

**REPUBLIC OF TURKEY
YILDIZ TECHNICAL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES**

**FMCW LPI RADAR SIGNAL DETECTION AND PARAMETER
EXTRACTION METHODS BASED ON WIGNER HOUGH
TRANSFORM**

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TRANSFORM**

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LIST OF SYMBOLS

Ω	radial frequency
T_m	modulation time
f_c	center frequency
B	bandwidth
d	distance between signal components
T_D	dwel time
T_C	coherent integration time
γ	non-coherent integration efficiency
T_I	non-coherent integration time
δ	power ratio
dc	duty cycle
B	transmission modulation bandwidth
ΔR	range resolution
τ	time lag of the signal with respect to itself
M	FFT length
N	number of signal samples
T_{obs}	observation time
f_s	sampling frequency
Th_{WD}	WVD threshold
Th_{XWD}	WVD threshold for XWHT
ζ	detection threshold
θ	chirp-rate
P_{CRE}	chirp rate estimation performance
P_{fa}	probability of false alarm
P_d	probability of detection

LIST OF ABBREVIATIONS

ADC	Analog to Digital Converter
DCR	Digital Chirp Rate
ESR	Electronic Support Receiver
EW	Electronic Warfare
FMCW	Frequency Modulated Continuous Waveforms
FrFT	Fractional Fourier Transform
FT	Fourier Transform
GLRT	Generalized Likelihood Ratio Test
HT	Hough Transform
LFM	Linear Frequency Modulation
LPI	Low Probability of Intercept
MC	Monte-Carlo
MWHT	Modified Wigner Hough Transform
QMFB	Quadrature Mirror Filter Bank
RAT	Radon Ambiguity Transform
RCS	Radar Cross Section
RT	Radon Transform
RWR	Radar Warning Receiver
STFT	Short Time Fourier Transform
TF	Time-Frequency
VLP	Ventricular Late Potentials
WVD	Wigner Ville Distribution
WHT	Wigner Hough Transform
WHRT	Wigner Hough Radon Transform
XWVD	Cross Wigner-Ville Distribution
XWHT	Cross Wigner Hough Transform

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ABSTRACT

FMCW LPI RADAR SIGNAL DETECTION AND PARAMETER EXTRACTION METHODS BASED ON WIGNER HOUGH TRANSFORM

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PhD. Thesis

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With more Low Probability of Intercept (LPI) radars coming into use, the signals of interest are changing at a rapid pace. Electronic Support Receivers (ESR) currently in use are not well optimized for LPI radar detection. The techniques used by conventional ESRs either cannot detect LPI waveforms or they can detect them only from a range much less than LPI radars' instrumented range, and this gives LPI radar platforms a range advantage. ESRs used for LPI radar detection shall use; time-frequency (TF) signal processing techniques, correlation techniques and algorithms to overcome the advantages of LPI radars.

LPI radars use different waveforms for different purposes of the radar. In this thesis we focused on Frequency Modulated Continuous Waveforms (FMCW). Wide-band linear FMCW is one of the most important and common waveform structures used to achieve LPI operations. It is important for ESRs to autonomously detect and analyze FMCW and other LPI modulations due to their time-critical mission.

In this thesis, TF space and parameter space are used for FMCW signal detection and parameter extraction. TF space is derived by calculating the Wigner Ville Distribution (WVD) of signal samples and parameter space is derived by applying Hough / Radon (H/R) transform to TF space points that exceed TF threshold. This combination of WVD and H/R transform (WHT) spaces makes it possible to redefine the problem of detecting FMCW components in WVD as detecting the high intensity points in parameter space. The detection decision is given whenever the amplitude of any point in parameter space exceeds the detection threshold.

WHT based methods in literature are known to be computationally expensive since they use the entire WVD space and calculate HT for all the possible chirp rates. LFM signals above or below certain chirp rates are useless to detect any kind of real targets. Calculating the HT for only a limited set of chirp rates and for WVD pixels exceeding the threshold will reduce the required processing power and complexity.

In this thesis, new WHT methods that reduce the complexity yet keeping detection and parameter extraction performance above a certain level are proposed. The performance of proposed WHT based detection and parameter extraction algorithms is compared in terms of transform speed, parameter extraction and detection performance. Also, the effect of chirp rate on FMCW signal detection using proposed WHT based methods is analysed. Following the chirp rate effect analysis, digital chirp rate adaptation method is proposed to increase WHT based methods' detection performance.

The analyses provided throughout the thesis show that the proposed methods are candidates that can improve ESRs automatic signal detection and analysis capabilities with their speed and performance.

Key words: FMCW, Hough Transform, Electronic Support Receiver, Wigner Ville Distribution, Cross-term.

WIGNER HOUGH DÖNÜŞÜMÜ TABANLI FMCW YOD RADAR TESPİTİ VE PARAMETRE ÇIKARTIMI YÖNTEMLERİ

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Elektronik ve Haberleşme Anabilim Dalı

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Her geçen gün daha fazla Yakalanma Olasılığı Düşük (YOD) radarın kullanılmaya başlamasıyla, sinyal türleri de hızlı bir şekilde değişmektedir. Geleneksel ED sistemleri YOD radarları tespit edemezler ya da ancak radarın sınırlandırılmış menziline çok daha az bir menzilden tespit edebilirler. Bu durum YOD radarlara büyük bir menzil avantajı kazandırır. YOD radar tespiti için kullanılan ED sistemlerinin; zaman-frekans (ZF) işaret işleme teknikleri, korelasyon teknikleri ve YOD radarların diğer avantajlarının üstesinden gelecek algoritmalara sahip olmaları gerekir.

YOD radarlar farklı fonksiyonları için farklı dalgaformları kullanırlar. Bu tez kapsamında frekans modülasyonlu sürekli dalgaformları (FMSD) üzerinde durulmuştur. Geniş band lineer FMSD, YOD fonksiyonlarını elde edebilmek için kullanılan en önemli ve yaygın dalgaformu yapılarından biridir. ED sistemlerinin zaman-kritik görevlerinden dolayı FMSD ve diğer YOD radar modülasyonlarını otomatik olarak tespit edip analiz edebilmeleri önemlidir.

Bu tezde FMSD işaret tespiti ve parametre çıkartımı için ZF uzayı ve parametre uzayı kullanılmıştır. ZF uzayı, işaret örneklerinin Wigner Ville Dağılımı (WVD) hesaplanarak elde edilmiştir. Parametre uzayı ise ZF uzayında, ZF eşiğini geçen noktalara Hough / Radon (H/R) dönüşümü uygulanarak elde edilmiştir. WVD ve H/R dönüşümünden oluşan bu kombinasyon (WHD) sayesinde WVD uzayında FMSD bileşenlerini tespit etme problemi, parametre uzayında yüksek genlikli noktaların bulunması problemi haline dönmüş olur. Tespit kararı, parametre uzayındaki bir noktanın tespit eşiğini geçmesi üzerine verilir.

Literatürdeki WHD temelli metodlar, tüm WVD uzayını kullandıkları ve mümkün olan tüm cıvıltı oranları için Hough dönüşümünü (HD) hesapladıkları için hesaplama açısından maliyetlidirler. Belirli cıvıltı oranının üzerindeki ve altındaki FMSD işaretleri gerçek hedeflerin tespiti için kullanılamazlar. HD'yi sadece belirli limitlerdeki cıvıltı oranları için ve WVD eşiğini aşan noktalar için hesaplamak, ihtiyaç duyulacak işlemci gücünü ve karmaşıklığı azaltacaktır.

Bu tez kapsamında; işaret tespiti ve parametre çıkartımı performansını belirli bir seviyenin üstünde tutmak kaydıyla, karmaşıklığı azaltacak yeni WHD metodları önerilmektedir. Önerilen WHD temelli işaret tespiti ve parametre çıkartımı metodlarının dönüşüm uzayı oluşturma hızı, parametre çıkartma ve tespit performansları karşılaştırılarak analiz edilmiştir. Ayrıca cıvıltı oranının, önerilen WHD temelli metodların tespit ve parametre çıkartma performansı üzerindeki etkileri de incelenmiştir. Cıvıltı oranı etkisi analizinden sonra, WHD temelli metodların tespit performansını arttırmak üzere sayısal cıvıltı oranı uyarlama metodu önerilmiştir.

Tez kapsamında yapılan analizler, önerilen metodların sağladığı hız ve performans özellikleri nedeniyle, ED sistemlerinin otomatik işaret tespiti ve analiz kabiliyetlerini geliştirmeye aday metodlar olduğunu göstermiştir.

Anahtar Kelimeler: FMSD, Hough Dönüşümü, Elektronik Destek Sistemi, Wigner Ville Dağılımı, Çapraz Terim.

1.1 Literature Review

Military radar systems need to detect before being seen in order to gain an operational superiority over the opponent's non-cooperative intercept receiver [1]. However, most of the conventional pulsed radar systems are easily detected by advanced electronic support receivers (ESR) and radar warning receivers (RWR) due to their use of high power transmissions. Therefore frequency modulated continuous waveforms (FMCW) and other low probability of intercept (LPI) radar waveforms are widely used in advanced radar systems because of their low power and wide-band properties [2].

Considering the detection (interception) of radar signals; equipments recording the intercepted radar signals usually record the changes in the signal vs. time. Therefore in the analysis of these intercepted signals, time domain representation and frequency spectrum, derived by Fourier Transform (FT), are the most common signal representations. For the detection of FMCW signals; different techniques have been proposed in frequency-domain, time-domain and joint time-frequency domain. Adaptive time domain methods based on recursive least squares and least mean square algorithms showed poor performance under low SNR conditions [3]. On the other hand, if the modulation parameters of the searched waveform are known a-priori; matched filters followed by a threshold comparison can be used as an optimal time-domain detector in white Gaussian noise environments [4]. In the matched filter technique, the linear frequency modulated (LFM) waveform is passed through a matched filter to derive the autocorrelation function. The performance of the matched filter techniques can be determined by; Neyman-Pearson criterion or likelihood ratio test for deterministic signals, and generalized likelihood ratio test for multiple deterministic

signals [5]. The main drawback of the matched filter techniques is the decrease in the peak value of the autocorrelation function when there is a mismatch between the filter and the signal's chirp rate.

In the frequency domain; FT is the projection of the signal on to infinite length $e^{-j\Omega t}$ waves (where t denotes time and Ω is the radial frequency). The frequency spectrum of the signal provides information about the frequency components of the signal along with its phase and amplitude. But, the spectrum does not show how the frequency content of the signal changes with time. To analyze the non-stationary signals, whose frequency content change with time, single dimensional signal representations do not suffice, we need two dimensional representations. Although we can gather all the signal information we need, using time and frequency representations separately, a combined representation will be more illustrative and compact. Time-Frequency (TF) signal processing techniques show the energy density of the signal both in time and frequency domain simultaneously [6] and they allow us to determine if a signal is multicomponent or not. Therefore, they are most suitable in the analysis of FMCW and other non-stationary signals. Figure 1.1 is presented below to compare the level of information provided by a TF technique, Wigner-Ville Distribution (WVD), with that of a single dimensional signal representation, frequency spectrum, of an example FMCW signal.

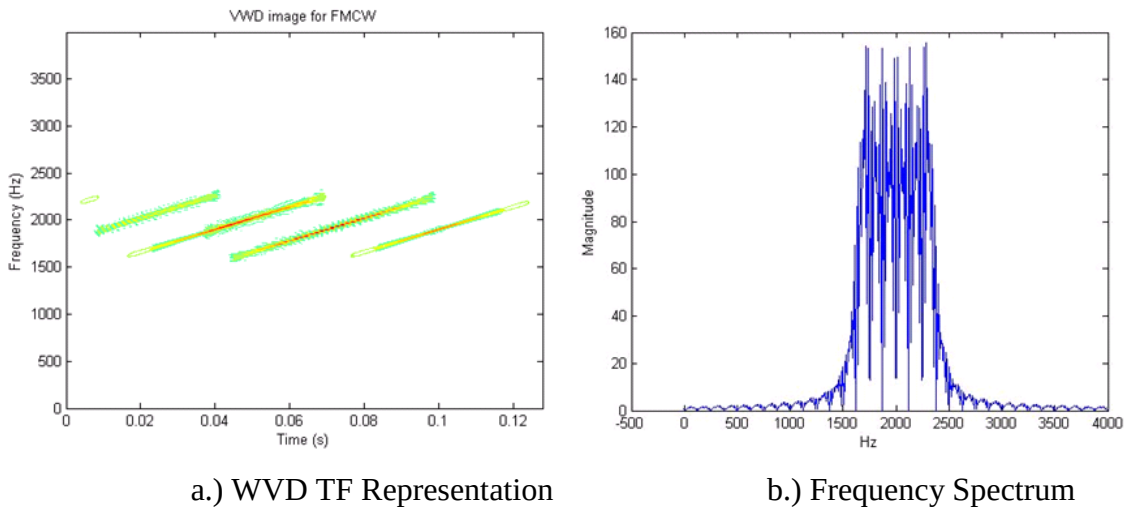


Figure 1.1 FMCW signal

Some FMCW detection and parameter extraction algorithms utilizing TF distributions have been conceptually presented in [7], [8], [9], [10]. In these works, WVD was chosen as the TF distribution since; it provides the marginal conditions and gives the highest energy concentration in the TF plane [11]. The implementation details of these

algorithms will be presented in the following chapters. WVD is widely used in detection and estimation of LFM signals [12], the optimality of WVD for detection of LFM signals was explained in [13] and statistical analysis of WVD with applications on wideband signal detection was performed in [14].

WVD differs from other known linear techniques with its high resolution both in time and frequency domain. Most of the time, a mere visual inspection of the WVD of the intercepted signal is enough for the detection decision. However, sometimes the background noise level makes it difficult to visually detect a signal in a noisy intercept's WVD. In the study of radar, signal detection and communication systems, the assumption of white Gaussian noise is frequently used to simplify their analysis. In [13], Kay stated that the best method to detect LFM signals in complex Gaussian noise is to integrate the WVD of the intercept along the instantaneous frequency line of the signal. In his work, Kay paved the way to use WVD along with Hough / Radon type transforms for the detection of LFM signals by stating that; if the position and sweep rate of the linear chirp is not known, to perform Generalized Likelihood Ratio Test (GLRT) one should integrate WVD along all the possible lines in the TF plane and the resulting peak value shall be compared against a threshold.

In [15], Barbarossa followed the footsteps of Kay with his Wigner Hough Transform (WHT) and stated that; by the combination of WVD and Hough Transform (HT), one can transform signals from time domain to a parameter space in which detection and parameter extraction of signals can be performed simultaneously.

In the literature there are many other novel approaches for LFM signal analysis and detection; a combination of Teager Huang and HT was used for analysis of multicomponent LFM signals in [16]. The results of the method were compared with pseudo-WHT. Exploiting sparsity and compressive sensing applications for LFM signal analysis are also relatively new research areas. A sparse detection application to FMCW radar signals was investigated in [17]. In [18], chirplet transform was used for detection and segmentation of FMCW radar signals and in [19], multi-channel digital deramping method was investigated for LPI signal detection. Radon Ambiguity Transform (RAT) is another effective TF analysis method. This method was used for efficient signal parameter estimation of wideband signals in [20] and it was used in combination with second-order difference to improve detection probability of LFM signals in low SNR cases [21].

After this general literature review, each chapter in the thesis includes a more focused literature review about the topic of that chapter.

1.2 Objective of the Thesis

ESRs require minimum operator interaction and autonomous methods for the detection and analysis of radar signals due to their time-critical missions. ESRs currently in use are not well optimized for LPI radar detection. There are many types of legacy analog intercept receiver architectures such as square-law, wideband and channelized receivers. However, interception of LPI radar signals requires sophisticated receivers that use TF signal processing, correlation techniques and algorithms to overcome the processing gain of the LPI radar. Therefore, almost all current ESRs are designed as digital receivers to provide these capabilities.

Radio receivers that perform the signal processing after the received signal is digitized by the analog-to-digital conversion (ADC) process are called as digital receivers. Figure 1.2 is presented here to show the general steps of electronic warfare (EW) using a conceptual digital ESR architecture.

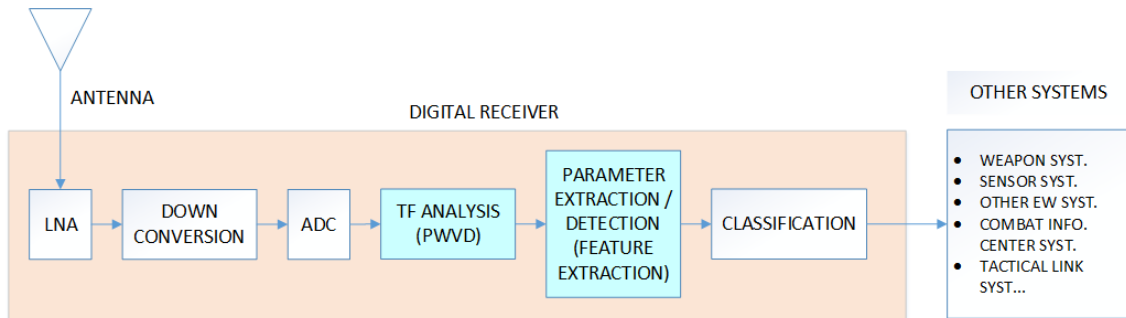


Figure 1.2 Conceptual digital ESR block diagram

The signal is received by the antenna and then it is amplified by an LNA (Low Noise Amplifier). The signal frequency is usually downconverted to an intermediate frequency before digitization by an ADC. **Our goal in this thesis is to propose candidate methods to be used in ESRs after this step. We focus on TF signal processing (PWVD) and image processing (HT) methods to detect and extract the FMCW waveform parameters.** Our focus areas are shown as “TF ANALYSIS” and “PARAMETER EXTRACTION” boxes in Figure 1.2. Once the signal parameters are extracted, these are used to sort the signals. Although it is outside the scope of this thesis, parameter extraction is followed by “classification” process. The result of

classification is sent to other systems. These other systems may use the classification result to fuse it with other classification results, to assign another sensor on the signal or even to launch a guided missile.

1.3 Hypothesis

In this work we suggest that; we can create new WHT methods for FMCW signal detection and parameter extraction, we can make use of WVD cross-terms in HT space to detect FMCW signals, we can perform FMCW signal parameter extraction using Cross Wigner Hough (XWHT) transform cross-term vector and WHT signal component vector, we can define the “Digital Chirp Rate” concept and use it to increase our FMCW signal detection performance.

1.4 Principal Contributions and Thesis Outline

In chapter 2 we provide a review of LPI radar concept. We explain the LPI radar along with its advantages, characteristics and waveforms. We also give common fielded LPI radar examples and summarize the TF analysis approach for LPI radar detection

Chapter 3 explains the WVD in detail. We elaborate on WVD cross-terms since one of the main proposals in this thesis depends on utilizing them. We provide explanations of WVD properties and use of WVD as an FMCW TF analysis tool. When using WVD as a TF analysis tool, time-frequency resolution and WVD aliasing considerations shall be taken into account. TF resolution and aliasing considerations have only been partly mentioned in the thesis. But we are working on these issues in parallel studies [22].

Chapter 4 explains the general approach for Wigner Hough based FMCW detection and summarizes the legacy Hough and Radon transforms. Also in this chapter we propose; WHT [8], Wigner Hough-Radon (WHRT) [9], Modified Wigner Hough (MWHT) [10] and XWHT transforms for FMCW signal detection and new FMCW signal parameter extraction methods based on these transforms. In these works we aim to estimate the critical FMCW signal parameters like center frequency (f_c), modulation time (T_m) and bandwidth (B) of the signal components.

In WHRT we aim to overcome the deficiencies of WHT and Wigner Radon Transform (WRT) [7] by combining the advantageous parts of each. The MWHT method makes use of the oscillating characteristic of the cross terms intensity values and keeps them

from building up in the transform space. XWHT method also makes use of the WVD cross-terms, instead of trying to get rid of them the method uses only a negative threshold value to create a HT space consisting only of cross-terms. As it is explained later, the cross-terms in WVD have the same chirp rate as the signal components. So, the method detects cross-terms and their chirp-rate, and declares FMCW signal detection with the same chirp-rate.

Parameter extraction methods using WHT, WHRT and MWHT process the transform space vector with signal components. In the XWHT parameter extraction method; once the detection and chirp-rate estimation of the FMCW signal is performed, HT is calculated for only at the chirp-rate angle to derive a vector, than XWHT vector where the detection is performed is subtracted from this vector. Considering that HT vector is the same vector as XWHT detection vector with signal components, the subtraction will cancel out the cross terms. The resulting cross-term free and smooth vector can then be used for parameter extraction.

Chapter 5 defines the WVD threshold for Hough based transforms and compares the transform methods in terms of parameter extraction performance and transform speed, assuming detection is declared.

In chapter 6, we first perform the detection analysis of FMCW signals by XWHT. Then we compare the performance of WHT based detection methods. The effect of chirp rate on FMCW signal detection using XWHT, MWHT and WHT is also analysed. After the chirp rate effect analysis, we propose digital chirp rate adaptation method [23], [24] to increase WHT based methods' detection performance.

After these detailed analysis studies, chapter 7 concludes the work by stating that these proposed methods are candidates that can improve ESRs automatic signal detection and analysis capabilities with their speed and performance.

CHAPTER 2

LPI RADAR

In the military operational area there are radars trying to detect targets without being detected by ESRs and there are threats trying to operate without being detected by the radars. Radar – target – ESR geometry in an example operational scenario is presented in Figure 2.1 to explain the LPI radar idea. Considering the platform carrying the radar in the figure, different methods can be used to reduce the platform’s probability of detection by attack aircraft’s (“TARGET” in Figure 2.1) radar including; platform surface shaping techniques and using the advances in material science. Other than these, intentional and unintentional emissions may be controlled to try and reduce platform’s probability of being detected by the ESR (“INTERCEPT RECEIVER” in Figure 2.1). Although the use of passive sensors seems like an easy solution to reduce emissions, it is necessary to detect, track and classify targets to gain situational awareness, and this is better achieved with active sensors. Deterministic waveforms used by conventional active radars are easily detected and classified by ESRs, therefore it is necessary to consider the probability of detection by ESRs when using active sensors.

The scenario depicted above, forced the radars to have LPI properties. The intercept receiver and the radar together as a system bring out the LPI property. The main goal of an LPI radar is to escape detection by the ESR. However, the LPI performance depends on both the radar and the ESR. LPI radars implement power management, use low transmit power, have low side lobe antenna patterns, use wideband and enhanced modulation techniques to avoid being detected by ESRs [1]. But even with these properties, a radar might be LPI for one type of ESR, but not for another type of ESR. Therefore when claiming a radar as having LPI properties, it is best to give the descriptions of the types of ESRs for which the radar is claimed to be LPI. Also, it has

to be remembered that no radar can completely escape detection, since after a certain range between the ESR and the radar, the ESR detection threshold is exceeded [25].

In Figure 2.1, ESR's function is to alert the attack aircraft when it is targeted by the radar. Interception of the radar transmissions by the ESR before the aircraft is detected by the radar, gives the attack aircraft advantage for any kind of offensive actions. On the contrary, if the radar detects the attack aircraft before the ESR detects its transmissions, the radar platform gains advantage to deploy offensive actions or to reduce its vulnerability to attacks.

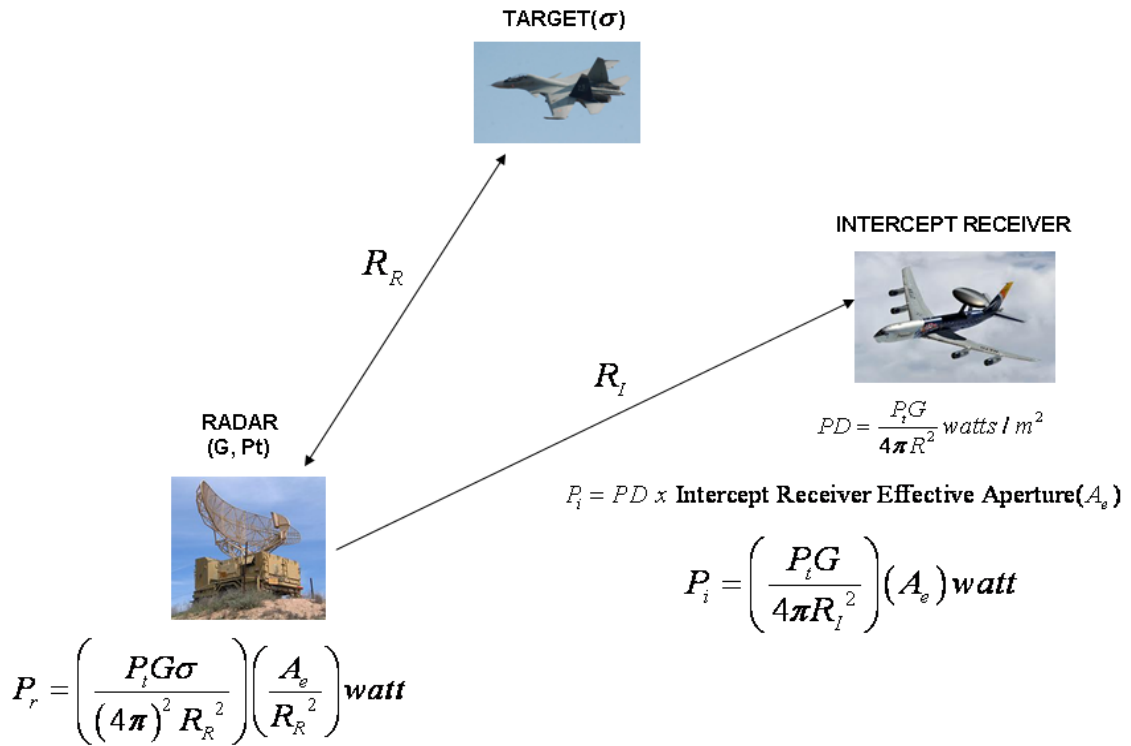


Figure 2.1 Operational area: radar, intercept receiver and target

In Figure 2.1, radar detection range (R_R) shall be bigger than intercept receiver range (R_I) for the radar to detect before being detected. P_i shows the power at the ESR and P_r shows the power at the radar returning from the target. The equation for P_i is also called one-way RF equation. The basic radar range equation is given by (2.1) and the result of this equation changes depending on random variables of radar cross section (RCS), noise and clutter [26].

ESRs are affected by clutter and noise but not by RCS. As can be seen from Figure 2.1, the power at ESR is at least R^2 times more than the power at radar. So, if we consider only the tx power, ESRs have range advantage over radars.

$$\frac{S}{N} = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 k T_0 B F L R^4}$$

P_t = Peak Tx power, G = Antenna gain, λ = Wave length,
 σ = Radar Cross Section, k = Boltzmann const.,
 T_0 = Temperature, B = Noise bandwidth, F = Noise factor,
 L = System losses, R = Range

(2.1)

2.1 LPI Radar Advantage

We've mentioned about the range advantage of the ESR, here we try to explain the "LPI advantage". We can take R_R as the range that the radar can detect the target and, R_I as the range that the radar signal can be intercepted by the ESR. The ratio of these two ranges is given as [27];

$$\frac{R_I}{R_R} = R_R \left[\frac{4\pi}{\delta} \cdot \frac{1}{\sigma} \cdot \frac{G_{TI} \cdot G_I}{G_T \cdot G_R} \cdot \frac{L_I}{L_R} \right]^{1/2} \quad (2.2)$$

Where:

$$\delta = \frac{P_I}{P_R} = \frac{\text{power required by ESR to detect a signal}}{\text{power required by radar to detect a target}} \quad (2.3)$$

G_{TI} : radar antenna gain towards ESR

G_I : ESR antenna gain towards radar

G_R : radar antenna gain towards target

L_I, L_R : losses in the ESR and radar

In (2.2), the LPI advantage is mainly related with δ . This power ratio can be given in terms of the products of the minimum SNR required at the input of intercept receiver (SNR_{inI}) and radar receiver (SNR_{inR}) and noise level in the input bandwidth of each receiver ($B_{I,R}$) [28];

$$\delta = \frac{(SNR)_{inI}}{(SNR)_{inR}} \cdot \frac{kTB_I F_I}{kTB_R F_R} \quad (2.4)$$

We know that for any receiver;

$$SNR_{out} = SNR_{in} \cdot G_p, \quad G_p : \text{processing gain} \quad (2.5)$$

Assuming that similar detection performances are required for each receiver, we can take $SNR_{outI} = SNR_{outR}$. With some simplification and assuming that noise factors of radar and ESR are also equal, (2.4) can be rewritten as;

$$\delta = \frac{(SNR)_{outI}}{(SNR)_{outR}} \cdot \frac{B_I F_I}{B_R F_R} \cdot \frac{G_{pR}}{G_{pI}} \cong \frac{B_I}{B_R} \cdot \frac{G_{pR}}{G_{pI}} \quad (2.6)$$

So, the ratio of radar receiver sensitivity and ESR sensitivity is related with the pre-detection bandwidths and processing gains of the receivers ($G_{pR, pI}$). The radar receiver bandwidth is chosen in accordance with the required range resolution, and the ESR bandwidth is generally much wider since the ESR is supposed to intercept a variety of signals. We get the maximum radar processing gain if we perform coherent integration during the dwell time, T_D . This gives us $G_{pR} = B_R T_D$. If the radar uses both coherent and non-coherent integration, the radar processing gain is reduced by the following formula,

$$G_{pR} = \frac{B_R}{T_C} \cdot \left(\frac{T_D}{T_C} \right)^\gamma \quad (2.7)$$

where T_C is coherent integration time, and γ is the non-coherent integration “efficiency” which is between 0.5-0.8 [29].

On the ESR side we try to perform non-coherent integration for a time T_I which is close to T_D . This gives the ESR processing gain as

$$G_{pI} = (B_I T_I)^\gamma \quad (2.8)$$

We can see the LPI radar advantage if we combine (2.6), (2.7) and (2.8).

$$\delta = (B_I T_C)^{1-\gamma} \left(\frac{T_D}{T_I} \right)^\gamma \quad (2.9)$$

If the ESR is able to make $T_I = T_D$ and $B_I = B_R$ then δ can be expressed as the time-bandwidth product of the radar waveform raised to the $(1-\gamma)$ power.

$$\delta_{\min} = (B_R T_C)^{1-\gamma} \quad (2.10)$$

Since B_R is determined by the required range resolution, radars usually increase the integration time to increase δ and to decrease transmit power [28]. This means radars have the control over the power ratio δ . As can be seen from the above formula, the best we can get on the ESR side is when we can match the radar receiver bandwidth and non-coherent integration time.

2.2 LPI Radar Characteristics

The LPI capability is required due to modern ESRs increased capability to detect and locate radar emitters. The detection of the radar is usually followed by electronic attack and / or attack by anti-radiation missiles under warfare conditions. The following properties separate LPI radars from conventional radars and combinations of these properties help the LPI radar to hide its emissions from ESRs.

ESRs use different techniques to determine scan properties, angle of arrival, f_c , T_m , and B . These properties can be used to identify LPI radars, and randomly changing any of these parameters provide confusion to the ESR [1].

2.2.1 Low side-lobe antennas

ESRs can detect the emissions of a radar from the side lobe structures of the antenna pattern. Therefore it is necessary for an LPI radar to have a transmit radiation pattern with low side lobes. With phased array antenna apertures the excitation parameters of each antenna element can be controlled separately, and this makes it possible to apply a desired tapered excitation with low side lobes. Generally speaking -20 dB side lobe level is acceptable, but if we are talking about LPI radars, ultra-low side lobe levels at -45 dB are required [1].

2.2.2 Random antenna scan patterns

Radars need to move their antenna beams in a particular pattern to search and / or track targets. This systematic movement is called scanning. Antenna scan characteristics determine the total time the radar looks towards the ESR. LPI radars use different scan methodologies to confuse ESR identification process, even if detection occurs.

2.2.3 LPI waveform considerations

LPI radars need to use high duty-cycle, wideband, low peak power emissions and continuous waveforms with large time-bandwidth products. Conventional radars that use coherent pulse trains usually have high peak-to-average power ratio. The detection characteristics of the radar are related with the average power. If the radar is using short pulses to get high range resolution, it also needs to use high peak power to get high average power. However, the high peak power transmissions can easily be intercepted by non-cooperative ESRs. The duty cycle (dc) is a measure of the percentage of time energy is emitted from the transmitter and it can be expressed by one of the following two ratios [1];

$$dc = \frac{\text{average power}}{\text{peak power}} = \frac{P_{avg}}{P_t} \quad \text{or} \quad dc = \frac{\text{pulse width}}{\text{pulse repetition interval}} = \frac{\tau}{PRI} \quad (2.11)$$

A typical duty cycle value for conventional navigation radars is $dc = 0.001$. For modulated CW signals $dc = 1$, and this allows to use a lower transmit power to get the same detection performance as the coherent pulse train radar. LPI radar waveforms spread the radar signal energy over a wide band and time to reduce transmit peak power (on the order of 1W). In Figure 2.2 we can see a comparison of a pulse radar waveform and a CW radar waveform.

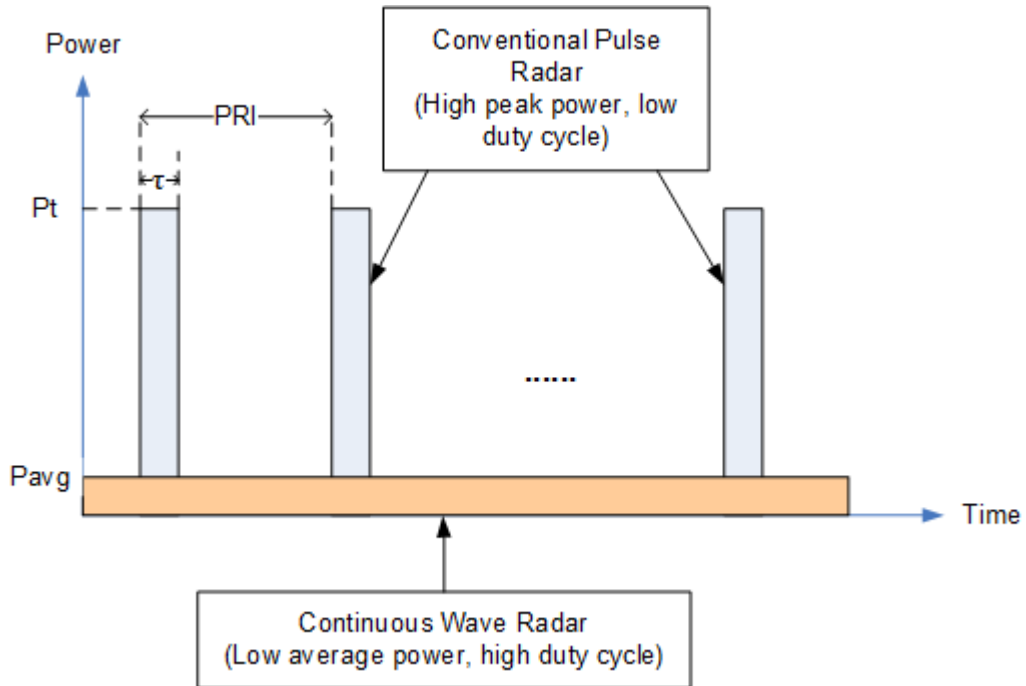


Figure 2.2 Pulse radar waveform vs. CW radar waveform

Conventional ESRs are generally tuned to detect pulse and CW waveforms, and LPI radars try to avoid interception by mismatching their waveform to these waveforms [25]. Since a single tone CW signal is easily detected with narrowband ESRs and cannot provide range information, LPI radars use periodically modulated CW signals. These signals provide us with large bandwidths and small resolution cells, and they are well suited for pulse compression. Pulse compression is the most common LPI radar technique and it can be accomplished by [30];

- Linear / nonlinear frequency modulation,
- PSK Techniques,
 - Binary PSK,
 - Barker Codes
 - Concatenated Barker Codes
 - Polyphase Codes
 - Frank Codes
 - P1, P2, P3, P4 Codes
 - Polytime Codes
 - T1(n), T2(n), T3(n), T4(n) Code
- FSK Techniques,
 - Costas Codes
 - Hybrid PSK/FSK Technique
 - Target Adaptive PSK/FSK Technique
- Noise modulation.

