

ROUGHNESS INDUCED TRANSITION

by

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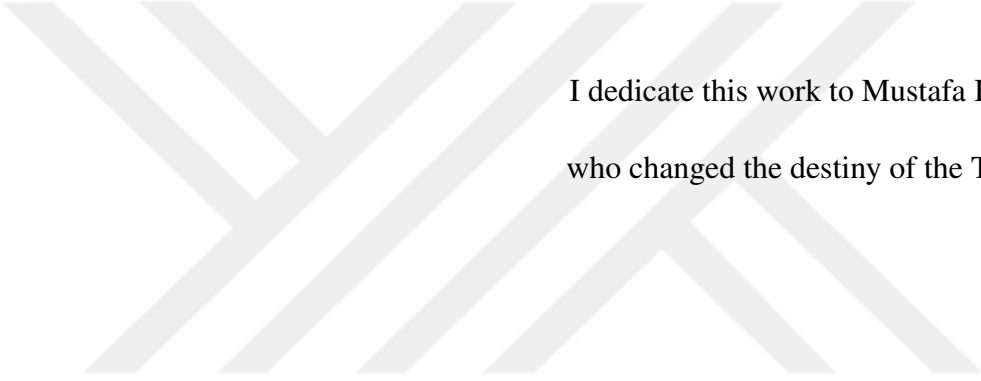
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I dedicate this work to Mustafa Kemal Atatürk,
who changed the destiny of the Turkish Nation.

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List of Abbreviations

A	Area
A, B, n	Hotwire calibration constants
D	Diameter
e_{f_0}	Unsteady disturbance energy in a frequency band centered at f_0
E	Electrical potential
$E^{U,V,W}$	Steady disturbance energy for U , V and W
f	Frequency
h	Convective heat transfer coefficient
H	Shape factor, δ^*/θ
H_n	Discrete Fourier coefficients, $\sum_{j=0}^{N-1} u'_k e^{2\pi i j n/N}$
i	$\sqrt{-1}$
i, j, k, n	Dummy index variables
k	Roughness height, thermal conductivity, hotwire yaw coefficient
N	Number of samples
Nu	Nusselt number, hD/k
PSD	Temporal Power Spectral Density, $\text{PSD}(f_n) = 2 H_n ^2/(f_{\text{samp}}W_{ss})$

$PSD^{U,V,W}$	Spatial Power Spectral Density, $PSD(\lambda_n^{-1}) = 2 H_n ^2\lambda_k/N^2$
Q	Heat flux
R	Electrical resistance
Re_x	Reynolds number, $U_\infty x/\nu$
Re'	Unit Reynolds number, U_∞/ν
Re_k	Roughness height Reynolds number, $\overline{U}(k)k/\nu$
St	Strouhal number of unsteady vortex shedding
t	Time
T	Temperature
Tu	Turbulence intensity, u'_{rms}/U_∞
U,V,W	Velocity components in x,y,z
$\overline{U},\overline{V},\overline{W}$	Spanwise-invariant basic state velocity components in x,y,z
U',V',W'	Steady velocity perturbation in x,y,z
u',v',w'	Fluctuating velocity perturbation in x,y,z
V_e	Effective cooling velocity
$\tilde{V}$	Freestream velocity during calibration
W_{ss}	Hann window squared and summed, $\sum_{j=0}^N \{0.5[1 - \cos(2\pi j/N)]\}^2$
x,y,z	Cartesian coordinate directions
x_k	Streamwise location of roughness array

Greek Symbols

α	Hotwire yaw angle; streamwise growth rate, $\delta(x_k)\partial(\ln e_{f_0})/\partial x$
β	Dimensionless spanwise wavenumber, $\beta^*\delta$; flow angle, $\tan^{-1}(W/U)$
β^*	Spanwise wavenumber, $2\pi/\lambda_k$
δ	Boundary layer dimension, $(\nu x/U_\infty)^{1/2}$
δ^*	Displacement thickness, $\int_0^\infty (1 - U/U_\infty)dy$
$\delta_{.99}$	Boundary layer thickness, 5.0δ
η	Dimensionless boundary layer coordinate, y/δ
θ	Momentum thickness, $\int_0^\infty U/U_\infty(1 - U/U_\infty)dy$
λ	Spanwise wavelength
λ_k	Roughness spacing
μ	Dynamic viscosity
ν	Kinematic viscosity, μ/ρ
ρ	Density
σ	Standard deviation, $([\sum(x_i - \bar{x})^2]/(N - 1))^{1/2}$

Subscripts

c	Centerline
b	Bottom
crit	Critical
k	Roughness
n	Normal (perpendicular)

rms	Root-mean-square
s	Shedding
samp	Sampling
t	Tangential, top
vle	Virtual leading edge
y	Derivative with respect to wall-normal coordinate
w	Wire
λ	Wavelength components; λ_k , $\lambda_k/2$, $\lambda_k/3$ and $\lambda_k/4$
∞	Freestream

Roughness Induced Transition

Abstract

by

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Surface roughness has numerous different effects on boundary layer stability. For roughness amplitudes lower than a critical level, the velocity disturbances roughness creates in laminar boundary layers experience a transient algebraic growth followed by exponential decay without leading to transition. When the critical level is exceeded unsteady disturbances are amplified and the flow rapidly transitions to turbulence. Whether the flow remains laminar or becomes turbulent is solely determined by the receptivity of the initial disturbance. Two experiments are performed that address two fundamental problems related to roughness induced disturbances. One examines the disturbances produced by subcritical elements. The other identifies the instability mechanism responsible for breakdown in the wake of supercritical roughness elements. For subcritical conditions, multiple component velocity measurements are obtained in a flat plate boundary layer in the wake of an array of subcritical cylindrical elements. These data reveal the topology of the steady horseshoe vortex that exists in the wake of roughness elements and leads to transient growth of streamwise and spanwise velocity disturbances. A theoretical technique that quantifies the receptivity of velocity disturbances to surface roughness requires the complete velocity

field as input and the data obtained here are suitable for this purpose. For supercritical conditions, velocity measurements are obtained above and below the critical roughness height Reynolds number, Re_k . The steady disturbance field reveals local shear layers in the wall-normal and spanwise directions and the unsteady disturbance field reveals evidence of hairpin vortices observed in flow visualization studies. The locations of maximum fluctuation intensity correspond to the locations of local inflection points of the steady velocity field suggesting that the fluctuations result from a Kelvin–Helmholtz-type instability. Rapid transition takes place at $Re_k = 334$ but not below this value. The disturbances’ energy growth rates indicate that the transition scenario can be understood as a competition between the unsteady disturbances’ growth and the rapid relaxation of the steady flow that tends to stabilize these disturbances.

Chapter 1

INTRODUCTION

1.1 Motivation for Research

Laminar-to-turbulent transition in two dimensional (2D) Blasius boundary layers has been studied extensively for nearly a century. The view that has emerged is that transition takes place because disturbances in the environment such as freestream turbulence, acoustic fluctuations and surface irregularities are received into and amplified by the laminar boundary layer. The level of amplification depends on the composition of the disturbances from these sources.

When the freestream turbulence and surface roughness levels are low, the input disturbances are mainly 2D in character (e.g., freestream acoustic waves and surface vibrations) and transition is brought about by unsteady, spanwise invariant disturbances known as Tollmien–Schlichting (TS) waves. This transition scenario is well understood. TS waves at very low amplitudes are produced by 2D disturbances, these grow exponentially to finite amplitudes at which point nonlinear interactions lead to breakdown and, finally, to turbulent flow. In physically realistic situations no surface is perfectly smooth and roughness can also play an important role in this process. However, despite its impact on boundary layer, roughness effects are still not well understood, because most research effort has focused

on the growth of TS waves and the link between TS waves and roughness is not clear. For a long time, TS waves were thought to be more important than the steady and three dimensional (3D) roughness induced disturbances, because Squire's theorem states that the 2D waves become unstable earlier and grow faster than the 3D disturbances. As a result, even today, a solid, mechanism-based transition prediction method does not exist for surfaces with significant roughness levels. There are some transition prediction methods for rough surfaces using empirical relations, but these do not supply insight on the underlying physics of the problem (Klebanoff et al. 1955; von Doenhoff and Braslow 1961; Tani 1969; Klebanoff et al. 1992).

It is imperative to understand, predict and control transition because of its impact on performance in engineering applications. Transition control can mean suppression or tripping of early transition. Whether the objective is tripping or suppression, intelligently designed roughness elements can have a variety of effects on the boundary layer behavior.

In some cases making an otherwise laminar flow turbulent can enhance mixing. Unlike laminar flow, turbulent flow has a disorganized, chaotic structure that is characterized by fluctuating velocity components that provide large amounts of mixing. So, in engineering applications where chemical reactions and heat transfer are involved, mixing enhancement increases the combustion efficiency and heat transfer coefficient. Similarly, in internal aerodynamics, the recent trend in low-pressure turbine design tends toward higher blade loadings and strong decelerations on the blades' suction side. Such blades are prone to laminar boundary layer separation at cruise conditions. But, inducing early boundary-layer

transition using surface roughness can result in 20% to 40% reductions in total pressure loss coefficients (Lake et al. 1999). Another possible application of roughness is related to wind engineering. One major challenge in the wind power industry is to harness the wind energy at low wind speeds. According to Corten and Veldkamp (2001), controlled surface roughness can be used to improve the efficiency of the wind turbine blades with an improved operation speed range. On the other hand, uncontrolled roughness (dead insects) on the leading edge of wind turbine blades can cause production losses up to 25%.

In other cases, early transition is not desired because the total skin friction on a streamlined object is the sum of laminar and turbulent regions' contributions of local shear stress. In the turbulent region, the shear stress at the wall is significantly higher than the laminar region and the transition location is of vital importance in determining the total drag on this object. The steady, decaying disturbances created by high-amplitude roughness arrays have been predicted to reduce TS growth rates and may therefore delay transition (Cossu and Brandt 2002; Cossu and Brandt 2004). This finding has far reaching implications in terms of aircraft performance and fuel economy, and underlines the importance of understanding roughness-induced disturbances. On the other hand, high-amplitude roughness may itself cause rapid transition, so it is important to know what kind or how big of a roughness element can be used for control that will not itself bring about transition.

Stability theories predict the growth and decay rates of disturbances that exist in boundary layers. However, they do not address how the initial disturbances are formed by a disturbance source. This process is called receptivity (Morkovin 1969). Once an

initial disturbance is formed, the subsequent evolution (growth, decay, or both) is solely determined by that particular initial disturbance. This means the most important piece of the picture is missing; the receptivity of roughness-induced disturbances. In general, receptivity can be explained as the process by which an external disturbance source (such as a surface irregularity on a smooth surface or turbulence in the freestream flow) manifests itself as deviations from the basic state velocity and pressure field in the boundary layer. In particular, different roughness conditions create different disturbance fields and very little is known on how the geometrical irregularities on the surface become velocity and pressure disturbances. The possibility of solving the receptivity problem implies great improvements in predicting and, therefore, controlling transition. In fact, all the engineering applications discussed above are directly affected or can be improved by a better understanding of roughness receptivity.

The theoretical foundation for the roughness receptivity problem has been addressed by using a biorthogonal multimode decomposition method (Tumin 2003). A biorthogonal formulation allows the decomposition of an initial disturbance to its constituent modes including both the discrete and continuous spectra. This way, the initial disturbances can be quantified and linked to the source that generates them. In general this method requires full experimental knowledge of the flow field. However, under certain regularization assumptions, the application of this method requires the experimental data for the complete velocity disturbance information in the near wake.

An improved understanding of roughness effects (receptivity and growth) on boundary

layer behavior will result in better transition prediction and control methods in physically realistic situations. With this in mind, the primary motivation of this work is to gain a better understanding on two fundamentally important problems in fluid mechanics; (i) the receptivity of roughness-induced disturbances in Blasius boundary layers and (ii) the mechanism that is responsible for early transition in the wake of large amplitude ($k \sim \delta$ or larger) 3D roughness elements.

1.2 Transient Growth: Theory, Experiments and Simulations

The traditional approach to modelling transition in 2D subsonic boundary layers has focused on TS wave growth. This approach includes the use of the well-known Orr–Sommerfeld (OS) and Squire equations that represent an eigenvalue problem for the spatial or temporal growth of low-amplitude 2D disturbances. These equations lead to a famous theorem by Squire, stating that 2D, streamwise-travelling disturbances are destabilized at lower Reynolds numbers than oblique (i.e., 3D) waves. This led to the conclusion that the 2D waves are more important to transition than their 3D counterparts. As a result, much research effort was focused on 2D TS waves. This delayed the emergence of transient growth theory, the theory that is now thought to correctly address the behavior of 3D, low frequency and steady disturbances.

Transient growth was first identified by Ellingsen and Palm (1975) who showed that under certain initial disturbance conditions, a streamwise disturbance velocity can grow algebraically in time. Until that point, exponential growth was exclusively thought to be

the main reason for transition. Hultgren and Gustavsson (1981) investigated the temporal evolution of 3D disturbances in a boundary layer and found that the initial algebraic growth identified by Ellingsen and Palm is followed by a viscous decay. The physical picture of spatial transient growth is now described using Landahl's (1980) "lift-up" mechanism, in which a pair of stable, counter-rotating streamwise-oriented vortices transfer momentum across the shear layer, creating a velocity perturbation of the streamwise flow that can be much larger than the initial vortex pair. After the decay of this weak vortex pair, the streamwise velocity perturbation decays exponentially due to viscous forces. The creation of high- and low-speed longitudinal streaks that are observed in the last stages in the transition is thought to be due to this redistribution of the streamwise mean velocity.

The mathematical foundation of transient growth has been described by Schmid and Henningson (2001) and Reshotko (2001). Transient growth arises because the linearized disturbance equations are not self-adjoint and, therefore, have nonorthogonal eigenmodes. In boundary layers, the correct representation of disturbances is the superposition of the discrete and continuous eigenmodes of the OS equation. In classical linear stability analysis, the most amplified discrete eigenmode represents the dominant disturbance observed in an experiment. However, in the subcritical region where all eigenmodes are damped, superposition of continuous spectrum modes (each of which has a different decay rate) can exhibit a brief period of algebraic growth before the exponential decay manifests itself.

For a given frequency and streamwise wavenumber, a particular combination of decaying modes represent the initial disturbance that experiences the maximum amount of

growth. Such a disturbance is called the optimal disturbance (Farrell 1988) and many investigators have contributed to the modelling of spatially growing optimal disturbances (Andersson et al. 1999; Luchini 2000; Tumin and Reshotko 2001). The common finding in these works is that for a Blasius boundary layer, the optimal initial disturbance is a spanwise periodic array of stationary streamwise vortices with a dimensionless spanwise wavenumber of 0.45. These vortices decay but lead to a large growth of streamwise velocity perturbations.

There have been attempts to experimentally verify transient growth. These experiments can be collected under two categories based on the 3D disturbance source: freestream turbulence and surface roughness. The algebraic growth of disturbances generated by freestream turbulence measured by Westin et al. (1994) and Matsubara and Alfredsson (2001), are in agreement with the optimal growth studies of Andersson et al. (1999) and Luchini (2000). This finding is surprising because the streaky structures observed downstream of the initial freestream disturbance input — which was far from optimal — were in striking resemblance with those generated by optimal perturbations. The implication is that almost any initial upstream 3D disturbance will develop into a streamwise streak downstream. Freestream turbulence is important because it exists in many engineering applications. From an experimental standpoint, however, controlled initial disturbances cannot be produced and, therefore, detailed comparisons with theoretical predictions such as growth rates cannot be performed.

In terms of surface roughness, it is important to mention an experiment by Kendall

(1981), that revealed many signs of transient growth prior to the full development of transient growth theory. Kendall considered a variety of carefully designed roughness configurations including an isolated 3D cylindrical roughness element that could be raised above a flat surface by using a solenoid. Kendall obtained streamwise velocity measurements in wall-normal/spanwise planes for cylinder-present and cylinder-absent conditions. Along the lateral centerline, Kendall observed a strong deceleration in the near wake and acceleration in the far wake. The far-wake accelerated region along the centerline was accompanied by decelerated regions on each side of the lateral centerline. He attributed this effect to a possible wall-normal momentum transport mechanism induced by the trailing vortex. Kendall stated that “roughness induced a downward flow of faster-moving fluid at the centerline and an upward flow of slower fluid at off-axis locations.” He further concluded that the trailing legs of the horseshoe vortex transports fluid toward the plate along the centerline and away from the surface at the two spanwise locations of decelerated flow. This observation of Kendall is perhaps the first experimental validation of the Landahl’s lift-up mechanism.

Nearly two decades after Kendall, White and coworkers (White and Reshotko 2002; White 2002; White and Ergin 2003; White et al. 2005; Ergin and White 2005a) at Case Western Reserve University (Case) have conducted a number of experiments on the behavior of roughness-induced transient disturbances. In these works, steady hotwire velocity measurements are obtained in the boundary layer of a flat plate and disturbances are generated by a spanwise array of cylindrical roughness elements. The main data analysis tech-

nique in these works is to decompose the kinetic energy associated with the steady disturbance into spanwise wavelength components by performing a spanwise Fourier transform, computing the energy of each and examining the downstream evolution of these energies.

The experiment conducted by White (2002) was intended to verify the existence of optimal disturbances but instead found important discrepancies between roughness-induced disturbances and optimal disturbances. Specifically, the streamwise location of maximum kinetic energy growth was found to be upstream of what is predicted by the optimal theory. Therefore, the key finding in White (2002) is that roughness-induced disturbances show suboptimal behavior. Fransson et al. (2004) confirm this observation and attribute the mismatch to differences between the initial disturbance profiles used in optimal disturbance studies and those induced by surface roughness. Optimal initial disturbances are streamwise vortices that are not confined to the boundary layer. Unlike the case of freestream turbulence, however, disturbances generated by low amplitude surface roughness are thought to be mostly confined within the boundary layer.

White and Ergin (2003) investigated how the steady disturbance energy in spatial wavelengths scales with roughness amplitude. In this study, it was found that the disturbance energy approximately scales with Re_k^2 (i.e., k^4), and this scaling is further verified by White et al. (2005) for Re_k ranging from 16 to 195. Rice (2004) extends the range to 254. Fig. 1 shows the transient growth of disturbance kinetic energy contained in the wavelength that is equal to one third the roughness spacing ($E_{\lambda_k/3}$) for selected values of Re_k . In this figure, the solid lines denote the least squares fits to the data using the model func-

tion $a(x - x_k)\exp[-(x - x_k)/b]$ proposed by Ergin (2003). The energy contained in $\lambda_k/4$ also experience transient growth and scale with Re_k^2 , therefore the growth of $E_{\lambda_k/4}$ is very similar to that of $E_{\lambda_k/3}$ in Fig. 1. However, White et al. (2005) find that a model function with a quadratic energy growth term, i.e., $a(x - x_k)^2\exp[-(x - x_k)/b]$ better represents the growth of $E_{\lambda_k/4}$, rather than a linear energy growth term. This finding is important, because simplified transient growth models, such as the one proposed by Boberg and Brosa (1988) expect quadratic energy growth as observed for $E_{\lambda_k/4}$. The reason for different growth behavior between $E_{\lambda_k/3}$ and $E_{\lambda_k/4}$ remains to be addressed and, once again, underlines the importance of the receptivity problem.

White et al. (2005) also considered the effect of roughness diameter on the disturbance energy and found that even a small change in roughness diameter has a dramatic effect on the disturbance spectrum, both quantitatively and qualitatively. The effect of roughness diameter on the downstream evolution of $E_{\lambda_k/3}$ is shown in Fig. 2, where the solid line is an interpolation of 6.35-mm diameter for $Re_k = 177$. In this figure, the 6.35- and 7.62-mm-diameter roughness produce transient growth, 4.76- and 5.08-mm-diameter roughness produce an initial decay followed by transient growth, and 3.81-mm-diameter roughness produce pure decay. These findings indicate that both height and shape are equally important in the receptivity of roughness-induced transient disturbances.

With the increase of the computing power, there have been attempts to numerically simulate transient growth. Fischer and Choudhari (2004) have carried out direct numerical simulations for various roughness configurations that enable detailed comparisons with the

measurements of White and Ergin (2003). Fischer and Choudhari also examined the downstream evolution of the relevant spanwise harmonics of the disturbance field and found good agreement between the experimentally measured and numerically computed modal energy magnitudes and their streamwise evolution. In order to give an idea about this agreement, Fig. 3 compares the downstream evolution of steady disturbance energy, E_{rms} obtained by experiments and DNS for $Re_k = 119$. Here, the spanwise-rms disturbance energy at each x location is computed by integrating the spanwise variance of the steady velocity disturbance from the wall to the freestream.

Receptivity is a critical first stage of the boundary layer transition process. Disturbances in a laminar boundary layer may follow many paths to turbulence and this path is entirely defined by the receptivity of the initial disturbance. With this idea in mind, Morkovin (1969) developed a general receptivity and transition road map which illustrates the various paths to transition. The latest version of this road map (Morkovin et al. 1994) which includes the transient growth mechanism is illustrated in Fig 4. Path A is the traditional transition scenario and is well-understood. It represents the exponential growth of infinitesimal TS waves (primary mode growth) which leads to secondary instabilities (secondary mode growth) and finally breakdown to turbulence. Paths B, C and D represent the cases where transient growth mechanism is involved in the evolution of the disturbance, and are not fully understood. Path B represents the growth of small amplitude transient disturbances, where primary and secondary mode growth is observed before breakdown. With increasing disturbance level, algebraic disturbance growth might bypass the primary

growth mechanism (Path C) or both the primary and secondary growth mechanisms (Path D). It is not known whether or not transient growth can lead to breakdown through its own secondary instability. Therefore, the situation requires the introduction of the so-called “bypass mechanisms”, which is a general name given for instability mechanisms that have not been explained (Reshotko 2001). As a result, in path D transient growth mechanisms lead to certain bypass mechanisms for breakdown. With still increasing disturbance level, the disturbances follow path E, where primary mode, secondary mode and transient growth mechanisms are all bypassed for breakdown to turbulence. The importance of receptivity to transient disturbances is twofold: First, paths B, C and D all involve the transient growth mechanism and represent most of the realistic flow conditions, and second, the same mechanism (TG) can produce different effects by selecting different paths depending on the engineering application.

To address the receptivity problem for spatially growing perturbations, Tumin (2003) proposed a multimode decomposition technique that includes the contributions of both discrete and continuous spectrum of OS and Squire modes. Quantification of the initial disturbance profile for comparison with this theory requires the measurement of the complete disturbance field, meaning all three velocity components and the pressure perturbation. From an experimental standpoint, it is very difficult to measure the wall-normal velocity component and almost impossible to measure the pressure perturbation in a boundary layer. However, Tumin suggests that the biorthogonal decomposition of the flow field based on partial disturbance data might be possible under additional assumptions about the pressure

perturbation. To address this, recent efforts at Case have been focused on obtaining the spanwise velocity field and Ergin and White (2005a) took the first step in obtaining the complete velocity field by measuring the spanwise and streamwise velocity components.

Another theoretical analysis of perturbations generated by a roughness element was presented in Tumin and Reshotko (2004) who considered the linearized Navier–Stokes equations and linearized boundary conditions. The method is based on an expansion of the solution into the normal modes of the continuous spectrum and amplitudes were calculated using the biorthogonal eigenfunction system (Tumin 2003). In order to assess the differences between the linear versus nonlinear receptivity mechanisms and optimal versus realistic disturbances, Ergin et al. (2005) compare the numerical results for a $Re_k = 119$ roughness array using linear theory, DNS and optimal perturbations. The finding is that the velocity perturbations computed using linear receptivity theory are localized closer to the wall than the DNS perturbations and that the DNS perturbations are closer to the wall in comparison with the conventional optimal perturbation. The exceptional agreement between DNS and experiments suggest that DNS is the closest approximation to the measured realistic disturbances and that the receptivity mechanism is nonlinear. This finding is in agreement with the results of White (2002) and Fransson et al. (2004), and confirms the suboptimality of roughness-induced disturbances.

1.3 Objectives and Organization

There are two main objectives of this work. The first is to identify the vortical structure that initiates transient growth by obtaining all three velocity components in the near wake region of the roughness element. According to Landahl's (1980) "lift-up" mechanism, transient growth results from the transfer of streamwise momentum by means of a streamwise oriented vortex. Optimal growth results from a streamwise oriented vortex, and the suboptimality of realistic disturbances is attributed to the fact that realizable vortices are more confined in the boundary layer than their optimal counterparts. However, the initial vortex structure has not been directly measured to verify this. Besides simply verifying the initial vortices' spatial extent, the method introduced by Tumin (2003) may permit a rigorous receptivity analysis. Under certain regularization assumptions, the complete multicomponent velocity data of the initial vortex system can be sufficient to perform this analysis. Actually performing this analysis is a daunting task and beyond the scope of the present work.

The work's second objective is to identify the roughness height limit for transition and the underlying mechanism that leads to breakdown when this height is exceeded. The motivation for this is the recent finding that large-amplitude optimal disturbances can suppress TS-wave growth and possibly delay transition. However, large amplitude roughness may itself lead to transition. Therefore, it is important to know the height limit that can generate large-amplitude subcritical disturbances. More importantly, the mechanism responsible for roughness-induced transition still remains to be identified.

This thesis is organized as follows. This introductory chapter is intended to summarize

the motivation for studying roughness-induced transition, to present the existing theoretical knowledge and experimental results on transient growth, and the specific objectives of this work. In Chapter 2, general information on the experimental facility, setup, and instrumentation are presented. In Chapter 3 the experiments that measure the multiple velocity components of the vortex system and their results are described. Chapter 4 describes the experiments that measure the unsteady velocity fields resulting from large-amplitude roughness elements. An overall summary and conclusions drawn from the results of both experiments are presented in Chapter 5 along with suggestions for future work.

Chapter 2

FACILITY AND METHODS

2.1 Case Wind Tunnel Facility

The experiments described here were performed in the Case Western Reserve University (Case) subsonic wind tunnel. The tunnel is designed to be completely modular and mobile and this allows a variety of different experimental configurations to be realized. Currently, the facility has three test sections dedicated for three different experimental setups: a straight section for flat plate boundary-layer flow measurements, a straight section for turbulence and turbulent mixing measurements and an angled cascade section for boundary layer flow measurements around turbine blades. Recently, an active grid turbulence generator was also added to provide the capability of producing controlled freestream turbulence intensity levels. All these sections can be easily interchanged, and this increases the time efficiency of the facility.

The tunnel is an open-return facility with a maximum speed of 25 m/s and is designed to provide good flow quality. The incoming flow treatment consists of a 150-mm radius, well-rounded inlet fairing followed by a 100-mm-long aluminum honeycomb flow straightener with a 6.25-mm mesh size. Downstream of the honeycomb, up to four 65% open-area-ratio, stainless steel screens with 0.85-mm mesh size (30 wires per inch) are

used to break down the large-scale turbulent eddies of the incoming flow, improve the mean-flow uniformity and to minimize dust in the following sections. The screens are kept under high tension to minimize vibrations and are separated by 300-mm-long standoffs. Rubber gaskets are used to seal each standoff/screen interface. The number of screens and standoffs can be changed depending on the experiment; four screens are generally used when good flow quality is required whereas two screens are sufficient to capture dust for turbulence experiments where the incoming flow quality is not as important. Downstream of the inlet flow treatment section a 6.25:1 area-ratio square contraction cone accelerates the flow to test-section velocity. The contraction's surface is a fifth-degree polynomial as recommended by Reshotko et al. (1997).

Without the active grid turbulence generator in place, the tunnel provides a very low freestream turbulence level, making it suitable for highly sensitive boundary-layer-stability experiments. White (2002) reports a total turbulence intensity level of $Tu = 0.35\%$ at a freestream speed of 12 m/s. This turbulence intensity level includes the contributions of low-frequency fluctuations caused by the breathing effect, a common problem encountered in open-return wind tunnels. When a high-pass filter of 1 Hz is applied 93% of the fluctuation power is eliminated reducing the Tu level to about 0.09%. Alternatively, separating contributions of acoustic and turbulent fluctuations in a manner suggested by (Reshotko et al. 1997), White shows that turbulence intensity of the flow is $Tu = 0.05\%$.

