

EUROPEAN UNIVERSITY OF LEFKE
INSTITUTE of GRADUATE STUDIES and RESEARCH
DEPARTMENT of CIVIL ENGINEERING
MASTER OF SCIENCE THESIS

EVALUATING THE SHEAR BEHAVIOR OF REINFORCED
CONCRETE DEEP BEAMS WITHOUT SHEAR
REINFORCEMENT USING STRUT AND TIE MODEL

Rozelin TÜRK

SUPERVISOR

Asst. Prof. Dr. Shahram DEROGAR

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ABSTRACT

Reinforced concrete (RC) deep beams are used mainly as transfer girders, wall footings, and foundation pile caps in building structures. The shear failure is usually the dominant behaviour in RC deep beams due to its relatively small shear span to effective depth ratio. Existing empirical shear design provisions such as ACI 318, AASHTO LRFD and EC 2, overestimate the shear capacity of deep beams and are not suggested to be used for deep beams. Strut and tie model (STM) is depended on the lower bound theorem of plasticity theory and it is implemented to ACI, EC 2 and AASHTO standards for the design of reinforced concrete (R.C.) deep beams. This study aims to evaluate the existing STM shear design provisions in design codes such as ACI 318, EC 2 and AASHTO LRFD with an extensive experimental database on reinforced concrete deep beam with no shear reinforcement. Collected database used in this thesis were tested under point loads and do not contain shear reinforcement. It was shown that the accuracy of shear resistance prediction according to STM model are dependent on various assumptions such as types of struts, nodes and effectiveness factor for concrete strength for inclined strut. However, available codes do not ensure detailed guideline on the use for STM in design. The shear strength predictions by STM adopted in various codes with the extensive experimental database is also evaluated in this study. It was observed that the use of current STM provisions in design codes can lead to an unconservative calculation of nominal capacities for some test specimens.

Keywords: RC deep beams, shear failure, shear behaviour, strut-and-tie model, codes of practice.

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LIST OF ABBREVIATIONS

a	: Shear span
d	: Effective depth
a/d	: Shear span to effective depth
b_w	: Beam width
f_c'	: Effective concrete compressive strength
f_y	: Yield strength of steel reinforcement
ρ_w	: Longitudinal reinforcement ratio
f_{ce}	: Effective concrete compressive strength
h	: Beam height
ν, β, λ	: The strut efficiency factor
α	: Angle between the compressive strut and the tie
φ	: Reduction factor
ε_s	: The tensile strain in the concrete in the direction of the tension tie
ε_l	: Principal tensile strain
f_{cu}	: The usable compressive strength
P_n	: Nominal resistance of a compressive strut
A_{cs}	: Effective cross-sectional area of strut
f_{ck}	: Specified concrete cylinder strength
σ_{Rd}	: Allowable stress

f_{cd} : Design compressive strength

γ : Safety factor

F_{ns} : Nominal compressive strength of strut

F_{nt} : Nominal strength of a tie

F_{nn} : Nominal compressive strength of a nodal zone

A_{nz} : Area of the face of the nodal zone

CHAPTER 1

INTRODUCTION

One of the main problems in Reinforced Concrete (RC) deep beam is shear failure due to its relatively small shear span to the effective depth ratio. In general, deep beams are preferred to be used in tall buildings, long span bridge and off shore structure. The ultimate strength of deep beams have been complex to calculate concrete capacity rather than steel capacity because the reinforcement capacity of a beam is depend on only yield strength and cross section area of bars. However, to evaluate concrete strength of deep beam, more parameters should be considered and effect of each one also is observed. Therefore, to clarify strength of concrete, the Strut and Tie Method (STM) is recommended for evaluating shear capacity of RC deep beam without shear reinforcement according to U.S. and European code provisions. Existing empirical shear design provisions such as ACI 318, AASHTO LRFD and EC 2, cannot predict accurately the shear capacity of deep beams and are not suggested to be used for deep beams. This thesis aims to evaluate shear behaviour of deep beams by gathering the prior knowledge on the strut and tie method and it is performed a literature study. To do this, the predictions of existing STM are evaluated according to shear design provisions which is shown by researchers and codes with an extensive experimental database on RC deep beams without shear reinforcement. Also, for this research experimental databases were collected from of Collins, Bentz, Sherwood, Ismail and Todisco.

1.1 Aim and Objectives

This study is based on three main aims, the first one is to explain shear behaviour of deep beams with different transfer technics and mechanism. Since humanity began civilization, they need to accommodation with different type of structural members. The safety and complex one is reinforced concrete structures. However researcher and design engineers often encounter portions of concrete structures that are subjected to shear stresses. A strut and tie model (STM) offers an alternative to such methods. A rational method for the analyzing and designing of deep beams, the strut and tie model, that has been developed by Ritter (1899) and Morsch (1902).

The second aim of the research is to gather the previous knowledge on the strut and tie method and it is performed a literature study.

The third aim of the research is to examine the effect of the parameters on shear strength predictions of existing STM with an extensive experimental database on RC deep beams without shear reinforcement. Also, for this research, experimental databases were collected from of Collins, Bentz, Sherwood, Ismail, and Todisco.

1.2 Thesis Structure

This thesis consists in total of six chapters, as well as chapter 1 introduction and chapter 6 conclusion of the work done, after the introduction, shear behaviour of deep beam without shear reinforcement is showed in chapter 2. This chapter includes the definition and application of deep beams, shear transfer mechanism and failure modes.

In chapter 3, details of the earlier studies on shear and existing STM experiments which is based on influencing parameters were shown. Also this chapter includes, strut and tie model procedures for ACI 318, EC 2 and AASHTO provisions.

In chapter 4, collected databases were evaluated by comparing with codes and statistical analysing were shown. The database used in the paper includes only RC deep beam without shear reinforcement. For each RC deep beam test, data for material strength, cross section geometry, loading and support boundary conditions, flexural reinforcement and the measured failure load were collected.

Chapter 5 contains the discussion of the parametric study from collected databases. The influence parameters are plotted to show effect of each parameter on the safety factor.

At the end of the thesis, conclusion and summary part is showed in chapter 6, a summary of all the work is presented, along with the conclusions that could be observed from the understanding to shear behaviour, parametric study and statistical analyse of deep beams without shear reinforcement. Lastly, the earlier research studies are inspired to evaluate and discuss influence of the each parameter on the shear strength of deep beams.

CHAPTER 2

SHEAR BEHAVIOUR OF DEEP BEAMS

2.1 Overview

One of the main problems in reinforced concrete (RC) deep beams is shear which has confused since first construction fails because of concrete's brittle and sudden behaviour. Therefore, the important issue or complex part for shear capacity evaluation is concrete effect rather than steel effect. RC deep beam have many application in high rise building, bridge and offshore structure. Because the deep beam has small shear span to the effective depth ratio, shear failure is dominant behavior in RC deep beams. Available shear design provisions in codes can not forecast exactly and reliably the shear strength of RC deep beams and also they are not safe in some cases (Ismail, 2016). According to codes of practice, shear strength calculation of deep beams can be categorized as sectional and strut and tie provisions (Collins et al., 2008). Sectional design provision consists of empirical formulae and it is not suggested for RC deep beams in terms of safety and economically. However, strut and tie model (STM) is based on rational approach and it gives more conservative results than sectional provision because it takes into account more parameters to assess shear capacity. This chapter goals is to improve a better understanding of the shear behaviour of deep beams without shear reinforcement.



Figure 2.1 Deep beams location and uses (Khair, 2005)

2.2 Deep Beam

Reinforced concrete beams are classified according to load location, if the located load point is less than $2.5d$ from the support, nonlinear stress distribution available in that region (D-region or Discontinuity) so it is named as deep beam and if the load location is bigger than $2.5d$, it is slender beam and it is based on Bernoulli assumption (B-region) which plane section remain plane after deformation and also linear stress distribution is available in that region (B-region) (Tuchscherer et al. 2011).

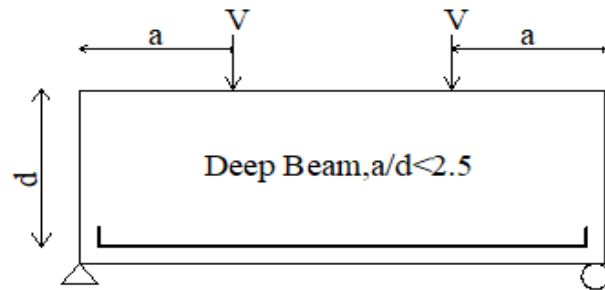


Figure 2.2 Definition of RC deep beam

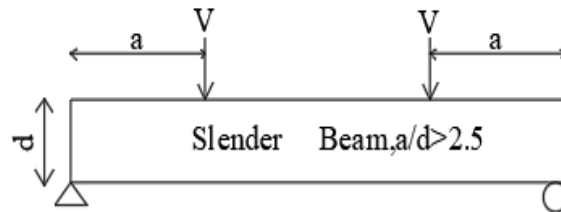


Figure 2.3 Definition of RC slender beam

The strain distribution along the depth of a section in deep beam becomes nonlinear with the decrease of the shear span to depth ratio (Collins and Mitchell 1991). Deep beams were observed by many researchers and summarized that the sectional model failed to assume the accurate behavior in case of a beam having shear span to the effective depth ratio is less than 2 (Fig. 2.5) (Rogowsky 1986, Collins and Mitchell 1991).



Figure 2.4 Shear failure of deep beams in laboratory experiment by Tuladhar, 2014

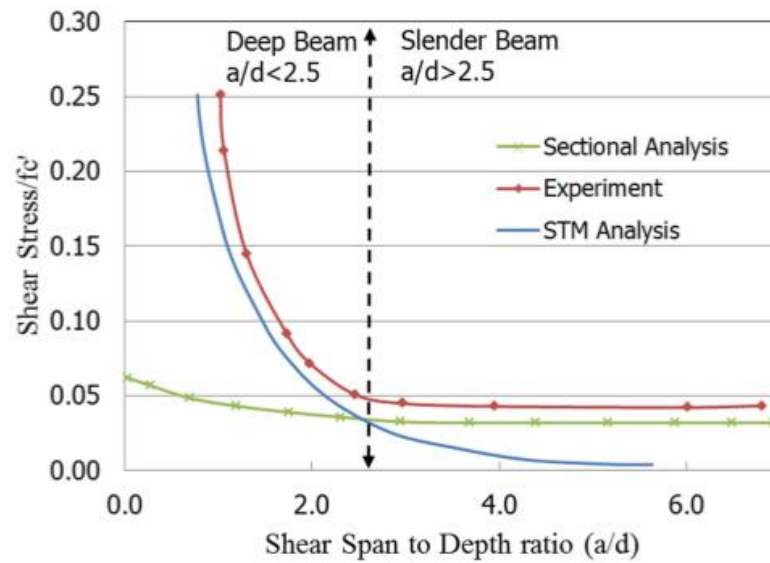


Figure 2.5 Variation of shear stress with a/d ratio (Collins and Mitchell, 1991)

Table 2.1 Comparison of Deep Beam and Slender Beam

DEEP BEAM	SLENDER BEAM
After deformation, plane section does not remain plane	After deformation, plane section remains plane
Shear deformation is significant compared to flexure deformation	Shear deformation is neglected
Shear span to the effective depth ratio is less than 2.5	Shear span to the effective depth ratio is bigger than 2.5
The resulting strain is non linear	The strain is linear

Reinforced concrete (RC) deep beams have been used mainly as transfer girders, wall footings, and foundation pile caps in building structures. The shear failure is usually the dominant behaviour in RC deep beams due to its relatively small shear span to effective depth ratio.



Figure 2.6 Examples of “Deep Beam” applications in constructions (Source: <https://www.emporis.com>, Ball, 2016)

2.3 Shear Transfer Mechanism

2.3.1 Fundamental Mechanism

For beams which do not have a shear reinforcement, the shear applied is spread over the surface of the inclined crack through a combination of shear in the compression zone (v_c), a vertical component of aggregate interlock (v_{ai}) and dowel action (v_d) (MacGregor et al., 2012). These three components contribute specifically to the shear resistance mechanism. Since decades, each of these elements has been the subject of study and remains a subject of discussion. In fact, some shear is transferred over the crack by residual tension in addition to these three shear resistant mechanics; nevertheless, this is relatively small particularly in cases of large cracks (Ismail, 2016).

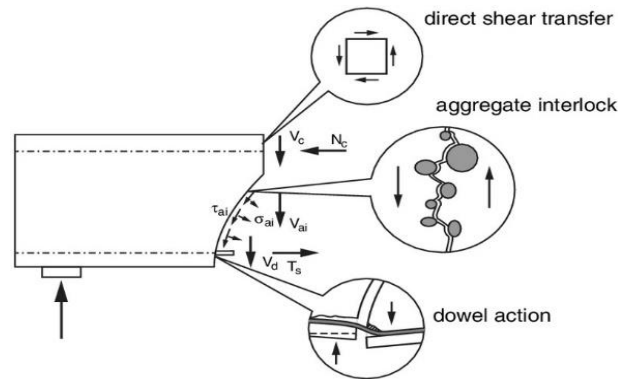


Figure 2.7 The forces transferring shear across an inclined crack in a beam without stirrups, by Yang et al. (2017)

2.3.2 Truss Mechanism

According to Ritter (1899) and Morch (1902), a truss action is enabled if the applied load is far from the support. The top is a longitudinal compression chord and the bottom of the tension chord is formed by a longitudinal tension reinforcement. Because of this mechanism, shear forces are transmitted up and down through beam depth by inclined compressive forces that can be transmitted by ties and concrete.

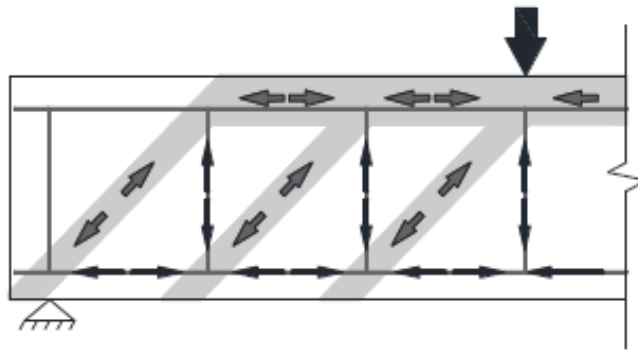


Figure 2.8 Truss mechanism, by Yang et al. (2017)

2.3.3 Modified Compression Field Theory (MCFT)

In the 1970s, Compression Field Theory (CFT) was developed by Mitchell and Collins (1974) and then this theory is improved as Modified Compression Field Theory (MCFT) to build on the by means of a definition of the behavior of RC components under pure torsion. The main distinction between MCFT and CFT is the use of MCFT's concrete tensile strength. Vecchio and Collins (1986) also found that cracking concrete can carry a significant amount of stress in the main tensile direction, based on stress and strain of the tested components. Hence, the tensile strength of the cracked concrete that was ignored before and then it was added to the fundamental material models.

Vecchio and Collins (1986) improved MCFT on the basis of the generalized hypothesis which the average direction of the principal compressive stress in the fractured concrete is correlated to the average direction of the principal tensile strain and that inclination of the critical crack is parallel to the direction of the main compressive stress. In fact, this theory suppose that there is only one related strain state for any stress state and concrete and reinforcement are perfectly connected together. Figure 2.9 summarizes the MCFT's equilibrium and compatibility equations, and stress-strain relationship.

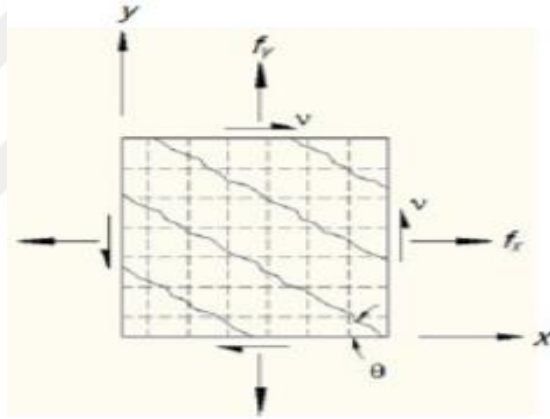
Equilibrium:

Average Stresses:

1. $f_x = \rho_x f_{xx} + f_1 - v \cot \theta$
2. $f_y = \rho_y f_{yy} + f_1 - v \tan \theta$
3. $v = (f_1 + f_2) / (\tan \theta + \cot \theta)$

Stresses at cracks:

4. $f_{xxcr} = (f_x + v \cot \theta + v_{ci} \cot \theta) / \rho_x$
5. $f_{xycr} = (f_y + v \tan \theta - v_{ci} \tan \theta) / \rho_y$

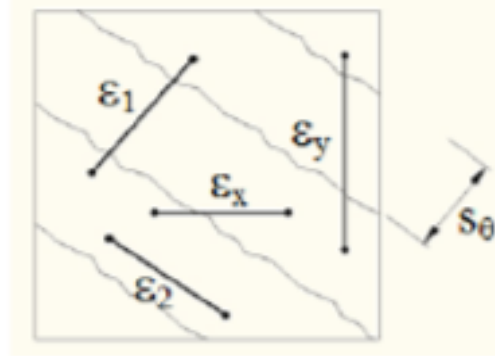
**Geometric Conditions:**

Average Strains:

6. $\tan^2 \theta = \frac{\epsilon_x + \epsilon_2}{\epsilon_y + \epsilon_2}$
7. $\epsilon_1 = \epsilon_x + \epsilon_y + \epsilon_2$
8. $\gamma_{xy} = 2(\epsilon_x + \epsilon_2) \cot \theta$

Crack Width:

9. $w = s_\theta \epsilon_1$
10. $s_\theta = 1 / \left(\frac{\sin \theta}{s_x} + \frac{\cos \theta}{s_y} \right)$



Stress-Strain Relationships:

Reinforcement:

$$11. f_{sx} = (E_s \varepsilon_x \leq f_{ys})$$

$$12. f_{sy} = (E_s \varepsilon_y \leq f_{yy})$$

Concrete:

$$13. f_2 = \frac{f'_c}{0.8 + 170 \varepsilon_1} \left(2 \frac{\varepsilon_2}{\varepsilon_c} - \left(\frac{\varepsilon_2}{\varepsilon_c} \right)^2 \right)$$

$$14. f_1 = 0.33 \sqrt{f'_c} / (1 + \sqrt{500 \varepsilon_1})$$

Shear Stress on Crack

$$15. v_{ci} \leq \frac{0.18 \sqrt{f'_c}}{0.31 + \frac{24w}{a_g + 16}}$$

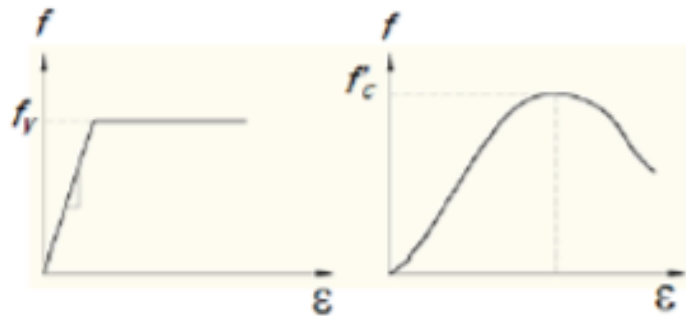


Figure 2.9 Equations of modified compression field theory.

2.3.4 Strut and Tie Model (STM)

The sectional method was not found to produce significant effects for some beams. Thus most design codes replaced deep members' sectional design methods by STM. Strut and tie model is very useful method for discontinuity or disturbed (D) regions in concrete structures and it states complex stress patterns as triangulated or trapezoidal shapes (Fig. 2.10). STM is depended on truss analogy and it can be applicable to many elements of concrete structures (Schlaich et al., 1987). There are three truss elements which consist of strut, tie and node to transfer forces from applied load and support. If the member carries compression forces, these are called struts, while the force paths are in tension, these are tie and intersection point of strut and tie is named node (Bayrak et al. 2006).

