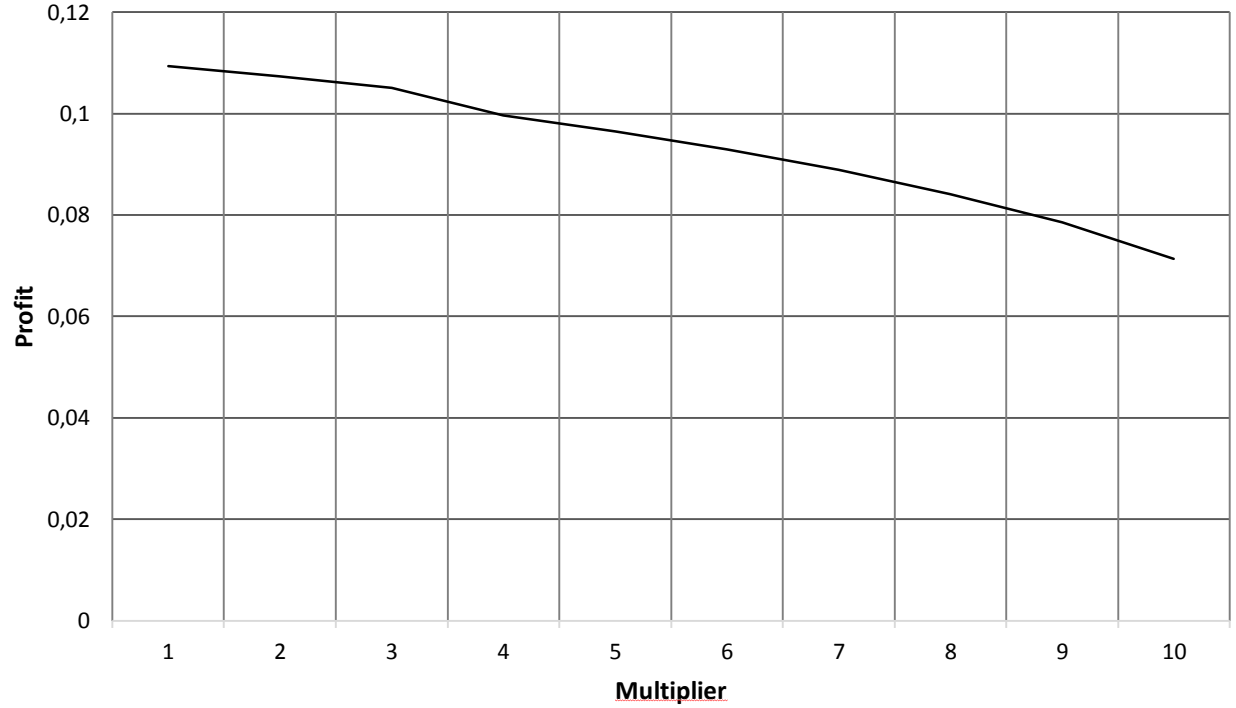

Figure 3.17: Relation between multiplier and profit

In figure 3.16 we observe that there is no observable pattern for order time while multiplier increases.

In figure 3.17 we observe that increase in multiplier decreases profit value.

3.1.1.2 Discounted Cost Dynamic Programming Model

Here are the definitions of notations we use in this section:

p(t): price depends on time

p:price independent of time

J(q,t): Value function with respect to the inventory amount and time

ß: Discount factor

A: Ordering Cost

c: Cost per Product

λ(t): Demand function depends on t

We used p(t) as price function that depends on our decision in time t, and use p as a pre-decided price constant. J(q,t) is the value function of dynamic equation which depends on inventory amount q and time t. ß is the notation which is used to discount value function, A is the amount we pay for a new order, c is the price of each product that we pay for and λ(t) is the demand function that applies in each small time slot.

3.1.1.2.1 Discounted Cost Dynamic Programming Model with Constant Price:

We formulate the problem as an infinite horizon discounted cost dynamic program with constant price.

$$J(q,t)=\max\{Order,Continue\} \quad (3\text{-}38)$$

$$=\max\{f(q,t),f_0\}$$

$$f(q,t)=\lambda(t).(p+\beta.J(q-1,t+1)) +(1-\lambda(t)).\beta.J(q,t+1) \quad (3\text{-}39)$$

$$f_0=-A-c.Q+\lambda(1).(p+\beta.J(Q-1,1))+(1-\lambda(1)).\beta.J(Q,1)) \quad (3\text{-}40)$$

Here, we try to maximize profit value by deciding "continue" or "order" in each time interval and inventory amount. At any state, if we choose to "continue", with probability $\lambda(t)$ one unit of demand arrives and we multiply it with selling price p and new value function $J(q-1,t+1)$ multiplied by discount factor β when time passed by 1 and inventory decreased by 1 too. Therefore first part will be $\lambda(t).(p+\beta.J(q-1,t+1))$. Second part consists of not having a demand situation, so the probability will be $(1-\lambda(t))$ and we multiply it with new value function $J(q,t+1)$ multiplied by β in which inventory amount stays constant and time passes by 1. Because there is no selling, we will get no money. Therefore the total value equation, if we choose to continue, will be $\lambda(t).(p+\beta.J(q-1,t+1)) +(1-\lambda(t)).\beta.J(q,t+1)$.

For the second part of the equation, we consider "ordering" for that time unit and inventory amount. In that case, we pay a constant amount A for fixed

ordering cost, and c for each product we ordered. Then, for these newly ordered products, age of the products is updated to "1" and with probability (1-λ(1)) no demand arrives and the new profit value function will be J(Q,1) which indicates an inventory amount of Q and time will be reset to 1, beginning of the cycle. With probability λ(1), because it is the beginning of cycle, one unit of demand arrives and the new profit value function will be J(Q-1,1), which indicates an inventory amount of Q-1 at age 1. The selling price of one product, p, is also added.

3.1.1.2.2 Discounted Cost Dynamic Programming Model with Variable Price:

We formulate the problem similar to the constant price model, however the only difference in here is taking price as a variable, dependent to the inventory amount on hand and time.

$$J(q,t)=\max\{Order,Continue\} \quad (3\text{-}41)$$

$$=\max\{f(q,t),f_0\}$$

$$f(q,t)=\lambda(p(q,t),t).(p(q,t)+\beta.J(q-1,t+1))+(1-\lambda(p(q,t),t)).\beta.J(q,t+1) \quad (3\text{-}42)$$

$$f_0= -A-c.Q+\lambda(p(Q,0),1).(p(Q,1)+\beta.J(Q-1,1))+(1-\lambda(p(Q,0),1)).\beta.J(Q,1)) \quad (3\text{-}43)$$

Here, we try to maximize average profit value by deciding "continue" or "order" decisions for each time and inventory amount, but now we insert p(q,t) instead of p to all equations that we observe in constant price model.

3.1.2 Analytical Approach

With the lead of the results in dynamic programming approach, we observe, in proposition 3 and 6 that the optimal ordering time only depends on a time threshold independent of the inventory at hand. We think that this structure of the optimal solution might lead to easier ways than dynamic programming to determine the optimal results. Thus, we design another approach in this section, using this structure of the optimal solution to come up with an analytical equation that defines the optimal solution. We also aim to develop easier to apply heuristics and approximation algorithms that can be used by the managers of these systems, rather than employing a dynamic programming algorithm, which requires to keep track of every possible state. Thus, we develop an analytical model that shows different cases that might occur before the ordering time threshold. In this analysis, we first consider the constant price model and then we analyze the variable price case. We calculate the expected profit of a cycle in these models which will be repeated infinitely, considering the demand process as "non-homogeneous Poisson process". We also develop approximation algorithms and heuristics for the cases in which finding the optimal solution becomes cumbersome.

