

The University of Southampton

2021/22

Faculty of Social Sciences Southampton Business School



MSc Dissertation

Distributionally Robust Portfolio Optimization for Radioactive Waste Fund

ERGO number: 75702

Student number: 32387334

Presented for MSc Business Analytics & Finance

This project is entirely the original work of student registration number 32387334. I declare that this dissertation is my own work, and that where material is obtained from published or unpublished works, this has been fully acknowledged in the references. This dissertation may include material of my own work from a research proposal that has been previously submitted for assessment for this programme.

Word Count: 13500 words

Abstract

While the benefits obtained with nuclear technologies, especially nuclear energy, continue to make our lives easier, the radioactive wastes should be stored as hazardous waste for many years. Therefore, their storage and decommission cause various costs. A radioactive waste fund can be named a security fund established to cover all costs arising from radioactive waste in the current or future. The main problem for this fund is that the fund, established to cover future costs, loses its value over time due to reasons such as inflation. For this reason, risky investments and efficient portfolio management are a requirement for radioactive waste funds. Because it will not be efficient to determine a framework for the investments of funds due to the different risk approaches of each country, it will be more constructive and effective to suggest an up-to-date portfolio optimization method to be used in investments. In this study, an optimum portfolio selection model has been studied with the Distributionally Robust Optimization method, which is up-to-date and continues to develop. In order to demonstrate the efficiency of the model, comparisons have been made based on the investment principles and results of the Swedish nuclear waste fund, which shares the data openly. In the comparison made for 2021, the model calculated the optimum portfolio by using historical data until the end of 2020. At the end of 2021, the real returns of the indexes are observed. As a result, the optimum portfolio weights determined by the model can get higher yield than the return rates published by the Swedish nuclear waste fund for 2021.

Acknowledgments

First of all, I would like to express my sincere gratitude to The Supreme Republic of Türkiye for providing me with this educational opportunity. It is a great honor for me to make even a small contribution with my study to the domestic and national energy movement that will liberate Türkiye economically and politically. Moreover, I give endless thanks to my wife Büşra who has always stood with me and supported me during this hazardous period, and my daughter Erva. I would also like to thank my dissertation supervisor Wei Wang and my personal academic tutor Jaw-Kai Wang, who helped me during the research and education, and of course, the University of Southampton.

Bin atlı akınlarda çocuklar gibi şendik...

Table of Contents

Abstract.....	II
Acknowledgments	III
Table of Contents.....	IV
Chapter 1: Introduction	1
1.1 Problem Statement and Project Objective.....	1
1.2 Motivation and Significance of Study	2
1.3 Structure of Contents	3
Chapter 2: Background	4
2.1 The Radioactive Waste Management.....	4
2.2 The Radioactive Waste Management Fund	5
2.3 Why Portfolio Optimization?	8
Chapter 3: Literature Review.....	11
3.1 Evolution of Portfolio Optimization Methods	11
3.2 Stochastic Programming and Robust Optimization.....	13
Chapter 4: Methodology.....	16
4.1 Sweden Nuclear Waste Fund.....	17
4.2 Data Collection and Processing	19
4.3 The Distributionally Robust Portfolio Optimization Model.....	20
Chapter 5: Results and Analysis	25
5.1 Empirical Distribution	26
5.2 Uncertainty Size and Ambiguity Set	29
5.3 Constraints	30
5.4 Application and Results	31
5.5 Analysis of Changing the Uncertainty Size	33
Chapter 6: Discussion	35
Chapter 7: Conclusion.....	39
References	41

List of Tables

Table 1 The Base Portfolio Weight Limits and The Comparison Index's Weights (Kärnavfallsfonden, 2020)	18
Table 2 Annual Return Rates of The Base Portfolio and The Comparison Index.....	18
Table 3 The Skewness and Kurtosis Values of The Indexes' Return Rates	27
Table 4 The Skewness and Kurtosis Values of The Indexes' Return Rates (with outlier cleaning)	27
Table 5 Mean, Variance and Standard Deviation of The Indexes' Return Rates obtained by the MLE.....	28
Table 6 Covariance Matrix of The Worst-Case Distribution.....	32
Table 7 Optimum Portfolio Weights obtained by The Model and Indexes' Actual Return Rates for 2021	32
Table 8 Results according to changed uncertainty size limits	33

List of Figures

Figure 1 Illustrative time projection of funding costs (IEAE, 2007)	6
Figure 2 The historical prices between 2011 and 2021	25
Figure 3 Histograms of The Indexes' return rates	26
Figure 4 Plots of The Empirical Distributions of Rates of Return For Indexes	30



Chapter 1: Introduction

1.1 Problem Statement and Project Objective

Developing nuclear technologies are taking more and more place in human life day by day. Nuclear technology is used in nuclear medicine, aircraft engines, smoke detectors and various technological devices, especially nuclear power plants. These benefits unfortunately cause radioactive wastes that are dangerous and need to be kept under control. The safe storage or disposal of these wastes brings various responsibilities together. As a matter of course, these responsibilities cause costs due to the storage of these wastes, which have the danger of radiation for a long time, is also high and long-term.

Many countries have established a radioactive waste fund in order to cover the costs that may arise or may arise, taking into account the advice given by the international atomic energy agency (International Atomic Energy Agency [IAEA] 2007). Because the purpose of these funds is mainly to cover future costs instead of current costs, money needs to be managed by considering time value and protected against inflation. In order to do this, some countries prefer completely risk-free investments, while others prefer to add risky assets to the fund's preferences by using effective portfolio management. In this way, it is aimed to protect the fund better and even to gain more return. Eventually, effective portfolio management is essential for the protection of the fund or for the targeted rate of return to be achieved in a controlled manner. In order to achieve effective portfolio management, it is important to get support from constantly updated statistical approaches and portfolio optimization methods.

In this thesis, we will examine the Distributionally Robust Portfolio Optimization method, which is up-to-date and continuing its development. The main purpose of examining this method is to present a reference optimization model for the established radioactive waste funds and their portfolios. Portfolio optimization methods, which started with the Mean-Variance method introduced by Henry Markowitz (1952), have made great improvements until today. Although Markowitz's mean-variance model remains popular today, concerns over its assumptions have driven portfolio experts to seek new approaches. As a branch

of Robust optimization, which is an advanced stochastic model, the Distributionally Robust optimization method emerges as an efficient optimization method. Thanks to this model, an efficient optimization method will be presented for the radioactive waste fund by exhibiting a more reliable risk approach.

In order to prove the efficiency of portfolio optimization, the model will be applied to a portfolio belonging to Swedish Nuclear Waste Fund. The main reason for choosing this country is that the Sweden Nuclear Waste Fund has been clearly publishing its investment rules and policies. In addition to this, since the fund publishes the returns of its portfolios annually, we will have the opportunity to compare our results. Results obtained from this implementation will be compared with what the fund has achieved, and the efficiency of the Distributionally Robust Portfolio Optimization method will be interpreted.

1.2 Motivation and Significance of Study

Although the demand for nuclear technologies and energy increases due to the great opportunities it provides, the hazardous wastes caused by these systems are too dangerous to ignore. Although these great responsibilities legally belong to governments and organizations that cause radioactive wastes, both academics and researchers have duties for this developing technology and its results. One of these responsibilities is radioactive waste management and a fund that will cover the expenses of radioactive waste management. Therefore, from the point of view of optimization and finance, it is a necessity to put forward various tools that will enable this fund to be managed effectively.

Moreover, Turkey has focused on establishing nuclear power plants like other developed or developing countries to meet its energy needs. These nuclear power plants, which are under construction, will need waste facilities in the future. Therefore, there will be a need for a radioactive waste fund in order to meet the costs that will arise in the future, both for the decommissioning of nuclear power plants and for the radioactive waste facilities to be established. It is important to support and provide a base for these future demands by conducting research. In parallel with this, our main motivation can be counted as benefiting from the experiences of other countries that have had nuclear facilities for many

years and conducting research that will support the management of the fund with current methods. Consequently, we will evaluate the distributionally robust portfolio optimization model, which is a current optimization method not only for the waste fund to be established in Turkey but also for the waste funds belonging to other countries around the world.

1.3 Structure of Contents

This study consists of 6 chapters. Chapter 1 identifies the problem to be studied and specifies the approach to be applied. It also gives information about why this research was conducted or, in other words, about the main motivation underlying the study. In Chapter 2, it will be mentioned basic information about the radioactive waste fund and published reports. Moreover, why effective portfolio management is needed will be explained by some references. After that, While the historical development of the portfolio optimization model to be applied and the weak aspects of other models will be discussed in Chapter 3, The Distributionally Robust Optimization model to be used will be defined in detail and a template model will be constructed in Chapter 4. While we will show the implementation of this model on a portfolio and its results In Chapter 5, these results will be further analyzed and discussed by getting support from arguments in the literature. Finally, Chapter 7 will summarize the results of the implementation and recommendations for future work will be given.

Chapter 2: Background

2.1 The Radioactive Waste Management

Nuclear science has been used in many applications around the world for a long time by improving day by day. Undoubtedly, the most well-known of these applications is the nuclear energy systems used for electricity production. In addition to these, it provides great contributions to human life by being used in fields such as medicine, the defense industry, spacecraft, general industry, agriculture, etc. On the other hand, in addition to these great benefits, end-of-life nuclear power plants and radioactive wastes as a result of nuclear processes continue to be dangerous for a long time. Therefore, these systems impose a responsibility on the government and all humanity, especially on the producers (Andra, 2020).

Almost all areas of production and consumption cause some wastes that need to be managed. All levels of waste arising from household wastes or chemical producers must be collected and disposed of in certain legal frameworks. Therefore, facilities producing radioactive waste are not in a different position if we look from only a management perspective. The main difference between radioactive waste and other wastes, on the other hand, is that their dangerous level is very high, so they must be collected and disposed of with much stricter rules. As long as these rules are properly applied and inspected, nuclear technologies will continue to take place in our lives as clean energy that greatly facilitates. Nuclear energy, which is evaluated by the International Energy Agency as the backbone of low-carbon electricity generation provides stable and efficient energy. Therefore, Nuclear energy can be an effective energy source to achieve the pace of CO₂ emissions reductions in line with the Paris Agreement and to combat global warming in the future (Biol, 2019).

Various international conventions on radioactive waste management have been drawn up to regulate these responsibilities and take into account the burden not only for today but also for future generations (IAEA, 2009). The standards specified in these conventions and agreements are regulated and updated by international organizations

such as the International Atomic Energy Agency (IAEA) and the Nuclear Energy Agency (NAE) according to technological developments and experiences.

The main purpose of Radioactive Waste Management is to minimize the damage of radiation to the environment and human health by ensuring that hazardous radioactive wastes are stored and disposed of safely (Rangarajan, 2014). The main challenge of radioactive waste management is the longevity of these hazardous wastes. While low-level radioactive wastes lose their ability to emit harmful radiation in a short time, they constitute 90% of the total radioactive wastes by volume. Medium-level wastes are subjected to various simple protective processes and stored or disposed of for a while. It also constitutes 7% of radioactive waste. High-level radioactive wastes are the highest level in terms of danger and difficulty. It needs to be protected and isolated with special covers in suitable environments for many years. Although these wastes constitute 3% of all radioactive wastes by volume, they cause 95% of the total radioactivity (World Nuclear Association, 2022).

