FUZZY ANALYTIC HIERARCHY PROCESS: A COMPARISON OF THE EXISTING ALGORITHMS WITH NEW PROPOSALS

by,

FARAN AHMED

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ABSTRACT

Fuzzy Analytic Hierarchy Process: A Comparison of the Existing Algorithms with New Proposals

Faran Ahmed

M.S. in Industial Engineering

Thesis Supervisor: Dr. Kemal Kılıç

Keywords: *Analytic Hierarchy Process, Fuzzy Analytic Hierarchy Process, Performance Analysis, Fuzzy Extent Analysis, Logarithmic Least Square Method.*

In a multiple-criteria decision analysis, prioritizing and assigning weights to each criteria with reference to set of available alternatives is key to effective decision making. Analytic Hierarchy Process (AHP) is one such technique through which experts provide pairwise comparisons and this information is processed in a comparison matrix to calculate priority vector which ranks the available alternatives. Original AHP as proposed by Thomas L. Saaty used crisp numbers to represent pairwise comparisons. However, human judgments are often vague and traditional 1-9 scale is not capable to incorporate the inherent human uncertainty into pairwise comparisons. In order to address this issue, fuzzy set theory is being used along side original AHP where human judgments are recorded in the form of fuzzy numbers and thus comparison matrices are formed in such a way that its elements are fuzzy numbers.

Various algorithms have been proposed over the past three decades through which priority vector can be calculated from fuzzy comparison matrices. This study performs an extensive review of the most common algorithms proposed in fuzzy AHP (FAHP) and conducts a performance analysis of nine algorithms, out of which five are existing FAHP algorithms namely Logarithmic Least Square Method (LLSM), Modified LLSM, Fuzzy Extent Analysis (FEA), modified FEA and Buckley's Geometric Mean method, while four models are introduced in this study which includes Geometric Mean method, Arithmetic Mean method, Row Sum method and Inverse of Column Sum method. A separate algorithm is also proposed to construct fuzzy comparison matrices of varying sizes, level of fuzziness and inconsistency, so as to carry out performance analysis of the selected nine FAHP algorithms.

We found that Geometric Mean method discussed in this study performs significantly better than other algorithms, while FEA is the worst performing algorithm. Although at high inconsistency levels, performance of FEA method improves however, even at high inconsistency levels, Geometric Mean method performs significantly better. Modification to FEA method (Row Sum method) proposed in this study significantly improves its performance and this modified FEA method is the second best performing algorithm among the selected nine FAHP models.

In addition, we also conducted a comparative analysis based on popularity, computational time, applicability of fuzzy numbers, ease of understanding and ease of implementation. Through this study, we attempt to consolidate the existing literature on FAHP algorithms and identify the best performing methodologies to calculate priority vector from fuzzy comparison matrices.

ÖZET

Bulanık Analitik Hierarşi Süreci: Bilimsel Yazında Yeralan ve Yeni Önerilen Algoritmaların Performanslarının Kıyaslanması

Faran Ahmed
Yüksek Lisans, Endüstri Mühendisliği
Tez Danışmanı: Dr. Kemal Kılıç

Anahtar Sözcükler: *Analitik Hierarşi Süreci, Bulanık Analitik Hierarşi Süreci, Performans Kıyaslama, Bulanık Boyut Analizi, Logaritmik En Küçük Kareler Yöntemi.*

Çok kriterli karar analizinde, kriterlere doğru ağırlıklar atanması iyi kararlar verilmesi için oldukça çok önemlidir. Analitik Hiyerarşi Süreci (AHS) uzmanlar tarafından kriterlerin ikişerli kıyaslamaları neticesinde oluşturulan bir kıyas matrisinin kullanılarak, söz konusu ağırlıkların (başka bir değişle öncelik vektörünün) bulunmasının sağlanmasında yaygın olarak kullanılan bir yöntemdir. Thomas L. Saaty tarafından geliştirilmiş olan AHSde, önceleri söz konusu ikili kıyaslamaların, tam sayılar kullanılarak ölçülmesi önerilmiştir. Halbuki, insani yargılar genellikle belirsizdir ve geleneksel 1den 9a tam sayılardan oluşan ölçülendirme sistemi, doğal insani belirsizlikleri ikili kıyaslamalara dahil edebilmek için yeterli olmayabilir. Zaman içerisinde, bu problemin üstesinden gelebilmek için, insani yargıların bulanık sayılar formunda tutulduğu ve bu yüzden kıyas matrislerinin elemanlarının da bulanık sayılardan oluştuğu Bulanık AHS (BAHS) bilimsel yazında daha sık olarak kullanılmaya başlandı.

Son otuz yılda, öncelik vektörünün bulanık kıyas matrisleri kullanılarak hesaplandığı çeşitli algoritmalar üretildi. Bu çalışmada, BAHS alanında ortaya atılmış algoritmalar geniş çaplı bir araştırma ile tarandı ve aralarında bunların beşinin ve yeni önerilmekte olan dört algoritmanın, yani toplam olarak dokuz algoritmanın performansları karşılaştırıldı. Bilimsel yazından kaynaklı olan beş algoritmayı sıralamak gerekirse; Logaritmik En Küçük Kareler Yöntemi (LEKKY), İyileştirilmiş LEKKY, Bulanık Boyut Analiz (BBA), İyileştirilmiş BBA ve Buckleynin Geometrik Ortalama Yöntemi (BGOY)dir. Diğer dört algoritma da bu çalışmada ortaya atılan Geometrik Ortalama Yöntemi (GOY), Aritmetik Ortalama Yöntemi (AOY), Satır Toplamı Yöntemi (STY) ve Sütun Toplamının Tersi Yöntemi (STTY)dir. Tez kapsamında ayrıca söz konusu dokuz BAHS algoritmasının performanslarının karşılaştırılabilmesi

amacıyla çeşitli büyüklüklerde, bulanıklık derecelerinde ve tutarsızlıklarda bulanık kıyas matrislerinin oluşturulmasını sağlayan bir çerçeve de geliştirilmiştir.

Analizler sonucunda, GOYun diğer yöntemlerden çok daha iyi sonuçlar verdiği, bununla birlikte BBAnın ise en kötü sonuçları verdiği görülmüştür. Yüksek tutarsızlık seviyelerinde, BBA yönteminin performasının artmasına rağmen GOYun çok daha iyi çalıştığı sonucuna da ulaşılmıştır. Bu çalışmada ortaya atılan, BBA yöntemine yapılan bir değişikliğin (yukarıda STY olarak adlandırılmış olan) algoritmanın performansını büyük ölçüde artırdığı belirlenmiş ve bu yeni yöntem karşılaştırılmış olan dokuz BAHS algoritması arasından en iyi ikinci sonucu veren algoritma olarak bulunmuştur.

Bütün bunlara ek olarak; popülerlik, hesaplama süresi, bulanık sayıların uygulanılabilirliği, anlaşılma kolaylığı ve uygulama kolaylığına göre karşılaştırmalı bir analiz yapılmıştır. Özetle, bu çalışma sürecinde varolan literatürü pekiştirmeye ve bulanık kıyaslama matrisleri kullanılarak öncelik vektörünü hesaplamak için en iyi çalışan yöntemler belirlenmiş, bilimsel yazında daha önce yer alamayan yeni yöntemler önerilmiştir.

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This work is dedicated to

My Beloved Parents

&

My Brother

Whose support, guidance and

encouragement have been the

source of inspiration throughout

completion of this project

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List of Abbreviations and Symbols

AHP	Analytic Hierarchy Process
FAHP	Fuzzy Analytic Hierarchy Process
LLSM	Logarithmic Least Square Method
FEA	Fuzzy Extent Analysis
FMCG	Fast Moving Consumer Goods
MCDM	Multiple Criteria Decision Making
TOPSIS	Technique for Order of Preference by Similarity to Ideal Solution
I.C.S	Inverse of Column Sum
A.M	Arithmetic Mean
G.M	Geometric Mean
R.S	Row Sum
R.M	Row Multiplication
C.I	Consistency Index
C.R	Consistency Ratio
R.I	Random Index
λ	Eigenvalue
μ	Membership Function
α	Fuzzification Parameter
β	Inconsistency parameter

Chapter 1

Introduction

In both the corporate work environment as well as our daily routine life, decisions are being made which are rarely straightforward due to multiple factors that have to be considered. For example, an FMCG company, while choosing its supplier for a certain chemical will not only compare the prices, but also the quality of the product being offered, supplier image, transportation means and other miscellaneous factors would be considered. When choosing a university to pursue postgraduate studies, a student would consider the rank of the university, tuition fees, living conditions, and perhaps how far away it is from home. However, usually these criteria are conflicting with each other and hence it is not possible to choose an alternative that is best in terms of all of the criteria. Therefore, tradeoff has to be made, while the relative importance of the criteria with respect to each other is also considered.

There are number of different techniques available in the literature which prioritize and rank the available criteria. One such technique is Analytic Hierarchy Process (AHP) proposed by Thomas L. Saaty [1] and is one of the most popular methods in Multi Criteria Decision Analysis (MCDM) [2] . In this technique, experts are asked to provide their opinions through pairwise comparisons and these opinions are recorded in a comparison matrix. Afterwards, criterion weights can be extracted, for which number of different techniques have been developed over the years. Some of the most common techniques includes but are not limited to Saaty's eigenvector procedure, arithmetic mean approach, geometric mean approach, etc. Note that these techniques can be used both to extract the relative importance of the criteria and to determine the individual priorities of each alternative with respect to each criterion.

However, one of the major challenges being faced in AHP is to accurately transform expert opinions into comparison ratios, with various weighing scales proposed by different authors. Original AHP uses crisp numbers (Scale of 1-9) to represents expert judgments, however in reality these judgments are vague, and the given scale cannot incorporate the inherent uncertainty in human observation. To estimate more accurate weights, fuzzy set theory is extensively incorporated into the original AHP in which the weighing scale is composed of fuzzy numbers. Zadeh [3] introduced fuzzy set theory to address vagueness of human behavior in which fuzzy sets are represented by a continuum grade of membership called membership function which ranges from zero to one. Keeping in view the complexity of the decision making problem, not incorporating fuzziness of the human behavior into the decision analysis may lead to wrong decisions [4]. Judgment scale in Fuzzy Analytical Hierarchy Process (FAHP) is represented by fuzzy numbers and consequently the comparison matrix is also formed in such a way that its elements are fuzzy numbers and thus aim of FAHP is to extract weights from these fuzzy comparison matrices.

Review of the existing literature on FAHP shows that various algorithms have been proposed over the last three decades, with each claiming to estimate more accurate weights. Therefore, there is a need to review the most common algorithms proposed in the domain of FAHP and conduct a performance analysis to validate their accuracy claims. Until now such a review and comparison of FAHP algorithms is not available in the literature.

In order to conduct performance analysis of selected FAHP algorithms, we first propose an algorithm to construct fuzzy comparison matrices of varying sizes, level of fuzziness and inconsistency. Total of nine algorithms are investigated in this study out of which five are already implemented in the literature, while we add four new algorithms in the pool of existing FAHP literature. Two of these models have been extensively used in original AHP (Geometric Mean and Arithmetic Mean) and therefore, we replicate the same methodology in FAHP. We introduce a modified version of Fuzzy Extent Analysis method which was originally proposed by Chang [5] and found that the modified Fuzzy Extent Analysis performs significantly better than the original model. The fourth model is Inverse of Column Sum (I.C.S) which is proposed for the first time in this study. In addition we also compare these models with reference to popularity, computational time, applicability of fuzzy numbers, ease of understanding and ease of implementation.

Rest of the thesis is arranged as follows. Next chapter provides a comprehensive review of both original AHP as well as FAHP algorithms. In chapter 3, the design of experimental analysis is provided, while in chapter 4, results of performance analysis are summarized. In the final part of this thesis, conclusion and future research areas are highlighted.

Chapter 2

Analytic Hierarchy Process and its Fuzzy Extension

In a decision making environment, mechanism through which priorities are derived from a comparison matrix is of critical importance. In order to ensure effective decision making, these priorities should be unique and must capture the dominance of the judgments expressed by the experts [6]. Analytical Hierarchical Process (AHP) is such a technique through which priority scales can be derived by utilizing pairwise comparisons acquired through judgment of experts. AHP can also be used to determine the scores of the alternatives in terms of each criterion. Note that, in the rest of the thesis we will refer to the relative priority of the criteria, however, the same classification applies to individual scores of each criteria as well. The whole process consists of three main stages: decomposition of the main problem in a hierarchical structure consisting of sub problems, pairwise comparisons of the criteria with respect to each other a well as with reference to available alternative and in the final step weights or priority vector is estimated from the given comparison matrix.

The process starts by defining a fundamental objective and its associated hierarchy of sub objectives as well as available alternatives to achieve that fundamental objective which forms the final hierarchical structure as illustrated in Figure 2.1. In the previous example of a university student, the objective is to pursue post graduate studies in the best possible institution with the number of different universities that student can apply are referred as the available alternatives. At the intermediate level, we have various sub-objectives that are relevant to attain the overall objective which are referred to as the criteria. In the stated example, criteria could be rank of the university, tuition fee, living conditions etc. The aim of AHP is to systematically incorporate these criteria into decision making process by assigning weights to the criteria which will help rank and prioritize available alternatives.

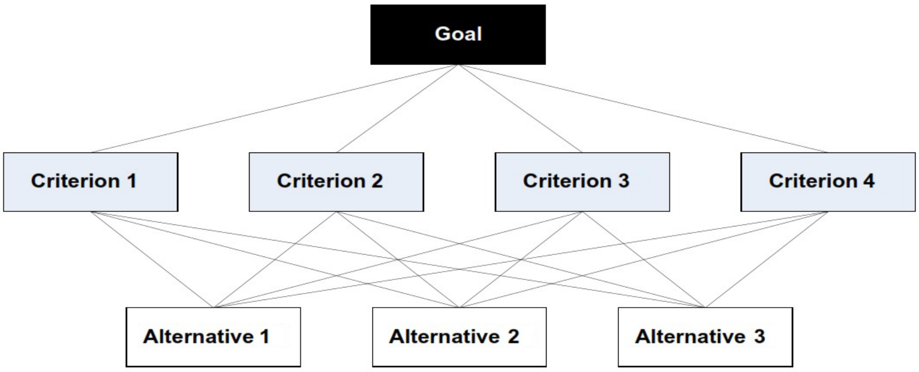


Figure 2.1: AHP hierarchical structure

Based on the judgment scale used AHP can be categorized into two; crisp AHP (i.e., the original AHP) which is based on 1-9 scale of crisp number as tabulated in Table 2.1, or the Fuzzy AHP (i.e., FAHP) where the judgment scales are fuzzy numbers. In the remainder of this chapter, we will first discuss the literature on the original AHP in detail. Later we will, briefly introduce the basics of the fuzzy set theory and fuzzy arithmetic, which will prepare the readers to the last sections which discuses the existing literature of FAHP and the new FAHP algorithms that are proposed.

2.1 Original Analytical Hierarchical Process (AHP)

The original AHP introduced by Thomas L. Saaty is based on the judgment scale that utilizes the crisp numbers as tabulated in Table 2.1. The judgments of the decision maker(s) are assessed through a process which is based on pair wise comparisons and a comparison matrix is constructed as a result. Suppose that for the decision maker(s), a_{ij} is the relative importance of criterion i with respect to criterion j. The comparison matrix that would be constructed will be as follows:

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \tag{2.1}$$

Table 2.1: Crisp AHP scale

Scale	**Representation**
1	Equal importance
2	Weak or slight
3	Moderate importance
4	Moderate plus
5	Strong importance
6	Strong plus
7	Very Strong or demonstrated importance
8	Very, very strong importance
9	Extreme Importance

Once the comparison matrix is formed, then there are number of different techniques through which weights or priority vector w_i can be calculated. The aim is to calculate set of priority vector $w_1, w_2, ..., w_n$, such that w_i/w_j match the comparisons matrix element a_{ij}. However, this is only possible if the expert opinions are perfectly consistent meaning that the comparison matrix holds the transitivity rule i.e. $a_{ik} = a_{ij}.a_{jk}$. Practically this is impossible and thus leads to inconsistent comparison matrix. The issue of consistency will be addressed later in this section. Following we provide a brief overview of the most popular methods employed to calculate priority vectors from comparison matrix.

2.1.1 Eigenvector

The original method proposed by Satty [1] was that of eigenvector. Let us briefly provide an overview of eigenvalues and eigenvectors. Assuming a Matrix A is multiplied with a nonzero vector x, then if the resultant vector Ax is in the same direction as x then we say that x is an eigenvector of the matrix A. Whenever, such a matrix is multiplied with its eigenvector x, then the resultant vector is λ (i.e., the corresponding eigenvalue) times the original vector x. Provided we have a fully consistent comparison matrix and multiply it with the column priority vector (which we are trying to identify) we end up with following:

$$\begin{pmatrix} w_1/w_1 & w_1/w_2 & \cdots & w_1/w_n \\ w_2/w_1 & w_2/w_2 & \cdots & w_2/w_n \\ \vdots & \vdots & \ddots & \vdots \\ w_n/w_1 & w_n/w_2 & \cdots & w_n/w_n \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix} = n \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix} \tag{2.2}$$

Therefore, provided we have a comparison matrix A, we can solve for the priority vector such that $A \times p = n \times p$, where n is the eigenvalue. Note that as a general rule, sum of the eigenvalues of a $n \times n$ matrix A is equal to the trace (i.e., sum of the diagonal elements) of A. Due to the special structure of the fully consistent comparison matrix (i.e., the transitivity rule holds and as a result the rank of such a matrix is 1), it has only one eigenvalue and its value is n (the sum of the diagonal elements, $\sum_{i=1}^{n} 1 = n$).

In reality, we do not encounter a perfectly consistent comparison matrix that is assessed from the decision maker(s). Therefore, the comparison matrix yields multiple number of eigenvalues with values that are not equal to n. Saaty proposes to use the maximum eigenvalue among the set of the eigenvalues that would be obtained from a inconsistent comparison matrix, which would be closer to the theoretical value of n obtained from a fully consistent comparison matrix. Furthermore, the deviation of the maximum eigenvalue and the theoretical value (i.e., n) can be used as a measure for the inconsistency of the comparison matrix. We will discuss this issue in more detail in subsection 2.6.1. Mathematical formulation for estimating maximum eigenvalues is given by Equation 2.3.

$$A \times p = \lambda_{max} \times p \tag{2.3}$$

where $\lambda_{max} \approx n$. As explained earlier, in case of a perfectly consistent matrix $\lambda_{max} = n$. Once the eigenvector corresponding to the maximum eigenvalue is calculated, it is normalized to estimate the final priority vector.

2.1.2 Arithmetic and Geometric Mean:

After the original proposal of Saaty, various other techniques that is not based on the eigen vector procedure is proposed in the literature. Arithmetic mean and the geometric mean approaches are among the most common ones. These two techniques originates from the properties of a fully consistent comparison matrix. Recall that a fully consistent comparison matrix is as follows:

$$W' = \begin{pmatrix} w_1/w_1 & w_1/w_2 & \cdots & w_1/w_n \\ w_2/w_1 & w_2/w_2 & \cdots & w_2/w_n \\ \vdots & \vdots & \ddots & \vdots \\ w_n/w_1 & w_n/w_2 & \cdots & w_n/w_n \end{pmatrix} \tag{2.4}$$

In the first step, we sum up each column which results in;

$$\frac{w_1 + w_2, \cdots, w_n}{w_1}, \frac{w_1 + w_2, \cdots, w_n}{w_2}, \cdots, \frac{w_1 + w_2, \cdots, w_n}{w_n}, \tag{2.5}$$

As $w_1 + w_2, \cdots, w_n = 1$, therefore column sums are equivalent to

$$\frac{1}{w_1}, \frac{1}{w_2}, \cdots, \frac{1}{w_n}, \tag{2.6}$$

Next we divide each element of the comparison Matrix with its corresponding column sum. We end up with n vectors as following.

$$W = \begin{pmatrix} w_1 & w_1 & \cdots & w_1 \\ w_2 & w_2 & \cdots & w_2 \\ \vdots & \vdots & \ddots & \vdots \\ w_n & w_n & \cdots & w_n \end{pmatrix} \tag{2.7}$$

That is to say for a fully consistent matrix, if one applies the above described normalization process, the resulting matrix W is composed of column vectors which are equal to each other, and they are all equal to the weights vector, i.e. $(w_1, w_2, \cdots, w_n)$. However, since in practice the comparison matrix obtained from the decision makers are rarely consistent, the resulting matrix of the weight vectors would not be composed of same column vectors and they would be different from each other. Since each column is a candidate for the weight vector, and the source of the inconsistency cannot be detected, a reasonable thing to do is to average the columns of the normalized matrix W. The average can be obtained by either arithmetic means or the geometric means approaches. Equations 2.8 and 2.9 represents these two approaches.

$$A.M = \frac{\sum_{i=1}^{n} w_j}{n} \quad for\ j = 1, 2, \cdots, n \tag{2.8}$$

$$G.M = \left[\prod_{i=1}^{n} w_j\right]^{1/n} \quad for\ j = 1, 2, \cdots, n \tag{2.9}$$

2.1.3 Row Sum

We start with the similar perfectly consistent comparison matrix given as follows;

$$W = \begin{pmatrix} w_1/w_1 & w_1/w_2 & \cdots & w_1/w_n \\ w_2/w_1 & w_2/w_2 & \cdots & w_2/w_n \\ \vdots & \vdots & \ddots & \vdots \\ w_n/w_1 & w_n/w_2 & \cdots & w_n/w_n \end{pmatrix} \tag{2.10}$$

We first take sum of all elements in i^{th} row and assign it to $R.S_i$. The sum of each row is as follows;

$$R.S_1 = w_1\left(\frac{1}{w_1} + \frac{1}{w_2} + \cdots + \frac{1}{w_n}\right) \tag{2.11}$$

$$R.S_2 = w_2\left(\frac{1}{w_1} + \frac{1}{w_2} + \cdots + \frac{1}{w_n}\right) \tag{2.12}$$

$$\vdots$$

$$R.S_n = w_n\left(\frac{1}{w_1} + \frac{1}{w_2} + \cdots + \frac{1}{w_n}\right) \tag{2.13}$$

The sum of all $R.S_i$ is given as follows;

$$\sum_{i=1}^{n} R.S_i = (w_1 + w_2 + \cdots + w_n).\left(\frac{1}{w_1} + \frac{1}{w_2} + \cdots + \frac{1}{w_n}\right) \tag{2.14}$$

where as $(w_1 + w_2 + \cdots + w_n) = 1$. In the last step, we calculate the priority vector by normalizing each $R.S_i$ by dividing it by $\sum_{i=1}^{n} R.S_i$. The priority vector is given as under;

$$\frac{R.S_1}{\sum_{i=1}^{n} R.S_i} = w_1, \frac{R.S_2}{\sum_{i=1}^{n} R.S_i} = w_2, \cdots \frac{R.S_n}{\sum_{i=1}^{n} R.S_i} = w_n \tag{2.15}$$

As comparison matrix is assumed to be perfectly consistent, therefore, the priority vector is given by $W = w_1, w_2, \cdots, w_n$

2.1.4 Row Multiplication:

The only difference between row multiplication method with that of row sum method is that in row multiplication instead of taking row sums, each element of the row is multiplied and its n^{th} root is taken.

$$R.M_1 = (w_1/w_1 \times w_1/w_2 \times \cdots w_1/w_n)^{1/n} = \frac{w_1}{(w_1 \times w_2 \cdots w_n)^{1/n}} \tag{2.16}$$

$$R.M_2 = (w_2/w_1 \times w_2/w_2 \times \cdots w_2/w_n)^{1/n} = \frac{w_1}{(w_1 \times w_2 \cdots w_n)^{1/n}} \tag{2.17}$$

$$\vdots$$

$$R.M_n = (w_n/w_1 \times w_n/w_2 \times \cdots w_1/w_n)^{1/n} = \frac{w_n}{(w_1 \times w_2 \cdots w_n)^{1/n}} \tag{2.18}$$

Afterwards, we calculate sum of all $R.M_i$ which is as follows;

$$\sum_{i=1}^{n} R.M_i = \frac{w_1 + w_2 + \cdots + w_n}{(w_1 \times w_2 \cdots w_n)^{1/n}} = \frac{1}{(w_1 \times w_2 \cdots w_n)^{1/n}} \tag{2.19}$$

Finally, normalization is done and priority vector is calculated as follows;

$$\frac{R.M_1}{\sum_{i=1}^{n} R.M_i} = w_1, \frac{R.M_2}{\sum_{i=1}^{n} R.M_i} = w_2, \cdots \frac{R.M_n}{\sum_{i=1}^{n} R.M_i} = w_n \tag{2.20}$$

Which is exactly the same as the weights assigned initially due to the fact that comparison matrix is perfectly consistent.

2.1.5 Integrated AHP

Another strategy to utilize AHP is to integrate it with some other supporting tool and make the decision making process more effective. Some of the tools that can be integrated with the AHP includes mathematical programming, quality function deployment (QFD), meta-heuristics, SWOT analysis, and data envelopment analysis (DEA). A comprehensive review is provided by Ho (2008) [7] in which he concludes that AHP integrated with goal programming and AHP integrated with QFD are the most commonly used integrated AHP methods, while logistics and manufacturing are the two main applications where integrated AHP technique has been used. However, integrated AHP is out of the scope of our research and interested readers are referred to this literature review as a starting point.

2.1.6 Consistency in AHP

A matrix is considered to be consistent if and only if $a_{ik} \times a_{kj} = a_{ij}$ for all i, j, k. As stated before, many of the AHP methodologies originates from consistent comparison matrices, such as arithmetic mean approach and geometric mean approach. Note that AHP results are based on subjective comparisons assessed from the experts. Humans are very good at comparing two concepts and providing a preferential ordering. However, they are not that good at associating a score on a particular concept and hence in practice comparison matrices are always inconsistent to some degree. Saaty [1] introduces an approach where the consistency of a matrix can be measured by;

$$C.I. = \frac{\lambda_{max} - n}{n - 1} \tag{2.21}$$

where λ_{max} is the maximum eigenvalue and n is the number of available criteria. Calculating the maximum eigenvalue has already been explained in the previous section. Recall that totally consistent comparison matrix theoretically has only one eigenvalue which is equals to n. As a result, deviation from this theoretical value is used as an indication of inconsistency.

In addition, a random index (R.I) is used to calculate consistency ratio (C.R). Random index is generated randomly and depends on the number of elements to be compared. For details on how to generate R.I, readers are referred to Saaty [1].

$$C.R = \frac{C.I}{R.I} \tag{2.22}$$

If $C.R \leq 0.1$ then the given comparison matrix has a reasonable amount of consistency, otherwise if $C.R \geq 0.1$ then the level of inconsistency is on higher side and comparison matrix should be reformed by consulting the experts again.

These are the most common techniques in original AHP to calculate priority vector or weights. Next, we will provide a brief overview of the fuzzy logic and then provide an extensive review of some of the most common FAHP algorithms proposed in the literature.

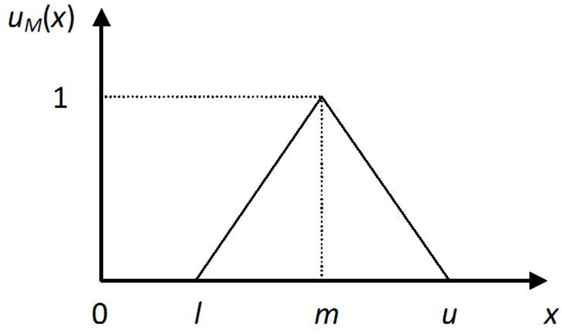
2.2 Introduction to Fuzzy Logic

One of the major concerns in the original AHP is to transform human judgments, which are usually natural language phrases such as "significantly more", "slightly more" etc, into a numerical scale. In order to address this issue, fuzzy sets have been employed which can record the imprecision arising in human judgments which are neither random nor stochastic [8]. Instead of a single value, fuzzy number represents a set of possible values each having its own membership function between zero and one. A triangular fuzzy number is represented by *[lower value, mean value, upper value]*, i.e., $[l \;\; m \;\; u]$ where as trapezoidal number is represented by $[l \; m \; n \; u]$ with membership functions μ_M given by;

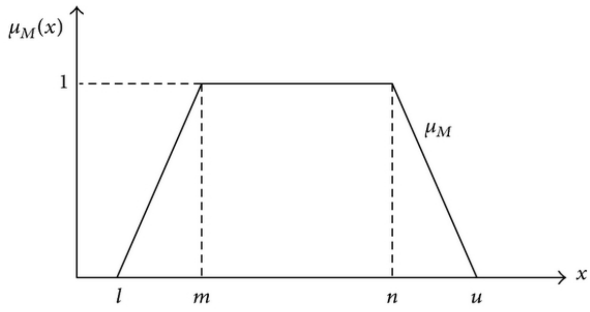
$$\mu_M(x) = \begin{cases} \frac{x}{m-l} - \frac{l}{m-l}, & x \in [l \; m] \\ \frac{x}{m-u} - \frac{u}{m-u}, & x \in [m \; u] \\ 0, & \text{otherwise} \end{cases} \tag{2.23}$$

Note that the membership function defined in Equation 2.23 is for triangular fuzzy numbers.

For trapezoidal numbers, membership function in the interval $[m\ n]$ is equal to one. The same is graphically illustrated in Figure 2.2.



(a) Triangular Fuzzy Number



(b) Trapezoidal Fuzzy Number

Figure 2.2: Membership function of fuzzy numbers

2.2.1 Fuzzy Arithmetic

Let $(l_1\ m_1\ u_1)$ and $(l_2\ m_2\ u_2)$ be two triangular fuzzy numbers and $(l_1\ m_1\ n_1\ u_1)$ be a trapezoidal fuzzy number, than the basic fuzzy arithmetic operations are listed in Table 2.2.

Table 2.2: Fuzzy arithmetic

Operation	**Result**
Addition	$(l_1\ m_1\ u_1) \oplus (l_2\ m_2\ u_2) = (l_1 + l_2 \quad m_1 + m_2 \quad u_1 + u_2)$
Multiplication	$(l_1\ m_1\ u_1) \odot (l_2\ m_2\ u_2) = (l_1.l_2 \quad m_1.m_2 \quad u_1.u_2)$
Scalar Multiplication	$(\lambda\ \lambda\ \lambda) \odot (l_1\ m_1\ u_1) = (\lambda.l_2 \quad \lambda.m_2 \quad \lambda.u_2)$
Inverse (Triangular Fuzzy Number)	$(l_1\ m_1\ u_1)^{-1} \approx (1/u_1 \quad 1/m_1 \quad 1/l_1)$
Inverse (Trapezoidal Fuzzy Number)	$(l_1\ m_1\ n_1\ u_1)^{-1} \approx (1/m_1 \quad 1/l_1 \quad 1/u_1 \quad 1/n_1)$

2.3 FAHP Algorithms

Over the years, various FAHP algorithms have been proposed with each claiming to estimate more accurate weights from a fuzzy comparison matrix. Among these various algorithms, the Logarithmic Least-Squares Method (LLSM) [9], Geometric Mean Method [10] and Fuzzy Synthetic Extent Analysis Method or in short the Fuzzy Extent Analysis (FEA) [5] are the most well known algorithms [11]. Major contributions from various authors in these three main methodologies are tabulated in Table 2.3. We will review these methods in detail for the rest of the section.

Table 2.3: Fuzzy AHP Algorithms

FAHP Approaches	**Authors**
Logarithmic Least Square Method (LLSM)	Original Model proposed by Van Laarhoven & Pedrycz (1983) Modification proposed by Boender. et. al (1989) Modified LLSM model based on constrained non-linear optimization proposed by Wang et. al (2006)
Geometric Mean Method	Original Model proposed by Buckley (1989)
Fuzzy Synthetic Extent Analysis	Original Model proposed by Chang (1996) Modification to normalization proposed by Wang et.al (2008)

2.3.1 Logarithmic Least Squares Method:

Van Laarhoveen and Pedrycz suggested one of the first models in the domain of Fuzzy AHP, which utilizes fuzzy logarithmic least squares method (LLSM) and formulated an unconstrained optimization model to obtain triangular fuzzy weights [9]. However, subsequent research point out some of the irregularities in the original model, especially related to the normalization procedure, and thus proposed modifications accordingly [12, 13]

First let us briefly provide an overview of the original LLSM Model proposed by van Laarhoven and Pedrycz. Let's assume that w_i and w_j are weights to be estimated while a_{ij} is the comparison ratio provided by the expert while comparing criterion i with criterion j. Due to the inherent inconsistency in human judgments, comparison ratio a_{ij} will differ from the corresponding set of weights. Therefore, the goal is to estimate such a combination of weights that minimizes the total deviation between comparison ratios provided by the expert and the ratio of the corresponding weights which can be achieved by minimizing following equation.

$$min \sum_{i<j} (\ln a_{ij} - (\ln w_i/w_j))^2 \tag{2.24}$$

The Equation 2.24 is valid when comparison ratios are provided by single expert and can be rewritten for multiple experts as follows.

$$min \sum_{i<j} \sum_{k=1}^{\delta_{ij}} (\ln a_{ijk} - (\ln w_i/w_j))^2 \tag{2.25}$$

where, δ_{ij} are the number of comparison ratios assessed from different experts available for a certain criteria. The Equation 2.25 is simplified by replacing $y_{ijk} = \ln a_{ij}$, $x_i = \ln w_i$ and $x_j = \ln w_j$;

$$min \sum_{i<j} \sum_{k=1}^{\delta_{ij}} (y_{ijk} - x_i + x_j)^2 \tag{2.26}$$

To minimize the Equation 2.26, we take partial derivatives with respect to x_i and equate them to zero. Following is the resultant set of equations.

$$x_i \sum_{\substack{j=1 \\ j\neq i}}^{n} \delta_{ij} - \sum_{\substack{j=1 \\ j\neq i}}^{n} \delta_{ij} x_j = \sum_{\substack{j=1 \\ j\neq i}}^{n} \sum_{k=1}^{n} y_{ijk} \tag{2.27}$$

The above system is composed of linearly dependent equations which can be simultaneously solved to calculate all x_i's. Afterwards, to convert the system into its original form, exponential of the solution are taken and then normalized to estimate final weights.