Pace made a thorough analysis of these waveforms in [1]. These modulation techniques can also be used to group LPI radars [31].

2.2.4 Adaptive power management

The main goal of power management is to dynamically control the transmitter power to maintain the minimum required SNR. As mentioned before, use of passive sensors is the easiest way to control emissions, but to achieve situational awareness we need active sensors. Therefore LPI radars need to manage the power they radiate because this also affects the distance at which the signal can be detected by the ESR. The emissions shall also be limited in time. With the use of wideband, pulse compression CW waveforms mentioned above, LPI radars only need to transmit a few watts.

2.2.5 Carefully selected carrier frequency

The frequency regions for maximum atmospheric attenuation are presented in Figure 2.3. The LPI radar RF frequency can be chosen within these attenuation regions to mask the transmit signal and avoid interception by a hostile ESR. The radar target detection will be influenced by the attenuation as well, therefore this technique is limited to short range systems. Choosing the LPI radar carrier frequencies outside of the common ESR working band (0.5 - 20 GHz) is another option [32].

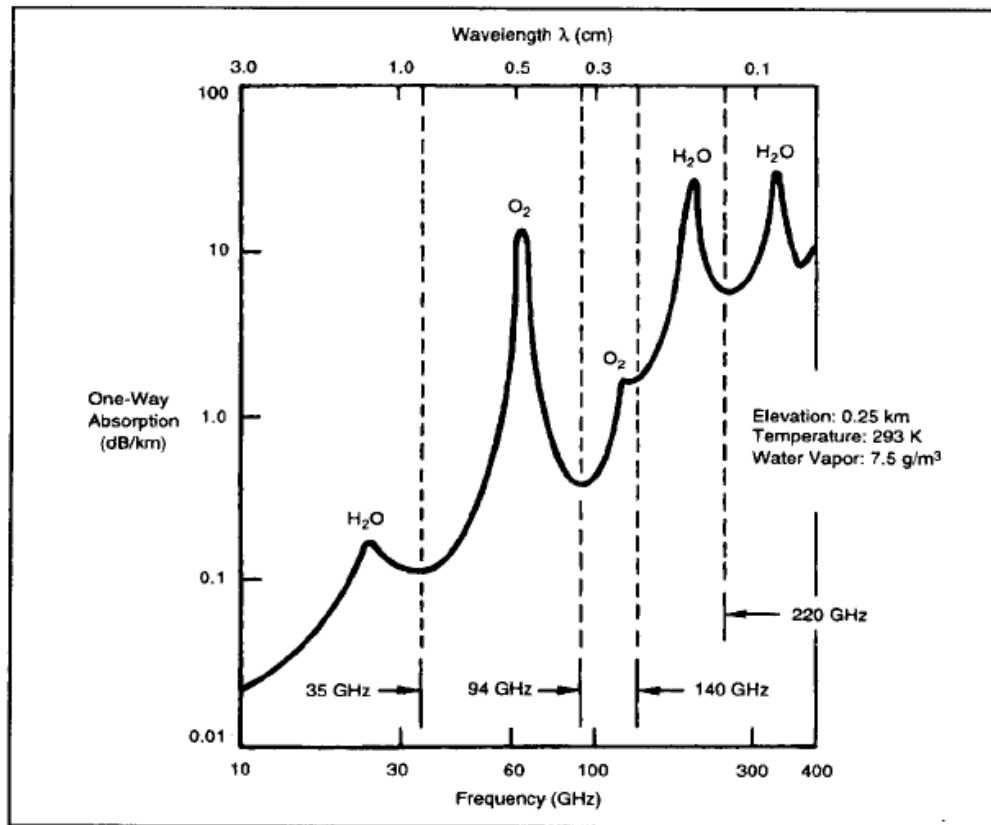


Figure 2.3 Maximum atmospheric absorption in the millimeter wave spectrum [33]

2.2.6 High processing gain

LPI radars get high processing gain because they know the signal modulation and bandwidth they are expecting. The intercept receiver does not know these parametric details a-priori and must accept a wide range of signals from a wide range of frequencies and make detailed analysis to identify the type of signal it is receiving [34].

2.3 FMCW LPI Radar Waveform

Different LPI waveform techniques mentioned in section 2.2.3 can be used together or separately for different purposes of the radar. In this thesis we focused on FMCW LPI radars. Wide-band linear FMCW is one of the most important and common waveform structures used to achieve LPI operations because of its ease in implementation, its versatility, and its suitability for fast FT processing to obtain range measurements [35]. Other advantages of FMCW waveforms are; their high time bandwidth product makes them very resistant to interception by ESRs, their deterministic nature provides that the form of the return signal can be predicted and this helps resistance to jamming since any signals not expected are suppressed. The characteristics of the most popular type of this modulation, triangular FMCW, are presented in Figure 2.4. This waveform consists of positive and negative slope LFM parts.

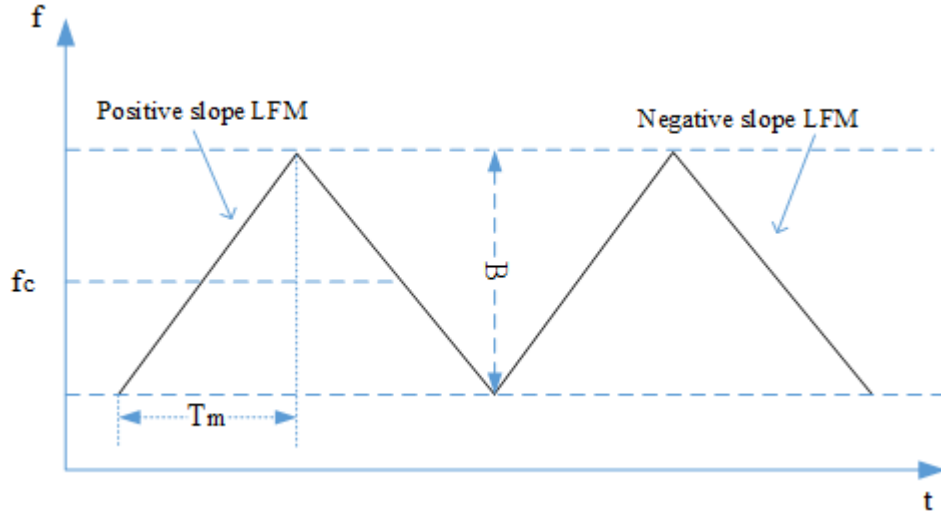


Figure 2.4 FMCW signal

Frequencies of positive and negative parts are given as [36];

$$f_{p,n} = f_c \mp \frac{B}{2} \pm \frac{B}{T_m} t, \quad 0 < t < T_m, \quad t = 0 \text{ otherwise} \quad (2.12)$$

where f_c is the RF carrier frequency, B is the transmission modulation bandwidth and T_m is the modulation period. The transmit signal expressions for positive and negative parts are given as [1];

$$s_{p,n}(t) = a_0 \cos \left(\underbrace{2\pi \left(\left(f_c \mp \frac{B}{2} \right) t \pm \frac{B}{2T_m} t^2 \right)}_{\phi_{p,n}} \right) \quad (2.13)$$

where $\phi_{p,n}$ represents the phase of the transmitted signal. The chirp rate of FMCW signals changes according to the radar's instrumented range and desired range resolution. Although the chirp rate is generally represented by the ratio ($\beta = \frac{T_m}{B}$), we represent it as an angle, defined in chapter 3, in this thesis. Radars tend to use long pulses to make use of average power and they need pulse compression for good range resolution, ΔR ;

$$\Delta R = \frac{c}{2B} \quad (2.14)$$

The waveform modulation bandwidth is chosen according to the desired range resolution and T_m is chosen according to radar's desired maximum detection range. For better doppler resolution, T_m is usually chosen as 2-3 times more than maximum return time, t_d , of the signal.

$$t_d = \frac{2R_{\max}}{c} \text{ sec} \quad (2.15)$$

Our goal in this work is to detect FMCW waveforms and extract their parameters, whereas the goal of radars using FMCW waveforms is to detect targets before being detected by the ESRs. Generally the chirp rate of FMCW signals cannot be too steep or horizontal to be used for realistic range and resolution requirements. Therefore in this work, we assume that the chirp rate of FMCW signals are between $10^\circ - 80^\circ$ and thus the transform spaces will be created between $10^\circ - 80^\circ$.

2.4 Common Commercial FMCW LPI Radars

The first radar defined as LPI is the PILOT radar that was integrated on a Swiss SPICA-II class fast patrol boat in 1987. The research and development efforts continued after the first PILOT radar; Netherlands Signaal company produced FMCW SCOUT [37] and Swiss CelciusTech produced FMCW PILOT MK2 [38]. The FMCW LPI radar market expanded in the following years and many other companies produced LPI radars. Some

examples are; SPN-730 (SELEX) [39], INDERA CX-3AH (RCS) [40], INDERA MX-2/3AH (RCS) [41], ALPER (ASELSAN) [42], ARIES (INDRA) [43] and Broadband 3G/4G (SIMRAD) [44].

Table 2.1 is given here to present some characteristic parameters of real radars to give an idea of what type of signals we are trying to detect on the ESR side. These parameters can be used as a starting point in the design of digital intercept receivers. We can see that, high range resolution is provided for short ranges therefore, we can expect narrower bandwidths as we get far from the radar.

Table 2.1 FMCW LPI radar characteristic parameters

	Freq. (GHz.)	Range Resolution (m.)	Bandwidth (MHz.)	Instrumented Range (NM.)	Modulation Time (us.)
PILOT	X (I) Band	24	6.25	24	296.32
		6	25	6	74.08
		2.4	62.5	2.40	29.63
SCOUT	X (I) Band	--	--	24	296.32
		--	--	12	148.16
		3	50	6	74.08
INDERAX-MX	X (I) Band	72	2.08	20	246.93
		0.9	166.66	0.25	3.08

2.5 LPI Radar Detection: TF Analysis Approach

The techniques used by conventional ESRs either cannot detect LPI waveforms or they can detect them only from a range much less than LPI radars' instrumented range, and this gives LPI radar platforms a big range advantage. Due to wide band characteristics of LPI radars, their detection requires high processing gain on the ESR side. The use of wide band signals spreads the signal energy over a wide band and creates a power spectral density lower than noise level at the input of the ESR. To be able to perform detection, this wide band signal shall be integrated over a long observation time while trying not to integrate the noise as much [45].

ESRs used for LPI radar detection shall use; TF signal processing techniques, correlation techniques and other algorithms to overcome the advantages of LPI radars. These processes require high processing power and storage of high volumes of data. Real-time or near real-time implementation of any of the LPI radar detection algorithms require very intricate design efforts. The hardware implementation considerations of the LPI radar detection methods are not thoroughly investigated in the scope of this thesis however; we have started to work on investigating algorithms to implement the methods

we define in this thesis [46]. The time and frequency resolution requirements, threat emitter characteristic parameters, the mission critical role of the ESR are all important issues when performing the hardware implementation.

As mentioned before, we need TF analysis methods due to the non-stationary nature of the received LPI radar signals. WVD will be explained in detail throughout the thesis; in this section some of the other common TF analysis techniques that are used in LPI radar detection are summarized. More details about TF analysis techniques that can be used in LPI radar signal detection can be found in [1].

2.5.1 Short Time Fourier Transform

In FT, we use complex sinusoidal functions as fundamental functions for signal analysis. Local information about the signal, like sudden changes, spread over all frequencies due to the infinite length of FT. Short Time Fourier Transform (STFT) is used to overcome this problem. To create time relation in FT, $x(u)$ signal is windowed around time t , FT is calculated for the windowed part, and this is repeated for each time t . STFT is represented as;

$$F_x(t, v; w) = \int_{-\infty}^{\infty} x(u) w^*(u-t) e^{-j2\pi vu} du \quad (2.16)$$

here, $w(t)$ is the short time analysis window at $t=0$ and $v=0$. When the signal is multiplied by $w^*(u-t)$, the parts of the signal around $u=t$ are suppressed and so STFT gives the local spectrum of the signal. STFT uses small time windows $w(t)$ to localize the signal in time.

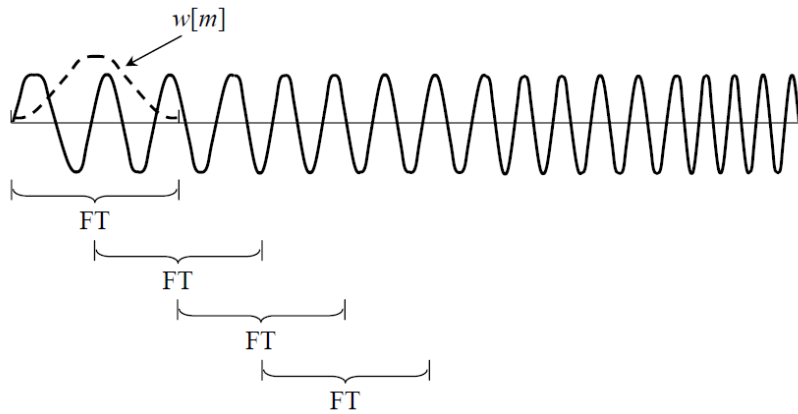


Figure 2.5 Windowing and time-shifting for STFT of a non-stationary signal

Time and frequency resolution of STFT are defined by the length of $w(t)$. To improve time resolution we need a short analysis window, whereas to get better frequency resolution we need a long analysis window to act as a narrow-band filter. However we cannot get good resolution in both domains at the same time due to Haisenberg-Gabor inequality.

If the signal under analysis is shifted in time by t_0 and / or in the spectrum by w_0 the STFT is also shifted by the same amounts in each domain;

$$\begin{aligned} y(t) &= x(t)e^{j2\pi v_0 t} \Rightarrow F_y(t, v; w) = F_x(t, v - v_0; w) \\ y(t) &= x(t - t_0) \Rightarrow F_y(t, v; w) = F_x(t - t_0, v; w)e^{j2\pi v t_0} \end{aligned} \quad (2.17)$$

Signal energy density, spectrogram, can be estimated from the squared modulus of the STFT [47]. The energy of the spectrogram is equal to the energy of the signal, if the energy of the windows is selected to be one.

$$spectrogram_x(t, w) = |STFT_x(t, w)|^2 \quad (2.18)$$

STFT was used as the main TF method before WVD and other TF methods have shown to provide better time-frequency resolutions. STFT is still commonly used since it is implemented easily and it provides the level of resolution required most of the time. STFT was used for LFM signal detection in low SNR cases along with wavelet denoising in [48]. STFT was used at the first step of the LFM signal parameter extraction method proposed in [49]. It was used for a coarse search then in the second stage; the fine search for correct parameters of LFM was implemented in the FRFT domain.

2.5.2 Wavelet Transform and Quadrature Mirror Filter Bank

STFT uses the same window for all frequencies; therefore it gives the same resolution for the whole TF space. It is possible to get better resolution in time or frequency domains by changing the window type. Very short windows can be used to detect discontinuities in the signal or very long basic functions can be used for detailed frequency analysis. Wavelet transform basic functions are derived by shifting or scaling the prototype $h_{ab}(t)$ wavelet as follows;

$$h_{ab}(t) = \frac{1}{\sqrt{a}} h\left(\frac{t-b}{a}\right) \quad (2.19)$$

here, a is a positive real number, b is a real number. When a is big, we get wavelets bigger than the prototype (to be used for low frequencies), when it is small we get wavelets smaller than the prototype (to be used for high frequencies). Wavelet transform is defined by the following formula;

$$X_w(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} h\left(\frac{t-b}{a}\right) x(t) dt \quad (2.20)$$

Once proper basic function set is determined by wavelet techniques, we can use Quadrature Mirror Filter Bank (QMFB) to derive signal properties with high time and frequency resolutions [50]. A basic two-channel QMFB structure is shown in Figure 2.6.

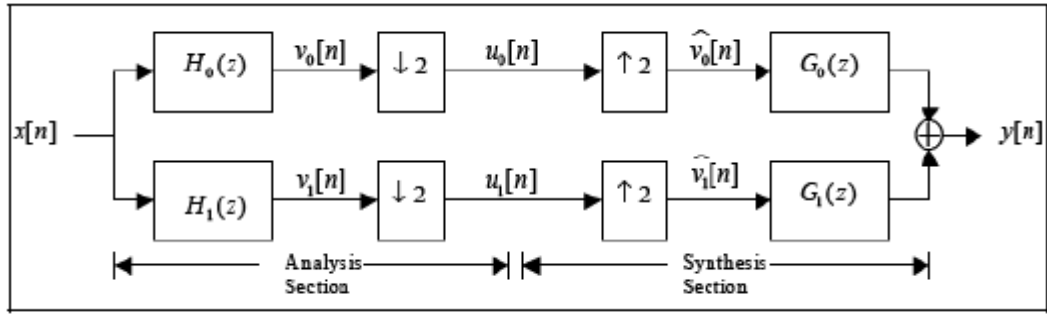


Figure 2.6 Two Channel QMFB structure [After 53]

In another study, the performance of a quadrature mirror filter (QMF) in detecting two LPI waveforms: a triangular FMCW and a P-4 polyphase code CW waveform was investigated in [51]. The results of the study showed that QMF processing can provide good detection of LPI waveforms even in the presence of a moderate amount of noise.

2.5.3 Fractional Fourier Transform

Fractional Fourier Transform (FrFT) is a generalization of FT. FrFT of a signal $x(t)$ can be expressed as;

$$X_g(u) = \int x(t) K_g(t,u) dt \quad (2.21)$$

In the above expression, $K_g(t,u)$ is the kernel function of FrFT, and expressed as;

$$K_{\vartheta}(t,u)=\sqrt{\frac{1-j\cot\vartheta}{2\pi}}\times e^{\left(j\frac{t^2+u^2}{2}\cot\vartheta-jtucsc\vartheta\right)} \quad (2.22)$$

FrFT rotates the signal $x(t)$ in TF space by angle ϑ , and takes it to an intermediate region. When $\vartheta = \pi/2$, FrFT becomes FT [52].

FrFT is one of the most common TF analysis methods for LPI radar signal detection. An analysis of detectors of LFM signals based on FrFT was given in [53]. Another approach was proposed to realize a digital channelized receiver in the fractional Fourier domain for signal intercept applications in [54].

CHAPTER 3

THE WIGNER DISTRIBUTION

The main motivation of TF distributions is to devise a joint function of time and frequency that will describe the energy of the signal accurately in both time and frequency. The behavior of TF analysis methods changes dramatically from one problem to the next and each method has special properties. A TF method may not behave as we desire in all respects or conditions but, it may still be useful for a particular property [55]. The choice of the distribution depends on the nature of the signals to be analyzed and on issues like the mathematical properties required and limitations in computation and storage. A successful application of TF distributions assumes some degree of expertise on the part of the user and prior knowledge about the signal so that the most suitable distribution is selected [55]. As mentioned before, WVD was chosen as the TF distribution in this work since; other than satisfying marginal conditions, WVD gives the highest energy concentration in TF plane and exhibits the non-stationary properties of the signals [11].

WVD was first proposed by Wigner with the motivation to devise a joint distribution that gave the quantum mechanical distributions of position and momentum simultaneously [56]. But it was Ville who introduced WVD into signal analysis with his work in [57]. And in 1980, Claasen and Mecklenbrauker published their comprehensive approach about WVD in their papers [58], [59], [60], and originated many new ideas well suited for TF analysis [61].

3.1 Continuous WVD Representation

The calculation of WVD is based on the use of autocorrelation function. The autocorrelation function compares the signal to its shifted versions.

$$r_{ss}(\tau) = \int s(t)s(t+\tau)dt \quad (3.1)$$

where τ represents the time lag of the signal with respect to itself. As can be seen from (3.1), autocorrelation is independent of time and is a function of the time lag τ . WVD uses a time dependent variation of autocorrelation function $R_{ss}(t, \tau)$. WVD of a signal $s(t)$ can be calculated with the following integral;

$$W_s(t, w) = \frac{1}{2\pi} \int \overbrace{s(t - \frac{1}{2}\tau)s^*(t + \frac{1}{2}\tau)}^{R_{ss}(t, \tau)} e^{-j\tau w} d\tau \quad (3.2)$$

where w is the frequency variable and t is the time variable. As can be seen from (3.2), WVD is bilinear since it uses the signal twice in its calculation. Basically, to calculate the WVD at time t we multiply a part of the signal at a past time by another part of the signal at a future time and we add up the results, the time into the past is equal to the time into the future. In another words, to determine the properties of the signal at time t , we fold the left part of the signal over to the right. Then, the resulting properties become the properties of the signal at time t [61]. We can think of WVD as a kind of STFT using the shifted version of the signal as the window. It compares the information of the signal with its own information at different times.

3.2 The WVD Cross-Terms

Despite all the good properties it has, WVD suffers from the cross-terms it produces. The cross-terms will be explained using the continuous WVD representation at this early part of the thesis because they are mentioned frequently and the main method proposed in this thesis is based on exploiting the properties of the cross-terms.

The cross-term issue of the WVD was first pointed out in [62]. If the signal has more than one component; the signal auto-terms are observed as positive amplitudes along their instantaneous frequency lines in the signal's WVD, the cross-terms are observed at the geometrical midpoint of auto-terms [15] and the cross-terms may also overlap with auto-terms. The creation of cross-terms can be explained as two points of the TF space interfering to create a contribution on another point at their geometrical midpoint. These cross-terms have positive and negative amplitude values, they are amplitude modulated, zero mean [12] and their amplitudes are higher than the signal components' amplitudes [63].

Assuming signal $s(t)$ is composed of two signal components $s(t) = s_1(t) + s_2(t)$, WVD of $s(t)$ can be calculated as follows [61];

$$\begin{aligned}
 W_s(t, w) = & \frac{1}{2\pi} \left[\overbrace{\int s_1(t - \frac{1}{2}\tau) s_1^*(t + \frac{1}{2}\tau) d\tau}^{W_{s_1s_1}(\tau, w)} + \right. \\
 & \overbrace{\int s_1(t - \frac{1}{2}\tau) s_2^*(t + \frac{1}{2}\tau) d\tau}^{W_{s_1s_2}(\tau, w)} + \overbrace{\int s_2(t - \frac{1}{2}\tau) s_1^*(t + \frac{1}{2}\tau) d\tau}^{W_{s_2s_1}(\tau, w)} \\
 & \left. + \overbrace{\int s_2(t - \frac{1}{2}\tau) s_2^*(t + \frac{1}{2}\tau) d\tau}^{W_{s_2s_2}(\tau, w)} \right] \int e^{-j\tau w} d\tau
 \end{aligned} \tag{3.3}$$

In this expression $W_{s_1s_2}(t, w)$ and $W_{s_2s_1}(t, w)$ terms are called the Cross Wigner-Ville distribution (XWVD) [64]. Since $W_{s_1s_2} = W_{s_2s_1}^*$, we can rearrange WVD expression as [61];

$$W_s(t, w) = W_{s_1s_1}(t, w) + W_{s_2s_2}(t, w) + 2 \text{Re}\{W_{s_1s_2}(t, w)\} \tag{3.4}$$

where the last term corresponds to the cross-terms caused by the XWVD. The relation between the auto-terms and cross-terms can be seen by using the “interference formula”

$$|W_s(t, w)|^2 = \frac{1}{2\pi} \iint W_{s_1}(t - \frac{1}{2}\tau, w - \frac{\nu}{2}) W_{s_2}(t + \frac{1}{2}\tau, w + \frac{\nu}{2}) d\tau d\nu \tag{3.5}$$

If W_{s_1} is around (t_{s_1}, w_{s_1}) and W_{s_2} is around (t_{s_2}, w_{s_2}) then the cross-term lies midway between these signal terms [65].

$$(t_x, w_x) = \left(\frac{t_{s_1} + t_{s_2}}{2}, \frac{w_{s_1} + w_{s_2}}{2} \right) \tag{3.6}$$

Under low SNR conditions, WVDs of multi-component signals create unreadable TF images which make it hard to detect signals and perform parameter extraction. In the literature, there are many studies to suppress the cross-terms and enhance the TF resolution; Barbarossa was the first to use the amplitude modulation of the cross-terms to reduce them in WHT [66]. In a relatively recent study, Poyil et.al. have detected the LFM signal components in HT, then using the parameters of these components they’ve used back estimation to filter out only the signal components in WVD [67]. Another

study investigated the use of time frequency-based blind source separation technique for elimination of cross-terms in WVD [68]. In [69], researchers proposed a method called standardization of the pseudo quadratic form to eliminate the cross-terms. The method was basically based on the signal that was decomposed by matching pursuit algorithm. Another method combined image processing techniques with WVD in [70]. In this method, the WVD was rotated by some angle and rotated again in the reverse direction until the cross-terms are defined. This new TF image is then used as a mask to suppress the cross-terms.

The existence of cross-terms is not always to be avoided; there are several studies in the literature that make use of WVD cross-terms in different research areas. For example; it was used in WVD-based radar processors to detect the presence of targets in [71]. In this study WVD was used as a Doppler filter bank in an adaptive moving target detection scheme. When the target approaches the radar the WVD produces the cross-term using the non-stationary clutter returns and the target returns. And this helps the radar processor to detect the target better than the FFT-based radar processor. In a medical application, WVD cross-terms were used to detect ventricular late potentials (VLP) in patients who have suffered heart attacks. Similar to [71], WVD cross-terms were used to detect presence of very small signal terms, in this case VLPs. The researchers made use of the geometric and amplitude properties of the cross-terms in TF plane of high resolution electrocardiographic signals [72].

However the use of WVD cross-terms, especially on HT space, has not been implemented in radar signal interception methods. In chapter 4, we will propose the XWHT method that makes use of the properties of the cross-terms in the effort of FMCW signal detection and parameter extraction.

To illustrate the properties of the cross-terms, we present sample WVDs of single and two-tone signals. To be able to focus only on the cross-term effect, the tones were created without any noise.

3.2.1 Single Tone Signal

The parameters of the single tone signal were chosen as; carrier frequency $f_c = 2\text{kHz}$., sampling frequency $f_s = 8\text{kHz}$., observation time $T_{obs} = 32\text{msec}$. and $T_m = 32\text{msec}$. The input signal was created in the form of $x = I + jQ$ where

$$I = \cos(2\pi f_c t) \text{ and } Q = \sin(2\pi f_c t) \quad (3.7)$$

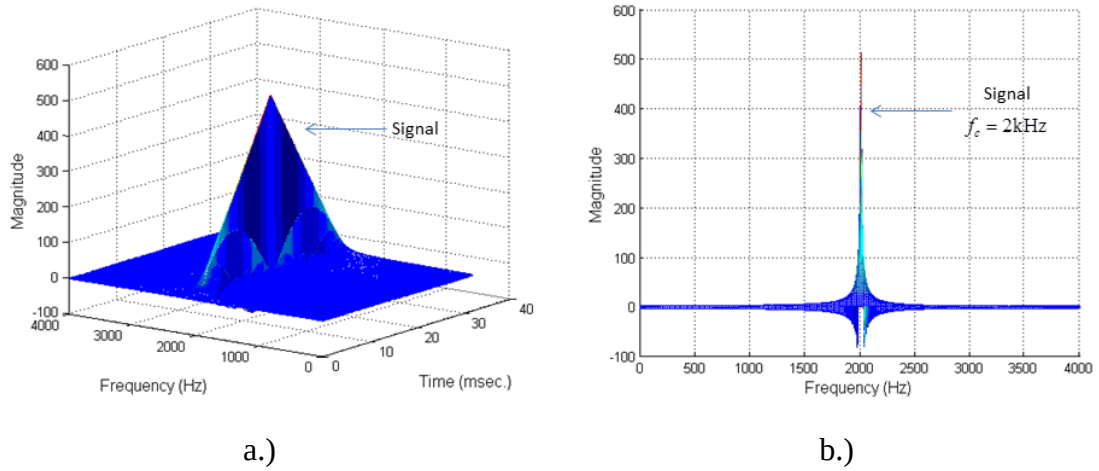


Figure 3.1 Single-tone signal a.) 3D mesh plot b.) Frequency vs. magnitude plot

3D mesh plot in Figure 3.1.a shows the energy of the signal in both time and frequency domain. In Figure 3.1.b, we see positive peak magnitude exactly on f_c , and no negative magnitude on the carrier frequency.

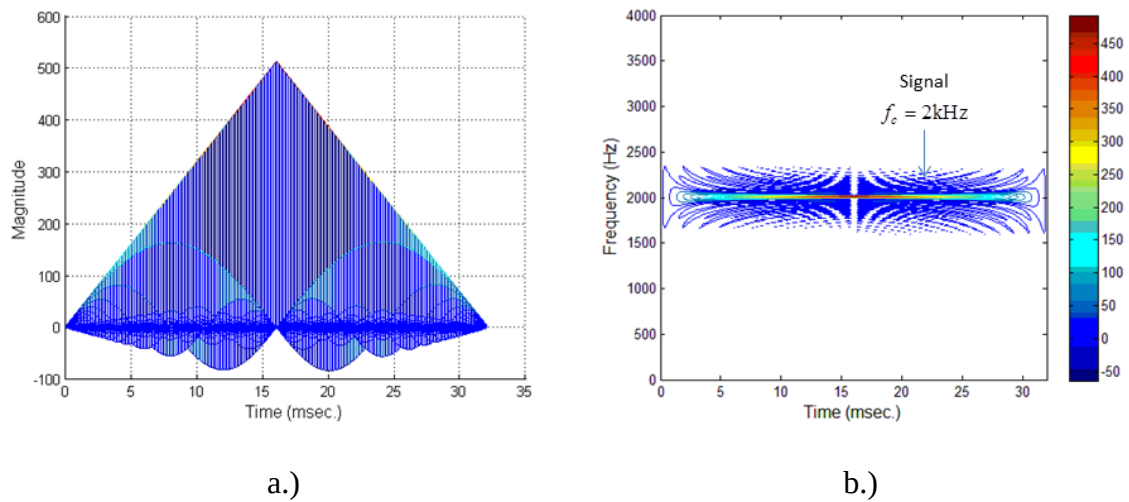


Figure 3.2 Single-tone signal a.) Time vs. magnitude plot b.) Time vs. frequency contour plot

Figure 3.2.a presents the time vs. magnitude plot and in Figure 3.2.b we can see the tone during the observation time from the contour plot. As expected, we don't see any cross-term effect in the single tone case.

3.2.2 Two Tone Signal

To observe the cross-terms, we created two tones at two different carrier frequencies; $f_{c1} = 1\text{kHz}$ and $f_{c2} = 2\text{kHz}$, using the same $f_s = 8\text{kHz}$, $T_{obs} = 32\text{msec.}$ and $T_m = 32\text{msec.}$ The input signal is in the form of $x = x_1 + x_2$ where $x_1 = I_1 + jQ_1$ and $x_2 = I_2 + jQ_2$.

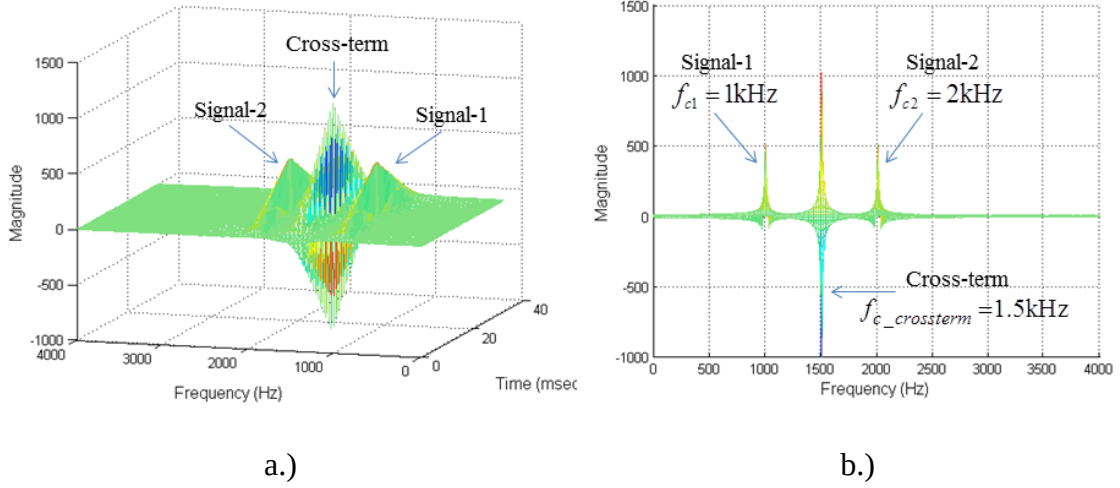


Figure 3.3 Two-tone signal a.) 3D mesh plot b.) Frequency vs. magnitude plot

Figure 3.3.a shows the energy of the signal components and cross-terms simultaneously on time and frequency domains. From Figure 3.3.b, we see that we get positive magnitudes exactly on carrier frequencies and the cross-term builds right between these two signal components at $f_{crossterm} = \frac{f_{c1} + f_{c2}}{2}$, with negative and positive magnitudes. Note that the cross-term magnitudes (both positive and negative) are much higher (twice) than the signal component magnitudes.