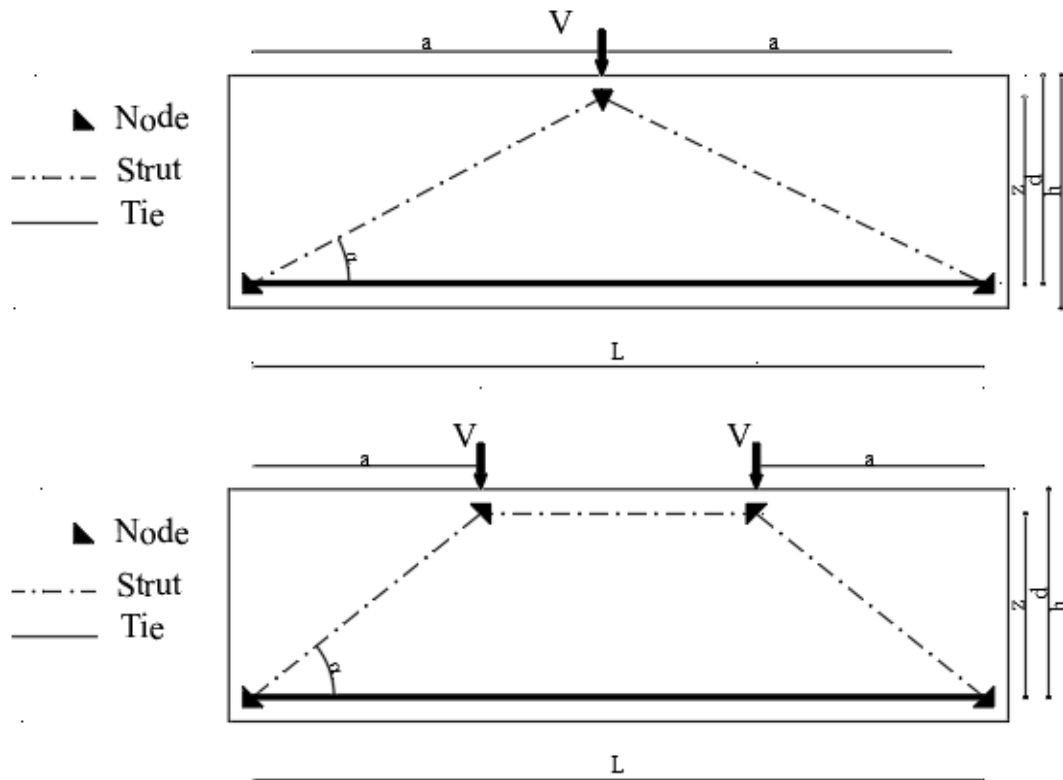


Figure 2.10 Stress patterns as triangulated or trapezoidal shapes in RC deep beam

2.3.4.1 Strut

Struts are divided into three types (Fig. 2.11) according to their widths and applied load type. These are prismatic strut which has constant width under point loads, bottle shaped strut which has wider width at center than two ends when applied point loads and also bottle stress fields are a feature of elastic stress field analysis (Brown et. al, 2008) and fan shaped strut which stress flows from large area to small area under distribution loads. Most of the struts are bottle shaped which spread laterally along their length in a two-dimensional STM. The lateral extension of a bottle shaped strut presents tensile stresses transverse to the strut (Vancoillie, 2018). Such tensile stresses result in premature failure and breaking along the length of the strut. Struts often are idealized as prismatic in STM in the form of bottles. Nonetheless, this simplification does not eliminate the risk of longitudinal splitting and still being in the shape of bottles. In the compression zone of the flexural region of a beam, prismatic struts occur. The other type of strut is fan shape strut which is under distribution load.

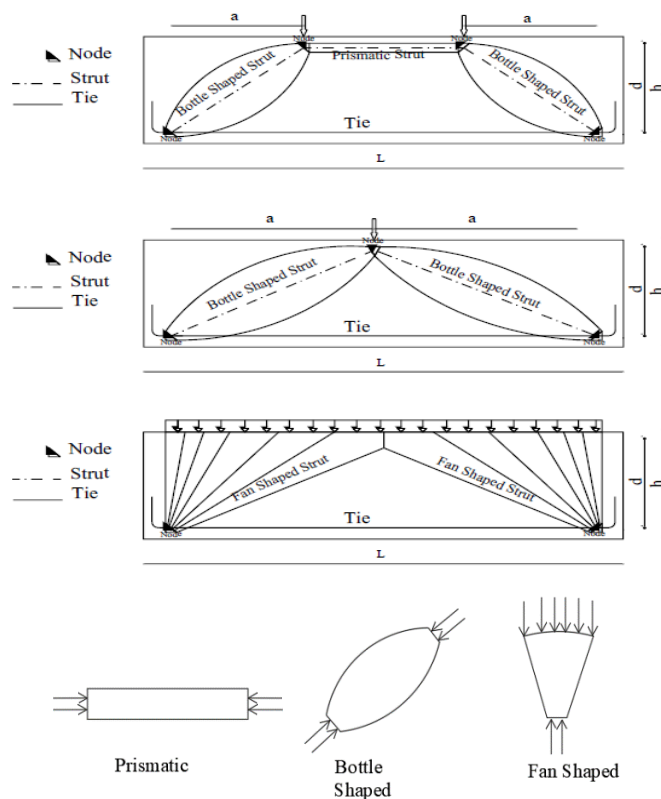


Figure 2.11 Types of struts

Effective compressive strength of strut;

$$\phi f_{ce} = 0.85 \beta_s f'_c \quad \text{eq.1}$$

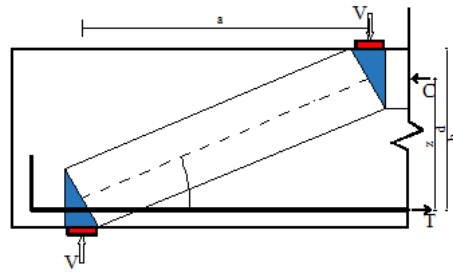
Strut strength;

$$\phi F_{ns} = \phi f_{ce} A_{cs} \quad \text{eq.2}$$

F_{ns} is the nominal compressive strength of the concrete in a strut, A_{cs} is the smallest effective cross-sectional area of a strut, f_{ce} is the effective compressive strength, β_s and ϕ are strength reduction factors for struts.

2.3.4.2 Tie

Reinforcing steel is generally placed in an STM at tie locations. The reinforcement should be spread to coincide with the position of the tie in its centroid. It is easy to select and place reinforcement for STM. For selection and positioning of the reinforcement, details like bar spacing, distribution and anchoring are the most important factors. The most deserving of consideration are proper bar distribution, spacing and development. The connections must be adequately anchored behind the nodal zones to develop the reinforcing steel.

**Figure 2.12** Development length of a tie.

2.3.4.3 Nodal Zones

Nodes are called according to the type of interconnected members, such as, if all of the members in node has compression forces it is C-C-C node, and only one force is under tension the other is under compression it is C-C-T node (Fig. 2.13). Moreover, the dimensions of nodes are frequently determined by the widths of load and support plates as well as the position of ties (Vollum et al., 2010). The forces within truss can be calculated using static equilibrium if the truss is statically determinate.

When there are more than 3 forces intersecting at a point, certain forces often have to be overcome to finish with three. The three node forms used in an STM are identified in Fig. 2.14.

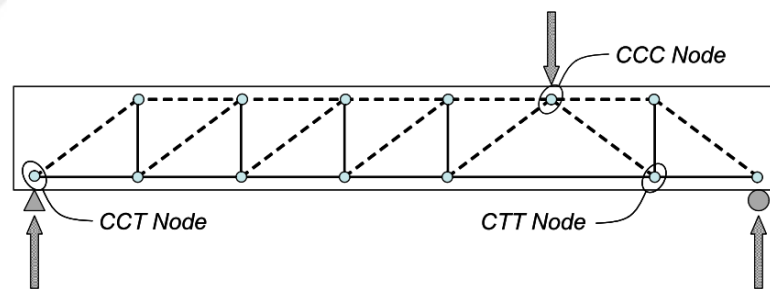


Figure 2.13 Nodal zones typically employed in STM, by Ismail, 2016

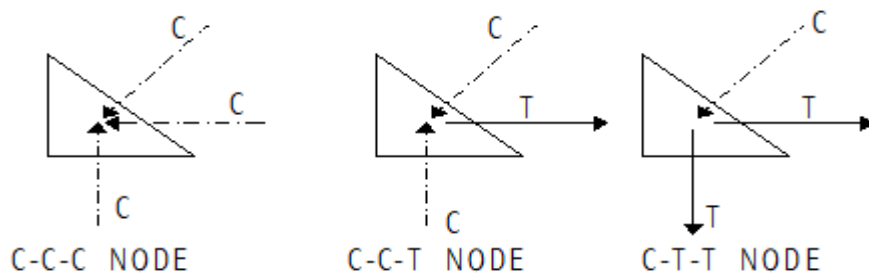


Figure 2.14 Types of nodes

Effective compressive strength of node;

$$f_{ce} = 0.85 \beta_n f'_c \quad \text{eq.3}$$

β_n is strength reduction factors for nodes. Nodes can be proportional so that the stresses on all faces are same. If the stresses are same on all faces, the ratio of the area of the side face is proportional to the applied force. In this case, the node is named a hydrostatic node (Bayrak et al., 2006). Principal stresses are same on all faces of a hydrostatic node; therefore, shear stresses do not occur within the node. However, the requirement of equal stresses on all faces of a node is rarely realized in practice as the requirement is impractical or inconvenient to accomplish. Therefore, most nodal regions are non-hydrostatic.

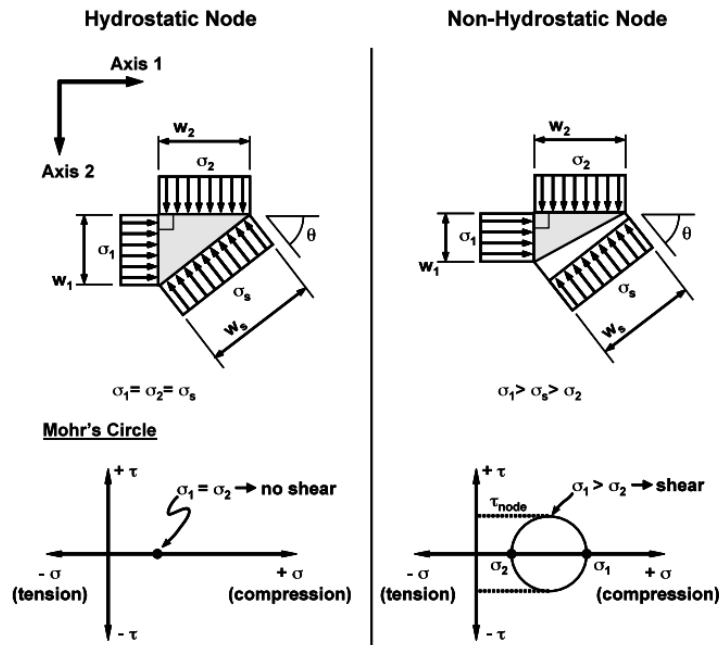


Figure 2.15 Types of nodal regions, by Bayrak et al., 2006

2.4 Failure Modes

Generally, flexural cracks formed initial in constant bending moment area out of the shear span and later spread into shear span. As the applied load increased a) diagonal crack or cracks grew rapidly in the middle of the shear span and their orientation was controlled primarily by the shear span to depth ratio, b) the flexural cracks continued to spread into the beam compression side until the total beam depth reached about two thirds and c) initially, diagonal cracks were propagated to the load, and then back to the support. In most situations, a second main inclined crack improved parallel to the initial crack and together they caused failure. (Ismail, 2016)

There are 4 different modes of failure, these are;

- a. **Diagonal Splitting failure:** It exists when the diagonal cracks grew firstly towards the load then the support. Also compared to the shear failure modes of other specimens, this failure was less brittle. It will occur when the horizontal component of the diagonal splitting force is less than the capacity of the tensile reinforcement and less than the compression capacity. If the compression capacity greater than the capacity of tensile reinforcement, the diagonal crack may penetrate into tensile flange. (Kong, 1990)



Figure 2.16 Diagonal Splitting failure, by Ismail, 2016

- b. **Shear Compression failure:** It occurs after diagonal cracks have developed in the shear span that cause high stress in the compression zone above the cracks

tip, that lead to an explosive failure (Ismail, 2016). Also, this is crushing of concrete near the tip of the inclined crack in compression zone. That is caused by inadequate concrete strength.

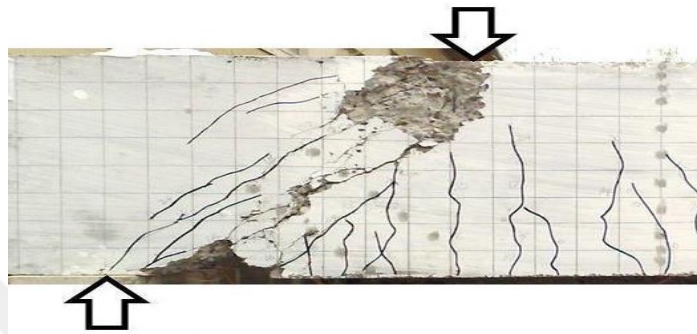


Figure 2.17 Shear-Compression failure, by Ismail, 2016

- c. **Strut compression failure:** This form of failure occurred because of the concrete crush in the shear span between inclined cracks. It has observed for shear span to depth ratio less than one. (Ismail, 2016)

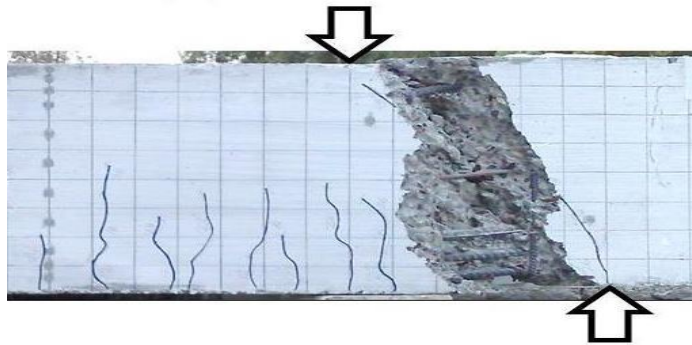


Figure 2.18 Strut compression failure, by Ismail, 2016

- d. **Flexural Compression failure:** This type of failure happened after yielding of the flexural reinforcement because of concrete crush in the constant moment area. (Ismail,2016)

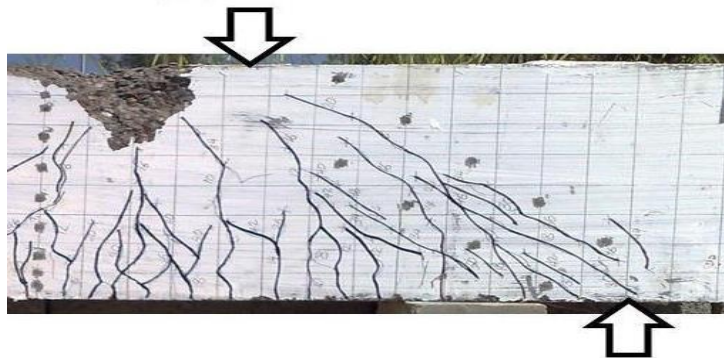


Figure 2.19 Flexural Compression failure, by Ismail, 2016

CHAPTER 3

LITERATURE REVIEW

3.1 Overview

In this chapter earlier studies about shear and deep beams according to STM are shown. And then ACI 318, EC 2 and AASHTO provisions are explained and shear capacity calculation is shown step by step.

3.2 Earlier Research Studies on Shear

For the past years, reinforced concrete design was based on limiting stresses at service loads and the ACI 318 (1951) shear provisions expressed that if the shear stress caused by service loads which did not exceed $0.03f_c'$ and stirrups were not required. However, one of the big shear disappointment occurred in Ohio in 1955. Failures of 914 mm deep beams in Air Force warehouses and it showed that these provisions could be unsafe, and

the resulting surge in shear research caused the revisions proposed by Joint ACI-ASCE Committee 326 (1962) that still form the basis of the current ACI shear provisions.



Figure 3.1 Shear failure of 914 mm deep beams in Wilkins Air force warehouse (Ohio,1955)

The aim of this proposed changes was to review the basic philosophy of shear design and due to this failure shear stress was changed to be a function not only of the concrete strength but also of the coexisting moment expressed in terms of the parameter $M/(\rho_w Vd)$ (Joint ACI-ASCE Committee 326, 1962). This parameter was chosen because it was assumed that failure shear stress v would decrease as the longitudinal steel stress f_s increased and this steel stress equals approximately $M/(\rho_w Vd)$ times v . When a short distance from the load applied to the closest reaction point (i.e. the shear span) was observed, the components failed at 100 percent more load than the diagonal load. Nevertheless, since it was known that this post-diagonal cracking force could not be predicted accurately, it was presumed that the failure load was equal to the diagonal cracking load. A considerable amount of research was conducted in the 25 years after 1960 on the mechanisms by which cracked reinforced concrete beams transmitted shear. Shear force in a region shows that the moment varies along the member's length. Fenwick and Paulay (1968) noted that this moment could change either by a tension in reinforcement changing that is known as beam action or by a change in length of the inner lever, known as arc action. In order that arch action carries the whole shear in a region with a constant shear, an inclined strut from the load to the support needs to compose the concrete compression zone. It has been shown that due to the current two mechanisms' geometric incompatibility with beam action usually being much stiffer than

arch action, nearly all the shear would be conducted by beam action until this mechanism failed. After beam mechanism fails, the internal redistribution of stresses could occur and if the distance between the load applied and the support was short, the remaining arch mechanism could hold even higher shears. By using detailed measurements of cracked beams and direct measurements on subassemblies, Fenwick and Paulay (1968) concluded that around 70% of vertical shear on a flexurally cracked section is carried across the flexural crack and that it is the breakdown of aggregate interlock action which causes the shears failure for slender beams without stirrups. The main role played by aggregate interlock was confirmed later by Taylor (1970), Kani et al. (1979), and Sherwood et al. (2007). Walraven's (1981) basic studies indicated that the shear stress that can be transmitted across a crack by aggregate interlock reduces as crack width increases and as aggregate size decreases. Vecchio and Collins (1986) have improved modified compression field (MCFT) theory that uses equilibrium, compatibility and stress-strain relations in order to predict the shear response of cracked reinforced concrete elements by testing the reinforced concrete elements subject to pure shear in a specially designed test system. The theory predicts that failure can occur for members with longitudinal reinforcement only when the shear stress on the crack faces necessary to maintain an equilibrium is the highest shear stress that can be transferred on with the aggregate interlock. As the expected width of the inclined crack increases, the expected failure shear stress reduces. Therefore, shear failure decreases with the increasing tensile strain in longitudinal reinforcement, which is known the strain effect, and as the increase crack distance nearly the mid-depth, the size effect. Kani et al. (1979) performed 362 shear tests on reinforced concrete beams without shear reinforcement in 1960, in an attempt to change systematically all major variables affecting shear strength. Kani's experiments (1966) have shown that the fundamental ACI equation has overestimated the effect of changing concrete strength and the influence of the increase in $M/(\rho_w Vd)$ parameter. He also summarized (1967) that as the depth of the member increased, shear stress at failure decreased and it is important to

realize that strain in the flexural tension reinforcement depends not only on moment but also on shear.

3.3 Experiments of Deep Beam Using Strut and Tie Model

The shear capacity determines strength of deep beams but the behavior of shear cannot be explained clearly. Thus some of the researchers also concentrate on this aspect and try to improve the models, formulae etc. with respect to the influencing parameters on shear capacity. The main purpose of this study was to provide a collection of latest guidance and improvements not to provide the best approach or model for the design of deep beams.

De Pavia & Siess (1965), has carried out an experimental study on the shear strength and behavior of some deep concrete beams. In the experimental investigation main parameters considered include tension reinforcement, concrete strength, amount of web reinforcement, shear span to the depth ratio. They concluded that reinforced concrete deep beams without web reinforced beams that had a high ability to crack beyond diagonal cracking and had little effect on ultimate strength by the addition of vertical stirrups and inclined bars.

The experiments on deep beams with loading top and bottom were also recorded by Leonhardt and Walther (1966). The simply supported specimens had a height /span ratio of 1.5. They chose to use well supported support bars and horizontal hooks to provide the most appropriate reinforcing alternative to anchor. They suggested also to spread the main reinforcement over less than 20 percent of the beam height. Stirrups were suggested to be extended to a height equivalent to span. Near spacing (< 400 mm), stirrups were suggested with vertical stirrups reaching the full height of the beam in order to reduce crack distance.

Gergely (1969) developed an experimental model to test the effect of aggregate interlocking and dowel action on reinforced beam cracking capacity without web reinforcement. He assessed contribution of aggregate interlock to be 40-60 percent and

dowel action was assessed to be 20-25 percent of total shear. Also, Taylor (1970) conducted several experiments to investigate same effect (aggregate interlocking, dowel action and compression zone). He found that compression zone 20-40 percent, aggregate interlock 33-50 percent, dowel action 15-25 percent of total shear.

Birrcher et al. (2014) focused on strength of reinforced concrete deep beams due to different section depths. Because of this, they observed three different a/d ratio (1.2, 1.85 and 2.5) to compare them with each other. The result shows highest a/d ratio (2.5) gave unconservative result.

The concrete compressive strength and stress limits in nodal areas were improved by Tuchscherer et al. (2014). They made recommendations for current deep beam and D-regions of STM design procedures. The three codes ACI 318-11, Fib and AASHTO LRFD are taken into consideration to ensure the proposed changes are compatible with the current design processes.

Su & Looi (2016), with different angles and concrete strength, studied the compressive strength of struts with a deep beam without stirrups. In order to achieve this, variations between different codes were analyzed in the strut efficiency factor and some proposals were made by the authors. Authors added a single constant for the unreinforced strut efficiency factor, this is also the case in the ACI 318-11. The difference between the authors suggestion and the ACI 318-11 code provision is that the proposed constant is less conservative.