Definitions:

f_d (Q,x): Probability of selling Q amount of demand at time x

N(t): Observed number of demand at time t

m(x): Integrated demand parameter

λ(p, t): Demand rate for price p and time t

$$f_d(Q,x) = \frac{e^{-m(x)}(m(x))^Q}{Q!} \tag{3-44}$$

$$m(x) = \int_0^x \lambda(s)ds \tag{3-45}$$

$$P(N(t) - N(0) = n) = \frac{e^{-m(x)}(m(x))^n}{n!} \tag{3-46}$$

The general equation that we try to maximize is;

$$\max E[Profit\ per\ unit\ time] = \lim_{n\to\infty} \frac{[P_1 + P_2 + P_3 \dots P_n]}{[T_1 + T_2 + T_3 \dots T_n]}$$

$$= \lim_{n\to\infty} \frac{\frac{[P_1 + P_2 + P_3 \dots P_n]}{n}}{\frac{[T_1 + T_2 + T_3 \dots T_n]}{n}}$$

$$= \frac{E[Profit\ per\ Cycle]}{E[Length\ of\ Cycle]}$$

P_n= Profit in nth cycle

T_n= Time spent in nth cycle

3.1.2.1 Constant Price Model

We consider problem depending on the possible cases that we may observe in each horizon. For the constant price model there are only two cases which occur depending on selling all inventory on hand before or after threshold time, t. We develop expected profit and time functions for each case and observe expected profit per time function for constant price model, and try to maximize;

$$\max \frac{E[Profit\ per\ Cycle]}{E[Length\ of\ Cycle]} = \frac{E[P]}{E[T]}$$

$$E[P] = E[P \mid Case1].P(Case1) + E[P \mid Case2].P(Case2)$$

$$= P_1 + P_2$$

$$E[T] = E[T \mid Case1].P(Case1) + E[T \mid Case2].P(Case2)$$

$$= T_1 + T_2$$

Case 1:

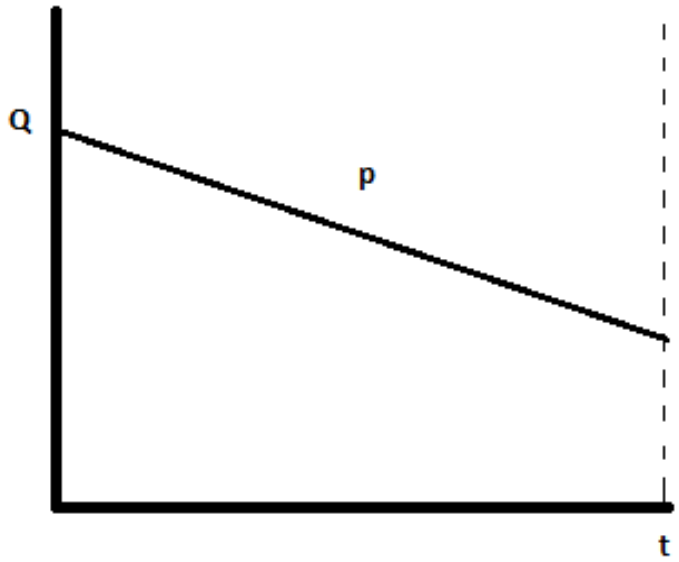

Figure 3.17: Constant Price Case 1

$$P1 = \sum_{q=0}^{Q-1} \frac{e^{-\int_0^t \lambda(p,s)ds} \cdot \left(\int_0^t \lambda(p,s)ds\right)^q}{q!} \cdot (pq - A - cQ) \quad (3\text{-}47)$$

$$T1 = \sum_{q=0}^{Q-1} \frac{e^{-\int_0^t \lambda(p,s)ds} \cdot \left(\int_0^t \lambda(p1,s)ds\right)^q}{q!} \cdot t \quad (3\text{-}48)$$

First case for the constant price is that Q amount of order is not sold before the threshold time t. We calculate the expected profit and expected time for this case as above. Expected profit equation (3-49) consists of all the money we get and pay until threshold time, t. In summation, we sum probability of having q amount of demand which we limit from zero to Q-1 for the sake of Case 1 properties. We multiply each probability to related money on pocket value which is q amount of product with price p, the money we get from selling,

payment A, the ordering cost, and Q amount of unit cost c, payment for each product we order.

Expected time equation (3-50) is similar to the equation for expected profit in this case because in each probability, we always give the order in time t. Therefore in summation, we sum probability of having q amount of demand which we constrained from zero to Q-1 for the sake of case 1 properties, and we multiply this summation to threshold time, t.

Case 2:

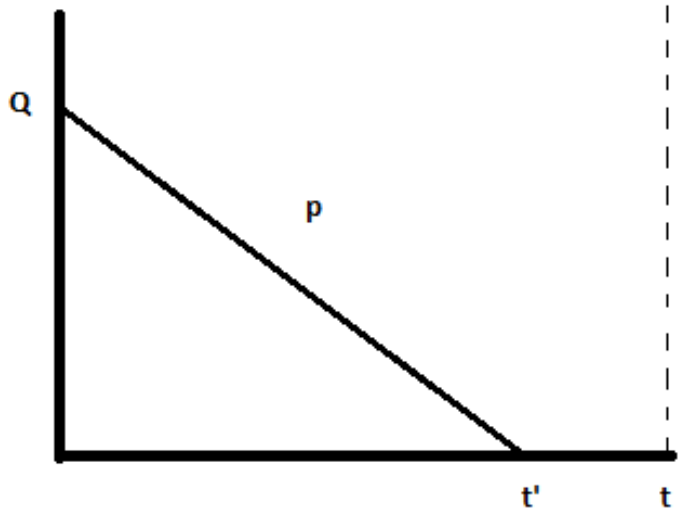

Figure 3.18: Constant Price Case 2

$$P2 = \sum_{q=Q}^{\infty} \frac{e^{-\int_0^t \lambda(p,s)ds} . \left(\int_0^t \lambda(p,s)ds\right)^q}{q!} . (pQ - A - cQ) \quad (3\text{-}49)$$

$$T2 = \sum_{i=1}^{t} \frac{e^{-\int_0^i \lambda(p,s)ds} . \left(\int_0^i \lambda(p,s)ds\right)^{Q-1}}{Q-1!} . (i) . \lambda(p,i) \quad (3\text{-}50)$$

Second case for the constant price is that Q amount of order were sold before the threshold time t, we observed expected profit and expected time for this case.

Expected profit equation (3-46) consists of all the money we get and pay until selling Q amount of product on time t'. In summation, we sum probability of having q amount of demand which we limited from Q to infinity for the sake of case 2 properties. We multiply each probability to related money on pocket value which is Q amount of products with price p, the money we get from selling, payment A, the ordering cost, and Q amount of unit cost c, payment for products we ordered.