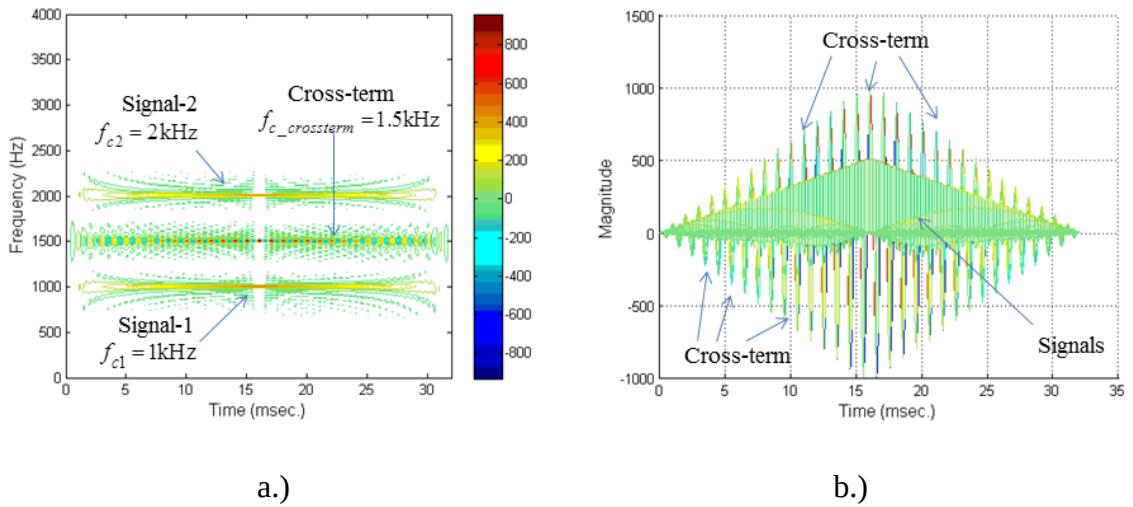


Figure 3.4 Two-tone signal a.) Contour plot b.) Time vs. magnitude plot

The contour plot in Figure 3.4.a shows the two signal components as positive magnitude lines on their carrier frequencies and the cross-term as positive and negative magnitude dots midway between the two carrier frequencies. To observe the magnitude behavior of the cross-term and signal components better, we plot only the vectors from the WVD matrix containing the cross-term and signal components in Figure 3.5. The oscillating nature of the cross-term magnitude and positive nature of the signal component magnitudes are obvious here.

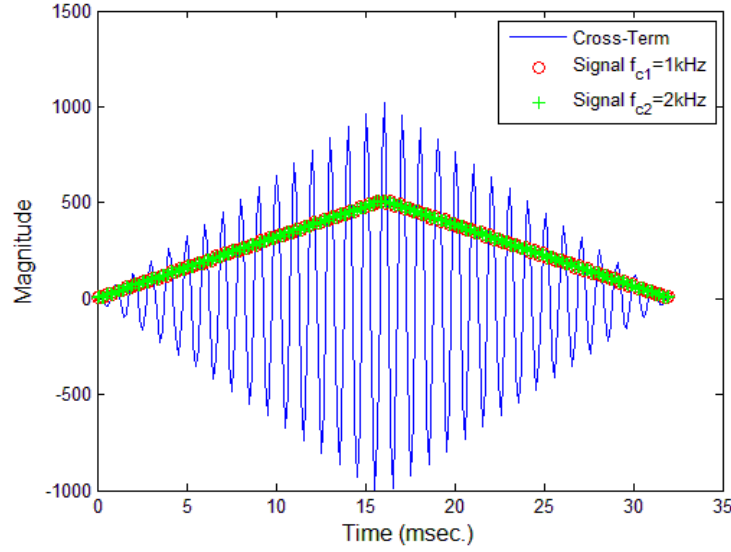


Figure 3.5 Cross-term and signal component vectors.

3.2.3 The Oscillation of the Cross-Terms

To illustrate the oscillation properties of the cross-terms, we can take signal $s(t)$ around $(0,0)$ in TF plane, $s_1(t)$ as $s_1(t) = a \cdot s(t-t_1)e^{jw_1t}$ and $s_2(t)$ as $s_2(t) = b \cdot s(t-t_2)e^{jw_2t}$, where a and b are complex constants. The WVDs of $s_1(t)$ and $s_2(t)$ can be given as [58];

$$\begin{aligned} W_{s_1}(t, w) &= |a|^2 W_s(t-t_1, w-w_1) \\ W_{s_2}(t, w) &= |b|^2 W_s(t-t_2, w-w_2) \end{aligned} \quad (3.8)$$

and the cross-term is calculated as;

$$2 \operatorname{Re}\{W_{s_1 s_2}(t, w)\} = 2 \cdot |ab| \cos[w_d(t-t_m) - t_d(w-w_m) + \varphi_0] \times W_s(t-t_m, w-w_m) \quad (3.9)$$

where $(t_d, w_d) = (t_2 - t_1, w_2 - w_1)$ and $(t_m, w_m) = \left(\frac{t_1 + t_2}{2}, \frac{w_1 + w_2}{2}\right)$.

From these expressions we see that the cross-term is a modulated version of the signal term shifted to (t_m, w_m) . The cross-terms oscillate perpendicular to the line connecting the points that created them. The frequency of this oscillation is proportional to the distance between the two points [15]. These properties can be used in radar signal interception efforts. We've investigated the oscillation properties of the cross-terms using the signals defined in Table 3.1.

Table 3.1 Signal database to be used in cross-term oscillation studies

<i>Tone 1 (Hz)</i>	<i>Tone 2</i>	<i>f_s(Hz)</i>	<i>T_m (msec)</i>
500	Chosen from 1000 Hz to 3500 Hz in 250 Hz steps	8000	32

In Figure 3.6 we try to illustrate the oscillation properties using two signal pairs; the first pair has two tones with frequency difference 500 Hz. (tone1: 500 Hz – tone2: 1000 Hz) and the other pair has frequency difference of 3000 Hz. (tone1: 500 Hz – tone2: 3500 Hz).

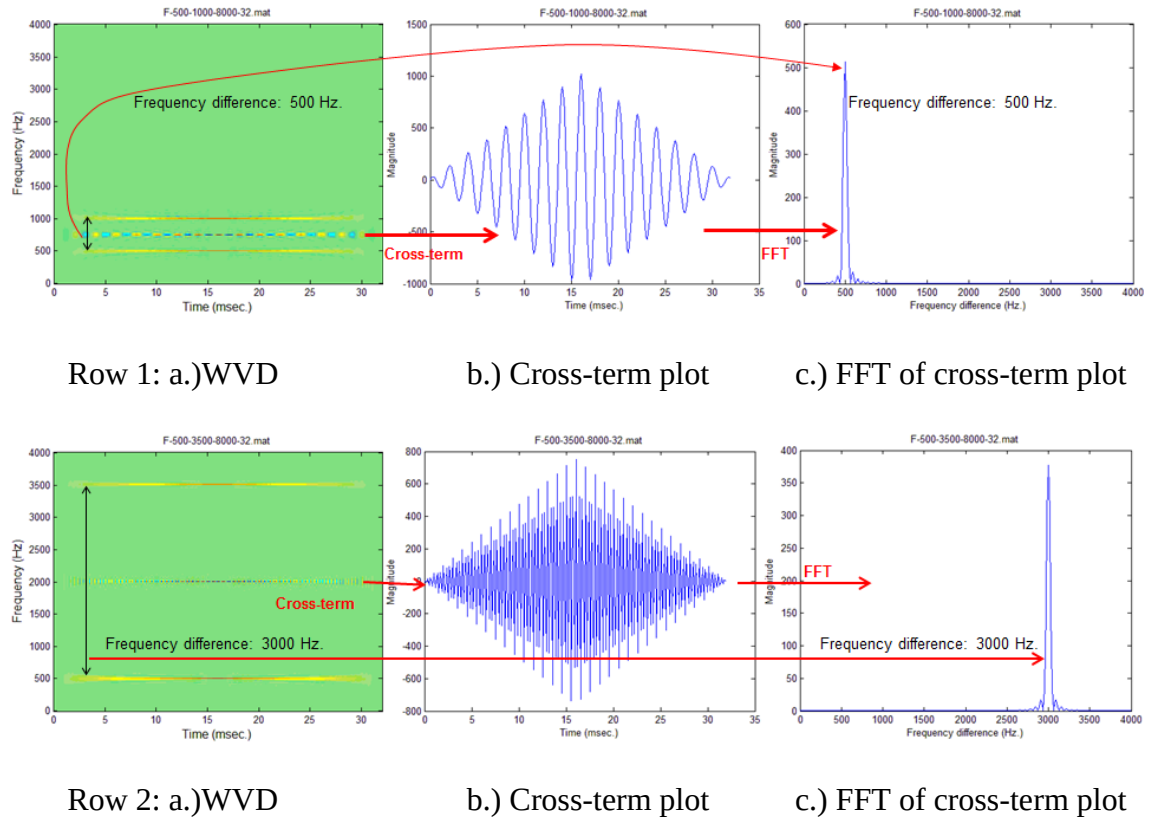


Figure 3.6 Row 1: Frequency difference 500 Hz. Row 2: Frequency difference 3kHz.

For both signal pairs we show the WVD of the two-tone signal, the plot of the cross-term vector and the FFT of the cross-term vector. As can be observed from the figure, the oscillation frequency shows the frequency difference between the tones.

We've also checked the behavior of the FFT magnitude as we increase the distance between the tones. Figure 3.7 shows the magnitudes of the cross-term FFTs for frequency differences starting from 500 Hz. to 3000 Hz. in 250 Hz. steps. We can see that the FFT magnitudes decrease as the frequency differences increase.

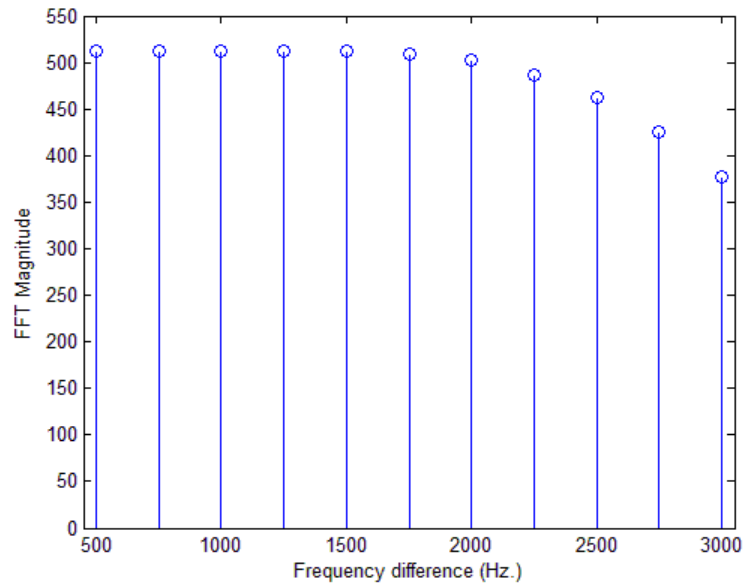


Figure 3.7 FFT magnitude change vs. frequency difference between tones.

These results have shown that; the WVD magnitude properties of the signal components and cross-terms are quite different from each other, the cross-terms have unique properties and these properties can be exploited in the signal analysis efforts used in ESRs. The cross-term properties have been shown using tones, however these same properties exist for LFM signals, tone and LFM mixed signals as well. Further investigation and exploitation of the cross-term properties are left for future work.

3.3 General WVD Properties

The properties of the WVD are summarized below. These properties will be used in the approach for detecting FMCW signals using WHT based algorithms.

3.3.1 Weak Finite Support

If we consider the calculation of the WVD for an infinite duration signal, the WVD is always nonzero since the folding of the right with the left parts will give us a nonzero answer. For the case of a signal that has a beginning in time, the WVD will be zero for times before the signal starts, since there is nothing to the left to fold. Likewise, for a signal that ends, the WVD will be zero after that time. The same arguments hold in the spectral domain as well; for a band limited signal, the WVD is zero for frequencies outside the band. Therefore, the WVD satisfies the weak finite support properties in time and frequency, i.e;

$$\begin{aligned} W(t, w) &= 0, \quad |t| > T \quad \text{if} \quad s(t) = 0, \quad |t| > T \\ W(t, w) &= 0, \quad |w| > B \quad \text{if} \quad S(w) = 0, \quad |w| > B \end{aligned} \quad (3.10)$$

The WVD has “weak” finite support because; the WVD of a signal that is zero for a finite period during its finite duration is not zero [61]. This property is also known as “wide-sense support conservation”.

3.3.2 Real-valued

The WVD is always real across time and frequency, even if the signal being analyzed is complex [61].

$$W(t, w) \in \mathbb{R}, \quad \forall t, w \quad (3.11)$$

3.3.3 Symmetry

For symmetric spectra, the WVD is symmetric in frequency domain. For real spectra, the WVD is symmetric in time [61].

$$W(t, w) = W(t, -w) \quad \text{for real signals (symmetrical spectra),} \quad S(w) = S(-w) \quad (3.12)$$

$$W(t, w) = W(-t, w) \quad \text{for real spectra (symmetrical signals),} \quad s(t) = s(-t) \quad (3.13)$$

3.3.4 Marginal Properties

The WVD satisfies the time and frequency marginal distributions. The instantaneous power and the energy spectral density are obtained as marginal distributions as follows

$$\begin{aligned}\int W(t, w)dw &= |s(t)|^2 \\ \int W(t, w)dt &= |S(w)|^2\end{aligned}\tag{3.14}$$

3.3.5 Total Energy Conservation

The total energy conservation is also satisfied [61]. We can obtain the energy of the signal $s(t)$ by integrating the WVD of $s(t)$ over TF space,

$$E = \iint W(t, w)dw dt = \int |s(t)|^2 d\tau = 1\tag{3.15}$$

3.3.6 Time and Frequency Shifts

If the signal under analysis is shifted in time by t_0 and / or shifted in the spectrum by w_0 the WVD is also shifted by the same amounts in each domain [61];

$$s(t) \rightarrow e^{jw_0 t} s(t - t_0) \quad \text{then} \quad W(t, w) \rightarrow W(t - t_0, w - w_0)\tag{3.16}$$

3.3.7 Unitarity

This property shows the conservation of the scalar product from time-domain to the time-frequency domain [47]. The formula given below is also known as the **Moyal's Formula**, and it will be used in chapter 4 to explain why the combination of WVD and HT is optimum for FMCW signal detection.

$$\left| \int s(t)x^*(t)dt \right|^2 = \iint W_s(t, w)W_x^*(t, w)dt dw\tag{3.17}$$

The existence of cross-terms is important for WVD's unitarity property (i.e. Moyal's formula) to hold [73].

3.3.8 Compatibility with Filterings

If a signal $x(t)$ is the convolution of $s(t)$ and $h(t)$, than the WVD of $x(t)$ can be represented as the time convolution between the WVD of $s(t)$ and WVD of $h(t)$ [47].

$$x(t) = \int s(\tau)h(t - \tau)d\tau \Rightarrow W_x(t, w) = \int W_s(\tau, w)W_h(t - \tau, w)d\tau\tag{3.18}$$

3.3.9 Compatibility with Modulations

If $x(t)$ is the modulation of $s(t)$ by $m(t)$, then the WVD of $x(t)$ can be expressed as the frequency convolution between the WVD of $s(t)$ and WVD of $m(t)$ [47].

$$x(t) = s(t)m(t) \Rightarrow W_x(t, \omega) = \int W_s(t, \theta) W_m(t, \omega - \theta) d\theta \quad (3.19)$$

3.4 Pseudo WVD

To calculate the WVD with (3.2) we need $R_{ss}(t, \tau)$ from $t|_{-\infty}^{\infty}$ and this is a practical problem. Therefore a regular window $h(t)$ is introduced into (3.2) and we get the expression for Pseudo WVD [47];

$$PWVD(t, \omega) = \frac{1}{2\pi} \int s(t - \frac{1}{2}\tau) s^*(t + \frac{1}{2}\tau) h(\tau) e^{-j\tau\omega} d\tau \quad (3.20)$$

This windowing operation provides frequency smoothing of the WVD.

$$PWVD(t, \omega) = \int H(\omega - \theta) W(t, \theta) d\theta \quad (3.21)$$

where $H(\omega)$ is the FT of the window $h(t)$. As expected, this frequency smoothing property attenuates the effects of the cross-terms due to their oscillating nature. As a compromise, the PWVD doesn't provide; marginals, unitarity and finite-support properties [47]. The discrete implementation of PWVD is explained in the following chapter.

3.5 Discrete WVD Representation

We can modify the continuous WVD representation for the discrete time signal $s(n)$, n being discrete time index in the interval $[-\infty, \infty]$. To compute the discrete WVD of the analyzed signal, first the oldest and newest samples of the signal are multiplied, then FTs of these multiplications are calculated. These FTs are brought together to form WVD matrix. Discrete WVD is actually FT of time dependent auto-correlation function of digital signal $s(n)$ and described as [1];

$$W(n, \omega) = 2 \sum_{m=-\infty}^{\infty} s(n+m) s^*(n-m) e^{-j2\omega m} \quad (3.22)$$

To reduce the effect of cross-terms we can use real window functions with length $2N$, to calculate discrete PWVD, where $w(0) = 1$.

$$W(n, \omega) = 2 \sum_{m=-N}^{N-1} s(n+m)s^*(n-m)w(m)w(-m)e^{-j2\omega m} \quad (3.23)$$

Larger N will give us much better TF resolution and smoother results, however computational complexity constraints often limit the length of the windows [22]. Taking $w(m)$ as a rectangular window with amplitude one will result in [74]

$$W(n, \omega) = 2 \sum_{m=-N}^{N-1} s(n+m)s^*(n-m)e^{-j2\omega m} \quad (3.24)$$

We can define the kernel function $f_n(m)$ as

$$f_n(m) = s(n+m)s^*(n-m) \quad (3.25)$$

and the WVD becomes

$$W(n, \omega) = 2 \sum_{m=-N}^{N-1} f_n(m)e^{-j2\omega m} \quad (3.26)$$

(3.26) is computationally expensive so, Boashash et al. presented an efficient algorithm to compute the PWVD [75]. They sampled the continuous frequency variable

$$w = \frac{\pi k}{2N}, \quad k = 0, 1, 2, \dots, 2N-1 \quad (3.27)$$

and adjust the kernel indexes to compute the WVD using the standard FFT algorithms, and (3.26) becomes;

$$W(n, \frac{\pi k}{2N}) = 2 \sum_{m=0}^{2N-1} g_n(m) \exp(-\frac{j2\pi nk}{2N}) \quad (3.28)$$

Where

$$g_n(m) = \begin{cases} f_n(m), & 0 \leq m \leq N-1 \\ 0, & n = N \\ f_n(m-2N), & N+1 \leq m \leq 2N-1 \end{cases} \quad (3.29)$$

Since the kernel is a symmetric function

$$f_n(m) = f_n^*(-m) \quad (3.30)$$

the DFT of the kernel is always real. As a result, we get the discrete PWVD as

$$W(n, k) = 2 \sum_{m=0}^{2N-1} f'_n(m) \exp\left(-\frac{j\pi nk}{N}\right) \quad (3.31)$$

3.6 WVD as an FMCW TF Analysis Tool

We need to consider resolution and aliasing issues when using WVD as a TF analysis tool for signal detection and parameter extraction.

3.6.1 Resolution Considerations

Geometric relations of FMCW modulation parameters on a two-component upchirp-FMCW signal's WVD is presented in Figure 3.8. WVD is actually an $N \times M$ matrix, elements of this matrix can be represented by;

$$W(n_i, w_j), \quad i = 1, 2, \dots, N; \quad j = 1, 2, \dots, M \quad (3.32)$$

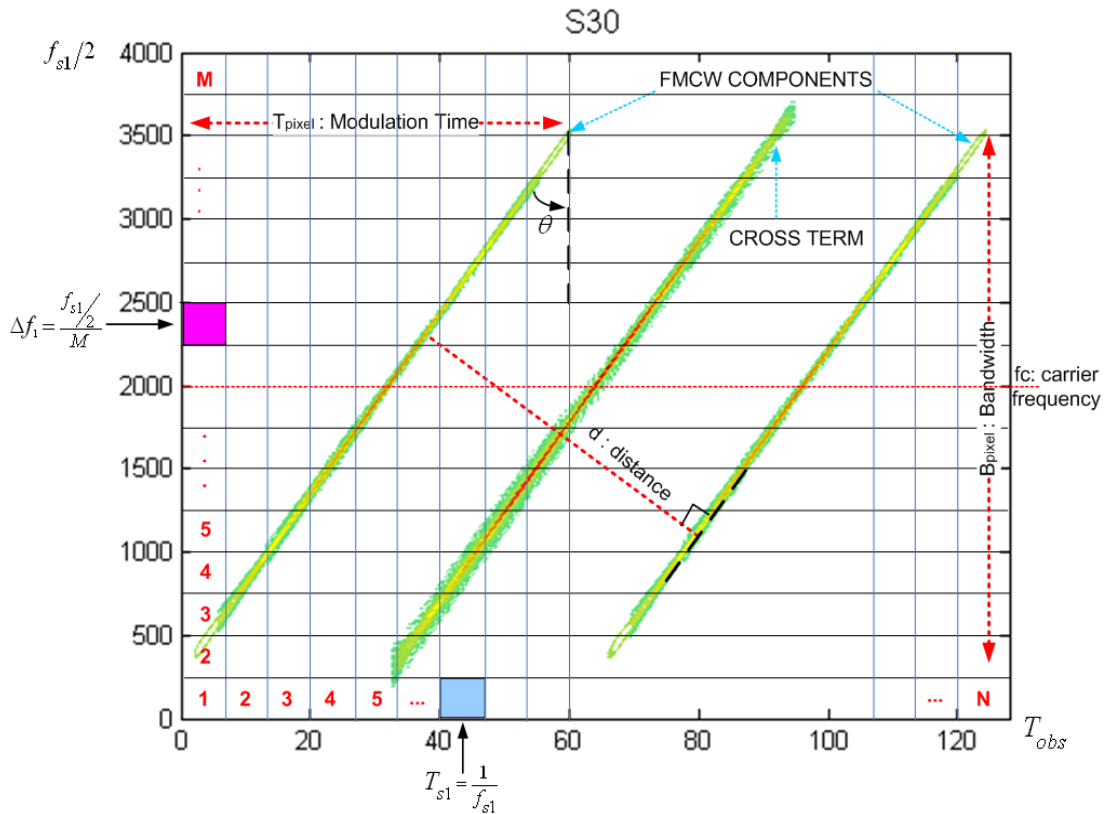


Figure 3.8 Geometric relations of a two component upchirp FMCW waveform parameters on WVD ($f_s = f_{s1}$)

We take $2M$ as the FFT length, N as the number of signal samples collected in observation time (T_{obs}), f_{s1} as the sampling frequency and T_{s1} as the sampling interval.

$$\Delta f_1 = \frac{f_{s1}/2}{M} \quad (3.33)$$

S30

$\Delta f_2 = \frac{f_{s1}}{M}$

T_{obs}

$T_{s2} = \frac{T_{s1}}{2} = \frac{1}{2f_{s1}}$

T_{pixel} : Modulation Time

B_{pixel} : Bandwidth

f_c : carrier frequency

1 2 3 4 5 ... N

36

As expected, in Figure 3.9 we represent two times more frequency content in each frequency bin and half the length of time in each time bin compared to Figure 3.8. Another important issue to notice here is the change of the chirp rate of the FMCW component due to f_s change. The effect of this change will be investigated and exploited in chapter 6.

One of the good properties of WVD is that, one can change frequency and time resolution of the distribution without affecting the other. For example here we could have increased the FFT length as well to make up for the decreased frequency resolution however we assume fixed FFT length for hardware implementation considerations. Another thing to consider is that the storage requirement for the signal samples will increase since we'll take more samples with increased sampling frequency.

3.6.2 WVD Aliasing Considerations

Assuming that we have a single component up-chirp FMCW signal with $T_{pixel} = T_{obs}$;

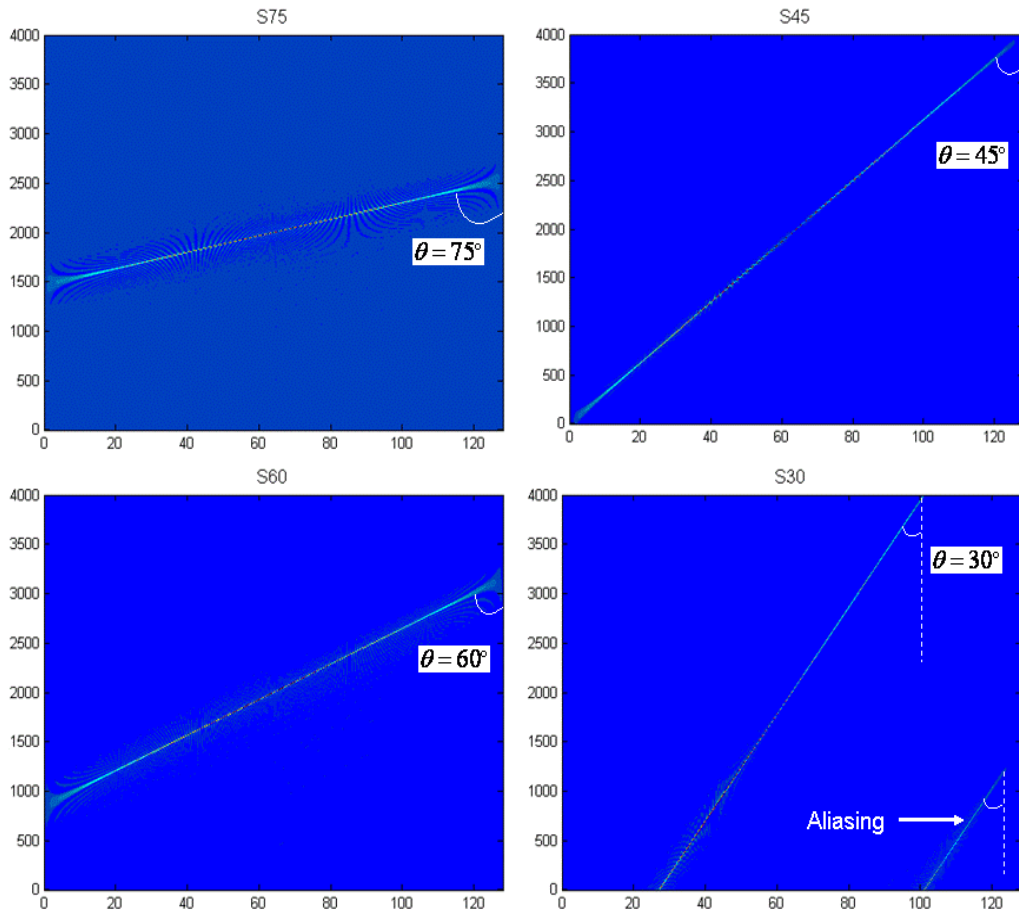


Figure 3.10 Single FMCW up-chirp component with changing chirp rate

we can represent this signal without aliasing if $0 < B \leq f_s/2$, since y-axis is limited by $f_s/2$. And the chirp rate limits will be between $90^\circ \geq \theta \geq 45^\circ$ as shown in Figure 3.10. This is because; with maximum $B = f_s/2$, and with $T_{pixel} = T_{obs}$ we have $\max(T_{pixel}) = \max(B_{pixel}) = M = N$. As a result

$$\tan(\theta) = \frac{\max(T_{pixel})}{\max(B_{pixel})} \Rightarrow \theta = \tan^{-1}\left(\frac{\max(T_{pixel})}{\max(B_{pixel})}\right) = \tan^{-1}(1) = 45^\circ \quad (3.34)$$

We can see the aliasing effect on signal S30 in Figure 3.10 where the $B_{S30} > f_s/2$, and $\theta = 30^\circ$ (Note: The titles of the WVDs (S30, S60 etc.) in Figure 3.10 and Figure 3.11 show the chirp rate of the presented signal). We can also note that the aliasing terms have the same chirp rate as the main signal components. If we fit more than one signal component in our observation time with fixed T_m , the chirp rate range that can be represented will change accordingly.

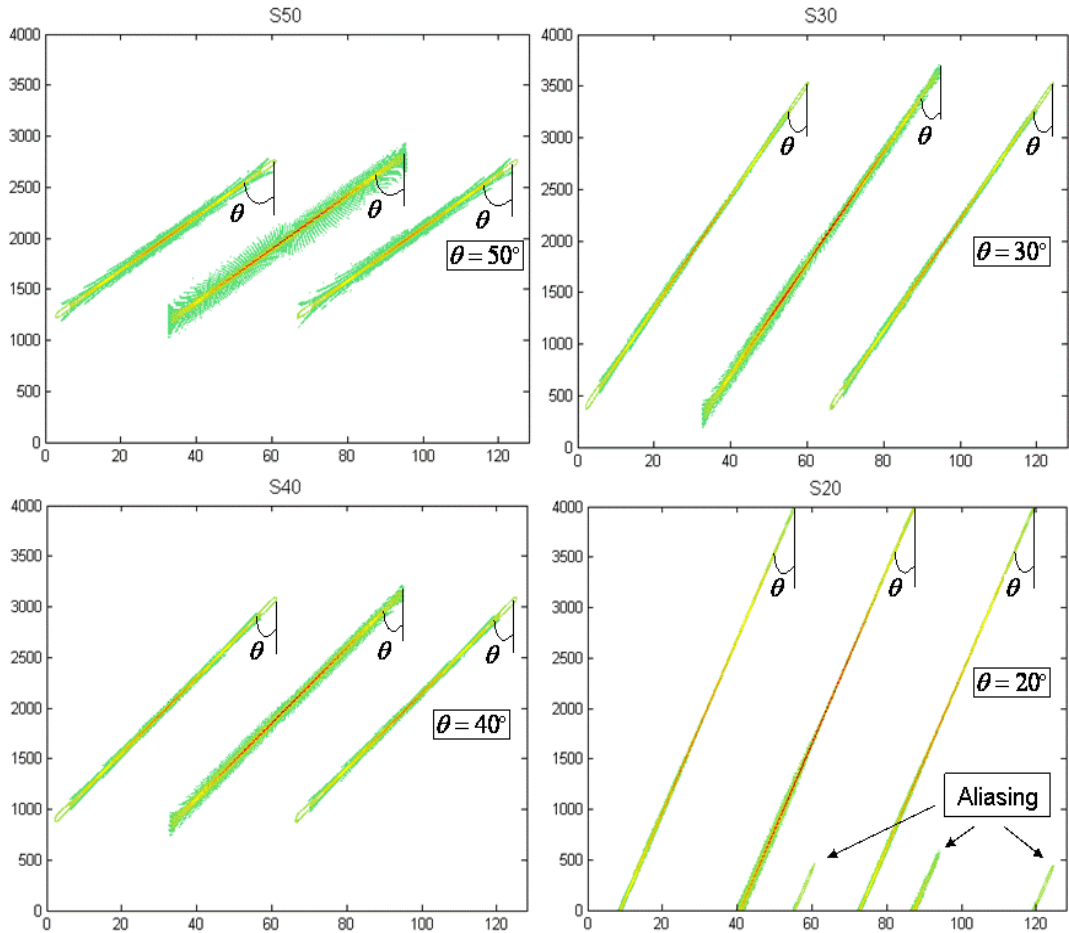


Figure 3.11 Two component FMCW up-chirp with changing chirp rate

For example; if we fit two FMCW up-chirp components in the same T_{obs} , i.e.

$2T_{pixel} = T_{obs}$, we can still represent signals with $0 < B \leq f_s/2$ without aliasing, however

the chirp rate range will change. This is because; with maximum $B = f_s/2$, and with

$2T_{pixel} = T_{obs}$ we have $\max(T_{pixel}) = T_{obs}/2 = N/2$ and $\max(B_{pixel}) = M$. As a result, given $M = N$;

$$\tan(\theta) = \frac{\max(T_{pixel})}{\max(B_{pixel})} \Rightarrow \theta = \tan^{-1}\left(\frac{N/2}{M}\right) = \tan^{-1}(0.5) = 26.56^\circ \quad (3.35)$$

So, as a rule of thumb; the chirp rate range that can be represented by WVD without aliasing is lower bounded as

$$90^\circ \geq \theta \geq \tan^{-1}\left(\frac{1}{K}\right) \quad (3.36)$$

where K is the number of FMCW up-chirp components we want to fit in our observation time. We can see the aliasing effect on S20 in Figure 3.11 where the

$B_{S20} > f_s/2$, and $\theta = 20^\circ$.

As mentioned before for realistic purposes, the chirp rate of the FMCW components should be upper bounded as well. These boundaries help us define the limits of transform space calculations, which improve hardware implementation performance.

CHAPTER 4

WIGNER HOUGH TRANSFORM BASED FMCW DETECTION AND PARAMETER EXTRACTION

ESRs need to detect radar signals under noisy conditions and among many other signals. In our detection scheme we use two different spaces; TF space and parameter space. TF space is derived by calculating the WVD of signal samples and parameter space is derived by applying Hough / Radon (H/R) transform to TF space points that exceed TF threshold, Th_{WD} . This combination of WVD and H/R transform spaces makes it possible to redefine the problem of detecting FMCW components in WVD as detecting the high intensity points in H/R space. The detection decision is given whenever the amplitude of any point in H/R parameter space exceeds the detection threshold, ζ . For deterministic LFM signals, we know the point at which the peak of parameter space will occur. But in our case, we try to detect LFM signals with unknown parameters; therefore we need to perform peak searching in all of the transform space.

WHT based methods in literature are known to be computationally expensive since they use the entire WVD space and calculate HT for all the possible chirp rates. Certain LFM chirp rates are useless to detect any kind of real targets. Calculating the HT for only a limited set of chirp rates and for WVD pixels exceeding Th_{WD} will reduce the required processing power and complexity. In this chapter we propose new WHT methods that reduce the complexity yet keeping detection and parameter extraction performance above a certain level. In all the methods described below, we control multiple detections declared from neighboring parameter space cells to reduce the false alarms. Since these are usually from the same FMCW component.

Autonomously detecting and analyzing FMCW and other LPI modulations can eliminate the need for human intervention and enable near real-time detection and

analysis of signal modulations. For FMCW signal parameter extraction, we need orthogonal distance (d) between FMCW signal components and the chirp-rate (θ) of the signal components. These two parameters were illustrated in Figure 3.8. The extracted parameters can later be compared with existing emitter parameters in a database for identification. In this chapter we also present our work on autonomous parameter extraction algorithms for FMCW signals. We use TF images obtained by the WVD and, derive the significant modulation parameters of FMCW waveforms by using different WHT methods. We first explain the general approach for WHT based FMCW detection, give short descriptions of HT and RT and then propose new methods to improve the WHT methods in the literature known to be computationally expensive.

4.1 General Approach for Wigner Hough Based FMCW Detection

The intercept receiver has the advantage of listening as much as it needs to intercept radar signals. In the $0 \leq t \leq T_{obs}$ interval the receiver receives complex white Gaussian noise $n(t)$ ($\mathcal{N}(0, \sigma_n^2)$; 0 : zero-mean, σ_n^2 : variance)) and the signal $s(t)$, if it exists. The receiver declares detection by considering which of the following two hypotheses is true.

$$\begin{aligned} H_0 : \quad r(t) &= n(t) && \text{no signal is present (only noise)} \\ H_1 : \quad r(t) &= s(t) + n(t) && \text{signal is present (in additive noise)} \end{aligned} \quad (4.1)$$

In the case of H_1 , where signal and noise are intercepted, we want to be able to declare detection with a high probability. For H_0 , which means only noise is intercepted, we don't want to declare any detection for low probability of false alarm. A false alarm is, when one of the points in the parameter space exceeds ζ in the absence of LFM signal and we declare detection. Correct detection is when one of the points in the parameter space exceeds ζ in the presence of LFM signal and we declare detection.