The experiments for the current work were conducted in the flat-plate test section which is a 710 mm $\times$ 710 mm $\times$ 2700 mm rectangular duct. This test section includes

an externally mounted, stepper-motor-driven traverse mechanism that can move hotwire velocity sensors in three perpendicular directions. The traverse provides 1700 mm of x (streamwise) travel with $6.4\ \mu\text{m}$ resolution, 100 mm of y (plate-normal) travel with $3.2\ \mu\text{m}$ resolution, and 172 mm of z (spanwise) travel with $3.2\ \mu\text{m}$ resolution. Hotwire probes are supported on a sting that is shaped as a thin symmetric airfoil with a sharp trailing edge. This design minimizes probe vibrations induced by vortex shedding. The sting accesses the interior of the test section through a window that slides in the vertical (spanwise) direction and has a slot that runs along the length of the streamwise travel. Flow through the slot is reduced by two dense nylon brushes. During calibration and experiments, these brushes are also sealed with duct tape to ensure isolation of the tunnel from possible inflow.

Two different sting designs are used for two experiments of this work. The first sting is used during unsteady velocity measurements and it holds two hotwires; one for boundary layer measurements and one for freestream velocity measurements. The sensors are separated by a distance of 150 mm so that the freestream hotwire is always far away from the boundary layer edge. The boundary layer hotwire probe holder is placed at an angle of 6° towards the flat plate with respect to the flow direction, so that the hotwire can be positioned arbitrarily close to the plate surface without having the sting contact the plate.

The second sting design is used during the multicomponent velocity measurements. This sting includes a shaft that rotates in a streamlined casing and allows angular calibration of slanted hotwires. Outside the test section, the shaft is connected to an angle indicator and the sting assembly is rigidly attached to the traverse. Inside the test section, the hotwire

sensors are placed perpendicular to the shaft's axis through a multiple-hotwire holder. The angular position of the sensors can be adjusted by rotating the spindle of the angle controller with a precision of 0.2° . The multiple-hotwire holder is designed to allow probe separation, yaw, pitch and roll adjustments, and can carry up to four hotwires. One hotwire is a straight sensor in the freestream; the remaining three sensors are in the boundary layer. The boundary-layer sensors are positioned in a trident design with one straight sensor in the middle and two slanted hotwire sensors mounted on either side of the the center probe. This sting assembly with the angle controller is seen in Figure 5.

The aluminum flat plate model and the roughness array is illustrated in Figure 6. An array of cylindrical roughness elements are placed 300 mm downstream of the leading edge. The plate is mounted vertically and is located 0.5 m downstream of the test section inlet with its polished test side located approximately 250 mm from the front wall. It is 9.5 mm thick, 635 mm in span, 1100 mm in length and has a 25-mm-long elliptical leading edge. A 0.5-mm-deep groove located between 100 mm and 200 mm downstream of the leading edge extends across the plate in the spanwise direction. The grove is used so that adhesive-backed sand paper can be installed with its surface flush with the plate's top surface in experiments related to distributed surface roughness. In this work the groove is filled with a smooth metal shim whose top edge is flush with the plate's surface. In a previous experiment conducted in the same facility (White and Reshotko 2002) 180-grit sand paper was reported to produce negligible stationary transient disturbances. Therefore, the effect of the metal shim is assumed to be negligible. A 300-mm-long trailing-edge flap

is used to control the leading-edge stagnation point's location and for the alignment of the flat plate.

The flat plate is supported on four adjustable brackets, that are attached to the top and bottom walls of the test section. The vertical alignment of the flat plate with respect to the traverse mechanism and the longitudinal alignment to establish a zero-pressure-gradient boundary layer on the test side is achieved by adjusting the position of the brackets. The plate alignment procedures are explained in detail later in Section 2.6. The flow between the test side and the back side of the plate is prevented by placing metal inserts between the flat plate and the test section's top and bottom walls.

The motor and diffuser sections are located at the downstream end of the tunnel. The 1-m-diameter, 16-rotor, 12-stator fan can draw air up to $20 \text{ m}^3/\text{s}$. The fan is driven by an externally mounted 20-hp DC motor using V-belts. Low-resonant-frequency, pneumatic isolation mounts are used for the motor section and test sections to reduce mechanical vibrations. In order to further avoid vibration propagation between the sections of the wind tunnel, no two sections of the tunnel are rigidly connected.

As mentioned above, the most significant flow quality limitation of the Case wind tunnel is low-amplitude, low-frequency unsteadiness, an effect that is encountered in all open-return wind tunnels. Maintaining a given test speed is very important for sensitive boundary layer stability experiments and the low-frequency unsteadiness is small enough that the test section velocity can be maintained within 0.1% of its set point. While this is an acceptable level of fluctuation in the mean flow, the effect in the boundary layer is not

desired. The adverse impact of these fluctuations is reduced by normalizing the boundary layer velocities by the freestream velocity on an instantaneous basis.

2.2 Roughness Arrays

The experiments of the present work are designed to gain a better understanding of spatially growing transient disturbances induced by surface roughness. Transient growth operates mainly on 3D disturbances; therefore, the spanwise behavior of disturbances is also important as well as streamwise evolution. The idea is to produce a boundary layer that contains spanwise periodic 3D disturbances. This is achieved using a spanwise array of cylindrical roughness elements.

Among many potential roughness configurations, spanwise arrays of identical roughness elements have certain advantages. First, it is a simple, repeatable configuration that can be easily reproduced in computational simulations with well-defined boundary conditions. Second, roughness arrays establish a clear definition of disturbance input. The multicomponent experiments are aimed to identify the vortical structures that generate transient growth and provide initial disturbance conditions for theoretical methods that investigate the receptivity of transient disturbances. Receptivity links the initial disturbance created by a certain roughness element with specified shape, height and location. Therefore, it is imperative to establish a clear definition of disturbance source to investigate receptivity. Mathematically speaking, type, shape and height of roughness impose a spatial initial condition for the governing equations. For spanwise roughness arrays, the location of disturbance in-

put, spanwise periodicity and roughness shape is well-specified. Third, roughness arrays allow excitation of controlled steady and unsteady disturbances, which are necessary ingredients for sensitive boundary layer stability experiments. Roughness arrays of cylindrical elements provide considerable flexibility in varying the controlled disturbance inputs, via variations in the streamwise location (x_k), height (k), spacing (λ_k), and diameter (D) of the roughness elements. These geometrical parameters can easily be measured, reproduced and altered, which allows controlled experiments. The spacing and the diameter of the roughness arrays determine the characteristic length scale in the spanwise direction and these spanwise scales are known to affect the growth rate of transient disturbances. Theoretical studies that consider optimal transient disturbances quantify the optimal disturbance by its spanwise wave number. By changing the spacing and diameter of the roughness array, controlled disturbances with particular spanwise wavenumbers can be generated and these can be compared to theoretical predictions. Finally, for data analysis purposes, the spacing can be selected as multiples of the roughness diameter to obtain cleaner spatial Fourier spectra in the spanwise direction. Additionally, various averaging techniques, such as spatial phase-locked averaging, can be utilized to improve the signal-to-noise characteristics during data analysis. This eliminates the random but steady error in the velocity measurement caused by the uncontrollable disturbance inputs. Therefore, surface roughness is used extensively in the experiments at Case.

In the experiments of this work, a spanwise array of cylindrical roughness elements is located 300 mm downstream of the leading edge. The same diameter and spacing but

different materials were used for the multicomponent and unsteady experiments. For the unsteady experiments, layers of adhesive-backed paper labels were stacked on top of one another to give the desired roughness height. Then, 6.35-mm-diameter circular discs were extracted using a punch. However, for the multicomponent experiments the 6.35-mm-diameter discs were machined from aluminum and were applied using a spray adhesive. The roughness elements are spaced $\lambda_k = 3D = 19$ mm apart in the spanwise (z) direction. This spacing is chosen because it affords the greatest number of elements within the spanwise range accessible by the hotwire traverse while maintaining sufficient distance between the roughness elements that they can be considered to act as isolated roughness elements in the near-wake region (von Doenhoff and Braslow 1961). This spacing puts eight elements within reach of the traverse, but the entire array consists of twelve elements to ensure that spanwise periodicity is maintained beyond the traverse's range. The application of the roughness array with the exact spacing was performed carefully with the help of a metal template placed on the plate.

The roughness amplitude is quoted in terms of the roughness Reynolds number, Re_k , which is defined as $Re_k = \overline{U}(k)k/\nu$ where $\overline{U}(k)$ is the velocity of the unperturbed boundary layer at the roughness height, k , and ν is the kinematic viscosity. In the calculation of Re_k the velocity at roughness height can be approximated as $\overline{U}(k) = \overline{U}_y(0)k$, where $\overline{U}_y(0)$ is the velocity gradient at $y = 0$. This value is found by taking the derivative of the Blasius similarity solution. After performing the above substitutions Re_k can be approximated with

the following formula:

$$Re_k = 0.332 \left[\frac{(Re')^3 k^4}{x - x_{vle}} \right]^{1/2} \quad (2.1)$$

where Re' is the unit Reynolds number U/ν and x_{vle} is the location of virtual leading edge. For the multicomponent experiments the height of each roughness element is $724 \mu\text{m}$ which gives a subcritical Re_k value of 209. For the unsteady measurements three configurations were used with $712 \mu\text{m}$, $813 \mu\text{m}$ and $914 \mu\text{m}$ resulting in Re_k values of 202, 264 and 334, respectively.

2.3 Automation and Instrumentation

Boundary-layer stability experiments generally require the experimentalist to track a certain type of velocity disturbance over the body of interest in order to gain an understanding of the disturbance's evolution. If a technique such as Particle Image Velocimetry is employed, the duration of the experiment can be short because the global flow field can be observed almost instantly. However, there are certain limitations for these techniques, especially when the problem of interest is a boundary-layer flow over roughness. These methods cannot guarantee that a sufficient number of seeding particles will enter the boundary layer to provide well-resolved (in time or space) disturbance measurements. Using hotwire anemometry avoids this problem because hotwires have an excellent frequency response and can be positioned anywhere. Additionally, the nature of the voltage signals that hotwire anemometers produce makes it possible to perform amplification and filtering processes that provide excellent sensitivity to small disturbances. However, because hotwires pro-

vide only single-point measurements, using hotwires leads to long-duration experiments because the probe has to visit a tremendous number of locations in order to follow a given disturbance over the domain of interest. This necessitates a repeatability and consistency of the experimental conditions that, in turn, demands a completely automated system. Full automation is also demanded by the sheer volume of data that is required to capture the evolution of 3D disturbances.

All the experiments described in this work were controlled by a Dell Dimension 800 personal computer with an Intel Pentium III processor running the Microsoft NT operating system. This system is responsible for controlling the experiment, as well as acquiring, storing and processing data. The data acquisition system includes a National Instruments PCI-MIO-16XE-10 data acquisition card, featuring a 16-bit resolution analog-to-digital converter. The card has an input range $\pm 10\text{V}$ and provides a maximum sampling rate of 100 kHz.

All the measurement devices convert the measured quantity to electrical voltage and these voltage values are transferred through co-axial cables to several input-output (I/O) devices which are located in the measurement rack. The I/O environment consists of several National Instruments devices; SCXI 1140, SCXI 1102B and SCXI 1163R which provide multiplexing and amplification. The fluctuating voltage signals are transferred through and conditioned (low-pass filtered and amplified) by the SCXI 1102B. The computer is also responsible for controlling the traverse's stepper motors and the fan motor. The traverse is equipped with a Velmex 9000 controller that is connected to the main control computer via

an RS-232 cable. Similarly, the fan motor is controlled using a Reliance Electric FLEX-PAK 3000 motor controller and communication with this is through device SCXI-1163R via digital and analog control lines. The channels of these I/O devices are assigned and managed with National Instruments Measurement and Automation Explorer software.

The data acquisition programs are written with LabVIEW software and are designed to automate various tasks including hotwire calibration, tunnel speed control, sensor movement control, turbulence measurements, near-wall velocity measurements and storage, and data analysis. These programs feature numerous sub-programs which are responsible for smaller tasks, such as acquiring hotwire velocities, freestream temperature and pressure, etc. Special attention is given to design a user-friendly interface for the programs so that non-experienced users can gain experience easily and quickly become familiar with the experiments.

The tunnel's mean velocity is measured by using a Pitot tube. The tube is located upstream of the flat plate leading edge, 280 mm below the top wall and 160 mm away from the rear wall of the test section. It is located behind the test side so that its wake does not interfere with the hotwire measurements. The static and dynamic pressures are measured using a 1000-Torr MKS Baratron Type 627B absolute pressure transducer and a 10-Torr MKS Baratron Type 120A differential pressure transducer, respectively. The measurements taken by these pressure transducers are insensitive to environmental temperature changes because both of these instruments have their own internal temperature controllers. The temperature in the test section is measured using an Omega DP119 resistance temper-

ature device. The boundary layer and freestream velocity measurements are obtained using constant temperature hotwire anemometers. The hotwire system consists of an A. A. Labs AN-1003 hotwire anemometer and Dantec design hotwire probes that feature 1 mm long, 2.5- μm - diameter and 5.0- μm -diameter tungsten wires. The prongs of the boundary layer probes are offset 3 mm from the probe axis to allow better access to the plate surface. The freestream sensor has straight prongs and the slanted sensors have yaw angles of 45°. The hotwire anemometer system is capable of measuring velocity through ten channels and up to four of these channels are used simultaneously during the experiments to measure freestream and boundary layer velocities.

2.4 Hotwire Principles and Calibration

Boundary layer and freestream velocity measurements in this work are obtained using constant temperature hotwire anemometry because of their fine spatial resolution in the wall-normal direction. The characteristic wall-normal length scale is the boundary-layer thickness, $\delta = (\nu x / U_\infty)^{1/2}$, which is on the order of 1 mm in the experiments of this work. This necessitates the fine resolution provided by hotwires and the hotwire traverse. The 2.5- μm -diameter boundary layer hotwires provide the best spatial resolution in the wall-normal direction among all possible velocity measurement devices suitable for open-flow configurations. The length of the sensor is about 1 mm. The two principal spanwise scales of the present experiments are the roughness diameter, $D = 6.35$ mm, and the roughness spacing, $\lambda_k = 19$ mm. Therefore, the resolution of the sensor in the spanwise direction is sufficient.

Additionally, the sensor behaves as a spatial low-pass filter in spanwise direction; small wavelengths (large wavenumbers) in this direction are filtered because of aliasing. Luckily, this is advantageous for measurement of low spanwise wavenumbers.

Apart from geometrical reasons, hotwire anemometers are useful because of their accuracy and fast temporal response. When properly calibrated, hotwire sensors are able to accurately sense very small fluctuations about the mean flow. The square wave test indicates that a frequency response of 10 kHz is achieved. Because the stability of boundary layers is governed by the evolution of initially small fluctuations, hotwire anemometry becomes almost inevitable for such experiments. The smallest velocity fluctuation is restricted by the range of hotwire anemometer and by the resolution of the data acquisition card. The test speed of the experiments is 12.2 m/s and the hotwire anemometer can be calibrated in a $\pm 10V$ range. With the existing 16-bit resolution data acquisition card and signal gain values, the smallest measurable velocity fluctuation is on the order of 5×10^{-6} m/s. This value is significantly smaller than any other uncertainty present in the measurement, therefore uncertainty associated with the hotwire anemometry can be neglected as an error source. Therefore, the only source of error in the present velocity measurements are due to bias error resulting from the calibration against the Pitot tube. The uncertainty associated with the dynamic pressure measurement is 1.2% of the full scale, i.e., $\sigma_{\Delta P} = 0.012\Delta P$. For the experiments' nominal freestream speed of $U_\infty = 12.2$ m/s the unit Reynolds number, $Re' = (U_\infty/\nu)$, is $(764 \pm 4) \times 10^3 \text{ m}^{-1}$, where the uncertainty in Re' is given by $\sigma_{Re'} = Re' \sigma_{U_\infty}/U_\infty$.

The working principle of the hotwire anemometer is based simply on an energy balance: the heat generated in the wire due to electrical resistance is dissipated by the forced convection cooling by air moving across the wire. Ohm's Law and the convective heat loss are related by the heat flux Q by

$$Q = \frac{E^2}{R} = hA(T_w - T_\infty), \quad (2.2)$$

where T_∞ is the ambient fluid temperature, T_w is the wire temperature, A is the surface area, R is the resistance, and E is the voltage across the wire. An increase in the fluid velocity increases the convective heat transfer coefficient, h , and hence the heat flux from the wire. This results in an instantaneous drop in the wire's temperature and an instantaneous decrease in the wire's resistance. The resistance, and therefore the temperature of the wire, is held constant by monitoring R using a feedback-controlled Wheatstone Bridge circuit that varies the wire's voltage, E . As a result, any change in the fluid velocity is matched by an appropriate input voltage that maintains a constant wire temperature. The voltage is therefore directly related to the velocity so long as $(T_w - T_\infty)$ remains constant. The temperature difference between the wire and the fluid can be adjusted by the overheat ratio, which is defined as the resistance ratio, R_s/R_f , where R_s is the resistance of the heated sensor at its operating temperature, and R_f is the resistance of the sensor at ambient fluid temperature. Typical values of overheat ratio for air velocity measurement range between 1.2 and 3.0. If a higher value is selected, the system's bandwidth increases and the probe becomes more sensitive to changes in fluid velocity. On the other hand, large overheat ratios create

thermal fatigue, decreasing the effective life of the probe. Also, wire cooling by natural convection can become significant at low velocities as the temperature difference ($T_w - T_\infty$) increases. As a result, the measured voltage can overestimate the actual fluid velocity. For the measurements performed in this work, an overheat ratio of 1.8 was used.

All the variables in Eq. (2.2) except the heat transfer coefficient, h , can be measured directly. The heat transfer coefficient depends on the flow velocity vector, which can be characterized by the magnitude and the wire's angular orientation with respect to the incident flow. These two parameters can be lumped in a single variable by defining an effective cooling velocity, V_e of the wire. The relationship between h and V_e is determined experimentally through dimensionless parameters, Reynolds number, $Re_D = V_e D / \nu$, and Nusselt number, $Nu = hD/k$, where D is the wire diameter, ν is the kinematic viscosity and k is the thermal conductivity of the fluid. Inserting these dimensionless parameters in the energy balance given by Eq. (2.2), and lumping all the constants, the governing equation for hotwire response is obtained:

$$Nu = A' + B' Re_D^{1/2}. \quad (2.3)$$

Equation (2.3) is also known as King's Law and the constants A' and B' are determined empirically. At constant temperature, this relation can be rewritten in terms of potential applied across the wire and the effective cooling velocity, which gives the final form of King's Law used in this work.

$$V_e = (A + BE^2)^n \quad (2.4)$$

The exponent $1/2$ from Eq. (2.3) is replaced by the variable $1/n$, so that better functional relationships can be obtained. Eq. (2.4) directly links the effective cooling velocity to the hotwire voltage, and the constants in this expression are provided by proper calibration.

A successful calibration necessitates an accurate measurement of flow speed. The calibration reference is provided by the Pitot tube which is an excellent calibration standard for steady flows. To measure flow speed, static and dynamic pressures and fluid density at test temperature are required and the measurement devices used to obtain these quantities have good accuracy, precision and long-term stability.

One important aspect of hotwire use is that over time the sensor response changes. This requires periodical calibration of the instrument against a stable velocity reference. Therefore, calibration is performed daily before the experiments in order to minimize the effects of long-term variations in the hotwire response. Because ambient temperature is assumed to be constant during the calibration, using Eq. (2.4) to calculate the velocity can lead to errors when temperature changes significantly. Therefore, recalibration is performed whenever laboratory temperature variations greater than 0.5°C occur.

Before calibration, the hotwire sting is moved to its most-forward position (upstream of the flat plate) so that the hotwire sensors are in a plane close to the Pitot tube. Two hotwire calibration programs — one for the multicomponent experiments and the other for the unsteady experiments — are responsible for the automatic calibration of all hotwire sensors. Both programs run the wind tunnel motor at specified velocities and, at each set point, records the Pitot tube velocity and the corresponding hotwire voltages.

Once data has been obtained across the velocity range of interest, the program uses the Levenberg–Marquardt algorithm (Press et al. 2001) to estimate best-fit values for the calibration constants A , B and n using a nonlinear least-squares approach. The uncertainty in effective cooling velocity associated with the nonlinear fit depends on the uncertainty associated with voltage measurements and errors made in calculating the calibration constants. The idea behind the Levenberg–Marquardt algorithm is to minimize the uncertainty of the velocity fit by adjusting the calibration constants. The algorithm itself and error analysis is thoroughly explained by Press et al. (2001). Typically, the mean standard error of the calibration is 1.5% of the edge velocity. However, it should be emphasized that the differential pressure transducer does not have good accuracies at very low pressures associated with velocities below 3 m/s and, therefore, the uncertainty is significantly larger closer to the flat surface, where the fluid velocity is smaller. Other calibration details and data analysis related to multicomponent and unsteady experiments will be presented in their respective chapters.

There are some difficulties associated with hotwire velocity measurements. Perhaps the biggest disadvantage of hotwire anemometry is that the direction of the velocity cannot be distinguished by the sensors. This issue becomes especially important for multicomponent measurements and potentially reversed flows that may exist in the near wakes of the roughness elements. Additionally, thermal effects between the wire and the plate become important when the probe is located close to the wall, particularly where the velocity is less than 15% of the freestream velocity. At these locations some fraction of the heat dissipated

by the hotwire is conducted to the metal surface and this produces erroneous results. Because of this, the velocity measurements are obtained only at locations where the boundary layer velocity is greater than 16% of the freestream velocity (around $300\text{ }\mu\text{m}$ away from the wall).

2.5 Boundary Layer Scans

Understanding the behavior of roughness-induced disturbances in a flat plate boundary layer require velocity measurements in a 3D experimental domain in the wake of roughness. The streamwise, spanwise and wall-normal extent of this domain is determined by the geometry of the problem and experimental limitations. (The schematic of the right-handed coordinate system used in the experiments is illustrated in Figure 6.) Measurements can be safely obtained between $x = 307.5\text{ mm}$ and $x = 900\text{ mm}$. No velocity measurements are taken upstream of the roughness array. The wall-normal measurement range extends from several millimeters outside the boundary layer to within $300\text{ }\mu\text{m}$ of the wall. The spanwise range is limited by the vertical travel (172 mm) of the sliding window and extends such that up to eight of the 16 total roughness elements can be included in the measurements. The measurement domain is a thin 3D volume close to the flat surface. The measurements are obtained in y - z planes perpendicular to the flow direction, at different streamwise (x) locations. One of these planes is shown in Fig. 6. Within each of these planes, hotwire sensors are positioned at y and z stations resulting in a discretized 2D velocity field. The collection of measurements at different x locations produces the desired 3D data that captures the

evolution of the roughness-induced disturbance field.

There are two possible scanning techniques to discretize the y - z plane at a particular x location. First, the hotwire can be moved towards the flat surface, measuring the velocity at different y stations to obtain a single boundary layer profile until a cutoff velocity is reached. Then, the same movement can be repeated at different z stations, until the measurement range is covered. Alternatively, the hotwire can measure velocities along a spanwise line with this same movement repeated at different wall-normal locations until a certain cutoff velocity is reached. Both methods scan the 2D measurement plane and provide the same data but do so in different ways. Both methods are employed in this work depending on the type of experiment. White and Ergin (2004) give a detailed discussion of the advantages and limitations of each technique; key points are summarized below.

2.5.1 Slice

The first scanning technique named “Slice” moves the hotwires in a manner similar to the movement of the needle in a sewing machine. The hotwire located close to the plate surface is moved into the boundary layer in the $-y$ direction until the cutoff velocity is reached and, at that point, it is moved out of the boundary layer, is advanced one z step and the process repeats. This scanning technique is used during the data acquisition of unsteady experiments.

While the basic concept behind the Slice program is simple, there are many details that must be properly included for its correct implementation. To begin, the boundary layer

sensor is placed along the midpoint of two neighboring elements (will be referred to as ‘midpoint z station’), several millimeters outside of the boundary layer edge before the program is executed. The program acquires the velocities from the boundary layer and freestream sensors and calculates the local normalized velocity by dividing the boundary layer velocity by the freestream value. This value is stored together with the measurement point’s y -location. Because the plate’s precise wall-location is not known in traverse coordinates, the first point acquired is assigned an uncorrected y location of 0.0 mm. As the hotwire progresses towards the plate the subsequent points are assigned increasingly negative uncorrected y locations.

For the first several points obtained outside the boundary layer the local normalized velocity equals unity. When this occurs, the hotwire sting moves towards the plate by a specified initial distance depending on the x location. Eventually, the sensor closer to the flat surface enters the boundary layer and measures a local normalized velocity that is less than unity. At this point and beyond, the initial inward y step is multiplied by the local normalized velocity raised to a power (1.8 is used for laminar boundary layers) to determine the next y -step toward the plate. By doing so, the y steps taken toward the plate become progressively smaller as the boundary layer sensor approaches the surface. This enables taking large steps outside the boundary layer where good resolution is not necessary and smaller steps in the boundary layer where it is.

The sensors approach the wall until the measured local normalized velocity becomes less than a specified cutoff velocity. This check is performed only at the midpoint z stations,

where the boundary layer is nearly the Blasius profile. Once the cutoff is passed, the program moves the sensors away from the plate to their initial starting point, completing a local boundary layer profile at a location. Then, the sensors are moved one step in the z direction and the same process is repeated with one difference: At the non-midpoint z stations, the cutoff velocity is not used to check the boundary layer sensor's position with respect to the surface. Instead, the total y travel at the midpoint z stations are stored and the sensors travel is compared against this value.

The sting is moved to the next z location by an amount of z step, such that an integer number of steps are taken per wavelength. The same procedure is repeated for the consecutive z locations until the spanwise range is covered for the given x location.

2.5.2 Dice

The second scanning technique is named “Dice”. This scanning technique is used during the multicomponent velocity measurements. Dice uses a velocity measurement and wall approach procedure equivalent to Slice, but the Dice program moves the sensors first in the spanwise direction until the roughness array is covered. When the sensor completes velocity measurements on the first spanwise line, the sting moves back to its initial spanwise position, and takes a step towards the flat surface. This sequence is repeated until the near-wall cutoff velocity is reached.

Slice and Dice do not attempt to find the wall location during the experiment. The wall-fit and wall-normal coordinate corrections are performed after the experiment is com-

plete using a separate data analysis program. The wall location is estimated during data analysis only for the midpoint z stations and the two neighboring stations of the midpoint, where the velocity profiles are most representative of the Blasius profile. For example, during the unsteady experiments with eight roughness wavelengths and 12 measurement locations in z per wavelength, the wall location is extrapolated for the 1st, 2nd, 12th, 13th, 14th... 95th and 96th z stations.

To compute the wall location, a linear extrapolation is performed using the the measured velocities and the uncorrected y positions and by assuming a no-slip boundary condition. The slope of the Blasius boundary layer's velocity profile is essentially constant for velocities less than 35% of the edge velocity and this permits the programs to use a linear least squares fit for normalized velocities measured between cutoff velocity and 0.35. The data analysis program uses the points that are closest to the surface to establish dU/dy and extrapolates to zero velocity to estimate the wall location in terms of uncorrected y values. The wall locations of non-midpoint z stations are computed by performing a quadratic polynomial fit along the span to the wall locations of the midpoint z stations. After the wall is located along the span, the y values are corrected individually at all the z stations to form the final estimate of the 2D cut through the velocity field.

2.6 Flat Plate Alignment

Prior to beginning roughness experiments, significant attention was given to both stream-wise and spanwise alignment of the smooth flat plate model. Correct alignment is not the

geometrical alignment of the plate with respect to incoming flow. Rather, it is an alignment that achieves a zero-pressure-gradient condition on the test side of the model. If the surface of the plate were perfectly aligned with the test section walls and with the incoming flow, the flow in the duct would be slightly accelerated and would feel a favorable pressure gradient as the developing boundary layers on the test section walls and the flat plate decrease the test section's effective cross-sectional area. Although these boundary layers are present no matter how the plate is aligned, the plate can be adjusted so that the cross sectional area on the test side gradually increases in the flow direction. This increasing area compensates for boundary layer thickness growth and provides a zero-pressure-gradient flow. The displacement of the trailing edge to satisfy zero-pressure-gradient condition induces a small angle of attack which leads to a different problem for the flow over the elliptical leading edge. As the trailing edge is displaced backwards the stagnation point moves from the tip of the leading edge towards the back of the plate. This situation is undesired because it can lead to separated flow on the test side of the plate and this leads to large disturbances and frequently turbulent flow in the boundary layer. To prevent this, a 300-mm-long trailing edge flap is used to ensure the stagnation point location be on the test side of the plate.

In order to verify that the zero-pressure-gradient condition is established, a measure of zero-pressure gradient condition is necessary. The best measurement for this purpose is the shape factor of laminar boundary layer which is a unique characteristic of Blasius flow. The shape factor, H is determined by the ratio of the displacement thickness, δ^* , to the momentum thickness, θ , and has a value of 2.59 for a zero-pressure-gradient flow.