The strut efficiency factor is important since it is directly connected to the shear capacity of a deep beam and this is again connected to the strength of the deep beam itself. Tuchscherer et al. (2016) specified that the AASHTO should modificate the efficiency factor, Su and Looi (2016) revised the value of the nominal strut efficiency factor of the ACI 318-14. The most interesting ones were the improvements Mohamed et al. (2016) and Ismail et al. (2018), they introduced new formulas to calculate strut efficiency factor. These mentioned formulas comprised the parameters the authors thought to be the most influential, namely the compressive strength, tensile strain and shear span depth

ratio. It makes for more precise and accurate results. In the available codes, all three parameters are not taken into account with together. Code provisions and future studies will take advantage of these performance considerations in order to produce more precise and accurate results. My conclusion, regarding the strut efficiency factor, is that at the moment these are the most important improvements that could be made and even investigated in the future. Because they have such a great impact on the shear capacity of a deep beam.

Chen et al. (2018) suggested an efficiency factor but for the broken part of the strut in his cracking STM model. Aggregate interlock action and the sum of dowel action of the longitudinal reinforcement and the tensile strength in web reinforcement combined in the efficiency factor, so it was able to accurately and safely predict the shear resistance of a deep beam.

All the ideas are explained and discussed can be used for future study as a basis to adopt ideas for other researches.

3.4 Available Shear Provisions

The worldwide codes and guidelines may contain the many shear strength models and design formulations. Code designs are generally depended on the truss analogy model that was prior by Ritter (1899) and Morsch (1902). They are used to predict the shear strength of current members in very conservative underestimates of the actual shear strength (Verderame et al., 2013). Eurocode 2, ACI 318 and AASHTO LRFD shear design procedures for RC members are the most used. Actually they are not effective tool to evaluate shear capacity because they are not consider more parameters.

3.4.1 ACI-318 Provisions

The American Concrete Institute (ACI) published Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14), and it was adopted by the 2015 International Building Code (IBC) (ICC, 2015). ACI 318 joint design provisions are depended on the assumption that joint shear strength is provided mainly

by a diagonal compression strut that develops across the joint. ACI 318 code supplies simple equation to calculate the shear strength at RC beams. Shear strength is carried in a member with no shear reinforcement by concrete web of the member and it is calculated using Eq. 4. This equation only take account for the effect of concrete compressive strength. The shear equations of ACI 318-14 are depend on the shear force instead of the ultimate shear force. Nevertheless, reinforced concrete beams do not restrict the ultimate shear capacity to the load that causes initiation of diagonal cracking. (Ismail, 2016)

$$V_c = 0.17 \sqrt{f'_c} b_w d \quad \text{eq.4}$$

Where, V_c is shear capacity of concrete, b_w is beam width and d is effective depth of beam.

3.4.2 EC 2 Provisions

Eurocode 2 offers an empirical equation for predicting RC beams ' ultimate shear strength with no shear reinforcement. Below equation take account for the effect of main flexural reinforcement, concrete compressive strength and size effect.

$$V = (C_{Rd,c} (100 \rho_l f_{ck})^{1/3} (1 + \sqrt{\frac{200}{d}})) b_w d \quad \text{eq.5}$$

The values of $C_{Rd,c}$, for use in a Country may be found in its National Annex. The suggested value for $C_{Rd,c}$ is $0.18 \gamma_c$, f_{ck} is specified concrete cylinder strength and ρ_l is the flexural reinforcement ratio.

3.4.3 AASHTO LRFD Provisions

The general procedure of the AASHTO LRFD specifications' sectional design model for shear was derived from the MCFT. AASHTO LRFD specifications' procedure accounts for the influence of concrete compressive strength and factor indicating ability of diagonally cracked concrete to transmit tension and shear, β .

$$V_c = 0.083\beta\sqrt{f'_c}b_vd_v \quad \text{eq.6}$$

3.5 STM Code Provisions

The strut and tie model which is a rational approach method is depend on the lower bound theorem of plasticity theory. Also it is used for designing discontinuous regions or D-regions of RC members. Actually, it is an alternative model to forecast shear strength of RC beams. Available codes of practice suggest to use this model for designing RC deep beams, for which shear is critical failure type. Current codes of practice such as ACI 318, EC2, and AASHTO, however, provide very little guidance in using STM, and the information supplied is mainly restricted to define the design concrete strength for concrete struts and nodes. The design provisions of deep beams using the STM method are discussed in this section.

3.5.1 ACI-318 Provisions

ACI 318 code added an Appendix regarding to STM in 2002 and then in 2005 modifications were applied which was mostly adopted from European code documents. The Strut and Tie model (STM) proposed by the American Concrete Institute (ACI) 318 – 2014, gives the following equations for the estimation of the shear strength of a deep beam based on the strength of the struts, ties or nodes developed in the beam due to loading. (ACI 318, 2014)

$$F_{ns} = f_{ce}A_{ce} \quad \text{eq.7}$$

$$F_{nt} = f_yA_{ts} \quad \text{eq.8}$$

$$F_{nn} = f_{ce}A_{nz} \quad \text{eq.9}$$

F_{ns} is the nominal compressive strength of the concrete in a strut, F_{nt} is the nominal strength of a tie, F_{nn} is the nominal compressive strength of a nodal zone, A_{cs} is the smallest effective cross-sectional area of a strut, A_{ts} is the area of non-prestressed reinforcement, A_{nz} is the area of the face of the nodal zone. Actually, there are 3 types

of struts. However, two efficiency factors are associated with bottle shaped struts so 4 classes of struts are listed in table 3.1. Two of these factors are based on the reinforcement within the strut. The stress is developed when the compression spreads out from the support. The compression is applied vertically and the induced tension is horizontal. When the generated tensile stress exceeds the concrete's tensile strength, a vertical crack develops. The strut could break without any horizontal reinforcement resulting in brittle failure. This is the basis for the split cylinder test often used to assess concrete tensile strength. However, if adequate transverse reinforcement is available, brittle failure could be avoided, and the strut can continue to transfer load beyond the cracking load. (Brown et. al,2008) Moreover, the angle between the strut and tie should not be less than 25 degrees in ACI 318. When the angle between the strut and the tie is less than 25, the strength of the strut also approaches zero (Collins et al. 2008).

3.5.2 EC 2 Provisions

Strut and tie model in Eurocode in 2004 follows the lower bound theory of plasticity. EC-2 gives design concrete strengths for struts which are based on simple criteria that account for the effects of cracking and confinement. According to EC 2, struts are checked at their ends where the cross sectional area is defined by the node dimensions. The concrete effectiveness factor, that is based on concrete compressive strength and it does not take account the effect of shear reinforcement and transverse tensile strain, is specified in equations as follows;

$$v' = 0.6 \left(1 - \frac{f_{ck}}{250} \right) \quad \text{eq.10}$$

$$\sigma_{Rd} = v f_{cd} \quad \text{eq.11}$$

$$f_{cd} = \frac{0.85 f_{ck}}{1.5}$$

eq.12

Where; v is effectiveness factor, f_{ck} is specified concrete cylinder strength σ_{Rd} is allowable stress and f_{cd} is design compressive strength. Also the angle between the strut and tie should not be less than 31 degrees in EC2.

3.5.3 AASHTO LRFD Provisions

The strut and tie mode were introduced into AASHTO LRFD Bridge Design Specifications code in 1994 and developed based on mechanical models derived from tests of reinforced concrete panels (AASHTO, 1998) Also, it is based on the modified compression field theory (MCFT) (Reineck 2002). Like ACI 318-14, AASHTO LRFD code provision calculates the strength of the struts of a strut and tie model as a specified compressive strength of the concrete;

$$f_{cu} = \lambda f_c' \leq 0.85 f_c'$$

eq.13

$$\lambda = \frac{1}{0.8 + 170\varepsilon_l}$$

eq.14

$$f_{cu} = \frac{f_c'}{0.8 + 170\varepsilon_l} \leq 0.85 f_c'$$

eq.15

$$\varepsilon_l = \varepsilon_s (\varepsilon_s + 0.002) \cot^2 \alpha$$

eq.16

$$P_n = f_{cu} A_{cs}$$

eq.17

Where; λ is the strut efficiency factor, α is angle between the compressive strut and the tie, ε_s is the tensile strain in the concrete in the direction of the tension tie, f_{cu} is the usable compressive strength, P_n is nominal resistance of a compressive strut, ε_l is principal tensile strain. AASHTO bases the efficiency on an average strain in the

concrete at the location of a tie. There is no limit the angle between the strut and tie in the AASHTO LRFD STM provisions and also if the angle between the strut and the tie close to zero, the strength of the strut also close to zero. Although very small angles are permitted by AASHTO, they become impractical because of the reduced efficiency factor of the strut. The reduced efficiency factors and the associated reductions in the permissible strength of struts inspire the design engineer to pursue a more refined truss mechanism without such shallow angles. For the nodal stress limits, AASHTO simply specifies factors depended on the type of node (Table 2.1). It should be noted that the AASHTO nodal efficiency factors are multiplied by f_c' directly rather than $0.85f_c'$ as specified in ACI 318-14, Appendix A. (Brown et. al,2008). It can be observed from the above equations that the concrete effective compressive stress, f_{cu} , will increase with the increase of angle between strut and tie. As mentioned above AASHTO does not specify any limitation on the value of angle however Rogowsky and MacGregor (1986) recommended a value for angle from 25 degrees to 65 degrees.

AASHTO LRFD provisions calculated concrete contribution to shear capacity V_c is the ability of the cracked concrete in order to carry diagonal tension in the web of member. This contribution is controlled by the level of longitudinal strain in the member, the reserve capacity of the longitudinal reinforcement at a crack location, and the shear-slip resistance of the concrete. Therefore, AASHTO LRFD provisions' concrete contribution is often considered to be controlled by interface shear transfer (aggregate interlock).

Table 3.1 Strut and node efficiency factors for codes

Types of strut and node	ACI-318-14 Efficiency Factor, β	AASHTO LRFD-2012 Efficiency Factor, λ	EC2-2004 Efficiency Factor, v
Prismatic strut	1	$\frac{1}{0.8+170\varepsilon_l}$	1
Bottle shaped strut with transverse reinf.	0.75	$\frac{1}{0.8+170\varepsilon_l}$	$v' = 0.6(1 - \frac{f_{ck}}{250})$

Bottle shaped strut without transverse reinf.	0.6	$\frac{1}{0.8+170\varepsilon_l}$	$v' = 0.6(1 - \frac{f_{ck}}{250})$
Compression fan strut	0.6	$\frac{1}{0.8+170\varepsilon_l}$	$v' = 0.6(1 - \frac{f_{ck}}{250})$
CCC node	1	0.85	1
CCT node	0.8	0.75	0.85
CTT node	0.6	0.65	0.75

3.6 Flowchart Design

Some authors suggest some flowcharts to show the different steps in STM designing. The below table (Table 3.2) is formed mixing of different proposes. Firstly truss shape is determined and then geometry part is calculated. Then STM calculation part is completed. Also, some parameters in this table are shown in fig. 3.2 and 3.3.

Table 3.2 Shear capacity calculation according to STM step by step

1. Determine truss shape of STM
• <i>trapezoidal or triangular</i>
2. Calculate lever arm (moment arm)
• $z = h - w_s/2 - w_t/2$

3. Calculate the angle between strut and tie

- $\alpha = \arctan(z)$

4. Calculate width of strut

- $w_{st} = l_b \sin \alpha + w_s \cos \alpha$

5. Calculate effective strength in struts

- $F_{ns} = f_{ce} A_{cs}$

6. Calculate effective strength in tie

- $F_{nt} = A_{st} f_y$

7. Calculate effective strength in nodes

- $F_{nn} = f_{ce} A_{cs}$

8. Calculate shear capacity

- $V_c = \min(F_{ns}, F_{nn}, F_{nt}) \sin \alpha$

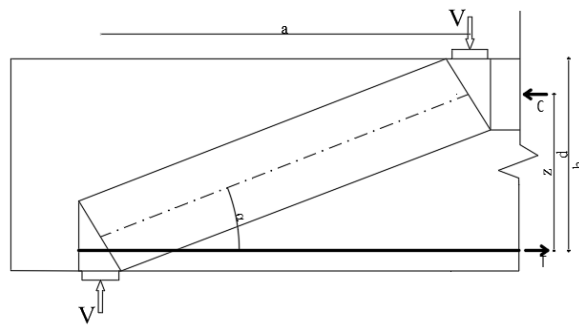


Figure 3.2 Free body diagram of deep beam

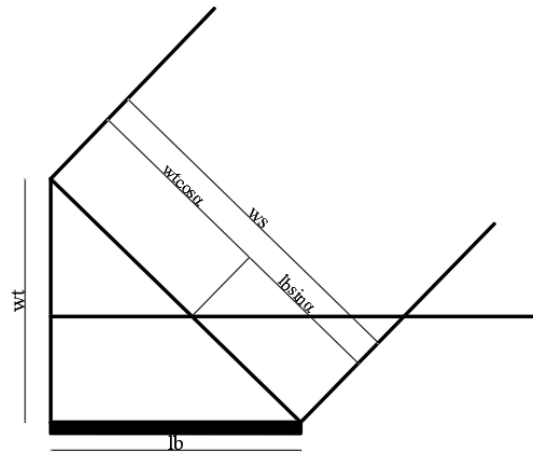


Figure 3.3 Free body diagram of node

CHAPTER 4

DEVELOPMENT OF SHEAR DATABASE

4.1 Overview

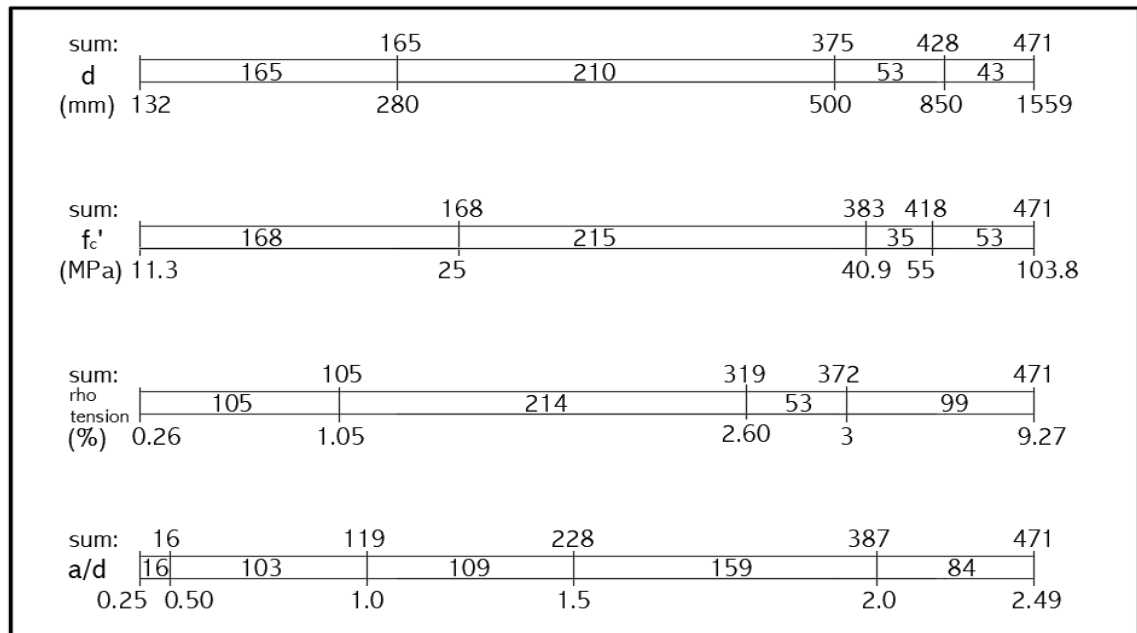
In this chapter, 471 reinforced concrete deep beam without shear reinforcement databases were collected from Collins, Bentz, Sherwood, Ismail, Todisco and the other

research papers. These test evaluated with respect to effective depth, concrete compressive strength, flexural reinforcement ratio, shear span to depth ratio and investigated statistically.

4.2 Collection of database

The database in this research was collected from database of Collins, Bentz, Sherwood, Ismail, and Todisco. The databases used in the paper includes only RC deep beam without shear reinforcement. For each RC deep beam test, data for material strength, cross section geometry, loading and support boundary conditions, flexural reinforcement and the measured failure load were collected. There were no limit on concrete strength, yield strength and geometrical member size for selecting database. Only point loads applied on simply supported or continuous deep beam which has rectangular or T cross section were considered in this database. Also, the data collected for the shear database was reported in SI units. The experimental results were compiled to form relationships among various factors affecting the shear strength. The variables used in the experiments were beam width (b), height (h), effective depth (d), shear span to effective depth ratio (a/d), the compressive strength of concrete (f_c') and longitudinal reinforcement ratio (ρ_l). The shear span to depth ratio, (a/d), of the deep beam in the database are between 0.25 and 2.49. In fact, 25.3 percent (119 out of 471) had values $a/d \leq 1$, 56.9 percent (268 out of 471) had values $1 < a/d \leq 2$ and 17.8 percent (84 out of 471) had $a/d > 2$. The distribution of parameters of 471 tests are shown in Table 4.1. Effective depth, main flexural reinforcement ratio, concrete compressive strength and shear span to depth ratio are used to describe the distribution of the test data.

Table 4.1 Distribution of parameters



4.3 Procedure for Comparison

The ratio of the measured ultimate shear strength, $V_{u, test}$, to the calculated value from STM code Equations, $V_{u, cal}$, is used for the comparison and is defined as the safety factor, γ , in this paper. The safety factor, γ , is given in Eq. (5).

$$\gamma = \frac{V_{test}}{V_{calc}} \quad \text{eq.15}$$

The predicted value of a given test is considered to be conservative when $\gamma > 1$. The material properties introduced in the formula were the average values obtained from the test reports.

4.4 Statistical Analysis

Table 4.2 summarizes the experimental shear load and the calculated shear strength load, V_{test}/V_{calc} , in terms of mean value, standard deviation (SD) and the coefficient of variation (COV) for comparison using STM Code provisions of ACI 318, EC 2 and

AASHTO LRFD. All reduction factors for strength and material are set to be equal to 1 when the STM code provisions were used to predict the failure load in RC deep beam experiments in the database. The mean, Standard Deviation (SD) and Coefficient of Variation (COV) of the ratio V_{test}/V_{calc} is used to examine the accuracy of the code provisions. Besides, the proposed classification adopted in Derogar (2013) is used to examine the distribution of V_{test}/V_{calc} . It is concluded that ACI and EC 2 predictions are categorized as conservative while the predictions by AASHTO is categorized as approximately safe. Table 4.2 Summary of the experimental shear load and the calculated shear load (V_{test}/V_{calc})

Table 4.2 Summary of the experimental shear load and the calculated shear load (V_{test}/V_{calc})

Code	ACI 318-14	EC 2-2004	AASHTO LRFD (2008)
RC Deep Beams without Shear Reinforcement			
Mean	1.30	1.49	1.15
S.D.	0.39	0.51	0.47
C.O.V.	0.15	0.26	0.22

4.5 Evaluation of the Performance of Code Provisions

The predictions obtained from the EC 2, AASHTO and ACI 318 and the empirical test results are shown in Figure 4.1 respectively for comparison. It was observed that the AASHTO has the lowest mean for the ratio of V_{test}/V_{calc} , however, a detail investigation on the distribution of V_{test}/V_{calc} reveals that AASHTO provides very unsafe predictions when compared to EC 2 and ACI STM provisions. The EC 2 STM shear design provision gave the most conservative predictions for RC deep beam members without shear reinforcement with a mean and COV of 1.49 and 0.26 respectively.