The choice of thresholds (Th_{wD}, ζ) is very important to get a good probability of detection performance with a low false alarm rate. These thresholds shall be defined such that, we can detect real signals with high efficiency and avoid false-alarms. Detection is usually followed by parameter extraction phase [35]. To detect the signal $s(t; \chi)$ in white Gaussian noise, ESR shall filter the intercepted $r(t)$ signal with a

matched filter whose impulse response $h(t; \chi)$ is determined by signal parameters. Here, $s(t)$, $r(t)$, $h(t)$ are continuous time domain representations of signal, intercept (signal and noise) and filter impulse responses, respectively. Actually, filtering is calculating the correlation of the intercepted signal with a sample of the searched signal. If the parameters of the signal, given with χ vector, are not known a-priori, maximum filter output is searched as a function of signal parameters and time delay. This maximum output is then compared against a threshold to decide detection [15].

$$\max_{t, \theta} \int h(t - \tau; \chi) r(\tau) d\tau \geq \zeta \quad (4.2)$$

In the above formula, t and χ values which give us the maximum value are the maximum likelihood estimates of time delay and signal parameters, respectively. Considering that we have no prior information about signal parameters, we have to investigate all the possible values of χ vector to find the maximum value of (4.2). Instead of using this rather unpractical approach, Barbarossa, in [15], used Kay's approach in [13] and assumed time-delay is known (i.e., $t = 0$). With this assumption, to declare signal detection, the expression below can be compared against ζ .

$$\max_{\theta} \left| \int h(\tau; \chi) r(\tau) d\tau \right|^2 \quad (4.3)$$

By using the unitarity property of the WVD, time domain square modulus of the scalar product in the above expression can be represented as scalar product of the WVDs in TF domain using Moyal's formula [15].

$$\max_{\theta} \iint W_h(t, f; \chi) W_r(t, f) dt df \quad (4.4)$$

If we consider LFM signals in additive white Gaussian noise (AWGN), WVD of the impulse response of the filter to be used ($W_h(t, f; \chi)$) will have the form of LFM. In this case, the double integral in the above expression can be calculated using a line integral. Since the parameters of this line are not known a-priori, the integral needs to be calculated for all the possible lines on WVD [15] and this can be done by calculating HT or RT of the WVD.

4.2 Hough Transform

HT is a pattern recognition technique that is widely used in image processing to detect geometric shapes in images. In an $f(x,y)$ image, we can represent a radial line l with its orthogonal distance r to the origin and with its normal's angle α , as shown in Figure 4.1.

NOTE: In this thesis, we've defined the chirp-rate as an angle and we use $\theta = 90^\circ - \alpha$ to represent the chirp-rate. The x-axis values of our HT and RT images are also adjusted to show the chirp rate value. Therefore we use θ in the formulations below instead of α .

The line l , corresponds to an (r, θ) point in HT space and transformed by the following formula (polar parameterization of a line).

$$r = x \sin \theta + y \cos \theta \quad (4.5)$$

Here, r -axis is perpendicular to l . We can look at (4.5) as stepping through θ from 0° to 180° and calculating the corresponding r . After some trigonometric manipulation (4.5) becomes [76];

$$\psi = \arctan \frac{x}{y}, \quad r = \sqrt{x^2 + y^2} \cdot \cos(\alpha + \psi) \quad (4.6)$$

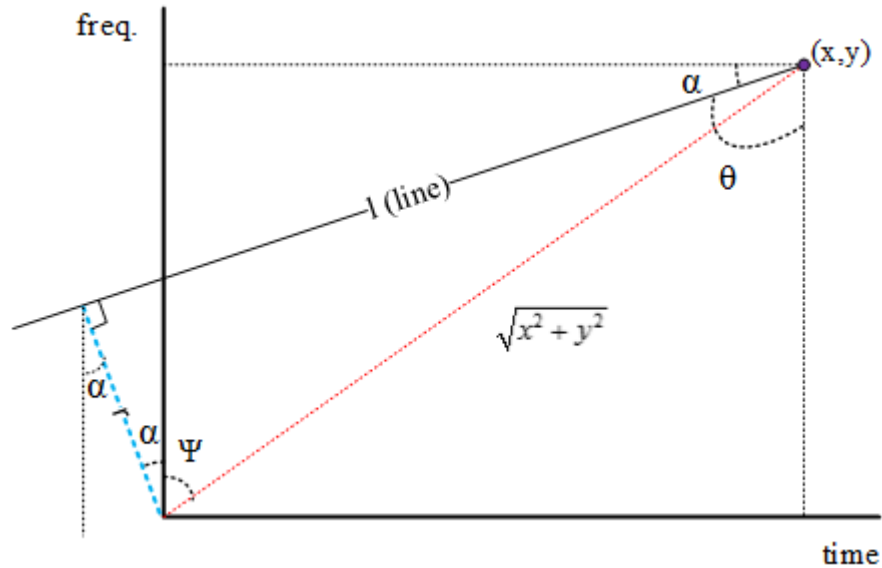


Figure 4.1 HT geometry shown on an $f(x,y)$ image

In (4.6), r becomes a sinusoid with amplitude and phase dependent on the coordinates of the (x_i, y_j) data point (where $i = 1, 2, \dots, N$ and $j = 1, 2, \dots, M$).

All the lines passing over $(x_i, y_j) \in \{f(x_i, y_j) \geq Th_{WD}\}$ points of $f(x, y)$ image are calculated and transferred to (r, θ) space. Figure 4.2 and Figure 4.3 are used to illustrate the HT method. In Figure 4.2 we see an FMCW upchirp component's WVD.

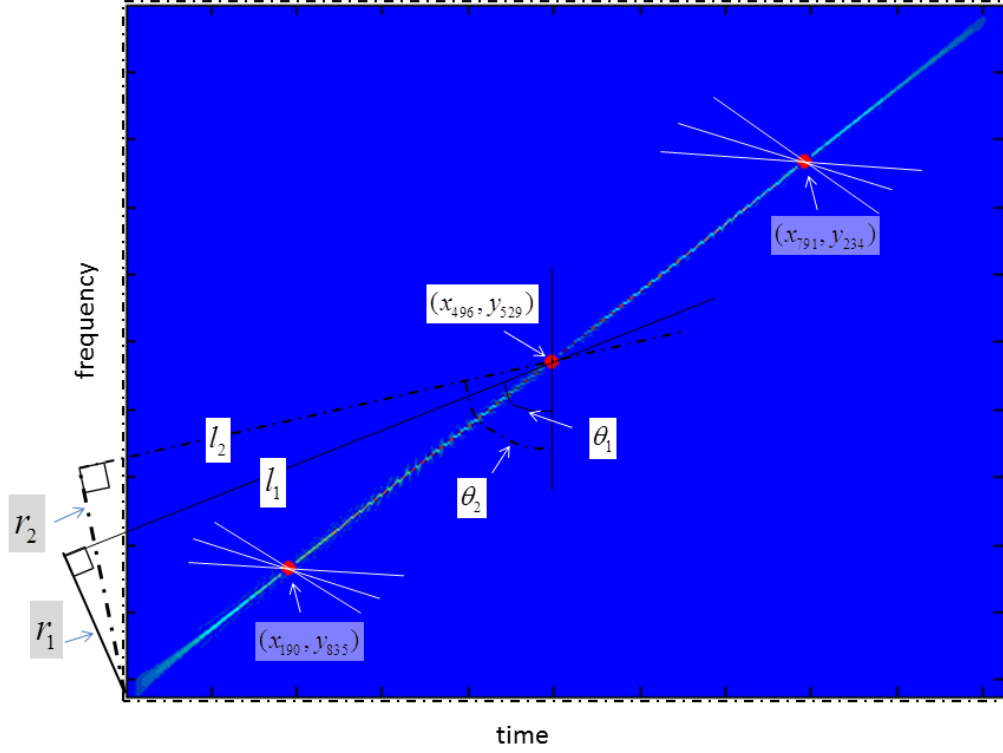


Figure 4.2 Three WVD points to be transformed to Hough space

On the upchirp, we show three $(x_i, y_j) \in \{f(x_i, y_j) \geq Th_{WD}\}$ points to be transferred to HT space. Countless lines can cross over each of these points, each line is transferred to HT space and represented by an (r, θ) parameter pair. At the end, these (r, θ) parameter pairs will create a curve for each (x_i, y_j) point. Figure 4.3 shows three curves created for the three points in Figure 4.2.

For points on the same line, one of the calculated (r, θ) coordinates, shown as $(r, \theta)_{UPCHIRP}$ in Figure 4.3, will be common. This common (r, θ) coordinate will be observed as a high intensity point in HT space. For all the (r, θ) coordinates calculated for an (x_i, y_j) point, we can use the intensity of the (x_i, y_j) point and this method will be explained as Wigner Hough-Radon Transform in this thesis. The different colors of

the curves in Figure 4.3 are used to represent the different intensity levels of the corresponding points.

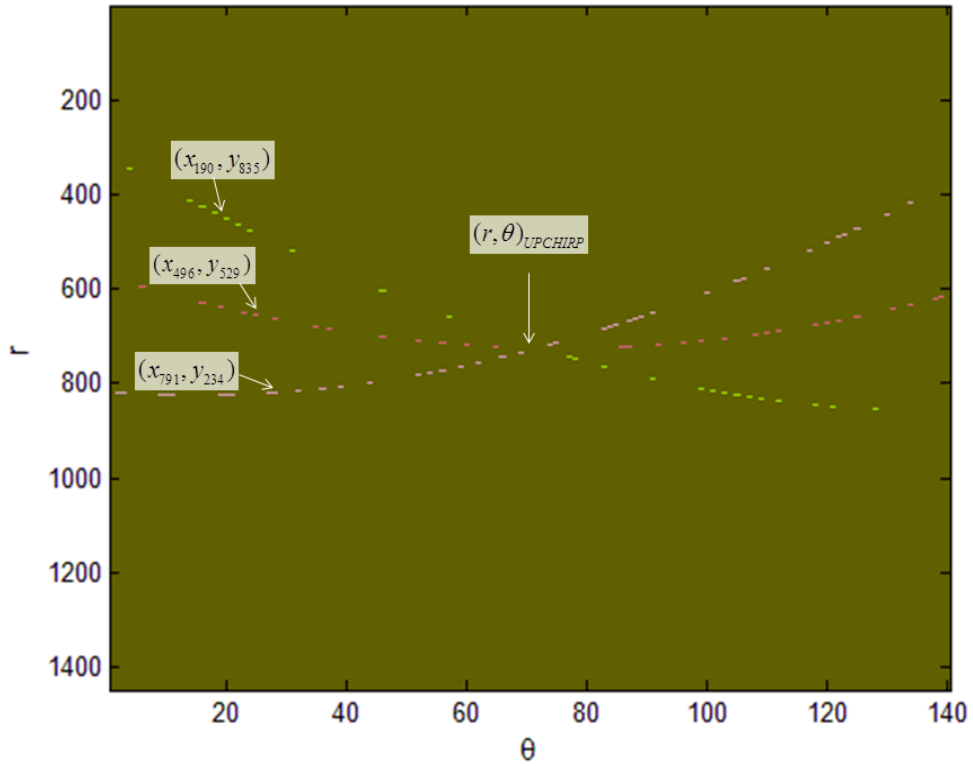


Figure 4.3 Hough transform of the points in Figure 4.2

4.3 Radon Transform

RT is the projection of image (WVD) intensity along a radial line, l , oriented at a specific angle. RT transforms a 2-D image with line-trends into a domain of line parameters. In RT, linear lines are represented with the same (r, θ) parameters as in HT. Using line description in (4.5), the RT of an $f(x, y)$ image can be given as [9], [77],

$$R(r, \alpha) = \int_{-\infty}^{+\infty} f(r \cos \alpha - l \sin \alpha, r \sin \alpha + l \cos \alpha) dl \quad (4.7)$$

where $(0 \leq \alpha < \pi)$ and l can be calculated as

$$l = y \cos \alpha - x \sin \alpha \quad (4.8)$$

(4.7) is the integral of $f(x, y)$ along the line described by (4.5) [78].

4.4 General Simulation Conditions for the Thesis

After this section we'll propose new WHT based FMCW detection and parameter extraction methods and analyse their performances with simulations. The simulations will be performed under the following general conditions. Additional simulation conditions will be mentioned in the corresponding section when applied.

1. Discrete PWVD as TF method,
2. Low sampling frequency for ease of simulations ($f_s = 8\text{kHz}$),
3. $N = 1024$ sample length signals, with sampling interval $t_s = 1/f_s$,
4. $M = 1024$ FFT bins (positive frequency bins) for WVD frequency localization,
5. $N \times M$ TF image dimensions,
6. At least two FMCW signal components in N sample length time window,
7. Different SNR levels to simulate different ranges,
8. Complex AWGN as thermal noise,
9. MATLAB software tool will be used on a PC with Intel(R) Core(TM) i7 CPU @ 2.2GHz x64-based processor.

4.5 Wigner Hough Transform

We can apply HT to the WVD of the LFM signal $s(t)$ during T_{obs} and we obtain WHT, with expression [12]:

$$\begin{aligned} WHT_s(w_0, \beta) &= \frac{1}{2\pi} \int_{T_{obs}} W_s(t, w_0 + \beta t) dt \\ &= \frac{1}{2\pi} \int_{T_{obs}} \int s(t - \frac{1}{2}\tau) s^*(t + \frac{1}{2}\tau) e^{-j\tau(w_0 + \beta t)} d\tau dt \end{aligned} \quad (4.9)$$

This expression is actually derived from the basic line formula, where chirp-rate $\beta = T_m/B$ and $f_1 = f_0 + \beta t_1$. (Note: In our studies we use an angle value for chirp rate $\arctan(\beta)$). A basic line, representing an up-chirp signal with β is shown in an example T-F space in Figure 4.4 to illustrate this expression. The WHT is compared against ζ and the estimates of (w_0, β) are derived from the coordinates of the detected peak in the Hough space. The polar coordinates (r, θ) of the detected peak are transformed to Cartesian coordinates (w_0, β) , in other terms (t, w) .

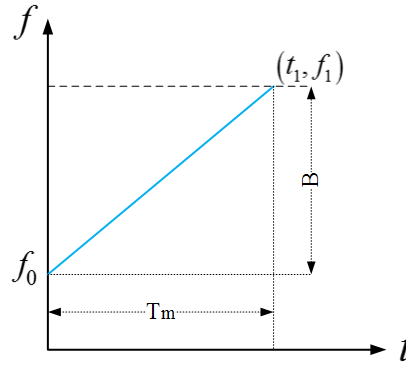


Figure 4.4 An example LFM signal in an example T-F space.

In our application, the number of lines transferred to (r, θ) space is kept in counters, each time we have the same (r, θ) pair for a given point in $f(x, y)$ image we increase the value of the counter by 1 for that (r, θ) point. For a line passing through $(x_i, y_j) \in \{f(x_i, y_j) \geq Th_{WD}\}$ point with $\alpha = \alpha_l$ angle, we can use equation $x_i \cos \alpha_l + y_j \sin \alpha_l = r_k$ and increase the counter by one [8].

$$C(r_k, \theta_l) = C(r_k, \theta_l) + 1 \quad (4.10)$$

4.5.1 FMCW Parameter Extraction Using WHT

Finding the coordinates of the maximum intensity points in HT space is equal to finding FMCW signal components in WVDs. Figure 4.5 shows an example WVD of a two component up-chirp FMCW signal and its corresponding HT. Investigating the HT space, it can be seen that the FMCW components have the same θ angles but different r values.

In WHT method first, a coarse θ value is calculated and then it is fine tuned in 0.1° resolution in order to increase the computational efficiency and the detection sensitivity of the chirp rates. (r, θ) values of FMCW components are found by locating the maximum value in the HT space and the angle where maximum value occurs is denoted by θ_0 .

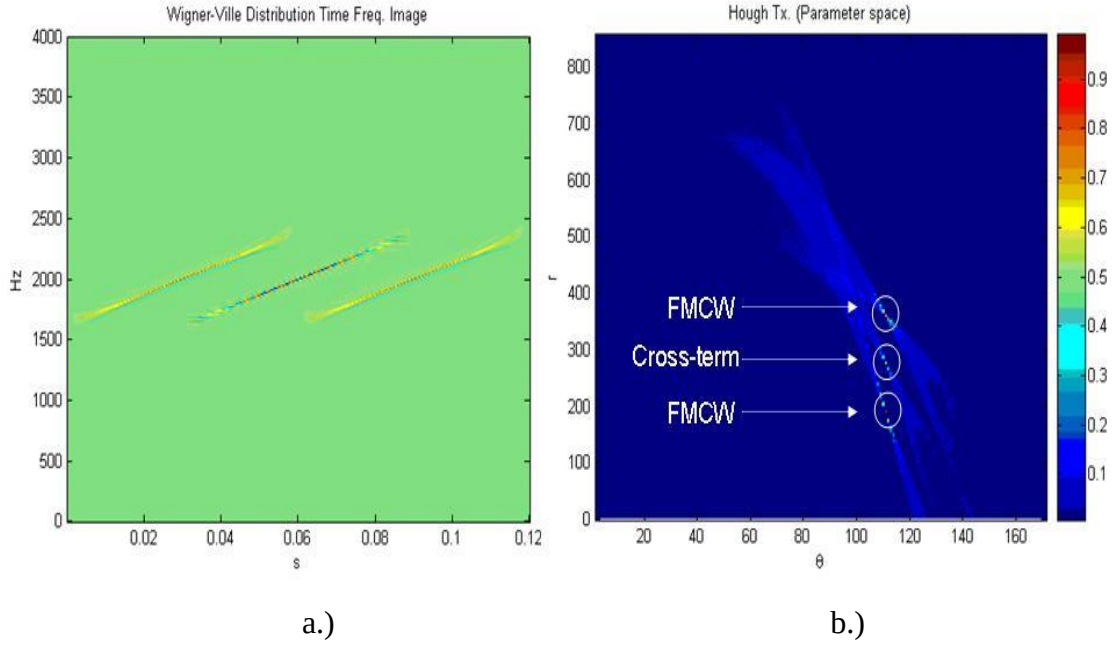


Figure 4.5 FMCW signal a.) WVD TF representation b.) HT image

The column at θ_0 is projected to a vector as shown in Figure 4.6 and the distance (d) between maximum peaks (FMCW components) is measured.

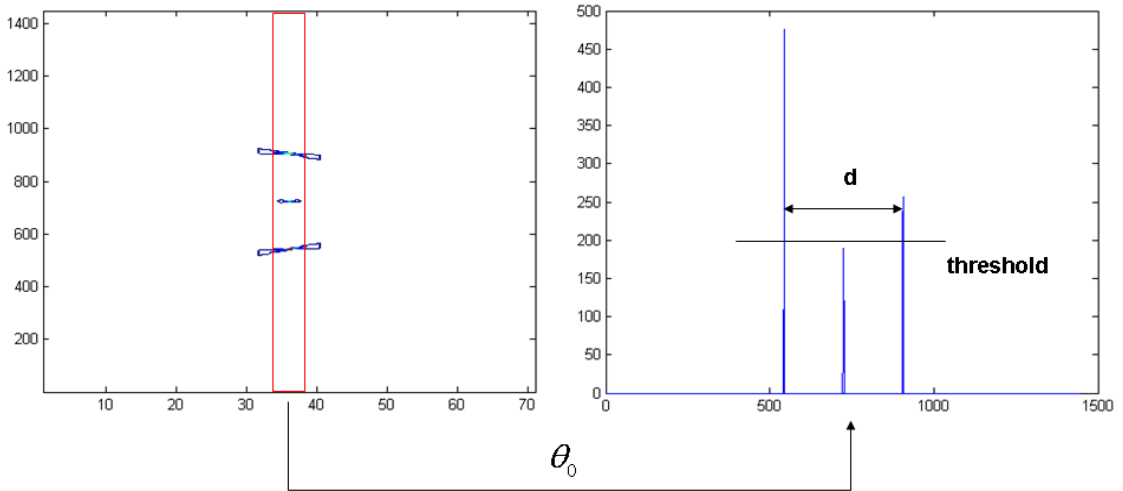


Figure 4.6 Vector cropping at θ_0

A heuristic threshold is used here to eliminate the peak representing the cross term. Once θ_0 and d are calculated as defined above, the remaining parameters are derived by using the geometric relations given in Figure 3.8. B and T_m parameters can be calculated by the following formulas;

$$\alpha = 90 - \theta_0$$

$$B = \frac{d}{\cos \alpha}, (\theta_0 > 90^\circ) \quad \text{or} \quad B = \frac{d}{\sin(\alpha - 90)}, (\theta_0 < 90^\circ) \quad (4.11)$$

$$T_m = \frac{B}{\tan(\alpha)}$$

The carrier frequencies of FMCW waveforms are extracted directly from the WVDs. The maximum intensity levels in WVD of FMCW waveforms occur at points corresponding to carrier frequencies. These points may occur on either the auto-term or on the cross-term. The flow diagram of the WHT based FMCW detection and parameter extraction algorithm is presented in Figure 4.7 [8].

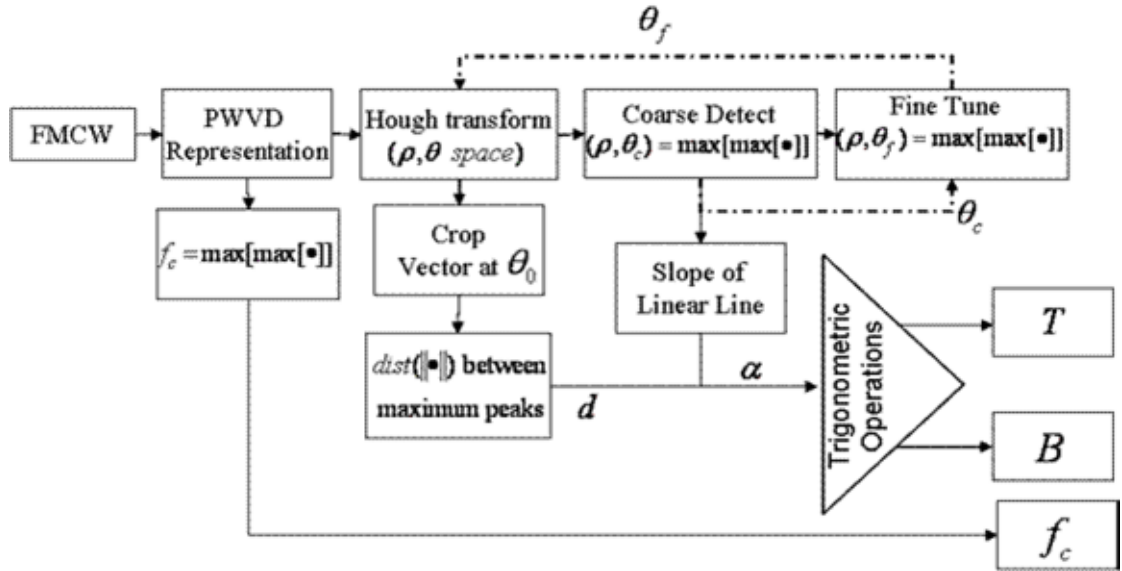


Figure 4.7 Flow diagram of WHT based FMCW detection and parameter extraction

The algorithm is tested with signals in Table 4.1 [8].

Table 4.1 Signal database to be used in parameter extraction studies using WHT

Signal	fc (Hz)	B (Hz)	T (ms)
F1	1500	1050	60
F2	1500	1380	40
F3	1500	720	50
F4	1930	1050	40
F5	1930	1380	50
F6	1930	720	60
F7	2145	1050	50
F8	2145	1380	60
F9	2145	720	40

To check the robustness of the algorithm against noise, signals are created with -6, -3, 0, 3 and 6 dB SNR level. The simulation results presented in Table 4.2 showed that the algorithm proved well for SNR levels down to -6 dB.

Table 4.2 FMCW parameters extracted by WHT and relative error rates [8]

Signal	SNR (dB)	Extracted Parameters			Error Percentage		
		fc (Hz)	B (Hz)	T (s)	fc_err	B_err	T_err
F1	6	1501	1057	0.0603	0.10	0.68	0.47
F2		1501	1376	0.0401	0.10	0.32	0.13
F3		1501	727	0.0504	0.10	1.00	0.73
F4		1951	1075	0.0401	1.09	2.40	0.36
F5		2002	1402	0.0504	3.73	1.60	0.72
F6		1935	729	0.0604	0.28	1.19	0.70
F7		2162	1047	0.0502	0.81	0.28	0.48
F8		2119	1375	0.0601	1.20	0.38	0.18
F9		2178	722	0.0400	1.53	0.23	0.00
F1	0	1501	1045	0.0603	0.10	0.47	0.55
F2		1501	1387	0.0401	0.10	0.49	0.24
F3		1501	712	0.0502	0.10	1.11	0.46
F4		1928	1057	0.0402	0.12	0.68	0.43
F5		1982	1382	0.0500	2.72	0.15	0.02
F6		1935	724	0.0603	0.28	0.54	0.57
F7		2162	1047	0.0502	0.81	0.28	0.48
F8		2068	1390	0.0601	3.57	0.71	0.17
F9		2178	734	0.0400	1.53	1.90	0.04
F1	-6	1451	1069	0.0602	3.29	1.83	0.38
F2		1501	1367	0.0399	0.10	0.93	0.13
F3		1501	724	0.0499	0.10	0.49	0.25
F4		1877	1028	0.0403	2.75	2.07	0.87
F5		1963	1363	0.0500	1.70	1.21	0.02
F6		1935	720	0.0600	0.28	0.05	0.01
F7		2162	1075	0.0502	0.81	2.35	0.45
F8		2186	1396	0.0602	1.90	1.19	0.28
F9		2201	750	0.0403	2.63	4.22	0.70

The error between the extracted parameters and the true values were relatively too small, providing high accuracy rates in average. If a_m is a measured value and a_e is the exact value of a quantity, the absolute value of the relative error $\mathcal{E}_r$ is defined by [7]

$$\mathcal{E}_r = \left| \frac{a_m - a_e}{a_e} \right| = \left| \frac{\text{Error}}{\text{True value}} \right| \quad (4.12)$$

4.6 Wigner Hough-Radon Transform

We showed the similarity between HT and RT. The main difference between the two transforms is: in HT we only calculate the lines passing through points above a certain threshold and count the number of points on the same line; in RT we integrate the

intensity values of points on the same line regardless of any threshold. When calculating RT for a given angle, all the pixels in the image are used and this is a costly process.

HT is more efficient in terms of processing cost, but since it uses only a limited number of WVD points, cross-terms created by WVD may cause detection errors under low SNR conditions. The deficiencies of these two methods can best be overcome by combining the advantageous parts of each [79].

In RT, the cross-terms in the WVD integrate to a relatively small value due to negative and positive oscillations present in their structure. If HRT is applied to the WVD to integrate only the values above a given threshold Th_{WD} , cross-term minimization property of RT will not be exploited. In order to maintain this advantage of RT, WHRT is applied to integrate the intensity (I) values both above Th_{WD} and below $-Th_{WD}$.

$$\begin{aligned} &\text{if } (x_i, y_j) \in \left\{ f(x_i, y_j) \geq Th_{WD} \text{ or } f(x_i, y_j) \leq -Th_{WD} \right\} \text{ then} \\ &I(r_k, \theta_l) = I(r_k, \theta_l) + I_{new}(r_k, \theta_l) \end{aligned} \quad (4.13)$$

4.6.1 FMCW Parameter Extraction Using WHRT

In [7], Gulum (et al.) managed to extract polyphase radar modulation parameters using a computationally expensive Wigner-Radon method and in the previous section we used WHT as a computationally more efficient tool for FMCW parameter extraction. This latter method performed well down to SNR levels of -6 dB, however noise levels made it hard to distinguish between WVD cross-terms and auto terms below -6 dB SNR. In this section, we use a combination of Hough and Radon transforms to identify modulation parameters. This algorithm can extract FMCW modulation parameters at low SNR levels, such as -9 dB, efficiently [9]. Figure 4.8 shows WVD of a two component upchirp FMCW at -9 dB SNR, with $f_c = 1050$ Hz, $B = 1050$ Hz, $T = 60$ ms and $f_s = 8000$ Hz. Parameter extraction algorithm using WHT was not successful enough under challenging TF images like this. The algorithm using WHRT provides the required improvement on detecting the signal components even at -9 dB SNR levels. The improvement provided by the WHRT over WHT, can be observed by comparing Figure 4.9 and Figure 4.10. In Figure 4.9, it is difficult to distinguish between the auto-terms and cross-terms whereas in Figure 4.10, the cross-terms are evidently suppressed and the auto-terms are emphasized.

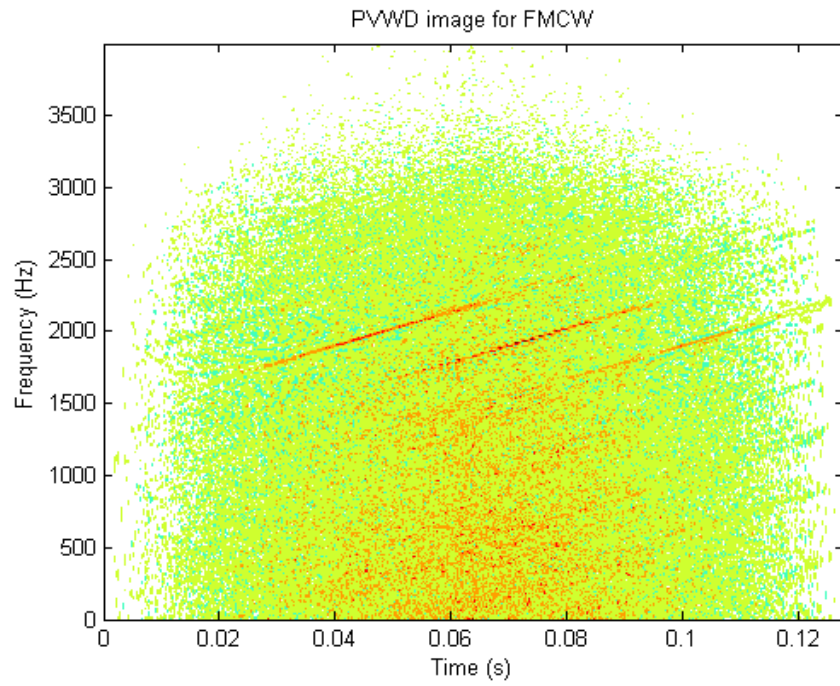


Figure 4.8 FMCW WVD at -9 dB SNR [9]

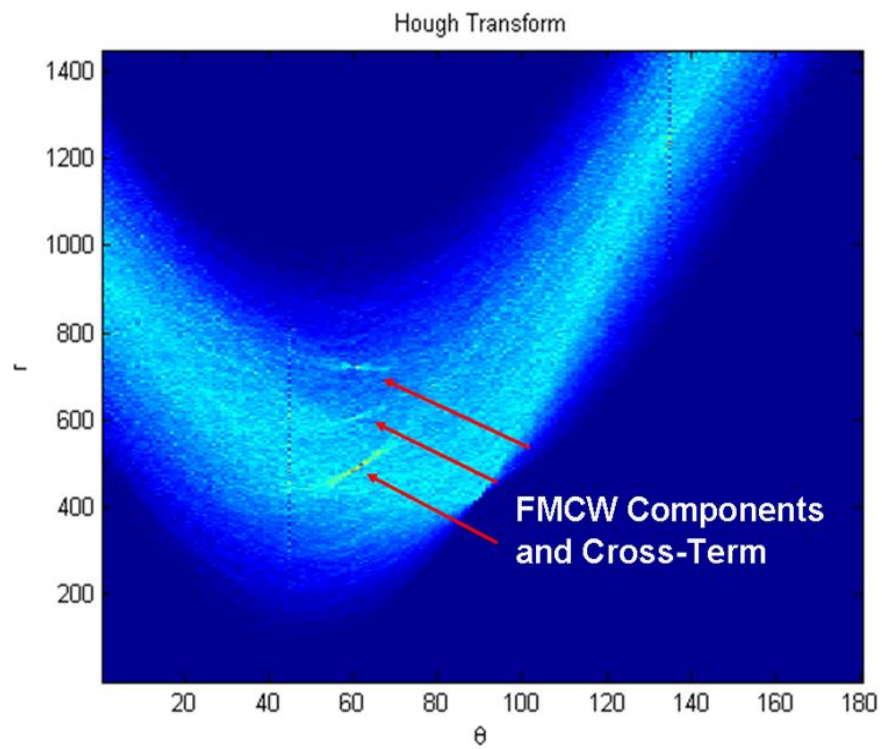


Figure 4.9 WHT of Figure 4.8 [9]

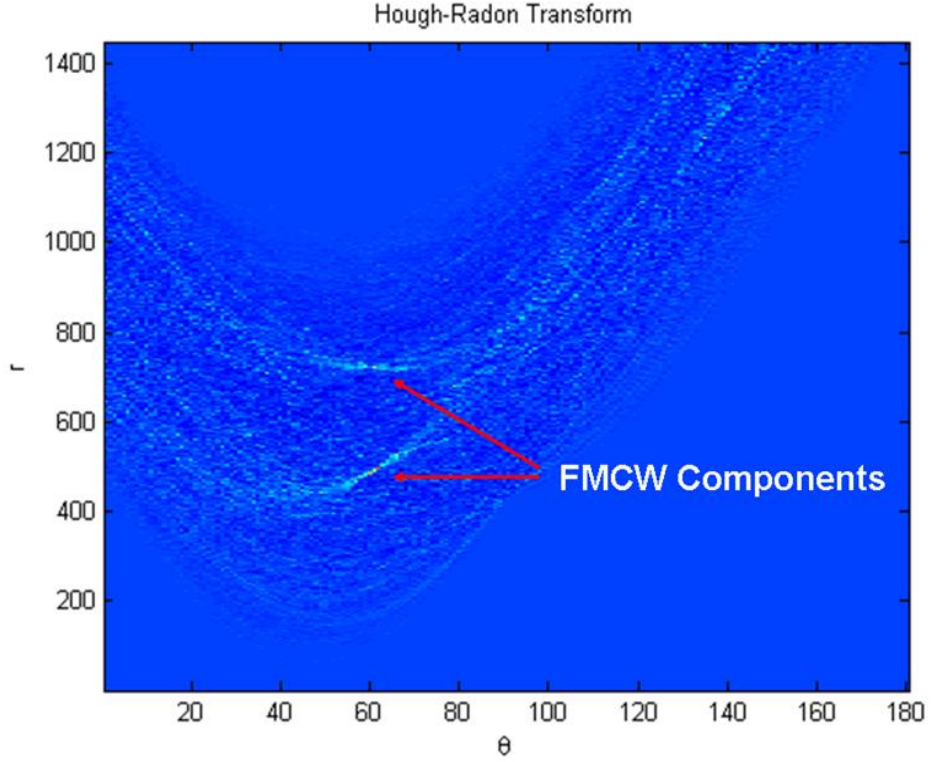


Figure 4.10 WHRT of Figure 4.8 [9]

As mentioned before, finding the (r, θ) coordinates of maximum intensity points in WHRT plane is equal to finding FMCW signal components in WVD. In HT, the maximum intensity points may correspond to either an auto-term or a cross-term, in WHRT however, maximum intensity points directly correspond to auto-terms.