According to Saric (1996), shape factor variations of ± 0.04 around 2.59 are acceptably close to zero-pressure-gradient behavior. Both thicknesses are integrals of the normalized boundary layer velocity profile over the wall-normal coordinate y :

$$\delta^* = \int_0^\infty (1 - U^*) dy \quad \text{and} \quad \theta = \int_0^\infty U^*(1 - U^*) dy \quad (2.5)$$

where U^* is the streamwise basic state velocity profile made dimensionless by U_∞ . Both integrals are performed numerically using Simpson's formula. Since the velocity is equal to the freestream velocity outside the boundary layer, one can change the limits of integration from zero to an arbitrary location outside the boundary layer.

The process of flat plate alignment is an iterative process. To ensure streamwise alignment, boundary layer scans are performed using the Slice approach to obtain the boundary layer velocity profiles at several x locations on the flat plate before roughness is applied. Based on the shape factors calculated from these profiles, the trailing edge is displaced so that the plate is at a certain angle of attack that balances boundary layer growth. Then, the trailing edge flap is adjusted to ensure that the stagnation streamline is on the test side of the plate. These two processes are performed iteratively until reasonably satisfactory shape factors are obtained. The spanwise-average shape factors calculated at different locations in the streamwise measurement domain are given in Fig. 7 and indicate a slightly accelerated flow over the surface. The streamwise average of these values is $H = 2.54$, which is slightly less than the lower acceptable value for H suggested by Saric (1996) for a zero-pressure-gradient condition. The displacement and momentum thickness variations

in the x direction over the smooth plate are given in Fig. 8. It should be emphasized that all x locations cited in this work and shown in the figures are with respect to the geometrical leading edge. However, in calculations that include the boundary-layer thickness such as the calculation of Re_k , streamwise coordinate (x) is corrected with the virtual leading edge information. In Fig. 8, the virtual leading edges (vle) calculated by performing nonlinear least squares fits to both displacement and momentum thickness data are also indicated. The virtual leading edge appears to be 4 ± 2 mm downstream of the physical leading edge according to displacement thickness data and 14 ± 1 mm upstream of the physical leading edge according to momentum thickness data (The uncertainties of the vle values are associated with the asymptotic standard errors resulting from the least squares fits in Fig. 8). The average of these values, $x_{vle} = -5$ mm is used during the calculations and the minus sign indicates upstream of the leading edge. The uncertainties of Re' and x_{vle} produce a 2% uncertainty in δ , even for the smallest x locations considered in this work.

After an acceptable streamwise alignment is obtained, a spanwise alignment is performed. Since the traverse's position is fixed to the test section, the plate is rotated along its x axis to achieve the spanwise alignment between the traverse and the plate. First, the tunnel is operated at constant speed and the sensors are moved to an upstream position on the plate. At this upstream position, two boundary layer measurements are taken, one on each extreme of the spanwise range accessible by the traverse. At the first location, the sensors are positioned so that the boundary layer velocity is half the freestream value. Then the sensors are moved in the spanwise direction to the other end. Due to the misalignment, the

normalized boundary layer velocity at the second spanwise location is different than 50%. The misalignment in y is determined by moving the sensors in the wall-normal direction until this value is reached. Finally, the brackets that hold the plate in place are adjusted carefully to account for the difference. The adjustment procedure is repeated at a downstream location and both adjustments are iterated until an acceptable spanwise alignment is achieved. The spanwise alignment of the flat surface and the traverse is especially crucial during the multicomponent experiments because there are multiple sensors in the boundary layer that need to be aligned with the flat surface. With the final adjustments in the spanwise direction, the experiments are performed with an average misalignment less than $25\text{ }\mu\text{m}$ in 76 mm spanwise travel.

Chapter 3

STEADY MULTIPLE COMPONENT VELOCITY MEASUREMENTS

3.1 Background

This section addresses the first objective of this work: to investigate the precise initial conditions of transient growth and to provide inputs for receptivity analysis. In the literature, many investigators attribute the cause for transient growth to a steady, decaying vortex, the axis of which is aligned in the streamwise direction. This vortex transfers momentum in the wall normal direction, creating local velocity perturbations and spanwise gradients in the boundary layer as explained by Landahl's (1980) "lift-up" mechanism. Another goal is to measure the decay rate of the streamwise vorticity, which has never been measured. 3D Roughness may generate streamwise vorticity under certain flow conditions and, therefore, is a suitable disturbance source for transient growth investigations.

The effects of low-amplitude surface roughness ($k \sim \delta$ or smaller) on boundary layer flows are summarized in a survey article by Sedney (1973) and more recently by Ergin and White (2005b). Sedney (1973) reviews the experimental data for laminar subsonic, turbulent subsonic, laminar supersonic and turbulent supersonic flows. Regardless of the flow type or the detailed shape of 3D roughness elements, the article points out three important observations: "(1) In the upstream separated flow a system of vortices are formed. (2)

These are stretched around the [windward side] of the protuberance in a horseshoe vortex fashion and persist as streamwise vortices far downstream. (3) Spiral vortices rise up from the surface in the near wake. The details of the downstream disturbance depend on whether or not transition takes place in a laminar boundary layer.” Identifying these vortical structures in the boundary layer requires the measurement of multiple velocity components.

The present work considers a cylindrical protuberance in a subsonic laminar boundary layer. The first systematic study of laminar-to-turbulent transition induced by 3D surface roughness was performed by Gregory and Walker (1956), who used smoke and china-clay flow visualization techniques around a cylindrical “pimple”. They provide a detailed description of the steady flow field around and downstream of the cylindrical roughness element and a key figure that describes these details is adapted from that work in Fig. 9. At low Reynolds numbers, Gregory and Walker (1956) observed only a pair of “spiral smoke filaments” that spiral upwards (normal to the plate) close behind the cylinder. As these filaments reached to the roughness height, they were convected downstream very close to one-another along both sides of the centerline. As the tunnel speed was increased, these filaments were undisturbed and a horseshoe vortex appeared wrapping around the windward side of the cylinder. At still larger Reynolds numbers, the vortex filaments were observed to break down as they reached the top of the cylinder and eddies were shed.

According to Gregory and Walker, “the boundary layer produces a static pressure gradient along the front stagnation generator of the cylinder.” This pressure gradient is responsible for the birth of the horseshoe vortex system and can be explained by the fact that

the boundary layer velocity just below the roughness height is larger than the velocity just above the surface in front of the cylinder. Therefore, the static pressure around roughness height is larger than that close to the surface. The consequence is a vertical flow towards the surface on the windward side and a subsequent horizontal reversed flow along the flat surface. (See side view on the left in Fig. 9.) Subsequently, the fluid “rolls up into a continuously-generated horseshoe shaped vortices” which wrap around the windward side of the cylinder and these trail downstream as “multi-ply” vortex pairs with axes oriented in the streamwise direction. The number of plies increase with increasing speed and as many as three horseshoe vortices were observed in that study.

No reason for the creation of spiral vortex sheets was given by Gregory and Walker (1956). However, these filaments explain another intriguing flow feature in Fig. 9. In the near wake, the spiral vortex filaments continuously removes fluid mass from the separation bubble behind the cylinder. In order to satisfy mass conservation some kind of inflow must be present to this region. Indeed, the smoke layer indicates that the inflow to the near wake is supplied by the upstream flow through two converging channels on both sides underneath the two legs of the horseshoe vortex.

Gregory and Walker’s (1956) main intention was to investigate transition based on the characteristic turbulent wedge that appears somewhere downstream of a large roughness element. Although a transitional flow configuration will be presented in the next chapter, one important observation regarding the turbulent wedge can be presented here. First and foremost, a turbulent wedge does not appear for subcritical roughness configurations and

the horseshoe vortex system decays leaving a disturbed laminar flow. However, with increasing freestream speed, the apex of the turbulence wedge appears first at some distance downstream of the roughness location, connected to the protuberance by the trailing legs of the horseshoe vortex. The authors observed that the location of the apex gradually moves upstream towards the roughness element with increasing roughness height. This means that although considerably distorted, the flow can remain laminar for a certain distance before breakdown to turbulence.

Gregory and Walker's results have been enhanced by numerous other flow-visualization studies that culminate with the experiments of Acarlar and Smith (1987). However, these works do not provide adequate quantitative information about these vortices. For instance, transient disturbances are believed to be a result of a decaying steady vortex. The aforementioned studies do not provide any quantitative information neither on the streamwise evolution of these vortices nor on their spanwise distribution. The current experiments aim to provide a quantitative description of the flow topology in the roughness wake.

Receptivity is the second crucial factor that requires the measurement of multiple velocity components. Tumin (2003) addressed the challenges of analyzing the receptivity of transient disturbances in a boundary layer in a rigorous manner. One of the challenges is related with the continuous and discrete spectrum. As mentioned before, in boundary layers the temporal or spatial disturbance evolution is most accurately characterized by the discrete and continuous spectrum of eigenmodes of the OS and Squire operators. A detailed analysis of continuous spectrum concept can be found in Ashpis and Reshotko (1998).

The modes of the discrete spectrum are TS type perturbations and are localized within the boundary layer. On the contrary, the eigenmodes of the continuous spectrum, which are responsible for transient growth, are bounded outside of the boundary layer solutions of the linearized disturbance equations. Additionally, the continuous spectrum is made up of infinite number of modes; therefore these modes must be discretized for comparison with experimental disturbance data. Perhaps the most important challenge is associated with the nonorthogonality of the eigenmodes. For a nonorthogonal equation system, an orthogonality condition among the eigenmodes cannot be formulated and therefore a conventional orthogonality solution is not possible. According to Tumin (2003), a definition of an orthogonality condition is possible when the solutions of the adjoint OS and Squire equations are also included in the formulation. This “biorthogonality condition” allows the decomposition of a certain perturbation into modes with known amplitudes. This casts the roughness receptivity problem to computation of several hundreds of eigenmode constants that represent a discretization of the continuous spectrum.

Tumin’s (2003) decomposition method presents a serious challenge to the experimentalist: It requires knowledge of the complete disturbance field including the pressure disturbance, all three velocity components’ disturbances and their spatial derivatives. Obtaining the complete disturbance information is trivial in computational studies because this information exists as part of the numerical solution. However, in an experiment, it is extremely difficult to measure the wall-normal velocity disturbances and perhaps impossible to measure wall-normal pressure-disturbance profiles. Fortunately, partial disturbance field infor-

mation, excluding pressure, can be used to decompose the flow field into a finite number of discretized eigenmodes if the disturbance consists of downstream vorticity eigenmodes only (Tumin 2003). For stationary transient disturbances in the far wake these are the only modes that are thought to be important.

The experiments described in this chapter addresses the two points explained above: (1) the identification of the streamwise vortex that is responsible for transient growth and (2) the generation of partial disturbance field data suitable as an input to biorthogonal decomposition technique. To accomplish this, the experiments are designed to obtain the streamwise and spanwise velocity components with high spatial resolution and the wall-normal velocity component is estimated using the continuity equation.

3.2 Data Acquisition and Analysis

Measuring the spanwise velocity requires the placement of at least two slanted hotwires with different angular orientations at the same measurement location. For this purpose, the hotwire sting design in Fig. 5 is used. One hotwire is a straight sensor in the freestream; the remaining three sensors are in the boundary layer. The boundary-layer sensors are positioned in a trident design with one straight sensor in the middle and two slanted hotwire sensors mounted 9.5 mm one either side of the the center probe. Because the flat plate is mounted vertically in the test section the slanted boundary layer sensors are seen on the upper and the lower side of the normal sensor by an observer. Therefore, the slanted sensors are named the “top slanted” and the “bottom slanted” sensors, and the normal sensor in the

middle is simply called the “boundary layer” sensor. Using a separation that is an integer multiple of the spanwise step (0.5 mm for multicomponent experiments) permits all three boundary layer sensors to be placed at the same measurement location at three different spanwise traverse steps. Naturally, this requires excellent alignment of the flat surface with the traverse plane and the sensors with respect to each other and great care is exercised to achieve this alignment.

Slanted hotwire angle calibration is performed following the recommendations of Bruun (1995). First, the yaw angle of each wire is determined by gradually rotating the sensor in the freestream and monitoring the bridge voltage. When the bridge voltage is a maximum, the maximum cooling rate is achieved because the impinging velocity is perpendicular to the wire axis. With respect to the incoming flow, the wire angle for both sensors is found as 40° with an accuracy of 0.5° . Then, velocity calibration of all four sensors is performed simultaneously. First, the sting is positioned so that all hotwires are located in the freestream at a tunnel cross section which also includes the Pitot tube. Second, it is rotated 5° clockwise from its position during an experiment and a velocity calibration is performed. Third, the sting is rotated 10° counterclockwise (5° counterclockwise from its position during an experiment) and another velocity calibration is performed for the same tunnel speed range. Finally, data from both velocity calibrations at two different angles are combined and the calibration constants are computed by using the King’s Law and Hinze’s (1959) formula:

$$V_e = (A + BE^2)^n = \tilde{V} (\cos^2 \alpha + k^2 \sin^2 \alpha)^{1/2} \quad (3.1)$$

Table 1: Typical calibration constants for multicomponent experiments.

Sensor	A	B	n	k
Freestream	-1.591	0.582	2.258	—
Boundary Layer	-1.419	0.538	2.377	—
Top Slanted	-1.546	0.551	2.328	0.160
Bottom Slanted	-1.879	0.665	2.209	0.180

In these equations A , B and n are calibration constants, $\tilde{V}$ is the freestream speed (during the calibration procedure only), V_e is the effective cooling velocity, E is the hotwire voltage, α is the sensor's yaw angle and k is the sensor's yaw coefficient. The yaw coefficient is assumed to be constant (Jørgensen 1971). According to Bruun, Hinze's formula predicts the effective cooling velocity to within 1% for yaw angles between 0° and 70° and the error reaches only 15% for a yaw angle of 90° .

A nonlinear least-square fit is performed to find A , B , n and k values for each sensor. Typical values of calibration constants obtained during the experiments of the present work are presented in Table 1. The precise determination of the yaw coefficient, k requires a very accurate knowledge of the wire angle, α . Typical k values for single yawed sensors reported in the literature are around 0.1. According to a study reported in Bruun (1995) the value of the calculated yaw coefficient k^2 is very sensitive to an assumed yaw angle error, $\Delta\alpha$: An uncertainty as low as -1° can produce negative values for k^2 , which is not physically possible. As noted above, the 40° yaw angle found from the angle calibration has an accuracy of 0.5° , so there could be considerable error in k . However, the individual uncertainties from α and k cancel each other in Hinze's formula, reducing the combined

Table 2: Parameter summary for multicomponent experiments.

Re_k	Re' [1/m]	k [mm]	x_k [mm]	x_{vle} [mm]	λ_k [mm]	Δz [mm]	D [mm]
209	$(764 \pm 4) \times 10^3$	0.724	300	-5 ± 2	19	0.5	6.35

uncertainty on the right-hand-side of Eqn. 3.1 to less than 1% for $-1^\circ \leq \Delta\alpha \leq 1^\circ$.

For the multicomponent velocity measurements, the roughness array consists of aluminum discs which are applied to the flat surface at $x = 300$ mm using spray glue. The elements were $711.2 \mu\text{m}$ (0.0280") thick and the glue added a thickness of $12.7 \mu\text{m}$ (0.0005"), resulting a total thickness of $724 \mu\text{m}$. Although 12 elements are applied on the surface, measurements are taken in the wake of 4 elements in the middle of the array. The parameters used for multicomponent experiments are summarized in Table 2. During data acquisition, the voltage readings from four sensors are converted to effective cooling velocities using the calibration constants in Eqn. 3.1. The effective cooling velocities from all boundary layer sensors are normalized by the freestream velocity and these are recorded for further analysis together with the yaw coefficients of slanted sensors.

Multicomponent data is analyzed by decomposing the steady streamwise and spanwise velocity fields into a steady, spanwise-invariant basic state and a steady disturbance as

$$\begin{aligned}
 U(x, y, z) &= \overline{U}(x, y) + U'(x, y, z) \\
 W(x, y, z) &= \overline{W}(x, y) + W'(x, y, z)
 \end{aligned} \tag{3.2}$$

Although the basic state for a flat plate is the Blasius boundary layer profile, the present work uses an appropriate experimental basic state which is the average of the boundary layer profiles at the midpoint of two roughness elements. This experimental basic state profile is most representative of the theoretical Blasius profile and is 2D, therefore, $\overline{W} = 0$.

The decomposition of effective cooling velocity measured using the slanted sensors requires a careful treatment of the experimental data. First and foremost, because the sensors sweep different (y, z) planes, the data are shifted such that the slanted sensors' spanwise locations match with those of the the middle sensor. Despite the effort to align the boundary layer sensors with respect to the flat surface, the misalignment between the sensors cannot be avoided completely. This implies that each sensor is sweeping the boundary layer at slightly different heights from the flat surface. To account for any misalignment of sensors in the wall-normal direction, the wall location is estimated for each sensor at between-roughness z stations by performing a linear extrapolation to zero velocity. This means that the effective cooling velocity data recorded by each sensor has a different y array, and the V_e data obtained from all three sensors must be interpolated onto a single array of heights. This is accomplished by interpolating the V_e data onto the y array of the middle sensor. Once these are completed, the spanwise and streamwise velocities are resolved from the effective cooling velocity data obtained from the slanted sensors.

The effective cooling velocity of a hotwire yawed in a 2D flow is given by Hinze's formula, $V_e^2 = U_n^2 + k^2 U_t^2$, where U_n and U_t are the normal and tangential velocity components on each slanted wire. In the present case, the flow has U and W components with

$U_n = (U \cos \alpha - W \sin \alpha)$ and $U_t = (U \sin \alpha + W \cos \alpha)$. The two slant wires have $\alpha = \pm 40^\circ$, therefore, combining measurements of a single point from the two slant wires provides U and W . Substituting the expressions for U_n and U_t in Hinze's formula and defining a flow angle $\beta = \tan^{-1}(W/U)$, one arrives at the following quadratic equation, which can be solved for $\tan \beta$:

$$(R^2 a_b - a_t) \tan^2 \beta + (R^2 b_b - b_t) \tan \beta + (R^2 c_b - c_t) = 0 \quad (3.3)$$

$$\text{where } a_{t,b} = \tan^2 \alpha_{t,b} + k_{t,b}^2, \quad b_{t,b} = 2 \tan \alpha_{t,b} (k_{t,b}^2 - 1),$$

$$c_{t,b} = \tan^2 \alpha_{t,b} k_{t,b}^2 + 1 \quad \text{and} \quad R = \frac{V_{e,t} \cos \alpha_b}{V_{e,b} \cos \alpha_t}$$

A second order perturbation expansion in R indicates that the solution with the minus sign represents the correct solution. Once the flow angle is determined, the determination of U and W follows a simple substitution.

With the trident design used in the multicomponent experiments, U can be measured directly from the normal boundary layer sensor in the middle, as well as indirectly from the decomposition of effective cooling velocity measured by the slanted sensors. This allows the comparison of the streamwise velocity field obtained from the slanted sensors with that obtained from the middle sensor to check the quality of the data analysis. The agreement between the two sources is found satisfactory. The streamwise velocity data reported in the results section of the multicomponent experiments is obtained from the middle boundary layer sensor.

With the available streamwise and spanwise velocity components, the wall-normal velocity component, V , or the wall-normal velocity perturbation, V' can be obtained by using the appropriate form of the continuity equation. The first order derivatives in this equation are approximated by central differences. Assuming constant step size in x and z , the perturbation continuity equation takes the following form:

$$\left(\frac{\partial V'}{\partial y}\right)_j = -\left(\frac{U'_{i+1} - U'_{i-1}}{2\Delta x} + \frac{W'_{k+1} - W'_{k-1}}{2\Delta z}\right) \quad (3.4)$$

where i , j and k are the grid indices in the x , y and z directions respectively. Omitting the primes, one obtains the continuity equation for the wall-normal velocity. In either case one obtains the wall-normal velocity gradient, rather than the velocity itself. Using the no-slip boundary condition on the wall, V (or V') can be calculated. This is the actual procedure followed, and V' is computed during data analysis.

Generating derivatives from experimental data is extremely dangerous and one has to consider the amount of accuracy of such a calculation. Forming Taylor series expansions for $f(x_{i+1})$ and $f(x_{i-1})$ around x_i and subtracting the two one obtains the following relationship:

$$\left(\frac{\partial f}{\partial x}\right)_{x_i} = \frac{f(x_{i+1}) - f(x_{i-1}))}{2\Delta x} - \frac{1}{6} \frac{\partial^3 f}{\partial x^3} \Delta x^2 + O(\Delta x^4) \quad (3.5)$$

This means that the central differencing scheme is second order accurate in approximating the first order derivative, where third order derivative is multiplied by the step size squared. Therefore, in order to decrease the uncertainty in the calculation of $\partial V/\partial y$, one has to

Table 3: Streamwise spacing, Δx values used in computation of $\partial V/\partial y$.

Δx [mm]	x [mm]
2.5	310, 312.5, ..., 330
5	400
7.5	332.5
10	340, 350
20	330
25	375, 400 , 425, 450, 475
40	360
50	500, 550

decrease the step sizes Δx and Δz as much as possible. For this purpose, Δx is smaller in the near wake where the flow properties are changing rapidly, (i.e. where third order derivatives are important) and is gradually increased in the far wake where third order derivatives are negligible. The Δx values used for the individual x locations are listed in Table 3. The boldface x locations, $x = 330$ mm and $x = 400$ mm are repeated calculations with a coarser resolution. In the wall-normal direction the U and W velocity data are interpolated onto a y array of $100 \mu\text{m}$ resolution. The multicomponent experiments are performed with a constant Δz of 0.5 mm, which provides sufficient resolution and accuracy in the z direction. In order to determine the vortical structures in the boundary layer, the vorticity in the streamwise direction is calculated using $\omega_x = (\partial W/\partial y - \partial V/\partial z)$ with a similar central difference discretization scheme.

The roughness-induced disturbances of the multicomponent experiments are quantified by the steady disturbance energy and its constituents in certain spanwise wavelengths. In the previous experimental works at Case (see, e.g., White and Ergin 2003; White et al.

2005) only streamwise velocity component, U , was measured and the steady disturbance energy was defined based on the spanwise root-mean-square of this component. However, in the present work, V and W components are also available and a similar steady disturbance energy definition is possible for each component, and we distinguish between each energy definition by a superscript. Note that the U , V and W velocities in the following equations are made dimensionless by U_∞ and the (*) superscripts of $U^* = U/U_\infty$, $V^* = V/U_\infty$ and $W^* = W/U_\infty$ are omitted from these equations.

$$\begin{aligned} E_{\text{rms}}^U(x) &= \int_0^\infty [U'_{\text{rms}}(x, \eta)]^2 d\eta, \\ E_{\text{rms}}^V(x) &= \int_0^\infty [V'_{\text{rms}}(x, \eta)]^2 d\eta, \\ E_{\text{rms}}^W(x) &= \int_0^\infty [W'_{\text{rms}}(x, \eta)]^2 d\eta \end{aligned} \quad (3.6)$$

where spanwise rms disturbance of the U , V and W velocities are obtained from the deviation of phase-lock-averaged velocity profiles from the experimental basic state.

$$\begin{aligned} U'_{\text{rms}}(x, \eta) &= \left\{ \frac{1}{N-1} \sum_{k=0}^{N-1} [U(x, \eta, z_k) - \bar{U}(x, \eta)]^2 \right\}^{1/2}, \\ V'_{\text{rms}}(x, \eta) &= \left\{ \frac{1}{N-1} \sum_{k=0}^{N-1} [V(x, \eta, z_k) - \bar{V}(x, \eta)]^2 \right\}^{1/2}, \\ W'_{\text{rms}}(x, \eta) &= \left\{ \frac{1}{N-1} \sum_{k=0}^{N-1} [W(x, \eta, z_k) - \bar{W}(x, \eta)]^2 \right\}^{1/2} \end{aligned} \quad (3.7)$$

Then, the total rms disturbance energy, E_{rms}^T can be defined as:

$$E_{\text{rms}}^T = E_{\text{rms}}^U + E_{\text{rms}}^V + E_{\text{rms}}^W \quad (3.8)$$

Note that, E_{rms}^U , E_{rms}^V , E_{rms}^W , and E_{rms}^T are all dimensionless parameters that represent disturbance energy for a particular streamwise location. Also of interest is the spanwise spectrum of the energies that constitute E_{rms}^T . For this purpose, the steady velocity perturbations over four roughness array wavelengths are Fourier transformed from z to λ so that spanwise power spectral density (PSD) profiles can be generated. The Fourier transformation of the experimental data is performed according to the recommendations of Press et al. (2001). The dimensionless complex Fourier coefficients of discretely sampled N velocity data points, U'_k is given by:

$$H_n^U = \sum_{k=0}^{N-1} U'_k e^{2\pi i k n / N} \quad (3.9)$$

There are many different ways to construct a PSD definition. In the present work, the PSD normalization is selected such that at any height in the boundary layer Parseval's Theorem is satisfied as

$$U_{\text{rms}}'^2 = \int_0^{1/2\Delta z} \text{PSD}^U(\lambda) d(\lambda^{-1}) \quad \text{where} \quad \text{PSD}^U = \frac{2|H_n^U|^2 \lambda_k}{N^2} \quad (3.10)$$

where $1/2\Delta z$ is the appropriate Nyquist frequency in the inverse wavelength domain. Similar expressions can be written for $V_{\text{rms}}'^2$ and $W_{\text{rms}}'^2$. The single-sided PSD in this equation

has the units of meters and once it is integrated over λ^{-1} , the integral is dimensionless, as its equal, $U_{\text{rms}}'^2$ on the left-hand-side. Finally, the steady disturbance energy contained in a dominant spanwise wavelength is defined as the integral of PSD profiles of that wavelength in the η direction. For the streamwise velocity component:

$$E_{\lambda}^U = \int_0^{\infty} \text{PSD}^U(\eta, \lambda) d(\eta). \quad (3.11)$$

Similar expressions can be written for E_{λ}^V and E_{λ}^W , and all these quantities have the units of meters.

3.3 Results

The results of the multicomponent experiments include spatially phase-lock averaged U , V , W , and ω_x data in a 3D rectangular domain. The streamwise range is $310 < x < 550$ mm which corresponds to a Reynolds number range of $2.41 \times 10^5 < Re_x < 4.24 \times 10^5$. The wall-normal and spanwise domains are $0 < y < 4.7$ mm and $-9.5 < z < 9.5$ mm. The grid has variable spacing in the streamwise direction with Δx as low as 2.5 mm in the near wake, gradually increasing to as high as $\Delta x = 50$ mm in the far wake. In the present work $310 < x < 350$ mm will be referred to as the near wake, and $350 < x < 550$ mm will be referred to as the far wake. The grid spacing is constant in the wall-normal and spanwise directions with $\Delta y = 0.1$ mm and $\Delta z = 0.5$ mm. The velocity and vorticity data is presented at each location as a combination of contour lines, colored contours and vectors.

The phase-lock averaged steady flow properties in the roughness wake can be seen in

Figures 10 through 21. In these figures the flow is into the page, the abscissa is the span-wise direction and the ordinate is the wall-normal direction, both displayed in mm's. The roughness element that generates the disturbance extends between $-3.17 < z < 3.17$ mm and $0 < y < 0.724$ mm. The green contour lines indicate 10% increments of dimensionless streamwise velocity U , the vector field represents the dimensionless V and W , and the colored contours represent the streamwise vorticity intensity. The length of the heavy horizontal lines in the figures' top right corner is equal to the length of a vector representing V , W pair equal to 10% of U_∞ . The colored contour levels that represent ω_x are shown on the right-hand side of each figure and have the units of m^{-1} . The limits of the color spectrum is selected such that the contour levels are the same for all figures. Warmer colors (yellow, orange, red etc...) represent a positive streamwise vorticity (into the page), indicating a clockwise fluid rotation in these figures. Conversely, the colder colors (blue, turquoise etc...) represent a negative streamwise vorticity (out from the page), indicating a counter-clockwise fluid rotation in the figures. White color indicates zero streamwise vorticity.