However, system of Equation 2.27 is applicable only when the given comparison ratios are in the form of crisp numbers. It can be transformed for triangular fuzzy weights while following the rules for fuzzy arithmetic operations presented earlier in Table 2.2. This transformation is given as follows;

$$l_i \sum_{\substack{j=1 \\ j\neq i}}^{n} \delta_{ij} - \sum_{\substack{j=1 \\ j\neq i}}^{n} \delta_{ij} u_j = \sum_{\substack{j=1 \\ j\neq i}}^{n} \sum_{k=1}^{n} l_{ijk} \tag{2.28}$$

$$m_i \sum_{\substack{j=1 \\ j\neq i}}^{n} \delta_{ij} - \sum_{\substack{j=1 \\ j\neq i}}^{n} \delta_{ij} m_j = \sum_{\substack{j=1 \\ j\neq i}}^{n} \sum_{k=1}^{n} m_{ijk} \tag{2.29}$$

$$u_i \sum_{\substack{j=1 \\ j\neq i}}^{n} \delta_{ij} - \sum_{\substack{j=1 \\ j\neq i}}^{n} \delta_{ij} l_j = \sum_{\substack{j=1 \\ j\neq i}}^{n} \sum_{k=1}^{n} u_{ijk} \tag{2.30}$$

where $l_i = \ln w_{il}$, $m_i = \ln w_{im}$ and $u_i = \ln w_{iu}$.

Same procedure is followed to convert the system into its original form by taking exponential of the solutions and then normalizing to estimate final fuzzy weights;

$$\tilde{w}_i = \left(\frac{exp(l_i)}{\sum_{i=1}^{n} exp(u_i)}, \frac{exp(m_i)}{\sum_{i=1}^{n} exp(m_i)}, \frac{exp(u_i)}{\sum_{i=1}^{n} exp(l_i)} \right) \tag{2.31}$$

As the set of Equations 2.28 - 2.30 are linearly dependent (hence yields infinitely many solutions) and the solution is generally given by

$$x_i = (l_i + p_1, m_i + p_2, u_i + p_1)$$

2.3.1.1 Modifications to Original LLSM Model

Subsequent research on this model identifies various irregularities and appropriate modifications are proposed. In the original LLSM Model, normalization process eliminates the optimality in the sense that the normalized solution violates the first order optimality conditions and thus normalized weights do not minimize the objective function. A modified version of the normalization procedure is proposed by Boender et al. [12] as follows;

$$\tilde{w_i} = \left(\frac{exp(l_i)}{\sqrt{\sum_{i=1}^{n} exp(l_i). \sum_{i=1}^{n} exp(u_i)}}, \frac{exp(m_i)}{\sum_{i=1}^{n} exp(m_i)} \frac{exp(u_i)}{\sqrt{\sum_{i=1}^{n} exp(l_i). \sum_{i=1}^{n} exp(u_i)}} \right) \quad (2.32)$$

However, Wang et al. [13] further criticized some other aspects of the original LLSM Model. These criticism are summarized below.

2.3.1.2 Incorrect Normalization

Fuzzy weights calculated after normalization procedure must satisfy the following conditions [14].

$$\begin{aligned} \sum_{i=1}^{n} w_i^U - \max_j(w_j^U - w_j^L) \geq 1 \\ \sum_{i=1}^{n} w_i^M = 1 \\ \sum_{i=1}^{n} w_i^L - \max_j(w_j^U - w_j^L) \leq 1 \end{aligned} \quad (2.33)$$

Although normalization procedure modified by Boender provides optimal weights, however, Wang et al. [13] shows a counter example in which normalized fuzzy weights violate the conditions presented in Equation 2.33.

2.3.1.3 Incorrectness of Triangular Fuzzy Weights

As mentioned above, solution to given system of equations can be represented as $(l_i{+}p_1, m_i{+}p_2, u_i + p_1)$. It was stated by van Laarhoven and Pedrycz [9] that arbitrary parameters p_1

and p_2 cannot be always chosen in a way that will ensure that the following condition is satisfied;

$$l_i + p_1 \leq m_i + p_2 \leq u_i + p_1, \quad for\ i = i, ..., n$$

After taking exponential and normalizing, fuzzy weights are again in the correct order. However, this claim was found not true as counter example was shown in which the normalized solution violated the given condition of a triangular fuzzy number [13]. Such issues are highlighted in the literature, but no proper recommendations are proposed to solve such issues yet.

2.3.1.4 Uncertainty of fuzzy weights for incomplete comparison matrices

In case of a comparison matrix in which some of the values/ratios are missing, the system of equations formed may contain free variables. Therefore, different configurations of free variables have to be formed with each configurations leading to a different weights. In Boender et al. [12] numerical example such a situation is faced however no justifications is provided for choosing a specific configuration. Such an uncertainty in estimating fuzzy weights exists in all incomplete fuzzy comparison matrices [13]. Kwiesielewicz and van Uden [15] suggest a minimum norm method to calculate values of free variables. This method is based on minimizing the following Euclidean norm.

$$\| lnW \| = \sqrt{\sum_{i}^{n} (l_i^2 + m_i^2 + u_i^2)}$$

However, Wang et al. [13] reports that this method is hard to explain and reason for minimizing the Euclidean norm of $\ln W$ is not clear at all.

2.3.2 Modified Fuzzy LLSM Model

Based on the discussion above, a modified fuzzy LLSM approach consisting of a constrained nonlinear optimization model is suggested by Wang et al. [13] which addresses all of the issues identified previously. The model is stated below:

$$minJ = \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} \sum_{k=1}^{\delta_{ij}} (\ln w_i^L - \ln w_j^U - \ln a_{ijk}^L)^2 + (\ln w_i^M - \ln w_j^M - \ln a_{ijk}^M)^2 + (\ln w_i^U - \ln w_j^L - \ln a_{ijk}^U)^2$$

$$\text{Subject to} \quad \begin{cases} w_i^L + \sum_{j=1, j \neq i}^{n} w_j^U \geq 1 \\ w_i^U + \sum_{j=1, j \neq i}^{n} w_j^L \leq 1 \\ \sum_{i=1}^{n} w_i^M = 1 \\ \sum_{i=1}^{n} (w_i^L + w_i^U) = 2 \\ w_i^U \geq w_i^M \geq w_i^L \end{cases} \tag{2.34}$$

Solution to this mathematical model is normalized fuzzy weights for both complete and incomplete comparison matrices. First three constraints in equation 2.34 satisfies normalization conditions of fuzzy numbers, while fourth constraint ensures a unique solution and the last constraint ensures that the condition $l < m < u$ is always satisfied.

2.3.3 Fuzzy Extent Analysis

Provided that $X = \{x_1, x_2, \cdots, x_n\}$ represents an object set and $G = \{g_1, g_2, \cdots, g_n\}$ represents a goal set, then as per the extent analysis method [16], for each object, extent analysis for each goal g_i is performed. Applying this theory in fuzzy comparison matrix, we can calculate value of fuzzy synthetic extent with respect to the i^{th} object as follows;

$$S_i = \sum_{j=1}^{m} M_{g_i}^j \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{g_i}^j \right]^{-1} \tag{2.35}$$

Where

$$\sum_{j=1}^{m} M_{g_i}^j = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j \right) \tag{2.36}$$

Note that Equation 2.35 resembles the process of the Row Sum approach discussed earlier in Subsection 2.1.3 for the crisp AHP. Recall that, in the case for a fully consistent comparison matrix, for all i the weight would be obtained as the result of this process. For the case where fuzzy triangular numbers are utilized in the judgment scale, the result would be a fuzzy triangular weight value as indicated in Equation 2.36.

Later in the decision making process (i.e., choosing the best alternative) we need to determine a crisp weight from these fuzzy triangular weights. A naive approach would be just using the means (i.e., mean of each fuzzy weight obtained from Equation 2.35). However, as opposed to the straight forward ordering of crisp numbers, the orderings of the fuzzy numbers are not that simple and one should be more careful. Chang [5] suggest utilizing the concept of comparison of fuzzy numbers in order to determine crisp weights from the fuzzy weights. In their approach, for each fuzzy weight, a pair wise comparison with the other fuzzy weights are conducted, and the degree of possibility of being greater than these fuzzy weights are obtained. The minimum of these possibilities are used as the overall score for each criterion i. Finally these scores are normalized (i.e., so that they sum up to 1), and the corresponding normalized scores are used as the weights of the criteria. That is to say by applying the comparison of the fuzzy numbers, the degree of possibility is obtained for each pair wise comparison as follows:

$$V(M_2 \geq M_1) = hgt(M_1 \cap M_2) = \mu_{M_2}(d) = \begin{cases} 1, & \text{if } m_2 \geq m_1 \\ 0, & \text{if } l_1 \geq u_2 \\ \frac{l_1 - u_2}{(m_2-u_2)-(m_1-l_1)}, & \text{otherwise.} \end{cases}$$

The same is illustrated in the Figure 2.3.

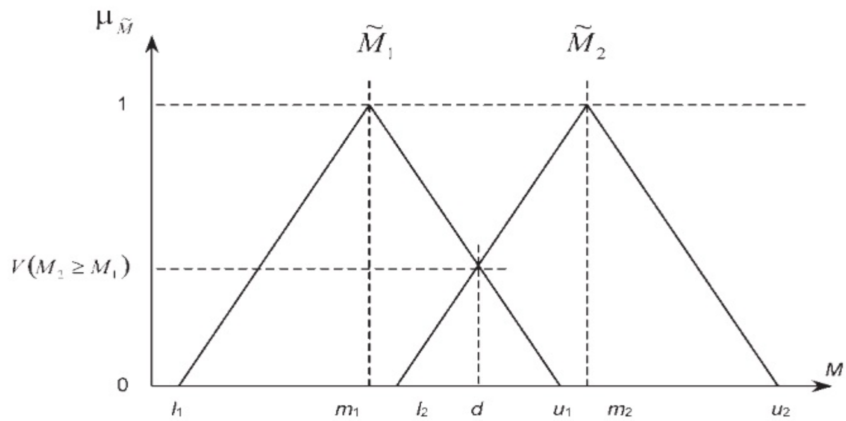


Figure 2.3: Degree of possibility

Degree of possibility for a convex fuzzy number to be greater than k convex fuzzy numbers is given by;

$$V(M \geq M_1, M_2, \cdots, M_k) = V[(M \geq M_1)\text{and}(M \geq M_2), \cdots, (M \geq M_k)] \quad (2.37)$$

$$= \min V(M \geq M_i), \quad i = 1, 2, \cdots, k \quad (2.38)$$

Assuming that $w'_i = \min V(M_i \geq M_k)$ then weight vector is given by

$$W' = w'_1, w'_2, \cdots, w'_n \quad (2.39)$$

Normalizing the above weights gives us the final priority vector $w_1, w_2, \cdots, w_n$

2.3.3.1 Criticism of Fuzzy Extent Analysis

Wang et.al [13] criticized fuzzy extent analysis technique and through an example showed that this method cannot estimate true weights from fuzzy comparison matrix. His main criticism revolves around the fact that this method may assign a zero as criterion weight which disturbs the whole decision making hierarchy. The basis of extent analysis theory is that it provides a degree to which one fuzzy number is greater than another fuzzy number, and this degree of greatness is considered as criterion weights. Therefore, if two fuzzy numbers do not intersect then the degree of greatness of one fuzzy number to the other is 100 percent and therefore it will assign 1 as weight to that criterion while the other criteria will be assigned as zero weight. In light of above Wang et.al summarized the main problems with this method as under;

- Once a criteria is assigned a zero weight, it will not be considered in the decision making process.
- This method may lose some useful information in the form of judgment ratios in the fuzzy comparison matrices as some of the criterion are assigned zero weight.
- It was shown that weights calculated through this method may not represents the true relative importance of that criteria.
- This method might select the worst decision alternative as the best one and thus leads to wrong decision making

2.3.4 Buckley Geometric Mean Method

The original model based on geometric mean was proposed by Buckley in which trapezoidal numbers were used to represent fuzzy numbers [10]. Trapezoidal numbers are defined by $(l/m, n/u)$ where $0 < l \leq m \leq n \leq u$. The membership function of a trapezoidal fuzzy number is explained in figure 2.2. Expert judgment is recorded in a comparison matrix by fuzzy ratio $a_{ij} = (l_{ij}/m_{ij}, n_{ij}/u_{ij})$ whereas $l, m, n, u \in \{1, 2, \cdots, 9\}$. Following calculations are required in order to estimate final weight vector.

$$l = \sum_{i=1}^{n} l_i \quad \text{where as} \quad l_i = \left[\prod_{j=1}^{n} \right]^{1/n} \tag{2.40}$$

$$m = \sum_{i=1}^{n} m_i \quad \text{where as} \quad m_i = \left[\prod_{j=1}^{n}\right]^{1/n} \tag{2.41}$$

$$n = \sum_{i=1}^{n} n_i \quad \text{where as} \quad n_i = \left[\prod_{j=1}^{n}\right]^{1/n} \tag{2.42}$$

$$u = \sum_{i=1}^{n} u_i \quad \text{where as} \quad u_i = \left[\prod_{j=1}^{n}\right]^{1/n} \tag{2.43}$$

The final priority vector is given by $\left(\frac{l_i}{u}, \frac{m_i}{n}, \frac{n_i}{m}, \frac{u_i}{l}\right)$ and the corresponding membership function of the resulting trapezoidal fuzzy number is given by;

$$f_i(y) = \left[\prod_{j=1}^{n}((m_{ij} - l_{ij})y + l_{ij})\right]^{1/n} \tag{2.44}$$

$$g_i(y) = \left[\prod_{j=1}^{n}((n_{ij} - u_{ij})y + u_{ij})\right]^{1/n} \tag{2.45}$$

These are the most common FAHP algorithms implemented in the literature and thus will be included in the performance analysis. Buyukozkan [17] provides a comparison analysis of these models which is summarized in Table 2.4. Next, we will introduce four new models which we will be included in our performance analysis of FAHP algorithms.

2.4 Four Additional Models

In our performance analysis, we add four more FAHP algorithms which were not discussed in the previous sections. They are outlined as follows;

2.4.1 Arithmetic Mean and Geometric Mean

These two algorithms are simply the extension of corresponding algorithms used in original AHP. Same procedure will be followed to replicate these two models in FAHP while following fuzzy arithmetic operation laws.

2.4.2 Row Sum

In the model proposed by Chang [5], values of fuzzy synthetic extent analysis are basically the row sums of fuzzy comparison Matrix. Afterwards, rather than using principal

Table 2.4: FAHP comparison analysis

Sources	Main Characteristics	Advantages (A)/ Disadvantages (D)
Van Laarhoven and Pedrycz (1983)	Direct extension of Saatys AHP method with triangular fuzzy numbers	(A) The opinions of multiple decision makers can be modeled in the reciprocal matrix
	Lootsmas logarithmic least square method is used to derive fuzzy weights and fuzzy performance scores	(D) There is not always a solution to the linear equations
		(D) The computational requirement is tremendous, even for a small problem
		(D) It allows only triangular fuzzy numbers to be used
Buckley (1985)	Extension of Saatys AHP method with trapezoidal fuzzy numbers	(A) It is easy to extend to the fuzzy case
	Uses the geometric mean method to derive fuzzy weights and performance scores	(A) It guarantees a unique solution to the reciprocal comparison matrix
		(D) The computational requirement is tremendous
Boender et al. (1989)	Modies van Laarhoven and Pedryczs method	(A) The opinions of multiple decision makers can be modeled
	Presents a more robust approach to the normalization of the local priorities	(D) The computational requirement is tremendous
Chang (1996)	Synthetical degree values	(A) The computational requirement is relatively low
	Layer simple sequencing	(A) It follows the steps of crisp AHP.It does not involve additional operations.
	Composite total sequencing	(D) It allows only triangular fuzzy numbers to be used

of comparison based on degree of possibility, centroid defuzzification is used to defuzzify weights. Similar technique was discussed while discussing methodologies to derive priorities in original AHP.

2.4.3 Inverse of Column Sum

We create an algorithm which is intuitive and require very few arithmetic operations. Column sum of each column in a fuzzy comparison matrix is calculated which is given as follows;

$$\frac{w_1 + w_2, \cdots, w_n}{w_1}, \frac{w_1 + w_2, \cdots, w_n}{w_2}, \cdots, \frac{w_1 + w_2, \cdots, w_n}{w_n}, \tag{2.46}$$

As $w_1 + w_2, \cdots, w_n w_1 = 1$ therefore column sums are equivalent to

$$\frac{1}{w_1}, \frac{1}{w_2}, \cdots, \frac{1}{w_n}, \tag{2.47}$$

When we take the inverse of column sum, we end up with the same priority vector $w_1, w_2, \cdots, w_n$. We will also add this algorithm in our performance analysis.

2.5 Summary

In this section, we have in detail discussed various priorities deviation techniques in original AHP as well as FAHP and the objective of this research is to provide a comprehensive performance evaluation of these techniques. In the next chapters, we will outline the methodology through which we carry out this analysis and in final chapters results of this analysis will be summarized.

Chapter 3

Design of Experimental Analysis

If multiple algorithms are proposed in a specific research domain, then a comprehensive performance analysis is often required so as to critically evaluate one algorithm over the others. Review of the existing literature shows that there exists such studies in original AHP. Golany and Kress [18] provides an analysis among six methods in which they used minimum violation, total deviation, conformity and robustness as criteria for performance analysis. They concluded that Modified Eigenvalue (MEV) is the most ineffective method, while among the remaining five algorithms, each have their own weaknesses and advantages. Another comparative analysis is performed by Ishizaka and Lusti [19] in which they used Monte Carlo simulations to compare and evaluate four priorities deviation techniques which includes right eigenvalue method, left eigenvalue method, geometric mean and the mean of normalized values and conclude that number of contradictions increases with increase in the inconsistency as well as the size of the matrix. Some other similar studies are also available in the literature [20] [21] [22] [23] [24], however, all of them evaluates priorities deviation techniques in original AHP.

The only comparative study among FAHP algorithms was carried out by Buyukozkan [17] which provides main characteristic of selected few algorithms and list down their advantages and disadvantages (Table 2.4). However, they fail to make a performance analysis similar to the ones provided in original AHP techniques. Therefore, in this study we attempt to carry out a detailed performance analysis of selected nine FAHP algorithms.

3.1 Algorithm to Generate Random Fuzzy Comparison Matrix:

For our comparative study, we need comparison matrices of varying sizes, level of fuzziness and inconsistency. Golany and Kress [18] provides a methodology which generates comparison matrices with various levels of consistency levels, however this technique is valid only when judgment ratios are in the form of crisp numbers and thus it cannot be replicated for comparison matrices consisting of fuzzy numbers. Therefore, we propose an algorithm through which random fuzzy comparison matrices can be generated with varying parameters mentioned above. This algorithm is step by step explained as follows;

Step 1: Assuming we have n criterion, we randomly generate crisp weights $w_1, w_2, \cdots, w_n$ and normalize them.

Step 2: Through these weights we can generate a perfectly consistent comparison matrix as follows

$$W = \begin{pmatrix} w_1/w_1 & w_1/w_2 & \cdots & w_1/w_n \\ w_2/w_1 & w_2/w_2 & \cdots & w_2/w_n \\ \vdots & \vdots & \ddots & \vdots \\ w_n/w_1 & w_n/w_2 & \cdots & w_n/w_n \end{pmatrix}$$

Step 3: Once the comparison matrix is generated, each element of the matrix is converted into a triangular fuzzy number $[l'\ m'\ u']$ with a fuzzification parameter α such that $l' = w_i/w_j - \alpha$, $m' = w_i/w_j$ and $u' = w_i/w_j + \alpha$.

Step 4:: As stated before, in reality human judgments are rarely consistent and thus comparison matrices formed through these judgments are also not consistent. Therefore, we introduce different levels of inconsistency in the matrices through the inconsistency parameter β. Depending on this parameter, an interval $[a\ b]$ is generated for each l' of the triangular fuzzy number such that $a = l' - l'(\beta)$ and $b = l' + l'(\beta)$. Same procedure is followed to create inconsistency intervals for m' and u'. Afterwards, a number is randomly selected from each one of these intervals and is correspondingly assigned as the lower, modal and upper value of the triangular fuzzy number i:e., $[l\ m\ u]$. However, once inconsistency parameter is increased, there is a possibility that the interval $[a\ b]$ generated for each element of the triangular fuzzy number intersects and the numbers are randomly chosen in such a way that they violates the condition $l < m < u$. We address this issue as follows;

Whenever the inconsistency intervals intersect, they are shrunk in such a way that for each lower value of the triangular fuzzy number, the right endpoint of the interval is readjusted such that it is the mid point of the right end point of the interval of lower value and the left end point of the interval generated for modular number. Similarly, both end points of the inconsistency interval of modular number are readjusted and the left endpoint of the inconsistency interval of upper number is readjusted. Numbers randomly chosen from these intervals will always satisfy the condition of $l < m < u$. This part of the algorithm is graphically explained below for clarity.

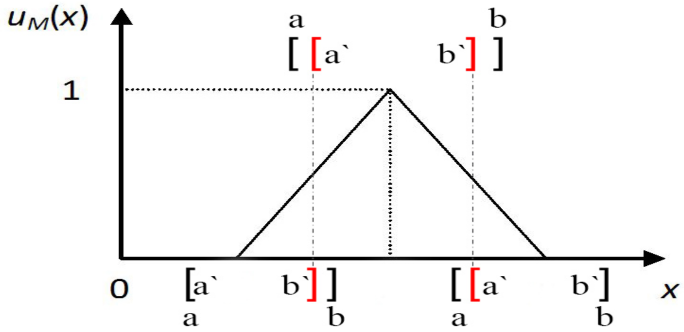


Figure 3.1: Interval formation

3.2 Data Set and Performance Criterion

Previous comparative analysis of methodologies in the original AHP shows that level of inconsistency and size of the matrix are two important criteria which directly affects the performance of a certain technique. In fuzzy AHP, the weighing scale is composed of fuzzy numbers and thus we add a third performance evaluation criteria which is level of fuzziness. Therefore, the aim of our analysis will be to not only investigate performance measure of each algorithm in general but also change in performance as we change these three parameters.

Through the algorithm outlined in the previous section to construct fuzzy comparison matrices, we generated a total of six hundred matrices while changing performance parameters which are size of the matrix, fuzzification parameter and inconsistency levels. A total of ten random matrices were replicated for each one combination of the performance parameters. Note that we ran some pilot experiments before finalizing the values of each parameter.

During pilot experiments, inconsistency levels were considerably low to the order of 5% and 10%, and it was found that these levels do not add any inconsistency and thus matrices constructed are near to consistent. Similarly, for level of fuzziness, once we increased values beyond 0.15 there were instances where the condition $l < m < u$ was violated. Therefore, after running pilot experiments, we finalized the values of the parameters which are given in Table 3.1.

Table 3.1: Parameters for random fuzzy comparison matrices

Parameter	**Values**
Size of the Matrix, n	3, 7, 11, 15
Level of Fuzziness, α	0.05, 0.10, 0.15
Level of Inconsistency, β	0%, 50%, 100%, 150%, 200%

Based on the literature review presented in Chapter 2, we select nine FAHP models for performance analysis. These models are listed in Table 3.2.

Table 3.2: Selected FAHP algorithms for performance analysis

Model Name	**Description**
Chang	Fuzzy Synthetic Extent Analysis
Wang	Fuzzy Synthetic Extent Analysis with modified normalization
Laarhoven	Logarithmic Least Square Method
Boender	Logarithmic Least Square Method with modified normalization
Buckley	Geometric Mean as proposed by Buckley
Arithmetic Mean	Similar to Arithmetic Mean in Original AHP
Geometric Mean	Similar to Geometric Mean in Original AHP
Row Sum	Fuzzy Synthetic Extent Analysis with Centroid Defuzzification
I.C.S	Inverse of Column Sums

Therefore, our performance analysis will be among nine selected FAHP models based on three parameters.

3.3 Performance Analysis

Note that fuzzy comparison matrices were constructed by assuming normalized crisp weights. Afterwards, each of the nine FAHP algorithms included in our comparative analysis was applied to these random fuzzy comparison matrices and priority vector or weights are calculated. Among all the fuzzy AHP algorithms, only Chang [5] computes the defuzzified weights through principle of comparison of fuzzy numbers based on degree of possibility, while rest of the algorithms calculate weights in the form of fuzzy numbers. Therefore, these fuzzy weights are defuzzified using centroid defuzzification technique which is one of the most popular technique for defuzzification [25].

In light of above, we can conduct a performance evaluation by simply calculating the difference between the initial weights and calculated weights. In order to carry out statistical testing, two error terms were calculated, i:e., average error and maximum absolute error. These error terms were than used to conduct Analysis of Variance (Anova) test, for which results are summarized in the next chapter.

Chapter 4

Computational Results and Discussions

In this chapter we present results of the performance analysis conducted on selected nine FAHP algorithms. Furthermore, we will also present comprehensive comparative study based on computational times, popularity among researchers, applicability of triangular or trapezoidal fuzzy numbers, ease of understanding and ease of implementation. This discussion is summarized below;

4.1 Performance

This is the primary analysis of our study in which performance of each algorithm is investigated. After weights are extracted from all the matrices using selected FAHP algorithm, error is calculated by subtracting calculated weights from the weights through which comparison matrices are constructed. Average error and maximum errors is calculated for each fuzzy comparison matrix and Analysis of Variance (Anova) test is performed to evaluate the results. Anova not only allows to compare means of more then two populations but also compare populations at several levels or subgroups, hence this test will greatly help us effectively evaluate the available FAHP algorithms based on different criteria mentioned before. The main finding from the experimental analysis are summarized below;

- Change in size of the matrix and inconsistency levels have significant effect on the overall performance of each algorithm, while change in the level of fuzziness do not have any significant effect (Table 4.1 - 4.2). Note that algorithm proposed in this study to construct fuzzy comparison matrices restricted us to choose a wider range for fuzzification parameter. Therefore, change in fuzzification parameter do not significantly effect the mean average error values. Although difference in mean absolute error is significant, however, once we go deeper into analysis, we conclude that change in this parameter do not significantly effect the overall performance of the algorithm.

Table 4.1: Effect of changing parameters (Average Error)

Source	Type III Sum of Squares	Degree of Freedom	Mean Square	F Value	Sig.
n	0.969	3	0.323	935.818	0.000
Alpha	0.000	2	0.0000	0.464	0.629
Beta	0.459	4	0.115	332.192	0.000

Table 4.2: Effect of changing parameters (Maximum Error)

Source	Type III Sum of Squares	Degree of Freedom	Mean Square	F Value	Sig.
n	1.304	3	0.435	228.907	0.000
Alpha	0.037	2	0.018	9.716	0.000
Beta	3.838	4	0.959	505.238	0.000

- Overall, Geometric Mean method discussed in this study performs significantly better than the rest of the FAHP algorithms, except for the Arithmetic Mean, Boender and Row Sum method, against which the performance is better but the differences between mean average errors and mean absolute errors is not significant (Table 4.3 - 4.4). As statistical results shows better performance of Geometric Mean method, therefore, we investigate this model further and calculate the percentage of instances when it performs better with respect to other models (Table 4.5).

Table 4.3: Overall performance of Geometric Mean method (Average Error)

Model	Mean Difference	Significance
Arithmetic Mean	-0.000515203	0.631
Boender	-0.002138968	0.046
Buckley	-0.064583270	0.000
Chang	-0.047093993	0.000
I.C.S	-0.004562945	0.000
Laarhoven	-0.003805695	0.000
Row Sum	-0.000479569	0.655
Wang	-0.053058594	0.000

Table 4.4: Overall performance of Geometric Mean method (Maximum Error)

Model	Mean Difference	Significance
Arithmetic Mean	-0.000939228	0.709
Boender	-0.003714015	0.140
Buckley	-0.011481600	0.000
Chang	-0.107025901	0.000
I.C.S	-0.113518520	0.000
Laarhoven	-0.005917900	0.019
Row Sum	-0.001088813	0.665
Wang	-0.118683267	0.000

Table 4.5: Percentage of instances for which Geometric Mean method performs better

Model	Average Error	Maximum Error
Arithmetic Mean	66%	62%
Boender	56%	57%
Buckley	84%	81%
Chang	97%	96%
I.C.S	80%	79%
Laarhoven	51%	53%
Row Sum	57%	53%
Wang	90%	89%

- We also evaluate the percentage of instances when Geometric Mean method performs better at different matrix sizes, level of fuzziness and inconsistency. As the size of the matrix is increased, the number of instances of better performance of Geometric Mean method also increases except for Laarhoven (Figure 4.1). However, note that as the size of the matrix increases, values of the starting normalized weights gets smaller and hence the error term is also small. Therefore this better performance cannot be associated with any feature of the FAHP algorithm. Results are consistent for both the mean average errors as well as mean absolute maximum errors (Appendices A.6 - A.9, B.6 - B.9). We do not observe any change in performance at different fuzzification levels. As stated before, algorithm devised to construct fuzzy comparison matrices restricts us from choosing wider range of fuzzification parameter, and hence we do not observe any variations in performance with change in fuzzification levels (Figure 4.2).

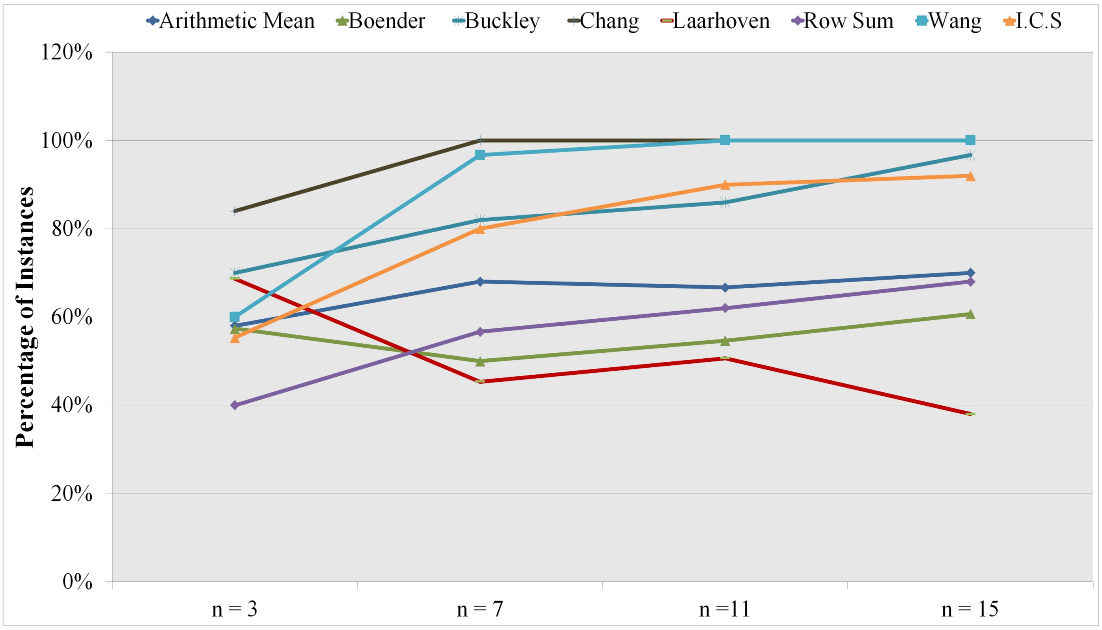


Figure 4.1: Percentage of instances when Geometric Mean method performs better at different matrix sizes (Average Error)

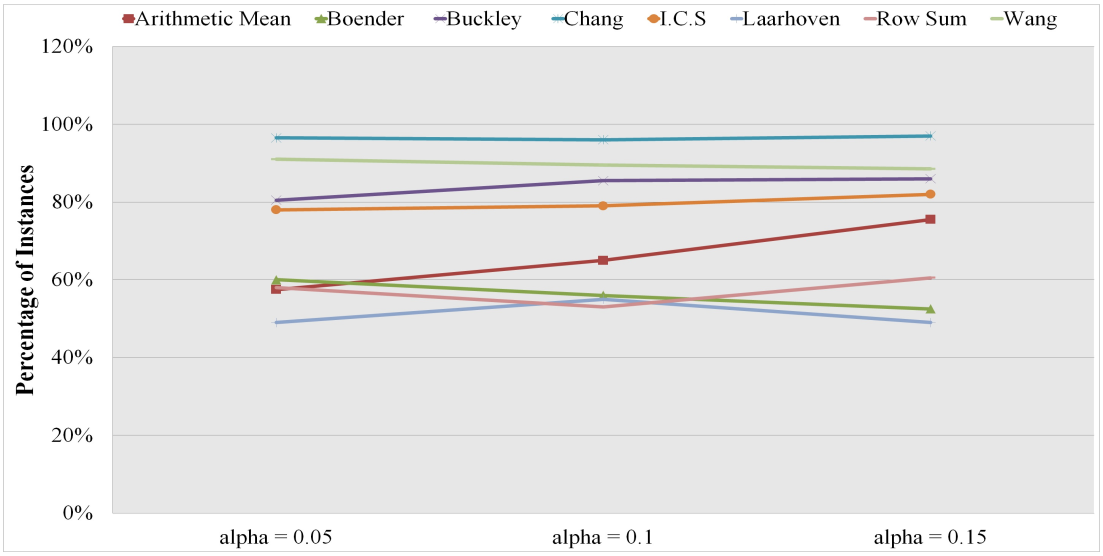


Figure 4.2: Percentage of instances when Geometric Mean method performs better at different level of fuzziness (Average Error)

- Generally, performance of all algorithms decreases as the inconsistency levels are increased except for FEA method for which performance increases with increase in inconsistency (Figure 4.6). However, even at high inconsistency levels, Geometric Mean method performs significantly better as compared to other FAHP algorithms (Table 4.6 - 4.7). At low inconsistency level, there are only 10 percent of the instances when Geometric Mean method performs better than Row Sum Method (Figure 4.3). However, as the inconsistency level increases, performance of Geometric Mean method against Row Sum method improves while for the rest of the algorithms, performance is better through out at different inconsistency levels.