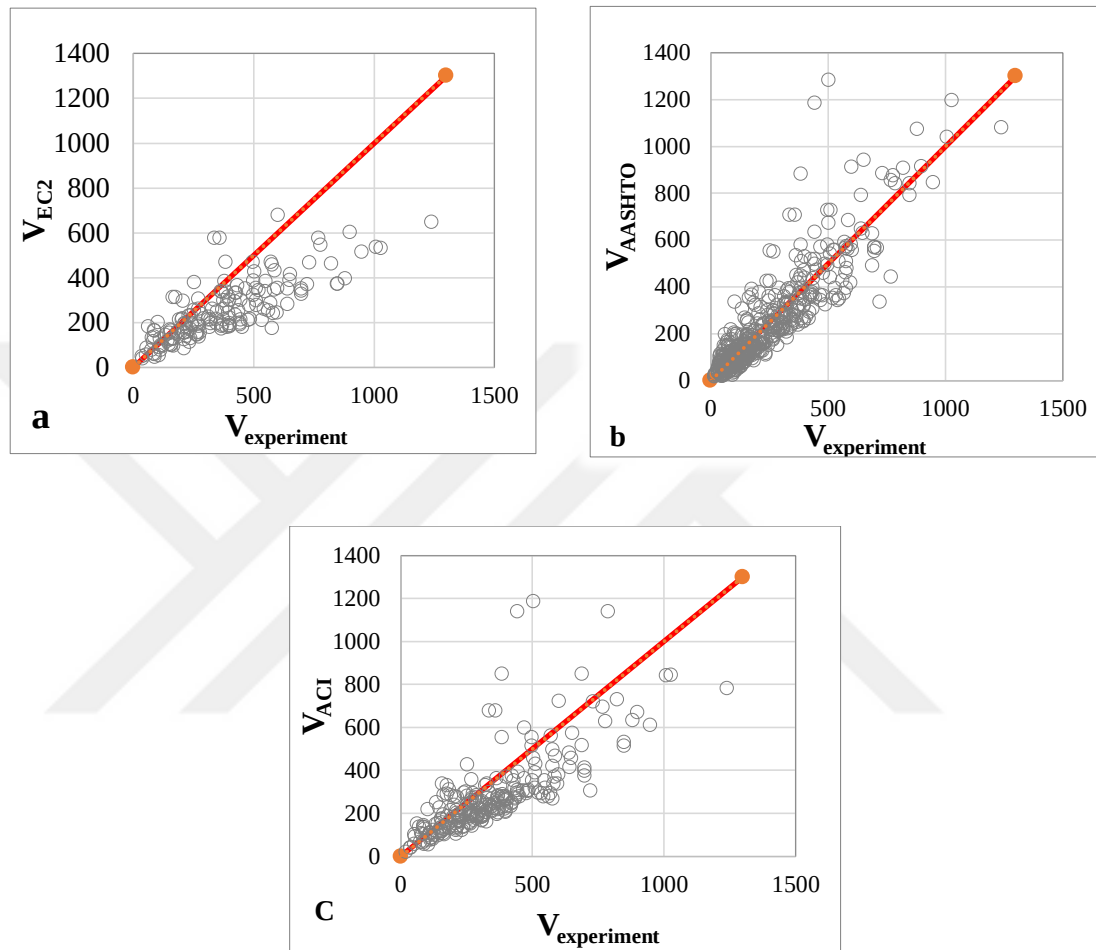


Figure 4.1 Comparison of Code STM procedures with empirical tests for RC deep beams without shear reinforcement: a) EC 2, b) AASHTO, c) ACI 318 predictions.

CHAPTER 5

PARAMETRIC STUDY ON SHEAR STRENGTH OF DEEP BEAM

5.1 Overview

It is worth noting that the large number of interrelated variables present in the reported experiments makes the isolation of individual parameters very challenging. It is particularly very difficult to eliminate the effect of other parameters when the influence of one specific variable is of concern. In fact, the influence of major variables shear span to depth ratio (a/d), concrete compressive strength (f'_c), flexural (tensile) reinforcement ratio (ρ_l) and effective depth (d) are still not accurately understood. To evaluate the influence of variables such as a/d , concrete compressive strength and flexural reinforcement ratio on STM shear prediction in different codes, particular experiments that have the same experimental configuration and the same test equipment with only one variable were selected. STM used with the provisions of all three codes (ACI 318, EC 2, AASHTO) of practice to forecast the shear strength of all deep beams contained in the database (for AASHTO 471, ACI 250, EC 2 162) without shear reinforcement. The quality of STM predictions according to EC2-2004, AASHTO LRFD (2008), and ACI 318-14 (2014) codes are evaluated using the safety factor, γ , given in Eq. 15. Safety factor, γ , is plotted versus the main parameters which affect the shear capacity of deep beams (concrete compressive strength, shear span to depth ratio, flexural reinforcement ratio and depth effect) in the following section.

5.2 Effect of concrete compressive strength (f'_c)

The compressive strength of concrete is one of the main important parameter which affect shear behaviour of members with no shear reinforcement. It is known that concrete compressive strength enhances the shear capacity of deep beams, where this enhancement is attributed by the effect of arching action. Figure 5.1 represents the

comparison for the experiments from Xie et al. (1994), Oh and Shin (2004), and Morrow and Viest (1957) with the predictions of EC 2-2004, AASHTO LRFD, ACI318-14 codes. Safety factor, γ , is plotted versus concrete compressive strength. Figure 5.1 shows unsafe prediction for the beams with high concrete compressive strength when the beam is designed according to ACI 318-14 and AASHTO LRFD. According to Figure 5.1, the shear strength of the deep beams are well predicted for EC2-2004. Eurocode provisions include the contribution of concrete compressive strength on the effectiveness factor of the inclined strut, however, ACI and AASHTO ignore this effect.

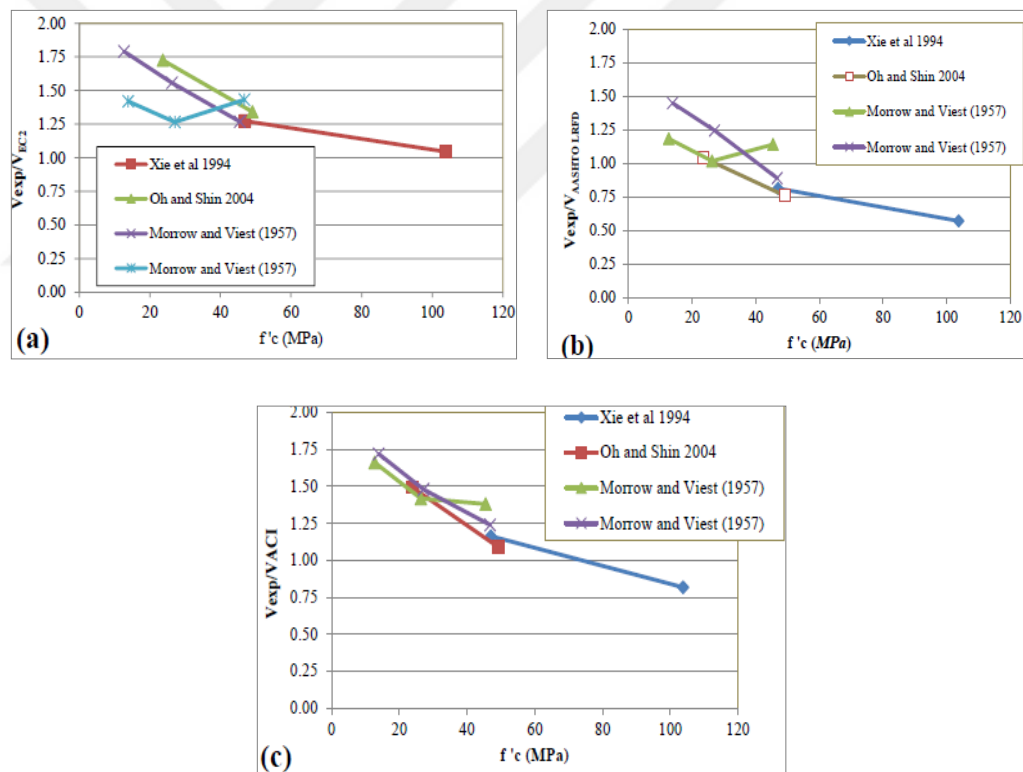


Figure 5.1 Effect of concrete compressive strength on the safety factor $\gamma = V_{exp}/V_{calc}$ for a) EC 2 predictions, b) AASHTO LRFD predictions, c) ACI 318 predictions plotted versus concrete compressive strength

Figure 5.2 shows the comparison for predictions of EC 2, AASHTO LRFD, ACI 318 codes with respect to concrete compressive strength. Safety factor, γ , is plotted versus concrete compressive strength. The effect of concrete compressive strength was observed based on AASHTO group of 471, EC 2 162, ACI 250 RC beams without shear reinforcement as shown in Figure 5.2. According to EC 2, AASHTO and ACI the value of concrete compressive strength in this group ranged from 11.3 to 103.8 MPa. According to Figure 5.2, the shear strength of the deep beams are well predicted for EC 2 rather than AASHTO and ACI 318 because EC 2 has low shear span to depth ratio. (Fig 5.5)

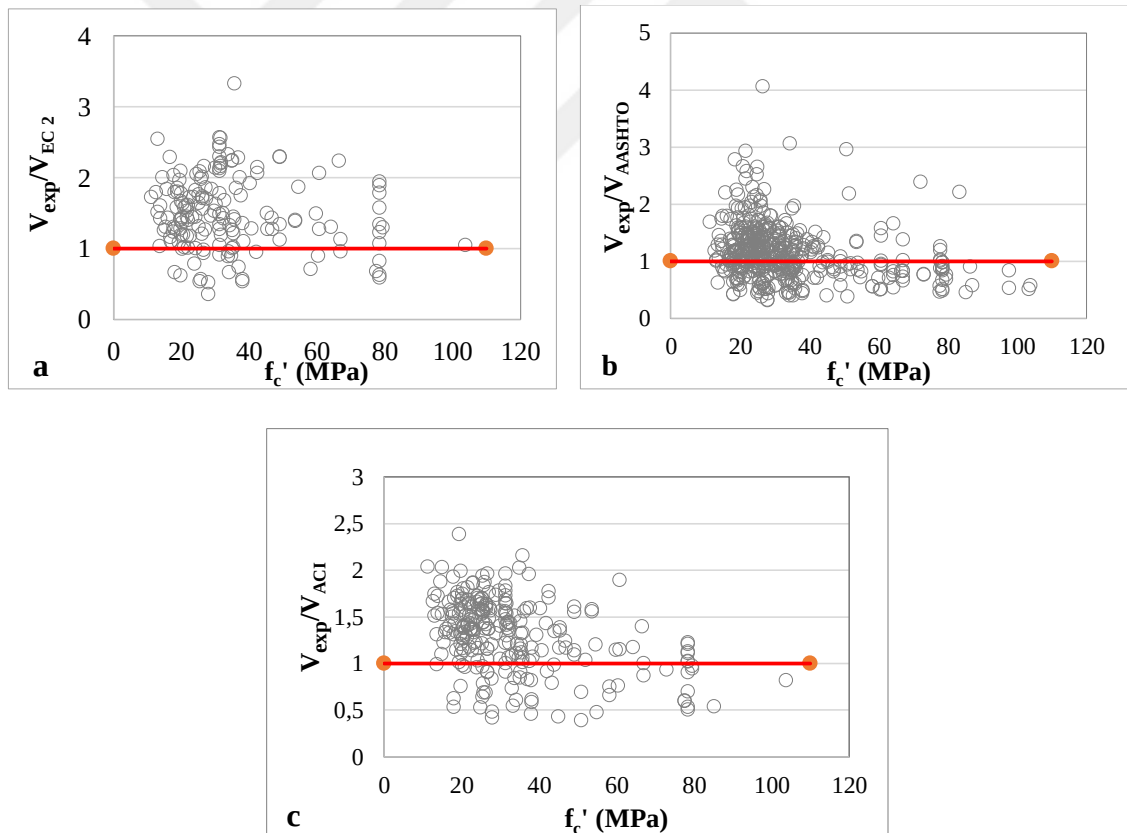


Figure 5.2 Shear strength prediction by STM with a) EC 2 provisions b) AASHTO LRFD provisions c) ACI 318 provisions versus concrete compressive strength

Fig. 5.3 shows effect of concrete compressive strength on shear strength by STM codes. This graph shows that shear strength increases when concrete compressive strength increases. However, highest value of concrete compressive strength gives unsafe and unconservative result for all codes.

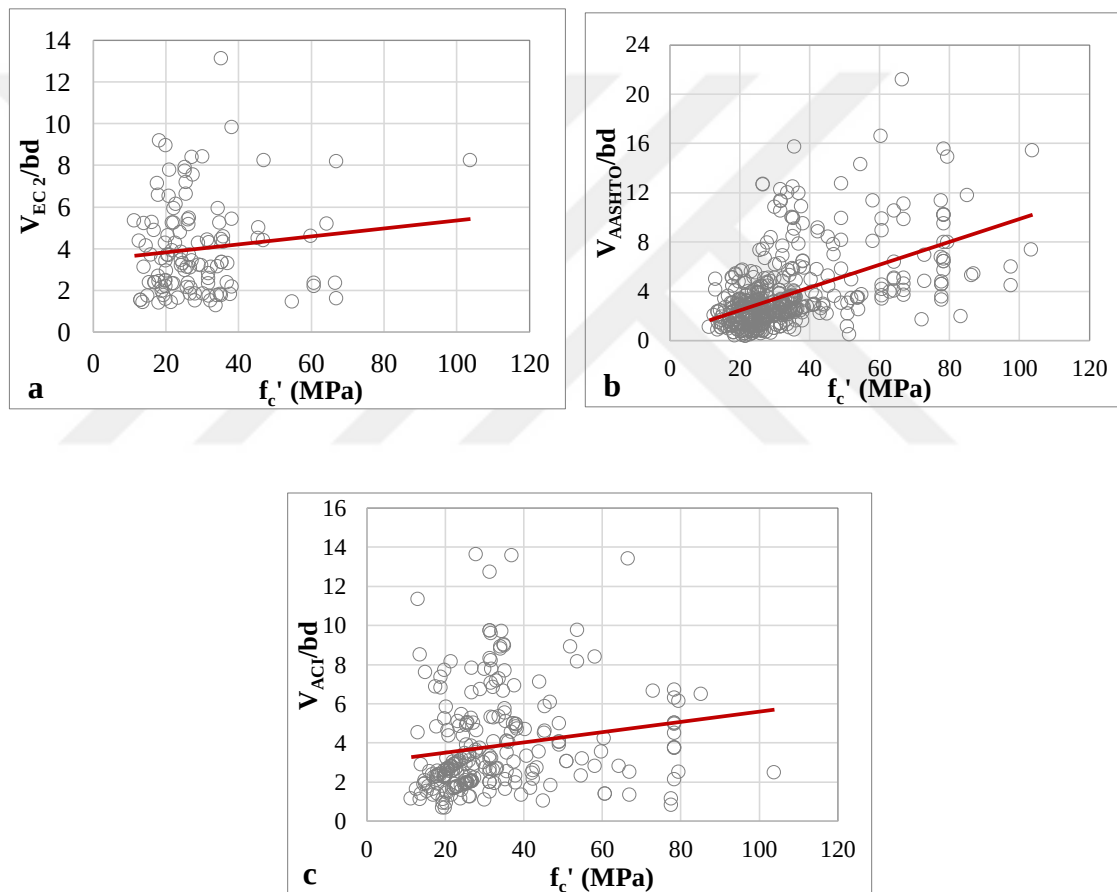


Figure 5.3 Effect of concrete compressive strength on shear strength by STM with a) EC 2 provisions b) AASHTO LRFD provisions c) ACI 318 provisions versus concrete compressive strength

5.3 Effect of shear span to depth ratio (a/d)

It is shown experimentally that the shear span to depth ratio (a/d) is an important parameter that affects the shear strength of RC deep beams as this ratio decreases, the shear strength capacity of deep beams increase. This is due to the fact that the load

directly transfers to the support through a single strut. Experiments from Oh and Shin (2001), Nakazawa (1980), Manuel (1979), and Kani et al. (1979) are selected where a/d parameter was the only variable and other parameters were kept constant.

Figure 5.4 represents the comparison for the prediction of the experiments from Oh and Shin (2001), Nakazawa (1980), Manuel (1979), and Kani et al. (1979) with EC 2-2004, AASHTO LRFD, ACI318 code provisions respectively. Safety factor, γ , is plotted versus shear span to depth ratio. It is observed from Figure 5.4 that ACI provides un-conservative results whereas EC 2 and AASHTO provides conservative results. It is also shown that as a/d increases, the predictions by EC2 and AASHTO gives less conservative results.

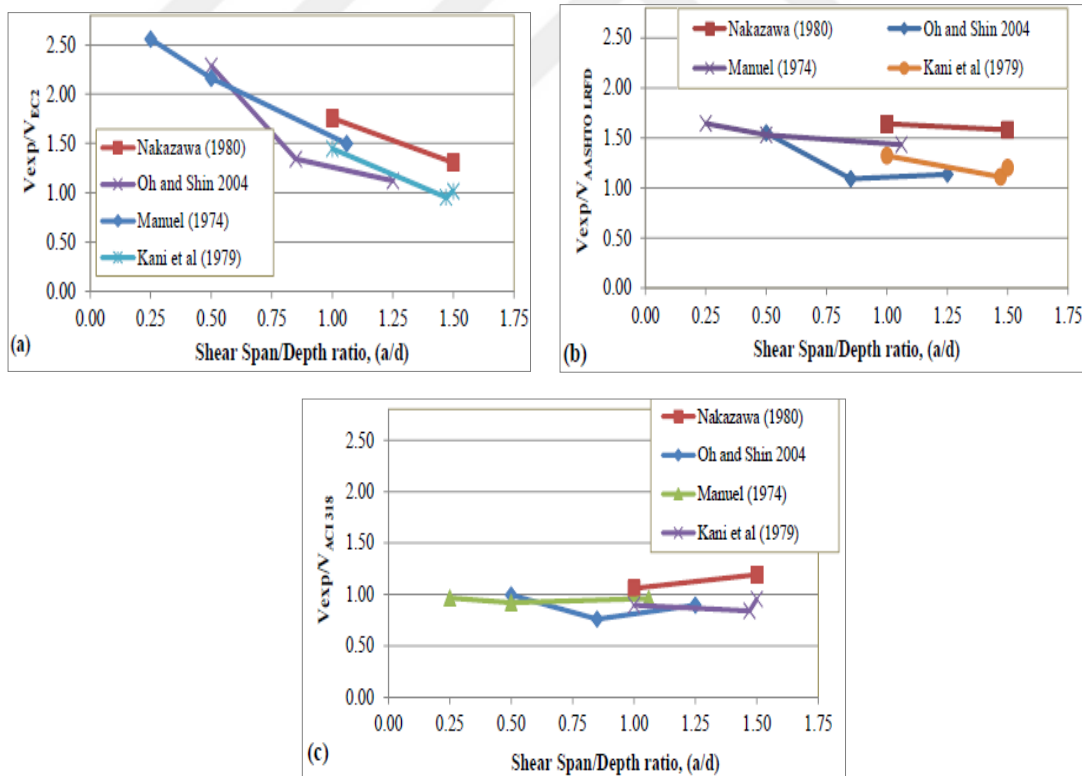


Figure 5.4 Effect of shear span to depth ratio on the safety factor $\gamma = V_{exp} / V_{calc}$ for a) EC2-2004 predictions, b) AASHTO LRFD predictions, c) ACI 318-14 predictions plotted versus shear span to depth ratio

Figure 5.5 shows the comparison for predictions of EC 2-2004, AASHTO LRFD, ACI318-14 codes with respect to shear span to depth ratio. Safety factor, γ , is plotted versus shear span to depth. The effect of shear span to depth ratio was examined based on AASHTO group of 471, EC 2 162, ACI 250 RC beams without shear reinforcement as shown in Figure 5.4. According to EC 2, AASHTO and ACI the value of shear span to depth ratio in EC 2 group ranged from 0.3 to 1.5 MPa, in AASHTO group ranged from 0.3 to 2.5, in ACI 318 group ranged from 0.3 to 2.0 MPa. According to Figure 5.5, the shear strength of the deep beams are well predicted for EC 2, then ACI 318 is well but AASHTO is not well predicted because it has high shear span to depth ratio.