First, the results in the near wake is presented. Immediately downstream of the roughness array, at $x = 310$ mm, the steady flow properties are shown in Fig. 10. First, the streamwise velocity contours indicate a decelerated region in the wake of the roughness element. The deceleration is symmetric around the roughness centerline and compact in a region directly behind the roughness element, lower in the boundary layer. The locations higher in the boundary layer are not effected by the presence of roughness as much as the

lower portions. Second, the V and W velocities are directed downward (towards the plate) along the centerline ($z = 0$). As they approach the plate, they are directed sideways, away from the centerline. Then they change direction one more time around $z = \pm 3.5$ mm and turn upwards (away from the plate) creating two streamwise vortex cores at approximately $y \approx 1.0$ mm. An interesting observation can be made about the spanwise location of the vortex cores: These appear to be around $z \approx \pm 2.75$ mm, which is not directly along the edges of the roughness ($z \approx \pm 3.17$ mm), but just a small distance closer to the centerline. This is consistent with the observation by Gregory and Walker (1956) (Fig. 9) that the horseshoe vortex wraps around the windward side of the roughness and moves towards the centerline close behind the roughness, then diverges from the centerline and leaves a pair of trailing legs parallel to each other. The rotation direction also agrees with the trailing legs of a horseshoe vortex indicated in Fig. 9. From this point forward, this vortex pair will be called the horseshoe pair. Each vortex core of the horseshoe pair is accompanied by an expected local ω_x intensity region with opposite intensities in the lower parts of the boundary layer, that terminate at the wall. Expected symmetries can also be seen in Fig. 10; there is little or no spanwise velocity along the centerline, or along both edges of the spanwise wavelength ($z = \pm 9.5$ mm). The vector magnitudes are less than 5% and only one pair of vortex eyes is distinctly visible.

At $x = 320$ mm, the steady streamwise velocity contours display accelerated flow regions along both sides ($z \approx \pm 3$ mm) of the roughness, and decelerated flow regions along the centerline (Fig. 11). These findings are in good agreement with previous findings of

White and Ergin (2003), and White et al. (2005). The V , W field is similar to that in Fig. 10, with increased velocity vector magnitudes and increased streamwise vorticity intensities. It is interesting to note that a secondary pair of vortices is starting to form outside of the existing horseshoe pair. The rotation of the secondary vortex pair is opposite in direction to that of the horseshoe pair, and this can be observed from the crossflow velocity vectors and the ω_x intensities. These observations remain in agreement with Gregory and Walker's (1956) results who identified up to three plies of vortex pairs in their flow-visualization study. Additionally, the rotation directions indicated in the section cut in Fig. 9 are similar to the measured directions in Fig. 11. A careful investigation of Fig. 9 reveals that the secondary vortex pair appears at a certain downstream distance and not directly in the wake of roughness. In the results of the present work, the secondary vortex is not present at $x = 310$ mm, (Fig. 10), is barely visible at $x = 320$ mm (Fig. 11), and will become more obvious further downstream. This finding is in complete agreement with Gregory and Walker's (1956) results.

At $x = 330$ mm and $x = 340$ mm (Figs. 12 and 13) the steady U contours are similar to those in Fig. 11, with one small difference: The decelerated region in the wake along the centerline is gradually accelerating, because of the downward flow along the centerline produced by the two legs of the horseshoe vortex. The $-V$ velocity along the centerline carries the high momentum fluid from higher parts in the boundary layer to the lower parts, as explained by Landahl's lift-up mechanism. This effect is more visible for the 90% U_∞ contour line in this figure and will become clearer later in this section. At $x = 330$ mm, the

secondary vortex pair is completely visible, with its associated vortex cores at $y \approx 1.0$ mm and $z \approx \pm 3.75$ mm. The spanwise positions of the cores of the horseshoe pair gradually approach $z \approx \pm 2.0$ mm at $x = 340$ mm, which indicates that they have shifted closer to the centerline. More importantly, the magnitudes of V , W , and ω_x are gradually increasing at $x = 330$ mm and $x = 340$ mm; and ω_x intensity reaches its maximum value at $x = 340$ mm.

The final x location in the near wake is at 350 mm and the steady flow properties at this location are presented in Fig. 14. In this figure, the general U , V , W and ω_x properties are similar to Figs. 11, 12 and 13, however, the measured V , W , and ω_x magnitudes are somewhat less than those at $x = 340$ mm. It can be concluded that the vortices generated by the roughness elements develop and get stronger in the first 40 mm and start their decay approximately 50 mm downstream of the array location.

The steady flow properties in the far wake are shown in Figs. 15 through 21. In these figures, the streamwise velocity continues its gradual acceleration first in the upper parts of the boundary layer, and then in the lower parts further downstream. This is due to the downward flow along the centerline and explains the observations of Kendall (1981) on the decelerated region in the near wake and the accelerated region in the far wake. The roughness element generates an initial deceleration directly behind itself which is gradually reduced by the downward flow from both legs of the horseshoe vortex system. The streamwise velocity disturbance also gradually diffuses in the spanwise direction with increasing downstream distance, covering a larger spanwise region in the far wake. V , W , and ω_x all continue their monotonic decrease in the far wake. Additionally, four counter-

rotating vortex cores at $y \approx 1.0$ mm, and their associated counterparts in the lower part of the boundary layer, all seem to cover a larger region with increasing streamwise distance.

In order to check the accuracy of the reported steady velocity properties, the V and ω_x calculations were performed with both a high and a low resolution in the x direction. In the near wake, Figures 22 and 23 show two different calculations of the V and ω_x fields at $x = 330$ mm with two different spacings; $\Delta x = 2.5$ mm and $\Delta x = 20$ mm, respectively. The match of the two figures indicate that a spacing of $\Delta x = 2.5$ mm is sufficient for calculation of V and ω_x in the near wake. Similarly in Figures 24 and 25, V and ω_x calculations in the far wake are shown with $\Delta x = 5$ mm and $\Delta x = 25$ mm, respectively. These figures indicate that, in the far wake there is virtually no difference between results with a $\Delta x = 5$ mm spacing and a $\Delta x = 25$ mm spacing.

The global distribution of the streamwise velocity component, U , is visualized in Fig. 26. In this figure, two surfaces of constant velocity are shown in a perspective view in the 3D measurement domain. The lower (blue) surface represents the 20% of U_∞ and the upper (yellow) surface represents 80% of U_∞ . In the near wake, there is a decelerated region directly behind the roughness element and this region is more emphasized on the lower surface, than the upper surface. The accelerated regions along both edges of the roughness element appear at a very short downstream distance in the near wake and these lead to the streaky structures further downstream. The streaky structures are initially narrow and gradually cover a larger spanwise region. This finding is in complete agreement with transient growth theory, because smaller spanwise wavenumbers are expected to grow

for a larger distance. The velocity along $z = \pm 9.5$ mm represents the basic state and the boundary layer growth is visible on both ends of the wavelength for the upper surface. The accelerated region in the far wake is more apparent in this figure: The basic state profile attains $80\% U_\infty$ at a height of $y \approx 2.0$ mm at $x = 550$ mm. However, directly behind the roughness centerline, $80\% U_\infty$ is attained at a smaller distance from the flat surface clearly indicating an accelerated profile.

A careful investigation of Figs. 10 through 21 reveal another interesting feature of the flow field: There is a thin wall-normal region between $y = 0.4$ mm and $y = 0.6$ mm in the boundary layer, where the streamwise vorticity is nearly zero. A better visualization of this no-vorticity region is presented in Figs. 27, 28 and 29, where contours of U and ω_x are shown at (x,z) planes 0.3 mm, 0.55 mm and 0.8 mm above the flat surface, respectively. The ω_x contour levels in these figures are similar to those in Figs. 10 through 21. In Fig. 27 the ω_x intensity in the lower part of the boundary layer follow $15\% U_\infty$ lines, and the decay of the ω_x intensity in the streamwise direction is clearly visible. The no-vorticity plane appears to be approximately 0.55 mm above the flat surface (Fig. 28). The downstream evolution of ω_x intensity of the counter-rotating horseshoe and the outer secondary vortex pairs are shown in Fig. 29. In this figure the inner horseshoe vortex pair starts at $z \approx \pm 3.0$ mm, gradually approaches towards the centerline in the near wake for a certain streamwise distance and remains parallel afterwards. This finding is agreement with Gregory and Walker's (1956) results and can be compared with Fig. 9. On the other hand the outer secondary vortex pair slowly diverges from the centerline and remains parallel after-

wards. These results indicate that the roughness-induced steady vortices slowly diffuse in the spanwise direction with increasing streamwise distance. These pairs are aligned with the constant U velocity lines with 35-40% U_∞ at $y = 0.8$ mm. The white regions between the counter-rotating pairs represent the regions where only wall-normal velocity exists.

A final point can be made on the wall-normal extent of the perturbations. One expects zero U' and zero W' at the edge of the boundary layer, and the wall-normal extent of the current experiments are sufficient to display this behavior. However, in Figs. 10 through 21 the V' and ω_x intensities reach the edge of the boundary layer. This rises two concerns about the experimental data: First, the fact that V' perturbation is nonzero at the boundary layer edge implies that several energy definitions for V defined in Sec. 3.3, E_{rms}^V and E_λ^V , could contain significant error. Therefore, these are not presented in the results. This can be explained by an insufficient wall-normal resolution in the upper parts of the boundary layer during data acquisition. As mentioned before, the Dice boundary layer scanning technique uses an uneven measurement grid in the y direction, with larger wall-normal steps outside the boundary layer and gradually decreasing steps within the boundary layer. It is possible that a large Δy was used in the region where negative $\partial V / \partial y$ values could have been measured during the experiments, and these data points were missed because of the large wall-normal step.

Second concern is related with the location of the vortical structures within the boundary layer. This point is of extreme importance, because the quantitative and qualitative differences between realizable and optimal disturbances are attributed to the differences in

wall-normal locations of the vortex core generated by roughness and the computed vortex core that produces an optimal disturbance (Fransson et al. 2004). However, the fact that ω_x perturbation is nonzero at the boundary layer edge is most likely an artifact of the non-zero $\partial V/\partial z$ magnitudes in the upper parts of the boundary layer. Fortunately, the cores of the vortices in Figs. 10 through 21 are distinctly visible at $y \approx 1$ mm, so that the vortices can be assumed to be localized within the lower parts of the boundary layer. The resolution of the fact that V and ω_x have non-zero values at the edge of the boundary layer requires further experimental investigation with higher wall-normal resolution in the upper parts of the boundary layer and with an increased wall-normal range.

The streamwise evolution of several disturbance energy measures are of interest. Figure 30 shows the streamwise evolution of E_{rms}^U and E_{rms}^W . In Fig. 30, E_{rms}^W experience algebraic growth followed by exponential decay. On the other hand, E_{rms}^U experience a rapid initial decay before transient growth. The streamwise behavior of E_{rms}^U is in good agreement with the previous results of White and Ergin (2003) and White et al. (2005). Several important conclusions about the flow field can be drawn from Fig. 30. First, Landahl's lift-up mechanism suggests that a small V' perturbation produces a much larger U' perturbation. Furthermore, in a streamwise oriented vortex, one expects that the V' and the W' perturbation to be the same order of magnitude. This translates into "a small W' perturbation produces a much larger U' perturbation". Indeed, the energy of W' perturbation is smaller than the energy of U' perturbation in Fig. 30, and this is in complete agreement with the lift-up concept.

Second, the streamwise growth characteristics of E_{rms}^U and E_{rms}^W reveal further physics about the flow field. A large E_{rms}^U value in the immediate near wake is expected, because of the presence of a separated flow region directly behind the roughness element, where the U velocity profile significantly deviates from the Blasius profile. With increasing streamwise distance the separation bubble terminates and the U profiles start to become fuller, and this gradually decreases the amount of deceleration and the amount of disturbance energy along the centerline. This is the possible explanation of E_{rms}^U 's rapid decay in the near wake region.

In the region where E_{rms}^U decays rapidly, E_{rms}^W experiences algebraic growth. The fact that E_{rms}^W is growing means that the strength of the vortex is increasing, and that the horseshoe vortex is constantly transporting higher U velocities from the upper parts of the boundary layer to the lower parts, eventually creating accelerated profiles along the centerline. In the mean time, the disturbance energy in the accelerated regions along both edges start to increase, contributing to the total magnitude of E_{rms}^U . At this point, (around $x = 325$ mm in Fig. 30) E_{rms}^U starts its algebraic growth. Since the roughness-induced vortices start to decay due to viscous forces after $x = 350$ mm, the growth of E_{rms}^U is transient and E_{rms}^U eventually experiences exponential decay starting around $x = 400$ mm. This suggests that the lift-up mechanism is in motion and the streamwise-oriented counter rotating vortices are in complete control of the streamwise velocity field.

Another important aspect of U and W disturbances is their distribution among their spanwise modes. For this purpose, a spanwise Fourier transform is performed on the U

and W data and the resulting PSD profiles are integrated to identify the disturbance energy distribution in particular wavelengths using Eqn. 3.11. Since the PSD profiles indicate that the disturbance energy is concentrated in the harmonics of λ_k (i.e. λ_k , $\lambda_k/2$, $\lambda_k/3$, and $\lambda_k/4$) and in the infinite wavelength (DC component), only the energy contained in these wavelengths is investigated extensively.

Figure 31 shows the streamwise evolution of E_{DC}^U and E_{DC}^W . First observation is that W perturbation contain virtually no energy in the infinite wavelength. On the other hand, E_{DC}^U contains significant energy in the immediate wake of roughness array which undergoes a sharp decrease in the next 40 mm's reaching to zero disturbance level, and starts a monotonic increase in the remaining streamwise measurement domain.

The distribution of disturbance kinetic energy contained in the harmonics of λ_k is shown in Figs. 32 and 33. In both figures, the points represent the experimental data and the solid curves represent the individual nonlinear least-squares fits performed on these values. The empirical function used in the performed fits is a modified version of what is proposed by Ergin (2003):

$$f(x) = a(x - x_k)e^{-(x-x_k)/b}. \quad (3.12)$$

In this Eqn. a is the algebraic growth rate and has the units of mm^{-1} . On the other hand, b is called the decay length, because it determines the location of maximum energy obtained by this function measured from the roughness location, x_k . Both b and x_k have the units of mm's. For these fits x_k is replaced with c in order to obtain better results for the virtual location of the initial disturbances.

In general, the results associated to the streamwise evolution of E_{λ}^U in Fig. 32 is in excellent agreement with the previous experimental findings of White and Ergin (2003) and White et al. (2005). The streamwise evolution of $E_{\lambda_k}^U$ in Fig. 32, is very similar to that of E_{DC}^U explained above: Immediately behind the roughness array $E_{\lambda_k}^U$ contains significant power when compared to $E_{\lambda_k/2}^U$, $E_{\lambda_k/3}^U$ and $E_{\lambda_k/4}^U$. Similar to E_{DC}^U , $E_{\lambda_k}^U$ experiences a sharp decrease to zero-disturbance level at approximately $x = 340$ mm and experiences growth for the rest of the streamwise measurement domain. A careful comparison between the initial decay rates of E_{DC}^U and $E_{\lambda_k}^U$ reveal that the initial sharp decrease of E_{DC}^U and $E_{\lambda_k}^U$ is almost identical. This brings us to an important conclusion about the E_{rms}^U 's sharp decay in Fig. 30: the initial sharp decay of E_{rms}^U in the near wake is due to E_{DC}^U and $E_{\lambda_k}^U$. This once again confirms the discussion related to the growth of E_{rms}^U and E_{rms}^W in Fig. 30, because the single broad decelerated region immediately behind roughness contains most power in λ_k and infinite wavelengths.

In Fig. 32, the energy contained λ_k 's first harmonic, $E_{\lambda_k/2}^U$, experience a rather fast decay in the near wake followed by a slower decay in the far wake. No attempt was made to fit and empirical function to $E_{\lambda_k/2}^U$, since no transient growth is observed for this component. Similarly, in Fig. 33 $E_{\lambda_k}^W$ and $E_{\lambda_k/2}^W$ contain very little energy with little streamwise activity; thus fits are not performed for these data. On the other hand, the energy contained in $\lambda_k/3$ and $\lambda_k/4$ wavelengths experience unmistakable transient growth for U and W velocity perturbations in Figs. 32 and 33, respectively. The nonlinear least squares fits to these data are performed using $f(x)$ in Eqn. 3.12 with three parameters, and the best fit values for a ,

Table 4: Best fit parameters obtained for $E_{\lambda_k/3}^U$, $E_{\lambda_k/3}^W$, $E_{\lambda_k/4}^U$ and $E_{\lambda_k/4}^W$.

	$a \pm \sigma_a$	$b \pm \sigma_b$	$c \pm \sigma_c$
$E_{\lambda_k/3}^U$	$2.51 \times 10^{-5} \pm 2.6\%$	$79.3 \pm 2.0\%$	$306.9 \pm 0.2\%$
$E_{\lambda_k/3}^W$	$7.99 \times 10^{-6} \pm 4.4\%$	$76.7 \pm 3.3\%$	$309.1 \pm 0.3\%$
$E_{\lambda_k/4}^U$	$1.34 \times 10^{-5} \pm 3.2\%$	$53.7 \pm 2.4\%$	$311.0 \pm 0.1\%$
$E_{\lambda_k/4}^W$	$6.18 \times 10^{-6} \pm 6.4\%$	$51.5 \pm 4.8\%$	$312.7 \pm 0.3\%$

b and c are reported in Table 4. The quality of these fits using $f(x)$ in Figs. 32 and 33 is reflected by the associated uncertainty values presented in Table 4. Several comparisons can be made from these results. First, within the same wavelength ($\lambda_k/3$ or $\lambda_k/4$), E^U 's growth rate is always larger than E^W 's growth rate, i.e., $a_{\lambda_k/3}^U > a_{\lambda_k/3}^W$ and $a_{\lambda_k/4}^U > a_{\lambda_k/4}^W$. This fact manifests itself in the peak energies obtained by these disturbance measures in Figs. 32 and 33, i.e., $E_{\lambda_k/3,\max}^U > E_{\lambda_k/3,\max}^W$ and $E_{\lambda_k/4,\max}^U > E_{\lambda_k/4,\max}^W$.

Second comparison is related to the decay lengths. It is observed that $\lambda_k/3$'s decay length is 78 mm, and is noticeably larger than $\lambda_k/4$'s decay length which is 53 mm. This indicates that the shorter wavelengths reach their maxima sooner than the longer wavelengths which is exactly what is predicted for transient disturbances.

A final comparison is made regarding the virtual initial streamwise locations of the disturbance measures in Table 4. Similar to the decay length, there is no noticeable difference in c values between $\lambda_k/3$ and $\lambda_k/4$ wavelength components. The virtual location of disturbance initiation appears to be between 307 mm and 313 mm, a short distance downstream of the roughness array. This is probably because it takes a short while for the

disturbances to take their initial forms and amplitudes upon leaving the roughness location which in reality extends from 296.8 mm to 303.2 mm. It is expected that there are regions of separated flow on the leeward side of the roughness elements and the streamwise extent of these regions could account for the difference between the calculated c values and the roughness location.



Chapter 4

UNSTEADY STREAMWISE VELOCITY MEASUREMENTS

4.1 Motivation and Background

Recent computational studies by Cossu and Brandt (2002, 2004) show that large-amplitude, stationary transient disturbances may be capable of suppression of the growth of TS waves, which could possibly delay the transition to turbulence. In Cossu and Brandt's numerical studies, spanwise-invariant disturbances in the TS-unstable frequency band are introduced into a boundary layer that is modulated by stationary, spanwise-periodic optimal disturbances. As the amplitude of the transient disturbances is increased, the unsteady TS-like disturbances become increasingly modulated in the spanwise direction and experience progressively slower growth in the streamwise direction. Once the amplitude of the transient disturbances reaches about 20% of U_∞ at their maximum-growth location, the unsteady disturbance growth is completely suppressed. Large-amplitude transient disturbances may be generated using three-dimensional (3D) surface roughness for suppressing or possibly delaying the TS-based transition scenarios. This control approach would be similar in appearance to the crossflow-dominated transition control approach using subcritically spaced roughness elements developed by Saric et al. (1998) but would be quite different in terms of the instability mechanisms involved.

Implementing the control approach inspired by Cossu and Brandt's (2002, 2004) calculations involves introducing the largest possible stationary transient disturbances because as the transient disturbances' amplitudes are increased there is progressively less TS growth. However, one means of introducing stationary disturbances — placing arrays of 3D roughness elements on the surface — will itself lead to transition if the roughness elements' amplitudes exceed $Re_{k,crit}$. Another danger is that the stationary transient disturbances may become unstable to a high-frequency secondary instability once their amplitude exceeds 26% of U_∞ (Andersson et al. 2001). Therefore, there is considerable uncertainty as to whether the control approach is feasible. This underlines the most important problem to be addressed in transient growth investigations, the disturbance receptivity to surface roughness.

The role of isolated 3D roughness elements was initiated in the 1950's by the flow visualization studies of Gregory and Walker (1956) who established a basic understanding of the horseshoe and hairpin vortex structure that exists around a moderate- to large-amplitude roughness element in a boundary layer. Gregory and Walker's results have been enhanced by numerous other flow-visualization studies that culminate with the experiments of Acarlar and Smith (1987). These studies lead to the conclusion that the topology of the steady flow about an isolated 3D roughness element in a boundary layer consists of a steady horseshoe vortex that wraps around the upstream side of an isolated roughness element and trails two or more steady counter-rotating legs downstream. The sense of rotation is such that along the line directly behind the roughness element the vertical velocity induced by the

primary vortices is directed down toward the surface. The steady aspects of this flow topology is presented in the previous chapter.

The present chapter investigates the unsteady flow in the wake of larger-amplitude cylindrical roughness elements. In terms of the unsteady flow, flow visualization experiments agree that at sufficiently high values of Re_k , unsteady hairpin vortices originate periodically from the separated region just aft of the roughness element. The heads of these vortices move up and away from the roughness element and are convected downstream. Acarlar and Smith (1987) show that shedding originates from hemispherical roughness elements at around $Re_k \approx 120$ and that a Strouhal number based on the shedding frequency, roughness height, and the unperturbed boundary layer velocity at the roughness height, $St = fk/\overline{U}(k)$, increases from below 0.05 at the lowest Re_k values that produce shedding to values between 0.2 and 0.4 as Re_k exceeds 1200. Hotwire measurements by Klebanoff et al. (1992) give similar results but Klebanoff et al. cast their data in terms of a Strouhal number based on displacement thickness, $St = f\delta^*/\overline{U}(k)$, and find that $St \approx 0.3$ from $Re_k = 300$ to 1700 for hemispherical roughness elements. Klebanoff et al. further find that cylindrical elements with $D = k$ are found to produce somewhat lower Strouhal numbers that increase slightly from $St \approx 0.2$ at $Re_k = 400$ to $St \approx 0.25$ at $Re_k = 1200$.

Regarding $Re_{k,crit}$, the critical roughness height Reynolds number for immediate roughness induced transition, the general finding is that the effect of isolated 3D roughness elements is “more critical” than 2D roughness strips (Dryden 1959; Smith and Clutter 1959; Tani 1961; von Doenhoff and Braslow 1961; Tani 1969; Sedney 1973). By “more critical”

it is meant that that as $Re_{k,crit}$ is exceeded, the transition location moves forward rapidly but that below $Re_{k,crit}$, an isolated 3D element has little or no effect on transition. This is different from the behavior of boundary layers with 2D roughness that tend to show transition gradually moving forward with increasing Re_k .

Studies on 3D elements originating with those by Klebanoff et al. (1955) establish that $Re_{k,crit}$ lies between 600 and 900 for isolated elements whose height-to-diameter ratio is approximately unity. The most recent and detailed studies of $Re_{k,crit}$ are also by Klebanoff et al. (1992) who find somewhat lower values of $Re_{k,crit}$ than previous studies (e.g., Tani et al. (1962)) but have a somewhat less-stringent definition of transition. A key finding of Klebanoff et al. (1992) is that even at limiting cases of high Re_k , there is always a finite distance between the roughness element and the resulting turbulent flow. The implication is that at high Re_k 's transition occurs as the result of an instability of the distorted flow in the roughness element's wake that requires some finite distance to develop. Other key findings of the early transition correlation work are that $Re_{k,crit}$ scales roughly as $(k/D)^{2/5}$ meaning that transition occurs for progressively lower $Re_{k,crit}$ values as the roughness diameter is increased (von Doenhoff and Braslow 1961; Tani 1969) and that elements in spanwise arrays behave as isolated elements when their spacing is three times their diameter or larger (von Doenhoff and Braslow 1961).

While transition correlations are useful, they do not reveal the detailed mechanism for transition nor do they assist in designing transition control strategies besides simply placing design limits on acceptable roughness levels. This requires detailed flow studies which

consider both steady and unsteady disturbances that can be used in conjunction with stability theories. However, there have been relatively few studies of the detailed flow field in the wake of isolated roughness elements that can be used to inspire or validate mechanisms-based theories. Notable exceptions among the early literature are by Mochizuki (1961), Tani et al. (1962), Kendall (1981) and Klebanoff et al. (1992). Recent measurements by White and coworkers (White and Ergin 2003; White and Ergin 2004; White, Rice, and Ergin 2005) on the receptivity of transient disturbances to surface roughness provide additional data. All of these studies find complicated distortions of the steady flow in the roughness elements' wakes.

Vortex-shedding in the wake of moderate- to large-amplitude elements supports the notion that transition beyond $Re_{k,crit}$ is due to an instability of elements' distorted wake flow. However, there are only a few studies that correlate the fluctuation intensities to particular locations in the wake. One of these is a series of u'_{rms} measurements by Klebanoff et al. (1992) in the near wake of a hemispherical roughness element. The measurements were obtained across a range of span locations for various heights in the boundary layer. High in the boundary layer there is a single peak of u'_{rms} centered at the roughness location while lower in the boundary layer there are two peaks that are located on either side of the roughness centerline. These results hint at a complicated instability associated with the various features of the distorted wake flow.

The main objective of the unsteady experiments is to link the steady and unsteady disturbance fields and to use this link to reveal the instability mechanism whose existence

is suggested by Klebanoff et al.'s (1992) data. The hope is that a detailed understanding of this mechanism will lead to an improved sense of why $Re_{k,crit}$ values are what they are and how large-amplitude transient disturbances may be created for TS-suppression purposes.

4.2 Data Acquisition and Analysis

In the present experiments, measurements of the steady and unsteady streamwise velocity fields were performed using two single-probe hotwire sensors with their probes oriented normal to the freestream. One sensor was in the freestream and one was in the boundary layer. The sting used in the unsteady experiments is the one used in the works of White and Reshotko (2002); White and Ergin (2003); and White et al. (2005). The calibration of the single normal sensors is less complicated than that of slanted sensors. For normal-sensor calibration the sting is positioned so that both hotwires are located in the freestream at a tunnel cross section that includes the Pitot tube. Then, both sensors are calibrated simultaneously by performing a velocity calibration, during which the tunnel is operated at different speeds while recording the Pitot tube velocity and hotwire voltages. The calibration constants are determined using a simplified version of Eqn. 3.1:

$$V_e = (A + BE^2)^n = \tilde{V} \quad (4.1)$$

where the effective cooling velocity is simply equal to the freestream air speed. Typical values of calibration constants obtained during the experiments of the present work are similar to those for the normal probes in Table 1.

For the unsteady velocity measurements, the roughness elements consist of 6.35-mm-diameter, 102- μ m-thick adhesive-backed paper labels that are stacked to provide desired roughness heights. Seven, eight or nine layers are used to provide Re_k values of 202, 264 and 334, respectively. These values are selected because they include both subcritical and supercritical values that reveal the differences between unsteady fluctuations that do and do not lead to roughness-induced transition in the elements' near wake. Based on the k/D values of these three roughness heights, $Re_{k,crit}$ is expected to lie between 250 and 275. Rice (2004) has previously observed roughness-induced breakdown in the Case wind tunnel under similar conditions at $Re_k = 254$. Although 12 elements are applied on the surface, measurements are taken in the wake of 8 elements in the middle, and detailed results are presented in the wake of 4 elements. The boundary layer scanning technique is Slice (see Sec. 2.5.1). During the scans the 19 mm spanwise wavelength, λ_k is covered in 12 hotwire steps. During data acquisition, the voltage readings from both sensors are converted to effective cooling velocities using Eqn. 4.1. The effective cooling velocity of the boundary layer sensor is normalized by the freestream velocity and these are recorded for further analysis.

Of interest in this work are the steady disturbances of the Blasius boundary layer that are known to undergo transient growth in the streamwise direction, the unsteady disturbances associated with vortex shedding and roughness-induced transition and the possible connections between both types of disturbances. To generate a quantitative measure of these disturbances and their growth, unsteady data is analyzed by decomposing the stream-

wise velocity field into a steady, spanwise-invariant basic state, a steady disturbance and an unsteady disturbance:

$$U(x, y, z, t) = \overline{U}(x, y) + U'(x, y, z) + u'(x, y, z, t). \quad (4.2)$$

As for the multi-component measurements, $\overline{U}$ is an appropriate experimental basic state that is the average of the boundary layer profiles at the midpoints between roughness elements. This approach means that U' includes a contribution due to the spanwise-invariant part of the disturbance. All velocities are made dimensionless using U_∞ .