The collection, transportation and storage of all these wastes, as well as the decommissioning of nuclear facilities when they reach the end of their life, are the main duties of radioactive waste management. These processes, which are carried out in the presence of strict safety rules that require special tools and facilities, cause high costs. In order to meet these necessary costs, countries have created Radioactive or Nuclear Waste Funds.

2.2 The Radioactive Waste Management Fund

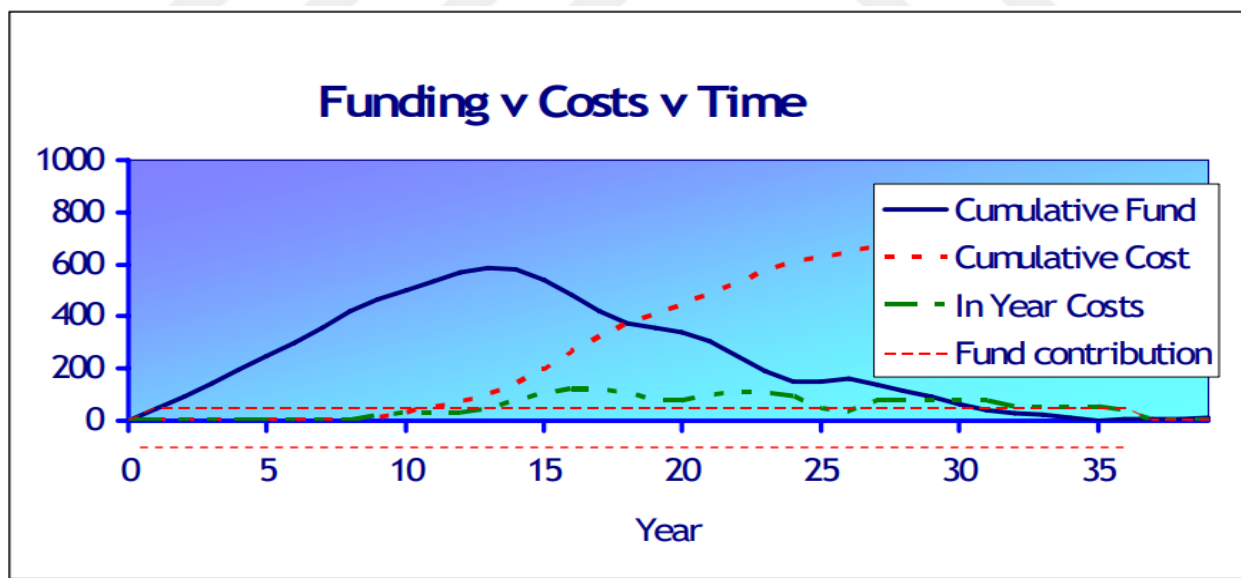
Generally, radioactive waste management is financed on the principle of "polluter pays". Namely, the costs incurred for the management of radioactive waste are a cost that nuclear facilities must cover. Therefore, it can be said that nuclear energy is one of the rare industries that takes full responsibility for its waste (Rangarajan, 2014).

An appropriate financing mechanism can be determined individually by the countries with nuclear facilities according to the amount of waste, waste management strategy, disposal or storage options, technological and physical infrastructure, and legal legislation. In most

of these countries, there are three types of organizations involved in the radioactive waste management mechanism. These can be listed as the producers responsible for providing the financial resources, the radioactive waste management organization responsible for the organization of waste management from start to finish, and the regulatory and supervisory institutions or authorities that will inspect both the radioactive waste producer and the radioactive waste management organization (IAEA, 2007).

Ensuring and maintaining the standards set for the storage and disposal of radioactive waste causes various operational and administrative costs in an ordinary way. Therefore, providing sustainable and adequate financial resources is one of the key requirements of waste management programs (IEAE, 2020). In order to meet these requirements from a reliable source, radioactive waste funds have been created. Moreover, the radioactive waste fund provides an equitable sharing of risk between the different stakeholders: producers, radioactive waste management organizations, and governments.

Figure 1 Illustrative time projection of funding costs (IEAE, 2007)



A radioactive waste fund, as mentioned earlier, generates income by receiving contributions from the producers and sometimes government budgets. The revenues generated must be available to cover the costs occurring throughout the life cycle of the facilities and the organization. In the figure above, the contributions received and costs emerging by the facility during its life cycle are roughly illustrated (IAEA, 2007). As can

be seen, it is shown that while the size of the fund increases cumulatively at the beginning, it will begin to decrease as a result of the costs incurred in the future and will eventually be reset. While the fund is enlarged with a fixed contribution by the producer or the government, it is decreased to zero as a result of the facility costs and dismissal costs that start to increase later on. These costs will inevitably change during the long lifecycle due to time value and various price changes around the world. Therefore, it is not possible to cope with these changes without evaluating the radioactive waste fund with various investments.

Contributions provided by the facilities that cause radioactive waste to the fund differ according to the preferences of the countries. Some countries, such as Germany, require a lump-sum payment of the fund by calculating future costs when installing facilities (KENFO, 2022). Some countries, such as Sweden, charge a tax or tariff on the amount produced (Kärnavfallsfonden, 2021). Of course, it is also possible to combine both systems according to preferences. Nevertheless, while it may be appropriate to provide funds from electricity generation plants that cause a continuous and high amount of waste, it is more appropriate to contribute to the fund through tariffs from producers that cause low levels and low amounts of waste (IAEA, 2007).

Some countries with nuclear power plants have chosen to support the fund with a system of guarantees, insurance policies or other guaranteed provisions to cover the costs, including unexpected situations that may arise during the long-life cycle of the facilities. The main reason for this is that only a part of the fund can be ready for use, especially at the beginning (European Commission, 1999). The increase in warehouse costs that will occur later and changes in prices can be counted among other important reasons. In this way, such guaranteed provisions will support the fund in case of higher-than-expected costs or unexpected situations. There will be a need for a cautious approach by the fund management against unexpected costs that may arise, especially when long-term investments are in question. Therefore, this attitude will prevent the larger part of the fund from being evaluated by investments. As a result, it can be said that these supports will allow the fund to be used in various investments.

On the other hand, the purpose of a radioactive waste management fund is to provide all the cash flow that will be needed in the future. The warranties should be regarded as an assurance against the unexpected. Generally, it is expected that the financial mechanism of the fund has already created the necessary budget to cover all the costs to be compensated. In this way, the necessary cash flow will be provided to fund management or waste facilities without being exposed to risks such as variable interest rates. As a result, the fund should be designed in such a way that it can provide uninterrupted financing to the Radioactive Waste Management Organization, provided the correct cost analyses are made and major unexpected situations are not encountered.

In conclusion, a radioactive waste fund is formed by being financed by the manufacturers as the government prefers. Its main purpose is to cover all costs that may arise during the life cycle of radioactive wastes and facilities. In order to provide this assurance, it is necessary to provide an effective cost calculation, a well-planned cash flow and various external assurances against unexpected situations. In this way, an effective cash flow will be ensured for the facilities and radioactive waste management organization in all possible situations.

2.3 Why Portfolio Optimization?

The purpose of Radioactive Waste Fund Management is not just keeping an amount of money under security as cash. While the gains from nuclear power are immediate, the resulting responsibilities spread for much longer years. In other words, while we can use electrical energy today obtained from nuclear power plants, we have to bear the storage and disposal costs for years and its decommissioning costs finally. These costs cannot be met by only keeping money as cash in a bank account. Therefore, the established fund must be properly managed for future costs by investing in industrial or public activities, including company shares, and corporate or governmental bonds.

As in every fund, all financial risks are valid in the radioactive waste fund. However, the main risks for the radioactive waste fund are specified and detailed in the IAEA (2007)'s report as return and liquidity risks. Return risk arises from the risk inherent in an investment portfolio. It should be managed within the framework of an investment

strategy, within the framework of certain rules. Some member countries use these investment decisions in favor of low-yield government bonds by applying a low-risk limit, while others focus on higher yields and use them in broader bonds. Liquidity risk arises from the uncertainty of cost estimates and the timing of plant installation or decommissioning. In these uncertain times, the size of the money to be withdrawn from the fund and the uncertainty of the time create a great constraint for long-term investments. The basic approach recommended against these risks is low-risk investments that will ensure that the fund does not lose value against inflation (IAEA, 2005).

There are two main risks of the fund arising from the two main duties of the fund. One of the main issues to be managed for the fund is the cost estimation and the other is the fund allocation. The dates of the facilities to be established, the dismissal times and their costs cause uncertainty in the cost estimation. Liquidity risk is one of the main risks as the fund must be kept ready to respond to these large liquidity demands. The return risk, which is the other main risk, is the risk that must be taken into account for the efficient allocation of medium and long-term investments. If the fund fails to deliver the expected return and loses value against inflation, it will lead to shortfalls for future costs and failing fund management will arise.

Although asset investment guidelines provided by IAEA suggest only low-risk investments for NAE countries, such guidelines only have a limited validity (2005) For instance, government bonds were traditionally regarded as safe assets on these guidelines. Even if it suggests very low or zero rates of return on government bonds, this will not protect the value of the obtained funds without a doubt (Keppler, 2021).

In addition, this problem was mentioned in European Commission written at the time of the Greek debt crisis, notes:

“Recent events in the financial markets have put serious doubts on the general safety of all types of government bonds. It may be advisable to revisit the implicit assumptions underlying the existing legislation in this respect and take into account the risk of state bankruptcies.” (European Commission, 2013).

Another negative example of keeping funds in only a currency was experienced in Germany. According to the statements made by Anja Mikus, the boss of the Nuclear Waste Disposal Fund, a large part of the 24 billion Euros belonging to the fund was in Bundesbank accounts. Because of the negative interest environment, the fund lost about 70 million Euros in 2018, the first year of the Nuclear Waste Disposal Fund (Reuters, 2018).

Each country and its fund managers have their own risk approach. It is not a good idea to assume that all countries with a radioactive waste fund have the same investment opportunities and the same economic conditions. Even an investment that seems risk-free for one country can turn into a risk, while not taking risks for another country causes losses. The results revealed that risky investments are a requirement for a radioactive waste fund. Parallel to this, investment recommendations made by international nuclear agencies or committees have lost their value.

As a result, it is a more efficient method for international organizations to direct countries to tools to be used for effective fund management instead of drawing a framework for investments in countries. Countries that have to make risky investments in order to protect the value of the fund aim to generate returns by creating various investment portfolios. Optimizing the portfolios used for funds during investments gains importance here. Portfolio optimization methods that can be adapted according to the risk approach of countries or various investment restrictions can be effective tools in this regard. Namely, by evaluating the current portfolio optimization methods, it will be an efficient method to advise and encourage the radioactive waste fund management to use these methods by international organizations.

Chapter 3: Literature Review

3.1 Evolution of Portfolio Optimization Methods

The main difficulty faced by management in the decision-making process around the world is the uncertainty over the parameters. This uncertainty may arise from limited data available, on measurements, estimation errors, etc. Likewise, the uncertainty of the return on investments to be made in portfolio management is seen as the main risk factor. The main purpose of portfolio optimization is to provide a balance of risk and return for investments at the optimum point. An investment portfolio represents a set of stocks, currencies, financial derivatives, bonds, etc., owned by the government, banks, and investment institutions, in short, any investor. Diversification has come to the fore as the basic approach to reducing risk in portfolio management. Diversification aims to reduce unsystematic risk in a portfolio by neutralizing the negative performance of some investments with the positive performance of others. The important thing here is to decide how to combine assets while reducing risk through diversification. For example, according to Best and Zhang (2022), the continued incorporation of weakly correlated assets can keep the portfolio's efficient frontier at risk. Therefore, the correlation of assets is one of the main factors for the return-risk balance to be achieved in portfolio management.

