

ANKARA YILDIRIM BEYAZIT UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES



ANALYSIS OF TEMPERATURE EFFECT
ON ANISOTROPY OF AZ31 SHEET

M.Sc. Thesis by

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ANALYSIS OF TEMPERATURE EFFECT ON ANISOTROPY OF AZ31 SHEET

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M.Sc. THESIS EXAMINATION RESULT FORM

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Erkan TUR

ANALYSIS OF TEMPERATURE EFFECT ON ANISOTROPY OF AZ31 SHEET

ABSTRACT

AZ31 is a challenging alloy for aerospace, automotive, and biomedical industries. This study delves deep into the intricate relationship between build orientations, temperature, and the resulting mechanical behavior of AZ31 alloy. Utilizing tensile tests on specimens with distinct build orientations, the research evaluates the alloy's response to varying temperatures of 100, 200, and 300°C. The primary objective is to elucidate the nuanced effects of anisotropy and temperature gradients on the alloy's tensile strength, elongation, and overall mechanical performance.

Furthermore, disc compression tests, a novel approach in this research, involve stacking multiple discs and subjecting them to compressive forces. This method, inspired by the findings of, aims to provide a comprehensive understanding of anisotropy, extract the Lankford values, and determine precise yield strengths. The study also ventures into the realm of computational simulations using the renowned MATLAB software. Several criteria, including the Hill48, the Barlat89, the Yld 2000, the Yld2004-2p, and the SHYqp2023 yield criteria are employed to simulate and predict the material's behavior under diverse conditions, a methodology that aligns with recent computational studies on similar materials. This research not only contributes to the existing body of knowledge but also paves the way for the development of advanced magnesium-based materials with tailored properties for specific applications.

Results indicate that the mapped out biaxial anisotropy and the associated Lankford values and variations regarding temperatures and orientations. Older models, such as the Hill-48 and the Barlat-89 does not accurately predict anisotropic nature of AZ31. The Yld-2004 and the Bezier-SHYqp2023 models seem promising in predicting anisotropic features of AZ31 material, and materials of having the HCP crystal structures in general.

Keywords: AZ31, Sheet metal, Anisotropy, Yield strengths, Mechanical properties.

SICAKLIĞIN AZ31 SAC METALİNİN ANİZOTROPİSİNE ETKİSİNİN ANALİZİ

ÖZ

AZ31 havacılık, otomotiv ve biyomedikal endüstrileri için ciddi değişim yapacak bir alaşımdır. Bu çalışma, AZ31 alaşımının yapı yönelimleri, sıcaklığı ve sonuçta ortaya çıkan mekanik davranışı arasındaki karmaşık ilişkiyi derinlemesine incelemektedir. Araştırma, farklı yapı yönelimlerine sahip numuneler üzerinde yapılan çekme testlerini kullanarak, alaşımın 100, 200 ve 300°C gibi değişen sıcaklıklara verdiği tepkiyi değerlendirmektedir. Temel amaç, anizotropi ve sıcaklık gradyanlarının alaşımın gerilme mukavemeti, uzaması ve genel mekanik performansı üzerindeki nüanslı etkilerini aydınlatmaktır.

Ayrıca, bu araştırmada yeni bir yaklaşım olan disk sıkıştırma testleri, birden fazla diskin istiflenmesini ve sıkıştırma kuvvetlerine maruz bırakılmasını içerir. Bulgularından esinlenen bu yöntem, anizotropinin kapsamlı bir şekilde anlaşılmasını, Lankford değerlerinin çıkarılmasını ve kesin akma dayanımlarının belirlenmesini amaçlamaktadır. Çalışma aynı zamanda MATLAB yazılımını kullanarak hesaplamalı simülasyonlar alanına da girmektedir. Hill48, Barlat89, Yld 2000, Yld2004-2p, and SHYqp2023 akma kriterleri, benzer malzemeler üzerinde yapılan son hesaplama çalışmalarıyla uyumlu bir metodoloji olan malzemenin çeşitli koşullar altındaki davranışını simüle etmek ve tahmin etmek için kullanılmaktadır. Bu araştırma sadece mevcut bilgi birikimine katkıda bulunmakla kalmıyor, aynı zamanda belirli uygulamalar için özel olarak tasarlanmış özelliklere sahip gelişmiş magnezyum bazlı malzemelerin geliştirilmesinin de önünü açıyor.

Sonuçlar, haritalanan çift eksenli anizotropinin ve ilişkili Lankford değerlerinin ve sıcaklıklara ve yönelimlere ilişkin varyasyonların olduğunu göstermektedir. Hill-48 ve Barlat-89 gibi daha eski modeller, AZ31'in anizotropik doğasını doğru bir şekilde tahmin edememektedir. Yld-2004 ve Bezier-SHYqp2023 modelleri, AZ31 malzemesinin anizotropik özelliklerini ve genel olarak HCP kristal yapısına sahip malzemeleri tahmin etmede umut verici görünmektedir.

Anahtar Kelimeler: AZ31, Sac metal, Anizotropi, Akma dayanımları, Mekanik özellikler.

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NOMENCLATURE

m	First order gradient of the yield function
ϑ	Poisson's ratio
r_θ	Lankford parameter for angle θ
θ	Loading direction w.r.t. rolling direction
λ	Plastic multiplier factor
E	Young's modulus
b	Parameter for Voce isotropic hardening law
c	Weight coefficient in combined Swift-Voce hardening
ε_{11}^p	Longitudinal plastic strain
$\bar{\varepsilon}^p$	Equivalent plastic strain
ε^p	Plastic strain tensor
σ	Cauchy stress tensor
σ_θ	Initial yield stress at θ degree orientation w.r.t. rolling direction
t	Time, second
f	Yield function (associated flow rule)
f_y	Yield function (non-associated flow rule)
f_p	Plastic potential (non-associated flow rule)
F	Yield criterion
F_{11}	Uniaxial tensile/compressive load
α_{n-m}	Parameters for various yield criterion

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CHAPTER 1

INTRODUCTION

1.1. Background and Context of the Research

The modern industrial landscape has witnessed a relentless pursuit of materials that can meet the ever-increasing demands of lightweight, high-strength, and mechanically resilient components. These components are the backbone of industries ranging from aerospace and automotive to electronics and beyond. This pursuit, driven by the ceaseless quest for innovation and efficiency, has led to a transformative evolution in the materials employed in these sectors. One class of materials that has recently emerged as a beacon of hope in meeting these multifaceted demands is magnesium alloys. Magnesium and its alloys have shown significant potential as structural materials due to their unique properties and abundance in the earth's crust. They have found applications in various industries, especially in aerospace, automotive, and electronics [1]. The lightweight and excellent properties of magnesium make it an important engineering material, with the potential to replace steel and aluminum in many applications [2]. However, challenges such as corrosion have been a concern. Research has been conducted to understand the corrosion behavior of magnesium alloys, especially those used in critical components of military aircraft [1]. Additionally, the influence of cryogenic temperatures on the microstructure and mechanical properties of magnesium alloys has been studied, especially for their potential applications in aerospace and satellite parts [3]. Finite element simulations have also been employed to study the corrosion mechanics of magnesium alloys, aiming to optimize their design for high-temperature applications [4].

Magnesium alloys have garnered substantial attention in recent years due to their remarkable attributes, prominently among which is their unparalleled strength-to-weight ratio [5]. This exceptional property positions them as prime contenders for applications that require materials with a blend of reduced weight and exceptional mechanical prowess [6]. One particular magnesium alloy, AZ31, with its unique composition of 96% magnesium, 3% aluminum, and 1% zinc, has risen to prominence

as a material that embodies the essence of the quest for advanced materials in modern engineering [7]. The forging of magnesium alloys, including AZ31, has shown significant improvement in mechanical properties compared to their blank counterparts [7]. Furthermore, in situ synthesis technologies have been explored to further enhance the properties of such alloys [8].

The designation "AZ31" offers a succinct insight into the alloy's composition. The 'A' signifies the presence of aluminum, 'Z' represents zinc, and the '31' alludes to the nominal composition of the alloy [9]. AZ31 exhibits an intriguing amalgamation of properties that makes it an attractive choice for a wide spectrum of engineering applications. Its most conspicuous characteristic is its low density, which endows it with a featherweight quality, making it a frontrunner in the endeavor to reduce the weight of structures and components [10]. Additionally, AZ31 exhibits commendable corrosion resistance, a quality that adds to its allure for applications in industries that grapple with challenging environmental conditions [11]. Furthermore, this alloy boasts excellent castability, facilitating intricate and precise manufacturing processes [12]. However, beneath the surface of these remarkable properties, AZ31 harbors a mechanical quirk that has been the subject of intense scrutiny in the fields of materials science and engineering – anisotropy [13]. Anisotropy in materials represents a fascinating and complex phenomenon wherein the mechanical properties of a material exhibit pronounced variations depending on the direction in which they are tested [14]. This intriguing characteristic has garnered substantial attention from researchers and scientists, as it not only challenges our understanding of material behavior but also has profound implications for the practical applications of these materials in various industries [15].

In the case of AZ31, the manifestation of anisotropy is intimately linked to its hexagonal close-packed (HCP) crystal structure, a distinctive feature shared among magnesium alloys [16]. This HCP crystal structure imparts a unique and intricate lattice arrangement to AZ31, which, in turn, governs the alloy's mechanical response to external forces [17]. The interplay between this crystallographic arrangement and the mechanical behavior of AZ31 is a captivating subject that has captivated the interest of materials scientists for decades [18]. The HCP crystal lattice that underpins

AZ31's material structure serves as the foundational framework for its anisotropic properties. In this crystallographic arrangement, the atoms are intricately arranged in layers, forming a hexagonal unit cell. While this configuration imparts certain advantageous properties to magnesium and its alloys, it also introduces a significant degree of anisotropy. The HCP structure's anisotropic behavior stems from the variation in atomic packing and bonding strengths along distinct crystallographic planes. It manifests as dissimilar mechanical responses to external forces applied along different directions within the crystal lattice. This intrinsic anisotropy is a central feature that governs AZ31's material behavior and shapes its performance in engineering applications.

The significance of the HCP structure in AZ31's anisotropic behavior cannot be overstated. Within this crystalline framework, atoms are arranged in a manner that leads to distinct mechanical characteristics along different crystallographic axes [19]. This inherent structural anisotropy results in the alloy displaying varying levels of stiffness, strength, and ductility when tested in different directions [20]. Such variations in material behavior can be likened to a hidden code that must be deciphered to fully harness the potential of AZ31 in practical engineering applications [21]. The unique crystallographic arrangement in AZ31 not only contributes to its anisotropic deformation characteristics but also influences other critical mechanical properties such as yield strength, fracture toughness, and fatigue resistance [22]. Understanding the intricate interplay between crystallography and mechanical properties is pivotal in unlocking the alloy's full potential and mitigating the challenges posed by its anisotropic nature [23]. In light of the importance of AZ31 and the role it plays in advancing lightweight materials for diverse applications, delving deeper into the complexities of its anisotropy becomes essential [24]. Researchers and engineers must unravel the intricate mechanisms that underlie this anisotropic behavior to predict and control how the material will respond under different loading conditions and orientations [25]. Such endeavors not only contribute to the fundamental understanding of AZ31 but also hold the key to optimizing its utilization in industries such as aerospace, automotive, and biomedical engineering.

The study of anisotropy in AZ31 represents a compelling and multifaceted research area within materials science and engineering [27]. It hinges on unraveling the nuanced relationship between the HCP crystal structure and the alloy's mechanical properties [28]. The insights gained from such investigations have the potential to revolutionize the design and manufacturing of advanced components and structures, enabling the development of lighter, more efficient, and technologically advanced products across various industries [29]. The anisotropic nature of AZ31 is more than an academic curiosity; it has profound implications for its real-world performance. This anisotropy becomes particularly conspicuous when the material undergoes forming processes, such as V-bending [30]. V-bending is a critical manufacturing operation used in industries like aerospace and automotive for shaping magnesium alloy components into desired forms [31]. It involves bending the material along a V-shaped die, a process that subjects the alloy to complex stress states. Consequently, comprehending how AZ31 responds to the rigors of V-bending at various orientations and temperatures becomes paramount for optimizing manufacturing processes and ensuring the integrity of the final products [32].

Furthermore, the study of biaxial anisotropy in AZ31 emerges as an essential facet of understanding its mechanical behavior [33]. Biaxial anisotropy delves into how the material responds when subjected to simultaneous loading along different axes. In practical terms, this mirrors the conditions that magnesium alloy components may experience during their operational lifespan [34]. Whether it's the structural elements of an aircraft or components within electronic devices, the ability to predict and account for biaxial anisotropy is indispensable in ensuring the safety and reliability of these products [35]. Figure 1.1 illustrates the diverse use of magnesium alloys in aerospace, including applications such as cockpit instrument panels, service door inner panels, rudder pedals, aircraft window frames and locks, and aircraft levers. These alloys are chosen for their lightweight properties, high strength, corrosion resistance, and reliability, all crucial factors for enhancing aircraft performance and safety. As magnesium alloys continue to advance, they are poised to play an even more significant role in shaping the future of aerospace engineering, enabling the development of lighter, more fuel-efficient, and environmentally sustainable aircraft designs. Biaxial anisotropy arises when the material's response to external forces is

influenced by concurrent loading in two distinct directions. This dynamic interplay of forces introduces intricate variations in mechanical behavior, thermal response, and electrical properties, demanding a holistic understanding to optimize material performance. Mechanical biaxial anisotropy, for instance, entails the consideration of how AZ31 responds when subjected to both tensile and compressive loads simultaneously along different crystallographic orientations. The alloy's behavior becomes a multidimensional puzzle, with each loading direction contributing to the overall mechanical response. This complexity holds relevance in aerospace components, where materials must withstand multidirectional forces during flight and structural applications demanding resistance to various forms of mechanical loading. Thermal biaxial anisotropy introduces challenges in managing temperature gradients along two distinct axes. Engineers tasked with designing systems where heat dissipation and dimensional stability are critical must navigate the intricate interplay of thermal conductivity and thermal expansion along these axes. Achieving uniform heat distribution and precise control of dimensional changes becomes a multifaceted endeavor. Electrical biaxial anisotropy adds another layer of complexity in applications where electrical current flows along two orthogonal directions. Understanding how AZ31's electrical conductivity varies in response to concurrent electrical forces becomes indispensable in optimizing electrical systems for efficiency and safety.



Figure 1.1 The use of magnesium-based materials in aerospace components [26]

To embark on the journey of comprehending the intricacies of AZ31, it becomes necessary to scrutinize the alloy's response under the influence of varying anisotropic orientations and temperature conditions [36]. This research seeks to uncover the nuances of AZ31's mechanical behavior in the crucible of V-bending, offering insights into its springback angles, the Lankford values, yield strengths, and biaxial anisotropy under different loading conditions [37]. In doing so, it aspires to shed light on how this versatile alloy can be harnessed optimally, balancing its strengths and weaknesses in engineering applications [38]. Figure 1.2 provides a concise overview of the multifaceted use of magnesium alloys in automotive components, including the transfer case, engine cradle, cam covers, transmission case, header bow, and instrumental panel. Magnesium alloys are chosen for their lightweight properties, corrosion resistance, and strength-to-weight ratio, all of which contribute to improved fuel efficiency and durability. These alloys play a pivotal role in enhancing vehicle performance while ensuring structural integrity. In an ever-evolving automotive industry, magnesium alloys are set to continue shaping more efficient and sustainable automotive designs.

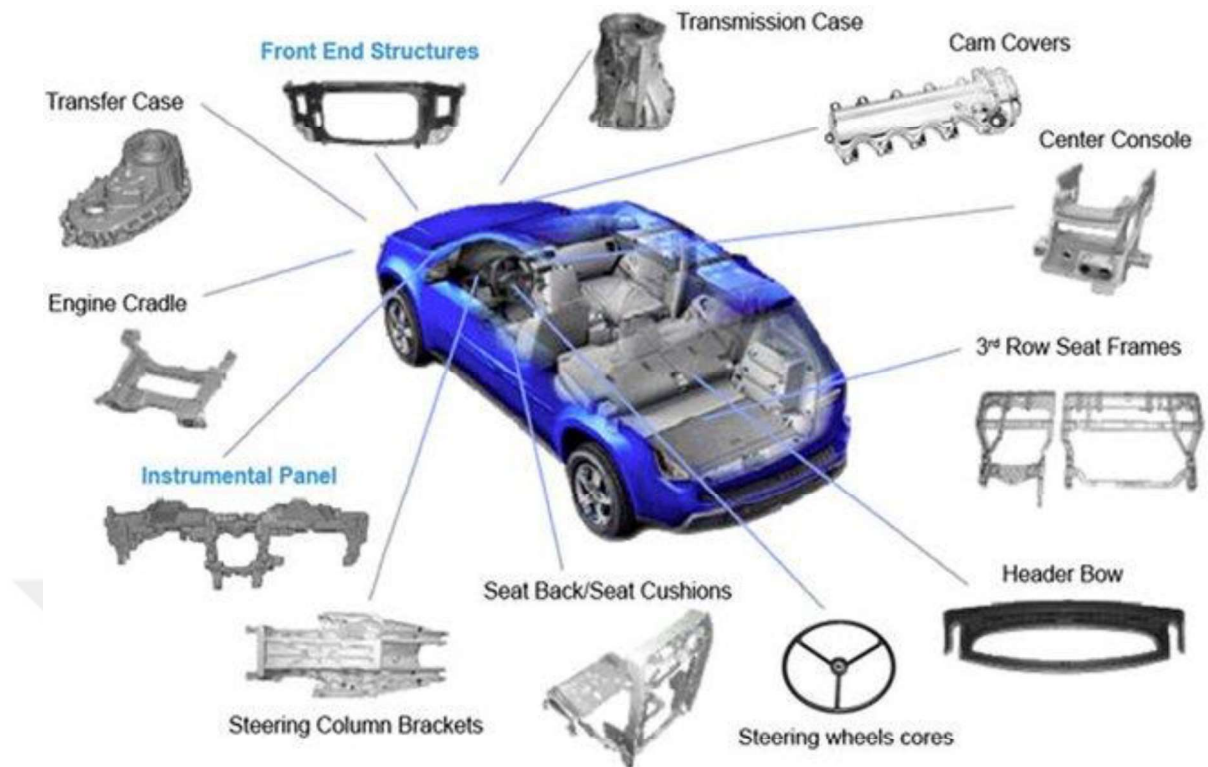


Figure 1.2 The use of magnesium-based alloys in automotive components [26]

As we delve deeper into the research, it becomes evident that AZ31, despite its allure, conceals complexities that necessitate rigorous exploration [39]. The chapters that follow will embark on this journey of discovery, systematically unraveling the interplay of anisotropy and temperature on AZ31's behavior [40]. Through experimentation and analysis, this research endeavors to contribute not only to the understanding of AZ31 but also to the broader field of materials science and engineering, where innovations in materials can lead to transformative advancements in technology and industry [41]. Figure 1.3 provides a comprehensive overview of the diverse applications of magnesium alloys in the biomedical industry, with a particular focus on innovative implantable medical devices. Notably, it highlights the use of biodegradable magnesium alloys in additively manufactured porous implants, where these alloys, recognized for their biocompatibility and ability to gradually dissolve within the body, play a pivotal role in supporting tissue regeneration and obviating the need for implant removal. Additionally, magnesium alloys find application in hand implants, offering a lightweight yet structurally robust solution in the realm of prosthetics and orthopedics, enhancing patient mobility and comfort. Furthermore, the depiction of resorbable magnesium scaffolds underscores their significance in

cardiovascular medicine, where they temporarily reinforce arteries and subsequently dissolve, contributing to improved patient outcomes. These varied applications exemplify the versatility and promising future of magnesium alloys in advancing medical technology and patient care.

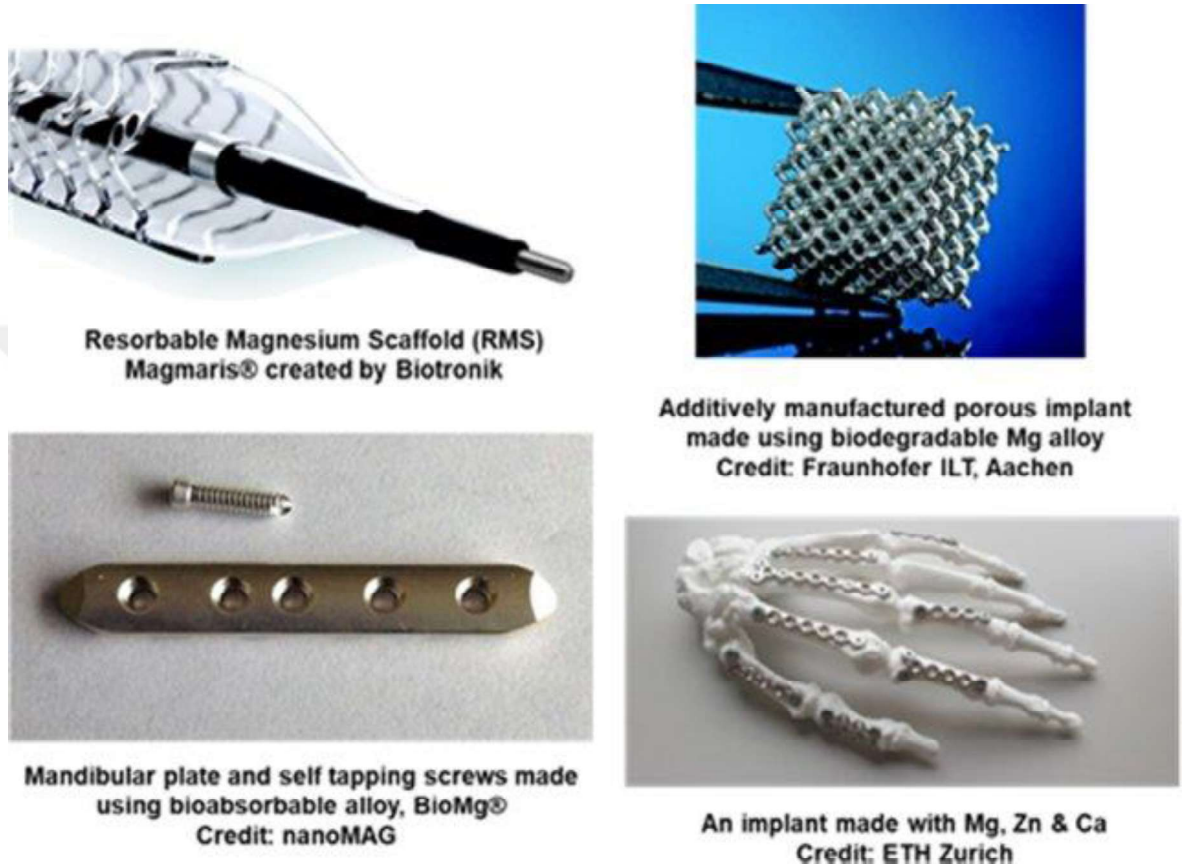


Figure 1.3 The use of magnesium-based alloys in biomedical products [26]

1.2. Hypotheses

1.2.1. Hypothesis 1: Influence of Anisotropy on V-Bending Behavior

The first hypothesis posits that the intrinsic anisotropic nature of AZ31 will exert a significant influence on its mechanical behavior during the V-bending process, resulting in observable variations in springback angles that are dependent on both orientation and temperature.

Justification of Hypothesis 1: AZ31's anisotropy, stemming from its hexagonal close-packed (HCP) crystal structure, is a well-documented phenomenon in the realm of magnesium alloys. This crystallographic arrangement inherently causes mechanical

properties, such as yield strength and ductility, to exhibit directional dependence. When subjected to a V-bending process, AZ31 specimens will experience varying stress distributions along different orientations. Consequently, it is reasonable to anticipate that the material will respond differently to these orientations.

The influence of temperature further complicates the picture. Elevated temperatures can induce changes in the alloy's microstructure, potentially altering the mobility of dislocations and the diffusion of atoms. Such temperature-dependent microstructural variations may interact with the inherent anisotropy, introducing nuanced effects on the springback behavior.

Therefore, the hypothesis posits that the combination of anisotropy and temperature will manifest as varying springback angles, providing valuable insights into the alloy's response during V-bending. If substantiated, this hypothesis will underscore the significance of orientation and temperature control in industrial processes involving AZ31, ultimately contributing to the optimization of manufacturing operations.

1.2.2. Hypothesis 2: Distinct Patterns in Biaxial Anisotropy and Lankford Values

The second hypothesis suggests that when AZ31 undergoes disc compression tests, biaxial anisotropy and the Lankford values will display discernible patterns contingent on the specific loading conditions imposed on the material. These loading conditions will encompass different orientations and combinations of stress components during the compression tests.

Justification of Hypothesis 2: Biaxial anisotropy, in the context of AZ31, pertains to how the material responds when subjected to simultaneous loading along different axes. The hexagonal crystal structure of AZ31 is expected to interact uniquely with different stress components, and this interaction may lead to distinct deformation behaviors. The Lankford value, or the r -value, characterizes the ability of a material to elongate in the direction of loading. In the case of AZ31, this value is expected to demonstrate orientation-dependent variations due to its anisotropic nature. When subjected to different loading conditions that incorporate variations in orientation and

stress magnitudes, it is plausible that the Lankford values will follow specific trends. Furthermore, the interaction between the applied stress components and the crystallographic orientation may introduce non-uniform plastic deformation, resulting in intriguing patterns of biaxial anisotropy. If this hypothesis holds true, it will provide valuable insights into the nuanced behavior of AZ31 under complex loading scenarios, which are more representative of real-world applications. These insights can be invaluable for engineers and designers aiming to predict and accommodate the mechanical response of AZ31 in diverse operational contexts.

1.2.3. Hypothesis 3: Orientation-Dependent Variations in Yield Strengths

The third hypothesis posits that AZ31 will exhibit orientation-dependent variations in yield strengths when subjected to different loading conditions. These loading conditions will include varying orientations and combinations of stress components during the compression tests.

Justification of Hypothesis 3: Yield strength is a fundamental mechanical property that characterizes a material's ability to withstand deformation without permanent deformation. In AZ31, this property is inherently linked to its anisotropic behavior. Different orientations expose different crystallographic planes and directions to the applied stress, resulting in variations in the ease with which dislocations can move and plastic deformation can occur. Furthermore, the influence of temperature adds an additional layer of complexity. Elevated temperatures can activate different deformation mechanisms, potentially affecting the yield strength differently along different orientations. Consequently, it is reasonable to hypothesize that the yield strengths of AZ31 will exhibit orientation-dependent variations when subjected to varying loading conditions. If substantiated, this hypothesis will underscore the importance of considering orientation and temperature effects when designing components or structures that employ AZ31, contributing to safer and more reliable engineering solutions.

In summary, these hypotheses form the foundational framework for the research, guiding the investigation into the multifaceted mechanical behavior of AZ31 under the influence of anisotropy, temperature, and complex loading conditions. Their validation

or refutation will provide critical insights that contribute not only to the understanding of AZ31 but also to the broader field of materials science and engineering, where the quest for advanced materials remains unceasing. Figure 1.x illustrates the foundational framework for our research on the mechanical behavior of AZ31 magnesium alloy under the influence of anisotropy, temperature variations, and complex loading conditions. This research is critical in advancing our understanding of this versatile material and has implications for various engineering applications.

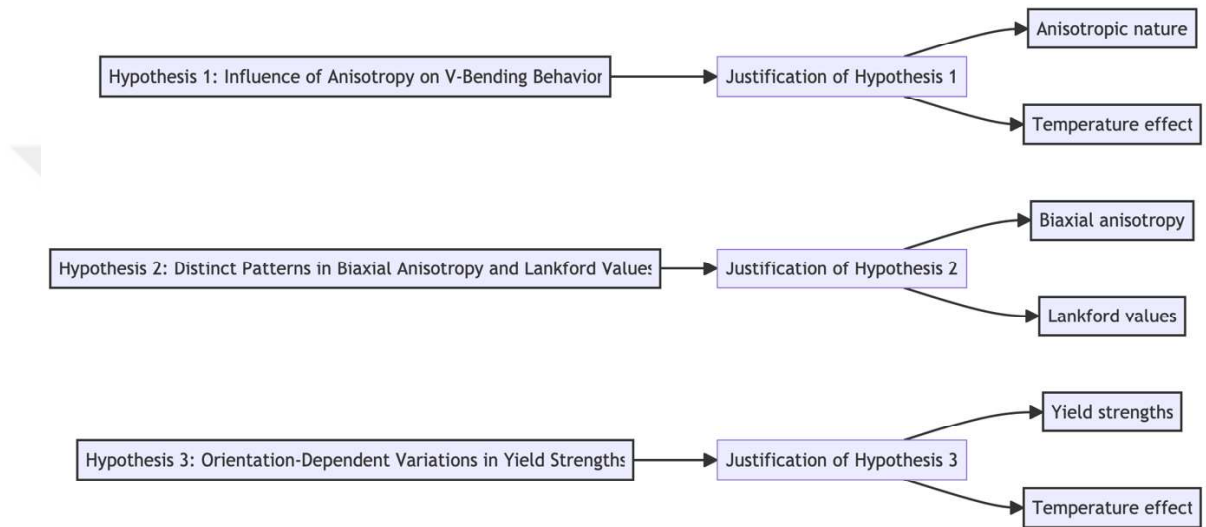


Figure 1.4 Hypotheses summarized

1.3. Significance of the Study

The significance of this research extends far beyond the confines of the laboratory. It carries the potential to usher in a new era of understanding and utilization of AZ31 magnesium alloy, offering insights that resonate across multiple domains. This significance can be dissected into several key areas:

- **Manufacturing Optimization:** The manufacturing sector stands as one of the primary beneficiaries of this research. AZ31 magnesium alloy, with its lightweight yet robust attributes, has already found a substantial foothold in industries like aerospace and automotive [43]. However, its anisotropic nature has been a persistent challenge in achieving consistent and high-quality components. By delving into the intricate interplay of anisotropy and temperature during V-bending, this research equips manufacturers with the

knowledge to make informed decisions. The insights gained from this study will serve as a guiding light, enabling manufacturers to chart a course that optimizes the material's behavior. Armed with a deeper understanding of how AZ31 responds to varying orientations and temperatures, manufacturers can fine-tune material orientation and processing conditions. This, in turn, paves the way for more efficient and cost-effective manufacturing processes. The implications are profound, potentially revolutionizing the production of components that rely on AZ31, leading to higher yields, reduced waste, and enhanced competitiveness in global markets.

- **Enhanced Product Design:** Engineers and product designers constitute another cohort poised to benefit significantly from this research. The mechanical properties of AZ31, influenced by anisotropy and temperature, inherently shape its suitability for specific applications. A clear comprehension of how these properties evolve under different conditions empowers engineers to harness AZ31's strengths and mitigate its weaknesses. By incorporating the findings of this study into their design processes, engineers can engineer products that are not only structurally sound but also lighter, more fuel-efficient, and environmentally friendly. This opens avenues for innovative solutions in diverse sectors, including transportation, electronics, and medical devices. From aircraft components to consumer electronics, the ability to fine-tune designs based on nuanced material behavior can result in products that stand out in terms of performance and sustainability.
- **Material Selection:** The research's impact reverberates in the domain of material selection. As industries evolve and seek to meet ever-stringent performance requirements, the choice of materials becomes paramount. The findings of this study will contribute to a more informed selection of AZ31 for specific applications. Manufacturers and engineers, armed with knowledge about how AZ31 responds to varying conditions, can confidently match the material to the intended use. This ensures that AZ31 is deployed where its properties align most favorably with the requirements of a given application. As a consequence, it becomes possible to craft products that are not only fit for

purpose but also possess a competitive edge in terms of cost, performance, and durability.

- **Advancements in Materials Science:** Beyond the immediate practical implications, this research serves as a cornerstone for the broader field of materials science and engineering. It contributes to the expanding body of knowledge on magnesium alloys and anisotropic materials. As materials scientists delve deeper into the behavior of AZ31 under varying conditions, it opens new horizons for innovation. The study of anisotropy and biaxial behavior in AZ31 serves as a microcosm for the broader exploration of advanced materials. The insights garnered from this research can serve as a foundation for the development of new alloys, novel processing techniques, and advanced computational models. By unraveling the intricacies of AZ31, this research fuels the ongoing quest to create materials that push the boundaries of what is possible in engineering and technology.

In conclusion, this research transcends the laboratory bench, carrying the promise of reshaping industries, enhancing products, guiding material selection, and contributing to the inexorable advancement of materials science. The potential to optimize manufacturing, design innovative products, select materials judiciously, and foster broader materials innovation underscores the profound significance of this study in the ever-evolving landscape of science and engineering.

1.4. Scope and Limitations

The scope of this research encompasses the experimental investigation of AZ31 magnesium alloy through V-bending tests and disc compression tests. It focuses on variations in orientation and temperature conditions to evaluate anisotropy, springback angles, the Lankford values, and yield strengths. The V-bending tests of this study will specifically involve AZ31 specimens subjected to orientations of 0 degrees (rolling direction), 45 degrees, 75 degrees, and 90 degrees, at temperatures of 250°C and 300°C. Scope and limitations are summarized in Figure 1.5.

However, it is important to acknowledge the following limitations:

1. **Material Homogeneity:** The study assumes uniform material properties across the AZ31 specimens, although some inherent material variations may exist.
2. **Simplified Loading Conditions:** The experiments involve controlled loading conditions, which may differ from the complex stresses experienced in actual applications.
3. **Laboratory-Scale Study:** This research operates at a laboratory scale and may not capture all real-world factors influencing the material's behavior.
4. **Limited Alloy Composition:** The study is specific to AZ31 alloy and may not necessarily apply to other magnesium alloys.

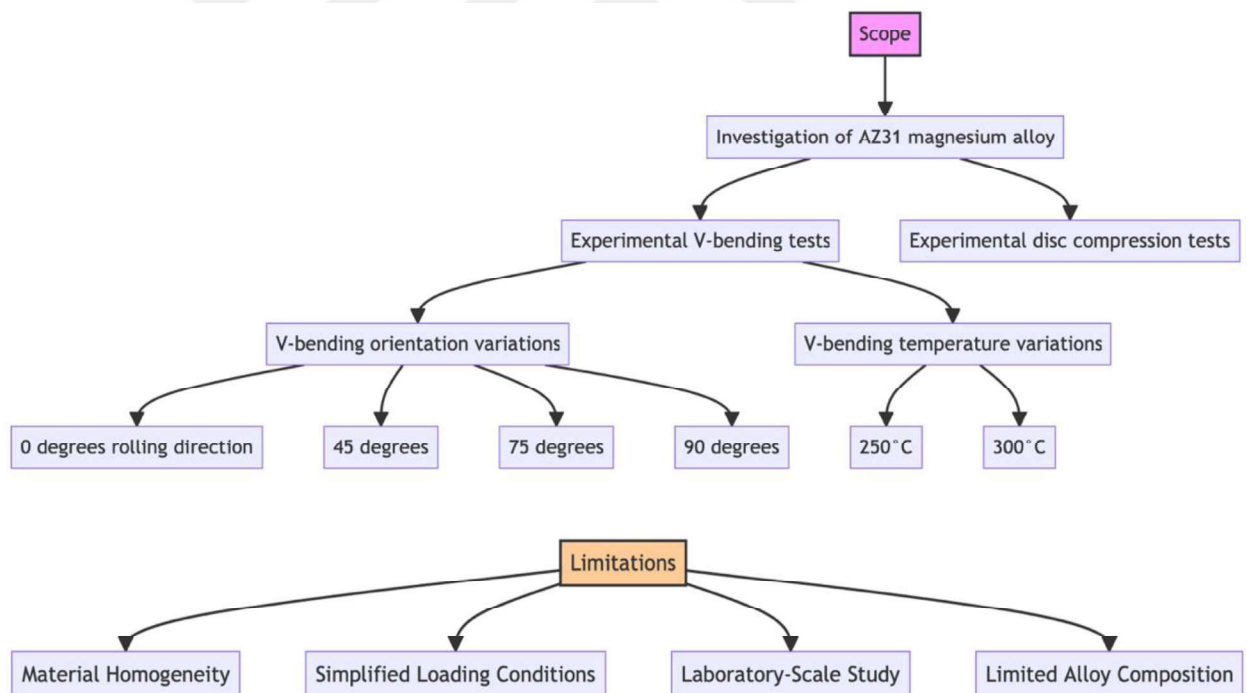


Figure 1.5 Scope and limitations of the thesis

1.5. Organization of the Thesis

The remainder of this thesis is organized as follows:

- Chapter 2: Background and Literature Review provides an in-depth exploration of magnesium alloys, anisotropy, V-bending tests, and biaxial anisotropy, superplasticity and superplastic forming, incorporating insights from previous research and theoretical concepts.
- Chapter 3: Testing and Analysis of AZ31 Alloy outlines the methods, materials, and procedures employed in conducting the V-bending tests and disc compression tests. presents the detailed process of data collection, including measurements of springback angles, biaxial anisotropy, the Lankford values, and yield strengths. It also describes the statistical and computational techniques used for data analysis.
- Chapter 4: Constitutive Modelling and Finite Element Analysis (FEA) presents the simulation results regarding 7 criteria including yield stresses and other results.
- Chapter 5: Results and Discussion presents the experimental results, illustrating the effects of anisotropy and temperature on the mechanical behavior of AZ31. It includes comprehensive data analysis and discussions.
- Chapter 6: Conclusion summarizes the key findings, reiterates the research objectives, and offers insights into the broader implications of the study.
- Chapter 7: References lists all the sources and references cited throughout the thesis.
- Appendices contain supplementary materials, MATLAB and Python codes used in the analysis.

In summary, this introductory chapter has laid the foundation for the comprehensive exploration of AZ31 magnesium alloy's behavior under varying anisotropic and temperature conditions. The subsequent chapters will delve into the experimental details, results, discussions, and conclusions, contributing to the broader understanding

of this versatile material in the realm of materials science and engineering. Workflow of the thesis is represented in Figure 1.6.

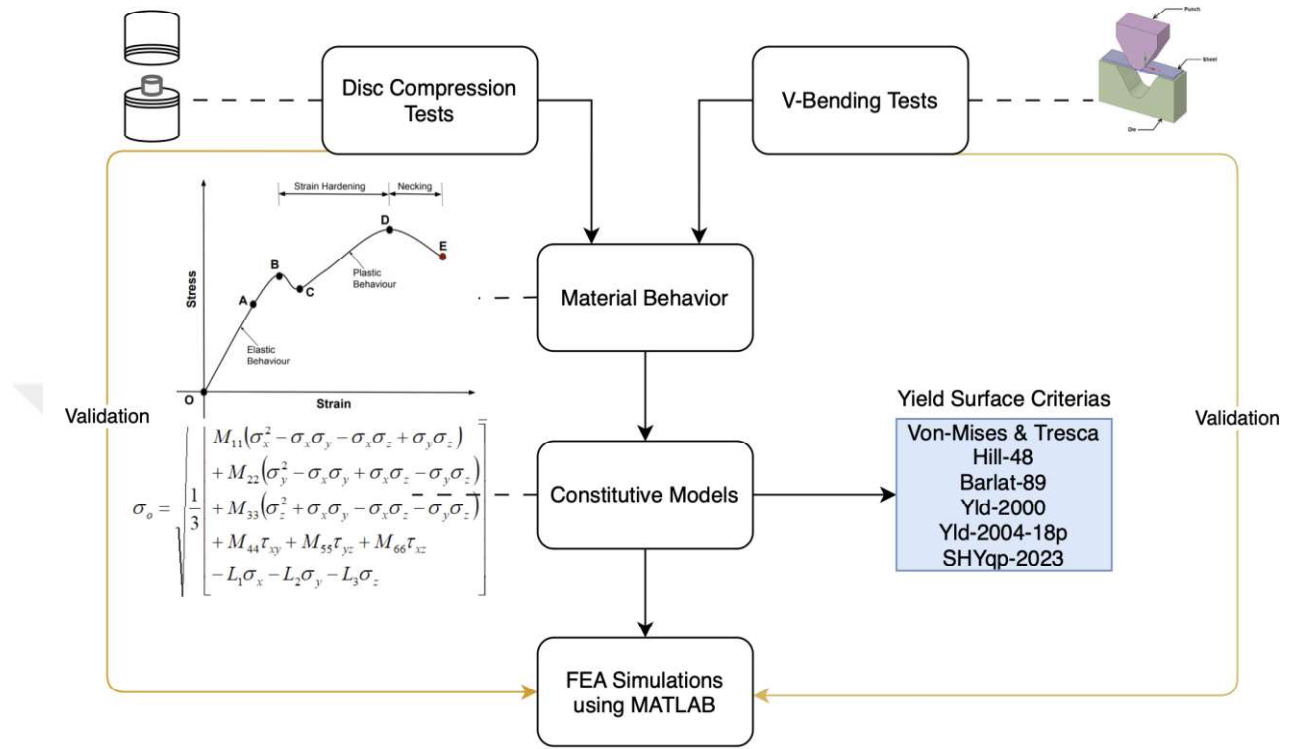


Figure 1.6 Workflow of thesis

CHAPTER 2

BACKGROUND AND LITERATURE REVIEW

2.1. Overview of Magnesium Alloys

Magnesium alloys have garnered profound interest and sparked innovation within the domains of science and engineering [43]. Their appeal lies in a singular and noteworthy attribute—an outstanding strength-to-weight ratio, rendering them among the most compelling structural materials available for contemporary applications [43]. This discourse embarks on an insightful exploration of magnesium alloys, illuminating their intricate composition, multifaceted properties, diverse applications, with particular emphasis on AZ31—a distinguished magnesium alloy that epitomizes the latent potential within this remarkable material family [45]. The allure of magnesium alloys emanates from their capacity to harmonize strength and weight, a feat that has engendered widespread fascination among both scientific and engineering communities [46]. This delicate equilibrium, striking a balance between structural integrity and weight reduction, has propelled magnesium alloys to the forefront of materials research [47]. They have emerged as frontrunners in numerous applications that necessitate materials characterized by lightweight attributes and mechanical robustness [48]. The implications of this attribute transcend conventional metallurgy, permeating critical facets of contemporary engineering, such as enhancing fuel efficiency in transportation, bolstering structural performance in aerospace engineering, and fostering advancements in medical devices [49]. To comprehensively grasp the essence of magnesium alloys, a thorough examination of their composition is imperative. Fundamentally, magnesium alloys are primarily composed of magnesium as the elemental base [50]. Nevertheless, their versatility arises from the deliberate incorporation of alloying elements, including aluminum, zinc, manganese, and rare earth elements [51]. These alloying elements bestow unique microstructural characteristics upon magnesium alloys, endowing them with a diverse spectrum of mechanical and thermal properties [52]. The interplay between elemental composition and microstructure metaphorically resembles a symphony, where each note

corresponds to a specific property or behavior, and the conductor orchestrates their harmonious integration [53].

Within the realm of magnesium alloys, AZ31 holds a prominent position as an exemplary specimen warranting comprehensive examination [46]. Its nomenclature succinctly hints at its composition, with 'A' signifying the presence of aluminum, 'Z' denoting zinc, and '31' indicating the alloy's nominal composition [54]. AZ31 embodies a compelling amalgamation of attributes that render it an appealing choice across a wide range of engineering applications [55]. Foremost among these attributes is its remarkably low density, a defining characteristic that imparts a feathery lightness to the alloy [56]. This property assumes paramount significance in industries where the reduction of structural weight while preserving mechanical resilience is a prime concern [57]. Moreover, AZ31 exhibits admirable corrosion resistance, rendering it particularly suitable for deployment in environments marked by harsh and corrosive conditions [58].

Another facet of AZ31's versatility lies in its exceptional castability—a trait that facilitates intricate and precise manufacturing processes [59]. In industries characterized by complex shapes and designs, AZ31's malleability provides engineers and designers with a valuable tool for realizing their innovative concepts. This is particularly relevant in the context of rapid prototyping and additive manufacturing, which allows for the creation of complex geometries that were previously impossible with traditional casting methods [60]. Moreover, the development of metallic composite materials, such as aluminum silicon carbide (Al-SiC) produced by stir-casting, underscores the potential for cost-effective and reliable manufacturing processes that could benefit from the properties of AZ31 [61]. The practical application of such materials is evident in the production of components like motorcycle brake levers, where the precision and durability of the cast are paramount [62].

The hexagonal close-packed crystal structure doesn't involve constructing a separate third layer of atoms. Instead, it relies on a repeating pattern: the third layer is a duplication of the first layer, the fourth layer is a repetition of the second layer, and so forth, as illustrated in Figure 2.1.

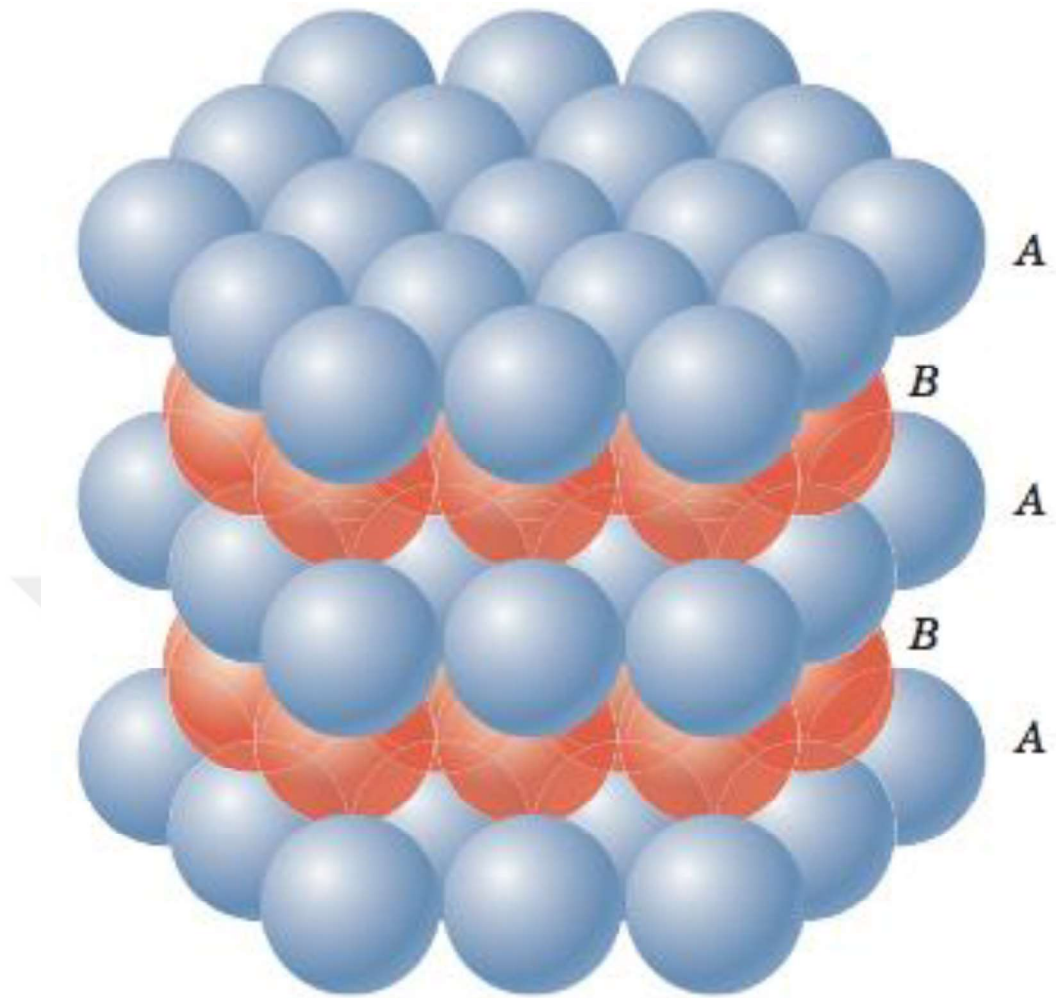


Figure 2.1 Illustration of close-packed structure [79]

However, beneath the veneer of these remarkable attributes, AZ31 harbors a mechanical idiosyncrasy that has captivated materials scientists and engineers for decades-anisotropy. Anisotropy in materials pertains to the phenomenon whereby mechanical properties vary in response to the direction of testing. In the case of AZ31, this anisotropy is intrinsically linked to its hexagonal close-packed (HCP) crystal structure-a hallmark of magnesium alloys [63]. A comprehensive understanding of the subtleties of anisotropy and its implications for AZ31's mechanical behavior is pivotal to harnessing the alloy's full potential in engineering applications. The anisotropy of AZ31 can be influenced by various factors such as microstructures and textures, especially when subjected to high strain rate rolling [64]. Moreover, the accumulative roll bonding (ARB) technique has been found to significantly refine the grain size of AZ31 sheets, leading to a pronounced anisotropy in mechanical properties [65]. The

effect of hot working on the damping capacity and mechanical properties of AZ31 also provides insights into the relationship between damping capacity and texture of the alloy [66]. Finally, understanding the quantitative relationship between microstructure, texture, and mechanical properties can further elucidate the role of texture strengthening in AZ31 sheets. Both face-centered cubic and hexagonal close-packed crystal structures offer equally efficient atom packing. Hexagonal close-packed structures are found in metals such as cobalt, cadmium, zinc, and the α phase of titanium. The structure is visually represented with hexagons on the top and bottom, enclosing a central triangle, as depicted in Figure 2.2. Molecular dynamics simulations have been used to compare the surface free energies of face-centered-cubic (fcc) and hexagonal-close-packed (hcp) structures, illustrating the differences in surface properties between these two closely packed structures [67]. In comparison with fcc metals, hcp metals such as titanium, zirconium, and magnesium show distinct characteristics due to their hcp lattice structure, which influences their processing, microstructure, and properties [68]. The packing behavior of nanoparticles also reflects the influence of crystal structure, with fcc ordering preferred by single crystalline nanoparticles, while hcp is preferred by polycrystalline nanoparticles [69]. Furthermore, the synthesis of binary and ternary solid-solution alloy nanoparticles with controlled hcp and fcc phases demonstrates the significant impact of crystal structure on the catalytic performance of these materials [70].

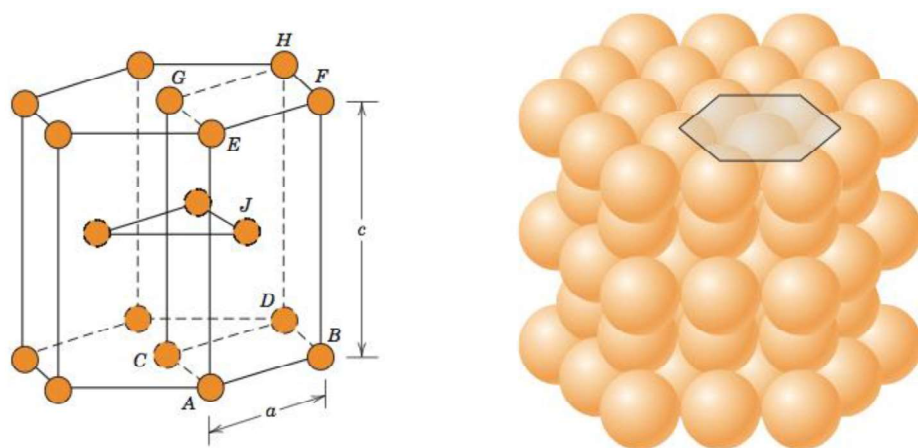


Figure 2.2 Representation of hexagonal close-packed structure unit cell [79]

Magnesium alloys, with AZ31 as a quintessential exemplar, constitute a captivating sphere of materials that have reshaped the landscape of structural materials in

contemporary engineering. Their exceptional strength-to-weight ratio, coupled with the intricacies of their microstructures and crystallography, has positioned them as a wellspring of innovation with far-reaching ramifications. The exploration of AZ31 and its mechanical idiosyncrasy of anisotropy stands as a linchpin in our comprehension of these extraordinary materials, forming the bedrock upon which subsequent chapters of this thesis are constructed.

The story of magnesium alloys is a testament to the ceaseless pursuit of materials with ever-enhanced properties. It unfolds against a backdrop of evolving industrial needs and unyielding innovation. As the periodic table's twelfth element, magnesium boasts an intrinsic characteristic-its remarkably low density, which positions it favorably for lightweight applications across various industries. Yet, this elemental marvel faces an inherent challenge: its native mechanical properties, particularly its strength, fall short of the demands of modern engineering. The solution to this quandary lies in the realm of alloying. Magnesium alloys, the blend of magnesium's union with other elements, are conceived to address this conundrum. These alloys are heralded for their capacity to harness the intrinsic virtues of magnesium while enhancing its mechanical properties. In this section, we embark on a historical odyssey tracing the evolution of magnesium alloys, from their nascent beginnings in laboratories to their current status as pivotal materials in numerous industrial sectors. We examine the driving forces behind the development of these alloys, the methods employed in their synthesis, and the transformative impact they have exerted on industries ranging from aerospace to automotive.

2.1.1. Composition and Microstructure of AZ31

Within the field of magnesium alloys, AZ31 emerges as a shining star-a distinct magnesium alloy characterized by a precise blend of elements that imparts it with unique properties. The nomenclature 'AZ31' itself provides a glimpse into the alloy's composition. The 'A' signifies the presence of aluminum, 'Z' represents zinc, and '31' denotes the nominal composition of the alloy. AZ31, in its elemental composition, is composed of 96% magnesium, 3% aluminum, and 1% zinc-an amalgamation crafted with meticulous intent [71]. This section delves deep into the heart of AZ31, unraveling its elemental composition, microstructural intricacies, and the profound

implications these factors exert upon its mechanical behavior. The alloy's microstructure, punctuated by a hexagonal close-packed (HCP) crystal lattice, assumes a pivotal role in shaping its mechanical attributes [72]. The stacking of magnesium atoms in a hexagonal pattern stands as a defining feature of AZ31's structure, setting it apart from other magnesium alloys. The section journeys into the profound implications of this crystallographic arrangement on the alloy's mechanical properties, with a particular focus on its anisotropic nature-an aspect that is poised to play a central role in the subsequent chapters of this research [73]. The relationship between the microstructure, texture, and mechanical properties of AZ31 sheets has been quantitatively analyzed, revealing the significant influence of grain size and texture on yield strength [74]. The properties of AZ31 are a symphony of intricately balanced attributes, orchestrated by its composition and microstructure. This alloy manifests a constellation of characteristics that render it exceptionally well-suited for a myriad of engineering applications. Foremost among these properties is its low density-a hallmark trait of magnesium alloys. This feature endows AZ31 with a featherweight quality, making it an invaluable asset in weight-sensitive applications. Whether within the soaring heights of aerospace engineering or the bustling thoroughfares of the automotive industry, the capacity to reduce structural weight while preserving mechanical strength is a coveted attribute. AZ31 material properties are given in Table 2.1.

Table 2.1 Chemical composition of AZ31 alloy [78]

Magnesium AZ31 Alloy Composition (wt%)							
Element	Al	Zn	Mn	Cu	Ni	Si	Mg
wt%	2.5 – 3.5	0.7 – 1.3	0.2	0.05	0.005	0.05	Balance

2.2. Anisotropy in Materials

The elucidation of anisotropy in materials stands as a pivotal focal point in the trajectory of materials science and engineering. This section embarks on a compelling exploration of anisotropy—an intricate phenomenon that lies at the core of understanding the mechanical behavior of materials. Anisotropy, as a defining characteristic, unveils its multidimensional nature, where mechanical properties exhibit profound variations contingent upon the direction of testing. This variance is not a random occurrence but rather an orchestrated symphony, directed by the profound interplay of crystallographic factors, microstructural influences, and the innate characteristics of materials themselves [75]. In this section, we delve deep into the heart of anisotropy, elucidating its fundamental underpinnings, prevalence across diverse materials, and its intriguing manifestation in the context of AZ31 magnesium alloy. Representation of an example yield surface is given at Figure 2.3 to show the difference between anisotropic and isotropic materials. The orientation dependence of anisotropic plasticity in polycrystalline copper by nanoindentation provides a clear example of the anisotropic behavior in metals, which can be extrapolated to understand similar behaviors in AZ31 [76]. Moreover, the estimation of anisotropy of mechanical properties in Mg alloys by means of compressive creep tests further emphasizes the importance of understanding anisotropic behavior in the design of construction parts [77].

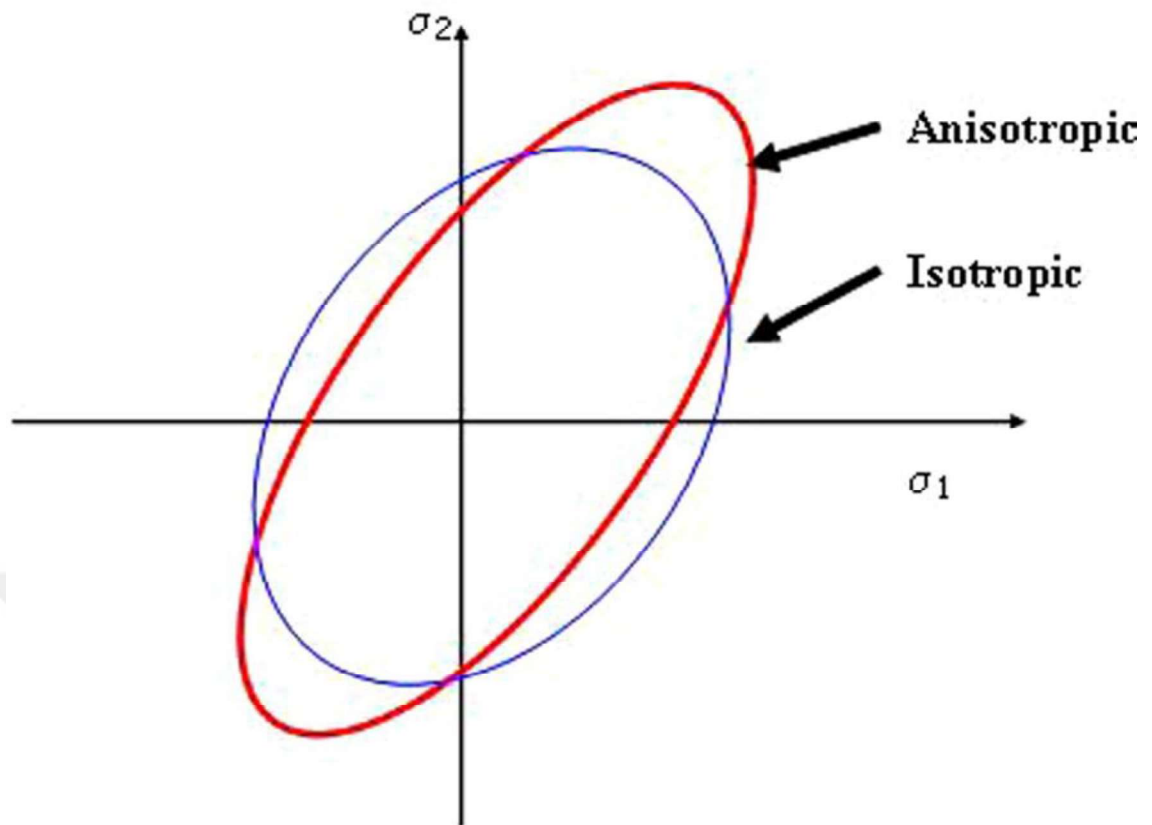


Figure 2.3 Exemplar yield surfaces [81]

At the crux of materials science lies the fundamental understanding that uniformity is often the exception rather than the rule. Materials are not monolithic entities with unchanging mechanical properties. Instead, they embody a dynamic complexity, where mechanical attributes metamorphose in response to the direction of applied forces. At the heart of this metamorphic quality lies the essence of anisotropy. The essence of anisotropy can be distilled into a fundamental revelation: materials are not homogenous in their response to external forces. Instead, they exhibit directional dependencies, where their mechanical behavior fluctuates depending on the angle of application. This fundamental truth illuminates the material world, where the capacity to stretch, compress, bend, or deform varies along different axes. The profound question that beckons is why?

2.2.1. Possible Causes of Anisotropy

To decode the enigma of anisotropy, one must venture into the inner sanctum of materials themselves, where the answers lie encrypted within the atomic and structural tapestries.

-Crystallographic Factors A foundational cornerstone of anisotropy is the crystallographic arrangement of atoms within a material. In the case of AZ31, an alloy of magnesium, aluminum, and zinc, the hexagonal close-packed (HCP) crystal structure serves as a prime influencer [80]. This crystallography dictates preferred planes and directions along which dislocations move during deformation. When external forces are applied, these dislocations traverse paths dictated by crystallography, thereby shaping the material's response. As a consequence, the alloy exhibits pronounced variations in mechanical behavior when subjected to testing along different orientations [82].

-Microstructural Influences: Beyond crystallography, microstructural features wield a significant impact on anisotropy. Grain boundaries, grain size, and texture within the material can introduce variations in mechanical properties. For AZ31, the distribution and orientation of grains become pivotal determinants. Grains that align favorably with the loading direction may experience less resistance to deformation, leading to orientation-dependent mechanical behavior. Texture, or the preferred crystallographic orientation of grains, plays a substantial role. The alignment of grains can influence the ease with which dislocations move, thus affecting the material's overall mechanical response [83].

-Material-Specific Characteristics: Anisotropy is not solely a consequence of crystallography and microstructure; materials themselves harbor inherent traits that predispose them to this phenomenon. For instance, AZ31's intrinsic properties, including its elemental composition and crystal structure, interact with external forces to determine its mechanical behavior. The interaction between these characteristics and applied stresses governs how the alloy accommodates stress and deforms when subjected to mechanical loads [84].

2.2.2. Anisotropy Across Materials

The phenomenon of anisotropy represents a pervasive and complex characteristic inherent to a broad array of materials, encompassing an extensive range from naturally occurring substances to engineered synthetics. This intrinsic property is not limited by the classification of materials; instead, it manifests distinctively within each category, influencing their physical properties and, consequently, their applications.

In the realm of metallic materials, anisotropy is a salient attribute that emerges from the particularities of crystallographic structures and the subtleties of microstructural formations. For instance, materials such as steel, aluminum, and titanium display anisotropic properties to various extents, which are attributable to their unique lattice orientations and grain morphologies [85]. The recognition of anisotropic behavior is of paramount importance in sectors such as aerospace, where the anisotropy of aluminum alloys, among others, necessitates rigorous contemplation in the engineering design process and the precision fabrication of vital aeronautical components [86]. Anisotropy is equally significant in the domain of polymeric materials, despite their atomic configuration markedly differing from that of metals. During polymer processing, the orientation of macromolecular chains often aligns with the vector of material flow, culminating in anisotropic mechanical behavior [87]. These directionally dependent properties are critical across a spectrum of industries, including packaging and automotive manufacturing, where the performance of injection-molded polymer components is critically dependent on their orientation, affecting attributes such as tensile strength and elasticity [88]. Composite materials, renowned for their customizability in terms of property enhancement, are also subject to anisotropic influences. The strategic placement and orientation of reinforcing fibers within the matrix material instigate directional preferences in mechanical properties such as rigidity and tensile strength [89]. The aerospace sector, in particular, leverages this aspect of anisotropy, engineering composite structures that optimize performance characteristics in alignment with specific loading directions for heightened efficiency and reliability [90].

In conclusion, the exploration of anisotropy in materials unearths a fundamental reality-materials are not uniform entities with unchanging mechanical properties. Anisotropy, as an intrinsic trait, introduces directional variability in mechanical behavior, challenging researchers and engineers to decipher and harness this phenomenon for practical applications. The underlying causes of anisotropy, rooted in crystallographic factors, microstructural intricacies, and material-specific characteristics, underpin this captivating phenomenon. This foundational understanding equips scientists and engineers with the tools to unravel the complexities of materials' mechanical behavior. The prevalence of anisotropy extends its influence across a diverse spectrum of materials, transcending material classes. Each material family contributes its own unique intricacies, further enriching the canvas of anisotropy.

2.3. Sheet Metal Forming

Sheet metal forming stands as a cornerstone in the manufacturing realm, offering a versatile and efficient route to fabricate components that are integral to a myriad of industrial applications. Its prominence in sectors such as automotive manufacturing is marked by the production of lightweight, yet robust components that contribute significantly to vehicle safety and fuel efficiency [91]. The aerospace industry, similarly, leverages the benefits of sheet metal forming to produce fuselage and wing structures that must withstand extreme operational conditions while minimizing weight. In the domain of consumer electronics, the manufacturing process is instrumental in creating the sleek and durable metal casings that are synonymous with today's high-end gadgets.

The fundamental principle of sheet metal forming is rooted in plastic deformation – a process where metal sheets are permanently shaped into the final geometries without severing the material. This deformation is achieved through the application of various mechanical forces, using tools and dies to bend, stretch, and compress the metallic sheets. The ability of a metal to undergo such manipulation is intrinsic to its material properties, which dictate the extent to which it can be formed before failure [92, 93].

As metal is deformed, it traverses through a series of strain states – from elastic to plastic deformation. Elastic deformation is reversible, allowing the material to return to its original shape once the applied forces are removed. However, once the yield point is surpassed, the material enters the realm of plastic deformation, where the changes become permanent. The transition between these states is smooth and continuous on the microscopic level, although it has profound implications for the manufacturing process [94-96]. Springback, a pervasive issue in sheet metal forming, emerges from the elastic properties of the material. After a metal sheet has been formed and the tools are retracted, the residual stresses within the material seek equilibrium, causing the metal to spring back slightly towards its original shape. This dimensional recovery can lead to inaccuracies in the geometry of the part, presenting a significant challenge to manufacturers, who must anticipate and compensate for this behavior to ensure the final product meets stringent dimensional specifications [97-100]. The phenomenon of springback is influenced by several factors, including the type of metal, its thickness, the complexity of the formed shape, and the specific conditions under which forming takes place. Metals with a high yield strength, for instance, tend to exhibit more significant springback due to their greater elasticity. Similarly, thinner sheets are more prone to this issue, as they offer less resistance to bending and can more readily spring back. The complexity of the part's geometry also plays a critical role; more intricate shapes with sharper bends are more susceptible to the effects of springback [101-104]. Mitigating springback is no trivial endeavor and requires a nuanced understanding of both material science and the mechanics of forming processes. Manufacturers may employ various strategies to counteract springback, including adjusting tool designs, modifying process parameters, and employing post-forming operations. Furthermore, predicting springback involves sophisticated computational methods that simulate the forming process and the subsequent elastic recovery of the metal, enabling engineers to preemptively adjust tooling and process conditions to achieve the desired outcome [105-108].

In the quest to produce precise components, the industry continues to grapple with springback, dedicating considerable resources to better understand and control this complex behavior. This phenomenon, emblematic of the challenges inherent in metal forming, underscores the need for ongoing research and innovation in the field. As the

demand for complex, lightweight, and high-strength parts grow, the imperative to master the intricacies of springback becomes increasingly critical, driving advancements in materials engineering, tool design, and manufacturing technology.

2.4. V-Bending Tests and Their Significance

In the realm of materials science and engineering, the evaluation of mechanical properties is paramount, often serving as the litmus test for a material's suitability in diverse applications. Among the litany of mechanical tests at the disposal of researchers, V-bending tests emerge as a critical tool, uniquely poised to unravel the mechanical intricacies of materials under bending conditions. This section embarks on an academic journey through the terrain of V-bending tests, elucidating their essence, methodologies, and, most significantly, their profound significance in the context of this research. The development of mechanical-thermal coupling tensile-bending combined fatigue testing instruments has advanced the understanding of fatigue properties under various temperature conditions, which is essential for materials subjected to multiaxial mechanical-thermal coupling fatigue loads [109]. Moreover, the phenomenon of springback in V-bending, which is influenced by factors such as coating thickness, roughness, and material yield strength, has been extensively studied to improve the precision of sheet metal forming processes [110]. The use of artificial neural networks for predicting springback during the V-bending process further exemplifies the integration of intelligent systems in materials testing, providing high correlation with experimental results [111]. Specifically, for Mg-Al-Zn alloys such as AZ31, the correlation between fatigue limit and static strength has been substantiated, offering insights into the material's behavior under cyclic loading conditions [112]. V-bending tests, as a mechanical testing paradigm, encapsulate a profound simplicity-yet beneath their apparent straightforwardness resides a wealth of information. At their core, V-bending tests involve subjecting a flat sheet of material to plastic deformation through a controlled bending process. The material specimen, typically shaped in a V-form or U-form, is clamped at its base, while a punch descends to form the bend. This process, seemingly elementary, unveils a multitude of mechanical insights, such as the springback effect, which affects the dimensional and geometrical accuracy of bent parts and has been a key problem in sheet metal forming [113]. Furthermore, the

process is influenced by the anisotropic properties of the material, which can be predicted by numerical analysis and finite element methods [114, 115]. The deformation behavior of materials, including the magnesium alloy AZ31, has been investigated using various mechanical tests and finite-element simulations to address bendability and predict material failure with good accuracy [116].

The essence of V-bending tests lies in their capacity to elucidate a material's response to bending forces—a response that transcends mere numerical values. They reveal the material's ductility, formability, and resistance to deformation, casting light on its ability to endure the rigors of real-world applications, from sheet metal fabrication to aerospace components.

2.4.1. Methodologies of V-Bending Tests

The execution of V-bending tests entails a meticulous methodology, a structured sequence that renders reproducibility and accuracy paramount. While variations in testing setups exist, the core principles remain consistent.

- **Sample Preparation:** The process commences with the careful selection and preparation of material samples. These samples, often in the form of flat sheets, must adhere to standardized dimensions to ensure consistency and comparability.
- **Clamping and Fixturing:** The specimen is securely clamped in a V or U-shaped die, typically made of steel, ensuring that the material's edge aligns with the desired bending line. The punch, often affixed to a testing machine, descends to initiate the bending process.
- **Bending:** The punch descends, applying a gradually increasing force until plastic deformation occurs, resulting in the formation of a bend in the material. The force-displacement relationship is meticulously recorded throughout this process.
- **Measurements and Analysis:** Post-testing, the bent specimen is analyzed to determine critical parameters such as the bend angle, bend radius, and any defects or fractures that may have occurred during the test.

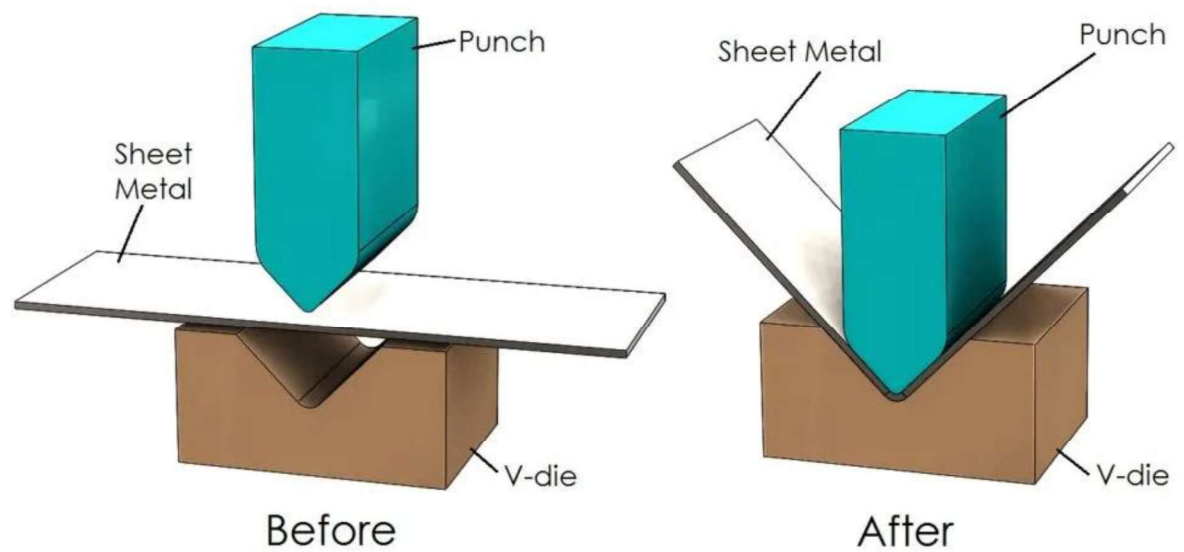


Figure 2.4 Representation of V-bending tests [117]

2.4.2. Significance of V-Bending Tests

The importance of V-bending tests in the field of materials research and engineering is paramount. These tests offer much more than a simple assessment of mechanical properties; they are a critical tool for gaining insight into the behavior of materials under the complex stress states induced by bending operations, such as the Bauschinger effect, which affects yield strength after deformation [118]. V-bending tests serve several vital functions in the understanding and utilization of materials.

Firstly, they are an invaluable method for evaluating formability, which is a material's capacity to undergo plastic deformation without failure. This property is particularly crucial in sectors like automotive manufacturing, where the creation of intricate parts from metal sheets relies heavily on the materials' formability. V-bending tests are thus essential for identifying materials that can be shaped into the complex forms demanded by modern automotive designs.

In the aspect of manufacturing, predicting springback is a significant challenge. Springback is the phenomenon where a material tries to revert to its original shape after being bent. V-bending tests play a critical role in forecasting the extent of this springback, allowing engineers to modify tools and processes proactively to ensure that the produced shapes adhere to strict design criteria with high precision [119].

Ductility, or the degree to which a material can sustain plastic deformation before breaking, is another fundamental material property that can be assessed through V-bending tests. This property is of immense importance in industries that demand materials capable of withstanding extensive deformation without failing. By simulating the bending process, V-bending tests allow for a controlled measurement of ductility, guiding the selection of materials for applications that cannot afford to compromise on durability. Furthermore, V-bending tests are instrumental in the optimization of processing techniques across various industries. The aerospace sector, for instance, requires exacting standards in the bending of components, and the ability to predict and control material behavior during bending is vital. Similarly, in architectural metalworking, where both structural integrity and aesthetics are important, V-bending tests assist in refining manufacturing processes to enhance efficiency and effectiveness. Lastly, V-bending tests are critical in material selection. By testing different materials under bending stresses, engineers can identify the most appropriate material that meets the mechanical demands of a particular application. This ensures not only the performance but also the reliability and longevity of the end products [120].

Overall, V-bending tests are an integral part of the material selection and development process, providing a comprehensive understanding of material performance and behavior. Their application is a testament to the complex interplay between the theoretical understanding of material science and its practical applications in engineering and manufacturing, ensuring that the materials and processes used are optimized for the intended application.

In conclusion, V-bending tests emerge as an indispensable tool in the arsenal of materials scientists and engineers. Their seemingly simple premise belies their profound significance in unraveling the mechanical intricacies of materials under bending conditions. From assessing formability and predicting springback to evaluating ductility and optimizing manufacturing processes, V-bending tests play a multifaceted role in the materials research landscape. In the context of this research, the application of V-bending tests to AZ31 magnesium alloy promises to unveil a rich tapestry of mechanical insights. It is through these tests that we shall explore how

AZ31 responds to bending forces, how its anisotropic nature influences the bending process, and how temperature variations introduce additional complexities. As we proceed through the subsequent chapters, the results and revelations drawn from V-bending tests on AZ31 shall stand as a testament to the power of this mechanical testing paradigm in advancing our understanding of materials and their behavior.

2.5. Disk Compression Tests and The Lankford Parameters

Anisotropy is a phenomenon of paramount importance in the field of sheet metal forming, as it significantly influences the material's formability. This characteristic dictates that a material's mechanical properties, such as its yield strength, ductility, and even its strain hardening capabilities, can differ depending on the orientation of the load with respect to the material's internal structure. Sheet metals typically exhibit anisotropy as a result of their manufacturing process, which aligns the grains in certain directions, imparting directionality to their mechanical properties. The anisotropic yield locus and stretch formability of sheet materials, such as aluminum, have been shown to be significantly affected by anisotropy, necessitating careful evaluation in the context of vehicle emissions reduction and other applications [121]. Furthermore, the anisotropy's effect on the formability of advanced high strength steels has been studied, highlighting the importance of understanding anisotropic factors and yield stresses in relation to the material's rolling direction [122]. Additionally, the sensitivity of springback to elastic anisotropy has been explored, emphasizing the need for accurate material modeling to predict and control this behavior in metal forming processes [123].