In Figure 4.11, we can see the effect of noise on HT, and Figure 4.12 shows the improvement provided by WHRT, which makes the signal components distinct from noise in the cropped vector. The number of FMCW components in the WVD can easily be detected from both Figure 4.10 and Figure 4.12.

For parameter extraction; the projection vector (cropped vector) is smoothed with a Wiener filter as in [7]. Following smoothing, the projection vector is thresholded to obtain a noise free vector which allows a practical distance measurement. Once θ_0 and d are calculated, the remaining parameters are derived by using the geometric relations as defined in section 4.5.1.

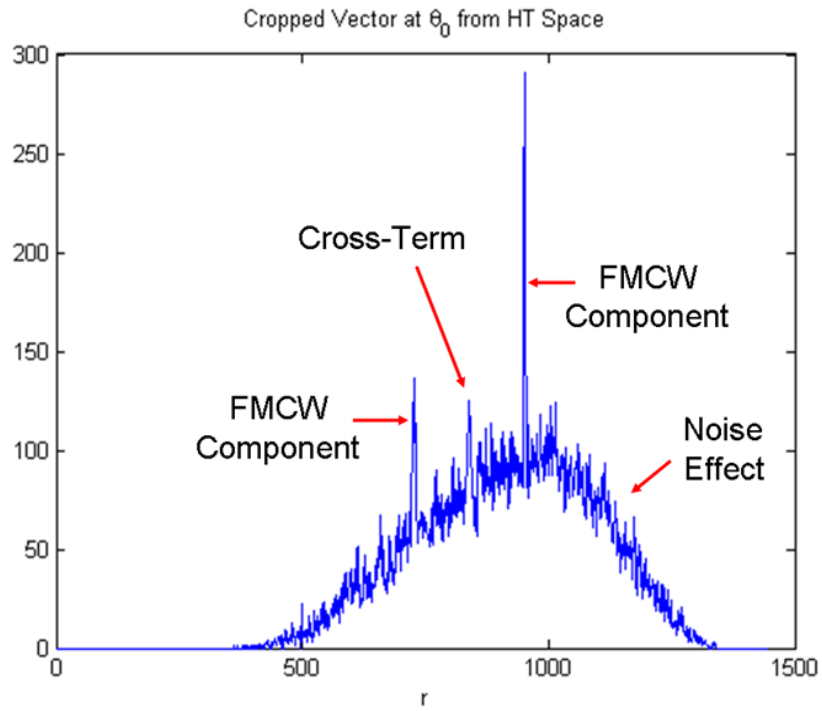


Figure 4.11 Cropped vector from WHT space in Figure 4.9 [9]

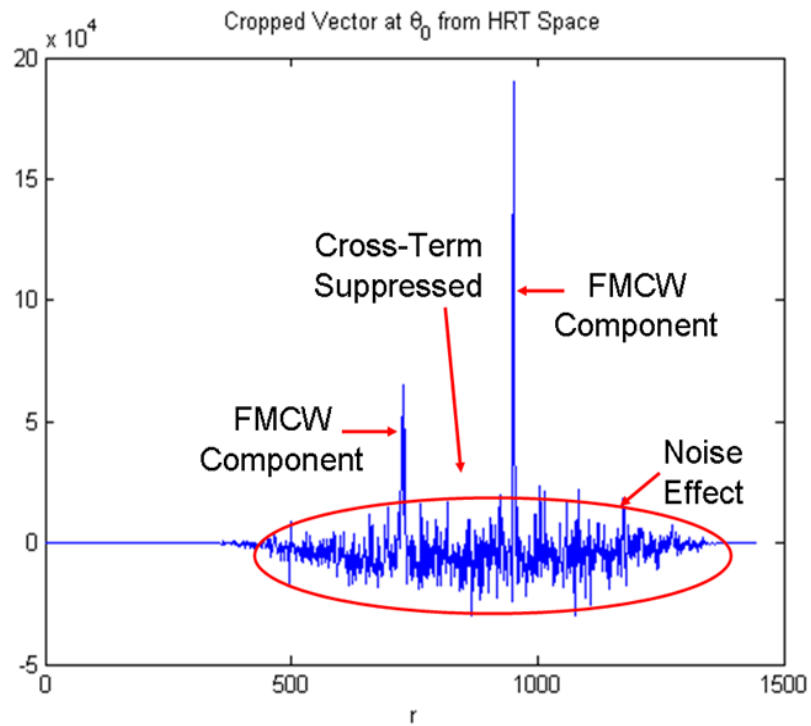


Figure 4.12 Cropped vector from WHRT space in Figure 4.8 [9]

The flow diagram of the WHRT based FMCW detection and parameter extraction algorithm is presented in Figure 4.13 [9].

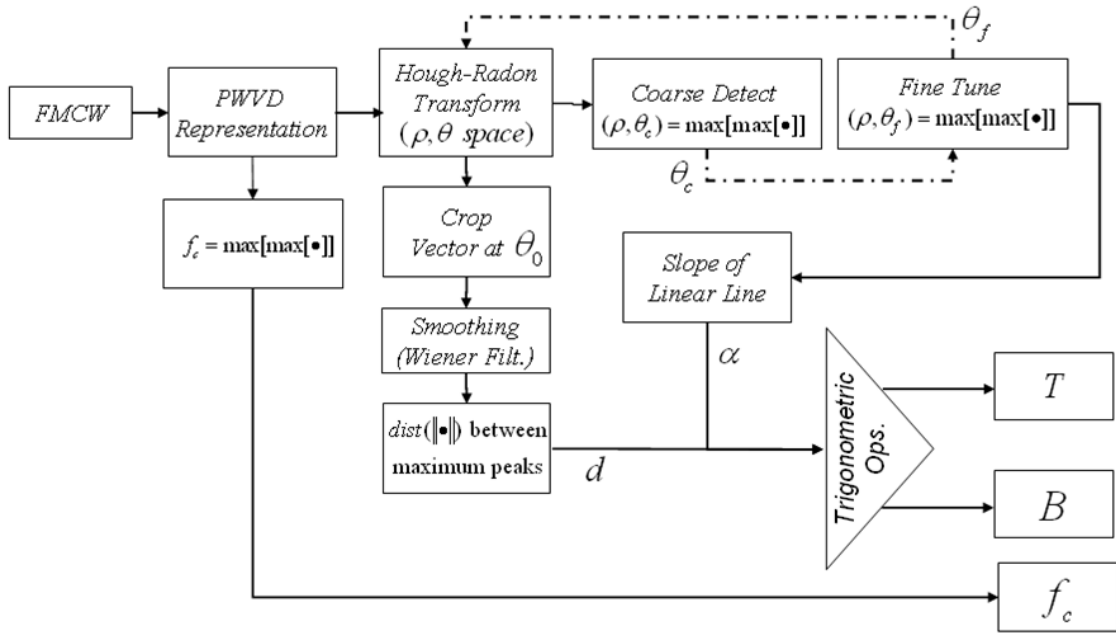


Figure 4.13 Flow diagram of WHRT based FMCW detection and parameter extraction

This algorithm is tested with the same signal data base given in Table 4.1, to be able to observe the improvement provided by WHRT. The robustness of the algorithm against noise is tested for -9, -6, -3, 0, 3 and 6 dB SNR levels. The simulation results are presented in Table 4.3 for 3, -3 and -9 dB SNR levels. Both the extracted parameter values and the relative error percentages are presented. The results showed that, the algorithm proved well even at low SNR levels of -9 dB for B and T . The error between the extracted parameters and the true values were relatively small providing high accuracy rates in average. However, at -9 dB SNR this approach produces inconsistent results for f_c . Due to the high noise level present in the WVD, an exact estimation based on the maximum intensity level in WVDs becomes very hard.

Table 4.3 FMCW parameters extracted by WHRT and relative error rates [9].

Signal	SNR (dB)	Extracted Parameters			Error Percentage		
		fc (Hz)	B (Hz)	T (s)	fc_err	B_err	T_err
F1	3	1501.5	1057.1	0.0603	0.10	0.68	0.47
F2		1501.5	1397.9	0.0401	0.10	1.30	0.34
F3		1501.5	716.89	0.0503	0.10	0.43	0.68
F4		1927.7	1075.2	0.0401	0.12	2.40	0.36
F5		1982.4	1391.7	0.0500	2.72	0.84	0.03
F6		1935.5	724.35	0.0601	0.28	0.60	0.12
F7		2142.7	1042.4	0.0500	0.11	0.73	0.03
F8		2134.9	1389.8	0.0601	0.47	0.71	0.17
F9		2127.1	727.67	0.0400	0.84	1.07	0.02
F1	-3	1517.1	1058.2	0.0601	1.14	0.78	0.16
F2		1501.5	1370.3	0.0402	0.10	0.70	0.45
F3		1501.5	716.89	0.0503	0.10	0.43	0.68
F4		1939.4	1063.8	0.0403	0.49	1.31	0.70
F5		1982.4	1377.9	0.0502	2.72	0.15	0.38
F6		1935.5	709.42	0.0604	0.28	1.47	0.66
F7		2162.3	1047	0.0502	0.81	0.28	0.48
F8		2002	1389.8	0.0601	6.67	0.71	0.17
F9		2127.1	690.27	0.0403	0.84	4.13	0.86
F1	-9	609.97	992.85	0.0593	59.34	5.44	1.25
F2		1051.8	1389.4	0.0400	29.88	0.68	0.08
F3		50.831	702.35	0.0500	96.61	2.45	0.02
F4		1876.8	1070.5	0.0404	2.75	1.95	0.98
F5		739	1380	0.0501	61.71	0.00	0.18
F6		1935.5	695.54	0.0602	0.28	3.40	0.28
F7		0	1037.7	0.0498	100.00	1.18	0.42
F8		2002	1381.4	0.0602	6.67	0.10	0.29
F9		2177.9	727.67	0.0400	1.53	1.07	0.02

4.7 Modified Wigner Hough Transform

In Modified Wigner Hough Transform (MWHT), the counter condition mentioned in WHT is changed considering the properties of WVD and FMCW signal to be detected. Other than counting the number of points above a threshold on the line, we subtract 1 from the counter for each point (on the line) whose intensity value is below negative threshold. This helps us increase the counter for auto-terms, and preserves the counter for cross-terms from increasing due to their oscillating intensity values. In this method the counter is updated as follows [10];

$$C(r_k, \theta_l) = \begin{cases} C(r_k, \theta_l) + 1 & \text{if } (x_i, y_j) \in \{f(x_i, y_j) \geq Th_{WD}\} \\ C(r_k, \theta_l) - 1 & \text{if } (x_i, y_j) \in \{f(x_i, y_j) \leq -Th_{WD}\} \end{cases} \quad (4.14)$$

4.7.1 FMCW Parameter Extraction Using MWHT

MWHT was proposed in [10] as a method to suppress cross-terms in WVD. Figure 4.14 shows the HT of a two component up-chirp FMCW signal and Figure 4.15 shows the MWHT of the same signal.

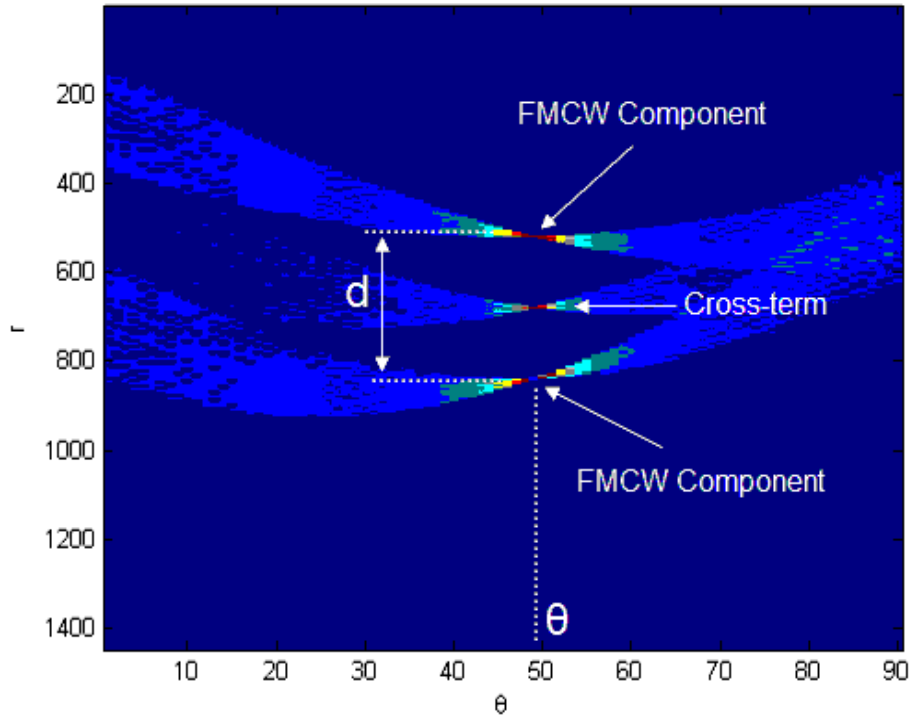


Figure 4.14 Sample two component up-chirp FMCW signal WHT [10]

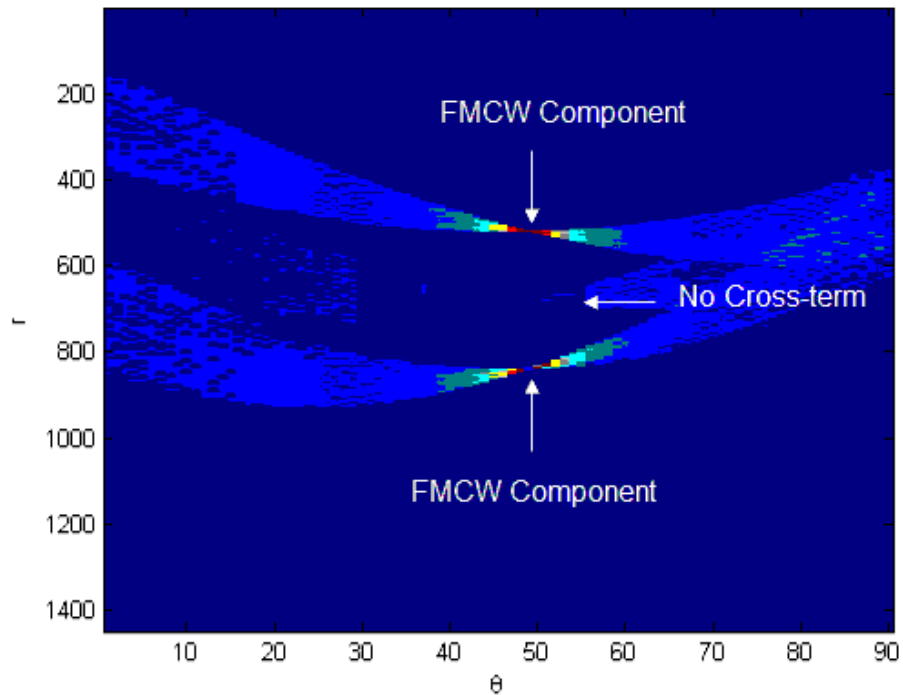


Figure 4.15 Sample two component up-chirp FMCW signal MWHT [10]

The MWHT suppresses the cross-term, therefore parameter extraction can be performed without sophisticated thresholding and smoothing techniques. The parameter extraction algorithm flow diagram using the MWHT is almost the same as in Figure 4.7. The analysis of this method will be performed by comparing it with the other methods in chapter 5.

4.8 Cross Wigner Hough Transform

We've seen that, if the signal to be analyzed is composed of more than one component, WVD creates cross-terms. Although the cross-terms are considered as undesirable, they are necessary for unitarity property to hold. And unitarity property, i.e. Moyal's formula, is important to derive a closed-form solution for LFM signal analysis based on WHT methods [13]. The existence of cross-terms is not always to be avoided, as we have presented in section 3.2, they also obtain valuable information that can be used in radar signal interception efforts.

The literature is rich in terms of studies to suppress the cross-terms and enhance the TF resolution however; little work has been performed in radar signal detection area on the utilization of information contained in cross-terms. In [80] cross-terms were deliberately formed between an analyzed signal and a reference signal. Since the geometric locations of the cross-terms follow certain rules, the presented method used these geometrical properties of the cross-terms in order to estimate the instantaneous frequency of the components of the analyzed signal. The study also tried to optimize the spread parameters of the reference signal to get a well localized signal in time and frequency. A similar method was used in [81], to suppress the cross-terms by multiplying the WVD with a 2-D mask. The mask was designed by exploiting the geometric properties of the cross-terms created between the analyzed signal and circularly symmetric Gaussian atom functioning as a reference signal.

The methods mentioned above use the properties of the cross-terms to exploit them on WVD space. We haven't seen any method utilizing the properties of the cross-terms to detect or unveil the parameters of the LFM signals on HT space. In this thesis, we are proposing the XWHT method that will utilize the properties of the cross-terms in HT space for radar signal detection and parameter extraction purposes.

The creation of cross-terms and their unique properties have been explained in chapter 3. In Figure 4.16 we present WVD mesh plot of a two-component FMCW upchirp

signal in 6 dB and -6 dB SNR cases. In both cases the high intensity cross-terms between two FMCW signal components can easily be distinguished.

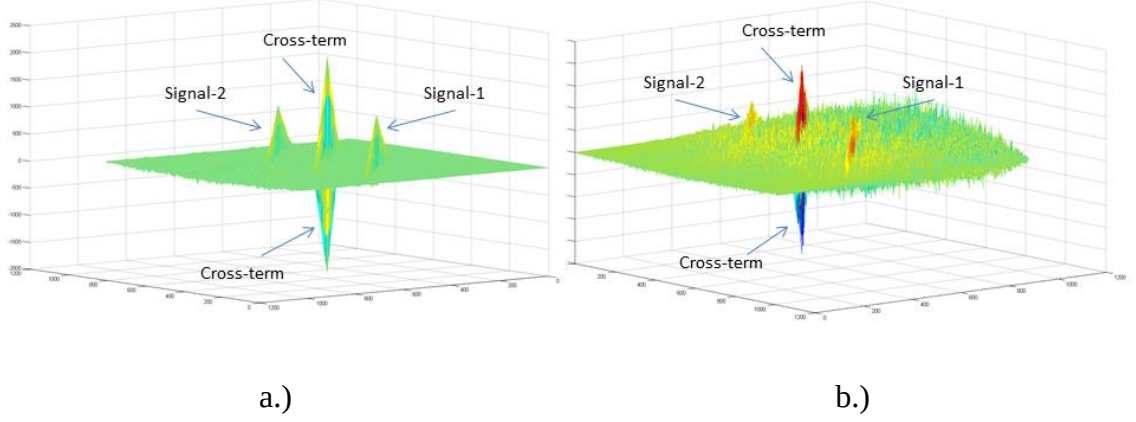


Figure 4.16 WVD Mesh plot of a 2 Component FMCW up-chirp signal a.) at 6 dB b.) at -6 dB

XWHT uses the existence of these higher intensity cross-terms created by WVD as an advantage. In the WVD of LFM signals the cross-terms will have the same slope as the signal components. Considering the properties of cross-terms, we propose to create a counter for only negative intensity values so that, the cross-terms inherent in WVD protrude significantly in HT. We increase the counter by one for each line crossing a pixel below the negative threshold. In this case if $x_i \cos \alpha_l + y_j \sin \alpha_l = r_k$ than the counter is updated as;

$$C(r_k, \theta_l) = C(r_k, \theta_l) + 1 \quad \text{if } (x_i, y_j) \in \{f(x_i, y_j) \leq -Th_{XWD}\} \quad (4.15)$$

In Figure 4.17 we can see that the negative intensity parts of the WVDs are more appropriate for putting a threshold both for low and high SNR cases. The positive intensity parts contain both the cross-term and signal components; therefore more attention is required in the detection process if this part will be used. On the other hand, we know that the negative intensity part contains only the cross-term and this is more obvious in Figure 4.17.b. With the choice of a correct threshold, the counted lines will be those passing through points on the cross-terms. The process of finding the correct threshold is explained in chapter 5.

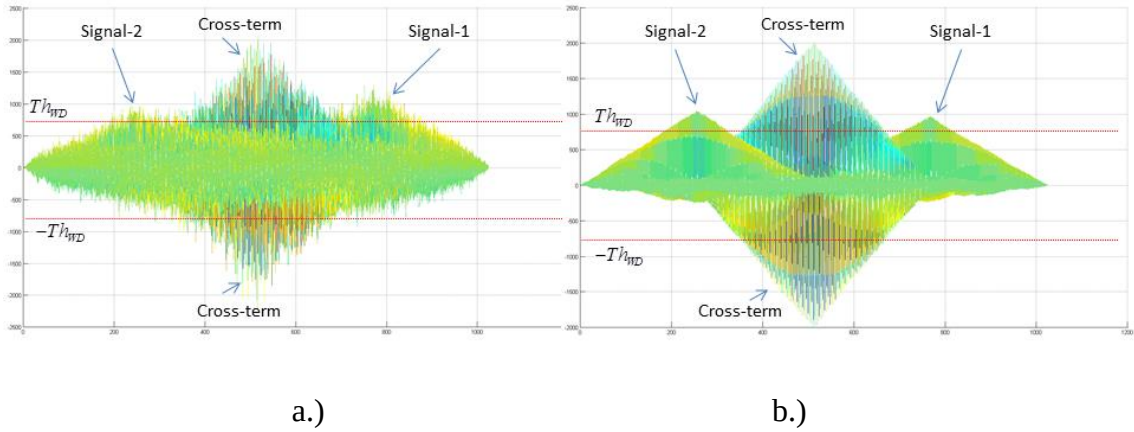


Figure 4.17 X-Z view of WVD mesh plot of a 2 Component FMCW up-chirp signal a.) at -6dB b.) at 6dB

4.8.1 FMCW Parameter Extraction Using XWHT

For parameter extraction from any of the transform methods mentioned so far, we crop the vector at θ holding the peak value of the transform space and measure d . A heuristic threshold is used to eliminate the peak representing the cross term. In this part of the thesis we propose another parameter extraction algorithm using the XWHT. We analyze the performance of the method using the signals given in Table.4.4.

Table 4.4 Test signal database

	θ°	$B(\text{Hz})$	$f_s(\text{Hz})$	$f_c(\text{Hz})$	$T_m(\text{msec})$
Signal 1	30	3464	8000	2000	64
Signal 2	45	2000			
Signal 3	60	1155			
Signal 4	75	536			

All the signals are modeled in 10 different SNR levels (9, 6, 3, 0, -3, -6,-7,-8,-9,-10dB). At low SNR levels we model the signals in 1 dB steps to investigate the performance with finer resolution. In order to fit at least two signal components in 1024 sample length time, $T_m = 64\text{msec}$ is used for all signals. Each signal at each SNR level is created 100 times using complex AWGN.

In this method we make use of the θ vectors. In Figure 4.18 we present the examples of these θ vectors derived from all transform methods at -9 dB for S3(•) (Signal 3). Each of these vectors is just one example of one hundred simulations for S3(•) at -9 dB SNR.

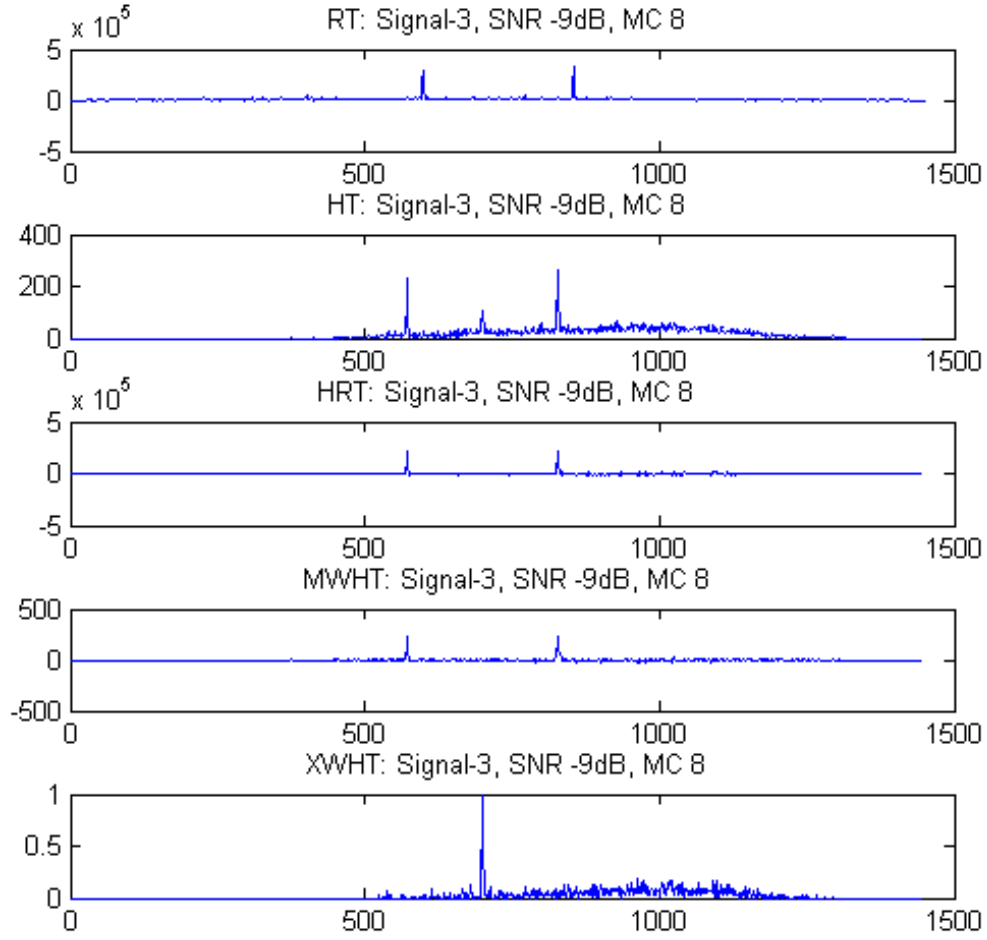


Figure 4.18 θ detection vectors of each transform method for Signal-3

As can be seen from the second plot of Figure 4.18, noise affects HT more than the other methods. WRT, WHRT and MWHT are robust to noise effects since they also use negative amplitudes to alleviate the effects of cross-term negative peaks. The main reason for presenting this figure is to show that with XWHT, just a simple thresholding method can be used for detection decision.

The flow diagram of the algorithm proposed in this section, which automatically detects FMCW signal components and performs parameter extraction is shown in Figure 4.19. Our goal in this method is to find θ with XWHT as soon as possible and then calculate d . After detecting the FMCW signal and its chirp-rate with the help of the cross-terms, we need the signal components to calculate d . At this point, HT can be employed for only one specific chirp-rate determined by XWHT at the detection stage. Then we obtain two vectors at the same chirp-rate θ , XWHT vector holding the cross-term and HT vector holding both the signal components and cross-term.

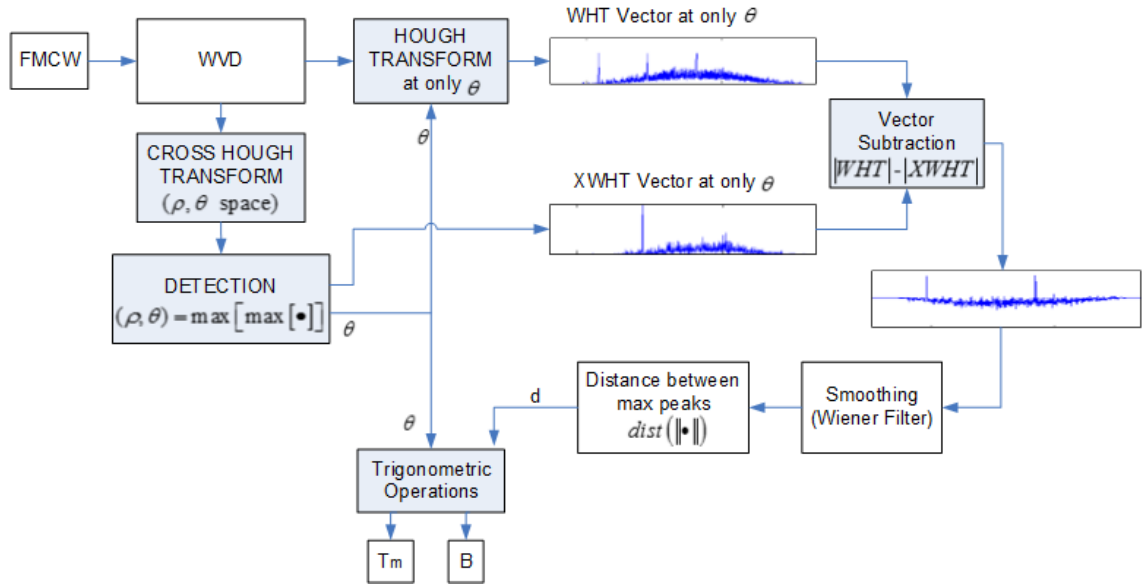


Figure 4.19 Proposed detection and parameter extraction algorithm flow diagram using XWHT

Figure 4.20 shows the effect of noise and cross-terms on the HT vector we want to use in parameter extraction. Obviously the correct calculation of d depends on avoiding the effects of noise and cross-terms.

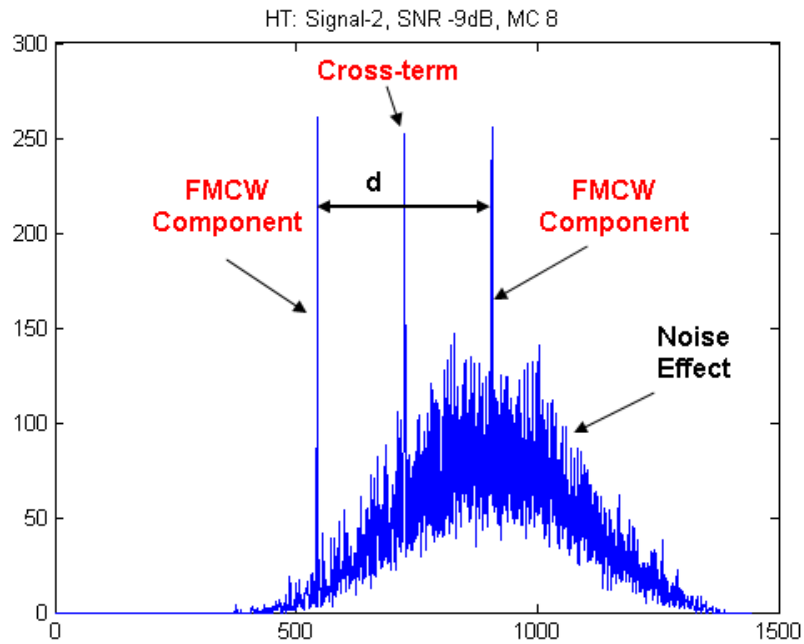


Figure 4.20 Signal components, cross-term and noise effect on HT θ detection vector at -9 dB SNR

Considering that XWHT is actually HT with only negative threshold, subtracting XWHT vector from HT vector suppresses cross-term and noise effect and it gives us a

rather clear vector including mostly signal components, for parameter extraction. Figure 4.21 illustrates the result of this vector subtraction for $S1(\cdot)$ at -12 dB SNR.

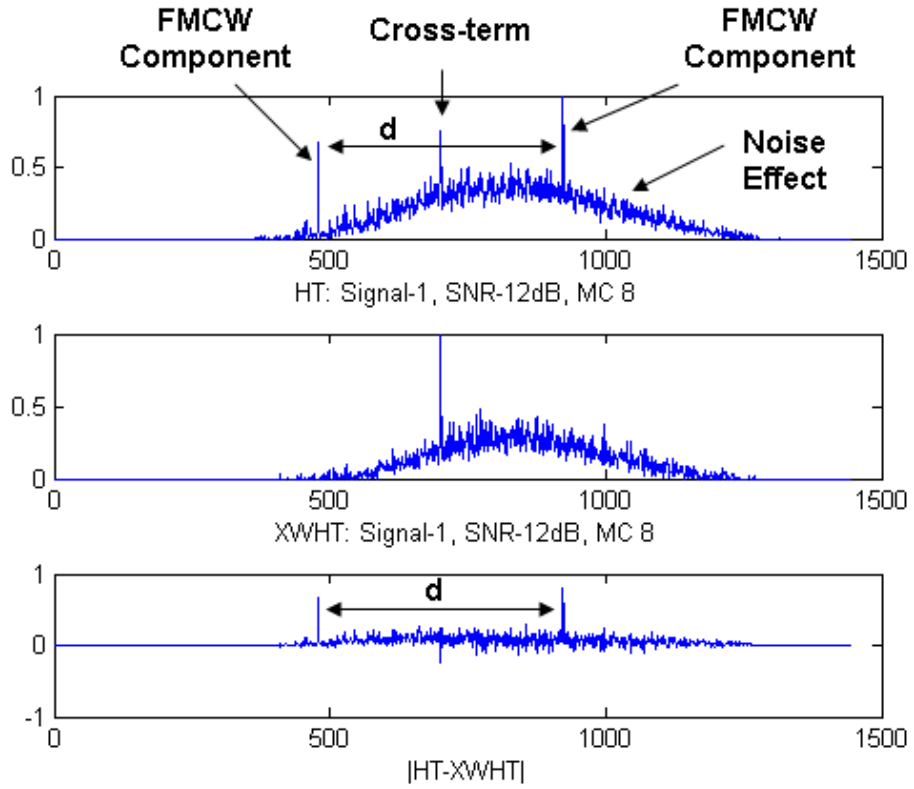


Figure 4.21 $|HT-XWHT|$ Vector for $S1(\cdot)$ at -12dB SNR

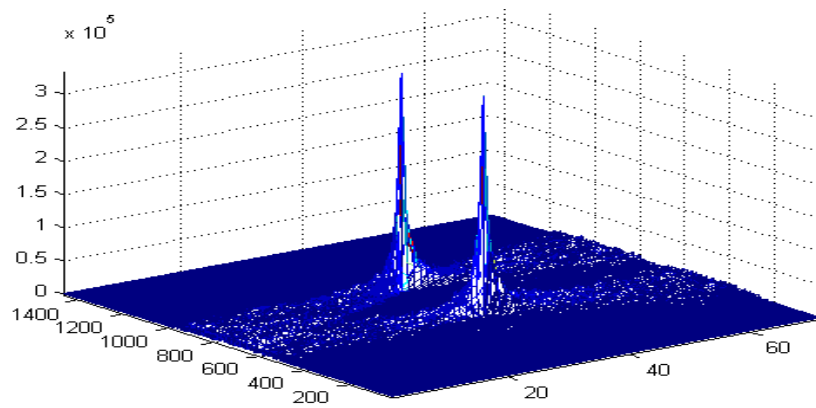
The analysis of this method will be performed by comparing it with the other methods in chapter 5.