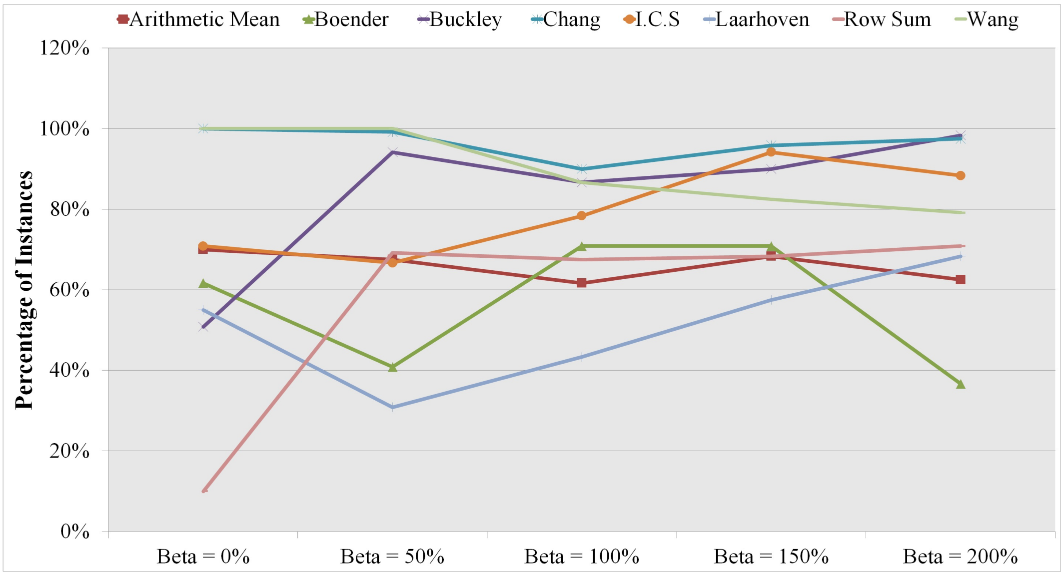


Figure 4.3: Percentage of Instances when Geometric Mean method performs better at different consistency levels (Average Error)

Table 4.6: Performance of Geometric Mean method at $\beta = 200\%$ (Average Error)

Model	Mean Difference	Significance
Arithmetic Mean	-0.000949545	0.717
Boender	-0.002363694	0.367
Buckley	-0.018852758	0.000
Chang	-0.015356391	0.000
I.C.S	-0.00884552	0.001
Laarhoven	-0.007134093	0.007
Row Sum	-0.000964185	0.713
Wang	-0.008491371	0.001

Table 4.7: Performance of Geometric Mean method at $\beta = 200\%$ (Maximum Error)

Model	Mean Difference	Significance
Arithmetic Mean	-0.002376188	0.549
Boender	-0.003239027	0.414
Buckley	-0.032845129	0.000
Chang	-0.027348781	0.000
I.C.S	-0.024531226	0.000
Laarhoven	-0.011239837	0.005
Row Sum	-0.001937068	0.625
Wang	-0.015508737	0.000

- Among the selected FAHP algorithms, Fuzzy Extent Analysis method proposed by Chang [5] is the worst performing algorithm (Table 4.8- 4.9). This is consistent as we change the performance criteria as well as these results hold true for both the mean average error as well as mean maximum absolute error (Appendices A.5 , B.5).

Table 4.8: Overall Performance of Chang FEA Method (Average Error)

Model	Mean Difference	Significance
Arithmetic Mean	0.04657879	0.000
Boender	0.044955025	0.000
Buckley	0.040635666	0.000
Geometric Mean	0.047093993	0.000
I.C.S	0.042531048	0.000
Laarhoven	0.043288298	0.000
Row Sum	0.046614425	0.000
Wang	-0.00596401	0.000

Table 4.9: Overall Performance of Chang FEA Method (Maximum Error)

Model	Mean Difference	Significance
Arithmetic Mean	0.106086673	0.000
Boender	0.103311886	0.000
Buckley	0.095544301	0.000
Geometric Mean	0.107025901	0.000
I.C.S	0.095674049	0.000
Laarhoven	0.101108001	0.000
Row Sum	0.105937088	0.000
Wang	-0.011657366	0.000

- Row Sum method proposed in this study is the modified version of Fuzzy Extent Analysis method in which degree of possibility is replaced with centroid defuzzification. Analysis of the results shows that this modification leads to improved performance. Results from Table 4.10 - 4.11 and Figure 4.1 - 4.3 shows that performance of Row Sum method is almost as good as the Geometric Mean method. Evaluating the Maximum error terms, Row Sum method performs better than Geometric Mean method however this difference is not significant (Table 4.11). Therefore, we can conclude that Row Sum method is the second best performing algorithm among all the FAHP algorithms evaluated in this study.

Table 4.10: Overall Performance of Row Sum Method (Average Error)

Model	Mean Difference	Significance
Arithmetic Mean	-0.00003563450	0.974
Boender	-0.0016594	0.122
Buckley	-0.005978759	0.000
Chang	-0.046614425	0.000
Geometric Mean	0.000479569	0.655
I.C.S	-0.004083377	0.000
Laarhoven	-0.003326127	0.002
Wang	-0.052579025	0.000

Table 4.11: Overall Performance of Row Sum Method (Maximum Error)

Model	Mean Difference	Significance
Arithmetic Mean	0.000149585	0.953
Boender	-0.002625202	0.297
Buckley	-0.010392787	0.000
Chang	-0.105937088	0.000
Geometric Mean	-0.001088813	0.665
I.C.S	-0.010263039	0.000
Laarhoven	-0.004829087	0.055
Wang	-0.117594455	0.000

- We have three main performance criteria in our performance evaluation namely the size of the matrix, level of fuzziness and inconsistency. As stated before, size of the matrix and different inconsistency levels have a significant impact on the overall performance while the fuzzification parameter do not significantly effect this performance. Some comments are made regarding these parameter which are as follows;

  1. Although increase in size of the matrix has a significant effect on performance of all the FAHP algorithms as illustrated in Figure 4.4, however, this improved performance can be misleading. As we increase the size of the matrix, the values of the starting normalized weights are decreased and hence the final error term is also low which depicts improvement in performance. Therefore, this improved performance cannot be associated with any of the FAHP algorithm.

  2. Change in the fuzzification parameter α do not significantly effect the performance of any FAHP algorithm (Figure 4.5). As the algorithm proposed in this study to construct fuzzy comparison matrices restricts us to very limited range

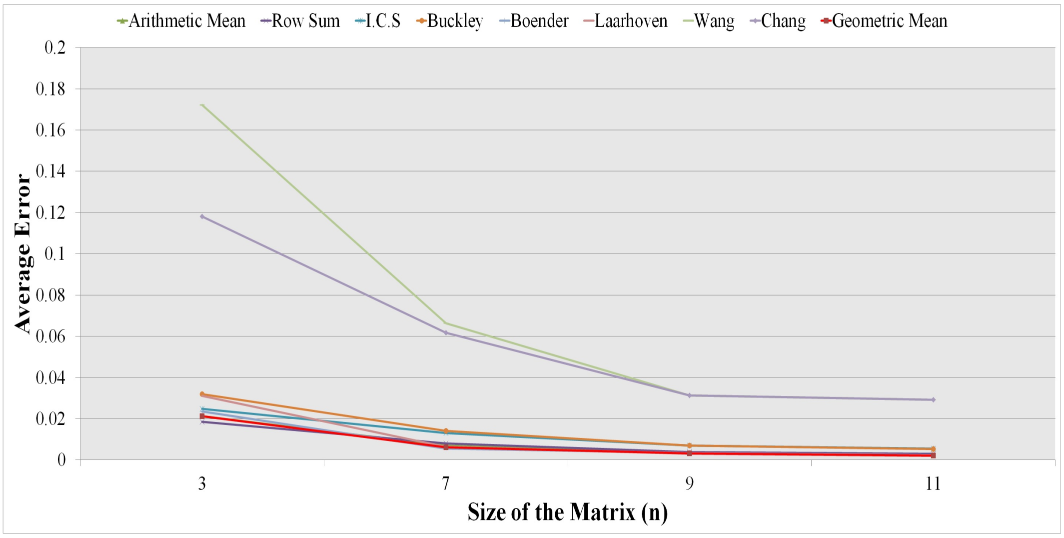


Figure 4.4: Change in performance w.r.t change in size of the matrix

of fuzzification parameter, so we may conclude that range of values chosen for α were not wide enough and thus we cannot investigate its impact on the overall performance of each algorithm.

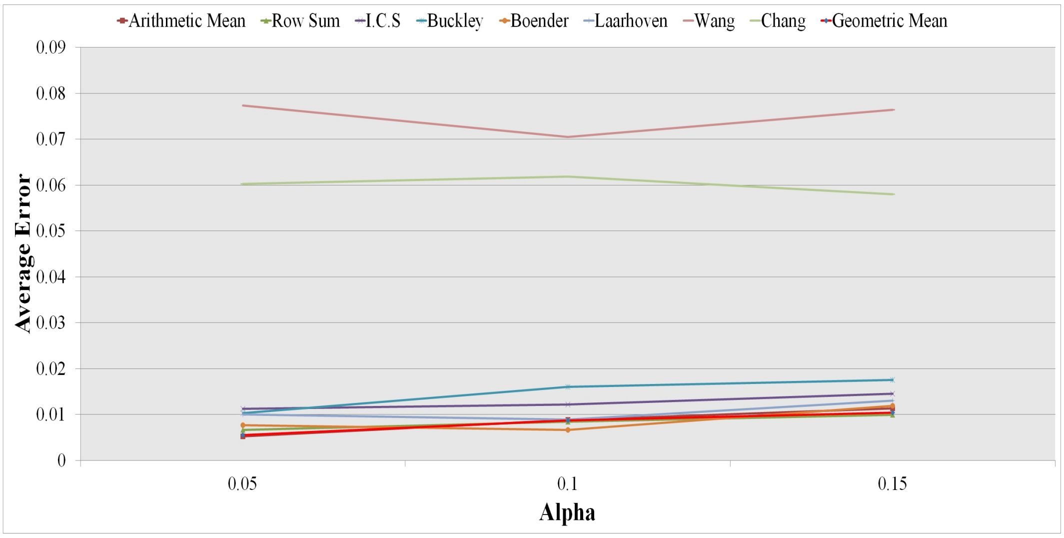


Figure 4.5: Change in performance w.r.t change in fuzzification parameter

3. As we increase the inconsistency factor, performance of most of the algorithms is decreased except for Chang [5] for which performance increases as we increase the inconsistency (Figure 4.5). However, this increase in performance is not enough and even at high inconsistency levels, this is the worst performing algorithm and Geometric Mean is the best performing algorithm among all the FAHP algorithms.

This concludes the performance analysis where we observe that Geometric Mean method proposed in this study performs significantly better as compared to rest of the FAHP algo-

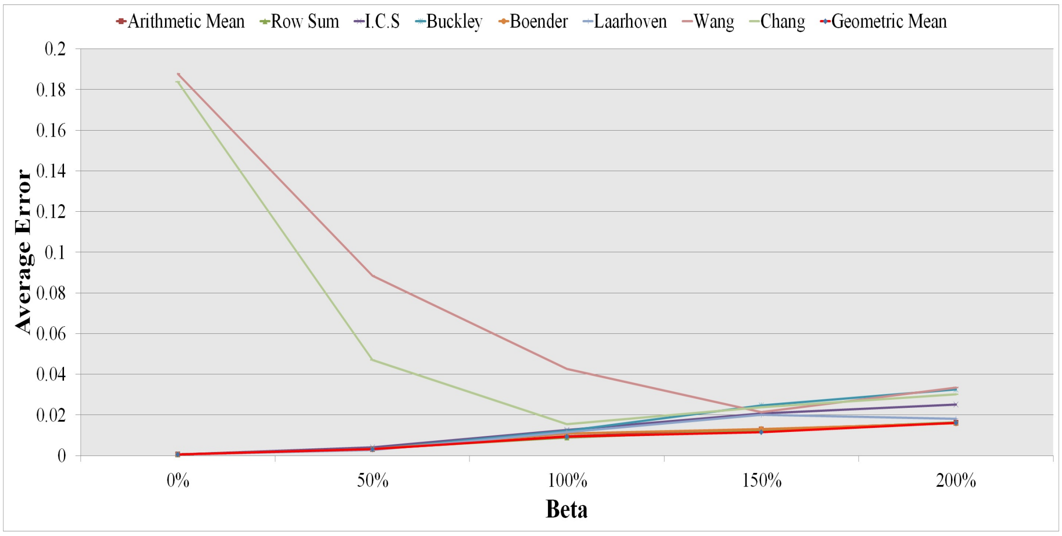


Figure 4.6: Change in performance w.r.t change in inconsistency

rithms at various experimental settings. Row Sum method which is the modified version of original FEA method is also among the best performing algorithms while the original FEA method is the worst performing algorithm. Overall, three out of four algorithms introduced in this study are the best performing algorithms among the selected nine FAHP algorithms. The performance of Inverse of Column Sum method is not satisfactory and is among the worst performing algorithm after FEA method.

4.2 Computational Times

In this study, computational time is defined as the time taken to generate all six hundred matrices, apply them to the selected FAHP algorithm and estimate the priority vector or weights. Computational times of all nine algorithms are tabulated in Table 4.12;

Table 4.12: Computational times

Algorithm	**Time**
Wang (2008)	2.2666 sec
Chang (1996)	2.3000 sec
Row Sum (2014)	2.5380 sec
Inverse Column Sum - I.C.S (2014)	4.3750 sec
Geometric Mean (2014)	5.8730 sec
Arithmetic Mean (2014)	6.5910 sec
Buckley (1985)	6.974 sec
Laarhoven (1983)	8.812 sec
Boender (1989)	34.368 sec

This shows that Fuzzy Extent Analysis method which includes Wang (2008), Chang (1996) and Row Sum (2014), perform considerably better while Logarithmic Least Square Method which includes Laarhoven (1983) and Boender (1989) performs worst among the selected FAHP algorithms. However, these computational times are very small and thus difference in computational times is not a big factor.

4.3 Popularity

Review of the available literature on application of FAHP algorithms shows that Fuzzy Extent Analysis is the most frequently used method [26]. Based on the assumption that whenever a fuzzy AHP algorithm is implemented, it always cites Chang Fuzzy Synthetic Extent Analysis method, we reviewed a total of 582 research papers on Thomas Reuters Web of Science which cites Chang's study. Among these research articles, total of 148 articles used original model as proposed by Chang while 92 articles used Chang Extent Analysis integrated with some other technique such as TOPSIS etc., Only six articles implements Buckleys Geometric mean method. Note that this analysis is based on the fact that all the papers which were selected were cited by Chang, however, one can safely conclude that model proposed by Chang is the most popular method among all FAHP algorithms available in the literature.

4.4 Applicability of Fuzzy Numbers

Among the selected nine FAHP algorithms, either traingular fuzzy numbers can be used or trapezoidal fuzzy number are used. We list down for each algorithm, which type of fuzzy numbers can be used.

Table 4.13: Applicability of fuzzy numbers

Model Name	**Type of Fuzzy Number**
FEA (Chang and Wang)	Triangular Fuzzy Number
LLSM (Laarhoven and Boender)	Triangular Fuzzy Number
Buckley	Trapezoidal Fuzzy Number
Arithmetic Mean	Both Triangular and Trapezoidal Fuzzy Number
Geometric Mean	Both Triangular and Trapezoidal Fuzzy Number
Row Sum	Both Triangular and Trapezoidal Fuzzy Number
I.C.S	Both Triangular and Trapezoidal Fuzzy Number

In this section, we have in detail presented the results of performance analysis as well as provided a comprehensive comparative study of some of the most common FAHP algorithms. Based on this performance analysis and comparative study, we summarize our findings in Table 4.14.

Table 4.14: Summary of results

Model	Performance	Computational time	Popularity	Ease of Understanding	Ease of Implementation
Arithmetic Mean	High	Medium	N/A	High	High
Boender	High	Low	Medium	Low	Low
Buckley	Medium	Medium	Medium	High	High
Chang	Low	High	High	Medium	Medium
Geometric Mean	High	Medium	N/A	High	High
I.C.S	Low	High	N/A	High	High
Laarhoven	High	Low	Medium	Low	Low
Row Sum	High	High	N/A	High	High
Wang	Low	High	High	Medium	Medium

Chapter 5

Conclusions and Future Research

This study is aimed at consolidating the existing literature related to FAHP algorithms and provide a detailed performance analysis. In a pool of existing FAHP models, we added total of four more models including Arithmetic Mean, Geometric Mean, Row Sum and Inverse of Column Sum method. The first two techniques are one of the most common methodologies to derive priority vector in original AHP and we replicated the same methodology in FAHP. Row Sum method is basically an extent analysis method proposed by Chang [5] however, in the original model, principal of comparison of fuzzy numbers based on degree of possibility is used to defuzzify weights, while in Row Sum method proposed in this stuy, we defuzzify weights using centroid defuzzification. While the Inverse of Column Sum method is used for the first time in this study.

Although there exists a methodology to construct crisp comparison matrices [18] but this cannot be replicated for the case of fuzzy comparison matrices. Therefore, we also proposed an algorithm to construct fuzzy comparison matrices with varying parameters such as matrix size, fuzziness and inconsistency. Using this algorithm we constructed six hundred instances of fuzzy comparison matrices and applied total of nine FAHP algorithm on it to investigate performance of each algorithm. The main conclusions of this study are summarized below;

- Geometric Mean algorithm model performs significantly better under different experimental settings as compared to other FAHP algorithms. Note that this model already existed in original AHP but was never replicated in the FAHP.
- Performance of Chang [5] which is often mistakenly categorized as Fuzzy Extent Analysis method is worst among all the nine selected FAHP models. Fuzzy Extent Analysis is just the first part of this model while the other part is the defuzzification which is

based on principal of comparison of fuzzy numbers based on degree of possibility. Row Sum method, which can be categorized as Fuzzy Extent Analysis method with centroid defuzzification performs significantly better than the original model proposed by Chang. Note that our study reveals that model proposed by Chang is one of the most frequently used among all the FAHP algorithms proposed over the past three decades. Therefore, in future, care should be taken by the decision makers while choosing a methodology to rank alternatives using FAHP.

- Among the four algorithms added in the pool of existing fuzzy AHP, Arithmetic Mean, Geometric Mean and Row Sum performs significantly better than rest of the algorithms with Geometric Mean as the best performing algorithm among all the selected FAHP Algorithms. The Inverse of Column Sum Model also performed significantly better than model proposed by Chang.

In addition we also performed a comparative analysis of selected FAHP algorithms with respect to other aspects highlighted in the previous chapter. In this study we have made an attempt to not only consolidate the existing literature on fuzzy AHP but also added to it some of the techniques which were previously only used in the original AHP (Arithmetic and Geometric Mean). Furthermore, we modified the Fuzzy Extent Analysis and proposed another algorithm which is Inverse of Column Sum.

5.1 Future Research

This study can be further extended as follows;

- Using the same algorithm to construct fuzzy comparison matrices, other less common AHP models available in the literature can be investigated.
- The column sum model devised in this study is very intuitive. This should be further investigated, criticized and improved.
- Parameters used in this study can be further extended and different parameters can be used to further investigate the differences.
- We propose an algorithm to construct a fuzzy comparison matrix. This algorithm can be further improved so that greater levels of fuzziness can be incorporated in performance analysis.

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Appendices

Appendix A

Anova Results - Mean Average Error

Table A.1: Between group analysis

Dependent Variable: AverageError

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	9.141[a]	539	.017	49.127	.000
Intercept	2.320	1	2.320	6721.600	.000
Model	2.139	8	.267	774.572	.000
n	.969	3	.323	935.818	.000
Beta	.459	4	.115	332.192	.000
Alpha	.000	2	.000	.464	.629
Model * n	.545	24	.023	65.822	.000
Model * Beta	3.662	32	.114	331.488	.000
Model * Alpha	.020	16	.001	3.666	.000
n * Beta	.065	12	.005	15.615	.000
n * Alpha	.001	6	.000	.469	.832
Beta * Alpha	.013	8	.002	4.831	.000
Model * n * Beta	1.214	96	.013	36.648	.000
Model * n * Alpha	.007	48	.000	.430	1.000
Model * Beta * Alpha	.026	64	.000	1.170	.168
n * Beta * Alpha	.006	24	.000	.676	.879
Model * n * Beta * Alpha	.014	192	7.375E-005	.214	1.000
Error	1.678	4860	.000		
Total	13.139	5400			
Corrected Total	10.818	5399			

a. R Squared = .845 (Adjusted R Squared = .828)

Table A.2: Analysis as the size of the matrix increases

Dependent Variable: AverageError
LSD

(I) n	(J) n	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
3	7	.0230598769*	.0007151239	.000	.0216579107	.0244618431
	11	.0308664601*	.0007151239	.000	.0294644939	.0322684263
	15	.0344161648*	.0007151239	.000	.0330141986	.0358181311
7	3	-.023059877*	.0007151239	.000	-.0244618431	-.0216579107
	11	.0078065832*	.0007151239	.000	.0064046170	.0092085494
	15	.0113562879*	.0007151239	.000	.0099543217	.0127582541
11	3	-.030866460*	.0007151239	.000	-.0322684263	-.0294644939
	7	-.007806583*	.0007151239	.000	-.0092085494	-.0064046170
	15	.0035497047*	.0007151239	.000	.0021477385	.0049516710
15	3	-.034416165*	.0007151239	.000	-.0358181311	-.0330141986
	7	-.011356288*	.0007151239	.000	-.0127582541	-.0099543217
	11	-.003549705*	.0007151239	.000	-.0049516710	-.0021477385

Based on observed means.
The error term is Mean Square(Error) = .000.

*. The mean difference is significant at the 0.05 level.

Table A.3: Analysis as the level of fuzziness increases

Dependent Variable: AverageError
LSD

(I) Alpha	(J) Alpha	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
.05	.10	.0005207750	.0006193155	.400	-.0006933634	.0017349133
	.15	.0005124594	.0006193155	.408	-.0007016790	.0017265977
.10	.05	-.0005207750	.0006193155	.400	-.0017349133	.0006933634
	.15	-.0000083156	.0006193155	.989	-.0012224540	.0012058228
.15	.05	-.0005124594	.0006193155	.408	-.0017265977	.0007016790
	.10	.0000083156	.0006193155	.989	-.0012058228	.0012224540

Based on observed means.
The error term is Mean Square(Error) = .000.

Table A.4: Analysis as the inconsistency increases

Dependent Variable: AverageError
LSD

(I) Beta	(J) Beta	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
.00	.50	.0233741095*	.0007995328	.000	.0218066636	.0249415554
	1.00	.0262659216*	.0007995328	.000	.0246984757	.0278333675
	1.50	.0197752224*	.0007995328	.000	.0182077765	.0213426683
	2.00	.0157641436*	.0007995328	.000	.0141966977	.0173315894
.50	.00	-.023374109*	.0007995328	.000	-.0249415554	-.0218066636
	1.00	.0028918121*	.0007995328	.000	.0013243663	.0044592580
	1.50	-.003598887*	.0007995328	.000	-.0051663329	-.0020314412
	2.00	-.007609966*	.0007995328	.000	-.0091774118	-.0060425200
1.00	.00	-.026265922*	.0007995328	.000	-.0278333675	-.0246984757
	.50	-.002891812*	.0007995328	.000	-.0044592580	-.0013243663
	1.50	-.006490699*	.0007995328	.000	-.0080581451	-.0049232533
	2.00	-.010501778*	.0007995328	.000	-.0120692239	-.0089343322
1.50	.00	-.019775222*	.0007995328	.000	-.0213426683	-.0182077765
	.50	.0035988871*	.0007995328	.000	.0020314412	.0051663329
	1.00	.0064906992*	.0007995328	.000	.0049232533	.0080581451
	2.00	-.004011079*	.0007995328	.000	-.0055785247	-.0024436330
2.00	.00	-.015764144*	.0007995328	.000	-.0173315894	-.0141966977
	.50	.0076099659*	.0007995328	.000	.0060425200	.0091774118
	1.00	.0105017780*	.0007995328	.000	.0089343322	.0120692239
	1.50	.0040110789*	.0007995328	.000	.0024436330	.0055785247

Based on observed means.
The error term is Mean Square(Error) = .000.

*. The mean difference is significant at the 0.05 level.

Table A.5: Analysis within FAHP models

Dependent Variable: AverageError
LSD

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Arithmetic Mean	Boender	-.0016237654	.0010726858	.130	-.0037267148	.0004791839
	Buckley	-.005943124*	.0010726858	.000	-.0080460738	-.0038401751
	Chang	-.046578790*	.0010726858	.000	-.0486817397	-.0444758410
	Geometric Mean	.0005152030	.0010726858	.631	-.0015877463	.0026181523
	I.C.S	-.004047742*	.0010726858	.000	-.0061506916	-.0019447929
	Laarhoven	-.003290492*	.0010726858	.002	-.0053934415	-.0011875429
	RowSum	.0000356345	.0010726858	.974	-.0020673148	.0021385838
	Wang	-.052543391*	.0010726858	.000	-.0546463403	-.0504404417
Boender	Arithmetic Mean	.0016237654	.0010726858	.130	-.0004791839	.0037267148
	Buckley	-.004319359*	.0010726858	.000	-.0064223083	-.0022164097
	Chang	-.044955025*	.0010726858	.000	-.0470579742	-.0428520756
	Geometric Mean	.0021389684*	.0010726858	.046	.0000360191	.0042419178
	I.C.S	-.002423977*	.0010726858	.024	-.0045269262	-.0003210275
	Laarhoven	-.0016667268	.0010726858	.120	-.0037696761	.0004362225
	RowSum	.0016593999	.0010726858	.122	-.0004435494	.0037623492
	Wang	-.050919626*	.0010726858	.000	-.0530225749	-.0488166762
Buckley	Arithmetic Mean	.0059431244*	.0010726858	.000	.0038401751	.0080460738
	Boender	.0043193590*	.0010726858	.000	.0022164097	.0064223083
	Chang	-.040635666*	.0010726858	.000	-.0427386152	-.0385327166
	Geometric Mean	.0064583275*	.0010726858	.000	.0043553781	.0085612768
	I.C.S	.0018953822	.0010726858	.077	-.0002075672	.0039983315
	Laarhoven	.0026526322*	.0010726858	.013	.0005496829	.0047555816
	RowSum	.0059787589*	.0010726858	.000	.0038758096	.0080817083
	Wang	-.046600267*	.0010726858	.000	-.0487032159	-.0444973172
Chang	Arithmetic Mean	.0465787903*	.0010726858	.000	.0444758410	.0486817397
	Boender	.0449550249*	.0010726858	.000	.0428520756	.0470579742
	Buckley	.0406356659*	.0010726858	.000	.0385327166	.0427386152
	Geometric Mean	.0470939933*	.0010726858	.000	.0449910440	.0491969427
	I.C.S	.0425310481*	.0010726858	.000	.0404280987	.0446339974
	Laarhoven	.0432882981*	.0010726858	.000	.0411853488	.0453912474
	RowSum	.0466144248*	.0010726858	.000	.0445114755	.0487173741
	Wang	-.005964601*	.0010726858	.000	-.0080675500	-.0038616513
Geometric Mean	Arithmetic Mean	-.0005152030	.0010726858	.631	-.0026181523	.0015877463
	Boender	-.002138968*	.0010726858	.046	-.0042419178	-.0000360191
	Buckley	-.006458327*	.0010726858	.000	-.0085612768	-.0043553781
	Chang	-.047093993*	.0010726858	.000	-.0491969427	-.0449910440
	I.C.S	-.004562945*	.0010726858	.000	-.0066658946	-.0024599959
	Laarhoven	-.003805695*	.0010726858	.000	-.0059086446	-.0017027459
	RowSum	-.0004795685	.0010726858	.655	-.0025825179	.0016233808
	Wang	-.053058594*	.0010726858	.000	-.0551615433	-.0509556447
I.C.S	Arithmetic Mean	.0040477423*	.0010726858	.000	.0019447929	.0061506916
	Boender	.0024239768*	.0010726858	.024	.0003210275	.0045269262
	Buckley	-.0018953822	.0010726858	.077	-.0039983315	.0002075672
	Chang	-.042531048*	.0010726858	.000	-.0446339974	-.0404280987
	Geometric Mean	.0045629453*	.0010726858	.000	.0024599959	.0066658946
	Laarhoven	.0007572501	.0010726858	.480	-.0013456993	.0028601994
	RowSum	.0040833768*	.0010726858	.000	.0019804274	.0061863261
	Wang	-.048495649*	.0010726858	.000	-.0505985980	-.0463926994
Laarhoven	Arithmetic Mean	.0032904922*	.0010726858	.002	.0011875429	.0053934415
	Boender	.0016667268	.0010726858	.120	-.0004362225	.0037696761
	Buckley	-.002652632*	.0010726858	.013	-.0047555816	-.0005496829
	Chang	-.043288298*	.0010726858	.000	-.0453912474	-.0411853488
	Geometric Mean	.0038056952*	.0010726858	.000	.0017027459	.0059086446
	I.C.S	-.0007572501	.0010726858	.480	-.0028601994	.0013456993
	RowSum	.0033261267*	.0010726858	.002	.0012231774	.0054290760
	Wang	-.049252899*	.0010726858	.000	-.0513558481	-.0471499494
RowSum	Arithmetic Mean	-.0000356345	.0010726858	.974	-.0021385838	.0020673148
	Boender	-.0016593999	.0010726858	.122	-.0037623492	.0004435494
	Buckley	-.005978759*	.0010726858	.000	-.0080817083	-.0038758096
	Chang	-.046614425*	.0010726858	.000	-.0487173741	-.0445114755
	Geometric Mean	.0004795685	.0010726858	.655	-.0016233808	.0025825179
	I.C.S	-.004083377*	.0010726858	.000	-.0061863261	-.0019804274
	Laarhoven	-.003326127*	.0010726858	.002	-.0054290760	-.0012231774
	Wang	-.052579025*	.0010726858	.000	-.0546819748	-.0504760761
Wang	Arithmetic Mean	.0525433910*	.0010726858	.000	.0504404417	.0546463403
	Boender	.0509196256*	.0010726858	.000	.0488166762	.0530225749
	Buckley	.0466002665*	.0010726858	.000	.0444973172	.0487032159
	Chang	.0059646007*	.0010726858	.000	.0038616513	.0080675500
	Geometric Mean	.0530585940*	.0010726858	.000	.0509556447	.0551615433
	I.C.S	.0484956487*	.0010726858	.000	.0463926994	.0505985980
	Laarhoven	.0492528988*	.0010726858	.000	.0471499494	.0513558481
	RowSum	.0525790255*	.0010726858	.000	.0504760761	.0546819748

Based on observed means.
The error term is Mean Square(Error) = .000.

*. The mean difference is significant at the 0.05 level.

