# FAILURE MODE AND EFFECT ANALYSIS USING INTUITIONISTIC FUZZY MARCOS METHOD

# SEZGİSEL BULANIK MARCOS YÖNTEMİ KULLANILARAK HATA MODU VE ETKİ ANALİZİ

**DİLARA AKKUŞ**

**PROF. DR. ÖZLEM MÜGE TESTİK**

**Supervisor**

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# ABSTRACT

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**Dilara AKKUŞ**

**Master of Science, Department of Industrial Engineering**

**Supervisor: Prof. Dr. Özlem Müge TESTİK**

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Risk analysis plays an essential role for both production and service sectors in terms of safety. Among many risk analysis methods, one of the most well-known method is Failure Mode and Effect Analysis (FMEA). The purpose of FMEA is to prevent existing Failure Modes (FMs) as well as to eliminate possible FMs that are not in the current situation at the source, and to prevent the effects that will arise in the event of these FMs. In this method, data is obtained by taking evaluations from field experts, and subjectivity arising from the experiences, value judgement and personal opinions of them directly affects the results. Therefore, when it is being utilized, usage of linguistic expressions instead of crisp numbers accelerates the data collection phase in practice and reduces the error. Due to the fact that FMEA method contains incomplete, doubtful, approximate and imprecise data sets, it is recommended to implement intuitionistic fuzzy set approach to the traditional FMEA method. This is primarily because intuitionistic fuzzy systems are pretty beneficial in problems that inhere ambiguity and hesitation. In this study, while prioritizing risks a relatively new method, Measurement Alternatives and Ranking according to the COmpromise Solution (MARCOS) method, will be used and it will be

used for the first time for risk analysis under Intuitionistic Fuzzy (IF) environment. In addition, intuitionistic fuzzy weights will be assigned to each expert based on their evaluations of each other, and these weights will be taken into account when evaluating Risk Factors (RFs) and FMs.

**Keywords:** Failure mode and effect analysis, Intuitionistic fuzzy sets, Intuitionistic fuzzy MARCOS method, Multi-criteria decision making

# ÖZET

## SEZGİSEL BULANIK MARCOS YÖNTEMİ KULLANILARAK HATA MODU VE ETKİ ANALİZİ

**Dilara AKKUŞ**

**Yüksek Lisans, Endüstri Mühendisliği Anabilim Dalı**

**Tez Danışmanı: Prof. Dr. Özlem Müge TESTİK**

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Risk analizi, gerek üretim gerekse hizmet sektörleri için güvenlik açısından önemli bir rol oynamaktadır. Birçok risk analizi yöntemi arasında en etkili yöntemlerden biri Hata Modu ve Etkisi Analizi (Failure Mode and Effect Analysis-FMEA)'dir. FMEA'nın amacı, mevcut Hata Modlarının (Failure Modes-FMs) önlenmesinin yanı sıra mevcut durumda olmayan olası FM'leri kaynağında ortadan kaldırmak ve bu FM'lerin ortaya çıkmaları durumundaki etkilerini önlemektir. Bu yöntemde alan uzmanlarından değerlendirmeler alınarak veriler elde edilir ve deneyimlerinden, değer yargılarından ve kişisel görüşlerinden kaynaklanan öznellik, sonuçları doğrudan etkiler. Bu nedenle, kullanıldığında kesin sayılar yerine dilsel ifadelerin kullanılması, pratikte veri toplama aşamasını hızlandırmakta ve hatayı azaltmaktadır. FMEA yönteminin eksik, şüpheli, yaklaşık ve belirsiz veri kümeleri içermesi nedeniyle, geleneksel FMEA yöntemine sezgisel bulanık küme yaklaşımının uygulanması önerilmektedir. Bunun başlıca nedeni sezgisel bulanık sistemlerin belirsizlik ve tereddüt içeren problemlerde oldukça faydalı olmasıdır. Bu çalışmada, risklerin önceliklendirilmesinde nispeten yeni bir yöntem olan Uzlaşma Çözümüne Göre Ölçüm Alternatifleri ve Sıralama (MARCOS) yöntemi Sezgisel Bulanık (IF) ortamda ilk kez risk analizi için kullanılacaktır. Ayrıca, her uzmana

birbirlerine verdikleri güven skorlarına dayalı olarak sezgisel bulanık ağırlıklar atanacak ve Risk Faktörleri (RFs) ve Hata Modları (FMs) değerlendirilirken bu ağırlıklar dikkate alınacaktır.

**Anahtar Kelimeler:** Hata türü ve etki analizi, Sezgisel bulanık kümeler, Sezgisel bulanık MARCOS metodu, Çok kriterli karar verme

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# SYMBOLS AND ABBREVIATIONS

## Symbols

| | |
|---|---|
| $\delta_d$ | The weight of $d^{th}$ DM |
| $\eta$ | The exponent expression in the formula of hierarchical DM weighting |
| $\varphi^+$ | The distance measure to IFPIS |
| $\varphi^-$ | The distance measure to IFNIS |
| $\omega_j$ | The weight of $j^{th}$ criterion |
| $\text{Ꝑ}_j$ | The aggregated evaluation score of $j^{th}$ criterion |
| $P_{jd}$ | The evaluation score of $d^{th}$ DM for the $j^{th}$ criterion |
| $\text{Ꝑ}_i$ | The aggregated evaluation score of $i^{th}$ alternative |
| $P_{id}$ | The evaluation score of $d^{th}$ DM for the $i^{th}$ alternative |
| $X_{ij}$ | The evaluation score of the $i^{th}$ alternative according to $j^{th}$ criterion |
| $\varsigma_{ij}$ | The normalized value for the $i^{th}$ alternative with $j^{th}$ criterion |
| $\vartheta_{ij}$ | The weighted value for the $i^{th}$ alternative with $j^{th}$ criterion |
| $K_i^-$ | Utility degree of $i^{th}$ alternative for AID |
| $K_i^+$ | Utility degree of $i^{th}$ alternative for ID |
| $f(K_i^+)$ | Utility function of the ideal solution |
| $f(K_i^-)$ | Utility function of the anti-ideal solution |
| $f(K_i)$ | Utility function of the $i^{th}$ alternative |

## Abbreviations

| | |
|---|---|
| AHP | Analytic Hierarchy Process |
| AID | Anti-IDeal solution |
| ARAS | Additive Ratio Assessment |
| BOM | Bill Of Materials |
| BWM | Best Worst Method |
| CC | Closeness Coefficient |
| D | Detection |

| | |
|---|---|
| DEMATEL | DEcision-MAking Trial and Evaluation Laboratory |
| DM | Decision Maker |
| EDAS | Evaluation based on Distance from Average Solution |
| FC | Failure Cause |
| FE | Failure Effect |
| FM | Failure Mode |
| FMEA | Failure Mode and Effect Analysis |
| FMECA | Failure Mode, Effects, and Criticality Analysis |
| GM | General Manager |
| ID | IDeal solution |
| IFRPN | Intuitionistic Fuzzy Risk Priority Number |
| IF | Intuitionistic Fuzzy |
| IFHWED | Intuitionistic Fuzzy Hybrid Weighted Euclidean Distance |
| IFN | Intuitionistic Fuzzy Number |
| IFNIS | IF negative ideal solution |
| IFPIS | IF positive ideal solution |
| IFS | Intuitionistic Fuzzy Set |
| IFWA | Intuitionistic Fuzzy Weighted Averaging |
| MABAC | Multi-Attributive Border Approximation area Comparison |
| MARCOS | Measurement Alternatives and Ranking according to the Compromise Solution |
| MCDM | Multi-Criteria Decision Making |
| ME | Manufacturing Engineer |
| O | Occurrence |
| OWA | Ordered Weighted Averaging |
| PIPRECIA | Pivot Pairwise Relative Criteria Importance Assessment |
| PE | Production planning and control Engineer |
| PM | Production department Manager |
| RPN | Risk Priority Number |

| | |
|---|---|
| S | Severity |
| SAW | Simple Additive Weighting |
| SWOT | Strengths, Weaknesses, Opportunities, Threats |
| TOPSIS | Technique for Order Preference by Similarity to Ideal Solution |
| VIKOR | Vısekriterijumska optimizacija i Kompromisno Resenje |
| WASPAS | Weighted Aggregated Sum Product Assessment |

# 1. INTRODUCTION

Risk analysis is imperative to maintain quality, reliability and occupational health and safety. Among many risk assessment methods used in risk analysis, the most prevalent one is FMEA because of its specialty to determine potential failures and their impacts in a quite straightforward manner (Yener and Can, 2021). FMEA is supposed to specify FMs and to relieve their effects in case of being happened through suggesting corrective and preventive actions so that they do not harm processes of production, quality of products, gratification of customers, so on. It can be called a proactive approach since it may strive to prevent FMs before coming true, yet when a process, product or service is needed improvement, then FMEA can also be implemented to currently existing FMs. It is utilized for various industries, such as aviation, automotive, chemical, mechanical, electronics, software, and nuclear.

During FMEA methodology, expert judgments are used based on their experience, field knowledge, value judgement and personal opinions, which inherently involve uncertainty and hesitation. Therefore, applying an extension approach such as Intuitionistic Fuzzy Sets (IFSs) to FMEA may result in more efficient results than traditional FMEA.

Traditional FMEA uses multiplication of the components Occurrence (O), Severity (S) and Detection (D) in calculating RPNs (Risk Priority Numbers) to rank the associated FMs. Though FMEA technique is practised commonly, there are many flaws in calculating these RPN scores. These shortcomings of conventional FMEA method are given below (Chang and Cheng, 2010; Chang, Cheng and Chang, 2010; Liu et al., 2015; Yener and Can, 2021).

1. FMEA is a tool that includes human evaluation; therefore, in account of uncertainity and hesitation of human judgment, it seems pointless to give a crisp risk number between 1-10 for each FMs by people who are working in different fields and with different experiences. Instead of those numbers, one had better use linguistic variables such as medium, very high, low important, etc. to evaluate FMs more accurately.

2. It can be obtained the same RPN score with different ratings of the RFs due to the fact that the multipliers of RPN scores are taken as if they are of equal importance. Assuming

the two FMs will be compared, if the first FM's O, S and D values are 5, 10, 2, respectively (RPN = 5 × 10 × 2 = 100) and for the other if they are 4, 3, 9, respectively (RPN = 4 × 3 × 9 = 108), then the second FM seems more dangerous. However, the first FM should have a higher precedence due to the S factor.

3. RPN can get 1 to 1000 score which of only 120 among 1000 numbers is unique. Because RPN scores are computed in a discrete manner, there are several duplicate numbers.

Fuzzy systems have appeared pretty useful to overcome these drawbacks since fuzzy approach to FMEA provides the components being evaluated linguistically. In this thesis, it is aimed to construct a developed method under IF environment to deal with the limitations of traditional FMEA. The assessments of Decision Makers (DMs) are taken as linguistically and are transformed into Intuitionistic Fuzzy Numbers (IFNs). After the evaluation step, IF-MARCOS methodology will implemented in order to prioritize the FMs. While conducting the method, Intuitionistic Fuzzy Weighted Averaging (IFWA) operator is used to aggregate the assessments of DMs. The proposed approach is practised to a real world problem, the production system in a defense company, and the conclusions are interpreted.

# 2. LITERATURE REVIEW

In this part of the thesis, FMEA related papers from the literature will be given, at first. Then, what MARCOS is used for and related studies will be delivered. In addition, at the end, contributions and aspects that differ from others of our study will be mentioned.

## 2.1. Studies Related to FMEA Method

Many studies have been carried out in the literature to deal with the aforementioned disadvantages in the method of computing RPN scores and to improve the outcomes in the traditional FMEA method. One can find a comprehensive literature review as to FMEA in the research of Liu, Liu and Liu (2013) and Liu et al. (2019). From the studies so far, it can be seen that fuzzy logic is generally used to deal with the flaws, subjectivity and vagueness of FMEA. In these review articles, future search ideas were also presented to enhance this subject (Liu, Liu and Liu, 2013; Liu et al., 2019).

Xu et al. (2002) practised fuzzy FMEA to a turbocharger system of a diesel engine to evaluate its design FMs. According to them, since FMEA related evaluation involves linguistic statements, fuzzy set approach was the best straightforward way to deal with it. At the end, they concluded that using fuzzy numbers in FMEA makes results more practical and flexible (Xu et al., 2002).

Chang, Cheng and Chang (2010) underscored drawbacks of traditional RPN computation, and proposed an IF ranking algorithm for ordering FMs in Failure Mode, Effects, and Criticality Analysis (FMECA). In the case study, the TFT-LCD process, which includes a dangerous material (silane), was taken into consideration to convince the validity of the approach. First, the authors set up a FMECA table for the aforementioned system. Then conventional RPN, fuzzy RPN, and Intuitionistic Fuzzy Sets (IFSs) were implemented to the problem respectively so as to rank the FMs, and these three techniques were compared with each other. It was pointed out that RPN method had not been sufficient to rank the failures with same RPN number but with different risk levels, and fuzzy RPN method was unable to distinguish FMs with the similar linguistic measure. However, when IFSs were used, it had not been faced with an issue as much as the other methods had (Chang, Cheng and Chang, 2010).

Another study that had been done to overcome the drawbacks of traditional FMEA was the paper of Chang and Cheng (2010). They proposed an integrated approach consisting of IFS and Decision-Making Trial and Evaluation Laboratory (DEMATEL). In the paper, a case study was conducted in a semiconductor manufacturing company in Taiwan. In the studied process, they found twenty-two potential FMs and their ranking orders. In addition, the combined technique was compared with traditional RPN method and solely DEMATEL method to proof the reliability of the suggested method (Chang and Cheng, 2010).

Chang and Cheng (2011) used fuzzy Ordered Weighted Averaging (OWA) with DEMATEL to implement FMEA. They conducted their methodology to the company where LCD products are produced by determining 11 FMs. They reduced the negative effects of conventional RPN computation by this study (Chang and Cheng, 2011).

Kutlu and Ekmekçioğlu (2012) carried out a fuzzy FMEA study through the fuzzy Analytic Hierarchy Process (AHP) and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) integrated method. They used fuzzy AHP to weight each RFs of FMEA which are S, O and D. As for the evaluation of potential FMs, fuzzy TOPSIS method was practised for ordering potential FMs together with weights coming from fuzzy AHP results. In the study, potential FMs were identified by a group of specialists from the automotive sector (Kutlu and Ekmekçioğlu, 2012).

Mandal and Maiti (2014) contributed to the literature through a new approach that combines fuzzy similarity value measurement and possibility theory to rank FMs. Two case studies were conducted to prove their proposed method is more reliable than other defuzzification based techniques. According to the methodology, after fuzzy RPN of each FM calculated, similarity measurement was used to rank fuzzy RPN partially. Possibility theory, in which is a new way to use, was used to obtain a comparison value between fuzzy RPN and the conformance standards (Mandal and Maiti, 2014).

Liu, Liu and Li (2014) and Liu et al. (2015) used Intuitionistic Fuzzy Weighted Averaging (IFWA) and OWA operators to conduct FMEA. While Liu, Liu and Li (2014) prioritized FMs according to the distances, Liu et al. (2015) ranked them by conducting IF hybrid

TOPSIS method. Both studies conducted case studies practising their proposed approach (Liu, Liu and Li, 2014; Liu et al., 2015).

Can (2018) proposed a new IF FMEA and Weighted Aggregated Sum Product Assessment (WASPAS) approach to cope with the shortcomings of conventional FMEA method. In addition to the generally used S, O and D, cost, system security and exposure time were also added as RFs. Can (2018) gave weights to each members of the expert team according to their year of experience and ranked the corrective-preventive strategies which were taken as the alternatives based on each FM which were taken as criteria. The efficiency of the proposed methodology was proved by practising it to a real assembly line. For the process, three experts determined nine FMs. After IFRPN values for each FM had been calculated through with taking into account the Decision Makers (DMs) weight scores, twelve corrective-preventive strategies were identified by the experts. At the end, these stratagies were ordered based on each FMs by implementing fuzzy WASPAS method (Can, 2018).

In the study that was done by Boral et al. (2020), an FMEA problem handled by practising fuzzy AHP and fuzzy MARCOS. Aforementioned FMEA problem was taken from (Kutlu and Ekmekçioğlu, 2012) with the aim of making a comparison between that paper's methodology (fuzzy AHP and TOPSIS) and their own methodology. There were three experts who evaluate eight FMs linguistically in the problem. At the end, they pointed out that their proposed method results in more robust solution than the study from which they take the problem (Boral et al., 2020).

Kushwaha, Panchal and Sachdeva (2020) implemented IF-FMEA and IF-TOPSIS techniques to the cutting system in a sugar mill to rank the FMs. These methods were compared with each other at the end of the study. In the first stage of the application, IFRPN scores were computed so that they could be ordered the failures in a decreasing manner. In the second stage, IF-TOPSIS method was practised, and the first two steps of the method (namely, weightining the RFs) is the same with IF-FMEA method. After the other steps had been finished to implement, the FMs ranked (Kushwaha, Panchal and Sachdeva, 2020).

