

**RELIABILITY ASSESSMENT BASED ON
STRUCTURAL HEALTH MONITORING DATA
AND BAYESIAN UPDATING OF STRUCTURAL
MODELS**

**A Thesis Submitted to
the Graduate School
İzmir Institute of Technology
in Partial Fulfillment of the Requirements for the Degree of
DOCTOR OF PHILOSOPHY
in Civil Engineering**

**by
Ertuğrul Türker UZUN**

**June 2024
İZMİR**

We approve the thesis of **Ertuğrul Türker UZUN**

Examining Committee Members:

Prof. Dr. Engin AKTAŞ

Department of Civil Engineering, İzmir Institute of Technology

Prof. Dr. Ufuk YAZGAN

Earthquake Engineering and Disaster Management Institute, İstanbul Technical University

Assoc. Prof. Dr. Gürsoy TURAN

Department of Civil Engineering, İzmir Institute of Technology

Assoc. Prof. Dr. Emre ERCAN

Department of Civil Engineering, Ege University

Assoc. Prof. Dr. Sadık Can GİRGIN

Department of Civil Engineering, Dokuz Eylül University

25 June 2024

Prof. Dr. Engin AKTAŞ

Supervisor,
Department of Civil Engineering
İzmir Institute of Technology

Assoc. Prof. Dr. Çağlayan HIZAL

Co-Supervisor
Department of Civil Engineering,
Ege University

Prof. Dr. Cemalettin DÖNMEZ

Head of the Department of Civil
Engineering

Prof. Dr. Mehtap EANES

Dean of the Graduate School

ACKNOWLEDGEMENTS

First of all, I would like to thank my advisor, Prof. Dr. Engin AKTAŞ, for his motivation, patience, constant support, and spending his valuable time to help with the work done in the thesis. I would like to thank my co-advisor, Assoc. Prof. Dr. Çağlayan HIZAL, for his support, predictions, and assistance throughout my thesis work.

Secondly, I would like to express gratitude to the doctoral committee members, Prof. Dr. Ufuk YAZGAN and Assoc. Prof. Dr. Gürsoy TURAN, for their kind comments and suggestions during the progress of the thesis. Their important contributions revealed the main objectives of this study. Additionally, I would like to thank examination committee members Assoc. Prof. Dr. Emre ERCAN and Assoc. Prof. Dr. Sadık Can GİRGİN for contributing their valuable comments to my thesis defense.

I would like to express my greatest gratitude to my family, who have always supported me financially and morally throughout my educational life. I would like to thank my father Hakan UZUN and my mother Aysun UZUN, who were my first teachers, my biggest supporters, and who made the biggest contribution to my upbringing, for everything they did. I would like to thank my sister, Şebnem UZUN, for always being with me and for her endless love and care. I am grateful to my wife, Zahide UZUN, for always being there for me and providing her unlimited support at every opportunity. I could not have published this thesis without her support. I would like to thank my grandfather and grandmother for their attention, love, and endless support at every moment of my life. Finally, I would like to thank my son, Hakan Yağız UZUN, who is the new addition to our family and my driving force in writing this thesis, for the happiness you have given me.

ABSTRACT

RELIABILITY ASSESSMENT BASED ON STRUCTURAL HEALTH MONITORING DATA AND BAYESIAN UPDATING OF STRUCTURAL MODELS

Finite element (FE) models are commonly used in numerical modeling of structures, but their assumptions can lead to inaccuracies and uncertainties. To address this, FE model update methods have been developed, calibrating the model based on structural health monitoring (SHM) data. However, a general framework for realistic life cycle performance assessment of structures using monitored data has not yet been presented. Bayesian modeling can characterize uncertain structural parameters as random variables, but it is complex and time-consuming. Metamodeling techniques, which are effective stochastic predictors, can be used to decrease the computational burden of model updating. Adapting a Polynomial-Chaos-Kriging (PCK) metamodeling technique to Bayesian model updating in order to reduce uncertainty and circumvent computational challenges using SHM data in order to assess the reliability of structures more precisely is the objective of this research. Therefore, the effectiveness of the proposed method has been tried and demonstrated through experimental and numerical studies. An experimental study of a bridge column is used to evaluate the reliability of structures subjected to various corrosion effects. As a result, the proposed solution method reduces computational costs and enables an updated FE model to be closer to real structure measurements. The updated models are found to be more reliable in reliability evaluations, providing more accurate predictions on issues like structure safety, service life, and maintenance cost compared to non-updated models.

ÖZET

YAPISAL SAĞLIK İZLEME VERİLERİNE VE YAPISAL MODELLERİN BAYES GÜNCELLEMESİNE DAYALI GÜVENİLİRLİK DEĞERLENDİRMESİ

Sonlu elemanlar (FE) modelleri yapıların sayısal modellenmesinde yaygın olarak kullanılmaktadır ancak varsayımları yanlışlıklara ve belirsizliklere yol açabilmektedir. Bu sorunu çözmek için, modeli yapısal sağlık izleme (SHM) verilerine dayalı olarak kalibre eden FE modeli güncelleme yöntemleri geliştirilmiştir. Ancak izlenen verileri kullanan yapıların gerçekçi yaşam döngüsü performans değerlendirmesine yönelik genel bir çerçeve henüz sunulmamıştır. Bayes modellemesi belirsiz yapısal parametreleri rastgele değişkenler olarak karakterize edebilir, ancak karmaşık ve zaman alıcıdır. Etkili stokastik tahminciler olan meta-modelleme teknikleri, model güncellenmenin hesaplama yükünü azaltmak için kullanılabilir. Yapıların güvenilirliğini daha kesin bir şekilde değerlendirmek için SHM verilerini kullanarak belirsizliği azaltmak ve hesaplama zorluklarını aşmak amacıyla Polynomial-Chaos-Kriging (PCK) meta-modelleme tekniğini Bayezyen model güncellemesine uyarlamak bu araştırmanın amacıdır. Bu nedenle önerilen yöntemin etkinliği deneysel ve sayısal çalışmalarla denenmiş ve gösterilmiştir. Çeşitli korozyon etkilerine maruz kalan yapıların güvenilirliğini değerlendirmek için bir köprü kolonunun deneysel bir çalışması kullanılır. Sonuç olarak, önerilen çözüm yöntemi hesaplama maliyetlerini azaltır ve güncellenmiş bir FE modelinin gerçek yapı ölçümlerine daha yakın olmasını sağlar. Güncellenen modellerin güvenilirlik değerlendirmelerinde daha güvenilir olduğu, yapı güvenliği, servis ömrü, bakım maliyeti gibi konularda güncellenmeyen modellere göre daha doğru tahminler sağladığı görüldü.

TABLE OF CONTENTS

LIST OF FIGURES	ix
LIST OF TABLES.....	xi
CHAPTER 1. INTRODUCTION	xii
1.1. Background	1
1.2. Research Problems.....	4
1.3. Aim and Objectives of the Thesis	4
1.4. Importance of the Thesis.....	5
1.5. Scope of the Thesis	6
1.6. Thesis Outlines.....	6
CHAPTER 2. LITERATURE REVIEW	8
2.1. Introduction.....	8
2.2. Overview of SHM.....	8
2.3. Overview of FE Model Updating.....	11
2.3.1. FE Model Updating Definitions in Literature.....	12
2.3.2. Classification of FE Model Updating Methods	13
2.3.2.1. Iterative Deterministic Methods	13
2.3.2.2. Iterative Stochastic Methods	16
2.4. Summary and Implications	22
CHAPTER 3. THEORETICAL BACKGROUND	24
3.1. Introduction.....	24
3.2. Classification of Modal Identification Techniques.....	24
3.2.1. The Pick Picking Method (PPM)	26
3.2.2. Least Square Complex Exponential Method (LSCEM).....	26
3.2.3. A Flowchart for Identification of Modal Properties	27
3.3. Bayesian Model Updating.....	29
3.4. Metamodeling Techniques.....	32
3.4.1. Basics of Kriging.....	33
3.4.1.1. Correlation Functions	33

3.4.1.2. Types of Trend.....	34
3.4.1.3. Error Estimation	34
3.4.2. Basics of PCE.....	35
3.4.2.1. Constructing Polynomial Basis	35
3.4.2.2. Computation of Coefficients	37
3.4.2.3. Error Estimation	37
3.4.3. Basics of PCK	39
3.4.3.1. Error Estimation	39
3.4.4. Sampling Strategy in Metamodeling.....	40
3.5. Reliability Assessment of Structures	41
3.5.1. Estimation of pf	43
3.5.2. Subset Simulation.....	43
3.5.3. Target Reliability Index	44
3.6. Summary of Proposed Bayesian Model Updating with Metamodeling Techniques	46
 CHAPTER 4. SHM-BASED BAYESIAN MODEL UPDATING WITH METAMODELING TECHNIQUES.....	49
4.1. Introduction.....	49
4.2. Numerical Examples	51
4.2.1. A Truss Bridge Example.....	51
4.2.2. Irvine Bridge Column	60
4.2.2.1. Finite Element Model of Irvine Bridge Column	63
4.2.2.2. Modal Parameter Identification of Irvine Bridge Column Based on Monitoring Data.....	64
4.2.2.3. Results of Bayesian FE Model Updating of the Irvine Bridge Column	66
4.3. Conclusion	69
 CHAPTER 5. RELIABILITY ASSESSMENT OF STRUCTURES WITH BAYESIAN MODEL UPDATING ACCELERATED via POLYNOMIAL-CHAOS- KRIGING METAMODELING	70
5.1. Introduction.....	70
5.2. Irvine Bridge Column	71

5.2.1. Finite Element Model of Irvine Bridge Column	71
5.2.2. Results of Bayesian FE Model Updating of the Irvine Bridge Column.....	74
5.2.3. Calculation of The Compressive Strength Based on Elastic Modulus and Density	77
5.2.3.1. Modeling Concrete and Reinforcement Steel.....	78
5.3. Time-Dependent Resistance Deterioration of Bridge Column	81
5.4. Time-Dependent Resistance Deterioration of Bridge Column	82
5.4.1. Deterioration Models of Reinforcement and Concrete	84
5.5. The Strength of Confined Concrete Columns Under Uniaxial Compression.....	88
5.6. Deformation Relationship of Bridge Column	92
5.7. Moment Capacity of Bridge Column.....	98
5.8. Reliability Assessment of Bridge Column.....	99
5.8.1. Drift Capacity Model for Bridge Column.....	100
5.8.2. Under Different Vehicle Loads	102
5.9. Conclusion	106
CHAPTER 6. CONCLUSIONS	108
REFERENCES	110

LIST OF FIGURES

<u>Figure</u>	<u>Page</u>
Figure 3.1. Flow chart for identification of modal properties	28
Figure 3.2. Plots of the different sampling strategies are shown for comparison.....	40
Figure 3.3. Characteristic flowchart for reliability assessment of structures.....	44
Figure 3.4. Traditional and proposed methodologies for probabilistic analysis of FE models	47
Figure 3.5. Proposed framework for SHM-based Bayesian model updating accelerated by metamodeling technique and reliability assessment	48
Figure 4.1. Truss bridge model.....	52
Figure 4.2. Comparison FE% for Case-1 after and before model updating	56
Figure 4.3. Comparison MAC for Case-1 after and before model updating	57
Figure 4.4. Comparison FE% for Case-2 after and before model updating	57
Figure 4.5. Comparison MAC for Case-2 after and before model updating	58
Figure 4.6. Comparison FE% for Case-3 after and before model updating	58
Figure 4.7. Comparison MAC for Case-3 after and before model updating	59
Figure 4.8. Bridge column (a) The illustration of the test configuration (LANL, 1997) (b) dimensions (cm) and (c) cross section and reinforcements layout	61
Figure 4.9. (a) Accelerometer locations (as cm) of Irvine bridge column test and (b) typical input power spectrum.....	62
Figure 4.10. FE model of Irvine bridge column	63
Figure 4.11. The stabilization diagram from the vibration data by LSCEM	65
Figure 4.12. Comparison of Mode Shapes from experiment and FE model analysis	65
Figure 4.13. Comparison FE% after and before model updating	68
Figure 4.14. Comparison MAC after and before model updating	68
Figure 5.1. FE model of Irvine bridge column	72
Figure 5.2. Comparison of Mode Shapes from experiment and FE model analysis	73
Figure 5.3. Comparison FE% after and before model updating	76
Figure 5.4. Comparison MAC after and before model updating	76
Figure 5.5. Stress-strain model for confined and unconfined concrete	78
Figure 5.6. Original bridge column and retrofitted jacket	80

<u>Figure</u>	<u>Page</u>
Figure 5.7. Stress-strain of concrete blocks (a) before updating (b) after updating	80
Figure 5.8. Stress-strain of reinforcements	81
Figure 5.9. Possible places of bridge column for surface chloride ion concentration, C_{sc}	83
Figure 5.10. Probable (a) uniform and (b) non-uniform corrosion effect on bridge column	85
Figure 5.11. The change in strength of concrete, f_{tc}/f_{c0} due to corrosion depends on different zones and volume of air-born salt, C_{sc}	87
Figure 5.12. The confined concrete (a) core (b) spiral (c) spiral stress layout	89
Figure 5.13. The axial load capacity change ratio, $N_{cc}(t)/N_c$ due to uniform corrosion	91
Figure 5.14. The axial load capacity change ratio, $N_{cc}(t)/N_c$ due to non-uniform corrosion	91
Figure 5.15. Cantilever column bending moment distribution and approximated curvature distribution	92
Figure 5.16. Idealized plastic hinge analysis for uniform corrosion	93
Figure 5.17. Idealized plastic hinge analysis for non-uniform corrosion	96
Figure 5.18. Ultimate drift ratio, $DR_u\%$ for uniform corrosion	97
Figure 5.19. Ultimate drift ratio, $DR_u\%$ for non-uniform corrosion (a) at splash zone (b) at tidal zone	97
Figure 5.20. Distribution of strain and force in circular cross-section subjected to the moment (a) original column (b) retrofitted part.....	99
Figure 5.21. Reliability index values, β for uniform corrosion cases	101
Figure 5.22. Reliability index values, β for non-uniform corrosion case (a) at splash zone (b) at tidal zone.....	101
Figure 5.23. Graphic illustration for vehicle type.....	102
Figure 5.24. Reliability index values, β before updating for uniform corrosion cases	104
Figure 5.25. Reliability index values, β before updating for non-uniform corrosion cases (a) at splash zone (b) at tidal zone	104
Figure 5.26. Reliability index values, β after updating for uniform corrosion cases...	105
Figure 5.27. Reliability index values, β after updating for non-uniform corrosion cases (a) at splash zone (b) at tidal zone	105

LIST OF TABLES

<u>Table</u>	<u>Page</u>
Table 3.1. MCMC: Metropolis-Hasting algorithm (Mao et al., 2020)	32
Table 3.2. Target reliability index for standards EN 1990 and ISO 2394 at different time periods (Nadolski et al., 2019).....	45
Table 3.3. Target reliability indices (JCSS, 2001).....	45
Table 3.4. Target reliability indices for load conditions (not include earthquake) (ASCE/SEI 7-10, 2016)	46
Table 4.1. Nominal values test and FE model	52
Table 4.2. Identified frequencies and MAC values FEM for Case-1	53
Table 4.3. Identified frequencies and MAC values FEM for Case-2	53
Table 4.4. Identified frequencies and MAC values FEM for Case-3	54
Table 4.5. Updated model parameters for Case-1 with meta-models.....	55
Table 4.6. Updated model parameters for Case-2 with meta-models.....	55
Table 4.7. Updated model parameters for Case-3 with meta-models.....	56
Table 4.8. Comparison for modal properties from vibration test and initial FEM.....	65
Table 4.9. Statistical information of selected parameters used in analysis of the bridge column simulation.....	66
Table 4.10. Updated model parameters with metamodeling techniques	67
Table 4.11. Comparison FE% and MAC after model updating.....	67
Table 5.1. Comparison for modal properties from LANL and this study	72
Table 5.2. Comparison for modal properties from vibration test and initial FE model..	73
Table 5.3. Statistical information of selected parameters used in analysis of the bridge column simulation.....	74
Table 5.4. Updated model parameters by PCK	75
Table 5.5. Comparison FE% and MAC after model updating.....	75
Table 5.6. Compressive strength of concrete, f_c' before and after updating	78
Table 5.7. Statistics of loads and reinforced concrete strength	81
Table 5.8. Surface chloride ion concentration, C_{sc} (kg/m ³) by JSCE (2010)	83
Table 5.9. Chloride induced corrosion rates, $icor$ for various exposures	84

Table 5.10. Random variables for axle weights and axle spacing (dimensions for axle weights and axle spacing are in kN and meter, respectively).....	103
--	-----



CHAPTER 1

INTRODUCTION

1.1. Background

Engineering structures such as bridges, tunnels, industrial structures, power plants, coastal structures, etc. play an important role in providing resources for communities. Due to the importance, use, and sensitivity of such structures, there is a need for both proper implementations of maintenance and repair plans and effective evaluation procedures. In this context, economically sustainable maintenance practices for such structures are needed over time. Besides, it is necessary to understand the safety and serviceability of critical components.

Although it was very difficult to examine such structures in the past, it is now possible with developing technologies. Especially in the last decade, there have been significant advances in modeling and analysis methods for engineering structures exposed to loadings from natural disasters. Finite element (FE) models of important engineering structures can predict damage and failure mechanisms that develop gradually in a severe loading environment or over time. This means that while the fidelity of the numerical models of the studied structures has increased, these models have become more complex, and the computational times required to carry out an analysis have become very large. Additionally, due to the various assumptions used to create numerical models, these models may not always reveal the actual behavior of structures. Combining experimental studies and FE model updating methods can be considered to decrease the differences between the actual and predicted structural behavior of these structures. Measuring errors and uncertainties related to model assumptions is important in determining the numerical model's reliability and accuracy. This has led to the need to develop FE model updating methods that target adjusting the numerical model based on the actual behavior of the structure.

In the FE modeling process, modeling errors that occur due to many different reasons, when taken together, are generally called aleatory and epistemic uncertainties. In a general sense, aleatory uncertainty denotes an inherent randomness of natural

occurrences governed by probabilistic models, while epistemic uncertainty is induced by a lack of complete knowledge such as insufficient data, inaccuracies in measurements, inadequate models, etc. At this point, real-time data obtained for a structural health monitoring (SHM) procedure can be manipulated to manage the uncertainty. Increased information may reduce the epistemic uncertainty and make the predictions more reliable. In addition, even if a structure is designed to withstand all possible loadings throughout its life, various unforeseen situations may be encountered during the design phase. SHM technologies have the ability to address these concerns and attract the attention of researchers. However, a general framework that offers a realistic life cycle performance assessment of structures by efficiently incorporating monitored data into structural identification has not yet been perfected and remains a subject of research. In general, FE model update methods are based on deterministic or Bayesian approaches. In the deterministic model update, computational optimization techniques are used to guess the unknown parameters in a way that keeps the difference between the measurements and the guess FE model response as small as possible. However, this method cannot precisely account for all uncertainties, resulting in overfitting and inaccurate parameter estimates. It cannot measure the estimation uncertainty for unknown parameters.

In contrast, Bayesian modeling techniques can characterize uncertain structural parameters as random variables and attract more attention in the field of FE model updating. Here, the parameters of the probability density function (PDF) are treated as random variables in the Bayesian approach, and unknown parameters are modeled as random variables. The prior information is combined using a probability distribution and updated to a posterior distribution that takes into account the measurement data. The posterior distribution encodes the remaining forecast uncertainty after the model is updated. Hence, this process can provide a systematic methodology for integrating a probabilistic SHM framework into the model updating procedure.

In contrast, when dealing with actual structural systems, the determination of joint and marginal PDFs necessitates the consideration of numerous parameters. This creates high-dimensional integrals that need to be solved using sampling or approximation methods like the Markov Chain Monte Carlo (MCMC) methods. These approaches require iterative assessment of the likelihood function, which in turn requires iterative assessment of the FE model. Therefore, sampling the high-dimensional joint posterior containing FE models of complex engineering structures would be extremely computationally costly.

Despite their successful applications, these methods often lead to complex, time-consuming, and computationally costly processes, causing limitations in their application to real structures. Consequently, the computational cost of Bayesian FE model updating for complex engineering structures is likely to become unaffordable and unfeasible. In order to address this challenge, a mathematical model is implemented in place of the FE model, which approximates the relationship between the preselected inputs and outputs of the model. By making modifications to the initial finite element (FE) model, an analytically and computationally more convenient proxy is produced. The metamodeling techniques (or surrogate modeling), which are effective stochastic predictors, decrease the computational burden in the model updating process. In this context, some variants of metamodeling techniques, such as the polynomial-based response surface method, the Kriging predictor, etc., are widely used in the literature. Among these, Kriging can be simply defined as a statistical interpolation method that uses Gaussian processes to interpolate various complex functions. Moreover, polynomial chaos expansion (PCE), another metamodeling technique, aims at providing a functional approximation of a computational model through its spectral representation of polynomial functions on a properly constructed basis. While Kriging interpolates local variations of the computational model output as a function of neighboring experimental design points, PCE well approximates the global behavior of this computational model output. As a general intention in the literature, it can be observed that the studies dealing with these approaches are implemented for model updating separately. However, a more accurate meta-model can be obtained by combining the global and local approaches of these two techniques. Polynomial-Chaos-Kriging (PCK), based on the well-established metamodeling techniques PCE and Kriging, is to be seen as the most prominent metamodeling algorithm. PCK uses the regression-type PCE to capture the computational model's global behavior, as well as the interpolation-type Kriging to capture local variations. This integration results in a much more efficient metamodeling technique.

In conclusion, there are two main problems encountered in FE model updating. The heavy computational burden and different sources of uncertainty must be addressed, especially when it comes to complex models of real structures. Probabilistic approaches like Bayesian, which are more reasonable than deterministic ones, have been presented in this context, but most of these techniques are still computationally expensive. Lately, the use of metamodeling techniques has become widespread to achieve this goal. However, since this technique is a relatively new research field, its performance and

applicability to Bayesian FE model updating in different types of real structures are rarely studied in the literature. In the structural definition of the updated FE model, environmental factors and/or operational variables that have an effect on the model must also be accounted for. Few approaches have thus far been capable of addressing all of these challenges simultaneously. Consequently, a comprehensive framework applicable to complex engineering structures remains essential.

1.2. Research Problems

Based on the previous explanations, it has been seen that creating an FE model with high reliability throughout the service life with an effective and sustainable FE model updating approach can enable different goals to be met in the future, such as an accurate performance evaluation of structures under different situations and the development of the structural design concept. In creating the updated FE model, different sources of uncertainty should be taken into account, reducing them by integrating structural health monitoring data into the comprehensive framework as well as reducing the computational burden. In this context, the following research problems are defined and subject to examination:

- How can the two main challenges encountered in FE model updating (computational costs and uncertainties from different sources) be solved?
- How can data from SHM be incorporated into the FE model updating framework?
- Although there have been successful applications with the Bayesian FE model update, how can the need for different applications in this field be met by facilitating applications on real structures?
- How can the data obtained from SHM using the FE model update framework be more effectively used in the reliability assessment of structures?

1.3. Aim and Objectives of the Thesis

This study aims at adapting a PCK-based methodology to Bayesian model updating to overcome the computational difficulties and reduce different sources of

uncertainty with SHM data, then making more reasonable reliability assessments for complex engineering structures. The proposed method for updating the FE model is aimed at decreasing the inconsistency between the measured responses of structures and the simulated responses from FE models by increasing the accuracy of the structural definition and handling the impacts and reliability assessments of the structures throughout their lifespan effectively, sustainably, and practically. In this context, the research objectives are listed as follows:

- Development of a Bayesian approach that will include external effects on the structure in the model updating process with data obtained through SHM in order to obtain a more effective and reliable structural definition.
- Determination of the method to be used in the development of the FE model update, which is low in terms of computational cost, provides ease of use, and can be applied to real engineering structures, taking into account the different uncertainties encountered in modeling.
- Incorporating data obtained from SHM into an accelerated Bayesian FE model update using metamodeling techniques in structural reliability assessment.
- The aim is to demonstrate the degree of reduction in inconsistencies between the measured values of real structures and the updated FE models that result from incorporating various metamodeling techniques commonly used in the literature into the proposed solution method.
- Applying the proposed method to the real structure and performing structural reliability assessments as a result of different external effects such as loading, environmental conditions, etc.

1.4. Importance of the Thesis

The method presented in the study is important because it is both low in terms of computational cost and can overcome basic problems by taking into account different uncertainties, significantly reducing the inconsistency between FE models of real engineering structures and measurement values obtained from SHM. In addition, it is important in terms of providing a framework for transforming the data obtained from

SHM into meaningful information when performing the reliability assessment of structures.

Although metamodeling techniques are a relatively new research area in the literature due to the ease they provide in calculations, their applicability to Bayesian FE model updating of complex engineering structures is rarely studied and is open to development. This serves as a focal point of the study and highlights another significant aspect of the research.

1.5. Scope of the Thesis

By using the information obtained through SHM, it is expected to reduce epistemic uncertainty and make predictions about the structure more reliable. A Bayesian approach can provide a systematic methodology for integrating a probabilistic SHM framework into the model updating procedure. However, when considering real structural systems, a large number of parameters are required for the calculation of joint and marginal PDFs, leading to high-dimensional integrals that require the use of approximate measurements or sampling methods to solve. These processes are extremely costly in terms of calculations and make it difficult to update FE models of complex engineering structures. To overcome this difficulty, the solution method should include metamodeling techniques, which are effective stochastic estimators.

In this context, the different metamodeling techniques are used with the proposed solution methodology. A truss bridge example and an experimental study using the vibration data of the Irvine bridge column project at Los Alamos National Laboratory are used to show the effectiveness of the solution methodology. A reliability assessment of a bridge column is made when it is exposed to different types of corrosion effects. The results are given comparatively for updated and non-updated FE models of bridge column. This is how the study's general scope is designed.

1.6. Thesis Outlines

The main research objectives of the study are examined under four main headings, as follows:

- Chapter 2 provides information about the literature studies on the subject. The methods and developments used to update FE models are examined in detail. Furthermore, the effects on structural reliability are also investigated. The issues that the researchers focused on are examined, and the developments are discussed chronologically. While the subject's developments and applications are mentioned, the points that are considered deficient and open to improvement are emphasized.
- In Chapter 3, the classification of modal identification techniques and the theoretical background of the proposed Bayesian model updating with metamodeling techniques are presented. A framework for SHM-based Bayesian model updating accelerated by metamodeling techniques and reliability assessment is proposed.
- In Chapter 4, one numerical study (truss bridge) and one experimental study (Irvine bridge column) are conducted in this section in order to show the effectiveness of the proposed methodology. Different metamodeling techniques on the same examples are considered, and results are compared with each other.
- In Chapter 5, the Irvine Bridge column is updated throughout the proposed methodology. Then, corrosion effects for different conditions are applied to the bridge column, and reliability assessments are performed before and after FE model updating.

CHAPTER 2

LITERATURE REVIEW

2.1. Introduction

Developments about SHM-based Bayesian FE model updating and reliability assessment of structures, which are the subject of the study, are discussed in this section. The developments in the SHM and FE model updates are discussed separately, and information is also given about the studies that use them together. As a result, in this section, the developments that are needed in the literature and constitute the subject of research have been tried to be given as a result of the reviews. The difficulties and research gaps encountered in updating the FE model of structures are given in detail. In addition, the effects of developments in these applications on structural reliability assessments are also investigated.

2.2. Overview of SHM

Throughout its service life, a structure is expected to meet safety, durability, and serviceability requirements. However, in this process, the structure may be exposed to different external effects (natural disasters, corrosion, fatigue, etc.) and may experience slow or sudden damage to its performance or even collapse. At this point, a decision can be made by analyzing the responses obtained regarding the behavior of the structure (such as a building, a dam, a bridge, or a simple beam) with SHM methods and evaluating its current mechanical condition. Thus, SHM methods enable the evaluation of the safety, durability, and serviceability of the structure throughout its service life.

In the literature, it has been seen that the first applications of SHM were used in the aviation field for damage detection of aircraft, and in the late 1970s, it was used to investigate offshore platforms. The use of SHM in civil engineering applications dates back to the 1990s (Gharehbaghi et al. 2022).

Collins et al. (1974) made it possible to statistically define a structure with the method presented in their study. For this purpose, experimental measurements of natural

frequencies and mode shapes were used to change the structural parameters (like stiffness and mass characteristics) of the finite element model.

Beck and Katafygiotis (1992) presented a probabilistic identification approach for monitoring the global health of the structure by detecting significant changes in the stiffness distribution using vibration measurements.

The Bayesian probabilistic methodology for monitoring structural health was introduced by Vanik, Beck, and Au (2000). The study suggests a way to figure out the odds that the updated model stiffness parameter values between the starting values and the values that are constantly being updated are less than a certain percentage of the starting values. This is done by using a set of defined modal parameter datasets. Achieving a high probability of reduction in model stiffness is considered representative of damage at the corresponding point of the structure.

Catbas, Susoy, and Frangopol (2008) emphasized that integrating data obtained from SHM into reliability analyses of system and component reliability is important for the management and decision-making processes of a bridge-type structure and that this is a needed research topic. The main focus of their work is to evaluate how structural health monitoring can be used to minimize uncertainties related to difficult-to-model events, for which results are presented on a selected bridge structure.

Hosser, Klinzmann, and Schnetgöke (2008) underlined that SHM can help to ensure the safety of damaged structures, and for this purpose, a framework for probabilistic safety assessment of structures is presented. It was developed by incorporating data obtained from SHM into the framework.

Frangopol, Strauss, and Kim (2008) emphasized that structural health monitoring concepts have become important in intervention planning for new and existing structures, but there is still a strong need for the effective use of data obtained from structural health monitoring in reliability assessment and prediction models. Intervention strategies are affected by updating prediction models based on monitoring data, and since these strategies involve a certain cost, monitoring systems are expected to help spend existing budgets efficiently. Therefore, the study shows that the demand for the effective use of monitoring data is important both in terms of structural reliability and cost. The main aim of the study is to present an approach to effectively incorporating monitoring data into the structural reliability assessment process and further using this data to improve prediction models.

Chen and Omenzetter (2013) underlined that integrating SHM data into reliability analysis procedures can provide a useful and practical methodology for evaluating constructs. Therefore, in their study, they propose an approach to integrating SHM data into a reliability assessment framework.

Since reliability is an important performance index and reliability-based procedures have the capacity to eliminate uncertainties in structural models, responses, loads, and monitoring data, studies have often suggested integrating SHM data into reliability analysis procedures, and solution frameworks have been presented for this. Different types of building applications for this are also discussed in the literature.

Ciang, Lee, and Bang (2008) mentioned the importance of obtaining monitoring data on wind turbines to improve safety issues, minimize downtime, reduce the frequency of sudden failures and the associated large maintenance costs, and recommend SHM systems because they ensure the integrity of the system.

Okasha, Frangopol, and Decò (2010) present an approach to integrating data from SHM in the life cycle performance assessment of ship structures under uncertainty, and the life cycle performance of the ship structure is measured in terms of structural reliability.

Li et al. (2012) utilized data based on SHM technology for the reliability analysis approach of long-span cable-stayed bridges in their studies. For this reason, the framework of structural reliability analysis is presented based on SHM data. Using the monitoring data of a cable-stayed bridge, vehicle and environmental motion models and extreme values of internal force were obtained statistically.

There is increasing interest in SHM as a knowledge-based assessment tool for effective seismic protection and vulnerability reduction of cultural heritage buildings and sites, as well as for measuring and reducing uncertainties regarding their structural performance. Lorenzoni et al. (2016) report on the application of SHM techniques and methodologies for the structural safety and reliability assessment of historic buildings and monuments, aiming to determine the structural behavior of historic buildings with a high level of confidence.

During their service life, structures may be exposed to different external effects, and these effects can significantly affect the performance of the structure. SHM systems are frequently used to observe the effects that will occur over time on the performance of structures under these effects.

Newhook and Edalatmanesh (2013) demonstrated how reliability and SHM concepts can be integrated into bridge evaluation processes, and a steel-free concrete bridge deck system was selected for review. The bridge evaluation model focused on fatigue cracking issues associated with wheel loads resulting from heavy truck traffic. With the methodology they presented, Heitner et al. (2019) wanted to update the estimated corrosion damage level using short-term health monitoring measurement data. For this, a reinforced concrete (RC) slab was examined.

When the studies in the literature are examined in general, it is seen that structural performance assessments are a tool for monitoring the safety of a structure throughout its lifecycle and play an important role in determining efficient and long-term maintenance strategies. The reliability index is frequently and widely used in structural performance evaluations for civil engineering structures. Structural reliability analyses enable decision-making regarding the investigation of the safety of the structure and the creation of long-term maintenance strategies by estimating the expected probability of damage.

2.3. Overview of FE Model Updating

FE models are frequently employed in the numerical modeling of various structural systems. These models have proven to be highly effective in the identification of structural damage, estimation of service life and reliability, and determination of suitable maintenance strategies. However, with the increasing need for numerical modeling of structures, the need to meet the requirements regarding the accuracy and reliability of the model and the results obtained has also emerged. Therefore, it is necessary to quantify errors and uncertainties associated with model assumptions, which frequently result in inaccuracies and uncertainties, in order to ascertain the numerical model's degree of dependability and precision. The need for this has resulted in the creation of FE model updating techniques that utilize data acquired by SHM methods to calibrate the numerical model.

When updating finite element models, two types of potential uncertainty arrive. One pertains to the predicted finite element model, while the other concerns data obtained experimentally. Inconsistencies between the predicted structure and the behavior of the numerical model are examples of uncertainties associated with the FE model. It is critical to emphasize that while this form of error may be mitigated through practice, it is

unattainable to entirely eradicate. Modeling uncertainties are generally associated with uncertainties resulting from incorrect assumptions of model parameters such as material or section properties. Additionally, incorrect assumptions about geometric shape and structural behavior can also cause uncertainty in the model structure. The model updating methods are applied to obtain more accurate FE models, and the data obtained as part of SHM is most preferred for updating FE models. Obtaining a more accurate FE model will positively affect the results of structural reliability analyses by allowing more accurate predictions about the structure.

2.3.1. FE Model Updating Definitions in Literature

The definition of FE model updating in literature studies is not based on a precise definition. The prominent definitions in the literature are as follows:

Modak, Kundra, and Nakra (2002) state that updating the FE model can be considered an attempt to use the best features of both experimental and analytical models. Marwala, Boulkaibet, and Adhikari (2016) define that the model updating is developed to correct and improve the FE model of the structure based on real behavior. Schommer et al. (2017) emphasize that in her studies, the aim of model updating was to minimize the deviation between simulated results and actual measured values. Shahbaznia, Dehkordi, and Mirzaee (2020) define the FE model updating as the process of updating the initial model of the structure so that it can better reproduce the measured responses of the real structure. Kim and Park (2004) state that updating the FE model generally arises as an optimization problem and is a procedure aimed at minimizing the differences between analytical and experimental results. Girardi et al. (2020) describe the updating of the FE model as a procedure. This procedure aims to calibrate the FE model of the structure to match the numerical and experimental results. Jin et al. (2014) define model updating as the process of changing the initial FE model to represent the dynamic behavior of structures in order to evaluate the condition of existing structures or to realize the dynamic behavior required for structural change.

In general, there is no uniform definition for the FE model updating in different literature works. However, the basis of the definitions is that the FE model updating process is the updating of the numerical model based on experimental data to obtain the actual structural behavior numerically.

2.3.2. Classification of FE Model Updating Methods

FE model updating is divided into two basic headings, such as direct (non-iterative) and indirect (iterative), that can be defined as follows:

Non-iterative model updating methods, the oldest method for updating numerical models, are computationally efficient and accurate, using matrix methods to calculate results and modify structural stiffness and mass matrices. They include matrix update, optimal matrix methods, eigen-structure assignment methods, and Lagrange multiplier methods. An important shortcoming of this approach is that the updated mass and stiffness parameter matrices may not be physically meaningful. Since there is no direct change in the physical parameters of the FE model, the importance of the numerical model decreases, meaning the simulation ability is reduced. That is, the result obtained with these methods cannot reflect the real changes in structural properties while meeting the mathematical problem. Despite the studies on calculation efficiency in the literature, the use of this method has decreased and the use of indirect (iterative) methods has increased.

The construction of an objective function between the input structural parameters and the responses of a numerical model occurs in iterative methods. By adjusting parameters iteratively, the aim is to decrease the discrepancy between the experimental output associated with the numerical model and the output of the model itself. When the deviation from model responses and experimental data remains constant in subsequent iterations, the iteration process concludes. Although these methods have high computational costs, they can represent the behavior of the structure more realistically. In this context, these techniques have other applications in structural engineering, and in the field of civil engineering, iterative methods are preferred for updating FE models of engineering structures. These methods are examined in two groups: deterministic and stochastic methods.

2.3.2.1. Iterative Deterministic Methods

The maximum likelihood method is employed in deterministic approach engineering applications. By employing this approach, the challenge of updating the model is transformed into an optimization problem in which the main aim is to reduce the error between the predicted model output and the observed responses at a different

estimate of the parameter. This procedure is executed through the optimization of an objective function that has been constructed utilizing various forms of experimental and numerical data. The dataset may comprise either static data, structural dynamic properties (e.g., mode shapes and natural frequencies), or a combination of the two.