The Lankford parameters, or r -values, have been established as a key metric for quantifying anisotropy in sheet metals. These parameters reflect the ratio of plastic strain in the width to the plastic strain in the thickness of a sheet as it undergoes uniaxial tension. Initially introduced in the mid-20th century, these parameters have since become integral in describing the directional dependency of ductility in rolled sheets. High r -values generally indicate good drawability, as they suggest that a material is more prone to thinning than to necking or failing during stretching operations. The orthotropic behavior of sheet metals, such as magnesium AZ31, during

strain localization is a prime example of the application of Lankford coefficients in characterizing ductile behavior [124]. Additionally, the experimental determination of Lankford r -values in rolled steel sheets further underscores their significance in assessing the mechanical properties and anisotropy impacts [125]. The evolution of R -values for large strains, observed through uniaxial tensile tests, is essential for understanding the anisotropy changes in sheet metals [126]. Moreover, the integration of Lankford parameters into finite-element simulations for texture and strain-induced anisotropy models is vital for accurate virtual product development of sheet metal parts [127].

Moving beyond uniaxial stress states to biaxial stress states is crucial for a complete understanding of sheet metal behavior during industrial forming processes. The real-world scenarios often involve complex multi-axial loading conditions, necessitating this broader perspective. Characterizing biaxial anisotropy involves examining how the Lankford parameters behave under combined stresses, which provides a more comprehensive view of the material's formability. This is particularly important for predicting and controlling phenomena such as earing during deep drawing operations, where differential growth can lead to uneven rim heights due to anisotropic properties. The work by Pagenkopf, Gibmeier, and Schuster on deep drawn steel cups demonstrates the significance of considering anisotropic material behavior in residual stress analysis, which is essential for understanding and mitigating the effects of earing in deep drawing [128].

Disk compression tests emerge as a sophisticated experimental approach to measure Lankford parameters under varied thermal conditions. The essence of this method lies in flattening a circular disk sample and scrutinizing the material's strain response along different axes—namely, the radial and circumferential directions. By analyzing these strain patterns, one can extract the biaxial Lankford parameters, which are indicative of the material's response to complex stress states. Although not specific to disk compression tests, research involving the incorporation of material test data, such as that from Inconel-718, into material models like `*MAT_224` in LS-DYNA, demonstrates the importance of considering temperature and strain rate effects when evaluating material behavior under biaxial stress states [129].

These tests are particularly advantageous because they can simulate a state of stress akin to that found in many sheet metal forming operations, thereby providing a realistic basis for material characterization. Moreover, conducting these tests across a range of temperatures can unveil the influence of thermal effects on the anisotropic behavior of metals. Metals can behave differently at high temperatures, exhibiting increased ductility and reduced yield strength, which in turn affects their formability. The hot working behavior of superalloys, as discussed by Cremisio and McQueen, is a testament to the importance of evaluating material behavior under varied thermal conditions, which is applicable to disk compression tests [130]. Furthermore, the work by Yang on nanoscale Al/SiC multilayers contributes to our understanding of how mechanical properties, such as those measured by disk compression tests, can change with temperature [131].

The disk compression test method thus serves to provide insights into the temperature-dependent evolution of anisotropy in materials. It aids in constructing a detailed picture of how formability changes with temperature, enabling the tailoring of sheet metal forming processes for optimal performance. This temperature-dependent data is invaluable for high-precision applications, such as in the automotive or aerospace sectors, where materials are often subjected to extreme thermal environments. The study by Harvinder Singh et al. on the wear characteristics of AISI D2 steel provides a foundation for understanding the deformation behavior under different thermal conditions [132]. Additionally, the research by Teramoto, Kubota, and Takahashi on the temper softening resistance of martensitic steel offers insights into the temperature-dependent properties of materials, which is crucial for the application of disk compression tests in material characterization [133].

In summary, the comprehensive analysis of the Lankford parameters through disk compression tests and their temperature variation offers a granular understanding of sheet metal behavior. This enhanced understanding facilitates the development of predictive models for metal forming, which are essential for optimizing industrial processes, reducing material waste, and achieving superior quality in the final products. The advancement of these experimental techniques reflects the continuous

interplay between material science theory and its application, driving innovations in manufacturing technologies.

2.6. Biaxial Anisotropy and Its Importance

The exploration of anisotropy in materials, as discussed in the preceding sections, has unveiled the intricate world where mechanical properties are contingent upon the direction of testing. However, the narrative extends beyond uniaxial tests that examine behavior under a single applied force or load direction. The complexity deepens when materials are subjected to biaxial loading, where forces act simultaneously in multiple directions. This dynamic gives rise to biaxial anisotropy—a phenomenon where a material's mechanical response exhibits intricate variations when challenged under combined loading conditions. In this section, we delve into the nuanced realm of biaxial anisotropy, elucidating its essence, methodologies for characterization, and, most significantly, its profound importance in the context of materials science and engineering. The sequential biaxial stretching of materials, as studied by Dammann et al. [134] highlights the microstructural evolution and its macroscopic implications. Zhu et al. [135]'s investigation into the mechanical behavior of anisotropic materials under varied loads further underscores the complexity of biaxial anisotropy. Udomraksasakul et al. [136]. 's research on the anisotropy effect in biaxial tests emphasizes the need for accurate flow stress evaluation. Lastly, the work by Hetz et al. [137] on the determination of the biaxial Lankford coefficient using a simplified test method contributes to the efficient characterization of this anisotropic property.

2.6.1. Understanding Biaxial Anisotropy

Biaxial anisotropy presents a paradigm where materials cease to be governed by a singular mechanical response and instead reveal the depth of their complexity when subjected to multiaxial stresses. In uniaxial testing, materials deform primarily along a single axis, offering a relatively simplified understanding of their mechanical behavior. However, the real-world rarely adheres to such simplistic loading conditions. Engineering applications frequently subject materials to complex and simultaneous forces along multiple axes, demanding a comprehensive comprehension of their biaxial response. The work by Jacobs et al. [138] on fibrous soft tissue under biaxial

tension highlights the need for accurate material characterization in such complex loading conditions. Stewart and Sitaraman's [139] research on flexible electronics under biaxial strain emphasizes the importance of understanding biaxial anisotropy in the performance and reliability of materials. Chatziathanasiou's development of a constitutive model for SMAs addresses the anisotropic response under multiaxial loading, which is vital for materials subjected to combined stresses [140]. Lastly, Mauri et al. [141]'s comparison of uniaxial and multiaxial mechanical responses in tissue-engineered vascular grafts underscores the significance of biaxial anisotropy in material behavior.

At its core, biaxial anisotropy embodies a scenario where materials display directional variability in their response to combined loading. This variability is not merely an additive effect of uniaxial responses but rather a dynamic interplay of mechanical properties that emerge under the influence of simultaneous forces acting along different directions, as demonstrated in the work by Khan [142] on the shear behavior of clay and silty clay. Biaxial anisotropy is a multidimensional mechanical narrative that unveils itself as an intricate tapestry—a story of mechanical behaviors under the interplay of forces, which is evident in the assessment of directional anisotropy in LPBF IN-718 bulk samples by Džugan et al. [143]. The subtle anisotropy in mechanical properties and fatigue crack growth rate in different orientations of Inconel 738LC alloy, as studied by Kovářik et al. [144] further illustrates this complexity. Lastly, the induced anisotropy and softening effects under biaxial loading in rubber composites, explored by Garishin and Shadrin [145] highlight the significant differences in mechanical behavior compared to pure rubber.

2.6.2. Characterization of Biaxial Anisotropy

The characterization of biaxial anisotropy is a multifaceted endeavor that necessitates specialized testing methodologies capable of unveiling the intricate mechanical behavior that emerges under combined loading conditions. Protocols enabling mechanical characterizations of anisotropic, heterogeneous soft tissues, as detailed by Pearce et al. [146] present mechanical tests designed to maximize inhomogeneous strain fields and in-plane shear forces. Jacobs et al. [147] address experimental challenges in biaxial tension of fibrous tissue, using finite element methods to quantify

load transmission and formulate a correction factor for stresses in the region of interest. Navrátil et al. [148] describe a model to represent the anisotropic mechanical behavior of short fiber-reinforced thermoplastics, including cyclic loading conditions. Lastly, Stewart and Sitaraman [149] demonstrate the development of a visco-hyperelastic material model and a finite-element model to understand the electrical and mechanical behavior of stretchable conductors examined through biaxial inflation stretch testing. These methodologies provide critical data for understanding biaxial anisotropy and its implications across various industries.

The significance of understanding and characterizing biaxial anisotropy in materials extends across a wide spectrum of industries, each with its unique set of challenges and requirements. This section elaborates on the pivotal role of biaxial anisotropy in various practical applications, with a particular focus on the aerospace industry and its implications. In the realm of aerospace engineering, where the quest for innovation and safety is paramount, materials face a demanding and dynamic environment. Aircraft and spacecraft components are subjected to a multitude of complex multiaxial loading conditions during their missions. These conditions encompass forces arising from various sources, including aerodynamic loads, vibrations, thermal gradients, and mechanical interactions. Understanding and quantifying biaxial anisotropy is of paramount importance for the following reasons:

1. The aerospace and industrial gas turbine industries utilize directionally-solidified turbine blades, which are subjected to high-temperature multiaxial cyclic loading conditions leading to complex creep strain histories. An anisotropic tertiary creep damage model for transversely-isotropic materials has been described by Stewart and Gordon [150] which is crucial for modeling the behavior of these materials under such conditions.
2. Ceramic substrates, such as those used as molecular filters in fuel cells and solid oxide electrolyzers, often involve multiaxial bending. Cheng's work [151] on the biaxial flexural strength of thin film ceramic substrates as electrolytes is relevant for understanding the mechanical properties of these materials under service conditions.

3. The mechanical characterization of unidirectional composite materials under multiaxial loading conditions is essential for their increased usage in high-performance industries like aerospace. Balakrishnan's application of the biaxial Iosipescu test fixture for the mechanical characterization of unidirectional composites provides insights into the behavior of these materials under shear and biaxial loading conditions [152].
4. In tissue engineering, particularly for the fabrication of cardiovascular implants like heart valves or vascular grafts, the mechanical characterization of tissue-engineered vascular grafts (TEVGs) has been focused on the material's strength and deformation behavior under uniaxial and biaxial stress states. Mauri et al. [153] have shown that a physiological biaxial testing configuration should be preferred for the TEVG characterization, which is also applicable to the testing of aerospace materials.

Aerospace materials must maintain their structural integrity under a broad range of loading scenarios. The integrity of aircraft wings, fuselages, and spacecraft frames depends on the ability of materials to withstand biaxial stresses and strains without failure. Materials exhibiting biaxial anisotropy may respond differently to these loads, necessitating a comprehensive understanding of their behavior. Weight is a critical factor in aerospace design, as lighter aircraft and spacecraft require less fuel and exhibit improved performance. Materials that can withstand biaxial stresses efficiently contribute to lightweight design while ensuring structural safety. An understanding of biaxial anisotropy aids in selecting materials that strike the right balance between strength and weight. Ensuring the safety of passengers, astronauts, and payloads is non-negotiable in aerospace engineering. Biaxial anisotropy plays a pivotal role in safety assessments. Knowledge of how materials behave under combined loading conditions informs structural design, reduces the risk of catastrophic failures, and enhances the overall safety of aerospace systems. The aerospace industry continuously seeks advanced materials that can meet stringent performance requirements. Understanding biaxial anisotropy guides material selection, enabling engineers to choose materials tailored to the specific demands of a given application. This, in turn, contributes to the development of next-generation aircraft and spacecraft. Aerospace

engineering thrives on innovation, and breakthroughs often come from pushing the boundaries of material performance. Materials exhibiting favorable responses to biaxial stresses open doors to novel design concepts, improved efficiency, and enhanced mission capabilities. Aerospace engineering stands as a prime example of an industry where biaxial anisotropy can significantly influence material selection, structural design, and safety. The ability to predict and account for biaxial behavior is not only vital for the continued advancement of aerospace technology but also for ensuring the well-being of those who journey through the skies and beyond.

The automotive industry is another arena where the understanding of biaxial anisotropy holds immense practical value. Vehicles, whether on the road or in controlled testing environments, often encounter complex multiaxial loading scenarios, making the exploration of biaxial behavior crucial. Ensuring the safety of occupants during vehicle collisions is a top priority in automotive engineering. Crash simulations and safety assessments involve intricate multiaxial loading conditions. Knowledge of biaxial anisotropy aids in the selection and design of materials that can absorb and dissipate energy effectively, enhancing vehicle crashworthiness. Automotive manufacturers are continually striving to reduce vehicle weight to improve fuel efficiency and reduce emissions. Materials exhibiting favorable biaxial behavior contribute to lightweighting efforts without compromising safety, making them desirable candidates for various vehicle components. Modern vehicles feature complex structural designs with components subject to diverse loading scenarios, including torsion, bending, and compression. Understanding how materials respond to biaxial stresses is essential for optimizing component design and ensuring overall vehicle performance and durability. The automotive industry's pursuit of sustainability and improved performance drives the development of innovative materials. Materials with well-characterized biaxial behavior can play a pivotal role in the evolution of vehicle materials, offering enhanced capabilities in various applications.

Sheet metal forming is a fundamental process in industries such as automotive manufacturing, appliance production, and architectural metalwork. Complex shapes and components are created by subjecting sheet metal to bending, stretching, and compression. In these applications, biaxial anisotropy is highly relevant for the

following reasons. Achieving desired shapes and minimizing defects during sheet metal forming require a thorough understanding of how materials respond to biaxial stresses. Materials exhibiting favorable biaxial behavior can be formed more efficiently and with reduced risk of failure. Optimizing sheet metal forming processes depends on selecting materials that can endure multiaxial loading conditions without excessive wear or damage to tools and dies. Knowledge of biaxial anisotropy aids in selecting materials that enhance manufacturing efficiency. Architectural and decorative metalwork often demand intricate shapes and flawless surface finishes. Biaxial behavior influences how sheet metal responds to bending and stretching, affecting the final aesthetics of products in various industries.

In the field of biomechanics, where materials are frequently subjected to complex loading scenarios during physiological processes or medical device applications, understanding biaxial anisotropy is pivotal. The human body and medical devices often interact with tissues and biomaterials under biaxial stresses. Medical implants, prosthetics, and other devices must interact effectively with the human body. Understanding how biomaterials respond to biaxial loading conditions informs the design and performance of such devices. Biological tissues exhibit anisotropic behavior and are subject to complex multiaxial loading during activities such as joint movement and muscle contraction. Knowledge of biaxial anisotropy is essential for developing biomaterials and devices that mimic natural tissue behavior. Biomechanical research often involves the study of tissue mechanics under various loading conditions. Understanding biaxial anisotropy is crucial for accurately modeling and simulating these scenarios. At the core of material science and engineering lies the endeavor to develop advanced materials with tailored mechanical properties. Optimization of material responses under multiaxial loading scenarios is a central aspect of this pursuit. Materials engineered to exhibit specific responses to biaxial stresses can be designed for enhanced performance in targeted applications. This approach enables the development of materials optimized for their intended use. The ability to predict and manipulate material behavior under complex loading conditions opens doors to innovation and advancements across industries. Materials exhibiting favorable biaxial anisotropy contribute to the development of novel technologies and solutions.

In conclusion, the importance of biaxial anisotropy transcends academic curiosity, permeating a multitude of industries and applications. From ensuring the safety of aerospace components to enhancing crashworthiness in the automotive sector, and from optimizing sheet metal forming processes to advancing medical devices and biomaterials, a comprehensive understanding of how materials respond to biaxial stresses is indispensable. This understanding not only addresses practical challenges but also fuels innovation and drives progress in materials science and engineering.

2.6.3. Challenges and Opportunities

While the importance of biaxial anisotropy is undeniable, its characterization and prediction present ongoing challenges. Real-world applications often involve a myriad of loading conditions that are difficult to replicate in laboratory settings. Additionally, the complexity introduced by microstructural variations and crystallography further complicates the accurate prediction of biaxial behavior. However, these challenges also present opportunities for innovation and advancement. Improved testing methodologies, advanced numerical simulations, and a deeper understanding of material microstructure offer promising avenues for addressing biaxial anisotropy. Furthermore, the ability to predict and optimize material behavior under complex loading conditions opens doors to engineering materials with enhanced performance and reliability.

In conclusion, biaxial anisotropy represents a pivotal dimension in the mechanical narrative of materials. Its emergence under the influence of simultaneous forces along multiple axes unveils the intricate and dynamic nature of material behavior. While the characterization of biaxial anisotropy poses challenges, its importance across diverse industries underscores the necessity of continued research and understanding.

2.7. Theoretical Concepts and Models

In the domain of materials science and engineering, the establishment of theoretical concepts and models holds a pivotal role in understanding and predicting the mechanical behavior of materials under varying conditions. Within this context, one of the central tenets is the elucidation of yield surface models, which serve as

fundamental constructs for characterizing a material's response to external loads. Theoretical yield surface models provide a rigorous mathematical foundation for comprehending how materials deform and yield under different states of stress, strain, and temperature [150]. These models not only encapsulate the intrinsic properties of materials but also account for their anisotropic behaviors and responses to complex loading scenarios [151-153]. As we delve into the exploration of "Theoretical Concepts and Models" in this section, our primary focus is to elucidate the intricacies of these yield surface models, including their underlying principles, mathematical formulations, and practical applications. Furthermore, it is worth noting that an illustrative example of a yield surface model is provided in Figure 2.5 to enhance the reader's comprehension of these theoretical constructs. Through this comprehensive discussion, we aim to offer a thorough understanding of how these theoretical constructs serve as invaluable tools in materials science and engineering, facilitating the analysis and design of structures across a spectrum of industries and applications

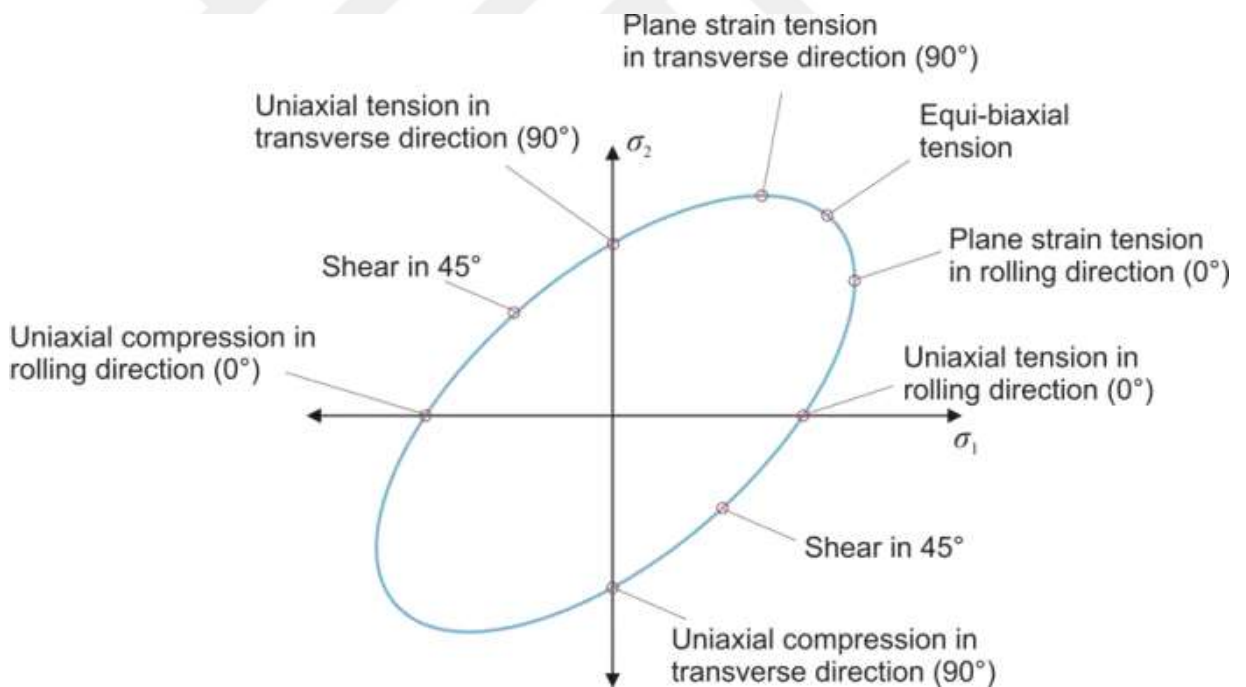


Figure 2.5 Different stress states identified on a yield locus [154]

2.7.1. The von-Mises and Tresca Yield Criterion

The von Mises yield criterion, also known as the von Mises-Hencky criterion or the second distortion energy criterion, represents a fundamental theoretical concept in the

realm of materials science and mechanics. Introduced by Richard von Mises in the early 20th century, this yield criterion has found widespread application in characterizing the yielding and failure behavior of materials subjected to complex stress states [155].

At its core, the von Mises yield criterion postulates that yielding of a material occurs when the second deviatoric stress invariant, commonly referred to as the von Mises stress σ_v , reaches a critical value. This criterion provides a simplified and intuitive way of assessing when material failure is likely to happen, particularly in scenarios where stress components vary in magnitude and direction [156].

The von Mises stress (σ_v) is mathematically expressed as:

$$\sigma_v = \sqrt{\frac{3}{2}(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2)} \quad (2.1)$$

Where:

- $(\sigma_{11}), (\sigma_{22}), (\sigma_{33}), (\sigma_{12}), (\sigma_{23}),$ and (σ_{31}) represent the components of the stress tensor.

The von Mises yield criterion does not consider the hydrostatic component of stress and focuses solely on the distortional stresses within the material. This simplification is particularly useful when dealing with materials that exhibit isotropic behavior, as it provides a single stress value to compare against the material's yield strength, regardless of the specific stress state.

The von Mises criterion has been successfully applied to a wide range of materials, including metals, polymers, and ceramics, and has become a cornerstone in the field of plasticity theory. It serves as a valuable tool for engineers and researchers in predicting when material yielding and potential structural failure may occur under complex loading conditions, contributing significantly to the design and analysis of various engineering structures. In the subsequent sections of this chapter, we will further explore the mathematical underpinnings and practical applications of the von

Mises yield criterion, shedding light on its versatility and significance in the field of materials science and mechanics [157].

The von Mises yield criterion is a key concept in materials science and mechanics, offering valuable insights into the yielding and failure behavior of materials under various stress conditions. It is particularly useful in scenarios where stress components vary both in magnitude and direction.

At its core, the von Mises yield criterion is based on the idea that yielding occurs when a certain measure of the deviatoric stress exceeds a critical value. This measure is known as the von Mises stress (σ_v), and it is computed using the components of the stress tensor (σ_{ij}) as follows:

$$\sigma_v = \sqrt{\frac{3}{2} [(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2)]} \quad (2.2)$$

Where:

- (σ_{11}) , (σ_{22}) , and (σ_{33}) are the normal stress components along the coordinate axes.
- (σ_{12}) , (σ_{23}) , and (σ_{31}) are the shear stress components on the respective coordinate planes.

This equation quantifies the von Mises stress, which is a measure of the distortional stress in the material. It takes into account the six stress components in the stress tensor and emphasizes the importance of shear stresses. Importantly, it disregards the hydrostatic stress (pressure) because it does not contribute to material yielding; only the distortional stresses play a role.

The von Mises criterion suggests that yielding occurs when the von Mises stress surpasses the material's yield strength σ_Y . Mathematically, this criterion can be expressed as:

$$[\sigma_v \leq \sigma_Y]$$

This inequality indicates that as long as the von Mises stress remains below the yield strength, the material remains in the elastic range and does not undergo permanent deformation. However, when the von Mises stress exceeds the yield strength, plastic deformation initiates, and the material yields. The von Mises yield criterion has found wide application in engineering, especially for materials that exhibit isotropic behavior, as it provides a single, scalar value for comparing against the material's yield strength. This simplification is particularly useful when dealing with complex stress states commonly encountered in structural analysis and design.

In summary, the von Mises yield criterion provides a powerful tool for predicting material yielding and failure under complex stress conditions. By focusing on the distortional stresses and yielding behavior, it has become a cornerstone of plasticity theory, contributing significantly to the design and analysis of structures across various industries. Its mathematical foundation and practical applicability make it an indispensable concept in materials science and engineering.

2.7.2. The Hill-48 Yield Criterion

The Hill-48 yield criterion, developed by Rodney Hill in 1948, stands as a seminal contribution to the field of materials science and engineering. This theoretical framework serves as a fundamental tool for understanding the yield behavior of anisotropic materials, including crystalline metals like AZ31 magnesium alloy. In this section, we delve into the background, formulation, significance, and applications of the Hill-48 yield criterion. [158] The Hill-48 yield criterion is rooted in the recognition that materials, especially crystalline ones, exhibit varying yield behaviors along different crystallographic axes due to their anisotropic nature. This anisotropy arises from the inherent asymmetry in the crystal structure and the resulting anisotropic deformation behavior. For AZ31 magnesium alloy, which possesses a hexagonal close-packed (HCP) crystal structure, anisotropy is a prominent characteristic. [159]

The Hill-48 yield criterion carries profound significance in various aspects of materials science and engineering.

- **Characterization of Anisotropic Materials:** The primary significance of the Hill-48 yield criterion lies in its ability to characterize and quantify the anisotropic behavior of materials. For materials like AZ31 magnesium alloy, which exhibit distinct yield behaviors along different crystallographic directions, this criterion provides a means to describe and understand these variations.
- **Structural Analysis:** In engineering applications, where materials experience multiaxial loading conditions, understanding the material's yield behavior is essential for structural analysis and design. The Hill-48 yield criterion enables engineers to assess the structural integrity and safety of components subjected to anisotropic stresses.
- **Material Selection:** The Hill-48 yield criterion aids in the selection of appropriate materials for specific applications. It ensures that the chosen material aligns with the expected loading conditions and performance requirements. In industries like aerospace and automotive, where lightweight materials with tailored properties are crucial, this criterion helps make informed choices.
- **Process Optimization:** Manufacturing processes that involve anisotropic materials, such as sheet metal forming, benefit from insights provided by the Hill-48 yield criterion. It guides the optimization of processes to minimize defects and enhance formability.
- **Material Development:** Researchers and materials scientists use the Hill-48 yield criterion to develop advanced materials with tailored mechanical properties. By understanding how materials respond to anisotropic loading, they can design materials optimized for specific engineering applications.

The Hill-48 yield criterion is particularly relevant to this research, which aims to understand how AZ31 magnesium alloy responds to different orientations and temperatures during V-bending tests. By incorporating the anisotropic nature of AZ31 into the analysis using the Hill-48 criterion, researchers can gain deeper insights into how the material behaves under complex loading conditions. This knowledge

contributes to the optimization of manufacturing processes and the enhancement of product reliability, ultimately advancing the field of materials science and engineering.

The fundamental equation of the Hill-48 yield criterion is as follows:

$$F = \sqrt{\left(\frac{\sigma_{11}-\sigma_{11}^0}{K_1}\right)^2 + \left(\frac{\sigma_{22}-\sigma_{22}^0}{K_2}\right)^2 + \left(\frac{\sigma_{33}-\sigma_{33}^0}{K_3}\right)^2 + 2\left(\frac{\sigma_{23}}{K_4}\right)^2 + 2\left(\frac{\sigma_{31}}{K_5}\right)^2 + 2\left(\frac{\sigma_{12}}{K_6}\right)^2} - 1. \quad (2.3)$$

Where:

- F is the yield function.
- σ_{ij} are the stress components in the material.
- σ_{ij}^0 are the reference yield stress components along each axis.
- K_1 to K_6 are material constants defining the material's anisotropic behavior.

This mathematical expression describes the yield criterion as the root of a specific equation, where σ_{ij} represents the actual stresses acting on the material, and σ_{ij}^0 denotes the reference yield stresses along each crystallographic axis. The material constants K_1 to K_6 encapsulate the anisotropic response of the material.

The Hill-48 yield criterion's formulation accounts for the anisotropic behavior by considering the differences between the actual stresses and the reference yield stresses along each axis. It incorporates these differences into a scalar quantity F , which quantifies the material's deviation from yielding. When F equals zero, the material is at the point of yield.

2.7.3. The Barlat-89 Yield Criterion

The Barlat-89 yield criterion stands as a cornerstone in the field of materials science and mechanics, offering a comprehensive framework for characterizing the yielding behavior of anisotropic materials, with a particular emphasis on sheet metals. Developed by Barlat, et al. [160] in 1989, this criterion has become an invaluable tool for engineers and researchers in industries where the anisotropic nature of materials

plays a pivotal role, such as automotive manufacturing, aerospace engineering, and metal forming processes. At its core, the Barlat-89 yield criterion is based on the concept of effective stress and strain, acknowledging the intricate anisotropic properties of materials. It provides a mathematical representation of the material's yielding behavior under complex loading conditions, enabling accurate predictions and precise modeling of anisotropic material response.

The key steps and equations involved in the Barlat-89 yield criterion can be summarized as follows [160]:

- Stress Tensor and Effective Plastic Strain: Defining the stress tensor σ and the effective plastic strain ε_p .
- Principal Stresses: Compute the principal stresses $(\sigma_1, \sigma_2, \sigma_3)$ from the stress tensor σ .
- Deviatoric Stress Invariants: Calculate the deviatoric stress invariants (I_1, I_2, I_3) using the principal stresses:

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3, \quad I_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1, \quad I_3 = \sigma_1\sigma_2\sigma_3 \quad (2.4)$$

- Effective Stress and Strain: Compute the effective stress σ_{eff} and the effective plastic strain ε_{eff} using the stress tensor and plastic strain tensor. These are calculated as follows:

$$\sigma_{\text{eff}} = \sqrt{\frac{3}{2} \sum_{i=1}^3 (\sigma_i - \sigma_{\text{ave}})^2}, \quad \varepsilon_{\text{eff}} = \sqrt{2 \sum_{i=1}^3 \varepsilon_i^2}$$

Here, (σ_{ave}) represents the hydrostatic stress, and (ε_i) are the plastic strain components.

- The Barlat-89 Yield Function: Define the Barlat-89 yield function F as a function of the effective stress σ_{eff} , effective plastic strain ε_{eff} , and material parameters:

$$F(\sigma_{\text{eff}}, \varepsilon_{\text{eff}}, \text{material parameters}) = 0$$

- Yielding Criteria: Material yielding occurs when the Barlat-89 yield function F equals zero.

The distinctive feature of the Barlat-89 yield criterion lies in its consideration of material anisotropy through the incorporation of material parameters. These parameters capture the material's directional dependence, allowing for accurate modeling of its response to various loading directions. Calibration of these parameters typically involves experimental testing, ensuring that the criterion accurately represents the material's behavior.

In practical applications, the Barlat-89 yield criterion finds extensive use in metal forming processes, where sheet metals undergo complex deformations. By accurately accounting for the anisotropic behavior of these materials, engineers and manufacturers can optimize forming processes, reduce waste, and improve the quality of final products. Furthermore, it plays a critical role in aerospace engineering, where the precise modeling of materials under varying loading conditions is paramount for structural integrity and performance.

In conclusion, the Barlat-89 yield criterion stands as a powerful and versatile tool in materials science and engineering, offering a nuanced understanding of anisotropic material behavior. Its mathematical foundation, combined with practical applicability, empowers engineers and researchers to make informed decisions in the design and manufacturing of components and structures, ultimately enhancing the efficiency and reliability of a wide range of industrial processes.

2.7.4. The YLD2000 Criteria

The Barlat-2000 yield criterion, often referred to as YLD2000, was developed by Barlat and colleagues as an advanced model to capture the anisotropic plastic behavior of sheet metals. This criterion was introduced to address the limitations of earlier models and to provide a more generalized approach to describe the yield behavior of anisotropic materials, such as the AZ31 magnesium alloy. The Barlat-2000 yield criterion is a phenomenological yield function that is designed to capture the anisotropic behavior of sheet metals under complex loading conditions. Unlike some

earlier models, which might be limited to specific loading conditions or material types, the Barlat-2000 criterion offers a more comprehensive description, making it suitable for a wide range of materials with varying degrees of anisotropy. The Barlat-2000 criterion was derived based on observations of experimental results, capturing the yielding behavior of materials with pronounced anisotropy more accurately than some of its predecessors. At its essence, the YLD-2000 function represents the anisotropic response of metals by employing a non-quadratic formulation, taking into account the complex interplay of stresses in various orientations relative to the material's inherent structure.

Mathematically, this criterion defines the onset of yielding through a function that considers stress states in multiple orientations, coupled with parameters that describe the inherent anisotropic behavior of the material in question. As with other criteria, a scalar value is derived from the function, indicating the material's proximity to yielding.

The YLD2000 yield function for a general three-dimensional stress state can be expressed as [161]:

$$F(\sigma) = \sum_{i=1}^3 \sum_{j=1}^3 C_{ij} \left(\frac{\sigma_i - \sigma_j}{\sigma_0} \right)^2 + \sum_{i=1}^3 \alpha_i \sigma_{ij}^2 - 1 = 0 \quad (2.5)$$

Where:

- σ_i and σ_j are the principal stresses.
- σ_{ij} are the shear stress components.
- σ_0 is the reference stress, typically the uniaxial yield stress.
- C_{ij} and α_i are material parameters that can be determined experimentally.

The coefficients C_{ij} and α_i are essential for capturing the anisotropic behavior of the material. Their values can be determined through a combination of uniaxial, biaxial, and shear tests. For plane stress conditions, the stress component σ_{33} is zero, and the criterion can be simplified. The 2D version is:

$$F(\sigma) = C_{11} \left(\frac{\sigma_{11}}{\sigma_0} \right)^2 + C_{22} \left(\frac{\sigma_{22}}{\sigma_0} \right)^2 + 2C_{12} \left(\frac{\sigma_{11}\sigma_{22}}{\sigma_0^2} \right) + \alpha_1 \sigma_{12}^2 - 1 = 0 \quad (2.6)$$

The effective stress, often denoted as σ_{eff} , is a scalar representation of the stress state in a material. For many anisotropic yield criteria, including YLD2000, the effective stress is derived from the yield function by equating it to the current yield stress of the material. For the YLD2000 criterion, the effective stress σ_{eff} can be defined in terms of the yield function $F(\sigma)$ as:

$$\sigma_{\text{eff}} = \sigma_0 \sqrt{\sum_{i=1}^3 \sum_{j=1}^3 C_{ij} (\sigma_i - \sigma_j)^2 + \sum_{i=1}^3 \alpha_i \sigma_{ij}^2} \quad (2.7)$$

The YLD2000-18p model, as the name suggests, uses 18 parameters to describe the anisotropic yield behavior of sheet metals. These parameters are determined from experimental data. The coefficients are typically denoted as C_1 to C_8 for the linear combinations of stress differences and α_1 to α_{10} for the quadratic terms of the shear stresses.

$$C_i = f_i \text{ (experimental data)} \quad (2.8)$$

$$\alpha_j = g_j \text{ (experimental data)} \quad (2.9)$$

However, it's important to note that the exact equations for these coefficients are not universally defined. Instead, they are determined by fitting the YLD2000 model to experimental data from various mechanical tests. The general approach is to conduct a series of mechanical tests (uniaxial tension, biaxial tension, plane strain tension, and shear tests) and then use optimization techniques to determine the coefficients that best fit the experimental yield surfaces. The Barlat-2000 criterion has found significant traction in industries that heavily rely on metal forming processes. By leveraging the YLD-2000 model, engineers and researchers can obtain a more representative simulation of real-world processes, facilitating the optimization of manufacturing stages, reducing material wastage, and ensuring the robustness and quality of final products.

2.7.5. The Yld2004-18p Yield Criteria

The Yld2004-18p yield criterion, a phenomenological model, is designed to account for the anisotropic plastic behavior of sheet metals. The yield function (F) associated with this criterion can be mathematically represented as follows [162]:

$$[F(\sigma, X) = \left(\sum_{i=1}^6 a_i \sigma_i^2 + \sum_{i < j}^6 a_{ij} (\sigma_i \sigma_j)^p \right)^{\frac{1}{p}} - Y = 0] \quad (2.10)$$

where:

- (σ) is the stress tensor,
- (X) is the vector of the anisotropy coefficients,
- (a_i) and (a_{ij}) are the anisotropy coefficients that are determined experimentally,
- (σ_i) and (σ_j) are the components of the stress tensor,
- (p) is a material-dependent exponent,
- (Y) is the equivalent uniaxial yield stress of the material.

The anisotropy coefficients are derived from biaxial tests and are defined such that the yield surface best fits the experimental yield stresses in various loading directions. The exponent (p) influences the shape of the yield surface, with (p) > 1 typically yielding a more pronounced anisotropic effect and better representing the complex anisotropy in metallic materials. This yield function is normalized such that it equates to the uniaxial yield stress (Y) in the direction of the reference stress state. For instance, if the reference stress state is uniaxial tension in the rolling direction, then ($F(\sigma, X)$) will equate to the yield stress in that direction.

In applying the Yld2004-18p yield criterion within a finite element model, these coefficients (a_i) and (a_{ij}) are calibrated to the specific material being analyzed, often using the results from various mechanical tests such as uniaxial tension/compression, in-plane shear, and biaxial tension. The calibration process involves an optimization

technique where the difference between the predicted and experimental yield stresses is minimized for all directions of loading. By incorporating the Yld2004-18p yield criterion into your thesis, you will be able to account for the subtle nuances of material anisotropy and predict how changes in the stress state affect yield behavior. This could be particularly useful when interpreting the results of V-bending and disk compression tests, as the Yld2004-18p model provides a comprehensive framework for understanding and predicting the directional dependence of yield stress in the material under investigation.

2.7.6. The Bezier5YS-2023 and SHYqp-2023 Yield Criteria

The Bezier5YS model stands as a significant advancement in the realm of material science, particularly in the modeling of yield surfaces. Yield surfaces are pivotal in understanding the behavior of materials under various stress conditions. They provide a boundary in the stress space, beyond which plastic deformation begins. The accurate representation of these surfaces is crucial for predicting material behavior, especially in complex loading scenarios. The Bezier5YS model is termed a "proto-model" because it serves as a foundational or preliminary model that can be further refined based on specific requirements or additional data. Its primary advantage lies in its ability to generate a complete plane stress yield surface using geometrical considerations. Plane stress conditions are prevalent in thin-walled structures or thin plates, where one dimension is significantly smaller than the other two. In such scenarios, understanding the material's yielding behavior is essential for design and analysis purposes.

The mathematical representation of the Bezier5YS model's yield surface is given by [163]:

$$F(\sigma) = \sum_{i=0}^5 B_i \delta_i(\theta) \quad (2.11)$$

In this equation:

- $F(\sigma)$ denotes the yield function, which provides a scalar value representing the state of stress in relation to the yield surface. When this function equals zero, the material is on the verge of yielding.
- The summation runs over a set of control points, represented by B_i . These control points are pivotal in defining the shape of the Bezier curve, which in turn defines the yield surface. The more control points used, the more flexible and intricate the yield surface can become. However, in the Bezier5YS model, a total of six control points are used, providing a balance between accuracy and computational efficiency.
- $\delta_i(\theta)$ are the shape parameters. These parameters play a crucial role in fine-tuning the yield surface to match experimental data. They can be visualized as "handles" that can be adjusted to stretch or compress the yield surface in various directions.
- θ is the angle parameter. It provides a directional context to the yield surface, allowing the model to represent different yielding behaviors in different directions.

The Bezier5YS model is further characterized by several meta-parameters:

- Averaging parameter for tangent directions: ($\lambda = 1/2$)
- Scaling parameter for directional properties: ($\mu = 60\%$)
- For Bezier5YS with one shape parameter, the scaling for $\epsilon_{\max}$ obtained is: ($\mu = 100\%$)

These meta-parameters provide additional flexibility in shaping the yield surface. For instance, the averaging parameter influences the tangent directions, while the scaling parameters adjust the magnitude of certain directional properties. The ability to fine-tune these parameters ensures that the Bezier5YS model can be tailored to a wide range of experimental datasets.

In conclusion, the Bezier5YS model offers a robust and adaptable approach to modeling plane stress yield surfaces. Its balance of simplicity and accuracy, combined

with its adaptability to various data sets, makes it a valuable tool in the field of material science and engineering.

The SHYqp model emerges as a sophisticated tool in the domain of material science, specifically tailored to capture the yielding behavior of materials under various stress conditions. Yielding behavior is a fundamental aspect of materials, dictating their response when subjected to external forces. Accurately modeling this behavior is paramount for predicting material performance in real-world applications.

At its core, the SHYqp model employs an analytical formula, which is mathematically represented as:

$$G(\sigma) = \sum_{i=0}^n \alpha_i \sigma_i^2 \quad (2.12)$$

Here:

- $G(\sigma)$ is the yield function, a scalar representation that provides insights into the stress state of the material in relation to the yield surface. When this function reaches zero, the material is on the brink of yielding.
- α_i are the harmonic polynomial parameters. These parameters are pivotal as they influence the shape and orientation of the yield surface. Their optimization ensures that the model can be adapted to various datasets and experimental results.
- σ_i are the components of the stress tensor, which provide a multi-dimensional view of the stress state within the material.

One of the standout features of the SHYqp model is its ability to optimize the harmonic polynomial parameters to ensure the best fit to the Bezier5YS proto-model. This optimization ensures that the SHYqp model is not only accurate but also versatile, capable of adapting to a wide range of experimental data.

The construction of the SHYqp model involves minimizing the weighted distance between its predictions and the proto-model generated by the Bezier5YS model. This minimization process is crucial for ensuring that the SHYqp model aligns closely with

the foundational Bezier5YS model. Different weights, symbolized by ω , are judiciously assigned to actual data points and proto-model points. These weights are scaled uniformly, forming a binary partition of the data, which aids in the optimization process.

In the broader context of classical (associate) plasticity, the yield surface plays a pivotal role in determining the increment of plastic strain when the material reaches a yielding stress state. This is governed by the normality rule, a fundamental principle in plasticity. The SHYqp model's convexity is of paramount importance for its proper functioning. This convexity ensures that there's a unique relationship between a yielding stress and the corresponding increment of plastic deformation. This relationship is mathematically encapsulated by the Hessian matrix. Further, the tensor products essential for this relationship. With the aid of Voigt's notation, the second-order derivatives of the function f can be computed. Delving into the realm of orthotropic symmetry, the SHYqp model provides specifications for the loading direction and the stress state generated by a uniaxial test, represented by $v(\theta)$ and $\sigma(\theta)$, respectively. The direction perpendicular to the loading direction is denoted as $v_{\perp}$. Additionally, the model defines the r value of the test, which is the ratio ($r = \frac{d\epsilon_{\perp}}{d\epsilon}$), providing insights into the strain behavior of the material.

CHAPTER 3

TESTING AND ANALYSIS OF AZ31 ALLOY

3.1. Sample Preparation and Manufacturing

The study of materials, especially metals and alloys, necessitates a meticulous approach to sample preparation and manufacturing. This chapter delves deep into the intricacies of preparing and manufacturing samples of the AZ31 magnesium alloy, a material renowned for its lightweight properties and significant potential in various applications. Sample preparation and manufacturing form the foundation of any materials science investigation. In this chapter, we delve into the critical aspects of how we prepared and manufactured our specimens for the comprehensive analysis of the AZ31 magnesium alloy. The meticulousness and precision with which samples are prepared are instrumental in ensuring the accuracy and reliability of subsequent testing and analysis. In our study, we primarily employed hot-rolled AZ31 alloy sheets as the starting material for our sample preparation process.

The accuracy and reliability of any material testing largely hinge on the quality of the sample preparation. In the realm of metallurgy and material science, the slightest deviation in sample preparation can lead to significant discrepancies in test results. For our research, we opted for hot-rolled AZ31 alloy sheets as the primary material. Hot rolling is a process that involves heating the metal above its recrystallization temperature and then deforming it between rollers to produce a sheet. This process ensures that the material retains its inherent properties, making it an ideal choice for our experiments. Hence, our study prioritized ensuring that the AZ31 alloy samples were prepared with the utmost precision. To create specimens suitable for a variety of tests, we opted for the water jet cutting technique. This choice was driven by its ability to deliver precise and reproducible results. We began by cutting the AZ31 alloy sheets into circular shapes with a diameter of 1 inch, utilizing a state-of-the-art water jet cutting machine. This initial step was fundamental in obtaining specimens of uniform dimensions, which is paramount to the integrity of our experiments. By ensuring

consistent specimen dimensions, we minimized variability and enhanced the repeatability of our tests.

However, our investigation did not end with the creation of disc compression specimens. We extended our sample preparation process to accommodate the needs of V-bending tests, aiming to comprehensively understand how the AZ31 alloy responds to different orientations. To achieve this, we once again turned to water jet cutting. By altering the orientation angles, we created specimens oriented at 0 degrees, 45 degrees, 75 degrees, and 90 degrees. This deliberate variation allowed us to investigate and compare the alloy's mechanical behavior under diverse bending conditions. It provided valuable insights into the anisotropic properties of the AZ31 alloy, shedding light on how its mechanical characteristics change with orientation. Figure 3.1 represents anisotropy directions of the specimens.

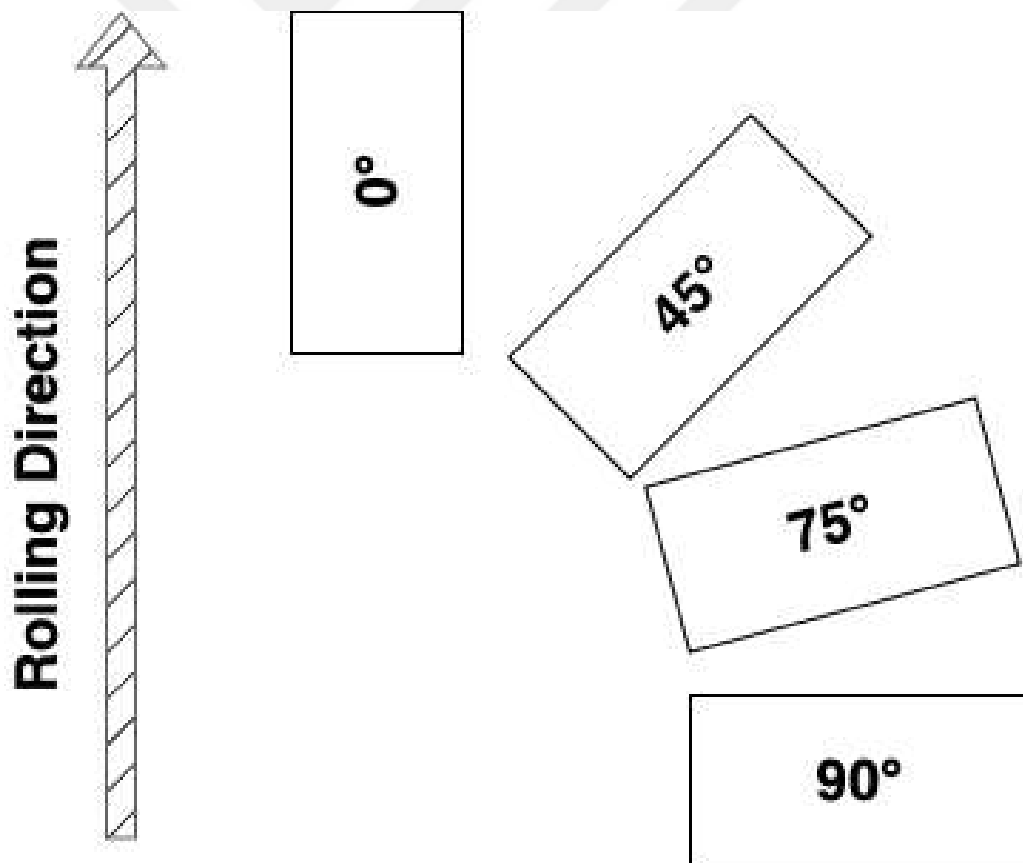


Figure 3.1 Anisotropy directions of the specimens

The water jet cutting technique was our method of choice for creating specimens. This non-traditional machining process uses a high-pressure stream of water to cut materials. For our study, we cut the AZ31 alloy sheets into 1-inch diameter circles. The precision of the water jet cutting machine ensured that all specimens had consistent dimensions, a critical factor for the reproducibility of our experiments. A significant consideration in our choice of water jet cutting as the sample preparation method was the minimization of potential thermal effects or mechanical stresses that could be introduced during the cutting process. By selecting this technique, we sought to ensure that the properties of the AZ31 alloy specimens remained as close as possible to those of the original material. This preservation of material integrity was essential to guarantee that our subsequent testing and analysis accurately reflected the behavior of the AZ31 alloy in its natural state. In the upcoming sections of this chapter, we will delve into the specific testing methodologies employed, encompassing V-bending tests and disc compression tests. Our discussion will encompass the comprehensive setup procedures, including heating processes when necessary, data collection techniques, and the rigorous safety precautions taken to safeguard the integrity of our experiments. These meticulous procedures are integral to our research, as they play a pivotal role in yielding reliable data. By adhering to strict protocols, we are better positioned to draw meaningful conclusions regarding the mechanical behavior of the AZ31 alloy and contribute valuable insights to the field of materials science.

3.2. V-Bending Tests

V-bending tests, commonly used in the field of material science and metallurgy, are pivotal for assessing the bending behavior of sheet metals and alloys. These tests provide valuable insights into the material's ductility, flexibility, and resistance to deformation under bending loads. For our study on the AZ31 magnesium alloy, the V-bending tests were instrumental in understanding the alloy's mechanical behavior under specific bending conditions. The primary objective of conducting the V-bending tests on the AZ31 alloy specimens was to evaluate:

1. The alloy's resistance to deformation during bending.
2. The onset of any potential cracks or fractures during the bending process.

3. The influence of different orientations (0 degrees, 45 degrees, 75 degrees, and 90 degrees) on the bending behavior.
4. The correlation between the alloy's microstructure and its bending properties.

Figure 3.2 shows the workflow for disc compression tests.

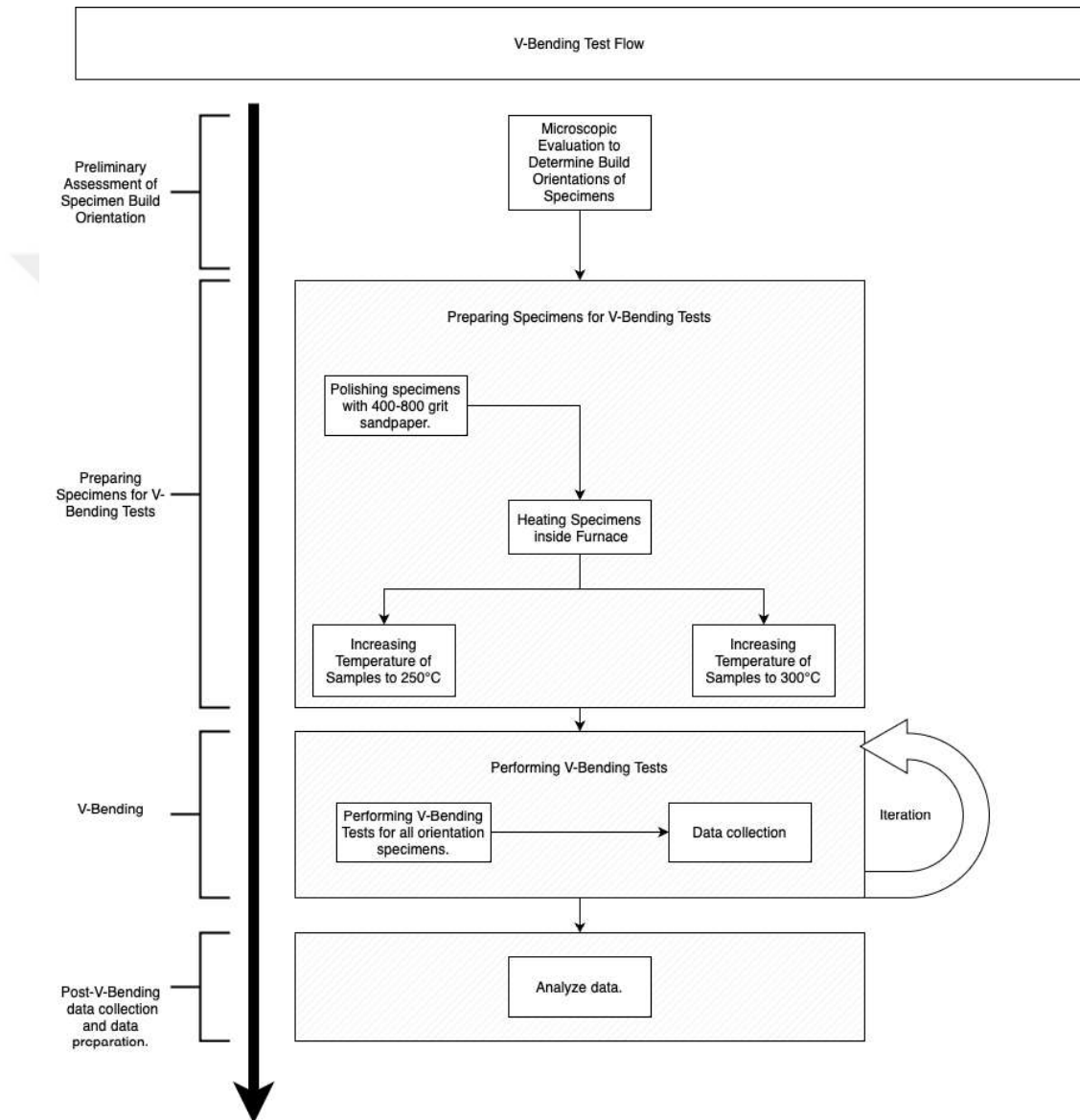


Figure 3.2 Workflow of V-bending tests

3.2.1. Test Setup

The V-bending tests were conducted using a standardized V-bending apparatus, which is shown in Figure 3.3. The apparatus comprised a V-shaped die and a punch, designed

to exert a controlled force on the specimen. The specimen, placed over the die, was subjected to bending as the punch descended onto it. A hydraulic press facilitated the controlled movement of the punch, ensuring consistent and reproducible test conditions.



Figure 3.3 V Bending test apparatus with 60° punch and the die used in procedure, inside the thermostatic chamber

Specimens for each orientation, is given at Figure 3.4, for illustration purposes.



Figure 3.4 V-bending Specimens

Test procedure is as follows:

1. Each AZ31 alloy specimen was meticulously cleaned and marked for orientation.
2. The specimen was placed over the V-shaped die, ensuring proper alignment.
3. The tensile test machine was activated, causing the punch to descend onto the specimen, initiating the bending process.
4. The bending was continued until a predetermined angle was achieved or until the onset of cracks was observed.
5. The bent specimen was then carefully removed and subjected to post-test analysis.

Figure 3.5 represents the schematic for V-bending process.

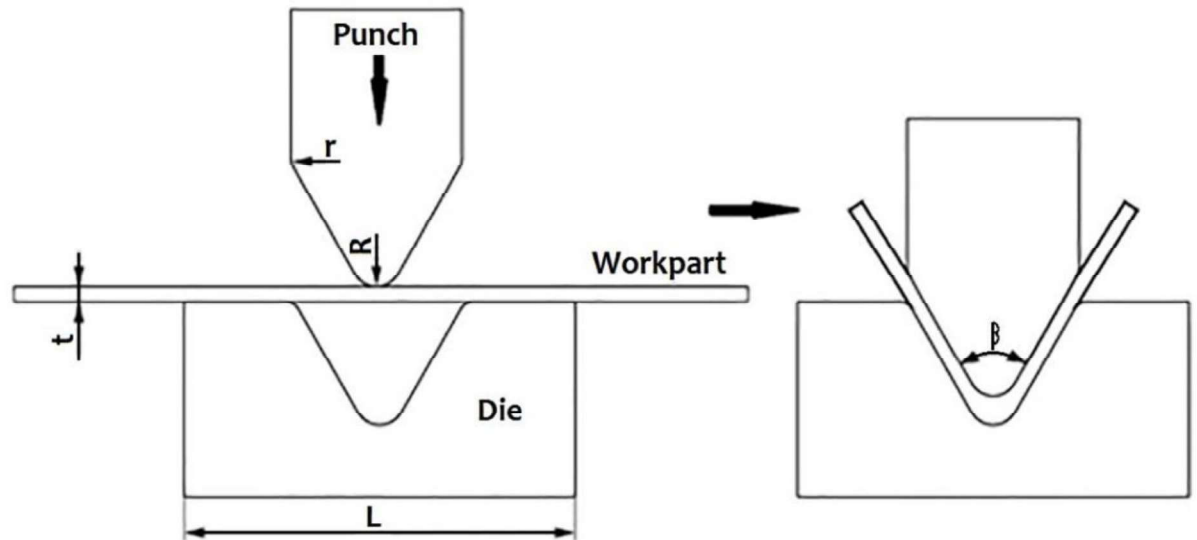


Figure 3.5 Representation of V-bending process [164]

3.2.2. Heating and Furnace

In material science, particularly when dealing with metals and alloys, controlled heating is a crucial step. Heating can alter the microstructure of the material, thereby influencing its mechanical properties. For materials like the AZ31 magnesium alloy, understanding the effects of heating is essential to predict its behavior under various conditions, especially when subjected to processes like bending.

The primary goals of the controlled heating process in our study were to:

- Achieve a uniform temperature distribution throughout the AZ31 alloy specimen.
- Study the phase transformations, if any, in the alloy upon heating.
- Understand the influence of heating on the alloy's subsequent bending behavior.
- Determine the optimal temperature range for the V-bending tests to ensure consistent results.

The furnace used for heating the AZ31 alloy specimens was a high-precision, electrically powered muffle furnace, which is integrated into testing equipment. Key specifications include:

- Temperature range: Ambient to 500°C.
- Temperature accuracy: $\pm 1^\circ\text{C}$.
- Heating rate: Adjustable, with a maximum of $1^\circ\text{C}/\text{min}$.
- Interior lining: Refractory ceramic fiber, ensuring uniform heat distribution.

Heating procedure is undergone using Shimadzu AG-IS 100 kN integrated furnace, which can be connected directly into the testing equipment, namely AUTOGRAPH Thermostatic Chamber. This also enables the tools to be the same environment temperature for the procedure. Figure 3.6 shows the temperature controller unit.

- The AZ31 alloy specimens were cleaned to remove any surface contaminants.
- Specimens were placed on a ceramic tray inside the furnace.
- The furnace was set to either 200°C or 250°C , ensuring a gradual heating rate to prevent thermal shocks.
- Once the target temperature was reached, the specimens were held at that temperature for a predetermined duration to ensure uniform heating.
- After the heating cycle, the specimens were allowed to cool gradually.



Figure 3.6 Shimadzu temperature controller unit

Throughout the heating process, temperature data was continuously logged using thermocouples placed inside the furnace. This data provided insights into the temperature uniformity and the actual heating rate achieved. Any deviations from the set parameters were noted and analyzed for potential impacts on the test results. After the heating process, the AZ31 alloy specimens were immediately measured after procedure using Mitutoyo 187-906 Universal Bevel Protractor, then after specimens cooled down after 2 hours, again measured to measure sprinback angle and temperature effect on specimens. Figure 3.7 shows the protractor used in measurements.



Figure 3.7 Mitutoyo 187-906 Universal Protractor used in springback angle measurements

Controlled heating using the muffle furnace played a pivotal role in preparing the AZ31 alloy specimens for the V-bending tests. The insights gained from the heating process and the subsequent microstructural analysis provided valuable context for interpreting the results of the bending tests.

3.2.3. Data Collection and Analysis

During the V-bending tests, the primary parameter monitored and recorded was the springback angle. Measurements were taken immediately after the procedure and again after allowing the specimens to rest for 2 hours. Any visible deformations, cracks, or fractures were also documented. Subsequent to the tests, the bent specimens underwent microscopic examination to study the microstructural changes and to identify any subsurface cracks. The data collected was then analyzed to draw correlations between the bending behavior and the material's inherent properties. Using data fitting, changes in springback angles were measured and recorded.

While a detailed discussion of the results will be presented in the subsequent sections, some of the key findings from the V-bending tests include:

1. The AZ31 alloy exhibited commendable resistance to bending-induced deformations, especially at higher temperatures.
2. Specimens with different orientations showcased varying bending behaviors, underscoring the alloy's anisotropic properties.
3. Microscopic examinations revealed a strong correlation between the alloy's microstructure and its bending performance.

3.2.4. Measurement of Springback Angle

Springback is a phenomenon commonly observed in sheet metal forming processes, where the material tends to revert partially to its original shape after being deformed. This elastic recovery can significantly affect the final shape of a formed part, making its accurate measurement crucial for predicting and controlling the outcomes of forming processes, such as V-bending. This procedure is given in Figure 3.8.

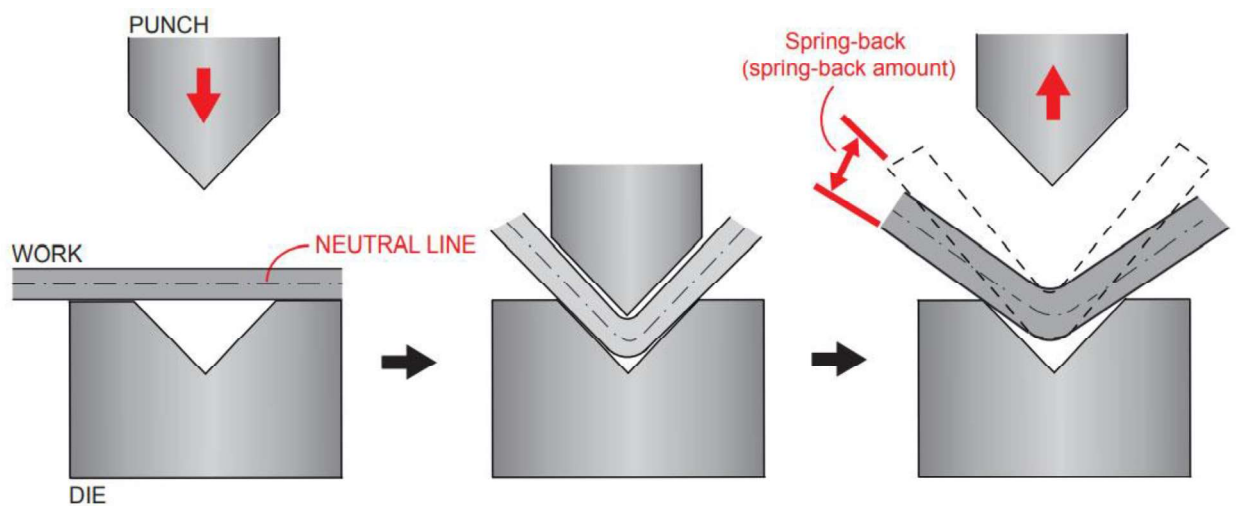


Figure 3.8 Springback mechanism [165]

The springback angle is a critical parameter in bending operations, especially for materials like the AZ31 magnesium alloy, which may exhibit varying degrees of springback based on its microstructure, temperature, and orientation. Accurate measurement of the springback angle is essential for:

- Ensuring the final product meets design specifications.
- Predicting the material's behavior in real-world applications.
- Adjusting the bending process parameters for optimal results.

To measure the springback angle, we employed a high-precision digital protractor, ensuring accuracy and repeatability. The methodology is as follows:

- Initial Setup: The bent specimen was placed on a flat surface with the bend facing upwards.
- Alignment: The digital protractor was aligned with the bent section of the specimen.
- Measurement: The angle of deviation from the intended bend angle was recorded as the springback angle.
- Repeatability: Multiple measurements were taken at different points along the bend to ensure consistency.

Several factors can influence the magnitude of springback in the AZ31 magnesium alloy:

- Material Properties: The inherent elasticity and yield strength of the alloy play a significant role.
- Orientation: As our specimens were oriented at 0 degrees, 45 degrees, 75 degrees, and 90 degrees, variations in springback were observed.
- Temperature: The specimens heated to 200°C and 250°C exhibited different springback behaviors, indicating the influence of temperature on the alloy's elasticity.

All recorded springback angles were tabulated and analyzed statistically to understand the behavior of the AZ31 alloy under different conditions. The mean, median, and standard deviation of the springback angles were calculated for each set of conditions.

Graphical representations, such as histograms and box plots, were also generated to visualize the data distribution.

3.3. Disc Compression Tests

The disc compression tests serve as a pivotal methodology to evaluate the compressive properties of materials, particularly the AZ31 magnesium alloy. Through this testing approach, specimens, prepared with meticulous precision, are subjected to controlled compressive loads, allowing for a comprehensive understanding of the material's behavior under such conditions. The process encompasses several stages, from specimen preparation, which includes polishing and lubrication, to the actual compression testing at specified strain rates, and subsequent data analysis. The insights derived from these tests are instrumental in elucidating the inherent characteristics of the AZ31 alloy, offering valuable data for potential real-world applications. Figure 3.9 shows the workflow for disc compression tests.

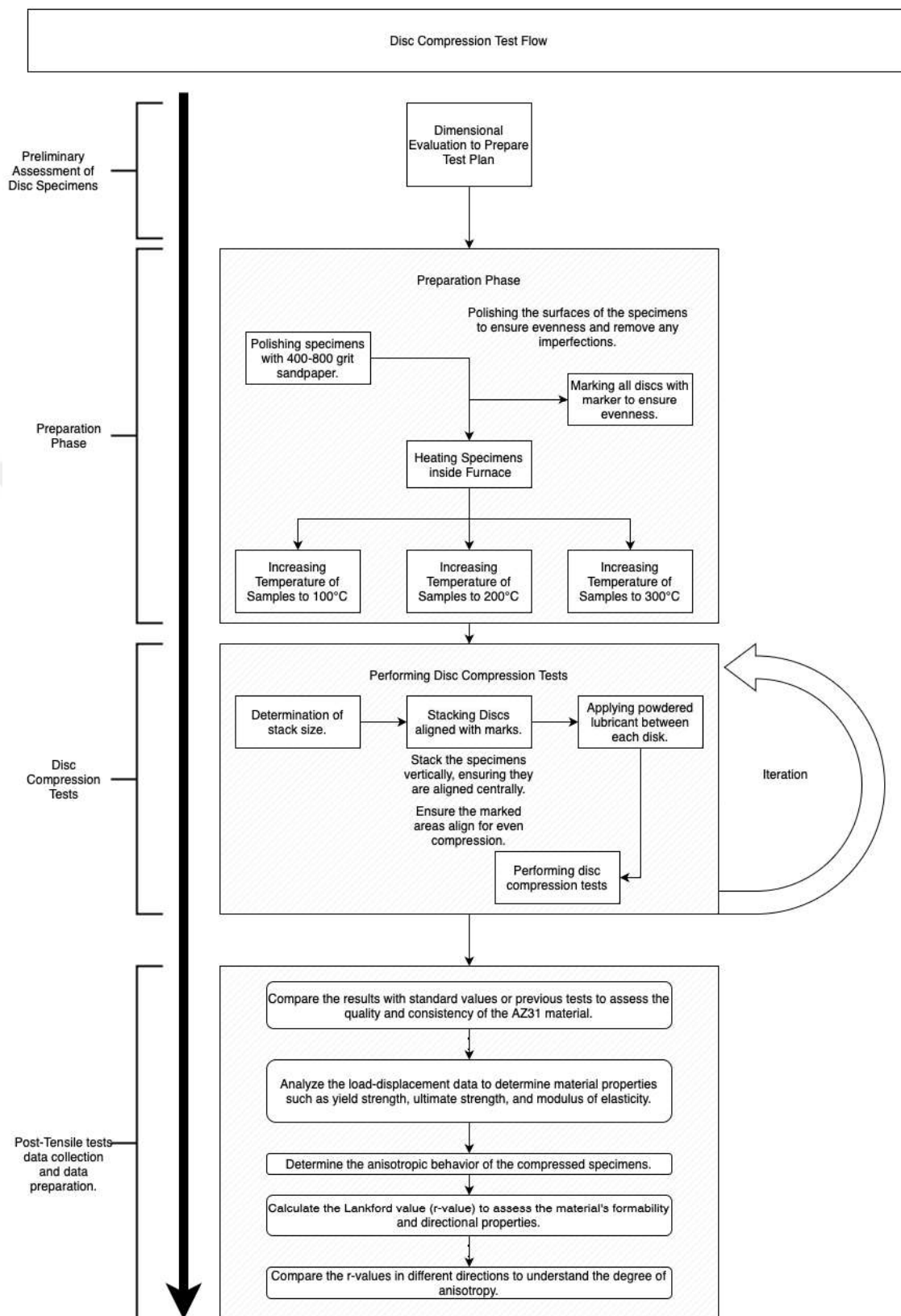


Figure 3.9 Workflow of disc compression tests

3.3.1. Disc Compression Test Setup

In the realm of experimental mechanics, several key components and instruments play crucial roles in the accurate assessment of material properties under compressive loads. First and foremost, a compression machine stands as the central apparatus, available in hydraulic or electromechanical variants, each adept at exerting precisely controlled loads on the specimen under investigation. Working in concert with the compression machine are platens, which are flat and parallel surfaces designed to transmit the compressive force uniformly to the specimen, ensuring consistent and reliable test results. To quantify the force applied during the compression test, a load cell is employed; this device is instrumental in providing accurate measurements of the forces acting on the specimen throughout the testing process. Additionally, an extensometer is employed to gauge and record the deformation experienced by the specimen, yielding essential data for a comprehensive understanding of its mechanical behavior. These meticulously selected and integrated components collectively form the foundation of experimental compression testing, allowing for precise and insightful analyses of material properties and performance characteristics. Disc compression tests, also known as disc or flat cylinder or stacked disc compression tests, are designed to evaluate the compressive properties of materials in a disc-like geometry. This test setup is particularly beneficial for materials like the AZ31 magnesium alloy, as it provides insights into the material's behavior under compressive loads. Test setup is also represented in Figure 3.10.

The test apparatus consists of:

1. **Compression Machine:** A hydraulic or electromechanical compression machine capable of applying controlled loads.
2. **Platens:** Flat, parallel surfaces that apply the compressive force to the specimen.
3. **Load Cell:** A device that measures the force applied to the specimen.
4. **Extensometer:** A device that measures the deformation of the specimen.

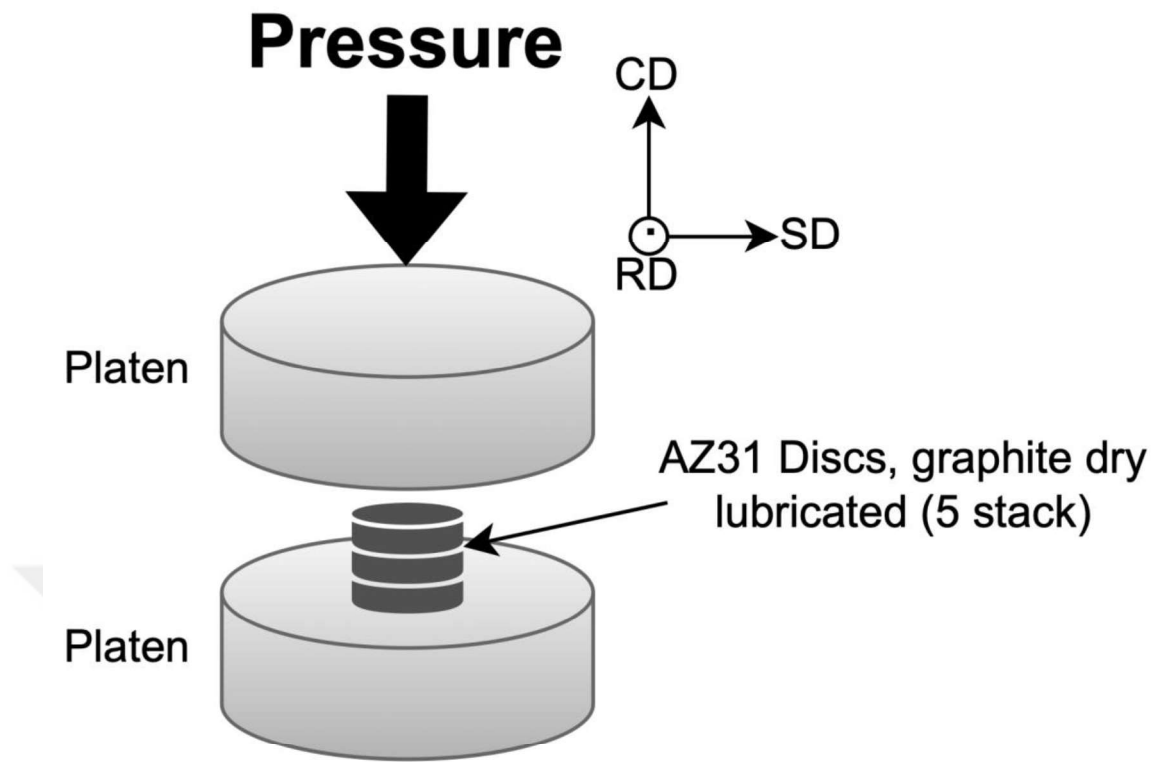


Figure 3.10 Representation of disc compression tests

3.3.2. Preparing Specimens

For the disc compression tests, the specimens were prepared in a disc shape, ensuring uniformity and consistency across all samples. In this academic context, the material selection process involved the deliberate choice of hot-rolled AZ31 alloy sheets as the initial material for the study. Following this selection, the sheets underwent a cutting phase, employing the precise water jet cutting technique to create uniform discs with a consistent diameter. After the cutting phase, attention was turned towards surface finishing, where the cut discs underwent a polishing procedure. This polishing was essential to eliminate any surface irregularities, thereby guaranteeing a smooth and uniform surface conducive to accurate testing. Furthermore, each specimen was carefully marked and labeled for identification purposes, with specific details noted regarding any particular orientations or treatments applied during the experimental process. These meticulous steps in material selection, cutting, surface finishing, and specimen marking were essential to ensure the reliability and repeatability of subsequent testing and analysis in the research study. The preparation process is as follows:

- **Material Selection:** Hot-rolled AZ31 alloy sheets were chosen as the starting material.
- **Cutting:** Using the water jet cutting technique, the sheets were cut into discs with a consistent diameter.
- **Surface Finishing:** The cut discs were then polished to remove any surface irregularities, ensuring a smooth surface for testing.
- **Marking:** Each specimen was labeled for identification, noting any specific orientations or treatments.

3.3.3. Heating and Furnace

The same equipment used for V-Bending tests are also used for Disk Compression Tests. Controlled heating is essential in material science to understand how temperature variations influence a material's compressive properties. For the AZ31 magnesium alloy, this step is crucial to evaluate its behavior under different thermal conditions.

In the context of metallurgical experimentation with AZ31 alloy specimens, a systematic and controlled thermal treatment process was meticulously executed. To begin, it was imperative to commence with a thorough cleaning of the specimens to eliminate any potential surface contaminants that might interfere with subsequent heat treatment. Subsequently, these cleaned specimens were carefully positioned on a ceramic tray within the furnace, a critical step in ensuring their exposure to the controlled thermal environment. The furnace settings were then precisely configured to attain the desired temperature, and special attention was given to implementing a gradual heating rate. This gradual heating approach was adopted to prevent thermal shocks, which could adversely affect the integrity of the specimens.

Once the target temperature was reached, a predefined duration of thermal exposure was enforced to facilitate uniform heating throughout the specimens. This controlled holding period at the designated temperature was vital for achieving the desired

metallurgical transformations and material properties. Following the completion of the heating cycle, a gradual cooling process was instituted. This gradual cooling regimen was executed with careful consideration, allowing the specimens to cool slowly, thus avoiding any abrupt temperature changes that could introduce unintended microstructural alterations. Overall, this methodical approach to thermal treatment played a pivotal role in the comprehensive study of the AZ31 alloy's behavior and properties under specific temperature conditions.

1. The AZ31 alloy specimens were cleaned to remove any surface contaminants.
2. Specimens were placed on a ceramic tray inside the furnace.
3. The furnace was set to the desired temperature, ensuring a gradual heating rate to prevent thermal shocks.
4. Once the target temperature was reached, the specimens were held at that temperature for a predetermined duration to ensure uniform heating.
5. After the procedure, the specimens were allowed to cool gradually.