Yener and Can (2021) conducted IF based risk evaluation study through FMEA. There were several brandly new developed methods used at the stages of the study. Firstly, a new IF weighting technique, which took into account the affinities between RFs, was utilized while calculating the weights of RFs. Secondly, the scholars proposed a Modified IF Multi Attribute Border Approximation area Comparison (MIF-MABAC) for ranking the FMs. While MIF-MABAC method was being used, the extended Haussdorf distance function was employed to differentiate experts' opinions. Finally, two fuzzy goal programming models were constructed to assess the risks including real company restrictions. In addition, they interpreted the differences of the model, and which one was more accurate to find out the most essential FM. A case study was conducted in a factory where metal shielded cells are manufactured. In the assembly line four experts specified nine FMs which are assessed by four main RFs plus fifteen subfactors under those main ones (Yener and Can, 2021).

Fan, Wang and Wu (2021) combined D-Best Worst Method (BWM), the distance-based method of D numbers and D-MARCOS method to apply FMEA method. They computed the weight of experts with different experience and knowledge by distance-based method of D numbers. Then D-BWM approach is used for weighting the RFs before conducting D-MARCOS method to prioritize FMs. At last, they concluded that using D numbers and MARCOS method are quite useful for dealing with uncertainity (Fan, Wang and Wu, 2021).

Testik and Unlu (2023) compared FMEA results with fuzzy FMEA results, and they proved that these two results are quite close to each other. In the study, possible failures observed within the 91 assessments in a Test and Calibration Laboratory for FMEA were analyzed. They computed the weights of O, S and D based on the 5 DMs evaluations by taking the geometric mean of these evaluations. Finally, FMs' FRPN scores were calculated and ranked using Mamdani fuzzy inference method and triangular fuzzy numbers (Testik and Unlu, 2023).

Consequently, we reviewed the studies including methodologies to overcome the drawbacks of FMEA in the literature in this part. In this thesis, it is aimed to enhance traditional FMEA by combining it in concert with IFSs and MARCOS method.

MARCOS method is a Multi-Criteria Decision Making (MCDM) technique and the studies including MARCOS are given below.

## 2.2. MARCOS Method and Related Studies

MCDM problem is a problem that includes several conflicting requests or restrictions, and in order to solve such problems, in the literature, there are many methods that have been developed and many approaches having combined and that have been used so far. Among the studies that include MCDM methods, many of them use fuzzy logic and its offshoots (Liu and Wang, 2007; Boran et al., 2009; Chang and Cheng, 2010; Awasthi et al.,2011; Kutlu and Ekmekcioglu, 2012; Kang, Jang and Park, 2016; Can, 2018; Kushwaha, Panchal and Sachdeva, 2020; Ozkan et al., 2020; Stankovic et al., 2020; Stevic et al., 2020; Bakır and Atalık, 2021; Celik and Gul, 2021; Yener and Can, 2021).

In this dissertation, it will be applied IF-MARCOS MCDM methodology to rank the FMs finding in the phase of FMEA study. Therefore, in this division of the thesis, MARCOS MCDM method will be defined and related studies to MARCOS are given.

MARCOS, also known as Measurement Alternatives and Ranking according to the Compromise Solution, developed by Stevic et al. (2020), try to solve a multi-criteria problem by searching a compromise solution. A feasible solution that is closest to the ideal, and farthest from the anti-ideal solution is called compromise solution that seeks to compromise between opposing goals and constraints. Stevic et al. (2020) made sustainable vendor selection in medical industry by using this new developed method calling it MARCOS. Since it was the first time that MARCOS method was used, detail steps of the technique were given before the case study. In the case study, there were three DMs who assessed eight supplier alternatives with respect to three main sustainability criteria (economic, social and environmental) and twenty-one sub-criteria under those main ones. After evaluation was carried out by MARCOS, the best supplier was selected for a polyclinic in Bosnia. In order to justify the method, a comprehensive sensitivity analysis had been made at the end of the paper. They compared MARCOS with other MCDM methods that alike and frequently used in the literature which are MABAC, Simple Additive Weighting (SAW), Additive Ratio Assessment (ARAS), WASPAS, TOPSIS, Evaluation based on Distance from Average Solution (EDAS), and proved its

validity, stability, reliability, effectiveness and robustness. Sensitivity analysis also included alteration of the measurement scale to 1-5, and twenty-one scenarios with changes of criterion weights (Stevic et al., 2020).

After the invention of MARCOS methodology, Stankovic et al. (2020) extented it to fuzzy world to assess risk on the roads more accurately. First of all, they gave the details of the fuzzy MARCOS method, and the new linguistic evaluation scale they created. Then, six potential criteria were determined to assess thirty-eight alternative sections of the road. While they were practising fuzzy MARCOS method to this real life example, they used the fuzzy Pivot Pairwise Relative Criteria Importance Assessment (PIPRECIA) method to weight the criteria. They ranked thirty-eight alternative sections according to their road risk. At the end of the study, they conducted validation analyzes to justify the proposed methodology's stability and effectiveness by comparing it with other methods (fuzzy SAW and fuzzy TOPSIS), and by considering alteration of the significance of input parameters through thirty scenarios (Stankovic et al., 2020).

In the article of Bakır and Atalık (2021), an assessment study was conducted in the airline industry in terms of e-service quality by employing fuzzy AHP and fuzzy MARCOS integrated method. Since it is an evaluation, they used fuzzy extensions of the methods to handle the uncertainty of human judgment. First of all, they established a comparison matrix for fuzzy AHP method by eleven DMs to provide criteria weights for the next step. Then, they practised fuzzy MARCOS method to evaluate alternatives according to decision matrix constituted from three hundred and ninety five consumers' answers to the survey. Finally, a sensitivity analysis was applied to verify the efficacy of the combined approach. In the analysis, the impact of changing criteria weights across thirteen scenarios was observed. In addition, five other proven methods in the literature and proposed method were compared with each other, and they interpreted the correlation between them (Bakır and Atalık, 2021).

Celik and Gul (2021) suggested an integrated approach including two MCDM methods which are BWM and MARCOS to order risks for dam construction under fuzzy environment. In the paper, by using interval type two fuzzy sets, subjectivity of the hazard assessment was overcame. Firstly, BWM was utilized to give weighs to probability and

severity of risks. Subsequently, MARCOS was used to prioritize them. At the end of the study, it was implemented a sensitivity analysis, as well (Celik and Gul, 2021).

Buyukozkan, Havle and Feyzioglu (2021) proposed an integrated methodology to rank digital transformation strategies, and they carried out a case study for an airline industry in Turkey. Firstly, a Strengths, Weaknesses, Opportunities, Threats (SWOT) analysis was conducted to determine digital transformation strategies and factors through experts' opinions. Then, the fuzzy AHP method was employed to weight each SWOT factor by pairwise comparison. After determining the importance of each factor, fuzzy MARCOS was used to pick the top digital transformation strategy. Towards the end of the study, in order to justify the results of the proposed method, the researchers compared fuzzy MARCOS with two different selection method which are fuzzy TOPSIS and fuzzy Vısekriterijumska optimizacija i Kompromisno Resenje (VIKOR) and practiced a sensitivity analysis to weights of SWOT factors (Buyukozkan, Havle and Feyzioglu, 2021).

Fuzzy MARCOS method was proceed one step further, and Ecer and Pamucar (2021) conducted an evaluation based study by using IF MARCOS technique for the first time in literature. They first gave the steps of IF MARCOS method, and the tables that are used for linguistic variables by the experts in detail. Then in the case study, ten insurance companies were assessed by five experts according to seven predetermined criteria. Through the end of the paper, in oder to justify IF MARCOS methodology, the authors practised a sensitivity analysis, as well (Ecer and Pamucar, 2021).

To the best of my knowledge, FMEA has not been combined with IF based MARCOS MCDM method before. Besides, contributions of my study can be stated as the ability of expression ambiguity, hesitation, etc. of DMs' evaluation, practising a group decision making process, taking into consideration of DMs' experience level, specifying FMs and ranking FMs.

# 3. METHODOLOGY

In this section, it is introduced related concepts and the proposed method for this dissertation. Accordingly, the description of IFS and IFWA operator, the steps of IF MARCOS technique and, at last, how IF MARCOS method will be used on FMEA are given respectively.

## 3.1. Intuitionistic Fuzzy Sets

Zadeh (1965) developed fuzzy set theory to model and solve the problems containing subjective, missing and/or imprecise statements. While dual logic, 0 or 1, is used in classical sets, fuzzy sets can take all values between 0 and 1. Since there are no sharp boundaries in real life, fuzzy sets represent real life much better than crisp sets. This flexibility is generally represented by linguistic adjectives, such as very low, low, moderate, high, very high. The linguistic expressions used for real life problems are converted into fuzzy numbers and after methodology conducted, the solution is reached. According to the theory of fuzzy sets, an element can be part of a set to some extent.This degree of belonging is defined by membership functions. Membership functions give the extent to which an element is included in the specified set (Zadeh, 1965).

IFS, which has been developed by Atanassov (1986), is an extended version of fuzzy sets. Atanassov (1986) argues in his article that the sum of membership and non-membership functions may not be 1 for real life problems. There may be missing of data or there may be hesitation, lack of knowledge/experience. Thus, he defines a hesitation degree to overcome this lackness (Atanassov, 1986). So, IFSs are a convenient way to deal with hesitation besides uncertainty and subjectivity (Boran et al., 2009). Due to the computational easiness and effectiveness, IFSs have been used in various studies in the literature (Boran et al., 2009; Chang and Cheng, 2010; Can, 2018; Kushwaha, Panchal and Sachdeva, 2020; Ecer and Pamnucar, 2021; Yener and Can, 2021). Definition of IFSs and the main operations of them are as follows (Atanassov, 1986).

<u>Definition 1</u>: Letting X be a finite set, an IFS $\tilde{A}$ in X is described as $\tilde{A} = \{ \langle x, \mu_{\tilde{A}}(x), \upsilon_{\tilde{A}}(x), \pi_{\tilde{A}}(x) \rangle \mid x \in X \}$ where $\mu_{\tilde{A}}(x), \upsilon_{\tilde{A}}(x) : X \rightarrow [0,1]$ are the membership and non-membership functions respectively, providing that $0 \leq \mu_{\tilde{A}}(x) + \upsilon_{\tilde{A}}(x) \leq 1$. IFSs have also

third parameter $\pi_{\tilde{A}}(x)$, the hesitation degree, in the event of $\pi_{\tilde{A}} = 1 - \mu_{\tilde{A}}(x) - \upsilon_{\tilde{A}}(x)$. The more the hesitation margin of $x$, intuitionistic index of $x$, the more uncertainty there is about the value of $x$ (Atanassov, 1986).

### 3.2. IFWA Operator

The IFWA operator will be used for aggregating the opinions of all individual decision makers while conducting IF MCDM method to obtain a consensus among them. Let S be the set of all IFNs, IFWA operator is decribed in Xu (2007) as follows.

Definition 4: Let $\alpha_i = (\mu_{\alpha i}, \upsilon_{\alpha i}, \pi_{\alpha i})$, $i = 1, 2,\ldots, n$ be a series of IFNs and let IFWA : $S^n \rightarrow S$, then (Xu, 2007):

$$\text{IFWA}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \omega_1\alpha_1 + \omega_2\alpha_2 + \cdots + \omega_n\alpha_n = \left(1 - \prod_{i=1}^{n}(1 - \mu_{\alpha i})^{\omega i}, \prod_{i=1}^{n}(\upsilon_{\alpha i})^{\omega i}, \prod_{i=1}^{n}(1 - \mu_{\alpha i})^{\omega i} - \prod_{i=1}^{n}(\upsilon_{\alpha i})^{\omega i}\right) \quad (1)$$

is called an IFWA operator of dimension n, where $\omega = (\omega_1, \omega_2,\ldots, \omega_n)^T$ is the weight vector of $\alpha_i$ ($i = 1, 2,\ldots, n$), with $\omega_i \in [0, 1]$ and $\sum_{i=1}^{n} \omega_i = 1$.

### 3.3. IF MARCOS Method

MARCOS, as mentioned earlier, is an MCDM method aiming to reach a compromise solution. Due to the usage of ideal and anti-ideal solutions, it is similar to TOPSIS method. However, according to Stevic et al. (2020), MARCOS method is more reliable, since, unlike TOPSIS method, MARCOS method is able to pay attention the closeness between the reference point and the ideal solution as well as the relative importance of the distances of reference point to ideal and anti-ideal solution.

MARCOS method is merged with fuzzy sets so that it can represent inconsistent, vague, imprecise, missing data information (Stankovic et al., 2020, Bakır and Atalık, 2021). There is no FMEA study by implementing IF MARCOS method, and it will be handled in this thesis. The steps of IF MARCOS MCDM methodology is briefly explained below (Ecer and Pamucar, 2021).

Step 1) Supposing we have $d$ = 1,…, $p$ DMs, $i$ = 1,…, $m$ alternatives and $j$ = 1,…, $n$ criteria, at first we can calculate the weights of DMs according to the predetermined assessment table, and then we put corresponding assessment IFNs in the formula (2).

$$\delta_d = \frac{\mu_d + \Pi_d\left(\frac{\mu_d}{\mu_d + \upsilon_d}\right)}{\sum_{d=1}^{p}\left(\mu_d + \Pi_d\left(\frac{\mu_d}{\mu_d + \upsilon_d}\right)\right)} \tag{2}$$

Step 2) Criteria are evaluated by DMs according to given rating scale so that we can get IF decision matrix. At this assessment scale given as a table, there are linguistic expressions (e.g. important, medium important) and corresponding IFNs.

Step 3) The aggregated IF decision matrix is created according to the criteria evaluation of each DM's. The IFWA operator formula (3) including DMs' weights ($\delta_d$) as weight vector is used to perform aggregation operation.

$$P_j = \text{IFWA}\,(P_{j1}, P_{j2}, \ldots, P_{jp}) = \left(1 - \prod_{d=1}^{p}(1 - \mu_{P_{jd}})^{\delta_d}, \prod_{d=1}^{p}(\upsilon_{P_{jd}})^{\delta_d}, \left(1 - \prod_{d=1}^{p}(1 - \mu_{P_{jd}})^{\delta_d}, - \prod_{d=1}^{p}(\upsilon_{P_{jd}})^{\delta_d}\right), \quad j = 1,\ldots, n \tag{3}$$

where $P_{jd}$ is the evaluation score of $d^{th}$ DM for the $j^{th}$ criteria.

Step 4) The distance measures ($\varphi^+$ and $\varphi^-$) are calculated in the event of IF positive ideal solution (IFPIS) is $\tau^+$ = (1, 0, 0) and IF negative ideal solution (IFNIS) is $\tau^-$ = (0, 1, 0). Then Closeness Coefficient (CC) of each criterion is calculated by using $\varphi^+$ and $\varphi^-$. Finally, CC values of each criteria are normalized to obtain the weights of criteria. All of these values are calculated using convenient formulas which are given in equations (4), (5), (6) and (7) (Ecer and Pamucar, 2021).

$$\varphi_j^+ = \sqrt{[(\mu_{P_j} - \tau^+)^2 + (\upsilon_{P_j} - \tau^+)^2 + (\pi_{P_j} - \tau^+)^2]} \tag{4}$$

$$\varphi_j^- = \sqrt{[(\mu_{P_j} - \tau^-)^2 + (\upsilon_{P_j} - \tau^-)^2 + (\pi_{P_j} - \tau^-)^2]} \tag{5}$$

$$CC_j = \varphi_j^- \div (\varphi_j^- + \varphi_j^+) \tag{6}$$

$$\text{Normalized weight } (\omega_j) = CC_j \div \sum_{j=1}^{n} CC_j \tag{7}$$

Step 5) Alternatives are evaluated by decision makers according to criteria and given scale. The scale consists of linguistic expressions (e.g. good, very bad, medium good).

Step 6) IFNs of the alternatives are aggregated per criterion by using IFWA operator (8).

$$\dot{P}_i = \text{IFWA}(P_{i1}, P_{i2}, \ldots, P_{ip}) = (1 - \prod_{d=1}^{p}(1 - \mu_{P_{id}})^{\delta_d}, \prod_{d=1}^{p}(\upsilon_{P_{id}})^{\delta_d}, (1 - \prod_{d=1}^{p}(1 - \mu_{P_{id}})^{\delta_d}, -\prod_{d=1}^{p}(\upsilon_{P_{id}})^{\delta_d}), \quad i = 1, \ldots, m \tag{8}$$

where $P_{id}$ is the evaluation score of $d^{th}$ DM for the $i^{th}$ alternative.

Step 7) CC of each evaluation of alternatives according to the criteria ($X_{ij}$) is calculated by using $\varphi^+$ and $\varphi^-$ when IFPIS is $\tau^+ = (1, 0, 0)$ and IFNIS is $\tau^- = (0, 1, 0)$. $\varphi^+$, $\varphi^-$ and CC values are computed by using equations (4)-(6).