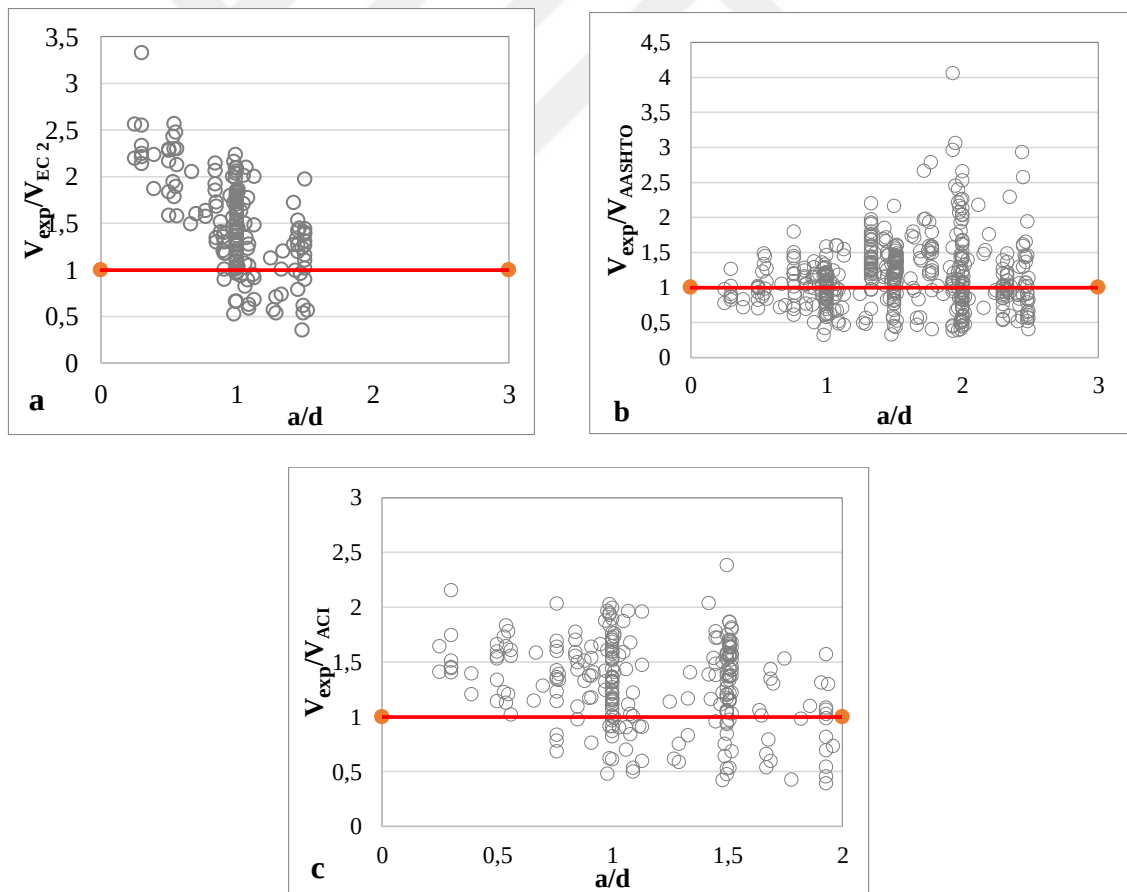


Figure 5.5 Shear strength prediction by STM with a) EC 2 provisions b) AASHTO LRFD provisions c) ACI 318 provisions versus shear span to depth ratio

Fig. 5.6 shows effect of shear span to depth ratio on shear strength by STM codes. EC 2 graph shows that shear strength increases when shear span to depth ratio increases because of relation between concrete compressive strength and shear span to depth ratio. However, for ACI 318 and AASHTO codes, shear strength decreases when a/d ratio decreases.

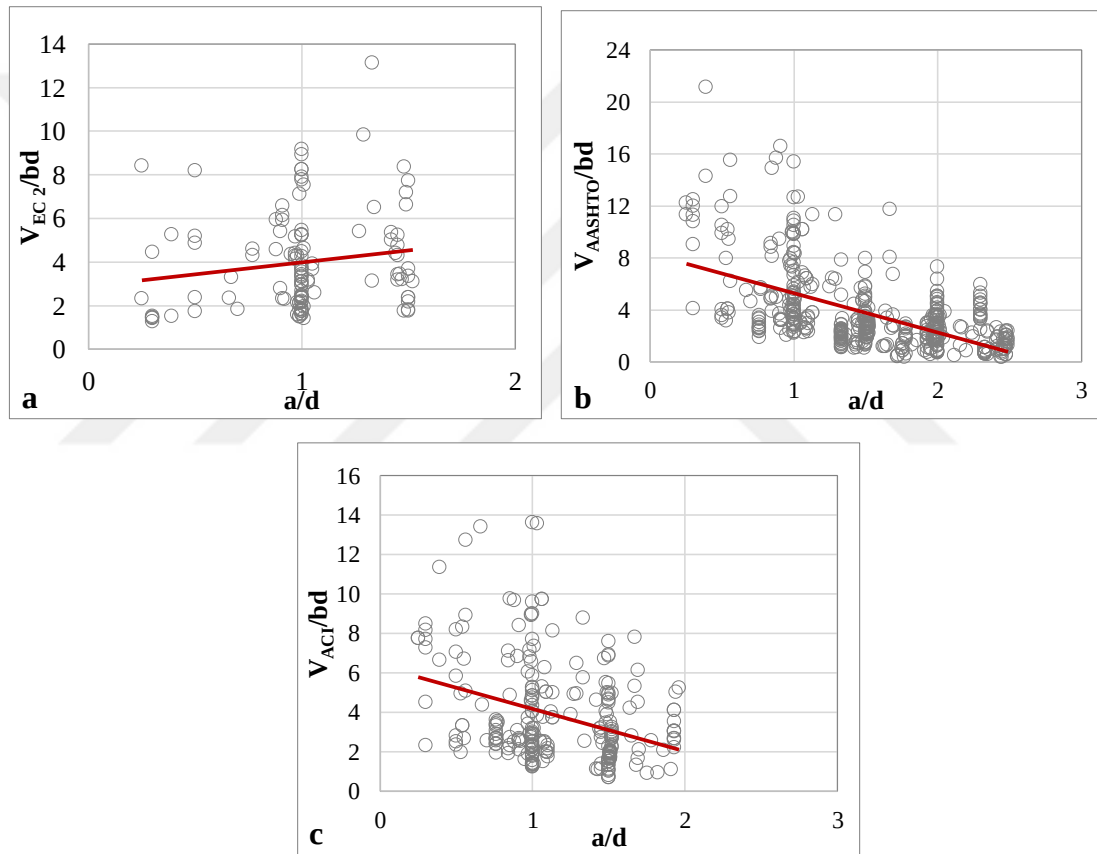


Figure 5.6 Effect of shear span to depth ratio on shear strength by STM with a) EC 2 provisions b) AASHTO LRFD provisions c) ACI 318 provisions versus concrete compressive strength

5.4 Effect of flexural reinforcement ratio (ρ_l)

It is known that the flexural reinforcement can influence the shear strength capacity of beams due to the dowel action. In order to assess the effect of flexural reinforcement ratio on the shear strength of deep beams, experiments from Morrow and Viest (1957), Nakazawa (1980), and Nakazawa (1980) were selected. In those experiments, only

flexural reinforcement ratio was variable while all other parameters such a/d , concrete compressive strength, effective depth were kept constant.

Figure 5.7 represents the comparison for the prediction of the experiments from Morrow and Viest (1957), Nakazawa (1980), and Nakazawa (1980) with EC 2-2004, AASHTO LRFD, ACI 318 code provisions respectively. Safety factor, γ , is plotted versus flexural reinforcement ratio. It is observed from Figure 5.7 that AASHTO provisions provides unconservative results whereas EC2 and ACI predictions provide almost safe results. It is also noticed that for beams with higher flexural reinforcement ratios, the predictions by EC 2 and ACI are more conservative.

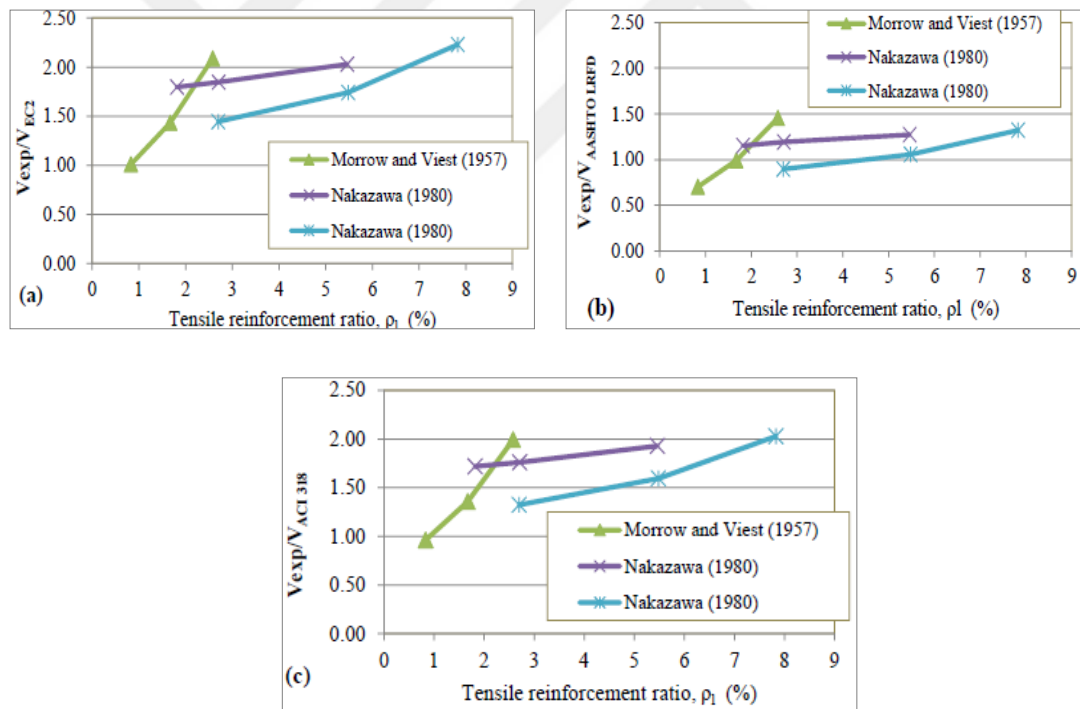


Figure 5.7 Effect of tensile reinforcement ratio on the safety factor $\gamma = V_{exp}/V_{calc}$ for a) EC2-2004 predictions, b) AASHTO LRFD predictions, c) ACI 318-14 predictions plotted versus tensile reinforcement ratio

Figure 5.8 shows the comparison for predictions of EC 2-2004, AASHTO LRFD, ACI318-14 codes with respect to flexural reinforcement ratio. Safety factor, γ , is plotted versus flexural reinforcement ratio. The effect of flexural reinforcement ratio

was observed based on AASHTO group of 471, EC 2 162, ACI 250 RC beams without shear reinforcement as shown in Figure 5.6. According to EC 2, AASHTO and ACI the value of flexural reinforcement ratio in EC 2 group ranged from % 0.3 to % 7.9, in AASHTO group ranged from % 0.3 to % 9.3, in ACI 318 group ranged from % 0.3 to % 7.9 According to Figure 4.8 the shear strength of the deep beams are well predicted for EC 2, rather than ACI 318 and AASHTO. From an analysis of Figure 5.8 it shows that predictions are not safe for members with flexural reinforcement ratio in EC 2 less than 0.04, in AASHTO less than 0.07, in ACI 318 less than 0.05.

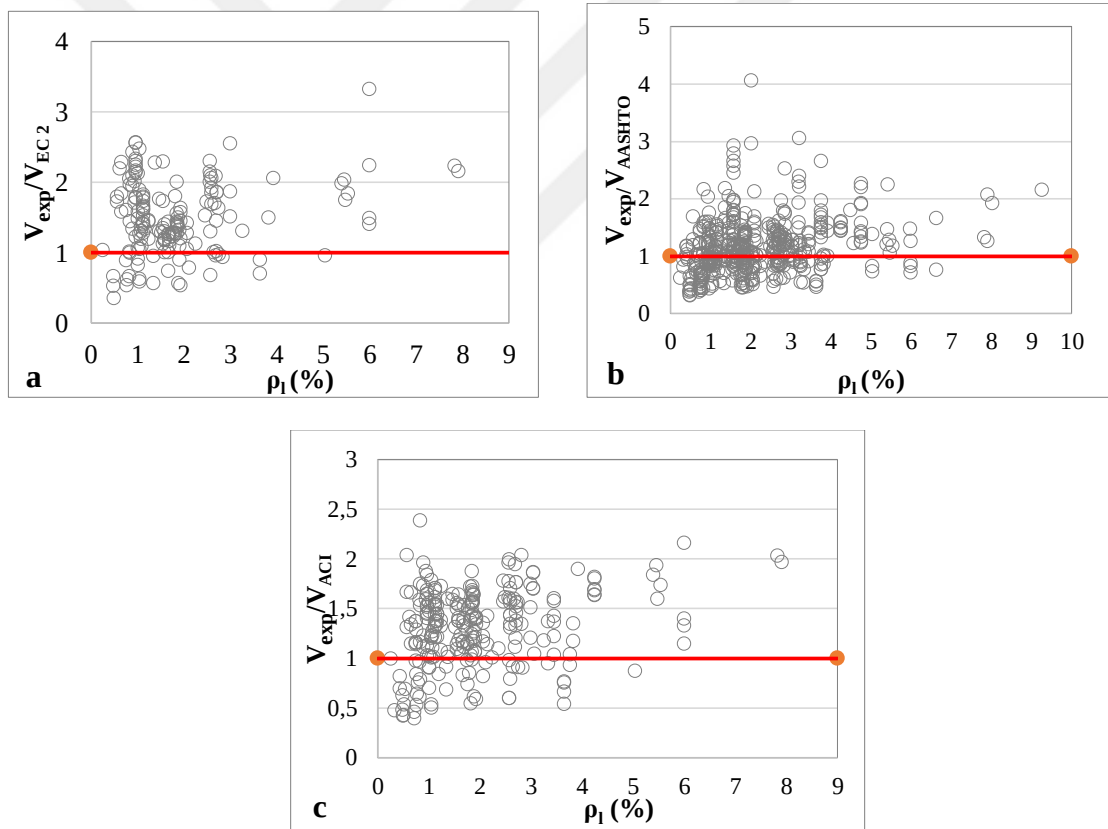


Figure 5.8 Shear strength prediction by STM with a) EC 2 provisions b) AASHTO LRFD provisions c) ACI 318 provisions versus tensile reinforcement ratio

Fig. 5.9 shows effect of flexural reinforcement ratio on shear strength by STM codes. EC 2 graph show that shear strength increases when flexural reinforcement ratio

decreases. However, for ACI 318 and AASHTO codes, shear strength increases when flexural reinforcement ratio increases generally.

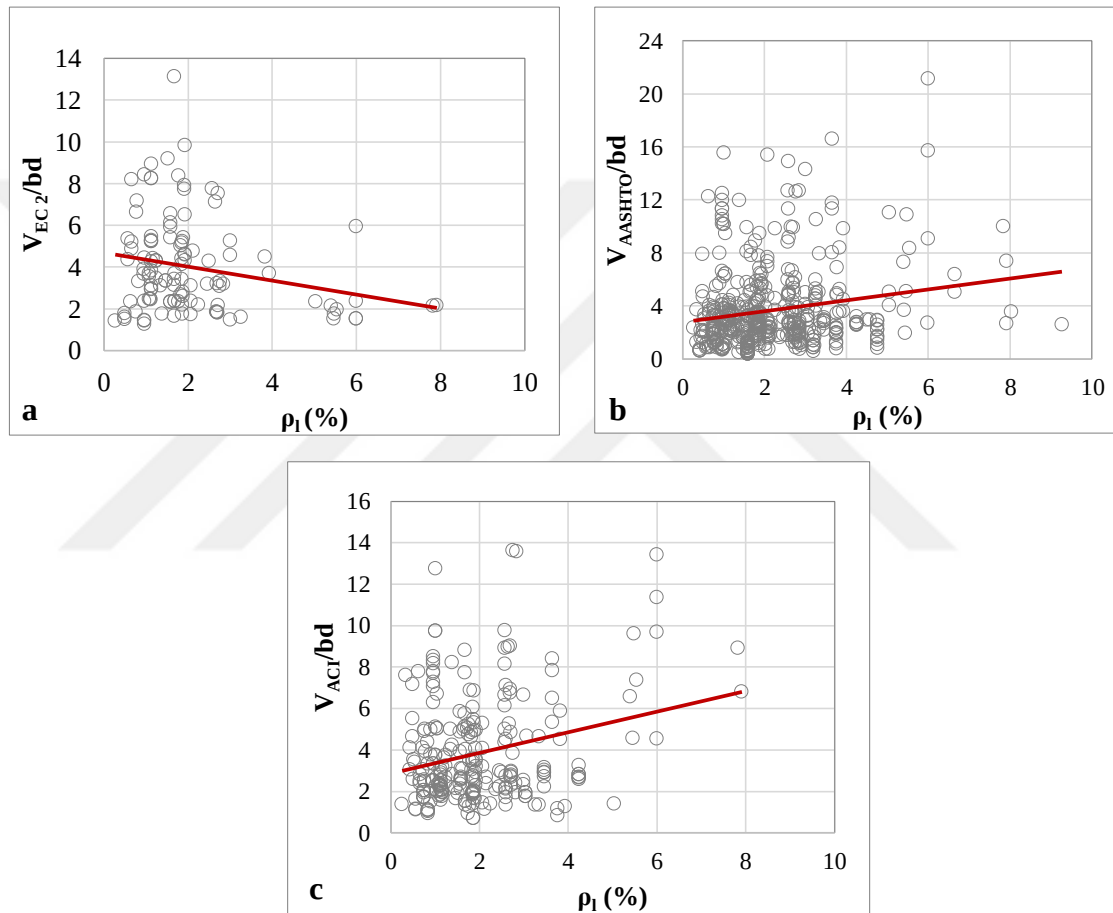


Figure 5.9 Effect of flexural reinforcement ratio on shear strength by STM with a) EC 2 provisions b) AASHTO LRFD provisions c) ACI 318 provisions versus concrete compressive strength

5.5 Effect of effective depth (d)

The size effect is the result of a reduction in average shear strength due to the increasing effective depth of the member. To assess the effect of effective depth on the shear strength of deep beams, experiments from Walraven & Lehwalter (1994), Zhang & Tan (2007) and Lertsrisakulrat et al. (2002) were selected. It is verified that the shear strength decreases by increasing the beam depth and this is also reflected in Figure 5.10.

Both Walraven and Lehwalter and Zhang and Tan scaled the length of the bearing plates to the depth of the member. However, upon comparison of both graphs, the findings of the researchers are not consistent. The source of the difference is likely due to the high bearing stresses at the single applied load at midspan of the Walraven and Lehwalter tests.

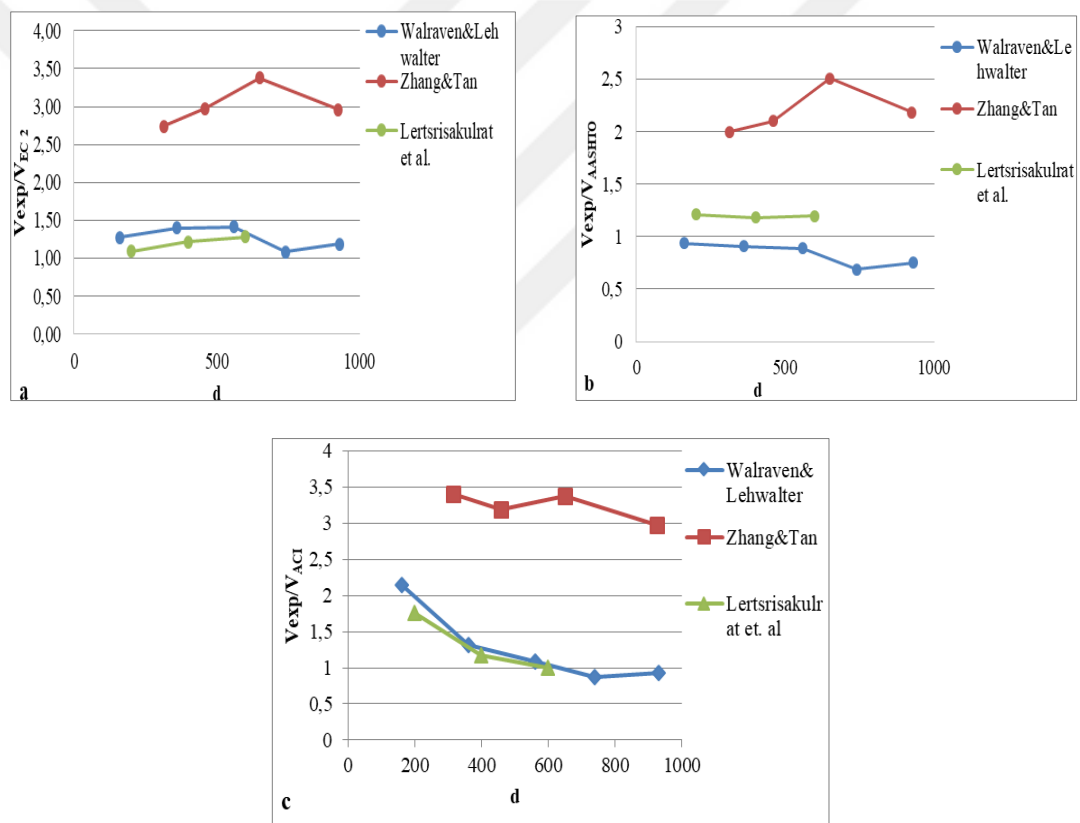


Figure 5.10 Effect of effective depth on the safety factor $\gamma = V_{exp}/V_{calc}$ for a) EC2-2004 predictions, b) AASHTO LRFD predictions, c) ACI 318-14 predictions plotted versus effective depth

Figure 5.11 shows the comparison for predictions of EC 2-2004, AASHTO LRFD, ACI318-14 codes with respect to effective depth. Safety factor, γ , is plotted versus effective depth. The effect of effective depth was examined based on AASHTO group of 471, EC 2 162, ACI 250 RC beams without shear reinforcement as shown in

Figure 5.11. According to Figure 5.11 the shear strength of the deep beams are well predicted for EC 2 and ACI 318 rather than AASHTO.