Expected time equation (3-47) is now dependent to the exact time that Q amount o products were sold, therefore in summation, we sum probability of having Q amount of demand in exact time t which we constrained from one to t and adjusted function (3-42) inside of the function depending on the i in summation for the sake of case 2 properties. The important twist in the equation is taking to the power of Q-1 instead of Q because it may have been sold any time before indicated ith time. Than we multiply each of these probabilities with one unit time demand $\lambda(p,i)$ for Qth sold product and time i for the exact time of this sell.

$$\text{Objective: } \max \frac{P1+P2}{T1+T2} \qquad (3\text{-}48)$$

After computations we calculate total expected profit divided by total expected time to observe expected profit per time and maximize it.

3.1.2.1 Variable Price Model

We consider problem depending on the possible cases that we may observe in each horizon similar to the constant price model. For the variable price model we develop four different heuristics to eliminate complexity of solution of the problem. We make price change ones, at the half of the order time in Heuristic 1, and make price change ones at the half of the order amount in Heuristic 2. Heuristic 3 is an extension of Heuristic 1 that we can make price change at the optimal time, and Heuristic 4 is an extension of Heuristic 2 that we can make price change at the optimal order amount. Then, we observe expected profit and time functions for each heuristic by diving them into cases.

3.1.2.1.1 Heuristic 1

There are three different cases that we may observe during a horizon in Heuristic 1; first one is selling all products in first half, second one is selling all in second half, and third one is that still having product on hand after time threshold.

$$\max \frac{E[Profit\ per\ Cycle]}{E[Length\ of\ Cycle]} = \frac{E[P]}{E[T]}$$

$$E[P] = E[P \mid Case1].P(Case1) + E[P \mid Case2].P(Case2) + E[P \mid Case3].P(Case3)$$

$$= P_1 + P_2 + P_3$$

E[T] =E[T | Case1].P(Case1) + E[T | Case2].P(Case2)+ E[T | Case3].P(Case3)

$=T_1 + T_2 + T_3$

Case 1:

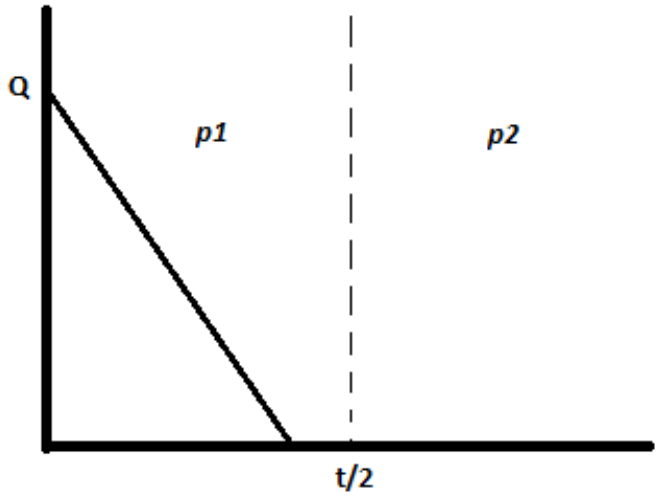

Figure 3.19: Heuristic 1 Case 1

$$P1 = \sum_{q=Q}^{\infty} \frac{e^{-\int_0^{\frac{t}{2}} \lambda(p1,s)ds} \cdot \left(\int_0^{\frac{t}{2}} \lambda(p1,s)ds\right)^q}{q!} \cdot (pQ - A - cQ) \quad (3\text{-}49)$$

$$T1 = \sum_{i=1}^{\frac{t}{2}} \frac{e^{-\int_0^{i} \lambda(p1,s)ds} \cdot \left(\int_0^{i} \lambda(p1,s)ds\right)^{Q-1}}{Q-1!} \cdot (i).\lambda(p,i) \quad (3\text{-}50)$$

First case for the Heuristic 1 is that Q amount of order were sold before the half of the threshold time, t/2, we observe expected profit and expected time for this case.

Expected profit equation (3-49) is pretty similar to the equation in constant price case 2 (3-46). All of the components of the equation are the same but instead of taking time interval between zero and t, because of the structure of the case we take the upper limit for demand as t/2.

As we observe for the expected profit equation, same similarities are valid for the expected time too when we compare equations (3-50) and (3-47). The only difference now is changing upper limit of the summation from t to t/2 because we only consider half of the observed pattern in case 1.

Case 2:

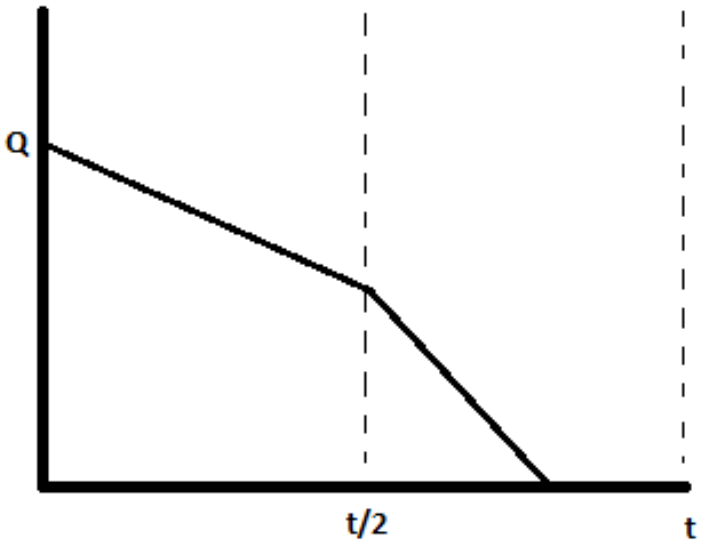

Figure 3.20: Heuristic 1 Case 2

$$P2 = \sum_{q=0}^{Q-1} \frac{e^{-\int_0^{t/2} \lambda(p1,s)ds} \cdot \left(\int_0^{t/2} \lambda(p1,s)ds\right)^q}{q!} \left[\sum_{z=Q-q}^{\infty} \frac{e^{-\int_{t/2}^{t} \lambda(p2,s)ds} \cdot \left(\int_{t/2}^{t} \lambda(p2,s)ds\right)^z}{z!} \cdot (p1q + p2z - A - cQ)\right] \quad (3\text{-}51)$$

$$T2 = \sum_{q=0}^{Q-1} \frac{e^{-\int_0^{t/2} \lambda(p1,s)ds} \cdot \left(\int_0^{t/2} \lambda(p1,s)ds\right)^q}{q!} \left[\sum_{i=t/2}^{t} \frac{e^{-\int_{t/2}^{i} \lambda(p2,s)ds} \cdot \left(\int_{t/2}^{i} \lambda(p2,s)ds\right)^{Q-q-1}}{Q-q-1!} . \lambda(p2,i).i \right] \quad (3\text{-}52)$$

Second case for the Heuristic 1 is that Q amount of order were not sold before the half of the threshold time, t/2, but rest of the products will be sold in between t/2 and t, and we observed expected profit and expected time for this case.

Expected profit equation (3-51) consists of all the money we get and pay until selling Q amount of product before t. In first summation, we sum probability of having q amount of demand which we limited from 0 to Q-1in between time intervals zero and t/2 for the first half of the case. For the second half, we sum probability of having rest of the Q amount of demand which we limited from Q-q to infinity in between time intervals t/2 to t. We multiply each probability to related money on pocket value which is q amount of products with price p1 in first half and z amount of products with price p2 in second half, the money we get from selling, payment A, the ordering cost, and Q amount of unit cost c, payment for products we ordered.