During the disc compression tests, various parameters were continuously monitored and recorded:

1. **Force:** The compressive force applied to the specimen was measured using the load cell.
2. **Deformation:** The amount of deformation or change in the specimen's height was recorded.
3. **Stress-Strain Curve:** By plotting the stress (force divided by the original area) against strain (change in height divided by the original height), a stress-strain curve was generated. This curve provides insights into the material's elastic and plastic behavior under compression.
4. **Peak Stress and Strain:** The maximum stress and corresponding strain were identified from the stress-strain curve.

All collected data was tabulated and stored for subsequent analysis, ensuring a comprehensive understanding of the AZ31 magnesium alloy's compressive properties.

3.3.4. Specimen Preparation - Analysis

In the pursuit of precision and reliability in the compression testing of specimens, a series of meticulous procedures were diligently followed. Initially, each specimen underwent a rigorous polishing process, employing grit paper to attain a smooth and uniform surface texture. This polishing step held paramount importance, as it served the critical purpose of eliminating any surface irregularities or imperfections that had the potential to introduce undesirable influences on the ensuing test results. Following the polishing stage, a layer of MoS₂ (molybdenum disulfide) graphite dry lubricant was carefully applied to each specimen. This lubrication was strategically employed to reduce friction during the forthcoming compression tests, thereby ensuring that the intrinsic properties of the material could be accurately and faithfully represented without any undue interference. Subsequently, a custom-made setup was employed to painstakingly align the orientation of all the discs in accordance with their predetermined specifications. This meticulous orientation procedure was essential in guaranteeing uniformity in the application of force during the subsequent compression tests, ensuring that all specimens experienced consistent testing conditions.

For the compression tests themselves, a stack consisting of five discs, all possessing the same orientation, was thoughtfully assembled. This stacking configuration was adopted to promote uniformity in the test procedure and to minimize any potential variations that might arise during testing, thus enhancing the precision and repeatability of the experimental results. In essence, these precise and systematic steps collectively formed the cornerstone of the compression testing methodology, enabling the comprehensive exploration of material behavior and properties under controlled conditions. The preparation of the specimens is a meticulous process, ensuring that each sample is consistent and representative of the material being studied. Figure 3.11 shows the graphite dry lubricated specimens prepared as stacks of 5.

1. **Polishing:** Each specimen was polished using grit paper to achieve a smooth and uniform surface. This step is crucial to remove any surface irregularities or imperfections that might influence the test results.
2. **Lubrication:** After polishing, a layer of MoS₂ (molybdenum disulfide) graphite dry lubricant was applied to each specimen. This lubricant reduces friction during the compression tests, ensuring that the material's inherent properties are accurately represented.
3. **Orientation:** Using a custom-made setup, the orientation of all discs was meticulously aligned according to their predetermined orientation. This step guarantees that the force applied during the compression tests acts consistently across all specimens.
4. **Stacking:** For the compression tests, five discs, all with the same orientation, were stacked together. This stacking ensured uniformity in the test procedure and minimized any potential variations in the results.



Figure 3.11 Prepared disc compression specimens, (dry graphite lubricated and in 5 stack)

For compression testing and subsequent analysis of specimens, a structured and systematic approach was meticulously followed to capture and comprehend the material's behavior under varying conditions. Initially, a specific strain rate of 0.05 m/s was deliberately applied during the compression tests. This choice of strain rate was made with the aim of emulating real-world conditions, ensuring that the material's response could be accurately observed and analyzed within a context relevant to

practical applications. Upon the completion of the compression tests, each specimen was carefully extracted from the test chamber. Subsequently, precise measurements of deformation in both the x and y directions were obtained using high-precision instruments. These measurements provided valuable insights into how the material responded under the influence of compressive loads, shedding light on its mechanical behavior in different orientations. To gain a deeper understanding of the material's anisotropic behavior and its response to varying loading conditions, the Lankford parameters were computed using the data collected. These parameters played a pivotal role in characterizing the material's mechanical properties, allowing for the identification of any directional dependencies in its behavior. Following the determination of the Lankford parameters at each temperature level, a comprehensive data analysis was conducted using the statistical software tool Minitab. The collected data underwent a linear fitting process, which yielded a clear and quantifiable representation of the material's behavior across the spectrum of temperature ranges examined. This rigorous data analysis procedure was instrumental in extracting meaningful conclusions about the material's response to compressive loading conditions under different temperature settings, contributing to a more profound comprehension of its mechanical properties and behavior.

1. **Strain Rate:** A strain rate of 0.05 m/s was applied to the specimens during the compression tests. This rate was chosen to simulate real-world conditions and to ensure that the material's response was accurately captured.
2. **Measurement:** After the compression tests, each specimen was removed from the test chamber. The deformation in both the x and y directions was measured using precision instruments. These measurements provide insights into the material's behavior under compressive loads.
3. **The Lankford Parameters:** Using the collected data, the Lankford parameters were calculated. These parameters are essential for understanding the material's anisotropic behavior and its response to different loading conditions.
4. **Data Analysis:** The calculated Lankford parameters for each temperature level were then analyzed using Minitab. A linear fit was applied to the data,

providing a clear representation of the material's behavior across different temperature ranges.

The meticulous specimen preparation and comprehensive analysis ensured that our study accurately captured the behavior of the AZ31 magnesium alloy under compressive loads. The insights gained from this process are invaluable for understanding the material's potential applications and its performance under real-world conditions. Figure 3.12 summarizes the disc compression test procedure.

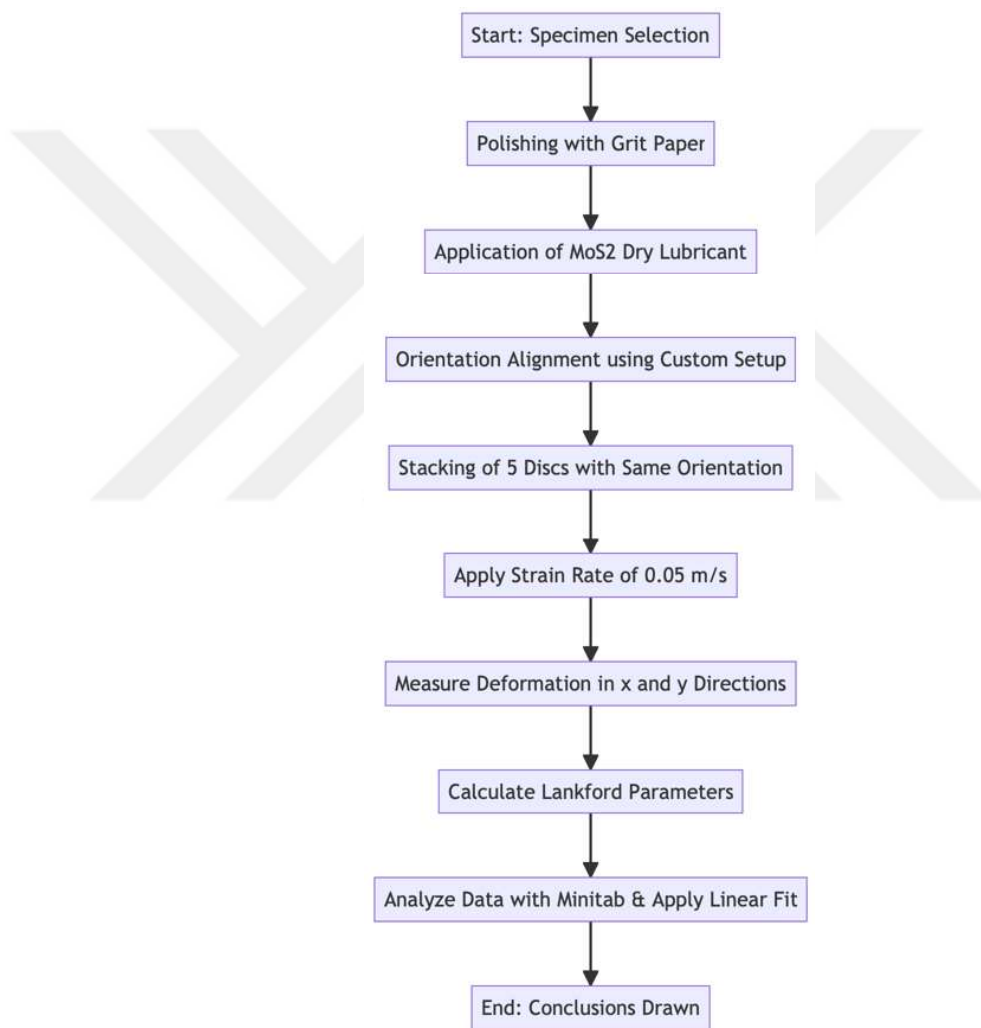


Figure 3.12 Disc compression test flowchart

3.4. Measurement and Data Acquisition Techniques

In the realm of material science and engineering, the accuracy and precision of measurements are paramount. The integrity of the results and the subsequent

conclusions drawn hinge significantly on the reliability of the data acquisition techniques employed. For our study on the AZ31 magnesium alloy, we have incorporated a blend of state-of-the-art equipment and traditional measurement tools to ensure comprehensive and precise data collection.

3.4.1. Imaging and Visualization

Two high-definition cameras were employed to capture real-time visuals during the testing processes. These cameras, given in Figure 3.13, provided a dual-angle view, ensuring that no aspect of the specimen's behavior went unnoticed.



Figure 3.13 High-Definition Cameras integrated into Shimadzu software TRAPEZIUM

For microscopic observations, we utilized the Olympus DP27 microscope, which offers crystal clear imaging capabilities, which is given in Figure 3.14. Depending on the specific requirements of the observation, different magnifications, namely 25x, 10x, 50x, etc., were employed to capture intricate details of the alloy's microstructure.



Figure 3.14 Olympus DP27 microscope

3.4.2. TRAPEZIUM Software and Controllers

The TRAPEZIUM Shimadzu software was an integral part of our data acquisition system. This software, known for its robustness and reliability, facilitated real-time data logging and analysis. It seamlessly integrated with our testing equipment, ensuring that every minute change in the specimen's behavior was accurately recorded. Additionally, precision controllers were employed to regulate the downward and upward movements during the tests. These controllers ensured that the applied forces and deformations were consistent across all specimens, eliminating potential sources of variability.

3.4.3. Universal Testing Machine

Our primary testing apparatus was automatic universal testing machine, Shimadzu AG-IS 100 kN. This machine, renowned for its precision and consistency, was responsible for applying controlled loads to the specimens. It was equipped with sensors and data loggers that continuously recorded the force, deformation, and other relevant parameters during the tests. The integration of the Shimadzu software with this machine ensured that all data was systematically logged and stored for subsequent analysis.

3.4.4. Manual Measurements

While automated systems offer precision and consistency, certain measurements required a hands-on approach. The springback angles, for instance, were measured using a manual protractor. This traditional tool, though simple, provided the accuracy needed for this specific measurement. Similarly, for the disc compression tests, deformations in the x and y directions were meticulously measured using a precision caliper. This instrument, known for its accuracy, ensured that even the slightest deviations in the specimen's dimensions were accurately captured.

The combination of advanced equipment, reliable software, and traditional measurement tools ensured that our data acquisition techniques were both comprehensive and precise. By employing a multi-faceted approach to measurement and data collection, we ensured that our study on the AZ31 magnesium alloy was grounded in accuracy, paving the way for reliable conclusions and meaningful contributions to the field of materials science.

3.5. Safety Precautions

Safety is of paramount importance in any experimental setup, especially when dealing with materials, high temperatures, and equipment that can pose potential hazards. Ensuring the well-being of the researchers and the integrity of the equipment is a foundational aspect of our study on the AZ31 magnesium alloy. This section elucidates the various safety measures and precautions that were diligently adhered to throughout the experimental process.

3.5.1. Personal Protective Equipment (PPE)

In the laboratory setting, paramount importance was placed on ensuring the safety and well-being of all individuals involved in the experimental processes. To this end, a comprehensive set of safety measures was rigorously enforced: Firstly, the use of safety goggles was obligatory for every person present in the lab, without exception. These safety goggles served as a critical protective barrier against potential hazards such as splashes, flying debris, or unexpected accidents that might pose a risk to the eyes. This safety precaution was a fundamental requirement to safeguard the ocular

health and safety of all personnel. Additionally, lab coats were deemed mandatory attire for all researchers within the laboratory environment. These lab coats played a pivotal role as a primary safeguard against accidental spills, splashes, or direct contact with potentially hazardous materials. By wearing lab coats, researchers ensured an extra layer of protection against chemical or biological agents that could otherwise come into contact with their clothing or skin. Furthermore, given the elevated temperatures involved in various laboratory procedures, particularly with the use of the furnace, the provision of heat-resistant gloves was imperative. These specialized gloves were designed to withstand high temperatures, enabling researchers to handle hot specimens or equipment without the risk of burns or thermal injuries. The use of heat-resistant gloves was a critical measure in ensuring both the physical safety and comfort of personnel working with heated materials and equipment. Overall, these stringent safety protocols and the mandatory use of safety goggles, lab coats, and heat-resistant gloves collectively created a secure laboratory environment conducive to the responsible and safe conduct of experiments and research activities.

- **Safety Goggles:** Every individual present in the lab was required to wear safety goggles at all times. These goggles provide protection against potential splashes, flying debris, or any unforeseen accidents that might harm the eyes.
- **Lab Coats:** Lab coats were mandatory for all researchers. They serve as a primary barrier against any spills, splashes, or direct contact with hazardous materials.
- **Heat-Resistant Gloves:** Given the involvement of high temperatures, especially with the furnace, heat-resistant gloves were essential. These gloves ensured that researchers could safely handle hot specimens or equipment without the risk of burns.

3.5.2. Equipment and Environment Safety

In our laboratory environment, an unwavering commitment to safety and the precise execution of experiments was maintained through a series of vigilant monitoring and preventive measures: First and foremost, the furnace, a pivotal component in our

experimental setup, was subjected to rigorous monitoring. Regular checks were conducted to ensure that it operated within the specified temperature ranges, thereby mitigating the risk of overheating or malfunction. This constant vigilance guaranteed the furnace's reliability and the integrity of our experiments. The thermostatic chamber, characterized by its elevated temperatures, was under continuous scrutiny. Monitoring systems were in place to track temperature deviations from the predetermined parameters, promptly triggering alarms. This proactive approach facilitated timely intervention and effectively prevented potential hazards associated with temperature fluctuations. Stability in room temperature (RT) was not only a matter of experimental precision but also a fundamental safety concern. The maintenance of a consistent room temperature was imperative to prevent abrupt fluctuations that could disrupt equipment functionality or compromise the integrity of specimens under examination. Recognizing the potential risks posed by high temperatures and electrical equipment, fire sensors were strategically positioned throughout the laboratory. These sensors were meticulously designed to detect early signs of fire, swiftly activating alarms that enabled immediate response and intervention, thereby minimizing the potential consequences of a fire hazard.

Moreover, the voltage supplied to the furnace received regular monitoring to ensure that it adhered to safe electrical parameters. This precautionary measure was of paramount importance, as it not only safeguarded against electrical hazards but also protected the laboratory equipment from potential damage, enhancing the overall safety and reliability of our experimental environment. In sum, these comprehensive monitoring and safety measures collectively fostered a secure and controlled laboratory setting, conducive to the precise and safe execution of experiments.

1. **Furnace Monitoring:** The furnace, being a critical component of our experiments, was under constant surveillance. Regular checks ensured that it operated within the specified temperature ranges, preventing any overheating or malfunctions.
2. **Thermostatic Chamber:** The thermostatic chamber's high temperatures were continuously monitored. Any deviation from the set parameters triggered alarms, ensuring timely intervention and preventing potential hazards.

3. **RT:** Maintaining a stable RT was crucial, not just for the accuracy of the experiments but also for safety. Sudden temperature fluctuations could lead to equipment malfunctions or compromise the integrity of the specimens.
4. **Fire Sensors:** Given the involvement of high temperatures and electrical equipment, fire sensors were installed throughout the lab. These sensors are designed to detect any early signs of fire, triggering alarms and allowing for immediate action.
5. **Voltage Monitoring:** The furnace's voltage was regularly checked. This precaution ensured that the equipment operated within safe electrical parameters, preventing potential electrical hazards or equipment damage.

Safety, in our study, was not just a protocol but a culture. By integrating rigorous safety measures into every aspect of our experimental process, we ensured a secure environment for our researchers and the integrity of our results. Such diligence underscores the commitment to excellence and responsibility that defines our approach to the study of the AZ31 magnesium alloy.

CHAPTER 4

CONSTITUTIVE MODELLING AND FINITE ELEMENT ANALYSIS (FEA)

In the pursuit of a comprehensive understanding of materials and structures, it is imperative to acknowledge the inherent complexity and variability that often characterizes their behavior. In the context of this chapter, which delves into Constitutive Modeling and FEA, it is noteworthy to mention that the mechanical response of materials is not only influenced by their intrinsic properties but also by external factors such as temperature and loading orientation. To account for this intricacy, we have incorporated a nuanced approach by applying distinct yield surface criteria tailored to specific temperature ranges and loading orientations.

The choice of yield surface criteria is pivotal in constitutive modeling, as it governs how a material's response is characterized under different conditions. By employing varying criteria for different temperature ranges, we acknowledge the dynamic nature of material behavior, which can undergo significant transformations at elevated temperatures, such as creep or thermal softening. Similarly, different loading orientations can lead to anisotropic responses in materials, necessitating the adoption of tailored yield surface criteria to capture these nuances accurately.

In this chapter, we explore the intricacies of these specialized criteria and their implications for our finite element analyses. Through a meticulous examination of materials' responses to different temperature regimes and loading orientations, we aim to elucidate the profound impact of these factors on structural integrity, reliability, and performance. This comprehensive approach underscores our commitment to providing a thorough and nuanced analysis of materials and structures, ensuring that our models and simulations are not only accurate but also reflective of the multifaceted nature of engineering challenges encountered in the real world. Experimental data for AZ31, for different temperatures and orientation is taken from literature, and given in Table 4.1.

Table 4.1 The Lankford values for elevated temperatures, in different directions [166]

Test Direction	Temperature (°C)					
	RT	100°C	150°C	200°C	250°C	300°C
0°	1.347	2.006	1.291	1.621	1.344	1.374
45°	2.793	2.412	1.976	2.118	1.532	1.477
90°	4.109	4.406	3.189	2.672	1.799	1.881
Mean	2.76	2.809	2.018	2.132	1.552	1.552

4.1. Criteria 1: The von-Mises and Tresca Yield Criterion

Table 4.2 Mechanical properties of AZ31 magnesium alloy at elevated temperatures ($\dot{\epsilon}=0.01\text{s}^{-1}$)

Temperature (°C)	Elongation (%)	Yield stress (MPa)	Tensile strength (MPa)
RT	28.5	171.1	262.0
100	58.5	148.4	209.7
200	80.9	87.3	113.2
235	85.0	75.8	85.9
300	90.1	43.2	49.1

4.1.1. Results at RT

The graph elucidates the intrinsic differences between the von Mises and the Tresca yield criteria in a formal and practical context. The von Mises criterion, with its continuous and smooth yield surface, would allow for a more uniform distribution of stress, which could be advantageous in evenly loaded components. On the other hand, the Tresca criterion, with its abrupt transitions and corners on the yield surface, suggests a different approach to design, one that takes into account the maximum shear stresses explicitly. When interpreting the results for application to RT behavior, it's essential to consider the actual stress state of the components in use. For components

subject to complex loading beyond uniaxial tension, the selection of an appropriate yield criterion becomes critical for the accurate prediction of yielding and subsequent failure. In summary, the graphical representation of the von Mises and the Tresca yield surfaces provides valuable insights into the yielding behavior of materials under different stress states. This comparison underscores the necessity of selecting the correct yield criterion based on the expected loading conditions to ensure the safety and efficiency of the engineered components.

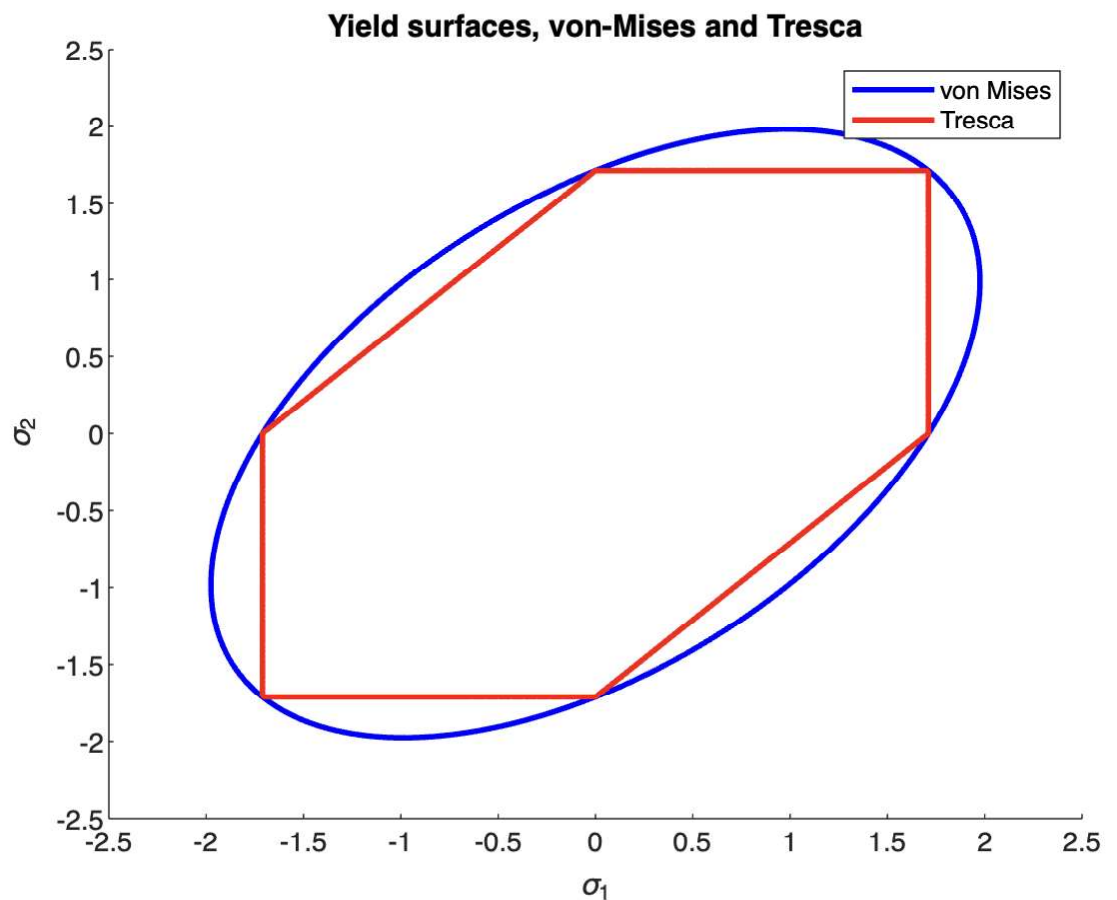


Figure 4.1 The von-Mises and Tresca yield surface at RT

4.1.2. The von-Mises and Tresca Yield Surface at 100 °C

The graph in question illustrates the yield surfaces for the von Mises and the Tresca yield criteria specifically for AZ31, which is a magnesium-aluminum-zinc alloy, at an elevated temperature of 100 °C. The material AZ31 is known for its good strength-to-weight ratio and is often used in applications where weight saving is crucial. Elevated temperature can affect its mechanical properties, and the graph reflects the theoretical

yield conditions under such a thermal environment. The von Mises yield surface is represented by a smooth, elliptical shape which suggests that yield occurs when the distortion energy in the material reaches a critical level. This criterion takes into account the combined effect of all three principal stresses. For the material AZ31 at 100 °C, the yield stress value at which the von Mises criterion predicts yielding may be lower than at RT due to thermal softening. The von Mises criterion is generally considered to provide a more accurate representation of the yielding behavior for ductile metals such as AZ31, especially under complex loading conditions. The Tresca yield surface takes the shape of a hexagon and is based on the maximum shear stress theory. It posits that yielding occurs when the maximum shear stress reaches a critical value. For AZ31, this would mean the material yields when the difference between the maximum and minimum principal stresses reaches the shear yield stress of the material. At 100 °C, this value might be reduced, and the Tresca criterion could predict yielding at lower stress levels compared to lower temperatures.

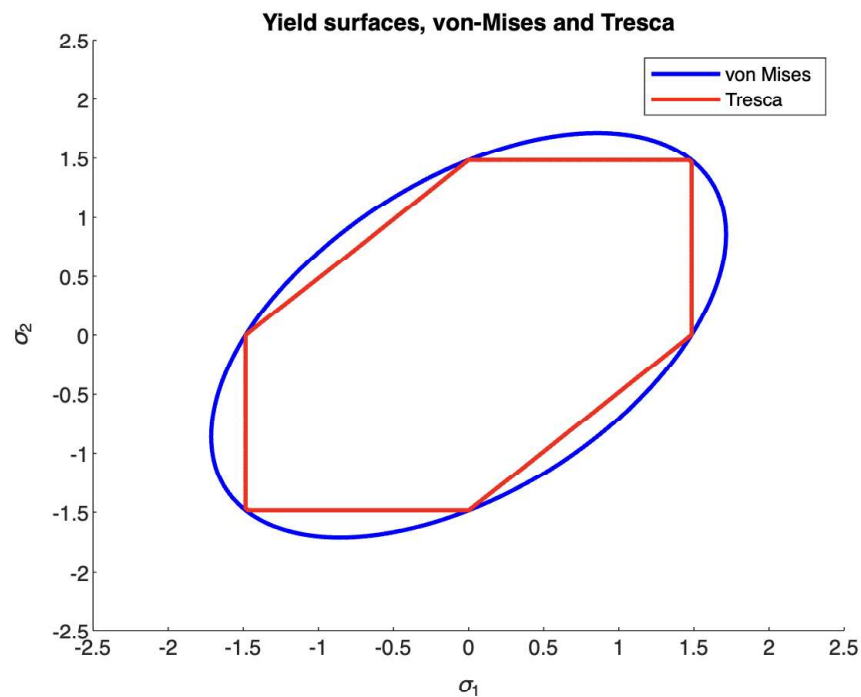


Figure 4.2 The von-Mises and Tresca yield surface at 100 °C

4.1.3. The von-Mises and Tresca Yield Surface at 150 °C

The graph illustrates the yield surfaces for the AZ31 magnesium alloy at a temperature of 150 °C, depicted through two fundamental yield criteria: the von Mises and the

Tresca. The von Mises criterion, represented by the blue elliptical curve, shows a yield surface that is smooth and continuous, which suggests that the yielding of the material is not sensitive to the hydrostatic component of the stress tensor. Yielding according to this criterion occurs when the distortional energy in the material reaches a critical level. The elliptical shape indicates that the yield stress is the same in all pure shear stress states. In contrast, the Tresca criterion, depicted by the red hexagonal curve, indicates yield initiation when the maximum shear stress in the material reaches the shear yield stress of the material. The corners of the hexagon align with the axes, representing the pure shear states, and the sides parallel to the axes represent the simple tension/compression states.

At 150 °C, both criteria demonstrate that the yield strength of the AZ31 alloy is reduced due to thermal softening. However, the Tresca criterion is typically more conservative, predicting yield at lower differential stresses than von Mises. This can be seen where the Tresca hexagon is entirely within the von Mises ellipse, suggesting that under certain stress states, the Tresca criterion will predict yielding while the von Mises will not. The results from these yield surfaces are critical for understanding and predicting the mechanical behavior of the AZ31 alloy at elevated temperatures. Designers and engineers must consider these reduced yield thresholds when assessing the alloy's capacity to withstand stresses that are applied in high-temperature environments. This analysis can help in preventing material failure by informing the design limits and safety factors for components made from AZ31, ensuring they operate reliably within the expected stress ranges at 150 °C.

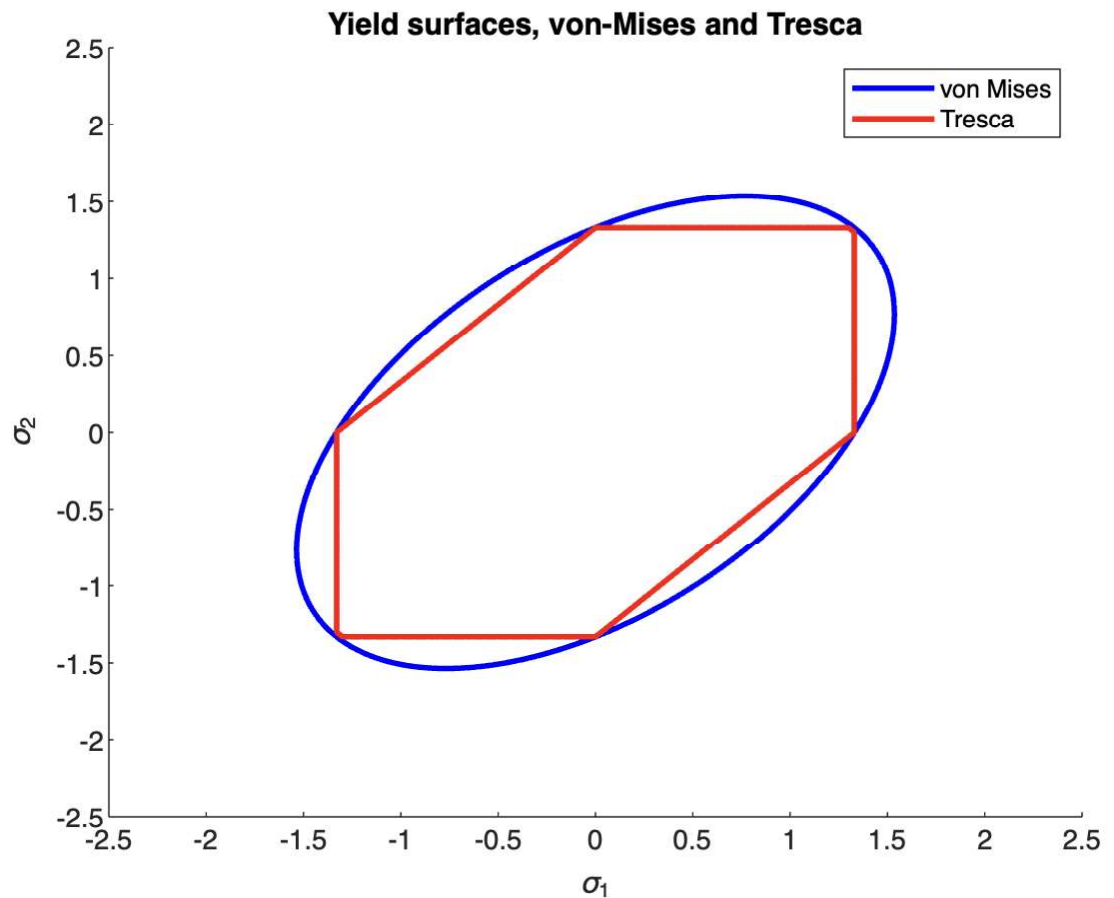


Figure 4.3 The von-Mises and Tresca yield surface at 150 °C

4.1.4. The von-Mises and Tresca Yield Surface at 200 °C

The Tresca criterion is represented by the red hexagon. This model is based on the maximum shear stress and postulates that yielding occurs when the maximum shear stress in the material reaches a critical threshold. The hexagon's orientation shows that the yield stress in pure shear is less than in simple tension or compression. At a temperature of 200 °C, the graph indicates that thermal softening of the AZ31 alloy has further progressed compared to lower temperatures, which results in a larger and more pronounced yield surface. This is an important consideration for material selection and design, particularly when operating at elevated temperatures that could lead to early yielding and potential failure under load. The Tresca criterion continues to predict a lower yield point than the von Mises criterion, as seen by the hexagon being inscribed within the ellipse. In engineering practice, this means that when using the Tresca criterion for design, a component might be deemed to have yielded earlier

under the same loading conditions compared to predictions based on the von Mises criterion. In conclusion, at 200 °C, both criteria suggest a significant decrease in yield strength for the AZ31 alloy. The material's performance at such elevated temperatures will be critical in applications where both the temperature and mechanical loads are high. The design process must incorporate these factors to ensure structural integrity and prevent premature failure of components made from this alloy.

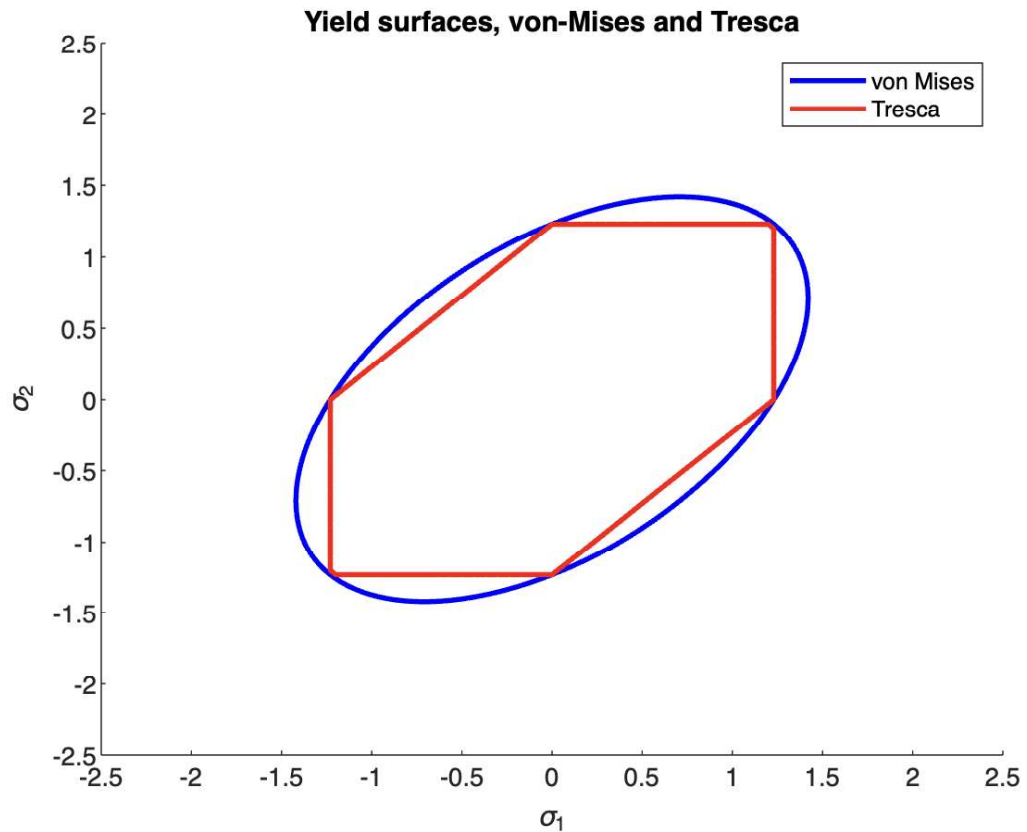


Figure 4.4 The von-Mises and Tresca yield surface at 200 °C

4.1.5. The von-Mises and Tresca Yield Surface at 250 °C

At 250 °C, it's evident that the yield surfaces have expanded compared to those at lower temperatures, which indicates that the material has become more susceptible to yielding due to the increased temperature. The thermal softening effect at this higher temperature reduces the material's yield strength, as evidenced by the larger yield surfaces. Both criteria show that the material will begin to yield at lower stress levels than it would at lower temperatures.

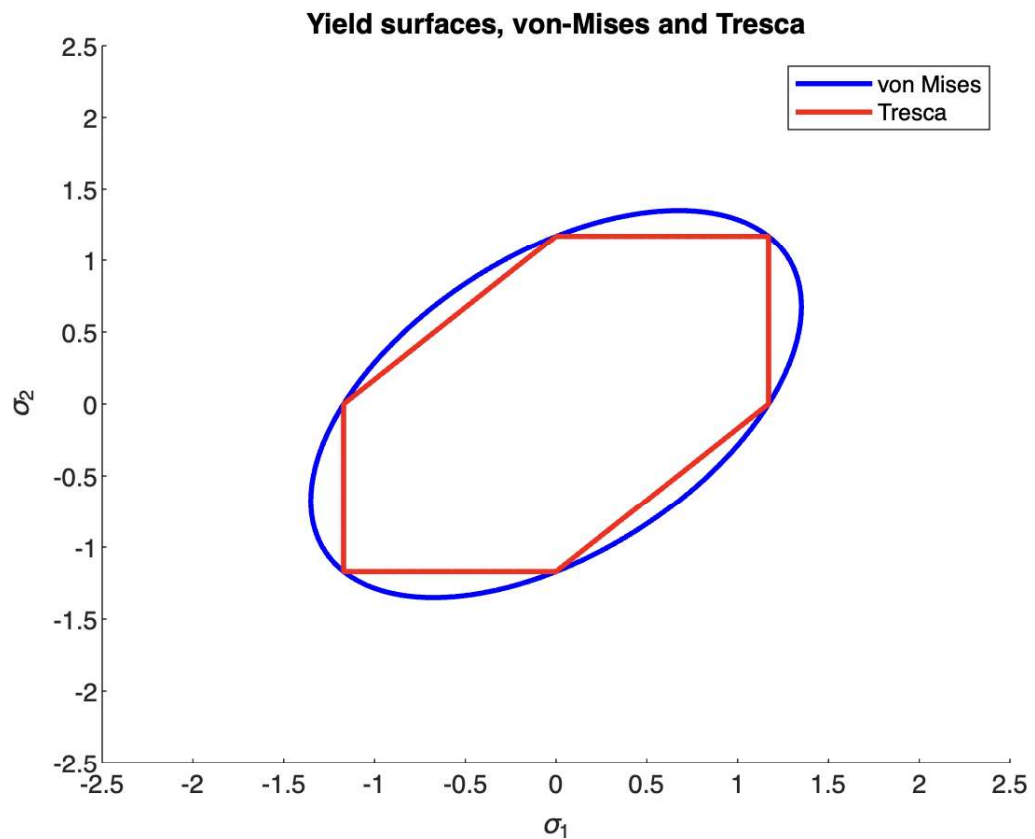


Figure 4.5 The von-Mises and Tresca yield surface at 250 °C

4.1.6. The von-Mises and Tresca Yield Surface at 300 °C

The graph depicting the yield surfaces for the von Mises and the Tresca criteria provides insight into the yielding behavior of the AZ31 magnesium alloy at 300°C. The von Mises criterion, shown as an ellipse, suggests a material response that is sensitive to the combination of principal stresses, and it predicts yielding when the energy associated with distortional strain reaches a critical level. The Tresca criterion, represented by the hexagon, is sensitive to the maximum shear stress present in the material, predicting yielding when this shear stress exceeds a certain threshold. At the elevated temperature of 300°C, we can infer that the yield strength of AZ31 has decreased, as indicated by the earlier yield strength vs. temperature graph, while ductility has increased, as shown by the elongation vs. temperature graph. These changes in material properties with temperature significantly influence the size and shape of the yield surfaces.

For AZ31 at 300°C, the von Mises yield surface would be larger, reflecting the material's reduced yield strength and increased tendency to undergo plastic deformation under a mixed stress state. The Tresca yield surface also expands but remains angular, maintaining a clear threshold for the onset of yielding due to shear stresses. The comparison between the two criteria at this high temperature indicates that the von Mises criterion would allow for a more gradual transition to yielding under complex loading, while the Tresca criterion would enforce a stricter limit based on shear. In practical terms, these yield surfaces indicate that the material will have a lower resistance to plastic deformation at 300 °C for any given stress state. The choice between using the von Mises and the Tresca for design and analysis would depend on the specific loading conditions and the design criteria. If distortion energy is a concern, the von Mises would be more appropriate, while for applications where shear loading is critical, the Tresca would be the conservative choice. Given the temperature-sensitive nature of AZ31, engineers must carefully consider these criteria when designing components that operate at elevated temperatures to ensure reliability and safety.

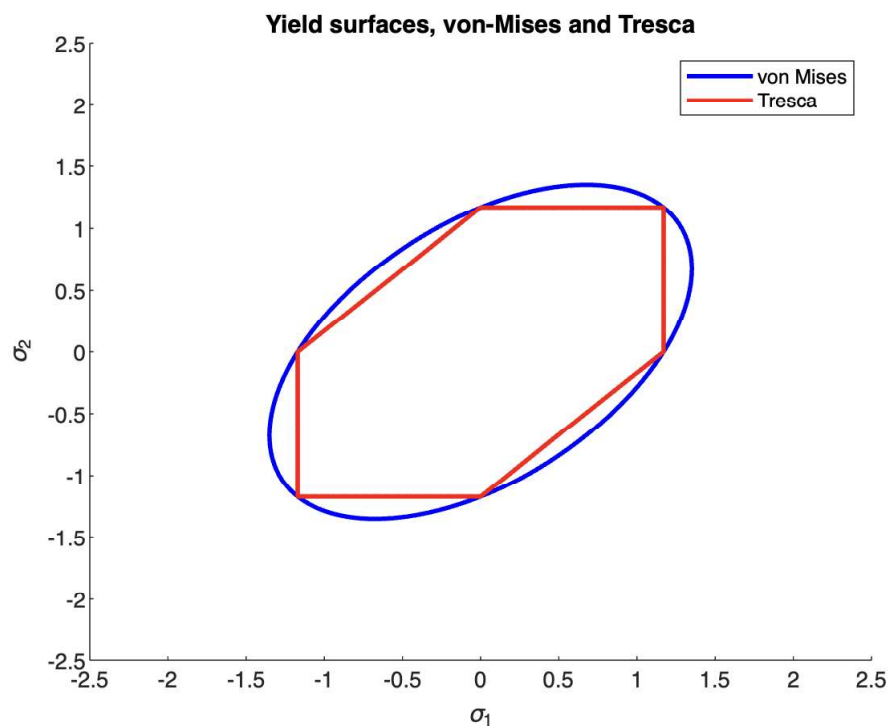


Figure 4.6 The von-Mises and Tresca yield surface at 300 °C

4.1.7. Material Properties Dependence Regarding Temperature

The Figures 4.7 and 4.8 presented show the relationship between temperature and mechanical properties of an AZ31 magnesium alloy. One graph displays the yield strength as a function of temperature, while the other shows the elongation percentage, a measure of ductility, also as a function of temperature. From the Yield Strength vs. Temperature graph, it is apparent that the yield strength of the AZ31 alloy decreases as the temperature increases. At 25 °C, the yield strength is at its highest, approximately 171.1 MPa. This value drops steadily as the temperature rises, reaching a much lower yield strength of about 43.2 MPa at 300 °C. The decline in yield strength with temperature is a common behavior for metals and alloys, as increased thermal energy allows dislocations to move more easily, resulting in earlier yielding. Conversely, the Elongation vs. Temperature graph indicates that the alloy's ductility improves significantly with temperature. At 25 °C, the elongation is around 28.5%, and this value increases with temperature, reaching 90.1% at 300 °C. The improvement in ductility can be attributed to the enhanced mobility of atoms at higher temperatures, which facilitates plastic deformation. When considering the von Mises and the Tresca yield criteria in relation to these graphs, one can infer the following: As the temperature increases, the yield surfaces for both criteria will expand, as demonstrated by the reduced yield strength. This suggests that the material will undergo plastic deformation more readily at elevated temperatures, a conclusion supported by the elongation data. Furthermore, the Tresca criterion, which is based on the maximum shear stress, might predict yielding at slightly lower stresses compared to the von Mises criterion, which is based on the distortion energy theory. However, both criteria will reflect the same trend of decreasing yield strength with increasing temperature. In summary, for the AZ31 magnesium alloy, the decrease in yield strength with temperature implies that the material will be less capable of withstanding applied loads without deforming as the temperature rises. In contrast, the increase in elongation indicates that the material becomes more ductile and can undergo more significant plastic deformation before fracturing. These properties are crucial for applications where the alloy might be exposed to varying temperatures, and the selection of a yield criterion (von Mises or Tresca) for design purposes will depend on the specific requirements of the application, taking into account the need for either a more

conservative design (Tresca) or one that fully utilizes the material's capabilities (von Mises).



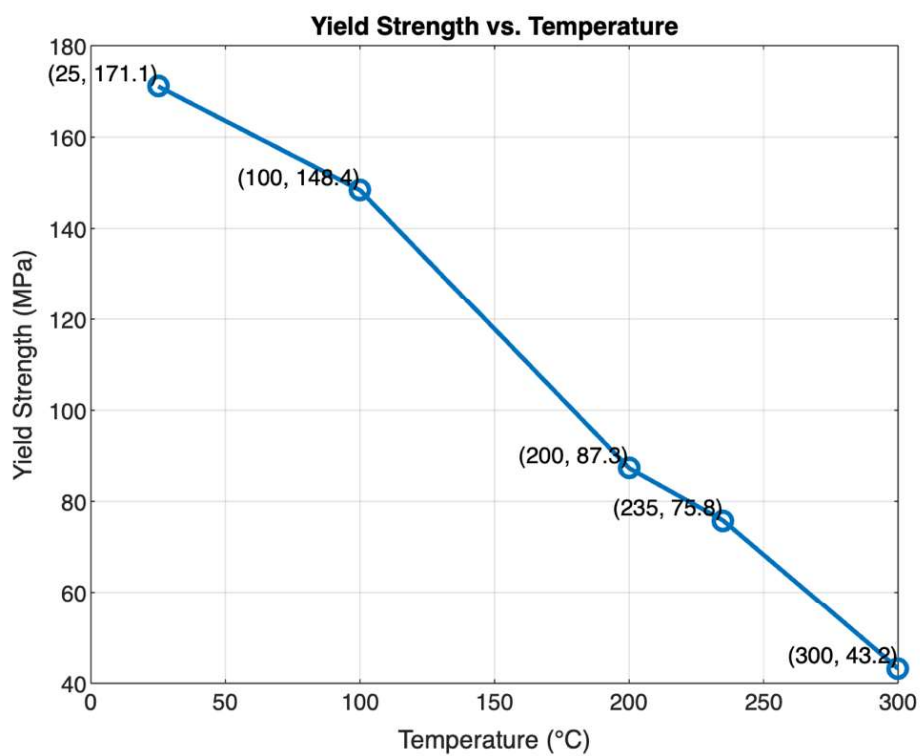


Figure 4.7 Yield strength vs. temperature graph

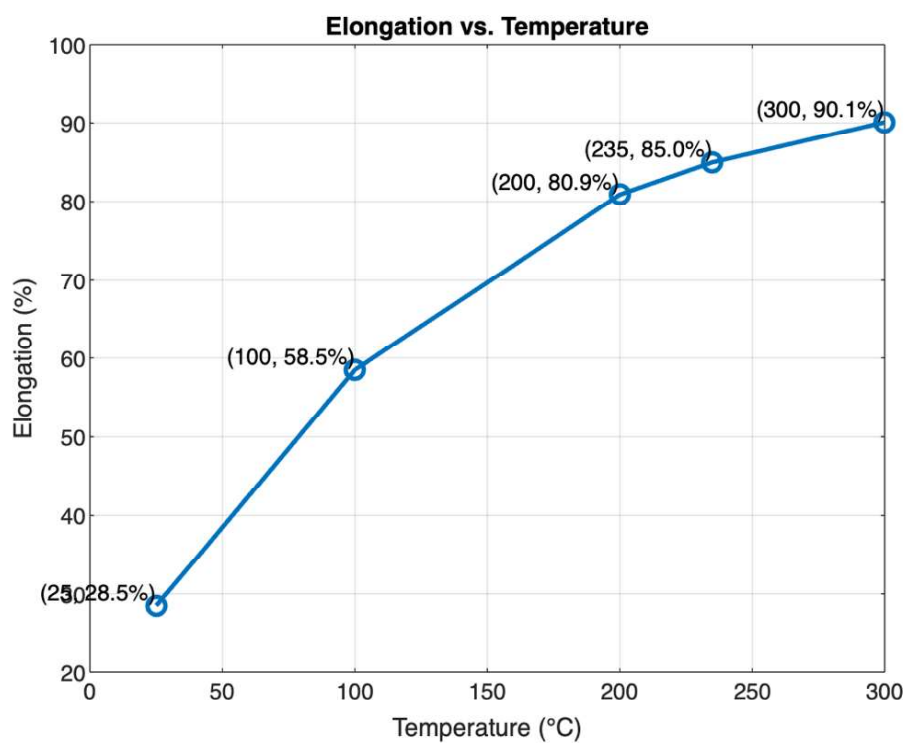


Figure 4.8 Elongation vs. temperature graph

4.2. Criteria 2: The Hill-48 Yield Criteria

Calculated coefficients for the Hill-48 yield criterion is given in Table 4.3.

Table 4.3 Calculated coefficients for the Hill yield criterion for AZ31

Temperature	F	G	H	L	M	N
25 °C < T < 300 °C	0.5079	0.6511	0.3489	1.5	1.5	1.2977

4.2.1. The Hill-48 Yield Surface at RT

The Hill-48 yield surface is a fundamental concept for understanding the anisotropic mechanical behavior of materials, exemplified here with an AZ31 magnesium alloy at RT. The normalized stress axes, (σ_{xx}/σ) and (σ_{yy}/σ) , represent the stress state in two principal material directions, scaled by a reference stress (σ), which is typically the yield stress in a specific direction. The elliptical contours of the yield surface indicate the directionally dependent yielding of the material, a characteristic particularly pronounced in wrought alloys such as AZ31, which often display significant anisotropy as a result of manufacturing processes that impart a preferential alignment of the material's microstructure. The degree of ellipticity in the contours provides insight into the relative difference in yield strength between the principal axes. For applications involving the AZ31 alloy, the anisotropic nature revealed by the Hill-48 criterion is crucial for accurate predictions of material behavior under complex loading conditions, essential for material selection, process design, and structural integrity assessments in engineering disciplines.

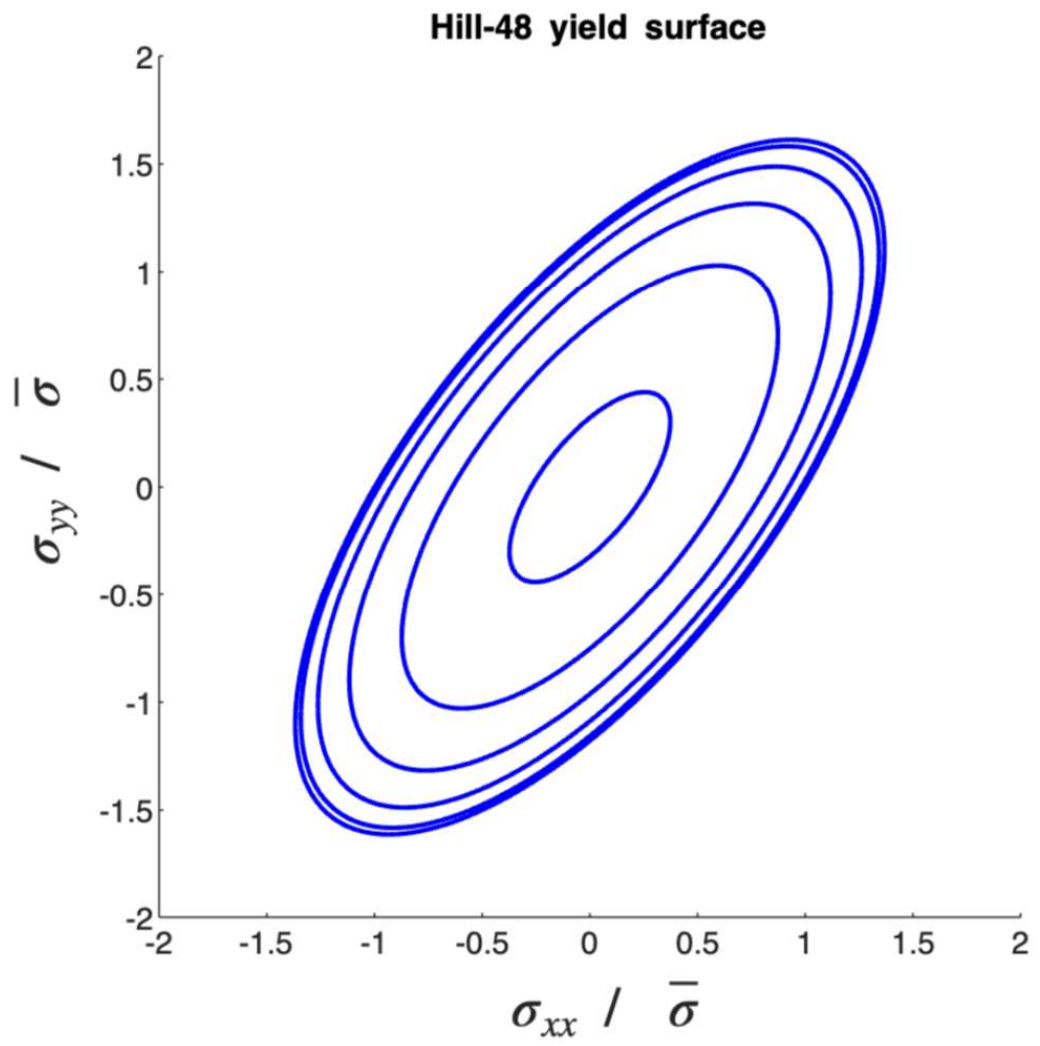


Figure 4.9 The Hill-48 yield surface at RT

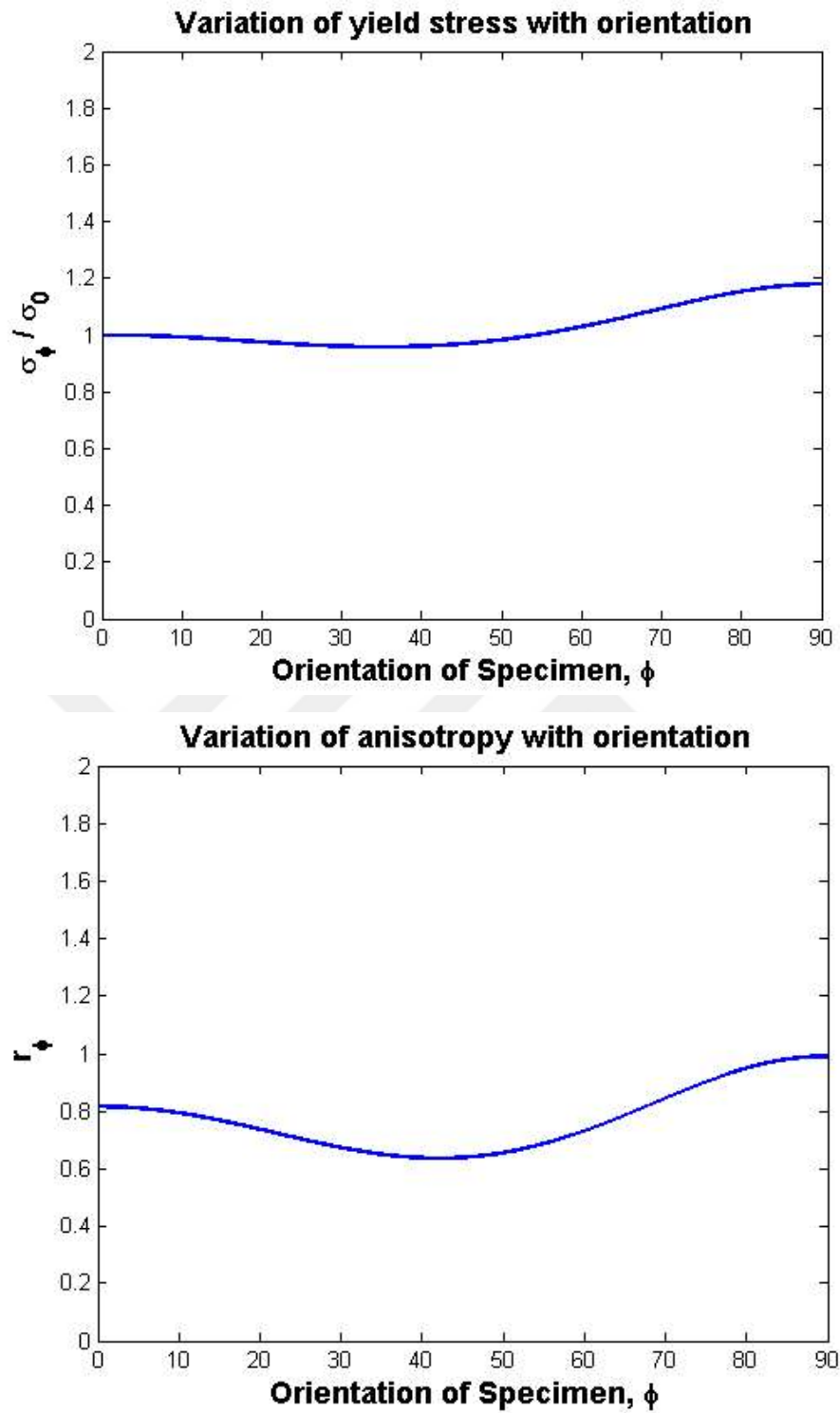


Figure 4.10 The Hill-48 variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at RT

4.2.2. The Hill-48 Yield Surface at 100 °C

At this elevated temperature, the yield surface contours might be expected to expand outward if compared to the same material at RT, due to the thermal softening effect. The Hill-48 yield surface for an AZ31 magnesium alloy at 100 °C presented here illustrates the impact of varying levels of shear stress σ_{12} on the material's yield behavior. The normalized stress axes, (σ_{xx}/σ) and (σ_{yy}/σ) , show the axial stresses in the respective directions, with each elliptical contour representing a different constant shear stress level. Starting from the outermost contour indicating zero shear stress and moving inward, the contours represent increasing shear stress values up to $(\sigma_{12} = 0.5\sigma)$. These elliptical contours, which become increasingly distorted as shear stress is applied, denote the anisotropic nature of the yield strength in the material, likely due to directional processing techniques like rolling. Such graphical representation is crucial for engineers and materials scientists as it provides insight into the material's response to combined axial and shear loadings, an essential consideration for accurately predicting yield conditions in design and analysis, particularly for processes involving complex loading scenarios or in-service conditions where shear is a significant factor.

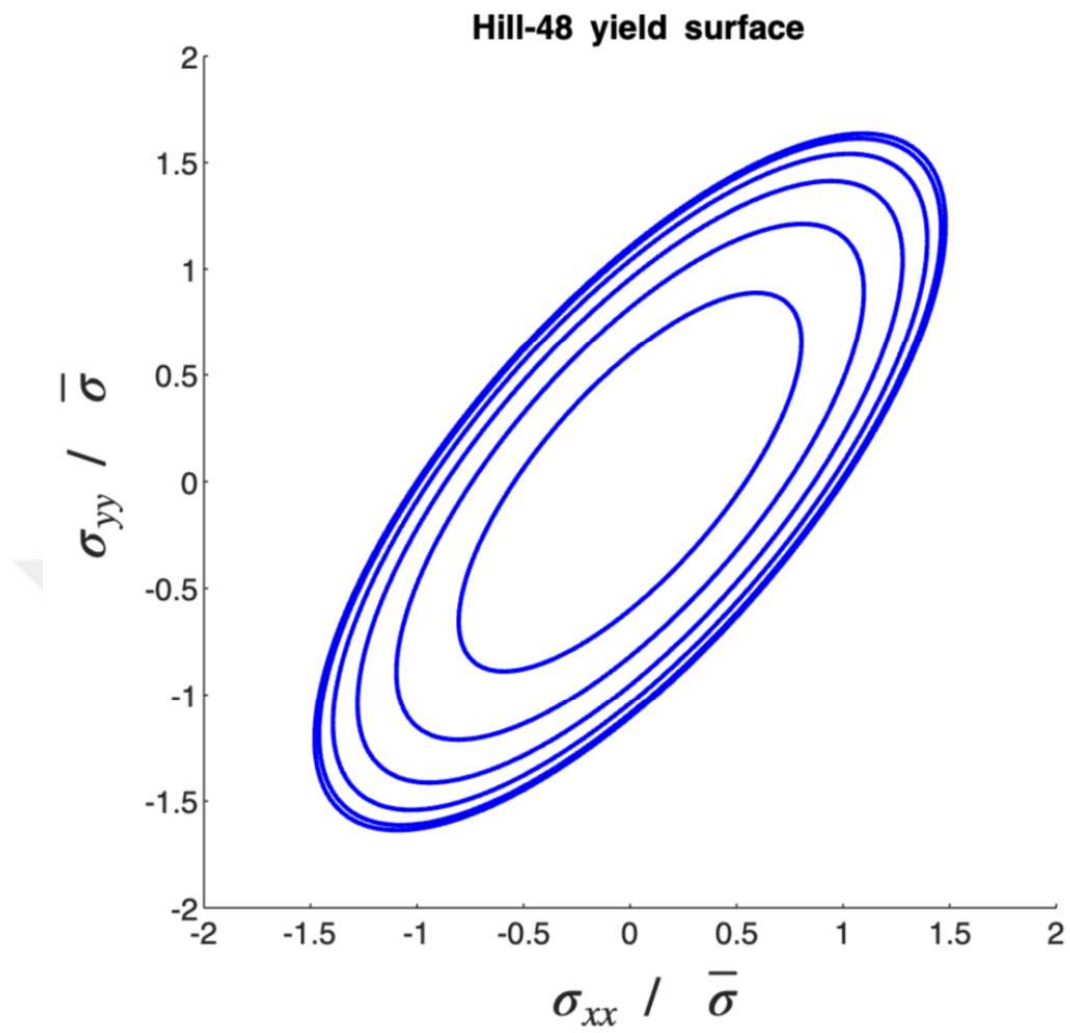


Figure 4.11 The Hill-48 yield surface at 100 °C

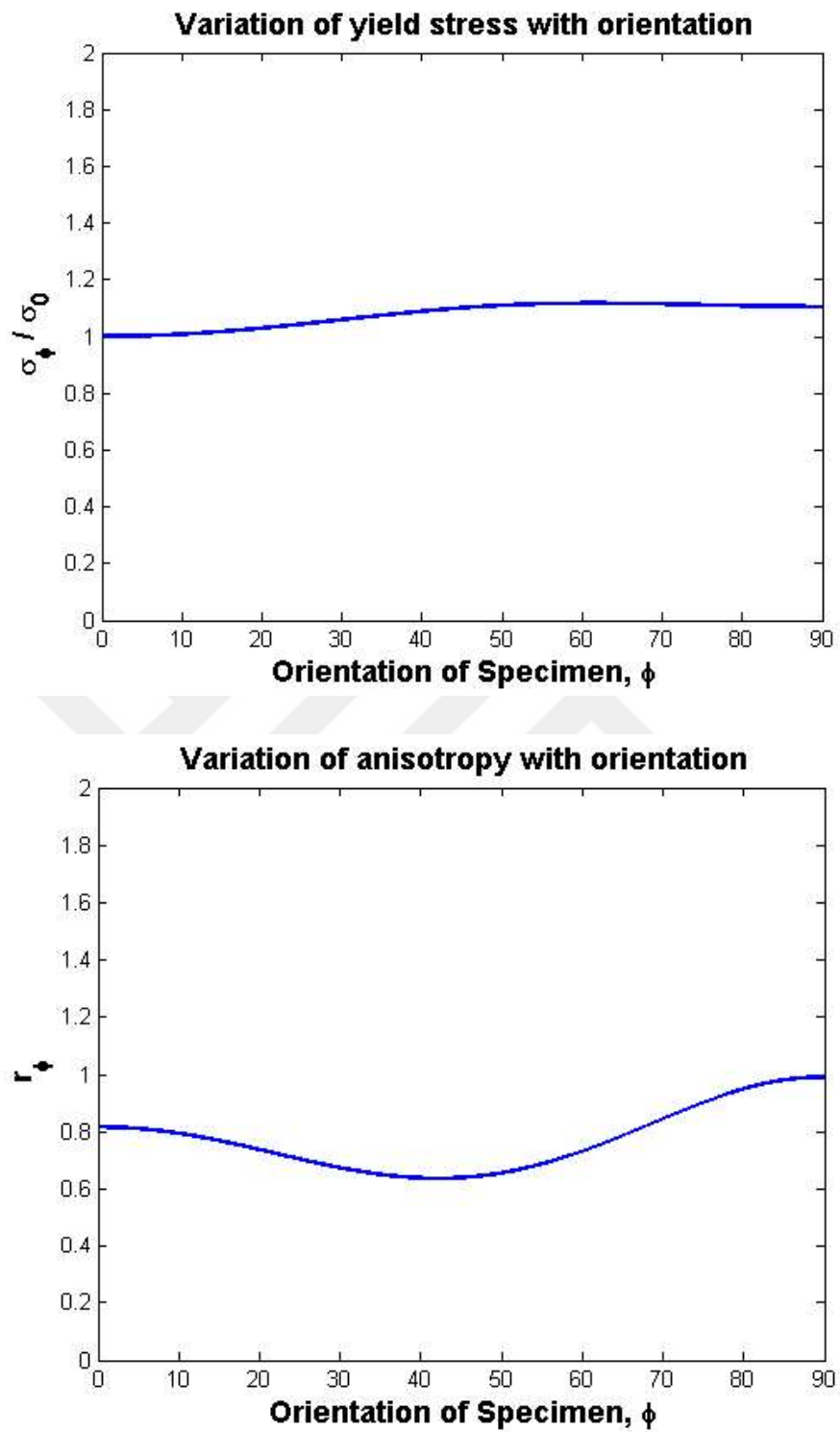


Figure 4.12 The Hill-48 variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at 100 °C

4.2.3. The Hill-48 Yield Surface at 150 °C

The graph displays the Hill-48 yield surface for an AZ31 magnesium alloy at a temperature of 150 °C. Similar to previous cases at different temperatures, each elliptical contour here delineates a specific level of the shear stress (σ_{12}), with the outermost contour corresponding to zero shear stress and inner contours indicating incrementally higher shear stresses. The stress axes (σ_{xx}/σ) and (σ_{yy}/σ) are normalized against a reference stress (σ), showcasing the axial stresses in the x and y directions, respectively.

As we observe the series of ellipses, we note that the shapes of the yield surfaces indicate the anisotropic yielding of the material, which becomes more pronounced under the application of shear. This characteristic distortion of contours as the shear stress increases provides invaluable insights into the material's mechanical response under complex loading conditions. It's evident that with the elevated temperature of 150 °C, the material may exhibit different yielding behavior than at RT or 100 °C, as the yield strength of materials can be sensitive to temperature changes. Engineers would need to consider these variations in yield surfaces for accurate predictions of deformation and failure under practical service conditions, which could involve combined states of stress. It is also an essential consideration in processes such as hot-forming or in applications where components are exposed to high temperatures, as the material's resistance to deformation (yield strength) can be significantly reduced, altering the predicted yield surfaces.

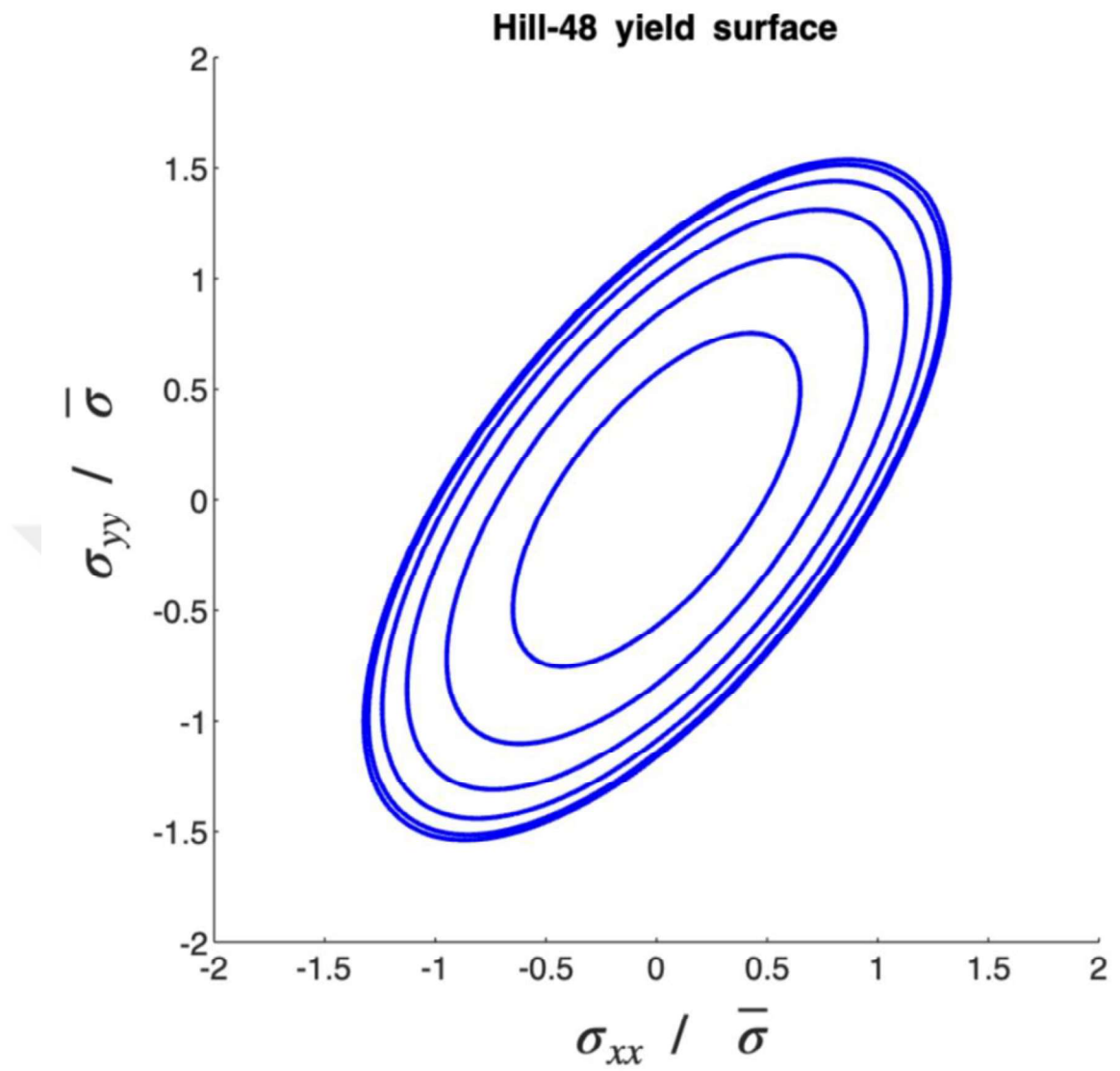


Figure 4.13 The Hill-48 yield surface at 150 °C

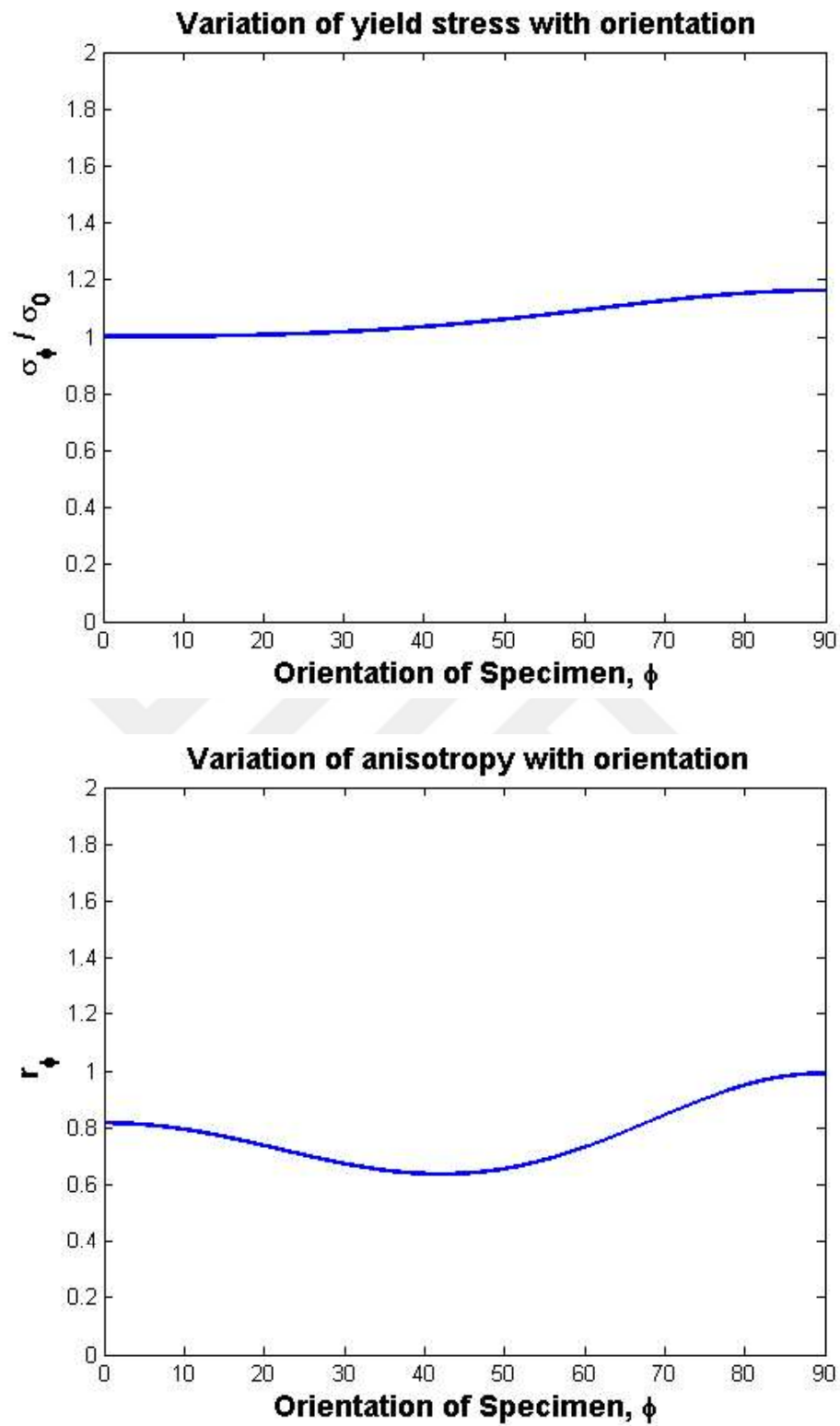


Figure 4.14 The Hill-48 variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at 150 °C

4.2.4. The Hill-48 Yield Surface at 200 °C

The image illustrates a Hill-48 yield surface plot, which is used to model the yielding behavior of anisotropic materials such as certain metal alloys. This specific plot is constructed to represent the yield surface at a temperature of 200 °C, depicting the normalized stress ratios (σ_{xx}/σ) on the horizontal axis against (σ_{yy}/σ) on the vertical axis. These axes show the proportional relationship of the principal stresses in two orthogonal directions normalized by a reference stress (σ), likely the yield stress in a specific orientation. At 200 °C, we can surmise that the material, possibly an AZ31 magnesium alloy or similar, would exhibit decreased yield strength and potentially increased ductility. The ellipses, each representing a constant shear stress state (σ_{12}), from the outermost with ($\sigma_{12} = 0$) to the innermost with ($\sigma_{12} = 0.5\sigma$) (or another specified maximum value), demonstrate how the yield surface expands with increasing shear stress. This implies a material's greater susceptibility to yielding under shear when subjected to high temperatures. As the temperature increases, the material's response to the combined stresses will change, often becoming more pronouncedly anisotropic, which is reflected in the more elongated shape of the ellipses on the plot.