Table A.6: Performance analysis among models when n = 3

Dependent Variable: AverageError
LSD

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Arithmetic Mean	Boender	-.0073925079	.0070859614	.297	-.0212932836	.0065082677
	Buckley	-.0092006355	.0070859614	.194	-.0231014111	.0047001401
	Chang	-.077670500*	.0070859614	.000	-.0915712759	-.0637697246
	Geometric Mean	.0010351423	.0070859614	.884	-.0128656333	.0149359180
	I.C.S	-.0043096686	.0070859614	.543	-.0182104443	.0095911070
	Laarhoven	-.0136210860	.0070859614	.055	-.0275218616	.0002796896
	RowSum	.0021636062	.0070859614	.760	-.0117371695	.0160643818
	Wang	-.099002200*	.0070859614	.000	-.1129029761	-.0851014248
Boender	Arithmetic Mean	.0073925079	.0070859614	.297	-.0065082677	.0212932836
	Buckley	-.0018081276	.0070859614	.799	-.0157089032	.0120926481
	Chang	-.070277992*	.0070859614	.000	-.0841787680	-.0563772167
	Geometric Mean	.0084276502	.0070859614	.235	-.0054731254	.0223284259
	I.C.S	.0030828393	.0070859614	.664	-.0108179364	.0169836149
	Laarhoven	-.0062285781	.0070859614	.380	-.0201293537	.0076721976
	RowSum	.0095561141	.0070859614	.178	-.0043446615	.0234568897
	Wang	-.091609693*	.0070859614	.000	-.1055104682	-.0777089169
Buckley	Arithmetic Mean	.0092006355	.0070859614	.194	-.0047001401	.0231014111
	Boender	.0018081276	.0070859614	.799	-.0120926481	.0157089032
	Chang	-.068469865*	.0070859614	.000	-.0823706404	-.0545690891
	Geometric Mean	.0102357778	.0070859614	.149	-.0036649978	.0241365535
	I.C.S	.0048909669	.0070859614	.490	-.0090098088	.0187917425
	Laarhoven	-.0044204505	.0070859614	.533	-.0183212261	.0094803251
	RowSum	.0113642417	.0070859614	.109	-.0025365340	.0252650173
	Wang	-.089801565*	.0070859614	.000	-.1037023406	-.0759007893
Chang	Arithmetic Mean	.0776705003*	.0070859614	.000	.0637697246	.0915712759
	Boender	.0702779923*	.0070859614	.000	.0563772167	.0841787680
	Buckley	.0684698648*	.0070859614	.000	.0545690891	.0823706404
	Geometric Mean	.0787056426*	.0070859614	.000	.0648048670	.0926064182
	I.C.S	.0733608316*	.0070859614	.000	.0594600560	.0872616073
	Laarhoven	.0640494143*	.0070859614	.000	.0501486386	.0779501899
	RowSum	.0798341064*	.0070859614	.000	.0659333308	.0937348821
	Wang	-.021331700*	.0070859614	.003	-.0352324759	-.0074309246
Geometric Mean	Arithmetic Mean	-.0010351423	.0070859614	.884	-.0149359180	.0128656333
	Boender	-.0084276502	.0070859614	.235	-.0223284259	.0054731254
	Buckley	-.0102357778	.0070859614	.149	-.0241365535	.0036649978
	Chang	-.078705643*	.0070859614	.000	-.0926064182	-.0648048670
	I.C.S	-.0053448110	.0070859614	.451	-.0192455866	.0085559647
	Laarhoven	-.014656228*	.0070859614	.039	-.0285570040	-.0007554527
	RowSum	.0011284638	.0070859614	.873	-.0127723118	.0150292395
	Wang	-.100037343*	.0070859614	.000	-.1139381185	-.0861365672
I.C.S	Arithmetic Mean	.0043096686	.0070859614	.543	-.0095911070	.0182104443
	Boender	-.0030828393	.0070859614	.664	-.0169836149	.0108179364
	Buckley	-.0048909669	.0070859614	.490	-.0187917425	.0090098088
	Chang	-.073360832*	.0070859614	.000	-.0872616073	-.0594600560
	Geometric Mean	.0053448110	.0070859614	.451	-.0085559647	.0192455866
	Laarhoven	-.0093114174	.0070859614	.189	-.0232121930	.0045893583
	RowSum	.0064732748	.0070859614	.361	-.0074275008	.0203740505
	Wang	-.094692532*	.0070859614	.000	-.1085933075	-.0807917562
Laarhoven	Arithmetic Mean	.0136210860	.0070859614	.055	-.0002796896	.0275218616
	Boender	.0062285781	.0070859614	.380	-.0076721976	.0201293537
	Buckley	.0044204505	.0070859614	.533	-.0094803251	.0183212261
	Chang	-.064049414*	.0070859614	.000	-.0779501899	-.0501486386
	Geometric Mean	.0146562283*	.0070859614	.039	.0007554527	.0285570040
	I.C.S	.0093114174	.0070859614	.189	-.0045893583	.0232121930
	RowSum	.0157846922*	.0070859614	.026	.0018839165	.0296854678
	Wang	-.085381114*	.0070859614	.000	-.0992818901	-.0714803388
RowSum	Arithmetic Mean	-.0021636062	.0070859614	.760	-.0160643818	.0117371695
	Boender	-.0095561141	.0070859614	.178	-.0234568897	.0043446615
	Buckley	-.0113642417	.0070859614	.109	-.0252650173	.0025365340
	Chang	-.079834106*	.0070859614	.000	-.0937348821	-.0659333308
	Geometric Mean	-.0011284638	.0070859614	.873	-.0150292395	.0127723118
	I.C.S	-.0064732748	.0070859614	.361	-.0203740505	.0074275008
	Laarhoven	-.015784692*	.0070859614	.026	-.0296854678	-.0018839165
	Wang	-.101165807*	.0070859614	.000	-.1150665823	-.0872650310
Wang	Arithmetic Mean	.0990022005*	.0070859614	.000	.0851014248	.1129029761
	Boender	.0916096926*	.0070859614	.000	.0777089169	.1055104682
	Buckley	.0898015650*	.0070859614	.000	.0759007893	.1037023406
	Chang	.0213317002*	.0070859614	.003	.0074309246	.0352324759
	Geometric Mean	.1000373428*	.0070859614	.000	.0861365672	.1139381185
	I.C.S	.0946925318*	.0070859614	.000	.0807917562	.1085933075
	Laarhoven	.0853811145*	.0070859614	.000	.0714803388	.0992818901
	RowSum	.1011658067*	.0070859614	.000	.0872650310	.1150665823

Based on observed means.
The error term is Mean Square(Error) = .004.

*. The mean difference is significant at the 0.05 level.

Table A.7: Performance analysis among models when n = 7

Dependent Variable: AverageError
LSD

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Arithmetic Mean	Boender	.0008925057	.0037164532	.810	-.0063981890	.0081832004
	Buckley	-.0068994612	.0037164532	.064	-.0141901559	.0003912335
	Chang	-.050935092*	.0037164532	.000	-.0582257863	-.0436443968
	Geometric Mean	.0007324903	.0037164532	.844	-.0065582044	.0080231850
	I.C.S	-.0047223933	.0037164532	.204	-.0120130880	.0025683014
	Laarhoven	.0002573963	.0037164532	.945	-.0070332984	.0075480910
	RowSum	-.0005655699	.0037164532	.879	-.0078562646	.0067251248
	Wang	-.052567467*	.0037164532	.000	-.0598581615	-.0452767721
Boender	Arithmetic Mean	-.0008925057	.0037164532	.810	-.0081832004	.0063981890
	Buckley	-.007791967*	.0037164532	.036	-.0150826616	-.0005012722
	Chang	-.051827597*	.0037164532	.000	-.0591182920	-.0445369026
	Geometric Mean	-.0001600154	.0037164532	.966	-.0074507101	.0071306793
	I.C.S	-.0056148990	.0037164532	.131	-.0129055937	.0016757957
	Laarhoven	-.0006351094	.0037164532	.864	-.0079258042	.0066555853
	RowSum	-.0014580756	.0037164532	.695	-.0087487703	.0058326191
	Wang	-.053459973*	.0037164532	.000	-.0607506673	-.0461692778
Buckley	Arithmetic Mean	.0068994612	.0037164532	.064	-.0003912335	.0141901559
	Boender	.0077919669*	.0037164532	.036	.0005012722	.0150826616
	Chang	-.044035630*	.0037164532	.000	-.0513263251	-.0367449357
	Geometric Mean	.0076319515*	.0037164532	.040	.0003412568	.0149226462
	I.C.S	.0021770679	.0037164532	.558	-.0051136268	.0094677626
	Laarhoven	.0071568575	.0037164532	.054	-.0001338373	.0144475522
	RowSum	.0063338913	.0037164532	.089	-.0009568034	.0136245860
	Wang	-.045668006*	.0037164532	.000	-.0529587004	-.0383773109
Chang	Arithmetic Mean	.0509350916*	.0037164532	.000	.0436443968	.0582257863
	Boender	.0518275973*	.0037164532	.000	.0445369026	.0591182920
	Buckley	.0440356304*	.0037164532	.000	.0367449357	.0513263251
	Geometric Mean	.0516675819*	.0037164532	.000	.0443768872	.0589582766
	I.C.S	.0462126982*	.0037164532	.000	.0389220035	.0535033930
	Laarhoven	.0511924878*	.0037164532	.000	.0439017931	.0584831825
	RowSum	.0503695217*	.0037164532	.000	.0430788270	.0576602164
	Wang	-.0016323753	.0037164532	.661	-.0089230700	.0056583194
Geometric Mean	Arithmetic Mean	-.0007324903	.0037164532	.844	-.0080231850	.0065582044
	Boender	.0001600154	.0037164532	.966	-.0071306793	.0074507101
	Buckley	-.007631952*	.0037164532	.040	-.0149226462	-.0003412568
	Chang	-.051667582*	.0037164532	.000	-.0589582766	-.0443768872
	I.C.S	-.0054548836	.0037164532	.142	-.0127455784	.0018358111
	Laarhoven	-.0004750941	.0037164532	.898	-.0077657888	.0068156007
	RowSum	-.0012980602	.0037164532	.727	-.0085887549	.0059926345
	Wang	-.053299957*	.0037164532	.000	-.0605906519	-.0460092625
I.C.S	Arithmetic Mean	.0047223933	.0037164532	.204	-.0025683014	.0120130880
	Boender	.0056148990	.0037164532	.131	-.0016757957	.0129055937
	Buckley	-.0021770679	.0037164532	.558	-.0094677626	.0051136268
	Chang	-.046212698*	.0037164532	.000	-.0535033930	-.0389220035
	Geometric Mean	.0054548836	.0037164532	.142	-.0018358111	.0127455784
	Laarhoven	.0049797896	.0037164532	.180	-.0023109051	.0122704843
	RowSum	.0041568234	.0037164532	.264	-.0031338713	.0114475182
	Wang	-.047845074*	.0037164532	.000	-.0551357682	-.0405543788
Laarhoven	Arithmetic Mean	-.0002573963	.0037164532	.945	-.0075480910	.0070332984
	Boender	.0006351094	.0037164532	.864	-.0066555853	.0079258042
	Buckley	-.0071568575	.0037164532	.054	-.0144475522	.0001338373
	Chang	-.051192488*	.0037164532	.000	-.0584831825	-.0439017931
	Geometric Mean	.0004750941	.0037164532	.898	-.0068156007	.0077657888
	I.C.S	-.0049797896	.0037164532	.180	-.0122704843	.0023109051
	RowSum	-.0008229661	.0037164532	.825	-.0081136609	.0064677286
	Wang	-.052824863*	.0037164532	.000	-.0601155578	-.0455341684
RowSum	Arithmetic Mean	.0005655699	.0037164532	.879	-.0067251248	.0078562646
	Boender	.0014580756	.0037164532	.695	-.0058326191	.0087487703
	Buckley	-.0063338913	.0037164532	.089	-.0136245860	.0009568034
	Chang	-.050369522*	.0037164532	.000	-.0576602164	-.0430788270
	Geometric Mean	.0012980602	.0037164532	.727	-.0059926345	.0085887549
	I.C.S	-.0041568234	.0037164532	.264	-.0114475182	.0031338713
	Laarhoven	.0008229661	.0037164532	.825	-.0064677286	.0081136609
	Wang	-.052001897*	.0037164532	.000	-.0592925917	-.0447112023
Wang	Arithmetic Mean	.0525674668*	.0037164532	.000	.0452767721	.0598581615
	Boender	.0534599726*	.0037164532	.000	.0461692778	.0607506673
	Buckley	.0456680057*	.0037164532	.000	.0383773109	.0529587004
	Chang	.0016323753	.0037164532	.661	-.0056583194	.0089230700
	Geometric Mean	.0532999572*	.0037164532	.000	.0460092625	.0605906519
	I.C.S	.0478450735*	.0037164532	.000	.0405543788	.0551357682
	Laarhoven	.0528248631*	.0037164532	.000	.0455341684	.0601155578
	RowSum	.0520018970*	.0037164532	.000	.0447112023	.0592925917

Based on observed means.
The error term is Mean Square(Error) = .001.

*. The mean difference is significant at the 0.05 level.

Table A.8: Performance analysis among models when n = 11

Dependent Variable: AverageError
LSD

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	Upper Bound
Arithmetic Mean	Boender	.0000271917	.0021980875	.990	-.0042848725	.0043392559
	Buckley	-.004887859*	.0021980875	.026	-.0091999231	-.0005757947
	Chang	-.033234026*	.0021980875	.000	-.0375460899	-.0289219614
	Geometric Mean	.0001889484	.0021980875	.932	-.0041231158	.0045010127
	I.C.S	-.0039851387	.0021980875	.070	-.0082972029	.0003269255
	Laarhoven	.0000901706	.0021980875	.967	-.0042218936	.0044022348
	RowSum	-.0008749807	.0021980875	.691	-.0051870449	.0034370836
	Wang	-.034004829*	.0021980875	.000	-.0383168936	-.0296927651
Boender	Arithmetic Mean	-.0000271917	.0021980875	.990	-.0043392559	.0042848725
	Buckley	-.004915051*	.0021980875	.026	-.0092271148	-.0006029864
	Chang	-.033261217*	.0021980875	.000	-.0375732816	-.0289491531
	Geometric Mean	.0001617567	.0021980875	.941	-.0041503075	.0044738210
	I.C.S	-.0040123304	.0021980875	.068	-.0083243946	.0002997338
	Laarhoven	.0000629789	.0021980875	.977	-.0042490853	.0043750431
	RowSum	-.0009021724	.0021980875	.682	-.0052142366	.0034098919
	Wang	-.034032021*	.0021980875	.000	-.0383440853	-.0297199568
Buckley	Arithmetic Mean	.0048878589*	.0021980875	.026	.0005757947	.0091999231
	Boender	.0049150506*	.0021980875	.026	.0006029864	.0092271148
	Chang	-.028346167*	.0021980875	.000	-.0326582310	-.0240341025
	Geometric Mean	.0050768073*	.0021980875	.021	.0007647431	.0093888715
	I.C.S	.0009027202	.0021980875	.681	-.0034093441	.0052147844
	Laarhoven	.0049780295*	.0021980875	.024	.0006659652	.0092900937
	RowSum	.0040128782	.0021980875	.068	-.0002991860	.0083249424
	Wang	-.029116970*	.0021980875	.000	-.0334290347	-.0248049063
Chang	Arithmetic Mean	.0332340256*	.0021980875	.000	.0289219614	.0375460899
	Boender	.0332612173*	.0021980875	.000	.0289491531	.0375732816
	Buckley	.0283461668*	.0021980875	.000	.0240341025	.0326582310
	Geometric Mean	.0334229741*	.0021980875	.000	.0291109099	.0377350383
	I.C.S	.0292488869*	.0021980875	.000	.0249368227	.0335609511
	Laarhoven	.0333241962*	.0021980875	.000	.0290121320	.0376362604
	RowSum	.0323590450*	.0021980875	.000	.0280469808	.0366711092
	Wang	-.0007708037	.0021980875	.726	-.0050828679	.0035412605
Geometric Mean	Arithmetic Mean	-.0001889484	.0021980875	.932	-.0045010127	.0041231158
	Boender	-.0001617567	.0021980875	.941	-.0044738210	.0041503075
	Buckley	-.005076807*	.0021980875	.021	-.0093888715	-.0007647431
	Chang	-.033422974*	.0021980875	.000	-.0377350383	-.0291109099
	I.C.S	-.0041740872	.0021980875	.058	-.0084861514	.0001379771
	Laarhoven	-.0000987779	.0021980875	.964	-.0044108421	.0042132864
	RowSum	-.0010639291	.0021980875	.628	-.0053759933	.0032481351
	Wang	-.034193778*	.0021980875	.000	-.0385058420	-.0298817136
I.C.S	Arithmetic Mean	.0039851387	.0021980875	.070	-.0003269255	.0082972029
	Boender	.0040123304	.0021980875	.068	-.0002997338	.0083243946
	Buckley	-.0009027202	.0021980875	.681	-.0052147844	.0034093441
	Chang	-.029248887*	.0021980875	.000	-.0335609511	-.0249368227
	Geometric Mean	.0041740872	.0021980875	.058	-.0001379771	.0084861514
	Laarhoven	.0040753093	.0021980875	.064	-.0002367549	.0083873735
	RowSum	.0031101581	.0021980875	.157	-.0012019062	.0074222223
	Wang	-.030019691*	.0021980875	.000	-.0343317549	-.0257076264
Laarhoven	Arithmetic Mean	-.0000901706	.0021980875	.967	-.0044022348	.0042218936
	Boender	-.0000629789	.0021980875	.977	-.0043750431	.0042490853
	Buckley	-.004978029*	.0021980875	.024	-.0092900937	-.0006659652
	Chang	-.033324196*	.0021980875	.000	-.0376362604	-.0290121320
	Geometric Mean	.0000987779	.0021980875	.964	-.0042132864	.0044108421
	I.C.S	-.0040753093	.0021980875	.064	-.0083873735	.0002367549
	RowSum	-.0009651512	.0021980875	.661	-.0052772155	.0033469130
	Wang	-.034095000*	.0021980875	.000	-.0384070642	-.0297829357
RowSum	Arithmetic Mean	.0008749807	.0021980875	.691	-.0034370836	.0051870449
	Boender	.0009021724	.0021980875	.682	-.0034098919	.0052142366
	Buckley	-.0040128782	.0021980875	.068	-.0083249424	.0002991860
	Chang	-.032359045*	.0021980875	.000	-.0366711092	-.0280469808
	Geometric Mean	.0010639291	.0021980875	.628	-.0032481351	.0053759933
	I.C.S	-.0031101581	.0021980875	.157	-.0074222223	.0012019062
	Laarhoven	.0009651512	.0021980875	.661	-.0033469130	.0052772155
	Wang	-.033129849*	.0021980875	.000	-.0374419129	-.0288177845
Wang	Arithmetic Mean	.0340048294*	.0021980875	.000	.0296927651	.0383168936
	Boender	.0340320211*	.0021980875	.000	.0297199568	.0383440853
	Buckley	.0291169705*	.0021980875	.000	.0248049063	.0334290347
	Chang	.0007708037	.0021980875	.726	-.0035412605	.0050828679
	Geometric Mean	.0341937778*	.0021980875	.000	.0298817136	.0385058420
	I.C.S	.0300196906*	.0021980875	.000	.0257076264	.0343317549
	Laarhoven	.0340949999*	.0021980875	.000	.0297829357	.0384070642
	RowSum	.0331298487*	.0021980875	.000	.0288177845	.0374419129

Based on observed means.
The error term is Mean Square(Error) = .000.

*. The mean difference is significant at the 0.05 level.

Table A.9: Performance analysis among models when n = 15

Dependent Variable: AverageError
LSD

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Arithmetic Mean	Boender	-.0000222512	.0015440472	.989	-.0030512621	.0030067597
	Buckley	-.0027845422	.0015440472	.072	-.0058135531	.0002444687
	Chang	-.024475544*	.0015440472	.000	-.0275045547	-.0214465330
	Geometric Mean	.0001042310	.0015440472	.946	-.0029247799	.0031332418
	I.C.S	-.003173768*	.0015440472	.040	-.0062027793	-.0001447575
	Laarhoven	.0001115503	.0015440472	.942	-.0029174606	.0031405611
	RowSum	-.0005805177	.0015440472	.707	-.0036095285	.0024484932
	Wang	-.024599067*	.0015440472	.000	-.0276280781	-.0215700564
Boender	Arithmetic Mean	.0000222512	.0015440472	.989	-.0030067597	.0030512621
	Buckley	-.0027622910	.0015440472	.074	-.0057913019	.0002667199
	Chang	-.024453293*	.0015440472	.000	-.0274823035	-.0214242818
	Geometric Mean	.0001264821	.0015440472	.935	-.0029025287	.0031554930
	I.C.S	-.003151517*	.0015440472	.041	-.0061805281	-.0001225063
	Laarhoven	.0001338015	.0015440472	.931	-.0028952094	.0031628123
	RowSum	-.0005582665	.0015440472	.718	-.0035872773	.0024707444
	Wang	-.024576816*	.0015440472	.000	-.0276058269	-.0215478052
Buckley	Arithmetic Mean	.0027845422	.0015440472	.072	-.0002444687	.0058135531
	Boender	.0027622910	.0015440472	.074	-.0002667199	.0057913019
	Chang	-.021691002*	.0015440472	.000	-.0247200125	-.0186619908
	Geometric Mean	.0028887731	.0015440472	.062	-.0001402377	.0059177840
	I.C.S	-.0003892262	.0015440472	.801	-.0034182371	.0026397847
	Laarhoven	.0028960925	.0015440472	.061	-.0001329184	.0059251033
	RowSum	.0022040245	.0015440472	.154	-.0008249863	.0052330354
	Wang	-.021814525*	.0015440472	.000	-.0248435359	-.0187855142
Chang	Arithmetic Mean	.0244755438*	.0015440472	.000	.0214465330	.0275045547
	Boender	.0244532926*	.0015440472	.000	.0214242818	.0274823035
	Buckley	.0216910016*	.0015440472	.000	.0186619908	.0247200125
	Geometric Mean	.0245797748*	.0015440472	.000	.0215507639	.0276087856
	I.C.S	.0213017754*	.0015440472	.000	.0182727646	.0243307863
	Laarhoven	.0245870941*	.0015440472	.000	.0215580833	.0276161050
	RowSum	.0238950261*	.0015440472	.000	.0208660153	.0269240370
	Wang	-.0001235234	.0015440472	.936	-.0031525343	.0029054874
Geometric Mean	Arithmetic Mean	-.0001042310	.0015440472	.946	-.0031332418	.0029247799
	Boender	-.0001264821	.0015440472	.935	-.0031554930	.0029025287
	Buckley	-.0028887731	.0015440472	.062	-.0059177840	.0001402377
	Chang	-.024579775*	.0015440472	.000	-.0276087856	-.0215507639
	I.C.S	-.003277999*	.0015440472	.034	-.0063070102	-.0002489885
	Laarhoven	.0000073193	.0015440472	.996	-.0030216915	.0030363302
	RowSum	-.0006847486	.0015440472	.657	-.0037137595	.0023442622
	Wang	-.024703298*	.0015440472	.000	-.0277323091	-.0216742873
I.C.S	Arithmetic Mean	.0031737684*	.0015440472	.040	.0001447575	.0062027793
	Boender	.0031515172*	.0015440472	.041	.0001225063	.0061805281
	Buckley	.0003892262	.0015440472	.801	-.0026397847	.0034182371
	Chang	-.021301775*	.0015440472	.000	-.0243307863	-.0182727646
	Geometric Mean	.0032779994*	.0015440472	.034	.0002489885	.0063070102
	Laarhoven	.0032853187*	.0015440472	.034	.0002563078	.0063143295
	RowSum	.0025932507	.0015440472	.093	-.0004357601	.0056222616
	Wang	-.021425299*	.0015440472	.000	-.0244543097	-.0183962880
Laarhoven	Arithmetic Mean	-.0001115503	.0015440472	.942	-.0031405611	.0029174606
	Boender	-.0001338015	.0015440472	.931	-.0031628123	.0028952094
	Buckley	-.0028960925	.0015440472	.061	-.0059251033	.0001329184
	Chang	-.024587094*	.0015440472	.000	-.0276161050	-.0215580833
	Geometric Mean	-.0000073193	.0015440472	.996	-.0030363302	.0030216915
	I.C.S	-.003285319*	.0015440472	.034	-.0063143295	-.0002563078
	RowSum	-.0006920680	.0015440472	.654	-.0037210788	.0023369429
	Wang	-.024710618*	.0015440472	.000	-.0277396284	-.0216816067
RowSum	Arithmetic Mean	.0005805177	.0015440472	.707	-.0024484932	.0036095285
	Boender	.0005582665	.0015440472	.718	-.0024707444	.0035872773
	Buckley	-.0022040245	.0015440472	.154	-.0052330354	.0008249863
	Chang	-.023895026*	.0015440472	.000	-.0269240370	-.0208660153
	Geometric Mean	.0006847486	.0015440472	.657	-.0023442622	.0037137595
	I.C.S	-.0025932507	.0015440472	.093	-.0056222616	.0004357601
	Laarhoven	.0006920680	.0015440472	.654	-.0023369429	.0037210788
	Wang	-.024018550*	.0015440472	.000	-.0270475604	-.0209895387
Wang	Arithmetic Mean	.0245990672*	.0015440472	.000	.0215700564	.0276280781
	Boender	.0245768161*	.0015440472	.000	.0215478052	.0276058269
	Buckley	.0218145251*	.0015440472	.000	.0187855142	.0248435359
	Chang	.0001235234	.0015440472	.936	-.0029054874	.0031525343
	Geometric Mean	.0247032982*	.0015440472	.000	.0216742873	.0277323091
	I.C.S	.0214252988*	.0015440472	.000	.0183962880	.0244543097
	Laarhoven	.0247106175*	.0015440472	.000	.0216816067	.0277396284
	RowSum	.0240185496*	.0015440472	.000	.0209895387	.0270475604

Based on observed means.
The error term is Mean Square(Error) = .000.
*. The mean difference is significant at the 0.05 level.

Table A.10: Performance analysis among models when $\alpha = 0.05$

Dependent Variable: AverageError
LSD

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Arithmetic Mean	Boender	-.0022269023	.0042064538	.597	-.0104769755	.0060231710
	Buckley	-.0056996366	.0042064538	.176	-.0139497098	.0025504367
	Chang	-.052315385*	.0042064538	.000	-.0605654580	-.0440653116
	Geometric Mean	.0001948353	.0042064538	.963	-.0080552380	.0084449085
	I.C.S	-.0049928090	.0042064538	.235	-.0132428822	.0032572642
	Laarhoven	-.0032146635	.0042064538	.445	-.0114647367	.0050354097
	RowSum	-.0005733566	.0042064538	.892	-.0088234299	.0076767166
	Wang	-.059451112*	.0042064538	.000	-.0677011853	-.0512010388
Boender	Arithmetic Mean	.0022269023	.0042064538	.597	-.0060231710	.0104769755
	Buckley	-.0034727343	.0042064538	.409	-.0117228075	.0047773389
	Chang	-.050088483*	.0042064538	.000	-.0583385557	-.0418384093
	Geometric Mean	.0024217376	.0042064538	.565	-.0058283357	.0106718108
	I.C.S	-.0027659067	.0042064538	.511	-.0110159799	.0054841665
	Laarhoven	-.0009877612	.0042064538	.814	-.0092378345	.0072623120
	RowSum	.0016535457	.0042064538	.694	-.0065965276	.0099036189
	Wang	-.057224210*	.0042064538	.000	-.0654742830	-.0489741365
Buckley	Arithmetic Mean	.0056996366	.0042064538	.176	-.0025504367	.0139497098
	Boender	.0034727343	.0042064538	.409	-.0047773389	.0117228075
	Chang	-.046615748*	.0042064538	.000	-.0548658215	-.0383656750
	Geometric Mean	.0058944718	.0042064538	.161	-.0023556014	.0141445451
	I.C.S	.0007068276	.0042064538	.867	-.0075432457	.0089569008
	Laarhoven	.0024849731	.0042064538	.555	-.0057651002	.0107350463
	RowSum	.0051262799	.0042064538	.223	-.0031237933	.0133763532
	Wang	-.053751475*	.0042064538	.000	-.0620015487	-.0455014022
Chang	Arithmetic Mean	.0523153848*	.0042064538	.000	.0440653116	.0605654580
	Boender	.0500884825*	.0042064538	.000	.0418384093	.0583385557
	Buckley	.0466157482*	.0042064538	.000	.0383656750	.0548658215
	Geometric Mean	.0525102201*	.0042064538	.000	.0442601468	.0607602933
	I.C.S	.0473225758*	.0042064538	.000	.0390725026	.0555726490
	Laarhoven	.0491007213*	.0042064538	.000	.0408506481	.0573507945
	RowSum	.0517420282*	.0042064538	.000	.0434919549	.0599921014
	Wang	-.0071357273	.0042064538	.090	-.0153858005	.0011143460
Geometric Mean	Arithmetic Mean	-.0001948353	.0042064538	.963	-.0084449085	.0080552380
	Boender	-.0024217376	.0042064538	.565	-.0106718108	.0058283357
	Buckley	-.0058944718	.0042064538	.161	-.0141445451	.0023556014
	Chang	-.052510220*	.0042064538	.000	-.0607602933	-.0442601468
	I.C.S	-.0051876443	.0042064538	.218	-.0134377175	.0030624290
	Laarhoven	-.0034094988	.0042064538	.418	-.0116595720	.0048405745
	RowSum	-.0007681919	.0042064538	.855	-.0090182651	.0074818813
	Wang	-.059645947*	.0042064538	.000	-.0678960206	-.0513958741
I.C.S	Arithmetic Mean	.0049928090	.0042064538	.235	-.0032572642	.0132428822
	Boender	.0027659067	.0042064538	.511	-.0054841665	.0110159799
	Buckley	-.0007068276	.0042064538	.867	-.0089569008	.0075432457
	Chang	-.047322576*	.0042064538	.000	-.0555726490	-.0390725026
	Geometric Mean	.0051876443	.0042064538	.218	-.0030624290	.0134377175
	Laarhoven	.0017781455	.0042064538	.673	-.0064719277	.0100282187
	RowSum	.0044194524	.0042064538	.294	-.0038306209	.0126695256
	Wang	-.054458303*	.0042064538	.000	-.0627083763	-.0462082298
Laarhoven	Arithmetic Mean	.0032146635	.0042064538	.445	-.0050354097	.0114647367
	Boender	.0009877612	.0042064538	.814	-.0072623120	.0092378345
	Buckley	-.0024849731	.0042064538	.555	-.0107350463	.0057651002
	Chang	-.049100721*	.0042064538	.000	-.0573507945	-.0408506481
	Geometric Mean	.0034094988	.0042064538	.418	-.0048405745	.0116595720
	I.C.S	-.0017781455	.0042064538	.673	-.0100282187	.0064719277
	RowSum	.0026413069	.0042064538	.530	-.0056087664	.0108913801
	Wang	-.056236449*	.0042064538	.000	-.0644865218	-.0479863753
RowSum	Arithmetic Mean	.0005733566	.0042064538	.892	-.0076767166	.0088234299
	Boender	-.0016535457	.0042064538	.694	-.0099036189	.0065965276
	Buckley	-.0051262799	.0042064538	.223	-.0133763532	.0031237933
	Chang	-.051742028*	.0042064538	.000	-.0599921014	-.0434919549
	Geometric Mean	.0007681919	.0042064538	.855	-.0074818813	.0090182651
	I.C.S	-.0044194524	.0042064538	.294	-.0126695256	.0038306209
	Laarhoven	-.0026413069	.0042064538	.530	-.0108913801	.0056087664
	Wang	-.058877755*	.0042064538	.000	-.0671278287	-.0506276822
Wang	Arithmetic Mean	.0594511121*	.0042064538	.000	.0512010388	.0677011853
	Boender	.0572242098*	.0042064538	.000	.0489741365	.0654742830
	Buckley	.0537514755*	.0042064538	.000	.0455014022	.0620015487
	Chang	.0071357273	.0042064538	.090	-.0011143460	.0153858005
	Geometric Mean	.0596459473*	.0042064538	.000	.0513958741	.0678960206
	I.C.S	.0544583031*	.0042064538	.000	.0462082298	.0627083763
	Laarhoven	.0562364485*	.0042064538	.000	.0479863753	.0644865218
	RowSum	.0588777554*	.0042064538	.000	.0506276822	.0671278287

Based on observed means.
The error term is Mean Square(Error) = .002.

*. The mean difference is significant at the 0.05 level.