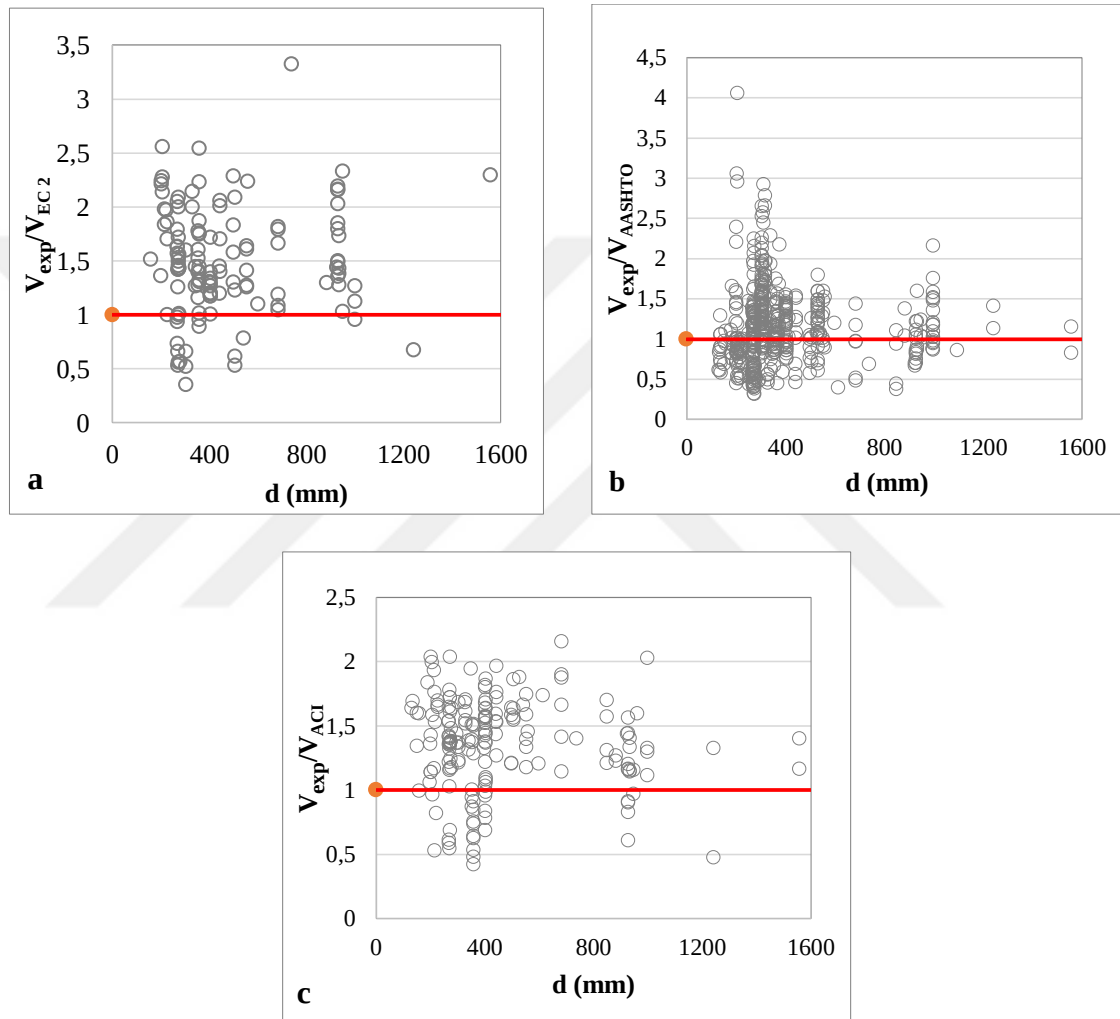


Figure 5.11 Shear strength prediction by STM with a) EC 2 provisions b) AASHTO LRFD provisions c) ACI 318 provisions versus tensile reinforcement ratio

Fig. 5.12 shows effect of effective depth on shear strength by STM codes. These graphs shows that shear strength decreases when effective depth decreases for all codes.

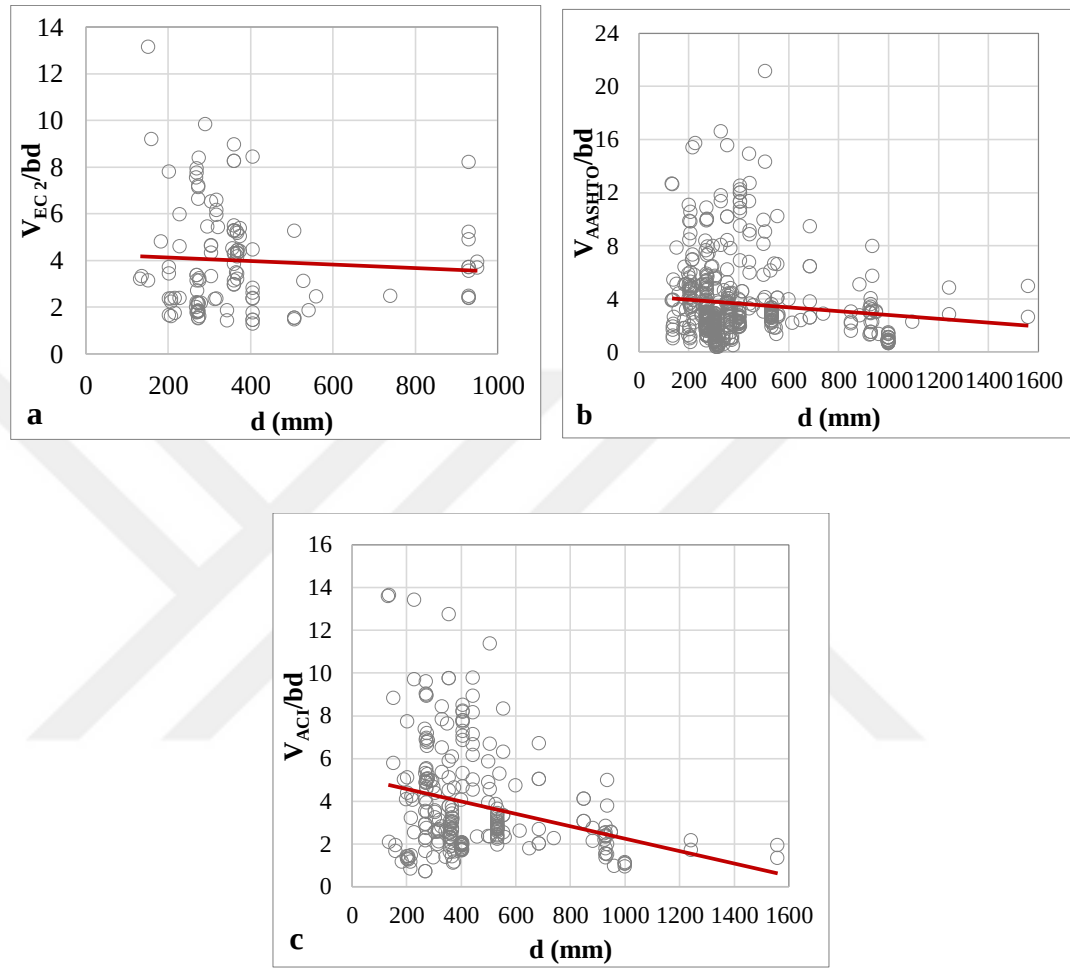


Figure 5.12 Effect of effective depth on shear strength by STM with a) EC 2 provisions b) AASHTO LRFD provisions c) ACI 318 provisions versus concrete compressive strength

CHAPTER 6

CONCLUSION

6.1 Summary

This thesis aimed to understand shear behavior of deep beams and evaluate the accuracy of code provisions using STM predictions on the shear capacity of RC deep beams.

A literature survey including experiments on shear capacity of deep beams were presented in this thesis. Numerical models developed by different researchers were presented and main parameters affected the shear capacity of deep beams were highlighted. It was shown that most of the codes do not consider the concrete compressive strength, tensile strain and shear span depth ratio of the beams in calculating the shear capacity of such beams. Furthermore, a database on shear tests of R.C. deep beams were developed from literature and the database was used to evaluate the accuracy of code provisions such as ACI 318, 2004, EC 2-2004 and AASHTO 2008. Main parameters such as span to depth ratio, concrete compressive strength, flexural reinforcement ratio, and effective depth influencing the shear capacity of deep were investigated using the collected database. Experimental databases consist of 471 tests on RC deep beams and the distribution of parameters were represented.

6.2 Code Prediction with Parametric Study

It was shown that ACI 318-14 and AASHTO LRFD 2008 provides unsafe predictions with high scatter on beams with high concrete compressive strength. The shear strength predictions of the deep beams according to EC 2-2004 are more safe since EC2 includes the contribution of concrete compressive strength on the effectiveness factor of the inclined strut, whereas, ACI and AASHTO ignore this effect.

ACI provisions provides non conservative predictions when shear span to depth ratio is considered while EC 2 and AASHTO provides conservative results. It is also observed

that as span to depth ratio (a/d) increases, the predictions by EC2 and AASHTO gives less conservative results.

When compared to AASHTO predictions, EC2-2004 and ACI 318-14 provisions provide more conservative results as the amount of flexural reinforcement ratio in deep beams are increased.

ACI 318-14 provides non conservative predictions on shear capacity of beams with higher depths the shear capacity of deep beams while EC2 and AASHTO provides conservative prediction on shear capacity of beams on all ranges of depths.

Lots of research works carried out in literature to assess the size effect on shear strength of deep beams. For instance, both Walraven and Lehwalter (1994) and Zhang and Tan (2007) scaled the length of the bearing plates to the depth of the member. However, upon comparison of depth effect graphs, no consistency was observed in those works. The source of the difference is likely due to the high bearing stresses at the single applied load at mid-span of those experiments. The shear strength of the deep beams are well predicted for EC 2 and ACI 318 rather than AASHTO provisions.

6.3 Future Research

The shear capacity of deep beams according to STM were investigated in this thesis. However, the proposed shear equations from researchers and codes need to be verified including different influencing parameters on shear strength. Based on parametric studies, it was shown that few parameters have no clear trend on the shear strength of RC deep beam due to lack of more experiments on that range. Therefore, either more experimental works could be performed or the gap among the data points could be filled using the FEM.

The codes of equations are using effectiveness factor which are empirically developed and this leads to poor relation against the collected experimental results, with large scatter. Thus, there is a need to modify the effectiveness factor for better predictions.

To understand stress and strain distribution in the shear span and depth of the compression zone in reinforced concrete deep beams, more studies should be done. Because this identifies the size of compression zone and inclined struts in the STM. In fact, strain energy could be clear after this because stress-strain relation cannot be ignored during this act.



APPENDIX

Author and Notes	Year	Beam Name	b _w (mm)	h (mm)	d (mm)	a (mm)	a/d ratio	Bear (mm)	ρ _w (%)	f _c ' (MPa)	f _y (MPa)	V _{exp} (kN)	V _{predict} (kN)	V _{exp} / V _{predict}
Moody Viest Elstner and Hognestad ("Part I" paper)	1954	III-24a	178	610	533	813	1,52	203	2,72	17,8	483	296,5	216,30	1,37
	1954	III-24b	178	610	533	813	1,52	203	2,72	20,6	483	303,2	250,33	1,21
	1954	III-25a	178	610	533	813	1,52	203	3,46	24,3	483	267,6	295,29	0,91
	1954	III-25b	178	610	533	813	1,52	203	3,46	17,2	483	289,8	209,01	1,39
	1954	III-26a	178	610	533	813	1,52	203	4,25	21,7	483	421,1	263,70	1,60
	1954	III-26b	178	610	533	813	1,52	203	4,25	20,6	483	396,6	250,33	1,58
	1954	III-27a	178	610	533	813	1,52	203	2,72	21,4	483	347,7	260,05	1,34
	1954	III-27b	178	610	533	813	1,52	203	2,72	22,9	483	356,6	278,28	1,28
	1954	III-28a	178	610	533	813	1,52	203	3,46	23,3	483	303,2	283,14	1,07
	1954	III-28b	178	610	533	813	1,52	203	3,46	22,4	483	341,0	272,20	1,25
	1954	III-29a	178	610	533	813	1,52	203	4,25	21,7	483	389,9	263,70	1,48
	1954	III-29b	178	610	533	813	1,52	203	4,25	25,0	483	436,6	303,80	1,44
Ferguson	1956	F3	103	210	183	267	1,45	25	2,12	24,0	483	36,9	44,79	0,82
	1956	F5	95	210	181	356	1,97	25	2,33	25,9	483	34,2	30,89	1,11
	1956	F7	102	210	184	356	1,93	25	2,14	27,2	483	33,9	33,47	1,01
Moody Viest Elstner Hognestad ("part II paper")	1955	I-g	178	381	298	813	1,36	203	0,95	30,6	483	133,4	112,10	1,19
	1955	I-h	178	381	298	813	1,36	203	1,47	24,4	483	132,0	89,39	1,48
	1955	I-i	178	381	305	813	1,33	203	2,10	22,9	483	146,8	87,25	1,68
	1955	I-1a	178	381	305	813	1,33	203	2,86	17,3	483	114,2	65,91	1,73
	1955	I-1b	178	381	305	813	1,33	203	2,86	19,3	483	129,0	73,53	1,75
	1955	I-2a	178	381	305	813	1,33	203	3,76	16,3	483	111,2	62,10	1,79
	1955	I-2b	178	381	305	813	1,33	203	3,76	18,8	483	140,9	71,63	1,97
	1955	I-2c	178	381	305	813	1,33	203	3,76	26,1	483	139,4	99,44	1,40
	1955	I-3a	178	381	305	813	1,33	203	4,76	15,8	483	132,0	60,20	2,19
	1955	I-3b	178	381	305	813	1,33	203	4,76	20,5	483	149,7	78,10	1,92
	1955	I-j	178	381	305	813	1,33	203	1,47	33,4	483	155,7	127,25	1,22
	1955	I-k	178	381	305	813	1,33	203	2,10	26,6	483	161,6	101,34	1,59
	1955	I-4a	178	381	305	813	1,33	203	2,86	29,8	483	148,3	113,54	1,31
	1955	I-4b	178	381	305	813	1,33	203	2,86	27,9	483	132,0	106,30	1,24
	1955	I-5a	178	381	305	813	1,33	203	3,76	28,0	483	177,9	106,68	1,67
	1955	I-5b	178	381	305	813	1,33	203	3,76	27,9	483	163,1	106,30	1,53
	1955	I-6a	178	381	305	813	1,33	203	4,76	31,4	483	170,5	119,63	1,43
	1955	I-6b	178	381	305	813	1,33	203	4,76	24,6	483	177,9	93,72	1,90
	1955	I-l	178	381	305	813	1,33	203	1,47	35,2	483	158,6	134,11	1,18
	1955	I-m	178	381	305	813	1,33	203	2,10	30,3	483	155,7	115,44	1,35
	1955	I-7a	178	381	305	813	1,33	203	2,86	33,0	483	170,5	125,73	1,36
	1955	I-7b	178	381	305	813	1,33	203	2,86	34,5	483	148,3	131,44	1,13
	1955	I-8a	178	381	305	813	1,33	203	3,76	33,0	483	215,0	125,73	1,71
	1955	I-8b	178	381	305	813	1,33	203	3,76	32,3	483	163,1	123,06	1,33
	1955	I-9a	178	381	305	813	1,33	203	4,76	36,3	483	192,7	138,30	1,39
	1955	I-9b	178	381	305	813	1,33	203	4,76	32,1	483	192,7	122,30	1,58
	1955	I-n	178	381	305	813	1,33	203	1,47	36,1	483	175,0	137,54	1,27
	1955	I-o	178	381	305	813	1,33	203	2,10	34,8	483	189,8	132,59	1,43
	1955	I-p	178	381	305	813	1,33	203	2,86	41,2	483	197,2	156,97	1,26
	1955	I-q	178	381	305	813	1,33	203	3,76	33,7	483	192,7	128,40	1,50
	1955	I-r	178	381	305	813	1,33	203	4,76	40,9	483	207,6	155,83	1,33
	1955	II-a	178	610	533	813	0,76	203	0,54	26,3	483	192,7	319,60	0,60
	1955	II-b	178	610	533	813	0,76	203	0,84	25,7	483	215,0	312,30	0,69
	1955	II-c	178	610	533	813	0,76	203	1,20	27,9	483	249,1	339,04	0,73
	1955	II-d	178	610	533	813	0,76	203	1,63	23,7	483	311,4	288,00	1,08
	1955	II-17a	178	610	533	813	0,76	203	2,15	18,3	483	278,7	222,38	1,25
	1955	II-17b	178	610	533	813	0,76	203	2,15	20,7	483	252,1	251,54	1,00

	1955	II-18a	178	610	533	813	0,76	203	2,82	15,0	483	326,2	182,28	1,79
	1955	II-18b	178	610	533	813	0,76	203	2,82	18,6	483	266,9	226,03	1,18
	1955	II-19a	178	610	533	813	0,76	203	3,46	20,9	483	357,3	253,98	1,41
	1955	II-19b	178	610	533	813	0,76	203	3,46	22,3	483	324,7	270,99	1,20
	1955	II-20a	178	610	533	813	0,76	203	4,25	19,9	483	348,4	241,82	1,44
	M	II-20b	178	610	533	813	0,76	203	4,25	20,4	483	369,2	247,90	1,49
	1955	IV-g	178	381	305	1219	2,00	203	0,95	23,4	483	80,1	39,52	2,03
	1955	IV-h	178	381	305	1219	2,00	203	1,47	25,9	483	89,0	43,74	2,03
	1955	IV-i	178	381	305	1219	2,00	203	2,10	24,1	483	86,4	40,70	2,12
	1955	IV-j	178	381	305	1219	2,00	203	2,86	24,8	483	105,5	41,88	2,52
	1955	IV-k	178	381	305	1219	2,00	203	3,76	25,0	483	111,8	42,22	2,65
	1955	IV-l	178	381	305	1219	2,00	203	4,76	27,0	483	102,9	45,59	2,26
	1955	V-b	178	381	305	813	1,33	203	1,47	26,0	483	94,9	99,06	0,96
	1955	V-d	178	381	305	813	1,33	203	2,86	24,8	483	113,4	94,49	1,20
	1955	V-f	178	381	305	813	1,33	203	4,76	23,3	483	110,5	88,77	1,24
	1955	VI-a	178	381	305	813	1,78	203	0,95	28,2	483	102,7	107,44	0,96
	1955	VI-b	178	381	305	813	1,78	203	1,47	28,7	483	172,1	109,35	1,57
	1955	VI-c	178	381	305	813	1,78	203	2,10	24,7	483	146,8	94,11	1,56
	1955	VI-d	178	381	305	813	1,78	203	2,86	26,9	483	157,5	102,49	1,54
	1955	VI-e	178	381	305	813	1,78	203	3,76	28,4	483	170,8	108,20	1,58
	1955	VI-f	178	381	305	813	1,78	203	2,10	38,4	483	186,8	146,30	1,28
	1955	VI-g	178	381	305	813	1,78	203	2,86	38,1	483	173,5	145,16	1,20
	1955	VI-h	178	381	305	813	1,78	203	3,76	36,6	483	206,8	139,44	1,48
	1955	VI-i	178	381	305	813	1,78	203	4,76	41,5	483	194,8	158,11	1,23
Morrow and Viest	1957	B14-B2	305	406	368	356	0,97	102	1,85	14,6	483	367,0	272,89	1,34
	1957	B14-E2	305	410	375	356	0,95	102	0,57	12,7	483	278,0	234,91	1,18
	1957	B14-A4	305	406	362	356	0,98	102	2,50	22,6	483	511,5	438,94	1,17
	1957	B14-B4	305	406	368	356	0,97	102	1,85	26,3	483	500,4	491,58	1,02
	1957	B14-E4	305	406	368	356	0,97	102	1,24	28,9	483	511,5	540,18	0,95
	1957	B14-A6	305	406	356	356	1,00	102	3,83	45,4	483	900,7	912,19	0,99
	1957	B14-B6	305	406	368	356	0,97	102	1,85	46,8	483	778,4	874,76	0,89
	1957	B21-B2	305	406	367	533	1,45	102	1,86	13,9	483	238,5	164,53	1,45
	1957	B21-A4	305	406	368	533	1,45	102	2,46	29,8	483	523,1	350,37	1,49
	1957	B21-B4	305	406	368	533	1,45	102	1,85	27,1	483	396,4	318,63	1,24
	1957	B21-E4	305	406	365	533	1,46	102	1,24	24,2	483	423,0	290,16	1,46
	1957	B21-E4R	305	406	368	533	1,45	102	1,24	31,9	483	434,2	375,06	1,16
		B21-E2	305	406	375	533	1,42	102	0,57	11,3	483	211,7	126,01	1,68
		B21-F4	305	406	370	533	1,44	102	1,17	31,4	483	467,6	364,03	1,28
	1957	B21-A6	305	406	356	533	1,50	102	3,83	45,3	483	578,8	570,01	1,02
	1957	B21-B6	305	406	375	533	1,42	102	1,82	45,5	483	578,7	507,38	1,14
	1957	B28-B2	305	406	362	711	1,96	102	1,88	14,7	483	201,2	115,36	1,74
	1957	B28-E2	308	406	372	711	1,91	102	0,57	13,7	483	130,0	101,31	1,28
	1957	B28-A4	305	406	368	711	1,93	102	2,46	27,5	483	323,5	207,68	1,56
	1957	B28-B4	305	406	368	711	1,93	102	1,85	32,3	483	256,8	243,93	1,05
	1957	B28-E4	305	406	368	711	1,93	102	1,24	33,1	483	267,9	249,98	1,07
	1957	B28-A6	308	406	353	711	2,02	102	3,83	47,2	483	334,7	389,96	0,86
	1957	B28-B6	305	406	368	711	1,93	102	1,85	43,9	483	323,5	331,54	0,98
Chang & Kesler	1958	IB1	102	152	137	330	2,42	64	1,86	27,6	328	19,6	26,49	0,74
	1958	IIC1	102	152	137	330	2,42	64	2,89	17,7	328	17,8	16,99	1,05
	1958	IIIA2	102	152	137	254	1,86	64	2,37	14,9	328	20,7	23,08	0,90
	1958	IIIC1	102	152	137	330	2,42	64	2,89	14,9	328	18,4	14,30	1,29
Watstein and Mathey	1958	B-18-1	203	457	404	610	1,51	89	3,05	25,4	267	311,4	266,69	1,17
	1958	B-18-2	203	457	404	610	1,51	89	3,05	23,0	267	309,1	241,49	1,28
	1958	C-18-1	203	457	404	610	1,51	89	1,85	25,6	490	289,1	210,64	1,37
	1958	C-18-2	203	457	404	610	1,51	89	1,88	26,4	466	311,4	222,40	1,40
	1958	D-18-1	203	457	404	610	1,51	89	1,17	25,7	724	266,9	172,34	1,55
	1958	D-18-2	203	457	404	610	1,51	89	1,16	27,0	669	266,9	189,28	1,41
	1958	E-18-1	203	457	404	610	1,51	89	0,75	22,4	686	220,8	154,86	1,43