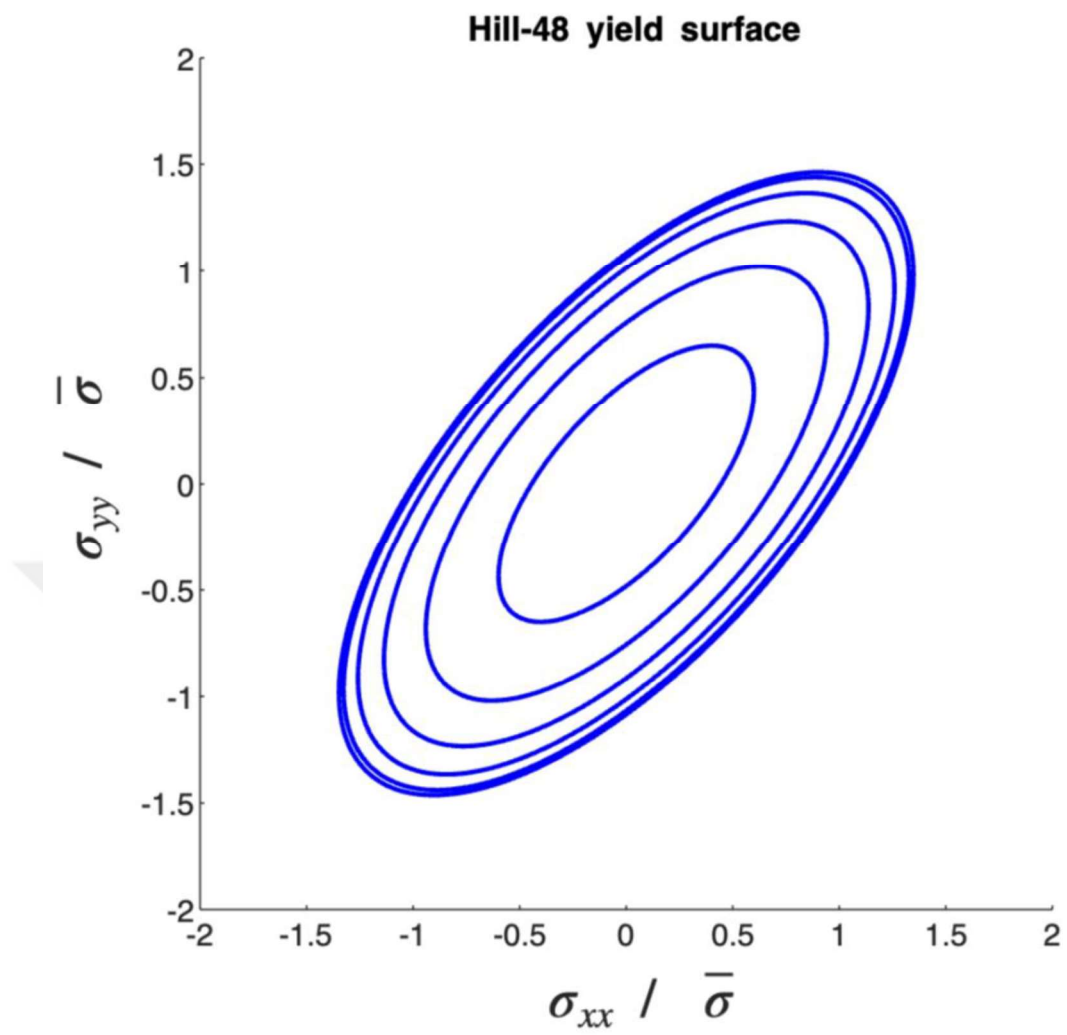


Figure 4.15 The Hill-48 yield surface at 200 °C

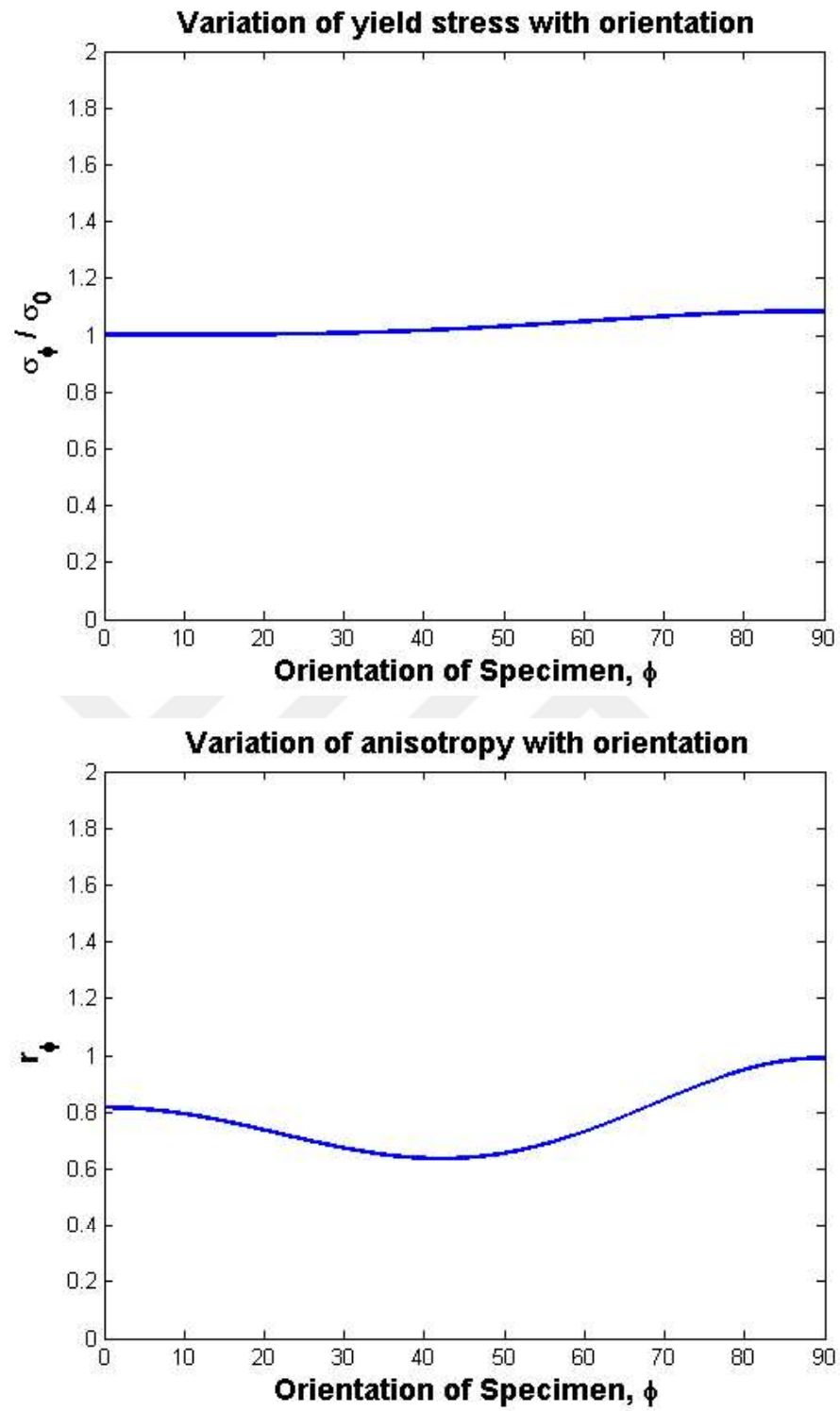


Figure 4.16 The Hill-48 variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at 200 °C

4.2.5. The Hill-48 Yield Surface at 250 °C

Figure 4.17 showcases the Hill-48 yield surface model at a temperature of 250 °C. This model is a critical tool for understanding the plastic deformation of anisotropic materials, which is particularly important for metals and alloys that have direction-dependent properties. The yield surface is depicted in terms of normalized stress ratios, with (σ_{xx}/σ) plotted along the horizontal axis and (σ_{yy}/σ) on the vertical axis, where (σ) is a reference stress, generally taken as the yield stress in a particular direction. At the elevated temperature of 250 °C, it is expected that the material properties to change significantly compared to RT. Specifically, the material could exhibit lower yield strength and greater plasticity, allowing for more extensive deformation before failure. The series of ellipses represent contours of constant shear stress (σ_{12}) , incrementally increasing from the outermost ellipse to the innermost one, illustrating how the yield criteria changes with increasing shear. With temperature elevation to 250 °C, the ellipses become more pronounced, indicating that the yield surface expands and the material becomes more yielding under the combined action of normal and shear stresses. This expansion can be attributed to thermal softening, where increased temperature leads to a reduction in material strength and an increase in ductility. This understanding is pivotal when designing components intended to operate at high temperatures, as it impacts how the material will behave under various loading conditions. Accurate prediction of these behaviors ensures that components are designed with appropriate safety factors and perform reliably under the expected service conditions.

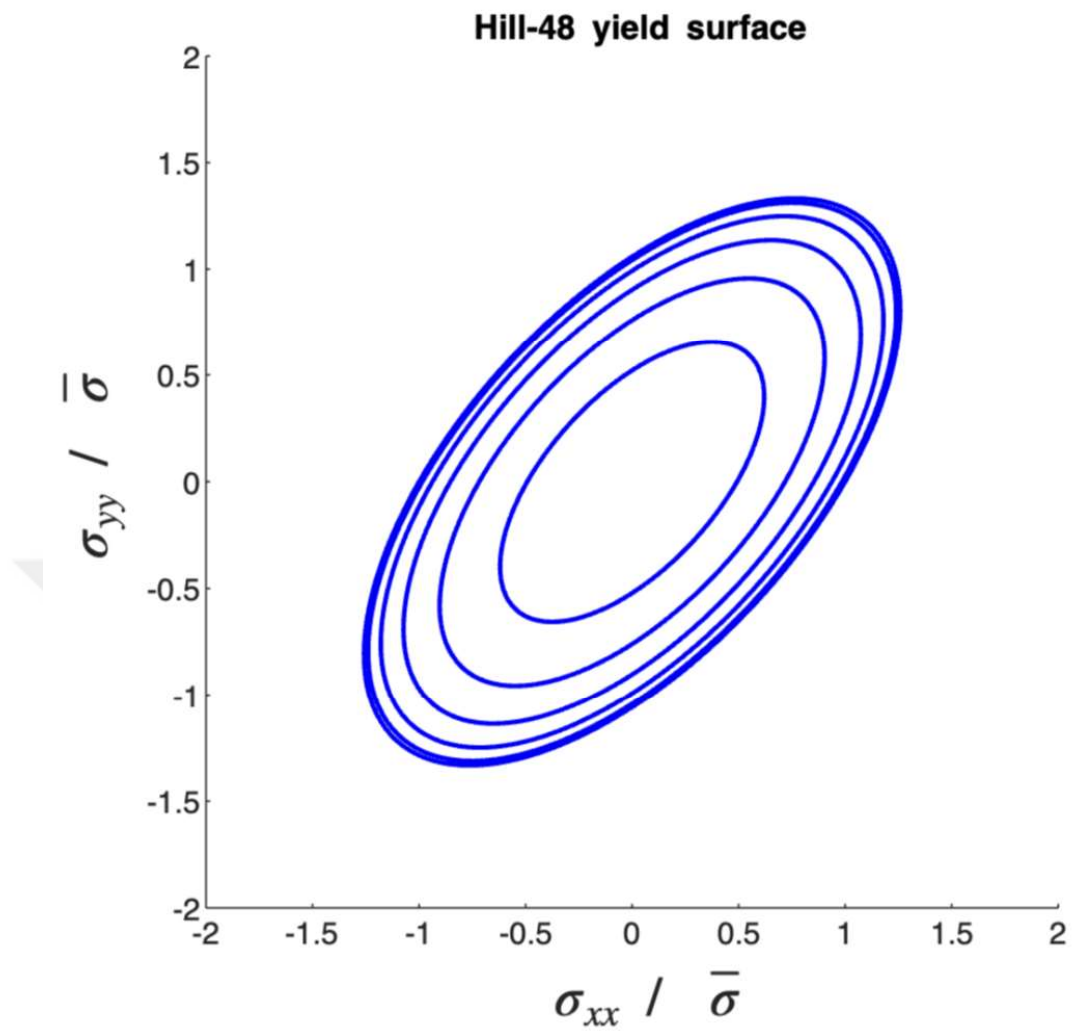


Figure 4.17 The Hill-48 yield surface at 250 °C

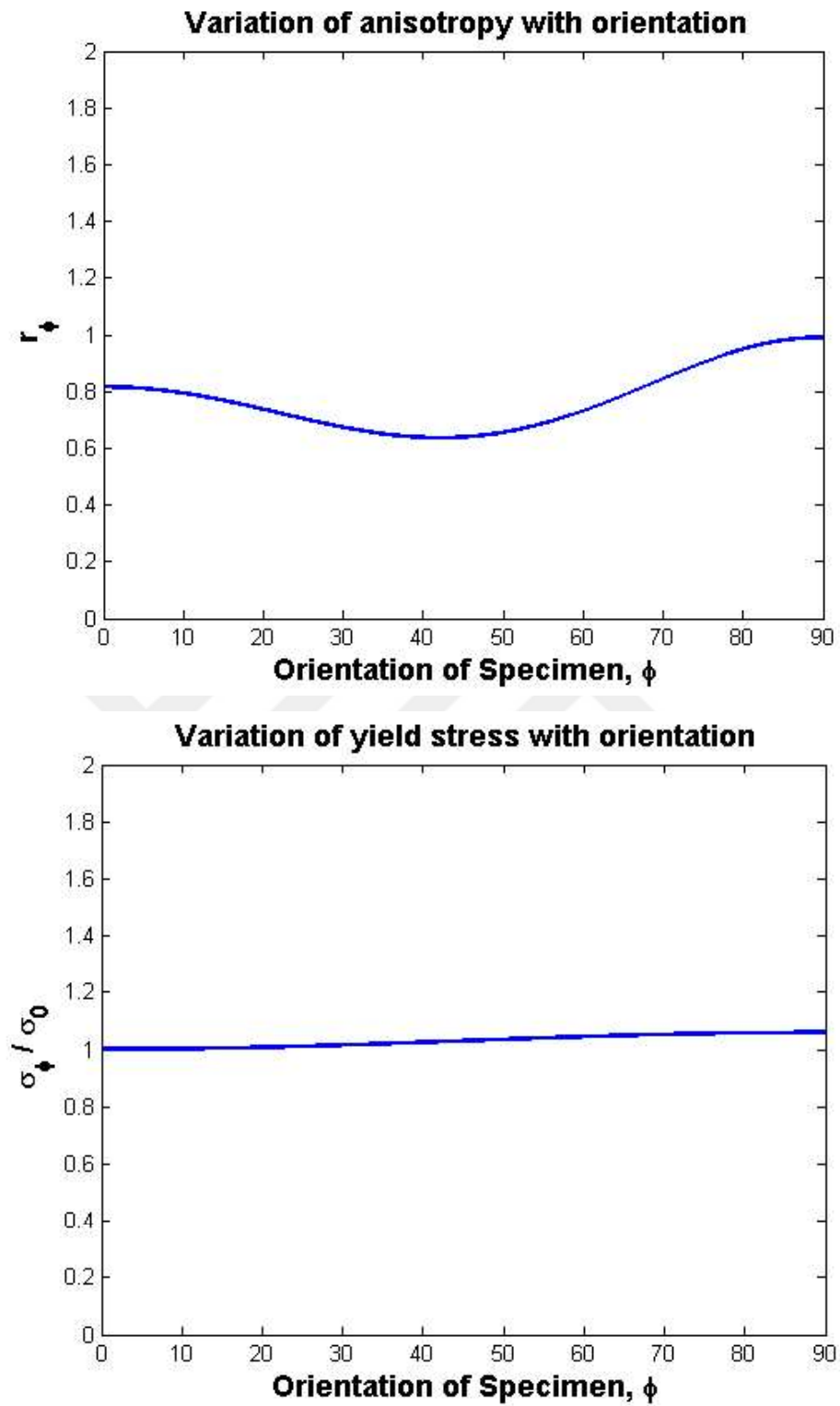


Figure 4.18 The Hill-48 variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at 250 °C

4.2.6. The Hill-48 Yield Surface at 300 °C

At 300 °C, the representation of the Hill-48 yield surface for an anisotropic material would likely undergo significant changes compared to the yield surface at RT. Due to thermal softening, the material's resistance to deformation (yield strength) decreases with an increase in temperature, which would result in an expansion of the yield surface in the (σ_{xx}) vs. (σ_{yy}) stress space, as depicted in the image. This indicates that the material will begin to yield under lower levels of applied stress in both the (σ_{xx}) and (σ_{yy}) directions. Additionally, the contours of the yield surface might appear more circular at elevated temperatures, suggesting a trend toward isotropic behavior where the material's response becomes less dependent on the direction of loading. The exact nature of these changes would be contingent on the material's specific thermal properties, and assessing them accurately would require empirical data or a sophisticated material model that accounts for temperature effects on the yield criteria.

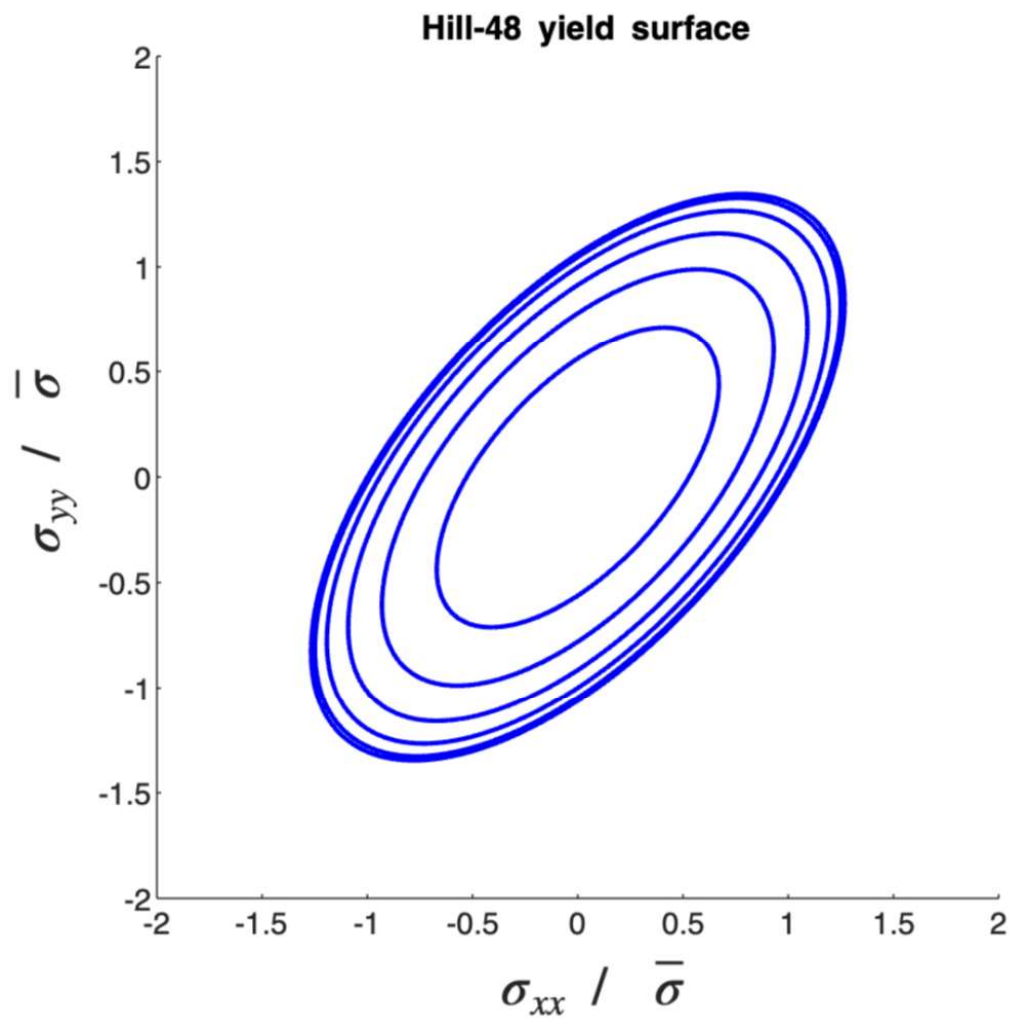


Figure 4.19 The Hill-48 yield surface at 300 °C

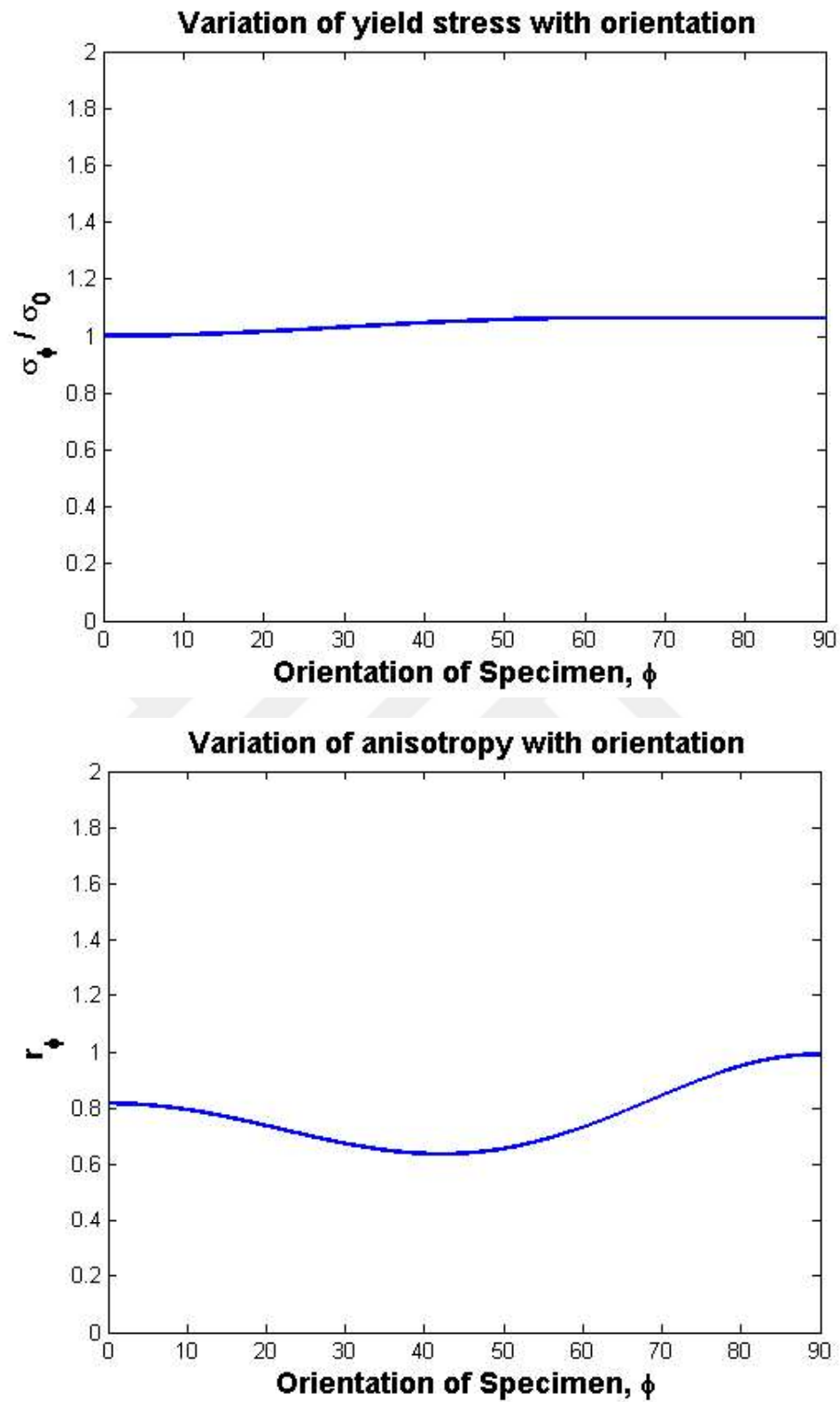


Figure 4.20 The Hill-48 variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at 300 °C

4.3. Criteria 3: The Barlat-89 Yield Criteria

Calculated coefficients for the Barlat-89 yield criteria is given in Table 4.4

Table 4.4 Calculated coefficients for the Barlat-89 criteria

Temperature	a	c	l	Q
RT	1.142	0.738	0.967	0.934
100 < T < 200	0.967	1.032	0.978	0.955
200 < T < 300	0.886	1.114	0.992	0.977

4.3.1. The Barlat-89 Yield Surface at RT

The graph presented illustrates the Barlat-89 yield surface for the AZ31 magnesium alloy material at RT. Yield surfaces are pivotal in describing the onset of plastic deformation in materials under complex loading conditions. The Barlat-89 model is an advanced, anisotropic yield criterion that accommodates the anisotropic plastic response often observed in metallic materials, particularly those that have been rolled or otherwise mechanically processed. The axes of the graph, (σ_{xy}/σ) and (σ_{xx}/σ) , are normalized stress components, where (σ) typically represents the equivalent stress or flow stress of the material. Normalization helps in generalizing the yield condition, making it applicable to various stress states and simplifying comparisons between materials or loading conditions.

The series of closed curves represents the locus of points at which the AZ31 material initiates yielding for different values of the shear stress component, (s_{12}) , ranging from 0 to 0.5. As we move from the innermost curve to the outermost curve, the value of (s_{12}) increases. This increase indicates that the material can sustain higher combinations of (σ_{xx}) and (σ_{xy}) stresses without yielding when subjected to greater shear stress.

Each curve's shape suggests that AZ31 exhibits anisotropic yielding behavior, where the yield strength varies with the direction of the applied stress. This anisotropy is

particularly notable due to the non-circular shape of the curves. For isotropic materials, these yield curves would be expected to form concentric circles. The elongated shape along the σ_{xx} axis implies that the yield strength is higher in the direction of normal stress, which is typical for rolled metals where the grain structure can influence directional strength. In an academic discussion, one might reflect on the implications of these yield surfaces for forming processes. For instance, when considering the drawing or rolling of AZ31 sheets, the anisotropy depicted here would be critical in predicting potential areas of failure and in designing process parameters that avoid those conditions. Moreover, the gradual change in yield surface shape with increasing shear stress, as illustrated by the progression of curves, is indicative of the continuous evolution of the material's yield condition, which must be considered in numerical simulations of metal forming that aim to capture material behavior with high fidelity.

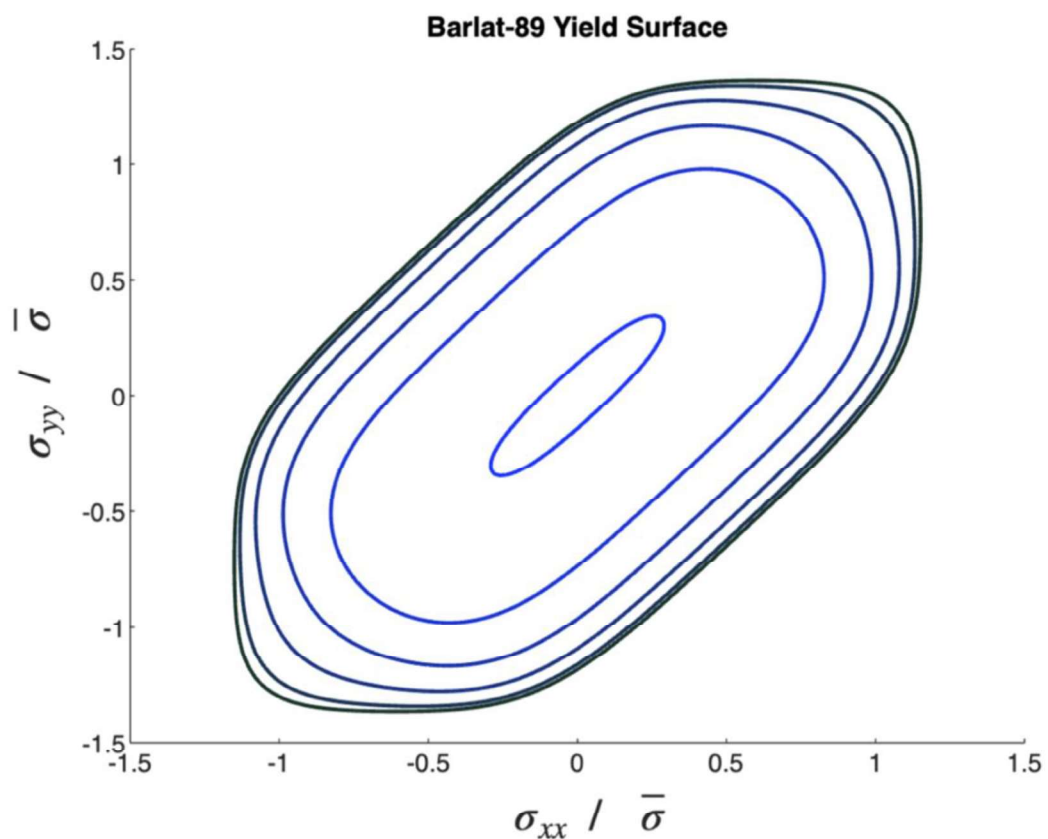


Figure 4.21 The Barlat-89 yield surface at RT

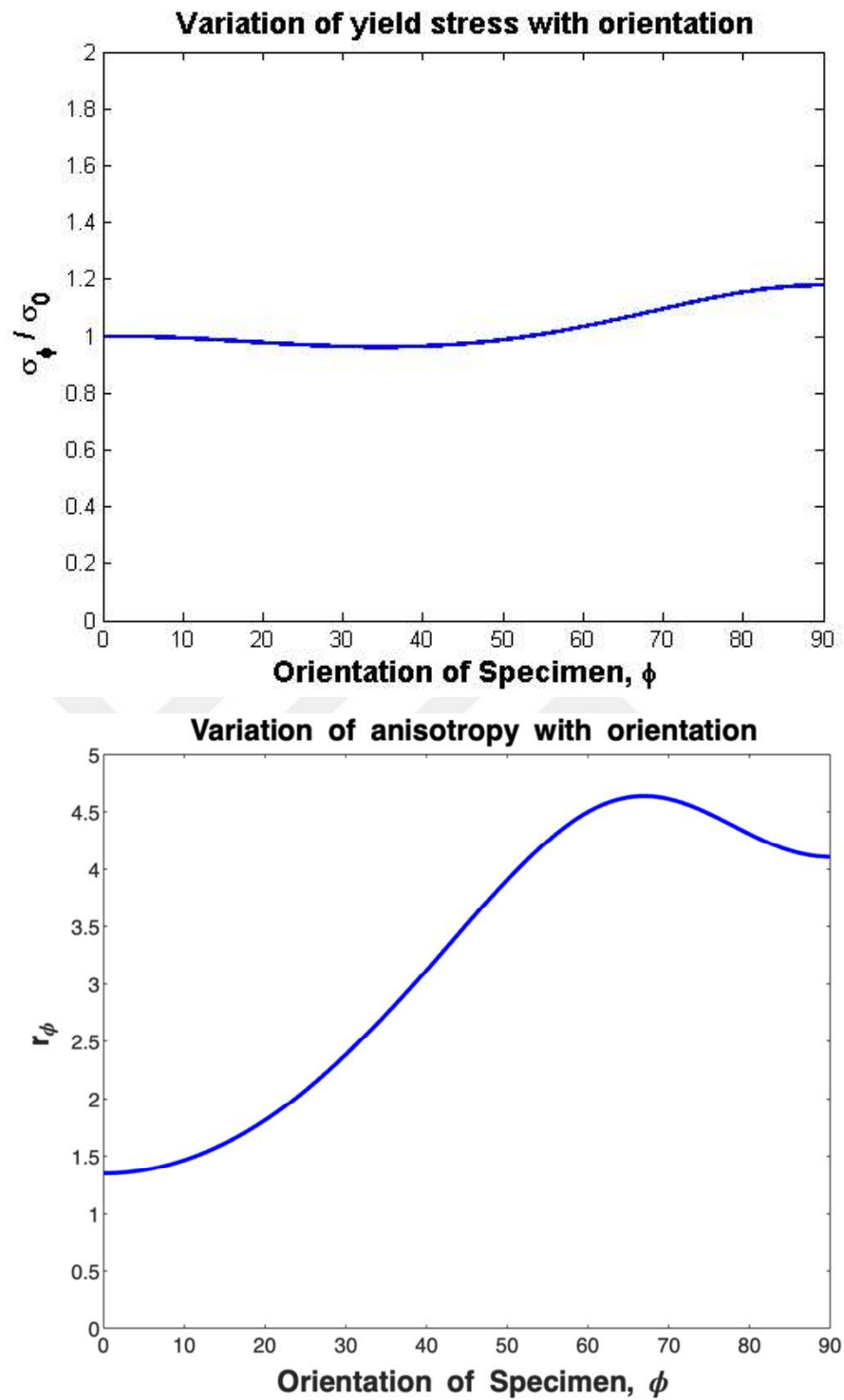


Figure 4.22 The Barlat-89 variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at RT

4.3.2. Barlat-89 Yield Surface at 100 °C

The graph shown is a representation of the Barlat-89 yield surface for the AZ31 material, this time characterized at an elevated temperature of 100 °C. Yield surfaces are critical in the field of material science and engineering as they represent the boundary within which a material can sustain elastic deformation, beyond which plastic deformation begins. At 100 °C, the material properties may differ from those at RT due to thermal softening effects that typically reduce the yield strength of materials. The Barlat-89 model is particularly useful for capturing these temperature-dependent changes in yield behavior, especially in anisotropic materials like the AZ31 magnesium alloy.

The axes of the graph, normalized stress components (σ_{xy}/σ) and (σ_{xx}/σ), maintain consistency with the previous RT graph, facilitating comparison between the two temperature states. The normalization by (σ) allows for a dimensionless representation of stress states, making it a more universal form of depiction.

The contours presented from inner to outer reflect increasing values of the shear component (s_{12}), from 0 to 0.5, similar to the RT case. It is evident that the yield surface has expanded outward as compared to the RT case, suggesting that the material can endure higher stress ratios before yielding at 100 °C. This expansion could be attributed to the material's thermal softening, where increased temperature reduces its resistance to deformation.

The shape of the yield loci implies an anisotropic yield behavior which seems consistent with the behavior at RT, though the expansion of the curves might hint at a more pronounced anisotropy at elevated temperatures. In the context of material processing, this information is vital. For example, when heat treatments or high-temperature forming processes are involved, the material's yield criteria under these conditions must be well-understood to optimize the processes and prevent failure. From an academic standpoint, analyzing the Barlat-89 yield surface at different temperatures allows us to understand the influence of temperature on the anisotropic plastic deformation of materials. Such an understanding is instrumental in the development of constitutive models that are used in FEA for the simulation of forming

processes, crash tests, and other applications where materials are subjected to complex loading conditions at various temperatures. In conclusion, the yield surface diagram for the AZ31 material at 100 °C provides an expanded view of the yield criteria relative to RT, illustrating the decreased yield strength and the potential for greater plastic deformation due to thermal softening. This temperature-dependent behavior is essential for accurate predictions and designs in engineering applications involving AZ31 and similar materials.

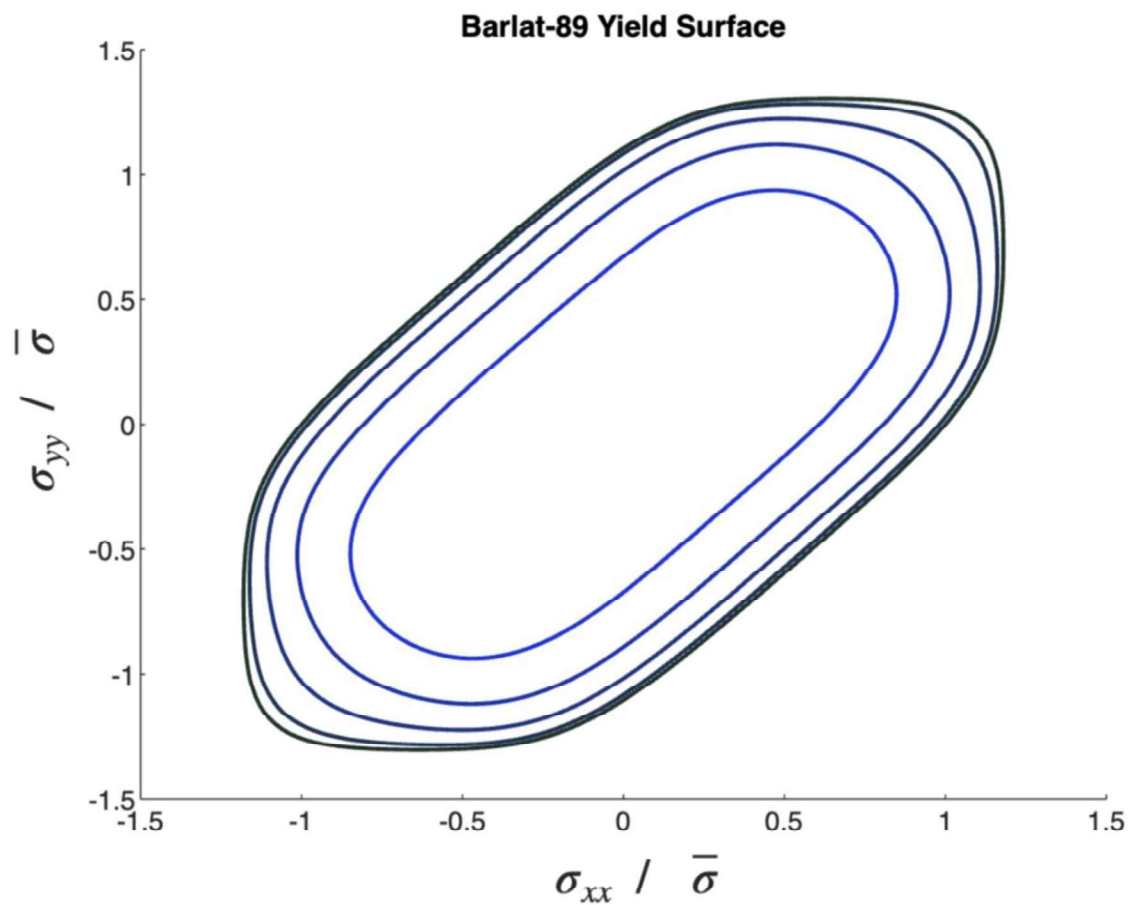


Figure 4.23 The Barlat-89 yield surface at 100 °C

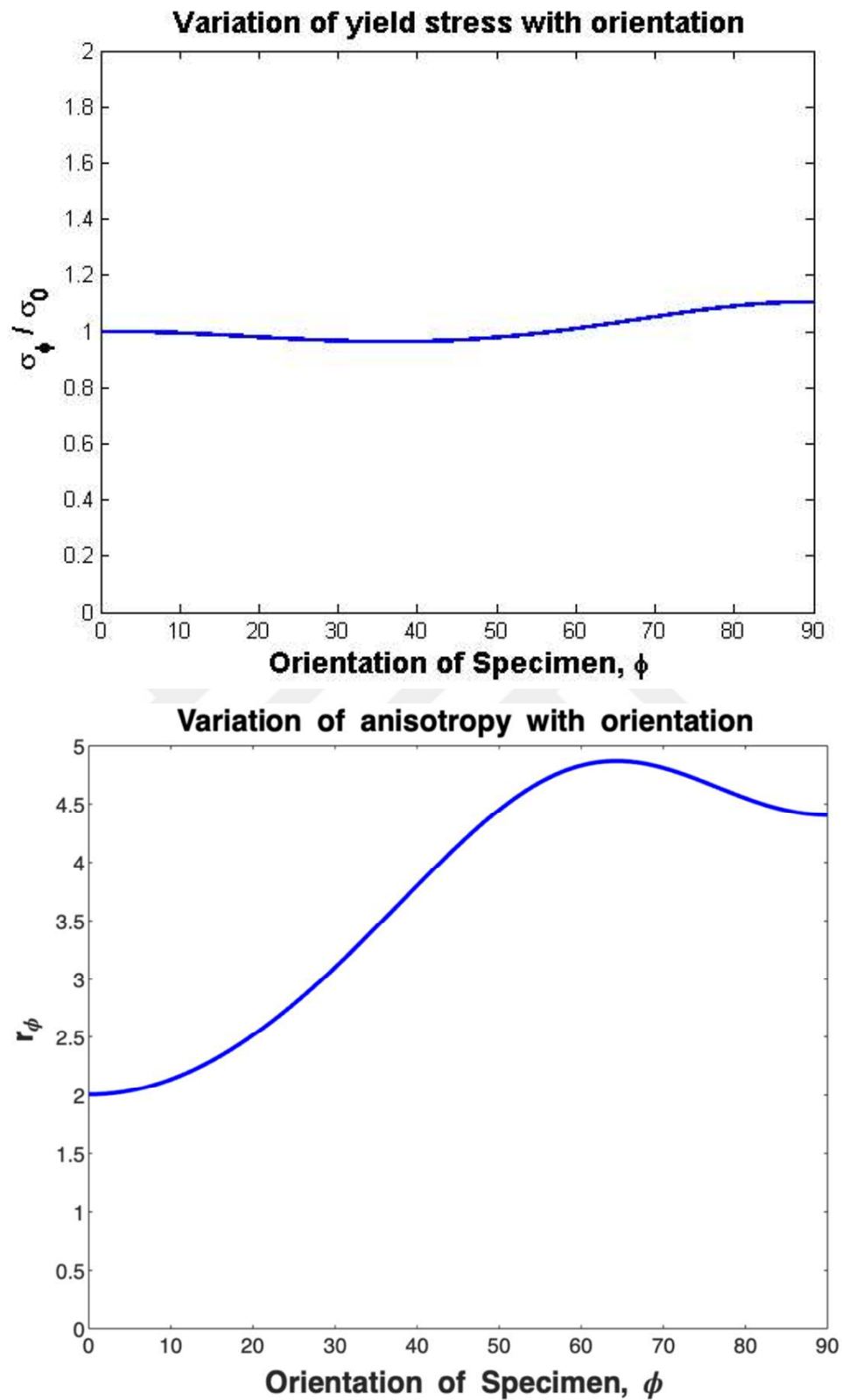


Figure 4.24 The Barlat-89 variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at 100 °C

4.3.3. The Barlat-89 Yield Surface at 150 °C

This graph presents the Barlat-89 yield surface for AZ31 material at an elevated temperature of 150 °C. As in the previous examples, the yield surface provides valuable insight into the material's response to stress under these specific thermal conditions. The axes represent the normalized stress components, (σ_{xy}/σ) and (σ_{xx}/σ) , allowing comparison across different stress states and temperatures.

At 150 °C, we would expect further softening of the material compared to the yield surfaces at RT and 100 °C, as higher temperatures generally lead to a reduction in yield strength and an increase in ductility. The contours of the yield surface, which depict increasing values of shear stress component (s_{12}) from 0 to 0.5, appear more expanded than those at lower temperatures, confirming this expectation.

This outward expansion of the yield loci with increasing temperature suggests that the material's capacity to undergo plastic deformation has increased. The yield surface's elongated shape in certain stress directions indicates an anisotropic plastic response, which is characteristic of magnesium alloys like AZ31, often due to their hexagonal close-packed (HCP) crystal structure.

In the context of material processing or structural applications where the AZ31 alloy would be subjected to elevated temperatures, the understanding of how the yield surface evolves with temperature is crucial. For instance, in sheet metal forming, temperature control is essential to prevent tearing or failure, and in predicting the buckling of structural components under thermal loads, knowledge of the yield surface at relevant temperatures is critical.

An academic discussion of the Barlat-89 yield surface at 150 °C would note that the change in yield behavior with temperature underscores the need for dynamic material models that can accurately represent the mechanical behavior under a range of thermal conditions. Such models must incorporate the temperature-dependent yield criteria to simulate the performance of structures or components over their operational temperature range effectively. To conclude, the 150 °C Barlat-89 yield surface for AZ31 illustrates a pronounced thermal softening effect, reflected in the expanded yield

loci. This information is imperative for engineers and material scientists who are concerned with the design and analysis of components made from this material, as it provides critical data for predicting the onset of plastic deformation under elevated temperature conditions.

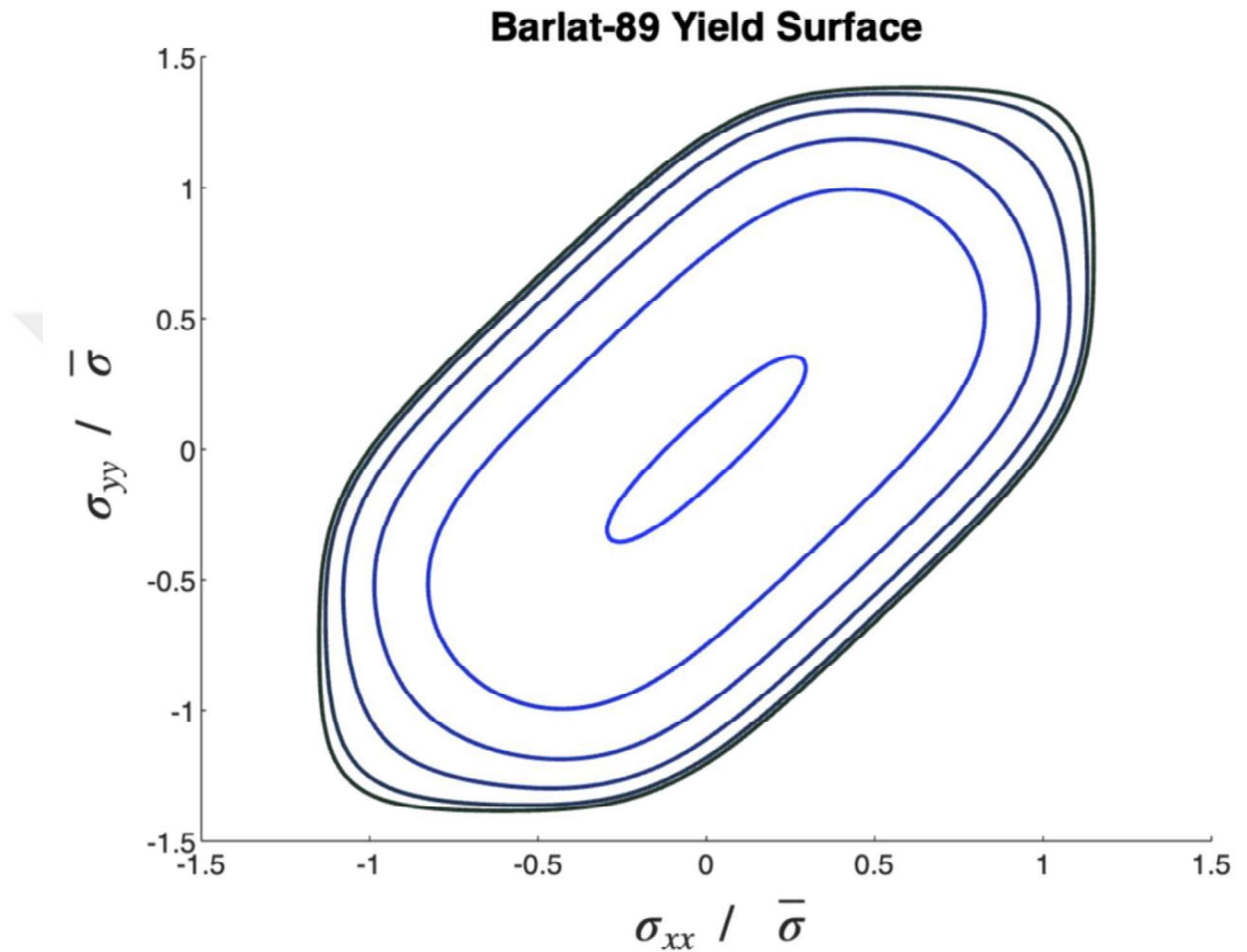


Figure 4.25 The Barlat-89 yield surface at 150 °C

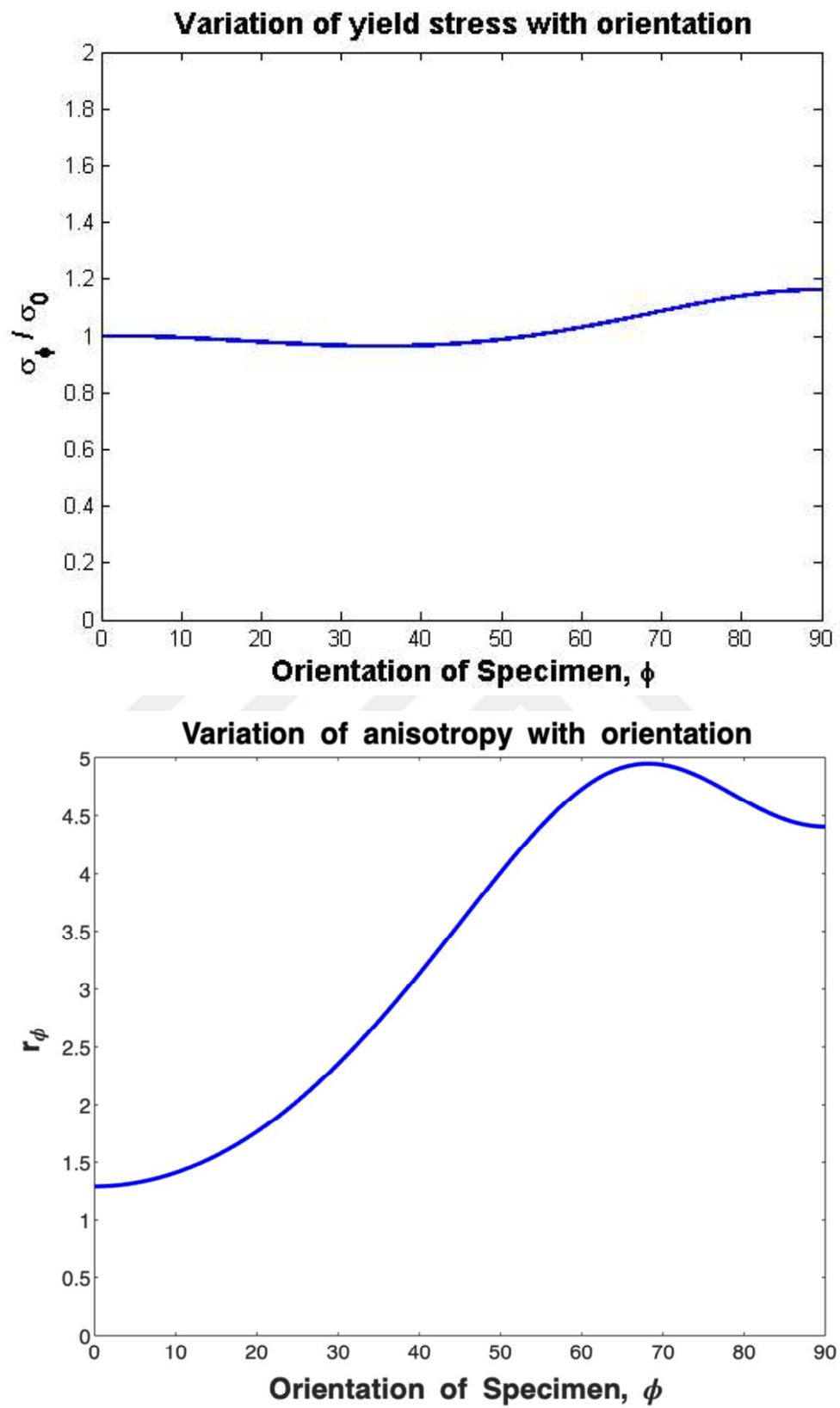


Figure 4.26 The Barlat-89 variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at 150 °C

4.3.4. The Barlat-89 Yield Surface at 200 °C

The graph illustrating the Barlat-89 yield surface for AZ31 at 200 °C provides insightful details about the material's response to stress at elevated temperatures. As temperature increases, we observe significant thermal softening, evident from the expanded contours of the yield surface, indicating a greater propensity for plastic deformation. The material's anisotropic properties are likely more pronounced, with the yield surface suggesting varying strengths and ductilities along different axes. This elevated temperature behavior is crucial for simulations and real-world applications where accurate predictions of material performance under thermal conditions are required. Such data are indispensable in engineering fields, particularly in the aerospace and automotive sectors, where materials are frequently subjected to high operational temperatures, ensuring that designs and manufacturing processes account for the temperature-induced changes in material properties.

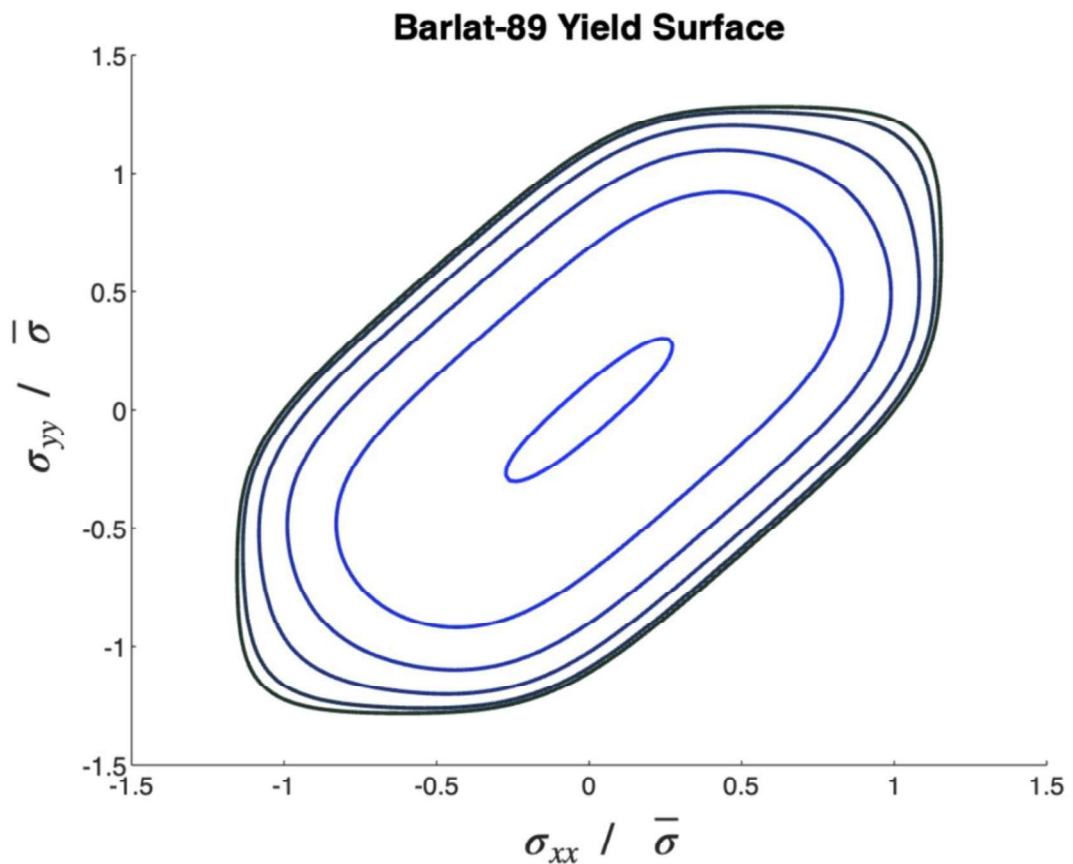


Figure 4.27 The Barlat-89 yield surface at 200 °C

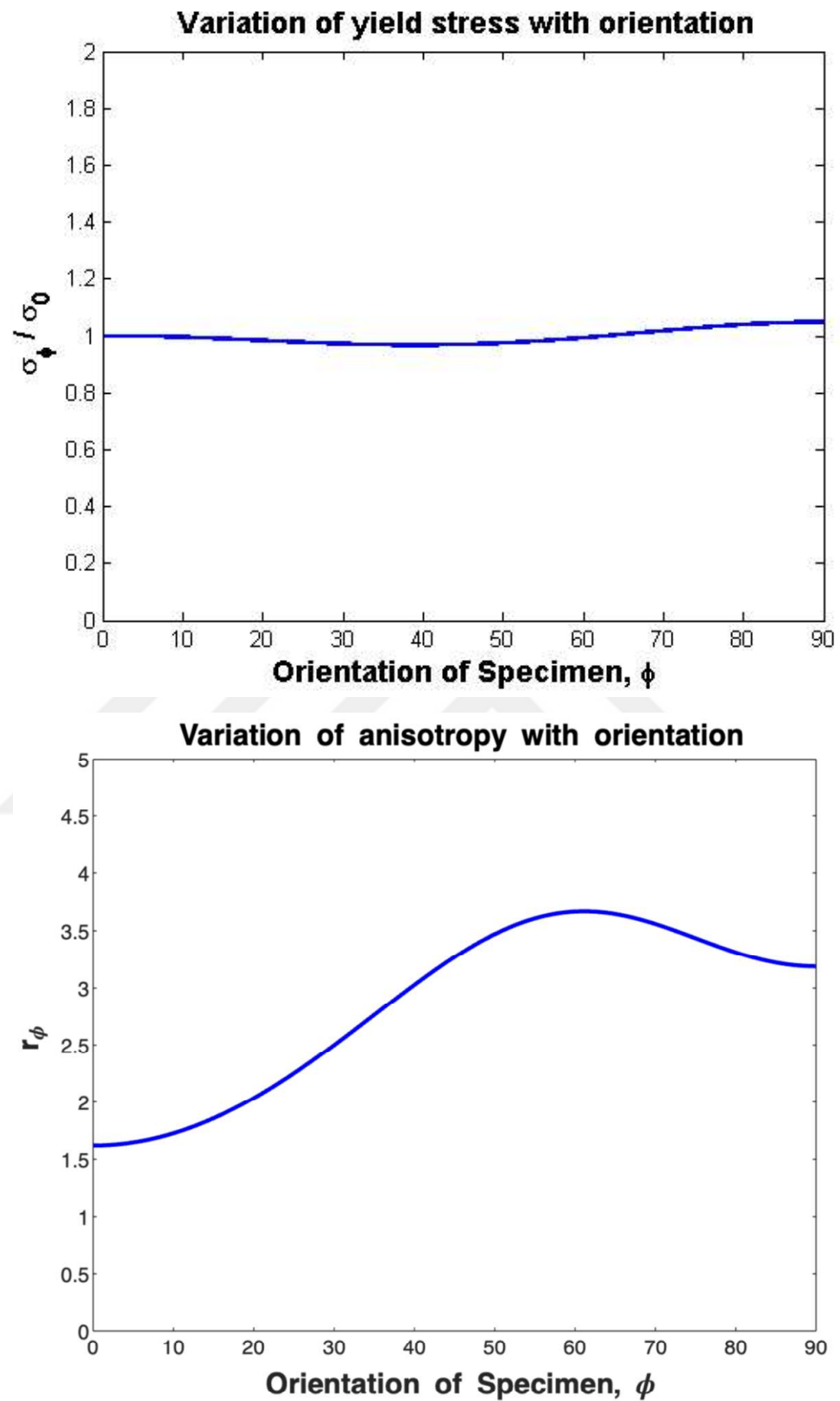


Figure 4.28 The Barlat-89 variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at 200 °C

4.3.5. The Barlat-89 Yield Surface at 250 °C

The graph illustrating the Barlat-89 yield surface for AZ31 at 250 °C provides insightful details about the material's response to stress at elevated temperatures. As temperature increases, we observe significant thermal softening, evident from the expanded contours of the yield surface, indicating a greater propensity for plastic deformation. The material's anisotropic properties are likely more pronounced, with the yield surface suggesting varying strengths and ductilities along different axes. This elevated temperature behavior is crucial for simulations and real-world applications where accurate predictions of material performance under thermal conditions are required. Such data are indispensable in engineering fields, particularly in the aerospace and automotive sectors, where materials are frequently subjected to high operational temperatures, ensuring that designs and manufacturing processes account for the temperature-induced changes in material properties.

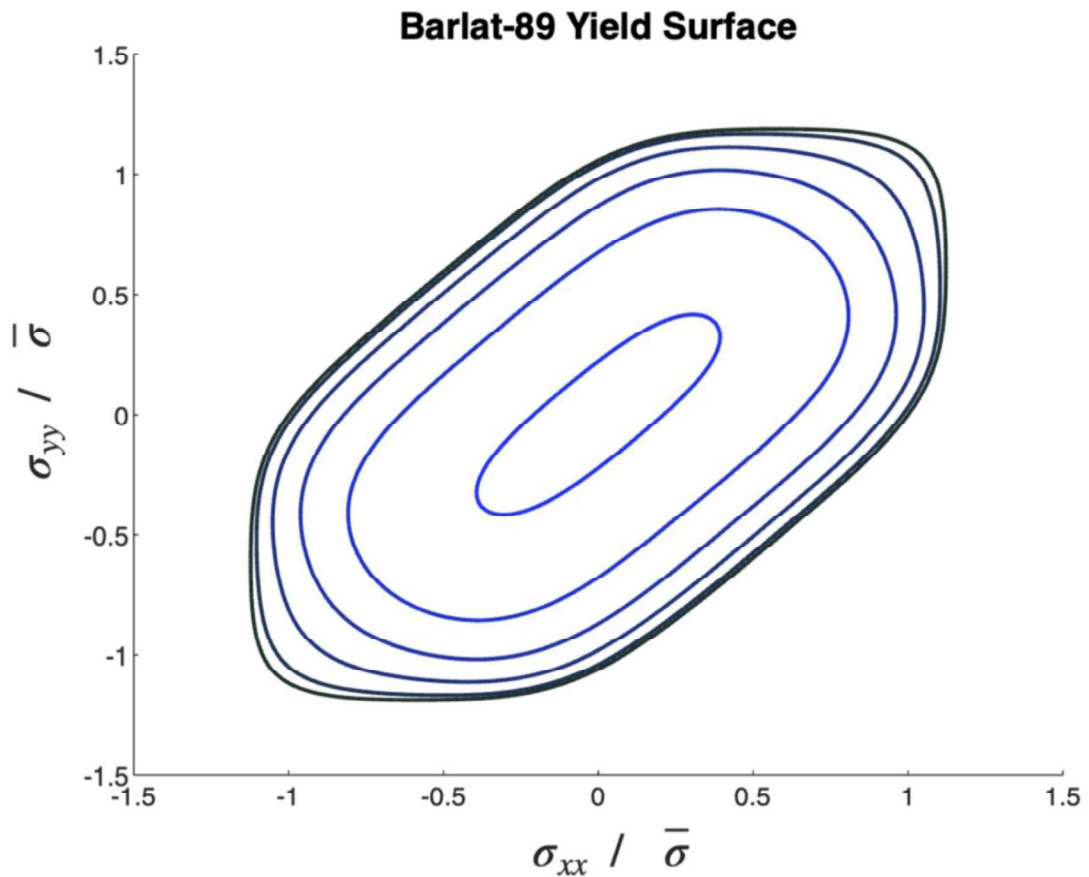


Figure 4.29 The Barlat-89 yield surface at 250 °C

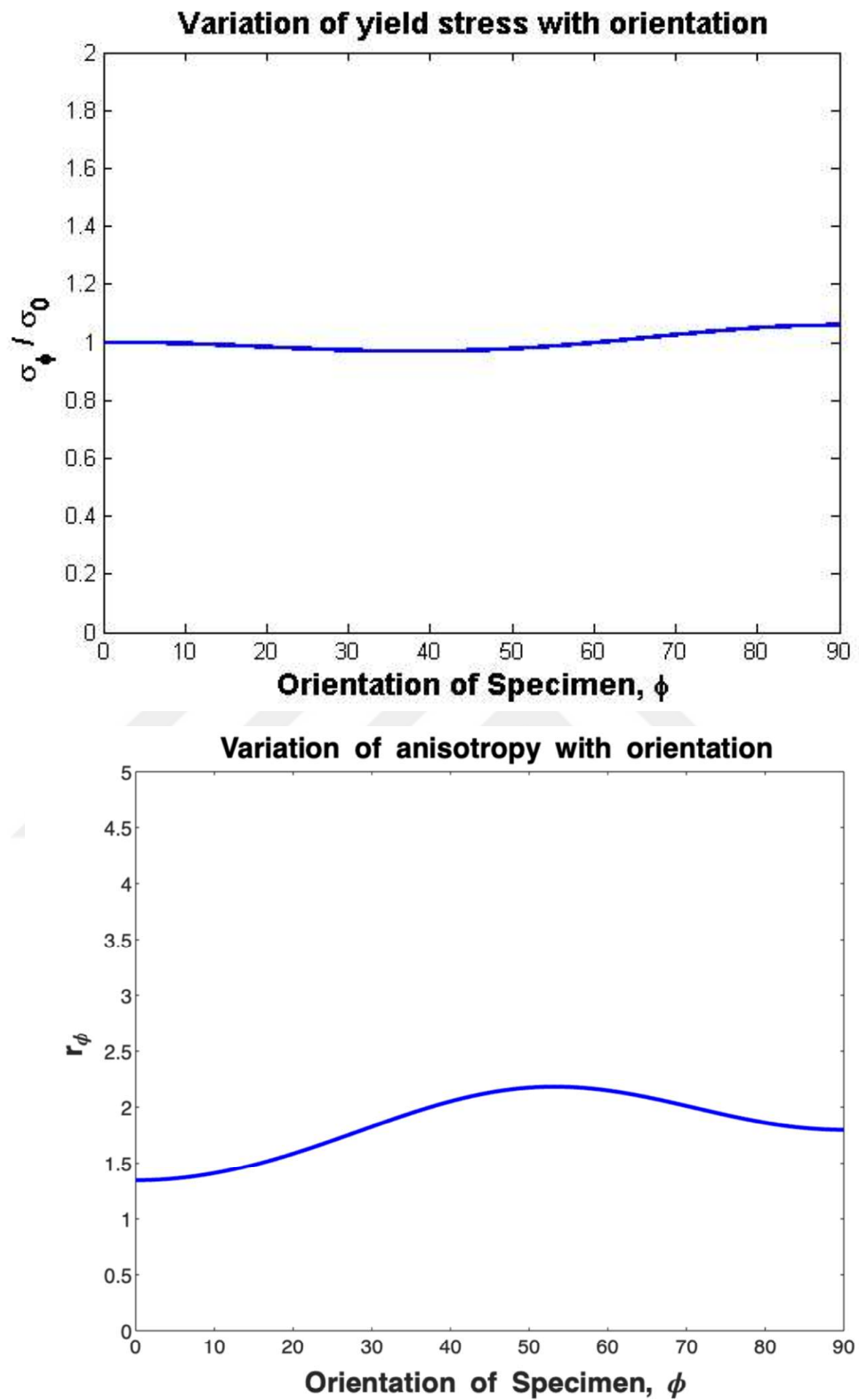


Figure 4.30 The Barlat-89 variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at 250 °C

4.3.6. The Barlat-89 Yield Surface at 300 °C

The latest graph showcases the Barlat-89 yield surface for the material at an elevated temperature of 300 °C. This temperature increase has led to further thermal softening, which is demonstrated by the continued growth and change in the shape of the yield surface contours. The effects of temperature on material properties such as yield strength and ductility are even more evident at this higher temperature, with the yield surface expanding outward. This indicates that the material has become more ductile and can undergo more significant plastic deformation before failure. The distinct shape of the yield surface at this elevated temperature also underscores the anisotropic nature of the material's mechanical properties. This type of data is crucial for industries where materials are expected to perform under high thermal stress, such as in the manufacture of engine components or heat shields, where understanding the impact of temperature on material behavior ensures the reliability and safety of the final product.

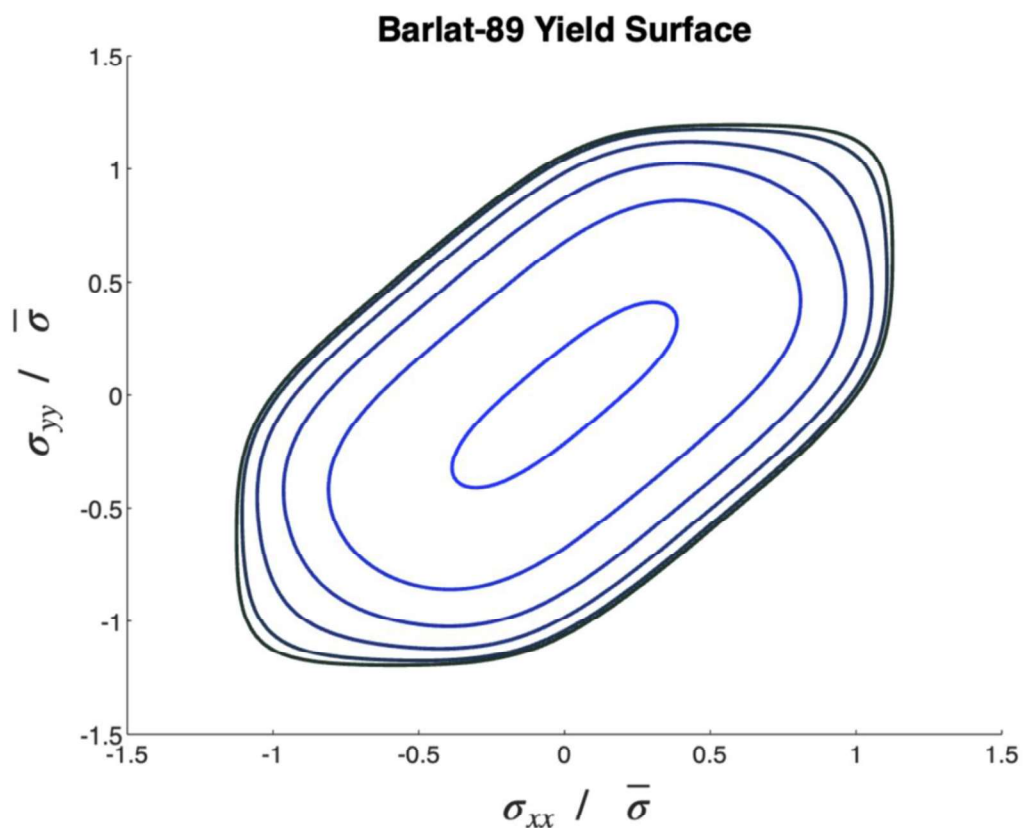


Figure 4.31 The Barlat-89 yield surface at 300 °C

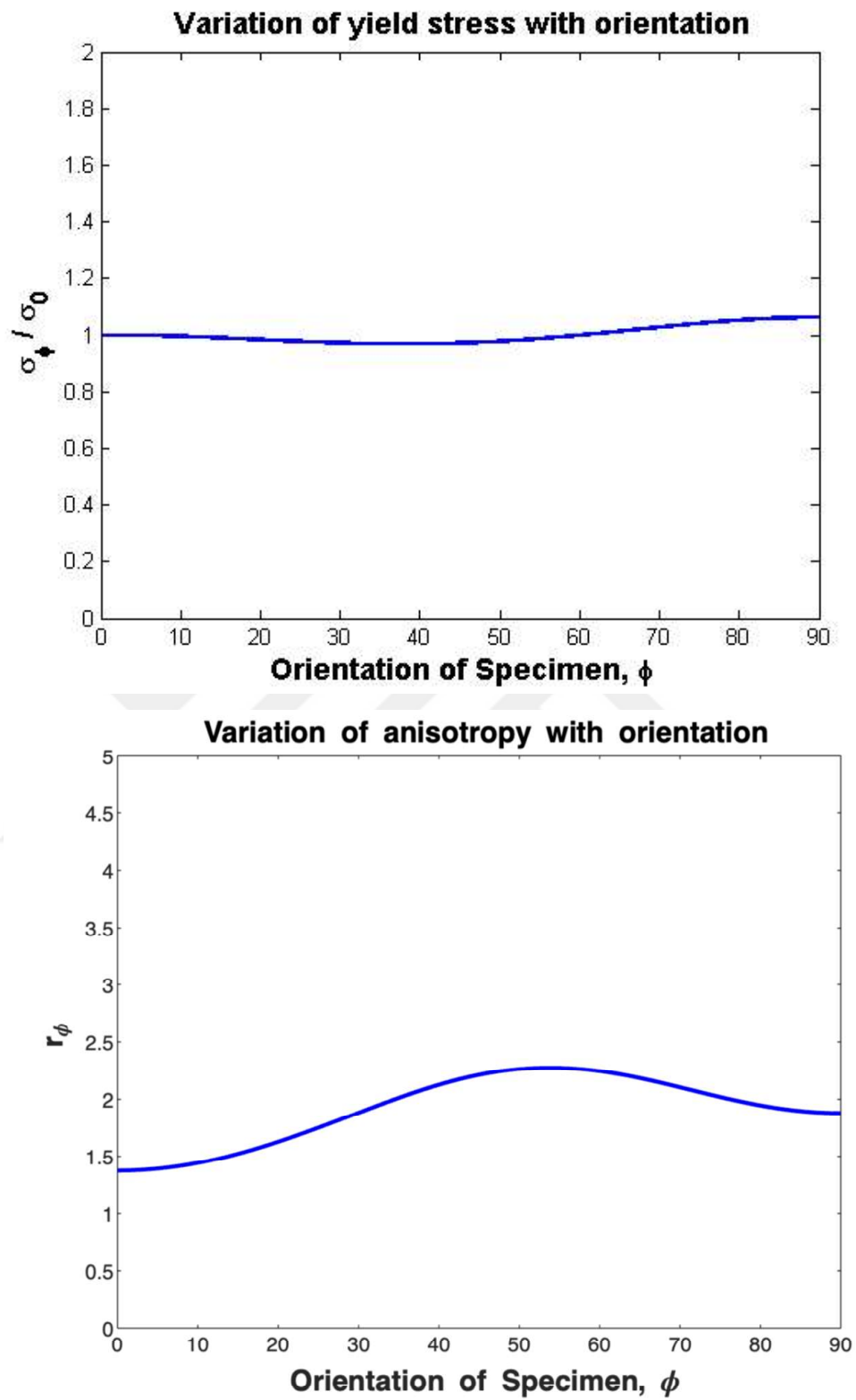


Figure 4.32 The Barlat-89 variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at 300 °C

4.4. Criteria 4: The Yld-2000 Yield Criteria

4.4.1. The Yld2000 Yield Surface at RT

The displayed graph depicts the Yld2000-2p yield surface at RT. This model represents an advanced description of the yield surface for anisotropic materials, accommodating planar anisotropy effects. The contours on the graph represent different levels of equivalent plastic strain, outlining the stress states under which the material begins to yield. At RT, the material demonstrates a tight and symmetrical yield surface, with S_{11} and S_{22} as the normal stress components on the horizontal and vertical axes, respectively. The yield surface's relatively narrow shape suggests that the material has a higher yield strength at RT when compared to how it might behave at elevated temperatures, which usually exhibit broader yield surfaces due to increased ductility and decreased yield strength. Each contour encircles the origin, showing the characteristic convexity of a yield surface, which is a fundamental postulate of plasticity theory. The material's response is likely to be quite predictable and uniform in all directions at this standard temperature, as indicated by the uniform spacing and symmetry of the contours. The data from this graph is fundamental for engineers and designers who need to predict how a material will behave under different loading conditions at RT. Understanding the yield surface at this base temperature is crucial for manufacturing processes, structural design, and safety evaluations where thermal effects are negligible or not present.