Step 8) The decision matrix is constructed using the CC values of the alternatives from the previous step (Xij) and the criteria weights' (ωj).

Step 9) IDeal solutions (ID) and Anti-IDeal solutions (AID) can be determined according to the given rule down below, and then extended IF decision matrix will be formed.

- If $j$ is a benefit type criterion, then ID = max $X_{ij}$
- If $j$ is a cost type criterion, then ID = min $X_{ij}$
- If $j$ is a benefit type criterion, then AID = min $X_{ij}$
- If $j$ is a cost type criterion, then AID = max $X_{ij}$.

Step 10) The previous table will be normalized in this current step. The normalization formulas is as following: (9) and (10).

$$\varsigma_{ij} = \frac{X_{ij}}{X_{ID}} \quad \text{, if } j \text{ is a benefit type criterion} \tag{9}$$

$$\varsigma_{ij} = \frac{X_{ID}}{X_{ij}} \quad \text{, if } j \text{ is a cost type criterion} \tag{10}$$

Step 11) Normalized values are weighted by using criteria weights ($\omega_j$) and equation (11).

$$\vartheta_{ij} = \varsigma_{ij}\, \omega_j \tag{11}$$

Step 12) Utility degrees of alternatives ($K_i^-$, $K_i^+$) are computed by utilizing row sums ($S_i$) of the previous $\vartheta_{ij}$ values and formulas (12) and (13).

$$K_i^- = \frac{S_i}{S_{AID}} \tag{12}$$

$$K_i^+ = \frac{S_i}{S_{ID}} \tag{13}$$

Step 13) Finally, utility function of the ideal ($f(K_i^+)$) and anti-ideal ($f(K_i^-)$) solution, and utility function of alternatives ($f(K_i)$) are calculated. Alternatives are ranked with respect to the values of utility functions.

$$f(K_i^+) = \frac{K^-}{K^- + K^+} \tag{14}$$

$$f(K_i^-) = \frac{K^+}{K^- + K^+} \tag{15}$$

$$f(K_i) = \frac{K_i^+ + K_i^-}{1 + \frac{1 - f(K_i^+)}{f(K_i^+)} + \frac{1 - f(K_i^-)}{f(K_i^-)}} \tag{16}$$

Let's see a simple example with 2 DMs, 2 criteria, 3 alternatives, where we apply the IF MARCOS MCDM method.

**Example**

Step 1) Calculate the weights of 2 DMs by using Table 3.1 (Yener and Can, 2021).

Table 3.1. Linguistic variables and IFN values for weighing DMs

| Linguistic Terms | IFNs |
|---|---|
| Very experienced | (0.90, 0.05, 0.05) |
| Experienced | (0.75, 0.20, 0.05) |
| Moderately experienced | (0.50, 0.40, 0.10) |
| Inexperienced | (0.25, 0.60, 0.15) |
| Very inexperienced | (0.10, 0.80, 0.10) |

Table 3.2. The weights of DMs'

| | DM 1 | DM 2 |
|---|---|---|
| Linguistic expression | Experienced | Very experienced |
| Weight ($\delta_d$) | 0.45 | 0.55 |

$$\delta_1 = \frac{0.75 + 0.05\left(\frac{0.75}{0.75+0.20}\right)}{0.75 + 0.05\left(\frac{0.75}{0.75+0.20}\right) + 0.90 + 0.05\left(\frac{0.90}{0.90+0.05}\right)} = 0.45$$

$$\delta_2 = \frac{0.90 + 0.05\left(\frac{0.90}{0.90+0.05}\right)}{0.75 + 0.05\left(\frac{0.75}{0.75+0.20}\right) + 0.90 + 0.05\left(\frac{0.90}{0.90+0.05}\right)} = 0.55$$

Step 2) 2 Criteria are evaluated by DMs according to the Table 3.3 (Yener and Can, 2021), and evaluations are given in Table 3.4.

Table 3.3. Linguistic variables and IFN values for weighing criteria

| Linguistic Terms | Abbreviation | IFN |
|---|---|---|
| Very Important | VI | (0.90, 0.05, 0.05) |
| Important | I | (0.75, 0.20, 0.05) |
| Medium | M | (0.50, 0.40, 0.10) |
| Unimportant | UI | (0.25, 0.60, 0.15) |
| Very Unimportant | VU | (0.10, 0.80, 0.10) |

Table 3.4. The lingustic evaluations of criteria by DMs

| | $C_1$ | $C_2$ |
|---|---|---|
| DM 1 | I | M |
| DM 2 | VI | M |

Step 3) In order to get only one score for each criteria, the evaluations of DMs are aggregated by the IFWA operator (3) and the results are given in the Table 3.5.

Table 3.5. Aggregated IF decision matrix for criteria

| | $\mu$ | $\upsilon$ | $\pi$ |
|---|---|---|---|
| $C_1$ | 0.850 | 0.090 | 0.060 |
| $C_2$ | 0.500 | 0.400 | 0.100 |

$\text{Ꞓ}_1$= IFWA ($P_{11}$, $P_{12}$) = $(1 - [(1 - 0.75)^{0.45} * (1 - 0.90)^{0.55}], 0.20^{0.45} * 0.05^{0.55}, (1 - 0.75)^{0.45} * (1 - 0.90)^{0.55} - 0.20^{0.45} * 0.05^{0.55})$

= (0.850, 0.090, 0.060)

$\text{Ꞓ}_2$= IFWA ($P_{21}$, $P_{22}$) = $(1 - [(1 - 0.50)^{0.45} * (1 - 0.50)^{0.55}], 0.40^{0.45} * 0.40^{0.55}, (1 - 0.50)^{0.45} * (1 - 0.50)^{0.55} - 0.40^{0.45} * 0.40^{0.55})$

= (0.500, 0.400, 0.100)

Step 4) IFN values for criteria computing in Table 3.5 are used to give weights criteria (Table 3.6).

Table 3.6. The criteria weights

| | $\varphi^+$ | $\varphi^-$ | $CC_j$ | Normalized weights ($\omega_j$) |
|---|---|---|---|---|
| $C_1$ | 0.185 | 0.046 | 0.200 | 0.820 |
| $C_2$ | 0.648 | 0.030 | 0.044 | 0.180 |

$\varphi_1^+ = \sqrt{[(0.85 - 1)^2 + (0.09 - 0)^2 + (0.06 - 0)^2]}$ = 0.185

$\varphi_2^+ = \sqrt{[(0.50 - 1)^2 + (0.40 - 0)^2 + (0.10 - 0)^2]}$ = 0.648

$\varphi_1^- = \sqrt{[(0.85 - 0)^2 + (0.09 - 1)^2 + (0.06 - 0)^2]}$ = 0.046

$\varphi_2^- = \sqrt{[(0.50 - 0)^2 + (0.40 - 1)^2 + (0.10 - 0)^2]}$ = 0.030

$CC_1$ = 0.0464 / (0.1849 + 0.0464) = 0.200

$CC_2$ = 0.03 / (0.6481 + 0.03) = 0.044

$\omega_1$ = 0.2 / (0.2 + 0.044) = 0.820

$\omega_2$ = 0.044 / (0.2 + 0.044) = 0.180

Step 5) Alternatives are evaluated by DMs according to the Table 3.7 (Liu et al., 2015), and Table 3.8 gives these evaluations.

Table 3.7. Linguistic variables and IFN values to evaluate alternatives

| Linguistic Terms | Abbreviation | IFN |
|---|---|---|
| Extremely Good | EG | (1, 0, 0) |
| Very Good | VG | (0.90, 0.10, 0) |
| Good | G | (0.75, 0.10, 0.15) |
| Medium Good | MG | (0.60, 0.25, 0.15) |
| Medium | M | (0.50, 0.45, 0.05) |
| Medium Bad | MB | (0.40, 0.50, 0.10) |
| Bad | B | (0.25, 0.60, 0.15) |
| Very Bad | VB | (0.10, 0.75, 0.15) |
| Extremely Bad | EB | (0, 0.90, 0.10) |

Table 3.8. The linguistic assessments of the alternatives

| Alternatives | DMs | $C_1$ | $C_2$ |
|---|---|---|---|
| $A_1$ | $DM_1$ | G | M |
| | $DM_2$ | VG | MG |
| $A_2$ | $DM_1$ | VG | M |
| | $DM_2$ | VG | G |
| $A_3$ | $DM_1$ | VG | MG |
| | $DM_2$ | MG | MB |

Step 6) In order to obtain only one IFN for each alternative, the DMs' assessments are collected by the IFWA operator (8) per criterion and the results are given in the Table 3.9.

Table 3.9. Aggregated IF decision matrix for alternatives

| | $C_1$ | | | $C_2$ | | |
|---|---|---|---|---|---|---|
| | $\mu$ | $\upsilon$ | $\pi$ | $\mu$ | $\upsilon$ | $\pi$ |
| $A_1$ | 0.850 | 0.100 | 0.050 | 0.560 | 0.330 | 0.110 |
| $A_2$ | 0.900 | 0.100 | 0.000 | 0.660 | 0.200 | 0.140 |
| $A_3$ | 0.790 | 0.170 | 0.040 | 0.500 | 0.370 | 0.130 |

For $C_1$;

$$Ꝑ_1 = \text{IFWA}(P_{11}, P_{12}) = (1 - [(1-0.75)^{0.45} * (1-0.90)^{0.55}], 0.10^{0.45} * 0.10^{0.55}, (1-0.75)^{0.45} * (1-0.90)^{0.55} - 0.10^{0.45} * 0.10^{0.55})$$

$$= (0.850, 0.100, 0.050)$$

$$Ꝑ_2 = \text{IFWA}(P_{21}, P_{22}) = (1 - [(1-0.90)^{0.45} * (1-0.90)^{0.55}], 0.10^{0.45} * 0.10^{0.55}, (1-0.90)^{0.45} * (1-0.90)^{0.55} - 0.10^{0.45} * 0.10^{0.55})$$

$$= (0.900, 0.100, 0.000)$$

$$Ꝑ_3 = \text{IFWA}(P_{31}, P_{32}) = (1 - [(1-0.90)^{0.45} * (1-0.60)^{0.55}], 0.10^{0.45} * 0.25^{0.55}, (1-0.90)^{0.45} * (1-0.60)^{0.55} - 0.10^{0.45} * 0.25^{0.55})$$

$$= (0.790, 0.170, 0.040)$$

For $C_2$;

$$Ꝑ_1 = \text{IFWA}(P_{11}, P_{12}) = (1 - [(1-0.50)^{0.45} * (1-0.60)^{0.55}], 0.45^{0.45} * 0.25^{0.55}, (1-0.50)^{0.45} * (1-0.60)^{0.55} - 0.45^{0.45} * 0.25^{0.55})$$

$$= (0.560, 0.330, 0.110)$$

$$Ꝑ_2 = \text{IFWA}(P_{21}, P_{22}) = (1 - [(1-0.50)^{0.45} * (1-0.75)^{0.55}], 0.45^{0.45} * 0.10^{0.55}, (1-0.50)^{0.45} * (1-0.75)^{0.55} - 0.45^{0.45} * 0.10^{0.55})$$

$$= (0.660, 0.200, 0.140)$$

$$Ꝑ_3 = \text{IFWA}(P_{31}, P_{32}) = (1 - [(1-0.60)^{0.45} * (1-0.40)^{0.55}], 0.25^{0.45} * 0.50^{0.55}, (1-0.60)^{0.45} * (1-0.40)^{0.55} - 0.25^{0.45} * 0.50^{0.55})$$

= (0.500, 0.370, 0.130)

Step 7) IFN values for alternatives computing in Table 3.9 are used to compute CC values ($X_{ij}$ values in the decision matrix), which are given in the Table 3.10.

Table 3.10. The CC values of alternatives

| | $C_1$ | | | $C_2$ | | |
|---|---|---|---|---|---|---|
| | $\varphi^+$ | $\varphi^-$ | $CC_i$ | $\varphi^+$ | $\varphi^-$ | $CC_i$ |
| $A_1$ | 0.187 | 1.239 | 0.869 | 0.561 | 0.880 | 0.611 |
| $A_2$ | 0.141 | 1.273 | 0.900 | 0.419 | 1.047 | 0.714 |
| $A_3$ | 0.273 | 1.147 | 0.808 | 0.635 | 0.815 | 0.562 |

For $C_1$;

$\varphi_1^+ = \sqrt{[(0.85 - 1)^2 + (0.10 - 0)^2 + (0.05 - 0)^2]}$ = 0.187

$\varphi_2^+ = \sqrt{[(0.90 - 1)^2 + (0.10 - 0)^2 + (0 - 0)^2]}$ = 0.141

$\varphi_3^+ = \sqrt{[(0.79 - 1)^2 + (0.17 - 0)^2 + (0.04 - 0)^2]}$ = 0.273

$\varphi_1^- = \sqrt{[(0.85 - 0)^2 + (0.10 - 1)^2 + (0.05 - 0)^2]}$ = 1.239

$\varphi_2^- = \sqrt{[(0.90 - 0)^2 + (0.10 - 1)^2 + (0 - 0)^2]}$ = 1.273

$\varphi_3^- = \sqrt{[(0.79 - 0)^2 + (0.17 - 1)^2 + (0.04 - 0)^2]}$ = 1.147

$CC_1$ = 1.239 / (1.239 + 0.187) = 0.869

$CC_2$ = 1.273 / (1.273 + 0.141) = 0.900

$CC_3$ = 1.147 / (1.147 + 0.273) = 0.808

For $C_2$;

$\varphi_1^+ = \sqrt{[(0.56 - 1)^2 + (0.33 - 0)^2 + (0.11 - 0)^2]}$ = 0.561

$\varphi_2^+ = \sqrt{[(0.66 - 1)^2 + (0.20 - 0)^2 + (0.14 - 0)^2]}$ = 0.419

$\varphi_3^+ = \sqrt{[(0.50 - 1)^2 + (0.37 - 0)^2 + (0.13 - 0)^2]}$ = 0.635

$\varphi_1^- = \sqrt{[(0.56 - 0)^2 + (0.33 - 1)^2 + (0.11 - 0)^2]}$ = 0.880

$\varphi_2^- = \sqrt{[(0.66 - 0)^2 + (0.20 - 1)^2 + (0.14 - 0)^2]}$ = 1.047

$\varphi_3^- = \sqrt{[(0.50 - 0)^2 + (0.37 - 1)^2 + (0.13 - 0)^2]}$ = 0.815

$CC_1$ = 0.88 / (0.88 + 0.561) = 0.611

$CC_2$ = 1.047 / (1.047 + 0.419) = 0.714

$CC_3$ = 0.815 / (0.815 + 0.635) = 0.562

Step 8) $X_{ij}$ (CC values of alternatives found at the previous step) and $\omega_j$ are placed in the decision matrix (Table 3.11).

Table 3.11. IF decision matrix

| $\omega_j$ | [ 0.820 | 0.180 ] |
|---|---|---|
| $A_1$ | 0.869 | 0.611 |
| $A_2$ | 0.900 | 0.714 |
| $A_3$ | 0.808 | 0.562 |

Step 9) Extended IF decision matrix is generated in Table 3.12, which includes ID and AID solutions. Let's assume $C_1$ is a cost type and $C_2$ is a benefit type criterion when determining these solutions.

Table 3.12. Extended IF decision matrix

| | [ 0.82 | 0.18 ] |
|---|---|---|
| $A_1$ | 0.869 | 0.611 |
| $A_2$ | 0.900 | 0.714 |
| $A_3$ | 0.808 | 0.562 |
| ID | 0.808 | 0.714 |
| AID | 0.900 | 0.562 |

Step 10) According to the equations (9) and (10) the values of Extended IF decision matrix are normalized (Table 3.13).

Table 3.13. Normalized IF decision matrix

| | $C_1$ (0.82) | $C_2$ (0.18) |
|---|---|---|
| $A_1$ | 0.930 | 0.856 |
| $A_2$ | 0.898 | 1.000 |
| $A_3$ | 1.000 | 0.787 |
| ID | 1.000 | 1.000 |
| AID | 0.898 | 0.787 |

$\varsigma_{11}$ = 0.808 / 0.869 = 0.930

$\varsigma_{21}$ = 0.808 / 0.90 = 0.898

$\varsigma_{31}$ = 0.808 / 0.808 = 1.000

$\varsigma_{ID1}$ = 0.808 / 0.808 = 1.000

$\varsigma_{AID1}$ = 0.808 / 0.90 = 0.898

$\varsigma_{12}$ = 0.611 / 0.714 = 0.856

$\varsigma_{22}$ = 0.714 / 0.714 = 1.000

$\varsigma_{32}$ = 0.562 / 0.714 = 0.787

$\varsigma_{ID2}$ = 0.714 / 0.714 = 1.000

$\varsigma_{AID2}$ = 0.562 / 0.714 = 0.787

Step 11) Normalized values are weighted with $\omega_j$ in Table 3.14.