At each measurement location, the velocity is sampled at 10 kHz, so frequencies up to 5 kHz can be resolved by a Fourier transform. The time signal is divided into segments of 1024 points and the number of segments increases from 8 to 12 to 18 as the boundary layer sensor approaches to the flat surface. This permits the measurements closest to the wall to be well resolved without requiring that all points be sampled as extensively. The average of the individual segments' transforms at each position gives the one-sided, magnitude-averaged, temporal power spectral density (PSD) of that particular measurement location. Similar to Eqn. 3.9, the dimensionless complex Fourier coefficients for the discrete time signal in each segment is given by the following summation:

$$H_n = \sum_{k=0}^{N-1} u'_k e^{2\pi i k n / N} \quad (4.3)$$

For the unsteady experiments the PSD normalization is such that the Parseval's Theorem is

satisfied in the time and frequency domains using

$$u_{\text{rms}}'^2 = \int_0^{f_{Ny}} \text{PSD}(f) d(f) \quad \text{where} \quad \text{PSD} = \frac{2|H_n|^2}{f_{\text{samp}} W_{ss}}. \quad (4.4)$$

In Eqn. 4.4 f_{Ny} is the Nyquist frequency and f_{samp} is the sampling frequency.

The PSD definition in Eqn. 4.4 is different than the PSD^U definition in Eqn. 3.10. This is because data windowing is applied on $u'(t)$ to prevent PSD amplitude “leakage” to neighboring frequency bins. According to Press et al. (2001), when a function is non-zero at the endpoints of the data window, there is a danger leakage to neighboring frequency bins. The solution is data windowing, modifying the original signal with a window function. The steady perturbations $U'(z)$, $V'(z)$ and $W'(z)$ measured during the multicomponent experiments are zero on both ends of the spatial domain and, therefore, windowing is not used during multicomponent data analysis. However, the value of the unsteady velocity perturbation, $u'(t)$ is not necessarily zero on both ends of the segment and is prone to considerable leakage. To avoid this the segments are multiplied by a Hann window function, $\omega_j = 0.5[1 - \cos(2\pi j/N)]$, which starts and ends at zero and changes smoothly in between. Since the original function is multiplied with the Hann window, the power of the windowed function is less than the power of the original function. That is why the PSD amplitude in Eqn. 4.4 is divided by the term, W_{ss} , which is the squared and summed window function.

Using this velocity decomposition, the disturbance energy associated with the steady streamwise disturbance is given by Eqn. 3.6. The total disturbance energy associated with

the unsteady fluctuations is obtained from the temporal u'_{rms} data using

$$e_{\text{rms}}(x) = \int_{\eta=0}^{\infty} \int_{z=-1/2}^{1/2} [u'_{\text{rms}}(x, \eta, z)]^2 d\eta dz \quad (4.5)$$

where z is made dimensionless using λ_k . The unsteady disturbance energy of 100-Hz bands centered on $f_0 = 360$ Hz, 720 Hz and 1440 Hz (the frequencies that will be of interest below) are computed using

$$e_{f_0}(x) = \int_{\eta=0}^{\infty} \int_{z=-1/2}^{1/2} \int_{f=f_0-\Delta f}^{f_0+\Delta f} \text{PSD}(x, \eta, z, f) d\eta dz df \quad (4.6)$$

where $\Delta f = 50$ Hz. The PSD normalization discussed above makes it possible to interpret e_{360} , e_{720} and e_{1440} as fractions of e_{rms} . Because U'_{rms} is generated using data from spanwise lines that extend across eight roughness wavelengths, E_{rms}^U is representative of the stationary disturbance that exists across the entire spanwise field. However, e_{rms} , e_{360} , e_{720} and e_{1440} are computed across a single roughness wavelength that is not phase-lock averaged. The reason for this is that the unsteady disturbances are somewhat variable from element to element and therefore, it is most appropriate to consider each separately.

4.3 Results

The lowest-disturbance configuration for the unsteady experiments is the roughness configuration with $Re_k = 202$. This value is below $Re_{k,\text{crit}}$ but is large enough to produce unsteady fluctuations of significant amplitude. The steady flow produced by this roughness

array is illustrated by Figs. 34 and 35 which give the spatially phase-lock averaged steady streamwise velocities at $x = 310$ mm and 330 mm, respectively. In each plot, the rectangle indicates the approximate projection of the $Re_k = 202$ roughness element onto the η, z plane. The $\overline{U}(\eta)$ profiles indicated by heavy solid lines are in excellent agreement with the Blasius profile. The two larger-amplitude roughness arrays with $Re_k = 264$ and 334 generate nearly identical $\overline{U}(\eta)$ profiles and $U'_{\text{rms}}(\eta)$ profiles whose amplitudes scale roughly as Re_k .

The evolution of the steady disturbance energy generated by the three roughness array configurations is shown in Fig. 36. In all configurations the total disturbance energy decreases rapidly just aft of the roughness elements and undergoes transient growth farther downstream. In the $Re_k = 202$ and 264 configurations the algebraic growth rate decreases with increasing x and, beyond $x = 400$ mm, the total disturbance energy remains roughly constant for the remaining portion of the measurement domain. This behavior is consistent with what is observed in other roughness-induced transient growth experiments of White (2002), White and Ergin (2003), and White et al. (2005). However, for $Re_k = 334$, E_{rms} undergoes a second rapid decrease beginning at $x = 425$ mm. This occurs because the roughness is supercritical and trips turbulence. The resulting turbulent mixing reduces the spanwise velocity gradients and, consequentially, the energy of the steady disturbances. The flow is observed to be completely turbulent downstream of $x = 475$ mm.

4.3.1 Non-Transitional Configuration

An understanding of the unsteady disturbances' evolution is obtained by monitoring the downstream evolution of the fluctuation intensity. Figure 37 shows contours of the steady and unsteady streamwise velocities for the $Re_k = 202$ configuration at $x = 310, 350, 400$ and 500 mm. Although data is available behind eight roughness elements only one half of the spanwise domain is plotted. The colored contours in this figure indicate the temporal u'_{rms} velocity fluctuation intensity. (The contour levels follow an exponential distribution so that wide variations in u'_{rms} can be visualized.) The maximum value of u'_{rms} in this figure is 0.4% of U_∞ . Although great attention is given to produce identical roughness elements, there are significant variations among roughness elements' fluctuation intensities. Therefore, the spanwise phase-locked averaging technique is not used for unsteady fluctuation intensities and data in the wake of the second roughness element centered at $z/\lambda_k = -0.5$ will be used for analysis.

In the near wake at $x = 310$ mm, the fluctuations are weak and concentrated mostly in small lobes directly downstream of the roughness elements' centerline ($z/\lambda_k = -1.5, -0.5, 0.5$ and 1.5) around $\eta = 1.5$. Note that these are the locations where the local velocity profile has an inflection point in the η direction, $\partial^2 U / \partial \eta^2 = 0$. This can be seen in Fig. 38 where the phase locked centerline velocity profile, U_c , the derivative of this profile, $\partial U_c / \partial \eta$ and the basic state profile are plotted at $x = 310$ mm. In this figure, the $\partial U_c / \partial \eta$ profile is not drawn in the region close to the surface where the velocity cannot be measured. The fact that the inflection point and the maximum u'_{rms} intensity occur at the same point indi-

cates that the fluctuating lobes are likely the result of a Kelvin–Helmholz (KH) instability mechanism of a local shear layer in the η direction. Lobes of increased fluctuation intensity at these points will be referred to as heads. The power spectra obtained at the heads (points indicated by A, B, C and D in Fig. 37) are compared in Fig. 39. The spectra look similar even though a lobe is not visible behind the second element at B due to the selection of the contour levels.

The second roughness element's fluctuation intensity is concentrated downstream of its right edge instead of downstream of its centerline, possibly because the local shear layer in the z direction is stronger than in the other roughness wakes. This is clearly illustrated in Fig. 40 in which U and $\partial U / \partial z$ are plotted for $\eta = 1.25$. The streamwise velocity deficits behind roughness are clearly visible. The local inflection points in spanwise direction ($\partial^2 U / \partial z^2 = 0$) appear exactly at the locations where the fluctuations are largest and this indicates a KH instability generated by a local shear layer in the z direction. Lobes of increased fluctuation intensity at these points will be referred to as legs. Although other elements' $\partial U / \partial z$ maxima are comparable to the second element's wake, the lobe appears only in the second element's wake, at point E because only the fluctuations near E are greater than the the minimum contour level.

The power spectra obtained at the head and in the legs can be compared to a reference point that is not in the wake of elements. For this purpose a point located between the elements at $\eta = 1.25$ is selected and will be referred to as the midpoint. The power spectra obtained at the head, right leg and midpoint (points B, E, and F, respectively, in Fig. 37) are

compared in Fig. 41. Peaks at 360 Hz and 720 Hz are visible as is a band of frequencies from 300 Hz to 800 Hz at the head and the right leg. Additionally, low-frequency fluctuations centered at 35 Hz are also present in the leg. As expected, the magnitudes at the midpoint are noticeably less than the magnitudes at the head and in the right leg.

Moving downstream to $x = 350$ mm, the fluctuations are stronger and cover a wider spanwise region. The lobes have evolved into inverted V-shaped structures that are roughly symmetric about the roughness elements' centerlines. This implies that the local shear layers in the z direction on both sides of the element join with the local shear layer in η direction along the centerline, forming the complex 3D shear layer that is hinted at by Klebanoff et al.'s (1992) data. The shear layer is in the wall-normal direction along the roughness centerline and is in the spanwise direction closer to the surface on both sides. Between these locations the shear layer's direction gradually changes forming a curved locus of inflection points. The inverted V structures could be interpreted as elongated hairpin vortices tethered to the roughness elements on their spanwise edges. It is suspected that the locus of inflection points constitutes the axis of the unsteady hairpin vortex.

Figures 42 and 43 are equivalent to Figs. 38 and 40; they show local shear layers in the η and the z directions at $x = 350$ mm. In Fig. 42 the $\partial U_c / \partial \eta$ profile has a broad peak extending from $\eta \approx 1$ to $\eta \approx 2$. This translates into more extended fluctuation lobes in the wall-normal direction and these are clearly visible in Fig. 37. Similar to $x = 310$ mm, the local inflection points in spanwise direction appears exactly at the location where the fluctuation intensity is maximum reinforcing the notion that the fluctuations in the legs are

generated by a KH instability associated with shear in the z direction.

Figure 44 shows the power spectra obtained at the head, right leg and the midpoint at $x = 350$ mm, which corresponds to points G, H and I, respectively. Compared to Fig. 41, the power spectrum of the midpoint remains essentially unchanged, whereas the spectra at the head and the right leg show a band of amplified frequencies extending from 300 Hz to 800 Hz as well as a sharp peak near 360 Hz. For the experiment's conditions TS unstable frequencies are between 100 Hz to 200 Hz and, therefore, the observed fluctuations are not associated with the TS mechanism. In order to link the observed frequencies in the power spectra with the proposed KH instability mechanism, the local shear layer in the z direction is considered. In an attempt to obtain a representative frequency for the local shear layer between spanwise locations of G and H we calculate $[U(z_G) - U(z_H)]/(z_H - z_G)$ at constant $\eta = 1.25$ and find a frequency of approximately 750 Hz. In Fig. 44 this frequency, and frequencies of a similar order contain significant power at point H, supporting that the fluctuations are not unsteady transient disturbances which would exist at frequencies below TS frequencies but are results of a KH instability that arises at the spanwise inflection points of the streamwise velocity. A similar calculation in the η direction around the wall-normal inflection point, G results a frequency of 5500 Hz, a frequency which is outside the measured frequency domain.

At $x = 400$ mm the fluctuation intensity decreases and the structures split into as many as three lobes that cover an even larger spanwise region. It is possible that the structures split into four lobes but that two or three lobes are sometimes observed because of the

unsteady measurements' spanwise resolution. At $x = 500$ mm, in the wake of the second roughness element, the u'_{rms} distribution is split into four nearly identical lobes entered exactly at the $(\partial^2 U / \partial z^2)$ inflection points. Although the fluctuation intensity is greatly decreased from upstream, the pattern continues to indicate that the fluctuations are likely to be the still-decaying remnants of the KH instability. Figure 45 shows the power spectra obtained at $\eta = 2$, locations indicated by the labels J, K and L in Fig. 37. The power spectra obtained at J and L are essentially identical while the power spectrum at K, where u'_{rms} intensity and $\partial U / \partial z$ is a maximum, contains more power at low frequencies but no significant power in the expected high-frequency band. This leads us to the conclusion that the fluctuations between 300 and 800 Hz have decayed for this subcritical configuration.

The preceding discussion for $Re_k = 202$ configuration applies as a whole to the second subcritical roughness configuration with $Re_k = 264$ except for the fact that the $Re_k = 264$ configuration produces somewhat larger fluctuation intensities. For this reason a detailed description of the $Re_k = 264$ configuration is omitted for brevity.

4.3.2 Transitional Configuration

Figure 46 shows the distribution and evolution of u'_{rms} for the $Re_k = 334$ configuration with the same U and u'_{rms} contour levels used in Fig. 37. It is immediately apparent that both the unsteady fluctuation intensity and the steady disturbance levels are higher than those of the baseline configuration and that this roughness configuration leads to rapid transition to turbulence.

Downstream of the roughness array at $x = 310$ mm, the unsteady velocity fluctuations are concentrated in three regions, one directly above the roughness centerline around $\eta = 1.5$ and two on both sides of the roughness element. These regions are most distinct and symmetric for third and fourth elements from the left. Again, the two lobes located downstream of either side of the roughness element are associated with local shear layers in the z direction and the lobe directly downstream of the centerline is associated with the local shear layer in the η direction. This can be seen in Figs. 47 and 48, that show the inflection points in the η and in the z directions, respectively. In Fig. 47, the inflection point in the wall-normal direction is again at $\eta = 1.5$, but $\partial U_c / \partial \eta$ is significantly larger than that of the $Re_k = 202$ configuration in Fig. 38. In Fig. 48 the streamwise velocity deficits behind the elements are also observed to be larger than their counterparts of the baseline configuration. Again the spanwise locations of the $\partial U / \partial z$ maxima perfectly match the spanwise locations of the legs in Fig. 46.

The power spectra obtained at the head, right leg and midpoint (points M, N, and O, respectively in Fig. 46) of the transitional configuration are compared in Fig. 49. These points are equivalent to points B, E, and F in Fig. 37 whose power spectra are shown in Fig. 41. As before, a broad band of amplified frequencies from 300 Hz to 800 Hz is visible in the head and the leg. However, the fluctuation intensities in this band are higher and are shifted towards higher frequencies, than those of the baseline configuration.

The lobes centered around points M and N in the wake of the second roughness element in Fig. 46 are very close to one another, and the fluctuations may influence the

neighboring lobe. However, the head and leg regions are most distinct and isolated for third and fourth elements from the left. In an effort to better understand the frequency distribution in each region, Fig. 50 shows the power spectra obtained at locations indicated by P, Q and R in Fig. 46. Points P and R are located downstream of left and right legs of the rightmost roughness element, both at $\eta = 0.75$. Point Q is located in the head, and the spectra at point Q in Fig. 50 can be compared to that at point M in Fig 49. As expected, the power spectra obtained in the legs, (i.e., at points P and R) are essentially identical as the configuration's symmetry requires. The legs contain more power in the low frequency region around 35 Hz compared to the head at point Q. On the other hand, the head's spectrum contains more power in the high frequency region. This suggests that this frequency band originates from the top of the roughness element and are associated with the wall-normal shear layer. Conversely, the low frequency fluctuations in the legs may be associated with the spanwise shear layer on both sides.

The fact that the peaks are observed at 360 and a 720 Hz is unfortunate because these values are harmonics of 60 Hz. However, contour plots of these bands' integrated intensities (not shown in the present work) are similar to those in Figs. 37 and 46 and it is concluded that these frequencies are associated with the disturbance input and not electronic noise.

At $x = 350$ mm, the unsteady fluctuations are greatly intensified compared to the same streamwise location for the baseline configuration. Additionally, the region covered by the fluctuations is increased such that the second and third roughness elements' unsteady wakes

are joined and the second roughness element's fluctuations reach the edge of the boundary layer. The local shear layers in the η and the z directions at $x = 350$ mm for the transitional configuration are similar to those of baseline configuration with larger amplitudes.

The power spectra obtained at the head, right leg and midpoint for $x = 350$ mm (points S, T, and U, respectively in Fig. 46) are compared in Fig. 51. These points are equivalent to points G, H, and I in Fig. 37 and the power spectra can be compared to those in Fig. 44. Immediately apparent in Fig. 51 are the amplified fluctuations in almost all of the low frequency region. The unsteady disturbance power contained in the head is larger than the power contained in the right leg, which contains more power than the midpoint. This matches with the contour levels shown in Fig. 46. The power contained at point U is noticeably higher than the midpoint O at $x = 310$ mm due to the merging of the elements' unsteady wakes. Sharp peaks at 360 Hz and 720 Hz are visible in the head and the right leg, but only 360 Hz is visible at the midpoint. When compared to the $Re_k = 202$ configuration in Fig. 44 and $x = 310$ mm in Fig. 49, the 300 Hz-800 Hz frequency band has amplified several orders of magnitude in the transitional configuration shifting towards higher frequencies of this band. Fig. 51 also shows the formation and growth of the harmonics of the broad band of amplified frequencies centered at 720 Hz. The shedding frequency of $f_s = 720$ Hz obtained here corresponds to $St = 0.17$ which is consistent with a reasonable extrapolation of Klebanoff et al.'s (1992) data.

The growth of the harmonics of the frequency band centered at 720 Hz is shown in Fig. 52 [Ergin and White's (2005a) Fig. 7] in more detail, where the downstream evolution

of the temporal power spectrum in the second roughness elements' wake for the transitional configuration is reported. Although the PSD profiles in that figure were reported to be along the centerline, they actually are along the spanwise location between right leg and the the head. This was due to an indexing mistake overseen in the previous work. However this figure is still useful in explaining the formation and growth of the 720 Hz frequency band's harmonics in the high-frequency region, and gradually transforming the spectrum into a turbulent one.

Klebanoff et al. (1992) performed similar experiments using isolated 3.2-mm-high hemispherical and cylindrical roughness elements with diameters of 6.3 mm and 3.2 mm, respectively. Those authors report that bypass transition is observed at $Re_k = 325$ for hemispherical roughness elements and at $Re_k = 450$ for cylindrical roughness elements. In that study, the transition is defined as the first occurrence of turbulent bursts in the oscilloscope with a hotwire positioned on the roughness centerline close to the surface. Klebanoff et al. suggest that the Strouhal number, St , is not a strong function of Re_k , when calculated as $St = f_s \delta^* / U_k$, where U_k is the flow velocity at roughness height. For hemispherical and cylindrical roughness elements, Klebanoff et al. report Strouhal numbers of 0.3 and 0.25, respectively. However, Klebanoff et al.'s Fig. 23 indicates that for cylindrical roughness elements, St does decrease somewhat with decreasing Re_k as Re_k approaches values used in the current work. Moreover, the height-to-diameter aspect ratio of the cylindrical roughness elements is quite a bit lower in the current work than those used by Klebanoff et al. and this may be another reason for the somewhat lower Strouhal number.

At $x = 400$ mm, abrupt changes occur in both the steady and unsteady disturbances. The unsteady wakes of all roughness elements shown in this figure are joined and most of the boundary layer is contaminated by turbulent fluctuations. The mixing downstream of the middle two roughness elements is most severe and the steady velocity contours begin to deviate from their usual laminar appearance. By $x = 475$ mm, the flow is completely contaminated by unsteady fluctuations with high turbulence levels. The steady velocity contours are strongly affected by turbulent mixing and are well on their way toward a fully developed turbulent boundary layer profile. The maximum u'_{rms} in Fig. 46 is 16% of U_{∞} .

4.3.3 Unsteady Energy's Streamwise Evolution

A quantitative assessment of the unsteady disturbances' evolution can be made by computing e_{rms} , the unsteady disturbance energy contained in a single roughness-array wavelength. The downstream evolution of e_{rms} generated by the roughness element at $z/\lambda_k = -0.5$ is shown in Fig. 53. With increasing downstream distance, the unsteady disturbance energy barely rises above the noise floor for the $Re_k = 202$ and 264 configurations. For $Re_k = 334$, however, it experiences substantial growth and then saturation as the flow becomes turbulent.

Because there is significant energy at low frequencies, the evolution of e_{rms} does not provide detailed information on the growth of the unstable band between 300 Hz and 800 Hz that is associated with the KH instability. In order to better understand this growth and how it does or does not trigger breakdown, e_{360} and e_{720} are plotted in Figs. 54

Table 5: Spatial growth rates of frequency band energies.

Re_k	α_{360}	α_{720}	α_{1440}
202	0.073 ± 0.006	0.037 ± 0.007	—
264	0.093 ± 0.009	0.063 ± 0.004	—
334	0.137 ± 0.020	0.192 ± 0.034	0.089 ± 0.025

and 55. These frequencies are both inside the band of amplified frequencies and are more revealing than e_{rms} because the low-frequency contributions are excluded from consideration. For the $Re_k = 202$ and 264 configurations, e_{360} grows until about $x = 350$ mm and e_{720} grows until $x = 330$ mm. Downstream of their respective maxima, they decay and remain constant for the rest of the streamwise domain. For the $Re_k = 334$ configuration, both bands experience exponential growth as they do at lower Re_k 's in the near wake. However, both continue to grow past the x locations where their growth ceases at lower Re_k 's. Finally, e_{360} and e_{720} saturate where the flow becomes fully turbulent.

Figure 56 shows the growth of the high-frequency harmonic, which is centered around 1440 Hz. This band does not contain significant energy for the $Re_k = 202$ and 264 configurations, but, for the $Re_k = 334$ configuration, e_{1440} experiences exponential growth starting at $x = 320$ mm. The growth rates of these bands' energies are listed in Table 5. The growth rates, α are obtained by performing a linear fit to the natural logarithm of each band's energy between $x = 310$ and $x = 330$ mm. In this calculation the streamwise length is made dimensionless using δ at x_k . Because the growth rates are decreasing with increasing x these are only approximate values representative of the early behavior.

Examining Figs. 54 and 55 it is apparent that unsteady disturbances are unstable in the roughness array's near wake for all Re_k values considered. Although the disturbances initially grow in the $Re_k = 202$ and 264 configurations, the entire unstable band has stabilized by about $x = 350$ mm, harmonics do not appear and the fluctuations do not lead to transition. However, in the $Re_k = 334$ configuration, the unsteady fluctuations grow more rapidly than in the other cases, harmonics do appear and transition results.

An explanation of the unsteady disturbances' behavior lies with the spatial evolution of the basic state provided by the roughness elements' steady wakes. It has been established that the velocity deficit region behind each roughness element supports a KH-type instability and that the KH mechanism is the source of the unsteady disturbances observed in each case. This explains why the growth rates of the unsteady fluctuations increases with increasing Re_k : larger velocity deficits are more unstable than smaller deficits. However, the E_{rms} data (Fig. 36) show that in the near wake region where unsteady fluctuations grow, the magnitude of the mean flow variations that lead to the instability are rapidly decreasing. This means that as x increases the flow becomes progressively less unstable and decreasing growth rates are observed at all Re_k in Figs. 54 and 55. However, in the transitional case, $Re_k = 334$, the initial growth rate is sufficiently large and E_{rms} , although rapidly decaying, remains sufficiently high that the unsteady fluctuations reach nonlinear amplitudes and breakdown is initiated before the basic state becomes stable to the high-frequency disturbances.

Stated another way, the “critical” behavior noted in the earliest experiments on transi-

tion behind 3D roughness elements results from a race between unsteady fluctuation growth and the rapid relaxation of the basic state toward a spanwise-uniform Blasius flow. When $Re_k < Re_{k,crit}$ the unsteady disturbances stabilize before they reach nonlinear amplitudes and decay without causing transition. However, when $Re_k > Re_{k,crit}$ the unsteady disturbances are able to reach transitional amplitudes before the basic state relaxes to a condition at which they would become stable so transition results. In this supercritical case the two features that allow the unsteady disturbances to grow to nonlinear amplitudes are the larger initial growth rates and the longer streamwise extent of an unstable basic state.

Chapter 5

SUMMARY AND CONCLUSIONS

The present work investigates two different roughness effects that are significant in boundary layer transition. The first is the receptivity of stationary transient disturbances to roughness; the second is the mechanism for rapid transition in the wake of large-amplitude roughness elements. The experiments are performed in a subsonic, laminar flat-plate boundary layer with disturbances generated by a spanwise array of cylindrical roughness elements and velocity measurements performed using constant temperature hotwire anemometry.

The first experiment concerning receptivity consists of obtaining measurements of three velocity components. The measurements are intended to quantitatively identify the streamwise-oriented vortices that play a critical role in Landahl's (1980) lift-up concept. Time-averaged streamwise and spanwise velocity components are measured using a combination of straight and slanted sensors. The experiments are performed with high spatial resolution so velocity derivatives can be generated and the wall-normal velocity component is estimated from these values using a discretized approximation of the continuity equation. The resulting complete steady velocity disturbance field will be used in future theoretical studies on the receptivity of roughness-induced disturbances using Tumin's (2003) biorthogonal decomposition technique.

The multicomponent velocity measurements reveal many important aspects of transient disturbances. First, in agreement with previous findings, strongly decelerated velocity profiles are observed in the near wake of the roughness elements and these profiles gradually become accelerated in the far wake. High-speed streaks appear a short distance downstream of the roughness array aligned with the spanwise edges of the roughness elements. Second, the V , W vector field and the streamwise vorticity indicate the trailing legs of the primary horseshoe vortex and the secondary vortex. The sense of the vortices' rotation directions and the fact that the secondary pair appears at a short distance downstream of the roughness array is in good agreement with the flow visualization studies performed by Gregory and Walker (1956). No trailing legs of spiral vortex filaments are observed.

The streamwise vorticity associated with the horseshoe vortex pairs is most intense near $y \approx 1$ mm, indicating that the roughness-induced transient disturbances are localized in the lower portions of the boundary layer. Interestingly, the intensity of both pairs experience transient growth in the streamwise direction.

A number of different disturbance kinetic energy definitions are made for the steady flow field and their streamwise evolution is monitored. E_{rms}^W is observed to experience immediate transient growth, whereas E_{rms}^U experiences a sharp initial decay and then undergoes transient growth. Spanwise Fourier transforms are performed on the velocity and the streamwise vorticity data to obtain the distribution of disturbance kinetic energy in various wavelength components. The sharp initial decay of E_{rms}^U is attributed to its DC and λ_k components that have similar streamwise evolution characteristics. $E_{\lambda_k/3}^U$, $E_{\lambda_k/4}^U$, $E_{\lambda_k/3}^W$

and $E_{\lambda_k/4}^W$ consistently experience transient growth in the streamwise direction. Nonlinear least squares fits are performed on these data sets using the empirical function proposed by Ergin (2003). The associated algebraic growth rates calculated for $\lambda_k/3$ or $\lambda_k/4$ indicate that E^U 's growth rate is always larger than E^W 's growth rate and, that E^U 's maximum amplitude is always larger than E^W 's maximum amplitude. The exponential decay lengths indicate that the shorter wavelengths reach their maxima more upstream than the longer wavelengths, as predicted by the transient growth theory.

Unsteady velocity measurements are performed to investigate the steady and unsteady velocity fields associated with high amplitude 3D roughness in the critical Re_k limit. Empirical correlations give a critical Re_k for breakdown, but provide no further insight to transition physics. The unsteady velocity measurements specifically intend to identify the mechanism associated with breakdown. Additionally, transient disturbances generated by surface roughness may suppress the growth of Tollmien–Schlichting waves but large disturbance amplitudes would be required. This necessitates the behavior generated by large-amplitude roughness elements to be well understood. With this in mind, the second experiment consists of detailed measurements of the steady and unsteady streamwise velocity fields in the high Re_k limit. The velocity is decomposed into a steady spanwise-invariant basic state, a steady disturbance and an unsteady disturbance. Due to significant variations in the unsteady disturbance magnitudes from element to element, phase locked averaging is not performed in the analysis of the unsteady data.