CHAPTER 5

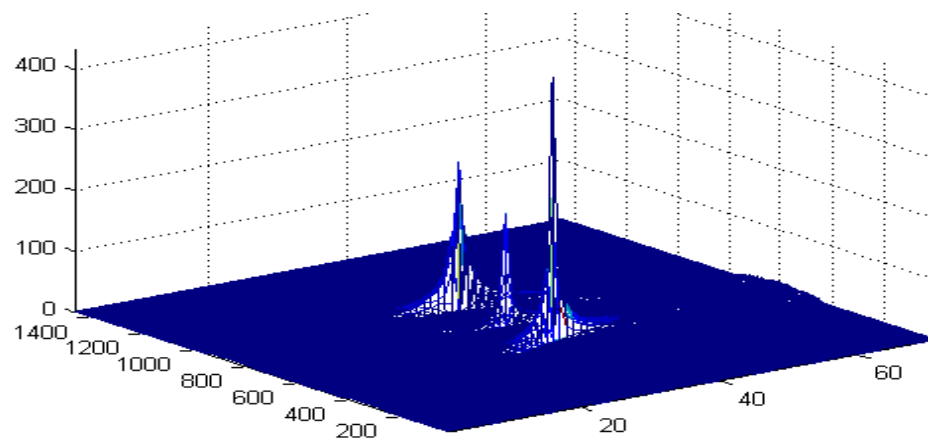
PERFORMANCE COMPARISON of WHT BASED PARAMETER EXTRACTION METHODS

Individual parameter extraction performance analysis of transform methods have been performed in the previous chapter. In this chapter, we analyze the efficiency of WHT based algorithms by comparing the methods with each other in terms of parameter extraction performance and transform speed.

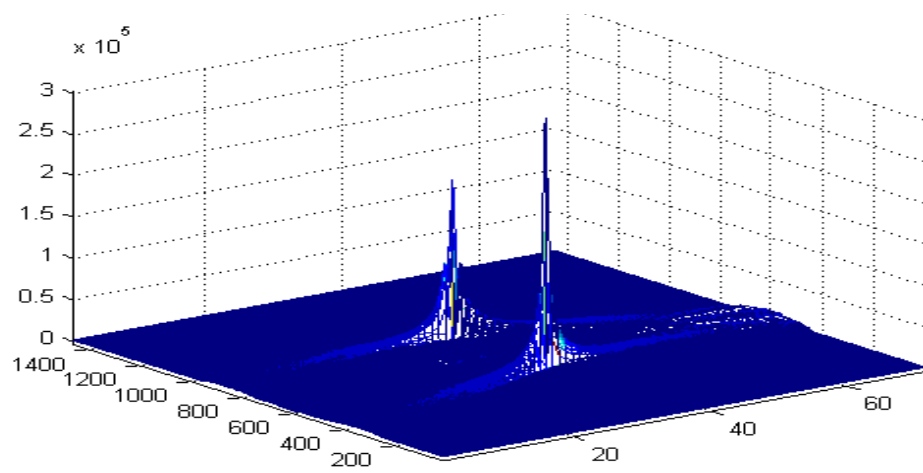
Figure 5.1 shows examples of 3D plots of transform spaces for $S2(\cdot)$ in 0 dB SNR. Figure 5.1.a shows the computationally expensive, WRT space. Figure 5.1.b shows WHT space with the cross-term between auto-terms, Figure 5.1.c and Figure 5.1.d show that the WHRT and MWHT methods suppress cross-terms respectively and Figure 5.1.e shows the cross-term only 3D plot derived by XWHT.



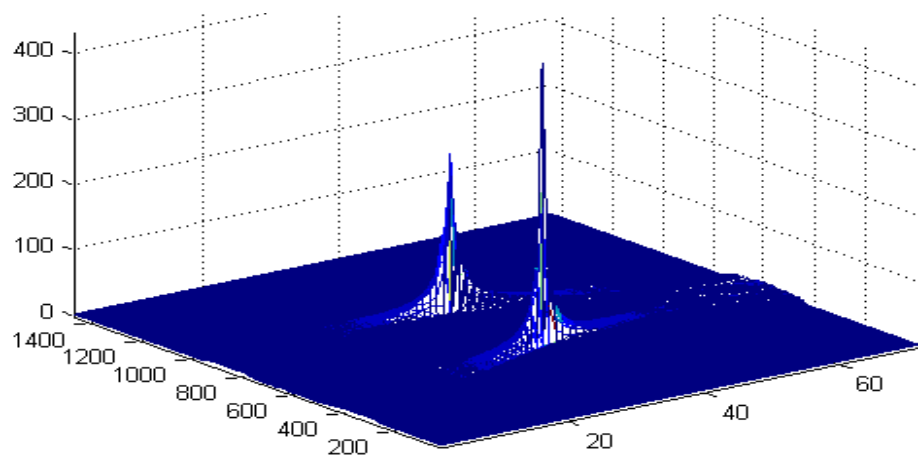
(a) WRT Space



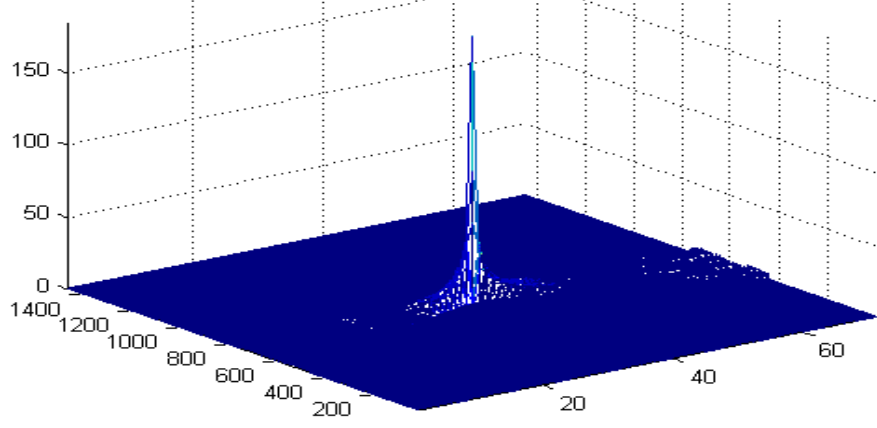
(b) WHT Space



(c) WHRT Space



(d) MWHT Space



(e) XWHT Space

Figure 5.1 Transform spaces 3D Plots

For the comparative analysis, it is assumed that the intercept includes both signal and noise, i.e., detection is declared and proper thresholds are chosen for each method. The general simulation conditions mentioned in section 4.4 were used with 1° angle resolution for HT/RT. The next two sections explain how we've found Th_{wD} for WHT based transforms and Th_{xwD} for XWHT.

5.1 Defining the WVD Threshold for Hough Based Transforms

Th_{wD} is one of the important parameters defining the efficiency of WHT based algorithms. In the previous chapter, this threshold was chosen heuristically for each method. Th_{wD} determines the number of points to be used in transformations, so it also determines processing time. Before comparing the transform methods in terms of parameter extraction performance and transform speed, we determined the optimum Th_{wD} for WHT based algorithms. This optimum threshold will also reduce the simulation time in the analysis of detection performance of the proposed methods in the next chapter.

The optimum threshold interval to be applied on WVD and the effect of threshold on extracting FMCW signal parameters using WHT based methods are investigated below [10]. In this investigation; correct calculations of d , and chirp rate were used as metrics. These two parameters are illustrated in Figure 5.2 using a HT contour plot.

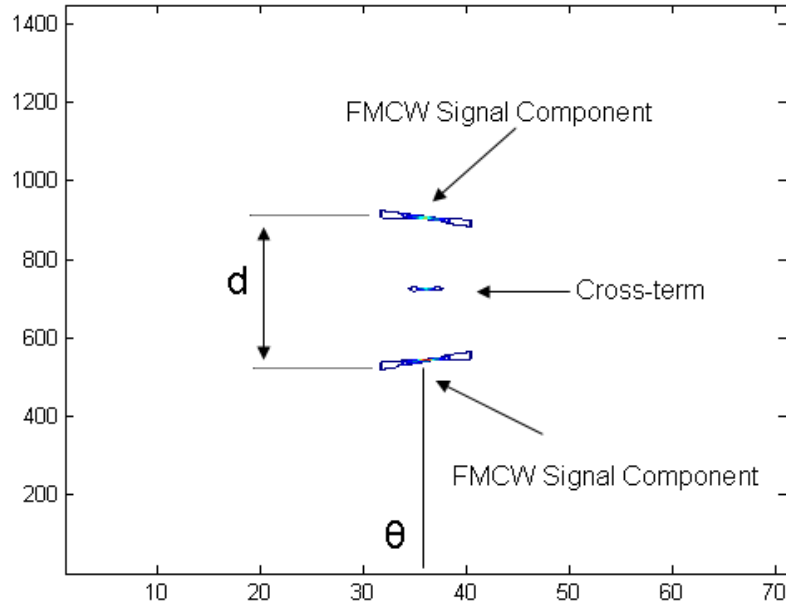


Figure 5.2 Contour plot of Hough transform of a two-component up-chirp FMCW signal

The test signal database shown in Table 5.1, (a simulated intercept database) consisting of FMCW signals with different bandwidths, modulation times, carrier frequencies and SNRs is used.

Table 5.1 Signal database to be used in WVD threshold studies

f_c	B	Tm
1000	1380	30, 36, 51
1400	2000	52
1700	850	19, 23, 32
1930	1050	23, 27
2145	1650	36, 61

It is shown in Figure 5.3 that; increasing Th_{wD} decrease transform space creation time exponentially and lower SNR levels increase transform space creation time. Even though high threshold levels exponentially decrease transform space calculation time, these threshold values are upper and lower bounded by breakpoints. Finding these upper and lower breakpoints and determining a threshold value between these bounds is critical for FMCW signal detection, because outside of these bounds signal detection and parameter extraction cannot be performed. These upper and lower breakpoints are shown in Figure 5.4.

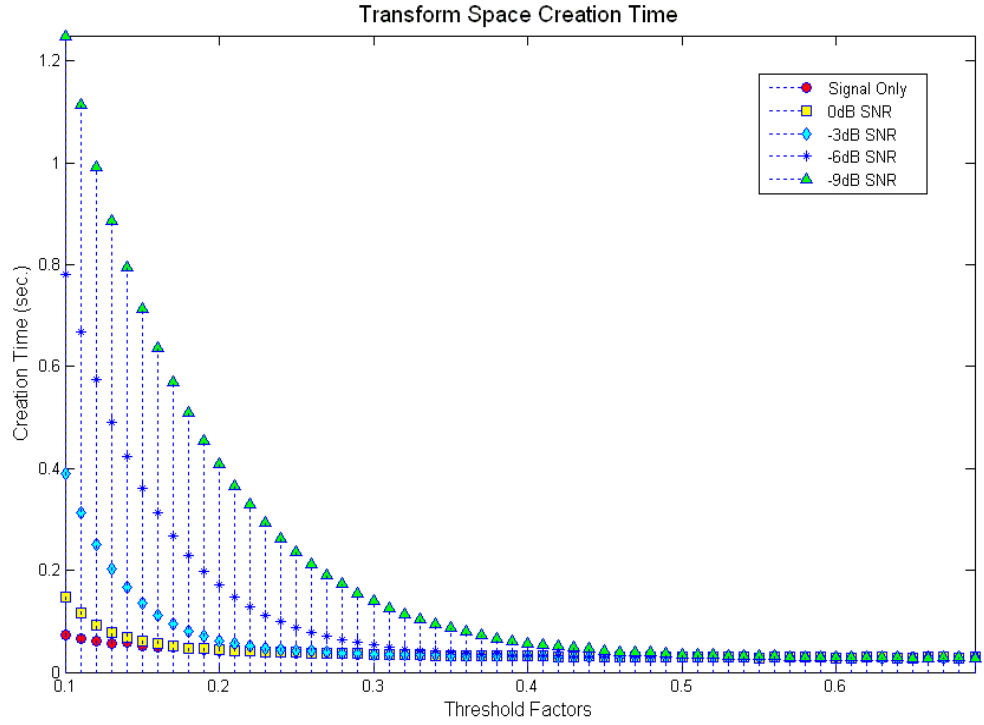


Figure 5.3 Threshold levels vs. transform space creation time [After 14]

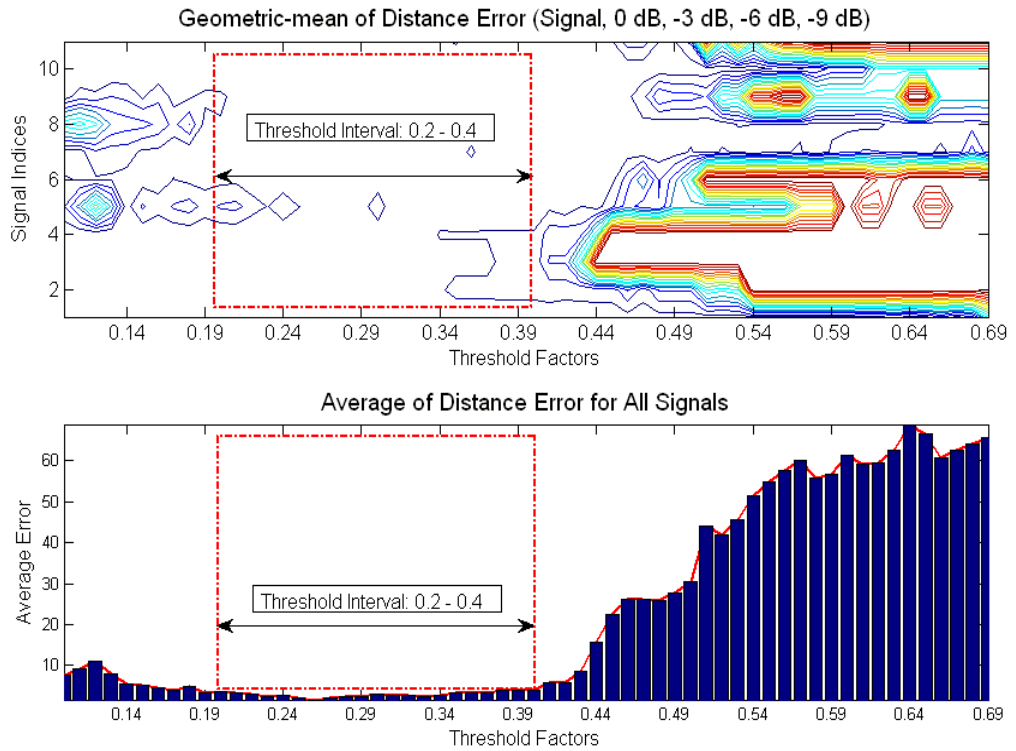


Figure 5.4 (a) Geometric-mean of distance error (b) Average of distance error for all signals

As can be seen from Figure 5.4, WHT based algorithms perform well with Th_{wD} values chosen between 20%-40% of the maximum WVD intensity levels of the signal. Figure

5.4 shows the relative errors made in calculation of d . y-axis shows the indices of 11 signals from Table 5.1. At low threshold levels the algorithm does not perform well due to the effect of noise. At high threshold levels some of the pixels on the FMCW signal components do not pass the threshold and so they are not counted in the transform space calculation. This decrease in the number of points in transform space calculation causes (r, θ) points -representing FMCW signal components- to have less amplitude in HT space. Therefore the algorithm shows poor performance at high SNR levels with high threshold levels.

For the comparisons Th_{WD} is set to 25% of maximum intensity level of WVD for all transform methods as a starting point.

5.2 Optimum WVD Threshold for XWHT

As explained before, XWHT tries to increase the intensities of cross-terms more than auto-terms' in HT space. So, considering the properties of cross-terms we can use a higher negative threshold on WVD to increase the probability of detection of cross-terms and to decrease transform space creation, i.e., detection time.

To find the optimum Th_{WD} for optimum parameter extraction performance using XWHT; we change the threshold in the interval of 0.2 - 0.65 factors in 0.05 steps and try to find the optimum threshold factor value giving the best P_{CRE} for signals at -10 dB SNR level. In Figure 5.5, x-axis shows threshold factors and y-axis shows P_{CRE} values. As can be seen from the figure, the previously proposed threshold factor of 0.25 is not a good choice for XWHT and threshold factor 0.4 makes P_{CRE} 100% for all signals at -10 dB SNR, which is a rather low SNR level. Therefore we can say that for XWHT, we can use $-Th_{XWD} = -0.4 \times \max[WVD]$ to get better P_{CRE} for all signals, at all chirp rates.

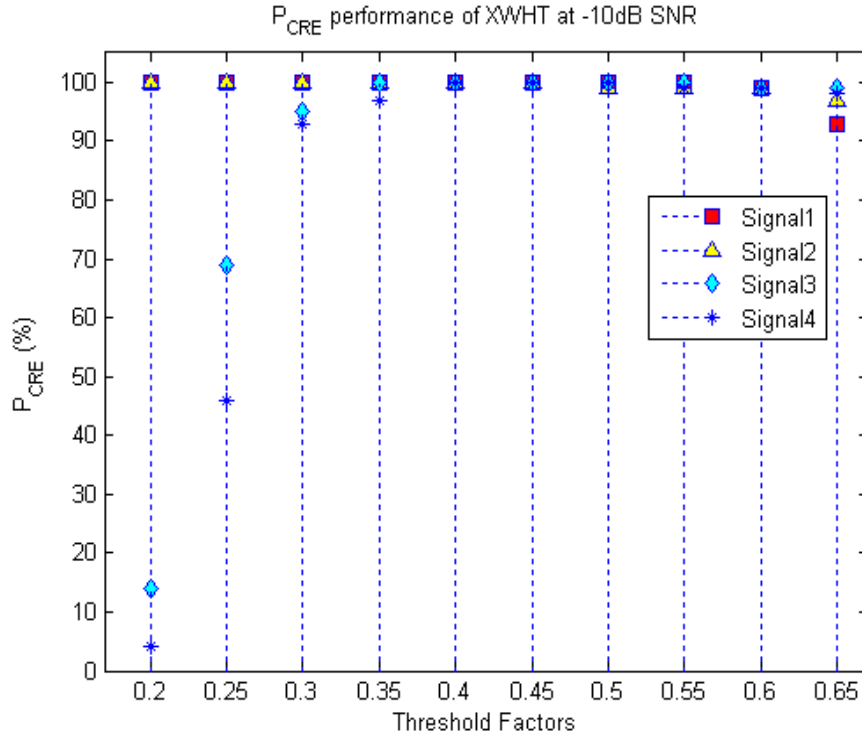


Figure 5.5 P_{CRE} Performance of XWHT vs. varying WVD threshold factors

The effect of $-Th_{XWD}$ for higher SNR levels (0dB to -9dB) is also investigated. Figure 5.6 shows that for SNR levels -9 dB and higher, we get $P_{CRE}=100\%$ when threshold factor level exceeds 0.35.

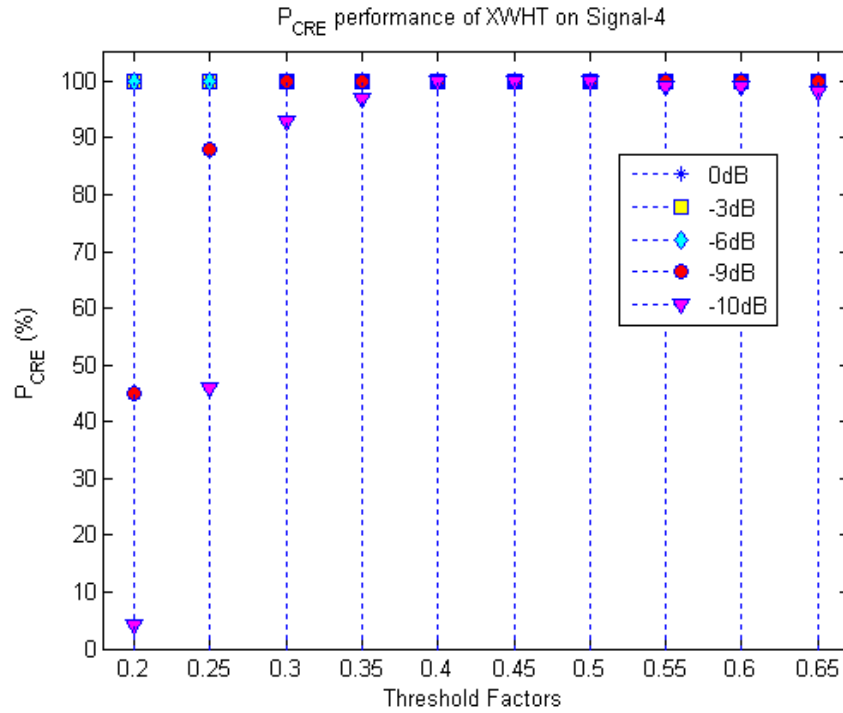


Figure 5.6 P_{CRE} Performance of XWHT for different SNRs vs. varying threshold $S4(\cdot)$

In Figure 5.6, we use $S4(\cdot)$ as an example, which is a harder to detect signal with XWHT due to its chirp rate however, the analysis holds true for the other signals as well. So, we verify that threshold factor level 0.4 can be used both for low and high SNR levels. Table 5.2 shows the increase in P_{CRE} of XWHT due to new Th_{XWD} for all signals.

Table 5.2 Increase in P_{CRE} performance of XWHT with new threshold

SNR	Threshold	S1($\cdot$) P_{CRE} (%)	S2($\cdot$) P_{CRE} (%)	S3($\cdot$) P_{CRE} (%)	S4($\cdot$) P_{CRE} (%)
-10 dB	0.25	100	100	69	46
	0.4	100	100	100	100
-9 dB	0.25	100	100	93	88
	0.4	100	100	100	100

5.3 Orthogonal Distance Calculation Performance

We test the parameter extraction algorithms with the signal database defined in Table 4.4 and compare the results. All signals are simulated one hundred times between -10dB and 9dB. Figure 5.7 shows the distance calculation performance of all the methods for $S1(\cdot)$. Number of correct distance calculations (values in a given tolerance) is counted.

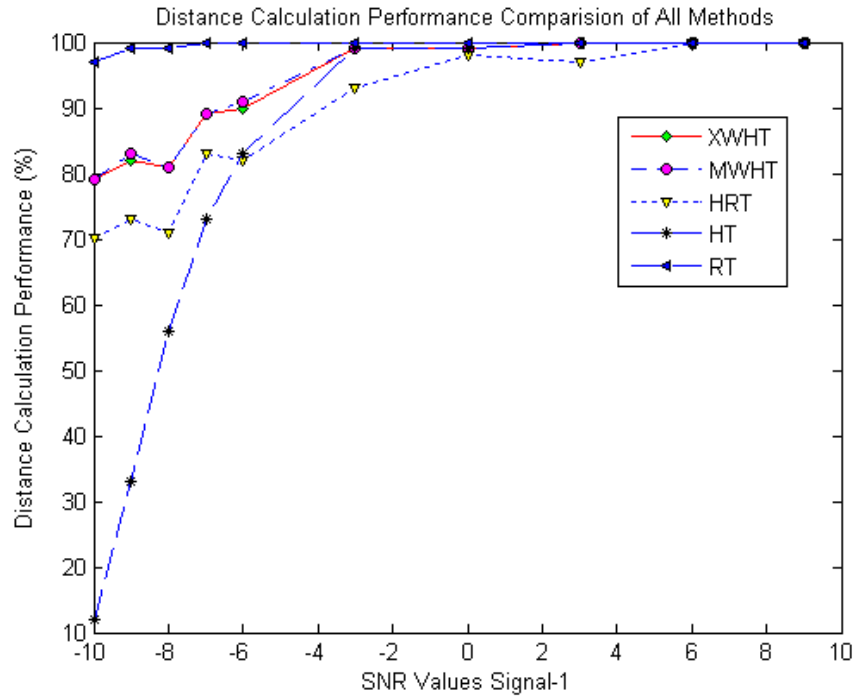


Figure 5.7 The orthogonal distance calculation performance of transform methods

Figure 5.7 shows that WRT gives us the best parameter extraction performance and XWHT and MWHT algorithms are next.

5.4 Chirp Rate Estimation Performance

This section compares the transform methods in terms of chirp rate estimation performance, P_{CRE} . Table 5.3 lists each method's P_{CRE} . Finding the chirp rate in $\pm 1^\circ$ tolerance is accepted as successful estimation.

Table 5.3 P_{CRE} performance of transform methods

SNR	Methods	S1($\cdot$) P_{CRE} (%)	S2($\cdot$) P_{CRE} (%)	S3($\cdot$) P_{CRE} (%)	S4($\cdot$) P_{CRE} (%)
-10dB $\leq$	WHRT	100	100	100	100
	MWHT	100	100	100	100
	XWHT	100	100	100	100
	HT	100	100	100	100
	RT	100	100	100	100

Orthogonal distance calculation performance analyses presented by Figure 5.7 favors WRT but, chirp rate estimation performance analyses shows that all methods are successful in finding chirp rate for $SNR \geq -10dB$. However we have to keep in mind that ESR systems need to operate very fast due to their time-critical missions. Therefore we need a very fast method with reasonable performance metrics. So we need to compare the transform methods in terms of their transform space creation time performance as well.

5.5 Transform Speed Comparison

Transform speed can actually be considered as the related method's signal detection speed. Because for signal detection, we need to search for the maximum intensity points in the transform spaces and this takes the same amount of time for all transforms, once the transform space is created. So, transform space creation time defines the signal detection speed of the methods.

The transform space creation times are investigated using the signals defined in Table 4.4. Table 5.4 and Figure 5.8 show the transform space creation times of methods for S1($\cdot$) for SNR levels (-10dB to 9dB).

As expected, RT space creation time is considerably higher than the others'. Similar graphs are derived for all the test signals, which mean that the transform space creation times are not related to chirp rate. Figure 5.8 shows that, the fastest transform spaces are created by XWHT method. Since MWHT and WHRT use the same number of points, their transform space creation times are equal.

Table 5.4 Transform space creation times for S1(·)

	WHT (s)	WHRT(s)	WRT(s)	MWHT(s)	XWHT(s)
-10	0,4062	0,7993	2,1534	0,7713	0,0757
-9	0,3370	0,6511	2,1657	0,6278	0,0564
-8	0,2434	0,4650	2,2003	0,4491	0,0364
-7	0,1748	0,2821	2,1680	0,2688	0,0234
-6	0,0833	0,1524	2,1544	0,1434	0,0185
-3	0,0368	0,0525	2,2247	0,0506	0,0171
0 to 9 dB	~0,0348	~0,0486	~2,1751	~0,0469	~0,0173

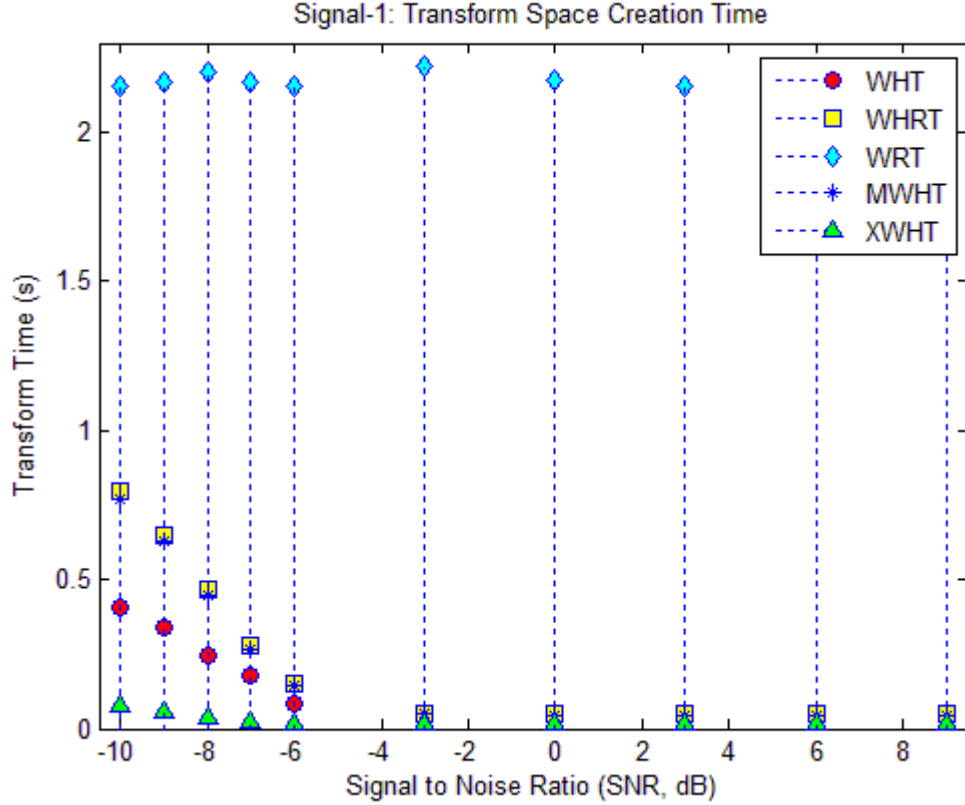


Figure 5.8 Transform space creation times for S1(·)

Transform space creation time results can be supported by the computational complexity calculations. The signal length is taken as S , number of FFT bins is taken as $2S$ and the number of angles for transforms is taken as F . Since WRT uses all the S^2 pixels in the WVD image, the computational complexity of WRT will be $O(FS^2)$. The computational complexity for WHT and WHRT is $O(FK)$ where $K < S^2$, because we only use $(x_i, y_j) \in \{f(x_i, y_j) \geq Th_{WD}\}$. For MWHT the computational complexity is $O(FL)$ where $K < L < S^2$, because we use $(x_i, y_j) \in \{f(x_i, y_j) \geq Th_{WD} \text{ and } f(x_i, y_j) \leq -Th_{WD}\}$. And for XWHT, since we use only $(x_i, y_j) \in \{f(x_i, y_j) \leq -Th_{XWD}\}$

the computational complexity is $O(FJ)$ where $J < K < L < S^2$ and $J \ll S^2$, because only the cross-term magnitudes exceed $-Th_{XWD}$.

As a result of the analysis performed in this chapter we can conclude that, XWHT method is a much faster transform method with a very good parameter extraction performance.

CHAPTER 6

FMCW SIGNAL DETECTION

In the previous chapter we analyzed the performance of transform methods assuming that the signal was present in the intercepted data. The analyses were performed with a comparison focus and they showed that XWHT produced results much faster than the other transform methods mentioned in this work. In this chapter we try to derive the detection performance metrics of Wigner Hough Based FMCW detection methods; probability of detection (P_d) for a given probability of false alarm (P_{fa}).

Although there are some studies in the literature to determine performance standards of ESRs [82], [83] there is still no globally accepted performance standard for ESR development. Also, as it is stated in [84], we see a large difference between the theoretical and practical performance metrics of ESRs if we use generally accepted performance metrics like Cramer-Rao bound. This is because; generally accepted receiver performance metrics are suitable for communication receivers where signal properties like the frequency, modulation and bandwidth of the received signal are known and an efficient receiver design is in place for the cooperative incoming signal. However this is not the case for ESRs where the parameters of the incoming signal are not known. Thus, it is not possible to design an optimum receiver. Since theoretical analysis does not give relevant performance metrics, ESR designs are usually started based on experience from previous system performances. Another problem about ESR performance is intentionally or unintentionally reported, misleading performance results [84].

As explained above, the ESRs cannot be designed as optimum receivers, therefore it is difficult to specify the requirements of a receiver without designing it first [84]. Since

the performance metrics will be receiver dependent, we choose to perform numerical analysis for the WHT based detection performance analysis instead of deriving analytic expressions. In order to avoid any unintentional misleading performance report, we only perform performance analysis by comparing our own methods with each other.

6.1 XWHT Probability of Detection

XWHT proposes to perform FMCW detection on HT space using only the cross-terms. In this section we first want to see the XWHT FMCW detection performance individually. The comparative detection performance analysis of the methods will be performed in the next sections.

We use the same set of signals given in Table 4.4. First, we try to find the maximum value of the XWHT spaces of all intercepts to normalize the intercept XWHT spaces, including noise only cases. We assume that, in our simulated operational area the most powerful signals are intercepted at 9 dB SNR. So, for each signal in Table 4.4 we run 100 Monte-Carlo (MC) simulations at 9 dB SNR and find the peak value of XWHT space of each simulation. The average of peak intensity values for each signal are found to be; $S1(\cdot)=147$, $S2(\cdot)=150$, $S3(\cdot)=90$, $S4(\cdot)=81$. So, all the intercepts are normalized by 150, assuming that this is the highest intensity value for our simulated ESR.

As mentioned before, if we don't have a-priori information about the signal parameters, we simply compare the highest intensity point in the transform space against a threshold (ζ) to declare "detection". So, defining the detection threshold is the most important step for any detection method. As a requirement for our simulated ESR, we want to have $P_{fa}=10^{-4}$. To determine the ζ for this P_{fa} we simulate 10^4 "noise only" intercepts. The histograms of these 10^4 intercepts are calculated to see the maximum intensity value that noise only intercepts can get. Figure 6.1 shows the average of these histograms. If we take a closer look at the zoomed part of Figure 6.1, we can see that one transform space point, out of 10^4 transform spaces reaches the relative intensity value of 0.404. This means that if we set ζ at this relative intensity value, we can declare detection with a $P_{fa}=10^{-4}$.

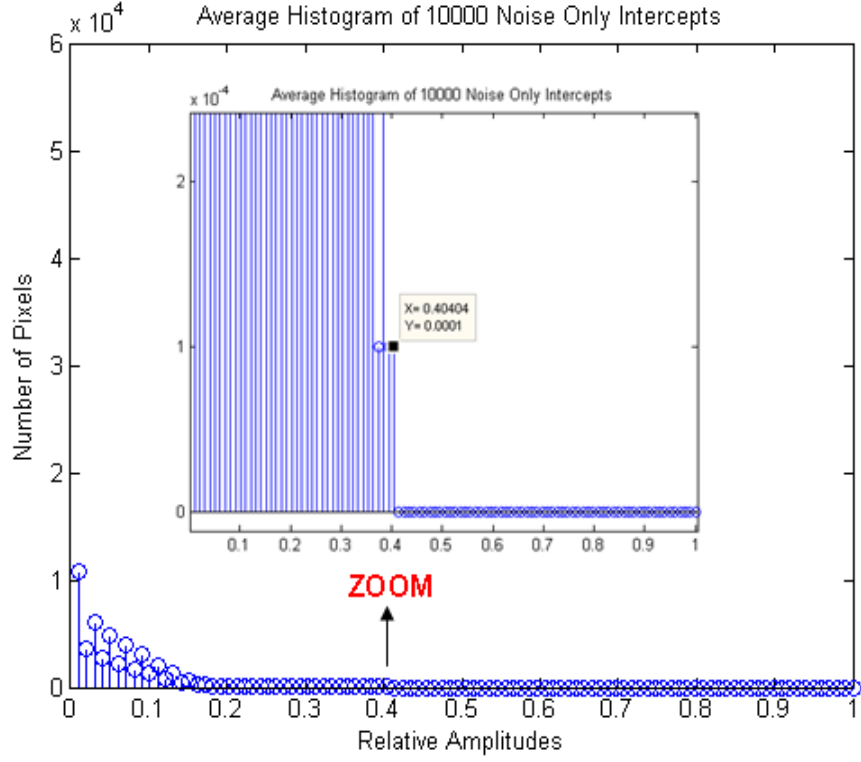


Figure 6.1 Average histogram of 10000 noise only intercepts

To investigate P_d performance of XWHT for detection threshold factor 0.404 (i.e. $P_{fa}=10^{-4}$), we tried to derive P_d curves using 1000 intercept simulations for each signal in Table 4.4, at each SNR level. Figure 6.2 shows these P_d curves and as can be observed from it, we can claim that XWHT has a very successful P_d performance down to -10 dB SNR levels. Another thing we can see from Figure 6.2 is that XWHT's detection performance is directly related to chirp rate.