In this context, Sanayei et al. (1997) use both static displacement and strain measurements in order to successfully evaluate the unknown stiffness parameters of the structural components. Jaishi and Ren (2005) present a practical FE model updating technique for real structures using ambient vibration test results. (Schommer et al. 2017) combine static and dynamic characteristics of a structure in a model-updating procedure using a finite element model. In addition, the formulation of the target created using the frequency response function (FRF) in FE model updating is preferred because it offers some advantages in practice. (Sipple and Sanayei 2014) present a FRF based FE model updating method. This method is used to perform parameter estimation of the University of Central Florida Grid Benchmark Structure. The initial finite element model using measured frequency response functions from the undamaged, intact structure is calibrated by proposed method in this study. Wu et al. (2020) propose a model updating method using FRF based on the Sherman–Morrison formula. In this study, only the initial FRFs and parameter perturbations are employed to calculate the sensitivity, avoiding repeated FE analyses and improving the computational efficiency. By using FRF, errors due to modal fit can be avoided in the FE model updating, so no fit between the predicted and measured mode shapes is required.

Generally, initial indicators in studies are structural dynamic properties (natural frequencies and mode shapes), which are considered the most accurate predictors of the structure's actual behavior. The rationale behind this is that alterations in the structure induce modifications in the stiffness of the structure, subsequently resulting in adjustments to the dynamic properties of the structure. In addition, the use of FRF in FE model updating is preferred due to its ability to adequately reproduce dynamic features.

The modal strain energy can be used to describe the objective function. Srinivas, Ramanjaneyulu, and Jeyasehar (2011) proposed a multi-stage approach for the identification of damage and its quantification using a genetic search algorithm combined with the modal strain energy method. This proposed method aims to overcome the disadvantages associated with the genetic algorithm (GA)-based damage identification approach, especially since the solution parameters will be large in large-scale structures.

Yang et al. (2020) present a new modal strain energy-based model updating method using the Gaussian smoothing technique and a laser scanning vibrometer.

An objective function formulates discrepancies between the FE model and modal features such as natural frequencies and mode shapes, which are considered as reference features obtained from the structure. Depending on the number of modal features considered, the same number of residual come into play, and since this set of modal features are of different nature, the question arises of how to weigh the impact of each residual. Therefore, two approaches have been adopted depending on whether a single-objective function (SOF) or a multi-objective function (MOF) is considered. An optimization technique with SOF is used in traditional FE model updating, and its general formulation can be expressed as follows (Jin et al. 2014):

$$\begin{aligned}
& \min \sum_{i=1}^n w_i F_i(X) = w^T F(X) \\
& \text{subject to } g_j(X) < 0, \quad j = 1, 2, \dots \\
& \quad \quad \quad h_k(X) = 0, \quad k = 1, 2, \dots \\
& \quad \quad \quad X^L \leq X \leq X^U, \quad \sum w_i = 1, \quad w_i \geq 0
\end{aligned} \tag{2.1}$$

where X is an updating parameter vector; $F_i(X)$ is the sub-objective function that returns a scalar value; n is the number of the sub-objective functions; w_i is the corresponding weighting factor to each sub-objective function; $g_j(X)$ and $h_k(X)$ are inequality and equality constraints, respectively; and X^L and X^U are lower and upper constraint vectors of the updating parameter vector, respectively.

The MOF approach's optimization problem, which permits unassigned weighting factors to be utilized while exploring all objective function spaces independently, is as follows (Jin et al. 2014):

$$\begin{aligned}
& \min(F_1(X), F_2(X), \dots, F_M(X)) \\
& \text{subject to } g_j(X) < 0, \quad j = 1, 2, \dots \\
& \quad \quad \quad h_k(X) = 0, \quad k = 1, 2, \dots \\
& \quad \quad \quad X^L \leq X \leq X^U
\end{aligned} \tag{2.2}$$

In the SOF approach, a single objective is optimized with different weighting factors assigned to balance the effect of each residual, while in the MOF approach, different terms of a multi-objective function are optimized. A multi-objective optimization technique is introduced by Kim and Park (2004) to simultaneously overestimate several objective terms. Additionally, it is emphasized that the selection of parameters is important for a successful FE model updating and that the number of parameters should be kept as small as possible.

In addition to the use of these approaches in engineering applications, there are also studies comparing their performances. Jin et al. (2014) present a new approach to the FE model updating using the MOF and the comparison between the single and multi-objective approaches. The modal properties of a highway bridge are determined through the implementation of an ambient vibration test. A study comparing the performances of these methods was also conducted by Jiménez-Alonso et al. (2020). The study is directed at considering a laboratory model of a footbridge using both a single and a multi-objective function.

According to scholarly research that characterizes FE model updating as an optimization problem, the multi-objective approach is deemed to be the more favorable alternative. This is due to the fact that this methodology enables an extensive search space, reduces computational time, and provides a more equitable distribution of the impact of the residue set under consideration.

2.3.2.2. Iterative Stochastic Methods

When employing iterative stochastic methods to update finite element (FE) models, this is regarded as a statistical issue that centers on the quantification of uncertainty. Here, uncertainty in model parameters, modeling errors, and uncertainties arising from observation errors form the basis of the uncertainties that arise in the model prediction (Ereiz, Duvnjak, and Fernando Jiménez-Alonso 2022). The comprehensive relationship between structural responses and the numerical model can be expressed as follows:

$$z = D(x; \theta) + \varepsilon \quad (2.3)$$

$x \in R^i$ and $z \in R^j$ are measured (or observed) input and output responses of the structure, respectively. i and j denote the number of input and output measurements. Parameterized by the unknown parameter vector θ , $D(x; \theta) \in R^j$ refers to the model prediction made by the FE model at the locations of the sensors measuring the output response to the input x . ε denotes the error caused by measurement errors and/or model parameter uncertainties.

With the FE model updating, it is tried to find the value range of the unknown parameter vector, θ that is stable with the measurement data, (x, z) . In deterministic model updating, the unknown parameter vector, θ is estimated to minimize the error. Stochastic methodology, on the other hand, can accurately explain various sources of uncertainty in the model updating process with the probabilistic framework it offers. The probability distribution functions (PDF) (such as Bernoulli, uniform, binomial, normal, Poisson, or exponential distributions) are used to model uncertainties. (Marwala, Boulkaibet, and Adhikari 2016) define that uniform and normal distributions are generally used in FE model updating of civil engineering applications. A prior probability density function is assigned to each set of parameters, and they contain prior knowledge or information about the value of the structural parameters.

Here, Bayes' theorem can be used to update the quantile uncertainty of the prior PDF, defined as follows (K. Yuen 2010):

$$p(\theta|x, z) = \frac{p(z|x, \theta) \times p(\theta)}{p(z|x)} \quad (2.4)$$

In Bayesian methodology, the unknown parameters vector, θ is modeled as a random variables. The prior distribution, $p(\theta)$ is updated to a posterior distribution, $p(\theta|x, z)$ taking into account the measurement data, (x, z) . $p(z|x, \theta)$ is the likelihood function of the parameters, and $p(z|x)$ described as a normalization constant and ensures that $p(\theta|x, z)$ integrates to one. Therefore, Equation (2.4) can be described as follows:

$$p(\theta|x, z) \propto p(z|x, \theta) \times p(\theta) \quad (2.5)$$

It is possible to use Equation (2.5) to update the PDFs of the model parameters based on the results of the experiment after finding the prior PDF and likelihood function.

After the posterior PDF has been computed or estimated, it is possible to deduce the degree to which the parameters' uncertainty has been diminished by utilizing the observed data and existing prior knowledge. In general, the resultant posterior can be estimated using a variety of techniques, including sampling methods, analytical approximations, and numerical evaluation. Numerical evaluation may not be accurate to use when the posterior distribution is very large (Khodaparast, Mottershead, and Friswell 2008).

Analytical solution applications for the resulting posterior distribution can be used when the posterior and prior are from the same distribution family; these are called conjugate distributions. The prior is defined as conjugate for the likelihood function (Diaconis and Ylvisaker 1979). While the posterior PDF can be determined numerically when a conjugate prior is used, it can be determined analytically when the number of parameters is limited. In general, binomial likelihood and beta prior are used for discrete situations, and normal likelihood and prior are used for continuous situations. In real engineering structure investigations, posterior and prior non-conjugate distributions are often encountered, and the calculation of joint and marginal PDFs involves a large number of parameters. This situation causes the formation of high-dimensional integrals whose solution is quite complex. Approximate measures or sampling techniques, such as the Markov Chain Monte Carlo (MCMC) method, are necessary to solve the problem (Beck and Au 2002).

In this context, the development of the Bayesian model updating approach can be summarized as follows:

Beck and Katafygiotis (1998) propose to solve the problem of updating a structural model and its associated uncertainties with a Bayesian statistical framework using dynamic response data. The aim here is to both provide more accurate response estimates for predicted dynamic loadings and to provide a quantitative evaluation of this accuracy.

Katafygiotis, Papadimitriou, and Lam (1998) work on the general Bayesian statistical model updating framework presented in Beck and Katafygiotis (1998). Considered here are the computational challenges associated with applying this general methodology in an unidentified situation where the asymptotic approximations presented in Beck and Katafygiotis (1998) are not valid. The study discovered that the posterior PDF was centered around a long, very complicated manifold in the parameter space. It was suggested that this manifold could be rounded off to a set of points using an algorithm.

Papadimitriou, Beck, and Katafygiotis (2001) define the concept of robust reliability as taking into account uncertainties in structural modeling in addition to the uncertain excitation that a structure will experience throughout its life. The study presents a general framework for robust measurement of structural safety and reliability and a methodology for updating it using dynamic test data. To update robust reliability using measured response data, the Bayesian statistical framework proposed by Beck and Katafygiotis (1998) is integrated with probabilistic structural dynamics tools for system identification.

Asymptotic approximation can be used to approximate posterior PDFs when working with sufficiently large amounts of data (Diaconis and Ylvisaker 1979). In addition to being approximated by a Gaussian PDF at the maximum posterior point, the posterior PDF is also defined by a covariance matrix. However, when the amount of data is smaller, the accuracy of the asymptotic approximation is questionable or problematic. For these cases, a stochastic simulation approach based on the Markov chain Monte Carlo algorithm was proposed by Beck and Au (2002).

In a Bayesian probabilistic framework for robust system identification, structural response estimates and performance reliability can be improved by updating structural test data with the predictions of a set of possible structural models, weighted by their updated probabilities (Beck and Au 2002). The evaluation of the integral in question proves challenging for this application due to the impracticality of direct numerical integration on account of the parameter space's size and the fact that the posterior PDF is confined to a small region and is only known up to a scaling constant. In order to assess the intended integral, an adaptive Markov chain Monte Carlo simulation method is suggested. This method is founded upon the Metropolis-Hastings algorithm and a notion akin to simulated annealing (Beck and Au 2002).

For high-dimensional problems, the numerical integration required for Bayesian updating is computationally prohibitive, according to K. V. Yuen, Beck, and Au (2004). In order to address this issue, they suggested a Markov chain Monte Carlo simulation method based on the Metropolis-Hastings algorithm, in addition to an adaptive concept for obtaining information about crucial regions of the updated probability distribution in an efficient manner.

Despite its successful applications, the stochastic simulation approach proposed by Beck and Au (2002) has a significant limitation in that it is only effective for low-dimensional problems. Considering this shortcoming, Ching, Muto, and Beck (2006)

present a new approach based on the Gibbs sampler for updating the Bayesian model of linear structural models containing missing modal data. Additionally, Ching and Chen (2007) present a newly developed simulation-based approach for updating the Bayesian model called the transitive Markov chain Monte Carlo approach (TMCMC). This approach is based on the Monte Carlo Markov chain and is inspired by the adaptive Metropolis-Hastings method developed by Beck and Au (2002). The main goal with this method is to avoid the problem of sampling difficult target probability density functions (PDFs) from a series of intermediate PDFs that approximate the target PDF and are easier to sample.

The difference between the TMCMC algorithm and the approach of Beck and Au (2002) is that in implementing the Metropolis-Hastings algorithm, instead of using a global proposal PDF based on a kernel density constructed from samples at the previous level, a local proposal PDF is essentially a local random walk in the parameter space. In this context, Muto and Beck (2008) discussed a study focusing on the application of the TMCMC algorithm to Bayesian updating of model parameters.

To address the issue of updating Bayesian models, several Markov chain Monte Carlo simulation methods have been devised in the studies examined thus far. However, the size of the model parameter space generally makes these proposed methods less useful. For this purpose, Cheung and Beck (2009) investigate the Hybrid Monte Carlo method and tried to show how it can be used to solve high-dimensional Bayesian model updating problems.

The Bayesian statistical system identification framework can make full use of information from measurements. However, because they are so expensive to run, most Bayes-based model updating studies have only used simple structural systems to prove or validate their results. In their work, H. F. Lam, Peng, and Au (2014) meticulously derived the Bayesian probabilistic structural model updating methodology and applied it to a full-scale civil engineering structure. For this purpose, a pragmatic data-exchange algorithm is employed to integrate the Bayesian model updating method.

Though Bayesian updating is a preferred method to learn and calibrate models with data and observations, high-dimensional integrals must be calculated for Bayesian updating. For this purpose, MCMC sampling methods are developed and their successful applications are mentioned. However, the obvious disadvantage of these developed, MCMC methods is the difficulty of ensuring stationarity of the Markov chain. Therefore, Straub and Papaioannou (2015) present an alternative called Bayesian updating with

structural reliability methods (BUS), which is especially effective in updating mechanical and other computational models. In this method, structural reliability methods are used to calculate the posterior distribution of uncertain model parameters and model outputs. BUS, proposed by Straub and Papaioannou (2015) reinterprets the Bayesian updating problem as a structural reliability problem. Then, this method is developed with Subset Simulation by Betz et al. (2018). In this way, it can efficiently handle a large number of uncertain parameters and estimate very small properties. To improve Bayesian updating techniques, Z. Wang and Shafieezadeh (2020) propose a new approach called BUAK, which combines the BUS method developed by Straub and Papaioannou (2015) with adaptive Kriging metamodeling. With this integration, new capabilities are gained in reducing the number of evaluations of costly performance functions and simultaneously training very accurate meta-models.

It has been noted that the majority of existing research is devoted to numerical and simple experimental models; therefore, there is a need for additional studies that utilize field monitoring of large-scale, actual engineering structures.

The Bayesian updating method is employed by Mao, Wang, and Li (2020) to update the initial finite element (FE) model of a suspension bridge. The objective is to align the numerical modal properties with the outcomes obtained from field monitoring. To reduce the computational burden of model updating, a Kriging estimator is built as a surrogate model using experimental samples generated by the Latin hypercube sampling algorithm.

Hariri-Ardebili and Sudret (2020) state that studies using FE models of dam structures as input to develop surrogate meta-models for the prediction and classification of these structures are quite limited. This study looks at how polynomial chaos expansion (PCE), a good way to replace complicated deterministic models with a meta model to cut down on high computing costs, can be used to solve different dam engineering issues.

Martakis et al. (2022) use a Bayesian model updating framework to perform a seismic assessment using modal data from masonry buildings. The PCE technique was used to reduce the computational burden that frequently occurs in Bayesian model updating.

Ramancha et al. (2022) say that surrogate modeling techniques are a fairly new area of study and that their use for Bayesian updating of FE models of large structures has not been extensively researched in the past. Consequently, the primary objective of their research is to assess the benefits and drawbacks associated with the utilization of

surrogate models in the process of model updating. Additionally, it is aimed at filling the gap in the literature in the context of updating FE models of large-scale structural systems using surrogate models. For this purpose, the miter gate structural system is investigated to show procedures for surrogate model construction and FE model updating.

The literature review reveals that the most significant challenge encountered during model updating process pertains to the management of various sources of uncertainty. Deterministic methods strive to yield a singular response in order to minimize the discrepancy that may exist between the outputs of the model and the measured data. As a consequence, establishing a reliable correlation between the numerical model and real data is challenging owing to the intrinsic uncertainties in the structure. However, probabilistic approaches do not accept these parameters as fixed numbers due to the existence of uncertainties in the updating process; instead, they consider a realistic statistical distribution for each parameter. For this reason, probabilistic approaches have been found to be more reliable in most cases. Nevertheless, the deficiencies and problems encountered during the model updating are still open to development and research. There is a need to develop applications to solve the computational difficulties encountered in making more accurate structural reliability assessments, especially in real and complex engineering structure applications.

2.4. Summary and Implications

The difficulties and research gaps encountered in updating the FE model of large-scale structural systems are tried to be given in detail:

- Integrating SHM into FE model updating and reliability analysis of structures emerges as an important and much-needed research topic in structure decision-making processes.
- As of yet, there is no comprehensive framework in place that effectively integrates SHM data into structural description while also delivering an accurate evaluation of the life cycle performance of structures.
- FE model updating can be used with SHM methods, but it has been observed that neglecting existing uncertainties in model updating can significantly affect the reliability of structural descriptions and future evaluations of structures.

- Consideration can be given to these uncertainties using probabilistic methods, such as the Bayesian framework. Nevertheless, a probabilistic method that comprehensively tackles uncertainty in research is still absent. The disadvantage of using these probabilistic methods in FE model updating of real and large-scale engineering structures is their high computational load. This makes the application impractical and disrupts the examination of these structures.
- Due to computational difficulties, it can be seen that there are deficiencies in model updating and structural reliability examination of real engineering structures that are worth investigating.
- Metamodeling (or surrogate) techniques, a relatively new area of research in Bayesian updating of FE models of large-scale engineering structures, have rarely been studied. However, they can be effective in solving many difficulties encountered in applications.
- Polynomial-Chaos-Kriging (PCK), based on the combination of well-established metamodeling techniques such as Polynomial Chaos Expansions (PCE) and Kriging, has not yet been found to be used in Bayesian FE model updating procedures to overcome computational challenges and provide more accurate reliability assessment for structures.
- In addition, while the literature focuses mostly on updating the FE models of structures in the most accurate way, it has been observed that the structural reliability evaluations of these updated models are missing. A more accurate FE model will enable more reliable performance evaluations of structures and, therefore, safer decisions regarding maintenance and structural safety.

CHAPTER 3

THEORETICAL BACKGROUND

3.1. Introduction

In this section, the theoretical background of the proposed solution method is given to eliminate the deficiencies of the required study area and produce solutions to them. First, the methods of obtaining modal properties are mentioned, and the integration of these obtained data into an accelerated Bayesian FE model updating with metamodeling techniques is explained. Then, the procedures to be followed in evaluating the reliability of the resulting model are mentioned.

3.2. Classification of Modal Identification Techniques

In civil engineering applications, experimental modal analysis is widely used to extract structural modal parameters such as natural frequencies, damping ratios, and mode shapes from vibration measurements. The methods that are used for modal analysis can be divided into two main categories, such as time and frequency domains (Zrayka and Mucchi 2019). Frequency response functions (FRFs) in the frequency domain or impulse response functions (IRFs) in the time domain often form the basis of system identification algorithms (He et al. 2009).

The dynamic equilibrium equation for N -degree of freedom system with viscous damping can be described as follows:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{f\} \quad (3.1)$$

where $[M]$, $[C]$, and $[K]$ are the mass, damping and stiffness matrices, respectively.

A matrix, $[H]$ can be used to establish the relationship between complex response amplitudes and the amplitudes of applied forces in steady-state conditions and in the case of harmonic excitation. $[H]$ is named as the frequency response function matrix such that $\{\bar{X}\} = [H]\{f\}$. Each element of $[H]$ is called the reception frequency response function,

which relates the response at coordinate, i to a force at coordinate, j and is expressed as (Ewins 2000):

$$H_{ij}(\omega) = \sum_{r=1}^N \left(\frac{A_{ij}^r}{\omega_r \xi_r + i(\omega - \omega_r \sqrt{1 - \xi_r^2})} + \frac{A_{ij}^{r*}}{\omega_r \xi_r + i(\omega - \omega_r \sqrt{1 - \xi_r^2})} \right) \quad (3.2)$$

where A_{ij}^r , ω_r and ξ_r are the residue, natural frequency, and viscous damping ratio of mode, r .

The behavior of the structure between selected points is expressed in the time or frequency domain with this equation. Fourier transforms are used to move from one domain to another (Maia and Silva 2001). Also, the IRFs are calculated from FRFs via an inverse Fourier transform.

The two methods used in modal identification are examined in more detail, and their advantages and disadvantages are stated here. In time domain methods, signals are tracked using only output responses; in frequency domain methods, the frequency response function (FRF) is determined, but this usually requires the input excitation data to be detailed. The time-domain methods can also recognize two modes when they are very close to each other. Because of these properties, time-domain methods are considered very useful for experimental tests. In frequency domain methods, the accuracy of the results depends on the number of orders. The lower this number, the more accurate the results. However, there are certain issues with fast Fourier transform (FFT) analysis, such as leakage. As a result, the time domain and frequency domain perform differently depending on whether the damping is low or high. At low damping, the time domain works better than the frequency domain, while at high damping, the frequency domain works better (Zrayka and Mucchi 2019).

In line with the number of input and output locations, input-output classifications can also be made for each domain (Maia and Silva 2001). One response and many responses due to one force can be described as single-input, single-output (SISO) and single-input, multiple-output (SIMO), respectively. One response and many responses due to various forces can be described as multiple-input single-output (MISO) and multiple-input multiple-output (MIMO), respectively. In this context, the following common modal analysis methods are explained in detail:

3.2.1. The Pick Picking Method (PPM)

PPM, also called the peak amplitude method, is a single-input, single-output (SISO) method. It is the simplest of the modal parameter estimation methods that work in the frequency domain (Ewins 2000). It consists of separating each mode to determine modal parameters, and in this way, the natural frequency, damping factor, and residuals are calculated as follows:

$$\omega_r = \max(FRF) \quad (3.3)$$

$$\xi_r \cong \frac{\omega_a - \omega_b}{2\omega_r} \quad (3.4)$$

$$A_{ij}^r = 2|\tilde{a}|_{ij}\omega_r^2\xi_r \quad (3.5)$$

where ω_a and ω_b are the frequencies on the half-power points.

3.2.2. Least Square Complex Exponential Method (LSCEM)

LSCEM is a single-input multiple-output (SIMO) method and works in the time domain. In this method, the FRF reception of a general multi-degree of freedom (MDOF) system is used with a general viscous damping, and then the impulse response function is obtained by inverse Fourier transform as follows (Zrayka and Mucchi 2019):

$$\alpha_{ij}(k\omega) = \sum_{r=1}^{2N} \frac{A_{ij}^r}{k\omega - \lambda_r} \rightarrow \mathcal{F}^{-1} \rightarrow h_{ij}(t) = \sum_{r=1}^{2N} A_{ij}^r e^{\lambda_r t} \quad (3.6)$$

where $k\omega$ is Laplace variable, $\alpha_{ij}(k\omega)$ is receptance FRF, A_{ij}^r and λ_r are modal constant and system pole for mode, r , $h_{ij}(t)$ is impulse response function corresponding $\alpha_{ij}(k\omega)$.

The LSCEM method computes the impulse response corresponding to each FRF and fits to the response a set of complex damped sinusoids using Prony's method (Brandt 2023). For the n samples:

$$h_n = \sum_{r=1}^{2N} A_{ij}^r e^{\lambda_r t} = \sum_{r=1}^{2N} a_r x_r^n \quad (3.7)$$

where a_r is amplitudes and x_r is the poles. Prony's method expresses a sampled function h_n as a superposition of $N/2$ modes and thus N amplitudes and poles:

$$\begin{aligned} h(0) &= a_1 x_1^0 + a_2 x_2^0 + \dots + a_N x_N^0 \\ &\vdots \\ h(N-1) &= a_1 x_1^{N-1} + a_2 x_2^{N-1} + \dots + a_N x_N^{N-1} \end{aligned} \quad (3.8)$$

The poles are the roots of a polynomial with coefficients, $\Omega_0, \Omega_1, \dots, \Omega_{N-1}$ can be calculated as follows:

$$\Omega_0 x_r^0 + \Omega_1 x_r^1 + \dots + \Omega_{N-1} x_r^{N-1} + x_r^N = 0 \quad (3.9)$$

The coefficients are found using an autoregressive model of $L = 2N$ samples of h :

$$\begin{bmatrix} h(0) & \dots & h(N-1) \\ \vdots & \ddots & \vdots \\ h(L-N-1) & \dots & h(L-2) \end{bmatrix} \begin{bmatrix} \Omega_0 \\ \vdots \\ \Omega_{N-1} \end{bmatrix} = - \begin{bmatrix} h(N) \\ \vdots \\ h(L-1) \end{bmatrix} \quad (3.10)$$

After the poles are determined, it is possible to determine the frequencies and damping factors by identifying the imaginary and real parts of the pole logarithms. The final step is solving for the amplitudes and reconstructing the impulse response using:

$$\begin{bmatrix} h(N) \\ \vdots \\ h(L-1) \end{bmatrix} = \begin{bmatrix} x_1^0 & \dots & x_N^0 \\ \vdots & \ddots & \vdots \\ x_1^{N-1} & \dots & x_N^{N-1} \end{bmatrix} \begin{bmatrix} \Omega_0 \\ \vdots \\ \Omega_{N-1} \end{bmatrix} \quad (3.11)$$

3.2.3. A Flowchart for Identification of Modal Properties

In summary, experimental modal analysis is a process that describes a structure in terms of its natural characteristics, such as frequency, damping, and mode shapes, known as its dynamic properties, based on monitoring data that requires evaluation of

measurements from the actual structure. In this context, the flow chart of the transactions to be made has been presented in Figure 3.1.

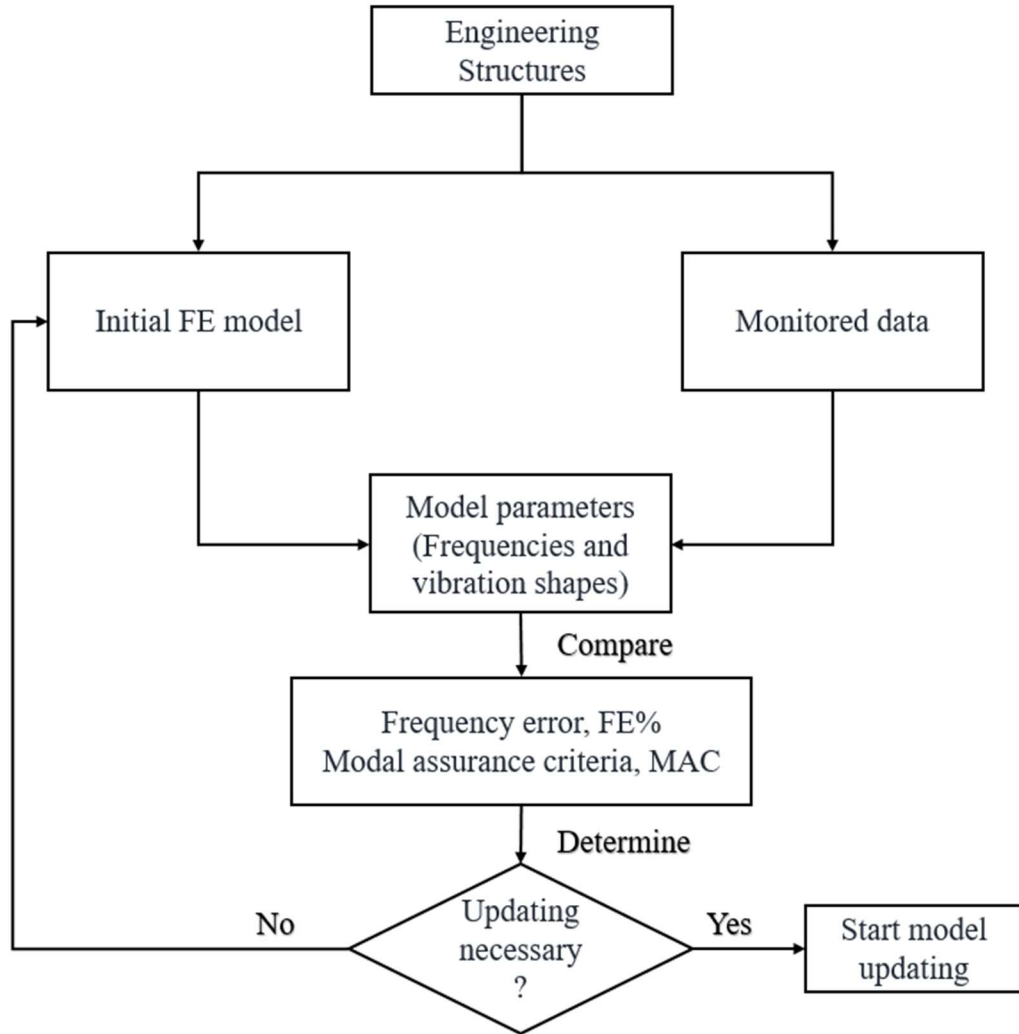


Figure 3.1. Flow chart for identification of modal properties

This framework primarily aims at identifying the modal properties of the measured structure to achieve a reasonable fit to the results obtained from the FE model (Mao, Wang, and Li 2020). In this sense, generally the natural frequencies and mode shape vectors are used to search an optimal fit between the observed data and constructed FE model. Here, the discrepancy between the identified frequencies, f_v and analytical values obtained from the FE model analysis, f_{fem} can be defined as an error function as shown below:

$$FE \% = \frac{|f_v - f_{fem}|}{f_{fem}} \quad (3.12)$$

where f_v is identified frequencies from monitoring data and f_{fem} is calculated frequencies from FE model analysis.

To measure the discrepancy between the measured, ϕ_t , and analytical mode shapes, ϕ_f , the probability distribution of modal assurance criterion (MAC) can be considered. In this sense, the MAC value for complex mode shapes can be defined as follows (Pastor, Binda, and Harčarik 2012):

$$MAC(\phi_f, \phi_t) = \frac{|\phi_f' \phi_t|^2}{(\phi_f' \phi_f)(\phi_t' \phi_t)} \quad (3.13)$$

where ϕ_f is analytical or FEM modal vector and ϕ_t is test modal vector comes from experimental modal analysis.

According to the results from here, it will be possible to conclude whether the model parameters need updating or not. In cases of difference, it can be concluded that the initial FE model of the structure needs to be updated.

3.3. Bayesian Model Updating

In Bayesian FE model updating procedures, it is essentially aimed at estimating the unknown parameters, θ of the FE model given the probability distribution of the measured data. Here, it can be considered a computational model M that allows estimation of certain amounts of interest collected in the vector, y as a function of the input parameters, x . Thus, a mathematical representation of the system, $M(x)$ can be described as follows:

$$M : x \in R^{N_x} \rightarrow y = M(x) \in R^{N_y} \quad (3.14)$$

In deterministic approaches, the unknown parameters, θ are estimated by minimizing the error equation between the output measurements, y and FE response, $M(x)$. Although successful estimating can be obtained, the deterministic approaches do not attempt to measure the estimation uncertainty of the unknown parameter vector, θ . Therefore, it might be rather problematic to ingrate the FE model updating results into the reliability

problem. From this aspect, the Bayesian approaches take the advantage of the fact that the unknown parameter vector, θ is modelled as a random variable. Thus, the propagation of uncertainties can be tracked from FE model updating to reliability estimation process. Once the input data for the observed structural system is available, the output measurements can be directly correlated as an indicator of statistical error to obtain the probability distribution of the structural model parameters. Accordingly, a prediction error can be described as follows:

$$w = y - M(x) \quad (3.15)$$

where x and y represent the input and output data respectively. Additionally, M represents a computational model which is defined as a function of the input parameters, and w indicates the prediction error term which can be assumed to follow a zero mean Gaussian distribution with a covariance matrix of Σ :

$$w \sim N(0, \Sigma) \quad (3.16)$$

The probability distribution of the structural model parameter vector, θ by the Bayes' theorem, can be estimated as follows:

$$f(\theta|x, y) = \frac{L(y|x, \theta)f(\theta)}{Z} \quad (3.17)$$

where $L(y|\theta, M)$ is the likelihood function which defines the probability distribution of observed data given the model parameters, $f(\theta)$ indicates the prior probability distribution of θ and Z denotes the evidence term. Here, Z can be considered as a normalizing constant to ensure that the posterior distribution integrates to 1. Neglecting the normalizing term, Equation (3.17) can also be written as follows:

$$f(\theta|x, y) \propto L(y|x, \theta)f(\theta) \quad (3.18)$$

In most cases, it is not practical to assume that the prediction error term is perfectly known. Instead, the measured modal parameters can be attributed as the output data, and $M(x)$ can be considered as the FE model based modal information. Thus, the prediction error term can be obtained in terms of modal parameters such as natural frequency and

modal shape vector. The modal frequency can be assumed to follow a zero mean Gaussian distribution and described as follows:

$$w_{fr_i} = \widehat{fr_i^2} - fr_i^2(\theta) \sim N(0, \sigma_{fr_i}^2) \quad (3.19)$$

where fr_i and w_{fr_i} are the modal frequency and the prediction error for the i^{th} mode, respectively.

It can be taken as Gaussian with zero mean standard deviation, $\sigma_{fr_i} \widehat{fr_i^2}$. At this point, σ_{fr_i} is accepted as the unknown parameter and included in the parameter set, θ . Moreover, the prediction error for modal shape vectors can be defined in terms of the MAC between the observed and model predicted mode shapes as below:

$$w_{MAC_i} = 1 - MAC_i(\theta) \quad (3.20)$$

In the literature, it is shown that the MAC variable may not be represented by a Gaussian distribution. Instead, it can be assumed that the MAC values follows a Chi-square (Hızal 2023) or truncated-Gaussian distribution (Argyris et al. 2020). In this context, prediction error, w_{MAC_i} , is assumed to be follows a univariate Truncated distribution between the range of $[0,1]$, with a standard deviation of σ_{MAC_i} . Here, the standard deviation term can be defined as depending on the number of mode shape components, N is shown below (Argyris et al. 2020):

$$\sigma_{MAC_i} = \sigma_{MAC_i}' \left(1 + \frac{1}{\sqrt{N}} \right) \quad (3.21)$$

where σ_{MAC_i}' is the standard deviation deduced from the data. The second term in Equation (3.21) depends on the number of sensors; therefore, one can observe the larger the number of sensors the smaller the standard deviation of MAC. The likelihood function for modal prediction errors given the structural model parameters can be defined as below:

$$L(y|x, \theta) = \prod_{i=1}^m f(w_{fr_i}|\theta) \prod_{i=1}^m f(w_{MAC_i}|\theta) \quad (3.22)$$

where m represents the number of considered modes.

To reduce the computational effort, the high-dimensional joint posterior can be estimated by Markov chain Monte Carlo sampling techniques (Jin, Ju, and Jung 2019) such as Metropolis-Hasting, Gibbs Sampling algorithm etc. However, plotting the samples directly from any target distribution, $p(x)$ through a Markov chain might be a challenging problem. Therefore, it might be necessary to design a transition operator for the Markov chain that matches the stationary distribution of the chain to the target distribution. In this context, it should be stated that the Metropolis-Hasting algorithm (Beck and Au 2002) conducts a simple heuristic process to implement such a transition operator.

At this point, a pseudo-code for the implemented Metropolis-Hastings algorithm is presented in Table 3.1. Here, a potential new state x^* is firstly proposed based on a previous state, $x^{(t-1)}$ according to the proposal distribution $q(x_i \setminus x_i^{(t-1)})$. The acceptance criteria of the proposed state are based on the density of the target distribution, $p(x)$. An additional correction factor, c , is used to differentiate Metropolis (Metropolis et al. 1953) and Metropolis-Hastings algorithms (Hastings 1970). In this way, some asymmetric proposal distributions can be used.

Table 3.1. MCMC: Metropolis-Hasting algorithm (Mao, Wang, and Li 2020)

Set $t = 0$ and starting an initial state $x^{(0)} = (x_1^{(0)}, \dots, x_n^{(0)})$ iterate $t = 1, 2, \dots$	
1. Generate proposal state, x^* from $q(x_i \setminus x_i^{(t-1)})$	
2. Calculate the proposal correction factor, $c = \frac{q(x_i^{(t-1)} \setminus x_i^*)}{q(x_i^* \setminus x_i^{(t-1)})}$	
3. Calculate acceptance probability, $\alpha = \min(1, \frac{p(x_i^*, x_j^{(t-1)})}{p(x_i^{(t-1)}, x_j^{(t-1)})} x c)$	
4. Draw a random number $u \in [0, 1]$ from uniform probability density	
5. if $u \leq \alpha$, then	$x_i^{(t)} = x_i^*$
else	$x_i^{(t)} = x_i^{(t-1)}$
end if	

3.4. Metamodeling Techniques

The evaluation of finite element models in engineering applications might be a challenging and time-consuming process for complex structures. Therefore,

metamodeling techniques specifically focus on reducing computational costs in FE models. In this sense, the PCK algorithm is derived as a combination of the widely known and well-established metamodeling techniques PCE and Kriging. The working principle of PCK in the computational model is that it uses PCE to capture general behavior and interpolation-type Kriging to capture local variations. This working principle turns PCK into a more effective and efficient metamodeling technique than PCE and Kriging considered separately (Schöbi, Sudret, and Wiart 2015).