Many models have been improved that aim to provide the optimum matching of assets in a way that provides minimum risk or maximum return. Foremost among these is the Modern Portfolio Theory, which was published by Markowitz (1952), who was the first to come up with a systematic mean-variance method to evaluate the return and risk of the portfolio. However, this model is known to be extremely sensitive to estimation errors for risk and return balance, as well (Michaud and Michaud, 2007). Although it has computational difficulty and estimation errors arising from its assumptions, the model is still successfully applied today and encourages the development of finance (Qin, 2015). Moreover, after the modern portfolio theory, a number of studies and models that have been published have often taken this model as a reference or made improvements to this model, as well.

After Markowitz's portfolio theory, many approaches have been propounded for portfolio management. While some of them reveal various risk aversion approaches that Markowitz's mean-variance model cannot take into account because of its assumptions, some are optimization approaches that are built on Markowitz's theory or produced from a different perspective. The safety-first ratio put forward by Roy (1952) is an approach that determines the required minimum return at a certain risk level. In this way, investors were allowed to compare their portfolio investments based on the probability that their portfolio returns would fall below the desired minimum return threshold. Since William Sharpe laid the foundation of the Sharpe ratio in 1966, it has been one of the most widely used risk/return measures in finance. In addition to this, much of its popularity is attributed to its simplicity and easily calculable.

Another fundamental method, the Capital Asset Pricing Model, as known as CAPM, revolutionized modern finance. That was gradually developed in the early 1960s by William Sharpe (1964), and Jan Mossin (1966). The model provided the first consistent framework for relating the required return on investment to the risk of that investment (Perold, 2004). The Capital Asset Pricing Model, which was developed as a way to measure systematic risk, has become a fundamental model used in finance to price risky securities and generate expected returns for assets, taking into account the risk and cost of assets. William Sharpe published two more main approaches to provide an indicator for investments' risk-benefit rates. One of them is The Sharpe Ratio provides a simple way of observing how much return can be made for the extra volatility incurred when adding a riskier asset to the portfolio. The other one means absolute deviation (Sharpe, 1971). In this approach, a portfolio's characteristic line relates its rate of return to a "market portfolio". It has been calculated that it is more efficient to obtain such a line by minimizing the sum of the absolute deviations rather than the sum of the squares of the deviations around the mean. A problem that has been overlooked by assumptions with Markowitz's theory is also brought up with the Prospect Theory. The Prospect Theory explains the attitudes of investors in assessing their losses and their asymmetrical attitudes (Kahneman and Tversky, 1979). This study has ensured that the impact of aversion is also taken into account in portfolio selection. As a variation of the Sharpe ratio, The Sortino Ratio, which was introduced later, is published in 1994 by Sortino and Price.

This ratio uses the negative deviation of the asset instead of the total standard deviation of the portfolio returns, taking an approach that separates the harmful volatility from the total overall volatility. Another well-known method introduced is optimizing The Value at Risk as known as VaR, which remains popular today. The Value at Risk is a statistical method that measures the extent of possible financial losses within a portfolio or position over a given period of time (Rockfeller and Ursayev, 1999). However, optimization of The Value at Risk leads to a non-convex problem in which it lacks sub-applicability and convexity (Artzner et al. 1999). To address these disadvantages, Rockafellar and Uryasev (2001) proposed The Conditional Value at Risk, known as CVaR by calculating the uncertainty in the tail part of The Value at Risk more precisely in order to provide effective risk management. The Conditional Value at Risk is obtained by taking the weighted average of “extreme” losses in the tail of the distribution of possible returns, beyond The Value at Risk cut-off point.

Although the developed models, theories, and approaches enable us to obtain results closer to reality, the basic assumptions bring potential errors for these models. The main shortcoming of these portfolio optimization methods, which are created with various variations, is that the distribution obtained from historical data is assumed to be the true distribution, and therefore the results are overly sensitive to the estimated problem parameters (Best and Grauer 1991). For this reason, it is obvious that a model that will not ignore the errors that may arise from assumptions will give results closer to reality.

3.2 Stochastic Programming and Robust Optimization

The main problem in decision-making processes in the fields of Engineering and Management is uncertainty. Parameters' uncertainty in these processes may arise from noisy data, limited observability, estimation errors, etc... Stochastic and robust optimization methods aim to provide better results by modelling these uncertainties with certain assumptions (Rahimian and Mehrotra, 2019). The main difference between the two optimization methods is the assumption about the distribution belonging to the random parameter. Stochastic optimization assumes that the probability distribution of random parameters obtained from historical data is known exactly (Birge and Louveaux,

2011). Robust optimization, on the other hand, assumes that the probability distribution of random parameters is not known, but a range of the parameter is determined by the decision maker (Ben-Tal and Nemirovski, 2002). Nevertheless, any estimate or assumption is subject to error. Therefore, it would be helpful if the portfolio optimization problem could handle inputs given in other forms such as statistical distributions (Bertsimas et al., 2011).

Stochastic portfolio theory is a method designed to analyze portfolio and market behaviors with a modular structure. It was first introduced in detail by Fernholz (2002) in a monograph called the Stochastic Portfolio Theory. This theoretical tool is consistent with observable data of portfolios and markets. Moreover, it is a more advanced mathematical theory that aims to minimize the errors caused by the normative assumptions that support earlier famous theories such as Markowitz's mean-variance theory. It is stated that this theoretical tool, which approaches the stock market structure and arbitrage problems with a different evaluation, supports successful investments for many years (Fernholz and Karatzas, 2008).

The Monte Carlo simulation is one of the well-known examples of using stochastic models. The simulation model can simulate the movements of stocks or various assets based on probability distributions. It has been shown that models created as a result of various approaches give better results than Markowitz's mean-variance model and Naive's equal asset weight models (Nascimento et al., 2021)

This assumption of traditional methods that evaluate the inputs of the observational data from a deterministic point of view is not accepted in Robust Optimization. The Robust Optimization approach suggests that these inputs are observed or estimated with errors (Fabozzi et al., 2007). Namely, Robust Optimization assumes that the distribution of parameters cannot be known exactly. Therefore, it prefers to focus on the parameter rather than the distribution. As an objective, Robust Optimization takes the approach of estimation over a worst-case range or uncertainty set belonging to the parameter. Moreover, the optimized portfolio weights' results are able to be more stable and less sensitive against changes in model parameters, unlike traditional models. The

development of robust methods for optimization is seen as a major achievement within modern financial modeling theories (Xidonas et al., 2020). However, due to the nature of the methodology, robust optimal solutions are highly conservative and aim to guard against all possible worst-case uncertainty realizations. In some cases, this can lead to very conservative solutions (Hertog and Ross, 2017; Tütüncü and Koenig, 2004).

The extremely conservative results of Robust optimization have led to the emergence of Distributionally Robust Optimization as an alternative. While the distribution is unknown in robust optimization, distributionally robust optimization defines probability distribution as uncertain instead of unknown. However, it assumes that the probability distribution lies in an ambiguous set that includes all probability distributions (Rahimian and Mehrotra, 2019). Therefore, it can be said that The Distributionally Robust Optimization focuses on the uncertainty of the distribution of the parameters rather than the uncertainty of the parameters. In other words, while the robust model tries to find the worst case in an uncertainty set determined for the parameter, The Distributionally Robust Optimization focuses on finding the worst-case distribution within an uncertainty set (Gao et al, 2017). As a result, The Distributionally Robust Optimization paradigm bridges the gap between stochastic optimization and robust optimization. To do that, it assumes that the probability distribution's type is partially known, and this distribution lies in an ambiguity set that contains all the same type distributions that are close to the empirical or reference distribution with the true distribution. While creating this uncertainty set, which is the distinct feature of The Distributionally Robust Optimization model, various methods have been published and advised. Moreover, these methods are still improving such as the Kullback-Leibler divergence (Chen et al, 2018) and the Wasserstein distance metric (Blanchet et al, 2018). The main purpose of these methods is to specify a distance around the empirical distribution.

In conclusion, the stochastic programming approach has provided a new and revolutionary perspective for portfolio optimization. Robust optimization models, which do not want to ignore the errors that may be caused by the assumptions of the Stochastic Portfolio Theory, showed higher success compared to the traditional models. But these models also tend to give very conservative results. The Distributionally Robust

Optimization model, on the other hand, filled the gap between these two models and emerged as a model that does not ignore risks but is not unnecessarily conservative. Therefore, Distributionally Robust Optimization models, which are current and continue to develop, have the potential to be an effective tool for Radioactive waste funds.

Chapter 4: Methodology

The purpose of the method we will apply is to examine an optimization model for portfolios belonging to radioactive waste funds. This model will assist decision-makers in estimating the optimal portfolio weights among the investment options determined for the portfolio in an uncertain situation. As a result of the literature review, we decided to evaluate the Distributionally Robust Portfolio Optimization model as a current portfolio optimization management. For this evaluation, we will implement the model on a radioactive waste fund and its portfolio. These results will be compared with the actual return of the funds. In order to make this comparison, the historical data up to that date will be used, assuming that investment decisions are made at the beginning of 2021. Then, we will observe the return of the model over the prices realized at the end of 2021. After That, we will compare the results with the return obtained by the radioactive waste fund worked on. Thanks to this application, the importance of using portfolio optimization methods on radioactive waste funds will have been examined.

As stated in the previous sections, countries with radioactive waste funds have different approaches to the management of these funds. These differences generally arise from differences in risk approach or economic conditions. Generally, these countries focus on effectively managing the fund and evaluating the fund with investments at various risk levels. Therefore, each country preferred to create its specific investment policy. Some European countries such as England, Germany, and Sweden preferred to manage the fund by dividing it in order to diversify risk. In this way, they can keep money in the fund to provide a constant cash flow against unexpected costs, while at the same time they can provide extra revenue or increase the value of the fund thanks to various higher risky investments.

In order to clearly evaluate The Distributionally Robust Optimization model, information on the fund to be compared and its investments must be accessible and reliable. While some countries have shared some information about the fund's structure or investment policies, they have generally not chosen to report all results publicly. In this regard, the Swedish Nuclear Waste Fund stands out as the most efficient resource in terms of both clearly stating the investment rules and sharing the investment results with annual reports. The Sweden nuclear waste fund named Kärnavfallsfonden (<http://www.karnavfallsfonden.se/>) officially publishes most of the information about the aims, objectives, and investments of the radioactive waste fund on its website.

4.1 Sweden Nuclear Waste Fund

Sweden prefers to manage its nuclear waste fund investments using two different portfolios with different investment rules. One of them is the Base Portfolio, which can be invested in predetermined indexes with minimum risk. The other is the so-called Long-Term Portfolio, which includes investments based on a relatively higher risk and returns approach (Kärnavfallsfonden, 2021).

The portfolio, in which we will apply our model and compare the results, is the Base Portfolio because this portfolio has simply and clearly defined limits. There are three basic indexes, named the Real Return Bond Index, the Treasury Bond Index, and The Mortgage Bond Index among the investment options of the fund. These indexes' abbreviations are respectively, OMRXREAL, OMRXTBOND, and OMRXMORT. The weights of these indexes within the portfolio are subject to certain limits by investment policy decisions. Although these limits have changed over the years, the same investment policy and limits have been adopted since 2018 (Kärnavfallsfonden, 2022).