Two non-transitional ($Re_k = 202$ and $Re_k = 264$) and one transitional ($Re_k = 334$)

configuration are examined in detail. Common to all configurations is the inverted V-shaped structures observed in the unsteady fluctuation intensities in the near wake. These structures are interpreted as the unsteady hairpin vortices that are tethered to the roughness elements on both spanwise edges (Acarlar and Smith 1987). For the non-transitional configurations, the hairpin vortices' fluctuation intensities first increase in the near wake, then decrease and evolve into fringe structures further downstream. For the transitional configuration, the unsteady fluctuation intensities of the hairpin vortices become unstable and spread outward in the spanwise direction contaminating the complete spanwise domain. This finding is in agreement with Gregory and Walker's (1956) observations on the vertex of turbulent wedge. For the unsteady experiments the turbulent wedge appears at $x \approx 400$ mm for $Re_k = 334$. This is an expected $Re_{k,crit}$ value when the (k/D) ratio is taken into consideration (von Doenhoff and Braslow 1961; Tani 1969).

Fourier transform of the unsteady data in the time domain permits the investigation of the growth and decay of the individual frequency bands. This reveals a broad band of frequencies centered between 300 Hz and 800 Hz that grows and decays for the non-transitional configurations. TS unstable frequencies are between 100 Hz and 200 Hz and, therefore, the fluctuations are generated by the roughness elements. The Strouhal number $St = f\delta^*/\bar{U}(k)$ calculated using the 720 Hz fluctuation is 0.17, which is in good agreement with an extrapolation of Klebanoff et al.'s (1992) data. For the transitional case, additional frequency bands centered around 1440 Hz and other harmonics of 720 Hz are observed to grow and lead the flow to breakdown. The harmonics give a characteristic rippled appear-

ance in the high frequency region that is an indication of nonlinear disturbance growth.

The results of the unsteady velocity measurements also reveal the likely transition mechanism that is active in this case. A comparison of the steady and the unsteady disturbance fields reveal that the unsteady fluctuation intensities appear exactly at the locations of maximum spanwise or wall-normal shear of the streamwise velocity. Additionally, a representative frequency for the local shear layer in the z direction is on the order of 750 Hz, in agreement with the measured frequency bands in the experiment. These facts indicate that the instability is likely of the Kelvin–Helmholtz (KH) type. It is suspected that the axes of the hairpin vortices are located on the locus of local inflection points of the steady flow.

The unsteady disturbances' growth rates indicate that the transition scenario can be understood as a competition between the unsteady disturbances' growth and the rapid relaxation of the steady flow that tends to stabilize these disturbances. It has been established that the velocity deficit region behind each roughness element supports a KH-type instability and that the KH mechanism is the source of the unsteady disturbances observed in each case. The growth rates of unsteady fluctuations increase with increasing Re_k , because larger velocity deficits are more unstable than smaller deficits in the context of KH instability mechanism. However, the mean flow variations that lead to the instability are rapidly decreasing in the near wake region where unsteady fluctuations grow. This means that as x increases, the steady flow becomes progressively less unstable. In the transitional case, E_{rms} is rapidly decaying yet remains sufficiently high and the initial growth rate is sufficiently large that the unsteady fluctuations reach nonlinear amplitudes and cause breakdown before

the basic state becomes stable to the high-frequency disturbances. The turning point of this race is determined by $Re_{k,crit}$. When $Re_k < Re_{k,crit}$ the unsteady disturbances stabilize before they reach nonlinear amplitudes and decay without causing transition. However, when $Re_k > Re_{k,crit}$ the unsteady disturbances are able to reach transitional amplitudes before the basic state relaxes to a condition at which they would become stable and transition results.



Appendix A

FIGURES

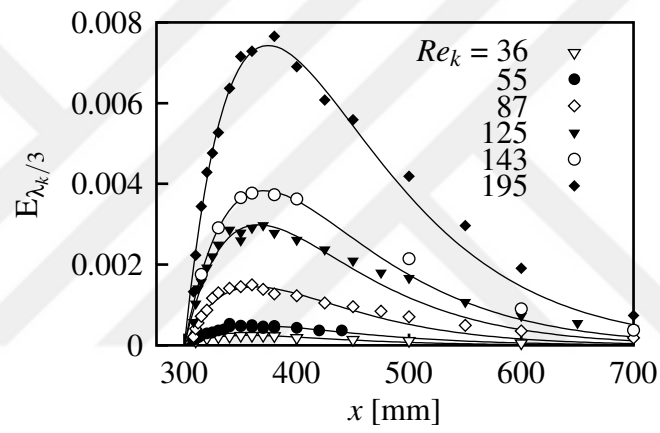


Figure 1: Transient growth of disturbance kinetic energy contained in $\lambda_k/3$ for various roughness heights, $d = 6.35$ mm.

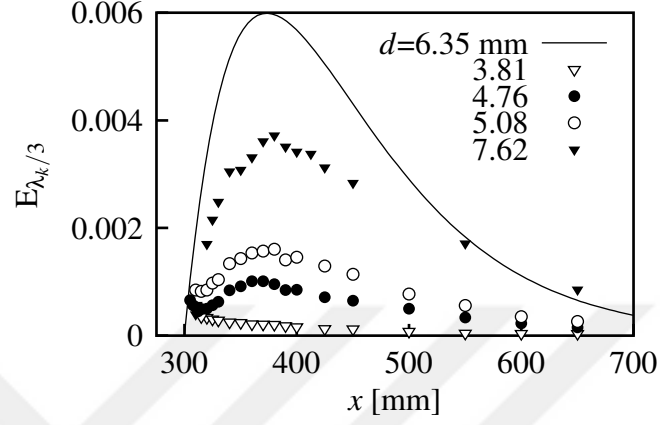


Figure 2: Evolution of disturbance kinetic energy contained in $\lambda_k/3$ for various roughness diameters, $Re_k = 177$.

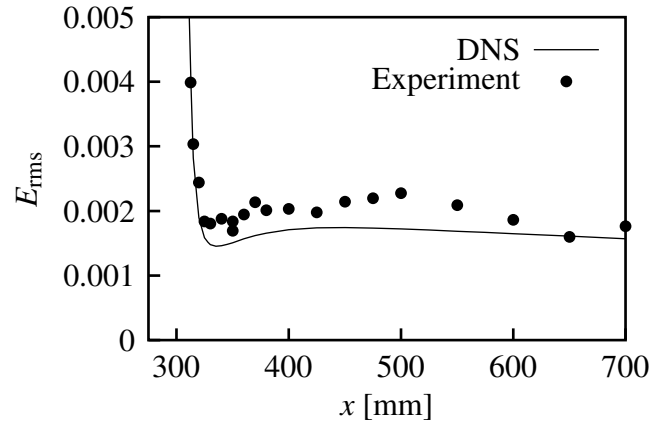


Figure 3: Downstream evolution of steady, spanwise-rms disturbance energy, E_{rms}^U for $Re_k = 119$. From Ergin et al. (2005).

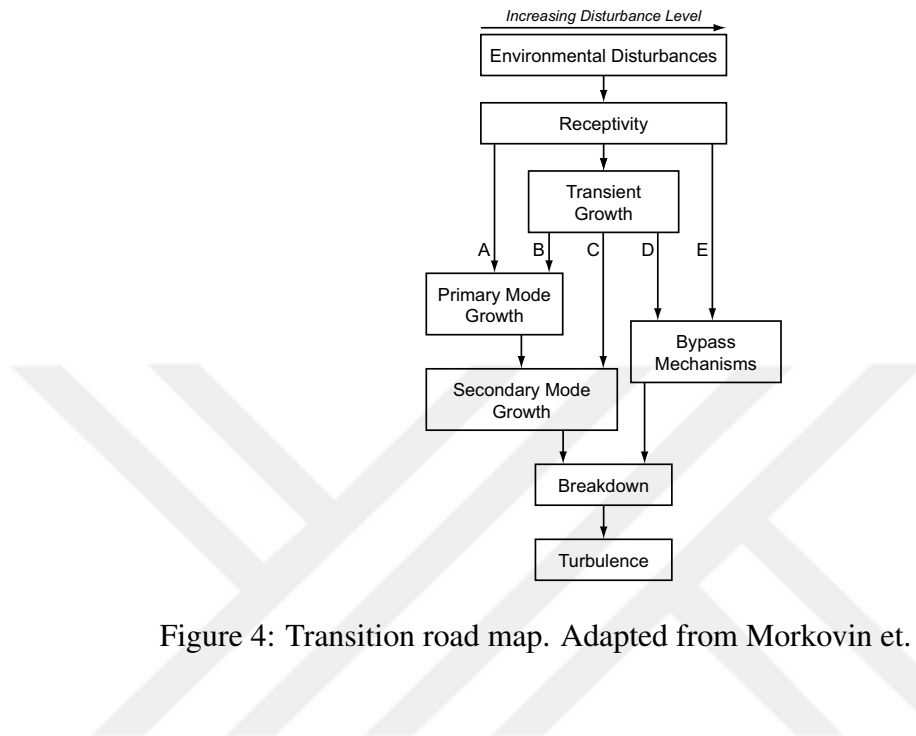


Figure 4: Transition road map. Adapted from Morkovin et. al. (1994)



Figure 5: Sting design and angle controller that allows for angle calibration.

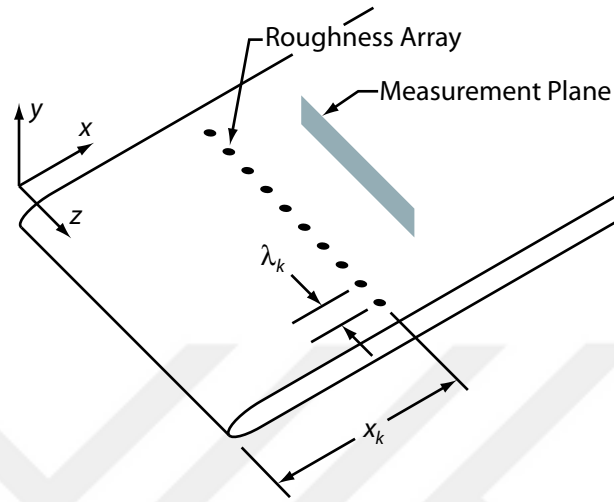


Figure 6: Schematic illustration of the flat plate and the roughness array.

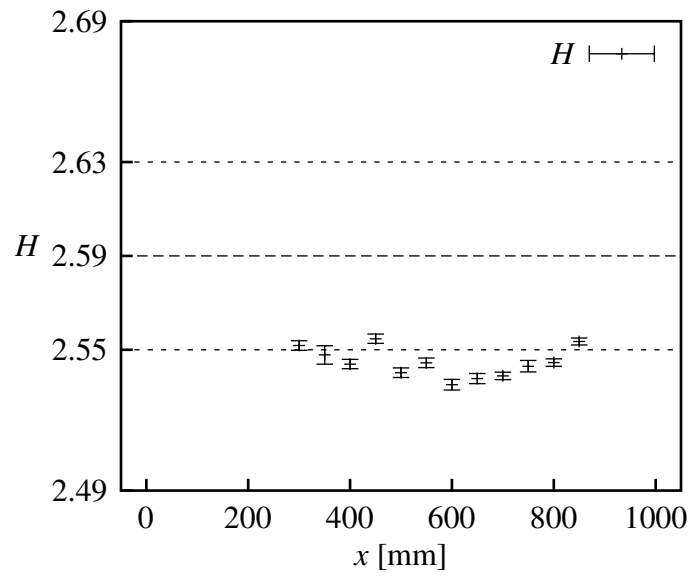


Figure 7: Shape factor variation in x direction.

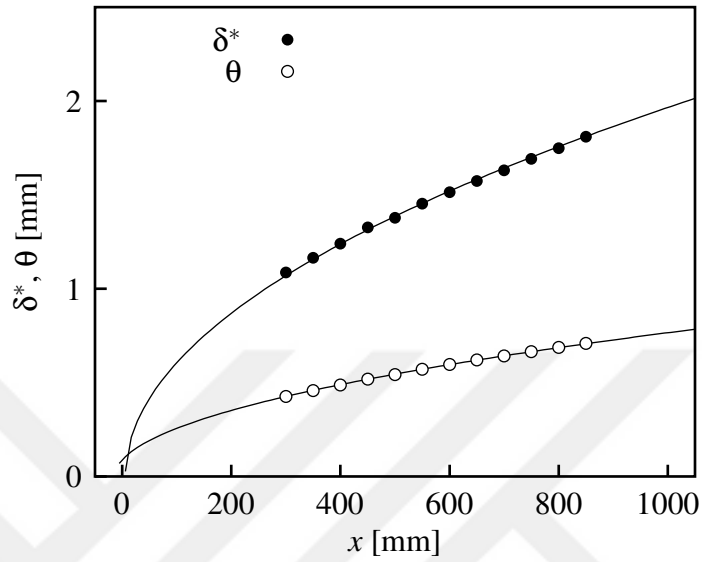


Figure 8: Displacement and momentum thickness variations in x direction.

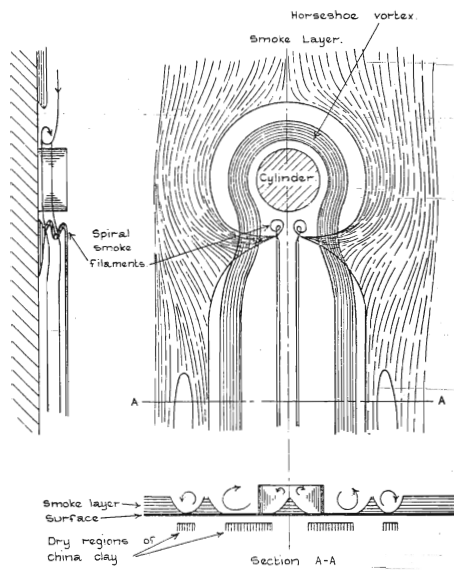


Figure 9: Flow topology in the vicinity of a cylindrical roughness element. Adapted from Gregory and Walker (1956).

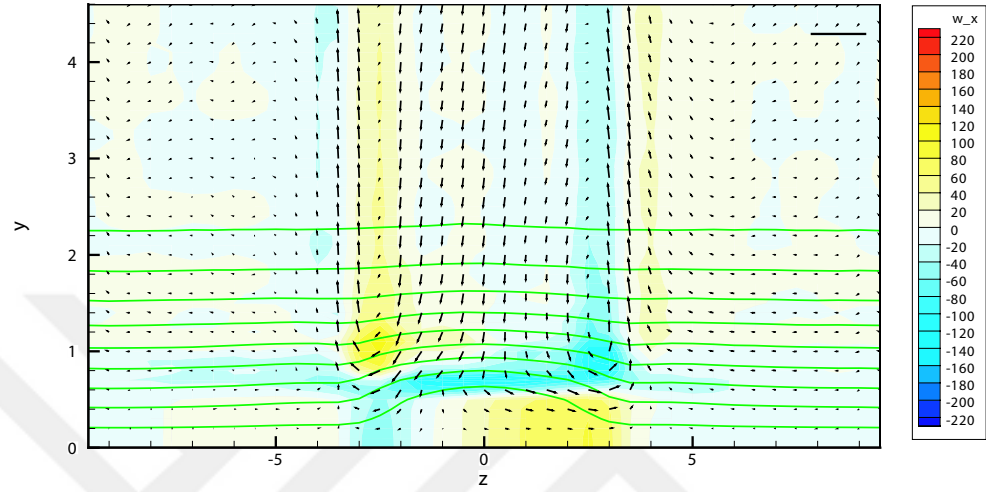


Figure 10: Steady flow properties in the near wake at $x = 310$ mm.

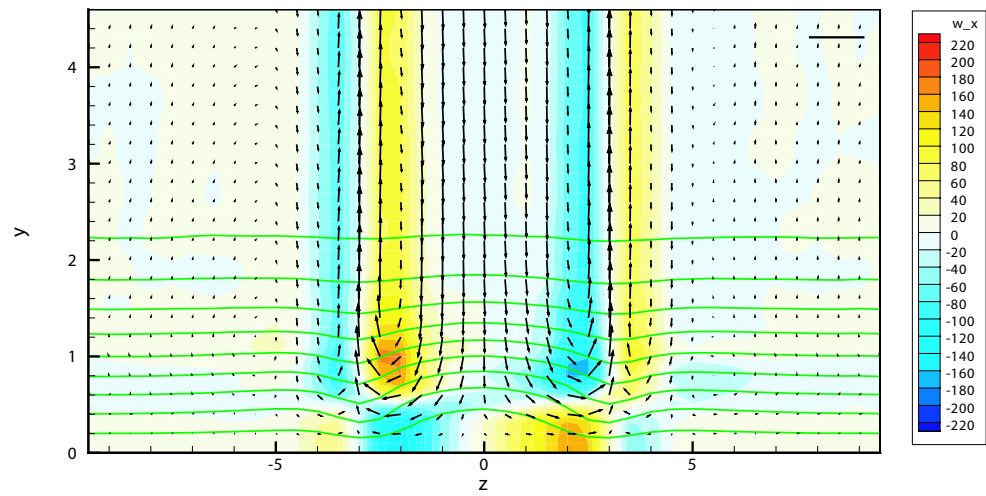


Figure 11: Steady flow properties in the near wake at $x = 320$ mm.

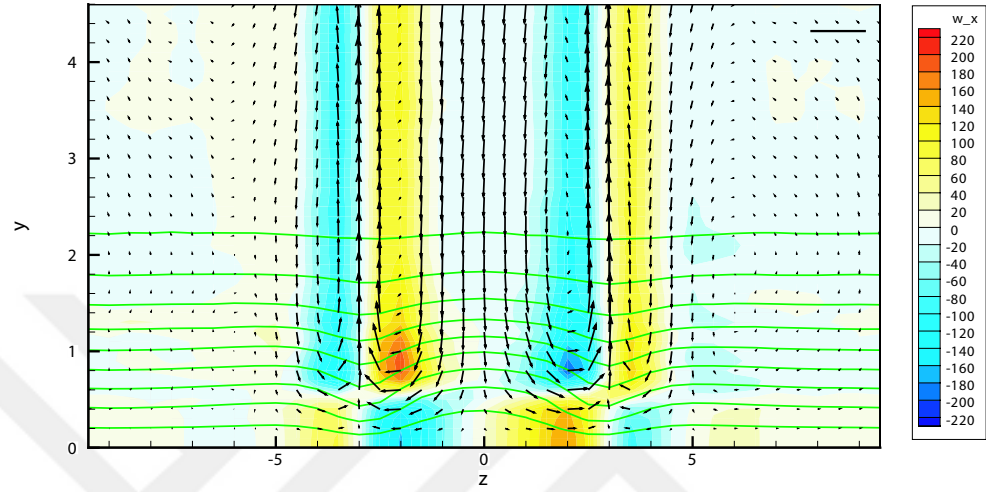


Figure 12: Steady flow properties in the near wake at $x = 330$ mm.

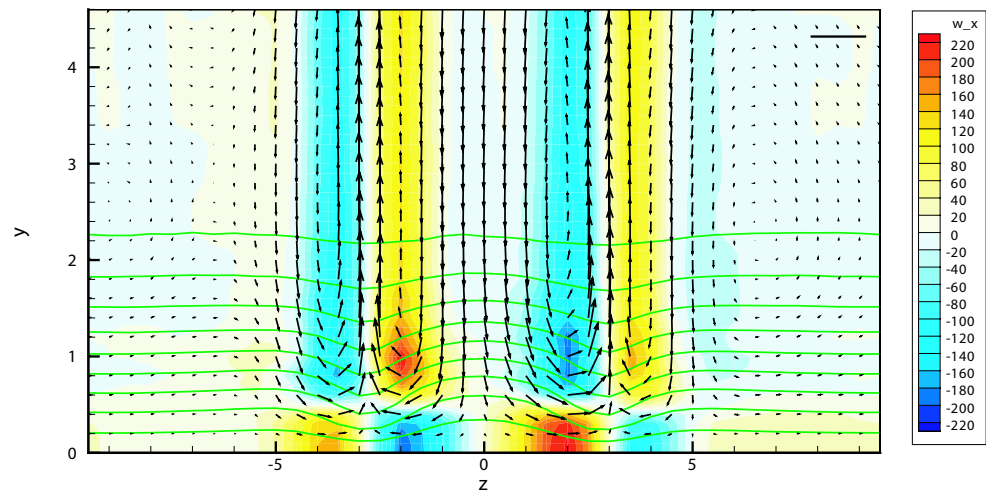


Figure 13: Steady flow properties in the near wake at $x = 340$ mm.

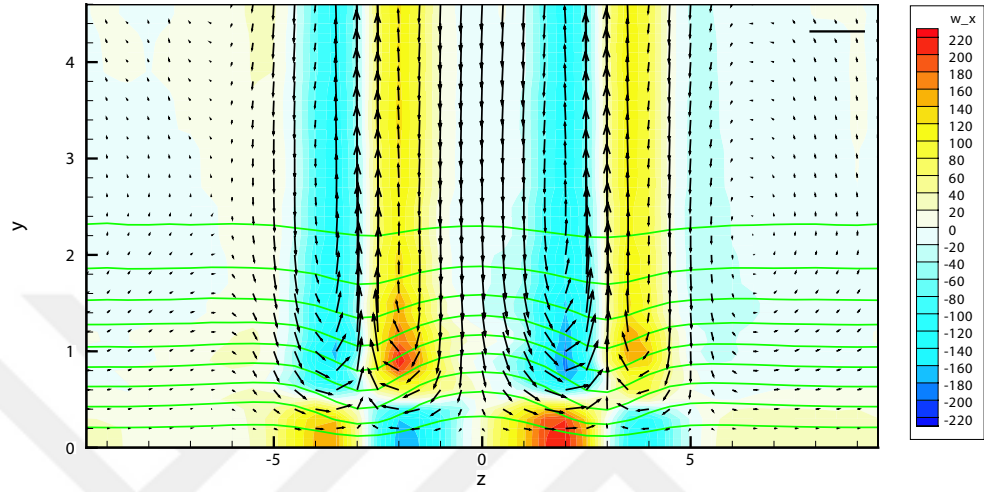


Figure 14: Steady flow properties in the near wake at $x = 350$ mm.

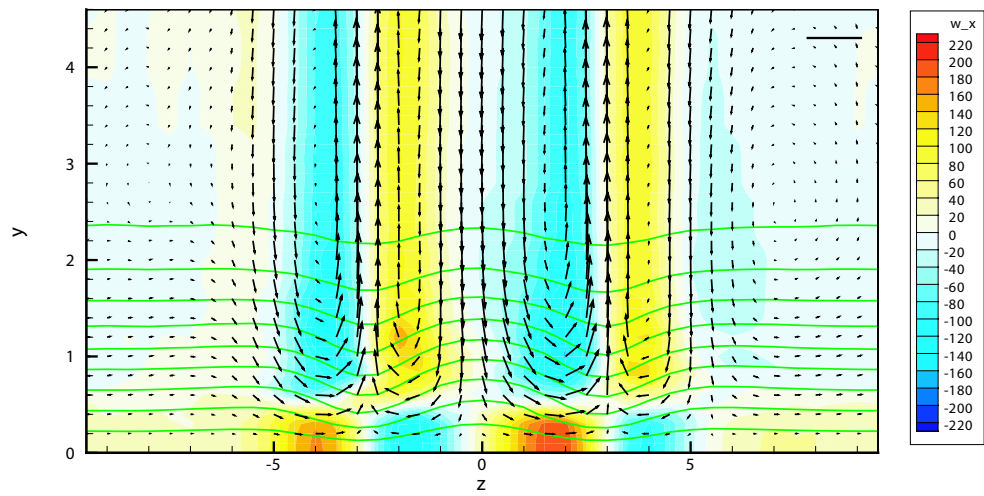


Figure 15: Steady flow properties in the far wake at $x = 375$ mm.

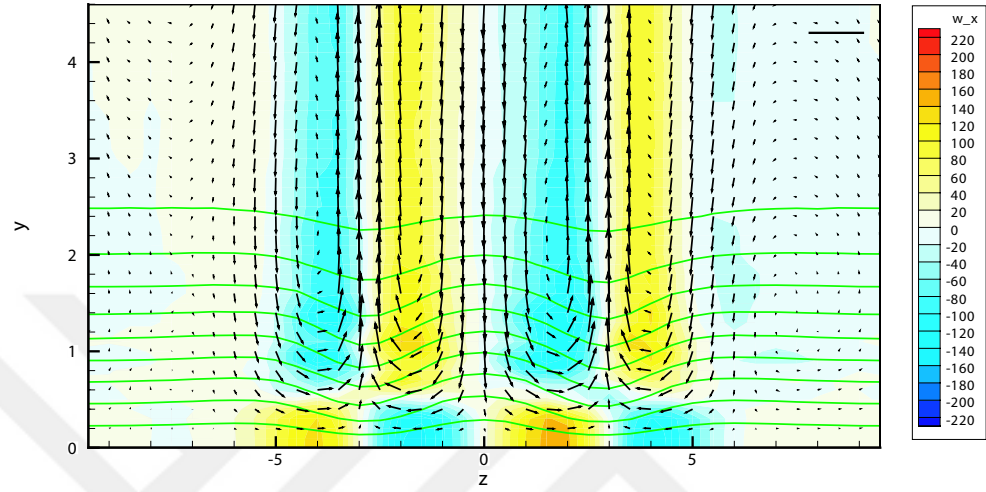


Figure 16: Steady flow properties in the far wake at $x = 400$ mm.

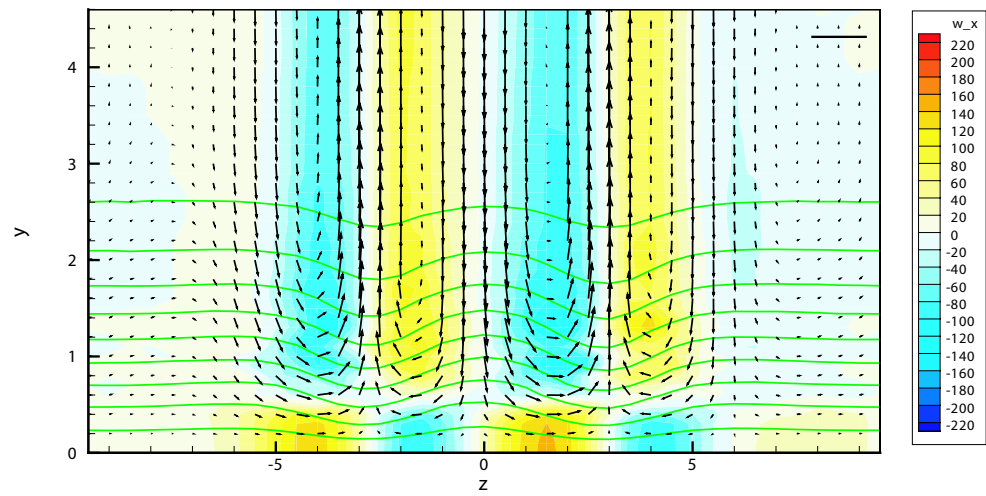


Figure 17: Steady flow properties in the far wake at $x = 425$ mm.

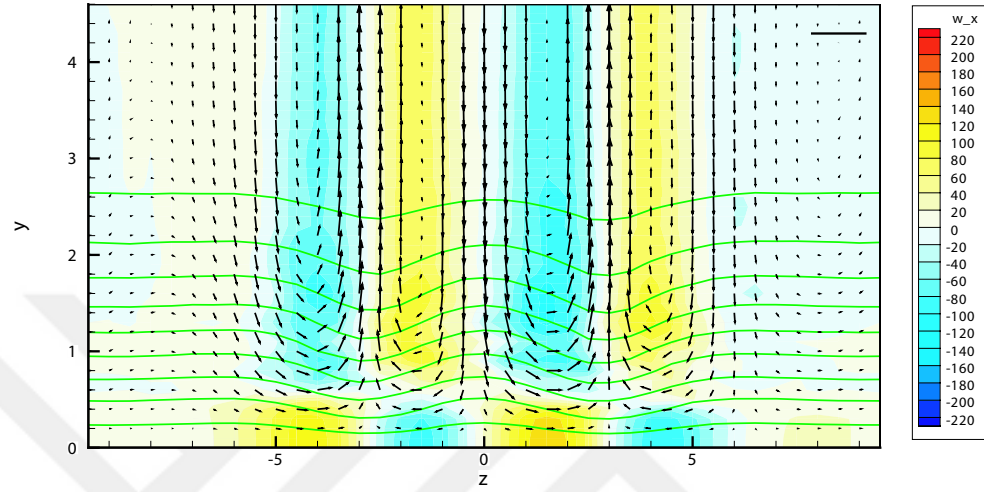


Figure 18: Steady flow properties in the far wake at $x = 450$ mm.