Table 3.14. Weighted IF decision matrix

| | $C_1$ | $C_2$ |
|---|---|---|
| $A_1$ | 0.763 | 0.154 |
| $A_2$ | 0.736 | 0.180 |
| $A_3$ | 0.820 | 0.142 |
| ID | 0.820 | 0.180 |
| AID | 0.736 | 0.142 |

Step 12) $K_i^-$, $K_i^+$ (Utility degrees of alternatives) are calculated by using $S_i$ (row sums in Table 3.15) of weighted IF decision matrix and equations (12) and (13).

Table 3.15. Row sums of weighted IF decision matrix

| | $C_1$ | $C_2$ | $S_i$ |
|---|---|---|---|
| $A_1$ | 0.763 | 0.154 | 0.917 |
| $A_2$ | 0.736 | 0.180 | 0.916 |
| $A_3$ | 0.820 | 0.142 | 0.962 |
| ID | 0.820 | 0.180 | 1.000 |
| AID | 0.736 | 0.142 | 0.878 |

$K_1^- = 0.917 / 0.878 = 1.044$ $\quad$ $K_1^+ = 0.917 / 1 = 0.917$

$K_2^- = 0.916 / 0.878 = 1.043$ $\quad$ $K_2^+ = 0.916 / 1 = 0.916$

$K_3^- = 0.962 / 0.878 = 1.096$ $\quad$ $K_3^+ = 0.962 / 1 = 0.962$

Step 13) Finally $f(K_i^+)$, $f(K_i^-)$ (utility function of the ideal and anti-ideal solution), and $f(K_i)$ (utility function of alternatives) are given in Table 3.16.

Table 3.16. Utility degress and utility function of alternatives

| | $K_i^-$ | $K_i^+$ | $f(K_i^-)$ | $f(K_i^+)$ | $f(K_i)$ | Rank |
|---|---|---|---|---|---|---|
| $A_1$ | 1.044 | 0.917 | 0.468 | 0.532 | 0.650 | 2 |
| $A_2$ | 1.043 | 0.916 | 0.468 | 0.532 | 0.649 | 3 |
| $A_3$ | 1.096 | 0.962 | 0.468 | 0.532 | 0.682 | 1 |

$f(K_1^-) = 0.917 / (0.917 + 1.044) = 0.468$ $\quad$ $f(K_1^+) = 1.044 / (0.917 + 1.044) = 0.532$

$f(K_2^-) = 0.916 / (0.916 + 1.043) = 0.468$ $\quad$ $f(K_2^+) = 1.043 / (0.916 + 1.043) = 0.532$

$f(K_3^-) = 0.962 / (0.962 + 1.096) = 0.468$ $\quad$ $f(K_3^+) = 1.096 / (0.962 + 1.096) = 0.532$

$f(K_1) = (1.044 + 0.917) / (1 + ((1- 0.532) / 0.532) + ((1- 0.468) / 0.468) = 0.650$

$f(K_2) = (1.043 + 0.916) / (1 + ((1- 0.532) / 0.532) + ((1- 0.468) / 0.468) = 0.649$

$f(K_3) = (1.096 + 0.962) / (1 + ((1- 0.532) / 0.532) + ((1- 0.468) / 0.468) = 0.682$

In our example, we order the values of utility function of alternatives in descending order, which means the greater the utility value, the more important the alternative becomes.

### 3.4. Proposed FMEA Technique

As being conducted conventional FMEA, predetermined FMs are evaluated by taking into consideration of the RFs which are O, S and D. O is the frequency of an FM, namely how many times it may occur, S is the affects of a FM if it happens, D is the ability of detecting a FM before it happens (Liu et al., 2019). FMEA team, which compose of the field experts and DMs, assesses FMs according to the RFs by using ten-point scale (1-10). Then RPN is calculated by multiplying RFs' scores;

$$RPN = O \times S \times D \tag{17}$$

RPN scores of FMs' are ordered in a descending manner. The higher the score is, the more essential the associated FM is (Kushwaha, Panchal and Sachdeva, 2020).

We have already mentioned that fuzzy sets are handy tools to used in problems that incorporate assessment stages. Because FMEA counts also as evaluation of FMs with respect to the RFs, the usage of fuzzy sets will give more realistic and more accurate results. In this dissertation, IFSs will be used while DMs evaluating FMs and weighting RFs, and IF MARCOS method will be used when FMs are ranked. The structure of the steps of the proposed methodology used for FMEA is given below.

Step 1) FMs are determined for the company, and FMEA expert crew (DMs) is formed.

Step 2) The weights of DMs are computed. By using Table 3.17 and Eq. (2), assign a crisp weight ($\delta_d$) to each DM, where $W_d = (\mu_d, \upsilon_d, \pi_d)$ ; in the set $d = \{1, 2, \ldots, p\}$, providing that $\sum_{d=1}^{p} \delta_d = 1$ (Can, 2018). Differently in this proposed method, a linguistic variable table will be used that measures how much the decision makers trust the people in the same hierarchy with them and the people in the lower hierarchy in terms of FM evaluation.

Table 3.17. IFNs for the DMs' trust level, $W_d$

| Linguistic terms | Abbreviation | $W_d$ (IFN) |
|---|---|---|
| Extremely low | EL | (0, 0, 1) |
| Very low | VL | (0.02, 0.09, 0.89) |
| Low | L | (0.07, 0.15, 0.78) |
| Relatively low | RL | (0.13, 0.20, 0.67) |
| Moderate | M | (0.22, 0.22, 0.56) |
| Relatively high | RH | (0.33, 0.22, 0.45) |
| Moderately high | MH | (0.46, 0.20, 0.34) |
| High | H | (0.62, 0.15, 0.23) |
| Very high | VH | (0.79, 0.09, 0.12) |
| Extremely high | EH | (1, 0, 0) |

In this study, apart from and as a contribution to the literature, the assignment of weights to DMs are not provided by the authors. Instead, they are asked to evaluate each other and their weights are calculated according to these evaluations. DMs give trust levels to each other in accordance with the hierarchy, the following rules are proposed as:

Rule 1: In order to assign a single IFN weight to a DM with more than one rating, the ratings must be aggregated using the IFWA operator.

For $DM_a$ and $DM_d$ being two decision makers in the expert crew, IFWA operator will be achieved as given in Eq. (18).

$$\text{Ꝑ}_d = \text{IFWA}\ (P_{d1},\ P_{d2},\ \ldots,\ P_{dp}) = (1 - \prod_{d=1}^{p}(1 - \mu_{P_{da}})^{\eta},\ \prod_{d=1}^{p}(\upsilon_{P_{da}})^{\eta}\ ,\ \prod_{d=1}^{p}(1 - \mu_{P_{da}})^{\eta} - \prod_{d=1}^{p}(\upsilon_{P_{da}})^{\eta}),\qquad j = 1,\ldots, n \tag{18}$$

where $P_{da}$ is the evaluation score that $a^{th}$ DM give to $d^{th}$ DM.

The exponent expressions in the formula (η) are determined according to the number of people in order to give hierarchical importance to the people.

Rule 2: If the person at the top of the Hierarchy is one person, he/ she will be labeled "Extremely high" and have an IFN (1,0,0).

Rule 3: If only one evaluation is allowed in any DM according to the hierarchy, that evaluation will be based on what it is.

Rule 4: If a DM is rated extremely high (1,0,0) or extremely low (0,0,1) by a DM who can rate him/her, don't use their actual score in order not to ignore other DMs' ratings if any. In this study, in order to take into consideration of all DMs, instead of using sharp ratings for "extreme" evaluations, we used (0,99; 0,005; 0,005) for "extremely high", and (0,005; 0,99; 0,005) for "extremely low".

Rule 5: If there is a person in the lowest hierarchy, his exponent ($\eta$) will be unused since he cannot evaluate anyone anyway.

After computing one IFN score for each DMs, a crisp weight value will be assigned by utilizing Equation (2).

Step 3) The RFs are assessed by DMs by using Table 3.18 (Ecer and Pamucar, 2021).

Table 3.18. IFNs evaluation for RFs weights

| Linguistic Terms | Abbreviation | IFN |
|---|---|---|
| Very Important | VI | (0.88, 0.08) |
| Important | I | (0.75, 0.20) |
| Medium | M | (0.50, 0.45) |
| Unimportant | UI | (0.35, 0.60) |
| Very Unimportant | VU | (0.08, 0.88) |

Step 4) Aggregated IFN values of RFs that are linguistically evaluated by each DM are found, and equation (3) (IFWA operator) is used to do that.

Step 5) $\varphi^+$, $\varphi^-$, $CC_j$ and $\omega_j$ values for RFs are calculated by using (4)-(7).

Step 6) Basen on each RF, FMs are assessed by using linguistic variables coming from Table 3.19 – Table 3.21 (Can, 2018).

Table 3.19. IFNs for O evaluation of FMs

| Failure's Probability | Occurence Ratio | $\mu_{ij}$ | $\nu_{ij}$ | $\pi_{ij}$ |
|---|---|---|---|---|
| Incredibly High | ≥1 in 2 | 1 | 0 | 0 |
| Too high | 1 in 3 | 0.79 | 0.09 | 0.12 |
| Reoccurring failure | 1 in 8 | 0.62 | 0.15 | 0.23 |
| High | 1 in 20 | 0.46 | 0.2 | 0.34 |
| Moderately high | 1 in 80 | 0.33 | 0.22 | 0.45 |
| Moderate | 1 in 400 | 0.22 | 0.22 | 0.56 |
| Relatively low | 1 in 2.000 | 0.13 | 0.2 | 0.67 |
| Low | 1 in 15.000 | 0.07 | 0.15 | 0.78 |
| Remote | 1 in 150.000 | 0.02 | 0.09 | 0.89 |
| Almost not possible | ≥1 in 1.500.000 | 0 | 0 | 1 |

Table 3.20. IFNs for S evaluation of FMs

| Severity | Magnitude of Impact | $\mu_{ij}$ | $\nu_{ij}$ | $\pi_{ij}$ |
|---|---|---|---|---|
| Dangerous | Failure includes dangerous consequences. | 1 | 0 | 0 |
| Very Extensive | Failure is critical and may happen without prior warning. | 0.79 | 0.09 | 0.12 |
| Extensive | The product is non-functional with loss of its primary purpose. The system is out of order. | 0.62 | 0.15 | 0.23 |
| Huge | Product performance is strongly impaired, but it works. The system may not run. | 0.46 | 0.2 | 0.34 |
| Significant | Product performance is reduced. Comfort or convenience functionality may not perform. | 0.33 | 0.22 | 0.45 |
| Medium | A medium level of effect on product performance. The product needs to be fixed. | 0.22 | 0.22 | 0.56 |
| Low | Negligible affect on product performance. The product does not need maintenance. | 0.13 | 0.2 | 0.67 |
| Minor | Little influence on product or system performance. | 0.07 | 0.15 | 0.78 |
| Very Minor | Very little influence on product or system performance. | 0.02 | 0.09 | 0.89 |

| | | | | |
|---|---|---|---|---|
| Nothing | No impact. | 0 | 0 | 1 |

Table 3.21. IFNs for D evaluation of FMs

| Identification | Detectability | $\mu_{ij}$ | $\nu_{ij}$ | $\pi_{ij}$ |
|---|---|---|---|---|
| Definite vagueness | The safety design check does not detect a potential cause of failure or successive failure mode. | 0 | 0 | 1 |
| Too distant | It is very unlikely that design control will determine a potential cause of failure or consequent failure mode. | 0.02 | 0.09 | 0.89 |
| Distant | It is unlikely that design control will determine a potential cause of failure or consequent failure mode. | 0.07 | 0.15 | 0.78 |
| Very small | There is a very small probability that design control will spot a possible cause of failure or ensuing failure | 0.13 | 0.2 | 0.67 |
| Small | There is a small probability that design control will spot a possible cause of failure or ensuing failure | 0.22 | 0.22 | 0.56 |
| Medium | The probability that the design check will detect a potential cause of failure or next failure mode is mid- | 0.33 | 0.22 | 0.45 |
| Medium High | The probability that the design check will detect a potential cause of failure or next failure mode is | 0.46 | 0.2 | 0.34 |
| High | It is likely that design control will determine a potential cause of failure or consequent failure mode. | 0.62 | 0.15 | 0.23 |
| Very high | It is very likely that design control will determine a potential cause of failure or consequent failure mode. | 0.79 | 0.09 | 0.12 |
| Nearly certain | The design check will almost certainly detect a potential cause of failure or subsequent failure mode. | 1 | 0 | 0 |

Step 7) For each DM's evaluation of FM based on RFs are aggregated by using equation (8) (IFWA operator).

Step 8) $\varphi+$, $\varphi-$ and CC values for FMs are calculated by using (4)-(6).

Step 9) Decision matrix is builded with FMs on the rows, and RFs on the columns, just like Table 3.11.

Step 10) Extended IF decision matrix is formed by determining ID and AID solutions and adding them at the bottom of the matrix as in the Table 3.12.

Step 11) The previous table is normalized by using (9) and (10).

Step 12) Normalized table is weighted with RFs weights found in step 3 by utilizing equation (11).

Step 13) Utility degrees of FMs are computed by using equation (12) and (13).

Step 14) The utility function of ideal and anti-ideal solutions and the utility function of FMs are obtained by applying Eq. (14)-(16). Finally, FMs are ordered according to the utility functions in ascending order.

Since the IF-MARCOS method is a MCDM method, it has a logic that the higher the utility function of an alternative is, the more it should be selected while it is normally applied. When FMEA is applied, we need to think in reverse of this logic since FMs are negative in nature. That is, when the IF-MARCOS method is used for FMEA, it should be considered that the lower the utility function of the FMs (alternatives), the more important and the higher priority should be given to that FM.

# 4. APPLICATION

The proposed FMEA methodology (IF-MARCOS FMEA) was applied in the production department of a defense industry company located in Ankara.

The company offers mission-critical systems and solutions for the defense industry and the civil sector. It carries out integration and assembly activities in many fields such as voice communication systems, sensors, electronic systems, radars, etc. The company produces project-based products. Together with the customer, a project schedule is determined through a contract and the production process starts according to the schedule. The company produces products for more than one project in a certain period and the shelves in its warehouse are divided according to project names. The general production process of the company is as follows:

1. Through tendering, a project contract is signed with the client and technical requirements are defined in the contract.

2. Infrastructure is built, which is defined in the contract and by whom.
    a. If the infrastructure is to be built by the customer, information about the infrastructure is provided by the company.
        i. The customer builds the infrastructure suitable for the desired infrastructure with its own means.
    b. If the infrastructure setup will be done by the company, the design team designs the infrastructure setup.
        i. First, the client's approval for exploration is obtained and a preliminary field investigation is carried out. The technical design of the project may change as a result of the investigation.
        ii. After the investigation, work begins for the construction of infrastructure suitable for the field and the infrastructure is constructed in accordance with the project.

3. The project plan is made by project engineers. Bill of Material (BOM) and production documents are prepared by system engineers in accordance with the project requirements.

4. The BOM, design and production documents are examined, approved and finally promulgated by quality control department.

5. Material requirement planning is done by the production planning and control engineer according to the BOM.

a. The condition of the sub-parts in the warehouse is evaluated. Materials not in the warehouse are ordered.

6. As the materials are completed, production starts to be done piece by piece (Sometimes it may not be possible to proceed with one production step for a Project due to supply problems and/or project timeline).

a. The related materials of the sub-assemblies to be produced are withdrawn from the warehouse.
b. Production is carried out in accordance with the production document, technical requirements document and BOM.

7. The produced product waits in the warehouse until all its sub-parts are completed and is taken to the test area or the project warehouse according to the demand.

a. The software of the completed products coming to the test area is loaded by the software engineers.
b. Technical tests are carried out by the test team.
c. Factory acceptance test of the project (customer comes to the company and examines the product) is done.

8. The work order is closed and the production planning engineer is informed about the materials used during production.

9. The product is packaged and delivered to the customer. If there is a clause in the contract, the field setup of the product is also done by the production teams.

In accordance with its quality policies, the Company aims to fulfill customer demands and expectations and to continuously improve the quality management system's performance in an effort to increase efficiency and quality. In this direction, it is aimed to determine the ongoing and future FMs in the production department of the company and to find the order of importance of these failures.

FMs are caused by a problem in the past and they effect the future. In other words, today's FMs that are due to the Failure Causes (FCs) result in Failure Effects (FEs). FMEA investigates failures in a system, how to improve them, and suggests corrective and preventive actions. An FMEA was conducted for the company by applying the proposed technique using the R Studio program. One can see all the steps in the implementation phase below.