Expected time equation (3-52) can be divided into first half and second half by half of the time threshold, t/2. First part includes selling less than q amount in between time intervals zero and t/2. In the second part rest of the Q amount will be sold in time i, in between time intervals t/2 and t, and the same twist that is valid for constant price case 2, so we multiply having total Q-1 demand with probability of having one demand in time i, $\lambda(p2,i)$, and time I itself

Case 3:

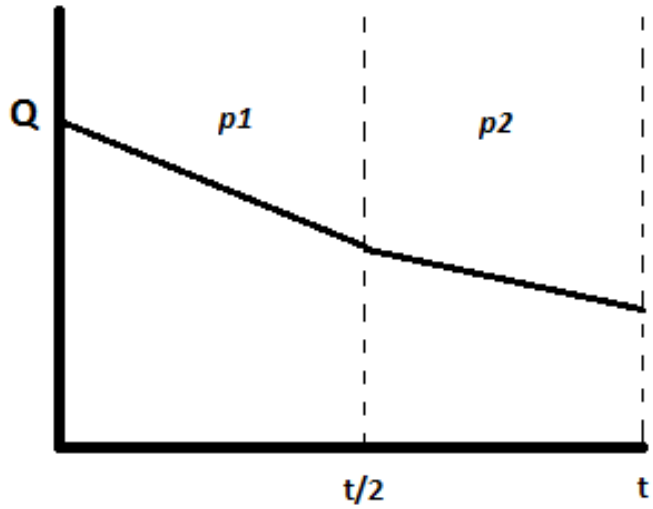

Figure 3.21: Heuristic 1 Case 3

$$P3 = \sum_{q=0}^{Q-1} \frac{e^{-\int_0^{t/2} \lambda(p1,s)ds} \cdot \left(\int_0^{t/2} \lambda(p1,s)ds\right)^q}{q!} \left[\sum_{z=0}^{Q-q} \frac{e^{-\int_{t/2}^{t} \lambda(p2,s)ds} \cdot \left(\int_{t/2}^{t} \lambda(p2,s)ds\right)^z}{z!} . (p1q + p2z - A - cQ)\right] \quad (3\text{-}53)$$

$$T3 = \sum_{q=0}^{Q-1} \frac{e^{-\int_0^{t/2} \lambda(p1,s)ds} \cdot \left(\int_0^{t/2} \lambda(p1,s)ds\right)^q}{q!} \left[\sum_{z=0}^{Q-q} \frac{e^{-\int_{t/2}^{t} \lambda(p2,s)ds} \cdot \left(\int_{t/2}^{t} \lambda(p2,s)ds\right)^z}{z!} . t\right] \quad (3\text{-}54)$$

Third case for the Heuristic 1 is that Q amount of order were not sold until the end of threshold t, and we observed expected profit and expected time for this case.

Expected profit equation (3-53) consists of all the money we get and pay until threshold time t. In first summation, we sum probability of having q amount of demand which we limited from 0 to Q-1in between time intervals zero and t/2

for the first half of the case. For the second half, we sum probability of having less than rest of the Q amount of demand which we limited from 0 to Q-q in between time intervals t/2 to t. We multiply each probability to related money on pocket value which is q amount of products with price p1 in first half and z amount of products with price p2 in second half, the money we get from selling, payment A, the ordering cost, and Q amount of unit cost c, payment for products we ordered.

Expected time equation (3-54) can be divided into first half and second half by half of the time threshold, t/2. First part includes selling less than q amount in between time intervals zero and t/2. In the second part rest of the Q amount will not be sold in time t, in between time intervals t/2 and t, therefore second part will be similar to the one in expected profit, so we multiply having total Q-1 demand with probability of having one demand in time i, $\lambda(p2, i)$, and time i itself

$$\text{Objective: } \max \frac{P1+P2+P3}{T1+T2+T3} \qquad (3\text{-}55)$$

After computations we calculate total expected profit divided by total expected time to observe expected profit per time and maximize it.

3.1.2.1.2 Heuristic 2

There are three different cases that we may observe during a horizon in Heuristic 2; first one is still having more than half of the inventory we ordered after threshold time, second one is still having less than half of the inventory we ordered after threshold time, and third one is selling all of the products before threshold time.

$$\max \frac{E[Profit\ per\ Cycle]}{E[Length\ of\ Cycle]} = \frac{E[P]}{E[T]}$$

$$E[P] = E[P \mid Case1].P(Case1) + E[P \mid Case2].P(Case2) + E[P \mid Case3].P(Case3)$$

$$= P_1 + P_2 + P_3$$

$$E[T] = E[T \mid Case1].P(Case1) + E[T \mid Case2].P(Case2) + E[T \mid Case3].P(Case3)$$

$$= T_1 + T_2 + T_3$$

Case 1:

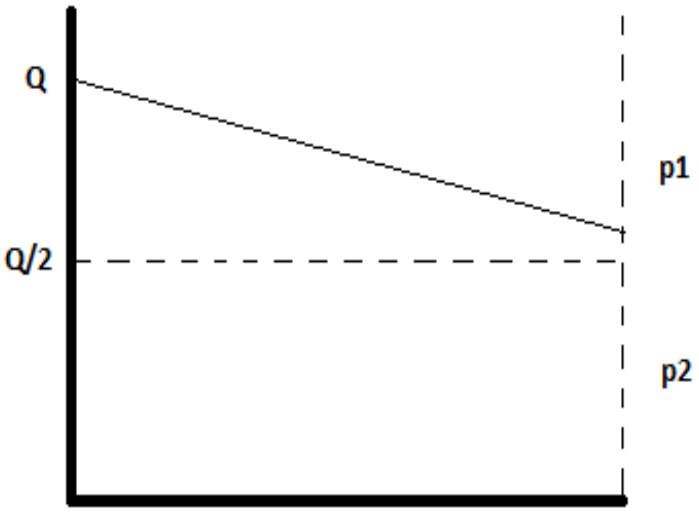

Figure 3.22: Heuristic 2 Case 1

$$P1 = \sum_{q=0}^{Q/2} \frac{e^{-\int_0^t \lambda(p,s)ds} \cdot \left(\int_0^t \lambda(p,s)ds\right)^q}{q!} \cdot (pq - A - cQ) \qquad (3\text{-}56)$$

$$T1 = \sum_{q=0}^{Q/2} \frac{e^{-\int_0^t \lambda(p,s)ds} \cdot \left(\int_0^t \lambda(p1,s)ds\right)^q}{q!} \cdot t \qquad (3\text{-}57)$$

First case for the Heuristic 2 is that Q/2 amount of order were not sold before the threshold time, t, we observed expected profit and expected time for this case.

In Expected profit equation (3-56), summation part is limited between 0 and Q/2 amount, and demand time interval is restricted until the end of the

threshold time, t. We multiply it q amount of price, p, deduct order cost, A, and Q amount of sold units with price c.

Like we observed for the expected profit equation, same similarities are valid for the expected time too, when we compare equations (3-56) and (3-57), probability of having threshold time are the same, so we multiply same probability equation with time, t.