The goal or success criterion for the base portfolio is based on a benchmark portfolio called the comparison index. This index consists of the same three indices we mentioned before and their weights in the portfolio are constant according to the investment policy. The purpose of the base portfolio is to earn 0.25% more return than the specified comparison index during five-year periods.

Table 1 The Base Portfolio Weight Limits and The Comparison Index's Weights (Kärnavfallsfonden, 2020)

%	Lowest Share	Comparison Index (Reference portfolio)	Highest Share
OMRX REAL (Index-linked treasury bonds)	20	30	40
OMRX TBOND (Fixed-income treasury bonds)	10	20	30
OMRX MORT (Covered bonds)	35	50	60
Cash (Treasury bills and cash)	0	-	35

The weights of the comparison index or reference portfolio and the weight limits of the portfolio for predefined indexes are mentioned in the above table. The reference portfolio consists of OMRXREAL, OMRXTBOND, and OMRXMORT with respectively 30%, 20%, and 50%. The Base Portfolio can contain OMRXREAL index in the range of 20%-40%, OMRXT BOND index between 10%-30%, and OMRXMORT index between 35%-60%. While the base portfolio has specific lower and upper limits to investing for each index, there is one more option specified as "Cash". This option can be evaluated as a noninvestment part of the fund. Namely, according to this option, 35 per cent of the Base Portfolio can be kept as cash. These weight limits specified for the portfolio will appear as constraints to be considered for optimization while the mathematical model is being set up.

Table 2 Annual Return Rates of The Base Portfolio and The Comparison Index

Year	Return	Comparison Index
2017	1.48	0.39
2018*	1.97	1.81
2019	1.84	0.97
2020	0.93	0.20
2021	0.36	0.82

The Swedish Nuclear Waste Fund regularly reports the return of the fund every year. The table below is taken from the annual report published for the end of 2021 by the fund. The annual return provided by the base portfolio and the return that can be obtained as a result of investing in indexes at the rates in the reference portfolio are shown in the table. The detail specified for 2018* is due to the change in the weights of the comparison index. Until 2018, the comparison index for the base portfolio included OMRXREAL and OMRXTBOND with 30% and 70% respectively. After 2018 it was updated as

OMRXREAL, OMRXTBOND, and OMRXMORT with new weights determined as 30%, 20%, and 50%, respectively.

4.2 Data Collection and Processing

Before constructing the model, we need to collect the data and then make it available for use in the model. In addition, the parameters of the data to be used in the model should be estimated or determined by various approaches. In the application, historical prices of three indexes, OMRXREAL, OMRXTBOND, and OMRXMORT, which are specified in the Sweden Nuclear Waste Fund investment policy as the base portfolio will be used. The daily closing prices of the indexes specified for the 10-year period between 2011 and 2021 are taken. As a source, the data belonging to “nasdaqomxnordic.com” has been used.

After obtaining the data, various approaches and mathematical operations will be done to make it ready for use. Support will be received from SPSS, Excel and Excel Solver, both in the preparation phase and in the application phases of the model.

As a first step, the historical data at hand will be handled in a way that will show the daily return. These price changes in the indexes are random variables for the model as the return rate. The return rate formula was used for each index to obtain daily returns. In the formula below, while “R” means return rate, “ V_f ” and “ V_i ” mean the final and initial value for the index, respectively.

$$R = \frac{V_f - V_i}{V_i}$$

The second step will be to observe the distribution type of the data we will use. By observing the daily return rates obtained with histograms or other graphic methods, we will evaluate which distribution may be appropriate. Later, in order to evaluate this estimation, we will examine various tests or values of distributions such as skewness and kurtosis. We will explain the model's process on the basis of the normal distribution, which is a general assumption. However, the fact that the data has a different distribution may require different approaches. For example, while the parameters to be estimated from the

sample for the normal distribution are mean and standard deviation, degrees of freedom are also required for the t-distribution.

Empirical distributions will be the last thing we need before the data is used in the model. In order to fit the historical data to the determined distribution type, it is necessary to estimate the parameters of each index. Under the assumption of normal distribution, these parameters are the mean and standard deviation. We will use the Maximum Likelihood Estimation method to calculate and estimate the specified parameters of each index. If the empirical data have a normal distribution, this method can fit changes in historical prices to the most likely Gaussian bell curve. It can be said that the maximum likelihood estimator is here looking for a curve that maximizes the probability of historical data given a set of curve parameters. In other words, using this method maximizes the probability of the data while also maximizing the probability of a curve. As a result, the following formula will allow us to find the mean and standard deviation under the assumption of a normal distribution by finding the maximum likelihood value.

$$L(\mu, \sigma | x_1, x_2, \dots, x_n) = \prod_i^n \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x_n - \mu)^2 / 2\sigma^2} = -\frac{1}{2} \ln(2\pi) - \ln(\sigma) - \frac{1}{2\sigma^2} \sum_i^n (x_n - \mu)^2$$

The empirical distributions are obtained by adapting the daily price changes of each index to the normal distribution using the values obtained by the Maximum Likelihood Estimation method. The formula below is used to fit daily return values to a normal distribution. With these distributions, all the data that the model would need was obtained. What to do next will continue as constructing the model, determining the constraints, and applying the model to data.

$$y = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x_n - \mu)^2 / 2\sigma^2}$$

4.3 The Distributionally Robust Portfolio Optimization Model

In this section, we will explain a Distributionally Robust Portfolio Optimization model. One of the important differences of the Distributionally Robust Optimization model is that it has an inner problem. Before optimizing the portfolio weights for minimum risk, it is necessary

to find the distribution that gives us the worst-case in the uncertainty set, which is determined with an ambiguity set. In other words, this problem is denoted as inner maximization and our main problem is defined as a minimax distributionally robust optimization problem. After finding the worst-case in the ambiguity set by inner maximization, we will solve the minimization problem to optimize portfolio weights for minimum risk in the feasible region, where arranged according to constraints for the model.

As mentioned in the previous sections, Markowitz's mean-variance portfolio selection model is one of the main methods that is still used and accepted today by financial managers. For this reason, the Distributionally Robust Portfolio Optimization model can also be explained more clearly based on this model. The basic formulation for Markowitz's Modern Portfolio Theory is given by (Blanchet et al., 2021);

$$\min_{\omega \in H} \{ \omega^T Var_{P^*}(R) \omega : \omega^T \mathbf{1} = 1, \omega^T E_{P^*}(R) = \gamma \}$$

The Modern Portfolio Theory's model aims to find minimum portfolio variance which is considered as a risk value for the portfolio. In addition to this, the model has to meet the targeted rate of return during minimization of the portfolio variance. In the model given by the above formula, H denotes the unit simplex, $E_{P^*}(R)$ represents the expected return of the portfolio, and $Var_{P^*}(R)$ indicates the covariance of the return rates under P^* . P^* is a probability measure underlying the distribution of R . " $\omega^T Var_{P^*}(R) \omega$ " is the variance of the portfolio. The model considers it as a risk value for the portfolio. Namely, the main purpose of the model is to find the minimum " $\omega^T Var_{P^*}(R) \omega$ " by optimizing the weights of the portfolio " ω " and meeting the targeted expected return " γ ". As specified as " $\omega^T \mathbf{1} = 1$ ", it explains that the sum of the portfolio weights should be 100% or 1 for optimal portfolio.

One of the well-known weaknesses of modern portfolio theory is that it is very sensitive to the estimation of mean and covariance calculated from historical data because it assumes that these values belong to the true distribution. Therefore, this assumption causes estimation errors because of noisy data and insufficient data. The Distributionally Robust Optimization model can overcome these estimation errors and obtain a more

accurate estimation in compared with the modern portfolio theory by using an uncertainty size created by various ambiguity set methods. These methods for ambiguity set aim to provide a distance measurement for the limits of the uncertainty size where the worst-case distribution is chosen. Moreover, it is assumed that this uncertainty set contains true distribution and all distributions which are the same type as the empirical distribution. Although there are a few current models in the literature to acquire an efficient ambiguity set or uncertainty size, the parametric-based ambiguity set will be used in the model in order to make the application of the study simpler and clearly understandable.

The main objective function for the distributionally robust portfolio optimization, in other words, the formulation of the distributionally robust optimization for the mean-variance problem is given by the formula below (Blanchet et al., 2021).

$$\min_{\omega \in F_{\varepsilon_1, \varepsilon_2, \gamma}(n)} \max_{P \in U_{\varepsilon_1, \varepsilon_2}(P_n)} \{\omega^T Var_P(R) \omega\}$$

In this formula, ω and $Var_P(R)$ have the same meaning as in Markowitz's model. These symbolize portfolio weights and covariance of returns respectively. However, this covariance is not obtained from historical data under P^* . It symbolizes the artificial true distribution's covariance under the artificial true distribution is denoted by P assumed within the determined ambiguity set. Actually, this is the main distinction of the distributionally robust optimization method because the distributionally robust model does not limit its estimation to only historical data or empirical distribution. Instead of this, it uses a larger uncertainty size containing the empirical and prospective true distributions with an assumption. The parametric-based ambiguity set formulation to be used to determine the limits of the uncertainty size is shown by the formula below.

$$U_{\varepsilon_1, \varepsilon_2}(P_n) = \left\{ N(\mu, \sigma) \in U : \frac{|\mu - \mu'|}{\mu'} \leq \varepsilon_1 \text{ and } \frac{|\sigma - \sigma'|}{\sigma'} \leq \varepsilon_2 \right\}$$

Thanks to this ambiguity set, we will create an uncertainty size to solve the inner maximization problem. In other words, we will seek the distribution giving us the worst-case scenario. While probable mean μ and standard deviation σ belong to normal distributions and prospective true distributions in the ambiguity set, μ' and σ' are mean

and standard deviation which belong to the empirical distribution. This set provides new distributions in uncertainty size " U " limited by ε_1 and ε_2 and it surrounds the empirical distribution P_n . Thanks to this formulation, we can define uncertainty size that can be proportion of this uncertainty size contains all normal distributions within these distance limits denoted by ε_1 and ε_2 . The aim here is to assume that the probable true distribution has the same type of distribution as the empirical distribution. Moreover, it will be assumed that the ambiguity set, expanded around the empirical distribution, contains the true distribution, as well. Thanks to this, we will have been taken into account errors arising from considering only empirical distribution by using an ambiguity set.

There are two key parameters, ε_1 and ε_2 , in the ambiguity set formulation. These parameters draw a bound around the empirical distribution. While ε_1 restrain the distance between the mean of the empirical μ' and the artificial true distribution μ , ε_2 limits the distance between the standard deviations of the empirical and the artificial true distribution, respectively σ and σ' .

To provide an efficient uncertainty size, these parameters need to be chosen carefully. The importance of the ambiguity set limits is to provide a relatively larger cluster with a boundary created near calculated empirical distribution, rather than a huge cluster. As stated before, this way, the estimation errors that will be made with only historical data will be avoided. If these limits, ε_1 and ε_2 , are determined too large, the model uncertainty will be too high. Therefore, the portfolio selection will be unnecessarily conservative, and the available data will become less relevant. If ε_1 and ε_2 , are designed too small, using the distribution function, which is the main goal of the distributionally robust method, will become inefficient for the model.

The equations and formulations we have explained so far belong to the objective function and ambiguity set. Now, we will define the constraints that the model must meet. The feasible region denoted by F defines these constraints together. Therefore, what will be explained here are the constraints that the model must meet while determining the portfolio weights that will give us the minimum variance. Two of these will belong to portfolio weights. Because the sum of all the weights in our portfolio has to be 1.