	1958	E-18-2	203	457	404	610	1,51	89	0,75	26,7	688	222,4	184,29	1,21
Rodriguez Bianchini Viest Kesler	1959	E6N1	152	368	318	432	0,91	102	1,59	22,1	483	211,1	154,78	1,36
	1959	E6N2	154	368	318	432	0,91	102	1,59	17,9	483	162,5	127,02	1,28
	1959	E6N3	154	368	318	432	0,91	102	1,59	22,7	483	184,6	161,08	1,15
	1959	C6N1	156	368	321	432	0,90	102	1,59	26,1	483	210,4	185,41	1,13
	1959	C6N2	152	368	318	432	0,91	102	1,59	22,1	483	151,8	154,78	0,98
	1959	C6N3	154	368	314	432	0,92	102	1,59	21,7	483	184,8	156,01	1,18
	1959	E3N1	152	368	318	813	1,65	102	1,59	25,2	483	109,5	64,07	1,71
	1959	E3N2	152	368	318	813	1,63	102	1,59	23,3	483	102,3	59,24	1,73
	1959	C3N1	152	368	318	813	1,62	102	1,59	23,1	483	65,9	58,73	1,12
	1959	C3N2	154	368	318	813	1,64	102	1,59	22,7	483	104,8	58,48	1,79
	1959	E2N1	157	368	318	1295	1,78	102	1,59	29,0	483	55,3	31,01	1,78
	1959	E2N2	152	368	318	1295	1,72	102	1,59	20,8	483	57,2	21,54	2,66
	1959	E2N3	156	368	321	1295	1,72	102	1,59	23,0	483	47,5	24,15	1,97
	1959	C2N1	156	368	318	1295	1,73	102	1,59	26,1	483	54,6	27,73	1,97
	1959	C2N2	152	368	318	1295	1,77	102	1,59	18,6	483	53,5	19,26	2,78
Baldwin & Viest	1959	0B28	305	406	368	711	1,93	102	1,83	37,6	519	289,1	273,92	1,06
	1959	0F28	305	406	368	711	1,93	102	1,83	33,3	313	135,7	304,10	0,45
de Cossio & Seiss	1960	L-1	152	305	252	508	2,01	152	3,36	21,0	303	116,1	108,99	1,07
	1960	A-1	152	305	254	508	2,00	305	0,98	28,1	459	73,4	169,78	0,43
	1960	A-11	152	305	254	508	2,00	305	3,33	28,3	341	103,4	195,53	0,53
Bower + Viest	1960	IA-1a	152	356	306	762	2,49	102	1,59	22,8	324	59,2	72,43	0,82
	1960	IA-1b	152	356	314	762	2,42	102	1,59	30,0	323	76,9	92,66	0,83
	1960	IA - 2a	152	356	311	1016	2,45	102	1,59	24,8	308	73,1	46,13	1,58
	1960	IA - 2b	152	356	311	1016	2,45	102	1,59	24,4	319	39,7	44,73	0,89
	1960	IA - 3a	152	356	311	1270	2,45	102	1,59	27,7	326	52,0	32,39	1,61
	1960	IA - 4b	152	356	311	1524	2,45	102	1,59	21,9	319	46,0	17,90	2,57
	1960	IA - 5a	152	356	308	1270	2,48	102	1,59	23,9	317	41,4	28,65	1,45
	1960	IA - 5b	152	356	308	1270	2,48	102	1,59	21,5	323	49,4	25,57	1,93
	1960	IA - 8a	152	356	311	1100	2,45	102	1,59	24,1	307	49,4	38,45	1,28
	1960	IB-1	152	356	305	813	2,00	102	1,59	22,2	326	56,7	62,67	0,90
	1960	IB-2	152	356	305	1016	2,00	102	1,59	21,2	330	65,2	39,08	1,67
	1960	IB-3	152	356	305	1016	2,00	102	1,59	22,8	319	60,2	42,64	1,41
	1960	IIa-1a	152	356	308	914	1,49	102	1,59	22,8	319	61,7	51,77	1,19
	1960	IIa-1b	152	356	318	914	1,44	102	1,59	25,5	324	93,7	54,82	1,71
	1960	IIa-2	152	356	312	1219	1,95	102	1,59	21,6	319	67,3	27,54	2,44
	1960	IIA - 3	152	356	312	1524	2,44	102	1,59	21,7	320	51,5	17,62	2,92
Guralnick	1960	IA-1M	178	387	319	914	1,43	152	1,37	26,9	552	139,7	75,78	1,84
	1960	IC-1M	178	387	316	914	1,44	152	1,38	33,5	552	155,2	93,72	1,66
Hanson	1961	8B3	152	305	267	660	2,48	102	1,25	30,1	334	46,0	81,63	0,56
de Cassio & Seiss		D-5	152	305	276	419	1,52	152	1,35	25,8	361	90,5	174,31	0,52
Leonhardt & Walther	1962	1	190	320	270	270	1,00	100	2,07	32,4	465	388,5	392,95	0,99
	1962	2	190	320	270	400	1,48	100	2,07	32,4	465	260,0	245,83	1,06
	1962	3	190	320	270	540	2,00	100	2,07	32,4	465	147,2	153,88	0,96
	1962	4L	190	320	270	670	2,48	100	2,07	32,4	465	81,6	105,20	0,78
	1962	4R	190	320	270	670	2,48	100	2,07	32,4	465	87,0	105,20	0,83
		11/1	190	340	290	375	1,29	100	1,93	38,1	465	165,8	352,29	0,47
		11/2	190	346	296	375	1,27	100	1,89	38,1	465	177,0	362,80	0,49
Mathey and Watstein	1963	I-1	203	457	403	610	1,51	89	3,05	25,4	267	312,9	268,31	1,17
	1963	I-2	203	457	403	610	1,51	89	3,05	23,0	267	310,7	242,96	1,28
	1963	II-3	203	457	403	610	1,51	89	1,88	21,9	466	261,8	185,56	1,41
	1963	II-4	203	457	403	610	1,51	89	1,88	26,4	466	312,9	223,69	1,40
	1963	III-5	203	457	403	610	1,51	89	1,85	25,7	490	288,5	212,69	1,36
	1963	III-6	203	457	403	610	1,51	89	1,85	25,6	490	290,7	211,86	1,37
	1963	IV-7	203	457	403	610	1,51	89	1,86	24,1	443	290,8	208,99	1,39
	1963	IV-8	203	457	403	610	1,51	89	1,86	24,9	443	304,0	215,92	1,41
	1963	V-9	203	457	403	610	1,51	89	1,16	23,1	698	224,0	159,04	1,41

	1963	V-10	203	457	403	610	1,51	89	1,16	27,0	698	268,4	185,89	1,44
	1963	VI-11	203	457	403	610	1,51	89	1,17	25,4	698	224,0	174,88	1,28
	1963	VI-12	203	457	403	610	1,51	89	1,17	25,7	698	268,4	176,94	1,52
	1963	V-13	203	457	403	610	1,51	89	0,75	22,4	712	222,4	152,50	1,46
	1963	V-14	203	457	403	610	1,51	89	0,75	26,7	712	224,0	181,77	1,23
	1963	VI-15	203	457	403	610	1,51	89	0,75	25,5	712	179,5	173,60	1,03
	1963	VI-16	203	457	403	610	1,51	89	0,75	22,8	712	188,6	155,22	1,22
de Pavia	1965	G23S-11	51	330	305	203	0,67	98	0,83	24,6	483	89,8	85,80	1,05
	1965	G33S-11	76	229	203	203	1,00	98	1,67	23,3	483	85,3	86,10	0,99
	1965	G33S-31	76	229	203	203	1,00	98	2,58	19,9	483	106,9	73,53	1,45
	1965	G34S-11	76	229	203	203	1,00	98	1,67	35,2	483	109,8	130,07	0,84
	1965	G43S-11	102	178	152	203	1,33	98	1,67	24,2	483	76,9	79,51	0,97
	1965	G44S-11	102	178	152	203	1,33	98	1,67	37,0	483	83,6	121,56	0,69
	1965	G43S-11	102	178	152	203	1,33	98	1,67	24,2	483	76,9	79,51	0,97
Krefeld & Thurston	1965	G44S-11	102	178	152	203	1,33	98	1,67	37,0	483	83,6	121,56	0,69
	1966	II-4A3	203	457	390	915	2,35	178	2,06	30,6	400	109,9	188,43	0,58
	1966	II-5A3	203	457	390	915	2,35	178	3,09	29,9	400	170,4	184,12	0,93
Kani	1964	24	152	305	271	407	1,50	152	1,87	27,9	396	182,0	191,46	0,95
	1964	25	152	305	271	543	2,00	152	1,87	24,6	396	104,1	103,44	1,01
	1964	26	152	305	271	543	2,00	152	1,87	27,1	396	78,1	113,95	0,69
	1967	41	152	152	141	340	2,41	152	2,61	27,2	381	51,5	56,67	0,91
	1967	45	151	152	133	272	2,05	152	2,83	25,5	392	64,6	77,69	0,83
	1967	46	151	152	136	272	2,00	152	2,76	25,5	392	69,1	80,12	0,86
	1967	53	151	152	132	136	1,03	152	2,84	26,7	392	155,3	252,79	0,61
	1967	54	151	152	136	136	1,00	152	2,76	26,7	392	157,8	259,55	0,61
	1967	61	156	610	542	1085	2,00	76	2,55	26,8	349	163,3	152,71	1,07
	1967	65	150	610	552	1359	2,46	152	2,82	27,0	374	112,4	111,56	1,01
	1967	67	157	610	528	543	1,03	152	2,75	30,3	407	548,0	508,05	1,08
	1967	69	155	610	542	543	1,00	229	2,67	27,4	373	585,6	558,81	1,05
	1967	72	152	610	549	1087	1,98	152	2,71	24,8	384	196,9	153,81	1,28
	1966	85	154	305	274	272	0,99	64	2,69	25,5	381	233,6	182,43	1,28
	1966	87	154	305	269	272	1,01	64	2,72	27,2	366	239,6	207,36	1,16
	1966	94	153	305	273	543	1,99	64	2,77	25,3	352	110,6	78,54	1,41
	1966	95	153	305	275	678	2,47	64	2,75	25,3	338	72,8	53,64	1,36
	1966	98	153	305	275	679	2,47	64	2,68	26,2	366	76,3	53,53	1,43
	1966	100	153	305	270	544	2,02	64	2,75	27,2	366	111,9	85,07	1,32
	1966	102	153	305	269	543	2,02	152	0,76	25,3	423	48,8	104,11	0,47
	1966	109	153	305	271	407	1,50	152	0,76	25,0	457	71,9	162,15	0,44
	1966	111	154	305	272	678	2,49	152	0,76	27,0	368	43,3	77,74	0,56
	1966	112	153	305	273	678	2,48	152	0,76	27,0	368	39,4	77,07	0,51
	1966	113	152	305	274	408	1,49	152	0,77	25,5	486	87,2	158,48	0,55
	1966	114	153	305	270	544	2,02	152	0,80	25,5	486	61,4	97,75	0,63
	1966	129	155	305	275	407	1,48	152	1,78	17,6	348	143,3	128,99	1,11
	1966	131	151	305	274	679	2,48	152	1,85	18,1	401	49,6	48,78	1,02
	1966	134	154	305	273	544	1,99	152	1,81	17,4	419	59,9	71,81	0,83
	1966	135	149	305	274	544	1,98	152	1,86	17,4	411	76,8	69,97	1,10
	1966	141	151	305	270	544	2,01	152	0,81	19,3	382	48,7	81,73	0,60
	1966	142	156	305	276	544	1,97	152	0,77	19,3	382	58,3	83,57	0,70
	1966	147	152	305	287	678	2,36	152	0,70	16,8	417	42,3	42,22	1,00
	1966	148	152	305	274	408	1,49	152	0,79	19,9	382	79,9	137,52	0,58
	1966	150	153	305	273	678	2,48	152	0,77	18,0	380	46,2	50,65	0,91
	1966	151	154	305	273	679	2,49	152	0,78	19,3	382	35,6	54,37	0,65
	1966	162	153	305	272	543	1,99	152	0,77	34,3	377	59,0	148,16	0,40
	1966	163	156	305	273	678	2,49	152	0,76	35,4	378	40,5	101,80	0,40
	1966	181	154	305	272	543	2,00	152	1,79	33,9	390	65,2	145,21	0,45
	1966	184	154	305	271	407	1,50	152	1,80	35,1	394	163,3	244,56	0,67
	1966	188	153	305	277	543	1,96	152	1,76	33,1	384	92,6	140,36	0,66
	1966	193	153	305	278	678	2,44	152	1,80	34,6	352	56,7	99,13	0,57

	1966	197	150	305	274	679	2,48	152	1,84	36,0	376	60,1	99,24	0,61
	1966	199	152	305	273	544	1,99	152	1,83	36,0	410	76,8	148,10	0,52
	1966	201	155	305	274	272	0,99	152	2,65	35,2	381	253,7	422,85	0,60
	1966	204	152	305	275	543	1,97	152	2,69	34,8	369	147,1	149,89	0,98
	1966	205	153	305	275	544	1,98	152	2,66	35,2	381	125,5	150,01	0,84
	1966	215	154	305	274	679	2,48	152	2,67	36,0	412	88,1	97,70	0,90
	1966	248	153	305	282	678	2,41	152	0,49	27,6	400	37,2	73,36	0,51
	1966	251	154	305	276	544	1,97	152	0,48	26,2	391	41,9	110,86	0,38
		249	153	305	276	270	0,98	152	0,49	28,0	376	104,1	334,63	0,31
		250	152	305	274	406	1,48	152	0,50	28,0	376	62,8	196,24	0,32
	1966	265	153	305	269	407	1,51	152	0,52	18,1	400	53,0	124,71	0,42
		269	154	305	274	270	0,99	152	0,49	18,1	396	89,0	214,59	0,41
	1966	266	153	305	272	673	2,48	152	0,50	18,1	396	32,5	50,83	0,64
	1966	270	152	305	273	542	1,99	152	0,50	20,1	396	41,4	84,56	0,49
	1967	3041	152	1219	1097	2195	2,00	432	2,72	26,9	377	326,2	380,82	0,86
	1968	5A	203	406	370	914	2,47	10	1,03	29,9	350	80,5	75,92	1,06
	1968	5B	203	406	370	914	2,47	10	1,03	29,9	350	80,5	75,92	1,06
	Ramakrishnan & Ananthanarayana			76	381	343	241	0,70	76	0,76	21,5	320	90,1	121,31
			79	381	343	345	1,01	76	0,26	13,7	320	39,4	63,93	0,62
Manuel, Slight and Suter	1971	1	102	457	406	122	0,30	152	0,97	33,8	410	444,8	496,56	0,90
	1971	2	102	457	406	122	0,30	152	0,97	35,2	410	444,8	517,13	0,86
	1971	3	102	457	406	122	0,30	152	0,97	30,1	392	367,0	448,43	0,82
	1971	4	102	457	406	122	0,30	152	0,97	31,9	410	400,3	468,65	0,85
Manuel, R.F., "Fail"	1974	4	102	457	406	102	0,25	152	0,97	31,7	404	453,7	469,69	0,97
		1	102	457	406	102	0,25	152	0,63	31,7	308	389,2	506,95	0,77
		5	102	457	406	203	0,50	152	0,97	31,7	401	402,5	436,50	0,92
		11	102	457	406	432	1,06	152	1,88	32,3	374	273,6	284,49	0,96
	1974	7	102	457	406	203	0,50	152	1,38	36,9	432	487,1	495,46	0,98
	1974	10	102	457	406	366	0,90	152	1,88	38,1	374	293,6	392,33	0,75
Kani Continuous	1979	806	152	305	272	1087	2,00	152	2,77	35,7	428	69,8	35,53	1,96
	1979	807	152	305	275	813	1,48	152	2,75	36,2	419	88,5	66,45	1,33
	1979	808	152	305	275	544	0,99	152	2,75	36,2	421	123,7	146,57	0,84
Kani T-Beams	1979	4849	154	305	273	272	0,99	152	5,46	17,9	379	273,1	214,53	1,27
	1979	4856	156	305	273	544	1,99	152	5,43	19,0	379	186,4	83,07	2,24
	1979	4829	153	305	272	544	2,00	152	2,72	19,0	401	128,5	79,60	1,61
	1979	4835	155	305	272	272	1,00	152	2,71	19,0	403	267,3	224,40	1,19
	1979	4809	155	305	267	544	2,04	152	1,86	18,2	406	107,2	77,12	1,39
	1979	4811	155	305	271	544	2,00	152	1,80	19,3	408	97,6	81,38	1,20
	1979	4817	155	305	271	272	1,00	152	1,82	19,0	392	262,0	227,23	1,15
	1979	465	154	305	280	274	0,98	152	7,91	26,8	361	400,3	318,59	1,26
	1979	468	156	305	274	544	1,98	152	7,91	25,9	366	236,9	114,74	2,06
	1979	442	155	305	252	544	2,16	152	5,98	24,4	361	156,3	106,29	1,47
	1979	445	153	305	268	272	1,01	152	5,54	29,0	395	403,0	343,08	1,17
	1979	452	154	305	275	272	0,99	152	5,40	25,8	374	370,7	309,32	1,20
	1979	421	153	305	276	406	1,47	152	2,69	34,1	373	202,4	240,44	0,84
	1979	422	155	305	276	544	1,97	152	2,65	28,0	386	145,0	119,92	1,21
	1979	423	153	305	276	678	2,46	152	2,68	27,0	386	108,5	74,81	1,45
	1979	402	156	305	273	544	1,99	152	1,78	29,4	438	154,8	120,40	1,29
	1979	403	152	305	275	678	2,47	152	1,80	27,4	417	120,3	72,99	1,65
	1979	408	155	305	280	544	1,94	152	1,74	31,7	417	154,8	129,62	1,19
	1979	416	153	305	274	681	2,49	152	1,78	26,5	379	83,0	73,82	1,12
	1979	4891	154	305	273	544	1,99	152	8,03	35,0	388	286,9	149,53	1,92
	1979	4899	155	305	274	272	0,99	152	7,83	35,0	369	562,2	425,41	1,32
	1979	4881	155	305	274	544	1,99	152	5,41	35,4	366	228,2	155,83	1,46
	1979	4886	155	305	271	272	1,00	152	5,48	37,7	379	482,2	456,62	1,06
	1979	4871B	155	305	272	544	2,00	152	2,69	35,7	403	139,7	151,17	0,92
	1979	4875	154	305	272	272	1,00	152	2,70	35,4	404	371,9	415,01	0,90
	1979	4876	154	305	273	409	1,50	152	2,70	35,4	404	230,9	241,57	0,96