Case 2:

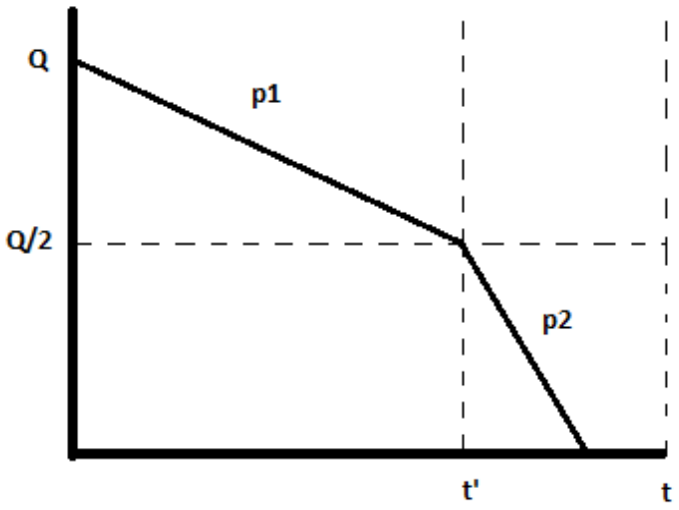

Figure 3.23: Heuristic 2 Case 2

$$P2 = \sum_{i=0}^{t-1} \frac{e^{-\int_0^i \lambda(p1,s)ds} \cdot \left(\int_0^i \lambda(p1,s)ds\right)^{Q/2}}{Q/2!} \left[\sum_{k=i}^{t-k} \frac{e^{-\int_i^k \lambda(p2,s)ds} \cdot \left(\int_i^k \lambda(p2,s)ds\right)^{\frac{Q}{2}}}{\frac{Q}{2!}} \cdot ((p1+p2)(\frac{Q}{2}) - A - cQ)) \right] \quad (3\text{-}58)$$

$$T2 = \sum_{i=0}^{t-1} \frac{e^{-\int_0^i \lambda(p1,s)ds} \cdot \left(\int_0^i \lambda(p1,s)ds\right)^{Q/2}}{Q/2!} \left[\sum_{k=i}^{t-k-1} \frac{e^{-\int_i^k \lambda(p2,s)ds} \cdot \left(\int_i^k \lambda(p2,s)ds\right)^{\frac{Q}{2}}}{Q/2!} . k \right] \qquad (3\text{-}59)$$

Second case for the Heuristic 2 is that Q/2 amount of order were sold before the threshold time, t, in exact time t', and rest of the products will be sold in between t' and t, and we observed expected profit and expected time for this case.

Expected profit equation (3-58) consists of all the money we get and pay until selling Q amount of product before t. In first summation, we sum probability of having Q/2 amount of demand between time intervals zero and t-1, and summation limits will be the same for the first part of the case. For the second part, we sum probability of having rest Q/2 of demand in between time intervals i that we observe in the first part, the exact time that we sold Q/2, to the exact moment of selling them, k. We multiply each probability to related money on pocket value which is Q/2 amount of products with both price p1 and price p2, the money we get from selling, payment A, the ordering cost, and Q amount of unit cost c, payment for products we ordered.

Expected time equation (3-59) can be divided into two parts similar to the expected profit equation. Equations inside first and second part will be the same, the only difference between these two equations is multipliers. Now, we have to multiply it with the observed time which comes from the upper limit of the second equation, k.

Case 3:

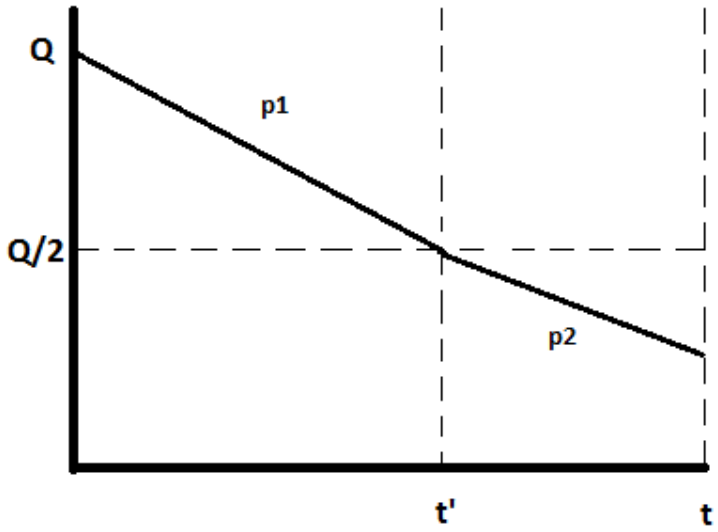

Figure 3.24: Heuristic 2 Case 3

$$P3 = \sum_{i=0}^{t-1} \frac{e^{-\int_0^i \lambda(p1,s)ds} \cdot \left(\int_0^i \lambda(p1,s)ds\right)^{Q/2}}{Q/2!} \left[\sum_{q=0}^{\frac{Q}{2}} \frac{e^{-\int_i^t \lambda(p2,s)ds} \cdot \left(\int_i^t \lambda(p2,s)ds\right)^{q}}{q!} \cdot ((p1\left(\frac{Q}{2}\right) + p2q - A - cQ))\right] \quad (3\text{-}60)$$

$$T3 = \sum_{i=0}^{t-1} \frac{e^{-\int_0^i \lambda(p1,s)ds} \cdot \left(\int_0^i \lambda(p1,s)ds\right)^{Q/2}}{Q/2!} \left[\sum_{q=0}^{\frac{Q}{2}} \frac{e^{-\int_i^t \lambda(p2,s)ds} \cdot \left(\int_i^t \lambda(p2,s)ds\right)^{q}}{q!} \cdot t\right] \quad (3\text{-}61)$$

Third case for the Heuristic 2 is that Q/2 amount of order were sold before the threshold time, t, in exact time t', and rest of the products will not be sold in between t' and t, and we observed expected profit and expected time for this case.

Expected profit equation (3-60) consists of all the money we get and pay until selling products before t. In first summation, we sum probability of having Q/2 amount of demand between time intervals zero and t-1, and summation limits will be the same for the first half of the case. For the second part, we sum probability of having q amount of demand with upper limit Q/2 in between time interval i that we observe in the first part, and the threshold time, t. We multiply each probability to related money on pocket value which is Q/2 amount of products with price p1 and q amount of product with price p2, the money we get from selling, payment A, the ordering cost, and Q amount of unit cost c, payment for products we ordered.

Expected time equation (3-59) can be divided into two parts similar to the expected profit equation. Equations inside first and second part will be the same, the only difference between these two equations is multipliers. Now, we have to multiply it with the observed time which is threshold time, t.

$$\text{Objective: } \max \frac{P1+P2+P3}{T1+T2+T3} \qquad (3\text{-}62)$$

After computations we calculated total expected profit divided by total expected time to observe expected profit per time and maximize it.

3.1.2.1.3 Heuristic 3

Similar to the Heuristic1, now instead of making price change at the half of order time, we decide both when to order and when to change price optimally, so, t, t' , Q, p1 and p2 values will be decided by searching via MATLAB. Results will be provided in next periods.