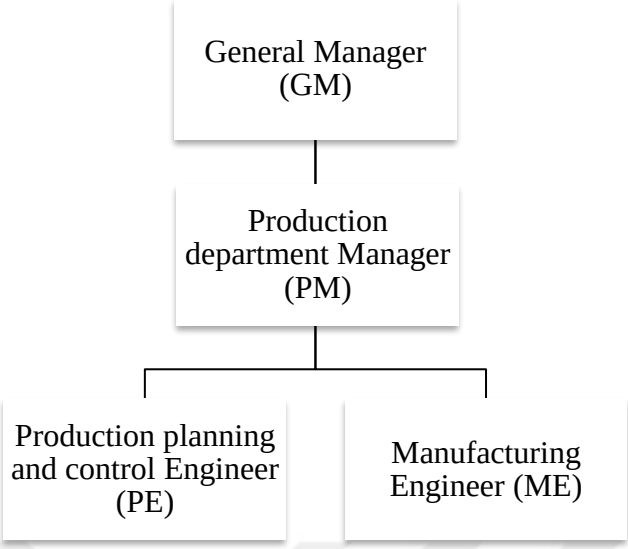
Step 1) FMs determined are given in Table 4.1, and an expert group consisting of 4 DMs has been established. The expert crew consists of a General Manager (GM), a Production department Manager (PM), a Production planning and control Engineer (PE) and a Manufacturing Engineer (ME). The hierarchical structure between DMs is given in the Figure 4.1.

Table 4.1. FMs of the production department

| | FMs |
|---|---|
| 1 | Inability to access up-to-date and accurate information needed in production planning and production processes. |
| 2 | Defective electronic cards. They are not subjected to functional tests during the incoming quality control process and post-production tests are not carried out in sufficient detail. |
| 3 | Errors in the production document and insufficient detail of the production steps in the document. |
| 4 | Existence of some products/materials with supply problems (not available on time). |
| 5 | When material requirements planning is done by the production planning engineer, the SAP program does not suggest alternative materials and some materials are defined as duplicates in SAP software. |

| | |
|---|---|
| 6 | Some products that are no longer produced by the relevant manufacturer are still active in SAP software and can be added to the product trees. Additionally, these products are mentioned in the product trees of some other systems. |
| 7 | SAP software does not support shelf information within the scope of warehouse stocking. |
| 8 | The obligation to take the materials that are not available in their own storage from the storage of another project in order to complete the project. |
| 9 | Incorrect or missing information on some Product Identification Forms (PIFs). |
| 10 | Detailed data of products/materials defined before the transition to SAP software are included in the PIFs in SVN software, while those defined after the transition to SAP software are included in the SAP database. |
| 11 | The obligation to start production before all products/materials required for production have been procured. |
| 12 | The obligation to set up the product in the integration and test environment before production is completed. |
| 13 | Existence of duplicate products in the SAP system, multiple different part numbers defined for the product with the same model number. |
| 14 | The configuration serial number given to the products that are production items and the serial numbers that have to be entered into the SAP software do not match. |
| 15 | Request by project management to place orders for some products before the BOM is finalized. |

Figure 4.1. Hierarchical structure of DMs




Step 2) DMs are asked to give a trust level to the DMs appropriate to their hierarchical position using Table 3.17 and weights are derived from the rules given in step 2 of the suggested method. The following Table 4.2 gives the evaluations of DMs, and the appropriate boxes of DMs that cannot hierarchically evaluate another DM are shaded in dark gray.

Table 4.2. DMs' evaluations of each other

| | GM | PM | PE | ME |
|---|---|---|---|---|
| GM | - | EH | VH | VH |
| PM | | - | EH | EH |
| PE | | | - | VH |
| ME | | | H | - |

The table below, Table 4.3, shows the IFN transformed versions of the linguistic evaluations given in table 4.2. The exponent values (η) of each DM is also given in the last column of the Table 4.3.

Table 4.3. The IFNs of DMs' evaluations of each other and their exponents

| | GM | PM | PE | ME | η |
|---|---|---|---|---|---|
| GM | - | (0.990, 0.005, 0.005) | (0.790, 0.090, 0.120) | (0.790, 0.090, 0.120) | 0.3 |
| PM | | - | (0.990, 0.005, 0.005) | (0.990, 0.005, 0.005) | 0.2 |

| | | | | | |
|---|---|---|---|---|---|
| PE | | | - | (0.790, 0.090, 0.120) | 0.1 |
| ME | | | (0.620, 0.150, 0.230) | - | 0.1 |

In accordance with the rules, GM will directly have the EH trust level due to his hierarchical position according to Rule 2 and will receive IFN value of (1, 0, 0). PM will only take a trust level based only on what GM evaluates of him (Rule 3). Since the GM evaluates PM with EH, Rule 4 should also apply, so PM gets an IFN of (0.99, 0.005, 0.005). PE and ME received evaluations from GM, PM and each other. Therefore, according to the Rule 1, these scores should be aggragated using IFWA operator (18).

$$P_{PE} = \text{IFWA}\ (P_{PE\text{-}GM}, P_{PE\text{-}PM}, P_{PE\text{-}ME}) = (1 - [(1 - 0.79)^{0.3} * (1 - 0.99)^{0.2} * (1 - 0.62)^{0.1}], 0.09^{0.3} * 0.005^{0.2} * 0.15^{0.1}, (1 - 0.79)^{0.3} * (1 - 0.99)^{0.2} * (1 - 0.62)^{0.1} - 0.09^{0.3} * 0.005^{0.2} * 0.15^{0.1})$$
$$= (0.774, 0.139, 0.087)$$

$$P_{ME} = \text{IFWA}\ (P_{ME\text{-}GM}, P_{ME\text{-}PM}, P_{ME\text{-}PE}) = (1 - [(1 - 0.79)^{0.3} * (1 - 0.99)^{0.2} * (1 - 0.79)^{0.1}], 0.09^{0.3} * 0.005^{0.2} * 0.09^{0.1}, (1 - 0.79)^{0.3} * (1 - 0.99)^{0.2} * (1 - 0.79)^{0.1} - 0.09^{0.3} * 0.005^{0.2} * 0.09^{0.1})$$
$$= (0.787, 0.132, 0.081)$$

After obtaining an IFN for all DMs, DM weights are calculated according to Eq. (2) and they are given in Table 4.4.

Table 4.4. Weights of DMs

| DMs | Weights |
|---|---|
| GM | 0.270 |
| PM | 0.269 |
| PE | 0.229 |
| ME | 0.232 |

Step 3) RFs were assessed by 4 DMs according to Table 3.18 and the IFN versions of these evaluations are as follows (Table 4.5).

Table 4.5. DMs' evaluations of RFs

| | O | | | S | | | D | | |
|---|---|---|---|---|---|---|---|---|---|
| DMs | μ | ν | π | μ | ν | π | μ | ν | π |
| GM | 0.750 | 0.200 | 0.050 | 0.880 | 0.080 | 0.040 | 0.500 | 0.450 | 0.050 |
| PM | 0.500 | 0.450 | 0.050 | 0.880 | 0.080 | 0.040 | 0.750 | 0.200 | 0.050 |
| PE | 0.500 | 0.450 | 0.050 | 0.880 | 0.080 | 0.040 | 0.750 | 0.200 | 0.050 |
| ME | 0.750 | 0.200 | 0.050 | 0.880 | 0.080 | 0.040 | 0.350 | 0.600 | 0.050 |

Step 4) The 4 evaluation scores for each RF were aggregated with the IFWA operator (Equation (3)), which takes the DM weights as exponent, to obtain a single IFN. These obtained IFNs at the end are given in Table 4.6.

Table 4.6. Aggragated versions of IFNs of RFs

| AGGR. | μ | ν | π |
|---|---|---|---|
| O | 0.647 | 0.300 | 0.054 |
| S | 0.880 | 0.080 | 0.040 |
| D | 0.624 | 0.321 | 0.055 |

Step 5) Weights of RFs are found by using Equaitons (4)-(7) and given in Table 4.7.

Table 4.7. Weights of RFs

| | φ+ | φ- | CC | Normalized weights (ωj) |
|---|---|---|---|---|
| O | 0.466 | 0.955 | 0.672 | 0.303 |
| S | 0.150 | 1.274 | 0.895 | 0.404 |
| D | 0.498 | 0.924 | 0.650 | 0.293 |

Step 6) Table 4.8. shows DMs' views on FMs according to RFs. DMs took Tables 3.19, 3.20 and 3.21 into account when making their assessments. Accordingly, the IFN versions of the verbal variable assessments are given here.

Table 4.8. Evaluations of FMs by DMs

| | | O | | | S | | | D | | |
|---|---|---|---|---|---|---|---|---|---|---|
| FMs | DMs | μ | ν | π | μ | ν | π | μ | ν | π |
| FM1 | GM | 0.220 | 0.220 | 0.560 | 0.330 | 0.220 | 0.450 | 0.790 | 0.090 | 0.120 |
| | PM | 0.620 | 0.150 | 0.230 | 0.220 | 0.220 | 0.560 | 0.620 | 0.150 | 0.230 |
| | PE | 0.990 | 0.005 | 0.005 | 0.790 | 0.090 | 0.120 | 0.790 | 0.090 | 0.120 |
| | ME | 0.620 | 0.150 | 0.230 | 0.460 | 0.200 | 0.340 | 0.620 | 0.150 | 0.230 |
| FM2 | GM | 0.620 | 0.150 | 0.230 | 0.330 | 0.220 | 0.450 | 0.790 | 0.090 | 0.120 |
| | PM | 0.990 | 0.005 | 0.005 | 0.620 | 0.150 | 0.230 | 0.990 | 0.005 | 0.005 |
| | PE | 0.790 | 0.090 | 0.120 | 0.790 | 0.090 | 0.120 | 0.790 | 0.090 | 0.120 |
| | ME | 0.990 | 0.005 | 0.005 | 0.620 | 0.150 | 0.230 | 0.790 | 0.090 | 0.120 |
| FM3 | GM | 0.460 | 0.200 | 0.340 | 0.220 | 0.220 | 0.560 | 0.460 | 0.200 | 0.340 |
| | PM | 0.790 | 0.090 | 0.120 | 0.620 | 0.150 | 0.230 | 0.790 | 0.090 | 0.120 |
| | PE | 0.790 | 0.090 | 0.120 | 0.790 | 0.090 | 0.120 | 0.790 | 0.090 | 0.120 |
| | ME | 0.790 | 0.090 | 0.120 | 0.330 | 0.220 | 0.450 | 0.790 | 0.090 | 0.120 |
| FM4 | GM | 0.790 | 0.090 | 0.120 | 0.330 | 0.220 | 0.450 | 0.620 | 0.150 | 0.230 |
| | PM | 0.790 | 0.090 | 0.120 | 0.790 | 0.090 | 0.120 | 0.990 | 0.005 | 0.005 |
| | PE | 0.790 | 0.090 | 0.120 | 0.460 | 0.200 | 0.340 | 0.620 | 0.150 | 0.230 |
| | ME | 0.790 | 0.090 | 0.120 | 0.460 | 0.200 | 0.340 | 0.790 | 0.090 | 0.120 |
| FM5 | GM | 0.790 | 0.090 | 0.120 | 0.460 | 0.200 | 0.340 | 0.620 | 0.150 | 0.230 |
| | PM | 0.330 | 0.220 | 0.450 | 0.220 | 0.220 | 0.560 | 0.620 | 0.150 | 0.230 |
| | PE | 0.460 | 0.200 | 0.340 | 0.330 | 0.220 | 0.450 | 0.790 | 0.090 | 0.120 |
| | ME | 0.620 | 0.150 | 0.230 | 0.330 | 0.220 | 0.450 | 0.220 | 0.220 | 0.560 |
| FM6 | GM | 0.330 | 0.220 | 0.450 | 0.220 | 0.220 | 0.560 | 0.460 | 0.200 | 0.340 |
| | PM | 0.620 | 0.150 | 0.230 | 0.220 | 0.220 | 0.560 | 0.620 | 0.150 | 0.230 |
| | PE | 0.460 | 0.200 | 0.340 | 0.790 | 0.090 | 0.120 | 0.790 | 0.090 | 0.120 |
| | ME | 0.460 | 0.200 | 0.340 | 0.330 | 0.220 | 0.450 | 0.330 | 0.220 | 0.450 |
| FM7 | GM | 0.330 | 0.220 | 0.450 | 0.130 | 0.200 | 0.670 | 0.790 | 0.090 | 0.120 |
| | PM | 0.460 | 0.200 | 0.340 | 0.130 | 0.200 | 0.670 | 0.460 | 0.200 | 0.340 |
| | PE | 0.460 | 0.200 | 0.340 | 0.130 | 0.200 | 0.670 | 0.620 | 0.150 | 0.230 |
| | ME | 0.990 | 0.005 | 0.005 | 0.220 | 0.220 | 0.560 | 0.220 | 0.220 | 0.560 |
| FM8 | GM | 0.220 | 0.220 | 0.560 | 0.130 | 0.200 | 0.670 | 0.790 | 0.090 | 0.120 |
| | PM | 0.620 | 0.150 | 0.230 | 0.220 | 0.220 | 0.560 | 0.330 | 0.220 | 0.450 |
| | PE | 0.790 | 0.090 | 0.120 | 0.220 | 0.220 | 0.560 | 0.620 | 0.150 | 0.230 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | ME | 0.790 | 0.090 | 0.120 | 0.220 | 0.220 | 0.560 | 0.220 | 0.220 | 0.560 |
| FM9 | GM | 0.460 | 0.200 | 0.340 | 0.330 | 0.220 | 0.450 | 0.620 | 0.150 | 0.230 |
| | PM | 0.460 | 0.200 | 0.340 | 0.460 | 0.200 | 0.340 | 0.460 | 0.200 | 0.340 |
| | PE | 0.790 | 0.090 | 0.120 | 0.790 | 0.090 | 0.120 | 0.790 | 0.090 | 0.120 |
| | ME | 0.620 | 0.150 | 0.230 | 0.620 | 0.150 | 0.230 | 0.620 | 0.150 | 0.230 |
| FM10 | GM | 0.790 | 0.090 | 0.120 | 0.070 | 0.150 | 0.780 | 0.790 | 0.090 | 0.120 |
| | PM | 0.790 | 0.090 | 0.120 | 0.220 | 0.220 | 0.560 | 0.460 | 0.200 | 0.340 |
| | PE | 0.990 | 0.005 | 0.005 | 0.460 | 0.200 | 0.340 | 0.790 | 0.090 | 0.120 |
| | ME | 0.990 | 0.005 | 0.005 | 0.330 | 0.220 | 0.450 | 0.790 | 0.090 | 0.120 |
| FM11 | GM | 0.790 | 0.090 | 0.120 | 0.330 | 0.220 | 0.450 | 0.790 | 0.090 | 0.120 |
| | PM | 0.790 | 0.090 | 0.120 | 0.620 | 0.150 | 0.230 | 0.620 | 0.150 | 0.230 |
| | PE | 0.990 | 0.005 | 0.005 | 0.330 | 0.220 | 0.450 | 0.620 | 0.150 | 0.230 |
| | ME | 0.990 | 0.005 | 0.005 | 0.330 | 0.220 | 0.450 | 0.790 | 0.090 | 0.120 |
| FM12 | GM | 0.790 | 0.090 | 0.120 | 0.330 | 0.220 | 0.450 | 0.790 | 0.090 | 0.120 |
| | PM | 0.790 | 0.090 | 0.120 | 0.620 | 0.150 | 0.230 | 0.620 | 0.150 | 0.230 |
| | PE | 0.790 | 0.090 | 0.120 | 0.790 | 0.090 | 0.120 | 0.620 | 0.150 | 0.230 |
| | ME | 0.990 | 0.005 | 0.005 | 0.330 | 0.220 | 0.450 | 0.220 | 0.220 | 0.560 |
| FM13 | GM | 0.620 | 0.150 | 0.230 | 0.220 | 0.220 | 0.560 | 0.620 | 0.150 | 0.230 |
| | PM | 0.330 | 0.220 | 0.450 | 0.220 | 0.220 | 0.560 | 0.620 | 0.150 | 0.230 |
| | PE | 0.220 | 0.220 | 0.560 | 0.220 | 0.220 | 0.560 | 0.790 | 0.090 | 0.120 |
| | ME | 0.460 | 0.200 | 0.340 | 0.330 | 0.220 | 0.450 | 0.130 | 0.200 | 0.670 |
| FM14 | GM | 0.790 | 0.090 | 0.120 | 0.130 | 0.200 | 0.670 | 0.790 | 0.090 | 0.120 |
| | PM | 0.990 | 0.005 | 0.005 | 0.330 | 0.220 | 0.450 | 0.620 | 0.150 | 0.230 |
| | PE | 0.790 | 0.090 | 0.120 | 0.460 | 0.200 | 0.340 | 0.790 | 0.090 | 0.120 |
| | ME | 0.620 | 0.150 | 0.230 | 0.330 | 0.220 | 0.450 | 0.220 | 0.220 | 0.560 |
| FM15 | GM | 0.620 | 0.150 | 0.230 | 0.220 | 0.220 | 0.560 | 0.620 | 0.150 | 0.230 |
| | PM | 0.790 | 0.090 | 0.120 | 0.330 | 0.220 | 0.450 | 0.790 | 0.090 | 0.120 |
| | PE | 0.790 | 0.090 | 0.120 | 0.330 | 0.220 | 0.450 | 0.620 | 0.150 | 0.230 |
| | ME | 0.790 | 0.090 | 0.120 | 0.070 | 0.150 | 0.780 | 0.220 | 0.220 | 0.560 |

Step 7) Those evaluations given in Table 4.8 are aggragated for each FMs by using IFWA operator (8). Aggregated IFN values for RFs are given in Table 4.9.