3.4.1. Basics of Kriging

In Kriging, also known as Gaussian process modeling, it is assumed that the model output $y = M(x)$ represents the realization of a Gaussian process indexed by x . Here, the Kriging model can be defined as (Schöbi, Sudret, and Wiart 2015):

$$y \approx M^K(x) = \beta^T f(x) + \sigma^2 Z(x, \varepsilon) \quad (3.23)$$

where $\beta^T f(x) = \sum_{j=1}^P \beta_j f_j(x)$ and σ^2 shows the mean value and variance of the Gaussian process, respectively. Additionally, $Z(x, \varepsilon)$ represents a zero mean stationary Gaussian process and is determined by auto-correlation function between two input sample points $R(x, x') = R(|x - x'|; \theta)$ due to stationarity, θ describe hyper-parameters.

3.4.1.1. Correlation Functions

The correlation function establishes the degree of similarity between new and observed data points, or the degree of similarity, by calculating the distance between the input and target points. It is a critical component of the Kriging estimator in this context, as it contains assumptions regarding the approximation function. It is anticipated that input points in close proximity will yield comparable outputs or responses. There are different correlation functions, such as linear, exponential, and Gaussian, and their expressions are given in Equations (3.24)-(3.26), respectively (Fu et al. 2020).

$$R(x, x'; \theta) = \max\left(0, 1 - \frac{|x - x'|}{\theta}\right) \quad (3.24)$$

$$R(x, x'; \theta) = \exp\left(-\frac{|x - x'|}{\theta}\right) \quad (3.25)$$

$$R(x, x'; \theta) = \exp\left(-1/2 \left(\frac{|x - x'|}{\theta}\right)^2\right) \quad (3.26)$$

3.4.1.2. Types of Trend

In Equation (3.23), $\beta^T f(x)$ is expressed as the trend part, which refers to the mean of a Kriging meta-model. Three different Kriging meta-models have been defined in the literature. In simple Kriging, the trend has a known constant value as $\beta^T f(x) = \beta_0$. In ordinary Kriging, the trend has a constant but unknown value as $P = 1$ and $f_1(x) = 1$. In universal Kriging, the trend is composed of a sum of P pre-selected functions, $f_j(x)$ and defined as follows:

$$\beta^T f(x) = \sum_{j=1}^P \beta_j f_j(x) \quad (3.27)$$

Here, universal Kriging is more general and flexible formulation whereas simple and ordinary Kriging are special cases of universal Kriging.

3.4.1.3. Error Estimation

Kriging metamodeling eliminates empirical error through interpolation; therefore, a straightforward global error metric cannot be employed to assess the meta-model's accuracy. Leave-one-out (LOO) cross-validation error is a metric employed to evaluate the predictive accuracy of a model developed using Kriging metamodeling on a novel dataset. LOO cross-validation error can be calculated on the initial experimental design, X and its corresponding responses, $Y = M(X)$ as follows:

$$Err_{LOO}^{(Kriging)} = \frac{1}{N} \left[\frac{\sum_{i=1}^N \left(M(x_i) - \mu_{\hat{Y},(i)}(x_i) \right)^2}{var[Y]} \right] \quad (3.28)$$

where $\mu_{\hat{Y}(i)}(x_i)$ denotes the Kriging meta-model that is obtained using all the points of the experimental design, X , except x_i .

3.4.2. Basics of PCE

Utilizing a properly constructed basis to represent polynomial functions spectrally in order to approximate the functionality of a computational model, this is the primary aim of PCE. To be more precise, PCE specifies how a random variable is represented by a polynomial function comprising additional random variables. Here, a random vector is considered with independent components of $X \in R^M$ described by joint PDF, f_X . Then, a random variable Y can be defined with finite variance as a function of an M dimensional random vector X .

A computational model, $Y = M(X) \in R$ with finite variance can be obtained by using Equations (3.29) and (3.30).

$$E[Y^2] = \int_{D_X} M^2(x) f_X(x) dx < \infty \quad (3.29)$$

$$Y = M(X) = \sum_{\alpha \in N^M} y_\alpha \Psi_\alpha(X) \quad (3.30)$$

where y_α and $\Psi_\alpha(X)$ are corresponding coefficients and multivariate polynomials orthonormal to f_X . In more realistic applications, Equation (3.30) needs to be truncated to a finite sum below.

$$M^{PCE}(X) \approx M(X) = \sum_{\alpha \in N^M} y_\alpha \Psi_\alpha(X) \quad (3.31)$$

3.4.2.1. Constructing Polynomial Basis

The polynomial basis of $\Psi_\alpha(X)$ is usually performed starting from a set of univariate orthonormal polynomials $\phi_k^i(x_i)$ which satisfy as follow:

$$\langle \phi_j^i(x_i), \phi_i^i(x_i) \rangle = \int_{D_{X_i}} \phi_j^i(x_i) \phi_i^i(x_i) f_{X_i}(x_i) dx_i \quad (3.32)$$

where i identifies the input variable with respect to which they are orthogonal as well as the corresponding polynomial family, j corresponding polynomial degree and $f_{X_i}(x_i)$ is the i^{th} input marginal distribution.

Then, the multivariate polynomials $\Psi_\alpha(X)$ can be accumulated as:

$$\Psi_\alpha(x) = \prod_{i=1}^M \phi_{\alpha_i}^i(x_i) \quad (3.33)$$

where $\phi_{\alpha_i}^i$ is the univariate orthogonal polynomial in the i^{th} variable of α_i where orthogonality is meant with respect to the probability density function, f_{X_i} of this i^{th} variable.

Substituting an infinite series of polynomials for the computational model is the fundamental concept underlying PCE. To be used in the context of real-world problems, the summation in $M^{PCE}(X) \approx M(X)$ should be limited to a finite sum as follows:

$$M^{PCE}(X) = \sum_{\alpha \in \mathcal{A}} y_\alpha \Psi_\alpha(X) \quad (3.34)$$

A truncation set, $(\mathcal{A})$ can be chosen a priori in a number of different ways. Standard and hyperbolic truncation schemes are the two primary types. The standard truncation scheme operates by selecting all polynomials in M input random variables that have a maximum total degree of p (the maximum degree of any polynomial). The cardinality of the set $\mathcal{A}$ is defined as follows:

$$\mathcal{A}^{M,p} = \frac{(M+p)!}{M! p!} \quad (3.35)$$

where $\mathcal{A}^{M,p} = \{\alpha \in N^M : |\alpha| \leq p\}$

The hyperbolic truncation scheme uses the parametric q-norm to define the truncation:

$$\mathcal{A}^{M,p,q} = \{\alpha \in \mathcal{A}^{M,p} : \|\alpha\|_q \leq p\} \quad (3.36)$$

where $\|\alpha\|_q = (\sum_{i=1}^m \alpha_i^q)^{1/q}$

3.4.2.2. Computation of Coefficients

There exist multiple techniques for determining the coefficients, y_α , including regression and projection. Projection methods compute the coefficients through numerical integration by utilizing the orthogonality of the basis functions. Regression methods formulate $\sum_{\alpha \in N^M} y_\alpha \Psi_\alpha(X)$ as a system of linear equations and solve the system by standard linear regression approaches. Ordinary least-squares regression is one such method; to implement it, the system of linear equations must be overdetermined. In this context, the infinite series in $\sum_{\alpha \in N^M} y_\alpha \Psi_\alpha(X)$ can be written as a sum of its truncated version $\sum_{\alpha \in \mathcal{A}} y_\alpha \Psi_\alpha(X)$ and a residual:

$$Y = M(X) = \sum_{j=0}^{P-1} y_j \Psi_j(X) + \delta_p \equiv y^\top \Psi(X) + \delta_p \quad (3.37)$$

where $P = \mathcal{A}^{M,p}$, δ_p is truncation error, $y_\alpha = \{y_0, \dots, y_{P-1}\}^\top$ is a vector containing the coefficients, and $\Psi(X) = \{\Psi_0(X), \dots, \Psi_{P-1}(X)\}^\top$ is the vector that assembles the values of all orthonormal polynomials in X . The least-square minimization problem can then be set-up as:

$$\hat{y} = \arg \min \mathbb{E}[(y^\top \Psi(X) + M(X))^2] \quad (3.38)$$

3.4.2.3. Error Estimation

Once a meta-model has been constructed, its predictive quality and accuracy can be validated by calculating the relative generalization error as follows:

$$Err_{Gen}^{(PCE)} = \mathbb{E} \left[(M(X) - M^{PCE}(X))^2 \right] / \text{Var}(Y) \quad (3.39)$$

When an independent set of inputs and outputs, referred to as the validation set, $[X_{val}, Y_{val} = M(X_{val})]$ is present in addition to the training set utilized to build the meta-model, the validation error can be computed as follows:

$$Err_{val}^{(PCE)} = \frac{N-1}{N} \left[\frac{\sum_{i=1}^N \left(M(x_{val}^{(i)}) - M^{PCE}(x_{val}^{(i)}) \right)^2}{\sum_{i=1}^N \left(M(x_{val}^{(i)}) - \hat{\mu}_{Y_{val}} \right)^2} \right] \quad (3.40)$$

where $\hat{\mu}_{Y_{val}} = 1/N \sum_{i=1}^N M(x_{val}^{(i)})$ is the sample mean of the validation set response.

If no validation set is available, as is commonly the case with expensive computational models, there are two main ways to estimate, $Err_{Gen}^{(PCE)}$ is the normalized empirical error and the leave-one-out cross-validation error. As an estimator of the generalization error, the normalized empirical error, $Err_{Norm}^{(PCE)}$ is computed. It is determined by the precision with which the meta-model replicates the experimental design model evaluations, Y . It is characterized as:

$$Err_{Norm}^{(PCE)} = \frac{\sum_{i=1}^N \left(M(x^{(i)}) - M^{PCE}(x^{(i)}) \right)^2}{\sum_{i=1}^N \left(M(x^{(i)}) - \hat{\mu}_Y \right)^2} \quad (3.41)$$

where $\hat{\mu}_Y$ is the sample mean of the experimental design response.

The leave-one-out (LOO) cross-validation error, $Err_{LOO}^{(PCE)}$ is designed to overcome the over-fitting limitation of $Err_{Norm}^{(PCE)}$ by using cross-validation, a technique developed in statistical learning theory. It consists in building, N meta-models, $M^{PCE \setminus i}$, each one created on a reduced experimental design, $X \setminus x^{(i)} = \{x^{(j)}, j = 1, \dots, N, j \neq i\}$ and comparing its prediction on the excluded point $x^{(i)}$ with the real value, $y^{(i)}$. Then, $Err_{LOO}^{(PCE)}$ can be described as follows:

$$Err_{LOO}^{(PCE)} = \frac{\sum_{i=1}^N \left(M(x^{(i)}) - M^{PCE \setminus i}(x^{(i)}) \right)^2}{\sum_{i=1}^N \left(M(x^{(i)}) - \hat{\mu}_Y \right)^2} \quad (3.42)$$

3.4.3. Basics of PCK

Kriging interpolates local variations of Y as a function of neighboring experimental design points in PCK; PCE provides a good approximation of the overall behavior of Y ; and a more accurate meta-model can be obtained by combining these methods. PCK is thus defined as a universal Kriging model whose trend is represented by the following set of orthonormal polynomials:

$$y \approx M^{PCK}(x) = \sum_{\alpha \in A} y_{\alpha} \Psi_{\alpha}(X) + \sigma^2 Z(x, \varepsilon) \quad (3.43)$$

where $\sum_{\alpha \in A} y_{\alpha} \Psi_{\alpha}(X)$ is a weighted sum of orthonormal polynomials describing the trend of the PCK model, σ^2 and $Z(x, \varepsilon)$ denote the variance and the zero mean, unit variance, stationary Gaussian process, respectively. The PCK model is created by determining the optimum set of polynomials in the trend and calibrating the Kriging model.

The PCK model is constructed in two parts; the determination of the optimal set of polynomials contained in the trend and the calibration of the Kriging model (determining the parameters $\{\theta, \sigma^2, y_{\alpha}\}$).

3.4.3.1. Error Estimation

Using the so-called validation error, the predictive accuracy of the PCK meta-model on new data can be evaluated once it has been established. The relative generalization error is computed using a distinct set of inputs and outputs. $[X_{val}, Y_{val} = M(X_{val})]$:

$$Err_{val}^{(PCK)} = \frac{N-1}{N} \left[\frac{\sum_{i=1}^N \left(M(x_{val}^{(i)}) - M^{PCE}(x_{val}^{(i)}) \right)^2}{\sum_{i=1}^N \left(M(x_{val}^{(i)}) - \hat{\mu}_{Y_{val}} \right)^2} \right] \quad (3.44)$$

When comparing the performance of various surrogate models on the same validation set, this error metric is useful.

3.4.4. Sampling Strategy in Metamodeling

Basically, an attempt is made to express the entire parameter space through metamodeling. In this case, space-filling or exploratory designs are preferred as a sampling strategy. The aim is to spread the sample points as uniformly as possible throughout the parameter space (Ramancha et al. 2022). An example shows how it works to plot samples using various sampling strategies. The probabilistic input model consists of two uniform random variables: $X_i \sim \mathcal{U}(0,1), i = 1,2$. Then, different samples from the input object are drawn using various sampling strategies, such as Monte Carlo sampling, Latin hypercube sampling, and Sobol's sequence sampling, as shown in Figure 3.2.

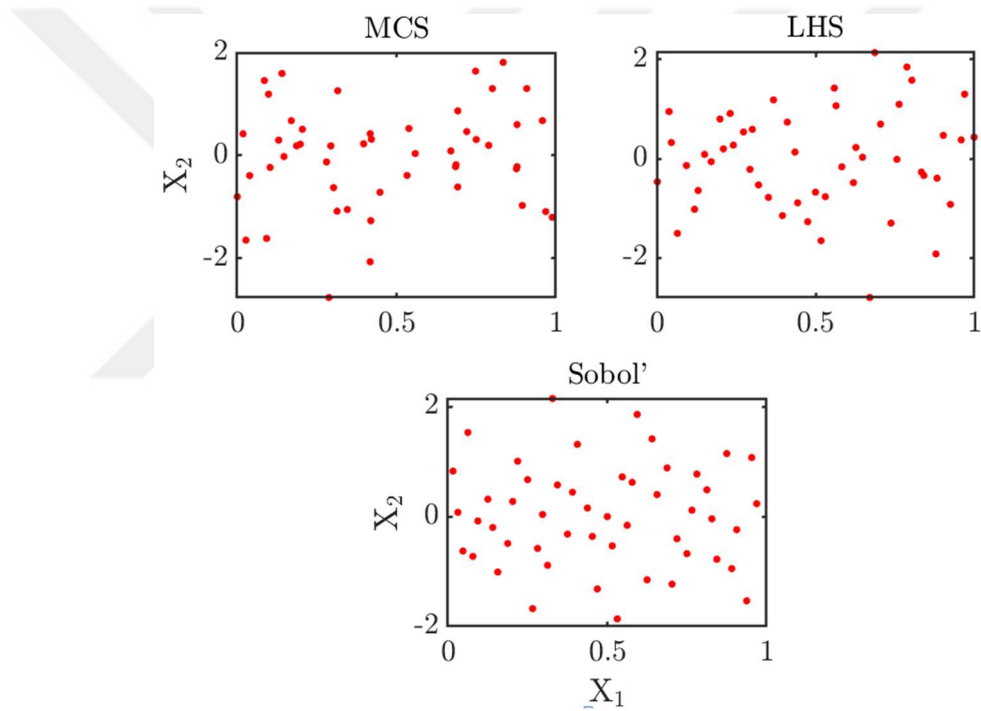


Figure 3.2. Plots of the different sampling strategies are shown for comparison

According to the plots, one can observe that the results by Sobol' method are much more spread out and have no clustering effects compared to the other methods. At this point, if the desired result cannot be achieved in the model, an experimental design must be enriched with new examples. Therefore, more points may need to be added to the existing sample set if necessary. The overall procedure can be follows:

- Selected materials of structure are set as design parameters and follow proper distribution.
- The initial training samples are obtained through sampling methods based on the properties of the random variable.
- The structural responses are obtained using the FE model of the structure and the training samples. Then, the input-output mapping relationship of the initial training samples is obtained.

As stated, the main objective is to provide an approach to efficiently integrate monitoring data into the model updating process and to improve the development of the prediction model. To this end, the design parameters are selected assuming they follow a suitable probability distribution. According to the properties of the random variables, the initial training samples are evaluated via sampling strategies and used to create metamodeling. Based on this procedure, in this study, PCK is adapted to the Bayesian model updating problem using the UQLab software framework (Marelli and Sudret 2014).

3.5. Reliability Assessment of Structures

By imposing safety constraints on the structural system, specific functionality is preserved. During the design phase of the system, these constraints must be considered, along with the environmental impacts to which it may be subjected. The operational range of the structure may be exceeded if the material properties or environmental impacts of the system being evaluated are subject to uncertainty. In this particular scenario, a structural failure is a distinct possibility. A limit state beyond which the system is unable to meet a particular performance metric is also regarded as a system failure, irrespective of the particular criterion chosen.

A system whose state is represented by a random vector of variables, $X \in D_X$ can be defined as two distinct domains, corresponding to the state space safe, D_s and failure, D_f regions of D_X , respectively. With this classification, it is possible to create a limit state function, $g(X)$, that assumes that positive values are a safety region and negative values are a failure region, which is defined as:

$$D_f = \{X|g(X) \leq 0\} \quad (3.45)$$

$$D_s = \{X|g(X) > 0\} \quad (3.46)$$

When the random vector of variables, X is defined by the joint probability density function (PDF) $X \sim f_X(x)$, the failure probability expression, p_f can be described as:

$$p_f = \mathbb{P}(g(X) \leq 0) \quad (3.47)$$

It expresses the probability of the system being in a failed state, given the uncertainties of the state parameters mentioned above. The failure probability, p_f is then calculated as:

$$p_f = \int_{D_f} f_X(x) dx = \int_{\{x:g(x) \leq 0\}} f_X(x) dx \quad (3.48)$$

It should be noted here that the field to be obtained from equation (3.48) is only implicitly defined by equations (3.45) and (3.46). This makes a direct prediction of the expression almost impossible. This limitation can be overcome by introducing the indicator function of the failure area, which is given in equation (3.49). With this specified function, equation (3.48) can be expressed in equation (3.50).

$$1_{D_f}(x) = \begin{cases} 1 & \text{if } g(x) \leq 0 \\ 0 & \text{if } g(x) > 0 \end{cases}, x \in D_X \quad (3.49)$$

$$p_f = \int_{D_f} 1_{D_f}(x) f_X(x) dx = \mathbb{E}[1_{D_f}(X)] \quad (3.50)$$

where $\mathbb{E}[\cdot]$ is the expectation operator with respect to the $f_X(x)$. This reduces the calculation of p_f to the estimation of the expectation value of $1_{D_f}(x)$.

Presently, the objective of structural reliability analysis is to quantify the likelihood that such failures will transpire, in accordance with a model that incorporates uncertainty in load, environmental, and structural parameters. This evaluation is utilized in structural reliability analysis to quantify risks via the reliability index and, by extension, to assess the repercussions of failure. An associated generalized reliability index, β can be defined as:

$$\beta = -\phi^{-1}(p_f) \quad (3.51)$$

3.5.1. Estimation of p_f

There are two types of methods, such as approximation and simulation-based, for evaluating p_f . Approximation methods, including the second-order reliability method (SORM) and the first-order reliability method (FORM), approximate the limit-state function at a reference point on a local scale. In situations where calculating, p_f requires a relatively small number of model evaluations, these methods may prove to be highly efficient. However, when complex and non-linear limit-state functions are present, their dependability tends to diminish. Simulation methods are based on sampling the joint distribution of the state variables, X . Despite the computationally intensive nature of these methods, their convergence behavior in p_f estimates is well-defined. Examples of sampling-based algorithms are Monte-Carlo simulation, importance sampling, and subset simulation.

3.5.2. Subset Simulation

When the value of p_f is small, multiple evaluations of the limit state function by the Monte Carlo simulation may be required to converge to an acceptable level of accuracy. In order to address this constraint, (Au and Beck 2001) introduce the subset simulation approach, which resolves a sequence of less complex reliability issues characterized by intermediate failure thresholds.

If a sequence of failure domains $D_1 \supset D_2 \supset \dots D_m = D_f$ is considered, such sequence can be built with a series of decreasing failure threshold, $t_1 > \dots > t_m = 0$ with the traditional definition of limit state function. The corresponding intermediate failure domains include: $D_k = \{x: g(x) \leq t_k\}$. After that, the probability mass of each intermediate failure zone created is combined through conditional probability and can be defined as (Au and Beck 2001):

$$p_f = \mathbb{P}(D_m) = P\left(\bigcap_{k=1}^m \mathbb{P}(D_k)\right) = \mathbb{P}(D_1) \prod_{i=1}^{m-1} \mathbb{P}(D_{i+1} \mid D_i) \quad (3.52)$$

In order to estimate p_f , it is necessary to estimate intermediate probabilities, $\mathbb{P}(D_1)$ and conditional probabilities, $\mathbb{P}(D_{i+1} \setminus D_i, i = 1 \text{ to } m - 1)$. $\mathbb{P}(D_1)$ can be readily estimated from a sample of size, N_s with given initial threshold, t_1 and described as:

$$\mathbb{P}(D_1) \approx \hat{P}_1 = \frac{1}{N_s} \sum_{k=1}^{N_s} 1_{D_f}(x^{(k)}) \quad (3.53)$$

The remaining conditional probabilities can subsequently be calculated in the same manner. Here, a characteristic flowchart for reliability assessment is presented in Figure 3.3.

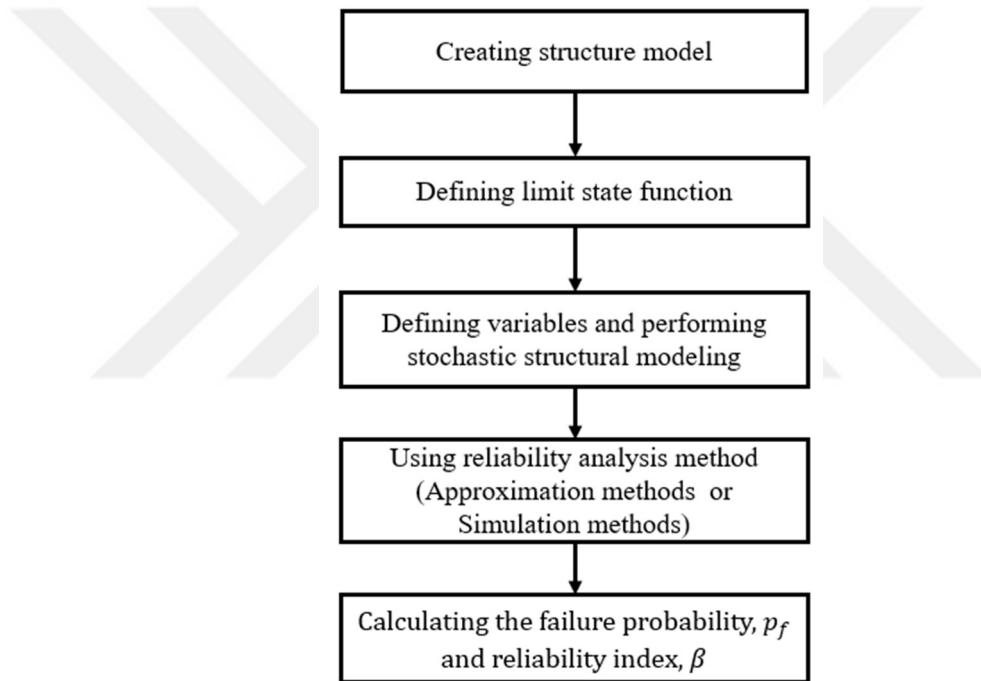


Figure 3.3. Characteristic flowchart for reliability assessment of structures

3.5.3. Target Reliability Index

Target levels for the reliability index may often differ, considering issues such as the cost of safety measures, failure consequences, reference period, or operating life of the design (Holicky, Diamantidis, and Sykora 2015). When determining target reliabilities, comparisons with existing satisfactory design practices are generally made, human safety criteria and economic optimization are made by focusing on the life cycle

costs of representative structures (Nadolski, Rózsás, and Sýkora 2019). In Table 3.2, target reliability indices, including different reference periods, are given according to the selected standards.

Table 3.2. Target reliability index for standards EN 1990 and ISO 2394 at different time periods (Nadolski, Rózsás, and Sýkora 2019)

Standard	Failure consequence	Time period	β
EN 1990 (EN 1990:2002 2002)	Medium	1 year	4.7
EN 1990	Medium	50 years	3.8
ISO 2394 (ISO2394:1998 1998)	Moderate	Life-time	3.1*
ISO 2394	Moderate	1 year	4.2**

*For moderate relative cost of safety measures

**For normal relative cost of safety measures

The standard JCSS (2001) proposes the target reliability values for ultimate limit states and is given in Table 3.3. The values given are obtained based on public cost-benefit analysis and are compatible with calibration studies and statistical observations.

Table 3.3. Target reliability indices (JCSS 2001)

Relative cost of safety measure	Minor consequences of failure	Moderate consequences of failure	Large consequence of failure
Large	$\beta = 3.1$	$\beta = 3.3$	$\beta = 3.7$
Normal	$\beta = 3.7$	$\beta = 4.2$	$\beta = 4.4$
Small	$\beta = 4.2$	$\beta = 4.4$	$\beta = 4.7$

In ASCE/SEI 7-10 (2016), target reliability levels are grouped by the four risk categories. These are based on the risk to people's life, health, and welfare that comes with damage or failure because of how they are used. Risk categories are associated with category (I) with few people to category (IV) with tens of thousands. The target reliability indices given in Table 3.4 are provided for a reference period of 50 years.

Table 3.4. Target reliability indices for load conditions (not include earthquake)
(ASCE/SEI 7-10 2016)

	Risk category			
	I	II	III	IV
Failure that is not sudden and does not lead to widespread progression of damage	$\beta = 2.5$	$\beta = 3.0$	$\beta = 3.25$	$\beta = 3.5$
Failure that is either sudden or lead to widespread progression of damage	$\beta = 3.0$	$\beta = 3.5$	$\beta = 3.75$	$\beta = 4.0$
Failure that is sudden and results in widespread progression of damage	$\beta = 3.5$	$\beta = 4.0$	$\beta = 4.25$	$\beta = 4.5$

In this context, target reliability levels must be met in the design to ensure that certain levels of safety are maintained. For this reason, target reliability levels are used in the study to determine the reliability of the structure as it is exposed to corrosion over time. Considering all the above-mentioned propositions, it is decided that it would be appropriate to take the target reliability index between $\beta_l = 3.5$ and $\beta_u = 4.7$. The results will be evaluated accordingly.

3.6. Summary of Proposed Bayesian Model Updating with Metamodeling Techniques

In Figure 3.4, a comparison of traditional and proposed methodologies used in probabilistic evaluation of structures is given. The basic steps to be followed for the proposed methodology are given sequentially. Firstly, an initial FE model is created based on design drawings, vibration data is gathered, and modal properties such as natural frequencies, mode shapes, etc. are identified. Finally, the proposed methodology is used to update the FE model of structure. The probabilistic analysis of interest can be applied to the updated FE model.

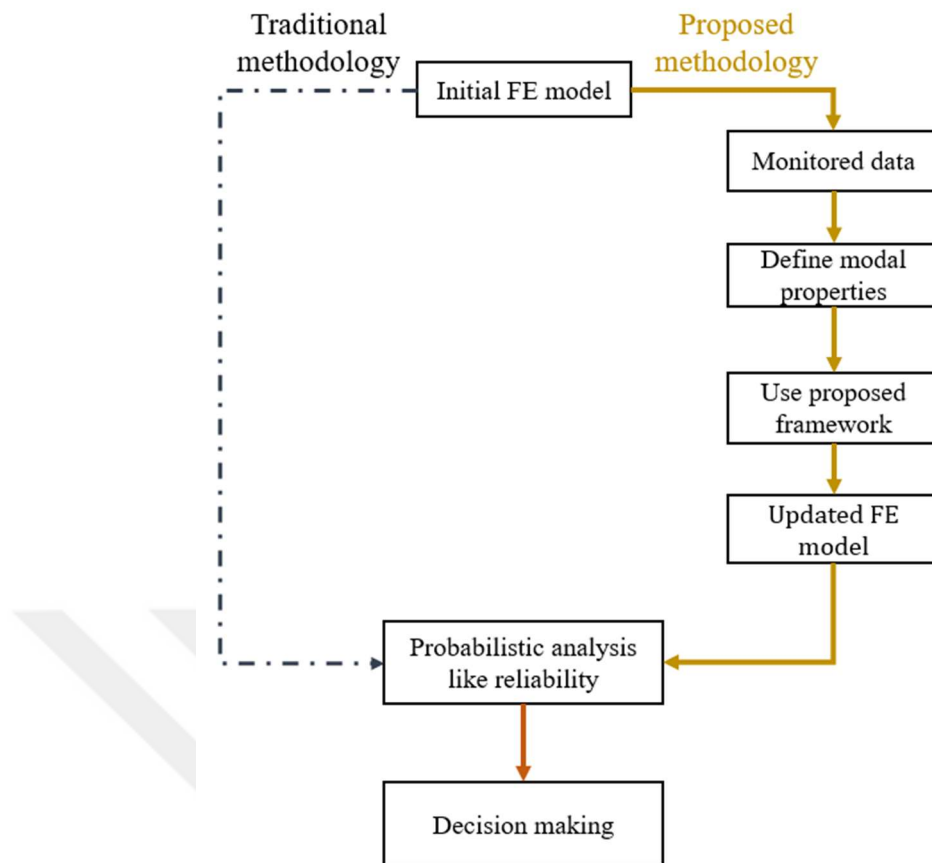


Figure 3.4. Traditional and proposed methodologies for probabilistic analysis of FE models

A detailed examination and explanation of the proposed framework are given in previous sections. As stated previously, the main objective of this study is to provide a PCK-based methodology to efficiently integrate monitoring data into the model updating process and to improve the quality of the prediction model. In this context, a summary of the proposed computational algorithm is presented in Figure 3.5.

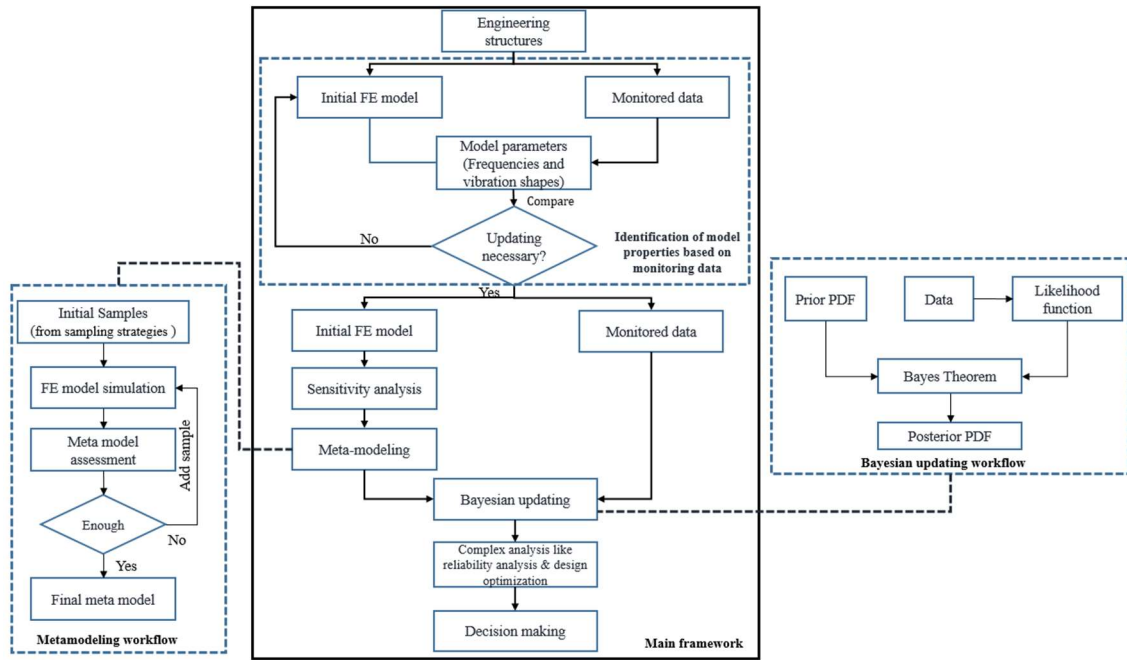


Figure 3.5. Proposed framework for SHM-based Bayesian model updating accelerated by metamodeling technique and reliability assessment

Disregarding prevailing uncertainties can have a substantial impact on the dependability of structural descriptions and subsequent assessments of structures. To address this, the Bayesian framework can be applied to the FE model updating process, incorporating the uncertainties into the data acquired with SHM. The computational burden during the specified process is a significant problem, but it can be reduced by metamodeling techniques. Essentially, computational models are replaced by cheap proxies to compensate for the increase in computational cost of stochastic modeling. Thus, it can significantly reduce the computational burden of FE model updating procedures for large-scale structural systems.

CHAPTER 4

SHM-BASED BAYESIAN MODEL UPDATING WITH METAMODELING TECHNIQUES

4.1. Introduction

In the analysis and design procedures of civil engineering structures, it is frequently referred to as FE approaches to predict structural responses effectively and economically. In the FE approach, however, modeling errors are inevitable for many different reasons, such as simplified models of real structures, construction errors, and uncertainties in the computational material properties. These errors are globally named as aleatory and epistemic uncertainties if they are considered collectively (Dolšek 2012). In a general sense, aleatory uncertainty represents an inherent randomness of natural occurrences governed by probabilistic models, while epistemic uncertainty is induced by a lack of complete knowledge such as insufficient data, inaccuracies in measurements, inadequate models, etc. At this point, real-time data obtained for a SHM procedure can be manipulated to manage the uncertainty (Catbas, Susoy, and Frangopol 2008). Increased information may reduce the epistemic uncertainty and make the predictions more reliable. Besides, the model updating concept is used to provide more accurate FE model-based performance estimates. In general, FE model updating methods deal with deterministic errors. Due to limitations in the data from monitoring devices used for SHM and limitations of analysis techniques, however, the inferred dynamic information from observations also contains uncertainty. Accordingly, the deterministic model updating methods may not produce accurate results in applications by ignoring these uncertainties.

Bayesian modeling techniques can characterize uncertain structural parameters as random variables and attract more attention in the field of FE model updating (H.-F. Lam, Yang, and Au 2015). Using the Bayesian approach, modeling uncertainties associated with the underlying structural system can be estimated efficiently. Here, the parameters of the probability density function (PDF) are treated as random variables in the Bayesian approach; therefore, it is possible to use prior and posterior knowledge of the model. Hence, this process can provide a systematic methodology for integrating a probabilistic

SHM framework into the model updating procedure. Due to the high complexity of Bayesian inference problems, sampling methods such as Markov chain Monte Carlo (MCMC) are generally used to estimate the posterior PDF. Despite their effectiveness, the MCMC methods have some limitations since they may be computationally intensive due to the large number of iterations needed for better convergence. Consequently, the computational cost of Bayesian FE model updating for complex engineering structures is likely to become unaffordable and unfeasible. To overcome this problem, meta-modeling techniques (or surrogate modeling) which are effective stochastic predictors to decrease the computational burden in the model updating process Mao, Wang, and Li (2020) can be integrated to the MCMC methods. It should be stated that one of the leading studies (Straub and Papaioannou 2015; Hariri-Ardebili and Sudret 2020; Simwanda, De Koker, and Viljoen 2021; Z. Wang and Shafieezadeh 2020) proposes the use of meta-modeling to further reduce the number of model evaluations. In this context, some variants of metamodeling techniques, such as the polynomial-based response surface method, Kriging predictor, etc., are widely used in the literature. Among these, Kriging can be simply defined as a statistical interpolation method that uses Gaussian processes to interpolate various complex functions. It is introduced in the context of metamodeling and computer experiments in the work of Sacks et al. (1989) and Kriging is used to represent the input/output mapping of an expensive computational model. Moreover, the polynomial chaos expansion (PCE), another metamodeling technique, aims at providing a functional approximation of a computational model through its spectral representation of polynomial functions on a properly constructed basis. While Kriging interpolates local variations of the computational model output as a function of neighboring experimental design points, PCE well approximates the global behavior of this computational model output. As a general intention in the literature, it can be observed that the studies dealing with these approaches are implemented for model updating separately (Martakis et al. 2022; Ramancha et al. 2022). However, a more accurate meta-model can be obtained by combining the global and local approaches of these two techniques.

Polynomial-Chaos-Kriging (PCK), based on the well-established metamodeling techniques PCE and Kriging, is to be seen as the most prominent metamodeling algorithm. PCK makes use of the regression-type PCE to capture the global behavior of the computational model as well as the interpolation-type Kriging to capture local variations. This combination results in a metamodeling technique that is more efficient than PCE and Kriging separately. Therefore, one of the biggest difficulties in structural

reliability analysis, reducing the number of calls made to the studied numerical model and reducing the computation time, has been tried to be achieved by using PCK in structural reliability analysis (Okoro, Khan, and Ahmed 2021; Zhou and Peng 2022; Zhou et al. 2023). In addition, Rossat et al. (2022) has suggested the implementation of PCK in Subset simulation (SuS) within the framework of the Bayesian updating with Structural Reliability developed by Straub and Papaioannou (2015) to speed up the SuS sampling procedure. In order to decrease the computational burden, which is a frequently encountered problem in Bayesian inference applications, it is recommended to use the PCK meta-model (García-Merino et al. 2023).