	1979	4861	154	305	271	544	2,01	152	1,84	35,4	429	166,6	144,91	1,15
	1979	6049	152	305	275	544	1,98	152	9,27	24,9	349	235,5	109,42	2,15
	1979	6029	152	305	274	544	1,99	152	4,50	28,7	364	223,1	124,18	1,80
Taylor, M. A., El-Ha	1980	1	216	635	561	1219	2,17	356	1,77	30,9	483	491,0	322,34	1,52
Nakazawa, T., "Th	1980	10	150	300	270	540	2,00	50	0,70	23,2	319	46,6	65,28	0,71
	1980	15	150	300	270	270	1,00	50	1,47	25,3	358	166,8	157,47	1,06
	1980	16	150	300	270	405	1,50	50	1,47	25,3	358	122,6	102,83	1,19
	1980	17	150	300	270	540	2,00	50	1,47	24,2	358	90,7	64,97	1,40
	1980	22	150	300	270	270	1,00	50	1,91	34,4	358	166,8	214,11	0,78
	1980	23	150	300	270	405	1,50	50	1,91	34,8	358	112,8	141,44	0,80
	1980	24	150	300	270	540	2,00	50	1,91	34,3	358	93,2	92,08	1,01
	1980	(1)-1	180	400	340	1200	1,76	275	3,20	23,6	343	97,5	65,62	1,49
Fukuhara, M., Kok	1980	(2)-1	180	400	340	1200	1,76	275	3,20	31,8	368	104,6	85,65	1,22
	1980	(2)-2	180	400	340	1200	1,76	275	3,20	31,8	368	101,0	85,65	1,18
	1980	(3)-1	180	400	340	1600	2,35	275	3,20	23,6	343	81,1	35,47	2,29
	1980	(4)-1	180	400	340	1200	1,76	275	3,20	19,5	343	85,7	54,22	1,58
	1980	(4)-2	180	400	340	1200	1,76	275	3,20	19,5	343	104,6	54,22	1,93
	1980	(4)-2	180	400	340	1200	1,76	275	3,20	19,5	343	104,6	54,22	1,93
Niwa, J., Maekawa	1981	T1	100	300	228	200	0,88	140	6,00	35,8	364	314,5	358,01	0,88
	1981	T2	100	600	507	200	0,39	100	3,00	54,7	389	514,0	724,85	0,71
	1981	T3	100	300	228	200	0,88	100	3,00	13,1	364	114,0	114,05	1,00
	1981	T4	100	600	507	150	0,30	140	3,00	13,1	389	212,5	208,96	1,02
	1981	T5	100	600	507	200	0,39	140	6,00	66,6	389	881,5	1072,35	0,82
	1981	T6	100	600	507	150	0,30	100	6,00	35,8	389	577,5	459,34	1,26
	1981	T7	100	300	228	150	0,66	100	6,00	59,9	364	446,0	632,23	0,71
Smith and Vantsiotis [1]		0A0-44	102	356	305	235	0,77	102	1,93	20,5	431	159,5	174,54	0,91
		0A0-48	102	356	305	235	0,77	102	1,93	20,9	431	156,1	177,94	0,88
		0B0-49	102	356	305	308	1,01	102	1,93	21,7	431	149,0	149,72	1,00
		0C0-50	102	356	305	408	1,34	102	1,93	20,7	431	115,7	104,47	1,11
Mphonde & Frantz [49]		AO-3-1	152	337	298	448	1,50	50	3,34	23,1	414	116,1	104,79	1,11
		AO-15-1a	152	337	298	448	1,50	50	3,34	79,5	414	275,7	360,64	0,76
Rogowsky & MacGregor	1986	BM1A/1.0	200	1000	950	1000	1,05	200	0,95	26,4	367	600,0	579,84	1,03
		BM1/1.0/T2	200	1000	950	1000	1,05	200	0,95	26,1	380	699,0	565,98	1,24
Mansur, M.A., Ong	1986	A1	150	225	200	400	2,00	10	1,34	24,2	463	60,0	37,46	1,60
Ahmad and Lue	1987	A4	127	254	203	467	2,30	10	3,93	60,8	586	93,4	93,30	1,00
	1987	A5	127	254	203	406	2,00	10	3,93	60,8	586	166,8	116,05	1,44
	1987	A6	127	254	203	203	1,00	10	3,93	60,8	586	400,3	254,13	1,58
	1987	A10	127	254	208	478	2,30	10	1,77	60,8	586	82,3	88,72	0,93
	1987	A11	127	254	208	416	2,00	10	1,77	60,8	586	55,6	109,87	0,51
	1987	A12	127	254	208	208	1,00	10	1,77	60,8	586	222,4	235,70	0,94
	1987	B4	127	254	202	464	2,30	10	5,04	67,0	586	143,2	103,97	1,38
	1987	B5	127	254	202	403	2,00	10	5,04	67,0	586	106,8	129,57	0,82
	1987	B6	127	254	202	202	1,00	10	5,04	67,0	586	205,5	283,92	0,72
	1987	B10	127	254	208	478	2,30	10	2,25	67,0	586	64,1	97,77	0,66
	1987	B11	127	254	208	416	2,00	10	2,25	67,0	586	122,3	121,07	1,01
	1987	B12	127	254	208	208	1,00	10	2,25	67,0	586	213,5	259,74	0,82
	1987	C4	127	254	184	424	2,30	10	6,64	64,3	483	89,0	118,36	0,75
	1987	C5	127	254	184	368	2,00	10	6,64	64,3	483	246,9	149,26	1,65
	1987	C10	127	254	207	475	2,30	10	3,26	64,3	483	56,9	105,52	0,54
	1987	C11	127	254	207	413	2,00	10	3,26	64,3	483	106,8	130,68	0,82
	1987	C12	127	254	207	207	1,00	10	3,26	64,3	483	244,6	277,00	0,88
Lehwalt PhD	1988	V111	250	1000	930	465	0,50	232	0,66	16,8	420	849,0	839,11	1,01
	1988	V121	250	1000	930	465	0,50	232	0,66	18,8	420	654,0	939,01	0,70
	1988	V122	250	1000	930	465	0,50	232	0,66	15,8	420	642,0	789,17	0,81
	1988	V221	250	1000	930	930	1,00	232	1,08	20,4	420	588,0	683,69	0,86
	1988	V222	250	1000	930	930	1,00	232	1,08	15,4	420	430,0	516,12	0,83
	1988	V311	250	1000	930	1395	1,50	232	1,69	17,0	420	369,0	350,44	1,05
	1988	V321	250	1000	930	1395	1,50	232	1,69	16,9	420	387,0	348,37	1,11
	1988	V322	250	1000	930	1395	1,50	232	1,69	15,0	420	375,0	309,21	1,21

	1988	V811	250	200	160	240	1,50	40	1,90	20,6	420	140,0	127,65	1,10
Adebar and Collins	1989	ST17	290	410	378	1600	2,12	10	1,37	51,5	536	118,9	54,69	2,17
Thorenfeldt, Drang	1990	B12	151	251	221	508	2,30	25	1,82	54,0	500	70,8	83,94	0,84
	1990	B15	154	254	207	476	2,30	25	3,24	54,0	500	107,1	110,96	0,97
	1990	B22	150	250	221	508	2,30	25	1,82	77,8	500	102,7	117,46	0,87
	1990	B25	150	250	207	476	2,30	25	3,24	77,8	500	175,8	148,08	1,19
	1990	B45	150	250	207	476	2,30	25	3,24	86,4	500	148,3	164,45	0,90
	1990	B52	150	250	221	508	2,30	25	1,82	97,7	500	77,7	147,51	0,53
	1990	B55	150	250	207	476	2,30	25	3,24	97,7	500	156,1	185,96	0,84
	1990	B62	300	500	442	1017	2,30	25	1,82	77,8	500	438,7	435,42	1,01
	1990	B65	300	500	414	952	2,30	25	3,24	77,8	500	576,3	564,08	1,02
	1990	B65R	300	500	414	952	2,30	25	3,24	77,8	500	710,3	564,08	1,26
Tan, K.H., Muruga	1993	1	60	375	290	580	2,00	10	4,58	32,8	460	63,2	51,69	1,22
Tan, K.H., and Man	1992	S13	150	400	350	525	1,5	140	0,34	55	1480	159	195,39	0,81
Tan, K.H., and Man	1992	S14	150	400	350	700	2	140	0,34	30,6	1480	62	66,62	0,93
Xie et al	1994	NNN-1	127	254	216	216	1,00	15	2,07	47,0	421	155,7	191,35	0,81
	1994	NNN-2	127	254	216	432	2,00	15	2,07	41,4	421	56,6	80,65	0,70
	1994	NHN-1	127	254	216	216	1,00	15	2,07	103,8	421	241,5	422,61	0,57
	1994	NHN-2	127	254	216	432	2,00	15	2,07	103,4	421	101,6	201,42	0,50
Walraven & Lehwa	1994	V011	250	400	360	360	1,00	90	1,13	16,1	420	226,0	241,40	0,94
	1994	V012	250	400	360	360	1,00	90	1,13	21,8	420	322,0	326,87	0,99
	1994	V013	250	400	360	360	1,00	90	1,13	22,1	420	344,0	331,37	1,04
		V014	250	400	360	360	1,00	90	1,13	24,3	420	425,0	364,35	1,17
		V034	250	400	360	360	1,00	90	1,13	26,4	420	437,0	395,84	1,10
	1994	V021	250	400	360	360	1,00	90	1,13	13,9	420	220,0	208,42	1,06
	1994	V023	250	400	360	360	1,00	90	1,13	20,1	420	347,0	301,38	1,15
	1994	V024	250	400	360	360	1,00	90	1,13	25,2	420	396,0	377,85	1,05
	1994	V031	250	400	360	360	1,00	90	1,13	20,0	420	323,0	299,88	1,08
	1994	V032	250	400	360	360	1,00	90	1,13	18,2	420	318,0	272,89	1,17
	1994	V033	250	400	360	360	1,00	90	1,13	19,8	420	246,0	296,88	0,83
	1994	V711	250	200	160	160	1,00	40	1,52	18,1	420	165,0	176,19	0,94
	1994	V022	250	400	360	360	1,00	90	1,13	19,9	420	270,0	298,38	0,90
	1994	V511	250	600	560	560	1,00	140	1,12	19,8	420	350,0	393,36	0,89
	1994	V411	250	800	740	740	1,00	185	1,10	19,4	420	365,0	531,17	0,69
	1994	V211	250	1000	930	930	1,00	233	1,08	20,0	420	505,0	672,04	0,75
Kim, J.-K., and Par	1994	A1.5-1	170	300	270	405	1,50	10	1,87	53,7	477	212,5	159,81	1,33
	1994	A1.5-2	170	300	270	405	1,50	10	1,87	53,7	477	215,3	159,81	1,35
Tan et al	1997	I-1/0.75	110	500	443	375	0,85	110	2,58	79,5	499	500,0	726,58	0,69
	1997	II-1/1.00	110	500	443	500	1,13	110	2,58	77,6	499	255,0	553,05	0,46
	1997	III-1/1.50	110	500	443	750	1,69	110	2,58	77,6	499	185,0	329,33	0,56
Islam M.S., Pam H	1998	M100-S2	150	250	203	800	1,97	10	3,22	83,3	532	131,1	59,41	2,21
	1998	M80-S2	150	250	203	800	1,97	10	3,22	72,2	532	123,0	51,50	2,39
	1998	M60-S2	150	250	207	800	1,93	10	2,02	50,8	554	103,8	35,13	2,95
	1998	M40-S2	150	250	205	800	1,95	10	3,22	34,4	320	96,1	31,45	3,06
	1998	M25-S2	150	250	207	800	1,93	10	2,02	26,6	350	94,2	23,24	4,05
Shin et al	1999	MHB1.5-0	125	250	215	323	1,50	10	3,77	52,0	414	112,9	132,56	0,85
	1999	MHB2.0-0	125	250	215	430	2,00	10	3,77	52,0	414	87,9	92,13	0,95
	1999	HB1.5-0	125	250	215	323	1,50	10	3,77	73,0	414	142,2	186,09	0,76
	1999	HB2.0-0	125	250	215	430	2,00	10	3,77	73,0	414	99,4	129,34	0,77
Tan and Lu	1999	1-500/0.50	140	500	444	250	0,56	140	2,57	49,1	520	850,0	790,50	1,08
	1999	1-500/0.75	140	500	444	375	0,84	140	2,57	42,5	520	700,0	550,40	1,27
	1999	1-500/1.00	140	500	444	500	1,13	140	2,57	37,4	520	570,0	370,89	1,54
Kuchma & Collins	1999	TTC	960	650	615	1092	1,78	110	0,51	45,0	490	505,0	1281,38	0,39
Kim & Kim & White	1999	1.5CNB	127	229	191	286	1,50	95	1,65	30,1	461	112,8	122,73	0,92
	1999	2CNB	127	229	191	381	2,00	95	1,65	30,1	461	70,1	76,97	0,91
Pendyala and Mendis	2000	inf(30)(2)	80	160	140	280	2,00	10	2,02	34,0	410	25,0	23,72	1,05
	2000	inf(60)(2)	80	160	140	280	2,00	10	2,02	63,0	410	31,0	43,94	0,71

	2000	inf(100)(2)	80	160	140	280	2,00	10	2,02	87,0	410	35,0	60,69	0,58
Adebar June	2000	DF-1	500	1090	1000	2330	2,33	150	0,42	21,0	550	429,0	367,83	1,17
	2000	DF-2	500	1090	1000	2330	2,33	150	0,42	18,4	550	315,0	322,29	0,98
	2000	DF-2R	500	1090	1000	2330	2,33	150	0,42	18,4	550	378,0	322,29	1,17
	2000	DF-3	500	1090	1000	2330	2,33	150	0,42	18,4	550	329,0	322,29	1,02
	2000	DF-4	500	1090	1000	2330	2,33	150	0,60	25,5	550	387,0	446,65	0,87
	2000	DF-5	500	1090	996	2400	2,41	150	0,66	25,5	550	381,0	432,83	0,88
	2000	DF-6	500	1090	1000	2200	2,20	200	0,98	21,0	550	771,0	440,52	1,75
	2000	DF-7	500	1090	1000	2330	2,33	150	0,98	20,6	550	435,0	360,82	1,21
	2000	DF-8	500	1090	1000	2330	2,33	150	0,98	22,4	550	531,0	392,35	1,35
	2000	DF-8R	500	1090	1000	2330	2,33	150	0,98	22,4	550	579,0	392,35	1,48
	2000	DF-9	500	1090	1000	2330	2,33	150	0,98	31,7	550	532,0	555,24	0,96
	2000	DF-10	500	1090	1000	2330	2,33	150	0,98	31,7	550	524,0	555,24	0,94
	2000	DF-10R	500	1090	1000	2330	2,33	150	0,98	31,7	550	605,0	555,24	1,09
	2000	DF-11	250	1090	1000	2000	2,00	150	0,84	19,5	550	330,0	219,96	1,50
	2000	DF-12	250	1090	1000	1500	1,50	150	0,84	19,5	550	723,0	334,88	2,16
	2000	DF-13	250	1090	1000	1500	1,50	150	0,84	20,3	550	550,0	348,62	1,58
Oh and Shin	2000	DF-14	250	1090	1000	1750	1,75	150	0,84	19,5	550	409,0	270,07	1,51
	2000	DF-15	250	1090	962	1751	1,82	150	1,75	20,3	550	330,0	325,18	1,01
	2000	DF-16	250	1090	1000	1430	1,43	150	0,84	20,3	550	380,0	370,71	1,03
	2001	N4200	130	560	500	425	0,85	100	1,56	23,7	483	265,2	254,91	1,04
	2001	H4100	130	560	500	250	0,50	100	1,56	49,1	483	642,2	645,26	1,00
Lertsrisakulrat, T.,	2001	H4200	130	560	500	425	0,85	100	1,56	49,1	483	401,1	528,10	0,76
	2001	H4300	130	560	500	625	1,25	100	1,56	49,1	483	337,4	375,99	0,90
	2001	H4500	130	560	500	1000	2,00	100	1,56	49,1	483	112,5	196,73	0,57
	2002	D200	150	250	200	200	1,00	50	1,91	38,4	1026	214,2	176,58	1,21
Yang et al	2002	D400	150	450	400	400	1,00	100	1,69	35,5	1004	285,3	240,52	1,19
	2002	D600	150	650	600	600	1,00	150	1,76	40,8	1006	424,5	354,73	1,20
	2003	L5-40	160	400	355	200	0,56	100	1,01	31,4	804	447,0	353,11	1,27
	2003	L5-60	160	600	555	300	0,54	100	0,97	31,4	804	535,0	362,26	1,48
	2003	L5-60R	160	600	555	300	0,54	100	0,97	31,4	804	479,0	362,26	1,32
	2003	L5-75	160	750	685	375	0,55	100	1,05	31,4	804	597,0	415,25	1,44
	2003	L5-100	160	1000	935	500	0,53	100	0,90	31,4	606	582,0	477,57	1,22
	2003	UH5-40	160	400	355	200	0,56	100	1,01	78,5	804	733,0	882,76	0,83
	2003	UH5-60	160	600	555	300	0,54	100	0,97	78,5	804	823,0	905,64	0,91
	2003	UH5-75	160	750	685	375	0,55	100	1,05	78,5	804	1010,0	1038,13	0,97
	2003	UH5-100	160	1000	935	500	0,53	100	0,90	78,5	605	1029,0	1194,77	0,86
	2003	L10-40	160	400	355	400	1,13	100	1,01	31,4	804	192,1	215,60	0,89
	2003	L10-40R	160	400	355	400	1,13	100	1,01	31,4	804	311,6	215,60	1,45
	2003	L10-60	160	600	555	600	1,08	100	0,97	31,4	804	375,3	235,22	1,60
	2003	L10-75	160	750	685	750	1,09	100	1,05	31,4	804	271,5	281,89	0,96
	2003	L10-75R	160	750	685	750	1,09	100	1,05	31,4	804	330,3	281,89	1,17
	2003	L10-100	160	1000	935	1000	1,07	100	0,90	31,4	605	543,9	341,14	1,59
		UH10-40	160	400	355	375	1,06	100	1,01	78,5	804	498,8	578,00	0,86
	2003	UH10-40R	160	400	355	375	1,06	100	1,01	78,5	804	385,1	578,00	0,67
	2003	UH10-60	160	600	555	600	1,08	100	0,97	78,5	804	573,3	588,05	0,97
Tan et al.[29]		*1-500/1.5	140	500	444	375	0,84	150	2,60	41,8	620	340,0	225,88	1,51
		*2-1000/1.5	140	1000	884	1500	1,70	150	2,60	39,4	620	470,0	341,63	1,38
		*3-1400/1.5	140	1400	1243	2100	1,69	150	2,60	44,1	620	690,0	489,03	1,41
		*4-1750/1.5	140	1750	1559	2625	1,68	150	2,60	43,4	620	470,0	569,50	0,83
Tan et al.[32]		*1-500/0.75	140	500	444	375	0,84	150	2,60	42,5	530	700,0	568,14	1,23
		*2-1000/0.75	140	1000	884	750	0,85	150	2,60	32,7	530	650,0	626,81	1,04
		*3-1400/0.75	140	1400	1243	1050	0,84	150	2,60	36,2	530	950,0	843,58	1,13
		*4-1750/0.75	140	1750	1559	1313	0,84	150	2,60	40,4	530	1240,0	1079,89	1,15

Guadagnini, M., Pil	2006	SB41	150	250	223	500	2,24	50	1,35	43,0	500	68,0	74,68	0,91
	2006	SB42	150	250	223	250	1,12	50	1,35	42,2	500	135,5	193,34	0,70
Brown & Bayrak	2006	2	203	457	406	610	1,50	203	3,07	26,8	469	285,1	320,12	0,89
	2006	4	203	457	406	914	2,25	203	3,07	26,9	469	151,7	159,40	0,95
Bentz and Collins	2007	AF8	300	1000	925	1994	2,16	152	0,76	33,8	562	249,2	357,24	0,70
	2007	AF11-r2	300	1000	925	1000	1,08	152	0,76	33,8	562	602,8	910,62	0,66
Ismail, K. S.		A1	100	400	330	550	1,67	100	3,65	85,2	364	176,6	388,50	0,45
		D1	100	400	330	550	1,67	100	3,65	58,2	364	148,2	265,39	0,56
		E1	100	400	330	425	1,29	100	3,65	58,2	364	207,5	374,35	0,55
		F1	100	400	330	300	0,91	100	3,65	60,5	364	270,8	547,39	0,49
		H2	150	375	328	540	1,65	80	1,38	35,8	500	158,0	167,45	0,94
		H3	150	250	219	360	1,64	80	1,38	35,8	500	127,0	128,96	0,98
NCSU MMFX tests	2007	G-1.9-51	460	915	850	1640	1,93	230	0,72	51,0	468	444,1	1182,96	0,38
	2007	M-1.9-51	460	915	850	1640	1,93	230	0,44	51,0	865	788,6	839,81	0,94
	2007	G-1.9-38	460	915	850	1640	1,93	230	0,72	38,0	468	385,1	881,42	0,44
	2007	M-1.9-38	460	915	850	1640	1,93	230	0,44	38,0	865	690,6	625,74	1,10

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