Table A.11: Performance analysis among models when $\alpha = 0.10$

Dependent Variable: AverageError
LSD

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Arithmetic Mean	Boender	-.0010768828	.0040261222	.789	-.0089732737	.0068195081
	Buckley	-.0057156643	.0040261222	.156	-.0136120552	.0021807267
	Chang	-.046109303*	.0040261222	.000	-.0540056935	-.0382129117
	Geometric Mean	.0005860671	.0040261222	.884	-.0073103239	.0084824580
	I.C.S	-.0038191265	.0040261222	.343	-.0117155174	.0040772644
	Laarhoven	-.0031145953	.0040261222	.439	-.0110109862	.0047817956
	RowSum	.0003826029	.0040261222	.924	-.0075137881	.0082789938
	Wang	-.050602185*	.0040261222	.000	-.0584985760	-.0427057942
Boender	Arithmetic Mean	.0010768828	.0040261222	.789	-.0068195081	.0089732737
	Buckley	-.0046387814	.0040261222	.249	-.0125351724	.0032576095
	Chang	-.045032420*	.0040261222	.000	-.0529288107	-.0371360289
	Geometric Mean	.0016629499	.0040261222	.680	-.0062334410	.0095593408
	I.C.S	-.0027422437	.0040261222	.496	-.0106386346	.0051541472
	Laarhoven	-.0020377125	.0040261222	.613	-.0099341034	.0058586784
	RowSum	.0014594857	.0040261222	.717	-.0064369052	.0093558766
	Wang	-.049525302*	.0040261222	.000	-.0574216932	-.0416289114
Buckley	Arithmetic Mean	.0057156643	.0040261222	.156	-.0021807267	.0136120552
	Boender	.0046387814	.0040261222	.249	-.0032576095	.0125351724
	Chang	-.040393638*	.0040261222	.000	-.0482900293	-.0324972474
	Geometric Mean	.0063017313	.0040261222	.118	-.0015946596	.0141981222
	I.C.S	.0018965377	.0040261222	.638	-.0059998532	.0097929287
	Laarhoven	.0026010689	.0040261222	.518	-.0052953220	.0104974599
	RowSum	.0060982671	.0040261222	.130	-.0017981238	.0139946580
	Wang	-.044886521*	.0040261222	.000	-.0527829118	-.0369901299
Chang	Arithmetic Mean	.0461093026*	.0040261222	.000	.0382129117	.0540056935
	Boender	.0450324198*	.0040261222	.000	.0371360289	.0529288107
	Buckley	.0403936384*	.0040261222	.000	.0324972474	.0482900293
	Geometric Mean	.0466953697*	.0040261222	.000	.0387989788	.0545917606
	I.C.S	.0422901761*	.0040261222	.000	.0343937852	.0501865670
	Laarhoven	.0429947073*	.0040261222	.000	.0350983164	.0508910982
	RowSum	.0464919055*	.0040261222	.000	.0385955146	.0543882964
	Wang	-.0044928825	.0040261222	.265	-.0123892734	.0034035084
Geometric Mean	Arithmetic Mean	-.0005860671	.0040261222	.884	-.0084824580	.0073103239
	Boender	-.0016629499	.0040261222	.680	-.0095593408	.0062334410
	Buckley	-.0063017313	.0040261222	.118	-.0141981222	.0015946596
	Chang	-.046695370*	.0040261222	.000	-.0545917606	-.0387989788
	I.C.S	-.0044051936	.0040261222	.274	-.0123015845	.0034911973
	Laarhoven	-.0037006624	.0040261222	.358	-.0115970533	.0041957285
	RowSum	-.0002034642	.0040261222	.960	-.0080998551	.0076929267
	Wang	-.051188252*	.0040261222	.000	-.0590846431	-.0432918613
I.C.S	Arithmetic Mean	.0038191265	.0040261222	.343	-.0040772644	.0117155174
	Boender	.0027422437	.0040261222	.496	-.0051541472	.0106386346
	Buckley	-.0018965377	.0040261222	.638	-.0097929287	.0059998532
	Chang	-.042290176*	.0040261222	.000	-.0501865670	-.0343937852
	Geometric Mean	.0044051936	.0040261222	.274	-.0034911973	.0123015845
	Laarhoven	.0007045312	.0040261222	.861	-.0071918597	.0086009221
	RowSum	.0042017294	.0040261222	.297	-.0036946615	.0120981203
	Wang	-.046783059*	.0040261222	.000	-.0546794495	-.0388866677
Laarhoven	Arithmetic Mean	.0031145953	.0040261222	.439	-.0047817956	.0110109862
	Boender	.0020377125	.0040261222	.613	-.0058586784	.0099341034
	Buckley	-.0026010689	.0040261222	.518	-.0104974599	.0052953220
	Chang	-.042994707*	.0040261222	.000	-.0508910982	-.0350983164
	Geometric Mean	.0037006624	.0040261222	.358	-.0041957285	.0115970533
	I.C.S	-.0007045312	.0040261222	.861	-.0086009221	.0071918597
	RowSum	.0034971982	.0040261222	.385	-.0043991927	.0113935891
	Wang	-.047487590*	.0040261222	.000	-.0553839807	-.0395911989
RowSum	Arithmetic Mean	-.0003826029	.0040261222	.924	-.0082789938	.0075137881
	Boender	-.0014594857	.0040261222	.717	-.0093558766	.0064369052
	Buckley	-.0060982671	.0040261222	.130	-.0139946580	.0017981238
	Chang	-.046491905*	.0040261222	.000	-.0543882964	-.0385955146
	Geometric Mean	.0002034642	.0040261222	.960	-.0076929267	.0080998551
	I.C.S	-.0042017294	.0040261222	.297	-.0120981203	.0036946615
	Laarhoven	-.0034971982	.0040261222	.385	-.0113935891	.0043991927
	Wang	-.050984788*	.0040261222	.000	-.0588811789	-.0430883971
Wang	Arithmetic Mean	.0506021851*	.0040261222	.000	.0427057942	.0584985760
	Boender	.0495253023*	.0040261222	.000	.0416289114	.0574216932
	Buckley	.0448865209*	.0040261222	.000	.0369901299	.0527829118
	Chang	.0044928825	.0040261222	.265	-.0034035084	.0123892734
	Geometric Mean	.0511882522*	.0040261222	.000	.0432918613	.0590846431
	I.C.S	.0467830586*	.0040261222	.000	.0388866677	.0546794495
	Laarhoven	.0474875898*	.0040261222	.000	.0395911989	.0553839807
	RowSum	.0509847880*	.0040261222	.000	.0430883971	.0588811789

Based on observed means.
The error term is Mean Square(Error) = .002.
*. The mean difference is significant at the 0.05 level.

Table A.12: Performance analysis among models when $\alpha = 0.15$

Dependent Variable: AverageError
LSD

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	Upper Bound
Arithmetic Mean	Boender	-.0015675112	.0038002173	.680	-.0090208372	.0058858149
	Buckley	-.0064140725	.0038002173	.092	-.0138673986	.0010392536
	Chang	-.041311684*	.0038002173	.000	-.0487650096	-.0338583575
	Geometric Mean	.0007647067	.0038002173	.841	-.0066886194	.0082180328
	I.C.S	-.0033312913	.0038002173	.381	-.0107846174	.0041220348
	Laarhoven	-.0035422178	.0038002173	.351	-.0109955439	.0039111082
	RowSum	.0002976572	.0038002173	.938	-.0071556688	.0077509833
	Wang	-.047576876*	.0038002173	.000	-.0550302018	-.0401235497
Boender	Arithmetic Mean	.0015675112	.0038002173	.680	-.0058858149	.0090208372
	Buckley	-.0048465613	.0038002173	.202	-.0122998874	.0026067647
	Chang	-.039744172*	.0038002173	.000	-.0471974984	-.0322908463
	Geometric Mean	.0023322179	.0038002173	.539	-.0051211082	.0097855439
	I.C.S	-.0017637801	.0038002173	.643	-.0092171062	.0056895459
	Laarhoven	-.0019747067	.0038002173	.603	-.0094280327	.0054786194
	RowSum	.0018651684	.0038002173	.624	-.0055881576	.0093184945
	Wang	-.046009365*	.0038002173	.000	-.0534626907	-.0385560385
Buckley	Arithmetic Mean	.0064140725	.0038002173	.092	-.0010392536	.0138673986
	Boender	.0048465613	.0038002173	.202	-.0026067647	.0122998874
	Chang	-.034897611*	.0038002173	.000	-.0423509371	-.0274442850
	Geometric Mean	.0071787792	.0038002173	.059	-.0002745469	.0146321053
	I.C.S	.0030827812	.0038002173	.417	-.0043705449	.0105361073
	Laarhoven	.0028718547	.0038002173	.450	-.0045814714	.0103251807
	RowSum	.0067117297	.0038002173	.078	-.0007415963	.0141650558
	Wang	-.041162803*	.0038002173	.000	-.0486161293	-.0337094772
Chang	Arithmetic Mean	.0413116836*	.0038002173	.000	.0338583575	.0487650096
	Boender	.0397441724*	.0038002173	.000	.0322908463	.0471974984
	Buckley	.0348976111*	.0038002173	.000	.0274442850	.0423509371
	Geometric Mean	.0420763903*	.0038002173	.000	.0346230642	.0495297163
	I.C.S	.0379803923*	.0038002173	.000	.0305270662	.0454337183
	Laarhoven	.0377694657*	.0038002173	.000	.0303161397	.0452227918
	RowSum	.0416093408*	.0038002173	.000	.0341560147	.0490626668
	Wang	-.0062651922	.0038002173	.099	-.0137185183	.0011881338
Geometric Mean	Arithmetic Mean	-.0007647067	.0038002173	.841	-.0082180328	.0066886194
	Boender	-.0023322179	.0038002173	.539	-.0097855439	.0051211082
	Buckley	-.0071787792	.0038002173	.059	-.0146321053	.0002745469
	Chang	-.042076390*	.0038002173	.000	-.0495297163	-.0346230642
	I.C.S	-.0040959980	.0038002173	.281	-.0115493240	.0033573281
	Laarhoven	-.0043069245	.0038002173	.257	-.0117602506	.0031464015
	RowSum	-.0004670495	.0038002173	.902	-.0079203755	.0069862766
	Wang	-.048341582*	.0038002173	.000	-.0557949085	-.0408882564
I.C.S	Arithmetic Mean	.0033312913	.0038002173	.381	-.0041220348	.0107846174
	Boender	.0017637801	.0038002173	.643	-.0056895459	.0092171062
	Buckley	-.0030827812	.0038002173	.417	-.0105361073	.0043705449
	Chang	-.037980392*	.0038002173	.000	-.0454337183	-.0305270662
	Geometric Mean	.0040959980	.0038002173	.281	-.0033573281	.0115493240
	Laarhoven	-.0002109265	.0038002173	.956	-.0076642526	.0072423995
	RowSum	.0036289485	.0038002173	.340	-.0038243775	.0110822746
	Wang	-.044245584*	.0038002173	.000	-.0516989106	-.0367922584
Laarhoven	Arithmetic Mean	.0035422178	.0038002173	.351	-.0039111082	.0109955439
	Boender	.0019747067	.0038002173	.603	-.0054786194	.0094280327
	Buckley	-.0028718547	.0038002173	.450	-.0103251807	.0045814714
	Chang	-.037769466*	.0038002173	.000	-.0452227918	-.0303161397
	Geometric Mean	.0043069245	.0038002173	.257	-.0031464015	.0117602506
	I.C.S	.0002109265	.0038002173	.956	-.0072423995	.0076642526
	RowSum	.0038398751	.0038002173	.312	-.0036134510	.0112932011
	Wang	-.044034658*	.0038002173	.000	-.0514879840	-.0365813319
RowSum	Arithmetic Mean	-.0002976572	.0038002173	.938	-.0077509833	.0071556688
	Boender	-.0018651684	.0038002173	.624	-.0093184945	.0055881576
	Buckley	-.0067117297	.0038002173	.078	-.0141650558	.0007415963
	Chang	-.041609341*	.0038002173	.000	-.0490626668	-.0341560147
	Geometric Mean	.0004670495	.0038002173	.902	-.0069862766	.0079203755
	I.C.S	-.0036289485	.0038002173	.340	-.0110822746	.0038243775
	Laarhoven	-.0038398751	.0038002173	.312	-.0112932011	.0036134510
	Wang	-.047874533*	.0038002173	.000	-.0553278591	-.0404212070
Wang	Arithmetic Mean	.0475768758*	.0038002173	.000	.0401235497	.0550302018
	Boender	.0460093646*	.0038002173	.000	.0385560385	.0534626907
	Buckley	.0411628033*	.0038002173	.000	.0337094772	.0486161293
	Chang	.0062651922	.0038002173	.099	-.0011881338	.0137185183
	Geometric Mean	.0483415825*	.0038002173	.000	.0408882564	.0557949085
	I.C.S	.0442455845*	.0038002173	.000	.0367922584	.0516989106
	Laarhoven	.0440346580*	.0038002173	.000	.0365813319	.0514879840
	RowSum	.0478745330*	.0038002173	.000	.0404212070	.0553278591

Based on observed means.
The error term is Mean Square(Error) = .001.
*. The mean difference is significant at the 0.05 level.

Table A.13: Performance analysis among models when $\beta = 0\%$

Dependent Variable: AverageError
LSD

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	Upper Bound
Arithmetic Mean	Boender	.0000116626	.0059760397	.998	-.0117144118	.0117377369
	Buckley	.0000476064	.0059760397	.994	-.0116784679	.0117736808
	Chang	-.163909796*	.0059760397	.000	-.1756358703	-.1521837217
	Geometric Mean	.0000474792	.0059760397	.994	-.0116785951	.0117735535
	I.C.S	-.0000631048	.0059760397	.992	-.0117891792	.0116629695
	Laarhoven	.0000030606	.0059760397	1.000	-.0117230138	.0117291349
	RowSum	.0001501248	.0059760397	.980	-.0115759496	.0118761991
	Wang	-.170473713*	.0059760397	.000	-.1821997872	-.1587476386
Boender	Arithmetic Mean	-.0000116626	.0059760397	.998	-.0117377369	.0117144118
	Buckley	.0000359439	.0059760397	.995	-.0116901305	.0117620182
	Chang	-.163921459*	.0059760397	.000	-.1756475329	-.1521953843
	Geometric Mean	.0000358166	.0059760397	.995	-.0116902577	.0117618910
	I.C.S	-.0000747674	.0059760397	.990	-.0118008417	.0116513069
	Laarhoven	-.0000086020	.0059760397	.999	-.0117346763	.0117174723
	RowSum	.0001384622	.0059760397	.982	-.0115876121	.0118645365
	Wang	-.170485375*	.0059760397	.000	-.1822114498	-.1587593011
Buckley	Arithmetic Mean	-.0000476064	.0059760397	.994	-.0117736808	.0116784679
	Boender	-.0000359439	.0059760397	.995	-.0117620182	.0116901305
	Chang	-.163957402*	.0059760397	.000	-.1756834768	-.1522313281
	Geometric Mean	-1.27230E-7	.0059760397	1.000	-.0117262016	.0117259471
	I.C.S	-.0001107112	.0059760397	.985	-.0118367856	.0116153631
	Laarhoven	-.0000445459	.0059760397	.994	-.0117706202	.0116815285
	RowSum	.0001025183	.0059760397	.986	-.0116235560	.0118285927
	Wang	-.170521319*	.0059760397	.000	-.1822473936	-.1587952450
Chang	Arithmetic Mean	.1639097960*	.0059760397	.000	.1521837217	.1756358703
	Boender	.1639214586*	.0059760397	.000	.1521953843	.1756475329
	Buckley	.1639574024*	.0059760397	.000	.1522313281	.1756834768
	Geometric Mean	.1639572752*	.0059760397	.000	.1522312009	.1756833495
	I.C.S	.1638466912*	.0059760397	.000	.1521206169	.1755727655
	Laarhoven	.1639128566*	.0059760397	.000	.1521867822	.1756389309
	RowSum	.1640599208*	.0059760397	.000	.1523338465	.1757859951
	Wang	-.0065639169	.0059760397	.272	-.0182899912	.0051621575
Geometric Mean	Arithmetic Mean	-.0000474792	.0059760397	.994	-.0117735535	.0116785951
	Boender	-.0000358166	.0059760397	.995	-.0117618910	.0116902577
	Buckley	1.27230E-7	.0059760397	1.000	-.0117259471	.0117262016
	Chang	-.163957275*	.0059760397	.000	-.1756833495	-.1522312009
	I.C.S	-.0001105840	.0059760397	.985	-.0118366583	.0116154903
	Laarhoven	-.0000444186	.0059760397	.994	-.0117704930	.0116816557
	RowSum	.0001026456	.0059760397	.986	-.0116234288	.0118287199
	Wang	-.170521192*	.0059760397	.000	-.1822472664	-.1587951178
I.C.S	Arithmetic Mean	.0000631048	.0059760397	.992	-.0116629695	.0117891792
	Boender	.0000747674	.0059760397	.990	-.0116513069	.0118008417
	Buckley	.0001107112	.0059760397	.985	-.0116153631	.0118367856
	Chang	-.163846691*	.0059760397	.000	-.1755727655	-.1521206169
	Geometric Mean	.0001105840	.0059760397	.985	-.0116154903	.0118366583
	Laarhoven	.0000661654	.0059760397	.991	-.0116599090	.0117922397
	RowSum	.0002132296	.0059760397	.972	-.0115128447	.0119393039
	Wang	-.170410608*	.0059760397	.000	-.1821366824	-.1586845337
Laarhoven	Arithmetic Mean	-.0000030606	.0059760397	1.000	-.0117291349	.0117230138
	Boender	.0000086020	.0059760397	.999	-.0117174723	.0117346763
	Buckley	.0000445459	.0059760397	.994	-.0116815285	.0117706202
	Chang	-.163912857*	.0059760397	.000	-.1756389309	-.1521867822
	Geometric Mean	.0000444186	.0059760397	.994	-.0116816557	.0117704930
	I.C.S	-.0000661654	.0059760397	.991	-.0117922397	.0116599090
	RowSum	.0001470642	.0059760397	.980	-.0115790101	.0118731385
	Wang	-.170476773*	.0059760397	.000	-.1822028478	-.1587506991
RowSum	Arithmetic Mean	-.0001501248	.0059760397	.980	-.0118761991	.0115759496
	Boender	-.0001384622	.0059760397	.982	-.0118645365	.0115876121
	Buckley	-.0001025183	.0059760397	.986	-.0118285927	.0116235560
	Chang	-.164059921*	.0059760397	.000	-.1757859951	-.1523338465
	Geometric Mean	-.0001026456	.0059760397	.986	-.0118287199	.0116234288
	I.C.S	-.0002132296	.0059760397	.972	-.0119393039	.0115128447
	Laarhoven	-.0001470642	.0059760397	.980	-.0118731385	.0115790101
	Wang	-.170623838*	.0059760397	.000	-.1823499120	-.1588977633
Wang	Arithmetic Mean	.1704737129*	.0059760397	.000	.1587476386	.1821997872
	Boender	.1704853755*	.0059760397	.000	.1587593011	.1822114498
	Buckley	.1705213193*	.0059760397	.000	.1587952450	.1822473936
	Chang	.0065639169	.0059760397	.272	-.0051621575	.0182899912
	Geometric Mean	.1705211921*	.0059760397	.000	.1587951178	.1822472664
	I.C.S	.1704106081*	.0059760397	.000	.1586845337	.1821366824
	Laarhoven	.1704767734*	.0059760397	.000	.1587506991	.1822028478
	RowSum	.1706238377*	.0059760397	.000	.1588977633	.1823499120

Based on observed means.
The error term is Mean Square(Error) = .002.

*. The mean difference is significant at the 0.05 level.

Table A.14: Performance analysis among models when $\beta = 50\%$

Dependent Variable: AverageError
LSD

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Arithmetic Mean	Boender	.0001124302	.0039443541	.977	-.0076271082	.0078519687
	Buckley	.0000554447	.0039443541	.989	-.0076840937	.0077949832
	Chang	-.034953317*	.0039443541	.000	-.0426928552	-.0272137783
	Geometric Mean	.0000705094	.0039443541	.986	-.0076690290	.0078100479
	I.C.S	-.0003631874	.0039443541	.927	-.0081027259	.0073763510
	Laarhoven	-.0000190948	.0039443541	.996	-.0077586333	.0077204436
	RowSum	-.0002890410	.0039443541	.942	-.0080285795	.0074504974
	Wang	-.062410043*	.0039443541	.000	-.0701495810	-.0546705041
Boender	Arithmetic Mean	-.0001124302	.0039443541	.977	-.0078519687	.0076271082
	Buckley	-.0000569855	.0039443541	.988	-.0077965239	.0076825530
	Chang	-.035065747*	.0039443541	.000	-.0428052854	-.0273262086
	Geometric Mean	-.0000419208	.0039443541	.992	-.0077814593	.0076976176
	I.C.S	-.0004756177	.0039443541	.904	-.0082151561	.0072639208
	Laarhoven	-.0001315251	.0039443541	.973	-.0078710635	.0076080134
	RowSum	-.0004014712	.0039443541	.919	-.0081410097	.0073380672
	Wang	-.062522473*	.0039443541	.000	-.0702620112	-.0547829343
Buckley	Arithmetic Mean	-.0000554447	.0039443541	.989	-.0077949832	.0076840937
	Boender	.0000569855	.0039443541	.988	-.0076825530	.0077965239
	Chang	-.035008762*	.0039443541	.000	-.0427483000	-.0272692231
	Geometric Mean	.0000150647	.0039443541	.997	-.0077244738	.0077546031
	I.C.S	-.0004186322	.0039443541	.915	-.0081581706	.0073209063
	Laarhoven	-.0000745396	.0039443541	.985	-.0078140780	.0076649989
	RowSum	-.0003444858	.0039443541	.930	-.0080840242	.0073950527
	Wang	-.062465487*	.0039443541	.000	-.0702050257	-.0547259488
Chang	Arithmetic Mean	.0349533168*	.0039443541	.000	.0272137783	.0426928552
	Boender	.0350657470*	.0039443541	.000	.0273262086	.0428052854
	Buckley	.0350087615*	.0039443541	.000	.0272692231	.0427483000
	Geometric Mean	.0350238262*	.0039443541	.000	.0272842877	.0427633646
	I.C.S	.0345901293*	.0039443541	.000	.0268505909	.0423296678
	Laarhoven	.0349342219*	.0039443541	.000	.0271946835	.0426737604
	RowSum	.0346642758*	.0039443541	.000	.0269247373	.0424038142
	Wang	-.027456726*	.0039443541	.000	-.0351962642	-.0197171873
Geometric Mean	Arithmetic Mean	-.0000705094	.0039443541	.986	-.0078100479	.0076690290
	Boender	.0000419208	.0039443541	.992	-.0076976176	.0077814593
	Buckley	-.0000150647	.0039443541	.997	-.0077546031	.0077244738
	Chang	-.035023826*	.0039443541	.000	-.0427633646	-.0272842877
	I.C.S	-.0004336968	.0039443541	.912	-.0081732353	.0073058416
	Laarhoven	-.0000896042	.0039443541	.982	-.0078291427	.0076499342
	RowSum	-.0003595504	.0039443541	.927	-.0080990889	.0073799880
	Wang	-.062480552*	.0039443541	.000	-.0702200904	-.0547410135
I.C.S	Arithmetic Mean	.0003631874	.0039443541	.927	-.0073763510	.0081027259
	Boender	.0004756177	.0039443541	.904	-.0072639208	.0082151561
	Buckley	.0004186322	.0039443541	.915	-.0073209063	.0081581706
	Chang	-.034590129*	.0039443541	.000	-.0423296678	-.0268505909
	Geometric Mean	.0004336968	.0039443541	.912	-.0073058416	.0081732353
	Laarhoven	.0003440926	.0039443541	.930	-.0073954459	.0080836310
	RowSum	.0000741464	.0039443541	.985	-.0076653920	.0078136849
	Wang	-.062046855*	.0039443541	.000	-.0697863935	-.0543073166
Laarhoven	Arithmetic Mean	.0000190948	.0039443541	.996	-.0077204436	.0077586333
	Boender	.0001315251	.0039443541	.973	-.0076080134	.0078710635
	Buckley	.0000745396	.0039443541	.985	-.0076649989	.0078140780
	Chang	-.034934222*	.0039443541	.000	-.0426737604	-.0271946835
	Geometric Mean	.0000896042	.0039443541	.982	-.0076499342	.0078291427
	I.C.S	-.0003440926	.0039443541	.930	-.0080836310	.0073954459
	RowSum	-.0002699462	.0039443541	.945	-.0080094846	.0074695923
	Wang	-.062390948*	.0039443541	.000	-.0701304861	-.0546514092
RowSum	Arithmetic Mean	.0002890410	.0039443541	.942	-.0074504974	.0080285795
	Boender	.0004014712	.0039443541	.919	-.0073380672	.0081410097
	Buckley	.0003444858	.0039443541	.930	-.0073950527	.0080840242
	Chang	-.034664276*	.0039443541	.000	-.0424038142	-.0269247373
	Geometric Mean	.0003595504	.0039443541	.927	-.0073799880	.0080990889
	I.C.S	-.0000741464	.0039443541	.985	-.0078136849	.0076653920
	Laarhoven	.0002699462	.0039443541	.945	-.0074695923	.0080094846
	Wang	-.062121001*	.0039443541	.000	-.0698605399	-.0543814631
Wang	Arithmetic Mean	.0624100425*	.0039443541	.000	.0546705041	.0701495810
	Boender	.0625224727*	.0039443541	.000	.0547829343	.0702620112
	Buckley	.0624654873*	.0039443541	.000	.0547259488	.0702050257
	Chang	.0274567257*	.0039443541	.000	.0197171873	.0351962642
	Geometric Mean	.0624805519*	.0039443541	.000	.0547410135	.0702200904
	I.C.S	.0620468551*	.0039443541	.000	.0543073166	.0697863935
	Laarhoven	.0623909477*	.0039443541	.000	.0546514092	.0701304861
	RowSum	.0621210015*	.0039443541	.000	.0543814631	.0698605399

Based on observed means.
The error term is Mean Square(Error) = .001.

*. The mean difference is significant at the 0.05 level.

Table A.15: Performance analysis among models when $\beta = 100\%$

Dependent Variable: AverageError
LSD

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Arithmetic Mean	Boender	-.0023086249	.0026371950	.382	-.0074832799	.0028660302
	Buckley	-.0012392039	.0026371950	.639	-.0064138590	.0039354511
	Chang	-.007924493*	.0026371950	.003	-.0130991480	-.0027498380
	Geometric Mean	.0003134963	.0026371950	.905	-.0048611588	.0054881513
	I.C.S	-.0023677077	.0026371950	.369	-.0075423627	.0028069474
	Laarhoven	-.0030832031	.0026371950	.243	-.0082578581	.0020914519
	RowSum	-.0003899599	.0026371950	.882	-.0055646150	.0047846951
	Wang	-.015421180*	.0026371950	.000	-.0205958354	-.0102465253
Boender	Arithmetic Mean	.0023086249	.0026371950	.382	-.0028660302	.0074832799
	Buckley	.0010694210	.0026371950	.685	-.0041052341	.0062440760
	Chang	-.005615868*	.0026371950	.033	-.0107905232	-.0004412131
	Geometric Mean	.0026221212	.0026371950	.320	-.0025525339	.0077967762
	I.C.S	-.0000590828	.0026371950	.982	-.0052337378	.0051155722
	Laarhoven	-.0007745782	.0026371950	.769	-.0059492333	.0044000768
	RowSum	.0019186650	.0026371950	.467	-.0032559901	.0070933200
	Wang	-.013112555*	.0026371950	.000	-.0182872105	-.0079379004
Buckley	Arithmetic Mean	.0012392039	.0026371950	.639	-.0039354511	.0064138590
	Boender	-.0010694210	.0026371950	.685	-.0062440760	.0041052341
	Chang	-.006685289*	.0026371950	.011	-.0118599441	-.0015106340
	Geometric Mean	.0015527002	.0026371950	.556	-.0036219548	.0067273552
	I.C.S	-.0011285037	.0026371950	.669	-.0063031588	.0040461513
	Laarhoven	-.0018439992	.0026371950	.485	-.0070186542	.0033306559
	RowSum	.0008492440	.0026371950	.747	-.0043254110	.0060238990
	Wang	-.014181976*	.0026371950	.000	-.0193566315	-.0090073214
Chang	Arithmetic Mean	.0079244930*	.0026371950	.003	.0027498380	.0130991480
	Boender	.0056158681*	.0026371950	.033	.0004412131	.0107905232
	Buckley	.0066852891*	.0026371950	.011	.0015106340	.0118599441
	Geometric Mean	.0082379893*	.0026371950	.002	.0030633342	.0134126443
	I.C.S	.0055567853*	.0026371950	.035	.0003821303	.0107314404
	Laarhoven	.0048412899	.0026371950	.067	-.0003333651	.0100159449
	RowSum	.0075345331*	.0026371950	.004	.0023598780	.0127091881
	Wang	-.007496687*	.0026371950	.005	-.0126713424	-.0023220323
Geometric Mean	Arithmetic Mean	-.0003134963	.0026371950	.905	-.0054881513	.0048611588
	Boender	-.0026221212	.0026371950	.320	-.0077967762	.0025525339
	Buckley	-.0015527002	.0026371950	.556	-.0067273552	.0036219548
	Chang	-.008237989*	.0026371950	.002	-.0134126443	-.0030633342
	I.C.S	-.0026812039	.0026371950	.310	-.0078558590	.0024934511
	Laarhoven	-.0033966994	.0026371950	.198	-.0085713544	.0017779557
	RowSum	-.0007034562	.0026371950	.790	-.0058781112	.0044711988
	Wang	-.015734677*	.0026371950	.000	-.0209093317	-.0105600216
I.C.S	Arithmetic Mean	.0023677077	.0026371950	.369	-.0028069474	.0075423627
	Boender	.0000590828	.0026371950	.982	-.0051155722	.0052337378
	Buckley	.0011285037	.0026371950	.669	-.0040461513	.0063031588
	Chang	-.005556785*	.0026371950	.035	-.0107314404	-.0003821303
	Geometric Mean	.0026812039	.0026371950	.310	-.0024934511	.0078558590
	Laarhoven	-.0007154954	.0026371950	.786	-.0058901505	.0044591596
	RowSum	.0019777477	.0026371950	.453	-.0031969073	.0071524028
	Wang	-.013053473*	.0026371950	.000	-.0182281277	-.0078788176
Laarhoven	Arithmetic Mean	.0030832031	.0026371950	.243	-.0020914519	.0082578581
	Boender	.0007745782	.0026371950	.769	-.0044000768	.0059492333
	Buckley	.0018439992	.0026371950	.485	-.0033306559	.0070186542
	Chang	-.0048412899	.0026371950	.067	-.0100159449	.0003333651
	Geometric Mean	.0033966994	.0026371950	.198	-.0017779557	.0085713544
	I.C.S	.0007154954	.0026371950	.786	-.0044591596	.0058901505
	RowSum	.0026932432	.0026371950	.307	-.0024814119	.0078678982
	Wang	-.012337977*	.0026371950	.000	-.0175126323	-.0071633222
RowSum	Arithmetic Mean	.0003899599	.0026371950	.882	-.0047846951	.0055646150
	Boender	-.0019186650	.0026371950	.467	-.0070933200	.0032559901
	Buckley	-.0008492440	.0026371950	.747	-.0060238990	.0043254110
	Chang	-.007534533*	.0026371950	.004	-.0127091881	-.0023598780
	Geometric Mean	.0007034562	.0026371950	.790	-.0044711988	.0058781112
	I.C.S	-.0019777477	.0026371950	.453	-.0071524028	.0031969073
	Laarhoven	-.0026932432	.0026371950	.307	-.0078678982	.0024814119
	Wang	-.015031220*	.0026371950	.000	-.0202058755	-.0098565654
Wang	Arithmetic Mean	.0154211804*	.0026371950	.000	.0102465253	.0205958354
	Boender	.0131125555*	.0026371950	.000	.0079379004	.0182872105
	Buckley	.0141819764*	.0026371950	.000	.0090073214	.0193566315
	Chang	.0074966874*	.0026371950	.005	.0023220323	.0126713424
	Geometric Mean	.0157346766*	.0026371950	.000	.0105600216	.0209093317
	I.C.S	.0130534727*	.0026371950	.000	.0078788176	.0182281277
	Laarhoven	.0123379773*	.0026371950	.000	.0071633222	.0175126323
	RowSum	.0150312204*	.0026371950	.000	.0098565654	.0202058755

Based on observed means.
The error term is Mean Square(Error) = .000.
*. The mean difference is significant at the 0.05 level.