In this context, this study aims at adapting a PCK based methodology to Bayesian model updating to overcome the computational difficulties encountered in the health monitoring of real and complex engineering structures. As previously stated in the literature, the primary objective of this study is not to develop new metamodeling techniques. Rather, it is to examine the benefits and drawbacks of utilizing meta-models for updating models and to contribute to the resolution of the gaps in updating finite element models. In addition, the effectiveness of the proposed solution method in this context is examined by comparing the results obtained from different meta-models on the same examples. In this context, the proposed method is applied to the numerical model of a truss bridge example. Further, an experimental study using the vibration data of the Irvine Bridge Column Project at Los Alamos National Laboratory (LANL 1997) is used.

4.2. Numerical Examples

To show the effectiveness of the proposed methodology, one numerical and one experimental study are conducted in this section. In the numerical example, a truss bridge is considered, considering different measurement configurations. Then, the model parameters of a previously investigated laboratory structure are assessed.

4.2.1. A Truss Bridge Example

The numerical model of a plane truss bridge, whose schematic representation is presented in Figure 4.1, is investigated in this section. The constructed numerical model comprises 20 nodes and 37 degrees of freedom (DOFs). The cross-section area of each

element is accepted as 0.0014 m^2 . Elastic modulus of 200 GPa, a mass density of $7.85 \times 10^3 \text{ kg/m}^3$, and Poisson's ratio of 0.3 are specified in the test model.

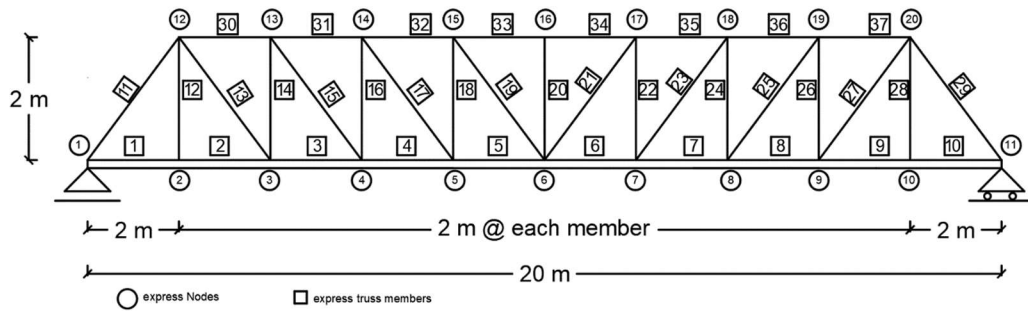


Figure 4.1. Truss bridge model

The excitation force is simulated using band-limited white noise generated in the MATLAB environment (MATLAB 2018). It has been considered that the test data will be affected by channel noise in the actual data measurement. Here, the root mean square (rms) of channel noise is arranged as 10% of the noise-free response rms. The simulated excitation force is applied in the vertical direction on the nodes 13, 16 and 19, respectively. Since the modulus of elasticity is one of the sensitive parameters in FE model simulation, it is selected as a parameter to be updated. As shown in Table 4.1, the actual value of modulus of elasticity is considered as 200 GPa and the initial FE model is manipulated by increasing it by 50%. The standard deviations of the priors (nominal values) are arranged so that the coefficient of variation, COV (%) is to be 30 %.

Table 4.1. Nominal values test and FE model

Parameter	Actual value	Error (%)	FEM value
Modulus of Elasticity, E (GPa)	200	50	300

The FE model of the truss bridge is generated using ANSYS software (ANSYS 2015). Then, the modal parameters of the simulated bridge are identified by using the Enhanced Frequency Domain Decomposition (EFDD) technique. In this context, three different cases (Case-1, Case-2, and Case-3) are considered depending on the number of sensors. In Case-1, the modal properties are estimated using vibration response data obtained by nodes 2, 3, 4, 6, 8, 9, and 10. In Case-2, the model parameter properties are tried to be estimated by removing nodes 3 and 9 and using vibration response data

obtained by nodes 2, 4, 6, 8, and 10. In Case-3, however, it is assumed that the vibration data of nodes 4, 6, and 8 are measured only. The identification results for the considered cases are presented in Tables 4.2, 4.3, and 4.2, respectively.

Table 4.3. Identified frequencies and MAC values FEM for Case-1

Identified (Hz)	Manipulated Initial FE model (Hz)	Difference with initial FE model (%)	MAC with initial FE model
12.207	15.053	18.905	0.964
34.424	42.532	19.064	0.638
45.166	55.878	19.171	0.875
77.881	97.842	20.401	0.952
100.342	128.448	21.881	0.746
114.502	148.511	22.900	0.586
133.057	175.454	24.164	0.873
145.019	194.505	25.442	0.628

Table 4.4. Identified frequencies and MAC values FEM for Case-2

Identified (Hz)	Manipulated Initial FE model (Hz)	Difference with initial FE model (%)	MAC with initial FE model
12.207	15.053	18.905	0.939
34.424	42.532	19.064	0.423
45.166	55.878	19.171	0.678
77.881	97.842	20.401	0.928
100.342	128.448	21.881	0.442
114.502	148.511	22.900	0.397
133.057	175.454	24.164	0.733
145.019	194.505	25.442	0.403

Table 4.5. Identified frequencies and MAC values FEM for Case-3

Identified (Hz)	Manipulated Initial FE model (Hz)	Difference with initial FE model (%)	MAC with initial FE model
12.207	15.053	18.905	0.846
34.424	42.532	19.064	0.259
45.166	55.878	19.171	0.409
77.881	97.842	20.401	0.390
100.342	128.448	21.881	0.193
114.502	148.511	22.900	0.234
133.057	175.454	24.164	0.424
145.019	194.505	25.442	0.198

The unknown model parameters θ_1 to θ_9 are considered as scaling coefficients for the elastic modulus of the members. Here, $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8$ and θ_9 represent the scaling coefficients for E_1 (members 1 to 3), E_2 (members 4 to 7), E_3 (members 8 to 10), E_4 (members 11 to 13), E_5 (members 14 to 20), E_6 (members 21 to 24), E_7 (members 25 to 29), E_8 (members 30 to 33) and E_9 (members 34 to 37). Thus, the actual values for the model parameters are considered 1.0, while their nominal values are selected as 1.5.

In the updating procedure, structural design parameters are assumed to be follow lognormal distributions as $X \sim LN(\mu, \sigma)$. In order to update the model parameters, the initial training samples are obtained by using the LHS method and applied to the properties of the random variable, which is based on each selected model parameter. Here, the analysis procedure is performed with 100 samples. The structural responses are obtained using training samples created in FE model. Then, the input-output of the initial training samples are matched. In this context, the achieved results are presented in Tables 4.5, 4.6 and 4.7, respectively.

Table 4.6. Updated model parameters for Case-1 with meta-models

Parameters	Updated by Kriging		Updated by PCE		Updated by PCK	
	μ	COV (%)	μ	COV (%)	μ	COV (%)
θ_1	1.15	14.62	1.05	13.27	1.05	11.26
θ_2	1.10	15.62	1.10	18.44	1.00	17.00
θ_3	1.50	14.54	1.15	16.67	1.10	14.69
θ_4	1.20	13.07	1.10	15.24	1.10	16.54
θ_5	1.15	18.25	1.10	18.00	1.05	17.00
θ_6	1.25	20.41	0.95	19.42	1.00	21.44
θ_7	1.25	17.58	1.25	13.46	1.10	17.88
θ_8	1.25	16.27	1.30	15.45	1.20	15.75
θ_9	1.10	20.53	0.95	18.68	0.95	19.58

Table 4.7. Updated model parameters for Case-2 with meta-models

Parameters	Updated by Kriging		Updated by PCE		Updated by PCK	
	μ	COV (%)	μ	COV (%)	μ	COV (%)
θ_1	1.15	13.81	1.05	13.33	1.10	13.33
θ_2	1.15	17.00	1.10	17.50	1.00	17.00
θ_3	1.50	15.91	1.20	16.67	1.15	16.36
θ_4	1.20	15.24	1.10	15.24	1.10	15.24
θ_5	1.15	16.50	1.10	18.00	1.05	17.00
θ_6	1.25	22.22	0.95	20.56	1.00	20.00
θ_7	1.25	17.62	1.35	17.14	1.10	17.14
θ_8	1.30	15.91	1.30	15.45	1.20	15.45
θ_9	1.10	20.53	0.90	20.00	0.90	20.00

Table 4.8. Updated model parameters for Case-3 with meta-models

Parameters	Updated by Kriging		Updated by PCE		Updated by PCK	
	μ	COV (%)	μ	COV (%)	μ	COV (%)
θ_1	1.15	13.33	1.05	13.81	1.10	12.86
θ_2	1.15	16.50	1.10	17.50	1.00	17.00
θ_3	1.55	16.67	1.20	16.36	1.15	16.36
θ_4	1.20	15.24	1.10	15.24	1.10	15.71
θ_5	1.15	17.00	1.15	16.67	1.10	17.00
θ_6	1.25	21.05	0.95	20.00	1.00	19.47
θ_7	1.25	17.62	1.35	17.14	1.10	17.62
θ_8	1.35	15.91	1.30	15.45	1.20	15.91
θ_9	1.10	21.05	0.90	21.58	0.95	20.53

The differences between frequencies and MAC values of updated and non-updated FE models for Case-1 are given in Figures 4.2 and 4.3, respectively.

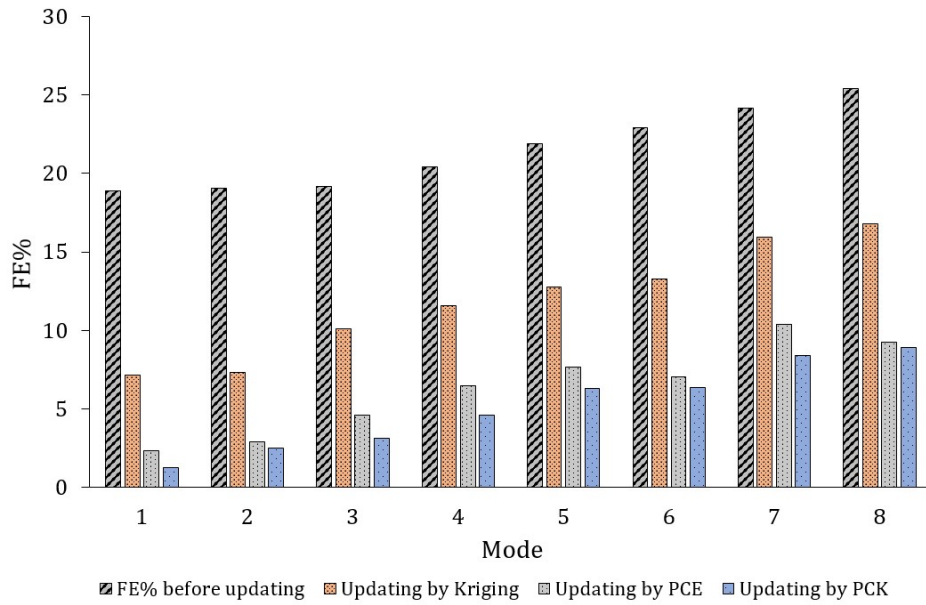


Figure 4.2. Comparison FE% for Case-1 after and before model updating

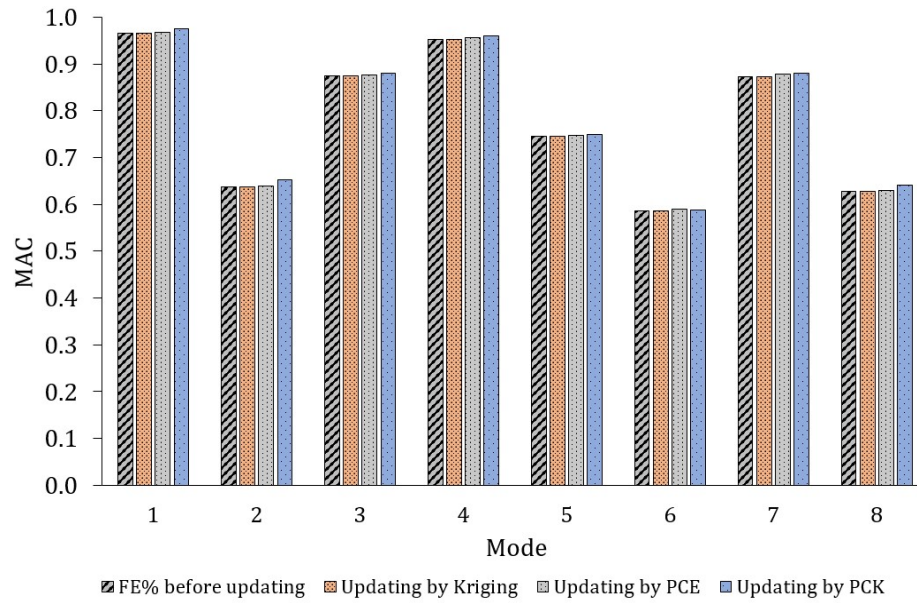


Figure 4.3. Comparison MAC for Case-1 after and before model updating

The differences between frequencies and MAC values of updated and non-updated FE models for Case-2 are given in Figures 4.4 and 4.5, respectively.

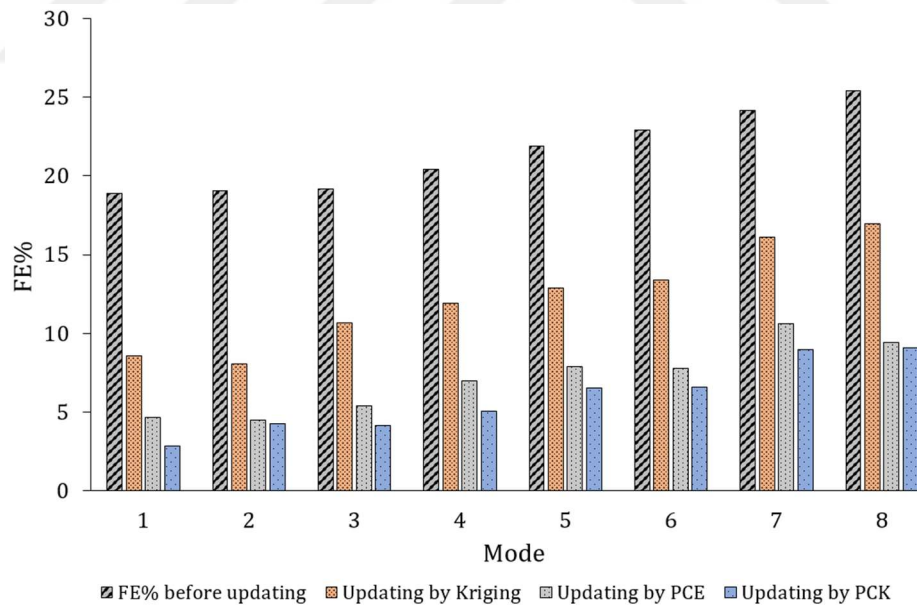


Figure 4.4. Comparison FE% for Case-2 after and before model updating

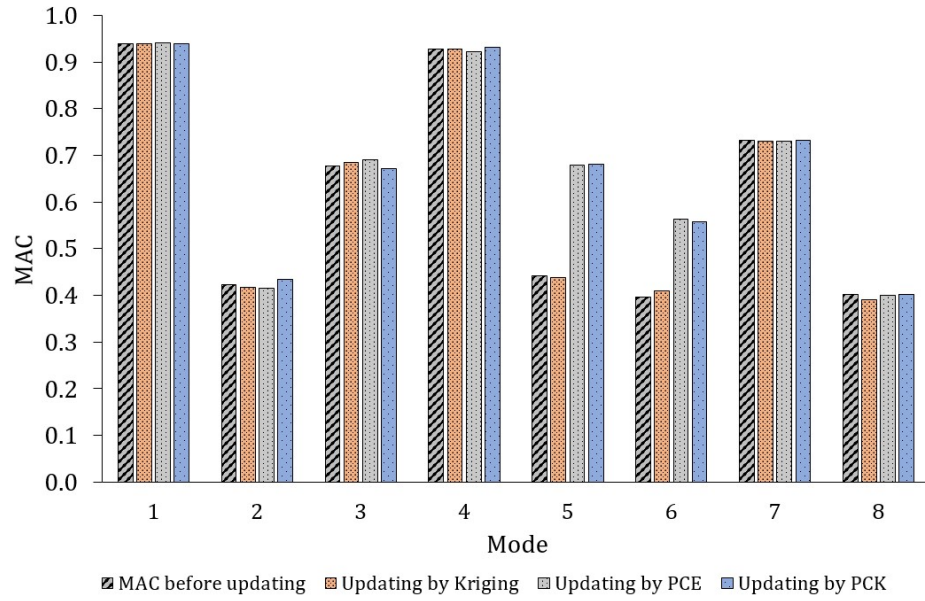


Figure 4.5. Comparison MAC for Case-2 after and before model updating

The differences between frequencies and MAC values of updated and non-updated FE models for Case-3 are given in Figures 4.6 and 4.7, respectively.

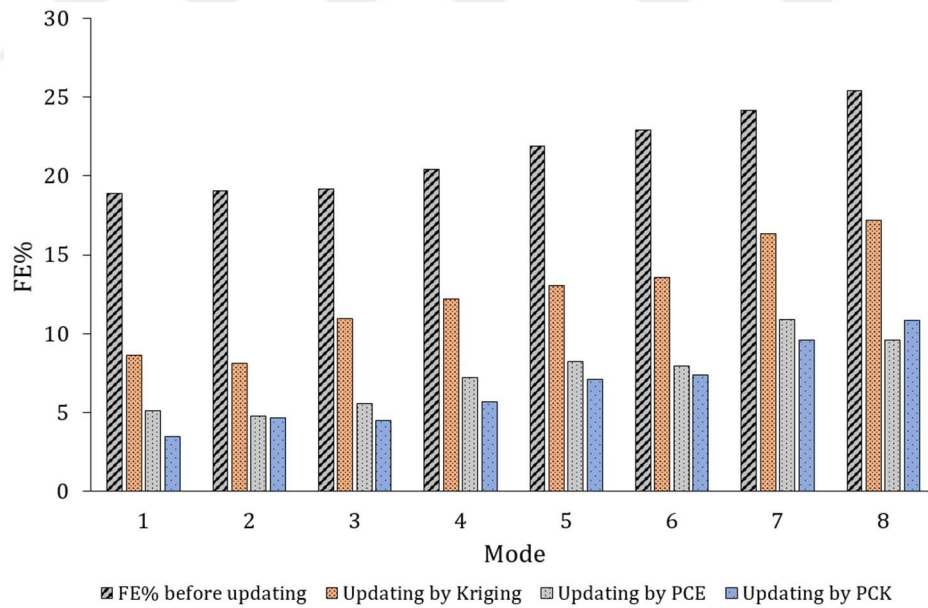


Figure 4.6. Comparison FE% for Case-3 after and before model updating

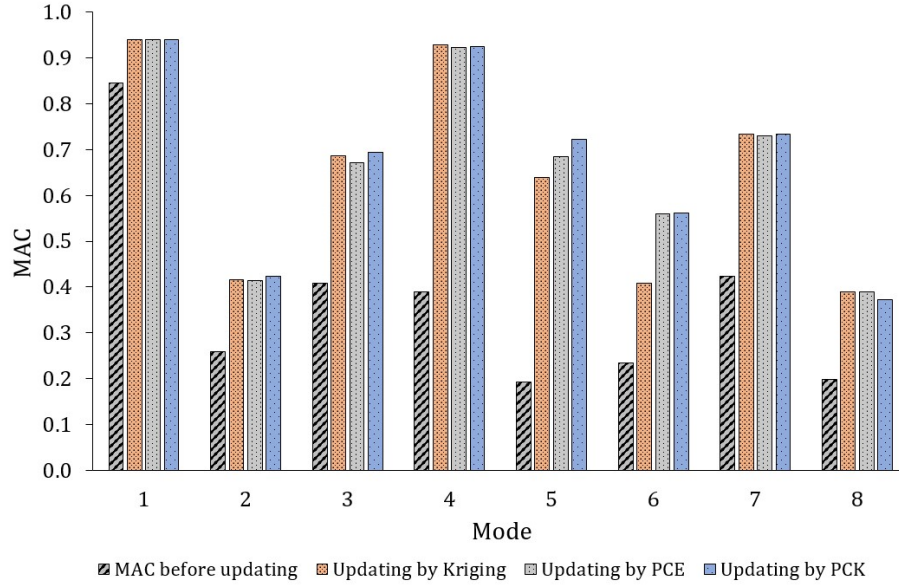


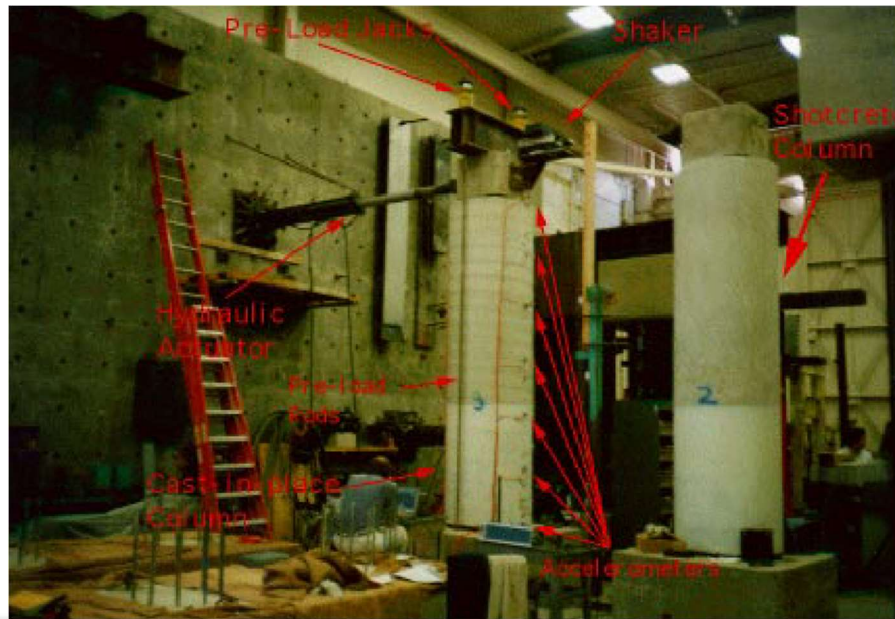
Figure 4.7. Comparison MAC for Case-3 after and before model updating

A significant improvement in FE% values is achieved for each examined Cases. Even though modal frequencies have gone down a lot, MAC values show that meta-models haven't made any big improvements over the original FEM, except for modes 5 and 6 for Case-2. As the number of measurement points decreases (for Case-3), however, a more remarkable increase can be observed for MAC values by metamodeling.

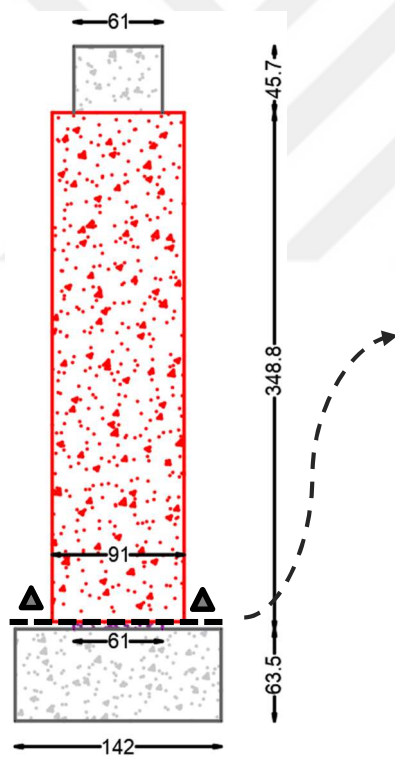
During the analysis, the number of steps is set to 2000 with a chain count of 500. Moreover, the MCMC sample size is prescribed as 10^6 , with approximately 200,000 accepted samples. The approximate computation time of the FEM simulation using a computer with an Intel (R) Core™ i5-10300H CPU @ 2.50 GHz and 16 GB of main memory is 45 seconds for a single estimate of the modal properties. So, it might take around 2500 hours (≈ 104.17 days) to update the Bayesian model with the FE model simulation if each sample used for the Markov chain over the FE model is used without using metamodeling. However, with the applied metamodeling techniques, a significant decrease can be observed in the overall FE model updating (approximately 1.25 hours). As a result, although it has been observed that the proposed PCK method can give better results with a slight difference, a more dramatic decrease is observed in the computational effort. Moreover, considering that PCK is a combination of two different metamodeling techniques, it can be stated that PCK provides better convergence to actual results, especially at the point where one method reaches a far greater value than the other.

4.2.2. Irvine Bridge Column

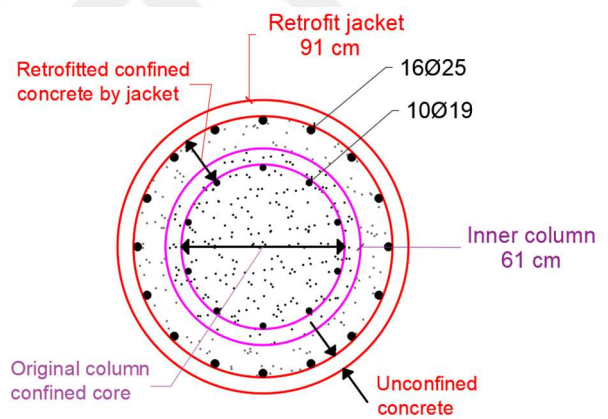
In this section, experimental results of the Irvine bridge column, which were conducted by the University of California, Irvine, at Los Alamos National (LANL 1997), are investigated. The illustration of the test configuration is shown in Figure 4.8. (a). The test structure consists of a 61-cm-diameter column, and it is retrofitted by placing concrete. Then the diameter of the retrofitted bridge column is 91 cm. The column has a 142-cm-square concrete foundation with a height of 63.5 cm. The concrete foundation of the column is bolted to the 0.61-meter-thick test floor in the UCI laboratory during the experiments so that the structure does not move after the test is started. The total length of the column is 348.8 cm, and the 91 cm portion of the column is 345 cm in length. The schematic of the bridge column is given in Figure 4.8. (b). It is specified by the LANL report that the original column consisted of 10Ø19 longitudinal rebars and was enclosed by a spiral cage of Ø13.5 rebar spaced at 18 cm. A 5-cm cover is provided for spiral reinforcement. The retrofitted jacket has 16Ø25 longitudinal rebars and is enclosed by a spiral cage of Ø19 rebar, spaced at 15 cm. Again, a 5-cm cover is provided for this reinforcement. The reinforcement layout of the bridge column is shown in Figure 4.8. (c).



(a)



(b)



(c)

Figure 4.8. Bridge column (a) The illustration of the test configuration (LANL 1997)
(b) dimensions (cm) and (c) cross section and reinforcements layout

For the data acquisition, forty accelerometers shown in Figure 4.9 (a) are used. Accelerometers are grouped as group 1 for the left and group 2 for the right side of the z direction. A 400 kN axial force is applied in order to replicate the dead loads that would be encountered by a real column. For experimental modal analysis, the excitation is supplied by an APS electro-magnetic shaker mounted off-axis at the top of the structure. The transfer of horizontal load from the shaker to the structure occurs via a frictional connection formed between the shaker's supports and the steel plate. A 0-400 Hz uniform random signal is sent from a source module in the data acquisition system to the shaker, and the typical input power spectrum is shown in Figure 4.9 (b).

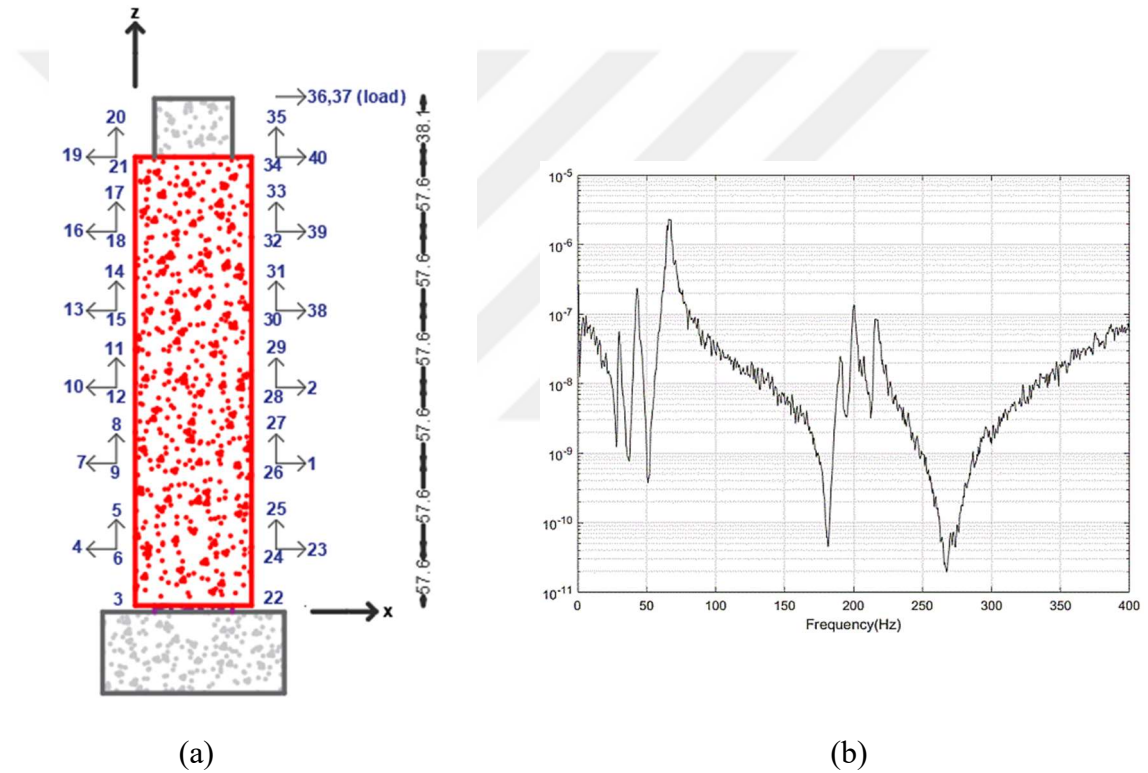


Figure 4.9. (a) Accelerometer locations (as cm) of Irvine bridge column test and (b) typical input power spectrum

PCB 302A accelerometers with a nominal sensitivity of 10 mV/g are used for 2, 39, and 40 accelerometers. Wilcoxon 736t accelerometers with a nominal sensitivity of 100 mV/g are placed in locations 33, 34, 35, 36, and 37. PCB 336C accelerometers with a nominal sensitivity of 1 V/g are used for other locations. Accelerometers 36 and 37 are located 20.3 cm off the axis in the y direction. The HP 3566A dynamic data acquisition system is used to sample and process data. Frequency response functions (FRFs) are

measured in the range of 0-400 Hz. Each spectrum is calculated from 30 averages of 2 s duration time histories with 2048 sampling points. This sampling rate produced a frequency resolution of 0.5 Hz.

4.2.2.1. Finite Element Model of Irvine Bridge Column

The FE model is developed in order to facilitate a comparison between the outcomes of experimental and analytical modal analyses. In the model, reinforcement is not incorporated. Elastic modulus of 24.8 GPa, mass density of $2.28 \times 10^3 \text{ kg/m}^3$, and Poisson's ratio of 0.2 are specified in this model for concrete columns. ANSYS software (ANSYS 2015) is used to establish the FEM of the Irvine column bridge. In Figure 4.10, the FE model of the column is presented.

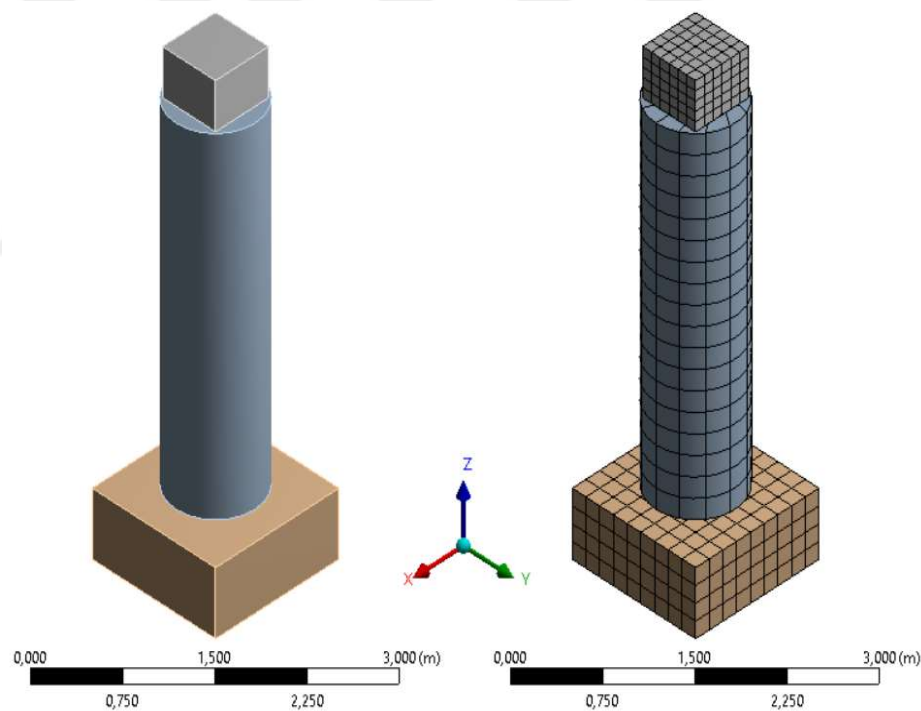


Figure 4.10. FE model of Irvine bridge column

By utilizing an undamaged column example, the experimental modal analysis yields the natural frequencies and mode shapes. The 61-cm concrete block and retrofitted part are considered separately. Elastic modulus and mass density values of these parts are assumed unknown and selected for the model update.

4.2.2.2. Modal Parameter Identification of Irvine Bridge Column Based on Monitoring Data

It is known that a coherence function that ranges between 0 and 1 is an indicator of the quality of the FRF and evaluates the consistency of the FRF from measurement to repeat of the same measurement. A value of 0 and 1 indicates that the FRF amplitude and phase are very repeatable or not from measurement to measurement. Not repeatable means a possible warning flag that there is an error in the measurement setup. Therefore, all of the nodes are first investigated by plotting coherence vs. FRF graphics. It is trying to determine whether a measurement has an error or not. Accelerometers at 2, 39, and 40 nodes are found not to be sensitive enough to measure the desired vibration quantities; the acceleration data from these sensors is excluded for modal analysis.

The pick-picking method (PPM) and the least squares complex exponential method (LSCEM) are used for modal parameter identification. It is observed that there are no considerable differences in the outcomes obtained as a result of the two methods applied. However, in PPM, the dynamic properties are obtained as a singular value from each vibration signal (Bayraktar et al. 2009). Therefore, results coming from the LSCE are taken into account in the model updating process. PPM results are presented here for comparison with LSCE. The stabilization diagram for the vibration signal using LSCE is given in Figure 4.11. Six modes of the bridge column are identified and shown in Figure 4.12. In Table 4.8, the modal parameters that are found by comparing the results from the FE model to the experimental modal analyses can be seen. Frequency errors, FE%, and MAC values are also given in the same table. The frequency values found in the literature are also presented here to compare the calculations.