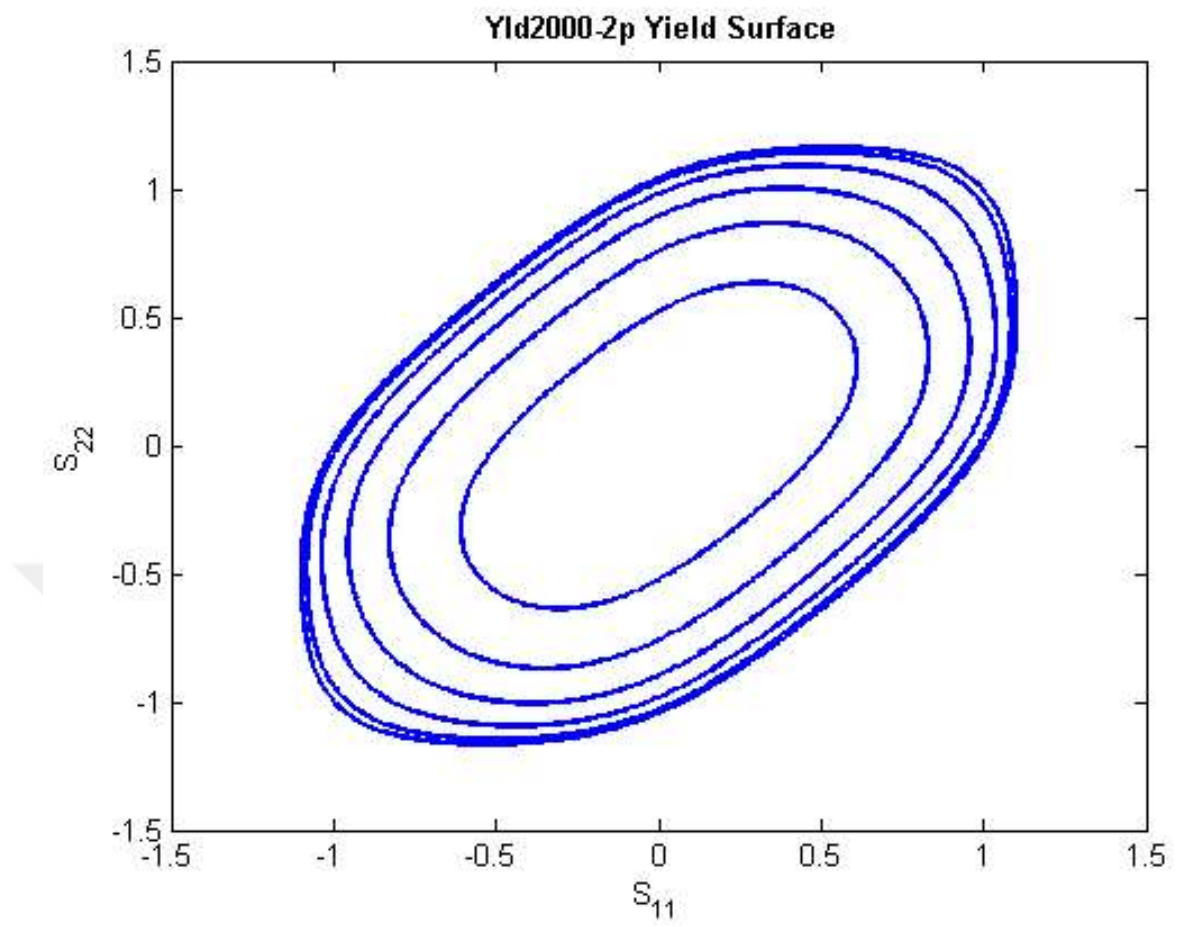


Figure 4.33 The Yld-2000 yield surface at RT

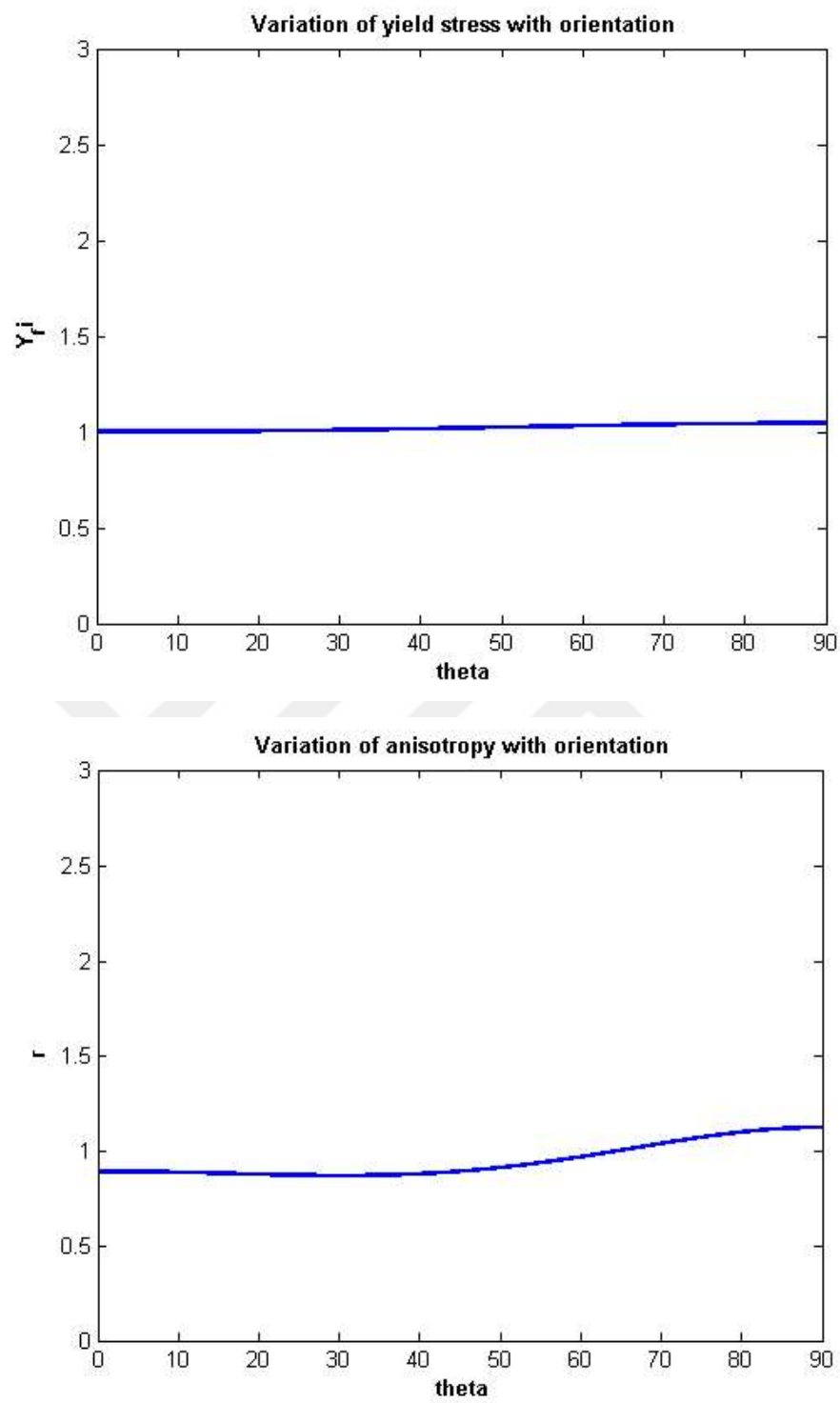


Figure 4.34 The Yld-2000 variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at RT

4.4.2. The Yld-2000 Yield Surface at 100 °C

At RT, the contours of the yield surface represented in this graph are smooth and symmetrical about the (S_{11}) axis, which suggests isotropic hardening behavior and a material with no directional preference in yielding under biaxial stress states (S_{11}) and (S_{22}). The largest elongation along the (S_{11}) axis suggests that the material can sustain more stress along this axis before yielding. As temperature increases to 100 °C, materials generally have decreased yield strength and increased ductility. This may cause the yield surface to expand outward, reflecting the material's ability to deform more before reaching its yield point under the same stress conditions. However, the exact change in the shape of the yield surface would depend on the specific material properties, which can be significantly different between materials and even between different treatments of the same material. If we were to hypothesize about the yield surface for this particular material at 100 °C, we might expect that the contours could become larger, indicating that the material yields at higher stress values due to thermal softening. Additionally, the contours may also change shape if the material exhibits different behaviors in different directions when exposed to high temperatures, possibly leading to anisotropic hardening or softening behaviors.

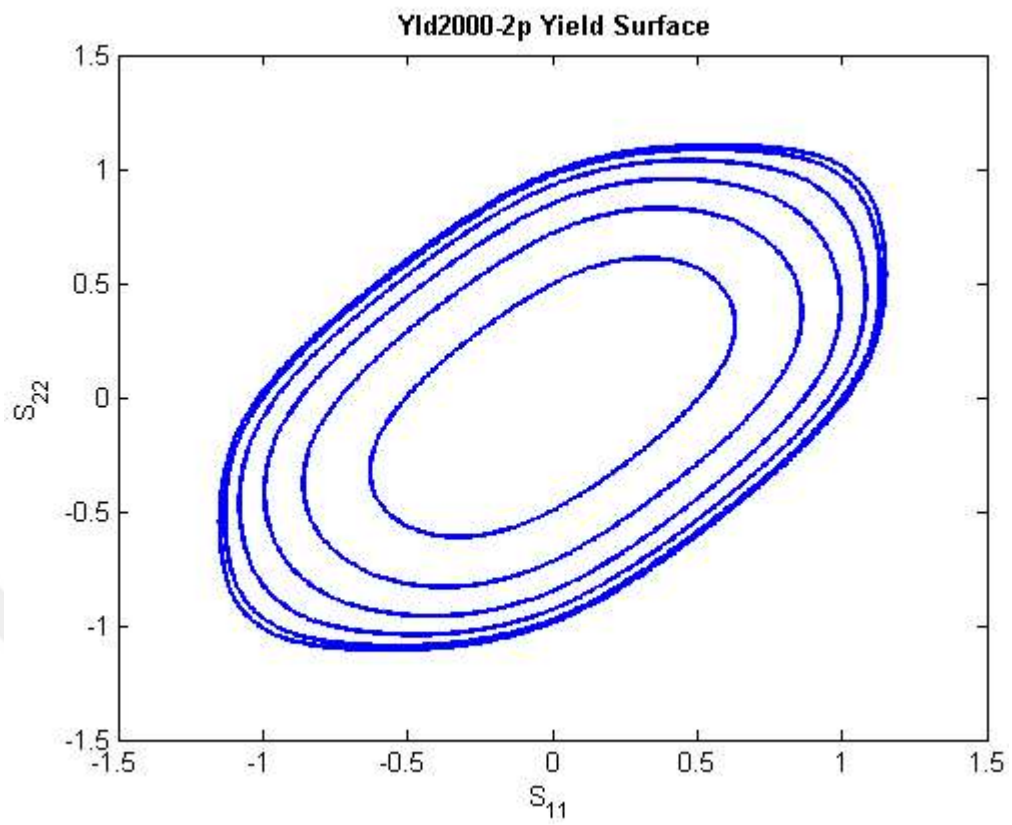


Figure 4.35 The Yld-2000 yield surface at 100 °C

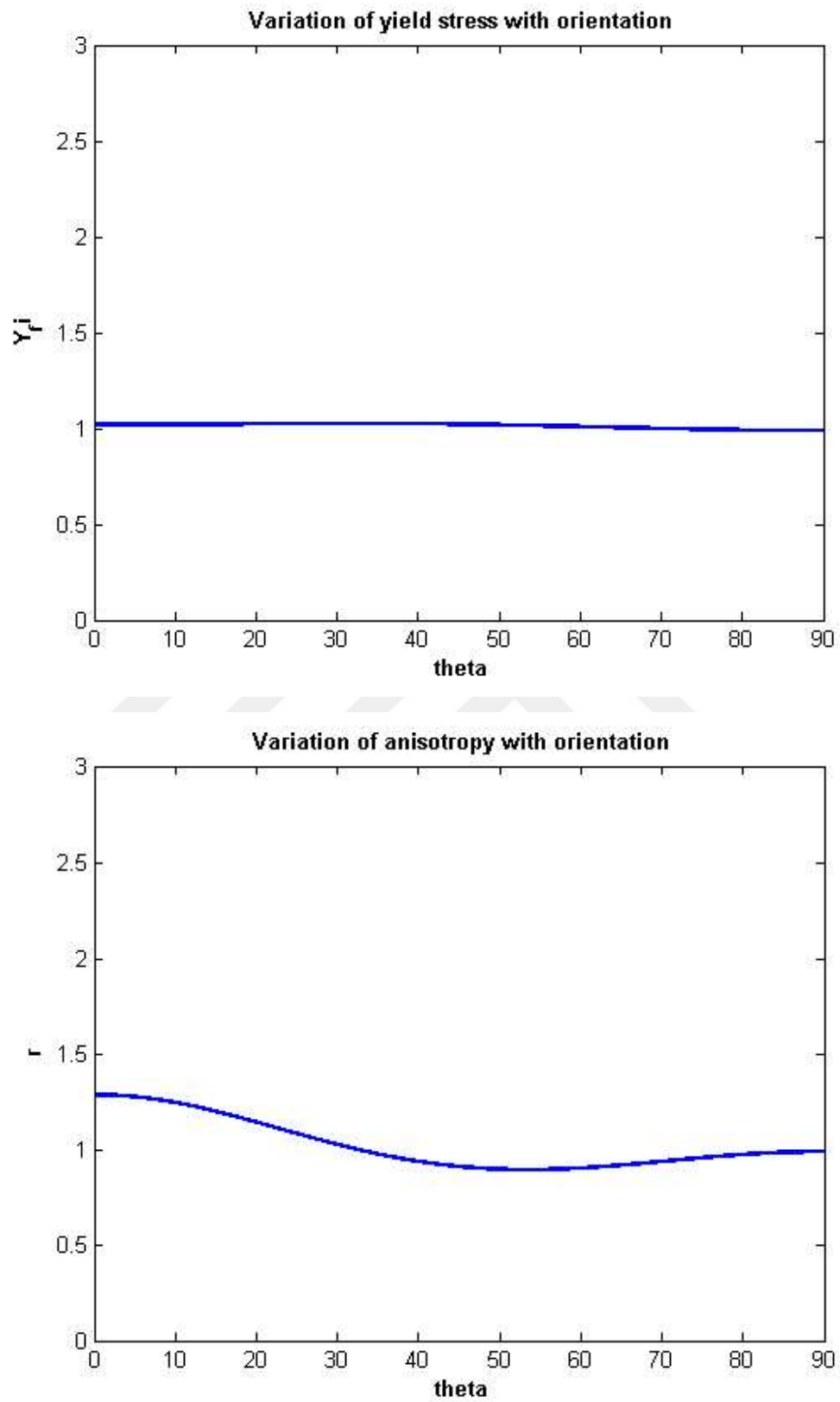


Figure 4.36 The Yld-2000 variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at 100 °C

4.4.3. The Yld-2000 Yield Surface at 150 °C

In an academic context, when examining the Yld2000-2p yield surface representation for a particular material at an elevated temperature of 150 °C, one must account for the thermally induced alterations in the mechanical response of the material. Predominantly, an escalation in temperature to this level typically results in a diminution of yield strength, evidenced by a contraction of the yield surface, as the material requires less stress to commence plastic deformation. Concurrently, the material's ductility is likely to augment, potentially elongating the yield surface along the axes of principal stresses, indicative of the material's enhanced propensity for plastic flow prior to failure. Additionally, thermal expansion can affect intrinsic stress states, thus modifying the yield surface configuration. Moreover, the temperature rise may influence the rate of work hardening, potentially leading to a less pronounced expansion of the yield surface with increasing strain. Furthermore, anisotropic characteristics of materials could be exacerbated at elevated temperatures, potentially leading to a distortion of the typically symmetrical yield surface. Rate sensitivity is another critical factor; at higher temperatures, the deformation rate may exert a more pronounced influence on the yield stress, thereby introducing a dynamic component to the yield surface which might not be represented in a static yield surface plot. Lastly, one must consider the possibility of phase transformations that can drastically alter the mechanical properties of a material at such temperatures, consequently reshaping the yield surface in a manner that can only be reliably determined through empirical experimentation specific to the material under investigation.

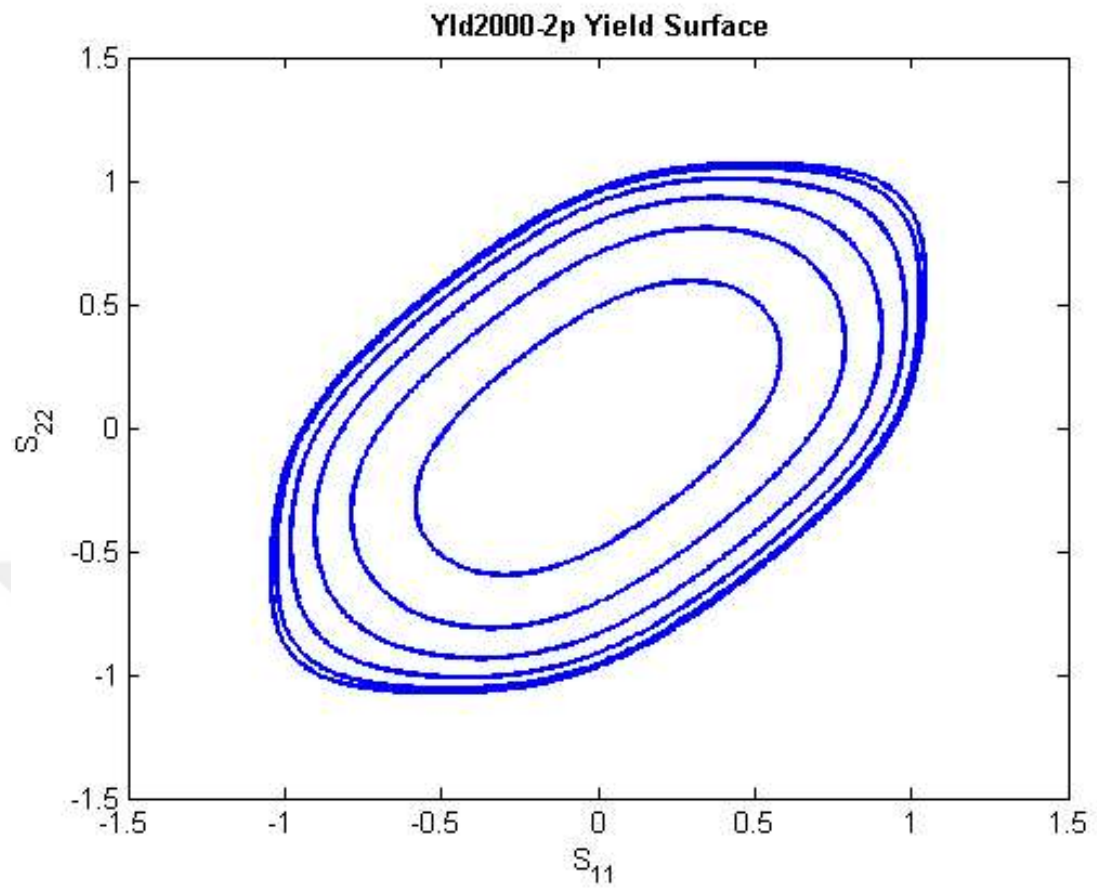


Figure 4.37 The Yld-2000 yield surface at 150 °C

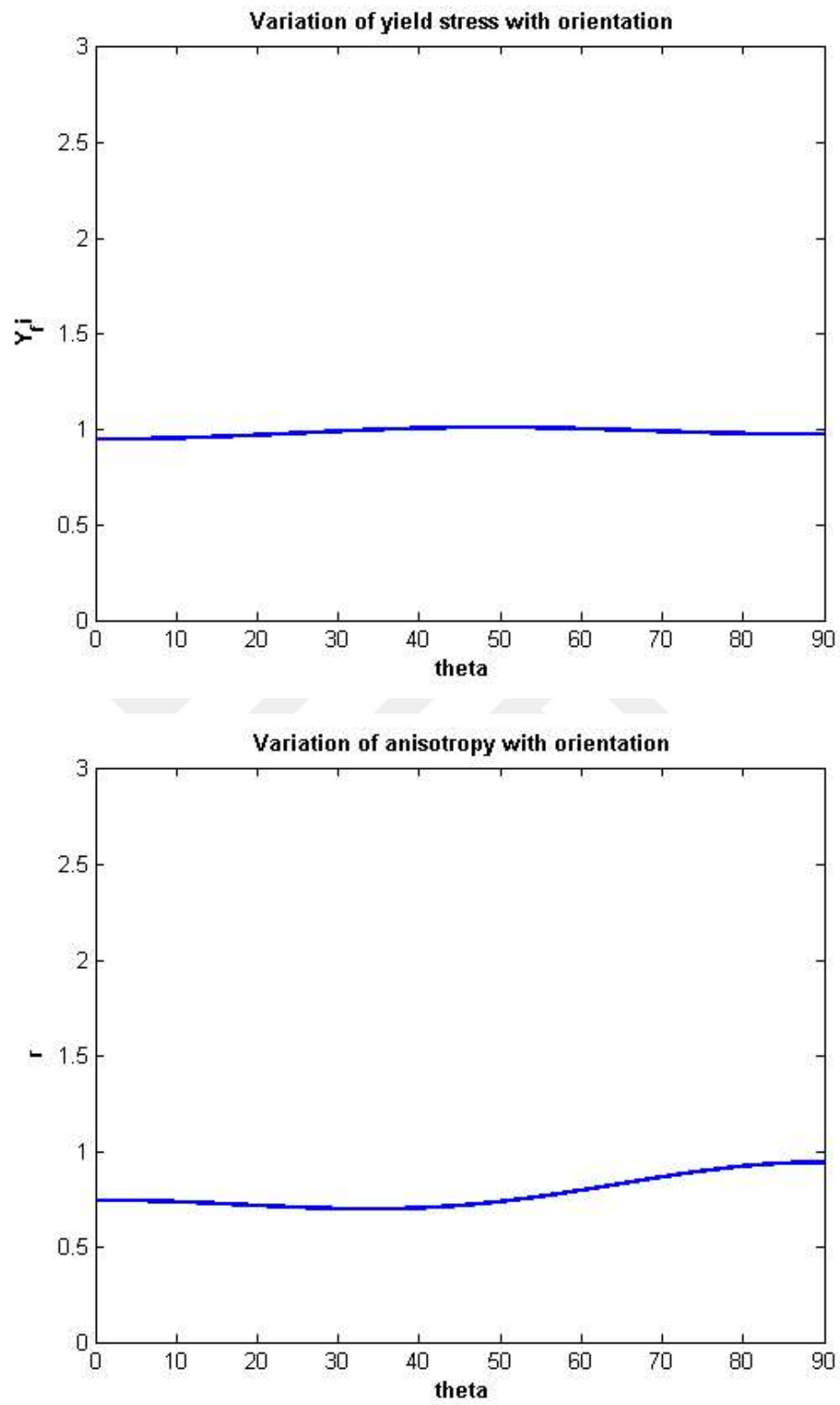


Figure 4.38 The Yld-2000 variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at 150 °C

4.4.4. The Yld-2000 Yield Surface at 200 °C

The Yld2000-2p yield surface diagram presented herein is indicative of a material's yield behavior under biaxial stress states, typically employed in the field of plasticity to model the onset of yielding. As the environmental temperature escalates to 200 °C, several material properties could undergo substantial changes, which would be reflected in the morphology of the yield surface.

At such an elevated temperature, it is plausible to expect a reduction in the yield stress threshold due to thermal softening, which typically causes the yield surface to expand. This phenomenon suggests that the material may begin to deform plastically under lower stress levels as compared to its behavior at RT. The high-temperature condition is also likely to enhance the material's ductility, which would further modify the yield surface contours, potentially resulting in a more pronounced elongation of the yield surface along the principal stress axes, thus reflecting the material's increased capacity to undergo plastic deformation.

Additionally, the thermal expansion of the material must be considered as it can alter the internal stress state, thereby potentially skewing the yield surface. Moreover, the rate of work hardening could decrease, leading to less stiffening of the material with progressive deformation, which in turn might be observed as less pronounced growth of the yield surface with strain.

Anisotropy effects could also become more marked at 200 °C. Materials that exhibit different properties in different directions might exhibit an asymmetric yield surface due to these directional dependencies being accentuated at higher temperatures. Rate sensitivity is another aspect that might become more apparent at elevated temperatures; the material may show a more pronounced rate-dependent yield behavior, which could introduce time-dependent effects into the yield criteria.

Lastly, the possibility of microstructural changes or phase transformations occurring at 200 °C cannot be disregarded, as they could lead to significant alterations in material properties, hence reshaping the yield surface. This transformation might not be visible

in a static yield surface plot and requires dynamic testing to fully understand its impact on the material's yielding behavior.

Experimental validation is imperative to accurately characterize the material behavior at this temperature, as many of the aforementioned factors can interact in complex ways, necessitating a comprehensive understanding of the material's thermal response to predict the behavior with precision.

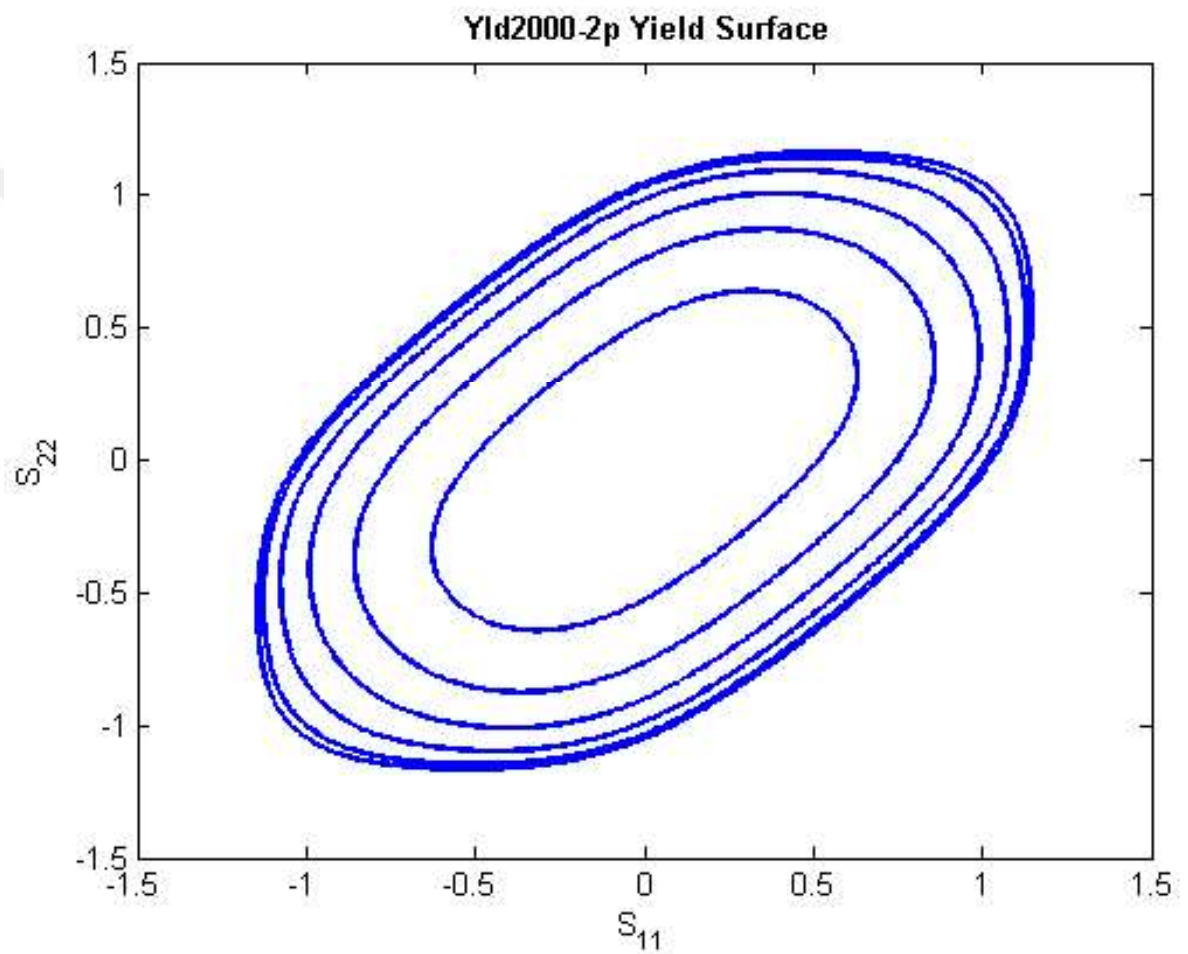


Figure 4.39 The Yld-2000 yield surface at 200 °C

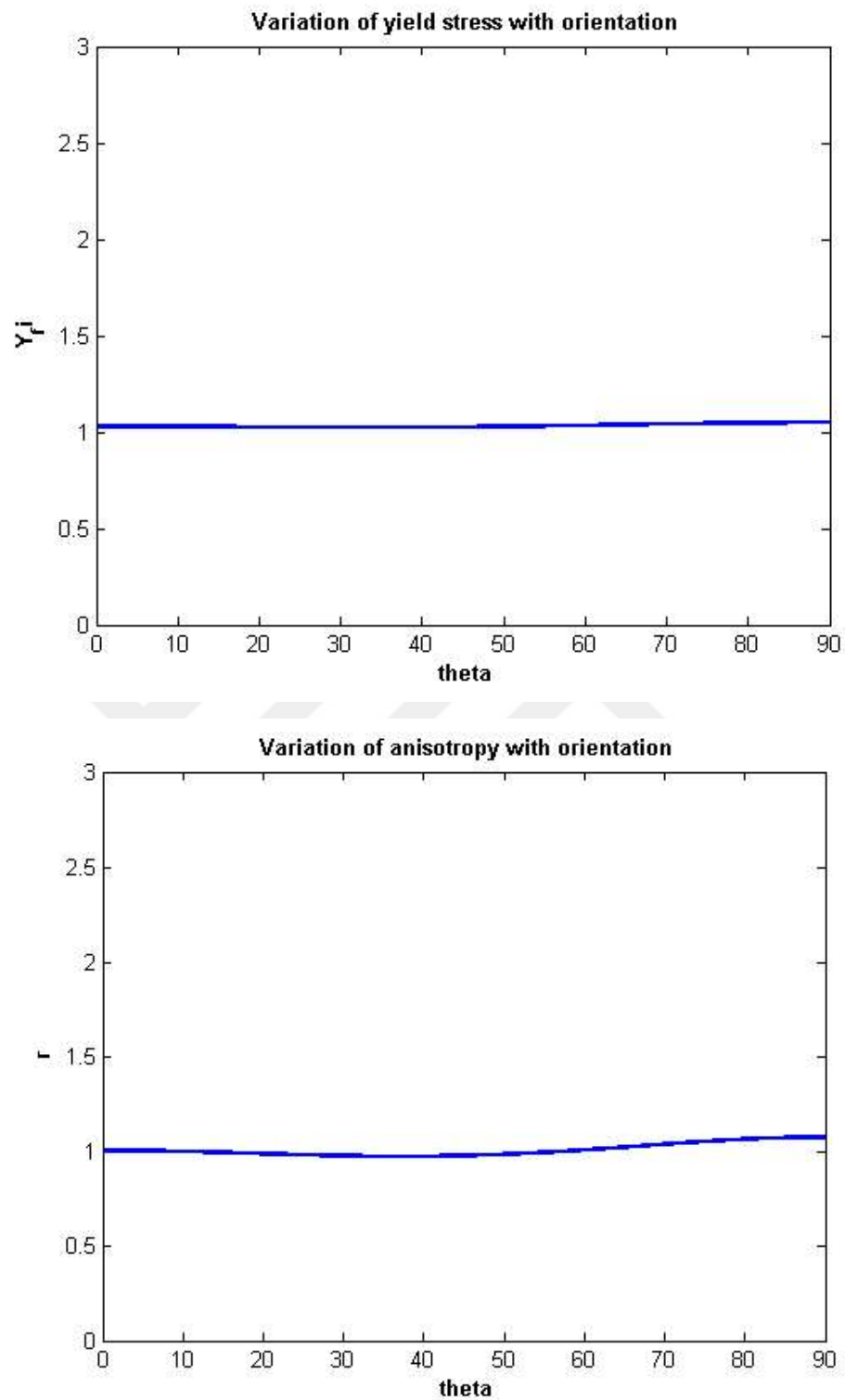


Figure 4.40 The Yld-2000 variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at 200 °C

4.4.5. The Yld-2000 Yield Surface at 250 °C

The graphical representation depicts a yield surface, specifically the Yld2000-2p model, which is designed to predict the onset of plastic deformation in materials under different stress states. Analyzing such a model at a temperature of 250 °C suggests a consideration of thermally induced material property alterations affecting the yield condition.

At 250 °C, the yield surface may exhibit significant changes when compared to ambient temperature conditions due to the pronounced effects of thermal softening, which reduces the resistance of the material to plastic deformation. This would likely cause an expansion of the yield surface in the stress space. The rate of thermal expansion might also be greater, leading to an increase in the internal stress levels, which could alter the stress state at which yielding initiates.

Ductility is generally expected to increase with temperature, and this can manifest in the yield surface as a more pronounced distension in the direction of the principal stresses, suggesting the material's capacity to undergo significant plastic deformation before failure. The high temperature could also influence the material's strain rate sensitivity, which may become more pronounced, indicating that the time-dependent aspects of deformation are more critical to consider at this elevated temperature.

Furthermore, the anisotropic features of the material's mechanical response can become more evident. The directional dependencies of yield strength and ductility can result in an asymmetric yield surface, a characteristic that is crucial to consider in the design and analysis of components expected to operate under such high temperatures.

The propensity for microstructural evolution or phase transformation at this elevated temperature is another aspect that must be factored into the interpretation of the yield surface. Such transformations can lead to an abrupt change in the mechanical behavior, which would not be directly observable from a static yield surface but would significantly affect the actual yielding behavior of the material.

It is imperative to note that the predictive capacity of the yield surface at elevated temperatures such as 250 °C depends critically on the accuracy of the material model to account for all thermally activated mechanisms. Validation through experimental data is crucial to confirm the model's assumptions and to ensure its reliability in predicting the yield behavior under such extreme conditions. As such, the yield surface at 250 °C serves as a valuable tool for understanding material behavior, informing the design process, and predicting failure modes in high-temperature applications.

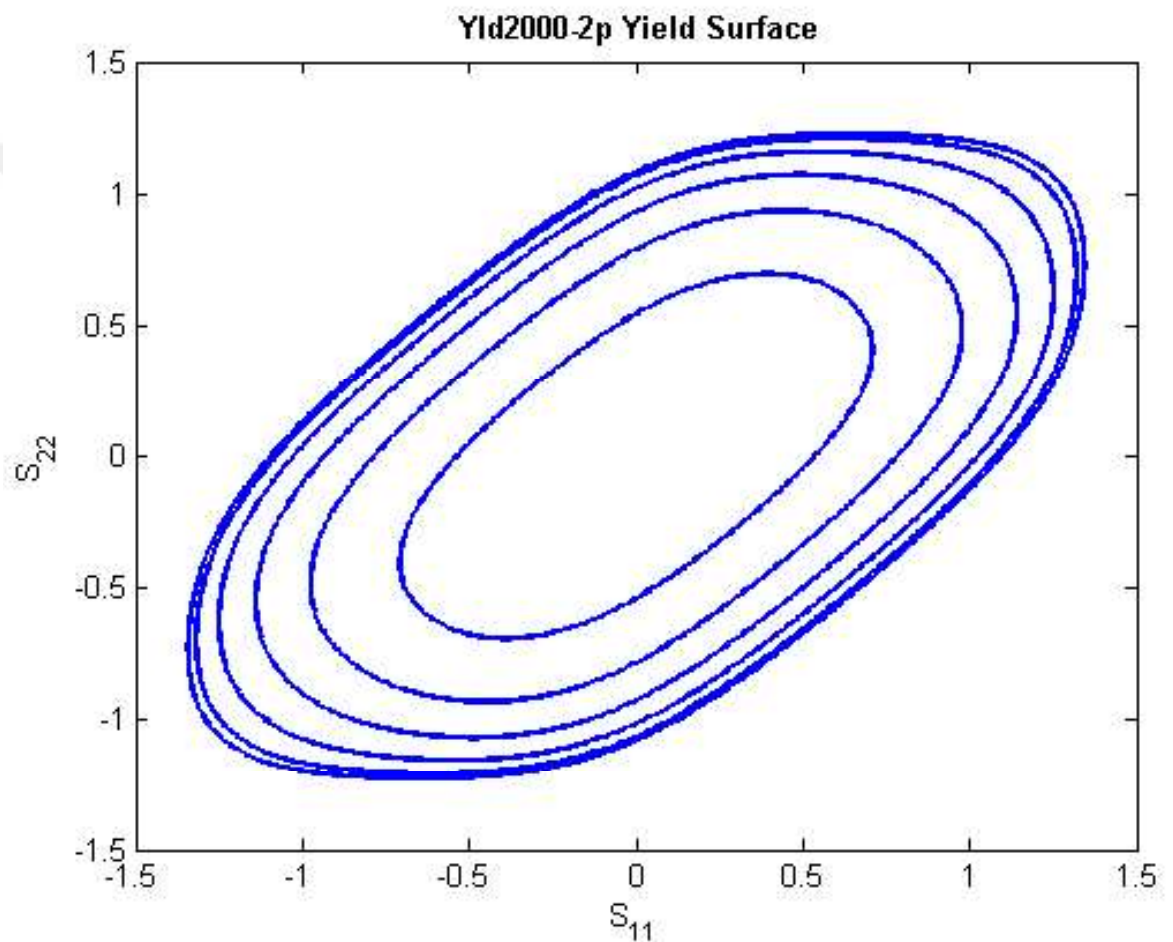


Figure 4.41 The Yld-2000 yield surface at 250 °C

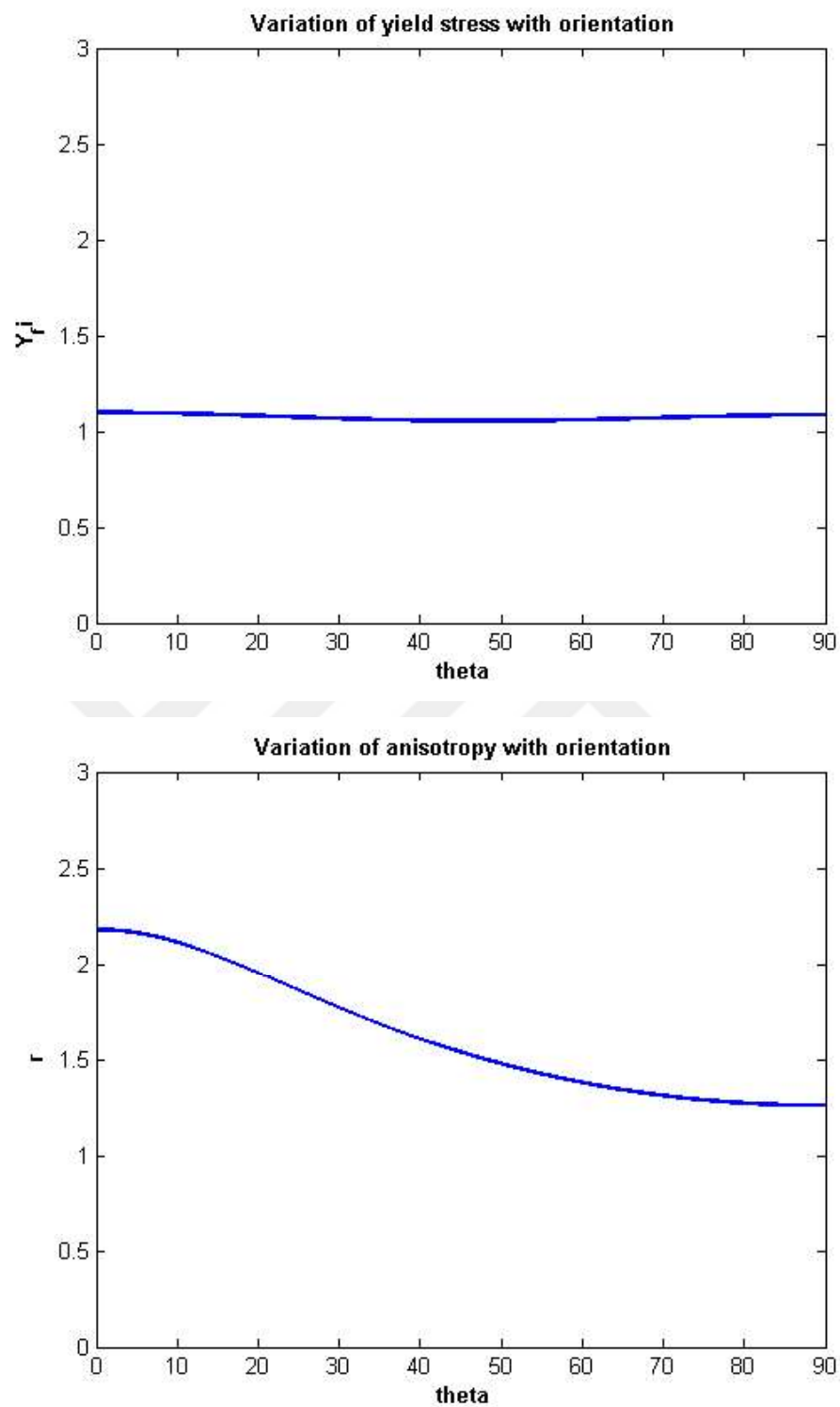


Figure 4.42 The Yld-2000 variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at 250 °C

4.4.6. The Yld-2000 Yield Surface at 300 °C

The yield surface depicted in the image, labelled as the Yld2000-2p Yield Surface, serves as a fundamental tool in the realm of plasticity theory for predicting when a material under multi-axial stress will begin to plastically deform. When examining this model at a temperature of 300 °C, we are considering the influence of high temperature on the material's yield behavior, which is essential for materials subjected to elevated thermal conditions, such as those found in aerospace or power generation applications.

At 300 °C, the material represented by this yield surface would likely undergo significant thermal softening, which decreases its yield strength, making it more susceptible to plastic deformation. As a result, the yield surface would expand, indicating that the material can sustain less stress before yielding in comparison to its behavior at RT. This thermal softening effect is especially relevant for materials like metals and alloys, where thermal activation can lead to an increased number of slip systems, making dislocation movement easier and thus reducing the critical resolved shear stress.

Furthermore, at such temperatures, recovery and recrystallization processes could be activated, which may lead to a change in the material's microstructure over time, affecting its yield and hardening behavior. The yield surface might also become more rounded due to the increased ductility, suggesting that the material can accommodate more plastic strain. This is particularly crucial for processes such as hot forming or high-temperature component service where maintaining structural integrity is essential.

The strain rate sensitivity of the material could also become more significant at this temperature. If the material exhibits viscoplastic behavior, the yield surface would be dynamic, shifting with the strain rate. Therefore, at 300 °C, the strain rate effects must be incorporated into the model for accurate predictions of the yield point under different loading rates.

Accounting for temperature-dependent properties, such as the Young's modulus and the thermal expansion coefficient, is critical as these would directly influence the stress

state within the material. The high temperature could cause the material to expand, inducing compressive stresses that would need to be factored into the yield condition.

The yield surface at 300 °C thus provides critical insight into the material's behavior under high thermal stress and is a valuable asset in predicting the lifecycle and safety of components operating under such extreme conditions. Experimental validation at this temperature range would be vital to confirm the material model's accuracy and to refine the yield surface for reliable application in engineering designs.

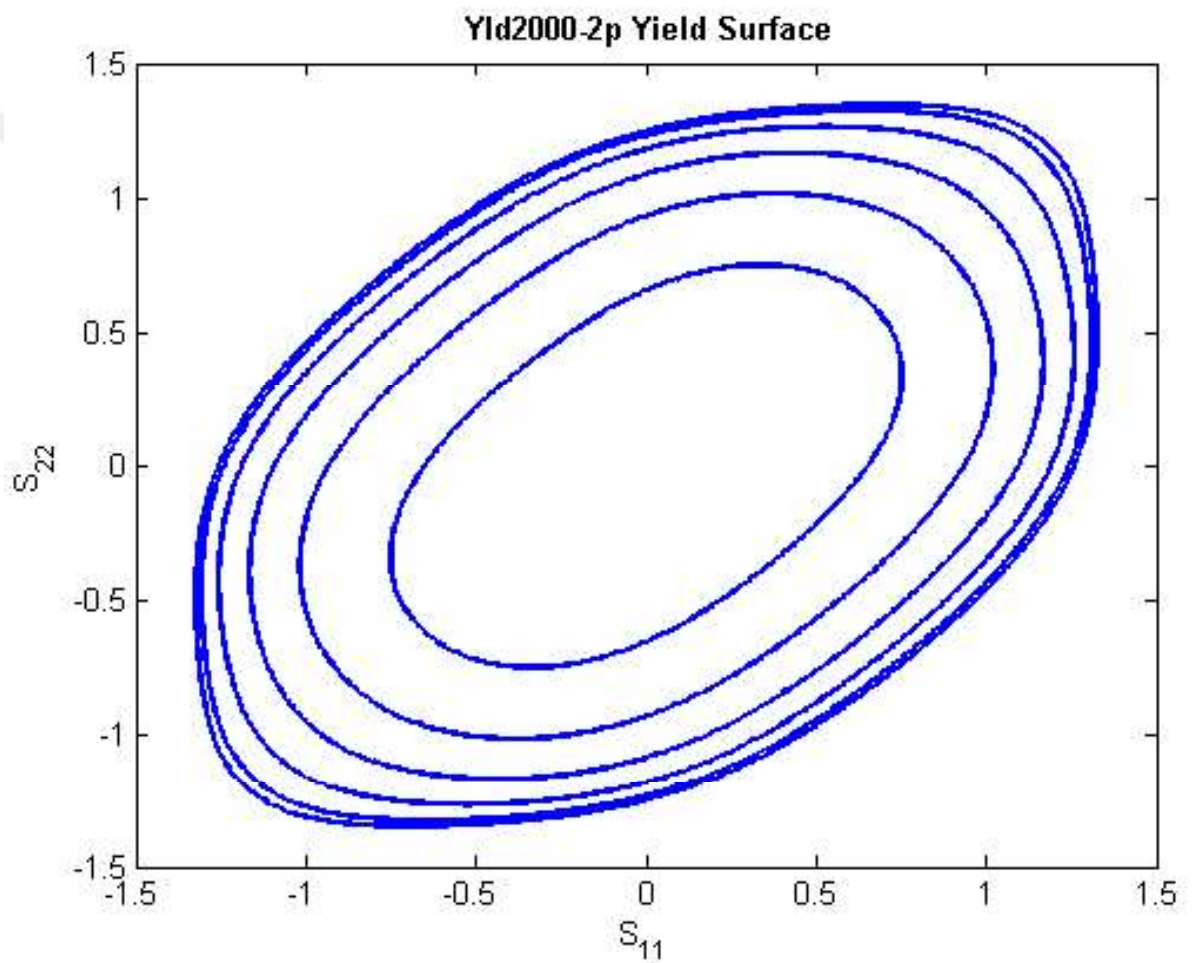


Figure 4.43 The Yld-2000 yield surface at 300 °C

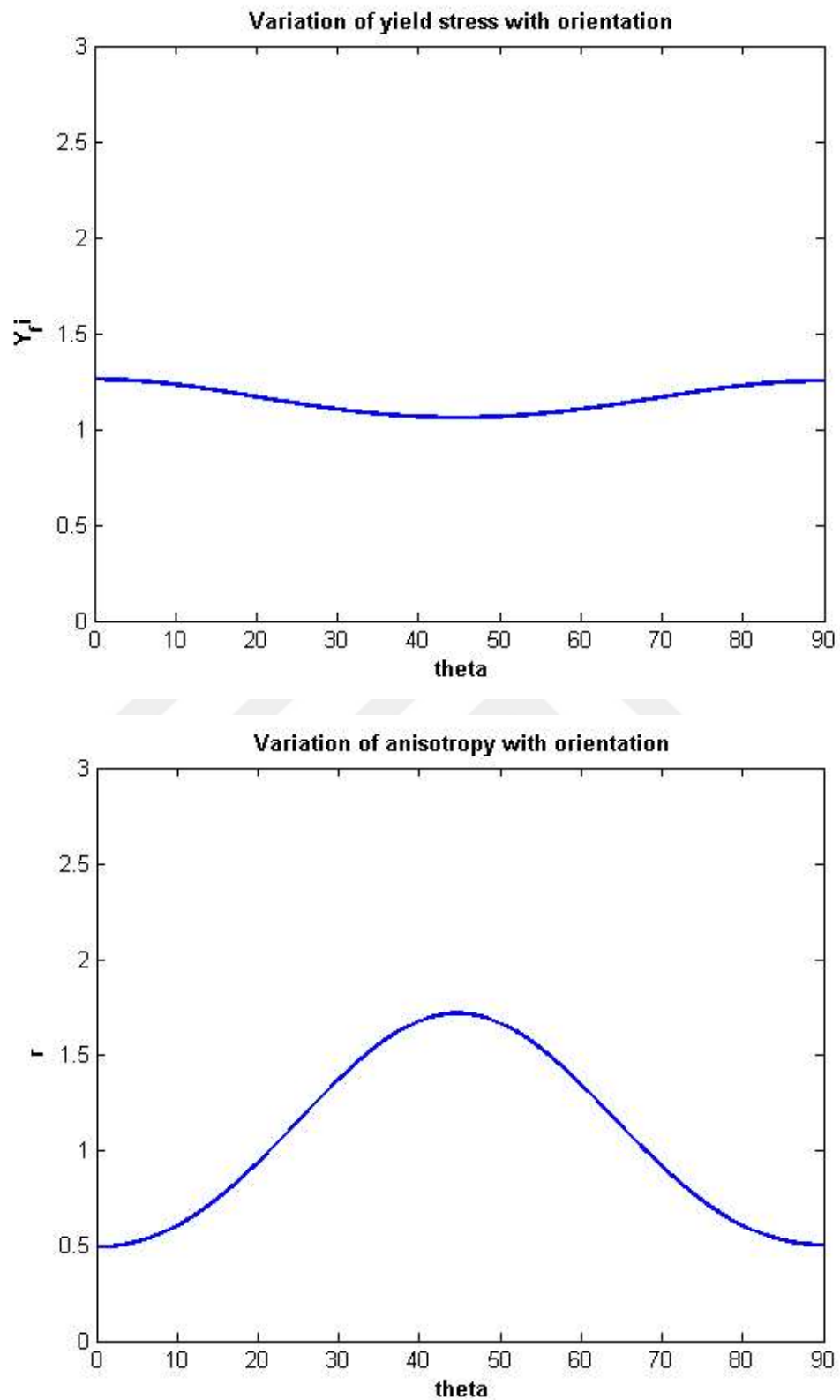


Figure 4.44 The Yld-2000 variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at 300 °C

4.5. Criterion 5: The Yld-2004-18p Yield Criteria

In this section, results of fifth criterion will be presented and discussed. Model coefficients are more complex in this model compared to the Yld2000, and they are given in Table 4.5. Same mechanical parameters are applied to this criteria given aforementioned in the study.

Table 4.5 Analysis parameters for the Yld2004-18p criteria

Analysis Parameters for the Yld2004-18p Yield Criterion			
Imposed strain rate: $\dot{\epsilon}$	Plastic work to extract stress points: w_c^p	Resolution of the strain grid: n_{pts}	Yield surface exponent: α
0.2	0.4	2	8

4.5.1. The Yld2004 Yield Surface at RT

In the presented suite of graphs, the mechanical anisotropy of the investigated material at RT is quantitatively depicted, utilizing the Yld2004-18p yield criterion framework. The yield surface plot exhibits an elliptical contour configuration, which distinctly underscores the anisotropic yielding behavior in the material, as evidenced by the varying response in the rolling and transverse directions (RD and TD, respectively). This anisotropy is further corroborated by the normalized yield stress distribution, which indicates a peak, suggesting a preferential orientation of the material's microstructure that corresponds to enhanced strength. Most notably, the Lankford coefficient's variation with the tensile direction provides a revealing insight into the material's formability. With its value fluctuating considerably between 0 and 90 degrees, it reveals an inherent directional dependency in the material's ability to resist thinning under tensile stress—a critical consideration for industrial forming processes. Collectively, these graphs are indicative of a pronounced directional sensitivity in the material's plastic deformation behavior, which must be meticulously accounted for to predict the performance and shape the design strategies in manufacturing processes.

involving this material. The yield surface graph provides a compelling visualization of the material's plastic anisotropy through the Yld2004-18p criterion at RT. The depicted contours represent levels of equivalent stress at which the material transitions from elastic to plastic behavior. The shape of the yield surface is not a perfect circle, signifying that the yield stress is not uniform in all directions. This non-uniformity indicates that the material will yield at different stress levels depending on the direction of loading, which is characteristic of anisotropic materials where the crystal structure, inclusions, or grain orientations affect mechanical responses.

Table 4.6 The Yld2004-18p parameters for RT

The Yld2004-18p Parameters for RT								
$\hat{c}'_{12}$	$\hat{c}'_{13}$	$\hat{c}'_{21}$	$\hat{c}'_{23}$	$\hat{c}'_{31}$	$\hat{c}'_{32}$	$\hat{c}'_{44}$	$\hat{c}'_{55}$	$\hat{c}'_{66}$
1.0000	1.0000	0.04948	-0.1254	-0.0055	-0.2510	-0.7587	1.0000	1.0000
$\hat{c}''_{12}$	$\hat{c}''_{13}$	$\hat{c}''_{21}$	$\hat{c}''_{23}$	$\hat{c}''_{31}$	$\hat{c}''_{32}$	$\hat{c}''_{44}$	$\hat{c}''_{55}$	$\hat{c}''_{66}$
0.4653	-1.4255	-0.5591	-1.0885	0.1874	-1.0769	0.1953	1.0000	1.0000

The elongation of the yield surface along the RD and TD axes implies that the material exhibits different yield points depending on whether it is pulled in the direction of rolling or transverse to it. This characteristic is particularly relevant for processes like sheet metal forming, where understanding the direction-dependent behavior is crucial for avoiding failures such as premature necking or tearing.

Moreover, the absence of sharp corners on the yield surface suggests that the material transitions smoothly from elastic to plastic deformation, without any abrupt changes in behavior, which is desirable for forming processes as it implies a more uniform and predictable strain distribution.

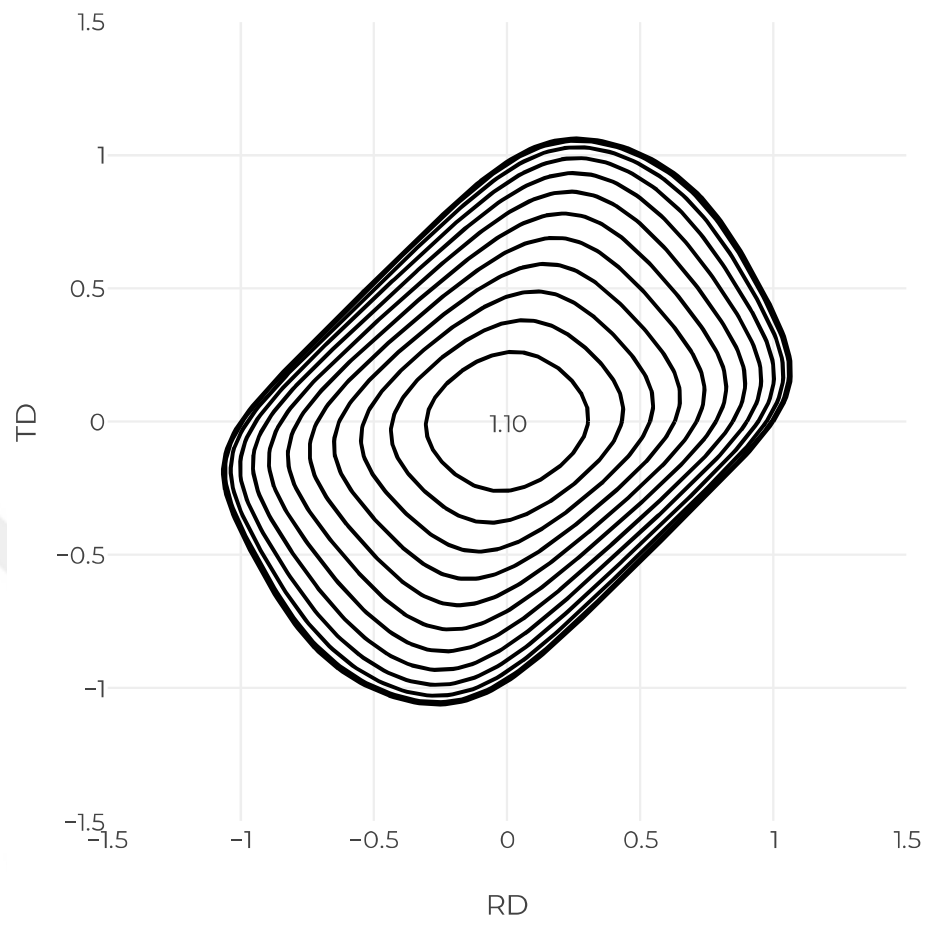


Figure 4.45 The Yld2004 yield surface at RT

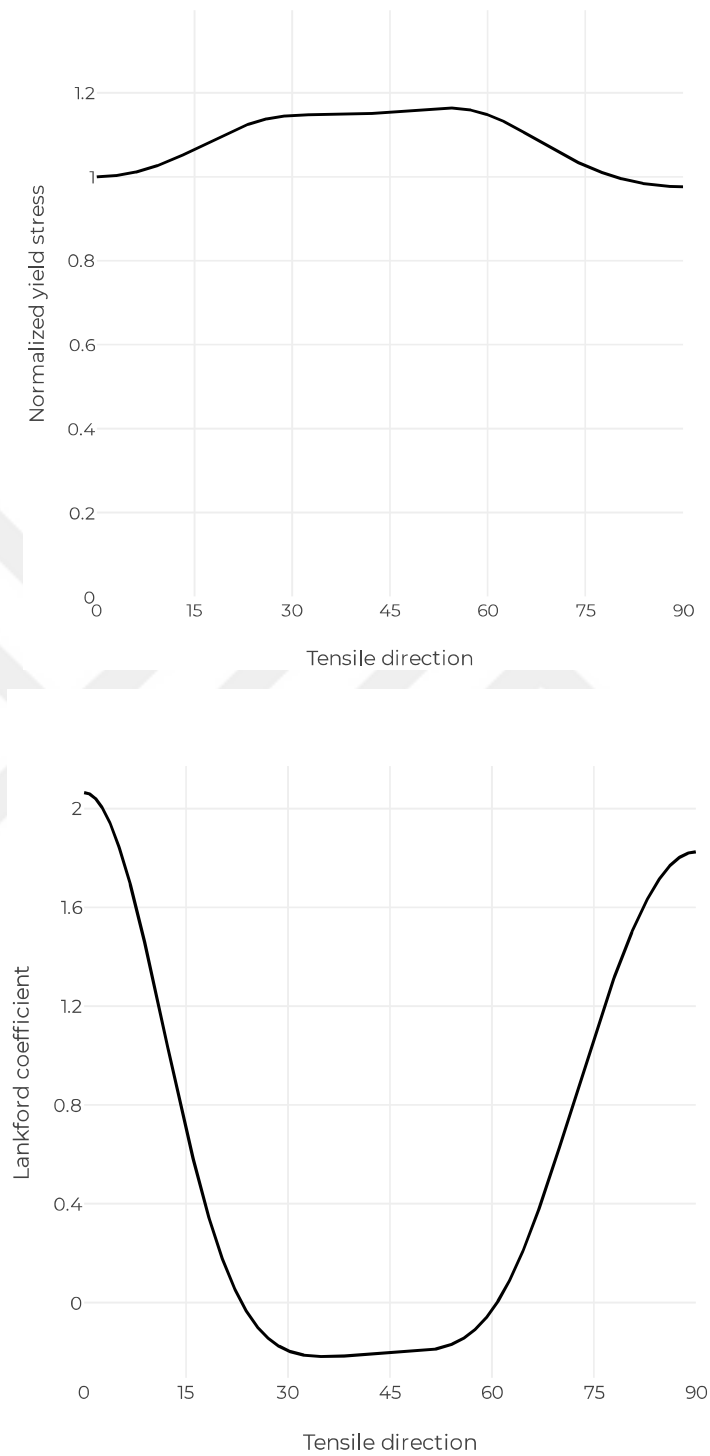


Figure 4.46 The Yld-2004 variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at RT

4.5.2. The Yld2004 Yield Surface at 100 °C

At 100 °C, the yield surface graph demonstrates a pronounced evolution from the RT condition, illustrating how thermal effects can alter the plastic behavior of the material. The contours of the yield surface remain elliptical, suggesting that the material maintains anisotropic behavior at elevated temperatures. However, any changes in the elongation or contraction of the yield surface axes would indicate a temperature-dependent variation in the material's response to stress in the rolling direction (RD) and transverse direction (TD).

Table 4.7 The Yld2004-18p parameters for 100 °C

The Yld2004-18p Parameters for 100 °C								
$\hat{c}'_{12}$	$\hat{c}'_{13}$	$\hat{c}'_{21}$	$\hat{c}'_{23}$	$\hat{c}'_{31}$	$\hat{c}'_{32}$	$\hat{c}'_{44}$	$\hat{c}'_{55}$	$\hat{c}'_{66}$
1.0000	1.0000	0.0441	0.0010	-0.0026	-0.1820	-0.6531	1.0000	1.0000
$\hat{c}''_{12}$	$\hat{c}''_{13}$	$\hat{c}''_{21}$	$\hat{c}''_{23}$	$\hat{c}''_{31}$	$\hat{c}''_{32}$	$\hat{c}''_{44}$	$\hat{c}''_{55}$	$\hat{c}''_{66}$
0.5460	-1.4413	-0.4161	-0.667	0.1306	-1.2146	0.1852	1.0000	1.0000

The analysis of the normalized yield stress graph at 100 °C would focus on any shifts in the peak values that represent the yield stress in different tensile directions. A flattening or a change in the peak position could suggest a reduced sensitivity to directional loading or a change in the material's internal structure at higher temperatures.

The Lankford coefficient graph at elevated temperatures is crucial to understanding the material's ductility and formability. Changes in the Lankford coefficient values could indicate alterations in the strain-hardening behavior and the propensity for thinning in sheet metal operations. If the curve flattens or the minimum and maximum values change, this would signal a variation in the material's ability to distribute strain

unevenly, which is pivotal for predicting the success of forming processes at higher temperatures.

Overall, these graphical analyses at 100 °C would provide essential insights into the thermal stability of the material's mechanical properties and would be critical for applications where the material is exposed to elevated temperatures, such as in the automotive or aerospace industries where materials often operate in high-temperature environments.

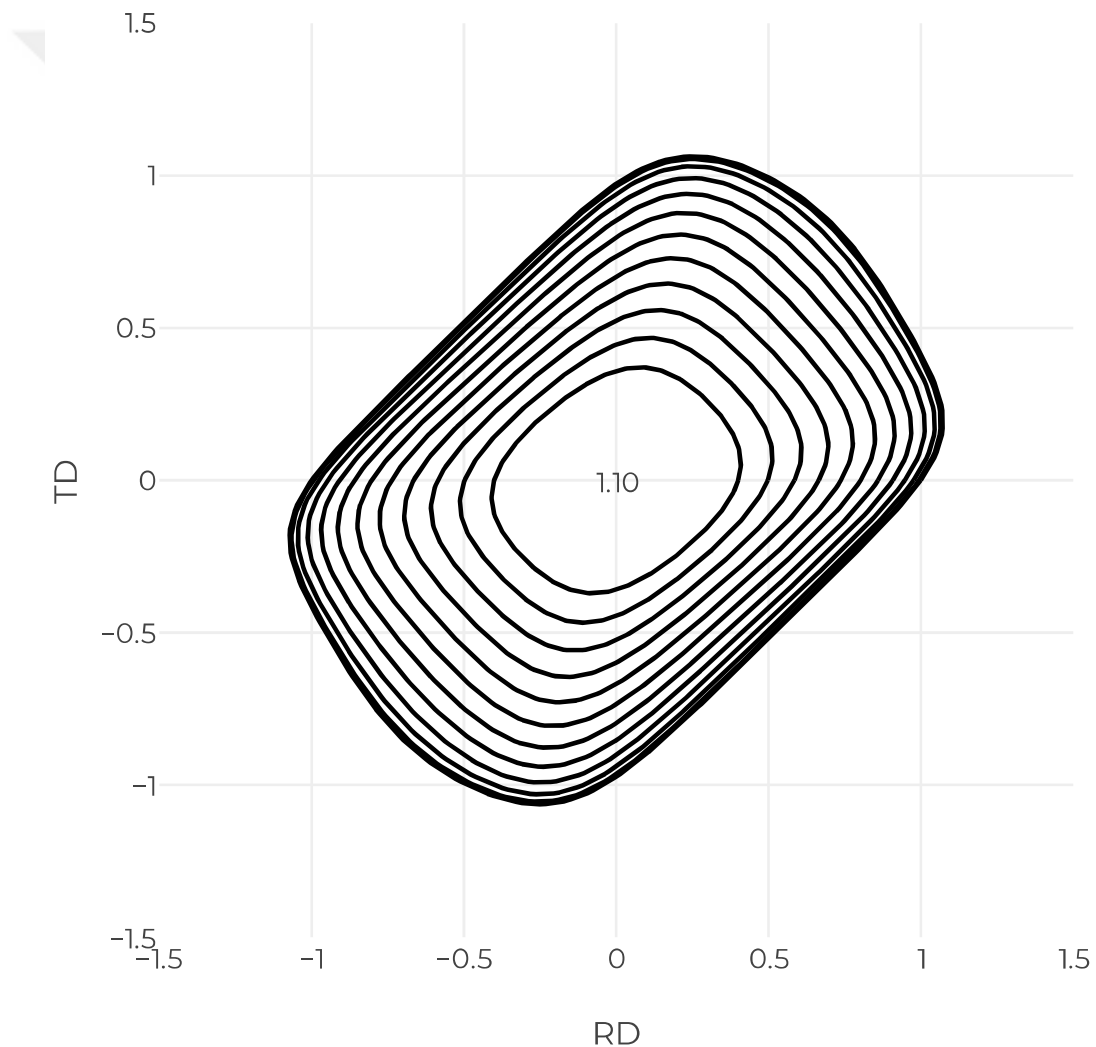


Figure 4.47 The Yld2004 yield surface at 100 °C

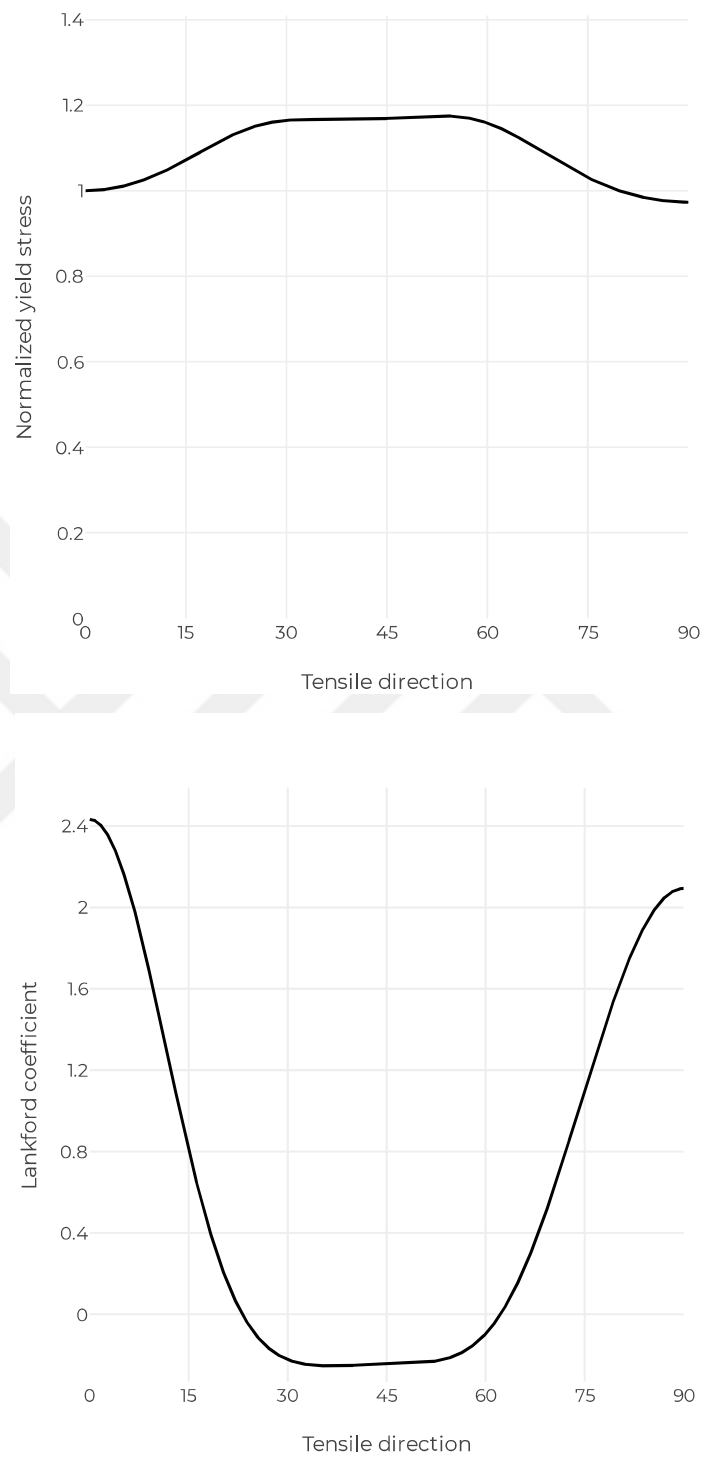


Figure 4.48 The Yld-2004 variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at 100 °C

4.5.3. The Yld2004 Yield Surface at 150 °C

For the AZ31 tested at 150 °C:

Table 4.8 The Yld2004-18p parameters for 150 °C

The Yld2004-18p Parameters for 150 °C								
$\hat{c}'_{12}$	$\hat{c}'_{13}$	$\hat{c}'_{21}$	$\hat{c}'_{23}$	$\hat{c}'_{31}$	$\hat{c}'_{32}$	$\hat{c}'_{44}$	$\hat{c}'_{55}$	$\hat{c}'_{66}$
1.0000	1.0000	0.0610	-0.0061	0.0026	-0.1599	-0.7552	1.0000	1.0000
$\hat{c}''_{12}$	$\hat{c}''_{13}$	$\hat{c}''_{21}$	$\hat{c}''_{23}$	$\hat{c}''_{31}$	$\hat{c}''_{32}$	$\hat{c}''_{44}$	$\hat{c}''_{55}$	$\hat{c}''_{66}$
0.3655	-1.3653	-0.3540	-0.7687	0.1552	-1.1224	0.3870	1.0000	1.0000

The Yld2004 model for 100 °C, the yield locus still appears to be centrosymmetric, indicating isotropy in the plastic behavior of the material at this temperature. The shape of the locus seems to be similar to the RT one, suggesting that the fundamental yield behavior of the material does not drastically change with the increase in temperature to 150 °C. The spread of the contour lines may differ slightly, indicating changes in the yield stresses under different stress states due to temperature effects. The normalized yield stress appears to follow a similar trend to the RT graph, with a peak at around the 45-degree direction. The peak's magnitude relative to the stresses at 0 and 90 degrees may have changed, which could be due to the material's response to elevated temperatures, potentially affecting dislocation movement or other mechanisms at play during yielding. The Lankford coefficient graph is different from the RT one. At 150 °C, the coefficient stays relatively low and flat from 0 to about 60 degrees, suggesting that the material's anisotropy in this range of tensile directions is less pronounced at this temperature. There is a sharp increase after 60 degrees, much steeper than observed at RT. This could indicate that at higher temperatures, the material becomes significantly more anisotropic in its plastic deformation characteristics, particularly in the orientations close to 90 degrees. This could be due to temperature-dependent texture development or grain boundary sliding, which

becomes more prominent at elevated temperatures. These graphs indicate how the material's mechanical properties change with temperature, which is critical for applications where the material will be subjected to high temperatures or varying thermal conditions. It's important to note that the absolute values of the stresses and coefficients would be necessary to make a quantitative assessment, as these graphs provide trends rather than specific property values.

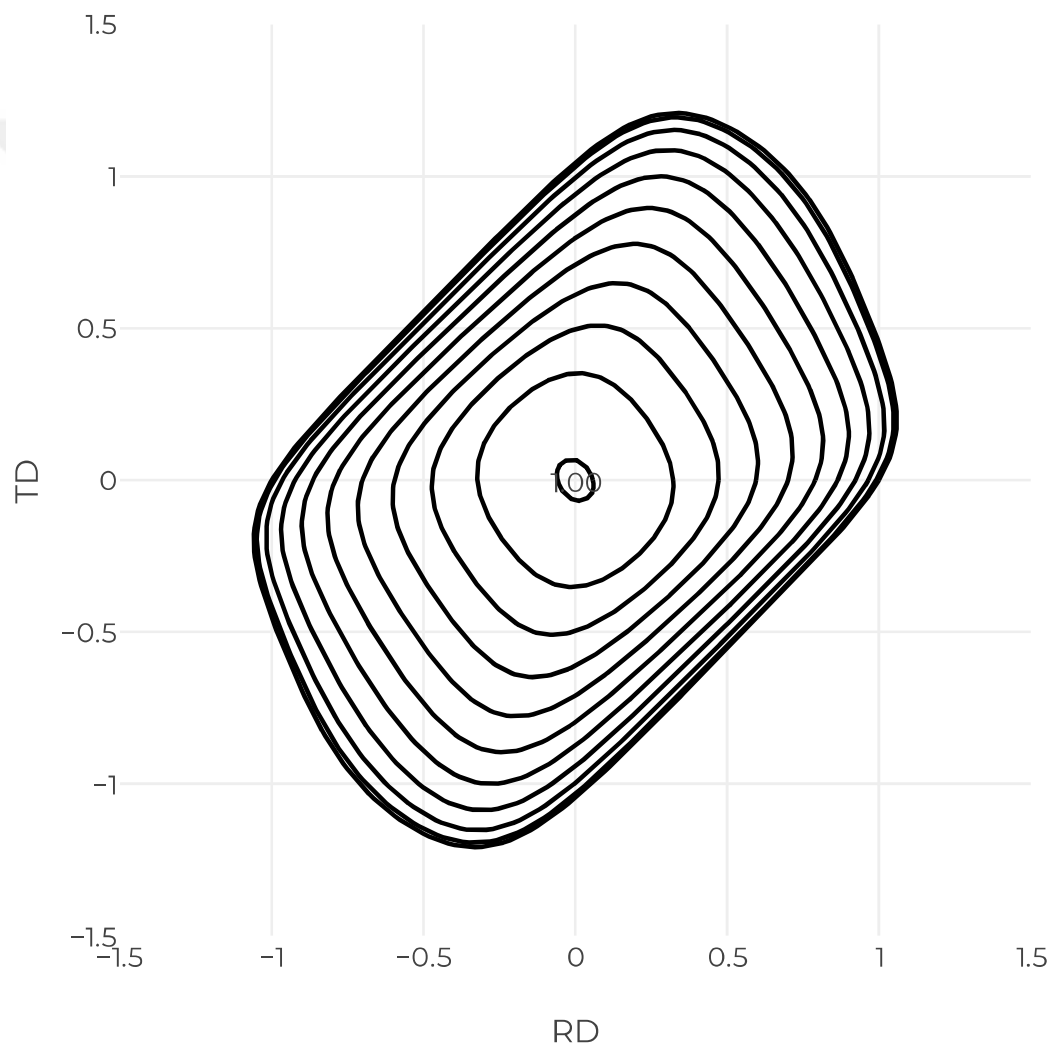


Figure 4.49 Yld2004 yield surface at 150 °C

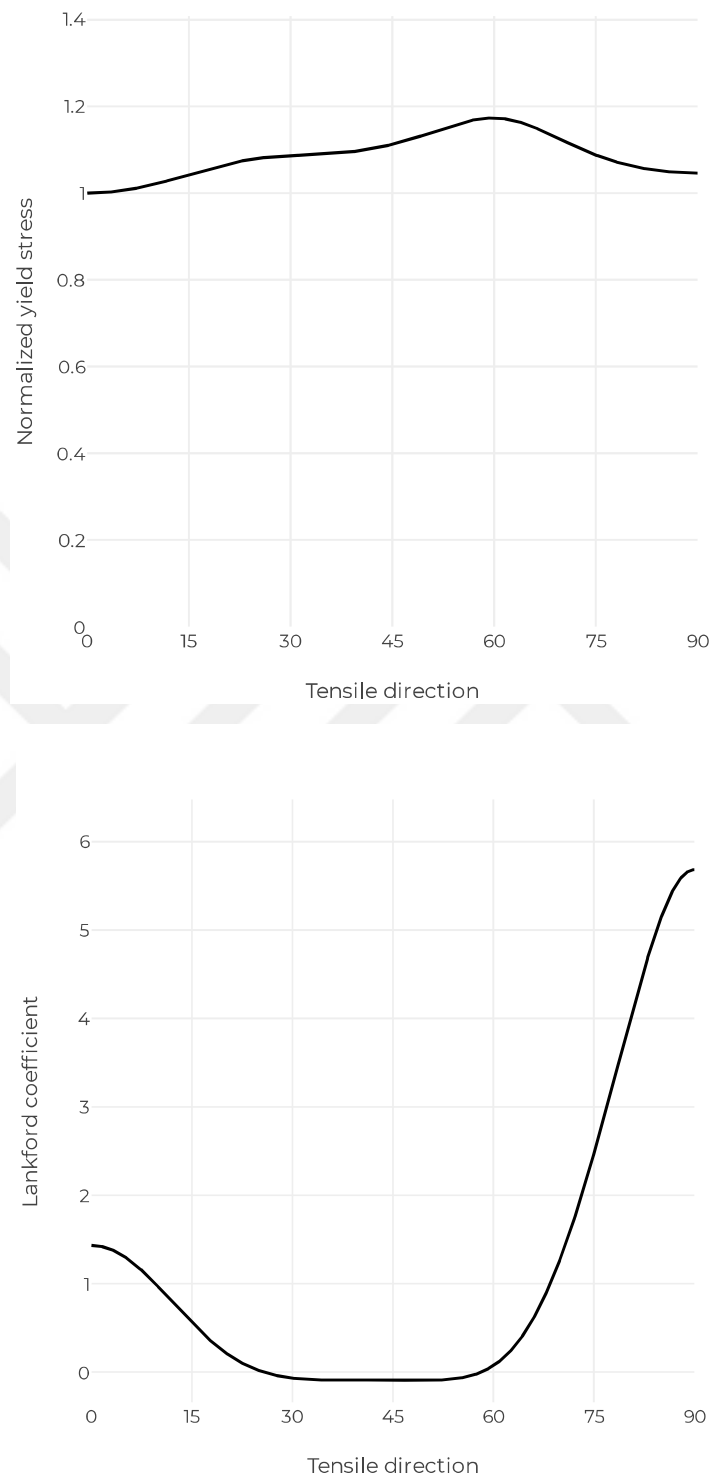


Figure 4.50 The Yld-2004 variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at 200 °C

4.5.4. The Yld2004 Yield Surface at 200 °C

Table 4.9 The Yld2004-18p parameters for 200 °C

The Yld2004-18p Parameters for 200 °C								
$\hat{c}'_{12}$	$\hat{c}'_{13}$	$\hat{c}'_{21}$	$\hat{c}'_{23}$	$\hat{c}'_{31}$	$\hat{c}'_{32}$	$\hat{c}'_{44}$	$\hat{c}'_{55}$	$\hat{c}'_{66}$
1.0000	1.0000	0.4745	-0.2811	0.1128	-0.8799	-0.3712	1.0000	1.0000
$\hat{c}''_{12}$	$\hat{c}''_{13}$	$\hat{c}''_{21}$	$\hat{c}''_{23}$	$\hat{c}''_{31}$	$\hat{c}''_{32}$	$\hat{c}''_{44}$	$\hat{c}''_{55}$	$\hat{c}''_{66}$
0.4270	-0.9591	-0.0060	0.5972	0.8363	0.1330	0.8391	1.0000	1.0000

The series of plots provided offers a comprehensive overview of the anisotropic mechanical properties of a material subjected to a temperature of 200 °C. Such a temperature can have a pronounced effect on the material's behavior, particularly if it is near critical transition points such as recrystallization temperatures for metals or the glass transition for polymers. At 200°C, the material's response to stress will likely differ from that at RT, with a general expectation of reduced yield stress, modified work-hardening rates, and altered anisotropic properties due to the increased mobility of dislocations and atoms. The contour plot distinctly outlines the anisotropic yield behavior, with the elliptical contour lines indicating how the yield stress varies between the rolling direction (RD) and the transverse direction (TD). This ellipticity is suggestive of the fact that the material properties are not uniform and depend greatly on the direction of the applied stress. Typically, such a contour pattern could be indicative of a planar anisotropy, which is a critical consideration for processes such as metal rolling and forming. At 200°C, the observed contours may have expanded or shifted when compared to ambient temperature conditions, signifying a more ductile material that yields more easily under applied stresses.