Table 4.9. Aggregated IFN values for FMs evaluation

| | O | | | S | | | D | | |
|---|---|---|---|---|---|---|---|---|---|
| | μ | ν | π | μ | ν | π | μ | ν | π |
| FM1 | 0.799 | 0.076 | 0.124 | 0.491 | 0.175 | 0.334 | 0.717 | 0.116 | 0.166 |
| FM2 | 0.946 | 0.024 | 0.029 | 0.613 | 0.148 | 0.239 | 0.907 | 0.041 | 0.051 |
| FM3 | 0.729 | 0.112 | 0.159 | 0.541 | 0.162 | 0.298 | 0.729 | 0.112 | 0.159 |
| FM4 | 0.790 | 0.090 | 0.120 | 0.556 | 0.166 | 0.278 | 0.875 | 0.053 | 0.071 |
| FM5 | 0.591 | 0.155 | 0.254 | 0.342 | 0.214 | 0.444 | 0.608 | 0.146 | 0.246 |
| FM6 | 0.479 | 0.190 | 0.331 | 0.443 | 0.179 | 0.378 | 0.584 | 0.158 | 0.258 |
| FM7 | 0.773 | 0.087 | 0.140 | 0.152 | 0.204 | 0.644 | 0.580 | 0.154 | 0.266 |
| FM8 | 0.649 | 0.131 | 0.220 | 0.197 | 0.214 | 0.589 | 0.555 | 0.158 | 0.287 |
| FM9 | 0.599 | 0.156 | 0.245 | 0.575 | 0.160 | 0.265 | 0.635 | 0.144 | 0.220 |
| FM10 | 0.948 | 0.024 | 0.028 | 0.274 | 0.194 | 0.532 | 0.729 | 0.112 | 0.159 |
| FM11 | 0.948 | 0.024 | 0.028 | 0.425 | 0.198 | 0.377 | 0.718 | 0.116 | 0.166 |
| FM12 | 0.896 | 0.046 | 0.058 | 0.559 | 0.162 | 0.279 | 0.618 | 0.143 | 0.240 |
| FM13 | 0.434 | 0.194 | 0.372 | 0.247 | 0.220 | 0.533 | 0.598 | 0.143 | 0.259 |
| FM14 | 0.894 | 0.047 | 0.060 | 0.316 | 0.210 | 0.475 | 0.666 | 0.127 | 0.207 |
| FM15 | 0.753 | 0.103 | 0.143 | 0.247 | 0.201 | 0.552 | 0.617 | 0.143 | 0.240 |

Step 8) Table 4.10 presents the CC values ($X_{ij}$ values for the decision matrix) of FMs.

Table 4.10. CC values of FMs

| | O | | | S | | | D | | |
|---|---|---|---|---|---|---|---|---|---|
| | φ+ | φ- | CC | φ+ | φ- | CC | φ+ | φ- | CC |
| FM1 | 0.248 | 1.228 | 0.832 | 0.633 | 1.016 | 0.616 | 0.348 | 1.150 | 0.768 |
| FM2 | 0.066 | 1.360 | 0.954 | 0.478 | 1.077 | 0.693 | 0.114 | 1.321 | 0.921 |
| FM3 | 0.334 | 1.160 | 0.777 | 0.571 | 1.041 | 0.646 | 0.334 | 1.160 | 0.777 |
| FM4 | 0.258 | 1.211 | 0.824 | 0.550 | 1.041 | 0.654 | 0.153 | 1.291 | 0.894 |
| FM5 | 0.505 | 1.062 | 0.678 | 0.823 | 0.965 | 0.540 | 0.485 | 1.077 | 0.689 |
| FM6 | 0.646 | 0.998 | 0.607 | 0.697 | 1.006 | 0.591 | 0.514 | 1.057 | 0.673 |
| FM7 | 0.281 | 1.204 | 0.811 | 1.084 | 1.035 | 0.488 | 0.521 | 1.059 | 0.670 |
| FM8 | 0.435 | 1.106 | 0.718 | 1.019 | 1.001 | 0.496 | 0.553 | 1.048 | 0.655 |
| FM9 | 0.495 | 1.064 | 0.682 | 0.526 | 1.052 | 0.667 | 0.450 | 1.088 | 0.708 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| FM10 | 0.063 | 1.361 | 0.956 | 0.921 | 1.004 | 0.522 | 0.333 | 1.160 | 0.777 |
| FM11 | 0.063 | 1.361 | 0.956 | 0.716 | 0.982 | 0.579 | 0.347 | 1.151 | 0.768 |
| FM12 | 0.127 | 1.310 | 0.911 | 0.546 | 1.046 | 0.657 | 0.473 | 1.083 | 0.696 |
| FM13 | 0.705 | 0.988 | 0.584 | 0.948 | 0.976 | 0.507 | 0.499 | 1.077 | 0.683 |
| FM14 | 0.130 | 1.308 | 0.909 | 0.859 | 0.974 | 0.532 | 0.413 | 1.117 | 0.730 |
| FM15 | 0.303 | 1.180 | 0.796 | 0.955 | 1.002 | 0.512 | 0.474 | 1.083 | 0.696 |

Step 9) Decision matrix which composes of CC values of FMs finding at Step 8, and weights of RFs finding at Step 5 is established in Table 4.11.

Table 4.11. Decision matrix

| | O | S | D |
|---|---|---|---|
| Weights | 0.303 | 0.404 | 0.293 |
| FM1 | 0.832 | 0.616 | 0.768 |
| FM2 | 0.954 | 0.693 | 0.921 |
| FM3 | 0.777 | 0.646 | 0.777 |
| FM4 | 0.824 | 0.654 | 0.894 |
| FM5 | 0.678 | 0.540 | 0.689 |
| FM6 | 0.607 | 0.591 | 0.673 |
| FM7 | 0.811 | 0.488 | 0.670 |
| FM8 | 0.718 | 0.496 | 0.655 |
| FM9 | 0.682 | 0.667 | 0.708 |
| FM10 | 0.956 | 0.522 | 0.777 |
| FM11 | 0.956 | 0.579 | 0.768 |
| FM12 | 0.911 | 0.657 | 0.696 |
| FM13 | 0.584 | 0.507 | 0.683 |
| FM14 | 0.909 | 0.532 | 0.730 |
| FM15 | 0.796 | 0.512 | 0.696 |

Step 10) Extended IF decision matrix is given in Table 4.12. by adding ID and AID solutions, which are determined according to below rule.

- Since O is a cost type RF, ID of that column will be min $X_{ij}$ value; AID will be max $X_{ij}$ value.
- Since S is a cost type RF, ID of that column will be min $X_{ij}$ value; AID will be max $X_{ij}$ value.

- Since D is a benefit type RF, ID of that column will be max $X_{ij}$ value; AID will be min $X_{ij}$ value.

Table 4.12. Extended decision matrix

| | O | S | D |
|---|---|---|---|
| Weights | 0.303 | 0.404 | 0.293 |
| FM1 | 0.832 | 0.616 | 0.768 |
| FM2 | 0.954 | 0.693 | 0.921 |
| FM3 | 0.777 | 0.646 | 0.777 |
| FM4 | 0.824 | 0.654 | 0.894 |
| FM5 | 0.678 | 0.540 | 0.689 |
| FM6 | 0.607 | 0.591 | 0.673 |
| FM7 | 0.811 | 0.488 | 0.670 |
| FM8 | 0.718 | 0.496 | 0.655 |
| FM9 | 0.682 | 0.667 | 0.708 |
| FM10 | 0.956 | 0.522 | 0.777 |
| FM11 | 0.956 | 0.579 | 0.768 |
| FM12 | 0.911 | 0.657 | 0.696 |
| FM13 | 0.584 | 0.507 | 0.683 |
| FM14 | 0.909 | 0.532 | 0.730 |
| FM15 | 0.796 | 0.512 | 0.696 |
| ID | 0.584 | 0.488 | 0.921 |
| AID | 0.956 | 0.693 | 0.655 |

Step 11) Extended decision matrix is normalized in Table 4.13 by using Equations (9) and (10).

Table 4.13. Normalized decision matrix

| | O | S | D |
|---|---|---|---|
| Weights | 0.303 | 0.404 | 0.293 |
| FM1 | 0.702 | 0.793 | 0.834 |
| FM2 | 0.612 | 0.705 | 1.000 |

| | | | |
|---|---|---|---|
| FM3 | 0.752 | 0.756 | 0.843 |
| FM4 | 0.708 | 0.746 | 0.971 |
| FM5 | 0.861 | 0.905 | 0.749 |
| FM6 | 0.961 | 0.827 | 0.731 |
| FM7 | 0.720 | 1.000 | 0.728 |
| FM8 | 0.813 | 0.985 | 0.711 |
| FM9 | 0.855 | 0.732 | 0.768 |
| FM10 | 0.611 | 0.936 | 0.844 |
| FM11 | 0.611 | 0.844 | 0.834 |
| FM12 | 0.640 | 0.743 | 0.756 |
| FM13 | 1.000 | 0.962 | 0.742 |
| FM14 | 0.642 | 0.919 | 0.793 |
| FM15 | 0.734 | 0.954 | 0.756 |
| ID | 1.000 | 1.000 | 1.000 |
| AID | 0.611 | 0.705 | 0.711 |

Step 12) The $X_{ij}$ values of Table 4.13 are weighted with weights of RFs by using equation (11) and these values are given in Table 4.14.

Table 4.14. Weighted decision matrix

| | O | S | D |
|---|---|---|---|
| Weights | 0.303 | 0.404 | 0.293 |
| FM1 | 0.213 | 0.320 | 0.244 |
| FM2 | 0.186 | 0.285 | 0.293 |
| FM3 | 0.228 | 0.305 | 0.247 |
| FM4 | 0.215 | 0.301 | 0.285 |
| FM5 | 0.261 | 0.365 | 0.220 |
| FM6 | 0.291 | 0.334 | 0.214 |
| FM7 | 0.218 | 0.404 | 0.213 |
| FM8 | 0.246 | 0.398 | 0.208 |
| FM9 | 0.259 | 0.296 | 0.225 |
| FM10 | 0.185 | 0.378 | 0.247 |

| | | | |
|---|---|---|---|
| FM11 | 0.185 | 0.341 | 0.245 |
| FM12 | 0.194 | 0.300 | 0.222 |
| FM13 | 0.303 | 0.389 | 0.218 |
| FM14 | 0.195 | 0.371 | 0.233 |
| FM15 | 0.222 | 0.385 | 0.222 |
| ID | 0.303 | 0.404 | 0.293 |
| AID | 0.185 | 0.285 | 0.208 |

Step 13) By utilizing equation (12) and (13) utility degress of each FMs are calculated which are given in Table 4.15.

Table 4.15. Utility degrees of FMs

| | $S_i$ | $K_i^-$ | $K_i^+$ |
|---|---|---|---|
| FM1 | 0.777 | 1.146 | 0.777 |
| FM2 | 0.763 | 1.125 | 0.763 |
| FM3 | 0.780 | 1.151 | 0.780 |
| FM4 | 0.801 | 1.180 | 0.801 |
| FM5 | 0.846 | 1.247 | 0.846 |
| FM6 | 0.839 | 1.238 | 0.839 |
| FM7 | 0.835 | 1.238 | 0.835 |
| FM8 | 0.853 | 1.258 | 0.853 |
| FM9 | 0.780 | 1.150 | 0.780 |
| FM10 | 0.810 | 1.195 | 0.810 |
| FM11 | 0.770 | 1.136 | 0.770 |
| FM12 | 0.716 | 1.056 | 0.716 |
| FM13 | 0.909 | 1.341 | 0.909 |
| FM14 | 0.798 | 1.177 | 0.798 |
| FM15 | 0.829 | 1.222 | 0.829 |

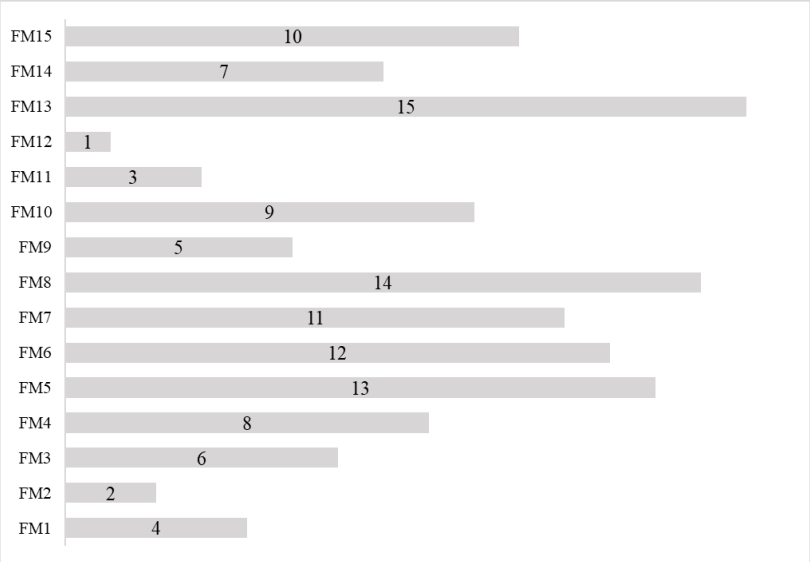
Step 14) Lastly, utility functions are found for each FMs by implementing equations (14)-(16), and ranked in ascending order (Table 4.16).

Table 4.16. Utility functions of FMs and their order

| | $f(K_i^-)$ | $f(K_i^+)$ | $f(K_i)$ | Rank |
|---|---|---|---|---|
| FM1 | 0.404 | 0.596 | 0.610 | 4 |
| FM2 | 0.404 | 0.596 | 0.599 | 2 |
| FM3 | 0.404 | 0.596 | 0.612 | 6 |
| FM4 | 0.404 | 0.596 | 0.628 | 8 |
| FM5 | 0.404 | 0.596 | 0.664 | 13 |
| FM6 | 0.404 | 0.596 | 0.659 | 12 |
| FM7 | 0.404 | 0.596 | 0.656 | 11 |
| FM8 | 0.404 | 0.596 | 0.669 | 14 |
| FM9 | 0.404 | 0.596 | 0.612 | 5 |
| FM10 | 0.404 | 0.596 | 0.636 | 9 |
| FM11 | 0.404 | 0.596 | 0.605 | 3 |
| FM12 | 0.404 | 0.596 | 0.562 | 1 |
| FM13 | 0.404 | 0.596 | 0.714 | 15 |
| FM14 | 0.404 | 0.596 | 0.626 | 7 |
| FM15 | 0.404 | 0.596 | 0.651 | 10 |

The ranking and related FMs according to the application results of the proposed method are given in the Table 4.17 below.

Table 4.17. FMs and their order of importance



FMEA is inherently accompanied by Failure Causes (FC), which explain why FMs are caused, and Failure Effects (FE), which tell what they cause. After ranking the FMs according to their significance, corrective and preventive actions are also suggested. When it is looked at the results from Table 4.17, it is seem that FM12 is the first FM to be prevented, followed by FM2, FM11 and so on. FC, FE and corrective and preventive actions are given in the table below (Table 4.18) for the top three FMs to be prevented. For the remaining FMs, the same kind of table is presented in the Appendix.

Table 4.18. FCs, FEs and corrective and preventive actions for the first three FMs

| | FMs | FCs | FEs | Corrective and preventive actions |
|---|---|---|---|---|
| 12 | The obligation to set up the product in the integration and test environment before production is completed. | Short project schedule and customer pressure. | Increased risk of error and labor force used during the production phase due to partial and interrupted production. | Determining project schedules by taking into account the delivery times of products/materials with global supply problems. |
| 2 | Defective electronic cards. They are not subjected to functional tests during the incoming quality control process and post-production tests are not carried out in sufficient detail. | Lack of suitable and sufficient test environment for electronic cards for incoming quality control and lack of test environment and procedures for these products in the production process. | Cost and time loss, especially in the production planning and production process, as defective products can only be detected during the integration and testing phase. | Establish a test environment and procedure for electronic boards to be operated in the incoming quality control and production process. |
| 11 | The obligation to start production before all products/materials required for production have been procured. | Short project schedules and consequent inability to procure all products/materials needed for production on time. | Increased risk of error and labor force used during the production phase due to partial and interrupted production. | Determining project schedules by taking into account the delivery times of products/materials with global supply problems. Establishing safety stocks of some parts in the product tree, conducting needs analysis and standardization. |

In order to measure the robustness of the IF MARCOS method on FMEA, 2% of the highest value of the RF weights was taken and allocated to the other two RFs and the ranking was found again. In the new ranking, only two FMs (ranked 5th and 6th) changed their ranking (6th and 5th respectively). The relationship between the rankings found with these two different weights in Minitab was found to be very close to each other with a Spearman's rank correlation of 0.996, which means that this method is robust.