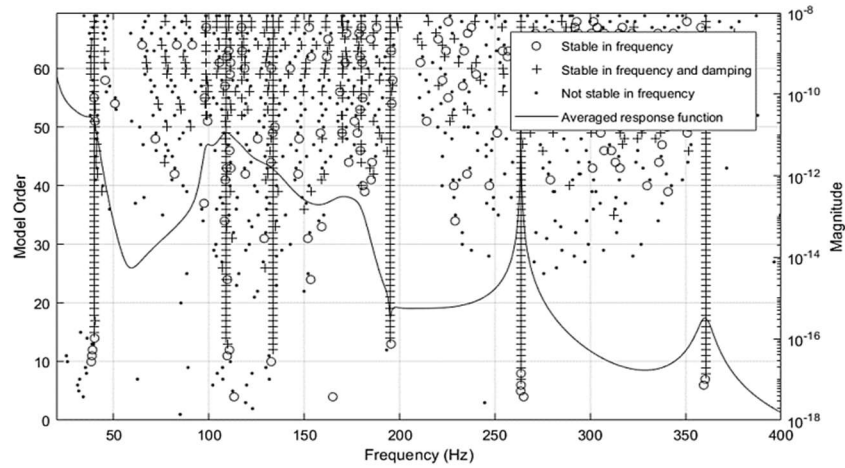


Figure 4.11. The stabilization diagram from the vibration data by LSCEM

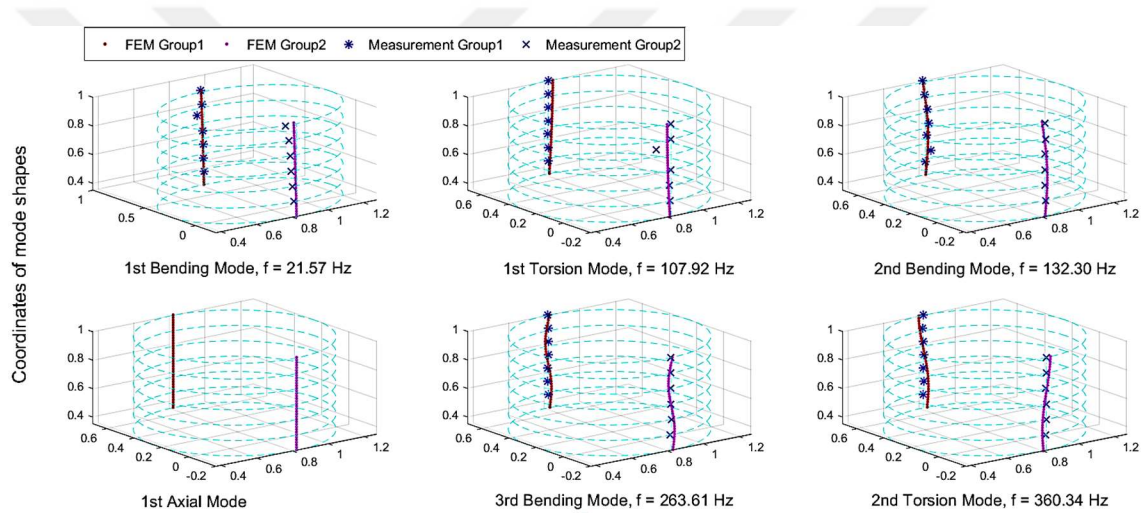


Figure 4.12. Comparison of Mode Shapes from experiment and FE model analysis

Table 4.9. Comparison for modal properties from vibration test and initial FEM

Frequencies from FEM (Hz)	Experimental frequencies (Hz)				
	PPM	LSCEM	Literature by (Sohn and Law 2000)	FE (%)	MAC
19.08	16.17	21.57	27.82	13.05	0.881
115.73	98.58	107.92	110.42	6.75	0.723
123.82	108.31	132.30	147.51	6.85	0.655
180.93	-	-	-	-	-
307.04	263.57	263.61	272.83	14.14	0.637
355.70	360.10	360.34	340.71	1.30	0.806

The FE model is to be updated by using the model parameter values obtained. For this, the method proposed in the study is used in a way that includes different metamodeling techniques. The frequency and MAC values of the 1st axial mode shape, which cannot be obtained from the experimental model analysis, are not included.

4.2.2.3. Results of Bayesian FE Model Updating of the Irvine Bridge Column

For prior PDF selection, Leyder, Chatzi, and Frangi (2017) suggested using JCSS (2002) for the selected parameters. H. Wang, Li, and Li (2010) said that the updating parameters are bounded mainly according to the engineering judgments if there are no test results for the nominal values. Accordingly, statistical information for the selected updating parameters is presented in Table 4.9.

Table 4.10. Statistical information of selected parameters used in analysis of the bridge column simulation

Component	Parameter	Abbreviation	Distribution	μ	COV (%)
Concrete block	Elastic modulus	E_c	Lognormal	2.48e10	30
	Mass Density	D_c	Normal	2.28e3	20
Retrofitted part	Elastic modulus	E_r	Lognormal	2.48e10	30
	Mass Density	D_r	Normal	2.28e3	20

Initial training samples are obtained by the LHS method and applied to the properties of the random variable, which is based on each selected model parameter. Here, three popular metamodeling techniques are used. The MCMC sample size is 10^6 , and an acceptance rate of 20% is achieved. The first 30% of the sample is discarded to assess the burn in. Accordingly, the updated FE model parameter values obtained are presented in Table 4.10. Following the Bayesian model updating process, the variance of the model parameters could be reduced after observation.

Table 4.11. Updated model parameters with metamodeling techniques

Component	Parameters	Updated by		Updated by		Updated by PCK	
		Kriging		PCE			
		μ	COV (%)	μ	COV (%)	μ	COV (%)
Concrete block	E_c (Pa)	2.8e10	17.50	3.2e10	16.88	3e10	16.67
	D_c (kg/m ³)	1.9e3	19.47	2.1e3	18.09	1.9e3	15.26
Retrofitted part	E_r (Pa)	2e10	25.50	1.9e10	22.11	2.1e10	19.05
	D_r (kg/m ³)	2.2e3	17.27	2.4e3	13.33	2.5e3	12.40

Table 4.11 shows the frequency and MAC values that are found after the structure is analyzed using the new FE model parameters. For clarity, FE% and MAC values are given in Figures 4.13 and 4.14.

Table 4.12. Comparison FE% and MAC after model updating

Test	Metamodeling techniques								
	Kriging			PCE			PCK		
	Frequency (Hz)	FE%	MAC	Frequency (Hz)	FE%	MAC	Frequency (Hz)	FE%	MAC
21.57	19.73	9.35	0.891	19.54	10.38	0.891	19.74	9.27	0.892
107.92	114.86	6.04	0.734	112.09	3.72	0.734	112.55	4.12	0.733
132.30	125.63	5.31	0.672	121.93	8.51	0.671	125.23	5.64	0.672
-	-	-	-	-	-	-	-	-	-
263.61	313.63	15.95	0.664	304.83	13.52	0.659	312.29	15.59	0.659
360.34	350.90	2.69	0.832	340.49	5.83	0.835	343.84	4.80	0.836

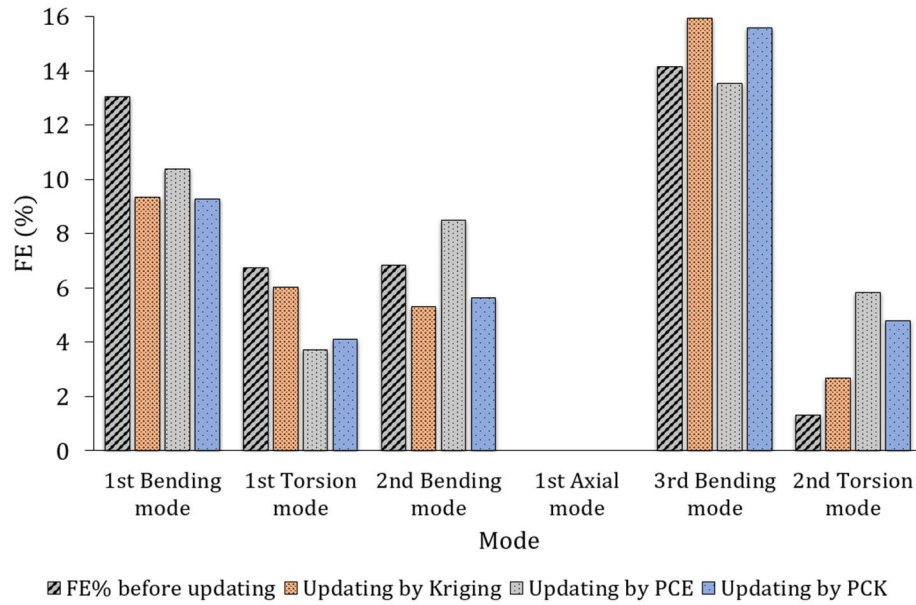


Figure 4.13. Comparison FE% after and before model updating

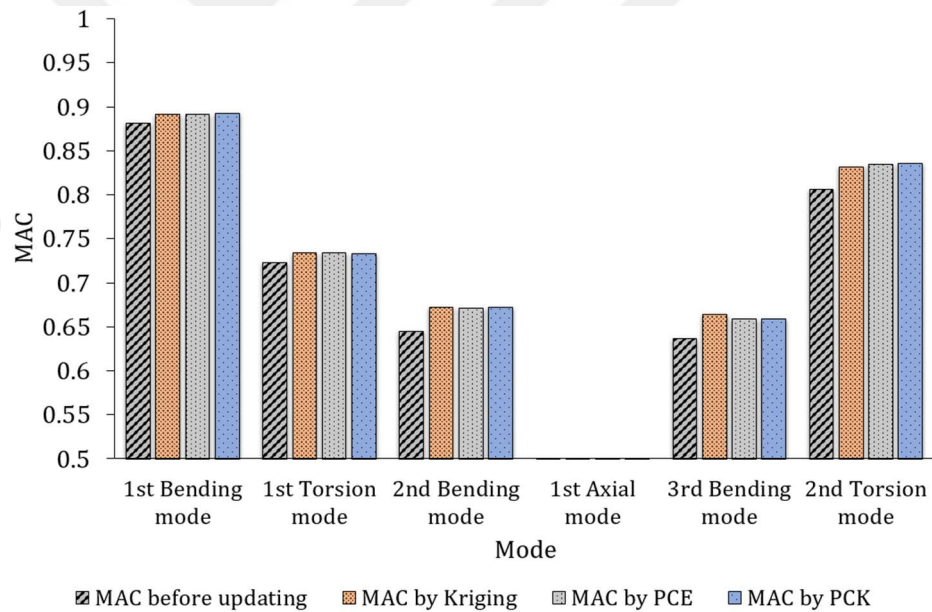


Figure 4.14. Comparison MAC after and before model updating

For the first three frequency values, FE% is significantly reduced, while MAC values are improved. While PCK showed the characteristic features of Kriging and PCE, it is able to give better results than them. In this context, it has been seen that it would be more advantageous to include PCK in the solution method.

4.3. Conclusion

In this study, not frequently encountered in the literature, different metamodeling techniques and the results of model updates with these methods are presented comparatively. For this purpose, the modal properties of a truss bridge and an Irvine bridge column are identified using system identification techniques. A framework for SHM-based Bayesian model updating accelerated by metamodeling is proposed. It is aimed at reaching more accurate results by adding PCK, which is a more effective solution method obtained by combining PCE and Kriging meta-models, which are frequently used in the literature. The identified values are then utilized with the proposed model updating framework to update the initial FE model of the structures investigated in this study. The summarized conclusion is described as follows:

- Bayesian model updating with metamodeling techniques reduces a significant amount of computational cost. In this way, it is thought that it can serve as a powerful strategy in model updating in terms of minimizing modeling errors and taking reasonable parameter values.
- The findings are shown next to the outcomes from various metamodeling methods used on the examples that were looked at. It is clear that the suggested SHM-based Bayesian model updating method combined with metamodeling has worked better.
- In the investigated numerical and experimental study, the PCK successfully showed the properties of the PCE and Kriging meta-models from which it is derived, and thus, the ineffectiveness of any method is able to catch the point where the other is successful. It also gives better results in many areas than the other two methods. In this way, it is expected that more optimum results will be obtained for the examined structures when considering PCK metamodeling in Bayesian model updating.
- In this context, it is expected that the proposed solution method will allow for more complex analyses, such as reliability analysis and design optimizations, that are considered to be made in the future.

CHAPTER 5

RELIABILITY ASSESSMENT OF STRUCTURES WITH BAYESIAN MODEL UPDATING ACCELERATED via POLYNOMIAL-CHAOS-KRIGING METAMODELING

5.1. Introduction

Reliability assessment should be implemented to calculate the probabilities of failure events, which can be defined as any limit state function such as ultimate, deflection, buckling, etc. However, one of the biggest difficulties in structural reliability analysis, reducing the number of calls made to the studied numerical model and reducing the computation time, has been tried to be achieved by using PCK in structural reliability analysis (Zhou et al. 2023). In addition, when working with structure models that are not updated, it negatively affects the decisions to be made regarding the safety of the structure, service life, maintenance cost, etc.

For instance, coastal bridges are exposed to high levels of sea salt and humidity, which causes corrosion-related damage during long-term service (A. Guo et al. 2018). Corrosion occurring in reinforcement can damage reinforced concrete structures in different ways (Akiyama, Frangopol, and Yoshida 2010). The areas of both longitudinal and transverse reinforcement decrease due to rust formation, which may cause tensile stresses to occur in the concrete and subsequently cracking of the concrete. As a result of the decrease in reinforcement cross-sections, stress concentrations may occur in the steel, resulting in a loss of ductility in the structure (Rao et al. 2017).

The goal of this study is to find a way to use a PCK-based method with Bayesian model updating to get around computational problems and then get more accurate assessments of how reliable structures are. This study focuses specifically on the effects of uniform and non-uniform corrosion along the height of the bridge column. Therefore, the Irvine bridge column project (LANL 1997) is selected, and results for updated and non-updated FE models of bridge columns are presented comparatively.

5.2. Irvine Bridge Column

Bridges are critical links within transportation networks that provide access between regions and cities. A bridge generally connects two land masses across a body of water, while bridge columns are structures that extend over water and are built perpendicular to the shoreline. Therefore, bridge columns are the basic components that provide the stability and load-carrying capacity of bridge structures.

External environmental conditions inevitably cause performance deterioration in engineering structures throughout their service lives. Since bridge columns are in direct contact with water, this exposes them to aggressive chloride-induced corrosion. It is also possible to see different corrosion effects (like uniform and non-uniform) depending on the water level to which the bridge column is exposed. This brings the corrosion damage to these structures to serious levels. For this reason, in the study, the Irvine bridge column constructed by the University of California, Irvine, at Los Alamos National Laboratory (LANL 1997) is investigated.

5.2.1. Finite Element Model of Irvine Bridge Column

FE model of the experimental models available in LANL report (LANL 1997). Here, an elastic modulus of 24.8 GPa, a mass density of $2.28 \times 10^3 \text{ kg/m}^3$ and Poisson's ratio of 0.2 is specified in concrete blocks. In the available FE model has been created in ABAQUS software environment without no reinforcement details. In this study, however, the FE model is constructed incorporating reinforcements using the ABAQUS software. While the material properties for concrete are taken as the same as in the report, an elastic modulus of 210 GPa, a mass density of $7.85 \times 10^3 \text{ kg/m}^3$ and a Poisson's ratio of 0.3 is accepted for reinforcements. Yield stresses for longitudinal rebars are considered as 516 MPa and 414 MPa for original and retrofitted part of the column, respectively. Yield stresses for spiral cage are 207 MPa and 414 MPa for original and retrofitted part of the column, respectively. ANSYS software (ANSYS 2015) is used to establish FE model of Irvine bridge column which is shown in Figure 5.1. Here, the base of the element that simulated the concrete foundation is subjected to fixed boundary conditions.

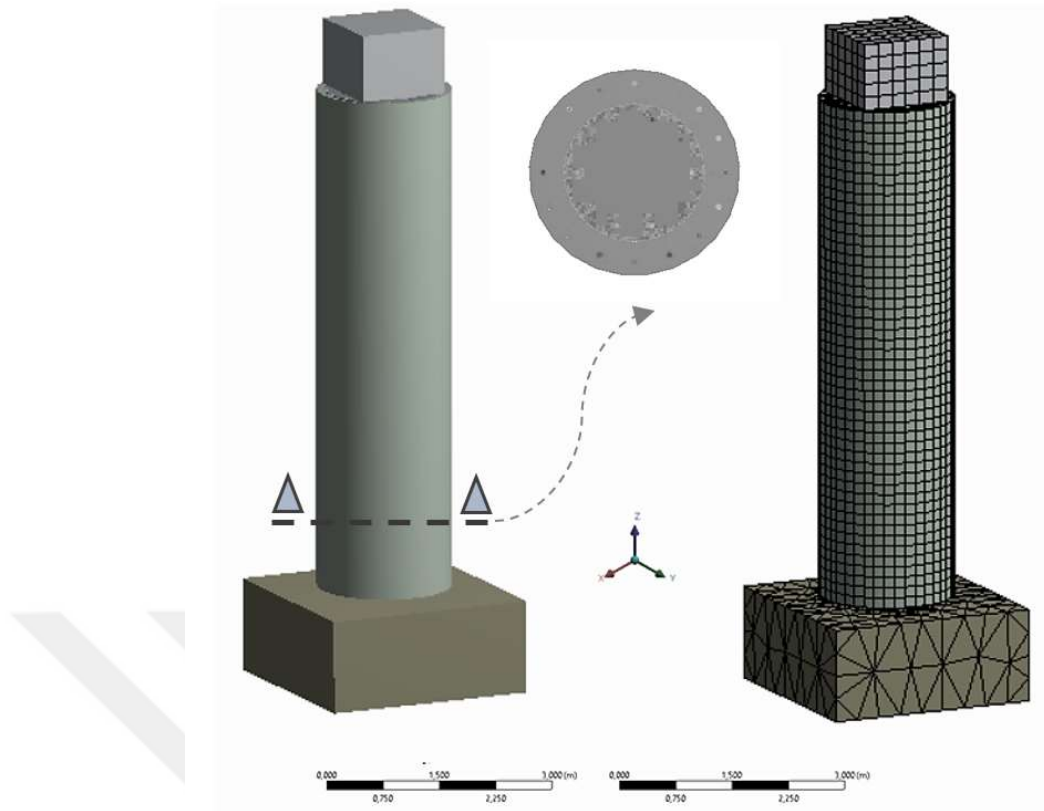


Figure 5.1. FE model of Irvine bridge column

For the consistency of the model, the modal frequencies obtained by this study and those reported by the LANL report are presented in Table 5.1. The obtained frequency values and mode shapes are used to update FE model of the bridge column. For this purpose, the elastic modulus and mass density values of concrete blocks and reinforcements are assumed as unknown parameters selected for the model updating.

Table 5.1. Comparison for modal properties from LANL and this study

Mode shape	Frequency of FE model (Hz)	
	Reported by LANL	In this study
1 st Bending	19.10	18.90
1 st Torsion	114.00	115.02
2 nd Bending	124.00	127.87
1 st Axial	181.00	183.99
3 rd Bending	306.00	314.55
2 nd Torsion	351.00	347.62

Identified modal parameters calculated through the experimental modal analyses and comparison with results from FE model are presented in Table 5.2. The least square complex exponential method (LSCEM) are used for modal parameter identification to describe dynamic properties as a singular value from collection of all vibration signals (Bayraktar et al. 2009). Frequency errors, FE%, and MAC values are also given in the same table. The frequency values found in the literature are also presented here to compare the calculations. Frequency and MAC values of the 1st axial mode shape, which cannot be obtained from the experimental modal analysis, are not included. Six modes of the bridge column shown in Figure 5.2 are utilized in the model updating process.

Table 5.2. Comparison for modal properties from vibration test and initial FE model

Frequencies from FEM (Hz)	Experimental frequencies (Hz)		FE%	MAC
	LSCEM	Literature by Sohn & Law (2000)		
18.90	21.57	27.82	14.07	0.87
115.02	107.92	110.42	6.22	0.75
127.87	132.30	147.51	3.46	0.67
183.99	-	-	-	-
314.55	263.61	272.83	16.19	0.62
347.62	360.34	340.71	3.66	0.78

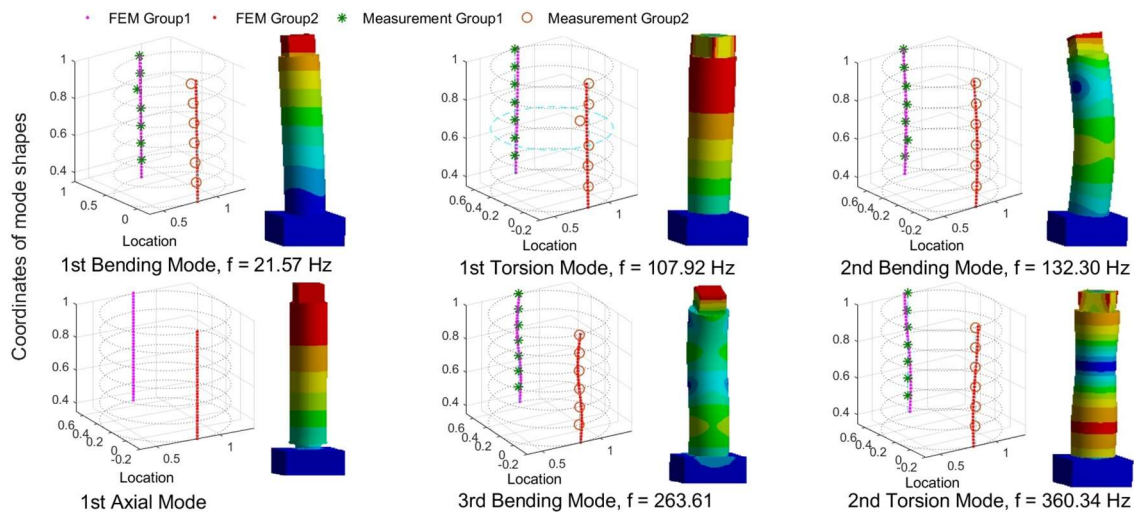


Figure 5.2. Comparison of Mode Shapes from experiment and FE model analysis

5.2.2. Results of Bayesian FE Model Updating of the Irvine Bridge Column

In Table 5.3, statistical information for the selected updating parameters and adopted as prior distribution based on domain knowledge and expert opinion are presented. These values are determined by taking into account the recommendations found in the literature (JCSS, 2002; H. Wang et al., 2010).

Table 5.3. Statistical information of selected parameters used in analysis of the bridge column simulation

Component	Parameter	Abbreviation	Distribution	μ	COV (%)
Concrete block	Elastic modulus	E_c	Lognormal	2.48e10	30
	Mass Density	D_c	Normal	2.28e3	20
Retrofitted part	Elastic modulus	E_r	Lognormal	2.48e10	30
	Mass Density	D_r	Normal	2.28e3	20
Reinforcement	Elastic modulus	E_s	Lognormal	2.1e11	20
	Mass Density	D_s	Normal	7.85e3	15

Since estimating the probability of the MCMC algorithm presumably may need excessive amount of model evaluations, it is chosen to replace the model with proxy using PCK metamodeling technique. Initial training samples according to the properties of the random variables for the specified design parameters are obtained using the Sobol method and then, FE model is evaluated with these samples to generate training data sets. The efficient set of orthonormal polynomials is determined by the PCK using sequential and optimal techniques. In this study, optimal techniques such as least-angle regression are employed to arrange sparse polynomial bases depending on their correlation to residuals. Each polynomial is individually added to the Kriging model trend, and the model with the smallest leave-one-out cross-validation error is chosen as the final model. The q-norm truncation scheme with 0.75 is implemented, and the maximum rank truncation is set at 3. The degree of the polynomials is between 1 and 10. The MCMC sample size is 10^6 with an acceptance rate of 20%-40%. The first 30% of the sample is discarded to assess

the burn in. Accordingly, the updated FE model parameters as well as their posterior coefficient of variation (COV) values are presented in Table 5.4.

Table 5.4. Updated model parameters by PCK

Component	Parameter	μ	COV (%)
Concrete block	E_c (Pa)	2.8e10	25.1
	D_c (kg/m3)	2.1e3	13.1
Retrofitted part	E_r (Pa)	2.5e10	22.8
	D_r (kg/m3)	2.3e3	16.9
Reinforcement	E_s (Pa)	2.1e11	14.8
	D_s (kg/m3)	7.8e3	10.5

It is observed that the concrete block parts of the bridge column dominated the structure behavior as expected. The obtained frequency error, FE% and MAC values for the updated FE model are presented in Table 5.5. For clarity, the results before and after the FE model updating for FE% and MAC are given comparatively in Figures 5.3. and 5.4, respectively.

Table 5.5. Comparison FE% and MAC after model updating

Frequency of updated			
Test (Hz)	FE model (Hz)	FE %	MAC
21.57	19.55	9.69	0.89
107.92	114.80	6.01	0.76
132.30	131.71	0.46	0.78
-	-	-	-
263.61	324.00	18.64	0.63
360.34	355.42	1.38	0.81

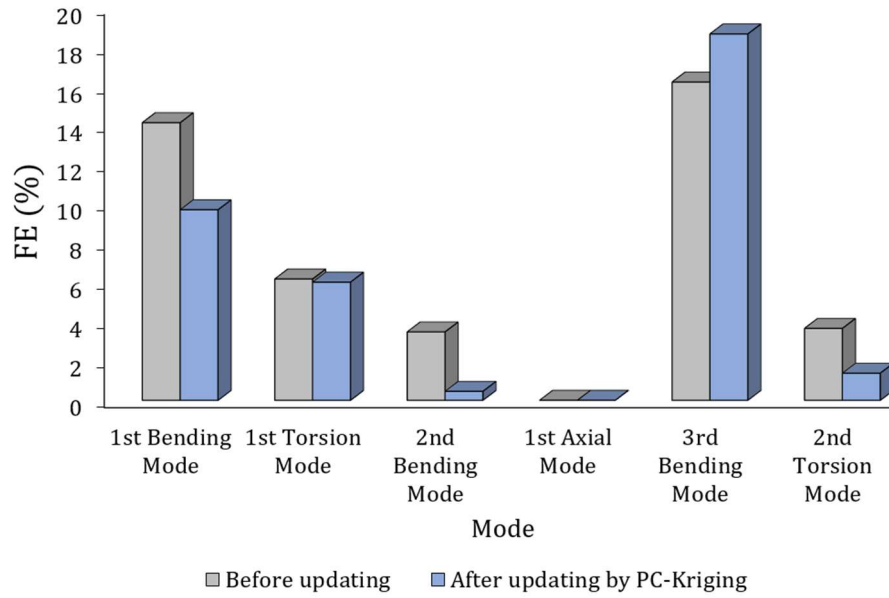


Figure 5.3. Comparison FE% after and before model updating

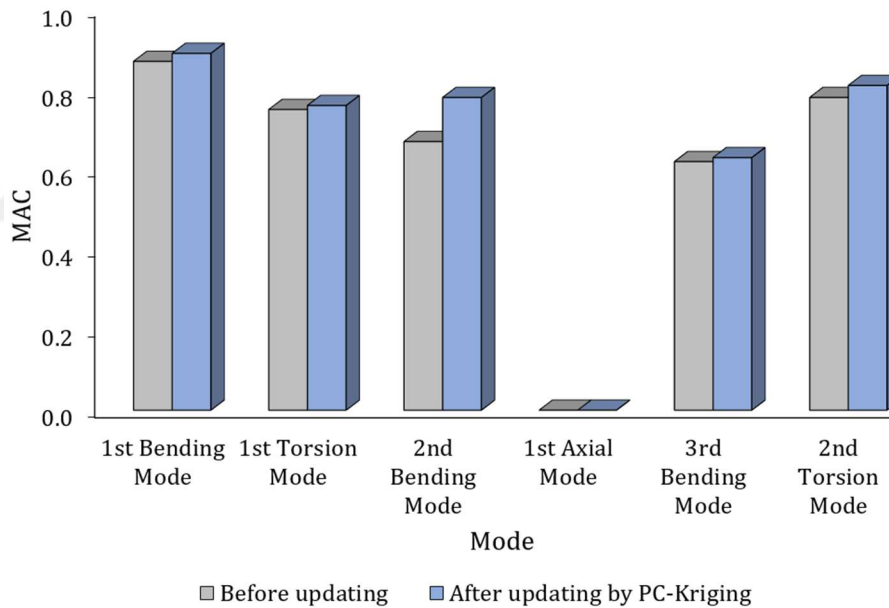


Figure 5.4. Comparison MAC after and before model updating

In a realistic model, numerous DOFs can be found, and there are many natural frequencies. However, all of them are not important on the same level. In order to determine the most effective natural frequencies and modes, the effective modal mass or participation factor is used to quantify these. High effective modal mass gives rise to a high reaction force at the base, but mode shapes with low associated effective modal mass are merely excited by the base acceleration and therefore give low reaction forces at the

base. Modes with a higher ratio of effective modal mass are the most important to focus on, while modes with a lower ratio of effective modal mass are less likely to be considered. According to the results, 1st bending mode and 2nd bending mode constitute an effective modal mass ratio of over 50 % ($\approx 58\%$). In this context, it has been observed that modal properties can be updated significantly, especially for modes with a high effective modal mass ratio.

During the performed analysis, the number of steps is set to 100 with a number of chains of 1000. Moreover, the MCMC sample size is prescribed as 10^5 , with approximately 40,000 accepted samples. For a single estimate of the modal properties, the FE model simulation on a computer with an Intel (R) Core™ i5-10300H CPU running at 2.50 GHz and 16 GB of main memory takes about 17 minutes to run. As a result, if each sample used for the Markov chain over the FE model is used without using metamodeling, it could take about 11333.3 hours (≈ 472.2 days) to update the Bayesian model with the FE model simulation. However, when considering metamodeling, a significant decrease can be observed in the time taken to update the FE model (approximately 7 hours). As a result, a significant reduction in computational effort is observed with the proposed methodology.

5.2.3. Calculation of The Compressive Strength Based on Elastic Modulus and Density

The calculation of modulus of elasticity, E_c of normal weight concrete based on compressive strength of standard test cylinders, f'_c described in ACI318-02 (2002) as:

$$E_c = 4.73\sqrt{f'_c} \quad (5.1)$$

when the density of concrete, ρ (kg/m³) is specified, the calculation of modulus of elasticity is;

$$E_c = 43\rho^{1.5}\sqrt{f'_c} \times 10^{-6} \quad (5.2)$$

Before model updating, the strength of concrete blocks has been calculated using Equation (5.2) based on modulus of elastic and density values, which are specified in the report of the University of California, Irvine, at Los Alamos National Laboratory (LANL

1997). After the model is updated, the strength of the original bridge column and the retrofitted part is calculated based on the updated modulus of elastic and density values. These compressive strengths of concrete, f'_c values are given in Table 5.6, comparatively.

Table 5.6. Compressive strength of concrete, f'_c before and after updating

Compressive strength, f'_c (MPa)	Before updating		After updating	
	Original column	Retrofitted jacket	Original column	Retrofitted jacket
	28.1	28.1	45.8	27.8

5.2.3.1. Modeling Concrete and Reinforcement steel

Confined and unconfined concrete models proposed by Mander et al. (1988) are used for the core and cover concrete, as shown in Figure 5.5.

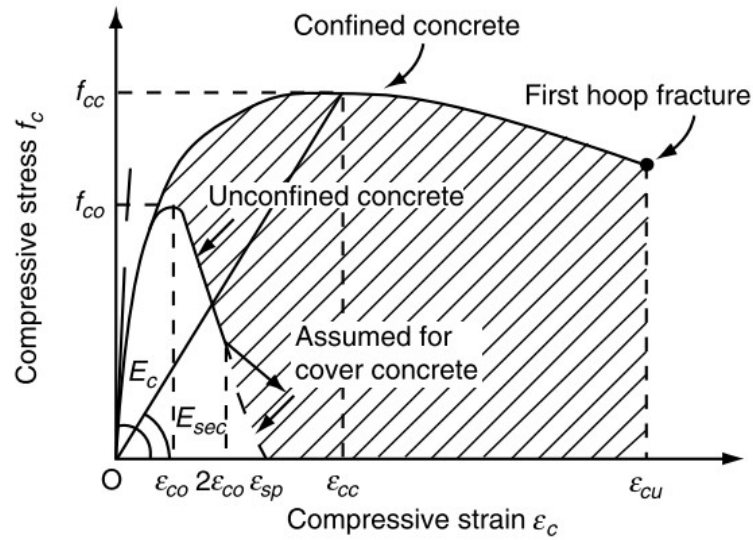


Figure 5.5. Stress-strain model for confined and unconfined concrete

The calculation of the maximum compressive strength, f'_{cc} , and the corresponding strain, ϵ_{cc} of the confined concrete is performed utilizing the theoretical stress-strain model presented in Figure 5.5. This model was proposed by Mander et al. (1988).

$$f'_{cc} = f'_{co} \left(-1.254 + 2.254 \sqrt{1 + \frac{7.94 f'_l}{f'_{co}}} - 2 \frac{f'_l}{f'_{co}} \right) \quad (5.3)$$

$$\varepsilon_{cc} = 0.002 \left(1 + 5 \left(\frac{f'_{cc}}{f'_{co}} - 1 \right) \right) \quad (5.4)$$

where f'_{co} is the unconfined concrete strength, f'_l is the confining lateral pressure provided by the transverse reinforcement and calculated as follows;

$$f'_l = \frac{1}{2} k_e \rho_s f_{yw} \quad (5.5)$$

$$k_e = \frac{1 - \frac{s'}{2d_s}}{1 - \rho_{cc}} \quad (5.6)$$

where f_{yw} is the yield strength of transverse reinforcement, k_e is a coefficient measuring confinement effectiveness, ρ_{cc} is the ratio of longitudinal rebar cross-sectional area to the area of the concrete core, s' is the clear spacing of transverse reinforcement and d_s is the diameter of the core.

Hafeez (2008) offered a model which is based on the calculation of double confinement on the base column with considering the existing model of Mander et al. (1988) to calculate the confined concrete strength. According to Hafeez (2008), the calculation of confinement effectiveness coefficient k_e for the core concrete of base column is the sum of confinement provided by the original spiral reinforcement of the base column, k_{e1} and the confinement effect due to the external jacket, k_{e2} and given in Equation (5.7).

$$k_e = k_{e1} + k_{e2} \quad (5.7)$$

Different parts of the examined retrofitted bridge column are shown in Figure 5.6. Based on the previously mentioned model, a stress-strain diagram of the concrete blocks is created and presented in Figure 5.7.

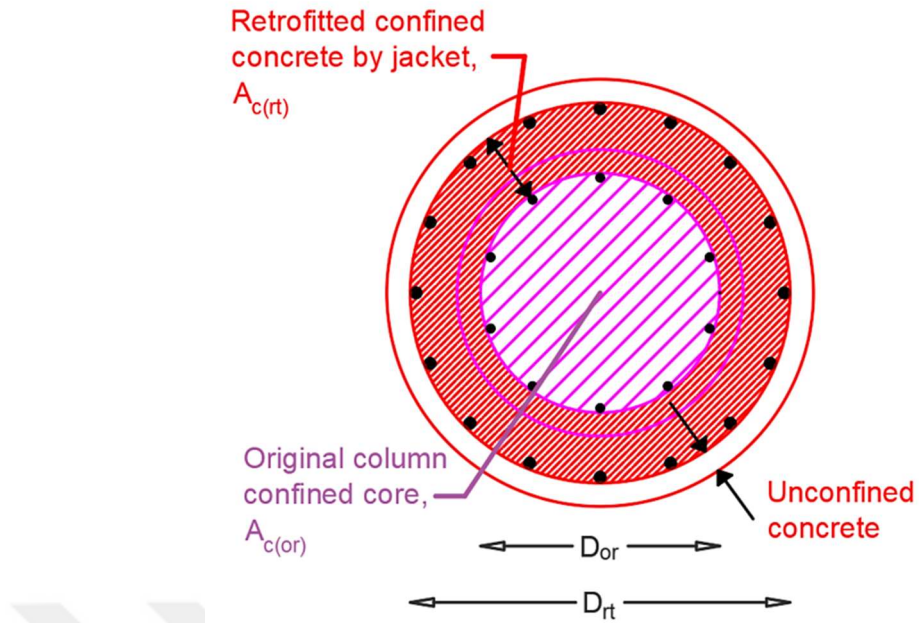


Figure 5.6. Original bridge column and retrofitted jacket

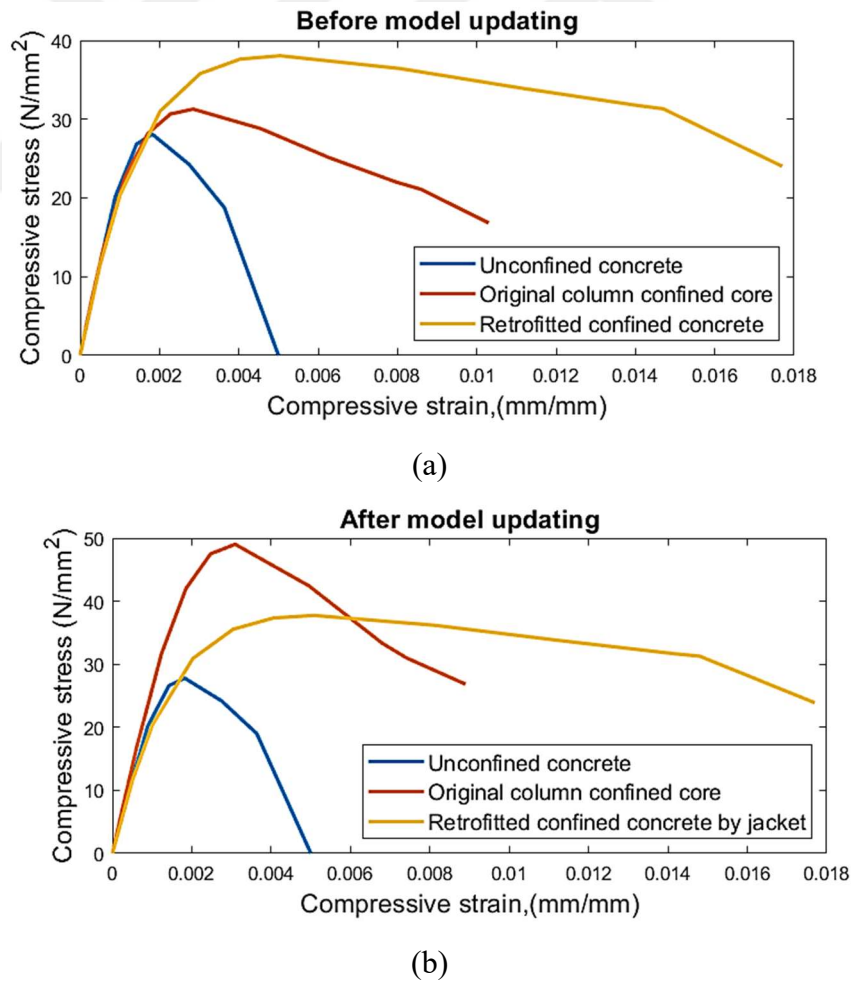


Figure 5.7. Stress-strain of concrete blocks (a) before updating (b) after updating

The stress-strain model for reinforcements is based on the previously defined values represented by a quadratic curve, as shown in Figure 5.8.