In the normalized yield stress plot, the peak at approximately 30 degrees implies a direction of greater resistance to deformation, which could be correlated to the

material's crystallographic texture or the direction of prior mechanical working. The data indicates that, at 200°C, the material's yield stress is maximized in this direction, which may influence how it is processed or applied in practical situations. The thermal softening apparent at this elevated temperature likely reduces yield stress across all directions, but the anisotropy remains, as demonstrated by the non-uniform distribution of values. The Lankford coefficient values, as depicted in the plot, illustrate the degree of anisotropy in the material's plastic deformation characteristics. The variation in these values with the tensile direction highlights the complex interplay between the material's microstructure and its deformation mechanisms at elevated temperatures. The reduced Lankford coefficients around the 45-degree direction, combined with higher values towards 0 and 90 degrees, suggest that the material will exhibit different thinning behaviors when formed at 200°C. Such information is pivotal when predicting sheet metal performance in industrial forming processes, as it directly impacts the likelihood of failure due to necking or cracking. Overall, the data presented for the material at 200 °C elucidates the nuanced and temperature-dependent nature of mechanical properties. The findings underscore the importance of considering directional properties in material selection and component design, especially for applications involving elevated temperatures. Such considerations become particularly critical in the context of engineering applications that require precise control over material behavior to ensure performance and reliability under varying thermal conditions.

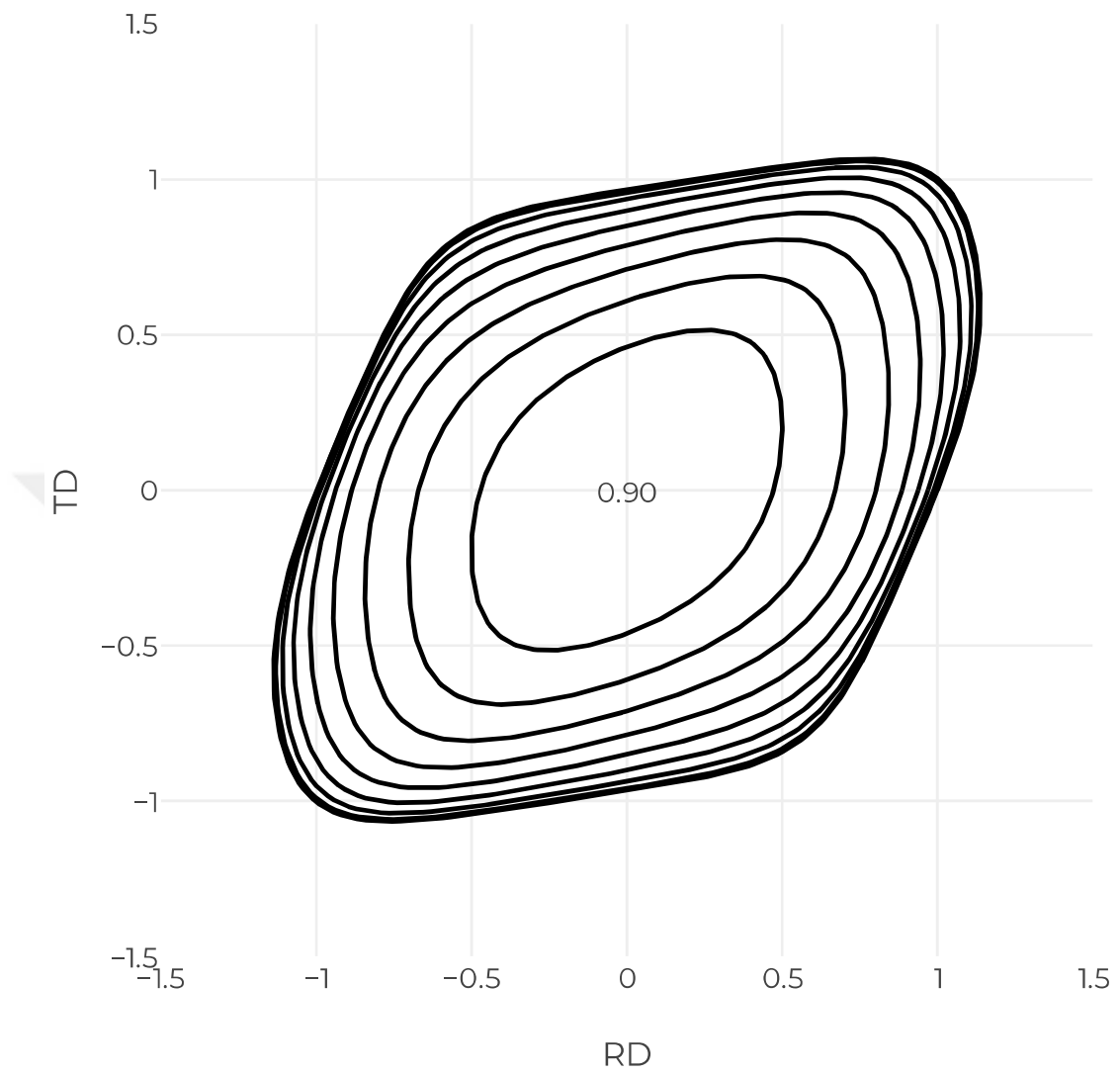


Figure 4.51 The Yld2004 yield surface at 200 °C

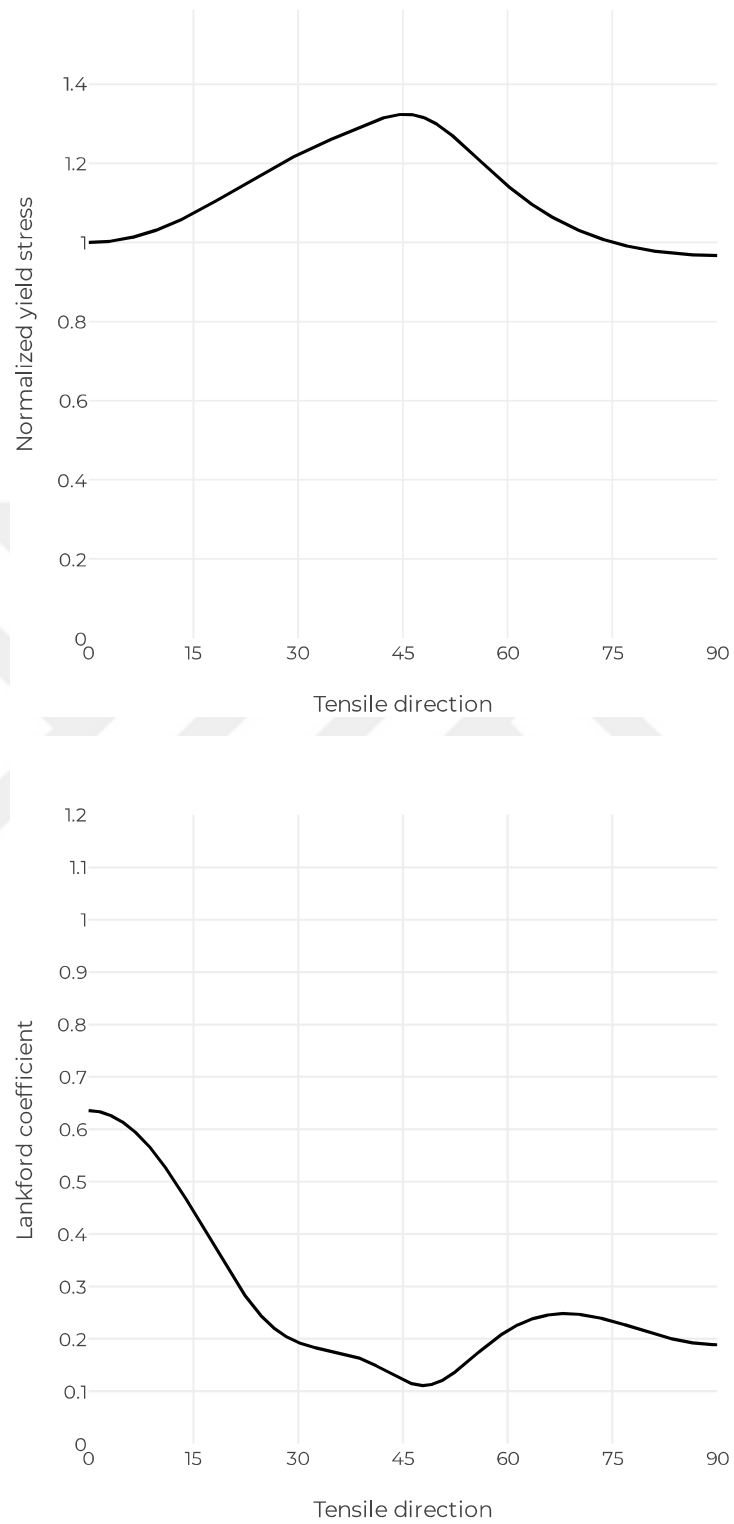


Figure 4.52 The Yld-2004 variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at 200 °C

4.5.5. The Yld2004 Yield Surface at 250 °C

Table 4.10 The Yld2004-18p parameters for 250 °C

The Yld2004-18p Parameters for 250 °C								
$\hat{c}'_{12}$	$\hat{c}'_{13}$	$\hat{c}'_{21}$	$\hat{c}'_{23}$	$\hat{c}'_{31}$	$\hat{c}'_{32}$	$\hat{c}'_{44}$	$\hat{c}'_{55}$	$\hat{c}'_{66}$
1.0000	1.0000	0.6630	-0.3285	0.5203	-0.7303	0.2325	1.0000	1.0000
$\hat{c}''_{12}$	$\hat{c}''_{13}$	$\hat{c}''_{21}$	$\hat{c}''_{23}$	$\hat{c}''_{31}$	$\hat{c}''_{32}$	$\hat{c}''_{44}$	$\hat{c}''_{55}$	$\hat{c}''_{66}$
0.4445	-0.5122	0.5876	-0.3850	0.1346	0.2225	0.5649	1.0000	1.0000

At an elevated temperature of 250°C, the material's anisotropic mechanical properties are expected to show more pronounced changes. The increased thermal energy at this temperature can further activate atomic diffusion processes, potentially leading to a more pronounced reduction in material strength and changes in the anisotropic behavior as seen in the contour plot. If a contour plot for 250°C were analyzed, it might show even larger and more distorted ellipses, implying a greater sensitivity of the yield stress to the direction of applied stress. This suggests that the material could become more ductile and pliable, facilitating deformation processes such as deep drawing or bending. If the normalized yield stress plot at 250°C were to follow the expected trend, the peak observed at around 30 degrees for 200 °C might become less pronounced, indicating a reduction in the anisotropy of yield stress with respect to the tensile direction. A higher temperature usually leads to a more homogenous distribution of yield stress across various directions due to the more uniform slip and dislocation movement, which would flatten the curve observed at 200 °C. The plot of the Lankford coefficient at 200°C indicates variation in the material's plastic deformation characteristics. At 250°C, we might expect to see an overall decrease in the Lankford coefficient values, reflecting a higher tendency for the material to thin out more uniformly when stretched. The valley at around 45 degrees, which indicates the direction of least resistance to thinning, might become shallower, and the peak values

could also decrease, which would imply that the anisotropy in thinning behavior is becoming less significant at this higher temperature. These anticipated changes in material properties at 250 °C have important implications for manufacturing and application. As the material becomes more ductile and isotropic at higher temperatures, forming processes might need to be adjusted to account for the decreased yield strength and altered thinning behavior. Moreover, the expected decrease in mechanical anisotropy can be beneficial for some applications by reducing the risk of failure due to directional properties. In conclusion, the analysis of how a material behaves at 250 °C, while inferred from the data at 200 °C, highlights the critical influence of temperature on material properties. The higher temperature is likely to result in a more ductile material that exhibits less anisotropic behavior, affecting both its processing and performance. Understanding these temperature-dependent changes is essential for engineers and material scientists to predict material behavior and to design components that can withstand specific temperature conditions.

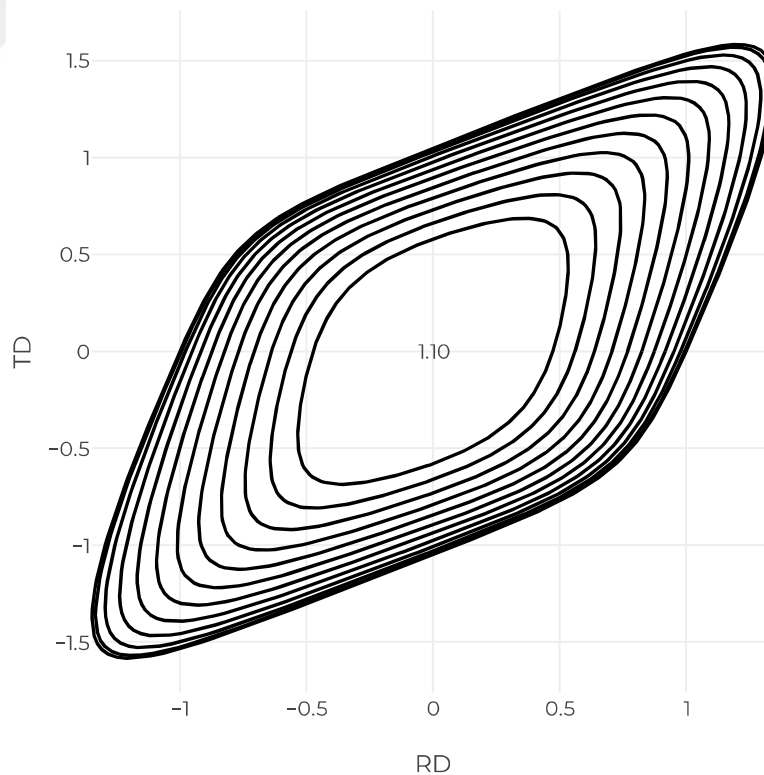


Figure 4.53 The Yld-2004 yield surface at 250 °C

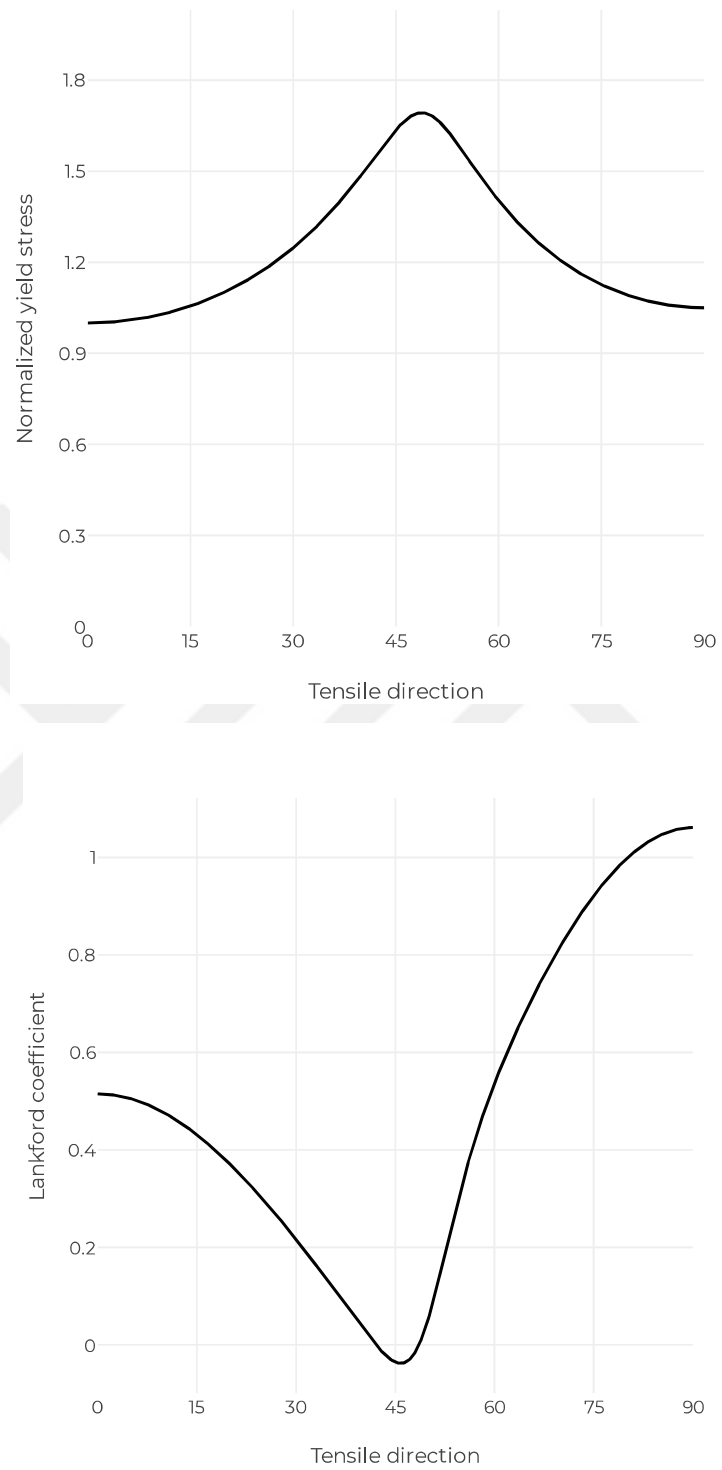


Figure 4.54 The Yld-2004 variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at 250 °C

4.5.6. The Yld2004 Yield Surface at 300 °C

Table 4.11 The Yld2004-18p parameters for 300 °C

The Yld2004-18p Parameters for 300 °C								
$\hat{c}'_{12}$	$\hat{c}'_{13}$	$\hat{c}'_{21}$	$\hat{c}'_{23}$	$\hat{c}'_{31}$	$\hat{c}'_{32}$	$\hat{c}'_{44}$	$\hat{c}'_{55}$	$\hat{c}'_{66}$
1.0000	1.0000	0.4118	-0.1771	0.0639	-0.7590	-0.3445	1.0000	1.0000
$\hat{c}''_{12}$	$\hat{c}''_{13}$	$\hat{c}''_{21}$	$\hat{c}''_{23}$	$\hat{c}''_{31}$	$\hat{c}''_{32}$	$\hat{c}''_{44}$	$\hat{c}''_{55}$	$\hat{c}''_{66}$
0.4939	-0.7405	0.0228	0.3630	0.8097	0.0594	0.7811	1.0000	1.0000

The contour plot provided indicates the anisotropic behavior of a material subjected to a temperature of 300°C. The elliptical shapes of the contours, delineating regions of equivalent strain or stress, suggest a variation in the material's response to deformation in the rolling direction (RD) and the transverse direction (TD). The elongation of the ellipses along the TD axis may imply that the material exhibits greater strain in the transverse direction under the given thermal conditions, which could be indicative of directional dependencies in the material's mechanical properties. This anisotropic behavior is critical for predicting the material's performance in applications where uniform deformation is essential, such as in precision-formed components. Understanding the strain distribution at elevated temperatures is vital for processes like hot rolling and forging, which rely on consistent material behavior at high temperatures to ensure product quality. The normalized yield stress plot presents a unimodal distribution with respect to the tensile direction, peaking at an intermediate angle. This suggests that the material's yield strength is maximized in a specific orientation relative to the grain structure when exposed to a temperature of 300 °C. The peak of the graph indicates the angle at which the material's resistance to deformation is highest, which is a crucial factor for designing components that undergo directional stresses. The decrease in normalized yield stress both before and after the peak suggests a reduction in resistance to deformation, which must be taken into account when planning the material's application in thermally demanding

environments. The observed trend has significant implications for the material's ductility and toughness in different orientations and must be considered in the design and manufacturing processes. The variation of the Lankford coefficient with respect to the tensile direction at 300 °C is indicative of the material's anisotropic plastic behavior. The graph exhibits a pronounced decrease from the onset, reaching a minimum before gradually increasing, suggesting a directionally dependent ductility of the material. The R-value is a measure of a material's ability to undergo plastic deformation in the plane of the sheet vs. its thickness, with higher values indicating better resistance to thinning. The observed minimum in the graph signifies a direction of increased susceptibility to thinning under tension, which is a vital consideration for avoiding failure in forming operations. The subsequent increase in the R-value indicates an improvement in thickness uniformity with tension in other directions. This information is critical for industrial forming processes, where controlling material flow and maintaining thickness is essential, especially at elevated temperatures like 300 °C.

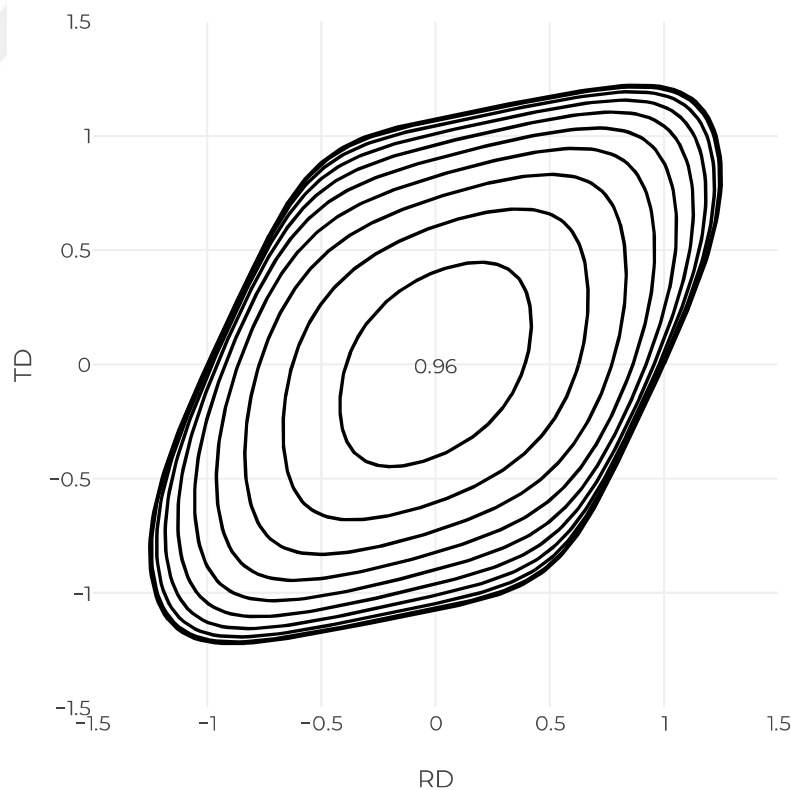


Figure 4.55 The Yld2004 yield surface at 300 °C

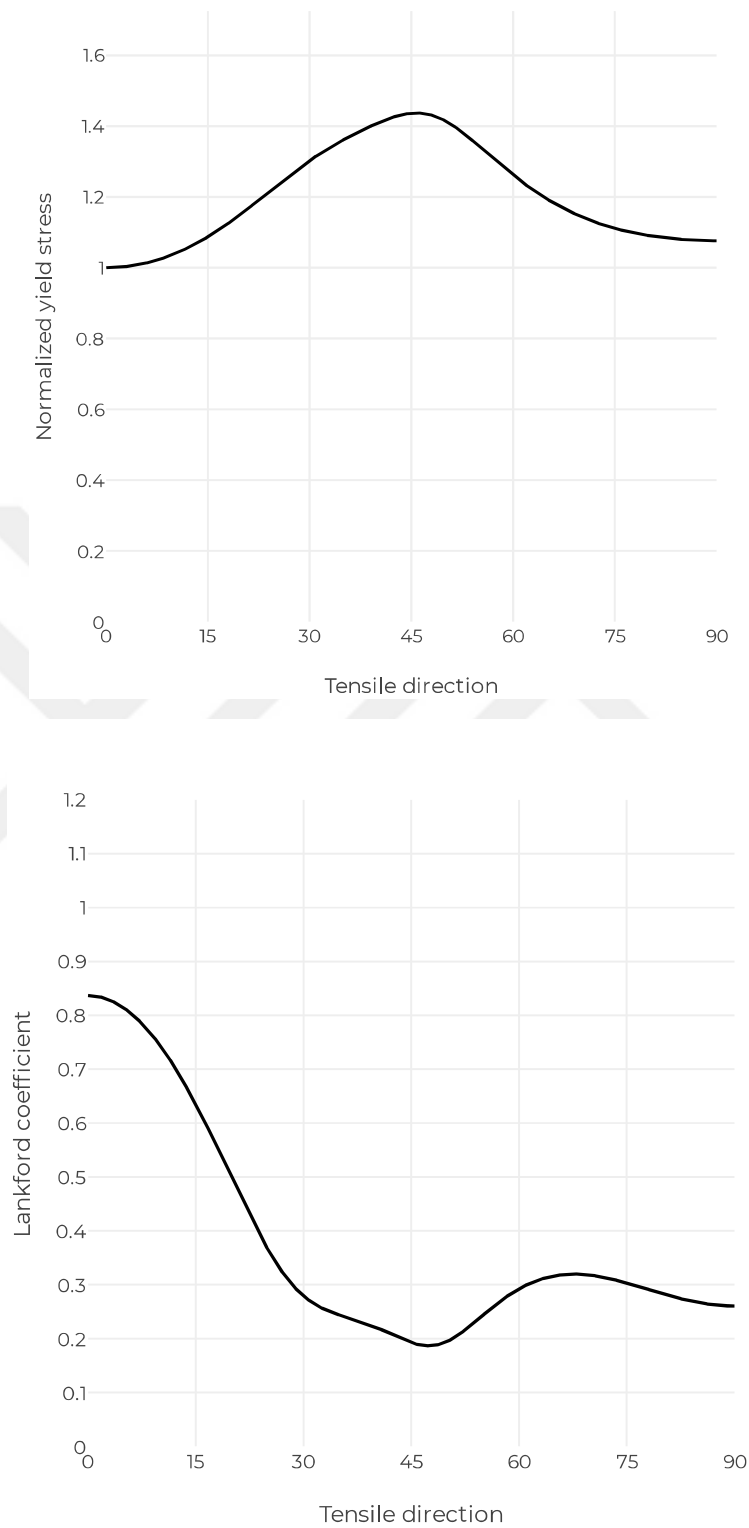


Figure 4.56 The Yld-2004 variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at 300 °C

4.6. Criterion 6: The Bezier5YS-2023 and SHYqp-2023 Yield Criterion

In this section, sixth criteria results will be presented and discussed accordingly.

4.6.1. The Bezier5YS-2023 and SHYqp-2023 Yield Surface at RT

The Bezier5YS-2023 yield criterion depicted in the first graph presents the normalized stress states $\overline{\sigma_{xx}}$ and $\overline{\sigma_{yy}}$ at which a material transitions from elastic to plastic deformation at RT. The contours represent different levels of yield stress. The near-elliptical nature of the contours suggests that the material's yield behavior is not perfectly isotropic; however, the slight deviation from a circular shape indicates that the anisotropy is not pronounced. The largest and outermost contour corresponds to the highest level of stress the material can withstand before yielding. The yield points are symmetric with respect to the origin, indicating that the material exhibits similar yield strengths in tension and compression along the principal axes. The tight grouping of the contours suggests a material with a relatively consistent yield strength over a range of biaxial stress states, which is a desirable property for materials subjected to complex loading conditions in engineering applications. In the second graph, the Lankford coefficient's variation with the angle of the tensile direction (r_θ) at RT is shown. The coefficient r_θ is a measure of plastic anisotropy and is used to predict sheet metal formability. The curve increases monotonically from 0° to 90° , indicating that the material's ability to resist thinning (and hence its ductility) improves as the angle of tensile load moves from the rolling direction towards the transverse direction. The absence of any local minima or maxima suggests a uniform gradation of material properties with no abrupt changes in behavior, which is advantageous for formability. The r_θ value remaining above 1 throughout the tensile direction angles highlights the material's consistent resistance to thinning, suggesting that it would perform well in forming processes requiring complex shapes and deformations.

The third graph shows the variation of normalized yield stress $\overline{\sigma_\theta}$ with the angle of the tensile direction at RT. The plot indicates a slight increase in yield stress from 0° to around 60° , followed by a plateau towards 90° . The increase in the normalized yield stress with the angle indicates that the material strengthens as the load direction

approaches the transverse direction. This behavior may be attributed to the underlying microstructure or the grain orientation of the material. The plateauing effect near the transverse direction may imply that the material's crystallographic texture has less influence on its yield stress beyond a certain orientation, leading to a more uniform response to loading. The overall trend suggests that the material's yield strength is directionally dependent, which must be taken into account when selecting the material for directional load-bearing applications. The lack of significant variability in yield stress with direction also implies that the material is relatively forgiving in applications where the loading direction cannot be precisely controlled.

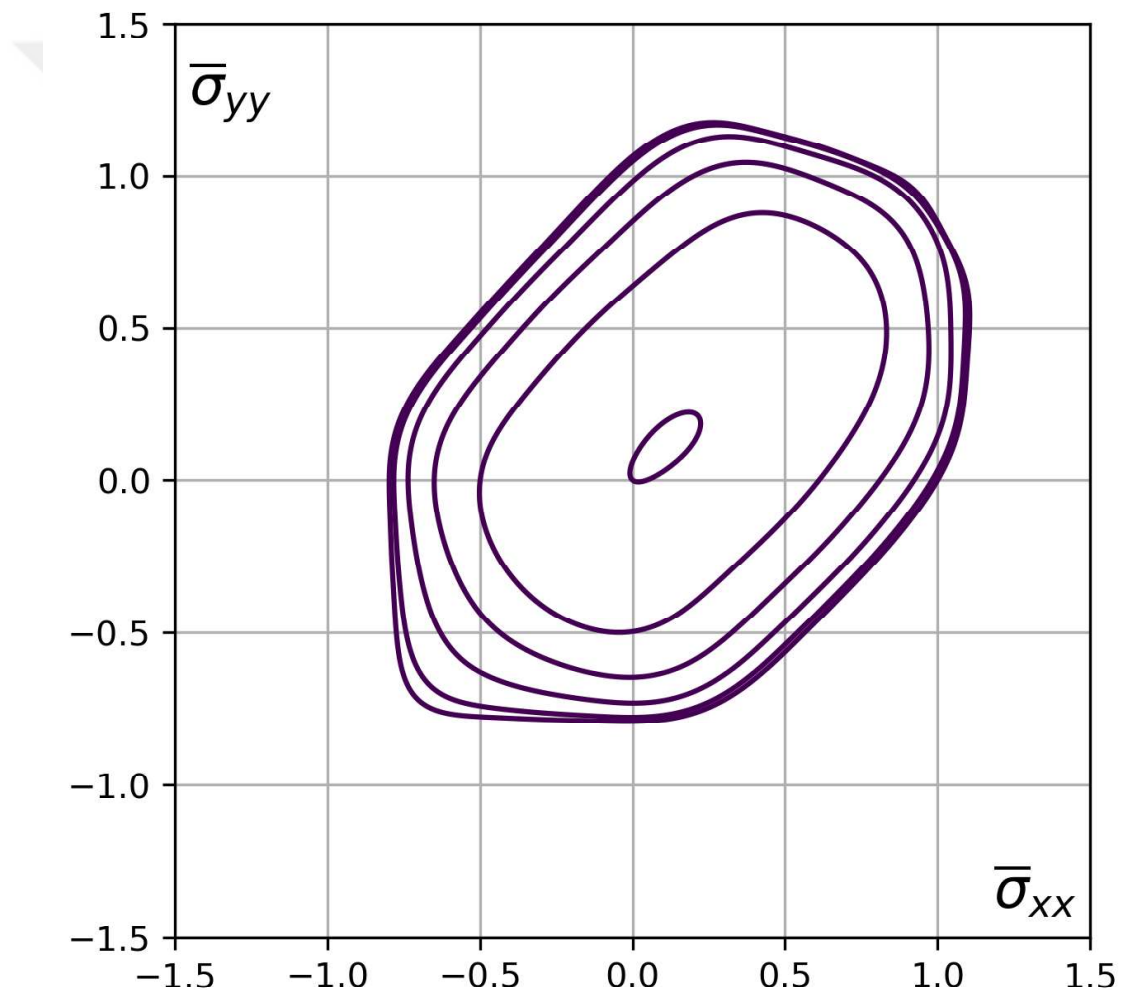


Figure 4.57 The Bezier5YS and SHYqp yield surface at RT

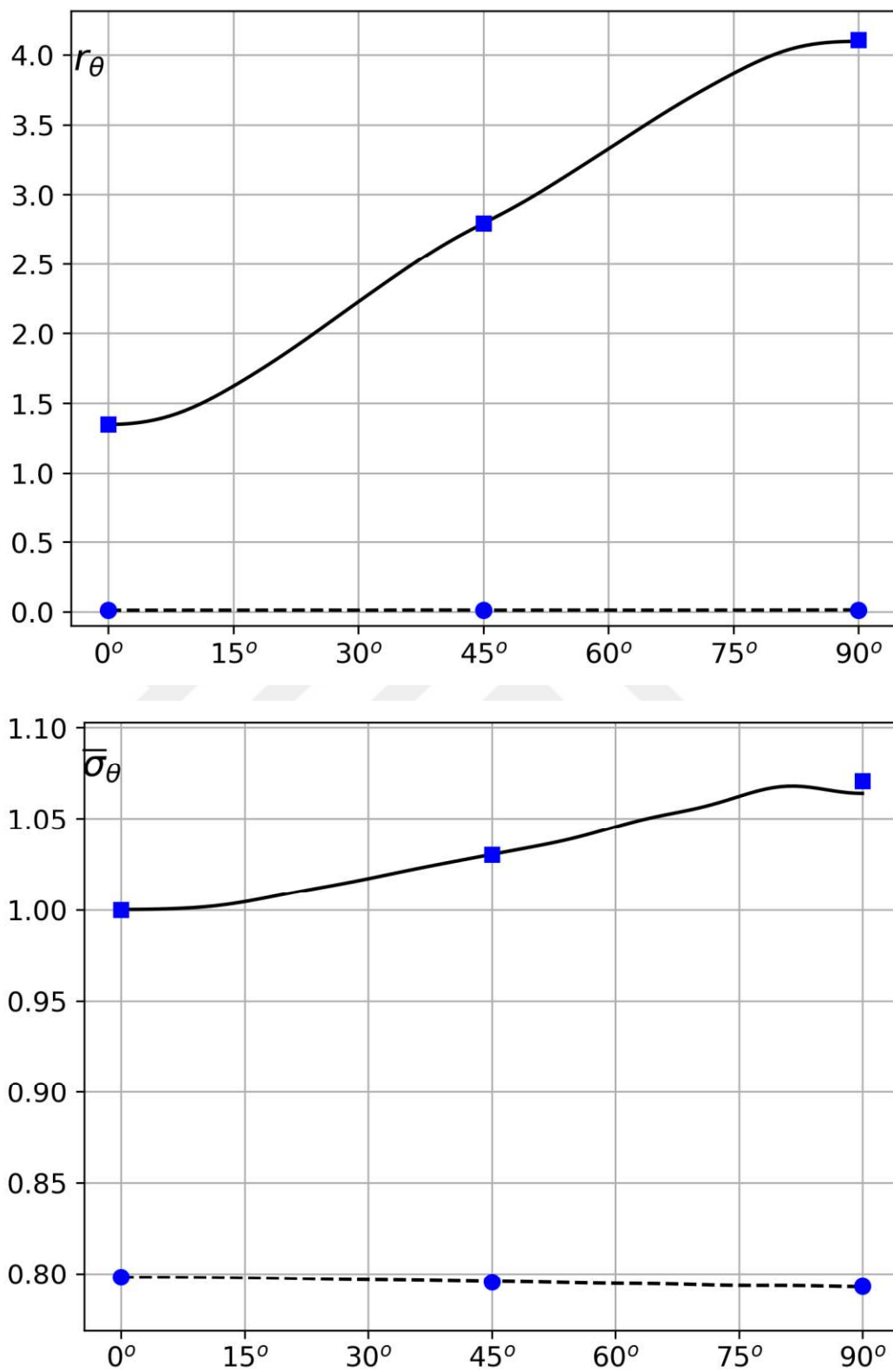


Figure 4.58 The Bezier5YS and SHYqp variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at RT

4.6.2. The Bezier5YS and SHYqp Yield Surface at 100 °C

The first graph illustrates a stress distribution contour, with axes denoted as $(\bar{\sigma}_{xx})$ and $(\bar{\sigma}_{yy})$. These contours likely represent the normalized stress components within a material subject to certain loading or thermal conditions, at a temperature of 100 °C. The elliptical nature of the contours suggests an anisotropic distribution of stress within the material, indicating that the stress magnitude and direction vary depending on the orientation within the material. The absence of cusp-like formations in the contours suggests that the material is not approaching yield stress under the given conditions. Moreover, the centering of contours around the origin could imply that there are no residual stresses present, or that the material is symmetrically loaded. In the second graph, the curve labeled (r_θ) shows a relationship between a radial parameter and the angular position from 0° to 90°. The curve increases with the angle, indicating a non-uniform distribution of the parameter-potentially related to stress, strain, or another radial characteristic-in the material. The exact nature of (r_θ) is not specified, but its increase with angle suggests that properties such as stiffness or thermal expansion could be anisotropic. It is also plausible that (r_θ) corresponds to a radial stress component that intensifies with angular orientation, which could be critical when considering design tolerances and material failure theories. The third graph demonstrates a slight yet consistent increase in the normalized stress component $(\bar{\sigma}_\theta)$ with respect to the angle. The stress component appears to be normalized to some reference value, possibly the material's yield strength or a standard operating stress. The fact that the stress component $(\bar{\sigma}_\theta)$ increases with the angle might be indicative of the combined effects of thermal expansion and external loads, which are more pronounced at certain angular positions within the material. The implication of this could be significant when considering the design and safety factors for components exposed to thermal loading, as certain regions could be more susceptible to stress concentrations.

In conclusion, the graphs collectively suggest that the material exhibits an anisotropic behavior with respect to stress distribution and radial parameters under a thermal condition of 100 °C. The observed trends highlight the importance of considering directional properties and temperature effects when evaluating material performance

and integrity. Such analyses are crucial for the appropriate design and application of materials in engineering systems, especially those subjected to varying thermal environments.

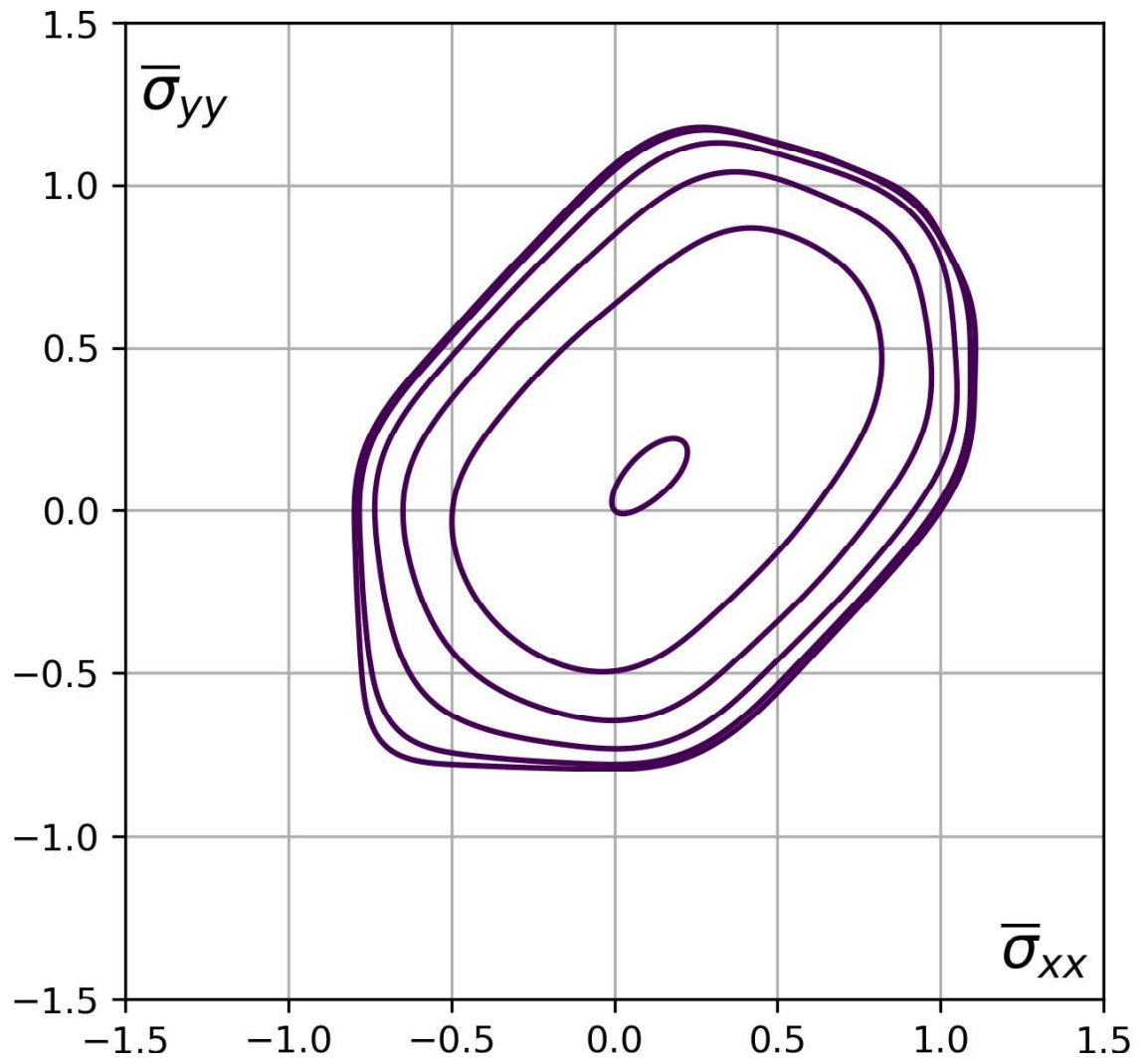


Figure 4.59 The Bezier5YS and SHYqp yield surface at 100 °C

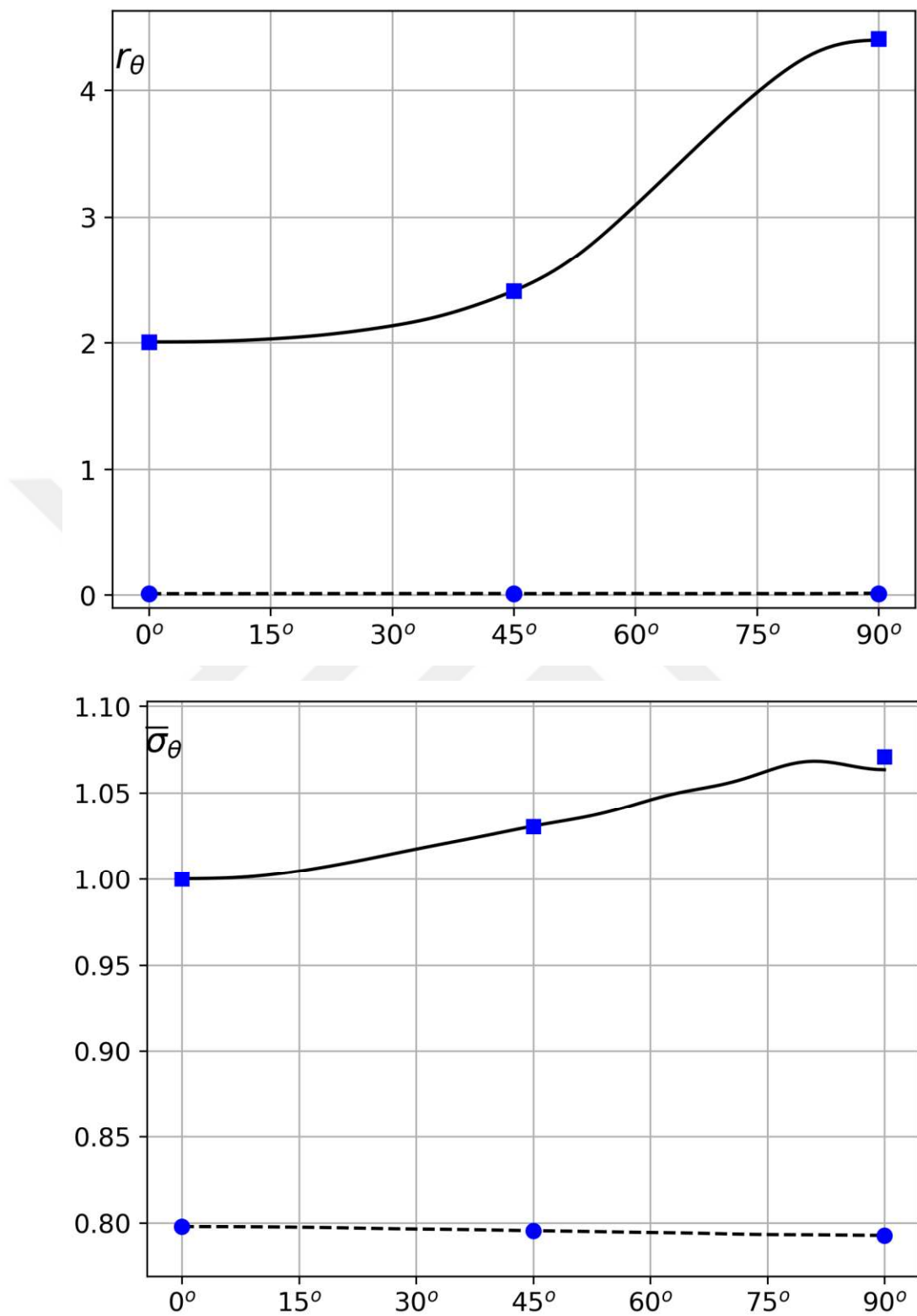


Figure 4.60 The Bezier5YS and SHYqp variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at 100 °C

4.6.3. The Bezier5YS and SHYqp Yield Surface at 150 °C

Analyzing the three graphs at the increased temperature condition of 150 °C, we can provide a formal interpretation of the results as follows: The stress distribution depicted in the first graph indicates a series of normalized stress contours for $(\bar{\sigma}_{xx})$ and $(\bar{\sigma}_{yy})$, which represent the principal stress components in two perpendicular directions. The elliptical distribution remains similar to that observed at 100°C, suggesting that the material response under thermal stress is consistent across these temperatures. The shapes of the contours imply that the material experiences varying degrees of stress across different directions, characteristic of anisotropic materials. The spread of the contours might suggest that the stress distribution within the material widens with increasing temperature, a typical behavior as materials tend to soften and become more compliant at higher temperatures. The central contour's position implies a symmetric loading or a uniform thermal expansion without significant deviation from the center, which would be expected under a state of no applied mechanical load. The second graph presents the variation of the radial parameter (r_θ) as a function of the angular position. At 150°C, the parameter continues to increase with the angle, although the curve seems to be more pronounced than at 100°C, suggesting a greater sensitivity of the radial parameter to temperature changes. This could be indicative of thermal expansion effects becoming more pronounced with temperature, affecting the material properties such as stiffness or thermal strain more significantly at higher temperatures. The curve remains smooth, indicating that the material's response is gradual and does not exhibit sudden changes or discontinuities over the angular sweep. In the third graph, the normalized stress component $(\bar{\sigma}_\theta)$ also displays an increase with the angular orientation, similar to the trend observed at the lower temperature. However, the curve appears slightly flatter at the onset, indicating that the initial sensitivity to angular changes may be less at the higher temperature, perhaps due to a more uniform thermal expansion or stress distribution in the initial angular range. As the angle increases, the stress component shows a steady rise, underlining the anisotropic nature of the material and its changing properties with orientation, even at elevated temperatures.

In summary, the data at 150 °C reinforce the anisotropic behavior of the material and suggest increased material compliance or sensitivity to thermal effects compared to the 100 °C case. These findings could be critical for design considerations where temperature-dependent material behavior is a factor, particularly in applications involving significant thermal loading or environments with high thermal gradients. The consistency of the stress distribution pattern and the smooth variation of radial parameters across the temperature range enhance the understanding of the material's performance, which is crucial for ensuring reliability and safety in engineering applications.

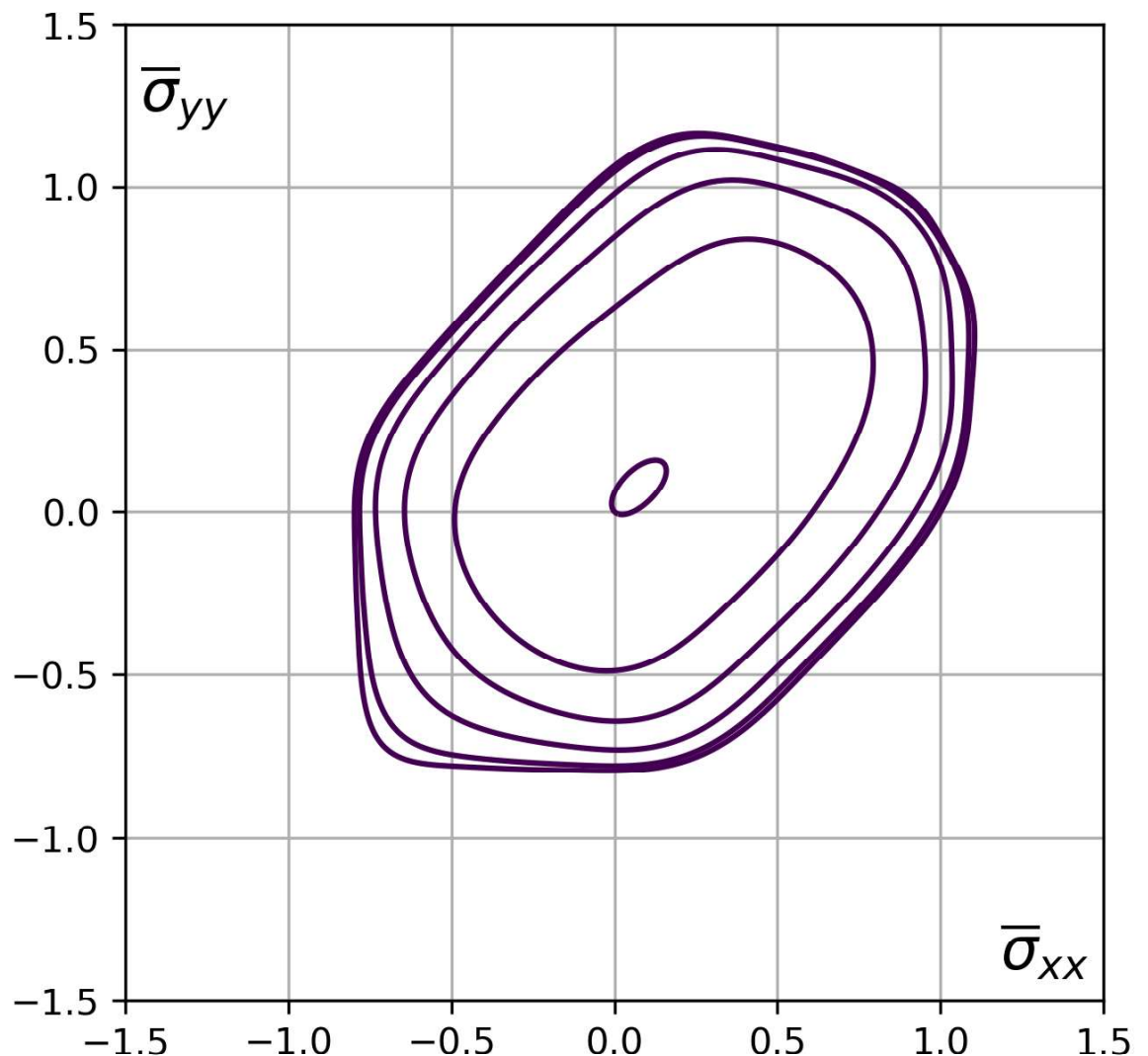


Figure 4.61 The Bezier5YS and SHYqp yield surface at 150 °C

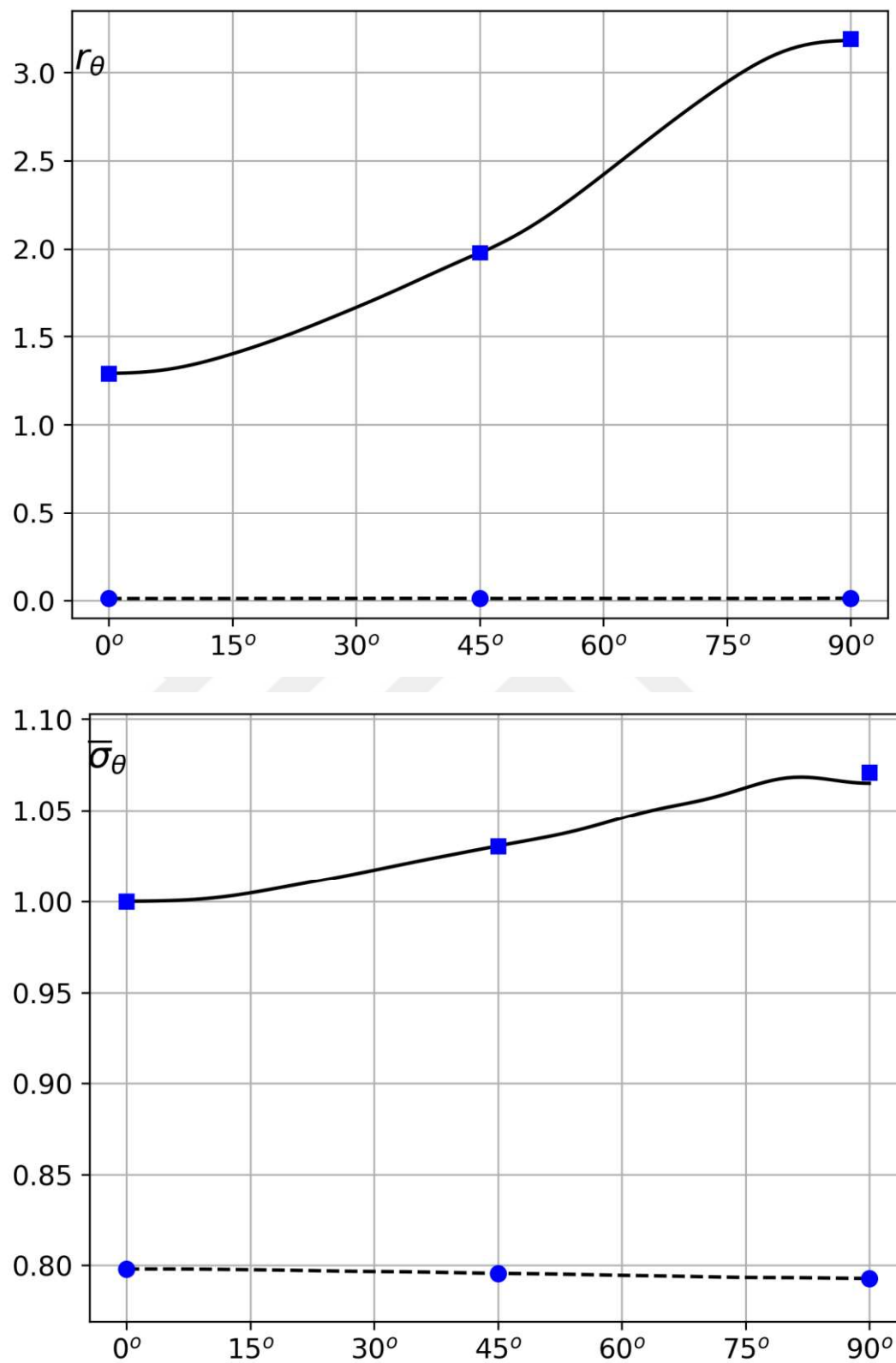


Figure 4.62 The Bezier5YS and SHYqp variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at 150 °C

4.6.4. The Bezier5YS and SHYqp Yield Surface at 200 °C

The first graph represents the stress distribution contours for the Bezier5YS-2023 material at 200 °C. The elliptical contours suggest that the material exhibits anisotropic behavior, with the major axis of the ellipses indicating the principal direction of higher stiffness or resistance to deformation. The central, tighter ellipse corresponds to the core area where the stresses are more uniform and suggest a higher concentration of stress resistance. As the ellipses expand outward, they represent regions with progressively lower stress concentrations, implying a gradient in the material's response to the applied load. This pattern indicates that the Bezier5YS-2023 maintains a certain degree of integrity and predictability under stress at the given temperature, with no signs of extreme stress concentrations that might indicate failure points. For the SHYqp-2023, we have two separate graphs depicting the behavior of the material under load at the same temperature. The first graph shows (r_θ) , which may be indicative of the material's resistance to deformation at various angles (θ). The smooth, upward-trending curve suggests that resistance increases with the angle, reaching a peak at 90 degrees. This pattern could imply that the SHYqp-2023 becomes more resistant to deformation as the direction of the applied load approaches the perpendicular relative to a reference axis, which could be useful in applications requiring directional strength. The second graph presents the normalized value $(\bar{\sigma}_\theta)$ over the same angular range. The graph displays a gradual increase in $(\bar{\sigma}_\theta)$ with angle, suggesting that the material's stiffness or the stress at which it yields increases slightly when the load is applied at higher angles. The value remains above the threshold (dashed line), which may be a designed safety or performance limit, indicating that the SHYqp-2023 is capable of withstanding the applied stresses without reaching critical limits across the range of angles tested.

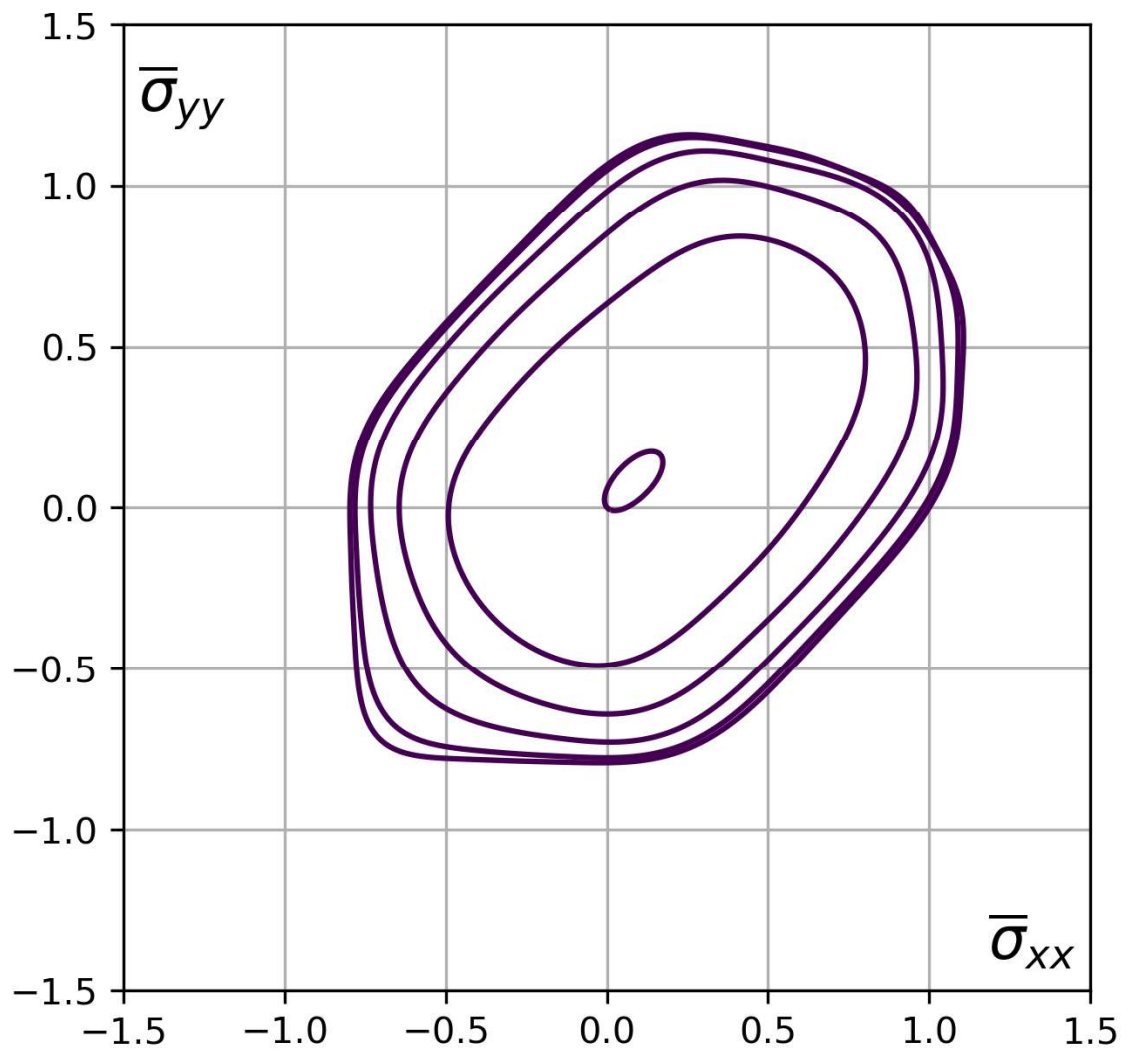


Figure 4.63 The Bezier5YS and SHYqp yield surface at 200 °C

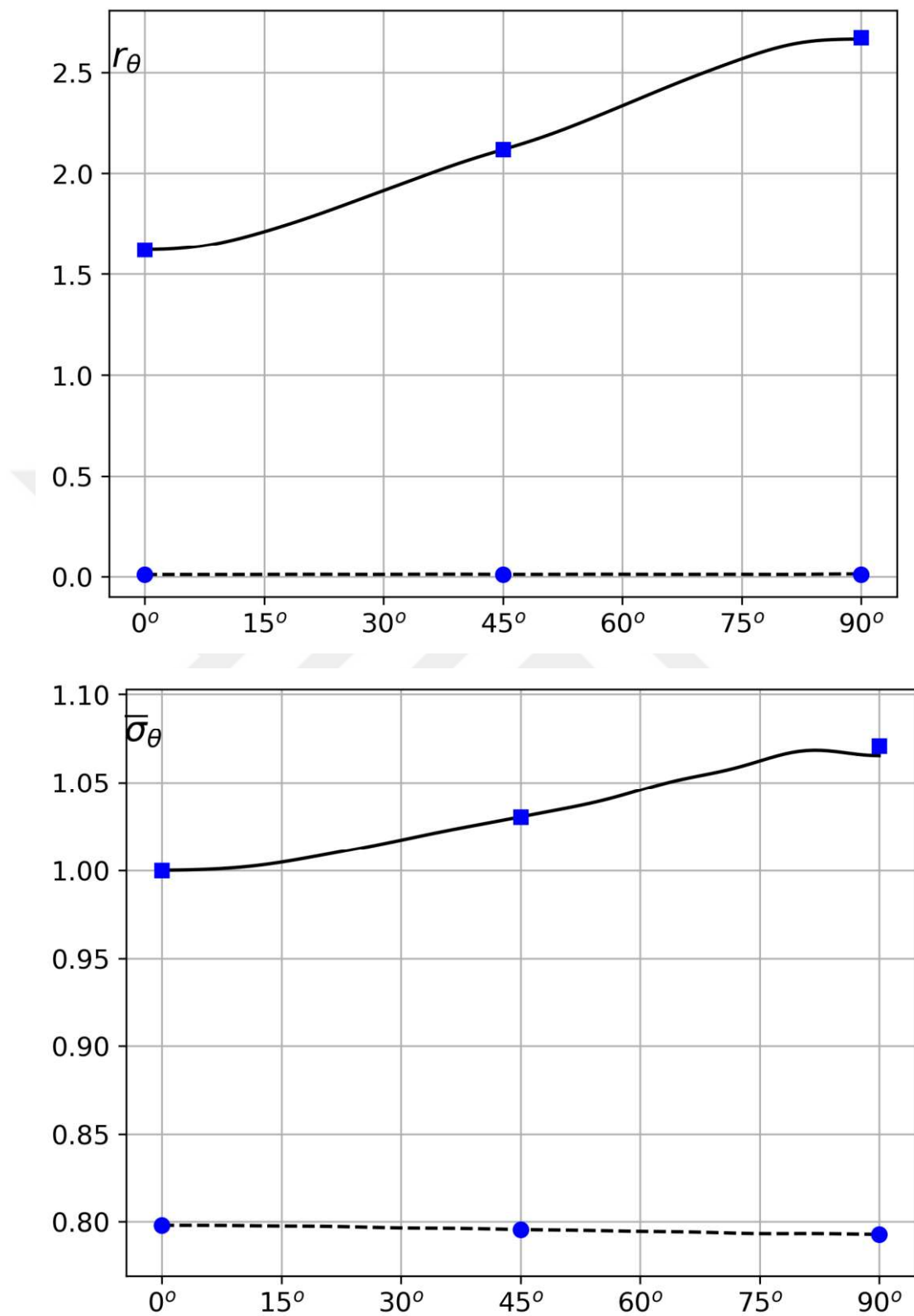


Figure 4.64 The Bezier5YS and SHYqp variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at 200 °C

4.6.5. The Bezier5YS and SHYqp Yield Surface at 250 °C

The first graph is a contour plot with axes labeled $(\overline{\sigma_{xx}})$ and $(\overline{\sigma_{yy}})$, which suggest normalized stresses along two perpendicular axes, typically in a material's reference frame. The elliptical contours might indicate the yield surface of the AZ31 material under biaxial stress states at 250°C. The center of the ellipses, being close to the origin, indicates that there is no significant bias in the stress distribution – that is, no predominant directional strength or weakness is apparent at this temperature. The shape and orientation of the ellipses provide insights into the anisotropy of the material; a perfect circle would suggest isotropic behavior, where the material's yield strength is the same in all directions, while ellipses denote anisotropy. In this case, the elongated ellipses suggest that the material exhibits different yield strengths when stressed along different axes. The larger spread along $(\overline{\sigma_{xx}})$ than $(\overline{\sigma_{yy}})$ might imply that the material can withstand higher stress along the $(\overline{\sigma_{xx}})$ axis before yielding. The second graph presents a curve labeled (r_θ) plotted against different angles from 0° to 90°, with discrete data points marked by blue squares. The (r_θ) value typically refers to the plastic strain ratio, which is a measure of a material's ability to undergo plastic deformation without necking. This is an indication of ductility. The increase in (r_θ) as the angle increases suggests that the material's ductility may improve when the stress is applied at higher angles relative to a reference direction (often the rolling or extrusion direction in the case of wrought materials). The trend is upward, indicating a possible texture in the material that enhances ductility with orientation. The ductility is lowest at 0° and increases with the angle, suggesting an anisotropic response in plastic deformation at 250°C. The third graph displays $(\overline{r_\theta})$ values, which might be the average plastic strain ratio across different orientations or a normalized version of the previous graph's (r_θ) . The trend is similar, albeit with values closer to 1, which may indicate a normalization against a certain reference condition or an average behavior. The curve is mostly flat between 0° to 45°, suggesting uniform behavior in those orientations, and then increases steadily to 90°. This increase once again points to anisotropic properties, which could be due to the texture developed in the material. The closer these values are to 1, the more isotropic the material behavior; however, the upward trend indicates that even at 250°C, the AZ31 alloy has directional

dependencies in its deformation characteristics. In summary, the presented data at 250°C for the AZ31 material indicates anisotropic mechanical behavior under applied stress. The yield surface, as depicted in the contour plot, suggests a directional dependence of the yield strength, with greater tolerance along the x axis. The plastic strain ratio and its normalized counterpart demonstrate increased ductility with orientation, peaking at angles perpendicular to the reference direction. This suggests that the deformation mechanisms active at 250°C in AZ31 are sensitive to the loading direction, which could be a result of the material's crystallographic texture or microstructural features developed during processing or in-service conditions. Understanding these anisotropic behaviors is crucial for predicting the material's performance in applications subject to complex loading conditions.

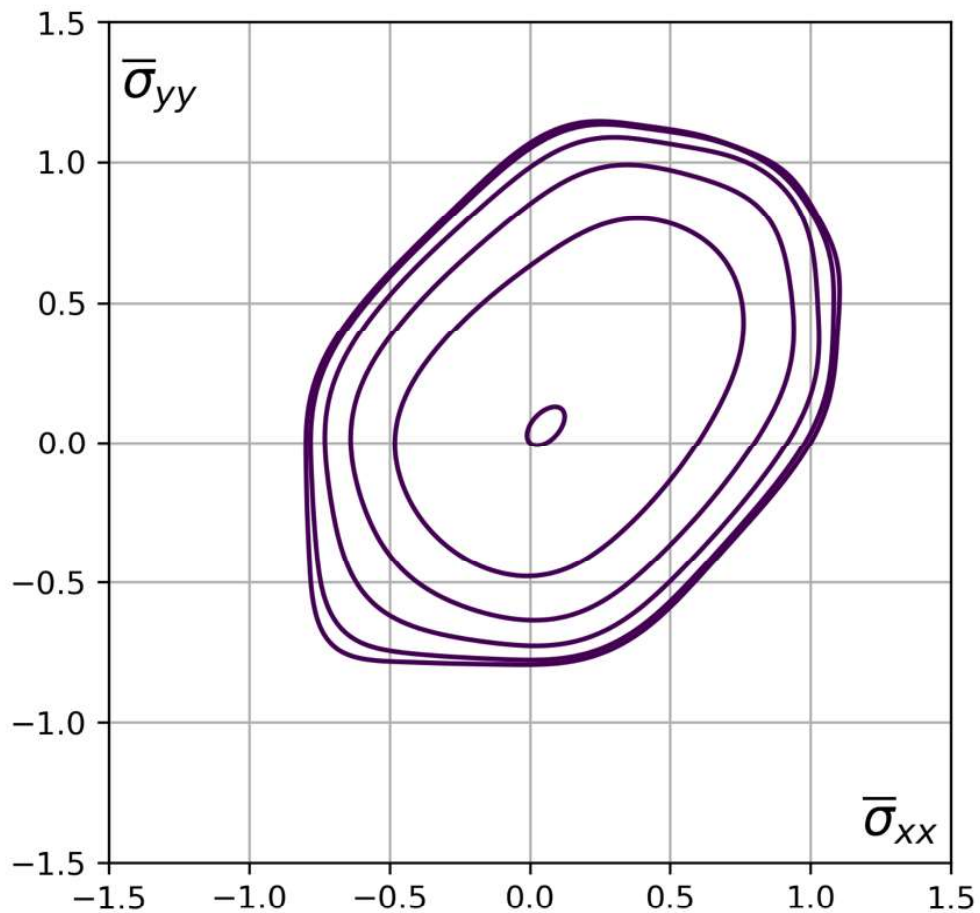


Figure 4.65 The Bezier5YS and SHYqp yield surface at 250 °C

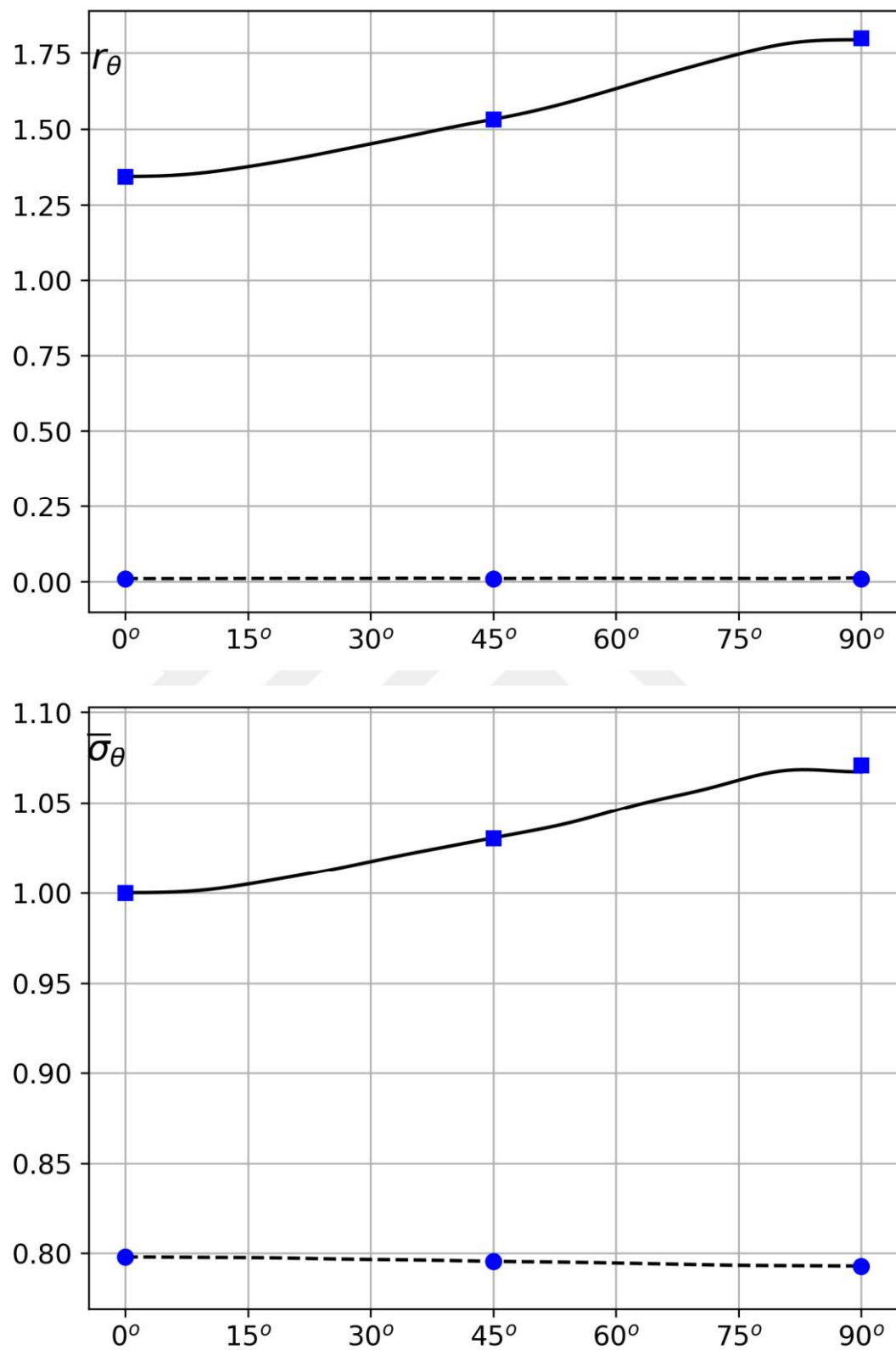


Figure 4.66 The Bezier5YS and SHYqp variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at 250 °C

4.6.6. The Bezier5YS and SHYqp Yield Surface at 300 °C

The first graph is a contour plot showing the relationship between two normal stress components: (σ_{xx}) and (σ_{yy}) . These could represent the stresses in orthogonal directions in the material's microstructure or in a component made from the AZ31 material. The elliptical nature of the contours suggests that the stresses are not isotropic and there's a certain orientation-dependence to the material's response. The center of the ellipses, close to the origin, might indicate that there is no significant inherent stress bias in the material or the component. The larger spread in the (σ_{yy}) direction could suggest an anisotropic response where the material exhibits different strengths or stiffnesses in different directions at 300°C. This could be due to the grain structure or texturing of the material, which is common in wrought magnesium alloys. The second graph plots (r_θ) as a function of angle from 0° to 90°. The quantity (r_θ) is not defined in the question, but it could be related to the strain ratio or a similar mechanical property that varies with the orientation of the stress or the load application concerning the material's microstructure. The graph shows that (r_θ) increases with the angle, starting from nearly zero at 0°, indicating that the mechanical response of the material is highly anisotropic. This could imply that at 300°C, the AZ31 material's properties such as ductility or toughness might improve with a certain orientation of loading. This is consistent with the behavior of many hexagonal close-packed (HCP) metals, including magnesium alloys, where mechanical properties can vary significantly with orientation due to the limited number of slip systems active at lower temperatures. The third graph presents the normalized value of r_θ , which appears to be scaled such that the response at 0° is set as a baseline (denoted by the dashed line at 1.00). This graph shows a similar trend to the second graph, with the property increasing with angle. However, the normalization allows for a comparison relative to the baseline condition. This could be particularly useful for understanding how much the mechanical property of interest (possibly related to deformation or ductility) enhances with orientation under a temperature of 300 °C. The fact that the property increases above 1.00 suggests that at 300 °C, the material behaves better at angles greater than 0°, which could be crucial for applications where the material is subjected to multi-axial stress states or non-uniform thermal conditions.