Table A.16: Performance analysis among models when $\beta = 150\%$

Dependent Variable: AverageError
LSD

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Arithmetic Mean	Boender	-.0045201463	.0024227326	.062	-.0092739872	.0002336946
	Buckley	-.010676256*	.0024227326	.000	-.0154300973	-.0059224155
	Chang	-.011699500*	.0024227326	.000	-.0164533405	-.0069456588
	Geometric Mean	.0011949850	.0024227326	.622	-.0035588559	.0059488259
	I.C.S	-.009548736*	.0024227326	.000	-.0143025770	-.0047948952
	Laarhoven	-.007168675*	.0024227326	.003	-.0119225163	-.0024148346
	RowSum	.0007216886	.0024227326	.766	-.0040321522	.0054755295
	Wang	-.006870193*	.0024227326	.005	-.0116240341	-.0021163523
Boender	Arithmetic Mean	.0045201463	.0024227326	.062	-.0002336946	.0092739872
	Buckley	-.006156110*	.0024227326	.011	-.0109099510	-.0014022692
	Chang	-.007179353*	.0024227326	.003	-.0119331942	-.0024255125
	Geometric Mean	.0057151313*	.0024227326	.019	.0009612904	.0104689722
	I.C.S	-.005028590*	.0024227326	.038	-.0097824307	-.0002747489
	Laarhoven	-.0026485291	.0024227326	.275	-.0074023700	.0021053117
	RowSum	.0052418350*	.0024227326	.031	.0004879941	.0099956758
	Wang	-.0023500469	.0024227326	.332	-.0071038878	.0024037940
Buckley	Arithmetic Mean	.0106762564*	.0024227326	.000	.0059224155	.0154300973
	Boender	.0061561101*	.0024227326	.011	.0014022692	.0109099510
	Chang	-.0010232432	.0024227326	.673	-.0057770841	.0037305977
	Geometric Mean	.0118712415*	.0024227326	.000	.0071174006	.0166250823
	I.C.S	.0011275203	.0024227326	.642	-.0036263206	.0058813612
	Laarhoven	.0035075810	.0024227326	.148	-.0012462599	.0082614219
	RowSum	.0113979451*	.0024227326	.000	.0066441042	.0161517860
	Wang	.0038060633	.0024227326	.116	-.0009477776	.0085599041
Chang	Arithmetic Mean	.0116994996*	.0024227326	.000	.0069456588	.0164533405
	Boender	.0071793533*	.0024227326	.003	.0024255125	.0119331942
	Buckley	.0010232432	.0024227326	.673	-.0037305977	.0057770841
	Geometric Mean	.0128944847*	.0024227326	.000	.0081406438	.0176483256
	I.C.S	.0021507635	.0024227326	.375	-.0026030774	.0069046044
	Laarhoven	.0045308242	.0024227326	.062	-.0002230167	.0092846651
	RowSum	.0124211883*	.0024227326	.000	.0076673474	.0171750292
	Wang	.0048293065*	.0024227326	.046	.0000754656	.0095831474
Geometric Mean	Arithmetic Mean	-.0011949850	.0024227326	.622	-.0059488259	.0035588559
	Boender	-.005715131*	.0024227326	.019	-.0104689722	-.0009612904
	Buckley	-.011871241*	.0024227326	.000	-.0166250823	-.0071174006
	Chang	-.012894485*	.0024227326	.000	-.0176483256	-.0081406438
	I.C.S	-.010743721*	.0024227326	.000	-.0154975620	-.0059898803
	Laarhoven	-.008363660*	.0024227326	.001	-.0131175014	-.0036098196
	RowSum	-.0004732964	.0024227326	.845	-.0052271373	.0042805445
	Wang	-.008065178*	.0024227326	.001	-.0128190191	-.0033113373
I.C.S	Arithmetic Mean	.0095487361*	.0024227326	.000	.0047948952	.0143025770
	Boender	.0050285898*	.0024227326	.038	.0002747489	.0097824307
	Buckley	-.0011275203	.0024227326	.642	-.0058813612	.0036263206
	Chang	-.0021507635	.0024227326	.375	-.0069046044	.0026030774
	Geometric Mean	.0107437211*	.0024227326	.000	.0059898803	.0154975620
	Laarhoven	.0023800607	.0024227326	.326	-.0023737802	.0071339016
	RowSum	.0102704248*	.0024227326	.000	.0055165839	.0150242657
	Wang	.0026785429	.0024227326	.269	-.0020752979	.0074323838
Laarhoven	Arithmetic Mean	.0071686755*	.0024227326	.003	.0024148346	.0119225163
	Boender	.0026485291	.0024227326	.275	-.0021053117	.0074023700
	Buckley	-.0035075810	.0024227326	.148	-.0082614219	.0012462599
	Chang	-.0045308242	.0024227326	.062	-.0092846651	.0002230167
	Geometric Mean	.0083636605*	.0024227326	.001	.0036098196	.0131175014
	I.C.S	-.0023800607	.0024227326	.326	-.0071339016	.0023737802
	RowSum	.0078903641*	.0024227326	.001	.0031365232	.0126442050
	Wang	.0002984823	.0024227326	.902	-.0044553586	.0050523232
RowSum	Arithmetic Mean	-.0007216886	.0024227326	.766	-.0054755295	.0040321522
	Boender	-.005241835*	.0024227326	.031	-.0099956758	-.0004879941
	Buckley	-.011397945*	.0024227326	.000	-.0161517860	-.0066441042
	Chang	-.012421188*	.0024227326	.000	-.0171750292	-.0076673474
	Geometric Mean	.0004732964	.0024227326	.845	-.0042805445	.0052271373
	I.C.S	-.010270425*	.0024227326	.000	-.0150242657	-.0055165839
	Laarhoven	-.007890364*	.0024227326	.001	-.0126442050	-.0031365232
	Wang	-.007591882*	.0024227326	.002	-.0123457227	-.0028380409
Wang	Arithmetic Mean	.0068701932*	.0024227326	.005	.0021163523	.0116240341
	Boender	.0023500469	.0024227326	.332	-.0024037940	.0071038878
	Buckley	-.0038060633	.0024227326	.116	-.0085599041	.0009477776
	Chang	-.004829306*	.0024227326	.046	-.0095831474	-.0000754656
	Geometric Mean	.0080651782*	.0024227326	.001	.0033113373	.0128190191
	I.C.S	-.0026785429	.0024227326	.269	-.0074323838	.0020752979
	Laarhoven	-.0002984823	.0024227326	.902	-.0050523232	.0044553586
	RowSum	.0075918818*	.0024227326	.002	.0028380409	.0123457227

Based on observed means.
The error term is Mean Square(Error) = .000.

*. The mean difference is significant at the 0.05 level.

Table A.17: Performance analysis among models when $\beta = 200\%$

Dependent Variable: AverageError
LSD

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Arithmetic Mean	Boender	-.0014141487	.0026169804	.589	-.0065491391	.0037208416
	Buckley	-.017903213*	.0026169804	.000	-.0230382034	-.0127682227
	Chang	-.014406846*	.0026169804	.000	-.0195418365	-.0092718558
	Geometric Mean	.0009495452	.0026169804	.717	-.0041854452	.0060845355
	I.C.S	-.007895975*	.0026169804	.003	-.0130309657	-.0027609849
	Laarhoven	-.006184548*	.0026169804	.018	-.0113195386	-.0010495579
	RowSum	-.0000146400	.0026169804	.996	-.0051496304	.0051203503
	Wang	-.007541826*	.0026169804	.004	-.0126768163	-.0024068356
Boender	Arithmetic Mean	.0014141487	.0026169804	.589	-.0037208416	.0065491391
	Buckley	-.016489064*	.0026169804	.000	-.0216240546	-.0113540739
	Chang	-.012992697*	.0026169804	.000	-.0181276878	-.0078577071
	Geometric Mean	.0023636939	.0026169804	.367	-.0027712965	.0074986842
	I.C.S	-.006481827*	.0026169804	.013	-.0116168169	-.0013468362
	Laarhoven	-.0047703995	.0026169804	.069	-.0099053899	.0003645908
	RowSum	.0013995087	.0026169804	.593	-.0037354816	.0065344991
	Wang	-.006127677*	.0026169804	.019	-.0112626676	-.0009926869
Buckley	Arithmetic Mean	.0179032130*	.0026169804	.000	.0127682227	.0230382034
	Boender	.0164890643*	.0026169804	.000	.0113540739	.0216240546
	Chang	.0034963668	.0026169804	.182	-.0016386235	.0086313572
	Geometric Mean	.0188527582*	.0026169804	.000	.0137177678	.0239877485
	I.C.S	.0100072377*	.0026169804	.000	.0048722474	.0151422281
	Laarhoven	.0117186648*	.0026169804	.000	.0065836744	.0168536551
	RowSum	.0178885730*	.0026169804	.000	.0127535826	.0230235633
	Wang	.0103613870*	.0026169804	.000	.0052263967	.0154963774
Chang	Arithmetic Mean	.0144068462*	.0026169804	.000	.0092718558	.0195418365
	Boender	.0129926975*	.0026169804	.000	.0078577071	.0181276878
	Buckley	-.0034963668	.0026169804	.182	-.0086313572	.0016386235
	Geometric Mean	.0153563913*	.0026169804	.000	.0102214010	.0204913817
	I.C.S	.0065108709*	.0026169804	.013	.0013758805	.0116458612
	Laarhoven	.0082222979*	.0026169804	.002	.0030873076	.0133572883
	RowSum	.0143922062*	.0026169804	.000	.0092572158	.0195271965
	Wang	.0068650202*	.0026169804	.009	.0017300298	.0120000106
Geometric Mean	Arithmetic Mean	-.0009495452	.0026169804	.717	-.0060845355	.0041854452
	Boender	-.0023636939	.0026169804	.367	-.0074986842	.0027712965
	Buckley	-.018852758*	.0026169804	.000	-.0239877485	-.0137177678
	Chang	-.015356391*	.0026169804	.000	-.0204913817	-.0102214010
	I.C.S	-.008845520*	.0026169804	.001	-.0139805108	-.0037105301
	Laarhoven	-.007134093*	.0026169804	.007	-.0122690838	-.0019991030
	RowSum	-.0009641852	.0026169804	.713	-.0060991755	.0041708052
	Wang	-.008491371*	.0026169804	.001	-.0136263615	-.0033563808
I.C.S	Arithmetic Mean	.0078959753*	.0026169804	.003	.0027609849	.0130309657
	Boender	.0064818266*	.0026169804	.013	.0013468362	.0116168169
	Buckley	-.010007238*	.0026169804	.000	-.0151422281	-.0048722474
	Chang	-.006510871*	.0026169804	.013	-.0116458612	-.0013758805
	Geometric Mean	.0088455205*	.0026169804	.001	.0037105301	.0139805108
	Laarhoven	.0017114271	.0026169804	.513	-.0034235633	.0068464174
	RowSum	.0078813353*	.0026169804	.003	.0027463449	.0130163256
	Wang	.0003541493	.0026169804	.892	-.0047808410	.0054891397
Laarhoven	Arithmetic Mean	.0061845482*	.0026169804	.018	.0010495579	.0113195386
	Boender	.0047703995	.0026169804	.069	-.0003645908	.0099053899
	Buckley	-.011718665*	.0026169804	.000	-.0168536551	-.0065836744
	Chang	-.008222298*	.0026169804	.002	-.0133572883	-.0030873076
	Geometric Mean	.0071340934*	.0026169804	.007	.0019991030	.0122690838
	I.C.S	-.0017114271	.0026169804	.513	-.0068464174	.0034235633
	RowSum	.0061699082*	.0026169804	.019	.0010349179	.0113048986
	Wang	-.0013572777	.0026169804	.604	-.0064922681	.0037777126
RowSum	Arithmetic Mean	.0000146400	.0026169804	.996	-.0051203503	.0051496304
	Boender	-.0013995087	.0026169804	.593	-.0065344991	.0037354816
	Buckley	-.017888573*	.0026169804	.000	-.0230235633	-.0127535826
	Chang	-.014392206*	.0026169804	.000	-.0195271965	-.0092572158
	Geometric Mean	.0009641852	.0026169804	.713	-.0041708052	.0060991755
	I.C.S	-.007881335*	.0026169804	.003	-.0130163256	-.0027463449
	Laarhoven	-.006169908*	.0026169804	.019	-.0113048986	-.0010349179
	Wang	-.007527186*	.0026169804	.004	-.0126621763	-.0023921956
Wang	Arithmetic Mean	.0075418260*	.0026169804	.004	.0024068356	.0126768163
	Boender	.0061276773*	.0026169804	.019	.0009926869	.0112626676
	Buckley	-.010361387*	.0026169804	.000	-.0154963774	-.0052263967
	Chang	-.006865020*	.0026169804	.009	-.0120000106	-.0017300298
	Geometric Mean	.0084913711*	.0026169804	.001	.0033563808	.0136263615
	I.C.S	-.0003541493	.0026169804	.892	-.0054891397	.0047808410
	Laarhoven	.0013572777	.0026169804	.604	-.0037777126	.0064922681
	RowSum	.0075271860*	.0026169804	.004	.0023921956	.0126621763

Based on observed means.
The error term is Mean Square(Error) = .000.

*. The mean difference is significant at the 0.05 level.

Appendix B

Anova Results - Mean Absolute Maximum Error

Table B.1: Between group analysis

Dependent Variable: MaxError

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	44.696[a]	539	.083	43.665	.000
Intercept	10.037	1	10.037	5285.358	.000
Model	10.997	8	1.375	723.854	.000
n	1.304	3	.435	228.907	.000
Alpha	.037	2	.018	9.716	.000
Beta	3.838	4	.959	505.238	.000
Model * n	.814	24	.034	17.868	.000
Model * Alpha	.286	16	.018	9.403	.000
Model * Beta	23.766	32	.743	391.070	.000
n * Alpha	.026	6	.004	2.239	.037
n * Beta	.333	12	.028	14.596	.000
Alpha * Beta	.221	8	.028	14.579	.000
Model * n * Alpha	.030	48	.001	.324	1.000
Model * n * Beta	2.014	96	.021	11.045	.000
Model * Alpha * Beta	.656	64	.010	5.400	.000
n * Alpha * Beta	.074	24	.003	1.618	.029
Model * n * Alpha * Beta	.301	192	.002	.825	.961
Error	9.230	4860	.002		
Total	63.963	5400			
Corrected Total	53.926	5399			

a. R Squared = .829 (Adjusted R Squared = .810)

Table B.2: Analysis as the size of the matrix increases

Dependent Variable: MaxError
LSD

(I) n	(J) n	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
3	7	.0135825785*	.0016773464	.000	.0102942211	.0168709360
	11	.0302310262*	.0016773464	.000	.0269426688	.0335193837
	15	.0406178398*	.0016773464	.000	.0373294824	.0439061973
7	3	-.013582579*	.0016773464	.000	-.0168709360	-.0102942211
	11	.0166484477*	.0016773464	.000	.0133600903	.0199368052
	15	.0270352613*	.0016773464	.000	.0237469039	.0303236188
11	3	-.030231026*	.0016773464	.000	-.0335193837	-.0269426688
	7	-.016648448*	.0016773464	.000	-.0199368052	-.0133600903
	15	.0103868136*	.0016773464	.000	.0070984562	.0136751711
15	3	-.040617840*	.0016773464	.000	-.0439061973	-.0373294824
	7	-.027035261*	.0016773464	.000	-.0303236188	-.0237469039
	11	-.010386814*	.0016773464	.000	-.0136751711	-.0070984562

Based on observed means.
The error term is Mean Square(Error) = .002.

*. The mean difference is significant at the 0.05 level.

Table B.3: Analysis as the level of fuzziness increases

Dependent Variable: MaxError
LSD

(I) Alpha	(J) Alpha	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	Upper Bound
.05	.10	.0044717882*	.0014526246	.002	.0016239871	.0073195893
	.15	.0062052892*	.0014526246	.000	.0033574881	.0090530902
.10	.05	-.004471788*	.0014526246	.002	-.0073195893	-.0016239871
	.15	.0017335010	.0014526246	.233	-.0011143001	.0045813020
.15	.05	-.006205289*	.0014526246	.000	-.0090530902	-.0033574881
	.10	-.0017335010	.0014526246	.233	-.0045813020	.0011143001

Based on observed means.
The error term is Mean Square(Error) = .002.
*. The mean difference is significant at the 0.05 level.

Table B.4: Analysis as the inconsistency increases

Dependent Variable: MaxError
LSD

(I) Beta	(J) Beta	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	Upper Bound
.00	.50	.0696225248*	.0018753303	.000	.0659460294	.0732990202
	1.00	.0733569343*	.0018753303	.000	.0696804389	.0770334297
	1.50	.0610540303*	.0018753303	.000	.0573775349	.0647305257
	2.00	.0535057267*	.0018753303	.000	.0498292313	.0571822221
.50	.00	-.069622525*	.0018753303	.000	-.0732990202	-.0659460294
	1.00	.0037344095*	.0018753303	.047	.0000579141	.0074109049
	1.50	-.008568494*	.0018753303	.000	-.0122449898	-.0048919990
	2.00	-.016116798*	.0018753303	.000	-.0197932934	-.0124403026
1.00	.00	-.073356934*	.0018753303	.000	-.0770334297	-.0696804389
	.50	-.003734410*	.0018753303	.047	-.0074109049	-.0000579141
	1.50	-.012302904*	.0018753303	.000	-.0159793993	-.0086264085
	2.00	-.019851208*	.0018753303	.000	-.0235277029	-.0161747121
1.50	.00	-.061054030*	.0018753303	.000	-.0647305257	-.0573775349
	.50	.0085684944*	.0018753303	.000	.0048919990	.0122449898
	1.00	.0123029039*	.0018753303	.000	.0086264085	.0159793993
	2.00	-.007548304*	.0018753303	.000	-.0112247990	-.0038718082
2.00	.00	-.053505727*	.0018753303	.000	-.0571822221	-.0498292313
	.50	.0161167980*	.0018753303	.000	.0124403026	.0197932934
	1.00	.0198512075*	.0018753303	.000	.0161747121	.0235277029
	1.50	.0075483036*	.0018753303	.000	.0038718082	.0112247990

Based on observed means.
The error term is Mean Square(Error) = .002.
*. The mean difference is significant at the 0.05 level.

Table B.5: Analysis within FAHP models

Dependent Variable: MaxError
LSD

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Arithmetic Mean	Boender	-.0027747868	.0025160196	.270	-.0077073230	.0021577494
	Buckley	-.010542372*	.0025160196	.000	-.0154749079	-.0056098355
	Chang	-.106086673*	.0025160196	.000	-.1110192093	-.1011541369
	Geometric Mean	.0009392279	.0025160196	.709	-.0039933083	.0058717641
	I.C.S	-.010412624*	.0025160196	.000	-.0153451600	-.0054800877
	Laarhoven	-.004978672*	.0025160196	.048	-.0099112080	-.0000461357
	RowSum	-.0001495849	.0025160196	.953	-.0050821211	.0047829513
	Wang	-.117744040*	.0025160196	.000	-.1226765757	-.1128115033
Boender	Arithmetic Mean	.0027747868	.0025160196	.270	-.0021577494	.0077073230
	Buckley	-.007767585*	.0025160196	.002	-.0127001211	-.0028350487
	Chang	-.103311886*	.0025160196	.000	-.1082444224	-.0983793501
	Geometric Mean	.0037140147	.0025160196	.140	-.0012185215	.0086465509
	I.C.S	-.007637837*	.0025160196	.002	-.0125703732	-.0027053009
	Laarhoven	-.0022038850	.0025160196	.381	-.0071364212	.0027286512
	RowSum	.0026252019	.0025160196	.297	-.0023073343	.0075577381
	Wang	-.114969253*	.0025160196	.000	-.1199017889	-.1100367165
Buckley	Arithmetic Mean	.0105423717*	.0025160196	.000	.0056098355	.0154749079
	Boender	.0077675849*	.0025160196	.002	.0028350487	.0127001211
	Chang	-.095544301*	.0025160196	.000	-.1004768375	-.0906117652
	Geometric Mean	.0114815996*	.0025160196	.000	.0065490634	.0164141358
	I.C.S	.0001297479	.0025160196	.959	-.0048027883	.0050622840
	Laarhoven	.0055636999*	.0025160196	.027	.0006311637	.0104962361
	RowSum	.0103927868*	.0025160196	.000	.0054602506	.0153253230
	Wang	-.107201668*	.0025160196	.000	-.1121342040	-.1022691316
Chang	Arithmetic Mean	.1060866731*	.0025160196	.000	.1011541369	.1110192093
	Boender	.1033118863*	.0025160196	.000	.0983793501	.1082444224
	Buckley	.0955443013*	.0025160196	.000	.0906117652	.1004768375
	Geometric Mean	.1070259009*	.0025160196	.000	.1020933648	.1119584371
	I.C.S	.0956740492*	.0025160196	.000	.0907415130	.1006065854
	Laarhoven	.1011080012*	.0025160196	.000	.0961754651	.1060405374
	RowSum	.1059370882*	.0025160196	.000	.1010045520	.1108696243
	Wang	-.011657366*	.0025160196	.000	-.0165899026	-.0067248303
Geometric Mean	Arithmetic Mean	-.0009392279	.0025160196	.709	-.0058717641	.0039933083
	Boender	-.0037140147	.0025160196	.140	-.0086465509	.0012185215
	Buckley	-.011481600*	.0025160196	.000	-.0164141358	-.0065490634
	Chang	-.107025901*	.0025160196	.000	-.1119584371	-.1020933648
	I.C.S	-.011351852*	.0025160196	.000	-.0162843879	-.0064193156
	Laarhoven	-.005917900*	.0025160196	.019	-.0108504359	-.0009853635
	RowSum	-.0010888128	.0025160196	.665	-.0060213490	.0038437234
	Wang	-.118683267*	.0025160196	.000	-.1236158036	-.1137507312
I.C.S	Arithmetic Mean	.0104126239*	.0025160196	.000	.0054800877	.0153451600
	Boender	.0076378371*	.0025160196	.002	.0027053009	.0125703732
	Buckley	-.0001297479	.0025160196	.959	-.0050622840	.0048027883
	Chang	-.095674049*	.0025160196	.000	-.1006065854	-.0907415130
	Geometric Mean	.0113518517*	.0025160196	.000	.0064193156	.0162843879
	Laarhoven	.0054339520*	.0025160196	.031	.0005014158	.0103664882
	RowSum	.0102630390*	.0025160196	.000	.0053305028	.0151955751
	Wang	-.107331416*	.0025160196	.000	-.1122639518	-.1023988795
Laarhoven	Arithmetic Mean	.0049786718*	.0025160196	.048	.0000461357	.0099112080
	Boender	.0022038850	.0025160196	.381	-.0027286512	.0071364212
	Buckley	-.005563700*	.0025160196	.027	-.0104962361	-.0006311637
	Chang	-.101108001*	.0025160196	.000	-.1060405374	-.0961754651
	Geometric Mean	.0059178997*	.0025160196	.019	.0009853635	.0108504359
	I.C.S	-.005433952*	.0025160196	.031	-.0103664882	-.0005014158
	RowSum	.0048290869	.0025160196	.055	-.0001034493	.0097616231
	Wang	-.112765368*	.0025160196	.000	-.1176979039	-.1078328315
RowSum	Arithmetic Mean	.0001495849	.0025160196	.953	-.0047829513	.0050821211
	Boender	-.0026252019	.0025160196	.297	-.0075577381	.0023073343
	Buckley	-.010392787*	.0025160196	.000	-.0153253230	-.0054602506
	Chang	-.105937088*	.0025160196	.000	-.1108696243	-.1010045520
	Geometric Mean	.0010888128	.0025160196	.665	-.0038437234	.0060213490
	I.C.S	-.010263039*	.0025160196	.000	-.0151955751	-.0053305028
	Laarhoven	-.0048290869	.0025160196	.055	-.0097616231	.0001034493
	Wang	-.117594455*	.0025160196	.000	-.1225269908	-.1126619184
Wang	Arithmetic Mean	.1177440395*	.0025160196	.000	.1128115033	.1226765757
	Boender	.1149692527*	.0025160196	.000	.1100367165	.1199017889
	Buckley	.1072016678*	.0025160196	.000	.1022691316	.1121342040
	Chang	.0116573664*	.0025160196	.000	.0067248303	.0165899026
	Geometric Mean	.1186832674*	.0025160196	.000	.1137507312	.1236158036
	I.C.S	.1073314157*	.0025160196	.000	.1023988795	.1122639518
	Laarhoven	.1127653677*	.0025160196	.000	.1078328315	.1176979039
	RowSum	.1175944546*	.0025160196	.000	.1126619184	.1225269908

Based on observed means.
The error term is Mean Square(Error) = .002.

*. The mean difference is significant at the 0.05 level.

Table B.6: Performance analysis among models when n = 3

Dependent Variable: MaxError
LSD

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Arithmetic Mean	Boender	-.0110887618	.0106289422	.297	-.0319399253	.0097624017
	Buckley	-.0138009531	.0106289422	.194	-.0346521166	.0070502103
	Chang	-.116505750*	.0106289422	.000	-.1373569138	-.0956545869
	Geometric Mean	.0015527136	.0106289422	.884	-.0192984499	.0224038771
	I.C.S	-.0064645029	.0106289422	.543	-.0273156664	.0143866606
	Laarhoven	-.0204316289	.0106289422	.055	-.0412827924	.0004195346
	RowSum	.0032454093	.0106289422	.760	-.0176057541	.0240965728
	Wang	-.148503301*	.0106289422	.000	-.1693544642	-.1276521373
Boender	Arithmetic Mean	.0110887618	.0106289422	.297	-.0097624017	.0319399253
	Buckley	-.0027121913	.0106289422	.799	-.0235633548	.0181389721
	Chang	-.105416989*	.0106289422	.000	-.1262681520	-.0845658251
	Geometric Mean	.0126414754	.0106289422	.235	-.0082096881	.0334926389
	I.C.S	.0046242589	.0106289422	.664	-.0162269046	.0254754224
	Laarhoven	-.0093428671	.0106289422	.380	-.0301940306	.0115082964
	RowSum	.0143341711	.0106289422	.178	-.0065169923	.0351853346
	Wang	-.137414539*	.0106289422	.000	-.1582657024	-.1165633755
Buckley	Arithmetic Mean	.0138009531	.0106289422	.194	-.0070502103	.0346521166
	Boender	.0027121913	.0106289422	.799	-.0181389721	.0235633548
	Chang	-.102704797*	.0106289422	.000	-.1235559607	-.0818536338
	Geometric Mean	.0153536667	.0106289422	.149	-.0054974967	.0362048302
	I.C.S	.0073364502	.0106289422	.490	-.0135147132	.0281876137
	Laarhoven	-.0066306758	.0106289422	.533	-.0274818392	.0142204877
	RowSum	.0170463625	.0106289422	.109	-.0038048010	.0378975259
	Wang	-.134702348*	.0106289422	.000	-.1555535111	-.1138511842
Chang	Arithmetic Mean	.1165057504*	.0106289422	.000	.0956545869	.1373569138
	Boender	.1054169886*	.0106289422	.000	.0845658251	.1262681520
	Buckley	.1027047972*	.0106289422	.000	.0818536338	.1235559607
	Geometric Mean	.1180584640*	.0106289422	.000	.0972073005	.1389096274
	I.C.S	.1100412475*	.0106289422	.000	.0891900840	.1308924109
	Laarhoven	.0960741215*	.0106289422	.000	.0752229580	.1169252849
	RowSum	.1197511597*	.0106289422	.000	.0988999962	.1406023232
	Wang	-.031997550*	.0106289422	.003	-.0528487139	-.0111463869
Geometric Mean	Arithmetic Mean	-.0015527136	.0106289422	.884	-.0224038771	.0192984499
	Boender	-.0126414754	.0106289422	.235	-.0334926389	.0082096881
	Buckley	-.0153536667	.0106289422	.149	-.0362048302	.0054974967
	Chang	-.118058464*	.0106289422	.000	-.1389096274	-.0972073005
	I.C.S	-.0080172165	.0106289422	.451	-.0288683800	.0128339470
	Laarhoven	-.021984343*	.0106289422	.039	-.0428355060	-.0011331790
	RowSum	.0016926957	.0106289422	.873	-.0191584677	.0225438592
	Wang	-.150056014*	.0106289422	.000	-.1709071778	-.1292048509
I.C.S	Arithmetic Mean	.0064645029	.0106289422	.543	-.0143866606	.0273156664
	Boender	-.0046242589	.0106289422	.664	-.0254754224	.0162269046
	Buckley	-.0073364502	.0106289422	.490	-.0281876137	.0135147132
	Chang	-.110041247*	.0106289422	.000	-.1308924109	-.0891900840
	Geometric Mean	.0080172165	.0106289422	.451	-.0128339470	.0288683800
	Laarhoven	-.0139671260	.0106289422	.189	-.0348182895	.0068840375
	RowSum	.0097099123	.0106289422	.361	-.0111412512	.0305610757
	Wang	-.142038798*	.0106289422	.000	-.1628899613	-.1211876344
Laarhoven	Arithmetic Mean	.0204316289	.0106289422	.055	-.0004195346	.0412827924
	Boender	.0093428671	.0106289422	.380	-.0115082964	.0301940306
	Buckley	.0066306758	.0106289422	.533	-.0142204877	.0274818392
	Chang	-.096074121*	.0106289422	.000	-.1169252849	-.0752229580
	Geometric Mean	.0219843425*	.0106289422	.039	.0011331790	.0428355060
	I.C.S	.0139671260	.0106289422	.189	-.0068840375	.0348182895
	RowSum	.0236770382*	.0106289422	.026	.0028258748	.0445282017
	Wang	-.128071672*	.0106289422	.000	-.1489228353	-.1072205084
RowSum	Arithmetic Mean	-.0032454093	.0106289422	.760	-.0240965728	.0176057541
	Boender	-.0143341711	.0106289422	.178	-.0351853346	.0065169923
	Buckley	-.0170463625	.0106289422	.109	-.0378975259	.0038048010
	Chang	-.119751160*	.0106289422	.000	-.1406023232	-.0988999962
	Geometric Mean	-.0016926957	.0106289422	.873	-.0225438592	.0191584677
	I.C.S	-.0097099123	.0106289422	.361	-.0305610757	.0111412512
	Laarhoven	-.023677038*	.0106289422	.026	-.0445282017	-.0028258748
	Wang	-.151748710*	.0106289422	.000	-.1725998736	-.1308975467
Wang	Arithmetic Mean	.1485033008*	.0106289422	.000	.1276521373	.1693544642
	Boender	.1374145390*	.0106289422	.000	.1165633755	.1582657024
	Buckley	.1347023477*	.0106289422	.000	.1138511842	.1555535111
	Chang	.0319975504*	.0106289422	.003	.0111463869	.0528487139
	Geometric Mean	.1500560144*	.0106289422	.000	.1292048509	.1709071778
	I.C.S	.1420387979*	.0106289422	.000	.1211876344	.1628899613
	Laarhoven	.1280716719*	.0106289422	.000	.1072205084	.1489228353
	RowSum	.1517487101*	.0106289422	.000	.1308975467	.1725998736

Based on observed means.
The error term is Mean Square(Error) = .008.
*. The mean difference is significant at the 0.05 level.

Table B.7: Performance analysis among models when n = 7

Dependent Variable: MaxError
LSD

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Arithmetic Mean	Boender	.0011398850	.0123456377	.926	-.0230789795	.0253587494
	Buckley	-.0144479315	.0123456377	.242	-.0386667959	.0097709330
	Chang	-.139224035*	.0123456377	.000	-.1634428999	-.1150051710
	Geometric Mean	.0015762328	.0123456377	.898	-.0226426317	.0257950972
	I.C.S	-.0119505965	.0123456377	.333	-.0361694609	.0122682679
	Laarhoven	.0003533485	.0123456377	.977	-.0238655160	.0245722129
	RowSum	-.0010237248	.0123456377	.934	-.0252425893	.0231951396
	Wang	-.146293772*	.0123456377	.000	-.1705126361	-.1220749072
Boender	Arithmetic Mean	-.0011398850	.0123456377	.926	-.0253587494	.0230789795
	Buckley	-.0155878164	.0123456377	.207	-.0398066809	.0086310480
	Chang	-.140363920*	.0123456377	.000	-.1645827849	-.1161450560
	Geometric Mean	.0004363478	.0123456377	.972	-.0237825166	.0246552123
	I.C.S	-.0130904815	.0123456377	.289	-.0373093459	.0111283830
	Laarhoven	-.0007865365	.0123456377	.949	-.0250054009	.0234323280
	RowSum	-.0021636098	.0123456377	.861	-.0263824742	.0220552547
	Wang	-.147433657*	.0123456377	.000	-.1716525210	-.1232147921
Buckley	Arithmetic Mean	.0144479315	.0123456377	.242	-.0097709330	.0386667959
	Boender	.0155878164	.0123456377	.207	-.0086310480	.0398066809
	Chang	-.124776104*	.0123456377	.000	-.1489949684	-.1005572395
	Geometric Mean	.0160241642	.0123456377	.195	-.0081947002	.0402430287
	I.C.S	.0024973350	.0123456377	.840	-.0217215295	.0267161994
	Laarhoven	.0148012799	.0123456377	.231	-.0094175845	.0390201444
	RowSum	.0134242066	.0123456377	.277	-.0107946578	.0376430711
	Wang	-.131845840*	.0123456377	.000	-.1560647046	-.1076269757
Chang	Arithmetic Mean	.1392240355*	.0123456377	.000	.1150051710	.1634428999
	Boender	.1403639204*	.0123456377	.000	.1161450560	.1645827849
	Buckley	.1247761040*	.0123456377	.000	.1005572395	.1489949684
	Geometric Mean	.1408002682*	.0123456377	.000	.1165814038	.1650191327
	I.C.S	.1272734390*	.0123456377	.000	.1030545745	.1514923034
	Laarhoven	.1395773839*	.0123456377	.000	.1153585195	.1637962484
	RowSum	.1382003106*	.0123456377	.000	.1139814462	.1624191751
	Wang	-.0070697362	.0123456377	.567	-.0312886006	.0171491283
Geometric Mean	Arithmetic Mean	-.0015762328	.0123456377	.898	-.0257950972	.0226426317
	Boender	-.0004363478	.0123456377	.972	-.0246552123	.0237825166
	Buckley	-.0160241642	.0123456377	.195	-.0402430287	.0081947002
	Chang	-.140800268*	.0123456377	.000	-.1650191327	-.1165814038
	I.C.S	-.0135268293	.0123456377	.273	-.0377456937	.0106920352
	Laarhoven	-.0012228843	.0123456377	.921	-.0254417488	.0229959801
	RowSum	-.0025999576	.0123456377	.833	-.0268188221	.0216189068
	Wang	-.147870004*	.0123456377	.000	-.1720888689	-.1236511400
I.C.S	Arithmetic Mean	.0119505965	.0123456377	.333	-.0122682679	.0361694609
	Boender	.0130904815	.0123456377	.289	-.0111283830	.0373093459
	Buckley	-.0024973350	.0123456377	.840	-.0267161994	.0217215295
	Chang	-.127273439*	.0123456377	.000	-.1514923034	-.1030545745
	Geometric Mean	.0135268293	.0123456377	.273	-.0106920352	.0377456937
	Laarhoven	.0123039450	.0123456377	.319	-.0119149195	.0365228094
	RowSum	.0109268717	.0123456377	.376	-.0132919928	.0351457361
	Wang	-.134343175*	.0123456377	.000	-.1585620396	-.1101243107
Laarhoven	Arithmetic Mean	-.0003533485	.0123456377	.977	-.0245722129	.0238655160
	Boender	.0007865365	.0123456377	.949	-.0234323280	.0250054009
	Buckley	-.0148012799	.0123456377	.231	-.0390201444	.0094175845
	Chang	-.139577384*	.0123456377	.000	-.1637962484	-.1153585195
	Geometric Mean	.0012228843	.0123456377	.921	-.0229959801	.0254417488
	I.C.S	-.0123039450	.0123456377	.319	-.0365228094	.0119149195
	RowSum	-.0013770733	.0123456377	.911	-.0255959378	.0228417911
	Wang	-.146647120*	.0123456377	.000	-.1708659846	-.1224282557
RowSum	Arithmetic Mean	.0010237248	.0123456377	.934	-.0231951396	.0252425893
	Boender	.0021636098	.0123456377	.861	-.0220552547	.0263824742
	Buckley	-.0134242066	.0123456377	.277	-.0376430711	.0107946578
	Chang	-.138200311*	.0123456377	.000	-.1624191751	-.1139814462
	Geometric Mean	.0025999576	.0123456377	.833	-.0216189068	.0268188221
	I.C.S	-.0109268717	.0123456377	.376	-.0351457361	.0132919928
	Laarhoven	.0013770733	.0123456377	.911	-.0228417911	.0255959378
	Wang	-.145270047*	.0123456377	.000	-.1694889113	-.1210511824
Wang	Arithmetic Mean	.1462937716*	.0123456377	.000	.1220749072	.1705126361
	Boender	.1474336566*	.0123456377	.000	.1232147921	.1716525210
	Buckley	.1318458402*	.0123456377	.000	.1076269757	.1560647046
	Chang	.0070697362	.0123456377	.567	-.0171491283	.0312886006
	Geometric Mean	.1478700044*	.0123456377	.000	.1236511400	.1720888689
	I.C.S	.1343431751*	.0123456377	.000	.1101243107	.1585620396
	Laarhoven	.1466471201*	.0123456377	.000	.1224282557	.1708659846
	RowSum	.1452700468*	.0123456377	.000	.1210511824	.1694889113

Based on observed means.
The error term is Mean Square(Error) = .011.