Moreover, any portfolio weight cannot be below zero or any negative number. Another constraint is the targeted return value symbolized by γ . Namely, the model will try to give us the minimum risk that will give us the return we want. Of course, these constraints should be met with the worst-case distribution in the ambiguity set. The specified feasible region is given cluster below (Blanchet et al, 2018).

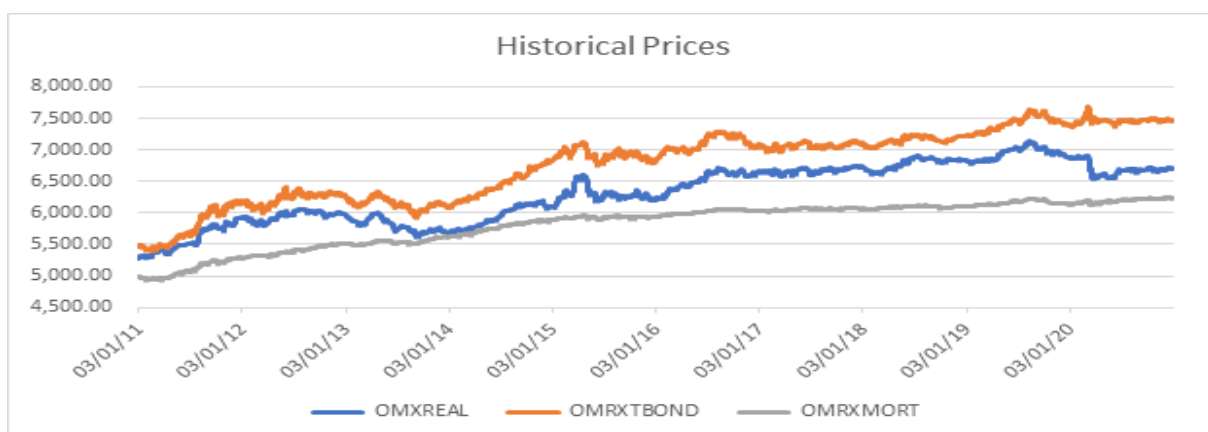
$$F_{\varepsilon_1, \varepsilon_2, \gamma}(n) = \left\{ \omega: \omega^T \mathbf{1} = 1, \omega_i \geq 0, \min_{P \in U_{\varepsilon_1, \varepsilon_2}(P_n)} E_P(\omega^T R) \geq \gamma \right\}$$

The optimization model was explained on the Modern Portfolio Theory accepted as a base model. As can be seen, the main feature of The Distributionally Robust Optimization is considering possible estimation errors arising from Markowitz's assumption about the true distribution. To overcome these errors, The Distributionally Robust Optimization method uses a cluster called the ambiguity set that surrounds the empirical distribution, which it assumes includes the true distribution. Although there are efficient models such as Wasserstein Distance or Kullback-Leibler divergence in the literature to determine the uncertainty set limits, in this study, a parametric-based uncertainty set was preferred to ensure clarity and easy explanation (Chen et al., 2018; Blanchet et al., 2018). As a first step, the inner maximization problem is solved, and the worst-case distribution is obtained. After defining other general portfolio constraints such as targeted return and sum of portfolio weights by the feasible region, the model optimizes portfolio weights to obtain minimum variance in the determined uncertainty set and the determined feasible region. Then, by observing the return rates of the indexes at the end of 2021, we will calculate the return we can achieve with the optimized weights. Finally, we will compare the return rate of 2021 published by the fund for the base portfolio with the return we can obtain as a result of optimization. Consequently, thanks to The Distributionally Robust Optimization approach, estimation errors arising from assumptions will be decreased, and, whether more efficient results can be gained will be observed for the fund's portfolio.

Chapter 5: Results and Analysis

The purpose of the optimization model we will implement is to create a portfolio with minimum risk that will meet the targeted return. The accepted risk measure for the portfolio is the standard deviation or variance of the return rates of the indexes. Markowitz's mean-variance method uses a covariance matrix directly derived from historical data to evaluate both the relationship between indexes and the standard deviation of indexes. The main difference in the Distributionally Robust model is its focus on distributions instead of this. First of all, the daily prices need to be obtained from the mentioned website's database and handled to convert to daily return rates. After that, the distribution type of historical data is observed and determined. Then, by creating an empirical distribution, an ambiguity set is created to determine a boundary around this distribution, and it is assumed that the true distribution is within this ambiguity set. The inner maximization problem will be solved to find the distribution which will give us the maximum variance among all distributions within the limits of the parameters specified in the ambiguity set. Furthermore, these distributions should have the same type as the empirical distribution. After the worst-case distribution is found in the uncertainty size, the portfolio weights will be optimized to obtain minimum risk value according to this distribution by meeting the fund and general portfolio optimization constraints.

Figure 2 The historical prices between 2011 and 2020



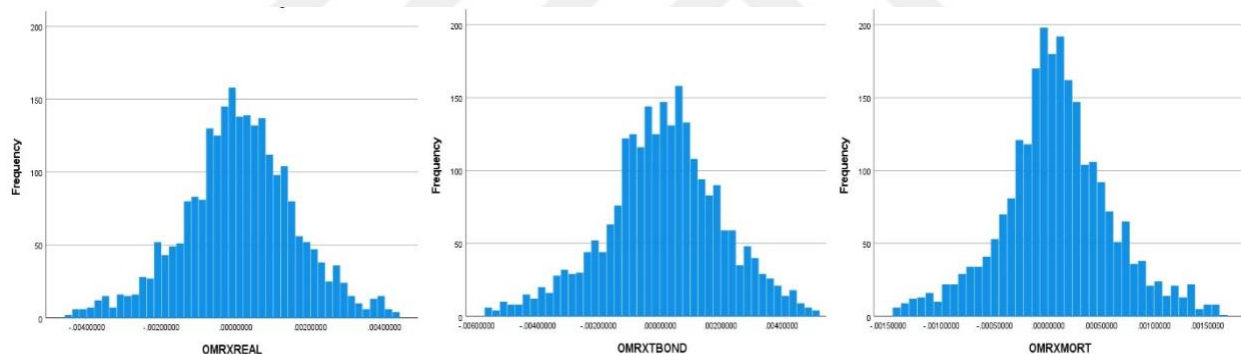
First, the data were obtained for each index to be included in our portfolio from the previously mentioned source. Historical prices belonging to the indexes can be seen on

the graph below. While all indexes have a general upward trend, OMRXREAL and OMRXTBOND have parallel each other and fluctuating pricing. However, the prices of OMRXMORT have a more stable increase. Therefore, OMRXMORT can be evaluated as a less risky index. These graphical evaluations will also be clearly seen numerically when standard deviation calculations are made.

5.1 Empirical Distribution

The next step is an observation of the distribution type of historical data before getting empirical distributions. Therefore, we need to first predict a similar distribution type by extracting the histograms of the return rates with the help of SPSS after the daily prices convert to daily return rates for each index. The histograms of the return rates for each index that we will work on are given in the figure below.

Figure 3 Histograms of The Indexes' return rates



We will observe the skewness and kurtosis values in order to assume a normal distribution. The range in which normality assumptions can be made in various references has been evaluated. While the range of +2 and -2 is the acceptable range for normal distribution assumptions in some papers (George and Mallery, 2010; Trochim and Donnelly, 2006; Field, 2009; Gravetter & Wallnau, 2014), Hair et al. (2010) and Bryne (2010) claimed that the data can be considered as a normal distribution if the skewness value is between -2 to +2 and the kurtosis value is between -7 to +7. The skewness and kurtosis values of the daily returns of each index are given in two tables by calculating in the SPSS. Table 4 shows the results of the raw data, while Table 5 shows the values obtained after outlier cleaning. For this outlier cleaning, 0.7% of the data in OMRXREAL,

0.3% in OMRXTBOND, and 0.7% in OMRXBOND were replaced with average values. The skewness values in both tables are in the acceptable range provided in the references. Kurtosis, on the other hand, complies with all references given with an outlier arrangement below 1 per cent, while it does not exceed +7 with raw data. Therefore, according to the references given, it can be assumed that the daily return values of the indexes are normally distributed. In order to provide more strong proof of the normality assumption, the values after the outlier arrangement are also shown. However, the raw data will be used in this optimization study.

Table 3 The Skewness and Kurtosis Values of The Indexes' Return Rates

Descriptive Statistics					
	N	Skewness		Kurtosis	
	Statistic	Statistic	Std. Error	Statistic	Std. Error
VAR00016	2508	-.545	.049	6.839	.098
VAR00017	2508	-.286	.049	5.914	.098
VAR00018	2508	.269	.049	5.310	.098

Table 4 The Skewness and Kurtosis Values of The Indexes' Return Rates (with outlier cleaning)

Descriptive Statistics					
	N	Skewness		Kurtosis	
	Statistic	Statistic	Std. Error	Statistic	Std. Error
OMRXREAL00	2508	-.214	.049	1.912	.098
OMRXTBOND00	2508	-.042	.049	1.569	.098
OMRXMORT00	2508	.318	.049	1.914	.098

When we examine the skewness and kurtosis values according to the values obtained from the data, we see that they are within the acceptable range for the normal distribution specified in the references. After the distribution type is determined as a normal distribution, the Maximum Likelihood Estimation method will be used to find the mean and variance. Assuming that the daily return rates have a normal distribution, we need mean and standard deviation parameters to fit our data to the normal distribution. We can find the mean and variance by calculating the maximum likelihood value. We make

calculations to find this value using the Solver and we reach the mean and variance. During these calculations, we will use the probability density function of normal distribution and its simplified formula to use in the maximum likelihood estimation method. These formulas are given below.

$$L(\mu, \sigma | R_1, R_2, \dots, R_n) = \prod_i^n \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_n - \mu)^2}{2\sigma^2}} q = -\frac{1}{2} \ln(2\pi) - \ln(\sigma) - \frac{1}{2\sigma^2} \sum_i^n (x_n - \mu)^2$$

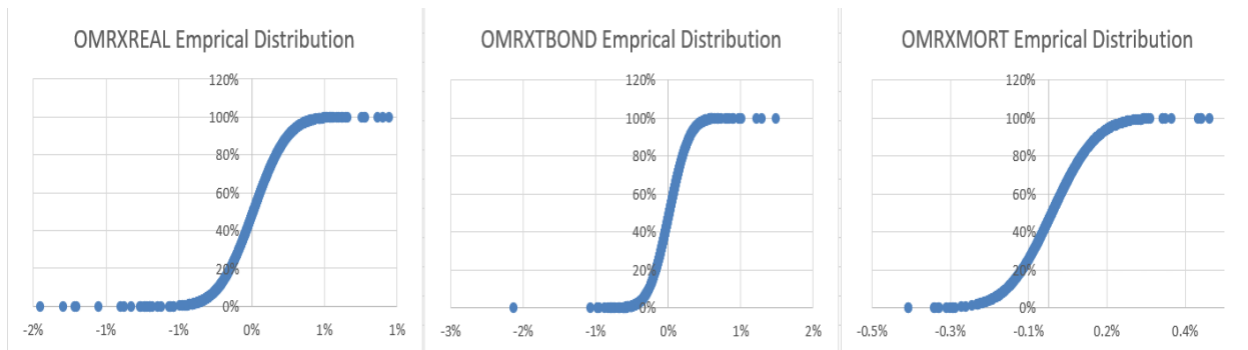
With the given formula, the maximized likelihood value and the mean, variance, and standard deviation values of the indexes were obtained. The results are as follows.