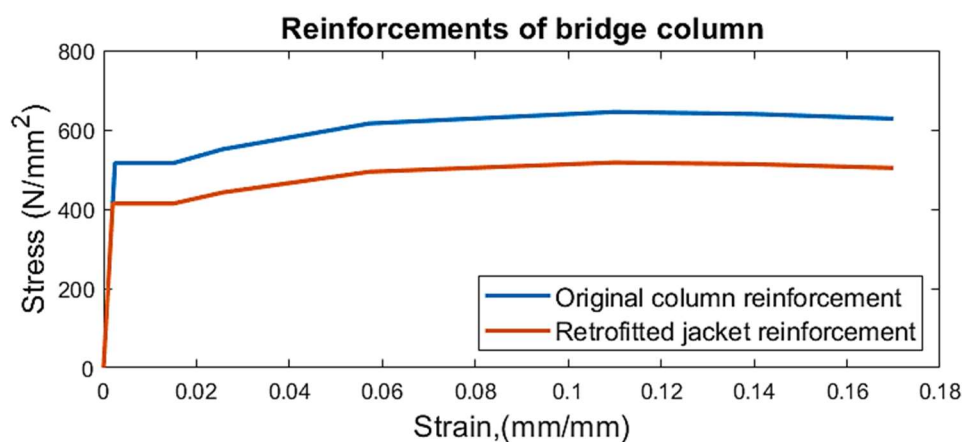


Figure 5.8. Stress-strain of reinforcements

5.3. Time-Dependent Resistance Deterioration of Bridge Column

Reliability analyses for structures models require probabilistic models of engineering variables and databases to characterize uncertainties. These data should be representative of expected values in a structure to reflect uncertainties due to inherent variability, modeling, prediction, and measurement process. Extensive databases describe probabilistic models from previous code studies, summarized in a non-dimensional form in Table 5.7 (N. Wang and Ellingwood 2015).

Table 5.7. Statistics of loads and reinforced concrete strength

Parameter	Distribution type	Bias	COV
Dead load, D_L	Normal	1.05	0.10
Live load, L_L	Normal	1.15	0.18
Yield stress, f_y	Lognormal	1.12	0.10
Compressive stress, f_c	Normal	1.20	0.18
Modelling error for columns	Normal	0.95	0.075

5.4. Time-Dependent Resistance Deterioration of Bridge Column

The diffusion theory is the foundation for the transfer of chloride ions in concrete. The diffusion equation, also known as Fick's second law, can be employed to predict the diffusion of chloride ions under suitable boundary conditions. The diffusion equation is denoted as (JSCE 2010):

$$C(x, t) = C_{sc} \left[1 - \operatorname{erf} \left(\frac{x}{2\sqrt{D_c \cdot t}} \right) \right] \quad (5.8)$$

where $C(x, t)$ is the chloride ion concentration at depth of x and time of t . C_{sc} is the surface chloride ion concentration at the surface of concrete ($x = 0$). D_c is the chloride diffusion coefficient, $\operatorname{erf}(\dots)$ is the error function. The following assumptions are supported by this model of chloride ion diffusion: the concrete element is isotropic, the medium is semi-infinite, and the parameters, D_c and C_{sc} are time-invariant. The concentration of chloride ions at the reinforcement level increases over time, as indicated by the model.

Reinforced concrete structures located in coastal areas are exposed to chloride ions coming from the sea. These ions pass through the concrete layer and penetrate the steel inside, and after reaching a certain level, they cause corrosion in the reinforcement steel. In this context, the corrosion initiation time, T_I can be calculated with the expression defined below (Bhargava, Mori, and Ghosh 2011):

$$T_I = \frac{c^2}{4D_c} \left[\operatorname{erf}^{-1} \left(\frac{C_{sc} - C_{tc}}{C_{sc}} \right) \right]^{-2} \quad (5.9)$$

where c is the depth of the concrete cover, D_c is the chloride diffusion coefficient, C_{sc} is the surface chloride ion concentration at the surface of concrete, and C_{tc} is the threshold concentration of chloride ions required to initiate the corrosion. $\operatorname{erf}(\dots)$ is the error function.

When previous studies are examined, it is seen that C_{tc} concentration tends to be in the range of 0.6-1.2 kg/m³ of concrete. This value corresponds to 0.024-0.048% of the weight of the concrete. The average of these values is used in the study (Amey and Dwayne A. Johnson, Matthew A. Miltenberger 1998). It has been stated that C_{sc} concentration varies depending on the distance from the coast. In JSCE Guidelines for

Concrete JSCE (2010), C_{sc} values that vary depending on areas where air-born salt is high or low and the distance of the structure from the seashore have been defined and given in Table 5.8.

Table 5.8. Surface chloride ion concentration, C_{sc} (kg/m³) by JSCE (2010)

	Distance from seashore (km)				
	Near the shoreline	0.1	0.25	0.5	1.0
Areas with a high volume of air-born salt	9.0	4.5	3.0	2.0	1.5
Areas with a low volume of air-born salt	4.5	2.5	2.0	1.5	1.0

In this study, C_{sc} values are chosen as 9.0, 4.5 and 2.0 in order to find structures at different distances from the seashore and to examine areas where air-born salt is high or low. Possible locations where the structure can be found are also shown in Figure 5.9.

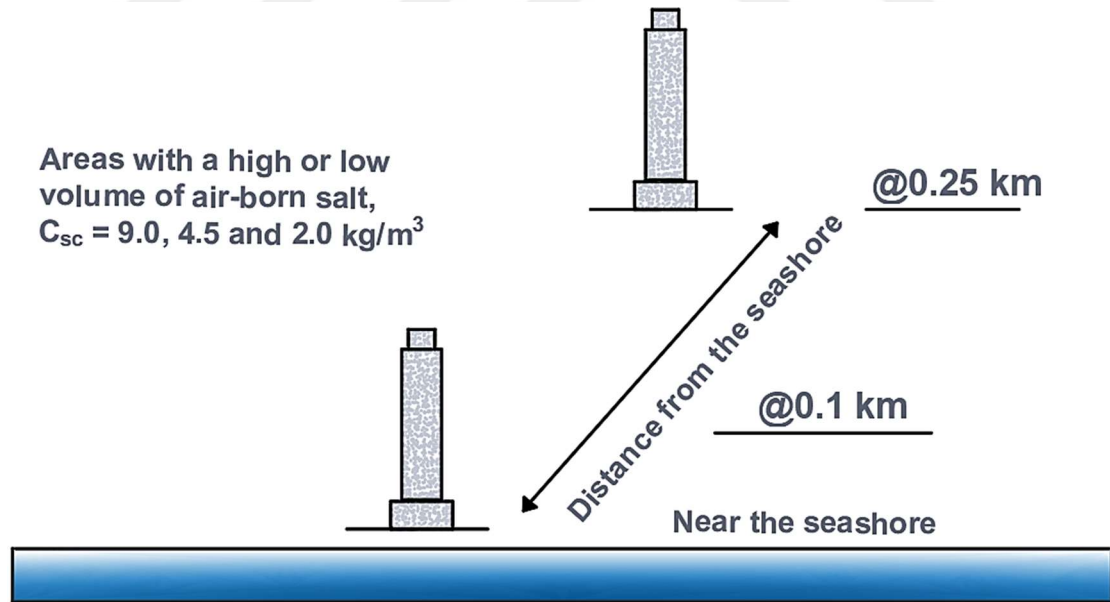


Figure 5.9. Possible places of bridge column for surface chloride ion concentration, C_{sc}

The chloride diffusion coefficient in concrete, D_c represents the permeability in concrete. This coefficient is affected by cement type, water-cement ratio, (w/c), and environmental conditions, etc. D_c expressed as (Lin 1991):

$$D_c = \left[1.249 - 5.051 \left(\frac{w}{c} \right) + 8.941 \left(\frac{w}{c} \right)^2 \right] * 10^{-8} \text{ cm}^2/\text{sn} \quad (5.10)$$

where (w/c) is the water–cement ratio. The relation between (w/c) and compressive strength is given in different studies and standards in the literature (IS 456, 2000; Singh et al., 2015). The (w/c) for a specific concrete strength, f'_c is calculated taking into account reference studies.

5.4.1. Deterioration Models of Reinforcement and Concrete

With the onset of corrosion, there is a continuous decrease in the yield stress, $f_y(t)$ and diameter, $d(t)$ of the reinforcement and can be expressed as follows (Yuan et al. 2023):

$$d(t) = \begin{cases} d_0 & \text{for } t \leq T_I \\ d_0 - 0.023i_{cor}(t - T_I) & \text{for } T_I < t \leq T_I + \frac{d_0}{0.023i_{cor}} \\ 0 & \text{for } t > T_I + \frac{d_0}{0.023i_{cor}} \end{cases} \quad (5.11)$$

$$f_y(t) = [1 - 0.005Q_{cor}(t)]f_{y0} \quad (5.12)$$

where f_{y0} and d_0 are the initial diameter and the yield stress of the reinforcement before corrosion; the corrosion current density, i_{cor} is presented in Table 5.9. and can be obtained from study of Stewart et al. (2011).

Table 5.9. Chloride induced corrosion rates, i_{cor} for various exposures

Exposure type (Chloride initiated corrosion)	Mean	Standard deviation	Distribution
Wet rarely dry	0.345 $\mu\text{A}/\text{cm}^2$ *	0.259 $\mu\text{A}/\text{cm}^2$	Lognormal
Cyclic wet dry	2.586 $\mu\text{A}/\text{cm}^2$	1.724 $\mu\text{A}/\text{cm}^2$	Lognormal
Airborne sea water	2.586 $\mu\text{A}/\text{cm}^2$	1.724 $\mu\text{A}/\text{cm}^2$	Lognormal
Submerged	Not expected	-	Lognormal
Tidal zone	6.035 $\mu\text{A}/\text{cm}^2$	3.448 $\mu\text{A}/\text{cm}^2$	Lognormal

* $\mu\text{A}/\text{cm}^2$: It is the intensity of corrosion current passing through 1 cm^2 surface area of the corroded metal.

According to BS6349-1:2000 (2003) suggests that mean corrosion rate for the atmospheric zone is 0.004 mm/yr ($0.345 \mu\text{A}/\text{cm}^2$), 0.03 mm/yr ($2.586 \mu\text{A}/\text{cm}^2$) for the splash zone, and 0.07 mm/yr ($6.035 \mu\text{A}/\text{cm}^2$) for the tidal zone. In this context, atmospheric zone values for uniform corrosion and splash and tidal zone values for non-uniform corrosion are taken into consideration in the study. Figure 5.10 shows the possible types of uniform corrosion and non-uniform corrosion that may occur. In the study, corrosion effects are examined based on the figures shown here.

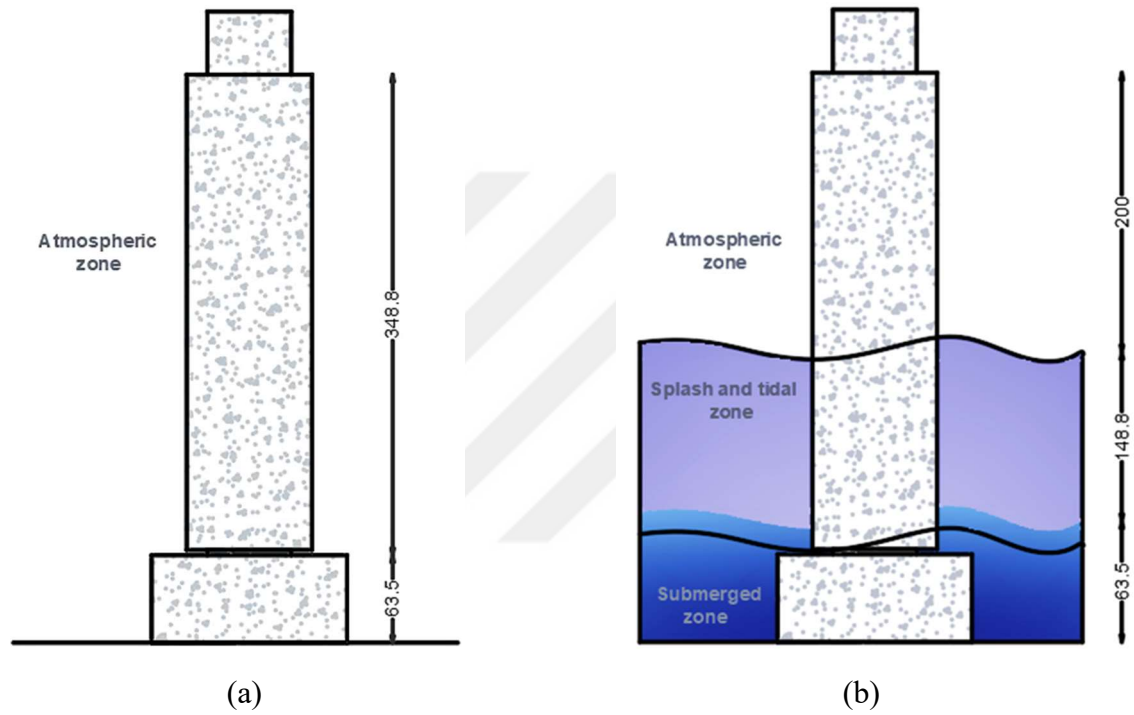


Figure 5.10. Probable (a) uniform and (b) non-uniform corrosion effect on bridge column

The mass loss ratio of corroded reinforcement, $Q(t)_{cor}$ is related to the service time of the structure and can be defined as:

$$Q(t)_{cor} = \left[1 - \left(\frac{d(t)}{d_0} \right)^2 \right] * 100 \quad (5.13)$$

The reinforcement volume gradually increases as the structure is exposed to corrosion. As a result, the force that will occur around the concrete surrounding the

reinforcement will cause cracking and spalling in the concrete. The strength resulting from damage to the concrete cover can be calculated as:

$$f(t)_c = \frac{f_{c0}}{1 + K \frac{\varepsilon(t)_1}{\varepsilon_{c0}}} \quad (5.14)$$

where K is the coefficient which is associated with the morphology of the reinforcement, and it is used as $K = 0.1$ in this study (A. Guo et al. 2018). f_{c0} and ε_{c0} are the strength and strain of the sound concrete and $\varepsilon(t)_1$ is the average value of the tensile strain of the corrosion-induced cracked concrete (Rao et al. 2017).

$$\varepsilon(t)_1 = \frac{b(t)_{cor}^* - b_0}{b_0} \quad (5.15)$$

where b_0 is the width of the column before corrosion cracking and $b(t)_{cor}^*$ is the column width increased by corrosion cracking and rust expansion.

$$b(t)_{cor}^* - b_0 = n_{bars} w_{cr} \quad (5.16)$$

where n_{bars} is the number of bars undergoing compression, and w_{cr} is the sum of the widths of all crack openings for a given corrosion level. w_{cr} can be calculated as (Molina, Alonso, and Andrade 1993):

$$w_{cr} = 2\pi t \quad (5.17)$$

where

$$t = (\gamma_{vc} - 1)R \quad (5.18)$$

where R is the depth of the corrosion attack (equal to the reduction in bar radius) and γ_{vc} is the ratio of the volume of the corroded steel bar including the expansive rust products to the volume of the virgin steel bar. Based on study of Molina et al. (1993), the value of γ_{vc} is assumed to be equal to 2.

Based on the time-dependent resistance deterioration methodology of bridge column due to corrosion, changes in strength of concrete are calculated and presented in Figure 5.11. These effects are expressed considering the structure's location in different

zones (also related with the uniform and non-uniform corrosion) and volume of air-bone salt. The change in concrete strength over time is expressed by dividing the strength that changes due to corrosion over time, $f(t)_c$ by the strength of the sound concrete, f_{c0} . The mean value of change in concrete strength, mean plus and minus standard deviation are plotted as prediction intervals in Figure 5.11. Prediction intervals are created for 95% confidence interval. For these obtained values, a curve is fitted that provided the best fit to a set of data points, subject to possible restrictions. Accordingly, significant decreases are observed in the strength of concrete in splash and tidal zone. In addition, the increase in air-bone salt volume causes significant decreases in capacity. Depending on the presence of air-bone salt volume or in splash and tidal zone, the strength decrease in the structure started to occur in the previous years.

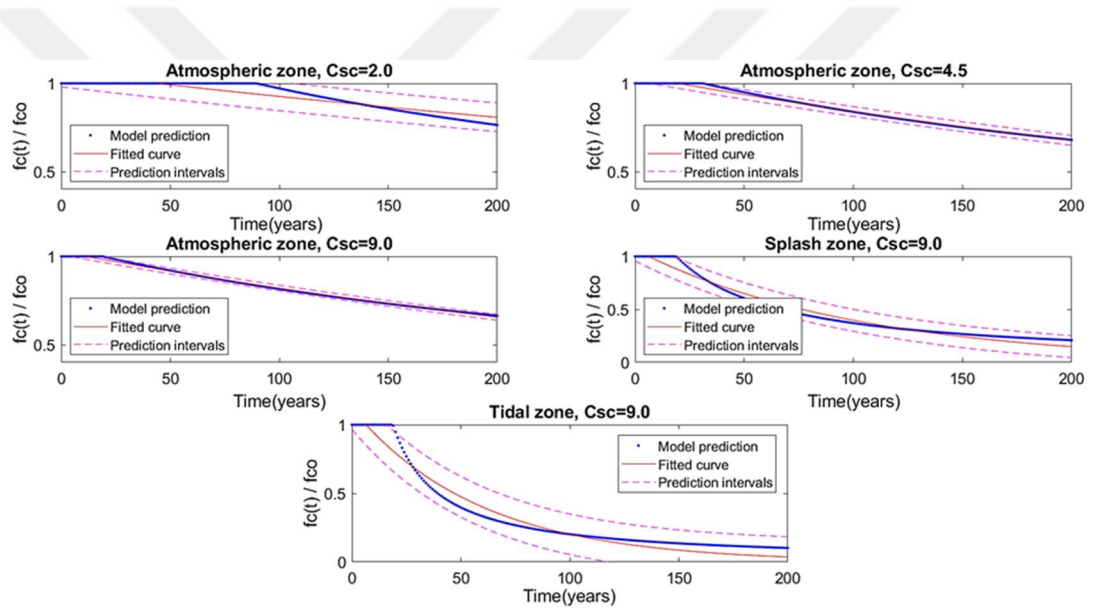


Figure 5.11. The change in strength of concrete, $f(t)_c/f_{c0}$ due to corrosion depends on different zones and volume of air-born salt, C_{sc}

Significant decreases are observed in the strength of concrete in the splash and tidal zones. In addition, the increase in air-bone salt volume causes significant decreases in capacity. Furthermore, depending on the presence of air-bone salt volume or in the splash and tidal zones, the strength decrease in the structure started to occur in the previous years.

5.5. The Strength of Confined Concrete Columns Under Uniaxial Compression

The nominal axial load capacity of the column is defined in ACI318-19 (2019) as follows:

$$N = 0.85f_c(A_g - A_{st}) + A_{st}f_y = 0.85f_cA_c + A_{st}f_y \quad (5.19)$$

where A_g is the gross area of concrete, A_{st} is the total area of longitudinal reinforcement, f_c is compressive strength of concrete, f_y is yield strength of reinforcement. A_c equals to $(A_g - A_{st})$.

In GB50367-2013 (2013), axial load capacity is specified as:

$$N = 0.9[(f_c + 4\sigma)A_c + A_{st}f_y] \quad (5.20)$$

where σ is the effective confinement pressure.

In the study of Hu et al. (2021), it is found that the predicted values of ultimate load-carrying capacity of confined RC columns using ACI formula is remains conservative compared to other regulations. According to the experiment results, the formula defined in GB50367-2013 (2013) gives more accurate results in order to predict the load-carrying capacity of confined specimens. Therefore, the confinement effect of the circular column on the axial load capacity has to be taken into account to get more realistic results. In Figure 5.12 (a) and (b), a spiral circular column is shown. When uniaxial pressure is applied to this column, the compressive stresses that will occur in concrete and steel will cause the member to shorten. While the lateral expansion tendency of the core concrete is prevented by spiral reinforcement, this causes radial pressure on the core concrete. The radial pressure, q shown in Figure 5.12 (c) can be described as σ .

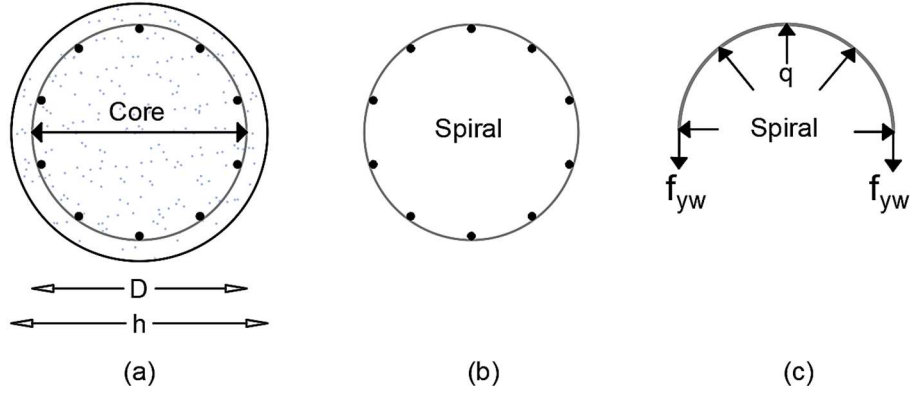


Figure 5.12. The confined concrete (a) core (b) spiral (c) spiral stress layout

The stress in a spiral column could be expressed by the following equation:

$$\sigma = \frac{2A_0f_{yw}}{D(s)} \quad (5.21)$$

where A_0 is core area since cover is lost, f_{yw} is yield strength of spiral reinforcement, s is spacing of spirals and D is core diameter.

The ultimate strength of uniaxial loaded column can be calculated using Equation (5.19). The unconfined concrete strength term ($0.85f_c$) is replaced as $(0.85f_c + 4\sigma)$ for the strength of confined concrete.

$$N = (0.85f_c + 4\sigma)A_c + A_{st}f_y \quad (5.22)$$

Substituting Equation (5.21) into Equation (5.22), the axial capacity of column is defined as:

$$N = 0.85f_cA_c + 8 * \frac{A_0f_{yw}}{D(s)}A_c + A_{st}f_y \quad (5.23)$$

In Figure 5.6. , confined and unconfined concrete areas of the bridge column are shown. The abbreviation “or” is used for the original column, and the abbreviation “rt” is used for the retrofitted part. The ultimate axial-load capacity of a retrofitted bridge column can be calculated using Equation (5.24). The detailed explanation of the formula is given in Equation (5.25).

$$N_c = N_{or} + N_{rt} \quad (5.24)$$

$$N_c = 0.85f_{c(or)}A_{c(or)} + 8 \frac{A_{0(or)}f_{yw(or)}}{D_{(or)}S_{(or)}} A_{c(or)} + A_{st(or)}f_{y(or)} + 0.85f_{c(rt)}A_{c(rt)} + 8 \frac{A_{0(rt)}f_{yw(rt)}}{D_{(rt)}S_{(rt)}} A_{c(rt)} + A_{st(rt)}f_{y(rt)} \quad (5.25)$$

Under the corrosion effect, the axial load capacity of the column is to be changed with time and defined as:

$$N(t)_{cc} = 0.85f(t)_c A(t)_c + 8 \frac{A(t)_0 f(t)_{yw}}{D(t)S} A(t)_c + A(t)_{st} f(t)_y \quad (5.26)$$

Here, it is thought that corrosion would first affect the outermost concrete cover, but the capacity change is formulated considering that the capacity of the inner column would change with the loss of this layer and presented in Equation (5.27). For this reason, the change in axial capacity of the bridge column is formulated by taking into account the difference in the radii (R_{or} and R_{rt}) of the two concrete elements. The time, t_i for $i \in \mathbb{N}$ refers to the time that the structure will be exposed to corrosion.

$$N(t_i)_{cc} = \begin{cases} \sum_{i=1}^n N_{or} + N(t_i)_{rt} & \text{for } R(t_i)_{rt} - R(t_i)_{or} > 0 \\ \sum_{i=n}^l N(t_i)_{or} + N(t_i)_{rt} & \text{for } R(t_i)_{rt} - R(t_i)_{or} < 0 \end{cases} \quad (5.27)$$

Based on the presented approaches, a capacity comparison is first made for the before and after model updating cases. The axial force capacity of the updated model is 1.2 times larger than that of the non-updated one. Then, it is accepted that there has been a decrease in capacity over time due to the effect of corrosion. The capacity decreases for each case are expressed as the $N_{cc}(t)/N_c$ ratio. The changes that will occur in the capacitance under different corrosion effects for two different models are presented in Figure 5.13 for uniform corrosion and Figure 5.14 for non-uniform corrosion.

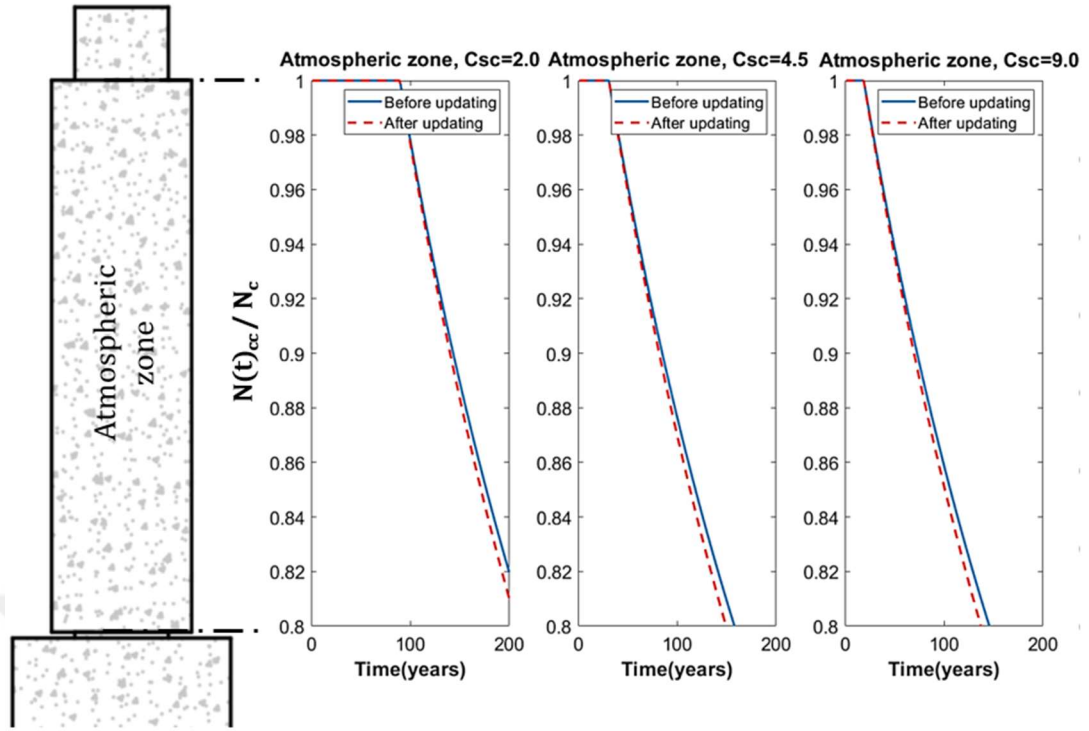


Figure 5.13. The axial load capacity change ratio, $N_{cc}(t)/N_c$ due to uniform corrosion

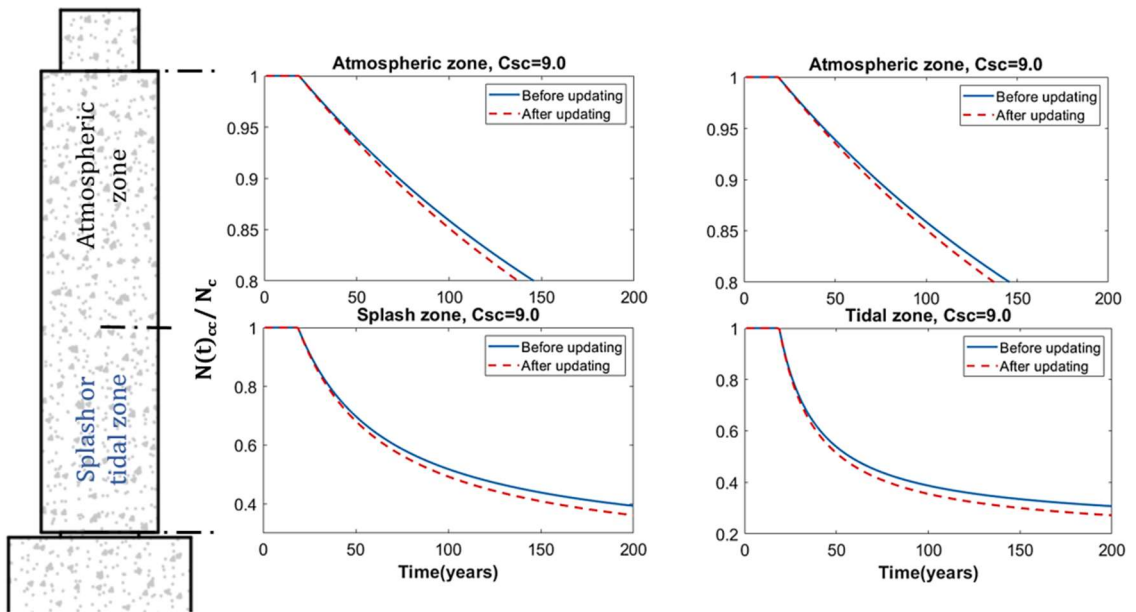


Figure 5.14. The axial load capacity change ratio, $N_{cc}(t)/N_c$ due to non-uniform corrosion

Although the capacity of the updated model is higher than that of the non-updated model, it has been observed that the rate of decrease in capacity is higher for the updated

model over time. The reason for this is that, while the material properties for the original and retrofit parts are the same at the first model acceptance, after model updating, it is determined that there are different material properties for these two parts.

5.6. Deformation Relationship of Bridge Column

Using the bending deformation formulation in Equation (5.27) and the well-known plastic hinge concept by Park & Paulay (1991) given in Figure 5.15. , the ultimate displacement, Δ of cantilever column can be determined as follows:

$$\Delta = \int_0^L \phi(x) x dx \quad (5.28)$$

where L is the height of the column, $\phi(x)$ is the curvature distribution at any distance x from the column top.

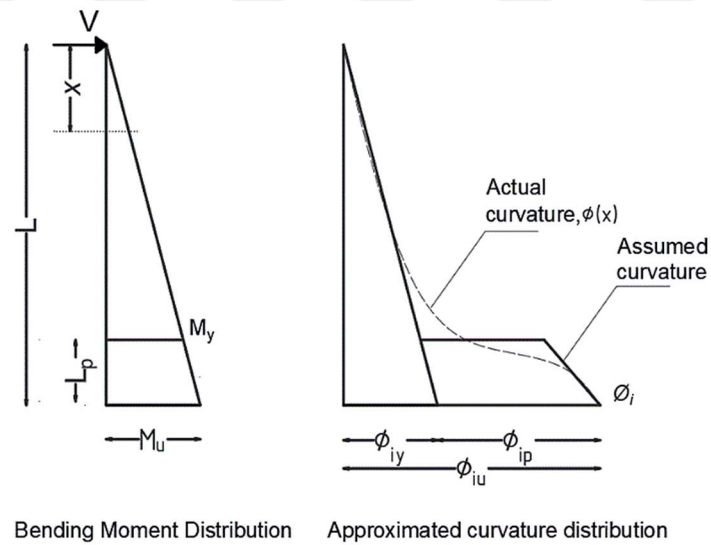


Figure 5.15. Cantilever column bending moment distribution and approximated curvature distribution

The ultimate displacement can be calculated based on the defined plastic hinge concept:

$$\Delta_i = \begin{cases} \frac{1}{3} \phi_i L^2 & \phi_i < \phi_{iy} \\ \frac{1}{3} \phi_{iy} L^2 + (\phi_{iu} - \phi_{iy}) L_p (L - 0.5 L_p) & \phi_i \geq \phi_{iy} \end{cases} \quad (5.29)$$

where ϕ_{iy} and ϕ_{iu} are the yield curvature and the ultimate curvature at any location of column, respectively. L is the length of the column and the L_p is the length of plastic hinge.

In this study, different corrosion characteristics along the height of the bridge column are investigated, and the ultimate deformations for uniform and non-uniform cases are conducted. For a uniform corrosion case, the idealized plastic hinge concept is given in Figure 5.16.

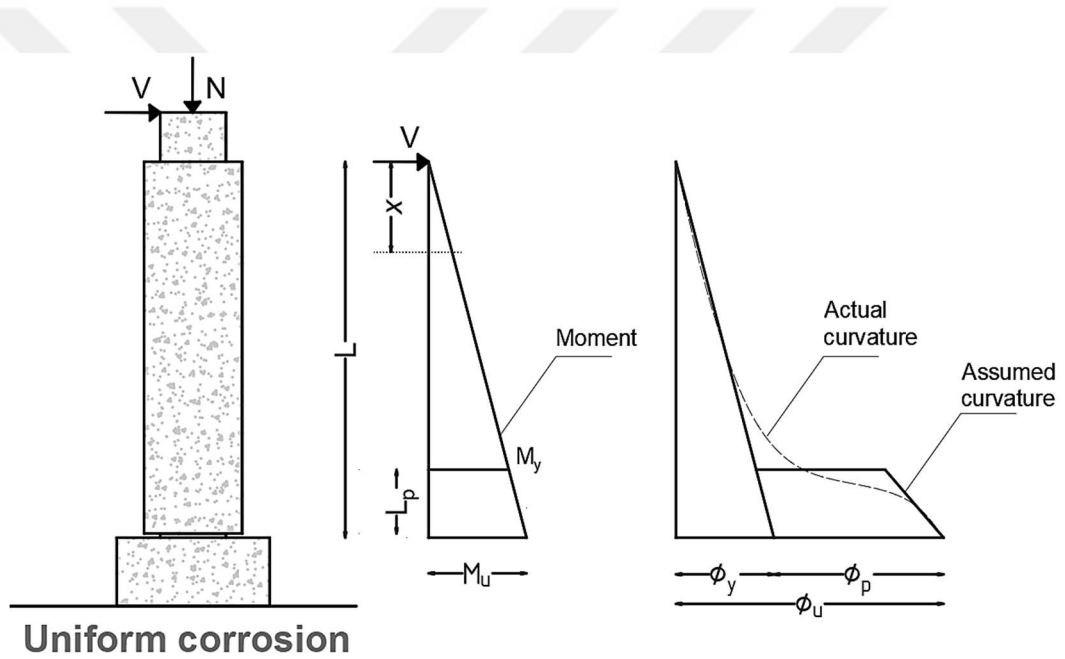


Figure 5.16. Idealized plastic hinge analysis for uniform corrosion

The plastic hinge concept and ultimate displacement, Δ_u are defined as follows with respect to uniform corrosion:

$$\Delta_u = \Delta_y + \Delta_p = \frac{\phi_y L^2}{3} + (\phi_u - \phi_y) L_p (L - 0.5 L_p) \quad (5.30)$$

where Δ_y is the yield displacement, Δ_p is the plastic displacement, ϕ_u is the ultimate curvature at the column base, ϕ_y is the yield curvature, and L and L_p are the lengths of

the column and the plastic hinge, respectively. The length of the plastic hinge, L_p can be estimated equation (5.30) proposed by Priestley & Park (1987):

$$L_p = 0.08L + 6d_b \quad (5.31)$$

where L is the length of the column, and d_b is the bar diameter.

The expression for the ultimate drift ratio, θ_u , is as follows:

$$\theta_u = \frac{\Delta_u}{L} = \frac{\phi_y L}{3} + \frac{(\phi_u - \phi_y)L_p(L - 0.5L_p)}{L} \quad (5.32)$$

It is possible to express the ultimate and yield curvature of a reinforced concrete column as (Gu et al. 2012):

$$\phi_u = \frac{\varepsilon_{cu}}{c} \quad (5.33)$$

$$\phi_y = \lambda \frac{\varepsilon_y}{D} \quad (5.34)$$

where c is the depth of the compression zone (or depth of the neutral axis) when the strain at the extreme compressive fiber reaches the ultimate strain, ε_{cu} . $\lambda = 2.45$ for spiral-reinforced columns, ε_y is the yield strain of the longitudinal reinforcement, and D is the diameter of the column.