*. The mean difference is significant at the 0.05 level.

Table B.8: Performance analysis among models when n = 11

Dependent Variable: MaxError
LSD

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Arithmetic Mean	Boender	-.0008744255	.0098785843	.929	-.0202535860	.0185047350
	Buckley	-.0092755239	.0098785843	.348	-.0286546844	.0101036366
	Chang	-.100074378*	.0098785843	.000	-.1194535382	-.0806952172
	Geometric Mean	.0002452354	.0098785843	.980	-.0191339251	.0196243959
	I.C.S	-.0123439581	.0098785843	.212	-.0317231186	.0070352024
	Laarhoven	-.0000970338	.0098785843	.992	-.0194761943	.0192821267
	RowSum	-.0020786804	.0098785843	.833	-.0214578409	.0173004801
	Wang	-.105580956*	.0098785843	.000	-.1249601160	-.0862017950
Boender	Arithmetic Mean	.0008744255	.0098785843	.929	-.0185047350	.0202535860
	Buckley	-.0084010984	.0098785843	.395	-.0277802589	.0109780621
	Chang	-.099199952*	.0098785843	.000	-.1185791128	-.0798207918
	Geometric Mean	.0011196609	.0098785843	.910	-.0182594997	.0204988214
	I.C.S	-.0114695327	.0098785843	.246	-.0308486932	.0079096278
	Laarhoven	.0007773917	.0098785843	.937	-.0186017688	.0201565522
	RowSum	-.0012042549	.0098785843	.903	-.0205834154	.0181749056
	Wang	-.104706530*	.0098785843	.000	-.1240856906	-.0853273695
Buckley	Arithmetic Mean	.0092755239	.0098785843	.348	-.0101036366	.0286546844
	Boender	.0084010984	.0098785843	.395	-.0109780621	.0277802589
	Chang	-.090798854*	.0098785843	.000	-.1101780144	-.0714196933
	Geometric Mean	.0095207593	.0098785843	.335	-.0098584012	.0288999198
	I.C.S	-.0030684343	.0098785843	.756	-.0224475948	.0163107263
	Laarhoven	.0091784901	.0098785843	.353	-.0102006704	.0285576506
	RowSum	.0071968435	.0098785843	.466	-.0121823170	.0265760040
	Wang	-.096305432*	.0098785843	.000	-.1156845921	-.0769262711
Chang	Arithmetic Mean	.1000743777*	.0098785843	.000	.0806952172	.1194535382
	Boender	.0991999523*	.0098785843	.000	.0798207918	.1185791128
	Buckley	.0907988538*	.0098785843	.000	.0714196933	.1101780144
	Geometric Mean	.1003196131*	.0098785843	.000	.0809404526	.1196987736
	I.C.S	.0877304196*	.0098785843	.000	.0683512591	.1071095801
	Laarhoven	.0999773439*	.0098785843	.000	.0805981834	.1193565044
	RowSum	.0979956974*	.0098785843	.000	.0786165369	.1173748579
	Wang	-.0055065778	.0098785843	.577	-.0248857383	.0138725827
Geometric Mean	Arithmetic Mean	-.0002452354	.0098785843	.980	-.0196243959	.0191339251
	Boender	-.0011196609	.0098785843	.910	-.0204988214	.0182594997
	Buckley	-.0095207593	.0098785843	.335	-.0288999198	.0098584012
	Chang	-.100319613*	.0098785843	.000	-.1196987736	-.0809404526
	I.C.S	-.0125891935	.0098785843	.203	-.0319683540	.0067899670
	Laarhoven	-.0003422692	.0098785843	.972	-.0197214297	.0190368913
	RowSum	-.0023239157	.0098785843	.814	-.0217030763	.0170552448
	Wang	-.105826191*	.0098785843	.000	-.1252053514	-.0864470304
I.C.S	Arithmetic Mean	.0123439581	.0098785843	.212	-.0070352024	.0317231186
	Boender	.0114695327	.0098785843	.246	-.0079096278	.0308486932
	Buckley	.0030684343	.0098785843	.756	-.0163107263	.0224475948
	Chang	-.087730420*	.0098785843	.000	-.1071095801	-.0683512591
	Geometric Mean	.0125891935	.0098785843	.203	-.0067899670	.0319683540
	Laarhoven	.0122469243	.0098785843	.215	-.0071322362	.0316260848
	RowSum	.0102652778	.0098785843	.299	-.0091138827	.0296444383
	Wang	-.093236997*	.0098785843	.000	-.1126161579	-.0738578369
Laarhoven	Arithmetic Mean	.0000970338	.0098785843	.992	-.0192821267	.0194761943
	Boender	-.0007773917	.0098785843	.937	-.0201565522	.0186017688
	Buckley	-.0091784901	.0098785843	.353	-.0285576506	.0102006704
	Chang	-.099977344*	.0098785843	.000	-.1193565044	-.0805981834
	Geometric Mean	.0003422692	.0098785843	.972	-.0190368913	.0197214297
	I.C.S	-.0122469243	.0098785843	.215	-.0316260848	.0071322362
	RowSum	-.0019816466	.0098785843	.841	-.0213608071	.0173975139
	Wang	-.105483922*	.0098785843	.000	-.1248630822	-.0861047612
RowSum	Arithmetic Mean	.0020786804	.0098785843	.833	-.0173004801	.0214578409
	Boender	.0012042549	.0098785843	.903	-.0181749056	.0205834154
	Buckley	-.0071968435	.0098785843	.466	-.0265760040	.0121823170
	Chang	-.097995697*	.0098785843	.000	-.1173748579	-.0786165369
	Geometric Mean	.0023239157	.0098785843	.814	-.0170552448	.0217030763
	I.C.S	-.0102652778	.0098785843	.299	-.0296444383	.0091138827
	Laarhoven	.0019816466	.0098785843	.841	-.0173975139	.0213608071
	Wang	-.103502275*	.0098785843	.000	-.1228814357	-.0841231146
Wang	Arithmetic Mean	.1055809555*	.0098785843	.000	.0862017950	.1249601160
	Boender	.1047065300*	.0098785843	.000	.0853273695	.1240856906
	Buckley	.0963054316*	.0098785843	.000	.0769262711	.1156845921
	Chang	.0055065778	.0098785843	.577	-.0138725827	.0248857383
	Geometric Mean	.1058261909*	.0098785843	.000	.0864470304	.1252053514
	I.C.S	.0932369974*	.0098785843	.000	.0738578369	.1126161579
	Laarhoven	.1054839217*	.0098785843	.000	.0861047612	.1248630822
	RowSum	.1035022752*	.0098785843	.000	.0841231146	.1228814357

Based on observed means.
The error term is Mean Square(Error) = .007.

*. The mean difference is significant at the 0.05 level.

Table B.9: Performance analysis among models when n = 15

Dependent Variable: MaxError
LSD

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Arithmetic Mean	Boender	-.0002758449	.0065414551	.966	-.0131084436	.0125567537
	Buckley	-.0046450784	.0065414551	.478	-.0174776771	.0081875202
	Chang	-.068542529*	.0065414551	.000	-.0813751274	-.0557099301
	Geometric Mean	.0003827297	.0065414551	.953	-.0124498689	.0132153284
	I.C.S	-.0108914379	.0065414551	.096	-.0237240366	.0019411607
	Laarhoven	.0002606269	.0065414551	.968	-.0125719718	.0130932255
	RowSum	-.0007413438	.0065414551	.910	-.0135739425	.0120912548
	Wang	-.070598130*	.0065414551	.000	-.0834307288	-.0577655315
Boender	Arithmetic Mean	.0002758449	.0065414551	.966	-.0125567537	.0131084436
	Buckley	-.0043692335	.0065414551	.504	-.0172018321	.0084633652
	Chang	-.068266684*	.0065414551	.000	-.0810992824	-.0554340851
	Geometric Mean	.0006585747	.0065414551	.920	-.0121740240	.0134911733
	I.C.S	-.0106155930	.0065414551	.105	-.0234481916	.0022170057
	Laarhoven	.0005364718	.0065414551	.935	-.0122961268	.0133690705
	RowSum	-.0004654989	.0065414551	.943	-.0132980975	.0123670998
	Wang	-.070322285*	.0065414551	.000	-.0831548838	-.0574896865
Buckley	Arithmetic Mean	.0046450784	.0065414551	.478	-.0081875202	.0174776771
	Boender	.0043692335	.0065414551	.504	-.0084633652	.0172018321
	Chang	-.063897450*	.0065414551	.000	-.0767300490	-.0510648517
	Geometric Mean	.0050278081	.0065414551	.442	-.0078047905	.0178604068
	I.C.S	-.0062463595	.0065414551	.340	-.0190789581	.0065862391
	Laarhoven	.0049057053	.0065414551	.453	-.0079268934	.0177383039
	RowSum	.0039037346	.0065414551	.551	-.0089288640	.0167363333
	Wang	-.065953052*	.0065414551	.000	-.0787856504	-.0531204531
Chang	Arithmetic Mean	.0685425287*	.0065414551	.000	.0557099301	.0813751274
	Boender	.0682666838*	.0065414551	.000	.0554340851	.0810992824
	Buckley	.0638974503*	.0065414551	.000	.0510648517	.0767300490
	Geometric Mean	.0689252585*	.0065414551	.000	.0560926598	.0817578571
	I.C.S	.0576510908*	.0065414551	.000	.0448184922	.0704836895
	Laarhoven	.0688031556*	.0065414551	.000	.0559705570	.0816357543
	RowSum	.0678011849*	.0065414551	.000	.0549685863	.0806337836
	Wang	-.0020556014	.0065414551	.753	-.0148882000	.0107769973
Geometric Mean	Arithmetic Mean	-.0003827297	.0065414551	.953	-.0132153284	.0124498689
	Boender	-.0006585747	.0065414551	.920	-.0134911733	.0121740240
	Buckley	-.0050278081	.0065414551	.442	-.0178604068	.0078047905
	Chang	-.068925258*	.0065414551	.000	-.0817578571	-.0560926598
	I.C.S	-.0112741676	.0065414551	.085	-.0241067663	.0015584310
	Laarhoven	-.0001221029	.0065414551	.985	-.0129547015	.0127104958
	RowSum	-.0011240735	.0065414551	.864	-.0139566722	.0117085251
	Wang	-.070980860*	.0065414551	.000	-.0838134585	-.0581482612
I.C.S	Arithmetic Mean	.0108914379	.0065414551	.096	-.0019411607	.0237240366
	Boender	.0106155930	.0065414551	.105	-.0022170057	.0234481916
	Buckley	.0062463595	.0065414551	.340	-.0065862391	.0190789581
	Chang	-.057651091*	.0065414551	.000	-.0704836895	-.0448184922
	Geometric Mean	.0112741676	.0065414551	.085	-.0015584310	.0241067663
	Laarhoven	.0111520648	.0065414551	.088	-.0016805339	.0239846634
	RowSum	.0101500941	.0065414551	.121	-.0026825045	.0229826928
	Wang	-.059706692*	.0065414551	.000	-.0725392909	-.0468740936
Laarhoven	Arithmetic Mean	-.0002606269	.0065414551	.968	-.0130932255	.0125719718
	Boender	-.0005364718	.0065414551	.935	-.0133690705	.0122961268
	Buckley	-.0049057053	.0065414551	.453	-.0177383039	.0079268934
	Chang	-.068803156*	.0065414551	.000	-.0816357543	-.0559705570
	Geometric Mean	.0001221029	.0065414551	.985	-.0127104958	.0129547015
	I.C.S	-.0111520648	.0065414551	.088	-.0239846634	.0016805339
	RowSum	-.0010019707	.0065414551	.878	-.0138345693	.0118306280
	Wang	-.070858757*	.0065414551	.000	-.0836913556	-.0580261584
RowSum	Arithmetic Mean	.0007413438	.0065414551	.910	-.0120912548	.0135739425
	Boender	.0004654989	.0065414551	.943	-.0123670998	.0132980975
	Buckley	-.0039037346	.0065414551	.551	-.0167363333	.0089288640
	Chang	-.067801185*	.0065414551	.000	-.0806337836	-.0549685863
	Geometric Mean	.0011240735	.0065414551	.864	-.0117085251	.0139566722
	I.C.S	-.0101500941	.0065414551	.121	-.0229826928	.0026825045
	Laarhoven	.0010019707	.0065414551	.878	-.0118306280	.0138345693
	Wang	-.069856786*	.0065414551	.000	-.0826893850	-.0570241877
Wang	Arithmetic Mean	.0705981301*	.0065414551	.000	.0577655315	.0834307288
	Boender	.0703222852*	.0065414551	.000	.0574896865	.0831548838
	Buckley	.0659530517*	.0065414551	.000	.0531204531	.0787856504
	Chang	.0020556014	.0065414551	.753	-.0107769973	.0148882000
	Geometric Mean	.0709808599*	.0065414551	.000	.0581482612	.0838134585
	I.C.S	.0597066922*	.0065414551	.000	.0468740936	.0725392909
	Laarhoven	.0708587570*	.0065414551	.000	.0580261584	.0836913556
	RowSum	.0698567863*	.0065414551	.000	.0570241877	.0826893850

Based on observed means.
The error term is Mean Square(Error) = .003.

*. The mean difference is significant at the 0.05 level.

Table B.10: Performance analysis among models when $\alpha = 0.05$

Dependent Variable: MaxError
LSD

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Arithmetic Mean	Boender	-.0037943342	.0101392502	.708	-.0236803383	.0160916698
	Buckley	-.0103975974	.0101392502	.305	-.0302836014	.0094884066
	Chang	-.128378734*	.0101392502	.000	-.1482647376	-.1084927295
	Geometric Mean	.0003751779	.0101392502	.970	-.0195108261	.0202611819
	I.C.S	-.0119659930	.0101392502	.238	-.0318519970	.0079200110
	Laarhoven	-.0048476239	.0101392502	.633	-.0247336279	.0150383801
	RowSum	-.0012892342	.0101392502	.899	-.0211752382	.0185967698
	Wang	-.143423231*	.0101392502	.000	-.1633092348	-.1235372267
Boender	Arithmetic Mean	.0037943342	.0101392502	.708	-.0160916698	.0236803383
	Buckley	-.0066032631	.0101392502	.515	-.0264892672	.0132827409
	Chang	-.124584399*	.0101392502	.000	-.1444704034	-.1046983953
	Geometric Mean	.0041695122	.0101392502	.681	-.0157164919	.0240555162
	I.C.S	-.0081716588	.0101392502	.420	-.0280576628	.0117143453
	Laarhoven	-.0010532897	.0101392502	.917	-.0209392937	.0188327144
	RowSum	.0025051000	.0101392502	.805	-.0173809040	.0223911041
	Wang	-.139628897*	.0101392502	.000	-.1595149006	-.1197428925
Buckley	Arithmetic Mean	.0103975974	.0101392502	.305	-.0094884066	.0302836014
	Boender	.0066032631	.0101392502	.515	-.0132827409	.0264892672
	Chang	-.117981136*	.0101392502	.000	-.1378671402	-.0980951322
	Geometric Mean	.0107727753	.0101392502	.288	-.0091132287	.0306587793
	I.C.S	-.0015683956	.0101392502	.877	-.0214543996	.0183176084
	Laarhoven	.0055499735	.0101392502	.584	-.0143360305	.0254359775
	RowSum	.0091083632	.0101392502	.369	-.0107776408	.0289943672
	Wang	-.133025633*	.0101392502	.000	-.1529116374	-.1131396294
Chang	Arithmetic Mean	.1283787336*	.0101392502	.000	.1084927295	.1482647376
	Boender	.1245843993*	.0101392502	.000	.1046983953	.1444704034
	Buckley	.1179811362*	.0101392502	.000	.0980951322	.1378671402
	Geometric Mean	.1287539115*	.0101392502	.000	.1088679075	.1486399155
	I.C.S	.1164127406*	.0101392502	.000	.0965267366	.1362987446
	Laarhoven	.1235311097*	.0101392502	.000	.1036451057	.1434171137
	RowSum	.1270894994*	.0101392502	.000	.1072034953	.1469755034
	Wang	-.0150444972	.0101392502	.138	-.0349305012	.0048415068
Geometric Mean	Arithmetic Mean	-.0003751779	.0101392502	.970	-.0202611819	.0195108261
	Boender	-.0041695122	.0101392502	.681	-.0240555162	.0157164919
	Buckley	-.0107727753	.0101392502	.288	-.0306587793	.0091132287
	Chang	-.128753911*	.0101392502	.000	-.1486399155	-.1088679075
	I.C.S	-.0123411709	.0101392502	.224	-.0322271749	.0075448331
	Laarhoven	-.0052228018	.0101392502	.607	-.0251088058	.0146632022
	RowSum	-.0016644121	.0101392502	.870	-.0215504161	.0182215919
	Wang	-.143798409*	.0101392502	.000	-.1636844127	-.1239124047
I.C.S	Arithmetic Mean	.0119659930	.0101392502	.238	-.0079200110	.0318519970
	Boender	.0081716588	.0101392502	.420	-.0117143453	.0280576628
	Buckley	.0015683956	.0101392502	.877	-.0183176084	.0214543996
	Chang	-.116412741*	.0101392502	.000	-.1362987446	-.0965267366
	Geometric Mean	.0123411709	.0101392502	.224	-.0075448331	.0322271749
	Laarhoven	.0071183691	.0101392502	.483	-.0127676349	.0270043731
	RowSum	.0106767588	.0101392502	.292	-.0092092452	.0305627628
	Wang	-.131457238*	.0101392502	.000	-.1513432418	-.1115712337
Laarhoven	Arithmetic Mean	.0048476239	.0101392502	.633	-.0150383801	.0247336279
	Boender	.0010532897	.0101392502	.917	-.0188327144	.0209392937
	Buckley	-.0055499735	.0101392502	.584	-.0254359775	.0143360305
	Chang	-.123531110*	.0101392502	.000	-.1434171137	-.1036451057
	Geometric Mean	.0052228018	.0101392502	.607	-.0146632022	.0251088058
	I.C.S	-.0071183691	.0101392502	.483	-.0270043731	.0127676349
	RowSum	.0035583897	.0101392502	.726	-.0163276143	.0234443937
	Wang	-.138575607*	.0101392502	.000	-.1584616109	-.1186896028
RowSum	Arithmetic Mean	.0012892342	.0101392502	.899	-.0185967698	.0211752382
	Boender	-.0025051000	.0101392502	.805	-.0223911041	.0173809040
	Buckley	-.0091083632	.0101392502	.369	-.0289943672	.0107776408
	Chang	-.127089499*	.0101392502	.000	-.1469755034	-.1072034953
	Geometric Mean	.0016644121	.0101392502	.870	-.0182215919	.0215504161
	I.C.S	-.0106767588	.0101392502	.292	-.0305627628	.0092092452
	Laarhoven	-.0035583897	.0101392502	.726	-.0234443937	.0163276143
	Wang	-.142133997*	.0101392502	.000	-.1620200006	-.1222479925
Wang	Arithmetic Mean	.1434232308*	.0101392502	.000	.1235372267	.1633092348
	Boender	.1396288965*	.0101392502	.000	.1197428925	.1595149006
	Buckley	.1330256334*	.0101392502	.000	.1131396294	.1529116374
	Chang	.0150444972	.0101392502	.138	-.0048415068	.0349305012
	Geometric Mean	.1437984087*	.0101392502	.000	.1239124047	.1636844127
	I.C.S	.1314572378*	.0101392502	.000	.1115712337	.1513432418
	Laarhoven	.1385756069*	.0101392502	.000	.1186896028	.1584616109
	RowSum	.1421339966*	.0101392502	.000	.1222479925	.1620200006

Based on observed means.
The error term is Mean Square(Error) = .010.
*. The mean difference is significant at the 0.05 level.

Table B.11: Performance analysis among models when $\alpha = 0.10$

Dependent Variable: MaxError
LSD

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Arithmetic Mean	Boender	-.0020018090	.0086089208	.816	-.0188863943	.0148827764
	Buckley	-.0099716449	.0086089208	.247	-.0268562302	.0069129404
	Chang	-.102126529*	.0086089208	.000	-.1190111147	-.0852419441
	Geometric Mean	.0010155529	.0086089208	.906	-.0158690324	.0179001382
	I.C.S	-.0108752453	.0086089208	.207	-.0277598306	.0060093400
	Laarhoven	-.0048577578	.0086089208	.573	-.0217423431	.0120268275
	RowSum	.0003211574	.0086089208	.970	-.0165634280	.0172057427
	Wang	-.110814189*	.0086089208	.000	-.1276987740	-.0939296034
Boender	Arithmetic Mean	.0020018090	.0086089208	.816	-.0148827764	.0188863943
	Buckley	-.0079698359	.0086089208	.355	-.0248544212	.0089147494
	Chang	-.100124720*	.0086089208	.000	-.1170093057	-.0832401351
	Geometric Mean	.0030173619	.0086089208	.726	-.0138672234	.0199019472
	I.C.S	-.0088734364	.0086089208	.303	-.0257580217	.0080111489
	Laarhoven	-.0028559488	.0086089208	.740	-.0197405342	.0140286365
	RowSum	.0023229663	.0086089208	.787	-.0145616190	.0192075516
	Wang	-.108812380*	.0086089208	.000	-.1256969650	-.0919277944
Buckley	Arithmetic Mean	.0099716449	.0086089208	.247	-.0069129404	.0268562302
	Boender	.0079698359	.0086089208	.355	-.0089147494	.0248544212
	Chang	-.092154885*	.0086089208	.000	-.1090394698	-.0752702992
	Geometric Mean	.0109871978	.0086089208	.202	-.0058973875	.0278717831
	I.C.S	-.0009036004	.0086089208	.916	-.0177881858	.0159809849
	Laarhoven	.0051138871	.0086089208	.553	-.0117706982	.0219984724
	RowSum	.0102928022	.0086089208	.232	-.0065917831	.0271773876
	Wang	-.100842544*	.0086089208	.000	-.1177271291	-.0839579585
Chang	Arithmetic Mean	.1021265294*	.0086089208	.000	.0852419441	.1190111147
	Boender	.1001247204*	.0086089208	.000	.0832401351	.1170093057
	Buckley	.0921548845*	.0086089208	.000	.0752702992	.1090394698
	Geometric Mean	.1031420823*	.0086089208	.000	.0862574970	.1200266676
	I.C.S	.0912512841*	.0086089208	.000	.0743666987	.1081358694
	Laarhoven	.0972687716*	.0086089208	.000	.0803841863	.1141533569
	RowSum	.1024476867*	.0086089208	.000	.0855631014	.1193322721
	Wang	-.0086876593	.0086089208	.313	-.0255722446	.0081969260
Geometric Mean	Arithmetic Mean	-.0010155529	.0086089208	.906	-.0179001382	.0158690324
	Boender	-.0030173619	.0086089208	.726	-.0199019472	.0138672234
	Buckley	-.0109871978	.0086089208	.202	-.0278717831	.0058973875
	Chang	-.103142082*	.0086089208	.000	-.1200266676	-.0862574970
	I.C.S	-.0118907983	.0086089208	.167	-.0287753836	.0049937871
	Laarhoven	-.0058733107	.0086089208	.495	-.0227578960	.0110112746
	RowSum	-.0006943956	.0086089208	.936	-.0175789809	.0161901897
	Wang	-.111829742*	.0086089208	.000	-.1287143269	-.0949451563
I.C.S	Arithmetic Mean	.0108752453	.0086089208	.207	-.0060093400	.0277598306
	Boender	.0088734364	.0086089208	.303	-.0080111489	.0257580217
	Buckley	.0009036004	.0086089208	.916	-.0159809849	.0177881858
	Chang	-.091251284*	.0086089208	.000	-.1081358694	-.0743666987
	Geometric Mean	.0118907983	.0086089208	.167	-.0049937871	.0287753836
	Laarhoven	.0060174875	.0086089208	.485	-.0108670978	.0229020729
	RowSum	.0111964027	.0086089208	.194	-.0056881826	.0280809880
	Wang	-.099938943*	.0086089208	.000	-.1168235287	-.0830543580
Laarhoven	Arithmetic Mean	.0048577578	.0086089208	.573	-.0120268275	.0217423431
	Boender	.0028559488	.0086089208	.740	-.0140286365	.0197405342
	Buckley	-.0051138871	.0086089208	.553	-.0219984724	.0117706982
	Chang	-.097268772*	.0086089208	.000	-.1141533569	-.0803841863
	Geometric Mean	.0058733107	.0086089208	.495	-.0110112746	.0227578960
	I.C.S	-.0060174875	.0086089208	.485	-.0229020729	.0108670978
	RowSum	.0051789152	.0086089208	.548	-.0117056702	.0220635005
	Wang	-.105956431*	.0086089208	.000	-.1228410162	-.0890718456
RowSum	Arithmetic Mean	-.0003211574	.0086089208	.970	-.0172057427	.0165634280
	Boender	-.0023229663	.0086089208	.787	-.0192075516	.0145616190
	Buckley	-.0102928022	.0086089208	.232	-.0271773876	.0065917831
	Chang	-.102447687*	.0086089208	.000	-.1193322721	-.0855631014
	Geometric Mean	.0006943956	.0086089208	.936	-.0161901897	.0175789809
	I.C.S	-.0111964027	.0086089208	.194	-.0280809880	.0056881826
	Laarhoven	-.0051789152	.0086089208	.548	-.0220635005	.0117056702
	Wang	-.111135346*	.0086089208	.000	-.1280199314	-.0942507607
Wang	Arithmetic Mean	.1108141887*	.0086089208	.000	.0939296034	.1276987740
	Boender	.1088123797*	.0086089208	.000	.0919277944	.1256969650
	Buckley	.1008425438*	.0086089208	.000	.0839579585	.1177271291
	Chang	.0086876593	.0086089208	.313	-.0081969260	.0255722446
	Geometric Mean	.1118297416*	.0086089208	.000	.0949451563	.1287143269
	I.C.S	.0999389434*	.0086089208	.000	.0830543580	.1168235287
	Laarhoven	.1059564309*	.0086089208	.000	.0890718456	.1228410162
	RowSum	.1111353460*	.0086089208	.000	.0942507607	.1280199314

Based on observed means.
The error term is Mean Square(Error) = .007.

*. The mean difference is significant at the 0.05 level.

Table B.12: Performance analysis among models when $\alpha = 0.15$

Dependent Variable: MaxError
LSD

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Arithmetic Mean	Boender	-.0025282173	.0078083871	.746	-.0178427242	.0127862897
	Buckley	-.0112578729	.0078083871	.150	-.0265723799	.0040566341
	Chang	-.087754756*	.0078083871	.000	-.1030692632	-.0724402493
	Geometric Mean	.0014269528	.0078083871	.855	-.0138875542	.0167414597
	I.C.S	-.0083966333	.0078083871	.282	-.0237111402	.0069178737
	Laarhoven	-.0052306338	.0078083871	.503	-.0205451408	.0100838732
	RowSum	.0005193221	.0078083871	.947	-.0147951849	.0158338291
	Wang	-.098994699*	.0078083871	.000	-.1143092061	-.0836801921
Boender	Arithmetic Mean	.0025282173	.0078083871	.746	-.0127862897	.0178427242
	Buckley	-.0087296556	.0078083871	.264	-.0240441626	.0065848513
	Chang	-.085226539*	.0078083871	.000	-.1005410460	-.0699120320
	Geometric Mean	.0039551700	.0078083871	.613	-.0113593370	.0192696770
	I.C.S	-.0058684160	.0078083871	.452	-.0211829230	.0094460910
	Laarhoven	-.0027024166	.0078083871	.729	-.0180169235	.0126120904
	RowSum	.0030475394	.0078083871	.696	-.0122669676	.0183620463
	Wang	-.096466482*	.0078083871	.000	-.1117809888	-.0811519749
Buckley	Arithmetic Mean	.0112578729	.0078083871	.150	-.0040566341	.0265723799
	Boender	.0087296556	.0078083871	.264	-.0065848513	.0240441626
	Chang	-.076496883*	.0078083871	.000	-.0918113903	-.0611823764
	Geometric Mean	.0126848257	.0078083871	.104	-.0026296813	.0279993326
	I.C.S	.0028612396	.0078083871	.714	-.0124532673	.0181757466
	Laarhoven	.0060272391	.0078083871	.440	-.0092872679	.0213417461
	RowSum	.0117771950	.0078083871	.132	-.0035373120	.0270917020
	Wang	-.087736826*	.0078083871	.000	-.1030513332	-.0724223192
Chang	Arithmetic Mean	.0877547563*	.0078083871	.000	.0724402493	.1030692632
	Boender	.0852265390*	.0078083871	.000	.0699120320	.1005410460
	Buckley	.0764968834*	.0078083871	.000	.0611823764	.0918113903
	Geometric Mean	.0891817090*	.0078083871	.000	.0738672020	.1044962160
	I.C.S	.0793581230*	.0078083871	.000	.0640436160	.0946726300
	Laarhoven	.0825241224*	.0078083871	.000	.0672096155	.0978386294
	RowSum	.0882740784*	.0078083871	.000	.0729595714	.1035885853
	Wang	-.0112399428	.0078083871	.150	-.0265544498	.0040745641
Geometric Mean	Arithmetic Mean	-.0014269528	.0078083871	.855	-.0167414597	.0138875542
	Boender	-.0039551700	.0078083871	.613	-.0192696770	.0113593370
	Buckley	-.0126848257	.0078083871	.104	-.0279993326	.0026296813
	Chang	-.089181709*	.0078083871	.000	-.1044962160	-.0738672020
	I.C.S	-.0098235860	.0078083871	.209	-.0251380930	.0054909209
	Laarhoven	-.0066575866	.0078083871	.394	-.0219720936	.0086569204
	RowSum	-.0009076307	.0078083871	.907	-.0162221376	.0144068763
	Wang	-.100421652*	.0078083871	.000	-.1157361588	-.0851071449
I.C.S	Arithmetic Mean	.0083966333	.0078083871	.282	-.0069178737	.0237111402
	Boender	.0058684160	.0078083871	.452	-.0094460910	.0211829230
	Buckley	-.0028612396	.0078083871	.714	-.0181757466	.0124532673
	Chang	-.079358123*	.0078083871	.000	-.0946726300	-.0640436160
	Geometric Mean	.0098235860	.0078083871	.209	-.0054909209	.0251380930
	Laarhoven	.0031659994	.0078083871	.685	-.0121485075	.0184805064
	RowSum	.0089159554	.0078083871	.254	-.0063985516	.0242304624
	Wang	-.090598066*	.0078083871	.000	-.1059125728	-.0752835589
Laarhoven	Arithmetic Mean	.0052306338	.0078083871	.503	-.0100838732	.0205451408
	Boender	.0027024166	.0078083871	.729	-.0126120904	.0180169235
	Buckley	-.0060272391	.0078083871	.440	-.0213417461	.0092872679
	Chang	-.082524122*	.0078083871	.000	-.0978386294	-.0672096155
	Geometric Mean	.0066575866	.0078083871	.394	-.0086569204	.0219720936
	I.C.S	-.0031659994	.0078083871	.685	-.0184805064	.0121485075
	RowSum	.0057499559	.0078083871	.462	-.0095645510	.0210644629
	Wang	-.093764065*	.0078083871	.000	-.1090785723	-.0784495583
RowSum	Arithmetic Mean	-.0005193221	.0078083871	.947	-.0158338291	.0147951849
	Boender	-.0030475394	.0078083871	.696	-.0183620463	.0122669676
	Buckley	-.0117771950	.0078083871	.132	-.0270917020	.0035373120
	Chang	-.088274078*	.0078083871	.000	-.1035885853	-.0729595714
	Geometric Mean	.0009076307	.0078083871	.907	-.0144068763	.0162221376
	I.C.S	-.0089159554	.0078083871	.254	-.0242304624	.0063985516
	Laarhoven	-.0057499559	.0078083871	.462	-.0210644629	.0095645510
	Wang	-.099514021*	.0078083871	.000	-.1148285282	-.0841995142
Wang	Arithmetic Mean	.0989946991*	.0078083871	.000	.0836801921	.1143092061
	Boender	.0964664818*	.0078083871	.000	.0811519749	.1117809888
	Buckley	.0877368262*	.0078083871	.000	.0724223192	.1030513332
	Chang	.0112399428	.0078083871	.150	-.0040745641	.0265544498
	Geometric Mean	.1004216519*	.0078083871	.000	.0851071449	.1157361588
	I.C.S	.0905980658*	.0078083871	.000	.0752835589	.1059125728
	Laarhoven	.0937640653*	.0078083871	.000	.0784495583	.1090785723
	RowSum	.0995140212*	.0078083871	.000	.0841995142	.1148285282

Based on observed means.
The error term is Mean Square(Error) = .006.
*. The mean difference is significant at the 0.05 level.