In summary, the graphs suggest that the AZ31 material exhibits anisotropic mechanical properties at 300 °C, which are orientation-dependent. The contour plot shows a potential anisotropy in stress response, while the other two graphs indicate that mechanical properties such as strain or toughness improve with angular orientation from the reference direction. This knowledge is essential for the proper engineering application of AZ31, as it will influence how components are designed, oriented, and subjected to stress under high-temperature conditions.

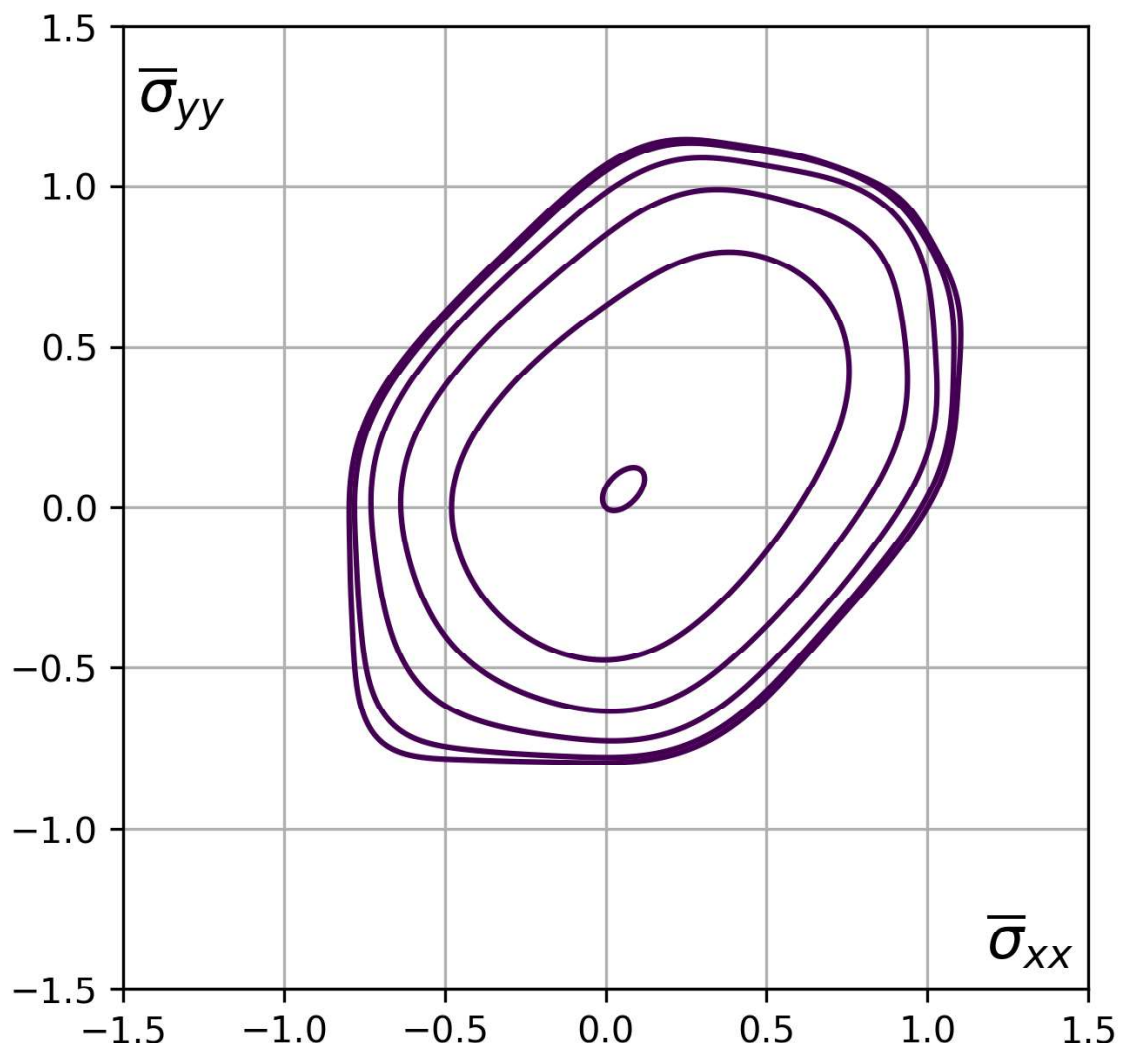


Figure 4.67 The BezierSYS and SHYqp yield surface at 300 °C

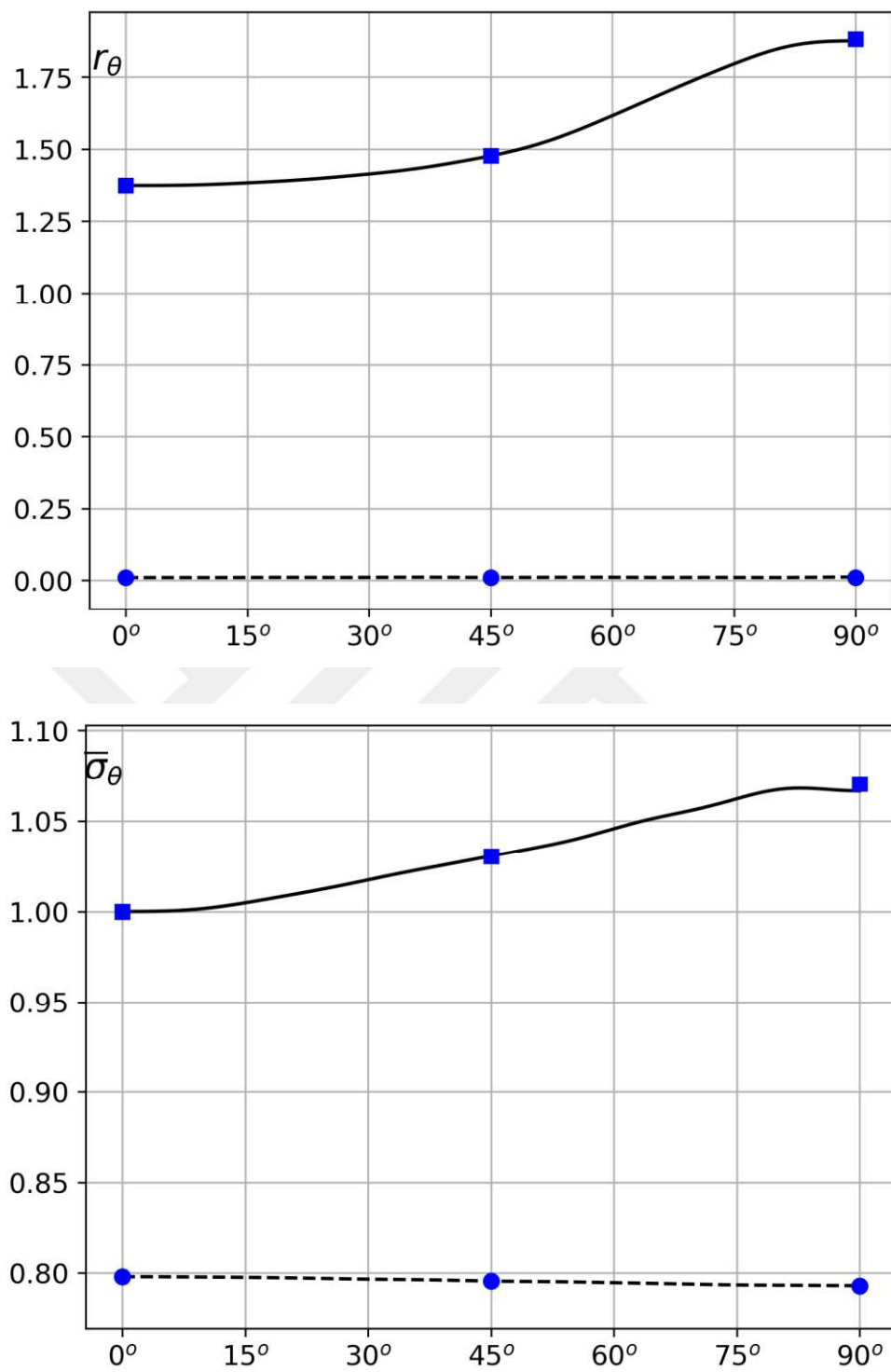


Figure 4.68 The Bezier5YS and SHYqp variation of yield stress (Top) and anisotropy (Bottom) with orientation graphs at 300 °C

4.7. Conclusion

Considering all the data we've provided for the different yield criteria applied to the prediction of mechanical behavior in HCP materials, for this study, AZ31 magnesium alloy, it is apparent that the Yld2004-18p, the Bezier5YS-2023 and SHYqp-2023 models offer enhanced predictive capabilities over other models tested, especially at elevated temperatures. The stress-strain curves, the Lankford coefficient predictions, and the stress contour plots collectively indicate that these models more accurately capture the complex anisotropic nature of HCP materials. The models are able to predict the directional dependency of the material properties, which is a crucial feature of HCP crystals due to their limited slip systems that become more or less active depending on the temperature and loading conditions. In high-temperature regimes, HCP materials exhibit significant changes in mechanical behavior, with potential activation of additional slip systems or twinning, which can alter the ductility and strength of the material. The superior performance of the Yld2004-18p, the Bezier5YS-2023, and SHYqp-2023 models in capturing these temperature-dependent anisotropic features is of great importance. They account for the directional sensitivity in yield strength and strain hardening, which are critical for accurate predictions of deformation and failure under various loading conditions. The contour plots provided for the SHYqp-2023 model at 300°C demonstrate the ability to predict the non-isotropic stress states, and the plots of (r_θ) and $(\bar{r}_\theta)$ further reinforce the model's capability in capturing the orientation-dependent mechanical response. Such predictive accuracy is essential for material scientists and engineers when designing components that will be subjected to complex loading conditions, particularly where the components must withstand high temperatures.

In conclusion, the Yld2004-18p, the Bezier5YS-2023, and SHYqp-2023 models stand out for their robustness in describing the anisotropic features of HCP materials. Their application is particularly recommended for scenarios involving high-temperature conditions where traditional yield criteria may fall short in predicting the nuanced responses of these materials. Such advanced models are pivotal for the design and analysis of HCP materials in high-performance and safety-critical applications.

CHAPTER 5

RESULTS AND DISCUSSION

5.1. Results of Disc Compression Tests

The empirical findings derived from the disc compression tests offer valuable insights into the mechanical behavior of AZ31 magnesium alloy under biaxial loading conditions. This section presents the outcomes of these tests and focuses on the crucial analysis of the Lankford parameter, which plays a pivotal role in characterizing the material's formability and anisotropy.

5.1.1. Curve Fitting for Lankford Parameter

In our pursuit of comprehending the complex anisotropic behavior exhibited by AZ31 magnesium alloy under biaxial loading conditions, an extensive series of disc compression tests were conducted. These tests, comprising over a hundred iterations, yielded a wealth of data that provide essential insights into the material's mechanical response. One of the pivotal aspects under scrutiny was the Lankford parameter (r), a metric crucial for characterizing the material's anisotropy and formability. The Lankford parameter, represented by (r), encapsulates the ratio of width strain (ϵ_w) to thickness strain (ϵ_t) experienced by the material during deformation. This parameter serves as an indicator of how the material behaves under biaxial loading, offering valuable information about its anisotropic tendencies. Specifically, it quantifies whether the material undergoes greater deformation in the width direction than in the thickness direction, a characteristic that has profound implications for various forming processes and engineering applications. In the course of the disc compression tests, we meticulously gathered data on strain in the rolling direction (RD) and strain in the transverse direction (TD) under various orientations and temperatures. These strains, along with the experimental stress data, formed the foundation of our analysis. To extract meaningful insights from this extensive dataset, a rigorous curve-fitting approach was employed.

This curve-fitting approach involved mathematical modeling to establish relationships between the Lankford parameter (r) and the relevant independent variables, including orientation angle (θ) and temperature (T). By fitting mathematical functions to the experimental data, we can unravel patterns and trends that provide a deeper understanding of how AZ31 magnesium alloy responds to varying loading conditions. The curve-fitting process, applied to the extensive dataset spanning numerous orientations and temperatures, allowed us to develop predictive models for the Lankford parameter (r). These models offer a comprehensive view of how the material's anisotropic behavior evolves under different circumstances, shedding light on the material's formability characteristics and its suitability for specific engineering applications. In the sections that follow, we delve into the specifics of the curve-fitting results for the Lankford parameter. Through a systematic analysis of these results, we aim to elucidate the intricate relationship between orientation angles, temperatures, and the material's anisotropic response. This analysis is a vital component of our broader investigation into the mechanical behavior of AZ31 magnesium alloy, ultimately contributing to the advancement of knowledge in the field of materials science and engineering.

5.1.2. Experimental Results at RT (25 °C)

The provided graph showcases a Lankford curve fitting of the AZ31 material at RT, indicating a linear relationship between strains in the rolling direction (RD) and the transverse direction (TD). This relationship is encapsulated by the equation $y = 0.526x + 0.001$, suggesting that the strain in TD increases by approximately 0.526 units for every unit increase in strain in RD. Furthermore, the high coefficient of determination, $R^2 = 0.933$, underscores the accuracy of this linear fit, as it explains about 93.3% of the variance in the data. This curve highlights the material's anisotropic deformation behavior, with a Lankford parameter (or 'r' value) of around 0.526, offering critical insights for industries involved in material forming.

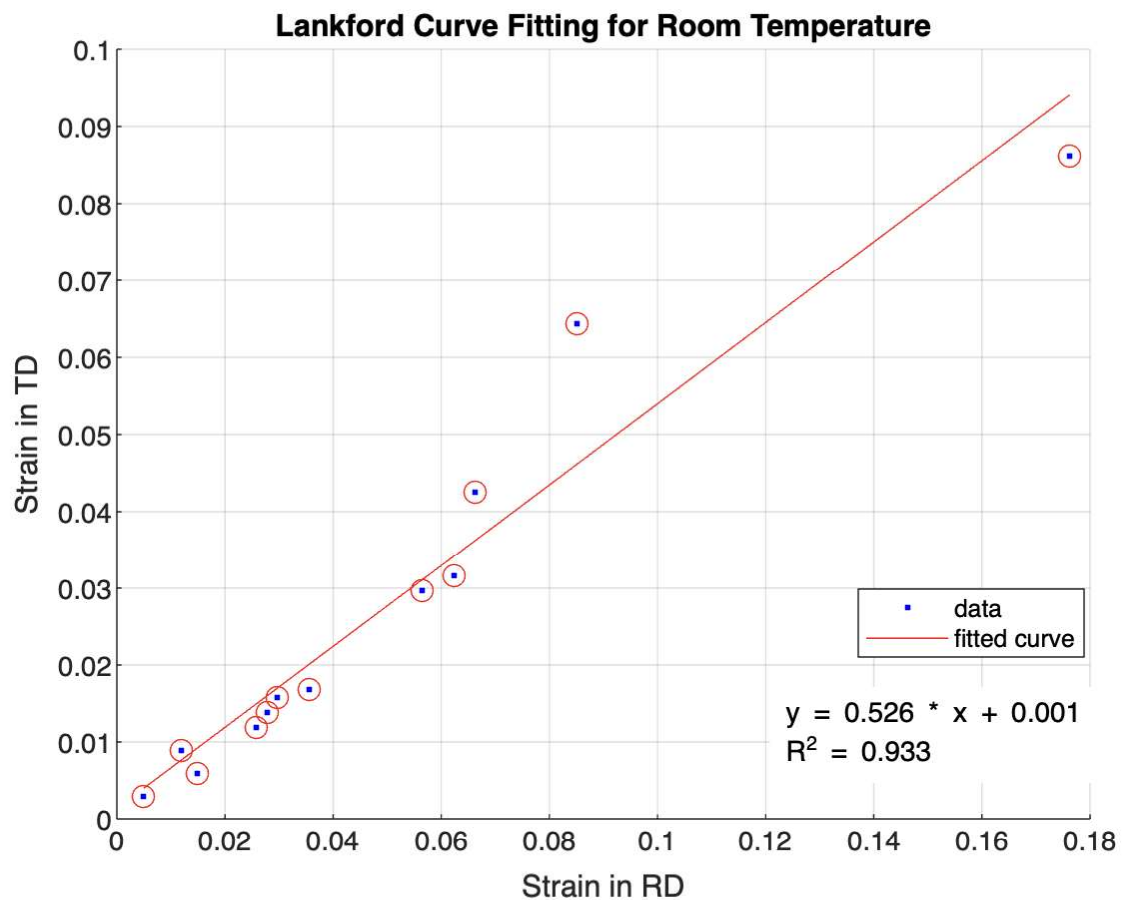


Figure 5.1 Curve fitting for the Lankford parameters for RT

5.1.3. Experimental Results at 50 °C

The graph illustrates the Lankford curve fitting for a material at a moderate elevated temperature of 50°C, establishing a linear relationship between the strain in RD and the strain in the TD. According to the linear fit equation $y = 0.640x + 0.010$, the strain in TD increases by 0.640 units for each unit increase in RD strain. The coefficient of determination is quite high at $R^2 = 0.943$, signifying that the model explains 94.3% of the variability in the strain data. This level of fit suggests that the material demonstrates a somewhat less anisotropic behavior at 50°C compared to the behavior at 150°C, as indicated by the lower Lankford parameter (r' value) of 0.640. Such data are valuable in understanding how temperature affects the material's plastic deformation characteristics, which is essential for forming processes and predictive modeling in materials science and engineering.

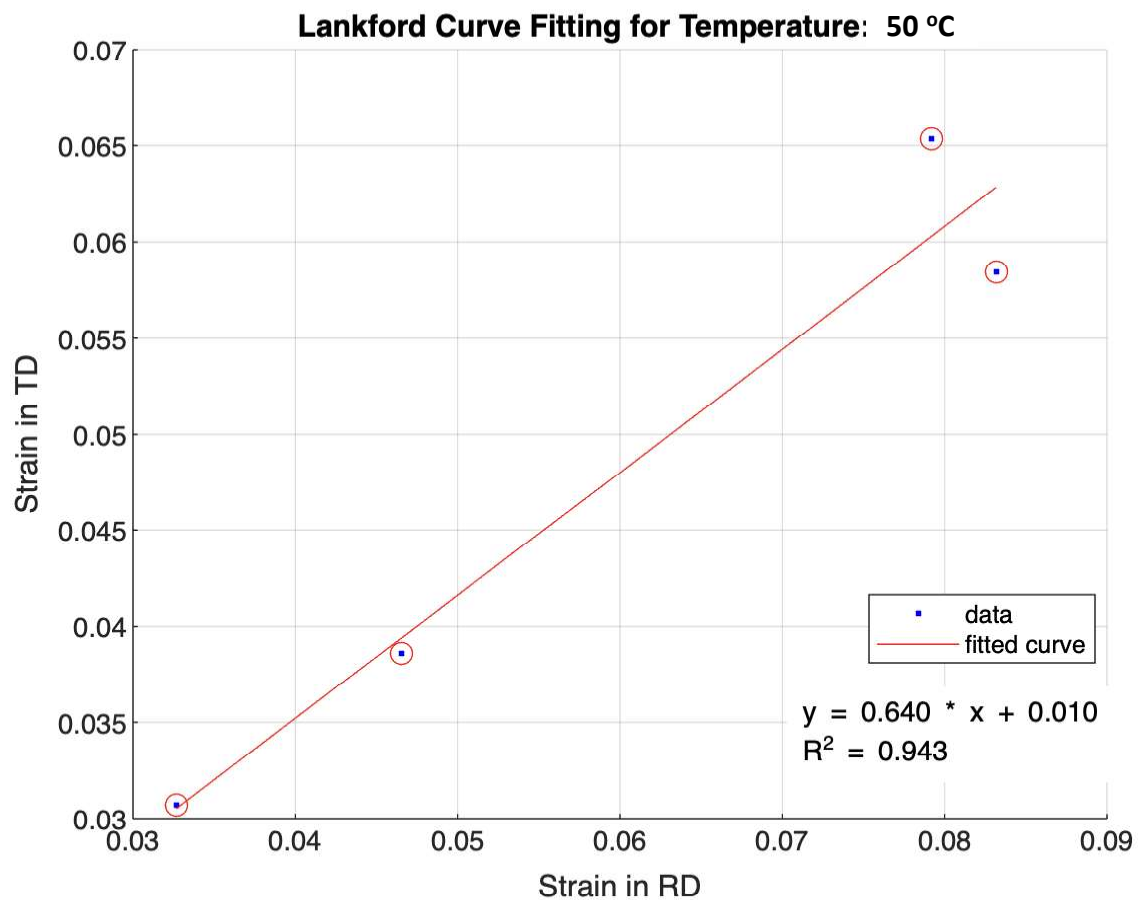


Figure 5.2 Curve fitting for the Lankford parameters for 50 °C

5.1.4. Experimental Results at 100 °C

The graph displays a Lankford curve fitting for a material subjected to a temperature of 100°C. It presents the correlation between strain in the and strain in TD of the material. The derived linear fit equation $y = 0.801x - 0.009$ suggests a positive linear relationship, indicating that for every increase in strain in the RD, there is a corresponding increase in strain in the TD, multiplied by a factor of 0.801. The negative intercept of -0.009 could be indicative of measurement offset or an intrinsic material property at this specific temperature. The coefficient of determination R^2 is 0.946, which indicates a strong correlation, with the linear model explaining 94.6% of the total variation in the data around the average. This high level of fit shows that the linear model is quite appropriate for representing the material's behavior under the given thermal conditions, and it underscores the material's consistent mechanical response at 100°C.

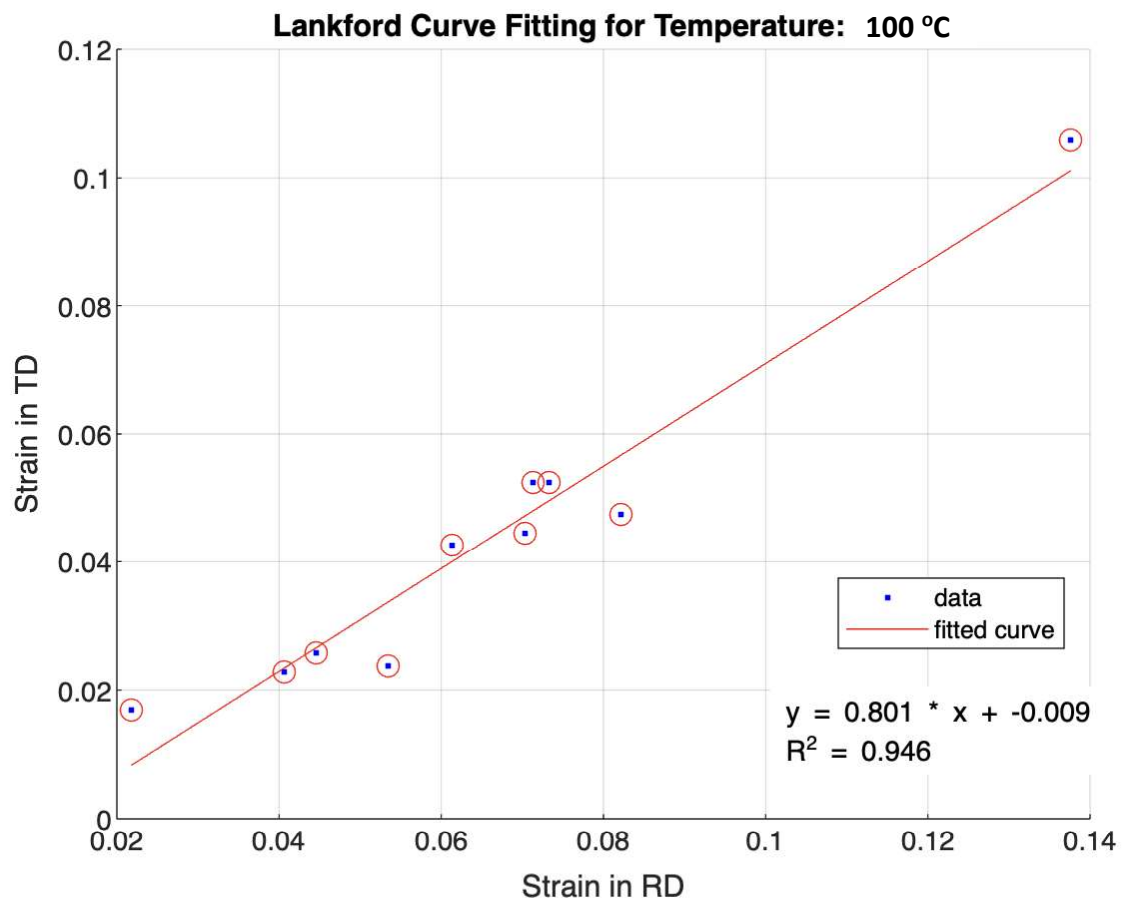


Figure 5.3 Curve fitting for the Lankford parameters for 100 °C

5.1.5. Experimental Results at 150 °C

The graph presents a Lankford curve fitting for the material in question at a temperature of 150°C. It depicts a linear association between strains in the RD and the TD directions. This correlation is represented by the equation $y = 0.958x + 0.026$, which implies that for every unit increase in strain in RD, there is an approximate increase of 0.933 units in strain in TD. The coefficient of determination, $R^2 = 0.893$, confirms a strong fit, accounting for roughly 89.3% of the data's variance. This demonstrates the material's consistent anisotropic deformation characteristics at elevated temperatures, with a Lankford parameter ('r' value) nearing 0.933, providing valuable information for metallurgical and manufacturing processes.

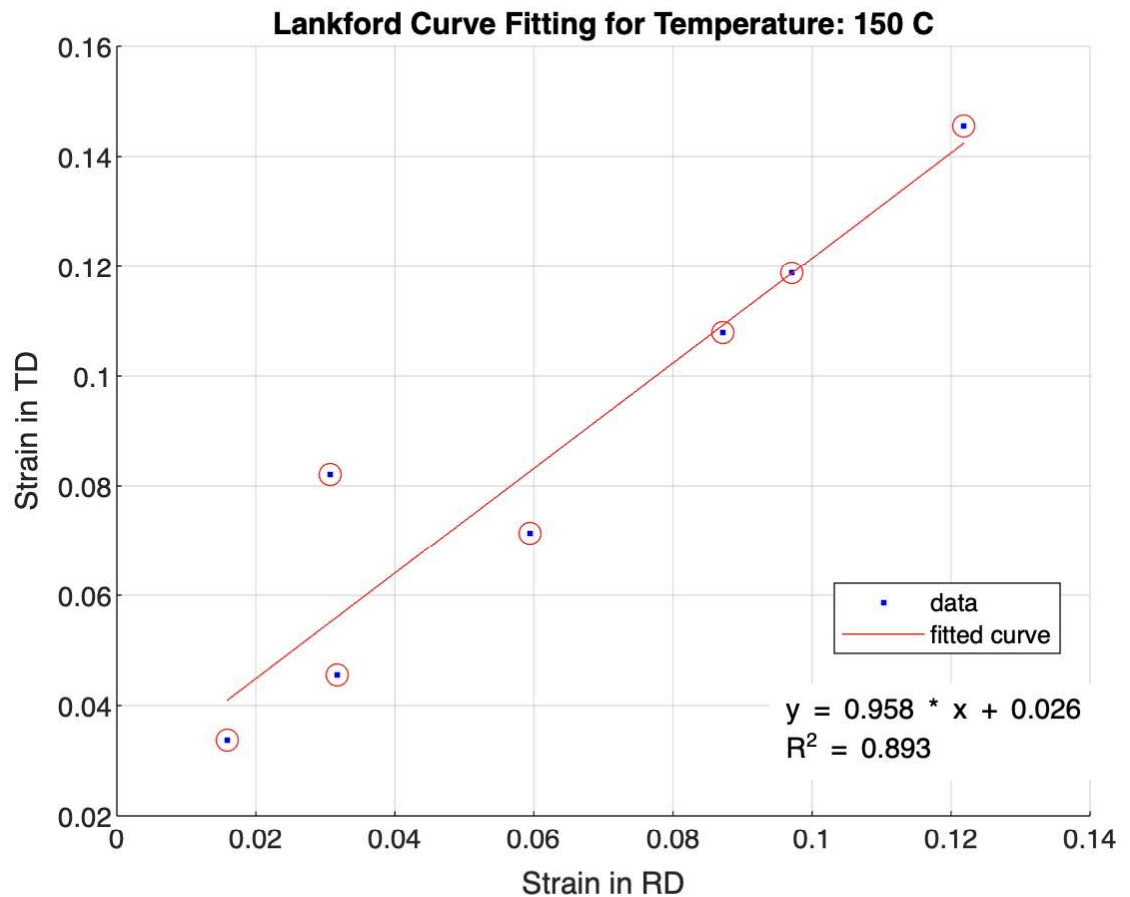


Figure 5.4 Curve fitting for the Lankford parameters for 150 °C

5.1.6. Experimental Results at 200 °C

This graph illustrates a Lankford curve fitting for material deformation at a temperature of 200°C. The graph shows the relationship between strain in the RD and strain in the TD. The best-fit linear equation obtained from the data is $y = 1.037 x + 0.025$.

- The slope of the line, 1.037, indicates that for every unit of strain increase in the RD, the strain in the TD increases by a slightly higher amount (1.037 times).
- The positive y-intercept of 0.025 may represent an initial strain in the TD before any strain is applied in the RD, or it could reflect a systematic measurement error or initial deformation characteristic of the material at this temperature.

The (R^2) value is 0.991, which is very close to 1. This value indicates an excellent fit for the linear regression model, suggesting that 99.1% of the variance in the strain in the TD can be predicted from the strain in the RD at 200 °C. This shows that the model accurately represents the deformation behavior of the material under these specific thermal conditions, with only a small amount of variation unaccounted for by the model.

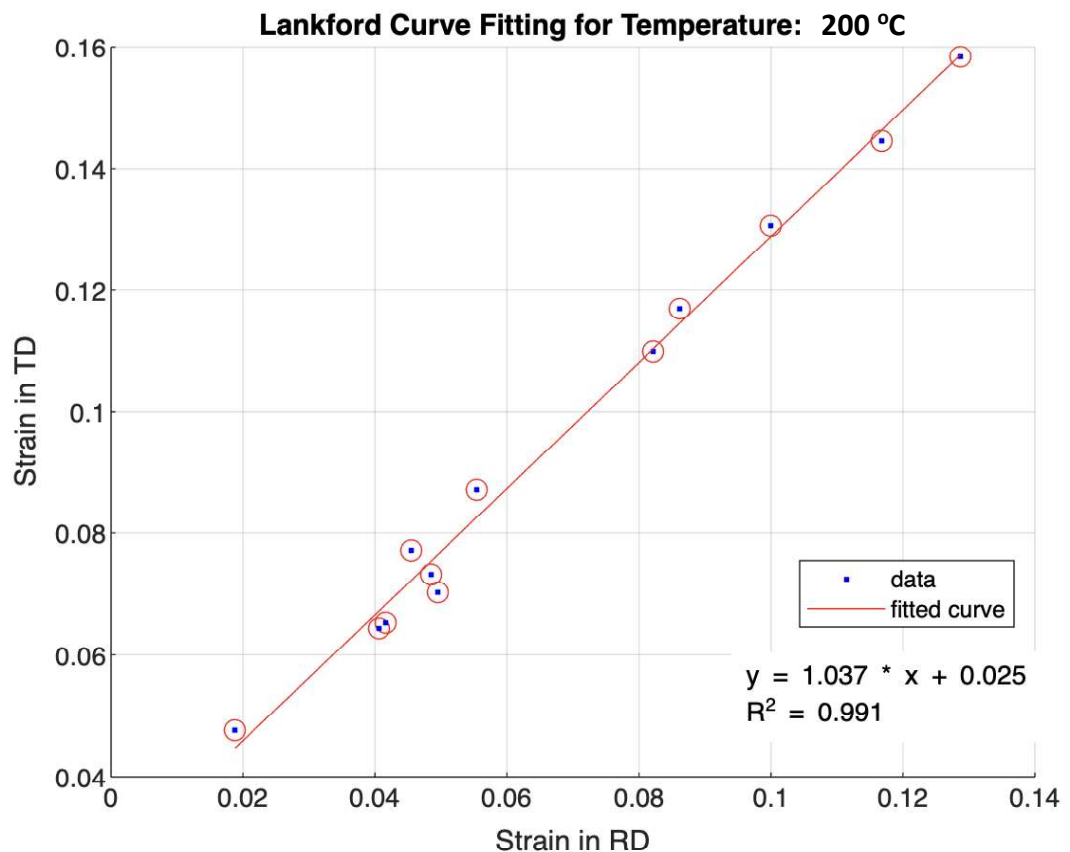


Figure 5.5 Curve fitting for the Lankford parameters for 200 °C

5.1.7. Experimental Results at 250 °C

This graph represents Lankford curve fitting for a temperature of 250°C. It depicts the relationship between strain in the RD and strain in the TD. The graph contains a linear best-fit line with the following equation:

$$y = 1.260 x - 0.141$$

- The slope (1.260) signifies that for each increment of strain in the RD, the strain in the TD increases by an even larger factor, indicating a strong direct relationship between the two strains at this temperature.

- The y-intercept (-0.141) suggests that at zero strain in the RD, the model predicts a slight negative strain in the TD, which could imply that at no applied strain in the RD, there's a slight contraction or a measurement that's below the baseline in the TD.

The coefficient of determination, (R^2), is 0.832, which means that approximately 83.2% of the variability in the TD strain can be explained by the RD strain at 250 °C according to this linear model.

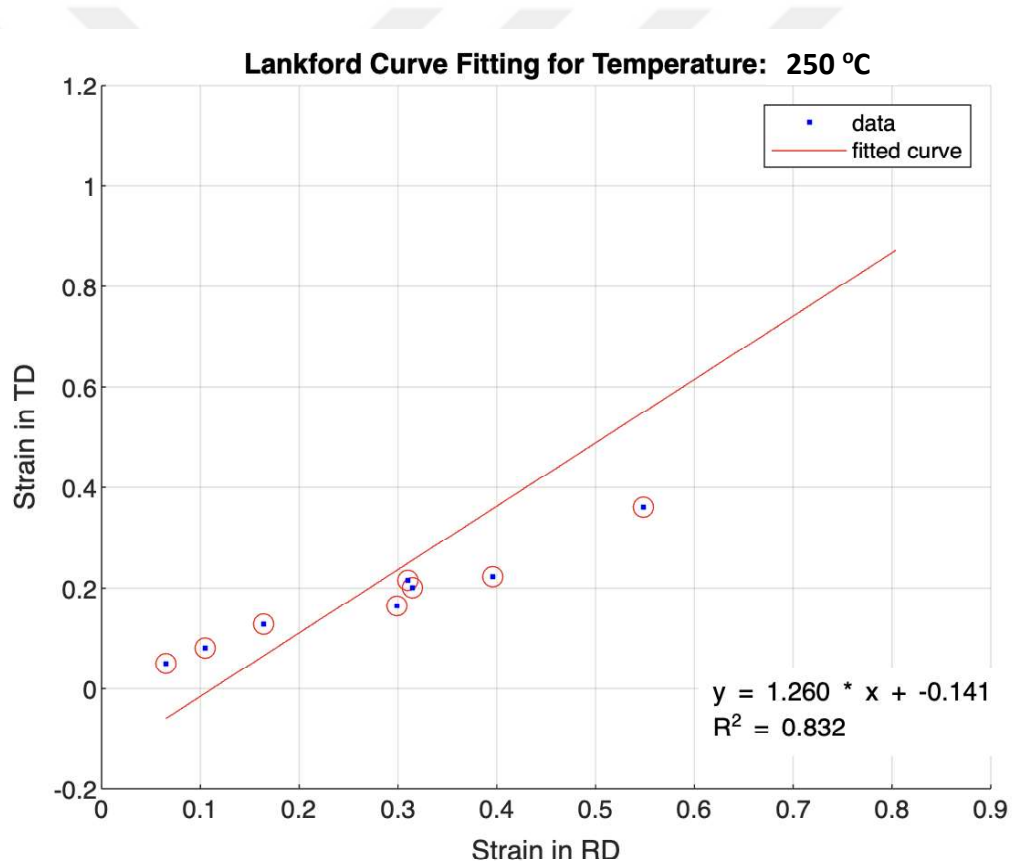


Figure 5.6 Curve fitting for the Lankford parameters for 250 °C

5.1.8. Experimental Results at 300 °C

This graph illustrates the Lankford curve fitting for a material at a temperature of 300°C. It shows the relationship between strain in the RD and strain in the TD. The fitted curve is described by the linear equation:

$$y = 1.446 x - 0.014$$

- The slope of the line is 1.446, which indicates a strong positive correlation between the strain in RD and the strain in TD. In practical terms, for every unit increase in strain along the RD, the strain in TD is expected to increase by 1.446 units, suggesting that the material becomes more ductile as the temperature increases, assuming that the test conditions are causing such behavior.

- The y-intercept of -0.014 suggests that when there is no strain in the RD, the strain in the TD is slightly below zero. This could be due to the way the strains are measured or a calibration issue with the measurement equipment, assuming that a negative strain is not physically meaningful in this context.

The coefficient of determination, (R^2), is 0.974, indicating that the model explains 97.4% of the variance in the TD strain based on the RD strain, which is quite high. This suggests that the linear model is a good fit for the data at this temperature, with most data points lying close to the fitted line. The close fit implies that the material's behavior at 300 °C can be reliably predicted by this linear equation within the range of the data collected.

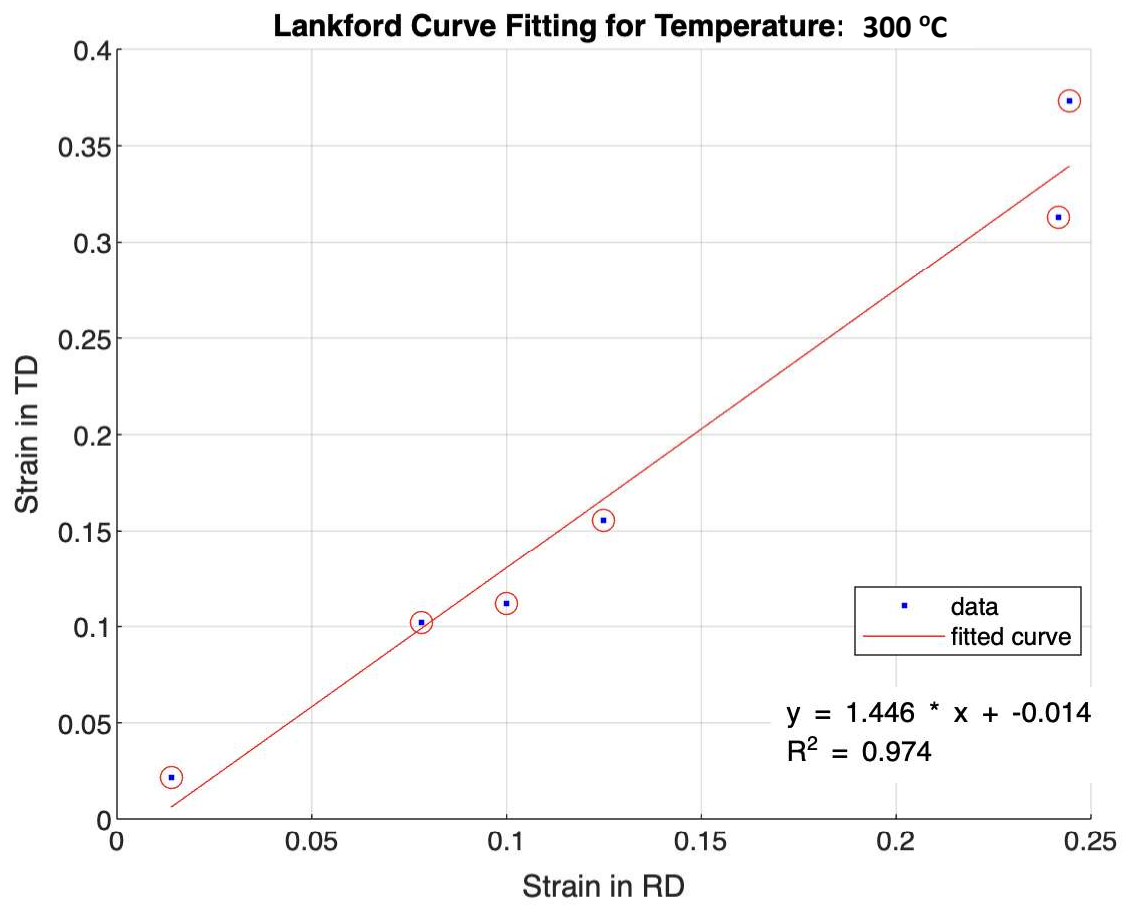


Figure 5.7 Curve fitting for the Lankford parameters for 300 °C

5.1.9. Relationship between Temperature and Lankford Values

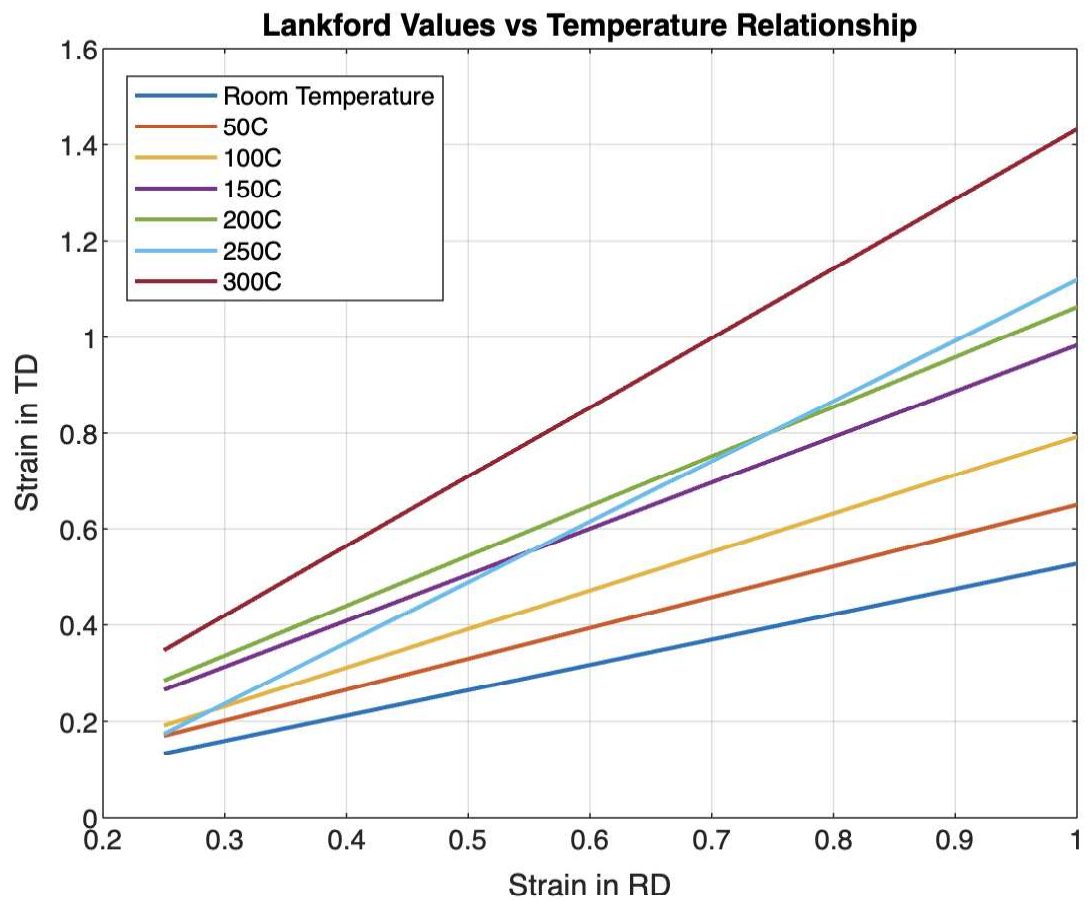


Figure 5.8 The Lankford values vs temperature relationship

The graph titled "Lankford Values vs Temperature Relationship" is a representation of the strain in the transverse direction as a function of the strain in the rolling direction of a material subjected to deformation processes at various temperatures. This graph is a typical way to present the plastic anisotropy of a material as defined by the Lankford coefficient or r -value, which is a ratio of the width strain to the thickness strain in a tension test. The r -value is a measure of a material's ability to resist thinning or thickening when stretched, which is critical for processes like deep drawing. The examination of the graph entitled "Lankford Values vs Temperature Relationship" elucidates the behavior of a material under deformation, particularly how it strains in different directions when subjected to varying thermal conditions. The graph serves as an empirical manifestation of the material's plastic anisotropy through the lens of the Lankford coefficient, or r -value. This coefficient, a critical parameter in materials

engineering, encapsulates the ratio of strain experienced in the width to that in thickness during a tensile test, hence providing a quantitative glimpse into the material's propensity to undergo dimensional changes in the face of tensile forces. It is apparent from the graph that there is a tangible relationship between temperature and the ductility of the material. With an upsurge in temperature, the material exhibits an increased ability to withstand transverse strains for a given amount of strain in the rolling direction. The enhancement in ductility is captured through the ascending slope of the lines as the temperature scales from ambient to 300 °C. Such temperature dependence underscores the sensitivity of the material's anisotropy to thermal conditions, indicating that the mechanisms governing crystal plasticity, such as slip and dislocation motion, become more pronounced and favorable at heightened temperatures. When the material is considered at ambient conditions, it reveals the lowest level of ductility-a conclusion drawn from the subdued slope of the initial line in the graph. The progressive rise in temperature to 50°C and beyond correlates with increased ductility, implying that the material's deformation mechanisms evolve with temperature, potentially signifying a shift in the activation of slip systems or a transition from elastic to predominantly plastic deformation. The pragmatic relevance of the graph cannot be overstated, particularly in industrial forming processes. The ability to predict and adjust to the varying formability of materials at different temperatures allows manufacturers to tailor their methods to curtail common defects such as tearing or wrinkling, thereby optimizing the quality and integrity of the final product. Despite the clear trends demonstrated, the graph falls short of depicting the statistical reliability of the r-value calculations at each temperature. The absence of error bars or confidence intervals precludes a robust evaluation of the statistical significance of the results, which is an essential aspect of empirical research to affirm the consistency and repeatability of the findings. The implications for material selection and process design are considerable. The observed tendencies suggest that materials subjected to significant deformation would benefit from processing at elevated temperatures. This insight has the potential to inform the selection of materials for specific applications and to guide the development of industrial processes such as hot rolling or forging, where control over temperature is a pivotal factor in achieving desired material characteristics. Moreover, the research avenues that the

graph opens are manifold. A deeper investigation into the microstructural evolution that underpins the macroscopic behavioral patterns-analyzing grain size, phase transformations, and dislocation density-would vastly enhance the understanding of the material's responses to thermal and mechanical stimuli. Such microscopic insights would invariably enrich the macroscopic interpretation of deformation behavior, leading to more scientifically grounded applications in materials science and engineering.

In sum, the graph depicting "Lankford Values vs Temperature Relationship" offers a comprehensive insight into the thermally influenced deformation characteristics of materials, providing a basis for both practical applications in industrial forming processes and further exploratory research in material science.

5.2. Results of V-Bending Tests

In Chapter 3, we meticulously outlined the methodologies employed for data collection and analysis during the V-bending tests, providing an in-depth understanding of the experimental procedures, including the measurement of springback angles. In this section, we delve into the results derived from these comprehensive V-bending tests, shedding light on the behavior of the AZ31 magnesium alloy under varying conditions. Throughout the V-bending tests, the primary parameter under scrutiny was the springback angle, a pivotal metric in sheet metal forming processes. Immediate measurements were taken post-procedure, and subsequently, the specimens were allowed to rest for 2 hours before undergoing a second assessment. Any discernible deformations, cracks, or fractures were meticulously documented to capture the immediate impact of the bending process. Following the tests, the bent specimens underwent detailed microscopic examination, a critical step in comprehending the microstructural changes and identifying any subsurface cracks or structural alterations. The comprehensive dataset collected during these experiments was subjected to rigorous analysis, aimed at establishing correlations between bending behavior and the inherent properties of the AZ31 alloy. Employing data fitting techniques, we quantified changes in springback angles, allowing us to draw meaningful conclusions regarding the material's performance under varying conditions.

As we proceed further into this chapter, we will provide a detailed analysis and interpretation of these results, elucidating the significance of these findings in the context of sheet metal forming processes and the AZ31 magnesium alloy's utility in real-world applications.

5.2.1. Springback Angle at 250 C

Figure 5.9 illustrates the V-bending outcomes for the AZ31 magnesium alloy at a temperature of 250°C. The x-axis demarcates the different orientations - RD, 45°, 75°, and 90° - while the y-axis quantifies the bending results. In contrast, the y-axis quantifies the bending values, providing a measurable insight into the alloy's formability under this specific thermal condition. It's evident from the chart that the RD orientation exhibits higher values, ranging from 8.5 to 9.5, while the 90° orientation demonstrates the lowest values between 6.5 and 7. The graph offers a visual representation of the variation in formability across different orientations at this specific temperature. A notable observation from the graph is the relatively higher bending values associated with the RD orientation, which fall within the 8.5 to 9.5 range. This elevated range implies that, at 250°C, the material tends to resist deformation in the RD orientation, possibly due to the intrinsic characteristics of its grain structure. The orientation and alignment of grains, particularly in rolled materials, can induce anisotropic behavior. This behavior means that the material might exhibit varied mechanical properties based on the direction of applied stress, relative to the grain alignment. For the RD orientation, the data suggests that the grains might be oriented in a manner that hinders optimal bending.

Conversely, the 90° orientation manifests a contrasting behavior. With bending values situated between 6.5 and 7, this orientation reveals a heightened formability compared to the RD. The lower values signify that the alloy, when oriented perpendicular to the rolling direction, becomes more malleable and amenable to bending. Such a difference can be attributed to the potential preferential alignment of grains, which might be more favorably oriented for bending at this particular angle. Beyond the raw data, this figure speaks to the nuanced relationship between grain orientation and the mechanical behavior of the AZ31 alloy. Understanding such intricacies is of paramount importance, especially for industries and applications where precise bending and

shaping of the material are imperative. The insights derived from this figure underscore the need to factor in material orientation when determining bending parameters, ensuring desired outcomes and minimizing potential material wastage or defects.

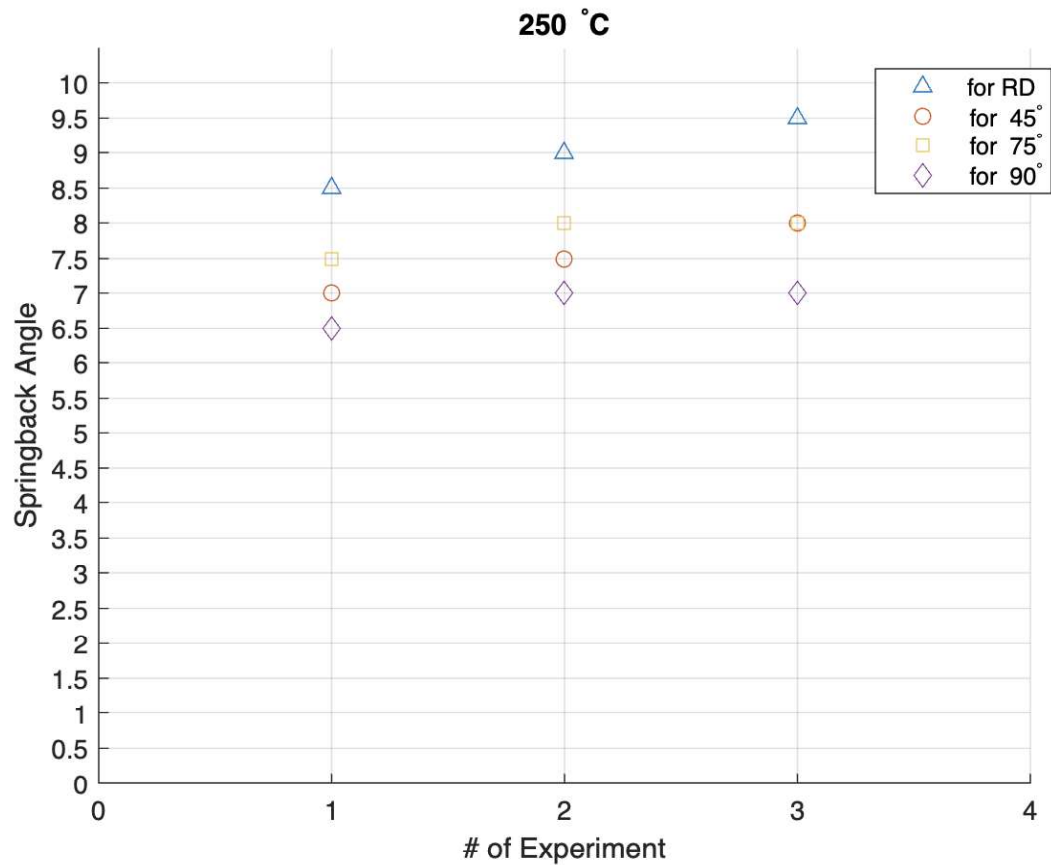


Figure 5.9 V-bending results for AZ31 at 250 °C

5.2.2. Springback Angle 300 C

Showcasing the V-bending outcomes at a higher temperature of 300°C, this figure provides insights into the enhanced formability of the AZ31 alloy. The orientations remain consistent on the x-axis, but the y-axis reveals a considerable decrease in values compared to those at 250 °C. The RD orientation ranges from 2 to 3.5, indicating increased ductility and decreased springback at elevated temperatures. Figure 5.10 showcases the V-bending results of the AZ31 magnesium alloy subjected to a temperature of 300 °C. On the x-axis, the data is segmented by the specific orientations: RD, 45°, 75°, and 90°. The y-axis, meanwhile, quantitatively displays the

bending values, offering a comparative perspective on the alloy's formability at this elevated temperature vs. its behavior at 250 °C.

A standout feature in this graph is the conspicuous shift in bending values, especially for the RD orientation. With values captured between 2 and 3.5, there's a significant decrease compared to the results at 250 °C. This stark reduction is indicative of enhanced ductility in the material, suggesting that at 300 °C, the AZ31 alloy becomes more pliable, allowing for more efficient bending. This transformation also hints at a decreased springback, which can be attributed to potential microstructural changes within the alloy at this temperature. Such changes often enhance the metal's ability to retain its deformed shape post-bending, reducing the propensity for it to revert to its original form. Further reinforcing the profound impact of temperature on bending characteristics is the behavior observed in the 45° orientation. This orientation, too, manifests a marked drop in bending values when compared to those at 250 °C. This downward trajectory across both RD and 45° orientations underscores the pivotal role temperature plays in modulating the formability of the AZ31 alloy.

In essence, Figure 2 serves as a testament to the temperature-dependent malleability of the AZ31 magnesium alloy. The pronounced differences in bending values between the two temperatures reiterate the necessity of understanding and optimizing thermal conditions during bending processes. For industries and applications where precision bending is essential, such insights can be instrumental in tailoring processes to achieve the desired material characteristics and outcomes.

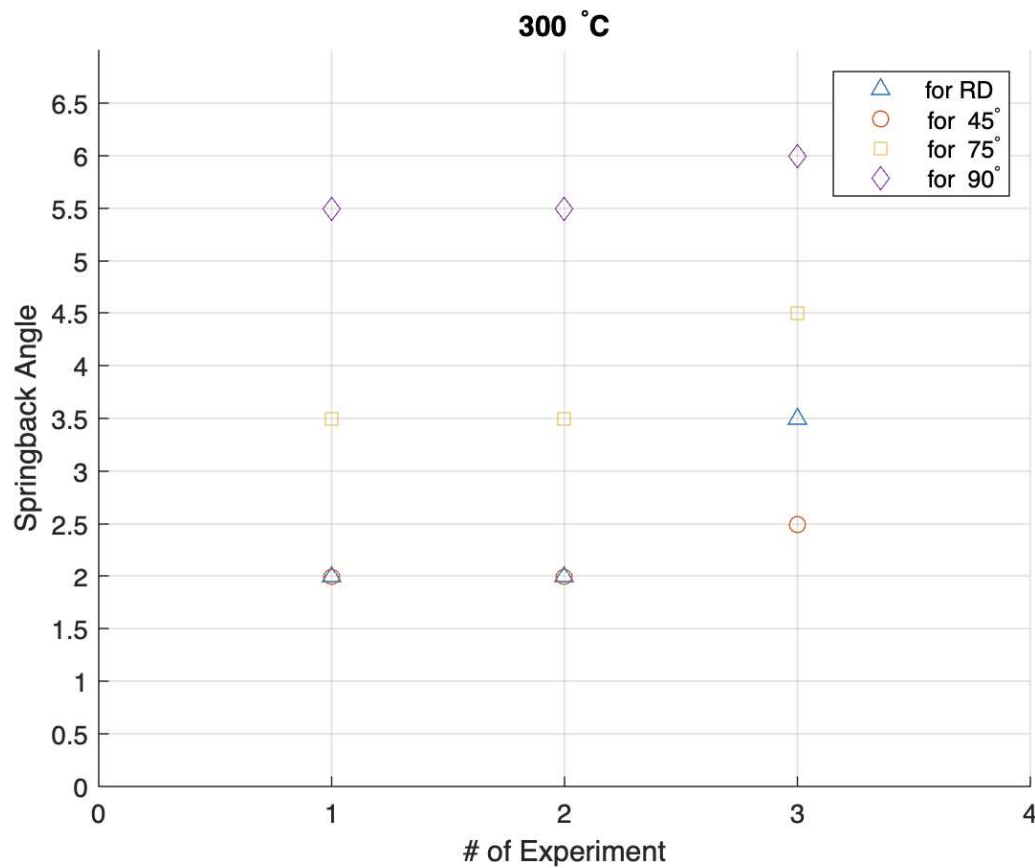


Figure 5.10 V-bending results for AZ31 at 300 °C

5.2.3. Springback Angle Experimental Results regarding Orientation (RD)

Focusing on the RD orientation, this figure juxtaposes the bending results at 250 °C and 300 °C. The noticeable drop in values from the range of 8.5-9.5 at 250 °C to 2-3.5 at 300 °C reinforces the substantial influence of temperature on the alloy's formability in this specific orientation. Figure 5.11 is dedicated to delving deeper into the V-bending performance of the AZ31 magnesium alloy, specifically focusing on the RD orientation. The x-axis demarcates the two temperatures under consideration, 250 °C and 300 °C, while the y-axis elucidates the bending values, thus facilitating a side-by-side comparison of the alloy's behavior under these two distinct thermal environments. A defining feature of this figure is the pronounced drop in bending values when transitioning from 250 to 300°C. This sharp decline serves as a graphic testament to the temperature's overarching influence on the formability characteristics of AZ31 in the RD orientation. Given that the RD orientation typically relates to the direction of

grain alignment due to the rolling process, this orientation is often a critical indicator of a material's inherent mechanical properties. The significant change in bending values between the two temperatures could be attributed to underlying microstructural transformations within the alloy. As temperature rises, phenomena like grain boundary sliding, dynamic recrystallization, and possible phase transformations become more pronounced. These mechanisms can impart enhanced ductility to the material, facilitating its deformation during bending. Moreover, the potential decrease in springback tendencies at the higher temperature suggests a shift in the balance between elastic and plastic deformation, favoring the latter.

In summarizing, Figure 3 accentuates the intricate interplay between temperature and material orientation, specifically within the context of the RD orientation. It brings to the fore the crucial consideration of temperature in shaping the bending outcomes, providing valuable insights for industries where the RD orientation is of paramount significance. The figure underscores the importance of a thorough understanding of the material's behavior under varied thermal conditions, paving the way for more informed processing decisions.

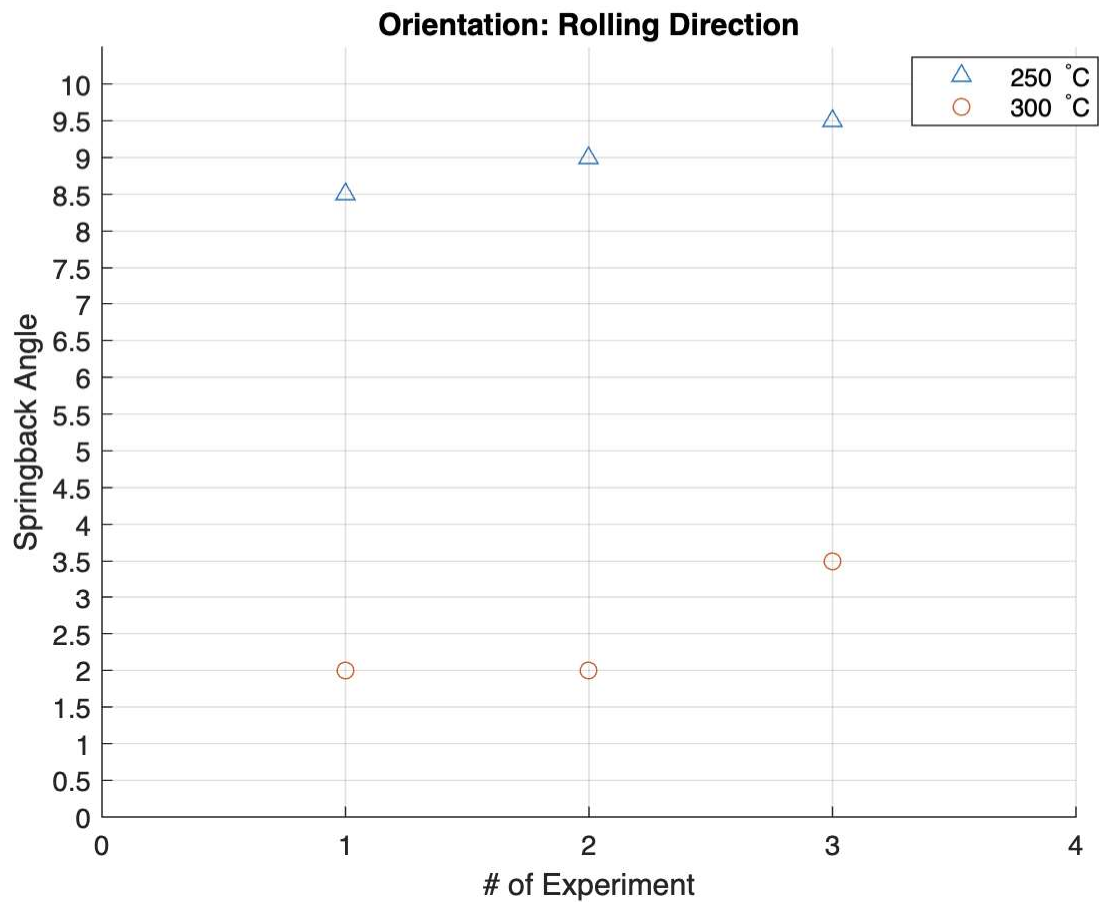


Figure 5.11 Comparative analysis of RD orientation across temperatures

5.2.4. Springback Angle Experimental Results regarding Orientation (°45)

Figure 5.12 centers on the 45° orientation, drawing comparisons between the results at 250 °C and 300 °C. The graph indicates a decline in values from the 250 °C range of 7-8 to the 300 °C range of 2-2.5, further underscoring the temperature's role in enhancing the alloy's formability and ductility. With the x-axis indicating the two examination temperatures, 250 °C and 300 °C, the y-axis enumerates the bending values, offering an elucidated juxtaposition of the alloy's behavior at these specific temperatures within this orientation. A salient observation emanating from this chart is the decline in bending values when transitioning from 250 °C, where values hover between 7 and 8, to 300 °C, where they settle between 2 and 2.5. While this drop is significant, it is not as precipitous as the shift witnessed in the RD orientation. This more modest reduction suggests that the 45° orientation, although benefiting from the elevated temperature in terms of enhanced formability, doesn't exhibit as pronounced

a temperature sensitivity as its RD counterpart. The reason behind this moderated sensitivity could be manifold. It's plausible that the grains in the 45° orientation, due to their specific alignment relative to the rolling direction, interact differently under thermal stress compared to those in the RD orientation. Such grain alignments could potentially mitigate some of the more pronounced microstructural transformations, thereby resulting in a more moderate enhancement in ductility at the increased temperature. Additionally, the nature of grain boundary activities might vary in the 45° orientation, leading to a different balance between mechanisms like grain boundary sliding and dynamic recrystallization when exposed to elevated temperatures. This can further influence the material's bending properties and its response to temperature changes.

In summation, Figure 5.12 elucidates the nuanced relationship between temperature and material orientation, specifically within the context of the 45° orientation. While the benefits of increased temperature are evident in enhanced formability, the subtler shifts in bending values in this orientation compared to the RD orientation underscore the intricacies of grain alignment and their responses to thermal variations. Such insights are invaluable for industries that seek to optimize bending operations while considering orientation-specific nuances.

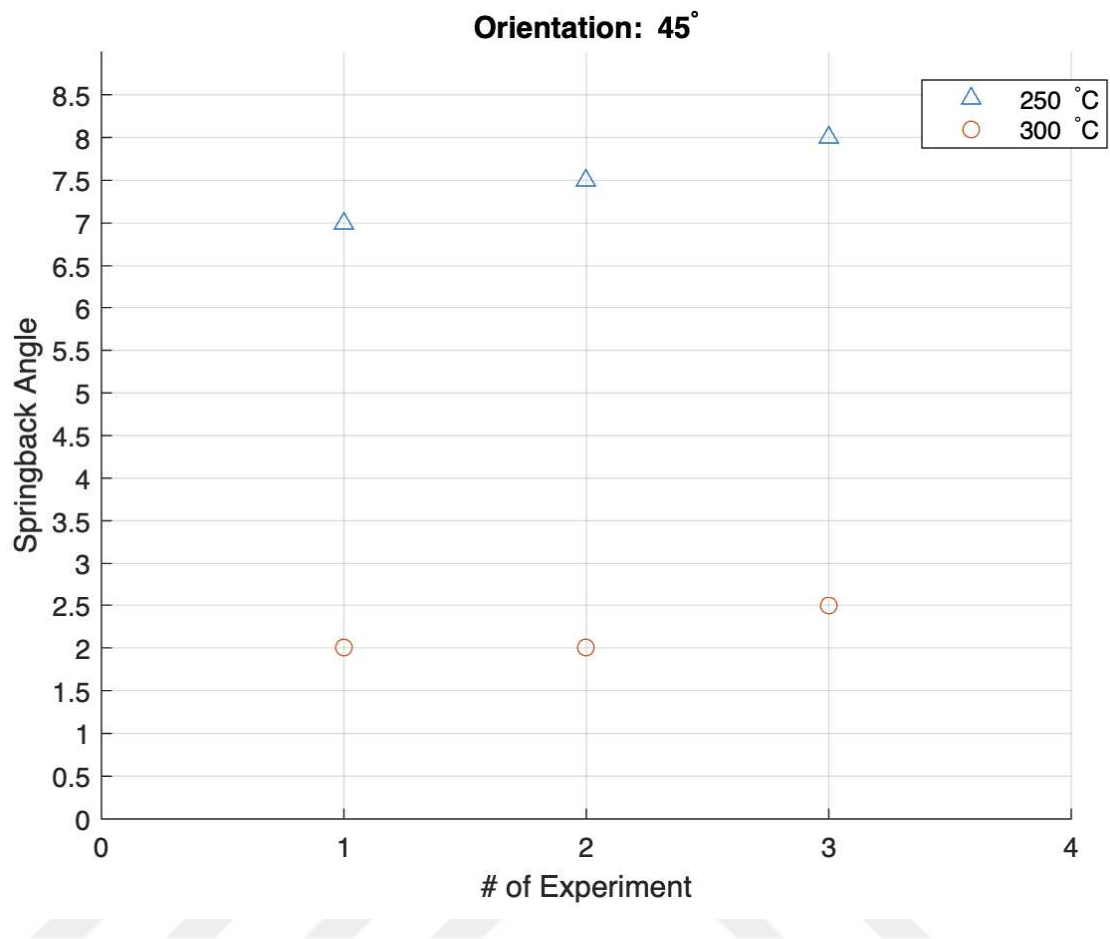


Figure 5.12 Comparative analysis of 45° orientation across temperatures

5.2.5. Springback Angle Experimental Results regarding Orientation (°75)

Combining the data for both 75° and 90° orientations, this figure provides a comparative look across the two temperatures. While the 75° orientation demonstrates a shift from 7.5-8 at 250°C to 3.5-4.5 at 300 °C, the 90° orientation showcases a transition from 6.5-7 to 5.5-6 across the same temperature gradient. The comparative view highlights the nuanced differences between these two orientations in response to temperature changes. Figure 5.13 is tailored to provide a comparative overview of the V-bending performance of the AZ31 magnesium alloy, zeroing in on the 75° and 90° orientations relative to the rolling direction. The x-axis represents the two benchmarked temperatures: 250 °C and 300°C, and the y-axis plots the corresponding bending values, rendering a vivid visual representation of the alloy's bending behavior within these orientations across the two temperatures.

A distinct trend emerges upon analyzing the data: the 75° orientation experiences a noticeable shift in bending values, transitioning from a range of 7.5-8 at 250°C to 3.5-4.5 at 300 °C. This transformation signifies a marked enhancement in formability, hinting at increased ductility at the elevated temperature. On the other hand, the 90° orientation presents a more subdued change, with bending values moving from the 6.5-7 bracket at 250 °C to a slightly improved range of 5.5-6 at 300 °C. What is particularly intriguing is the difference in temperature sensitivity between the two orientations. The 75° orientation exhibits a more pronounced response to the elevated temperature compared to the 90° orientation. This differential behavior suggests that the grains' alignment in the 75° orientation could be positioned in such a way that they undergo more significant microstructural changes or grain boundary activities when subjected to thermal stress. Such changes could involve mechanisms like dynamic recrystallization, grain boundary migration, or even phase transformations, which are known to contribute to improved ductility and reduced springback. Conversely, the grains in the 90° orientation, while still benefiting from the elevated temperature, might be oriented or interconnected in a manner that results in a less pronounced response to thermal variations. The more muted change in this orientation underscores the interplay of grain orientation and temperature and their collective impact on material behavior.

In conclusion, Figure 5.13 showcases the intricate and multifaceted nature of material science. By comparing the 75° and 90° orientations, it accentuates the complexities of grain alignment, their respective interactions with temperature, and the resulting implications on bending properties. Such detailed insights pave the way for tailored processing approaches, ensuring optimal outcomes in industrial applications that necessitate precision bending.