Cases:

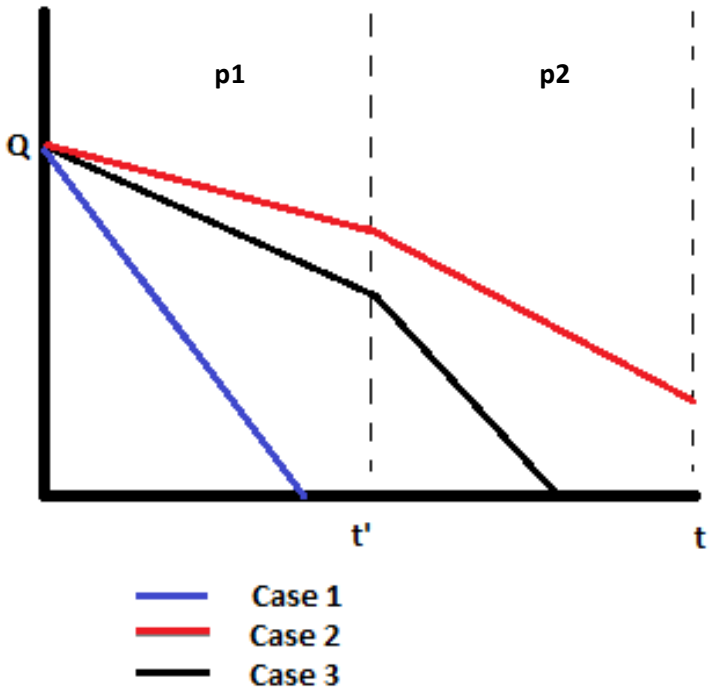

Figure 3.25: Heuristic 3

3.1.2.1.4 Heuristic 4

Similar to the Heuristic2, now instead of making price change at the half of order amount, we decide both when to order and in which amount of inventory to change price optimally by searching via MATLAB results will be provided in next periods.

Cases:

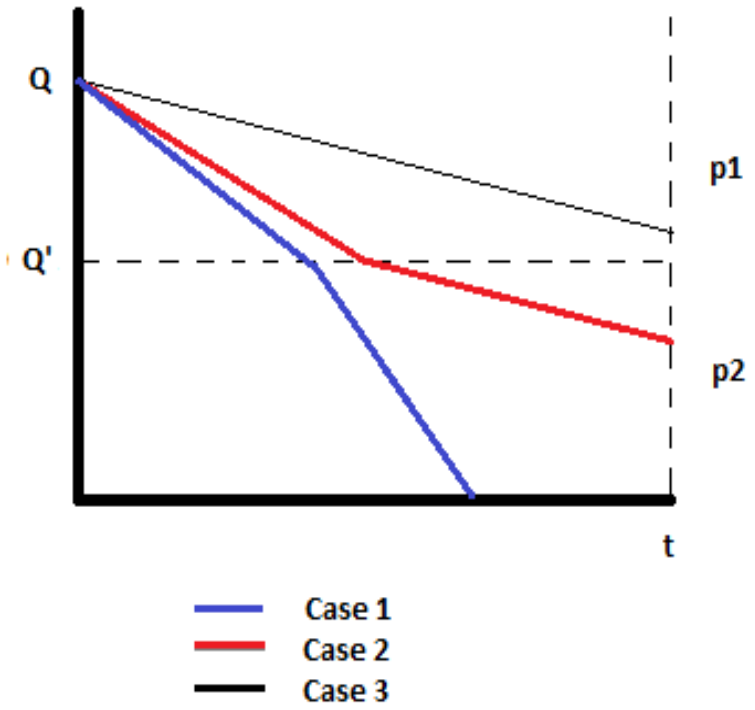

Figure 3.26: Heuristic 4

CHAPTER 4

Numerical Results

We conduct numerical results for different demand functions that fit the structure of demand function that reflects features of perishable product with uncertain and diminishing demand with an uncertain shelf time. We generalize a demand function with Poisson distribution such that;

$$f_d(Q,x) = \frac{e^{-m(x)}(m(x))^Q}{Q!} \quad (4\text{-}1)$$

$$m(x) = \int_0^x \lambda(s)ds \quad (4\text{-}2)$$

$$P(N(t) - N(0) = n) = \frac{e^{-m(x)}(m(x))^n}{n!} \quad (4\text{-}3)$$

Generalized demand function is indicated on (4-3), and as a result of that in each small amount of time, we are going to have a demand with λ(p,t) which depends on both price and time, and product is deteriorated randomly, so with uncertainty. We observe a demand function to use in our runs "α-$p^{\theta}.i.\epsilon$" and parameters,

α: Max demand constant

i:time

ϵ: Time multiplying Factor

θ:Price multiplying factor

The demand function for numerical results is: **0.03 - 0.0001 . (p^3 /216) . t**

4.1 Dynamic Programming Model Results

4.1.1 Constant Price Model

The demand function we consider is in the format of "α-$p^{\theta}.t.\epsilon$" because price is constant for this scenario, our demand function only depends on time and the results are interpreted according to that. We analyze the results for different prices, units ordered, unit cost and ordering cost. We obtain expected profit of infinite horizon and optimal ordering time for specified variables. We can only see the table results below, however for each line we obtain a strategy of pricing for each amount of inventory and time, however in constant price model, it is useless because of inability to manipulate prices. Order time is a time threshold as we prove in previous sections.

Table 4.1 Results for constant prices with dynamic approach

Parameters				*Results*	
Price	***Unit Cost***	***Order Cost***	***Order Amount***	***Order Time***	***Expected Profit***
5	2	5	4	281	0,0284
5	2	5	3	288	0,025
5	2	5	2	300	0,0097
5	2	5	5	284	0,0271
5	2	5	6	292	0,0236
6	2	5	4	178	0,0203
4	2	5	6	300	0,0213
7	2	5	4	300	0,0003
5	2	4	4	267	0,0343
5	2	3	4	253	0,0403
5	1	5	4	224	0,0529
5	3	5	4	300	0,0056
5	2	2	4	239	0,0465

5	0,5	5	4	194	0,0662
5	2	6	4	300	-0,453
5	2	5	7	300	0,0189
5	1,5	5	4	253	0,0403
5	2,1	5	4	286	0,0261
5	1,9	5	4	275	0,0307
5	1,8	5	4	270	0,0331
5	1,7	5	4	264	0,0355
5	1,6	5	4	259	0,0379
5	1,5	5	4	253	0,0403
5	1,4	5	4	248	0,0428
5	1,3	5	4	242	0,0452
5	1,2	5	4	236	0,0478
5	1,1	5	4	230	0,0503
5	2	4,9	4	279	0,029
5	2	4,8	4	278	0,0296
5	2	4,7	4	277	0,0302
5	2	4,6	4	275	0,0307
5	2	4,5	4	274	0,0313
5	2	4,4	4	273	0,0319
5	2	4,3	4	271	0,0325
5	2	4,2	4	270	0,0331
5	2	4,1	4	268	0,0337
6	1	1	2	118	0,1093
6	1	1	3	124	0,1057
6	1	1	4	133	0,1002
6	1	1	5	144	0,0942
6	1	1	6	154	0,0880
6	1	1	7	164	0,0821
5	1	1	2	197	0,0931

Table 4.2 Results for constant prices with dynamic approach

Parameters				*Results*			
Unit Cost	***Order Cost***	θ	ϵ	***Price***	***Order Time***	***Order Amount***	***Expected Profit***
1.5	1	3	1	6	142	2	0,0949
1.6	1	3	1	6	147	2	0,0922
1.7	1	3	1	6	151	2	0,0894
1.8	1	3	1	6	156	2	0,0867
1.9	1	3	1	6	160	2	0,0840
2	1	3	1	6	165	2	0,0813
1	2	3	1	6	192	3	0,0973
1	2	3	2	6	91	2	0,0716
1	2	3	3	6	72	2	0,0522
1	2	2	1	6	141	3	0,0955
1	2	4	1	6	141	3	0,0955
1	1.9	3	1	6	140	3	0,0982
1	1.8	3	1	6	137	2	0,0977
1	1.7	3	1	6	135	2	0,0991
1	1.6	3	1	6	133	2	0,1006
1	1.5	3	1	6	131	2	0,1020

4.1.2 Variable Price Model

We used demand function we mentioned for explaining general structure which is "α-$p^{\theta}.t.\epsilon$" The structure of results we obtain here is similar to constant price model. Only difference is now, we can change the prices in each time unit, which gives us a strategy for having optimal profit in long term. We analyzed the results for different prices, units ordered, unit cost and ordering cost. We obtained expected profit of infinite horizon and optimal ordering time for specified variables. We can only see the table results below, however for each

line we obtaine a strategy of pricing for each amount of inventory and time as we mentioned. Order time is a time threshold as we prove in previous sections.