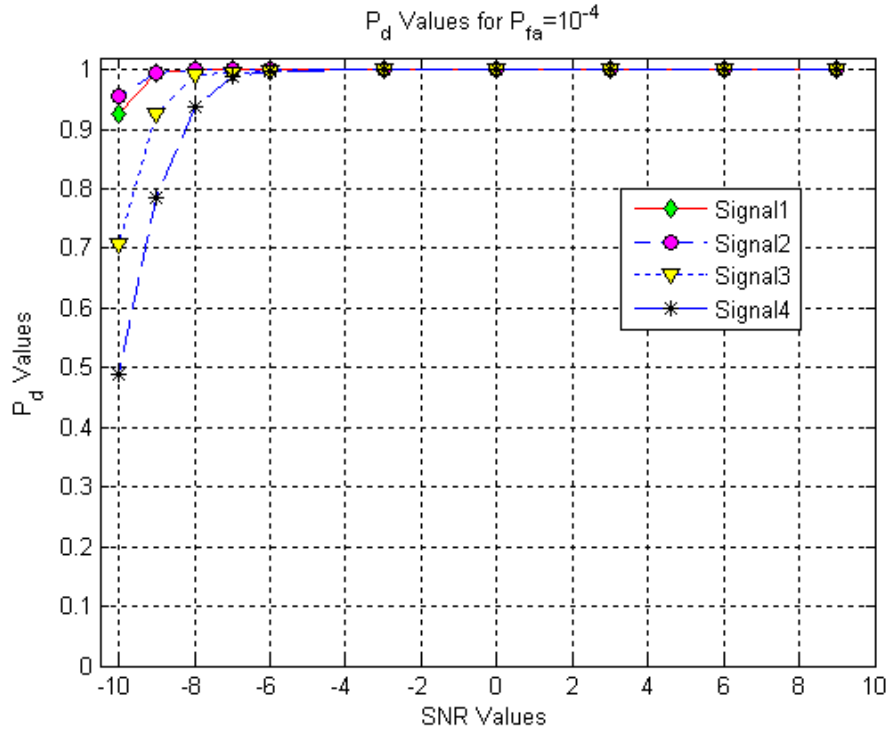
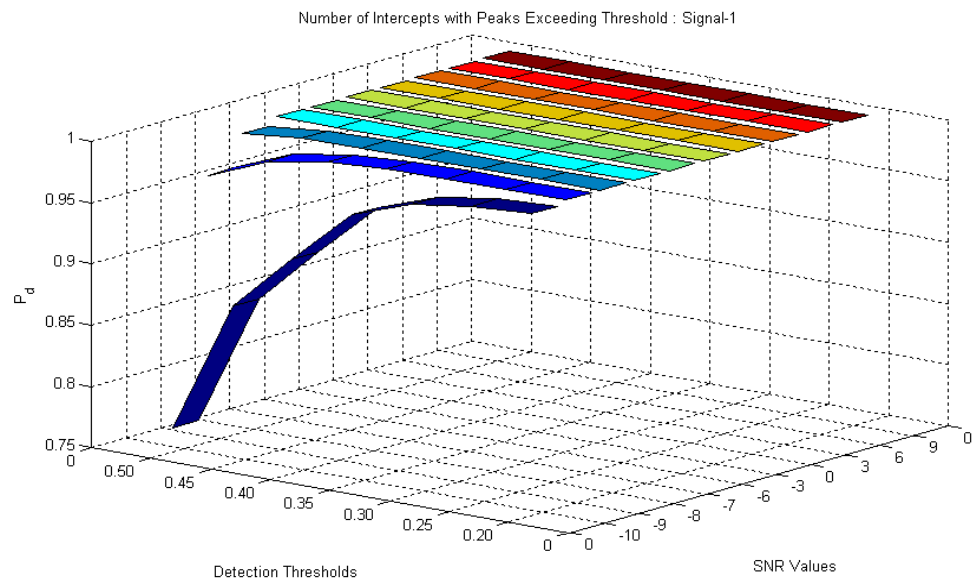


Figure 6.2 XWHT P_d curves for $P_{fa} = 10^{-4}$

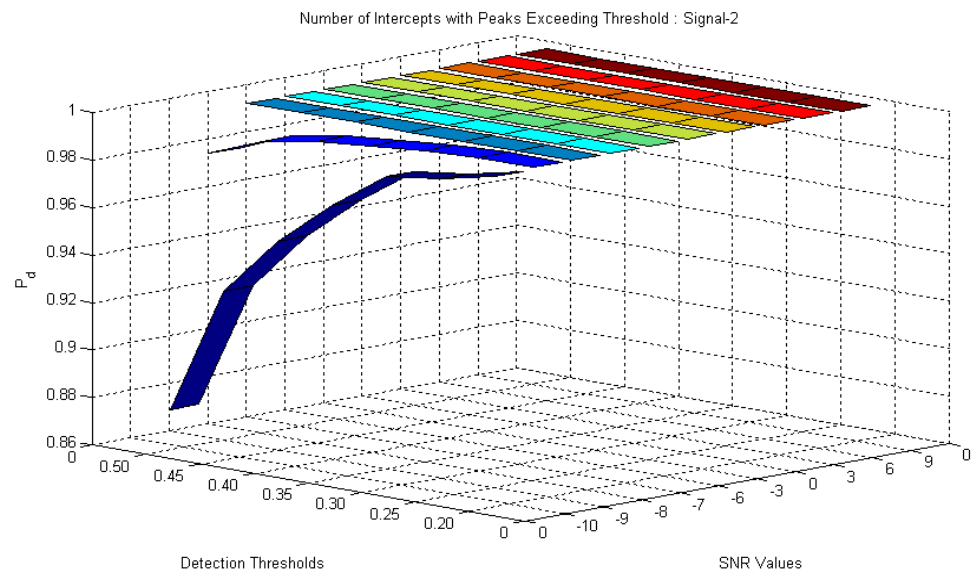
6.2 Optimum Detection Threshold for XWHT

Although we have very satisfying results down to -10dB SNR, we also investigate if we can increase our P_d performance at lower SNRs and still have acceptable P_{fa} performance. To do so, we want to see the maximum possible P_d values. We know that our intercept database consists of *signal + noise* simulations. First we assume that all intercept simulations pass ζ , meaning that we declare detection for every intercept in our database. Than we control how many of these detections are correct. Actually this has been shown in Table 5.3. In Figure 6.2 we have $P_d = 0.490$ for S4($\cdot$) at -10 dB SNR, but in Table 5.3 we read a P_{CRE} performance of 100% value for S4($\cdot$) at -10 dB SNR with threshold factor 0.40. This P_{CRE} performance of 100% value means that, if ζ were set low enough, than it would be possible for XWHT to detect 100% of S4($\cdot$) (at -10 dB SNR) correctly. This holds true for the other signals as well. Therefore, we can say that we can decrease ζ to let more intercepts to pass and be evaluated by XWHT. To find an optimum ζ , we use all our intercept simulations defined in Table 4.4 and we investigate the change in P_d performance while changing ζ . In Figure 6.3; x-axis is

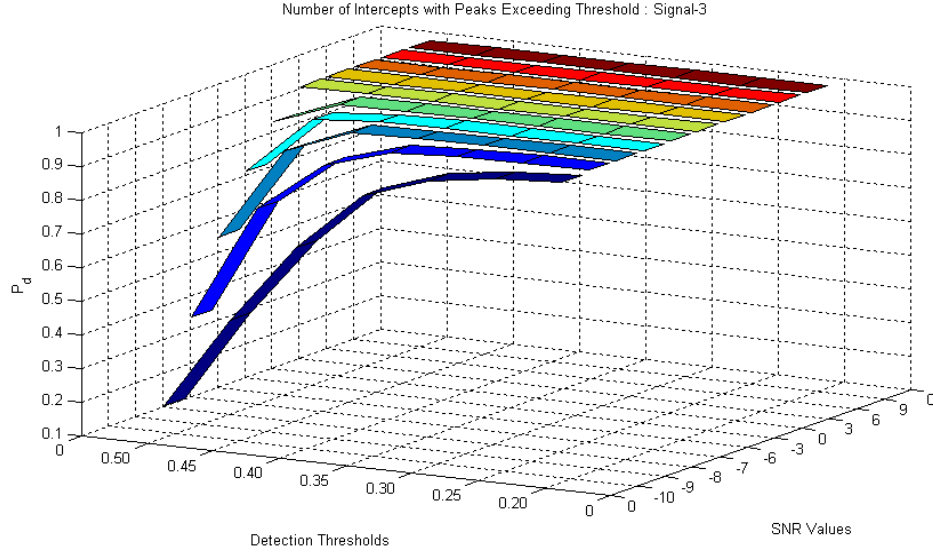
SNR values between 9 dB and -10 dB, y-axis is ζ values between 0.2 and 0.5 in 0.05 steps, z-axis is P_d for each “SNR- ζ ” pair.



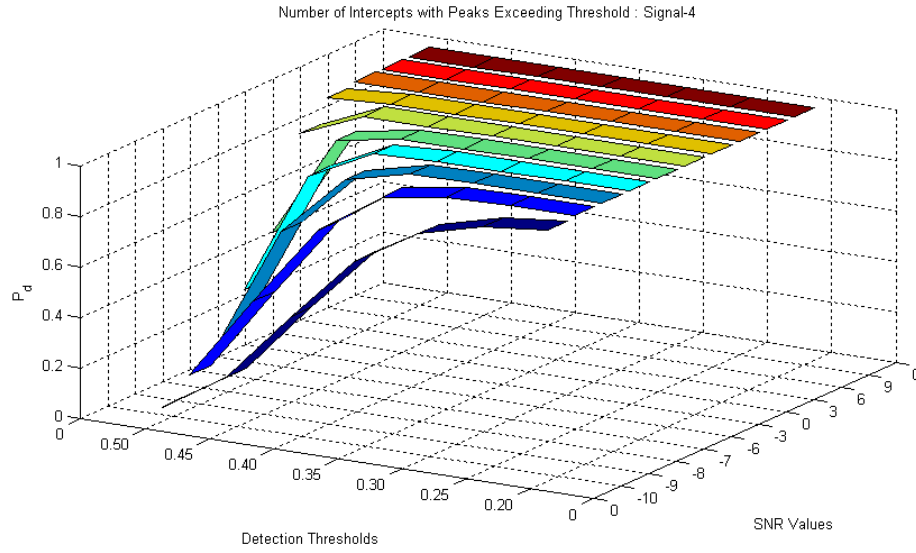
(a) Signal-1



(b) Signal-2



(c) Signal-3



(d) Signal-4

Figure 6.3 P_d values for changing SNR vs. changing detection thresholds

As we can see from Figure 6.3 (more obviously Figure 6.3.c and Figure 6.3.d) that, setting detection thresholds higher than factor 0.35 considerably deteriorates P_d performance at low SNR levels. Considering all the signals at all SNR levels, we can see that detection thresholds lower than factor 0.30 can increase P_d performance as desired. The important issue here is keeping P_{fa} at a reasonable level. P_{fa} behavior vs. detection threshold is investigated and illustrated in Figure 6.4.

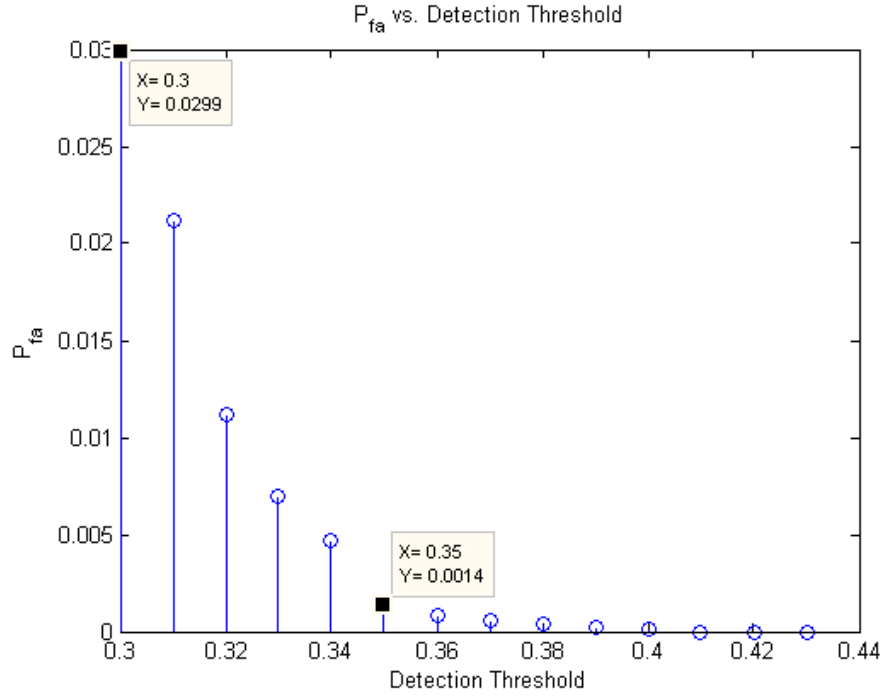


Figure 6.4 P_d values vs. changing detection thresholds

Figure 6.4 shows the increasing P_{fa} vs. decreasing detection threshold factors and we can see that detection threshold factors lower than 0.3 cause very high P_{fa} values. Figure 6.3 illustrated that, detection threshold factors between 0.30-0.35 can give us a much better P_d performance than we can get with detection threshold factor 0.404. Figure 6.4, on the other hand, illustrates that our P_{fa} values will increase to $300 \times 10^{-4} - 14 \times 10^{-4}$ for a detection threshold factor interval of 0.30-0.35. In conclusion, depending on the P_{fa} requirements we can set a matching detection threshold and XWHT will detect required number of signals.

6.3 Performance Comparison of WHT Based Detection Methods

FMCW LPI radars use different chirp rates to vary range resolution and doppler resolution. In the preceding chapters on FMCW signal interception and parameter extraction, we observed that chirp rate of the signal effects the ESRs intercept performance considerably, especially in low SNR cases. In this section, we investigate the FMCW detection performance of WHT based methods along with the chirp rate effects on the detection. The investigation is performed on ROC curves of the WHT

methods. The detection threshold levels of each transform method for $P_{fa}=10^{-4}$ were found using the following steps;

- The maximum intensity level that can be reached was found using all the signals at 9dB SNR level, (assuming the highest SNR level is at 9dB SNR)
- Detection of 10^4 noise only intercepts were simulated,
- All intercepts were normalized with the max. intensity level to limit the later calculated histogram dynamic range between 0-1.
- The histogram of all noise only intercepts are checked to find the detection threshold for $P_{fa}=10^{-4}$.
- The result we got from the histogram is verified by controlling the maximum value of all 10^4 noise simulations.

For the analysis, a more comprehensive FMCW signal database is used to include signals between chirp rates $30^\circ - 75^\circ$ in 5° steps. The signals are shown in Table 6.1.

Table 6.1 Signal database to be used in “chirp-rate effect studies” on detection performance

	θ°	B (in terms of f_s)	fc
Signal 1	30	0.433	$f_s/4$
Signal 2	35	0.357	
Signal 3	40	0.298	
Signal 4	45	0.25	
Signal 5	50	0.209	
Signal 6	55	0.175	
Signal 7	60	0.144	
Signal 8	65	0.117	
Signal 9	70	0.091	
Signal 10	75	0.067	

All the signals were modeled in 8 different SNR levels (-3,-4,-5,-6,-7,-8,-9,-10dB) with same T_m value. Higher SNR values were not modeled since we know that signals with high SNR values are detected with a very high performance regardless of their chirp rate [7]-[9], [23]. Monte Carlo (MC) runs are performed 1000 times for each SNR level.

6.3.1 XWHT Detection Performance Analysis

The detection performance of XWHT has already been investigated in section 6.2 but, the analysis is repeated here with a larger signal database to investigate the chirp rate

effect. Figure 6.5 shows the peak intensity values of XWHT spaces for each signal at 9 dB SNR level.

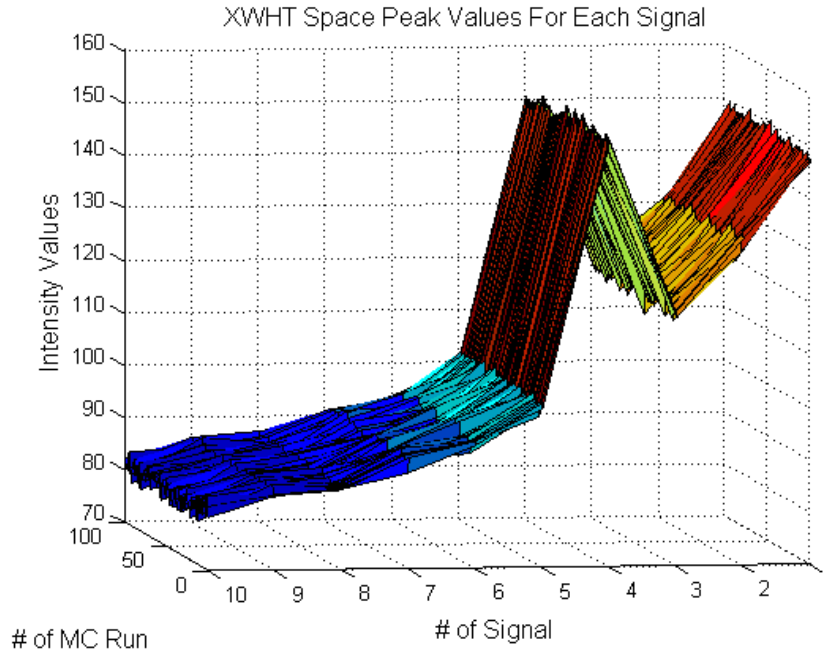


Figure 6.5 XWHT space peak values for each signal (SNR: 9 dB, Max:150)

In Figure 6.5, x-axis shows the number of signals beginning from 30° ($S1(\cdot)$) chirp rate to 75° ($S10(\cdot)$) chirp rate in 5° steps, y-axis shows the number of MC run and z-axis shows the peak intensity values of the transform space for each signal.

As can be seen here, signals with small θ values (steeper signals) give bigger Peak values. The highest peak value is derived at $\theta = 45^\circ$ as 150 and this value is used to normalize all detection spaces derived by XWHT. It is important to note here that, another set of 10^4 noise only intercepts are simulated here which are different from those used in section 6.2. As expected, we found a slightly different detection threshold and thus different ROCs for this new set.

The following figures Figure 6.6 – Figure 6.8 represent the results of steps performed for detection performance analysis. Figure 6.6 shows the sum of histograms and the maximum intensity value out of 10^4 noise simulations. Figure 6.7 verifies the maximum value. In Figure 6.8, we derived P_d curves for $P_{fa} = 10^{-4}$ using each signal in Table 6.1, at each SNR level. Detection threshold factor was set at 0.38, as found in Figure 6.7 and Figure 6.8.

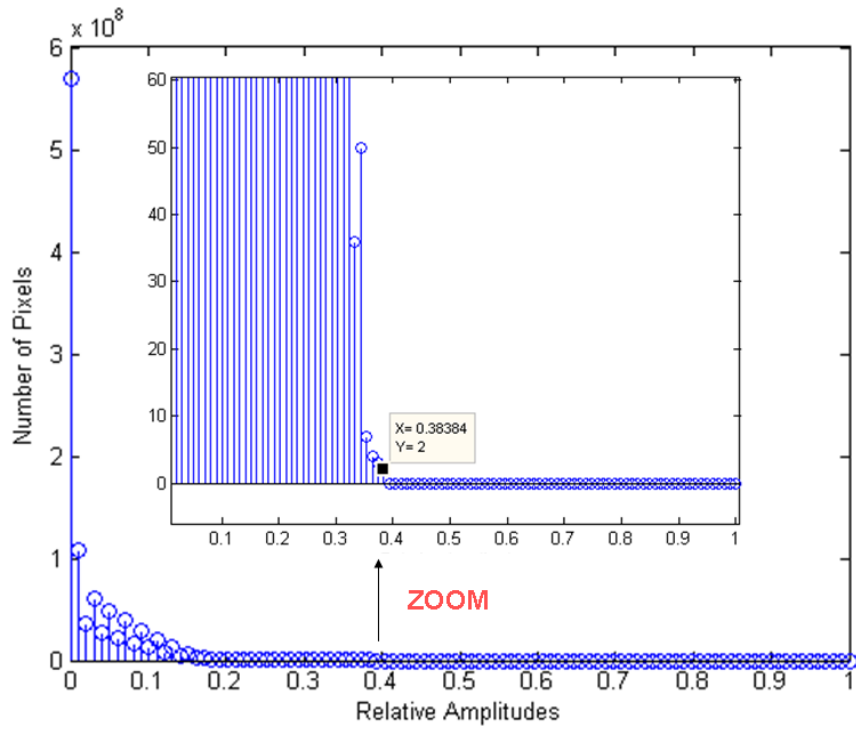


Figure 6.6 XWHT space histograms, 10^4 noise only simulations ($\zeta = 0.38$)

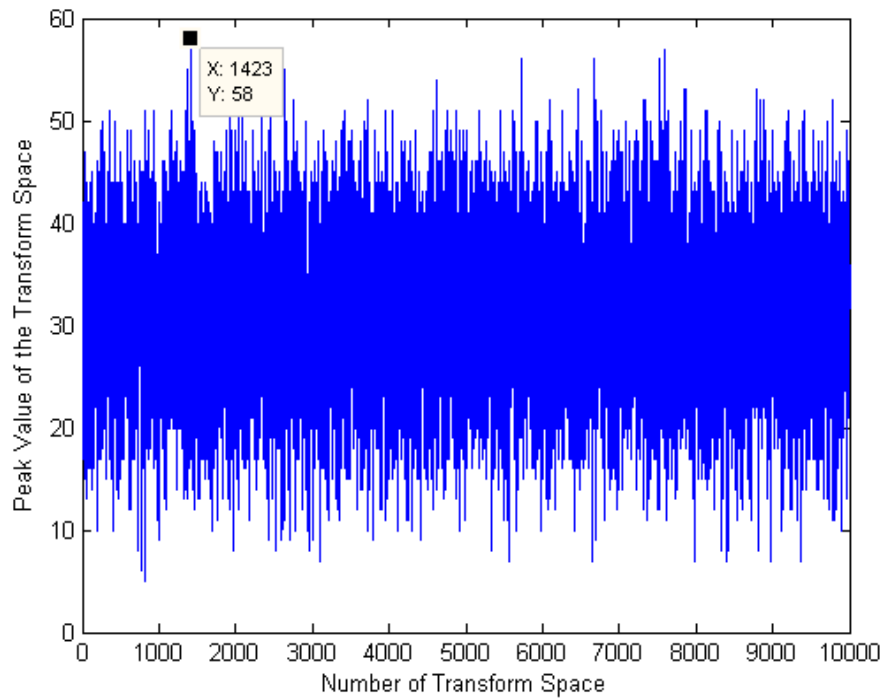


Figure 6.7 XWHT space peak values, 10^4 noise only simulations ($\zeta = 58/150 = 0.38$)

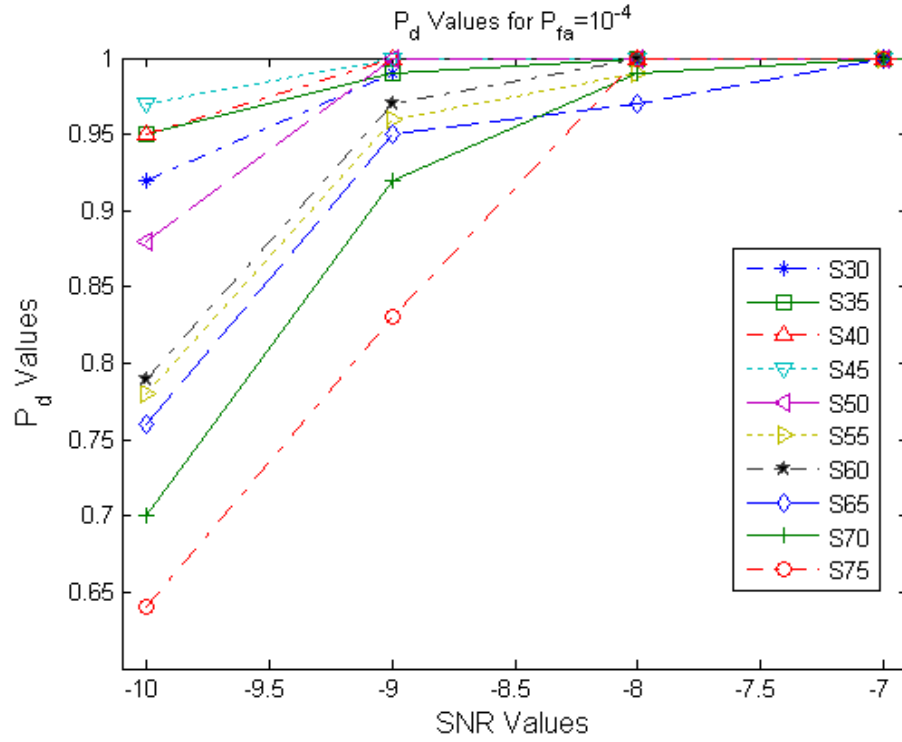


Figure 6.8 ROC : XWHT for $P_{fa} = 10^{-4}$

6.3.2 MWHT Detection Performance Analysis

The same analysis is performed for MWHT method and the related graphs are presented below.

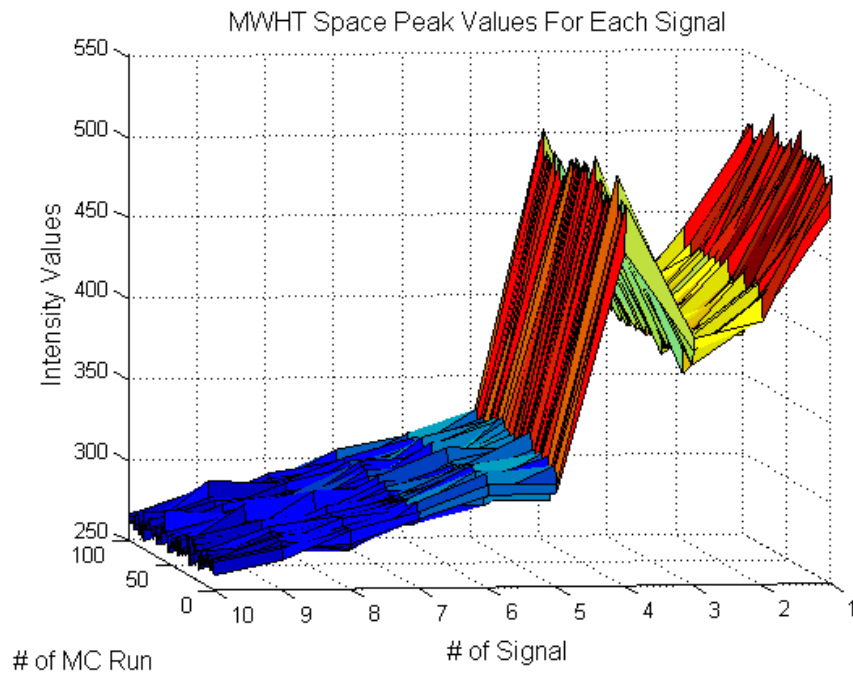


Figure 6.9 MWHT space peak values for each signal (SNR: 9 dB, Max:495)

The peak intensity value is derived at $\theta = 30^\circ$ as 495 and this value is used to normalize all detection spaces. The histogram and the peak values of the MWHT spaces are used to find the detection threshold as shown below.

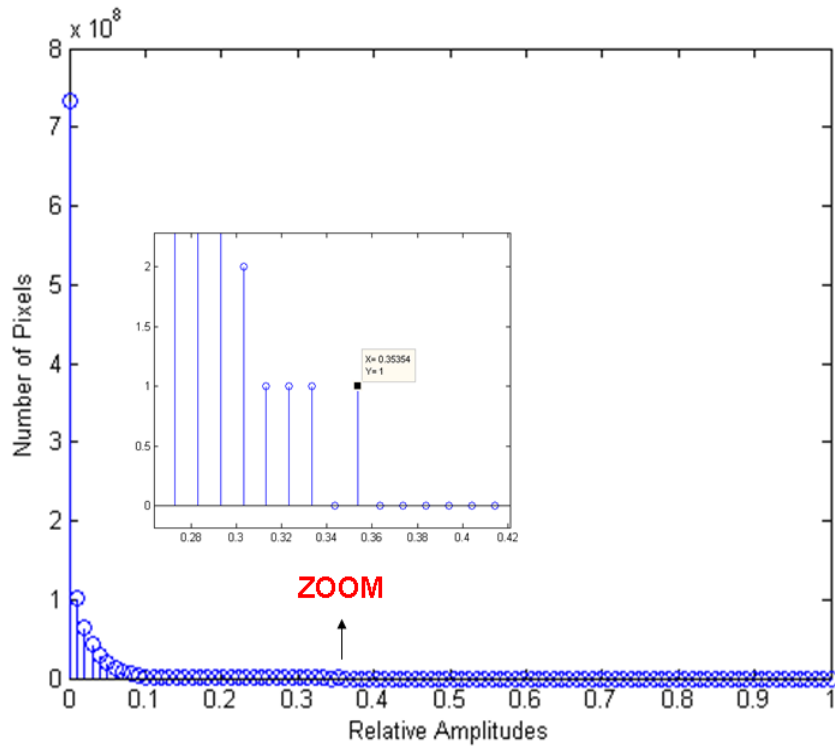


Figure 6.10 MWHT space histograms, 10^4 noise only simulations ($\zeta = 0.353$)

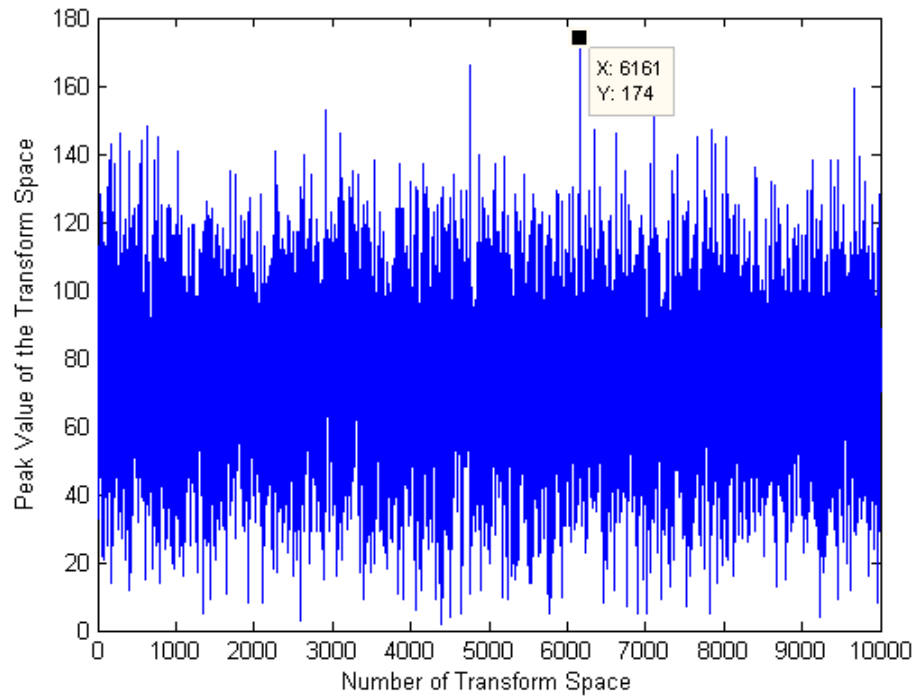


Figure 6.11 MWHT space peak values, 10^4 noise only simulations ($\zeta = 174/495 = 0.35$)

The ROCs for MWHT spaces for each signal in Table 6.1 is shown in Figure 6.12.

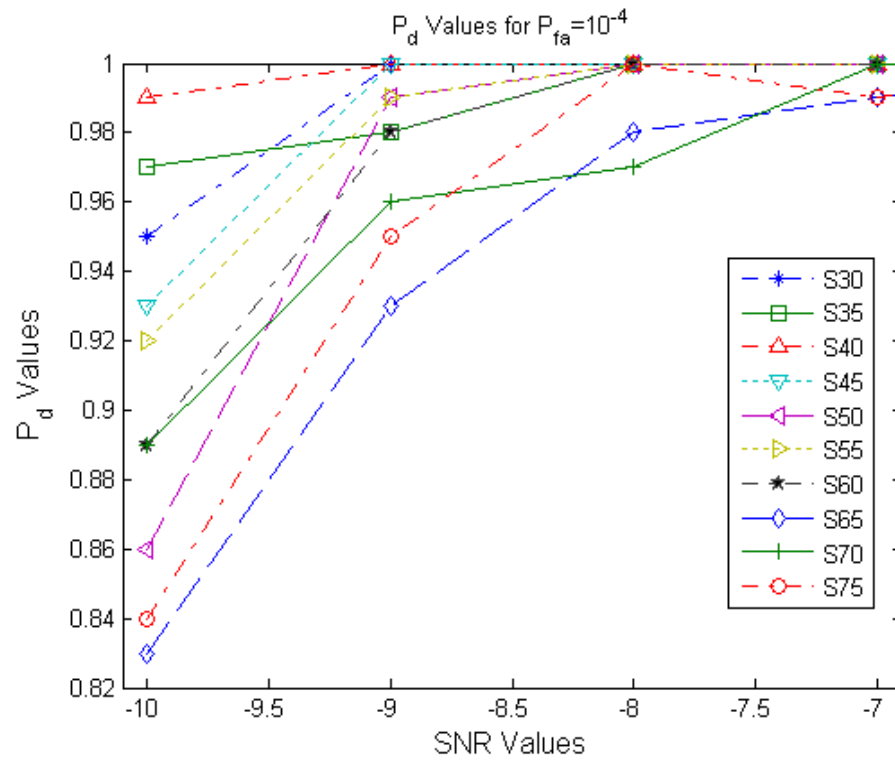


Figure 6.12 ROC : MWHT for $P_{fa} = 10^{-4}$

6.3.3 WHT Detection Performance Analysis

The analysis was carried on for HT as well.