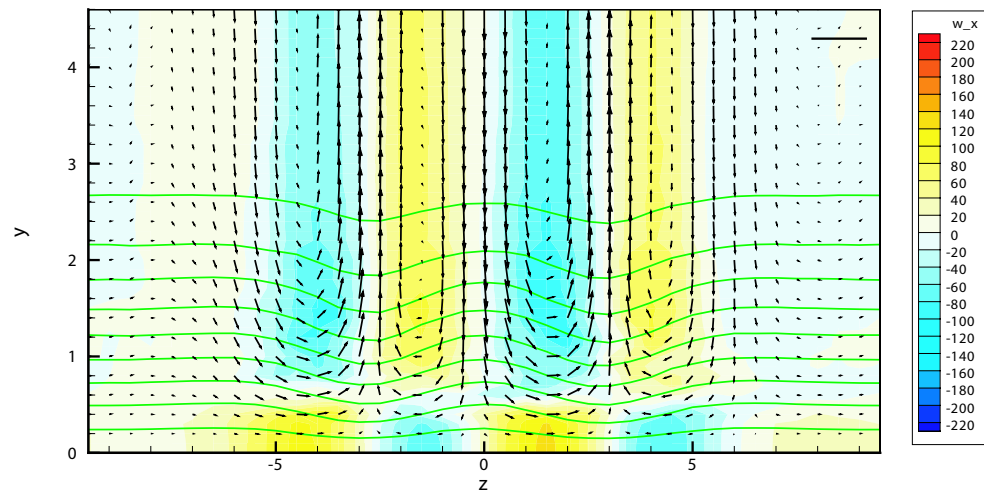


Figure 19: Steady flow properties in the far wake at $x = 475$ mm.

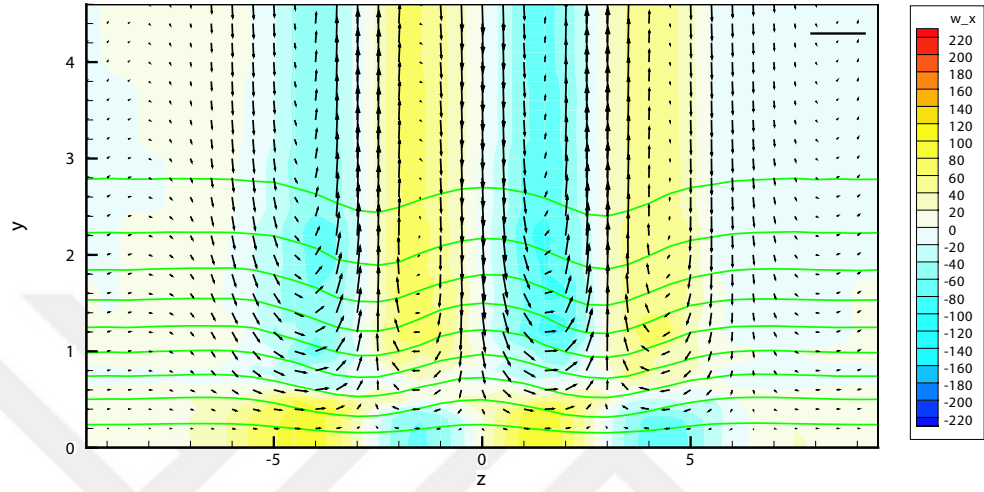


Figure 20: Steady flow properties in the far wake at $x = 500$ mm.

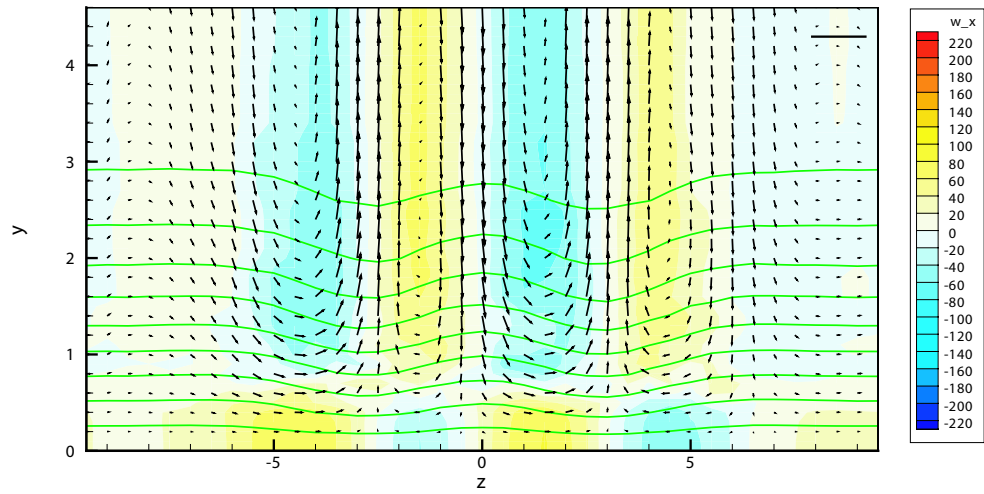


Figure 21: Steady flow properties in the far wake at $x = 550$ mm.

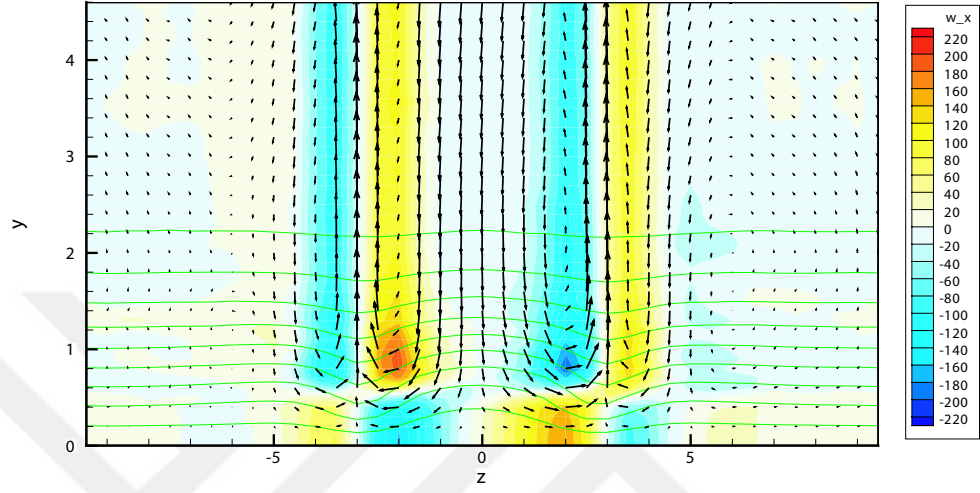


Figure 22: Steady flow properties at $x = 330$ mm with $\Delta x = 2.5$ mm resolution.

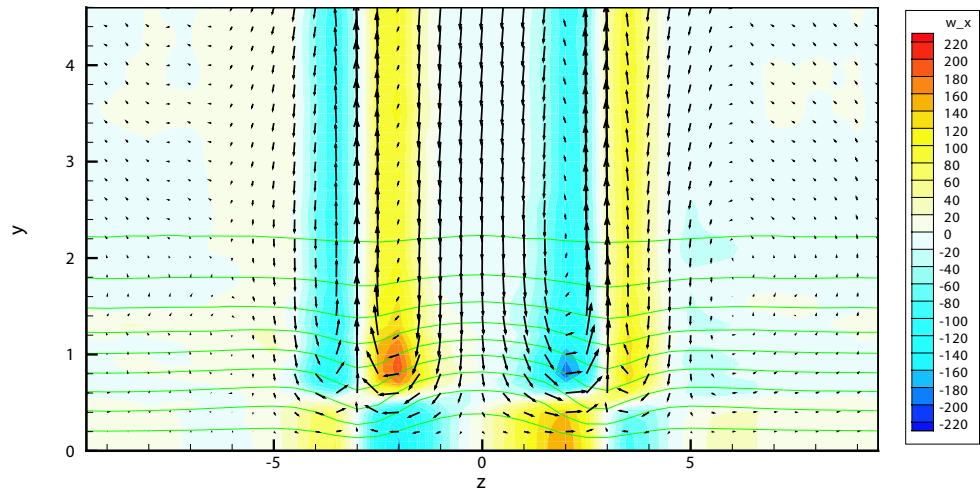


Figure 23: Steady flow properties at $x = 330$ mm with $\Delta x = 20$ mm resolution.

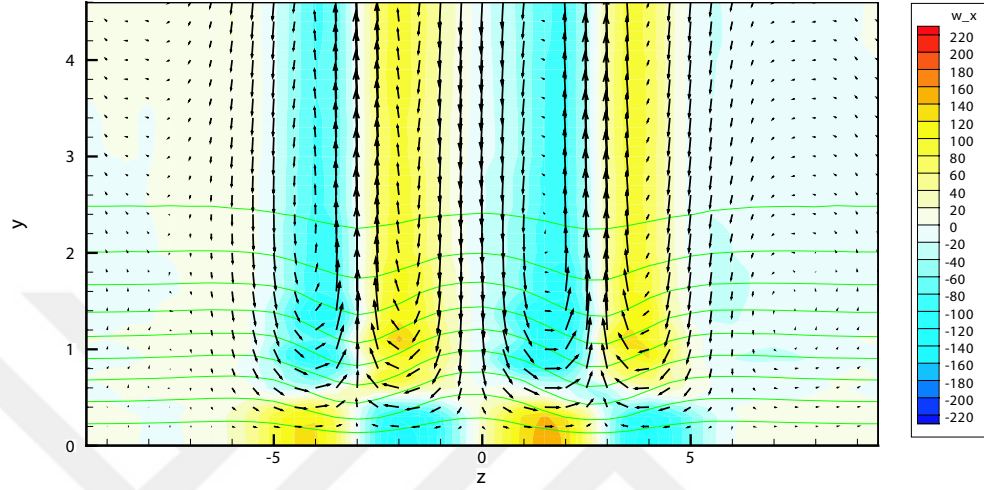


Figure 24: Steady flow properties at $x = 400$ mm with $\Delta x = 5$ mm resolution.

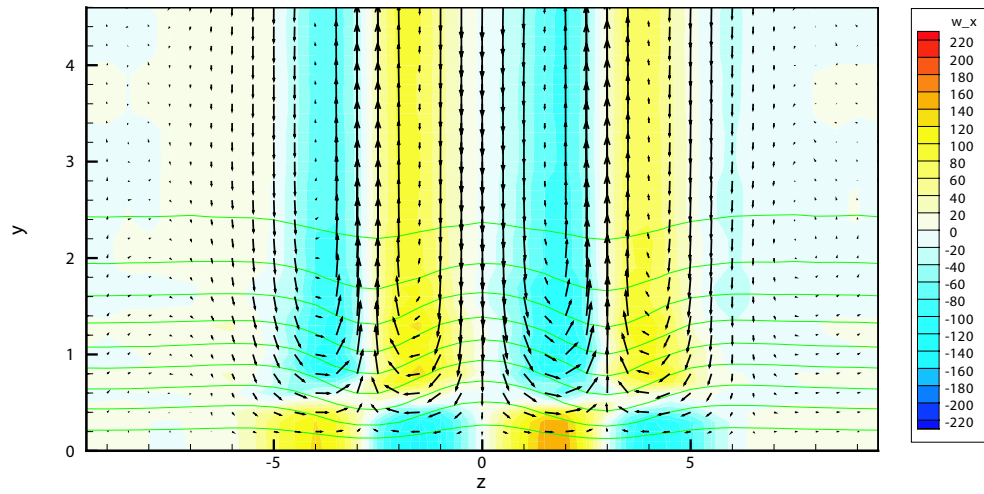


Figure 25: Steady flow properties at $x = 400$ mm with $\Delta x = 25$ mm resolution.

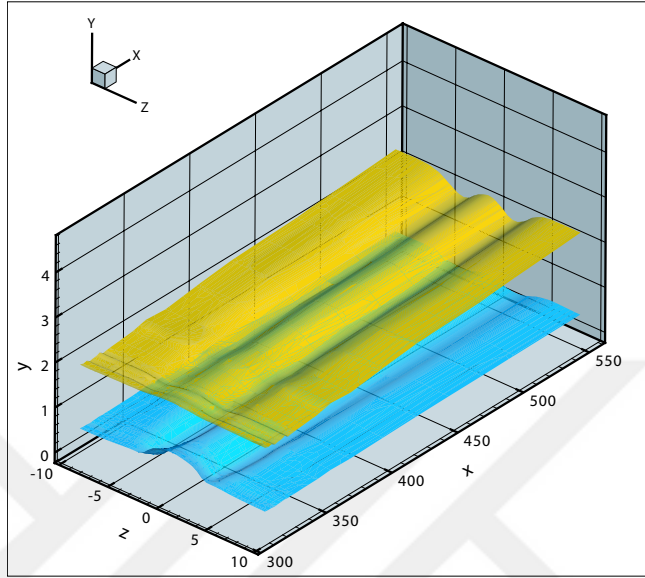


Figure 26: Iso-velocity surfaces of 20% and 80% of U_∞ .

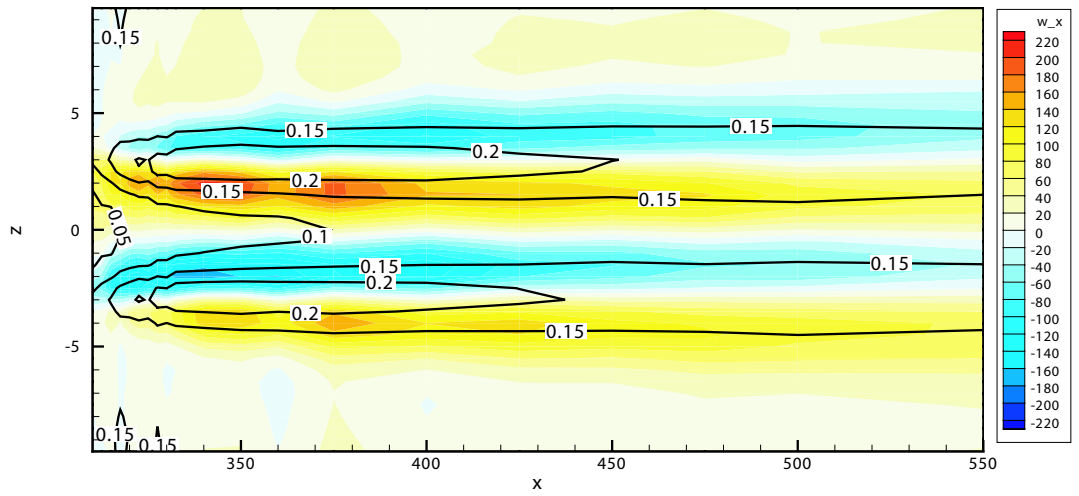


Figure 27: U and ω_x contours $300\ \mu\text{m}$ above the surface.

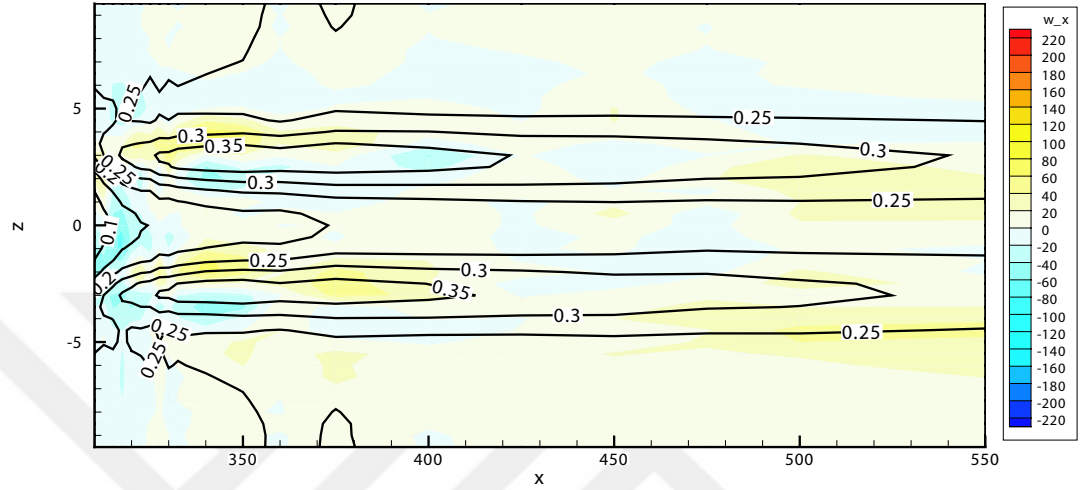


Figure 28: U and ω_x contours 550 μm above the surface.

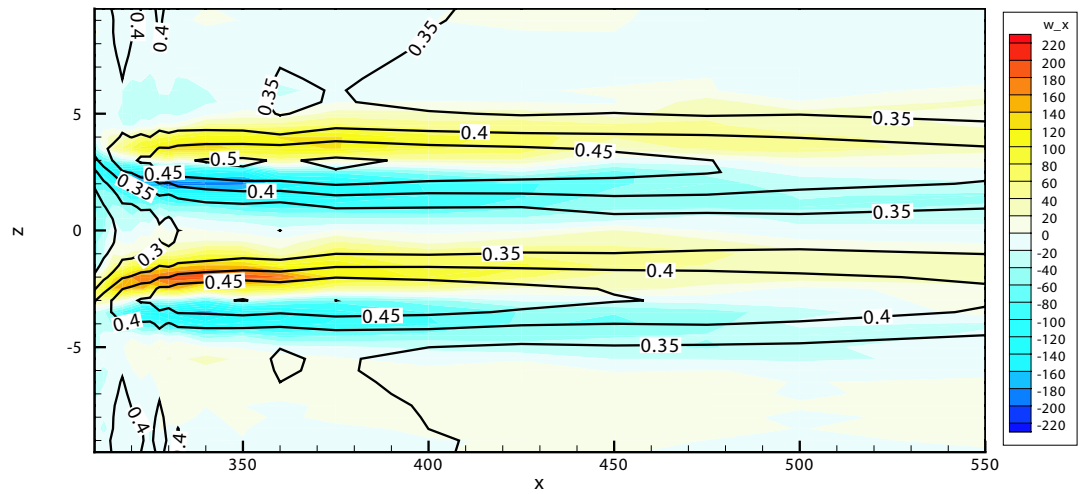


Figure 29: U and ω_x contours 800 μm above the surface.

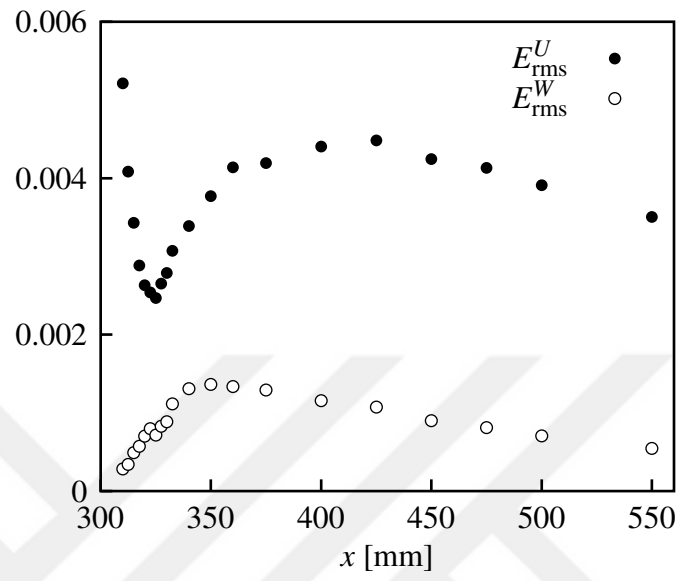


Figure 30: Streamwise evolution of rms disturbance energy for U and W .

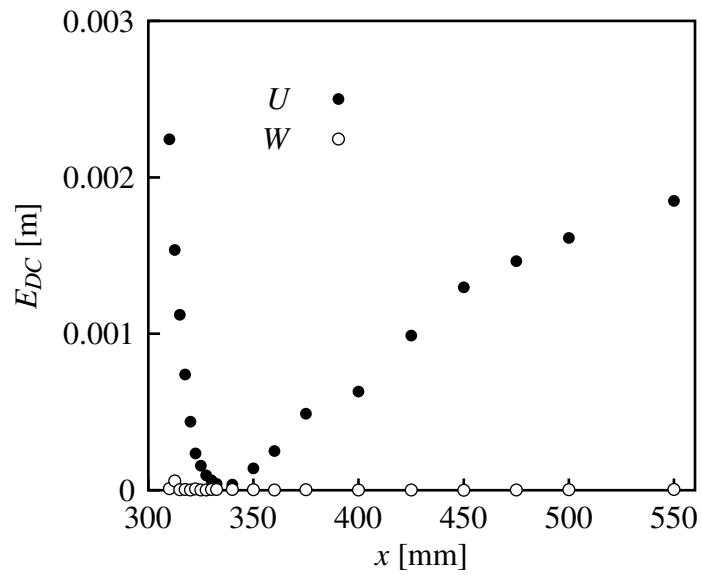


Figure 31: Streamwise evolution of disturbance energy contained in infinite wavelength.

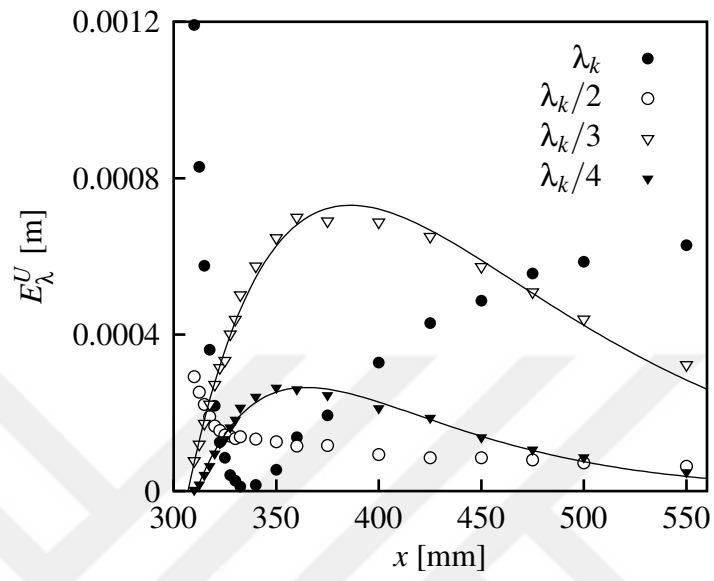


Figure 32: Wavelength components of streamwise disturbance energy.

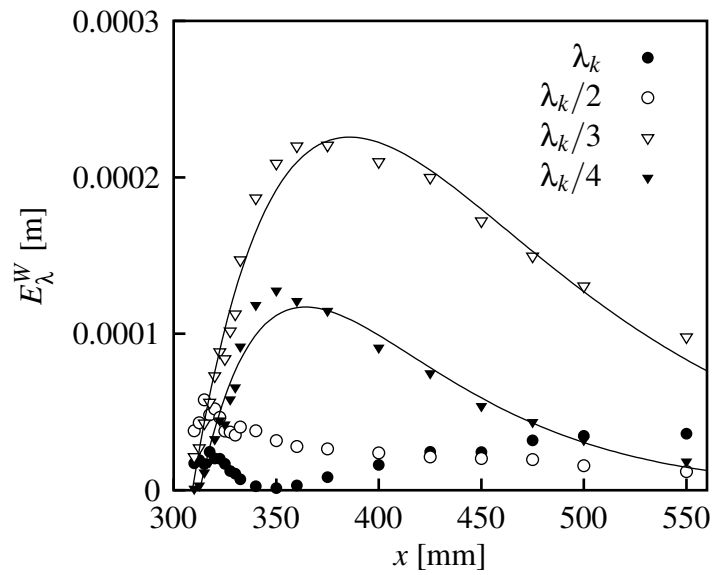


Figure 33: Wavelength components of spanwise disturbance energy.

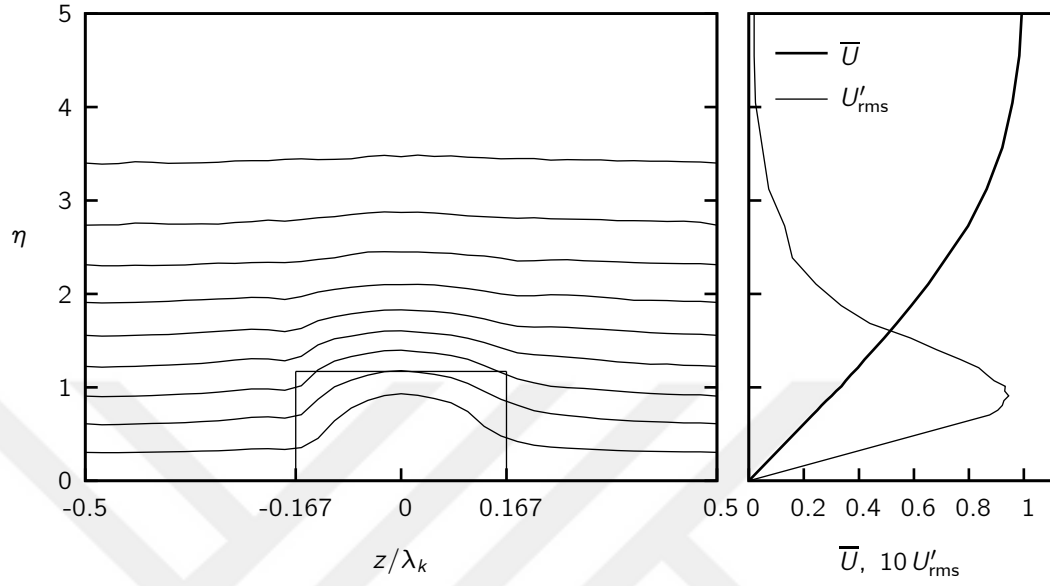


Figure 34: Streamwise velocity contours (left) and steady flow profiles (right) for $x = 310$ mm, $Re_k = 202$.

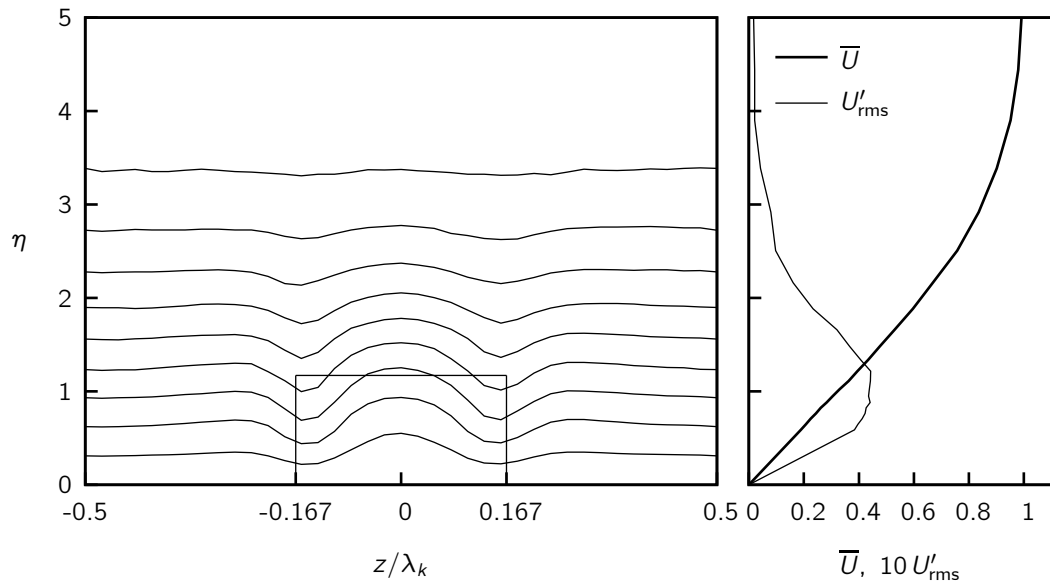


Figure 35: Streamwise velocity contours (left) and steady flow profiles (right) for $x = 330$ mm, $Re_k = 202$.

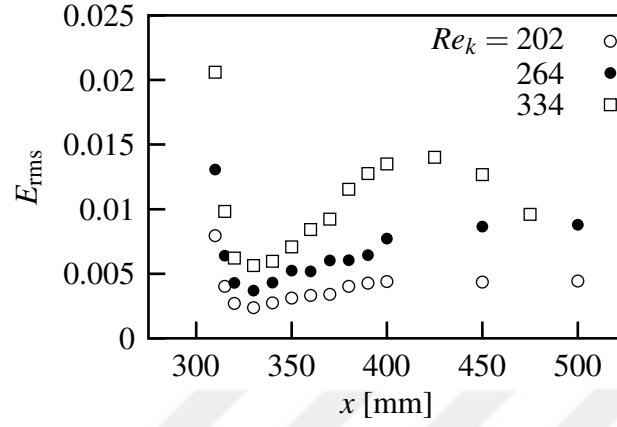


Figure 36: Streamwise evolution of the steady disturbance energy, E_{rms} .