Table 4.3 Results for variable prices with dynamic approach

Parameters				*Results*			
Unit Cost	***Order Cost***	θ	ϵ	***Price***	***Order Time***	***Order Amount***	***Expected Profit***
1.5	1	3	1	4-5-6	201	2	0,0961
1.6	1	3	1	4-5-6	195	2	0,0935
1.7	1	3	1	4-5-6	189	2	0,0910
1.8	1	3	1	4-5-6	183	2	0,0884
1.9	1	3	1	4-5-6	177	2	0,0859
2	1	3	1	4-5-6	174	2	0,0833
1	2	3	1	4-5-6	192	3	0,0973
1	2	3	2	4-5-6	176	2	0,0785
1	2	3	3	4-5-6	150	2	0,0667
1	2	2	1	4-5-6	157	3	0,0958
1	2	4	1	4-5-6	264	3	0,0992
1	1.9	3	1	4-5-6	169	2	0,0982
1	1.8	3	1	4-5-6	166	2	0,0991
1	1.7	3	1	4-5-6	177	2	0,1001
1	1.6	3	1	4-5-6	201	2	0,1014
1	1.5	3	1	4-5-6	224	2	0,1028

As we can see from Table.4.3 and Table 4.4 we observe price and ordering decisions for each inventory and time variable decision. Matlab gives this table as a result and we can gather how many inventory we should order ("4" as we can see in inventory column.) and dedicated prices between 4 and 6 in each cells. Ordering decisions were shown as "2" and non-ordering as "1".

Table 4.4 Pricing Decisions

					Time												
Inventory	***1***	***2***	***3***	**….**	***51***	***52***	***53***	***54***	***55***	***56***	**..**	***165***	***166***	***167***	***168***	**..**
1	6	6	6	6	6	6	6	6	5	5	5	5	5	4	6	6
2	6	6	6	6	6	6	6	5	5	5	5	4	4	4	6	6
3	6	6	6	6	6	5	5	5	5	5	5	4	4	4	6	6
4	6	6	6	6	5	5	5	5	5	5	5	4	4	4	6	6

Table 4.5 Ordering Decisions

					Time											
Inventory	***1***	***2***	***3***	**….**	***51***	***52***	***53***	***54***	***55***	***56***	**..**	***165***	***166***	***167***	***168***	**..**
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2

4.2 Analytical Approach Results

4.2.1 Constant Price Model

For constant price, we expect to have almost identical results which only can differ because of inability to reflect all ups and downs in function exactly. We consider the same function structure with the analyzed one in constant price which is "α-$p^{\theta}.t.\epsilon$"

Table 4.6 Optimal Results for constant price with analytical approach

Parameters				*Results*			
θ	ϵ	*Order Cost*	*Unit Cost*	*Price*	*Order Amount*	*Order Time*	*Expected Profit*
3	1	1	1.5	6	2	142	0,0939
3	1	1	1.6	6	2	147	0,0911
3	1	1	1.7	6	2	151	0,0884
3	1	1	1.8	6	2	156	0,0857
3	1	1	1.9	6	2	161	0,083
3	1	1	2	6	2	165	0,0804
3	1	2	1	6	3	141	0,0949
3	2	2	1	6	2	77	0,0706
3	3	2	1	6	2	70	0,0512
2	1	2	1	6	2	132	0,0945
4	1	2	1	6	2	132	0,0945
3	1	1.9	1	6	3	140	0,0959
3	1	1.8	1	6	3	138	0,0969
3	1	1.7	1	6	2	135	0,0981
3	1	1.6	1	6	2	132	0,0995
3	1	1.5	1	6	2	130	0,1009

4.2.2 Variable Price Heuristics

For the variable price, because of the complexity of the problem, it is hard to implement and solve the model by changing price instantly, and it is also cannot be applicable in real life for most of the systems in our range. Therefore we have results for the heuristics we developed considering same demand function as we used in variable price dynamic programming approach.

4.2.2.1 Heuristic 1 Results

For this part, we obtain results with two phased price using order cost and unit cost as dependent variable

Table 4.7 Optimal Results for Heuristic 1

Parameters				*Results*				
θ	ϵ	*Order Cost*	*Unit Cost*	*Price1*	*Price2*	*Order Amount*	*Order Time*	*Expected Profit*
3	1	1	1.5	6	5	2	189	0,0954
3	1	1	1.6	6	5	2	195	0,0928
3	1	1	1.7	6	5	2	201	0,0902
3	1	1	1.8	6	5	2	207	0,0876
3	1	1	1.9	6	5	2	209	0,085
3	1	1	2	6	5	2	209	0,0824
3	1	2	1	6	5	3	192	0,0973
3	2	2	1	6	5	2	140	0,0775
3	3	2	1	6	5	2	147	0,0655
2	1	2	1	6	5	3	234	0,0947
4	1	2	1	6	5	3	187	0,0972
3	1	1.9	1	6	5	3	183	0,0982
3	1	1.8	1	6	5	3	181	0,0991
3	1	1.7	1	6	5	3	719	0,1
3	1	1.6	1	6	5	3	177	0,1009
3	1	1.5	1	6	5	2	175	0,1021

Here we obtained that expected profit has a reverse impact on order cost such that when order cost is decreasing, expected profit increases. We are going to combine these results with other heuristics results.

4.2.2.2 Heuristic 2 Results

For Heuristic 2, we obtain results with two phased price in between 4-6 and observed optimal price and order sizes and display results for different unit cost, price, order amount and order cost.

Table 4.8 Optimal Results for Heuristic 2

Parameters				*Results*				
θ	ϵ	*Order Cost*	*Unit Cost*	*Price1*	*Price2*	*Order Amount*	*Order Time*	*Expected Profit*
3	1	1	1.5	6	6	2	142	0,0934
3	1	1	1.6	6	6	2	146	0,0907
3	1	1	1.7	6	6	2	151	0,088
3	1	1	1.8	6	6	2	156	0,0854
3	1	1	1.9	6	6	2	160	0,0828
3	1	1	2	6	6	2	165	0,0802
3	1	2	1	6	5	3	141	0,0973
3	2	2	1	6	6	2	166	0,0745
3	3	2	1	6	6	2	140	0,0625
2	1	2	1	6	6	3	147	0,0917
4	1	2	1	6	6	3	234	0,0952
3	1	1.9	1	6	6	2	139	0,0948
3	1	1.8	1	6	6	2	137	0,0961
3	1	1.7	1	6	6	2	134	0,0975
3	1	1.6	1	6	6	2	132	0,0989
3	1	1.5	1	6	6	2	130	0,1003

Same relationship is valid between expected profit and order cost in Heuristic 3, as we obtained in variable price dynamic approach

4.2.2.3 Heuristic 3 Results

For Heuristic 3, we obtained results with two phased price in between 4-6 by using order cost and unit cost as dependent variable.