The depth of the compression zone, c has been widely studied by researchers. Though various factors (axial load, longitudinal reinforcement, etc.) affect the value of c , the simplified equation provided has received approval for the assessment of c (Berry and Eberhard 2005).

$$\frac{D}{c} = \frac{\phi_c D}{\varepsilon_c} = \frac{G_0}{1 + G_1 * n} + \frac{\phi_y D}{\varepsilon_c} \quad (5.35)$$

where ϕ_c is the curvature at a given extreme compression fiber strain, ε_c , n is the axial load ratio and defined as $N/A_g f'_c$ where A_g is the gross cross sectional area, f'_c is the concrete compressive strength, and N is the axial force, and G_0 and G_1 are parameters depending on the ε_c . The values of G_0 and G_1 , which are derived from moment-curvature

analyses of flexure-dominant columns, are 5.3 and 9.4, correspondingly Berry & Eberhard (2005). Jiang et al. (2010) proposed Equation (5.36) for the same situation.

$$\frac{c}{D} = 0.189 + 0.898 * n \quad (5.36)$$

For RC columns with a significant confinement by steel stirrups, the ultimate failure strain is greater than 0.004 and closely related to the degree of confinement. Therefore, apart from the parameter, n in Equation (5.36), another parameter involving confinement should be included in the model. From moment–curvature analyses, the compression zone depth at ultimate failure can be calculated as (Jiang, Lu, and Kubo 2010):

$$\frac{c}{D} = 0.12 + (1.07 - 1.05k_e w_s) * n \quad (5.37)$$

where w_s is the confinement ratio of transverse steel, and k_e is the shape factor related to the effective area of confinement.

$$w_s = \frac{\rho_s f_{yh}}{f_{c0}} \quad (5.38)$$

Confinement effectiveness coefficient, k_e in circular concrete columns can be calculated as (Eid and Dancygier 2006):

$$k_e = \frac{(1 - \frac{S}{4a})^{n_c}}{1 - \rho_{cc}} \quad (5.39)$$

For non-uniform corrosion cases, deformation should be calculated piecewise. The idealized plastic hinge concept is given in Figure 5.17. According to the approximated curvature distribution, local deformations are calculated.

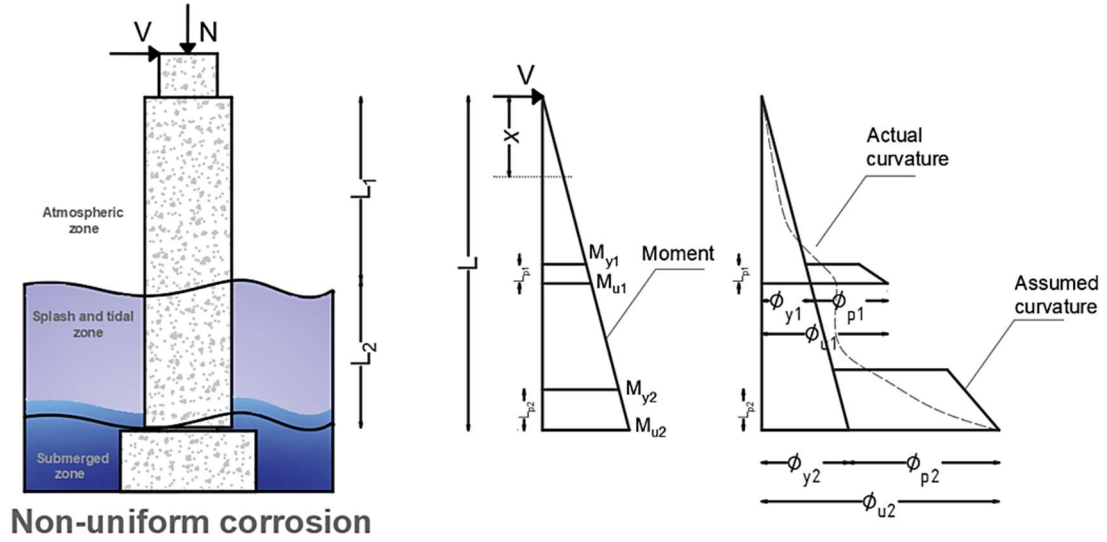


Figure 5.17. Idealized plastic hinge analysis for non-uniform corrosion

Local deformations of columns, Δ_{u1} and Δ_{u2} presented in atmospheric, splash, or tidal zones are used to calculate the ultimate deformation of the column as follows:

$$\Delta_{u1} = \begin{cases} \frac{\phi_{y1} L_1^2}{3} & \phi_1 < \phi_{y1} \\ \frac{\phi_{y1} L_1^2}{3} + (\phi_{u1} - \phi_{y1}) L_{p1} (L_1 - 0.5 L_{p1}) & \phi_1 \geq \phi_{y1} \end{cases} \quad (5.40)$$

$$\Delta_{u2} = \begin{cases} \frac{\phi_{y2} L_2^2}{L_1 + L_2} \left(\frac{L_1}{2} + \frac{L_2}{3} \right) & \phi_2 < \phi_{y2} \\ \frac{\phi_{y2} L_2^2}{L_1 + L_2} \left(\frac{L_1}{2} + \frac{L_2}{3} \right) + (\phi_{u2} - \phi_{y2}) L_{p2} (L_1 + L_2 - 0.5 L_{p2}) & \phi_2 \geq \phi_{y2} \end{cases} \quad (5.41)$$

Then the total deformation of the bridge column exposed to non-uniform corrosion is calculated as follows:

$$\Delta_u = \Delta_{u1} + \Delta_{u2} \quad (5.42)$$

The displacement capacity of the bridge column can be defined as the ultimate drift ratio, $DR_u(\%)$ (Aydemir, Eser Aydemir, and Arslan 2023).

$$DR_u(\%) = \frac{\Delta_u}{L} * 100 \quad (5.43)$$

In Figures 5.18 and 5.19, $DR_u(\%)$ for uniform and non-uniform cases after and before model updating are given.

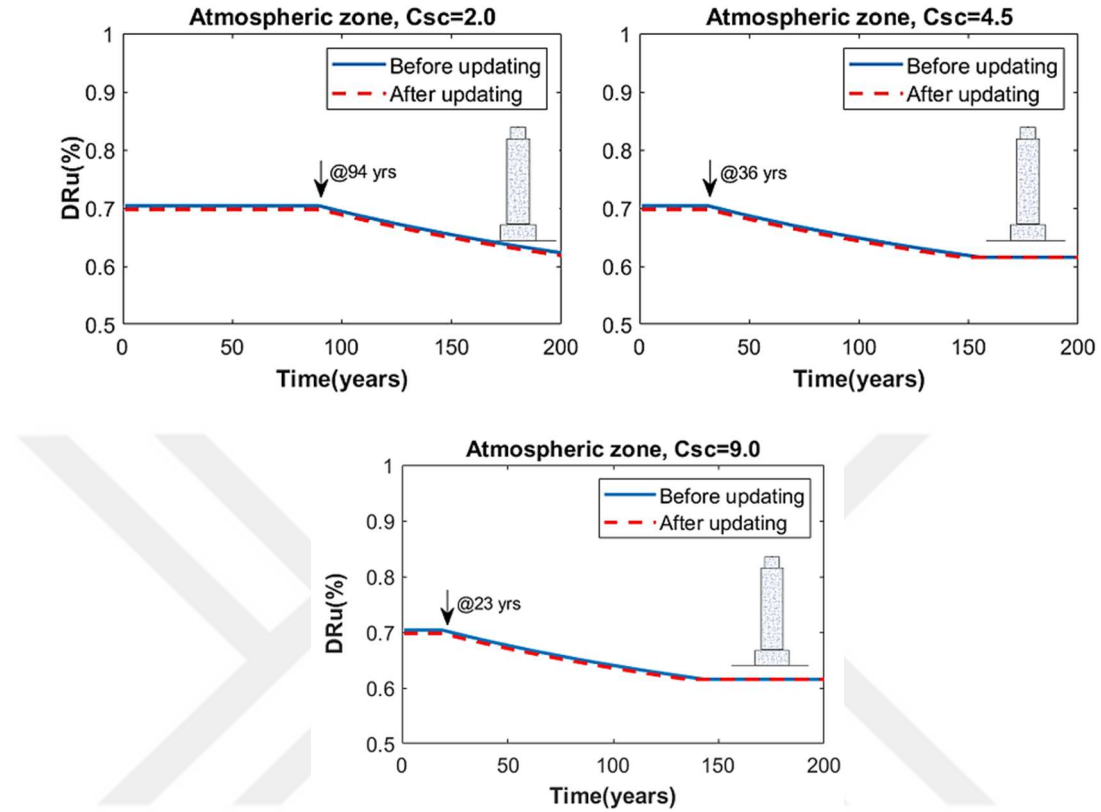


Figure 5.18. Ultimate drift ratio, $DR_u(\%)$ for uniform corrosion

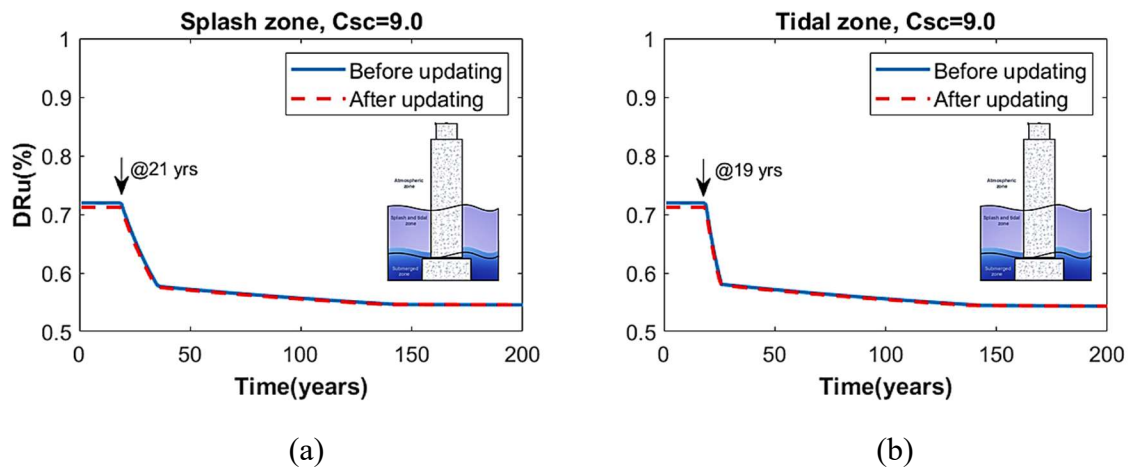


Figure 5.19. Ultimate drift ratio, $DR_u(\%)$ for non-uniform corrosion (a) at splash zone
(b) at tidal zone

In the uniform corrosion case, it has been observed that as surface chloride ion concentration, C_{sc} value increases, $DR_u(\%)$ value decreases. For coastal structures, a decrease in the $DR_u(\%)$ value began after 20 years, and dramatic decreases are observed in structures exposed to the splash and tidal zones in non-uniform cases. Since the retrofitted part is effective for ultimate drift and similar material values for this part are obtained after and before model updating, no major differences are observed in $DR_u(\%)$ values.

5.7. Moment Capacity of Bridge Column

The strength values given in Equations (5.44) and (5.46) are characteristic values for the design. Moment capacity, M_r can be calculated using Equation (5.46) after the depth of the compression zone, c is identified at the ultimate limit state using Equation (5.37).

$$N_r = (0.85f(t)_c + 4\sigma(t)) * A(t)_c + A(t)_{st}f(t)_y \quad (5.44)$$

where σ_{si} is reinforcement stress and calculated using Equation (5.44), A_{si} is reinforcement area. The cross sectional area of column (shaded area shown in Figure 5.20) is A_{cc} .

$$\sigma_{si} = 0.003E_s \left(1 + \frac{y(t)_i - y(t)_p}{c(t)} \right) \leq f(t)_y \quad (5.45)$$

$$\begin{aligned} M(t)_r &= N(t)_r * e \\ &= (0.85f(t)_c + 4\sigma(t))A(t)_{cc}(y(t)_p - \bar{y}(t)) \\ &\quad + \sum_{i=1}^n A(t)_{si}\sigma(t)_{si}y(t)_i \end{aligned} \quad (5.46)$$

where y_p is distance of column center, y_i is distance of column reinforcement to center, $\bar{y}$ is the location of centroid of the shaded area, e is the eccentricity.

For the examined bridge column, the characteristic circular column section subjected to axial load, N and flexural moment is shown in Figure 5.20. On the same figure, possible strain and force distributions are also illustrated. Basic assumptions (such as a perfect bond between steel and concrete, the tensile strength of concrete is neglected,

the foundation and column are connected correctly, etc.) are made in the ultimate strength theory.

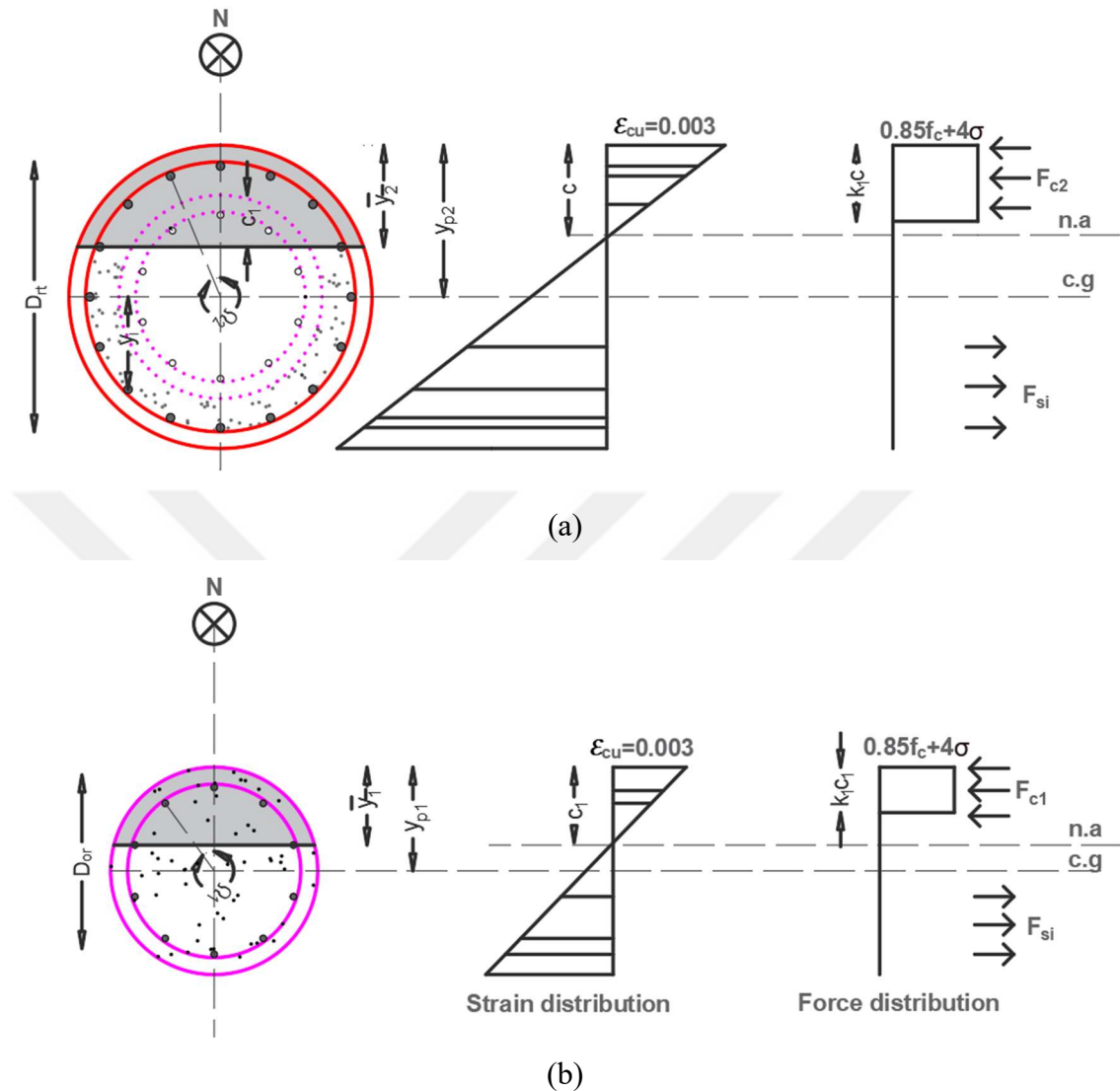


Figure 5.20. Distribution of strain and force in circular cross-section subjected to the moment (a) original column (b) retrofitted part

5.8. Reliability Assessment of Bridge Column

It is inevitable that the performance of bridge structures, which have an important place in transportation networks, will deteriorate during their service life under external environmental conditions. In particular, bridge columns are the basic components that ensure the stability and load-carrying capacity of bridge structures, and it is possible to see different corrosion effects (such as uniform and non-uniform) on these structures

depending on the water level they are exposed to. This necessitates the most accurate structural reliability assessments of such structures.

In this case, the proposed solution method is used to compare the structural reliability of the FE model of the updated and non-updated bridge columns. The results are then shown. It is expected that the updated model will positively affect the decision on issues such as structure safety, lifespan, maintenance cost, etc. compared to the non-updated model.

5.8.1. Drift Capacity Model for Bridge Column

The bending moment of a cantilever column under variable shear force and 10% of the total axial load density of the bridge column obtained by previously defined formulas is examined. V described as Gumbel distribution with mean equal to 700 kN and COV equal to 0.1. The failure damage state can be expressed as follows:

$$M_r \leq VL + N\Delta_u \quad (5.47)$$

Figures 5.21 and 5.22 show the reliability indices for uniform and non-uniform cases over the service times. These are used to compare the failure damage assessment results from the FE models that were updated before and after the changes are made.

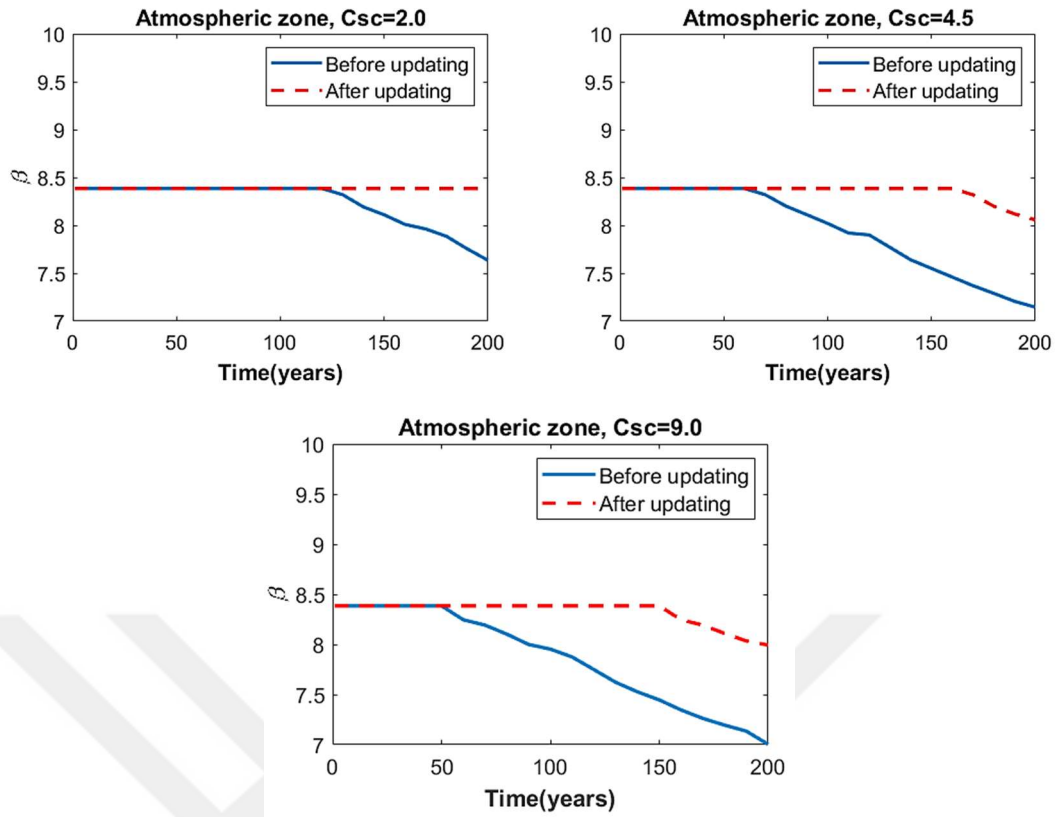


Figure 5.21. Reliability index values, β for uniform corrosion cases

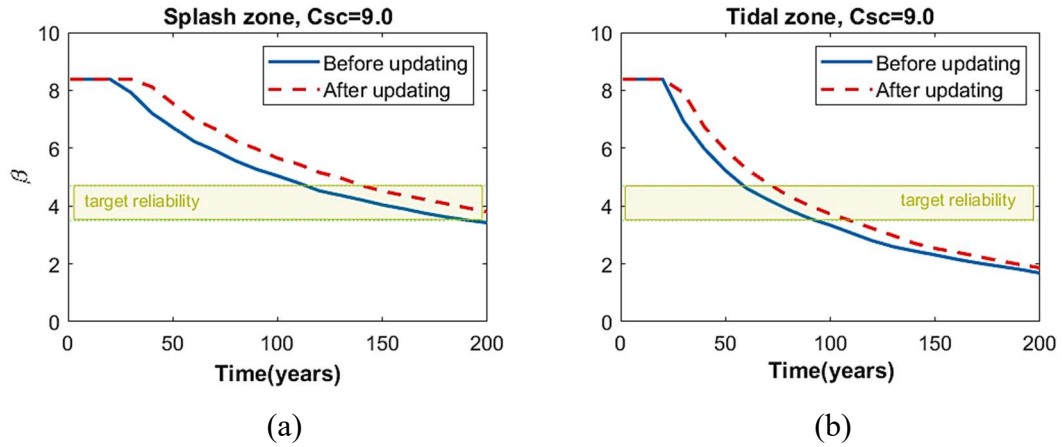


Figure 5.22. Reliability index values, β for non-uniform corrosion case (a) at splash zone (b) at tidal zone

As previously mentioned, the system failure event is assumed to occur when the target reliability index falls below. Obviously, the reliability indices of the bridge column decrease with increasing service life, which means that the probability of failure increases with increasing use of the bridge under any corrosion effect. In addition, the reliability index of the bridge column under non-uniform corrosion is smaller than in uniform

corrosion cases. It is also determined that the reliability index of the updated building model is significantly higher than the non-updated one.

5.8.2. Under Different Vehicle Loads

In Figure 5.23, the probability densities of axle weights and axle spacing are given. Variables are presented in Table 5.10. Here, vehicle types with a large portion of traffic volume are selected from the literature (T. Guo, Frangopol, and Chen 2012). In this examination, since the loading on the column is to be taken into account regardless of the girders, the loading that occurs when passing vehicles arrive at the centroid of the column has been considered.

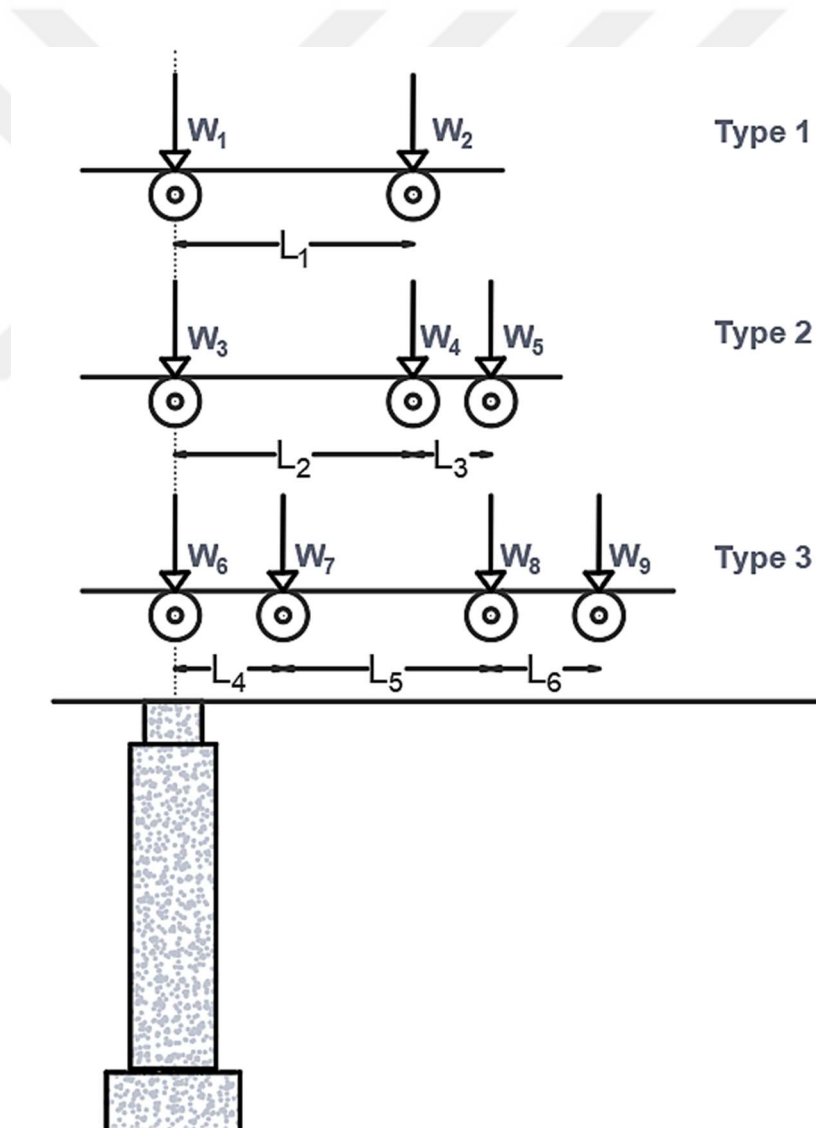


Figure 5.23. Graphic illustration for vehicle type

Table 5.10. Random variables for axle weights and axle spacing (dimensions for axle weights and axle spacing are in kN and meter, respectively)

Vehicle type	Variable	Distribution type	Mean, μ	Standard deviation, σ
1	W_1	Normal	38.22	9.74
	W_2	Normal	62.08	20.92
	L_1	Lognormal	6.036	1.044
2	W_3	Lognormal	48.06	8.51
	W_4	Lognormal	62.02	24.43
	W_5	Lognormal	56.92	21.99
	L_2	Lognormal	4.864	1.388
	L_3	Lognormal	1.310	0.052
3	W_6	Normal	47.31	6.68
	W_7	Lognormal	63.19	21.05
	W_8	Lognormal	59.70	20.44
	W_9	Lognormal	61.04	23.13
	W_{10}	Lognormal	59.37	27.24
	L_4	Normal	4.609	1.073
	L_5	Normal	1.323	0.033
	L_6	Normal	9.839	1.136
	L_7	Normal	1.258	0.049

The failure damage state under different vehicle loads can be expressed as follows:

$$M_r \leq \sum_{i=1}^n W_i * L_{Ti} \quad (5.48)$$

where M_r is the moment capacity of column, W_i is the axle weights, L_{Ti} is the total axle spacing of axle weights to centroid of the column.

The reliability indices of the non-updated model are shown in Figures 5.24 and 5.25. These show how the model behaves under uniform and non-uniform corrosion over the service life of the bridge column.

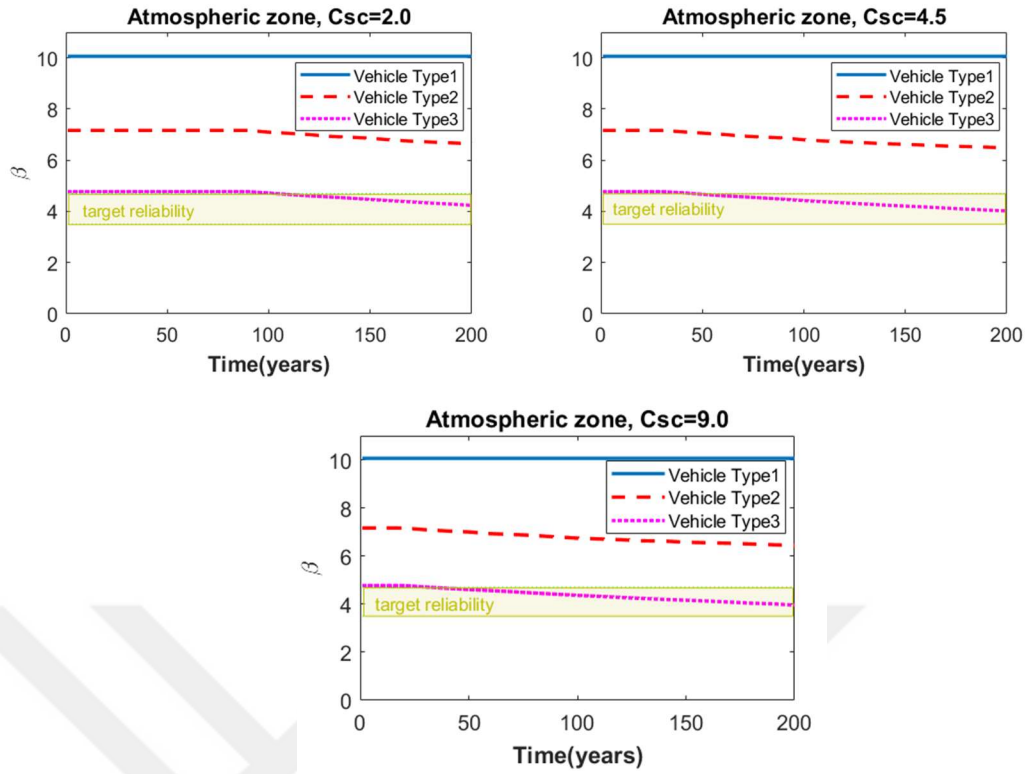


Figure 5.24. Reliability index values, β before updating for uniform corrosion cases

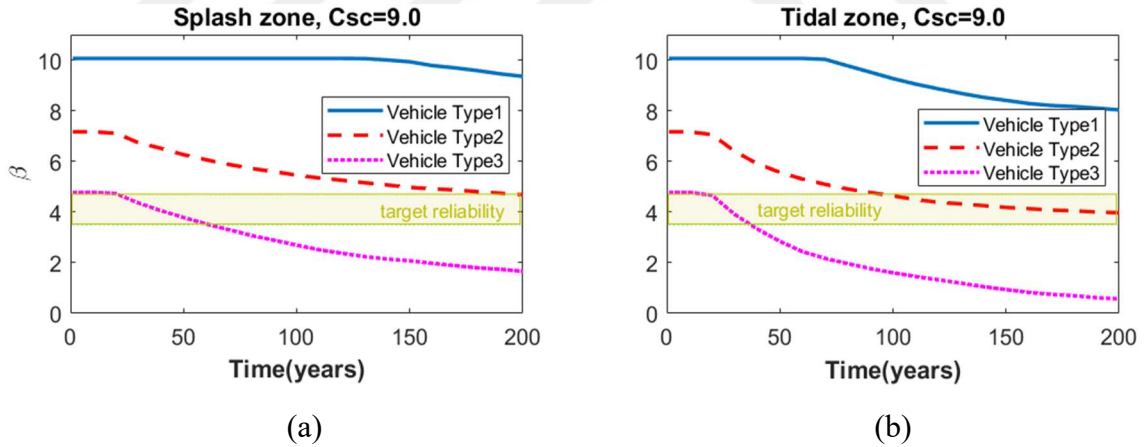


Figure 5.25. Reliability index values, β before updating for non-uniform corrosion cases (a) at splash zone (b) at tidal zone

Figures 5.26 and 5.27 are used to show the reliability indices of the updated model under uniform and non-uniform cases over the service times of the bridge column.

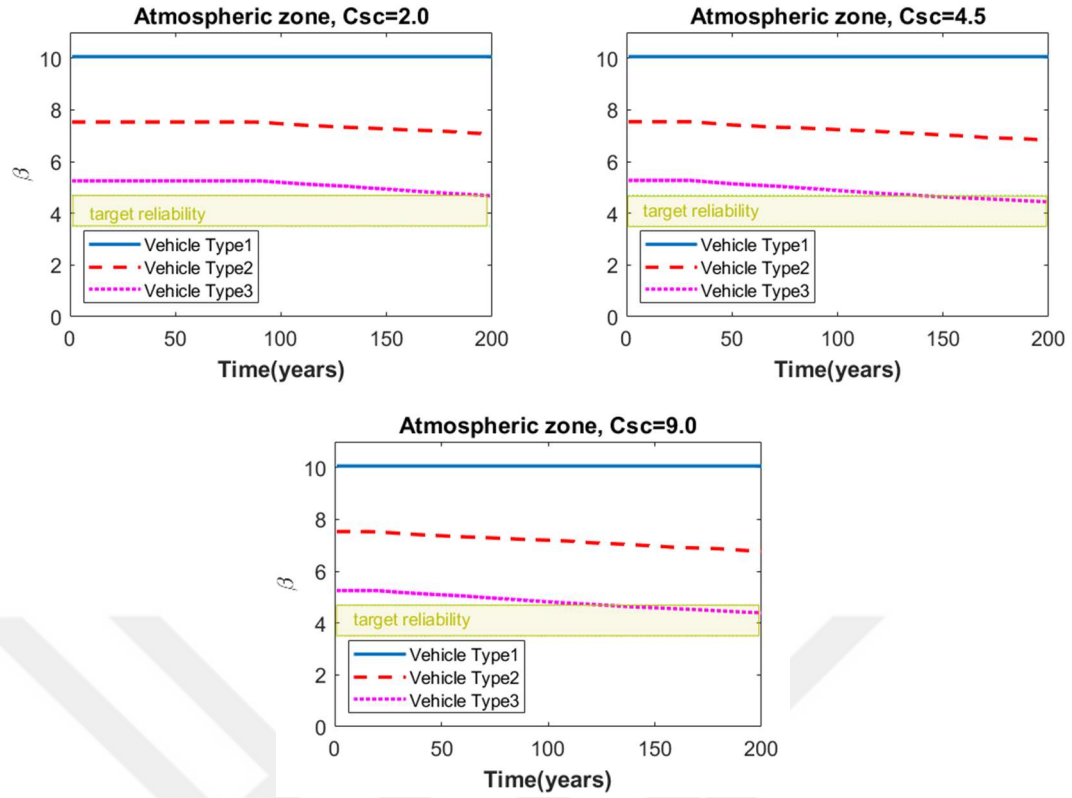


Figure 5.26. Reliability index values, β after updating for uniform corrosion cases

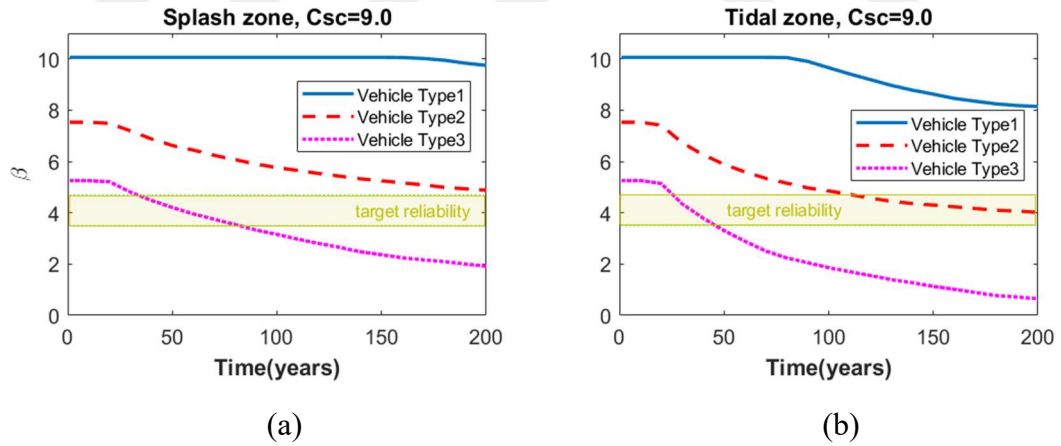


Figure 5.27. Reliability index values, β after updating for non-uniform corrosion cases
(a) at splash zone (b) at tidal zone

As a result, it is seen that the reliability index for the updated FE model is higher. The reliability index for vehicle Type1 and 2 loads remained within the target reliability index in the service life of the column for the both uniform and non-uniform cases. In non-uniform cases of non-updated model, the reliability index remained below the target reliability index before reaching 50 years. In non-uniform cases of updated model, while

the reliability index remains below the target reliability index between 50-100 years for the splash zone, it is close to 50-years for the tidal zone.

5.9. Conclusion

The main concern of the study is to provide a PCK based methodology to efficiently integrate monitoring data into FE model updating process and to improve the quality of the prediction model. In this way, the computational cost of updating Bayesian FE model for complex engineering structures is reduced and a reliability assessment of this type of structures can be made more accurate. In addition, decision about the safety of the structure, service-life, maintenance cost, etc. of updated model is to be positively affected compared to non-updated model.

In this context, the proposed methodology is implemented to an experimental study using the vibration data of Irvine bridge column project at Los Alamos National Laboratory. To investigate various damage modes of concrete cover, changes in time-dependent capacities of cross-sections of bridge column in the atmospheric zone and in the splash and tidal zone under different load combinations such as axial, lateral force and bending moment are presented. Reliability assessments of updated and non-updated FE models of bridge column are investigated and results are presented comparatively. The results obtained from the study are as follows:

- With the proposed solution method, the computational cost is reduced and an updated FE model closer to the measurement values taken from experiment of real structure.
- The updated model is found to be more reliable than the non-updated model in the reliability evaluations made by exposing it to different load effects and corrosion.
- It is thought that this will provide more accurate results for predictions on issues such as the safety of the structure, service-life, maintenance cost, etc.
- The bridge column is examined in detail depending on different corrosion types and intensity. As a result, it has been concluded that serious decreases can be observed in the capacities of structures in direct contact with water or in places where surface chloride ion concentration is high, in certain periods of time. In

this context, it is of great importance to determine the correct or closest time to the impact of this effect on the structure. In this regard, it is thought that the more accurate model updating will provide significant benefits for structure safety.