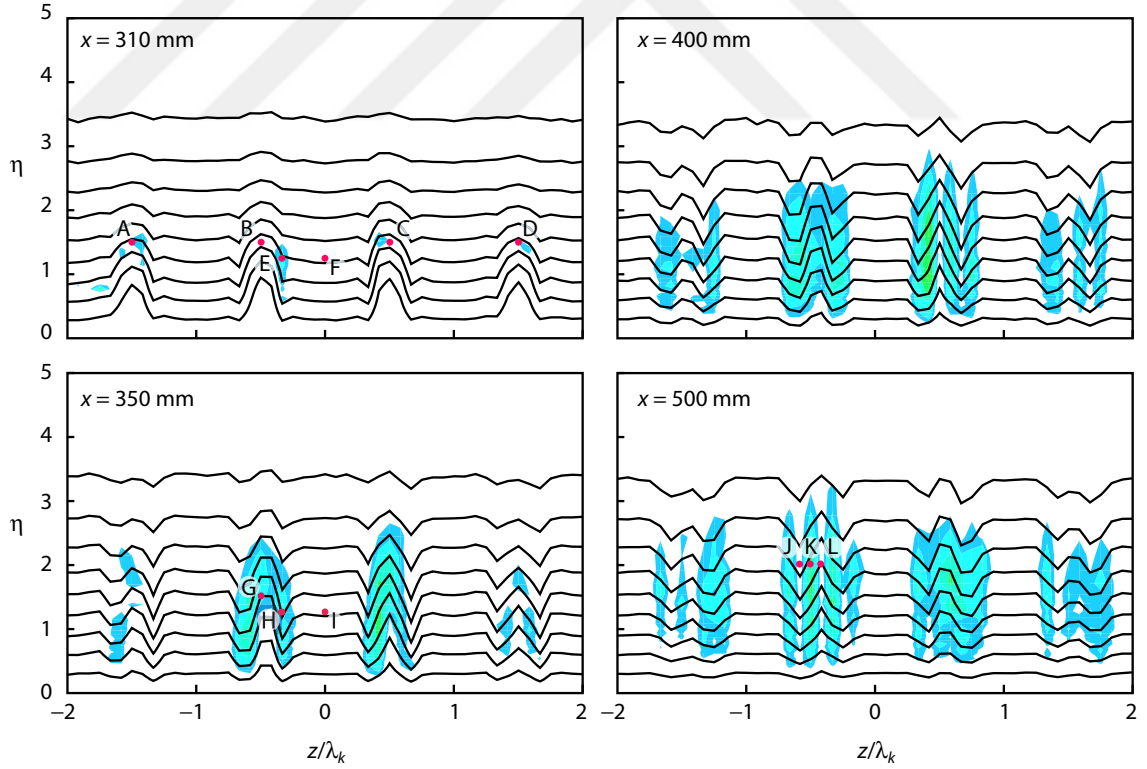


Figure 37: Contours of u'_{rms} and U for $Re_k = 202$. The black lines are 10% increments of U_∞ in U and the filled colored contours are exponentially distributed contours of u'_{rms} .

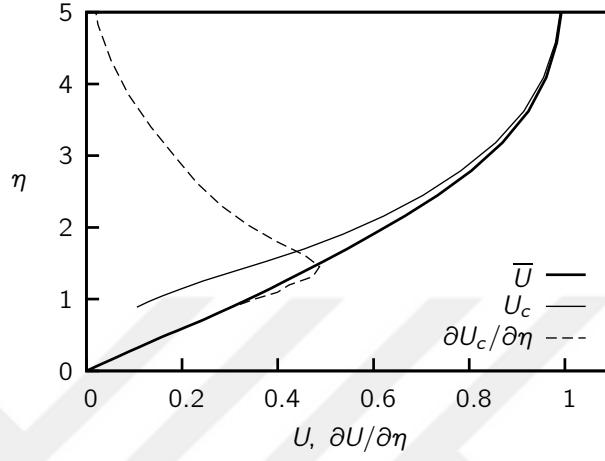


Figure 38: Basic state, centerline and $\partial U/\partial \eta$ profiles at $x = 310$ mm, $Re_k = 202$.

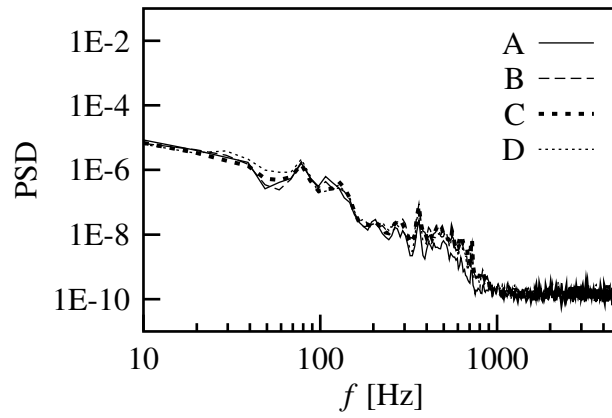


Figure 39: Power spectra obtained at local wall-normal inflection points of all roughness elements, $\eta = 1.5$.

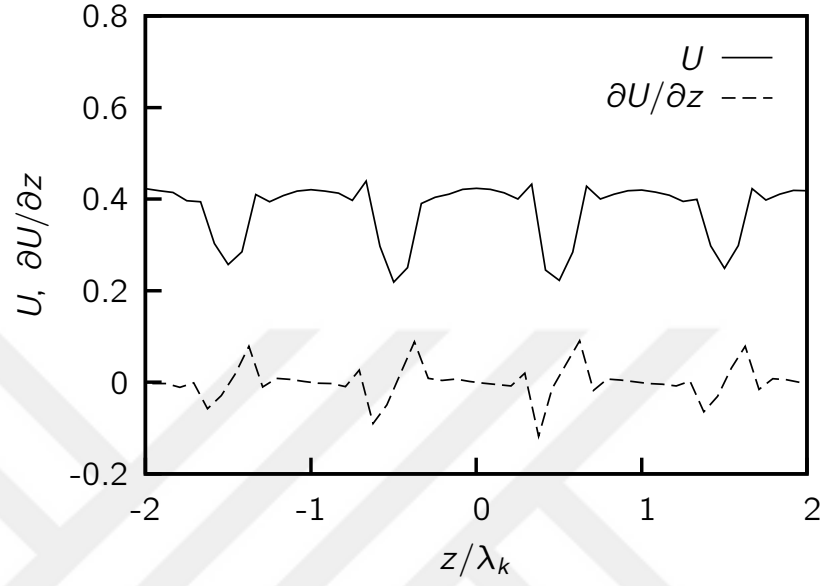


Figure 40: Spanwise variation of U and $\partial U / \partial z$ at $x = 310$ mm, $Re_k = 202$, $\eta = 1.25$.

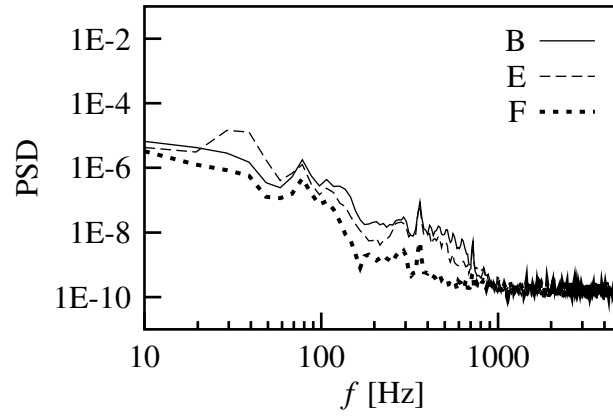


Figure 41: Power spectra obtained at the head, right leg and midpoint at $x = 310$ mm, $Re_k = 202$.

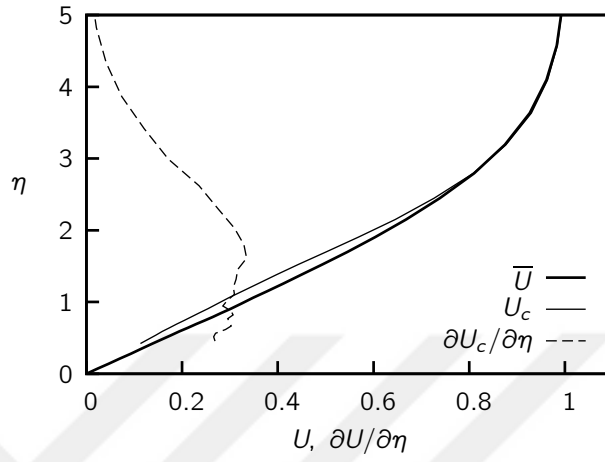


Figure 42: Basic state, centerline and $\partial U/\partial \eta$ profiles at $x = 350$ mm, $Re_k = 202$.

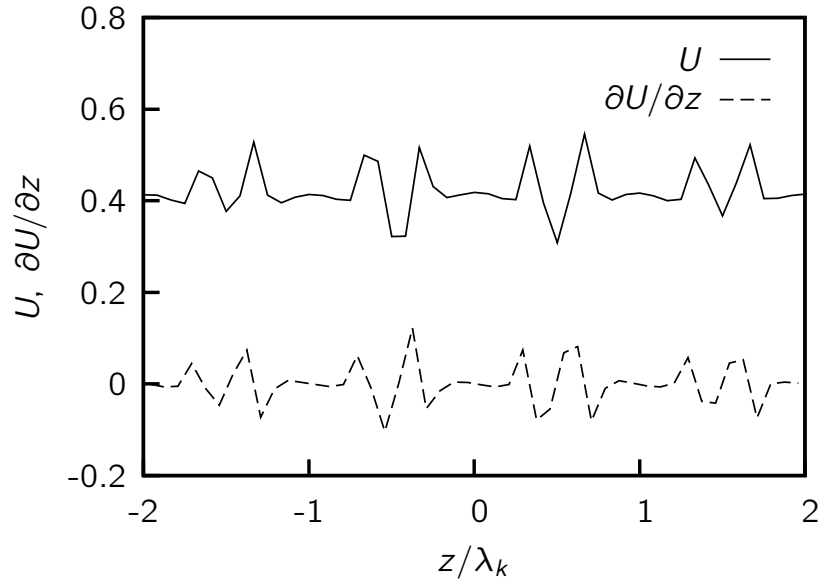


Figure 43: Spanwise variation of U and $\partial U/\partial z$ at $x = 350$ mm, $Re_k = 202$, $\eta = 1.25$.

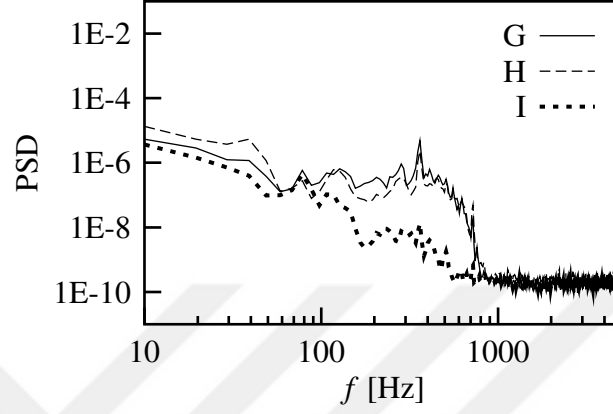


Figure 44: Power spectra obtained at the head, right leg and midpoint at $x = 350$ mm, $Re_k = 202$.

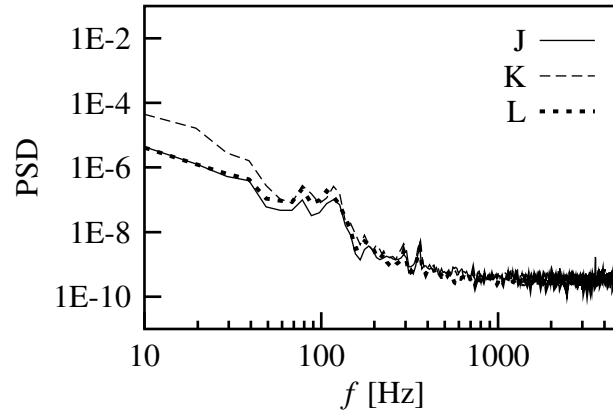


Figure 45: Power spectra obtained at the points J, K and L at $x = 500$ mm, $\eta = 2$, $Re_k = 202$.

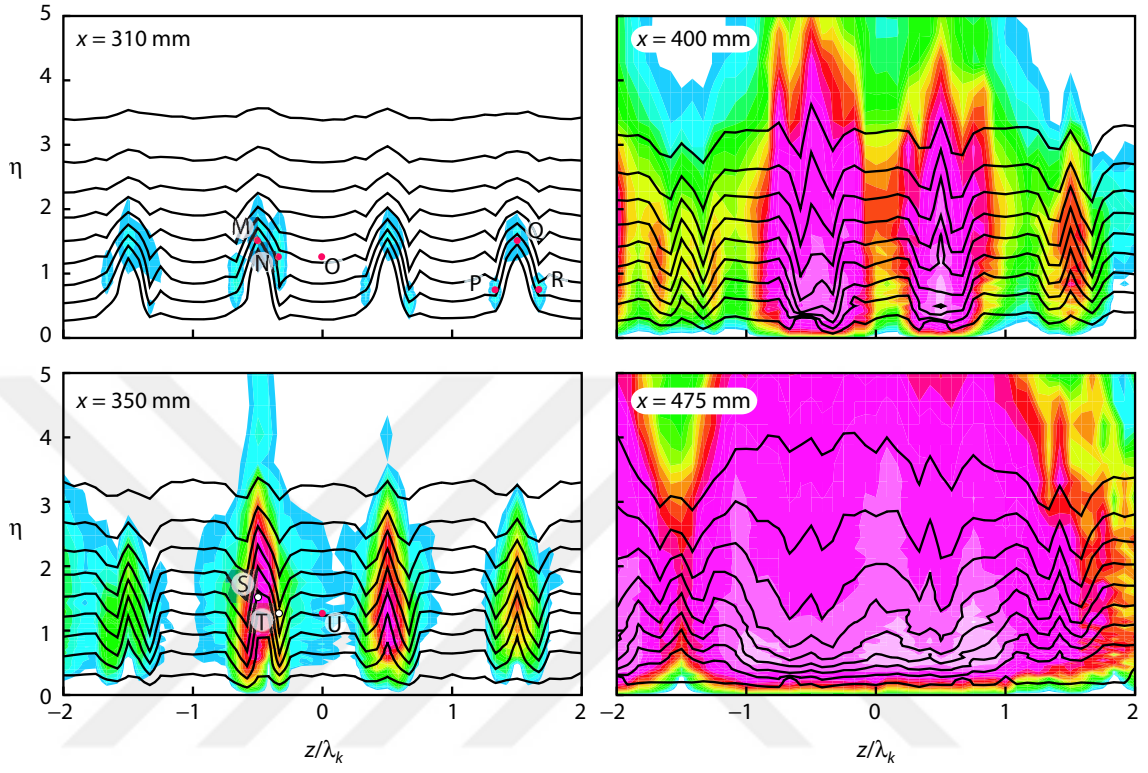


Figure 46: Contours of u'_{rms} and U for $Re_k = 334$. All contour levels are identical to Fig. 37.

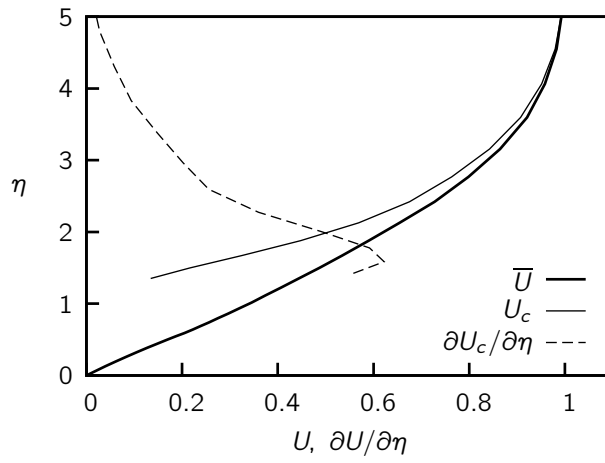


Figure 47: Basic state, centerline and $\partial U/\partial \eta$ profiles at $x = 310$ mm, $Re_k = 334$.

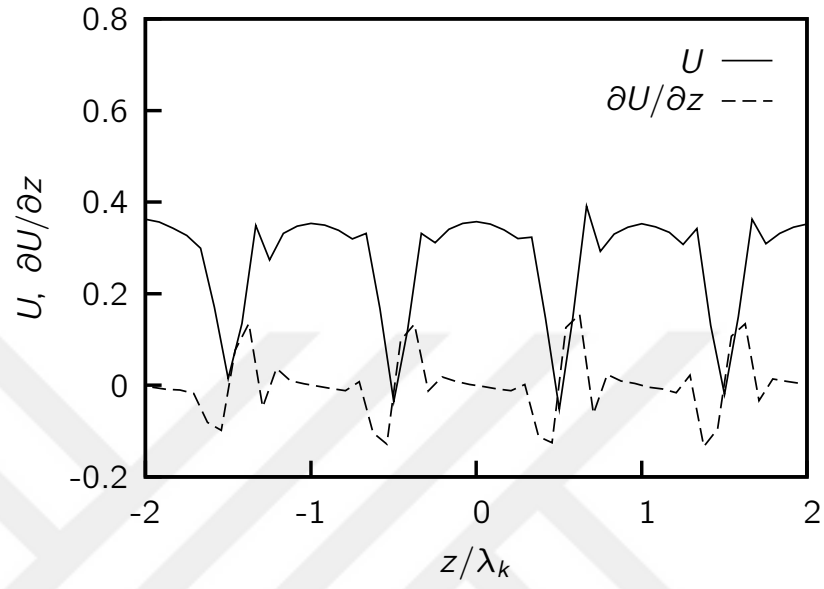


Figure 48: Spanwise variation of U and $\partial U/\partial z$ at $x = 310$ mm, $Re_k = 334$, $\eta = 1.25$.

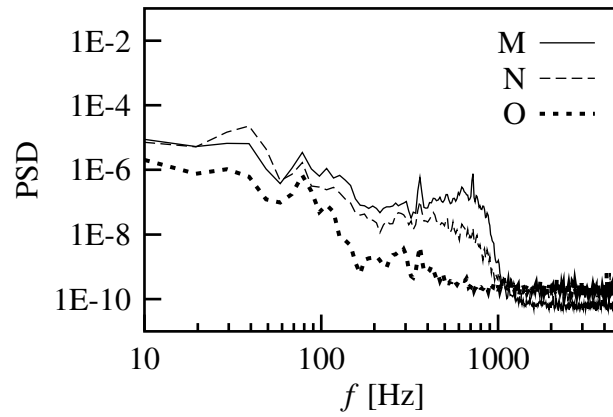


Figure 49: Power spectra obtained at the head, right leg and midpoint at $x = 310$ mm, $Re_k = 334$.

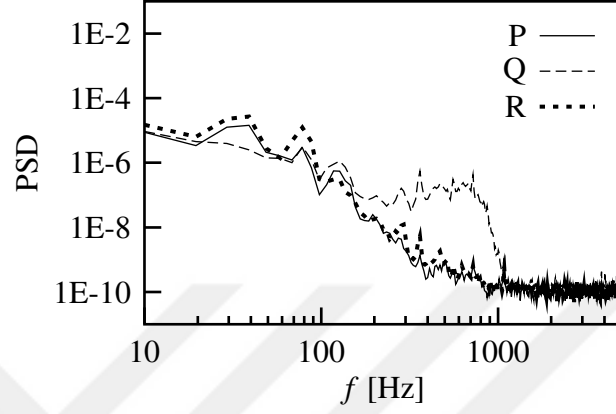


Figure 50: Power spectra obtained at points P, Q and R, $Re_k = 334$.

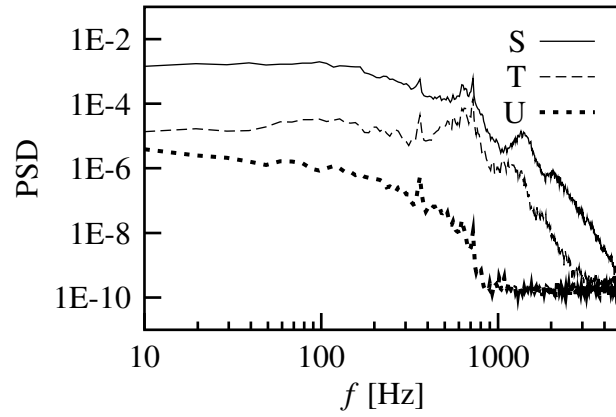


Figure 51: Power spectra obtained at the head, right leg and midpoint at $x = 350$ mm, $Re_k = 334$.

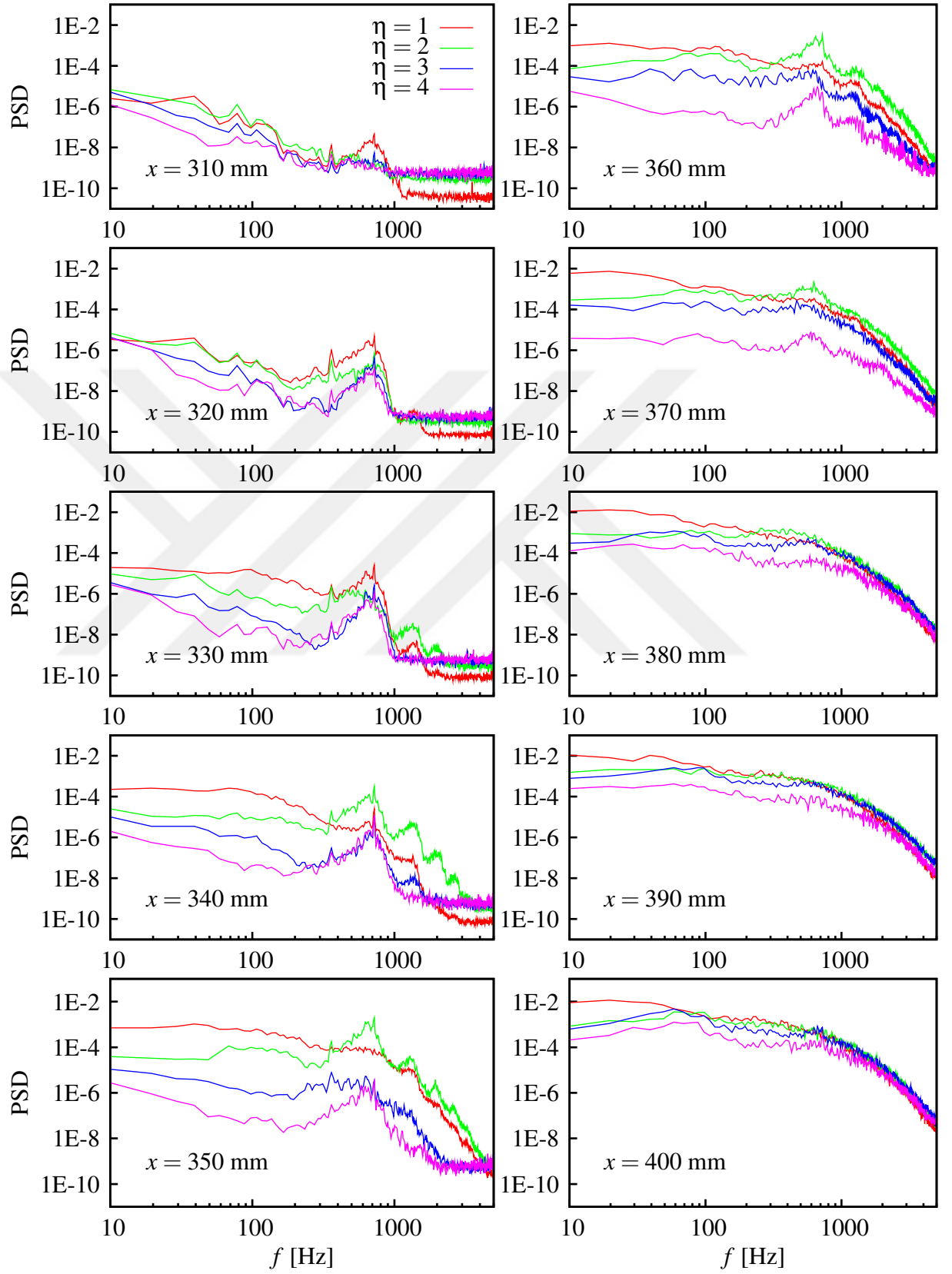


Figure 52: Temporal power spectra of the unsteady disturbances for $Re_k = 334$.

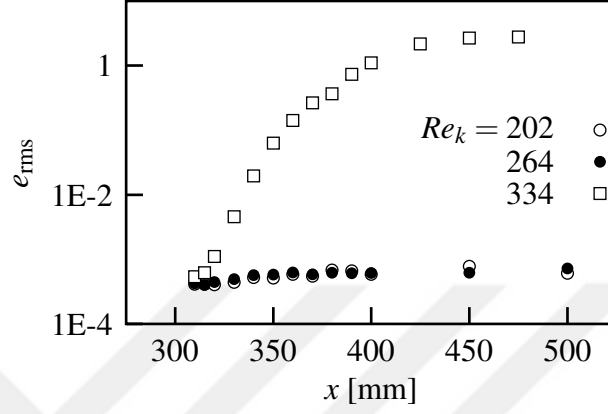


Figure 53: Streamwise evolution of the total unsteady disturbance energy, e_{rms} , in the wake of the second roughness element.

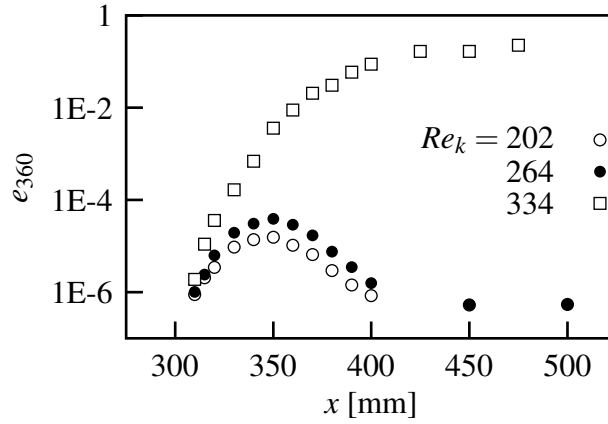


Figure 54: Streamwise evolution of e_{360} in the wake of the second roughness element.

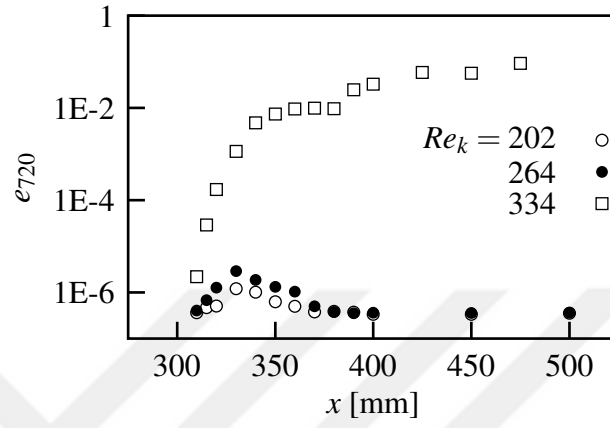


Figure 55: Streamwise evolution of e_{720} in the wake of the second roughness element.

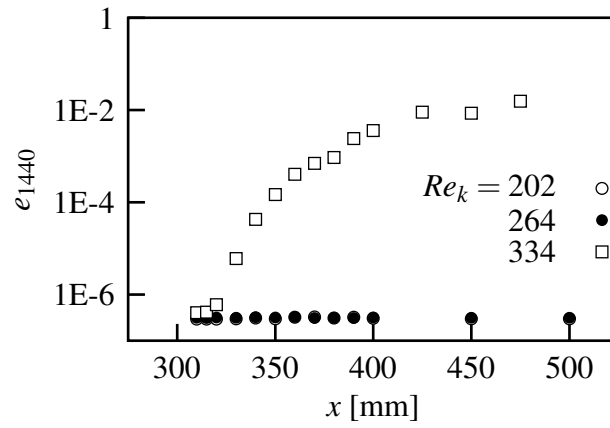


Figure 56: Streamwise evolution of e_{1440} in the wake of the second roughness element.

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