In order to make sure that the proposed method is much more effective and accurate, the FM rankings of the proposed method are compared with the classical FMEA method rankings. The RPN scores calculated for each DM according to the traditional FMEA method and the geometric means of these scores are given in Table 4.19.

Table 4.19. Classic RPN scores

| FMs | GM | PM | ME | PE | Geometric Means |
|---|---|---|---|---|---|
| 1 | 60 | 120 | 168 | 180 | 121 |
| 2 | 96 | 80 | 160 | 162 | 119 |
| 3 | 140 | 144 | 108 | 162 | 137 |
| 4 | 162 | 81 | 126 | 189 | 133 |
| 5 | 189 | 90 | 288 | 84 | 142 |
| 6 | 120 | 120 | 210 | 140 | 143 |
| 7 | 48 | 112 | 300 | 84 | 108 |
| 8 | 40 | 200 | 270 | 135 | 131 |
| 9 | 126 | 196 | 192 | 162 | 166 |
| 10 | 54 | 180 | 120 | 140 | 113 |
| 11 | 108 | 216 | 120 | 180 | 150 |
| 12 | 108 | 216 | 360 | 270 | 218 |
| 13 | 120 | 90 | 294 | 50 | 112 |
| 14 | 72 | 180 | 288 | 126 | 147 |
| 15 | 120 | 108 | 162 | 162 | 136 |

The higher the classic RPN score for an FM, the higher the priority for prevention. Accordingly, the ranking results of the average RPN scores calculated by the traditional method in descending order and the ranking results according to the proposed method are shown in Table 4.20.

Table 4.20. Traditional FMEA vs IF FMEA

| FMs | RPNs | FMEA | IF- FMEA |
|---|---|---|---|
| 1 | 121 | 11 | 4 |
| 2 | 119 | 12 | 2 |
| 3 | 137 | 7 | 6 |
| 4 | 133 | 9 | 8 |
| 5 | 142 | 6 | 13 |
| 6 | 143 | 5 | 12 |
| 7 | 108 | 15 | 11 |
| 8 | 131 | 10 | 14 |
| 9 | 166 | 2 | 5 |
| 10 | 113 | 13 | 9 |
| 11 | 150 | 3 | 3 |
| 12 | 218 | 1 | 1 |
| 13 | 112 | 14 | 15 |
| 14 | 147 | 4 | 7 |
| 15 | 136 | 8 | 10 |

In the rankings, it is observed that only 2 FMs (1st and 3rd FM) have the same ranking while the others are quite different. When the feedback from the company was received for the resulting rankings, it was concluded that the ranking of the proposed method was much more meaningful and accurate for all DMs.

The reason why the rankings obtained with the traditional method are not in the desired direction is that the crisp scores are calculated by multiplying them without weighting. In addition, in the traditional method, the opinions of DMs are treated as if everyone's opinion is equal without taking into account their experience, business knowledge, position and versatility. With the method proposed in this thesis, all these weights are taken into account and through the use of IFNs, each idea is assigned a hesitation and no one is given a precise score for any FM.

# 5. CONCLUSION AND DISCUSSION

In this thesis, it is aimed to integrate IFSs with the MARCOS method, which is used in a real FMEA application, and it was revealed that the results were more efficient, more realistic and more robust under intuitionistic fuzzy environment than crisp one.

The IF-MARCOS method, which is an MCDM method, used for FMEA to rank the FMs from most to least preventable by translating the linguistic assessments of the identified FMs given by DMs according to RFs into IFNs and running its algorithm. IF-MARCOS, a consensus solver, has been observed to work on FMEA in an intuitionistic fuzzy environment for the first time, and it has worked quite well. Another contribution of this proposed IF-MARCOS-FMEA method is that decision makers can easily express their opinions through linguistic variables and determine their weights among themselves by giving a level of trust to the opinions of other DMs in accordance with their hierarchical position.

The application was carried out in the production department of a defense industry company. After four DMs were selected and FMs were determined with the ideas of each, their FM evaluations were taken according to RFs. The received data were processed in the R studio program in accordance with the IF-MARCOS algorithm and FM importance orders were obtained. The codes written in R studio are given in Appendix.

When the importance ranking of the FMs is analyzed, it is seen that the most important issue to be prevented is "The obligation to set up the product in the integration and test environment before production is completed". The recommended corrective and preventive action for this FM, which is highly damaging and potentially damaging to the production system, is the preparation of project schedules by taking into account the delivery times of products/materials with global supply problems. In order to carry out this action, it should be ensured that the materials with supply problems among the materials used and their number in the warehouse should be determined, and then safety stocks should be created and monitored. Thus, products can be produced according to the project schedule, as they should be, rather than according to the time of material procurement. In fact, this solution is also capable of solving other FMs in the department:

FM11 (ranked 3rd importance), FM4 (ranked 8th importance), FM15 (ranked 10th importance) and FM8 (ranked 14th importance). This means that the company will be able to solve 5 FMs (one third of all FMs) by implementing this solution first.

Apart from this, it should be ensured that a test environment is allocated in the production areas and a formal procedure for these tests should be established. With this solution, the second FM problem will be solved. Finally, the two solutions proposed above will lead to the solution of the first three FMs and three more FMs in different order. The solution suggestions for the remaining 7 FMs are presented in the annex, so that it can get rid of its other problems by applying the solution suggestions given in the table.

For future research, researchers can determine DM weights and/ or RF weights by other methods. After the ranking of the proposed method, FMs can be divided into priority clusters using a clustering method. Then a firm cost analysis can then be performed to intervene in the clusters. Additionally, IF-FMEA can be recommended for application in areas other than the defense industry.

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# APPENDIX

**FCs, FEs and Corrective and preventive actions of each FMs**

| | FMs | FCs | FEs | Corrective and preventive actions |
|---|---|---|---|---|
| **1** | Inability to access up-to-date and accurate information needed in production planning and production processes. | Keeping up-to-date/correct information on different platforms, having different revisions of the same document, not coordinating with relevant departments and not following the revisions properly. | Cost and time loss due to incorrect or incomplete work. | Collecting the information, data and documentation of all projects on a single platform and tracking the revision of each document. |
| **2** | Defective electronic cards. They are not subjected to functional tests during the incoming quality control process and post-production tests are not carried out in sufficient detail. | Lack of suitable and sufficient test environment for electronic cards for incoming quality control and lack of test environment and procedures for these products in the production process. | Cost and time loss, especially in the production planning and production process, as defective products can only be detected during the integration and testing phase. | Establish a test environment and procedure for electronic boards to be operated in the incoming quality control and production process. |
| **3** | Errors in the production document and insufficient detail of the production steps in the document. | Failure to prepare design/production documents with sufficient attention to detail and insufficient time for this process. | Loss of time and labor due to faulty production or stoppage of production until the document is corrected. | Providing sufficient time for the preparation of design/production documents in accordance with project schedules and obtaining support and opinions from production engineers in the preparation of these documents. |

| | | | | |
|---|---|---|---|---|
| **4** | Existence of some products/materials with supply problems (not available on time). | Some products/materials have long lead times and cannot be procured on time. | Project schedule and other works are delayed accordingly or materials belonging to other projects are shifted to the seemingly prioritized project. | Determining project schedules by taking into account the delivery times of products/materials with global supply problems. Establishing safety stocks of some parts in the product tree, conducting needs analysis and standardization. |
| **5** | When material requirements planning is done by the production planning engineer, the SAP program does not suggest alternative materials and some materials are defined as duplicates in SAP software. | SAP software system not responding to employee needs. | Unnecessary purchase of some products and unnecessary storage space in the SAP software database due to the inability to identify alternative products even though they are available in warehouse stocks. | Overhauling the SAP software database and identifying alternative products one by one and recording them as "alternatives" in the system. |
| **6** | Some products that are no longer produced by the relevant manufacturer are still active in SAP software and can be added to the product trees. Additionally, these products are mentioned in the product trees of some other systems. | SAP software system not responding to employee needs. | Loss of time and cost during the production planning phase when notifying the relevant project management and system engineer that new products should be included in the product tree instead of products that can no longer be supplied. | Overhauling the SAP software database and identifying "end of life" products one by one and defining their substitutes to the system and revising the product trees accordingly. |
| **7** | SAP software does not support shelf information within the scope of warehouse stocking. | SAP software system not responding to employee needs. | The loss of time incurred during the search for a material that might be found in any project repository. | Making the SAP system support shelf information in warehouses or establishing another mechanism in this direction. |

| | | | | |
|---|---|---|---|---|
| **8** | The obligation to take the materials that are not available in their own storage from the storage of another project in order to complete the project. | Exceeding material procurement time due to short project schedule. | Increased workload and loss of time for the production planning engineer and warehouse personnel due to too many material shifts among projects. | Determining project schedules by taking into account the delivery times of products/materials with global supply problems. Establishing safety stocks of some parts in the product tree, conducting needs analysis and standardization. |
| **9** | Incorrect or missing information on some product identification forms (PIFs). | Problems in material definitions. | Disruption of the production process and loss of time due to " reject" decisions for some materials due to incomplete or incorrect information, especially in the incoming quality process. | All PIFs should be reviewed one by one to ensure that the correct and necessary information is included in the content of the PIF. |
| **10** | Detailed data of products/materials defined before the transition to SAP software are included in the PIFs in SVN software, while those defined after the transition to SAP software are included in the SAP database. | Dependence on two different platforms for material, stock and inventory management. | Loss of time and inability to carry out all transactions through SAP software. | Transferring all data in PIFs in SVN software to SAP software. |
| **11** | The obligation to start production before all products/materials required for production have been procured. | Short project schedules and consequent inability to procure all products/materials needed for production on time. | Increased risk of error and labor force used during the production phase due to partial and interrupted production. | Determining project schedules by taking into account the delivery times of products/materials with global supply problems. Establishing safety stocks of some parts in the product tree, conducting needs analysis and standardization. |

| | | | | |
|---|---|---|---|---|
| **12** | The obligation to set up the product in the integration and test environment before production is completed. | Short project schedule and customer pressure. | Increased risk of error and labor force used during the production phase due to partial and interrupted production. | Determining project schedules by taking into account the delivery times of products/materials with global supply problems. |
| **13** | Existence of duplicate products in the SAP system, multiple different part numbers defined for the product with the same model number. | Uncontrolled product/material definitions. | Unnecessary purchasing activities are carried out because some products are in stock with different part numbers although they are available in warehouse stocks, and when there is an urgent need for a product available in another project warehouse with a different part number, this need cannot be met. | Overhauling the SAP software database and eliminating duplicate identifications. Also, developing a mechanism for controlled product/material identifications. |
| **14** | The configuration serial number given to the products that are production items and the serial numbers that have to be entered into the SAP software do not match. | Due to the fact that the configuration lists were not prepared before the production process. Configuration lists are delayed by not being prepared until after the system design and BOM have been finalized. | Different serial numbers for the same product/system/subsystem in configuration lists and SAP software database. | Configuration lists are created immediately after the system is designed and the BOM is finalized, and updated as needed in the future. |
| **15** | Request by project management to place orders for some products before the BOMs are finalized. | Due to the short project schedule, purchase orders were placed before the design and BOM were finalized in order to meet production deadlines. | Increase in cost due to purchasing extra materials in case of design changes, irregularity in production planning and loss of labor force. | Determining project schedules by taking into account the delivery times of products/materials with global supply problems. |