Table 4.9 Optimal Results for Heuristic 3

Parameters				*Results*				
θ	ϵ	*Order Cost*	*Unit Cost*	*Price1*	*Price2*	*Order Amount*	*Order Time*	*Expected Profit*
3	1	1	1.5	6	5	2	188	0,0951
3	1	1	1.6	6	5	2	194	0,0925
3	1	1	1.7	6	5	2	198	0,0899
3	1	1	1.8	6	5	2	204	0,0873
3	1	1	1.9	6	5	2	210	0,0847
3	1	1	2	6	5	2	216	0,0821
3	1	2	1	6	5	2	207	0,0969
3	2	2	1	6	5	2	140	0,0779
3	3	2	1	6	5	2	147	0,0660
2	1	2	1	6	5	3	234	0,0953
4	1	2	1	6	5	3	187	0,0976
3	1	1.9	1	6	5	2	205	0,0978
3	1	1.8	1	6	5	3	202	0,0987
3	1	1.7	1	6	5	3	186	0,0996
3	1	1.6	1	6	5	3	186	0,1005
3	1	1.5	1	6	5	2	186	0,1017

Same relationship is valid between expected profit and order cost in Heuristic 3, as we obtained in variable price dynamic approach

4.2.2.4 Heuristic 4 Results

For Heuristic 4, we obtained results with two phased price in between 4-6 by using order cost and unit cost as dependent variable.

Table 4.10 Optimal Results for Heuristic 4

Parameters				*Results*				
θ	ϵ	*Order Cost*	*Unit Cost*	*Price1*	*Price2*	*Order Amount*	*Order Time*	*Expected Profit*
3	1	1	1.5	6	6	2	142	0,0934
3	1	1	1.6	6	6	2	146	0,0907
3	1	1	1.7	6	6	2	151	0,088
3	1	1	1.8	6	6	2	156	0,0854
3	1	1	1.9	6	6	2	160	0,0828
3	1	1	2	6	6	2	165	0,0802
3	1	2	1	6	5	3	187	0,0973
3	2	2	1	6	6	2	166	0,0750
3	3	2	1	6	6	2	140	0,0631
2	1	2	1	6	6	3	147	0,0929
4	1	2	1	6	6	3	234	0,0959
3	1	1.9	1	6	6	2	139	0,0948
3	1	1.8	1	6	6	2	137	0,0961
3	1	1.7	1	6	6	2	134	0,0975
3	1	1.6	1	6	6	2	132	0,0989
3	1	1.5	1	6	6	2	130	0,1003

4.3 Result Comparison

Here are the results for the heuristics we created and deviation shows optimal solution that we observed from variable price dynamic approach which we were able to change price immediately.

Table 4.11 Result Comparison of Optimal Price

Parameters				*Results*			*Heuristic 1*		*Heuristic 3*		*Heuristic 2*		*Heuristic 4*	
θ	ϵ	*Unit Cost*	*Order Cost*	*Price*	*Order Amount*	*Optimal Result*	*Result*	*Deviation*	*Result*	*Deviation*	*Result*	*Deviation*	*Result*	*Deviation*
3	1	1,5	1	4-5-6	2	0,0961	0,0954	**0,73%**	0,0951	**1,04%**	0,0934	**2,81%**	0,0934	**2,81%**
3	1	1,6	1	4-5-6	2	0,0935	0,0928	**0,75%**	0,0925	**1,07%**	0,0907	**2,99%**	0,0907	**2,99%**
3	1	1,7	1	4-5-6	2	0,0910	0,0902	**0,88%**	0,0899	**1,21%**	0,088	**3,30%**	0,088	**3,30%**
3	1	1,8	1	4-5-6	2	0,0884	0,0876	**0,90%**	0,0873	**1,24%**	0,0854	**3,39%**	0,0854	**3,39%**
3	1	1,9	1	4-5-6	2	0,0859	0,085	**1,05%**	0,0847	**1,40%**	0,0828	**3,61%**	0,0828	**3,61%**
3	1	2	1	4-5-6	2	0,0833	0,0824	**1,08%**	0,0821	**1,44%**	0,0802	**3,72%**	0,0802	**3,72%**
3	1	1	2	4-5-6	3	0,0973	0,0973	**0,00%**	0,0969	**0,41%**	0,0973	**0,00%**	0,0973	**0,00%**
3	2	1	2	4-5-6	2	0,0785	0,0775	**1,27%**	0,0745	**5,10%**	0,0779	**0,76%**	0,0750	**4,46%**
3	3	1	2	4-5-6	2	0,0667	0,0655	**1,80%**	0,0625	**6,30%**	0,0660	**1,05%**	0,0631	**5,40%**
2	1	1	2	4-5-6	3	0,0958	0,0947	**1,15%**	0,0917	**4,28%**	0,0953	**0,52%**	0,0929	**3,03%**
4	1	1	2	4-5-6	3	0,0992	0,0972	**2,02%**	0,0952	**4,03%**	0,0976	**1,61%**	0,0959	**3,33%**

3	1	1	1,9	4-5-6	2	0,0982	0,0982	**0,00%**	0,0978	**0,41%**	0,0948	**3,46%**	0,0948	**3,46%**
3	1	1	1,8	4-5-6	2	0,0991	0,0991	**0,00%**	0,0987	**0,40%**	0,0961	**3,03%**	0,0961	**3,03%**
3	1	1	1,7	4-5-6	2	0,1001	0,1	**0,10%**	0,0996	**0,50%**	0,0975	**2,60%**	0,0975	**2,60%**
3	1	1	1,6	4-5-6	2	0,1014	0,1009	**0,49%**	0,1005	**0,89%**	0,0989	**2,47%**	0,0989	**2,47%**
3	1	1	1,5	4-5-6	2	0,1028	0,1021	**0,68%**	0,1017	**1,07%**	0,1003	**2,43%**	0,1003	**2,43%**

CHAPTER 5

Conclusion

In our study we decide on the pricing and inventory strategies for perishable products with uncertain demand and shelf life. First, we develop approaches by using both dynamic programming and analytical tools, then we investigate properties of models and prove some observations that are valid for the equations we created. We analyze how our expected profit and order amount react to changes in unit price, ordering cost and unit cost and we visualize them by graphs.

We observed that with the lead of results in dynamic programming we can make our decision with the analytical model structure which easier to solve. The model with dynamic variable price is superior to all other models because it has no constraints for solution. For constant price model, analytical model is faster and easier to solve, so for companies that have millions of SKU's can prefer analytical model. Heuristics are developed by considering time constraints, so for huge companies, it is easily applicable and results are close enough to dynamic model.

We apply our models to daily observations and gather results for each of dynamic and analytical models with variable and constant price. Because of the non-convexity of the value function, it was not possible to find an optimal solution mathematically; therefore we develop heuristics to have a solvable approach in daily life strategic decision problems for perishable products. We

observe results for the demand function we create for the variable price problem and run the model for both heuristic and dynamic solutions and found the deviations between them.

For future research, assumptions may be limited and more than one product can be considered in same time. Substitute effect of customers may be added to the demand structure and how it affects the total profit can be observed. Salvage value and sunk costs are other addable factors to the problem, in a scenario that expired products can be sold and there is a cost burden for the expired and non-sellable products.

All in all, different from the literature we investigate optimal strategy of pricing and inventory for having maximum profit in infinite horizon, we show that our heuristics are easily applicable and results are close to optimal solution.

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