Table 5 Mean, Variance and Standard Deviation of The Indexes' Return Rates obtained by the MLE

	OMXREAL	OMRXTBOND	OMRXMORT
Mean	0.009708568%	0.011117427%	0.008898540%
Variance	0.000348684%	0.000478331%	0.000039986%
Standard Deviation	0.186730700%	0.218707806%	0.063234217%
Maximum Likelihood Value	12198.03	11696.62	14900.13

The values obtained by the Maximum likelihood estimation method and the data were fitted to the normal distribution and empirical distributions were created for each index. The graphs of these distributions are shown in the graphs below.

Figure 4 Plots of The Empirical Distributions of Rates of Return For Indexes



5.2 Uncertainty Size and Ambiguity Set

As we have mentioned before, the distinct feature of Distributionally Robust Optimization is the ambiguity set used by it. In this application, the method to be used to determine the uncertainty size will be the parametric-based ambiguity set. The parameters which will be limited are the mean and standard deviation of the distributions. Namely, a cluster will be obtained by expanding the mean and standard deviation of the empirical distribution with a distance in the determined ratio. The basic assumption here is that this resulting set contains the true distribution, and the true distribution has the same type of distribution as the empirical distribution. In other words, we draw a boundary around the empirical distribution and consider all normal distributions within this boundary. What we will do next is to find the worst-case normal distribution within this uncertainty size. The determined parametric-based ambiguity set is given expression below.

$$U_{\varepsilon_1, \varepsilon_2}(P_n) = \left\{ N(\mu, \sigma) \in U : \frac{|\mu - \mu'|}{\mu'} \leq \varepsilon_1 \text{ and } \frac{|\sigma - \sigma'|}{\sigma'} \leq \varepsilon_2 \right\}$$

While ε_1 and ε_2 symbolizes the distance limit between the means of the true distribution μ and empirical distribution μ' , likewise, ε_2 is mentioning a distance limit between the covariances. As mentioned in previous sections, these limits should be determined carefully. Otherwise, too small values will render the distributional robust method meaningless, while too large values will lead to very conservative results. We determined ambiguity set limits as 0.1 for each index. In other words, we will define the uncertainty size by extending the empirical distributions' mean and standard deviations by +10% and -10%. In order to evaluate the impact of these limits, we will again analyze the results by changing the epsilons as 0.001, 1, and 2 after the model is optimized first with 0.1.

The ability that the ambiguity set provides us with is to determine a cluster for the inner maximization problem. All normal distributions that can occur with the parameters we have limited will be included in this uncertainty size. What we will do here is to find the distribution that will maximize the variance within these limits we have determined. To do that, we keep the portfolio weights constant and ask the solver to find the normal distribution, which will give the maximum variance, by changing the mean and standard

deviation values within limits. The distributions that allow us to obtain the maximum variance will be our worst-case distributions. Then, we will define the mean and covariance matrix obtained from these distributions as expected return and risk values and use them in our minimization problem. Before proceeding to the main optimization problem, the constraints must be set for optimization calculation. Here, in addition to the basic optimization constraints, the other constraints set by the fund's specific investment policy will be added.

5.3 Constraints

After determining the uncertainty size, it is necessary to define the constraints for general portfolio optimization and the specific constraints of the Swedish nuclear waste fund. As a general portfolio optimization constraint, the sum of portfolio weights must be 1 and all weights must be greater than or equal to zero. The restrictions set by the Swedish nuclear waste fund for the portfolio are different for each index. While the investment limits have been determined for OMRXREAL, OMRXTBOND and OMRXMORT as a minimum of 20%, 10%, 35% and a maximum of 40%, 30%, 60% respectively. In addition, a part of the portfolio, excluding indexes, can be kept without investing in any index. This part's proportion can be between 0 and 35%. Therefore, apart from the weights of the indexes, one more weight called "Cash" will be added and restricted among the portfolio options. Moreover, it is considered as zero mean and zero standard deviation during portfolio optimization.

The last constraint is the targeted return for the portfolio. Essentially, the investment policy of Sweden Nuclear Waste Fund management does not set a specific limit for return and risk, instead preferring to set portfolio weights. In this study, as an example, we will consider the minimum targeted return rate for the portfolio as the risk-free rate in Sweden. This rate mentioned is determined as 0.9% for 2021 (Statista, 2022). Namely, the expected return should be equal to or greater than 0.9% as a limit for the model. In addition to all these, of course, all the constraints have to be met, not only feasible region's constraints but also ambiguity set limits. The feasible region definition below shows constraints together for the sum of portfolio weights and targeted return. The other specific weight constraints for the portfolio belonging to the Sweden Nuclear Waste Fund

are shown separately. All constraints, ambiguity set, and objective functions are given all together below.

$$\min_{\omega \in F_{\varepsilon_1, \varepsilon_2, \gamma}(n)} \max_{P \in U_{\varepsilon_1, \varepsilon_2}(P_n)} \{\omega^T \text{Var}_P(R) \omega\}$$

$$U_{\varepsilon_1, \varepsilon_2}(P_n) = \left\{ N(\mu, \sigma) \in U : \frac{|\mu - \mu'|}{\mu'} \leq 0.01 \text{ and } \frac{|\sigma - \sigma'|}{\sigma'} \leq 0.01 \right\}$$

$$F_{\varepsilon_1, \varepsilon_2, \gamma}(n) = \left\{ \omega : \omega^T 1 = 1, \omega_i \geq 0, \min_{P \in U_{\varepsilon_1, \varepsilon_2}(P_n)} E_P(\omega^T R) \geq 0.009 \right\}$$

$$20\% \leq \omega_{OMRXREAL} \leq 40\%$$

$$10\% \leq \omega_{OMRXTBOND} \leq 30\%$$

$$35\% \leq \omega_{OMRXMORT} \leq 60\%$$

$$0 \leq \omega_{CASH} \leq 35\%$$

5.4 Application and Results

The details of the necessary operations to be performed before performing optimization with the Excel Solver were explained in the previous section. To summarize, after making the daily return conversion for historical prices, empirical distributions were created, and uncertainty size was formulated by ambiguity set. After that, both general and specific constraints were defined for the portfolio. The last thing to do before the solver starts the calculation is to set a limit for the ambiguity set. These distance limits were defined as 0.1 for each asset's distribution parameters. We will solve this optimization problem in two steps. In the first step, as mentioned before, we will optimize the mean and standard deviations of the distributions in the uncertainty size to find the maximum portfolio variance. In the second optimization, the model will optimize portfolio weights to find minimum portfolio variance by meeting all constraints. Later, we will change the ambiguity set limits or the epsilons and make the same processes again and evaluate the impacts of these limits on the results.

After the determination of the uncertainty size limits to 0.1, distributions that would give the minimum variance were obtained by optimizing the mean and standard deviation with the support of the solver. The worst-case covariance matrix generated as a result of the optimization of the inner maximization problem is given in the table below.

Table 6 Covariance Matrix of The Worst-Case Distribution

	OMXREAL	OMRXTBOND	OMRXMORT
OMXREAL	0.000349269%	0.000344054%	0.000077963%
OMRXTBOND	0.000344054%	0.000519022%	0.000119929%
OMRXMORT	0.000077963%	0.000119929%	0.000040474%

Optimum portfolio weights were obtained as a result of applying the model by performing both stages, respectively. As we explained earlier, we will assume that the investments in these assets were made with obtained optimum portfolio weights at the beginning of 2021. In order to see the return obtained as a result of the investments made for 2021, we will use the return rates of the indexes at the end of 2021. Therefore, the table below shows the annual real return rates for each index in 2021 and the optimal portfolio weights obtained by means of the model.

Table 7 Optimum Portfolio Weights obtained by The Model and Indexes' Actual Return Rates for 2021

	OMXREAL	OMRXTBOND	OMRXMORT	Cash
Optimum Portfolio Weights	20%	10%	35%	35%
Actual Annual Return Rates for Each Asset in 2021	5.241%	-1.245%	-0.931%	0

According to the results in the above table, if the Distributionally Robust Portfolio Optimization model built has been used while making investment decisions for 2021, a 0.59% return could be achieved with the optimum portfolio weights obtained by means of the model. On the other hand, in the 2021 annual report published by Sweden Nuclear Waste Fund, the return rates for the Base Portfolio remained at 0.36%. Therefore, it can be claimed that the model established with the Distributionally Robust Portfolio Optimization achieved better results than the Sweden Nuclear Waste Fund's portfolio selection.

5.5 Analysis of Changing the Uncertainty Size

In the table below, the optimum portfolio weights obtained by changing the ambiguity set limits used were shown. As is known, the purpose of these limits is to limit the distance between empirical distribution and normal distributions in the uncertainty size. To provide this, we limit mean and standard deviation belonging to probable normal distributions. In this way, the uncertainty set will have been extended with the determined limit. We already made the previous optimization by assuming epsilons equal to 0.1 for mean and standard deviation. During these comparisons and analyses, the epsilons will be changed to 0.001, 1, and 2. Namely, we will extend the multiplier of the empirical distribution by 0.01%, 100% and 200%, respectively. After that, the optimization method will be applied again. By this means, we will evaluate the differences in expected return, expected variance, and optimum portfolio weights provided by the model.

Table 8 Results according to changed uncertainty size limits

Epsilons	Exp. Return	Exp. SD	Real Annual Return	Exp. Annual Return	OMRX REAL	OMRX TBOND	OMRX MORT	Cash
0.001	0.0063724%	0.0756214%	0.598%	2.353%	20%	10%	35%	35%
0.1	0.0069638%	0.0756301%	0.598%	2.574%	20%	10%	35%	35%
1	0.0125152%	0.0756709%	0.598%	4.674%	20%	10%	35%	35%
2	0.0186835%	0.0757205%	0.598%	7.057%	20%	10%	35%	35%

The results show that the increase in the limits causes the expected standard deviation to increase. In other words, as mentioned before, too large limits cause the model to give more conservative results. Nevertheless, these growing risk values did not change optimum portfolio weights. The main reason for this result, our model does not have a constraint for variance. If we had a risk constraint, when the model cannot meet the maximum variance constraint, it would have to change the portfolio weights. Namely, it would have to change the portfolio weights with less return and less risky assets. As a result, growing uncertainty size or ambiguity set limits cause higher risk value. This

situation arises from the inner maximization problem. Because we try to find maximum variance in the inner maximization, the model focuses on the more extreme limits as long as increase standard deviation limit. Therefore, this situation causes more conservative results as mentioned in the literature.

In conclusion, we preferred to implement the Distributionally Robust Portfolio Optimization model on a portfolio belonging to the Swedish Nuclear Waste Fund because this fund clearly provides information about its investments and investment policy. In order to evaluate the results of this model, we assumed that the portfolio had been optimized at the beginning of 2021. For this reason, we handled historical data between 2011 and 2020. Following that, the empirical distributions were created from these data. After that, we defined an uncertainty size by using a parametric-based ambiguity set. In this set, we optimized optimum portfolio weights by providing the minimum variance in the worst-case distribution. The results showed that the return provided by using this model is better than the return gained by the fund. Moreover, a comparison made by changing ambiguity-set limits has shown that an increase in limits caused more conservative expected values. Therefore, the model was forced to make risky investments to meet the expected targeted return because of these conservative values when the limits were increased. Based on these results, it can be claimed that using the Distributionally Robust Portfolio Optimization method for Radioactive Waste Funds can provide effective results. Nevertheless, using more current models for ambiguity sets such as Wasserstein Distance or Kullback-Leibler divergence finding in the literature can provide more efficient results for the portfolios.