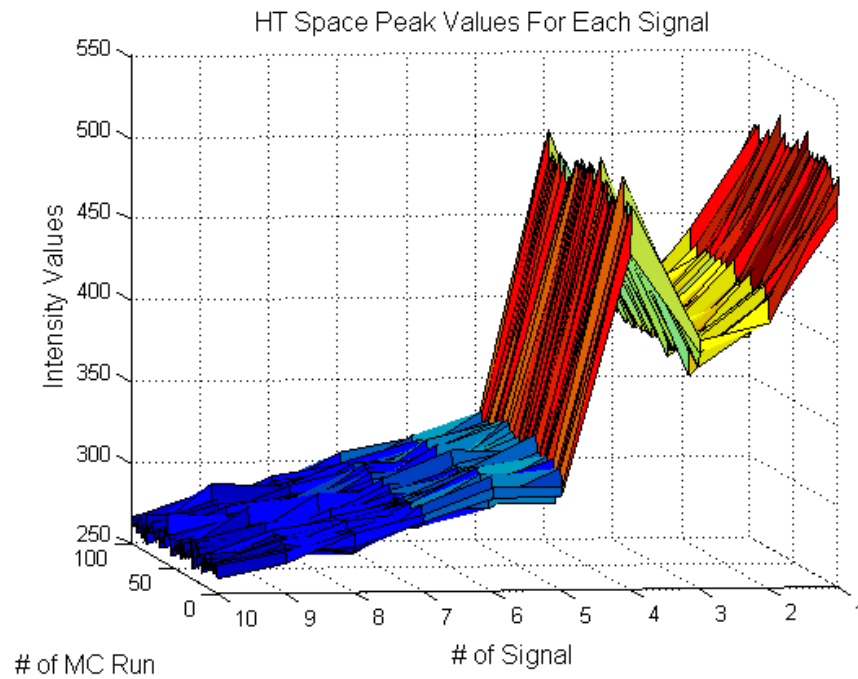


Figure 6.13 HT space peak values for each signal (SNR: 9 dB, Max:495)

As expected, the transform space peak intensity values for the signal intercept simulations were the same as MWHT space peak values because of the resemblance in the creation of both transform spaces. The histogram and the peak intensity values of the HT spaces for noise only intercepts are given in Figure 6.14 - 6.15, and the ROCs for WHT are plotted in Figure 6.16.

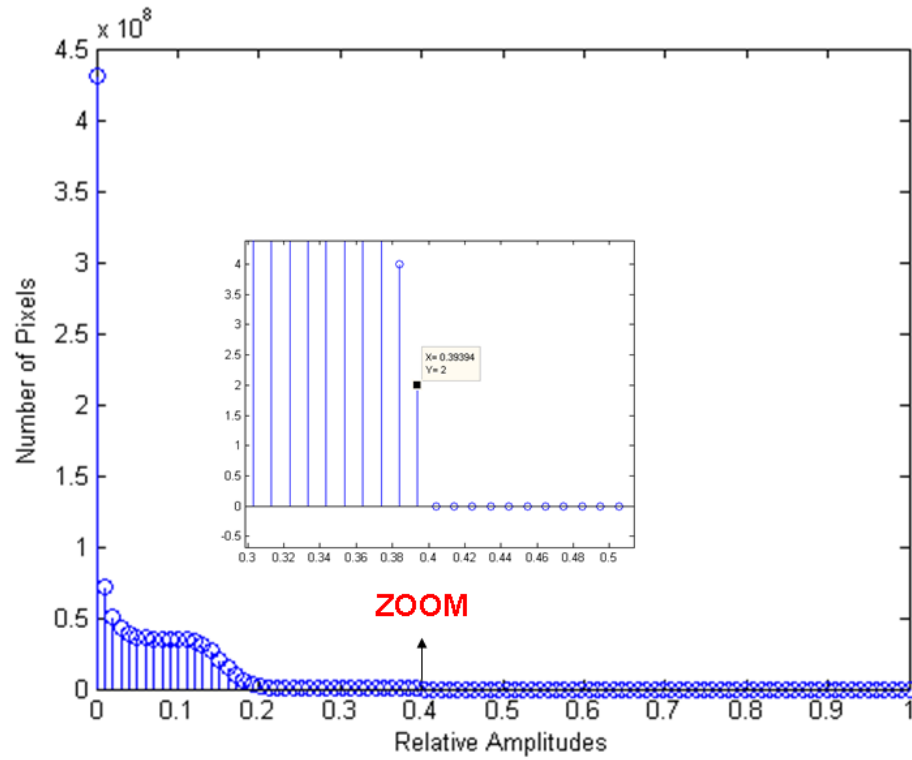


Figure 6.14 HT space histograms, 10^4 noise only simulations ($\zeta=0.393$)

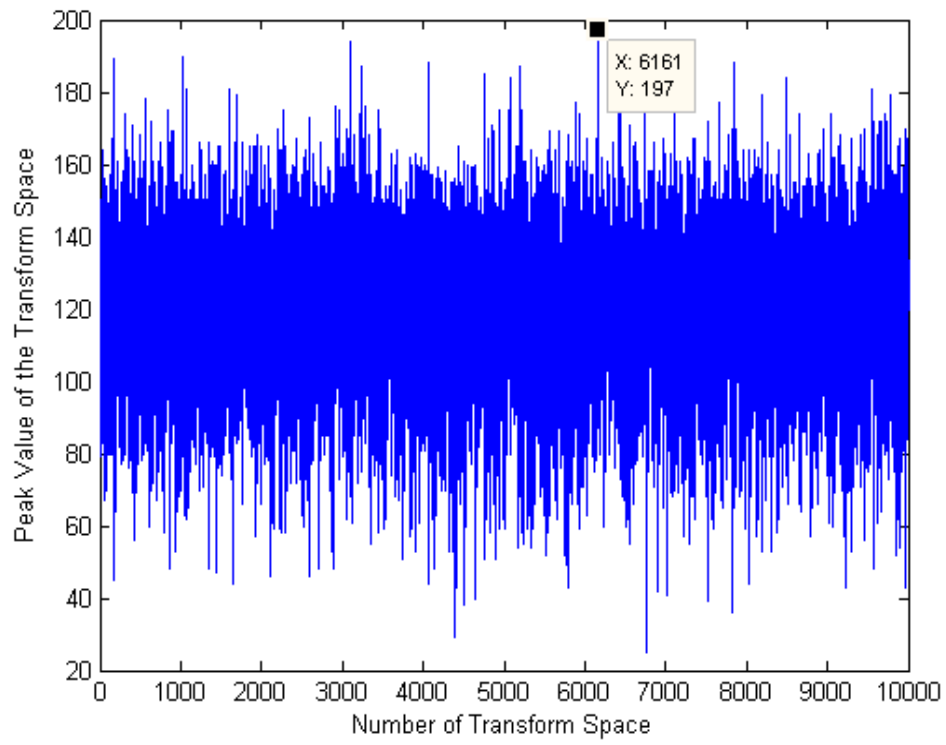


Figure 6.15 HT space peak values, 10^4 noise only simulations ($\zeta = 197/495 = 0.398$)

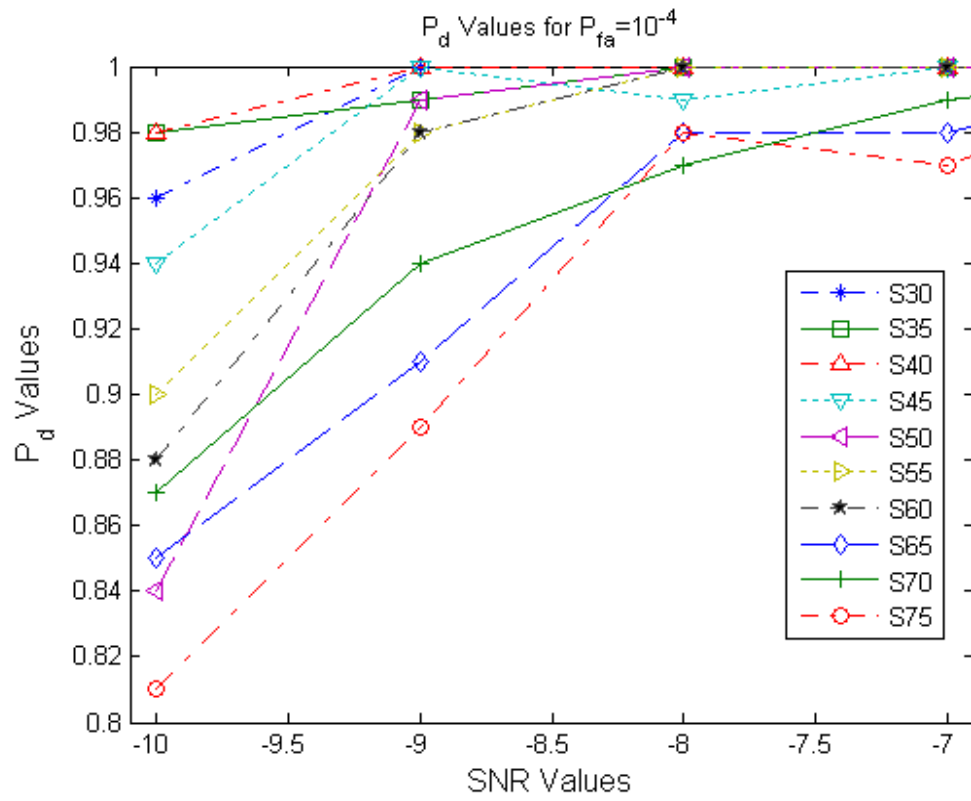


Figure 6.16 ROC : HT for $P_{fa} = 10^{-4}$

6.3.4 Performance Comparison

Individual ROC curves were plotted in the previous section for XWHT, MWHT and WHT. To create a more illustrative representation of the chirp rate effect, we show the P_d values of all HT based methods at -9 dB SNR on a same plot in Figure 6.17. We can clearly see that the P_d performance of HT based FMCW interception methods is considerably better between chirp rates $30^\circ - 50^\circ$.

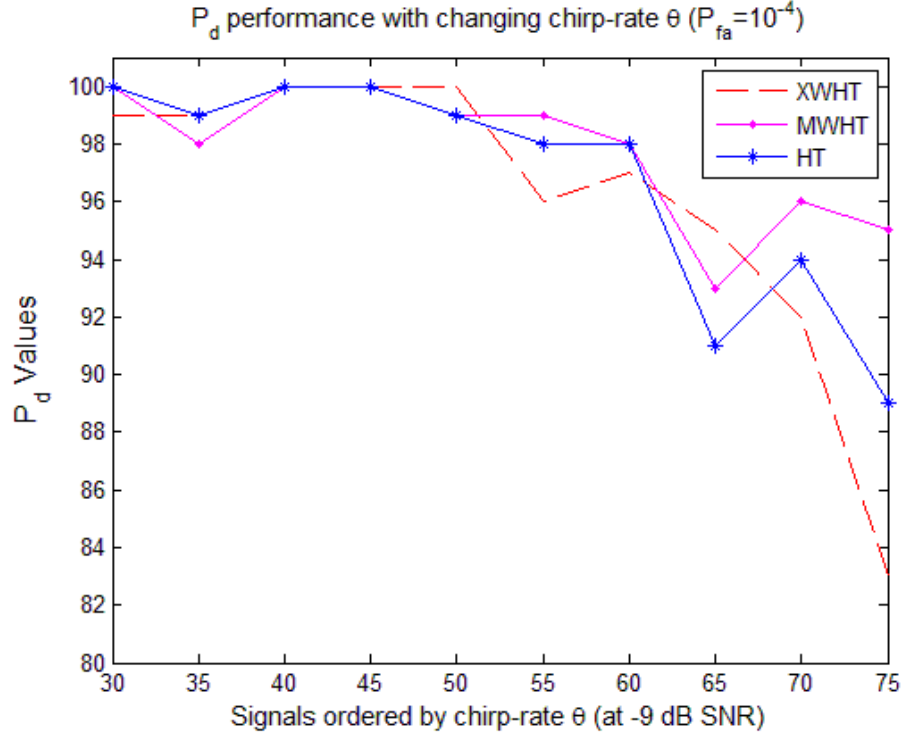


Figure 6.17 P_d performance vs. chirp rate (at -9dB)

6.4 Chirp Rate Adaptation for Improved FMCW Signal Detection

In this part of the thesis we define the ESR Digital Chirp Rate (DCR) concept. ESR's mission is to detect, locate and identify the threat emitters in their assigned observation area. Since the characteristics of the signals of interest are not under the control of the ESR; the system must be designed so as to make maximum use of signal properties. In the ESR side, we usually have some form of intelligence about the parameters of the threat emitters we want to detect i.e.; B , T_m and / or f_c . The threat emitter might use a certain chirp rate for its own requirements like range resolution, detection range. But in the ESR side, we can control the intercepted signals DCR and use it in our advantage for increased waveform interception performance. With Hough based transforms, we

have seen that we have better interception performance between $\theta = 30^\circ - 45^\circ$. So, we propose that if we can adjust the intercepted signal DCR to be in this interval we can improve our interception performance.

6.4.1 Electronic Support Receiver Digital Chirp Rate Concept

By using the geometric relations of the FMCW parameters given in Figure 3.8, the DCR of the intercepted signals can be defined by the equation below [24]:

$$\tan \theta = \frac{T_{pixel}}{B_{pixel}} = \frac{T_m \left(\frac{N}{T_{obs}} \right)}{B \left(\frac{M}{\frac{f_s}{2}} \right)} = \frac{T_m}{B} \cdot \frac{\left(\frac{N \cdot f_s}{N} \right)}{\left(\frac{2 \cdot M}{f_s} \right)} = \frac{T_m}{B} \cdot \frac{f_s^2}{M \cdot 2} \quad (6.1)$$

where T_m is in seconds and B is in Hz. In our work, we assume fixed T_m for each signal in our analysis and change B of the signals to derive different chirp rates.

6.4.2 Adjusting the Sampling Frequency to Achieve Required DCR

By looking at (6.1), we can see that the DCR of the received signals can be adjusted by changing either the FFT length, M , or f_s . Considering the hardware implementation, it is much easier and requires less resources to change the f_s for (near) real-time implementations. Figure 6.18 shows the effect of changing f_s on chirp rate using the two component FMCW up-chirp signal defined in Table 6.2.

Table 6.2 Example signal for changing chirp rate by changing f_s

	θ°	$f_s(\text{Hz})$	$B(\text{Hz})$	$f_c(\text{Hz})$	$T_m(\text{msec})$
Signal	30	8000	3464	$f_s/4$	64
	52	12000			
	67	16000			

Keeping the B, T, M, N fixed, the chirp rate of the signal is increased as we increase the f_s . As can be observed from Figure 6.18, doubling the f_s results in presenting only one FMCW component, because we fill our N time samples in half the observation time due to twice as many time samples.

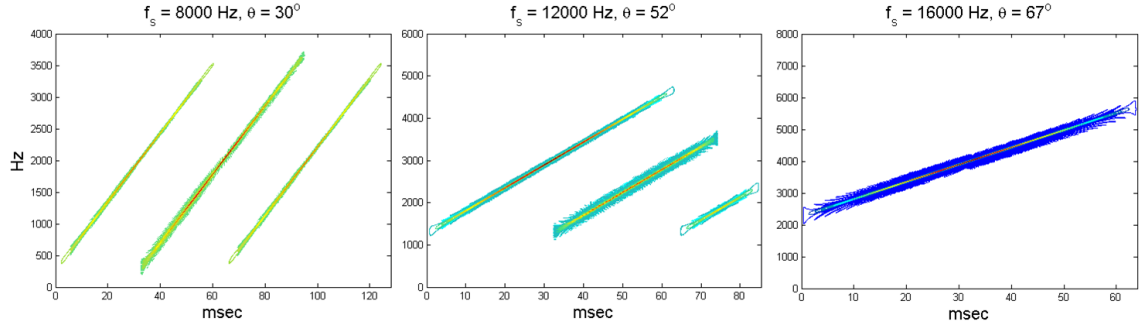


Figure 6.18 Effect of changing f_s on signal chirp rate

Increasing the f_s help us change the DCR of the intercepted signal at the cost of worse frequency resolution, but better time resolution. Previously we have seen that, we have a comparably worse P_d performance with $\theta > 45^\circ$ in low SNR cases. So, we suggest to adjust the DCR of the intercepted signal to our advantage by changing f_s . As an example, Figure 6.19 shows the required f_s to get DCR $\theta = 45^\circ$ for each signal in our database given in Table 6.1. Signals are ordered as S10($\cdot$) to S1($\cdot$) from left to right [24].

In section 6.3.4 we have seen that signals with bandwidths bigger than $f_s/4$ (last four signals on the right S4($\cdot$)-S1($\cdot$) in Figure 6.20) are intercepted successfully and thus no DCR adjustment is necessary. These signals would require sampling frequencies bigger than f_s if DCR adjustment were necessary.

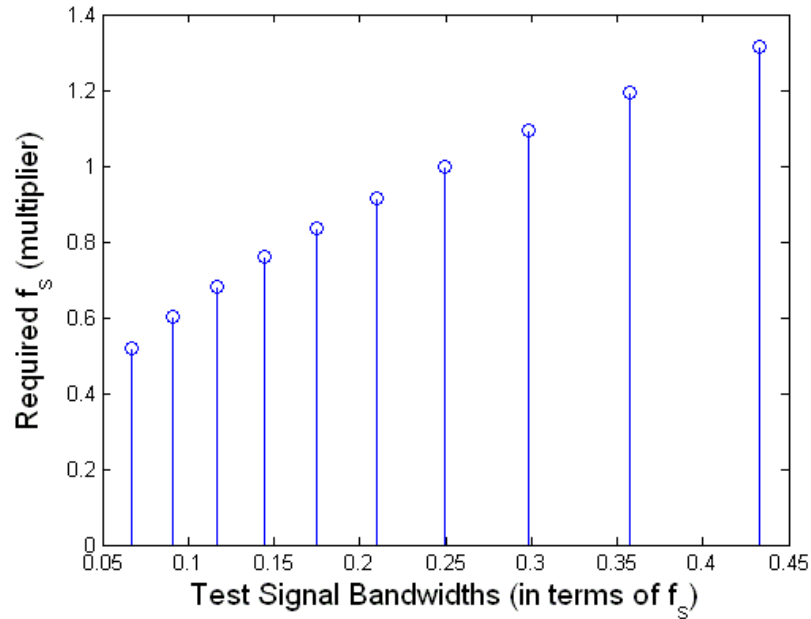


Figure 6.19 Required f_s for $\theta = 45^\circ$ for each test signal

Figure 6.19 shows us that we can adjust DCR by decreasing f_s for signals with $B < f_s/4$ to improve our intercept performance. Improving the interception performance of the ESR by using a slower sampling rate is another gain of the method. Table 6.3 shows the required f_s to make each signal's chirp rate 45° .

Table 6.3 Required f_s to make each signal's chirp rate 45°

	Original θ°	Original f_s (Hz)	New θ°	New f_s (Hz)	Bandwidth (Hz)	fc
Signal 5	50	8000	45	7328	1678	$f_s/4$
Signal 6	55			6694	1400	
Signal 7	60			6079	1155	
Signal 8	65			5463	933	
Signal 9	70			4826	728	
Signal 10	75			4141	536	

Figure 6.20 aims to explain that as we decrease f_s , we get better frequency resolution since we represent a smaller frequency span in a given frequency bin. Figure 6.20.a. shows the ratio of the signal bandwidth over the original sampling frequency (red circle marked line) and over the new sampling frequencies (blue diamond marked line). It is seen that the chirp rate adaptation also keeps us from oversampling. As an example, for S10(·) we use almost half of the f_s used in the original case. Figure 6.20.b shows the original frequency resolution (red circle marked line) and, increase in frequency resolution with decreasing sampling frequency (blue diamond marked line). These values mean that in WVD, we can represent the signals in more detail in frequency domain after chirp rate adaptation.

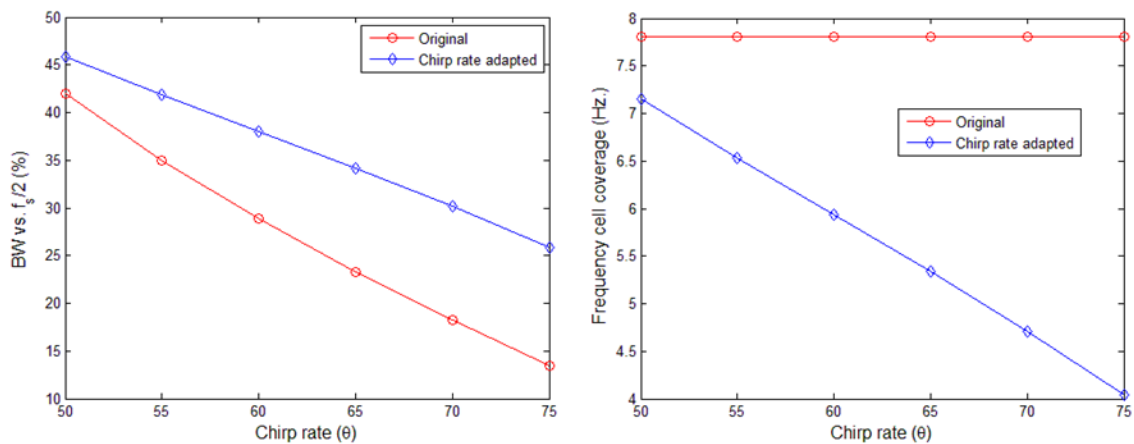


Figure 6.20 a.) Percent of BW ratio over $f_s/2$ b.) Frequency resolution increase as we decrease f_s .

In Figure 6.21 we can see that as we decrease sampling frequency, more time is covered in each time bin and more signal components can be observed in N time samples. These effects can be observed in Figures 6.22 and Figure 6.23 where we compare the original signal WVDs with chirp-rate-adapted signal WVDs.

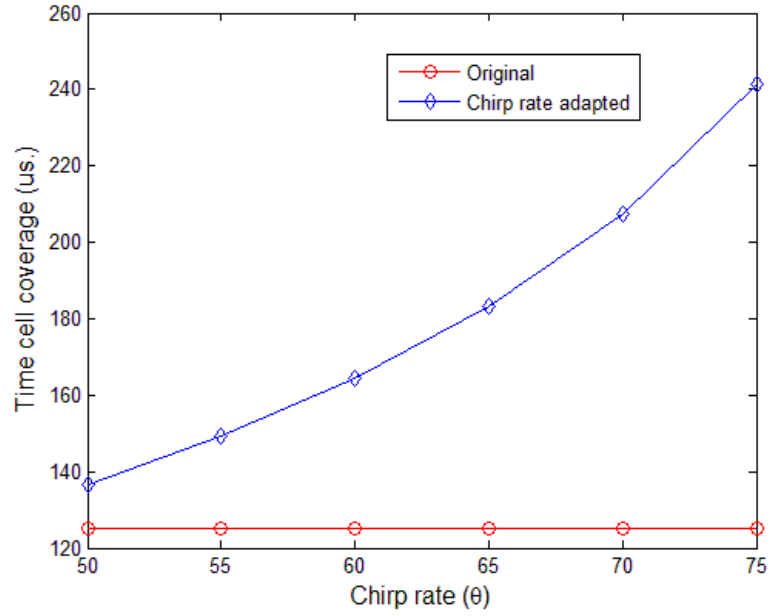


Figure 6.21 Time resolution decreases as we decrease f_s .

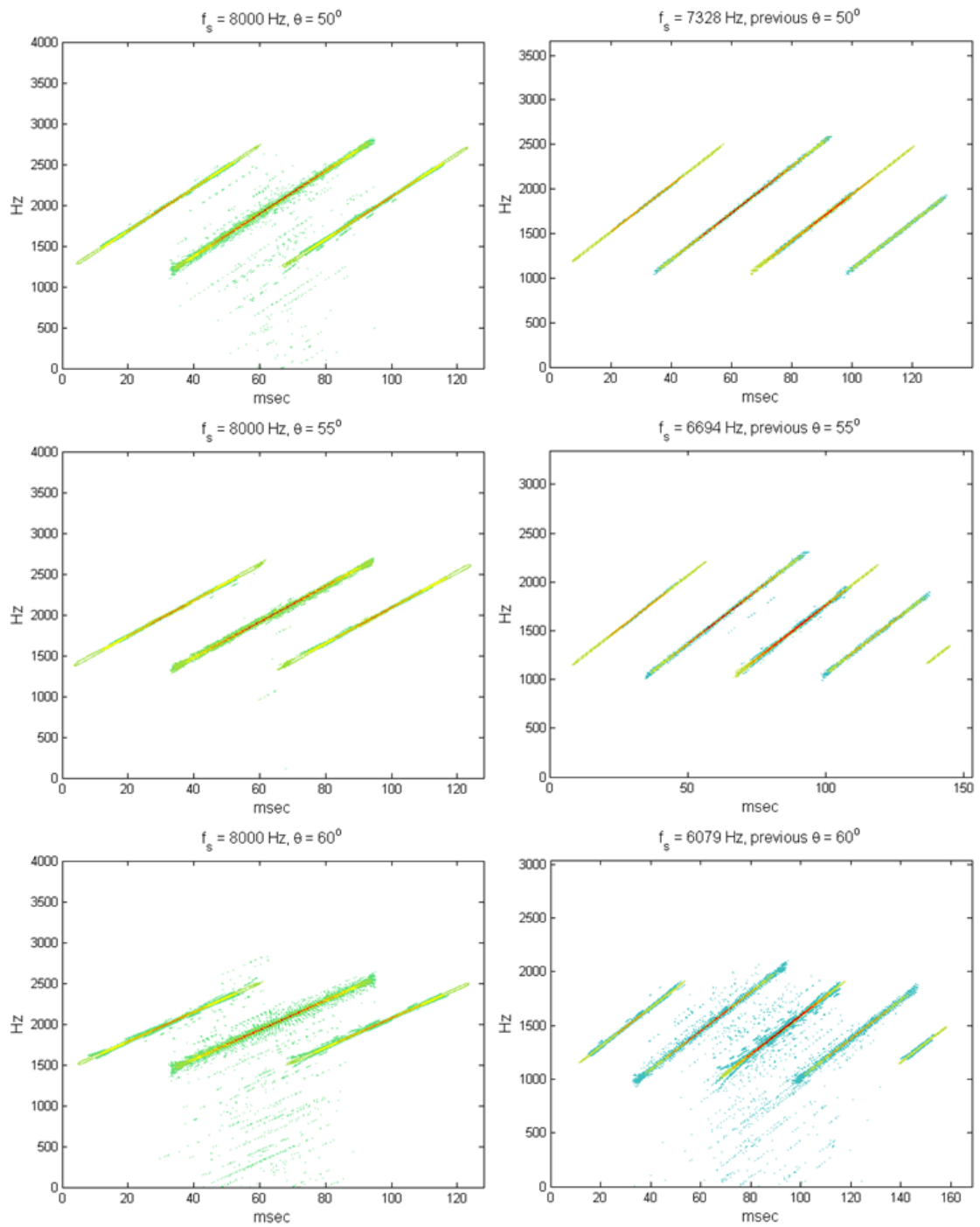


Figure 6.22 Comparing original signal WVDs (left) with chirp-rate-adapted WVDs (right)

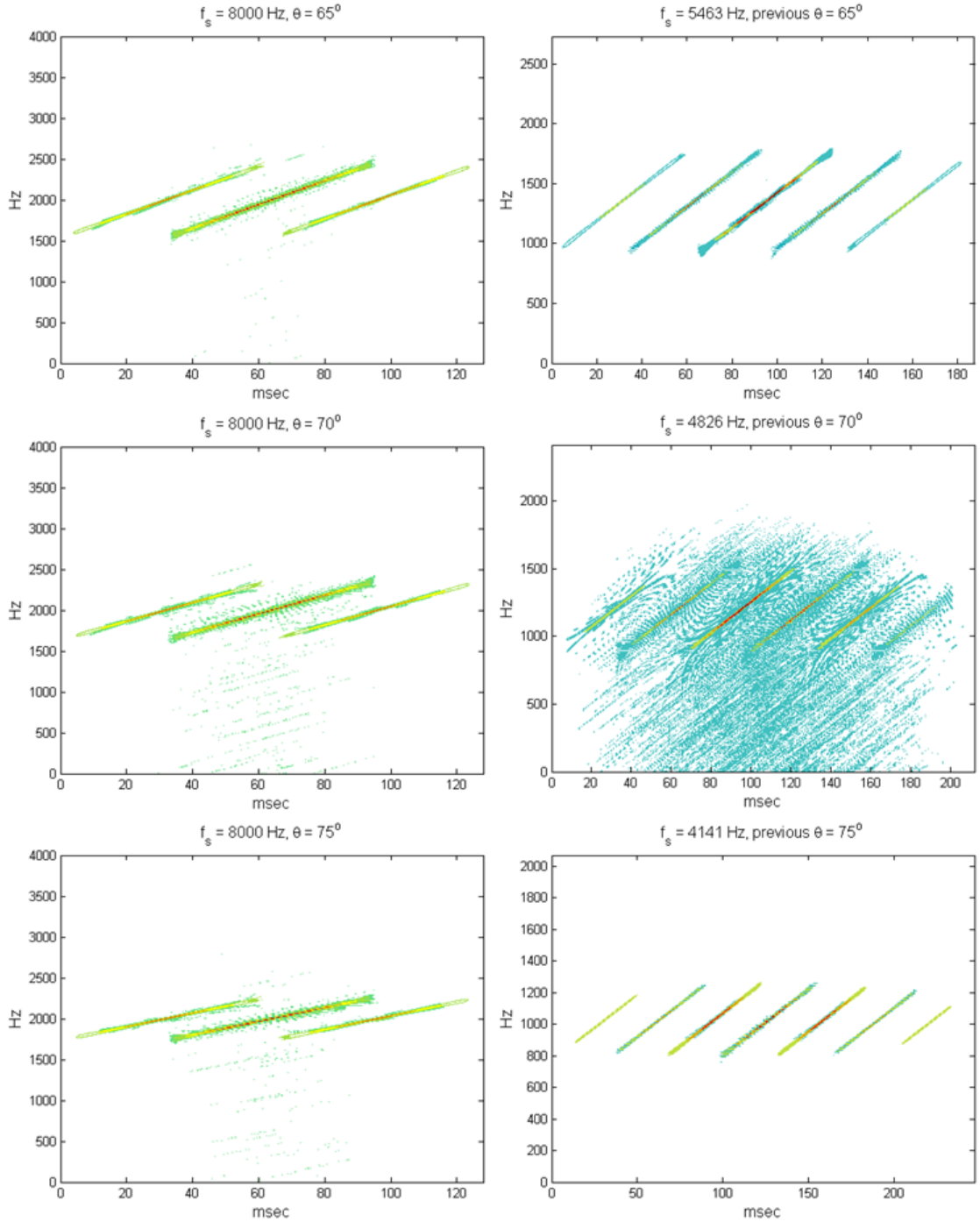


Figure 6.23 Comparing original signal WVDs (left) with chirp-rate-adapted WVDs (right)

6.4.3 Digital Chirp Rate Adaptation Implementation Issues

The idea of digital chirp rate adaptation is only mentioned here by using simulations. This method assumes that ESR is tuned to one signal each time, i.e., using a specific sampling frequency for each signal. The actual implementation requires more detailed work on many issues, and these are not covered within the scope of this thesis. At least, the issues listed below shall be considered before implementation;

- Cross-term effect on the auto-terms,
- New detection threshold for each sampling frequency,
- Choice of FFT length and time samples, i.e., how many signal components we want to observe,
- The effect of new detection threshold and WVD axis lengths on HT.

CHAPTER 7

CONCLUSION

With more LPI radars coming into use, the signals of interest are changing at a rapid pace. ESRs currently in use are not well optimized for LPI radar detection. Detection of LPI radar signals requires sophisticated receivers that use time-frequency signal processing, correlation techniques and algorithms to overcome the processing gain of the LPI radar. Ideally, it is desired to have an ESR with a high degree of accuracy, low P_{fa} , good signal processing capacity and 100% P_d . But in practice, the design of an ESR is usually a trade off of these requirements.

The performance of any particular ESR system should be defined by the purpose of the system. Most of the time, it is not necessary to have idealistic performance figures to achieve the dedicated task. It is more practical to specify performance working backwards from what is available to the system such as; the required time before intercept or identification [85].

Like communication receivers, ESRs process received signals over a period many receptions. And this makes it possible to perform detection without the need to have 100% P_d on every received signal. In this thesis, TF signal processing methods to be used in digital intercept receivers for FMCW LPI radar detection are proposed. It is shown that these methods can improve the performance of digital ESRs.

In this thesis TF space and parameter space are used for detection and parameter extraction of FMCW LPI signals. WVD is used to derive the TF space and Hough / Radon (H/R) type transforms are used to derive the parameter space. This combination of WVD and H/R transform spaces makes it possible to redefine the problem of detecting FMCW components in WVD as detecting the high intensity points in H/R transform space.

WHT based methods in literature are known to be computationally expensive since they use the entire WVD space and calculate HT for all the possible chirp rates. Calculating the HT for only a limited set of chirp rates and for WVD pixels crossing the threshold will reduce the required processing power and complexity. In this thesis we present our work on new WHT methods that reduce the complexity yet keeping detection and parameter extraction performance above a certain level. The proposed methods were compared in terms of speed and parameter extraction performance. The analyses of these methods were performed using correct calculation of the orthogonal distance between FMCW signal components, chirp rate estimation performance and the transform speed. We also derived the ROC curves of the detection methods.

The new FMCW detection methods and parameter extraction methods produced successful results under the simulation assumptions stated throughout the work. Considering the computational complexity and performance analysis results, we can conclude that XWHT method produced very successful results in a much less processing time compared to other methods which makes it a candidate method to be used in digital intercept receivers for LPI radar detection. Since the main goal in ESR receiver design is to process the received signal in near real time, the calculation speed of the processing methods is the main concern. In the EW area, the emphasis is always on techniques rather than theoretical discussions [84].

Another observation was that, the interception performance of WHT based methods is better between chirp-rates $\theta = 30^\circ - 45^\circ$. We have shown that, in the ESR side we can control the intercepted signals DCR to be in this interval and use it to our advantage for optimum waveform interception. DCR adaptation method for improved FMCW interception performance is also a candidate method to be used in ESRs for LPI radar interception. However, the actual implementation of this method requires more detailed analysis.

Future work will be on; trying the methods on a much bigger database including poly-phase, poly-time coded signals, and hardware implementation of the proposed methods with real radar waveform simulations.

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PUBLISHERMENTS

Conference Papers

1. Gulum, T.O., Erdogan, A.Y., Yildirim, T. and Durak-Ata, L., (2011). "Parameter extraction of FMCW modulated radar signals using Wigner-Hough Transform", 12th IEEE International Symposium on Computational Intelligence and Informatics, 21-22 November 2011, Budapest.
2. Gulum, T.O., Erdogan, A.Y. Yildirim, T., (2012). "An edge detection technique using hybrid Ant Colony Optimization-genetic algorithm", SIU 2012, 18-20 April 2012, Antalya.
3. Gulum, T.O., Erdogan, A.Y., Yildirim, T. and Pace, P.E., (2012). "A parameter extraction technique for FMCW radar signals using Wigner-Hough-Radon transform", IEEE RadarCon12, 7-11 May 2012, Atlanta.
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Projects

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AWARDS

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2. Awarded “2014 2214-A Yurt Disi Arastirma Burs Programi” scholarship by TUBITAK. (Unable to use the scholarship due to professional obligations.)