Table B.13: Performance analysis among models when $\beta = 0\%$

Dependent Variable: MaxError
LSD

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	Upper Bound
Arithmetic Mean	Boender	.0000257878	.0125325293	.998	-.0245653088	.0246168844
	Buckley	.0001010184	.0125325293	.994	-.0244900782	.0246921150
	Chang	-.408864866*	.0125325293	.000	-.4334559623	-.3842737691
	Geometric Mean	.0001008240	.0125325293	.994	-.0244902726	.0246919206
	I.C.S	-.0003164892	.0125325293	.980	-.0249075858	.0242746074
	Laarhoven	.0000353430	.0125325293	.998	-.0245557536	.0246264396
	RowSum	.0002861430	.0125325293	.982	-.0243049536	.0248772396
	Wang	-.432661428*	.0125325293	.000	-.4572525248	-.4080703315
Boender	Arithmetic Mean	-.0000257878	.0125325293	.998	-.0246168844	.0245653088
	Buckley	.0000752306	.0125325293	.995	-.0245158661	.0246663272
	Chang	-.408890653*	.0125325293	.000	-.4334817501	-.3842995569
	Geometric Mean	.0000750362	.0125325293	.995	-.0245160604	.0246661328
	I.C.S	-.0003422770	.0125325293	.978	-.0249333736	.0242488196
	Laarhoven	.0000095552	.0125325293	.999	-.0245815414	.0246006518
	RowSum	.0002603552	.0125325293	.983	-.0243307414	.0248514518
	Wang	-.432687216*	.0125325293	.000	-.4572783126	-.4080961194
Buckley	Arithmetic Mean	-.0001010184	.0125325293	.994	-.0246921150	.0244900782
	Boender	-.0000752306	.0125325293	.995	-.0246663272	.0245158661
	Chang	-.408965884*	.0125325293	.000	-.4335569807	-.3843747874
	Geometric Mean	-1.94392E-7	.0125325293	1.000	-.0245912910	.0245909022
	I.C.S	-.0004175076	.0125325293	.973	-.0250086042	.0241735890
	Laarhoven	-.0000656754	.0125325293	.996	-.0246567720	.0245254213
	RowSum	.0001851246	.0125325293	.988	-.0244059720	.0247762213
	Wang	-.432762447*	.0125325293	.000	-.4573535432	-.4081713499
Chang	Arithmetic Mean	.4088648657*	.0125325293	.000	.3842737691	.4334559623
	Boender	.4088906535*	.0125325293	.000	.3842995569	.4334817501
	Buckley	.4089658841*	.0125325293	.000	.3843747874	.4335569807
	Geometric Mean	.4089656897*	.0125325293	.000	.3843745931	.4335567863
	I.C.S	.4085483765*	.0125325293	.000	.3839572799	.4331394731
	Laarhoven	.4089002087*	.0125325293	.000	.3843091121	.4334913053
	RowSum	.4091510087*	.0125325293	.000	.3845599121	.4337421053
	Wang	-.0237965625	.0125325293	.058	-.0483876591	.0007945341
Geometric Mean	Arithmetic Mean	-.0001008240	.0125325293	.994	-.0246919206	.0244902726
	Boender	-.0000750362	.0125325293	.995	-.0246661328	.0245160604
	Buckley	1.94392E-7	.0125325293	1.000	-.0245909022	.0245912910
	Chang	-.408965690*	.0125325293	.000	-.4335567863	-.3843745931
	I.C.S	-.0004173132	.0125325293	.973	-.0250084098	.0241737834
	Laarhoven	-.0000654810	.0125325293	.996	-.0246565776	.0245256157
	RowSum	.0001853190	.0125325293	.988	-.0244057776	.0247764156
	Wang	-.432762252*	.0125325293	.000	-.4573533488	-.4081711555
I.C.S	Arithmetic Mean	.0003164892	.0125325293	.980	-.0242746074	.0249075858
	Boender	.0003422770	.0125325293	.978	-.0242488196	.0249333736
	Buckley	.0004175076	.0125325293	.973	-.0241735890	.0250086042
	Chang	-.408548376*	.0125325293	.000	-.4331394731	-.3839572799
	Geometric Mean	.0004173132	.0125325293	.973	-.0241737834	.0250084098
	Laarhoven	.0003518322	.0125325293	.978	-.0242392644	.0249429289
	RowSum	.0006026322	.0125325293	.962	-.0239884644	.0251937288
	Wang	-.432344939*	.0125325293	.000	-.4569360356	-.4077538423
Laarhoven	Arithmetic Mean	-.0000353430	.0125325293	.998	-.0246264396	.0245557536
	Boender	-.0000095552	.0125325293	.999	-.0246006518	.0245815414
	Buckley	.0000656754	.0125325293	.996	-.0245254213	.0246567720
	Chang	-.408900209*	.0125325293	.000	-.4334913053	-.3843091121
	Geometric Mean	.0000654810	.0125325293	.996	-.0245256157	.0246565776
	I.C.S	-.0003518322	.0125325293	.978	-.0249429289	.0242392644
	RowSum	.0002508000	.0125325293	.984	-.0243402966	.0248418966
	Wang	-.432696771*	.0125325293	.000	-.4572878678	-.4081056746
RowSum	Arithmetic Mean	-.0002861430	.0125325293	.982	-.0248772396	.0243049536
	Boender	-.0002603552	.0125325293	.983	-.0248514518	.0243307414
	Buckley	-.0001851246	.0125325293	.988	-.0247762213	.0244059720
	Chang	-.409151009*	.0125325293	.000	-.4337421053	-.3845599121
	Geometric Mean	-.0001853190	.0125325293	.988	-.0247764156	.0244057776
	I.C.S	-.0006026322	.0125325293	.962	-.0251937288	.0239884644
	Laarhoven	-.0002508000	.0125325293	.984	-.0248418966	.0243402966
	Wang	-.432947571*	.0125325293	.000	-.4575386678	-.4083564746
Wang	Arithmetic Mean	.4326614282*	.0125325293	.000	.4080703315	.4572525248
	Boender	.4326872160*	.0125325293	.000	.4080961194	.4572783126
	Buckley	.4327624465*	.0125325293	.000	.4081713499	.4573535432
	Chang	.0237965625	.0125325293	.058	-.0007945341	.0483876591
	Geometric Mean	.4327622521*	.0125325293	.000	.4081711555	.4573533488
	I.C.S	.4323449390*	.0125325293	.000	.4077538423	.4569360356
	Laarhoven	.4326967712*	.0125325293	.000	.4081056746	.4572878678
	RowSum	.4329475712*	.0125325293	.000	.4083564746	.4575386678

Based on observed means.
The error term is Mean Square(Error) = .009.

*. The mean difference is significant at the 0.05 level.

Table B.14: Performance analysis among models when $\beta = 50\%$

Dependent Variable: MaxError
LSD

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Arithmetic Mean	Boender	.0001959800	.0061512214	.975	-.0118738326	.0122657926
	Buckley	.0000758382	.0061512214	.990	-.0119939744	.0121456508
	Chang	-.058913066*	.0061512214	.000	-.0709828781	-.0468432530
	Geometric Mean	.0001070054	.0061512214	.986	-.0119628072	.0121768180
	I.C.S	-.0013723705	.0061512214	.823	-.0134421831	.0106974421
	Laarhoven	.0000192437	.0061512214	.998	-.0120505689	.0120890563
	RowSum	-.0005735812	.0061512214	.926	-.0126433938	.0114962314
	Wang	-.104657754*	.0061512214	.000	-.1167275663	-.0925879412
Boender	Arithmetic Mean	-.0001959800	.0061512214	.975	-.0122657926	.0118738326
	Buckley	-.0001201418	.0061512214	.984	-.0121899544	.0119496708
	Chang	-.059109046*	.0061512214	.000	-.0711788581	-.0470392330
	Geometric Mean	-.0000889746	.0061512214	.988	-.0121587872	.0119808380
	I.C.S	-.0015683505	.0061512214	.799	-.0136381631	.0105014621
	Laarhoven	-.0001767363	.0061512214	.977	-.0122465489	.0118930763
	RowSum	-.0007695612	.0061512214	.900	-.0128393738	.0113002514
	Wang	-.104853734*	.0061512214	.000	-.1169235463	-.0927839212
Buckley	Arithmetic Mean	-.0000758382	.0061512214	.990	-.0121456508	.0119939744
	Boender	.0001201418	.0061512214	.984	-.0119496708	.0121899544
	Chang	-.058988904*	.0061512214	.000	-.0710587163	-.0469190911
	Geometric Mean	.0000311673	.0061512214	.996	-.0120386453	.0121009799
	I.C.S	-.0014482087	.0061512214	.814	-.0135180213	.0106216039
	Laarhoven	-.0000565944	.0061512214	.993	-.0121264070	.0120132182
	RowSum	-.0006494194	.0061512214	.916	-.0127192320	.0114203932
	Wang	-.104733592*	.0061512214	.000	-.1168034045	-.0926637793
Chang	Arithmetic Mean	.0589130656*	.0061512214	.000	.0468432530	.0709828781
	Boender	.0591090456*	.0061512214	.000	.0470392330	.0711788581
	Buckley	.0589889037*	.0061512214	.000	.0469190911	.0710587163
	Geometric Mean	.0590200710*	.0061512214	.000	.0469502584	.0710898836
	I.C.S	.0575406950*	.0061512214	.000	.0454708824	.0696105076
	Laarhoven	.0589323093*	.0061512214	.000	.0468624967	.0710021219
	RowSum	.0583394843*	.0061512214	.000	.0462696717	.0704092969
	Wang	-.045744688*	.0061512214	.000	-.0578145008	-.0336748756
Geometric Mean	Arithmetic Mean	-.0001070054	.0061512214	.986	-.0121768180	.0119628072
	Boender	.0000889746	.0061512214	.988	-.0119808380	.0121587872
	Buckley	-.0000311673	.0061512214	.996	-.0121009799	.0120386453
	Chang	-.059020071*	.0061512214	.000	-.0710898836	-.0469502584
	I.C.S	-.0014793760	.0061512214	.810	-.0135491886	.0105904366
	Laarhoven	-.0000877617	.0061512214	.989	-.0121575743	.0119820509
	RowSum	-.0006805867	.0061512214	.912	-.0127503993	.0113892259
	Wang	-.104764759*	.0061512214	.000	-.1168345718	-.0926949466
I.C.S	Arithmetic Mean	.0013723705	.0061512214	.823	-.0106974421	.0134421831
	Boender	.0015683505	.0061512214	.799	-.0105014621	.0136381631
	Buckley	.0014482087	.0061512214	.814	-.0106216039	.0135180213
	Chang	-.057540695*	.0061512214	.000	-.0696105076	-.0454708824
	Geometric Mean	.0014793760	.0061512214	.810	-.0105904366	.0135491886
	Laarhoven	.0013916143	.0061512214	.821	-.0106781983	.0134614269
	RowSum	.0007987893	.0061512214	.897	-.0112710233	.0128686019
	Wang	-.103285383*	.0061512214	.000	-.1153551958	-.0912155706
Laarhoven	Arithmetic Mean	-.0000192437	.0061512214	.998	-.0120890563	.0120505689
	Boender	.0001767363	.0061512214	.977	-.0118930763	.0122465489
	Buckley	.0000565944	.0061512214	.993	-.0120132182	.0121264070
	Chang	-.058932309*	.0061512214	.000	-.0710021219	-.0468624967
	Geometric Mean	.0000877617	.0061512214	.989	-.0119820509	.0121575743
	I.C.S	-.0013916143	.0061512214	.821	-.0134614269	.0106781983
	RowSum	-.0005928250	.0061512214	.923	-.0126626376	.0114769876
	Wang	-.104676997*	.0061512214	.000	-.1167468101	-.0926071849
RowSum	Arithmetic Mean	.0005735812	.0061512214	.926	-.0114962314	.0126433938
	Boender	.0007695612	.0061512214	.900	-.0113002514	.0128393738
	Buckley	.0006494194	.0061512214	.916	-.0114203932	.0127192320
	Chang	-.058339484*	.0061512214	.000	-.0704092969	-.0462696717
	Geometric Mean	.0006805867	.0061512214	.912	-.0113892259	.0127503993
	I.C.S	-.0007987893	.0061512214	.897	-.0128686019	.0112710233
	Laarhoven	.0005928250	.0061512214	.923	-.0114769876	.0126626376
	Wang	-.104084173*	.0061512214	.000	-.1161539851	-.0920143599
Wang	Arithmetic Mean	.1046577538*	.0061512214	.000	.0925879412	.1167275663
	Boender	.1048537338*	.0061512214	.000	.0927839212	.1169235463
	Buckley	.1047335919*	.0061512214	.000	.0926637793	.1168034045
	Chang	.0457446882*	.0061512214	.000	.0336748756	.0578145008
	Geometric Mean	.1047647592*	.0061512214	.000	.0926949466	.1168345718
	I.C.S	.1032853832*	.0061512214	.000	.0912155706	.1153551958
	Laarhoven	.1046769975*	.0061512214	.000	.0926071849	.1167468101
	RowSum	.1040841725*	.0061512214	.000	.0920143599	.1161539851

Based on observed means.
The error term is Mean Square(Error) = .002.
*. The mean difference is significant at the 0.05 level.

Table B.15: Performance analysis among models when $\beta = 100\%$

Dependent Variable: MaxError
LSD

					95% Confidence Interval	
(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	Lower Bound	Upper Bound
Arithmetic Mean	Boender	-.0048414201	.0038790211	.212	-.0124527633	.0027699232
	Buckley	-.0021232098	.0038790211	.584	-.0097345531	.0054881334
	Chang	-.015741010*	.0038790211	.000	-.0233523530	-.0081296665
	Geometric Mean	.0002118207	.0038790211	.956	-.0073995226	.0078231639
	I.C.S	-.0061411816	.0038790211	.114	-.0137525249	.0014701616
	Laarhoven	-.0049912228	.0038790211	.198	-.0126025660	.0026201205
	RowSum	-.0014295735	.0038790211	.713	-.0090409167	.0061817698
	Wang	-.025232653*	.0038790211	.000	-.0328439961	-.0176213096
Boender	Arithmetic Mean	.0048414201	.0038790211	.212	-.0027699232	.0124527633
	Buckley	.0027182102	.0038790211	.484	-.0048931330	.0103295535
	Chang	-.010899590*	.0038790211	.005	-.0185109329	-.0032882464
	Geometric Mean	.0050532408	.0038790211	.193	-.0025581025	.0126645840
	I.C.S	-.0012997615	.0038790211	.738	-.0089111048	.0063115817
	Laarhoven	-.0001498027	.0038790211	.969	-.0077611459	.0074615406
	RowSum	.0034118466	.0038790211	.379	-.0041994966	.0110231899
	Wang	-.020391233*	.0038790211	.000	-.0280025760	-.0127798895
Buckley	Arithmetic Mean	.0021232098	.0038790211	.584	-.0054881334	.0097345531
	Boender	-.0027182102	.0038790211	.484	-.0103295535	.0048931330
	Chang	-.013617800*	.0038790211	.000	-.0212291432	-.0060064567
	Geometric Mean	.0023350305	.0038790211	.547	-.0052763127	.0099463738
	I.C.S	-.0040179718	.0038790211	.301	-.0116293150	.0035933715
	Laarhoven	-.0028680129	.0038790211	.460	-.0104793562	.0047433303
	RowSum	.0006936364	.0038790211	.858	-.0069177069	.0083049796
	Wang	-.023109443*	.0038790211	.000	-.0307207862	-.0154980997
Chang	Arithmetic Mean	.0157410098*	.0038790211	.000	.0081296665	.0233523530
	Boender	.0108995897*	.0038790211	.005	.0032882464	.0185109329
	Buckley	.0136177999*	.0038790211	.000	.0060064567	.0212291432
	Geometric Mean	.0159528304*	.0038790211	.000	.0083414872	.0235641737
	I.C.S	.0095998282*	.0038790211	.013	.0019884849	.0172111714
	Laarhoven	.0107497870*	.0038790211	.006	.0031384438	.0183611302
	RowSum	.0143114363*	.0038790211	.000	.0067000931	.0219227796
	Wang	-.009491643*	.0038790211	.015	-.0171029863	-.0018802998
Geometric Mean	Arithmetic Mean	-.0002118207	.0038790211	.956	-.0078231639	.0073995226
	Boender	-.0050532408	.0038790211	.193	-.0126645840	.0025581025
	Buckley	-.0023350305	.0038790211	.547	-.0099463738	.0052763127
	Chang	-.015952830*	.0038790211	.000	-.0235641737	-.0083414872
	I.C.S	-.0063530023	.0038790211	.102	-.0139643455	.0012583410
	Laarhoven	-.0052030434	.0038790211	.180	-.0128143867	.0024082998
	RowSum	-.0016413941	.0038790211	.672	-.0092527374	.0059699491
	Wang	-.025444473*	.0038790211	.000	-.0330558167	-.0178331302
I.C.S	Arithmetic Mean	.0061411816	.0038790211	.114	-.0014701616	.0137525249
	Boender	.0012997615	.0038790211	.738	-.0063115817	.0089111048
	Buckley	.0040179718	.0038790211	.301	-.0035933715	.0116293150
	Chang	-.009599828*	.0038790211	.013	-.0172111714	-.0019884849
	Geometric Mean	.0063530023	.0038790211	.102	-.0012583410	.0139643455
	Laarhoven	.0011499588	.0038790211	.767	-.0064613844	.0087613021
	RowSum	.0047116081	.0038790211	.225	-.0028997351	.0123229514
	Wang	-.019091471*	.0038790211	.000	-.0267028145	-.0114801280
Laarhoven	Arithmetic Mean	.0049912228	.0038790211	.198	-.0026201205	.0126025660
	Boender	.0001498027	.0038790211	.969	-.0074615406	.0077611459
	Buckley	.0028680129	.0038790211	.460	-.0047433303	.0104793562
	Chang	-.010749787*	.0038790211	.006	-.0183611302	-.0031384438
	Geometric Mean	.0052030434	.0038790211	.180	-.0024082998	.0128143867
	I.C.S	-.0011499588	.0038790211	.767	-.0087613021	.0064613844
	RowSum	.0035616493	.0038790211	.359	-.0040496939	.0111729926
	Wang	-.020241430*	.0038790211	.000	-.0278527733	-.0126300868
RowSum	Arithmetic Mean	.0014295735	.0038790211	.713	-.0061817698	.0090409167
	Boender	-.0034118466	.0038790211	.379	-.0110231899	.0041994966
	Buckley	-.0006936364	.0038790211	.858	-.0083049796	.0069177069
	Chang	-.014311436*	.0038790211	.000	-.0219227796	-.0067000931
	Geometric Mean	.0016413941	.0038790211	.672	-.0059699491	.0092527374
	I.C.S	-.0047116081	.0038790211	.225	-.0123229514	.0028997351
	Laarhoven	-.0035616493	.0038790211	.359	-.0111729926	.0040496939
	Wang	-.023803079*	.0038790211	.000	-.0314144226	-.0161917361
Wang	Arithmetic Mean	.0252326528*	.0038790211	.000	.0176213096	.0328439961
	Boender	.0203912327*	.0038790211	.000	.0127798895	.0280025760
	Buckley	.0231094430*	.0038790211	.000	.0154980997	.0307207862
	Chang	.0094916430*	.0038790211	.015	.0018802998	.0171029863
	Geometric Mean	.0254444735*	.0038790211	.000	.0178331302	.0330558167
	I.C.S	.0190914712*	.0038790211	.000	.0114801280	.0267028145
	Laarhoven	.0202414300*	.0038790211	.000	.0126300868	.0278527733
	RowSum	.0238030794*	.0038790211	.000	.0161917361	.0314144226

Based on observed means.
The error term is Mean Square(Error) = .001.
*. The mean difference is significant at the 0.05 level.

Table B.16: Performance analysis among models when $\beta = 150\%$

Dependent Variable: MaxError
LSD

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Arithmetic Mean	Boender	-.008391443*	.0035772245	.019	-.0154106063	-.0013722792
	Buckley	-.020296564*	.0035772245	.000	-.0273157271	-.0132774001
	Chang	-.021941831*	.0035772245	.000	-.0289609945	-.0149226675
	Geometric Mean	.0019003017	.0035772245	.595	-.0051188619	.0089194652
	I.C.S	-.022078039*	.0035772245	.000	-.0290972027	-.0150588756
	Laarhoven	-.011093074*	.0035772245	.002	-.0181122371	-.0040739101
	RowSum	.0005299678	.0035772245	.882	-.0064891957	.0075491314
	Wang	-.013035813*	.0035772245	.000	-.0200549770	-.0060166499
Boender	Arithmetic Mean	.0083914427*	.0035772245	.019	.0013722792	.0154106063
	Buckley	-.011905121*	.0035772245	.001	-.0189242844	-.0048859574
	Chang	-.013550388*	.0035772245	.000	-.0205695518	-.0065312247
	Geometric Mean	.0102917444*	.0035772245	.004	.0032725809	.0173109079
	I.C.S	-.013686596*	.0035772245	.000	-.0207057599	-.0066674329
	Laarhoven	-.0027016309	.0035772245	.450	-.0097207944	.0043175327
	RowSum	.0089214106*	.0035772245	.013	.0019022470	.0159405741
	Wang	-.0046443707	.0035772245	.194	-.0116635342	.0023747928
Buckley	Arithmetic Mean	.0202965636*	.0035772245	.000	.0132774001	.0273157271
	Boender	.0119051209*	.0035772245	.001	.0048859574	.0189242844
	Chang	-.0016452674	.0035772245	.646	-.0086644309	.0053738961
	Geometric Mean	.0221968653*	.0035772245	.000	.0151777018	.0292160288
	I.C.S	-.0017814755	.0035772245	.619	-.0088006390	.0052376880
	Laarhoven	.0092034900*	.0035772245	.010	.0021843265	.0162226535
	RowSum	.0208265315*	.0035772245	.000	.0138073679	.0278456950
	Wang	.0072607502*	.0035772245	.043	.0002415866	.0142799137
Chang	Arithmetic Mean	.0219418310*	.0035772245	.000	.0149226675	.0289609945
	Boender	.0135503883*	.0035772245	.000	.0065312247	.0205695518
	Buckley	.0016452674	.0035772245	.646	-.0053738961	.0086644309
	Geometric Mean	.0238421327*	.0035772245	.000	.0168229691	.0308612962
	I.C.S	-.0001362081	.0035772245	.970	-.0071553717	.0068829554
	Laarhoven	.0108487574*	.0035772245	.002	.0038295939	.0178679209
	RowSum	.0224717988*	.0035772245	.000	.0154526353	.0294909624
	Wang	.0089060176*	.0035772245	.013	.0018868540	.0159251811
Geometric Mean	Arithmetic Mean	-.0019003017	.0035772245	.595	-.0089194652	.0051188619
	Boender	-.010291744*	.0035772245	.004	-.0173109079	-.0032725809
	Buckley	-.022196865*	.0035772245	.000	-.0292160288	-.0151777018
	Chang	-.023842133*	.0035772245	.000	-.0308612962	-.0168229691
	I.C.S	-.023978341*	.0035772245	.000	-.0309975043	-.0169591773
	Laarhoven	-.012993375*	.0035772245	.000	-.0200125388	-.0059742117
	RowSum	-.0013703338	.0035772245	.702	-.0083894974	.0056488297
	Wang	-.014936115*	.0035772245	.000	-.0219552786	-.0079169516
I.C.S	Arithmetic Mean	.0220780391*	.0035772245	.000	.0150588756	.0290972027
	Boender	.0136865964*	.0035772245	.000	.0066674329	.0207057599
	Buckley	.0017814755	.0035772245	.619	-.0052376880	.0088006390
	Chang	.0001362081	.0035772245	.970	-.0068829554	.0071553717
	Geometric Mean	.0239783408*	.0035772245	.000	.0169591773	.0309975043
	Laarhoven	.0109849655*	.0035772245	.002	.0039658020	.0180041291
	RowSum	.0226080070*	.0035772245	.000	.0155888434	.0296271705
	Wang	.0090422257*	.0035772245	.012	.0020230622	.0160613892
Laarhoven	Arithmetic Mean	.0110930736*	.0035772245	.002	.0040739101	.0181122371
	Boender	.0027016309	.0035772245	.450	-.0043175327	.0097207944
	Buckley	-.009203490*	.0035772245	.010	-.0162226535	-.0021843265
	Chang	-.010848757*	.0035772245	.002	-.0178679209	-.0038295939
	Geometric Mean	.0129933753*	.0035772245	.000	.0059742117	.0200125388
	I.C.S	-.010984966*	.0035772245	.002	-.0180041291	-.0039658020
	RowSum	.0116230414*	.0035772245	.001	.0046038779	.0186422050
	Wang	-.0019427398	.0035772245	.587	-.0089619034	.0050764237
RowSum	Arithmetic Mean	-.0005299678	.0035772245	.882	-.0075491314	.0064891957
	Boender	-.008921411*	.0035772245	.013	-.0159405741	-.0019022470
	Buckley	-.020826531*	.0035772245	.000	-.0278456950	-.0138073679
	Chang	-.022471799*	.0035772245	.000	-.0294909624	-.0154526353
	Geometric Mean	.0013703338	.0035772245	.702	-.0056488297	.0083894974
	I.C.S	-.022608007*	.0035772245	.000	-.0296271705	-.0155888434
	Laarhoven	-.011623041*	.0035772245	.001	-.0186422050	-.0046038779
	Wang	-.013565781*	.0035772245	.000	-.0205849448	-.0065466177
Wang	Arithmetic Mean	.0130358134*	.0035772245	.000	.0060166499	.0200549770
	Boender	.0046443707	.0035772245	.194	-.0023747928	.0116635342
	Buckley	-.007260750*	.0035772245	.043	-.0142799137	-.0002415866
	Chang	-.008906018*	.0035772245	.013	-.0159251811	-.0018868540
	Geometric Mean	.0149361151*	.0035772245	.000	.0079169516	.0219552786
	I.C.S	-.009042226*	.0035772245	.012	-.0160613892	-.0020230622
	Laarhoven	.0019427398	.0035772245	.587	-.0050764237	.0089619034
	RowSum	.0135657813*	.0035772245	.000	.0065466177	.0205849448

Based on observed means.
The error term is Mean Square(Error) = .001.
*. The mean difference is significant at the 0.05 level.

Table B.17: Performance analysis among models when $\beta = 200\%$

Dependent Variable: MaxError
LSD

(I) Model	(J) Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Arithmetic Mean	Boender	-.0008628391	.0039665181	.828	-.0086458672	.0069201891
	Buckley	-.030468942*	.0039665181	.000	-.0382519699	-.0226859135
	Chang	-.024972593*	.0039665181	.000	-.0327556215	-.0171895652
	Geometric Mean	.0023761876	.0039665181	.549	-.0054068406	.0101592158
	I.C.S	-.022155039*	.0039665181	.000	-.0299380670	-.0143720107
	Laarhoven	-.008863650*	.0039665181	.026	-.0166466777	-.0010806214
	RowSum	.0004391193	.0039665181	.912	-.0073439089	.0082221474
	Wang	-.013132549*	.0039665181	.001	-.0209155776	-.0053495213
Boender	Arithmetic Mean	.0008628391	.0039665181	.828	-.0069201891	.0086458672
	Buckley	-.029606103*	.0039665181	.000	-.0373891308	-.0218230745
	Chang	-.024109754*	.0039665181	.000	-.0318927824	-.0163267261
	Geometric Mean	.0032390267	.0039665181	.414	-.0045440015	.0110220548
	I.C.S	-.021292200*	.0039665181	.000	-.0290752279	-.0135091716
	Laarhoven	-.008000810*	.0039665181	.044	-.0157838386	-.0002177823
	RowSum	.0013019584	.0039665181	.743	-.0064810698	.0090849865
	Wang	-.012269710*	.0039665181	.002	-.0200527385	-.0044866822
Buckley	Arithmetic Mean	.0304689417*	.0039665181	.000	.0226859135	.0382519699
	Boender	.0296061026*	.0039665181	.000	.0218230745	.0373891308
	Chang	.0054963484	.0039665181	.166	-.0022866798	.0132793765
	Geometric Mean	.0328451293*	.0039665181	.000	.0250621011	.0406281575
	I.C.S	.0083139028*	.0039665181	.036	.0005308747	.0160969310
	Laarhoven	.0216052921*	.0039665181	.000	.0138222640	.0293883203
	RowSum	.0309080610*	.0039665181	.000	.0231250328	.0386910891
	Wang	.0173363923*	.0039665181	.000	.0095533641	.0251194204
Chang	Arithmetic Mean	.0249725933*	.0039665181	.000	.0171895652	.0327556215
	Boender	.0241097543*	.0039665181	.000	.0163267261	.0318927824
	Buckley	-.0054963484	.0039665181	.166	-.0132793765	.0022866798
	Geometric Mean	.0273487810*	.0039665181	.000	.0195657528	.0351318091
	I.C.S	.0028175545	.0039665181	.478	-.0049654737	.0106005827
	Laarhoven	.0161089438*	.0039665181	.000	.0083259156	.0238919720
	RowSum	.0254117126*	.0039665181	.000	.0176286845	.0331947408
	Wang	.0118400439*	.0039665181	.003	.0040570158	.0196230721
Geometric Mean	Arithmetic Mean	-.0023761876	.0039665181	.549	-.0101592158	.0054068406
	Boender	-.0032390267	.0039665181	.414	-.0110220548	.0045440015
	Buckley	-.032845129*	.0039665181	.000	-.0406281575	-.0250621011
	Chang	-.027348781*	.0039665181	.000	-.0351318091	-.0195657528
	I.C.S	-.024531226*	.0039665181	.000	-.0323142546	-.0167481983
	Laarhoven	-.011239837*	.0039665181	.005	-.0190228653	-.0034568090
	RowSum	-.0019370683	.0039665181	.625	-.0097200965	.0058459598
	Wang	-.015508737*	.0039665181	.000	-.0232917652	-.0077257089
I.C.S	Arithmetic Mean	.0221550389*	.0039665181	.000	.0143720107	.0299380670
	Boender	.0212921998*	.0039665181	.000	.0135091716	.0290752279
	Buckley	-.008313903*	.0039665181	.036	-.0160969310	-.0005308747
	Chang	-.0028175545	.0039665181	.478	-.0106005827	.0049654737
	Geometric Mean	.0245312265*	.0039665181	.000	.0167481983	.0323142546
	Laarhoven	.0132913893*	.0039665181	.001	.0055083611	.0210744175
	RowSum	.0225941581*	.0039665181	.000	.0148111300	.0303771863
	Wang	.0090224894*	.0039665181	.023	.0012394613	.0168055176
Laarhoven	Arithmetic Mean	.0088636496*	.0039665181	.026	.0010806214	.0166466777
	Boender	.0080008105*	.0039665181	.044	.0002177823	.0157838386
	Buckley	-.021605292*	.0039665181	.000	-.0293883203	-.0138222640
	Chang	-.016108944*	.0039665181	.000	-.0238919720	-.0083259156
	Geometric Mean	.0112398372*	.0039665181	.005	.0034568090	.0190228653
	I.C.S	-.013291389*	.0039665181	.001	-.0210744175	-.0055083611
	RowSum	.0093027688*	.0039665181	.019	.0015197407	.0170857970
	Wang	-.0042688999	.0039665181	.282	-.0120519280	.0035141283
RowSum	Arithmetic Mean	-.0004391193	.0039665181	.912	-.0082221474	.0073439089
	Boender	-.0013019584	.0039665181	.743	-.0090849865	.0064810698
	Buckley	-.030908061*	.0039665181	.000	-.0386910891	-.0231250328
	Chang	-.025411713*	.0039665181	.000	-.0331947408	-.0176286845
	Geometric Mean	.0019370683	.0039665181	.625	-.0058459598	.0097200965
	I.C.S	-.022594158*	.0039665181	.000	-.0303771863	-.0148111300
	Laarhoven	-.009302769*	.0039665181	.019	-.0170857970	-.0015197407
	Wang	-.013571669*	.0039665181	.001	-.0213546969	-.0057886405
Wang	Arithmetic Mean	.0131325494*	.0039665181	.001	.0053495213	.0209155776
	Boender	.0122697103*	.0039665181	.002	.0044866822	.0200527385
	Buckley	-.017336392*	.0039665181	.000	-.0251194204	-.0095533641
	Chang	-.011840044*	.0039665181	.003	-.0196230721	-.0040570158
	Geometric Mean	.0155087370*	.0039665181	.000	.0077257089	.0232917652
	I.C.S	-.009022489*	.0039665181	.023	-.0168055176	-.0012394613
	Laarhoven	.0042688999	.0039665181	.282	-.0035141283	.0120519280
	RowSum	.0135716687*	.0039665181	.001	.0057886405	.0213546969

Based on observed means.
The error term is Mean Square(Error) = .001.
*. The mean difference is significant at the 0.05 level.