**R Studio Codes**

```
#DM AĞIRLIKLARI

dm <- data.frame(DMs_Onur)

GM_member <- as.numeric(1)

GM_nonmember <- as.numeric(0)

GM_hesit <- as.numeric(0)

PM_member <- as.numeric(0.99)

PM_nonmember <- as.numeric(0.005)

PM_hesit <- as.numeric(0.005)

PE_member <- 1-((1-as.numeric(dm[4,2]))^as.numeric(dm[5,11])*(1-
as.numeric(dm[4,5]))^as.numeric(dm[3,11])*(1-
as.numeric(dm[4,8]))^as.numeric(dm[2,11]))

PE_nonmember <-
as.numeric(dm[4,3])^as.numeric(dm[5,11])*as.numeric(dm[4,6])^as.numeric(dm[3,11])
*as.numeric(dm[4,9])^as.numeric(dm[2,11])

PE_hesit <- 1-PE_member-PE_nonmember

ME_member <- 1-((1-as.numeric(dm[5,2]))^as.numeric(dm[4,11])*(1-
as.numeric(dm[5,5]))^as.numeric(dm[3,11])*(1-
as.numeric(dm[5,8]))^as.numeric(dm[2,11]))

ME_nonmember <-
as.numeric(dm[5,3])^as.numeric(dm[4,11])*as.numeric(dm[5,6])^as.numeric(dm[3,11])
*as.numeric(dm[5,9])^as.numeric(dm[2,11])

ME_hesit <- 1-ME_member - ME_nonmember

GM_1 <- GM_member + GM_hesit*(GM_member/(GM_member+GM_nonmember))

PM_1 <- PM_member + PM_hesit*(PM_member/(PM_member+PM_nonmember))

PE_1 <- PE_member + PE_hesit*(PE_member/(PE_member+PE_nonmember))

ME_1 <- ME_member + ME_hesit*(ME_member/(ME_member+ME_nonmember))

Sum <- GM_1 + PM_1 +PE_1 + ME_1
```

```
#AĞIRLIKLAR:
GM_w <- GM_1 / Sum
PM_w <- PM_1 / Sum
PE_w <- PE_1 / Sum
ME_w <- ME_1 / Sum
#Alternatifler
fm <- data.frame(FMs_Onur)
FM_members <- function(x,y){
  1-((((((1-as.numeric(fm[x,y]))^GM_w))*(((1-as.numeric(fm[x+1,y]))^PM_w))*(((1-
as.numeric(fm[x+2,y]))^PE_w))*(((1-as.numeric(fm[x+3,y]))^ME_w)))))
}
FM_nonmembers <- function(x,y){
as.numeric(fm[x,y])^GM_w*as.numeric(fm[x+1,y])^PM_w*as.numeric(fm[x+2,y])^PE
_w*as.numeric(fm[x+3,y])^ME_w}
FM1_O_member <- FM_members(3,3)
FM1_O_nonmember <- FM_nonmembers(3,4)
FM1_O_hesit <- 1-FM1_O_member-FM1_O_nonmember
FM1_S_member <- FM_members(3,6)
FM1_S_nonmember <- FM_nonmembers(3,7)
FM1_S_hesit <- 1-FM1_S_member-FM1_S_nonmember
FM1_D_member <- FM_members(3,9)
FM1_D_nonmember <- FM_nonmembers(3,10)
FM1_D_hesit <- 1-FM1_D_member-FM1_D_nonmember
FM2_O_member <- FM_members(7,3)
FM2_O_nonmember <- FM_nonmembers(7,4)
FM2_O_hesit <- 1-FM2_O_member-FM2_O_nonmember
FM2_S_member <- FM_members(7,6)
```

```
FM2_S_nonmember <- FM_nonmembers(7,7)

FM2_S_hesit <- 1-FM2_S_member-FM2_S_nonmember

FM2_D_member <- FM_members(7,9)

FM2_D_nonmember <- FM_nonmembers(7,10)

FM2_D_hesit <- 1-FM2_D_member-FM2_D_nonmember

FM3_O_member <- FM_members(11,3)

FM3_O_nonmember <- FM_nonmembers(11,4)

FM3_O_hesit <- 1-FM3_O_member-FM3_O_nonmember

FM3_S_member <- FM_members(11,6)

FM3_S_nonmember <- FM_nonmembers(11,7)

FM3_S_hesit <- 1-FM3_S_member-FM3_S_nonmember

FM3_D_member <- FM_members(11,9)

FM3_D_nonmember <- FM_nonmembers(11,10)

FM3_D_hesit <- 1-FM3_D_member-FM3_D_nonmember

FM4_O_member <- FM_members(15,3)

FM4_O_nonmember <- FM_nonmembers(15,4)

FM4_O_hesit <- 1-FM4_O_member-FM4_O_nonmember

FM4_S_member <- FM_members(15,6)

FM4_S_nonmember <- FM_nonmembers(15,7)

FM4_S_hesit <- 1-FM4_S_member-FM4_S_nonmember

FM4_D_member <- FM_members(15,9)

FM4_D_nonmember <- FM_nonmembers(15,10)

FM4_D_hesit <- 1-FM4_D_member-FM4_D_nonmember

FM5_O_member <- FM_members(19,3)

FM5_O_nonmember <- FM_nonmembers(19,4)

FM5_O_hesit <- 1-FM5_O_member-FM5_O_nonmember
```

```
FM5_S_member <- FM_members(19,6)

FM5_S_nonmember <- FM_nonmembers(19,7)

FM5_S_hesit <- 1-FM5_S_member-FM5_S_nonmember

FM5_D_member <- FM_members(19,9)

FM5_D_nonmember <- FM_nonmembers(19,10)

FM5_D_hesit <- 1-FM5_D_member-FM5_D_nonmember

FM6_O_member <- FM_members(23,3)

FM6_O_nonmember <- FM_nonmembers(23,4)

FM6_O_hesit <- 1-FM6_O_member-FM6_O_nonmember

FM6_S_member <- FM_members(23,6)

FM6_S_nonmember <- FM_nonmembers(23,7)

FM6_S_hesit <- 1-FM6_S_member-FM6_S_nonmember

FM6_D_member <- FM_members(23,9)

FM6_D_nonmember <- FM_nonmembers(23,10)

FM6_D_hesit <- 1-FM6_D_member-FM6_D_nonmember

FM7_O_member <- FM_members(27,3)

FM7_O_nonmember <- FM_nonmembers(27,4)

FM7_O_hesit <- 1-FM7_O_member-FM7_O_nonmember

FM7_S_member <- FM_members(27,6)

FM7_S_nonmember <- FM_nonmembers(27,7)

FM7_S_hesit <- 1-FM7_S_member-FM7_S_nonmember

FM7_D_member <- FM_members(27,9)

FM7_D_nonmember <- FM_nonmembers(27,10)

FM7_D_hesit <- 1-FM7_D_member-FM7_D_nonmember

FM8_O_member <- FM_members(31,3)

FM8_O_nonmember <- FM_nonmembers(31,4)
```

```
FM8_O_hesit <- 1-FM8_O_member-FM8_O_nonmember

FM8_S_member <- FM_members(31,6)

FM8_S_nonmember <- FM_nonmembers(31,7)

FM8_S_hesit <- 1-FM8_S_member-FM8_S_nonmember

FM8_D_member <- FM_members(31,9)

FM8_D_nonmember <- FM_nonmembers(31,10)

FM8_D_hesit <- 1-FM8_D_member-FM8_D_nonmember

FM9_O_member <- FM_members(35,3)

FM9_O_nonmember <- FM_nonmembers(35,4)

FM9_O_hesit <- 1-FM9_O_member-FM9_O_nonmember

FM9_S_member <- FM_members(35,6)

FM9_S_nonmember <- FM_nonmembers(35,7)

FM9_S_hesit <- 1-FM9_S_member-FM9_S_nonmember

FM9_D_member <- FM_members(35,9)

FM9_D_nonmember <- FM_nonmembers(35,10)

FM9_D_hesit <- 1-FM9_D_member-FM9_D_nonmember

FM10_O_member <- FM_members(39,3)

FM10_O_nonmember <- FM_nonmembers(39,4)

FM10_O_hesit <- 1-FM10_O_member-FM10_O_nonmember

FM10_S_member <- FM_members(39,6)

FM10_S_nonmember <- FM_nonmembers(39,7)

FM10_S_hesit <- 1-FM10_S_member-FM10_S_nonmember

FM10_D_member <- FM_members(39,9)

FM10_D_nonmember <- FM_nonmembers(39,10)

FM10_D_hesit <- 1-FM10_D_member-FM10_D_nonmember

FM11_O_member <- FM_members(43,3)
```

```
FM11_O_nonmember <- FM_nonmembers(43,4)

FM11_O_hesit <- 1-FM11_O_member-FM11_O_nonmember

FM11_S_member <- FM_members(43,6)

FM11_S_nonmember <- FM_nonmembers(43,7)

FM11_S_hesit <- 1-FM11_S_member-FM11_S_nonmember

FM11_D_member <- FM_members(43,9)

FM11_D_nonmember <- FM_nonmembers(43,10)

FM11_D_hesit <- 1-FM11_D_member-FM11_D_nonmember

FM12_O_member <- FM_members(47,3)

FM12_O_nonmember <- FM_nonmembers(47,4)

FM12_O_hesit <- 1-FM12_O_member-FM12_O_nonmember

FM12_S_member <- FM_members(47,6)

FM12_S_nonmember <- FM_nonmembers(47,7)

FM12_S_hesit <- 1-FM12_S_member-FM12_S_nonmember

FM12_D_member <- FM_members(47,9)

FM12_D_nonmember <- FM_nonmembers(47,10)

FM12_D_hesit <- 1-FM12_D_member-FM12_D_nonmember

FM13_O_member <- FM_members(51,3)

FM13_O_nonmember <- FM_nonmembers(51,4)

FM13_O_hesit <- 1-FM13_O_member-FM13_O_nonmember

FM13_S_member <- FM_members(51,6)

FM13_S_nonmember <- FM_nonmembers(51,7)

FM13_S_hesit <- 1-FM13_S_member-FM13_S_nonmember

FM13_D_member <- FM_members(51,9)

FM13_D_nonmember <- FM_nonmembers(51,10)

FM13_D_hesit <- 1-FM13_D_member-FM13_D_nonmember
```

```
FM14_O_member <- FM_members(55,3)

FM14_O_nonmember <- FM_nonmembers(55,4)

FM14_O_hesit <- 1-FM14_O_member-FM14_O_nonmember

FM14_S_member <- FM_members(55,6)

FM14_S_nonmember <- FM_nonmembers(55,7)

FM14_S_hesit <- 1-FM14_S_member-FM14_S_nonmember

FM14_D_member <- FM_members(55,9)

FM14_D_nonmember <- FM_nonmembers(55,10)

FM14_D_hesit <- 1-FM14_D_member-FM14_D_nonmember

FM15_O_member <- FM_members(59,3)

FM15_O_nonmember <- FM_nonmembers(59,4)

FM15_O_hesit <- 1-FM15_O_member-FM15_O_nonmember

FM15_S_member <- FM_members(59,6)

FM15_S_nonmember <- FM_nonmembers(59,7)

FM15_S_hesit <- 1-FM15_S_member-FM15_S_nonmember

FM15_D_member <- FM_members(59,9)

FM15_D_nonmember <- FM_nonmembers(59,10)

FM15_D_hesit <- 1-FM15_D_member-FM15_D_nonmember

#Uzaklıklar ve CC değerleri

df_O_member <-
c(FM1_O_member,FM2_O_member,FM3_O_member,FM4_O_member,FM5_O_mem
ber,FM6_O_member,FM7_O_member,FM8_O_member,FM9_O_member,FM10_O_m
ember,FM11_O_member,FM12_O_member,FM13_O_member,FM14_O_member,FM
15_O_member)

df_O_nonmber <-
c(FM1_O_nonmember,FM2_O_nonmember,FM3_O_nonmember,FM4_O_nonmember
,FM5_O_nonmember,FM6_O_nonmember,FM7_O_nonmember,FM8_O_nonmember,
```

```
FM9_O_nonmember,FM10_O_nonmember,FM11_O_nonmember,FM12_O_nonmemb
er,FM13_O_nonmember,FM14_O_nonmember,FM15_O_nonmember)

df_O_hesit <-
c(FM1_O_hesit,FM2_O_hesit,FM3_O_hesit,FM4_O_hesit,FM5_O_hesit,FM6_O_hesit
,FM7_O_hesit,FM8_O_hesit,FM9_O_hesit,FM10_O_hesit,FM11_O_hesit,FM12_O_he
sit,FM13_O_hesit,FM14_O_hesit,FM15_O_hesit)

df_S_member <-
c(FM1_S_member,FM2_S_member,FM3_S_member,FM4_S_member,FM5_S_membe
r,FM6_S_member,FM7_S_member,FM8_S_member,FM9_S_member,FM10_S_memb
er,FM11_S_member,FM12_S_member,FM13_S_member,FM14_S_member,FM15_S_
member)

df_S_nonmber <-
c(FM1_S_nonmember,FM2_S_nonmember,FM3_S_nonmember,FM4_S_nonmember,
FM5_S_nonmember,FM6_S_nonmember,FM7_S_nonmember,FM8_S_nonmember,F
M9_S_nonmember,FM10_S_nonmember,FM11_S_nonmember,FM12_S_nonmember,
FM13_S_nonmember,FM14_S_nonmember,FM15_S_nonmember)

df_S_hesit <-
c(FM1_S_hesit,FM2_S_hesit,FM3_S_hesit,FM4_S_hesit,FM5_S_hesit,FM6_S_hesit,F
M7_S_hesit,FM8_S_hesit,FM9_S_hesit,FM10_S_hesit,FM11_S_hesit,FM12_S_hesit,F
M13_S_hesit,FM14_S_hesit,FM15_S_hesit)

df_D_member <-
c(FM1_D_member,FM2_D_member,FM3_D_member,FM4_D_member,FM5_D_mem
ber,FM6_D_member,FM7_D_member,FM8_D_member,FM9_D_member,FM10_D_m
ember,FM11_D_member,FM12_D_member,FM13_D_member,FM14_D_member,FM
15_D_member)

df_D_nonmber <-
c(FM1_D_nonmember,FM2_D_nonmember,FM3_D_nonmember,FM4_D_nonmember
,FM5_D_nonmember,FM6_D_nonmember,FM7_D_nonmember,FM8_D_nonmember,
FM9_D_nonmember,FM10_D_nonmember,FM11_D_nonmember,FM12_D_nonmemb
er,FM13_D_nonmember,FM14_D_nonmember,FM15_D_nonmember)

df_D_hesit <-
c(FM1_D_hesit,FM2_D_hesit,FM3_D_hesit,FM4_D_hesit,FM5_D_hesit,FM6_D_hesit
```

```
,FM7_D_hesit,FM8_D_hesit,FM9_D_hesit,FM10_D_hesit,FM11_D_hesit,FM12_D_he
sit,FM13_D_hesit,FM14_D_hesit,FM15_D_hesit)
df1 <-
data.frame(df_O_member,df_O_nonmber,df_O_hesit,df_S_member,df_S_nonmber,df_
S_hesit,df_D_member,df_D_nonmber,df_D_hesit)
FM_O_PU <- c()
for (i in 1:15){
  FM_O_PU[i] <- sqrt((df1[i,1]-1)^2 + (df1[i,2]-0)^2 + (df1[i,3]-0)^2)}
FM_O_NU <- c()
for (i in 1:15){
  FM_O_NU[i] <- sqrt((df1[i,1]-0)^2 + (df1[i,2]-1)^2 + (df1[i,3]-0)^2)}
FM_O_CC <- c()
for (i in 1:15){
  FM_O_CC[i] <- FM_O_NU[i]/(FM_O_NU[i]+FM_O_PU[i])}
FM_S_PU <- c()
for (i in 1:15){
  FM_S_PU[i] <- sqrt((df1[i,4]-1)^2 + (df1[i,5]-0)^2 + (df1[i,6]-0)^2)}
FM_S_NU <- c()
for (i in 1:15){
  FM_S_NU[i] <- sqrt((df1[i,4]-0)^2 + (df1[i,5]-1)^2 + (df1[i,6]-0)^2)}
FM_S_CC <- c()
for (i in 1:15){
  FM_S_CC[i] <- FM_S_NU[i]/(FM_S_NU[i]+FM_S_PU[i])}
FM_D_PU <- c()
for (i in 1:15){
  FM_D_PU[i] <- sqrt((df1[i,7]-1)^2 + (df1[i,8]-0)^2 + (df1[i,9]-0)^2)}
FM_D_NU <- c()
```

```
for (i in 1:15){
  FM_D_NU[i] <- sqrt((df1[i,7]-0)^2 + (df1[i,8]-1)^2 + (df1[i,9]-0)^2)}
FM_D_CC <- c()
for (i in 1:15){
  FM_D_CC[i] <- FM_D_NU[i]/(FM_D_NU[i]+FM_D_PU[i])}
OSD <-
data.frame(FM_O_PU,FM_O_NU,FM_O_CC,FM_S_PU,FM_S_NU,FM_S_CC,FM_D
_PU,FM_D_NU,FM_D_CC)
#Kriter ağırlıkları
Kriter_veri <- data.frame(Kriter_Onur)
O_member <- 1-((1-as.numeric(cr[3,2]))^GM_w*(1-as.numeric(cr[4,2]))^PM_w*(1-
as.numeric(cr[5,2]))^PE_w*(1-as.numeric(cr[6,2]))^ME_w)
O_nonmember <-
as.numeric(cr[3,3])^GM_w*as.numeric(cr[4,3])^PM_w*as.numeric(cr[5,3])^PE_w*as.
numeric(cr[6,3])^ME_w
O_hesit <- 1-O_member-O_nonmember
S_member <- 1-((1-as.numeric(cr[3,5]))^GM_w*(1-as.numeric(cr[4,5]))^PM_w*(1-
as.numeric(cr[5,5]))^PE_w*(1-as.numeric(cr[6,5]))^ME_w)
S_nonmember <-
as.numeric(cr[3,6])^GM_w*as.numeric(cr[4,6])^PM_w*as.numeric(cr[5,6])^PE_w*as.
numeric(cr[6,6])^ME_w
S_hesit <- 1-S_member-S_nonmember
D_member <- 1-((1-as.numeric(cr[3,8]))^GM_w*(1-as.numeric(cr[4,8]))^PM_w*(1-
as.numeric(cr[5,8]))^PE_w*(1-as.numeric(cr[6,8]))^ME_w)
D_nonmember <-
as.numeric(cr[3,9])^GM_w*as.numeric(cr[4,9])^PM_w*as.numeric(cr[5,9])^PE_w*as.
numeric(cr[6,9])^ME_w
D_hesit <- 1-D_member-D_nonmember
O_PU <- sqrt((O_member-1)^2 + (O_nonmember-0)^2 + (O_hesit-0)^2)
```

```
O_NU <- sqrt((O_member-0)^2 + (O_nonmember-1)^2 + (O_hesit-0)^2)

O_CC <- O_NU/(O_NU+O_PU)

S_PU <- sqrt((S_member-1)^2 + (S_nonmember-0)^2 + (S_hesit-0)^2)

S_NU <- sqrt((S_member-0)^2 + (S_nonmember-1)^2 + (S_hesit-0)^2)

S_CC <- S_NU/(S_NU+S_PU)

D_PU <- sqrt((D_member-1)^2 + (D_nonmember-0)^2 + (D_hesit-0)^2)

D_NU <- sqrt((D_member-0)^2 + (D_nonmember-1)^2 + (D_hesit-0)^2)

D_CC <- D_NU/(D_NU+D_PU)

SUM_CC <- O_CC + S_CC + D_CC

O_w <- O_CC/SUM_CC

S_w <- S_CC/SUM_CC

D_w <- D_CC/SUM_CC

kriter_agırlıkları <- data.frame("Olasılık ağırlığı" = O_w,"Şiddet ağırlığı"= S_w,"Fark
edilebilirlik ağırlığı"= D_w)

#IF Decision Matrix

ID_O <- min(OSD[1:15,3])

ID_S <- min(OSD[1:15,6])

ID_D <- max(OSD[1:15,9])

AID_O <- max(OSD[1:15,3])

AID_S <- max(OSD[1:15,6])

AID_D <- min(OSD[1:15,9])

#Normalized IF Decision Matrix

ideal <- c("O"=ID_O/ID_O,"S"=ID_S/ID_S,"D"=ID_D/ID_D)

antiideal <- c("O"=ID_O/AID_O,"S"=ID_S/AID_S,"D"=AID_D/ID_D)

ID_AID <- t(data.frame(ideal,antiideal))

Norm_IF <- data.frame("O"=ID_O/OSD[1:15,3],"S"=ID_S/OSD[1:15,6],"D"=
OSD[1:15,9]/ID_D)
```

```
Norm_IF_matrix <- rbind(Norm_IF,ID_AID)

#weighted IF Decision Matrix

Weighted_IF <-
data.frame("O"=Norm_IF_matrix[1:17,1]*O_w,"S"=Norm_IF_matrix[1:17,2]*S_w,"D
"=Norm_IF_matrix[1:17,3]*D_w)

Weighted_IF_matrix <- transform(Weighted_IF,sum=rowSums(Weighted_IF))

#Utility degrees and Utility functions

Utility <- data.frame("Row sums"= Weighted_IF_matrix[1:15,4], "Negatif Utility
degree"=Weighted_IF_matrix[1:15,4]/Weighted_IF_matrix[17,4], "Pozitif Utility
degree"=Weighted_IF_matrix[1:15,4]/Weighted_IF_matrix[16,4], "Negatif Utility
Func"=(Weighted_IF_matrix[1:15,4]/Weighted_IF_matrix[16,4])/((Weighted_IF_matri
x[1:15,4]/Weighted_IF_matrix[16,4])+(Weighted_IF_matrix[1:15,4]/Weighted_IF_mat
rix[17,4])), "Pozitif Utility
Func"=(Weighted_IF_matrix[1:15,4]/Weighted_IF_matrix[17,4])/((Weighted_IF_matri
x[1:15,4]/Weighted_IF_matrix[16,4])+(Weighted_IF_matrix[1:15,4]/Weighted_IF_mat
rix[17,4])))

Rank <- data.frame("Utility Function"=(Utility[1:15,3]+Utility[1:15,2])/(1+((1-
Utility[1:15,5])/Utility[1:15,5])+((1-Utility[1:15,4])/Utility[1:15,4])))

Order <- cbind(Rank,"Rank" = rank(Rank[1:15,1]))
```