Consequently, the proposed methodology is successfully applied on a real structure, and structural reliability evaluations are made by considering different corrosion effects. This study offers the opportunity to perform studies regarding the use of measured data in both reliability assessment models and for the development of reliability-based prediction models.



CHAPTER 6

CONCLUSIONS

At the end of each section, the results of the studies carried out in that section are given. In this section, a brief summary of the notable findings and general conclusions are presented for each section.

- In Chapter 1, the background, purpose, scope, and importance of the thesis are given under separate headings. Research problems that can be examined are defined and presented in this section. In addition, the main research objectives of the study are grouped under four main headings, and information about them is given.
- In Chapter 2, studies on the subject are given in detail in the literature. In particular, the deficiencies and gaps that emerged in the FE model updating and structural reliability evaluations of real engineering structures were identified and examined. In this context, the research topics needed in the literature are presented in items.
- In Chapter 3, taking the deficiencies mentioned in the previous section (in Chapter 2) as reference, the theoretical basis of the solution proposal that will both take into account the existing uncertainties and eliminate the calculation difficulties that will arise in the FE model updating process of real engineering structures is presented. In addition, the details of the solution scheme into which the proposed solution method is integrated in order to make more accurate structural reliability assessments are given in this section.
- In Chapter 4, a truss bridge example and a real bridge column example are selected and examined to demonstrate the effectiveness of the proposed solution method. As an application not frequently encountered in the literature, FE model updates are made by adding different metamodeling techniques to the proposed solution method, and the results are given comparatively. While a significant decrease in the computational cost is achieved, it is observed that the updated FE models could come close to the results of the data obtained

from the real structure. It is expected that the proposed solution method will enable more complex analyses, such as reliability analyses and design optimizations.

- In Chapter 5, the previously thought-about structural reliability assessment of the bridge column is done for situations in which it is subjected to different types of corrosion (uniform and non-uniform). In addition, the differences between the results of updated and non-updated FE models are highlighted, taking into account different loading situations. This study provides the opportunity to conduct studies on the use of measured data both in reliability assessment models and in the development of reliability-based prediction models. As a result, it was found that the evaluations made with the updated model were more accurate than the non-updated ones. In this way, more accurate results can be obtained in estimating issues such as the safety of the structure, its lifespan, maintenance costs, etc. In addition, the capacity changes that may occur in the bridge column as a result of different corrosion effects are discussed in detail, and the results are presented.

REFERENCES

- ACI318-02. 2002. "Building Code Requirements for Structural Concrete." Farmington Hills, MI.
- ACI318-19. 2019. "Building Code Requirements for Structural Concrete and Commentary." Farmington Hills, MI.
- Akiyama, Mitsuyoshi, Dan M. Frangopol, and Ikumasa Yoshida. 2010. "Time-Dependent Reliability Analysis of Existing RC Structures in a Marine Environment Using Hazard Associated with Airborne Chlorides." *Engineering Structures* 32 (11): 3768–79. <https://doi.org/10.1016/j.engstruct.2010.08.021>.
- Amey, Stephen L., Dwayne A. Johnson, Matthew A. Miltenberger, and Hamid Farzam. 1998. "Predicting the Service Life of Concrete Marine Structures: An Environmental Methodology." *ACI Structural Journal* 95 (2). doi: 10.14359/540
- ANSYS. 2015. "Version 15.0, Canonsburg, Ansys. Inc., 2015." Canonsburg, PA USA,: Ansys. Inc.
- Argyris, Costas, Costas Papadimitriou, Panagiotis Panetsos, and Panos Tsopela. 2020. "Bayesian Model-Updating Using Features of Modal Data: Application to the Metsovo Bridge." *Journal of Sensor and Actuator Networks* 9 (2). <https://doi.org/10.3390/JSAN9020027>.
- ASCE/SEI 7-10. 2016. "Minimum Design Loads for Buildings and Other Structures." Virginia, USA: American Society of Civil Engineers.
- Au, Siu-Kui, and James L. Beck. 2001. "Estimation of Small Failure Probabilities in High Dimensions by Subset Simulation" *Probabilistic engineering mechanics* 16: 263–77. [https://doi.org/10.1016/S0266-8920\(01\)00019-4](https://doi.org/10.1016/S0266-8920(01)00019-4).
- Aydemir, Cem, Müberra Eser Aydemir, and Güray Arslan. 2023. "Drift Capacity and Allowable Axial Load Level of RC Columns." *Structures* 48 (January): 1072–81. <https://doi.org/10.1016/j.istruc.2023.01.014>.
- Bayraktar, Alemdar, Ahmet Can Altunişik, Barış Sevim, and Temel Türker. 2009. "Modal Testing, Finite-Element Model Updating, and Dynamic Analysis of an Arch Type Steel Footbridge." *Journal of Performance of Constructed Facilities* 23 (2): 81–89. [https://doi.org/10.1061/\(asce\)0887-3828\(2009\)23:2\(81\)](https://doi.org/10.1061/(asce)0887-3828(2009)23:2(81)).
- Beck, James L. Beck, and Siu-Kui Au. 2002. "Bayesian Updating of Structural Models and Reliability Using Markov Chain Monte Carlo Simulation." *Journal of*

- Engineering Mechanics* 128 (4): 380–91. [https://doi.org/10.1061/\(asce\)0733-9399\(2002\)128:4\(380\)](https://doi.org/10.1061/(asce)0733-9399(2002)128:4(380)).
- Beck, James L., and Lambros S. Katafygiotis. 1998. “Updating Models and Their Uncertainties I: Bayesian Statistical Framework.” *Journal of Engineering Mechanics* 124 (4): 463–67. [https://doi.org/10.1061/\(ASCE\)0733-9399\(1998\)124:4\(455\)](https://doi.org/10.1061/(ASCE)0733-9399(1998)124:4(455)).
- Beck, James L., and Lambros S. Katafygiotis. 1992. “Probabilistic System Identification and Health Monitoring of Structures.” In *Earthquake Engineering, Tenth World Conference*. Balkema, Rotterdam.
- Berry, Michael P., and Marc O Eberhard. 2005. “Practical Performance Model for Bar Buckling.” *Journal of Structural Engineering* 131 (7): 1060–70. [https://doi.org/10.1061/\(asce\)0733-9445\(2005\)131:7\(1060\)](https://doi.org/10.1061/(asce)0733-9445(2005)131:7(1060)).
- Betz, Wolfgang, Iason Papaioannou, James L. Beck, and Daniel Straub. 2018. “Bayesian Inference with Subset Simulation Strategies and Improvements.” *Computational Structural Engineering Institute* 331: 72–93. <https://doi.org/10.1016/j.cma.2017.11.021>.
- Bhargava, Kapilesh, Yasuhiro Mori, and Aniruddha K. Ghosh. 2011. “Time-Dependent Reliability of Corrosion-Affected RC Beams. Part 3: Effect of Corrosion Initiation Time and Its Variability on Time-Dependent Failure Probability.” *Nuclear Engineering and Design* 241 (5): 1395–1402. <https://doi.org/10.1016/j.nucengdes.2010.06.039>.
- Brandt, Anders. 2023. *Noise and Vibration Analysis: Signal Analysis and Experimental Procedures*. John Wiley & Sons.
- BS6349-1:2000. 2003. “Marine Structures-Part 1: Code of Practice for General Criteria.”
- Catbas, F. Necati, Melih Susoy, and Dan M. Frangopol. 2008. “Structural Health Monitoring and Reliability Estimation: Long Span Truss Bridge Application with Environmental Monitoring Data.” *Engineering Structures* 30 (9): 2347–59. <https://doi.org/10.1016/j.engstruct.2008.01.013>.
- Chen, Xinghua, and Piotr Omenzetter. 2013. “A Framework for Reliability Assessment of an In-Service Bridge Using Structural Health Monitoring Data.” In *Key Engineering Materials*, 558:39–51. <https://doi.org/10.4028/www.scientific.net/KEM.558.39>.
- Cheung, Sai Hung, and James L. Beck. 2009. “Bayesian Model Updating Using Hybrid

- Monte Carlo Simulation with Application to Structural Dynamic Models with Many Uncertain Parameters.” *Journal of Engineering Mechanics* 135 (4): 243–55.
[https://doi.org/10.1061/\(ASCE\)0733-9399\(2009\)135](https://doi.org/10.1061/(ASCE)0733-9399(2009)135).
- Ching, Jianye, and Yi-chu Chen. 2007. “Transitional Markov Chain Monte Carlo Method for Bayesian Model Updating, Model Class Selection, and Model Averaging” *Journal of engineering mechanics* 133 (7): 816–32.
[https://doi.org/10.1061/\(ASCE\)0733-9399\(2007\)133:7\(816\)](https://doi.org/10.1061/(ASCE)0733-9399(2007)133:7(816)).
- Ching, Jianye, Matthew Muto, and James L. Beck. 2006. “Structural Model Updating and Health Monitoring with Incomplete Modal Data Using Gibbs Sampler.” *Computer-Aided Civil and Infrastructure Engineering* 21 (4): 242–57.
<https://doi.org/10.1111/j.1467-8667.2006.00432.x>.
- Ciang, Chia Chen, Jung Ryul Lee, and Hyung Joon Bang. 2008. “Structural Health Monitoring for a Wind Turbine System: A Review of Damage Detection Methods.” *Measurement Science and Technology* 19 (12).
<https://doi.org/10.1088/0957-0233/19/12/122001>.
- Collins, Jon D., Gary C. Hart, T. K. Hasselman, and Bruce Kennedy. 1974. “Statistical Identification of Structures.” *AIAA Journal* 12 (2): 185–90.
<https://doi.org/10.2514/3.49190>.
- Diaconis, Persi, and Donald Ylvisaker. 1979. “Conjugate Priors for Exponential Families.” *The Annals of Statistics* 7 (2): 269–81.
<https://www.jstor.org/stable/2958808>.
- Dolšek, Matjaž. 2012. “Simplified Method for Seismic Risk Assessment of Buildings with Consideration of Aleatory and Epistemic Uncertainty.” *Structure and Infrastructure Engineering* 8 (10): 939–53.
<https://doi.org/10.1080/15732479.2011.574813>.
- Eid, Rami, and Avi Dancygier. 2006. “Confinement Effectiveness in Circular Concrete Columns.” *Engineering Structures* 28 (13): 1885–96.
<https://doi.org/10.1016/j.engstruct.2006.03.015>.
- EN 1990:2002. 2002. *EUROCODE Basis of Structural Design*. Brussels: CEN;
- Ereiz, Suzana, Ivan Duvnjak, and Javier Fernando Jiménez-Alonso. 2022. “Review of Finite Element Model Updating Methods for Structural Applications.” *Structures* 41 (May): 684–723. <https://doi.org/10.1016/j.istruc.2022.05.041>.
- Ewins, Dawid. 2000. “Basics and State-of-the-Art of Modal Testing.” *Sadhana - Academy Proceedings in Engineering Sciences* 25 (3): 207–20.

- <https://doi.org/10.1007/BF02703540>.
- Frangopol, Dan M., Alfred Strauss, and Sunyong Kim. 2008. "Bridge Reliability Assessment Based on Monitoring." *Journal of Bridge Engineering* 13 (3): 258–70. [https://doi.org/10.1061/\(asce\)1084-0702\(2008\)13:3\(258\)](https://doi.org/10.1061/(asce)1084-0702(2008)13:3(258)).
- Fu, Chongbo, Peng Wang, Liang Zhao, and Xinjing Wang. 2020. "A Distance Correlation-Based Kriging Modeling Method for High-Dimensional Problems." *Knowledge-Based Systems* 206: 106356. <https://doi.org/10.1016/j.knosys.2020.106356>.
- García-Merino, J. C., Carmen Calvo-Jurado, Emilio Martínez-Pañeda, and Enrique García-Macías. 2023. "Multielement Polynomial Chaos Kriging-Based Metamodelling for Bayesian Inference of Non-Smooth Systems." *Applied Mathematical Modelling* 116: 510–31. <https://doi.org/10.1016/j.apm.2022.11.039>.
- GB50367-2013. 2013. *Code for Design of Strengthening Concret Structure*. China Planning Press: Beijing, China.
- Gharehbaghi, Vahid Reza, Ehsan Noroozinejad Farsangi, Mohammad Noori, T. Y. Yang, Shaofan Li, Andy Nguyen, Christian Málaga-Chuquitaype, Paolo Gardoni, and Seyedali Mirjalili. 2022. "A Critical Review on Structural Health Monitoring: Definitions, Methods, and Perspectives." *Archives of Computational Methods in Engineering* 29 (4): 2209–35. <https://doi.org/10.1007/s11831-021-09665-9>.
- Girardi, Maria, Cristina Padovani, Daniele Pellegrini, Margherita Porcelli, and Leonardo Robol. 2020. "Finite Element Model Updating for Structural Applications." *Journal of Computational and Applied Mathematics* 370: 112675. <https://doi.org/10.1016/j.cam.2019.112675>.
- Gu, Dong Sheng, Yu Fei Wu, Gang Wu, and Zhi Shen Wu. 2012. "Plastic Hinge Analysis of FRP Confined Circular Concrete Columns." *Construction and Building Materials* 27 (1): 223–33. <https://doi.org/10.1016/j.conbuildmat.2011.07.056>.
- Guo, Anxin, Wenting Yuan, Haitao Li, and Hui Li. 2018. "Structural Strength Deterioration of Coastal Bridge Piers Considering Non-Uniform Corrosion in Marine Environments." *Earthquake Engineering and Engineering Vibration* 17 (2): 429–44. <https://doi.org/10.1007/s11803-018-0451-z>.
- Guo, Tong, Dan M. Frangopol, and Yuwen Chen. 2012. "Fatigue Reliability Assessment of Steel Bridge Details Integrating Weigh-in-Motion Data and Probabilistic Finite Element Analysis." *Computers and Structures* 112–113: 245–

57. <https://doi.org/10.1016/j.compstruc.2012.09.002>.
- Hafeez, Luqman. 2008. "Development of Models for Strengthened Reinforced Concrete Columns." Ohio State University.
- Hariri-Ardebili, Mohammad Amin, and Bruno Sudret. 2020. "Polynomial Chaos Expansion for Uncertainty Quantification of Dam Engineering Problems." *Engineering Structures* 203 (June): 109631. <https://doi.org/10.1016/j.engstruct.2019.109631>.
- Hastings, W. Keith. 1970. "Monte Carlo Sampling Methods Using Markov Chains and Their Applications." *Biometrika* 57 (1): 97–109. <https://academic.oup.com/biomet/article-abstract/57/1/97/284580>.
- He, Xianfei, Babak Moaveni, Joel P. Conte, Ahmed Elgamal, and Sami F. Masri. 2009. "System Identification of Alfred Zampa Memorial Bridge Using Dynamic Field Test Data." *Journal of Structural Engineering* 135 (1): 54–66. [https://doi.org/10.1061/\(asce\)0733-9445\(2009\)135:1\(54\)](https://doi.org/10.1061/(asce)0733-9445(2009)135:1(54)).
- Heitner, Barbara, Eugene J. OBrien, Thierry Yalamas, Franck Schoefs, Cathal Leahy, and Rodrigue Décatoire. 2019. "Updating Probabilities of Bridge Reinforcement Corrosion Using Health Monitoring Data." *Engineering Structures* 190 (March): 41–51. <https://doi.org/10.1016/j.engstruct.2019.03.103>.
- Hızal, Çağlayan. 2023. "FDD Based Modal Identification of Structures Using Least Squares Approach." *Structures* 55 (February): 1071–83. <https://doi.org/10.1016/j.istruc.2023.06.092>.
- Holicky, Milan, Dimitris Diamantidis, and Miroslav Sykora. 2015. "Determination of Target Safety for Structures." *12th International Conference on Applications of Statistics and Probability in Civil Engineering, ICASP 2015*, no. Vrouwenvelder 2002: 1–8.
- Hosser, Daniela, Christoph Klinzmann, and Ralf Schnetgöke. 2008. "A Framework for Reliability-Based System Assessment Based on Structural Health Monitoring." *Structure and Infrastructure Engineering* 4 (4): 271–85. <https://doi.org/10.1080/00268970600740790>.
- Hu, Zhongjun, Quanheng Li, Hongfeng Yan, and Yuchuan Wen. 2021. "Experimental Study on Slender Cfrp-confined Circular Rc Columns under Axial Compression." *Applied Sciences (Switzerland)* 11 (9). <https://doi.org/10.3390/app11093968>.
- IS 456. 2000. "Plain Concrete and Reinforced - Code of Practice." New Delhi.
- ISO2394:1998. 1998. *General Principles on Reliability for Structures*. Geneve,

Switzerland.

- Jaishi, Bijaya, and Wei-Xin Ren. 2005. "Structural Finite Element Model Updating Using Ambient Vibration Test Results." *Journal of Structural Engineering* 131 (4): 617–28. [https://doi.org/10.1061/\(asce\)0733-9445\(2005\)131:4\(617\)](https://doi.org/10.1061/(asce)0733-9445(2005)131:4(617)).
- JCSS. 2001. "Probabilistic Model Code – Part 1: Basis of Design." *Joint Committee on Structural Safety*.
- JCSS. 2002. "Probabilistic Model Code – Part 3: Concrete Properties." *Joint Committee on Structural Safety*.
- Jiang, Huanjun, Xilin Lu, and Tetsuo Kubo. 2010. "Damage Displacement Estimation of Flexure-Dominant RC Columns." *Advances in Structural Engineering* 13 (2): 357–68. <https://doi.org/10.1260/1369-4332.13.2.357>.
- Jiménez-Alonso, Javier Fernando, Javier Naranjo-Perez, Aleksandar Pavic, and Andrés Sáez. 2020. "Maximum Likelihood Finite-Element Model Updating of Civil Engineering Structures Using Nature-Inspired Computational Algorithms." *Structural Engineering International*, 1–13. <https://doi.org/10.1080/10168664.2020.1768812>.
- Jin, Seung Seop, Soojin Cho, Hyung Jo Jung, Jong Jae Lee, and Chung Bang Yun. 2014. "A New Multi-Objective Approach to Finite Element Model Updating." *Journal of Sound and Vibration* 333 (11): 2323–38. <https://doi.org/10.1016/j.jsv.2014.01.015>.
- Jin, Seung Seop, Heekun Ju, and Hyung Jo Jung. 2019. "Adaptive Markov Chain Monte Carlo Algorithms for Bayesian Inference: Recent Advances and Comparative Study." *Structure and Infrastructure Engineering* 15 (11): 1548–65. <https://doi.org/10.1080/15732479.2019.1628077>.
- JSCE. 2010. "Standar Specifications for Concrete Structures-Maintenance." Tokyo, Japan.
- Katafygiotis, Lambros S., Costas Papadimitriou, and Heung-Fai Lam. 1998. "A Probabilistic Approach to Structural Model Updating." *Soil Dynamics and Earthquake Engineering* 17 (7–8): 495–507. [https://doi.org/10.1016/S0267-7261\(98\)00008-6](https://doi.org/10.1016/S0267-7261(98)00008-6).
- Khodaparast, Hamed Haddad, John E. Mottershead, and Michael I. Friswell. 2008. "Perturbation Methods for the Estimation of Parameter Variability in Stochastic Model Updating." *Mechanical Systems and Signal Processing* 22 (8): 1751–73. <https://doi.org/10.1016/j.ymssp.2008.03.001>.

- Kim, Gyeong Ho, and Youn Sik Park. 2004. "An Improved Updating Parameter Selection Method and Finite Element Model Update Using Multiobjective Optimisation Technique." *Mechanical Systems and Signal Processing* 18 (1): 59–78. [https://doi.org/10.1016/S0888-3270\(03\)00042-6](https://doi.org/10.1016/S0888-3270(03)00042-6).
- Lam, Heung-Fai., Huayi Peng, and Siu-Kiu Au. 2014. "Development of a Practical Algorithm for Bayesian Model Updating of a Coupled Slab System Utilizing Field Test Data." *Engineering Structures* 79: 182–94. <https://doi.org/10.1016/j.engstruct.2014.08.014>.
- Lam, Heung-Fai, Jiahua Yang, and Siu-Kui Au. 2015. "Bayesian Model Updating of a Coupled-Slab System Using Field Test Data Utilizing an Enhanced Markov Chain Monte Carlo Simulation Algorithm." *Engineering Structures* 102: 144–55. <https://doi.org/10.1016/j.engstruct.2015.08.005>.
- LANL. 1997. "University of California, Irvine Bridge Column Tests." https://www.lanl.gov/projects/national-security-education-center/engineering/ei-software-download/downloads/uc-irvine-bridge-data/uci_testreport.pdf.
- Leyder, Claude, Eleni Chatzi, and Andrea Frangi. 2017. "Vibration-Based Model Updating of a Timber Frame Structure." *Procedia Engineering* 199: 2132–39. <https://doi.org/10.1016/j.proeng.2017.09.141>.
- Li, Hui, Shunlong Li, Jinping Ou, and Hongwei Li. 2012. "Reliability Assessment of Cable-Stayed Bridges Based on Structural Health Monitoring Techniques." *Structure and Infrastructure Engineering* 8 (9): 829–45. <https://doi.org/10.1080/15732479.2010.496856>.
- Lin, Sheng H. 1991. "Chloride Diffusion in a Porous Cylindrical Concrete Column." *Chemical Engineering Science* 46 (4): 1189–1192. [https://doi.org/10.1016/0009-2509\(91\)85112-B](https://doi.org/10.1016/0009-2509(91)85112-B).
- Lorenzoni, Filippo, Filippo Casarin, Mauro Caldon, Kleidi Islami, and Claudio Modena. 2016. "Uncertainty Quantification in Structural Health Monitoring: Applications on Cultural Heritage Buildings." *Mechanical Systems and Signal Processing* 66–67: 268–81. <https://doi.org/10.1016/j.ymssp.2015.04.032>.
- Maia, Nuno Manuel Mendes, and Julio Martins M. Silva. 2001. "Modal Analysis Identification Techniques." *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* 359 (1778): 29–40. <https://doi.org/10.1098/rsta.2000.0712>.
- Mander, John B., Michael J. N. Priestley, and Robert Park. 1988. "Theoretical Stress-

- Strain Model for Confined Concrete.” *Journal of Structural Engineering* 114 (8): 1804–26. [https://doi.org/10.1061/\(asce\)0733-9445\(1988\)114:8\(1804\)](https://doi.org/10.1061/(asce)0733-9445(1988)114:8(1804)).
- Mao, Jianxiao, Hao Wang, and Jian Li. 2020. “Bayesian Finite Element Model Updating of a Long-Span Suspension Bridge Utilizing Hybrid Monte Carlo Simulation and Kriging Predictor.” *KSCE Journal of Civil Engineering* 24 (2): 569–79. <https://doi.org/10.1007/s12205-020-0983-4>.
- Marelli, Stefano, and Bruno Sudret. 2014. “UQLab: A Framework for Uncertainty Quantification in Matlab.” *Vulnerability, Uncertainty, and Risk: Quantification, Mitigation, and Management - Proceedings of the 2nd International Conference on Vulnerability and Risk Analysis and Management, ICVRAM 2014 and the 6th International Symposium on Uncertainty Modeling A*, no. July: 2554–63. <https://doi.org/10.1061/9780784413609.257>.
- Martakis, Panagiotis, Yves Reuland, Marco Imesch, and Eleni Chatzi. 2022. *Reducing Uncertainty in Seismic Assessment of Multiple Masonry Buildings Based on Monitored Demolitions. Bulletin of Earthquake Engineering*. Vol. 20. Springer Netherlands. <https://doi.org/10.1007/s10518-022-01369-0>.
- Marwala, Tshilidzi, Ilyes Boulkaibet, and Sondipon Adhikari. 2016. *Probabilistic Finite Element Model Updating Using Bayesian Statistics: Applications to Aeronautical and Mechanical Engineering*. John Wiley & Sons.
- MATLAB. 2018. “Version R2018a, Massachusetts, MATLAB Release. Natick, 2018.” Massachusetts, USA: MATLAB Release. Natick.
- Metropolis, Nicholas, Arianna W. Rosenbluth, Marshall N. Rosenbluth, Augusta H. Teller, and Edward Teller. 1953. “Equation of State Calculations by Fast Computing Machines.” *The Journal of Chemical Physics* 21 (6): 1087–92. <https://doi.org/10.1063/1.1699114>.
- Modak, Subodh V., T. K. Kundra, and Bahadur Chand Nakra. 2002. “Comparative Study of Model Updating Methods Using Simulated Experimental Data.” *Computers and Structures* 80 (5–6): 437–47. [https://doi.org/10.1016/S0045-7949\(02\)00017-2](https://doi.org/10.1016/S0045-7949(02)00017-2).
- Molina, Francisco Javier, C. Alonso, and C. Andrade. 1993. “Cover Cracking as a Function of Rebar Corrosion: Part 2-Numerical Model.” *Materials and Structures* 26 (9): 532–48. <https://doi.org/10.1007/BF02472864>.
- Muto, Matthew, and James L. Beck. 2008. “Bayesian Updating and Model Class Selection for Hysteretic Structural Models Using Stochastic Simulation.”

- JVC/Journal of Vibration and Control* 14 (1–2): 7–34.
<https://doi.org/10.1177/1077546307079400>.
- Nadolski, Vitali, Árpád Rózsás, and Miroslav Sýkora. 2019. “Calibrating Partial Factors - Methodology, Input Data and Case Study of Steel Structures.” *Periodica Polytechnica Civil Engineering* 63 (1): 222–42.
<https://doi.org/10.3311/PPci.12822>.
- Newhook, John P., and Rahman Edalatmanesh. 2013. “Integrating Reliability and Structural Health Monitoring in the Fatigue Assessment of Concrete Bridge Decks.” *Structure and Infrastructure Engineering* 9 (7): 619–33.
<https://doi.org/10.1080/15732479.2011.601745>.
- Okasha, Nader M., Dan M. Frangopol, and Alberto Decò. 2010. “Integration of Structural Health Monitoring in Life-Cycle Performance Assessment of Ship Structures under Uncertainty.” *Marine Structures* 23 (3): 303–21.
<https://doi.org/10.1016/j.marstruc.2010.07.004>.
- Okoro, Aghatise, Faisal Khan, and Salim Ahmed. 2021. “An Active Learning Polynomial Chaos Kriging Metamodel for Reliability Assessment of Marine Structures.” *Ocean Engineering* 235 (June).
<https://doi.org/10.1016/j.oceaneng.2021.109399>.
- Papadimitriou, Costas, James L. Beck, and Lambros S. Katafygiotis. 2001. “Updating Robust Reliability Using Structural Test Data.” *Probabilistic Engineering Mechanics* 16 (2): 103–13. [https://doi.org/10.1016/S0266-8920\(00\)00012-6](https://doi.org/10.1016/S0266-8920(00)00012-6).
- Park, Robert, and Thomas Paulay. 1991. *Reinforced Concrete Structures*. John Wiley & Sons, Inc.
- Pastor, Miroslav, Michal Binda, and Tomáš Harčarik. 2012. “Modal Assurance Criterion.” *Procedia Engineering* 48: 543–48.
<https://doi.org/10.1016/j.proeng.2012.09.551>.
- Priestley, Michael J. N., and R. Park. 1987. “Strength and Ductility of Concrete Bridge Columns under Seismic Loading.” *Structural Journal* 84 (1): 61–76. doi: 10.14359/2800
- Ramancha, Mukesh K., Manuel A. Vega, Joel P. Conte, Michael D. Todd, and Zhen Hu. 2022. “Bayesian Model Updating with Finite Element vs Surrogate Models: Application to a Miter Gate Structural System.” *Engineering Structures* 272 (July): 114901. <https://doi.org/10.1016/j.engstruct.2022.114901>.
- Rao, Anirudh S., Michael D. Lepech, Anne S. Kiremidjian, and Xiao Yan Sun. 2017.

- “Simplified Structural Deterioration Model for Reinforced Concrete Bridge Piers under Cyclic Loading.” *Structure and Infrastructure Engineering* 13 (1): 55–66. <https://doi.org/10.1080/15732479.2016.1198402>.
- Rossat, Donatien, Julien Baroth, Matthieu Briffaut, and F. Dufour. 2022. “Bayesian Inversion Using Adaptive Polynomial Chaos Kriging within Subset Simulation.” *Journal of Computational Physics* 455. <https://doi.org/10.1016/j.jcp.2022.110986>.
- Sacks, Jerome, William J Welch, Toby J Mitchell, and Henry P Wynn. 1989. “The Design and Analysis of Computer Experiments.” *Statistical Science* 4 (4): 409–35. <https://doi.org/10.2307/2246134>.
- Sanayei, Masoud, Gregory R Imbaro, Jennifer A. S. McClain, and Linfield C Brown. 1997. “Structural Model Updating Using Experimental Static Measurements.” *Journal of Structural Engineering* 123 (6): 792–98. [https://doi.org/10.1061/\(asce\)0733-9445\(1997\)123:6\(792\)](https://doi.org/10.1061/(asce)0733-9445(1997)123:6(792)).
- Schöbi, Roland, Bruno Sudret, and Joe Wiart. 2015. “Polynomial-Chaos-Based Kriging.” *International Journal for Uncertainty Quantification* 5 (2): 171–93. <https://doi.org/10.1615/Int.J.UncertaintyQuantification.2015012467>.
- Schommer, Sebastian, Viet Ha Nguyen, Stefan Maas, and Arno Zürbes. 2017. “Model Updating for Structural Health Monitoring Using Static and Dynamic Measurements.” *Procedia Engineering* 199: 2146–53. <https://doi.org/10.1016/j.proeng.2017.09.156>.
- Shahbaznia, Mahdi, Morteza Raissi Dehkordi, and Akbar Mirzaee. 2020. “An Improved Time-Domain Damage Detection Method for Railway Bridges Subjected to Unknown Moving Loads.” *Periodica Polytechnica Civil Engineering* 64 (3): 928–38. <https://doi.org/10.3311/PPci.15813>.
- Simwanda, Lenganji, Nico De Koker, and Celeste Viljoen. 2021. “Structural Reliability of Ultra High-Performance Fibre Reinforced Concrete Beams in Flexure.” *Engineering Structures* 244 (June): 112767. <https://doi.org/10.1016/j.engstruct.2021.112767>.
- Singh, Shamsheer Bahadur, Pankaj Munjal, and Nikesh Thammishetti. 2015. “Role of Water/Cement Ratio on Strength Development of Cement Mortar.” *Journal of Building Engineering* 4: 94–100. <https://doi.org/10.1016/j.jobbe.2015.09.003>.
- Sipple, Jesse D., and Masoud Sanayei. 2014. “Finite Element Model Updating of the UCF Grid Benchmark Using Measured Frequency Response Functions.” *Mechanical Systems and Signal Processing* 46 (1): 179–90.

- <https://doi.org/10.1016/j.ymssp.2014.01.008>.
- Sohn, Hoon, and Kincho Law. 2000. "Bayesian Probabilistic Damage Detection of a Reinforced-concrete Bridge Column." *Earthquake Engineering and Structural Dynamics*, no. 29: 1131–52. [https://doi.org/10.1002/1096-9845\(200008\)29:8<1131::AID-EQE959>3.0.CO;2-J](https://doi.org/10.1002/1096-9845(200008)29:8<1131::AID-EQE959>3.0.CO;2-J).
- Srinivas, Voggu, K. Ramanjaneyulu, and C. Antony Jeyasehar. 2011. "Multi-Stage Approach for Structural Damage Identification Using Modal Strain Energy and Evolutionary Optimization Techniques." *Structural Health Monitoring* 10 (2): 219–30. <https://doi.org/10.1177/1475921710373291>.
- Stewart, Mark G., Xiaoming Wang, and Minh N. Nguyen. 2011. "Climate Change Impact and Risks of Concrete Infrastructure Deterioration." *Engineering Structures* 33 (4): 1326–37. <https://doi.org/10.1016/j.engstruct.2011.01.010>.
- Straub, Daniel, and Iason Papaioannou. 2015. "Bayesian Updating with Structural Reliability Methods." *Journal of Engineering Mechanics* 141 (3): 04014134. [https://doi.org/10.1061/\(asce\)em.1943-7889.0000839](https://doi.org/10.1061/(asce)em.1943-7889.0000839).
- Vanik, M.W., James L. Beck, and Siu-Kui Au. 2000. "Bayesian Probabilistic Approach to Structural Health Monitoring." *Journal of Engineering Mechanics* 126 (7): 738–745. [https://doi.org/10.1061/\(ASCE\)0733-9399\(2000\)126:7\(738\)](https://doi.org/10.1061/(ASCE)0733-9399(2000)126:7(738)).
- Wang, Hao, Ai Qun Li, and Jian Li. 2010. "Progressive Finite Element Model Calibration of a Long-Span Suspension Bridge Based on Ambient Vibration and Static Measurements." *Engineering Structures* 32 (9): 2546–56. <https://doi.org/10.1016/j.engstruct.2010.04.028>.
- Wang, Naiyu, and Bruce R. Ellingwood. 2015. "Limit State Design Criteria for FRP Strengthening of RC Bridge Components." *Structural Safety* 56: 1–8. <https://doi.org/10.1016/j.strusafe.2015.03.004>.
- Wang, Zeyu, and Abdollah Shafieezadeh. 2020. "Highly Efficient Bayesian Updating Using Metamodels: An Adaptive Kriging-Based Approach." *Structural Safety* 84 (November 2019): 101915. <https://doi.org/10.1016/j.strusafe.2019.101915>.
- Wu, Yimeng, Rui Zhu, Zhifu Cao, Ying Liu, and Dong Jiang. 2020. "Model Updating Using Frequency Response Functions Based on Sherman-Morrison Formula." *Applied Sciences (Switzerland)* 10 (14). <https://doi.org/10.3390/app10144985>.
- Yang, Xiuming, Huajiang Ouyang, Xinglin Guo, and Shancheng Cao. 2020. "Modal Strain Energy-Based Model Updating Method for Damage Identification on Beam-Like Structures." *Journal of Structural Engineering* 146 (11): 1–13.

- [https://doi.org/10.1061/\(asce\)st.1943-541x.0002812](https://doi.org/10.1061/(asce)st.1943-541x.0002812).
- Yuan, Wenting, Xiangtong Wu, Yuren Wang, Zhenliang Liu, and Peng Zhou. 2023. “Time-Dependent Seismic Reliability of Coastal Bridge Piers Subjected to Nonuniform Corrosion.” *Materials* 16 (3). <https://doi.org/10.3390/ma16031029>.
- Yuen, Ka-veng. 2010. *Bayesian Methods for Structural Dynamics and Civil Engineering*. John Wiley & Sons (Asia) Pte Ltd.
- Yuen, Ka Veng, James L. Beck, and Siu-Kui Au. 2004. “Structural Damage Detection and Assessment by Adaptive Markov Chain Monte Carlo Simulation.” *Structural Control and Health Monitoring* 11 (4): 327–47.
<https://doi.org/10.1002/stc.47>.
- Zhou, Tong, Tong Guo, You Dong, and Yongbo Peng. 2023. “Polynomial Chaos Kriging-Based Structural Reliability Analysis via the Expected Margin Volume Reduction.” *Computers and Structures* 287 (August).
<https://doi.org/10.1016/j.compstruc.2023.107117>.
- Zhou, Tong, and Yongbo Peng. 2022. “Reliability Analysis Using Adaptive Polynomial-Chaos Kriging and Probability Density Evolution Method.” *Reliability Engineering and System Safety* 220 (December 2021).
<https://doi.org/10.1016/j.ress.2021.108283>.
- Zrayka, Abdel Kader, and Emiliano Mucchi. 2019. “A Comparison among Modal Parameter Extraction Methods.” *SN Applied Sciences* 1 (7): 1–11.
<https://doi.org/10.1007/s42452-019-0806-8>.

VITA

Name – Surname: Ertuğrul Türker UZUN

Educational Information:

<u>Graduation Degree</u>	<u>Institution</u>	<u>Graduation Year</u>
PhD.	İzmir Institute of Technology	2024
MSc.	İzmir Kâtip Çelebi University	2017
BSc.	Anadolu University	2014

Published Articles:

Uzun, E. T., & Aktaş, E. (2021). Reliability of corroded steel members subjected to elastic lateral torsional buckling. *International Journal of Steel Structures*, 21(4), 1478-1501.

Uzun, E. T., & Seçer, M. (2019). Lateral torsional buckling of doubly symmetric I-shaped steel members under linear moment gradient. *Pamukkale Üniversitesi Mühendislik Bilimleri Dergisi*, 25(6), 635-642.

Secer, M., & Uzun, E. T. (2019). Inelastic ultimate load analysis of steel frames considering lateral torsional buckling under distributed loads. *Periodica Polytechnica Civil Engineering*, 63(3), 872-881.

Submitted Articles:

Uzun, E. T., Hızal, Ç., & Aktaş, E. (2024). Reliability Assessment of Structures with Bayesian Model Updating Accelerated via Polynomial-Chaos-Kriging metamodeling. *Structure and Infrastructure Engineering*,