Chapter 6: Discussion

It is a well-known fact that, especially, high-level radioactive wastes are dangerous for human health and the environment. The storage or disposal of these wastes by removing them from hazards brings along various high costs. The fact that the financial responsibilities of these wastes belong almost entirely to the waste generator makes these energy facilities one of the rare facilities that are fully responsible for their own waste (Rangarajan, 2014). However, thanks to the funds created, it can be said that the manufacturer covers the calculated cost and transfers this responsibility to the fund management. Incorrect calculation of future costs, improper determination of rates in the payment system, or the loss of value of the fund as a result of incorrect investments may cause future costs to be not met. In some countries, with the approach of internal management of the fund, the energy supplier company was allowed to manage the fund in its own account and only audited it. Some countries, on the other hand, preferred to manage the fund by establishing a reliable radioactive waste fund with an external management approach (IAEA, 2005). In other words, while some countries have taken full control and responsibility of the fund, some countries have left the responsibility to the supplier company, with the management of the fund under the supervision of the supplier company. The reliability of both approaches is debatable. Therefore, limiting these approaches by international organizations and determining the ideal approach would be a more accurate solution. Otherwise, the responsibility of both the waste and the funds may lead to consequences as the result of conflicts of interest between countries and energy providers, as well as failure to manage the waste correctly.

It is a fact accepted in published reports that a fund that will only be used for risk-free investments will lose value over time. We can see this clearly, especially with the ongoing inflation 2021-2022 inflation surge around the world in recent years (International Monetary Fund [IMF], 2022). Against these rising inflation values, the income to be obtained from risk-free investments only allows the fund to lose less value. For example, while the inflation rate published by Sweden in the report of the 2021 nuclear waste fund was 3.9% (Karnavfallsfonden, 2022), the average risk-free rate in 2021 remained at 0.9% (Statista, 2022). Therefore, investments seen as risk-free in Sweden in 2021 can make

the fund lose 3% instead of 3.9%. As a result, the use of the fund in risky investments is a mandatory approach in order to protect the value of the fund.

When the investment results of the Swedish nuclear waste fund are observed, although the fund provided the expected return before 2021, it lagged behind the comparison index's return in 2021. The reason for this can be the economic impact of the coronavirus. The virus, which still has an impact around the world even in 2022, caused damage and imbalance in the financial order of the world in 2019. According to the report presented by the IMF (2022), although there was a temporary recovery in 2021 for some economic indicators, the expected risks were increasing the uncertainty and risk in the financial ratio. Not surprisingly, the realization of these risks in 2022 led to increasingly pessimistic developments. The world economy, weakened by the pandemic, took various blows, and caused extremely high inflation rates in developed countries, especially America and European countries. Therefore, the world has encountered a gloomy and more uncertain financial atmosphere. In parallel with that, the estimated return rates of the model we applied were also lower than the real rates. The main reason for this estimation problem probably arises from this uncertain atmosphere in the financial world. Although the expected return of the model we applied is not the same as the actual return for 2021, even in a year with such an uncertain atmosphere, yielding a better return by the model than the return obtained by Sweden Nuclear Waste Fund is another proof to show the efficiency of the model.

An interesting result came from the maximization of the Sharpe ratio. When the solver optimized the maximum Sharpe ratio for the portfolio, instead of the minimum variance of the portfolio, the optimum portfolio weights for OMRXREAL, OMRXTBOND and OMRXMORT became 20%, 10%, and 60%, respectively. If these weights multiply with the 2021 return rates, the return is equal to 0.36%. The interesting point is that this value is equal to the return published by the Sweden Nuclear Waste Fund in the 2021 annual report. From this, it can be deduced that the fund management also makes its investments using the Sharpe Ratio. Although this situation is likely to be a coincidence, it can be thought that the fund aims for higher returns with a maximum Sharpe ratio instead of minimum variance and takes more risk.

An ambiguity set is a tool, which is the main difference of The Distributionally Robust Optimization model. It is used to determine uncertainty size. Uncertainty size has been interpreted in a source as a power given to an adversary, and they have drawn an analogy that the larger the uncertainty size is, the larger the power is given to the adversary (Blanchet et al, 2018). This is mainly because The Distributionally Robust Optimization model always focuses on the worst case. Therefore, high limits allow the worst case to reach the extremes. The model, which decides according to this case, has to meet the constraints in the worst-case calculate optimum portfolio weights. In other words, the purpose of the model here is to provide the constraints of the parameters that are calculated according to the worst case and have constraints. Therefore, if we add a constraint as maximum variance, the model can struggle to meet this constraint and can have to change optimum portfolio weights with less return portfolio. In short, the limits cause changes in the expected values that the model evaluates when making-decision. The constraints placed on the model for these values affect the optimal portfolio results by changing the limits. In other words, the optimum portfolio remains at the same weight unless ambiguity set limits conflict with the parameter's constraint in the worst case.

This study, which was carried out in a short time, needs to be expanded in future studies. Although an example has been presented in this study, this evidence alone cannot demonstrate that the distributional robust portfolio optimization model is a perfect model for radioactive waste fund management. Examples should be multiplied by considering the funds from different countries, different investment rules, and different years. In this way, more data can be evaluated, and more efficient results can be obtained. The studies to be done for the development of the model should be about the uncertainty set. The determination of distances is one of the key elements of the distributional robust model. Initially, examining the studies evaluating various divergences on this subject (Ben-Tal et al., 2013) and then, as we mentioned before, Kullback-Leibler (Hu and Hong, 2012) and especially Wasserstein Distance (Blanchet et al., 2019; Gao et al., 2017) in the literature. Studies to be carried out on the subject will enable the use of a more effective distributional robust model. Both the evaluation and implementation of these studies and the following up-to-date models by following the literature will be guiding for future studies. Parallel to this, more reliable models will be established for radioactive waste funds.

As a result, our application has revealed the idea that effective portfolio management is required and this will be possible with a portfolio optimization method, based on the studies that indicate the necessity of risky investments for radioactive waste funds. However, this study presented in a short time cannot be sufficient alone to show that the Distributionally Robust Portfolio Optimization is the most suitable method for radioactive waste funds. Therefore, expanding the study will yield more efficient results. In other words, the results obtained by applying portfolio optimization applications to different funds in different countries should be evaluated. Moreover, the determination of ambiguity set limits is a subject that needs to be examined in more detail. In particular, choosing the distance too high may even cause the model to not work. By following the developments in the literature, more efficient results can be carried out with more current uncertainty set methods.

Chapter 7: Conclusion

This paper includes an implementation of a current optimization model for radioactive or nuclear waste funds. Thanks to this study, we have found an opportunity to observe the distributionally robust optimization method for a portfolio belonging to a radioactive waste fund. It was decided that this fund, which was recommended to be used with riskless investments by international nuclear energy agencies in the past, cannot be protected only by investing only risk-free assets. Namely, it was mentioned why the portfolio management needs to focus on risky investments for the specified funds. Due to each country having a different risk approach, it was thought that it will not be possible to draw up an investment framework and it would be more appropriate attitude to suggest using a portfolio optimization method instead. Portfolio optimization methods that have developed from The Modern Portfolio Theory introduced by Henry Markowitz (1952) until the present, were perused and studies were carried out on one of the most current methods, the Distributionally Robust Optimization model. Since distributionally robust portfolio optimization models focus on minimum risk, it will be an efficient portfolio optimization management for funds that aim to minimize risk, such as the radioactive waste fund. The main feature that distinguishes the model from other optimization methods is that it does not consider only historical data and empirical distribution as a true distribution like traditional portfolio optimization methods but also partially takes into account the errors or differences by means of an ambiguity set. Moreover, it can be said that the Distributionally Robust Optimization method, which is a two-stage minimax problem, can provide less risky and efficient portfolio management with values optimized for the worst case.

As a result of the studies based on the data of the Swedish Nuclear Waste Fund, which clearly shares its investment policy and results, it has been observed that the return which can be obtained by using The Distributionally Robust Optimization model provides more profit or return than investments made by Swedish Nuclear Waste Fund. Moreover, it was mentioned that for the ambiguity set, which is one of the key features of the model, it is possible to turn to more up-to-date and effective methods, which can yield more efficient results. Consequently, it is observed that using the up-to-date distributionally robust

portfolio optimization method and monitoring its development will provide efficient results for radioactive waste funds that want to act with minimum risk.



References

1. Ahmad, T., Haroon, Baig, M. and Hui, J. (2020) Coronavirus Disease 2019 (COVID-19) Pandemic and Economic Impact. *Pakistan journal of medical sciences*, 36(COVID19-S4), S73–S78. Available from: <https://doi.org/10.12669/pjms.36.COVID19-S4.2638> [Accessed 01 October 2022].
2. Andra (2020) National Inventory of Radioactive Materials and Waste. *Essentials*, January Available from: https://international.andra.fr/sites/international/files/2020-03/Andra-MAJ_Essentiels_2020_UK.pdf [Accessed 01 October 2022].
3. Artzner, P., Jean-Marc, E., Delbaen, F. and Heath, D. (1999) Coherent Measures of Risk. *Mathematical Finance*, 9(3), 203-228. Available from: <https://doi.org/10.1111/1467-9965.00068> [Accessed 01 October 2022].
4. Ben-Tal, A. and Nemirovski, A. (2002) Robust Optimization-Methodology and Applications. *Mathematical Programming*, 92(3), 453-480. Available from: <https://doi.org/10.1007/s101070100286> [Accessed 01 October 2022].
5. Ben-Tal, A., Hertog, D., Waegenaere, A. and Melenberg, A. and Rennen, B. G. (2012) Robust Solutions of Optimization Problems Affected by Uncertain Probabilities. *Management Science*, 59(2), 341-357. Available from: <https://doi.org/10.1287/mnsc.1120.1641> [Accessed 01 October 2022].
6. Bertsimas, D., Caramanis, C. and Brown, D. B. (2010) Theory and Applications of Robust Optimization. *Society for Industrial and Applied Mathematics*, 53(3), 464-501. Available from: <https://doi.org/10.1137/080734510> [Accessed 01 October 2022].
7. Best, M. J. and Grauer, R. R. (1991) On the Sensitivity of Mean-Variance-Efficient Portfolios to Changes in Asset Means: Some Analytical and Computational Results. *The review of financial studies*, 4(2), 315-342. Available from: <https://doi.org/10.1093/rfs/4.2.315> [Accessed 01 October 2022].
8. Best, M. J. and Xili, Z. (2012) The Efficient Frontier for Weakly Correlated Assets. *Computational Economics*, 40, 355-375. Available from: <https://doi.org/10.1007/s10614-011-9296-5> [Accessed 01 October 2022].
9. Birge, J.R. and Louveaux, F. (2011) *Introduction to Stochastic Programming*. Springer, Germany:Berlin. Available from: <https://optimization-online.org/?p=15942>