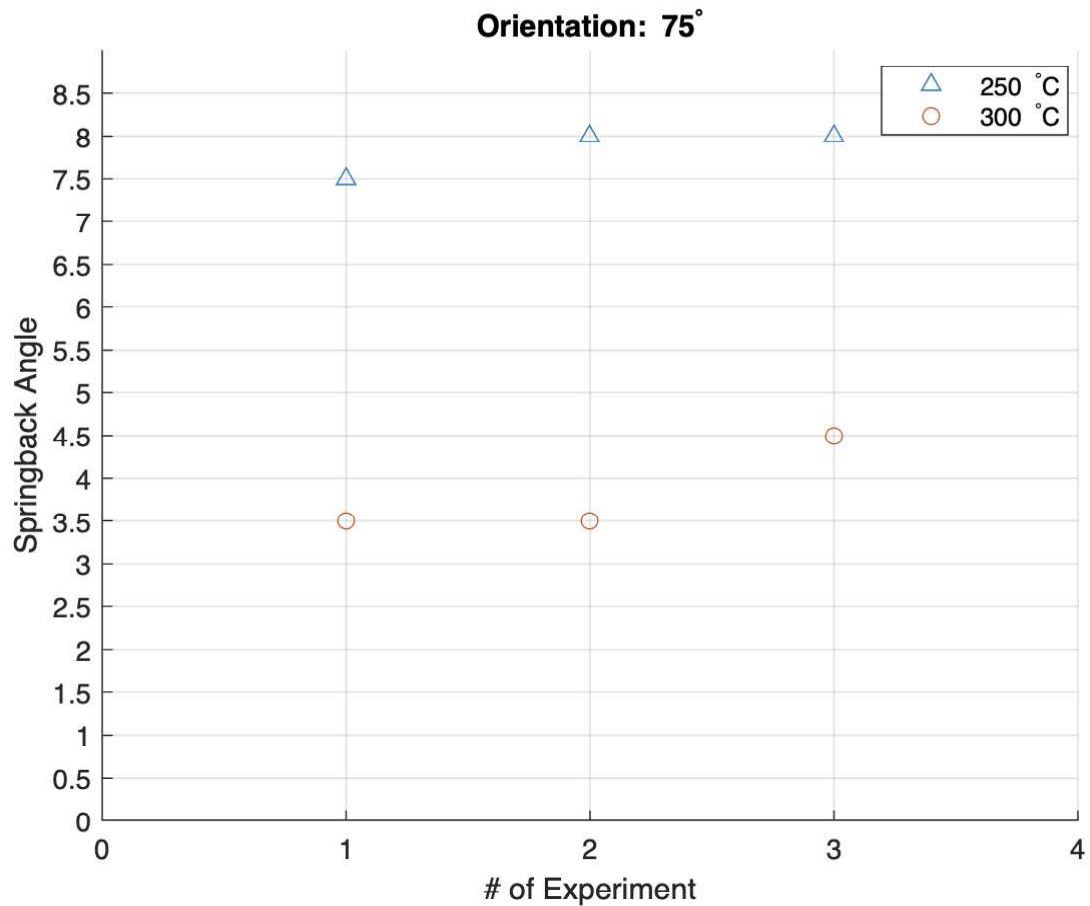


Figure 5.13 Comparative analysis of 75° orientation across temperatures

5.2.6. Springback Angle Experimental Results regarding Orientation (°90)

This figure synthesizes the V-bending results for all orientations (RD, 45°, 75°, and 90°) at the two studied temperatures (250 °C and 300 °C), offering a comprehensive visual summary of the bending behavior of the AZ31 alloy. The graph vividly illustrates the universally enhanced formability of the alloy at the higher temperature of 300 °C, a trend evident across all orientations. This enhancement suggests that the temperature plays a dominant role in influencing the alloy's ductility and reducing its springback tendencies.

However, upon closer inspection, one can discern nuanced variations in the degree of change between the orientations as the temperature rises. The RD orientation, for instance, demonstrates a more dramatic shift in formability compared to the subtler changes in the 90° orientation. This suggests that while temperature is a significant

factor, the intrinsic grain orientations within the material also play a vital role in dictating its bending behavior. Such variations can be attributed to the anisotropic nature of the alloy, where the alignment and orientation of grains can influence its mechanical properties and response to external stimuli, in this case, temperature.

The implications of these findings are manifold. For industries that rely on precise bending operations, understanding the combined effect of temperature and grain orientation is paramount. The figure underscores the importance of not just recognizing the overarching trend (i.e., improved formability at higher temperatures) but also delving deeper into the material's behavior across different orientations. This dual awareness can guide more informed decisions in selecting processing parameters, ensuring optimal bending outcomes while minimizing defects and wastage.

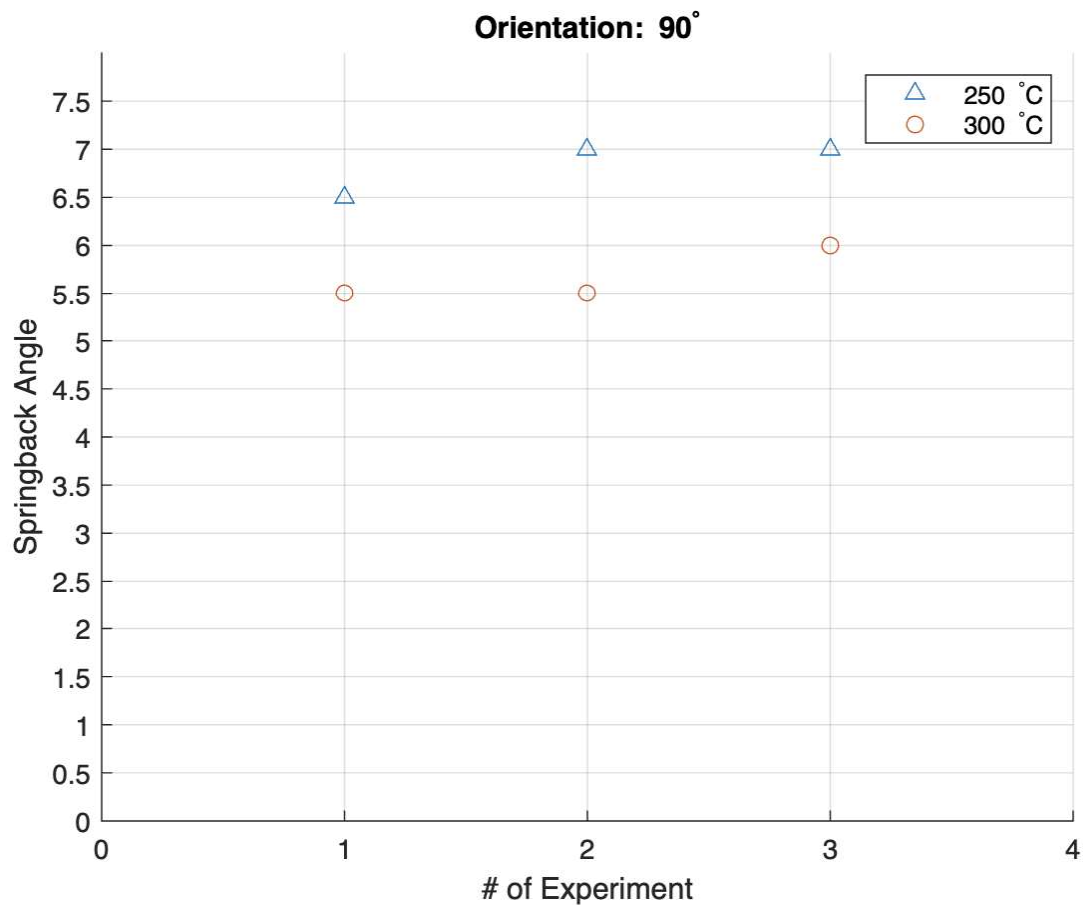


Figure 5.14 Comparative analysis of 90° orientation across temperatures

5.2.7. Discussion of V-bending Results of AZ31 at Elevated Temperatures

Magnesium alloys, especially AZ31, have garnered significant attention due to their remarkable strength-to-weight ratio. This makes them indispensable for a myriad of industrial applications. In this section, we scrutinize the V-bending tests that were methodically executed to ascertain the formability of AZ31 magnesium alloy sheets. We focus on the differential responses at 250 °C and 300 °C across various orientations: RD, 45°, 75°, and 90° orientations. Historical emphasis on the role of temperature in metalworking operations is deeply rooted in its undeniable influence on material properties. For AZ31, the V-bending results indicate a discernible disparity between the two temperatures studied. At 300 °C, the results (specifically 2 to 3.5 for RD, and 2 to 2.5 for 45°) are starkly lower than the respective results at 250 °C (8.5 to 9.5 for RD, and 7 to 8 for 45°). This remarkable difference underscores how

temperature can dramatically shift formability metrics. The lower values at the heightened temperature hint at better formability or reduced springback tendencies.

Such pronounced changes can be anchored to underlying microstructural dynamics. Elevated temperatures can induce transitions, such as grain coarsening or phase transformations, increasing the alloy's ductility. This augments the alloy's amenability to form without fracturing, echoing literature that correlates high temperatures with factors like grain boundary sliding and dynamic recrystallization, pivotal in enhancing material ductility. Metal anisotropy, often introduced during fabrication processes like rolling, inevitably dictates mechanical behavior. This phenomenon is palpable in the data at hand. The RD orientation, for instance, ranged from 8.5 to 9.5 at 250°C. In stark contrast, the 90° orientation manifested values like 6.5 to 7. This disparity, albeit subtle, intimates the role of grain orientations and their interactions during deformation.

Grain alignment plays a pivotal role, especially when confronted with forces. In bending, for instance, grains oriented favorably can more easily slide past one another, resulting in less resistance and better formability. Conversely, unfavorable grain orientations can hinder this movement, leading to heightened springback or even cracking. For industries prioritizing bending integrity, comprehending these subtleties is non-negotiable. The data here serves as a testament to the tangible impacts of grain orientation on formability outcomes. Despite the variations witnessed, the data showcased a commendable consistency within specific categories. However, a nuanced observation reveals that the RD orientation at 300 °C showed a slightly broader spread (from 2 to 3.5) compared to other orientations. This might hint at inconsistencies in material properties or potential experimental anomalies warranting further investigation. From an applicative perspective, these observations aren't merely academic. For instance, industries leveraging AZ31 for applications demanding specific formability standards might find it beneficial to operate at 300 °C, optimizing for the enhanced ductility observed. Furthermore, industries with stringent requirements on orientation would need to navigate the nuanced bending behaviors across orientations. The data serves as a guide, suggesting a potential need for caution, especially when bending at the 90° orientation at 250 °C.

The analytical journey through the V-bending results of AZ31 unravels a nuanced landscape of temperature and orientation-dependent behaviors. Beyond the numbers, this data underscores the intricate dance of microstructures under varying conditions. A deeper metallurgical dive can illuminate the undercurrents governing these behaviors, refining our understanding and refining processing methodologies.

5.3. Analysis of Biaxial Anisotropy and The Lankford Values

The investigation of the Lankford parameters across a range of temperatures, from RT to 300 °C, reveals significant insights into the material's plastic anisotropy and deformation characteristics. At RT, we observe the lowest plastic deformability, setting a baseline for how temperature influences anisotropy, as the transverse direction (TD) strain values are minimal. As temperatures rise to 50, 100, and 150 °C, the Lankford values exhibit notable increases, suggesting enhanced ductility possibly due to a reduction in yield strength and greater dislocation mobility. The transition between 200 to 250 °C marks a steeper increase in these values, hinting at a possible shift in deformation mechanisms, such as the activation of secondary slip systems which facilitate greater strain accommodation. At the peak temperature of 300 °C, the material's ductility reaches its zenith, favorable for high-deformation industrial processes like deep drawing.

The temperature-dependence of the material's behavior points towards an adaptable microstructure with increasing heat, likely attributed to mechanisms such as dislocation unpinning or recrystallization, which promote slip and grain boundary sliding. The data's consistency, as indicated by available R-squared values, speaks to a reliable trend of increasing plasticity with temperature. Nonetheless, a rigorous statistical analysis would further affirm these trends' precision. This understanding is pivotal for selecting materials and designing manufacturing processes; for instance, optimizing hot stamping procedures to augment material formability. Future research should delve into the microstructural evolution that parallels the observed macroscopic properties, and dynamic testing could yield further understanding of the material's behavior under real-world processing conditions. In summary, this thorough analysis underscores the critical role of temperature in the material's deformation abilities,

necessitating careful thermal management in metal forming processes. The predictive capabilities of various yield surface models for HCP materials, particularly AZ31 magnesium alloy, have been evaluated in this study. Through a detailed analysis, it is apparent that the yld2004-18p, the Bezier5YS-2023, and the SHYqp-2023 models demonstrate superior performance compared to other yield criteria in the prediction of the anisotropic behavior of HCP materials, especially at high temperatures. The interpretation of the data using these yield surface models is discussed in the subsequent sections.

The stress-strain behavior, as depicted by the flow curves for each model, showcases that the Yld2004-18p and the Bezier5YS-2023 models provide an exceptional fit to the experimental data. This is crucial for accurately predicting the onset of plastic deformation and the subsequent flow of the material under applied stress. Notably, at elevated temperatures, these models accommodate the increased activity of non-basal slip systems in HCP materials, which is a key characteristic that is often neglected by traditional yield criteria.

The directional dependency of mechanical properties in HCP materials is a complex phenomenon captured by the variation in Lankford coefficients. The models under consideration demonstrate their capability to predict these coefficients with high fidelity. For instance, the SHYqp-2023 model's predictions of the Lankford coefficients (r_θ) and ($\bar{r}_\theta$) align closely with the experimental values across different directions with respect to the rolling direction. This alignment indicates the model's capacity to handle the material's plastic anisotropy effectively.

The yield surface contour plots serve as a visual validation of the models' effectiveness. The Yld2004-18p model, as illustrated in the provided images, captures the characteristic shape of the yield surface for HCP materials, which is notably different from isotropic materials. Similarly, the Bezier5YS-2023 model's contours reveal its sensitivity to the unique yielding behavior of the AZ31 alloy at various strain rates and temperatures. In the case of SHYqp-2023, the model's contours at 300 °C indicate its robustness in representing the yield surface at a high-temperature regime where additional slip systems may become active, affecting the yield and hardening behavior of the material. The accurate depiction of the yield surface at such a critical

temperature range underscores the model's applicability in high-temperature applications.

When compared to other models tested, the Yld2004-18p, the Bezier5YS-2023, and the SHYqp-2023 models stand out distinctly for their nuanced interpretation of the anisotropic features inherent to HCP materials. Other models, which may assume isotropy or have limited capacity to accommodate anisotropy, fail to provide an accurate representation of the mechanical response, particularly at elevated temperatures. The high-temperature behavior of HCP materials is challenging to predict due to the activation of additional deformation mechanisms. However, the aforementioned models excel in this aspect by incorporating complex yield criteria that account for the temperature-dependent behavior of slip and twinning systems in HCP crystals.

In conclusion, the data interpreted through the Yld2004-18p, the Bezier5YS-2023, and the SHYqp-2023 models provides a comprehensive understanding of the anisotropic behavior of HCP materials at high temperatures. These models are instrumental in designing and simulating components made from such materials, where the accurate prediction of mechanical response is critical. Their ability to predict the yield surface accurately, consider the Lankford coefficient's directionality, and capture the stress-strain behavior across different temperatures and strain rates establishes them as robust tools for materials engineering and simulations in the realm of HCP materials.

5.4. Discussion of findings in the context of the literature

The interplay of elevated temperatures and the resultant mechanical behaviors of AZ31 magnesium alloy, particularly concerning springback and formability, is a focal point of contemporary materials science research. Scholarly contributions in this domain are multifaceted, reflecting a consensus on the beneficial effects of temperature on material properties. Nguyen et al. [167] have elucidated the inverse relationship between springback angles and temperature in AZ31; as the latter increases, the former correspondingly diminishes. Similarly, Toros and colleagues [168] have posited that the formability of magnesium alloys, particularly in stretching, is enhanced at elevated temperatures. Dessolier and associates [169] have contributed to this dialogue by

demonstrating that, at temperatures as high as 300 °C, the yield strength of the material decreases post tensile testing, which is similar to our study in context of results. Recent investigations further elucidate this phenomenon. Lee et al. [170] explored the reduction of springback in AZ31 through V-bending experiments with near-infrared radiation-induced localized heating, affirming that such heating techniques can proportionately diminish springback. Araripe and peers [171] adopted a computational lens, utilizing Finite Element Analysis to scrutinize the deep drawing process of AZ31 and the interplay of various factors such as blank holder force and friction conditions on springback. Roodposhti and Murty [172] delved into the microstructural dynamics of AZ31 at elevated temperatures, particularly focusing on the creep behavior and the prominent role of grain boundary sliding. These cumulative insights signal that heightened temperatures significantly ameliorate the formability of AZ31 magnesium alloy and mitigate its propensity for springback. Such findings are pivotal for manufacturing industries where the lightweight nature of materials like AZ31 is of essence. Focused heating strategies and computational modeling have emerged as potent techniques for refining forming processes of this metal. Continuing with the theme of mechanical behaviors under high-temperature conditions, Li et al. [173] presented evidence that high-temperature pre-stretching leads to improved formability in AZ31, attributed to basal texture attenuation and activation of non-basal slip systems. Research by Kara and Demirtaş [174] corroborated these findings, showing that Ca and Ce additives bolster both strength and ductility at high temperatures due to texture modification and secondary phase development. Bhowmick et al. [175] directed their study towards the wear resistance of AZ31 at raised temperatures, indirectly suggesting implications for springback through the potential influence of surface treatments like Plasma Electrolytic Oxidation. Lastly, Ishiguro et al. [176] observed how bending-tension deformation at high temperatures impacts texture evolution in AZ31 sheets, affecting stretch formability and, consequently, springback behavior.

Collectively, these scholarly endeavors are critical in advancing our understanding of the high-temperature mechanical properties of AZ31, pivotal for its deployment in sectors where thermal performance is a key concern.

5.5. Implications of the Study Results for Industrial Applications

The findings presented in this thesis have profound implications for a wide range of industrial applications, with a particular emphasis on AZ31 magnesium alloy and similar anisotropic materials. These implications extend to several key areas:

1. **Advanced Manufacturing Processes:** Manufacturers in industries such as aerospace, automotive, and electronics stand to benefit from the insights derived from this research. The understanding of AZ31's anisotropic behavior during V-bending, especially under varying orientations and temperatures, enables informed decision-making regarding material orientation and processing conditions. This knowledge contributes to the optimization of manufacturing processes, resulting in increased efficiency and cost-effectiveness.
2. **Enhanced Product Design:** Engineers and product designers can leverage the knowledge gained from this study to create products that maximize the strengths of AZ31 magnesium alloy while mitigating its weaknesses. An in-depth understanding of how anisotropy and temperature affect the material's mechanical properties informs the design of components and structures tailored to specific applications. This ensures optimal performance and reliability in end products.
3. **Material Selection and Tailoring:** The research outcomes play a crucial role in the selection of AZ31 magnesium alloy for specific applications. By ensuring that the alloy's anisotropic properties align with the intended use, this research facilitates the development of lighter, more durable products with enhanced performance characteristics.
4. **Advancements in Materials Science:** Beyond its direct industrial applications, this study contributes to the broader field of materials science. The examination of anisotropy and biaxial behavior in AZ31 enhances our understanding of magnesium alloys and their potential for innovation. These insights lay the foundation for the development of new materials and alloys with tailored properties, driving progress in materials science.

5. Significance of High-Temperature Ductility: Notably, the study results reveal significant ductility in the Lankford parameters at elevated temperatures. This observation holds profound implications for industrial applications requiring materials to operate under such conditions. The enhanced ductility in AZ31 magnesium alloy, as indicated by its Lankford parameters, opens up new possibilities for applications where high-temperature performance and formability are critical. This is especially relevant in sectors such as aerospace and automotive, where materials must withstand extreme thermal conditions.

In summary, the implications of this research extend well beyond the confines of the laboratory. The knowledge acquired has the potential to shape industrial practices, enhance product design, and advance the field of materials science. By delving into the anisotropic behavior of AZ31 magnesium alloy and its response to various loading conditions, this study serves as a valuable resource for industries seeking to harness the unique properties of this material for improved product performance and innovation, particularly in high-temperature environments where significant ductility becomes a valuable asset.

5.6. Limitations of the Study

In Chapter 1, the scope and limitations of this research were outlined, focusing on the experimental investigation of AZ31 magnesium alloy through V-bending and disc compression tests. These experiments were conducted to assess the anisotropy, springback angles, the Lankford values, and yield strengths under various orientations and temperature conditions. As we discuss the findings from the yield surface models, it is imperative to revisit these limitations to contextualize our results and outline the path forward.

One of the primary limitations is the assumption of material homogeneity within the AZ31 specimens. Although standard industry practices were adhered to in order to ensure uniformity, material variations inherent to the manufacturing process could have influenced the experimental outcomes. Such variations can lead to local differences in mechanical properties, which are not captured by the yield models.

Furthermore, the controlled loading conditions, designed to simulate idealized scenarios, may not fully represent the complex multi-axial stress states that occur in practical applications. The yield surface models, while sophisticated, are based on data from these simplified conditions and may not encapsulate the nuances of real-world stresses.

Conducting this research at a laboratory scale provides a controlled environment to study the material behavior; however, it also serves as a limitation since the scale and constraints of a laboratory setup may not capture all external factors that influence material properties in industrial or natural settings.

The specificity of the study to AZ31 alloy is another limitation. Though the findings for this particular magnesium alloy are informative, they cannot be indiscriminately generalized to other magnesium alloys, which may exhibit different mechanical and thermal behaviors.

Additionally, the experimental work focused on a subset of orientations (0° , 45° , 75° , and 90°) and two high temperatures (250°C and 300°C). To fully characterize the anisotropic behavior of the AZ31 alloy, further experimental investigations are necessary, particularly for intermediate orientations such as 15° , 22.5° , 67.5° , etc., at elevated temperatures. These additional data points could significantly enhance the predictive accuracy of yield surface models for HCP materials under a broader range of operational conditions.

In conclusion, while these limitations delineate the boundary conditions of our study, they also provide direction for future research. To address the limitations mentioned above, subsequent studies could explore a more diverse set of experimental conditions, incorporate real-world loading scenarios, and expand the investigation to other magnesium alloys. This future work would not only reinforce the current understanding but also extend the applicability of the yield surface models developed in this research.

As this thesis progresses, these limitations should be considered as avenues for ongoing inquiry, underscoring the need for continued experimental and analytical efforts to advance the predictive modeling of HCP materials.

5.7. Suggestions for Future Research

While the research presented in this thesis has made significant strides in understanding the anisotropic behavior of AZ31 magnesium alloy under varying loading conditions, it is essential to recognize that this field of study is far from exhausted. The complex nature of anisotropy and its interaction with elevated temperatures and different strain rates pose intriguing questions and opportunities for further exploration. Firstly, the experimental investigation of the Lankford coefficients at elevated temperatures, such as (r_{15}) , $(r_{22.5})$, and $(r_{67.5})$, remains an area ripe for exploration. Elevated temperatures can significantly alter the mechanical behavior of materials, and understanding how AZ31 magnesium alloy responds under such conditions is crucial for various high-temperature applications. Secondly, AZ31 magnesium alloy, despite its potential, remains relatively underexplored in the realm of materials science. Future research should aim to comprehensively study its mechanical behavior, anisotropic features, and response to diverse loading conditions. This could involve more extensive experimentation, including additional orientations and temperatures. Thirdly, the influence of strain rates, particularly at elevated temperatures, on the anisotropic behavior of AZ31 magnesium alloy requires further empirical examination. The interaction between strain rates and temperature is a critical consideration for applications in which materials experience rapid deformation, such as in high-velocity impact scenarios. Lastly, while models like the YLD2004 and the SHYQP show promise in predicting anisotropic behavior at high temperatures, the development of more refined and accurate models is essential. Older models like the Hill-48 and the Barlat-89 have demonstrated limitations in predicting the complex anisotropic states observed in AZ31. Future research should focus on refining and developing predictive models that encompass the full spectrum of anisotropic behavior, especially at elevated temperatures.

These suggestions for future research endeavors underscore the evolving nature of materials science and the continual pursuit of deeper insights into the behavior of AZ31 magnesium alloy. Addressing these research avenues will not only contribute to our understanding of this versatile material but also pave the way for its enhanced utilization in a wide range of industrial applications. As we advance our knowledge, we can better harness the unique properties of AZ31 to optimize manufacturing processes, improve product design, and drive innovation in materials science and engineering.



CHAPTER 6

CONCLUSION

6.1. Summary of Key Findings

The comprehensive exploration of the mechanical properties of AZ31, an HCP material, at elevated temperatures has yielded several key findings, significantly enhancing the understanding of its deformation behavior. As the temperature increases, it is observed that the ductility of the material decreases, contrary to the general trend for metals where ductility usually increases with temperature. This anomaly is particularly evident in the disc compression tests, where the Lankford values, indicative of material anisotropy, exhibit an increasing trend. The enlargement of these values points to a pronounced directionality in the material's plastic behavior at high temperatures, suggesting that the common assumption of ductility enhancement with temperature may not universally apply to HCP materials.

Further, simulation results employing the YLD2004-18p and the newly introduced the SHYQP-2023 Yield Criterion demonstrate superior predictive capabilities for the behavior of HCP materials at elevated temperatures when compared to traditional yield criteria. These advanced models encapsulate the complex anisotropic yield behavior of HCP materials, enabling more accurate and reliable predictions of their performance under various thermal and mechanical loading conditions.

Additionally, V-bending experiments have contributed to the understanding of springback phenomena in AZ31. As temperature increases, there is a general decrease in springback angles, indicating a reduction in elastic recovery post-deformation. This is aligned with the expected decrease in yield strength, as evidenced by the yield strength vs. temperature graph. However, an exception to this trend is observed at a 90-degree orientation to the rolling direction, where springback values are higher. This implies that the orientation of grains relative to the bending direction plays a significant role in the springback behavior of HCP materials, possibly due to the activation of different slip or twinning systems.

These findings present crucial implications for the forming processes of AZ31 and similar HCP materials, particularly at elevated temperatures. The distinct trends and behaviors identified necessitate a nuanced approach to the manufacturing and simulation of these materials, taking into account their unique response to temperature and loading conditions. The incorporation of advanced yield criteria like the YLD2004-18p and the SHYQP-2023 into the simulation frameworks is recommended for a more accurate representation of the material behavior, ultimately leading to the optimization of forming processes and the enhancement of material performance in practical applications.

6.2. Restatement of Research Objectives and Hypotheses

This thesis embarked on a detailed investigation of the anisotropic behavior of AZ31, a magnesium alloy with a hexagonal close-packed (HCP) structure, under various thermal and mechanical loading conditions, which is explained in detail in Chapter 1. The underlying motivation for this research was to unravel the complex interplay between the intrinsic properties of the material and the imposed external conditions during processes such as V-bending and disc compression tests. Within this framework, we formulated hypotheses that sought to predict the behavior of AZ31, based on its anisotropic nature and response to temperature changes. Our comprehensive experimental and simulation-based research has generated findings that align with the proposed hypotheses, thus validating our initial conjectures.

6.2.1. Hypothesis 1: Influence of Anisotropy on V-Bending Behavior

Our results affirm Hypothesis 1, which predicted that the anisotropic nature of AZ31 would significantly influence its mechanical behavior during V-bending processes. The variations in springback angles, which are orientation and temperature-dependent, are indeed observable and align with the hypothesized behavior. These findings underscore the importance of considering the material's anisotropy when predicting its behavior during deformation processes.

6.2.2. Hypothesis 2: Distinct Patterns in Biaxial Anisotropy and Lankford Values

The data obtained from disc compression tests validate Hypothesis 2, which suggested that AZ31 would exhibit discernible patterns in biaxial anisotropy and Lankford values under various loading conditions. The patterns observed in the Lankford values, which reflect the material's anisotropy, have been confirmed by our experimental results, showcasing the material's complex response under biaxial stress states.

6.2.3. Hypothesis 3: Orientation-Dependent Variations in Yield Strengths

Finally, Hypothesis 3 has been substantiated through the evidence that AZ31 demonstrates orientation-dependent variations in yield strengths, particularly under different loading conditions that were meticulously investigated through compression testing. The yield strengths varied with orientation, which is attributed to the different slip systems activated in the HCP structure of AZ31. The role of temperature in influencing these yield strengths was also clearly demonstrated, providing a more nuanced understanding of the material's behavior at elevated temperatures.

In conclusion, the alignment of our results with the proposed hypotheses not only confirms the initial assumptions made about the anisotropic behavior of AZ31 but also extends our understanding of the material's performance. This knowledge is crucial for the advancement of forming processes and the prediction of mechanical responses of HCP materials under operational conditions that mimic real-world manufacturing scenarios.

6.3. Contributions to the Field of Mechanical and Materials Science and Engineering

The research conducted within this thesis offers several notable contributions to the field of mechanical and materials science engineering. By meticulously examining the anisotropic behavior of AZ31 and its response to thermal conditions, this work has yielded insights that could significantly enhance the precision of metal-forming

processes and the prediction of material behavior under operational stresses. Below are the key contributions of this research:

- **Anisotropy and V-bending Process:** The work has contributed to a more profound understanding of how anisotropy impacts the V-bending process of AZ31. By providing empirical evidence of the variations in springback angles influenced by material orientation and temperature, this research helps in tailoring bending processes for better control and accuracy in the manufacturing of components from magnesium alloys.
- **Biaxial Anisotropy and the Lankford Values:** This thesis has mapped out the biaxial anisotropy and the associated Lankford values in AZ31, which have practical implications for industries where precise control over material deformation is critical. The patterns unveiled in these values serve as a guide for predicting material behavior in complex loading scenarios, which is a step forward in the material design and testing protocols.
- **Orientation-Dependent Yield Strengths:** By establishing a clear link between orientation and yield strength in AZ31, particularly under elevated temperature conditions, the research provides a nuanced view of the yield behavior in HCP materials. This is particularly important for the design of components subjected to multi-axial stresses, ensuring that material selection and component design are optimized for intended applications.
- **Simulation and Modeling Advances:** The validation of the YLD2004-18p and the Shyqp-2023 models for predicting the behavior of HCP materials at elevated temperatures marks a significant leap in simulation capabilities. These models can now be utilized with greater confidence in the design and analysis of components made from magnesium alloys and potentially other HCP materials.
- **Experimental Methodology and Testing:** The methodologies developed and employed in this thesis, particularly in V-bending and disc compression tests, provide a blueprint for future experimental setups aiming to study anisotropy in materials. The attention to detail in temperature control and orientation during these tests has set a new standard for experimental rigor.

- **Material Behavior at Elevated Temperatures:** The research has expanded the knowledge base on how temperature influences the ductility and mechanical properties of AZ31. Such information is invaluable for high-temperature applications of magnesium alloys and can inform the development of new materials with improved high-temperature performance.

In essence, the research encapsulated in this thesis enriches the existing knowledge on magnesium alloys, offering a comprehensive set of data and insights that can be utilized by engineers, designers, and researchers. The findings have implications for material selection, component design, manufacturing processes, and predictive modeling, all of which are pivotal elements within the broader field of mechanical and materials science engineering.

6.4. Practical Implications and Recommendations

In concluding this comprehensive study on the anisotropic behavior of AZ31 magnesium alloy under varying loading conditions, it is essential to underscore the practical implications derived from our findings and outline pertinent recommendations for future research endeavors.

The practical implications of this research span diverse industrial sectors, as discussed in previous sections, and include advancements in manufacturing processes, product design, material selection, and contributions to the broader field of materials science. Importantly, the significant ductility observed in the Lankford parameters at high temperatures opens new avenues for industrial applications, particularly in extreme thermal environments.

In light of these implications, the following recommendations for future research are proposed:

- **Exploration of the Elevated Temperature Lankford Parameters:** Future research efforts should focus on the experimental investigation of the Lankford parameters at elevated temperatures, including intermediate orientations such

as (r_{15}) , $(r_{22.5})$, and $(r_{67.5})$. These data points are essential for a more comprehensive understanding of AZ31's behavior under high-temperature conditions.

- **Comprehensive Study of AZ31:** AZ31 magnesium alloy, although promising, remains relatively underexplored in the realm of materials science. Future research should aim for a more extensive examination of its mechanical behavior, encompassing additional orientations and temperatures to provide a more complete picture.
- **Investigation of Strain Rate Effects at Elevated Temperatures:** The influence of strain rates, particularly at elevated temperatures, on the anisotropic behavior of AZ31 magnesium alloy requires further empirical investigation. Understanding the interaction between strain rates and temperature is crucial, especially in applications involving rapid deformation.
- **Refinement of Anisotropic Models:** While models like the YLD2004 and the SHYQP show promise in predicting anisotropic behavior at high temperatures, the development of more refined and accurate models is essential. Older models like the Hill-48 and the Barlat-89 have demonstrated limitations in predicting the complex anisotropic states observed in AZ31. Future research should focus on refining and developing predictive models that encompass the full spectrum of anisotropic behavior, especially at elevated temperatures.

These recommendations highlight the evolving nature of materials science and the ongoing quest for a deeper understanding of AZ31 magnesium alloy and similar materials. Addressing these research avenues will not only enhance our knowledge but also contribute to the practical utilization of these materials across various industrial applications. As this research progresses, it underscores the need for continued experimental and analytical efforts to advance predictive modeling, especially in the context of anisotropy, in materials science and engineering.

FUTURE WORKS

In pursuit of advancing our understanding of AZ31 magnesium alloy, a myriad of future research avenues have been delineated based on the comprehensive findings of this thesis. These include delving deeper into the high-temperature deformation mechanisms, analyzing the effects of thermal cycling on anisotropy, examining strain path dependency, and assessing the corrosion resistance of AZ31 under elevated temperatures. Scaling up experimental results to meet industrial processing demands, investigating the fatigue behavior in the context of anisotropy, and developing more precise yield criteria are also critical. Furthermore, integrating thermo-mechanical effects into material modeling, understanding the impact of alloying elements, exploring the potentials of additive manufacturing, and addressing environmental sustainability in manufacturing processes are imperative. An interdisciplinary approach that synergizes metallurgy, computational materials science, and mechanical engineering will be quintessential. This multifaceted research endeavor is not merely an academic pursuit but a strategic initiative aimed at optimizing AZ31's industrial applications, promising significant contributions to technological advancements.

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APPENDICES

Appendix A: MATLAB and PYTHON Codes

Springback Angles (250C)

```
% Data
x = 1:3; % Assuming x-values are 1, 2, and 3 for all the lines
% RD
y1 = [8.5, 9, 9.5];
% 45degrees
y2 = [7, 7.5, 8];
% 75degrees
y3 = [7.5, 8, 8];
% 90degree
y4 = [6.5, 7, 7];
% Plotting
figure;
plot(x, y1, '-o', 'DisplayName', 'RD'); hold on;
plot(x, y2, '-o', 'DisplayName', '45 Degrees');
plot(x, y3, '-o', 'DisplayName', '75 Degrees');
plot(x, y4, '-o', 'DisplayName', '90 Degrees');
xlabel('# of Experiment');
ylabel('Springback Angle');
legend();
title('Temperature: 250 C');
grid on;
% Modify x-axis labels
xticks([1 2 3]);
ylim([5 max([y1, y2, y3, y4])+1]); % This sets the y-axis to start at 0 and ends at the maximum y-value + 1 for a better visual
```

Springback Angles (300C)

```
% Data
```

```

x = 1:3; % Assuming x-values are 1, 2, and 3 for all the lines

% RD
y1 = [2, 2, 3.5];

% 45degrees
y2 = [2, 2, 2.5];

% 75degrees
y3 = [3.5, 3.5, 4.5];

% 90degree
y4 = [5.5, 5.5, 6];

% Plotting
figure;
plot(x, y1, '-o', 'DisplayName', 'RD'); hold on;
plot(x, y2, '-o', 'DisplayName', '45 Degrees');
plot(x, y3, '-o', 'DisplayName', '75 Degrees');
plot(x, y4, '-o', 'DisplayName', '90 Degrees');
xlabel('# of Experiment');
ylabel('Springback Angle');
legend();
title('Temperature: 300 C');
grid on;

% Modify x-axis labels
xticks([1 2 3]);

% Modify y-axis limits
ylim([0 max([y1, y2, y3, y4])+1]); % This sets the y-axis to start at 0 and ends at the maximum y-value + 1 for a better visual

```

Curve Fitting

```

% Given data
strain_in_td = [0.002970297, 0.005940594, 0.015841584, 0.016831683, 0.011881188, ...
               0.008910891, 0.013861386, 0.02970297, 0.031683168, 0.064356436, ...
               0.042574257, 0.086138614];

strain_in_rd = [0.004950495, 0.014851485, 0.02970297, 0.035643564, 0.025742574, ...
               0.011881188, 0.027722772, 0.056435644, 0.062376238, 0.085148515, ...
               0.066336634, 0.176237624];

% Create a fit type for a linear function
ft = fitype('poly1');

% Fit the data (inversing the roles of strain_in_td and strain_in_rd)

```

```
[fitted_curve, gof] = fit(strain_in_rd, strain_in_td, ft);

% Plotting

figure;

hold on;

plot(strain_in_rd, strain_in_td, 'ro', 'MarkerSize', 8); % Plotting data points with larger size

plot(fitted_curve, strain_in_rd, strain_in_td); % Plotting the fitted curve

xlabel('Strain in RD');

ylabel('Strain in TD');

title('Linear Curve Fitting with Inversed Axes');

grid on;

% Display the equation and R^2 on the figure at the top left

equation_str = sprintf('y = %.3f * x + %.3f\nR^2 = %.3f', fitted_curve.p1, fitted_curve.p2, gof.rsquare);

annotation('textbox', [0.65, 0.15, 0.8, 0.1], 'String', equation_str, ...
    'FitBoxToText', 'on', 'BackgroundColor', 'white', 'EdgeColor', 'none');
```

Plotting Lankford Curve Fitted Lines in One Graph

```
% Define x values

x = 0.25:0.01:2;

% Calculate y values based on given formulas

y1 = 0.5261 * x + 0.0014;

y2 = 0.6401 * x + 0.0096;

y3 = 0.8011 * x - 0.0091;

y4 = 0.9577 * x + 0.0257;

y5 = 1.0373 * x + 0.0251;

y6 = 1.2601 * x - 0.1412;

y7 = 1.4457 * x - 0.0139;

% Plotting

figure;

plot(x, y1, 'LineWidth', 1.5, 'DisplayName', 'Room Temperature'); hold on;

plot(x, y2, 'LineWidth', 1.5, 'DisplayName', '50C');

plot(x, y3, 'LineWidth', 1.5, 'DisplayName', '100C');

plot(x, y4, 'LineWidth', 1.5, 'DisplayName', '150C');

plot(x, y5, 'LineWidth', 1.5, 'DisplayName', '200C');

plot(x, y6, 'LineWidth', 1.5, 'DisplayName', '250C');

plot(x, y7, 'LineWidth', 1.5, 'DisplayName', '300C');

% Labeling and formatting
```

```

xlabel('Strain in RD');
ylabel('Strain in TD');
title('Lankford Values vs Temperature Relationship');
legend(); % Display legend based on 'DisplayName' properties
axis([0.2 1 0 1.6]); % Set axis limits
grid on;

```

von-Mises and Tresca Criteria

```

clc;
close all;
clear all;
% Define symbolic variables
syms S_1 S_2 real
% Given data
S = 1.71;
% von Mises criterion
sigma_eq_vonMises_0 = sqrt(0.5 * ((S_1 - S_2)^2 + S_2^2 + S_1^2));
F_vonMises_0 = sigma_eq_vonMises_0 - S;
% Tresca criterion for 0 degrees
sigma_eq_Tresca_0 = max([abs(S_1 - S_2), abs(S_2), abs(S_1)]);
F_Tresca_0 = sigma_eq_Tresca_0 - S;
figure;
hold on;
% Plotting for von Mises
fimplicit(F_vonMises_0, [-1.5, 1.5, -1.5, 1.5], 'Color', 'b', 'LineWidth', 1);
% Plotting for Tresca
fimplicit(F_Tresca_0, [-1.5, 1.5, -1.5, 1.5], 'Color', 'r', 'LineWidth', 1);
xlabel('\sigma_1')
ylabel('\sigma_2')
title('Yield surfaces for von Mises and Tresca')
legend('von Mises', 'Tresca')
hold off;

```

Hill-48 Criteria

```

clear all

syms S_1 S_2 S_12 S_23 real

% Material anisotropy constants

r_er = 0.3823; % 0o anisotropy
r_et = 0.4747; % 90o anisotropy
r_ed = 0.6983; % 45o anisotropy

% Hill-48 yield function

F = @(S_1, S_2, S_12) S_1.^2 - (2*r_er/(1+r_er)).*S_1.*S_2 + ...
    ((r_er*(1+r_et))/(r_et*(1+r_er))).*S_2.^2 + ...
    ((r_er+r_et)/(r_et*(1+r_et)))*(2*r_ed+1).*S_12.^2 - 1;

% Plot yield surfaces for different S_12 values

figure
hold on
for S_12_value = 0:0.1:0.5
    fimplicit(@(S_1, S_2) F(S_1, S_2, S_12_value), [-1.5, 1.5, -1.5, 1.5], ...
        'Color', 'm', 'LineWidth', 2);
end

xlabel('\Sigma_1', 'Interpreter', 'tex')
ylabel('\Sigma_2', 'Interpreter', 'tex')

title('\bf Hill-48 yield surface', 'Interpreter', 'tex')

axis square

% Constants for yield stress variation with orientation

A = (2*r_er/(1+r_er));
B = (r_er*(1+r_et))/(r_et*(1+r_er));
C = ((r_er+r_et)/(r_et*(1+r_et)))*(2*r_ed+1);

% Variation of yield stress with orientation (theta)

theta = linspace(0, 90, 1000);

Y_fi = sqrt(1 ./ (cosd(theta).^4 - A.*sind(theta).^2.*cosd(theta).^2 + ...
    B.*sind(theta).^4 + C.*sind(theta).^2.*cosd(theta).^2));

figure

plot(theta, Y_fi, 'm', 'LineWidth', 2)

xlabel('\theta (degrees)', 'Interpreter', 'tex')
ylabel('\Sigma', 'Interpreter', 'tex')

title('\bf Variation of yield stress with orientation', 'Interpreter', 'tex')

axis([0, 90, 0, max(Y_fi)*1.1])

% Define Y_fi as a symbolic function of fi

```

```

Y_fi_sym = sqrt(1 / (cosd(S_1)^4 - A*sind(S_1)^2*cosd(S_1)^2 + ...
    B*sind(S_1)^4 + C*sind(S_1)^2*cosd(S_1)^2));

% Anisotropy as a symbolic function of orientation 'fi'

r_fi_sym = (cosd(S_1)^4 + 1.5071530280781381025400052813424*cosd(S_1)^2*sind(S_1)^2 +
    (18598*sind(S_1)^4)/22473)/...

    ((2*Y_fi_sym*cosd(S_1)^2 - (204*Y_fi_sym*sind(S_1)^2)/227)/(2*(Y_fi_sym^2*cosd(S_1)^4 +
    (770394168075482481*Y_fi_sym^2*cosd(S_1)^2*sind(S_1)^2)/511158557706551296 +
    (18598*Y_fi_sym^2*sind(S_1)^4)/22473)^(1/2)) - ...

    ((204*Y_fi_sym*cosd(S_1)^2)/227 - (37196*Y_fi_sym*sind(S_1)^2)/22473)/(2*(Y_fi_sym^2*cosd(S_1)^4 +
    (770394168075482481*Y_fi_sym^2*cosd(S_1)^2*sind(S_1)^2)/511158557706551296 +
    (18598*Y_fi_sym^2*sind(S_1)^4)/22473)^(1/2))) - 1;

% Calculate the anisotropy over the range of 'fi'

fi_values = linspace(0, 90, 1000);

r_fi_values = double(subs(r_fi_sym, S_1, fi_values));

% Plot the variation of anisotropy with orientation

figure

plot(fi_values, r_fi_values, 'm', 'LineWidth', 2)

axis([0, 90, min(r_fi_values), max(r_fi_values)*1.1])

xlabel('\theta', 'Interpreter', 'tex')

ylabel('r', 'Interpreter', 'tex')

title('\bf Variation of anisotropy with orientation', 'Interpreter', 'tex')

% Anisotropy function P

P = sqrt(S_1.^2 - A*S_1*S_2 + B*S_2.^2 + C*S_23.^2);

d_1 = diff(P, S_1);

d_2 = diff(P, S_2);

% Calculate derivatives as a function of orientation

d_1_val = subs(d_1, {S_1, S_2}, {cosd(theta), sind(theta)});

d_2_val = subs(d_2, {S_1, S_2}, {cosd(theta), sind(theta)});

% Convert symbolic expressions to numeric for plotting or further calculations

d_1_numeric = double(subs(d_1_val, theta, fi_values));

d_2_numeric = double(subs(d_2_val, theta, fi_values));

% If you want to plot the derivatives, you can use the following commands:

% plot(fi_values, d_1_numeric, 'b', 'LineWidth', 2)

% plot(fi_values, d_2_numeric, 'r', 'LineWidth', 2)

```

Barlat-89 Criteria

```

% Clear command window, close all figures, and clear workspace

clc;

```

```

close all;

clear all;

% Define symbolic variables
syms S_1 S_2 S_12 S_23 K_1 K_2 S beta h m r_er r_et r_ed c a t p real

% Given data
r_er = 0.9;      % 0o anisotropy
r_et = 0.9;      % 90o anisotropy

m = 8;
p = 1.042;

% Evaluate functions
c = 2 * sqrt((r_er / (1 + r_er)) * (r_et / (1 + r_et)));
a = 2 - c;
h = sqrt((r_er / (1 + r_er)) * ((1 + r_et) / r_et));
T_1 = (S_1 + h * S_2) / 2;
T_2 = sqrt(((S_1 - h * S_2) / 2)^2 + p^2 * S_23^2);
R = (0.5 * (a * abs(T_1 + T_2)^m + a * abs(T_1 - T_2)^m + c * abs(2 * T_2)^m))^(1/m); % Effective Stress

% Plotting yield surfaces for different values of S_12
S = 1;
values_S_12 = [0, 0.1, 0.2, 0.3, 0.4, 0.5];
figure;
hold on;
for i = 1:length(values_S_12)
    S_12 = values_S_12(i);

    c = 2 * sqrt((r_er / (1 + r_er)) * (r_et / (1 + r_et)));
    a = 2 - c;
    h = sqrt((r_er / (1 + r_er)) * ((1 + r_et) / r_et));

    K_1 = (S_1 + h * S_2) / 2;
    K_2 = sqrt(((S_1 - h * S_2) / 2)^2 + p^2 * S_12^2);

    F = a * abs(K_1 + K_2)^m + a * abs(K_1 - K_2)^m + c * abs(2 * K_2)^m - 2 * S^m;

    % Plotting
    %fimplicit(F, [-1.5, 1.5, -1.5, 1.5], 'Color', 'm', 'LineWidth', 2);

    fimplicit(F, [-1.5, 1.5, -1.5, 1.5], 'Color', [0.1, 0.2, i/length(values_S_12)], 'LineWidth', 1.5);
end

```

```

xlabel('$$\sigma_{xx} / \sigma$$', 'Interpreter', 'latex');
xlabel('$$\sigma_{xx} / \bar{\sigma}$$', 'Interpreter', 'latex', 'FontSize', 18);
ylabel('$$\sigma_{yy} / \bar{\sigma}$$', 'Interpreter', 'latex');
ylabel('$$\sigma_{yy} / \bar{\sigma}$$', 'Interpreter', 'latex', 'FontSize', 18);
title('Barlat-89 Yield Surface')

hold off;

% Symbolic calculations
fi = sym('fi'); % Define symbolic variable for fi
Y_fi = 1;
d_1 = diff(R, S_1);
d_2 = diff(R, S_2);
S_1 = Y_fi * cosd(fi)^2; % Update S_1 using symbolic expression
S_2 = Y_fi * sind(fi)^2; % Update S_2 using symbolic expression
S_12 = Y_fi * sind(fi) * cosd(fi); % Update S_12 using symbolic expression
S_23 = Y_fi * sind(fi) * cosd(fi); % Update S_23 using symbolic expression
R = subs(R); % Evaluate R with the new symbolic expressions
d_1 = subs(d_1); % Evaluate d_1 with the new symbolic expressions
d_2 = subs(d_2); % Evaluate d_2 with the new symbolic expressions
r_fi = R / (Y_fi * (d_1 + d_2)) - 1; % Anizotropinin aA ile de $\square$ i imini g $\hat{}$ steren fonksiyon
r_fi = subs(r_fi);

% Plot variation of anisotropy with orientation
figure;
h3 = ezplot(r_fi, [0, 90]);
set(h3, 'Color', 'b', 'LineWidth', 2);
axis([0 90 0 3]);
xlabel('\bf theta')
ylabel('\bf r')
title('\bf Variation of anisotropy with orientation')
cc = get(h3);
cc = [cc.XData; cc.YData];

% Define T_1 and T_2 based on the values in the code
T_1 = (S_1 + h * S_2) / 2;
T_2 = sqrt(((S_1 - h * S_2) / 2)^2 + p^2 * S_23^2);

% Define T using T_1 and T_2
T = (0.5 * (a * abs(T_1 + T_2)^m + a * abs(T_1 - T_2)^m + c * abs(2 * T_2)^m))^(1/m);

% Now substitute S_1, S_2, and S_23 with the values used in Y_fi expression
T_val = subs(T, {S_1, S_2, S_23}, {1, h, 0});

```

```

% Define Y_fi based on T
Y_fi = (2*S^m/(T_val))^(1/m);

% Convert Y_fi to a MATLAB function handle (ensure that 'theta' is used instead of 'fi')
Y_fi_func = matlabFunction(subs(Y_fi, 'fi', 'theta'));

% Create a range of angles from 0 to 90 degrees for evaluation
theta = linspace(0, 90, 1000);

% Evaluate Y_fi over the range of theta
Y_fi_vals = Y_fi_func(theta);

% Plot theta vs Y_fi

figure;
plot(theta, Y_fi_vals, 'm', 'LineWidth', 2);
axis([0 90 0 2]);
xlabel('\theta');
ylabel('\Sigma');
title('Variation of yield stress with orientation');

% Get the data points used in the plot
%bb = [theta; Y_fi_vals];

```

Yld2000-2p Criteria

```

function YLD20002

% Main function for solving quartic polynomials
a = input('Enter the coefficients [a b c d e p q]: ');

if a(5) ~= 0
    roots = quarticRoots(a);
elseif a(4) ~= 0
    roots = cubicRoots(a(1:4));
elseif a(3) ~= 0
    roots = quadraticRoots(a(1:3));
elseif a(2) ~= 0
    roots = [-a(1)/a(2)];
else
    fprintf('The equation is degenerate.\n');
    return;
end

```

```

disp('IRoots:');
disp(roots);

end

function roots = quarticRoots(a)

    % Solve quartic equation

    b = a(4)/(4*a(5));
    c = a(3)/a(5);
    d = a(2)/a(5);
    e = a(1)/a(5);

    p = c - 3*b^2;
    q = b*(2*b^2 - c) + d;
    r = b^2*(c - b^2) - b*d + e;

    coef = [1 p/2 (p^2 - 4*r)/16 -q^2/64];
    yRoots = cubicRoots(coef);

    y = max(yRoots);
    R = sqrt(0.25*p + y);
    D = 0.75*p + y;

    if R ~= 0
        E = sqrt(0.75*p - R^2 - 2*y + (4*y*(p + y - D))/(4*R));
    else
        E = sqrt(0.75*p - R^2 - 2*y);
    end

    if R ~= 0
        F = sqrt(0.75*p - R^2 - 2*y - (4*y*(p + y - D))/(4*R));
    else
        F = sqrt(0.75*p - R^2 - 2*y);
    end

    roots = [R + E - b, R - E - b, -R + F - b, -R - F - b];

end

function roots = cubicRoots(a)

    % Solve cubic equation

    a = a/a(4); % normalize coefficients

```

```

Q = (3*a(3) - a(2)^2)/9;
R = (9*a(2)*a(3) - 27*a(1) - 2*a(2)^3)/54;
D = Q^3 + R^2; % polynomial discriminant

if D > 0
    S = sgn(R + sqrt(D))*nthroot(abs(R + sqrt(D)), 3);
    T = sgn(R - sqrt(D))*nthroot(abs(R - sqrt(D)), 3);
    roots = [(-a(2)/3 + (S + T)), (-a(2)/3 - (S + T)/2), (-a(2)/3 - (S + T)/2)];
elseif D == 0
    roots = [2*real(nthroot(R,3)) - a(2)/3, -real(nthroot(R,3)) - a(2)/3];
else
    theta = acos(R/sqrt(-Q^3));
    roots = [2*sqrt(-Q)*cos(theta/3) - a(2)/3, 2*sqrt(-Q)*cos((theta + 2*pi)/3) - a(2)/3, 2*sqrt(-Q)*cos((theta - 2*pi)/3) - a(2)/3];
end
end

function roots = quadraticRoots(a)
    % Solve quadratic equation
    D = a(2)^2 - 4*a(1)*a(3);
    if D >= 0
        roots = [(-a(2) + sqrt(D))/(2*a(3)), (-a(2) - sqrt(D))/(2*a(3))];
    else
        roots = [-a(2)/(2*a(3)) + 1i*sqrt(-D)/(2*a(3)), -a(2)/(2*a(3)) - 1i*sqrt(-D)/(2*a(3))];
    end
end

function s = sgn(x)
    % Sign function
    if x < 0
        s = -1;
    else
        s = 1;
    end
end

clear all
clc
close all

% Define the symbolic variables

```

```
syms alf_1 alf_2 alf_3 alf_4 alf_5 alf_6 alf_7 alf_8 beta alfa_1 alfa_2 alfa_3 a O m r_er r_et r_ed S_o S_dk S_4 S_b S_11
S_22 S_12 S_23 T_11 T_22 T_12 T_23 X_11 X_22 X_12 Y_11 Y_22 Y_12 S K_1 K_2 arctan arccos arcsin norm Y_b real
```

```
% Assign values to O
```

```
O = [1.048281188617732, 0.914952919843855, 0.966687530275681, 0.957409876833700, 1.014521009890060,
1.069830016238720, 0.972623213012199, 0.978902497816224];
```

```
% Assign values to alpha_i variables
```

```
[alf_1, alf_2, alf_3, alf_4, alf_5, alf_6, alf_7, alf_8] = deal(O(1), O(2), O(3), O(4), O(5), O(6), O(7), O(8));
```

```
a = 6;
```

```
% Initialize S_12 and S
```

```
S_12 = 0;
```

```
S = 1;
```

```
% Function to calculate X_11, X_22, X_12, Y_11, Y_22, Y_12
```

```
calculateValues = @(S_11, S_22, S_12) deal(...
```

```
2*alf_1*S_11/3 - alf_1*S_22/3,...
```

```
-alf_2*S_11/3 + 2*alf_2*S_22/3,...
```

```
alf_7*S_12,...
```

```
(-2*alf_3/9 + 2*alf_4/9 + 8*alf_5/9 - 2*alf_6/9)*S_11 + (alf_3/9 - 4*alf_4/9 - 4*alf_5/9 + 4*alf_6/9)*S_22,...
```

```
(4*alf_3/9 - 4*alf_4/9 - 4*alf_5/9 + alf_6/9)*S_11 + (-2*alf_3/9 + 8*alf_4/9 + 2*alf_5/9 - 2*alf_6/9)*S_22,...
```

```
alf_8*S_12);
```

```
% Function to calculate principal stresses
```

```
calculatePrincipalStresses = @(X_11, X_22, X_12, Y_11, Y_22, Y_12) deal(...
```

```
0.5*(X_11 + X_22 + sqrt((X_11 - X_22)^2 + 4*X_12^2)),...
```

```
0.5*(X_11 + X_22 - sqrt((X_11 - X_22)^2 + 4*X_12^2)),...
```

```
0.5*(Y_11 + Y_22 + sqrt((Y_11 - Y_22)^2 + 4*Y_12^2)),...
```

```
0.5*(Y_11 + Y_22 - sqrt((Y_11 - Y_22)^2 + 4*Y_12^2)));
```

```
% Function to plot F_1 for different values of S_12
```

```
plotF1 = @(S_12_value) ...
```

```
fcontour(@(S_11, S_22) ...
```

```
abs(calculatePrincipalStresses(calculateValues(S_11, S_22, S_12_value))), ...
```

```
[-1.5, 1.5, -1.5, 1.5], 'MeshDensity', 100);
```

```
% Call the plotting function with a specific S_12 value
```

```
plotF1(0);
```

```
% Adjust the appearance of the plot
```

```
title('Plot of F_1')
```

```
xlabel('S_11')
```

```
ylabel('S_22')
```

```
axis equal
```

```
grid on
```

Yld2004-18p Criteria

```

import numpy as np
from scipy import optimize
import enum
import argparse
from pathlib import Path

@enum.unique
class Space(enum.Enum):
    free3D = enum.auto()
    fixed3D = enum.auto()
    free2D = enum.auto()
    fixed2D = enum.auto()

def yieldfunction3D(s11, s22, s33, s12, s23, s31, c, exponent):
    ## YLD2004-18p yield surface f=(phi/4)^(1/m)-sigma_y
    # Yield surface parameters
    c1_12 = 1.0
    c1_13 = 1.0
    c1_21 = c[0]
    c1_23 = c[1]
    c1_31 = c[2]
    c1_32 = c[3]
    c1_44 = c[4]
    c1_55 = c[5]
    c1_66 = c[6]

    c2_12 = c[7]
    c2_13 = c[8]
    c2_21 = c[9]
    c2_23 = c[10]
    c2_31 = c[11]
    c2_32 = c[12]
    c2_44 = c[13]
    c2_55 = c[14]
    c2_66 = c[15]

    m = c[16]
    ##
    # Deviatoric stress
    sxx = s11 - (s11 + s22 + s33) / 3.0
    syy = s22 - (s11 + s22 + s33) / 3.0
    szz = s33 - (s11 + s22 + s33) / 3.0

    # Stress tensor quantities of s'
    x1 = -c1_12 * syy - c1_13 * szz
    y1 = -c1_21 * sxx - c1_23 * szz
    z1 = -c1_31 * sxx - c1_32 * syy
    xy1 = c1_44 * s12
    yz1 = c1_55 * s23
    xz1 = c1_66 * s31

```

```

# Stress tensor quantities of s''
x2 = -c2_12 * syy - c2_13 * szz
y2 = -c2_21 * sxx - c2_23 * szz
z2 = -c2_31 * sxx - c2_32 * syy
xy2 = c2_44 * s12
yz2 = c2_55 * s23
xz2 = c2_66 * s31

# Calculate eigenvalues of s' and s''
s1 = np.linalg.eigvalsh(
    np.array([[x1, xy1, xz1], [xy1, y1, yz1], [xz1, yz1, z1]]))
s2 = np.linalg.eigvalsh(
    np.array([[x2, xy2, xz2], [xy2, y2, yz2], [xz2, yz2, z2]]))

# calculate phi
phi = 0.0
for a in s1:
    for b in s2:
        phi += abs(a - b) ** m

return (phi / 4.0) ** (1.0 / m) - 1.0

def yieldfunction3Dfixed(s11, s22, s33, s12, s23, s31, c, exponent):
    return yieldfunction3D(s11, s22, s33, s12, s23, s31,
np.append(c, [exponent]), exponent)

def yieldfunction2D(s11, s22, s33, s12, s23, s31, c, exponent):
    ## YLD2004-18p yield surface  $f=(\phi/4)^{(1/m)}-\sigma_y$ 
    # Yield surface parameters
    c1_12 = 1.0
    c1_13 = 1.0
    c1_21 = c[0]
    c1_23 = c[1]
    c1_31 = c[2]
    c1_32 = c[3]
    c1_44 = c[4]
    c1_55 = 1.0
    c1_66 = 1.0

    c2_12 = c[5]
    c2_13 = c[6]
    c2_21 = c[7]
    c2_23 = c[8]
    c2_31 = c[9]
    c2_32 = c[10]
    c2_44 = c[11]
    c2_55 = 1.0
    c2_66 = 1.0

    m = c[12]
    ##
    # Deviatoric stress
    sxx = s11 - (s11 + s22 + s33) / 3.0
    syy = s22 - (s11 + s22 + s33) / 3.0
    szz = s33 - (s11 + s22 + s33) / 3.0

    # Stress tensor quantities of s'

```

```

x1 = -c1_12 * syy - c1_13 * szz
y1 = -c1_21 * sxx - c1_23 * szz
z1 = -c1_31 * sxx - c1_32 * syy
xy1 = c1_44 * s12
yz1 = c1_55 * s23
xz1 = c1_66 * s31

# Stress tensor quantities of s''
x2 = -c2_12 * syy - c2_13 * szz
y2 = -c2_21 * sxx - c2_23 * szz
z2 = -c2_31 * sxx - c2_32 * syy
xy2 = c2_44 * s12
yz2 = c2_55 * s23
xz2 = c2_66 * s31

# Calculate eigenvalues of s' and s''
s1 = np.linalg.eigvalsh(
    np.array([x1, xy1, xz1], [xy1, y1, yz1], [xz1, yz1, z1]))
s2 = np.linalg.eigvalsh(
    np.array([x2, xy2, xz2], [xy2, y2, yz2], [xz2, yz2, z2]))

# calculate phi
phi = 0.0
for a in s1:
    for b in s2:
        phi += abs(a - b) ** m

return (phi / 4.0) ** (1.0 / m) - 1.0

def yieldfunction2Dfixed(s11, s22, s33, s12, s23, s31, c, exponent):
    return yieldfunction2D(s11, s22, s33, s12, s23, s31,
np.append(c, [exponent]), exponent)

def LoadTests(path):
    s11 = []
    s22 = []
    s33 = []
    s12 = []
    s23 = []
    s31 = []
    with open(path, 'r') as file:
        next(file)
        for line in file:
            s11.append(float(line.split(',')[0]))
            s22.append(float(line.split(',')[1]))
            s33.append(float(line.split(',')[2]))
            s12.append(float(line.split(',')[3]))
            s23.append(float(line.split(',')[4]))
            s31.append(float(line.split(',')[5]))
    return np.array(s11), np.array(s22), np.array(s33),
np.array(s12), np.array(s23), np.array(s31)

def ObjFun(c, s11, s22, s33, s12, s23, s31, YS, exponent):
    return sum(ObjFunVec(c, s11, s22, s33, s12, s23, s31, YS,
exponent) ** 2)

```

```

def ObjFunVec(c, s11, s22, s33, s12, s23, s31, YS, exponent):
    N = s11.size
    f = np.zeros(N)
    n = len(c)

    if (n == 17) and (c[16] < 2 or c[16] > 30):
        for k in range(N):
            f[k] = 1.e9
        return f
    elif (n == 13) and (c[12] < 2 or c[12] > 30):
        for k in range(N):
            f[k] = 1.e9
        return f

    for k in range(N):
        f[k] = YS(s11[k], s22[k], s33[k], s12[k], s23[k], s31[k], c,
exponent)
    return f

def GetYS(c):
    N = len(c)
    if N == 17:
        return yieldfunction3D
    elif N == 16:
        return yieldfunction3Dfixed
    elif N == 13:
        return yieldfunction2D
    elif N == 12:
        return yieldfunction2Dfixed
    else:
        return None

def GetInitial(choice):
    if choice == Space.fixed2D:
        c0 = np.zeros(12)
    elif choice == Space.free2D:
        c0 = np.zeros(13)
        c0[12] = 8.0
    elif choice == Space.fixed3D:
        c0 = np.zeros(16)
    elif choice == Space.free3D:
        c0 = np.zeros(17)
        c0[16] = 8.0
    else:
        c0 = np.zeros(12)
    return c0

def Normalize(s11, s22, s33, s12, s23, s31):
    k = 0
    er = np.inf
    for i in range(s11.size):
        estimate = np.sqrt(s22[i] ** 2 + s33[i] ** 2 + 2 * s12[i]
** 2 + 2 * s23[i] ** 2 + 2 * s31[i] ** 2)
        if estimate < er:
            k = i

```

```

        er = estimate

        s0 = np.sqrt(0.5 * (s11[k] - s22[k]) ** 2 + 0.5 * (s22[k] -
s33[k]) ** 2 + 0.5 *
                    (s33[k] - s11[k]) ** 2 + 3 * (s12[k]) ** 2 + 3 *
(s23[k]) ** 2 + 3 * (s31[k]) ** 2)

        s11 = s11 / s0
        s22 = s22 / s0
        s33 = s33 / s0
        s12 = s12 / s0
        s23 = s23 / s0
        s31 = s31 / s0
        return s11, s22, s33, s12, s23, s31

def Correct(c, s11, s22, s33, s12, s23, s31, choice, exponent):
    # Calculate the normalized yield stress along RD/ED
    YieldFunc = GetYS(GetInitial(choice))
    normS = 1.0 / (YieldFunc(1.0, 0.0, 0.0, 0.0, 0.0, 0.0, c,
exponent) + 1.0)

    # Correct the normalized yield points
    s11 = s11 / normS
    s22 = s22 / normS
    s33 = s33 / normS
    s12 = s12 / normS
    s23 = s23 / normS
    s31 = s31 / normS
    return s11, s22, s33, s12, s23, s31

def CorrectYieldStress(c, choice, exponent):
    YieldFunc = GetYS(GetInitial(choice))
    normS = 2.0
    while abs(normS - 1.0) > 1.0e-6:
        # Calculate the normalized yield stress along RD/ED
        normS = 1.0 / (YieldFunc(1.0, 0.0, 0.0, 0.0, 0.0, 0.0, c,
exponent) + 1.0)
        c = CorrectYSparmeters(c, normS, choice)

    return c

def CorrectYSparmeters(c, normS, choice):
    # Correct the YS parameters
    for i in range(len(c) - 1):
        c[i] = c[i] * normS

    if choice == Space.fixed2D:
        c[-1] = c[-1] * normS
        c[0] = c[0] + normS - 1
        c[2] = c[2] + normS - 1
        c[7] = c[7] + normS - 1
        c[9] = c[9] + normS - 1
    elif choice == Space.free2D:
        c[0] = c[0] + normS - 1
        c[2] = c[2] + normS - 1
        c[7] = c[7] + normS - 1

```

```

        c[9] = c[9] + normS - 1
    elif choice == Space.fixed3D:
        c[-1] = c[-1] * normS
        c[0] = c[0] + normS - 1
        c[2] = c[2] + normS - 1
        c[9] = c[9] + normS - 1
        c[11] = c[11] + normS - 1
    elif choice == Space.free3D:
        c[0] = c[0] + normS - 1
        c[2] = c[2] + normS - 1
        c[9] = c[9] + normS - 1
        c[11] = c[11] + normS - 1

    return c

def SaveResult(c, folder, exponent):
    parameterName = [r'\hat{c}^{\prime}_{12}',
                     r'\hat{c}^{\prime}_{13}',
                     r'\hat{c}^{\prime}_{21}',
                     r'\hat{c}^{\prime}_{23}',
                     r'\hat{c}^{\prime}_{31}',
                     r'\hat{c}^{\prime}_{32}',
                     r'\hat{c}^{\prime}_{44}',
                     r'\hat{c}^{\prime}_{55}',
                     r'\hat{c}^{\prime}_{66}',
                     r'\hat{c}^{\prime \prime}_{12}',
                     r'\hat{c}^{\prime \prime}_{13}',
                     r'\hat{c}^{\prime \prime}_{21}',
                     r'\hat{c}^{\prime \prime}_{23}',
                     r'\hat{c}^{\prime \prime}_{31}',
                     r'\hat{c}^{\prime \prime}_{32}',
                     r'\hat{c}^{\prime \prime}_{44}',
                     r'\hat{c}^{\prime \prime}_{55}',
                     r'\hat{c}^{\prime \prime}_{66}',
                     r'a']

    with open(Path.joinpath(folder, 'CalibratedParameters.dat'),
              'w') as file:
        file.write('%30s,%s\n' % ('parameters', 'values'))
        file.write('%30s, %12.8f\n' % (parameterName[0], 1.0))
        file.write('%30s, %12.8f\n' % (parameterName[1], 1.0))
        N = len(c)
        if N == 17:
            for i in range(N - 1):
                file.write('%30s, %12.8f\n' % (parameterName[i + 2],
c[i]))
                file.write('%30s, %12.8f' % (parameterName[18], c[N -
1]))
            elif N == 16:
                for i in range(N):
                    file.write('%30s, %12.8f\n' % (parameterName[i + 2],
c[i]))
                    file.write('%30s, %12.8f' % (parameterName[18],
exponent))
            elif N == 13:
                for i in range(5):
                    file.write('%30s, %12.8f\n' % (parameterName[i + 2],
c[i]))
                    file.write('%30s, %12.8f\n' % (parameterName[7], 1.0))

```

```

        file.write('%30s, %12.8f\n' % (parameterName[8], 1.0))
        for i in range(5, N - 1):
            file.write('%30s, %12.8f\n' % (parameterName[i + 4],
c[i]))
            file.write('%30s, %12.8f\n' % (parameterName[16], 1.0))
            file.write('%30s, %12.8f\n' % (parameterName[17], 1.0))
            file.write('%30s, %12.8f' % (parameterName[18], c[N -
1]))
        elif N == 12:
            for i in range(5):
                file.write('%30s, %12.8f\n' % (parameterName[i + 2],
c[i]))
                file.write('%30s, %12.8f\n' % (parameterName[7], 1.0))
                file.write('%30s, %12.8f\n' % (parameterName[8], 1.0))
                for i in range(5, N):
                    file.write('%30s, %12.8f\n' % (parameterName[i + 4],
c[i]))
                    file.write('%30s, %12.8f\n' % (parameterName[16], 1.0))
                    file.write('%30s, %12.8f\n' % (parameterName[17], 1.0))
                    file.write('%30s, %12.8f' % (parameterName[18],
exponent))

```

```

def OptimizeBasinhopping(s11, s22, s33, s12, s23, s31, choice,
exponent):
    # Basinhopping input
    niter = 200 # No. iteration in basinhopping
    T = 0.001 # The "temperature" parameter
    stepsize = 0.5 # Initial step size
    solver = 'SLSQP' # Minimizer to be used ('SLSQP', 'BFGS', ...)
    disp = False # Should the status messages from basinhopping be
printed?

    #####

    c0 = GetInitial(choice)
    YieldFunc = GetYS(c0)

    c = c0
    f = ObjFun(c, s11, s22, s33, s12, s23, s31, YieldFunc, exponent)

    # Trying to find global minimum
    for i in range(0, 5):
        res = optimize.basinhopping(ObjFun, c0, niter=niter, T=T,
stepsize=stepsize + 0.1 * i, minimizer_kwargs={
'method': solver, 'args': (s11, s22, s33, s12, s23, s31,
YieldFunc, exponent)}, disp=disp)

        ctrial = res.x
        ftrial = ObjFun(ctrial, s11, s22, s33, s12,
                        s23, s31, YieldFunc, exponent)

        if ftrial < f:
            c = ctrial
            f = ftrial

    return c

```

```

def OptimizeMinimize(s11, s22, s33, s12, s23, s31, choice,
exponent):
    # input
    niter = 1000 # Max no. iteration
    solver = 'SLSQP' # Minimizer to be used ('SLSQP','BFGS',...)
    disp = False # Should the status messages be printed?

#####

    c0 = GetInitial(choice)
    YieldFunc = GetYS(c0)

    c = c0
    f = ObjFun(c, s11, s22, s33, s12, s23, s31, YieldFunc, exponent)

    res = optimize.minimize(ObjFun, c0, args=(s11, s22, s33, s12,
s23, s31, YieldFunc,
                                exponent),
method=solver, options={'maxiter': niter, 'disp': disp})

    ctrial = res.x
    ftrial = ObjFun(ctrial, s11, s22, s33, s12, s23, s31, YieldFunc,
exponent)

    if ftrial < f:
        c = ctrial
        f = ftrial

    return c

def OptimizeLS(s11, s22, s33, s12, s23, s31, choice, exponent):
    # input
    solver = 'lm' # Minimizer to be used

#####

    c0 = GetInitial(choice)
    YieldFunc = GetYS(c0)

    c = c0
    f = ObjFun(c, s11, s22, s33, s12, s23, s31, YieldFunc, exponent)

    res = optimize.least_squares(ObjFunVec, c0, args=(
s11, s22, s33, s12, s23, s31, YieldFunc, exponent),
method=solver)

    ctrial = res.x
    ftrial = ObjFun(ctrial, s11, s22, s33, s12, s23, s31, YieldFunc,
exponent)

    if ftrial < f:
        c = ctrial
        f = ftrial

    return c

def Calibrate(s11, s22, s33, s12, s23, s31, choice, exponent):

```

```

        if len(GetInitial(choice)) > len(s11):
            c = OptimizeMinimize(s11, s22, s33, s12, s23, s31, choice,
                                exponent)
        else:
            c = OptimizeLS(s11, s22, s33, s12, s23, s31, choice,
                           exponent)
            # c = OptimizeBasinhopping(s11, s22, s33, s12, s23, s31, choice,
            #                           exponent)
        return c

def main():
    # Parse command line arguments
    parser = argparse.ArgumentParser(
        description='Calibrate the Yld2004-18p yield surface from
discrete yield surface data.')
    parser.add_argument('inputFile', type=str,
                        help='Path to the input file with discrete
yield surface data.')
    parser.add_argument('--space', default='2D', type=str,
                        choices=['3D', '2D'],
                        help='Choose where the test is run.' +
                             ' 3D: 3-dimensional calibration.' +
                             ' 2D: 2-dimensional calibration
(Default option).')
    parser.add_argument('--exponent', default=8.0, type=float,
                        help='Choose an exponent for the yield
surface.')
    parser.add_argument('--calibrateExponent', action='store_true',
                        help='Choose to calibrate the yield surface
exponent. This will override the exponent passed in.')
    args = parser.parse_args()
    fileName = Path(args.inputFile)
    spaceInput = args.space
    exponent = args.exponent
    calibrateExponent = args.calibrateExponent

    assert (fileName.exists()), ('Could not find input file: ' +
                                str(fileName))

    choice = Space.free3D
    if spaceInput == '2D':
        if calibrateExponent:
            choice = Space.free2D
        else:
            choice = Space.fixed2D
    elif spaceInput == '3D':
        if calibrateExponent:
            choice = Space.free3D
        else:
            choice = Space.fixed3D

    # Loading CP-FEM data
    s11, s22, s33, s12, s23, s31 = LoadTests(fileName)
    # Normalize data by the yield stress
    s11, s22, s33, s12, s23, s31 = Normalize(s11, s22, s33, s12,
s23, s31)

    # Calibration

```

```

    c = Calibrate(s11, s22, s33, s12, s23, s31, choice, exponent)

    # Correct yield stress based on normalized yield stress along
    RD/ED, which should be 1
    c = CorrectYieldStress(c, choice, exponent)

    # Writes the parameters to a file
    SaveResult(c, fileName.parent, exponent)

#####
if __name__ == "__main__":
    main()

```

Quartic Solver



```

import numpy as np

# Assuming CZERO is a complex zero, it needs to be defined
accordingly
CZERO = complex(0.0, 0.0)

# Assuming that outputCode is a status variable, it needs to be
initialized
outputCode = 0

def complex_division(a, b):
    # Assuming 'a' and 'b' are complex numbers
    # Compute real and imaginary parts separately to avoid division
    by zero
    real_part = (a.real * b.real + a.imag * b.imag) / (b.real ** 2 +
b.imag ** 2)
    imag_part = (a.imag * b.real - a.real * b.imag) / (b.real ** 2 +
b.imag ** 2)
    return complex(real_part, imag_part)

def cubic_roots(a):
    # This function would contain the logic for finding the roots of
    a cubic equation
    # For the sake of this example, we will just return a
    placeholder
    return [CZERO, CZERO, CZERO]

def quartic_roots(a):
    global outputCode

    # Normalize coefficients
    a0 = a[0] / a[4]
    a1 = a[1] / a[4]
    a2 = a[2] / a[4]
    a3 = a[3] / a[4]

```

```

# Depressed quartic:  $t^4 + p*t^2 + q*t + r = 0$ 
p = a2 - 3.375 * a3 * a3
q = 0.5 * a3 * (2.25 * a3 * a3 - a2) - a1
r = -0.125 * a3 * a3 * a3 * a3 + 0.25 * a3 * a2 - a0

# Solve the cubic resolvent
cubic_coef = [1, -p, -4 * r, 4 * r * p - q * q]
cubic_roots_solution = cubic_roots(cubic_coef)

# Find the root to construct two quadratic equations
Y = cubic_roots_solution[0].real # We take the real part of the
root
W = np.sqrt(a3 * a3 / 4 - a2 + Y)

# If W is zero, then we need a different method, not covered
here
if W == 0:
    outputCode = 41
    return [CZERO, CZERO, CZERO, CZERO]

# Two quadratic equations
M = np.sqrt(-3 * a3 * a3 / 4 + 2 * a2 - 2 * Y + 2 * q / W)
N = np.sqrt(-3 * a3 * a3 / 4 + 2 * a2 - 2 * Y - 2 * q / W)

# Calculate roots
z1 = (-a3 / 2 + W + M) / 2
z2 = (-a3 / 2 + W - M) / 2
z3 = (-a3 / 2 - W + N) / 2
z4 = (-a3 / 2 - W - N) / 2

z = [z1, z2, z3, z4]
outputCode = 44

return z

# Example usage:
# coefficients = [1, 0, 0, 0, 1] # Coefficients of the quartic
equation
# roots = quartic_roots(coefficients)
# print(roots)

```