10. Birol, F. (2019) Nuclear Power in a Clean Energy System. The International Energy Agency [IEA], May. Available from: [https://iea.blob.core.windows.net/assets/ad5a93ce-3a7f-461d-a441-8a05b7601887/Nuclear Power in a Clean Energy System.pdf](https://iea.blob.core.windows.net/assets/ad5a93ce-3a7f-461d-a441-8a05b7601887/Nuclear_Power_in_a_Clean_Energy_System.pdf) [Accessed 01 October 2022].
11. Blanchet, J., Chen, L. and Zhou, X. Y. (2018) Distributionally Robust Mean-Variance Portfolio Selection with Wasserstein Distances. ArXiv preprint. Available from: <https://doi.org/10.48550/arXiv.1802.04885> [Accessed 01 October 2022].
12. Blanchet, J.H., Murthy, K. and Si, N. (2019) Confidence Regions in Wasserstein Distributionally Robust Estimation. *arXiv: Statistics Theory*, June. Available from: <https://doi.org/10.48550/arXiv.1906.01614> [Accessed 01 October 2022].
13. Byrne, B. M. (2010). *Structural equation modeling with AMOS: Basic concepts, applications, and programming*. New York: Routledge.
14. Chen, Y., Guo, Q., Sun, H., Li, Z. and Wu, W. (2018) A Distributionally Robust Optimization Model for Unit Commitment Based on Kullback–Leibler Divergence. *IEEE Transactions on Power Systems*, 33(5), 5147-5160. Available from: <https://doi.org/10.1109/TPWRS.2018.2797069> [Accessed 01 October 2022].
15. European Commission (1999) *Schemes for financing radioactive waste storage and disposal. Nuclear Safety and Environment*, Publications Office, Belgium:Brussel. Available from: <https://op.europa.eu/en/publication-detail/-/publication/304c7436-f13b-4419-8288-811ff6ca7f16/language-en> [Accessed 01 October 2022].
16. European Commission (2013) Financial Resources Earmarked For The Decommissioning of Nuclear Installations, Spent Fuel and Radioactive Waste. *Communication From the Commission to the European Parliament and The Council*, 8th March, Belgium:Brussel. Available from: <https://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=COM:2013:0121:FIN:EN:PDF> [Accessed 01 October 2022].
17. Field, A. (2000) *Discovering statistics using SPSS for Windows*. London-Thousand Oaks- New Delhi: Sage publications.
18. Fabozzi, F. (2007) *Fixed Income Analysis*. 2nd Edition, Wiley, Hoboken.

19. Fernholz R. and Karatzas, I. (2008) Stochastic Portfolio Theory: an Overview. *Mathematical Modeling and Numerical Methods in Finance*, 15, 89-167. Available from: <https://www.math.columbia.edu/~ik/FernKarSPT.pdf> [Accessed 01 October 2022].
20. Fernholz, R. (2002) *Stochastic Portfolio Theory*. Springer. Available from: <https://link.springer.com/book/10.1007/978-1-4757-3699-1> [Accessed 01 October 2022].
21. Gao, R., Chen, X. and Kleywegt, A.J. (2017) Wasserstein Distributionally Robust Optimization and Variation Regularization. *arXiv: Learning*. Available from: <https://arxiv.org/abs/1712.06050> [Accessed 01 October 2022].
22. George, D. and Mallery, M. (2010) *SPSS for Windows Step by Step: A Simple Guide and Reference*, 10th ed. Boston: Pearson.
23. Gravetter, F. and Wallnau, L. (2014) *Essentials of statistics for the behavioral sciences*, 8th ed. Belmont, CA: Wadsworth.
24. Hair, J., Black, W. C., Babin, B. J. and Anderson, R. E. (2010) *Multivariate data analysis*, 7th ed. Upper Saddle River, New Jersey: Pearson Educational International.
25. Hertog, D. and Roos, E. (2018) Reducing conservatism in Robust Optimization. *INFORMS Journal on Computing*, 32(4), 1109-1127. Available from: <https://doi.org/10.1287/ijoc.2019.0913> [Accessed 01 October 2022].
26. Hu, Z., & Hong, J.L. (2012) Kullback-Leibler Divergence Constrained Distributionally Robust Optimization. *Optimization Online*, November. Available at: <https://optimization-online.org/2012/11/3677/> [Accessed 01 October 2022].
27. International Atomic Energy Agency [IAEA] (2005) Financial aspects of decommissioning, IAEA Safety Standards Series, IAEA-TECDOC-1476, Vienna, November. Available from: https://www-pub.iaea.org/MTCD/publications/PDF/te_1476_web.pdf [Accessed 01 October 2022].
28. International Atomic Energy Agency [IAEA] (2007) Cost Considerations and Financing Mechanisms for the Disposal of Low and Intermediate Level Radioactive Waste, IAEA-TECDOC-1552, IAEA, Austria:Vienna Available from: https://www-pub.iaea.org/MTCD/Publications/PDF/te_1552_web.pdf [Accessed 01 October 2022].

29. International Atomic Energy Agency [IAEA] (2020) Costing Methods and Funding Schemes for Radioactive Waste Disposal Programmes. *IAEA Nuclear Energy Series*, November, Austria:Vienna. Available from: https://www-pub.iaea.org/MTCD/Publications/PDF/PUB1900_Web.pdf [Accessed 01 October 2022].
30. International Monetary Fund [IMF] (2022) Gloomy and More Uncertain. World Economic Outlook, July. Available from: <https://www.imf.org/en/Publications/WEO/Issues/2022/07/26/world-economic-outlook-update-july-2022> [Accessed 01 October 2022].
31. Kahneman, D. and Tversky, A. (1979) Prospect Theory: An Analysis of Decision under Risk. *Econometrica*, 47(2), 263–291. Available from: <https://doi.org/10.2307/1914185>
32. KENFO (2022) *The Fund*. Available from: <https://www.kenfo.de/en/about-us/about-kenfo> [Accessed 01 October 2022].
33. Keppler, J.H. (2021) Ensuring the Adequacy of Funding Arrangements for Decommissioning and Radioactive Waste Management. *Fundamentals of Sustainable Electricity Markets*, OECD. Available from: https://www.researchgate.net/publication/353224534_Ensuring_the_Adequacy_of_Funding_Arrangements_for-Decommissioning_and_Radioactive_Waste_Management [Accessed 01 October 2022].
34. Karnavfallsfonden (2021) *Investment Policy*. Sweden. Available from: <http://www.karnavfallsfonden.se/download/18.799ae30c17f44a711ed2673f/1648639301934/Investment%20policy%202022.pdf> [Accessed 01 October 2022].
35. Karnavfallsfonden (2022) The Nuclear Waste Fund Annual Report for 2021. Sweden. Available from: <http://www.karnavfallsfonden.se/download/18.799ae30c17f44a711ed2684a/1648639301864/Annual%20Report%202021.pdf> [Accessed 01 October 2022].
36. Markowitz, H. (1952) Portfolio Selection. *The Journal of Finance*, 7, 77-91. Available from: <https://doi.org/10.1111/j.1540-6261.1952.tb01525.x> [Accessed 01 October 2022].

37. Michaud, R.O. and Michaud R. (2007) Estimation Error and Portfolio Optimization: A Resampling Solution. *Journal of Investment Management*, 6(1), 8–28. Available from: <http://dx.doi.org/10.2139/ssrn.2658657> [Accessed 01 October 2022].
38. Mossin, J. (1966) Equilibrium in a Capital Asset Market. *Econometrica*, 34(4), 768–783. Available from: <https://doi.org/10.2307/1910098> [Accessed 01 October 2022].
39. Nascimento, I., Albuquerque, P. and Peng, Y. (2021) Stochastic Volatility Modelling in Portfolio Selection Via Sequential Monte Carlo Simulation. *International Journal of Portfolio Analysis and Management*, 2(3), 249-267. Available from: <https://doi.org/10.1504/IJPAM.2021.115633> [Accessed 01 October 2022].
40. Perold, A. F. (2004) The Capital Asset Pricing Model. *Journal of Economic Perspectives*, 18 (3), 3-24. Available from: <https://www.jstor.org/stable/3216804> [Accessed 01 October 2022].
41. Qin, Z. (2015) Mean-variance model for portfolio optimization problem in the simultaneous presence of random and uncertain returns. *European Journal of Operational Research*, 245(2) 480-488. Available from: <https://doi.org/10.1016/j.ejor.2015.03.017> [Accessed 01 October 2022].
42. Rangarajan, S. (2014) What Is Radioactive Waste Management?. *Chemical Business*, 28(3). Available from: <https://southampton.on.worldcat.org/oclc/5751422993> [Accessed 01 October 2022].
43. Rahiman, H. and Mehrotra, S. (2019) Distributionally Robust Optimization: A Review. arXiv preprint arXiv:1908.05659.
44. Reuters (2018) *Staatlicher Atomfonds verliert nach einem Jahr noch Geld*. Reuters, 10 July. Available from: <https://www.reuters.com/article/deutschland-atomfonds-idDEKBN1K01EP> [Accessed 01 October 2022].
45. Rockafellar, R. T. and Uryasev, S. (1999) Optimization of Conditional Value-At-Risk. *Journal of Risk*, 2(3), 21-42. Available from: <https://doi.org/10.21314/JOR.2000.038> [Accessed 01 October 2022].
46. Rockafellar, R. T. and Uryasev, S. (2001) Conditional Value-at-Risk: Optimization Approach. *Stochastic Optimization: Algorithms and Applications*, vol 54, 411-435. Available from: https://doi.org/10.1007/978-1-4757-6594-6_17 [Accessed 01 October 2022].

47. Roy, A. D. (1952) Safety First and the Holding of Assets. *Econometrica*, 20(3), 431–449. Available from: <https://doi.org/10.2307/1907413> [Accessed 01 October 2022].
48. Sharpe, W. F. (1964). Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. *The Journal of Finance*, 19(3), 425-442. Available from: <https://doi.org/10.2307/2977928> [Accessed 01 October 2022].
49. Sharpe, W. F. (1966) Mutual Fund Performance. *The Journal of Business*, 39(1), 119–138. Available from: <http://www.jstor.org/stable/2351741> [Accessed 01 October 2022].
50. Sharpe, W. (1971) Mean-Absolute-Deviation Characteristic Lines for Securities and Portfolios. *Management Science*, 18(2), B1-B115. Available from: <https://doi.org/10.1287/mnsc.18.2.B1> [Accessed 01 October 2022].
51. Sortino, F. A. and Price, L. N. (1994) Performance Measurement in a Downside Risk Framework. *The Journal of Investing Fall*, 3(3), 59-64. Available from: <https://doi.org/10.3905/joi.3.3.59> [Accessed 01 October 2022].
52. Statista (2022) Average risk free investment rate in Sweden 2015-2021. *Statista Research Department*, 23 May. Available from: <https://www.statista.com/statistics/885803/average-risk-free-rate-sweden/#:~:text=As%20of%202021%2C%20the%20risk,in%20Sweden%20was%200.9%20percent> [Accessed 01 October 2022].
53. Trochim, W. M. and Donnelly, J. P. (2006) *The research methods knowledge base*, 3rd ed. Cincinnati, OH: Atomic Dog.
54. Tütüncü, R. and Koenig, M. (2002) Robust Asset Allocation. *Annals of Operations Research*, 132(1), 157-187. Available from: <https://doi.org/10.1023/B:ANOR.0000045281.41041.ed> [Accessed 01 October 2022].
55. Xidonas, P., Steuer, R. and Hassapis, C. (2020) Robust portfolio optimization: a categorized bibliographic review. *Annals of Operations Research*, 292(1), 533-552. Available from: <https://doi.org/10.1007/s10479-020-03630-8> [Accessed 01 October 2022].
56. Xu, H., Liu, Y. and Sun, H. (2018) Distributionally robust optimization with matrix moment constraints: Lagrange duality and cutting plane methods. *Mathematical*

Programming, 169(2), 489–529. Available from: <https://doi.org/10.1007/s10107-017-1143-6> [Accessed 01 October 2022].

